

The Application of System and Control Theory to Monetary Policy

Development of a First-Principles LTI Model of the Economy

M. A. Vos

Master of Science Thesis

The Application of System and Control Theory to Monetary Policy

Development of a First-Principles LTI Model of the Economy

MASTER OF SCIENCE THESIS

For the degree of Master of Science in Systems and Control at Delft
University of Technology

M. A. Vos

May 20, 2019

Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of
Technology

DELFT UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF
DELFT CENTER FOR SYSTEMS AND CONTROL (DCSC)

The undersigned hereby certify that they have read and recommend to the Faculty of
Mechanical, Maritime and Materials Engineering (3mE) for acceptance a thesis
entitled

THE APPLICATION OF SYSTEM AND CONTROL THEORY TO MONETARY POLICY

by

M. A. Vos

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE SYSTEMS AND CONTROL

Dated: May 20, 2019

Supervisor(s):

Dr.ir. M. B. Mendel

Reader(s):

Dr.ir. T. Keviczky

Dr.ir. K. Hemmes

Abstract

Monetary policy is the process by which the central bank attempts to maintain price stability. Typically, this is done by manipulating the interest rate. This process is executed by the central bank of a country or union. Specifically, the policy makers of a central bank make policy decisions manually, based on advice and experience.

The process, of manipulating a variable to get the desired outcome, is a technology where system and control engineers are specialized in. Mostly, they focus on dynamical technological processes. Nevertheless, a new field in system and control theory pledges to widen this variety. Specifically, towards applying these system and control theories to economic processes: Economic engineering. Up to now, very little attention is paid to the possibility of applying system and control theory to monetary policy.

This work combines the current model of the European Central Bank (ECB), used to analyze monetary policy in the Eurozone, and the economic-engineering analog that can be used to model economics as if it were a mechanical system. It builds on these two theories by presenting monetary policy as a control problem. Firstly, the analog is extended in order to express money and interest in mechanical terms. With this extension, a first-principles model of the economy is developed. The results of the ECB's model are analyzed and an alternative is offered by the application of open loop transfer function characteristics. A PID controller is designed and the design conditions are interpreted as policy decisions. Finally, the effects of these design conditions are explained.

The model, developed in this thesis, adds to existing models because of its distinction between disturbances and the control input. This contrasts the existing models because they approach both disturbances and control input similar - i.e. so called shocks. The first-principles modeling approach offers the first model that presents the underlying dynamics of the system. These dynamics can now be analyzed in the form of the effects of disturbances. The model offers the possibility to better understand monetary policy. Policymakers can now simulate possible policy actions and the effects on different economic variables, such as wages, GDP and investment.

Table of Contents

1	Introduction and Background	1
1-1	Problem Description	1
1-1-1	Background in Monetary Policy	1
1-1-2	Application of Engineering Techniques	2
1-2	Problem Specification	4
1-2-1	Literature Research	4
1-2-2	Field Research	5
1-2-3	The Economic-Engineering Analog	5
1-2-4	The Research Questions	6
1-3	Thesis Outline	7
1-3-1	Contributions	7
1-3-2	Structure	8
2	Preliminaries	11
2-1	Definition of Economic Variables	11
2-2	Monetary Policy	12
2-2-1	Decision Support DSGE	17
2-2-2	Challenges DSGE Model	18
2-3	Economic-Engineering	20
2-3-1	Bond-Graph Modelling	20
2-3-2	Brewer: Precursor (1978)	24
2-3-3	Mendel: Current Research (2017)	24
2-3-4	Challenges Economic Engineering	26
2-4	Conclusions and Summary	27

3	Control Formulation of Monetary Policy	29
3-1	Monetary Policy Objective	29
3-2	Definition of the Signals	30
3-3	Discussion	31
3-4	Conclusions and Summary	32
4	Mechanical Analog for Money	35
4-1	Conjugate Pairs	35
4-2	Canonical Transformations	36
4-3	Discussion	39
4-4	Conclusions and Summary	40
5	First-Principles LTI Model	41
5-1	Motivation	41
5-2	Interacting Agents	43
5-2-1	Economics to mechanics	43
5-2-2	Model configuration	47
5-2-3	Model Validation	51
5-3	Discussion	56
5-4	Conclusions and Summary	57
6	Applications of the Model	59
6-1	Insight in the Dynamics of the System	59
6-1-1	Method	59
6-1-2	Results	61
6-1-3	Discussion	65
6-1-4	Conclusions and Summary	66
6-2	Monetary Policy Simulations	67
6-2-1	Monetary Policy Reaction Function	67
6-2-2	Price Stability	68
6-2-3	Discussion	69
6-2-4	Conclusions and Summary	71
6-3	Conclusions and Summary	71
7	Conclusions and Recommendations	73
7-1	Conclusions	73
7-2	Ethics	75
7-3	Future Research	76
A	Trial-and-Error	79
	Bibliography	89

List of Figures

1-1	Overview of the thesis structure per chapter.	8
2-1	Model structure of a DSGE model. The balloons present economic agents, variables are presented in black and economic modeling terms such as wage rigidities presented in grey. This figure is taken from [1]. The figure presents an abstract representation of interacting agents in the economy.	13
2-2	Plot directly copied from Smets-Wouters [2]. The plots present results of applying a labor supply shock to the system of eight variables. The x-label and y-label are not given. From Dr. R. Wouters it appeared the y-label is in quarters and the x-label depended on the variable. The graphs show the median response together with the 5 and 95 percentiles.	18
2-3	Tetrahedron of state, start from the force(top) integrating over time results in momentum (left), multiplying with a mass ($\mathcal{I}$) results in velocity (down), again integrating over time results in position, multiplying with the spring-coefficient($\mathcal{C}$) results again in the force. On the left, the mechanical terms are presented and on the right the generic term. The force is also related to the velocity by multiplying with the damping-coefficient ($\mathcal{R}$)	21
2-4	Single-port elements, from left to right: effort source, flow sink, intertance, capacitance and resistance element.	22
2-5	Two-port elements, left the transformer and right the gyrator.	22
2-6	Multi-port elements left the 0-junction and right the 1-junction.	23
2-7	The tetrahedron of state in mechanical terms from Figure 2-3 and the translation into the economic domain. From the cost (top) integrating over time results in the price (left), multiplying with the $\mathcal{I}$ element results in the quantity demanded (down), again integrating over time results in the quantity(right), finally multiplying with the $\mathcal{C}$ element results in the cost (down). The cost is also related to the quantity demanded through the $\mathcal{R}$ element.	25
3-1	The monetary-policy objective formulated as a control problem. The system is represented by the economy and the controller is represented by the Central Bank. The variables in grey present the system-and-control domain and the variables in black present the monetary-policy domain.	30

3-2	The DSGE model presented similarly to the control formulation in Figure 3-1. As a control engineer, the DSGE model is interpreted as an open-loop configuration with the economy and the central bank both included in the system.	31
4-1	The economic interpretation of 3 canonical coordinate pairs: momentum-position, action-angle and energy-tempus. These three coordinate configurations enable the modelling of economic concepts in terms of quantity (#), money (\$) and time (<i>yr</i>) and their corresponding price (conjugate pair).	39
5-1	Representation of the system - i.e. the economy - with the input - the interest rate - and the output - the inflation rate, subsection of the complete control formulation presented in Figure 2-1	42
5-2	The same model structure from a DSGE model [1] as Figure 2-1, added in black the variable couples defined in this section matched with the corresponding variables and agents from the DSGE model.	45
5-3	The four identified elements that describe the relationship between eight system variables. From left to right: The relation between labor and wage, the relation between the price of goods and output (production), the relation between rental of capital and capital and the relation between the price of stocks and investment.	46
5-4	Possible model configuration of the four elements from Figure 5-3 attached in a tetrahedron (3D). On the left the four 'generalized position' variables that can transact into one another, on the right the translation into a bond graph model.	48
5-5	Possible mechanical systems containing of masses and springs. The dynamical behavior is modelled and matched with possible behavior of economic variables.	48
5-6	Possible bond graph configuration of the four identified elements from Figure 5-3.	49
5-7	The circular flow of transactions between the four identified economic agents. The balloons from 5-2 convert to the agents in this figure and the corresponding economic variables that express the transactions between the agents.	50
5-8	Complete Bond Graph macro economic model consisting of 4 separate subsystems. The transactions between agents expressed in Figure 5-7 and their corresponding variables translates to the bond-graph model. Each junction with two-elements attached represent an agent. The dissipation of each agent is represented by an individual $\mathcal{R}$ element.	51
5-9	Changing eigenvalues when changing the parameters $\mathcal{C}_1$ and $\mathcal{C}_3$, expressing the relation between wage and labour, and rental on capital and capital respectively.	53
5-10	Changing eigenvalues when changing the parameters $\mathcal{I}_2$ and $\mathcal{I}_4$, expressing the relation between price and output (production), and price of stocks and investment respectively.	54
5-11	Mechanical representation of the economic concept of the marginal cost described by a mass, with two springs applying a force on this mass. The ratio between the force from one spring and the net force represents the marginal cost.	55
6-1	Result of a labor supply shock - i.e. a disturbance applied to the first state - to four system variables: labor, price of goods, capital and price of stocks.	65
6-2	Inflation decomposition as presented in Smets-Wouters. The contribution of three different types of shocks are presented in blue. The y-axis presents a rate in $\frac{\%}{yr}$ and the x-axis presents the years from 1970 to 2000. The amplitude of the shocks are expressed in $\frac{\%}{yr}$	68
6-3	Stochastic disturbance (Gaussian) distributed random signal on the investor market, simulating an equity premium (cost-push) shock	69

6-4	P, PI and PID control applied to the inflation variable, presented with the corresponding interest rate	70
A-1	Sketch of the government, the household, the bank and the firm interacting. . .	80
A-2	Mechanism of the household and the firm that result in inflation.	81
A-3	The monetary policy tools of a central bank expressed in an overview. The open-market-operations, the discount window and the reserve ratio are expressed. . .	82
A-4	Circular flow of income as expressed by Cantillon, interpreted as a dynamical system.	83
A-5	Possible bond graph model of the economy. Interacting variables are expressed in black, the bond graph elements are expressed in blue.	84
A-6	Sketch of the interaction of the economic variables labour, consumption, capital and money.	85
A-7	Sketch of economic variables expressed in bond graph elements. Four separate bond graph models are presented, upper-left the household with variables labour and wage, upper-right the firm with variables price and goods, left-down the entrepreneur with variables rental of capital and capital and left-right the bank with variables interest and money.	86
A-8	Sketch of variables corresponding to the bank, the firm and the household. Corresponding bond graph elements are presented.	87

List of Tables

1-1	The economic-engineering analog matching principle explained for the economic variable: consumption.	6
2-1	Overview of the variables presented in the DSGE model [2]	15
2-2	Overview of the parameters presented in the DSGE model [2]	16
2-3	The four general variables used in bond-graph modelling with the corresponding mechanical term and their symbol and dimension.	21
2-4	The three general elements used in bond-graph modelling with the corresponding mechanical element and their symbol and dimension.	22
2-5	The four mechanical variables used in bond-graph modelling translated to the economic domain. The same symbol is used for mechanics as for economics. . .	25
2-6	The translation of the three mechanical elements from Table 2-4 to the three economic elements. A mass is translated by demand, a spring by factor demand and a damper by consumption.	26
3-1	Translation of the variables of the control formulation in Figure 3-1 from the system-and-control domain to the monetary-policy domain. Both ε and η are used to represent stochastic shocks in the DSGE model.	31
4-1	In order to find the mechanical translation for the economic variable money the price and quantity variables and corresponding dimensions from Table 2-5 are multiplied, similarly for the dimensions of the corresponding mechanical dimensions. Which results in the mechanical translation for the economic variable money having dimensions $\frac{kgm^2}{s}$	37
4-2	Exact similar overview of the four mechanical variables translated into their economic analog. This table is repeated in order to get a complete overview of the possible coordinate configurations.	38
4-3	The four mechanical variables for action, angle and their derivatives energy and angular velocity (symbol taken from [3]) translated to their economic analog: money, return and their derivatives cash flow and rate of return. Interest is similar to return, interest is used in terms of savings where return is used in terms of investments.	38

4-4	The four mechanical variables for tempus (from [3]) and energy, with only the derivative of energy, power. The mechanical variables are translated to their economic analog: term, cash flow and their derivatives return and growth. A term could be for example an agreed term in a mortgage contract or bank lending contract with an investor, or the term specified in a (coupon) bond.	38
5-1	Overview of the variables taken from the DSGE model (Smets-Wouters) and the matched engineering analog presented in the generalized variable with its corresponding dimensions. For consumption both $\#/yr$ and $\$/yr$ can express the dimension of this variable which both corresponds to a generalized flow.	44
5-2	Overview of the four agents chosen to be in the first-principles model taken from the DSGE model, and the corresponding economic variables.	49
5-3	System variables of the first-principles model. x present the state variables and $\dot{x}$ the integral of the state variables, the other two columns are obtained by multiplication of the state variables with the system parameters.	52
5-4	System parameters of the first-principles model interpreted as parameters from the DSGE model, two parameters are not identified.	53
6-1	Categorization of the ten different shocks presented by Smets-Wouters in the three defined groups: cost-push shocks (forces), demand-pull shocks (velocities), build-in inflation (control inputs)	61
6-2	Shock variables from the DSGE model [2] influencing corresponding variables from the DSGE model interpreted to which system variable from the first-principles model it is acting on	62
6-3	The numerator of the transfer function is multiplied with one parameter to obtain the corresponding transfer function of the defined variable	64
6-4	Initial value theorem and Final value theorem applied to the transfer functions between the disturbance input d_1 and the four state variables	64

Chapter 1

Introduction and Background

Every month, the president of the European Central Bank (ECB), announces if the interest rates are going up or down. As phenomena in the world, such as the Brexit and the trade negotiations between the US and China, influence economic numbers such as inflation and economic growth, the ECB tries to manipulate these. This can be compared to a pilot trying to steer a plane. The technology in aviation is developed to a level that influences from wind can be corrected in order to maintain the flying direction of the plane. In economics, it is not always the case that the tools of the ECB have the desired effects.

