DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AEROSPACE ENGINEERING

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PERTURBATION SOLUTIONS FOR BLAST WAVES IN A LOW DENSITY GAS

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NOTATION

x - space coordinate

t - time coordinate

y - location of the shock front from the center

p - particle pressure in the perturbed gas

ρ - particle density in the perturbed gas

u - particle velocity in the perturbed gas

T - gas temperature in the perturbed state

R - universal gas constant

γ - ratio of specific heats

p_o - ambient gas pressure

ρ_o - initial gas density

e_o - initial internal energy of the gas

C_o - sound speed in the undisturbed gas

E_o - initial energy release per unit area

y_o - a characteristic length parameter

b - internal volume of the molecules

λ - non-ideal parameter

ξ - non-dimensional distance $\frac{x}{y}$

s - non-dimensional distance $\frac{y}{y_o}$, which is the perturbation parameter

V - shock velocity

θ - shock decay parameter

$\frac{V^2}{C_o^2}$ - shock Mach number

η - shock strength

f - normalized non-dimensional velocity

g - normalized non-dimensional pressure

h - normalized non-dimensional density

$M(s)$- an integral

p_1 - pressure immediately behind the shock front

u_1 - particle velocity immediately behind the shock front

ρ_1 - density immediately behind the shock front

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ABSTRACT

A solution for the propagation of blast waves in a low density gas in which, a virial power expansions in terms of the parameter $(b\rho_o)$ representing the product of the internal volume of the molecules which is assumed to be constant and the initial gas density, is used in the equation of state.

Analytical expressions are obtained by means of a perturbation technique showing the dependence of pressure, density and gas velocity on the distance from the origin of the explosion and the distance of the shock front. Computations are performed with these expressions for various values of λ $(= b\rho_o)$.

Its influence on the flow field and temperature distributions is studied at different stages of the explosion process. Shock velocity variations with the shock propagation distance and shock trajectories are also determined. A comparison is made with ideal gas solutions. It is found that the parameter λ slows down the decay of the shock and results in a significant temperature drop near the origin.

1. INTRODUCTION

The theory of blast waves is a fundamental one in gasdynamics and has been applied to a variety of problems in hypersonic aerodynamics [3, 6, 16], astrophysics [13, 17] and hypervelocity impact [9].
The term explosion in the context of the work, which is to follow, describes the transient gasdynamic dispersion of energy by the mechanism of a shock wave.
The physical problem we consider is as follows:
Initially a finite quantity of energy E_o is released in a finite usually spherical volume in a medium at rest, which generates a shock wave in the medium. At later times the shock expands dispersing its energy and hence it attenuates. This decaying shock wave is conventionally called a blast wave. The initial motion of the blast wave may be represented by a simple model of the shock wave at the front and a purely gasdynamic treatment of the gas inside. Similarity solutions have been widely used in blast wave theory, which requires that the energy of explosions tends to infinity in a particular manner so that the blast wave starts out with an infinite strength and remains so till it attenuates. In practice, the self similar profiles are only valid in the neighbourhood of the shock front and cannot adequately describe the flow in the region near the center.

The one dimensional self-similar problem of a strong point explosion was formulated and solved by Sedov [15] on the assumption that the initial pressure of the gas, which is small in comparison with the pressure at the front, can be neglected and that the initial density is constant. Strong explosions in a medium of varying initial density were considered in [5]. Departures from the classical self similar solutions due to counter pressure effects are accounted for by Sakurai [11, 12]. Landau [4], Whitham [18] and Sedov [15] with different approaches, obtained an asymptotic form of the solution in the weak regime, when the blast wave has propagated far from the source.

Considerable computational difficulties encountered in non-self similar problems have led to the appearance of several approximate methods. Sakurai [13, 14] attempted a perturbation technique with the inverse square of the Mach numbers as the perturbation parameter. The solution was thus rendered more accurate for greater distances from the point of explosion. However, due to the asymptotic nature of the perturbation expansions, they diverge rapidly and become inaccurate for low strength shock waves. The quasi-similar method of Oshima [7] yields a solution for any particular value of the shock Mach number specified. However, it gives fairly good results only for the intermediate shock strength regime, where the local similarity approximation is adequate. A serious drawback of the quasi-similar technique is that mass is not conserved. Hence flow distributions, particularly particle velocity trajectories, are poorly described as compared to quantities such as the shock trajectory. The density profile method proposed by Porsel [8] and developed by Rae [10] gives good results for the entire range of shock strengths. However, it lacks generality and can only be applied to problems, if the density distribution can adequately be described by a simple power law. The coordinate perturbation method developed by Sakurai [13, 14] and improved by Bach and Lee [2] is rigorous mathematically and yields excellent results for relatively high shock Mach numbers. However, due to the asymptotic nature of the perturbation expansions, they diverge rapidly and become again inaccurate for low strength shocks.

From statistical physics, for gases at low densities the equation of state can be defined by means of virial power expansion in the form [1]

$$p = \rho RT\ (1 + b\rho) \tag{1}$$

With the limitations that $b\rho_o << 1$, where ρ_o and b refer to the initial gas density and the internal volume of the molecules. The motive for the present study is to extend the non-similar analysis and to study the distribution of hydrodynamic variables in the gas with the equation of state defined in (1).

2. DYNAMIC EQUATIONS AND SHOCK CONDITIONS

The one dimensional equations of continuity, momentum and energy conservation for the plane adiabatic motion of a gas by considering the equation of state as defined in equation (1) are

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0$$

$$\rho \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0 \qquad (2)$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + p \left(\gamma + (\gamma - 1)\, b\rho + \frac{b\rho}{1 + b\rho}\right) \frac{\partial u}{\partial x} = 0$$

where x, t are the Cartesian space coordinate and the time respectively and u, p and ρ are particle velocity, pressure and density of the fluid in the order mentioned. The energy integral is given by

$$\int_0^y \left[\frac{1}{(\gamma - 1)} \frac{p}{(1 + b\rho)} + \frac{\rho u^2}{2}\right] dx - \int_0^y \frac{p_o}{(\gamma - 1)\,(1 + b\rho_o)}\, dx = E_o \qquad (3)$$

where E_o is the energy input per unit area at x = 0 and t = 0 and p_o is the ambient gas pressure. The first integral on the left hand side represents the total energy enclosed by the explosion products bounded by the front y(t) and the origin. The second integral denotes the initial thermal energy in the gas occupying the volume from x = 0 to x = y(t). Here we have assumed that the shock front originates at the origin x = 0.

The boundary conditions at the shock front are given by the following Rankine-Hugoniot relations:

$$\frac{\rho_o}{\rho_1} = 1 - \frac{u_1}{V}$$

$$p_1 + \rho_1 (V - u_1)^2 = p_o + \rho_o V^2 \qquad (4)$$

$$\frac{p_1}{\rho_1} + e_1 + \frac{1}{2}(V - u_1)^2 = \frac{p_o}{\rho_o} + e_o + \frac{1}{2}V^2$$

In the above equations the quantities p_1, ρ_1 and u_1 refer to the state immediately behind the shock.

Let V be the shock front velocity and y(t) the shock position at time t. We introduce the following dimensionless variables.

$$\xi = \frac{x}{y}, \quad s = \frac{y}{y_o}$$

where $y_o = \frac{E_o}{p_o}$ represents a scale connected with the effective range of the power of the explosion.

$$u = V\,f\,(\xi,\,s)\,, \quad p = \frac{\rho_o}{\gamma}V^2\,g\,(\xi,\,s)$$

$$\rho = \rho_o\,h\,(\xi,\,s)\,, \quad V = V(s) \tag{5}$$

the variable ξ has values between 0 and 1. $\xi = 0$ corresponds to the position at the origin and $\xi = 1$ to the position at the shock front.

In terms of the new variables defined in equations (5), the conservation equations (equations (2)) and the energy integral (equation (3)) become

$$h\frac{\partial f}{\partial \xi} + (f - \xi)\frac{\partial h}{\partial \xi} = -\,s\frac{\partial h}{\partial s}$$

$$h\,(f - \xi)\frac{\partial f}{\partial \xi} + \frac{1}{\gamma}\frac{\partial g}{\partial \xi} = -\,s\,h\frac{\partial f}{\partial s} - f\,h\frac{s}{V}\frac{dV}{ds} \tag{6}$$

$$g\left(\gamma + Jh + \frac{Jh}{(\gamma - 1) + Jh}\right)\frac{\partial f}{\partial \xi} + (f - \xi)\frac{\partial g}{\partial \xi} = -\,s\frac{\partial g}{\partial s} - \frac{2gs}{V}\frac{dV}{ds}$$

$$\frac{V^2}{C_o^{\,2}}\,s\int_0^1\left(\frac{g}{(\gamma - 1) + Jh} + \frac{\gamma}{2}f^2h\right)d\xi - \int_0^s\frac{1}{(\gamma - 1 + J)}\,ds = 1 \tag{7}$$

where $J = \lambda(\gamma - 1)$ in which

$$\lambda = b\rho_o \tag{8}$$

Making use of the equation of state defined in (1) in the shock relations (4) and simplifying one gets the following shock conditions at $\xi = 1$.

