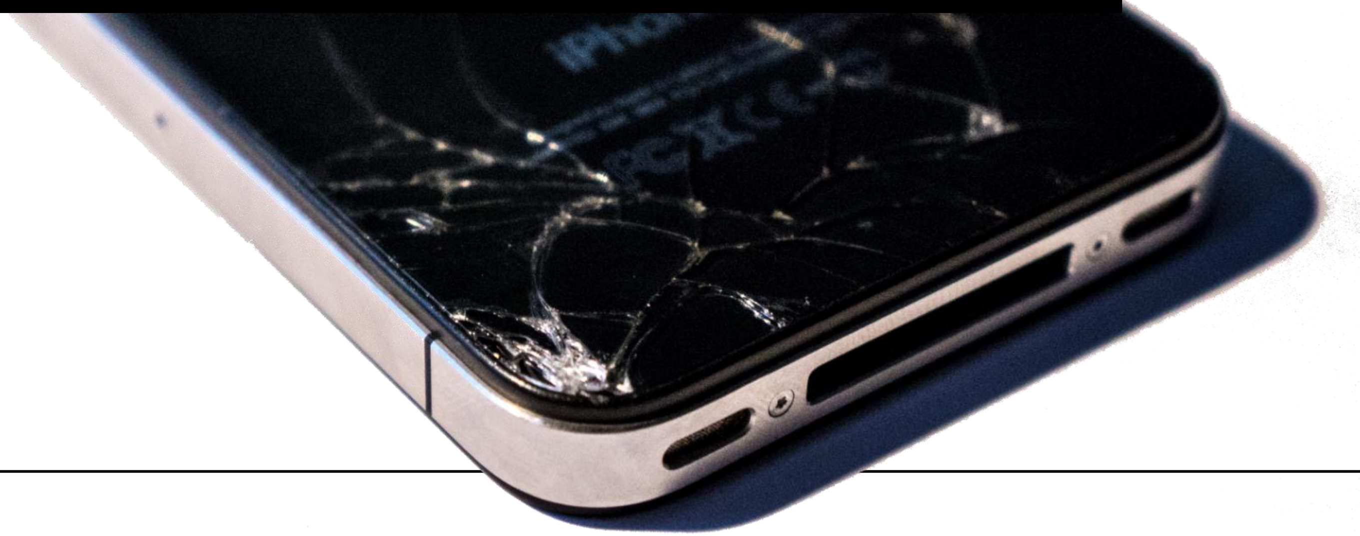
Impulse Based Substructuring Unravelled:

Simulation and coupling of structural dynamics in the time domain

Daniël D. van den Bosch

23 May 2014



Introduction

- Simulate coupled dynamics of a structure as a result of impact loading
  - Impact contains high frequency excitations
- Coupling techniques in the frequency domain exist
  - Not very efficient for this loadcase
- Simulate a coupled response in the time domain
  - Inherent to transient effects
  - Techniques are not mature yet

Contents

What do we need?

- ❑ Simulate structural dynamics in the time domain using "Impulse Response Functions"
- ❑ Obtaining Impulse Response Functions
- ❑ Coupling of structural dynamics
  - ❑ The concept
  - ❑ The application

"Simulate the coupled dynamics of components in the time domain"

Structural Dynamics

How to describe a structure's dynamics?

- Mass-, damping- and stiffness matrix

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{C}\,\dot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \mathbf{f}(t)$$

- The frequency domain approach
  - Dynamic stiffness $\mathbf{Z}(\omega)\,\mathbf{u}(\omega) = \mathbf{f}(\omega)$
  - Receptance / admittance $\mathbf{u}(\omega) = \mathbf{Y}(\omega)\,\mathbf{f}(\omega)$

Structural Dynamics

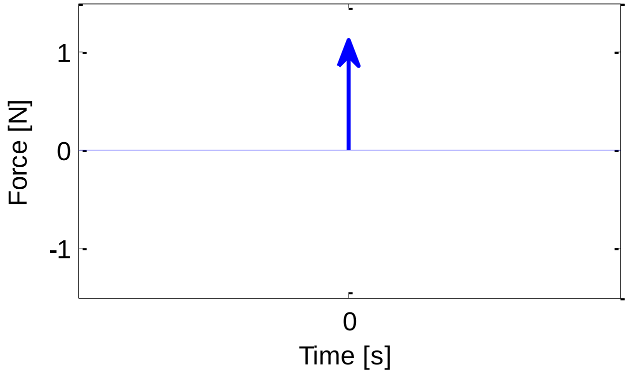
In the time domain!

- Convolution Product or Duhamel's integral

$$\mathbf{u}(t) = \int_0^t \mathbf{H}(t-\tau)\,\mathbf{f}(\tau)\,\mathrm{d}\tau$$

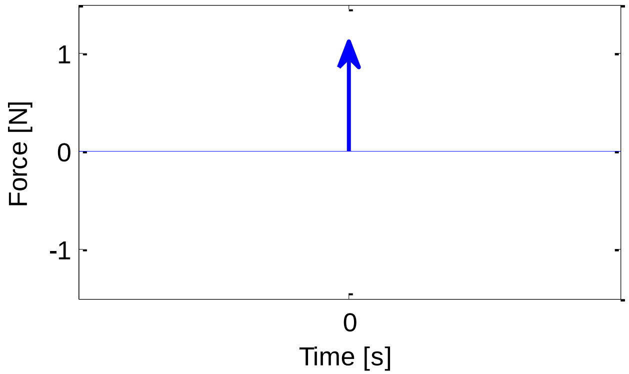
- Impulse Response Functions $\mathbf{H}(t)$
  - A structure's response to a Dirac impulse

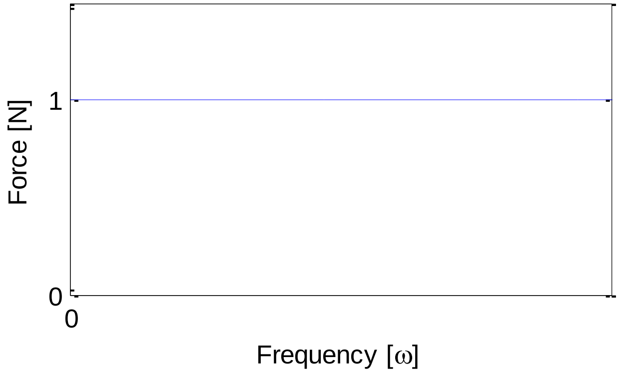
$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{C}\,\dot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \begin{bmatrix} \delta(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$



Towards building the impact load

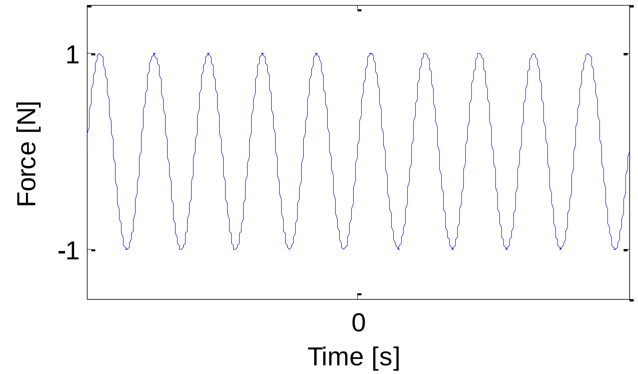
Dirac impulse

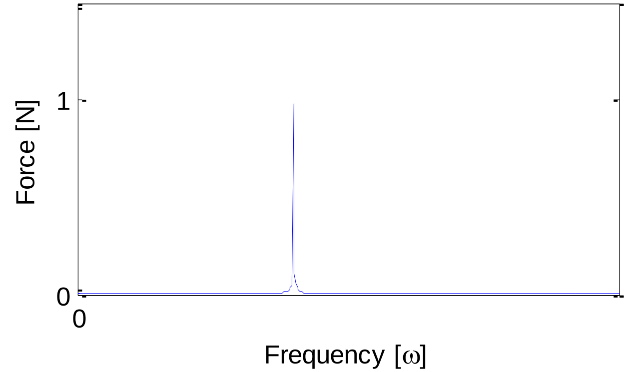




vs.

Harmonic excitation

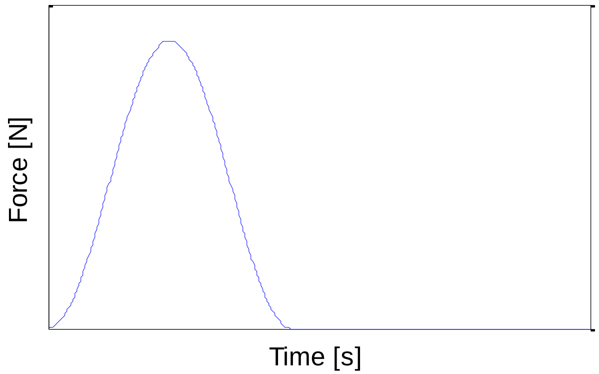


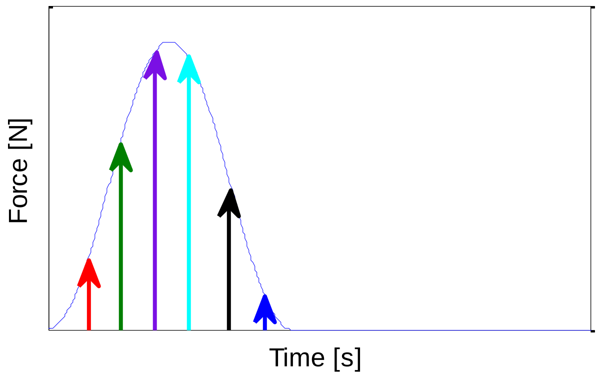


Building the impact load

Using superposition..

Dirac impulses





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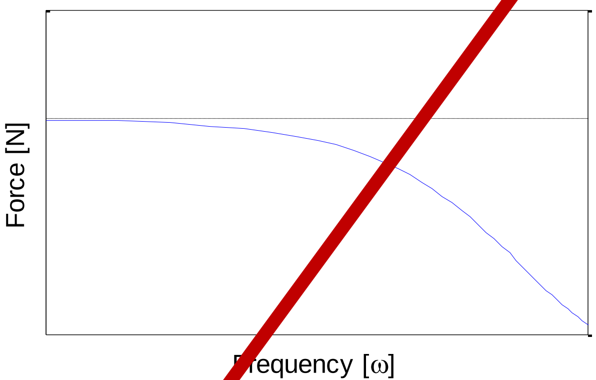
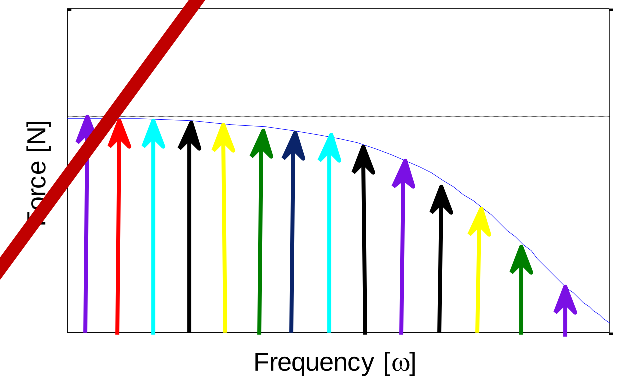
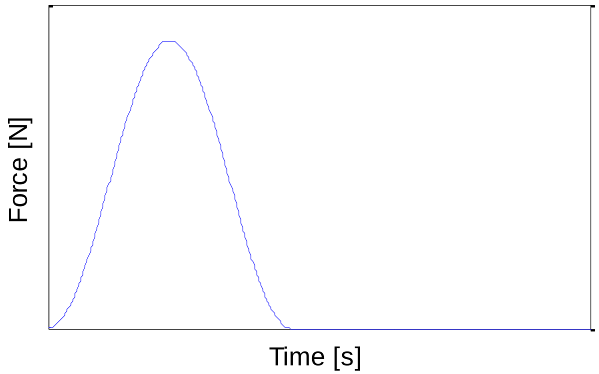




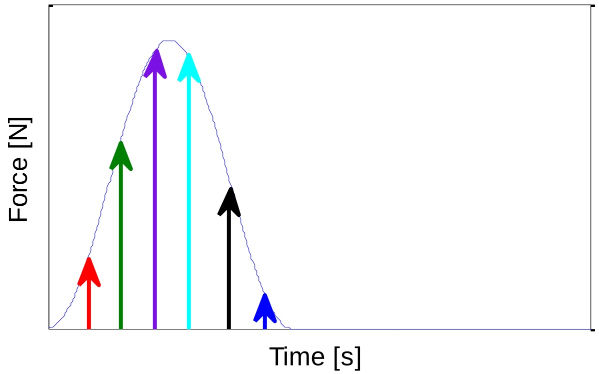
Convolution product explained

Discretisation

Dirac impulses



$$\mathbf{u}(t) = \int_0^t \mathbf{H}(t-\tau)\,\mathbf{f}(\tau)\,\mathrm{d}\tau$$



$$\mathbf{u}_n = \sum_{i=1}^{n} \mathbf{H}_{n-i}\,\mathbf{f}_i\,\Delta t$$

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"Simulate the coupled dynamics of components in the time domain"

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Impulse Response Functions

How to obtain them?