How come a pilot can steer a plane under influences of wind, but the ECB cannot always steer the economy under influences of shocks? Economic engineering is the field that believes that this dynamical behavior could also be described by engineering models. This thesis researches the niche of applying economic engineering theory to monetary policy.

The objective of this thesis combines the domains of engineering and economics. This chapter presents a thorough description of the origin, set-up and general method of this research. Section 1-1 provides a background in monetary policy and describes the problem motivation of this thesis work. Section 1-2 further defines the problem and specifies this problem into sub-problems. Furthermore, it provides the reader with the insight of the general set-up of this thesis which resulted in the research question. The chapter concludes with an outline of the thesis.

1-1 Problem Description

1-1-1 Background in Monetary Policy

Monetary policy is the process by which the central bank of a country controls the interest rate. The goal is to target an inflation rate to ensure price stability and general trust in the currency. The inflation rate is the change in the prices of consumer goods. The president of

a central bank executes this process. In Europe, Mr. Draghi is the president of the European Central Bank (ECB). Mr. Draghi manually makes policy decisions, based on experience and understanding of the monetary system.

In order to support these decisions, the ECB developed a Dynamic-Stochastic-General Equilibrium (DSGE) model [2]. A Dynamic-Stochastic-General-Equilibrium model is a method that attempts to explain economic phenomena, such as economic growth and business cycles and the effects of monetary policy. The model is based on applied general equilibrium theory and microeconomic principles. A general equilibrium theory attempts to explain the behavior of supply, demand, and prices in a whole economy with several or many interacting markets, by seeking to prove that the interaction of demand and supply will result in an overall general equilibrium. The microeconomic principles are the microeconomic behavior of individual agents. An agent is an actor and more specifically a decision maker in a model of some aspect of the economy. Typically, every agent makes decisions by solving an optimization problem. Most central banks have their own DSGE model, such as the Dutch Central Bank [4] and the Federal Reserve of the United States [1].

In practice, the DSGE model is estimated using an auto-regression method with historical data. To obtain results, the DSGE model includes so-called stochastic shocks. In economics, a stochastic shock represents an unexpected or unpredictable event that affects an economy, either positively or negatively. An example of a shock is the event of the Brexit. The effects, of the stochastic shocks on the variables in the model, are used as decision support for the policy actions of policy-makers. As it is the task of these policy-makers to maintain price stability, they could manually change the interest rate. To support these decisions, the DSGE model presents an analysis of the effects of these stochastic shocks to economic variables such as the inflation rate.

1-1-2 Application of Engineering Techniques

From the perspective of a control-engineer, there is a lot to gain in the modeling techniques used in monetary policy. Since 2017, the Economic-Engineering group from DCSC studies the application of system and control engineering techniques on such topics. This economic-engineering group attempts to gain a better understanding of the underlying dynamics of economic models. Specifically, economic engineering defines formalism's to model the economy as if it were a mechanical system. Economic engineering brings the disciplines of economics and engineering together, to see if these fields can strengthen each other and advantage of the developed techniques.

So far, in literature, not much is found on applying system and control theory to monetary policy specifically. This thesis presents the study to the applicability of system and control theory to monetary policy. Specifically, the thesis presents and applies a method to model the economy as if it were a mechanical system.

The study consists of some important steps. First, the monetary policy objective of a central bank is presented as a control problem. This control formulation is the basic structure from which the system can be modeled and controlled. Then, a dynamical model is developed

using a first-principles technique. First-principles means that it starts directly at the level of established laws of physics and does not make assumptions such as the empirical model and fitting parameters. This will result in a system where all information is available. Hence, if success-full, this opens the doors to applying all sorts of tested system and control theories. Finally, two types of system and control theories are applied to present the effectiveness in terms of supporting decisions in monetary policy.

The benefits of applying the economic-engineering approach to monetary policy are the following: The control formulation distinguishes disturbances and the control input. This contrasts the DSGE model that approaches these both as shocks. Therefore, the model cannot make a distinction between what can be manipulated and what can not. Furthermore, the first-principles model offers insight into the underlying dynamics of the system. All operations that can be applied to models of physical systems, can be applied to this model of the economy. The basic formalism's used to develop the model, make it possible to divide the model into sub-models and can be understood with knowledge of basic calculus. Hence, a larger group of academics can use the model, than only monetary policy experts. From the mathematics, the model can not only be expressed in an elegant way it also works self-containing; if the math doesn't work out, the addition on the model is not correct.

In terms of decision-support to monetary policymakers, the developed model is an addition to the existing models. The relations between all variables included in the model and all types of shocks - i.e. disturbances, can be expressed using the transfer function. Hence, the characteristics that describe responses, such as overshoot and steady-state error, can be expressed. These characteristics will help a monetary policymaker understand the effects of economic phenomena on economic variables such as consumption and investments. This contrasts the DSGE model that only offers observations based on empirical results. Finally, the control formulation offers the possibility to design a controller to mitigate undesired disturbance effects. This contrasts the DSGE model.

To conclude, the research of this thesis can be formulated in the following research question:

How to apply system and control theory to monetary policy?.

Since this main problem is rather vague the next section makes this more concrete by specifying the problem into sub-problems.

1-2 Problem Specification

This section presents the problem specification of the research. This information allows another researcher to build on the study. In practice, a cyclic process of reading literature, discussing and brainstorming with academics and experts and then applying the economic-engineering theory preceded the thesis work. This process results in five research questions that together answer the main objective of this thesis.

1-2-1 Literature Research

In order to understand whether the domains of economics and engineering can fit together, literature from both fields is studied. The engineering principles can be applied to economics more appropriately if the economic models are understood. The other way around, the engineering theory is refreshed and amplified to research theory best fitting to economic principles. A summary of the work significantly contributing to the research is given below:

Economics

The book 'Macroeconomics in the Global Economy' written by Sachs and Larrain [5] is studied to gain insight into the working principles of macroeconomics. The circular flow between households and firms was researched including economic variables such as labor and consumption. Furthermore, ch. 8; money demand and ch. 9; money supply process offers insight in central bank tools such as open market operations, the discount window and reserve ratio's.

The book 'Advanced Macroeconomics' by David Romer [6] has been an important source of a capital model presented in sect. 8.1 The cost of capital. Furthermore, understanding of the role of economic variables such as consumption, investment, and inflation and economic agents such as the household and the central bank was obtained from ch. 1 Solow-Growth, ch. 4 Real Business Cycle Theory, ch. 7 Consumption, ch. 8 Investment and ch. 9 Inflation and Monetary policy.

Different work from P. Samuelson is studied. Among with 'Economics' written together with William D. Nordhaus [7] and a bundle of Samuelson's works 'A collection of scientific papers' written by Joseph Stiglitz [8]. Especially, the paper 'Law of Conservation of the Capital-Output Ratio' that compared this ratio to a mass-spring system and its harmonic motion. The work of Samuelson appears to relate the most with engineering practices in terms of the use of math and his comparisons of economic concepts with mechanics.

Mechanics

The book 'Mechanics' by Landau and Lifshitz [9] presented a complete overview of classical mechanics. Especially ch. 1 'The equations of motion', ch. 2 'Conservation Laws' and ch. 7 'The Canonical Equations'.

The book 'Mathematical Methods of Classical Mechanics' by Arno'd [10] has contributed.

Specifically, ch. 8 'Symplectic Manifolds', ch. 9 'Canonical Formalism' and ch. 10 'Introduction to Perturbation Theory'.

The book 'Classical Mechanics' by Corben [11] has contributed insight into the complex definition of canonical coordinates and action-angle in particular. This is presented in ch. 11 'The Hamilton-Jacobi Method', specifically section 62 'Action-angle variables - periodic systems', this is originally from Max Born: 'The Mechanics of the Atom' [12].

1-2-2 Field Research

Weekly meetings took place among the economic-engineering group, prior to and during the study. Dr. M. Mendel leads this group. Furthermore, it consisted of the master students that graduate in the economic-engineering section. The group discusses theory that contributes to the economic-engineering research. This consists of economic theory, mechanics theory, with complex analysis in particular (i.e. complex Hamiltonian and Lagrangian) and the economic-engineering theory that is developed so far.

Weekly meetings took place between the researcher and the supervisor Dr. M. Mendel. During these meetings, brainstorm sessions were held about new insights and material that was supplied by the researcher. This was economic, mechanics or combined economic-mechanical research material.

To gain more insight into economics, experts from different central banks were consulted. Mail conversations and meetings took place with a former board member of the Dutch Central Bank (DNB) Prof. Dr. L. Hoogduin, Dr. R. Wouters researcher at the Bank of Belgium and Prof. Dr. F. Smets, Director General of the Directorate General Research of the European Central Bank en Counsellor to the President and Co-ordinator of the Counsel to the Executive Board. One meeting took place with Prof. Dr. L. Hoogduin, together with Dr. M. Mendel. During this meeting, the economic-engineering research was discussed and specifically the subject of monetary policy. Prof. Dr. L. Hoogduin introduced the DSGE model as the state-of-the-art in monetary policy. Dr. R. Wouters and Prof. Dr. F. Smets are the authors of the DSGE model that is used as the most important source for this thesis. Two meetings took place with Dr. R. Wouters, one to discuss the working principles of the DSGE model and one to discuss the results of the study presented in this thesis. This contact was of great importance in the development of this research and contributed to the development of the first-principles model of the economy presented in this thesis.

1-2-3 The Economic-Engineering Analog

The theory of Mendel is presented in short to specify what is necessary to use it and build on it for the field of monetary policy.

The initial task of the research is to apply the economic-engineering analog to an existing economic model. In order to do this, the literature about economics and the literature about mechanics are combined. Therefore, a large part of the research, presented in the thesis, consists of the interpretation of economic principles, variables, parameters and theories to

mechanical principles, variables, parameters, and theories. In other words, it is a matching course.

The matching course expands on an existing economic-engineering framework developed by Mendel [13]. This thesis assumes that this existing framework is correct (The framework is presented in section 3.2). This is based on success-full results from two preceding thesis reports [14], [15]. The most important assumptions are the linearity of demand, factor demand, and consumption. Furthermore, the analog between position in mechanics and quantity in economics and the analog between momentum in mechanics and price per quantity in economics.

Economics		Mechanics
Consumption	←	Velocity
↓ $\frac{\#}{yr}$ ↓		↑ $\frac{m}{s}$ ↑
Quantity Demanded	→	Velocity

Table 1-1: The economic-engineering analog matching principle explained for the economic variable: consumption.

In order to expand on the existing framework, economic variables are matched with the mechanical variables accordingly. The translation of an economic variable to a mechanical variable needs to fit in the existing framework. An example is given in Table 1-1. The economic variable consumption has units $\frac{\#}{yr}$, therefore it is interpreted as a quantity demanded. According to Mendel's theory, a quantity demanded in economics is interpreted as a velocity in mechanics. Hence, a velocity has units $\frac{m}{s}$. Concluding, the consumption variable is interpreted as a velocity in mechanics. Hence, the matching course.

1-2-4 The Research Questions

Based on the initial research, five research questions are formulated. In order to apply system and control theory to monetary policy, a working model is necessary. Therefore, the current model, the DSGE model, is thoroughly researched. The DSGE model, together with the existing economic-engineering framework, is the academic work this thesis builds on.

Both the DSGE model and the economic-engineering framework have challenges to overcome. From the perspective of a control engineer, the DSGE model is difficult to use in practice. Shortcomings have to be challenged: First, there is no explicit definition of a controller. Second, the equations cannot be reproduced into a working model. The model lacks mathematical expression of all variables and parameters in the difference equations. This thesis attempts to remedy the current mismatch between this economic model and system and control models. The economic-engineering framework offers a solution to this. Nevertheless, the economic-engineering framework is not yet developed to express economic variables in terms of money \$ and return or interest %. In order to model the economy as if it were a mechanical system, this is a shortcoming that needs to be resolved.