$$\frac{\rho_1}{\rho_o} = \frac{1}{z}$$

$$\frac{u_1}{V} = 1 - z \tag{9}$$

$$\frac{\gamma p_1}{\rho_o V^2} = \gamma\ (1 - z) + \eta$$

At the origin we impose the following zero particle velocity criteria:

$$f_o(o) = 0\ ,\quad f_1(o) = 0\ ,\quad f_2(o) = 0$$

In the above conditions $\eta = \frac{C_o^2}{V^2}$, C_o being the sound velocity at the origin and:

$$z = \frac{(\gamma - 1 - J + 2\eta) + \sqrt{(\gamma - 1 - J + 2\eta)^2 + 4(\gamma + 1)\ J\left(1 + \frac{2A\eta}{\gamma}\right)}}{2(\gamma + 1)}$$

where

$$A = \frac{(\gamma + J)}{(\gamma - 1 + J)}$$

3. METHOD OF SOLUTION

For the initial motion of the blast wave we seek the solutions for the flow field in power series of s in the following form:

$$f(\xi, s) = f_o + f_1 s + f_2 s^2 + \dots..$$

$$g(\xi, s) = g_o + g_1 s + g_2 s^2 + \dots.. \qquad (10)$$

$$h(\xi, s) = h_o + h_1 s + h_2 s^2 + \dots..$$

where f_i, g_i and h_i are functions of ξ only. Substituting the perturbation expressions given in equations (10) into the energy integral (7) leads to the following expression, which determines the shock velocity

$$\frac{V^2}{C_o^{\,2}} = \frac{1 + n_1 s}{sM(s)} \qquad (11)$$

where

$$M(s) = \int_o^1 \left(\frac{g}{(\gamma - 1 + Jh)} + \frac{\gamma}{2} f^2 h\right) d\xi$$

and

$$n_1 = \frac{1}{(\gamma - 1 + J)}$$

Making use of equations (10), M(s) can be expanded in power series of s as

$$M(s) = \Sigma\, \alpha_i\, s^i \qquad i \geqslant 0$$

where in particular

$$\alpha_o = \int_o^1 \left(\frac{\gamma}{2} f_o^{\,2} h_o + \frac{g_o}{C}\right) d\xi$$

$$\alpha_1 = \int_o^1 \left(\frac{\gamma}{2} f_o^2 h_1 + \gamma f_o h_o f_1 + \frac{g_1}{C} - \frac{J}{C^2} g_o h_1\right) d\xi$$

$$\alpha_2 = \int_o^1 \frac{1}{C} \left(g_2 - \frac{J}{C} g_1 h_1 + g_o \left(\frac{J^2}{C^2} h_1^2 - \frac{J}{C} h_2\right)\right) d\xi \tag{12}$$

$$+ \int_o^1 \left(\frac{\gamma}{2} f_o h_2 + \gamma f_o f_1 h_1 + \frac{\gamma}{2} h_o (f_1^2 + 2 f_o f_2)\right) d\xi$$

in which

$$C = (\gamma - 1) + Jh_o$$

Hence, the solution of the shock front velocity and the shock strength $\eta = \frac{C_o^2}{V^2}$ can be expressed as a power series in the distance coordinate s as

$$\frac{V^2}{C_o^2} = \frac{1}{\alpha_o s} \left[1 + \left(n_1 - \frac{\alpha_1}{\alpha_o}\right) s + \left(\frac{\alpha_1^2}{\alpha_o^2} - \frac{\alpha_2}{\alpha_o} - n_1 \frac{\alpha_1}{\alpha_o}\right) s^2 + \ldots\ldots\right] \tag{13}$$

$$\eta = \alpha_o s \left[1 + \left(\frac{\alpha_1}{\alpha_o} - n_1\right) s + \left(\frac{\alpha_2}{\alpha_o} + n_1^2 - n_1 \frac{\alpha_1}{\alpha_o}\right) s^2 + \ldots\ldots\right] \tag{14}$$

From the equation (14) one can readily determine an expansion for the shock decay parameter $\left(\theta = \frac{2s}{V} \frac{dV}{ds}\right)$ as

$$\theta = -1 + A_1 s + A_2 s^2 + \ldots\ldots \tag{15}$$

where the coefficients A_1 and A_2 are determined as

$$A_1 = n_1 - \frac{\alpha_1}{\alpha_o}, \quad A_2 = \frac{\alpha_1^2}{\alpha_o^2} - \frac{2\alpha_2}{\alpha_o} - n_1^2 \tag{16}$$

Substituting the perturbation expressions (equations (10)) into equations (6), we obtain the zeroeth, first and second order equations after equating coefficients of similar order in s.

Zeroeth order

$$h_o f_o' + (f_o - \xi) h_o' = 0$$

$$\gamma h_o (f_o - \xi) f_o' + g_o' = \frac{\gamma}{2} f_o h_o \qquad (17)$$

$$\gamma g_o G_o f_o' + (f_o - \xi) g_o' = g_o$$

First order

$$h_o f_1' + (f_o - \xi) h_1' = -h_1 (1 + f_o') - f_1 h_o'$$

$$\gamma h_o (f_o - \xi) f_1' + g_1' = -\gamma (h_o f_1 + h_1 (f_o - \xi)) + \frac{1}{2} (h_1 f_o - h_o f_1) - \frac{1}{2} f_o h_o A_1 \qquad (18)$$

$$\gamma g_o G_o f_1' + (f_o - \xi) g_1' = - (\gamma g_1 G_o + g_o G_1) f_o' - f_1 g_o' - g_o A_1$$

Second order

$$h_o f_2' + (f_o - \xi) h_2' = -h_1 f_1' - f_1 h_1' - h_2 f_o' - f_2 h_o' - 2h_2$$

$$\gamma h_o (f_o - \xi) f_2' + g_2' = -\gamma [h_1 (f_o - \xi) + h_o f_1] f_1' \qquad (19)$$

$$\gamma [h_2 (f_o - \xi) + h_1 f_1 + h_o f_2] f_o' -$$

$$\frac{A_1}{2} (h_1 f_o + f_1 h_o) - \frac{1}{2} (3f_2 h_o + h_1 f_1 - h_2 f_o) - \frac{A_2}{2} f_o h_o$$

$$\gamma G_o g_o f_2' + (f_o - \xi) g_2' = - (G_1 g_o + \gamma G_o g_1) f_1' - f_1 g_1'$$

$$- A_2 g_o - (G_2 g_o + G_1 g_1 + \gamma G_o g_2) f_o'$$

$$- f_2 g_o' - g_2 - A_1 g_1$$

where

$$G_o = 1 + \frac{Jh_o}{\gamma} + \frac{Jh_o}{\gamma C}$$

$$G_1 = Jh_1 + \frac{Jh_1}{C^2}(\gamma - 1) \qquad (20)$$

$$G_2 = Jh_2 + \frac{J}{C^2}(\gamma - 1)\left(h_2 - \frac{J}{C}h_1{}^2\right)$$

In these, the primes denote derivatives with respect to ξ. Boundary conditions to be satistied by the zeroeth, first and second order equations at the shock front $\xi = 1$ can be determined by substituting for η from equation (14) into the boundary conditions given in equations (9), expanding them in ascending power in s and sorting out coefficients of similar order in s, this yields for the zeroeth order

$$f_o(1) = (1 - \beta)$$

$$g_o(1) = \gamma\,(1 - \beta) \qquad (21)$$

$$h_o(1) = \frac{1}{\beta}$$

for the first order

$$f_1(1) = -\delta$$

$$g_1(1) = \alpha_o - \gamma\delta \qquad (22)$$

$$h_1(1) = -\frac{\delta}{\beta^2}$$

for the second order

$$f_2(1) = -\Gamma$$

$$g_2(1) = \alpha_1 - n_1 \alpha_o - \gamma\Gamma \tag{23}$$

$$h_2(1) = \frac{1}{\beta^3} (\delta^2 - \Gamma\beta)$$

where

$$\beta = \frac{1}{2(\gamma + 1)} [\gamma - 1 - J + (4B + C^2)^{\frac{1}{2}}]$$

$$\delta = \frac{\alpha_o}{\gamma + 1} \left[1 + (4B + C^2)^{\frac{1}{2}} K_1\right]$$

$$\Gamma = \frac{1}{(\gamma + 1)} \Big\{(\alpha_1 - n_1 \alpha_o) + (4B + C^2)^{\frac{1}{2}}$$

$$\Big[K_1(\alpha_1 - n_1 \alpha_o) + (K_2 - K_1^2) \alpha_o^2\Big]\Big\}$$

in which

$$K_1 = \frac{(\gamma C + 2BA)}{\gamma(4B + C^2)}$$

$$K_2 = \frac{1}{4B + C^2}$$

$$A = \frac{(\gamma + J)}{(\gamma - 1 + J)} \tag{24}$$

$$B = (\gamma + 1) J$$

$$C = (\gamma - 1 - J)$$

The solution of the zeroeth order equations with the corresponding boundary conditions yields the solution for an infinitely strong shock wave. Hence for very large values of the initiation energy E_o, the first and higher order conditions become zero and the shock remains strong throughout its history under this condition.