- Spatially discretised
  - Analytically
  - Numerically
- Spatially continuous
  - Experimentally
  - Analytically

Impulse Response Functions

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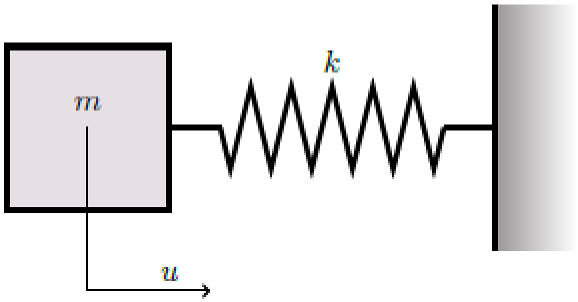
Analytical IRF

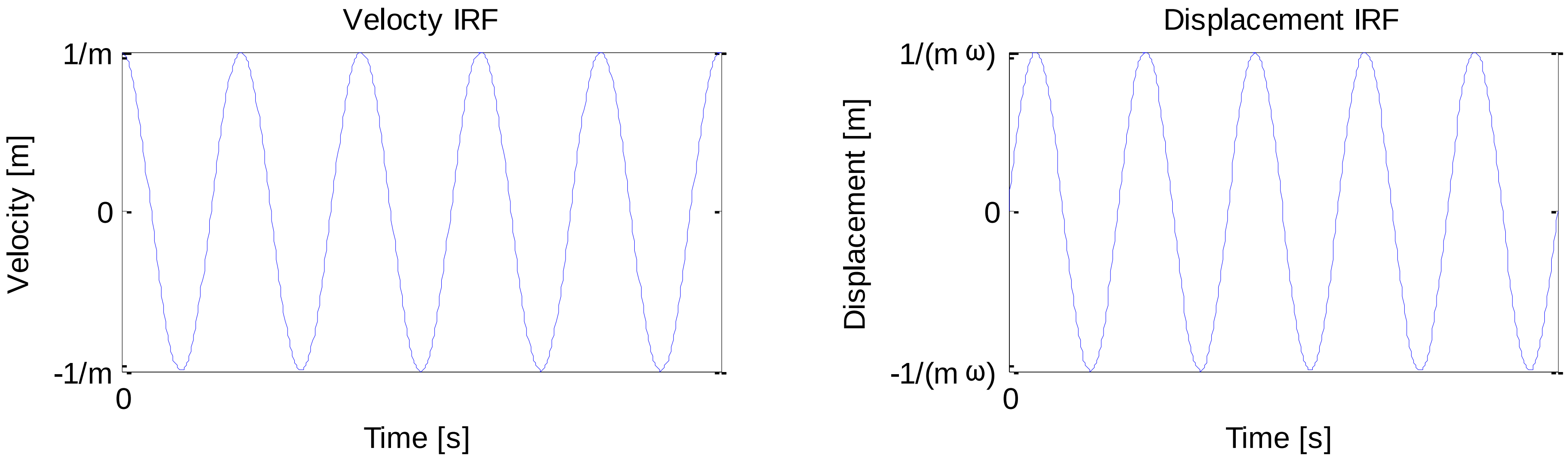
Single DoF



$$m\,\ddot{u}(t) + k\,u(t) = \delta(t)$$

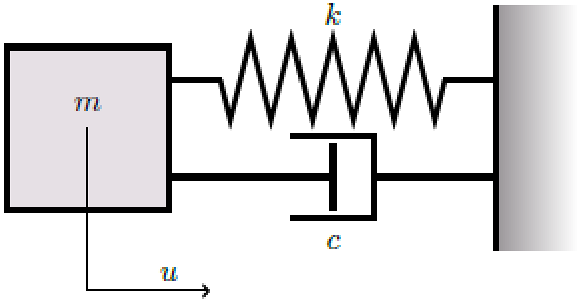
$$m\,\Delta\dot{u} = \int_0^{t+} \delta(t)\,\mathrm{dt} = 1$$

- Jump in velocity at time 0.



Analytical IRF

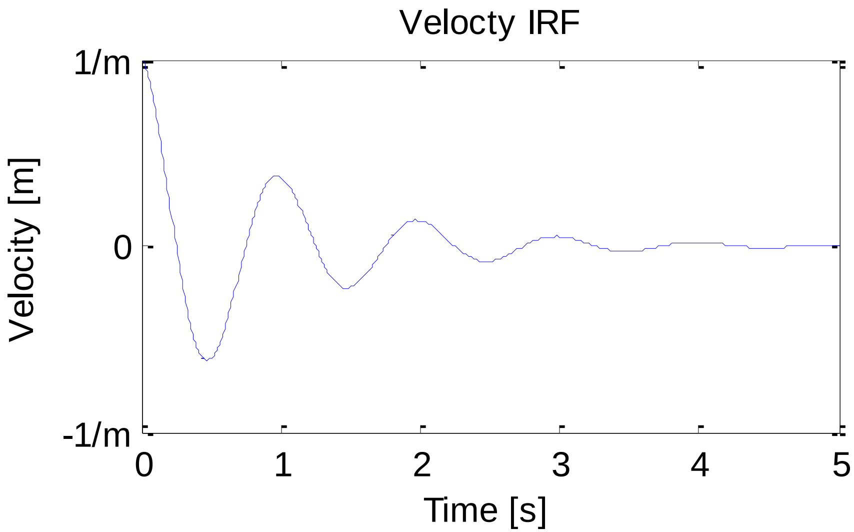
Single DoF

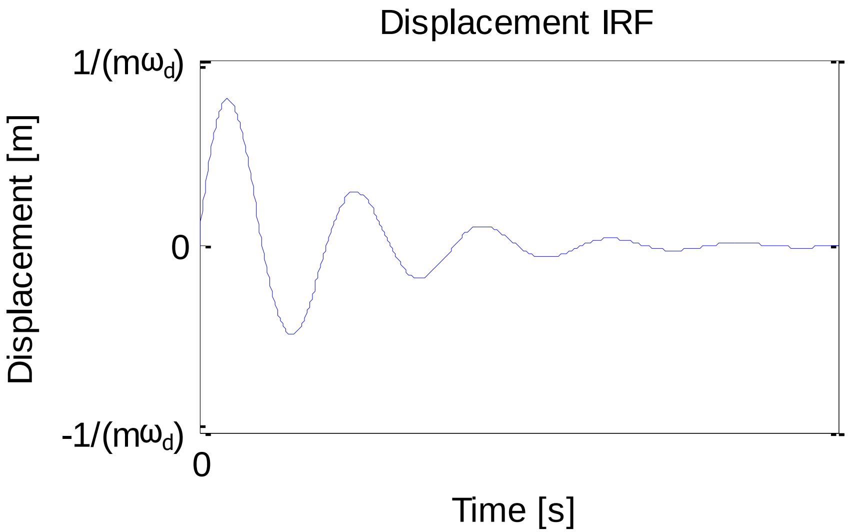


$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = \delta(t)$$

$$m\,\Delta\dot{u} = \int_0^{t+} \delta(t)\,\mathrm{dt} = 1$$

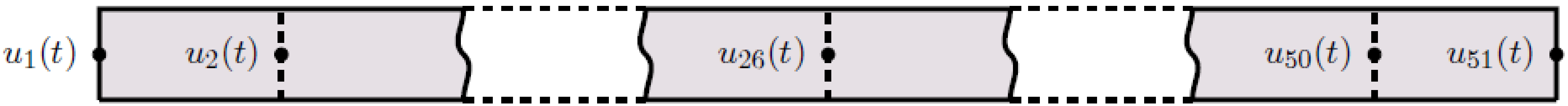
- Jump in velocity at time 0.





Analytical IRF

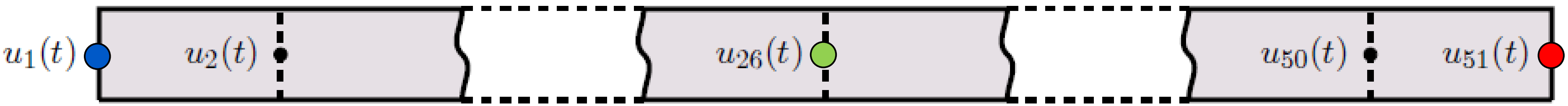
Multiple DoF

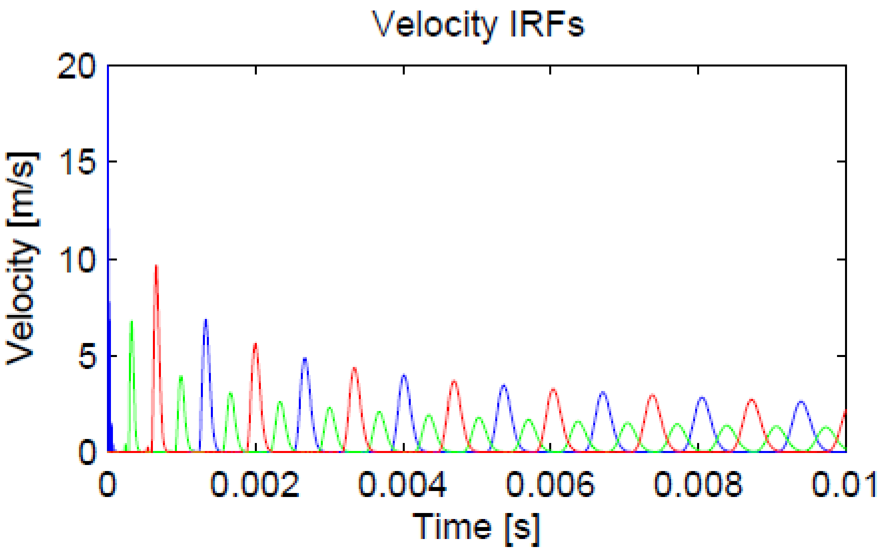


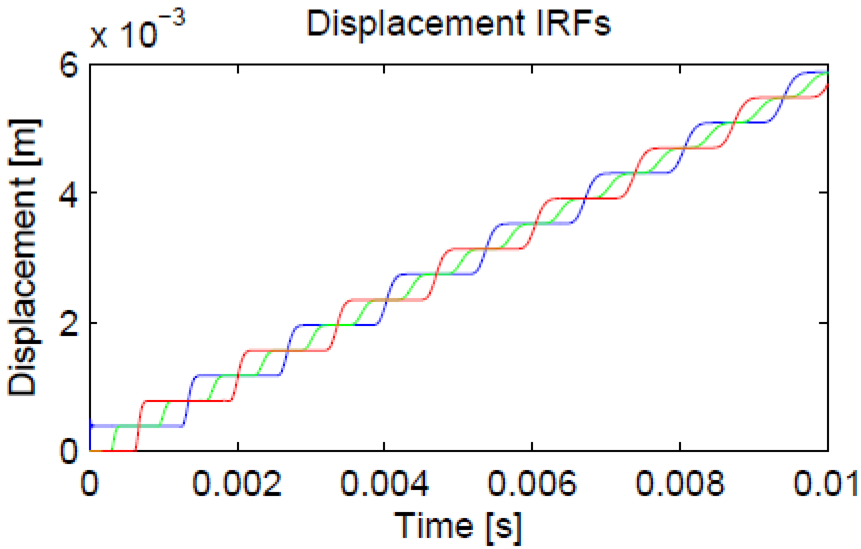
- The response of each *mode* for an impulse on a *node*

Analytical IRF

of a 50 element bar







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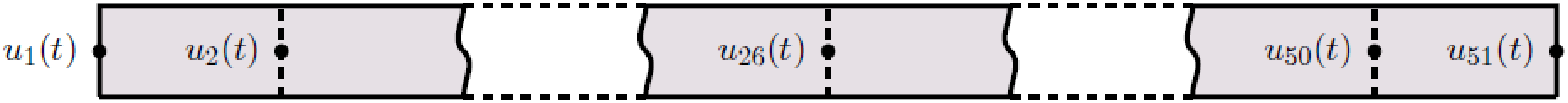
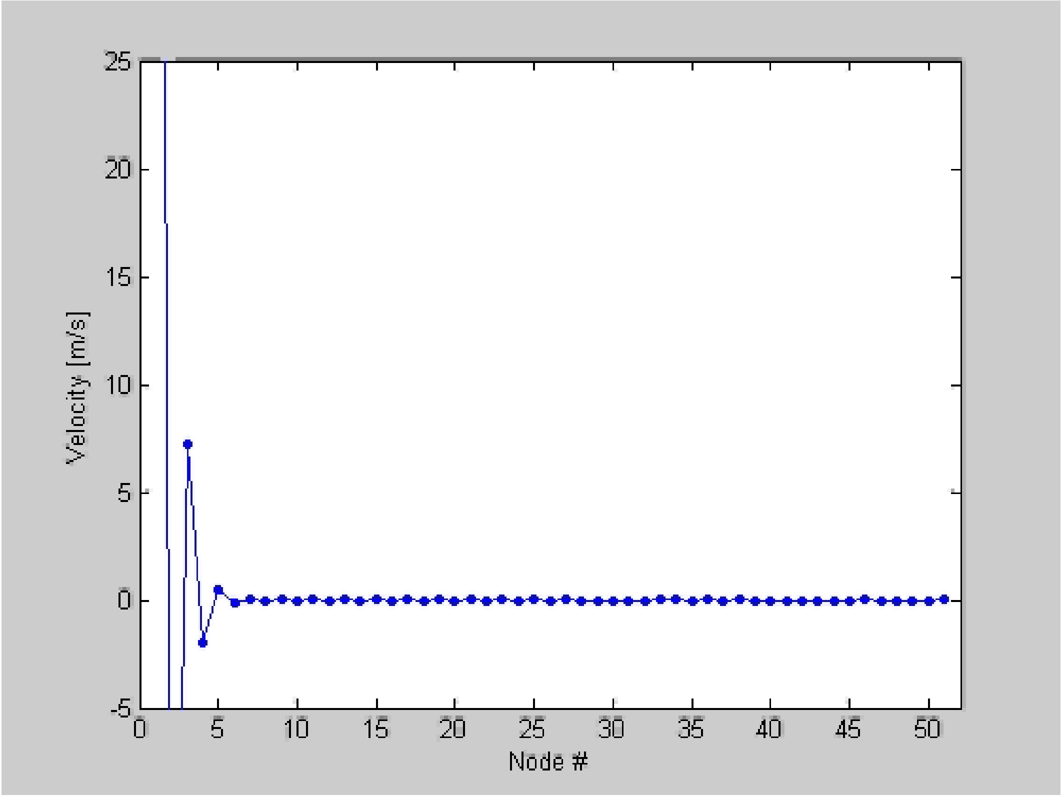




Impulse Response Functions

How to obtain them?

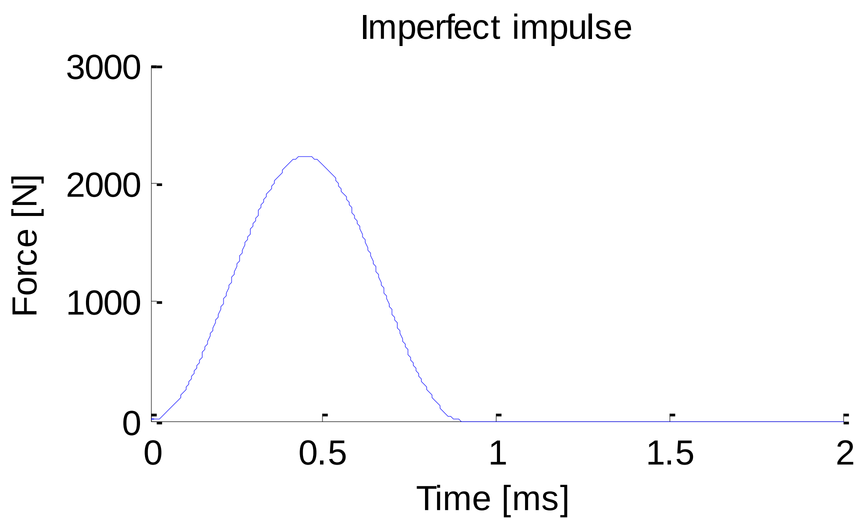
- Spatially discretised
  - Analytically
  - Numerically
- Spatially continuous
  - Experimentally
  - Analytically