This results in the first four research questions:

- What are the economic models used for monetary policy and what are their limitations for applying system and control theory to?
- How can the monetary policy problem of a central bank be formulated as a control problem?
- How can the economic concept of money be expressed in mechanical terms?
- How can the economy be modeled as if it were a mechanical system?

These research questions result in a working first-principles model that system and control theory can be applied to. The developed model is used in two case studies to present the effectiveness of applying system and control theory to monetary policy. The first case study offers an alternative to the analysis presented in the DSGE model. This analysis investigates the effects of the shocks. A systems theory is used to analyze the effects of disturbances on the system variables in a systematic manner, extending on the characteristics that can be analyzed, both in Laplace and time domain. This analysis could not be applied to the DSGE model as the equations cannot be reproduced properly. Furthermore, the system theory offers insight into the system parameters that contribute to the dynamic response characteristics. The actual value of the parameter can be better understood. The second case study offers an alternative to the manual policy that is applied by the ECB. Furthermore, it offers a tool to simulate the effects of possible policy actions. This is not possible in the existing DSGE model. The application of control theory offers a completely new approach that offers new insight into monetary policy decisions. This is formulated in the final research question:

- How can the developed model be used to support monetary policymakers with their decisions?

Together, the five research questions form the answer to the main research question:

How to apply system and control theory to monetary policy?

1-3 Thesis Outline

1-3-1 Contributions

This thesis presents a completely novel approach to the modeling of economic systems using system and control theory. Specifically, monetary policy is modeled as if the economy were a mechanical system and the central bank a controller. The contributions are specified as follows:

- Monetary Policy in the control formulation

- Economic-Engineering formalism to define the economic concept of money and interest mechanically
- Development of a first-principles model of the economy
- Laplace domain transfer function as an investigation tool for the effects of disturbances on the system based on the underlying dynamics of the system
- Monetary-policy interpretation of PID-control actions to stabilize the inflation variable to support policymakers

1-3-2 Structure

The structure of the thesis is presented in Figure 1-1. Each chapter discusses and answers one research question.

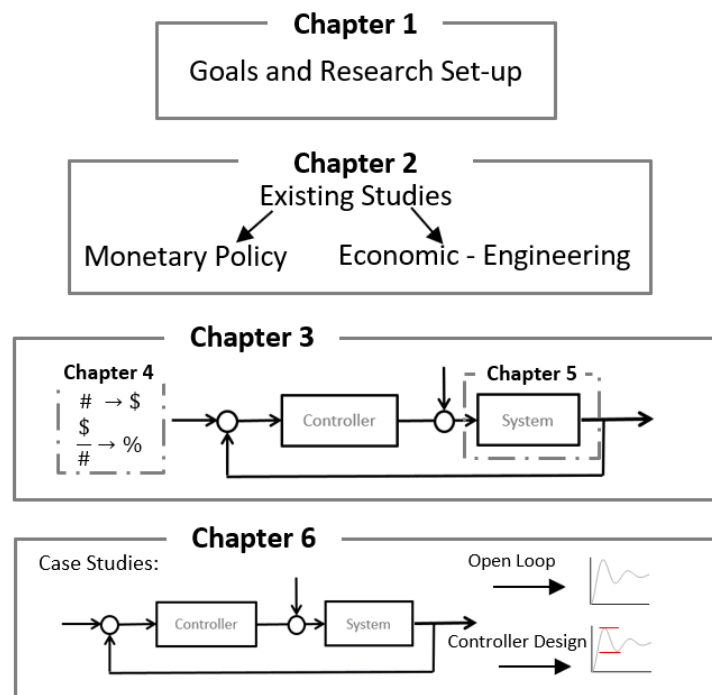


Figure 1-1: Overview of the thesis structure per chapter.

Chapter 2 introduces the background of both monetary policy models and economic engineering. It assesses the applicability of the current monetary policy model (DSGE model) and the challenges in the economic-engineering study to apply it to a monetary policy model. Chapter 3 interprets the monetary policy problem as a control problem. In this chapter, the framework is built on which the rest of the model can be developed. Chapter 4 presents the research in extending the economic-engineering theory in order to make it accessible to develop a monetary policy model. In chapter 5, the results from the previous chapters come together: The actual model is developed that captures the mechanics of the economy. This

model is necessary in order to apply system and control theory to monetary policy. Finally, chapter 6 presents two applications to the developed model in order to present the benefits of this economic-engineering approach to monetary policymakers. Chapter 7 presents a general discussion, conclusion and future recommendations.

Chapter 2

Preliminaries

The introduction presented the objective of this study; to apply system and control theory to monetary policy. This chapter presents the present-day model used in monetary policy. Section 2-2 answers the first research question:

What are the economic models used for monetary policy and what are their limitations to apply system and control theory to?

But first, section 2-1 provides an overview of the introduced economic terminology to help the reader. Then after introducing the economic preliminaries, section 2-3 introduces the economic-engineering framework. This will form the basis to transform the monetary policy model of economics in a model that can be used by engineers. The challenges in the economic-engineering framework, to be specifically applied to monetary policy, are identified.

2-1 Definition of Economic Variables

In order to understand the coming chapters, the economic variables are explained. This section presents a complete overview. The definitions are taken from [16]:

- **Labour:** Human beings as factors of production, i.e. hours worked.
- **Employment:** The state of being paid to work for someone else, i.e. the number of people employed.
- **Labour supply:** The supply of work effort.
- **Wage:** A level of pay laid down by law, for laborers.
- **Consumption:** Spending for survival or enjoyment.
- **Inflation:** A persistent tendency for prices to increase measured by the proportional change of a price index over time.

- **Price of goods:** A price index covering the prices of consumer goods.
- **Goods production:** The use of resources to make goods or services which have value.
- **Transaction cost:** Relative merits of conducting transactions within firms and between different firms using markets.
- **Capital :** Man-made means of production. Capital goods are goods designed to be used in production, for example, machinery.
- **Capital supply:** The process of increasing the stock of man-made means of production.
- **Rental on capital:** A payment for the service of an economic resource.
- **Depreciation:** Loss of value of capital goods due to wear and tear.
- **Return on stocks/equity/shares:** The increase of the price of stocks/equity/shares.
- **Price of stocks:** The price at which a stock can be traded.
- **Investment:** The process of adding to stocks of productive assets.
- **Interest :** Payment for a loan additional to repayment of the amount borrowed. The rate of interest is the extra payment per unit of the loan.

2-2 Monetary Policy

The ECB has developed a Dynamical-Stochastic-General-Equilibrium (DSGE) model to analyze the Eurozone. The goal of this model is to investigate the effects of shocks and their contribution to fluctuations in the euro area. In economics, a shock is defined as an unexpected or unpredictable event that affects an economy. This is their method to obtain or extract responses from the model. An example of a shock could be the event of the Brexit (2019) or the Oil crisis (1979). The shock responses try to simulate the possible effects of these types of events. The possible events are translated into ten different variables. For example, a labor supply shock or a productivity shock.

The model consists of agents that interact. Different variables capture this interaction. Figure 2-1 presents an overview of economic agents and variables used in DSGE models. This Figure is taken from the DSGE model from the Federal Reserve [1]. The household and the firm stand out in the figure, these are two important agents. Furthermore, this DSGE model distinguishes a final goods producer, capital producers, entrepreneurs, banks and the government. The figure presents the interaction between these different agents in an abstract manner. This is not unambiguously, as there is no strict formalism in economics to model this. The arrows express the interaction of the agents. The black terms, corresponding with the arrows, express an economic variable that captures this interaction. For example, the household interacts with the firm by providing labor. The grey term, in capitals, presents a stochastic shock that acts on this labor variable. Similarly, the variables that correspond with the interaction of the other agents are expressed in the figure.

The following paragraphs explain the DSGE model and the corresponding parameters, equations, and variables as stated in the DSGE model of Smets-Wouters.

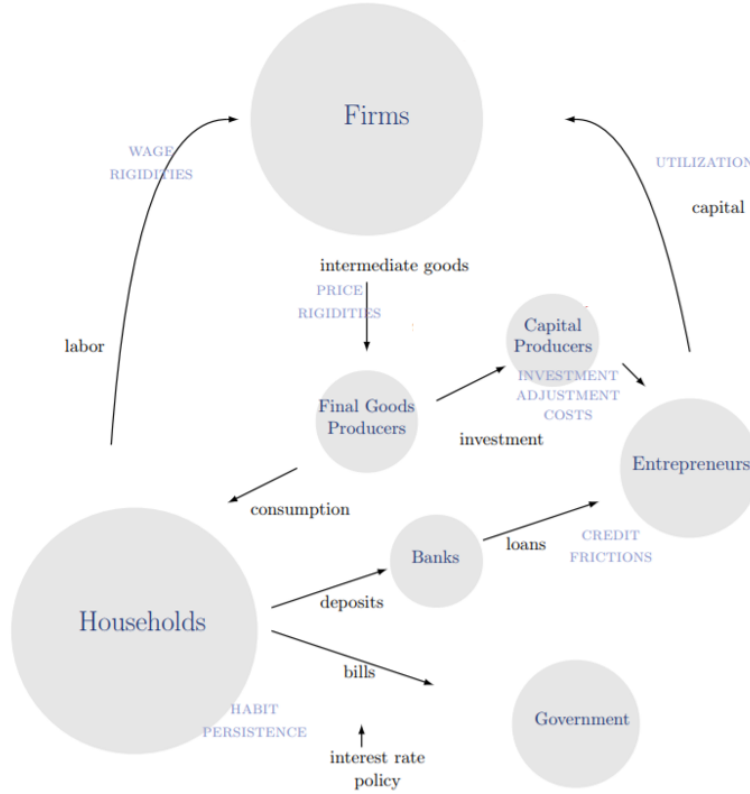


Figure 2-1: Model structure of a DSGE model. The balloons present economic agents, variables are presented in black and economic modeling terms such as wage rigidities presented in grey. This figure is taken from [1]. The figure presents an abstract representation of interacting agents in the economy.

The DSGE model starts with a household. A household consists of one or more people who live in the same residence and share meals. Smets-Wouters assumes that the household has the monopoly power over the supply of its labor and that they maximize a so-called intertemporal utility function (2-1) with three arguments: Goods, money, and leisure. The consumption variable C expresses the goods. The cash balance variable M expresses money. The labor supply variable L expresses leisure. The household optimizes this utility function subject to an intertemporal budget constraint (2-2). The household maximizes this utility function subject to the budget constraint, with respect to consumption and holdings of bonds. Variable B expresses the bonds. In economics, a bond is an instrument of indebtedness of the bond issuer to the holders. In other words, it represents a loan made by an investor to a borrower. A note of caution. This bond is different from the bond-graph modeling presented later in this chapter.

$$U_t = \varepsilon_t^B \left(\frac{1}{1 - \sigma_c} (C_t - H_t)^{1 - \sigma_c} - \frac{\varepsilon_t^L}{1 + \sigma_L} (l_t)^{1 + \sigma_L} + \frac{\varepsilon_t^M}{1 - \sigma_m} \left(\frac{M_t}{P_t} \right)^{1 - \sigma_m} \right) \quad (2-1)$$

$$\frac{M_t}{P_t} + b_t \frac{B_t}{P_t} = \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + Y_t - C_t - I_t \quad (2-2)$$

The households act as price-setters in the labor market. The households set their nominal wage to maximize their intertemporal utility function subject to the intertemporal budget constraint and the demand for labor. In simpler words, consumer behavior is a maximization problem: It means making the most of their limited resources to maximize their utility.

The capital stock is owned by households. The capital stock is defined as an asset that can enhance one's power to perform economically useful work and is presented as the variable capital K . Capital is a factor of production which households can rent out to the firm-producers at a given rental rate of r^k . Households can increase the supply of capital by investing in additional capital expressed as the investment variable I . Capital depreciates over time with the depreciation rate τ . The value of capital is expressed as the return on equity variable Q .

The firms produce a final good that is used for consumption and investment by the households. The price of the final good is presented as the variable P . A distinction is made between the production of intermediate and final goods. The firm produces a final good Y by using the intermediate goods y . In the model, the firm maximizes a profit equation (2-3). This profit equation depends on the price, marginal cost (2-4) and fixed cost Φ . The marginal cost is defined as the change in the total cost that arises when the quantity produced is incremented by one unit. A fixed cost is an expense or cost that does not change with an increase or decrease in the number of goods or services produced or sold. Equation 2-5 implies that the capital-labor ratio will be identical across intermediate goods producers and equal to the aggregate capital-labor ratio.

$$\pi_t = (p_t - MC_t) \left(\frac{p_t}{P_t} \right)^{\frac{1+\lambda_p}{\lambda_p}} Y_t - MC_t \Phi \quad (2-3)$$

$$MC_t = \frac{1}{\varepsilon_t^\alpha} W_t^{1-\alpha} r_t^{k\alpha} (\alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}) \quad (2-4)$$

$$\frac{W_t L_t}{r_t^k K_t} = \frac{1-\alpha}{\alpha} \quad (2-5)$$

The final goods market is in equilibrium if production Y equals demand by households for consumption C and investment I and the government G . The capital rental market is in equilibrium when the demand for capital by the intermediate producers equals the supply by the households. The labor market is in equilibrium if the firms' demand for labor equals labor supply at the wage level set by households.

