

# A Weak Form of the Conjugate Gradient FFT Method for Plate Problems

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**Abstract**—A number of electromagnetic field problems for planar structures can be formulated in terms of a hypersingular integral equation, in which a grad-div operator acts on a vector potential. The vector potential is a convolution of the free-space Green's function and some surface current density over the domain of interest. A weak form of this integral equation is obtained by testing it with subdomain basis functions defined over the plate domain only. As a next step, the vector potential is expanded in a sequence of subdomain basis functions and the grad-div operator is integrated analytically over the plate domain only. For the problem of electromagnetic scattering by a plate, the present method shows excellent numerical performance. The numerical difficulties encountered in some previous conjugate gradient fast Fourier transform (CGFFT) methods have been eliminated.

## I. INTRODUCTION

THE conjugate gradient fast Fourier transform (CGFFT) method has been applied to various electromagnetic field problems, where the conjugate gradient iterative method is combined with a fast Fourier transform technique for an efficient computation of the operator expressions with convolution kernels. For the two-dimensional scattering problem of a strip [1], [2] and the three-dimensional scalar scattering problem of a plate, the method leads directly to a rather efficient iterative scheme. The method has further been applied to the full three-dimensional electromagnetic scattering problem of a plate by a number of authors [3]–[7]. These authors have replaced the spatial derivatives of the grad-div operator in the hypersingular integral equation by simple algebraic manipulations in the spectral Fourier domain. This procedure leads to substantial numerical difficulties [7], and although the global error in the conjugate gradient iterative scheme decreases monotonically (but very slowly), the numerical results for the jump of the surface current densities through the plate exhibit erroneous results for an increasing (large) number of iterations. Only for a very fine mesh (about 50 points per wavelength), reasonable results for the current densities are obtained for a small number of iterations.

Several authors have improved the rate of convergence using expansion functions for the current density over the plate, e.g. rooftop functions [8], [9] and piecewise sinusoidal expansion functions [10]. In these "filtered" CGFFT methods the spectral values of the Green's function for larger spectral variables are suppressed. However, the resulting surface current densities exhibit a similar erroneous behavior as in the CGFFT methods [3]–[7] without using basis functions.

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Recently, the discrete Fourier transform method (DFTM) has been introduced [11] to obtain a better approximation of the spatial derivatives in the spectral Fourier domain. In this method, these derivatives are replaced by finite differences and the result is transformed to the spectral domain. Although an improvement in convergence is obtained, we show in this paper that somehow the same numerical difficulties occur as in the CGFFT methods [3]–[10].

The origin of all numerical difficulties in these methods [3]–[10] is the global differentiation (carried out via the spectral domain) over the edges of the plate, where the surface current is not continuously differentiable. Pearson [12] and Catedra *et al.* [13], however, have replaced the continuous equation by a discrete one, in such a way that never, neither explicitly nor implicitly, differentiations are carried out over the edges of the plate. They have expanded the current density in basis functions and carried out one differentiation with respect to this current density. To our knowledge these methods produce reliable results, but the number of FFT's needed in their work [12], [13] is larger than in the CGFFT methods [3]–[10].

In the present paper, we employ a weak form of the integral equation by testing the integral equation with subdomain basis functions defined only over the plate domain. Subsequently, a suitable expansion procedure of the vector potential in the integral equation is carried out. The first result is that the grad-div operator is integrated analytically over the domain of the plate only. The second result is that we have maintained the simple scalar form of the convolution structure of the vector potential (in fact two scalar convolutions). This means that the computation time of each iteration of our present scheme is even less than the one of all methods described previously. In our present method we have not observed the erroneous behavior of the surface currents present in the CGFFT method [3]–[10] and DFTM [11]. Even good results with a very coarse mesh are obtained, and increasing the number of iterations leads to stable numerical values for the surface current densities.

For the canonical problem of scattering of a plane wave by a perfectly conducting square plate the present method has been tested and both the numerical convergence rate and the numerical results of the surface current densities are compared to the ones of the CGFFT method [3]–[7], the filtered CGFFT method [8], [9] and the DFTM [11], respectively. We also present numerical results for a realistic plate configuration, the microstrip, with grazing incidence.