Since the higher order equations have the same determinant as the zeroeth order, there are no singularities involved in the solutions. The zeroeth order equations can be integrated immediately using the Runge-Kutta method and this yields the value of α_o by virtue of the first expression of (12). However, the first and second order equations cannot be integrated directly because of the presence of the constants A_1 and A_2. To obtain the solutions for these, we follow the procedure adopted by Sakurai [14]. At first the functions are split up into two parts as

$$f_1 = f_{1\sigma} + A_1 f_{1\mu}$$

$$g_1 = g_{1\sigma} + A_1 g_{1\mu} \qquad (25)$$

$$h_1 = h_{1\sigma} + A_1 h_{1\mu}$$

$$f_2 = f_{2\sigma} + A_2 f_{2\mu}$$

$$g_2 = g_{2\sigma} + A_2 g_{2\mu} \qquad (26)$$

$$h_2 = h_{2\sigma} + A_2 h_{2\mu}$$

In these, A_1 can be eliminated by substituting expressions (25) into equations (18) and grouping the terms with and without A_1. This leads, after some manipulation, to the following two pairs of coupled ordinary first order equations, the σ-subscripted equations and μ-subscripted equations.

$$h_o f'_{1\sigma} + (f_o - \xi) h'_{1\sigma} = -h_{1\sigma} f_o' - f_{1\sigma} h_o' - h_{1\sigma}$$

$$\gamma h_o (f_o - \xi) f'_{1\sigma} + g'_{1\sigma} = -\gamma (h_o f_{1\sigma} + h_{1\sigma} (f_o - \xi)) f_o' \qquad (27)$$

$$+ \frac{\gamma}{2} (h_{1\sigma} f_o - h_o f_{1\sigma})$$

$$\gamma g_o\ G_o\ f'_{1\sigma} + (f_o - \xi)\ g'_{1\sigma} = -\ (\gamma g_{1\sigma}\ G_o + g_o\ G_{1\sigma})\ f_o{}' - f_{1\sigma}\ g_o{}'$$

$$h_o\ f'_{1\mu} + (f_o - \xi)\ h'_{1\mu} = -\ h_{1\mu}\ f_o{}' - f_{1\mu}\ h_o{}' - h_{1\mu}$$

$$\gamma h_o\ (f_o - \xi)\ f'_{1\mu} + g'_{1\mu} = -\gamma \left[h_o\ f_{1\mu} + (f_o - \xi)\ h_{1\mu}\ \ f_o{}' \right] + \frac{\gamma}{2}\ (f_o\ h_{1\mu} - h_o\ f_{1\mu} - h_o\ f_o) \tag{28}$$

$$\gamma g_o\ G_o\ f'_{1\mu} + (f_o - \xi)\ g'_{1\mu} = -\ (\gamma g_{1\mu}\ G_o + g_o\ G_{1\mu})\ f_o{}' - f_{1\mu}\ g_o{}' - g_o$$

The corresponding boundary conditions are

$$f_{1\sigma}(1) = -\delta$$

$$g_{1\sigma}(1) = \alpha_o - \gamma\delta \tag{29}$$

$$h_{1\sigma}(1) = -\ \frac{\delta}{\beta^2}$$

and

$$f_{1\mu}(1) = 0$$

$$g_{1\mu}(1) = 0 \tag{30}$$

$$h_{1\mu}(1) = 0$$

By the same procedure A_2 can be eliminated, which leads to the following second order equations and the corresponding boundary conditions

$$h_o\ f'_{2\sigma} + (f_o - \xi)\ h'_{2\sigma} = -h_1\ f_1{}' - f_1\ h_1{}' - h_{2\sigma}\ f_o{}' - f_{2\sigma}\ h_o{}' - 2h_{2\sigma}$$

$$h_o\ (f_o - \xi)\ f'_{2\sigma} + \frac{1}{\gamma}\ g'_{2\sigma} = -\left[h_1\ (f_o - \xi) + h_o\ f_1\right] f_1{}'$$
$$- \left[h_1\ f_1 + h_o\ f_{2\sigma} + h_{2\sigma}\ (f_o - \xi)\right] f_o{}'$$
$$- \tfrac{1}{2}\ (3h_o\ f_{2\sigma} + f_1\ h_1 - h_{2\sigma}\ f_o) \qquad (31)$$
$$- \frac{A_1}{2}\ (h_1\ f_o + f_1\ h_o)$$

$$\gamma G_o\ g_o\ f'_{2\sigma} + (f_o - \xi)\ g'_{2\sigma} = -\ (G_1\ g_o + \gamma G_o\ g_1)\ f_1{}' - f_1\ g_1{}'$$
$$- (G_{2\sigma}\ g_o + G_1\ g_1 + \gamma G_o\ g_{2\sigma})\ f_o{}'$$
$$- f_{2\sigma}\ g_o{}' - g_{2\sigma} - A_1\ g_1$$

$$h_o\ f'_{2\mu} + (f_o - \xi)\ h'_{2\mu} = -\ h_{2\mu}\ f_o{}' - f_{2\mu}\ h_o{}' - 2h_{2\mu}$$

$$h_o\ (f_o - \xi)\ f'_{2\mu} + \frac{1}{\gamma}\ g'_{2\mu} = -\ (h_o\ f_{2\mu} + h_{2\mu}\ (f_o - \xi))\ f_o{}'$$
$$- \tfrac{1}{2}\ (3h_o\ f_{2\mu} - h_{2\mu}\ f_o) - \tfrac{1}{2}\ f_o\ h_o \qquad (32)$$

$$\gamma g_o\ G_o\ f'_{2\mu} + (f_o - \xi)\ g'_{2\mu} = -\ (G_{2\mu}\ g_o + \gamma G_o\ g_{2\mu})\ f_o{}'$$
$$- f_{2\mu}\ g_o{}' - g_{2\mu} - g_o$$

$$f_{2\sigma}(1) = -\Gamma$$

$$g_{2\sigma}(1) = \alpha_1 - n_1\ \alpha_o - \gamma\Gamma \qquad (33)$$

$$h_{2\sigma}(1) = \frac{1}{\beta^3}\ (\delta^2 - \Gamma\beta)$$

$$f_{2\mu}(1) = 0$$

$$g_{2\mu}(1) = 0 \qquad (34)$$

$$h_{2\mu}(1) = 0$$

The σ-subscripted and μ-subscripted first and second order equations with their corresponding boundary conditions can be integrated by the Runge-Kutta method. The constants A_1 and A_2 can then be determined based on the criteria that the correct solution must satisfy apart from the energy integral, the physical boundary condition of zero particle velocity at the origin and this leads to the following relations

$$A_1 = \frac{(n_1 \alpha_o - \alpha_{1\sigma})}{(\alpha_o + \alpha_{1\mu})} \tag{35}$$

$$A_2 = \frac{\left(-\frac{2\alpha_{2\sigma}}{\alpha_o} + \frac{\alpha_1^2}{\alpha_o^2} - n_1^2\right)}{\left(1 + \frac{2\alpha_{2\mu}}{\alpha_o}\right)} \tag{36}$$

Having found the various coefficients of the polynomial expansions of the dependent variables, one can determine the profiles by the use of the polynomial equations (10) for any fixed value of the distance parameter s.

For numerical integration schemes involving blast waves, one must note the inadequacy of the blast wave solution in describing the starting flow from an explosion of finite dimension. Hence, the integration was performed from $\xi = 1.0$ to $\xi = 0.001$. Also a step size should be selected such that the net effects of the truncation errors of the applied integration procedure and the round-off errors inherent in the computer used, are a minimum. Comparison of the results of the computer runs with 200 and 800 steps for various values of λ are given in Table 1.

Coefficients of the polynomial expansions for the decay parameter θ and the energy integral M(s) computed with $\gamma = 1.4$ for various values of λ are listed in Table 2.

λ	α_o (N = 200)	α_o (N = 800)	A_1 (N = 200)	A_1 (N = 800)	A_2 (N = 200)	A_2 (N = 800)
0	1.6960	1.6959	3.6360	3.6355	-30.354	-30.766
0.1	1.4408	1.4407	3.4584	3.4583	-29.543	-29.540
0.2	1.2705	1.2704	3.3339	3.3338	-28.408	-28.407
0.3	1.1410	1.1409	3.2046	3.2046	-26.636	-26.635

Table 1. Perturbation coefficients for step sizes 200 and 800.

λ	α_o	A_1	α_1	A_2	α_2
0	1.6959	3.6355	-1.9258	-30.766	21.882
0.1	1.4407	3.4583	-1.7081	-29.540	18.571
0.2	1.2704	3.3338	-1.5887	-28.407	16.281
0.3	1.1409	3.2046	-1.4621	-26.635	14.022

Table 2. Perturbation coefficients for the energy integral and the decay parameter.

