SYMPOSIUM "RESEARCH ON WAVE ACTION"

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INTRODUCTION

The Papers, Discussion Papers and General Reports as published in these Proceedings are photostatic copies of the manuscripts presented by the respective authors. Therefore the Organizing Committee does not bear any responsibility for the contents of these contributions. Only where errors or misprints have crept into the Papers and have been detected by the authors or during the discussions, have corrections been made by the Organizing Committee. Then the passage
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After ample consideration, the Organizing Committee has decided to present the Report on Discussions not in the form of verbatim minutes but rather as a substantial impression. However, in some cases which have been clearly indicated within: "....." participants in the discussions have been quoted literally. Where it was found necessary the Organizing Committee has supplemented or summarized the discussions, with the highly appreciated help of authors and reporters. As such, the Organizing Committee is responsible for the Report on Discussions except for the quoted passages.

The Discussion Paper by M.M. Lebreton and Cormault is specially mentioned here. The Organizing Committee is greatly indebted to these authors for their valuable and extensive contribution to the discussion. Because this Discussion Paper has the character of an independent contribution to the Symposium, it has been included in the Proceedings as a separate Paper (12 A).

Finally, the Organizing Committee wishes to thank all authors, chairmen, reporters and participants in the discussions for their efforts.

The Organizing Committee.

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WORD OF WELCOME

H.J. SCHOEMAKER

DIRECTOR

Delft Hydraulics Laboratory, The Netherlands

It is an honour and a pleasure for me to welcome you in this Symposium, and as a word of welcome I would like to begin with a brief explanation of its intention which will be different in a way from what is given in the participant's guide and different from the impression you got from the preprints.

This Symposium is meant as a kind of a marking-stone in an already existing exchange of knowledge and experience between the workers in near-shore oceanography in this part of the world.

This group of workers is by no means restricted to those in the scientific institutions; on the contrary, it has its members in all sorts of organizations which, in some aspect, have to deal with problems of the sea.

The situation within the group is even such that in the mutual contacts the actual limits of the organizations to which the members belong fade away in the common effort to gain knowledge and experience about the physics of the sea, its bottom and its shores.

We do have several international organizations which organize meetings and congresses on all aspects of the sea sciences. The oldest among them is without doubt the Permanent International Association for Navigation Congresses. The impressive series of proceedings of its many congresses, in particular on ocean navigation, is a living witness of the activities.

A brilliant initiative to organize conferences in the more specialized area of the engineering in the near-shore problems came from our American colleagues and resulted in the regular conferences on coastal engineering, nowadays under the safe management of the American Society of Civil Engineers.

For these activities I have a great respect, and I consider them to be indispensable for the promotion of science together with the direct practical applications.

But now let me say a few words about the procedure leading to, let us say, the presentation of a paper to a congress or conference: In general, remarkable results of site investigations, many times supplemented by laboratory tests or successful realizations of clever designs, are reported.

But what we as workers in this field also need in relation to publications or designs is a prenatal exchange of views with our colleagues working on similar objects, which means mutual communication within a relatively small group in the period preceeding the genesis of a publication or some other result. This is why regional conferences of relatively small numbers of young men actually working in these problems are so very useful.

In the totality of sciences and engineering, the people working on near-shore problems form a small group scattered over a number of organizations, institutes and governmental agencies, and are even more or less lost in it.

And here I come to the actual aim of our meeting: the attempt to contribute to the communication between the people facing the practical part of the problems connected with the water movement, and the deep-borers facing the basics of those problems in the sea.

To be effective in these contacts, we purposely limited the subject of this Symposium.

It is definitely not at all our aim to start a regional isolationism; on the contrary, we would like to see a sound base for world-wide communications, emanating from a lively regional activity. We have to keep our world-wide exchange on the highest level with all the means we have at our disposal.

In this context I wish to offer a special word of welcome to our guest from the U.S., Professor Robert Wiegel. We highly appreciate that he was willing to accept our invitation to contribute to our Symposium in a general lecture based on his broad experience in matters which are so important for our task.

My cordial welcome to our participants from Europe, up to the Urals, I would also like to combine with the wish for a continuation of our contacts, also along the unofficial channels. We already have many personal relations with each other; we shall continue them and - what is important in our community - we shall have to look after the young men who will take over our work in the future - nearby or more remote - because we must certainly bring them together.

Also a welcome to those coming from outside Europe. Although distance may be a handicap to frequent personal contacts, we know that many timelasying relations have been promoted by meetings like this.

In this welcome address I want to insert an expression of my gratitude to our hosts in the auditorium in which we are now.

The Delft Hydraulics Laboratory was established in 1933 to serve both the practical needs of the public works and the education of the coming hydraulic engineers. In this respect we feel ourselves very devoted to this University. We are grateful that we received permission to make use of the University's splendid facilities in this auditorium.

The attention of the Technological University is by no means passive: I feel it as an honour to see the active interest of the Board of Governors in the presence of its President and the Vice-President, a fact I wish to mention with great appreciation.

I do not dare to welcome our colleagues from the Netherlands, because we are here in the lucky circumstances that, in our collaboration, there is no distinction between us in relation with the organization or agency to which each of us belongs. I consider them as the hosts in this Symposium, in particular my colleagues of the Rijkswaterstaat and of the Department of Civil Engineering of this University.

The guidance in the management of the Delft Hydraulics Laboratory lies in the hands of our Board of Trustees. In great appreciation of their stimulating interest in our scientific work I welcome the Chairman and the Members of the Board who have shown their interest by their presence.

In our colleague Engineer Thijsse I welcome my predecessor who already in 1938 was the initiator of experimental research of wind-water phenomena.

No symposium or meeting can become a real success without the ladies, our wives who many times are suffering from our all too wavy hobbies. A special welcome to them, and thanks for their willingness to give our professional contacts a human accent.

I have already stressed that in our professional work we form a team without regard to our organizational background. In the Netherlands the responsibility for all activities in near-shore engineering, together with its basic studies, is in the hands of the Rijkswaterstaat, and we, as the Laboratory, the conveners of this Symposium, take part in it.

As the totality of the planning of the works along our coasts and estuaries, including the creation of facilities for special studies, is under the supreme guidance of the Director-General of the Rijkswaterstaat, Mr. J. van de Kerk, it is for the Delft Hydraulics Laboratory an honour that he was willing to open this Symposium and to initiate your visit this afternoon to our newest facility for studies on wave action.

Mr. van de Kerk, may I invite you to open this Symposium by your address.

OPENING SPEECH

J. van de KERK

DIRECTOR-GENERAL
Rijkswaterstaat, The Hague, The Netherlands

Ladies and Gentlemen,

After the warm welcome extended to us by our host, it is my privilege and pleasure to address a few words to you by way of opening this Symposium.

I am convinced that the Symposium is going to make a useful contribution to our particular speciality, and it is very appropriate that it is being held here in the Netherlands at this time and on this occasion. The geographical position of these low-lying lands, with their long exposed coastline, has meant that we have had to come to terms with the sea from the beginning of our history. Although we are inclined to regard hydraulic engineering as our speciality, we are always glad to hear how others are coping with similar problems. I trust that the present meetings will be very valuable and that much will be learned in the talks and discussions.

In this company I have no need to dwell on the many predictable factors which face us in the confrontation with the sea, and that we have always to take into account in our work. Nor do I need to point out the many doubtful factors, or unknown quantities, to which we have to devote special care and attention in the preparation and construction of our projects.

Whether we are concerned with a minor work that will be completed in a few months, or a major national project that will take years of planning, preparation and sheer hard work, the problems are essentially the same. We must have a quantitative knowledge of these problems, of the difficulties that await us, and every risk accepted must be a calculated one. It is impossible to devise a sound construction method without this basis of scientific knowledge.

But the engineer responsible for the design must decide for himself just how far this search after knowleage is to go. He must decide how much scientific research is necessary, what studies are needed, and where advice is to be sought. And if he is breaking entirely new ground and there are no specialists to consult, he must make the best of it and must become his own specialist. It is necessary to strike a reasonable balance between the increased cost caused by the extra safety margins and running a risk of which the extent, quantitatively speaking, cannot easily be determined.

But never must we be carried away by our subject to the extent that we are devoting ourselves to pure scientific research. We must never forget that ours is an applied science, that our job is essentially practical and that the purpose of our studies is to provide us with the tools to carry it out properly. The tools must be constantly improved and be perfectly adapted to the job in hand, but they are not an end in themselves.

Also, scientific research takes time. Time is a most precious commodity of which the engineer can never get enough. So we have to learn to strike a balance between the pursuit of knowledge for its own sake, -- the advancement of science if you like, thereby satisfying our natural and professional curiosity,-- and what I might call scientific opportunism: using whatever science has to offer at a given moment for our own ends.

I said just now that it is my considered opinion that it is very opportune to hold this Symposium here on this occasion and at this time.

You will appreciate that the vast hydraulic engineering works carried out in this country, the dikes and sea walls that protect our coasts, the great barrier dams, the harbour works that make our ports safe and accessible to all shipping, and our country-wide water management network, - all these have meant tremendous efforts on the part of Rijkswaterstaat. They have called for preparatory studies, painstaking observations of natural phenomena faithfully recorded and meticulously worked out, mathematical forecasts and designing of a great variety of structures.

Until recently this field — the invention and solution of original problems — was the exclusive property of Government departments such as Rijkswaterstaat. But now competitors have entered the arena in the form of the oil companies with their drilling platforms, huge monsters exploring the ocean bed for further supplies of what has become our most precious fluid, — second only, I need hardly say, to water. And in the future we expect to find more and more of this kind of equipment in the open sea - drilling platforms for gas and oil, pipelines and accommodation for loading and unloading ships.

Indeed, more and more attention has been paid to the needs of shipping in recent years. Incredible as it may sound, hydraulic engineers constructing a harbour entrance in years gone by concentrated on the structures of the harbour and virtually ignored the ships that were to use it. Similarly, ship model basins were mainly concerned with the behaviour of ships on the open sea and failed to consider the problems of navigation in shallow water. Nowadays when a harbour is designed there is three-way co-operation between the hydraulic engineer, the ship model basin and the shipping companies, and the problems of manoeuvring large vessels in and out of the ports at last get the attention they deserve.

This, too, requires study, and again for a practical purpose. We emphasize the importance of study to achieve as near perfection of design as possible, not only for economic reasons but because, in the Netherlands at least, it is quite literally a matter of life and death. In a country where large numbers of people are living below sea level, the consequences of faulty design in the sea defences are not difficult to imagine. Both the design and the choice of design criteria of our structures must be sound, particularly where the protection of our coast is concerned. At the same time — to make matters more complicated — the navigation channels must be kept deep enough for shipping to pass through and must be prevented from silting up. Extensive knowledge is required of changes in the sea bed, of winds and currents. A factor not to be overlooked is the danger of water pollution due to the population density in the coastal areas in the west of the country. Knowledge is required, therefore, of a wide range of disciplines, each with its own specialists.

The theme of the present Symposium at once focuses our attention on all these disciplines bearing on the dynamics of the sea: Meteorology, hydro-dynamics, sedimentology, to name but a few. Water movement and wave action have far-reaching effects on the sea bed and on the shore, and these are studies that are vital to on and off-shore engineering. The relation between water movement and its causes, meteorological and otherwise, is still imperfectly understood and no single discipline can tell us all we need to know. New and intensive research is called for in this field if really reliable data are to be produced on which to base design calculations.

We must not forget, either, that in recent years there has been a general tendency towards increasing the scale in almost all branches of engineering. For example, we have to keep pace with and remain abreast of the latest navigational developments, as well as those in shipbuilding and other industries. The great variety of interests we serve means that we cannot rely on empirical knowledge alone, nor on experience gained in other countries, and we are obliged to institute research as and when we need it.

This situation was foreseen by Rijkswaterstaat a long time ago, and the Ministry created special research departments for the purpose. These are responsible for scientific observations and the systematic collection and sifting of data, which they then make available to design engineers. They also provide an advisory service for the design engineers. Among the facilities recently acquired are analogue and digital computers. Extensive use is also made, I need hardly say, of the facilities of the Delft Hydraulics Laboratory. Laboratory experiments are indispensable, as we all know, at all stages of an engineering project, from preliminary research through the designing to the model of the construction itself. Frequent recourse is made to the model in the course of building operations whenever a particularly tricky stage is reached.

Turning now to the actual subject of this Symposium, Wave Action, it will perhaps be of interest if I give you a brief sketch of developments in the Netherlands.

Wave attack has been a factor in our calculations from the earliest times, the sea walls being strengthened and built to a height that left a safety margin above the design level to allow for wave action. At first this was done by trial and error, a somewhat hit-or-miss method which nevertheless worked most of the time on the theory that what was strong enough before would be strong enough again. Later it was realized that wave action, far from being constant, is affected by wind speed, length of fetch and orientation, and the problem began to be approached more scientifically.

But it was not until the period between the two world wars that the first really systematic studies were made in the Netherlands on the actual relation between wind, fetch and wave characteristics. This was at the time when the design work was being done on the barrier dam that was eventually to cut off the Zuyderzee from the open sea. In 1920, the first attempts were made to utilize a wind flume for research purposes, enabling engineers to reproduce systematically, so to speak, the generation of waves by the wind. The earliest experiments were made in the wind tunnel belonging to the Aeronautical Institute. A second experiment was set up at the Geological Institute of the University of Leiden in 1934.

By present-day standards, in the light of what we know now, these experiments were on too small a scale to be really useful. However, about 1936 the Delft Hydraulics Laboratory installed a wind-water flume specially designed for wave research on hydraulic structures. Directly responsible for this was then Director of the Laboratory, Professor Thijsse, whose devoted efforts to further systematic research on wave action have given us all cause to be grateful. The original flume, however, was soon found to be too short for really effective research on wind effects, and it was extended to a length of 50 metres. It has remained in use, suitably modernized, until quite recently. A larger one was built in 1956 for the solution of the numerous more practical problems that constantly arise.

During the war extensive research was carried out in Britain and in the United States on the relation between meteorological conditions and the ocean waves that result from them. Of course, in those days the main purpose was to aid the forecasting of wave conditions in connection with naval operations. But after the war hydraulic engineers were able to turn to good account the tremendous amount of observational data that had been amassed and to profit from the work of scientists of international repute. The value of a thorough knowledge of the dynamics of the sea for innumerable practical purposes came to be realized, and the research was continued all over the world, including, of course, in our own country.

The hydraulic models and mathematical aids now at our disposal are put to extensive use. The more that was learned of the essence of wave attack, the more the need was felt to make use of laboratory models reproducing actual conditions as closely as possible. A great step forward had already been made with the introduction into the model of wind-induced waves or mechanically-generated waves modulated by wind. As continuous observations became available, the technique of completely programmed mechanical wave generation was perfected and was combined with direct wind effect. It had been realized for some time that structures can only be accurately tested for stability if waves are reproduced realistically. Tests are now carried out under a number of extreme conditions whose probability of occurrence has been worked out statistically. The obsolete method of trial and error has been discarded, and it is now possible to approach the problem scientifically. The damage observed in the model, combined with its frequency of occurrence, present the engineer with the opportunity to make a choice of design justified on grounds.

Progress continues, but there is still much to be done. I am convinced we are on the right track, and a glance at what has been achieved in the hydraulic engineering field in recent years surely bears me out. I am a strong believer, too, in the value of the contacts made at meetings such as this Symposium and of the exchange of ideas that I am convinced will flow from them. I wish you all a successful and enjoyable week and I have much pleasure in declearing this Symposium open. Thank you.

GENERAL REPORT ON SESSION 1

A. PAAPE

Delft Hydraulics Laboratory, The Netherlands

The general lecture of Professor Wiegel is only discussed on the aspects referring to the general description of wave motion. The Discussion Papers have been included in these comments as far as they were available in time. By the whole of these contributions a fairly general outline is given of the methods being available now for the collection and elaboration of data on waves in nature and their reproduction in models for studies on wave attack, while some examples are given of the evaluation of results.

A major aspect of the Papers is, of course, the description of irregular wave motion. In research as well as in design work more sophisticated methods of describing such irregular wave motions can be applied nowadays. In order to promote the application of these methods, they must be easily accessible and suited to practical problems. Those active in applied research and consultants should preferably not be confronted with a confusion among specialists. For this reason it is felt necessary to start with some critical remarks on this subject.

For the description of irregular waves the energy density spectrum is generally accepted as one of the main characteristics. In trying to establish a definition which appeals to the physical understanding of the phenomena involved, it is often described as the distribution of wave energy density over the various frequencies in the actual wave motion. This leads to a definition of the energy spectral density S (ω):

$$S\ (\omega) = \tfrac{1}{2} \sum^{\Delta\omega} a_i^{\,2} / \Delta\omega \qquad (1)$$

corresponding to a wave motion:

$$\eta(t) = \sum_{i=1}^{n} \eta_i = \sum_{i=1}^{n} a_i \cos(\omega_i t + \varphi_i) \tag{2}$$

See: Wiegel, page 10, eq. (34) and following.

The energy in the whole frequency range is proportional to $\overline{\eta^2}$ and denoted by m_o :

$$m_o = \overline{\eta^2} = \tfrac{1}{2} \sum_{i=1}^{n} a_i^2 \tag{3}$$

And according to (1):

$$m_o = \overline{\eta^2} = \int_0^{\infty} S(\omega)\, d\omega \tag{4}$$

The determination of $S(\omega)$ in practice, however, is generally based on a mathematical concept. First, the auto-correlation function $R(\tau)$ of a function $\eta(t)$ is defined as:

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} \eta(t) \cdot \eta(t + \tau)\, d\tau \tag{5}$$

Subsequently, $S(\omega)$ is defined as the Fourier transform of $R(\tau)$:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R(\tau)\, e^{-i\omega\tau}\, d\tau \tag{6}$$

Wiegel, (eq. (26)), Svašek (page 3) and d'Angremond/van Oorschot, (eq. (2) and (3)) apply these definitions, using the frequency f instead of ω ($\omega = 2\pi f$).

From (6) it follows:

$$R(\tau) = \int_{-\infty}^{+\infty} S(\omega)\, e^{i\omega\tau}\, d\omega \tag{7}$$

$R(\tau)$ is an even function ($R(\tau) = R(-\tau)$). Consequently when writing $e^{i\omega\tau} = \cos\omega\tau + i \sin\omega\tau$, the integrals contaning $[i \sin\omega\tau]$ are zero.

Hence:

$$R(\tau) = \int_{-\infty}^{+\infty} S(\omega) \cos \omega\tau \; d\omega \tag{8}$$

The frequency interval is from $-\infty$ to $+\infty$! Although this may seem somewhat unrealistic, it is the result of the mathematical definition of $S(\omega)$ and it is recommended to apply the standard definition of the Fourier transform in a consistent way.

The nature of the information presented by $S(\omega)$ defined according to (1) and (6) is basically the same.
As $S(\omega)$ and $\cos \omega\tau$ are even functions, one may write:

$$R(\tau) = 2 \int_{0}^{\infty} S(\omega) \cos \omega\tau \; d\omega \tag{9}$$

although this may introduce confusion.

The moments of $S(\omega)$ with respect to the origin are defined as:

$$m_n = \int_{0}^{\infty} S(\omega) \,.\, \omega^n \; d\omega \tag{10}$$

According to this definition:

$$m_o = \int_{0}^{\infty} S(\omega) \; d\omega \tag{11}$$

similar to (4).

However, from (5) it follows that for $\tau = o$:

$$R(o) = \overline{\eta^2} \tag{12}$$

whereas (9) gives for $\tau = o$:

$$R(o) = 2 \int_{0}^{\infty} S(\omega) \; d\omega \tag{13}$$

Combining (11), (12) and (13) gives:

$$m_o = \tfrac{1}{2} \overline{\eta^2} \tag{14}$$

This introduces misunderstanding when compared with (4):

$$m_o = \overline{\eta^2} \tag{4}$$

Therefore:

if $S(\omega)$ is defined according to (6), it is recommended to define the moments m_n as:

$$m_n = \int_{-\infty}^{+\infty} S(\omega) \cdot |\omega|^n \, d\omega \tag{15}$$!

Then:

$$m_o = \int_{-\infty}^{+\infty} S(\omega) \, d\omega = R(o) = \overline{\eta^2} \tag{16}$$

If one prefers to attach a direct physical meaning to the energy density spectrum, excluding $\omega < o$, then it is recommended to introduce the function:

$$S'(\omega) = 2\,S(\omega) \quad \text{for } \omega \geqslant o$$
$$= o \quad \text{for } \omega < o$$

and consequently to replace $S(\omega)$ in eq. (1) by $S'(\omega)$.

Then eq. (15) and (16) should be written as:

$$m_n = \int_{o}^{\infty} S'(\omega) \cdot \omega^n \, d\omega \tag{15a}$$

$$m_o = \int_{o}^{\infty} S'(\omega) \, d\omega = R(o) = \overline{\eta^2} \tag{16a}$$

Prof. Wiegel has got round this difficulty by defining $S(\omega)$ according to (1) and applying the definition (6) only to $S(f)$. He introduces the relation:

$S(\omega) = \frac{1}{\pi} S(f)$ (page 10) which is in agreement with his eq. (26) and (34).

However, when replacing f by ω with $\omega = 2\pi f$, in his equation (26) one obtains an $S(\omega)$ which is different from the one defined in (34), and this again may introduce mistakes. In this context $S(\omega) = \frac{1}{\pi} S(f)$ should be replaced by $S'(\omega) = \frac{1}{\pi} . S(f)$. D'Angremond/van Oorschot have not succeeded in clarifying this by using a real and imaginary area of the spectrum. The use of these terms makes it worse, as (7) and (8) are real in the mathematical sense. The imaginary part is always zero. Svašek uses the same expression on pages 3 and 5.

The derivation of Battjes in this discussion paper is correct, provided that in this eq. (1) $S(\omega)$ has the meaning of $S'(\omega)$.

The problem outlined above has led to a mistake in Svašek's Paper. His definitions on page 3 are correct, they correspond, however, to the relation $m_o = \frac{1}{2} \overline{\eta^2}$, as found above, starting from (6). On page 5 his equation (3) seems to be confirmed by his equations (4) and (5) and his relationship (6) $H_s = 4 \sqrt{m_o}$ which is generally applied. However, in this relationship and his equation (5) m_o and $S(f)$ are defined so that $m_o = \overline{\eta^2}$! So in Svašek's eq. (3), H_s must be replaced by his H_u, or by $H_{eq.}$ as used by Battjes. The relation between H_s and H_u is:

$$H_s = \sqrt{2} . H_u. \tag{17}$$

Hence the energy of an irregular wave field with significant wave height H_s is only half the energy of a uniform (regular) wave field with wave height H_s.

Some remarks of the same nature have to be made on the use of the Rayleigh distribution. Svašek and d'Angremond/van Oorschot give in their Papers two of the many different ways in which this distribution is applied. Although there is in principle no difference, a standardized expression will stimulate the application. There is no need to introduce σ in eq.(1) of d'Angremond/van Oorschot with $\sigma = \sqrt{\overline{\eta^2}}$ if they use $\sqrt{m_o}$ in the graphs. In equation (8) of Svašek, H_{50} must be replaced by the average wave height $\overline{H}$.

What has been given above may seem to be hair-splitting. However, it should be kept in mind that only standardized methods can lead to a better use and exchange of the scarce data on natural waves. In this respect the paragraph on Wave Climates by Wiegel needs our fullest support in every respect.

The Papers of Svašek and d'Angremond/van Oorschot give together a valuable picture of the problems met in the collection and evaluation of data from nature, the available model techniques and the evaluation of results.

As stated by Svašek, the quality of present designs is mainly determined by the knowledge of boundary conditions. The importance of his contribution is found first of all in the attention which is given to the possibilities of dealing with complex and insufficient information on natural conditions in a typical "engineering approach". In combination with the discussion paper of Battjes, interesting information is given on the reference period for refraction. The concentration of energy around $T_r = 1.3\ T_m$ in Figure 3.4 makes Svašek's conclusion a very reasonable one. The addition by Battjes for short-crested waves in his eq. (10) and (11) is only valid if the directional energy distribution is the same for all frequencies. The formula is not applicable in case of swell and wind waves with appreciable difference in frequencies and different predominant directions.

The comparison between recorded wave heights and estimates from visual observations shows an appreciable difference. It is not completely clear from Figure 4.1 and 4.2b what relationship actually should be applied.

In the description of the boundary conditions for a dike (Chapter 6) it seems to be that H_s is no longer a stochastic variable due to breaking off-shore.

** Examining the results of Figure 6.2, the question arises whether it is justified to accept $H_s = 5$ m as the design condition.

In the paper of d'Angremond/van Oorschot the advantages of a programmed wave generator are clearly demonstrated. Other systems of wave generation in models apparently lead to considerable deviations from natural wave characteristics. Two interesting aspects of the results can be mentioned.

First, no significant difference has been traced up till now between the punch-tape and random-noise method. This would indicate that the statistical properties of the wave motion are completely prescribed by the energy density spectrum. It should be kept in mind, however, that only a number of statistical parameters have been used. The effect of certain types of non-linear interactions, like the formation of wave-trains, is not made clear in this way.

** Secondly it appears that the effect of wind is rather small, and that the values of Δ are close to 1. One may wonder if Δ is an adequate parameter, as $\frac{d\eta}{dt}$ might be affected considerably by small irregularities on the wave slopes.

In case of wind-generated waves with exaggerated wind velocities (Table 2), Δ is much greater. This may strongly affect the results of experiments, especially in case of studies on wave impact forces. It seems to be worth-while to discuss this problem in the session on wave impacts.

** Finally, it may be said that a comparison between the various spectra on Figs. 4.a, 5.a, 6.a and 7.a and the corresponding values of ε shows that ε is not an adequate measure for the width of the spectrum.

The question may rise whether laboratory facilities should be developed as soon as possible for the reproduction of directional wave spectra. In this respect it must be said that present knowledge on this matter is still rather poor. Professor Wiegel has outlined briefly the state of art. Much is based on assumptions yet, like eq. (45) in his Paper, these are limited in their applicability. Some aspects of this problem will be discussed by Mr. H. Botma.

Mr. Rogan has used the important problem of the destruction of a rubble mound breakwater for a comparitive study on the effect of regular and irregular waves. The influence of storm duration is rather small in case of conditions which cause only slight damage; however, it becomes important in the development of serious damage or destruction. A comparison between the development of such damage under regular and irregular waves is meaningful, as many designs are still based on regular wave experiments. The process of deformation which is given is typical for the behaviour of a rubble mound with single straight slope suffering severe damage. The relation between relative duration and wave conditions, however, raises some questions (eq. (1)). The wave conditions have been expressed in the dimensionless parameter: $\frac{H^2}{\nu T}$, which suggests an unexpected influence of the kinematic viscosity ν. In his Discussion Paper, van der Weide points out the relative importance of the various parameters, suggesting quite rightly the use of the parameter $\frac{gT^2}{H}$. The problem is not determined by this single parameter and $\frac{t}{T}$.

For a constant slope of the rubble mound, no overtopping and one type of armour blocks, constant water depth, the relative duration $\frac{t}{T}$ for destruction may be written as:

$$\frac{t}{T} = f\left(\frac{H}{gT^2}, \frac{H}{d}, \frac{H^2}{\upsilon T}\right) \qquad (d = \text{water depth})$$

If the influence of υ is small, as may be expected, then:

$$\frac{t}{T} = f\left(\frac{H}{gT^2}, \frac{H}{d}\right)$$

It is not justified that van der Weide expects a difference between regular and irregular waves if $\frac{t}{T}$ had been plotted against $\frac{H}{g\ T^2}$, as $\frac{H}{d}$ was not constant. The conclusion of Rogan that under the particular conditions of the tests regular waves H_r caused destruction in the same time as irregular waves H_s, provided that $H_r = H_s$, is still correct if these two conditions have indeed been tested. An explanation may then be that the destruction of a rubble mound requires much energy and cannot be effected by a few exceptional wave heights. For slight damage, however, these exceptional wave heights will be determinative to a larger extent. ** Consequently the conclusion of Rogan that H_s may be used as a project wave height is not justified.

The last-mentioned problem is related to the selection of design conditions. Interesting problems are introduced by Professor Wiegel when he mentions the "significant wave approach" and the "plateau-type design" (pages 2 and 6). In view of other Papers being presented, this might be a subject for discussion in another session.

REPORT ON DISCUSSIONS

Session: Morning, March 25, 1969.

Chairman: A. Brandtzaeg,
Technical University of Norway,
Trondheim, Norway

Reporter: A. Paape,
Delft Hydraulics Laboratory,
Delft, The Netherlands

Discussion on: General Lecture
Paper 1
Paper 2
Paper 3

Participants in the Discussion:

K. d'Angremond, Delft Hydraulics Laboratory, Delft, The Netherlands
J.A. Battjes, Delft University of Technology, Delft, The Netherlands
W.C. Bischoff van Heemskerck, Delft University of Technology, Delft, The Netherlands
R. Bonnefille, Electricité de France, Laboratoire National d'Hydraulique, Chatou, France
H. Botma, Delft Hydraulics Laboratory, Delft, The Netherlands
E.W. Bijker, Delft University of Technology, Delft, The Netherlands
P. Cormault, Electricité de France, Laboratoire National d'Hydraulique, Chatou, France
R. Dorrestein, Royal Netherlands Meteorological Institute, De Bilt, The Netherlands
J.P.Th. Kalkwijk, Delft University of Technology, Delft, The Netherlands
J.N. Svašek, Rijkswaterstaat, The Hague, The Netherlands
J.Th. Thijsse, The Hague, The Netherlands
A. Tørum, River and Harbour Laboratory, Trondheim, Norway
J. van der Weide, Delft Hydraulics Laboratory, De Voorst, The Netherlands
R.L. Wiegel, University of California, Berkeley, U.S.A.

Waves and Their Effects on Pile-supported Structures

by WIEGEL

BOTMA comments on directional wave spectra: "Wiegel states in his paper that Mobarek has compared experimental data on directional wave spectra, from laboratory and ocean, with the probability density of the circular normal distribution function and that the fit was excellent (See eq. 45 and 46). Mobarek has done this: « Since most natural phenomena follow the normal distribution and since wind generated waves is a natural phenomenon » [I. Mobarek: "Two dimensional spectrum of wind waves", Univ. of California, 1965, p. 43] .

My objection is that this way of treating directional wave spectra may lead to wrong interpretations by the users. The use of a probability density function as a theoretical directional spectrum suggests that probabilities of wave directions can be determined from these spectra. I have not found the statement in the paper of Wiegel that the wave direction is a stochastic variable, but it is suggested by the word probability density function and also by his Figure 8. There is only one stochastic variable in the model of the waves we are working with nowadays, and that is the phase: For one-dimensional wave spectra the model mostly used is given by eq. 35. For the directional wave spectra this model is extended to:

$$y_s\,(t,x,y) = \sum_{i,\,j} a_{i\,j} \cos\,(\omega_i t + k_i\, x\ \cos\ \varphi_j + k_i y\, \sin \varphi_j + \Phi_{ij})$$

in which:

k = wave number, φ = wave direction.

Of course, the estimates of the spectra are stochastic variables, due to the fact that the duration of a measurement is finite".

WIEGEL: Mobarek used this function because it simply happened to fit the data. The probability density function is a very general function and one concept of it is that it is a random function. It also describes, for example, the temperature distribution in a steel ingot submerged in a fluid as a function of time, although this is a deterministic problem. One has to be careful not to give only one interpretation to a particular function. The function is a solution of a differential equation and this equation may arise from many different physical phenomena. The distribution function for the directional wave spectra has nothing to do with probabilities. It merely is that this particular function seems to fit a certain class of data.

REPORTER: The distribution function (45) is not applicable in case of waves from two different sources, say: swell and wind waves with different predominant directions.

WIEGEL: The whole concept of obtaining directional spectra depends upon linear superposition. Hence we pretend that we are dealing with a linear phenomenon. Then one can also superimpose two distribution functions from two different sources.

The reverse procedure, the derivation of different sources from actual observations, may raise difficulties. A Paper will be presented in May (by Dr. N.F. Barber) in which, at least formally, the problem is solved how to get directional spectra from an array of wave recorders. There is a need for a simple model for the directional spectrum, in order to be able to define "design spectra". After having solved many difficult problems, there is a link now between the significant wave height, a reference period, and the one-dimensional spectrum. A similar development is to be expected for the directional spectrum.

DORRESTEIN comments on wave climates. "For many sites in restricted sea areas such as the North Sea or the Mediterranean, where a sufficient number of wind observations in the neighbourhood have been made in many years (e.g., by merchant ships), a first impression of the "wave climate" can be obtained quickly from our general knowledge about the wave heights having frequencies of being exceeded equal to those of the wind forces in such areas. [Wind and wave data of Netherlands lightvessels since 1949, Royal Netherl. Meteorol. Inst., Mededelingen en Verhandelingen no. 90 (1967), pages 16 and 36]".

PAPER 1

Statistical Evaluation of Wave Conditions in a Deltaic Area,

by SVAŠEK

SVAŠEK agrees with the remarks made by the Reporter on page 4 of the General Report. In this respect Svašek emphasizes the necessity of accurate and clear presentations, as one is often obliged to use results presented in terms of formulas etc. without being in a position to check their derivations in details.

For a number of problems in which wave energy is of importance the application of an equivalent monochromatic wave, having the same energy as the irregular waves, may be useful.

REPORTER: Figure 6.1 demonstrates what difficulties are met in practical applications. In view of the inaccuracies resulting therefrom, the question arises whether it is justified to accept, based on Figure 6.2, $H_s = 5$ m as a design condition.

SVAŠEK: Considerable changes in the bottom configuration off-shore the seawall are to be expected. Due to sedimentation, the maximum wave heights will decrease with time. For the period immediately after construction, a somewhat higher risk is accepted.

THIJSSE emphasizes , importance of the relation between the water depth and the maximum significant wave height that can occur. When constructing seawalls, for example, at locations with rather small depths and relatively long fetches off-shore, the existence of such a relation simplifies considerably the design and construction. One is no longer faced with the problem of establishing complex statistical evaluations of design conditions. The existence of a physical maximum of the wave heights, related to the water depth, is reflected in the diagrams for wave generation in shallow water (Sverdrup, Thijsse, Bretschneider). The relationships, however, have been based on a restricted number of observations which show appreciable scatter. There is no sound theoretical basis.

In the problem of wave breaking, the steepness of the wave is a major factor. The steepness of wind waves is most often in the same range

(1 : 15 to 1 : 20). A question is why the energy transmitted to the water results in an increase of wave height and wave length in such a way that the wave steepness is always in the same range. A more or less constant ratio H_s/d should then correspond to a more or less constant wave steepness. A plea is made to extend the data from measurements and to improve their reliability, and also make efforts for the development of a theoretical basis.

DORRESTEIN comments on visual wave observations. "From Figure 4.1 one gets the wrong impression that the wave heights visually observed on the lightship "Goeree" are on the avarage some 30 to 40% higher than the significant wave heights H_s. It should be taken into account, however, that the wave recording station "Triton" is situated much closer to the shore (only 2 miles compared with the lightship about 10 miles) and also is situated in shallower water. Hence the waves near "Triton" will be, on the average, lower than those near the lightship. In fact, comparisons between visual and instrumental observations, both from the lightship have shown that the visually observed heights are, on the average, perhaps only 10 % higher than the actual significant wave heights".

PAPER 2

Generation of Irregular Waves on Model Scales,

by D'ANGREMOND and VAN OORSCHOT

D'ANGREMOND agrees with the remarks of the Reporter on the definitions used in the description of wave energy spectra. With respect to the question whether punched tape natural wave records when used in programming the wave generator still have advantages compared to the random noise method, it should be kept in mind that the present paper does not cover all the statistical properties of wave motion which might be of interest. Investigations are still in progress. Phenomena like group formation have not yet been thoroughly studied. At present no definite answer can be given to the question whether a punch tape input is required for practical problems. Regarding the influence of small irregularities of the wave front on the value of $\frac{d\eta}{dt}$, we have recognized this point in the meantime, and at the moment $\frac{d\eta}{dt}$ is computed in a slightly different way:

$$\frac{d\eta}{dt} = \frac{\eta(t) - \eta(t+n\Delta t)}{n \Delta t}$$

n being 1, 2, 3 etc.

$\Delta t \approx 30 \cdot 10^{-3}$ sec for model records.

It is agreed that ε is not a very suitable parameter for describing the width of the energy spectrum. Maybe the Reporter overlooked the fact that according to its definition, the value of ε is not depending upon the energy density of the spectrum, but only on the frequency band where energy is found. And in this frequency band, the higher frequencies play a relatively important role.

REPORTER: This is exactly my point. Because if you compare the tests of your Figure 5a, they have almost the same ε and a rather different wave height distribution.

CORMAULT: When the coefficient ε was introduced by Longuet-Higgins it was meant to derive the probability density function p (η) of water level elevations from the spectrum. For a wide spectrum this resulted in a normal distribution function, for the narrow spectra in a Rayleigh function. I think it will be possible to check the validity of ε in this respect with the programmed wave generator, or have you done this already?

REPORTER: We have not tried this up to now. I agree with Cormault that the coefficient ε is more than only a parameter for the spectral width. It should be realized, however, that the relation between spectrum and distribution of water level elevations is of an academic character, because as engineers we are mainly interested in the distribution of wave heights p (H). Apparently for the same ε you can find different distribution functions of H. Then, for the engineer, something is wrong.

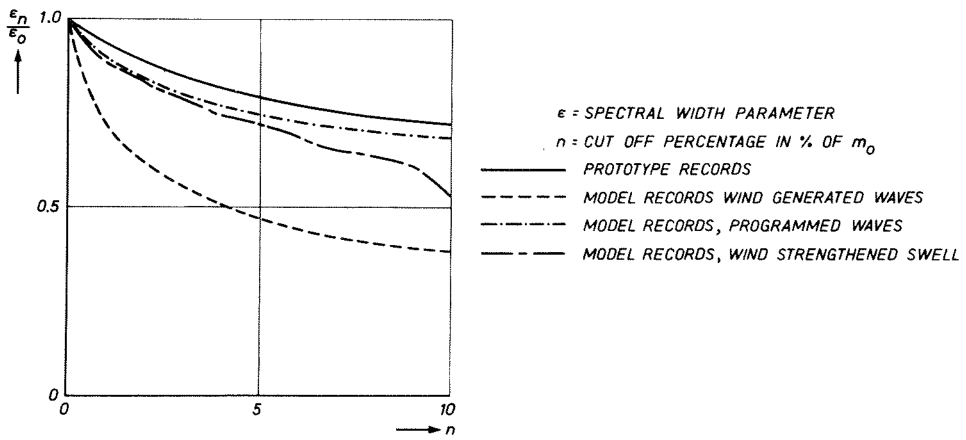
BATTJES: I would like to restore confidence in theory. Research as suggested by Cormault has been done applying electronic noise. In this way theory has been confirmed.

Further, I wonder why negative frequencies in the energy spectrum are strongly objected to. There is a small booklet of Barber on correlation functions, spectra, etc. which refers to the origin of the sine curve: the rotating wheel. In case of $\omega < 0$, there is an anti-clockwise rotation!

DORRESTEIN: "I strongly support the conclusion of the Reporter that it is unwise to use the parameter ε as a measure for the width of the spectrum. This width may be defined in much more direct and simple ways. The only useful meaning of ε is that $(1 - \varepsilon^2)^{-\frac{1}{2}}$ theoretically gives the ratio between the number of extrema and the number of zero crossings in a wave record".

BONNEFILLE: If you cut off the spectrum at another percentage of m_o , say at 1% , what is the influence on the value of ε? Does ε increase to 1?

D'ANGREMOND: No, in general ε did not increase to 1. We have varied the cut-off percentage, the result in dimensionless form is as shown below. One should realize that the values given are an average of several tests. Especially for the wind-generated waves and the wind-strenghthened swell, the scatter was considerable.



TØRUM comments on the wind-profile in a wind flume. Based on the experience in our wind flume, I tend to state that the wind profile is of minor importance for the growth of waves higher then about 20 cm being generated by the wave generator. The wind may have some influence on the higher frequencies in the energy spectrum. I feel refinements of the wind profile are of less importance compared with the fact that we still use two-dimensional instead of three-dimensional waves. Wind is of the utmost importance, however, when problems of splash and overtopping are studied.

BIJKER: I agree that with the modern wave-generating equipment we do not need wind to raise waves in a flume. But the wind is of importance to adjust the shape of the waves, and then, perhaps, the wind profile will also be of importance. Nevertheless, we have been able to reproduce the wind profile reasonably well.

KALKWIJK: During all tests the wind speed has been constant. Is large-scale turbulence influencing the steepness of waves and, if so, does it play a role in the velocity profile of the wind flume?

D'ANGREMOND: We have not studied the steepness of individual waves H/L; the only thing we have done is an analysis of the asymmetry Δ.

In private discussions it has been made clear that in the question of Kalkwijk this asymmetry was meant instead of the steepness.

It was concluded that, to a large extent, the small scale turbulent fluctuations of wind velocity in the flume are caused by the waves. It seems that the height of the wind channel is sufficient to prevent an influence of the ceiling upon these fluctuations. It should be realized, however, that no data from the prototype are available as a reference with respect to the turbulent velocity field above a waving sea to a height that can be compared with the height of the wind flume.

Turbulence of a still larger scale, for instance squalls with a duration of several minutes to half an hour, are known, but no data are available on the influence on short waves.

PAPER 3

A Comparison of Regular and Wind-generated Wave Action on Rubble-mound Breakwaters,

by ROGAN

Presentation by BONNEFILLE

BONNEFILLE: In general, I agree with the comments made by the Reporter, though I must say I am not responsible for the result of the investigations.

There is no doubt that the most important result of the investigation is the fact that the irregular waves with a significant wave height H_s destroyed the breakwater in the same time as regular waves with a height H equal to H_s.

The number of waves required for the destruction of the breakwater, t/T, was related to H/gT^2 and $H^2/\upsilon T$. The correlation coefficient r between t/T and H/gT^2 was 0.3, whereas this coefficient was 0.8 for the correlation between t/T and $H^2/\upsilon T$. It is not clear what is the reason of the importance of the viscosity υ. The surprising results may be explained by the fact that H/gT^2 was nearly constant during the tests.

VAN DER WEIDE: The main point of my Discussion Paper has been solved by the statement that H/gT^2 was kept nearly constant.

WIEGEL: Did you change viscosity or gravity?

BONNEFILLE: No.

WIEGEL: Then $\frac{H^2}{\upsilon T}$ represents the energy per unit wave H^2/T. And it is shown, in fact, that the greater the energy per wave, the faster the destruction, a result which could be expected.

KALKWIJK: In his analysis, the Reporter introduced a dimensionless parameter H/d, d representing the water depth. But also the dimensions of the stone D play a role, which should be taken into account as H/D for instance.

REPORTER: I would point out that on page 7 of the General Report it is stated that the analysis is valid for one size of stone and for one slope only. It refers only to the particular conditions of this series of tests by Rogan.

BIJKER: I think a bit of the misunderstanding or the not-clear results of the tests by Rogan is caused by the wish to use dimensional analysis. Perhaps it is known that I am not very fond of dimensional analysis, especially not in a relatively simple case like the wave attack on a breakwater, where there is no difficulty in scaling small materials. A breakwater is just a mound of big stones, which are still big stones on model scale.

In my opinion an energy flux towards the breakwater is responsible for the damage. So damage could be a function f (ρ, g^2, H^2, T, t). Or, if we wish to express this per wave, we can say:
t/T is a function of the energy per wave:

$$t/T = f\ (\rho, g^2, H^2, T^2)$$

Therefore, if we have a series of tests, I think it is worth-while to compare the relative duration of each test with the energy per wave.

BISCHOFF VAN HEEMSKERCK: Bijker thinks the appearance of υ is caused by dimensional analysis. I think somebody has made a factor dimensionless without analysis. Further, I wonder how he concludes that the damage is caused by energy flux only.

BIJKER: Damage of rubble-mound breakwaters is caused by stones falling down. The movement of material due to forces is just a question of energy. We could approach the problem even from another side: the only thing we know is that there is somehow an energy flux towards the coast which apparently results in damage to a breakwater. Therefore, I like to relate this energy and the damage.

BISCHOFF VAN HEEMSKERCK: This may be difficult because I am not sure what is causing the damage: is it the energy of the whole wave or only a part of it?

TØRUM comments on the conclusions that H_s is determinative for the design of a rubble-mound breakwater. At R.H.L., Trondheim, Norway, we have carried out tests on a number of breakwaters with regular and irregular waves. These tests show that the stability of a breakwater is a complex problem which can best be investigated in irregular waves. This is clearly shown in Paper 10 by Traetteberg and Berge and in the discussion paper by van Oorschot.

Also the depth is of importance. Probably it has not been taken into account by Rogan because the waves did not tend to break in front of the breakwater. Some years ago we did a similar study as Rogan, and the results were published at the Coastal Engineering Conference at Tokyo in 1966. The tests show that for a rough estimate one may use H_s as a design wave height. However, it has also been demonstrated clearly that the stability is depending not only upon the wave height but also upon the width of the spectrum. Run-up seems to be influenced even more by the width of the spectrum, though it is not clear in which way this influence is modified by the slope and the porosity or roughness of the breakwater.

REPORTER: I am worried that we are already discussing whether or not and to what extent the significant wave height or the energy is determinative for the stability of structures. Thus I think we are jumping at conclusions which are based upon a few incidental tests only and not on systematic investigations.

DORRESTEIN: The results of the study presented by Rogan suggest that the destruction of the breakwater is the result of a cumulative effect of many individual waves. Then, if one assumes that the effect of any wave is proportional to the n-th power of its height H and that the heights of the irregular waves obey a Rayleigh distribution, a simple calculation shows that n should be about 7 to arrive at the same conclusion as that of the authors.

The above assumption of the n-th power relationship will not hold strictly. It could be stated then that $n \approx 7$ will give the slope of a log-log plot of damaging effect of one wave with height H versus H in the region of $H \approx H_{1/3}$.

GENERAL REPORT ON SESSION 2

E.W. BIJKER

Delft University of Technology, The Netherlands

The four Papers may be divided into two groups of two Papers each. Those of Lundgren and Führböter deal with the physical phenomena of the wave impact, whereas Dietz and van Staal and Skladnev and Popov discuss design criteria based on the knowledge of wave impacts. The Paper of Lundgren itself consists of two parts, namely, that which deals with the actual wave impact and that in which the effects of the impact forces on the foundation and subsoil are discussed.

In this General Report the following subjects will be dealt with:

1. The physical aspects as discussed in the papers of Lundgren and Führböter.
2. The interaction between construction and foundation as treated by Lundgren.
3. The probability approach of Dietz and van Staal.
4. The results of wave impacts on slope protections and underlying filter layers of dikes as given by Skladnev and Popov.

1. Lundgren describes in a clear manner the different ways in which wave impacts can occur, viz., the ventilated shock, the compression shock and the hammer shock. For all three cases Lundgren gives acceptable approximations, but states that actual values should be obtained from model tests. In this case it is, however, essential that the model results can be translated to the prototype in the right way, so the scale for the pressures of the impact should be known. For this purpose it is indispensable that the basic equations which determine the problem are known.

Apart from difficulties which may arise with the pressure gauges and the model, the model scale for the hammer shock is relatively simple, ** because no air in great pockets is enclosed. When air is dissolved in the water in the form of very fine bubbles, corrections for the change in the celerity of propagation of sound in water should be made. Problems around pressure gauges and models will be discussed together with the interaction between shock forces and foundations.

For the compression shock and the ventilated shock the determination of the value of the scale for the shock pressure is much more complicated. It is a great point that Lundgren, Führböter and also Skladnev and Popov give procedures for the calculation of this scale or results of tests to determine this.

Führböter discusses the physical aspects of the impact pressure based upon his tests with a water jet striking suddenly a flat plate. Based upon his physical considerations, Führböter arrives at formulae to compute the impact pressures for all three type of shocks. These formulae give, moreover, the opportunity to determine the scales for the impact pressures. It will be very interesting to have further comments on this subject, because the two formulae given by Führböter in his paper lead to different scales, viz., the formula

$$p_{max} = \rho v c \cdot \sqrt[3]{\frac{c}{v}} \cdot \delta$$ leads to a pressure scale which is equal to 1

whereas the formula (17):

$$p_{max} = \rho v c \frac{1}{1 + (\frac{E}{E_a} - 1) \cdot \frac{D}{x}}$$

*

may result in a different scale for the pressures.

The values for the impact pressure scales as can be derived by Führböter's first formula and also by Richert's tests indicate that the scale is a power of the length scale smaller than 1. According to Lundgren the scale for the impact pressure may vary from $n_1^{\frac{1}{2}}$ to $n_1^{7/2}$ with increasing wave height.

Values for the scale of the impact pressure equal to the length scale, that is the scale for the wave heights, are also found by Skladnev and Popov in their scale tests for higher waves, which in their case is for relatively low values of wave height scales. In this General Report scales are determined as prototype over model values. Their conclusion that the scale for the wave impact pressure can be taken equal to the length scale may need further consideration after the information which has come available now. It will be interesting to know what scale values have been used in the tests that formed the base for the studies of Dietz and van Staal.

2. In the second part of his Paper Lundgren discusses the interaction between wave shock forces and the foundation. The principle of the kinematics of this motion is a simple mass-spring system. The same principle holds good also for modelling, and from this the characteristics of pressure gauges, strain gauges and models can be determined.

Lundgren does not introduce in his calculation the virtual mass of the ground which moves with the breakwater. This mass is very difficult to determine, but has an important influence on the response between breakwater and foundation, as may be demonstrated by the following equations for vertical motion:

** $$(m_{br} + m_{gr})\, z_{tt} - P(t) + C_{gr} z = 0$$

in which equation only the dynamic loads are written. In this equation is m_{br} = mass of breakwater, m_{gr} = virtual mass of subsoil, z_{tt} = second derivative of the vertical displacement of the breakwater to the time, P(t) the vertical load as a function of the time and C_{gr} the elasticity coefficient of the subsoil.

From this equation follows for the oscillating frequency of the total construction of breakwater and subsoil:

$$\omega = \left[C_{gr}/(m_{br} + m_{gr})\right]^{\frac{1}{2}}$$

The force as exerted by the breakwater on the subsoil is:

$$K = P(t) - m_{br}\, z_{tt} = P(t) \left[1 - m_{br} \cos \omega t/(m_{br} + m_{gr})\right]$$

The max. value by which P(t) has to be multiplied is:

$$\lambda = 1 + m_{br}/(m_{br} + m_{gr})$$

Together with the oscillating frequency ω and the form of the load P(t), this factor determines what pressures may occur in the subsoil.

The determination of the virtual mass of the subsoil may be performed via some energy discussions, but needs a close co-operation with soil mechanics specialists.

3. Dietz and van Staal describe in their Paper the procedure to compute the design loads on the weirs of the storm flood protection of the Eider Dam, starting from the measured impact pressures. These pressures have been measured by means of pressure gauges distributed over the surface of the construction. Since the impacts occur only over a limited area the probability factors have been determined by which the measured maximum impacts have to be multiplied to obtain the simultaneously acting pressures. These factors are substantially lower than 1.

To be able to determine design criteria for the construction, the probability of occurrence of water levels and wave heights has to be determined. The wave heights have been calculated from the wind velocities and the water depths above the shoals lying in front of the structure. Since sufficient wind data over a sufficient long time were not available, a rather great extrapolation had to be made. With the aid of a comparison between the known relation between water level and wind velocity and the probability distribution of the water levels, this extrapolation can, however, be made rather reliable.

Finally, the total forces and the design criteria as a function of the criterium of failure are discussed and the fact is mentioned that the measured loads have to be multiplied by an impact factor which is a function of the form of the wave impact and the characteristics of the structure.

4. Skladnev and Popov discuss in their Paper the tests they executed on wave impact forces exerted by breaking waves on various slope protections of dikes. They describe the basin in which waves are generated by means of a pneumatic generator. It will be interesting to obtain more information from the authors about the changes in the vertical distribution of the orbital motion along the rather long wave canal with this type of wave generator.

The authors give a lot of information about the relationship between the measured impact forces and the steepness, which tends to show a linear correlation between wave energy per wave and the impact force.

Another important point dealt with in this Paper is the influence of the joints between the slabs of the slope protection. It would be interesting to know if the authors can give additional information on the behaviour of the fill material of the dike under the filter layer with respect to the impact forces. Special attention in this respect will have to be paid to the scale effects which may occur in the filter layer.

Summarizing all four Papers, it can be stated that a good overall picture is obtained of the problems arising when impact loads on structures occur.

Since, as all authors state, it will be always necessary to determine the actual impact pressures by model tests, special attention will have to be paid to the scale on which these pressures are reproduced.

The interaction between construction and subsoil is still for a great part unknown and needs further basic research.

REPORT ON DISCUSSIONS

Session 2: Afternoon, March 25, 1969.

Chairman: D.M. McDowell,
University of Manchester,
Manchester, England

Reporter: E.W. Bijker,
Delft University of Technology,
Delft, The Netherlands

Discussion on: Paper 4
Paper 5
Paper 6
Paper 7

Participants in the Discussion:

W.T. Bakker, Rijkswaterstaat, The Hague, The Netherlands

J. Dietz, Bundesanstalt für Wasserbau, Karlsruhe, Germany

A. Führböter, Franzius-Institut für Grund- und Wasserbau der Technische Universität Hannover, Hannover, Germany

P. Gaillard, SOGREAH, Grenoble, France

G. Jarlan, C.G. Doris, Paris, France

H. Lundgren, Coastal Engineering Laboratory, Copenhagen, Denmark

A. Paape, Delft Hydraulics Laboratory, Delft, The Netherlands

W. Siefert, Forschungsgruppe Neuwerk, Cuxhaven, Germany

M.F. Skladnev, The B.E. Vedeneev All-Union Research Institute of Hydraulic Engineering, Leningrad, U.S.S.R.

G. van Staal, (formerly) Delft Hydraulics Laboratory, Delft, The Netherlands

W.A. Venis, Rijkswaterstaat, The Hague, The Netherlands

N.N.

PAPER 4

Wave Shock Forces: An Analysis of Deformations and Forces in the Wave and in the Foundation,

by LUNDGREN

In addition to the remarks in the General Report, the REPORTER discusses Chapter 4 of Lundgren's Paper on the compression shock: Lundgren stated that the initial area A_o of the air pocket is proportional to H^2. But, maybe, this relationship itself is influenced by the quantity of air involved. So I am not perfectly sure that the procedure used by Lundgren is completely watertight. Perhaps the best thing we can do at the moment is to apply the Froude model-law, because in that case the length scale n_L is used for the pressures, which is well in between the extremes mentioned $n_L^{\frac{1}{2}}$ and $n_L^{3\frac{1}{2}}$.

LUNDGREN does not agree with the Reporter that the model scale for the hammer shock is a relatively simple problem. He refers to Fig. 6 of his Paper, and concludes that small irregularities of the wave front are very important with respect to the maximum pressure recorded, and he expresses his doubts whether these small irregularities are the same in model and prototype.

Further, it has been pointed out in the Paper that the bubble content is playing an important role in the process of the hammer shock. Here also doubts arise with respect to scaling up the model results. For the compression shock, with its longer duration (a rising time 15 times as much as the hammer shock) the Reporter pointed very much to the bubbles. But I think that in model as well as in prototype the air content of the large pocket of air is much more important than the bubbles. Therefore, I do not expect the interpretation of the compression shock (according to the method given in my Paper) to be so conservative as it could be for the hammer shock.

The conclusion of the remarks on the different scale-laws for different shock types is that one has to interpret each decisive force in some very good manner and to decide for each shock separately which scale-law should be applied.

REPORTER: I think we agree in general, because what I denoted as a hammer shock was indeed not the shock you analyzed in Fig. 6 of your Paper. This in my opinion is more a compression shock, because there is air enclosed. But now you can say there are no hammer shocks. During our investigations for the Haringvliet evacuation sluices, however, we did the same type of analysis as you described and, as a matter of fact, we had a very few shocks causing the highest pressures without spouters occurring. So there was no air enclosed, and apparently this was a pure hammer shock.

For the rest, I agree completely with you that the case you analyzed was much more complicated than a compression shock.

GAILLARD: I would like to make some comments on the mathematical derivation of the rocking motion of a breakwater under the action of regular or irregular waves.

The motion of the structure is given, if we neglect damping forces, by the following set of differential equations:

$$M \begin{vmatrix} \ddot{u}_g \\ \ddot{v}_g \\ \ddot{\alpha} \end{vmatrix} + C \begin{vmatrix} u_g \\ v_g \\ \alpha \end{vmatrix} = \begin{vmatrix} F_x \\ F_y \\ M \end{vmatrix} \qquad (1)$$

where u_g and v_g are respectively the horizontal and vertical displacements of the centre of gravity G of the caisson, α its angle of rotation around G, F_x, F_y, M the resulting forces and moment of wave forces, C is a symmetric matrix accounting for the restoring forces and moments due to the elastic behaviour of the foundation, and M is a symmetric matrix accounting for the inertial effects of the caisson, the foundation and water body. I will neglect here the possible inertial effects of the foundation, which could be determined by geotechnical methods, and will only lay stress on the effect of water pressure on the vertical faces of the structure.

From Biesel's theory [1] of waves generated by a wave paddle, this effect can be derived. Calculations lead to the expression:

$$M = \begin{vmatrix} m_x & m_\alpha & \Theta_y \\ m_\alpha & m_y & \Theta_x \\ \Theta_y & \Theta_x & \Theta_\alpha \end{vmatrix} \qquad (2)$$

with

$$
\begin{aligned}
m_x &= m_s + m_e \\
m_y &= m_s \\
\Theta_d &= \Theta_G + \Theta_e + m_e(s-y_e)^2 \\
\Theta_x &= 0 \\
\Theta_y &= m_e(s-y_e) \\
m_\alpha &= 0
\end{aligned}
\qquad
\begin{aligned}
m_e &= 1{,}085\ \rho\ h^2 \\
\Theta_e &= 0{,}0312\ \rho\ h^4 \\
y_e &= 0{,}401\ h
\end{aligned}
\qquad (3)
$$

h is the water depth over the foundation, ρ is the mass density of water, s is the height of the centre of gravity of the caisson over the foundation, m_s the mass of the caisson; in this case, the body of water behaves like a mass m_e, of inertial moment Θ_e at a height y_e above the foundation.

In some cases, M is not given in such a straightforward manner. It can be derived by a numerical method based on potential flow theory such as the methods described by Kulmač [2] for deriving the added mass of a dam undergoing an earthquake and by John [3] for bodies oscillating in water.

As regards the rocking motion of the caisson, it is often assumed that the structure oscillates around an axis located below the foundation. In fact, it can be shown theoretically that the set of equations (1) can be transformed into the set of three independent differential equations:

$$\Theta_i \ \ddot{\varepsilon}_i + C_i \ \varepsilon_i = M_i \qquad i = 1, 2, 3 \qquad (4)$$

by means of an adequate linear transformation between (u_g, v_g, α) and $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$. This means that the motion can be considered as the linear superposition of oscillations around three axes of rotation, one located under the foundation, one above the centre of gravity and one at some distance from the structure.

The system (4) is convenient for calculating the motion of the structure with the use of Laplace transforms but, of course, the system (1) can be integrated directly by a numerical procedure such as the Runge-Kutta method.

<u>References</u>:

[1] BIESEL, F., and SUQUET, F., Les appareils générateurs de houle en laboratoire. La Houille Blanche, 1951 - 1952.

[2] KULMAČ, Effet hydrodynamique de l'eau sur un barrage fendant un seisure (in Russian). Isvestia Vniig im Vedeneeva, 1962, T 70, pp. 177 - 191.

[3] JOHN, F., On the motion of floating bodies I and II. Comm. Pure and Applied Math., 1949-1950.

LUNDGREN: We appreciate Gaillard's comments and most valuable mathematical approach. I shall refer to only one point, where he mentioned the damping from the water on the oscillations of a caisson. In this respect Gaillard lays stress upon the fact that the breakwater acts as a wave generator. It can be shown, however, by a simple physical analysis, that the forces involved in this process of wave generation are much smaller than the other forces involved.

With regard to the remarks on the foundation problem in the General Report, I have offered the main conclusion that the movements of the breakwater are so small that there is no feedback to the wave forces, not even in the case of the hammer shock, because the hammer shock impulse is so small that everything is over before the caisson starts moving. Further, special attention is drawn to the triaxial tests as described in Section 8.4 of my Paper. Due to the shock pressures, compression waves will occur in the ground. These compression waves will be mainly due to the compressibility of water, which is small. The celerity of the compression waves is in the order of 1400 m/sec. Assuming a duration of the shock of 0.1 sec., the compression wave travels a distance of 140 m in this time. This is far more than the dimensions of a caisson. This means that the reaction from the ground is very close to a static reaction.

REPORTER: You stated that a hammer shock is so short that the structure does not move. Now, I have a very short answer: when there is no movement, there are no forces. When the hammer shock is so short that the breakwater does not move, it does not exert any force on the bottom. We have analyzed in detail what will happen with a shock of 0.1 sec. And then indeed you see that even this short pressure will exert a force on the bottom via the breakwater. And if it exerts a force on the bottom it will create also a force on the groundwater. And - I am sorry - but also the water will move a little bit and consequently there will be a little bit of added mass or whatsoever.

JARLAN: I have some doubts about the use of Westergaard's principle as far as the aspect of waves is concerned.

Wave Forces on the Eider Evacuation Sluices,

by DIETZ and VAN STAAL

DIETZ: At first I must answer the written Discussion Paper by Siefert. Only prototype data were available covering a period of 10 years. Therefore, it is not possible to determine on the basis of these data the water level with a probability of less then 10^{-1} per year. Further, it appears that the water level of about N.N. + 5 m, measured in 1962, would have a probability of 10^{-2} per year, if the relation found on the basis of the 10 years of prototype records is right.

With respect to choosing a curve in Figure 7, we had in the first place the four points as indicated. Moreover, a wind velocity of zero is exceeded 365 days per year, and, on the other hand, experience at other places shows that a wind velocity of 35 m/sec will be exceeded approximately once per 10^3 years.

The curve of Figure 8 follows from the curves of Fig. 6 and Fig. 7. Because also we were of the opinion that the prototype data were not very comprehensive, we tried to find additional data to confirm this curve. For this purpose, the measurements from which the results are plotted in Fig. 8 have been used. The scattering of the values is caused, among other things, by the wind duration on the North Sea and in the Eider Mouth.

Regarding the first question of Venis, the reason for choosing a design criterion with a probability of exceedance of 10^{-2} per year is described in the Paper. The choice of this criterion, however, has been made by the Wasser und Schiffarts Verwaltung and not by the authors.

VAN STAAL: Beyond the question of the risk of the structure, Venis asked another two questions which are related directly to the model tests and their interpretation. The first question concerns the statistical distribution of the wave loads, and the second one refers to the deviations in the behaviour of the high pressures.

Question 1: The distribution of wave loads has been established on the basis of 400 series of measurements. Because of the fact that the wave force distribution is very important for the interpretation of the results, an extensive study was made into the characteristics of this distribution. The study of the 400 records of wave loads already gave us the opinion that the logarithmic distribution was the best one for the lower wave forces.

Question 2: To obtain a better idea about the statistical behaviour of the higher wave forces, another test was carried out. For this test a record of 10,000 wave forces has been considered. We agree with Venis when he says that more investigations on this point would have been useful; there was, however, a limit to the cost of the test programme. As the record of 10,000 wave loads was analized in our paper only very briefly, I should like to go further into detail now.

The record of 10,000 wave loads has been divided into 20 series of 500 loads, thus obtaining series with approximately the same length as the 400 series mentioned above. We plotted the statistical distribution of wave forces for all 20 series of 500 waves and we found that to a certain point the plotted points showed a straight line on logarithmic paper. Then we repeated the same procedure for 10 series of 1,000, 4 series of 2,500, 2 series of 5,000 and 1 series of 10,000 waves (See Figures 1 to 5 below). And every time the point from which deviations of the higher forces occurred, shifted to a lower frequency. This finally brought us to the idea that the logarithmic distribution can be used if a sufficient number of waves is taken into account. I agree that this procedure is not very clear from Figure 3 in our Paper, as it is not possible to distinguish the different points of the curve. Actually, it would have been better if we had given different symbols to all 20 couples of 500 waves, instead of having indicated them by one mark only.

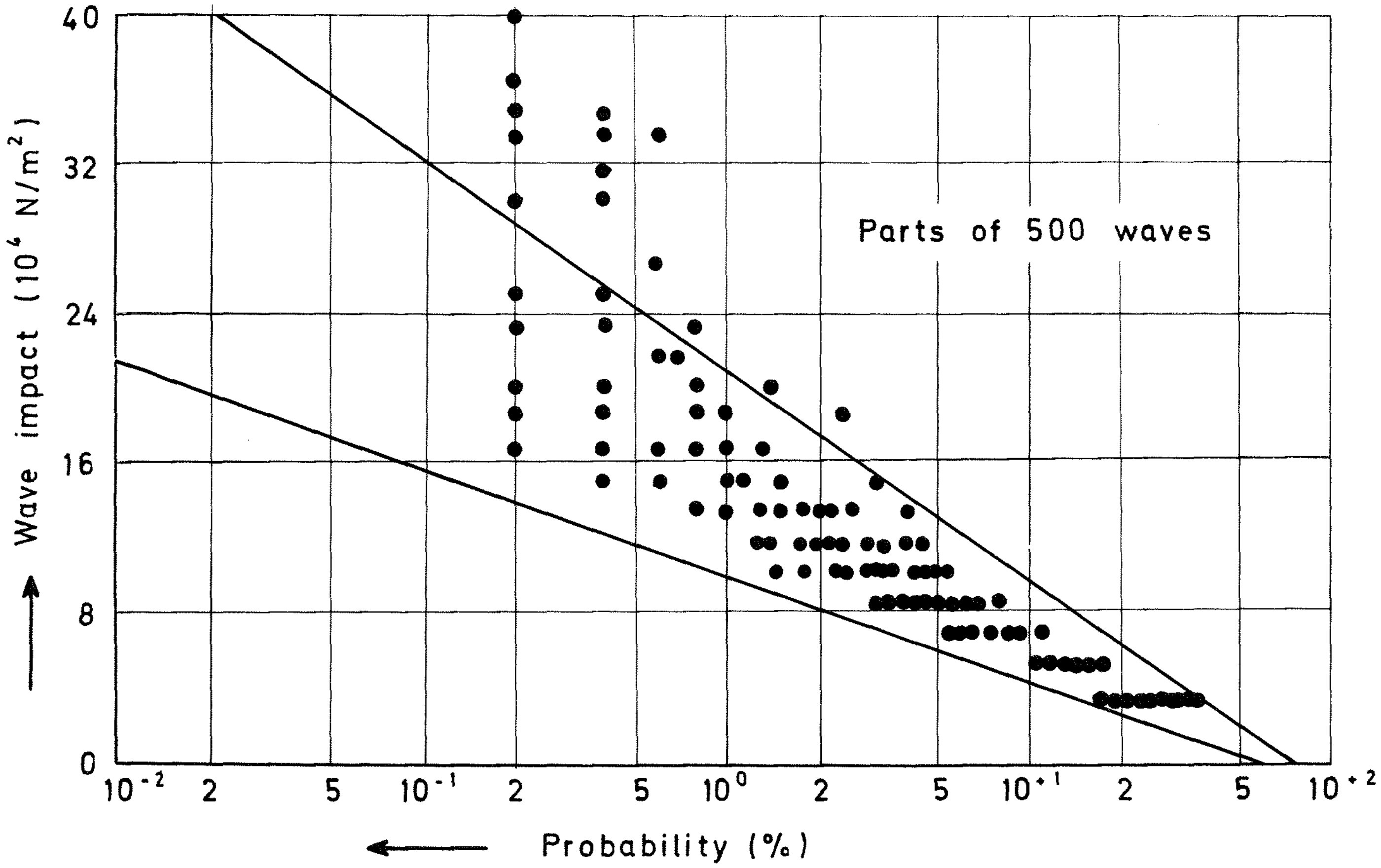
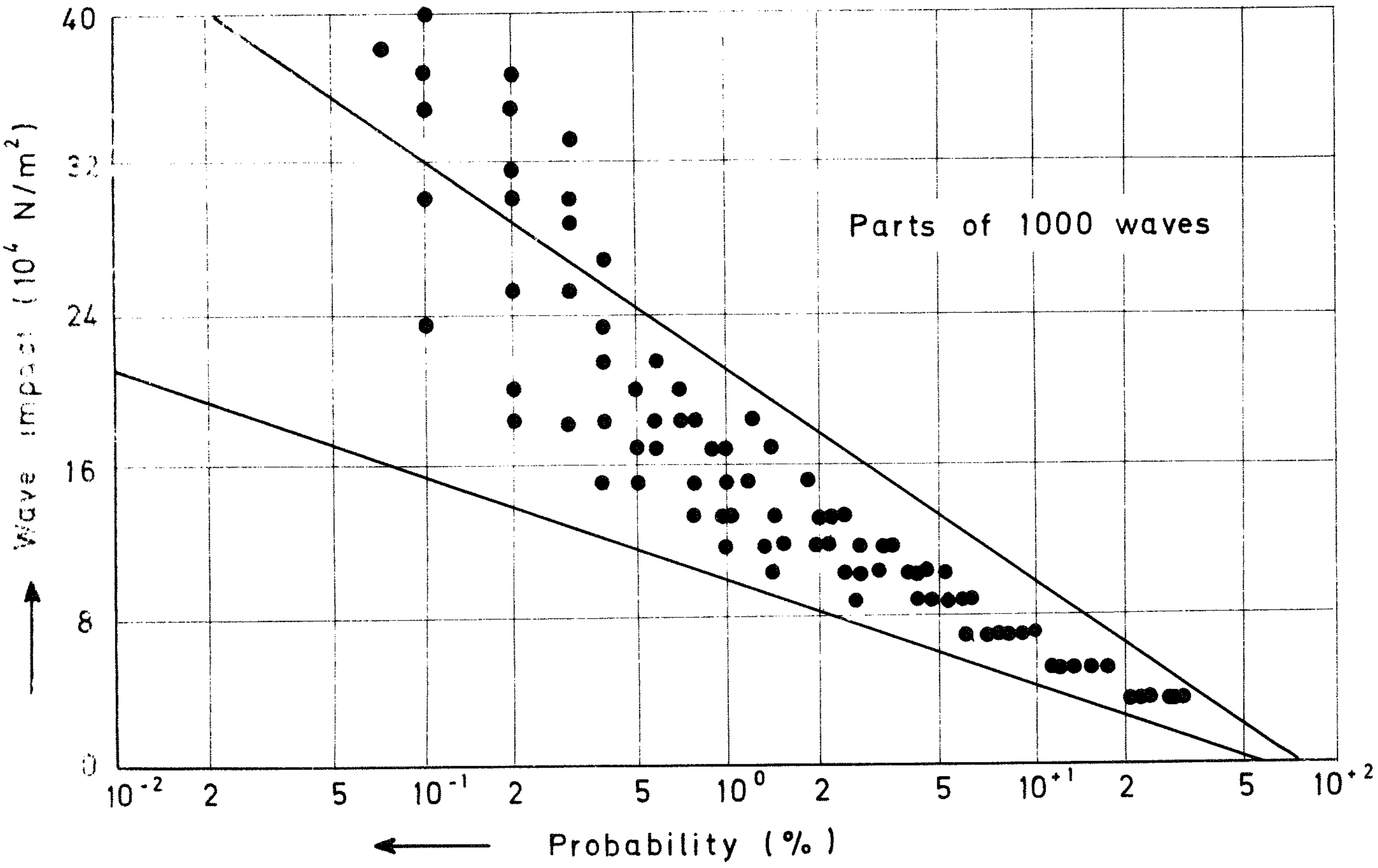
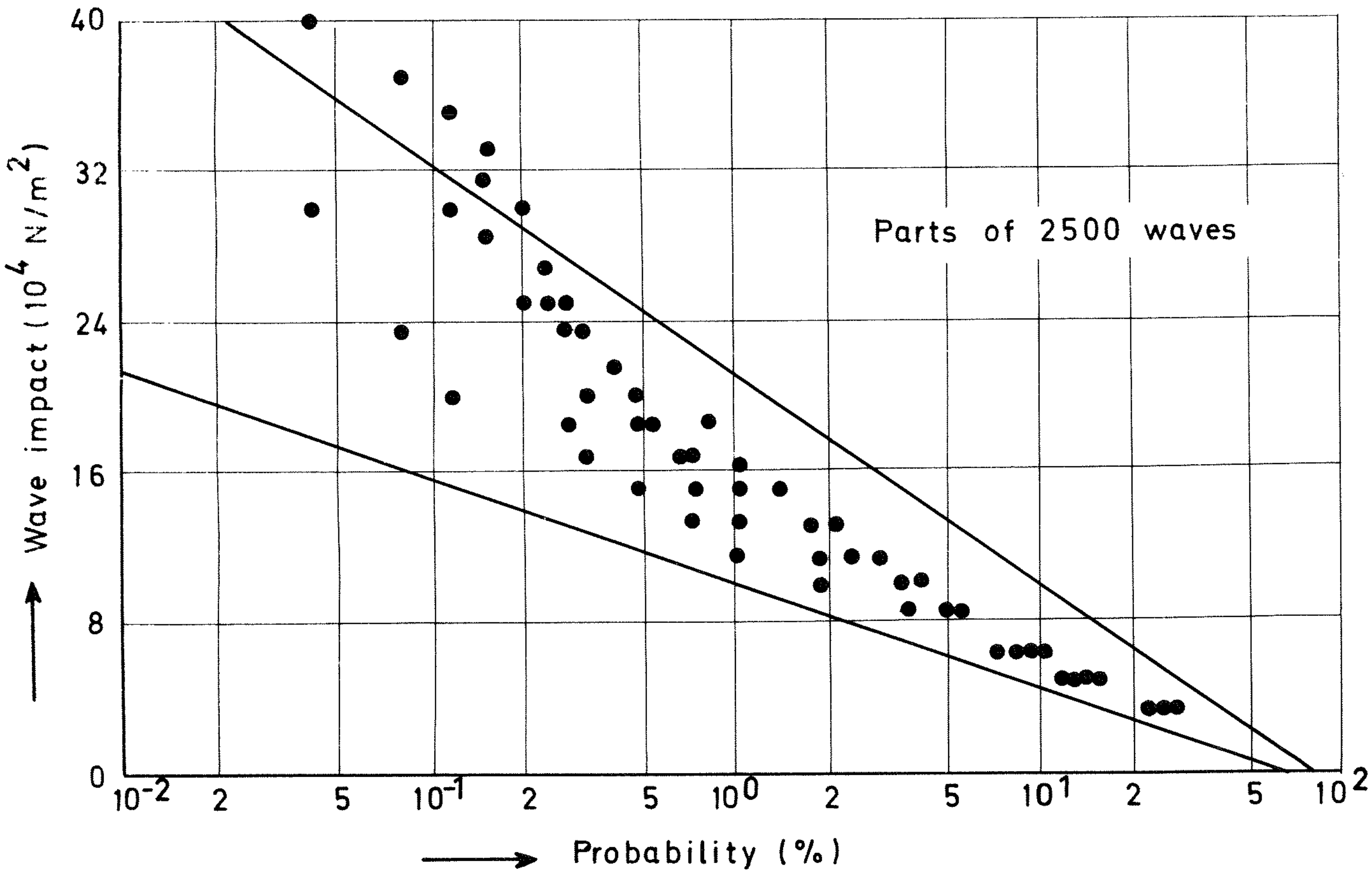


Fig. 1



Wave impact (10^4 N/m^2)
40
32
24
16
8
0
Parts of 1000 waves
10^-2 2 5 10^-1 2 5 10^0 2 5 10^+1 2 5 10^+2
Probability (%)

Fig. 2



Wave impact (10^4 N/m^2)
40
32
24
16
8
0
Parts of 2500 waves
10^-2 2 5 10^-1 2 5 10^0 2 5 10^1 2 5 10^2
Probability (%)

Fig. 3

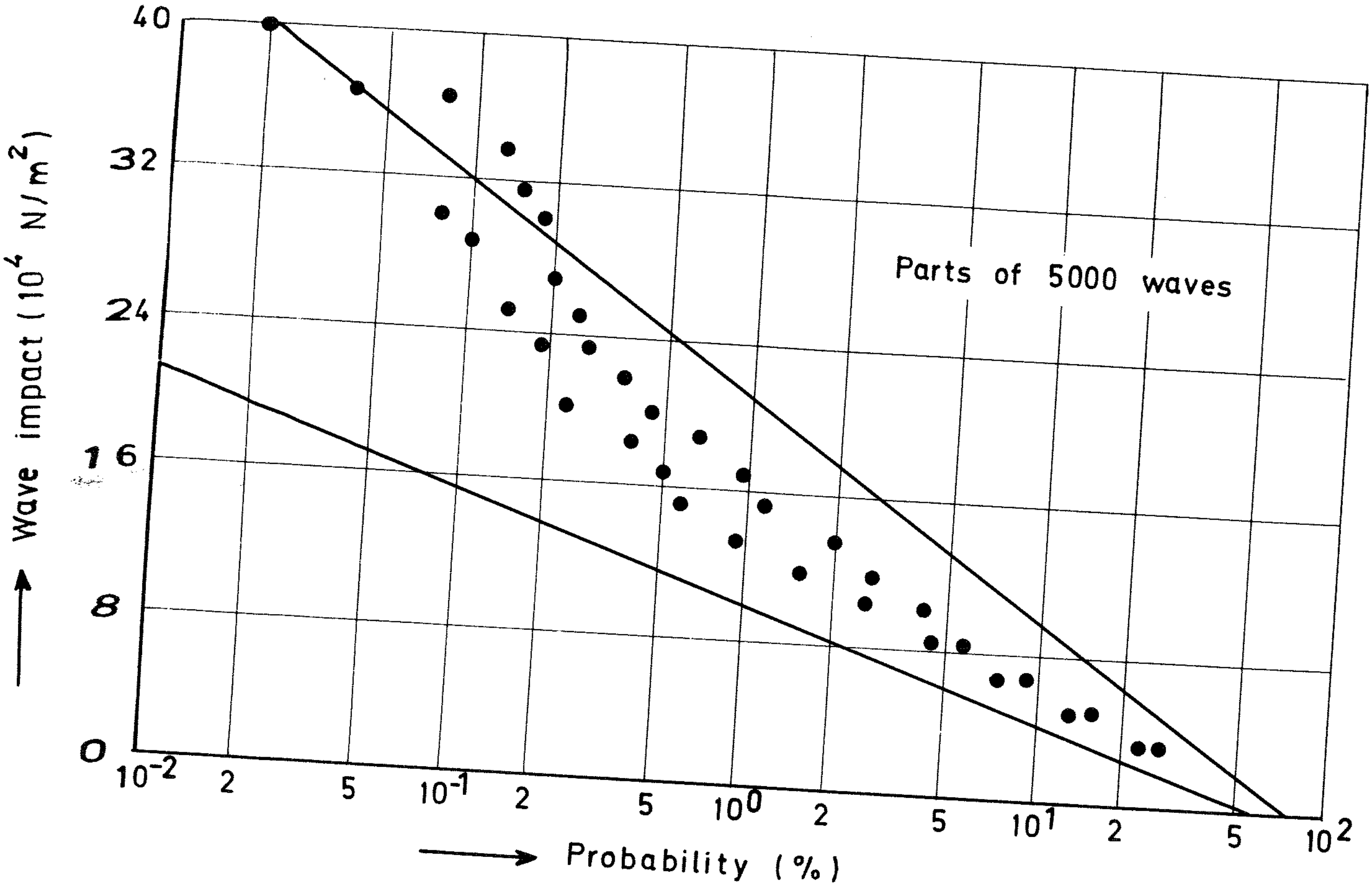

40
32
24
16
8
0
Wave impact (10^4 N/m^2)
Parts of 5000 waves
10^{-2} 2 5 10^{-1} 2 5 10^{0} 2 5 10^{1} 2 5 10^{2}
Probability (%)

Fig. 4

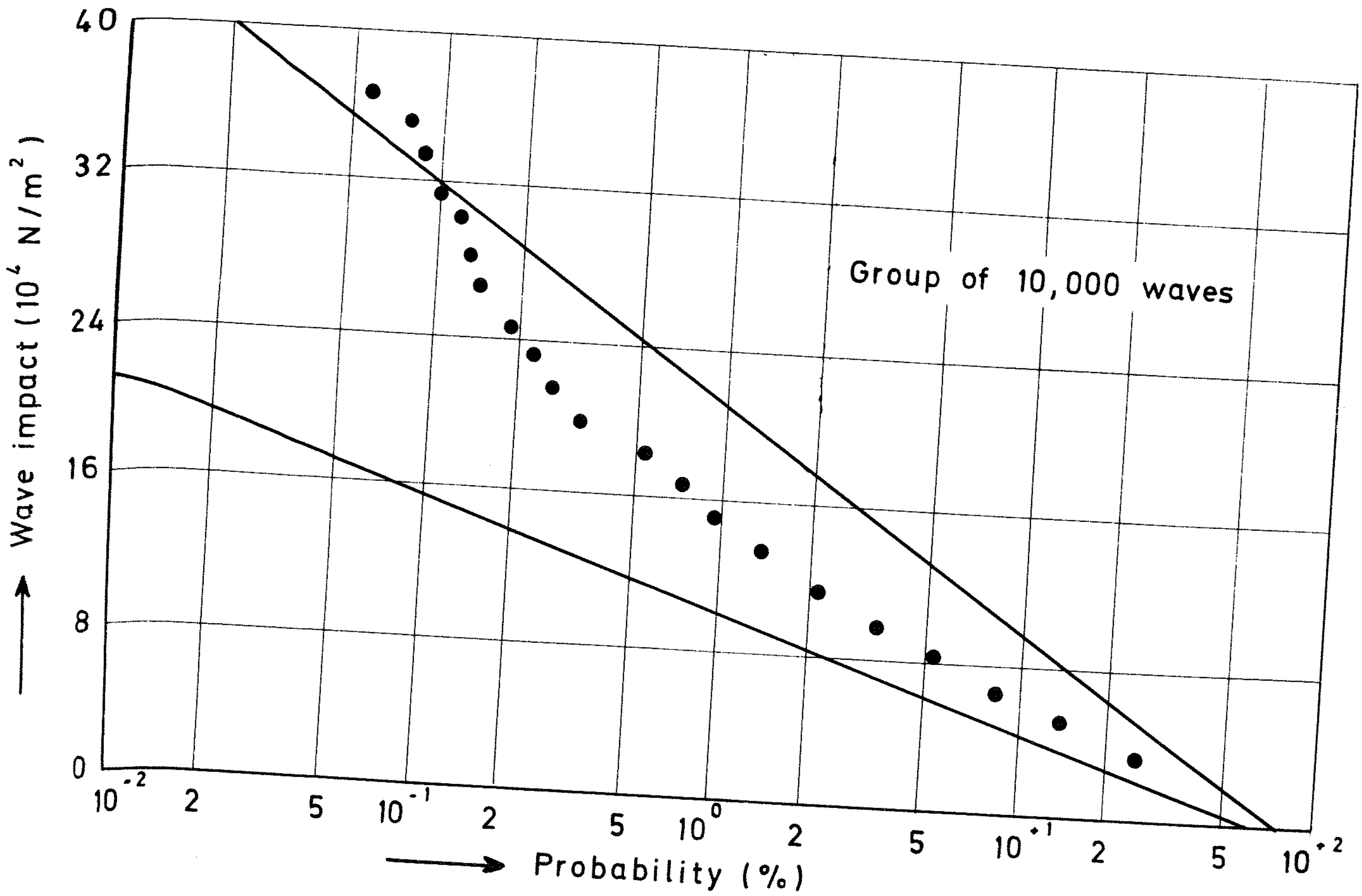

40
32
24
16
8
0
Wave impact (10^4 N/m^2)
Group of 10,000 waves
10^{-2} 2 5 10^{-1} 2 5 10^{0} 2 5 10^{+1} 2 5 10^{+2}
Probability (%)

Fig. 5

PAPER 6

Laboratory Investigation of Impact Forces,

by FÜHRBÖTER

FÜHRBÖTER: Consciously I avoid speaking about scale effects. At first I want to say now that formula (17)

$$p_{max} = \frac{1}{1 + (\frac{E}{E_a} - 1).D/x} \cdot \rho.v.c. \qquad (17)$$

cannot be applied for the purpose of scaling up. This formula has been used only to estimate the thickness of the air-cushion, necessary to damp the pure water-hammer pressures as measured both in model and in prototype.

The formula (7):

$$p_{max} = \rho.v^2.(\frac{R}{x})^2$$

would be applicable for scaling up if the factor R/x would be independent of v. Though Figure 10 covers only a range of impact velocity v from 4 to 8 m/sec, it is clear that within this range x/R is not constant.

But for scaling up, I have developed a transformation of this formula in a former paper:

$$p_{max} = \rho.v.c. \sqrt[3]{\frac{c}{v}} \; (\frac{E_a}{E} \cdot \frac{R}{D})^{2/3} \qquad (11)$$

The term $(\frac{E_a}{E} \cdot \frac{R}{D})^{2/3}$ is the dimensionless shock-pressure number δ. It can be seen that the hydraulic radius of the impact is of great importance as R appears in the formula with the power 2/3. Previously, this formula has been transformed into the following expression (see Ref. 4):

$$p_{max} = 42.\gamma \cdot H_B \sqrt[3]{\frac{y_B}{H_B} \cdot \frac{d^2}{H_B^2}} \cdot \sqrt[3]{\frac{1+2n^2}{1+n^2}}$$

Scaling up, according to this formula, would be possible with the Froude model-law if the shock-pressure number δ was not included in the factor 42.

A comparison of the values of δ, measured by means of the impact generator and from tests by Denny, has been made. The tests with the impact generator lead to a value of $\delta_{50} = 2.45 \cdot 10^{-3}$ for impact velocities of 8.3 m/sec. The first series of tests by Denny, performed with a purely smooth water surface, shows a $\delta_{50} = 5.63 \cdot 10^{-3}$, whereas the second series with a train of waves (i.e., a disturbed water surface) shows $\delta_{50} = 3.21 \cdot 10^{-3}$. Both series of tests have been made with the same impact velocities between 1 and 2 m/sec.

A similar trend can be observed from prototype observations by several authors. The observed pressures of wave impact are not much more than 50 m water column, whereas slamming tests for ships indicate pressures of more than 200 m water column. Apparently, there are three types of impact forces:

1. A rigid body falling on a smooth water surface.
2. A rigid body falling on a waving water surface.
3. A mass of water hitting a rigid body.

In the first two cases, very high pressures may occur. There seems in these cases to be a strong influence of the hydraulic radius of the impact, because ships with a U-shaped bow encounter much higher pressures due to slamming than ships with a V-shaped bow.

The third type of impact is the one we are faced with. There may be in this case (and this is only a supposition) a strong dependence between the aeration of the face of the impinging jet and the velocity. It may be that from a certain impact velocity onwards, the aeration is causing so many irregularities of the water face that this is a physical limit for the shock pressures.

Finally, I come to the scaling of models. I suppose we do not necessarily have to have a power scale; I think it is possible that there is an asymptotic scale in such a way that if the model is large enough we can scale-up after Froude. Maybe, the Paper of Skladnev and Popov already gives an indication where the limits are for the model scales.

<u>JARLAN</u>: Actually, my remarks also connect with Führböter's study. I did run myself series of tests in situ concerning the effect of breaking waves on vertical breakwaters at Le Havre. The results led me to believe that the effect of the air cushion, as far as the dampening effect is concerned, is probably true. In other words, we are in a situation where a bubble of air is somewhat acting as in case of a burst or an explosion which is in water, and whereby the bubble has a tendency, whenever it acquires a

certain momentum, to change its radius in such a way that to a certain extent the momentum will gradually be absorbed within the bubble itself. Consequently it is not surprising that when an air-bubble is interfering with the breaking wave, you will observe a certain tendency for a damping of the shock. The duration of the shock will be considerably longer, say 0.5 sec. instead of 0.005 sec.

I found also that for a wave of $4\frac{1}{2}$ to 5 m ready to break but not at the point of breaking exactly, the pressures were approximately 20 and 50 ton/m^2 for two shocks where no air cushion was involved. Whenever I observed the presence of an air-cushion (either by taking photographs or by listening), I found systematically that the pressures had a tendency to decrease. On the other hand, an important aspect of the hammer shock is the fact that the pressures are very localised timewise and spacewise, which makes detection difficult.

PAAPE: There has been a long discussion this afternoon about scaling of pressures, and I think it is very important that we know which scale-laws should be applied. But I really wonder what designers are going to do at the time when shock pressures can be exactly predicted as a function of time. Lundgren concluded (and it is quite reasonable) that the impulse of all types of shocks can be scaled up after Froude. Now, if the response of a structure on a single shock load is considered, it seems that for shock durations in the order of 0.1 to 0.01 sec, the response is almost independent of the shape of the shock and is mainly governed by the impulse of the shock. If this is so, both model engineers and designers could arrive at a compromise. Impulses can be measured more easily in the model and the computations of the dimensions of structures are also simplified.

It might be of great interest if designers would make experimental computations for various kinds of structures in order to analyze the response on shocks of various shape and equal impulse.

N.N.: I am in contact with colleagues in the U.S.S.R. and they are dealing with the same problems. They have calculated the resonance frequency of a breakwater, which was rather near the frequency of the wave load. So I agree, it is of great interest.

GENERAL REPORT ON SESSION 3

R.C.H. RUSSELL
Hydraulics Research Station, Wallingford, England

The only theme that connects the four Papers in this group - and a very fragile thread it is - is the research engineer's communication with his clients.

The theme of Mr. FERGUSON'S Paper, of which only a summary is available at the time of writing, is a regret that techniques employed by hydraulic investigators are now so specialized that designers may well find them difficult to understand. He rates the danger of misunderstandings particularly high when the phenomenon reproduced in the laboratory is complex. He is anxious lest the spectacle, for example of waves propagating in a flume, induced faith in similarity where, in fact, similarity may not exist and where in view of the difficulties of making measurements on site, similarity may be impossible to disprove.

Yet in a sense it is the simplifications that lead to the greatest misunderstandings. A regular train of waves in a channel is quite evidently to investigator and client alike a bad model of reality, and the client must take for granted what the investigator has in the past chosen for convenience to assume, namely, that the damaging properties of the regular waves are the same as those of an irregular sea of the same significant wave-height. There will be least opportunity for misunderstandings if complex phenomena are reproduced exactly, or as exactly as possible, with all their complexities left in.

The Paper by BERGE AND TREATTEBERG provides an example of how a designer who might have doubted whether a regular train of waves had similar damaging properties to a real sea, was correct in his doubts because it reveals that the more perfectly similar waves gave slightly different, and generally slightly more, damage. The differences are indeed rather subtle and not always consistent. Figures 12 and 17 show that with the widest spectrum entitled 'C' the damage was greatest and most different from that of regular waves having the same significant wave height. In the sense that the regular train of waves has the narrowest possible spectrum this result is that expected. The Neumann spectrum which was of intermediate width had damaging properties indistinguishable from those of regular waves, but the narrowest spectrum entitled 'B' was at least on one occasion more damaging than the regular waves (See Figure 13).

This inconsistency parallels that in the previous Paper on the subject by CARSTENS, TREATTEBERG AND TØRUM. Reference to Figure 5 of that Paper reveals that the widest spectrum, in this case the Neumann spectrum, was the least damaging, the narrowest random spectrum was the most damaging and that a regular train of waves had damaging properties lying between the other two.

A communication from VAN OORSCHOT discussing Paper No. 10 provides an interesting example of a major difference in behaviour arising from the uses of regular and irregular waves leading it would seem, to different optimum designs of the breakwater; while another paper in the Symposium, that by ROGAN AND BONNEFILLE, reports on a study in which the use of regular waves led to identical behaviour to that obtained with irregular waves of the same significant wave height.

The conclusion is evident. Regular significant waves can on occasions predict damage correctly: we cannot yet define these occasions.

Legitimate questions for the authors to answer include:

(a) What has happened to the plot in Fig. 4 of the data for the breakwater crest at 2.5 m?

(b) What was the size of the waves generated on the harbour side of this very low breakwater, and did the propagation of waves over the breakwater require the adjacent bank to be armoured?

(c) Is it possible that if the height distribution (Fig. 8) is similar to that of the prototype and the spectrum (Fig. 7) is similar that the correlation between wave height and period (Fig. 9) will inevitably be similar?
Is not a parameter defining long-crestedness likely to be more significant?

(d) Could the scatter in Fig. 19 in the results of apparently similar experiments be caused by differences in packing the rocks on the model-breakwater, that is to say by differences between two individual assessments of what constitutes a random packing?

The paper by JENSEN, written from a client's viewpoint, draws attention to the value of model studies and of computations. The Reporter being in a research laboratory would have dwelt far more on the limitations of the techniques; on their ability to answer ** very accurately idealised questions rather than to answer adequately the clients' real questions.

Wave-disturbance studies, for example results of which are presented in Figs. 2 and 3, have been the bread-and-butter of several hydraulic laboratories for many years, yet still they rarely answer the question that the client wishes to put. The question he wishes to ask is: How much will a moored ship move at a berth if a certain sea, propagating from a certain direction, is associated with a certain height of tide? He would be prepared to use average values or significants to describe the characteristics of the seas and of the movement of the ships. At the present time no answers are possible. The question that he is advised to ask is "What will the wave-height at a berth be if there are monochromatic waves, infinitely long-crested, propagating from a single direction and combined with a particular height of tide? because this is the question that can be accurately predicted by conventional studies. Unfortunately the answer is extremely sensitive to very minute changes of wave period, wave direction and height of tide, so that in order to get any sort of average the problem must be studied extremely intensely.

Furthermore, the information on wave-heights, even if it could by some means be converted to significant wave-heights in a complex sea, give him no indication of the movement of the ships. There is a total lack of information on the movement of ships moored in irregular swell, and the designer has frequently to fall back upon guesses regarding tolerable wave-heights at berths. Although the technique exhibits very severe limitations when used to predict disturbance in harbours on underdeveloped coastlines, it is of considerably more value if it can be used in a comparative way to relate the disturbance at a new quay to the disturbance at an existing berth, the adequacy of which has already been assessed by experience.

The gross simplifications involved in the conventional wave disturbance study prevent the client getting an accurate understanding of his harbour's performance: greater similarity with real waves, even if these were to involve much greater complexeties in describing the wave motion, will help rather than hinder the client's appreciation. Unfortunately no laboratory has designed a wave generator capable of producing a swell with a three - dimensional spectrum other than by the extremely expensive procedure of employing wind. And it is likely to continue difficult.

Legitimate questions for the author to answer include:

(a) Are not many irregularities in the iso-wave-height pattern shown in Fig. 3 omitted? Similar diagrams produced at the Hydraulics Research Station have always revealed a far more complex pattern of nodes and antinodes along the quays which render a general assessment of the adequacy of a berth difficult?

(b) Whose method of the several available in the literature was used to predict the coefficients shown in Figs. 2 and 5?

The Reporter congratulates BISCHOFF VAN HEEMSKERCK and BOOY on their paper "Optimization of Financial Investments". He has verified that no such procedure as that outlined is in use for maximising the benefits arising from a group of investments in sea-defences in the United Kingdom where the paper has been read with interest. He is not able to comment further.

The message that emerges from the group of Papers is for the need for close integration of the activities of designers and investigators. The establishment of criteria cannot be done in isolation: they need to be established in an understanding of the type of results expected and need to be continually revised in the light of the actual results. Fortunately the inevitable discussions that take place generally result in a mutual understanding of a fair measure of one another's problems.

REPORT ON DISCUSSIONS

Session 3: Morning, March 26, 1969.

Chairman: J.B. Schijf,
Rijkswaterstaat,
The Hague, The Netherlands

Reporter: R.C.H. Russell,
Hydraulics Research Station,
Wallingford, England

Discussion on: Paper 8
Paper 9
Paper 10
Paper 11

Participants in the Discussion:

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Application of Laboratory Tests in Harbour Design Works,

by JENSEN

JENSEN agrees with the position of the designer as illustrated in the General Report. This position asks for the results of a laboratory to be presented in such a way that the effect of minor changes (to which the designer may be forced) can be evaluated.

Regarding the inability of answering adequately the client's real questions with respect to wave-disturbance problems as suggested, the Reporter extends his criticism much more than I do. I am not sure that what we want to ask is: How much will a moored ship move at a berth? We will have to know first how much is allowed from other viewpoints, also in terms of statistics. The best approach at the moment is to use experience from existing similar harbours and then use the diagrams we get from the laboratory. In respect to this problem we have to take the laboratories as they are and not as we want them to have.

The Reporter asked me in his General Report, if there are not many irregularities in the iso-wave-height pattern shown in Fig. 3 omitted. I do not think so, but I have to pass on this question to Bijker, because it is a copy of his report.

As to the second question posed by the Reporter, about whose methods we used to predict the coefficients shown in Fig. 2, we used the graphs given by Johnson in the Proceedings of the 2nd Conference on Coastal Engineering.

It is interesting to see that the Norwegian breakwaters are as strong as they are. However, according to the Hudson formula I think you would need a slope of 1 : 3 or 1 : 4 for a no damage criterion, instead of the slopes of 1 : 1.25 as applied. Possibly the explanation is the stabilizing effect of moderate wave attack, especially on steep slopes where the gravity attempts to consolidate the packing of the front slope. Also the fact that blocks of various shapes might be used, giving a good interlocking, may be an explanation.

BRANDTZAEG agrees with the remarks of Jensen and adds that the use of this type of breakwaters has been imposed by the necessity of getting at least something when funds available are limited.

METTAM:"As a consulting engineer I would not be satisfied with a diffraction calculation as presented by Jensen. If there is no time available to do a test before construction starts, you have plenty of time to do it while you are building the breakwater. This will not hold up the work, and if you can make a small saving at the end of the breakwater, you can easily save far more than the cost of the model"

On the question of the relations between the designer and the laboratory METTAM suggests that many designers would like to spend more time in the laboratory discussing the problem while the experiments are going on. This is a difficult relationship because one can generate new ideas faster than one can test them. However, "I think the best laboratory is the one which is close to my office and which has open doors and where I can discuss the problem as it develops"

Special attention of the Laboratories is drawn to the need for reliable prototype wave recorders. I have dealt with five different types and they all gave trouble, particularly the pressure instruments, where the conversion depends on the wave period and the water depth. My experience is that the best one gives 25% of the records one expects to get. The wave recorder we want to have has to fulfil the following requirements:

- It has to be cheap and robust,
- it must not be a pressure instrument,
- it must not have a cable where one can hook an anchor around,
- it must send your records to you straight away so you know when it stops working, and
- it must also give you the direction of the waves.

About the question of movements of ships the Reporter touched on, it seems to me that the question is not how much will it move under certain conditions, but how much will it move and how often? It is a probability calculation to be used in an economic study. Again this depends on the wave measurements one is able to perform.

JENSEN: I agree that model tests should have been performed, but the position was forced by the commissioner who had to pay for the model tests. Another reason was that this lay-out had never been constructed as it was shown.

PAPER 9

The Use of Model Tests for the Design of Maritime Structures, with Special Regard to Problems of Wave Action

by FERGUSON

THORPE: Ferguson poses in his Paper that on the basis of preliminary tests it was concluded that the stability of specially-shaped interlocking blocks was less than expected. This is a very important decision in view of the large quantities of material involved. Can Ferguson tell us why no detailed tests were carried out from the very beginning with specially shaped interlocking units?

Ferguson continues that in order to attain a large output one has to consider that the blocks would have to be fabricated, handled and transported in an easy way; this consideration asked for a simple shape. This implies that any interlocking unit would be costly. I think this is not the case, and because it is such an important decision, I would like to hear more about it.

Finally, can Ferguson give us any rule as to the amount of money which ought to be spent on model tests as a percentage of the whole project?

FERGUSON answers that one was faced with the problem of a very large number of blocks while one did not know in advance what would be the equipment to bring them into position. It is very difficult to make a really calculated decision if one has not yet designed the equipment. This was the main reason why a block which is very simple to handle was chosen. However, after the decision was made we still wondered if there was a decisive difference between the stability of cubes and the specially-shaped blocks. If the difference had been really decisive, we would have thought of a special block, knowing that there would be complications. These considerations cannot be evaluated in various sharp figures, so the ultimate decision is always influenced by the feeling of the designing engineer who has to judge all these considerations. I think the decision was right and supported by the Delft Hydraulics Laboratory.

The cost of model investigations is in most cases a few percent. The estimate of cost of the hydraulic model studies for this project is 5 to 10 million guilders. The total cost is about 700 million guilders.

PAAPE: The saving in the amount of concrete by using specially-shaped blocks is 30% if you use the best block under optimal conditions. It is in many more practical cases 20 - 25%. In the case of the Europoort breakwaters with the low crest, wave attack concentrates especially on the inner crest line. Then interlocking is not so effective because legs of these blocks are protruding. It was our part in the advice that in this case the savings in the amount of concrete were reduced to 10-15%.

PAPER 10

Stability Tests of the Europoort Breakwater,

by BERGE and TRAETTEBERG

DICK asks if the wave spectra were measured with the breakwaters placed or not.

TRAETTEBERG: No, they were measured at the position of the breakwater without the breakwater.

DICK: Then how do you know that after the breakwater is in, the incident spectrum is still the same?

TRAETTEBERG: In this particular case the reflection was very small, due to the low crest. So very large differences are not to be expected.

DICK: Did you measure the reflection in a regular wave train case?

TRAETTEBERG: No, the reflection has not been measured in this way. But there are some indications, because waves were also measured in the control part of the flume, where there is no breakwater.

D'ANGREMOND adds that the regular waves mentioned in paper 10 were in fact not regular waves but wind-strengthened swell.

WIEGEL asks what the scatter was of the significant wave heights mentioned. Every time one measures the wave-height, one has a different sample and consequently gets a different answer.

TRAETTEBERG is not able to give exact figures but expects that they were pretty close, since the data were based on 200 successive waves.

WIEGEL: Is the use of regular waves with height H and period T equal to the significant height H_s and period T_s of a train of irregular waves in breakwater studies adequate for the designer, especially if the uncertainties in design conditions are taken into account?

TRAETTEBERG would not be satisfied with the wave spectrum only as a parameter for the wave motion, as suggested by the Reporter. The elctronic engineer may be satisfied because his noise is in conformity with the theory. As long as this has not been proved for gravity waves, one has to stick to wave-height distribution, joint distribution, etc.

The effect of long or short crestedness is not expected to be significant in the case of a section of a breakwater, because the blocks are not so much influenced by what is happening in another section.

As to the question of Prof. Wiegel about the apllicability of a regular wave train, Traetteberg expects that the possible differences are largest in shallow water, which acts as a filter, causing waves to break. However, one never knows how much the difference is before one has made the experiments.

RANCE gives some information on rip-rap studies performed at the Hydraulics Research Station at Wallingford, as a more general remark regarding the applicability of monochromatic waves.

As to the reproduction of the same damage, it seems that in one case H_s may be taken as the height for the regular waves and $H_{1\%}$ equally in another case. If one considers the simple case of a single stone that requires a certain wave height to be moved, one has in case of an irregular wave motion to wait for a wave of that particular height to move it. It is a matter of probabilities, and the probability depends on a parameter describing the average wave height or wave energy. From this point of view there is no reason why it should be related to H_s or $H_{1\%}$; if one is prepared to wait long enough the right size wave will come along. Applying regular waves in a more complicated situation of an armoured slope with various degrees of damage, one gets a damage curve (damage versus number of waves), the shape of which depends upon the grading. Instead of a simple probability (failure) one has to do with a "wait probability", which also means that one should perhaps not consider just the design storm, but also relatively mild conditions that precede this storm. This approach, however, tells you nothing of the maintenance costs involved, so this leads to an alternative, designing for a certain percentage of damage per year. Then one can balance the maintenance costs against the capitalization and come to a most economic solution.

Additionally one is referred to: J. van de Kreeke and A. Paape, "On optimum breakwater design", IXth Conf. on Coastal Engineering, Lisbon, 1964.

PAAPE remarks that regarding the question as to whether regular waves are adequate or not to give an answer on breakwater problems, one has to distinguish between two questions:

- Can irregular waves with a certain significant wave height H_s be replaced in the model by regular waves having a wave height equal to H_s?
- Is the significant wave height applicable or not in the design of structures, especially from the point of view of the designer.

The research people will most likely say no, because they are answering the first question and there are apparently situations also in the case of a rubble-mound breakwater where you cannot replace a train of irregular waves by a regular wave.

However, the designers will most likely say yes, because they emphasize the second question, and they cannot possibly tak into account, in their feasibility studies or economic decisions, all parameters describing the irregular sea.

This means that the laboratories have to apply irregular waves in their model studies, but provide the information on the relationship between wave conditions and damage in terms of simple parameters such as H_s. Maybe, instead of one formula of Hudson, they have to make two or three, each one valid for a certain type of structure or a certain type of wave conditions.

THIJSSE: Also as a consultant I cannot be satisfied by an infinite repetition of always the same wave. If so, it would not have been necessary to build this new wind flume.

PAAPE: You are considering the model results and as such you answer my first question.

Financial Optimization of Investments in Maritime Structures,

by BISCHOFF VAN HEEMSKERCK and BOOY

BISCHOFF VAN HEEMSKERCK: "In his written discussion Berdenis van Berlekom refers to a number of objections which could be made against the application of the method described in Paper 11. As the authors of this Paper do not agree with all his considerations, it is a pity that Berdenis van Berlekom could not attend this Symposium. However, we are sure that in spite of his absence he will be interested in our reply, wnich will be included in the Proceedings of this meeting.

First he states that «one group of people attaches great value to a very high income growth, another to a happy and healthy life».

Of course this is true and consequently there might be a tendency to consider this as a «political» decision only.

Apart from the fact that growth of income and happiness cohere, the question arises whether a policy maker will be able to solve this problem without a certain valuation of «happiness and health». If such a valuation must indeed be considered inevitable, one might as well introduce «happiness and health» in the calculations.

The second objection refers to the necessity of «a totalitarian authority, overruling the feeling and opinions of the one group in favour of the desires of the other».

With the respect to this objection we would like to answer that every decision implies the risk that the feelings of a group of people will be overruled.

The method described in our Paper only pretends to provide policy makers with tools which will enable them to make the proper decisions. Thus in our opinion the central authority recommended in the paper will rather prevent than promote governmental arbitrariness.

Furthermore, Berdenis van Berlekom states that «the riskfactor must in a large degree participate in the process of decision making and may upset or even distort pure economic reasoning. It would seem that this is a reason for the application sometimes of the Pay-Off Period as an investment criterion».

In special cases like political instability or economic depression, the pay-off period may indeed be useful, but this criterion can easily be introduced by capitalization of the benefits over a limited period T (see Chapter 8, page 11).

Then Berdenis van Berlekom suggests that ≪the economic system of free enterprise tends towards a similar goal. Economic theory claims that capital will flow towards those projects where the profitability is highest, so that more or less automatically the marginal benefit cost ratios $\frac{db}{dc}$ for all projects become equal≫.

This, indeed, is what one might expect theoretically when financiers are provided with the proper data. However, the fact that the usual optimization techniques include the error of maximizing the benefits per project, without any reference to the general policy of investment by the financier, already shows that many financiers are used to the habit of judging the merits of a project separately.

Finally, Berdenis van Berlekom states that ≪analyses like this one are nowadays very usual≫ , that ≪financiers are not content with $\frac{db}{dc} = 1$≫ and that we ≪may rest assured that international institutions such as the World Bank are prepared to lend money only after they are satisfied that the profitability complies with the criterion of opportunity cost, not of the simple market interest≫.

Answering these remarks, we would like to repeat that the usual optimization techniques aim at maximization of the benefits per project. From their experience also with international institutions the authors know that many designs in the past have been based on maximization of $(b_i - c_i)$ or $\frac{b_i}{c_i}$. As has been shown in Chapter 9 of paper 11, maximizing $(b_i - c_i)$ means the use of $\frac{db}{dc} = 1$, whilst maximizing $\frac{b_i}{c_i}$ is not consistent with an optimal policy of investment".

VASCO COSTA suggests that when making a decision based on the approach of Bischoff van Heemskerck one may risk more than one can justify. The value of money one gets decreases with the amount of money because also the utility decreases, even as it is the other way round when it comes to losing money. So the value of money that one risks depends on the amount of money one has to spend. In other words, this value function is not a linear one. The approach of Bischoff van Heemskerck is a wonderful one for an engineer to decide how to design a structure. However, we as engineers have to advise people like a board of a company or a harbour authority. They have to be very careful, because the consequences of adopting such a criterion can affect the future of a company or a town.

There is still another difficulty. One is never sure about the amount of money one is risking. The evaluation of the consequences of a flood, for instance, are extremely difficult. I am not able to solve this problem, but I stress that one cannot ignore that the problem is not as simple as it was presented in Paper 11.

BISCHOFF VAN HEEMSKERCK: Vasco Costa refers to a theory (evaluation of money) which I think does not interfere with the matters we described in the paper, but Booy will explain this later.

" In the Netherlands we have so much experience with floods that we know exactly what the cost are".And, more general, over the whole world there is a lot of experience. So I am not afraid to estimate the damage due to failure.

BOOY agrees with Vasco Costa that the appreciation of some amount of money is not a linear function of this amount. We are not interested in investments, nor benefit from investments, but in consumption (K). We want to maximize the total of the (subjective) value we assign to this consumption over the years or max $\int V(K)\, dt$. K is the result of the cash-flows as it is called of all projects, or

$K = \sum_i y_i(t)$ where y is benefit minus cost as a function of time.

The necessary condition for the optimum then is:

$$\int \frac{dV}{dK}\, \delta y_i \; dt \leqslant 0.$$

If a marginal investment of 1 dollar could result in a benefit of q dollars a year, we find that

$\frac{dV}{dK} = e^{-qt}$ times a constant.

So for each project we maximize $\int e^{-qt} y_i(t)\, dt$.

If we would have maximized B - C for each project, we would have max $\int e^{-rt} y_i(t)\, dt$. This analogy between r the interest, and q a factor which depends among others on technology, is as yet unexplained and it is also not known whether this q has anything to do with the q from our paper.

GENERAL REPORT ON SESSION 4

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The Papers discussed in this General Report are as follows:

Paper No.	Author and Title
12	J. SOMMET and Ph. VIGNAT, Complex wave action on submerged bodies,
13	P.J. RANCE, The influence of Reynolds number on wave forces,
14	H.J. ZUNDERDORP, The influence of wave forces on the design of offshore structures for the oil industry.

Progress achieved in theoretical and experimental basic research on the subject of wave-induced forces on structures at sea is due to a great extent to the development of offshore oil drilling. For this reason the Papers presented here deal more or less directly with the specific problems of this industry. The objectives aimed at are:

- The development of theoretical methods of calculation of hydrodynamic forces in regular waves. The Paper by Sommet and Vignat covers this field of investigation,
- The experimental study of forces which are still difficult to derive theoretically such as drag forces and lift or transverse forces on piles. The Paper by Rance deals with this problem.
- The development of statistical methods to analyse wave-induced forces and the resulting vibrations or oscillations of the structure. These methods are described in Zunderdorp's Paper and in the General Lecture by Wiegel.

In this Paper, Sommet and Vignat describe a numerical method for calculating wave-induced forces on fixed or freely moving submerged bodies, when no boundary layer separation occurs. This method is based on potential flow theory and the velocity field is described by a single-layer potential of the source-sink type. After a description of the basic assumptions and equations used, the authors prove the accuracy of the method on two examples where theoretical solutions are explicitly known. Finally, the method is applied here to the calculation of forces on an immersed caisson and of the resulting heave and pitch under the action of irregular waves.

By this procedure it is possible to determine the added mass and added inertia of deeply submerged bodies. The interference effects of neighbouring members of a composite structure may be studied as as a function of their spacing.

In the described form, the method does not allow for free-surface effects. To do so, the kernel $1/|MP|$ of integrals (2.5) and (2.6) should be replaced by John's formula (1):

$$G(MP) = \frac{1}{R} + \frac{1}{R'} + \int_0^{+\infty} \frac{2(w^2 + gk)e^{-kd} \cosh k(z+d) \cosh k(c+d)}{gk \sinh kd - w^2 \cosh kd} J_0(kr)\,dr \quad (1)$$

$$r = \left[(x-a)^2 + (y-b)^2\right]^{\frac{1}{2}} ; \qquad R = \left[r^2 + (z-c)^2\right]^{\frac{1}{2}} ;$$

$$R' = \left[r^2 + (2d+z+c)^2\right]^{\frac{1}{2}} .$$

(x, y, z) and (a, b, c) stand for the coordinates of points P and M in figure 1, J_0 is the Bessel function of first kind and zero order, w is the wave angular frequency, d the water depth.

With this more general form, the same numerical procedure can be used to calculate such surface effects as gravity waves reflecting from a fixed pile of a large diameter with respect to the wave length, and damping forces resulting from the radiation of surface waves by a freely floating structure. Details and applications of this method are given by Lebreton and Margnac (2). A simpler form of the velocity potential, valid only for vertical boundaries extending from the bottom to the water surface, has been derived by John and applied by Daubert and Lebreton (3).

These theoretical methods are only valid for waves of small amplitude, but they can give valuable information on the behaviour of different types of structures. The behaviour in high sea states can only be studied on laboratory scale models or in the field.

The method described here differs also from the above mentioned methods because it allows for the square velocity term in equation (2.10) and for the second-order interactions between components of the given sample wave spectrum. This is done in order to derive the force of attraction towards the water surface of submerged bodies and various force-motion coupling phenomena. This refinement greatly increases the time required for computation. Furthermore, statistical methods of analysis used with linear systems do not strictly apply in this case. For this reason, the statistical properties of the calculated forces and movements have to be derived from the analysis of time-history samples such as those given in Figure 15.

The authors mention that friction drag forces associated with unseparated boundary layers are now being considered in their mathematical model as forces proportional to the square of the relative velocity. Transverse forces and drag forces due to boundary layer separation and eddy shedding under wave action cannot be calculated at present. Improvements in numerical methods and a better knowledge of wake formation in oscillatory motion should allow the calculation of these forces in the future. McNown and Keulegan (4) give an example of an approximate theoretical derivation of the drag and virtual mass coefficients of a flat plate in oscillating flow, using the free streamline theory. These results are shown by a continuous line in Figure 15a of Wiegel's lecture together with experimental results in dashed line.

PAPER 14

In this Paper Zunderdorp makes a critical review of methods used in the design of offshore drilling structures.

Fixed platforms in shallow water have natural frequencies beyond the wave spectrum frequency range. Consequently a static analysis with a two-dimensional regular wave is used, forces being calculated with Morrison's formula.

Zunderdorp questions the usefulness of this formula for two reasons; first, because it requires the calculation of wave velocities and accelerations by means of very elaborate non-linear theories, and secondly because the associated drag and virtual mass coefficients are very crudely estimated. The use of some other formula relating directly the water-induced force to known variables is advocated.

Fixed platforms in deep water have natural frequencies in the range of the wave spectrum frequencies and due allowance has to be made for the structure's flexibility. Morrison's formula, linear wave theory and Borgman's method of analysis of wave forces are used to derive the spectral density of wave force from the known wave spectrum. Like Wiegel, the author draws our attention to the possibility of piles of large-diameter shedding eddies at a frequency equal to a natural frequency of the structure. As shown by Laird (5) on an inverted wave model (i.e., a cylinder oscillating in calm water) this generates vibrations and oscillating forces of high amplitude which may result in fatigue failure. A plea is made for more research on this point.

The spectrum of a floating platform's response can be determined in a way similar to the one used for wave-induced forces. The transfer functions needed for this purpose can be determined from laboratory scale models by two methods explained by the author.

I will now add here some personal comments on Morrison's formula. First considering regular waves, a rational analysis of C_D and C_M should be conducted on the basis of the relation which can be derived from Buckingham's π-theorem, i.e., if the basic variables of Keulegan and Carpenter (6) are considered:

$$\frac{F}{\rho\, gD^3} = f\left(\frac{u_m T}{D}, \frac{u_m D}{\upsilon}, \frac{u_m}{\sqrt{gD}}, \frac{z}{D}, \frac{t}{T}\right) \qquad (2)$$

where u_m is the maximum horizontal velocity of water particles

T wave period

D diameter of the cylindrical pile of given orientation

z distance of the test section below the water surface.

z/D is here considered to account for the wave velocity and acceleration gradients along a vertical.

t time.

If the wave period is replaced by the maximum horizontal acceleration A_m, dimensional analysis leads to a relation where Iversen's modulus appears:

$$\frac{F'}{\rho\, g D^3} = f\left(\frac{A_m D}{u_m^2},\ \frac{u_m D}{\upsilon},\ \frac{u_m}{\sqrt{gD}},\ \frac{z}{D},\ \frac{A_m t}{2\pi u_m} \right) \qquad (3)$$

It should not be forgotten that $\frac{A_m T}{\sqrt{gd}}$ and $\frac{u_m}{\sqrt{gd}}$ also depend on wave steepness $\frac{2\pi H}{gT^2}$ and on relative depth $2\pi d/gT^2$. If as suggested by Zunderdorp, we wished to work on the basis of easily known wave characteristics, we should have to determine the following relationship experimentally.

$$\frac{F}{\rho\, g D^3} = f\left(\frac{H}{D},\ \frac{HD}{\upsilon T},\ \frac{2\pi H}{gT^2},\ \frac{2\pi d}{gT^2},\ \frac{z}{d},\ \frac{t}{T} \right) \qquad (4)$$

Any parameter in these relations can, of course, be replaced by a combination of others in the set found to be more suitable by experiments. As can be seen, the difficulty in such a study comes from the great number of parameters involved. In past studies, attention has generally been focussed on a single parameter.

Keulegan and Carpenter's experiments with regular standing waves showed the influence of what they called the period parameter $u_m T/D$, but the influence of the Reynolds number $u_m D/\upsilon$ did not appear due to the narrow range investigated (from 5×10^3 to 3×10^4). The parameter z/D and the Froude number $u_m/\sqrt{gD}$ are ignored. As stated by Wilson (7), it is important to consider this last parameter for sections located near the water surface, where secondary gravity waves may be generated by the flow around the cylinder.

Investigations of C_D and C_M in irregular waves by Wiegel, Beebe and Moon (8) and by Agerschou and Edens (9) gave very scattered values. Their method of investigation, which is described in Zunderdorp's Paper, is thus seen to be not very suitable for randomly varying wave forces and surface elevations.

Several methods of statistical wave force analysis have since been devised by Borgman (10), Pierson and Holmes (11). Borgman's method based on the cross-spectral density between the measured wave force and wave surface profile seems to be the most interesting, since it shows the variation of C_D and C_M with angular frequency over the whole extent of the wave spectrum. Thus a comparison with results of regular wave experiments is possible. The other methods are based on:

- the moments of the force distribution,
- The least-square fitting of the co-variance function,
- the least-square fitting of the spectral densities,
- the moments of the peak force distribution, and
- a graphical fitting procedure of peak force distribution.

These methods are easier to work out than the first one, but only yield mean values of C_D and C_M over the range of spectrum frequencies. The few results given by Borgman and Jen (12) are promising and these methods should be applied more extensively to laboratory and field measurements by a digital or analog process. However, they assume that the linear wave theory applies; so efforts should be made to check their accuracy and find methods more suitable in non linear wave conditions.

PAPER 13

Rance stresses the influence of Reynolds number on wave-induced forces for cylinders of small diameter. His experiments were carried out in a pulsating water tunnel which could reach a Reynolds number $aD/\upsilon T$ of 6×10^5 (where a stands for the semi-orbit length). Because there was no free water surface involved, the Froude number and depth of submergence were eliminated, which enabled the study to be restricted to the following parameters:

$$\frac{F}{\rho \, gD^3} = f \left(\frac{a}{D} , \frac{aD}{\upsilon T} , \frac{t}{T} \right) \qquad (5)$$

The results given in Figure 1 show a marked decrease in wave forces due to the existence of a critical Reynolds number between $aD/\upsilon T = 10^4$ and $3.5 x 10^5$ in a manner similar to steady state flow. The scale effect resulting from the usual Froude scaling law is estimated from these results on a numerical example.

As few results are released here on the mean in-line forces, many questions remain unanswered. The following points for example, would be of great interest:

**- Whether the mean in-line force, at high values of a/D, could be derived from drag curves known for steady state flow with an appropriate definition of the characteristic velocity to use in the Reynolds number; and

**- whether the range of a/D where the transition occurs from predominant inertial forces to predominant drag forces varies with the Reynolds number. This transition can be seen from the shift of the phase lag between maximum wave force and maximum wave velocity.

If we compare the curves in Figure 1 with Figure 31 in Keulegan and Carpenter's Paper (assuming $u_m = 2\pi\ a/T$), it can be seen that the value of $F_{max}/\rho\ gD^3$ given by Keulegan for a Reynolds number $aD/\upsilon T$ of 10^3 to 5×10^3 is about 20 to 50% lower than the value given by Rance for $aD/\upsilon T = 10^4$. It would be greatly appreciated if Rance could tell us whether this difference is due to the Reynolds number.

In the present paper Rance also gives some information on the high-frequency in-line and transverse forces which are due to eddy shedding. His experiments show that:

- The transverse forces are of the same order of magnitude as the mean in-line forces at low Reynolds numbers, i.e., about 10^4 as can be seen by comparing Figs. 1 and 2. It is difficult to estimate the ratio of transverse to mean in-line forces for high Reynolds numbers from these figures. The frequency N of these forces is consistent with a Strouhal number ND/υ of 0.2 $\pm$ 6%.
- The in-line high-frequency components of the forces are about one half of the transverse forces in magnitude. They are generally less than 10% of the mean in-line force, but may be as high as 50% at low Reynolds number. They have no definite frequency.

It should be noted that the natural frequency of the testing device was much higher than the observed eddy shedding frequencies; this device could thus be considered as stiff. Consequently the features of forces described here are different from those to be expected at resonance for a flexible pile.

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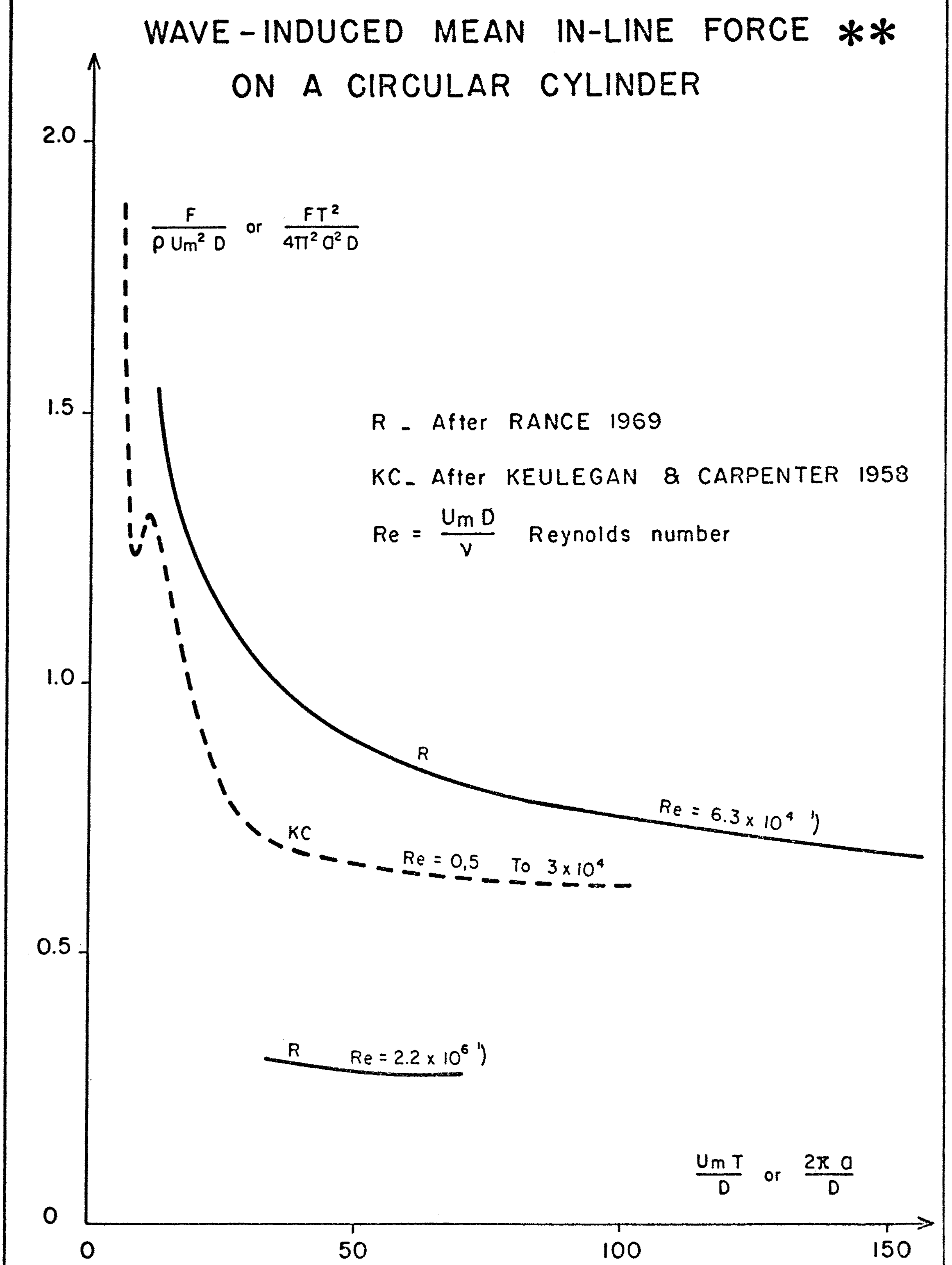
Fig. 1



¹) Corrected by Rance to 10^4 and 3.5×10^5 respectively

REPORT ON DISCUSSIONS

Session 4: Afternoon, March 26, 1969.

Chairman: G.H. Pesman,
Municipalty of Amsterdam,
Amsterdam, The Netherlands

Reporter: P. Gaillard,
SOGREAH,
Grenoble, France

Discussion on:
Paper 12
Paper 12 A
Paper 13
Paper 14
General Lecture

Participants in the Discussion:

H.N.C. Breusers, Delft Hydraulics Laboratory, Delft, The Netherlands
P. Holmes, University of Liverpool, Liverpool, England
S.U. Khan, Bechtel International, The Hague, The Netherlands
D.M. McDowell, University of Manchester, Manchester, England
P.J. Rance, Hydraulics Research Station, Wallingford, England
J. Sommet, SOGREAH, Grenoble, France
R.L. Wiegel, University of California, Berkeley, U.S.A.
H.J. Zunderdorp, Koninklijke/Shell Exploratie en Produktie Laboratorium, Rijswijk (ZH), The Netherlands.

PAPER 12

Complex Wave Action on Submerged Bodies

by SOMMET and VIGNAT

The REPORTER regrets that due to the late presentation he has not been able to make comments on the contribution of Lebreton and Cormault.

SOMMET thanks Lebreton and Miller for their suggestions to bring into account free surface effects in water of infinite depth. The introduction of extra terms, however, will increase the time required for the computations. Therefore a careful comparison must be made between the extra cost involved and the errors due to the simplification. From case to case a decision will have to be made, possibly on the basis of a simple computation test.

BREUSERS asks how Sommet motivates his conclusion that the calculation method is valid for bodies of any shape, with and without sharp edges. Especially in the case of separating flow, important deviations must be expected, as shown by Keulegan, who measured C_M values up to 3 or 4 for a strip, instead of the potential value of 1, a result which has been confirmed by our own measurements

SOMMET replies that in the calculation process sharp edges are replaced by a curved form with a small radius, which gives no significant difference with the sharp edge.

It was clarified in private discussions that the answer of Sommet referred to the computational problem connected with a sharp edge as a singular point. Breusers' question, however, was directed to the physical problem encountered in separated flow, which indeed is not solved in the proposed computation method.

PAPER 13

The Influence of Reynolds Number on Wave Forces,

by RANCE

RANCE regrets that he is not in a position to present all the results obtained from the investigation. Additional to the figures attached to his Paper, he shows a Figure presenting values of C_D in uni-directional flow versus values in oscillatory flow.

MCDOWELL asks whether the Reynolds number used in the graphs is $\frac{D.a}{T.\upsilon}$ and whether viscosity has been varied during the tests.

RANCE: The velocity introduced into the Reynolds number was neither $\frac{a}{T}$ as suggested by McDowell, nor $\frac{2\pi a}{T}$ which is the maximum velocity. Actually, the instantaneous velocity has been used. However, as the drag force is predominant, one is close to the maximum velocity condition ($\frac{2\pi a}{T}$).

The viscosity has not been changed, as water was used throughout the investigations.

MCDOWELL: If you refer to your Figure 2, is not the coefficient of lift C_L a variable which is in fact $\frac{2T}{a}$ divided by the Reynolds number? If viscosity has not been changed, I am really wondering whether the kind of curves you have got are not based on a reciprocal plotted against itself (1/x plotted against x).

RANCE: No, I think not.

BREUSERS: I am very grateful to Rance that he has cleared up the question in my discussion paper, by showing the unpublished Figures.

RANCE: A factor of 2 π has, in fact, crept in the graph presented by the Reporter. The Reynolds number I used had already included this factor. The values given in the General Report Fig. 1 (6.3×10^4 and 2.2×10^6) should be 10^4 and 3.5×10^5 respectively. This lies within the trend of Reynolds numbers established by Keulegan and Carpenter.

Keulegan and Carpenter did not find a distinct relationship with the Reynolds number due to the great scatter of results. I did not puranalysis of their results because I felt that possibly there was a Froude number effect coming in. Though their experiments were safe in this respect if you follow Lamb's derivation for the generation of a surface wave by a submerged cylinder, I am not convinced that this derivation applies here. Lamb was dealing with an ideal fluid, and I think wake effects will disturb the free surface. This fact may be responsible for the scatter in the results of Keulegan and Carpenter.

Another question raised by the Reporter is whether the uni-directional flow could be used, I think the answer is no. I must confess we have considered dragging a model through the water at high speed but we had to discard this idea.

The transition range from predominant inertial forces to predominant drag forces depends upon the a/D ratio. In fact, for the inertia and drag forces to be equally important, a/D has to be about 1.5. We did in fact go down to a/D ratios of 1.5.

REPORTER: Keulegan and Carpenter's results show that the transition range extended from about a/D = 0.5 to a/D = 5 for the Reynolds numbers investigated by them. I wanted to know if this transition region depends greatly on the Reynolds number and if it extends or becomes narrower for high Re numbers.

RANCE: I agree that we did not go down to an a/D ratio below 1.5, where acceleration forces and drag forces were equally important. But earlier I did a rough analysis. Assuming that one accepts an error of something like 10% of the total force, an a/D ratio as low as 0.25 has to be selected. On this argument the drag forces are still quite important also for very small a/D ratios. The indications are that the a/D ratio is far less important for the higher Reynolds numbers.

PAPER 14

The Influence of Wave Forces on the Design of Offshore Structures for the Oil Industry,

by ZUNDERDORP

SOMMET: Did the author have the opportunity to determine the transfer functions for floating platforms by both approaches described on pages 4 and 5 of his Paper and to compare the results?

ZUNDERDORP: For two platforms a comparison has been made. The deviations are of the same order of magnitude as the measuring errors in the model. A problem is always to reproduce the elasticity of the anchor system on model-scale. In view of these problems, the results are strikingly good.

KHAN: This is a question from a designer or a consulting engineer who has to face the problem in the field and not in the laboratory. It has been observed that in the open sea, one seldom meets a situation where waves and currents are not acting together. In the Morrison equation the velocity is taken as that produced by waves alone to calculate the drag force, and then the other current components are taken separately to calculate the drag force and added up in the end.

A big discrepancy is liable to occur, as the drag force is proportional to the square of the velocity. As there are two different kinds of flows (oscillatory and uni-directional), is it possible to have a linear summation and use that value in the Morrison equation? My second question, then, is whether it is justified to use the same coefficient of drag for both the oscillatory and the uni-directional flow.

ZUNDERDORP: I can only tell our way of doing in this field. In this method, first both velocities are added previous to squaring. And consequently one coefficient of drag is being used for both the contributions of uni-directional and oscillatory flow.

I am not convinced that this method is correct, but as long as there is nothing better than the Morrison equation, we have to work like this.

GENERAL LECTURE

Waves and their Effects on Pile-supported Structures,

by WIEGEL

HOLMES: When looking at the analysis of wave forces, one is faced with a set of alternatives.
The first one is: either Morrison or an alternative.

The Morrison equation ignores interference between the structure and the particle orbital velocities.

The desirable alternative would be a theory which included the disruption of orbital velocities. I know of none and it must be remembered that should such a theory be developed, it has to be applied to a random wave field.

Even when the concept of Morrison is accepted, there are several methods of approach:

a. Design wave,

b. numerical procedure as proposed by Dean, and

c. spectral and probabilistic approaches.

Ad. b

The method of Dean deals with actual time histories and could include varying values of C_D and C_M, as functions of time. This seems very laborious, but it should be realized in this respect that the capacity of computers to deal with this size of problem is increasing rapidly.

Ad. c

The goal of the probalistic and spectral method is not to compare random and uniform wave data. Ultimately, we have to come down to a design.