## II. THE INTEGRAL EQUATION

The vectorial position in space is denoted by  $\mathbf{x} = (x_1, x_2, x_3)$ , while the unit vectors are given by  $\mathbf{i}_1$ ,  $\mathbf{i}_2$ , and  $\mathbf{i}_3$ , respectively. We consider the field quantities in the temporal Laplace domain with the transform parameter  $s$ ,  $\text{Re}(s) > 0$ . Note that the frequency-domain results, with time factor  $\exp(-i\omega t)$ , are

obtained by taking the limit  $s \rightarrow -i\omega$ . We consider the problem of scattering by a perfectly conducting plate. This problem can be formulated as an integral equation of the first kind

$$-\mathbf{E}^i(\mathbf{x}) = (\gamma^2 - \text{grad div})\mathbf{A}(\mathbf{x}), \quad \text{for } x \in S, \quad (1)$$

where  $\gamma = s\sqrt{\epsilon\mu}$  and the vector potential  $\mathbf{A}(\mathbf{x})$  is given by

$$\mathbf{A}(\mathbf{x}) = \frac{1}{\epsilon S} \mathcal{F}^{-1}[\tilde{G}(\mathbf{k}) \mathcal{F}[\chi_S(\mathbf{x})\mathbf{J}(\mathbf{x})]], \quad (2)$$

in which  $\mathcal{F}$  and  $\mathcal{F}^{-1}$  denote the forward and inverse two-dimensional spatial Fourier transforms.  $\mathbf{E}^i$  denotes the tangential components of the incident electric field.  $\mathbf{J}$  denotes the jump of the electric surface current density through the plate domain  $S$ , while  $\chi_S(\mathbf{x}) = 1$  when  $\mathbf{x} \in S$  and  $\chi_S(\mathbf{x}) = 0$  elsewhere. Here  $\mathbf{x} = (x_1, x_2)$  denotes the two-dimensional position vector. Further, the spectral Green's function  $\tilde{G}$  is given by

$$\tilde{G}(\mathbf{k}) = \frac{1}{2}(\gamma^2 + k_1^2 + k_2^2)^{-1/2}, \quad \mathbf{k} \in \mathbb{R}^2. \quad (3)$$

### III. TESTING AND EXPANSION PROCEDURE

We first introduce a discretization in the spatial domain  $\mathbf{x} = (x_1, x_2)$ . We use a square mesh with a grid width of  $\Delta x$  in both directions. For our convenience the discrete values of  $\mathbf{x}$  are given by a staggered grid [13]

$$\mathbf{x}_{n,m}^{(1)} = \left( \left( n - \frac{1}{2} \right) \Delta x, m \Delta x \right),$$

$$\mathbf{x}_{n,m}^{(2)} = \left( n \Delta x, \left( m - \frac{1}{2} \right) \Delta x \right). \quad (4)$$

The closed contour  $\partial S$  of the discretized plate now consists of straight lines parallel to the  $x_1$ - or  $x_2$ -axis. The observation points  $\mathbf{x}_{n,m}^{(1)}$  on  $\partial S$  are located on the lines parallel to  $x_2$ -axis and the observation points  $\mathbf{x}_{n,m}^{(2)}$  on  $\partial S$  are located on the lines parallel to  $x_1$ -axis. To cope with the grad-div operator in (1), we test the two scalar equations of (1). Therefore we multiply both sides of (1) with a vectorial testing function  $\psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x})$ ,  $p = 1, 2$ , and integrate with respect to  $\mathbf{x}$  over a plate domain  $S$ . The testing vector function  $\psi^{(p)}(\mathbf{x}) = \psi^{(p)}(\mathbf{x})\mathbf{i}_p$  is a suitably chosen vector function that will be defined later. We then obtain

$$-\int_S \psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x}) E_p^i(\mathbf{x}) d\mathbf{x}$$

$$= \gamma^2 \int_S \psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x}) A_p(\mathbf{x}) d\mathbf{x}$$

$$+ \int_S \partial_p \psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x}) \text{div } \mathbf{A}(\mathbf{x}) d\mathbf{x},$$