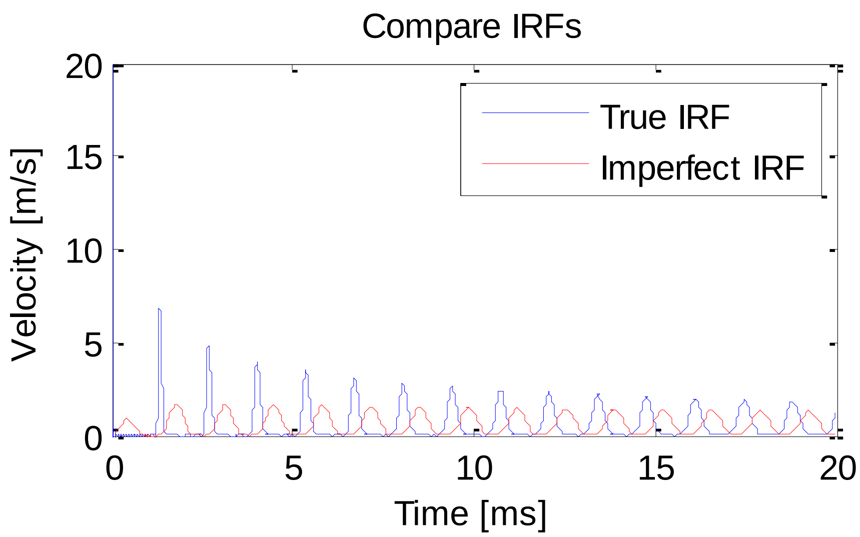
Measured IRF

Simulated using imperfect impulse

$$\tilde{h}(t) = \int_0^t h(t-\tau)\,\tilde{\delta}(\tau)\,\mathrm{d}\tau$$

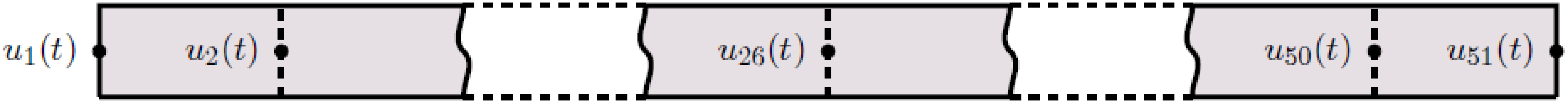
- Convolution with this load gives us the measured IRF

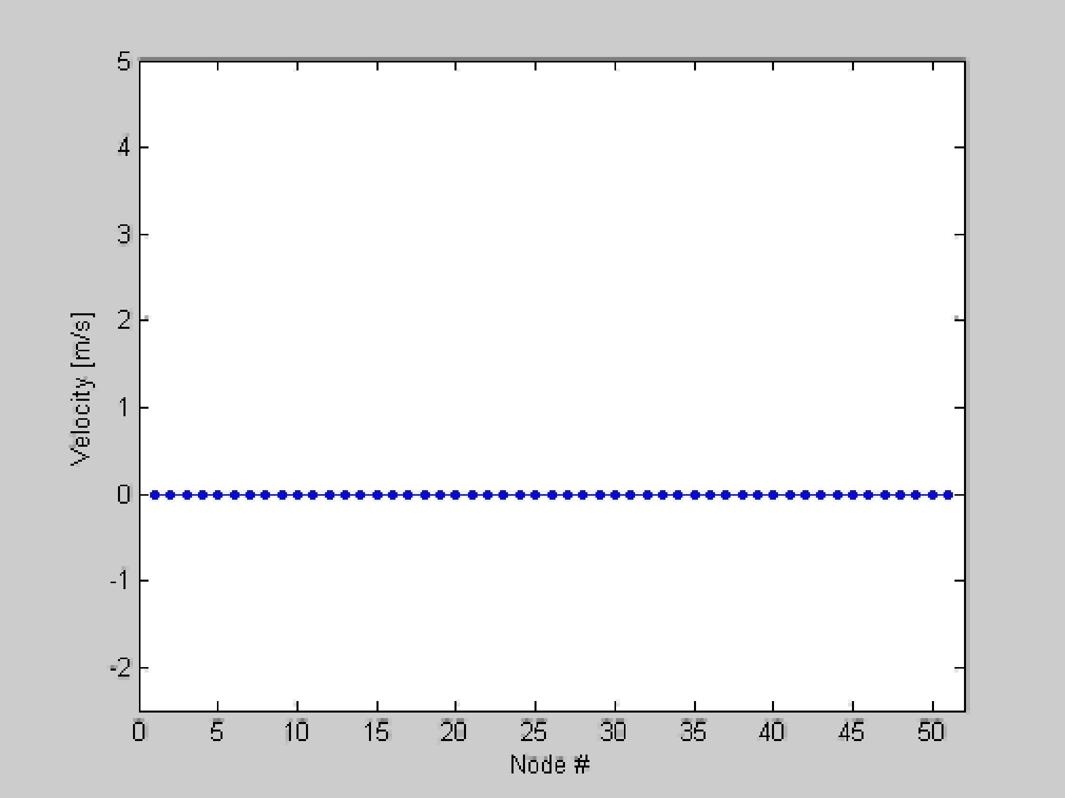




Measured IRF

Simulated using imperfect impulse

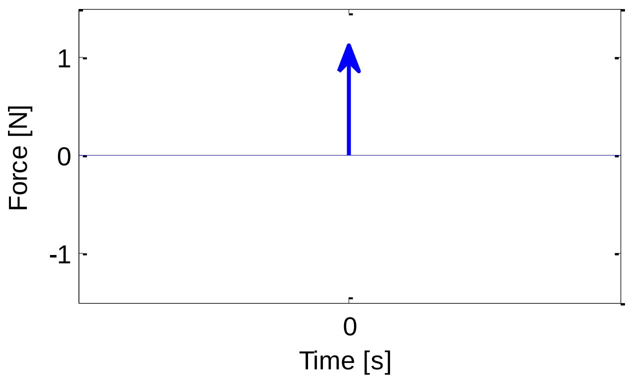


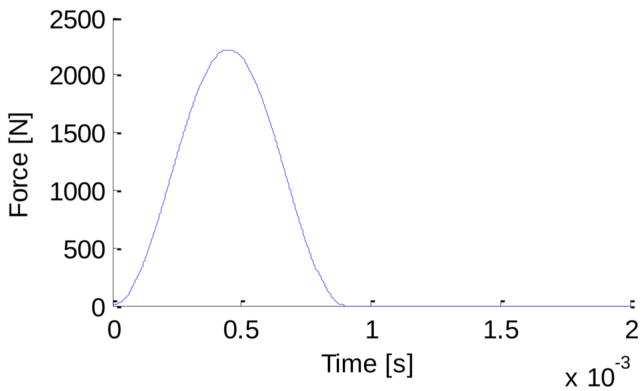


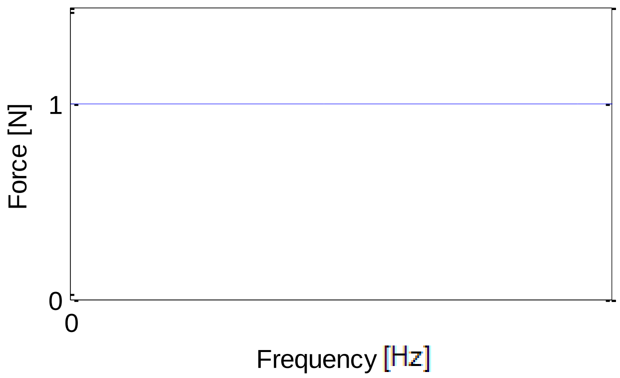
Measured IRF

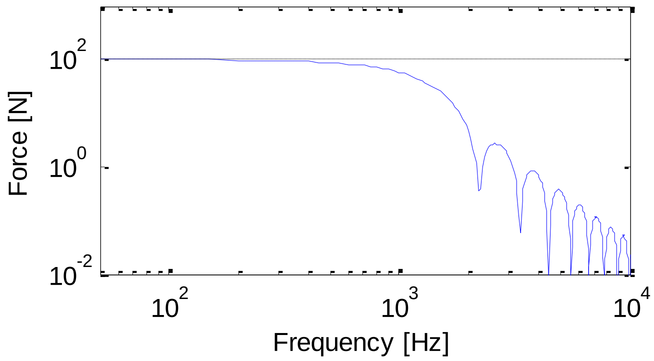
Comparison perfect and imperfect response

Perfect impulse vs. **Imperfect impulse**





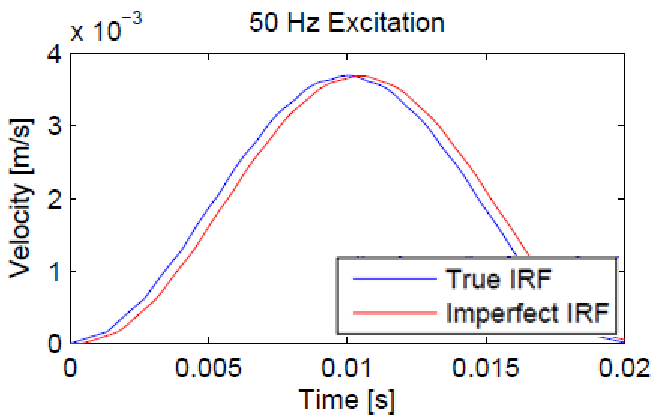


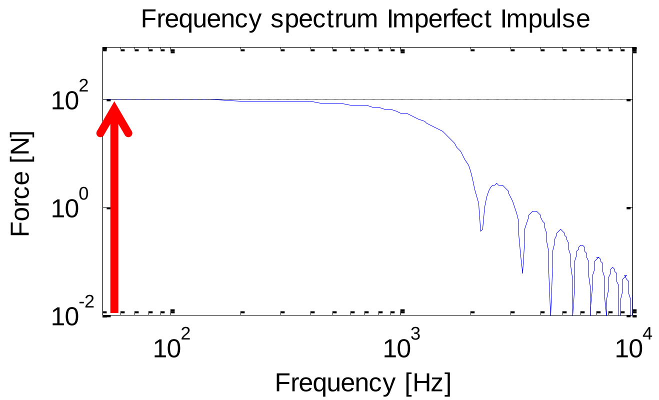


Response to a harmonic excitation

Simulation using imperfect Impulse

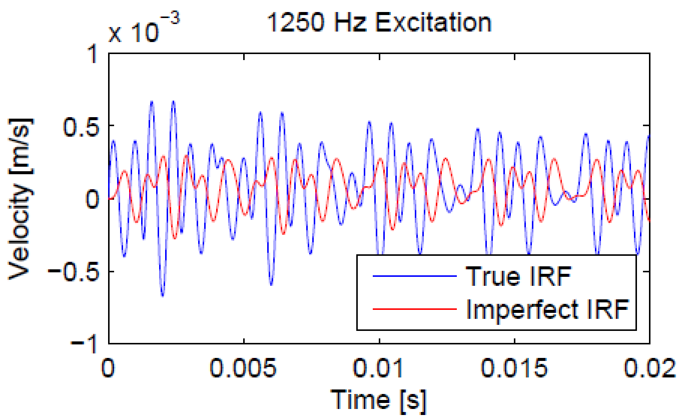
Low frequency

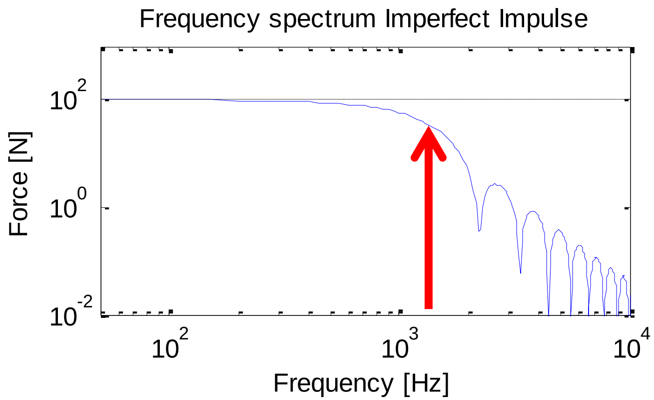




vs.

High frequency





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"Simulate the coupled dynamics of components in the time domain"

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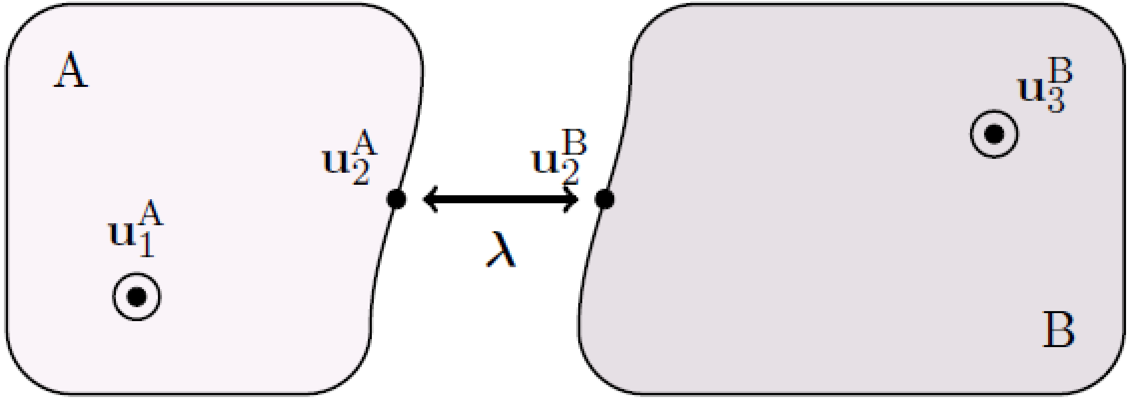
Coupling structural dynamic

Substructuring explained



- Compatibility condition: $$\mathbf{B}\,\mathbf{u} = \begin{bmatrix} \mathbf{0} & -\mathbf{I} & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1^A \\ \mathbf{u}_2^A \\ \mathbf{u}_2^B \\ \mathbf{u}_3^B \end{bmatrix} = \mathbf{0}$$
- Equilibrium condition: $$\mathbf{g} = -\mathbf{B}^T \boldsymbol{\lambda}$$

Coupling structural dynamic

Expanding the convolution product

- Expanded convolution product:

$$\begin{cases} \mathbf{u}(t) = \int_0^t \mathbf{H}(t-\tau) \left(\mathbf{f}(\tau) - \mathbf{B}^T \boldsymbol{\lambda}(\tau)\right) \mathrm{d}\tau \\ \mathbf{B}\,\mathbf{u}(t) = \mathbf{0} \end{cases}$$