The shock trajectory can be obtained from the definition of the shock velocity as

$$\tau = \int_o^s \frac{C_o}{V} \, ds \qquad (37)$$

Substituting the expansion for $\frac{C_o}{V}$ in terms of s from equation (14) into the above equation and integrating it up to second order in s, the resultant expression for the shock trajectory becomes

$$\tau = B_o \, s^{3/2} \, [1 + B_1 \, s + B_2 \, s^2 + \ldots..] \qquad (38)$$

where

$$B_o = \frac{2}{3} (\sqrt{\alpha_o})$$

$$B_1 = - \frac{3}{10} A_1$$

$$B_2 = \frac{3}{14} \left(\frac{\alpha_2}{\alpha_o} + n_1^{\,2} - n_1 \frac{\alpha_1}{\alpha_o} - \frac{A_1^{\,2}}{4} \right)$$

4. RESULTS AND DISCUSSION

Figure 1 and figure 2 depict the shock velocity variations and shock trajectories respectively for various values of the non-ideal parameter λ. These indicate that increase in the value of λ results in slowing down the decay of the shock. The velocity, pressure and density distributions of the flow behind the shock front correct up to the third approximation are given by

$$\frac{u}{C_o} = \frac{V}{C_o}(f_o + f_1 s + f_2 s^2)$$
$$\frac{p}{p_o} = \frac{V^2}{C_o^2}(g_o + g_1 s + g_2 s^2) \qquad (39)$$
$$\frac{\rho}{\rho_o} = h_o + h_1 s + h_2 s^2$$

and space profiles of these non-dimensional gas dynamic variables are presented in figures 3-11. Their changing behaviour with respect to time can be seen in their graphs against ξ for various values of s. Comparison of these results with those from the ideal gas solutions show that the distributions of dimensionless densities and pressures in the low density gas considered show the same qualitative behaviour as those in the medium of ideal gas. However, in a very small region closer to the origin the solutions of velocity profiles do not correspond to those of the ideal gas solutions, which reiterates the conclusions of the study of Anisimov and Spiner [1].

Normalized pressure and density distributions for the zeroeth, first and second order approximations are compared in figures 15-17. It is interesting to note that the physically incorrect maximum in the first order curve is corrected by the second order term.
In his work on hypersonic blunt-body flows Swigart [16] indicated that for blast wave Mach numbers below 3, the third order correction, particularly for the density is a significant percentage of the first and second order terms.

Distributions of pressure for $\lambda = 0.1$ at various stages of explosion are shown in figure 12. Also presented in figures 13 and 14 are the temperature distributions for various values of λ and for varying shock propagation distances. Shock velocity variations with s are given in Table 3 for varying λ.

s	MACHN = $\frac{V^2}{C_o^2}$ (λ = 0)	MACHN = $\frac{V^2}{C_o^2}$ (λ = 0.1)	MACHN = $\frac{V^2}{C_o^2}$ (λ = 0.2)	MACHN = $\frac{V^2}{C_o^2}$ (λ = 0.3)
0.05	13.675	15.977	18.026	19.979
0.1	7.523	8.7311	9.8149	10.856
0.15	5.2987	6.1124	6.851	7.576
0.2	4.572	4.6501	5.1987	5.7567

Table 3. Variations of shock Mach number with s for varying λ.

In conclusion, we may point out that a finite counter pressure in the undisturbed gas is associated with a finite sound velocity and this results in an increased shock velocity, particularly when the shock is weak. This when coupled with the effect of the parameter λ defining the non-ideal character of the medium results in slowing down the decay of the shock and effects in significant temperature drop at the center.

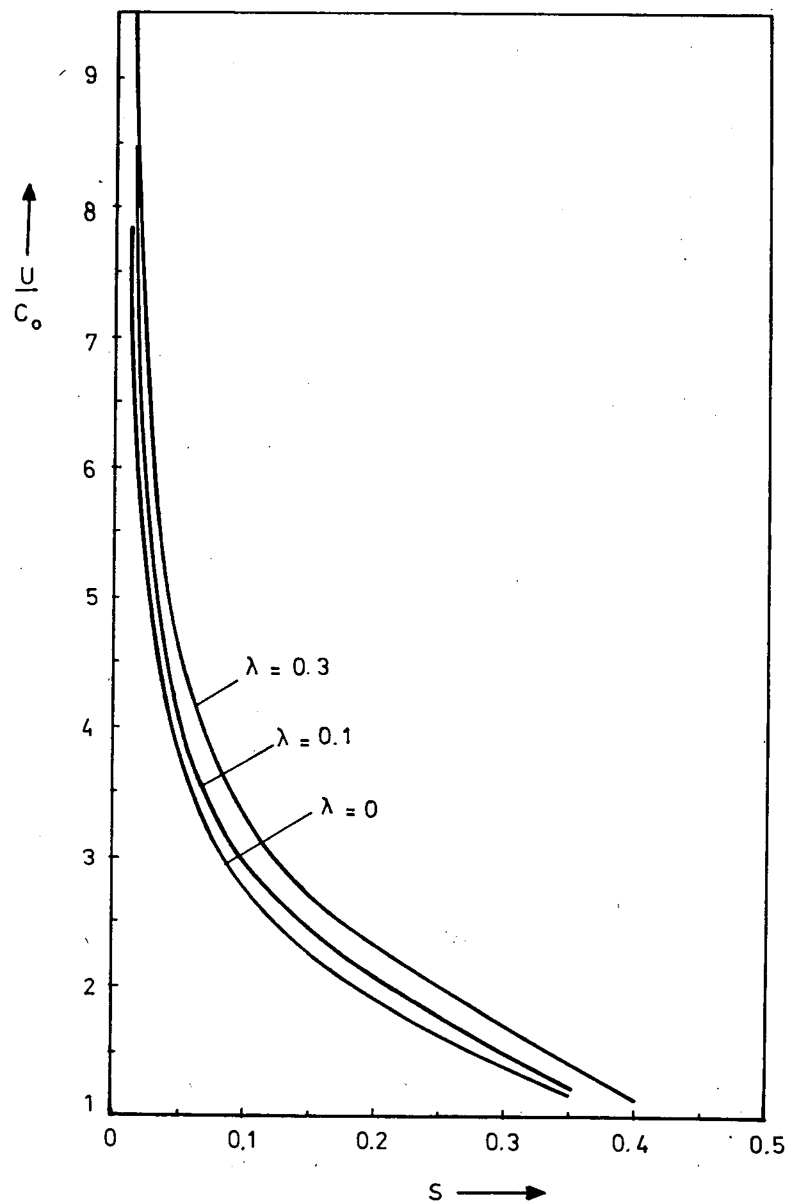

Figure 1. Shock propagation velocity as a function of the propagation distance s̄.

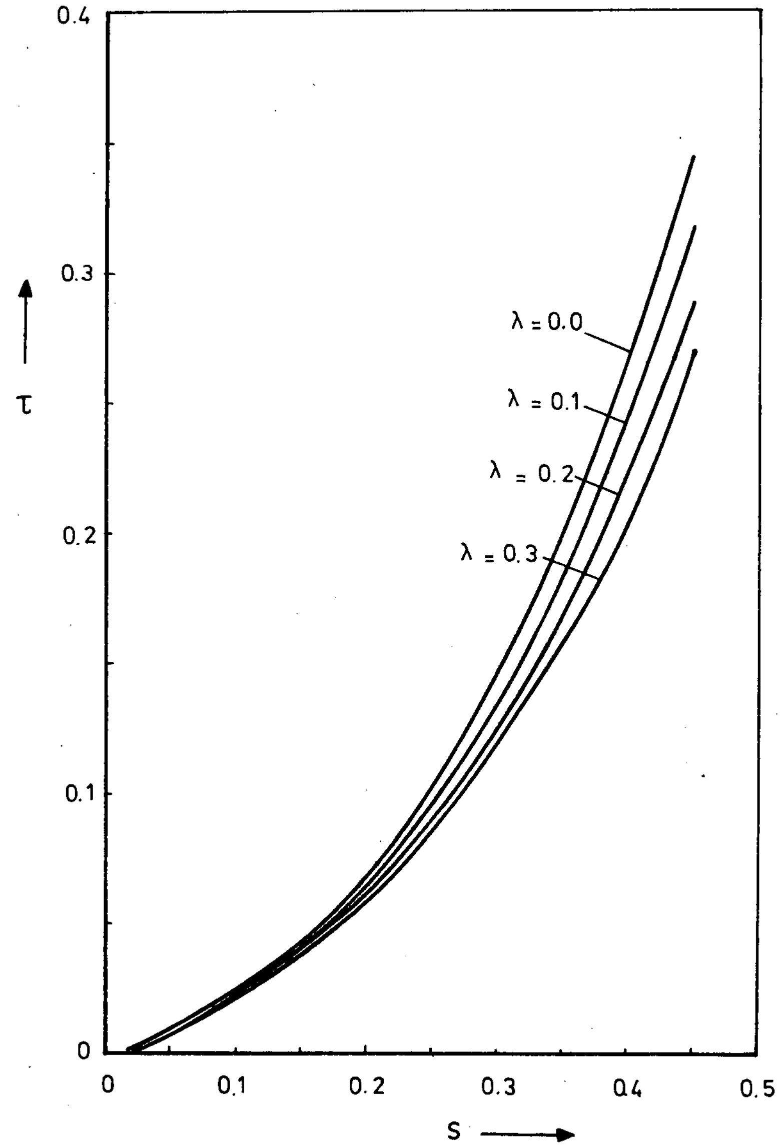

Figure 2. Shock trajectories for various values of λ.