$$\mathbf{x}_{n,m}^{(p)} \in S, \quad p = 1, 2, \quad (5)$$

where we have used Gauss' theorem. In the derivation of (5) we have assumed that  $\psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x})$  vanishes for  $\mathbf{x} \in \partial S$ . Further,  $\partial_p$  denotes the partial derivative with respect to  $x_p$ ,  $p = 1, 2$ . To maintain the simple convolution of the vector potential  $\mathbf{A}$ , we do not expand the current density but we expand the vector potential as

$$A_p(\mathbf{x}) = \sum_{k,l} a_{p;k,l} \psi^{(p)}(\mathbf{x} - \mathbf{x}_{k,l}^{(p)}). \quad (6)$$

Taking the divergence on this expression, substituting both the result and the expression for  $\mathbf{A}$  in the right-hand side of (5) and

interchanging the order of integration and summation, we obtain

$$-e_p^i(\mathbf{x}_{n,m}^{(p)}) = \sum_{k,l} \left[ \gamma^2 v^{(p)}(\mathbf{x}_{n,m}^{(p)}; \mathbf{x}_{k,l}^{(p)}) a_{p;k,l} \right.$$

$$\left. + \sum_{q=1}^2 w_{p,q}(\mathbf{x}_{n,m}^{(p)}; \mathbf{x}_{k,l}^{(q)}) a_{q;k,l} \right], \quad \mathbf{x}_{n,m}^{(p)} \in S, \quad p = 1, 2, \quad (7)$$

in which

$$e_p^i(\mathbf{x}_{n,m}^{(p)}) = \int_S \psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x}) E_p^i(\mathbf{x}) d\mathbf{x}, \quad (8)$$

$$v^{(p)}(\mathbf{x}_{n,m}^{(p)}; \mathbf{x}_{k,l}^{(p)}) = \int_S \psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x}) \psi^{(p)}(\mathbf{x} - \mathbf{x}_{k,l}^{(p)}) d\mathbf{x}, \quad (9)$$

$$w_{p,q}(\mathbf{x}_{n,m}^{(p)}; \mathbf{x}_{k,l}^{(q)}) = \int_S \partial_p \psi^{(p)}(\mathbf{x}_{n,m}^{(p)} - \mathbf{x})$$

$$\cdot \partial_q \psi^{(q)}(\mathbf{x} - \mathbf{x}_{k,l}^{(q)}) d\mathbf{x}. \quad (10)$$

In view of the partial derivatives in (10), the derivative with respect to  $x_p$  of the basis function  $\psi^{(p)}(\mathbf{x})$  must exist. Therefore we take the rooftop functions [13] as basis functions, viz.

$$\psi^{(1)}(\mathbf{x}) = \Lambda(x_1; 2\Delta x) \Pi(x_2; \Delta x),$$

$$\psi^{(2)}(\mathbf{x}) = \Pi(x_1; \Delta x) \Lambda(x_2; 2\Delta x), \quad (11)$$

in which  $\Lambda(x; 2\Delta x)$  is the triangle function with support  $2\Delta x$  and  $\Pi(x; \Delta x)$  is the pulse function with support  $\Delta x$ . The value of  $e_p^i$  of (8) can be integrated analytically for some particular choice of  $E_p^i$ . With (11) we are also able to compute the functions  $v^{(p)}(\mathbf{x})$  and  $w_{p,q}(\mathbf{x})$  analytically. Substitution of these results in (7) leads to

$$-e_1^i(\mathbf{x}_{n,m}^{(1)}) = \sum_{k=1}^3 d_k a_{1;n+k-2,m}$$

$$+ \sum_{k=1}^2 \sum_{l=1}^2 t_{kl} a_{2;n+k-1,m+l-2}, \quad (12)$$

$$-e_2^i(\mathbf{x}_{n,m}^{(2)}) = \sum_{k=1}^2 \sum_{l=1}^2 t_{kl} a_{1;n+k-2,m+l-1}$$

$$+ \sum_{l=1}^3 d_l a_{2;n,m+l-2}, \quad (13)$$

in which  $\mathbf{x}_{n,m}^{(p)} \in S$ . The coefficients  $d_k$  and  $t_{kl}$  are obtained as

$$d_k = \frac{\gamma^2 (\Delta x)^2}{6} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \quad k = 1, 2, 3, \quad (14)$$

and

$$t_{kl} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad k, l = 1, 2. \quad (15)$$