The interest rate is determined by the reaction function that describes monetary policy decisions. In order to maintain the money market equilibrium, the money supply adjusts endogenously to meet the money demand at those interest rates. A summary of the variables presented in Smets-Wouters is given in Table 2-1.

Variable	Symbol
Consumption	C
Cash balance	M
Labour supply	L
Rental rate of capital	r^k
Investment	I
Depreciation rate	τ
Return on equity	Q
Final goods price	P
Final goods production	Y
Intermediate goods production	y
Governmental spending	G
Bond	B
Marginal Cost	MC
Inflation and profit	π
Fixed cost	Φ

Table 2-1: Overview of the variables presented in the DSGE model [2]

Smets-Wouters presents a linearized model that is used to render results. It is linearized "above around the non-stochastic steady state" (citation from [2] page 17). Ten difference equations are presented. Variables dated at time $t + 1$, refer to the rational expectation of those variables. The consumption equation is given by:

$$C_t = \frac{h}{1+h}C_{t-1} + \frac{1}{1+h}C_{t+1} - \frac{1-h}{(1+h)\sigma_c}(R_t + \pi_{t+1}) + \frac{1-h}{(1+h)\sigma_c}(\varepsilon_t^b - \varepsilon_{t+1}^b) \quad (2-6)$$

The parameter σ_c is defined as the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution. The equation contains a preference shock ε^b , that represents a general shock to preference that affects the intertemporal substitution of households also called the preference shock.

The investment equation is given by:

$$I_t = \frac{1}{1+\beta}I_{t-1} + \frac{\beta}{1+\beta}I_{t+1} + \frac{\varphi}{1+\beta}Q_t + \beta\varepsilon_{t+1}^I - \varepsilon_t^I \quad (2-7)$$

The parameter β is defined as the discount factor and expressed as the following equation:

$$\beta = \frac{1}{1 - \tau + \bar{r}^k} \quad (2-8)$$

The equation contains the investment shock ε^I . The Q equation is corresponding to the I equation:

$$Q_t = -(R_t + \pi_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}Q_{t+1} + \frac{r^k}{1-\tau+\bar{r}^k}r_{t+1}^k + \eta_t^Q \quad (2-9)$$

The shock to the required rate of return on equity investment η^Q is introduced. Notice that both η and ε are used to express a shock. The capital accumulation equation is given by:

$$K_t = (1 - \tau)K_{t-1} + \tau I_{t-1} \quad (2-10)$$

Smets-Wouters presents the inflation equation as a general specification of the standard new Keynesian Phillips curve [17]:

$$\pi_t = \frac{\beta}{1 + \beta\gamma_p}\pi_{t+1} + \frac{\gamma_p}{1 + \beta\gamma_p}\pi_{t-1} + \frac{1}{1 + \beta\gamma_p} \frac{(1 - \beta\xi_p)(1 - \xi_p)}{\xi_p} (\alpha r_t^k + (1 - \alpha)w_t - \varepsilon_t^\alpha + \eta_t^p) \quad (2-11)$$

The parameter γ_p presents the price indexation of consumer goods. The parameter ξ_p presents the price flexibility of consumer goods. The parameter α presents the elasticity of output with respect to capital.

Parameter	Symbol
-not defined-	h
Coefficient of relative risk	σ_c
Discount factor	β
Depreciation rate	τ
-not defined-	$\bar{r}^k$
-not defined-	φ
Degree of price indexation	γ_p
Price flexibility	ξ_p
Elasticity of output wrt capital	α
Degree of wage indexation	γ_w
Wage flexibility	ξ_w
Elasticity of work effort wrt real wage	σ_L
Elasticity of money holdings wrt interest rate	σ_m
Degree of interest rate smoothing	ρ
-not defined-	r_π
-not defined-	r_Y
-not defined-	r_π
-not defined-	r
-not defined-	r_α
-not defined-	r_L

Table 2-2: Overview of the parameters presented in the DSGE model [2]

Real wage equation is given by:

$$w_t = \frac{\beta}{1+\beta}w_{t+1} + \frac{1}{1+\beta}w_{t-1} + \frac{\beta}{1+\beta}\pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} - \frac{1}{1+\beta} \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\gamma_w)\sigma_L}{\gamma_w})\xi_w} (w_t - \sigma_L L_t - \frac{\sigma_c}{1-h}(C_t - hC_{t-1}) - \varepsilon_t^L - \eta_t^w) \quad (2-12)$$

The parameter γ_w is the degree of wage indexation and the parameter ξ_w is the wage flexibility.

$$L_t = -w_t + (1+\psi)r_t^k + K_{t-1} \quad (2-13)$$

Parameter ψ is the inverse of the elasticity of the capital utilization cost function. A distinction is made between labor and employment, where employment is the hours worked per employer:

$$E_t = \beta E_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e} (L_t - E_t) \quad (2-14)$$

The parameter ξ_e is a constant fraction of firms to adjust employment to its desired total labor input. The goods market equilibrium condition can be written as:

$$Y_t = (1 - \tau k_y - g_y)C_t + \tau k_y I_t + g_y \varepsilon_t^G = \phi \varepsilon_t^\alpha + \phi \alpha K_{t-1} + \phi \alpha \psi r_t^k + \phi(1 - \alpha)L_t \quad (2-15)$$

With k_y as the steady state capital output ratio, g_y the steady-state government spending-output ratio and ϕ is one plus the share of the fixed cost in production. Finally, the model is closed by the monetary policy reaction function:

$$R_t = \rho R_{t-1} + (1 - \rho)(\bar{\pi}_t + r_\pi(\hat{\pi}_{t-1} - \bar{\pi}_t) + r_Y \hat{Y}_t) + r_\pi(\hat{\pi}_t - \hat{\pi}_{t-1}) + r_Y(\hat{Y}_t - \hat{Y}_{t-1}) - r_\alpha \eta_t^\alpha - r_L \eta_t^L + \eta_t^R \quad (2-16)$$

Parameter ρ captures the degree of interest rate smoothing.

2-2-1 Decision Support DSGE

The goal of the DSGE model is to investigate the effects of the stochastic shocks to the variables. Specifically, what their contribution is to business cycle fluctuations in the euro-area. To do this, Smets-Wouters presents a catalog of 54 shock responses. These responses describe the effect of economic variables, such as inflation and consumption, to the defined shocks that represent economic events. The observations of these shock responses try to explain these events in practise. This is described in the paper. One example is given:

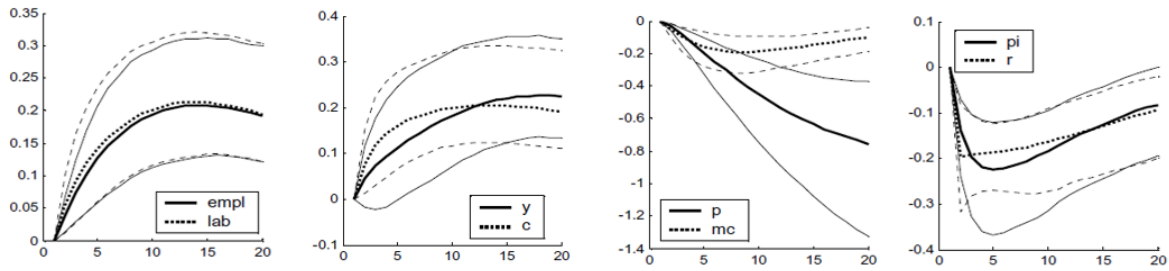


Figure 2-2: Plot directly copied from Smets-Wouters [2]. The plots present results of applying a labor supply shock to the system of eight variables. The x-label and y-label are not given. From Dr. R. Wouters it appeared the y-label is in quarters and the x-label depended on the variable. The graphs show the median response together with the 5 and 95 percentiles.

Figure 2-2 presents 4 plots taken from Smets-Wouters. These 4 plots are a subsection from a catalog of 54 plots. From the paper, it is unclear what is presented on the x-axis and the y-axis. Dr. R. Wouters explained that the y-axis is in time in quarters. The graphs plot the median response together with the 5 and 95 percentiles. This is a method used in statistics. Not much notion is paid to this in this thesis work. Smets-Wouters presents an analysis corresponding to the plots in terms of observations:

Citation: "Graph 4 shows the effects of a positive labor supply shock. The qualitative effects of this supply shock on output, inflation, and the interest rate are very similar to those of a positive productivity shock. Due to the higher persistence of the labor supply shock, the real interest rate is, however, not significantly affected. The main qualitative differences are that, first, employment also rises in line with output and, second, that the real wage falls significantly. It is this significant fall in the real wage that leads to a fall in the marginal cost and a fall in inflation." (Citation from Smets-Wouters page 28 [2])

2-2-2 Challenges DSGE Model

It appears that there is a big difference between economic models, and engineering models, and the way they are presented as academic work. In engineering papers, an important purpose is to inform academics and to enable them to build on the presented work. In economic papers, it appears that only economics from the specific field are able to understand the work. From the engineering perspective, it is not possible to build on this. It could be preferable to bring the economic field and engineering field closer. Not only to accommodate a possible synergy but also to get more understanding in the economic models for a larger audience then economics as their work has great influence worldwide. The following remarks on the Smets-Wouters paper can be given:

First, Smets-Wouters fails to specify the dimensions of the defined variables and parameters presented in their DSGE model. The discussion sessions with Dr. R. Wouters gave more insight. He explained that this is of lesser interest to them. Most variables were either in dimensions $\$/yr$ or $\%/yr$. From the engineering perspective, variables were summed that did not have matching dimensions. It could be questioned if this summation is possible in practice. From the discussion session with Dr. R. Wouters, it appears that sometimes dummy

parameters are used to solve for this. This results in a large catalog of parameters. In engineering, SI-units are used to derive all dimensions from. In economics, as they do not present dimensions of variables they also have no strict rules similar to SI-units. For example, time is sometimes measured in years and sometimes in quarters, but no strict rules are followed.

Second, the DSGE model presents results based on empirical studies. The data that is used to obtain these empirical results are not given. Without data, the model cannot be replicated and the results cannot be simulated and compared. Furthermore, the presented results cannot be interpreted properly, as the plots do not present x-label and y-label. Also, the input signal of the plot is not defined, only the term impulse response is used. It is not clear whether this impulse is the same impulse signal as is common in control engineering. It is possible that sometimes a step input is applied to the system but this distinction is not made in the paper. The possibility of the step input comes from the results converging to a new steady state which indicates a step input. Finally, the observations of the results are quite moderate. Also, no explanation is given why these results are expected or unexpected, what the risk is of the effect and what policy actions can mitigate for the risk.

Third, the DSGE model does not define an explicit controller. The stochastic shocks are interpreted as exogenous signals. Both the policy shocks and all other shocks are exogenous. In control terminology, this could be interpreted as if the model is open-loop with a disturbance acting on it. But the interference of the central bank is influenced and therefore not a disturbance, hence the definition of a controller would be more appropriate. The paper mentions that the linearized model is in the form of a state-space representation. But it is not clear what this representation captures. If a state space representation, known in control engineering, is to be used it is not clear what variable is fed-back into the system.

Fourth, a lot of parameters and variables are not defined and explained. For example ψ , r_π and r_Y . If the parameters are defined, it is unclear what it means and what the distinction is between these parameters. For example with the price indexation of consumer goods γ_p and price flexibility of consumer goods ξ_p . Similarly, with γ_w and ξ_w . The cash balance variable M , the bond variable B and the marginal cost variable MC are introduced and used in equations but do not return in the model or in the results. It is not clear if these variables are part of the model or not. If they are part of the model how they contribute to the results. Furthermore, the inflation variable and the profit variable are both expressed with π . It is not clear if inflation and profit result in the same behavior.

Finally, the DSGE model presented in Smets-Wouters appeared to be the most accessible DSGE model to understand as an engineer. Because this model presented a linearized version consisting of 10 equations. In other DSGE models [1], [4] it was even unclear what were the exact contents of the model.

Although, the DSGE model cannot be used directly to apply system and control theory to. Instead of starting from scratch, the parameters and variables are used as a starting point to develop a new model.

2-3 Economic-Engineering

The research area of economic-engineering tries to model, analyze and explain economic models and phenomena using engineering methods. Although, several attempts of similar approaches are found in literature ([18], [19], [20], [21], [22]), it did not result in a large and well-known field or research yet. Since 2017, Dr. M. B. Mendel has set-up the academic research group of 'Economic-Engineering' and tries to give this approach more ground and widen its applications. The economic-engineering group at DCSC models economic systems as causal dynamical systems so that the internal dynamics of economic systems are revealed and control theoretical formalism's can be applied [13]. The added value of these formalisms is that this can be understood and used by other academics. They can be built on, and be extended in further research.