If a spectrum of surface elevation, and hence force, is available, what is the value of that spectrum without the associated probability density function? Especially in the case of pile forces, we are interested in absolute extreme values. However, even for surface elevations, we do not know sufficient about the behaviour of an assumed Gaussian or Rayleigh distribution at extremes. The efforts which have been made with the probability density functions of force are to try and clarify what happens at extreme values. Wiegel has shown that to get the spectral density of force, it is possible to linearize in one way or another within quite acceptable limits. One has to be careful that this does not imply (and I am sure Wiegel does not) linearization of the problem in terms of probability densities, because if this is done, the discrepancies found at extreme values with measured force data are removed.

Unfortunately the data we used in our work on non-linear probability density functions will not be available before May. Apparently these data deserve still far more analysis.

It has been said that if we use the statistical approach (and let us recognize here that we are in a way lumping our ignorance in the probability density or a spectrum) no correlation is found between wave profile and wave force. I think here we are looking too much for correlation between what the wave is <u>now</u> and what the force is <u>now</u>. Gradually it is recognized that eddy-shedding and turbulent structure around the pile is of vital importance. The turbulent flow field which is being swept backward and forward past the pile is going to determine to a large extent the individual wave forces on the pile. Therefore the force that is measured should not essentially be directly related to the wave at the same moment, and such a correlation is not necessary. It could be related to what the wave is at the same moment plus its previous time-history.

<u>WIEGEL</u>: I have run through practically all of these questions that have been raised today and in my Paper I concluded that I have developed a design philosophy of a plateau design. There are so many uncertainties that I feel now that many of them we will never answer. I have concluded that we must put much redundancy in the design of our structures.

I must add some words here on the "economic design". If you lose an oil structure, in certain areas the repercussions are vast. If we, as engineers, wish to design too close, using information that we do not understand and we are subject to these almost disastrous losses, it is no use in saying "the insurance company covers".

So with caution I have finally decided I do not think we will ever come up with a relatively sharply tuned design. We are fooling ourselves if we think we can.

STATISTICAL EVALUATION OF WAVE CONDITIONS IN A DELTAIC AREA

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SUMMARY

Coastal engineering problems concerning wind waves and swell can be solved with the aid of hydraulic or mathematical models, The irregular wave field i.e. the state of the sea surface can be described in a sufficient way for engineering problems either by parameters such as significant wave height and mean wave period, or in the form of power spectrum function and the zero-moment thereof.

A semi-empirical method is developed using transfer functions in order to determine the boundary conditions from wave measurements on a limited number of stations in all important points within a shallow sea area.

An economical design is usually possible if the probability of occurrence of all parameters concerned is known. The extrapolation of multidimensional statistical distributions of such parameters is often based on a relatively short period of field observations. The accuracy of the conclusions drawn from these observations influence the methods applied in the modelstudies and the reliability of the economical decision.

In this respect, an analysis of the available data is made with reference to some engineering and navigational problems in the South-Eastern part of the North Sea.

1. INTRODUCTION

A practical solution for maritime and coastal engineering problems, derived from the results of studies based on uniform waves, was only possible in some exceptional cases. Consequently it was necessary to initiate the studies of irregular waves for which windflumes are a necessary tool; as most of the hydraulic laboratories have installed now.

An important improvement is the introduction of a fetch-independant generator of irregular waves. Except for three dimensional problems, most of the maritime structures, breakwaters and dikes can be designed by using this new laboratory facility.

The accuracy of present designs is mainly determined by the knowledge of adequate boundary conditions based on the statistical analysis of wave observations.

The following aspects of the statistical analysis of wave readings in a deltaic area are discussed:

- The definition of a limited number of statistical parameters describing a field of irregular waves given as a time series.
- The determination of the probability distribution of the parameters based on a large number of time series observations.

WAVES AND THEIR EFFECTS ON PILE-SUPPORTED STRUCTURES

R.L. WIEGEL

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SUMMARY

Three methods of presenting wave data are discussed: the significant wave (including the "design wave" concept), the wave spectrum, and the directional spectra. Their use in calculating wave forces on pile supported structures is described, with a discussion of the relative usefulness of the linear versus the non-linear approach. The concept of virtual mass is described, and how this leads to a type of non-linearity which is of great importance in the reversing flow field associated with wave motions. Finally, a plea is made for more wave data in order that adequate wave climates can be obtained for use by the design engineer, by the constructor, and by the operator.

INTRODUCTION

We are all aware of the tremendous forces exerted by hurricane and gale generated water waves on structures in the ocean. Man, since ancient times, has been constructing boats, breakwaters, and docks in a manner which he has hoped would be adequate to withstand these forces, often with success, but often failing. In recent years our knowledge of the physics of the phenomenon has been developed rather rapidly, permitting us to make better designs now than previously. Some concepts and details of the present state of our knowledge will be presented in this lecture.

Wind blowing over the ocean's surface drags water along with it, thus forming a current, while at the same time it generates waves. Many of the waves grow so steep that they become unstable and break, and in this breaking process they generate a substantial amount of turbulence. One of the most noticeable features of these waves is their irregularity, both in time and in space. Owing to the nature of the wind, the waves generated by the wind blowing over the water surface move in a continuous spread of directions, as measured from the direction of the mean wind velocity. Once the waves leave the generating area, they become smoother in appearance and are known as swell. Due largely to dispersion and angular spreading, the energy density decreases with distance travelled from the storm.

Three methods have been developed to represent these waves. The simplest method is to use the concept of a "significant wave" designated by a height (H_s), period (T_s) and direction (see Wiegel, 1964). Another method utilizes a "one-dimensional spectrum," that is, the wave energy density as a continuous function of both component wave frequency and direction. Both the one-dimensional and directional spectra are based upon the concept of linear superposition of component waves and assuming the statistical independence of phase angles amongst the frequency components. Although most of the wave data that are available have been obtained using the significant wave concept, a substantial amount of data is becoming available in the form of one-dimensional spectra.

Almost no directional spectra of ocean waves are available. Obtaining information of this type requires an array of wave gages, the use of an electronic analog to digital converter, and the use of a high-speed digital computer. Furthermore, the mathematical techniques necessary to obtain reliable directional spectra are difficult to use at the present time from a practical standpoint. However, it is expected that in the future many designs will be made which are based upon directional spectra, with the spectra being a few generalized types.

There are two principal reasons, beside the availability of data, which make the significant wave concept useful to the design engineer. One has to do with the problem of the conception of a design in the mind of an engineer, which, because of the large number of variables involved, requires a rather simple visualization of the variables. The second reason is that water waves are not a linear phenomenon, and in relatively shallow water where many structures are built, certain non-linearities are of controlling importance; the significant wave height, period and direction can be used together with the most appropriate non-linear theory for calculations. A variation of this concept is the use of the "design wave," a wave which has been estimated to be the most extreme which will be encountered during the life of a structure. Ultimately, it is expected that the mathematics of non-linear superposition will be developed sufficiently for the directional spectra concept to be used even in shallow water.

It is necessary to have information on the "wave climate" in the area of interest for the planning and design phases, and synoptic wave data for the construction and operation phases. Traditionally, the wave climate has been represented by "wave roses" or tables which have been obtained from visual observations, from wave recorders, or from hindcasts from weather maps. It would be of much greater benefit to the engineer to have wave data in the form of cumulative distribution functions in order to be able to make an economic design based upon the numerical probability of occurrence. In addition, it would also be better to have wave data in another form for use in planning construction and other operations; in the form of continuous observations, measurements, or hindcasts so that the statistical properties could be determined of the number of consecutive days the waves will be less than, or greater than, some safe or economic combination of height, period and direction. Continuous records would also permit the calculation of "wave spectra," and if an appropriate array were used, it would permit the calculation of "directional spectra" for a site.

Finally, a design philosophy is needed. Owing to the lack of statistical information, details of the forcing functions, and our inability to predict in advance our changing needs, it is usually necessary to develop a "plateau" type of design, rather than attempting to design for a sharply tuned optimum design.

LINEAR THEORY FOR PROGRESSIVE WAVES

Linear Wave Theory

The coordinate system usually used is to take x in the plane of the undisturbed water surface and y as the vertical coordinate, measured positive up from the undisturbed water surface. The undisturbed water depth is designated as d. Sometimes the vertical coordinate is taken as measured positive up from the ocean floor, being designated by S.

The wave surface is given by

$$S_s = y_s + d = \frac{1}{2} H \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) + d \qquad (1)$$

where H is the wave height, L is the wave length, T is the wave period, t is time, and the subscript $-_s$ refers to the wave surface. The wave length, L, and wave speed, C, are given by

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L} \qquad (2)$$

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L} \qquad (3)$$

where g is the acceleration of gravity. The horizontal component of water particle velocity, u, the local acceleration, $\partial u/\partial t$, and the pressure, p, are given by

$$u = \frac{\pi H}{T} \frac{\cosh 2\pi S/L}{\sinh 2\pi d/L} \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) \qquad (4)$$

$$\frac{\partial u}{\partial t} = \frac{2\pi^2 H}{T^2} \frac{\cosh 2\pi S/L}{\sinh 2 d/L} \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) \qquad (5)$$

$$p + \rho g y = \frac{1}{2} \rho g H \frac{\cosh 2\pi S/L}{\cosh 2\pi d/L} \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) \qquad (6)$$

where ρ is the mass density of the water.

Similar expressions are available for the vertical components, and expressions are available of the water particle displacements (see Wiegel, 1964).

Wave Forces on Piles

In a frictionless, incompressible fluid the force exerted on a fixed rigid submerged body may be expressed as (Lamb, 1945, p. 93)

$$F_I = (M_o + M_a) f_f = \rho B C_M f_f \qquad (7)$$

where F_I is the inertia force, M_o is the mass of the displaced fluid, M_a is the so-called added mass which is dependent upon the shape of the body and the flow characteristics around the body, and f_f is the acceleration of the fluid at the center of the body were no body present. C_M has been found theoretically to be equal to 2.0 for a right circular cylinder by several investigators (see, for example, Lamb, 1945). The product of the coefficient of mass, C_M, the volume of a body, B, and the mass density of the fluid, ρ, is often called the "virtual mass" of a body (i.e., $M_o + M_a$) in an unsteady flow (Dryden, Murnagham and Bateman, 1956, p. 97). C_M is sometimes expressed as

$$C_M = 1 + C_a \qquad (8)$$

where C_a is the coefficient of added mass.* The mass of the fluid displaced by the body enters into Eq. 7, with one part of the inertial force being due to the pressure gradient in the fluid which causes the fluid acceleration (or deceleration). This force per unit length of cylinder, F_p, is given by

$$F_p = \oint p dy = \rho \frac{dU}{dt} \int x dy = \rho A_o \frac{dU}{dt} \tag{9}$$

in which A_o is the cross sectional area of the cylinder and $\oint$ is a contour integral (McNown, 1957) which follows from the well-known relationship in fluid mechanics for irrotational flow

$$- \frac{1}{\rho} \frac{dp}{dx} = \frac{dU}{dt} \tag{10}$$

where dp/dx is the pressure gradient in the fluid in the absence of the body. In many papers on aerodynamic studies using wind tunnels F_p is called the "horizontal buoyancy" (see, for example, Bairstow, 1939). The added mass term, expressed by $C_a \rho A_o$ per unit length of cylinder, results from the acceleration of the flow around the body caused by the presence of the body. As the fluid is being accelerated around the body by the upstream face of the body (which requires a force exerted by the body on the fluid), the fluid decelerating around the downstream face of the body will exert a smaller or larger force on the downstream face, depending upon whether the flow is accelerating or decelerating. This concept can be seen more clearly for the case of a body being accelerated or decelerated, through a fluid. The force necessary to do this is proportional to the mass per unit length of the cylinder, M_c, plus the added mass, M_a,

$$F_I = (M_c + C_a \rho A_o) \frac{dU}{dt} = (M_c + M_a) \frac{dU}{dt} \tag{11}$$

The leading face of the cylinder pushes on the fluid causing it to accelerate, and the fluid decelerating on the rear side of the cylinder pushes on the cylinder (with the equivalent reaction of the cylinder). In accelerated motion, the reaction at the front must be greater than the reaction at the rear as the fluid decelerating at the rear was not accelerated as much, when it was at the front, as the fluid in front is being accelerated at that instant.

It is unfortunate that the terms added mass and virtual mass have entered the literature as they tend to confuse our concept of the phenomenon. MacCamy and Fuchs (1954; see also Wiegel, 1964, p. 273) solved the diffraction problem of waves moving around a vertical right circular cylinder extending from the ocean bottom through the water surface, using linear wave theory. They solved for the potential, obtained the distorted pressure field from this potential, and integrated the x-component of force around the pile which resulted from this pressure field. In our coordinate system, their solution is

$$F_{Ih}(S) = \frac{\rho g H L}{\pi} \frac{\cosh 2\pi S/L}{\cosh 2\pi d/L} f_A (D/L) \sin \left(- \frac{2\pi t}{T} - \beta\right) \tag{12}$$

where

$$f_A (D/L) = \frac{1}{\{[J_1{}' (\pi D/L)]^2 + [Y_1{}' (\pi D/L)]^2\}^{\frac{1}{2}}} \tag{13}$$

*In many papers the term virtual mass is used for the term added mass. Owing to this, care must be exercised in reading the literature on the subject.

in which J_1 and Y_1 are Bessel functions of the first and second kinds, respectively, and the prime indicates differentiation. β is the angle of phase lag, and will not be shown here as $\beta < 5°$ for values of $D/L < 1/10$, although it is very large for large values of D/L. When $D/L \to 0$, $f_A(D/L) \to \frac{1}{2}\pi\,(\pi D/L)^2$, and

$$F_{Ih}(S) \to 2\,\frac{2\pi^2 H}{T^2}\;\frac{\pi D^2}{4}\;\frac{\cosh 2\pi S/L}{\sinh 2\pi d/L}\;\sin\left(-\frac{2\pi t}{T} - \beta\right) \tag{14}$$

Neglecting β for small values of D/L, it can be seen that this is the commonly accepted equation for the inertial force, with $C_M = 2$.

In a real fluid, owing to viscosity, there is an additional force, known as the drag force, F_D. This force consists of two parts, one due to the shear stress of the fluid on the body, and the other due to the pressure differential around the body caused by flow separation. The most common equation used in the design of pile supported structures is due to Morison, O'Brien, Johnson and Schaaf (1953), and is:

$$F = F_D + F_I = \frac{1}{2}\,C_D\,\rho_w\,A\,|V|\,V + C_M\,\rho_w\,B\,\frac{dV}{dt} \tag{15}$$

where A is the projected area and B is the volume of the pile. As V and dV/dt vary with position, it is better to use the following equation where $F_h(S)$ is the force per unit length of a circular pile. Consider the case of a pile installed vertically in water of depth d, extending from the bottom through the surface. The water particles move in an orbit due to the waves, with both horizontal and vertical components of velocity and acceleration, u, v, du/dt and dv/dt, respectively. The horizontal component of wave induced force, per unit length of pile, is given by

$$F_h(S) = \frac{1}{2}\,C_D\,\rho_w\,D\,|u|\,u + C_M\,\rho_w\,\frac{\pi D^2}{4}\,\frac{du}{dt} \tag{16}$$

Here, du/dt is

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z} \tag{17}$$

If we consider only linear theory, the convective acceleration (the last three terms on the right-hand side of Eq. 17) can be neglected, leaving only the local acceleration; i.e., $du/dt \approx \partial u/\partial t$. u and $\partial u/\partial t$ are given by Eqs. 4 and 5. It can be seen that the drag and inertia forces are in quadrature, so that the maximum total force "leads the crest" of the wave. The larger the drag force relative to the inertia force, the closer will be the maximum total force to the passage of the wave crest past the pile. As will be pointed out in a later section, there is a relationship between C_D and C_M, so that Eq. 16 is quite complicated, although it is not usually treated as such.

If a circular structure is placed at an angle to the waves, the vertical component of wave induced force can be treated in a similar manner, using v and $\partial v/\partial t$ as well as u and $\partial u/\partial t$.

If strictly linear theory is used the total horizontal component of wave force acting on a vertical circular pile can be obtained by integrating $F_h(S)\,dS$ from 0 to d. Very often in practice, one integrates $F_h(S)$ from 0 to S_s, obtaining results which are somewhere between the results for linear wave theory and those for second order wave theory. A digital computer program for this operation is available for this purpose, as are graphs and tables of results (Cross, 1964; Cross and Wiegel, 1965).

Much time and money have been spent in obtaining prototype and laboratory values of C_D and C_M. Most of the work has been done by private companies and is not available.* Some data which are available for C_D are given in Fig. 1 (Wiegel, Beebe and Moon, 1957). It is evident that there is a considerable scatter of both C_D; this is also true for the values of C_M. One of the main reasons for this is that the analysis of the data was based upon two simplifications: First, that linear theory could be used to reduce the basic data, and second, that each wave (and force) of a series of irregular waves could be analyzed as one of a series of uniform waves having the height and period of the individual wave in the record.

Agerschou and Edens (1966) reanalyzed the published data of Wiegel, Beebe and Moon (1957) and some unpublished data of Bretschneider, using both linear theory and Stokes Fifth Order theory. They concluded that for the range of variables covered, the fifth-order approach was not superior to the use of linear theory. They recommended for design purposes, if linear theory is used, that C_D should be between 1.0 and 1.4, and that C_M should be 2.0, these values being obtained for circular piles 6-5/8, 8-5/8, 12-3/4, 16 and 24 inches in diameter. (It should be noted here that the theoretical value of C_M for a circular cylinder in potential flow is 2.0.) Wilson (1965; see also, Wilson and Reid, 1963) report average values of $C_D = 1.0$ and $C_M = 1.45$ for a 30-inch diameter pile. At a recent conference, one design engineer stated he used values of C_D ranging from 0.5 to 1.5 and C_M from 1.3 to 2.0, depending upon his client (Design and Analysis of Offshore Drilling Structures: Continuing Education in Engineering Short Course, University of California, Berkeley, California, 16-21 September 1968). The results reported above were obtained either as values of C_D and C_M at that portion of a wave cycle for which F_D = max and $F_I = 0$, and vice-versa, or for the best average values of C_D and C_M throughout a wave cycle, assuming C_D and C_M to be constant. Both of these methods of obtaining and reporting the coefficients should be refined, as the coefficients are dependent upon each other, and are also time dependent as well as dependent upon the flow conditions.

In the significant wave approach, the significant wave height, H_s, and significant wave period, T_s, are substituted for H and T in the above equations, treating the significant wave as one of a train of waves of uniform height and period. In the design wave approach, the chosen values of H_d and T_d are used in a similar manner.

**<u>One Dimensional Wave Spectra Approach</u>

Recently there have been several papers published on the study of wave forces exerted on circular piles, using probability theory. In these studies it was assumed that the continuous spectrum of component waves could be superimposed linearly, that the process was both stationary and ergodic, and that the phase relationship among the component waves was Gaussian.

Some years ago the author obtained both the wave and force spectral densities for a pile installed at the end of the pier at Davenport, California, as shown in Fig. 2. It was not evident why the form of the two spectral densities should be so similar considering the fact that the product $|u|\,u$ occurs in Eq. 16. Professor Leon E. Borgman (1966) studied this problem in detail and developed the following theory.

*It appears that the results of a long term prototype study of wave forces on piles, by a consortium of oil companies, will be released at the Offshore Technology Conference, to be held in Houston, Texas, 19-21 May 1969.

The basic wave force equation is Eq. 16, which may be expressed as a function of time as

$$F(t) = C_1 |V(t)|\ V(t) + C_2\ A(t) \tag{18}$$

Here F(t) is the time history of the horizontal component of force per unit length of circular pile at an elevation S above the ocean floor, and

$$C_1 = \frac{1}{2}\ \rho_w\ C_D\ D \tag{19a}$$

$$C_2 = \rho_w\ C_M\ \pi D^2/4 \tag{19b}$$

The theoretical covariance function for F(t) using ensemble averaging with the Gaussian random wave model is

$$R_{FF}(\tau) = C_1{}^2\ \sigma^4\ G\left[R_{VV}(\tau)/\sigma^2\right] + C_2{}^2\ R_{AA}(\tau) \tag{20}$$

where $R_{VV}(\tau)$ and $R_{AA}(\tau)$ are the covariance functions of the horizontal component water particle velocity, V(t), and local acceleration A(t) (i.e., u and $\partial u/\partial t$), where

$$\sigma^2 = 2 \int_{-\infty}^{\infty} S_{VV}(f)\ df \tag{21}$$

and

$$G(r) = [(2 + 4r^2)^2 \text{ arc sin } r + 6r \sqrt{1 - r^2}]/\pi \tag{22}$$

in which $G(r) = G(R_{VV}(\tau)/\sigma^2)$, and f is the frequency of the component wave ($f = 1/T$).

The covariance function $R_{VV}(\tau)$ and $R_{AA}(\tau)$ are calculated from the spectral densities $S_{VV}(f)$ and $S_{AA}(f)$ by use of the Fourier transforms

$$R_{VV}(\tau) = \int_{-\infty}^{\infty} S_{VV}(f)\ e^{i2\pi f\tau}\ df \tag{23a}$$

$$R_{AA}(\tau) = \int_{-\infty}^{\infty} S_{AA}(f)\ e^{i2\pi f\tau}\ df \tag{23b}$$

where

$$S_{VV}(f) = \frac{(2\pi f)^2 \cosh^2 2\pi S/L}{\sinh^2 2\pi d/L}\ S_{\eta\eta}(f) = T_V(f)\ S_{\eta\eta}(f) \tag{24a}$$

$$S_{AA}(f) = \frac{(2\pi f)^4 \cosh^2 2\pi S/L}{\sinh^2 2\pi d/L}\ S_{\eta\eta}(f) = T_A(f)\ S_{\eta\eta}(f) \tag{24b}$$

and

$$(2\pi f)^2 = \frac{2\pi g}{L} \tanh 2\pi d/L \tag{25}$$

The functions $T_V(f)$ and $T_A(f)$ are called transfer functions. The fundamental quantity $S_{\eta\eta}(f)$ is the spectral density of the water waves, and is obtained from the Fourier transform

$$S_{\eta\eta}(f) = \int_{-\infty}^{\infty} R_{\eta\eta}(\tau)\, e^{-i2\pi f\tau}\, d\tau \qquad (26)$$

in which $R_{\eta\eta}(\tau)$ is the averaged lagged product of $\eta(t)$ (i.e., average of $\eta(t)\,\eta(t+\tau)$) where $\eta(t)$ is the time history of the wave motion at the location of the pile (i.e., $\eta(t) = y_s(t)$).

Borgman found that Eq. 22 could be expressed in series form as

$$G(r) = \frac{1}{\pi}\left[8\,r + \frac{4r^3}{3} + \frac{r^5}{15} + \frac{r^7}{70} + \frac{5r^9}{1008} + \ldots\right] \qquad (27)$$

and that the series converges quite rapidly for $0 \leq r \leq 1$. He found that for $r = 1$, the first term $G_1(r) = 8r/\pi$ differed from $G(r)$ by only 15%, and that the cubic approximation $G_3(r) = (8r + 4r^3/3)/\pi$ differed from $G(r)$ by only 1.1%. Substituting the first term of the series into Eq. 20 results in

$$R_{FF}(\tau) = \frac{C_1^{\,2}\,\sigma^4}{\pi}\left[\frac{8\,R_{FF}(\tau)}{\sigma^2} + \ldots\ldots\right] + C_2^{\,2}\,R_{AA}(\tau) \qquad (28)$$

The Fourier transform of this is:

$$S_{FF}(f) = \frac{C_1^{\,2}\,\sigma^4}{\pi}\left[\frac{8\,S_{VV}(f)}{\sigma^2} + \ldots\ldots\right] + C_2^{\,2}\,S_{AA}(f) \qquad (29)$$

which is the desired force spectral density.

Borgman made a numerical analysis of the situation shown in Fig. 2. The *numerical integration of $S_{VV}(f)$ gave σ^2=1.203 ft^2/sec^2 and a least square fitting of the theoretical covariance of F(t) against the measured force covariance gave estimates of C_D = 1.88 and C_M = 1.73. The transfer functions $T_V(f)$ and $T_A(f)$ were calculated and plotted; it could be seen that $T_A(f)$ was nearly constant in the range of circular frequencies ($2\pi/T$) for which most of the wave energy was associated. The calculated and measured force spectral densities are shown in Fig. 3. The reason for the excellent fit is that for the conditions of the experiment $T_V(f)$ was nearly constant and the linear approximation to G(r), $G_1(r)$, was a reliable approximation.

Jen (1968) made a model study of the forces exerted by waves on a 6-inch diameter pile in the 200 ft. long by 8 ft. wide by 6 ft. deep wave tank at the University of California, Berkeley. In addition to using periodic waves, irregular waves were generated by a special wave generator using as an input the magnetic tape recording of waves measured in the ocean. The dimensions of the waves relative to the diameter of the pile were such that the forces were largely inertial. Jen found for the regular waves that $C_M \approx 2.0$, and using Borgman's method to analyze the results of the irregular waves tests found $C_M \approx 2.1$ to 2.2. The reason for this close agreement between theory and measurement of C_M is probably due to the small value of H/D, which resulted in quasi-potential flow (This will be discussed in a subsequent section).

Equation 29 permits the calculation of the force spectral density at a point. This is useful but the design engineer usually needs the total force on a pile, and the total moment about the bottom. In addition, the total force and the total moment on an entire structure is needed. These problems have been considered by Borgman (1966; 1967; 1968) and Foster (1968). In obtaining a solution to this problem, the integration of the force distribution is performed from the ocean bottom to the still water level as this is in keeping with linear wave theory. There is no difficulty in obtaining the solution for the inertia force, but cross product terms appear in the solution for **the drag force.* Borgman made use of the linearization of G(r) by restricting it to the first term of the series given by Eq. 27 to obtain the approximate solution for the total force spectral density $S_{QQ}(f)$.

$$S_{QQ}(f) \approx S_{\eta\eta}(f) \left\{ \frac{8}{\pi} \left[\frac{2\pi f\ C_1}{\sinh 2\pi\ d/L} \int_0^d \sigma(S) \cosh(2\pi S/L)\ dS \right]^2 + \left[\frac{(2\pi f)^2\ C_2}{\sinh 2\pi\ d/L} \int_0^d \cosh(2\pi S/L)\ dS \right]^2 \right\} \tag{30}$$

in which

$$\int_0^d \cosh(2\pi S/L)\ dS = \frac{\sinh 2\pi d/L}{2\pi/L} \tag{31}$$

The first integral in Eq. 30 cannot be prevaluated, but must be calculated for each sea-surface spectral density used.

The total moment about the bottom is

$$S_{MM}(f) \approx S_{\eta\eta}(f) \left\{ \frac{8}{\pi} \left[\frac{2\pi f\ C_1}{\sinh 2\pi\ d/L} \int_0^d S\ \sigma(S) \cosh(2\pi S/L)\ dS \right]^2 + \left[\frac{(2\pi f)^2\ C_2}{\sinh 2\pi d/L} \int_0^d S \cosh(2\pi S/L)\ dS \right]^2 \right\} \tag{32}$$

in which

$$\int_0^d S \cosh(2\pi S/L)\ dS = \frac{1}{(2\pi/L)^2} \left[1 - \cosh 2\pi d/L + (2\pi d/L)\cosh 2\pi d/L\right] \tag{33}$$

As in the case of Eq. 30, the first integral cannot be prevaluated.

Borgman (1967; 1968) has found this linearization of the drag term to be the equivalent of using $(V_{rms} \sqrt{8/\pi})\ V(t)$ in place of $|V(t)|\ V(t)$ in Eq. 18; the physical reason for this is not clear, however. It should be pointed out here, that another linearization has been used by nearly every investigator in the past, with essentially no discussion; that is, the use of $\partial u/\partial t$ rather than du/dt (see Eq. 17). Work is needed to determine the size of error introduced by this linearization compared with the size of the error introduced by the linearization of the drag term.

A relatively simple transfer function has been obtained by Borgman (1966; 1967) to calculate the total force and overturning moment the pile array of an offshore platform, and the reader is referred to the original work for information thereof.

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* *A solution to this problem has been obtained by A. Malhotra and J. Penzien, University of California, Berkeley, California, and is to be published soon.

One Dimensional Wave Spectra

There have been a number of papers published on one dimensional wave spectra (see, for example, National Academy of Sciences, 1963), and a large number of measured wave spectra have been published (see, for example, Moskowitz, Pierson and Mehr, 1963). There are several possible ways of using actual spectra, one being a simulation technique (Borgman, 1968) for a large number of spectra, or a large number of wave time histories reconstituted from spectra. Another way to use spectra is to develop a "standard" set of spectra. There have been a number of such standards suggested. One of these has been given by Scott (1965), who re-examined the data of Darbyshire (1959) and Moskowitz, Pierson and Mehr (1963), and then recommended the following equation as being a better fit of the ocean data

$$S(\omega)/H_s^2 = 0.214 \exp - \left[\frac{(\omega - \omega_o)^2}{0.065\,\{(\omega - \omega_o) + 0.26\}}\right]^{\frac{1}{2}} \quad (33a)$$

$$\text{for} \quad -0.26 < (\omega - \omega_o) < 1.65 \quad (33b)$$

$$\text{and, } = 0, \text{ elsewhere} \quad (33c)$$

where $\omega = 2\pi f$ (in radians per second), ω_o is the spectrum peak frequency, H_s is the significant wave height (in feet), and the energy spectral density $S(\omega)$ is defined by

** $$S(\omega) = \frac{1}{\pi} S_{\eta\eta}(f)$$

It is also defined by

** $$S(\omega) = \frac{1}{2} \sum^{\delta\omega} a_i^2/\delta\omega \quad (34)$$

in which the summation is over the frequency interval ω, $\omega + \delta\omega$, and a_i is the amplitude of the i^{th} component, with

$$y_s = \sum_{i=1}^{n} y_i = \sum a_i \cos(\omega_i t + \phi_i) \quad (35)$$

in which ϕ_i is the phase angle of the i^{th} component. The factor $\frac{1}{2}$ enters as $\sum^{n} a_i^2/2$ is the mean value of $y_s{}^2$ during the motion. The term $a_i^2/\delta\omega$ is used, as the concept of a_i tends to lose physical significance (i.e., $a_i^2 \to 0$) as $n \to \infty$, whereas $a_i^2/\delta\omega$ does not; hence the value of using the energy density as a function of frequency.

Scott also found, using linear regression, that

$$1/f_o = 0.19\,H_s + 8.5 \quad (36a)$$

$$1/\omega_o = 0.03\,H_s + 1.35 \quad (36b)$$

$$T = 0.085\,H_s + 7.1 \quad (36c)$$

where T is the average period (in seconds) of all waves in the record, and can be shown to be

$$T = 2\pi \, (m_o/m_2)^{\frac{1}{2}} \tag{37}$$

where

$$m_k = \int_o^\infty \omega^k \, S_{\eta\eta}(\omega) \, d\omega \tag{38}$$

For k = 0, we have the "variance," m_o, and for a narrow (i.e., "Rayleigh" spectrum) we have

$$H_s = 4 \, m_o^{\frac{1}{2}} \tag{39}$$

Using quadratic regression, Scott found

$$f_o = (0.501/T) + (1.43/T^2) \tag{40a}$$

$$\omega_o = (3.15/T) + (8.98/T^2) \tag{40b}$$

It is of considerable importance to the engineering profession to develop means by which the spectral approach can be studied in the laboratory. In studying some of the problems, it is necessary to know the relationship between the one-dimensional spectra in the ocean and the spectra generated in a wind-wave tank (Plate and Nath, 1968). Comparison of a number of wave spectra measured in the ocean, in lakes and in wave tanks have been made by Hess, Hidy and Plate (1968). Their results, shown in Figure 4, are fully developed seas' wind-wave energy density spectra. The high frequency portion of the spectra all tend to lie close to a single curve, with energy density being approximately proportional to ω^{-5} as predicted by the Phillips' equilibrium theory (see Wiegel and Cross, 1966, for a physical explanation of this). A close inspection of these data by Plate and Nath (1968) led them to conclude that the high frequency portion of the energy spectral density curve varies from the ω^{-5} "law," being proportional to ω^{-7} near the spectral peak, and being proportional to about ω^{-4} in the highest frequency range of the spectra. It would appear from the one example of Wiegel and Cross (1966), Figure 5, in which they compared a normalized measured laboratory wind-wave energy density spectrum with one calculated by use of Miles' theory, together with other physical reasoning, that a theoretically sound basis exists for the development of a "standard" set of spectra.

The argument for the high frequency portion of the energy density spectra being proportional to ω^{-5} is as follows (Wiegel and Cross, 1966). For a train of uniform periodic progressive waves, the maximum wave steepness is generally considered to be

$$\frac{H}{L} \approx \frac{1}{7} \tanh 2\pi \, d/L \tag{41a}$$

which, for deep water, reduces to

$$\frac{H}{L} = \frac{H}{(g/2\pi) \, T^2} \approx \frac{1}{7} \tag{42b}$$

and

$$H^2 \approx \frac{4\pi^2 g^2}{49(2\pi f)^2} \tag{43}$$

from which

$$H^2/\omega = H^2/2\pi f \approx \frac{4\pi^2 g^2}{49(2\pi f)^5} = \frac{4\pi^2 g^2}{49\ \omega^5} \tag{44}$$

If the energy spectral density is proportional to $(H/2)^2/\omega$, then it must also be proportional to ω^{-5}.

In order for the design engineer to use with confidence the work of the type proposed by Borgman, it would be desirable to measure $S_{VV}(f)$ and $S_{AA}(f)$ as a function of $S_{\eta\eta}(f)$ in both the ocean and in the laboratory to see how reliable the linear transfer functions are for different sea states.

Directional Wave Spectra

Before directional spectra can be used in the design of structures in relatively deep water it is necessary to have measurements of such spectra, and to understand them sufficiently to be able to choose a "design" directional spectra. Two sets of measurements have been made in the ocean (Chase, et al., 1957; Longuet-Higgins, Cartwright and Smith, 1963), a few in a bay (Stevens, 1965) and a few in the laboratory (Mobarek, 1965; Mobarek and Wiegel, 1967; Fan, 1968).

Mobarek (1965) checked several methods that had been suggested for obtaining the directional spectra from an array of wave gages, and found none of them too reliable. However, making use of simulated inputs, he was able to choose the most reliable method and to devise correction factors. Some of his measurements are shown in Figure 6. Values in the ordinate are in terms of the wave energy, E, rather than the energy density, $S_{\eta\eta}(f)$. When normalized, his laboratory results were found to be similar to normalized values of the measurements made in the ocean by Longuet-Higgins, et al. (1963), as can be seen in Figure 7. At the suggestion of Professor Leon E. Borgman, Dr. Mobarek compared the circular normal probability function (the solid curve in Figure 7) with the normalized data and found the comparison to be excellent.

** The probability density of the circular normal distribution function is given by (Gumbel, 1952 and Court, 1952):

$$P(\alpha, K) = \frac{1}{I_o(K)} \exp(K \cos \alpha) \tag{45}$$

where α is the angle measured from the mean $(\theta_m - \theta)$, K is a measure of the concentration about the mean, and $I_o(K)$ involves an incomplete Bessel function of the first kind of zero order for an imaginary argument. The larger K, the greater the concentration of energy; it is analogous to the reciprocal of the standard deviation of the linear normal distribution.

It has been found that much useful information on directional spectra can be obtained from the outputs of two wave recorders, through use of the co-spectra and quadrature spectra to calculate the linear coherence and the mean wave direction (Munk, Miller, Snodgrass and Barber, 1963; Snodgrass, Groves, Hasselman, Miller, Munk and Powers, 1966). It appeared to the author that if the directional spectra were represented by the circular normal distribution function it should be possible to obtain the necessary statistical parameters in a similar manner. It was believed that such a simplified approach could provide data of sufficient accuracy for many practical purposes. As a result of discussions with Professor Leon Borgman, a theory was developed by Borgman (1967) to do this, and tables were calculated to provide a practical means to obtain the required information.

Borgman (1967) used a slightly different representation of the directional spectra

$$S_{\eta\eta 2}(f, \alpha) = S_{\eta\eta 1}(f) \exp[-K \cos(\theta - \theta_m)]/2\pi I_o(K) \tag{46}$$

where the 2π in the denominator indicates an area under the curve of 2π rather than unity, f is the component wave frequency in cycles sec, and $S_{\eta\eta 1}(f)$ is the one-dimensional spectral density. The estimation of the parameters $S_{\eta\eta 1}(f)$, $K(f)$ and $\theta_m(f)$ is achieved by cross-spectral analysis based on a sea surface record at two locations. $S_{\eta\eta 1}(f)$ and the co- and quadrature spectral densities for the two recordings are computed by the usual time series procedures. The theoretical relations between measured and unknown quantities is

$$\frac{C(f)}{S_{\eta\eta 1}(f)} = \int_o^{2\pi} \frac{\exp[K \cos(\theta - \theta_m)]}{2\pi I_o(K)} \cos[k\overline{D} \cos(\theta - \overline{\beta})]\, d\theta \tag{47}$$

$$\frac{Q(f)}{S_{\eta\eta 1}(f)} + \int_o^{2\pi} \frac{\exp[K \cos(\theta - \theta_m)]}{2\pi I_o(K)} \sin[k\overline{D} \cos(\theta - \overline{\beta})]\, d\theta \tag{48}$$

where $\overline{D}$ is the distance between the pair of recorders, k is the wave number $(2\pi/L)$ and $\overline{\beta}$ is the direction from wave recorder #1 to wave recorder #2. For a given frequency, all quantities are known except θ_m and K. Hence these two equations represent two nonlinear equations with two unknowns. Borgman has prepared tables which enable one to solve for θ_m and K, given $C(f)/S_{\eta\eta 1}(f)$ and $Q(f)/S_{\eta\eta 1}(f)$. Two solutions, symmetric about the direction between the pair of recorders result. This ambiguity may be eliminated by using three wave gages instead of two, or in many applications using other information regarding the main direction of the directional spectra. The relationship between the parameter K and the directional width of the spectrum can be seen in Figure 8.

Using simulation techniques devised by Professor Leon Borgman, Dr. Fan (1968), continuing the work of Mobarek, made an extensive study of the effects of different lengths of data, lag numbers, wave recorder spacings, filters, and different samples on the calculation of directional spectra, using several methods, using a known circular normal distribution input. An example of the effect of gage spacings, relative to the component wave length, on the estimates can be seen in Figure 9. He then used the "best" combination to obtain the directional spectra of waves generated in a model basin by wind blowing over the water surface. As a result of this study it appears that, for the case of waves being generated in a nearly stationary single storm, the directional spectra can be approximated by two parameters and should be tested for use in the design of an offshore structure.

The results were sufficiently good to encourage Borgman and Suzuki to develop a new method for obtaining useful information on directional spectra by measuring the time histories of the x and y components of wave induced force on a sphere mounted a few feet above the ocean bottom, together with the wave pressure time history at the sphere. The results of this work (Suzuki, 1968) indicated that a practical method is available to the engineer for measuring the approximate directional spectra of ocean waves.

NON-LINEAR PROBLEMS

There are several types of non-linearities involved in the problem of wave induced forces on offshore structures. One, which is due to the term $|u|\ u$ of Eq. 16, is important in the wave spectra approach; a method of overcoming the handicap has been described in a previous section. A second enters through the term du/dt in Eq. 16, which has been linearized through the use of $\partial u/\partial t$ in place of du/dt. A third non-linearity enters through the generation of eddies, and will be discussed subsequently.

The most commonly considered non-linearity is associated with non-linear **wave theories. Two of these are the Stokes and the Cnoidal wave theories (see, for example, Wiegel, 1964). The first is best used for relatively deep water, and the second is best used for relatively shallow water. No attempt will be made to describe these theories in detail herein; rather a few equations will be given to indicate the general nature of the difference between these theories and the linear theory.

To the third order, the Stokes (Stokes, 1880; Skjelbreia, 1959) wave profile is given by

$$\frac{y_s}{L} = A_1 \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) + A_2 \cos 4\pi \left(\frac{x}{L} - \frac{t}{T}\right) + A_3 \cos 6\pi \left(\frac{x}{L} - \frac{t}{T}\right) \tag{49}$$

where the coefficients A_1, A_2 and A_3 are related to the wave height by

$$H/d = (L/d)\ [2A_1 + 2\pi^2 A_1^3 \quad F_3\ (d/L)] \tag{50}$$

where

$$A_2 = A_1^2 \cdot f_2\ (d/L), \quad A_3 = \pi^2 A_1^3 \cdot f_3\ (d/L) \tag{51}$$

with f_2 (d/L) and f_3 (d/L) being functions of d/L.

The waves have steeper crests and flatter troughs than linear waves, and there is a mass transport of water in the direction of wave advance. The equations for water particle velocities and accelerations will not be presented herein as extensive tables of functions are needed for their use (or the availability of a high speed digital computer).

When the wave length becomes quite long comapred with the water depth, about L/d > 10 (the value depending upon H/d as well), the Cnoidal wave theory is perhaps a better approximation than is the theory of Stokes waves. The theory was originally derived by Korteweg and de Vries (1895). To the first approximation the wave profile is given by S_s, measured from the ocean bottom

$$S_s = S_t + H\ cn^2\ [2\ K(k)\ (x/L - t/T),\ k] \tag{52}$$

where cn is the "cnoidal" Jacobian elliptical function and K(k) is the complete elliptic integral of the first kind of modulus k, S_t is the elevation of the wave trough above the bottom, and is given by

$$\frac{S_t}{H} - \frac{d}{H} + 1 = \frac{16\ d^3}{3L^2\ H}\ \{K(k)\ [K(k) - E(k)]\} \qquad (53)$$

where E(k) is the complete elliptic integral of the second kind of modulus k. The wave length is

$$L = \sqrt{\frac{16\ d^3}{3H}} \cdot kK(k) \qquad (54)$$

and the period is related to the modulus k through

$$T\sqrt{\frac{g}{d}} = \sqrt{\frac{16d}{3H}} \left\{ \frac{kK(k)}{\sqrt{1 + \frac{H}{d}\left[-1 + \frac{1}{k^2}\left(2 - 3\,\frac{E(k)}{K(k)}\right)\right]}} \right\} \qquad (55)$$

The equation for water particle velocities and acceleration and graphs which permit the use of the Cnoidal wave theory have been prepared by Wiegel (1964; see Masch and Wiegel, 1961 for tables of functions).

Professor Robert Dean (1968) has made analytical studies of the wave profiles predicted by these and other theories, including his "stream function wave theory," in order to determine the probable useful ranges of the theories. His results are shown in Figures 10 and 11.

It is necessary to be able to calculate the height of the wave crest above the water surface in order to determine the deck height on an offshore platform, and the work of Dean cited above is useful for this purpose. It is also important to be able to estimate the regions of reliability of the several theories in the prediction of water particle velocities and accelerations. Dr. Bernard Le Méhauté and his co-workers (Le Méhauté, Divoky and Lin, 1968) have made careful laboratory studies of the water particle velocities of "shallow water waves" for several values of H, T, and d and compared their measurements with predictions made using a number of linear and non-linear wave theories. An example of their results is shown in Fig. 12. They concluded, that while no theory was found to be exceptionally accurate, the Cnoidal wave theory of Keulegan and Patterson appeared to be most adequate for the range of wave parameters and water depths studied. It appears that much more work of this type is needed.

The water particle velocities and accelerations given by the most valid non-linear theory are used in Eq. 16 to calculate the force on a pile. These velocities and accelerations are usually calculated for the so-called "design wave," which is usually the wave considered by design engineers to be the largest wave the structure might encounter during its useful life.

Another reason for the variability of the data is associated with the wake. The formation of eddies in the lee of a circular cylinder in uniform steady flow has been studied by a number of persons. It has been found that the relationship among the frequency (cycles per second) of the eddies, f_e, the diameter of the cylinder, D, and the flow velocity, V, is given by the Strouhal number, N_s,

$$N_s\left(1 - \frac{19.7}{N_R}\right) = \frac{f_e\ D}{V} \approx N_s \qquad (56)$$

where N_R is the Reynolds number. Except in the range of laminar flow, the Reynolds number effect can be neglected. For flow in the sub-critical range

(N_R < about 2.0×10^5), $N_s \approx 0.2$. For $N_R > 2.0 \times 10^5$, there appears to be a considerable variation of N_s; in fact, it is most likely that a spectrum of eddy frequencies exists (see Wiegel, 1964, p. 268 for a discussion of this). The most extensive data on N_s at very high Reynolds numbers, as well as data on C_D and the pressure distribution around a circular cylinder with its axis oriented normal to a steady flow, has been given by Rosko (1961), some of which are shown in Figure 13.

What is the significance of N_s for the type of oscillating flow that exists in wave motion? Consider the horizontal component of water particle velocity as given by Eq. 4. For deep water, the equation is approximately

$$u = (\pi H/T) \cos 2\pi t/T \tag{57}$$

at x - 0. Then, using an average of u to represent V; i.e.,

$$V_w \approx u_{avg} \approx \pi H/2T \tag{58}$$

where V_w is the "average" horizontal component of water particle velocity due to a train of waves of height H and period T. For at least one eddy to have time to form it is necessary for

$$T > 1/f_e \approx 2DT/\pi H\, N_s \tag{59a}$$

And, if $N_s \approx 0.2$

$$H > 10\, D/\pi \tag{59b}$$

Keulegan and Carpenter (1958) studied both experimentally and theoretically the problem of the forces exerted on bodies in an oscillating flow. The oscillations were of the standing wave type in which the wave length was long compared with the water depth so that the horizontal component of water particle velocity was nearly uniform from top to bottom. Furthermore, the body was placed with its center in the node of the standing wave. They found that C_M and C_D depended upon the number $u_{max}\, T/D$ where $u = u_{max} \cos 2\pi t/T$. They observed that when $u_{max}\, T/D$ was relatively small, no eddy formed, that a single eddy formed when $u_{max}\, T/D$ was about 15, and that numerous eddies formed for large values of the parameter. It is useful to note that this leads to a conclusion similar to Eq. 59. For example, if one used the deep water wave equation for $u_{max} = \pi H/T$, then

$$u_{max}\, T/D > \pi H/D > 15 \tag{60}$$

and

$$H > 15D/\pi \tag{61}$$

It appears from the work described above that a high Reynolds number oscillating flow can exist which is quite different from high Reynolds number rectilinear flow unless the wave heights are much larger than the diameter of the circular cylinder. It would appear that the Keulegan-Carpenter number is of greater significance in correlating C_D and C_M with flow conditions than is Reynolds number (Wiegel, 1964, p. 259), and that the ratio H/D should be held constant to correlate model and prototype results, or at least should be the appropriate value to indicate the prototype and model flows are in the same "eddy regime" (see Paape and Breusers, 1967, for similar results for a cylinder oscillating in water).

When the Keulegan-Patterson number is large enough that eddies form, an oscillating "lift" force will occur with a frequency twice that of the wave frequency. For a vertical pile the "lift" force will be in the horizontal plane normal to the direction of the drag force. Essentially no information has been published on the coefficient C_L for water wave type of flow. In uniform recti-linear flows it has about the same numerical value as C_D.

Photographs taken of flow starting from rest, in the vicinity of a circular cylinder for the simpler case of a non-reversing flow, show that it takes time for separation to occur and eddies to form. The effect of time on the flow, and hence on C_D and C_M has been studied by Sarpkaya and Garrison (1963; see also Sarpkaya, 1963). A theory was developed which was used as a guide in analyzing laboratory data taken of the uniform acceleration of a circular cylinder in one direction. Figure 14 shows the relationship they found between C_D and C_M was found which was dependent upon ℓ/d, where ℓ is the distance traveled by the cylinder from its rest position and D is the cylinder diameter. They indicated the "steady state" (i.e., for large value of ℓ/D) values of $C_D = 1.2$ and $C_M = 1.3$.

The results shown in Figure 14 are different than those found by McNown and Keulegan (1959) for the relationship between C_D and C_M in oscillatory flow, Figure 15. They measured the horizontal force exerted on a horizontal circular cylinder placed in a standing water wave, with the cylinder being parallel to the bottom, far from both the free surface and the bottom, and with the axis of the cylinder normal to the direction of motion of the water particles. The axis of the cylinder was placed at the node of the standing wave so that the water particle motion was only horizontal (in the absence of the cylinder). Their results are shown in Figure 15. Here, T is the wave period and T_e is the period of a pair of eddies shedding in steady flow at a velocity characteristic of the unsteady flow. In their figure, the characteristic velocity was taken as the maximum velocity. They found that if T/T_e was 0.1 or less, separation and eddy formation were relatively unimportant, with the inertial effects being approximately those for the classical unseparated flow, and if T/T_e was greater than 10, the motion was quasi-steady.

WAVE CLIMATES

In preparing feasibility studies, in designing, in constructing and in operating coastal and offshore structures and facilities it is necessary to have reliable information on surface water waves. These structures and facili-ties include harbors, pipelines on the bottom, offshore oil structures and drilling vessels, buoys for use in mooring tankers, dredging for offshore min-eral recovery, lightering craft and equipment, and waste disposal systems.

** It is necessary to have information on the "wave climate" in the area of interest for the planning and design phases, and synoptic wave data for the construction and operation phases. Traditionally, the wave climate has been represented by "wave roses" or tables which have been obtained from visual observations, from wave recorders, or from hindcasts from weather maps. It would be of much greater benefit to the engineer to have wave data in the form of cumulative distribution functions in order to be able to make an economic design based upon the numerical probability of occurrence. In addition, it would also be better to have wave data in another form for use in planning construction and other operations; in the form of continuous observations, meas-urements, or hindcasts so that the statistical properties could be determined of the number of consecutive days the waves will be less than, or greater than, some safe or economic combination of height, period and direction.

As an example, the cumulative significant wave height distribution functions for swell and sea were constructed for one location in the Pacific Ocean, using information obtained from a wave hindcasting study which was made using a three year series of weather maps (Figure 16). The distribution functions are not too useful as both swell and seas must have occurred simultaneously on a number of days; the data were not reported in a manner that permitted the recovery of this information. The few data that are available on the ability of several types of floating structures to perform their functions in waves are given in Tables 1 and 2. These data are not too useful as the capability of a floating structure to work in waves depends upon the wave period, winds, currents and the crew as well as upon the wave height. However, in the absence of other data, these data must be used. Consider either a seaworthy suction hopper dredge, with a flexible suction tube, or a seaworthy tin dredge; an average workable wave height might be taken to be about 5 feet. This limitation on wave height, together with the significant wave height distribution functions given in Figure 16 indicate that these two types of dredges would not be usable for 24% of the time owing to swell and 18% of the time owing to seas that were too high. If one assumes that half of the time the seas were too high occurred simultaneously with swell that were too high, one would estimate that the site was "unworkable" about one-third of the time. Considering the time necessary to get a dredge from the work site to a harbor of refuge and back again the site would be "unworkable" for considerably more than one-third of the time

Similar data are necessary for the safer and more economic use of the oceans for transportation. These data are needed for improved ship design and routings, as well as for improved terminal facilities. In this regard, it should be emphasized that the design of unique ships or shipping techniques interacts with the harbors and offshore facilities of many countries. In a UNESCO report ("Marine Science and Technology: Surveys and Proposals," Report of the Secretary General, E/4487, 24 April 1968) it was pointed out that in 1966, alone, 112 ships larger than 1,000 gross tons were lost.

At the present time there are very few places in the world for which we have sufficient, or even barely adequate wave data. This is especially true of the little-traveled portions of the open oceans.

It is recognized that considerable advances have been made, and are continuing to be made, in our understanding of the basic phenomenon of the generation of waves by winds, and in the development and use of computer programs to calculate wave fields from meteorological inputs. It would be desirable if the programs could be developed in such a way, if this is not already the case, that the required wave data could be recalled at a reasonable cost for any geographical location for which the data became necessary.

Associated with the problems of transforming meteorological data to wave data are the phenomena of wave scattering, dispersion, energy dissipation, refraction, reflection, and diffraction. It is necessary to make reliable measurements of wave characteristics on an ocean-wide basis to obtain the data needed by the engineer to perform his job properly, and by the geophysicist to test and improve his theories. To be useful to the engineers the measurements should be made for a long period of time. Measurements should be made in the open ocean and along the coasts. It would be desirable, for use by both the engineer and the geophysicist, if directional spectra could be measured. At the other extreme it would still be useful to obtain consistent visual observations, especially in areas for which few measurements have been made (i.e., most areas). Much valuable data could be obtained from a study of newspapers, technical publications, harbor logs, etc., by investigators in each country. These data would be of greater value if damage resulting from wave and wind action could be summarized with the wave data.

Two international standards should be used for data reduction, one simplified and one rather sophisticated. It would appear that the standards proposed by L. Draper ("The Analysis and Presentation of Wave Data - A Plea for Uniformity," Proceedings of the Tenth Conference on Coastal Engineering, ASCE, pp. 1-11, 1967) should be used.

Table 1. Wave Heights Limiting Operations of Dredges and Barges
(after Santema, 1955)

Equipment and kind of work	Limiting wave height (feet)
1. Dredging with	
a. Seaworthy suction hopper dredge, with rigid suction tube and cutters	2–3
b. Seaworthy suction hopper dredge, with flexible suction tube	4½–6
c. Suction dredges of the nonpropelled, low pontoon type, rigid suction tube	1½–3
d. Bucket dredge nonpropelled, low pontoon type, hard bottom	1½
e. Seaworthy tin dredges	4½–6
2. Mooring barges alongside a dredge with barge discharge, or alongside a barge-unloading dredge	1½–2½
3. Dumping stones, sand, or clay with dump barge with bottom doors (up to 400 tons)	1½–3
4. Dumping stones and clay with self-tipping barges (up to 600 tons)	1½–2½
5. Transport and sinking fascine mattresses	1–1½
6. Pumping stones in layers on fascine mattresses from barges	1½–2½

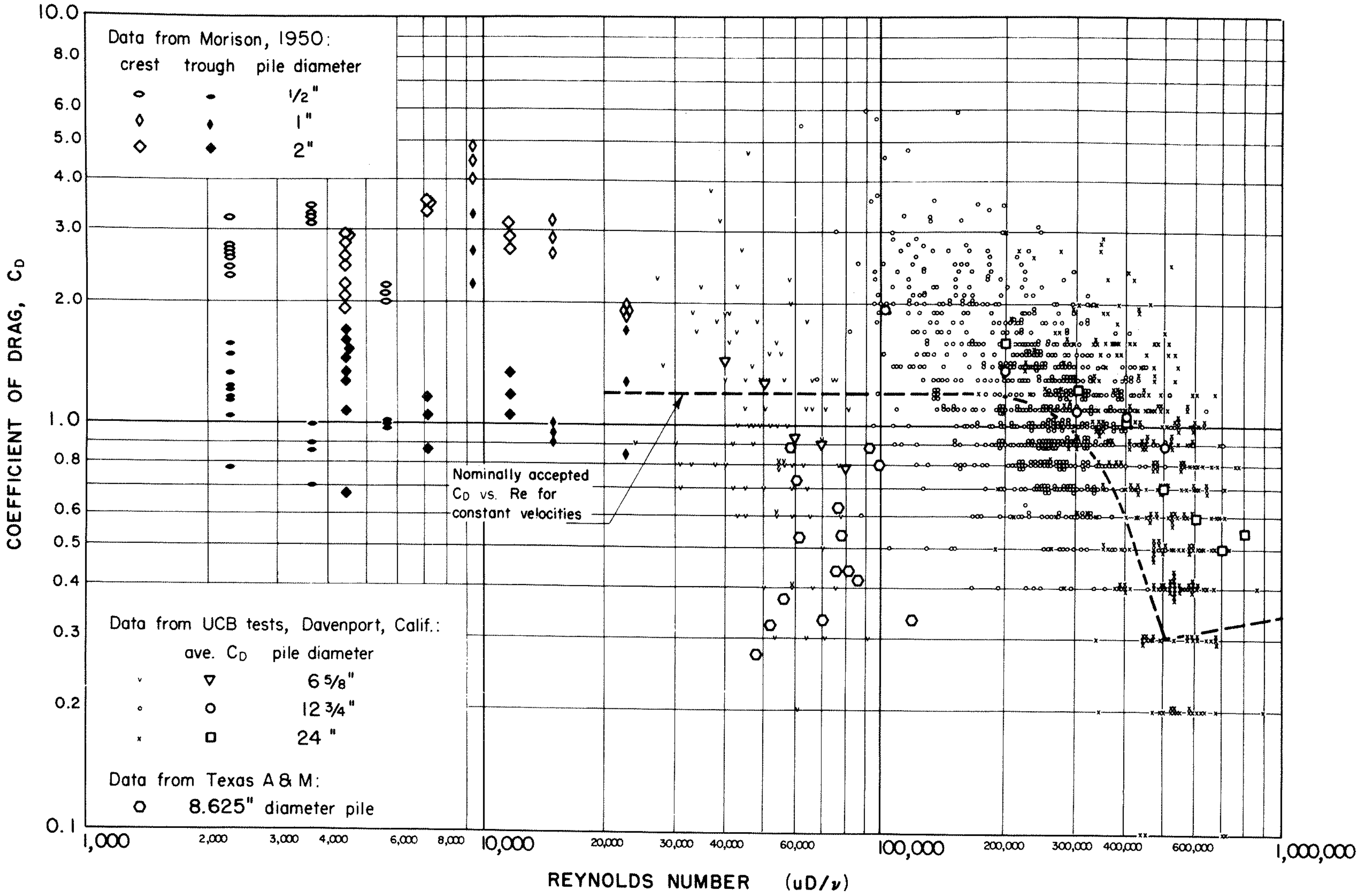
Santema, P., About the estimation of the number of days with favorable meteorological and oceanographical conditions for engineering operations on the sea coast and in estuaries, *Proc. Fifth Conf. Coastal Eng.*, Berkeley, Calif.: The Engineering Foundation, Council on Wave Research, 1955, pp. 405-410.

Glenn, A. H., Progress report on solution of wave, tide, current, and hurricane problems in coastal operations, *Oil Gas J.*, 49, 7 (June 22, 1950), 174-77.

Table 2. Generalized Performance Data for Marine Operations
(after Glenn, 1950)

Type of operation	Wave heights[a] (feet) for: Safe, efficient operation	Marginal operation	Dangerous and/or inefficient operation
Deep sea tug			
Handling oil and water barge	0–2	2–4	>4
Towing oil and water barge	0–4	4–6	>6
Handling derrick barge	0–2	2–3	>3
Handling and towing LST-type vessel	0–3	3–5	>5
Crew boats, 60–90 ft in length			
Underway	0–8	8–15	>15
Loading or unloading crews at platform	0–3	3–5	>5
Supervisor's boats, fast craft, 30–50 ft in length			
Underway at cruising speed	-2	2–4	>4
Loading or unloading personnel at platform or floating equipment	0–2	2–4	>4
LCT-type vessel and cargo luggers			
Underway	0–4	4–5	>5
Loading or unloading at platform	0–3	3–4	>4
Loading or unloading at floating equipment	0–4	4–5	>5
Buoy laying (using small derrick barge)	0–2	2–3	>3
Platform building			
Using ship-mounted derrick	0–4	4–6	>6
Using large derrick barge	0–3	3–5	>5
Pipeline construction	0–3	3–4	>4
Gravity-meter exploration using surface vessel (limiting conditions caused by instrument becoming noisy)	0–4	4–6	>6
Seismograph exploration using craft under 100 ft in length	0–6	6–8	>8
Large amphibious aircraft (PBY)			
Sea landings and take-offs	0–1.5	1.5–3	>3
Boat-to-plane transfer operations in water	0–1	1–2	>2
Small amphibious aircraft	0–1	1–2	>2

[a] Wave heights used are those of the average maximum waves. Height limits given are not rigid and will vary to some extent with locality, local wind conditions, experience of personnel, etc.



Coefficient of Drag for Circular Cylindrical Piles of Various Diameters

(from Wiegel, Beebe and Moon, 1957)

FIGURE 1

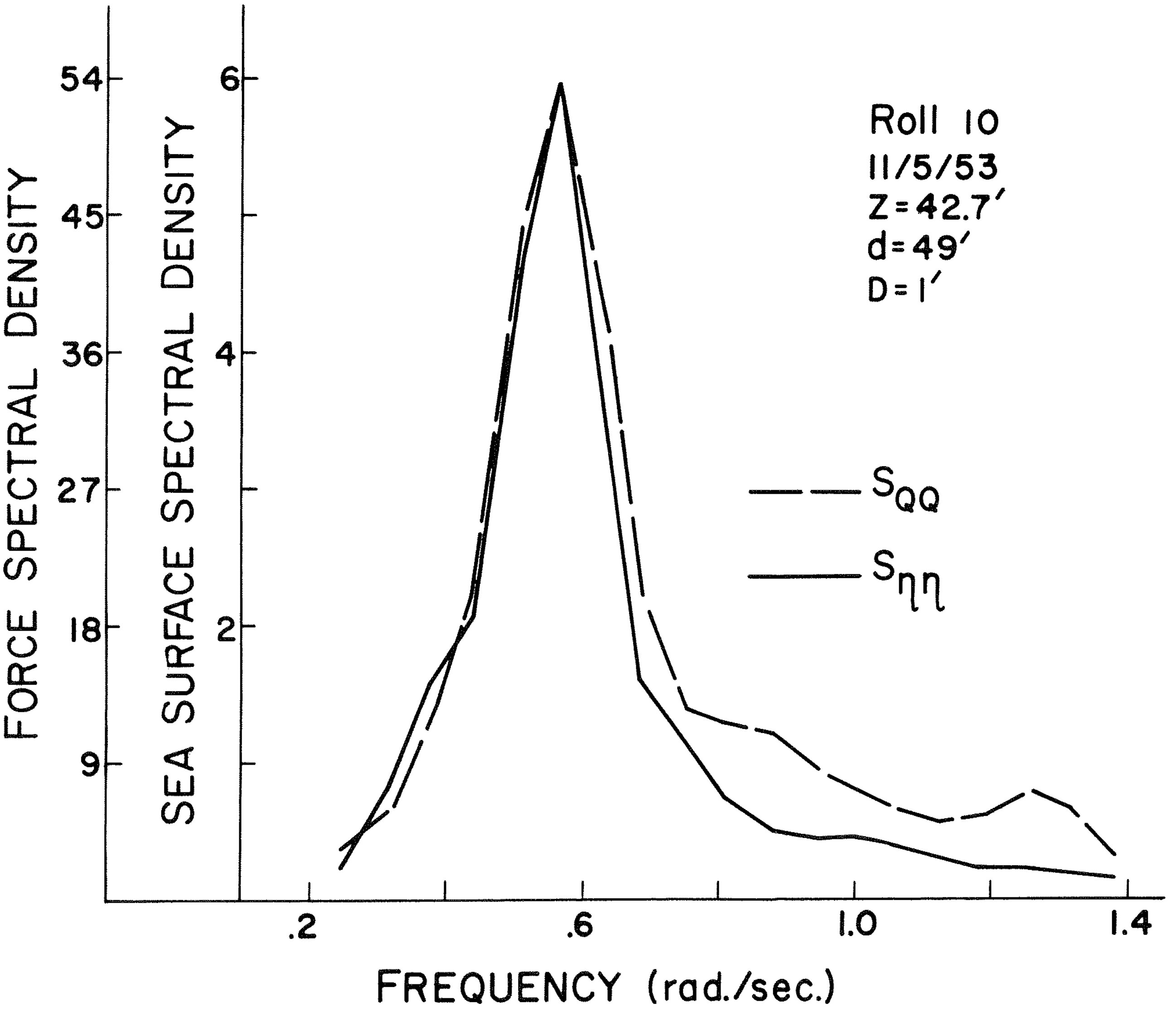


Figure 2. A comparison of force and sea-surface spectral densities for roll 10, Davenport data.

(From Borgman, 1966)

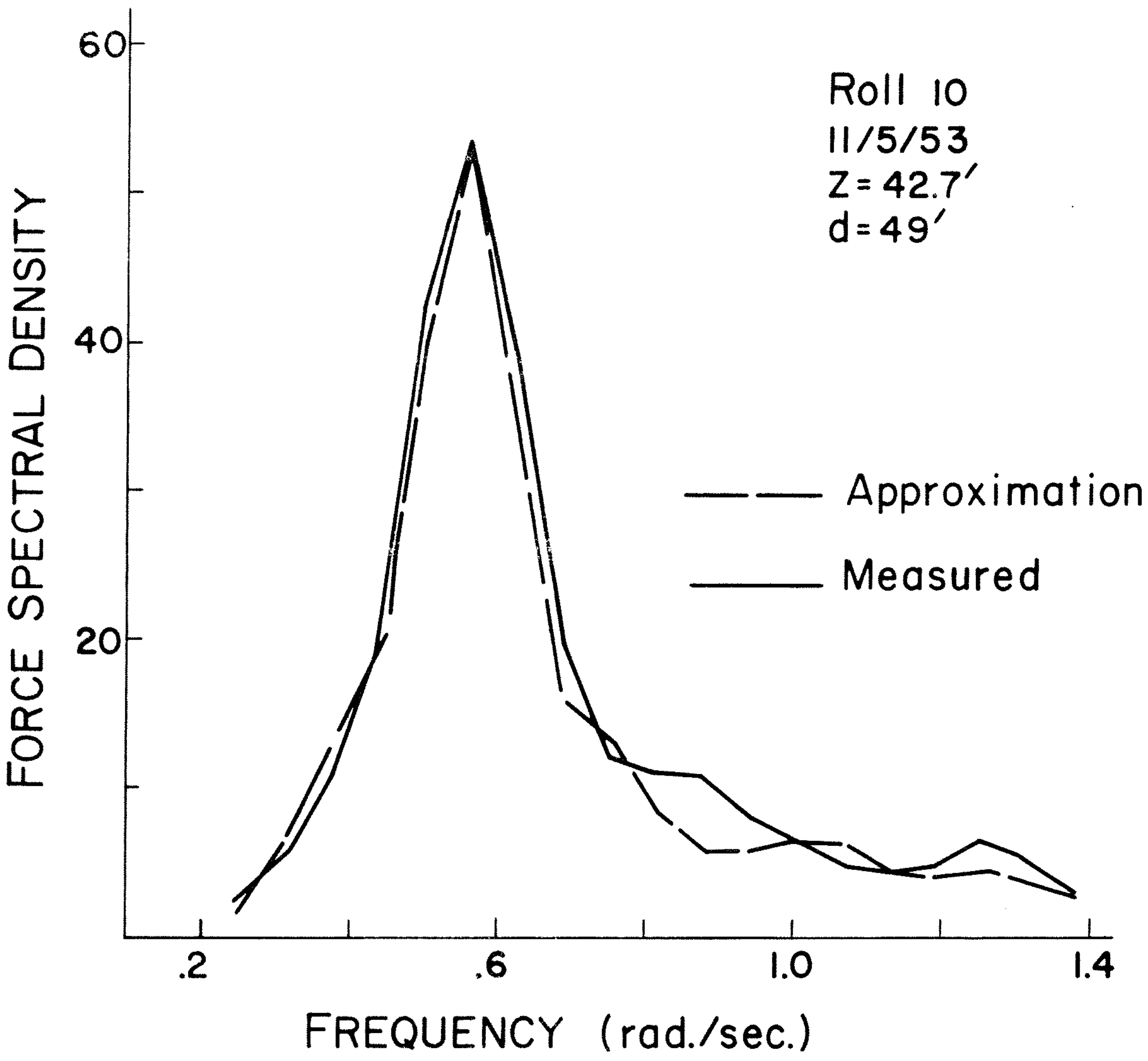
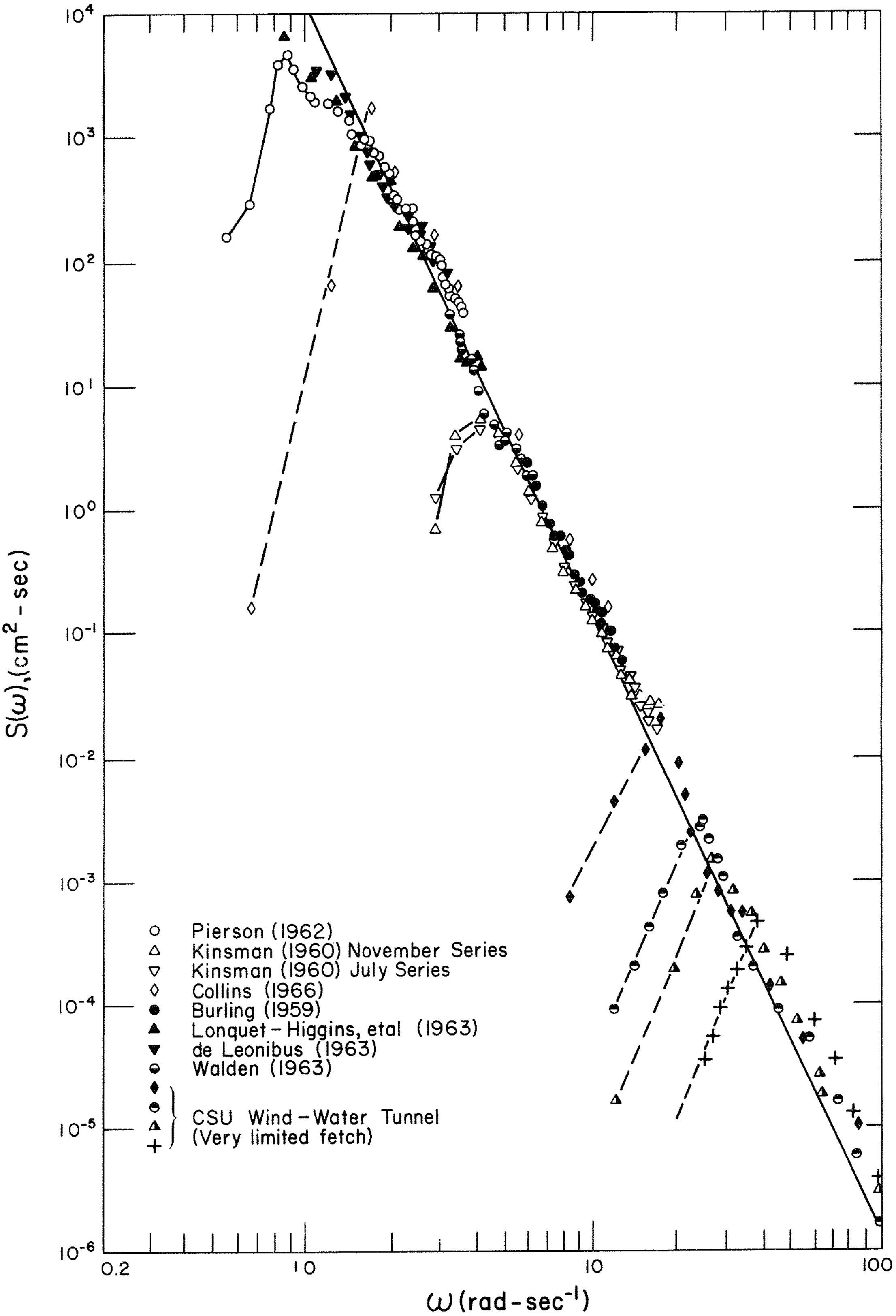
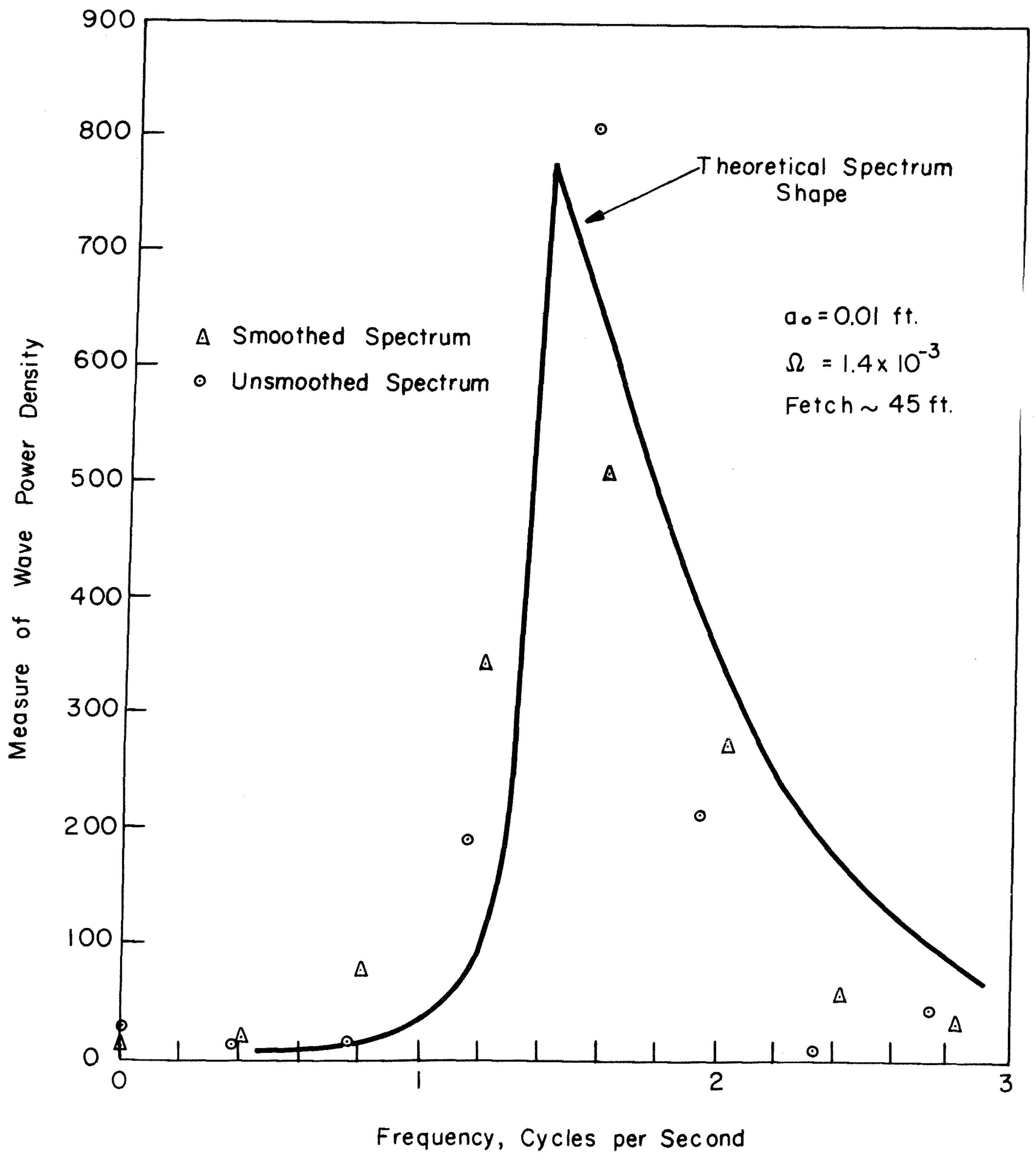


Figure 3. A comparison of the measured and computed force spectral density for roll 10, Davenport data.
(From Borgman, 1966)



(from Hess, Hidy and Plate, 1968)

Figure 4



COMPARISON OF SMOOTHED AND UNSMOOTHED WAVE POWER SPECTRA WITH SHAPE OF THEORETICAL SPECTRUM

(from Wiegel and Cross, 1966)

H-9426-A

FIGURE 5

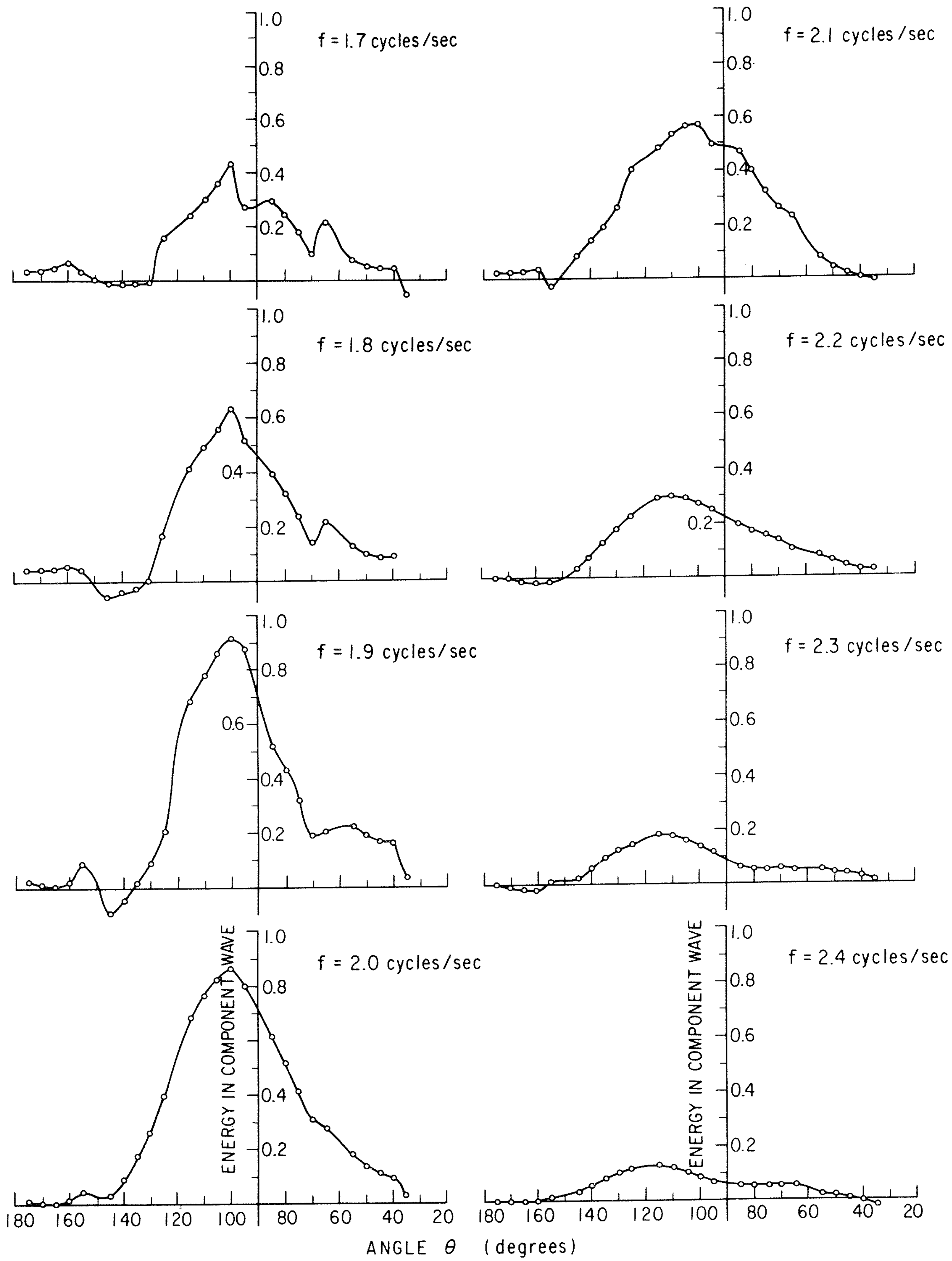


FIG. 6 DIRECTIONAL SPECTRA, DISCRETE ENERGY METHOD
(FROM MOBAREK 1965)

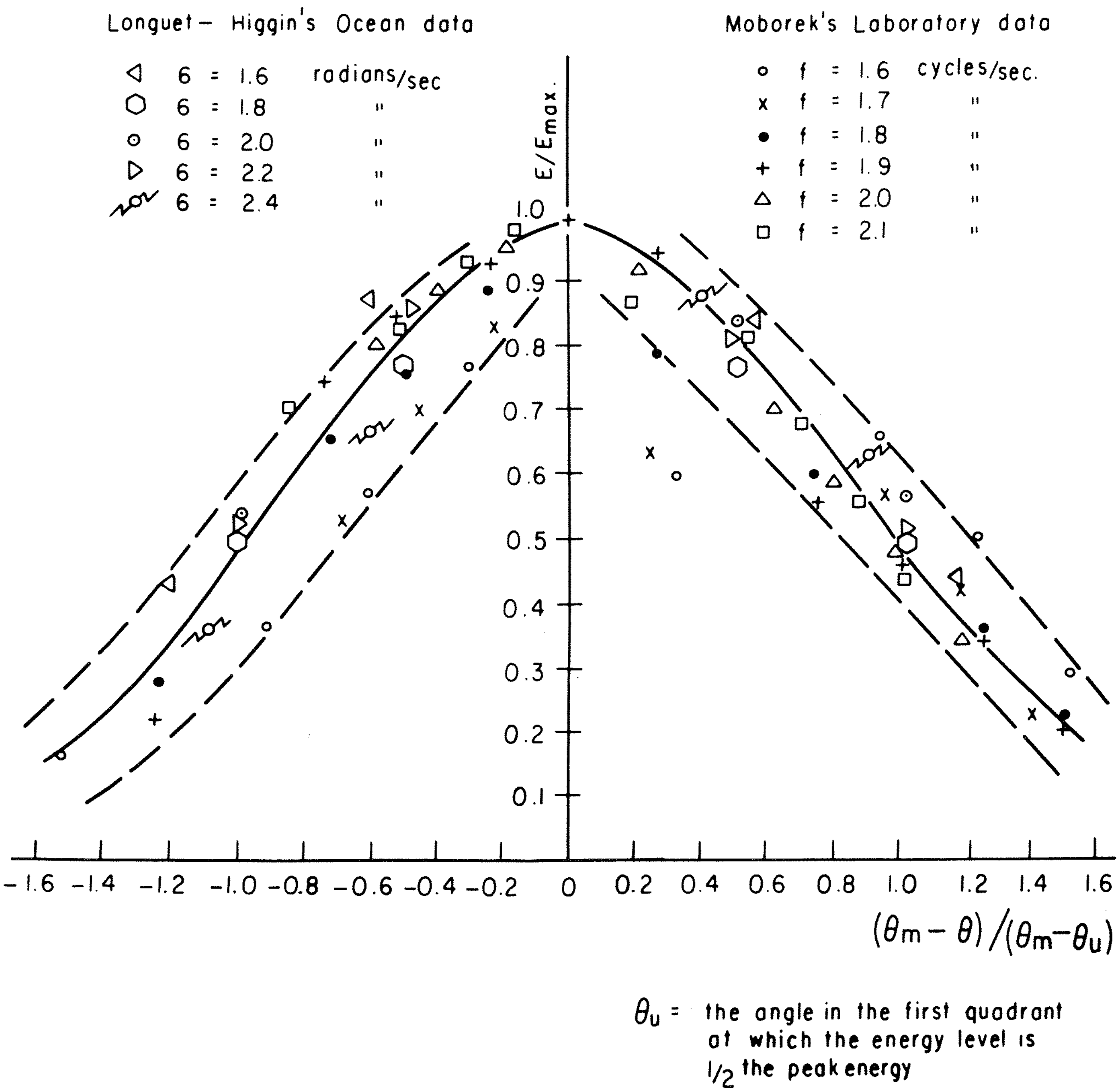


FIG. 7 NORMALIZED PLOT OF DIRECTIONAL SPECTRA
(FROM MOBAREK, 1965)

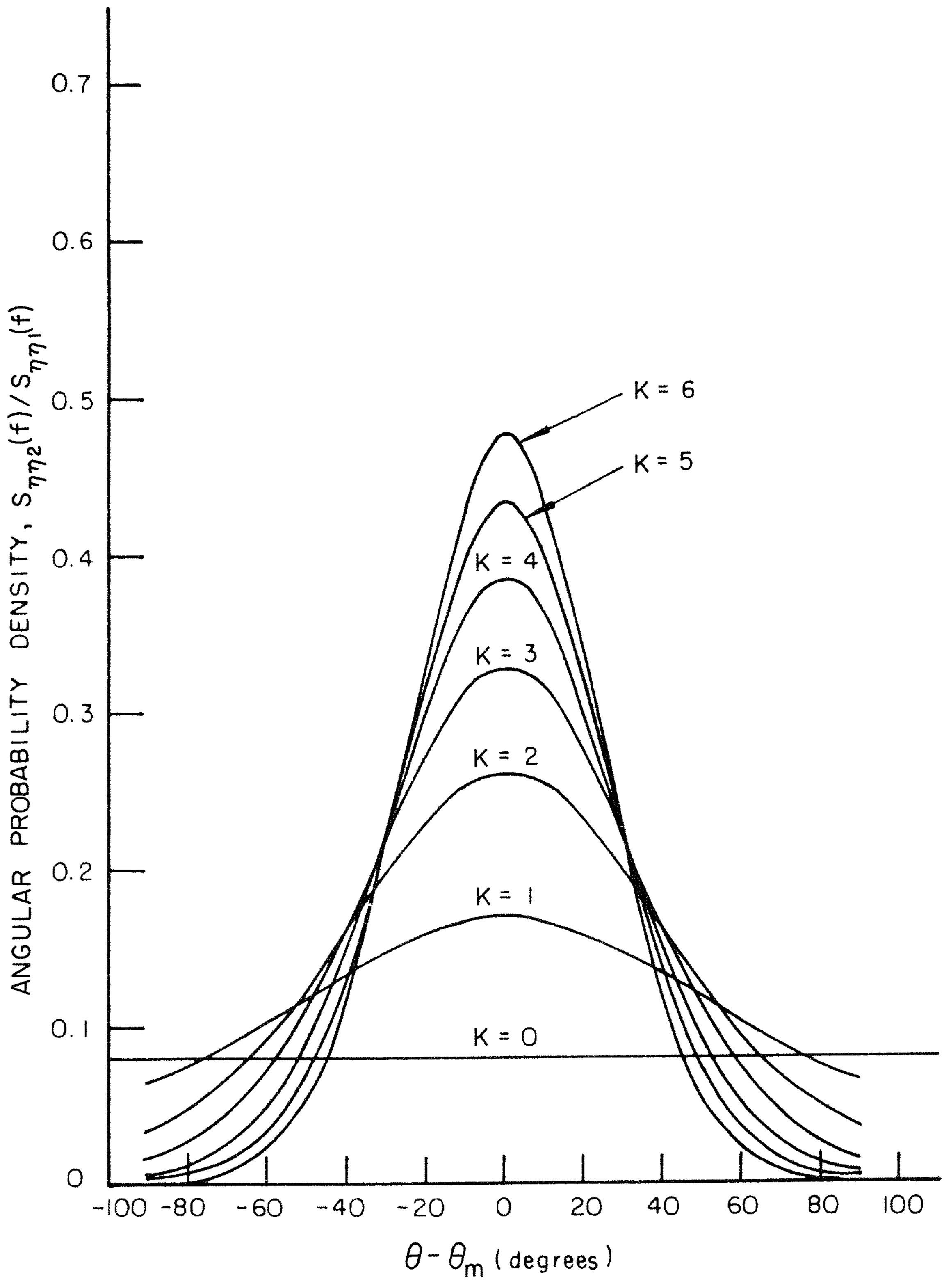


FIG. 8 THE CIRCULAR NORMAL DISTRIBUTION

(from Fan, 1968)

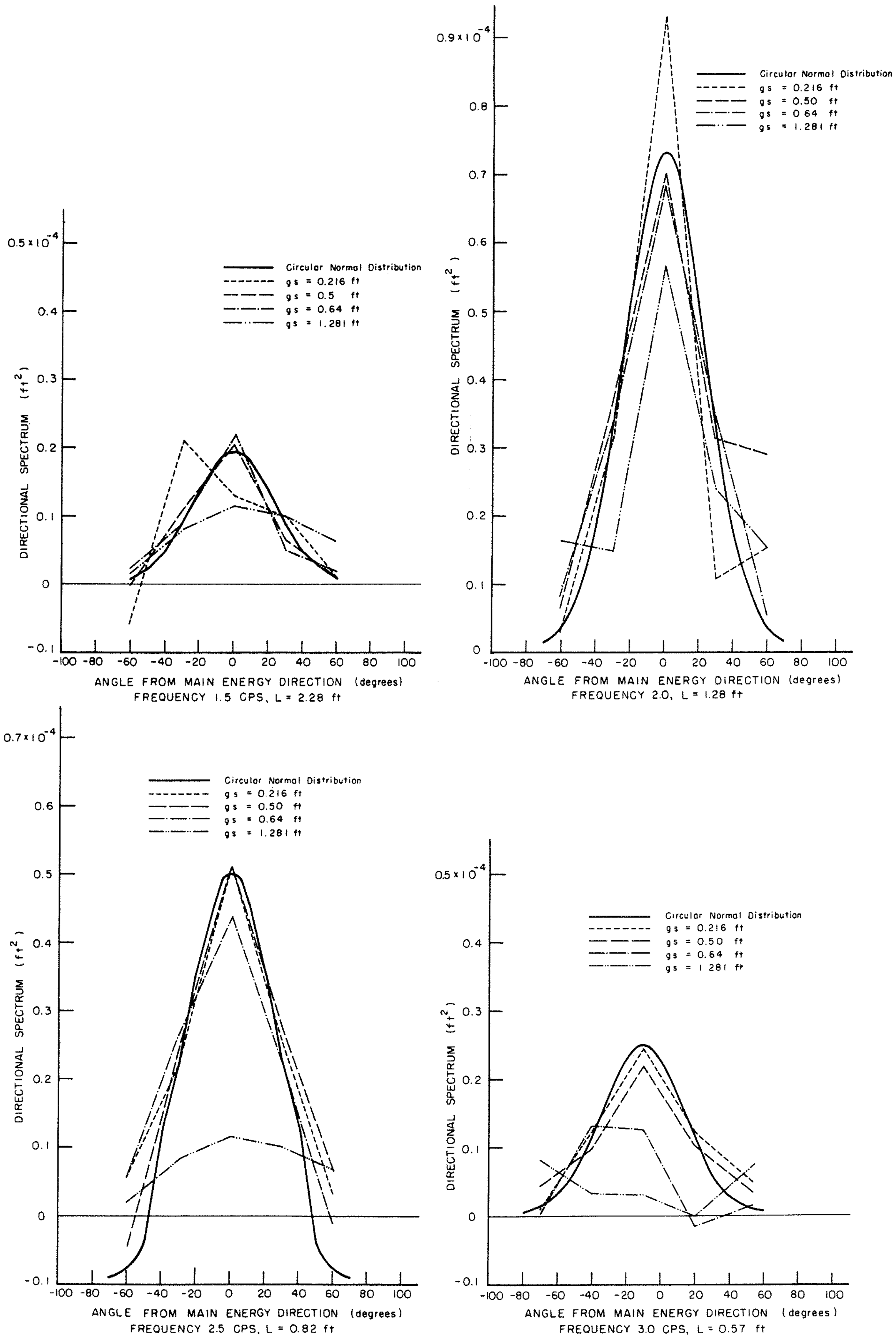


FIG. 9 COMPARISON OF DIRECTIONAL SPECTRAL ESTIMATES FOR VARIOUS GAGE SPACINGS (from Fan, 1968)

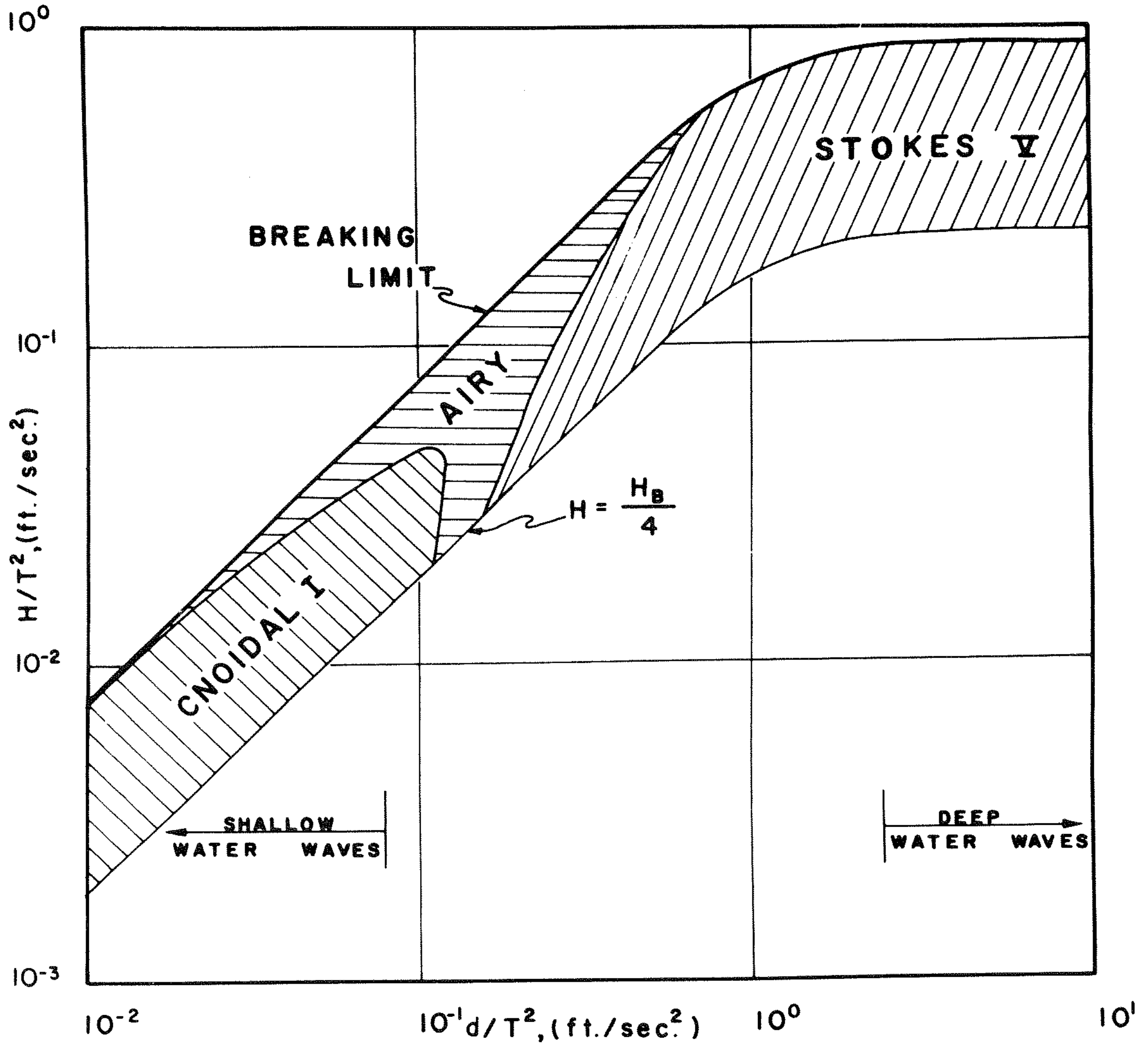


FIGURE 10a
PERIODIC WAVE THEORIES PROVIDING BEST FIT TO DYNAMIC FREE SURFACE BOUNDARY CONDITION (ANALYTICAL THEORIES ONLY)
(FROM DEAN, 1968)

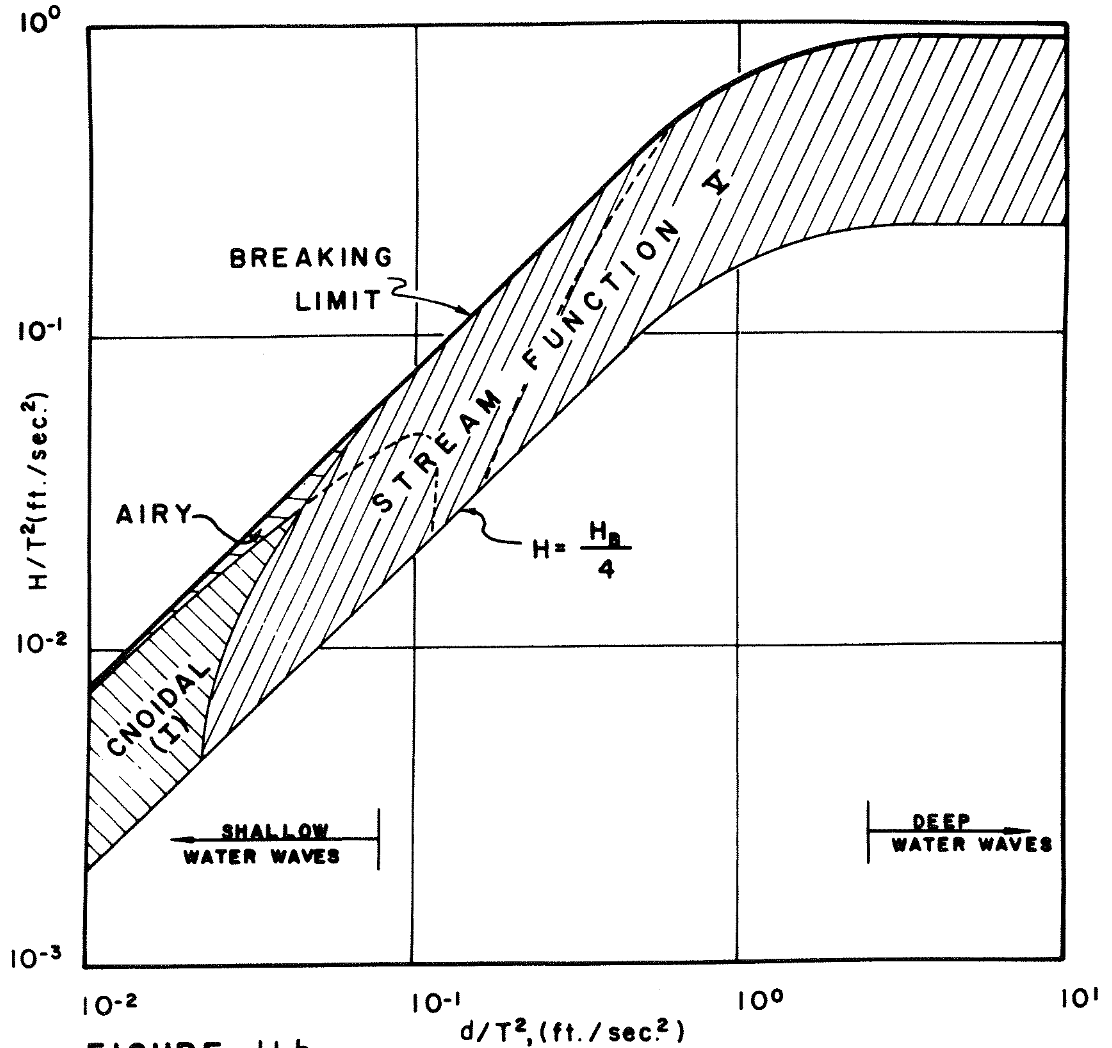


FIGURE 11 b
PERIODIC WAVE THEORIES PROVIDING BEST FIT TO DYNAMIC FREE SURFACE BOUNDARY CONDITION (ANALYTICAL AND STREAM FUNCTION V THEORIES)
(FROM DEAN, 1968)

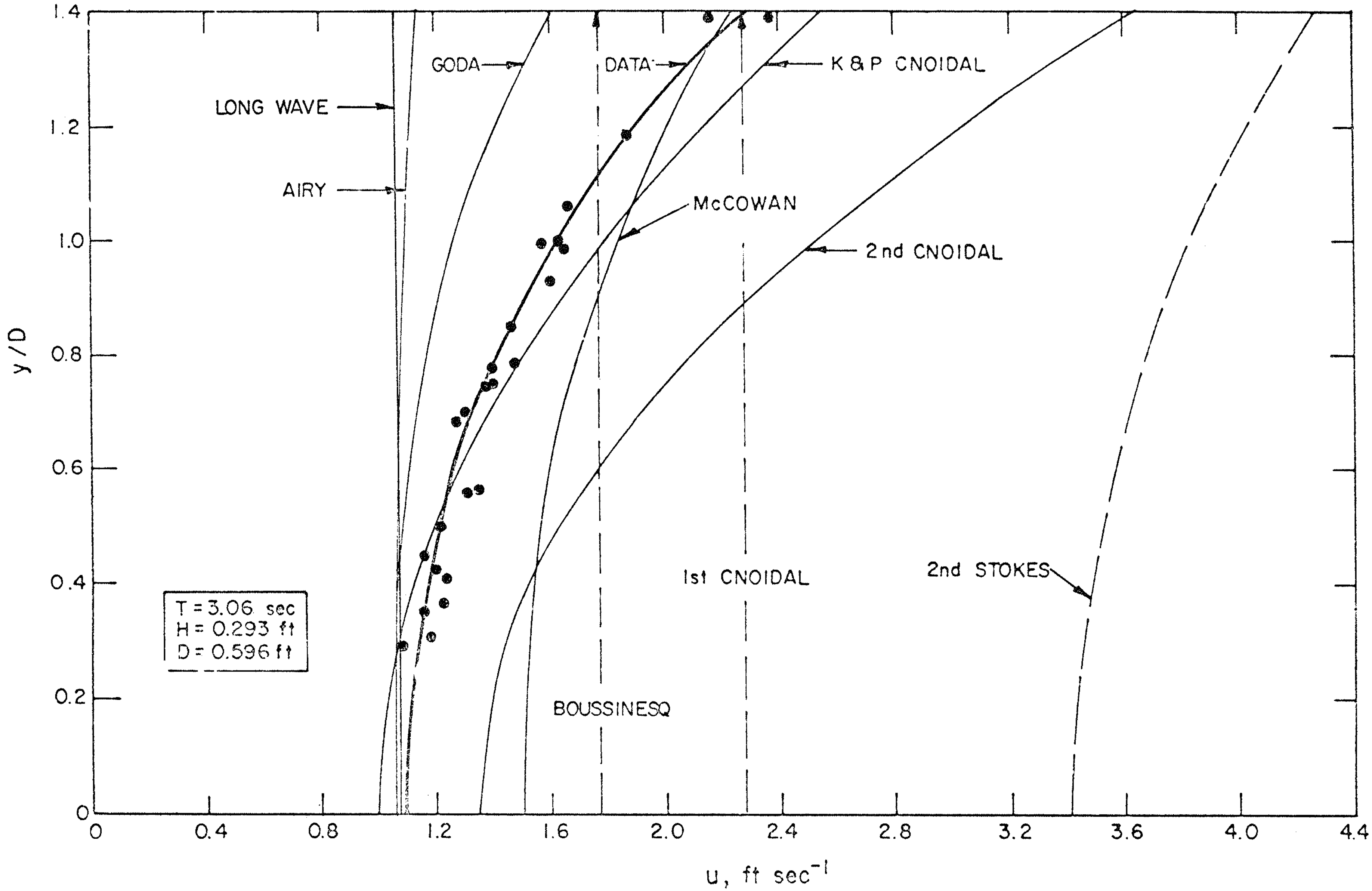
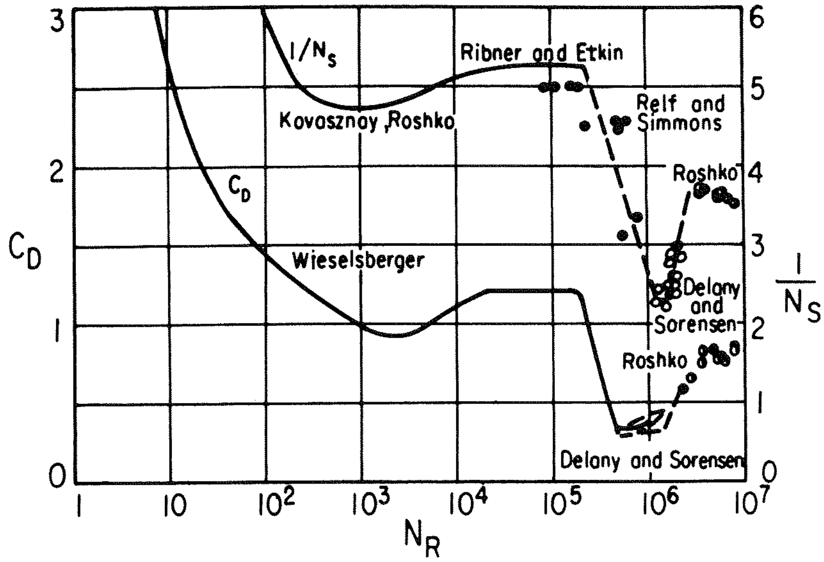
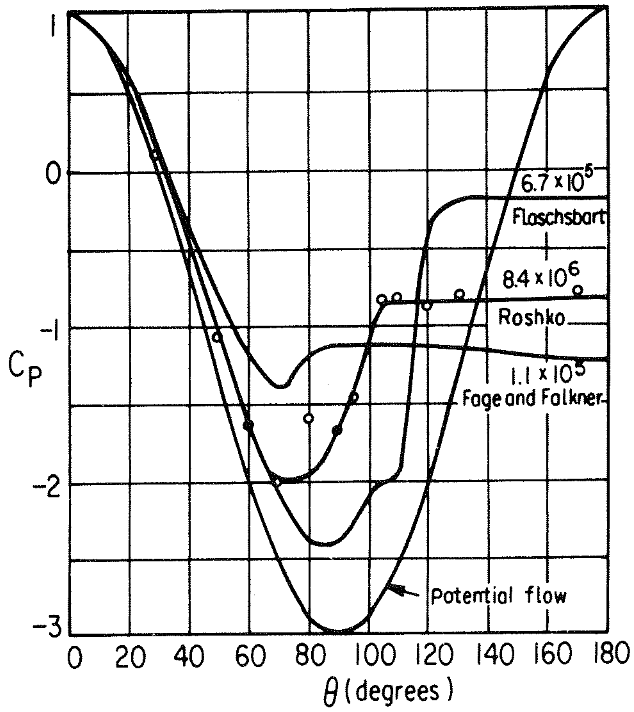


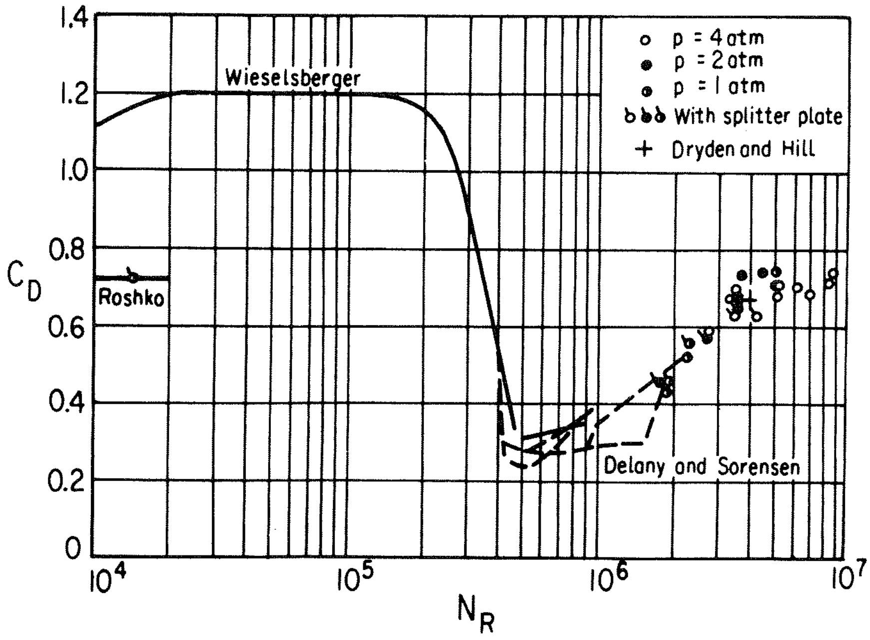
Figure 12 Horizontal Particle Velocity under the Crest - NEAR-BREAKING WAVE
(FROM LÉ MEHAUTÉ, DIVOKY AND LIN, 1968)



(a) Drag coefficient and reciprocal of Strouhal number



(b) Pressure distributions



(c) Drag coefficient

FIG. 13 STROUHAL NUMBER, DRAG COEFFICIENT AND PRESSURE COEFFICIENT AT HIGH REYNOLDS NUMBER FOR A CIRCULAR CYLINDER.

(From Roshko 1961)

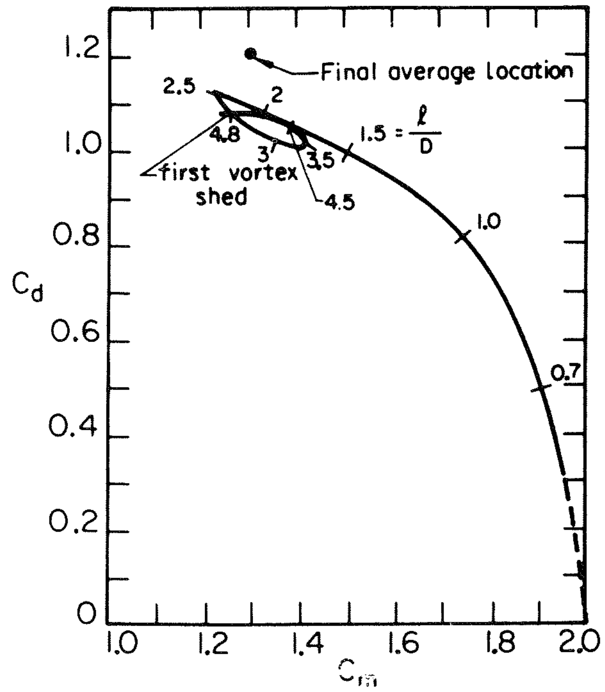
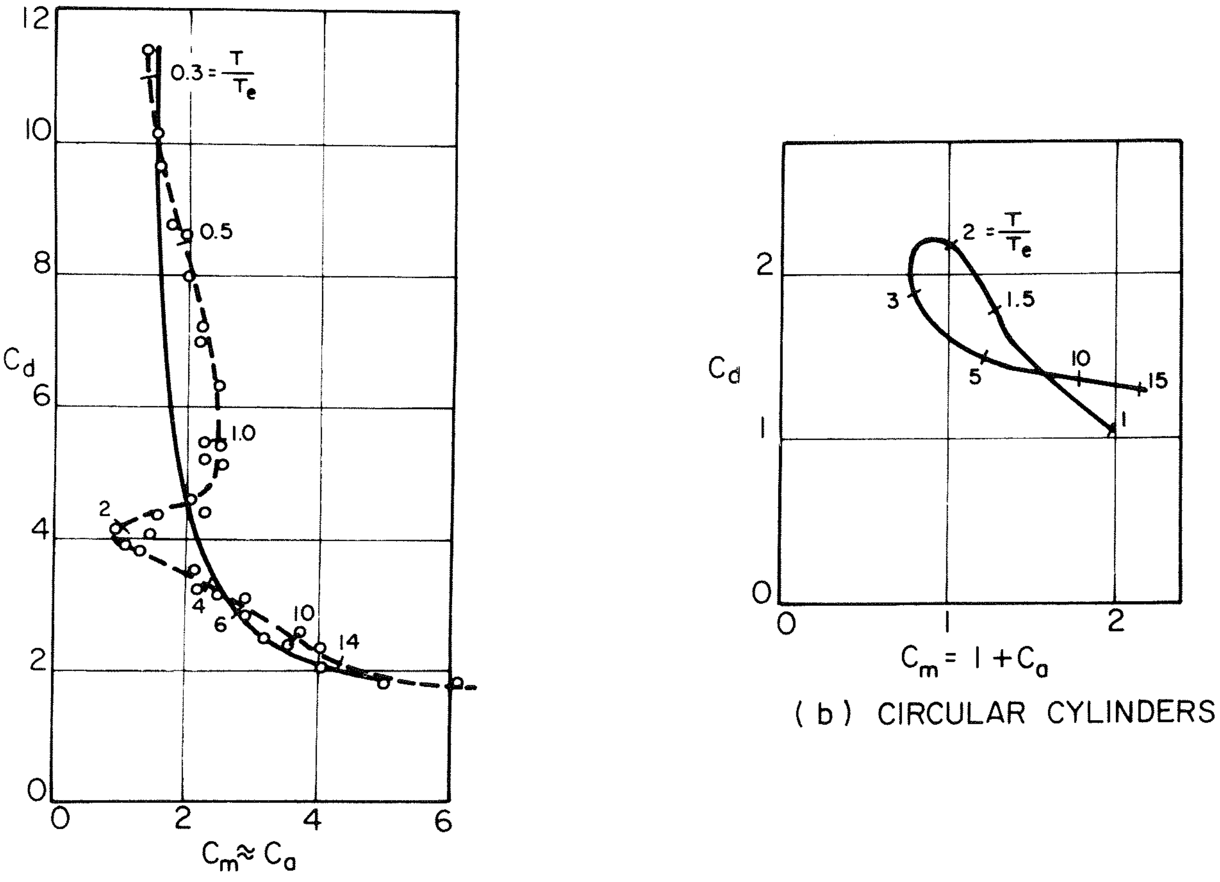


FIG. 14 CORRELATION OF DRAG AND INERTIA COEFFICIENTS
(From Sarpkaya and Garrison, 1963)



(a) PLATES

(b) CIRCULAR CYLINDERS

FIG. 15 INTER-RELATIONSHIP BETWEEN COEFFICIENTS OF COEFFICIENTS OF DRAG AND OF VIRTUAL MASS FOR (a) FLAT PLATES AND (b) CIRCULAR CYLINDERS (From Mc Nown and Keulegan, 1959)

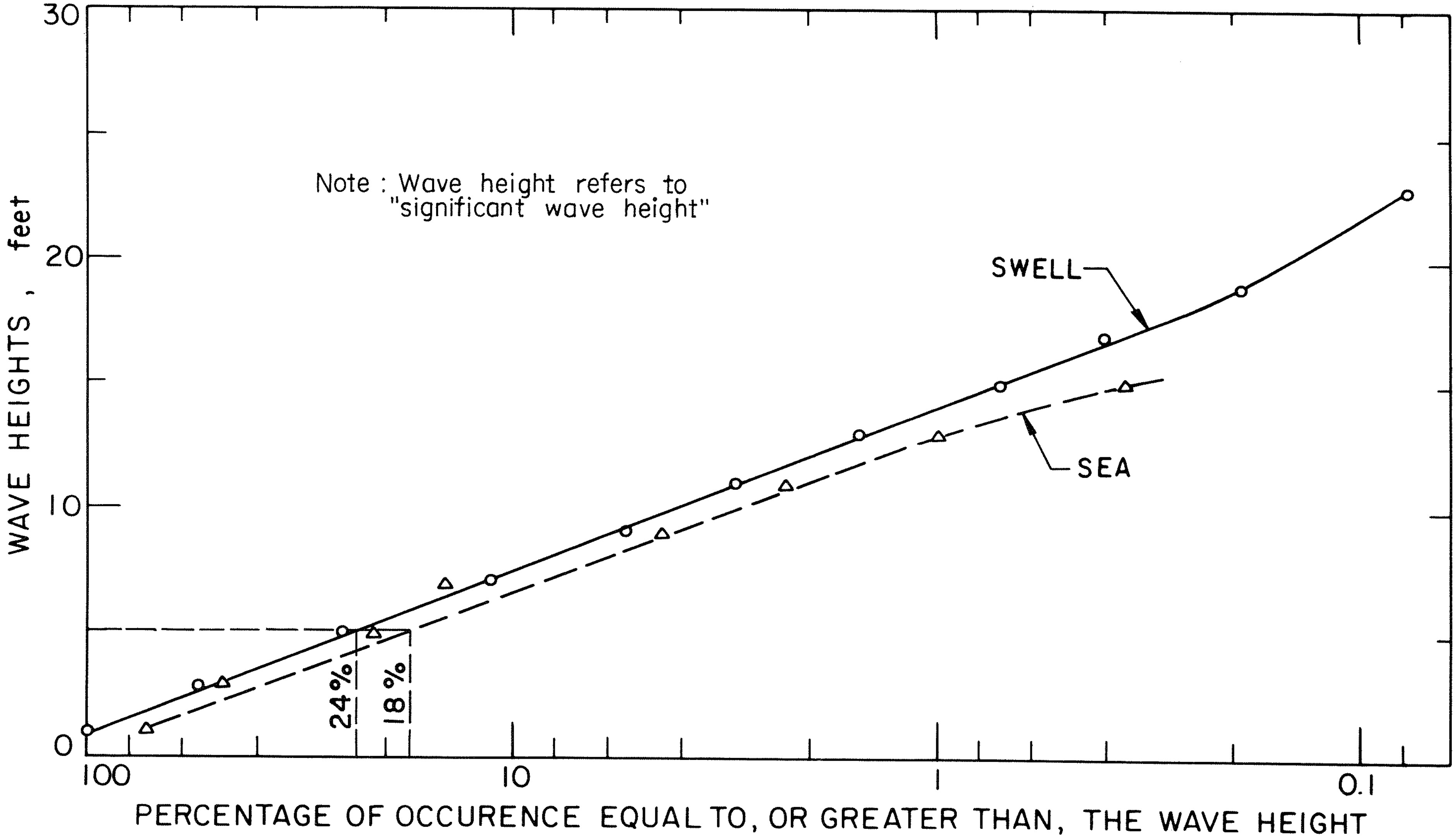


FIG. 16 EXAMPLE OF A WAVE CLIMATE

LIST OF SYMBOLS

a_i = Amplitude of wave component, feet

A = Projected area, $feet^2$

A_o = Cross sectional area of cylinder, $feet^2$

A_1, A_2, A_3 = Coefficients in Stokes' third order wave theory, dimensionless

B = Volume of submerged body, $feet^3$

cn = Jacobian cnoidal elliptical function, dimensionless

C_1 = $\frac{1}{2}\ \rho_w C_D D$; pound-$second^2/feet^3$

C_2 = $\rho_w C_m \pi D^2/4$, pound-$second^2/foot^2$

$C(f)$ = Co-spectra, $feet^2$-second

C_a = Coefficient of added mass, dimensionless

C_D = Coefficient of drag, dimensionless

C_m = Coefficient of mass, dimensionless

d = Water depth, with no waves present, feet

D = Diameter of pile, feet

$\overline{D}$ = Distance between a pair of wave recorders, feet

dp/dx = Pressure gradient, $pounds/foot^3$

du/dt = Horizontal component of water particle total acceleration, $feet/second^2$

dv/dt = Total acceleration, $feet/second^2$

$E(k)$ = Complete elliptic integral of the second kind of modulus k, dimensionless

f = Wave frequency, 1/T, cycles/second

f_2, f_3 = Functions in Stokes' third order wave theory, dimensionless

f_e = Eddy frequency, cycles/second

f_f = Acceleration of fluid in general case of unsteady flow, $feet/second^2$

f_o = Wave component frequence for which spectral density peak value occurs, 1/second

F = $F_D + F_I$, total force, pounds

$F(t)$ = Horizontal component of force, per unit length of pile, statistical theory, pounds/foot

F_D = Drag force, pounds

$F_h(s)$ = Horizontal component of force on a vertical pile, pounds

F_I = Inertia force, pounds

F_{Ih} = Horizontal component of inertia force, pounds

F_p = Integrated pressure force, pounds

g = Acceleration of gravity, feet/second^2

$G(r)$ = Defined by Equation 22, dimensionless

$G_1(r)$ = First term of series representing $G(r)$, dimensionless

H = Wave height, feet

H_d = Design wave period, seconds

H_s = Significant wave period, seconds

$I_o(k)$ = Incomplete Bessel function of the first kind of zero order for an imaginary argument, dimensionless

J_1 = Bessel function of the first kind, dimensionless

k = integer, dimensionless

K = Measure of concentration about the mean, circular normal distribution function, dimensionless

$K(k)$ = Complete elliptic integral of the first kind of modules k, dimensionless

L = Wave length, feet

l = Distance traveled by cylinder from position of rest, feet.

m_k = Statistical moment, defined by Equation 38, feet^2-ω^k

m_o = m_k for $k = 0$, variance of wave surface time history, feet^2

M_a = Added mass, slugs

M_c = Mass per unit length of cylinder, slugs/foot

M_o = Mass of a submerged body, slugs

N_s = Strouhal number, dimensionless

p = Pressure due to wave, at some point x, y, z in the water, pounds/feet ^{2}

$P(\alpha,K)$	=	Circular normal distribution function, dimensionless
$Q(f)$	=	Quadrature spectra, $feet^2$-second
r	=	In $G(r)$, $r = R_{vv}(\tau)/\sigma^2$, dimensionless
$R_{AA}(\tau)$	=	Covariance function of the horizontal component of water particle acceleration, $feet^2/second^4$
$R_{FF}(\tau)$	=	Covariance function of the horizontal component of force, $pounds^2/foot^2$
$R_{VV}(\tau)$	=	Covariance function of the horizontal component of water particle velocity, $feet^2/second^2$
$R_{\eta\eta}(\tau)$	=	Covariance function of wave surface, $feet^2$
S	=	Vertical coordinate, measured from the ocean bottom (i.e., $S=0$ at ocean bottom), feet
$S_{AA}(f)$	=	Wave water particle horizontal component of water particle local acceleration spectral density, $feet^2/second^3$
$S_{FF}(f)$	=	Wave force per unit length of pile (horizontal component) spectral density, $pounds^2$-$second/feet^2$
$S_{MM}(f)$	=	Total wave induced moment about pile bottom (horizontal component) spectral density, $foot^2$-$pounds^2$-second
$S_{QQ}(f)$	=	Total wave force (horizontal component) spectral density, $pounds^2$-second
S_s	=	Vertical distance from the ocean bottom to the water surface, feet
S_t	=	Elevation of wave trough above ocean bottom, feet
$S_{VV}(f)$	=	Wave water particle horizontal component of velocity, $feet^2$/second
$S_{\eta\eta}(f)$	=	Wave surface spectral density, $feet^2$-second
$S_{\eta\eta 1}$	=	One dimensional wave energy spectral density in directional spectra theory, $feet^2$-second
$S_{\eta\eta 2}$	=	Directional wave energy spectral density, $feet^2$-second/radian
$S(\omega)$	=	Wave spectral density, $S_{\eta\eta}(f)/\pi$, $feet^2$-second
t	=	time, seconds
T	=	Wave period, seconds

T_A = Transfer function, Equation 24b, 1/second^4

T_d = Design wave period, seconds

T_e = Eddy period, second

T_s = Significant wave period, seconds

T_V = Transfer function, Equation 24a, 1/second^2

U = Horizontal component of water particle velocity, feet/second

V = Velocity, feet/second

V_{rms} = Root mean square value of V, feet/second

V_w = An "average" horizontal component of water particle velocity, feet/second

x = Horizontal coordinate, x = 0 at wave crest, feet

y_s = Vertical distance from the undisturbed water surface to the water surface when waves are present, feet

Y_1 = Bessel function of the second kind, dimensionless

α = Angle measured from the mean, degrees

β = A phase angle, diffraction theory for wave force, radians

$\overline{\beta}$ = Direction between a pair of wave recorders, degrees

$\delta\omega$ = Circular frequency interval, cycles/second

θ = Horizontal angle in directional spectra, degrees

θ_m = Direction angle of peak value of directional wave energy spectral density, degrees

ρ = Mass density of a fluid slugs/foot^3

ρ_w = Mass density of water, slugs/foot^3

σ^2 = $2 \int_{-\infty}^{\infty} S_{VV}(f)df$, feet^2/second^2

τ = Lag in covariance function, seconds

ω = Circular frequency, $2\pi/T$, radians/second

ω_o = Circular frequency at which peak value occurs in wave spectrum, radians/second

$\partial u/\partial t$ = Horizontal component of water particle local acceleration, feet/second^2

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The determination of the boundary conditions at all points of interest within a deltaic area where the wave data are known for a limited number of reference stations only.

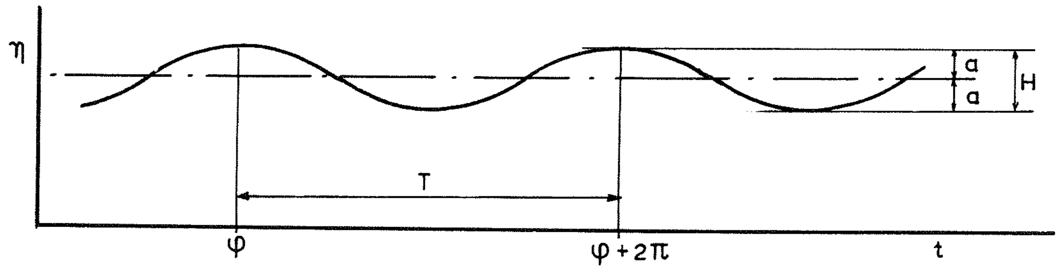
Two examples of the application of the statistical evaluations of the wave conditions to some problems in marine and coastal engineering are given.

In this general dealing with these examples the description of the method of application prevails over the details of the results.

2. DEFINITION OF SOME TERMS

2.1. Boundary conditions are the parameters or a system of parameters which are a basis for the determination of the design of a structure. Usually it is a multidimensional function of parameters comprising sea level, wave height and wave period. Very often, the mean wave direction is of importance. For different values of probability of exceedance of the parameters, studies in hydraulic models or computations have to be carried out in order to obtain physical data for the mathematical decision (Ref.1). For many problems the energy density spectrum function must be introduced replacing both wave height and wave period.

2.2. Uniform periodical waves, first order wave theory

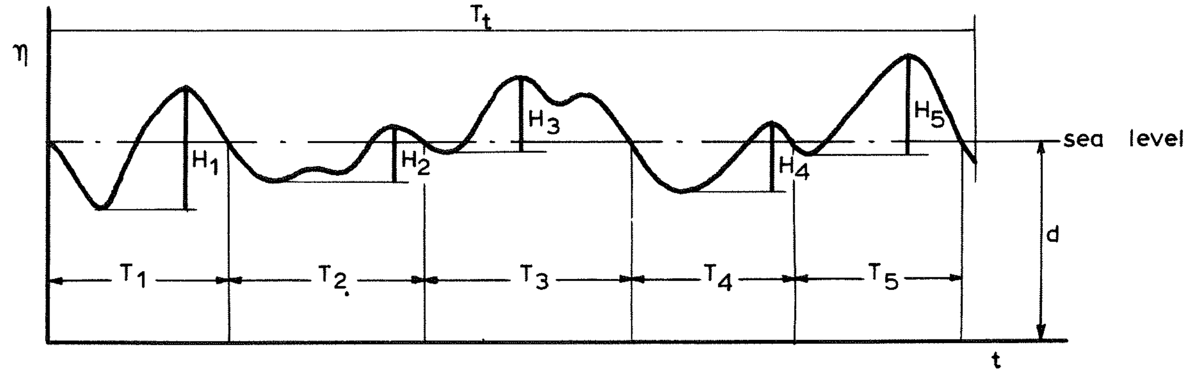


movement of the water level as a function of time t	$\eta = a \cos(\omega t - \varphi)$
amplitude	a
wave height	$H = 2a$
wave period i.e. distance between two maxima of	T
angular velocity	$\omega = \frac{2\pi}{T}$
wave frequency	$f = \frac{1}{T}$
wave phase	φ
water depth	d
wave celerity (propagation velocity of the wave form)	$C = C(\omega, d)$
wave energy per unity of sea surface	$E = \frac{1}{8} \cdot \rho g H^2$

energy propagation velocity (group celerity) — $C_g = n.C$ with $\frac{1}{2} \leq n(\omega, d) \leq 1$

energy transport (wave power) — $N = E.C_g$

2.3. Irregular wave field



movement of the water level (time series)	$\eta = \eta(t)$
length of the time series	T_t
individual period	T_i = distance between two crossings of the sea level for $\frac{d\eta}{dt} < 0$
individual wave height	H_i = distance between the maximum and minimum of η within the corresponding period T_i
number of individual waves	n
mean period	$T_m = \frac{T_t}{n}$
representative period	T_r
significant wave height	H_s
wave height corresponding with exceedance value q	H_q
exceedance probability q	q(Hq)=percentage of the number of individual wave heights which are higher than Hq.
frequency of "wave components" in a wave field	f
autocorrelation function	$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{1}{2}T}^{+\frac{1}{2}T} \eta(t).\ t+\tau)dt$
** energy density spectrum (total energy)	$S(f) = \int_{-\infty}^{+\infty} R(\tau) e^{2\pi i f \tau}\, d\tau = 2\int_{0}^{\infty} R(\tau).\cos 2\pi f\tau.d\tau$
potential energy of a time series	$E_p = \rho g m_o = \rho g \int_0^{\infty} S(f)\, df$
** zero moment of the real part of spectrum	$m_o = \int_0^{\infty} S(f) df$
total energy of wave field	$E_T = 2E_p = 2\rho g m_o$

3. STATISTICAL PARAMETERS DESCRIBING A WAVE FIELD

3.1. General approach

Most of the parameters which are generally used in order to define the wave movement in fluids are derived from the theory of periodical uniform waves. This theory, in which the mathematical relations between the parameters are determined, could be applied to many kinds of sharply defined and schematized problems. In deep water, the solution of the linearized differential equations is correct for first order harmonic waves of small amplitude. A number of solutions of non-linear equations exists for the waves of a special type in shallow water i.e. the trochoid wave, the third and higher order waves, the cnoidal wave etc. (Ref.2,3 and 4). The results of such theoretical studies are more or less verified by the results obtained by hydraulic model research in wave flumes using periodical wave generators. Such studies have contributed to the improvement of knowledge of the basic wave movement mechanism.

Many times, however, only approximate and sometimes even wrong solutions of engineering problems are obtained from the mathematical and model studies based on uniform waves. Moreover the variability of the seawaves limits the use of the very refined results of the uniform wave theory.

For irregular wave fields, the parameters defined for uniform waves, are to be expressed in statistical terms such as the significant wave height H_s, mean period T_m or reference wave celerity C_r. These parameters determined by empirical or semi-empirical distribution functions describe the behaviour of the "individual waves" of a wave field. However, an "individual wave" can only momentarily be defined. In fact it is a sum of many random disturbances travelling with different celerities. Obviously the mathematical relations between the different parameters of uniform waves does not hold for the "individual waves". Some times these relations may be used as a good approximation for the relations between the statistical parameters describing the wave field. The reliability of the results, however, depends on the proper choice amongst them.

The development in the field of the communication theory and especially the analysis of time series introduced by RICE in 1944 (Ref.5) opened new possibilities for study of the irregular wave fields. Until about 1957 the new statistical approach was mainly explored by oceanographers and mathematiciens.

The most important improvements in the application of the theory of irregular wave fields on engineering projects are the introduction of the spectral analysis of wave recordings and the Rayleigh-distribution function of wave heights based on the first order harmonic theory. (Ref.6).

3.2. The energy density spectrum is obtained from the auto-correlation analysis of wave records. It gives information about the potential energy $\rho g \frac{\Delta E}{\Delta f}$ within a narrow range of wave frequenties Δf as a function of the corresponding frequency f. Usually, this energy is divided by $\rho . g$ and has the dimensions of m^2

* $$\left(\frac{\Delta E}{\Delta f}\right)_j = F(f_j + \tfrac{1}{2}) \text{ for } j = 1,2,3, \ldots\ldots.. \qquad (1)$$

If $\Delta f \rightarrow o$, than $\frac{\Delta E}{\Delta f} \rightarrow \frac{dE}{df} = S(f)$ which is called the continuous ** energy density spectrum. In fact it is the real part of the fourier-transform of the autocorrelation function (Ref.2).

The energy density function is defined for stationary processes of long time series. The spectral analysis of times series offers the possibility for approximation of the irregular sea surface in a flume by means of the spectral components representing the energy in every class $(j+\frac{1}{2})$ of the frequency f. Two methods which are directly based on the energy density spectrum for the boundary condition in a flume are discussed in paper no.2 of this symposium.

The accuracy of this approach is better then the accuracy of the generation of waves in wind flumes depending on fetch. It is, however, still limited on account of the statistical character of the spectrum which is derived from a limited time series and usually represents only one sample. In other words the spectrum function does not determine the wave motion uniquelly.

It is important to note, that the definition of the total energy of spectra opens the possibility to determine the relation between the irregular wave motion and the uniform wave theory:
The total energy E_u of an uniform sinusoidal wave field per unity of surface may be expressed by the energy-height H_u of the uniform sinus wave:

$$E_u = \frac{1}{8} H_u^2 \qquad \left[m^2\right] \qquad (2)$$

The energy represented by the high frequency components of a spectrum tends to zero. For practical purposes the values of $j.\Delta f$ may be limited to $f_{max} = (n+\frac{1}{2}).\Delta f < 5$ Hz.
Consequently, the total energy of the wave field may be approximated by the energy of uniform sinusoidal wave components computed for n classes of $f(j+\frac{1}{2})$.

** $$E_t = \rho g \sum_{j=o}^{j=n} 2\left(\frac{\Delta E}{\Delta f}\right)_j = \rho g \sum_{i=o}^{i=n} \frac{1}{8} H_i^2 = \frac{1}{8} \rho g H_s^2 \qquad (3)$$

where H_i = individual wave height and
H_s = significant wave height, which is the wave height of an uniform sinus-wave with the same energy as the wave field.
In consequence,

$$H_s^2 = 16 \sum_{j=o}^{j=n} \left(\frac{\Delta E}{\Delta f}\right)_j \qquad (4)$$

and for f o

$$H_s^2 = 16 \int_o^{f_{max}} S(f)\, df = 16\, m_o \qquad (5)$$

in which m_o = zero moment of the real part of the spectral function.

Finally, it follows:

$$H_s = 4 \sqrt{m_o} \qquad (6)$$

3.3. The height of the "individual wave"

The statistical probability distributions of the different parameters related to the "individual waves" are a reliable basis for the comparison between different records, except for the extreme values in the low probability range. The practical treatment of this problem is dealt with here in after, by discussing the probability distribution function of the "individual wave heights".