With our particular choice of basic functions, the quantities  $a_{p;n,m}$  are directly related to the vector potential as

$$a_{p;k,l} = \frac{1}{\epsilon S} \text{DFT}^{-1}[\tilde{G} \text{DFT}[\chi_S \mathbf{J}]], \quad (16)$$

in which the discrete Fourier transforms (DFT) are computed with the two-dimensional FFT algorithms. Observe that the scalar form of the convolution structure (in fact two scalar

convolutions) has been maintained. Further, it is mentioned that we have not made any extra assumption on the expansion of the current density. The numerical approximation of the current density is only dictated by the projection of the continuous Fourier transform to the discrete FFT. This means that we have used a bandlimited representation for the current density. In our numerical test cases we have also used rooftop functions as expansion functions for the current density and no significant differences in the numerical results have been observed.

Note that (12), (13), and (16) represent an operator equation with unknown  $\mathbf{J}(\mathbf{x}_{n,m}^{(p)})$  on  $S$ . This operator equation is solved iteratively by a conjugate gradient scheme where it attempts to minimize the residual error over the domain  $S$  only (excluding the edge  $\partial S$ ). In each iteration, we assign a zero value to the residual and the normal component of the surface current at the points  $\mathbf{x}_{n,m}^{(p)}$  located on  $\partial S$ .

#### IV. NUMERICAL RESULTS

In this section we present the numerical results for the problem of scattering by a perfectly conducting plate. We consider the time-harmonic case with angular frequency  $\omega$ . The Laplace transform parameter is taken to be  $s = -i\omega(1 + 0.001i)$ , so that numerical difficulties with the branch-point of the Green's function are avoided. All computations were carried out on a VAX 8250 with the FFT numbers equal to  $128 \times 128$ .

For a square  $\lambda \times \lambda$  plate and a normally incident plane wave with electric field vector parallel to the  $x_1$ -axis, the performance of the present method is compared to the CGFFT [3]-[7], filtered CGFFT [8]-[10] and DFTM [11]. The numerical convergence rates of the different methods are compared by presenting the normalized root mean square error  $\text{err}$ , defined as

$$\text{err} = \frac{\|\mathbf{r}^{(n)}\|}{\|\mathbf{r}^{(0)}\|}, \quad (17)$$

in which  $\|\mathbf{r}^{(n)}\|$  denotes the norm of the residual error in the satisfaction of the operator equation of (12) and (13) over the domain  $S$  of the plate in the  $n$ th iteration. In all cases we have taken a zero initial estimate. Fig. 1 shows the numerical convergence rate for a spatial discretization of  $\Delta x = \lambda/17$  and  $\Delta x = \lambda/33$ , respectively. It is observed that the present method performs substantially better than both the CGFFT [3]-[7] and DFTM [11].

Figs. 2-4 present the jump of the surface current density through the  $\lambda \times \lambda$  plate for different discretizations and error tolerances. We have observed that the CGFFT method [3]-[7] and the filtered CGFFT method [8]-[10] both produce nearly the same results for the current density. Therefore, in Figs. 2-4 we present only one of them. Taking into account the physical smoothness of the surface current, it is observed from Fig. 2 that the (filtered) CGFFT methods [3]-[10] produce suspicious results for  $\Delta x = \lambda/17$  and a tolerance in the residual norm of 1%. Also it is noted that the surface current density  $|J_2|$  does not vanish at the edge, where it should be. It is observed that the DFTM [11] indeed produces better results than the (filtered) CGFFT methods [3]-[10] do. However, the present method produces a smoother result in  $|J_2|$ . Increasing the spatial discretization it is observed from Fig. 3 that the (filtered) CGFFT methods [3]-[10] and DFTM [11] now produce the expected surface current more accurately, but the present method again produce smoother surface currents for  $\Delta x = \lambda/33$  and a tolerance in the residual norm of 1%. Subsequently we have strengthened the tolerance in the residual norm to 0.1% and