The theory of economic engineering is based on dynamical analogs between economic elements and phenomena and physical elements (e.g. mechanical and electrical) elements and phenomena. To explain the working principles of the economic-engineering framework, bond-graph modeling is first explained. Then, the work of two researchers Brewer and Mendel are presented in the field of economic-engineering. The challenges of applying this framework directly to monetary policy in the control framework are presented in the form of a discussion.

2-3-1 Bond-Graph Modelling

Bond-graph modeling is a technique that constructs system models using a uniform notation that can be used in different domains (i.e. mechanic, electric, economic(new)). It allows one, to study the nature of the parts of the model and the manner in which the systems interact evident in a graphical format. Standard techniques allow the models to be translated into differential equations and computer simulation schemes.

The uniform notation makes it appropriate to use bond-graph modelling in the economic-engineering framework. Furthermore, since the economy is based on systems - e.g. agents - interact it is a convenient way of translating this into a model configuration.

A bond-graph is a graphical representation of a physical dynamic system [23]. The arcs in bond-graphs represent a bi-directional exchange of physical energy. bond-graphs are multi-energy domain and domain neutral. Each bond represents an instantaneous flow of energy or power. Not to confuse this bond with the economic bond which is defined as an instrument of indebtedness of the bond issuer to the holders. The flow in each bond is denoted by a pair of variables called power variables, whose product is the instantaneous power of the bond. The power variables are broken into two parts flow and effort. For example, for the bond of a mechanical system, the flow is the velocity while the effort is the force. By multiplying the velocity and the force you can get the instantaneous power of the bond. This bi-directional exchange in a bond is a good fit with economics, where a transaction is also bi-directional. For example, a consumer-good transacted for an amount of money.

The tetrahedron of state is a tetrahedron that graphically shows the conversion between effort and flow, see Figure 2-3. In this tetrahedron, the relations between the position, velocity,

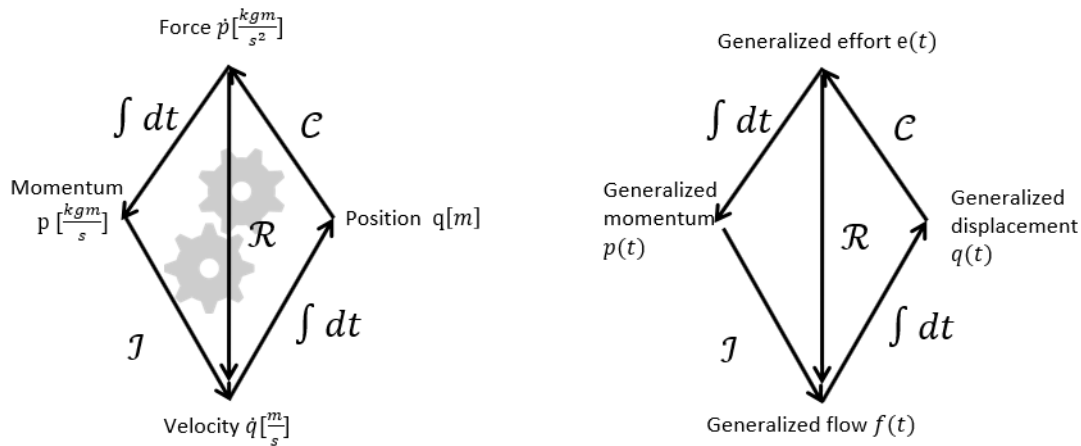


Figure 2-3: Tetrahedron of state, start from the force(top) integrating over time results in momentum (left), multiplying with a mass ($\mathcal{I}$) results in velocity (down), again integrating over time results in position, multiplying with the spring-coefficient($\mathcal{C}$) results again in the force. On the left, the mechanical terms are presented and on the right the generic term. The force is also related to the velocity by multiplying with the damping-coefficient ($\mathcal{R}$)

Generic	Mechanics	Sym.	Dim.
Generalized displacement	Position	q	m
Generalized flow	Velocity	$\dot{q}$	$\frac{m}{s}$
Generalized momentum	Momentum	p	$\frac{kgm}{s}$
Generalized effort	Force	$\dot{p}$	$\frac{kgm}{s^2}$

Table 2-3: The four general variables used in bond-graph modelling with the corresponding mechanical term and their symbol and dimension.

momentum, and force are presented. Position and momentum are a conjugate pair that together span the phase space of the system. The velocity and force are the derivatives of position and momentum variables. These four variables are presented in their generic form and in the domain of mechanics in Table 2-3. In mechanics, three laws define the relations between force, velocity and their integrals momentum and position. A mass describes the relationship between the velocity and momentum of a particle. Figure 2-3 presents this relation between momentum and velocity with an $\mathcal{I}$ ¹. A spring describes the relationship between the force and the position. Figure 2-3 presents this relation between force and position with a $\mathcal{C}$. A damper describes the relation between a force and a velocity. Figure 2-3 presents this relation between force and velocity with an $\mathcal{R}$. Similarly, in electronics an inductance relates a current and flux linkage, a capacitance relates a voltage and a charge, and a resistance relates a voltage and a current. As bond-graph modeling is domain neutral, inertance is the generalized term used to describe a mass and an inductance. Similarly, compliance is the generalized term to describe a spring and a capacitance and resistance is the generalized term to describe a damper and a resistance. This is summarized in Table 2-4. The symbols from Table 2-4 are

¹Because $\mathcal{R}$, $\mathcal{C}$ and $\mathcal{I}$ are already used as economic variables interest rate, consumption and investment the script letters are used.

used to show the conversion of the variables in the tetrahedron of state presented in Figure 2-3.

Generic	Mechanics	Sym.	Dim.
Inertance	Mass	$\mathcal{I}$	kg
Compliance	Spring	$\mathcal{C}$	$\frac{kg}{s^2}$
Resistance	Damper	$\mathcal{R}$	$\frac{kg}{s}$

Table 2-4: The three general elements used in bond-graph modelling with the corresponding mechanical element and their symbol and dimension.

The components that are used to describe a system using bond-graph modeling techniques are either a single-port element, a two-port element or a multi-port element.



Figure 2-4: Single-port elements, from left to right: effort source, flow sink, inertance, capacitance and resistance element.

There are five single-port elements (See Figure 2-4). A source represents the input for a system, which is either an effort (S_e) or a flow (S_f). A sink represents the output for a system, which also is either an effort or a flow. $\mathcal{I}$ denote an inertance element. They store kinetic energy and always have power flowing into them. The arrow contains an effort e (up), a flow f (down), and generalized momentum p (right). $\mathcal{C}$ denote a capacitance element. They store potential energy and also always have power flowing into them. The arrow contains an effort e (up), a flow f (down), and generalized position q (right). $\mathcal{R}$ denote a resistance element. They dissipate energy and always have power flowing into them. The arrow contains an effort e (up), a flow f (down). They do not store anything.

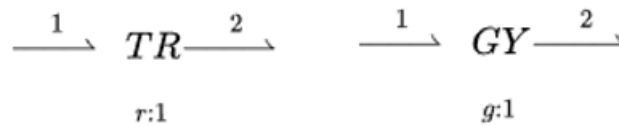


Figure 2-5: Two-port elements, left the transformer and right the gyrator.

There are two two-port elements. Two-port elements change the power between or within a system. When converting from one to another, no power is lost during the transfer. There are two two-port elements. A transformer (TF) applies a relationship between flow-in and flow-out, and effort-in and effort-out. An example is an electrical transformer or a lever.

$$f_1 \cdot r = f_2 \quad (2-17)$$

$$e_2 \cdot r = e_1 \quad (2-18)$$

A gyrator applies a relationship between flow-in and effort-out, and effort-in and flow-out. An example of a gyrator is a DC motor.

$$e_2 \cdot g = f_1 \quad (2-19)$$

$$e_1 \cdot g = f_2 \quad (2-20)$$



Figure 2-6: Multi-port elements left the 0-junction and right the 1-junction.

There are two multi-port elements. A multi-port element is also called a junction. A junction splits power across their ports. There are two junctions, a 0-junction, and a 1-junction. The 0-junction behave such that all efforts values are equal across bonds, but the sum of flow values in equals the sum of flow values out.

$$e_1 = e_2 = e_3 \quad (2-21)$$

$$f_1 = f_2 + f_3 \quad (2-22)$$

The 1-junction behave opposite of 0-junctions, such that all flow values are equal across bonds, but the sum of effort values in equals the sum the effort values out.

$$f_1 = f_2 = f_3 \quad (2-23)$$

$$e_1 = e_2 + e_3 \quad (2-24)$$

To conclude, these elements together can form a model. This model can describe phenomena in different domains, mechanics, electronics, economics, but still use the same elements. The conjugate pair of generalized position and generalized momentum forms the basis of the tetrahedron of state, which forms the basis of bond graph modelling. In physics, the conjugate pairs that are necessary to describe the different types of phenomena are available. In economics, this is an area that still needs to be explored. This is done in economic-engineering.

2-3-2 Brewer: Precursor (1978)

The first serious discussions and analyses of modeling economy like an engineer, emerged during the 1970s with Brewer [19] [20]. Brewer did not call his methods economic-engineering explicitly, but he is a precursor to the field. His study attempts to express the dynamics of a marketplace in terms of general laws. Not a lot of studies appeared ever since, which seems weird. It most definitely is no common knowledge among economists or engineers. A possible reason could be that economists and engineers or physicists do not communicate with each other and don't understand each other. As it seems to be a fruitful and promising idea.

His first paper "bond-graphs of microeconomic systems", published in the American Society of Mechanical Engineering, Houston (1975) could not be found. Therefore, "Progress in the bond-graph representations of economics and population dynamics" [19] is used to describe its contents. This paper defines the effort variable as the unit price $\$/\#$. The flow variable is defined as the flow of orders $\#/\text{period}$. They define that orders flow from buyer to seller flow in the same directions as the flow of cash.

$$p = \text{price per unit commodity} \quad (2-25)$$

$$f = \text{time rate of flow of orders} \quad (2-26)$$

In [20] the flow of money is denoted $\dot{V}$ and presented as:

$$\dot{V} = pf \quad (2-27)$$

The integral of the effort variable p price per unit commodity is defined as economic impulse γ . It is recognized that there seems no use-fullness in the conventional economics literature. Furthermore, the integral of the time rate of flow of order is q the inventory or accumulation of orders.

2-3-3 Mendel: Current Research (2017)

Since 2017, Dr. M.B. Mendel [13] has been working on the economic-engineering analog from scratch. Mendel proposes a theory for modeling economics as mechanics. His work originates from the combination of experience in finance as a trader in Boston and academic experience in engineering as a postdoc at MIT and UC Berkeley. An important idea of his work is the academic method. Other academics can understand and use the model but foremost expand on the model and build on it.

Mendel's study defines the value of a coordinate as the stock of some assets, consumer goods, resources such as land, labor or capital. This coordinate is also called the position in mechanics and represents a quantity in economics. The derivative of position, velocity or motion in mechanics represents the quantity demanded in economics. Momentum in mechanics is defined as a price in economics. The derivative of momentum, force in mechanics is defined

Mechanics	Sym.	Dim.	Economics	Sym.	Dim.
Position	q	m	Quantity	q	$\#$
Velocity	$\dot{q}$	$\frac{m}{s}$	Quantity Demanded	$\dot{q}$	$\frac{\#}{yr}$
Momentum	p	$\frac{kgm}{s}$	Price	p	$\frac{\$}{\#}$
Force	$\dot{p}$	$\frac{kgm}{s^2}$	Cost	$\dot{p}$	$\frac{\$}{\#yr}$

Table 2-5: The four mechanical variables used in bond-graph modelling translated to the economic domain. The same symbol is used for mechanics as for economics.

analogously to a cost in economics. Table 2-5 presents an overview of these four variables in mechanics and economics, including their dimensions.

Figure 2-7 presents the position, velocity, momentum, and force that are related in a tetrahedron of state. The analogs of these four variables for economics relate in a similar manner. In Newtonian mechanics, momentum relates to velocity by a mass. Newton stated: 'Quantity of motion if a measure of motion that arises from the velocity and the quantity of matter'. This quantity of matter presents a mass. The economist Marshall stated: 'Price is a measure of demand that depends on quantity demanded and the slope of the demand line jointly'. Hence, economic engineering defines the analog of mass in mechanics as demand in economics.