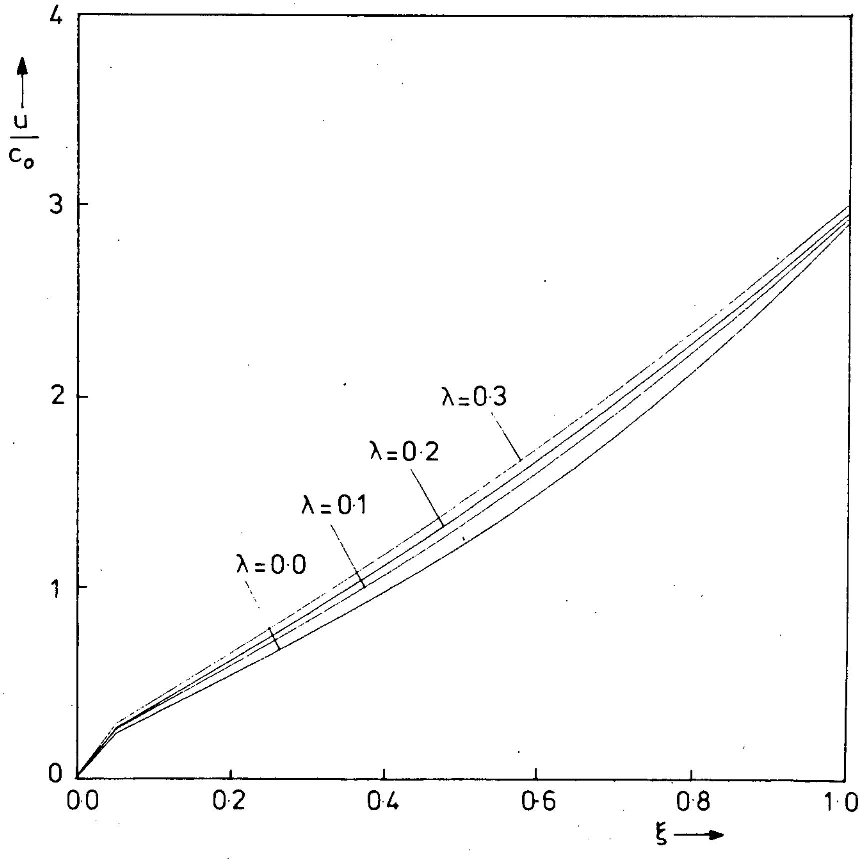

Figure 3. Particle velocity profiles for varying λ (s = 0.05).

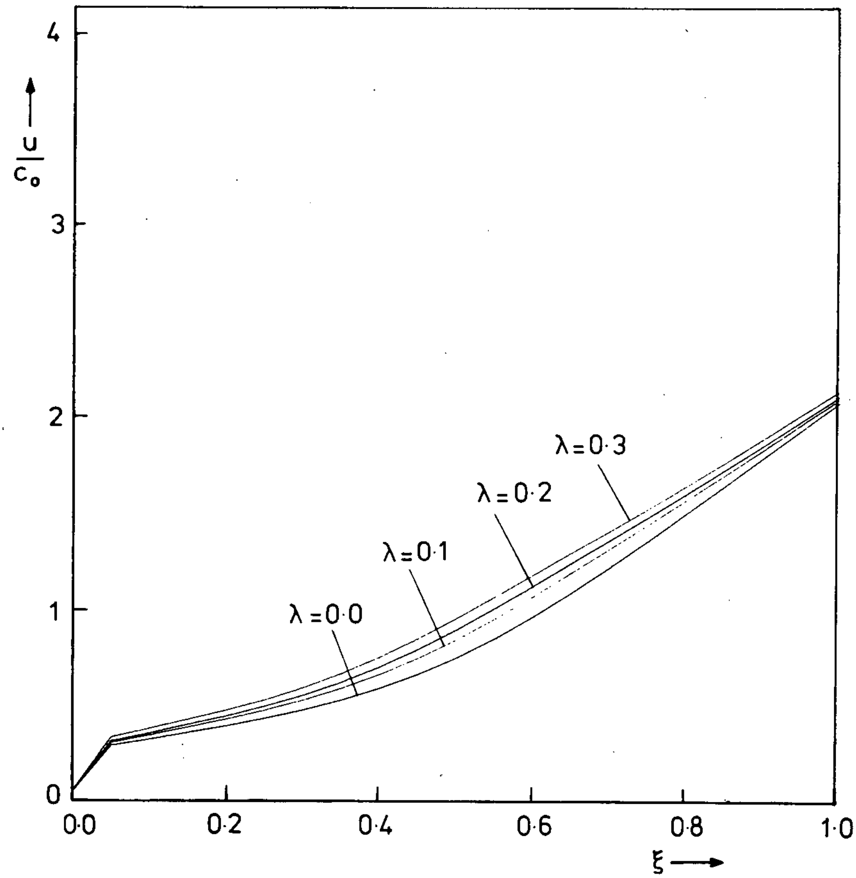

Figure 4. Particle velocity profiles for varying λ (s = 0.1).

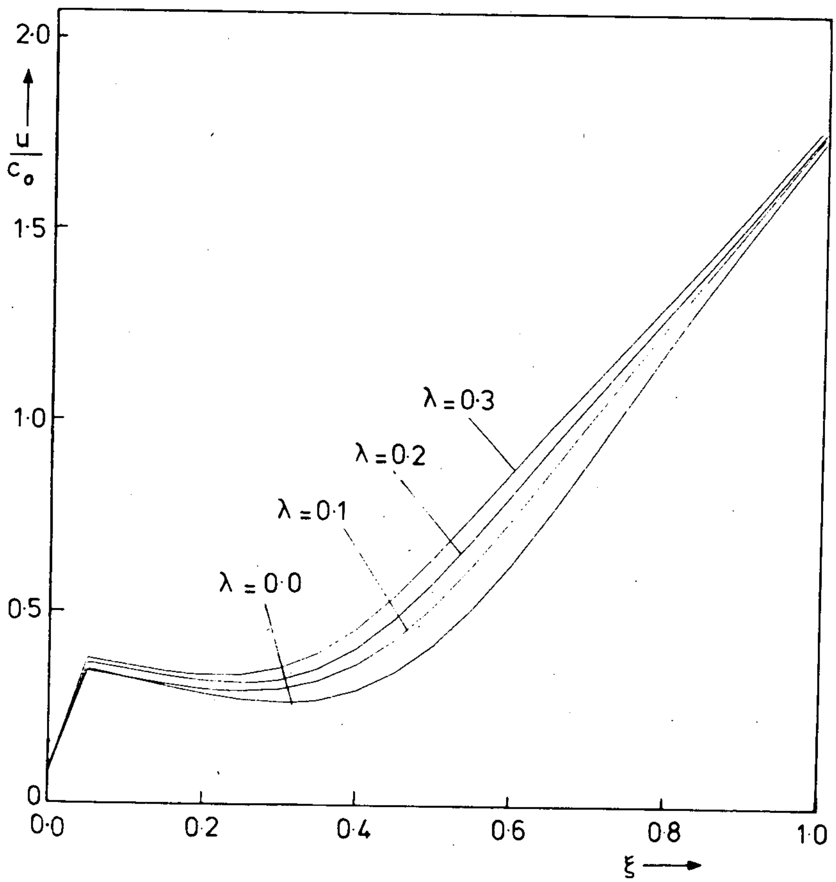

Figure 5. Particle velocity profiles for varying λ (s = 0.15).

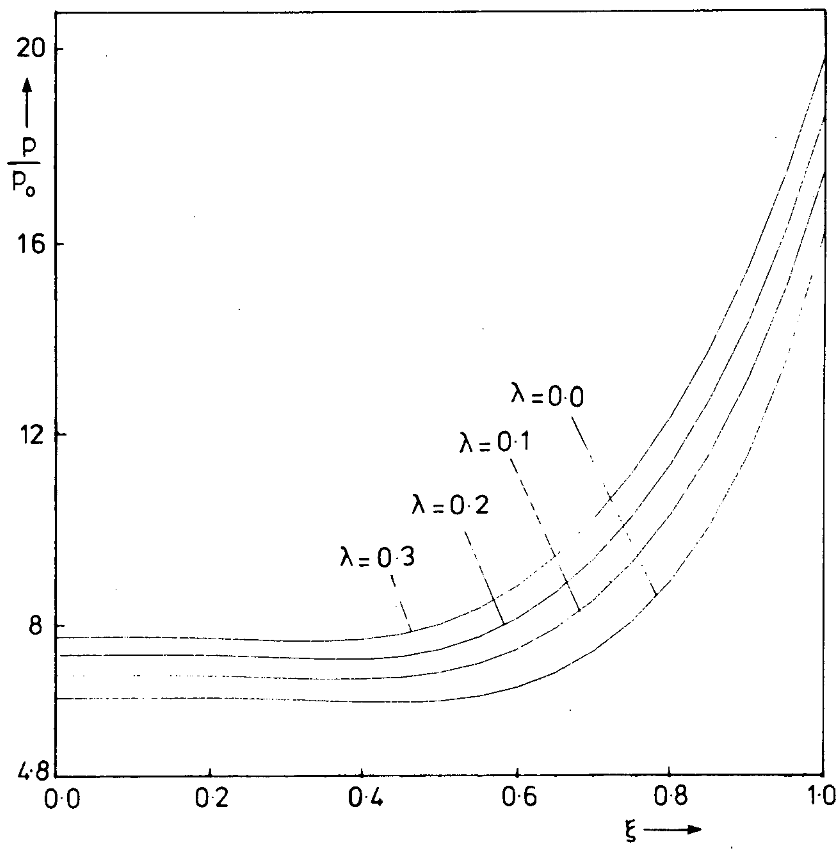

Figure 6. Pressure profiles for varying λ (s = 0.05).