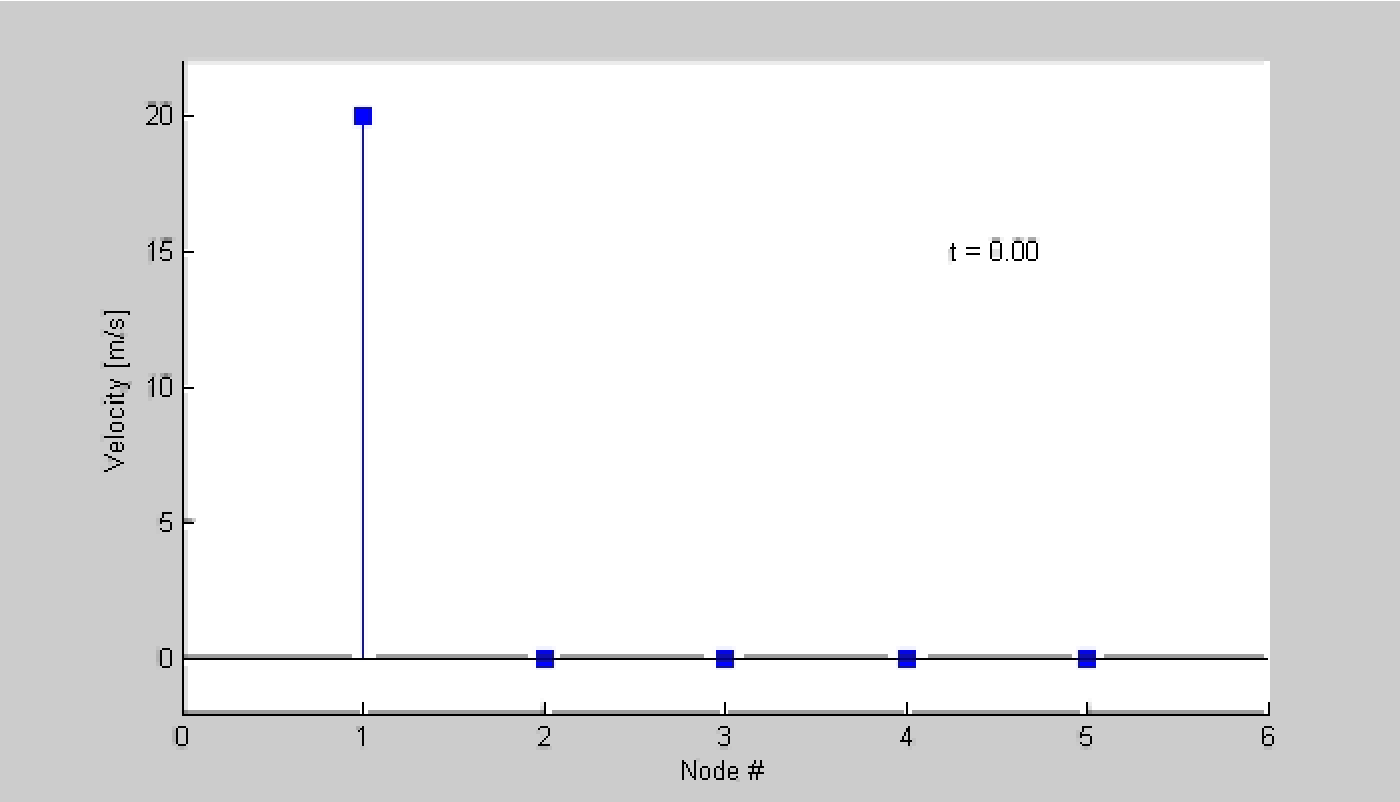
- And discretise:

$$\begin{cases} \mathbf{u}_n = \sum_{i=1}^{n} \mathbf{H}_{n-i} \left(\mathbf{f}_i - \mathbf{B}^T \boldsymbol{\lambda}_i\right) \Delta t \\ \mathbf{B}\,\mathbf{u}_n = 0 \end{cases}$$

IRF of a 5 node bar

To illustrate coupling phenomena

- Consider the following model and IRF:

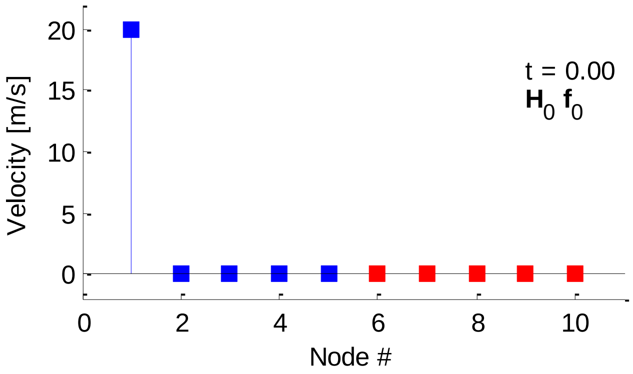


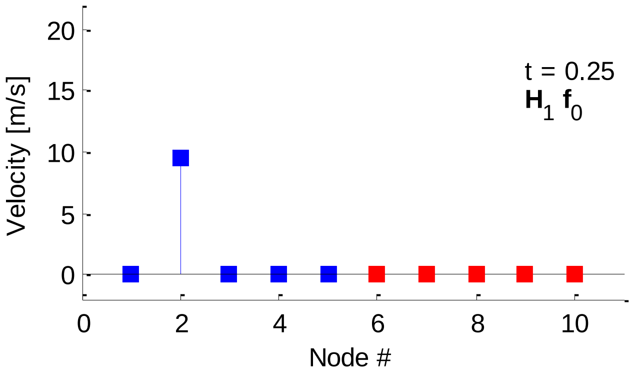
Coupling two bars

Behind the scenes

- Couple two of these bars..

And see what happens!





$$\mathbf{u}_n = \sum_{i=1}^{n} \mathbf{H}_{n-i} \left(\mathbf{f}_i - \mathbf{B}^T \boldsymbol{\lambda}_i \right) \Delta t$$

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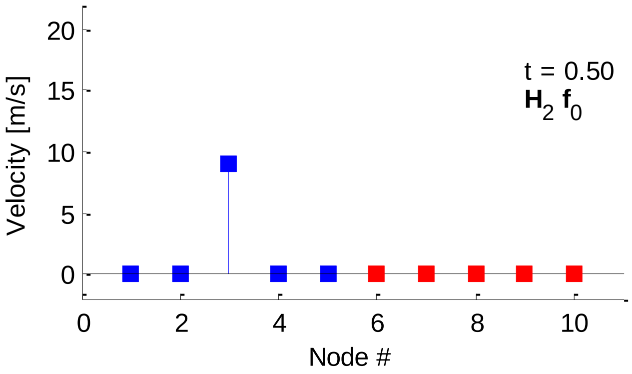

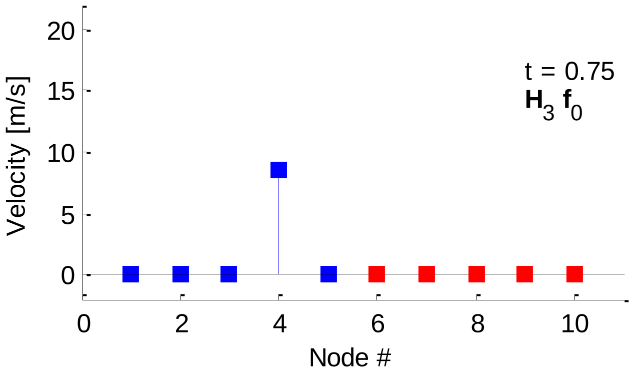

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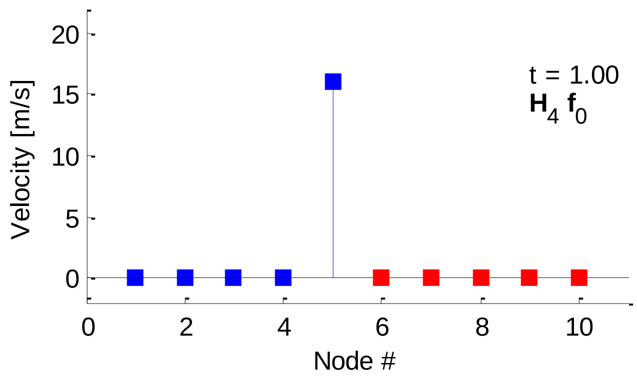
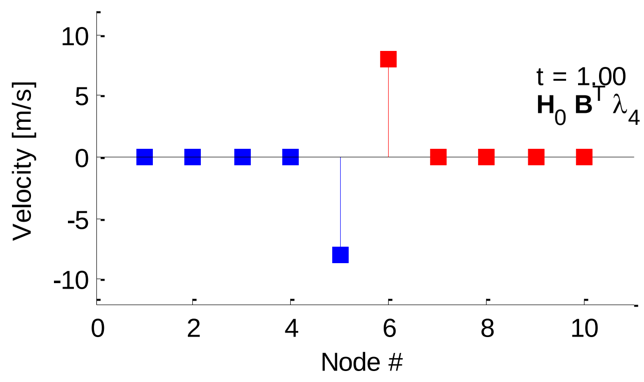
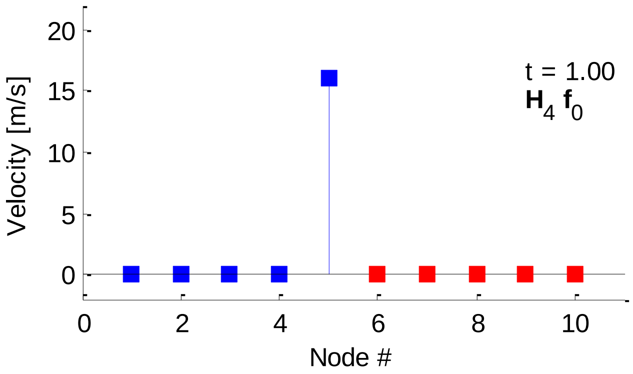




$$\mathbf{u}_n = \sum_{i=1}^{n} \mathbf{H}_{n-i} \, (\mathbf{f}_i \; - \mathbf{B}^T \, \boldsymbol{\lambda}_i) \Delta t$$

Coupling two bars

Behind the scenes



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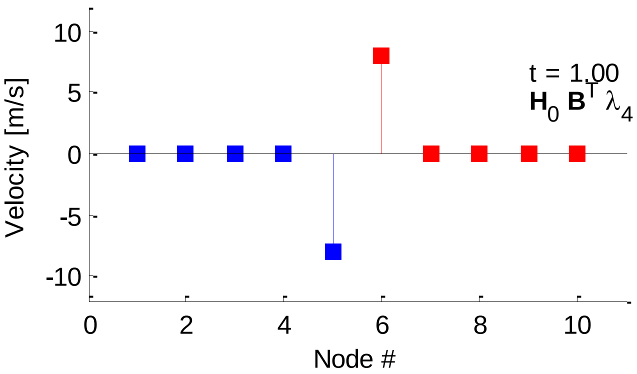

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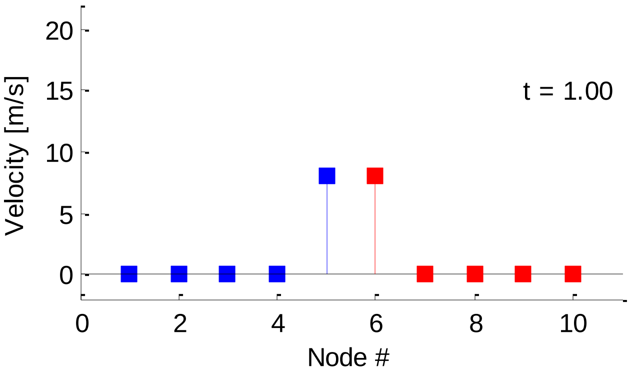

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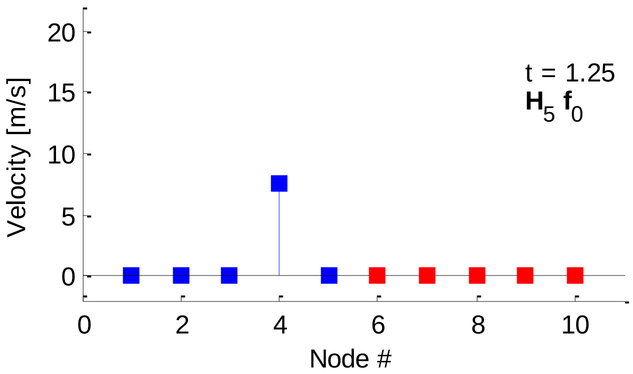
Coupling two bars

Behind the scenes



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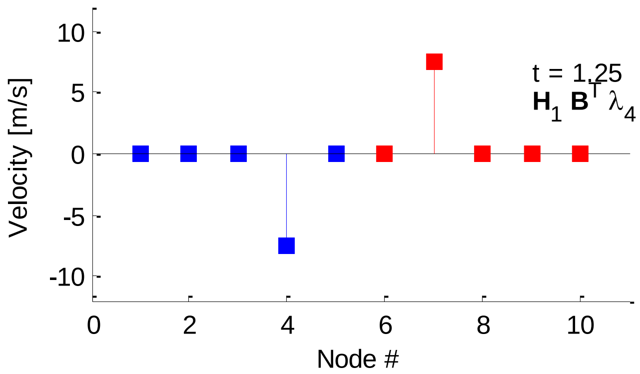

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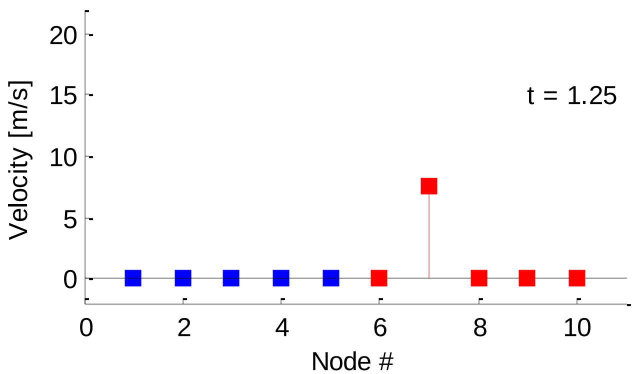
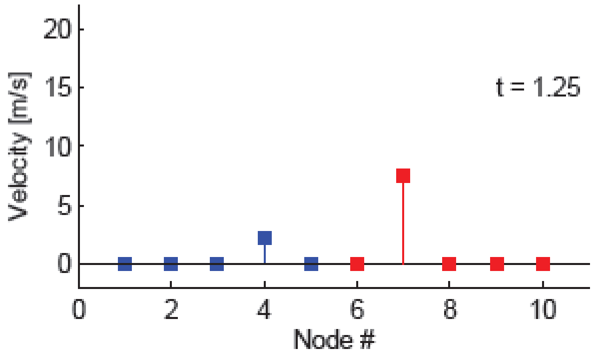


$$\mathbf{u}_n = \sum_{i=1}^{n} \mathbf{H}_{n-i}\,(\mathbf{f}_i\ - \mathbf{B}^T\,\boldsymbol{\lambda}_i)\Delta t$$

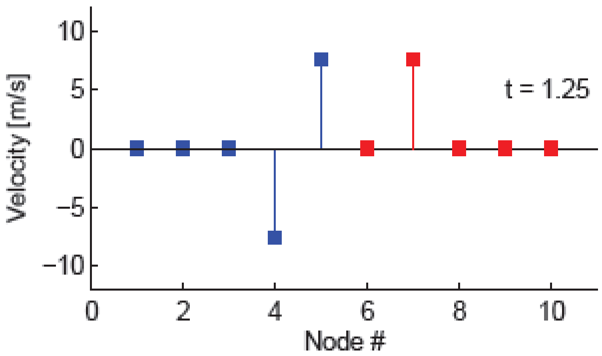
Coupling phenomena

What can go wrong?

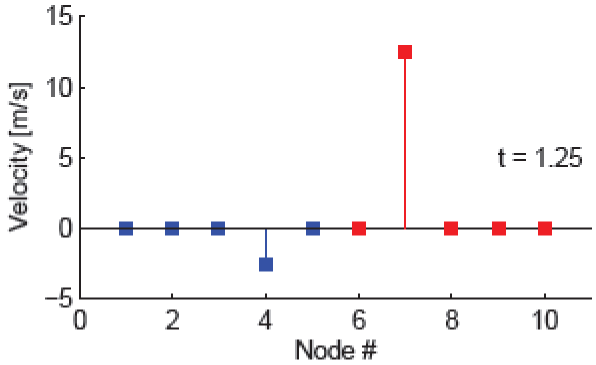
Disturbances



Delay



Non-causal



- What is required for stable and clean coupling?