As mentioned before, the mathematical approach of LONGUET-HIGGINS results in the Rayleigh probability distribution of wave heights. In an other publication, CARTWRIGHT and LONGUET-HIGGINS (Ref.7) show the relation of the Rayleigh function with a sharply peaked spectrum of which the energy is concentrated in a single narrow frequency band.

By this relation they show, that the individual wave height H_r which is representative for the total energy of the narrow spectrum equals:

$$H_r = 4 \sqrt{m_o} \qquad (7)$$

and the statistical parameter H_r of an irregular wave field appears to be equal to the sinusoidal wave height H_s determined from the relation between the energy of the uniform sinus wave and the total energy of the wave field.

The cumulative probability distribution is given by BRETSCHNEIDER (Ref.8) in the equation:

$$* \quad q\left(\frac{H_q}{\overline{H}}\right) = 1 - \exp.\left\{-\frac{\pi}{4}\left(\frac{H_q}{\overline{H}}\right)^2\right\} \qquad (8)$$

in which $\overline{H}$ = the "average" wave height

H_q = wave height with the cumulative probability value q. (Exceedance probability value = 1-q)

* $\overline{H}$ is related to the significant wave height as $\overline{H} = 0.625\ H_s$

Note, that this equation is expressed in dimensionless parameters. The exceedance probability of all other "individual" wave heights, including the representative wave height e.g. of the significant wave height can be computed from equation (8), on the condition that the assumptions of LONGUET-HIGGINS are valid. Moreover, the time series must remain stationary for a sufficiently long duration.

The extrapolation of the cumulative probability function to the extremely high waves is only correct for a limited time series when a range of significancy is considered (fig.3.1), as shown by GUMBEL (Ref.9). For the research in behalf of an engineering project the stationary process is often limited to the duration of a severe gale, which often are lasting only a few hours.

CARTWRIGHT and LONGUET-HIGGINS developed also a different category of probability functions by introducing a parameter describing the shape of the spectrum.

In the authors contention the practical value of theoretical refinements of the statistical analysis of timeseries is limited by the uncertainities of the extreme values mentioned by GUMBEL. This opinion holds also for the determination of the probability distributions of wave heights in shallow water with respect to the effect of the non-linear inter-action between the wave components.

The wave readings of limited duration, e.g. 30 min, at some stations in the shallow parts of the North Sea show only small deviations of the extreme values of wave heights from the Rayleigh-distribution. The wave heights at the exceedance probability level of 0,5% are almost never larger then 15% and/or smaller then 10% of the corresponding wave heights obtained from the Rayleigh-distribution. The Rayleigh distribution appears to be acceptable in shallow water as far as in the breaker zone (fig.3.2).

Moreover, there is an empirical relation between the significant wave height and the actual water depth:

$$H_s = (0,4 \pm 0,05)\ d \qquad (9)$$

This relation is of importance for the determination of the boundary conditions near the coast and in the deltaic areas where the actual depth usually depends on both the tidal movement and the wind effect during storms. Consequently, the boundary conditions for waves in this areas often are determined by these two factors.

3.4. The reference period of an irregular wave field

Wave observations, necessary for the determination of the boundary conditions in a deltaic, i.e. shallow, area cannot always be obtained from the most favourable locations due to the high cost of the measuring stations; another detrimental factor is the usually limited observation time available for sampling. Consequently, boundary conditions must be derived from known wave conditions, measured on stations situated some distance away from the area concerned, by means of refraction computations. Theoretically, such refraction computations should be carried out separately for all components of a directional spectrum and for all circumstances of importance.

This approach is very laborious and often also inpractical, because of the limitations set by the accuracy of both the statistical evaluation of the available wave data and the final research results. Therefore, a representative wave celerity C_r or a representative period T_r must be defined in order to approximate the refraction computations for an irregular wave field, if a much more simplified computation method, viz. the refraction of the first order waves has to be utilized.

The mean period T_m as well as the statistical distribution of all "individual" periods of a wave field can be computed from wave records by the well known "zero crossing" method.

A comparison of schematized refraction patterns, obtained by radar with the computed refraction patterns in areas in which also simultaneous wave records are available shows the following results:

A. The mean period T_R determined from the mean wave lengths measured on the radar photographs and from the corresponding depths, appears to be a linear function of the mean period T_m computed from the corresponding time series: $T_R \approx 1,2 \ T_m$ (fig.3.3). This result is of course only valid for that special type of radar and that particular position of the radar emmitor.

B. The best agreement between the observed and computed refraction patterns is obtained when this "radar period" T_r is used for the refraction computations.

The schematization of the wave pattern on a radar photograph is carried out manually and it is therefore not free of subjectivism. Results obtained from schematization of the same photograph by different persons, however, show only minor differences. The significant difference of T_r and T_m is caused by the geometry of the radar beam, because the smaller waves are invisible behind the larger ones.
The value of the coefficient, which is 1,2 in the above mentioned relation, depends on this geometry.
After discussing these results with the author, Ir. J.A. Battjes of the Delft Technological University derived a theoretical value of the coefficient from different types of spectra using the energy transport function of linear sinus waves as a weighting factor.

He found, that the values are in the range $1,2 \pm 0,1$ and explained the basis for the ressemblence between the observed and the computed refraction patterns using this coefficient.

The "radar period" correction must be determined by similar studies as mentioned above for each other types of radar or a different location of the emmitter.

Ir. L.A. Koelé of the Rijkswaterstaat studied the correlation between the wave heights and the periods of "individual waves" using a great number of time series from the North Sea (fig.3.4). His results which are not published, inspired the authors of the papers No. 3 and 10 of this Symposium to publish analogeous results of model experiments.

The correlation shows a statistically reliable correspondence between the significant wave height H_s and the period $T_r = (1,3 \pm 0,2)T_m$. However, the correlation is rather poor which fact is already stated by BRETSCHNEIDER (Ref.8).
This empirical result may be seen as an other argument for the application of the reference period $T_r = 1,2 \ T_m$ in refraction computations.

4. DETERMINATION OF THE BOUNDARY CONDITIONS FROM A REFERENCE STATION.

The deeper insight in the problems concerning the irregular sea waves asked for better laboratory techniques. Also the demand for observations are changed qualitatively and quantitatively so as to provide the boundary conditions for the extensive engineering projects in coastal areas. However, the engineering decissions have to be based on long term observations and especially for this purpose the data from the wave gauges are usually insufficient.

Moreover, it is not possible to construct a great number of stations in the sea on account of both the cost involved and the time consuming elaboration of the data. For large projects such as the Delta project or the Europoort project a limited number of reference stations have been set up.

Long series of data may be obtained e.g. from the visual observations on light vessels and/or the visual observations performed on a board of merchant ships or by using the computed wave data from the long series of meteorological observations of wind velocity and direction.

Single visual observation of wave height should be treated with certain reservation because of the inherent subjective character. The comparison with instrumental wave gauge readings may give an insight in the reliability (fig.4.1). Analogeous arguments hold true for the value of the wave data computed from the observed wind conditions.

The statistical distributions of the parameters defining the irregular wave fields i.e. the significant wave heights H_s, the representative periods T_r and the mean directions of the wave propagation θ_m, may be of importance for the determination of the boundary conditions which are required for many engineering problems. They must be derived from a long period of observations. There are, however, not many stations in the world with series of wave recording longer than 5 years. Consequently, the data have to be extrapolated for the extreme conditions from rather few samples.

The distribution functions can almost never be described by formulae and the extrapolation becomes a very rough approximation especially because of the significancy lines of GUMBEL (Ref.9).

An example of the discrepancy between the statistical data on wave heights obtained from the Lightvessel"GOEREE" is given in fig.4.2 for two observation periods. It is obvious from this figure, that a mathematical solution of a decision problem concerning an engineering project usually does not show a very significant minimum of the sum of initial- and maintenance costs if it is based on such data.

** In the same figure the probability distributions of wave heights computed from visual observations and from readings obtained from the wave gauge "TRITON" (5 km offshore Scheveningen) are combined in order to determine the errors which might be obtained from the direct comparison of both distributions.

5. DETERMINATION OF THE CRITICAL DRAFT OF SHIPS IN A NAVIGATION CHANNEL

The depth below mean sea level of navigation channels in shallow waters determines the economy of many large harbours. The vastly increasing dimensions of the oil tankers and bulk-carriers call for still deeper channels. The costs of initial- and maintenance dredging increase more than proportionally to an increase in depth.

The physical factors which primarily influence the choice of the safe depth for ships of a certain mean draft are:

- tidal variation of the sea level
- wind-effect on the sea level
- roll, pitch and heave of the ship
- squat of the ship

A statistical method resulting in empirical probability function for the determination of the safe depth in the channel is described at the International Navigation Congress in 1964 (Ref.10). In that paper, the authors contribution deals with the empirical probability function for the combination of the first three factors using a mean response factor for determining the movement of the ship in a wave field (fig.5.1 and 5.2).

Economic decisions can be made by using these figures. The necessary depth of a channel can be determined provided the costs of initial dredging, the costs of maintenance dredging, the statistics on the density of navigation and their economical value for different types of ships are known.

The result of this three dimensional statistical analysis is the probability function which allows to determine the mean value of the risk of touching the bottom for a ship with a given mean response factor $\bar{\alpha}$ to the waves. The response factor α however is a function of three other main factors: the frequency of the wave-components of the energy density spectrum, the direction of the wave propagation in respect to the ship's course and speed of the ship.

The response factor could be applied to a Rayleigh distribution of wave heights in order to compute approximately the corresponding probability distribution of the deepest draft of the ship's.

The position of the deepest point changes continuously. It is determined as a sum of the three possible motions i.e. roll, pitch and heave.

The behaviour of large ships subjected to different types of wave fields are studied in a model basin in which it is possible to generate irregular waves.

Another study is undertaken in order to check the model results on a larger scale (approximately 1:5) in the North Sea using a self propelled barge with dimensions roughly corresponding to those of a tanker. It was impossible to use an actual tanker for experiments under the critical conditions because of the financial risks.

Some preliminary results of this research are given in fig. 5.3.

This research opens the possibility for refining of such conclusions which are already given in the paper mentioned above. The risks of touching bottom for the largest ships, which want to approach the harbour under critical conditions, can be eliminated by the decision that the ship have to remain in deep water.

The economical loss caused by a delay of a tanker and the probability of the delay of a certain duration is a determining factor when loading the ships. The danger of an accident like the well known grounding of Torry Canion may very well determine the level of risks to be permitted by the harbour authorities. For both reasons, operational guidance is necessary. This guidance should be based on an on-line analysis of wave fields and water

levels and on a short time prediction of both.
The following method is in development for this purpose:

A. On line analysis of the record of a permanent wave station near the navigation channel will give an energy density spectrum of waves which is representative for the data sampling period depending on the importance of the lowest frequency components. This sampling period is about 30 min for the North Sea, so the mean value of the spectrum is obtained with a delay of 15 min. The mean direction of propagation of the wave fields will be derived from radar-observations.

B. Continuous registration of filtered variations of the sea-level will give the corresponding value of the reduction of the recente soundings in order to get the critical depth in the channel.

C. The energy density spectrum of the ships movement will be computed for the types of ships approaching the channel. The following data are necessary: the speed, the course and the type of the ship concerned and the response functions of the ship's movements corresponding with these data. It is to be expected, that there will be only a limited number of types of the very large ships. The companies concerned will provide these necessary data when detrimental effect and economic loss of eventual delay's become apparent.

D. The probability distribution of the maximum draft of the ship will be determined from the spectrum of the ship's movement, corrected by squat. The squat may be approximated by using the data mentioned above.

E. The check on the danger-probability level by comparing the results of the computations with the accepted risks-criteria determine the time necessary for warning the ships commander. The expected change in weather conditions and the state of sea must also be considered by cooperation with a meteorological service, and the expected delay can be passed to the commander.

6. BOUNDARY CONDITIONS ON WAVES IN DESIGN OF A DIKE

The sea dikes of the modern type in the Delta are designed so as to allow a limited overtopping by waves under conditions which never were observed before. These conditions are determined by the critical storm defined by the critical storm surge level. This level is the result of an economical decision study. The mean probability of occurrence of this storm or of a higher one within a century is fixed to the value of 1%.

Only 2% of the wave tongues are accepted to overtop the dike during the above critical stormconditions.

Delft Hydraulic Laboratory carried out experiments in the wind flume in which certain boundary conditions, heights and profiles of dikes were used.

Hence the choice of the design profile is strictly related to certain critical boundary conditions which, as we hope, will never be experienced in the nature. Moreover, the boundary conditions will change in the course of time by the changing morphology of the area to the seaward of the dike. The design of the dike and especially the determination of the height is almost impossible without further schematization of the problem.

The following assumptions were made when determining the profile of the dike in one of the delta estuaries i.e. "Brouwershavense Gat".

A. The mean direction of propagation of the waves on the North Sea is determined by the expected direction of the critical storm wind, which is North-West. The corresponding water level with the above mentioned exceedance probability is in this area about 5 m above mean sea lev l.

B. The waves in the estuary are smaller than the sea waves because of breaking on the outlaying shallow banks. The significant wave height is limited to: $H_s \approx 0,6$ d which is about 30% too high as compared to recent observation data which were not available at the time when the decision on the design of the dike had to be made (1964).

C. The wave height is further determined by the refraction coefficient computed from refraction computations using the reference period T_r = 12 sec. This period is probably too long according to our present knowledge.

D. The critical significant wave height can be approximated by $H_s = \sqrt{\Sigma H^2_k}$ in regions in which two or more significantly different directions of wave propagation Θ_k occur. H_k is the significant wave height corresponding with the direction Θ_k, k = 1,2,.....

E. The boundary condition is determined at the distance of about 200 m of the dike. This is the distance, which can be reproduced on scale in most of the laboratory flumes. Moreover, at this distance the reflection of irregular waves does not significantly affect the wave pattern induced by the wave generator.

The results of the construction of wave rays is shown in fig. 6.1. The significant wave heights between neighbouring rays in deep water is computed along their tracks using the theory of constant energy transport. Corrections of this significant wave height H_s are computed for each point where the depth is less then 0,6 H_s (Assumption B). For the determination of the boundary condition i.e. the significant wave height 200 m out of the dike, the principle of superposition is applied (Assumption D).

In fig.6.2, the significant wave height along the dike is shown (graph A). In order to check the breaking condition for the total result after superposition, the maximal significant wave height H_s = 1,3 d which is possible for depths along the line 200 m out of the dike is determined (graph B). For most of the points of the dike two values can be obtained from these graphs; the lowest value found is used for the critical significant wave height at that particular spot.

** The value H_s = 5 m is chosen to be the representative boundary condition for whole the dike.

A second group of computations were carried out based on the possible changed bottom conditions. This change is expected after the estuary has been closed. A comparison of the results showed that the critical significant wave heights in future cannot be higher than the chosen boundary conditions.

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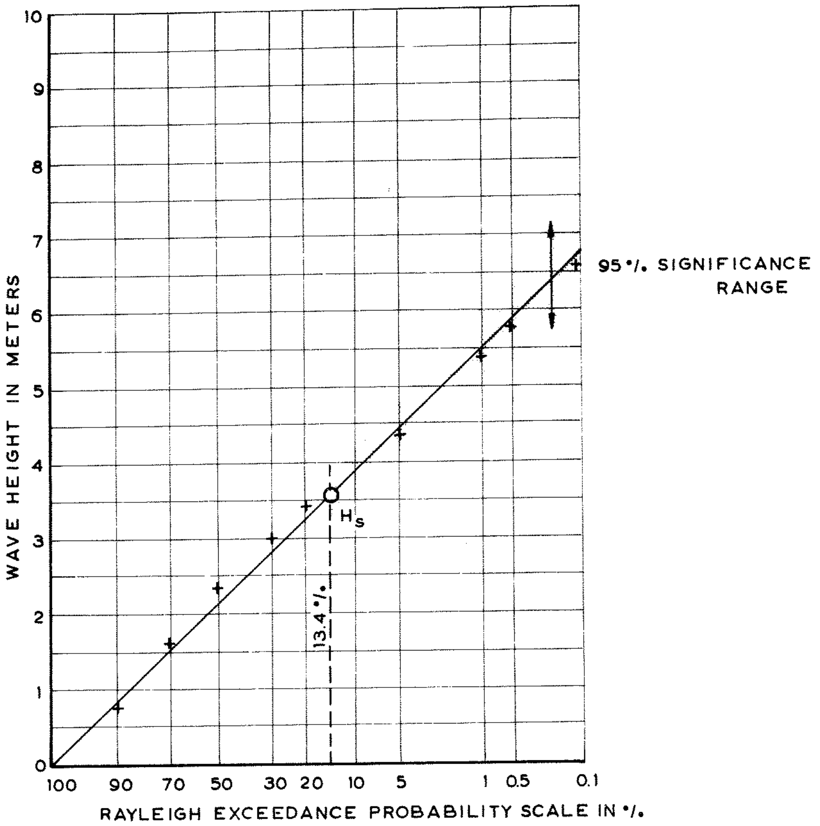
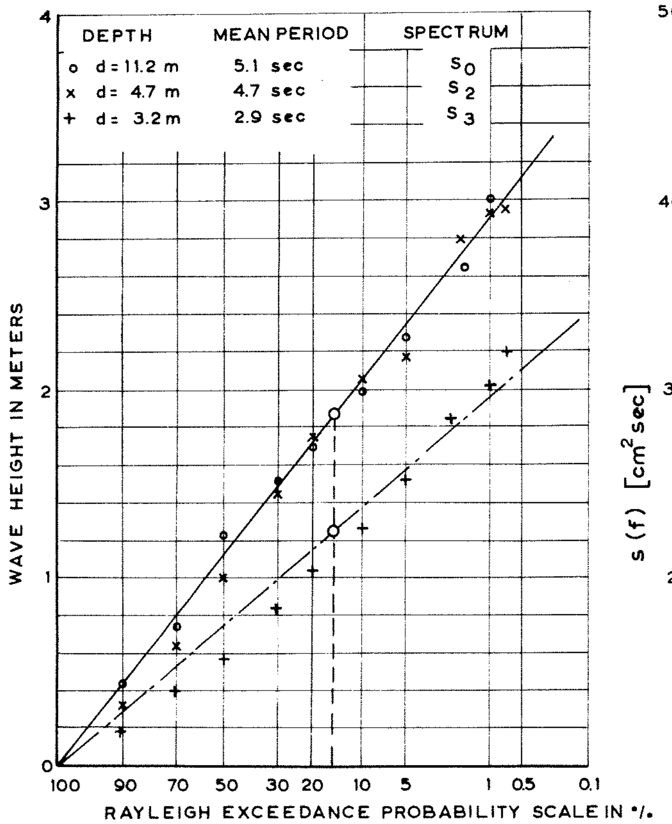


Fig. 3.1 EXAMPLE OF MEAN EXCEEDANCE PROBABILITY OF „INDIVIDUAL WAVES" OF A RECORD IN 16 m DEPTH



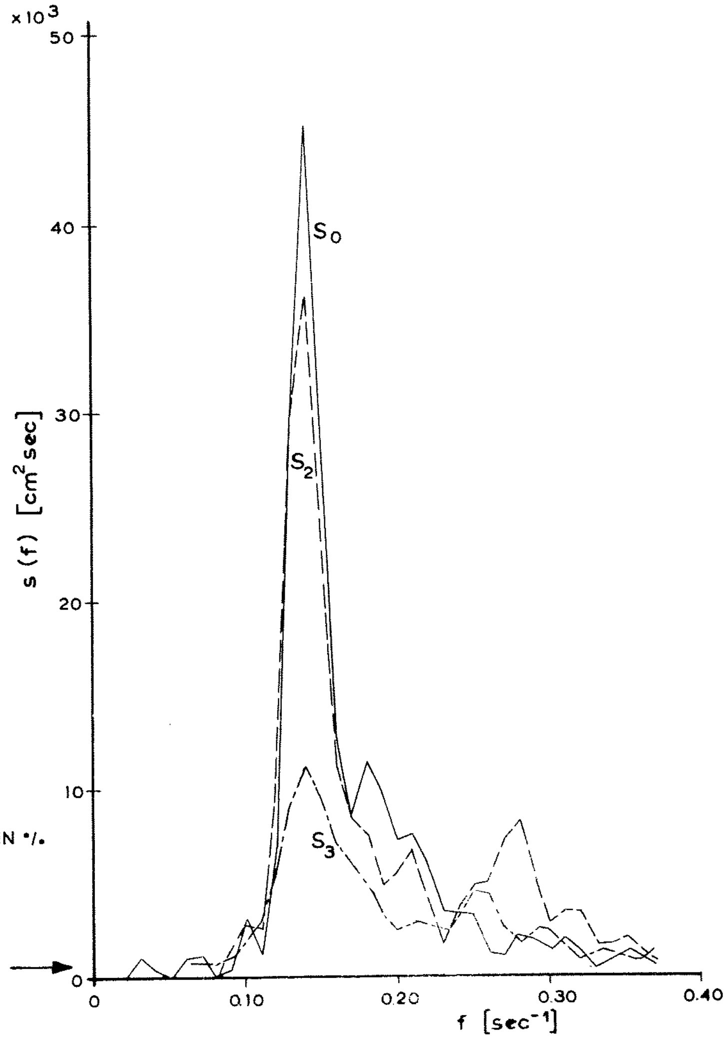


Fig. 3.2 EXAMPLES OF SIMULTANEOUS WAVE RECORDS IN SHALLOW WATER
A EXCEEDANCE PROBABILITY
B CORRESPONDING ENERGY DENSITY SPECTRA

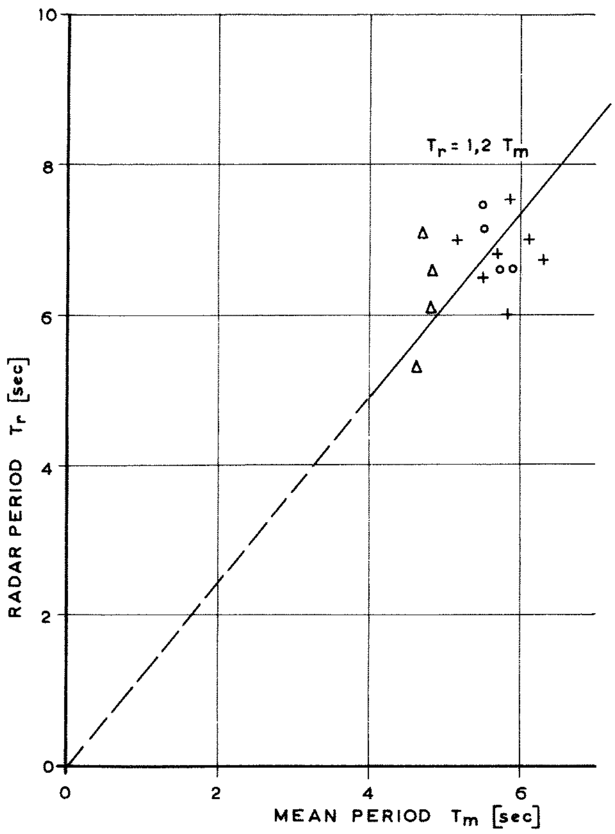


Fig. 3.3 COMPARISON BETWEEN „ RADAR PERIOD" T_r USED FOR REFRACTION COMPUTATIONS AND MEAN PERIOD T_m FROM TIME SERIES

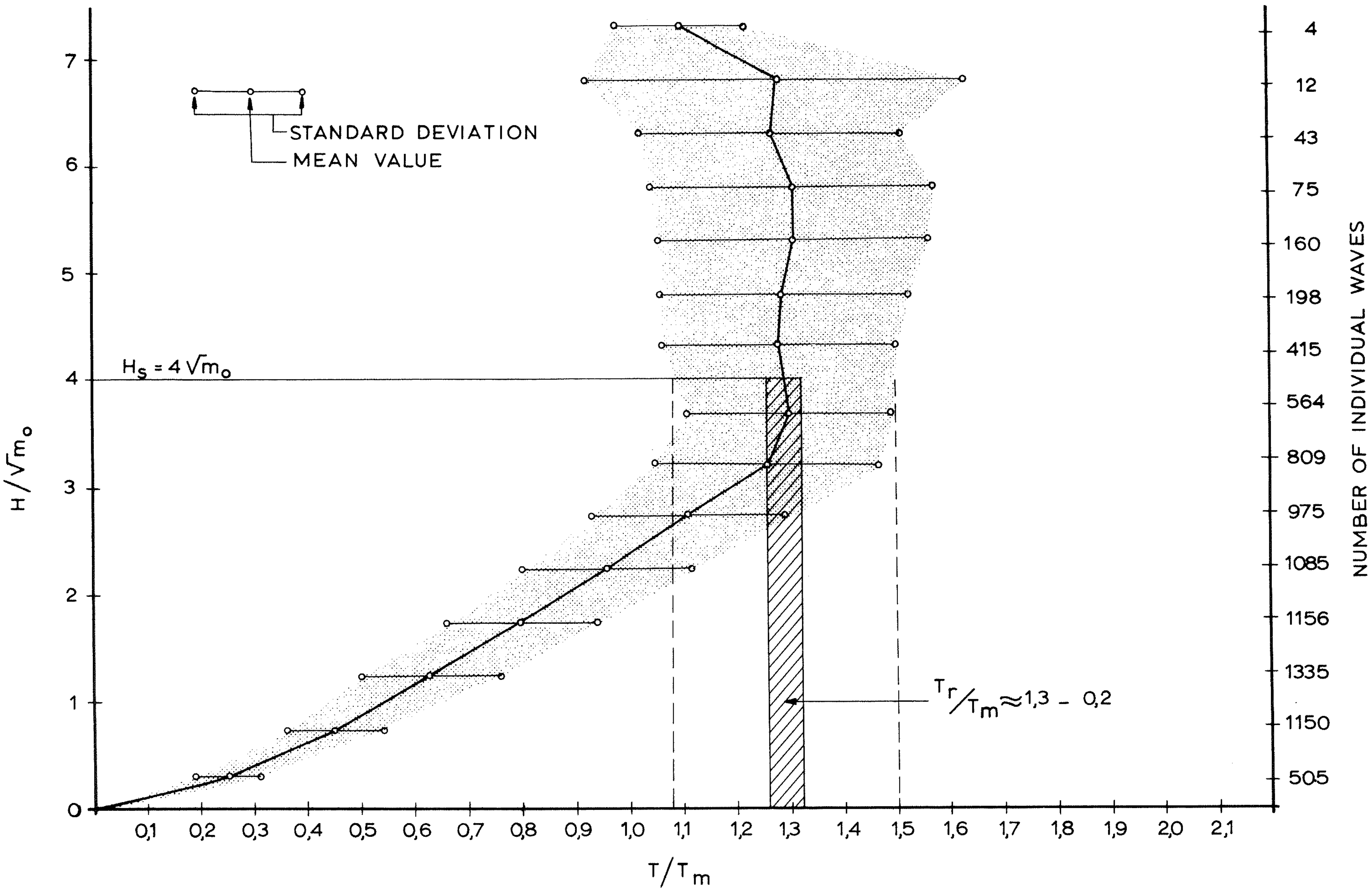
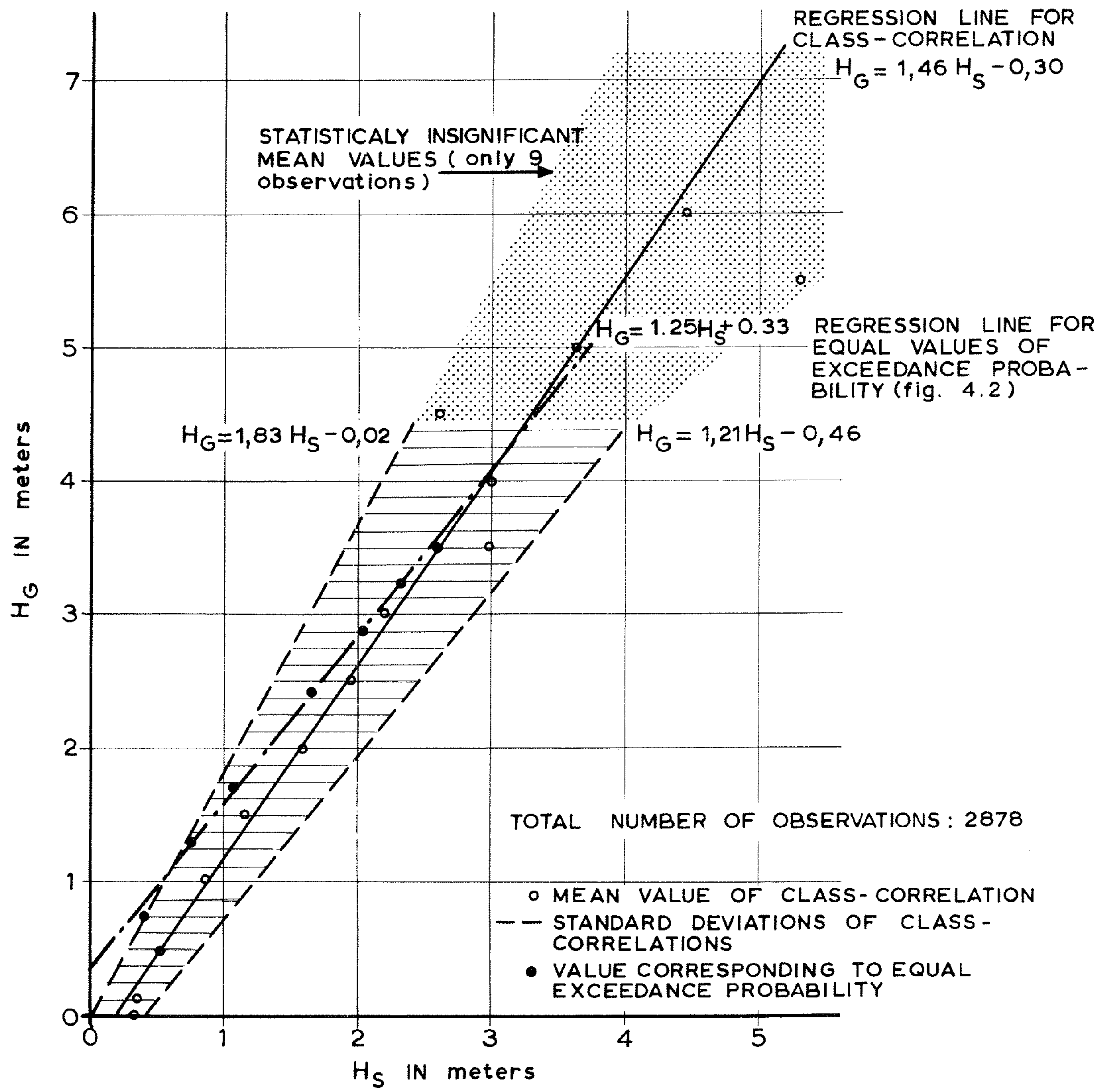


Fig. 3.4 CORRELATION BETWEEN THE HEIGHT H AND PERIOD T OF " INDIVIDUAL WAVES" ON THE NORTH SEA



* Fig. 4.1 CORRELATION BETWEEN VISUAL OBSERVATIONS OF WAVE HEIGHTS H_G (LIGHT VESSEL „GOEREE") GROUPED IN CLASSES OF 0,5 m AND SIGNIFICANT WAVE HEIGHTS H_S OBTAINED FROM RECORDS AT „TRITON" STATION

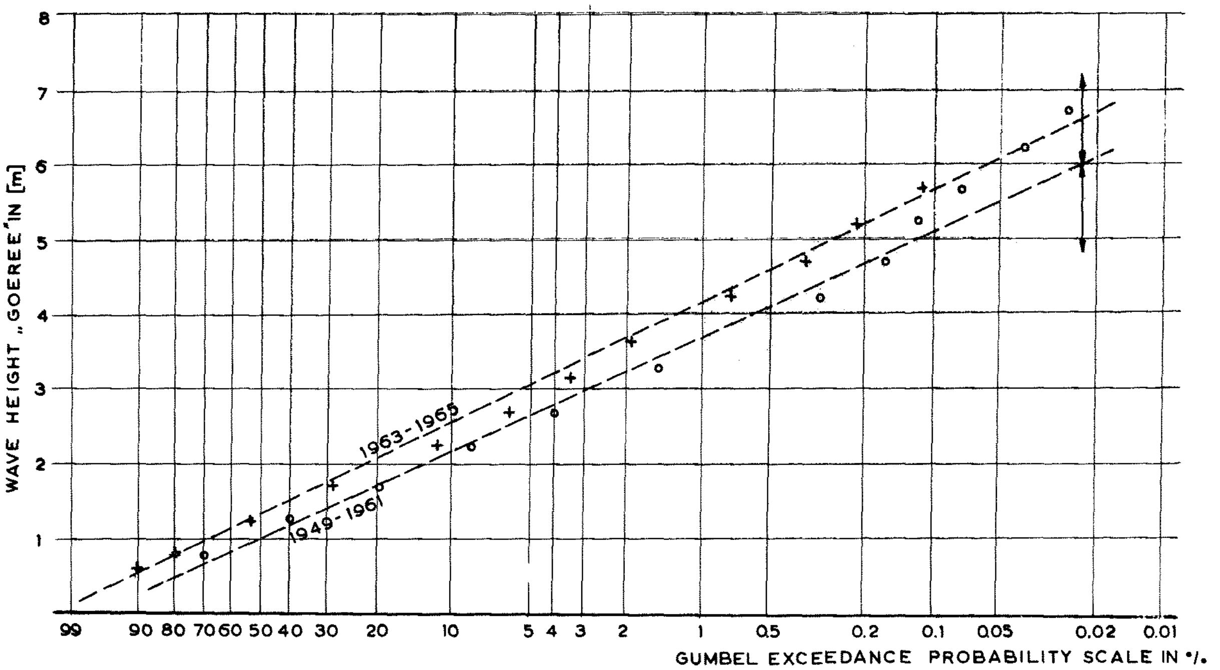
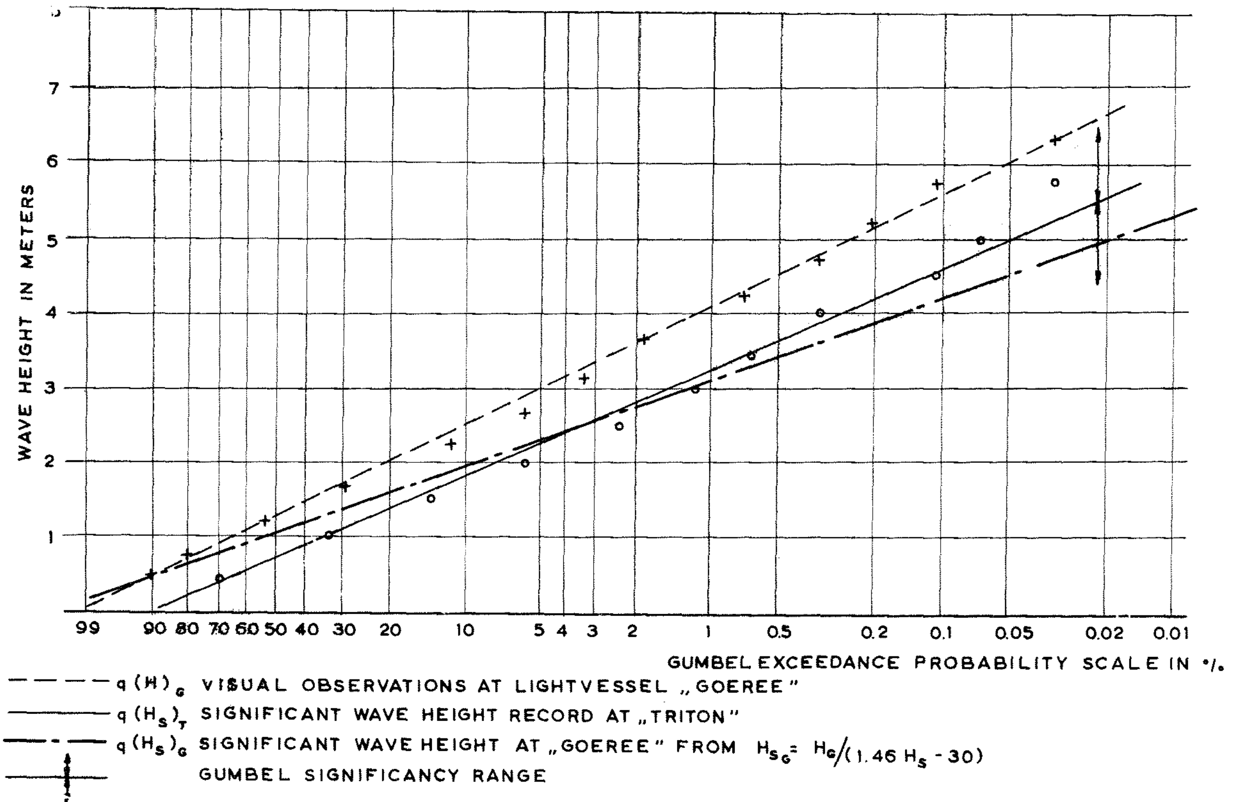
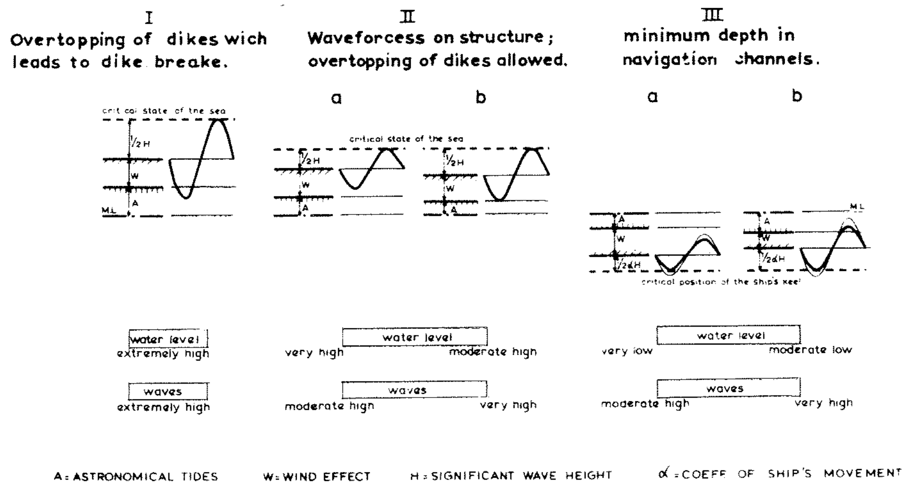


Fig. 4.2a COMPARISON BETWEEN TWO PERIODS OF VISUAL OBSERVATIONS AT LIGHTVESSEL GOEREE



* Fig. 4.2 b REDUCTION OF EXCEEDANCE PROBABILITY FROM CORRELATION SHOWN IN Fig 4.1.



* Fig 5.1 EXAMPLES OF CRITICAL CONDITIONS OF THE SEA – SURFACE

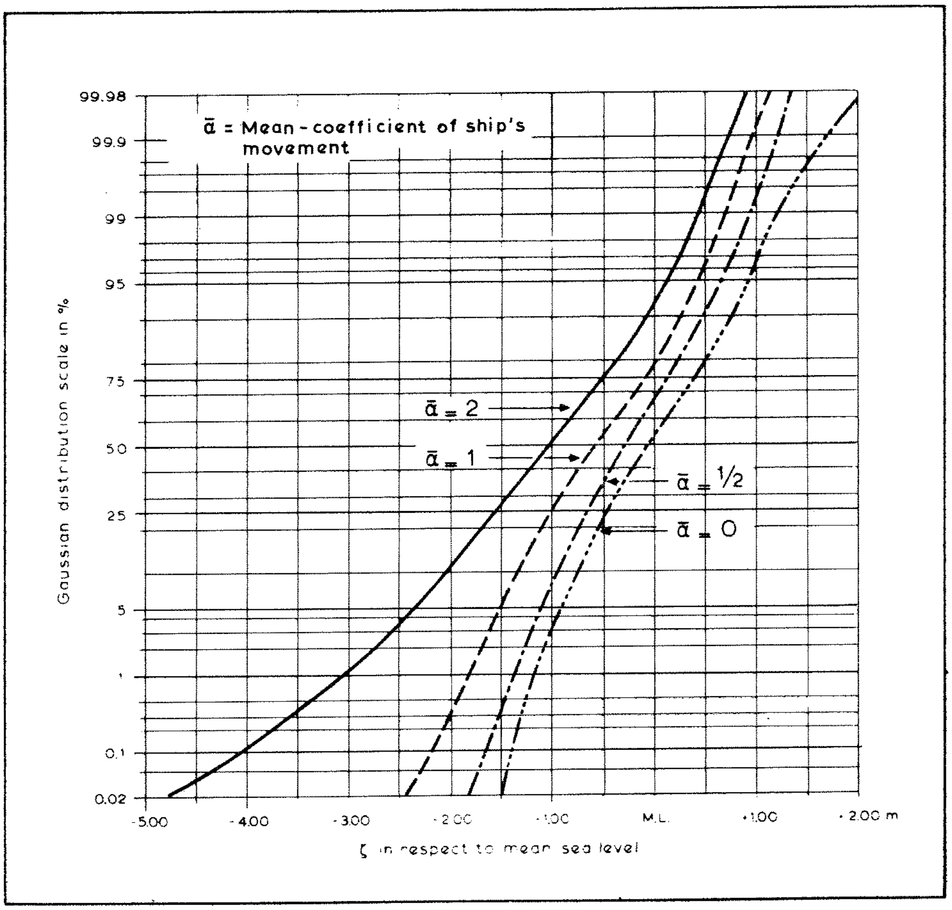


Fig. 5.2 RESULTING CUMULATIVE PROBABILITY DISTRIBUTIONS OF THE LOWEST POSITION ζ OF SHIP'S KEEL IN RESPECT TO MEAN SEA LEVEL M.L.

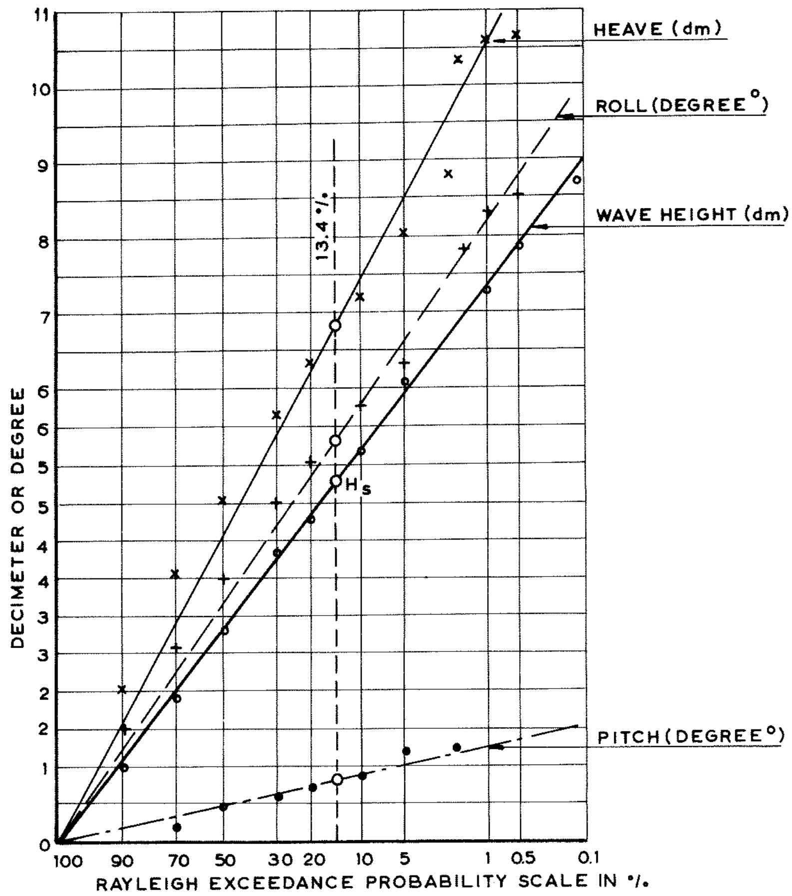


Fig. 5.3 COMPARISON BETWEEN SHIPS MOVEMENT AND RECORDED WAVES

REFRACTIEPATTERN FOR BROUWERSHAVENSE GAT

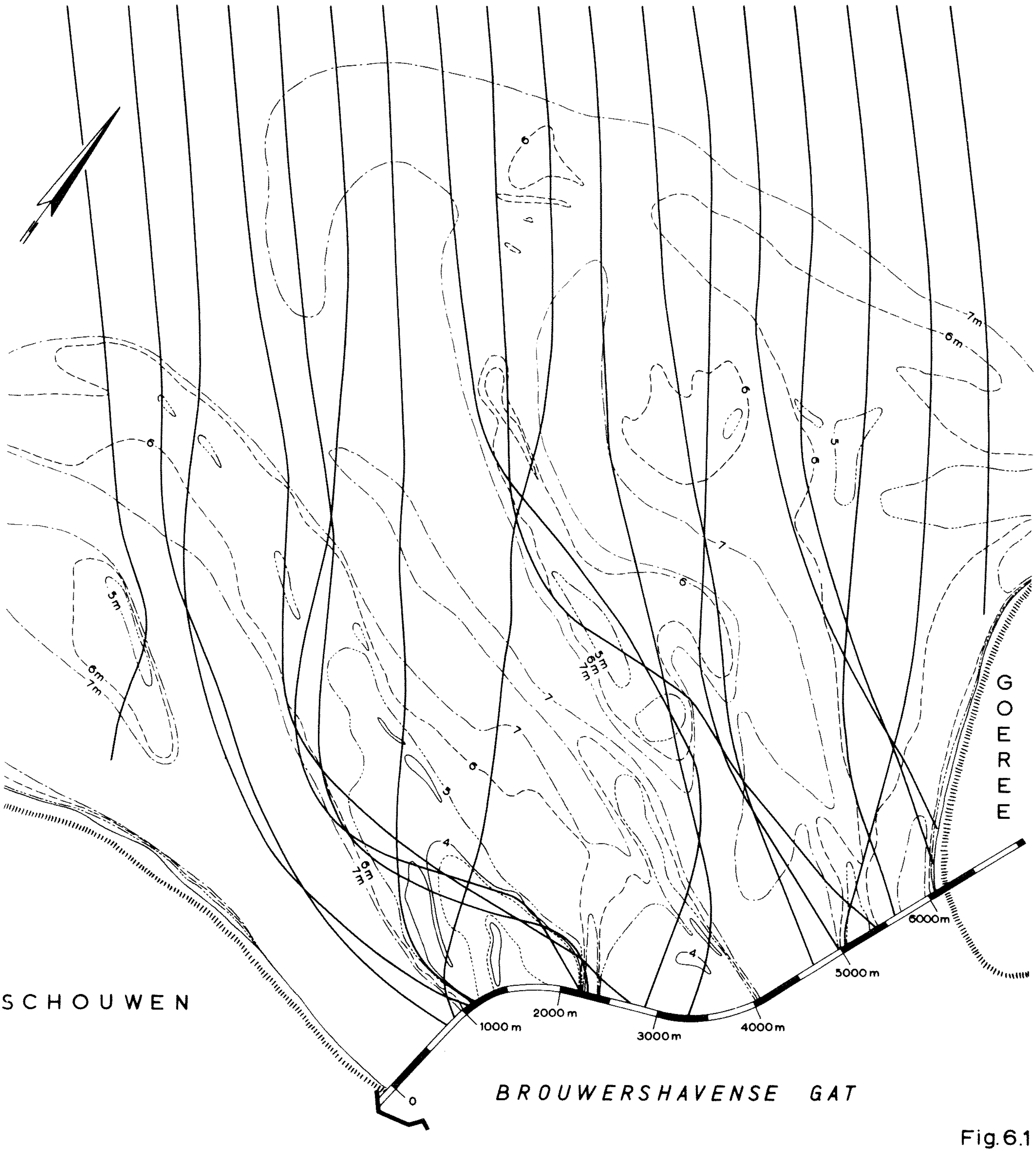
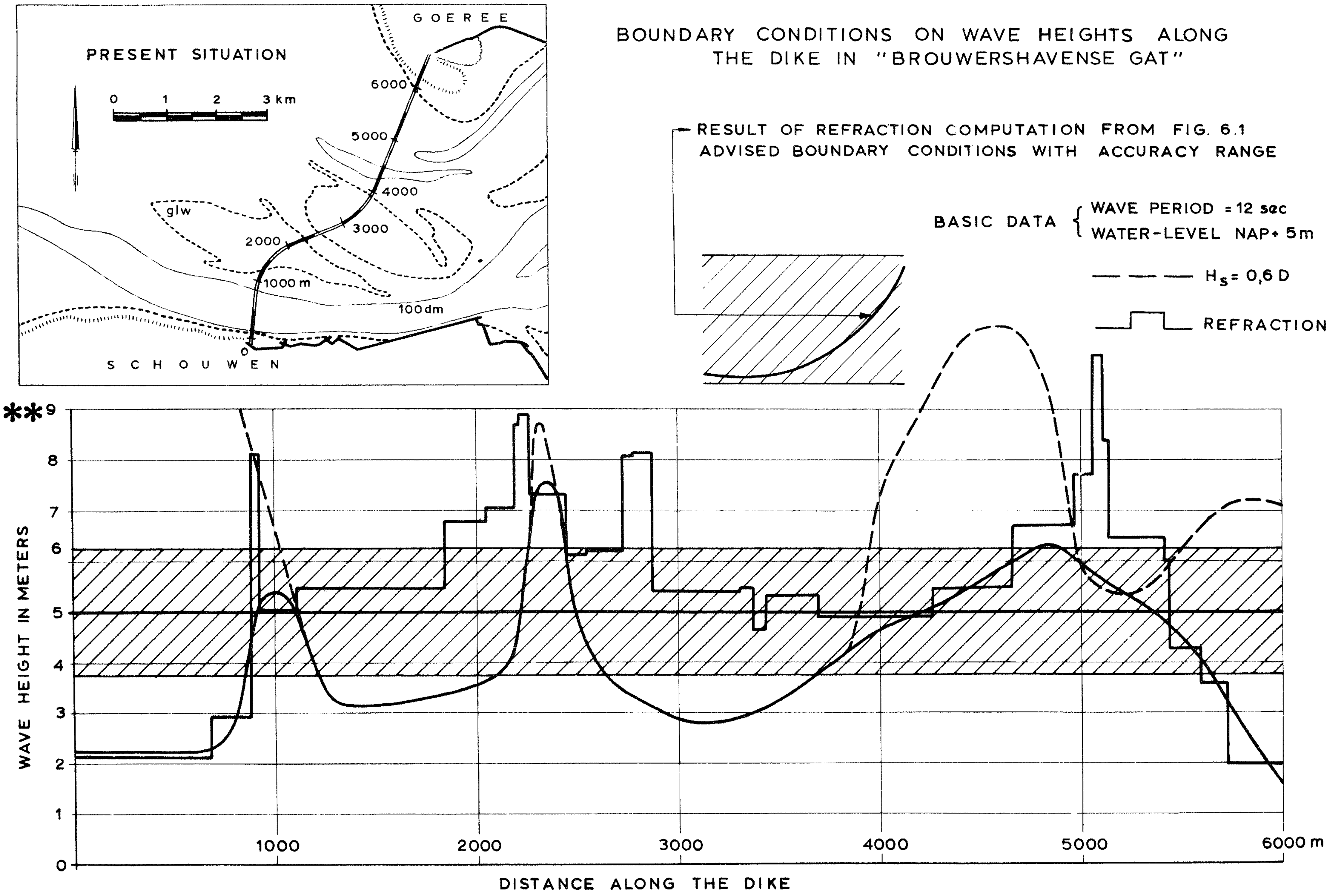


Fig. 6.1



BOUNDARY CONDITIONS ON WAVE HEIGHTS ALONG THE DIKE IN "BROUWERSHAVENSE GAT"
PRESENT SITUATION
0 1 2 3 km
GOEREE
SCHOUWEN
glw
100dm
0
1000 m
2000
3000
4000
5000
6000
RESULT OF REFRACTION COMPUTATION FROM FIG. 6.1
ADVISED BOUNDARY CONDITIONS WITH ACCURACY RANGE
BASIC DATA
WAVE PERIOD = 12 sec
WATER-LEVEL NAP+ 5m
H s= 0,6 D
REFRACTION
WAVE HEIGHT IN METERS
DISTANCE ALONG THE DIKE
0
1
2
3
4
5
6
7
8
**9
1000
2000
3000
4000
5000
6000 m

Fig. 6.2

DISCUSSION ON PAPER 1

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It is pointed out by Svašek that in certain computations the energy of a random wave field, with a continuous spectrum, is lumped at a single frequency. This procedure is sometimes applied in problems of refraction, even though this is a frequency-dependent phenomenon for the waves being considered. However, neither the shortcomings nor the merits of the procedure will be discussed here. The purpose of this discussion is merely to provide a criterion which may be used in the determination of an "equivalent" monochromatic wave train, and to give expressions for the "equivalent" height and period resulting therefrom. The criterion which has been adopted here states that in deep water the monochromatic wave shall have the same average energy and energytransport as the random wave field. The term "equivalent" refers to this condition only and in no way implies the correctness of the simplifying procedure.

At first, waves which are long-crested in deep water will be considered. Let $S(\omega)$ be the energy spectrum, with moments

**

$$m_j = \int_0^\infty \omega^j S(\omega)\, d\omega \tag{1}$$

The energy content per unit area is $\rho g m_0$, and the equivalent height of the monochromatic wave is given by

$$\frac{1}{8} H_{eq}^2 = m_0 \tag{2}$$

The condition of equal energy transport may be written as

$$\frac{1}{8} H_{eq}^2\, c_{g_{eq}} = \int_0^\infty c_g(\omega)\, S(\omega)\, d\omega \tag{3}$$

or

$$c_{g_{eq}} = \frac{\int_0^\infty c_g(\omega)\, S(\omega) d\omega}{\int_0^\infty S(\omega) d\omega} \tag{4}$$

The equivalent group velocity appears as the average group velocity of the spectral components, weighted with the spectral density $S(\omega)$.

In deep water, $c_g = \frac{g}{2\omega} = gT/4\pi$. It then follows from (4) that the equivalent period is given by

$$T_{eq} = 2\pi \frac{m_{-1}}{m_0} \tag{5}$$

It is convenient to compare this value with T_m, the mean zero up- or down-crossing period:

$$T_m = 2\pi \sqrt{\frac{m_0}{m_2}} \qquad (6)$$

Thus

$$\frac{T_{eq}}{T_m} = \frac{m_{-1}\, m_2^{1/2}}{m_0^{3/2}} \qquad (7)$$

If

$$S(\omega) = a\omega^{-b} e^{-c\omega^{-d}} \qquad (8)$$

then

$$\frac{T_{eq}}{T_m} = \frac{\Gamma(\frac{1}{d})\, \Gamma^{1/2}(\frac{b-3}{d})}{\Gamma^{3/2}(\frac{b-1}{d})} \qquad (9)$$

Numerical values are given in Table 1 for a spectrum which is similar to a Neumann spectrum, a Bretschneider spectrum or a Pierson-Moskowitz spectrum.

	b	d	T_{eq}/T_m
Neumann	6	2	1.23
Bretschneider, Pierson-Moskowitz	5	4	1.20

Table 1

It is possible to carry out an analogous averaging procedure for waves which are short-crested in deep water. The group velocities in Eqs. (3) and (4) should then be treated as vectors. For a numerical estimate the energy is assumed to be confined to components travelling within ± 90° from the resultant direction $\alpha = 0°$, and to be distributed according to $\cos^n \alpha$. Eq. (7) is then replaced by

** $$\frac{T_{eq}}{T_m} = f(n) \frac{m_{-1}\, m_2^{1/2}}{m_0^{3/2}} \qquad (10)$$

in which

$$f(n) = \frac{\int_0^{\pi/2} \cos^{n+1}\alpha \, d\alpha}{\int_0^{\pi/2} \cos^{n}\alpha \, d\alpha} \qquad (11)$$

Common values of n range from 4 to 10, with a corresponding range of $f(n)$ from 0.91 to 0.96. Thus, if the averaging procedure is applied not only to the frequencies but also to the directions, the values of T_{eq}/T_m listed in Table 1 should typically be reduced by 5% to 10% approximately.

GENERATION OF IRREGULAR WAVES ON MODEL SCALES

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SUMMARY

Wind-wave flumes have been added to the laboratory facilities since a long time, in order to simulate natural wind waves. Most wind-wave flumes have been equipped in addition with mechanical (regular) wave generators to avoid extreme small model scales. For some time past a programmed (irregular) wave generator has been installed in one of the existing wind flumes of the Delft Hydraulics Laboratory.

After a historical review of the development of wave generating facilities a comparison has been made of the various kinds of wave generation. Moreover, some records of North Sea wave conditions have been added to the comparison.

1. INTRODUCTION

Wind-wave flumes have been applied since a long time in model experiments for several purposes. Regarding the investigations, three subjects of particular interest with respect to problems in marine engineering can be distinguished:

1. Properties of wind profiles.
2. Wave generation by wind.
3. Wave attack on maritime structures exposed to irregular waves.

Originally small flumes were used in which waves were generated by wind blowing over the water surface. The investigations mentioned sub 1 and to a less extent those sub 2 did not require a substantial development of the model facilities.
Fetch and depth in these flumes limit wave heights to a few centimetres and wave periods to far less than 1 sec. However, valuable investigations have been and still are being produced in these relatively short and shallow flumes, which have grown in number only.

For investigations of the third type, waves of considerably greater height and period are indispensable to perform the investigations on suitable scales with respect to Reynolds and Weber number.

This study deals mainly with wave generation for the purpose of investigations of the third type, though the same considerations are of importance to investigations into the the wind-wave interaction.

Three methods of generating irregular waves, namely,
- wind,
- wind + monochromatic-wave generator, and
- wind + programmed wave generator

have been subjected to a critical analysis and results have been compared with prototype data in Section 4 of this study, while the methods of analysis are discussed in Section 3.

2. HISTORICAL REVIEW

The first experiments with irregular waves in the Netherlands date back as far as 1920 when in a wind-tunnel a provisional arrangement was made to study wave run-up. After similar investigations in 1933, the construction of a special wind-wave flume was started at the Delft Hydraulics Laboratory and put into use in 1936.

The dimensions of this flume (length 25 m; width 4 m; maximum water depth 0.45 m) were unique at that time. However, as the length was not sufficient to meet the requirements of wave height and period, it was extended to a length of 50 m in 1941 and equipped with a monochromatic-wave generator.

Investigations into wave run-up, wave overtopping, stability of rubble-mound breakwaters, wave impact forces and stability of floating structures have been successfully performed in this flume (Ref. 1, 2 and 3). Similar investigations in other Institutes confirmed the importance of the application of irregular waves (Ref. 4, 5 and 6).

It is interesting to notice that model investigations into wave generation, carried out during World War II, yielded good agreement with prototype data collected by Sverdrup and Munk (Ref. 7 and 8).

The interest in model experiments applying irregular waves was growing so fast that the Delft Hydraulics Laboratory decided to establish another wind flume in 1957 at "De Voorst". The length of this flume was 100 m, the width 4 m, and the maximum water depth 0.8 m. This flume was also equipped with facilities to generate waves, either by wind only or by a combination of wind and a mechanical (regular) wave generator.

The growing interest in irregular wave phenomena also resulted in an increasing number of observations in nature, and simultaniously forced the evaluation of elaborate statistical analysis conceiving the wave motion as a stochastical process. Application of the mathematical techniques to both model waves and prototype data have shown unacceptable discrepancies. Since 1962, therefore, the Delft Hydraulics Laboratory has been working on a system of wave generation which yields still a more realistic reproduction of natural wave conditions. A prototype of the installation has been installed in the existing wind flume at Delft. The installation comprises a wave board driven by a hydraulic servo system and generating waves according to an arbitrary programme. A similar installation, based upon this concept, has been realized at the River and Harbour Research Laboratory at Trondheim (Norway).

The new wind flumes, recently completed at Delft have also been equipped with programmed wave generators.

3. STATISTICAL ANALYSIS OF THE WAVE RECORDS

Descriptions and definitions will now be given of the different wave characteristics involded in the analysis of prototype as well as model records.

The characteristics applied in this study have by no means the pretension of giving a complete and satisfactory description. Moreover, the import of several of them is not clear in all respects. Their actual choice, however, is considered to be justified by the present state of research.

The individual wave heights and wave periods have been defined by the "zero-crossing method", in which each crossing of the surface elevation record $\eta(t)$ by the mean water level is termed a zero - crossing. Accordingly, the wave crest and wave trough are respectively the maximum (positive) and minimum (negative) value of $\eta(t)$ between two successive zero-crossings. The wave height H is termed the difference between the elevation of a wave trough and the next wave crest, and the wave period T: the time-lag between two zero - down crossings.

The zero - crossings method neglects the existence of positive minima or negative maxima. Also the Rayleigh distribution of wave heights (1) (presuming an energy spectrum with an infinitive small band width), similarly excludes the existence of negative maxima or positive minima (Ref. 9).

$$p(H) = H/4\sigma^2 . e^{-H^2/8\sigma^2} \qquad (1)$$

** σ is the standard deviation of the wave record, and by definition also equal to the square root of the total (imaginary and real) area of the energy spectrum. From the cumulative frequency distributions of H and T the values have been determined, exceeded by 2,15 and 50 percent of the waves respectively. They have been demoted by the subscript 2, 15 or 50. Also the significant wave height Hs (average of the highest one-third waves of the record) has been calculated.

Apart from the statistical distributions of H and T, statistical distributions of the extreme values of $\frac{d\eta}{dt}$, separately for the lee-side and the windward-side of each wave, have been determined. $\frac{d\eta}{dt}$ has been approximated by $\frac{\eta(t) - \eta(t+\Delta t)}{\Delta t}$. Cumulative frequency distributions of $\frac{d\hat{\eta}}{dt}$, separately for the lee-side, $(\frac{d\hat{\eta}}{dt})^-$, and the windward-side, $(\frac{d\hat{\eta}}{dt})^+$, of the waves, give some information about the asymmetry. Accordingly the ratio of absolute $\frac{d\hat{\eta}}{dt}$ values at 15 % of exceedance

$$\frac{(\frac{d\hat{\eta}}{dt})_{+15}}{(\frac{d\hat{\eta}}{dt})_{-15}}$$

has been adopted as a "ratio of asymmetry" Δ of the waves.

Though very small, the correlation between H and T seems to be not always zero and consequently wave heights and wave periods may not be considered as stochastical uncorrelated variables.

The relation between wave heights and wave periods has been expressed in H - T correlation curves, indicating the mean height and the standard deviation of waves having periods within a distinct period interval.

Besides the statistical characteristics just described, the energy spectrum of the waves has been computed for all cases. In accordance with the more or less standardized procedure as proposed, for instance, by Blackman and Tukey (Ref. 10), the auto-correlation function $R(\tau)$ of the wave record and subsequently the Fourier transform of the correlation function have been computed, and conform to the expressions (2) and (3),

** $$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{1}{2}T}^{\frac{1}{2}T} \eta(t) . \eta(t+\tau) . dt \qquad (2)$$

$$S(f) = \int_{-\infty}^{+\infty} R(\tau) . e^{2\pi i f \tau} . d\tau = 2 \int_{0}^{+\infty} R(\tau) . \cos 2\pi f \tau . d\tau \qquad (3)$$

$S(f)$ is the spectral density at the frequency f (cps).

The width of the spectrum is expressed by the parameter ε

$$\varepsilon^2 = \frac{m_o m_4 - m_2^2}{m_o m_4} \qquad (4)$$

in which $m_n = \int_{0}^{+\infty} S(f) . f^n . df \qquad (5)$

For n = 0 one obtains the total (imaginary and real) area of the spectrum m_o which follows by definition also from (2) for $\tau = 0$. (Though the real area of the spectrum is only $\frac{1}{2} m_o$, as can be derived from (2) and (3), this factor $\frac{1}{2}$ is usually neglected in practice).

Though in principle not a statistical parameter, in practical computations the wave spectrum has a statistical nature too, because under the assumption of stationarity and ergodicity a finite record of the wave motion is taken, which yields an estimate of the spectral density distribution only.

All computations have been performed on a digital computer, for which the continuous wave records have been converted to digitized punch - tape records. The sampling frequency of the prototype records was 5 cps and of the model records 32 cps (in conformity with the adapted length scale 1 : 45 and time scale 1 : $\sqrt{45}$). The length of each "time series" was at least 15.000 samples for the records of the model wind waves, and at least 30.000 samples for all other records.

The statistical distributions of wave heights, periods and slopes $(\frac{d\hat{\eta}}{dt})$ as well as the H - T correlations have been determined directly from the obtained time series. Regarding the spectral analysis, only one of every six samples in the series has been taken in order to get a sufficiently high frequency resolution. As a result the Nyquist frequency $(\frac{1}{2\Delta t})$ became 2.666 cps and the frequency lag between two adjacent spectral estimates $(\frac{1}{2\ m\ \Delta t})$ 0.0444 cps. In order to "smooth" the spectrum, the correlation function has been filtered by a triangular-screen filter.

** Inherent to the definition of ε, the higher frequencies have an disproportionately large influence on the calculated value of ε, whereas both the accuracy and the practical interest of the spectrum in this frequency range is small. Therefore, the part of the spectrum at the high frequency side, which contributes 2% to the total area of the spectrum, has been discarded in the calculation of ε, and ε calculated in this way has been denoted ε_2.

4. METHODS OF GENERATION

Three methods of wave generation have been compared on the basis of the parameters indicated in Section 3, and the results for each of the methods will be discussed separately. They have been compared with actually measured North Sea records and theoretical work of Cartwright and Longuet - Higgens (Ref. 9) as a reference.

4.1 Prototype records

Prototype records from the North Sea have been made availably by the Rijkswaterstaat. They have been recorded at the platform Triton, situated approximately 4 km off the Netherlands coast near Kijkduin, at a depth of 18 metres below M.S.L.

The wave height distributions, spectra and H - T correlations are shown in Fig. 6. Other parameters have been compiled in Table 1, together with data based upon Ref. 9.

4.2 Model records, wind only

Special measurements have been made in the wind-wave flume of the Delft Hydraulics Laboratory at "De Voorst". The fetch during these tests was 100 m, and water depths were 0.4 and 0.67 m. The average wind speeds $\overline{w}$ ranged from 6.6 to 22.5 m/sec. Wind profiles have been measured and related to the average wind speed (Figs. 1 a and b). Note: $\overline{w} = w_{0.3}$ to $w_{0.4}$. (Wind speed measured at a height of 0.3 to 0.4 m above M.S.L.). Additionally, measurements of Colonell and Prins have been used. (Ref. 3 and 11).

It appears that the significant wave height Hs is increasing only slowly with the fetch for fetches greater than circa 100 m (Fig. 2). The same holds for the wave period (Fig. 3).

Apart from the absolute magnitude of height and period, the statistical distributions of these quantities show a rather limited variation, especially for higher wind speeds (See Table 2). Hence the spectra (Fig. 4 a) are very narrow, which is also illustrated by the small values of ε ($\varepsilon_2 \approx 0.5$) and of T peak/T 50 in Table 2.
As the wind speeds in these tests have been exaggerated-in this sense that $\frac{gF}{w^2}$ is very small-to attain reasonable wave heights, the asymmetry of the waves can be expected to be high. This is confirmed by the tests, where Δ is found to be 1.15 to 1.77, whereas the prototype values varied from 0.99 to 1.15.

The H - T relation is given in Fig. 4 c, which shows both the average H - T relation of the six test runs and the average standard deviation with respect to the mean values.

4.3 Model records, wind strengthened swell

Measurements have been made in the same wind-wave flume as the tests mentioned in 4.2. The regular swell has been generated by a wave board situated at the beginning of the flume, and the period and height of the swell has been kept constant during each test run. Water depths were 0.4 and 0.67 m, and the periods of the swell 1.5 and 1.8 sec. respectively. Wave heights of the swell ranged from 5 to 15 cm. The average wind speeds varied between 6.6 and 12.8 m/sec. Wind profiles have not been measured separately.

The results of the tests have been summarized in Table 3 and Figure 5. It is evident that the spectra show a sharp and dominating peak at the frequency of the swell. The ε-values, however, do not indicate the narrow band width of the main part of the spectrum due to the second (wind) peak. Wave height distribution and period distribution are still less satisfactory then those of wind-generated waves. It is interesting to notice that also in this case the ratio of asymmetry Δ, for the higher wind speeds, is considerably larger than the average value of the prototype waves.

The H - T relation shown in Fig. 5 c again presents the average wave height and average standard deviation in the corresponding period intervals, of all five tests. The regular character of this type of wave motion is accentuated by the very small standard deviation for periods exceeding 0.8 T_{15}.

4.4 Programmed wave generator

The purpose of the wave generator is to reproduce wide spectrum ocean waves as closely as possible. To achieve this, the wave board is driven by two separate hydraulic actuators so as to permit both translatory and rotational movements. Each actuator is controlled by two servo valves. The most attractive way to simulate ocean waves is to have an actual prototype record reproduced. The surface elevation record, however, has to be transferred into a command signal for the actuators, i.e., the horizontal movement of the wave board. Such a transfer function has been calculated by Biesel (Ref. 12) for monochromatic waves. The transfer function is shown in Figure 11, and presents the required stroke of the wave board at the water surface and near the bottom as a function of wave height, wave period and water depth. Though the theory of Biesel has been derived for monochromatic waves only, test results show that the method is also applicable for irregular waves.

Actual wave records can be used as an input signal to the programmed wave generator by means of a punch-tape. An electric network reproduces the transfer function of Fig. 11 and supplies separate command signals for both actuators.

If no prototype record is available, a wave record is simulated by using a random-noise generator, the random noise being filtered by a set of analogue second order filters. As both the resonance frequency and the damping of these filters are variable, the noise can be transformed into a signal having any arbitrary spectrum.
The output signal of the filter unit is fed into the transfer network in the same way as the puch-tape record. An outline of this system is presented in Fig. 12.

The tests referred to in this paragraph have been carried out in the old wind flume of the Laboratory at Delft. The length of this flume is 50 m and the water depth was 0.4 m. The prototype records mentioned in 4.1 have been reproduced on a length scale of 1 : 45.

The time and velocity scales were consequently 1: $\sqrt{45}$. Average wind speeds during the tests ranged from 0 to 5.2 m/s; Wind profiles were not measured separately.

The results of the tests have been tabulated in Table 4 and are presented in Figs. 7, 8, 9 and 10.

It is clear that wave height and period distributions as well as the Δ-values are in very good agreement with the corresponding prototype data, both for the "punch-tape method" and the "random-noise method". The wave energy spectra also show satisfactory agreement with prototype data, though the energy density in the model is slightly lower in the high frequency range. In the meantime, this has been corrected by a modification of the electronic design.

The influence of wind speeds has been investigated extensively for run ST III. From Table 4 it appears that a variation of the average wind speed from 0 to 5.2 m/sec has no perceptible consequences for the statistical characteristics determined. As a result a small over- or underestimation of the wind speed in the model seems to have no consequences for the reproduction of the natural wave conditions. However, still insufficient information is available regarding the mechanism of the wind stress on the water surface and the shape of waves in exceptional conditions as to draw definite conclusions in respect to this.

The question might arise whether the application of programmed waves is worthwhile. In this respect attention is called to Refs. 6 and 13 where considerable influence of wave irregularity has been shown for wave run-up on smooth slopes and the stability of rubble-mound breakwaters.

5. SUMMARY AND CONCLUSIONS

Three methods of wave generation have been compared with prototype data on the basis of a number of statistical characteristics.

It has been shown that generating irregular waves by wind only has a serious drawback as flume lengths have to be very long. To limit the flume length, either wind speeds are increased and thus exaggerated with respect to the model scale, or swell is generated mechanically in addition.

Both methods affect the desired frequency distribution of wave heights and periods, and lead to relatively narrow energy spectra. The steepness of the wave fronts seems also to be rather high in comparison with prototype data.

An alternative method is found in the application of a hydraulic servo system, the programmed wave genrator. With this system it appears to be possible to reproduce actual prototype records or to simulate these records on the basis of their energy spectrum.

Also in the latter case it has been found that the frequency distributions of wave heights and periods are in good agreement with data obtained from prototype or theory. As to the shape of the wave, an attempt has been made to evaluate a parameter describing the asymmetry of the wave, and it appears that the application of wind is important in this respect.

During the evaluation of statistical parameters for the comparison of wave records it became clear that hardly any data are available on the detailed shape of prototype waves. Further research on this subject seems to be necessary as, for instance, the steepness of wave fronts is considered to be of great importance for the occurrence of impact forces. The use of the spectral width parameter ε has proved to be hazardous in some case, especially when double-peaked spectra occur.

TABLE 1. Prototype Records.

No. of record	ST III	165	224	Ref.
Date of record	30 XI 65	4 XII 64	14 II 65	
H_{15} (m)	3.85	3.95	3.15	-
H_{15}/H_{50}	1.64	1.68	1.57	1.62
H_2/H_{50}	2.30	2.34	2.23	2.37
T_{15} (sec)	10.3	10.7	9.1	-
T_{15}/T_{50}	1.46	1.49	1.48	-
T_2/T_{50}	1.67	1.94	1.98	-
T_{peak}/T_{50}	1.31	1.50	1.39	-
m_o (m^2)	1.03	1.06	0.73	-
$Hs/\sqrt{m_o}$	3.89	3.91	3.80	4.0
ε_2	0.64	0.65	0.64	-
Δ	1.135	0.99	1.15	-

TABLE 2. Wind-generated Waves, Fetch 100 m.

Test Run	T2 w	T3 w	T5 w	T6 w	T7 w	T8 w
$\overline{w}$ (m/sec)	6.6	12.8	16.5	16.3	6.6	22.5
d (m)	0.4	0.4	0.67	0.67	0.67	0.67
H_{15}(cm)	4.6	13.4	18.1	16.4	6.2	20.6
H_{15}/H_{50}	1.64	1.36	1.35	1.35	1.44	1.35
H_2/H_{50}	2.13	1.65	1.59	1.79	1.80	1.65
T_{15}(sec)	0.86	1.30	1.28	1.27	1.13	1.45
T_{15}/T_{50}	1.16	1.12	1.13	1.14	1.11	1.13
T_2/T_{50}	1.32	1.31	1.24	1.30	1.29	1.28
T_{peak}/T_{50}	1.05	1.01	0.98	1.03	0.99	1.00
m_o (cm^2)	1.39	12.9	22.8	19.6	2.5	30.0
$Hs/\sqrt{m_o}$	4.01	3.79	3.80	3.83	3.90	3.81
ε_2	0.50	0.51	0.49	0.50	0.43	0.56
Δ	1.15	1.22	1.38	1.28	1.23	1.77

TABLE 3. Wind-strengthened Swell, Fetch 100 m.

Test run	T.4	T.8	T.12	T.14	T.18
H_{swell} (cm)	5.0	10.0	7.5	7.5	15.0
T_{swell} (sec)	1.5	1.5	1.8	1.8	1.8
$\overline{w}$ (m/sec)	12.8	6.5	6.6	11.4	11.4
d (m)	0.4	0.4	0.67	0.67	0.67
H_{15}(cm)	13.4	13.4	10.2	17.3	21.2
H_{15}/H_{50}	1.31	1.08	1.16	1.34	1.10
H_2/H_{50}	1.67	1.16	1.27	1.61	1.22
T_{15} (sec)	1.78	1.70	1.94	1.89	1.96
T_{15}/T_{50}	1.25	1.07	1.04	1.66	1.04
T_2/T_{50}	1.37	1.10	1.08	1.80	1.11
T_{peak}/T_{50}	1.06	0.95	0.97	1.70	0.96
m_o (cm^2)	12.2	17.5	8.1	19.8	37.5
$Hs/\sqrt{m_o}$	3.92	3.24	3.58	3.95	3.55
ε_2	0.61	0.56	0.75	0.62	0.60
Δ	1.11	1.00	1.01	1.19	1.28

TABLE 4. Programmed Wave Generator.

Test run	ST III A	ST III B	ST III C	ST III R	224 P	165 P	165 R
Origin Input signal	punch-tape	punch-tape	punch-tape	random-noise	punch-tape	punch-tape	random-noise
$\overline{w}$ (m/sec)	3.0	0	5.2	3.0	3.0	3.0	3.0
H_{15} (cm)	8.6	8.4	8.9	8.5	7.3	8.3	8.5
H_{15}/H_{50}	1.62	1.64	1.59	1.63	1.62	1.57	1.54
H_2/H_{50}	2.17	2.18	2.14	2.30	2.16	2.15	2.00
T_{15} (sec)	1.57	1.56	1.52	1.60	1.47	1.76	1.81
T_{15}/T_{50}	1.27	1.27	1.32	1.28	1.36	1.46	1.39
T_2/T_{50}	1.52	1.55	1.56	1.61	1.80	1.87	1.78
T_{peak}/T_{50}	1.16	1.17	1.25	1.17	1.18	1.81	1.27
m_o(cm^2)	4.9	4.7	5.6	5.0	3.7	5.1	4.9
$Hs/\sqrt{m_o}$	3.92	3.91	3.88	3.89	3.86	3.80	3.90
ε_2	0.52	0.52	0.54	0.54	0.54	0.59	0.61
Δ	1.04	1.015	1.04	1.10	1.05	1.075	1.09

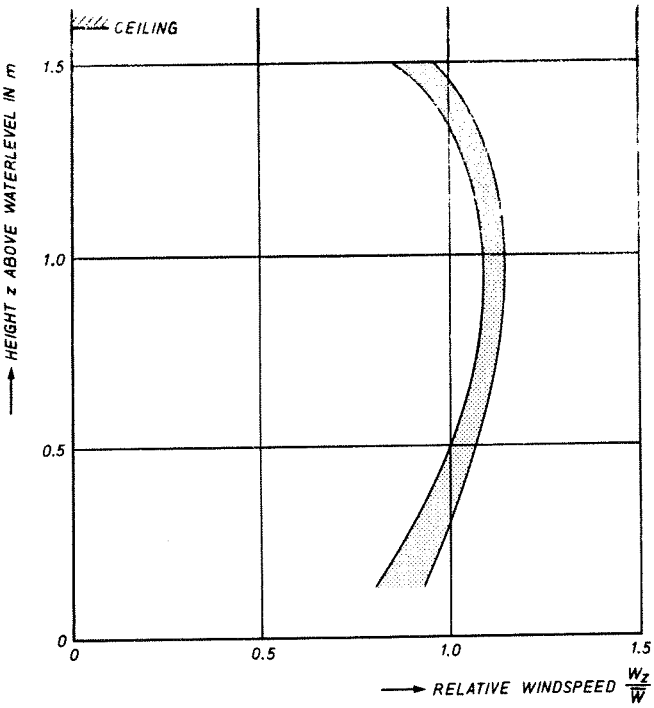

FIG. 1a RELATIVE WINDSPEED, WATERDEPTH 0.4 m

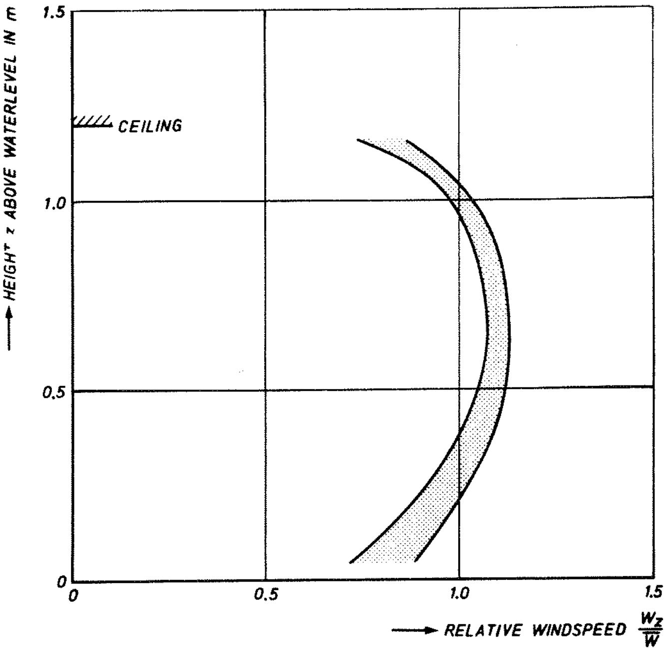

FIG. 1b RELATIVE WINDSPEED, WATERDEPTH 0.8 m

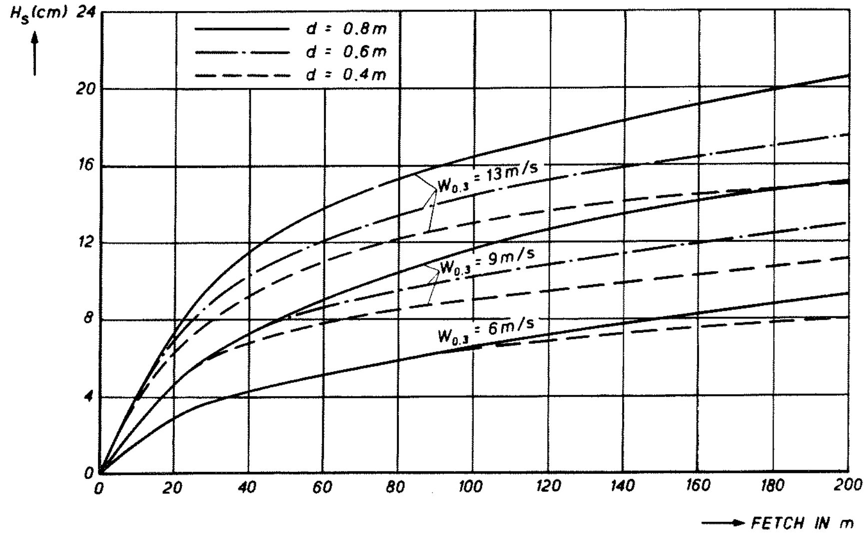

FIG. 2 SIGNIFICANT WAVE HEIGHT VERSUS FETCH

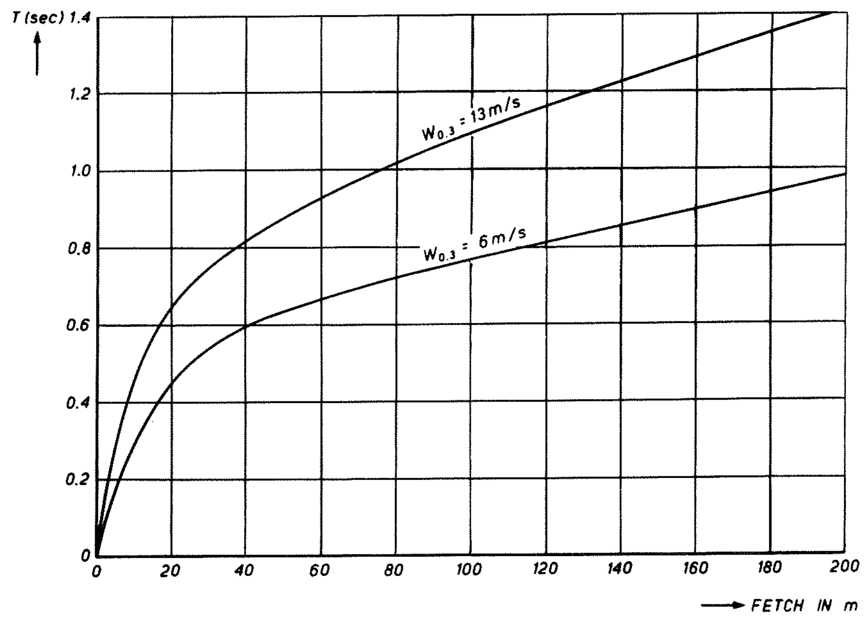

FIG. 3 WAVE PERIOD VERSUS FETCH

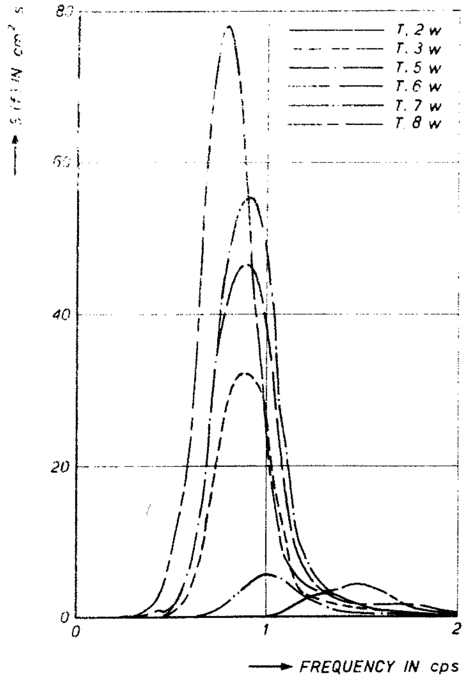

FIG. 4a WAVE-ENERGY SPECTRA, WIND GENERATED WAVES

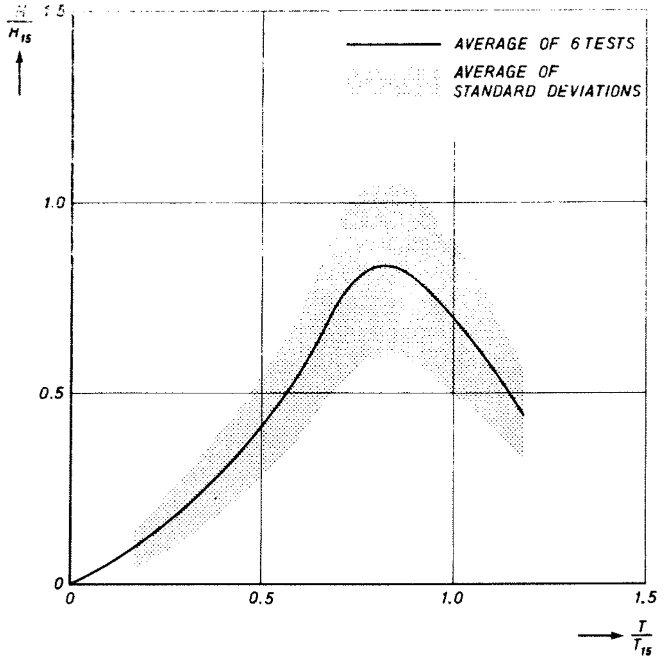

FIG. 4c H-T RELATION, WIND GENERATED WAVES

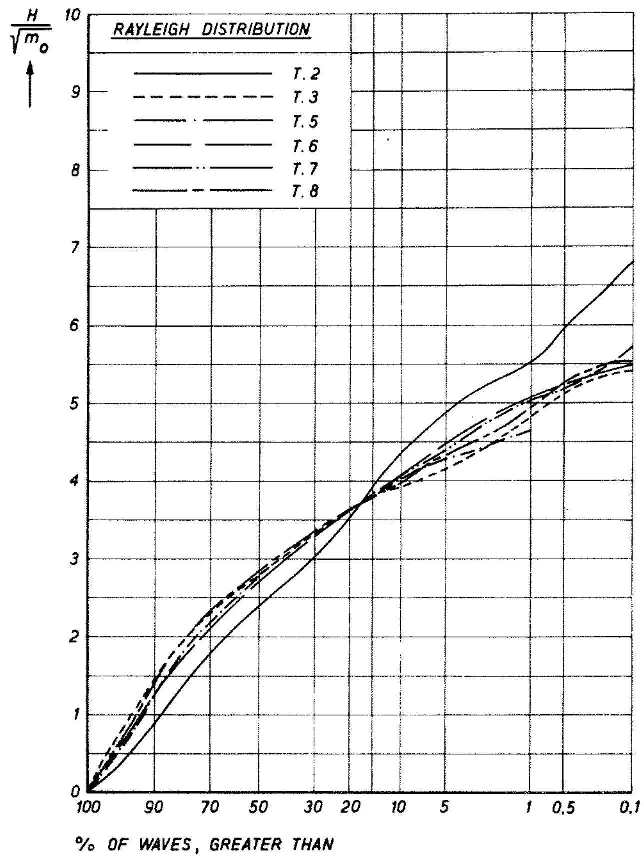

FIG. 4b WAVE HEIGHT DISTRIBUTIONS WIND GENERATED WAVES

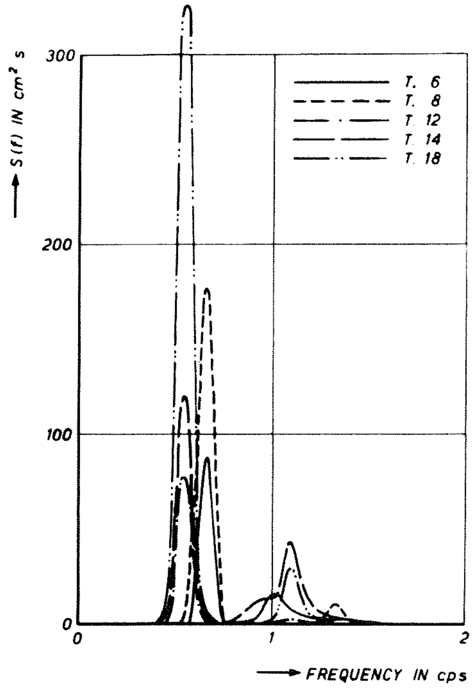

FIG. 5a WAVE-ENERGY SPECTRA, WIND STRENGTHENED SWELL

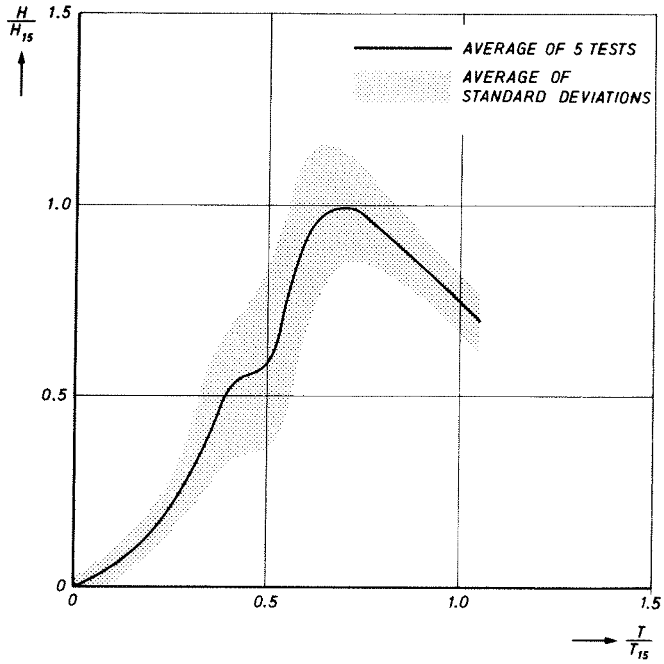

FIG. 5c H-T RELATION, WIND STRENGTHENED SWELL

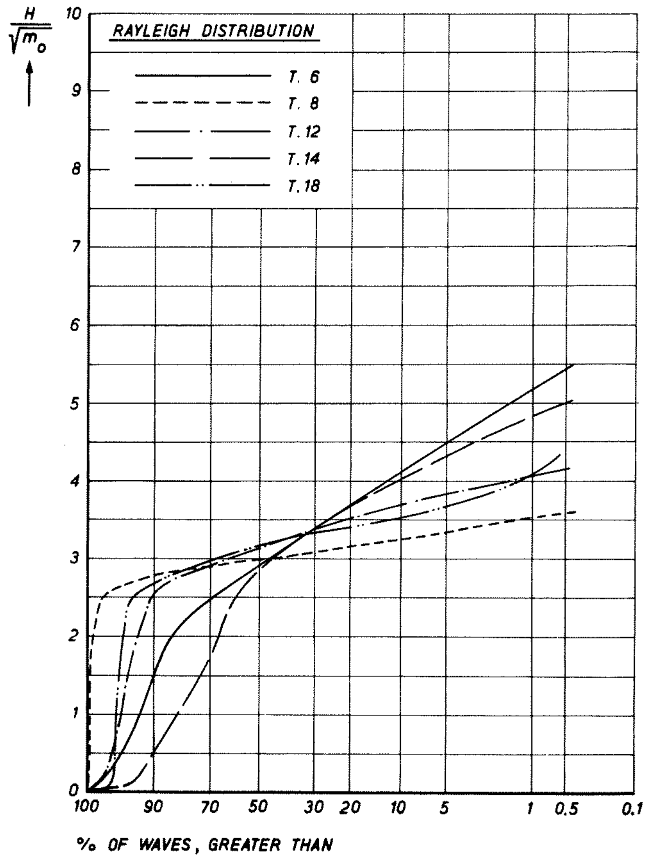

FIG. 5b WAVE HEIGHT DISTRIBUTIONS WIND STRENGTHENED SWELL

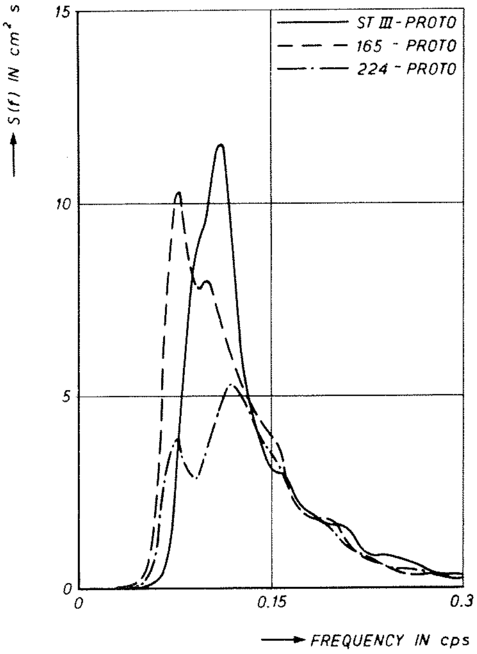

FIG. 6a WAVE-ENERGY SPECTRA PROTOTYPE WAVES

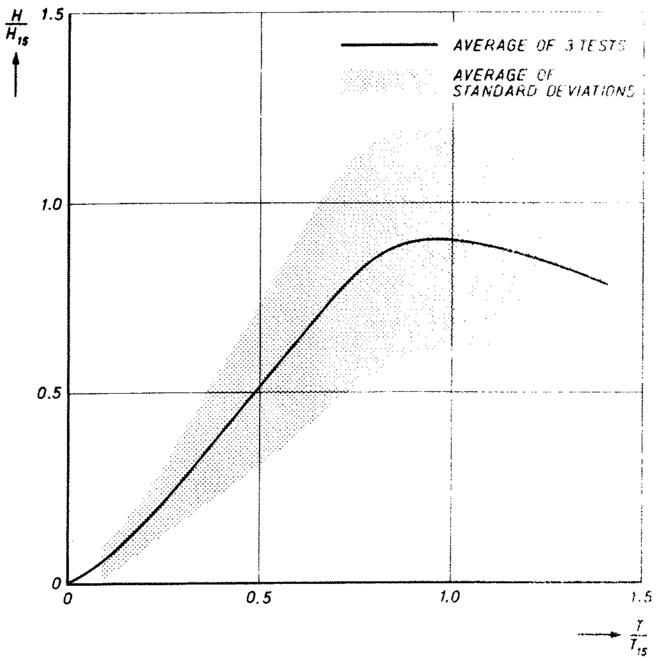

FIG. 6c H-T RELATION, PROTOTYPE RECORDS

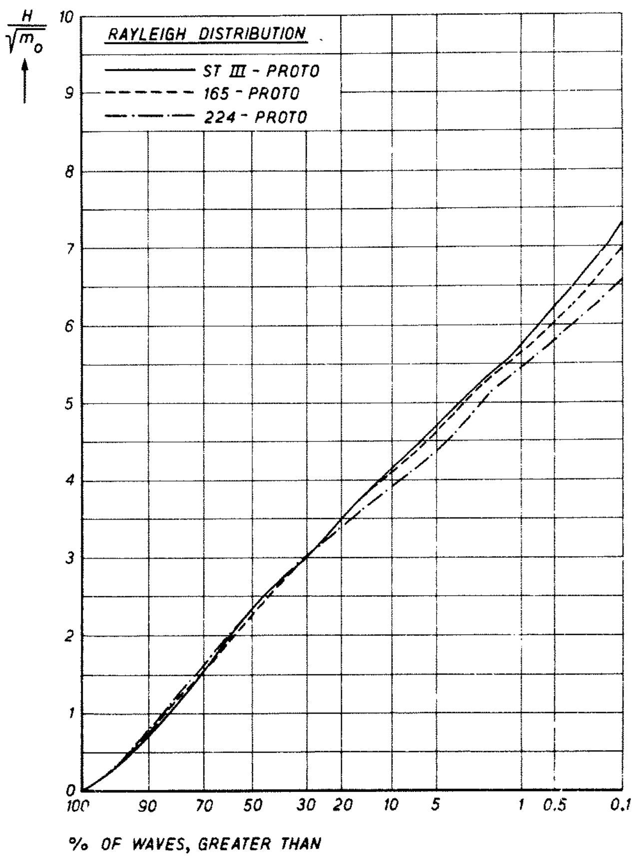

FIG. 6b WAVE HEIGHT DISTRIBUTIONS PROTOTYPE WAVES

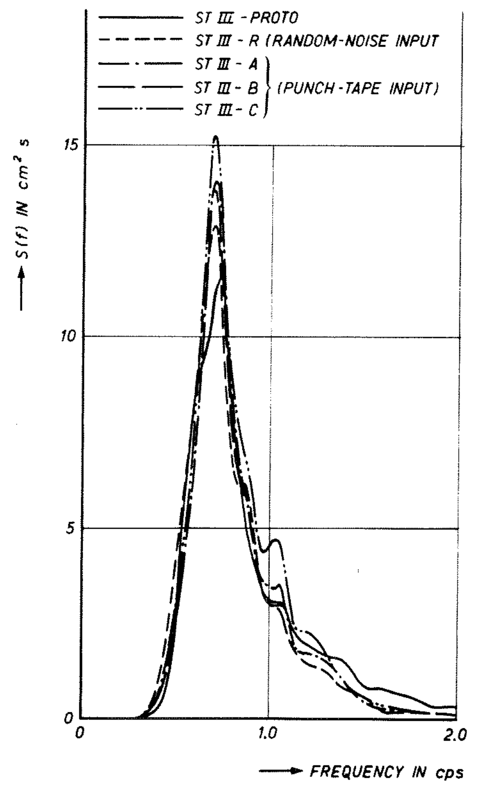

FIG. 7a WAVE – ENERGY SPECTRA, PROGRAMMED WAVES VERSUS PROTOTYPE WAVES

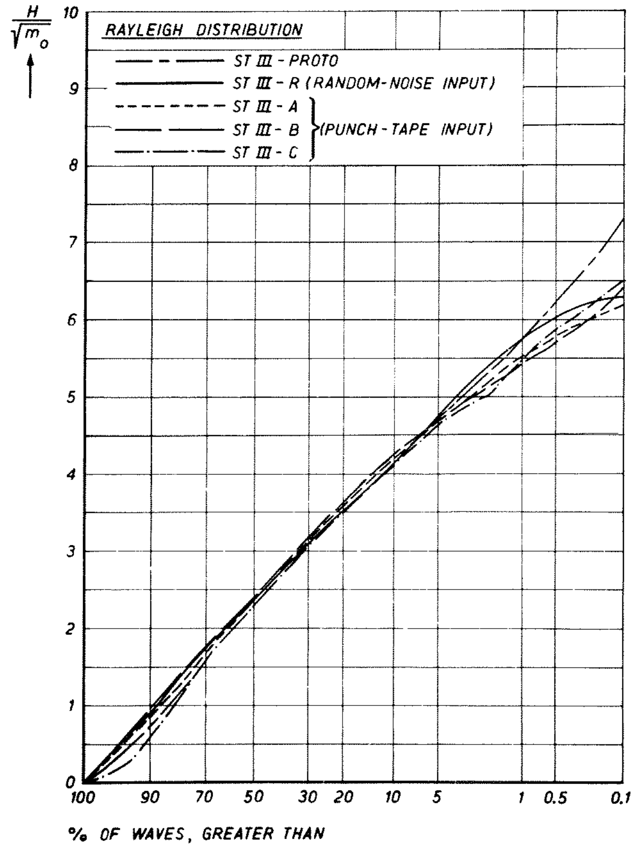

FIG. 7b WAVE HEIGHT DISTRIBUTIONS, PROGRAMMED WAVES VERSUS PROTOTYPE WAVES

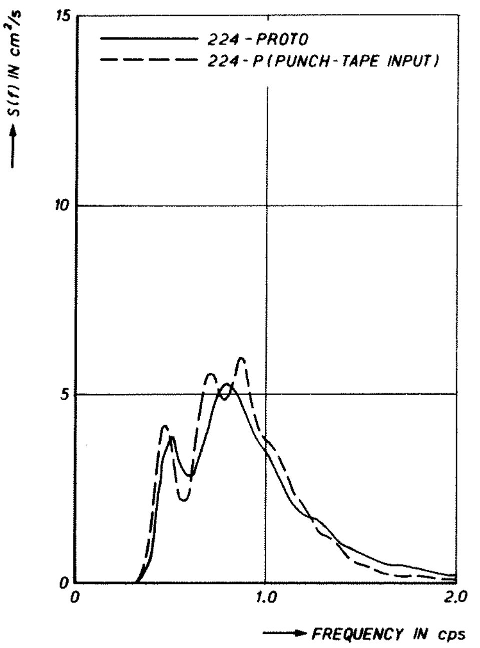

FIG. 8a WAVE – ENERGY SPECTRA, PROGRAMMED WAVES VERSUS PROTOTYPE WAVES

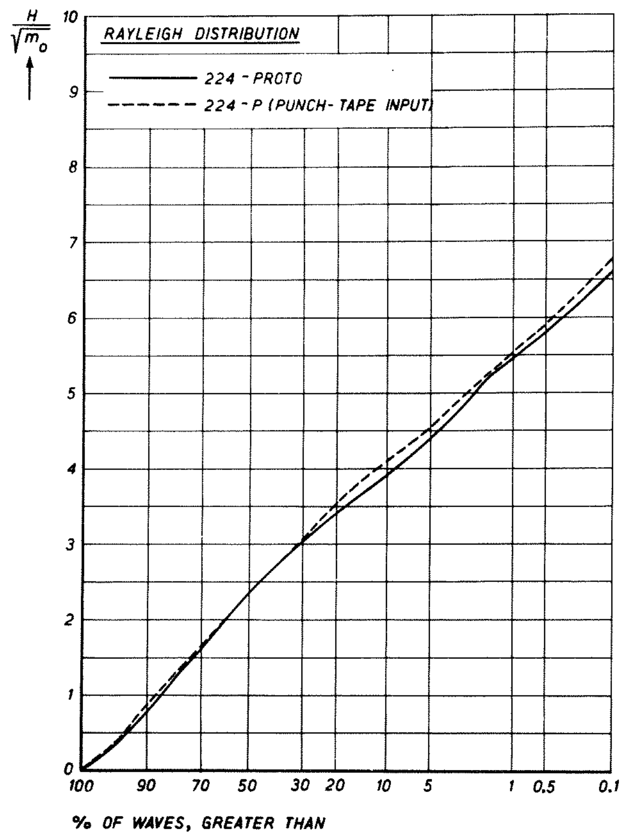

FIG. 8b WAVE HEIGHT DISTRIBUTIONS, PROGRAMMED WAVES VERSUS PROTOTYPE WAVES

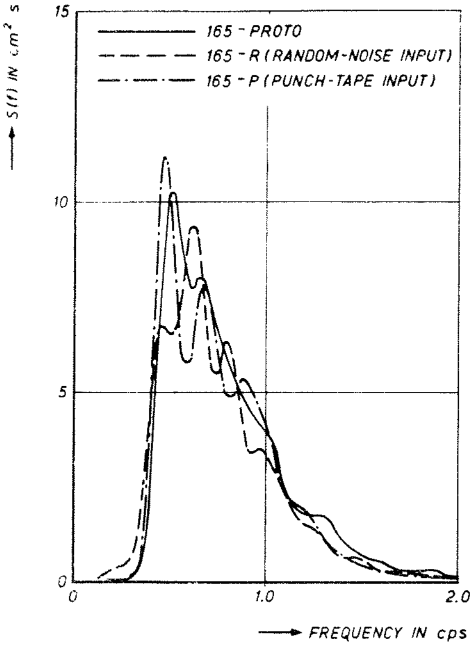

FIG 9a WAVE – ENERGY SPECTRA, PROGRAMMED WAVES VERSUS PROTOTYPE WAVES

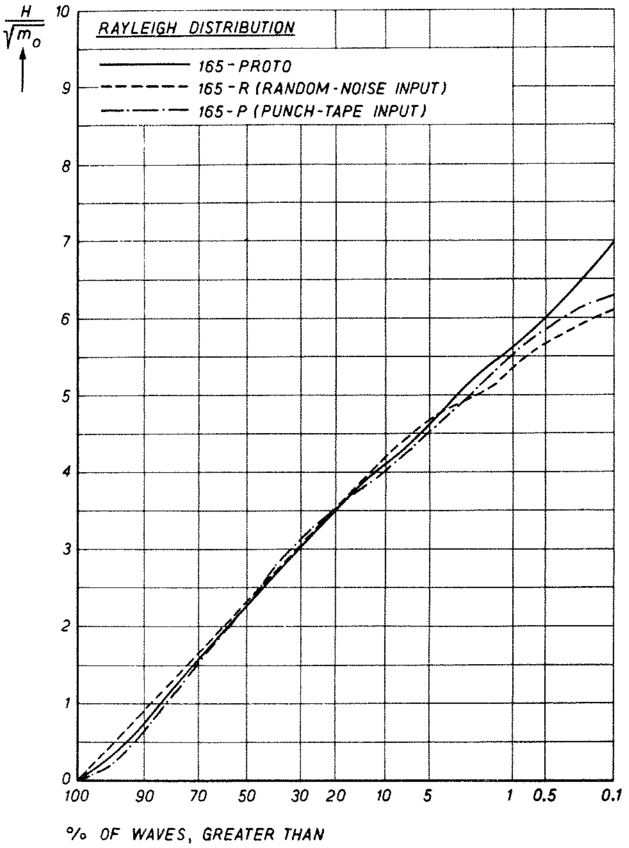

FIG. 9b WAVE HEIGHT DISTRIBUTIONS, PROGRAMMED WAVES VERSUS PROTOTYPE WAVES

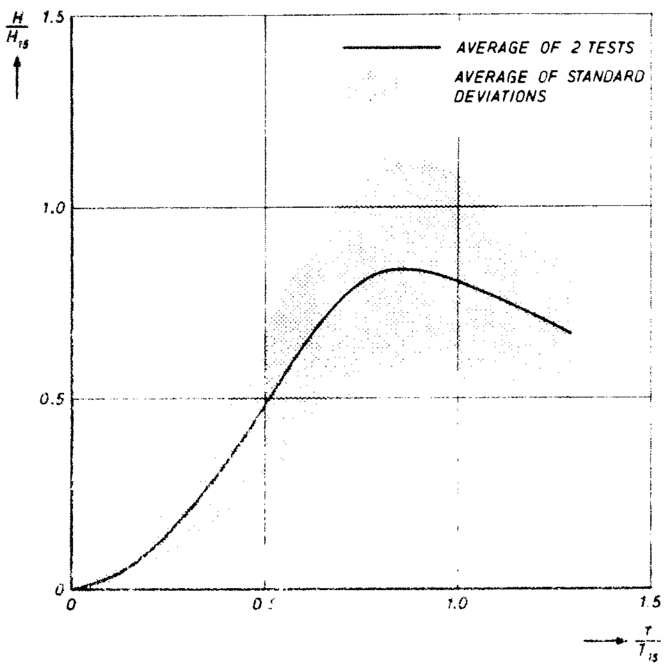

FIG 10a H-T [illegible]TION, P[illegible]GRAMMED WAVES [illegible]
[illegible]

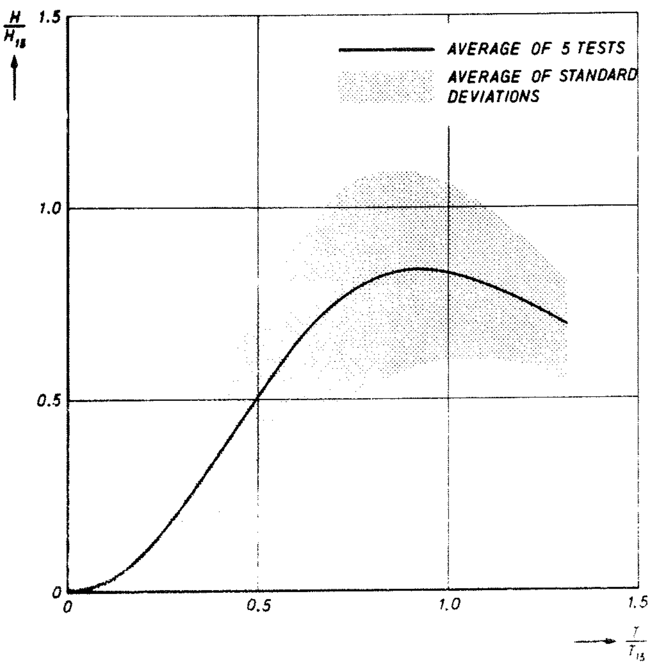

FIG 10b H-T RELATION, PROGRAMMED WAVES
[illegible]

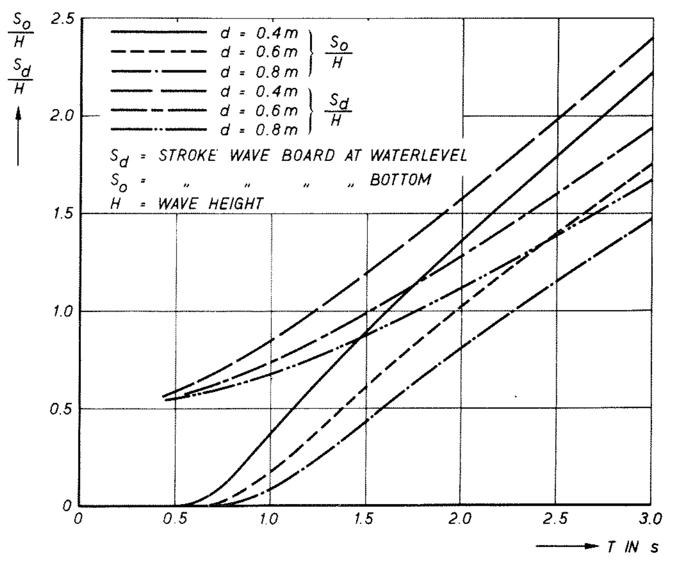

FIG. 11 TRANSFER FUNCTION WAVE HEIGHT TO WAVE BOARD

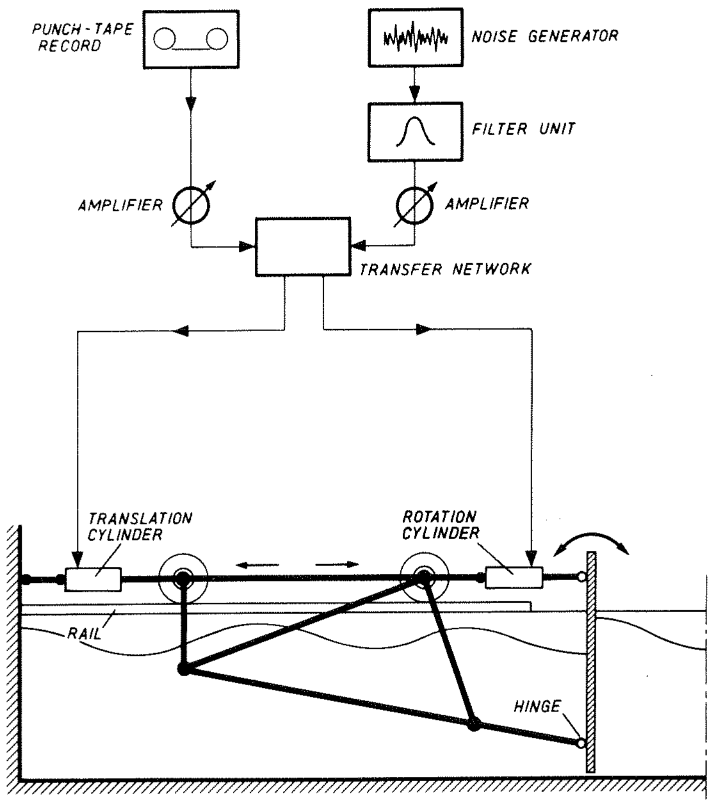

FIG. 12 SYSTEM SET-UP WAVE GENERATOR

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LIST OF SYMBOLS

d	water depth
Δ	ratio of asymmetry of the waves
ε	spectral width parameter
f	frequency in cps
H	wave height
Hs	significant wave height
H_n	wave height with an percentage of exceedance n
m	number of points of the auto-correlation function
m_n	n th moment of the energy spectrum
$p(H)$	probability density function of wave heights
$S(f)$	spectral density at frequency f
T	wave period
T_n	wave period with an percentage of exceedance n
T_{peak}	period of the wave spectrum with maximum energy density
t	time
$\eta(t)$	surface elevation record
Δt	time-lag between two samples in the time series
τ	time - shift
$R(\tau)$	auto - correlation function
w_z	wind speed measured at z metres above the mean still-water level
$\overline{w}$	average wind speed

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A COMPARISON OF REGULAR AND WIND-GENERATED WAVE ACTION ON RUBBLE-MOUND BREAKWATERS

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ABSTRACT

The general purpose of this research is to study wave action on rubble-mound breakwaters with periodic waves on the one hand, and random wind generated waves on the other hand, and to compare the effects of these two types of waves by use of the storm duration.

With the first serie of periodic waves experiments, we obtained the destruction of breakwater cover-layer for different storm durations t, and waves height H and period T. The risk criterion is :

** $$\frac{t}{T} = - A \log \left(\frac{H^2}{\nu T}\right) + B$$

A and B being constantes, ν the kinematic-viscosity.

With the second serie of tests, with random wind generated waves, we found that the destruction during the same storm duration was obtained for a significant wave height $H_{1/3}$ equal to the constant periodic wave height H. This is an experimental demonstration of the use of $H_{1/3}$ to study breakwaters on scale models.

MODEL

The studies (Ref.1) were made in two wave-flumes with the scale 1/40 (Fig.1) filled with water to the level 0.35 m (14 m in nature,

The breakwater profiles had three layers of stones of specific gravity 2.6 g/cm3, weighed one by one, and arranged always in the same way for all experiments. Their size distributions for each layer were (Fig.2) :

layer A : 50 - 80 g (3-5 tons in nature)
layer B : 20 - 50 g (1.5-3 tons in nature)
layer C : 5 - 25 g (0.32-1.6 tons in nature).

Four slopes of profiles were tested (30, 32, 34 and 36 degrees).

The storm duration t, corresponding to the destruction of breakwater, was the time of the test for which the complete destruction of profile was obtained. In fact when the cover-layer was destroyed, all the profile was radidly broken up.

EXPERIMENTS WITH PERIODIC WAVES

For each slope of profile, 16 tests were made with 4 wave periods : 0.948, 1.265, 1.581 and 1.897 s (6, 8, 10, 12 s in nature) and 4 wave heights : 0.05, 0.075, 0.10 and 0.125 m approximately (2,3,4 and 5 m in nature).

During the first minutes of every experiment the profile slope was transformed to a discontinious seaward profile (Fig.3), by moving of armor units from the upper part to the lower part of the slope, with the 3 angles : $\alpha_1 = 43°$ $\alpha_2 = 21°$, $\alpha_3 = 38°$ approximately. If the equilibrium profile did not reach the every layer, it was no risk of destruction ; the profile was stabilised and every test lasted 3 hours 45 minutes (24 hours in nature). If the second layer was reached the destruction was rapidly obtained, and then the storm duration was noticed.

The wave height H was obtained from a record of the clapotis along the channel.

The relationship among t, H and T is researched as a correlation between the number of waves $\frac{t}{T}$ required for destruction of the profile and the dimensionless parameter $\frac{H^2}{\nu T}$. We found (Fig.4) :

$$\frac{t}{T} = - A \log \left(\frac{H^2}{\nu T}\right) + B \qquad (1)$$

with the coefficient of correlation r = 0.796.

EXPERIMENTS WITH RANDOM WIND GENERATED WAVES

Random waves were induced by an air flow over the channel. By variation of fetches (up to 30 m) and cycles of starts-off and stops of wind (velocity 0 or 9.4 m/s), a sufficient variety of wave heights and periods were obtained. Surface elevations were measured during 4 minutes with a sonar every 0.1 s and punched on a paper tape. A resistance wave gauge gave a picture of waves.

The purpose of this experiments was to destroy the total cover-layer in approximately the same time as in test with periodic waves. A series of preliminary experiments showed that it was possible to obtain only 8 kinds of random waves producing the same effects among the 13 destructions by periodic waves.

The 8 tests were made again involving the following operations :

- construction of the model,
- choice of fetch and cycle of wind,
- regulation of wind deflector to prevent the direct effect of wind on the breakwater,
- starting of blower,
- records of waves by sonar and resistance wave gauge at the beginning and the end of the test,

During the random waves experiments the evolution of the equilibrium profile was little different as for periodic waves ; the three different slopes (Fig.3) were $\alpha_1 = 46°$, $\alpha_2 = 19°$, $\alpha_3 = 36°$.

Every tape record contained 2,400 values which were punched on cards and investigated using a CDC 6600 digital computer. Figures 5 and 6 give a example of autocorrelation function R(J) and spectral density SP(J). This values were obtained from the N = 2 200 discreetized observations X(I) for each record, using the following equations (Ref.2) :

- for autocorrelation function (J = 0, 1, 2 ... 200)

$$R(J) = \frac{\sum_{I=1}^{N-J} X(I).X(I+J)}{\sum_{I=1}^{N-J} X(I)^2} \qquad (2)$$

- for special density

first a first approximation

$$LP(J) = \frac{1}{N}\sum_{1}^{N} X(I)^2 + 2\sum_{K=1}^{K=N-1} \frac{1}{N-K}\left[\sum_{I=1}^{N-K} X(I).X(I+K)\right]\cos\frac{KJ\pi}{N}$$

$$+ \frac{1}{N-200}\left[\sum_{I=1}^{N-200} X(I).X(I+200)\right]\cos J\pi \qquad (3)$$

and finally after smoothing by Hamming

$$SP(J) = 0.23\ LP(J) + 0.54\ LP(J+1) + 0.23\ LP(J+2) \qquad (4)$$

The wave heights H_r and periods T_r for every sample were obtained using the zero-up-crossings method. The seiches were eliminated using a moving-mean over 75 points. H_r and T_r values were classed in increasing order, to evaluate the mean values $H_{n/m}$, $T_{n/m}$ ($n = 1, 2, 3$; $m = 1, 2 ..., 10$) and the joint distributions (example on Fig.7).

** The main result is that the significant wave height $H_{1/3}$ of random waves producing the destruction of the breakwater in the same time that periodic waves, is equal to the height H of this periodic waves. This is an experimental demonstration of the empirical and theoretical assumption that $H_{1/3}$ is the representative wave height and this of the justifiable use of $H_{1/3}$ as a project wave height.

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1 - Rogan A.J. ; Comportement des jetées en enrochements vis-à-vis de la houle ; Bulletin de la Direction des Etudes et Recherches, Electricité de France, série A, 1968, volume 3.

2 - Blakman R.B. and Tukey F. ; The measurement of power spectra from the point of view of communications engineering ; Dover Publ.Inc.; New York , 1969.

LIST OF SYMBOLS

A,B	constantes
$\alpha_1, \alpha_2, \alpha_3$,	slopes of profiles
H	height of periodic waves
H_r	height of random waves
$\overline{H} = H_{1/1}$	mean value of H_r in a record
$H_{1/3}$	significant wave height
T	period of periodic waves
T_r	period of random waves
$\overline{T}$	mean value of T_r in a record
t	storm duration
ν	kinematic viscosity
R(J)	autocorrelation function (equation 2)
LP(J)	first approximation of spectral density (equation 3)
SP(J)	spectral density (equation 4)
X(I)	discreetized observation in a record.

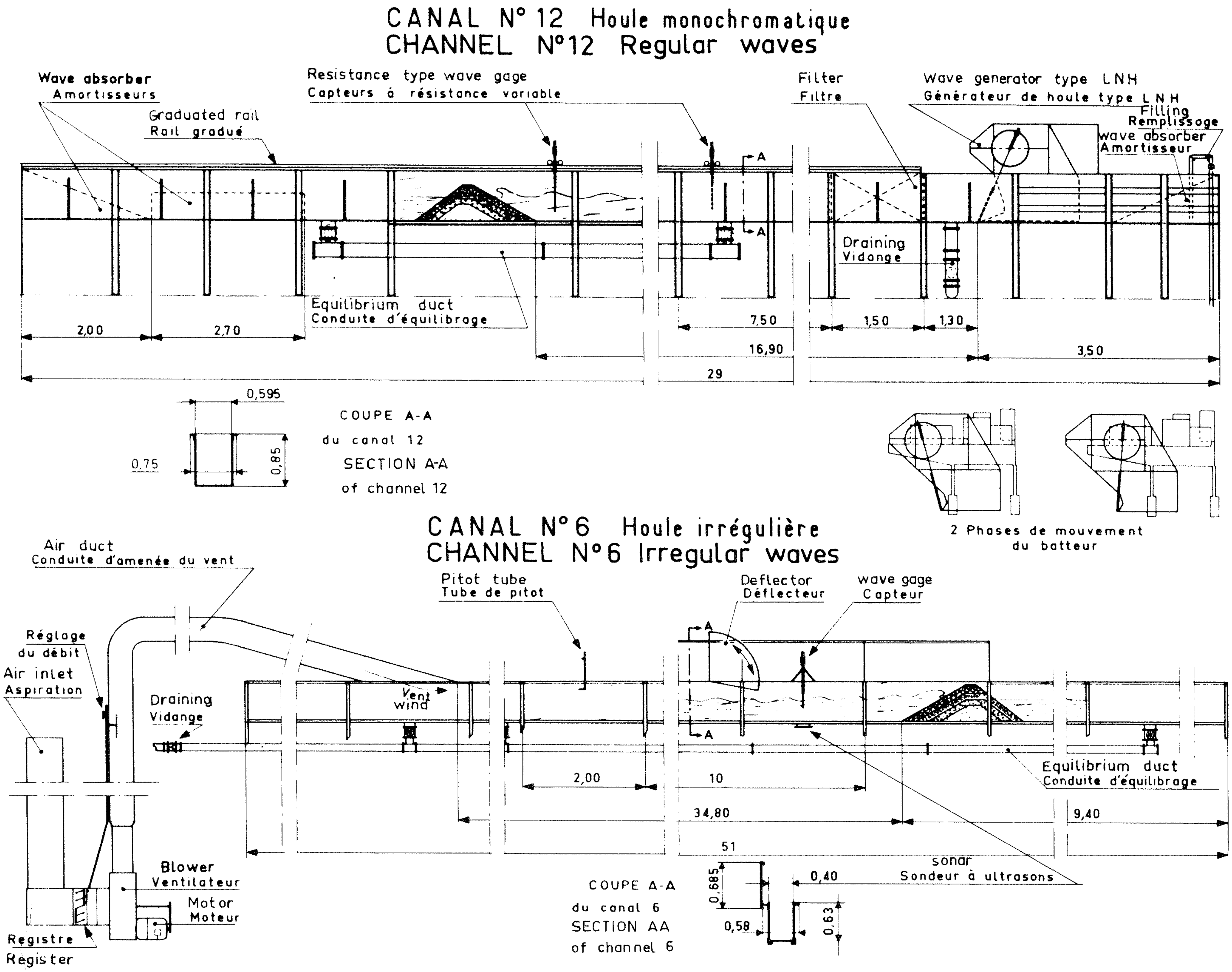

Fig. 1 Description of wave-flumes.

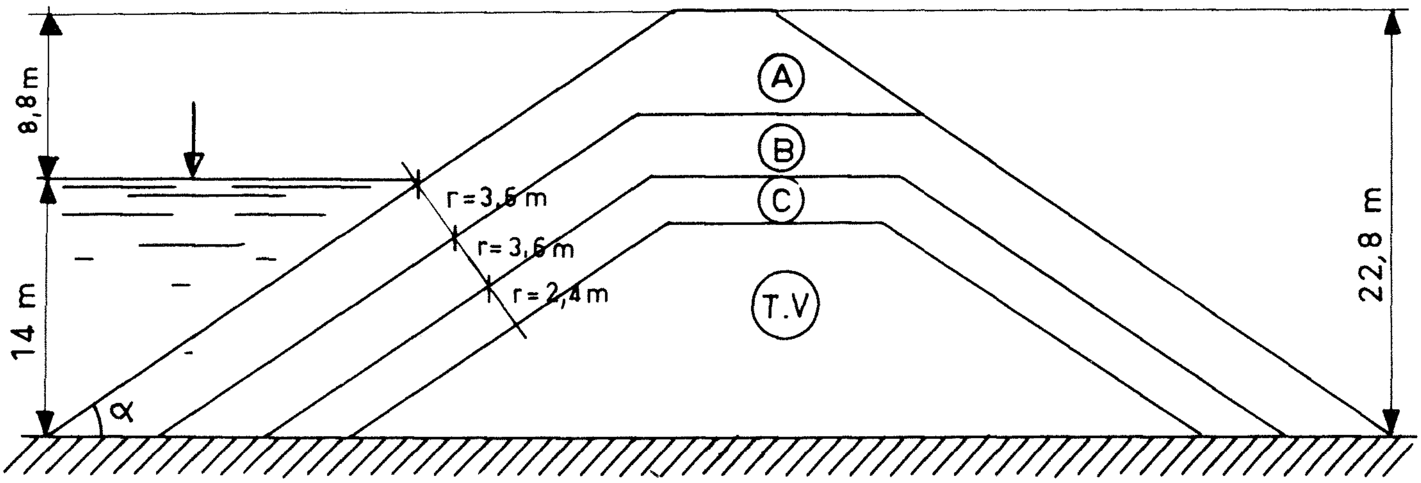

Fig. 2 Type of profile.

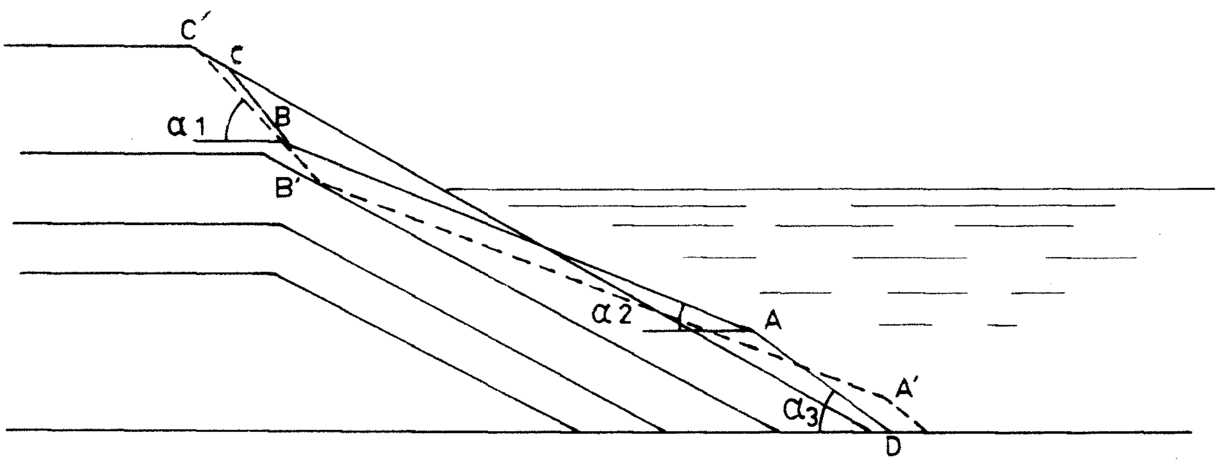

Fig. 3 Transformation of profile.

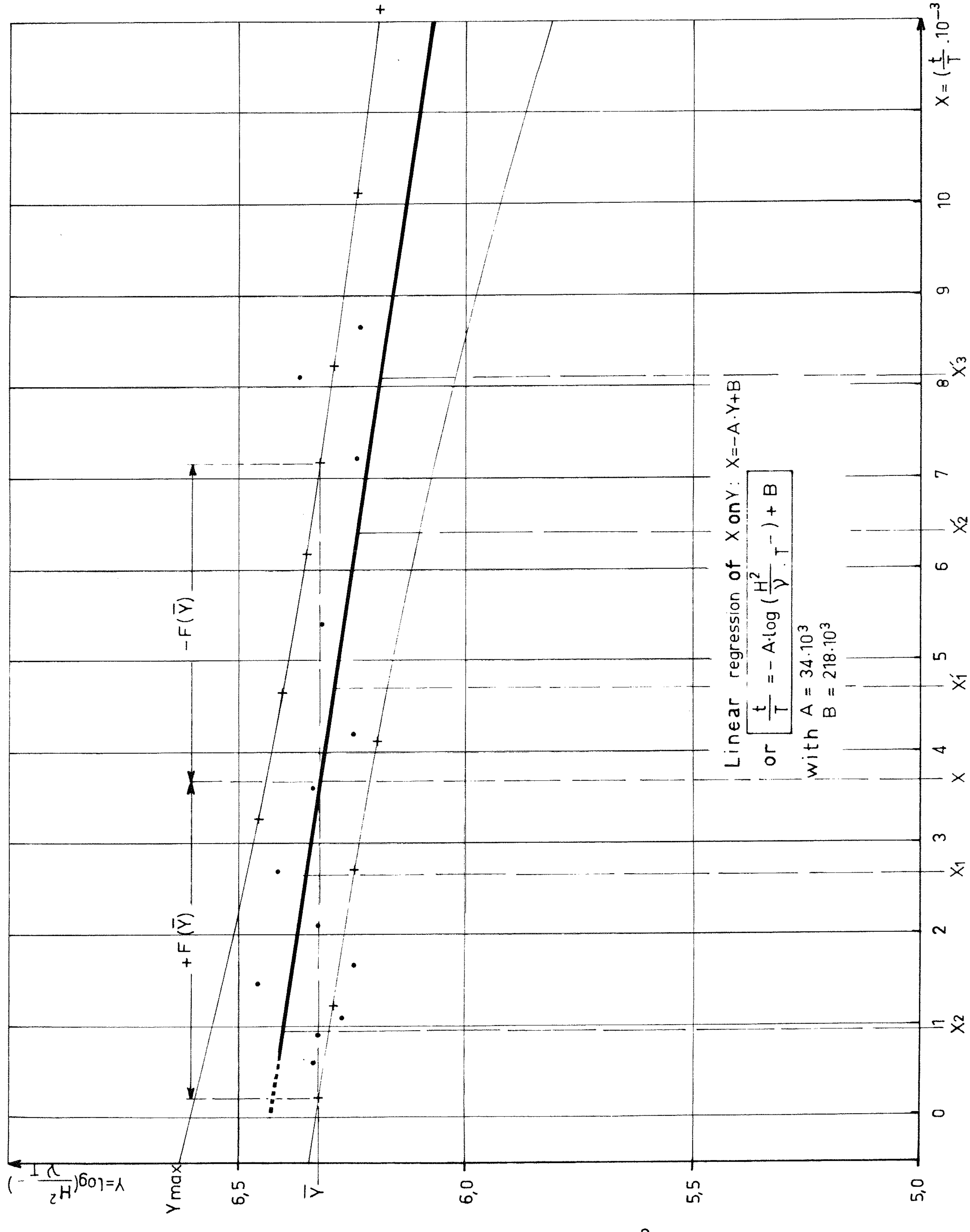

Fig.4 Correlation between $H^2/\nu T$ and t/T.

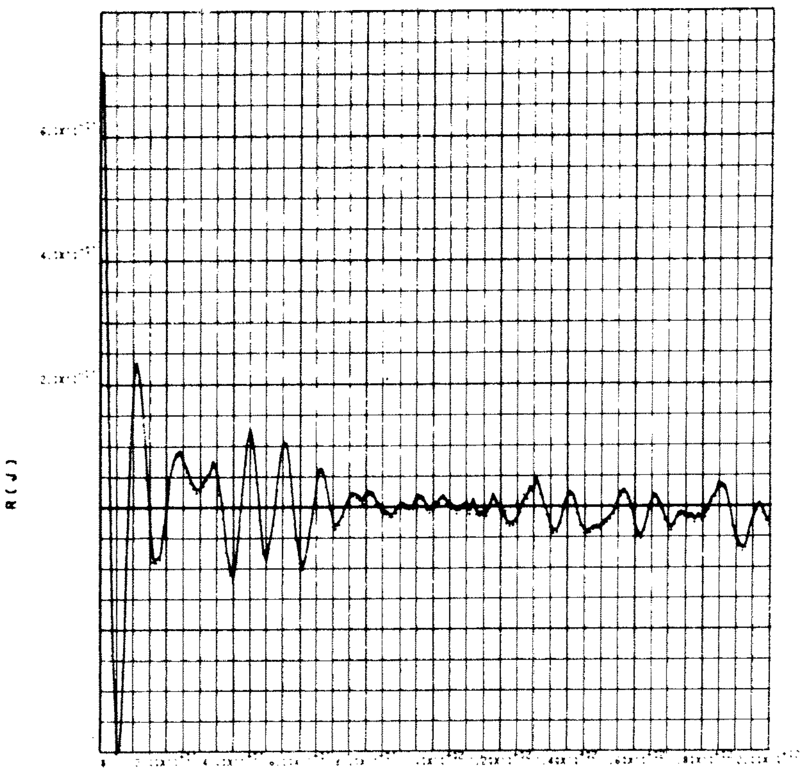

Fig. 5 Autocorrelation function R(J).