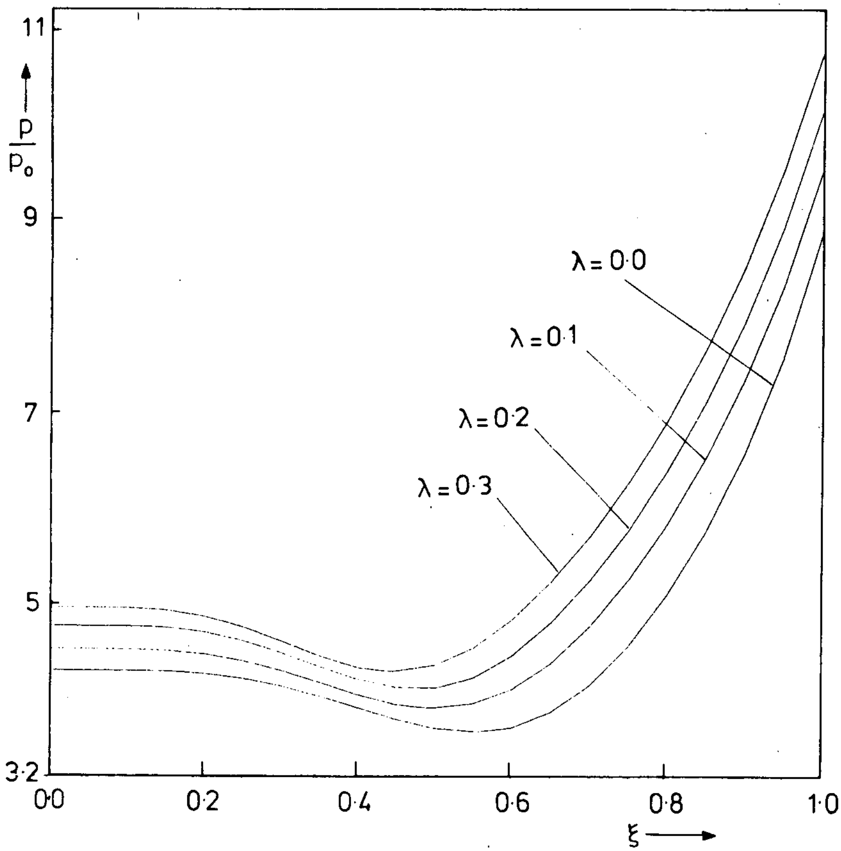

Figure 7. Pressure profiles for varying λ (s = 0.1).

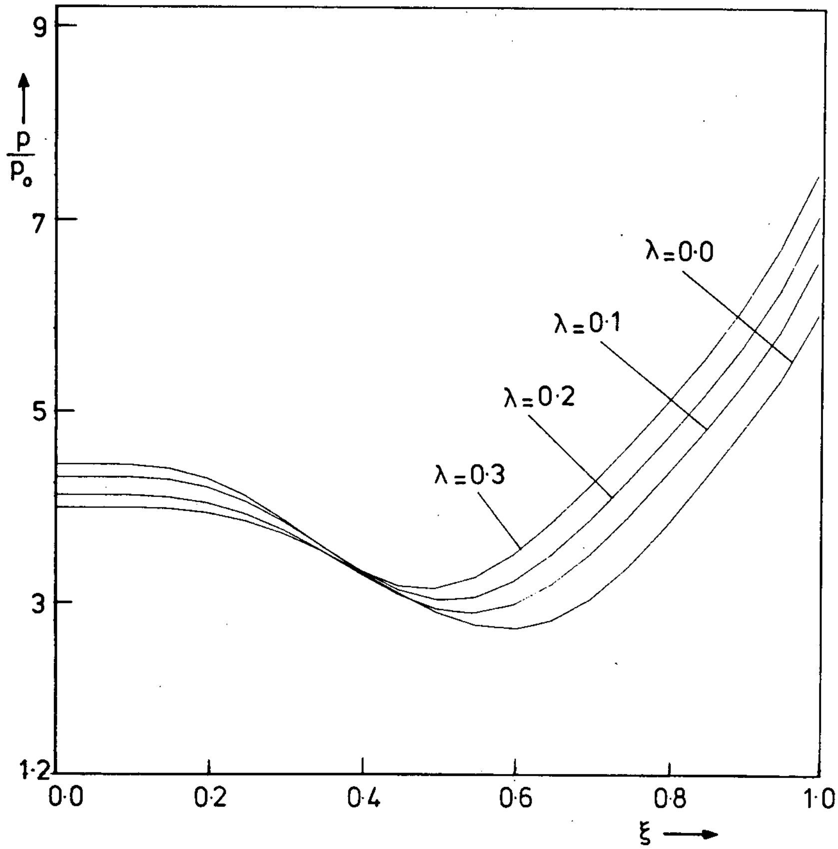

Figure 8. Pressure profiles for varying λ (s = 0.15).

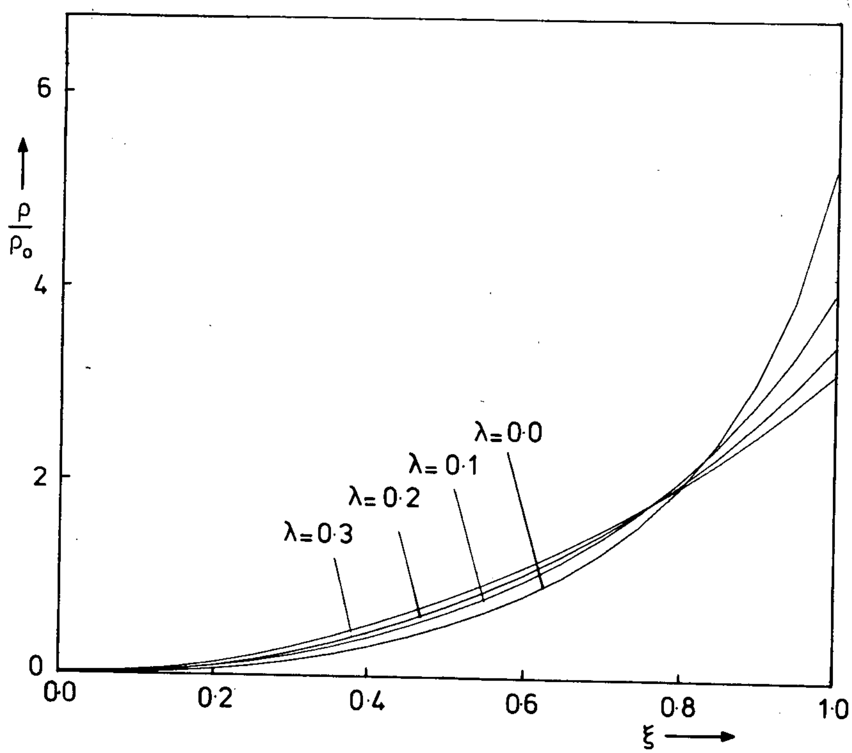

Figure 9: Density profiles for varying λ (s = 0.05).

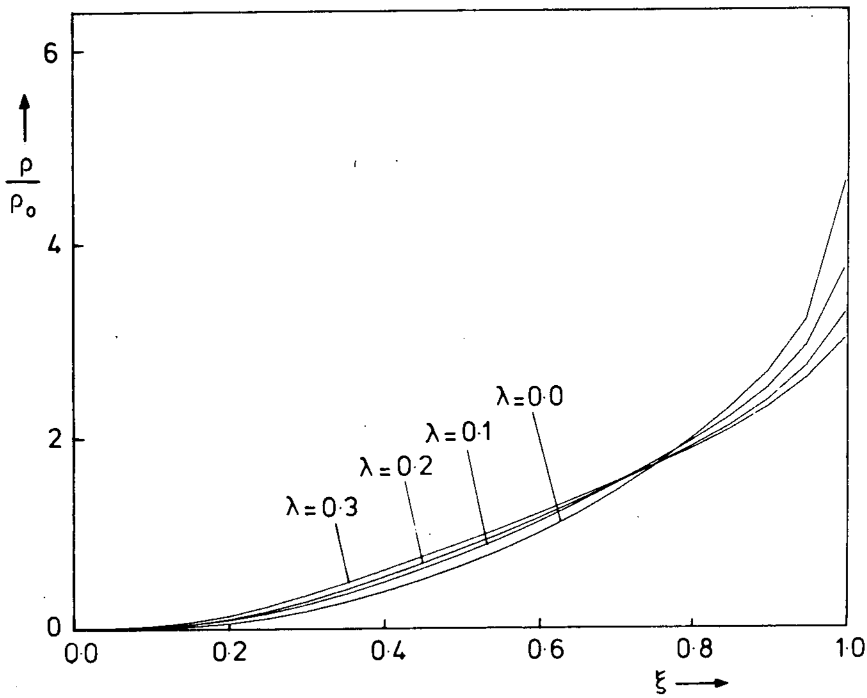

Figure 10. Density profiles for varying λ (s = 0.1).