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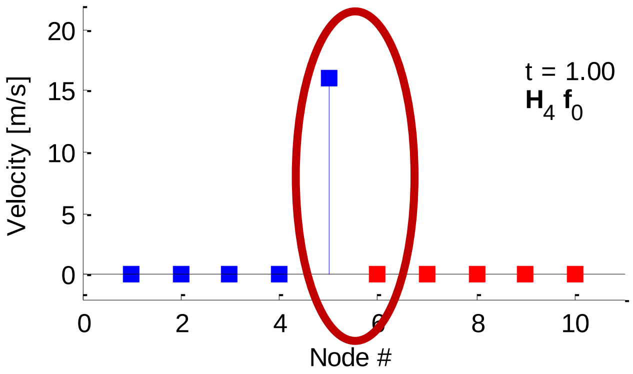
Classical discrete method

Recall this slide



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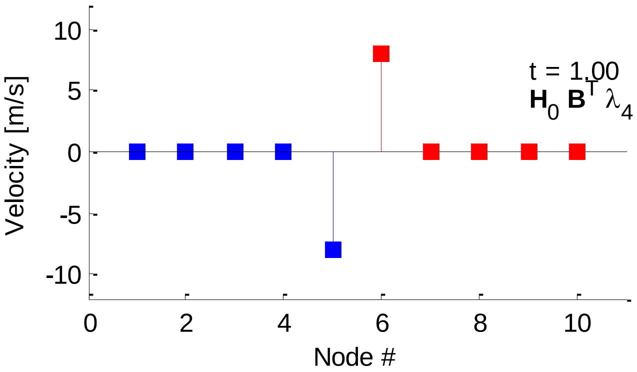

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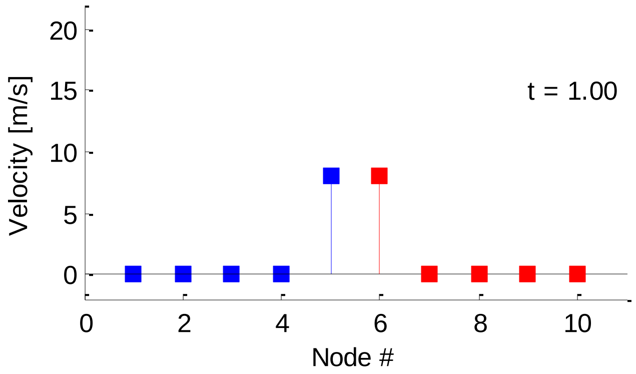

$$\mathbf{u}_n = \sum_{i=1}^{n} \mathbf{H}_{n-i} \left(\mathbf{f}_i - \mathbf{B}^T \boldsymbol{\lambda}_i \right) \Delta t$$

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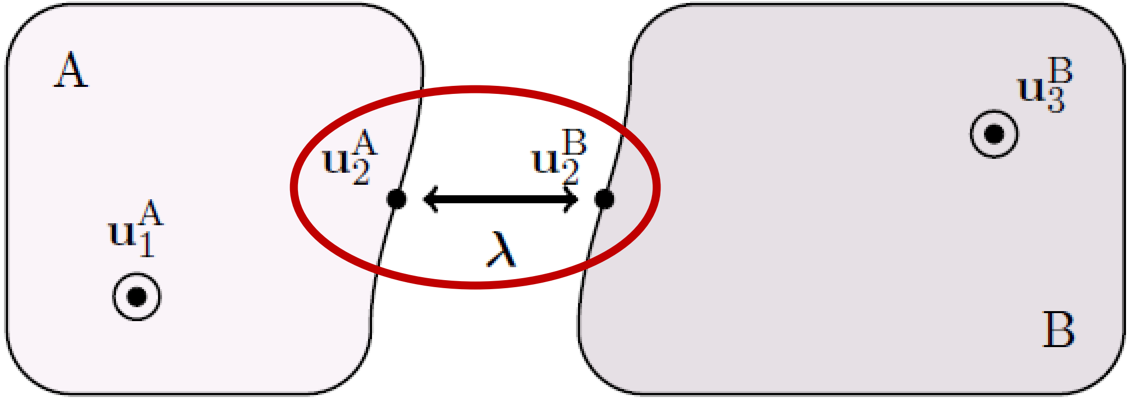
Inverse IRF Filter method

How to obtain the interface force?

$$\begin{cases} \mathbf{u}(t) = \mathbf{H}(t) * (\mathbf{f}(t) - \mathbf{B}^T \boldsymbol{\lambda}(t)) \\ \mathbf{B}\,\mathbf{u} = \mathbf{0} \end{cases}$$

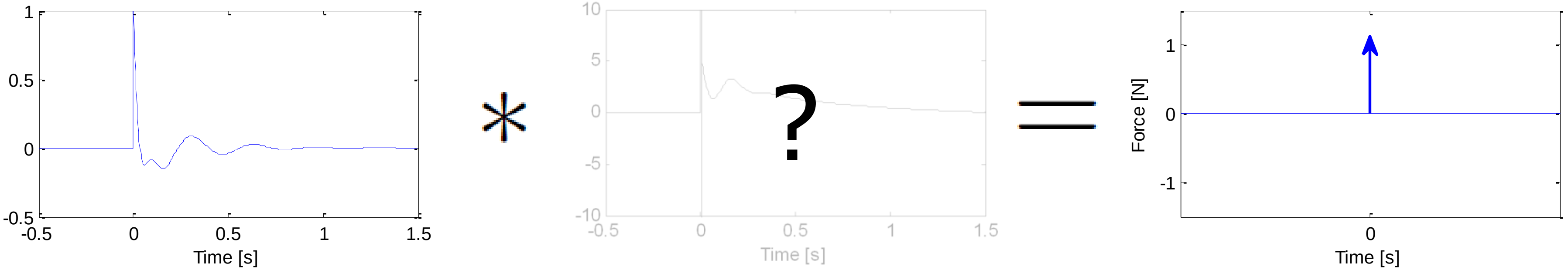
$$\mathbf{B}\,\mathbf{H}(t)\,\mathbf{B}^T * \boldsymbol{\lambda}(t) = \mathbf{B}\mathbf{H}(t) * \mathbf{f}(t)$$



Inverse IRF Filter method

How to obtain the interface force?

- An inverse filter: $$h(t) * h^{inv}(t) = \delta(t)$$



- Least squares approximation for the inverse filter
- Discrete → Dependancy on time step size

Comparison coupling methods

Classical discrete

Pro:
- Ensures compatibility

Con:
- Depends solely on $\mathbf{H}_0$

vs.

Inverse IRF filter

Pro:
- Depends on full IRF

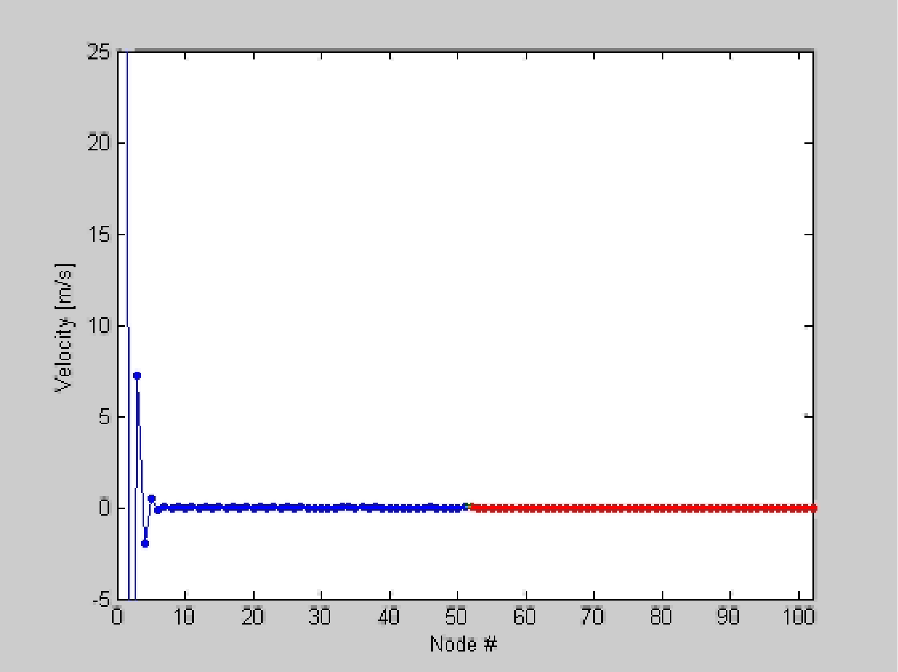
Con:
- Does not ensure compatibility

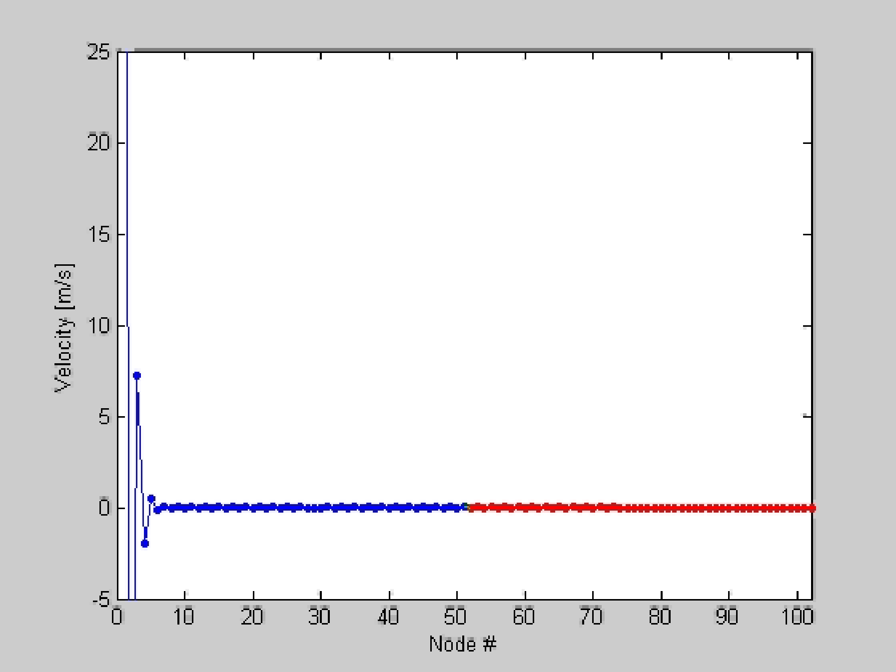
Coupled simulation

Classical discrete method vs. Inverse IRF filter method

Classical discrete:

Inverse IRF filter:



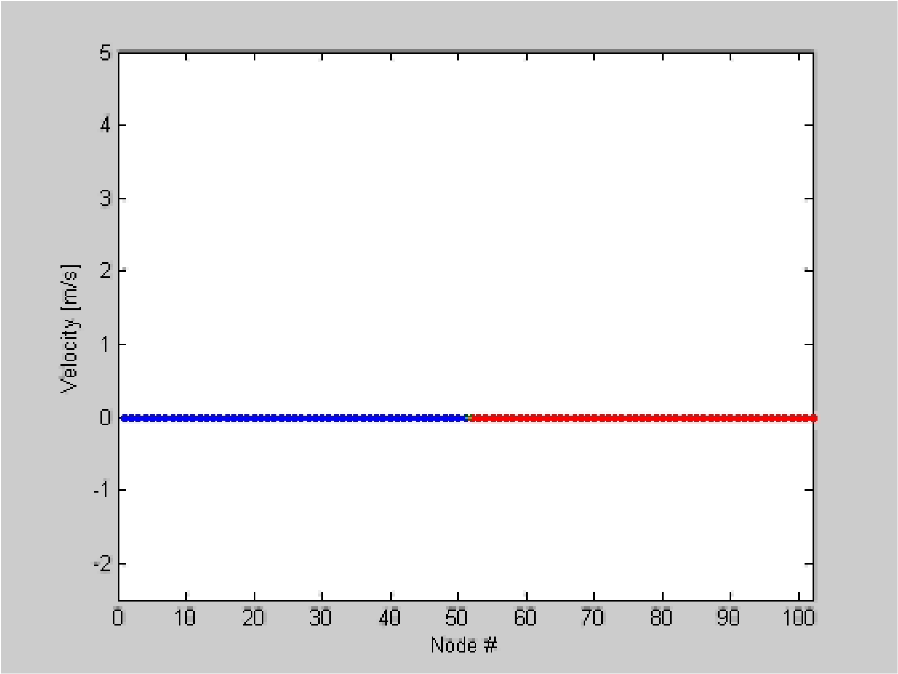


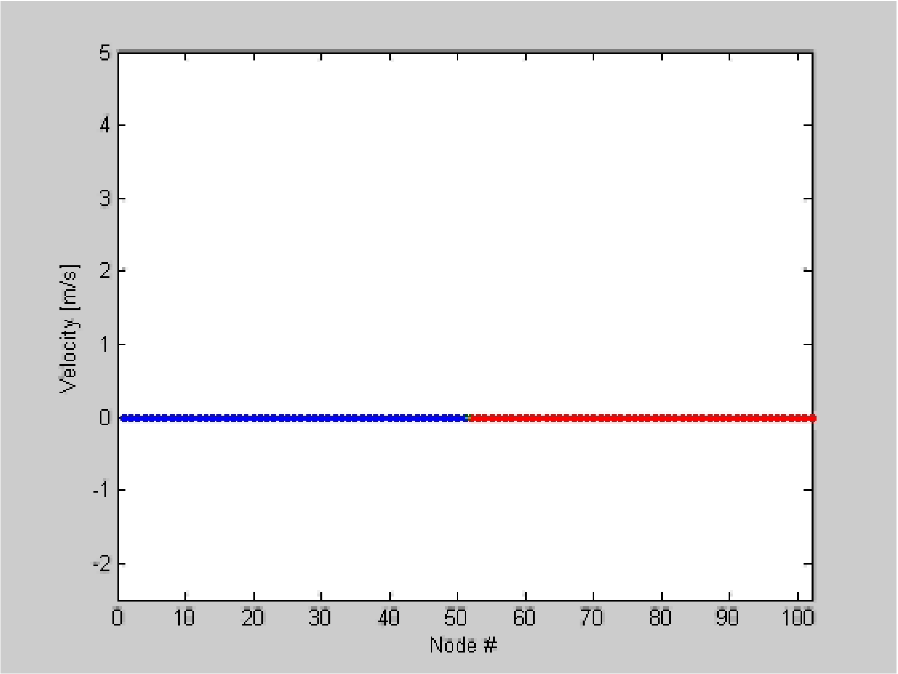
Coupled simulation

With an imperfect IRF

Classical discrete:

Inverse IRF filter:





Contents

What do we need?

- ✓ Simulate structural dynamics in the time domain using Impulse Response Functions
- ✓ Obtaining Impulse Response Functions
- ❑ Coupling of structural dynamics
  - ✓ The concept
  - ❑ The application

"Simulate the coupled dynamics of components in the time domain"

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"Simulate the coupled dynamics of components in the time domain"

Conclusions

First..