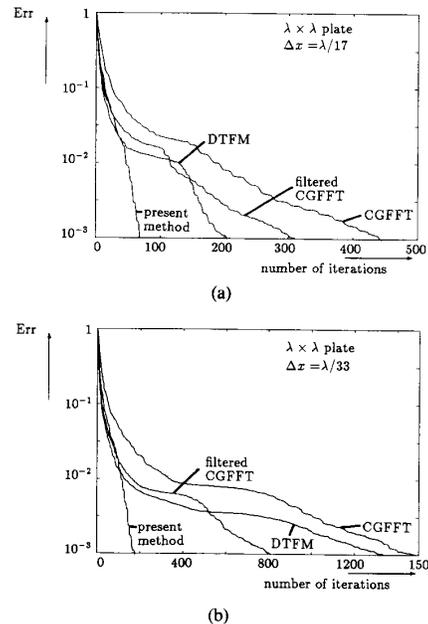


Fig. 1. Numerical convergence rate of the  $\lambda \times \lambda$  plate with normal incidence.

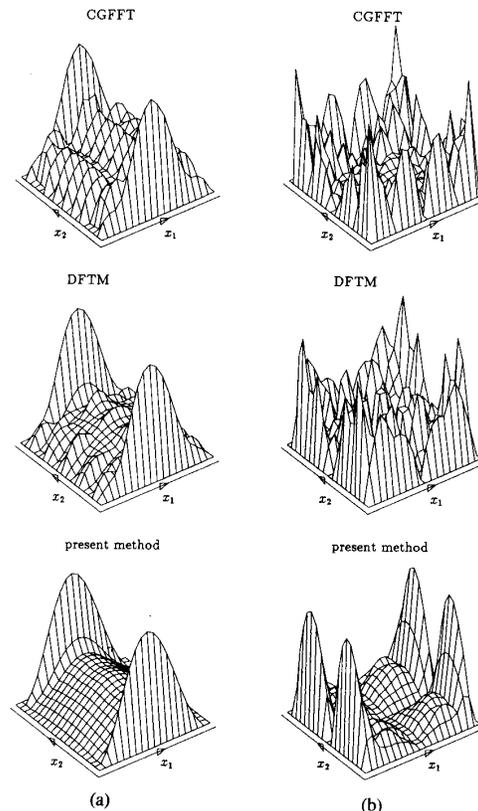


Fig. 2. (a) Surface current density  $|J_1|$  over a  $\lambda \times \lambda$  plate with normal incidence,  $\Delta x = \lambda/17$ ,  $\text{err} = 0.01$ . (b) Surface current density  $|J_2|$  over a  $\lambda \times \lambda$  plate with normal incidence,  $\Delta x = \lambda/17$ ,  $\text{err} = 0.01$ .

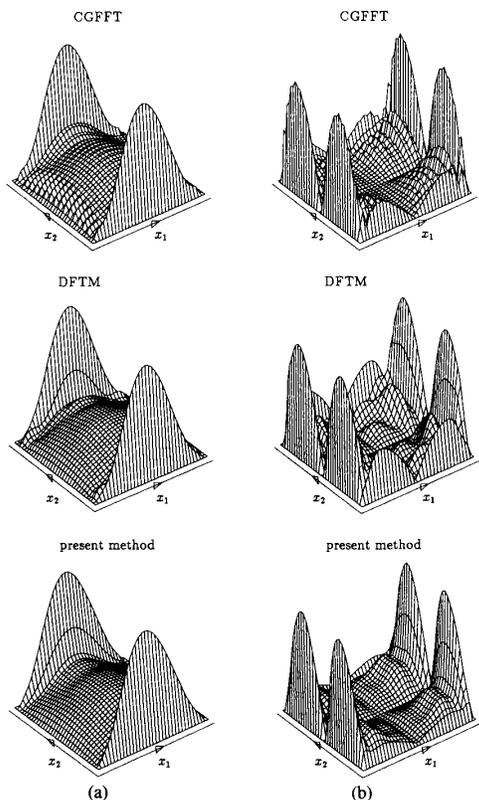


Fig. 3. (a) Surface current density  $|J_1|$  over a  $\lambda \times \lambda$  plate with normal incidence,  $\Delta x = \lambda/33$ , err = 0.01. (b) Surface current density  $|J_2|$  over a  $\lambda \times \lambda$  plate with normal incidence,  $\Delta x = \lambda/33$ , err = 0.01.