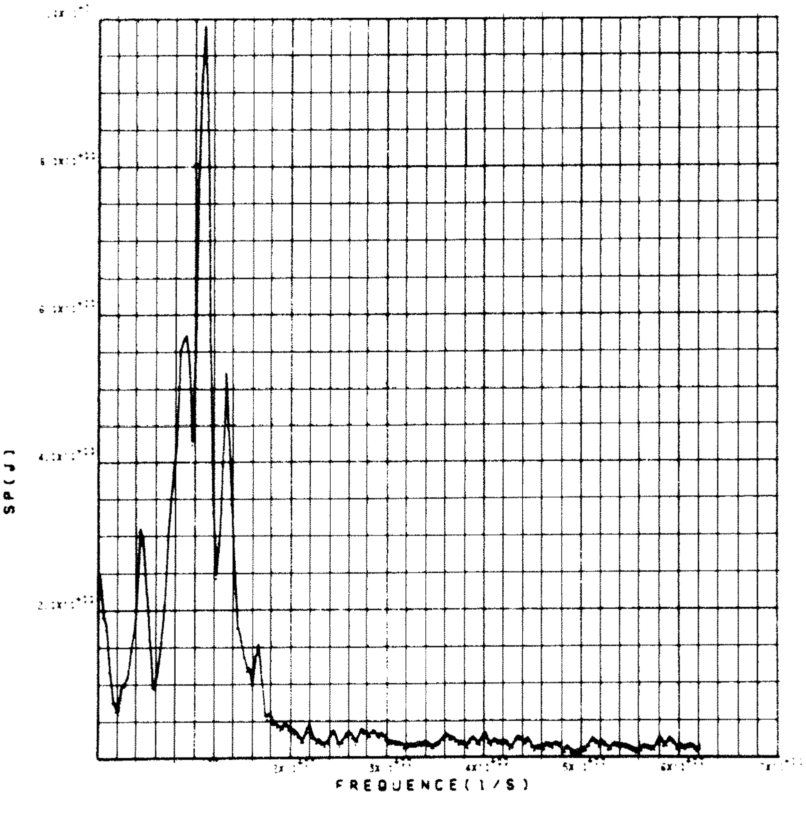

Fig. 6 Spectral density.

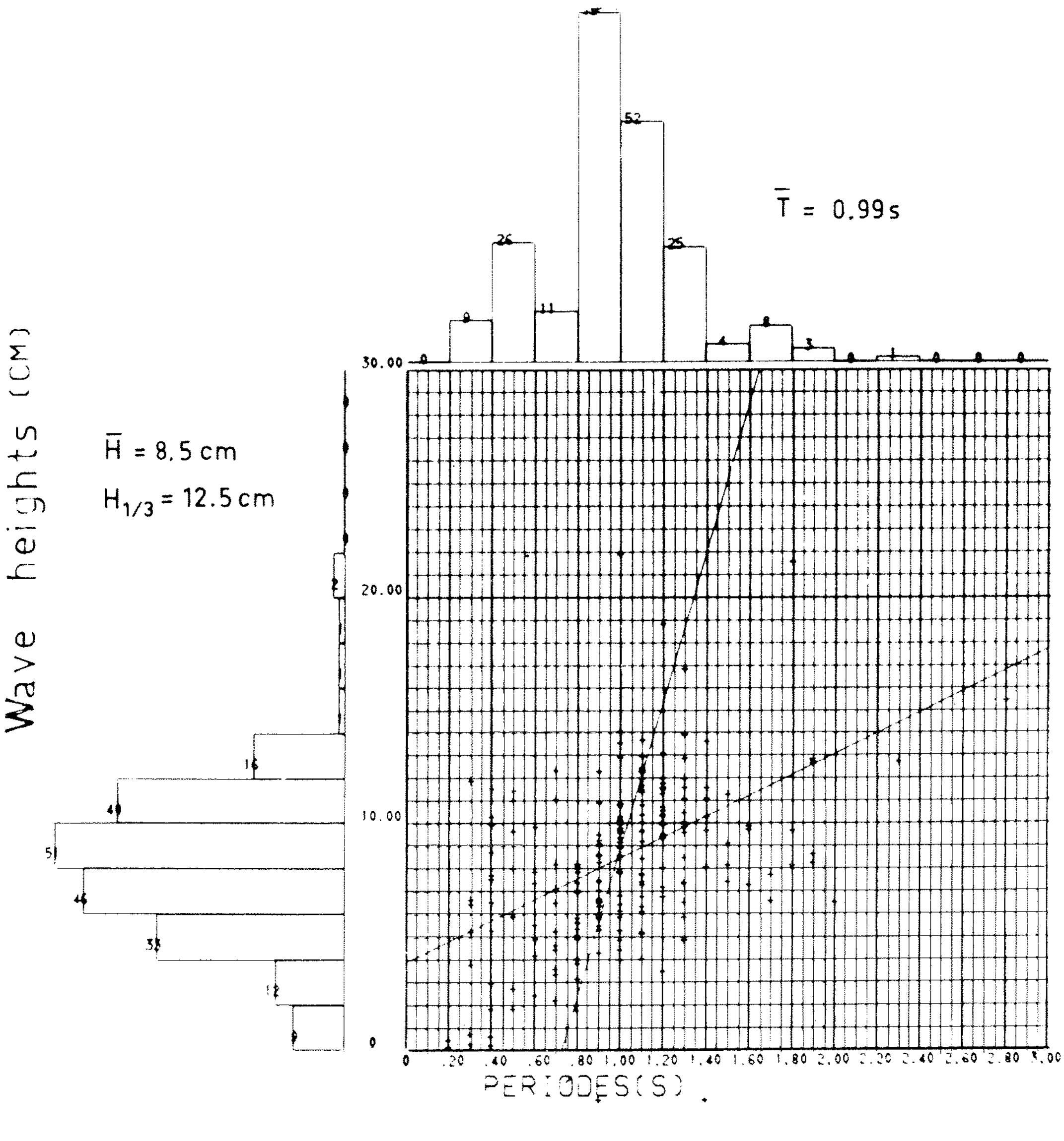

Fig. 7 Joint distribution of H_r and T_r.

DISCUSSION ON PAPER 3

J. van der WEIDE

Delft Hydraulics Laboratory, The Netherlands

When the well-known Navier Stokes equation

$$\rho \frac{\delta v}{\delta t} + \rho v \text{ grad } v = - \text{ grad } p + \rho g + \eta \nabla^2 v \qquad (1)$$

is plotted in a dimensionless way, dividing every term by a characteristic reference value, the following equation is obtained (Ref. 1):

$$Sr \frac{\delta \bar{v}}{\delta \bar{t}} + \bar{v} \text{ gr}\bar{\text{a}}\text{d } \bar{v} = - \frac{1}{\bar{\rho}} \frac{1}{Eu} \text{ gr}\bar{\text{a}}\text{d } p + \frac{1}{Fr} \bar{g} + \frac{1}{Re} \bar{\nu} \bar{\nabla}^2 \bar{v} \qquad (2)$$

with

$$Sr = \frac{L_*}{v_* t_*}$$

$$Eu = \frac{\rho_* v_*^2}{p_*}$$

$$Fr = \frac{v_*^2}{g_* L_*}$$

$$Re = \frac{v_* L_*}{\nu_*}$$

L_*, v_*, t_* etc. being the characteristic reference values and $\bar{v}$, $\bar{t}$ etc. being the dimensionless parameters.

Substituting

$$L_* = H \text{ (wave height)}$$

$$t_* = T \text{ (wave period)}$$

$$v_* = \frac{H}{T}$$

in equation (2) gives

$$\frac{\delta \bar{v}}{\delta t} + \bar{v} \text{ grad } \bar{v} = - \frac{1}{\varrho} \frac{p_* T^2}{\rho_* H^2} \text{ grad } \bar{p} + \frac{g\, T^2}{H} \quad \bar{g} + \frac{T\, \nu}{H^2} \quad \bar{\nu} \quad \bar{\nabla}^2 \bar{v} \qquad (3)$$

Accepting the average values for g and ν, it follows that the influence of gravity exceeds greatly that of viscosity provided velocity gradients are not too high.

Hydraulic phenomena, and hence damage as a result thereof, should be characterized by the parameter $\frac{g\, T^2}{H}$ rather than by the parameter $\frac{T\, \nu}{H^2}$ as was used by the authors.

If their relationship between $\frac{H^2}{\nu T}$ and $\frac{t}{T}$ is true, the equivalent time of demolition in prototype should be computed according to the Reynolds scale Law rather then according to the Froude's Law as was done by the authors.

** It is expected that results obtained when plotting $\frac{t}{T}$ against $\frac{g\, T^2}{H}$ will be different for regular waves and irregular wind-generated waves, since the value of $\frac{g\, T^2}{H}$, being a measure for the initial wave steepness, will be different in both cases.

Ref. 1: Vossers, Prof. dr. ir. G., "Inleiding tot de theorie van modellen en modelwetten", De Ingenieur, 1966, Dec. 2, pp. W 231 - W 238, (in Dutch with English summary).

WAVE SHOCK FORCES : AN ANALYSIS OF DEFORMATIONS AND FORCES IN THE WAVE AND IN THE FOUNDATION

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1. SUMMARY

At the inauguration of two advanced wind-wave flumes in the Netherlands, three decades after BAGNOLD's pioneer work, and one decade after the Haringvliet sluice investigation and the joint Dutch-Danish investigation of the Hanstholm breakwaters, it seems appropriate to summarize current knowledge on wave shock phenomena.

These may be classified into 3 types: Ventilated, compression and hammer shocks (Fig. 1). The following conclusions are reached: (1) Model impulses, $\int P\,dt$, can be translated to the prototype by Froude's law. - (2) Froude's law also applies to the maximum pressure of a well-ventilated shock, but will yield conservative values when the bubble content in prototype is high and the pressure rise is very rapid. - (3) The Compression Model Law, Fig. 5, applies approximately to the maximum pressure of a compression shock. - (4) Froude's law is valid for a normal hammer shock, but in some cases a composite effect, also involving compressibility, may occur.

It is suggested that future research be based on detailed studies in large, well instrumented models, and that the physical analysis be translated to prototype with due consideration of various scale effects such as difference in bubble contents.

The action of wave forces on foundations requires combined geotechnical/coastal research. It would seem that most foundations are so stiff that shock forces are transferred directly to the foundation with amplification factors up to 1.7.

The study of wave shock forces represents a particularly good example of a research field where the need of cross-scientific contacts cannot be stressed too much.

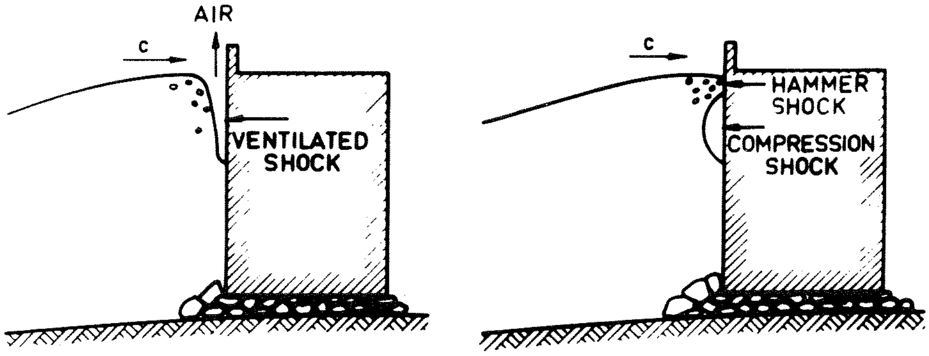

Fig. 1. Wave shock phenomena.

2. INTRODUCTION

When a wave train breaks in front of a structure, some of the waves will produce shock pressures, that is pressures which rise from a small value to a maximum within a time which is but a small fraction of the wave period. Because of the complexity of the phenomenon, a rigorous mathematical analysis appears to be unattainable, and hence for the design of structures, it is necessary to rely on model tests.

If gravitational and inertial forces alone were involved, the interpretation of the model tests would be simple and would depend only upon Froude's law. However, the following factors may also play an important role in the process:

(a) The pressures in an air pocket trapped between the structure and the concave wave front.

(b) The pressures in bubbles entrained in the breaking wave, as well as the concentration of entrained air.

(c) The pressures in the air cushion that is being expelled when the wave front collides with the structure.

(d) Interaction between the wave forces and forces induced in the underlying soil due to rocking motion of the structure on its foundations under wave action.

Hence, the interpretation of a model test with shock forces may be a very complicated affair. Special reference is made to point (b) above, because the amount of air entrained will be relatively larger in prototype than in a model, thereby introducing scale effects related to capillary action into the problem.

When a problem is complex, the first step towards its solution should always be a description of what actually happens combined with a magnitude analysis, that is an approximate, or rough, comparison of the deformations and forces involved. Such a magnitude analysis will in many cases enable us to exclude some of the factors involved as being relatively unimportant, and also to obtain some understanding of the plausibility of various possible assumptions. It is the purpose of this paper to provide an introduction to some of the physical aspects of wave shock forces.

Most of the analysis will not be given as formulae, but in terms of definite examples, often with reference to one specific case, for which the following characteristics have been chosen:

$$\begin{aligned} &\text{Water depth:} \quad h = 10 \text{ m.} \\ &\text{Wave period:} \quad T = 10 \text{ s.} \\ &\text{Height of breaking wave:} \quad H_b = 5 \text{ m.} \\ &\text{Wave celerity:} \quad c = 10 \text{ m/s.} \end{aligned} \tag{2.1}$$

For some problems, available test results will be used directly.

Two distinctly different types of vertical-face breakwaters will be involved in the discussion, cf. Fig. 2:

Type A. Breakwater with shallow or no rubble foundation: In this case the depth is 10 m in front of the vertical wall and increases gently outwards. A large percentage of the incoming wave energy is reflected, such that extensive breaking takes place far from the breakwater. Few of the waves give shock forces, say, about 5%.

Type B. Breakwater with high rubble foundation: In this case the depth in front of the vertical wall is substantially less than 10 m, say, 4 m, with the results that less energy is reflected and that a larger proportion of waves break directly in front of the wall causing shock effects.

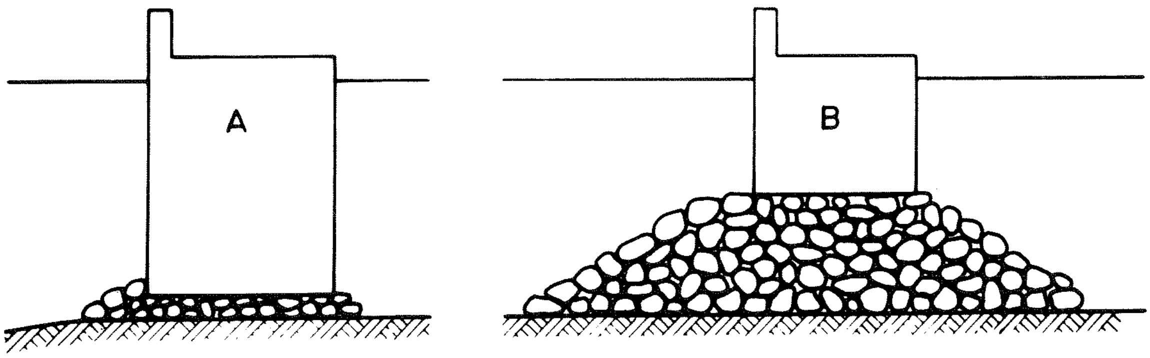

Fig. 2. Vertical-face breakwaters.

Finally, a definition of shock pressures is required: Assuming that the "parabolic" irregularity of the "vertical" front of the breaking wave is of the order of magnitude of $0.3\,H_b$, the time required to "flatten" this front against the vertical wall is of the order of magnitude of

$$t_{sh} = 0.2\ H_b/c \tag{2.2}$$

provided that the air actually escapes. Hence, a shock pressure is here defined as a pressure of substantial magnitude, the rising time of which is of the order of magnitude of t_{sh} or less.

For the numerical example (2.1) we find

$$t_{sh} = 0.1\ \text{s} \tag{2.3}$$

This figure should not prevent us from speaking of a shock pressure when the rising time is as much as 0.2 s, because this is still a small fraction of the rising time of pressure due to a complete or partial clapotis, where the time interval from still water level to wave crest is about 2 s.

3. VENTILATED SHOCK

In some cases the wave front approaches a vertical wall in such a manner that all, or nearly all of the air between the wave and the wall is able to escape as the front collides with the wall. This type of shock pressure will be called a ventilated shock. Ventilated shocks are relatively more frequent for breakwaters of type A than for breakwaters of type B.

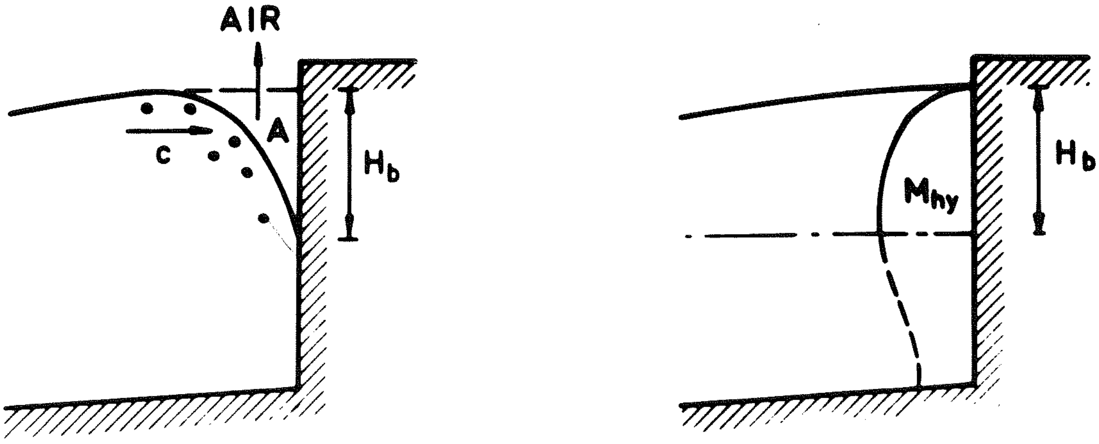

Fig. 3. Ventilated shock.

The air escape can take place in two different ways:

(a) The front has such a shape that the closure takes place "gradually" from the wave trough upwards, cf. Fig. 3.

(b) The horizontal projection of the wave front forms an angle with the wall, allowing the air to escape longitudinally.

The expulsion of air requires a pressure somewhat higher than that of the atmosphere. This matter is touched upon in Art. 7 below. The excess pressure delays the advance of the front slightly, but, for a "well-ventilated" shock, this delay is negligible compared with the rising time of pressure having regard to the irregularity of a wave front. Thus the excess pressure may be neglected in the interpretation of the test results.

If A is the volume of air, per unit length of the breakwater, contained between the wave front and the wall at the moment when the toe of the breaker reaches the wall, the rising time of pressure is approximately

$$t_{rising} = \frac{A}{H_b\ c} \tag{3.1}$$

The maximum shock pressure will vary over the height H_b. Let p_{max} be the average over this height. Then the pressure may be assumed to vary as

* $$p = \frac{1}{2}\ p_{max}\ (1 - \cos \pi\ t/t_{rising}) \tag{3.2}$$

from $t = 0$ to $t = 2\ t_{rising}$. Hence the total impulse delivered onto the wall over the height H_b is

$$I = \int H_b\ p\ dt = H_b\ p_{max}\ t_{rising} \tag{3.3}$$

The impulse equals the horizontal momentum removed from the upper water. Roughly, it is assumed that all water behind the breaking front moves with the velocity c, so that the impulse (3.3) can stop an equivalent hydrodynamic mass M_{hy} which is defined by the equation

$$I = c\ M_{hy} \tag{3.4}$$

Westergaard (Ref. 8) determined the hydrodynamic mass pertaining to the horizontal acceleration of a vertical wall (earthquake oscillation of a storage dam) to be

$$M_{hy} = 0.54\ \varrho\ h^2 \qquad (3.5)$$

where h is the water depth. The distribution of this mass, i.e. the distribution of the pressure corresponding to an acceleration a, constant over the depth h, is approximately a quarter of an ellipse with semi-axes h and 0.7 h, cf. Fig. 3, right.

For the wave shock the whole problem is more complicated than Westergaard's for the following reasons: (1) The water velocities, called c, are not constant over H_b. (2) The area A, Fig. 3, is not uniformly distributed over H_b. (3) If the acceleration a were constant over the height H_b, the hydrodynamic mass pertaining to H_b would be somewhat less than Westergaard's, with $h = H_b$, because the acceleration of the water can spread also over the water mass beneath the dash-dotted line in Fig. 3 which, in Westergaard's problem, constitutes the bottom of the reservoir. (4) During the time 2 t_{rising} there is also a shock pressure on the wall below H_b, this pressure being symbolically represented by the dotted curve in Fig. 3. (It is not possible to speak of a hydrodynamic mass for this part of the wall because, initially, there is no air gap and, hence, no acceleration of the wall relative to the water.)

In spite of all these complications, we shall, for the present purpose, accept (3.5) and crudely assume that

$$M_{hy} = 0.5\ \varrho\ H_b^2 \qquad (3.6)$$

and introduce this value in (3.4). Then with the use of (3.1-3) it follows that the maximum pressure is

$$p_{max} = \frac{I}{H_b\ t_{rising}} = 0.5\ \varrho\ \frac{c^2}{A}\ H_b^2 \qquad (3.7)$$

For the illustrative example given in (2.1), assuming $A = 0.2\ H_b^2$, we find $p_{max} = 25\ t/m^2$ and $t_{rising} = 0.1$ s.

In Westergaard's problem the horizontal acceleration of the wall produces horizontal as well as vertical accelerations in the water. In the wave shock problem the shock impulse stops the horizontal movement of M_{hy}, transferring its kinetic energy into a vertical motion. (For a small value of t_{rising}, the loss of kinetic energy to turbulence is negligible within the duration of the shock.) Naturally, this vertical motion is maximum at the wall and decreases with the distance from the wall. At a distance of H_b it is negligible.

Westergaard found in his problem a singularity at the point where the water surface touches the wall. The singularity consists of an infinite vertical acceleration. In terms of the wave shock problem this singularity means that the finite velocity c may, under circumstances, result in very high vertical velocities at the water surface when the breaking front has come into complete contact with the wall. This is probably one of the reasons for the well-known high-splash directly at the wall.

When the horizontal kinetic energy of M_{hy} has been transferred into a vertical motion, the nearly horizontal flow of water following behind the mass M_{hy} continues towards the wall as a part of the oscillatory wave motion. This flow results in a
* gradual rise of the water surface at the wall.

In all the considerations above, it has been assumed that pressures are transferred instantaneously from the wall to other parts of the water, i.e. the sound velocity c_e has been assumed to be infinite. In Fig. 7 c_e is given as a function of the concentration of bubbles in the water. As will be seen, it is realistic to assume that $c_e = 200$ m/s. Then, for the example discussed above, a small pressure increment at the wall can travel the distance 20 m in the time $t_{rising} = 0.1$ s. With $H_b = 5$ m the average "thickness" of M_{hy} is 2.5 m according to (3.6). This is a fairly small fraction of the travel 20 m, but it will be understood that, in some cases of ventilated shocks, the pressure distribution may be intermediate between incompressible and compressible flow.

With the exception of the last-mentioned effect, it will be seen that the process of the ventilated shock is governed by inertial and gravitational forces only. Hence we have come to the following conclusion for a well-ventilated shock: The impulses, as well as the pressures, can be transferred from the model to the prototype by means of Froude's law. If the bubble content in the breaking front is high, and if the time of pressure rise is short, Froude's law will give prototype pressures that are somewhat on the conservative side, whereas the value of the impulse is unaffected. In this connection it should be remembered that the bubble concentration may be considerably higher in the prototype.

4. COMPRESSION SHOCK

If the breaking wave front approaching the vertical wall is concave (Fig. 4, left), the wave crest may hit the wall first, entrapping an air pocket and producing a compression shock. This effect is found more frequently in connection with breakwaters of type B (Fig. 2) than with A.

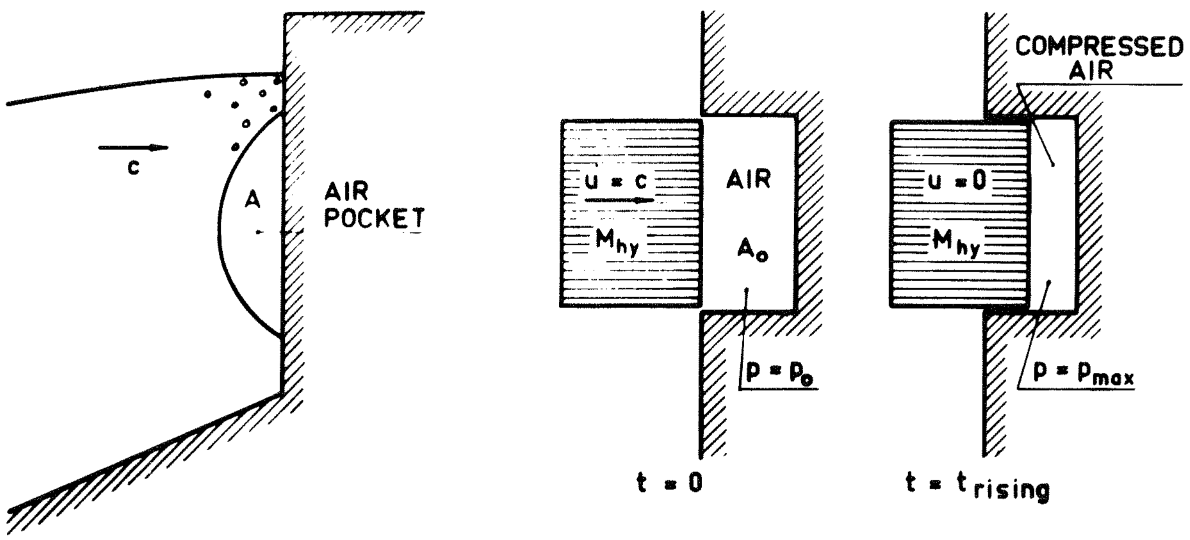

Fig. 4. Compression shock.

The air pocket acts as a spring the compression of which stops the horizontal movement of water. If the water velocities before "closure" are called c, the total impulse acting on the air pocket from $t = 0$ to $t = t_{rising}$ (maximum pressure in the air pocket) will equal the reduction

$$c\ M_{hy}$$

of horizontal momentum of the water. M_{hy} is again the equivalent hydrodynamic mass.

The process of stopping the water mass M_{hy} is rather similar to that described in Art. 3 with reference to Westergaard's formula (3.5), although there are some (minor) geometrical differences.

In halting the forward motion, some of the kinetic energy of M_{hy} is transformed into pneumatic energy of the air pocket, while the rest is transferred to vertical motion of the water. This vertical motion is maximum at the wall and is quite small at a distance of, say, a water depth.

A continued vertical motion, also resulting in a gradual rise of the water surface in front of the wall, is due to the nearly horizontal flow of water following behind the decelerated mass M_{hy}.

In addition to these vertical motions, that have their analogies in the ventilated shock, the air that escapes through the thin wave crest may give an "explosive" water splash along the wall.

With reference also to the bubble content in the water, it will be seen that the whole process is an entanglement of velocity fields, acceleration fields, compressibilities, bubble rising and capillary forces, making an exact translation from model to prototype impossible.

In order to obtain an approximate idea of the model law that should be applied, the water piston model in Fig. 4, right, is considered. This model is originally due to Bagnold (Ref. 1). A water piston of mass M_{hy} and velocity $u = c$ enters at the time $t = 0$ a cylinder filled with air of atmospheric pressure $p = p_o$. After the time t_{rising}, the pressure in the pocket has reached its maximum $p = p_{max}$, and the piston is stopped. During the short rising time, the loss of hydraulic energy into turbulence is sufficiently small to be neglected in an approximate shock theory. Hence, the value of p_{max} is most easily found by the energy equation.

The purpose of the analysis below is to compare the values of p_{max} in the model and in the prototype. At the instant of closure, $t = 0$, geometric similarity between model and prototype can be assumed to exist. When p_{max} is reached, the relative compression of the two air pockets is somewhat different but, since the volume of the air pocket is only a fraction of the total mass of water involved, the different compressions induce only a minor distortion of the geometric similarity at p_{max}.

The mass, M_{hy}, of the water piston is proportional to H^2, where H is a measure representing the wave height (in the model or in the prototype). Hence, we find for the kinetic energy of the water piston entering the air chamber

$$E_{kin} \sim \varrho H^2 c^2 \sim \varrho H^2 g H = \gamma H^3 \qquad (4.1)$$

The initial area A_o of the air pocket (air chamber) at atmospheric pressure p_o is

$$A_o \sim H^2 \qquad (4.2)$$

**

If the area is A at the pressure p, it follows from the adiabatic compression law that

$$p_o A_o^{1.4} = p A^{1.4} \qquad (4.3)$$

The maximum pneumatic energy stored in the air chamber is

$$E_{pneu} = -\int_{p_o}^{p_{max}} (p - p_o) \, dA \qquad (4.4)$$

where $(p - p_o)$ is the pressure difference between the front and the back of the water piston. Introducing the pressure ratio

$$r = p_{max}/p_o \qquad (4.5)$$

and, from (4.3),

$$A = A_o (p/p_o)^{-5/7} \qquad (4.6)$$

the integration in (4.4) yields

$$E_{pneu} = \frac{5}{2} p_o A_o \left[r^{2/7} - \frac{7}{5} + \frac{2}{5} r^{-5/7} \right] \qquad (4.7)$$

By expressing the assumption that the ratio of E_{kin} from (4.1) and E_{pneu} from (4.7) is the same for model and prototype, we find, with the use of (4.2),

$$r^{2/7} - 1.4 + 0.4 r^{-5/7} = k \frac{\gamma H}{p_o} = H^* \qquad (4.8)$$

where k is a dimensionless constant, and H^* a dimensionless wave-height.

Eq. (4.8) will be called the compression model law. Fig. 5 shows H^* as a function of $(p_{max} - p_o)/p_o$. Eq. (4.8) was first derived by Mitsuyasu (Ref. 5) by integration of the momentum equation. (Bagnold, Ref. 1, showed some examples of pressure-time curves obtained by graphical integration of the momentum equation.)

Fig. 5 may be used in the following manner for the interpretation of a model test: The diagram is entered with the value $p_{max,M}$ in the model, giving the dimensionless height H_M^* in the model. The corresponding value H_P^* for the prototype is obtained from H_M^* by multiplication with the linear scale ratio $H_P:H_M$. Finally, the diagram is entered with the value H_P^* yielding the value $p_{max,P}$ in the prototype.

It appears from Fig. 5 that around $H^* = 0.2$ the slope of the curve is 45^o, and hence the maximum shock pressure (in excess of the atmospheric pressure) is proportional to the wave height. Therefore, in a region around $H^* = 0.2$ the compression model law gives the same result as the Froude law. For small values of H^*, the maximum shock pressure is proportional to $H^{1/2}$, and for the highest values of H^* it is proportional to $H^{7/2}$.

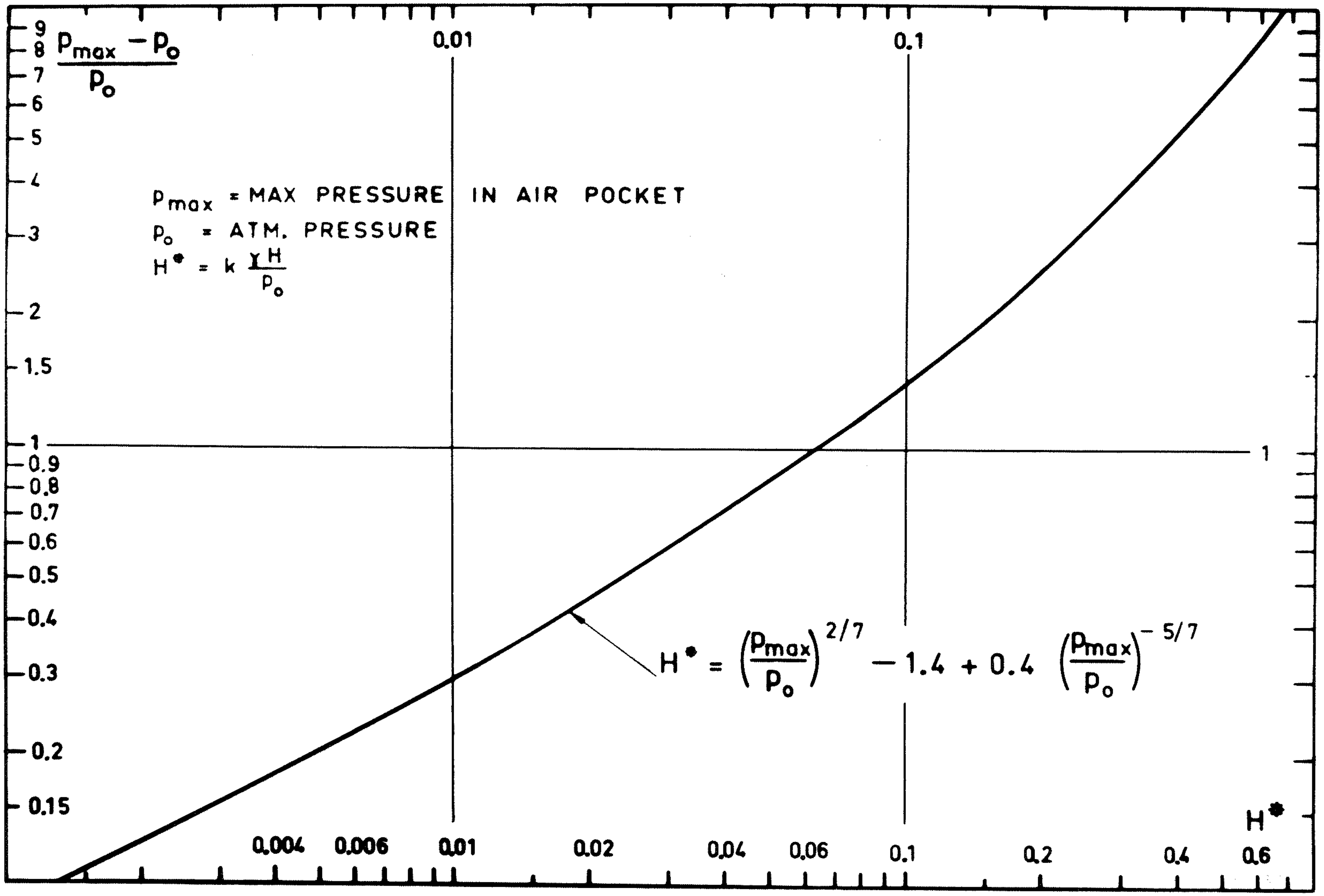

Fig. 5. Compression model law.

The table below gives two examples of the application of Fig. 5. In one example, the model peak pressure is measured to be 1 t/m² (= 205 lbs/sq.ft.). In the other, the model peak pressure is 5 t/m² (= 1025 lbs/sq.ft.). The values in the table are the ratios of the peak pressures in the prototype to the peak pressures in the model.

TABLE: Pressure Scale as a Function of Linear Scale

Linear scale	Pressure scale for a model peak pressure	
	1 t/m²	5 t/m²
10	3.5	6.0
20	5.4	12.4
50	10.0	48

Most of the values in the table are considerably smaller than corresponding to Froude's law (= linear scale).

After the maximum has been reached, the pressure in the air pocket drops off again for the following two reasons: (1) Some air escapes through the covering water. (2) The air pocket expands. The expansion may go so far as to create a negative pressure in the pocket, i.e. a pressure less than corresponding to still water level. Some model tests with compression shocks have shown several (strongly damped) oscillations of the pressure in the pocket.

For the interpretation of model tests the following conclusion on compression shocks is offered: Unless a more detailed analysis is carried out, the values of impulses can be transferred from model to prototype by means of Froude's law, whereas the compression model law applies approximately to the maximum pressures. This will give pressures a little on the conservative side if the concentration of bubbles entrained in the water is higher in the prototype than in the model.

5. HAMMER SHOCK

When the forward pointing crest of a plunging breaker hits the wall, as the introduction to a compression shock, it looks as if the wall is struck by a hammer (Fig. 4). Hence this type of shock will be called a hammer shock.

** A hammer shock is shown in greater detail in Fig. 6, which has been taken from a test by Hayashi (Ref. 2). The record of pressure cell B gave: $p_{max} = 2.8$ t/m² and $t_{rising} \sim 0.001$ s. Hence the total impulse, per unit area, delivered by the sharp-pointed hammer shock is about $i = 0.003$ ts/m².

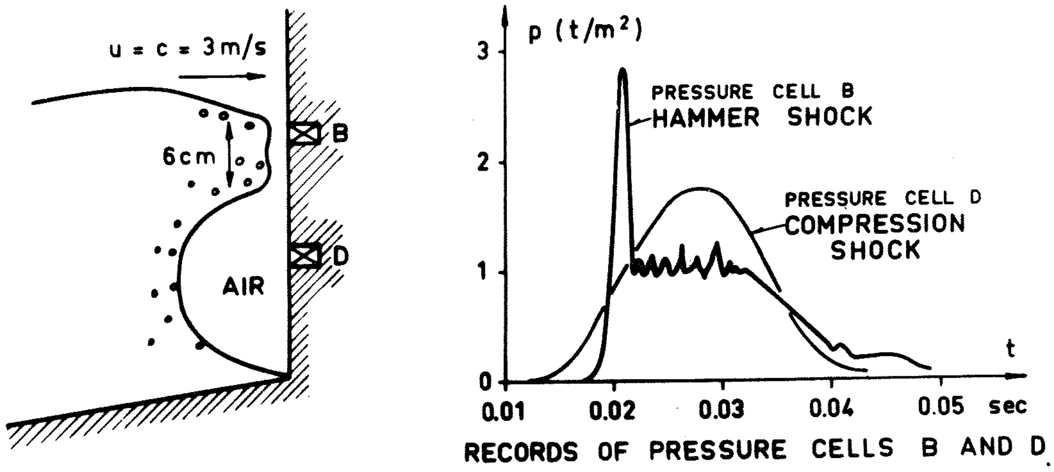

Fig. 6. Hammer shock.

The water velocity was measured to be $u = c = 3$ m/s. Thus the impulse has been able to stop a hydrodynamic mass, per unit area, of

$$m_{hy} = \frac{i}{u} = 0.001 \text{ t s}^2/\text{m}^3 \tag{5.1}$$

With $\varrho = 0.1$ t s²/m⁴, the "thickness" of the hydrodynamic mass is

$$b = m_{hy}/\varrho = 0.01 \text{ m} = 1 \text{ cm} \tag{5.2}$$

According to (3.5), the average thickness of the hydrodynamic mass of the Westergaard effect is 0.5 h and, because of the symmetry of the "hammer", $h = \frac{1}{2}$ 6 cm = 3 cm, giving

$$0.5 \text{ h} = 1.5 \text{ cm} \tag{5.3}$$

This value corresponds well to (5.2), pressure cell B being placed rather near the upper water surface.

It follows from the rising time that the irregularity of the front face of the hammer has been of the order of magnitude of 1 cm.

In extremely rare cases it is imaginable that the front face of the wave crest is so plane that a real water hammer occurs, i.e. an elastic wave in the bubble-containing water. According to von Kármán's formula (Ref. 7), the maximum pressure in a water hammer corresponding to the water velocity u is

$$p_{max} = \varrho \, u \, c_e \tag{5.4}$$

where c_e is the sound velocity. If, for the case in Fig. 6, we assume that $c_e = 100$ m/s (cf. Fig. 7), the water hammer pressure would be found to be

$$p_{max} = 0.1 \cdot 3 \cdot 100 = 30 \text{ t/m}^2 \tag{5.5}$$

which is an order of magnitude higher than that recorded on pressure cell B.

In less rare cases it is conceivable that a composite Westergaard/water hammer effect will occur.

6. SOUND VELOCITY IN WATER WITH AIR BUBBLES

For a medium of density ϱ and bulk modulus E the sound velocity is $c_e = \sqrt{E/\varrho}$. At 10°C and 35 ‰ salinity, an air concentration of ε in water gives

* $$\varrho = (1-\varepsilon)\,\varrho_w + \varepsilon\,\varrho_a = \varrho_w\,(1-\varepsilon+\frac{\varepsilon}{825}) \quad (6.1)$$

and

* $$\frac{1}{E} = \frac{1-\varepsilon}{E_w} + \frac{\varepsilon}{E_a} = \frac{1}{E_w}\,(1-\varepsilon+22{,}500\,\varepsilon) \quad (6.2)$$

where $E_w = 23{,}200$ kp/cm² corresponds to adiabatic compression, while $E_a = 1$ atm corresponds to isothermal compression because heat developed in bubbles is absorbed by the surrounding water within $t_{rising} = 0.1$ s for bubble sizes up to several centimeters.

Fig. 7 gives c_e as a function of the air percentage.

The above calculation of c_e based upon uniformly distributed air is justified when $c_e \cdot t_{rising} >>$ the bubble distance.

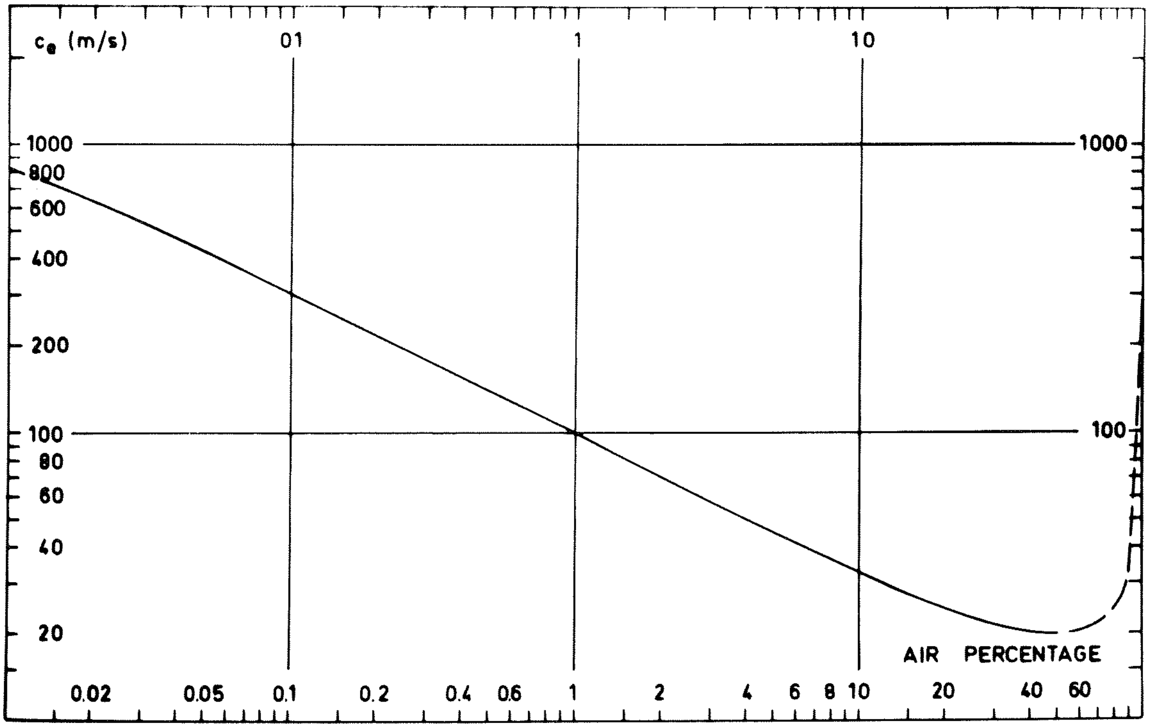

Fig. 7. Sound velocity c_e in water with air bubbles.

7. EXPULSION OF AIR

The expulsion of air from between a water face and a solid body approaching each other is a complicated phenomenon which can involve several physical aspects, such as: Subsonic and supersonic flow of the air, potential flow of the water due to the varying pressures in the air cushion, and compressibility of the water. This complex problem is of great interest in many fields, for example ship slamming, missiles entering water, etc.

Fig. 8 illustrates two examples of air expulsion. Both examples are based upon a high degree of idealization as compared to the problem of wave shock pressures.

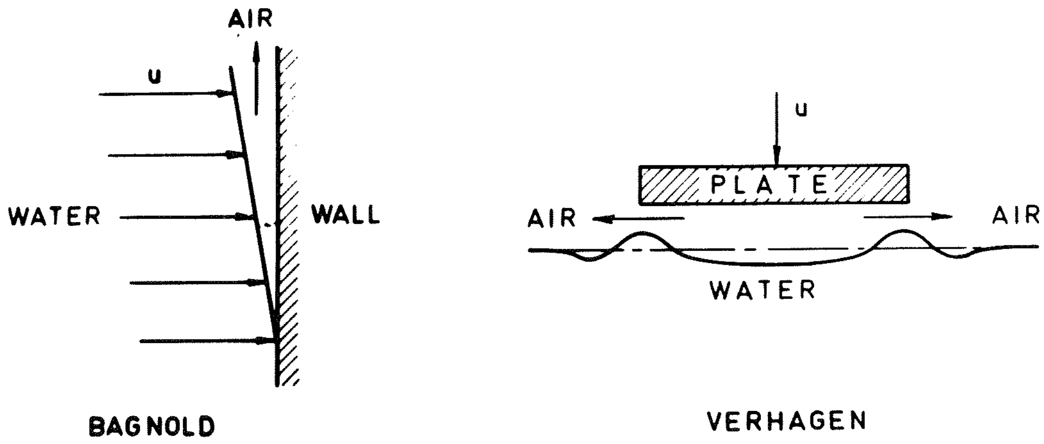

Fig. 8. Expulsion of air.

In 1939 Bagnold (Ref. 1), whose ideas have for 30 years had a predominant influence in the field of gravity wave shock pressures, discussed the example to the left. The free water front was assumed to be straight, and the water to move in horizontal filaments with constant velocity u, until the filaments are stopped by hitting the wall. Bagnold made some basic reasoning about this problem. Actually its complete solution would require the assistance from several chapters of Mathematical Physics.

In 1967 the example to the right in Fig. 8 was discussed by Verhagen (Ref. 6). The water is initially at rest and the underside of the falling plate is completely horizontal. Verhagen has analyzed this problem with great physical and mathematical care. He gives two numerical examples where the theoretical pressure-time histories are corroborated by experiments within an error of 25% on the pressures and 0.5 milliseconds on the times!

Both of the foregoing simplified approaches have been valuable in shedding light on some of the physical processes involved, but due to the extreme irregularity of natural breaking waves, there seems to be little prospect of achieving a detailed theory at the present time without painstaking experimentation. For this reason the present author is unable to contribute to the discussion of expulsion of air.

8. INTERACTION BETWEEN WAVE SHOCK FORCES AND FOUNDATION

This article will discuss the effect of the wave forces on the foundation soil, the resulting rocking of the breakwater, and the feed-back effect this rocking might have on the shock forces. This subject is so extensive that the author's conclusions must be tentative, and he would be only happy if future, joint coastal/soils research would "rock" his present conclusions.

It is quite clear that the dimensions of the waves are so large compared with the horizontal motions of the breakwater that the "normal", slowly varying wave forces, for example from a clapotis, are absolutely independent of these motions. However, it is the main conclusion of the considerations below that, for all practical purposes, the wave shock forces are also independent of the motions of the breakwater, perhaps with the exception of a minor influence in the case of breakwaters founded on soft clays and silts.

Some of the most important aspects of breakwater rocking have been discussed by Hayashi (Ref. 3), who has also developed pertinent mathematical theories. This article is much less ambitious: We shall mainly attempt to enumerate the hydrodynamical and geotechnical factors involved.

8.1 Kinematics of Rocking

The movement of a breakwater under the action of a train of irregular waves is a complicated stochastic process. However, because of the relatively small motions and the reversible character of the soil deformations under repeated loadings, it will often be permissible to linearize the response of the breakwater to the forces. Thus the stochastic processes of wave forces may be substituted by their spectra.

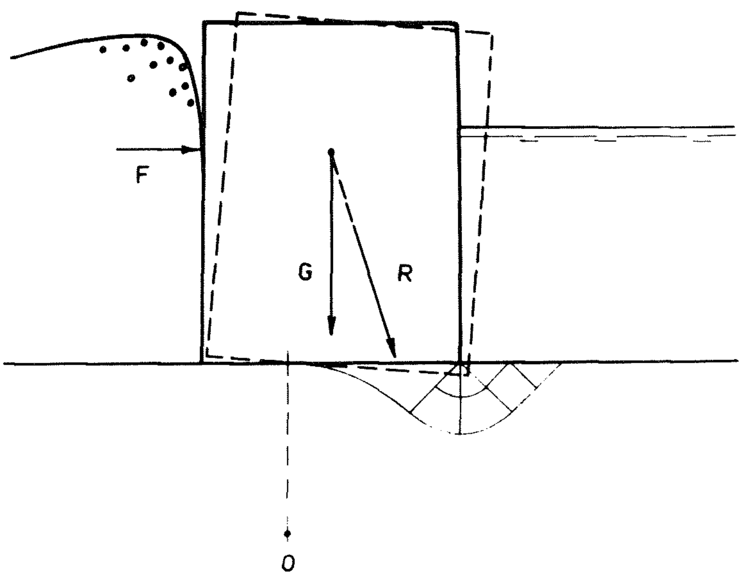

Fig. 9. Rocking of breakwater under wave action.

Fig. 9 shows this main oscillation symbolically: Under the action of the varying wave force F the breakwater rotates about the point O, the dashed cross section showing the extreme position to the right. Since the vertical stresses on the ground from F are larger than the horizontal ones, the vertical motions of the lower edges of the breakwater will be larger than their horizontal motion. The position of O must be determined from geotechnical considerations. At the nominal failure load it lies approximately on the vertical line through the point where the rupture line is tangential to the (rough) base.

8.2 Forces Involved in Rocking

** The following is a list of forces involved in the rocking of a breakwater. In individual cases these have to be estimated to ensure that all important forces are included in the mathematical analysis.

(1) Wave forces: The resultant wave force (horizontal component, vertical component, overturning moment) can be recorded in a model test by means of a strain-gauge dynamometer that carries a whole section of the breakwater. By means of the pertinent model laws, the record is converted to prototype dimensions.

(2) Inertial forces of breakwater: These consist of the horizontal and vertical components of a force acting through the centre of gravity, together with a moment of rotation.

(3) Inertial forces from the water: For an arbitrary motion of the cross section, the normal concept of a "hydrodynamic mass" is not sufficiently clear. For each spectral component, the motion can be assumed to be a small harmonic oscillation, to which the inertia of the water reacts through harmonically varying pressures on the two sides of the breakwater, these pressures being maximum at the two extreme positions, where the accelerations have their maxima. These inertial water pressures can be determined graphically by means of a so-called acceleration net. An acceleration net is completely analogous to the flow nets used in investigating seepage through soils. (Both problems are governed by Laplace' equation, because - in the breakwater case - the water velocities are so small that the convective terms are negligible.)

(4) Damping forces from the water: For each sinusoidal oscillation, the breakwater acts as a wave generator (to both sides). The corresponding pressures on the breakwater are proportional to the angular velocity in the rotation. For long-period oscillations, for example corresponding to the wave period, the damping forces are much larger than the inertial forces under (3) above. For oscillations of shock-force "periods", the damping forces are negligible.

(5) "Slit" pressures under the base: While the wave pressures transmitted to the pore water under the base are included in (1) above, special water pressures may occur in the "slit" that opens and closes on the seaward side when the breakwater moves. The importance of these "slit" pressures will depend on the permeability of the foundation. The slit pressures will be negligible if the structure rests on a permeable rubble foundation or on very permeable rock, as well as if there is an effective seal at the seaward face or a drain (not filled with sand!) at the rear face. If, on the other hand, underwater concrete is cast on impervious rock, the base suction created by a rapid opening of the slit is very high. Special pre-

cautions may have to be taken against the following possibility: The "slit" opens under maximum wave pressure, but the outflow of water is hampered when the wave load reduces again, with the result that the breakwater comes to rest on a "water sheet", sliding backwards.

(6) Reactions from the ground: The distribution of the reactions depends upon the foundation conditions. Because of the hysteresis, most soils will give some damping effect which, however, is believed in most cases to be insignificant.

8.3 Response of Breakwater to the Forces

For the first few loadings of high waves, the foundation soils will undergo initial, irreversible deformations, whereas the deformations from later loadings will be reversible, i.e. much smaller than the first loading, both for sands and clays. Hence, under the action of shock forces, the first high wave will normally be less dangerous because the natural frequency of the breakwater is lower than for the subsequent waves.

Because of the many factors involved, it is an extremely difficult task to calculate the natural frequency of a breakwater, and it must be stated that a correct calculation requires geotechnical investigations and advice of highest quality.

It should also be remembered that foundation conditions usually vary a good deal along a breakwater, so that approximate calculations only are justified. If such calculations show that the natural frequency of the breakwater roughly corresponds to one of the "periods" of large shock pressures, it should be noticed that the amplification factor of a triangular impulse, cf. pressure cell B in Fig. 6, may be as large as 1.7 at "resonance" conditions. If a factor of 1.7 is applied to a triangular shock force, it would seem reasonable to reduce somewhat the normal factor of safety.

With due consideration of all the factors mentioned above, the safety of the breakwater against failure can be investigated. It is well known that this investigation falls in two parts:
(1) Rupture in the foundation under the resultant R, Fig. 9.
(2) Sliding of the breakwater (Ref. 4). With a view to the latter, the underside of a breakwater resting on shingle should always be rough over the area where R is acting.

8.4 Breakwaters on Sand

On coasts with littoral drift, breakwaters will often be founded on sand. When waves pass over a sand deposit, they induce seepage gradients that vary in size and rotate, with the results that the sand becomes very dense, except near the sea bottom. As an example, it may be mentioned that the triaxial in-situ frictional angle was found to be $\varphi = 43^{o}$ for the Kish Bank lighthouse outside Dublin, Ireland.

It will be easily understood that, after a few large waves, the reversible deformations of such dense materials are very small, giving a high natural frequency of the breakwater. As a matter of fact, the geotechnical investigation of these dense materials requires special laboratory equipment.

The loading time from shock pressures is so short that the major part of the sand mass involved can be assumed to be un-drained.

In order to obtain an idea of the behaviour of an undrained sand, The Danish Geotechnical Institute was requested to run a triaxial test on the much studied, so-called G-12-sand. Excessive experimental difficulties were avoided by choosing the void ratio as large as $e = 0.66$, corresponding to a triaxial angle of only $\varphi = 32^{\circ}$. Even under these circumstances, it was not possible to reach the so-called end point of failure lines defined by K. H. Roscoe et al. (see p. 29 of the paper "On the yielding of soils", Géotechnique, vol. 8, p. 22-53, 1958), although the deviator stress reached the value $\sigma_1 - \sigma_3 = 110$ t/m². The potential dilatation during the extensive yielding from 8 to 110 t/m² was eliminated by the consolidation under the increasing stresses.

From the experience with this test and with a large series of tests on dense boulder clays, it can be concluded that the ultimate failure of a dilatant undrained sand will correspond to a frictionless soil of shear strength c, where

$$2\,c = \sigma_1 - \sigma_3 \qquad (8.1)$$

corresponds to the end point of failure lines, in the plane case. Hence, for a shock-force failure, the rupture lines in Fig. 9 intersect at right angles.

From present experience, it is also tentatively concluded that, after the loading from several large waves, the negative pore water pressure produced by the maximum wave at the end point of failure lines is a fraction only of the effective σ_3. This means that a longer duration of the loading does not reduce the bearing capacity of the sand too much.

8.5 Breakwaters on Preconsolidated Clays

For preconsolidated clays the ultimate failure circumstances are rather similar to those mentioned above for undrained sand.

8.6 Breakwaters on Rocks

In addition to the "normal" rock problems of engineering geology, it may be mentioned that the repeated shock loading from waves may produce undesirable effects in brittle materials, such as certain sandstones.

As a special example, it can be mentioned that the Hanstholm breakwaters, Denmark, are founded on chalk, part of which is indurated (hardened), being highly siliceous. In many areas, however, the unindurated chalk dominates. According to certain experiences, it is believed that the "connections" between the grains of this material are very brittle. This was one of several reasons for choosing a design with a chamfered edge, thus eliminating shock pressures almost exclusively (Ref. 9).

9. ACKNOWLEDGEMENTS

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The author is also much indebted to his friend, Dr. Bent Hansen, of The Danish Geotechnical Institute, for a most stimulating exchange of viewpoints with respect to the response of saturated sands to wave forces, however, responsibility for any geotechnical mistake in this paper is exclusively the author's.

Finally, my best thanks are due to K. L. Philpott for his assistance in editing this paper.

10. SYMBOLS

A	(m^2)	Air volume per unit length of breakwater
a	(m/s^2)	Acceleration
c	(m/s)	Wave celerity
c_e	(m/s)	Sound velocity
E_{kin}	(t)	Kinetic energy per unit length of breakwater
E_{pneu}	(t)	Pneumatic energy per unit length og breakwater
F	(t/m)	Wave force per unit length of breakwater
G	(t/m)	Effective weight per unit length of breakwater
h	(m)	Water depth
H	(m)	Waveheight
H_b	(m)	Height of breaking wave
H*		Dimensionless waveheight
I	(t·s/m)	Total impulse per unit length
i	($t\cdot s/m^2$)	Impulse per unit area of breakwater face
k		Dimensionless constant
M_{hy}	($t\cdot s^2/m^2$)	Hydrodynamic mass per unit length of breakwater
m_{hy}	($t\cdot s^2/m^3$)	Hydrodynamic mass per unit area of breakwater face
p	(t/m^2)	Pressure
p_o	(t/m^2)	Atmospheric pressure
p_{max}	(t/m^2)	Maximum shock pressure
R	(t/m)	Resultant force per unit length of breakwater
r		Pressure ratio p_{max}/p_o
T	(s)	Wave period
t	(s)	Time
t_{sh}	(s)	Upper bound of duration of shock pressure
t_{rising}	(s)	Rising time of shock pressure
u	(m/s)	Velocity of water
γ	(t/m^3)	Specific weight of water
ϱ	($t\cdot s^2/m^4$)	Density of water
σ	(t/m^2)	Normal stress in soil
τ	(t/m^2)	Shear stress in soil
φ		Frictional angle

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WAVE FORCES ON THE EIDER EVACUATION SLUICES

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SUMMARY

For a safe and efficient dimensioning of the Eider Evacuation Sluices it was necessary to know the magnitude and probability of the occurring wave forces.

To determine those data a model investigation has been carried out in one of the wind flumes of the Delft Hydraulics Laboratory in charge of and in co-operation with the Bundesanstalt für Wasserbau in Karlsruhe.

For this purpose it was necessary to consider all combinations of wave conditions and waterlevels in nature which can lead to important wave forces, taking into account their probability of occurrence. It was on these grounds that the conditions for the model tests were chosen.

The results of the model investigation had to be translated into probabilities of exceedance of the wave forces per year.

Taking into account the results of the model tests as well as the general knowledge about the distribution of the wave forces, suitable load figures have been determined especially for the dimensioning of the structure. Only this extensive investigation could provide the guarantee of a safe and efficient dimensioning of the structure against the impacts of breaking waves.

1. INTRODUCTION

The improvement of the water-levels and the conditions for the navigation in the Eider, which is threatened by a regularly continuing sedimentation, as well as the safety at stormflood conditions and the protection of the coast make it necessary to dam up the tidal Eider on the West coast of Schleswig-Holstein (Figure 1).

The projected damming-up consists essentially of a dike with a length of about 4 km, to give safety against storm-floods and of a complex of evacuation sluices with five openings, each with a span of 40 m. These evacuation sluices will be generally open, so that stream caused by the tide can pass freely.

Only at storm-flood the sluice complex will become an enclosure structure, if the gates are closed in time.

The sluice complex is to be built of reinforced concrete, and because the ground which has sufficient ability to bear is only found at rather great depth, the structure will be founded on piles.

To give double safety, two segment gates are used in each opening, each fixed to a tensioned concrete bridge with a span of 40 m (Figure 2).

Because of the function of the evacuation sluices and their situation being strongly exposed to waves from the open sea, the possibility has to be reckoned with that with high wind velocities breaking waves can attack the sluices through the existing channel or through a channel which will possibly be built in the mouth of the Eider. For this reason the construction will have to withstand not only quasihydrostatic forces, which have a progress in time corresponding to the wave period, but also very high wave impacts.

2. MODEL TESTS

For a secure and efficient dimensioning of the evacuation sluices, it was necessary to know the magnitude and the accessory probability distribution of the occurring

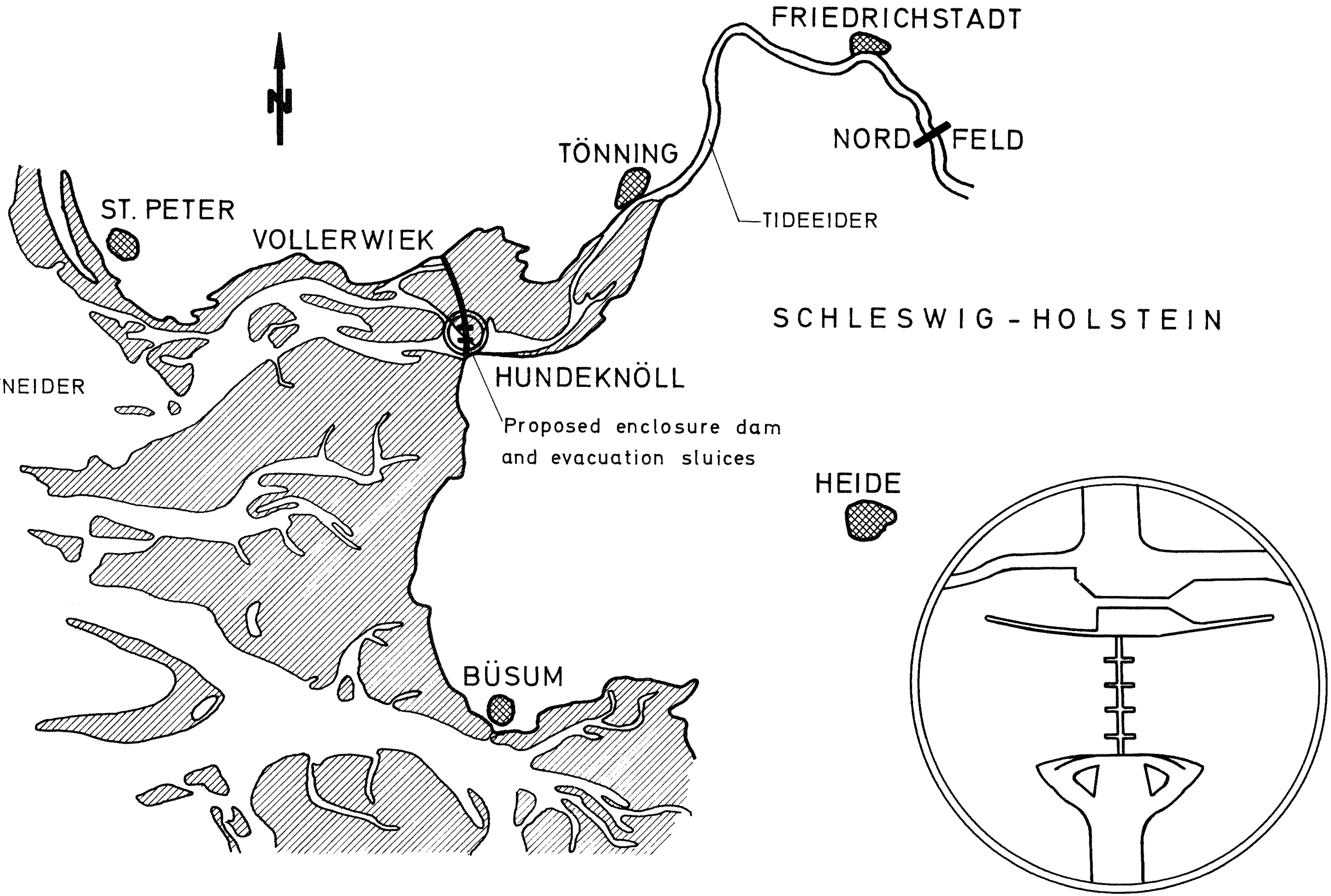
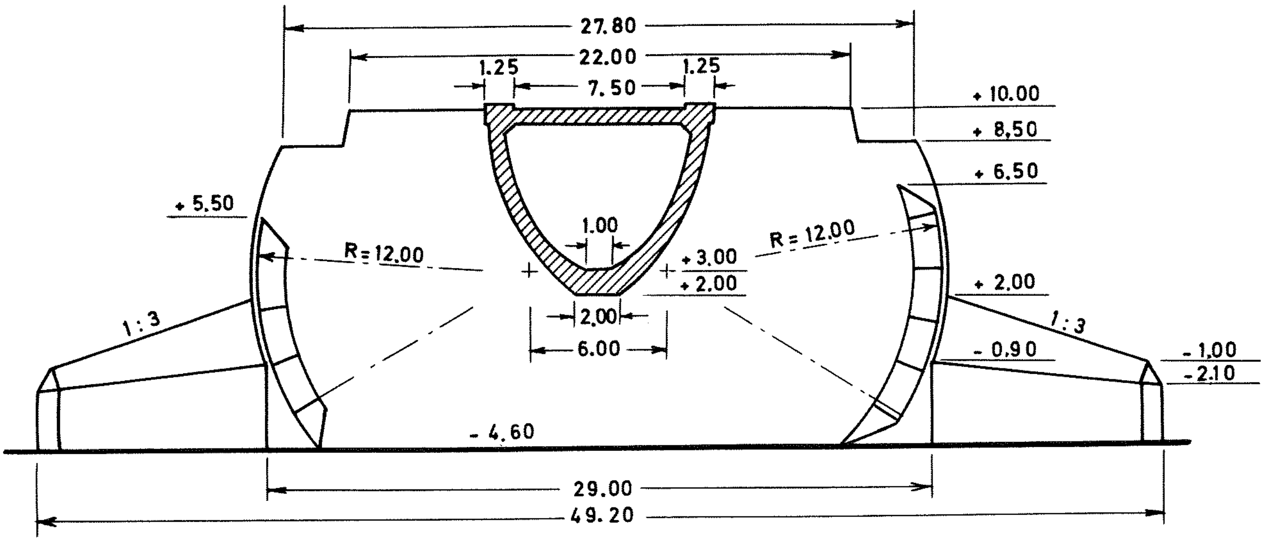


Fig. 1 General plan of the Eidermouth



Dimenensions related to protoype (m).
All heights are related to NN

Fig. 2 Cross-section of the evacuation sluices

wave impacts. This has been determined by model tests (Ref. 6). The wave impacts do not depend only on the shape of the structure and the bottom configuration in front of it, but also and in an important way, on the specific characteristics of the waves, including the breaker phenomena.

Because it is possible to imitate these phenomena very well by wind-generated waves, the model investigation has been carried out in the wind flume of the Laboratory "De Voorst" Section of the "Delft Hydraulics Laboratory". The investigation was commissioned by the Bundesanstalt für Wasserbau in Karlsruhe and has been made in close co-operation between the Bundesanstalt für Wasserbau and the Delft Hydraulics Laboratory.

For the determining of the programme of the model tests, which were carried out on a scale 1 : 25, it was taken into account that it is important for the dimensioning of both the segment gates and the tensioned concrete bridge, including the supports, to know both the local wave impacts and the sum forces which occur at the same time on different parts of the structure. In connection with this, different situations of loading have to be distingnuised:

A = Outer gate closed
Wm = Concrete bridge when outer gate has been closed
Im = Inner gate, when outer gate has been closed
Wo = Concrete bridge, when outer gate is omitted
Io = Inner gate, when outer gate is omitted

It is possible that the outer gate will be missing such as when it will be taken out for repair. In this case, the waves will directly attack the concrete bridge and attack the inner gate.

The situation that the outer gate has been left can be kept out of consideration, because it will always be possible to close this gate by its own weight.

The loads were determined by the use of pressure cells, by which for practical reasons at most the signals of fice pressure cells, along with the overall sum or the sum of a number of cells were recorded. The sum force which acts on the total surface of a segment gate or the concrete bridge was determined with the help of two measurements which were carried out separately. During one of them the pressure cells were placed in a horizontal position and during the other one in a vertical position. The method will be further described later on.

The pressures in the model were measured on a light-beam recorder. The paper velocity was relatively low, so that it was only possible to determine the magnitude and the number of the wave impacts for all pressure cells, but no idea was obtained about the time-pressure history. For this reason it was decided that for important cases the pressures were also recorded on tape. This made it possible to reproduce these impacts in more detail.

Because of the curved shape of both the segment gates and the concrete bridge, the slope of the front face of the concerned part of the structure at water-level depends on the water-level. For this reason the water-level was chosen as a variable.

To determine the way by which the wave impacts are influenced by the dimensions of the waves, at each combination of situation and water-level two different wave spectra were tested. This gave the possibility to interpolate and extrapolate to other wave-heights if necessary.

For all relevant combinations of situation, water-level and wave characteristics the pressures were measured in the vertical in the middle of the structure and in the horizontal. For the horizontal the level was chosen at which the greatest wave impacts occurred during the measurements in the vertical.

To obtain data on the pressures near the corner for important cases also measurements were made in the vicinity of piles.

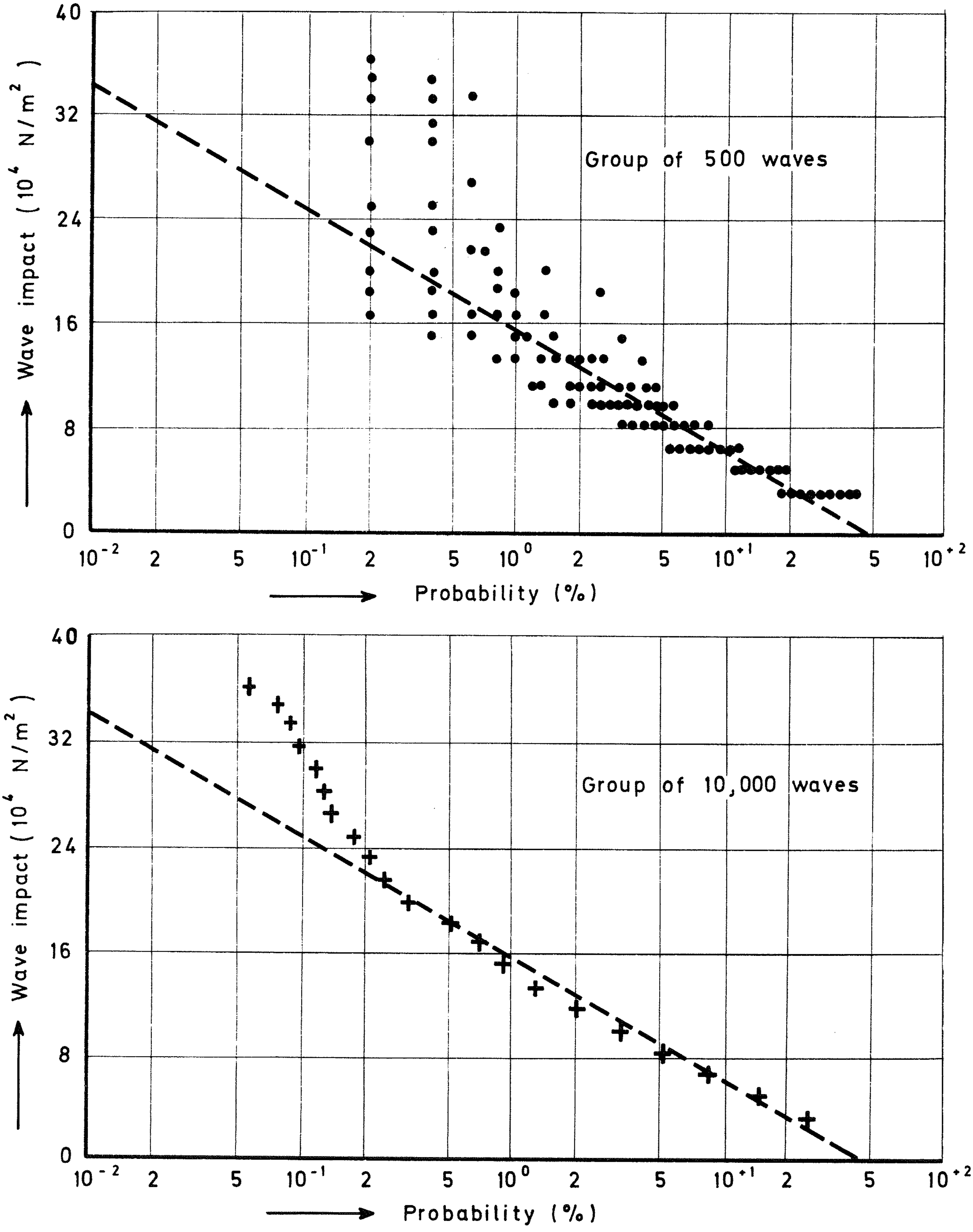
The magnitudes of the wave impacts measured show a very irregular character, as could be expected in view of other tests carried out in the past. For this reason the results at each measuring point were reproduced in terms of probability distributions, related to the number of the applied waves, for all combinations of situation, water-level and wave characteristics tested. This led to about 400 different curves.

** To obtain a better knowledge of these probability distributions, especially for small probabilities, a test of long duration was carried out for one of the combinations of the conditions measured. The results of these test, for which a duration of more than 10,000 waves was chosen, are shown in a compressed shape in Figure 3.

From the results of this test the following conclusion can be drawn:

1. From the comparison of the results of the tests with 500 and 10,000 waves, and with the latter also separated into groups of 500 - 1,000 - 2,500 - 5,000 and 10,000 waves, it appeared that the scatter in the magnitude of wave impacts decreases according by as they grow in number.

2. The scatter of the greatest wave impacts of a group is rather great as has been found also by other investigators. (Ref.3)



** Fig. 3 Relation between the wave impacts and their propability, related to the number of waves

Based on the results, a logarithmic relation between the impact and the accessory percentage of exceedings was established. According to this relation, all the necessary extrapolations to numbers of more than 500 or 1,000 waves were carried out.

* A point of contact for the difficulties about the
** extrapolation can be found in literature.

As already explained from the model results both the design local pressure and the design sum force had to be determined.

The maximum local pressure is defined as the greatest impact, and this one occurs often about at the average water-level or something above.

The sum force is obtained from the mean value of the impacts occurring on the total surface of the segment gates or the concrete bridge at the same time. This was determined from the probability distribution curves of the measurements in the horizontal and in the vertical. The method which has been used was as follows:

The separate notations can become clear from Figure 4 and the list of symbols.

The sumload can be written as

$$P\ (x) = 1 \cdot h^{1} \cdot \frac{\bar{P}h(x)}{P^{+}(x)} \cdot \bar{P}_{v}\ (x) \qquad (1)$$

Herein $\frac{\bar{P}h(x)}{P^{+}(x)}$ is called α, which is a distribution factor (2). The factor α being smaller than unity shows that the wave impacts which occur in the vertical do not necessarily occur at the same time over the whole span of the gate. This leads to a three dimensional result of the investigation.

From the tests it has been found that the factor α varies considerably with the shape of the structure. This can be seen from an example shown in Figure 5. Attention has been paid also to the variability of the factor α with the percentage of exceedance of the waves. This factor increases when the

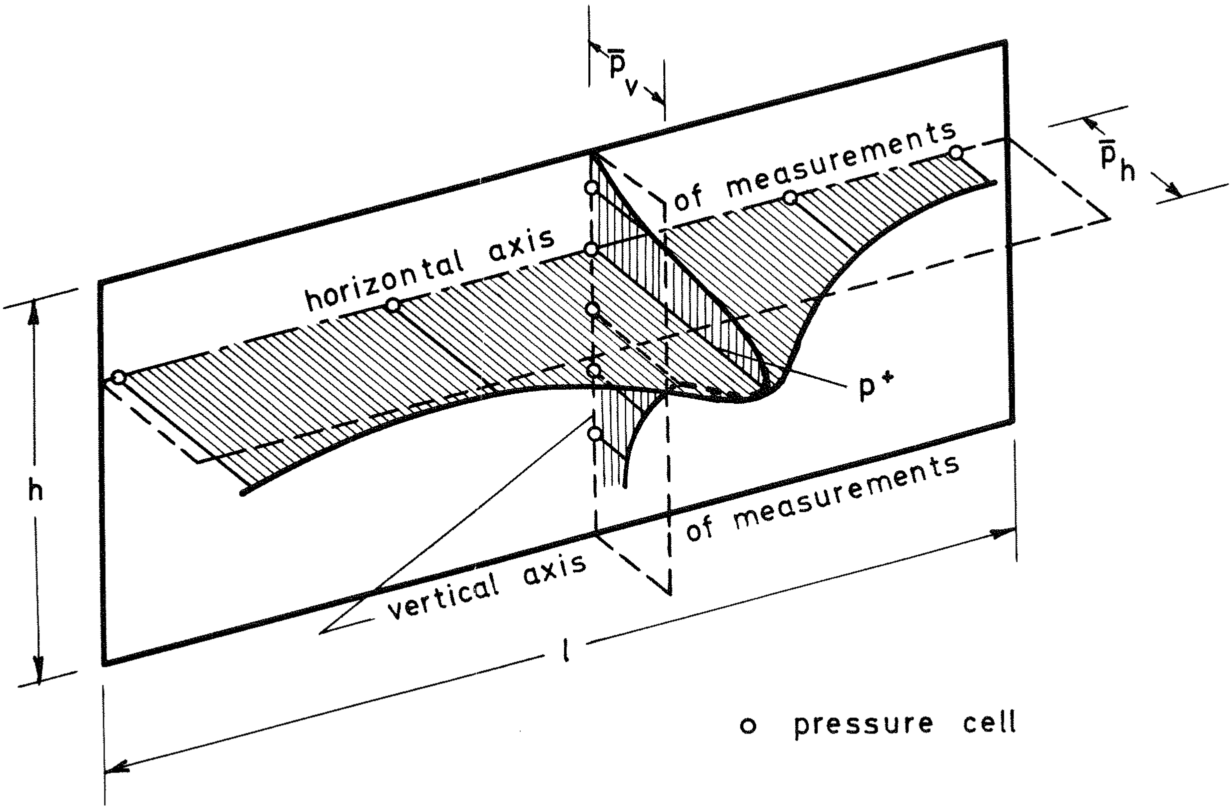


Fig. 4 Example of the three-dimensional view on the load distribution over the surfaces

percentage of exceedance increases.

It reaches theoretically the value 1 if the wave crests are precisely parallel to the front of the structure in which case the same pressure occur along the whole gate or bridge at the same time.

From the model tests for the outer gate it was found furtheron that the factor α increases when the water-level is decreasing. The reason for this can probably be found in the curved shape of the outer gate.

3. CALCULATION OF PROBABILITIES

For the dimensioning of the evacuation sluices and the bridge the starting-point has to be that the structure must resist the wave forces which occur with a chosen probability. To meet this requirement, it was necessary to make a calculation of probabilities.

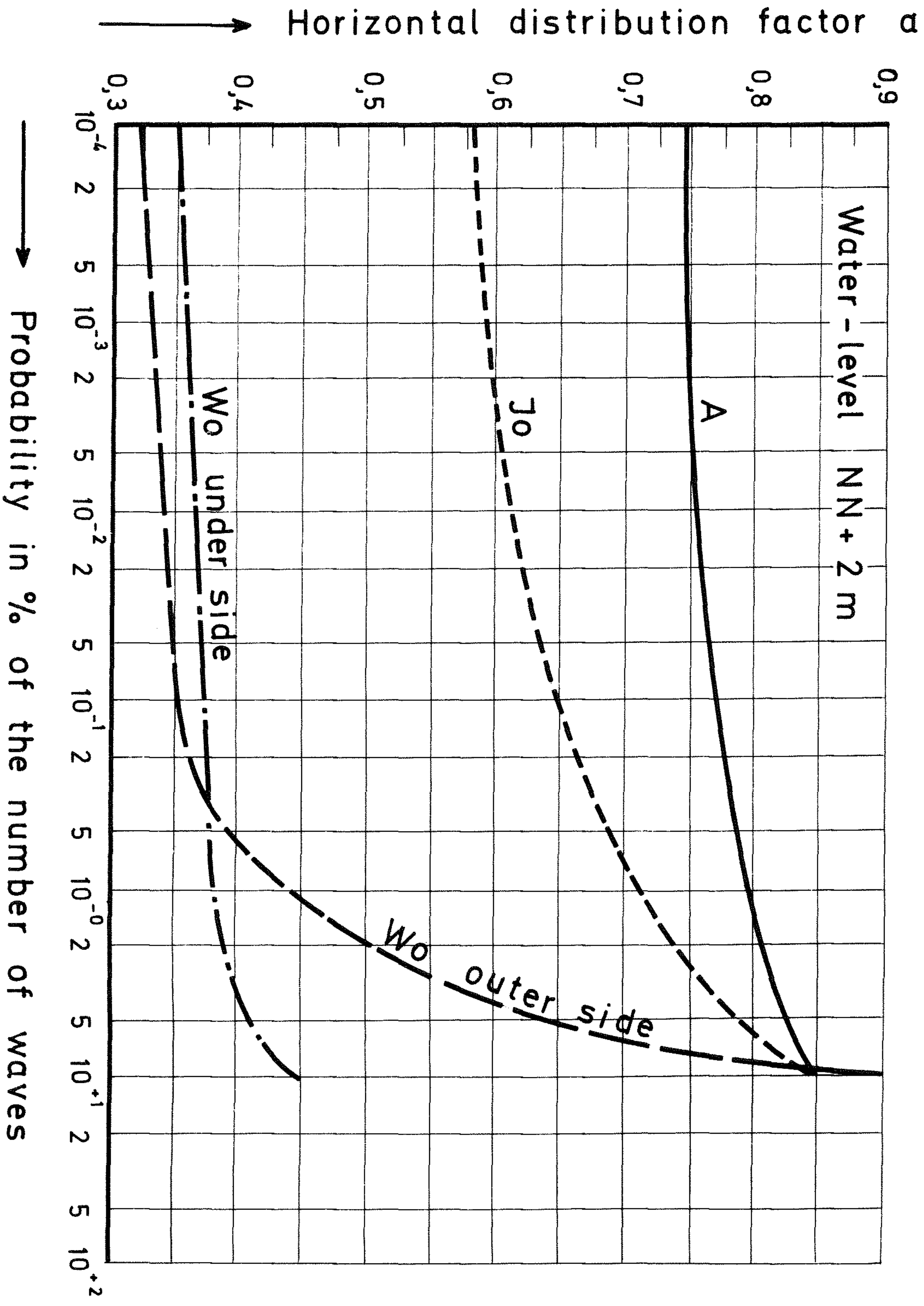


Fig. 5 Examples of the horizontal distribution factor

In this a probability distribution of the pressures was determined, taking into account for each situation both the probability of exceeding of the pressures at fixed wave spectra and the probabilities of the different occurring combinations of water-level and wave characteristics for the concerned situations. These are caused by the tide and the velocity, the direction and the duration of the wind.

For this purpose prototype measurements were available but the short time during which these measurements have been carried out made it necessary to extrapolate the prototype data to the small probabilities required.

** The probability distribution of the water-level is shown in Figure 6. This has been determined by measurements at the tide gauge at Tönning carried out from 1951 upto and including 1960. The short measuring period also made it necessary to extrapolate as the water-level with a probability of 10^{-3} per year is needed. This extrapolation is facilitated somewhat by the high water of 1962, with a probability of exceedingance of about 10^{-2} per year.

With respect to the probability distribution of the wind velocities from a westerly direction which determine the wave ** forces, only few prototype data were available (Figure 7), so that it was necessary to estimate more or less the probability distribution needed for an extrapolation to 10^{-3} per year.

A valuable control was obtained from the following considerations.

A relation is know between the high water-levels, occurring for the wind direction west, and their probability of exceed. A relation between the high water-level and the wind velocity was wanted for the wind direction west, which can be representative also for the directions W.S.W. and W.N.W. If the last relation can be found the water-level can be eliminated from the two relations, so that the probability curve of the wind velocities is obtained.

For the prototype data on the relation between the high water-level and the wind velocity measurements at Tönning were available.

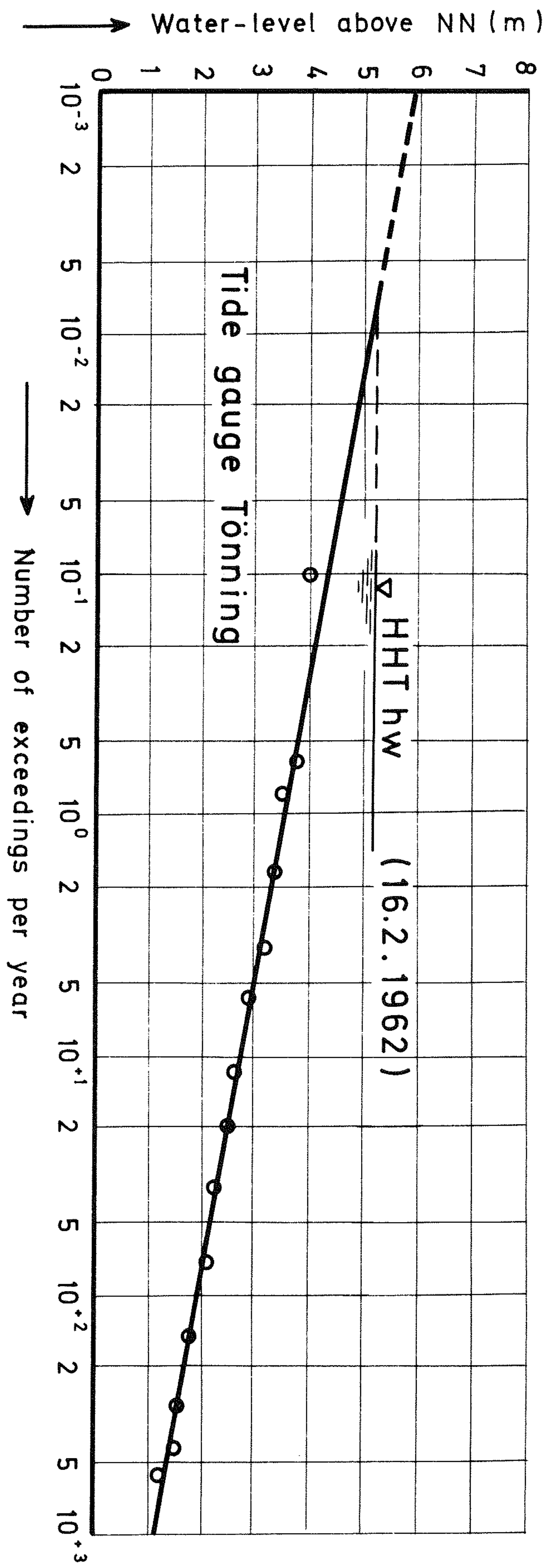


Fig. 6 Probability of the high water-levels

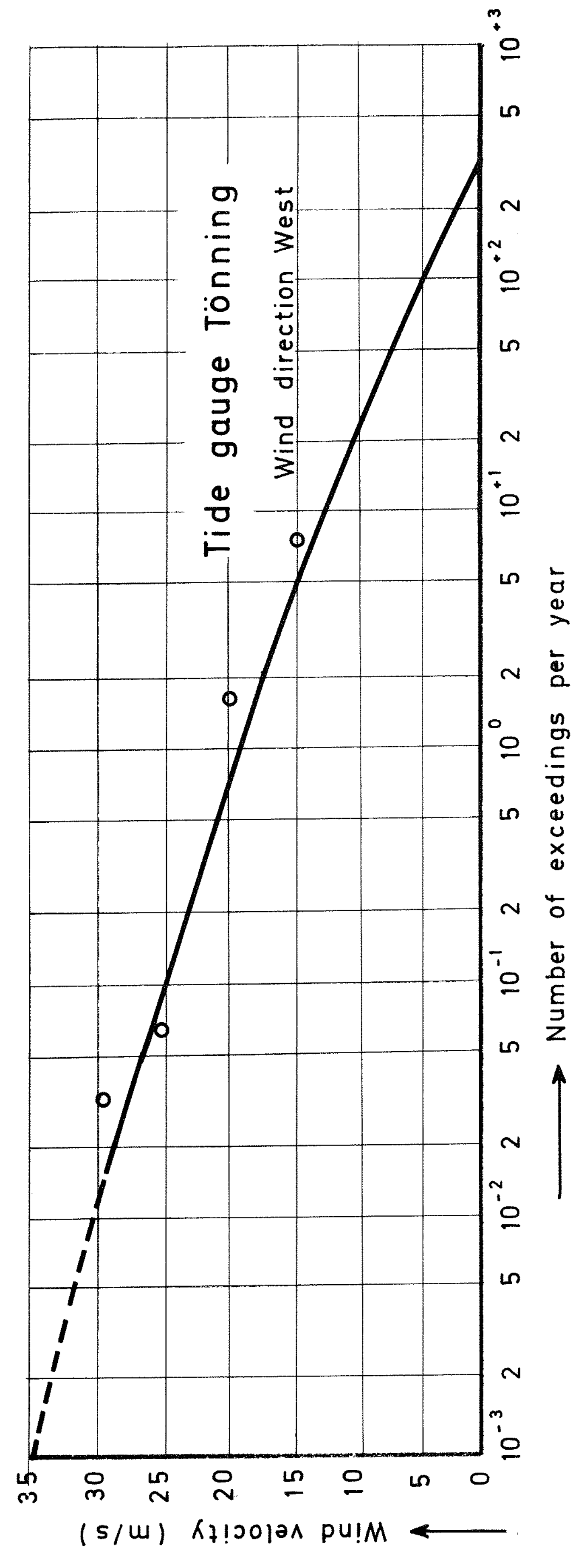


Fig. 7 Probability of the wind velocities

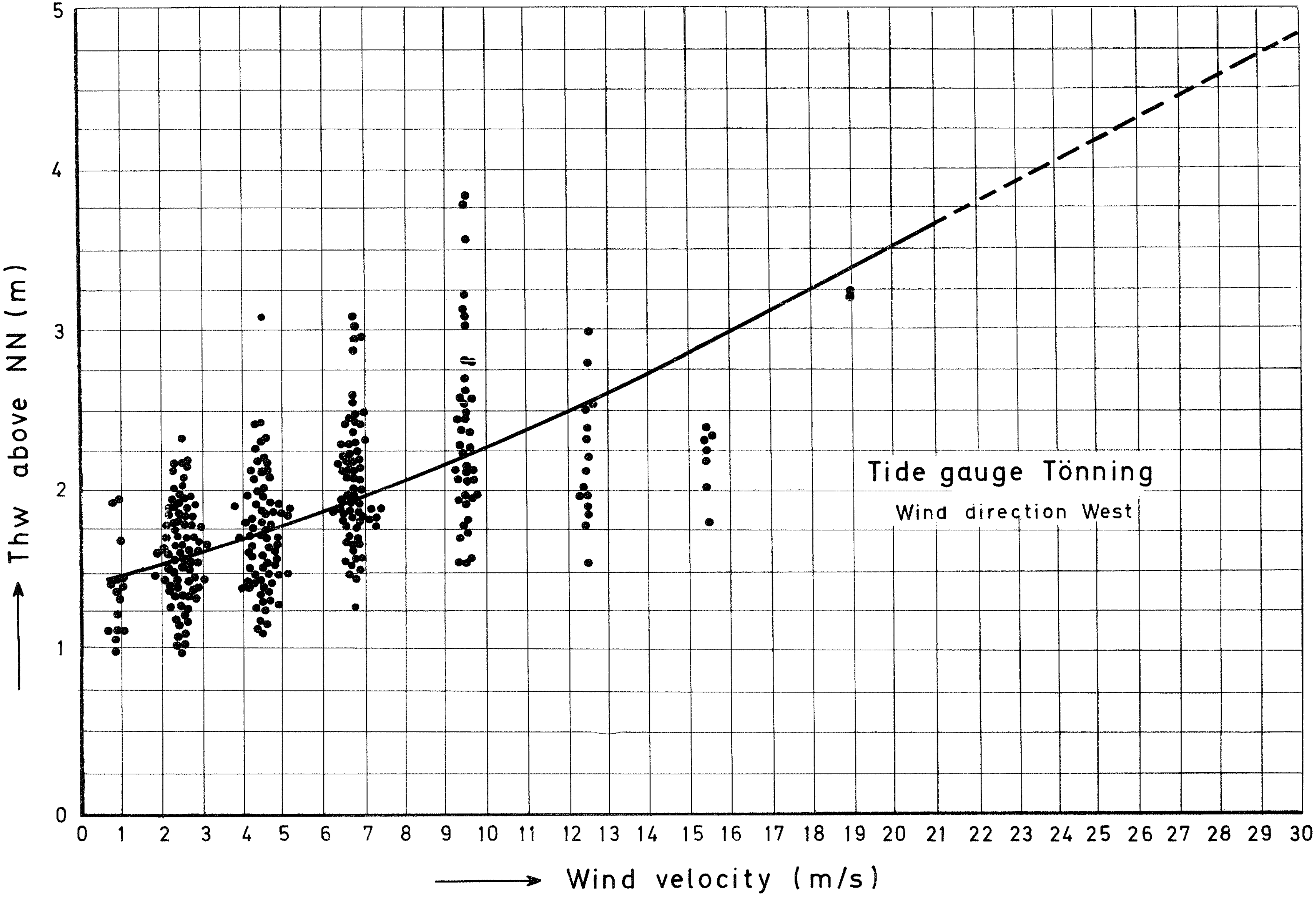


Fig. 8 Relation between high water and wind velocity

** From Figure 8 it appears that a great scatter exists. The reason for this can be found in the fact that the wind set up in Tönning is not only caused by the wind in this region but also by the wind in a region that lies essentially more seaward. The duration of the wind in both the regions can have shifted. In spite of the great scatter, still average relation between the high water-level and the windvelocity can be determined.

The results of this correspond rather well with the relation obtained before. It has been concluded, for instance, that the water-level NN + 5 m corresponds to a mean wind velocity of 30 m/s and the probability of both of them equals 10^{-2} per year.

As also the latest measurements during the storms on the February 23 and October 17, 1967, agree rather well with this correlation, the extrapolation to a probability of 10^{-3} per year seems to be reasonable.

Also sufficient prototype data were not available about the waves especially for high wind velocities. Moreover, the influence of the expected change of the channel system and the shoals on the seaward side of the structure after it has been put into use was necessarily lacking. For this reason, the wave conditions were calculated with the aid of the theories about wave generation by wind and the knowledge about the change of waves by changing water depth (Refs. 1,2,5).

In the present case it had to be considered that possibly waves from the North Sea enter the shallow region in front of the structure. These waves are reduced by the restricted water depth and can later grow again by wind from a westerly direction in the straight channel which will be eventually formed in front of the structure. In this case the maximum possible wave characteristics are still restricted by the water depth in the channel concerned. Information about this can be found in literature.

The assumption of the building-up of a straight channel in front of the structure and its water depth, is of great importance.

The probability calculation for the critical westerly wave direction was carried out with the aid of the scheme shown in Figure 9.

Here the probability calculations had also to be carried out separately for the different situations and for the different critical water-levels. As the intervals of the water-level and the wave heights obtained from the probability calculation did not agree with those of the model investigation for all cases, linear interpolations of the measured wave impact distributions to other water-levels and wave heights were carried out.

The method of the probability calculation can be written in the following mathematical way:

$$W\ (h,P) = \sum_{\text{high water intervals}} n\ (h,H_{(1)},\ T) \times O\ (h,H_{(1)},T,P) \times 10^{-2} \quad (3)$$

Herein

P	=	An arbitrarily chosen value of the wave force
$W\ (h,P)$	=	The probability with which the wave force P occurs or is exceeded at the water level h
$n\ (h,H_{(1)},T)$	=	The number of waves occuring per year at the water-level h and with the characteristics $(H_{(1)},\ T)$.
$O\ (h,H_{(1)},T,P)$	=	The percentage of the waves with which the load P occurs or is exceeded at the water-level h and with the wave characteristics $(H_{(1)},T)$.

$$n\ [B,H_{(1)},T] = [W(ThW = NN + A - 0.5\ m) - W(ThW = NN + A + 0.5m)] \times \frac{D\ (A \cdot B)}{T} \times 3{,}600. \quad (4)$$

Herein

$n\ [B,H_{(1)},T]$	=	The average number of waves per year with the characteristics $H_{(1)}$ and T occurring at the chosen interval B of the water-level

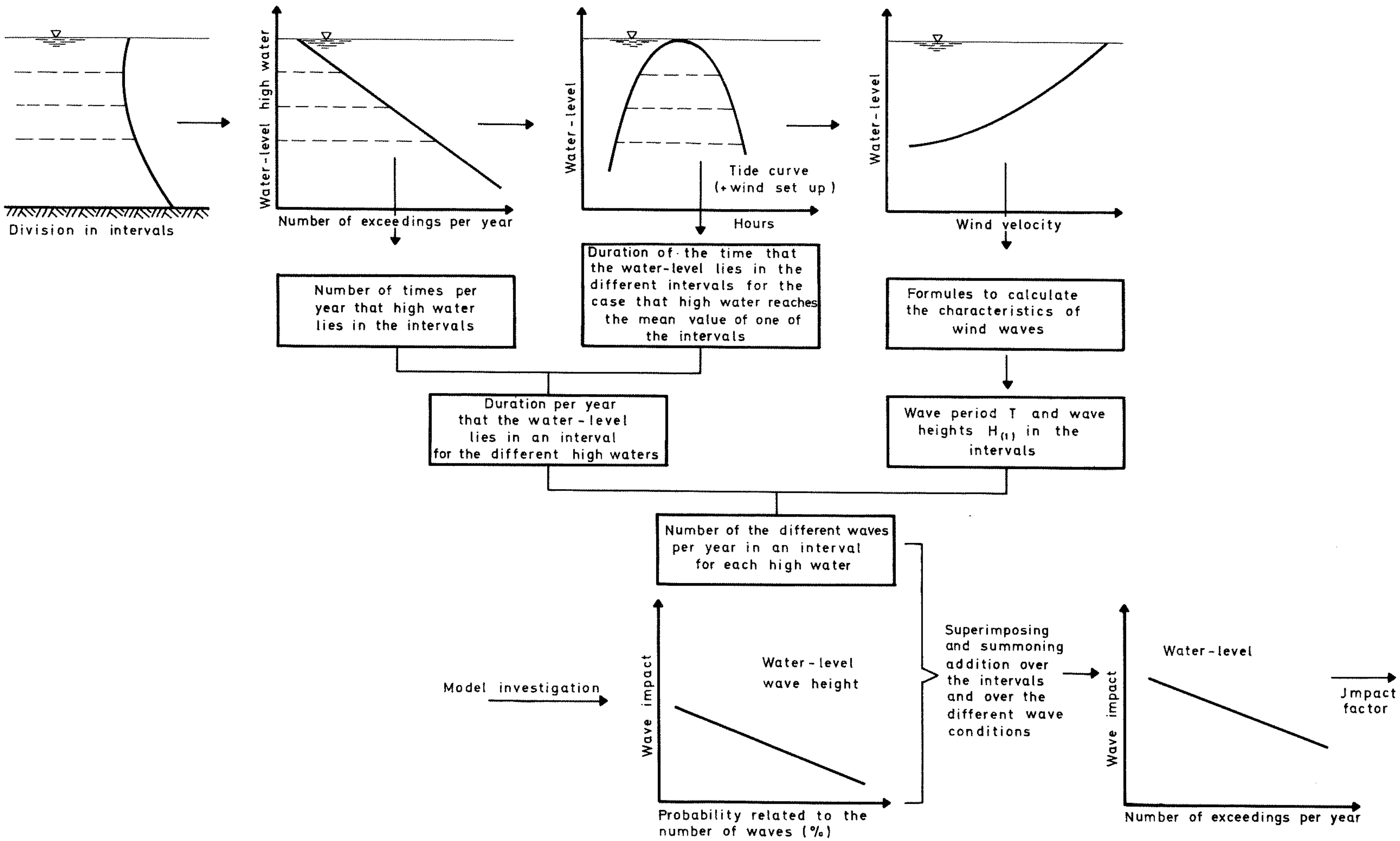


Fig. 9 Scheme of the probability calculation

$W[(ThW = NN + A - 0.5m)] - W(ThW = NN + A + 0.5)$ = The number of times that high water flood-tide averagely lies in the interval NN + A - 0.5 to NN + A + 0.5 m.

D (A,B) = The time during which the water-level h lies in interval B at a tide with a high water of NN + A.

It has to be noted that the wave characteristics $H_{(1)}$ and T which occur in interval B are determined by the wind velocity which belongs to the high water flood-tide NN + A. From (3) and (4) it can be concluded that

$$W[B,P] = 36.\sum_{\text{High water intervals}} [W(Thw = NN + A - 0.5\ m) - W(Thw=NN+A+0.5m)] \times \frac{D(A,B)}{T} \times O[B,H_{(1)},T,P] \qquad (5)$$

From the probability calculation for all water-levels, a representation of the probability curves of the loads related to prototype could be determined. In the investigation the water-levels between NN + 1.00 and NN + 6.00 m were considered. As an example, the probability curves of the maximum local pressures and of the sum forces at the water-level NN + 2.00 m are shown in Figures 10 and 11.

The most dangerous wave attack often exists at relatively low water-levels, because of the high frequency of occurrence of these conditions. Hence the probability curves have to be extrapolated to smaller probabilities for the lower water-levels (Figure 3).

The results of the test show that the maximum pressures occur about at the mean water-level or somewhat above. Only for the outer gate did the maximum pressures occurre somewhat below the mean water-level for the higher water-levels. The reason is probably the convex shape of this gate.

It can be seen in Figure 10 that considerably high wave impacts occur on the front side and the under side of the bridge, the latter being even greater than the former. However, it has to be noticed that the accessory distribution factor is rather small.

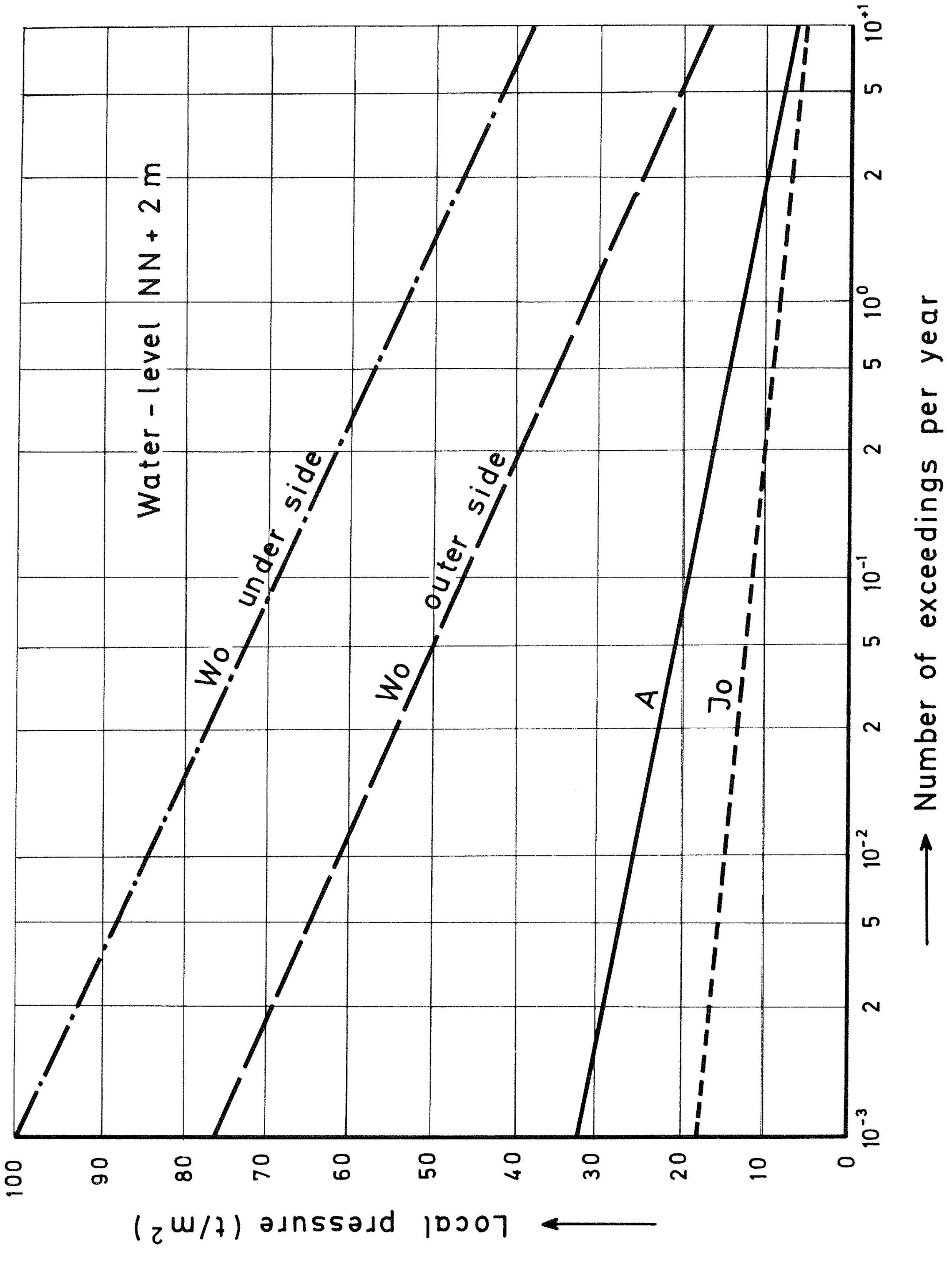


Fig.10 Examples of the probability curves of the maximum local pressures

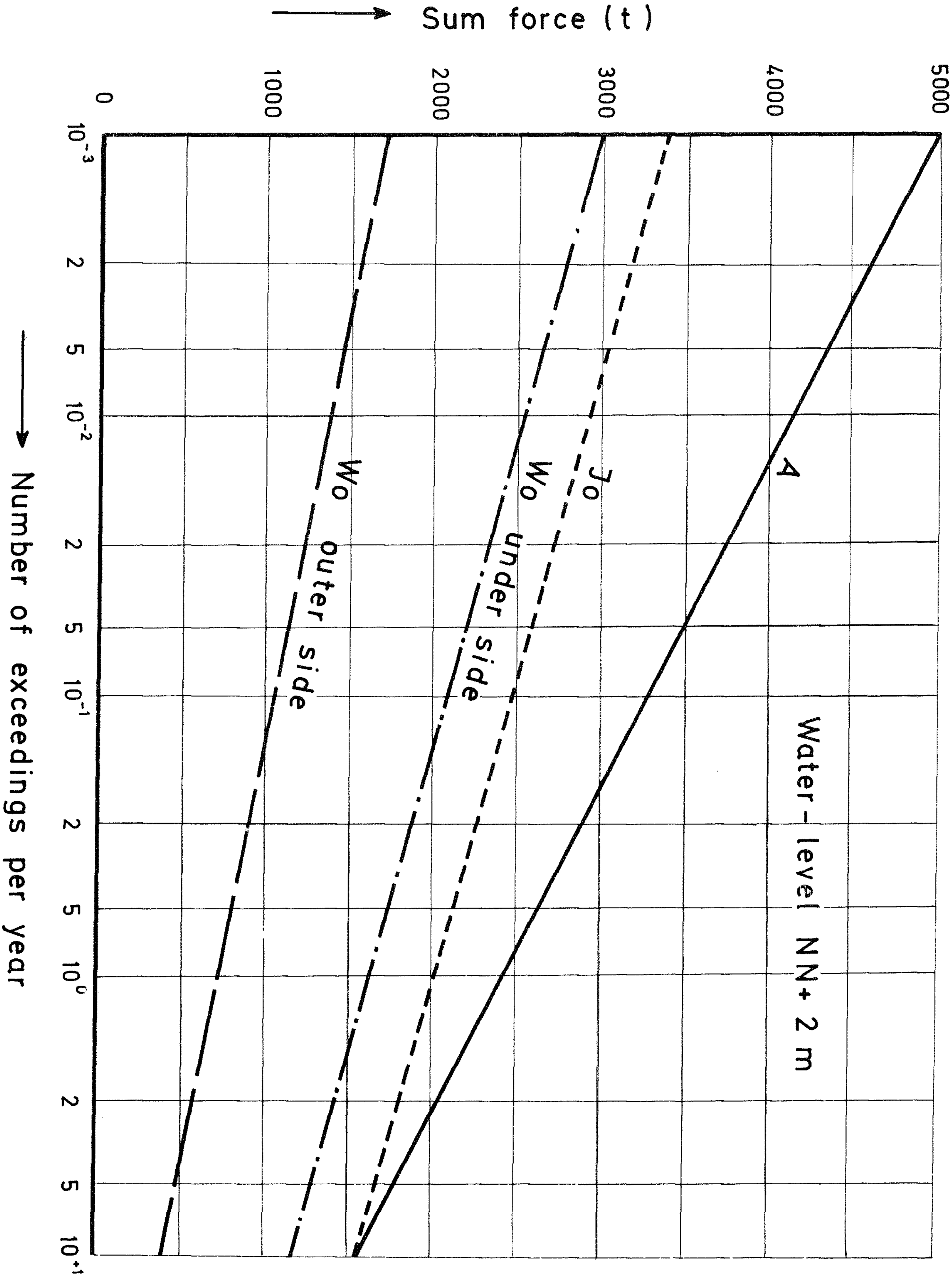


Fig.11 Examples of the probability curves of the sum forces

The maximum wave impacts do not occur over the total area of the structure at the same time, but hit the structure in places and irregularly. This conclusion can be drawn also from the sum force, shown in Figure 11. Here the sum force on the bridge is even smaller than on both the gates.

4. METHOD OF CHOOSING THE CRITICAL LOADS FOR THE STRUCTURE

To determine the dimensions of the gates and the bridge from the model results, an investigation still had to be done into the acceptable probability of circumstances under which failure of the structure is permitted, and into the vertical and horizontal load distributions, and the dynamical response of the structure.

The determination of the criterium of failure can most of the time not be solved in an easy way, and it is also not possible to give a general approach to that problem, because the choice is dependent on the problems of safety, efficiency, local circumstances and accuracy of the model investigation and calculations which have to be judged separately for each case. In this, the structure of the building-up and the settlements of the district which will be protected by the structure are of decisive significance.

In the present case of the Eider sluices, the abovementioned conditions are not critical, because the dike which protects the district now will form a second barrier after the damming up of the Eider has been finished. In effect, a large empty reservoir is being created, the volume of which is about 20,000,000 m3. Damage to the structure during one period of storm will at most result in flood water penetrating the reservoir, which is sufficiently large and which is uninhabited.

** Taking these arguments into account and after a comprehensive consideration of the safety factors involved in the horizontal and vertical distributions of the loads, as described later on, a failure probability of 10^{-2} per year has been chosen, especially as the connecting dikes have been designed to meet the same conditions.

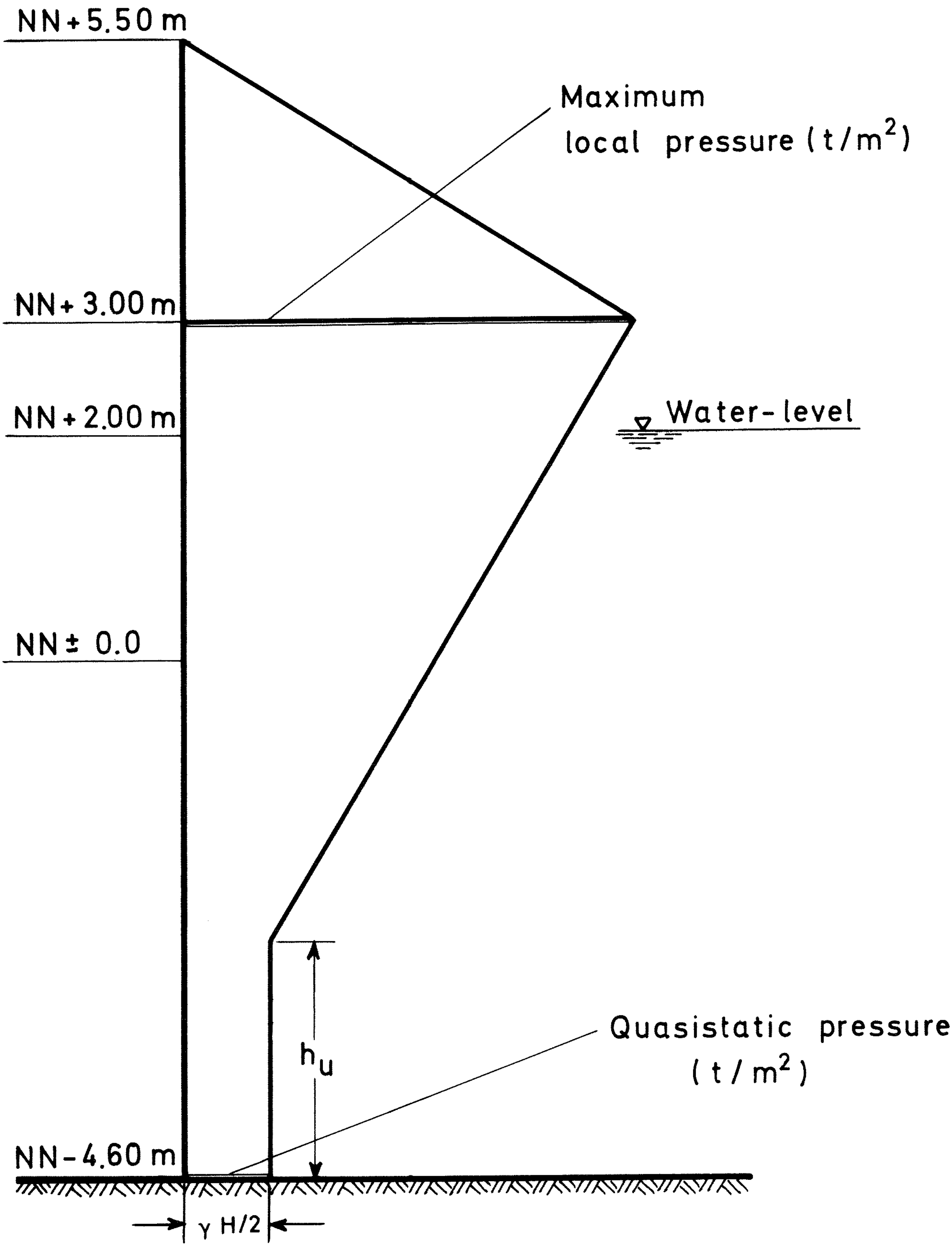


Fig. 12 Vertical load distribution

Figure 12 shows an example of the method of determining the vertical load distribution on the outer gate for the critical water-level NN + 2m. The maximum local pressure, determined as already explained was assumed as occurring at the height of NN + 3 m, as followed also from the model tests. Further on it was assumed that the pressure decreases upward linearly to zero at the upper side of the gate. The way in which the pressure decreases downward to the quasistatic load caused by the wave motion was determined applying the following suppositions:

1. The total surface of the vertical loading figure, formed as already described, must be equal to the load per running meter distributed uniformly, as can be calculated from the sum load taking the horizontal distribution factor into account.

2. At the bottom the quasistatic force was assumed to be $\frac{H}{2}$ when $\gamma = 1$ (Ref.4).

The height h_u at which only a pressure $\frac{H}{2}$ occurs can now be calculated.

The method can also be used for determining the vertical load distribution of the other cases.

Further on, the vertical loading distribution determined in this way was assumed to occur at the same time on a certain length of the structure, fixed by the horizontal distribution factor α and the length of the structure, as shown also in Figure 13.

To be safe, the factor α was applied for 10^6 waves.

According to the constructive requirements a choice has to be made between both the loading figures, as shown in Figure 13.

This implies further safety, because the wave impacts do not occur at the same time on such a length of the structure. Nevertheless, the horizontal loading distribution gives an advantage, because the application of the distribution factor, as determined with the aid of the model investigation, leeds to an efficient method.

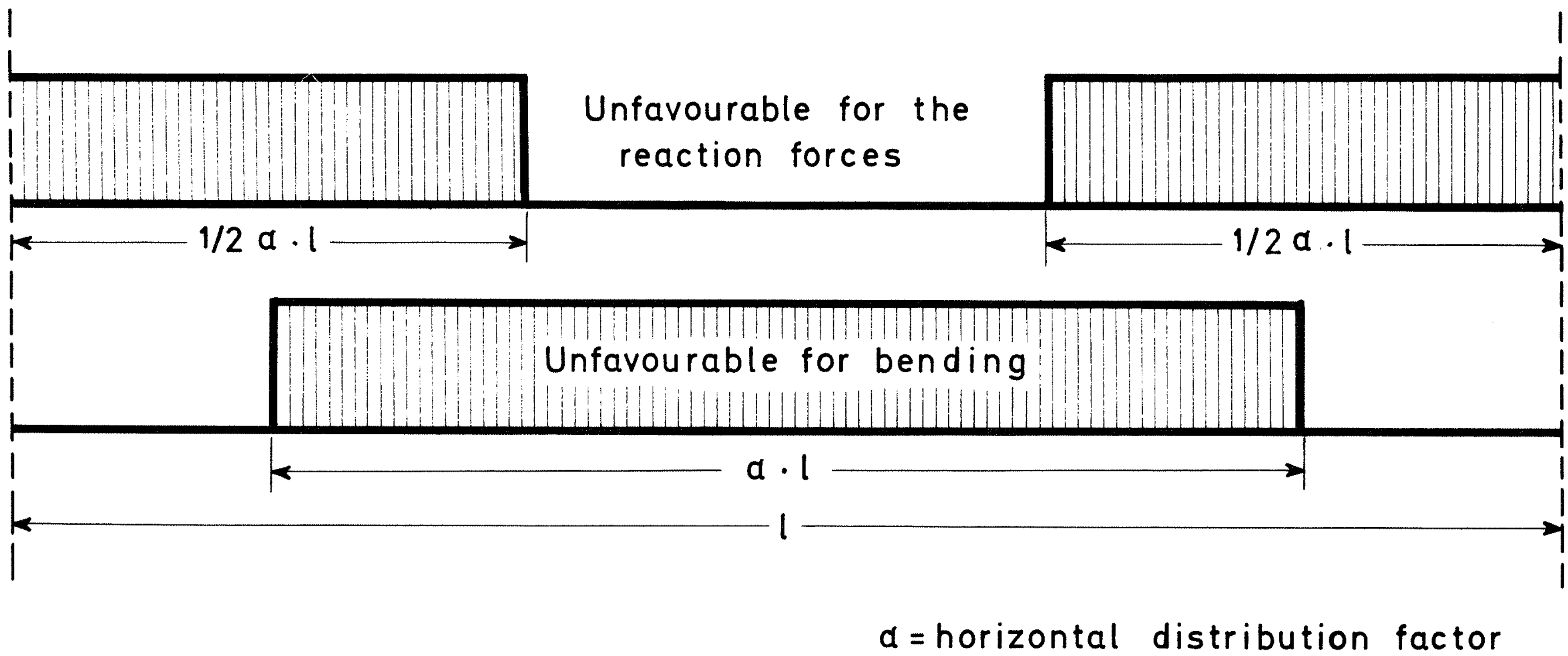


Fig. 13 Horizontal load distribution

After all these investigations it is still necessary to multiply all the wave impact loads which were measured, by an impact factor. This factor depends for every impact on its time-force relationship and on the natural frequencies of the structure or parts of it.

Starting from about 300 typical impulses of wave impacts, as found from the signals on tape at high paper velocity, impact factors were calculated for both the whole structure and for parts of it. In doing this the shape and the construction of the structure had to be taken into account.

The impact factor lies between 1.0 and 1.4. For the turnbearings and the arms of the gates, even an impact factor of 1.55 has to be applied.

All this explanation shows that extensive model tests were carried out to determine the wave forces, and which only together guarantee a safe and efficient dimensioning of the structure against the impacts of the waves.

LIST OF SYMBOLS

A	=	Outer gate (closed)
a	=	Horizontal distribution factor (For the definition see relation 2)
$H_{(1)}$	=	Wave height, exceeded by 1% of the number of the waves attacking the structure
HHThW	=	The highest known water-level
h^1	=	Height of the sluice gate or of the concrete bridge
I_m	=	Inner gate (outer gate closed)
I_o	=	Inner gate (outer gate omitted)
l	=	Span of sluice opening (length of a structure part)
NN	=	Mean sea-level
Px	=	Sum load
$\overline{P}v(x)$	=	Mean wave impact over the vertical
$\overline{P}h(x)$	=	Mean wave impact over the horizontal
$P^+(x)$	=	Local wave impact at the concerned point
T	=	Mean wave period
Thw	=	High water flood-tide
Wm	=	Concrete bridge (outer gate closed)
Wo	=	Concrete bridge (outer gate omitted)
Index(x)	=	Percentage of exceedings, related to the number of waves.

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Fig. 2 Cross-section of the evacuation sluices

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Fig. 9 Scheme of the probability calculation

Fig.10 Examples of the probability curves of the maximum local pressures

Fig.11 Examples of the probability curves of the sum forces

Fig.12 Vertical load distribution

Fig.13 Horizontal load distribution

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DISCUSSION ON PAPER 5

W. SIEFERT
Forschungsgruppe Neuwerk, Cuxhaven, Germany

Model tests are normally based on prototype data. As in the Eider mouth wave recordings are not available the authors tried to get some knowledge of the waves - necessary for the model tests - by evaluating tide and wind measurements. The relatively small number of data was completed by assumptions and extrapolations, i.e. probability of tide levels, and mean wind velocity of 30 m/s corresponding to a water level NN + 5 m. Thus these assumptions rule the "prototype" data taken as the basis of the model tests. However, by using fig. 6 - 8 it seems as if the prototype data given there are of great importance for the model tests. So therefore it is worth while to investigate the analysis.

Fig. 6 shows only 2 data (prob. 10^{-1} and Febr. 16th, 1962) with a probability less than $5 \cdot 10^{-1}$. This is not enough for extrapolating the curve to the probability 10^{-3}. As time of observation more than 10 years have to be chosen for an extrapolation like this one. Another point is that even Mr. Wemelsfelder, who first used this way of connecting water levels and probability, demands to be careful in extrapolating.

With the 4 points of fig. 7 a straight line can better be constructed to settle also the combination of fig. 6 and 8 as given in the text. The reasons for taking a curve are missing.

The large scattering of the values in fig. 8 is obviously referring to the astronomic influences on the tide, for even for extremely slow wind the data scatter more than

50 cm to each side. Furtheron the curve of fig. 8 does not link the centres of the scattering points of the different wind velocities. There is no reason for taking a curve like that in fig. 8. The curve even seems to show that with high wind velocities the water level goes up quicker than with low velocities. A fact to mention is the situation of the weather station of Tönning. The velocities taken at this point are lower than that of the wave-generating wind outside the coast. Besides, another important fact is omitted, i.e. the duration of the wave-generating wind taken from fig. 6 and 8. And above all the used theories about wave generation by wind have not yet shown considerable results in the North Sea.

It would have been necessary to have taken the differences between measured and predicted water levels as functions of the wind in order to get some practicable statements; after all the chosen time of observation (10 years) is too short for getting sufficient data.

Thus in the case discussed approximate estimations would have been more effective than functions that only seem to be the results of scientific research.

DISCUSSION ON PAPER 5

W.A. VENIS

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In their paper Dietz and Van Staal give a description of the determination of the design-load on the Eider sluices from laboratory and field observations. Reading this paper my attention was drawn by the close resemblance between the general shape of the Eider sluices and of the Haringvliet sluices in the Netherlands. It was with great pleasure that I learned, that there is a resemblance just as close between the procedure, applied by the authors for determining the design-load and that, applied by the Hydraulics Division of the Deltaworks for determining the wave load on the Haringvlietsluices.

The last mentioned procedure is described in: "Determination of the wave attack anticipated upon a structure from laboratory and field observations", which paper I presented at the seventh Congress on Coastal Engineering, held at the Hague in 1960. However there is a difference in the conclusions in the two papers. Therefore, I want to put some questions to the authors of the present paper.

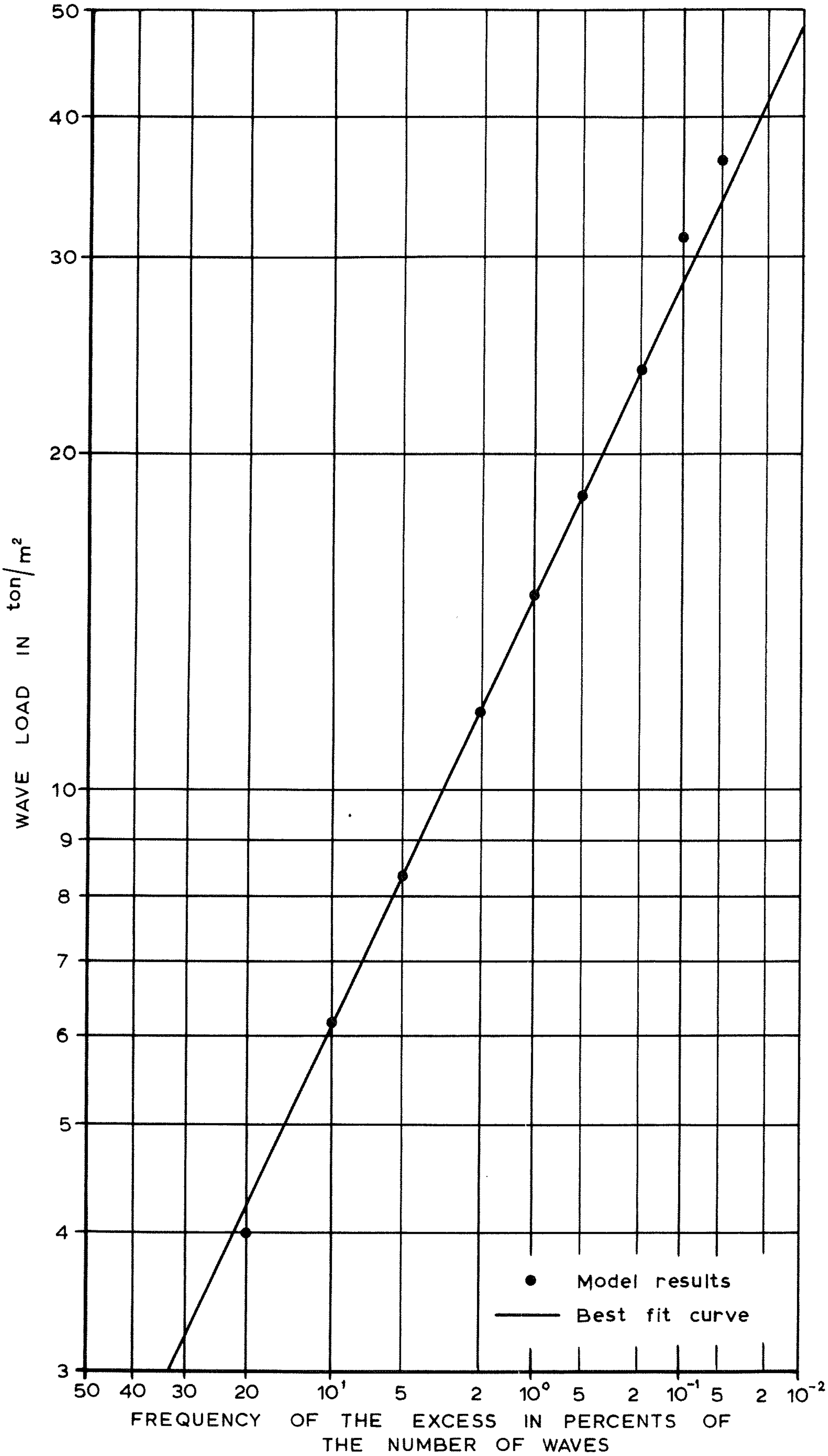
My first question concerns the probability of occurrence of a failure, mentioned in the last paragraph on page 21. The determining frequency of excess has been chosen at 10^{-2} per year, which implies that if the lifetime of the structure will be 100 years, there is a probability of about 60%, that the structure will collapse at least once during that period. This result cannot be based on calculations, related to the econometric decision problem. Therefore my first question is, which factors justify the acceptance of such a high risk.

My second question is related to the first one, more or less. On page 21 the authors discuss the accuracy of the model investigation. I would like to add, that the interpretation of the frequency-curves, based on the model results and especially their extrapolation, has to be taken into account as well. In figure 3 (page 7) the authors present the frequency-distribution of the wave loads. Looking at this figure I doubt the authors statement, that a logarithmic distribution function provides the best fit to the model results. A representation of these results on log-normal paper gives a far better fit, as my figure 1 shows. The article by Führböter, that the authors refer to on page 8, also learns that the log-normal distribution provides the best fit to the frequency distribution of the wave loads, recorded in his schematized model. Using a logarithmical distribution the authors neglect the statistical behaviour of the higher wave loads, which have a more local character than the lower ones, as stated on page 9.

To me this appears unfavourable, regarding the risk of 10^{-2} per year, accepted for the whole structure.

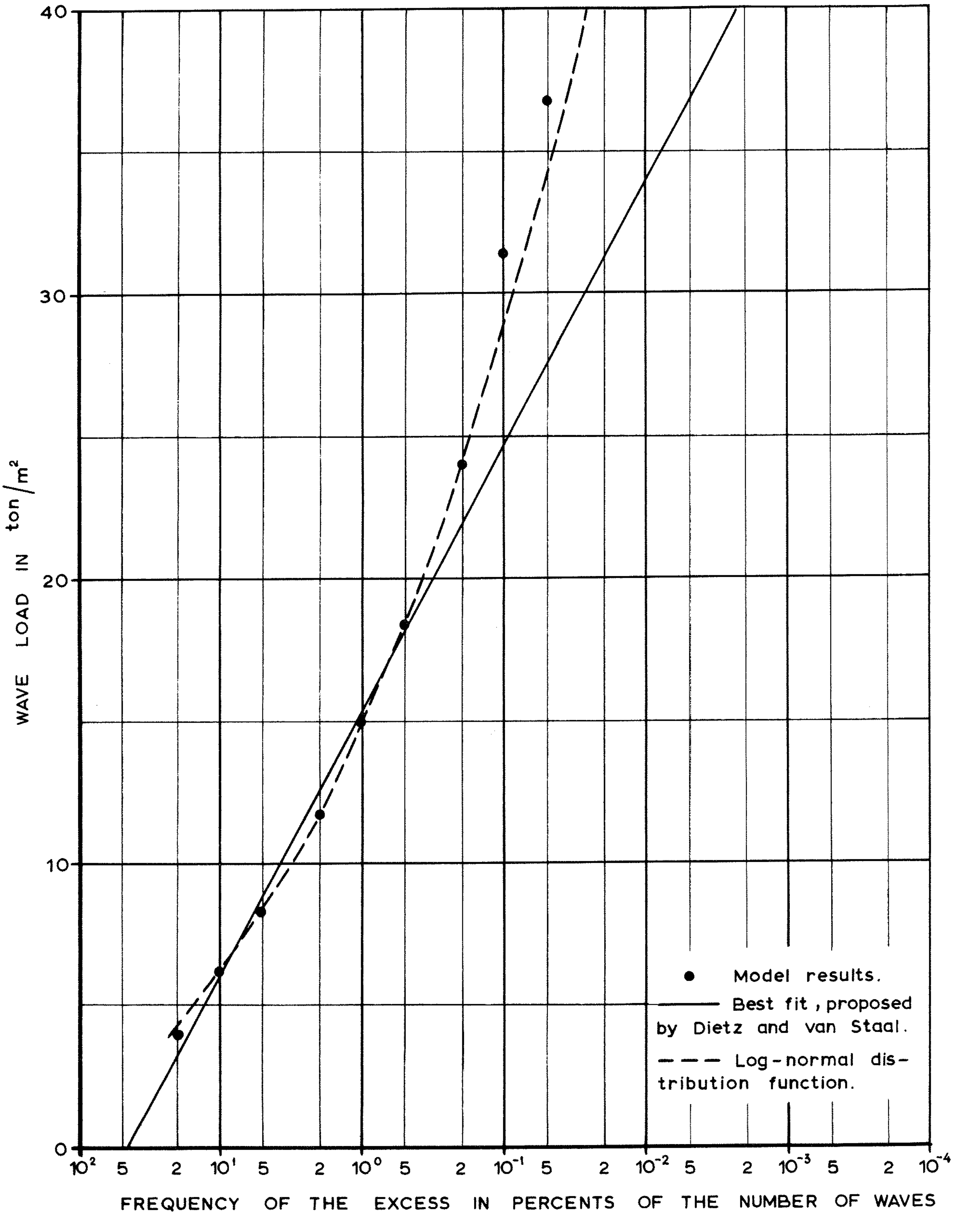
I also would like to call attention to the fact the extreme values of the wave loads have a frequency of excess, which is greater, than follows from the best fit probability function, both on logarithmic paper and on log-normal paper. I found the same deviation when interpreting the model records for the Haringvlietsluices. Generally this is not in accordance with theories regarding the statistical behaviour of extreme values. Then it was concluded, that these extreme values are connected with a physical background, different from that of the lower values of the wave loads. Therefore I want to put the following questions:

a. Will the authors clearly indicate, why they chose the logarithmic distribution as the best fit to the statistical behaviour of the local loads.
b. Have the authors any idea, why the statistical behaviour of the extreme values of the wave loads is different from that of the lower values.