- Obtain Impulse Response Functions analytically using Modal Super Position

- A structure’s response can be obtained by convolving its IRF with the excitation
    - Discrete algorithms exist approximating the continuous convolution

Conclusions

Secondly..

- Coupling requires a causal IRF
  - For the bar, a counter impulse has to exist.
  - The measured bar → not causal
- Coupling using the classical discrete method
  - Guarantees interface compatibility
  - Prone to errors in the first time step
- Coupling using inverse the IRF filter
  - Allows incompatibility

Recommendations

For the future..

- IRF using travelling waves
  - Rather than standing waves (MSP)
  - Analytically for spacially continuous model
- Requirements description for IRF
  - Guaranteeing stable and clean coupling
- Combining multiple findings and techniques to succesful coupling of experimentally obtained IRF's

Impulse Based Substructuring Unravelled:
Simulation and coupling of structural dynamics in the time domain

Daniël D. van den Bosch

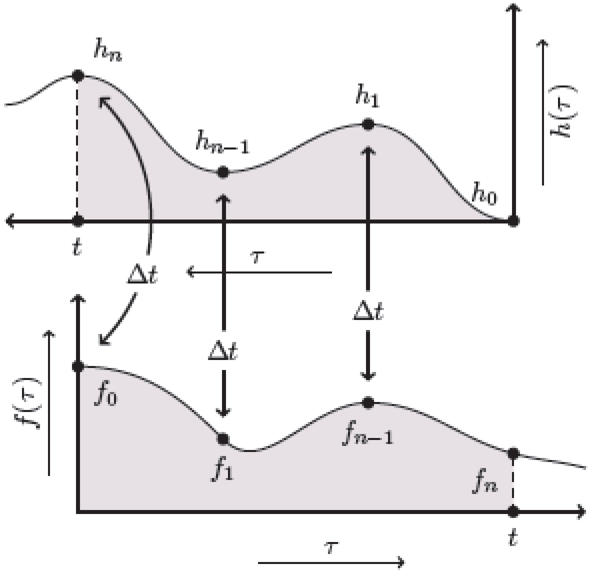
23 May 2014

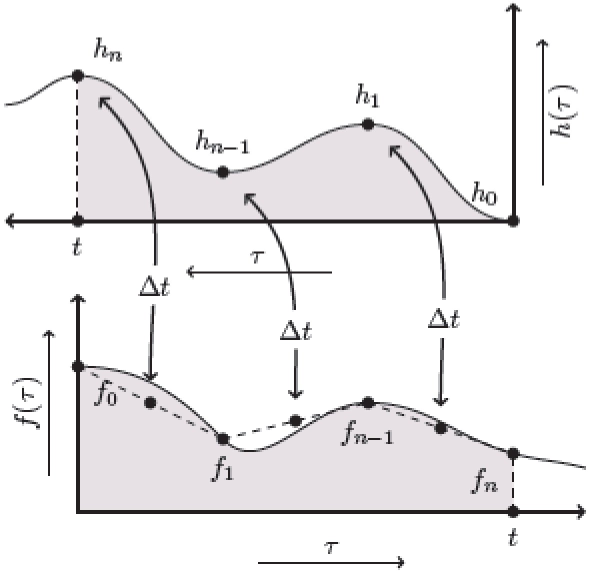
The big book of backup slides

Daniël D. van den Bosch

23 May 2014

Discretisation algorithms

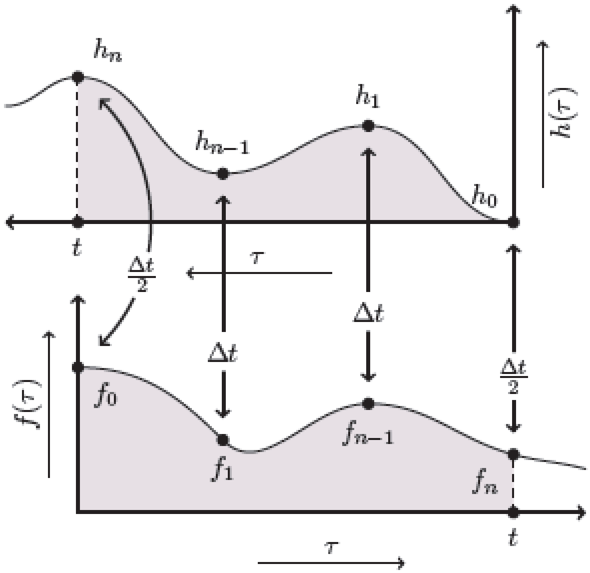




$$\mathbf{u}_n = \sum_{i=0}^{n-1} \mathbf{H}_{n-i}\, \mathbf{f}_i\, \Delta t$$

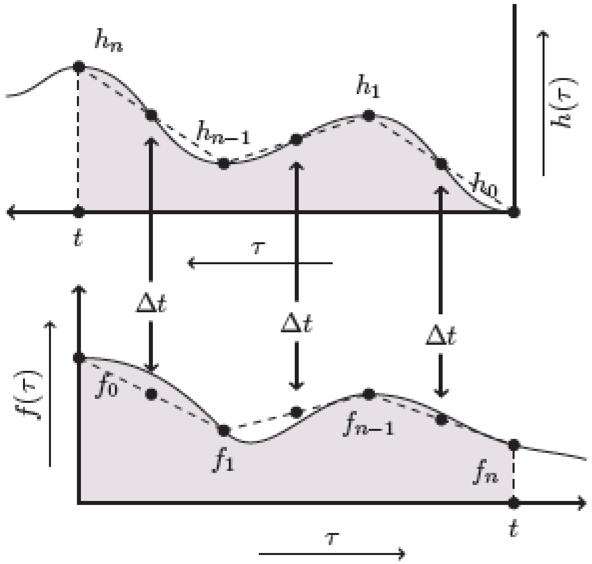
$$\mathbf{u}_n = \sum_{i=0}^{n-1} \mathbf{H}_{n-i}\, \frac{\mathbf{f}_i + \mathbf{f}_{i+1}}{2}\, \Delta t$$

Discretisation algorithms

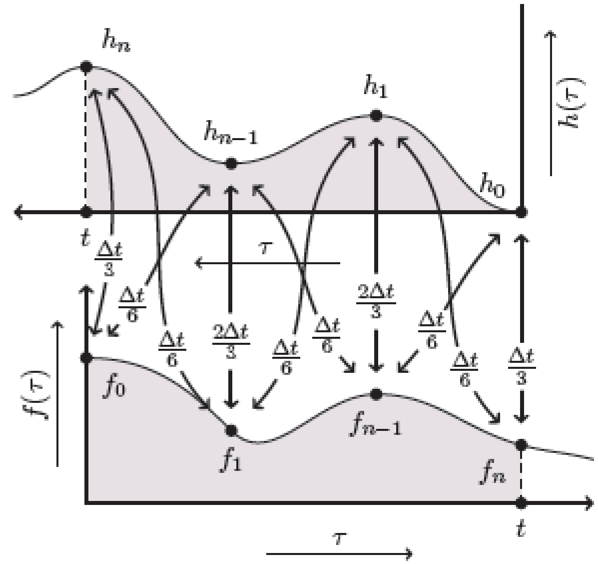


$$\mathbf{u}_n = \mathbf{H}_n\, \mathbf{f}_0\, \frac{\Delta t}{2} + \left(\sum_{i=1}^{n-1} \mathbf{H}_{n-i}\, \mathbf{f}_i\, \Delta t \right) + \mathbf{H}_0\, \mathbf{f}_n\, \frac{\Delta t}{2}$$

Discretisation algorithms



(c) Product of function averages, the result of $\chi = \frac{1}{2}$, $\psi = \frac{1}{2}$



(d) Product of piecewise linear functions, the result of $\chi = \frac{1}{3}$, $\psi = \frac{1}{2}$

$$\mathbf{u}_n = \frac{1-\chi}{2}\Delta t(\mathbf{H}_0\,\mathbf{f}_n + \mathbf{H}_n\,\mathbf{f}_0) + \sum_{i=1}^{n-1}(1-\chi)\,\Delta t\,\mathbf{H}_{n-i}\,\mathbf{f}_i$$

$$+\sum_{i=1}^{n}\chi\,\Delta t(\psi\,\mathbf{H}_{n-i}\,\mathbf{f}_{i-1} + (1-\psi)\mathbf{H}_{n-i+1}\mathbf{f}_i)$$

Courant's criterion

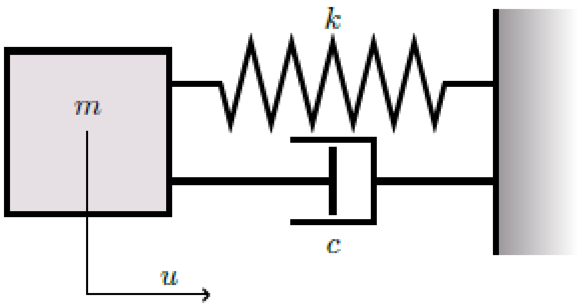
Involving the Courant's number

$$C = \frac{c' \, \Delta t}{\Delta x} \leq C_{\max}$$

$$\boxed{C = \frac{\omega_{cr} \, \Delta t}{2} \leq C_{\max}}$$

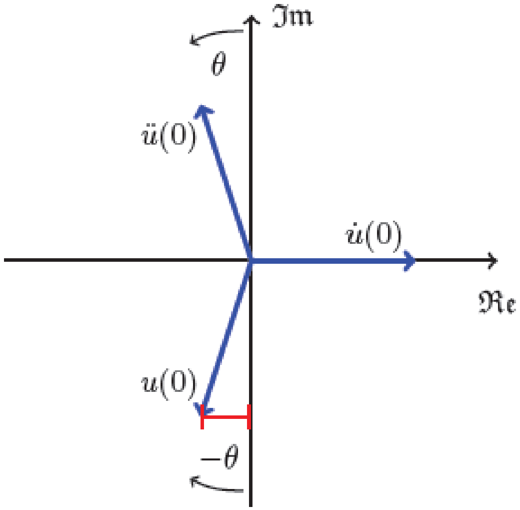
Analytical IRF

Single DoF damped system or mode



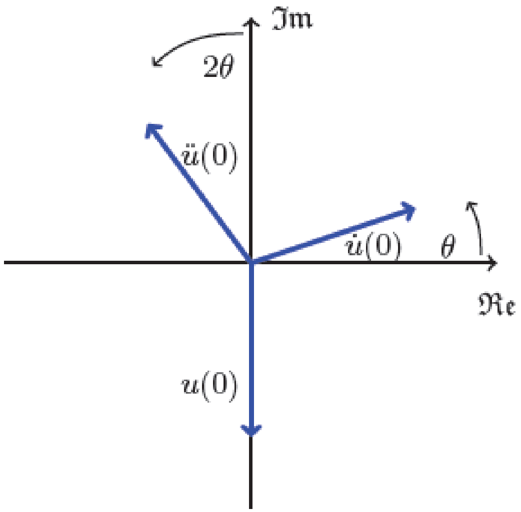
$$\dot{u}(t) = \hat{\dot{u}} \cos(\omega_d t)\, e^{-\zeta \omega_n t}$$

$$u(t) = \hat{\dot{u}} \frac{\sin(\omega_d t - \theta)}{\omega_n} e^{-\zeta \omega_n t}$$

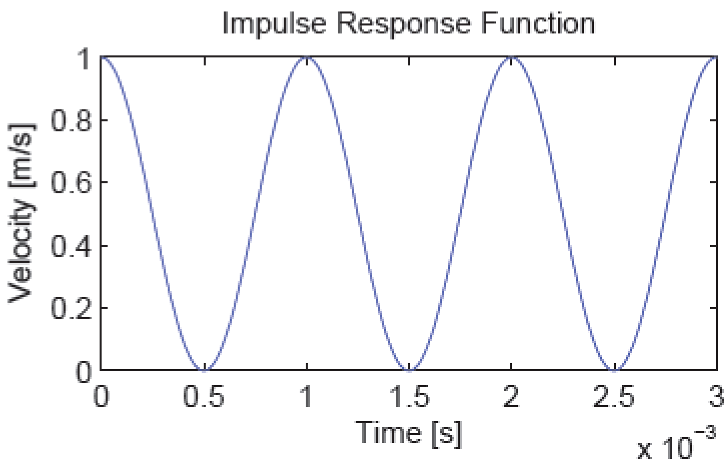


$$\dot{u}(t) = \frac{\omega_n}{m\,\omega_d} \cos(\omega_d t + \theta)\, e^{-\zeta \omega_n t}$$

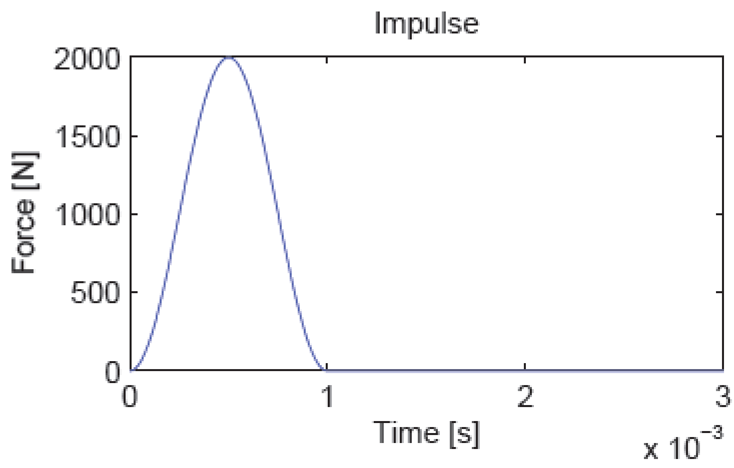
$$u(t) = \frac{\sin(\omega_d t)}{m\,\omega_d} e^{-\zeta \omega_n t}$$



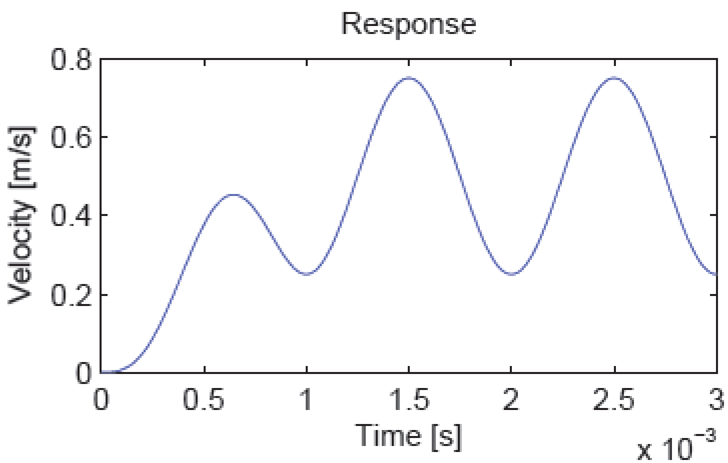
The first peak explained

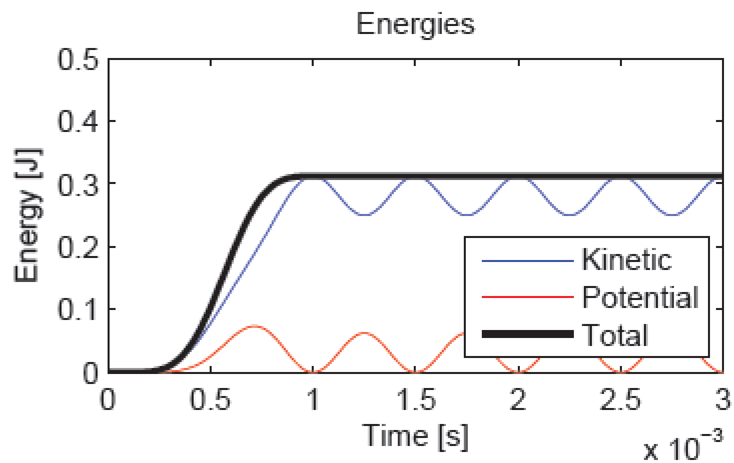


(a) Sample IRF with $T = 1\,[ms]$.