from Fig. 4 we observe that the (filtered) CGFFT methods [3]–[10] and DFTM [11] again produce suspicious surface current densities. We have increased the tolerance in the residual error further than 0.1% (up to  $10^{-5}$ !) and we have observed that the gross errors in the surface currents become larger and larger, while the numerical results of our weak formulation converge to a stable solution. Comparing our present results of Figs. 3(b) and 4(b) it is observed that an error criterion of 1% is too weak. At least an error criterion of 0.1% is needed.

In Fig. 5, we present the surface current densities for a realistic configuration, the microstrip, of  $\lambda \times 3\lambda$ , excited by a grazing incident plane wave propagating parallel to the  $x_2$ -axis and the electric field vector parallel to the  $x_1$ -axis. The results are obtained using our present method. Again smooth surface current densities are arrived at.

We have also implemented the CG-scheme based on the discrete cyclic convolution [14]. In this procedure our spectral Green's function is replaced by a spectral cyclic one as follows. We transform the spectral Green's function with large FFT numbers to the spatial domain, truncate it and transform it back to the spectral domain with a FFT pad twice the size of the plate in each dimension. Exactly the same results have been obtained with considerable decrease of computation time of each iteration in the CG-scheme.

V. CONCLUSION

From Figs. 2–4 it is observed that the (filtered) CGFFT methods [3]–[10] and DFTM [11] exhibit an erroneous behavior

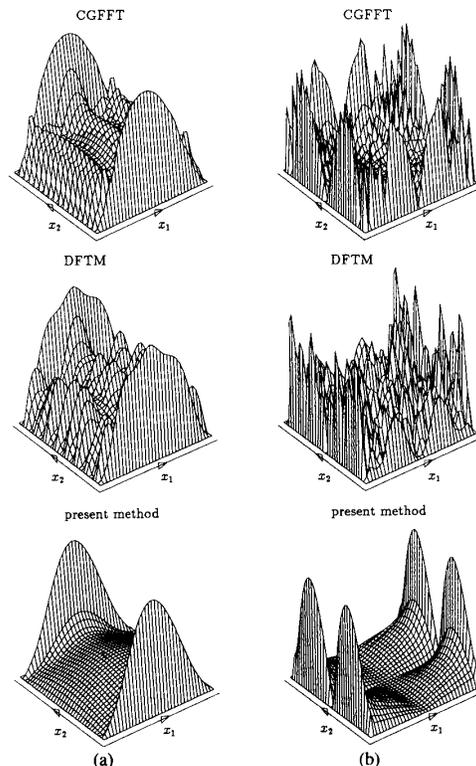


Fig. 4. (a) Surface current density  $|J_1|$  over a  $\lambda \times \lambda$  plate with normal incidence,  $\Delta x = \lambda/33$ , err = 0.001. (b) Surface current density  $|J_2|$  over a  $\lambda \times \lambda$  plate with normal incidence,  $\Delta x = \lambda/33$ , err = 0.001.