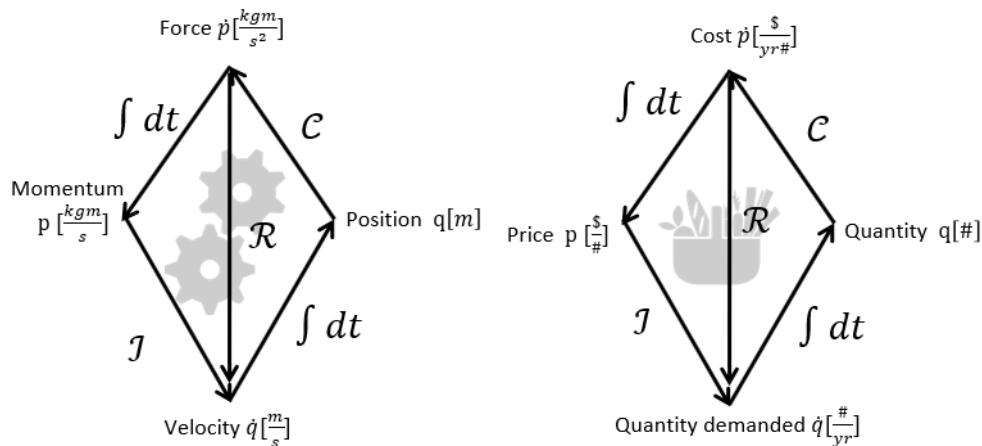


Figure 2-7: The tetrahedron of state in mechanical terms from Figure 2-3 and the translation into the economic domain. From the cost (top) integrating over time results in the price (left), multiplying with the I element results in the quantity demanded (down), again integrating over time results in the quantity (right), finally multiplying with the C element results in the cost (down). The cost is also related to the quantity demanded through the R element.

Where momentum relates to the velocity through a mass, force relates to the position through a spring. Dr. M. B. Mendel defines factor demand analogous to a spring: a spring stores potential energy in the same way an economic factor of production is stored by an agent. This factor of production can be for example labor, capital or land. Finally, a force also relates to velocity by a damper. Dr. M. B. Mendel defines consumption analogous to a damper: a damper acts to reduce the velocity of a mass the same way that consumption removes goods from circulation or relieves the displacement of the spring in the same way as depreciation reduces the amount of the stock. This is summarized in Table 2-6.

Sym.	Mechanics	Dim.	Economics	Dim.
$\mathcal{I}$	Mass	kg	Demand	$\frac{\$yr}{\#^2}$
$\mathcal{C}$	Spring	$\frac{kg}{s^2}$	Factor Demand	$\frac{\$}{yr\#^2}$
$\mathcal{R}$	Damper	$\frac{kg}{s}$	Consumption	$\frac{\$}{\#^2}$

Table 2-6: The translation of the three mechanical elements from Table 2-4 to the three economic elements. A mass is translated by demand, a spring by factor demand and a damper by consumption.

Economic-engineering develops linear-time-invariant models. These models define a variable and a parameter similar to engineering practices. A variable changes or evolves over time. $x(t)$ is taken as a state vector, a set of variables representing the configuration of the system at time t . The parameters are coefficients which are constant. $\mathcal{I}$, $\mathcal{C}$ and $\mathcal{R}$ are the three parameters that are used in modeling.

2-3-4 Challenges Economic Engineering

The objective of the study is to apply the economic-engineering framework to a monetary policy model. The study uses the framework developed by Dr. M. B. Mendel as a primary source to build upon. Since the study is performed under his supervision. Nevertheless, similar studies were analyzed. Brewer is found to present the most extensive research in the field of economic engineering. It is surprising that this study originates from 1970, but not many follow-up studies are found.

The attempt in 1970, to bring the engineering and economics domain closer, seems like a fruit-full idea. Especially, when analyzing the current DSGE model, used by the ECB, it seems that the symbiosis of these two disciplines has a lot of added value to offer. To show this added value to both engineers and economics, will add to the development of this movement.

Brewer found that the force in mechanics is analog to a price of a good in economics. However, this outcome is contrary to that of Mendel. Mendel found that momentum in mechanics is analog to a price of a good in economics. Both studies do not explain the reasoning behind this choice. Nevertheless, the choice of Mendel seems most reasonable for the following reasons. The price and quantity together form a conjugate pair because according to Mendel they are analogous to momentum and position. The momentum-position pair describes the phase space of a system. In economics, phase space is the most natural and convenient manner to describe the state of a system. Brewers momentum-position pair translates to a quantity and the economic impulse variable. Which, according to Brewer, there seems no recognition of the usefulness of this variable in economic literature. Also, the interpretation of a pair suits well in economics with the quantity-price. Each quantity, a good, a service, a man-hour, has a price. The specific price of a quantity is connected to that quantity. Hence, the pair.

Despite the importance of money and interest in economic models, there is remarkably little study towards this concept in the economic-engineering framework. Although, both Brewer

and Mendel include a cash-flow or money flow as power and energy respectively. This cannot be applied directly to develop a first-principles model describing monetary-policy. Since, the price of money, interest, cannot be expressed. The current state of economic-engineering theory is not enough to cover the area of monetary policy and therefore needs to be expanded.

2-4 Conclusions and Summary

The aim of this chapter was to examine the applicability of the DSGE model to system and control theory. The second aim was to examine the applicability of the economic-engineering framework to model monetary policy. Furthermore, the chapter serves as a background to understand the rest of the thesis work.

It can be concluded that generally the DSGE model cannot be used to apply system and control theory to. In summary; the data is unknown, the dimensions of the parameters and variables are unknown, some of the parameter values are unknown, there is no definition of a controller and it is unclear why some variables are part of the model and why others are not.

Hence, in order to apply system and control theory to monetary policy, a new model needs to be developed. In the development of the model, the economic agents, variables, and parameters from the DSGE model will be used. The DSGE model can only be understood and used by economics in the specific field of monetary policy. But as monetary policy largely influences economics in general, it could be preferable that a model will be developed that can be understood by a broader audience. Hence, the policy actions and the corresponding risks can become more transparent.

The results presented in Smets-Wouters are supposed to support monetary policymakers in their decisions. This support was found to be limited. As these decisions have a large influence, the expansion of the analysis of shock effects can help to improve risk-assessment of both shock effects and policy effects. The results from Smets-Wouters will be used as a starting point to apply system and control theory to the newly developed model, in order to present alternative decision making support.

The economic-engineering theory has, in the present-day, not covered the field of monetary policy. The framework cannot express money and interest as a conjugate pair. This conjugate pair is necessary as initial state variables to express the dynamics of the economy in a bond-graph model. Hence, it first needs to be expanded to apply it to the development of a monetary policy model. The expansion of the theory and the application to monetary policy will give the field of economic engineering more attention. This attention is necessary in order to let economists and engineers work together and create a synergy.

Control Formulation of Monetary Policy

This chapter answers the second research question:

How can the monetary policy problem of a central bank be formulated as a control problem?

This chapter provides the structure in order to develop the model. It results in a control problem that enables the application of system and control theory to monetary policy.

3-1 Monetary Policy Objective

The method, to formulate monetary policy as a control problem, includes the following steps:

- Identify the system
- Identify the controller
- Identify the corresponding input and output signals
- Identify the disturbance signal
- Identify the reference signal

In order to formulate monetary policy as a control problem, the basic system and control theory is used [24]. The knowledge from the master's course 'Control Theory' is used to identify the system, the controller and corresponding signals.

In order to identify the system, the European Central Bank website [25] is analyzed. This website states the goal of monetary policy as 'price stabilization, employment, and economic

growth'. The output signal is identified as the three variables that are monitored by the European Central Bank. The reference signal is identified as the target set by the European Central Bank for the output signals.

In order to identify the controller, the question is asked if the system is autonomous and if not what external inputs are acting on the system. The external signals are divided into signals that can and cannot be manipulated. This will result in the control input and disturbance input respectively. The controller will be identified as the acting party of the control input.

To validate this method, experts in the field of monetary policy are questioned. The control formulation is compared with the DSGE model. Furthermore, the terminology and corresponding symbols that are presented in the DSGE model are used to express the results in this chapter and throughout this thesis.

3-2 Definition of the Signals

Figure 3-1 presents the control formulation of monetary policy which is the theoretic foundation of the model. Control engineers recognize this formulation as a standard formulation used to make logic decisions. This is the crucial step to applying system and control theory to monetary policy.

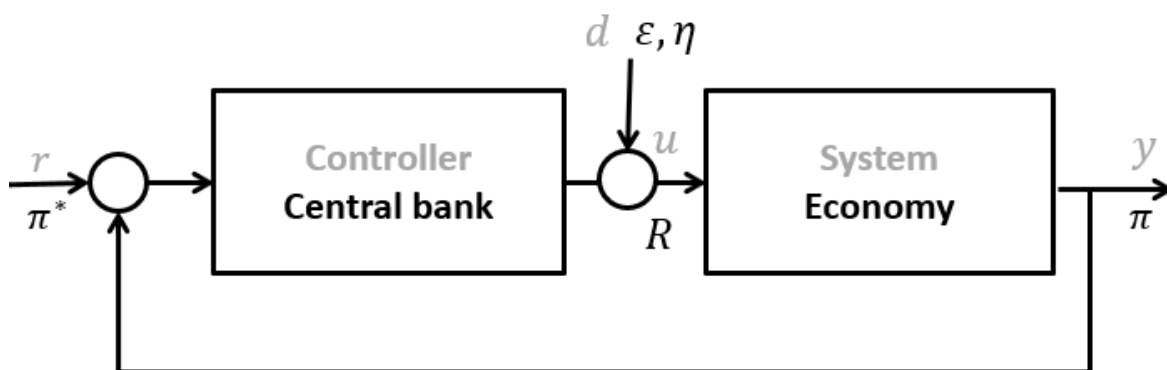


Figure 3-1: The monetary-policy objective formulated as a control problem. The system is represented by the economy and the controller is represented by the Central Bank. The variables in grey present the system-and-control domain and the variables in black present the monetary-policy domain.

The system (see Figure 3-1) represents the economy. This economy contains variables such as employment, prices for goods and GDP, similar to the DSGE model. The exact overview of variables 'inside' the economy are decided upon Chapter 5 that presents the development of a first-principles model for the economy.

The output signal could be price level, employment or GDP. In accordance with Dr. R. Wouters, the most important signal is the price level. To monitor this the inflation rate - the derivative of the price level - will be taken as the output signal.



Figure 3-2: The DSGE model presented similarly to the control formulation in Figure 3-1. As a control engineer, the DSGE model is interpreted as an open-loop configuration with the economy and the central bank both included in the system.

The reference signal is the target of the inflation rate of the ECB. According to the ECB website, this is set at 2 %/yr.

Control		Monetary Policy	
System output	y	Inflation variable	π
Control input	u	Interest rate variable	R
Disturbance input	d	Stochastic shock	η, ε
Reference signal	r	Target inflation	π^*

Table 3-1: Translation of the variables of the control formulation in Figure 3-1 from the system-and-control domain to the monetary-policy domain. Both ε and η are used to represent stochastic shocks in the DSGE model.

The system is non-autonomous because it is influenced externally. The external influences are twofold. For one, the central bank influences economic variables manually. The interest rate is a variable manually influenced by the central bank. Furthermore, the money supply (i.e. printing money), outstanding government bonds and loans with other central banks. The European Central Bank has a toolbox that could all result in optional control inputs. Second, external stochastic shocks influence the economy. These shocks are interpreted as disturbance inputs.

After these choices, the control problem is defined. This is summarized in Table 3-1, which presents the translation of monetary policy terminology to control terminology.

In comparison to the DSGE model, the control formulation presents a closed-loop system, where the DSGE model presents an open-loop system. Furthermore, the DSGE model includes both the economy and the central bank in the system. Changing the interest rate shock η^R is approached similarly as all other shocks that influence the system. Also, the DSGE model does not distinguish a reference signal explicitly.

3-3 Discussion

The DSGE model from Smets-Wouters showed that monetary policy can be modeled as one system, including both the central bank and the economy. This differs from the findings presented in this chapter, where the central bank is decoupled from the economy and identified

as the controller. This result may be explained by the fact that system and control theory is not a known theory by economists. A note of caution is due here since economists might not explicitly model monetary policy as feedback control but they could have similar ideas in mind.

The controller output is best represented by the central bank for this specific control objective. This finding is a modeling decision because any agent could be the controller. For example, if tax policy was to be modeled the government would be the controller, or if a firm was to be modeled the board or the CEO of the firm would be the controller. In practice, all these controllers are acting at the same time. In order to focus on monetary policy, it is chosen to only present the central bank as the controller.

To take the price stability as the output signal is chosen based on feedback from Dr. R. Wouters. In practice, monetary-policy focuses on price stability, employment, GDP and trust in the currency at the same time. According to Dr. R. Wouters, this is a simplification also done by economists. But the structure in the control formulation enables the extension to multi-output control in the future. Then, together with price stability, GDP, employment and trust in the currency can be added.