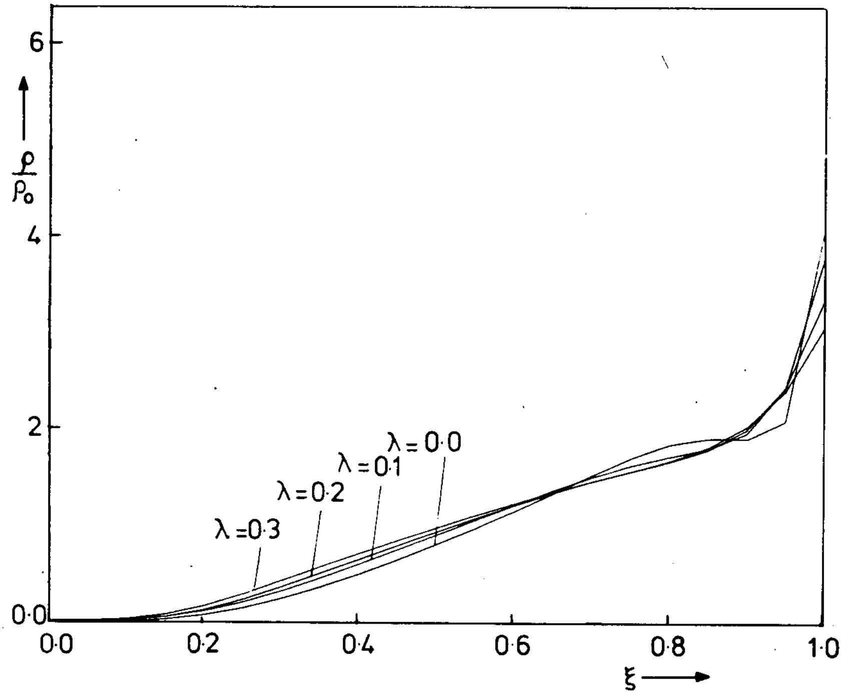

Figure 11. Density profiles for varying λ (s = 0.15).

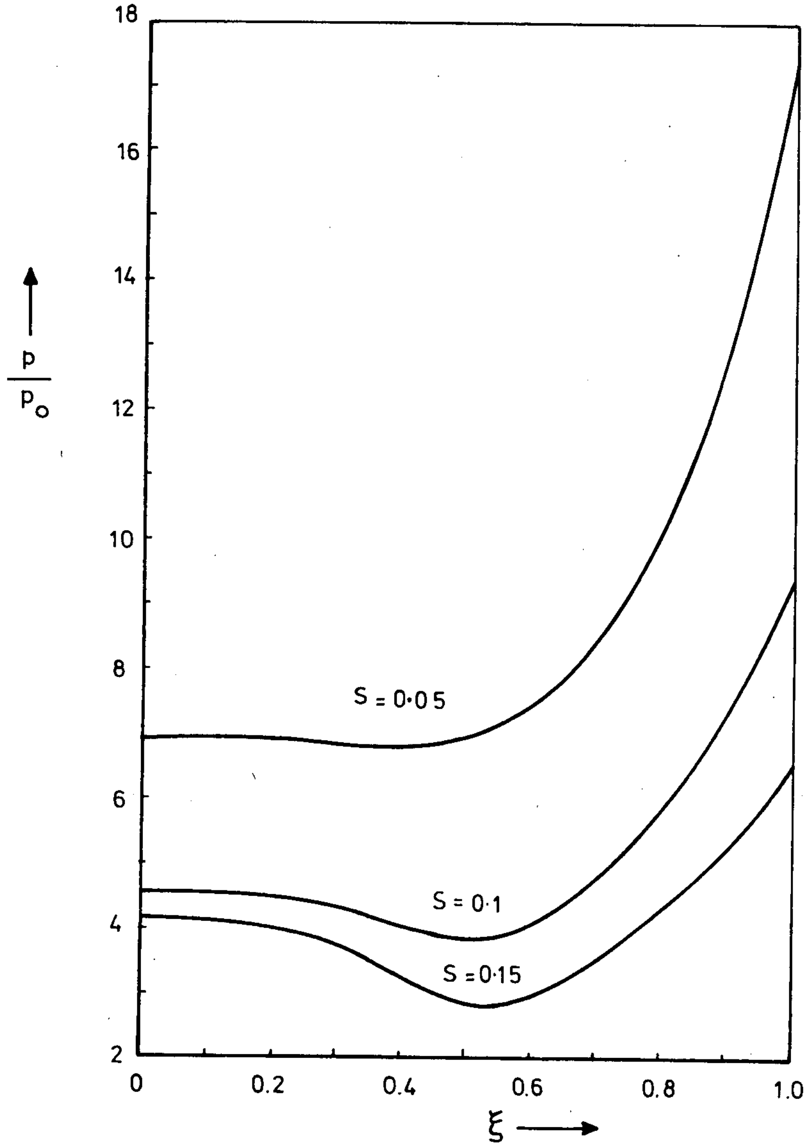

Figure 12. Pressure distributions for varying propagation distance s ($\lambda = 0.1$).

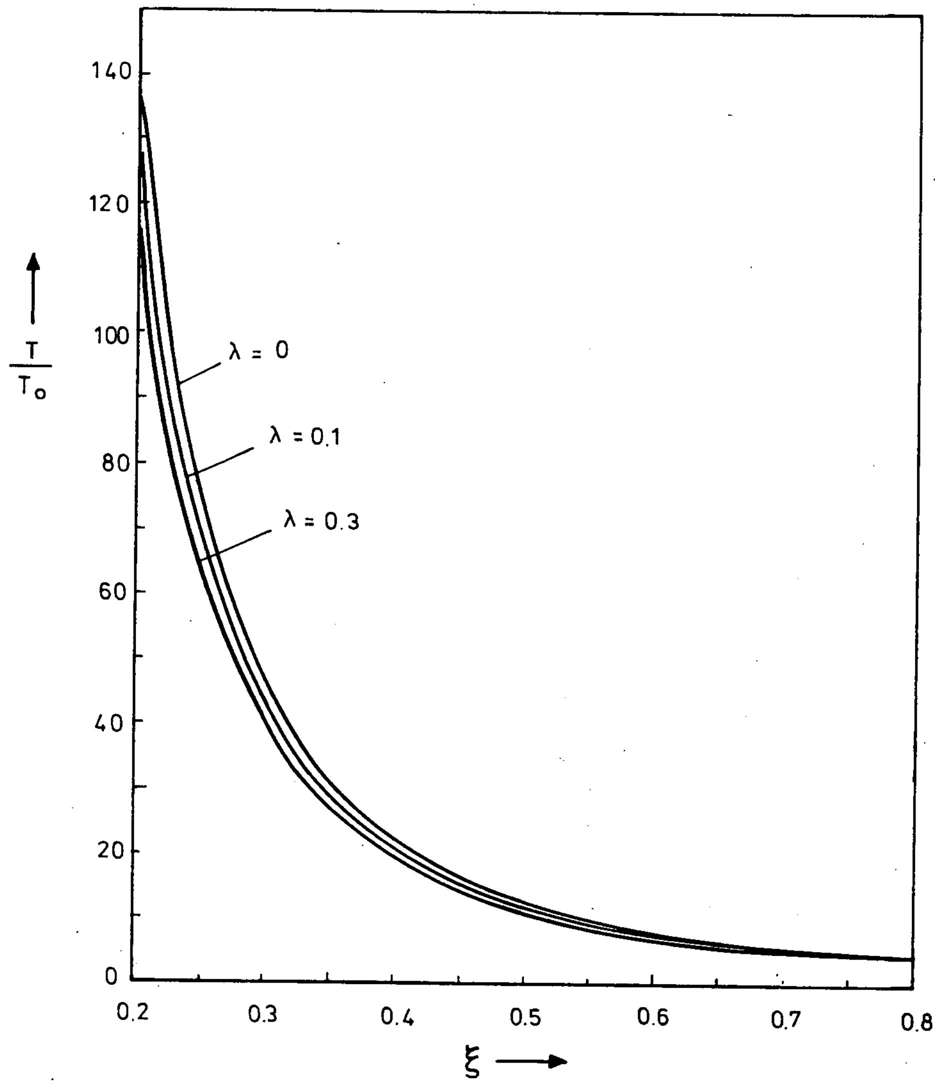

Figure 13. Temperature distributions for varying λ (s = 0.15).