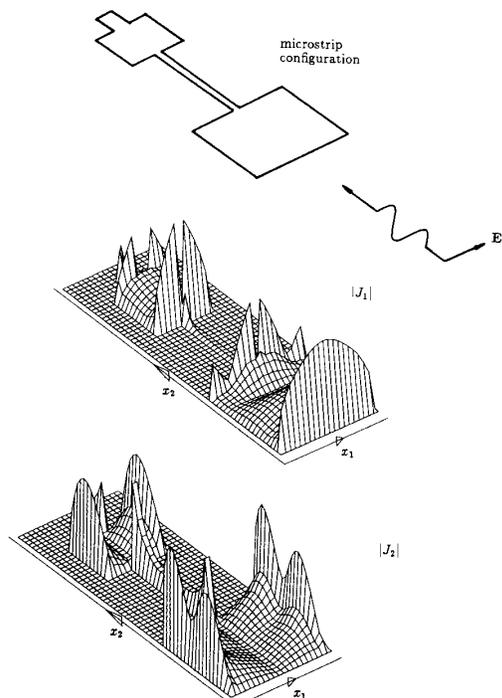


Fig. 5. Surface current densities  $|J_1|$  and  $|J_2|$  over a  $\lambda \times 3\lambda$  microstrip with grazing incidence,  $\Delta x = \lambda/19$ , err = 0.001, 240 iterations.

caused by a (global) differentiation over the edges of the plate, where the jump in the surface current is not continuously differentiable. At this point it is to be remarked that the methods based on the work of Pearson [12] and Catedra *et al.* [13] do not suffer from this problem.

In our weak formulation the grad-div operator is integrated analytically over the domain of the plate only. As a consequence we have maintained the simple convolution structure of the vector potential, we have no matrix-vector multiplications in the spectral domain. We only have matrix-vector multiplications in the spatial domain, but these ones are only over the domain occupied by the plate. This means that the computation time of each iteration of our present scheme is less than all previous methods.

## REFERENCES

- [1] P. M. van den Berg, "Iterative computational techniques in scattering based upon the integrated square error criterion," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 1063-1071, Oct. 1984.
- [2] —, "Iterative schemes based on the minimization of error field problems," *Electromagn.*, no. 5, pp. 237-262, 1985.
- [3] T. K. Sarkar, E. Arvas and S. M. Rao, "Application of the Fast Fourier Transform and the conjugate method for efficient solution of electromagnetic scattering from both electrically large and small conducting bodies," *Electromagn.*, no. 5, pp. 99-122, 1985.
- [4] —, "Application of FFT and the conjugate method for the solution of electromagnetic radiation from electrically large and small conducting bodies," *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 635-640, May 1986.
- [5] R. Kastner and R. Mittra, "Iterative analysis of finite-sized planar frequency selective surfaces with rectangular patches or perforations," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 372-377, Apr. 1987.
- [6] R. Kastner, "A conjugate gradient procedure for the analysis of planar conductors with alternating patch and aperture formulation," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1616-1620, Nov. 1988.
- [7] T. J. Peters and J. L. Volakis, "Application of a conjugate gradient FFT method to scattering from thin planar material plates," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 518-526, Apr. 1988.
- [8] T. A. Cwik and R. Mittra, "Scattering from a periodic array of free-standing arbitrarily shaped perfectly conducting or resistive patches," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 1226-1234, Nov. 1987.
- [9] P. Petre, "Application of the generalized biconjugate gradient FFT method for analysing arrays of resistive patches standing in free-space and in front of a ground plane covered by a dielectric layer," in *Proc. 1989 URSI Int. Symp. Electromagn. Theory*, Stockholm, Sweden, Aug. 14-18, 1989, pp. 338-340.
- [10] K. Barkeshli and J. L. Volakis, "Improving the convergence rate of the conjugate gradient FFT method using subdomain basis

functions," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 893-900, July 1989.

- [11] C., Y. Shen, K. J. Glover, M. I. Sancer and A. D. Varvatsis, "The discrete Fourier transform method for solving differential-integral equations in scattering theory," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 1032-1041, Aug. 1989.
- [12] L. W. Pearson, "A technique for organizing Large moment calculations for use with iterative solution methods," *IEEE Trans. Antennas Propagat.*, vol. AP-33, pp. 1031-1033, Sept. 1985.
- [13] M. F. Catedra, J. G. Cuevas, and L. Nuno, "A scheme to analyze conducting plates of resonant size using the conjugate gradient method and Fast Fourier transform," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1744-1752, Dec. 1988.
- [14] K. Barkeshli and J. L. Volakis, "On the implementation of the conjugate gradient Fourier transform method for scattering by planar plates," *IEEE Trans. Antennas Propagat. Mag.*, vol. 32, pp. 19-29, Apr. 1990.



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