50
40
30
20
10
9
8
7
6
5
4
3
WAVE LOAD IN ton/m²
Model results
Best fit curve
50 40 30 20 10¹ 5 2 10⁰ 5 2 10⁻¹ 5 2 10⁻²
FREQUENCY OF THE EXCESS IN PERCENTS OF THE NUMBER OF WAVES

Fig. 1



40
30
20
10
0
WAVE LOAD IN ton/m²
10² 5 2 10¹ 5 2 10⁰ 5 2 10⁻¹ 5 2 10⁻² 5 2 10⁻³ 5 2 10⁻⁴
FREQUENCY OF THE EXCESS IN PERCENTS OF THE NUMBER OF WAVES
Model results.
Best fit, proposed by Dietz and van Staal.
Log-normal distribution function.

Fig. 2

LABORATORY INVESTIGATION OF IMPACT FORCES

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Contents:

1. Introduction
2. Theoretical Consideration
3. Experimental Results
4. Discussion of the Results
5. List of Symbols
6. References

Summary:

A special impact generator was constructed in order to produce water impacts with velocities which are not available in scale models with waves.

The water impact was generated by a jet suddenly striking upon a measuring area.

Even under same conditions of impact, stochastic scattering of the peak pressures was observed; but for all test series the distribution of frequencies of the pressures was found to be normal-logarithmic.

The generated shock pressures by an impact velocity v came higher than 10 times the maximum pressure of steady flow of equal velocity v; but they were lower than 10 % of the water hammer pressure $\varrho \cdot v \cdot c$.

Even by a thin sheet of water on the measuring area the shock pressures were damped nearly completely.

Considerations about the effect of air content in connection with the effects of expansion show that shock pressures can be explained by a damping of water hammer pressure by a small air content. Some evaluations of the test material are given to this point.

1. INTRODUCTION

Most of all experimental investigations on the problems of shock pressures generated by wave impact have been done in model wave channels.

The advantage of these test arrangements is, that the connection between the wave characteristics and the impact condition can be studied directly. On the other side, it is not possible to control the impact conditions systematically; especially the velocity of the impact is limited by the size of the wave channels, for waves up to .5 m high the impact velocities only range between 1 and 2 m/sec.

Furthermore, it is well known after the comprehensive study by DENNY (3) that impact pressures only can be described by stochastic laws. Using a wave channel, it is only possible to measure the superposition of wave and impact statistics. Already in the classical work by BAGNOLD (2) he noted how sensitively the appearance of impact forces changed with very small differences in the wave generation.

In order to seperate between wave conditions and the dynamics of impact, it was felt necessary to construct a special impact generator. This impact generator should simulate the prototype conditions as nearly as possible.

Shock pressures by impact occur by a sudden stopping of a moving mass of water by a rigid wall. This process can be reconstructed in a laboratory by a jet which is deflected in a very short time upon a measuring area representing the rigid wall.

The present paper deals with such special tests with an impact generator. It is of interest that GAILLARD (5) as early as 1904 described experiments with a similar impact generator. His results, however, were, that by an impacting mass of water with the velocity v no higher pressures could be measured than by a steady flow of same velocity. The reason was that the spring pressure meters used by him could not indicate the short-time rise of pressure which is characteristic for all shock pressures; the lack of electronic devices was responsible for this result.

2. THEORETICAL CONSIDERATION

Taking into account only the elasticity of water (by the density ϱ and the velocity of sound c in pure water), von KARMAN (7) gave the simple solution for maximum pressure during an impact

$$p_{max} = \varrho \cdot v \cdot c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

BAGNOLD (2) first showed the high influence of entrapped air in the contact area between water and the rigid wall. The air in this contact area may occur in form of one or more cushions or bubbles; its influence on elasticity always can be reproduced by an average thickness D of a thin layer of air of equal volume.

For atmospheric pressure, the elasticity of water E stands in relation to the elasticity of air E_a like

$$\frac{E}{E_a} = 15500 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From this it can bee seen that the elasticity of the structure or of the wall in most cases can be neglected. Even a very thin layer of air gives a considerable damping to the pressure of impact.

Contrary to the phenomenon of water hammer effects in pipes, a free jet of water has no fixed boundaries on the sides. Therefore free expansion can take place at the circumference U of the impact area A; air entrainment and free expansion together provide shock pressures to rise till the magnitude of water hammer pressures.

In Fig. 1, the moment of impact of a free nappe of water is shown schematically for the case of a plane parallel front of the nappe before impact.

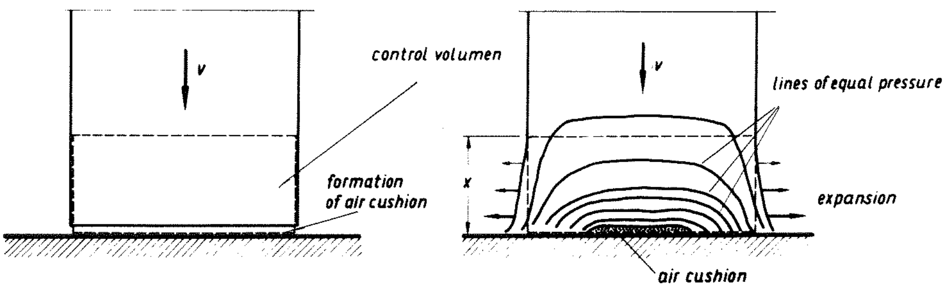

Fig. 1. Air entrainment and expansion during impact

With application of the law of continuity for each time element dt for the control volume in Fig. 1, it can be written (see also (4)).

$$A \cdot v \cdot dt = \left(\frac{A \cdot D}{E_a(t)} + \frac{A \cdot x(t)}{E}\right) dp + A_e(t) \cdot v_e \cdot dt \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(inflow) (compression of air and water) (outflow by expansion)

Here A is the area and v is the velocity of impact, D the representive thickness of the air cushion, $E_a(t)$ the adiabatic elasticity of the air corresponding to pressure and time, E the constant elasticity of water, x the unknown length of water in axis of the nappe compressed by dp; $A_e(t)$ the (average) area of expansion with the (average) outflow velocity v_e due to expansion.

From momentum equation for the direction parallel to the wall a relation between the expansion velocity v_e and the pressure p can be given by

$$K = p \cdot A_e = \varrho \cdot Q_e \cdot v_e = \varrho \cdot A_e \cdot v_e^2 \quad \text{or}$$

$$v_e = \sqrt{p/\varrho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

It shall be mentioned that, because of the nonuniform distribution of pressure p over the impact area A and the expansion velocity v_e over the expansion area A_e, equation (4) can give only an approximation for the average values.

Introducing equation (4) in equation (3), there is a differential equation for p(t):

$$A \cdot v \cdot dt = \left(\frac{A \cdot D}{E_a(t)} + \frac{A \cdot x(t)}{E}\right) dp + A_e(t) \cdot \sqrt{p/\varrho} \cdot dt \quad \ldots\ldots\ldots (5)$$

(inflow) (compression of air and water) (outflow by expansion)

A complete solution is not possible because of the many unknown variables; this complete solution, however, is not necessary when only the peak pressure p_{max} is desired; p_{max} is the maximum of p(t) during impact and is given by the condition

$$\frac{dp}{dt} = 0$$

which gives with equation (5) the simple relation

$$A \cdot v = A_e \cdot \sqrt{p_{max}/\varrho} \quad \text{(6)}$$

(inflow)(outflow by expansion)

For the moment of maximum pressure p_{max} during impact, inflow in the control volume is equal to outflow on the sides by expansion. Before maximum pressure, the outflow is lower than the inflow; after maximum pressure, outflow becomes higher than inflow (4).

Equation (6) can be solved for p_{max} and gives

$$p_{max} = \varrho \cdot v^2 \cdot \left(\frac{A}{A_e}\right)^2 \quad \text{(6)}$$

For $A_e(t)$ can be written $A_e(t)=U \cdot x(t)$ and for the time $p(t)=p_{max}$ $A_e = U \cdot x$; $A/U = R$ is the hydraulic radius of the impact area. So equation (6) becomes

$$p_{max} = \varrho \cdot v^2 \cdot \left(\frac{R}{x}\right)^2 \quad \text{(7)}$$

From the cross section of the impinging nappe, R is known; the only unknown variable in equation (7) is the length x, the length on which expansion takes place according to Fig. 1.

In equation (7) the thickness D of the air cushion does not directly appear. Considering the pressure rise between $p = 0$ (beginning of impact) and $p = p_{max}$, it can be shown easily that there is a close connection between the length of the expanding area and the thickness of the air cushion in a manner, that x increases with increasing D. For a higher air cushion, the pressure rise is lower than for a small one; therefore a greater area for the expansion effect can be built up which makes x increase.

Stochastic effects are introduced by the variables x and R, where x is mostly connected with the accidental air content in the contact area, R with irregularities in the face of the impinging nappe.

3. EXPERIMENTAL RESULTS

On Fig. 2, the experimental equipment can be seen, which

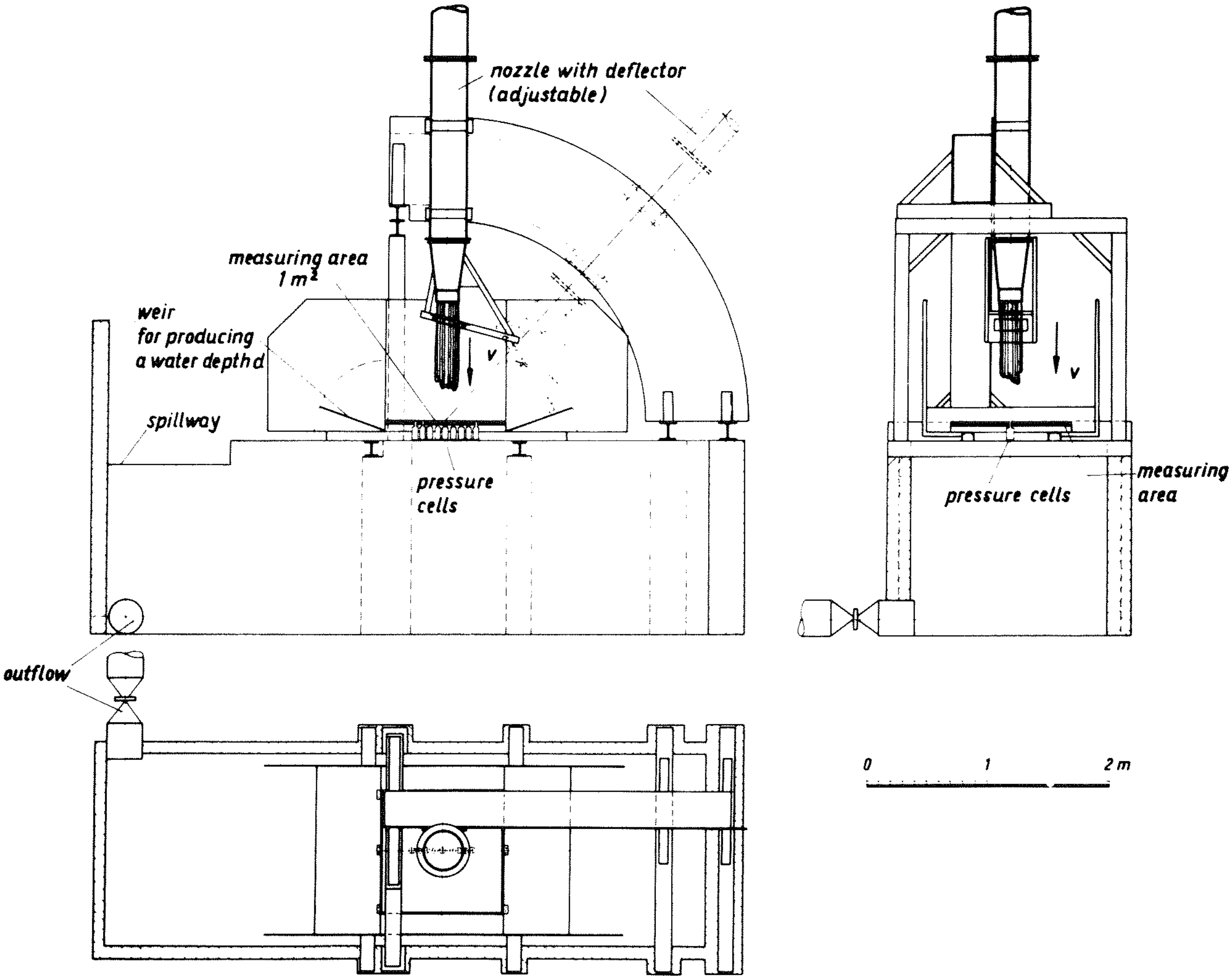

Fig. 2. Impact generator in the FRANZIUS-INSTITUT

was used to generate shock pressures by impact. The jet (diameter 200 mm) with the deflector mechanism for sudden opening was adjustable to any angle α between the jet axis and the measurement area, a strong plane steel plate with 8 electronic pressure cells in distances of 50 mm; the electronic equipments were selected so that single processes of only .001 sec and less could be recorded without damping (4).

For the front of the nappe, not only the jet angle α is of importance, but also the front angle ß which is formed by the short but not infinite short time of opening the deflector gate;

Fig. 3 shows these two angles at the face of the nappe.

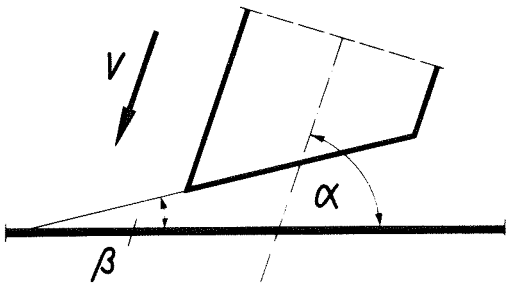

Fig. 3. Jet angle α and front angle β

For a constant velocity

$$v = 8.3 \text{ m/sec}$$

(this would correspond to the impact velocity of a wave about 3 m high) a series of 6 x 100 tests were carried out for different jet and front angles; the results are summarized in the table on page 8 and in Figs. 4 to 9 on the following pages.

Each Fig. 4 to 9 shows a series of 100 tests; from the 8 pressure records on the measuring area the highest pressure p_{max} was taken for the evaluation. Mostly the pressures were distributed uniformly over the measuring area and did not differ very much from one to another; only to the borders of the nappe also the peak pressures became lower.

If t_1 is the time for the pressure rise from zero to p_{max} and t_2 the time for the pressure drop from p_{max} to p_s (maximum pressure of the jet with steady flow with v), the records showed

t_1 between .001 and .002 secs
t_2 between .002 and .004 secs

according to a complete duration of shock pressure t_s

t_s between .003 and .006 secs,

the longer durations belonging to low, the shorter to high pressure peaks as already shown by BAGNOLD (2).

The maximum pressure p_s on an area under a jet of steady flow with the velocity v is

$$p_s = \rho \cdot \frac{v^2}{2} \quad \text{.................................} \quad (8)$$

and for v = const. = 8.3 m/sec.

$$p_s = 3.5 \text{ m (water column)}$$

for all angles of approach α. The results show that the highest

p_{max} m		$\alpha = 90°$ $\beta = +33.8°$	$\alpha = 82.5°$ $\beta = +23.7°$	$\alpha = 75°$ $\beta = +18°$	$\alpha = 60°$ $\beta = +3.6°$	$\alpha = 45°$ $\beta = -17.5°$	$\alpha = 30°$ $\beta = -35°$
from	to	Number	Number	Number	Number	Number	Number
1.0	1.9						
2.0	2.9						4
3.0	3.9						2
4.0	4.9						4
5.0	5.9						8
6.0	6.9						13
7.0	7.9	1					3
8.0	8.9	4				1	13
9.0	9.9	4	2	2	3	3	12
10.0	10.9	15	6	2	1	4	3
11.0	11.9	13	4	2	1	8	8
12.0	12.9	14	4	9	2	6	5
13.0	13.9	7	13	11	5	7	8
14.0	14.9	5	8	2	8	6	3
15.0	15.9	7	8	11	10	14	1
16.0	16.9	7	10	13	8	8	4
17.0	17.9	6	6	5	8	6	4
18.0	18.9	3	8	7	13	6	1
19.0	19.9	3	5	9	4	7	1
20.0	20.9	2	4	5	5	4	1
21.0	21.9		2	5	7	3	
22.0	22.9	3	2	5	5	6	1
23.0	23.9	2	3	4	4	1	
24.0	24.9	2	3	2	3	1	
25.0	25.9		1	2	2	4	
26.0	26.9	1	1	1	2		
27.0	27.9		2	1	3		1
28.0	28.9				1	1	
29.0	29.9		2	1	1	1	
30.0	30.9		2		1		
31.0	31.9	1		1	1		
32.0	32.9					1	
33.0	33.9		1				
34.0	34.9					1	
35.0	35.9						
36.0	36.9				1	1	
37.0	37.9						
38.0	38.9		2				
39.0	39.9				1		
40.0	40.9		1				
41.0	41.9						
42.0	42.9						
43.0	43.9						
44.0	44.9						
45.0	45.9						
46.0	46.9						
47.0	47.9						
48.0	48.9						
49.0	49.9						
Σ =		100	100	100	100	100	100

Table: Frequencies of maximum pressures p_{max}

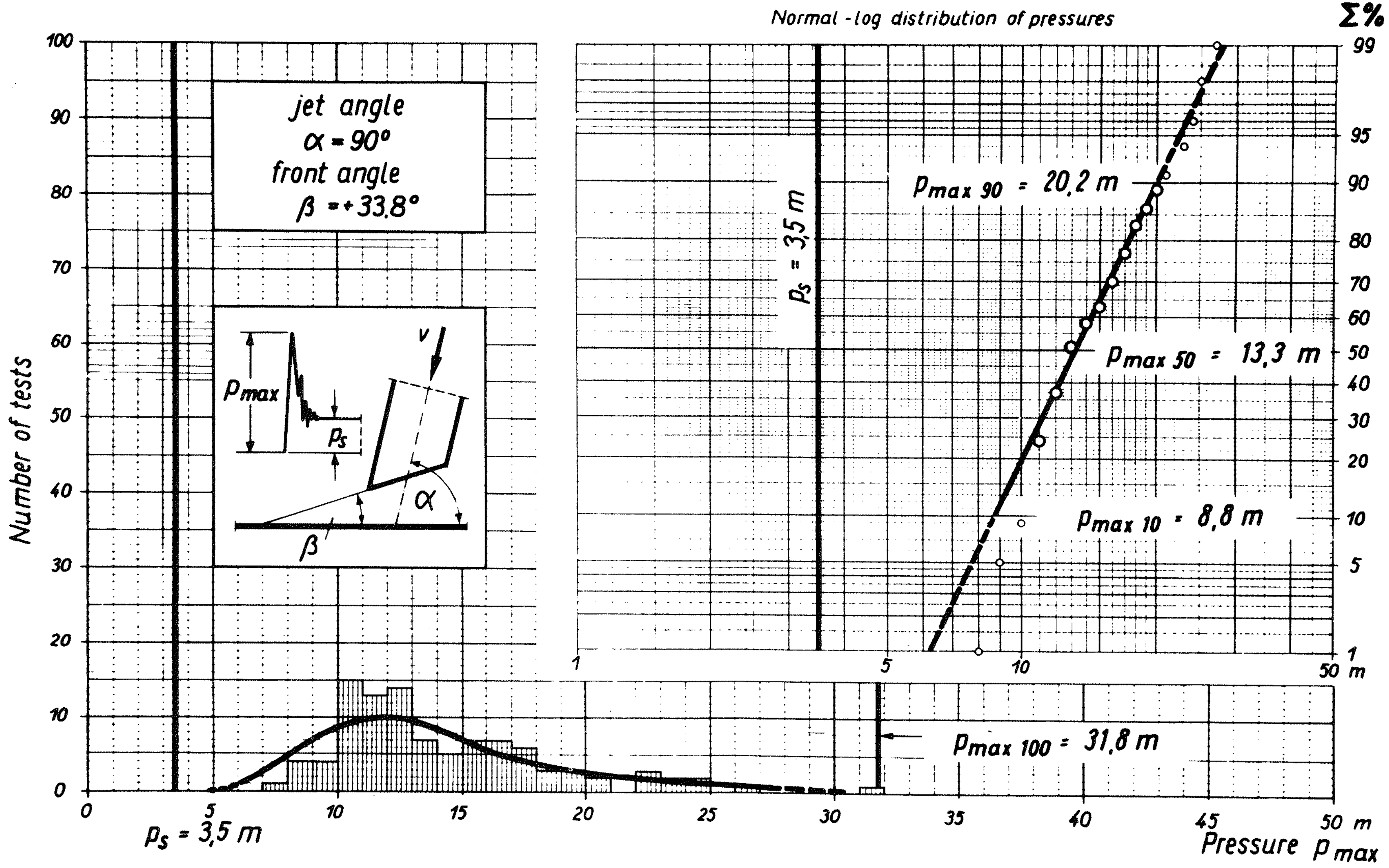

Fig. 4. Frequencies of maximum pressures p_{max}

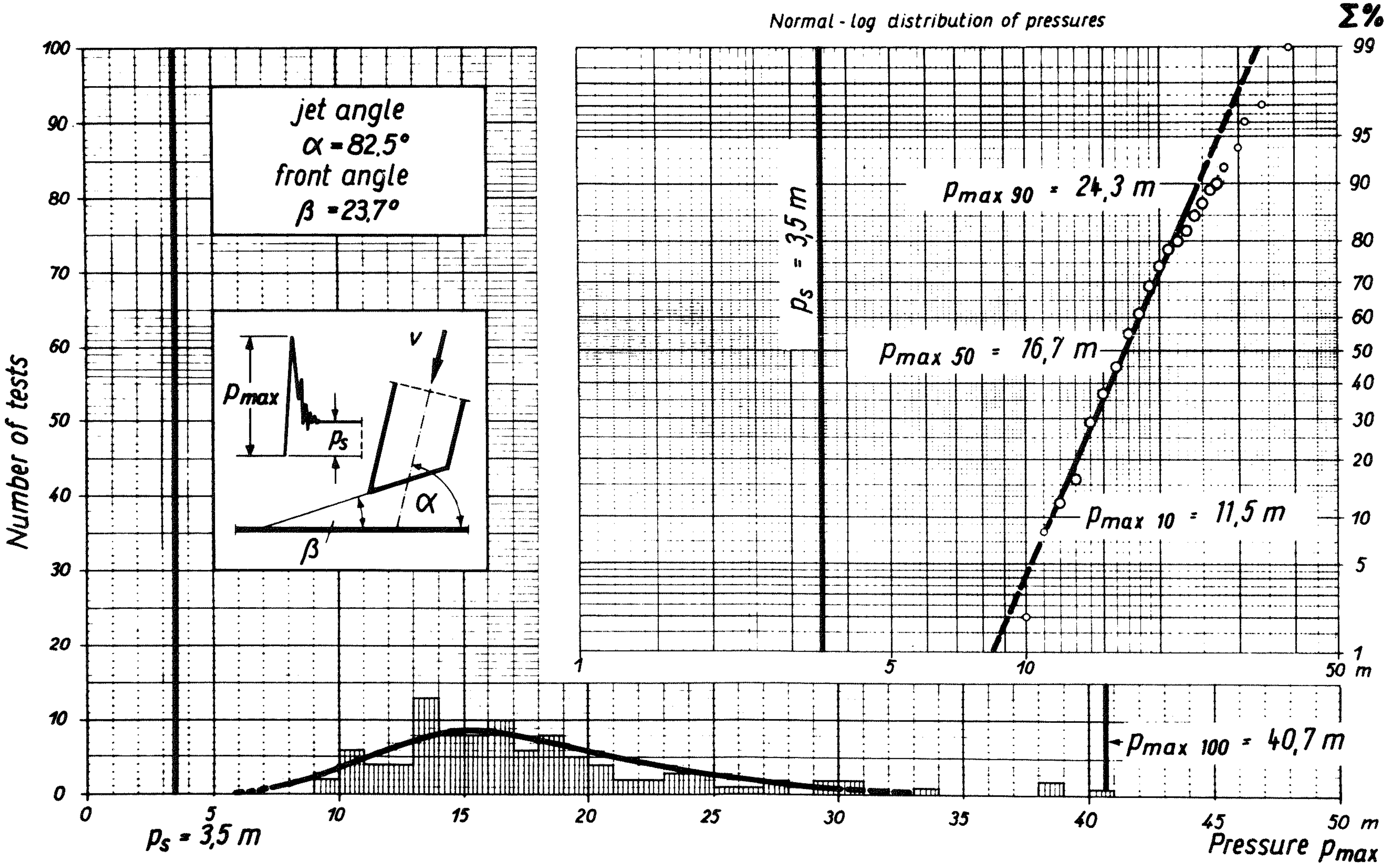

Fig. 5. Frequencies of maximum pressures p_{max}

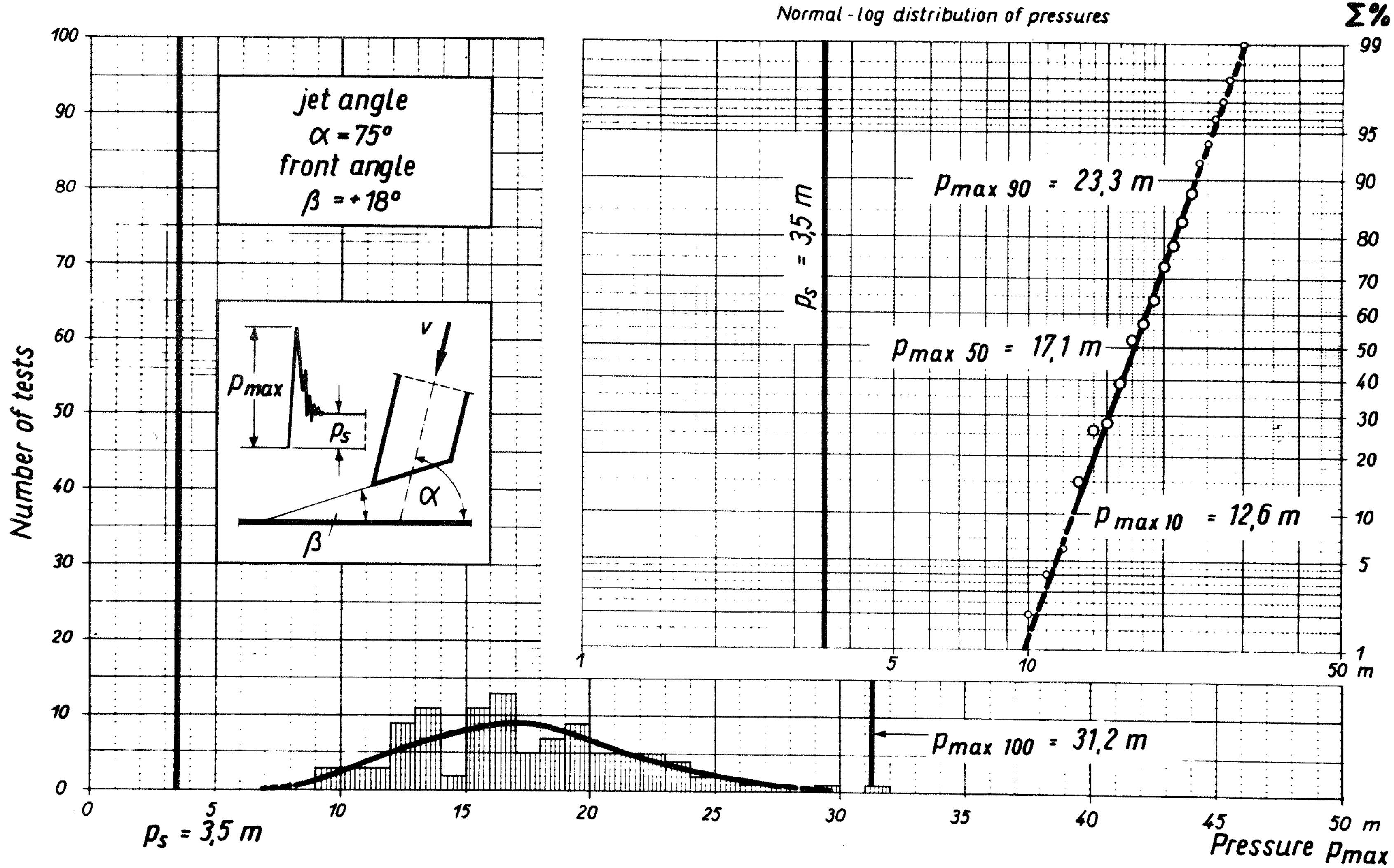

Fig. 6. Frequencies of maximum pressures p_{max}

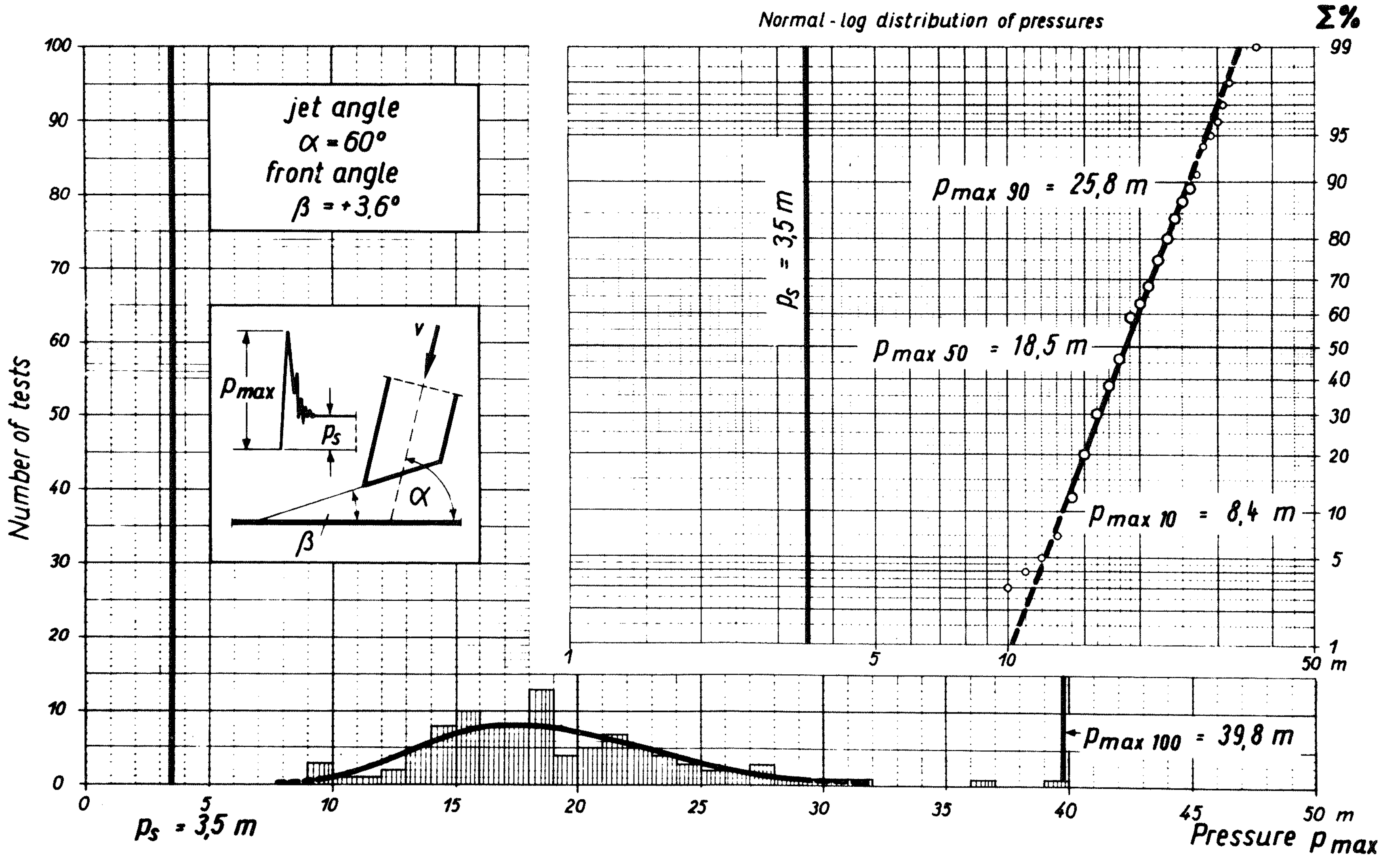

Fig. 7. Frequencies of maximum pressures p_{max}

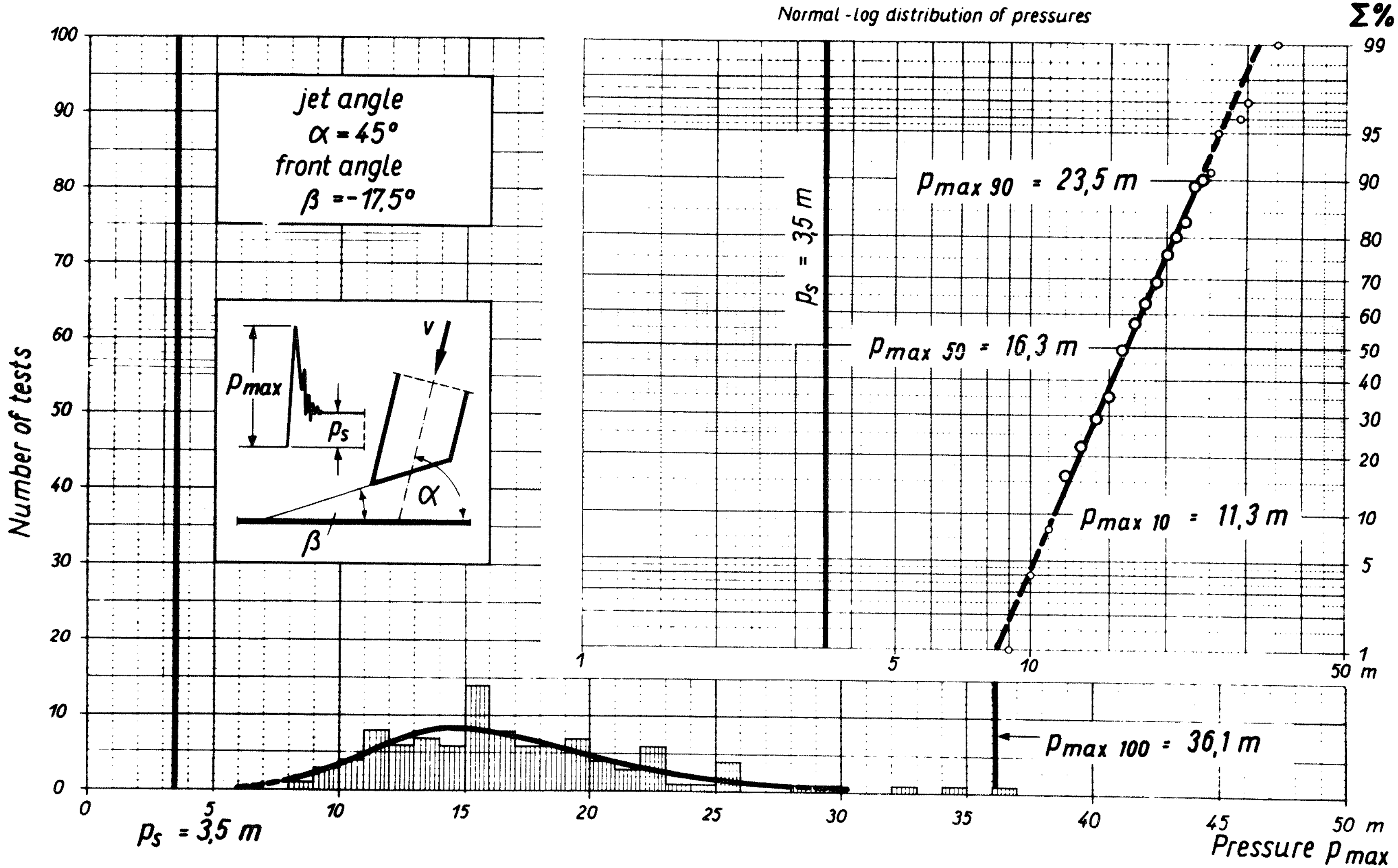

Fig. 8. Frequencies of maximum pressures p_{max}

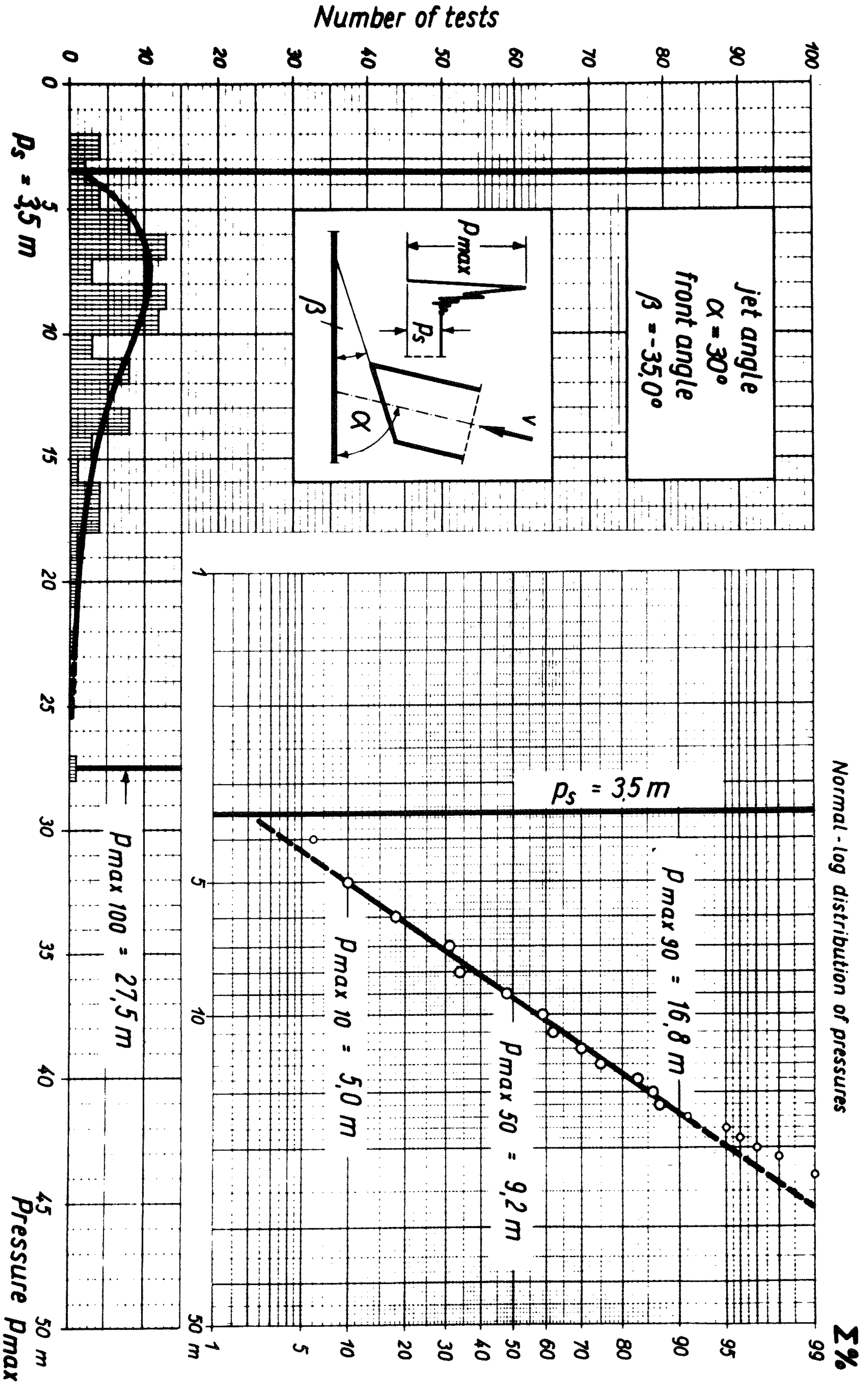

Fig. 9. Frequencies of maximum pressures p_{max}

pressure peaks are more than 10 times higher than p_s (highest pressure was p_{max} = 40.7 m, see Fig. 5).

In Figs. 4 to 9, the original histogram of the frequencies of p_{max} is to be seen as well as the integral function of it on special normal-log function paper. It can be seen from Figs. 4 to 9, that a normal-log distribution is in good agreement with the experimental results; it should be mentioned that the validity of this distribution must be limited by the water hammer pressure $\varrho \cdot v \cdot c$.

In Figs. 4 to 9 the values $p_{max\ 10}$, $p_{max\ 50}$ and $p_{max\ 100}$ are shown; these are the pressures, which are not exceeded by 10, 50 and 90 out of 100 tests; furthermore the highest pressure $\dot{p}_{max\ 100}$ measured during 100 tests is given on each of Figs. 4 to 9. When these pressures are evaluated by equation (7)

$$p_{max} = \varrho \cdot v \cdot \left(\frac{R}{x}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

for the dimensionless number x/R, they can be plotted against the impact velocity v perpendicular to the measuring plane, corresponding to the angle of approach α.

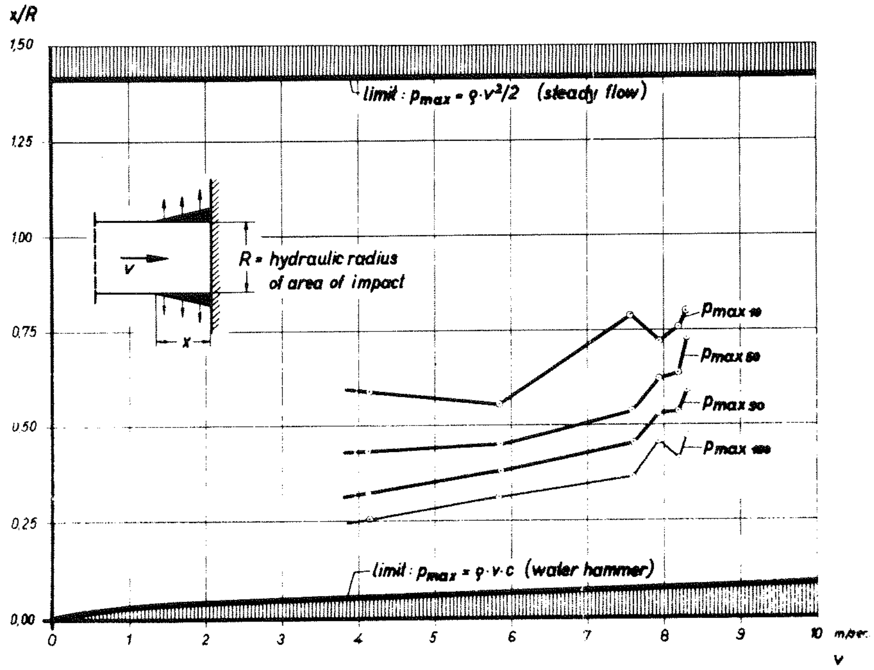

Fig. 10. Expansion factor x/R

The term x/R represents the relation between the length of the volume of expansion and the hydraulic radius R of the impact area; the higher the expansion factor x/R, the lower is the pressure peak. There are two limits for the expansion factor; from equation (8) and (7) follows for the case of steady flow

$$p_s = \varrho \cdot \frac{v^2}{2}; \quad \frac{x}{R} = \sqrt{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

and from equation (1) and (7) for the case of water hammer

$$p_{max} = \varrho \cdot v \cdot c; \quad \frac{x}{R} = \sqrt{\frac{v}{c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

These limits are also shown in Fig. 10.

It can be seen from Fig. 10, that the expansion factor x/R even for the highest pressures $p_{max\ 100}$ is much higher than for the water hammer, (equation (10)), but also lower than the constant value for steady flow (equation (9)). The hydraulic radius of a jet having a diameter of 200 mm is R = 5 cm; then lie x/R between the extremes .2 and .8 and the length of expansion x between 1 and 4 cm; for the average of pressures $p_{max\ 50}$ x ranges between 2.5 and 3 cm. It must be noted that x is the effective length of expansion only for the time of the maximum of pressure.

As the jet angle α is changing in the 6 series from 90° to 30°, the front angle ß from +33.8° to -35°, it is surprising that the results on Fig. 10 do not differ very much. There is a tendency of increase of x with the velocity v; it may be explained by higher disturbances at the face of the nappe with higher velocities.

Further experiments were conducted in order to study the effect of a water layer on the measuring area; this is the condition when a plunging breaker falls into the backrush water of the foregoing wave. In these experiments only one pressure cell was used in the center of the jet; the angle of approach was 90°.

As shown in Figs. 11 to 13, for 3 velocities (5.8 m/sec, 8.3 m/sec and 10.4 m/sec corresponding to steady flow pressures p_s of 1.7 m, 3.5 m and 5.5 m) the pressure distribution for 100 tests were compared for different depths of water on the (horizontal) impact area. It can be seen that even a thin layer of water is capable to give a high damping effect on the pressure maxima;

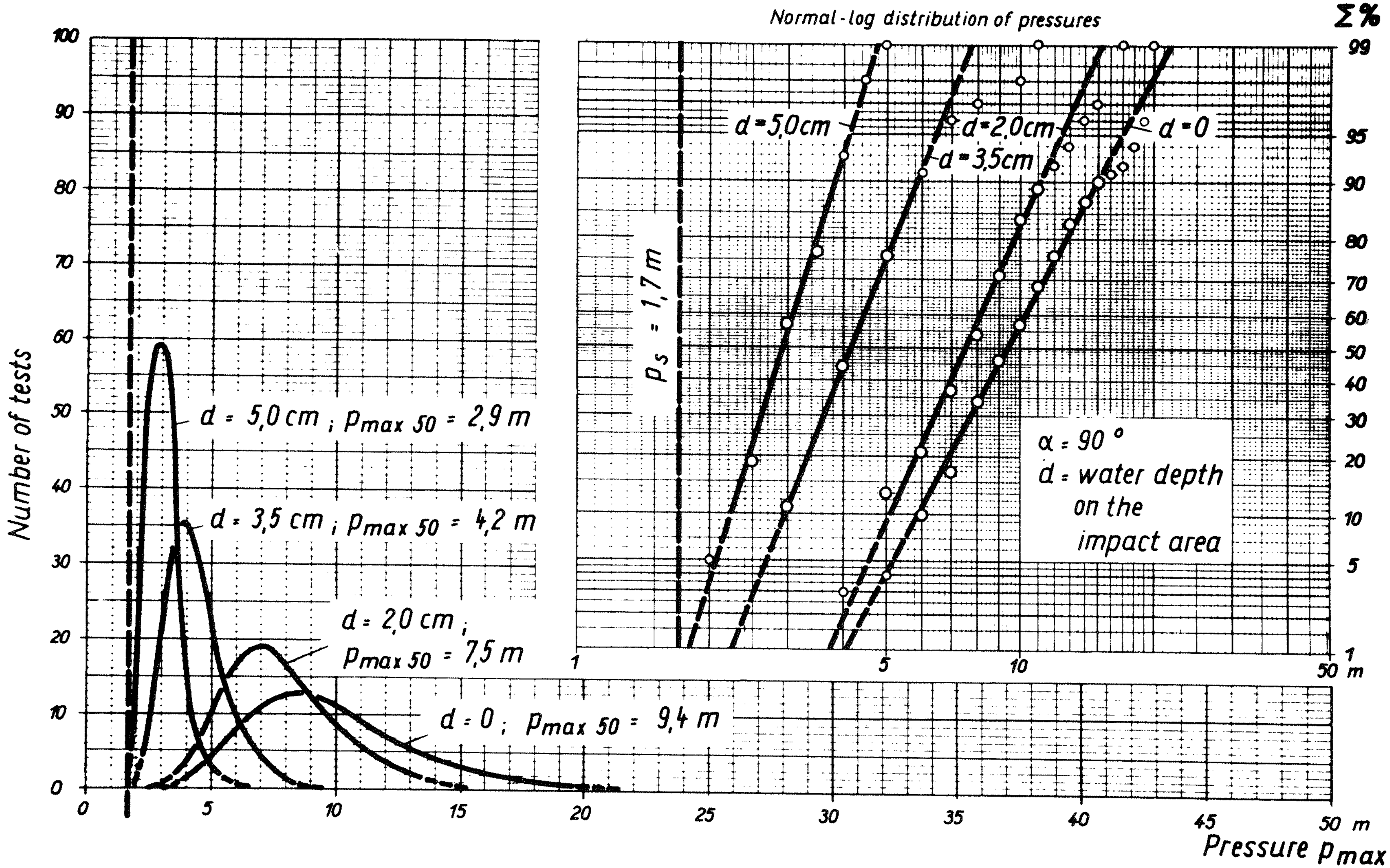

Fig. 11. Damping effect of water on the impact area

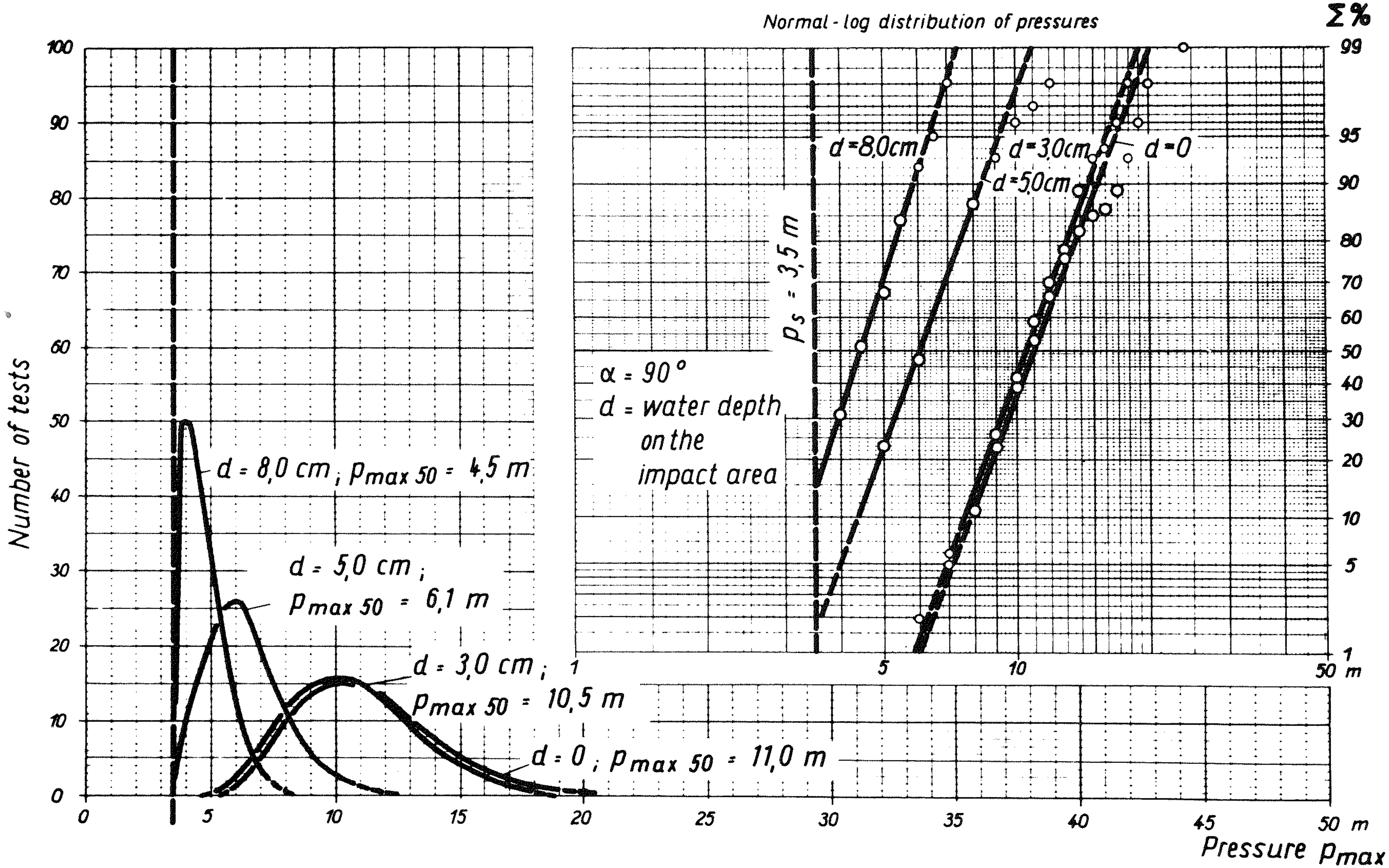

Fig.12. Damping effect of water on the impact area

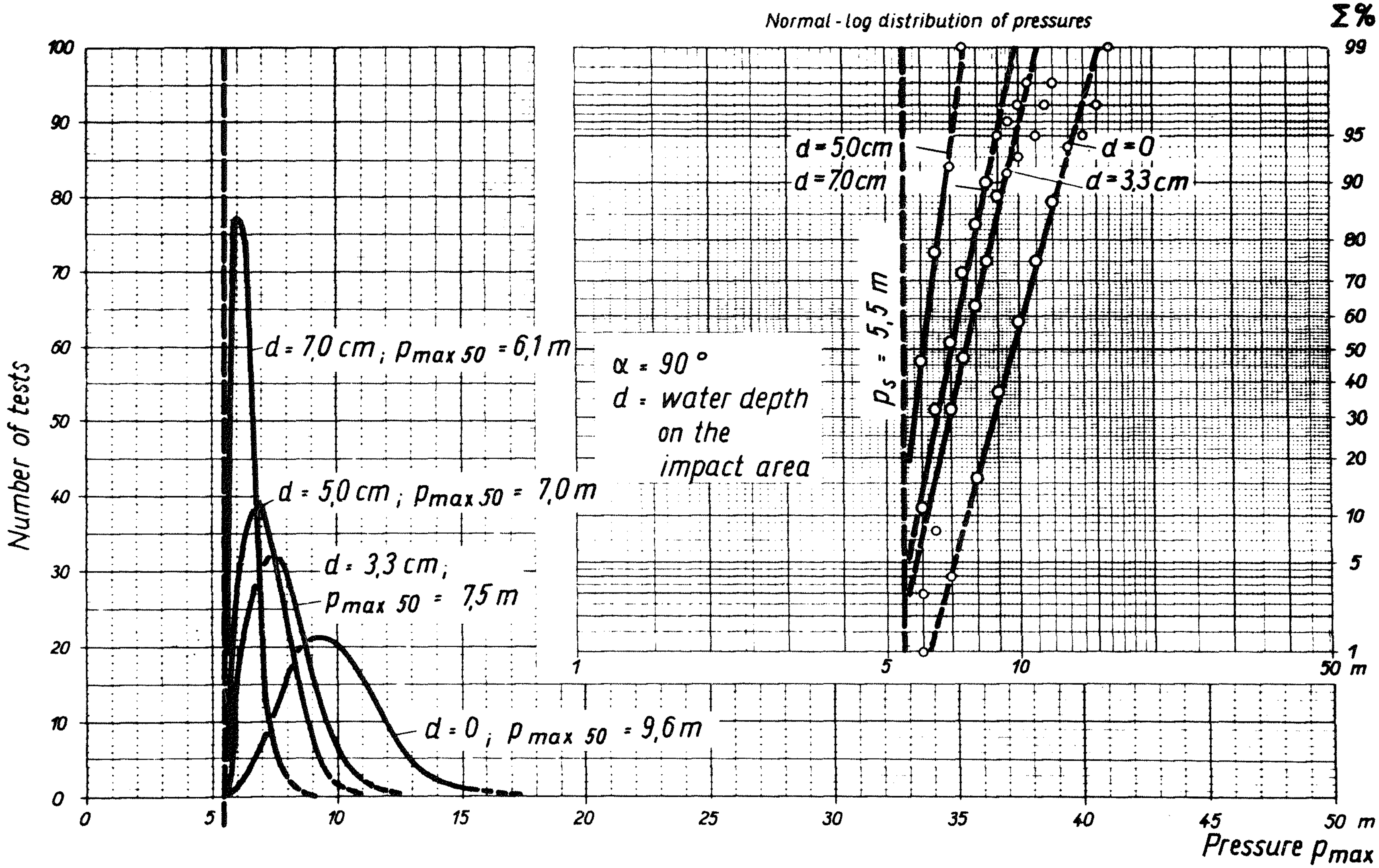

Fig. 13. Damping effect of water on the impact area

for water depths d more than 5 cm the higher pressures are reduced nearly completely.

That agrees with the results shown in Fig. 10; the length of the compressed volume of water and air is in the order of this water depth, therefore the pressure rise does not come till to the bottom formed by the measuring area.

Conspicuous is the fact that for d = v the median $p_{max\ 50}$ is not increasing with velocity; the distribution becomes more uniform for the upper velocities. Because in these series only one pressure cell was used, a direct comparison with the results of Figs. 4 to 9 is not possible, but it agrees with the tendency of x/R versus v in Fig. 10.

4. DISCUSSION OF THE RESULTS

For application to the problems of wave attack, the test material was evaluated in a previous paper (FÜHRBÖTER (4)) into a semi-empirical formula derived from equation (5)

** $$p_{max} = \varrho \cdot v \cdot c \cdot \sqrt[3]{\frac{c}{v}} \cdot \delta \quad \ldots\ldots (11)$$

with the dimensionless impact-number

$$\delta = \left(\frac{E_a}{E} \cdot \frac{R}{D}\right)^{\frac{2}{3}} \quad \ldots\ldots (12)$$

which was found from the tests to be for $p_{max\ 50}$

$$\delta_{50} = 0.00245 \quad \ldots\ldots (13)$$

with the relations corresponding to the normal-log distribution of p_{max}

$$p_{max\ 10} = 0.65 \cdot p_{max\ 50}$$
$$p_{max\ 50} = 1.00 \cdot p_{max\ 50}$$
$$p_{max\ 90} = 1.5 \cdot p_{max\ 50}$$
$$p_{max\ 99} = 2.1 \cdot p_{max\ 50}$$
$$p_{max\ 99.9} = 2.7 \cdot p_{max\ 50}$$

The time of pressure rise t_1 is given by

$$t_1 = \frac{R}{\sqrt[3]{v \cdot c^2 \cdot \sqrt{\delta}}} \qquad \dots\dots\dots\dots\dots\dots\dots \quad (15)$$

In this solution all the results of the 600 tests given in Figs. 4 to 9 are utilized.

Here, only the physical aspect of the results shall be taken into account, which is given by the fact, that from all tests till velocities up to 8.3 m/sec it was found, that the length of expansion (in axis of the jet) was of the same order of of magnitude of R:

$$x \sim R$$

with a tendency of increase for higher values of v.

NAGAI (9) found in his comprehensive tests in model wave tanks a length of 3 to 5 cm of water column which could be related by momentum equation to the shock pressure; this is in agreement with considerations of BAGNOLD (2) who found the length of the participating volume to be about .2 H_B; for waves with H_B of 20 cm therefore about 4 cm. In the tests of the FRANZIUS-INSTITUT the corresponding length x - here defined as the length of the expansion area A_e - also lies in the range between 1 and 4 cm from Fig. 10 with R = 5 cm.

It shall be mentioned here, that the hydraulic radius of impact areas of breaking waves is of the order of half the breaker height H_B. For model waves about 20 cm high the hydraulic radius is not different very much from R = 5 cm in the tests of the FRANZIUS-INSTITUT.

A simple explanation for the fact

$$x \sim R$$

can be given by Fig. 1. Because of the high velocity of sound c in water (compared with v), a build-up of pressure only can occur in a zone of a length x in the order of magnitude like R, because for longer distances from the wall the side expansion effect gives a pressure about 0 inside the jet during all phases of impact.

Contrary to the theory of NAGAI (9), also with the effect of expansion a water hammer pressure $\varrho \cdot v \cdot c$ would occur, when only the elasticity of water would govern the impact process; but it would appear only for a very short time in the order of $t_1 = R/c$ due to the beginning of expansion.

For the idealized case of a complete parallel front of the nappe to the wall, it can be shown, that the escaping of air out of the volume between the approaching front and the wall is limited by the velocity of sound in air c_a. After arriving to a certain distance from the wall, the escaping velocity of air v_a becomes equal c_a and remains constant for the last time till to the contact of the front of the nappe with the wall. From this idealized model of the process, it follows that a volume of air (under atmospheric pressure)

$$D \sim \frac{R}{2} \cdot \frac{v}{c_a} \quad \text{.............................} \quad (16)$$

must be included between the (parallel) front of the nappe and the wall.

For R = 5 cm, v = 8.3 m/sec and c_a = 331.6 m/sec equation (16) gives a value of .0012 m or 1.2 mm.

Because of irregularities and disturbances in the front of the jet, it may happen, that more air can escape than from the idealized case of a parallel front; also in opposite direction more air could be entrained by large cavities in the front.

This content of air of equation (16) seems to be very small, but taking into account the relation of elasticities or compressibilities of water and air given by equation (2)

$$\frac{E}{E_a} = 15500 \quad \text{................................} \quad (2)$$

it can be shown that this content of air in the compressed volume of the length x is able to explain the damping of water hammer pressures $\varrho \cdot v \cdot c$ to the values of observed shock pressures:

The relation between the compression of the volume of the length x may be related (neglecting the expansion volume) directly to the pressures in it, that is

$$\frac{\frac{A(x-D)}{E} + \frac{A\cdot D}{E_a}}{\frac{A\cdot x}{E}} = \frac{\varrho\cdot v\cdot c}{p_{max}} \quad \text{or}$$

** $$p_{max} = \frac{1}{1+(\frac{E}{E_a} - 1)\cdot\frac{D}{x}} \cdot \varrho\cdot v\cdot c \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Evaluating the pressures $p_{max\,10}$, $p_{max\,50}$, $p_{max\,90}$ and $p_{max\,100}$ on Figs. 4 to 9 by equation (17) with equation (2), Fig. 14 gives the results for the dimensionless ratio D/x between the thickness D of the air cushion and the length of expansion x.

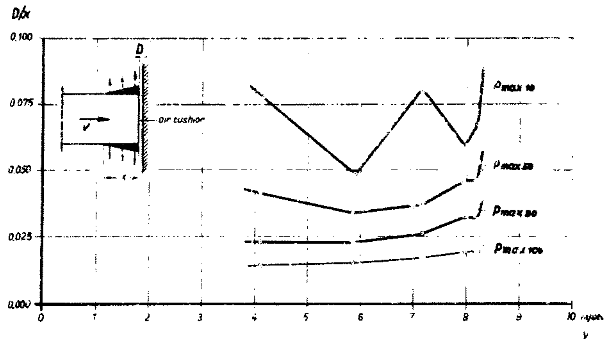

Fig. 14. Air content/expansion length = D/x

From Fig. 10, for the pressure $p_{max\,50}$, x was found about .5 R = 2.5 cm; with D/x about .04 to .05 from Fig. 14 it can be seen that a thickness D is necessary of

$$D \sim 1 \text{ mm for } p_{max\,50}$$

in order to explain the relation between observed shock pressure and water hammer pressure; for the highest observed pressures from 100 tests it gives with x/R about .3 from Fig. 10 and D/x about .02 from Fig. 14

$$D \sim .3 \text{ mm for } p_{max\,100}.$$

Here it is to be taken into consideration, that the factor of equation (2) is variable and decreases with the adiabatic rise of pressure. So equation (17) can only give an approximate approach, but there is a good agreement in the order of magnitude.

Fig. 10 shows for the equation (7) with $p_{max} = p_{max}$ (v^2) an increase of the values of x/R with v according to an increase of the pressure with a lower power of v than 2. From Fig. 14 it can be seen, that also the values of D/x indicate a slight increase with v; that means that the rise of peak pressure is even lower than the power 1 of v (equation (17)). The range of observation is too small to give a clear relation here; from both Fig. 10 (equation (7)) and Fig. 14 (equation (17)) can be seen that the scatter of results by stochastic effects is much higher than the dependence from v. It seems certain that there is also a correlation between x and D as mentioned before, as a high D also may give a higher value of x; by the superposition of the stochastic processes in both, it is not possible here to seperate them. Because the stochastic variable x in equation (7) as well as the stochastic variable D in equation (17) are in the denominator, the always stated normal-log distribution of p_{max} can be explained.

It seems to be sure that the shock pressures do not follow the law of FROUDE as already stated by ALLEN (1), BAGNOLD (2), JOHNSON (6) and MINIKIN (8); RICHERT (10) recently gives a theoretical approach for the scale-up of shock pressures in models; more experimental data are necessary also for this formula.

Because the surface tension of the water is the same in the model as in nature, it is to be expected that scale effects occur in a manner that small model waves with considerably smooth fronts have lower air content than larger waves in nature.

Especially for high impact velocities, there is a lack of information about the shock pressures produced by them. The present paper will give a contribution to this problem.

5. LIST OF SYMBOLS

A	=	Area of impact on the wall
A_e	=	Area of expansion at the sides of the jet
D	=	air content, represented by an uniform thickness on the area A
E	=	elasticity of water = $\varrho \cdot c^2$
E_a	=	elasticity of air = $\varrho_a \cdot c_a^2$
H_B	=	height of breaker
Q	=	inflow of the jet = $A \cdot v$
Q_e	=	outflow through the area of expansion $A_e = A_e \cdot v_e$
R	=	A/U = hydraulic radius of impact area
U	=	circumference of the area of impact
c	=	velocity of sound in water = 1485 m/sec for 0^o C and atmospheric pressure
c_a	=	velocity of sound in air = 331.6 m/sec for 0^o C and atmospheric pressure
d	=	water depth on the measuring area
g	=	gravitational acceleration = 9.81 m/sec^2
p	=	pressure
p_{max}	=	maximum of pressure during impact
$p_{max\ 10}$	=	pressure not exceeded by 10 % from 100 tests
$p_{max\ 50}$	=	pressure not exceeded by 50 % from 100 tests
$p_{max\ 90}$	=	pressure not exceeded by 90 % from 100 tests
$p_{max\ 100}$	=	highest pressure measured during 100 tests
p_s	=	maximum pressure of steady flow with the velocity v = $\varrho \cdot \frac{v^2}{2}$
t_1	=	time of pressure rise from p=o to $p=p_{max}$
t_2	=	time of pressure drop from $p=p_{max}$ to $p=p_s$
t_s	=	$t_1 + t_2$ = total duration of impact
v	=	velocity of impact, perpendicular to the measuring plane
v_e	=	velocity of water due to expansion on the sides of the jet
v_a	=	escaping velocity of air between the front of the jet and the wall
x	=	length of expansion area in axis of the jet
α	=	jet angle or angle of approach (Fig. 3)
β	=	front angle (Fig. 3)
δ	=	dimensionless number of impact given by equation (12)
ϱ	=	density of water
ϱ_a	=	density of air

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STUDIES OF WAVE LOADS ON CONCRETE SLOPE PROTECTIONS OF EARTH DAMS

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SUMMARY

A description is presented of a wave flume with a pneumatic wave productor, which can generate waves up to 2 m high. The principal experimental results are outlined of dynamic wave action on the slope protection of an earth dam, with the slope protection formed of slabs with open joints placed on an artificial filter layer. Data are cited on wave pressure distribution over the upper surface of the slope protection, and on the uplift pressure occurring under the slabs.

The effect of scale modelling on the wave pressure values in the breaking zone is estimated based on a series of scaled experiments, with wave heights ranging between 5 and 125 cm.

The erection of large modern hydraulic river projects is closely connected with the construction of large reservoirs where wind waves of over 3 m high may occur. Under these conditions the problem of protection from wave action of upstream slopes of earth dams and dykes, as well as protection of the natural shoreline from erosion and scour assumes a special importance.

The following types of slope and bank protection for hydraulic structures are extensively used at present in the construction practice of the U.S.S.R.:

1. Concrete protections:
   a) monolithic slabs concreted in situ,
   b) protection constructed of prefabricated slabs.
2. Asphalt concrete impervious coatings.
3. Stone protections:
   a) rock facing,
   b) dumped rockfill.

The main problems in designing the strength and stability of slope protection can be reduced to establishing two values:

a) the maximum hydrodynamic wave pressure on the slope protection
b) the maximum uplift pressure under the slab protection.

In the Soviet Union three main trends can be distinguished in the research into wave loads on slope protections.

1. Theoretical studies for determining the mean value of the hydrodynamic pressure in the breaking zone and establishing regularities in the propaga-

tion of the impact under the slab, both for open and closed joints between the slabs.
2. Models and special stands were used for investigations into the dynamic wave action on slabs, in particular, the establishment of relationships between wave parameters and wave loads under conditions of varying slope steepness, head, soil characteristics under the slabs, and the permeability factor of the protection.

3. Verifying of the experimental findings and the conclusions drawn at special experimental areas in the field.

It should be noted that the possibilities of theoretical solutions in designing slab strength and stability are essentially limited at present by the lack of data characterizing the complex mechanism of the interaction between the waves and the slope, in particular, such important characteristics as the duration of the wave impact, pressure pulsations in the impact zone, turbulence parameters in the run-up zone on the slope, the frequency of free oscillations of slabs, with virtual masses of water and soil taken into account, the decrement of damping of slabs oscillations et al. The above characteristics can be obtained only experimentally.

At the VNIIG when considering different procedures of experimental research into the dynamic action of waves on slope protections it was found expedient to use the scale series method with wave parameters closely approaching those in the prototype.

The necessity of conducting large-scale model experiments is mainly caused by the desire to get rid of the scale effect and establish conditions when the results of model investigations can be recalculated for the prototype according to the scale relations implied by the law of gravitational similitude, i.e. the Froude criterion.

At the Institute an experimental set-up was built consisting of a wave flume and a wave productor capable of generating waves up to 2 meters in height (Fig.1.).

The present paper is devoted to a short description of the set-up and some research findings on wave loads on slabs with open joints.

I. Description of the Experimental Set-Up.

The operation of the pneumatic wave productor is based on inducing oscillations in the liquid in an air tank with its open side submerged in

** water. The oscillations in the air tank are created by the varying air pressure in the part of the tank which is above the water level.

The air pressure parameters, the dimensions of the tank and the characteristics of the waves generated were established by utilizing the theoretical solution to the problem of wave generation caused by periodically varying pressures.

Using Lamb's solution for wave disturbances at an infinite depth E.F.Sakhno (R.1) received a more general solution of the problem for the case of waves at a limited depth under unit surface pressure equal to unity varying in accordance with the law:

$$p(x,t) = e^{i\sigma t} \cos k(x-a) \qquad /1/$$

where
- e - the base of natural logarithms.
- σ - circular frequency of pressure variation.
- T - wave period.
- t - time.
- λ - wave length.
- a - width of the air tank.
- x - abscissa with the origin of coordinates at the still water level at the reservoir wall.

Preliminary experiments showed that air pressure in the air tank is

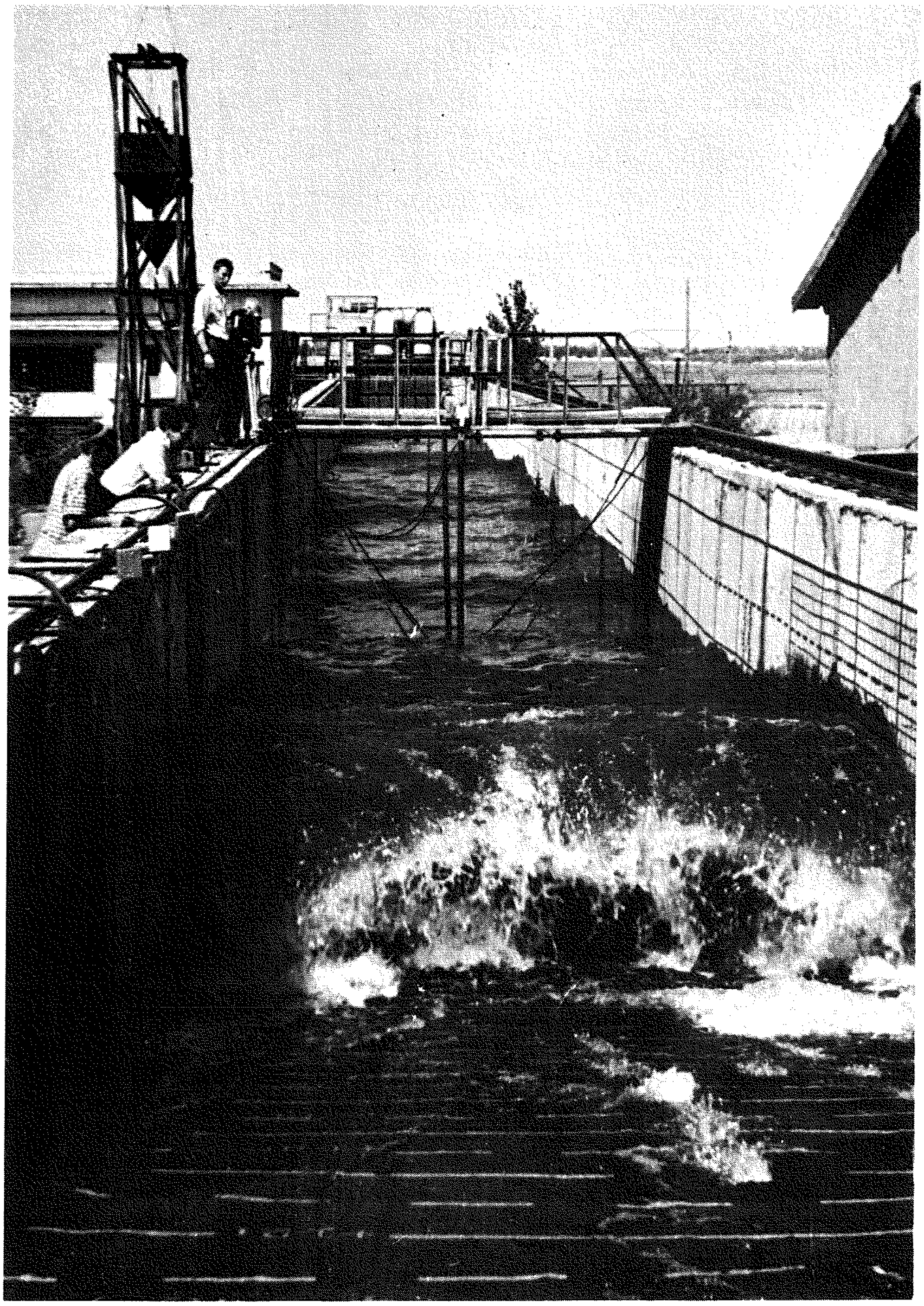


Fig.1. General view of the wave stand.

uniformly distributed across its width. In this case the equation for the waves generated by the wave productor can be written as :

$$y(x,t) = -\frac{2P_0}{\gamma} A e^{-2\pi b/\lambda} \sin 2\pi \frac{a}{\lambda} \sin\left(\sigma t - 2\pi \frac{x}{\lambda}\right) \qquad /2/$$

where
- y - wave surface ordinate.
- P_0 - amplitude of the air pressure in the air tank.
- γ - volume weight of water.
- A - factor taking account of the effect of shallows on the wave amplitude.

$$A = \frac{Sh\, 4\pi \frac{H}{\lambda}}{Sh\, 4\pi \frac{H}{\lambda} + 4\pi \frac{H}{\lambda}} \qquad /3/$$

- the depth of immersion of the tank in undisturbed water
- the depth of water in the flume.

The set-up is comprised of two principal parts: a wave flume and a pneumatic wave productor (Fig.2). The system of controlled air supply into the air tank /1/ incorporates two fans /2/, suction and delivery pipes /3,4/, an air distribution valve /5/ and butterfly throttles /6/.

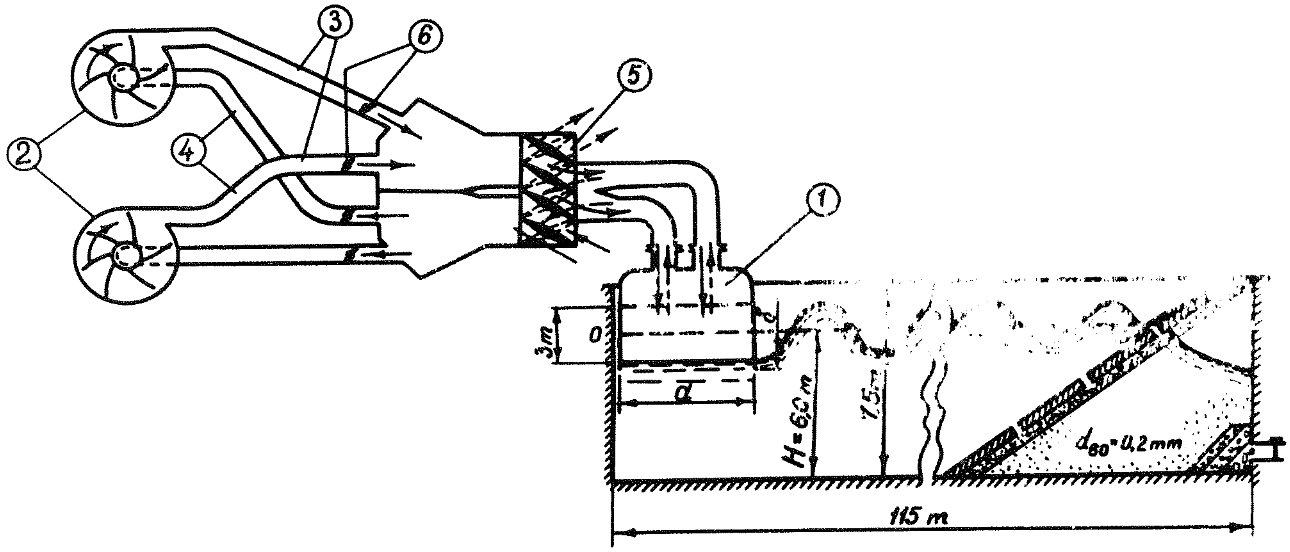


Fig.2. Layout of the wave stand with a pneumatic wave productor
1 - wave generating tank, 2 - fans, 3- and 4 - suction and delivery piping, 5 - air distributing valve, 6 - butterfly throttles.

The air distribution valve is designed to alternately connect the delivery and suction pipes to the atmosphere or to the part of the tank filled with air during the operation of the wave generator.

The wave height depends on the volume of air forced into and out of the tank. The air flow through the air pipes is regulated with butterfly throttles. The shutters of the air distribution mechanism are operated through a reducer and a crank drive by a direct-current motor, which allows to control the shut-off frequency of air pipes / wave frequency / within a wide range from 0.55 to 0.17 1/sec., which corresponds to the wave length of 5 to 40 m .

Characteristics of the experimental set-up:

1. Wave flume dimensions:
   Length - 115 m, Width -- 4 m, Depth - 7.5m.
2. Wave parameters:
   Height, h = 0.5 - 2.0 m ; Length, λ = 5 - 40 m ; Period, T = 1.8-5.8sec

3. Fan characteristics:

Overall capacity - 1400 m^3/min. Excess working pressure - 2900 mm of water column. Total output of electric motors for operation of fans - 800 kW. Air pipe diameter - 704 mm.

The wave productor tests have shown that:

- The profile of waves generated with a steepness $h/\lambda < 1/20$ is close to a sinusoid, and of those with a steepness $h/\lambda \geq 1/20$ is closer to a trochoid.
- The optimum depth of immersion of the air tank in undisturbed water, $b = (0.08 - 0.10)\lambda$.
- Wave generator efficiency factor is $\approx$ 40%.
- Steady -type waves are formed at a distance of 1.5λ from the wave generator.
- The maximum deviation of the height of the waves generated from the calculated value is $\pm$ 5%.

As compared to a mechanical wave productor a pneumatic one possesses a number of advantages: its higher efficiency results in reducing electric power consumption by the crank drive, the layout of the driving mechanism is more compact, wave generation takes place over a short section at the beginning of the flume and is performed with greater accuracy.

II. Distribution of Maximum and Minimum Wave Pressures over the Slope.

The term "wave pressure" is used here to denote a deviation of the pressure p at a given point on the slope protection from the hydrostatic pressure p_o , i.e. $\Delta p = p - p_o$.

The investigation on the distribution of the extreme values of wave pressure over the slope was aimed at determination of the maximum pressure value in the breaking zone, and evaluation of the degree of wave pressure damping above and below the breaking zone.

At the first stage of the study attention was concentrated on waves of a steepness 1/8 - 1/10 as most representative for river reservoir conditions.

The results of pressure measurements on a slope 1 : 4 for a wave steepness h/λ = 1/10 are plotted in Fig.3.

In this diagram along the y-axis maximum wave pressure is plotted in terms of the water column height against the wave height ratio $\frac{\Delta p_{max}}{\gamma h}$ and $\frac{\Delta p_{min}}{\gamma h}$ while along the x-axis is plotted the relative distance from the water edge. It was established that x/λ

1. The maximum pressure at the wave impact point is $\Delta p_{max} = 1.45\gamma h$
2. The maximum pressure point is at a distance of $x = 0.07\lambda$ from the water edge.
3. The minimum pressure is about $\Delta p_{min} = -0.25\gamma h$ and is applied at a distance of $(0.2 - 0.4)\lambda$ from the water edge.
4. The experimental points obtained at different wave heights / h = 50, 75, 100 and 125 cm / are concentrated about the curves $\frac{\Delta p_{max}}{\gamma h}$ and $\frac{\Delta p_{min}}{\gamma h}$ which indicates the existence of a stable functional relationship.

The second stage of the studies is devoted to determining the influence of wave steepness on the magnitude of wave pressure. Experiments conducted for $\varepsilon = h/\lambda$ = 1/10, 1/15, 1/20, 1/25, 1/30 showed that with a reduction in wave steepness, i.e. when wave length increases and wave height remains constant the relative pressure in the impact point increases linearly.

The relationship $\Delta p_{max}/\gamma h = f(\varepsilon)$ for the wave impact point is illustrated in Fig.4. The linear relationship between pressure and wave steepness seems to be caused by the linear dependence between wave energy and wave length when wave height remains constant.

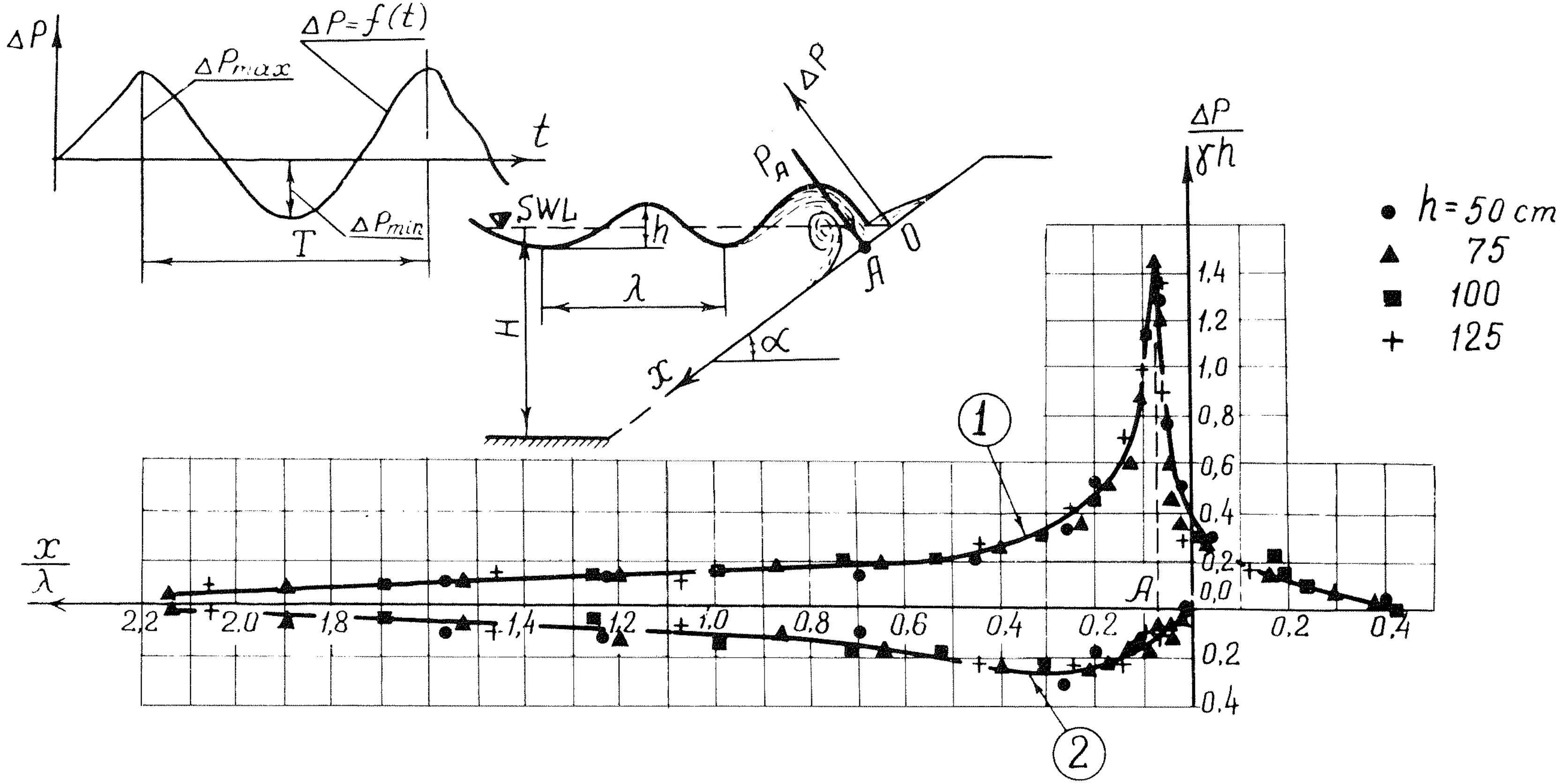


Fig.3. Distribution of extreme values of wave pressure on the upper surface of the slab protection (data based on experimental studies on the wave stand).

① $\frac{\Delta P_{max}}{\gamma h}$; ② $\frac{\Delta P_{min}}{\gamma h}$; $\varepsilon = \frac{h}{\lambda} = \frac{1}{10}$, $\frac{H}{\lambda} = 0{,}48 - 1{,}1$; $ctg\,\alpha = 4$.

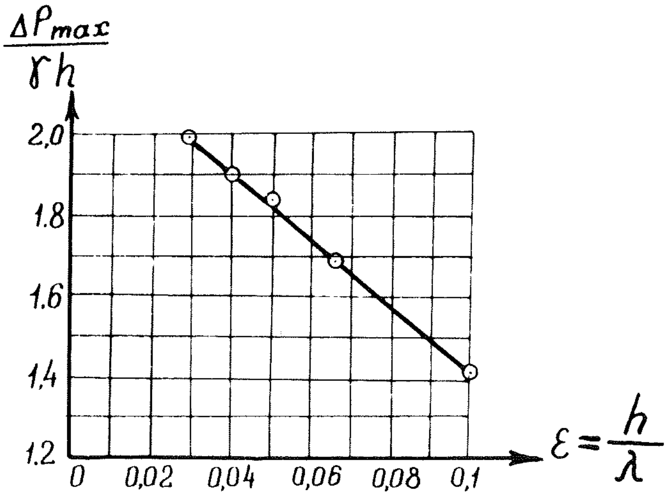


Fig.4. Effect of wave steepness on the wave pressure valve at the zone.

III. Resultant Wave Pressure on Slabs.

The strength of slabs is designed according to maximum wave pressures acting on their upper surfaces. The thickness of the slab ensuring its stability on the slope is determined by the maximum uplift value. Wave pressure on the upper surface of the slab depends on wave parameters, slope steepness, water depth above the structure. Besides, wave pressure is also affected by the perviousness factor of the protection (in the case under investigation by the number and dimensions of joints between slabs), the location of the wave impact point relative to the joint, and the seepage characteristics of the foundation. The modelling of hydrodynamic processes under the slab involves great difficulties. Therefore the establishment of regularities in the distribution of wave pressures under the slab required a series of scaled experiments, with wave heights approaching those in the prototype. Wave pressure and uplift studies were conducted on an earth dam model 7.5 m high, with a crest width of 5 m and the upstream batter 1 : 4. The dam was constructed of fine sand. The upstream slope of the dam was protected by concrete slabs 4 x 1.98 m, 15 cm thick, with the joints between the slabs 2 cm wide. The slabs were placed on a continuous inverted filter 25 cm thick. The
**largest particle diameter in the filler material was 70 mm.

Wave pressure and uplift were recorded by inductance meters with a resolution of up to 500 cps.

In the breaking zone the meters were spaced at 10 cm intervals; above and below the breaking zone the spacing varied between 50 and 100 cm. Two pickups were placed at every metering station, permitting to record wave pressure and uplift simultaneously. In the experiments wave heights varied between 50 and 1.25 cm, and wave steepness between 1/10 and 1/35. Synchronous recording of dynamic wave action on the slope for different wave phases with an interval 0.1 T led to obtaining curves of resultant pressure on slabs in the form of

$$\frac{\Delta P}{\gamma h} = \frac{\Delta P_1 + \Delta P_2}{\gamma h} = f\left(\frac{y}{\lambda}\right)$$

where ΔP_1 and ΔP_2 - wave pressure and uplift, respectively.
h and λ - height and length of wave.
x - distance of the point on the slope from the water edge.
T - wave period.

The most unfavourable combinations of loads on the slab were found to occur during two wave phases:

1. At the moment of breaking on the slope, which corresponds to maximum pressure on the upper surface of the slab.
2. At the moment preceding the breaking of the wave (0.20 - 0.25)T before the impact, when maximum uplift pressure occurs under the slab. The indicated moments and corresponding curves are accepted as design ones in calculating strength and stability of the slab protection under study.

Curves of resultant wave pressure for the indicated wave phases are presented in Figs 5-I and 5-II for the case when the wave impact zone lies in the central part of the slab. The location of the joint between slabs relative to the wave impact zone materially affects the magnitude and the distribution of the resultant wave pressure. In Fig.5-III the pressure curve is given for the case when the wave impact falls on the joint between the slabs. In this case due to instantaneous transfer of external pressure to the zone under the slab, the uplift pressure under the slabs above and below the impact zone increases and the resultant downward pressure value in the impact zone decreases (up to 40%).

The change in maximum ordinates of pressure and uplift depending on the location of the wave impact zone in relation to the joint is shown in Fig. 5-IV.

Experiments conducted led to the following conclusions:

1. Coincidence of pressure variation phases both for the upper and lower surfaces of the slab protection is achieved by making pervious joints between slabs placed on a continuous inverted filter.
2. Coincidence of pressure phases leads to a reduction (up to 40%) in the total wave load on the upper surface of the slabs and to a material increase in the uplift pressure under the slab protection with the joints open as compared to a continuous protection.
3. Periodical pressure variations under the slabs exert an adverse influence on the performance of the inverted filter, therefore its stability against piping and its percolation characteristics must be higher than those for a filter underlying slab protections with closed joints.
4. The problem of the permeability factor of the protection should be solved on the basis of an economic comparison between different design versions for the protection.

IV. Effect of Wave Dimensions on the Relative Pressure Value in the Wave Impact Zone.

In the course of experimental studies the researcher is often confronted with the problem of establishing the minimum model wave dimensions which allow to scale up experimental findings to the prototype on the basis of the Froude criterion. For dynamic wave action on a slope the automodelling region is not yet strictly defined. A series of experiments was conducted with wave heights ranging between 3 and 120 cm, with a constant steepness of 1/10 aimed at defining the scale effect of the absolute wave dimensions on the value of relative wave pressures in the impact zone, the run with 120 cm wave height being provisionally accepted as the prototype.

The other runs with wave heights $h_M <$ 120 cm were considered as

** models of the prototype. The results of the scaled series of experiments are

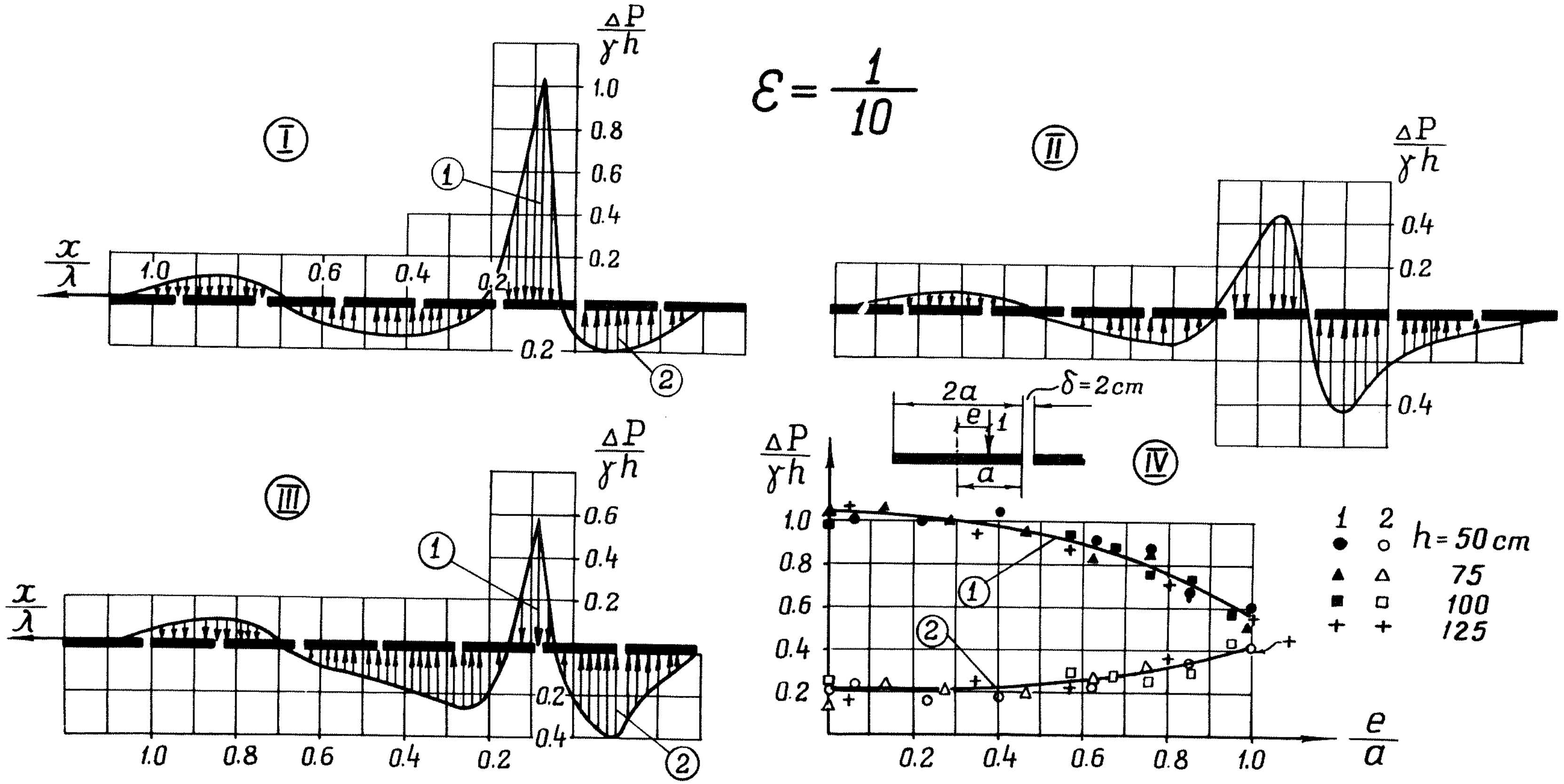


Fig.5. The resultant of the wave pressure on the slabs.
I - with t = 0.0 (moment of impact) } wave impact on the
II - with t = 0.2 T (before the impact) } central zone of the slab.
III - with t = 0.0 wave impact at the joint between the slabs.
IV - Variation of the resultant wave pressure depending on the location of the wave impact relative to the joint between the slabs.

presented in Fig.6 , where $K_p = \frac{\overline{\Delta P_M}}{\overline{\Delta P_N}}$; $\alpha = \frac{h_M}{h_N}$;

$$\overline{\Delta P_M} = \frac{\Delta P_M}{\gamma h_M} ; \overline{\Delta P_N} = \frac{\Delta P_N}{\gamma h}$$

Notation for the subscripts : M - model, N - prototype.

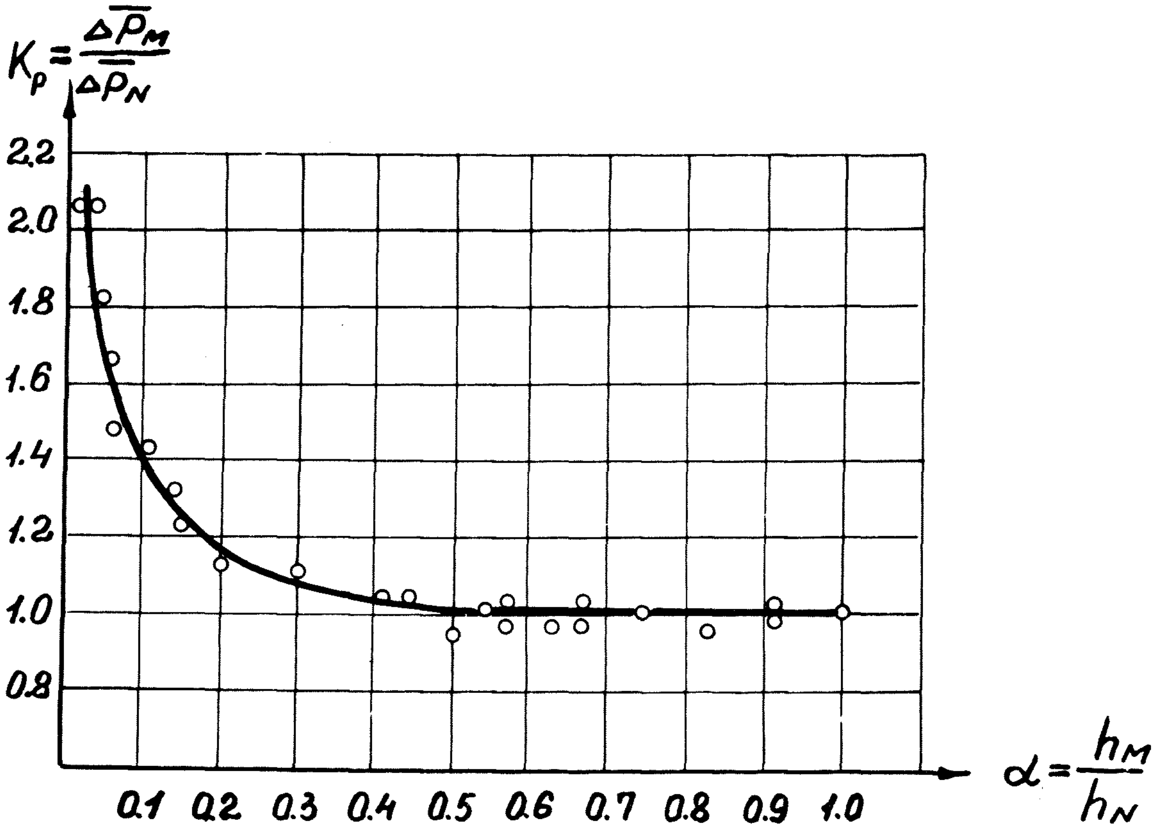


Fig.6. Effect of wave height on the relative wave pressure value in the wave impact zone.

As can be seen from the graph the value of relative wave pressure is practically independent from the absolute wave height only when $\alpha \geqslant 0,4$, i.e. at wave heights $h \geqslant$ 50 cm.

The data obtained from the scaled experimental series indicate, that on the basis of the Froude criterion it is not permissible to directly scale up to the prototype values of wave pressures measured on small scale models. One of the principal reasons of the great difference in pressures recorded at the impact zone seems to lie in the fact that with greater wave heights (h 50 cm) the jet falling from the wave crest is highly aerated and sprayed, the water cushion which transmits the impact pressure on the slab is also air saturated and the interaction between the wave and the slope materially differs from that of small height waves when aeration is negligible or does not occur at all.

Hence it follows that if the maximum wave height in reservoirs usually varies between 3.0 and 4.0 m, the minimum model scale, which enables to obtain reliable data on wave loads on protection, should not be less than 1/7 - 1/8 of the prototype dimensions.

List of Notations.

h - wave height.
λ - wave length.
T - wave period.
ε - wave steepness.
t - time.
e - base of natural logarithms.
y - wave surface ordinate.
$\mathcal{P}$ - amplitude of pressure variation in air tank.
$\mathring{P}$ - pressure on slabs with waves generated in the flume.
P_o - hydrostatic pressure on slabs (pressure with still water level in flume).
ΔP - wave pressure.
γ - volume weight of water.
P - resultant wave pressure.

Reference

1. Sakhno, V.F., "Theory and Design of Pneumatic Wave-Productors", Trudy TsNII im. Acad. A.N.Krylov, 1961, vyp.12, Sudpromgiz.

PAPER 7

Studies of Wave Loads on Concrete Slope Protections of Earth Dams,

by SKLADNEV and POPOV

Due to illness of the authors, there has been no discussion on Paper 7. One of the participants to the Symposium, Mr. Bakker, was so kind to make available a copy of his post symposium correspondence with the authors. The Organizing Committee considers this correspondence of such importance that it is reproduced here.

BAKKER: With much interest I read the paper of Mr. Popov and you.

In order to compute the needed thickness of asphalt revetments Rijkswaterstaat intends to use a computer program, developed by the Royal Dutch Shell Company, which enables to find the tension in the asphalt revetments, if the load, elasticity coefficients of asphalt and basement and other material constants are known.

Your paper can give a contribution about the knowledge of the loads by wave impact. However, not only the maximum wave pressure is of importance in this case, but also the distance along the slope over which the impact takes place.

According to your Figure 3, a reasonable, unfavourable approximation seems to be a uniform load of $1.4 \gamma h$ over a distance from $\frac{x}{\lambda} = .05$ till .10, if the slope is 1:4 and the wave steepness ε is $\frac{1}{10}$ (Fig.1).

However, although Fig. 4 gives the influence of smaller wave steepness on the wave pressures, I did not find the influence of the wave steepness on the distance along the slope, over which the impact takes place.

Although the horizontal axis in your Fig. 3 gives $\frac{x}{\lambda}$, it seems dangerous to extrapolate this result for other wave steepness. For in the case of Fig. 3, the impact takes place roughly over a distance $0.05\lambda = 0.05 \cdot 10\ h = 0.5\ h$, which sounds reasonable. However, for a wave steepness 0.02 this would be $0.05 \cdot 50\ h = 2.5\ h$, which is more difficult to imagine.

Summarizing, I would ask the following questions:

1. Do you think the scheme of Fig. 1, below, is reasonable in order to compute the maximum bending moments of an asphalt slab, in the case of

the conditions mentioned in your Fig. 3 ?

2. Can you send me similar Figures, as Fig. 3, for other wave steepness ?
3. Do you think the permeability of the slab you used effected the maximum wave pressure very much ?

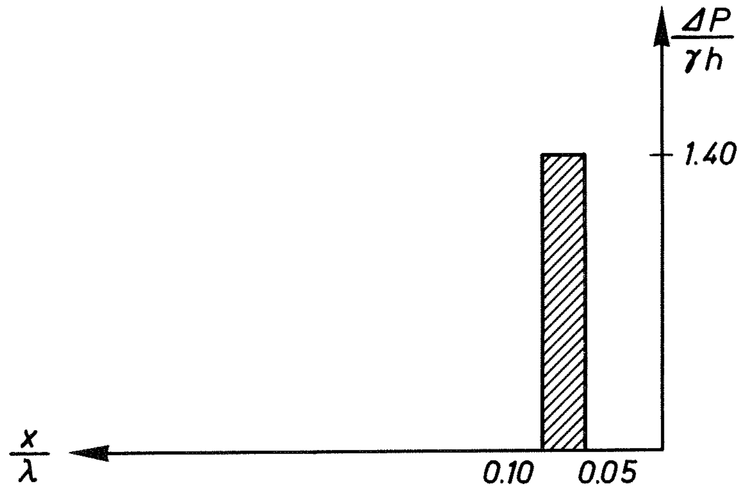


FIG. 1 SCHEMATISED WAVE PRESSURE

SKLADNEV: Unfortunately a prolonged illness prevented me from answering at the proper time your letter of March 28, 1969. Having resumed my duties at the Institute only at the end of May, I was unable to prepare earlier the materials for our joint reply with I.Ya. Popov to the questions you are interested in.

1. The diagram in Fig. 1 at the end of your question may be used as an approximate representation of wave impact pressure in the breaking zone with wave steepness 1/10. It must be borne in mind that the diagram $\frac{\Delta p}{\gamma h} = f\left(\frac{x}{\lambda}\right)$ shown in Fig. 3 of our report illustrates the distribution of extreme values of wave pressures over the surface of the slope. It goes without saying that extreme pressures do not occur simultaneously, but with a phase shift in time. However, taking into account the comparatively short length on which the impact pressure is applied, in our opinion, the phase shift can be neglected and, therefore, the scheme adopted by you may be considered acceptable.

2. According to our findings the length of the impact zone $\frac{x}{\lambda}$ changes only slightly with decreasing wave steepness. The change in the length of the impact zone with a varying $\frac{h}{\lambda}$ can be observed in the diagrams $\frac{\Delta p}{\gamma h} = f\left(\frac{x}{\lambda}\right)$ which correspond to $\frac{h}{\lambda} = \frac{1}{15}, \frac{1}{20}, \frac{1}{25}$ and $\frac{1}{35}$ (Figures B, C, D, and E). The above diagrams as well as the diagram in Fig 3 of our report are plotted without taking into account uplift pressure and are

valid only for impermeable continuous slightly deformable slab protections.

3. In the case you refer to for more accurate calculations can be applied the diagram of instantaneous wave pressure at wave impact with $\frac{h}{\lambda} = \frac{1}{35}$ (Fig. A). The "peak" value of $\frac{\Delta P_{max}}{\gamma h}$ for wave steepness $\frac{1}{50}$ can be obtained by extrapolation from Fig. 4 of our report.

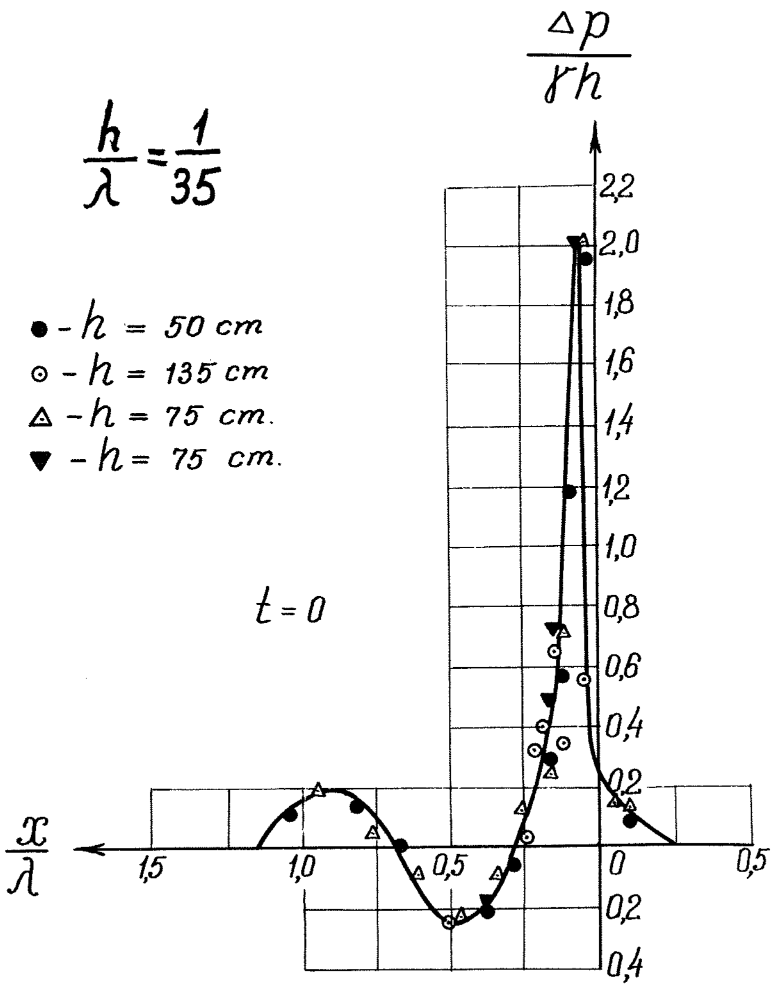


Fig.A Diagram of wave pressure at the wave impact.

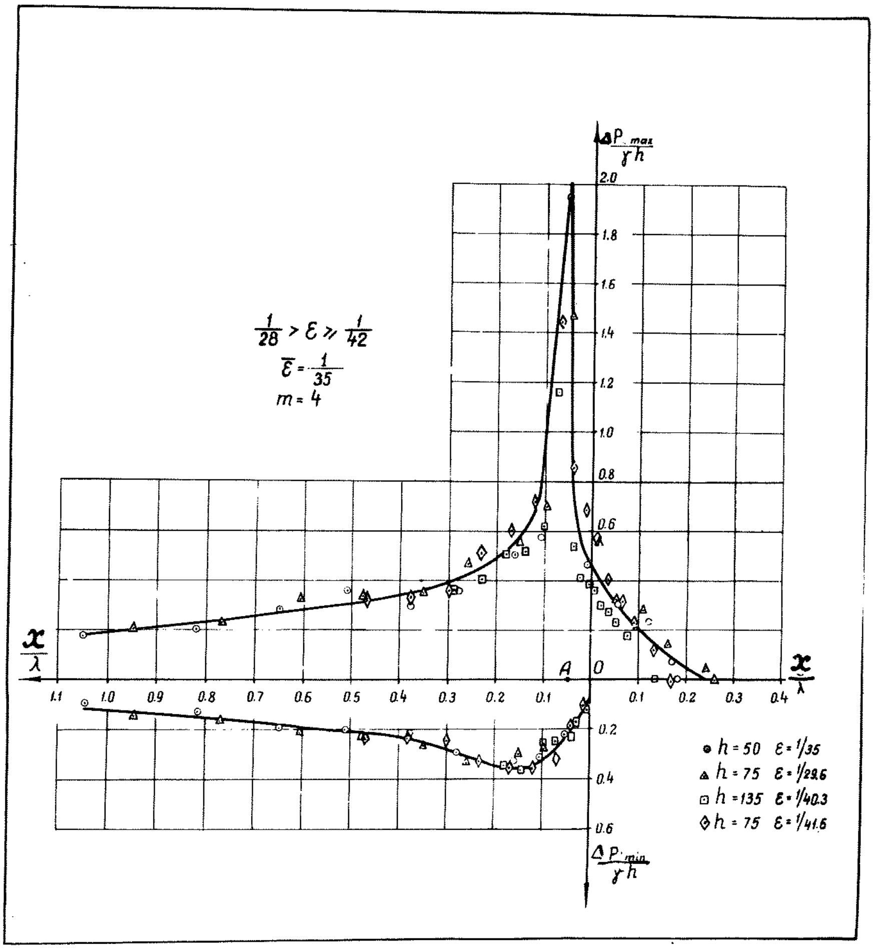

1/28 > ε ≫ 1/42
ε̄ = 1/35
m = 4
h = 50 ε = 1/35
h = 75 ε = 1/29.6
h = 135 ε = 1/40.3
h = 75 ε = 1/41.6

Fig. B

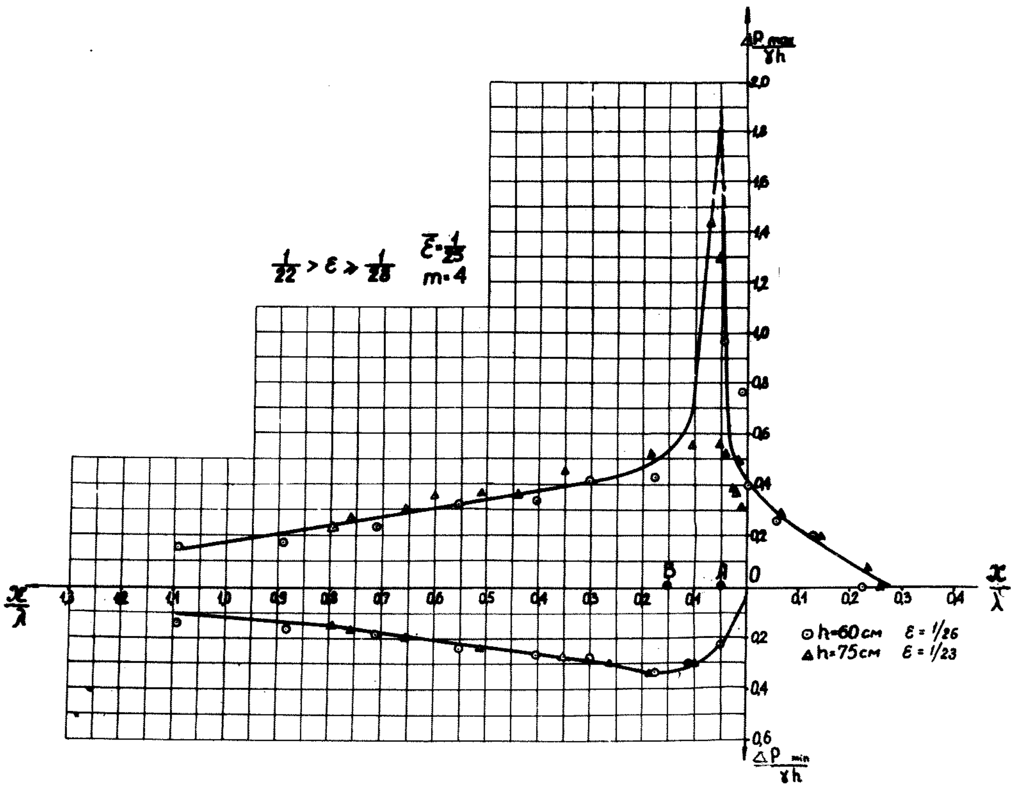
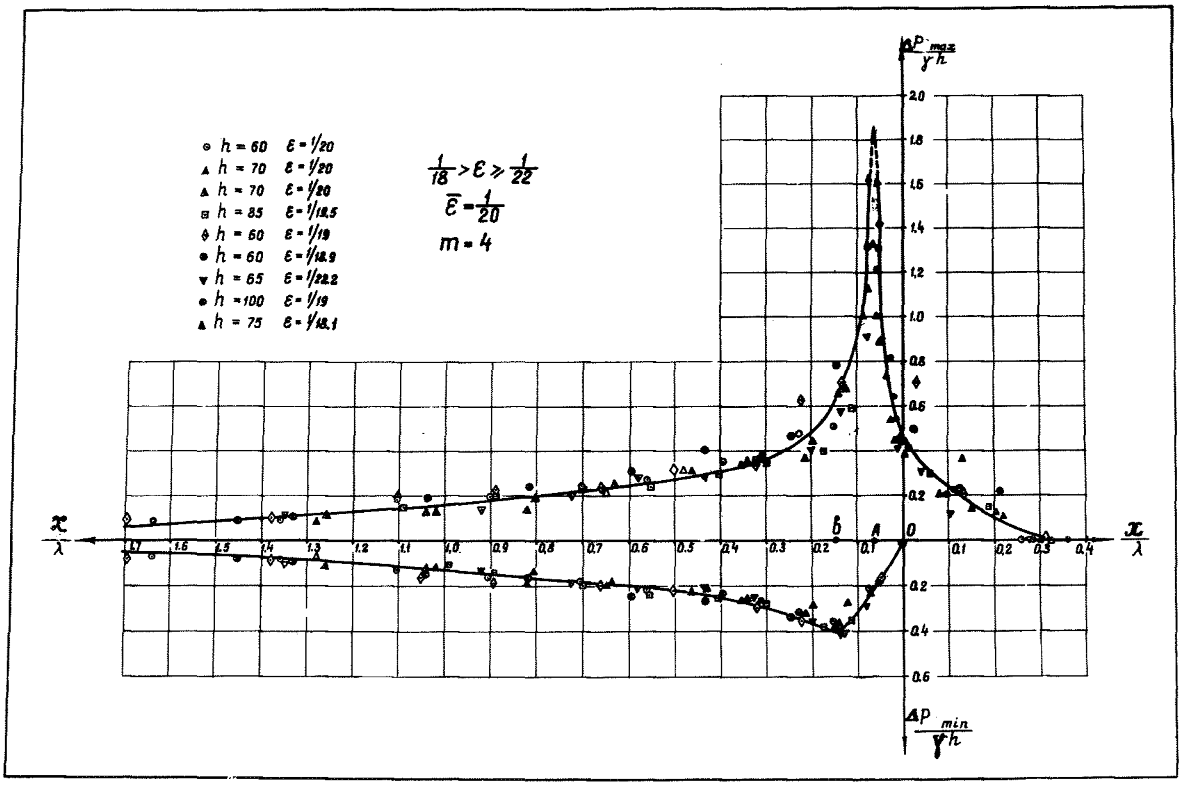
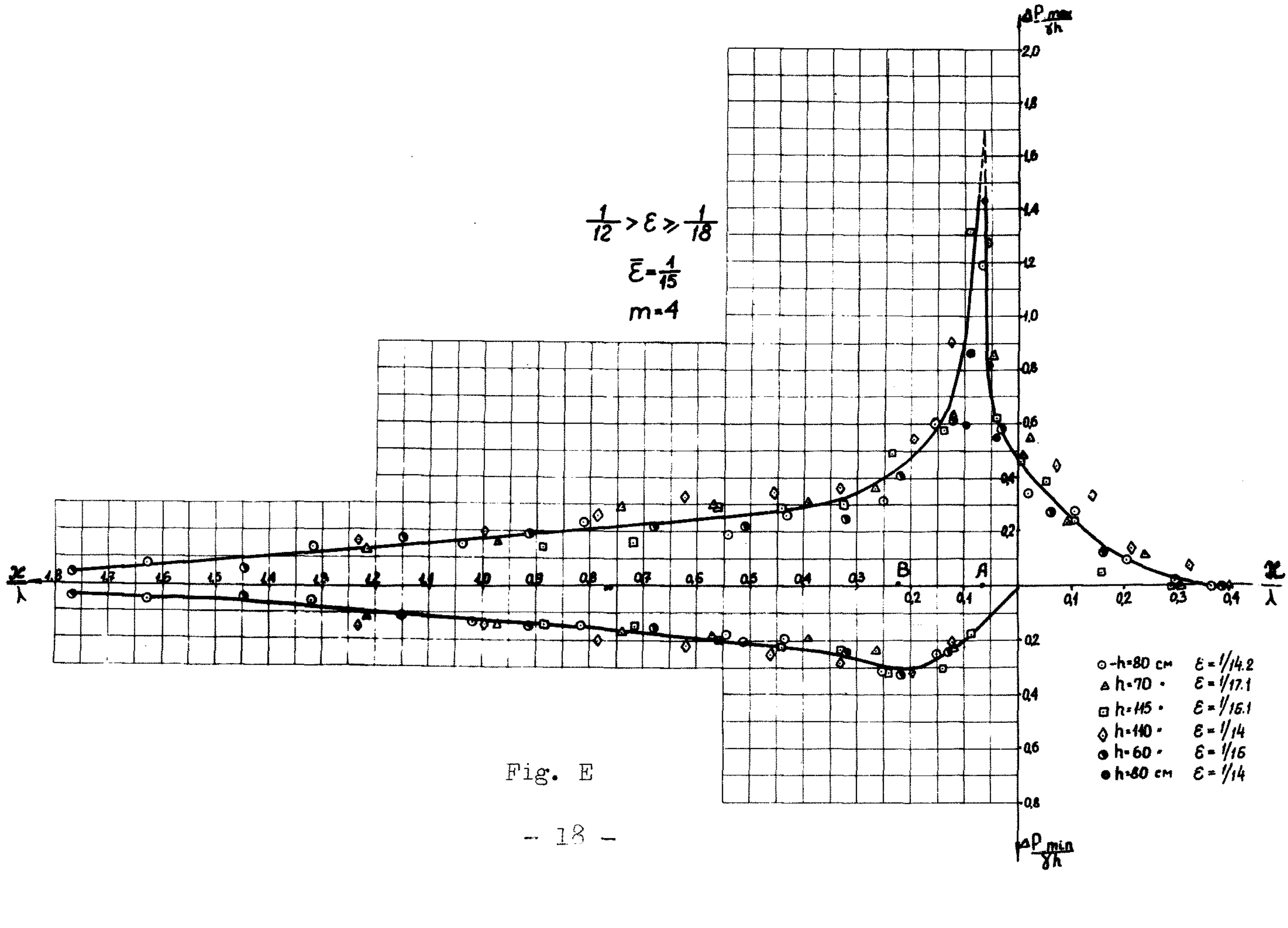

1/22 > ε > 1/28
ε̄ = 1/25
m = 4
h=60cm ε = 1/26
h=75cm ε = 1/23

Fig. C



$\Delta P_{max} / \gamma h$
h = 60 ε = 1/20
h = 70 ε = 1/20
h = 70 ε = 1/20
h = 85 ε = 1/18.5
h = 60 ε = 1/19
h = 60 ε = 1/18.9
h = 65 ε = 1/22.2
h = 100 ε = 1/19
h = 75 ε = 1/18.1
$\frac{1}{18} > \varepsilon \geq \frac{1}{22}$
$\bar{\varepsilon} = \frac{1}{20}$
m = 4
B
A
x/λ
$\Delta P_{min} / \gamma h$

Fig. D



$\Delta P_{max} / \gamma h$
$\frac{1}{12} > \varepsilon \geq \frac{1}{18}$
$\bar{\varepsilon} = \frac{1}{15}$
m = 4
B
A
x/λ
h = 80 cm ε = 1/14.2
h = 70 · ε = 1/17.1
h = 115 · ε = 1/16.1
h = 140 · ε = 1/14
h = 60 · ε = 1/16
h = 80 cm ε = 1/14
$\Delta P_{min} / \gamma h$

Fig. E

APPLICATION OF LABORATORY TESTS IN HARBOUR DESIGN WORKS

J.A.JENSEN

Kampmann, Kierulff and Saxild, Copenhagen, Denmark

SUMMARY

It is described why and to what extent the practicing engineer makes use of model tests in his design of rubble mound breakwaters.

Furthermore an example is shown from practice illustrating the application of models for investigation of wave penetration into harbours. Especially the way in which model tests can confirm and supplement theoretical investigations is described.

It is concluded that model tests are and most probably will remain an indispensable tool for harbour designs.

INTRODUCTION

Civil engineers concerned with the design and construction of harbours can nowadays obtain valuable assistance from laboratory work on models.

Admittedly, guidance can also be obtained from former practice, from observation of actual harbours, of damage that may have occurred, cases of insufficient shelter etc. As long as harbours are built along traditional lines and on a small scale, surprises do not occur too often. However, in modern engineering dealing with new types of structures and when harbours are constructed at more exposed locations there is obviously an increasing need that this former practice of, so to say, using full scale models is replaced by real model tests on a reduced scale on which thorough research can be carried out systematically and economically.

The typical design procedure starts with one or several concepts which are subjected to calculations, then follows an economic evaluation considering construction costs as well as maintenance costs. The result of these investigations will be one or several preliminary projects which can be examined by laboratory tests. Based on the results of these tests the optimum project is selected and probably revised where indicated from the tests and then detailed to form the final project.

During construction some revisions may arise mainly due to construction methods and possibly also from unexpected difficulties with the weather etc.
* say, from bad weather, subsoil deficiences etc.

The completed structure is also subject to observations which can supplement the information gained in the laboratory, and thus contribute to the increased experience to be applied in future design concepts.

The harbour designer benefits from laboratory tests in three different ways:

1) Calculated design can be confirmed or corrected.

2) Parts of the structure which are impossible to calculate or where calculation would be too costly and time consuming can be investigated.

3) New design ideas may incidentally be developed from observations on the model.

There are two principal types of problems in connection with harbour design i.e. strength problems mainly concerned with structures attacked by waves such as breakwaters and the problems of providing sufficient shelter in the basins and along quays. A third and often very serious type of problem is presented by siltation caused e.g. by littoral drift.

In the following some examples will be given of how model investigations are utilized in design works with special reference to rubble mound breakwaters and wave penetration into a harbour.

RUBBLE MOUND BREAKWATER

** As an example of a structure attacked by waves there is shown on Fig. 1 a rubble mound breakwater built up over a core of smaller stone material, quarry run or perhaps, for the lower part, sand fill or shingle. The core is protected against wave action by stone layers of various categories as required, and where necessary a stone filter layer is inserted to prevent the escape of core material through voids in the armour layer. The seaward armour is given a backing in the shape of a concrete superstructure which besides stabilizing the top also prevents percolation of wave motion through the voids in the top part of the breakwater and reduces overwash.

The shape of the core is to some extent determined by the construction procedure and the need for a certain stability at all construction stages. For instance the breakwater core is very often constructed mainly from the land by end-tipping and the crest width must therefore permit the traffic of trucks and dozers.

The slope of the rear side can often be about the natural slope of the material or only a little flatter but the front side is determined by the arrangement of armour and filter layers.

The part of the breakwater primarily exposed to wave attack is situated from water level down to a depth of H where H is the significant wave height measured from crest to trough. The attack is almost of constant intensity over this part and it is caused by the highly turbulent flow of water resulting from breaking of the waves.

Several attempts have been made to set up formulas for the design of this part of the structure, such as by Iribarren, Hedar, Hudson. They all are of the form:

$$W = K \times H^3 \times \gamma \times f(\gamma, \gamma w, a)$$

where W is the weight of the individual armour blocks, H is the wave height, γ the specific weight of the block material, γw specific weight of water and 'a,' the inclination of the armour slope.

As long as $2 < a < 4$ they all agree pretty well and the results are normally satisfactory in practice.

Moreover the coefficient K can as is done in the Hudson formula, be given different values corresponding to various degrees of damage after the wave attack. It is thus possible not only to determine a stone category resisting a given wave train without significant damage (0-2% by weight of armour layer) but also to calculate the degree of damage caused by higher waves, provided the corresponding K is known for the type of block in question. This gives valuable guidance for setting up an economical design basis considering maintenance as well as construction costs.

Nevertheless it is wise to have the design checked in the laboratory, as the actual effect of the wave on the armour is influenced also by the shape and character of the rest of the structure, the steepness of the waves, the shape of blocks, the interlocking of blocks etc. for which there is no allowance in the formulas but which can be reproduced in model tests at least to a reasonable extent.

The armour below -H is difficult to calculate although Iribarren indicates a procedure using his ordinary formula by inserting lower wave heights corresponding to the relevant depths but the results are not too reliable, especially not in the vicinity of berms and other slope discontinuities. In this case the only safe procedure is to design with excess safety or to have recourse to model tests.

On the sea bottom immediately in front of the breakwater there may be a danger of erosion, especially with another type of breakwater, that has vertical wall right down to the bottom. Fairly good guidance may be obtained by calculation, somewhat better is a model test at an ordinary laboratory scale so that it can be combined with the execution of other kinds of tests. A thorough investigation involves rather large-scale studies with model sand of light-weight material. This is expensive and instead the problem is often solved by a conservative design of the protective mat or stone layer, based upon calculation and the above mentioned simpler tests.

The part of the armour course situated above the calm water line is often calculated according to Hudson's or similar formulas but it has been revealed through a number of model tests that this portion of the slope can be made steeper than that below the calm water level. This is one of the cases where incidentally observation of the model and the stages of its collapse by overloading has led to new design features. This can lead to a considerable saving in materials, especially for breakwaters on deep water as it reduces the width of the whole breakwater profile.

Another point where design ideas have arisen during testing is at the rear side of the capping where the armour is rather exposed in case of overflow.

It has been suggested to protect the slope by extending the concrete superstructure to cover the slope and divert overflow to fall into the water without touching the armour blocks, which can then be reduced in size.

Whether this shall be done in practice depends on the cost of concrete and formwork versus armour blocks, chance of overflow etc.

The model investigations will also give an idea of the resistance of the superstructure against sliding although a more detailed study of the wave forces will require special studies on rather a large scale using special wave load meters. This is done not only in the laboratory but also in full-scale testing on prototypes.

However, the generally accepted massive concrete type of superstructure normally does not require or justify such large-scale tests as even a very severe wave attack cannot cause direct local damage to the concrete and the safety against sliding is considered secured by the ordinary tests.

The most significant local damage to a concrete superstructure is caused by rocks thrown against it from the armour layer and by uneven settlements in the supporting rubble.

These phenomena are not really suited for model testing.

Beside this testing of the completed structure there is often good reason to test also certain critical construction stages.

In connection with weather statistics this can give an idea of the contractors risk and, if wanted, precautions can be found to reduce it.

The above mentioned investigations aim at the strength of the structure and its components, Also problems like wave run-up and overtopping are often included in the laboratory work.

Even other types of breakwaters such as vertical walls and dikes are investigated, as especially the dynamic forces from breaking waves are impossible to calculate theoretically with reasonable approximation.

It should be mentioned here that the application in the later years of flumes for wind-generation of waves in the laboratories has greatly improved the ability to reproduce natural wave spectra and thereby investigate their effects on the structures.

Application of results

It goes without saying that the structure as revised and recommended by the laboratory as a result of the model investigations is in general adopted for practical use. During the testing period there normally is, and should be, close contact between the design engineers and the laboratory staff in order to ensure that changes and arrangements initiated in the laboratory, primarily from a scientific point of view, are in agreement with the engineer's ideas of design, in which also the possibilities in practice for an economical execution of the works play an important role.

However, after the laboratory work is finished it often happens that the engineer or the contractor wishes to introduce changes in the design.

It is desirable that such changes can be carried out without resuming model testing and this may be possible especially if the original testing programme has anticipated the need of a certain freedom later to chose between alternative solutions. This leads us to the very important problem how the test results should be presented in the report.

Presentation of results

In order to facilitate modifications as described above and to give a wider picture of the situation it is of great value to know not only the resistance of the structure against the design wave for which usually a "no-damage" criterion is adopted. Also resistance against higher waves is valuable and testing should normally terminate in increase of wave attack until destruction with description of the progression of collapse.

The term "total collapse" of a rubble mound breakwater is in practice often applied to a situation where the whole top of the breakwater is washed away down to a water depth of about 1/3 wave height or so, which means that the sheltering effect of the breakwater is seriously reduced, especially during more moderate wave conditions as the waves may then pass without breaking.

Such a collapse is on average caused by waves about 70% higher than design wave but the range of variation from this is large. When rather a low value like, say, 30% is found, this should give rise to reconsideration of the height of the design wave. If waves higher than design wave can be expected, as they most often can, there should be good reasons to adopt a less frequent but higher wave as design basis in such cases.

For harbours situated on exposed coasts facing deep water oceans of very long fetches the design wave is often taken as the one statistically to be exceeded with a frequency of once every 25 years or so, but in some cases, as mentioned above, a frequency of once every 50 or 100 years may give a more satisfactory solution. In judging this, the consequences of a breakdown of other structures sheltered by the breakwater must also be taken into consideration. In some cases, however, where water depths in front of the breakwater cause the breaking of waves higher than the design wave the factor of safety will be more securely fixed and a possible reduction of the "no-damage" design wave could be considered or the design wave could be maintained while adopting e.g. a 10% damage criterion.

WAVE PENETRATION INTO HARBOUR BASINS

** The other main problem connected with harbour design is how to secure sufficient shelter against wave penetration into the harbour basins.

Forecast of waves, their dimensions, direction, probability etc. is often outside the field of the hydraulic laboratory. The same applies to wave refraction in front of the harbour as this normally can be investigated theoretically in the office with sufficient accuracy, especially compared to the reliability of design basis. In cases where also diffraction occurs in front of the harbour, the situation is much more complex and laboratory tests can be justified. Quite recently efforts have been made to treat this problem of combined refraction and diffraction theoretically, and probably in the not too distant future computer programs will be set up for such problems, which may in many cases be cheaper and quicker to use than model testing.

The diffraction of the waves inside the harbour entrance might be calculated in simple cases to a reasonable degree of accuracy. But in practice the situation is often more complicated because diffraction at many successive steps, reflexion from quays, ships etc., resonance, sea currents and even refraction contribute to make impossible a theoretical calculation. Therefore problems concerned with agitation in harbours are very often treated by use of model investigations.

The procedure is perhaps best shown by an example from practice.

In Fig. 2 a preliminary project with indications of the calculated max. wave heights is shown. The calculation has been based upon four different directions of the wave rose but at almost all points it is the average direction with H = 3m, which is decisive.

In order to compare with the model test results, the wave heights are also shown in percent of the 3.0 m wave.

It should perhaps be mentioned that the maximum wave attack comes from SW in a line almost perpendicular to the outer part of the main breakwater. They are however of no significance with regard to agitation in the harbour.

The model test results are shown on Fig. 3. It appears that agitation is somewhat smaller than estimated but the general picture is in good agreement.

Based upon these results two main changes were introduced. The very low agitation in most of the harbour indicated that the main breakwater could be shortened.

However, compared to the rest, the southern basin was rather agitated, obviously due to waves reflected mainly from the outer part of the South quay. This quay was therefore shortened and a wave absorbing slope installed opposite the harbour entrance. Also the end of the southern pier was given a slope in order to reduce agitation in the outer harbour.

Test results on this modified layout are shown in Fig. 4.

After finishing the model investigations the whole harbour was, as a result of field surveys, soundings and geotechnical borings, shifted to a position somewhat further to the North as shown in Fig. 5.