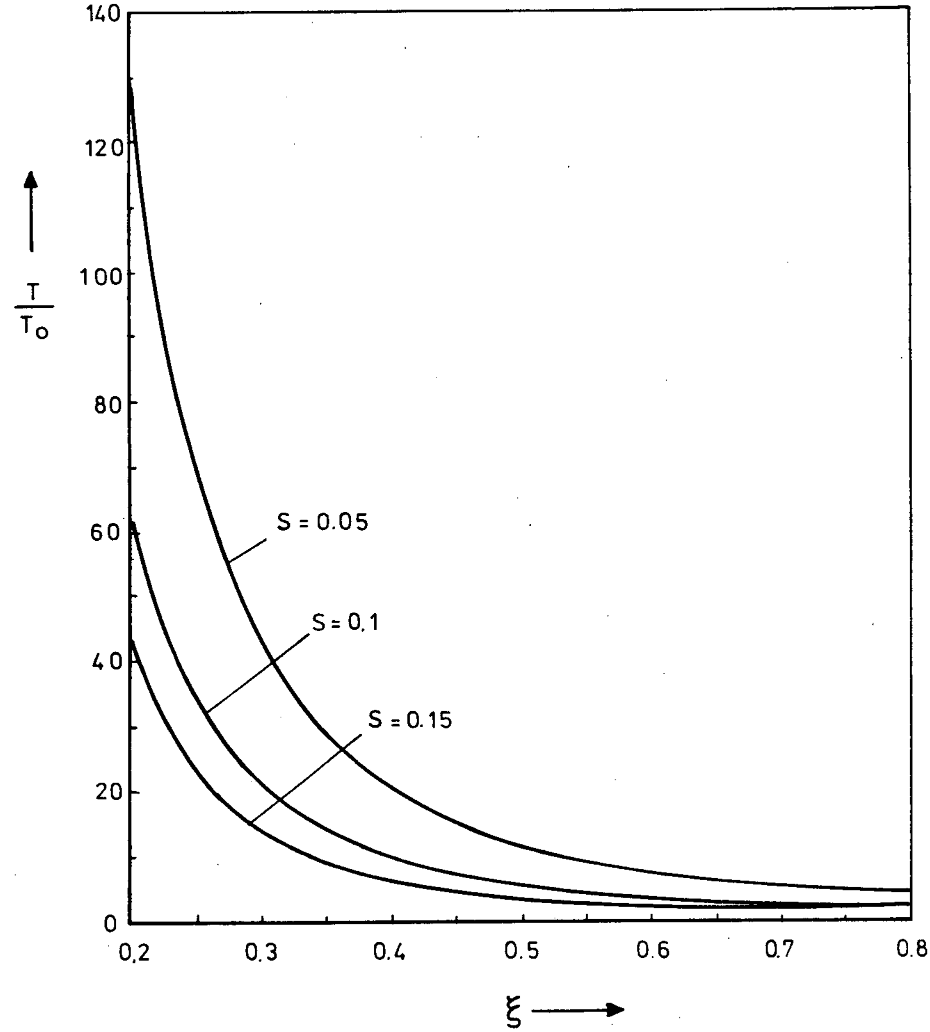

Figure 14. Temperature distributions for varying propagation distance s (λ = 0.1).

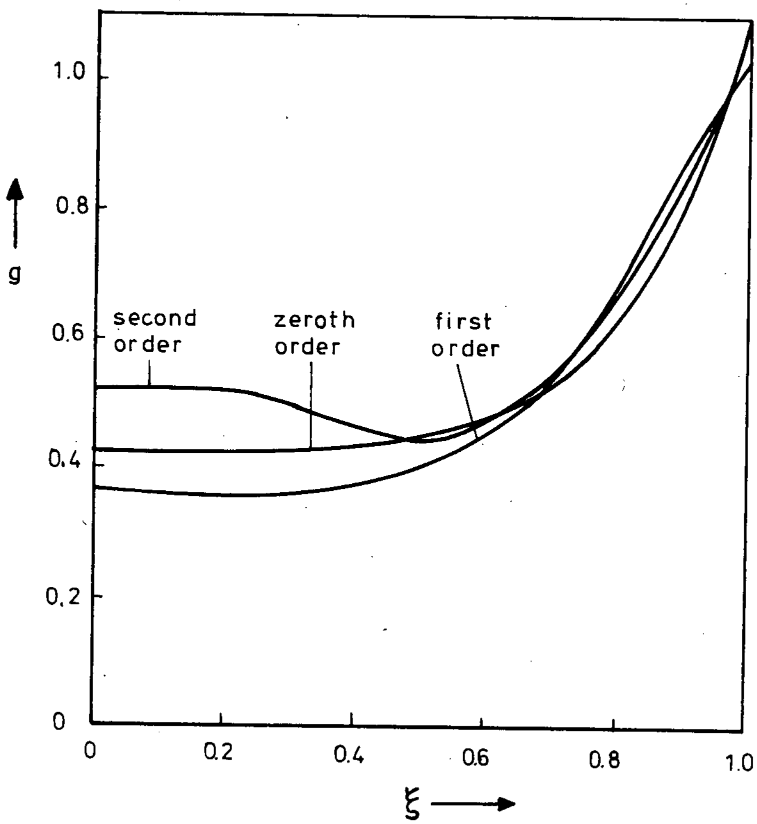

Figure 15. Normalised pressure profiles zeroeth, first and second order (λ = 0.1, s = 0.15).

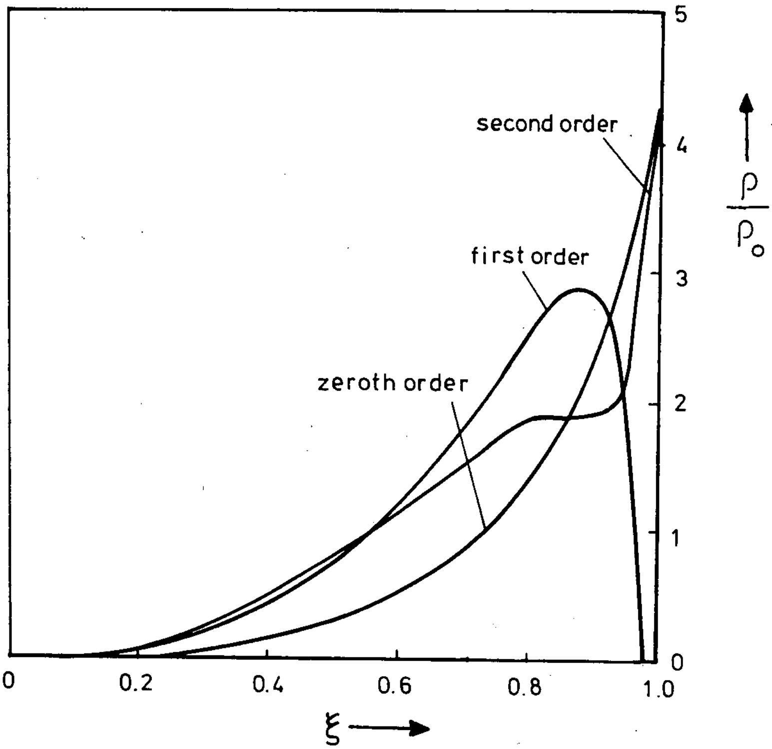

Figure 16. Normalised density profiles zeroeth, first and second order ideal gas solutions (s = 0.15).

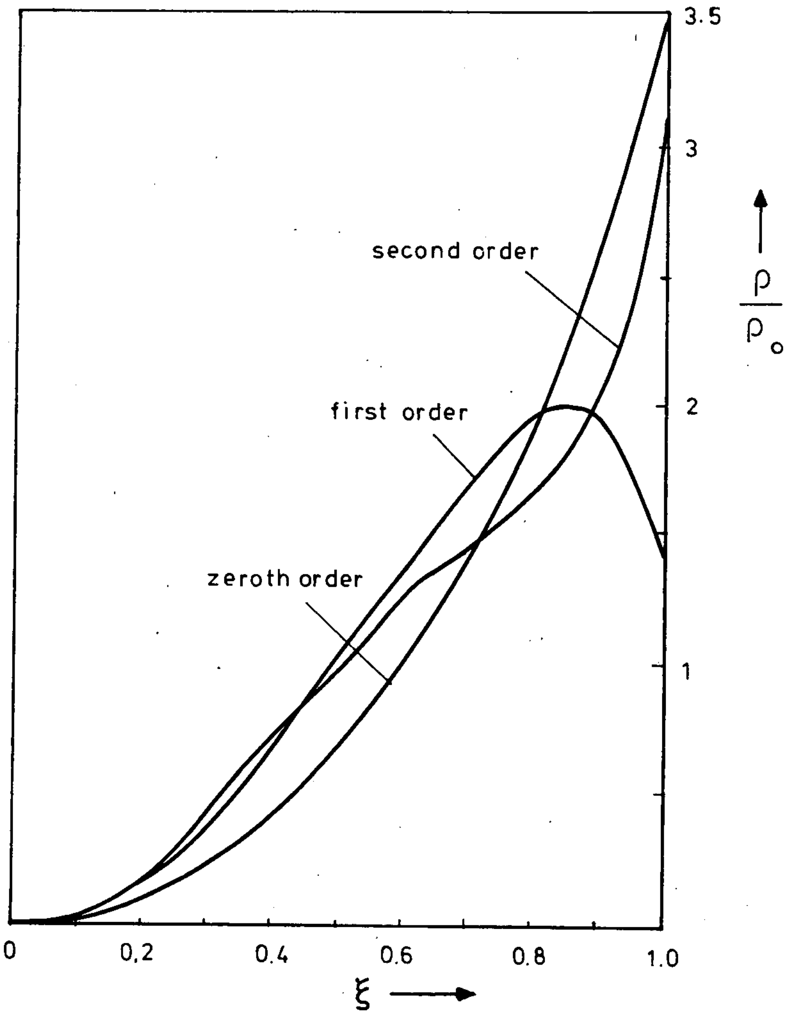

Figure 17. Normalised density profiles zeroeth, first and second order solutions ($\lambda = 0.2$, $s = 0.15$).

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