(b) Sample impulse using $T_{imp} = 1\,[ms]$

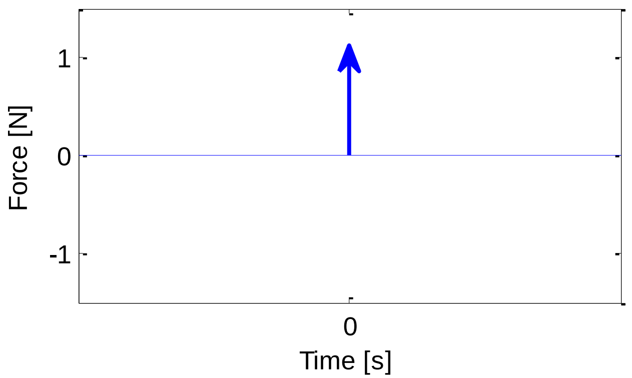


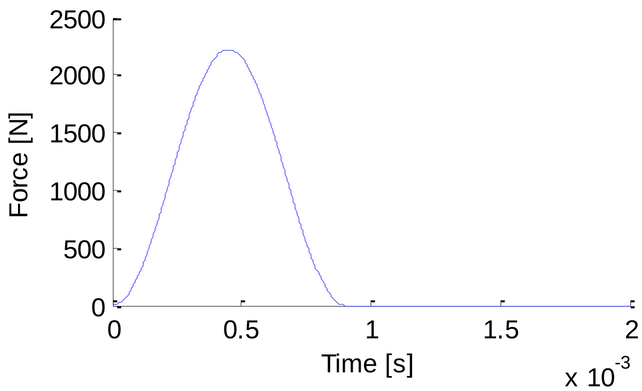


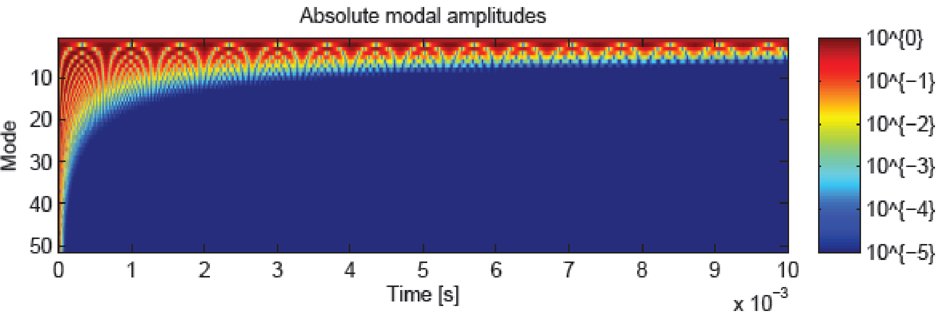
Modal content impulse

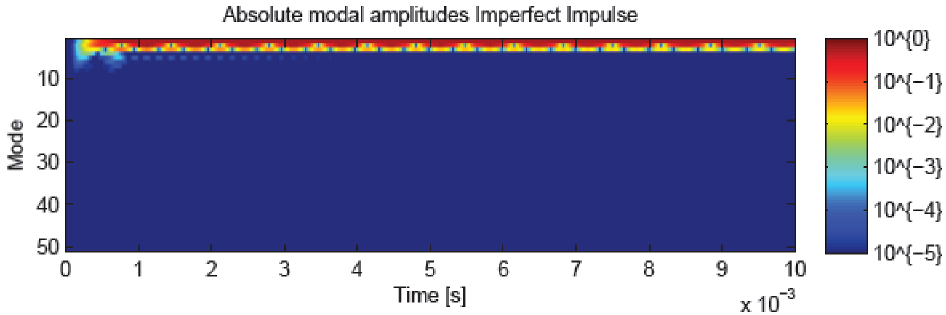
Perfect vs. Imperfect

Perfect impulse vs. **Imperfect impulse**





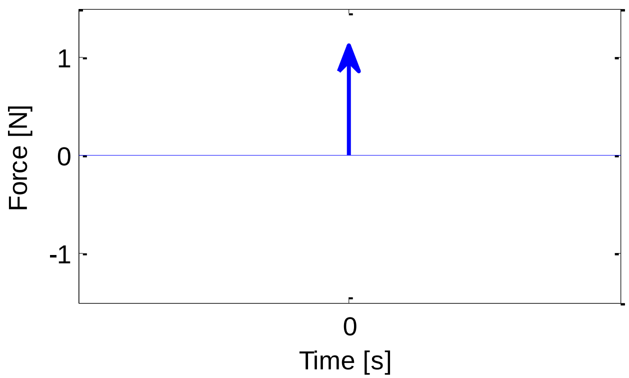


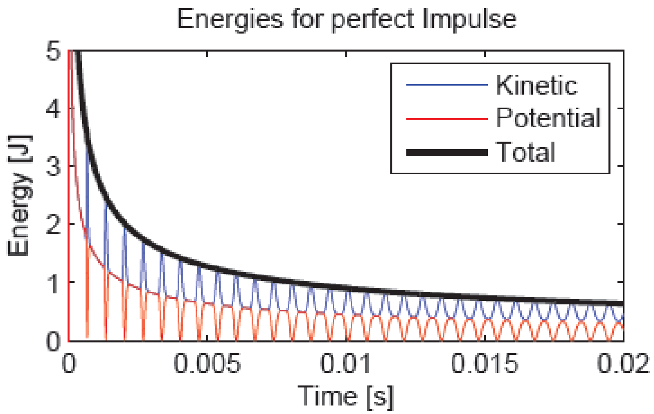


Energetic content

Perfect vs. Imperfect

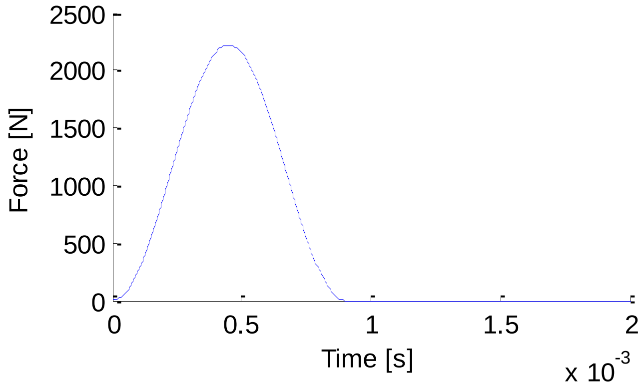
Perfect impulse

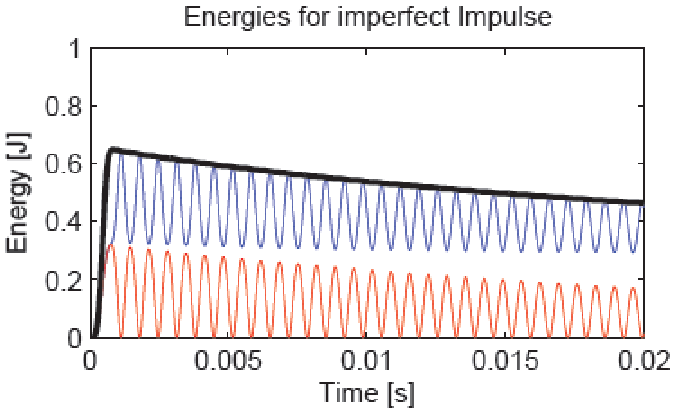




vs.

Imperfect impulse

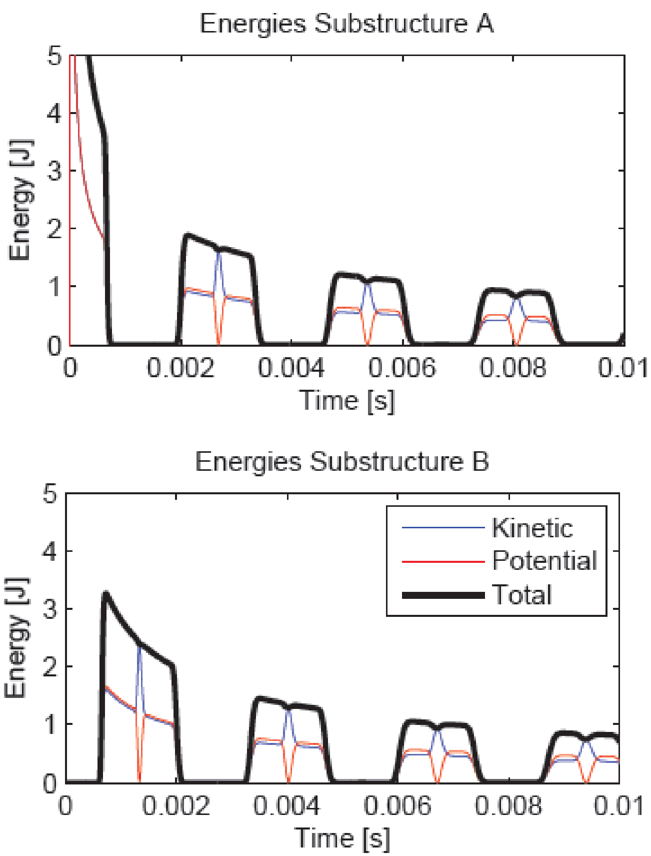


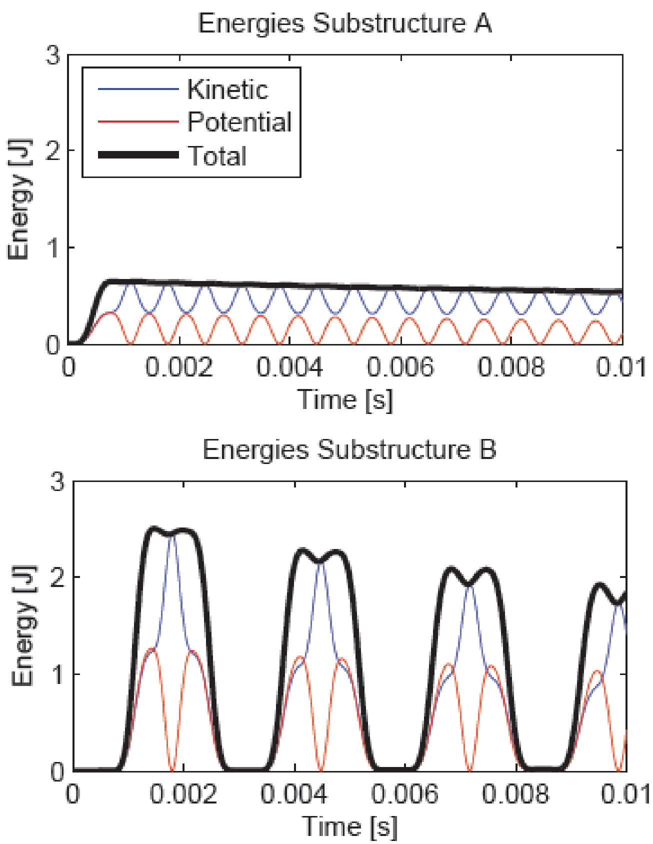


Energetic content coupled structure

Perfect vs. Imperfect

Classical discrete vs. **Inverse IRF filter**





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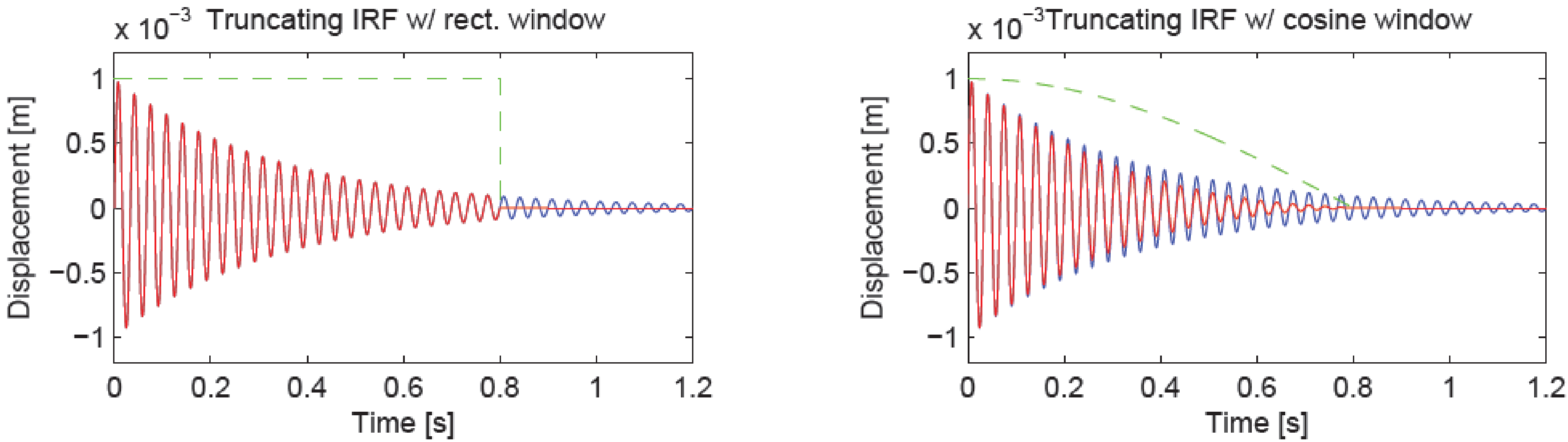
Truncating IRF

Enhancing computational performance



$$u(t) \simeq \int_{\max(0,t-t_c)}^{t} h(t-\tau)\, f(\tau)\, \mathrm{d}\tau$$

Splitting modal content

Enhancing computational performance

$$\mathbf{x}_r \in \begin{cases} \mathbf{X}^{\mathrm{RB}} & \text{for} \quad \omega_r = 0 \\ \mathbf{X}^{\mathrm{LF}} & \text{for} \quad 0 < \omega_r < \omega_c \\ \mathbf{V} & \text{for} \quad \omega_c \leq \omega_r \end{cases}$$

$$\mathbf{r}(t) = \int_0^t \left(\mathbf{P}\,\mathbf{H}(t-\tau)\right)\,\mathbf{f}(\tau)\,\mathrm{d}\tau = \int_0^t \mathbf{H}^{\mathrm{HF}}(t-\tau)\,\mathbf{f}(\tau)\,\mathrm{d}\tau$$