The controller is identified as the variable that is influenced by the central bank. It is assumed that the central bank only influences the interest rate. In practice, this is not true. The central bank also influences the money supply. They have different tools: An open-market operation includes the central bank's buying and selling of government securities in the open market in order to expand or contract the amount of money in the banking system. Furthermore, the reserve requirement ratio's are the amount of cash that banks must have in their vaults. Thereby, reserve requirements are another tool used by the Federal Reserve to increase or decrease the money supply in the economy. The interpretation of these tools as control actions can be researched in further studies. Similarly to the system output, the control input can be extended to multi-input. In comparison to the DSGE model, they did not extend to include these different tools in their model (yet) as well.

3-4 Conclusions and Summary

The aim of this chapter was to express monetary policy in the control formulation that control engineers are accustomed to, and system and control theory is based on. This is the first step to be able to apply system and control theory to monetary policy.

In contrast to the DSGE model, the central bank is identified as the controller and decoupled from the system. This configuration better fits monetary policy practices than the configuration of the DSGE model. This result does not only contribute to the possibility of using control theory. It also contributes to the way of looking at the central bank as a controller, and the logic in the working principles of the dynamics between the central bank and the economy it influences. Not only for monetary policymakers, but also publicly, it offers better insight. Instead of developing a new model, economists can research the implementation of this result in their DSGE model. This will open possibilities to simulate monetary policy actions differently, and closer to actual practices.

The decision to identify the central bank as the controller depends on which problem is addressed. As this is the first bond-graph model, not only of monetary policy but also of the economy, it is important to clearly define the modeling decisions. The same approach can be applied to a different type of economic models, such as tax policy. In general, this will create a better understanding of the role of economic agents and the effects of their actions.

The distinction between disturbances - i.e. shocks - and control input - i.e. policy actions, opens the possibility to simulate these separately. This contrasts the DSGE model, which simulates policy actions as if they can not be manipulated. In practice, they can. Even if economists prefer using their DSGE model, they can now add simulations of mitigating for undesired shock effects. An implication of this is the possibility that this will add to the risk analysis that is offered to monetary policymakers as decision-support. It enables awareness of risks of policy actions. Secondly, it increases insight on how to obtain the objective of maintaining price stability. This applies not only to monetary policymakers but also to the public.

Mechanical Analog for Money

This chapter answers the third research question:

What is the economic-engineering analog of money?

This theory builds on the existing economic-engineering analog from Mendel. The purpose of this theory is to make the analog applicable to model monetary policy as if it were a mechanical system. The chapter results in the extension from one to three conjugate pairs in the economic-engineering formalism.

4-1 Conjugate Pairs

In order to find the economic-engineering analog for money, a trial and error process was followed. This consisted of thinking about what a transaction is. Also, a lot of flow diagrams were made how the value of for example gold or a diamond would change. It included the way money could be exchanged for a good, for a service but also for time.

Like with all modeling, simplifications are required. The economy is seen as a whole. Hence, it was assumed that there is only one currency. Also, no difference between interest obtained for savings, for the demand deposit and no spread between savings and lending is assumed. Economics literature presents these assumptions or simplifications as well. Through the economic-engineering formalism in a further developed stadium, the simplifications can be undone by extending the theory. For example, in the future difference currencies can be implemented.

The study has to meet a few requirements:

- The dimensions, of the mechanical equivalent of the economic concept of money, need to fit in the existing framework of Mendel.

- The price of money needs to form a canonical conjugate pair, in a similar way goods and the price of goods does, with position and momentum.
- The dimensions, of the price of money, need to fit as well in the existing framework of Mendel.

The search is to find a conjugate pair that describes money, with price as its conjugate. The goal of this conjugate pair is to define the tetrahedron of state, in order to use this pair in bond graph modeling.

In physics, different conjugate pairs are described already. To determine the possible conjugate pairs that could fit in the framework and meet the requirements, the classical mechanic's literature, presented in Section 1-2-1, is read thoroughly. The uncertainty principle of Heisenberg presents possible conjugate pairs. In addition to classical mechanics, the work from Kobe [3], [26], [27] and [28] is read. Kobe wrote about an unconventional canonical coordinate pair. This pair is researched for its applicability for the economic-engineering analog.

The conjugate of money is researched, to decide which of the found possibilities will fit the mechanical analog. In order to find the conjugate of money, the meaning of an economy is re-analyzed. An economy is an area of the production, distribution, trade, and consumption of goods and services by different agents [5]. Economic agents can be individuals, businesses, organizations or governments. Economic transactions occur when two parties agree to the value or price of the transacted good or service expressed in money. The conjugate pair from the existing economic-engineering analog is price and quantity: A number of goods can be transacted for an amount of money that is expressed as the price.

The conjugate pairs are expressed in a similar way as Mendel; in terms of the generalized position, generalized momentum and its derivatives. This results in a tetrahedron that presents the relations between these four variables through three elements.

4-2 Canonical Transformations

The trial and error process consists of two approaches. Either, a flow diagram expresses an economic phenomenon to express it as a mechanical system or mechanical system simulations are tried to be matched with an economic phenomenon. An example of an economic situation was; a prehistoric situation in which only two hunters lived and either hunted or gathered. The mechanical variables; energy, momentum, force, velocity, position, were expressed in their economic analog. The change in the economic variables cash-flow, price, cost, quantity demanded and quantity were analyzed respectively. A couple of these process flows, tables and sketches are presented in Appendix A.

The trial and error process did not result in a final conclusion. Only intermediate results that helped in what worked and what did not.

The analog for the mechanical variables position and momentum into the economic variables; quantity and price, are presented in Table 4-1. These two analogs are taken from Mendel.

To identify the dimensions of the engineering analog to the economic concept of money, the dimensions of momentum and position are multiplied. This results in $\frac{kgm^2}{s}$ which is equal to Joule-second - Js . Table 4-1 presents this result, but the mechanical variable corresponding with the dimensions is missing.

Economics	Dim.	Mechanics	Dim.
Price	$\frac{\$}{\#}$	Momentum	$\frac{kgm}{s}$
Quantity	$\#$	Position	m
	$\downarrow$		
Money	$\$$	$\rightarrow$	$\frac{kgm^2}{s}$

Table 4-1: In order to find the mechanical translation for the economic variable money the price and quantity variables and corresponding dimensions from Table 2-5 are multiplied, similarly for the dimensions of the corresponding mechanical dimensions. Which results in the mechanical translation for the economic variable money having dimensions $\frac{kgm^2}{s}$

In physics, the joule-second describes the amount of action occurring in a physical system through a summation of energy over time. The joule-second is the unit of measure for action, angular momentum and the definition of Planck's constant. In classic mechanics, angular momentum describes the momentum of a rotating object. The Planck's constant is a physical constant that is the quantum of electromagnetic action, which relates the energy carried by a photon to its frequency. This constant is used in quantum mechanics.

Similar to a quantity with a price, money also has its price. If you need money, the price you pay is called interest. Interest is a payment from a borrower to a lender of an amount at a particular rate. The pair money-interest is similar to the pair quantity-price; a conjugate pair. Conjugate variables are pairs of variables mathematically defined in such a way that they become Fourier transform duals. In mathematical terms, conjugate variables are part of a symplectic basis. This symplectic basis ensures the conservation of energy when transforming from one conjugate variable pair to another. This transformation and corresponding conservation hold in bond-graph modeling. Therefore, it is important that the engineering equivalent for the economic concept of money is captured in a conjugate pair.

In physics, the momentum-position pair is a canonical conjugate pair. According to Heisenberg's [29] uncertainty principle there are more conjugate variable pairs. The angular momentum variable is conjugate to the orientation or angular position variable (angle). The energy variable is conjugate to the time variable (tempus). The orientation or angular position variable is measured in radians.

The action-angle coordinates can be expressed in a similar manner that Mendel used to express momentum-position in the economic-engineering analog (see Table 4-2). This results in the action variable being the money variable and the angle variable being the return or interest variable. The dimension analysis that translates the angle variable to the return variable results in dimensions %. Therefore, both the interest or a return can be measured. The energy and angular velocity variables are the derivatives of respectively action and angle. This results in the derivatives of money and the return, respectively cash flow and rate of return. A rate of return is similar to a rate of interest but applied to different economic

Mechanics	Sym.	Dim.	Economics	Sym.	Dim.
Position	q	m	Quantity	q	$\#$
Velocity	$\dot{q}$	$\frac{m}{s}$	Quantity Demanded	$\dot{q}$	$\frac{\#}{yr}$
Momentum	p	$\frac{kgm}{s}$	Price	p	$\frac{\$}{\#}$
Force	$\dot{p}$	$\frac{kgm}{s^2}$	Cost	$\dot{p}$	$\frac{\$}{\#yr}$

Table 4-2: Exact similar overview of the four mechanical variables translated into their economic analog. This table is repeated in order to get a complete overview of the possible coordinate configurations.

variables. A rate of return is applied to an investment where a rate of interest is applied to a loan or savings.

Mechanics	Sym.	Dim.	Economics	Sym.	Dim.
Action	J	Js	Money	M	$\$$
Energy	$\dot{J}$	J	Cash Flow	$\dot{M}$	$\frac{\$}{yr}$
Angle	ω	rad	Return, Interest	θ	$\%$
Angular Velocity	$\dot{\omega}$	$\frac{rad}{s}$	Rate of Return	$\dot{\theta}$	$\frac{\%}{yr}$

Table 4-3: The four mechanical variables for action, angle and their derivatives energy and angular velocity (symbol taken from [3]) translated to their economic analog: money, return and their derivatives cash flow and rate of return. Interest is similar to return, interest is used in terms of savings where return is used in terms of investments.

The energy-tempus coordinates can be expressed in a similar manner as the action-angle coordinates. This results in the energy variable being the cash flow variable and the tempus variable being the term variable. The derivatives of energy result in power, but the derivative of tempus is not defined in the used literature. The derivative of the cash flow results in growth. The derivative of the term results in a variable with dimensions $\%$. Therefore, it can be defined as the return, but also as the yield.

Mechanics	Sym.	Dim.	Economics	Sym.	Dim.
Tempus	T	s	Term	T	yr
-	$\dot{T}$	$-$	Return	$\dot{T}$	$\%$
Energy	E	J	Cash Flow	E	$\frac{\$}{yr}$
Power	$\dot{E}$	$\frac{J}{s}$	Growth	$\dot{E}$	$\frac{\$}{yr^2}$

Table 4-4: The four mechanical variables for tempus (from [3]) and energy, with only the derivative of energy, power. The mechanical variables are translated to their economic analog: term, cash flow and their derivatives return and growth. A term could be for example an agreed term in a mortgage contract or bank lending contract with an investor, or the term specified in a (coupon) bond.

The canonical transformation from the price-quantity/momentum-position pair to money-interest/action-angle and cash flow-term/energy-tempus results in an overview of three tetrahedron's of state as presented in Figure 5-2. The three tetrahedron's present the relations

of the four variables previously presented in subsection 2-3-1. The way a mass relates velocity and momentum, a cash-flow is related to return or interest. The way a spring relates a position and a force, money is related to a rate and a term is related to growth. The way a damper relates velocity and force, a cash-flow is related to a rate and return is related to growth.

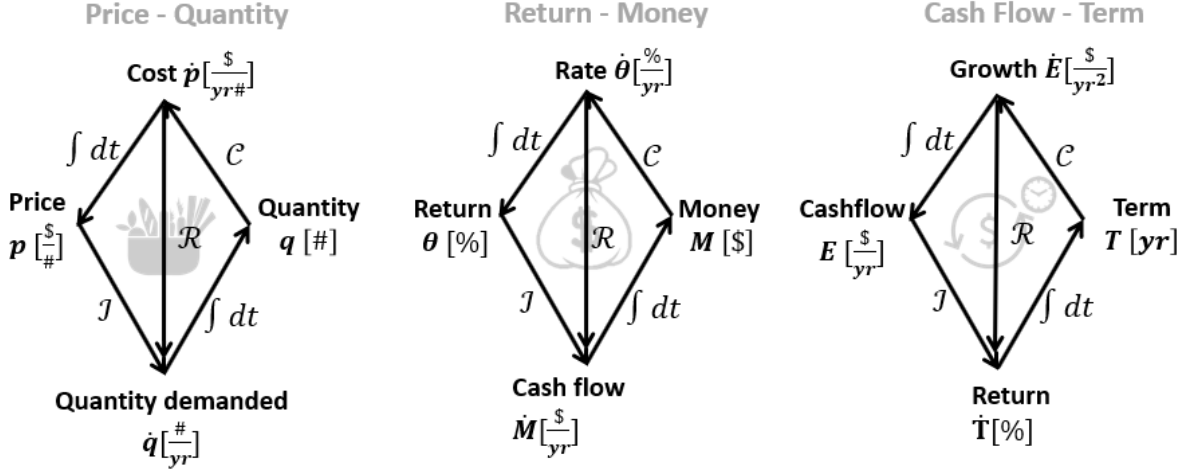


Figure 4-1: The economic interpretation of 3 canonical coordinate pairs: momentum-position, action-angle and energy-tempus. These three coordinate configurations enable the modelling of economic concepts in terms of quantity (#), money (\$) and time (yr) and their corresponding price (conjugate pair).