This led to another direction of approach channel which required a wider harbour entrance and, furthermore, the breakwaters were spaced somewhat wider apart so that one more pier could be placed. Finally the pier system was turned to be parallel with the secondary breakwater whereby navigation into the basins was facilitated and shelter improved, partly by this turning, partly by wave-absorbing slopes arranged at pier ends.

A new model investigation would of course have been preferable. However, construction was to start immediately and furthermore the layout was meant only as a general guide for future development as alone the Southern pier and the Northern basin were to be constructed in the first stage and the rest could eventually be changed anyhow.

** Encouraged by the fairly good agreement found between the model tests and the calculation in the first layout, it was decided to base the final design alone upon a theoretical investigation of the wave diffraction. It is felt that especially the high rate of wave-absorbing slopes surrounding the outer harbour will contribute to bring about good agreement between the calculated results and prototype or model tests, if these had been carried out.

A valuable guidance for the calculations was obtained from the model tests as not less than four different lengths of the main breakwater were investigated to illustrate the resulting effect. This served to adjust the calculated results.

This example shows how harbour layouts can in some cases be treated by calculation alone and possibly even made easier to calculate by measures such as absorbing slopes, though admittedly in most cases model tests are definitely to be preferred.

However, the value of model tests as well as of calculations will be governed by the degree of general knowledge of the wave situation on the site. This means that proper statistics and possibly specific field observations should be available.

Should this basis be defective for some reason model tests are sometimes omitted, as anyhow full advantage cannot be obtained of their high reliability.

CONCLUSION

Though it is impossible to see far into the future there are signs that in the coming years research in the laboratory will provide the designer with more efficient tools for the proper calculation of the projects so that these can be better prepared before testing. In certain fields such as refraction and diffraction investigations, electronic computers will probably be increasingly used and in other fields better formulas and design principles will be developed. It would thus be very useful if the Hudson formula could be modified to apply also to oblique wave attack and furthermore the general introduction of a series of damage coefficients representing the entire process of rupture from zero damage to total collapse would add much
*to the accuracy of calculations.

Through this work the laboratory will in a sense reduce its own field of application, but it is evident that complex cases and a steady stream of new design ideas and constructions will for many years to come still provide a rich and inspiring working field for those conducting laboratory investigations on harbour projects.

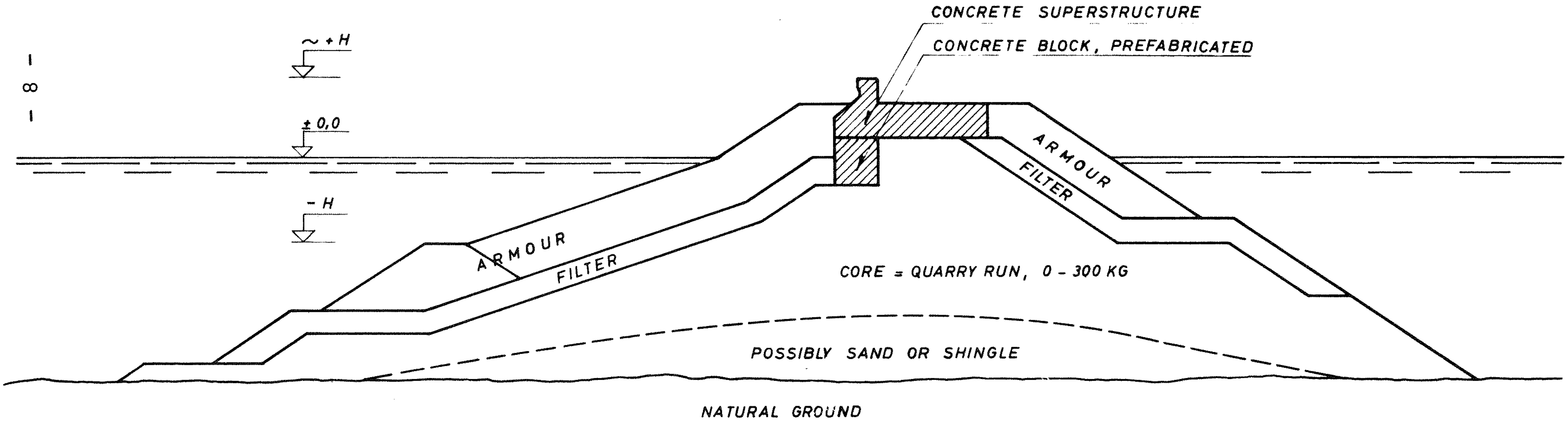
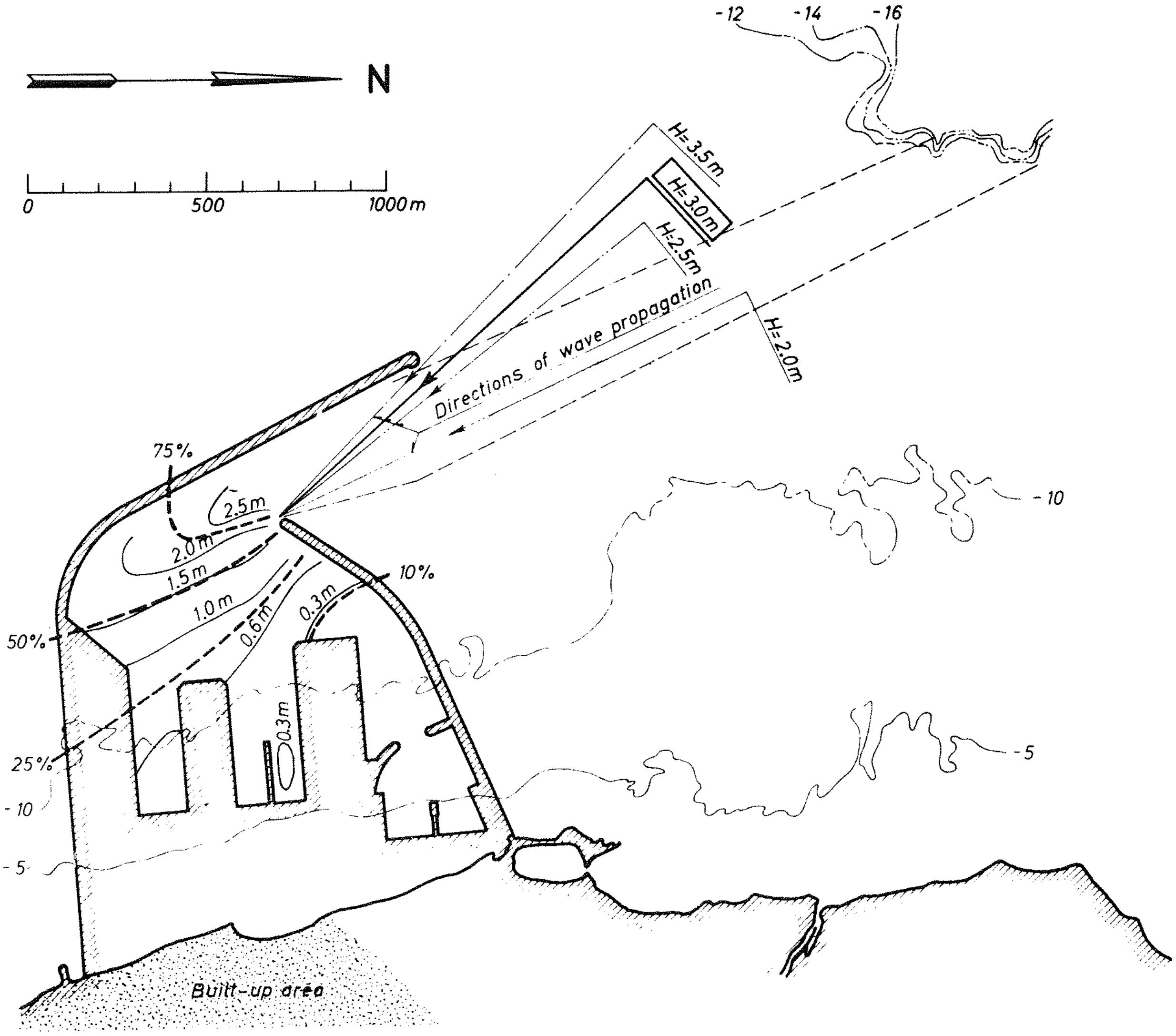
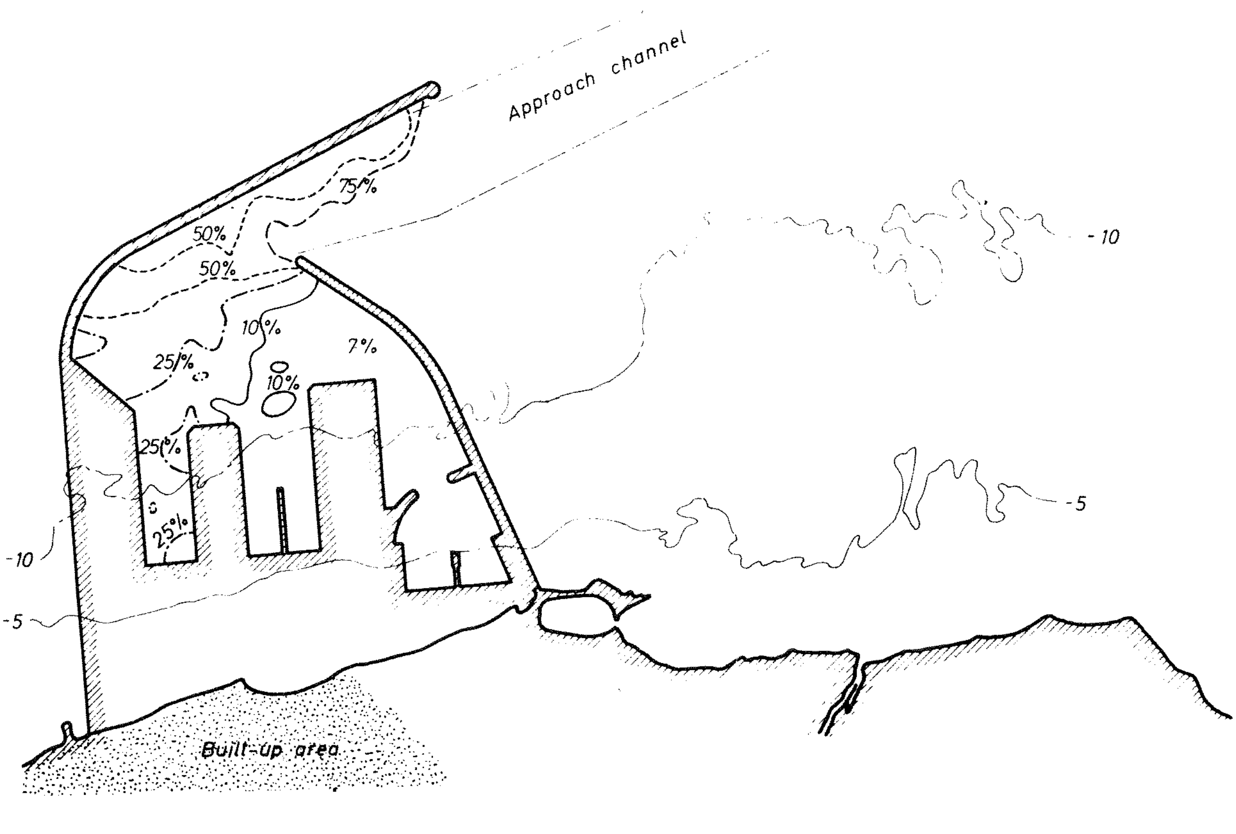


FIG. 1. RUBBLE MOUND BREAKWATER.



** FIG. 2. CALCULATED DIFFRACTION



** FIG. 3. MODEL TEST RESULTS

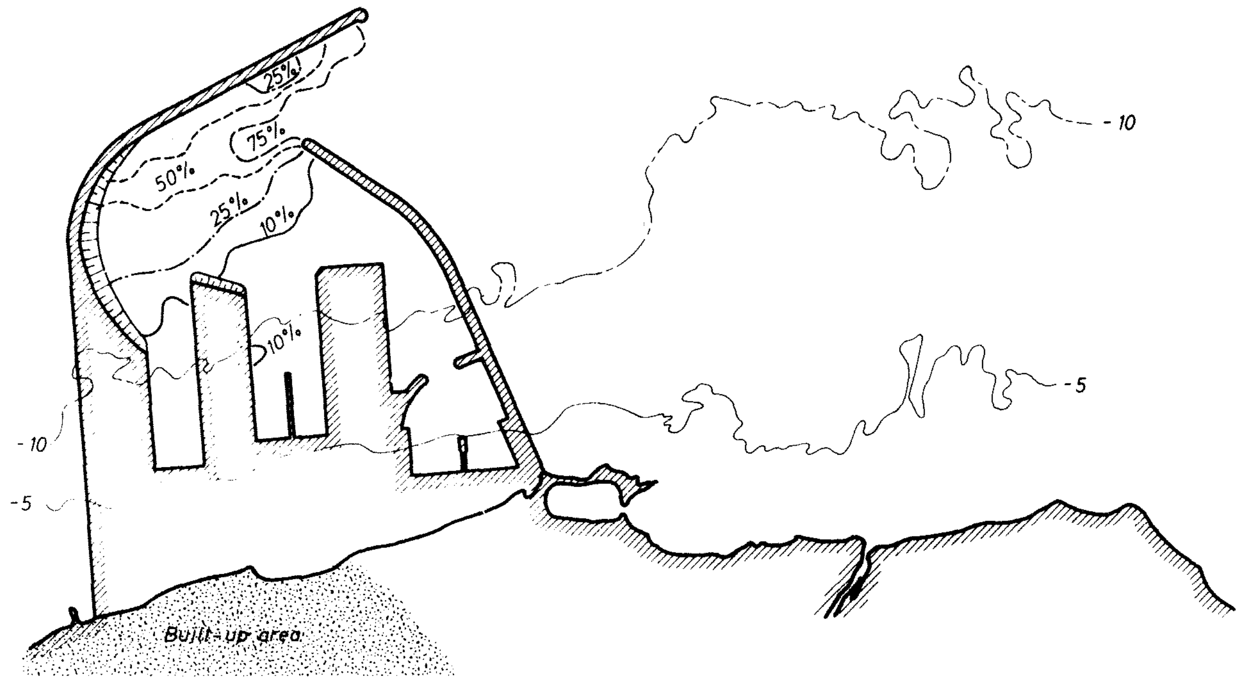
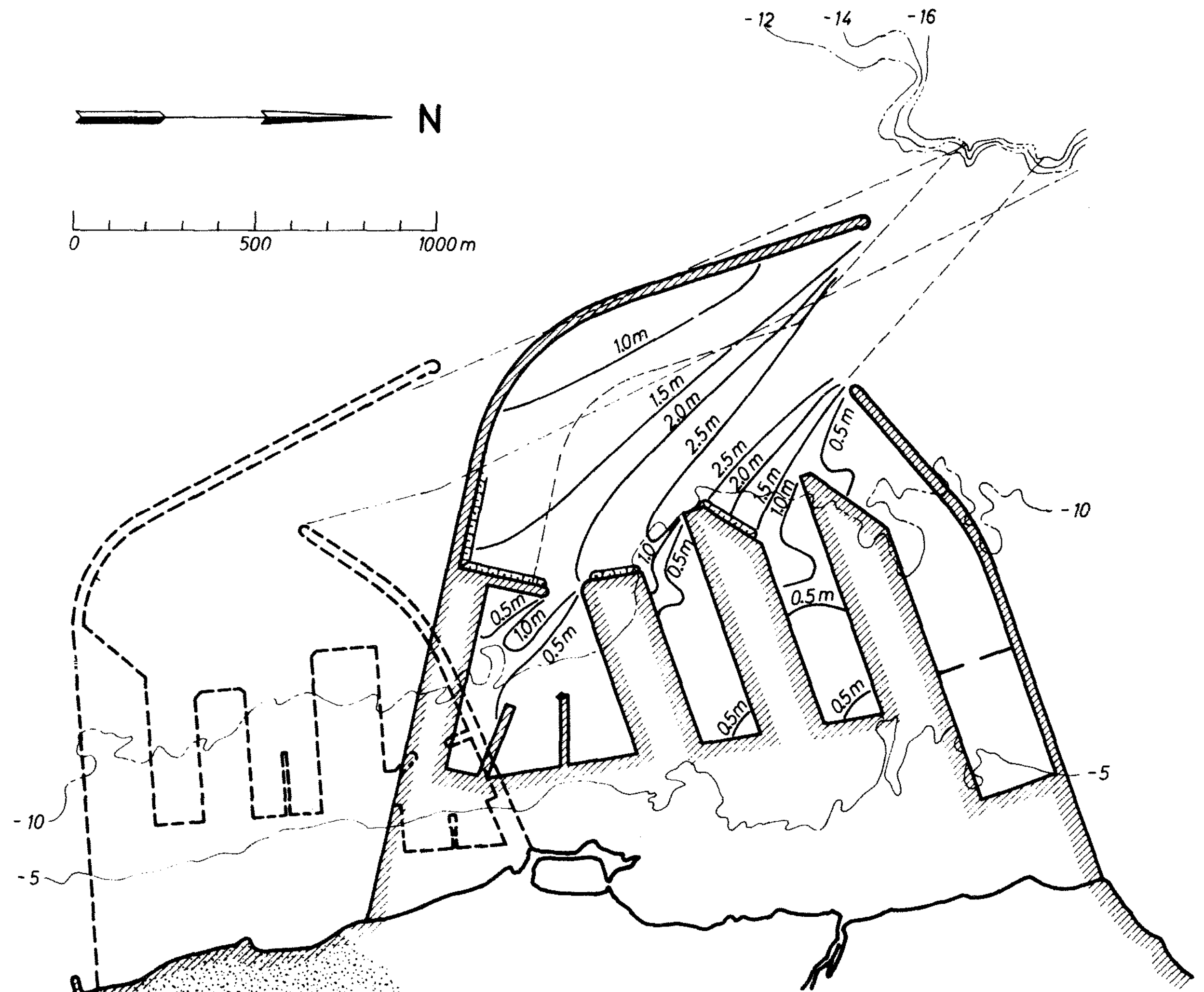


FIG. 4. MODIFIED LAYOUT – MODEL TEST RESULTS



** FIG. 5. NEW LOCATION - CALCULATED DIFFRACTION

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DISCUSSION ON PAPER 8

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Being mostly in full agreement with the views expressed in the paper, I want to offer a few comments on one point, the breakwater profile shown in Fig. 1, with the slope above Still Water Level (SWL) being steeper than that below. (The former seems indicated 1:1,5 and the latter as 1:3.) My comments are based mainly on Norwegian experience and practice.

**

As early as in 1899 a profile similar to that of Fig. 1 was considered in Norway in connection with one of our most difficult harbour projects. It was, however, not adopted, and for the last 50-60 years practically all bigger breakwaters in exposed locations have been built with more or less straight front slopes of 1:1,5 and often even of 1:1,25. Fig. 6 shows a typical profile from one of our more recent structures. Sometimes a convex break in the profile at SWL has been prescribed, with 1:1,5 above the break, and 1:1,25 below.

There are several reasons why a straight profile with rather steep slope seems logical:

1. Tests by Hedar, (1) pp 86-87, and at our laboratory in Trondheim (Fig. 7) have given strong indications that failure caused by downrushing waves on straight slopes occur above as well as below the SWL (Fig. 7 represents 65

tests of Series 1 and 55 tests of Series 2).

2. In coastal waters the SWL is varying, and a break in the profile, concave or convex, can not follow this variation. Where the tidal range approaches half the wave height, this variation becomes quite important.

3. The profile indicated in the author's Fig. 1 requires considerably more material than the straight profile shown in Fig. 6.

4. The construction of a breakwater with straight and steep sides is simpler than the building of more complex slopes. The construction can be carried out from the top of the breakwater itself, by dumping from high platform cars, as done in the case shown in Fig. 8, or by use of a mobile crane with sufficiently wide range as in Fig. 6. The breakwater represented in Fig. 8 is amongst our most heavily exposed ones. It was completed before World War II. Originally it was planned to have a broken profile as shown. This was not attainable by dumping, and soundings have shown the actual profile to be as indicated in the figure. The breakwater has suffered no serious damage until now. The heaviest cover blocks probably weigh about 20 tons.

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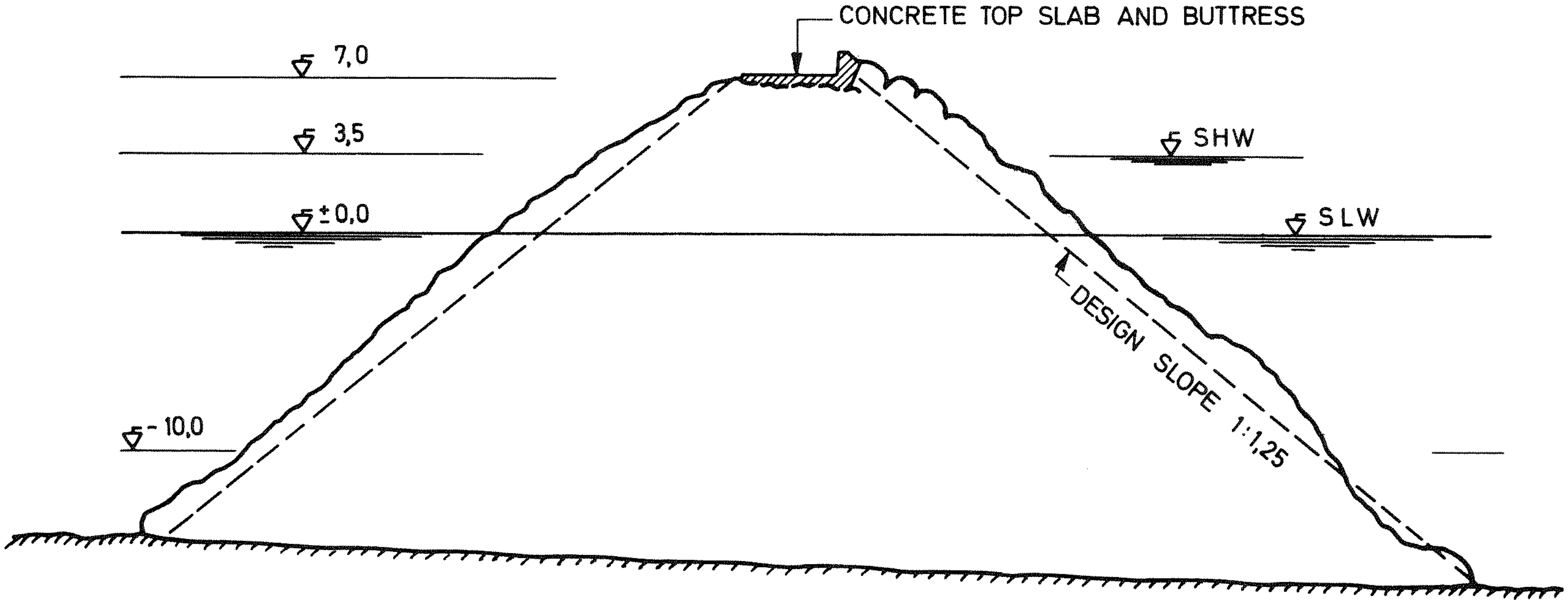
The author stresses the importance of considering critical construction stages. These should be taken into account already in the selection of the type of breakwater to use, whether vertical wall, rubble mound or other types.

The rubble mound with straight, rather steep sides seems to have the advantage of being the least susceptible to weather disturbance during construction.

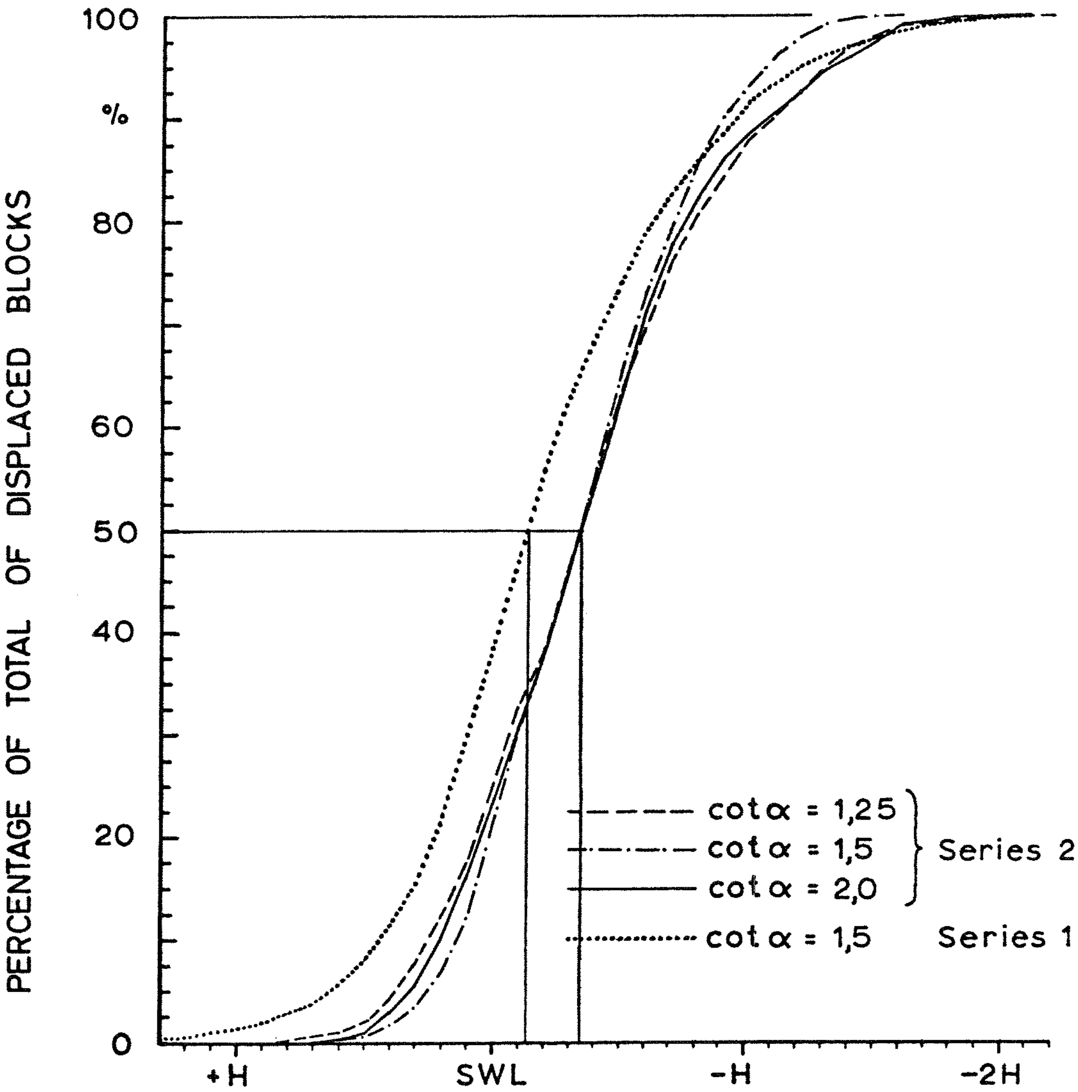
References:

(1) P.A. Hedar: "Stability of Rock-Fill Breakwaters", Göteborg, AB-Gumperts, 1960.



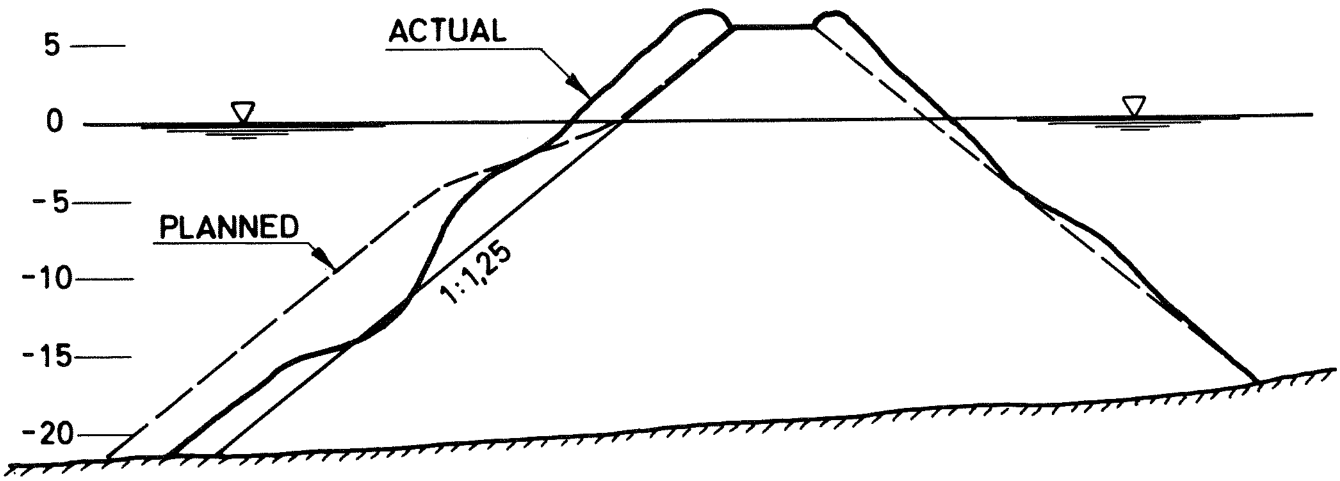
BREAKWATER PROFILE SVOLVÆR, NORWAY

FIG. 6



CUMULATIVE DISTRIBUTION OF LEVELS FROM WHICH BLOCKS WERE CARRIED AWAY BY DOWNRUSHING WAVES.

FIG. 7



BREAKWATER PROFILE

HALTEN, NORWAY

FIG.8

THE USE OF MODEL TESTS FOR THE DESIGN OF MARITIME STRUCTURES WITH REGARD TO WAVE ACTION

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1. General considerations

The hydraulic model is playing an increasing part in the process of hydraulic designing and it seems even to have taken over the dominant position that experience and tradition had held for such a long time. But, however important the model tests have become, they are not the only means available to the designer, nor does he have to rely solely upon them. Experience still holds a prominent place, while investigations in situ procure for the designer basic information he seldom can dispense with. A close interaction between investigation in model and prototype is a condition for obtaining reliable results.

Beside these relations in the hydraulic field, a design is influenced by other considerations, such as in the fields of technical construction and economy.

The model (hydraulic, mathematical or other) has its own place between several other components of the design.

As already stated, the influence of the hydraulic model in this interaction is increasing and is now tending to become the central point in the process of designing. This is to a large extent due to progress in modeltechniques: simulation, even of very intricate hydraulic phenomena, has been greatly improved by experience as well as through basic research, while also the instrumentation of the laboratory is steadily improving.

The other fields concerned with hydraulic design should keep pace with this progress. This applies especially to hydraulic investigation and research in the prototype. These field investigations should produce many of the basic data required for the determination of boundary conditions and for testing the model. As the technique of simulation improves, the need for better and more exact field data grows accordingly. But we seldom find the same facilities for field investigations in the prototype as the laboratory can offer. Measurements in nature are often restricted by practical circumstances and need a large amount of organization.

- Furthermore -

Furthermore, the repeatability is less, due to varying conditions, while the occasion to investigate under exceptional situations very seldom occurs. For these and other reasons, the investigations in the prototype lag behind these carried out in the laboratory, with the result that a gap of increasing width threatens to arise between these two fields of research.

The progress of the technique of running model tests challenges the designer with regard to the aspects of construction. He is bound to exploit to full extent the increasing possibilities of hydraulic research and of the ready information obtainable from this field. This requires at least some fundamental understanding of model tests, of the range of their validity, and of their limitations. The designer should be able to give the testresults their proper place and weight alongside his other hydraulic and non-hydraulic considerations. But as the technique of the model test has become more and more specialized, its scope has become less accesible to "outsiders", among whom often also the designer himself should be ranked. This may be
** the cause of a growing "mental distance" between the designer and the laboratory staff. This is a danger against which both sides should be on their guard, because such an alienation may lead either to an insufficient contribution of the model test to the design or, probably or more often, to a growing dictate of the model.

The designer should follow the progress of the test and should have a critical insight into what is done in the model. However, with the growing complexity of many models, it is often not easy for him to find his way, especially when - which too often happens - he has not had any special experience or training in this field. He should be on his guard against what may be called the suggestive power of hydraulic models; having water as medium they seem to be a true reproduction of the prototype, tempting the spectator to jump to rash conclusions by visual observation. This risk is not run with mathematical models and electric analogons. Nevertheless, in some cases the illustrative character of the hydraulic model has also its benefits.

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The improving thechnique of measuring hydraulic phenomena accurately in the model should be exploited as fully as possible. As the exact numerical evaluation of the information derived from the model is not always feasible, the designer often cannot make full use of the data which the model can supply him. On the one hand, this should restrain him from asking data from the model which he cannot take properly into account for his design, but on the other hand this lag should urge him to bring his criteria up to the level of the model.

These general considerations regarding the relations between model test and design hold good for all hydraulic designs but especially for those where the hydraulic phenomena involved are intricate and not easily accesible to observation and simulation. To this last category belong many of the phenomena of wave motion with which the designer of maritime structures has to deal. Though much progress has already been made in wavestudy and wavesimulation in models, there are still many problems to be solved in this field by both the laboratory and the designer.

2. Simulation of wavephenomena

In developing the technique of wavesimulation, two main practical problems are encountered: measurement of complex wavemotions in the prototype and the true reproduction of these data in the waveflume. Various attempts have been made to measure the effects of waveattack on structures in nature, but as already stated it is very difficult and wearisome to get reliable results of a sufficient scope from measurements in the prototype, especially of such a complex phenomenon as the breaking of waves on structures. And as the registration of the process of waveaction in the prototype is already a difficult matter under normal conditions, it is especially so during storms. This is one of the reasons why up till now it is still largely necessary to rely on a mathematical approach to the problem. But it is not possible to go far on this theoretical path without checking the assumptions by the facts observed and measured in the prototype. The same applies to the simulation of waves in a model. This can be based to some extent on scale laws, but if a close resemblance to nature is required measurements in the prototype are indispensable.

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For a long time we have had to be content with the approximation of the real wavemotion by more or less regular waves, not only because they could not be measured exactly in the protortype but also because it was technically impossible to reproduce them in the flume. Some declared that this simplified simulation of the wave spectra would serve the purpose, trying perhaps to make a virtue of this forced limitation. But the point has now been reached where it is possible to reproduce in the wave flume almost exactly the real wave spectrum. And the test results obtained so far seem to contradict to some extent the opinion of those, who trusted that tests with more or less regular waves would always give enough information on which to base a design.

As it has now become possible to simulate the wave spectrum truly in the wave flume, even though only unidirectionally, the need is felt to improve the investigation in the prototype accordingly, expecially with regard to the very complex phenomenon of the deformation of waves in the neighbourhood of structures.

We should aim here at a close cooperation between the research in the prototype and in the laboratory, especially with regard to the measuring of the forces exerted by wave impact against maritime structures.

With the simulation of a unidirectional spectrum in the wave flume, there still remains a restriction as compared with the prototype, because the aspects of the wave pattern are not yet brought into account. In the flume this pattern is almost regular that is, without the transversal differences that occur in the prototype, especially when there is question of a system of crossing wave trains. Though the neglect of this complication will often be permissible there may be cases where a closer investigation in this respect is desirable. This question arose, for instance, during the investigation of the wave impact against the gates of the Haringvliet sluices, where it was important to have information about the transversal extension of the wave impact in order to determine the total load on a gate. This problem deserves special consideration now that the simulation of wavemotions has been so much improved.

- 3. -

3. Evaluation of flume tests

As has already been observed, the designing engineer frequently has to cope with the problem that the criteria of failure of the structure are not well defined. Therefore it is often difficult to make full use of the impacts measured in the model when determining the strength of the structure to resist the wave attack. In these cases, they can only give him qualitative indications.

In addition, the designer has to give the proper weight to the hydraulic data in comparison with other non-hydraulic considerations, when dealing with questions of construction and economy. He may be fortunate enough to have all his considerations, hydraulic and nonhydraulic, point to the same direction. But very often there will be some contradiction between these different considerations. In those cases he has to decide which of them should prevail: whether the hydraulic evidence has to be decisive or some constructive or economic aspects should dominate. This all depends, of course, on the type and the character of the design.

Special attention will be given here to three problems:

- wave impact and wave run-up on the slope of a sea wall
- wave attack on sluice gates and
- wave attack on a rubble dam.

Some practical questions connected with the evaluation of model tests regarding these phenomena will now be discussed briefly.

a. Wave attack on the slope of a sea wall

This is a complex phenomenon consisting of the direct impact of the breaking wave on the revetment and, in addition, the run-up of a mass of water which eventually may overtop the crest of the wall. From observations in the prototype it is obvious that the process of a breaking wave can be affected by the backwash caused by the preceding wave. The reflux from a wave may smother the impact of the following wave and check its run-up. This may explain the fact that in model tests, carried out with irregular waves, a direct relationship between the run-up and the height of the idividual waves could not be detected. This was probably because the interaction of two succesive waves was not taken into account. The same difficulty should arise when trying to find the relation between the individual wave height and its impact on the slope.

- Further -

Further research into this problem is recommended, especially now that tests with a truly simulated irregular wave spectrum have become possible. However, for the time being the data available both on wave impact and wave run-up seem to be sufficient for the designing engineer, considering the vagueness of the criteria he applies.

The process of the wave impact on slopes has been measured in the model as well as in the prototype; reference may be made here as an example to the investigations carried out in this respect on the smooth concrete slope of the Westkappelle sea wall. (figure 1). These have given some insight into the process and the magitude of the impact. Furthermore, the results obtained from the measurements in the prototype seem to correspond to a fair extent with those obtained from similar model tests. But it is still very difficult to apply these data to the design of the revetment. The engineer can only take them into account very globally and has to content himself mainly with the knowledge that the impact on a slope decreases considerably with its gradient.

As to the wave run-up, the designer will especially be interested in the amount of overtopping that may be expected. This quantity of overtopping water, in relation to a certain wave spectrum and water-level, can be determined fairly correctly in a model for any sea wall design; data can be given regarding the overflow per unit length of seawall, as shown in figure 2. In order to take these data into account in a proper way the designer should know how much overflow his structure can stand and also how much overflow can be accepted on the hinterland behind the sea wall. But all too often he has to decide on these aspects without adequate information.

Investigations after the disastrous flood 1953, which broke so many dikes in Holland, showed that most damage was caused by overflow eroding the inner slopes of the seawalls. These slopes ordinarily are not protected by a stone revetment but simply by a grass cover which is only to some extent resistant against overflowing water. The exact cause of the dammage was found to be the sliding away of the protecting top layer of soil in which the grass was rooted, due to saturation by the overflowing water.

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It was found also that the resistance of the top layer can be considerably improved by drainage, making sure that this layer is in any case less permeable than the soil beneath, and further by decreasing the gradient of the inner slope. These rules were applied when rebuilding and reinforcing the dikes. But as this problem is not yet accessible to exact calculations, it is still impossible to assess the critical amount of overflow that can be accepted. So the engineer is still all a loss when he has to decide how much overtopping he may permit and for what duration.

This problem also exists when the crown and inner slope are protected by a more resistant material, for instance by an asphalt revetment. Though more overflow might be accepted here, no reliable criterion is available in this case either.

This uncertainty prevents an exact economical evaluation being made of the relation between the wave run-up obtained by model tests and the cross-section of a sea wall. And as there should be no taking of unknown risks, even very small overflow is only allowed in very exceptional cases. Normally the socalled 2% rule is applied in Holland, which means that 2% of the waves during design storm will reach the crown of the dike. This means that a point is chosen far on the left of the steep part of the curve.in figure 2. Obviously, looking at the shape of this curve, much could be gained economically, if more overtopping dare be allowed.

The importance of this problem depends on the situation. For instance, in dealing with a polder dike protecting low land, as shown in the upper picture of figure 3, a variation of the crestlevel will have more consequences than in the case of a dike protecting an industrial harbour site situated on a comparatively high level, like those being constructed in the sea near the Hook of Holland (Maasvlakte). When comparing these two examples it must also be pointed out, that in the case of the polder dike, the consequences involved, are much greater because the land to be protected here lies far below sea level. In assesing the freeboard of dikes, this aspect should also be taken into account.

Further investigations into the problem of the resistance of different types of revetments should be stimulated.

- b. -

b. Wave impact on the gates in the Haringvliet

The Haringvliet sluice, with its circularshaped segment gates is an interesting example of a design in which an important hydraulic desideratum had to yield to technical and economic considerations. With regard to the severe wave attack to which these gates will be exposed, their seaward inclination is very unfavourable. Wave impacts increase considerably with the angle of inclination, as is clearly showed in figure 4. However, the enormous weight of these gates (width 56 m, height 10 m), asked for a design that would keep the force required for the lifting of a gate within reasonable limits. Therefore the circular shape was chosen so that the resultant hydrostatic forces would act axially; the larger wave impacts due to this shape had to be accepted here as the lesser evil.

The process of the wave impact on this type of structure (figure 5) is of another character than on a flat slope as was shown in figure 1, and seems to be vary considerably with the shape of the wave. An eagerness to check the forces measured in the model by tests in the prototype resulted in several pressure recorders being built in one of the gates (figure 6). It will be interesting to follow the results of these investigations in the prototype and to compare them with the data obtained from the model.

4. Wave attack on the new breakwater at Hook of Holland.

a. Assessing type and weight of armour

The cross section of these breakwaters is shown in figure 7, They are composed of a core of quarry stones protected by an armour of concrete cubes. As the model tests that were carried out to check the stability of these blocks are dealt with in the paper of Norwegian colleagues, discussion here will be confined to someconsiderations in the fields of construction and economy that played an important part in assessing type and dimension of the blocks.

From the beginning it was without question that the strength of the wave attack that would have to be expected on the dams excluded the use of quarry stone for the armour and that consequently concrete blocks would be needed here.

- Starting -

Starting from this assumption, technological investigations were carried out in order to discover the maximum specific weight that could be obtained at reasonable cost. This turned out to be approximately 2.65. In addition, an approximate estimate had to be made of the maximum weight that could be handled by the tools that would be transported and put into place by ship.

** Next, to ascertain a large output, it had to be considered that the blocks would have to be fabricated, handled and transported in an easy way. This consideration asked for a simple shape, and therefore further tests were started with the cube, as this seemed to be the most advantageous shape in this respect.

Based on these facts and desiderata, stability tests were started in the model, where the relations between wave height, rate of damage, specific weight and special shapes of the blocks were investigated.

** One of the interesting results of these primary tests was that the stability of special more or less interlocking shapes (such as Akmons) was relatively less than had been expected. This was largely due to the fact that the rubble dam has its crest situated some 2 m below storm surge level. Consequently severe wave attack has also to be expected on the innerslope, where the interlocking systems are less adequate. Therefore -- although by using a special type of block, such as the Akmon, a certain reduction in block weight and total armour volume could be obtained -- the difference with cubes was too small in this case to be decisive.

Taking into account its lesser cost per unit weight and its easier handling, the final choice was made in favour of the cube.

The test that had to be performed to assess the dimension of the blocks were mainly based upon the rather arbitrary assumption that slight damage might be allowed only about once in 100 years, corresponding with a design wave of 8,5 m significant.

This criterion was checked on its econmical merits, trying to assess the optimum combination of initial and capitalized maintenance cost, and a fairly good agreement was obtained. The value of this check was limited, however, because it was not possible to assess accurately the relation between construction and maintenance cost for this type of structure in Holland. But as the construction cost was not appreciably influenced by the dimension of the blocks, the chosen weight of 43 tons that was regarded as approximately the maximum to be handled by the equipment without difficulty, appeared to be also about the most economical.

b. Measurements against erosion in front of the breakwater

The adjacent part of the coast where the breakwaters are built will be liable to considerable changes, not only due to the construction of the breakwaters but also to the Delta Works, e.g., the closure of the Haringvliet. As a consequence of these works, accretion may be expected along the first part of the southern breakwater which willconsist of a sanddam, and erosion along the most protruding part which runs almost parallel to the coast, and will be constructed as a rubble dam. The parts where accretion or erosion is expected are roughly indicated in figure 8, in which is also shown the approach channel that has to be dredged to give access to tankers up to 225.000 dwt.

It was necessary to anticipate these changes in the design, and especially the threat of erosion in front of the toe of the rubble dam. However, no reliable data could be obtained, either from the prototype or from the model, as to the ultimate extension of the eriosion and of the time it will take to develop. What could be done was to determine to what extent the erosion would be acceptable without endangering the stability of the breakwater, and then to plan what measures would have to be taken successively to keep the erosion within these bounds. The programme set up for that purpose is shown in figure 7.

In the first instance, a blanket of limited breadth consisting of gravel, will be placed before and under the dam, the sea bottom being locally excavated to the required depth for this foundation. Then, if erosion starts at the toe of this blanket, it will be extended horizontally for 40 à 50 m. If after this supplementary protection has been provided erosion still continues at the end of this berm, the blanket will be extended once more, this time sloping.

The ultimate profile as shown in figure 7 (No. 5) has been assessed in the model. The tests that were carried out with irregular waves indicated that although the boundary conditions of the wave motion were not changed, the impact on the rubble dam increased considerably when a certain depth in front of the breakwater was exceeded. The test showed, furthermore, that this phenomenon did not occur if a berm was kept in front of the dam as indicated in figure 7 (No. 3). Further increase of depth outside that berm did not worsen conditions.

This example shows that, although model tests cannot (yet) give reliable evidence as to the extension and pace of an erosion, it can nevertheless procure essential information with regard to the limits within which it should ultimately be kept for safety of the structure.

5. The use of model tests

In the preceding pages it has been pointed out that the results of model tests may have in many cases only a very limited value with regard to their quantitative interpretation. Sometimes this is due to certain limitations of the model, but often also to the circumstance that the criteria handled by the designer cannot be put into exact figures.

Therefore, though a certain quantitative evidence may sometimes be obtained from the model, its chief value lies in its contribution to the qualitative interpretation and appreciation of various solutions for the design.

The success of a model test is to a large extent dependent on the right choice of parameters. Their number should be restricted and limited to those whose influence is really important and can be evaluated by the engineer. For the choice of parameters, the engineer should not only be guided by their importance for the hydraulic effects on the design, but also by their influence on constructive and economic consequences.

The decision as to what boundary conditions should be applied is also very important. Special attention has to be paid here to the question whether in the prototype these conditions may be subject to changes. Such changes may arise from alterations of the topography of the foreshore, effected either by the structure itself or by natural hydrographic changes. This may be expecially occur along sandy coasts - like that of the Netherlands - liable to scour or siltation.
In these cases a structure should be tested in the model under different boundary conditions, corresponding with the changes that may be expected to occur in the prototype.

The boundary conditions may also be varied as parameter, in order to determine the design wave that goes with the most economic combination of construction cost and capitalized maintenance cost.

Finally, it can be asked to what extent the hydraulic model might be replaced by a mathematical one. This may be possible for those phenomena that can be simulated with sufficient precision by formulae based on theoretical considerations. But many wave phenomena are still too complex to be captured in a mathematical model. Some of these may be superficially represented by formulae derived empirically from hydraulic model tests, but as these formulae do not give a basic insight, it is dangerous to apply them on a design without considering if they hold good for that very case.

With the growing accuracy of simulation, the importance of the hydraulic model increases, especially for those designs which have an exeptional character.

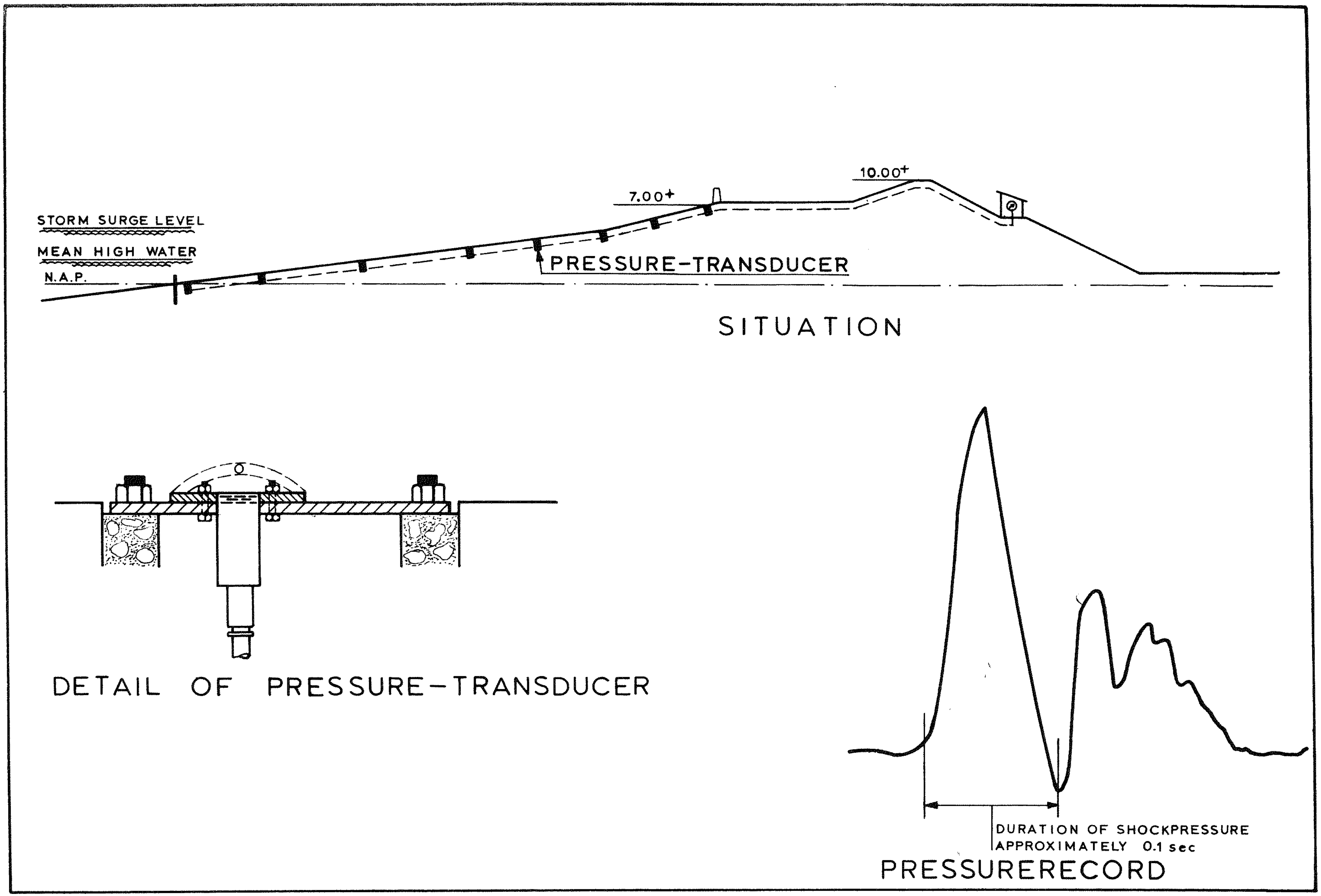


FIG.1. MEASUREMENTS OF WAVEPRESSURE ON THE WESTKAPELLE SEAWALL IN PROTOTYPE

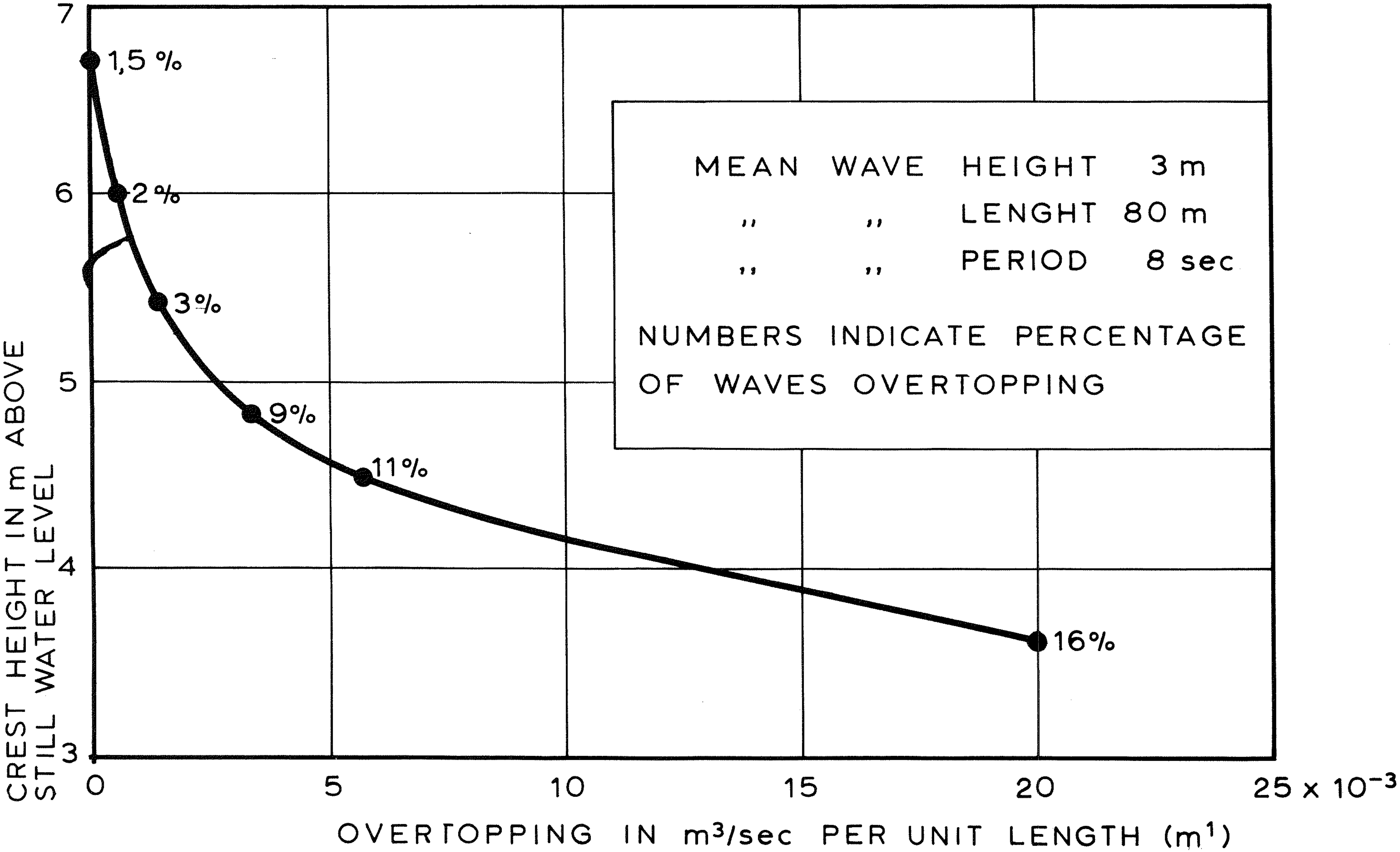


FIG.2. RESULTS OF MODELTESTS IN WAVE OVERTOPPING OF A SEAWALL

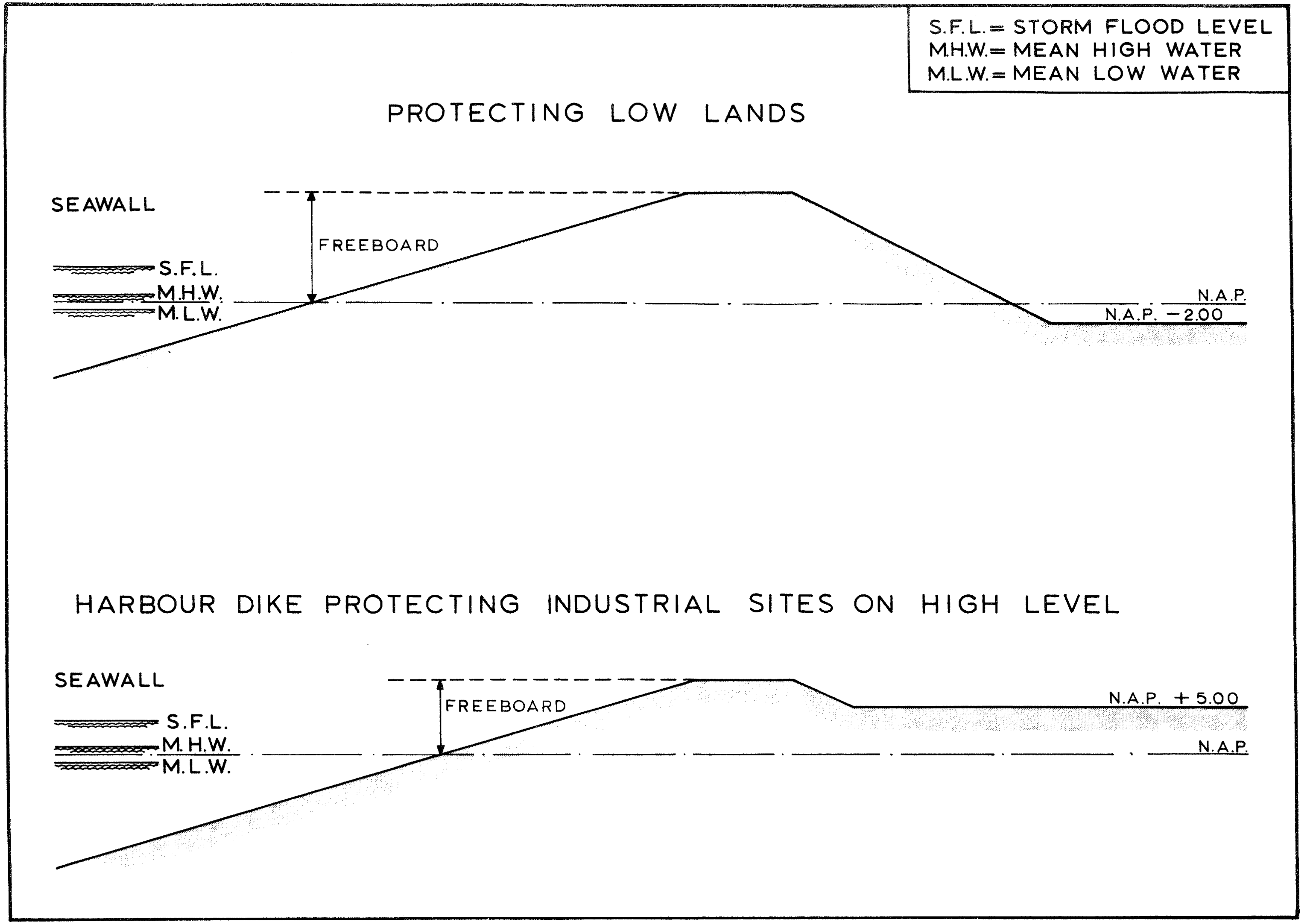


FIG. 3. FREEBOARD OF SEAWALLS IN RELATION TO LEVEL OF HINTERLAND.

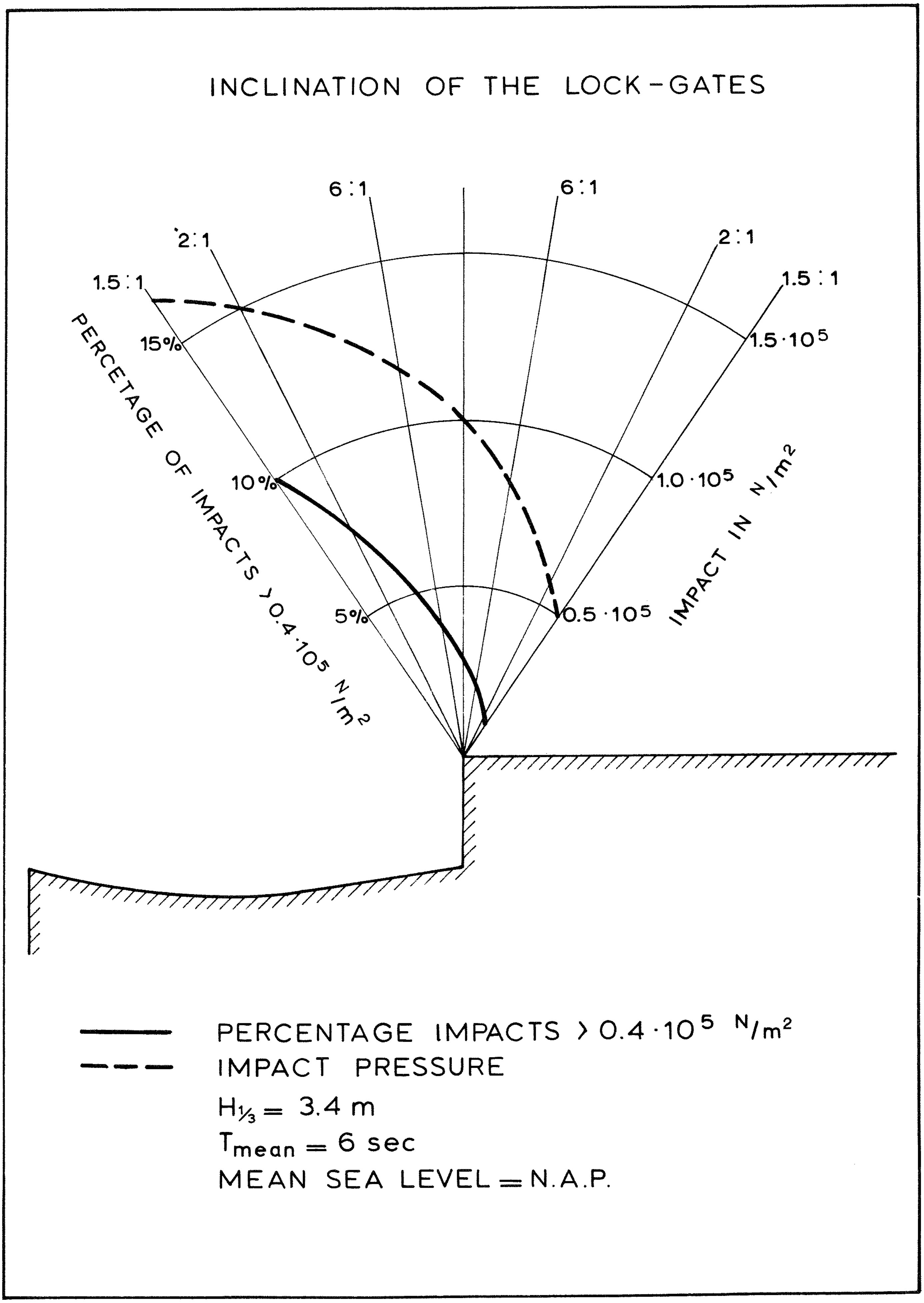


FIG. 4 IMPACT OF THE WAVES AS FUNCTION OF THE INCLINATION OF THE LOCK-GATE SEA-SIDE (HARINGVLIET SLUICES)

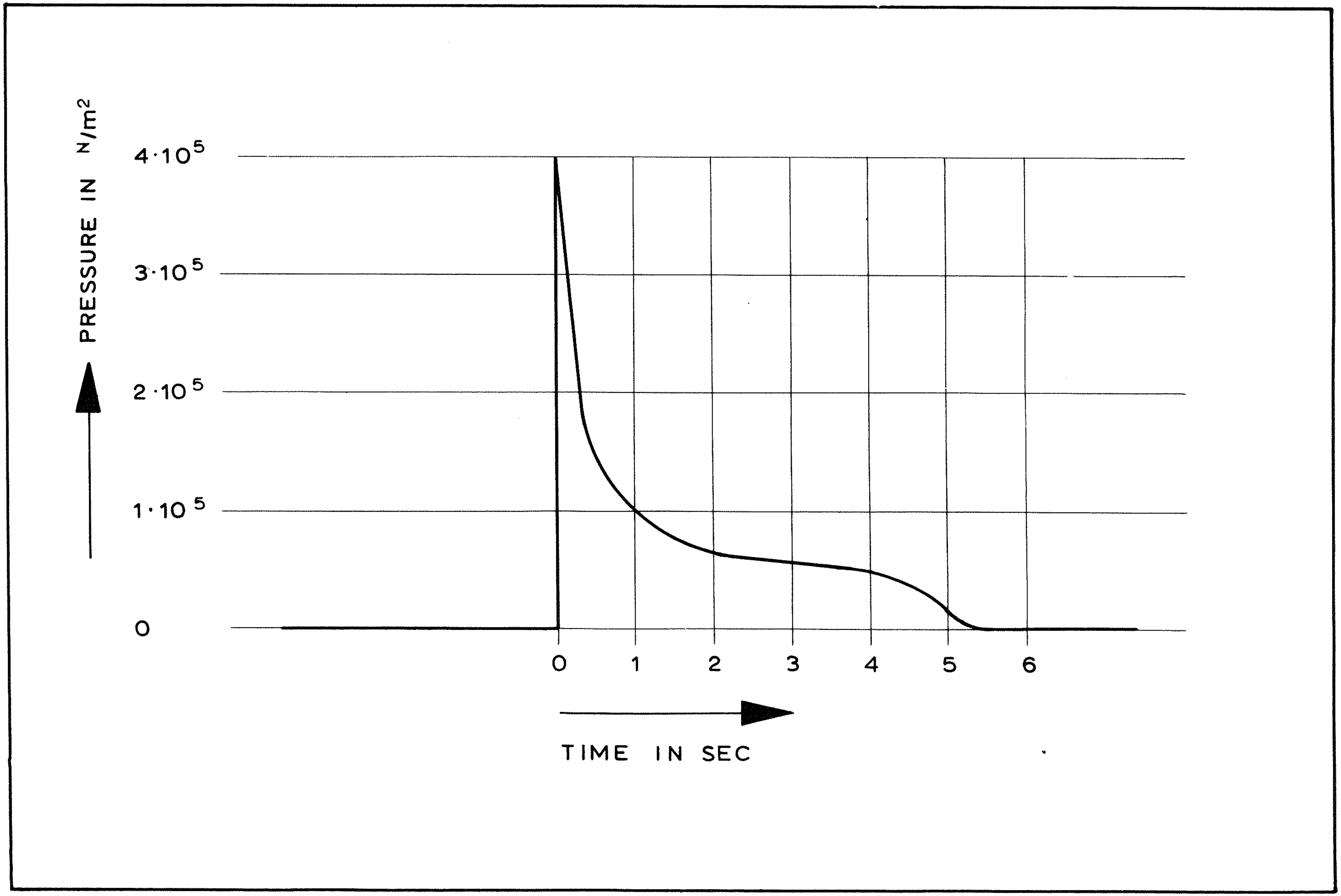
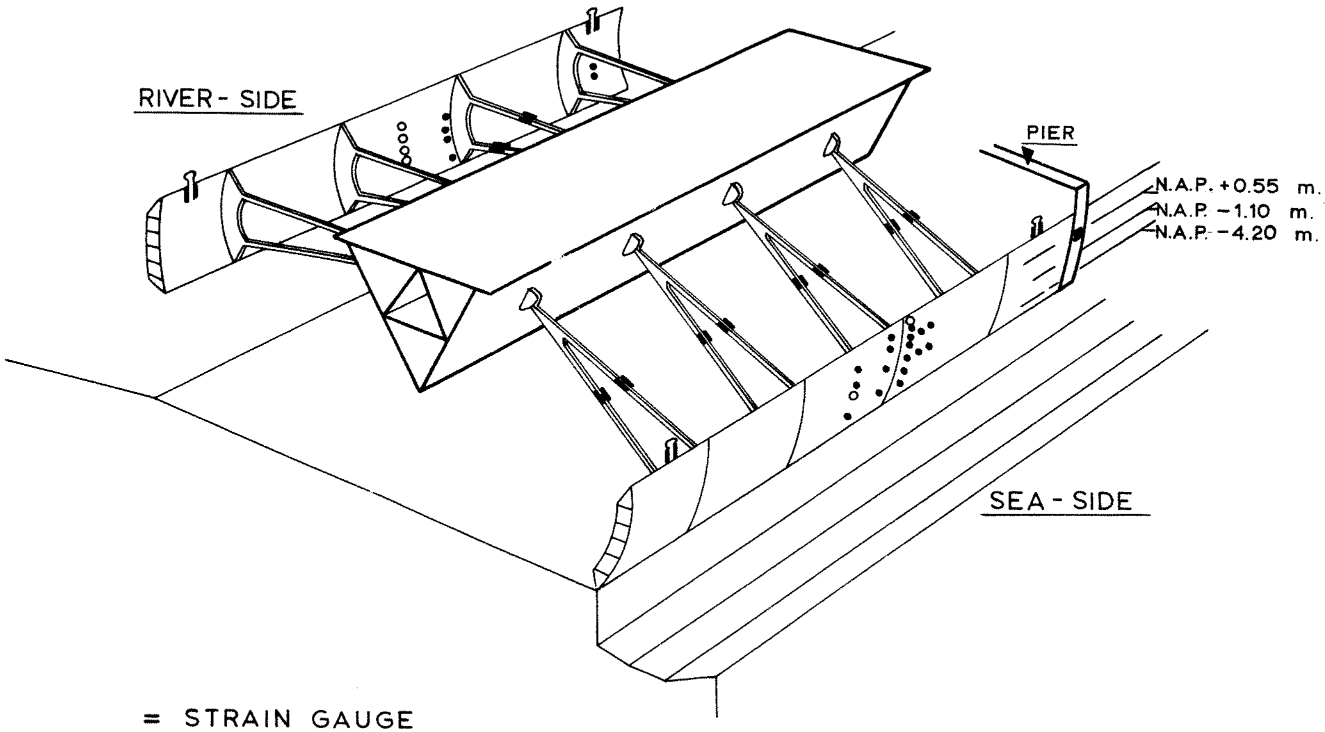


FIG. 5. RECORD TYPICAL OF THE PRESSURE ON THE LOCK-GATE SEA-SIDE (HARINGVLIET SLUICES)



= STRAIN GAUGE
• PRESSURE-TRANSDUCER OUTSIDE OF THE LOCK-GATES
∘ PRESSURE-TRANSDUCER INSIDE OF THE LOCK-GATES
▪ PRESSURE-TRANSDUCER OF THE PIER
N.A.P. = MEAN SEA LEVEL

FIG. 6. REVIEW OF MEASURING-SECTION WITH LOCALINDICATION OF STRAIN GAUGE AND PRESSURE-TRANSDUCERS (HARINGVLIET SLUICES)

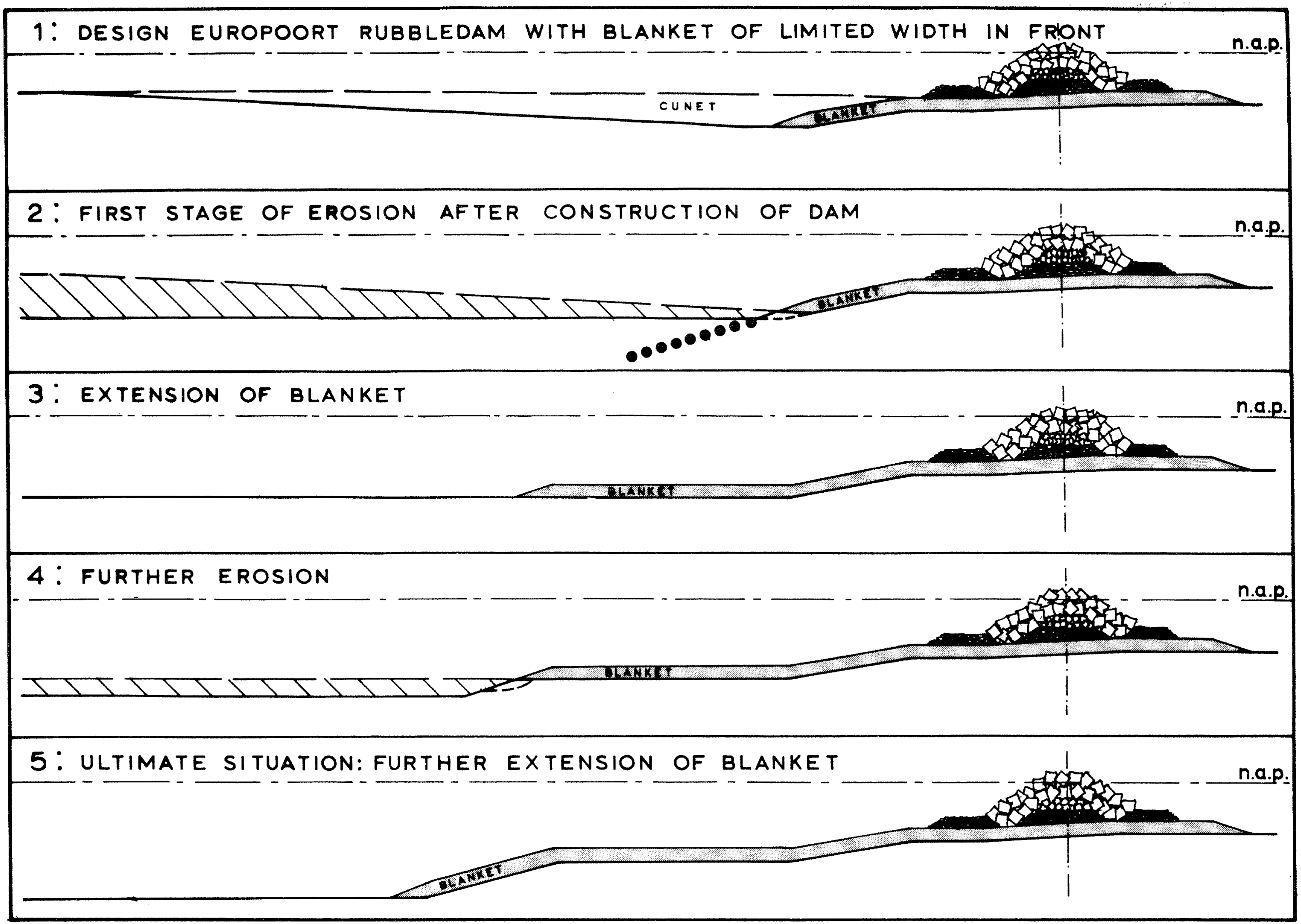


FIG. 7. PROGRAM FOR BOTTOMPROTECTION IN FRONT OF SOUTHERN EUROPOORT BREAKWATER

- 18 -

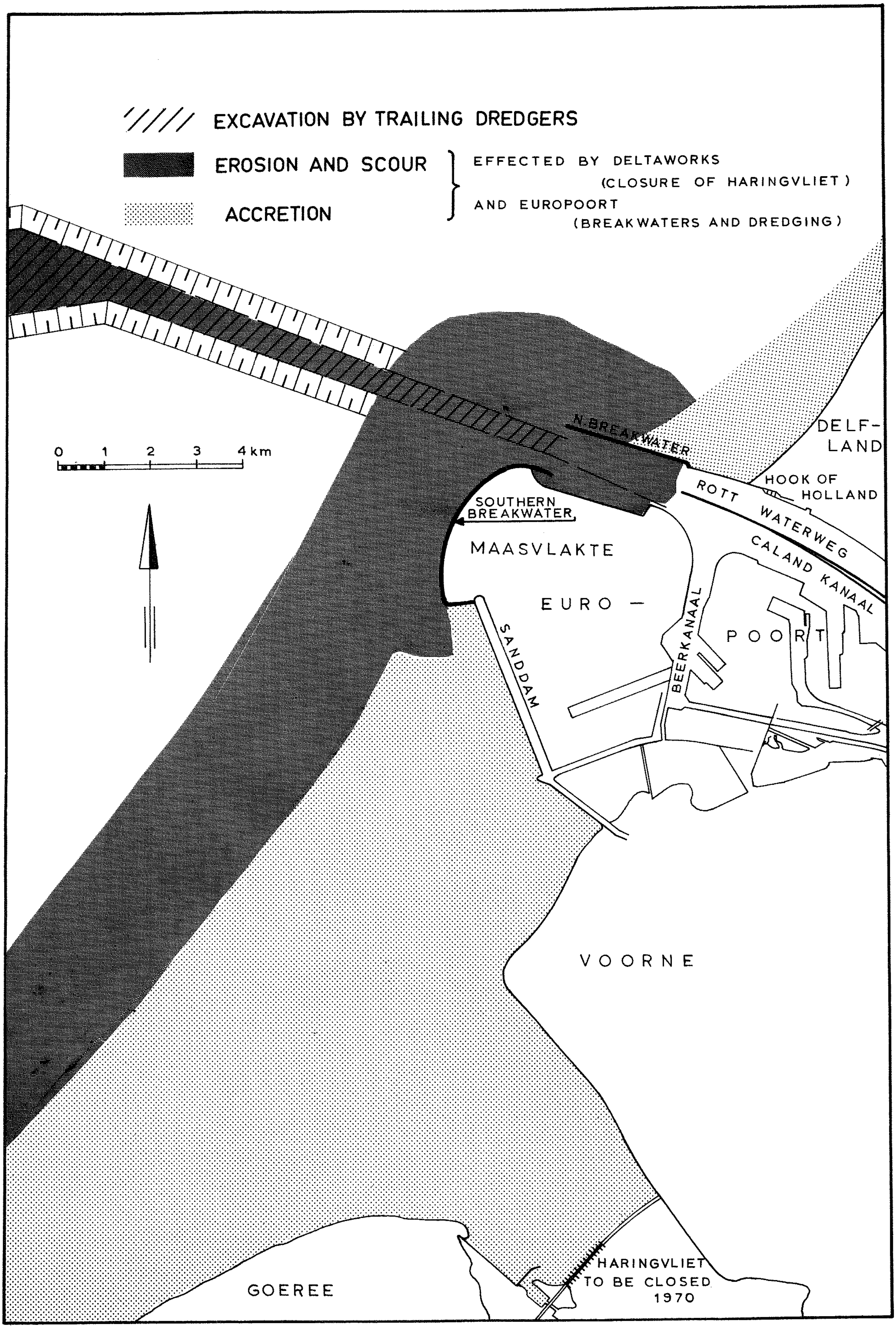


FIG. 8. EXPECTED COASTAL EROSION AND ACCRETION NEAR HOOK OF HOLLAND

STABILITY TESTS OF THE EUROPOORT BREAKWATER

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Summary.

This paper deals with model tests conducted at the Delft Hydraulics Laboratory (DHL) and the River and Harbour Laboratory at the Technical University of Norway (RHL) for the design of the Europoort Breakwater.

A series of tests with regular waves was conducted at the DHL from which the design of the breakwater was decided. The chosen design was tested with irregular waves based on in situ observations. Wave spectra, wave height distributions and the joint distribution of wave height and period were specified. These tests were conducted at the RHL, and some tests were repeated at the DHL.

It has been commonly assumed that the destructive effect of a train of regular waves on a breakwater in model is equal to the effect of irregular waves with a significant wave height corresponding to the height of the regular waves.

* The tests showed that for this particular breakwater the irregular waves represented a more severe wave attack than the regular waves.

INTRODUCTION

This paper deals with stability tests in model of the Europoort Breakwater conducted at the Delft Hydraulics Laboratory (DHL) and the River and Harbour Laboratory at the Technical University of Norway (RHL). The tests were run both with regular and irregular waves.

Fig. 1 shows the outer part of Europoort. An 8 km long breakwater extending from the south will protect new industrial areas with adjoining harbour basins. The dry land will be separated from the breakwater with a channel. This permits a considerable amount of overtopping, and the breakwater has been designed with a very low crest. Fig. 2 shows typical cross sections for the deeper and for the more shallow parts of the breakwater.

The sea bed consists of sand with a low stability against erosion and a low bearing capacity. The jetty is therefore constructed with a wide fill with flat slopes. The jetty is protected with cubical blocks in two layers placed pell mell. The design wave height is 8,5 metres.

** The model tests consisted of tests with regular waves in order to determine the general design of the breakwater and the necessary block weight. These tests were conducted at the DHL.

DHL had planned to check the results in a wave basin equipped with a new wave generator which could produce irregular waves, but it became apparent that the new equipment would not be in operation early enough to finish the tests before the deadline for the investigation.

At RHL equipment for producing irregular waves has been in operation since 1964, and RHL was asked to conduct these final tests.

This paper mainly deals with the tests which offer a possibility for comparison of results obtained with the use of regular and irregular waves, respectively.

** TEST WITH REGULAR WAVES

The tests were performed in scale 1:60 with a cross section as shown in Fig. 3. (All elevations given in this paper are referred to New Amsterdam Ordnance Datum (NAP)). The stability was investigated with cubes with different dimensions and specific densities. For every particular cube dimension the average wave period and water level were varied independently.

By way of example Fig. 4 shows the results of a series of tests. The damage is described in qualitative terms according * to a system of certain standard criteria used at the DHL. The system not only takes into account the number of blocks which are removed from the armour, but also from where in the armour the blocks are removed. Thus the damage is rated higher if a number of blocks are removed from a consentrated area on the breakwater than if they are removed from different places more evenly distributed over the whole armour surface.

From each series of tests the test giving the minimum stability, at a damage between "none" and "slight" was used to calculate the stability number, $K/f(\alpha)$ from the formula

$$G = \frac{K}{f(\alpha)} \frac{\gamma_b H^3 d}{\left(\frac{\gamma_b}{\gamma_w} - 1\right)^3}$$

The computed maximum values of $K/f(\alpha)$ (minimum stability) for each series of tests are listed in the table.

G tons	γb t/m^3	H_d m	H_{destr}/H_d	K/F (α)
27,5	2,2	5,5	1,68	$11,3 \cdot 10^{-2}$
21,5	2,2	5,4	1,82	$9,4 \cdot 10^{-2}$
23,0	2,6	6,9	-	$9,7 \cdot 10^{-2}$
20,3	2,7	6,8	1,73	$10,4 \cdot 10^{-2}$
20,3	2,7	6,8	-	$10,4 \cdot 10^{-2}$
13,0	2,8	5,8	1,72	$12,0 \cdot 10^{-2}$
8,0	2,8	5,5	1,59	$9,0 \cdot 10^{-2}$
25,0	2,8	7,3	1,70	$11,8 \cdot 10^{-2}$

Average values: $K/f(\alpha) = 10,5 \cdot 10^{-2} \pm 11\%$

$H_{destr}/H_d = 1,7 \pm 1,5\%$

According to these values the design block weight, G = 43 tons with a specific weight of 2,65 t/m^3, was selected for the design wave of 8,5 metres.

Also some tests with more shallow and less exposed parts of the breakwater were carried out in regular waves. In Fig. 5 is shown the respective average K/f(α) values for all cross sections concerned. Along the shallow part of the breakwaterscour scour is expected to occure in front of the breakwater. The graph shows the results under the condition of the original horizontal sea bed untouched and under the condition that scour has taken place. In the case of scour the models were tested with a 1:4 sloping bottom
* in front of the structure from the original level of -7 and -10 metres down to -10 and -14 metres respectively.

TESTS WITH IRREGULAR WAVES

Cross sections.

The breakwater cross sections in Fig. 2 are the ones selected on basis of the tests in regular waves, but for minor modifications as the result of the tests with irregular waves.

At RHL a number of 4 cross sections were tested in irregular waves, cross sections at depths -20, -15.5, -12 and -10 metres. The shift from the deep water to the shallow water design will be at depth -12 metres. At depths less than approximately -5 metres the breakwater is constructed entirely with sand.

Tests at RHL.

Test equipment. The RHL wave channel is shown in Fig. 6. The waves are generated by a paddle moved by two oil hydraulic pistons. The paddle movement is controlled by a servo system with a voltage reference input and position and velocity feedback. For generating irregular waves the input signal is synthesized on magnetic tape from a white noise generator connected to electronic filters. With the known transfer function of the wave channel the filters are adjusted to give a wanted wave spectrum in the channel.

The wave generator works within a range of wave periods of 0.5 - 5 sec. and can produce waves with a maximum height of approximately 0.5 metres.

The wave generating system also includes a fan capable of producing wind with a maximum velocity of 10 m/sec.

** Waves. At the site, waves have been recorded continously a considerable period, and a well specified wave programme could be put forward as a basis for the tests. These specifications consisted of wave spectra, wave height distributions and joint distributions of wave height and period.

Fig. 7 shows the two main types of spectra used in the model tests, a Neumann spectrum and a narrower one.

The spectra were run with three different peak frequencies corresponding to wave periods of 10, 12 and 14 second in the prototype.

In Fig. 8 the dimensionless wave height distributions of model and prototype are shown, while Fig. 9 shows a diagram which expresses the joint distribution of wave height and period. The H-T correlation is derived from a wave record by plotting the ratio of the mean apparent wave period, T_i, according to the zero upcrossing convention, in intervals of H/H_E = 0,5 and the mean apparent period of all waves, T_m, against the wave height parameter H/H_E. The two curves envelope H-T correlations of waves outside Europoort, and the plotted values are examples of H-T correlations of the model waves. All wave data from the model are obtained from records of 200 successive waves.

The prototype wave conditions were found to be reproduced satisfactorily in the model.

The models. The test arrangement is shown in Fig. 10. The tests were run using a scale of 1:36.

The wave basin is constructed for a water depth of approximately 1.0 metre, and a model bottom was constructed consisting of a slope 1:30 up to the correct sea bed level, whereafter the bed was kept horizontal.

Two cross sections with widths 1.0 metre were tested simultaneously, each positioned adjacent to the glass panels. With the different sea bed levels on each side of the basin the cross section became as symmetrical, but this did not have any significant
* influence on the wave conditions.

Test. For the deep water sections at depth -20 and 15.5 metres the stability of the given design was to be investigated.

For the breakwater in shallow water special precautions have to be taken against erosion, and the breakwater will be founded on a dredged bottom below the original sea bottom level. To keep the cost of dredging at a minimum the berms are to be as high as possible below the still water level. The criterion for the stability of the berm was that no rock be washed into the front armour or carried away from the berm on the harbour side. In addition to the study of the armour layer of 39 ton blocks an objective of the tests was to find the maximum crest elevation of the berms.

During a test the significant wave heights used were 4, 5, 6, 7, 8, 8.5, 9, 9.5 metres, each run for a period equal to 10 hours in prototype.

The tests were conducted with water levels of + 0.5 and + 1.5 metres.

Test at DHL.

Test equipement. The system for generating irregular waves in the DHL channel works approximately on the same principles as that of the RHL, i.e. the signal from a noise generator is filtered to give an input to a servo controlled wave paddle, which generates a wanted wave spectrum in the channel. The control system of the wave generator is also designed to accept a wave record as input signal.

Waves. In the tests three types of spectra were used, and each spectrum was run with peak frequencies corresponding to wave periods of 10, 12 and 14 seconds in prototype. The spectra are shown on normalized form in Fig. 11.

The A- and B spectra corresponds to the spectra used at the RHL, and in addition a very wide spectrum (C) was used.

Tests. The tests were run in scale 1:60 with two cross sections at depth -20.0 and -15.5 metres. The tests were run at water levels + 0.5 and + 1.5 metres.

TEST RESULTS

The results of the tests conducted at DHL and RHL are shown in Fig. 12-18. The line in the graphs, illustrating the effect of regular waves, is drawn on basis of the average values of $K/f(\alpha)$ and H_{destr}/H_d from the tests in regular waves which gave the lowest stability.

** As the structure was supposed to be stable up to the design wave height the damage was limited in all cases, and it is difficult to draw conclusions of a general nature about the influence of spectral shape, wave period etc. on the damage.

** Also, the conclusions about the stability are of course limited to the particular breakwater design and under the particular bottom conditions tested. However, at present very few tests on breakwaters have been conducted, in which the effect of regular and irregular waves can be compared, and it might be of some interest to discuss the results in some detail.

The typical development in the 43 ton armour layer during a test was as follows: Already at a wave height of 4-5 metres the blocks began to rock in the highest wave, resulting in a slow settling in the armour and core. This process continued for increased wave attack, and the thickness of the armour on the crest decreased. In most tests just a few blocks were removed from the armour layer, and after the completed test the breakwater seemed impaired only to a small degree.

As a result of the tests with irregular waves it was decided to compensate for settling in the core by increasing the initial crest elevation.

By comparing the tests which have been run both at DHL and RHL it seems that no systematic difference can be traced in tests performed in the scales 1:60 and 1:36.

* It has been commonly assumed that the destructive effect of a train of regular waves on a breakwater in model is equal to the effect of irregular waves with a significant wave height corresponding to the height of the regular waves.

** By comparing the results of tests run with regular and irregular waves it can be seen that in this case the damage appeared in the armour layer at a lower significant wave height than the height of the regular waves. This is apparently due to the few high waves present also for lower significant wave heights.

The difference in stability is somewhat greater than indicated by the average minimum curve as this curve is based on the results from the tests in regular waves giving the lowest stability. To illustrate this all results obtained in regular waves have been evaluated for the condition G = 43 t and γ_b = 2,65 t/m^3 and plotted in Fig. 19. In addition to the average minimum curve, the curve corresponding to the average of all results is drawn. In Fig. 20 all results obtained in irregular waves for the cross sections at depth -20 and -15,5 metres respectively, have been plotted for comparison with the average stability obtained in regular waves.

Also the stability of the berms seemed to be somewhat lower than observed in regular waves.

The increase of damage was, however, not considered to necessitate any change in the breakwater design.

In Fig. 21 is shown a diagram of damage vs. wave height for different wave spectra. The damage is expressed by the number of blocks which had to be placed on the model in order to restore the original shape. The number of blocks used for construction was approximately 250. In test of breakwater stability scatter is inherent, and the shaded area between results from two tests run under identical conditions illustrate the scatter in the test series. In view of the scatter, it is not possible to draw any conclusions on the effect of spectrum shape and peak frequencies on the stability.

FINAL COMMENTS

The tests described in this article were not meant to give results beyond those necessary to draw conclusions about this particular breakwater for a given range of wave dimensions. For this particular case, within the observed range of damage, irregular waves seemed to represent a more severe wave attack than regular waves with heights equal to the significant wave heights of the irregular waves.

In stability tests conducted at the RHL previously (Ref. 1, 2) relationships between damage and types of spectra have been indicated. It was shown that irregular waves, depending on the spectrum width, can be more or less dangerous than regular waves. Under certain conditions regular waves have been shown to be considerable more destructive.

On the basis of the sum of experience made so far, the conclusion seems to be that the factors which influence the stability of a breakwater are many and complex and vary within wide ranges from project to project. The best basis for breakwater design is still model testing, preferably with irregular waves.

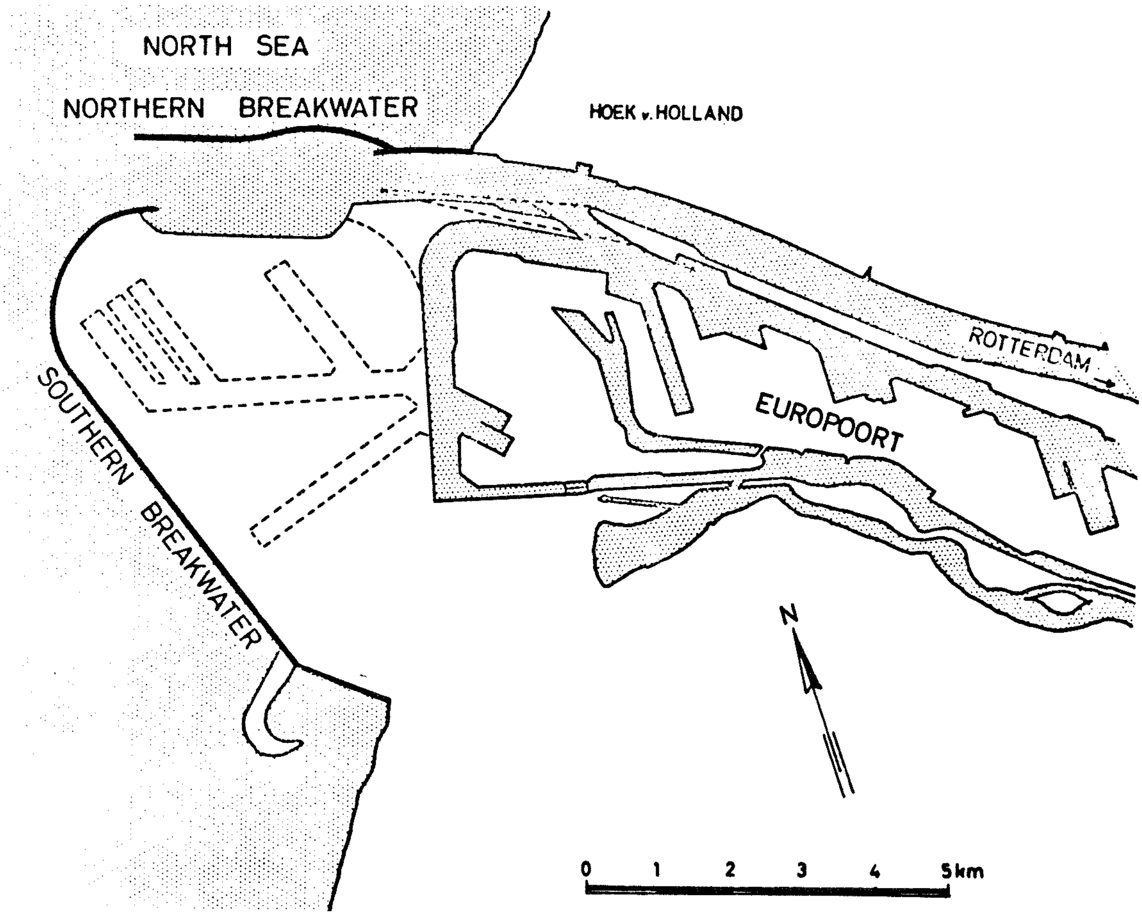


Fig. 1. Plan Europoort.

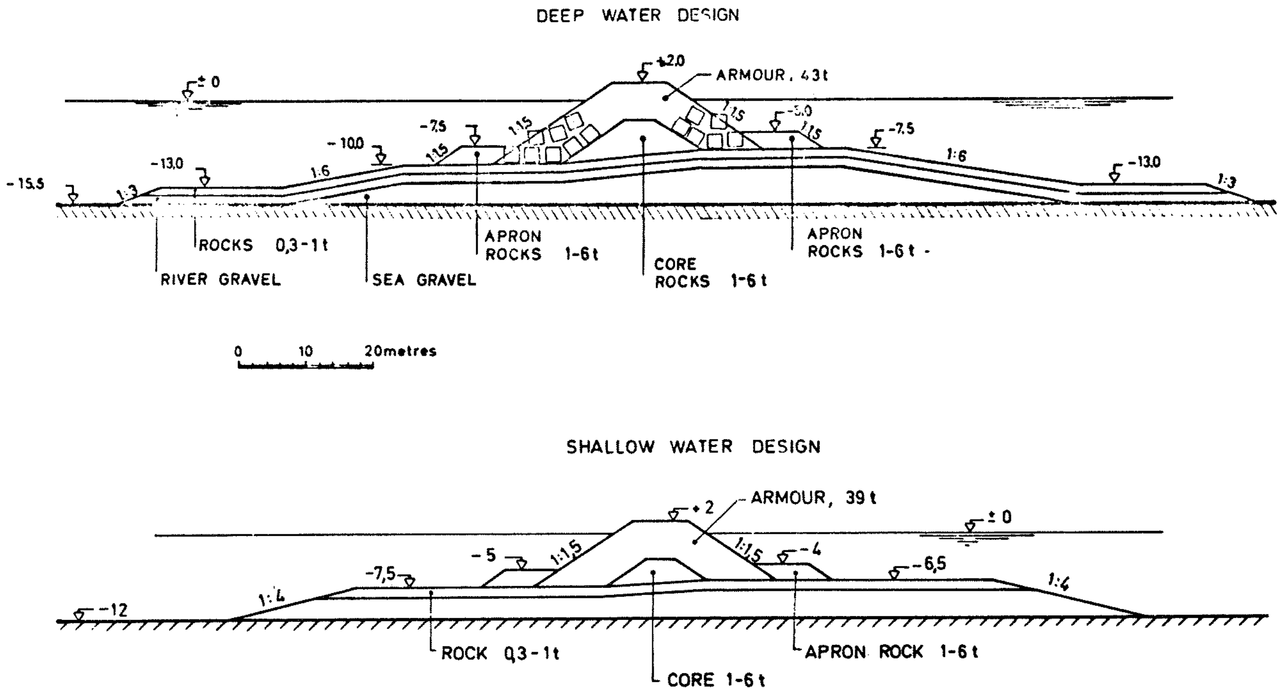


Fig. 2. Typical cross sections.

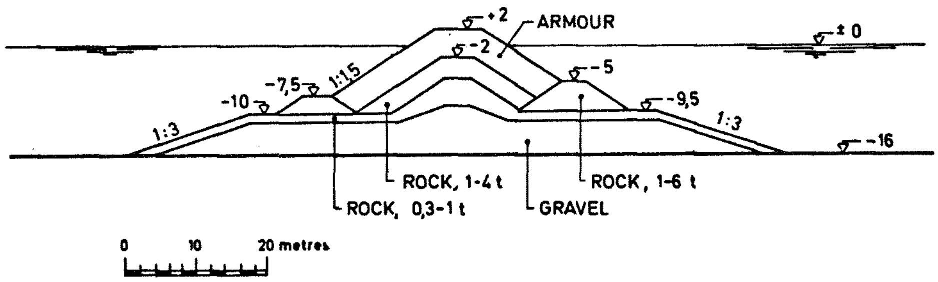
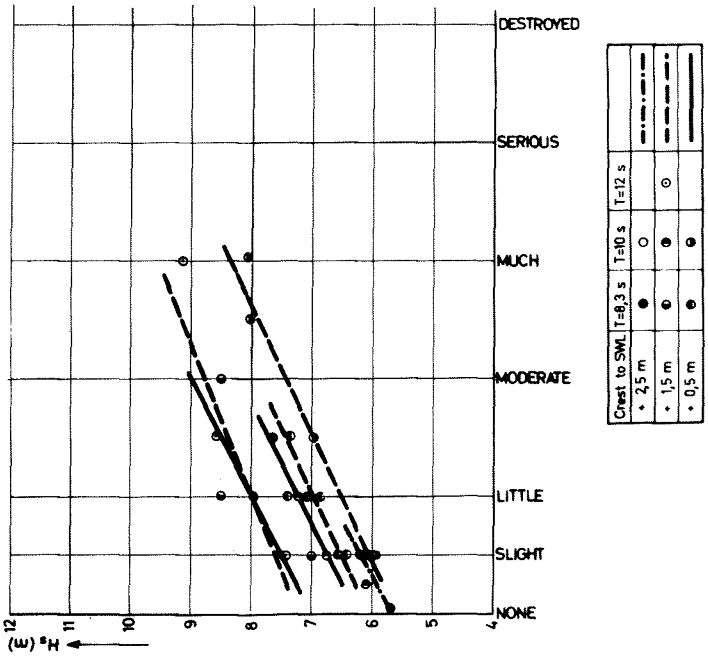


Fig. 3. Cross section. Tests with regular



** Fig. 4. Example of test results, regular waves. DHL.

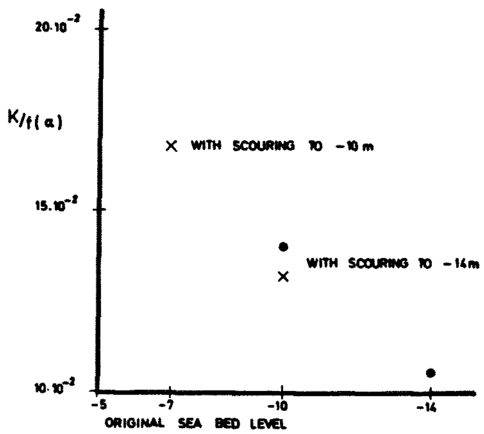


Fig. 5. Stability as a function of sea bed level. Regular waves. DHL.

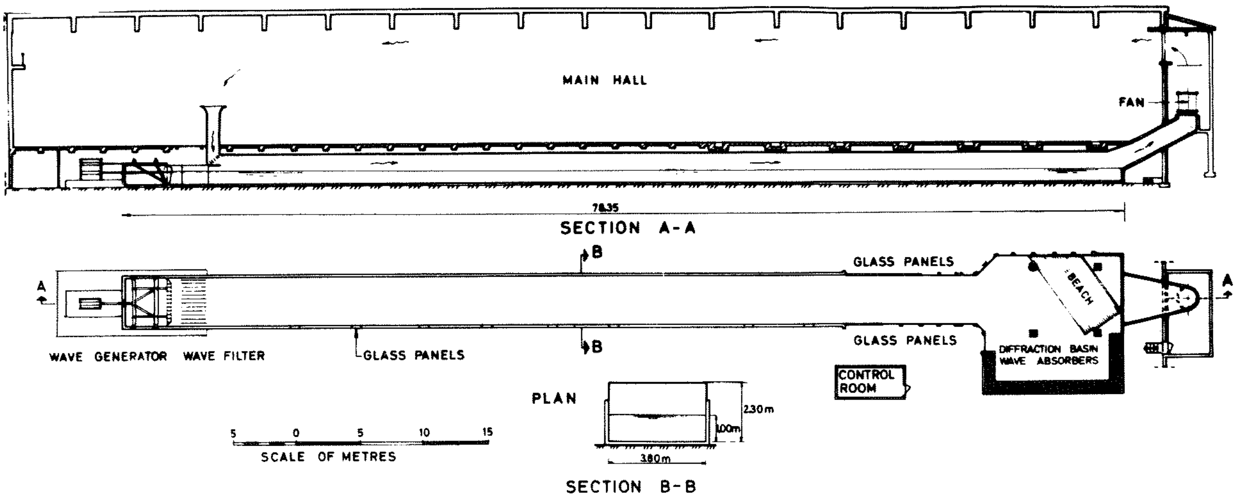
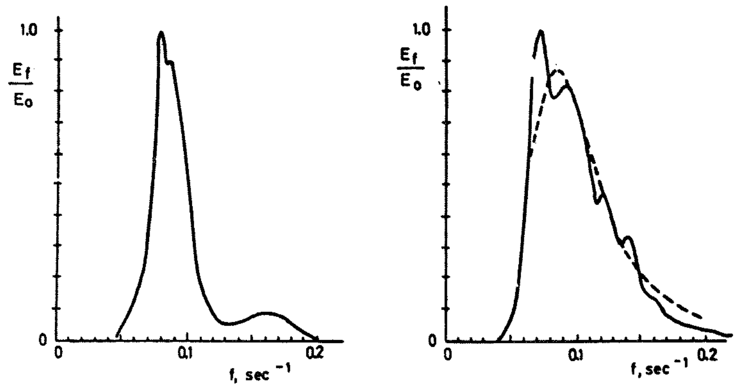
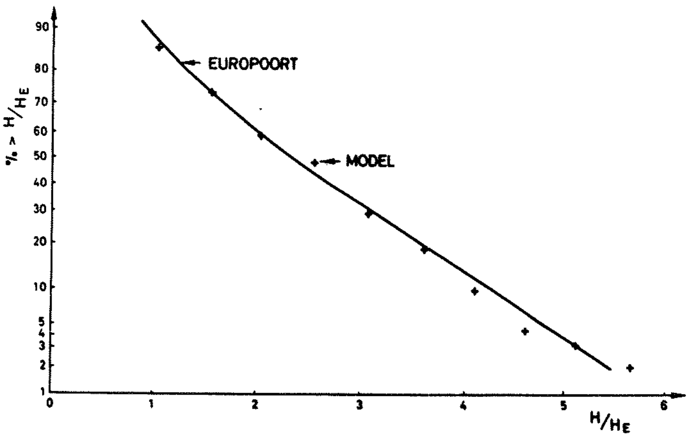


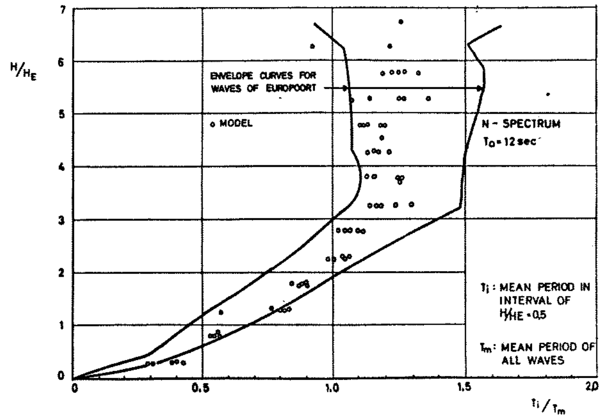
Fig. 6. Wave channel. RHL.



** Fig. 7. Wave spectra. RHL.



** Fig. 8. Wave height distribution. RHL.



** Fig. 9. Correlation between wave height and period. RHL.

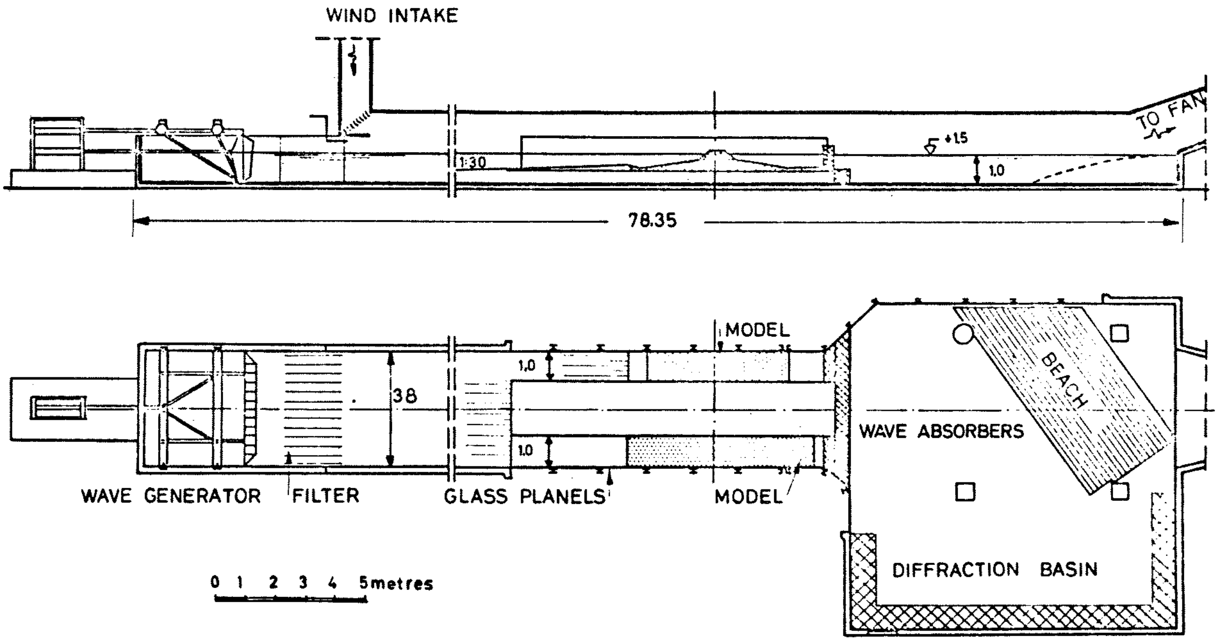


Fig. 10. Test arrangement. RHL.

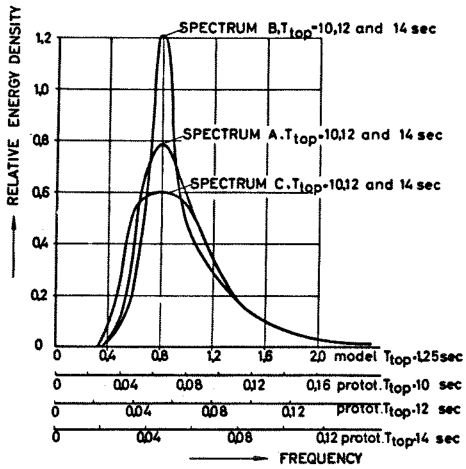
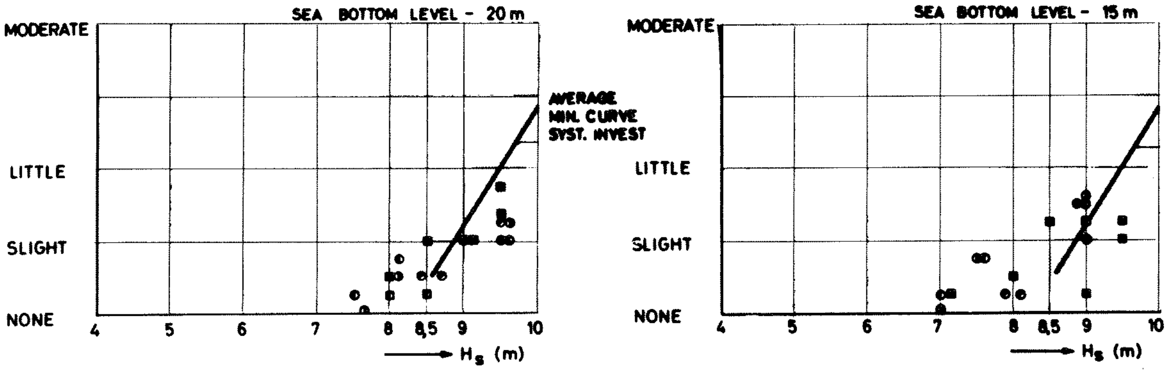


Fig. 11. Wave spectra. DHL.

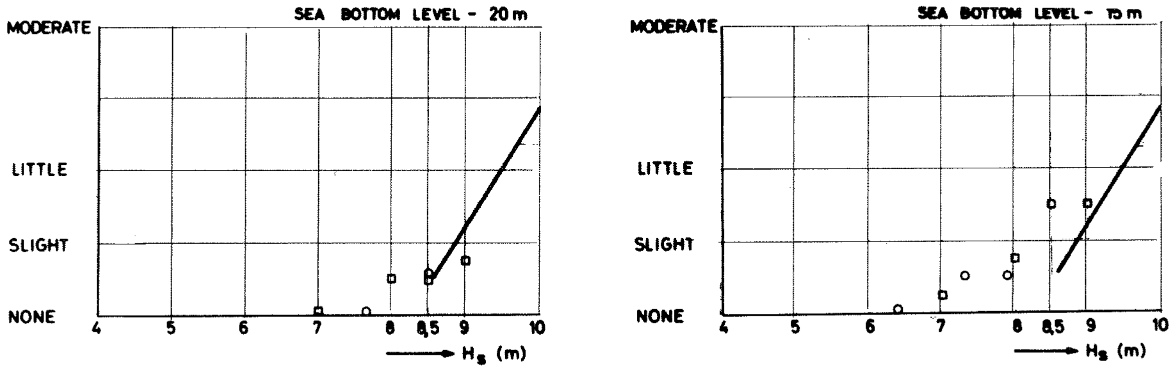


SPECTRUM : N

WATER LEVEL: + 0,50 m

T_0 sec	10	12	14	16
D H L			● ●	●
V H L		■	■	

Fig. 12. Test results. Irregular waves.

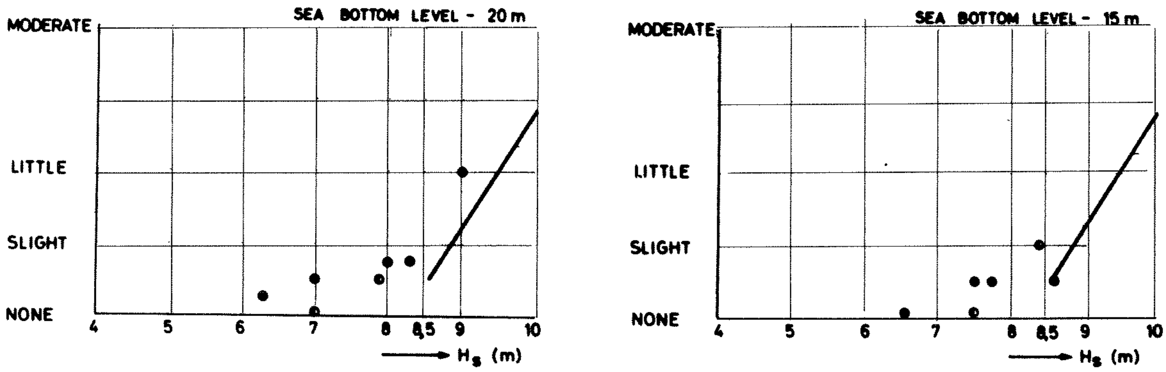


SPECTRUM : B

WATER LEVEL: + 0,50 m

T_0 sec	10	12	14	16
D H L	○			
V H L	□			

Fig. 13. Test results. Irregular waves.



SPECTRUM : C

WATER LEVEL: + 0,50 m

T_0 sec	10	12	14	16
D H L		●	●	
V H L				

Fig. 14. Test results. Irregular waves.

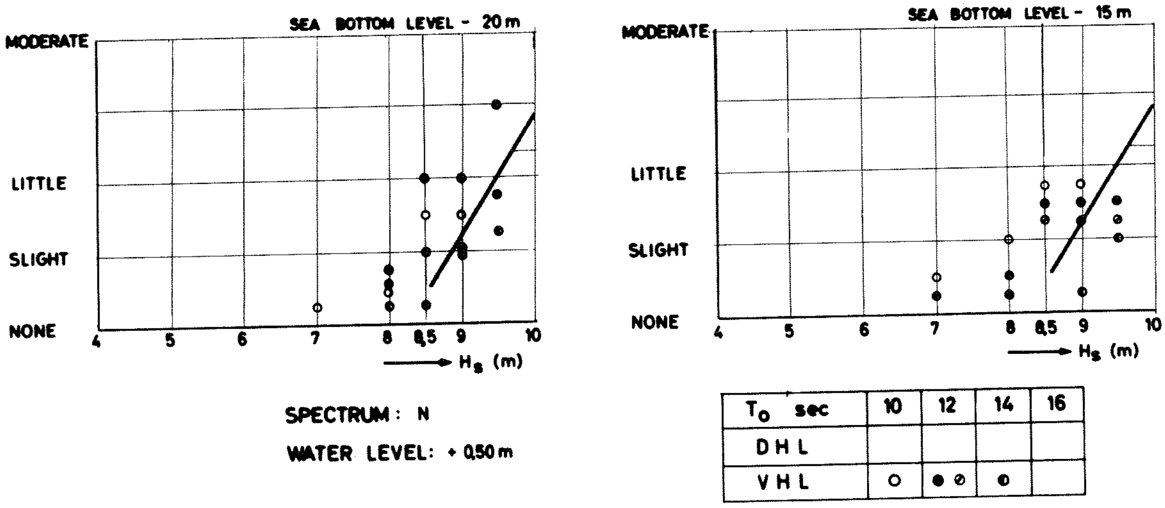


Fig. 15. Test results. Irregular waves.

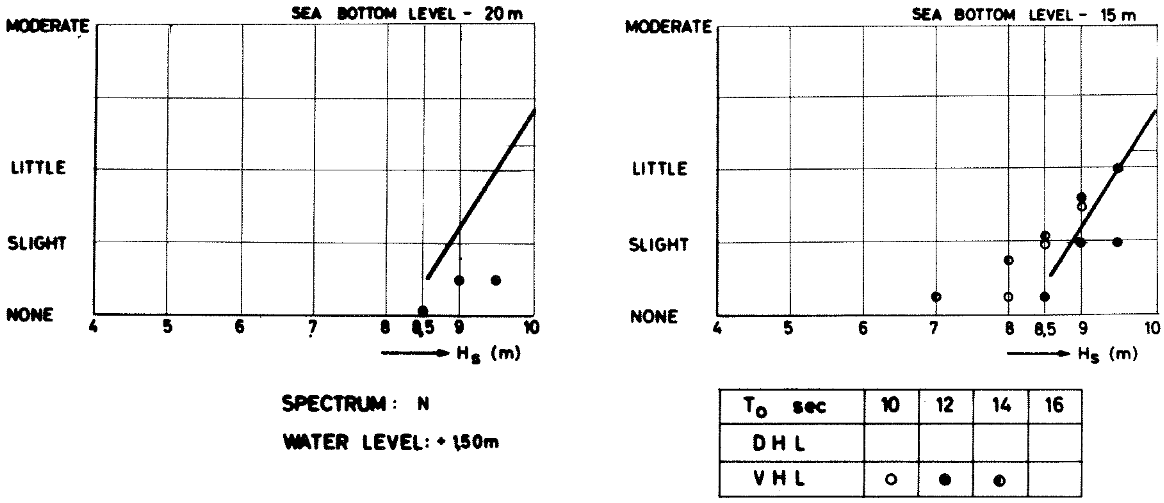


Fig. 16. Test results. Irregular waves.

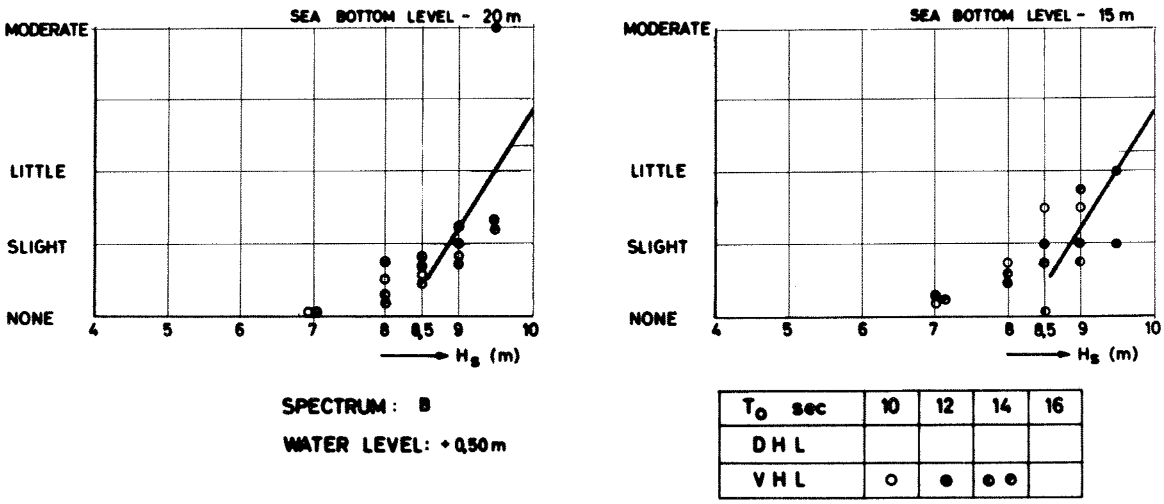
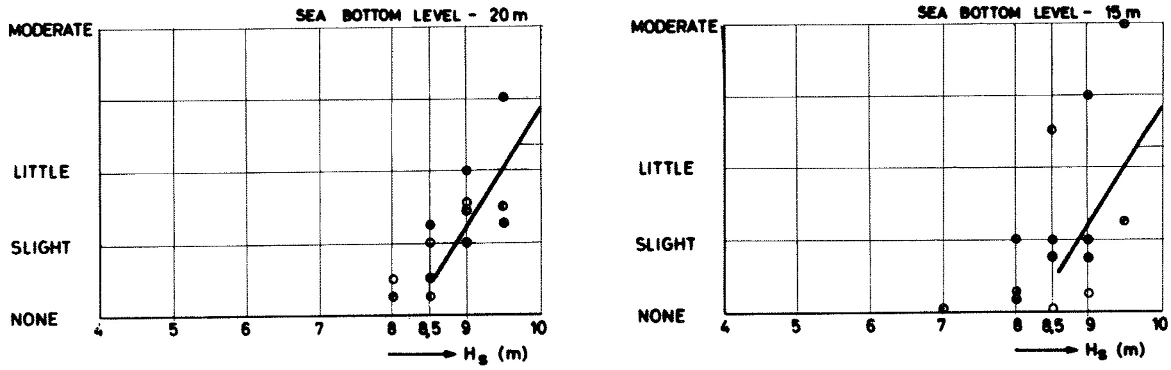


Fig. 17. Test results. Irregular waves.

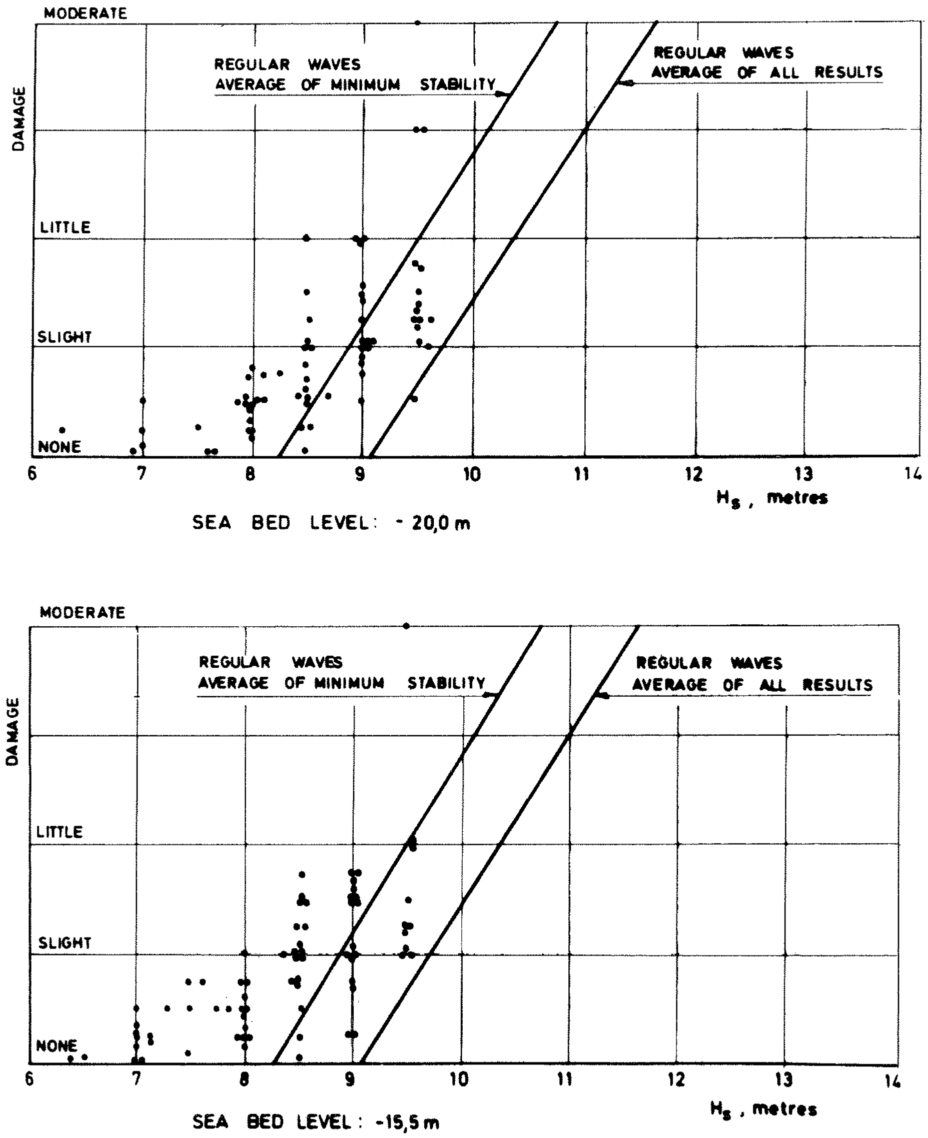


SPECTRUM : B

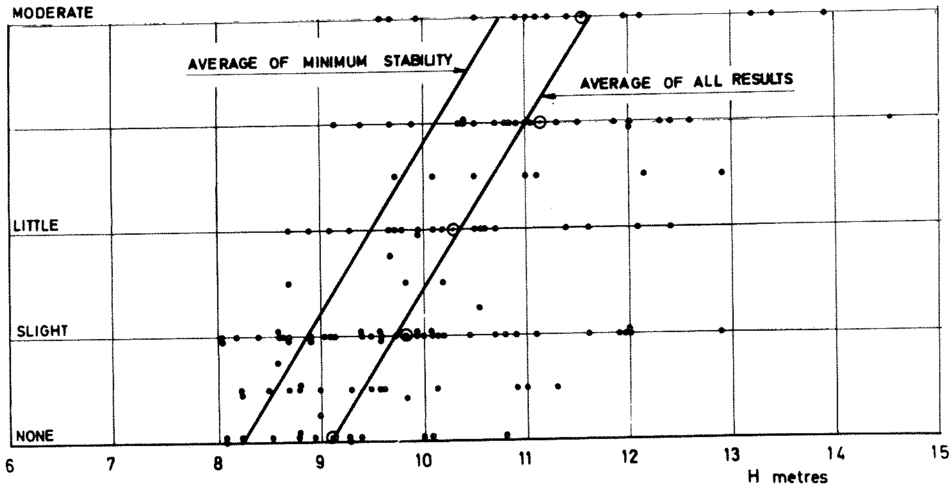
WATER LEVEL: +1,50m

T_0 sec	10	12	14	16
D H L				
V H L	○	●●	●	

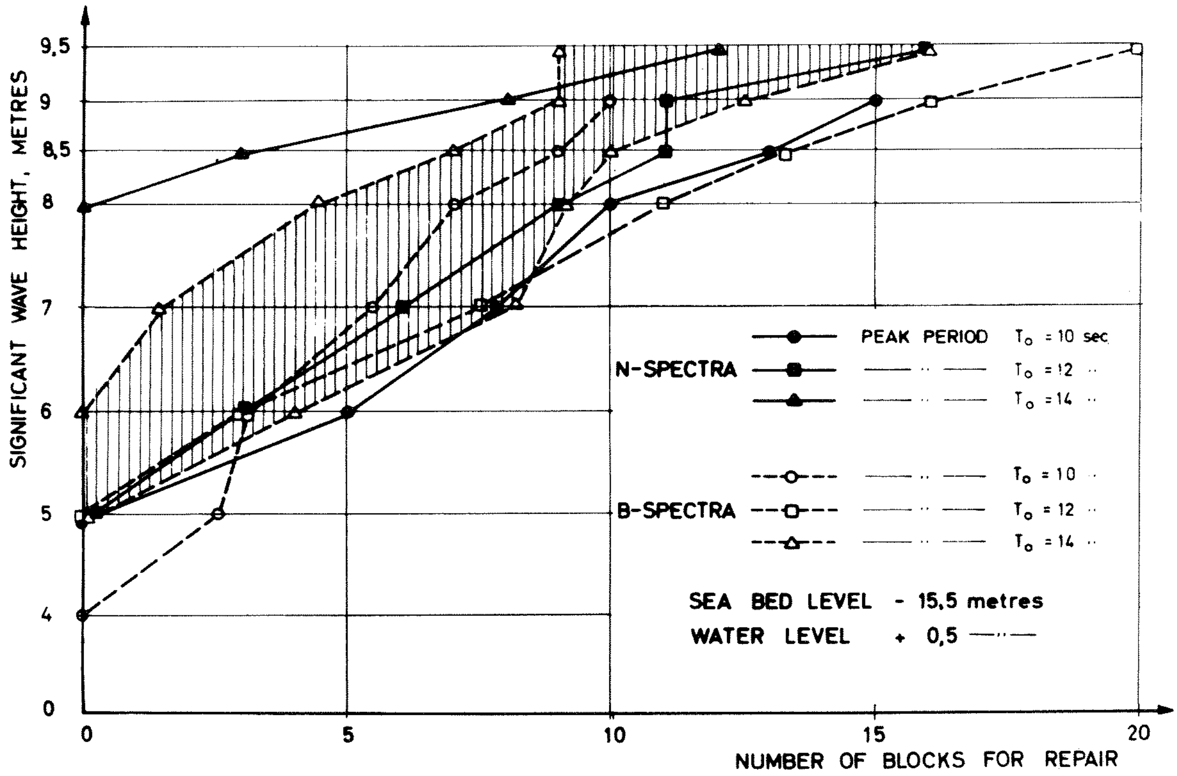
Fig. 18. Test results. Irregular waves.



* Fig. 19. Results from tests in regular waves evaluated for conditions $G = 43$ t, $\gamma_b = 2,65$ t/m^3.



* Fig. 20. Results in irregular waves.
All test data.



** Fig. 21. Test results. Irregular waves. RHL.

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2. Carstens, T., Trætteberg, A. and Tørum, A.
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DISCUSSION ON PAPER 10

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Delft Hydraulics Laboratory, The Netherlands

Berge and Treatteberg conclude, after having reviewed the results of experiments performed at the R.H.L.* and the D.H.L.** regarding the Europoort breakwaters, that "the factors which influence the stability of a breakwater are many and complex and vary within wide ranges from project to project. The best basis for breakwater design is still model testing, preferably with irregular waves".

Additional tests performed at the D.H.L. with a somewhat different cross-section have once again positively underlined the significance of this conclusion. Moreover, some results of these additional experiments are so interesting that a short discussion seems justified.

The cross-sections tested by R.H.L. and D.H.L. can be realized at those places where the original depth is about N.A.P. - 15 to - 20 m. However, parts of the Southern breakwater have to be constructed on a shallower bottom. After the construction is finished, the contraction of the tidal currents in combination with dredging work at the harbour entrance will cause a scouring in front of the bed protection of the breakwater up to N.A.P. - 20 m or even N.A.P. - 25 m. The influence of this bottom configuration on the stability of the breakwater has been extensively tested by the D.H.L. Though the differences in the cross-sections compared with those described by Berge and Treatteberg are minor, the stability appeared to be entirely different. With a proposed length of the apron of 20 m, serious damage occurred under design-wave conditions, whereas, considering the former experiments, only slight damage was expected. Moreover, damage increased rapidly with increasing wave height. After having considered this result, it was decided to vary the apron length in the model. Four lengths were taken: 3, 12.5, 20 and 50 m.

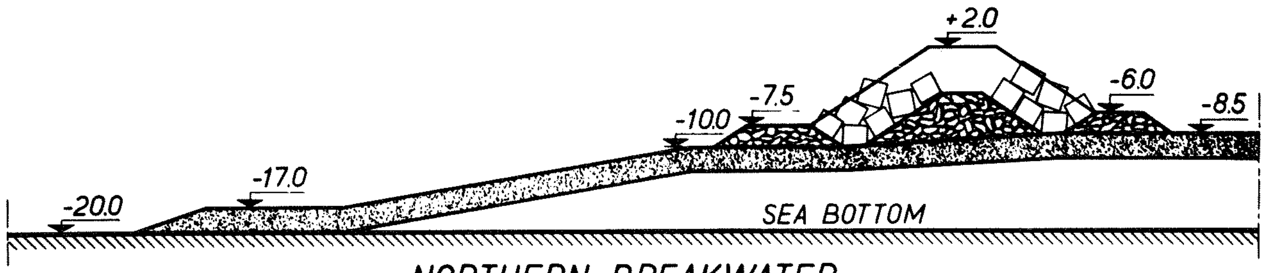
* River and Harbour Research Laboratory, Trondheim.

** Delft Hydraulics Laboratory.

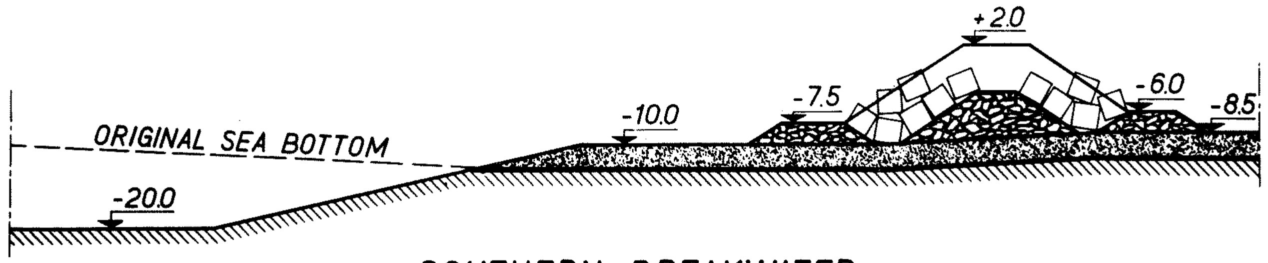
The results show the smallest damage for the 3 and 50 m aprons and a maximum damage for the originally selected length of 20 m. Visual observations of the model have created the impression that the character of the breaking wave is one of the causes of this phenomenon. Wave attack on the armour blocks got more the character of wave impact with increasing apron length up to 20 m, whereas at a length of 50 m a substantial amount of energy has already been dissipated by wave breaking in front of the breakwater.

One reference test applying regular waves showed hardly any damage at the design wave height of 8.5 m and an apron length of 20 m, whereas in the case of irregular waves the structure was seriously damaged at the corresponding significant wave height and period. Though for some specific problems regular-wave experiments may yield useful information, this result once more focusses attention on the risk of this method.

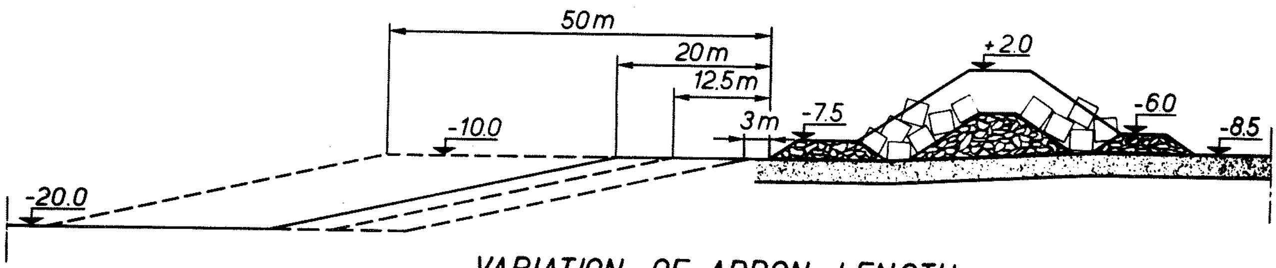
The amount of influence of the apron length was not predicted, and was discovered only thanks to the fact that extensive model studies had been performed for the Europoort project. As apparently minor factors have an important influence on model results, the extrapolation of data to other problems, which may look similar in a first approximation, has to be applied with the utmost care. It is evident that the same holds good even more for the use of stability formulae.