Matrix recurrence procedure

Enhancing computational performance

$$
\begin{bmatrix} \eta_{n+1} \\ \Delta t\, \dot{\eta}_{n+1} \end{bmatrix} = \begin{bmatrix} h^{(1)}(\Delta t) + 2\zeta\omega_n\, h^{(0)}(\Delta t) & \frac{h^{(0)}(\Delta t)}{\Delta t} \\ \Delta t\, h^{(2)}(\Delta t) + 2\zeta\omega_n \Delta t\, h^{(1)}(\Delta t) & h^{(1)}(\Delta t) \end{bmatrix} \begin{bmatrix} \eta_n \\ \Delta t\, \dot{\eta}_n \end{bmatrix}
$$

$$
+ \begin{bmatrix} h^{(-1)}(\Delta t) - \frac{h^{(-2)}(\Delta t)}{\Delta t} + \frac{c_2}{\Delta t} & c_1 + \frac{h^{(-2)}(\Delta t)}{\Delta t} - \frac{c_2}{\Delta t} \\ -h^{(-1)}(\Delta t) + \Delta t\, h^{(0)}(\Delta t) - c_1 & h^{(-1)}(\Delta t) + c_1 \end{bmatrix} \begin{bmatrix} \phi_n \\ \phi_{n+1} \end{bmatrix}
$$

Inverse IRF Filter method

Toeplitz notation

$$h(t) * h^{inv}(t) = \delta(t)$$

$$H\, h^{inv} = \delta$$

$$H \triangleq \underbrace{\begin{bmatrix} h_1 & 0 & \cdots & 0 \\ \vdots & h_1 & & \vdots \\ h_N & \vdots & \ddots & 0 \\ 0 & h_N & & h_1 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & h_N \end{bmatrix}}_{\text{M by Q=M-N+1}}, \qquad h^{inv} \triangleq \underbrace{\begin{bmatrix} h_1^{inv} \\ h_2^{inv} \\ \vdots \\ h_Q^{inv} \end{bmatrix}}_{\text{Q by 1}} \quad \text{and} \quad \delta \triangleq \underbrace{\begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_M \end{bmatrix}}_{\text{M by 1}}$$

Inverse IRF Filter method

Isolate the inverse IRF filter

$$h(t) * h^{inv}(t) = \delta(t)$$

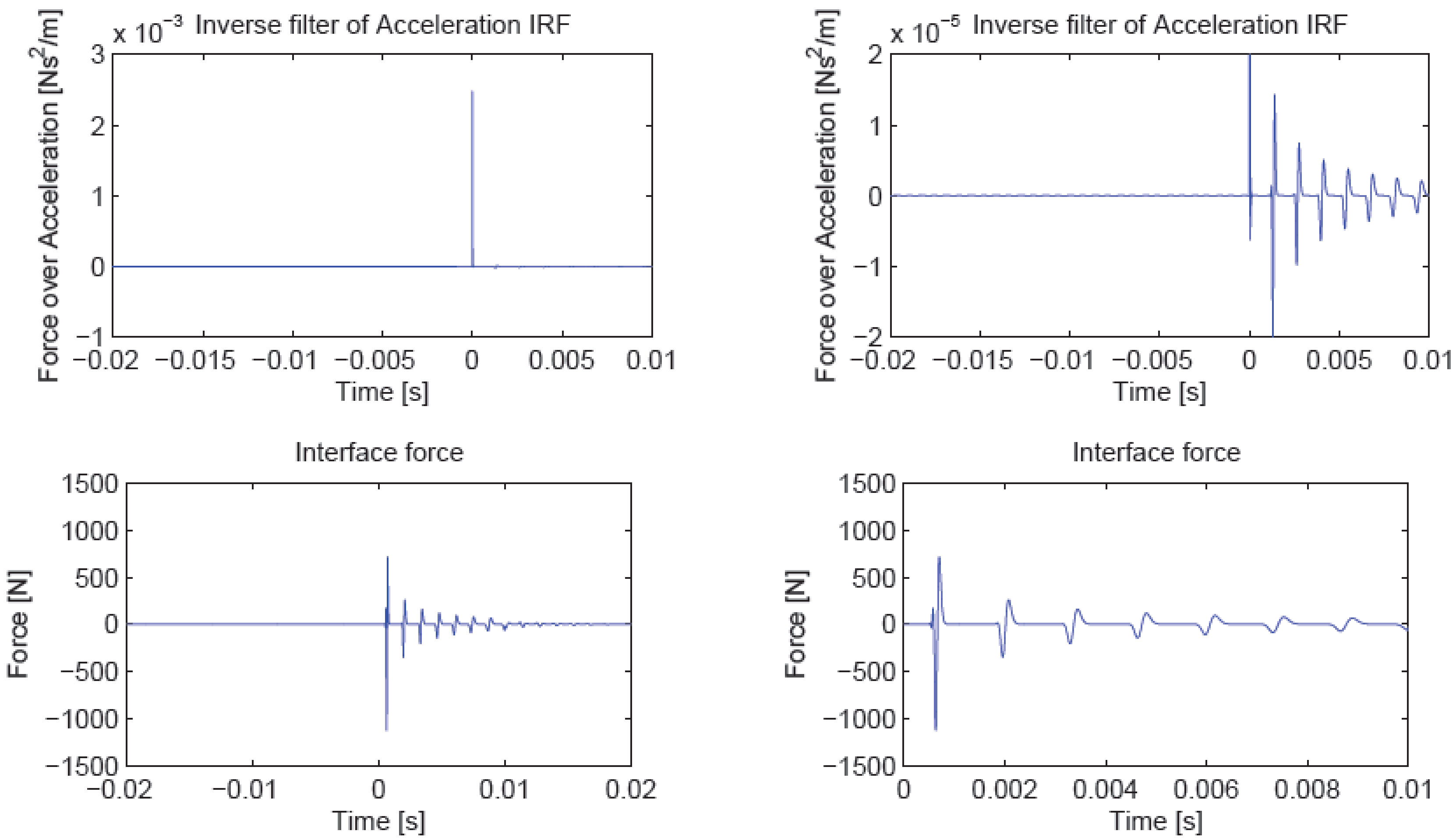
$$H\, h^{inv} = \delta$$

$$H^T\, H\, h^{inv} = H^T\, \delta$$

$$h^{inv} = (H^T\, H)^{-1}\, H^T\, \delta$$

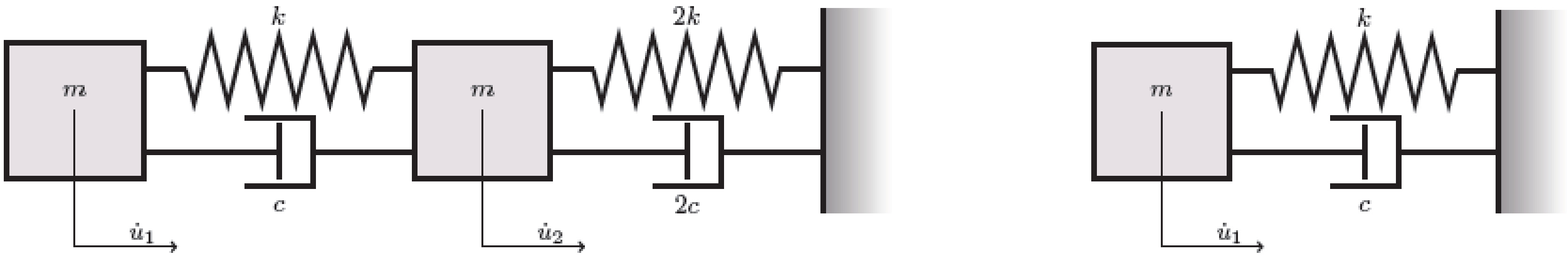
Inverse IRF Filter method

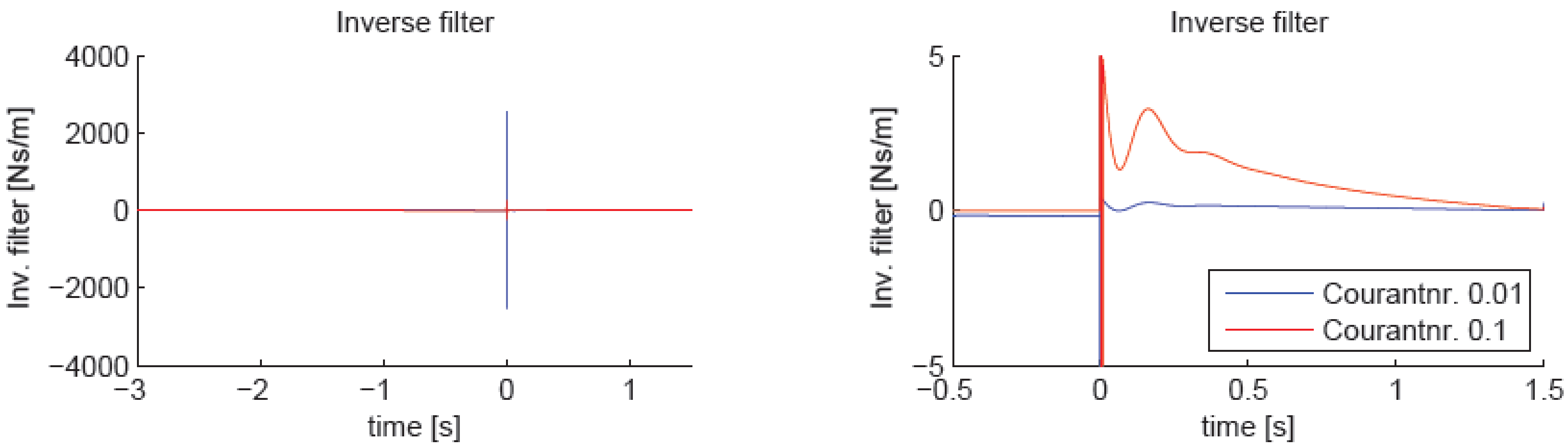
Causality



Inverse IRF Filter method

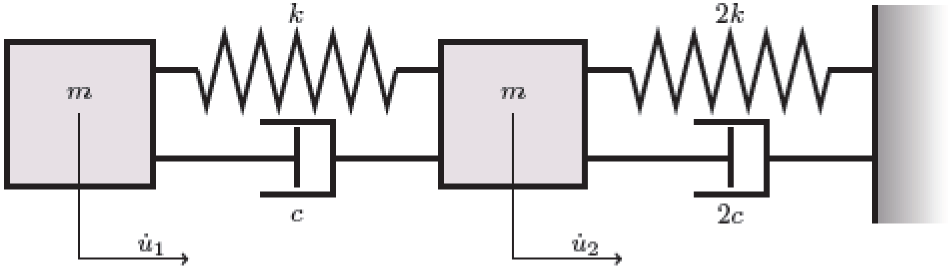
Time step dependency

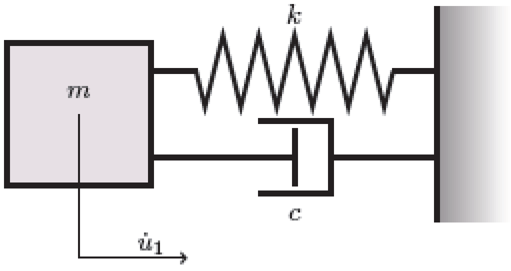


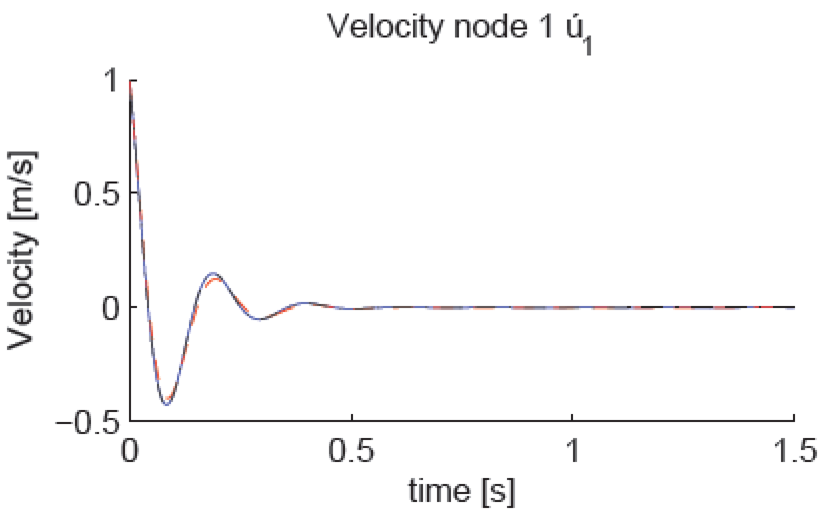


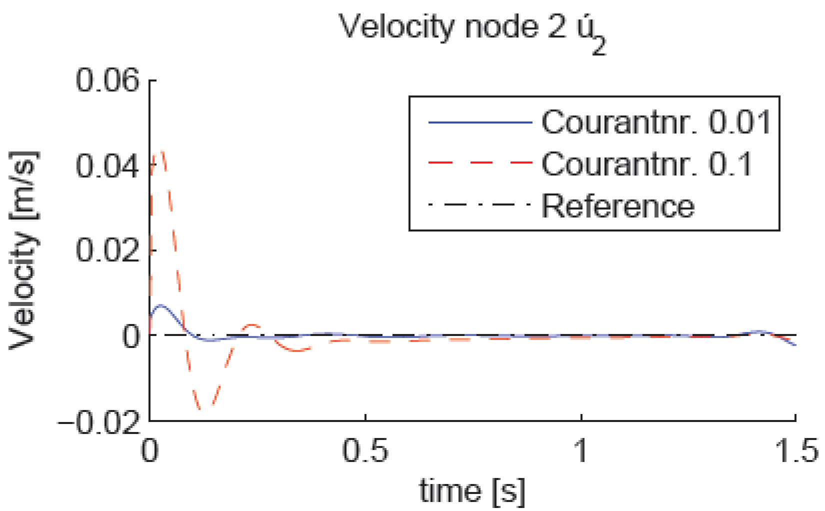
Inverse IRF Filter method

Time step dependency









Inverse IRF Filter method

Two DoFs

$$\mathbf{H}_{BB}^{inv}(t) * \mathbf{H}_{BB}(t) = \begin{bmatrix} \delta(t) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \delta(t) \end{bmatrix}$$

$$\begin{aligned} \mathbf{H}_{BB}^{inv}(t) * \mathbf{H}_{BB}(t) &= \left[\det{}^*(\mathbf{H}_{BB}(t))\right]^{inv} * \begin{bmatrix} h_{22}(t) & -h_{12}(t) \\ -h_{21}(t) & h_{11}(t) \end{bmatrix} * \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix} \\ &= \left[\det{}^*(\mathbf{H}_{BB}(t))\right]^{inv} * \begin{bmatrix} \det{}^*(\mathbf{H}_{BB}(t)) & 0 \\ 0 & \det{}^*(\mathbf{H}_{BB}(t)) \end{bmatrix} \\ &= \begin{bmatrix} \delta(t) & 0 \\ 0 & \delta(t) \end{bmatrix} \end{aligned}$$