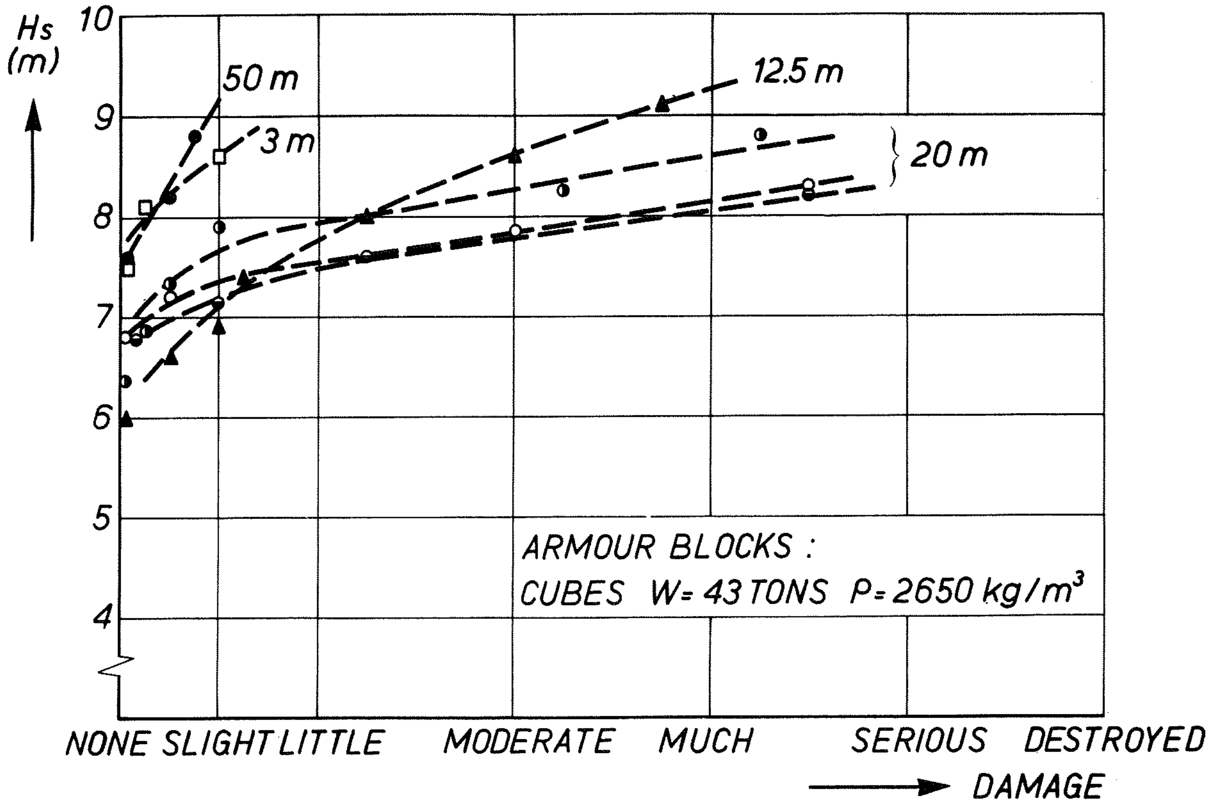
NORTHERN BREAKWATER



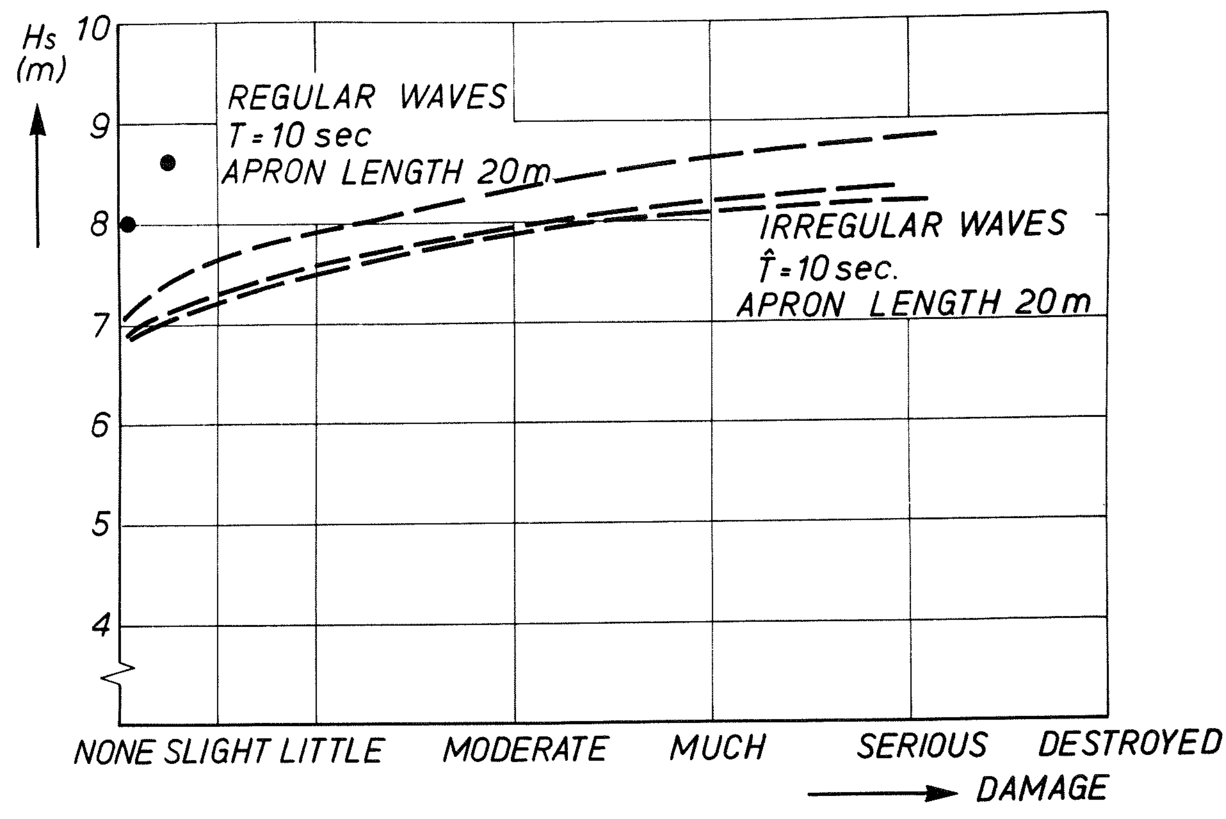
SOUTHERN BREAKWATER



VARIATION OF APRON LENGTH



DAMAGE ARMOUR-LAYER AS A FUNCTION OF WAVE HEIGHT AND APRON LENGHT.



COMPARISON OF DAMAGE CAUSED BY REGULAR AND IRREGULAR WAVES.

FINANCIAL OPTIMIZATION OF INVESTMENTS IN MARITIME STRUCTURES

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Delft University of Technology, The Netherlands

SUMMARY

The designing engineer often comes across the economic decision problem in which the "benefit" of a higher design criterium must be weighed against the "cost". Several methods have been developed to deal with this optimization problem; none of them, however, offer the possibility of introducing the appropriate considerations of a general policy of investment. Therefore this paper presents a solution for maximizing the total benefits of the total amount of investments made by one financier.

For each project with a benefit b_i and a cost c_i the difference (b_i - m c_i) must be maximized in order to fulfil the requirements to obtain a maximum total benefit (B) from the money available for investment (C). The factor m has to be so determined that

$$\sum_{i=1}^{i=n} c_i = C.$$

Beginning with the relationship between b_i and c_i for each project (see fig. 2, 6 and 7) the problem can be solved graphically. Maximizing (b_i - m c_i) for various values of m gives a graph in which the optimal investments c_i are set out against m (see fig. 3, 6 and 7). By summation of these data a graph giving the relationship between C(opt) and m can be drawn, from which the required value of m can be read. (see fig. 4 and 5)

1. INTRODUCTION

Optimization of investments plays a significant role in most decision making processes. Consequently the various optimization techniques are of interest to people in many professions and are no longer considered to be the exclusive speciality of economists and policy makers. For instance design engineers, in the execution of their duties, will often have to deal with the problem, that the critical conditions, which a structure should be able to withstand, are brought about by stochastical phenomena. In that case the design cannot be based on a maximum load, which will never be exceeded. For each design criterion it will only be possible to determine the mean frequency of exceedence and the design has to be based on the acceptance of a certain risk.

By choosing a safer design criterion the damage expectation may be reduced. On the other hand, however, the building costs will then increase. Thus the design engineer finds himself confronted with a problem of decision-making, which can only be solved by weighing the "benefits" - in this case the reduction of the damage expectation - against the "costs" - in this case the increase of the investment to be made. In doing so the engineer will usually aim at maximization of the net proceeds to be gained from his project. The investment is then referred to as being "optimal" if that object has been attained.

Nowadays various examples of this process of decision-making could be given, but in the Netherlands the first applications were related to maritime structures. The writers of this article, both of whom are engaged in hydraulic engineering, have gathered their experience exclusively in this field. However that does not alter the fact, that their considerations refer to a more fundamental problem, which will always be encountered, when optimizing the benefits of investments.

As engineers usually direct their attention to only one project at a time, many optimization techniques now available include the error of maximizing the benefits per project, without any reference to the general policy of investment by the financier. As a result of this omission seemingly unsolvable problems were encountered, which may be illustrated by a historical review of the evolution of the train of thought in the Netherlands.

2. HISTORICAL REVIEW

Investigations carried out after the catastrophic flood which ravaged the Netherlands in 1953 made it possible to draw two important conclusions:

a) All the damage done to the dikes was caused by overtopping of water. As a consequence determining the height of the dikes was considered to be a decisive problem.

b) It appeared impossible to determine a maximum water level, which would never be exceeded. It was possible, however, to draw graphs, in which stormflood levels were set out against their frequency of exceedence. From these graphs could be read that within practical limits every design criterion would include a certain risk.

Thus it became clear that, first of all, attention had to be focussed on the question as to what risk should be accepted. Consequently civil engineers began to develop techniques to find the proper answer to that question. This resulted in a method which, at least to civil engineers, was obvious. This method was founded on the principle that when building a dike two investments must be considered: firstly, the cost of building and secondly, an investment set aside to pay for all future maintenance and damage.

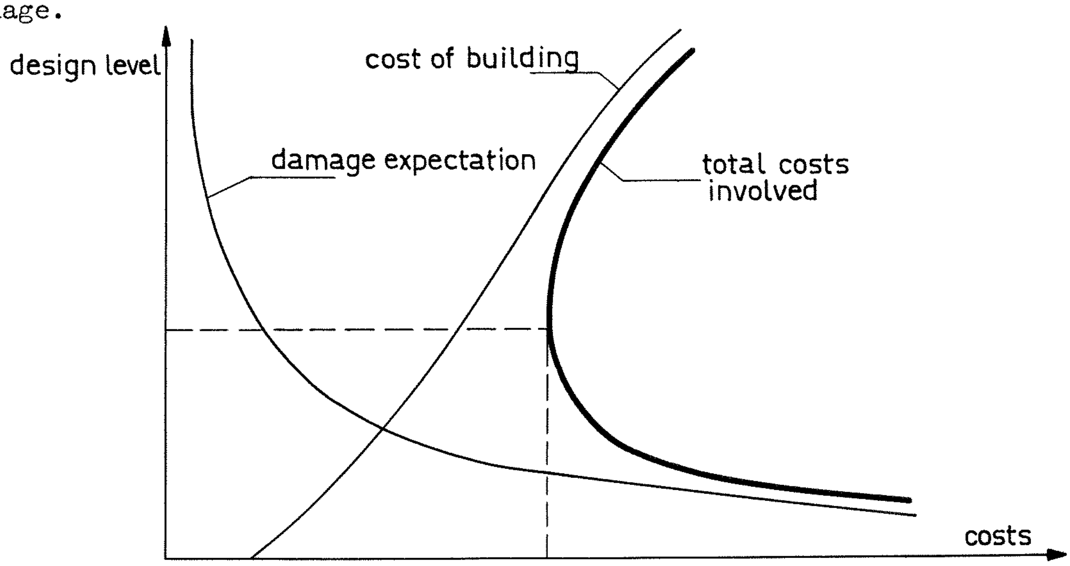

Fig. 1. Relation between total costs involved and design criterion.

** As has been mentioned in the introduction, the cost of building will increasė with the design criterion, whilst the safer the design criterion, the less the damage expectation. Both building costs and damage expectation can be computed for various design criteria and may be plotted in a graph. The relation between the total costs involved and the design criterion can be found easily by adding up the two investments. The optimal design criterion which corresponds to the minimum costs can be read from the relation thus achieved. (see fig. 1)

Actually this way of thinking led to the conclusion, that the design criterion of the Dutch dikes should have an average frequency of exceedence of 1/10.000 per year. Many assumptions had to be made to achieve this result and as a consequence some people objected to the use of the above mentioned method. These objections referred to a number of difficulties, which can be summarized as follows:

a) the accuracy of many data involved in this problem was only poor.
b) the loss of human lives had to be valued.
c) the method is valid only if a great number of independent problems can be treated in the same manner.
d) the rate of discount to be used in the calculations was a matter of great uncertainty.

A satisfactory solution was found to solve the problem of inaccuracy of the quantities to be introduced in the calculations. The problem of the human lives involved caused a lot of discussion but nowadays the ethical objections against the use of the method are no longer considered to be real obstacles. The same applies to the third problem which has not been solved fundamentally but the objections based on point c were unable to withstand the more or less philosophical arguments advanced in this respect.

This article only serves to draw the readers attention to the problem of valuation of the rate of discount. When economists were consulted to solve this problem they appeared to have a different opinion. Some of them preferred the long term discount rate of 3% to 6%, whilst others told us to use the rentability of the alternative projects, i.e. the projects which could not be realized, if more money was invested in the dikes. Consequently this group of economists advised the engineers to use a discount rate of 30%.

Moreover, the situation became even worse when some of them criticized the principles of the method described in the foregoing. They explained that in using this method the difference between benefits and costs (b - c) was made as large as possible, whilst actually the ratio benefits over costs (b/c) should be maximized. The latter, i.e. an effort to gain the greatest benefit per pound invested, their objection seemed to make sense. On the other hand the design engineers could not easily forget their original approach, which led to maximizing of (b-c). The basic principles underlying this result seemed to be practical as well as logical.

As stated in the introduction this contradiction was caused by the fact that both engineers and economists overlooked one important principle viz that one should not try to optimize the various projects separately. Therefore it is better to consider the general policy of investment by the financier as being the main object of optimization. This purpose may be pursued by beginning with the assumption, that we wish to achieve the greatest possible total benefits from the total amount of money available for investment.

3. CONDITIONS FOR INVESTMENT

Let us suppose that the amount of money available for investment is C and the capitalized benefits gained from it are B. Let us suppose furthermore, that we have n projects over which the capital to be invested may be spread out and that the amount of money invested in one project will be c_i (i being an arbitrary number in the series 1, 2, 3, ... n). Accordingly the capitalized benefits gained from one investment are indicated as b_i.
Our problem now is to determine c_i in such a way, that

$$B = \sum_{i=1}^{i=n} b_i \text{ reaches its maximum} \quad \ldots\ldots (1)$$

while

$$C = \sum_{i=1}^{i=n} c_i \quad \ldots\ldots (2)$$

Some conditions necessary to cope with formula 1 can be derived by changing c_i with a small amount δc_i and investigating whether the total capitalized benefits then increase or decrease. One may not choose the variations δc_i arbitrarily because the total amount C must remain the same. Hence it follows that:

$$C = \sum_{i=1}^{i=n} (c_i + \delta c_i) = \sum_{i=1}^{i=n} c_i + \sum_{i=1}^{i=n} \delta c_i.$$

So that in connection with (2):

$$\sum_{i=1}^{i=n} \delta c_i = 0 \quad \ldots\ldots (3)$$

The total capitalized benefits become:

$$B + \delta B = \sum_{i=1}^{i=n} (b_i + \delta b_i) = \sum_{i=1}^{i=n} b_i + \sum_{i=1}^{i=n} \delta b_i$$

Or after substitution of (1):

$$\delta B = \sum_{i=1}^{i=n} \delta b_i \quad \ldots\ldots (4)$$

Supposing that the capitalized benefits gained from each project (b_i) are a function of the investment in that projects (c_i) only, and confining ourselves provisionally to small variations δc_i and functions $b_i(c_i)$, which are differentiable for $c_i > 0$, we may write:

$$\delta b_i = b_i(c_i + \delta c_i) - b_i(c_i) = \frac{db_i}{dc_i} \delta c_i.$$

So that (4) may be written as:

$$B = \sum_{i=1}^{i=n} \frac{db_i}{dc_i} \delta c_i \quad \ldots\ldots (5)$$

Now the investments c_i will be optimal, if for every possible combination of values δc_i:

$$\delta B \leqslant 0 \quad \ldots\ldots \quad (6)$$

Thus optimal investment will be achieved, if two conditions are fulfilled, viz:

$$\sum_{i=1}^{i=n} \frac{db_i}{dc_i} \delta c_i \leqslant 0 \qquad \text{(from (5) and (6))} \quad \ldots\ldots \quad (7)$$

and

$$\sum_{i=1}^{i=n} \delta c_i = 0 \quad \ldots\ldots \quad (3)$$

Now let us suppose, that project nr. 1 will be realized. Condition (7) may then be written as:

$$\sum_{i=2}^{i=n} \frac{db_i}{dc_i} \delta c_i + \frac{db_1}{dc_1} \delta c_1 \leqslant 0 \quad \ldots\ldots \quad (8)$$

From (3) it follows:

$$\delta c_1 = - \delta c_2 - \delta c_3 \ldots\ldots \delta c_n, \text{ or}$$

$$\delta c_1 = - \sum_{i=2}^{i=n} \delta c_i \quad \ldots\ldots \quad (9)$$

Substitution of (9) in (8) then yields:

$$\sum_{i=2}^{i=n} \frac{db_i}{dc_i} \delta c_i - \sum_{i=2}^{i=n} \frac{db_1}{dc_1} \delta c_i \leqslant 0,$$

or:

$$\sum_{i=2}^{i=n} \left(\frac{db_i}{dc_i} - \frac{db_1}{dc_1}\right) \delta c_i \leqslant 0 \quad \ldots\ldots \quad (10)$$

In view of the fact that we have complied with (3), δc_i may vary independently in (10). This means, that if a project is realized ($c_i > 0$) δc_i may be positive as well as negative.
In order to cope with (10) we then must comply with:

$$\frac{db_i}{dc_i} = \frac{db_1}{dc_1}$$

or:

$$\frac{db_i}{dc_i} = \text{constant} = m \qquad \text{(condition for projects to be realized)} \ldots\ldots \quad (11)$$

The same reasoning may be applied to a project which will not be realized. In that case condition (10) must still be fulfilled.
c_i being zero we may conclude that $\delta c_i \geqslant 0$.
To cope then with equation (10) it is necessary that:

$$\frac{db_i}{dc_i} - \frac{db_1}{dc_1} \leqslant 0$$

from which it follows:

$$\frac{db_i}{dc_i} \leqslant m \qquad \text{(condition for projects not to be realized)} \ldots\ldots\ldots\ldots \quad (12)$$

The equations (11) and (12) present a more or less trivial conclusion which could also have been found by reasoning without any mathematical treatment.
This can easily be seen if two projects with different values of db_i/dc_i are compared. It is clear that the joint capitalized benefits can then be raised principally by withdrawing a relatively small amount of money from the project with the lowest value of db_i/dc_i and investing this in the project with the greatest value of db_i/dc_i.

Equations (11) and (12) having been derived by comparing various investments (c_i) with one investment c_1, which definitely will be made, only provide the conditions to be fulfilled if such an investment c_i will be made or not. The question as to which project should be invested in has not yet been answered.

4. SELECTION OF INVESTMENTS TO BE MADE.

In order to answer that question we now consider an investment c_n and compare the case that this investment will be made with the case that this investment will not be made.
If $c_n > 0$ condition (1) remains unchanged
If $c_n = 0$, B will decrease with the amount b_n. On the other hand however B may increase because more money becomes available for investment in the remaining projects.

As a result of both variations together B will increase with

$$\delta B = \sum_{i=1}^{i=n-1} \delta b_i - b_n \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Now it is provisionally supposed, that a much larger amount of money is involved with the investments to be made than with projects which fall through. In that case δc_i must again be small, so that by approximation:

$$\delta b_i = \frac{db_i}{dc_i} \delta c_i,$$

or after substitution of (11):

$$\delta b_i = m\, \delta c_i$$

Equation (13) now may be written as:

$$\delta B = m \sum_{i=1}^{n-1} \delta c_i - b_n,$$

or, c_n being the extra amount of money available for the remaining projects:

$$\delta B = m\, c_n - b_n.$$

B is the increase of B if the investment with the number n were not made. So this investment indeed should not be made, if $\delta B > 0$. Consequently the conditions are:

no investment if: $m\, c_n - b_n > 0$ or $\frac{b_n}{c_n} < m$

invest if: $m\, c_n - b_n < 0$ or $\frac{b_n}{c_n} > m$

indifferent if: $m\, c_n - b_n = 0$ or $\frac{b_n}{c_n} = m$

Summarizing, we may conclude, that for investments to be made the following conditions must be fulfilled:

**

$$\frac{db_i}{dc_i} = m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{b_i}{c_i} \geqslant m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (14)$$

All projects complying with these conditions must indeed be realized to gain the maximum benefits from the capital available.

5. MAXIMIZING $(b_i - m\, c_i)$.

The conditions (14) may both be complied with by maximizing the difference $(b_i - m\, c_i)$. In that case:

$$\frac{d}{dc_i}(b_i - m\, c_i) = 0,$$

or:

$$\frac{db_i}{dc_i} = m.$$

Maximizing $(b_i - m\, c_i)$ means that the second condition will automatically be fulfilled. This follows from the fact that for each value of $c_i > 0$ the difference $(b_i - m\, c_i) < 0$, if $b_i/c_i < m$. The optimum in that case will always be $c_i = 0$, which also means $b_i = 0$ and thus $(b_i - m\, c_i) = 0$. The optimal investments c_i now can be determined by means of a graphical construction if only m is known (see Fig. 2). In order to do so we draw the dotted line $b_i - m\, c_i = 0$ in a graph of b_i plotted against c_i. The optimal value of c_i will correspond to the spot where the function $b_i(c_i)$ reaches a maximum vertical distance above the line $b_i - m\, c_i = 0$.

If the line $b_i - m\, c_i = 0$ is entirely situated above the function $b_i(c_i)$ the second condition of (14) can only be fulfilled if $c_i = 0$.

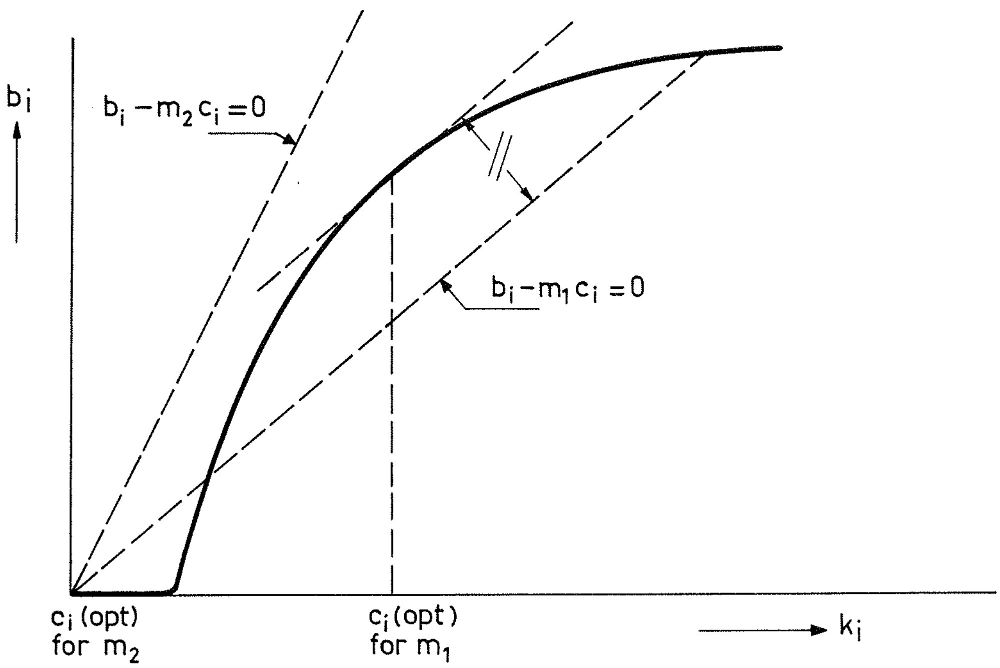

Fig. 2. Graphical determination of c_i(opt)

6. VALUATION OF THE MULTIPLIER m.

The multiplier m must be valuated such, that:

$$\sum_{i=1}^{i=n} c_i = C.$$

In order to comply with this condition for every project entitled to consideration, the function $b_i(c_i)$ must be made. These relations $b_i(c_i)$ should be sent to the central investment authority concerned. Beginning with various values of m this authority may determine the optimal value of c_i for each value of m. Thus for each project a graph can be made in which the optimal value of c_i is plotted against m (see Fig. 3). These functions may contain various discontinuities but they will always show a step, where it becomes impossible to comply with the second condition of (14).
The results thus achieved separately for various projects may then be combined in one graph, showing the function C (m). This has been done in fig. 4.

If the total amount of money available for investment is to be considered as an established datum the optimal value of m may be read from this graph as indicated in fig. 4. If neither m nor C are to be considered as an established datum one might wish to compare the results of various possible combinations of m and C (opt). This may be done by plotting the added benefits B(opt) in the same graph. (see fig. 5)
The amount of money available can now be subdivided into an amount to be invested I and an amount to be set apart for consuming purposes K. Varying the amounts I and K enables one to compare the actual results of various decisions.

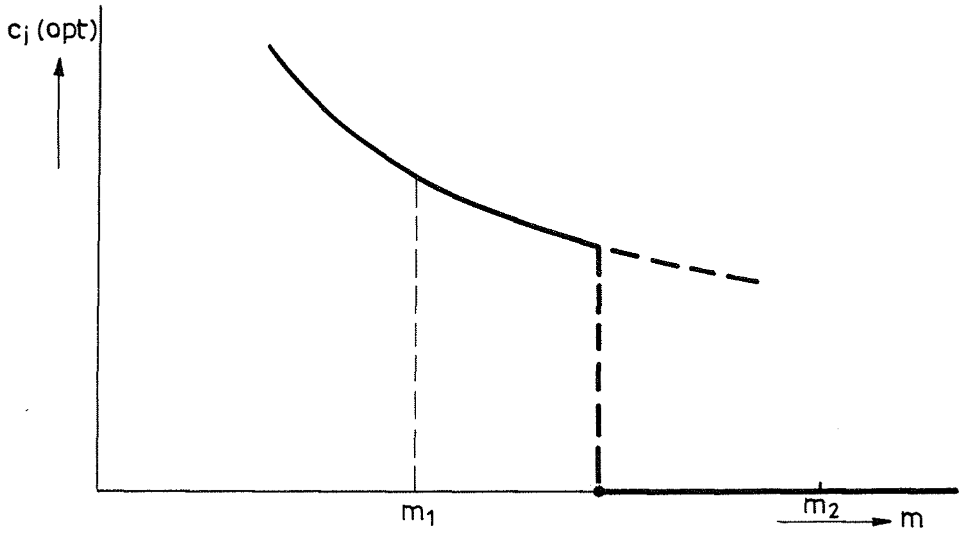

Fig. 3. Optimal value of c_i as a function of m.

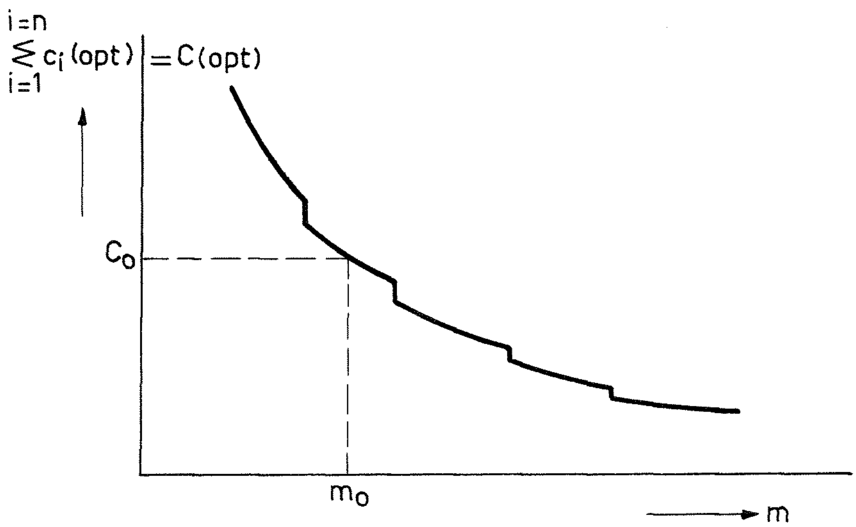

Fig. 4. Total of optimal investments as a function of m.

In this respect we would draw attention to the fact that the difference between B and C as shown in fig. 5 is, in fact, a measure for the future increase or decrease of the amount of money now available (C).

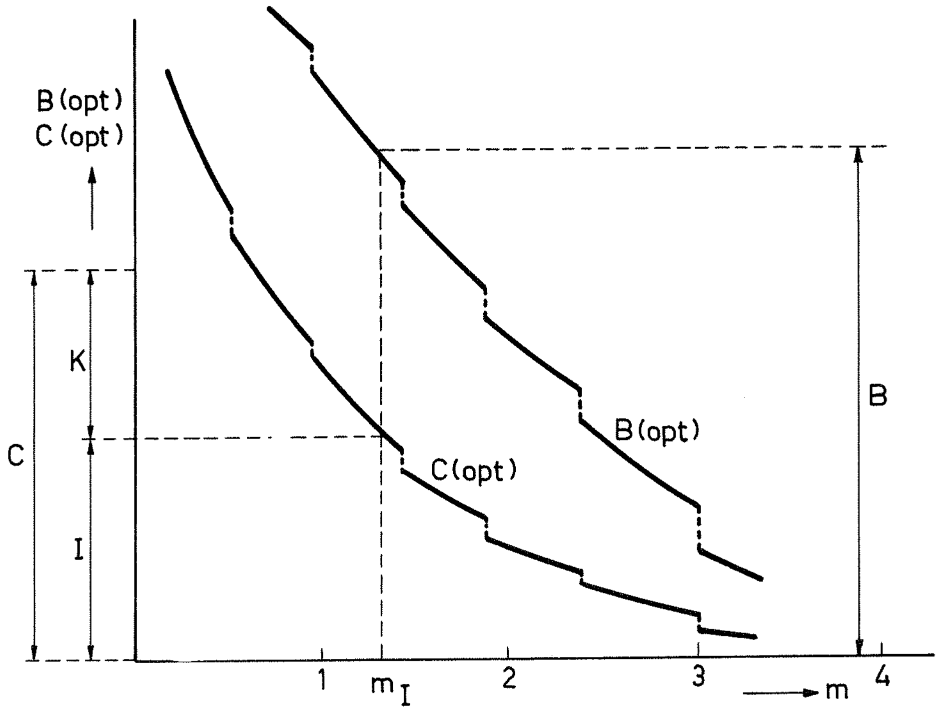

Fig. 5. Total of optimal investments and benefits as a function of m.

7. OPTIMIZING UNDIFFERENTIABLE FUNCTIONS $b_i(c_i)$.

In the foregoing the solution was confined to differentiable functions $b_i(c_i)$ and small amounts δc_i. However the results suggest that, in general, maximizing $(b_i - m\,c_i)$ might also led to optimal investment. If the various projects are mutually independent this appears to be true. In that case for every value of m to be established

$$\sum_{i=1}^{i=n} b_i - m \sum_{i=1}^{i=n} c_i$$

will be maximal, if $(b_i - m\,c_i)$ for each project is taken as large as possible.

The multiplier m having been determined such that

$$\sum_{i=1}^{i=n} c_1 = C$$

we may conclude that maximizing $(b_i - m\,c_i)$ corresponds to maximizing

$$\sum_{i=1}^{i=n} b_i - m\,C.$$

The added benefits:

$$B = \sum_{i=1}^{i=n} b_i$$

will then also reach a maximum.

Maximizing $(b_i - m\,c_i)$ for each project will indeed always result in the largest possible added benefits to be gained from the joint investments, as long as m is valuated such, that:

$$\sum_{i=1}^{i=n} c_i = C.$$

Thus the graphical solution described in fig. 2, 3, 4 and 5 may also be used for arbitrary functions $b_i(c_i)$. Discontinuities in these functions will in no way disturb the procedure mentioned above. This is demonstrated in fig. 6 and 7, in whicht the relation between c_i (opt) and m has been determined for two arbitrarily chosen functions $b_i^1(c_i)$. Such functions can be combined by a central office of the financier and then be used as has been explained in the discussion of fig. 4 and 5.

8. THE INFLUENCE OF THE RATE OF DISCOUNT.

Using a constant rate of discount r, the rentability of the marginal investment q follows from:

$$\frac{db_i}{dc_i} = \frac{q}{r},$$

substitution in (14) yields

$$m = \frac{q}{r}$$

Thus determining m means, that actually the optimal rentability of the marginal investment is established. As long as the annual benefits gained from the investments are independent of time the result will not be influenced by the value of the rate of discount r. The same applies if the annual benefits (w_i) of all the projects involved increase equally with a constant percentage per year (s).
Then we may write:

$$w_i(t) = w_i(o)\ e^{st}.$$

In that case the capitalized benefits will be:

$$b_i = \int_o^T w_i(o)\ e^{(s-r)t}\ dt,$$

if $T = \infty$ integration results in:

** $$b_i = \frac{w_i(o)}{r-s} = \frac{w_i(o)}{r'},$$

so that

$$\frac{db_i}{dc_i} = \frac{q(o)}{r'} = m.$$

q(o) being the initial rentability of the marginal investment ($dw_i(o) = q(o).d\ c_i$) and r' being the so called reduced rate of discount.

From this it may be concluded, that introduction of a reduced rate of discount solves the problem, as long as the annual benefits from each project have the same constant increase per year. If on the other hand the annual increase of benefits is not the same for each project, capitalization

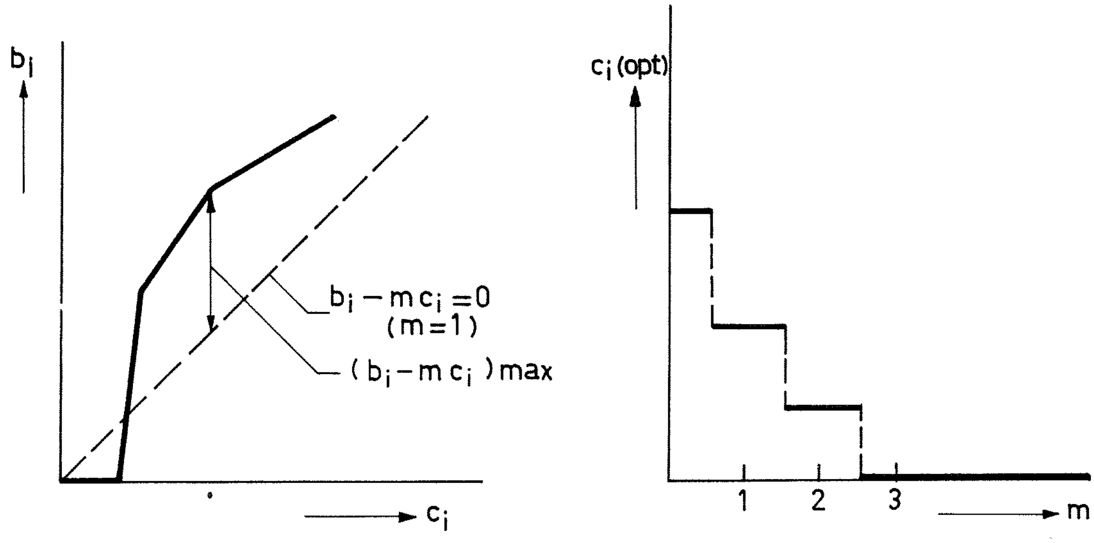

Fig. 6

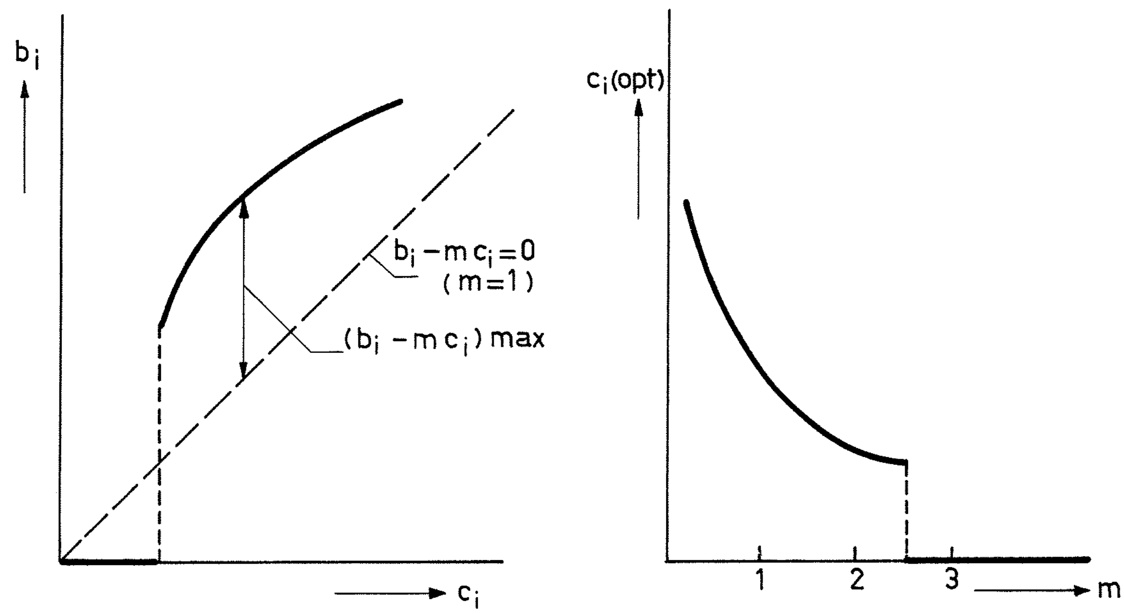

Fig. 7

Relation between c_i(opt) and m for two arbitrarily chosen functions $b_i(c_i)$.

still must be done with the reduced rate of discount, while at the same time the condition

$$\frac{db_i}{dc_i} = \frac{q(o)}{r'} = m$$

must be complied with.
In this case the results will indeed depend on the actual valuation of the rate of discount r.

The use of the "initial" rentability of the marginal investment q(o) may not be enlightning. In that case a mean value $\bar{q}$ might be introduced, which could be defined as follows:

$$\bar{q} = \frac{d(r.b_i)}{dc_i} .$$

The product $r.b_i$ in this formula is equivalent to the annual profit to be gained from the amount b_i, if the benefits of the investment c_i had been independent of time. Then:

$$m = \frac{\bar{q}}{r} = \frac{q(o)}{r'} .$$

In this case also, it is necessary to capitalize using the reduced rate of discount r', but the meaning of the multiplier m may be more comprehensive if $\bar{q}$ is introduced instead of q(o).

9. CRITICAL CONTEMPLATION OF SOME EXISTING OPTIMIZATION-METHODS.

As has been mentioned before most optimization methods available are not designed to optimize the general policy of investment of the financier. One of these methods aims at maximizing the ratio b_i/c_i. In that case:

$$d\left(\frac{b_i}{c_i}\right) = -\frac{b_i}{c_i^2}\,dc_i + \frac{1}{c_i}\,db_i = 0,$$

or:

$$\frac{db_i}{dc_i} = \frac{b_i}{c_i}\ (\max)$$

It would be unreasonable to assume, that b_i/c_i (max) would have the same value m for each project. Therefore we may conclude that, in general, maximizing b_i/c_i will not be in accordance with equation (14) and consequently the method will not be consistent with an optimal policy of investment.

The method described in the historical review (see fig. 1) means, in fact, the maximizing of $(b_i - c_i)$. However, from the foregoing it follows that $(b_i - m\,c_i)$ or $(b_i - \bar{q}/r\ c_i)$ should be maximized. Obviously maximizing $(b_i - c_i)$ includes the assumption $\bar{q} = r$ = long term rate of discount. As this is the lowest value of $\bar{q}$ which can be accounted for, maximizing $(b_i - c_i)$ seems acceptable as long as the method presented is not applied generally.

10. PRACTICAL APPLICATION.

From the foregoing it appears that the optimal value of m can be determined only by the central office of the financier, where the relation between benefits and costs of all relevant projects must be known. One might make the objection that it will not always be possible to determine this relationship. In this respect expenditures on defence are a well known example. However, the fact that some investments cannot be optimized does not imply that an optimal investment for the remaining projects should not be pursued.

In order to select the projects appropriate for optimal investment many problems must be solved. For instance when determining the optimal height of dikes, problems such as the estimation of the flood damage, the extrapolation of frequency curves and the valuation of human lives appeared to be solvable by rough approximation only. However, the final results obtained were such that the inevitable lack of accuracy is no longer considered to be a decisive obstacle for the financial optimization of civil engineering problems.

The usefulness of the method presented may be demonstrated by some results obtained for Dutch dikes. Using various values for m the optimal amounts of c_i, b_i, C and B have been calculated for a number of separate regions protected by dikes. The results of these computations have been given in table nr. 1.

Considering these results we may conclude firstly that the valuation of m in this example has only a minor influence on the extent of the investments to be made. Apparently this is a consequence of the relatively large damage expectation which for every reasonable value of m had already justified considerable investment in dikes. Therefore, it may be concluded that from an economical point of view the Dutch dikes were far to low. The actual values b_i/c_i in table nr. 1 confirm this conclusion. In all cases these values are much larger then the corresponding values of m (see conditions 14).

From a mutual comparison of the values of b_i/c_i within one column it may be furthermore concluded that the optimization technique presented in this paper also provides an important indication for the sequence of the various projects. Besides, the great differences between the values of b_i/c_i present an interesting problem as such. This problem will be studied thoroughly in the near future.

** Of course, financial optimization should not be confined to dike building or maritime structures only, but an example of a more general application, involving not only dikes, but also other expenditures such as roads, schools, health, etc. is not yet available. It is clear, that such an extension of the analysis presented, will be a difficult and time-consuming job, but the results obtained for the dikes already show how useful a more general application could be. At least one may expect, that every effort to extend the application will improve the understanding and sharpen the judgement of many decision makers.

$m = \frac{\bar{q}}{r}$ →	1			2			3		
region ↓	c_i	b_i	$\frac{b_i}{c_i}$	c_i	b_i	$\frac{b_i}{c_i}$	c_i	b_i	$\frac{b_i}{c_i}$
I	119	1678	14,1	106	1652,5	15,6	62	1536	24,8
II	65	4491	69,1	59	4483	76	57	4478	78,6
III	7	34,4	4,9	6,4	33,6	5,3	5,9	32,4	5,5
IV	87	798	9,2	84	793	9,4	83	790	9,5
V	46	539	11,7	43	535	12,4	41	528	12,9
VI	34,5	212,5	6,2	33	210	6,4	31,5	206,5	6,6
VII	51	245	4,8	47	238	5,1	44	231	5,3
VIII	46	246	5,3	44	242	5,5	42	238	5,7
	C	B	$\frac{B}{C}$	C	B	$\frac{B}{C}$	C	B	$\frac{B}{C}$
total	455,5	8243,9	18,1	422,4	8187,1	19,4	366,4	8039,9	21,9

Table I

Optimal amounts of b_i and c_i for a number of seperate regions protected by dikes (millions of Dutch guilders).

CONCLUSIONS.

1. Financially optimal investment can be achieved by maximizing the difference $(b_i - m\,c_i)$ per project.
   The only restriction is, that the benefits to be gained from each project are only a function of the investment in that project.

2. If the relation between the benefits and the costs are known for each project the factor m can easily be determined such that
   $$\sum_{i=1}^{i=n} c_i = C.$$
   A graphical solution for this problem is given in fig. 4 and fig. 5.

3. If the annual benefits do not change in time the results is independent of the rate of discount used in capitalization. If the annual benefits are dependent on time, the results may be influenced by the value of the rate of discount.

ACKNOWLEDGEMENT.

It will be clear to any reader, that the proverbial saying "every man to his own trade" has been disregarded in the above. Although this has undoubtably remained noticeable a serious attempt has been made to allow as much as possible for suggestions and criticism put forward by economists. Advisory remarks by Prof.dr. L.H. Klaassen and Drs. E.H. van de Poll contributed greatly to the approach presented in this paper whilst Prof.dr. W.J. van de Woestijne also provided the necessary directions for making the article acceptable to economists.

OPTIMIZATION OF FINANCIAL INVESTMENTS

LIST OF SYMBOLS

C	Total amount of money available for investment.
c_i	The amount of money invested in a certain project number i.
δc_i	A slight variation of c_i.
b_i	Cash value of profits given by c_i.
$b_i(c_i)$	b_i as a function of c_i.
δb_i	The variation of b_i resulting from the variation δc_i
B	Total cash value of profits given by C.
m	A factor - equal for each project - which occurs in the optimization process (e.g. multiplier).
w_i	The annual profit given by c_i.
r	Rate of discount.
s	Relative increase of the annual profit.
r'	Reduced rate of discount.
q	Rentability of the marginal investment in the case of annual profit independent of time.
q_o	Initial value of the rentability of the marginal investment.
$\bar{q}$	The average rentability - with respect to time - of the marginal investment.

DISCUSSION ON PAPER 11

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Papers like those of Bischoff van Heemskerck and Booy must certainly be very much welcomed and applauded because they show that economic thinking has penetrated into the guild of engineers. Nowadays, technical projects can no longer be considered independent from economics, and although it is realised that even the economic criteria are not under all circumstances the sole truth to be aimed at, at least they indicate what sacrifices are involved in a certain decision.

In this light the authors are to be thanked for their work to cross the border of technology and enter the transition zone between the technological and economic sciences, the area where both professions touch, viz, the question of: how far to go with the project. How high should a bridge be, how deep a navigation channel? How many berths are there to be built in a new port extension? and How long should your breakwater be to reduce the unworkable days in the port to an acceptable number? In the writer's experience as consultant, all these aspects clearly lend themselves for optimalization analyses, and that is what Bischoff van Heemskerck and Booy have done, on a macro-economic level so as to obtain the maximum benefit for the whole country.

The writer very much appreciates what they have said: too often are discussions made from a narrow viewpoint. But that not only refers to the decision of how far to go; in profitability calculations all kinds of community cost are (sometimes even willfully) forgotten so that the computed result hardly reflects the situation. A new factory for instance can cause quite a nuisance to neighbours through the created extra traffic congestion or through air pollution. Extremely difficult problems arise, sometimes leading to serious controversies between one part of the community and the other, and even between one authority and another. It often is a matter of evaluation and priority determination of the many elements that can not readily be expressed in quantitative terms. One group of people attaches great value to a very high income growth, another to a happy and healthy life. The authors mention, at the end of their paper, the choice in deciding whether to invest for the far future (in schools or in public health), for the near future (in securing new jobs to meet the population increase), or for the present (in housing, or in the production of consumer goods). A central agency that makes the decisions of where to invest will, to the writer's opinion, have to be more or less a totalitarion authority, overruling the feelings and opinions of the one group in favour of the desires of the other. It is hardly thinkable that such a proposition is realistic.

Moreover, many of the benefits or cost-elements, even the more readily quantifyable ones, are subject to uncertainty, based as they are upon forecasts and prognoses. For physical and natural phenomena, a statistic approach could indeed give at least the chance of occurrence which acts as a fairly reliable parameter in the calculations, but the matter becomes highly speculative as soon as human behaviour is involved, such as market response to a new product, or even population growth rates which often turn out to differ from expectations. This "risk" factor must in a large degree participate in the process of decision taking and may upset or even distort pure economic reasoning. It would seem that this is a reason for the application sometimes of the Pay-Off Period as an investment criterion, a criterion which has no clear relation with long term profitability.

For a number of investment decisions, however, a general macro-economic optimalization is indeed something that should be aimed at, and is to some extent possible, too, particularly in general facilities such as sea-defences and transportation infrastructure, where the Government acts as central agency already. If the "pressure groups" would cooperate, the method would certainly be effective, and the calculations of Bischoff van Heemskerck and Booy give a clear indication of the required criterion.

Fortunately, to some extent also the economic system of free enterprise tends towards a similar goal. Economic theory claims that capital will flow towards those projects where the profitability is highest, so that more or less automatically the marginal benefit cost ratio's $\frac{db}{dc}$ for all projects become equal. In his dealings with many such problems, the writer has learned that such profitability, represented by the opportunity cost of capital, seems to be statistically assessable with a fair accuracy.

In this light, it is clear that financiers are not content with a $\frac{db}{dc} = 1$; they demand a higher value. They may perhaps stop already at $\frac{db}{dc} = 2$, and seek other, better investment possibilities elsewhere.

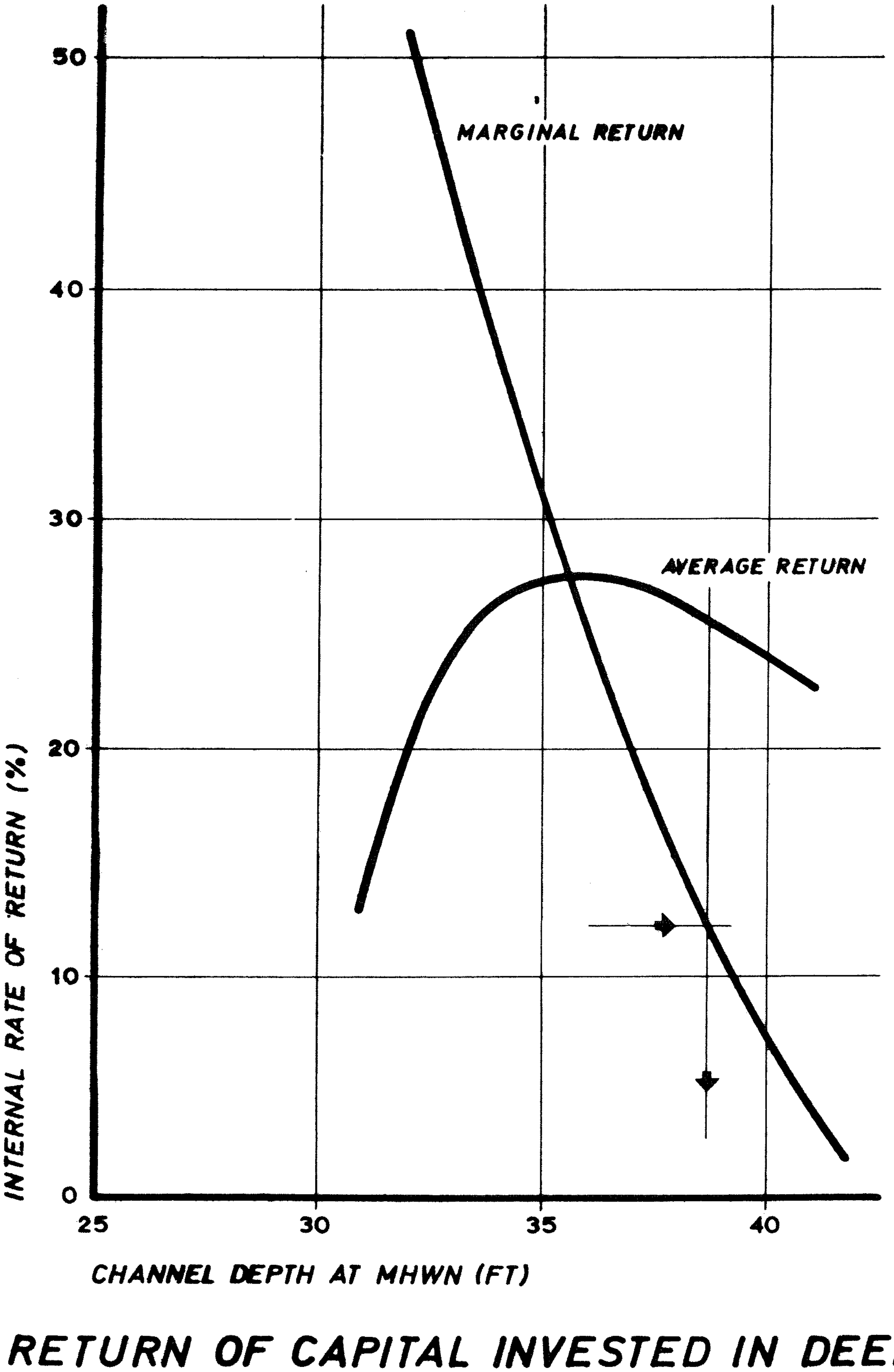
This, of course, implies that it is not always required that a central agency makes all the decisions: the conomic order has provisions to attain the same desired goal.

There are, of course, many factors that create a deviation from this ideal situation. An important factor is for instance the immobility of capital, such as the tendency to self-financing in large enterprises. But for large public projects, the writer believes that the authors are correct in stating that the criterion of investment should be, macro-economically, a higher return than the market interest.

Before a project is carried out, an economic analysis should therefore be made, whereby the return is to be assessed for various stages of investment, marginally as well as average, and the final decision should be taken on the basis of both criteria, expressed in formula (14) of Bischoff van Heemskerck and Booy. The writer gladly produces the attached Figure to serve as an illustration in this respect. In this graph which is taken from one of NEDECO's transportation studies, the economic return is plotted for a variable depth in an access channel towards a certain ocean port. In this case, the return is expressed as the "internal rate of return", but this does not essentially differ from the b/c method. From the graph it can be concluded that, for an opportunity cost of capital of, say, 12%, the optimum channel-depth is $38\frac{1}{2}$ ft, because at that depth is

the marginal return = 12 %, and
the average return = $25\frac{1}{2}\% > 12\%$.

** Analyses like this one are nowadays very usual, and Bischoff van Heemskerck and Booy may rest assured that international institutions such as the Worldbank are prepared to lend money only after they are satisfied that the profitability complies with the criterion of opportunity cost, not of the simple market interest.



RETURN OF CAPITAL INVESTED IN DEEPENING AN ACCESS CHANNEL

COMPLEX WAVE ACTION ON SUBMERGED BODIES

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SUMMARY

This paper describes a method of calculating exciting forces on free or fixed bodies in waves and its application to some examples. The method lays upon an accurate computation of the transitory pressures applied by a potential flow on a submerged body.

The calculation may be used for the case of a free body in complex waves.

First, we give the calculation hypothesis on flow conditions and the formulae which proceed from assumptions.

Second, we give the computation results on well known examples.

- On set parallel flow around a sphere,
- Fixed triaxial ellipsoïd in waves.

Third, we apply this method to a free caisson, steadied by schematic mooring device in stationnal waves computed with second orders term.

1. INTRODUCTION

Sogreah has investigated a wide variety of hydraulic problems during its lifetime, among which especially the effects of waves on marine structures and both immersed and non-immersed floating bodies.

With the advent of computers, digital computation proved itself a valuable adjunct to scale model research, and mathematical models are now being used instead of physical ones for certain applications.

Sogreah has developed a method for the calculation of flow around an immersed body and has used it to determine wave forces acting on a floating platform caisson.

The immersed body flow computation method described here is a conventional one assuming potential flow which can be represented by a single-layer potential. The original feature of the Sogreah investigation, however, is that it more specifically considered transient-state pressures and forces with a view to determining the behaviour of an immersed body under complex wave conditions. This method gives the response of a body immersed at a given depth to waves that are chromatic as regards height and phase, and it can be confirmed in this case by comparison with scale model tests. It is particularly useful as a means of studying the behaviour of an immersed body in complex wave conditions, and it scores over the physical model in that it enables any complex waves given by their spectrum to be investigated for either finite or infinite depth assumptions.

This note gives the confirmation of the method for simple bodies (sphere, ellipsoid) and describes its application to the motion of a single caisson suitably anchored for stability and exposed to Atlantic swell conditions.

2. DESCRIPTION OF THE METHOD - THEORETICAL STUDY

2.1 Remark

As the calculation method used in this study is quite conventional, the mathematical formulation of the problem will be discussed very briefly and only the basic formulae required to understand the method will be mentioned. We have applied this method to the special case of the determination of wave forces acting on an immersed body, but it is also suitable for other two - or three - dimensional flow problems.

2.2 Physical assumptions

We have supposed that the viscosity forces are low and do not perturb the flow around the body, and that the speeds and pressures on the body surface are identical both in ideal fluid and in viscous fluid.

This assumptions involves that the body motions are slow and of the same order of magnitude as the water motions. We will not take into account the wake effects which can occur in certain places of the body.

On the other hand, it is possible to take in account the drag and lift effects either as a whole, or with the aid of a shear term in each point of the body, this term being a function of the relative water body speed.

Then again, we have not taken in account the influence - on the flow characteristics - of the free surface distortion owing to the presence of ** the body.

This limitation leads us to the following point. This method is merely valid when the immersed body stands at a depht more than about twice the body height.

2.3 Type of flow

The water is considered as an incompressible fluid in irrotational motion, so that the flow is derived from a potential Φ which is the solution of the Laplace equation :

$$\Delta \Phi = 0 \qquad (2.1)$$

The fluid velocity at any point is :

$$\overline{V}_F = - \overline{\text{grad}\ \Phi} \qquad (2.2)$$

Without a body, the flow is simply the motion of the water, and the above assumptions require that we consider a wave scheme of potential Φ_H

Determination of the flow is then a matter of solving an exterior Neumann problem, i.e. the determination of a harmonic function Φ which is regular at infinity, knowing the normal derivative $d\Phi/dn$ on the body surface Σ .

2.4 Determination of potential Φ

The given condition $d\Phi/dn$ at point M on surface Σ is met when the normal velocity components for point M associated with the body and the fluid velocity at that point are equal, i.e. :

$$\frac{d\Phi}{dn} = - (\overline{V}_c + \overline{\Omega}_{\wedge}\overline{CM})\ \overline{n}\ (M) \qquad (2.3)$$

As the potential satisfies the Laplace equation we can apply the principle of superimposed flows and break down the overall potential Φ into the three following elementary potentials :

- Φ_H giving the flow of water without the body,
- Φ_{PH} giving the flow of water around the body, which is assumed to be stationary under the influence of Φ_H ,
- Φ_{PC} giving the flow of water due to the motion of the body in calm water.

hence :

$$\Phi = \Phi_H + \Phi_{PH} + \Phi_{PC} \qquad (2.4)$$

Potentials Φ_{PH} and Φ_{PC} are expressed conventionally by a single-layer potential of respective densities σ_{PH} and σ_{PC} , i.e. :

** $$\Phi_{PH}(P) = \iint_{\Sigma} \frac{\sigma_{PH}(M)}{|MP|} \, ds\,(M) \qquad (2.5)$$

** $$\Phi_{PC}(P) = \iint \frac{\sigma_{PC}(M)}{|MP|} \, ds\,(M) \qquad (2.6)$$

The source densities σ_{PH} and σ_{PC} are solutions of the Fredholm equation with the given condition $d\Phi/dn$:

$$2\pi\, \sigma_{PC}(P) + \iint_{\Sigma} \sigma_{PC}(M) \frac{\overline{MP}.\overline{n}(P)}{|MP|^3} \, ds\,(M) = [\overline{V}_c + \overline{\Omega}_{\Lambda}\, \overline{CM}]\, \overline{n}\,(P) \qquad (2.7)$$

$$2\pi\, \sigma_{PH}(P) + \iint_{\Sigma} \sigma_{PH}(M) \frac{\overline{MP}.\overline{n}(P)}{|MP|^3} \, ds\,(M) = -\, \overline{V}_H(P)\, \overline{n}\,(P) \qquad (2.8)$$

Potential Φ_{PC} only depends on the velocity of the body and can be expressed as a function of unit potentials φ_1 , φ_2 , φ_3 , χ_1 , χ_2 , χ_3 (ref 1)

$$\Phi_{PC} = u\varphi_1 + v\varphi_2 + w\varphi_3 + p\chi_1 + q\chi_2 + r\chi_3 \qquad (2.9)$$

Potentials φ_1 , φ_2 , φ_3 , χ_1 , χ_2 , χ_3 are calculated once and for all for the body surface area Σ . Potential Φ_{PH} is calculated for any moment of time in terms of the position of the body and wave conditions.

2.5 Determination of wave forces acting on the body

Knowing the overall flow potential the water pressure point - especially on the body surface - can be calculated by the following formula :

** $$P = Po + \rho \left(gz + \frac{\partial\Phi}{\partial t} - \frac{V_F^2}{2} \right) \qquad (2.10)$$

Integrating pressure over the surface area Σ gives the wave force and moment resultants on the body at any instant of time, i.e. :

$$\left.\begin{aligned} \overline{F}_H &= -\iint_\Sigma P(M)\,\overline{n}\,(M)\,ds\,(M) \\ \overline{M}_H &= -\iint_\Sigma P(M)\,[\overline{CM}_\Lambda\,\overline{n}\,(M)]\,ds\,(M) \end{aligned}\right\} \qquad (2.11)$$

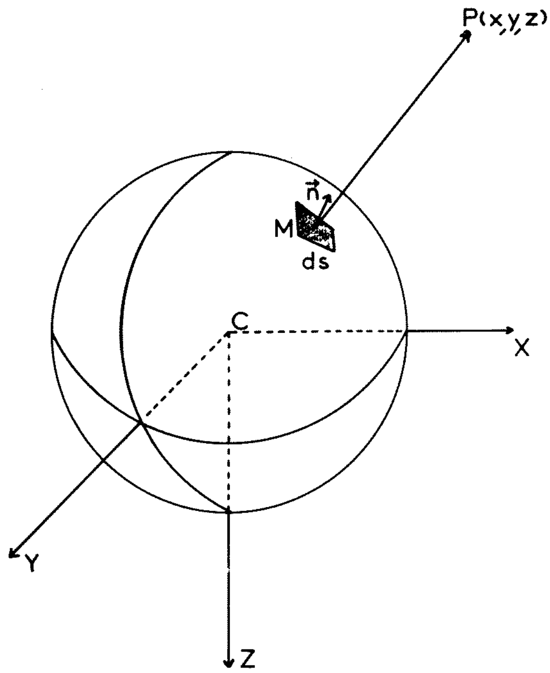

Figure 1 - The body surface. Notation used in describing the potential due to a surface source density distribution.

The forces are calculated along the body axes. Integration of the pressure term ρgz gives the buoyancy force and will not be carried out.

The pressure term $\partial\Phi/\partial t$ gives the forces at wave period and takes the motion of the immersed body into account. These forces are much greater than those due to the pressure term $V_F{}^2/2$ representing the surface attraction effect. In certain cases where only body motion at the wave period is considered the velocity term can be considered negligible compared to the pressure term $\partial\Phi/\partial t$.

Note : The pressure term $\partial\Phi/\partial t$ is the sum of derivatives :

$$\frac{\partial\Phi_H}{\partial t}, \quad \frac{\partial\Phi_{PH}}{\partial t} \quad \text{and} \quad \frac{\partial\Phi_{PC}}{\partial t},$$

the last of which is as follows :

$$\frac{\partial\Phi_{PC}}{\partial t} = u'\varphi_1 + v'\varphi_2 + w'\varphi_3 + p'\chi_1 + q'\chi_2 + r'\chi_3 \tag{2.12}$$

By integrating this term over surface area Σ the twenty-one added mass coefficients (ref 2) can be calculated, which are of the following form :

$$A = \iint_\Sigma \varphi_1 \quad \alpha \quad ds \tag{2.13}$$

If the added mass coefficients of the body are known, the calculation method used can be checked.

3. CALCULATION METHOD

The basic problem involved in determining the flow around an immersed body is to solve the Fredholm (2.7) and (2.8) equation, which generally defies analytical solution. Its digital solution method is conventional and consists in replacing the Fredholm integral by a linear system of n equations with n unknowns.

This system is obtained by replacing the continuous functions defined on the body surface Σ by their values at n points on the surface and by calculating the integrals by summation over the n considered points.

3.1 Discretisation - Approximate body definition

In order to solve the Fredholm equation (2.7) and(2.8) we divide the body surface area into n surface elements ("facets") (fig. 1) defined by the following :

(i) Facet area δs_ι

(ii) The vector normal to the facet $\overline{n}_\iota$.

The facet centre M_ι .

The ι-th facet is determined by its circumference Γ_ι ; the components of vector $\overline{n}_\iota$ and the facet area are then given by the following integral :

$$n_\iota \, \delta s_\iota = \iint_{\delta s_\iota} \overline{n} \, (M) \, ds \, (M) = \int_{\Gamma_\iota} \overline{CM}_\Lambda \, \overline{d\ell} \qquad (3.1)$$

The centre of the i-th facet (M_ι) has been assumed to coincide with the centre of gravity of the projection of the facet on a plane perpendicular to the mean normal given by formula (3.1), i.e. :

$$\overline{CM}_{\iota\Lambda} \, \overline{n}_\iota \, \delta s_\iota = \iint_{\delta s_\iota} \overline{CM}_\Lambda \, \overline{n} \, ds = -\frac{1}{2} \int_{\Gamma_\iota} |\overline{CM}|^2 \, \overline{d\ell} \qquad (3.2)$$

The integrals we have to use are of the following type :

$$\iint_\Sigma f \, (M) \, ds \, (M) \quad \text{or} \quad \iint_\Sigma f \, (M,P) \, ds \, (M) \qquad (3.3)$$

We shall calculate these integrals by summing over all n body facets, i.e. :

$$\left.\begin{aligned} \iint_\Sigma f \, (M) \, ds \, (M) &= \sum_{\iota=1}^{n} f \, (M_\iota) \, ds_\iota \\ \iint_\Sigma f \, (M,P) \, ds \, (M) &= \sum_{\iota=1}^{n} f \, (M_\iota, M_j) \, ds_\iota \end{aligned}\right\} \qquad (3.4)$$

3.2 Determination of the flow around the body

The characteristic flow quantities (source density, potential, velocity and pressure) are calculated at the centre of the n facets defining the body.

The Fredholm integral is calculated by summation and the source density at the centre of the facets is determined by solving the following linear system :

$$\sigma_\iota \, \delta s_\iota + \sum_{j=1}^{n} K_{\iota j} \, \sigma_j \, \delta s_j = \frac{1}{2\pi} \left[\overline{V}_c + \overline{\Omega}_\Lambda \, \overline{CM}_\iota - \overline{V}_{H\iota} \right] \overline{n}_\iota \, \delta s_\iota \qquad (3.5)$$

where $K_{\iota j}$ is the general term of a square matrix of rank n . This term solely depends on the body characteristics and is calculated in terms of the quantities defining each facet, i.e. :

$$K_{\iota j} = \frac{1}{2\pi} \frac{\overline{M_\iota M_j}}{|M_\iota M_j|^3} \, \overline{n}_\iota \, \delta s_\iota \qquad (3.6)$$

The linear equation system was solved by inverting the matrix $A = I + K$.

The quantity :

$$Q_\iota = \frac{1}{2\pi} \left[\overline{V}_c + \overline{\Omega}_\Lambda \, \overline{CM}_\iota - \overline{V}_{H\iota} \right] \overline{n}_\iota \, \delta s_\iota$$

can be accurately calculated for any instant of time and any point on the body.

Knowing the inverted matrix A^{-1} it is easy to find $\sigma_\iota \, \delta s_\iota$, and the values for σ_ι provide a practical means of determining the flow around the body, for the potential and velocities can be calculated from the source densities by simple summation over the body surface Σ , from which the pressure at each point on the body are then found.

3.3 Representation of waves

In order to determine the effect of waves on the body, we must introduce a wave scheme ensuring adequate representation of the motion of the water at any instant of time and at any point. A wave scheme of potential Φ_H was adopted for the purpose, in which complex waves are represented by a certain number of elementary waves whose heights and pulsations were selected to ensure adequate representation of the complex wave spectrum. By this method, given recorded waves can be reproduced.

The wave potential is given by the following formula :

$$\Phi_H = \sum_{\iota=1}^{I} \frac{A_\iota \omega_\iota}{K_\iota} e^{-K_\iota z} \sin(\omega_\iota t - K_\iota x + \psi_\iota)$$

$$- \sum_{\iota=1}^{I} \sum_{j=1}^{\iota-1} A_\iota A_j \omega_\iota e^{-(K_\iota - K_j)z} \sin[(\omega_\iota - \omega_j)t - (K_\iota - K_j)x + \psi_\iota - \psi_j] \qquad (3.7)$$

where I is the wave component number.

The (3.7) formula alows the explicit computation of the datas required for the Φ_{PH} potential computation.

The wave spectrum is parted in ten equal energy band. This sharing gives a satisfactory repruduction of the statistical properties of waves.

4. REMARKS ON THE COMPUTATION PROGRAMME

4.1 General considerations

- With the computation programme used to determine the effects of waves on an immersed caisson all the intermediate quantities required to calculated the forces can also be determined, i.e. source density, potential, water velocity and pressure. Our purpose in using this programme was to follow the various computation phases and to establish the degree of accuracy of the method by comparisons considering cases known by analytical calculation.

On the other hand, we intended to show how a method of this type can be used for very varied applications both for the investigation of transient wave effects as considered here and for the determination of water velocities at a given point of a fixed body immersed in a known flow.

We would like to draw attention to the following remarks regarding the application of the computation method in this paper to the case of an immersed body under wave condition :

Forces $\overline{F}_\varphi$ due to the $d\Phi/dt$ pressure term are linear functions of the Q_ι values.

The intermediate summations can be done once and for all and calculation of these forces boils down to the following summation :

$$\overline{F}_\varphi = \Sigma_{\iota=1}^{n} \overline{C}_\iota Q_\iota \tag{4.1}$$

Forces F_V due to $\frac{V_F^2}{2}$ pressure terms are not linear functions of the Q_ι values, however, which makes it necessary to also calculate the velocities in between.

$$\overline{V}_{Fj} = \sum_{\iota=1}^{n} \overline{B_{\iota j}} Q_\iota \tag{4.2}$$

The calculation of $\overline{F_V}$, therefore, will take about n times longer than for $\overline{F}_\varphi$.

Where $\overline{F_V}$ is negligible compared to $\overline{F}_\varphi$ and especially where second-order wave effects are not to be considered, the computation time cane be reduced considerably by neglecting F_V and considering a first-order wave formulation.

4.2 Features of the programme

The immersed body wave force computation programme was written in Fortran IV and is being used with IBM 360-65 equipment. All the computations are done with central storage and we have limited the number of facets (surface elements) defining the body to 190. A body with a place of symmetry can be divided up into 270 facets.

The mathematical model comprises three main programmes in the following sequence :

(i) The body characteristics computation programme, which calculates the matrix of the Fredholm equation K and gives the inverted form A^{-1} of the corresponding matrix $A = I + K$.

(ii) The programme for computing tables B and C from A^{-1} , which enables the velocities and forces to be calculated by formulas (4.1) and (4.2).

(iii) The programme to compute the wave forces and immersed body motion for various wave characteristics from tables B and C .

The computation times given below for these three phases are only a rough indication. For a body divided into 60 facets, these times are as follows :

- Computation and inversion of matrix A ...... 1 minute.
- Computation of tables B and C .... 3 minutes.
- Time to compute forces $\overline{F}_\varphi$ and to determine body motion, for a first-order approximation of complex waves represented by eight rays .... 0.02 second per time step
- Time to compute $\overline{F}_V$ and $\overline{F}_\varphi$ for a second-order approximation of complex waves represented by eight rays .. 0.5 second per time step, (i.e. very much longer than above).

5. COMPARISON BETWEEN COMPUTED AND ANALYTICAL DATA

In order to establish the accuracy of the method described in the previous section, we applied it to simple bodies for which some of the calculations can be done analytically.

In the comparison with analytical solutions, the flow itself (i.e. source density, potential, velocities, added mass coefficients) and wave effects on the body (heaving, rolling, pitching and yawing force coefficients were considered.

As a general rule we chose a number of facets giving and accuracy of one to two per cent for the calculated values, which we considered to be adequate for the wave calculations.

As the wave characteristics are approximate, it did not seem necessary to require more accurate computations. This enables us to achieve very short computation times, and so to represent the history of the studied phenomena during a time sufficiently long to reproduce their random aspects.

5.1 Study of a sphere

As flow around a sphere is a very well-known subject, this seemed a reasonable choice for the initial comparisons.

The sphere was divided up as shown in Fig. 2 , and though this is not the best method of subdivision, it is the one generally used for long bodies. In the considered example, the sphere was divided into 162 facets bounded by meridians and parallels of latitude every 20 degrees.

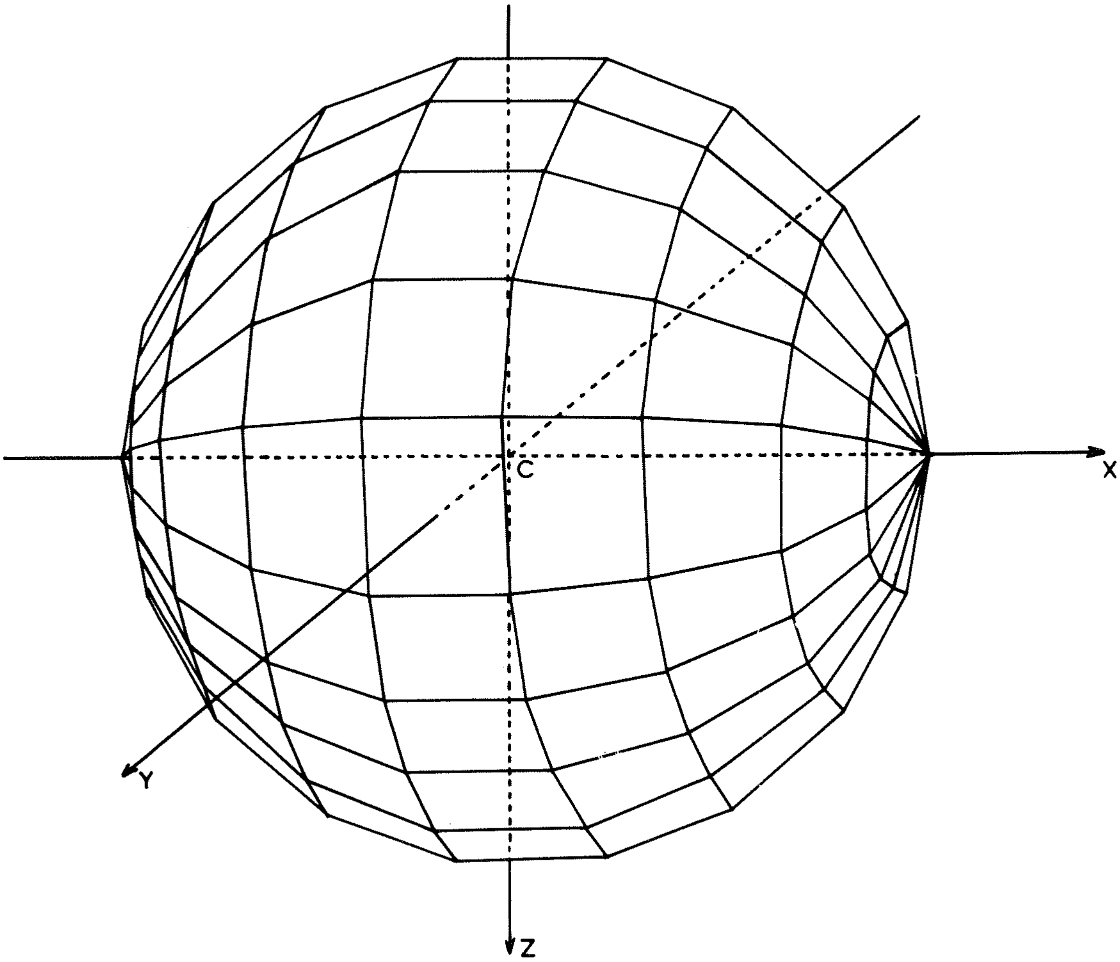

Figure 2 - The approximate representation of the sphere.

5.1.1 Sphere in uniform steady flow

Two uniform flows of unit velocity are considered, one along Cy and the other along Cz . We know the theoretical flow around the sphere in this case, and comparing this with the analytical solution in Fig. 3 we observe the following :

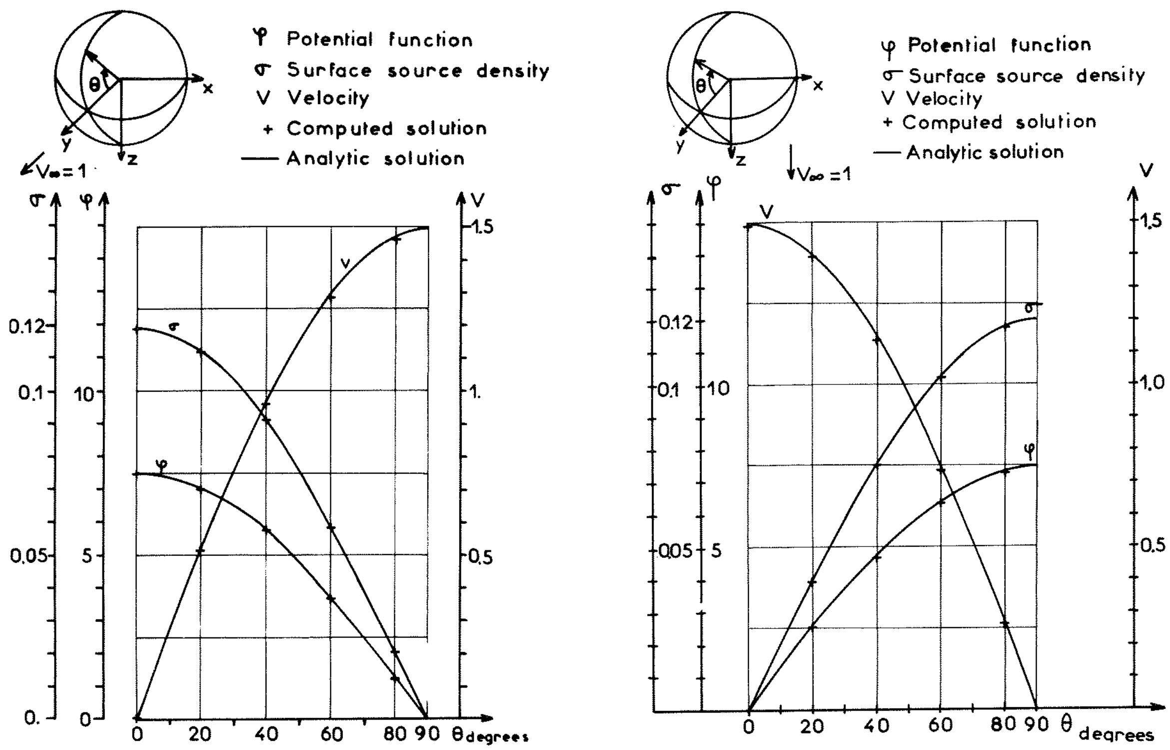

Figure 3 - Comparison of analytic and calculated values on a sphere for an onset uniform flow.

Source density and potential computation accuracy is satisfactory, there being less than 1 per cent error between computed and theoretical data throughout. The computed velocities are less satisfactory, however, as they differ from the theoretical values by as much as 5 per cent at certain points.

The difficulty of obtaining accurate velocity data is due to the $[M_i M_j]^3$ term in the denominator of the velocity computation formula.

We have not attempted to improve the velocity computation method yet as the corresponding force term is nearly always small enough to be neglected with respect to the forces at wave period.

Velocity computation accuracy can be improved either by increasing the number of facets or by improving the velocity integration formula by extrapolating the source densities. We intend to try out this second method for future problems as it does not result in an excessive increase in computation time.

5.1.2 Sphere in unsteady flow Added mass coefficients

The inertia tensor for the water set in motion by the body is symmetrical and defined by twenty-one coefficients (ref. 2).

For a body with three planes of symmetry this tensor becomes the main diagonal, and in the case of a sphere it is as follows :

$$\left.\begin{aligned} A &= B = C = \frac{\rho V}{2} \\ P &= Q = R = 0 \end{aligned}\right\} \qquad (5.1)$$

The theoretical and computed date compare well for a sphere with a radius of 5 metres, with differences alvays less than 2 per cent, as follows :

Quantity	Theoretical value	Computed value
$A = K_x \rho V$	$261.8 = 0,5\ \rho V$	$257.4 = 0,4916\ \rho V$
$B = K_y \rho V$	$261.8 = 0,5\ \rho V$	$256.8 = 0,4904\ \rho V$
$C = K_z \rho V$	$261.8 = 0,5\ \rho V$	$256.6 = 0,4901\ \rho V$

5.2 Tri-axial ellipsoid study

5.2.1 Some ellipsoid characteristics can be obtained by analytical methods.

Lamb (ref. 3) gives the values of Green's integrals which allow the computation of A, B, C, P, Q, R .

On the other hand, in the case of a tri-axial ellipsoid, Newman (Ref. 4) gives calculation formulae for the pressure term $\partial\Phi/\partial t$ forces, produced by monochromatic waves.

The comparison between our mathematical model and the analytical results is done for an ellipsoid determined by 120 facets.

5.2.2 Added mass coefficients

All the terms but those of the main diagonal of the inertia tensor are equal to zero. We have put the theoretical data computed from Lamb's formulae and the mathematical model data in a table (Fig. 4).

	ANALYTIC DATA	COMPUTED DATA
A	90 T = 0.0192 $V \times \rho$	88 T = 0.0188 $V \times \rho$
B	4880 T = 1.0443 $V \times \rho$	4970 T = 1.052 $V \times \rho$
C	4150 T = 0.8881 $V \times \rho$	4 270 T = 0.914 $V \times \rho$
P	275 T × m^2	264 T × m^2
Q	1 935 000 T × m^2	1991 000 T × m^2
R	2 281 000 T × m^2	2 344 000 T × m^2

Ellipsoid: $a_1 = 50{,}00$ m, $a_2 = 4{,}50$ m, $a_3 = 5{,}00$ m; $\rho = 1$, $V = 4710\ m^3$

Figure 4 - Added mass coefficients for tri-axial ellipsoid.

5.2.3 Waves forces on an ellipsoid - Exciting force coefficients

Newman gives theoretical formulae for the heaving, rolling, pitching, yawing coefficients (Cz, Cxx, Cyy, Czz) in the case of a triaxial ellipsoid. These formulae are more general than those of Havelock (Ref. 5) in which only a spheroid is taken in account.

In the latter case, Newman assumes that the body is in a fixed position. We have taken the same assumptions, i.e. for the ellipsoid computation :

- The major axis parallel to the wave direction with various wave period (6, 7, 8, 9, 10, 12 sec.) ;
- The major axis at 30, 60 and 90 degrees to the wave direction and with two wave periods (8 and 10 sec.) ;

The mathematical model gives the exciting forces on the ellipsoid in waves, from which we find the coefficients Cz, Cxx, Cyy and Czz using the following formulae :

$$\left.\begin{aligned} Fz &= \rho g\ VAK\ e^{-Kz}\ Cz \cos \omega t \\ Mx &= \rho g\ L\ VAK\ e^{-Kz}\ Cxx \sin \omega t \\ My &= -\rho g\ L\ VAK\ e^{-Kz}\ Cyy \sin \omega t \\ Mz &= -\rho g\ L\ VAK\ e^{-Kz}\ Czz \cos \omega t \end{aligned}\right\} \quad (5.3)$$

It is known that for λ/L tending to infinity the limit of Cz is :

$$1 + K_z = (\rho V + C)/\rho V$$

and the pitching coefficient Cyy decreases and tends to zero. Fig. 5 shows this very clearly.

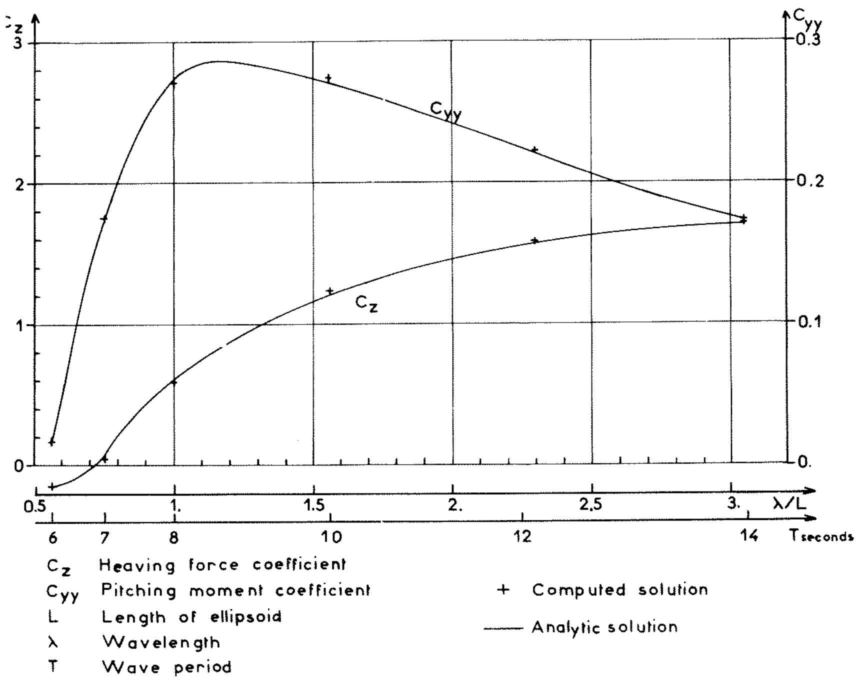

Figure 5 - Heaving force and pitching moment coefficients for varying λ/L

For the same reasons, the heaving force acting on an elongated ellipsoid broadside on to the waves is independent of wave period, so that the value of coefficient Cz is :

$$1 + K_z = 1 + \frac{C}{\rho V}$$

This property shows up well in the computations.

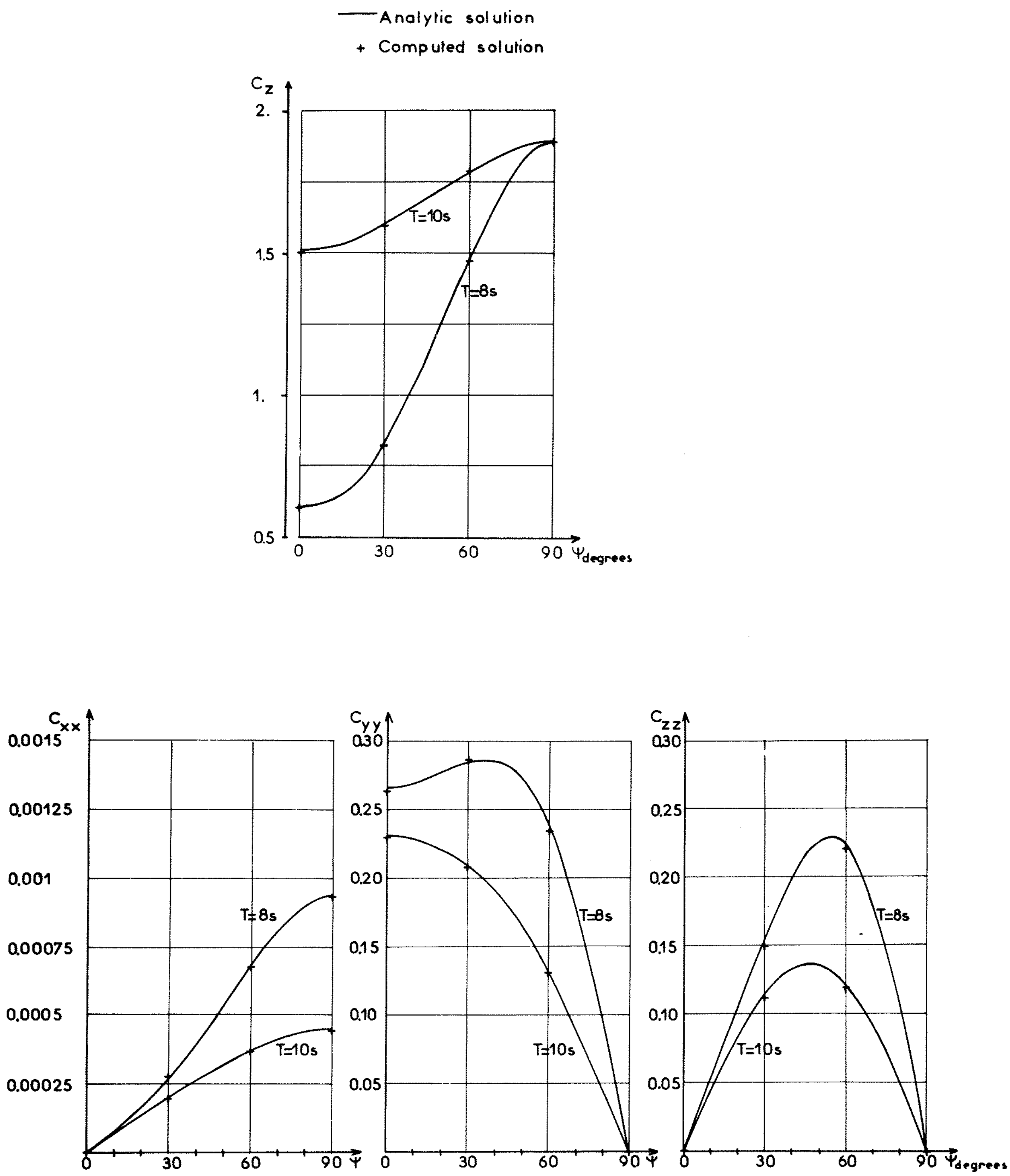

Figure 6 - Heaving force and rolling, pitching and yawing moment coefficients for varying directions.

The results are shown in figures 5 and 6. They show quite close agreement, the computation error being less than 3 per cent.

From the satisfactory agreement between of the various results and the theoretical data it can be concluded that our computation method is adequate for the $\partial\Phi/\partial t$ pressure term.

6. THE EFFECT OF WAVES ON AN IMMERSED CAISSON

6.1 The effect of waves on an immersed caisson can be considered from two aspects, as follows :

(i) An aspect associated with forces of the first order, which are periodic, have the same period as the waves and are proportional to wave height. These forces are of considerable magnitude and give rise to movements which cannot be eliminated by any form of anchoring or other stabilisation method. The corresponding movements are usually periodic, with the body oscillating about a mean position. The sole purpose of anchorings is to correct deviations from this mean position, but considerable deviations may nevertheless occur, even with a taut hawser, to the point of causing it to break. It is important to know whether such situations are likely to arise and to have a very sound statistical knowledge of these movements.

(ii) An aspect associated with second-order forces, by which we mean any forces that are non-periodic or with a period in excess of 30 seconds. The force of attraction on the surface, effects due to second-order terms in wave representation and various force and motion coupling cases are considered to come under this heading.

We have now seen the various aspects of wave action on a submerged body of any shape : the method we have described is equally suitable for the determination of forces of the first and second orders and provides a very thorough means of investigating wave action on an immersed body.

For the caisson discussed in this paper we have considered first-order forces and more specifically the motion of a free caisson under complex wave action.

The computation method can also be used to calculate forces on a caisson in forced motion, which is the case if the caisson submerged, is part of a complex structure such as a drilling platform.

6.2 Caisson characteristics

The outlines of the considered caisson are shown in Fig. 7.

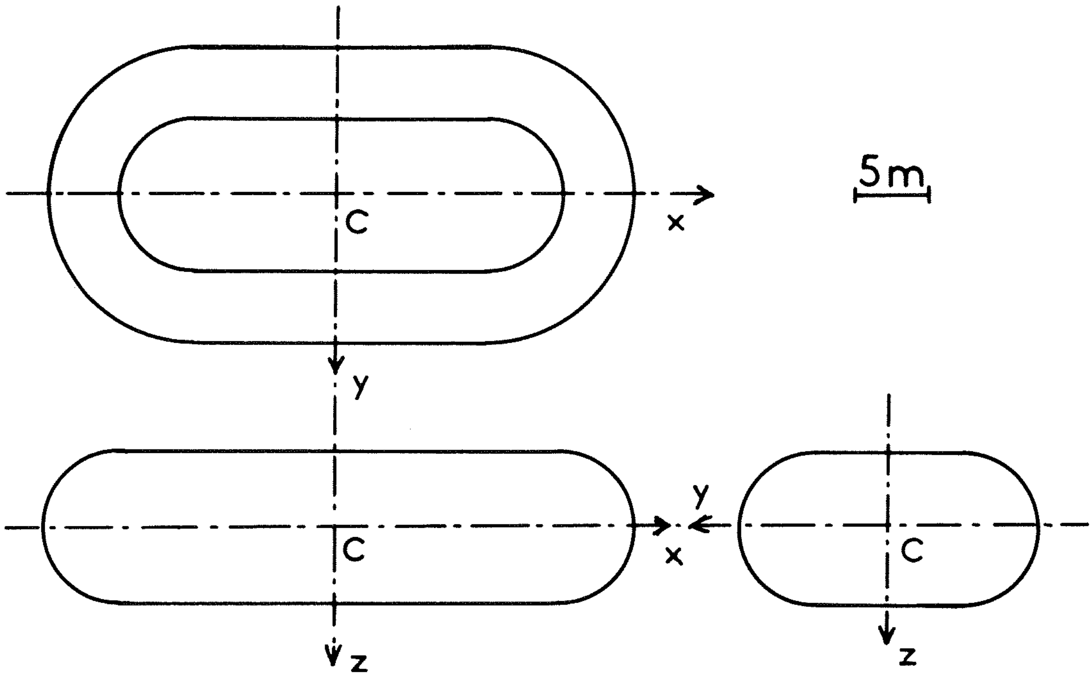

Figure 7 - Caisson outline

Its characteristic dimensions are as follows :

- Length : 40 metres
- Breadth : 20 metres
- Height : 10 metres
- Volume : 6110 cubic metres.

6.3 Added mass coefficients

As the considered caisson has three planes of symmetry only the coefficients of the main diagonal are not zero. From the results obtained for the sphere and ellipsoid it can be estimated that the values are accurate to within 3 per cent (Fig. 8)..

CAISSON

$V = 6110\ m^3$

$\rho = 1$

	COMPUTED DATA
A	9 3 1 T = 0.152 $V \times \rho$
B	2 4 0 0 T = 0.394 $V \times \rho$
C	7 4 8 0 T = 1.225 $V \times \rho$
P	47 100 $T \times m^2$
Q	414 900 $T \times m^2$
R	1 02 600 $T \times m^2$

Figure 8 - Added mass coefficients for caisson.

6.4 Computation of Cz , Cxx , Cyy and Czz - Computation of forces

Fig. 9 shows the force of attraction toward the surface and Fig. 10 and 11 the amplitudes of the first-order forces on the caisson due to 2 m waves (crest to through height). It will be noted that the attraction force is invariably less than 50 sthenes, which is negligible compared to the first-order forces.

The attraction force is due to the difference between flow velocities over the top and bottom caisson surfaces and is proportional to the difference between the squares of these velocities. It remains constant during a wave period ; its magnitude is proportional to wave height and varies with depth of submersion according to a e^{-2K_z} law. In calculating the forces the caisson is assumed to be held stationary at a depth of 15 metres below the surface.

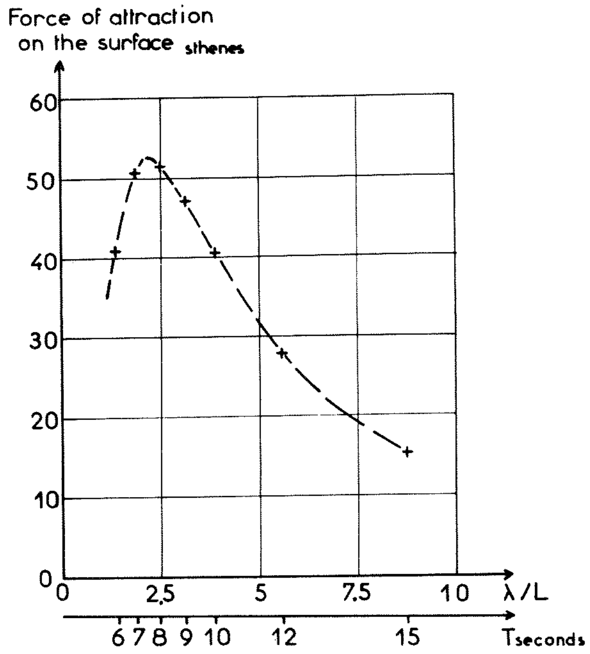

Figure 9 - Force of attraction toward the surface for varying λ/L

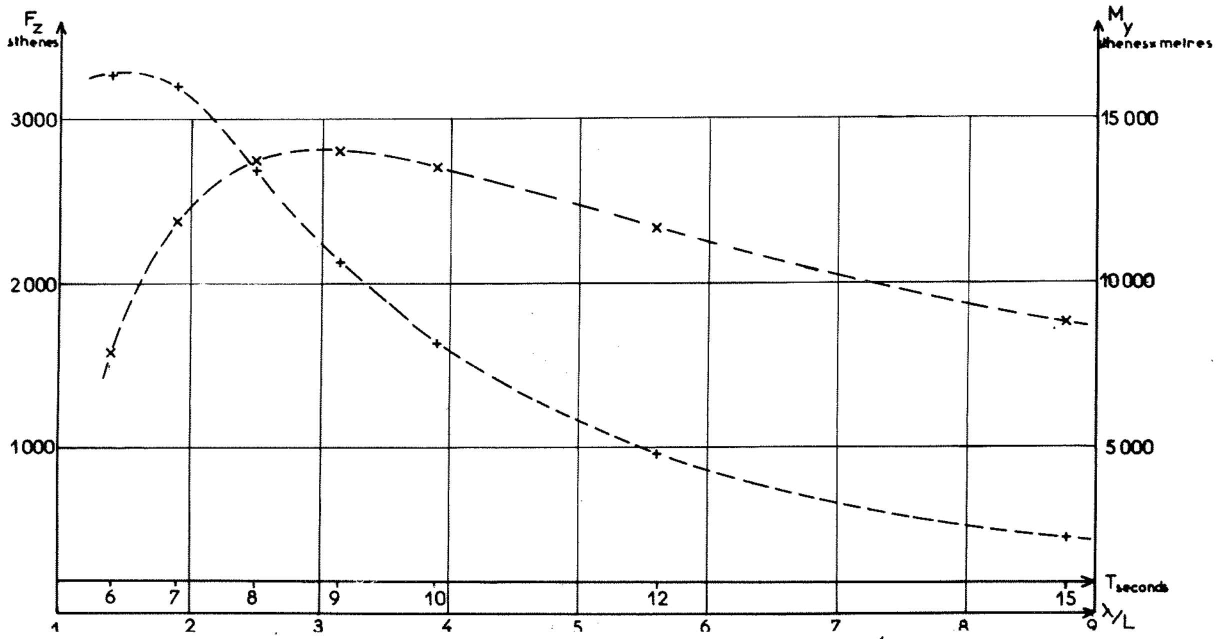

Figure 10 - Heaving force and pitching moment for varying λ/L

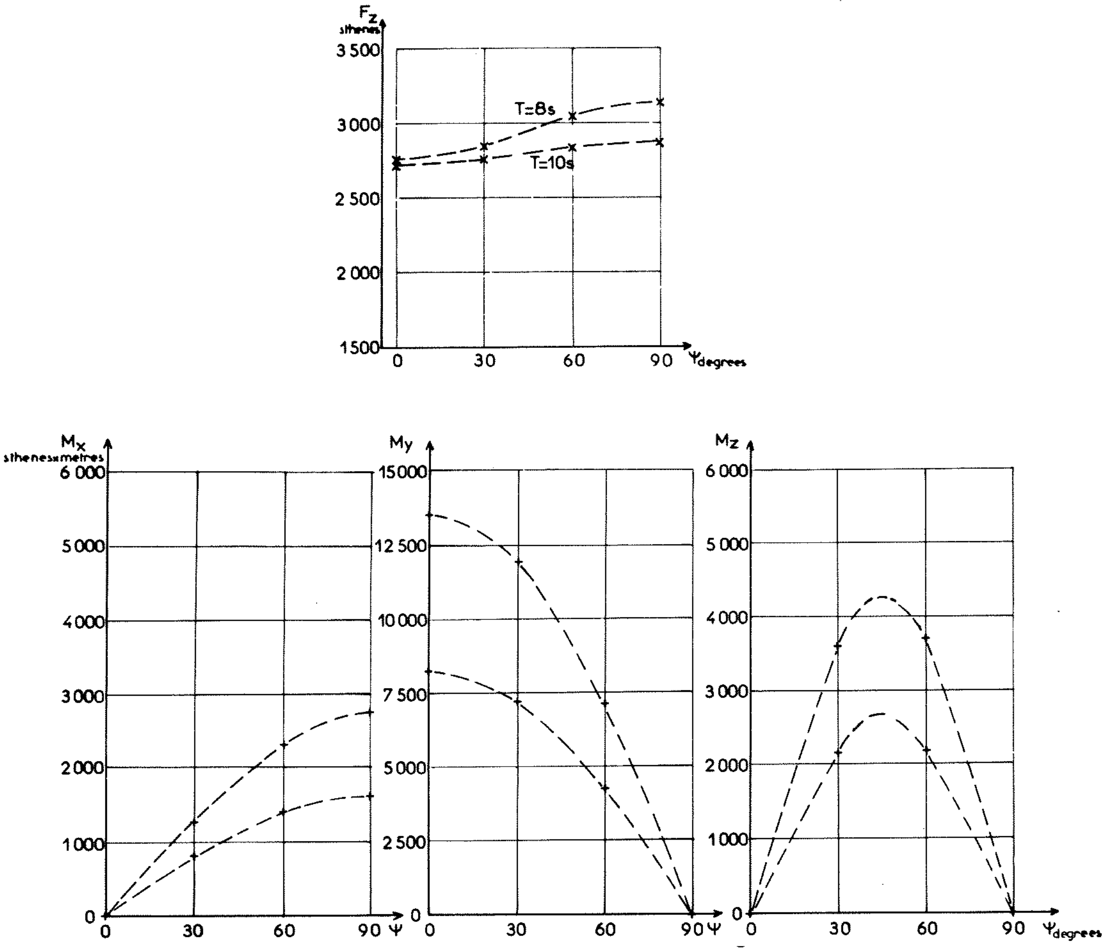

Figure 11 - Heaving force and rolling, pitching and yawing moments for various directions.

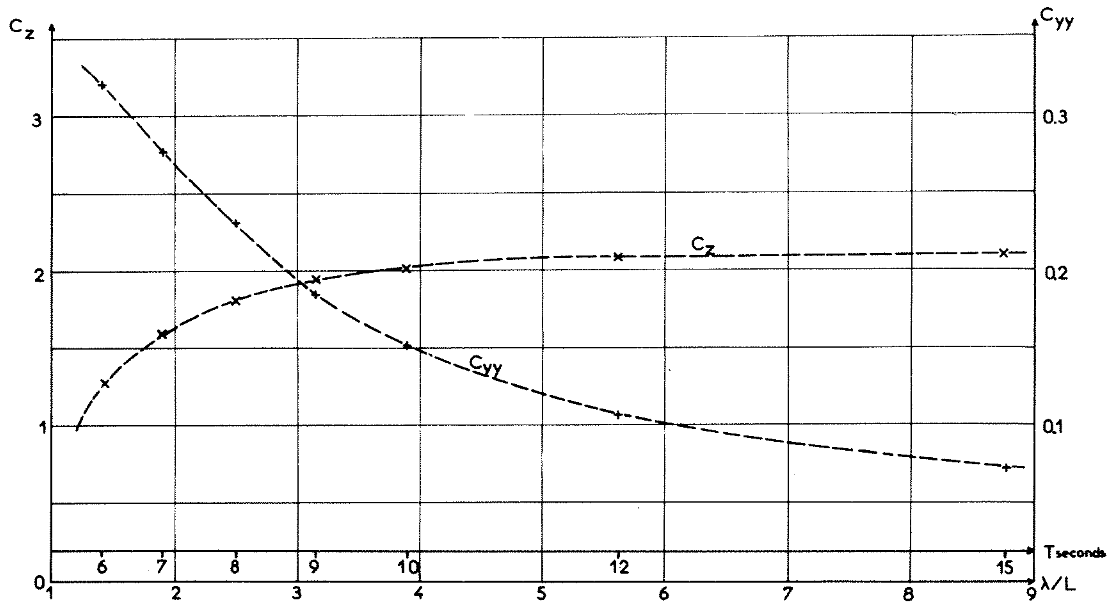

Figure 12 - Heaving force and pitching moment coefficients for various λ/L

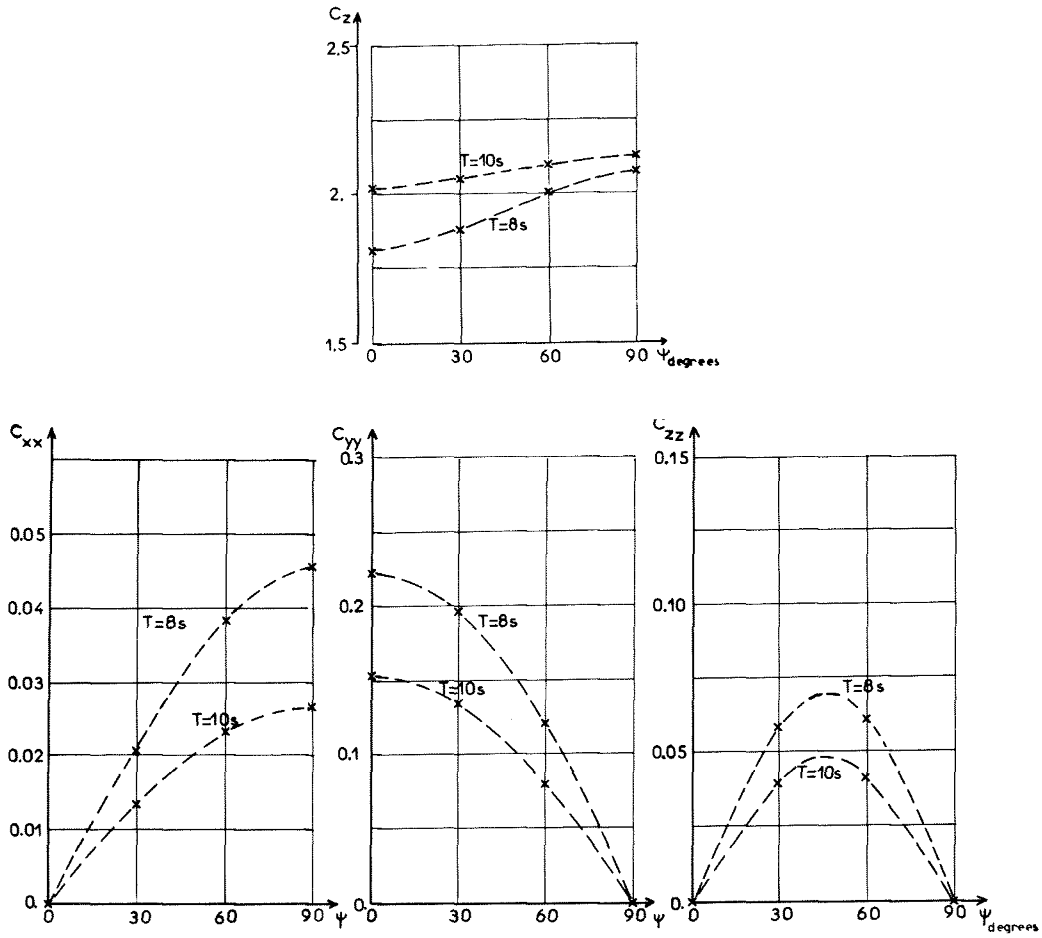

Figure 13 - Heaving force and rolling, pitching and yawing moment coefficients for various directions.

Coefficients Cz , Cxx , Cyy and Czz were calculated from the forces by formulae (5.3) and taking the biggest length of the caisson for L .

Figs. 12 and 13 show how these coefficients vary with the waves and caisson position.

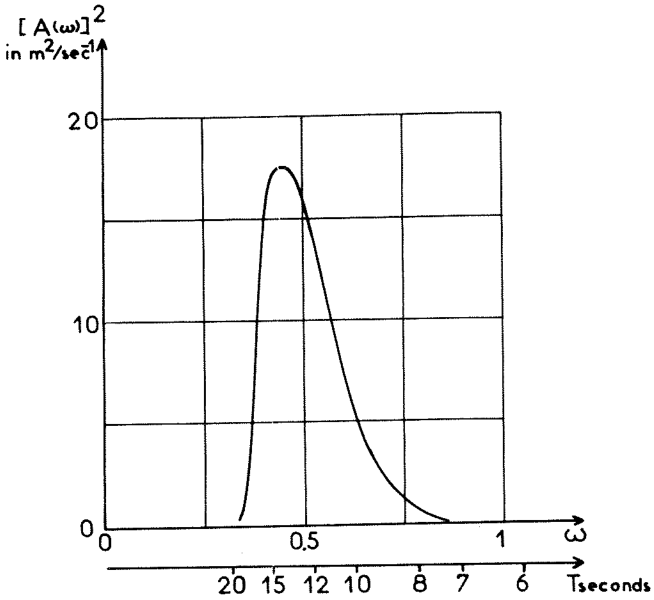

Figure 14 - Energy wave spectrum representing an Atlantic type swell.

6.5 Caisson motion when immersed under complex waves

The computation method described in this paper was u ed to determine the motion of a free caisson maintained at a depth of 15 metres by a schematic anchoring at its centre of thrust. The tension displacement relation ship for this anchoring is linear.

The considered complex waves are given by their energy spectrum (Fig 14) which is divided into ten constant-energy bands. This spectrum represents an Atlantic-type swell with an average period of 14 seconds.

Caisson heaving and pitching motion, corresponding wave forces and the difference in the free surface level vertically above the centre of thrust are all plotted in Fig. 15. It will be noted that as the caisson dimensions are small compared to the wave length, its motion is in phase with the wave motion. A low-frequency motion is superimposed upon the motion in phase with the coaves at a period close to the natural period of the system comprising the caisson and anchoring.

For this test, the caisson wa placed with its major axis in the wave direction and only one wave direction was considered. Use of the computation programme is not limited to this one case, however, and we have successfully applied it to the motion of a free caisson facing in any direction subjected to multi-directional waves.

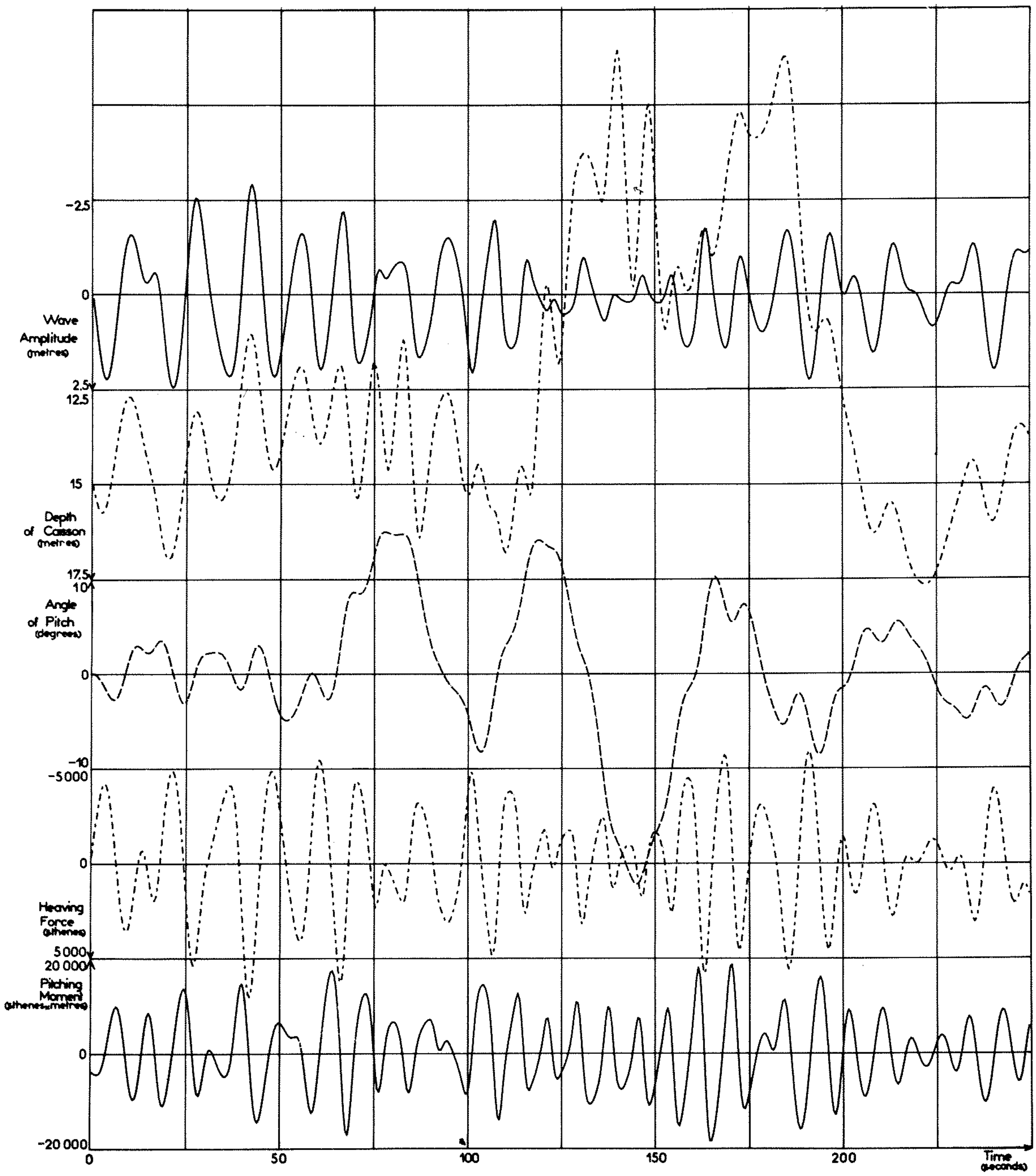

Figure 15 - Caisson heaving and pitching motion.

CONCLUSION

This study resulted in the design of a mathematical model for the computation of wave forces on body of any shape with and without sharp ** edges submerged at an adequate depth.

It is proposed to develop this model for calculations at any depth and allowing for free surface effects.

The model can already cope with viscosity forces which are computed from local friction coefficients and vary as the square of velocity.

Complete mathematical models of complex structures (e.g. semi-submersible drilling platforms) can thus be constructed for use in calculating real life wave forces and motion. Simplifying flow assumptions have to be made, however, especially as regards the mutual action of the structural members, and this leads to certain approximations which can then be narrowed down by carrying out a few tests on a model under monochromatic wave conditions.

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LIST OF SYMBOLS

$2A_i$	Wave height
A, B, C, P, Q, R.	Added mass coefficients
Cz	Heaving force coefficient
Cxx	Rolling moment coefficient
Cyy	Pitching moment coefficient
Czz	Yawing moment coefficient
$\overline{F}z$	Heaving force
$\overline{F}_\varphi$	Force due to the pressure term $\partial\Phi/\partial t$
$\overline{F}_V$	Force due to the pressure term $V_F^2/2$
g	Acceleration of gravity
K_i	Wave number
L	Length of body
$\overline{n}$	Unit vector normal to an element
$\overline{n}_i$	Unit vector normal to the i-th element
P(M)	Total pressure
Mx, My, Mz.	Rolling, pitching and yawing moments
t	Time
T_i	Wave period
k_x , k_y , k_z	Virtual inertia coefficients

V	Volume of immersed body
$\overline{V}_F$	Fluid velocity vector
$\overline{V}_C$	Buoyancy centre velocity vector
$\overline{V}_H$	Wave induced fluid velocity
u, v, w	Velocity components
p, q, r	Angular velocity components
z	Depth of submergence
x, β, γ	Components of the unit normal vector
δs_ι	Area of the i-th surface element
Γ_ι	Circumference of a surface element
θ	Angle of pitch
λ	Wavelength
ρ	Fluid density
Σ	Area of body surface
σ	Surface source density
Φ	Overall potential
Φ_H	Potential due to incident wave
Φ_{PH}	Potential due to the presence of a fixed body in waves
Φ_{PC}	Potential due to the body motion
ω_ι	Wave angular frequency
$\overline{\Omega}$	Angular velocity
$\varphi_1 , \varphi_2 , \varphi_3$ χ_1 , χ_2 , χ_3	Potentials associated to unit velocity components
ψ_ι	Wave phase angle
ψ	Heading of immersed body

USED UNITS

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Time	:	Second
Lenght	:	Metre
Mass	:	Metric ton
* Force	:	Sthene = 10^3 Newton

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DISCUSSION ON PAPER 12

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The authors neglect the influence of the body on the free surface motion and hence they have to assume that the depth of the body is more than twice its vertical height. This restriction can be removed if the source potential function is modified to satisfy the free surface boundary conditions.

We take horizontal co-ordinates x and y in the mean free surface and as vertical co-ordinate, z, measured positive downwards. A fluctuating source with strength varying as $\sigma \cos \omega t$ at x = a, y = b, z = f produces diverging waves at infinity. The potential which satisfies the boundary conditions for infinitesimal height waves is,

$$\phi(x, y, z, t) = \sigma\left[\frac{1}{r} - PV\int_0^{\infty}\frac{\upsilon+k}{\upsilon-k}\, e^{-k(z+f)}\, J_0(kR)\, dk\right] \cos\omega t + $$
$$+ \sigma\, \alpha\, \pi\, \upsilon\, e^{-\upsilon(z+f)}\, J_0(\upsilon R) \sin \omega t,$$

where $\upsilon = \sigma^2/g$,

J_0 is the Bessel function the first kind and order zero,

$R = \sqrt{(x-a)^2 + (y-b)^2}$,

$r = \sqrt{(x-a)^2 + (y-b)^2 + (z-f)^2}$,

and PV indicates that the Cauchy principal value of the integral is to be taken.

A similar expression can be obtained for the case of finite depth, see Thorne (1953).

This expression can be used in place of $1/|MP|$ in equation (2.5) and (2.6) of the paper. The solution for the source densities σ_{PH} and σ_{PC} would then be obtained from suitably modified versions of equations (2.7) and (2.8).

The Cauchy principal value integral can be evaluated by contour integration and this will increase the time taken to set up matrix A, but the time penalty incurred should not be too great.

The conditions under which the authors' solution is valid may be determined from this expression. If the depth of the source, f , is greater than half a wave length then the extra terms will be negligible. But at this depth we do not expect any appreciable wave motion, so it would appear that the extra terms should always be considered. However, the strength of the sources on a body must sum to zero, and if the body is sufficiently deep the potentials due to the sources will cancel at the surface, and it will not be necessary to consider the extra wave terms. This, of course, is the condition the authors impose.

Ref.: Thorne, R.C., Multipole expansions in the theory of surface waves, Proc. Camb. Phil. Soc., Vol. 49, 1953, pp. 707 - 716.

WAVE ACTION ON SLIGHTLY IMMERSED STRUCTURES, SOME THEORETICAL AND EXPERIMENTAL CONSIDERATIONS

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SUMMARY

This memorandum proposes a mathematical approach of the interactions between a non-viscous fluid with a free surface and an assembly of structures composed of shells, whether fixed or not, partially or totally immersed in the fluid.

Some fundamental aspects of the problem, such as the effects of the free surface and of the mutual influence of the structures on each other, have been developed, using widely the classical concepts of added masses, linear damping coefficients and transfer fonctions.

In the second part, purely experimental part, some results concerning actions on structures fixed to the bottom are given.

INTRODUCTION

The concept of added mass is used in most formulas giving the hydrodynamic force exerted on a fixed body placed in an incompressible non-viscous fluid.

When the field of acceleration vector is uniform on the whole volume of the body, the added mass is the same whether the body remains at rest in the fluid in motion or if the body is in motion in a fluid at rest. But when the acceleration field is no longer uniform, as it is the case for waves and when the dimensions of the body are of the same order as the wave length this reciprocity no longer exists and the added mass of the body in motion can no longer be used to compute the forces exerted by the waves on the fixed obstacle. The wave diffraction theory, while retaining the assumption of linear waves, enables us to compute the potential of the waves deformed by the presence of the body and is therefore a necessary tool for computing hydrodynamic forces exerted on fixed bodies whenever they are of large size or whenever their motion creates waves of the same length as the incident waves.

By giving a few examples, it was sought to better define the fields for which the method is necessary. The results, obtained by the facet theory, bring out the influence of the free surface and the mutual influence limits of several cylindrical piles.

Transfer functions, linking the wave spectrum to the hydrodynamic for spectrum or to that of structure displacement were derived directly from the computations and used to interpret the results.

FIRST PART - POSITION OF THE PROBLEM

Let us consider structures composed of shells of any form, immersed totally or partially in a fluid which is assumed to be infinite below a free surface.

This liquid medium, which represents the sea, is generally in motion either because of the existence of marine currents or because of waves propagating over the free surface. We shall limit ourselves to the second phenomena and shall assume that, as a preliminary approximation, the scheme of potential waves to the first order describes these waves correctly, provided, possibly, that several cycles and several directions are combined. When placed in the field of velocity due to these waves, the shells shall deform the initial potential and shall be subjected to forces. If they are rigidly connected to a fixed support, such as the bottom, both the forces and the resulting moments must be known. If, on the contrary, they are more or less free to be displaced, these forces and moments should still be computed, not only to deduce the stresses in the structures but also to obtain shell displacements in the fluid.

The model used here was based on the classical diffraction theory. We will therefore only recoll the basic principles and difficulties that might arise in its use.

The fluid is assumed to be non-viscous and its potential Φ is a sinusoidal function of time. This presupposes that the complex motion of the shells or of the fluid has been decomposed into simple waves and conversely that the superposition of elementary sinusoidal waves brings back the complex motion. This is true in the case of small motions of the fluid and of the shells.

The fundamental linearized equations defining Φ or φ are then :

$$\begin{cases} \Delta \varphi = 0 & \text{the Laplace equation} \\ \left.\dfrac{d\varphi}{d\eta}\right|_S = -i\sigma\,(X_G^{\,o} + \omega \wedge GM)\cdot\eta & \end{cases}$$

the condition of impermeability of shells with

$$\Phi = \mathrm{Re}\left\{ \varphi\, e^{-i\sigma t} \right\}$$

$$X_G = \mathrm{Re}\left\{ X_G^{\,o}\, e^{-i\sigma t} \right\} \qquad \text{linear displacement of the center of gravity G}$$

$$\Omega = \mathrm{Re}\left\{ \omega\, e^{-i\sigma t} \right\} \qquad \text{angular displacement}$$

η is the outside unit normal vector.

In order to close the preceding system, two vectorial equations, expressing the momentum theorem applied to the solid body S, must be added.

They are :

$$m\,\ddot{X}_G = -\oint_S p\,\eta\,dS$$

$$\bar{\bar{I}}\,\ddot{\Omega} = -\oint_S p\,(GM \wedge \eta)\,dS$$

The "linear" pressure being :

$$p = -\rho g Z - \rho \frac{\partial \Phi}{\partial t}$$

We did not introduce any boundary conditions into the preceding system in order to analyze them better, since they impart to this classic potential problem its highly particular character.

The fluid being infinite in every horizontal direction, a radiation condition must be satisfied for $r \longrightarrow \infty$ (r being the horizontal distance)

If the depth is finite, $\frac{d\varphi}{dZ} = 0$ shall be set for $Z = -d$ (d being the depth) ; if $d \longrightarrow \infty$, it is sufficient to have $\frac{d\varphi}{dZ} \longrightarrow 0$ for $Z \longrightarrow \infty$.

For, φ we must search for a solution satisfying to the surface condition :

$$-\sigma^2 \varphi + g \frac{\partial \varphi}{\partial Z} \Bigg|_{Z=0} = 0$$

This condition gives the effects of the free surface. It has always been adhered to in the following examples, so that the magnitude of these effects could be evaluated, particularly when the shells are slightly immersed.

INTRODUCTION OF ADDED MASS CONCEPT

In linear theory, the simplest fundamental method satisfying the shell impermeability condition :

$$\frac{d\varphi}{d\eta}\Bigg|_s = -i\sigma\,(X_G^{\,o} + \omega \wedge GM)\,.\,\eta\Bigg|_s$$

consists of superposing on the incident potential φ_∞ as many terms as may be necessary, it being understood that each term φ checks the equation $\Delta\varphi = 0$ and the bou dary conditions.

If the structures are fixed, it is sufficient to add to the term φ_∞ a term φ_d representing the diffracted wave and defined by :

$$\frac{d\varphi_d}{d\eta}\Bigg|_s = -\frac{d\varphi_\infty}{d\eta}\Bigg|_s$$

If the structures are free, there should be added the potential φ_e of the waves em tted by shell motion. φ_e shall be defined by :

$$\frac{d\varphi_e}{d\eta}\Bigg|_s = -i\sigma\,(X_G^{\,o} + \omega \wedge GM)\,.\,\eta\Bigg|_s$$

Finally, in the most general cases, the linear pressures shall have for resultants :

- the hydrodynamic forces (complex component) :

$$F_H = i\sigma\rho \oint_S (\varphi_\infty + \varphi_d + \varphi_e)\,\eta \; dS$$

- the hydrodynamic moment :

$$C_H = i\sigma\rho \oint_S \left\{ (\varphi_\infty + \varphi_d + \varphi_e)\, GM \wedge \eta \right\} dS$$

ADDED MASS AND ADDED MOMENT TENSORS

It is well known that φ_e is obtained by setting :

$$\varphi_e = -i\sigma \; (X_G^o \cdot \varphi_T + \omega \; \cdot \varphi\omega)$$

φ_T et φ_ω being two "potential vectors" defined by :

$$\begin{cases} \overline{\overline{\text{grad}}}\; \varphi_T \cdot \eta = \eta & \varphi_T = (\varphi_x, \varphi_y, \varphi_z) \\ \overline{\overline{\text{grad}}}\; \varphi\omega \cdot \eta = GM \wedge \eta & \varphi\omega = (\varphi_\alpha, \varphi_\beta, \varphi_\gamma) \end{cases}$$

φ_d and the components of φ_T and $\varphi\omega$ are obtained from the general form (1) (single layer potential) and by the solution of a Fredholm integral equation.

The added moments and masses tensor is then (by definition) obtained from the following table :

$$M' = \begin{bmatrix} -\rho \oint_S \varphi_T \otimes \eta \; dS, & -\rho \int \varphi\,\omega \otimes \eta \; dS \\ \\ -\rho \oint_S \varphi_T \otimes (GM \wedge \eta)\; dS, & -\rho \int \varphi_\omega \otimes (GM \wedge \eta)\; dS \end{bmatrix}$$

or a total of 36 complex coefficients of which the real parts are the added masses and the imaginary parts the linear damping coefficients.

In the case of fixed structures, as has already been said, the solution of the problem is in the form of :

$$\varphi = \varphi_\infty + \varphi_d$$

φ_d being defined by :

$$\left.\frac{d\varphi_d}{d\eta}\right|_S = -\left.\frac{d\varphi_\infty}{d\eta}\right|_S = -\left.\operatorname{grad}\varphi_\infty \cdot \eta\right|_S$$

Generally, grad φ_∞ is not a uniform vector field on the whole volume of shell S. This property becomes true again asymptotically when the wave length becomes very large with respect to the obstacle. Let us show that, at that time, the hydrodynamic force F_H can be expressed in the form of the product of the added mass coefficient and of the mean acceleration on the volume of the obstacle

$$F_H = -\rho \oint_S \frac{\partial}{\partial t}(\varphi_\infty + \varphi_d)\,\eta\; dS$$

Or :

$$F_H = -\rho \iint_v \operatorname{grad}\frac{\partial \Phi_\infty}{\partial t}\,dv - \rho \oint_S \frac{\partial \varphi_d}{\partial t}\,\eta\; dS$$

The first term may be written in function of the velocity vector V_∞ :

$$+\rho \iint_v \frac{\partial V_\infty}{\partial t}\,dv$$

or by introducing the mean acceleration $\dfrac{\partial V_\infty^m}{\partial t}$ and the volume $\forall$ of the shell :

$$+\rho \cdot \forall \frac{\partial V_\infty^m}{\partial t}$$

Let us, by definition :

$$-\rho \oint_S \frac{\partial \Phi_d}{\partial t}\,\eta\; dS = +\rho\,\forall\, C_H \frac{\partial V_\infty^m}{\partial t}$$

C_H is generaly a tensor which must not be confused with the added mass tensor. Therefore, there is no possible reciprocity unless it is assumed that the velocity field V_∞ becomes uniform on volume $\forall$. C_H is then identified with :

$$\frac{1}{\forall} \oint_s \varphi_T \otimes \eta \, dS$$

In summary, the hydrodynamic force on a fixed structure may be put in the form of :

$$F_H = - \rho \forall (1 + C_H) \frac{\partial V_\infty^m}{\partial t}$$

formula in which C_H is a tensor that we shall call the hydrodynamic tensor.

F_H may also be written :

$$F_H = - \rho \oint_s \frac{\partial}{\partial t} (\Phi_\infty + \Phi_d) \eta \, dS$$

II - EFFECTS OF IMMERSION ON THE HYDRODYNAMIC FORCES AND ON THE ADDED MASSES

The preceding considerations do not add any new elements. Nevertheless, they should call attention to the necessity of very accurate terminology for the concepts of added masses and hydrodynamics coefficients.

We have applied them to a very simple example, vis. a shell composed of elementary volumes like spheres, cylinders and cones (fig. 1) placed at different immersion depths.

A computation code based on the facet theory has been used on a CDC 6600 computer in order to evaluate the different hydrodynamic parameters.

The shell shown on figure slide 1 was decomposed into 120 plane facets in order to avoid prohibitive computation time and to compute the hydrodynamic elements for several periods.

The mathematical model supplied three categories of results :

a) the terms of the added mass tensors or 36 coefficients defined by their real and imaginary parts ;

b) the hydrodynamic forces and moments exerted on the shell, assumed to be fixed ;

c) the linear motion and the angular motion of the shell free to oscillate under the action of a given potential wave. Elastic connections may be introduced in order to stabilize the shell.

On the figure 2 appear :

- The added mass coefficient of the surge motion :

$$C_x + i\,H_x = -\frac{1}{\mathcal{V}} \oint_S \varphi_x \eta_x \, dS$$

- The added mass coefficient of the yaw motion :

$$C_y + i\,H_y = -\frac{1}{\mathcal{V}} \int_S \varphi_y \eta_y \, dS$$ appears on the figure 3

- The added mass coefficient of the heaving motion :

$$C_z + i\,H_z = -\frac{1}{\mathcal{V}} \oint_S \varphi_z \eta_z \, dS$$ appears on the figure 4

- The vertical hydrodynamic force F_z (in module), with the shell fixed :

$$\left| F_z \right| = \left| i \rho \sigma \oint_S (\varphi_\infty + \varphi_d)\, \eta_z \, dS \right|$$ appears on the Figure 5

The immersion d, referred to the length L of the shell, has taken the values :

$$\frac{d}{L} = 0{,}15 \; - \; 0{,}18 \; - \; 0{,}27 \; \text{and} \; 1.$$

The basic conclusions appear clearly on figures 2, 3, 4 and 5.

- The immersion influences considerably the added masses and the hydrodynamic forces as long as $\frac{d}{L} \leqslant 1$, and this holds true for practically all the wave lengths.

Therefore, the effects of the free surface are very large but become attenuated with depth, and it is remarkable that we find, for great depths, the theoretical values of cylinders in infinite fluid :

$$C_z = 1 \qquad C_y = 1 \qquad C_x \simeq 0,1 \qquad H_x = H_y = H_z = 0$$

- In the zone $0 < \frac{d}{L} < 1$, the coefficients vary with the wave length and are at a maximum for $\lambda \simeq 3\ L$.

- There exists a wave length for which the variation of the coefficients with the immersion is zero. Besides, the sens of the variation is reversed on either side of this wave length (fig. 6).

III - FIXED VERTICAL PILES - MUTUAL INFLUENCE

The diffraction theory has been applied to a cylindrical pile by Mac Camy and Fuchs.

The potential solution obtained is well known and agrees perfectly with the Morrison formula for very large wave lengths.

$$\begin{cases} B\ (ka) = \dfrac{1}{\sqrt{J'^{2}_{1}\ (ka) + Y'^{2}_{1}\ (ka)}} \\[2ex] tg\ \alpha = J'_1\ (ka)\ /\ Y'_1\ (ka) \end{cases}$$

We shall only use the force F_H in order to characterize the problem exactly :

$$T_H = \frac{|F_H|}{\rho g \pi a^2 (\eta)} = \frac{4}{\pi k^2 a^2} \text{ th } kd \; B\,(ka)$$

$$B\,(ka) = \frac{1}{\sqrt{J'^2_1 + Y'^2_1}}$$

η = elevation of the free surface with respect to the free surface at rest,

T_H is the transfer function permitting to pass from the spectrum of the free surface $S\eta\eta$ to the spectrum S_{HH} of the hydrodynamic forces when the only cause of these forces is the inertia :

$$S_{HH} = \rho^2 g^2 \pi^2 a^4 T_H^2 S_{\eta\eta}$$

Figure (7) gives the form of $\frac{T_H}{thkd}$. The crosses correspond to computation by the sources theory. The continuous line represents the Mac Camy and Fuchs computations.

On the basis of the same theory, we have sought the transfer function when two piles influence each other mutually (figs 8a and 8b).

The results indicated on figure 9 show that, for distances between pile centerlines on the order of 10 a, the transfer function is changed in the field of the usual wave lengths :

$$0{,}016 < \frac{2a}{\lambda} < 1$$

This effect is more marked for waves perpendicular to the plane of the pile centerlines as for waves parallel to this plane, the pile "two" being in the standing wave due to the pile "one".

It may therefore be deduced that the shells forming an integral part of the same structure and spaced 5 to 10 times their diameter cannot be considered as independent. The wave field deformations that they produce cannot be superimposed as the result of their interactions.

IV - CONCLUSIONS

In spite of the limitation of potential theory used in these few examples, we think that some conclusions can be deduced :

- Classical diffraction methods permit to introduce very simply the concept of added mass and linear damping coefficient ;
- It appears that added mass coefficient and hydrodynamic forces applied to fixed body are raised sometimes of 100 % in the case of slight immersion ($d <$ the greater dimension of the body) ;
- For distance of 5 or 10 diameter between two piles, potential theory gives interference effects in function of the position of one pile in the diffracted wave of the other. These phenomena add to the interference of wake behind the piles in the case of long waves.

These results show the interest of model lying upon facet theory for the simulation of floating platforms when inertia are predominant.

PART TWO

SOME EXPERIMENTAL RESULTS ABOUT WAVE ACTION ON SUBMERGED BODIES

As it was shown in part one, the hydrodynamic force acting upon a fixed structure can be given by the following expression :

$$F_H = \rho \forall C_M \frac{\partial V_\infty^m}{\partial t}$$

where :

$\forall$ is the body volume,

$\frac{\partial V_\infty^m}{\partial t}$ is the mean acceleration of the fluid within this volume.

C_M is the hydrodynamic coefficient (see part one).

This relationship holds for large bodies with strong predominance of inertia effects.

Dimensional analysis shows that the hydrodynamic coefficient depends of a great number of parameters, and must be expressed as :

$$C_M = f\left(\frac{D}{L}, \frac{H}{L}, \frac{d}{L}, \frac{HD}{\nu T}, \frac{\Delta}{d}, \frac{t}{T}\right)$$

where :

D, Δ are characteristic dimensions of the body

d the water depth,

H the wave height,

L the wave length,

T the wave period,

t the time.

Although it is obvious that this relationship is difficult to determine experimentally, due to the great number of parameters involved, the author believe that model tests carried out in well defined condition (i.g when drag effects are negligible) must provide to the design engineer useful estimates. This will be shown by the following example dealing with model experiments of horizontal forces acting upon offshore oil tanks laying on the bottom. This tanks were vertical cylinders, with a diameter D = 25 m and various heights. The scale was 1/50.

The test conditions are summarized in figure 10. The diffraction pattern of waves around such cylinders cutting the free surface is shown in figure 11. A sketch of symbols used is given by figure 12. Figure 13 shows a typical wave profile and the corresponding recorded force. The 90° phase lag exhibited establish the predominance of inertia effects.

In order to verify that the hydrodynamic coefficient could be derived from linear wave theory, it was first try to check the well known results for a vertical piling cutting the free surface. If the wave amplitude do not exceed 6 meters, figure 14 shows that the force is a linear function of wave amplitude, the best fit for the data giving a C_M value of 1.62.

The following figures show the corresponding results for a truncated pile. Figure 15 shows that for small amplitudes, the hydrodynamic coefficient is strongly reduced, as compared with the last case, and is roughly equal to 1.00. For higher amplitudes non linear phenomena occur, giving an important increase of the force. These non linear effects seem to initiate for wave heights reaching half the water height over the top of the pile.

These nonlinear effects disappear when the water height over the top of the pile becomes larger. But the hydrodynamic ooefficient still remain close to 1.

In conclusion, these tests seem to establish that the hydrodynamic coefficient C_M for horizontal forces upon submerged cylindrical bodies should be equal to 1, as soon as the water height overtopping the pile is more than one diameter of this pile. Some more work is needed to confirm these results.

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Epure de la coque

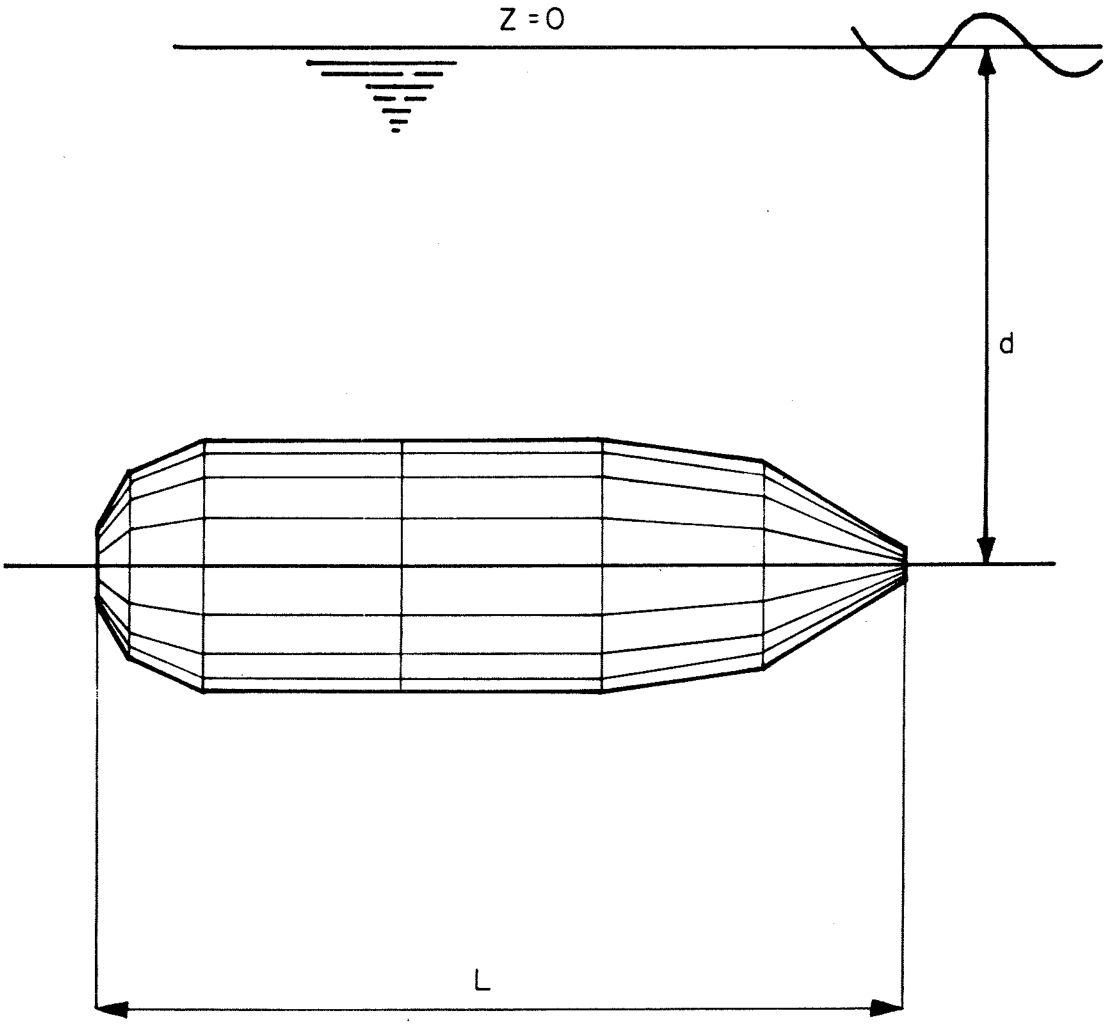


Fig. I

Variations du C_x et du H_x avec la longueur d'onde pour plusieurs immersions

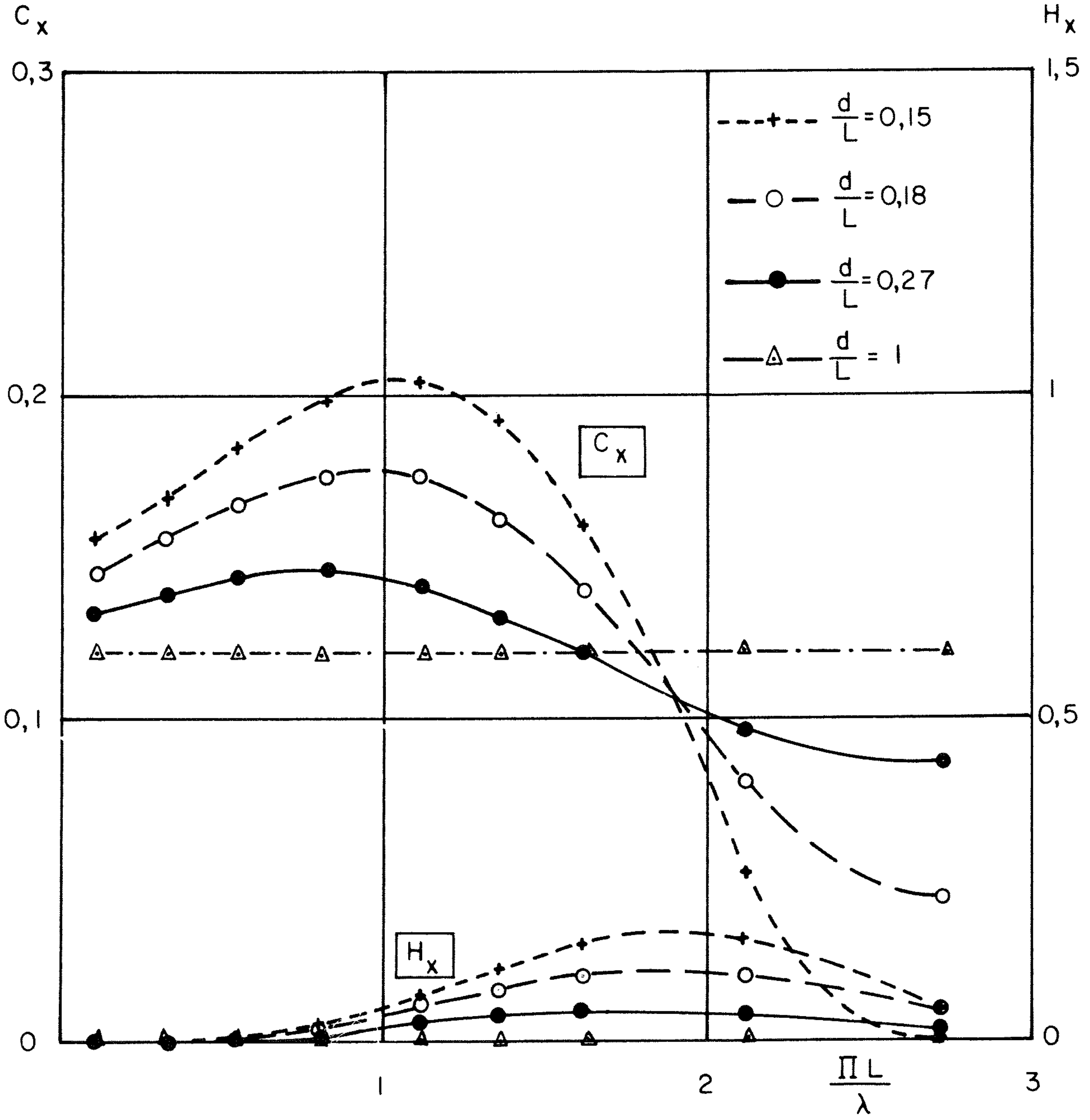


Fig.2

Variations du C_y et du H_y avec la longueur d'onde pour plusieurs immersions

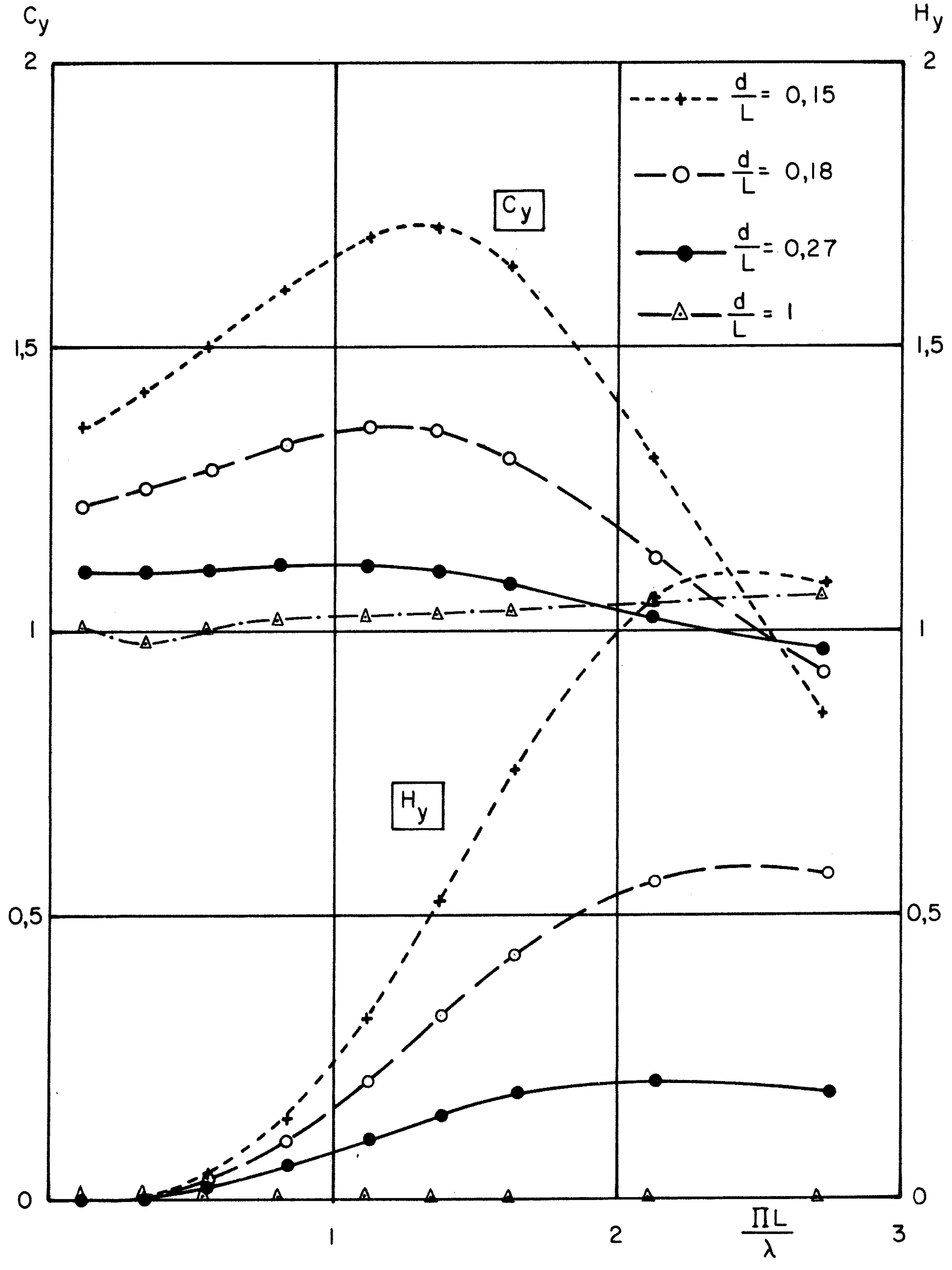


Fig. 3

Variations du C_z et H_z avec la longueur d'onde

pour plusieurs immersions

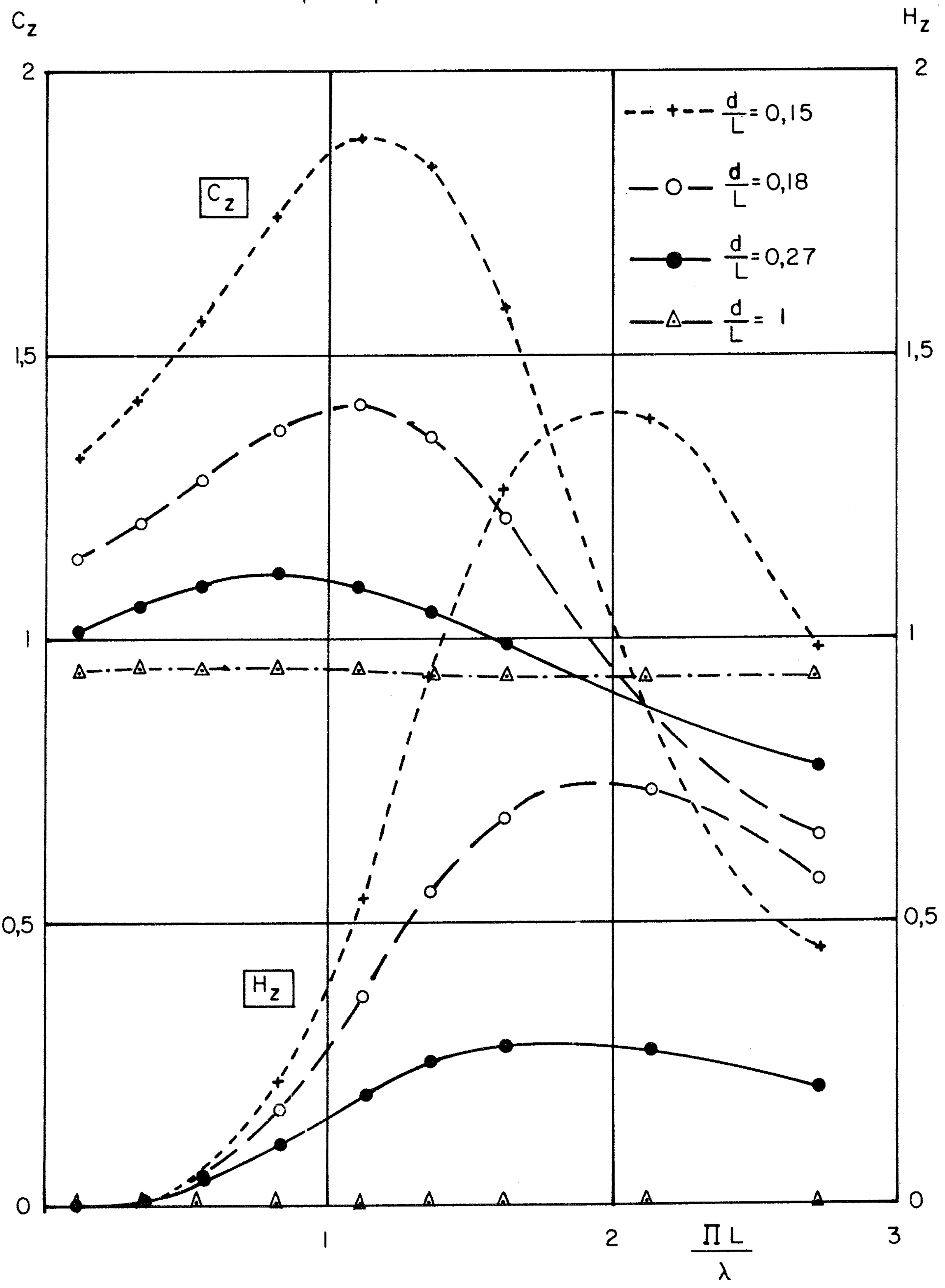


Fig.4

Variations de l'effort hydrodynamique vertical

avec la longueur d'onde et l'immersion

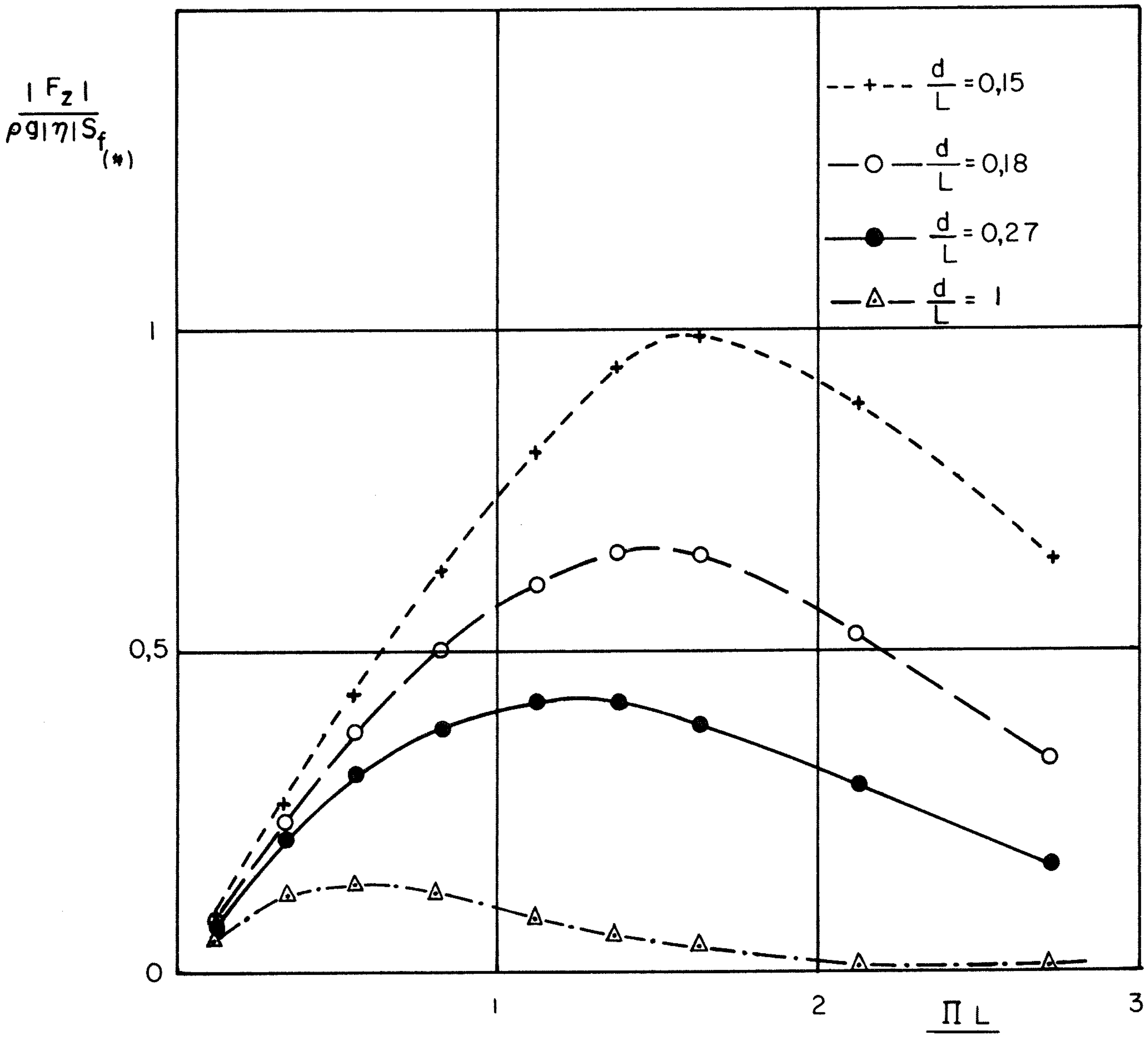


Fig. 5

* S_f = Surface du maitre_couple dans un plan horizontal

Variations du C_z avec l'immersion pour plusieurs longueurs d'onde

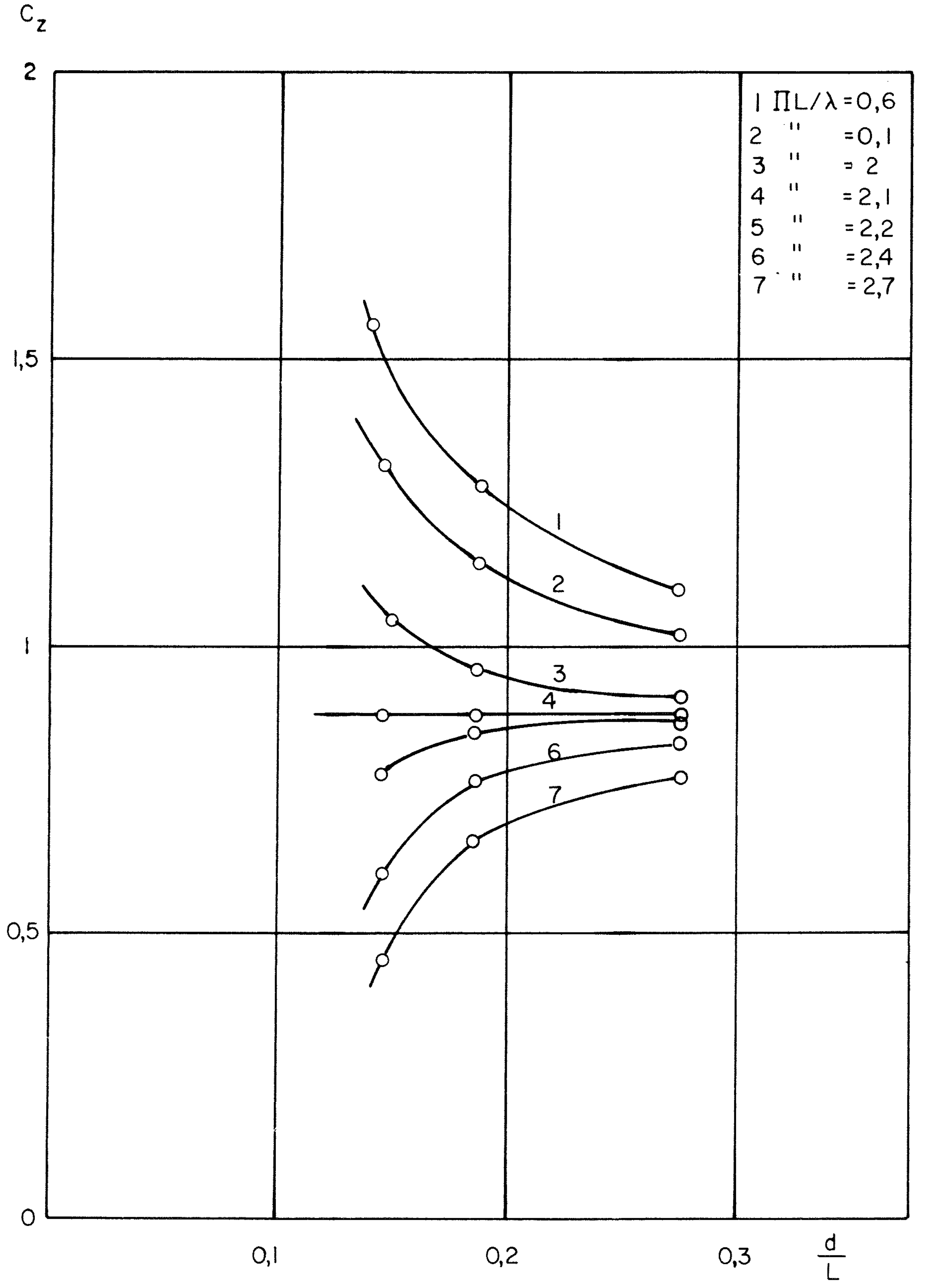


Fig. 6

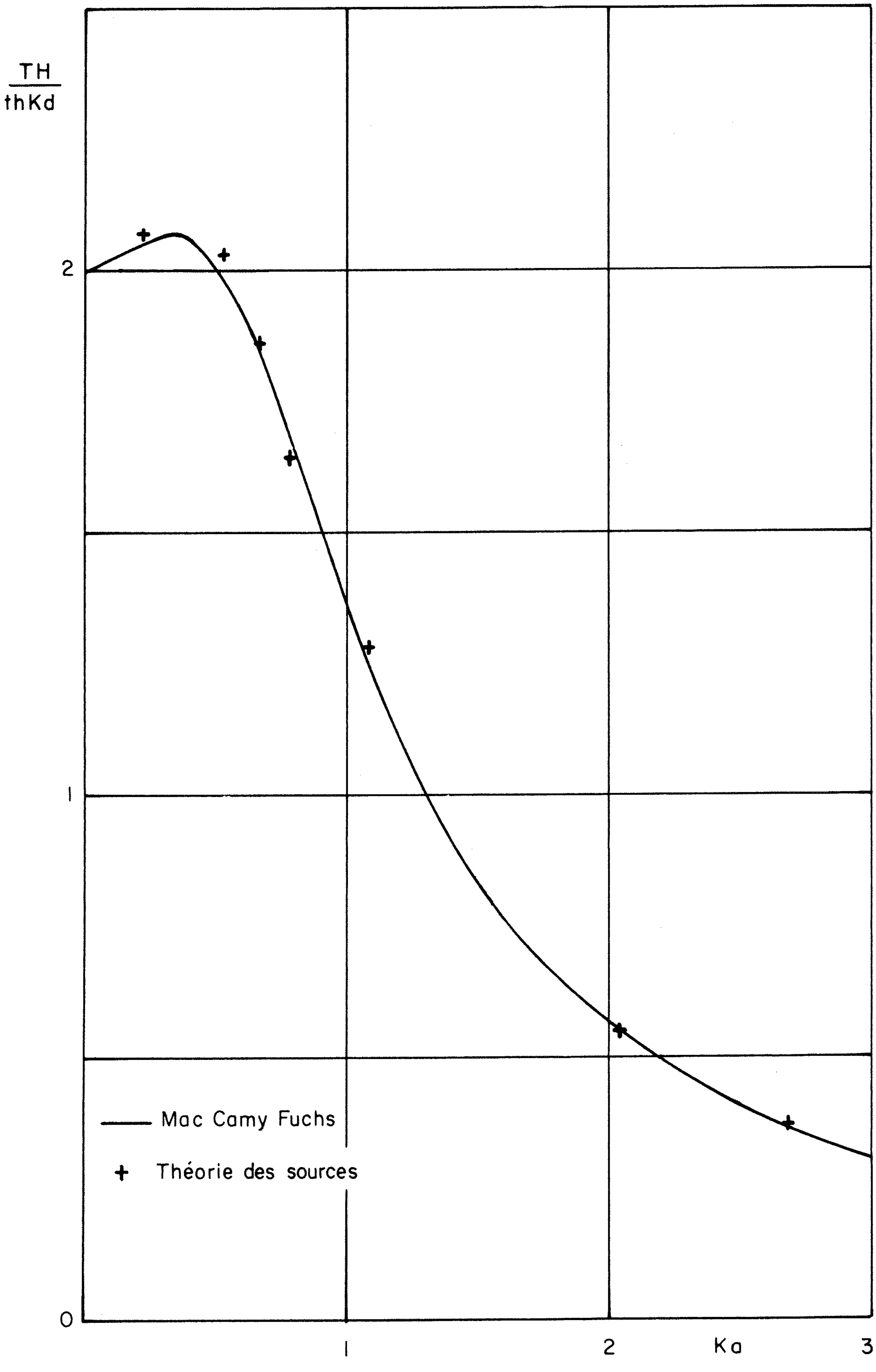


Fig. 7

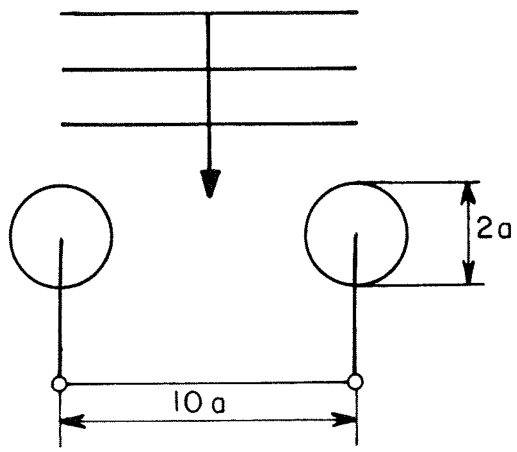


Fig.8 a

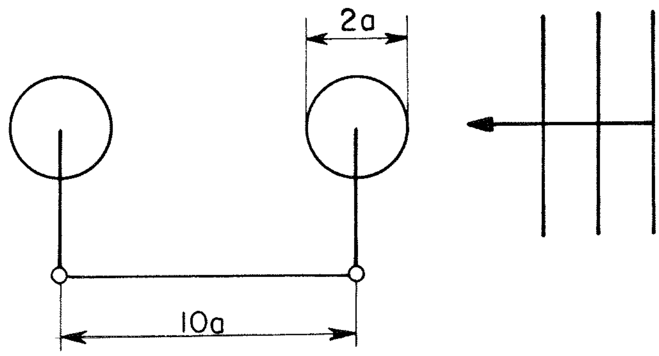
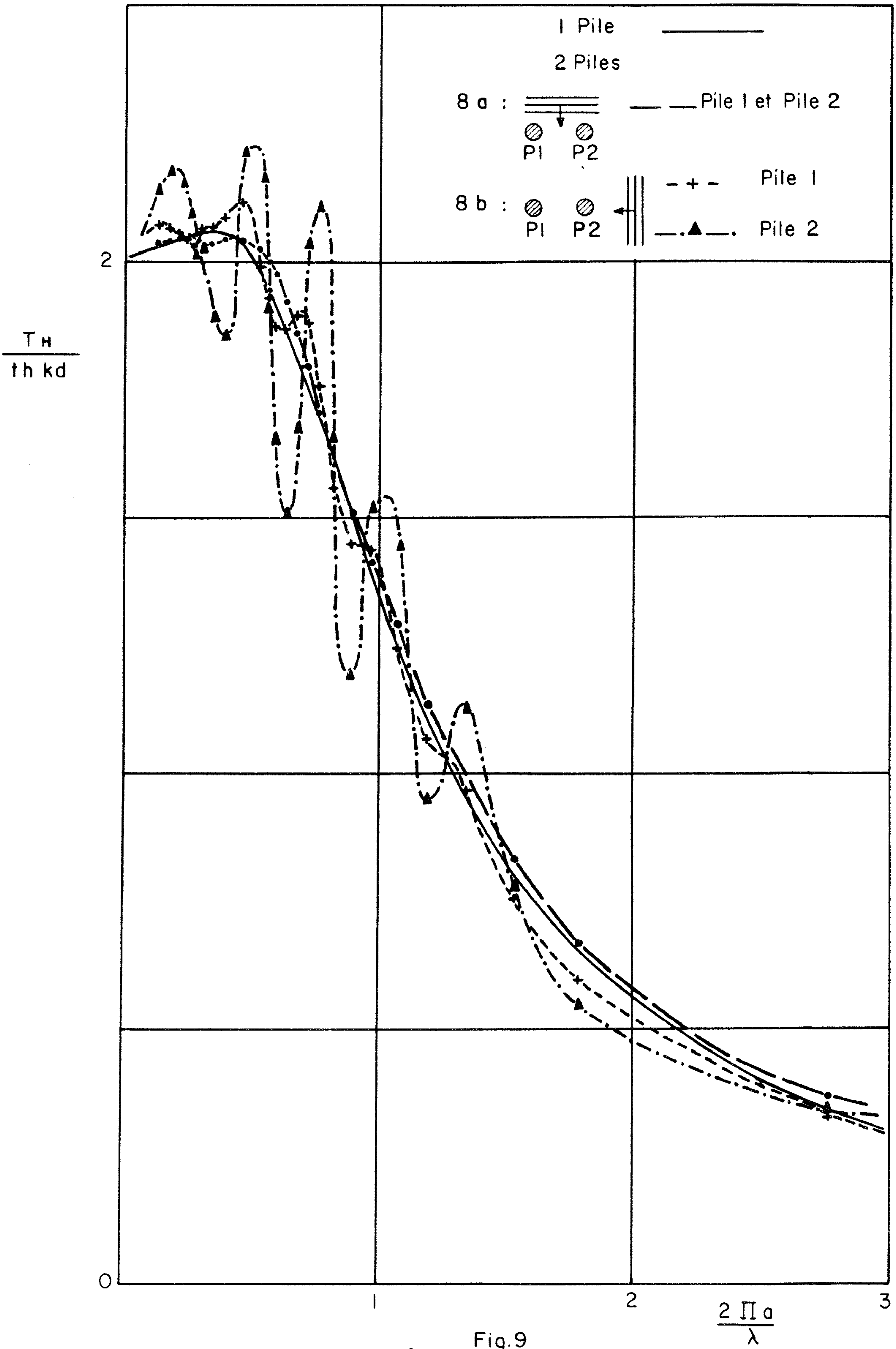
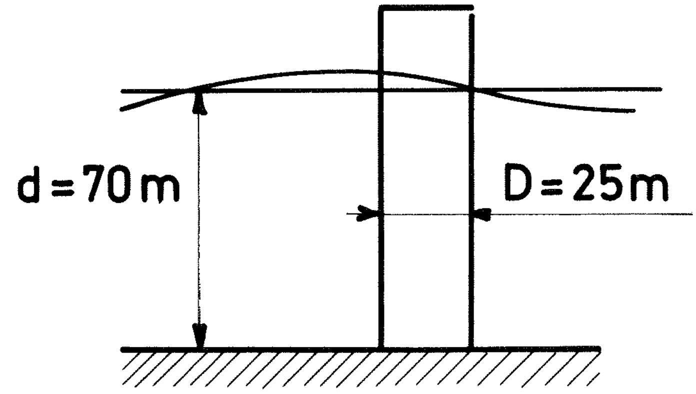


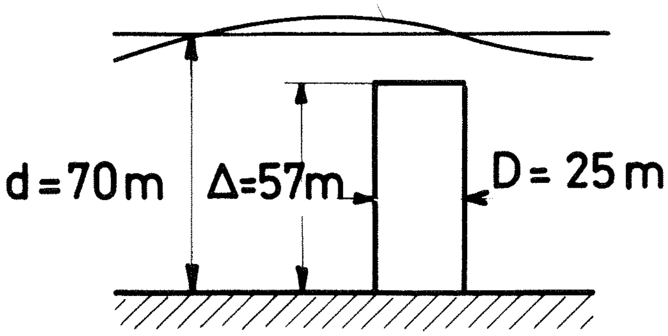
Fig.8 b


1 Pile
2 Piles
8 a :
Pile 1 et Pile 2
P1 P2
8 b :
P1 P2
Pile 1
Pile 2
2
0
Tн / th kd
1
2
3
2 Πa / λ
Fig.9



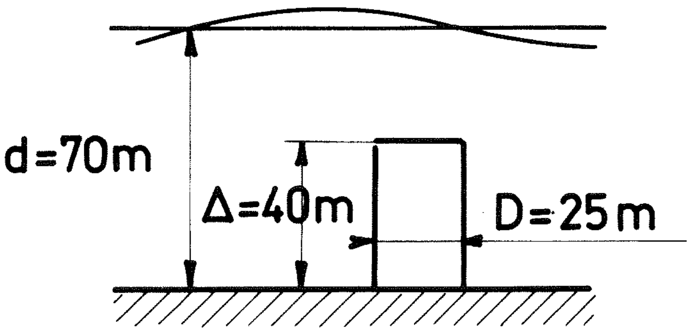
CAS 1

d/L	H/L	D/L
0,26	0,01 à 0,05	0,092



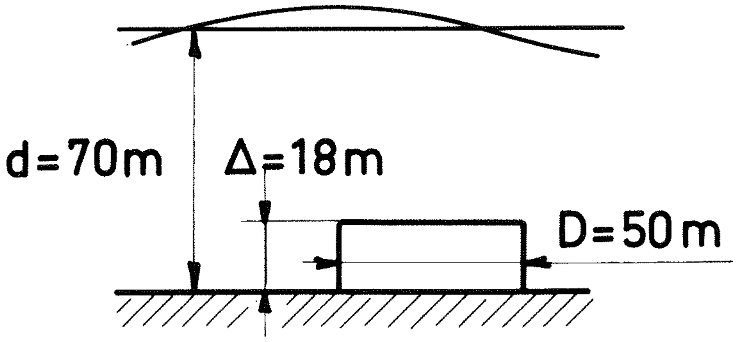
CAS 2

d/L	H/L	D/L
0,26	0,02 à 0,05	0,092



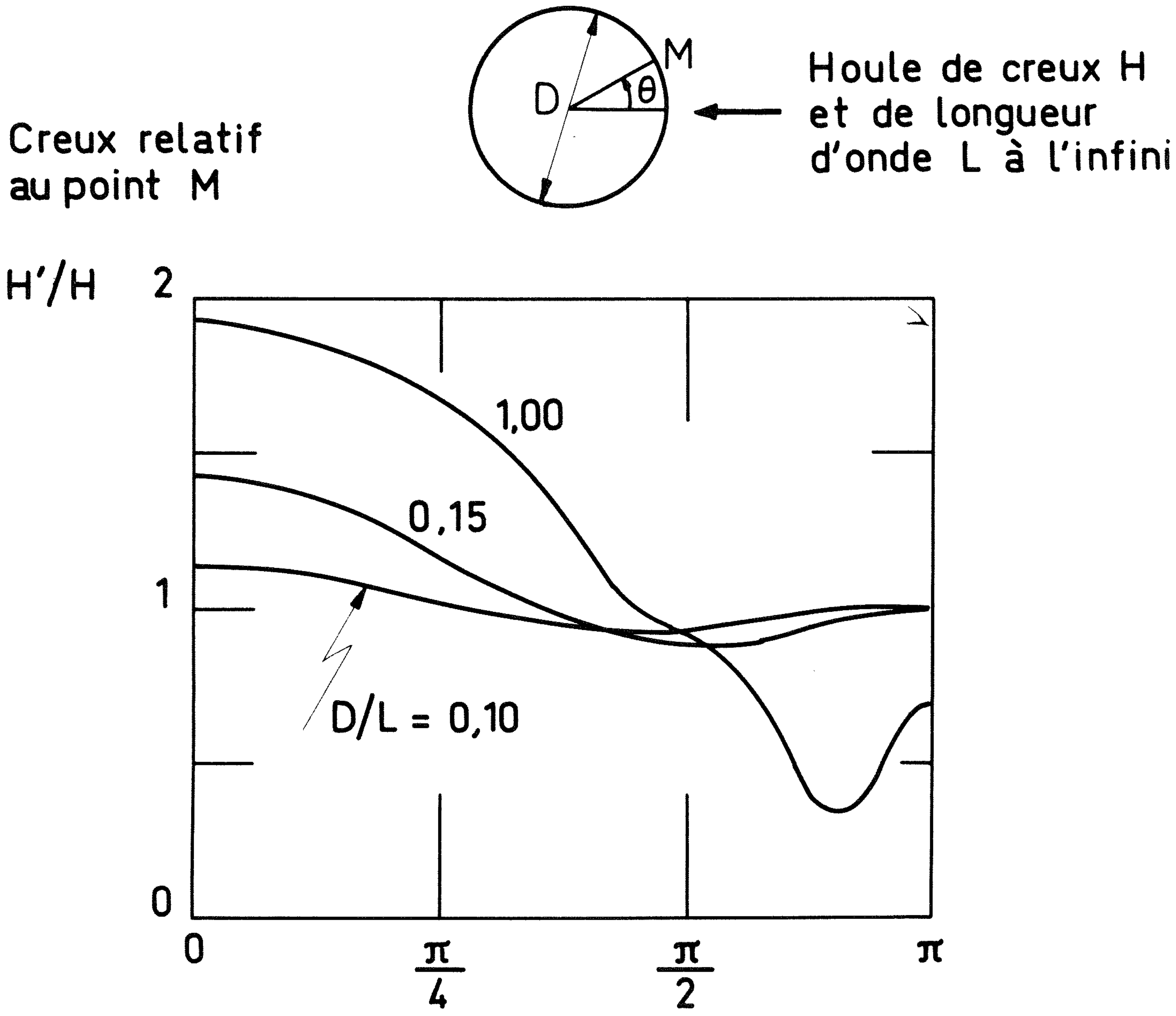
CAS 3

d/L	H/L	D/L
0,26 0,48	0,02 à 0,06	0,092 0,170

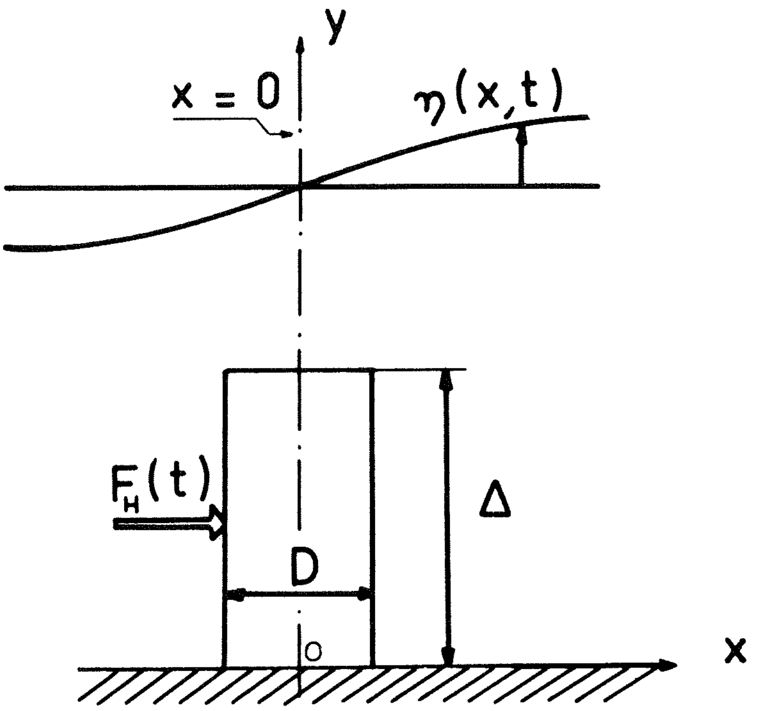


CAS 4

d/L	H/L	D/L
0,20	0,05	0,145
0,32	0,05	0,230
0,45	0,06	0,320



DIFFRACTION DE LA HOULE AUTOUR D'UN CYLINDRE A AXE VERTICAL ET A SECTION CIRCULAIRE



COTE DE LA SURFACE LIBRE

$$\eta(o,t) = a \sin\sigma t$$

ACCELERATION HORIZONTALE

$$\frac{\partial u}{\partial t}(o,y,t) = a\sigma^2 \frac{\cosh ky}{\sinh kd} \cos\sigma t$$

COMPOSANTE D'INERTIE

DE LA

FORCE HORIZONTALE

$$F_{HI}(t) = \rho \forall C_M \frac{\partial u_\infty^m}{\partial t}$$

avec : $\forall$ volume de la pile

$$\frac{\partial u_\infty^m}{\partial t} = \frac{1}{\forall} \iiint_\forall \frac{\partial u}{\partial t}\, dv$$

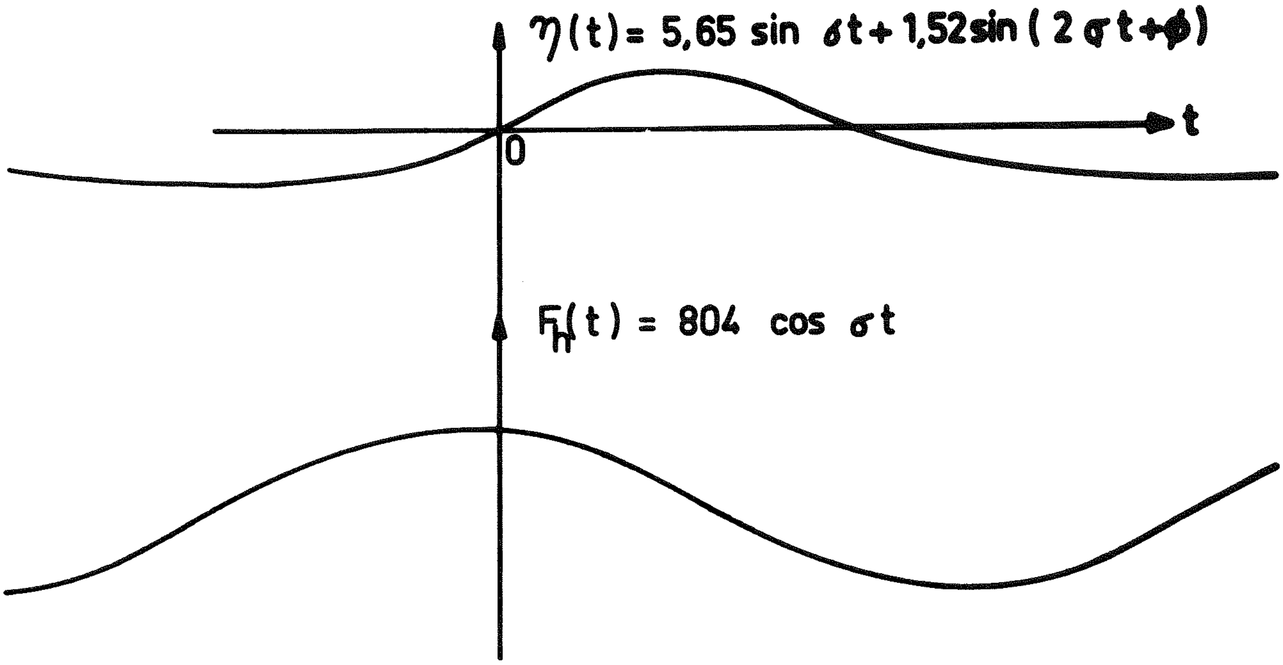
ou encore :

$$F_{HI}(t) = \rho g C_M \frac{\pi D^2}{4} a \frac{\sinh k\Delta}{\cosh kd} \cos\sigma t$$

12

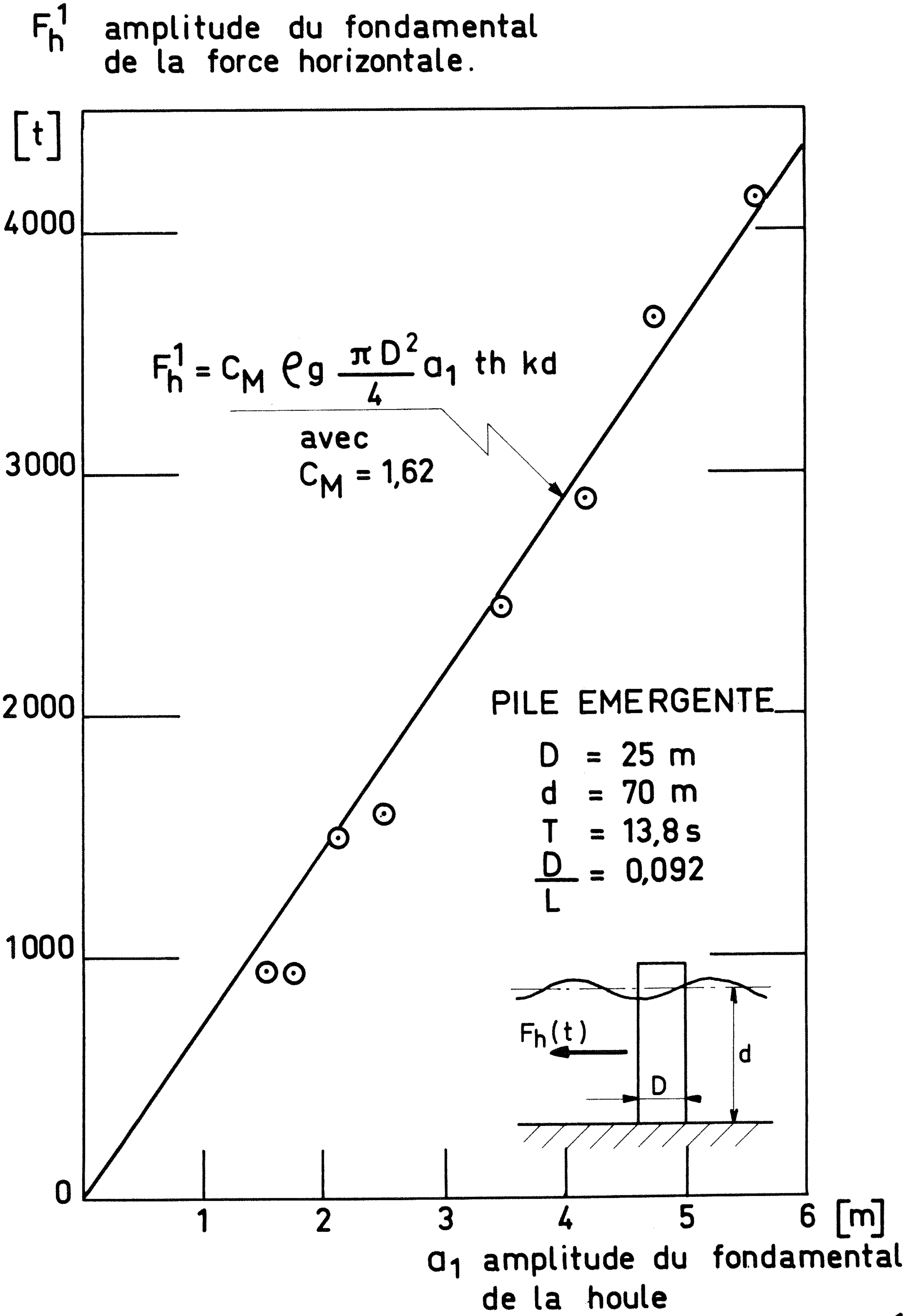
$D = 25\,m$
$d = 70\,m$
$\Delta = 40\,m$
$T = \frac{2\pi}{\sigma} = 9{,}8\,s$
$C_M = 1{,}06$

ESSAI 51



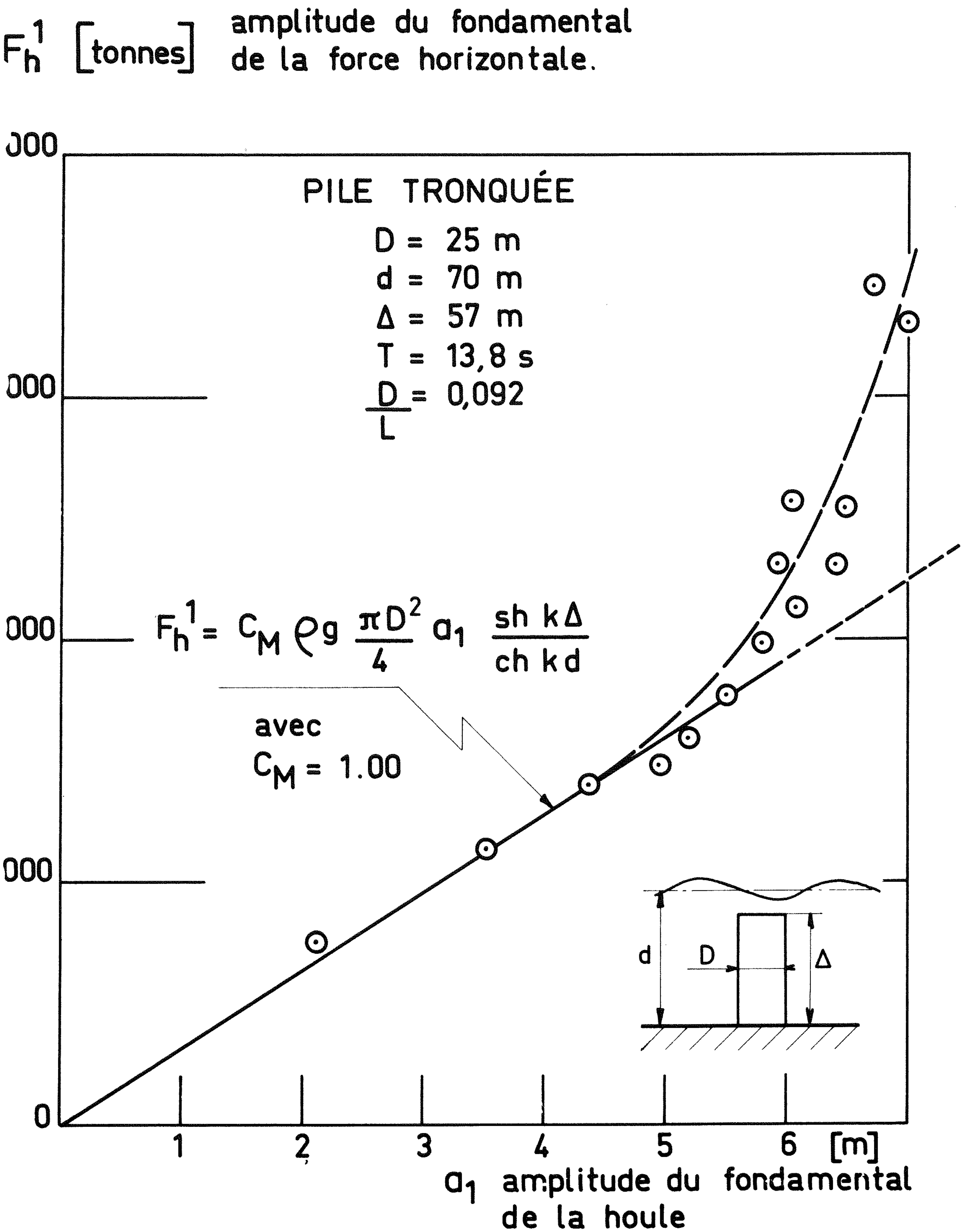
EXEMPLE D'ENREGISTREMENT.
LA FORCE HORIZONTALE EST EN PHASE
AVEC L'ACCELERATION HORIZONTALE
LA FORCE D'INERTIE EST PREPONDERANTE

13



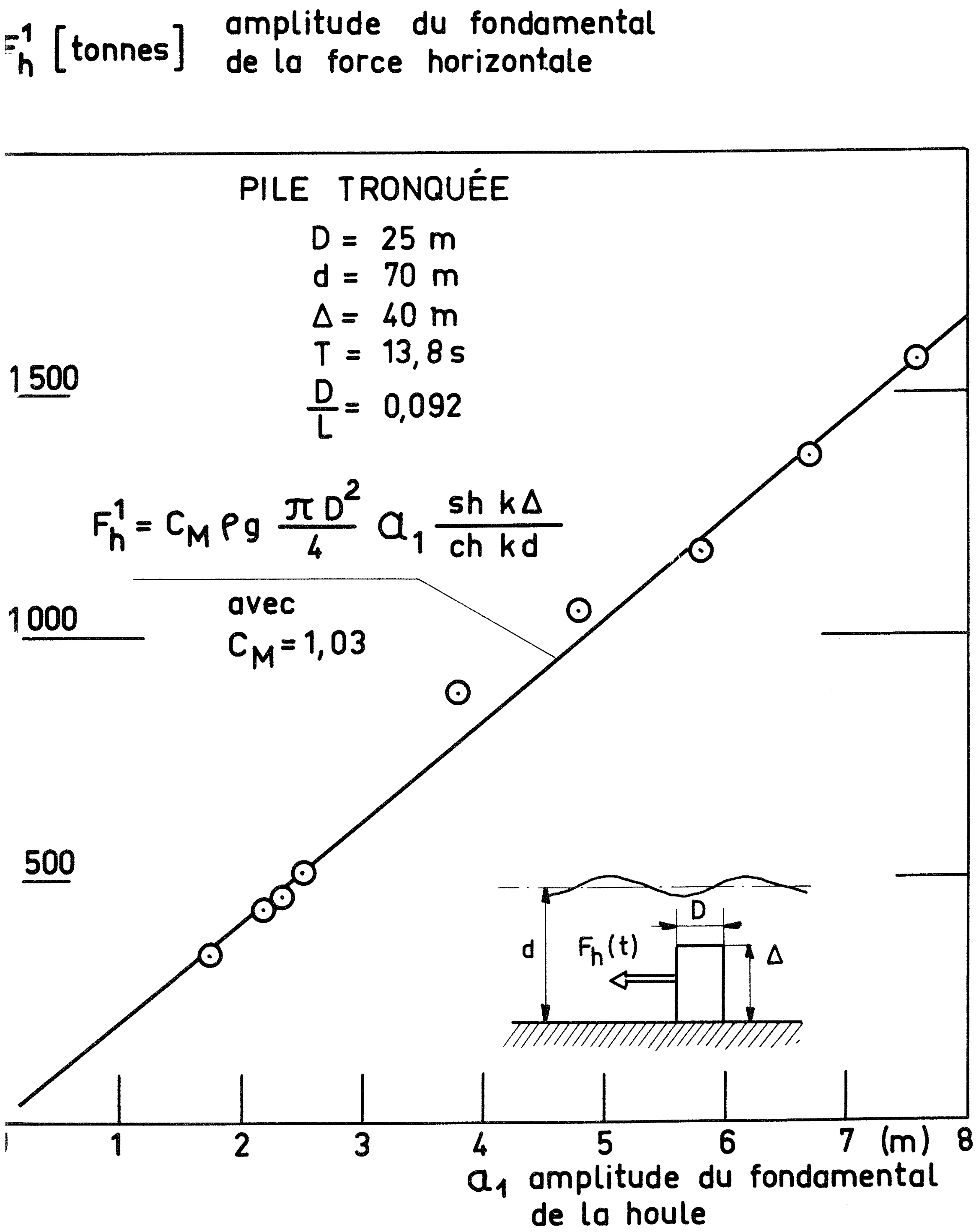
F_h^1 amplitude du fondamental de la force horizontale.
[t]
4000
3000
2000
1000
0
$F_h^1 = C_M \rho g \frac{\pi D^2}{4} a_1$ th kd
avec
C_M = 1,62
PILE EMERGENTE
D = 25 m
d = 70 m
T = 13,8 s
$\frac{D}{L}$ = 0,092
$F_h(t)$
d
D
1
2
3
4
5
6 [m]
a_1 amplitude du fondamental de la houle

14

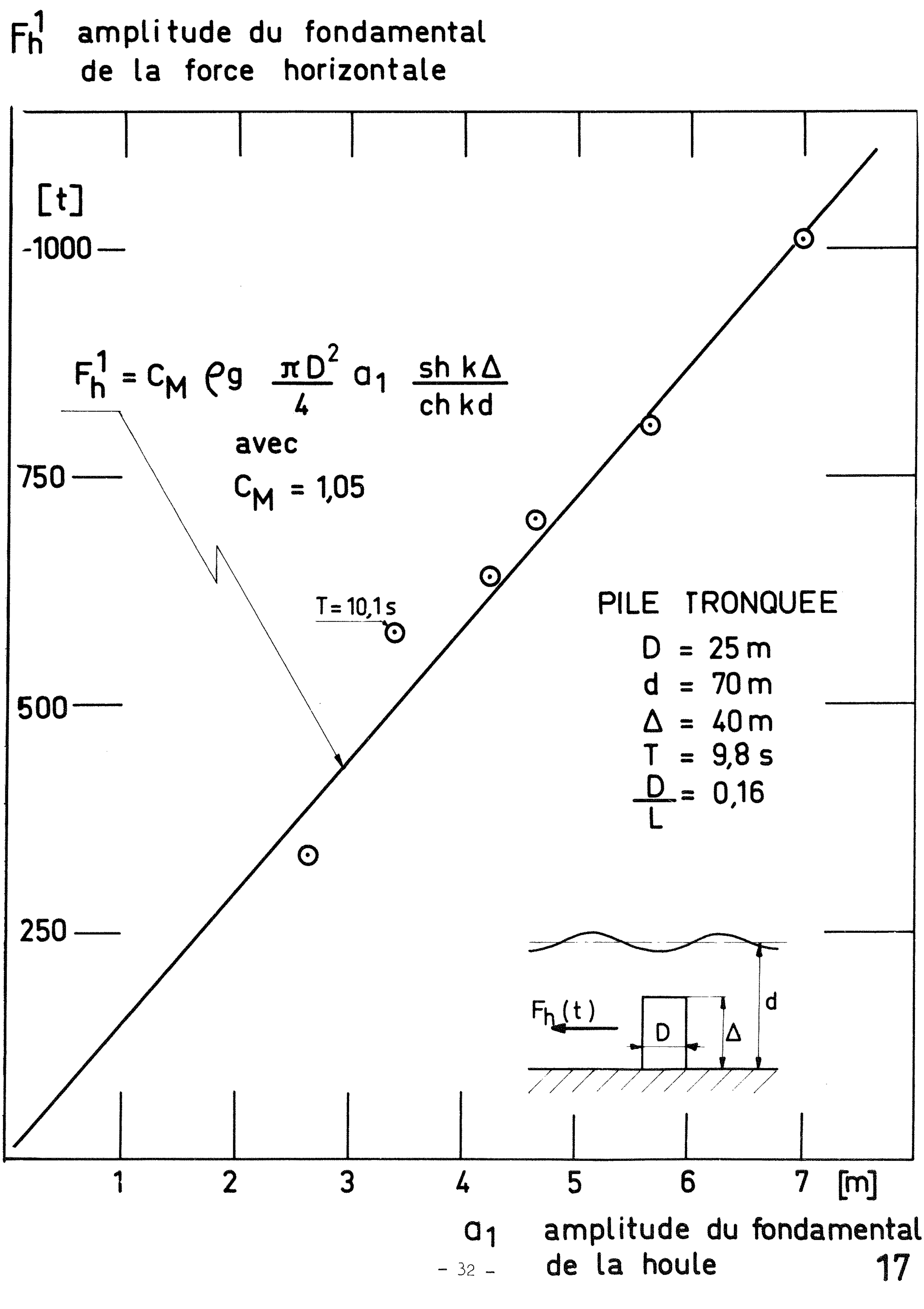


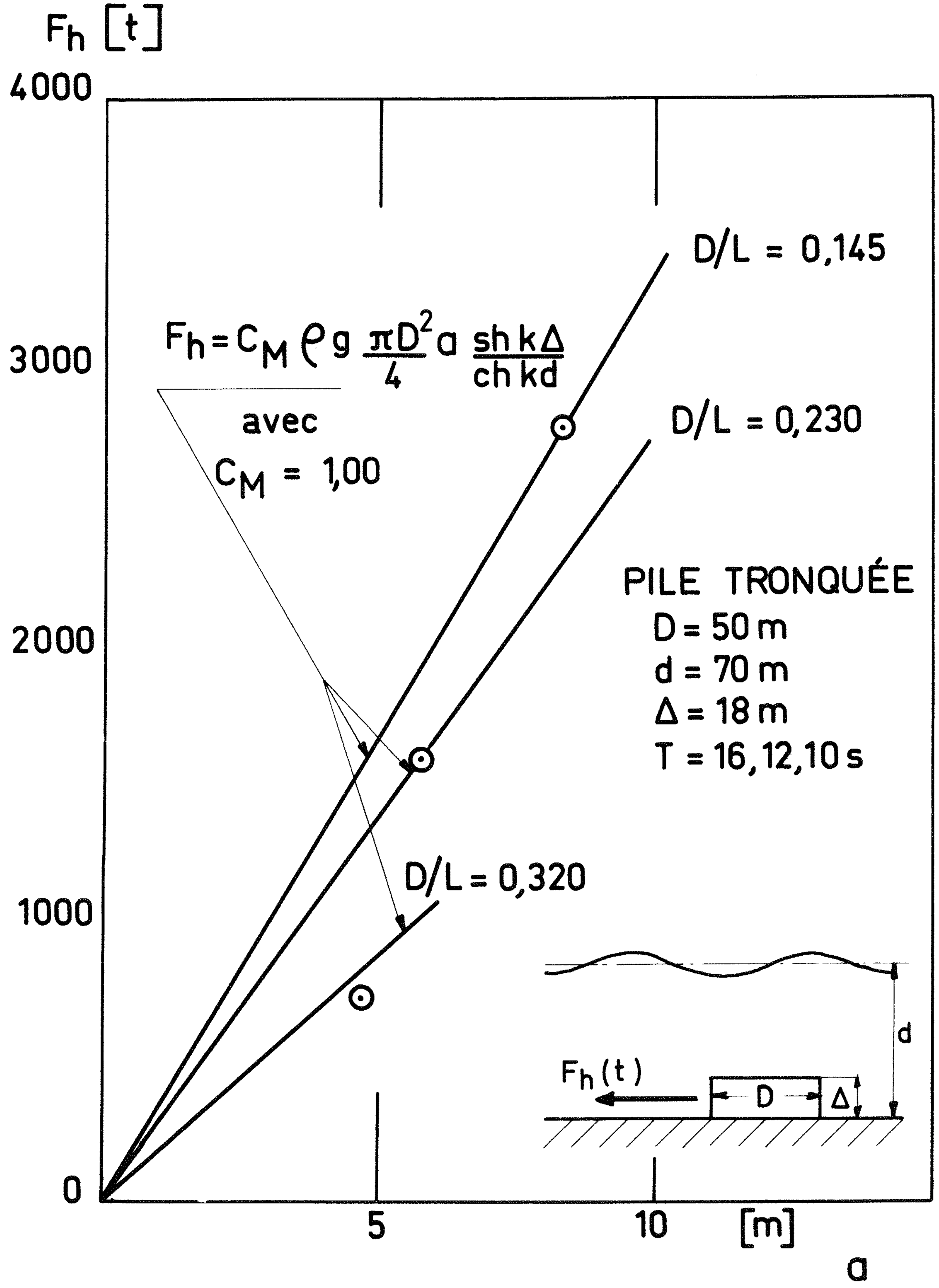
F_h^1 [tonnes] amplitude du fondamental de la force horizontale.
000
000
000
000
0
PILE TRONQUÉE
D = 25 m
d = 70 m
Δ = 57 m
T = 13,8 s
$\frac{D}{L}$ = 0,092
$F_h^1 = C_M \rho g \frac{\pi D^2}{4} a_1 \frac{sh\, k\Delta}{ch\, kd}$
avec
C_M = 1.00
d
D
Δ
1
2
3
4
5
6
[m]
a_1 amplitude du fondamental de la houle

15

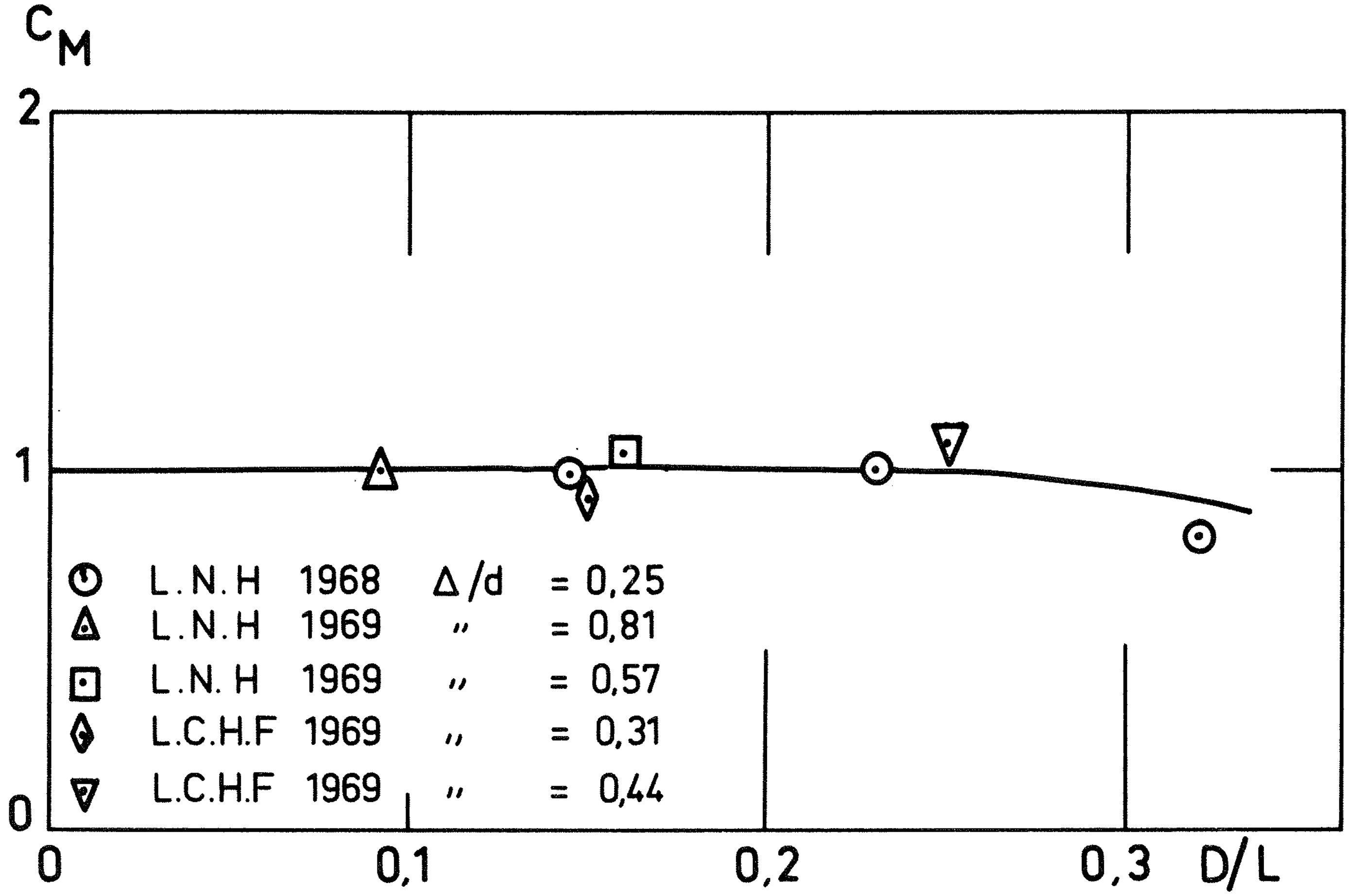

F_h^1 [tonnes] amplitude du fondamental de la force horizontale
PILE TRONQUÉE
D = 25 m
d = 70 m
Δ = 40 m
T = 13,8 s
$\frac{D}{L}$ = 0,092
$F_h^1 = C_M \rho g \frac{\pi D^2}{4} a_1 \frac{\text{sh } k\Delta}{\text{ch } kd}$
avec
$C_M = 1,03$
1 500
1 000
500
1
2
3
4
5
6
7 (m) 8
a_1 amplitude du fondamental de la houle
D
d
$F_h(t)$
Δ

16


F_h^1 amplitude du fondamental
de la force horizontale
[t]
1000
750
500
250
$F_h^1 = C_M \rho g \frac{\pi D^2}{4} a_1 \frac{sh\, k\Delta}{ch\, kd}$
avec
$C_M = 1{,}05$
T = 10,1 s
PILE TRONQUEE
D = 25 m
d = 70 m
Δ = 40 m
T = 9,8 s
$\frac{D}{L} = 0{,}16$
$F_h(t)$
D
Δ
d
1
2
3
4
5
6
7
[m]
a_1 amplitude du fondamental
de la houle
17


F_h [t]
4000
3000
2000
1000
0
5
10
[m]
a
D/L = 0,145
D/L = 0,230
D/L = 0,320
$F_h = C_M \rho g \frac{\pi D^2}{4} a \frac{sh\,k\Delta}{ch\,kd}$
avec
C_M = 1,00
PILE TRONQUÉE
D = 50 m
d = 70 m
Δ = 18 m
T = 16, 12, 10 s
$F_h(t)$
D
Δ
d

18



COEFFICIENT HYDRODYNAMIQUE C_M
POUR DES CYLINDRES VERTICAUX
TRONQUES DE HAUTEUR RELATIVE
$\Delta/d \leqslant 0{,}8$

19

THE INFLUENCE OF REYNOLDS NUMBER ON WAVE FORCES

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SUMMARY

Information, published to date, on the effect of viscosity on wave forces is very indefinite. A series of experiments carried out in the Pulsating Water Tunnel at the Hydraulics Research Station Wallingford under closely, controlled conditions clearly defined the variation of primary forces with Reynolds Numbers up to 6×10^5. Additionally secondary high frequency forces resulting from the shedding of eddies were observed. The findings of this work cast doubts on the validity of tests on model structures subjected to wave action.

INTRODUCTION

For the past two decades, since the proposition of the now familiar Morison Equation[1] there has been a great deal of discussion on the importance of the effects of viscosity on wave forces. Over this period the experimental evidence published has been very inconclusive.

This is understandable to some extent for to measure wave forces over a wide range of the governing parameters under controlled conditions has hitherto not been possible. Experiments on a large scale in the sea are subject to the random conditions found there presenting difficulties in the subsequent analysis. The early work of Harleman and Shapiro[2] assumed the combination of the diffraction theory of MacCormy and Fuchs with drag forces; the latter being based on the steady state values of drag coefficient Cd. In order to support this theory the forces on a pile were determined experimentally with indefinite results: this may have been due, in part, to the assumption of a constant coefficient throughout the cycle. Furthermore, if there is a significant variation of Cd with Reynolds Number the assumption of a constant coefficient with depth would lead to errors in analysis.

Keulegan and Carpenter[3] after measuring the forces on horizontal cylinders due to standing waves reached the conclusion that "a correlation between the coefficients and Reynolds number $U_m D/\nu$ does not appear to exist". It is possible that the lack of apparent correlation was due to the limitations of the experimental conditions. Again, since the cylinders were of significant size compared with the depth of water it is possible that there was a Froude Number effect.

On the other hand, Keim[4] carried out experiments under rectilinear acceleration conditions and found a relationship between the resistance coefficient and Reynolds Number within the very limited range of his experiments. The mass of field data collated by Wiegel, Beebe and Moon[5] gives no clear indication of the dependancy of drag coefficient upon Reynolds Number. The conclusion reached by these authors was no more than that the magnitude of the coefficients was of the same order as those for steady state flow.

Many researchers have felt that much of the scatter in results was due to using linear wave theory in the analysis and have resorted to higher order theories. However it has been suggested by Agerschou and Edens[6] that fifth order theory is not superior to first order theory. Indeed they found indications of the opposite being true.

Other researchers have approached the problem of analysis from the statistical rather than the deterministic point of view to bring order from chaos. This approach is not entirely convincing in that the methods used are subject to the validity of the same assumptions made by deterministic methods and are likely therefore to ignore the effects of an unrecognised parameter.

This brief and incomplete review of the subject of the influence of Reynolds Number on wave forces has been given to explain why it was considered necessary to carry out yet another series of experiments under controlled conditions. In 1967 the Hydraulics Research Station, Wallingford carried out a series of experiments in a pulsating water tunnel on behalf of a consortium of firms interested in off-shore structures, organised by the Construction Industry Research and Information Association. The tunnel, designed originally to study forces on a pipe-line resting on the sea bed, has a test section 2.3 metres high by 0.5 metres wide. It has a semi-orbit range of 0 - 2.5 metres and a period range of 4 - 14 seconds. Additionally, a uni-directional current of up to 0.6 m/s can also be superposed upon the oscillatory motion. This experimental facility eliminated two of the variables normally encountered i.e. gravity and depth thereby giving a closer control of the experiments. The Reynolds Number range was taken up to 6×10^5.

In addition to the measurement of drag forces the Hydraulics Research Station, on its own behalf, decided to study the effects of viscosity upon transverse forces. Wiegel, et al[5], mention considerable vibrations and the fatigue failure of a pile but on the whole there has been very little information published on this subject.

THE EXPERIMENTS

Eight cylindrical sections were tested with diameters ranging from 0.3 m down to 0.025 m. Each test piece was mounted in turn in the centre of the test section at mid-height. In order to avoid the boundary layer effects due to the walls only the centre 0.3 m of the test pieces were utilised for force measurements, the 0.1 m matching cheeks on either side being fixed to the tunnel walls. The centre section of the test piece was attached to one side cheek by a cantelever arrangement to which strain gauges were attached. The other side cheek was merely a dummy fixed to the wall and quite free of contact with the specimen. The natural frequency of the system (50-60 c/s) was designed to be well above the probable frequency of excitation (<20 c/s).

The measured forces were functions of five independent variables,

ℓ, the density of water

μ the viscosity of water

a, the semi-orbit length

T, the period of motion

and D, the diameter of the test piece

For the plotting of results these variables were grouped ** in two dimensionless parameters, $aD/T\nu$ and a/D. The former is a form of Reynolds Number whilst the latter may be interpreted in two ways. It may be looked upon as either describing the geometry of the experiment or as being a function of the ratio produced by dividing the drag force by the acceleration force. It is effectively the same as the reciprocal of Iversen's modulus $(du/dt)\, D/u^2$, and the Keulegan parameter U_mT/D.

In order to avoid making the assumption that the Morison equation was valid it was decided to use a force parameter rather than deduce drag and mass coefficients. The choice of a suitable force parameter was difficult. If the maximum forces were essentially drag forces then $FT^2/\ell a^2D$, where F is the maximum measured force per unit length, would be suitable. If on the other hand the maximum forces were principally acceleration forces, $FT^2/\ell aD^2$ would be a more suitable parameter. Since the experiments covered a wide range of conditions a compromise parameter $FT^2/\ell D^3$ was used. This may be considered as scaling the force in terms of test piece diameter and either unit velocity or unit acceleration i.e. $F/\ell D\,(T/D)^2$ or $F/\ell D^2\,(T/D)^2$. Although this might appear to offer a satisfactory compromise, when plotted against the parameter of a/D it does give undue weight to the test piece diameter. However its use was thought to be justified in that it enabled the Reynolds Number effect to be clearly defined. From the Morison Equation a simple relationship between $FT^2/\ell D^3$ and the mass and drag coefficients at the instant of maximum force is obtained i.e.

$$\frac{FT^2}{\ell D^3}\,\frac{1}{4\pi^2} = \frac{Cd}{2}\left(\frac{a}{D}\right)^2 + \frac{\pi^2}{32}\,\frac{Cm^2}{Cd}$$

The values of the force parameter were plotted against a/D for narrow bands of Reynolds Number. The scatter within each band was surprisingly small being largely attributable to the band width. From the individual plots a composite diagram was constructed which showed clearly the substantial effect that viscosity has on wave forces. In Fig 1 are shown two typical curves to form the basis of subsequent discussion.

Since transverse forces are primarily a function of velocity, the transverse force parameter was taken to be $C_L = LT^2/2\pi^2 a^2 D$ where L is the transverse force. This parameter plotted against Reynolds Number, Fig. 2 shows considerable scatter reflecting the variable nature of the **transverse forces.

These forces are cyclic, oscillating with a frequency much higher than that of the imposed frequency. The parameter for describing this vibration was taken to be the normal Strouhal Number S = ND/V where N is the frequency and V the velocity. It was found that the value of Strouhal Number showed remarkable consistency being 0.2 ± 6%. There was no variation with a/D or with Reynolds Number.

DISCUSSION

** The experimental programme demonstrated clearly the two effects of viscosity on wave forces viz. the primary forces in-line with the motion of the water and the higher frequency forces. Perhaps the most surprising outcome was the magnitude of the transverse forces in the lower Reynolds Number range where in some cases they equalled the in-line forces. It was almost as surprising to find that the frequency at which eddies were shed, this being the mechanism giving rise to the transverse forces, was apparently completely independant of acceleration forces.

In view of the clear definition of the transverse forces it was to be expected that similar high frequency forces should have appeared in the in-line forces at twice the Strouhal frequency. In fact the force records did show deviations about a mean force line but unfortunately it was found to be extremely difficult to determine the frequency of these deviations. The mean force line was the smoothed force curve obtained by ignoring the higher frequency oscillations. The deviations appeared to take place at several discrete frequencies, one superposed upon the others as distinct from a spectrum of frequencies.

The magnitude of the high frequency in-line forces was, generally speaking, low compared with the main in-line force being less than 10% of the mean measured force. Only at the low end of the Reynolds Number range did they reach the significant proportion of 50% of the mean. On the whole they were approximately one half of the strength of the transverse forces.

Perhaps the significance of these high frequency forces lies not so much in strength as in frequency. In the context of braced structures subject to wave action, if it so happened that the exciting frequency agreed with the natural frequency of a structural member, albeit a slender member, then fatigue is a possibility.

Returning to the subject of previous investigations it seems probable that the large degree of scatter in the values of published coefficients may be attributed to deviations about mean forces due to eddy shedding. This phenomenon would also explain the result of negative values of Cm, recorded by some authors. In the analysis of the results obtained from the H.R.S. experimental programme the maximum value of the mean force curve was used rather than the absolute maximum. Estimates of Cm at times of zero velocity and maximum force, again based on the mean force curve, both indicated values of 2.0.

The work described in this paper was made necessary by a proposal to build and test small scale models of braced structures and knowledge of the likely scale effects was required. It is fitting, therefore, to conclude by discussing the results in this context and the simplest way of doing this is to consider a hypothetical case.

A braced structure, standing in 60 m of water, is subjected to 9 m waves with a 10 s period. The maximum force on a 0.3 m diameter member 25 m below may be deduced as follows. The orbital velocity at this depth would be about 1 m/s and thus the Reynolds Number would be about 3.5×10^5. The semi-orbit length would be 1.7 m giving an a/D value of 5.6. From Fig. 1. the value of $FT^2/\ell D^3$ appropriate to these values of Reynolds Number and a/D is 400. Hence the maximum force on such a member would be about 12 Kg/m.

If a 1/10 scale model were built and subjected to corresponding conditions then the Reynolds Number would be about 0.1×10^5 and the value of $FT^2/\ell D^3$ 1250. In this case the force when scaled up to prototype would be about 37 Kg/m. Thus even with such a large, impracticable model the forecast of forces would be in error by a factor of 3. Smaller models would give greater errors.

The knowledge of this scale effect does not solve the problem of interpreting model results; since orbital velocities diminish with depth there will also be a varying scale effect with depth. Again, if a braced structure is being tested the variation in sizes of members at a particular depth will give rise to a corresponding scale effect at that depth. At the moment, therefore, it would seem as though model tests are not likely to be successful. However it is possible that interference effects between members will negate the effects of viscosity. The Hydraulics Research Station are now planning a programme of research on this aspect of the subject.

ACKNOWLEDGEMENTS

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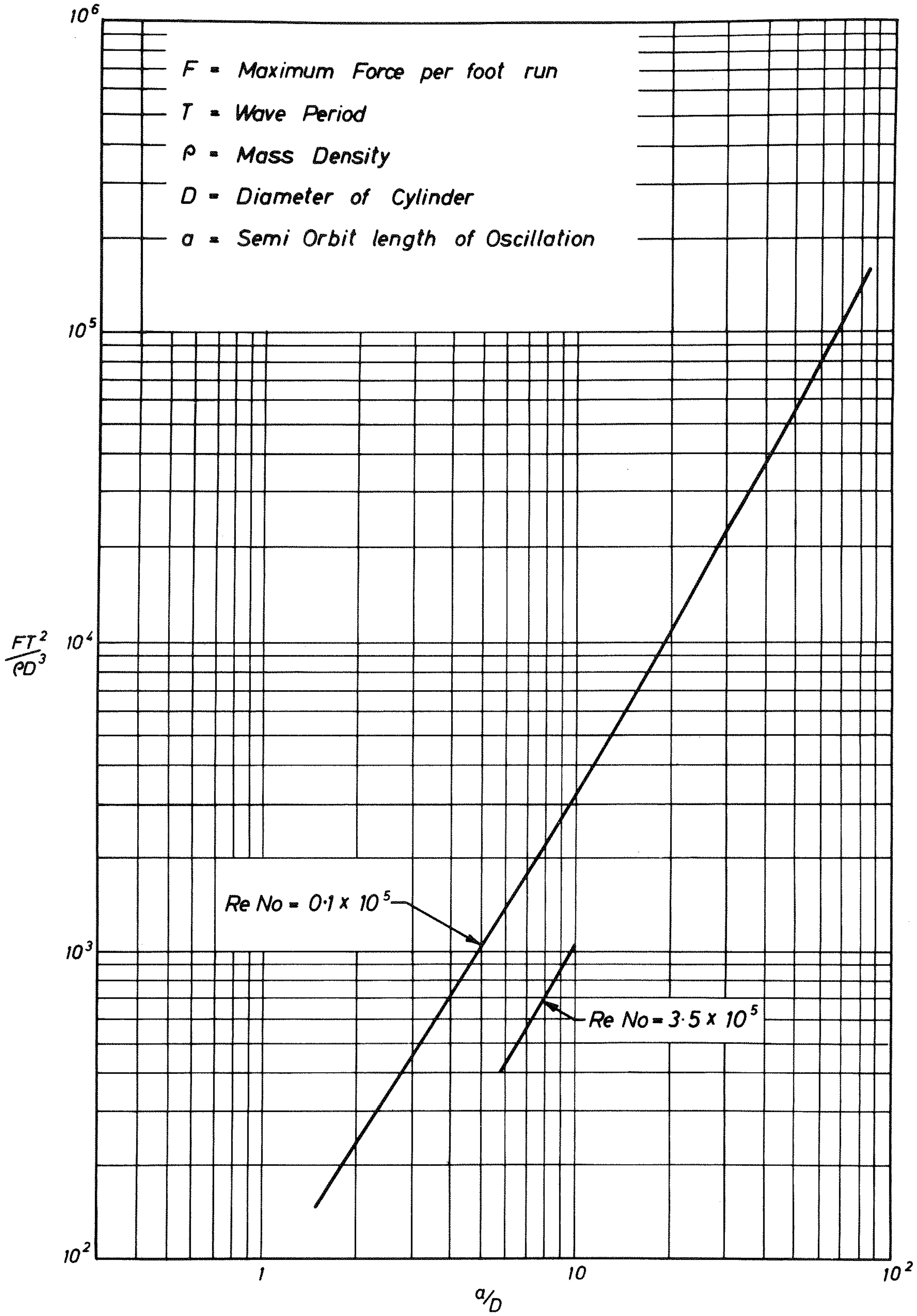
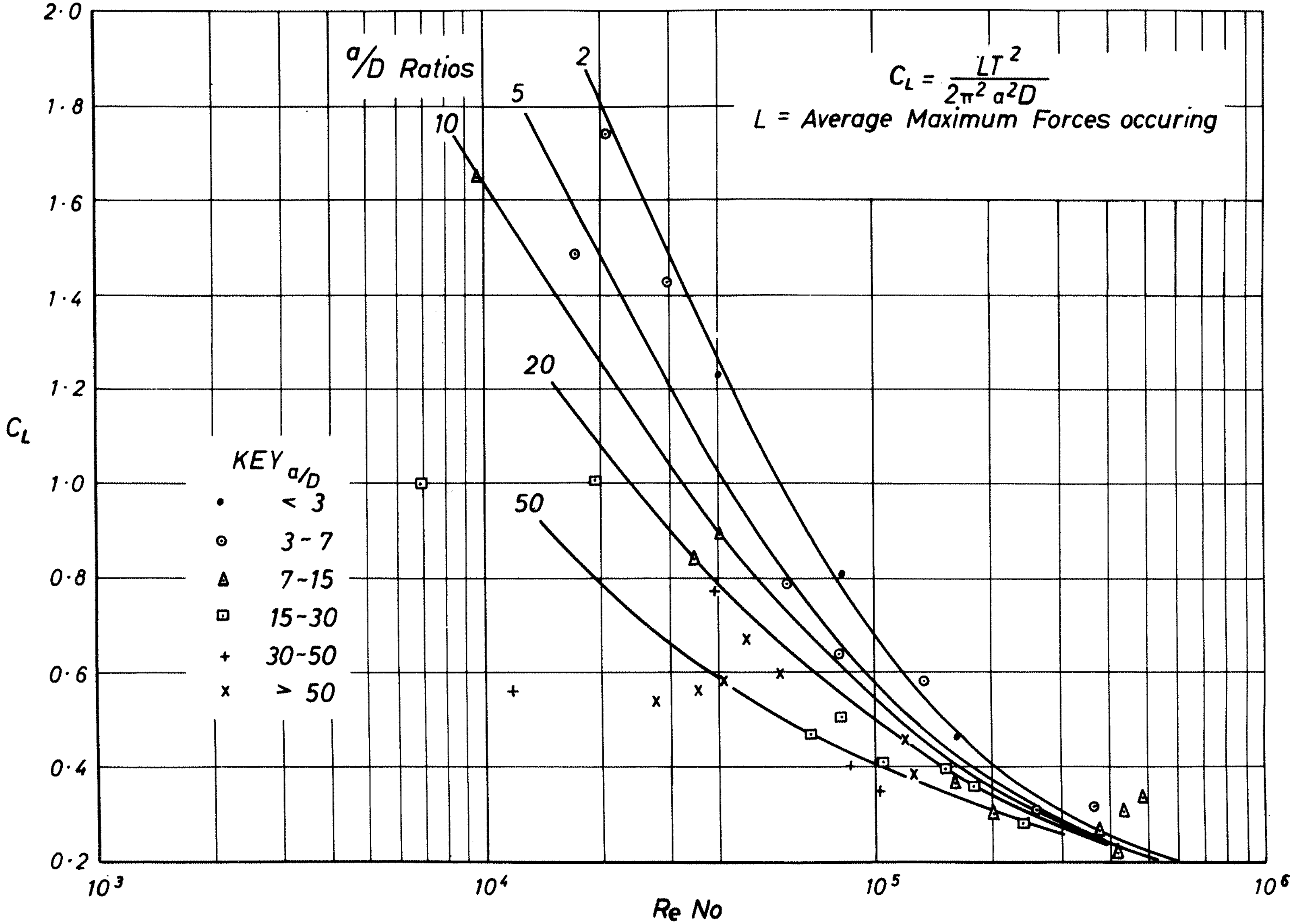


FORCES IN THE LINE OF MOTION

FIG 1



TRANSVERSE FORCES

FIG 2

DISCUSSION ON PAPER 13

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The influence of the Reynolds number on wave forces has been demonstrated for a range of a/D values in which the acceleration forces are negligible. As it is known that there is a strong dependency of the drag coefficient on the Reynolds number in uniform flow, it is not surprising that the same occurs in periodic flow. The values of $FT^2/\rho D^3$ given for a/D = 10 correspond to C_D values of 1.65 and 0.55 for Re = 0.1 . 10^5 and 3.5 . 10^5 respectively. These values are within the range given by WIEGEL. Although it is a good thing to be aware of scale effects, there is no need to mystify them.

THE INFLUENCE OF WAVE FORCES ON THE DESIGN OF OFFSHORE STRUCTURES FOR THE OIL INDUSTRY

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I. INTRODUCTION

Until after the second World War, the ocean was regarded by the oil industry only as a cheap means of transportation. Before the war some timber structures very close to the shoreline had been constructed, but drilling for oil on the continental shelf did not really start until 1947, when the first steel platform was erected in 20 ft. of water in the Gulf of Mexico. In the following twenty years, rapid progress took place not only in the number of constructed platforms, but also in the water depths in which these structures were placed. In 1955, 100' was reached, 200' in 1959, 285' in 1965 and in 1967 a platform was placed in 340' of water. This is certainly not the end. Platforms for 600' of water are technically feasible and are already on the drawing boards.

For exploration drilling, movable structures are preferred and these units have been constructed in great variety - jack-up units with from three to ten legs and submersible units, all still bottom supported when drilling. Floating units have also been developed - boat-shaped vessels, barges, catamarans and finally the semi-submersibles. Anyone who follows the patent literature will find a continuous stream of inventions in this field.

The designer of these structures is confronted with the problem of constructing them in such a way that they are able to withstand without significant damage the loads imposed by the environmental conditions. By "environmental conditions" we generally mean waves, winds and currents, but icefloes and earthquakes can also be included. In this paper we will limit ourselves mainly to wave forces.

In the first place the design limits have to be established, which means that we have to determine the maximum stresses to which the structure will be exposed. As regards the waves, we need the help of an oceanographer to predict the maximum wave height for the area in which the structure is to be placed. At the same time a policy decision has to be made, for, as we all know, the probability that a certain wave height will be exceeded increases with time. For this reason, we first have to decide on the risk factor we are prepared to accept. In the oil industry it is common to accept the so-called 100 year wave as the design wave. That means a wave with a probability of occurrence of once in a hundred years. Of course, we can never predict when this wave or an even higher one will occur. Statistics show that the encounter probability E, during a lifetime L for a return period T_1 can be derived from:-

$$E = 1 - \left(1 - \frac{1}{T_1}\right)^L$$

So when we design a structure with an expected lifetime of 20 years for a wave with a return period of 100 years, the probability that this wave will be encountered during this lifetime is $1-(1-\frac{1}{100})^{20}$ = 16% and this in fact is also the risk we are willing to accept.

Most structures are designed only for such maximum stresses, on the assumption that, if they can withstand these, they can withstand all others too. This belief is based on experience gained in the Gulf of Mexico, the birthplace of all offshore engineering techniques. Here hurricanes producing enormous waves alternate with very calm seas.

For more exposed areas, such as the North Sea, a second design criterion has to be taken into account - the lifetime of the structure under loads less than the maximum. In some cases, structures have succumbed to so-called low-cycle fatigue: material failure due to the repeated imposition of loads less than the maximum permissible.

In the following a short description will be given of the techniques used in offshore engineering. Perhaps this will give a rather pessimistic or even negative impression. This is not the intention, however. To date a large number of structures have been designed and in general they have performed satisfactorily. The number of failures is small, and only a small percentage of these is due to design faults. In this respect one can say that the offshore industry as a whole has done very well. When criticisms of methods or theories are made in this paper it should be remembered that the only purpose of this is to underline the difficulties the designer encounters in his day-to-day work and the problems with which he is faced when he is required to design yet larger structures for use in still deeper water.

The offshore industry is expanding rapidly and those working in this field have to meet the challenge that the demands on structures are becoming more and more stringent. To justify these structures economically, the designer has to approach the "optimum design" and this in turn only becomes possible if he can use more refined methods and theories. It is the need for these refinements that will be outlined in the following sections, and it is here that the offshore designer joins forces with colleagues working in the hydraulic and other laboratories.

II. FIXED PLATFORMS

Normally the forces resulting from a two-dimensional regular wave are used for the design. Much criticism of this procedure is heard, for in reality waves are neither unidirectional nor regular. This is true, but the use of such a "design wave" can be justified in various ways.

It is the task of the engineer to design the structure in such a way that it can fulfil its intended purpose and to do this in the quickest and easiest way. The use of idealised conditions simplifies the necessary calculations tremendously and speeds up the design. So the use of such idealised waves is justified as long as the calculated forces are equal to or higher than those ever met with in reality under the specified conditions. The calculated forces must however, be "realistic", since, if the values chosen for the forces to be withstood are too high, the design will become uneconomical. It is strange that although many hundreds of structures have been designed on these principles, very little effort has been made to check the underlying assumptions. To calculate the forces resulting from regular waves, the Morrison equation is generally used:

** $$\vec{F} = \tfrac{1}{2}.C_D.\rho.\ell.d.|\vec{v}|.\vec{v} + \pi.\rho.\frac{C_M}{4}.d^2.\ell.\frac{\partial \vec{v}}{\partial t}$$

This equation was suggested by Morrison et al[1] in 1951. To solve this equation one has to know the velocities $\vec{V}$ and the acceleration $\frac{\partial \vec{V}}{\partial t}$ of the fluid particles and the drag and mass coefficients C_D and C_m.

The water-particle velocities and accelerations are calculated with the aid of theories concerning unidirectional regular waves. The choice of the best wave theory is still a matter on which offshore engineers disagree. In Shell, the choice of the wave theory depends on the magnitude of two dimensionless parameters d/gT^2 or H/gT^2, d being the water depth, H the wave height and T the wave period. The theories used include: a modified solitary, a shallow water, Airy, Stokes-Struik third order, Stokes fifth order, Chappelear's numerical and a highest wave theory. The majority of waves used as design waves lie within the range of applicability of the Stokes fifth order or Chappelear's numerical theory.

The drag and inertia coefficients must also be determined. A method proposed by Wiegel[2] has been used to analyse a large quantity of storm-wave data. According to the two-dimensional wave theories, horizontal acceleration is zero at the time the crest of the wave passes the pile, so at that time the total force on the pile is determined by the drag term only. On the other hand when the wave surface passes the still-water line, the horizontal velocity is zero and the wave force is contributed by the inertia term.

A large number of storm waves have been analysed with the Wiegel method. There is a large spread in the coefficients so computed. Variations in drag coefficients ranging from 0.05 to 1.6 and variations in inertia coefficients ranging from 0.1 to 3.4 are found under seemingly similar conditions. It was also found that the drag coefficient decreases with increasing wave height and increasing wave period and also increases with distance below water level. No correlation with the Reynolds number was found. There are reasons for believing that the inertia coefficient is a function of the distance from the sea floor. A number of factors have contributed to these, at first sight, somewhat discouraging results.

In the first place the water particle velocities and accelerations are calculated with the aid of an appropritate wave theory for the measured wave height and wave period. In reality, however, a regular two-dimensional wave has never been present: the measured wave force has always been due to a complete spectrum of waves, ** approaching from various directions. Current velocity is also measured. It certainly does not go without saying that the same drag coefficient is applicable for a rectilinear flow as for an oscillatory flow. Moreover, it is permissible to wonder whether some element of error is not inherent to the Morrison equation. It does not, for example, take into account the dependence of the coefficients on the wave frequency, ω, although in other fields of engineering (aeronautics, naval architecture) this dependence is known to be very important. Wiegel[3] draws our attention to the formation of eddies, resulting in an oscillating flow with high Reynolds number, which is quite different from rectilinear flow with a high Reynolds number, especially when the pile diameter is not small with respect to the wave height.

As the water depths for which the platforms are intended become greater, the designs show a tendency to use piles of larger diameters. This being so, the problems just mentioned become more important and in our view more research is required.

One may wonder whether it is necessary to use an equation like Morrison's. No designer, in fact, is interested in water-particle velocities and accelerations requiring large computer programmes to calcutate them.

It may be possible that a transfer function can be given, directly relating the wave force to parameters depending on wave height, wave period, water depth, pile diameter and distance to the surface. For large-diameter piles where the drag term only makes a small contribution to the wave force and scale effects are consequently small, such transfer functions can perhaps be derived from the results of model tests.

Such transfer functions would also be of great value in dealing with irregular waves. Hitherto the designer was only little interested in irregular waves, since his structure was stiff and he was only concerned with maximum forces for the determination of the size of members. For platforms in deep water, however, dynamic effects have to be taken into account, so irregular waves have to be considered.

Knowing the transfer function as described above, one can easily calculate the spectral density of the wave force when the spectral density of the sea surface is known. Although measured wave spectra are available only for certain areas, theoretical wave spectra can replace them (Neumann, Fisher and Roll, Philips, Pierson). One can also calculate the transfer function starting from Morrison's equation. This is done by Borgmann (1968). The problem here is that the use of transfer functions assumes that the system is linear and that the superposition principle is valid. Because the term $\vec{V}/.\vec{V}$ appears in the drag term of Morrison's equation, these conditions are not fulfilled. A linear approximation of the drag term is allowable at any rate for large diameter piles. Such a process is described by Penzien[4]. Still, one has to keep in mind that by using spectral analysis theories, one treats the problem on a statistical basis, thereby losing the phase relations between wave profile and wave forces. Consequently, for designing the size of members this approach is less suitable than the design wave approach.

III. FLOATING PLATFORMS

The same problems as those described for fixed platforms are also encountered with floating platforms. Wave force calculations are complicated by the fact that the body itself moves so wave velocities and accelerations have to be compensated for the velocities and accelerations of the platform. The motion itself, however, is caused by the wave forces, so this problem can only be solved by an iterative process. The problem becomes even more complicated because there is mutual interference on the part of the members.

Semisubmersible units are mainly composed of large-diameter tubulars. This enables us to determine the motions and exciting forces by model experiments, without large errors due to scale effects.

Normally we determine transfer functions for the motion giving the relation between motion and wave height as a function of the frequency, as well in amplitude relation as in phase relation.

In principle there are two ways in which this relationship can be determined.

**In the first way, the floating body is subjected to the waves and both the motions and the waves are measured. This can be done for both regular and irregular waves. The irregular wave can be given as the superposition of a large number of regular waves

$$r = \sum_{n=1}^{\infty} r_n . \cos(\omega_n . t + \varepsilon_n)$$

For the heave motion we now may write

$$z = \sum_{n=1}^{\infty} |Y_{zr}(\omega_n)| . r_n \cos(\omega_n . t + \varepsilon_n)$$

in which $|Y_{zr}(\omega_n)|$ is the real part of the transfer function, or amplitude operator.

The mean square of the wave amplitude over a long period in the frequency band between ω and $d\omega$ equals

$$\sum_{\omega}^{\omega + d\omega} \tfrac{1}{2} r^2(\omega)$$

We know that this is equal to the spectral density

$$f_{rr}(\omega).d\omega = \sum_{\omega}^{\omega+d\omega} \tfrac{1}{2} r^2(\omega)$$

In the same way we find as the mean square of the heave response

$$f_{zz}(\omega).d\omega = \sum_{\omega}^{\omega+d\omega} |Y_{zr}(\omega)|^2 . r^2(\omega)$$

So the heave amplitude operator can be calculated as

$$|Y_{zr}(\omega)| = \sqrt{\frac{f_{zz}(\omega).d\omega}{f_{rr}(\omega).d\omega}}$$

** The second approach by which the transfer function can be calculated is by solving the differential equation of motion. The general equation for a body moving through a fluid is:

$$m\ (1-a)\ \ddot{s} - b\dot{s} - cs = F$$

in which s = a vector, indicating the displacement

$\dot{s}$ = first time derivate of that vector

$\ddot{s}$ = second " " " " "

m = mass of the body

a = added mass term coefficient

b = damping coefficient

c = restoring force coefficient

F = sum of all external forces acting upon the body.

The coefficients a, b and c can be determined by a forced oscillating technique in still water. The wave forces are measured on a restrained model vessel.

The transfer function can now be calculated as

$$\frac{z}{H/2} = \frac{F_s}{\frac{H}{2}\sqrt{\{c + \omega^2 m(1-a)\}^2 + b^2\omega^2}}$$

in which F_s is the wave force and H the wave height.

The figure is an amplitude calculator for heave, calculated in this way. To predict the heave motion, an irregular sea is used. For this sea, with a significant wave height of 8.65 m and a significant period of 11.54 seconds, a Neumann spectrum is used. From the resulting heave spectrum, it follows that the significant heave motion will be 2.54 m and the maximum heave motion 1.86 x 2.54 = 4.72 m.

IV. CONCLUSION

Hitherto offshore structures have been mainly designed by using regular wave * theories. The methods used yield acceptable results as long as the ratio of wave height to diameter of tubulars is large. For large-diameter tubulars, the validity of the methods is doubtful and further research on wave forces is required.

For the design of fixed structures in deep water and of floating bodies, dynamic effects have to be taken into account and this can only be done by considering irregular waves. In these studies model experiments will be necessary on a large scale. It is for this reason that those of us who are engaged in offshore work are delighted with these beautiful new facilities here at the Waterloopkundig Laboratorium.

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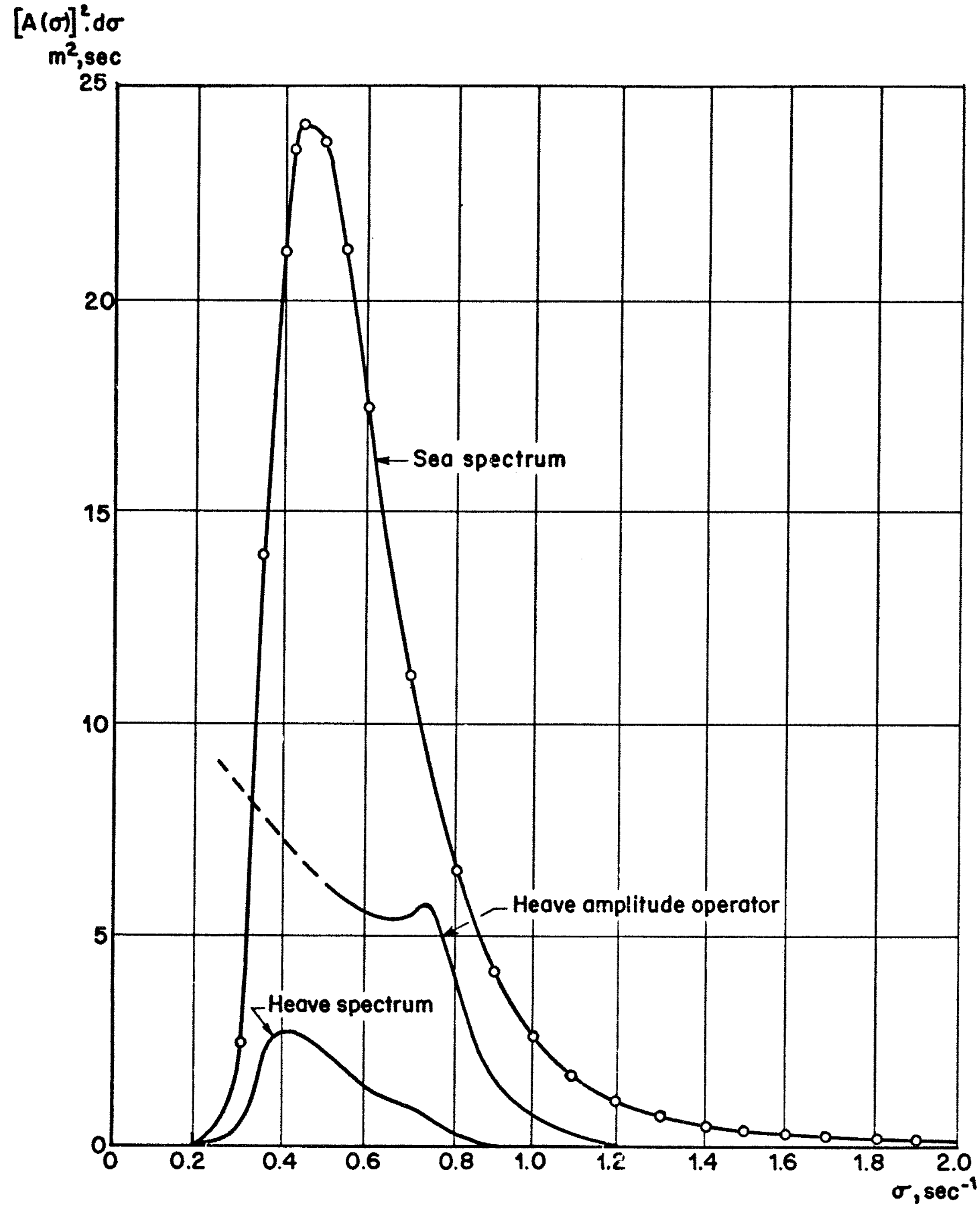

HEAVE OF A FLOATING DRILLING RIG IN IRREGULAR SEAS

NEW WIND-WAVE FLUMES AT DELFT

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Introduction

On March 24, 1969, the new wind-wave flumes at the Delft Hydraulics Laboratory (Figures 5 and 6) were officially opened by the Director-General of the Rijkswaterstaat. The design work started in 1964, and the building and installations were completed in November 1968. A general description of this equipment and some insight into its underlying principles are given below.

Wave generation

In the past basic research was undertaken in the Delft Hydraulics Laboratory on the generation of artificial wind-waves. Studies of the fetch necessary to obtain pure wind-waves of the desired dimensions in test facilities had, however, long showed that this required an unrealistic length of the flumes. So in the existing installations waves were generated by wind in combination with a monochromatic wave-maker.

By careful choice of the periods and amplitudes of the regular waves and, by using rather high wind speeds it was possible to get irregular waves of the required significant height, but it transpired that neither the wave-height distribution nor the energy spectrum as observed in nature could be reproduced in this way.

In 1966 a prototype of a new wave generator was installed in a flume at Delft, and the experience gained by this instrument led to the final choice of the wave-makers installed in both new flumes.

The installations comprise wave boards driven by a hydraulic servo system which generate waves according to an arbitrary programme. As an input signal, actual wave records from nature can be used by means of punch tape. On the other hand, arbitrary records can be simulated by means of a random noise generator (See paper 2: "Generation of irregular waves on model scales").

The signals are fed into two hydraulic drive actuators, one driving the upper edge of the wave-board, causing its rotation and the other attached to a carriage bearing the wave-board and thus causing its translatory motion.

By this equipment the parameters of the wave-energy spectrum can be varied over the total range required. But in addition to the wave-maker wind appeared to be still necessary to adjust the steepness of the wave fronts. Moreover, the wind itself can effect the attack on structures. Tests in the flumes at Delft and De Voorst showed that wind speeds exceeding 15 m/sec had an adverse effect on the wave pattern. Therefore this is the maximum wind speed in the new flumes. Normally, in combination with the wave-boards speeds not exçeeding 5 m/sec will do.

Dimensions

The length of the flumes is still determined in one way or the other by the necessary fetch in the flumes. It is clear that this length, using the wind mainly to adjust the geometrical shape of the waves, can be much shorter than by using the wind as the only wave-generating force. It was impossible, however, to give an exact prediction of the required length.

The most acceptable approximation was found by a rough extrapolation of the results attained in the old installation, using the required wave length as a parameter. As a result of this extrapolation, a length of 100 m was selected (Figure 1). The achievements of the installation since its completion indicate that this length is sufficient.

To find the necessary width of the new flumes, it was first tried to find a criterion in the desired length of the wave crest in the flumes. Studies were made comparing the crest length of wind-waves generated in a flume and similar waves found in nature, but this study did not provide the requirements for flume dimensions.

Moreover, because it was decided to install a wave-board as the main driving force it was clear that the length of the wave crests (being infinite under these conditions) was no longer a criterion at all. So the now chosen width of the flumes was mainly based on the practical requirement of testing breakwaters and dikes at arbitrary angles to the wave attack. The available width of 4 m in the flumes at Delft and De Voorst turned out to be too small for this purpose, whereas a basin at the end of a flume turned out to be profitable for studies on complex situations and for the reproduction of certain combinations of waves and currents. Based upon these considerations, one of the new flumes was given a width of 8 m with a widened "hammer-end" of 25 m, and the other one a width of only 2 m (Figure 2), which meant that in this flume only wave-attack perpendicular to the models can be simulated. It is expected that the series of 3 flumes now available, with widths of 2, 4 (De Voorst) and 8 m, will enable all problems arising in the near future to be dealt with.

The height of both flumes was fixed at 2.45 m, giving a reasonable possibility of passing through the flumes. The cross-sections of the flumes were minutely tested in a wind tunnel of the Delft University of Technology to ensure a reasonable velocity distribution in the wind profile and to prevent secondary wind currents, thus ensuring a uniform distribution of the shear stresses over the water surface, and also near the walls.

The study led to a cross-section of the 8 m flume in which the wind profile is about 1.50 m larger that the water surface (Figure 3).

As a systematic difference in water depth will give a difference in wave propagation, rigorous requirements to the accepted tolerances had to be laid down.

Control measurements afterwards showed that a random tolerance of less than 2 mm has been reached for the floors in both flumes.

Wind generation

Five axial ventilators, each capable of removing 60 m^3 air per second, are housed in the air-ducts below the flumes. Four of them serve the large flume, the fifth serves the small one. Adjustable D.C. motors make continuous adjustment of wind speeds possible within 2% of the desired value.

Model investigations in a wind tunnel led to the final shape of the diversion vanes, diffusors and stilling chambers in the air-ducts. Thus the air turbulences caused by the ventilators are smoothed and an acceptable velocity distribution above the water level is obtained, even when one ventilator of the large flume is not in operation.

Water circulation system

In both flumes a maximum steady current of 25 cm per second is required in a water depth of 0.5 m. This is done by 5 water-lubricated pumps of 0.2 m^3/sec each at a lift of 8 m, all housed in a pumping-room on the ground floor. Under normal conditions one pump serves the small flume, the other four the large one (Figure 4).

The water flows in and out through perforated pipes in specially-designed stilling channels in the bottom of the flumes. Investigations on small-scale models of these units were made by the laboratory to prevent distortion of the waves above the openings. The whole installation, including pipework, butterfly valves and pumps, is nylon-coated to prevent corrosion which would lead, for instance, to difficulties in making observations of the underwater parts of the models.

Auxiliary installations

Experience in the existing flumes at the Laboratory showed that the construction of models in the test section of the flumes interfered with the test programme on an unacceptable way. So in the new flumes these activities take place on an adjoining site. The completed models can be easily lowered into the test sections by crane.

After a model study on a rubble mound has been finished, the model can be hoisted out and all composing materials be dumped into a screen table on the ground floor from which they can be sorted out and stored again. The service crane for this purpose reaches the test sections of both flumes, the hammer-end and the construction site.

As an additional provision the flumes can be divided into sections by watertight gates, making it possible to empty the test sections, independently of the rest of the flumes, into the main water reservoir situated on the ground floor between the air-ducts.

Wave generators, ventilators, pumps and valves are all operated from control rooms near the respective flumes (Figure 7). These rooms contain, in addition to the main operation and control panels, also the different measuring instruments such as wave height meters, flow meters and strain gauge amplifiers.

Special instruments are available for "direct" evaluation of the statistical properties of measured quantities such as autocorrelation function, energy spectrum and exceedance probability. The control rooms are separated from the flumes by glass walls with doors, so that the models in the test sections can be easily observed and handled during test runs.

The electronic instruments in the control rooms need a low and constant air humidity not exceeding about 50%. To achieve this in rooms situated next to flumes in which an air humidity of some 100% can be reached during test runs, it was necessary to maintain a difference in temperature between the flumes and control rooms. Moreover, a simple air-conditioning unit installed in each control room dries the air entering.

The layout of the equipment was developed by the Laboratory in co-operation with the "Associatie van Ingenieurs en Architekten Buro op ten Noort-Blijdenstein" (consulting engineers), Utrecht, whose staff was responsible for the design and engineering of the plan and for the supervision and overall co-ordination of the construction.

Delft Hydraulics Laboratory: Wind-wave Flumes

completed	1938 (Delft)	1957 (De Voorst)	1968 (Delft)	1968 (Delft)
eff. length (m)	50	100	100	100
eff. width (m)	4	4	8	2
eff. height (m)	0.94/0.99	2.00	2.45	2.45
max. water depth (m)	0.45	0.80	0.80	0.80
max. wind velocity (m/s)	14	25	15	15
max. water circulation (m^3/s)	-	3	1.0	0.4

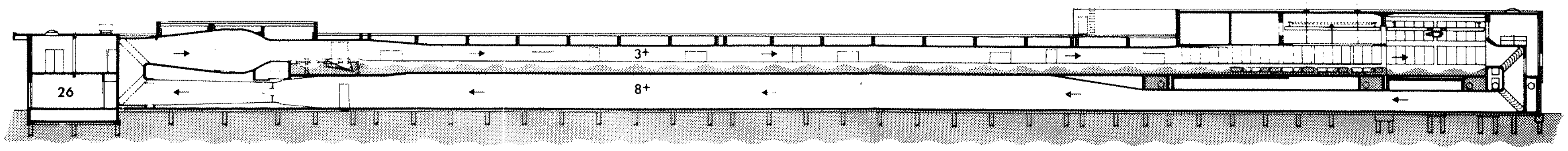

FIG. 1

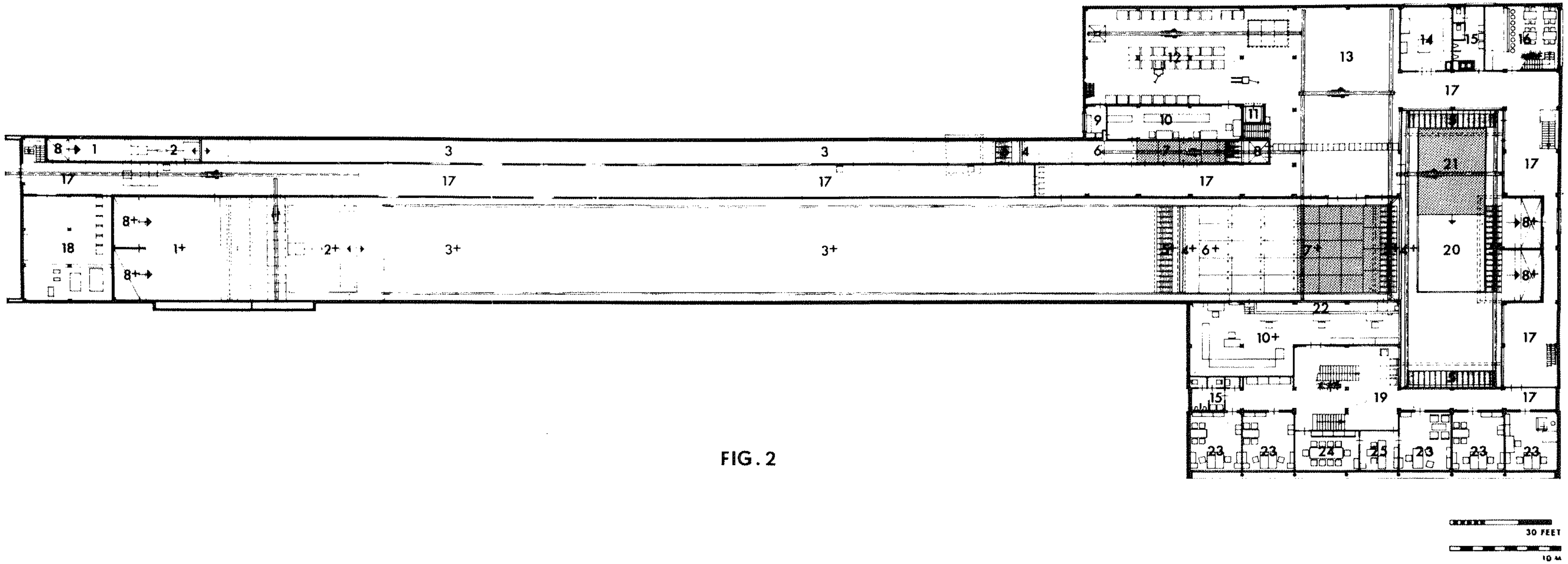

FIG. 2

Reference

1- 1+ Stilling chamber
2- 2+ Wave-generator
3- 3+ Wave-generation section
4- 4+ Movable barrier
5- 5+ Discharge channel
6- 6+ 'Adjustable sea-bed' section
7- 7+ Model section
8- 8+ Return channel
9 Room for control of material densities
10-10+ Control room
11 Elevator
12 Storage
13 Construction hall
14 Workshop
15 Toilets
16 Canteen
17 Corridor
18 Engine room
19 Entrance hall
20 Wave-basin
21 Movable floor
22 Recessed walkway
23 Offices
24 Conference room
25 Reception room
26 Transformator room

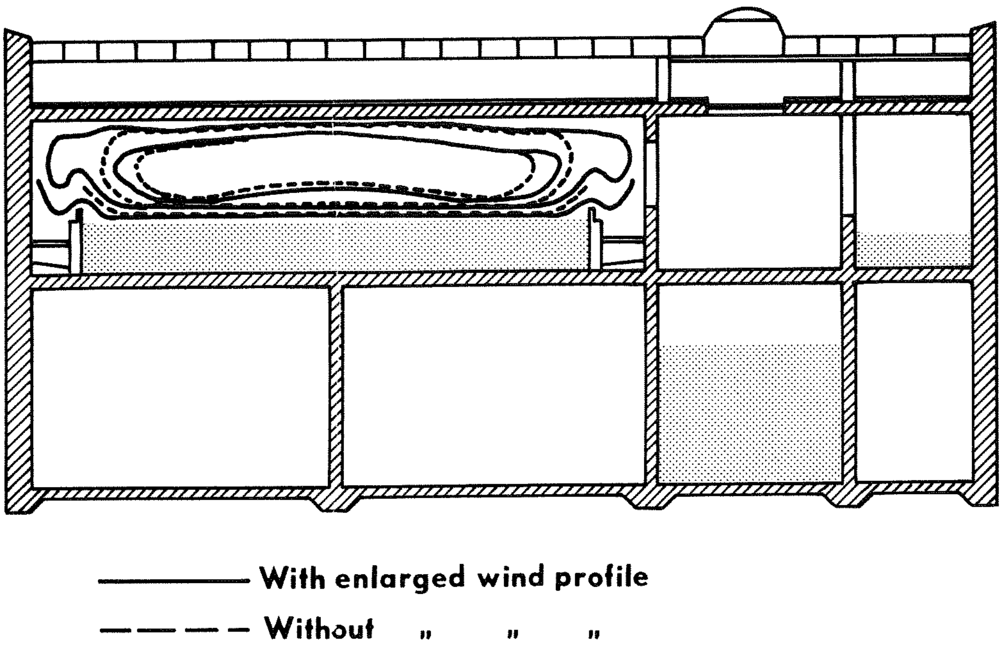

FIG. 3 CROSS SECTION WITH WIND-SPEED DISTRIBUTION

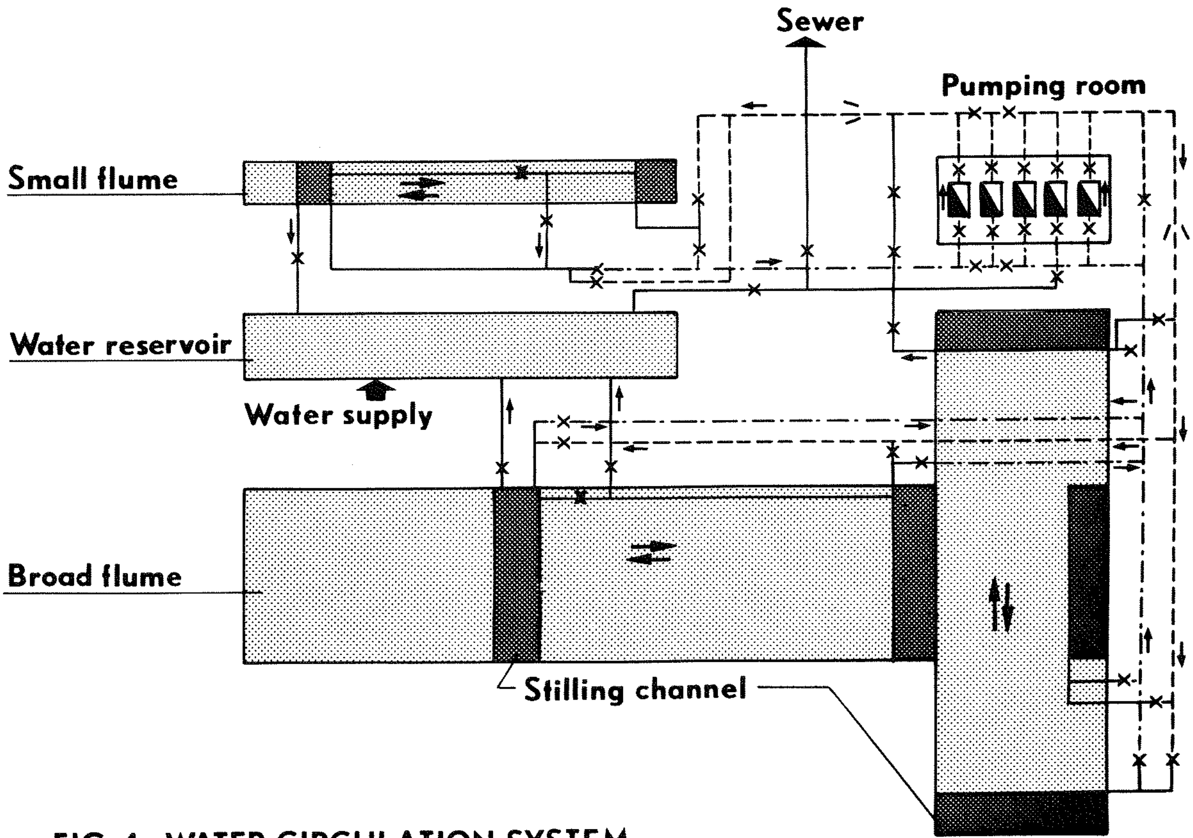

FIG. 4 WATER CIRCULATION SYSTEM

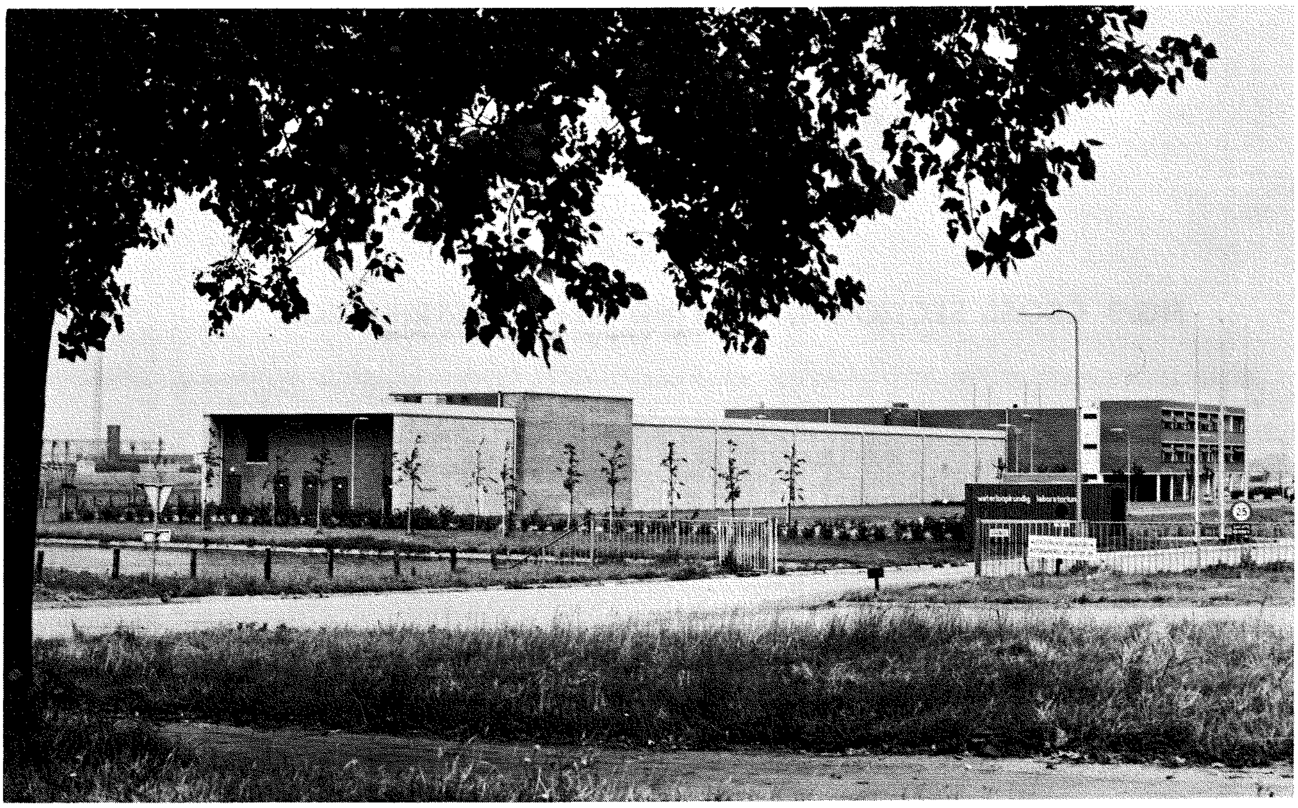

FIG.5 VIEW OF THE BUILDING

FIG. 6 LARGE FLUME AS SEEN FROM THE HAMMER-END

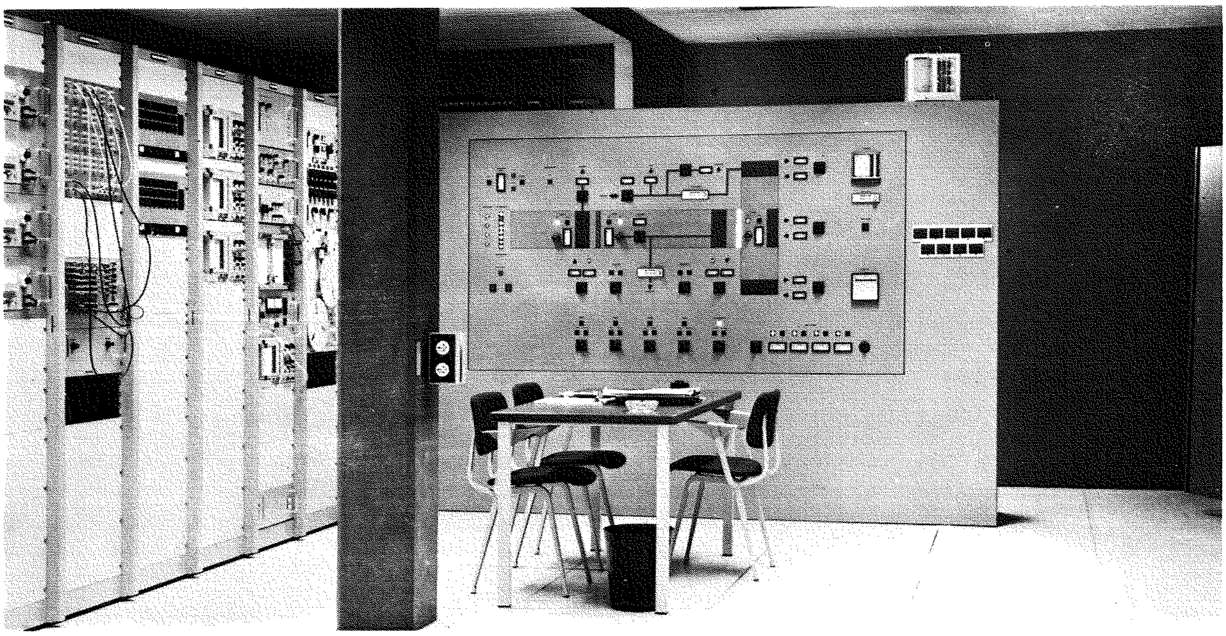

FIG. 7 CONTROL ROOM