4-3 Discussion

The current study found that money \$ can best be expressed as action kgm^2/s , specifically in the conjugate pair action-angle that expresses money-rates. This outcome is contrary to that of Machado et al. [30], who found that money \$ can be expressed as energy kgm^2/s^2 . Machado bases his results on a study from Brewer [19], [20]. Brewer defines economic analog for the force as price per unit commodity and the economic analog for velocity as the flow of orders. This results in the flow of money by multiplying the price per unit commodity by the flow of orders. Hence, the flow of money is the economic analog of power and money is the economic analog of energy. This discrepancy could be attributed by the definition of the economic analog for the force. The economic analog of the force differs one time integral, i.e. Brewer defines the force economically with the dimensions $\$/\#$ and Mendel defines the force economically with the dimensions $\$/\#yr$. The definition of Brewer results in a limitation when wanting to express an economic variable with dimensions $\$/\#yr$ because the integral of force does not exist in physics.

The results in this section indicate that the canonical transformation from one conjugate pair to another enables the modeling of different economic phenomena using the two new introduced economic-engineering analogs or the existing analog depending on the cause and nature of the economic model. The next chapter moves on to discuss the applicability of the action-angle analog to apply to a mechanical model for monetary policy.

As mentioned in the preliminaries (Monetary Policy Section 2-2), the DSGE model contains variables labor, capital, consumer goods (or production of consumer goods). Expressing variables in the price-quantity economic-engineering analog will, as a flash-forward to the coming chapter, result in the expression of labor, capital, and goods as a quantity. When economic agents are interacting, these quantities need to be transformed using gyrators or transformers following the bond graph modeling methodology. This will result in an over-complicated model. The money-interest analog will simplify this, because labor, capital, and goods are now expressed in terms of money. This leads to an elegant model, which is simple as possible, has no extra complexities and is therefore preferred.

The other way around, the mathematics that is used in this chapter can now be applied. This contrasts the energy-tempus theory from Kobe, which has not found many applications so far. Similarly, for the action-angle coordinates. The applications to economics can exploit this mathematical theory in a way it has not found an application before.

4-4 Conclusions and Summary

The aim of this chapter was to examine the economic-engineering analog of money, in order to model monetary policy using mechanics. This chapter has identified action to be the mechanical analog for the economic variable money. Hence, when modeling monetary policy, the initial state variables action and angle need to be chosen. In contrast with common practice choosing position and momentum as initial state variables. Without choosing these initial state variables, the model needs a lot of transformations. In theory, this is possible, but it makes it more complex. As another important objective is the understanding of the model, extra complexity is not desirable. Choosing the initial state variables as action-angle is not a technical choice, but an elegant choice.

The economic-engineering theory is extended to express economic models using three initial state variables: momentum-position (old), action-angle (new) and energy-tempus (new). Now, economic models can be expressed in quantity (#), money (\$) or time (*yr*). This extends the palette of economic models that can be expressed using the economic-engineering formalism's. Overall, this study strengthens the idea behind economic engineering, that economics can be modeled as a mechanical system. The choice of the framework depends on the economic model. The momentum-position expresses an economic model in prices \$/# and quantities #, which suits, for example, a production model. The action-angle expresses an economic model in return or interest % and money \$, which suits for macroeconomic models and specifically monetary policy. The energy-tempus expresses an economic model in cash flow \$/*yr* and a term *yr*, which suits for liability-driven investment (LDI) models with bonds (the economic bonds). This suggests using action-angle as the coordinate pair to develop the model for monetary policy in the following chapter.

A limitation of this study is the proof or validity of this approach. Despite its exploratory nature, the study will be used in further research in a proof-of-concept which will add to the argument of using the action-angle pair in the economic-engineering analog for money-interest.

First-Principles LTI Model

This chapter answers the fourth research question:

How can the economy be modeled as if it were a mechanical system?

This chapter presents the development of a first-principles model of the economy. Hence, it serves as a proof-of-concept study of the economic-engineering first-principles technique to an economic model. The technique starts directly at the level of established laws of physics and does not make assumptions such as the empirical model and fitting parameters. Furthermore, the development of a model for the system that can be used by system and control engineers is the crucial next step towards the thesis objective. Section 5-1 motivates the modelling decisions. The following sections present the development of the model. The chapter results in a fourth order model that represents the economy. In this economy four economic agents transact with each other; the household, the firm, the entrepreneur and the investor.

5-1 Motivation

There are different modeling approaches that can be chosen to model the economy. Figure 5-1 presents the system - i.e. the economy - that is modelled based on the control formulation from chapter 3. The control input u is the interest rate R and the system output y is the inflation rate π .

Three different modeling methods are considered:

- **Black box identification:** A black-box system identification approach can be used to estimate the model. However, the data used in Smets-Wouters is a major source of uncertainty. They fail to present the exact figures. Nevertheless, data can be gathered from open sources. But even if the interest-rate data and the inflation-rate data is found, it is also unclear if this will render to a usable result. The identification of a



Figure 5-1: Representation of the system - i.e. the economy - with the input - the interest rate - and the output - the inflation rate, subsection of the complete control formulation presented in Figure 2-1

system, including both the economy and the central bank, can be interpreted as closed-loop system identification [31]. The limitation of this approach is that, in the specific case of modeling monetary policy, the policy actions as a result of the controller are key. Hence, if the controller cannot be identified separately from the system the policy actions cannot be modeled.

- **Re-use the equations from DSGE :** In order to re-use these equations all parameters, variables and other mathematical terms need to be interpreted and understood. Discussion sessions with Dr. R. Wouters resulted in more insight into the dimensions of the parameters, the explanation of variables and of the difference equations. The main limitation of this approach is that the underlying dynamics of the DSGE model are not understood due to the regression approach. The regression approach results in a predefined mathematical structure that does not offer insight into the economic relations of the defined variables. This causes a limitation in understanding the effects of the shocks. In engineering, the pre-defining of equations and estimating the corresponding parameters are called grey-box estimation. But engineers use this approach when the physical system is known. This is not the case with the economy. Therefore, this approach is not preferable.
- **Economic-Engineering Analog :** The economy can be modeled similar to how an engineer models a mechanical system. Instead of measuring the elements (i.e. masses, springs) with a physical mechanical system, the elements are identified using the economic-engineering approach. If this approach would result in a usable model, this would reveal the underlying mechanisms of the economy. This opens the door to apply the whole toolbox of system and control theory since all states will be known. The disturbance responses applied to all state variables, that are included in the model, can be plotted and a controller can be designed on all state variables.

The economic-engineering analog is the best choice for the development of a model of the economy. Instead of identifying masses and springs directly from a physical model, the economic-engineering analog is used. The development of this model will be a proof-of-concept of the economic-engineering analog to develop LTI-models. The benefits of this approach in comparison to the black-box, grey-box approach, and the existing DSGE model are the following. First, the model is based on formalism's and expressed using common mathematics, therefore the model can be understood by anyone with knowledge of basic calculus. Secondly, the

model can be divided into subsystems and extra systems can be developed that add to the model. These two benefits contrast the existing DSGE model the first and second options.

5-2 Interacting Agents

The steps presented in the following paragraphs are an afterward description that presents the development of the model logically. These steps do not represent the actual development during the research. In practice, the steps were: Trial and error, brainstorm sessions, experiments in MATLAB and Simulink, and discussions with monetary experts. Decisions were mostly based on failed attempts. A model try-out was tested in Simulink and the behavior was compared to process-flows and sketches. If this did not result in slightly similar behavior there was no other option than to look for other methods. Therefore, three subsections will describe the modeling process. The first subsection will translate economics to mechanics. The second subsection will describe the puzzle of connecting the elements into a model that makes sense. The third subsection will validate the model and applies a control-engineering analysis.

The development builds on the following assumptions: The study assumes that Mendel's work is correct and linearity of demand, factor demand and consumption are correct. Furthermore, the study assumes that households have no distinction in savings accounts and demand deposit accounts. Also, the ownership of houses by the households is not taken into account.

5-2-1 Economics to mechanics

Method

There is no strict formalism, in monetary policy models, to which variables are describing the behavior of the economy. For example, the DSGE model of the DNB [4] takes into account the variable capital K but the DSGE model of the US [1] does not. Therefore, the variables described in the DSGE model of the ECB (Smets-Wouters [2]) are the starting point to develop the model. Furthermore, in order to compare the developed model with the DSGE model, the model needs to contain most variables as presented in the DSGE model (Equations 31-37 [2]).

The variables and parameters, presented in the DSGE model, are presented in an overview. Then, the variables are interpreted according to the economic-engineering analog. For example, the wage can be interpreted as a cost and the dimensions from a cost are applied to the wage - i.e. $\$/\#yr$. After interpreting all variables the parameters from the DSGE model can be interpreted in a similar manner. The equations presented in the DSGE model are used. For example, the wage is identified as a cost and labor as a quantity. Then the parameter σ_L multiplied by labor in the wage equation will have dimensions $\frac{\$}{\#^2 yr}$. The parameters can be interpreted as the economic equivalent of either a mass, a spring, or a damper. Hence, demand, factor demand or consumption respectively.

In order to find the relations between the variables, pairs are formed. For example, wage and labor are coupled. Because a wage is a result of performed labor. The couples are put

in an overview of the DSGE model. This is done to analyze the transactions or interactions between economic agents. For example, the household can transact with the shop where consumer goods are sold.

A decision is made, which variables are included and which not. These variables will be put in an overview. This is done in combination with the economic agents. Instead of only analyzing Smets-Wouters' DSGE model, also other DSGE models and economic theory are taken into account. In order to choose the economic agents represented by the system, different DSGE models are compared.

Results

The variables of the DSGE model from Smets-Wouters are presented in the first column of Table 5-1. The second column presents the corresponding symbol taken from Smets-Wouters. The third and fourth column presents the interpreted engineering analog of the economic variable. The choices are substantiated briefly.

Variable	Symbol	Dimensions	
Consumption	C	$\#/yr$ or $\$/yr$	Generalized Flow
Cash balance	M	$\$$	Generalized Position
Labour supply	L	$\#/yr$ or $\$/yr$	Generalized Flow
Labour	l	$\#$ or $\$$	Generalized Position
Wage	w	$\$/\#yr$ or $\%/yr$	Generalized Effort
Rental rate of capital	r^k	$\$/\#yr$ or $\%/yr$	Generalized Effort
Capital	K	$\#$ or $\$$	Generalized Position
Investment	I	$\$/yr$	Generalized Flow
Depreciation rate	τ	$\%/yr$	Generalized Effort
Return on equity	Q	$\%$	Generalized Momentum
Final goods price	P	$\$/\#$	Generalized Momentum
Final goods production	Y	$\#/yr$ or $\$/yr$	Generalized Flow
Intermediate goods production	y	$\#/yr$ or $\$/yr$	Generalized Flow
Governmental spending	G	$\$/yr$	Generalized Flow
Interest rate	R	$\%/yr$	Generalized Effort

Table 5-1: Overview of the variables taken from the DSGE model (Smets-Wouters) and the matched engineering analog presented in the generalized variable with its corresponding dimensions. For consumption both $\#/yr$ and $\$/yr$ can express the dimension of this variable which both corresponds to a generalized flow.

Five variables are interpreted analog to a generalized flow. Consumption is defined as using up a resource for the acquisition of utility [16]. Therefore, it is interpreted as a generalized flow variable. This corresponds to dimensions $\#/yr$. Nevertheless, economists measure consumption mostly as consumption expenditure in dimensions $\$/yr$. The action-angle framework makes it possible to model in this dimension. If applicable, column three presents both $\#/yr$ and $\$/yr$. Generalized flow is also interpreted analog to labor supply, investment, final and intermediate goods production and governmental spending. In the case of labor, the quantity

is a person. In terms of production, the labor supply is the supply of manpower over time that contributes to production. In the case of the final and intermediate goods production, the quantity represents produced stock. In the case of investment and government spending, it could be measured in terms of assets or stock but similarly to consumption it is measured in expenditure.

Three variables are interpreted analog to a generalized position: The cash balance, labor, and capital variable. These variables are not 'flowing' but express a stock or amount. Cash can only be expressed in \$, but labor and capital can be expressed in both # and \$.

Two variables are interpreted analog to a generalized momentum: Return on equity and final goods price. Since the general interpretation of generalized momentum is price, it is straightforward that final goods price is analog to a generalized momentum. Return on equity describes the price of a stock. If the stock price increases, this can be measured as a rate of return on equity. Therefore, the return on equity is interpreted as generalized momentum.

Four variables are interpreted analog to a generalized effort: The wage, the rental rate of capital, depreciation rate, and interest rate. The wage and the rental rate of capital are both measured in an amount per quantity - person or capital - per amount of time. Hence, \$/#yr which corresponds to a cost in the economic-engineering analog. In the action-angle framework, a rate is analog to a generalized effort. Hence, the interpretation of the depreciation rate and the interest rate as generalized efforts.

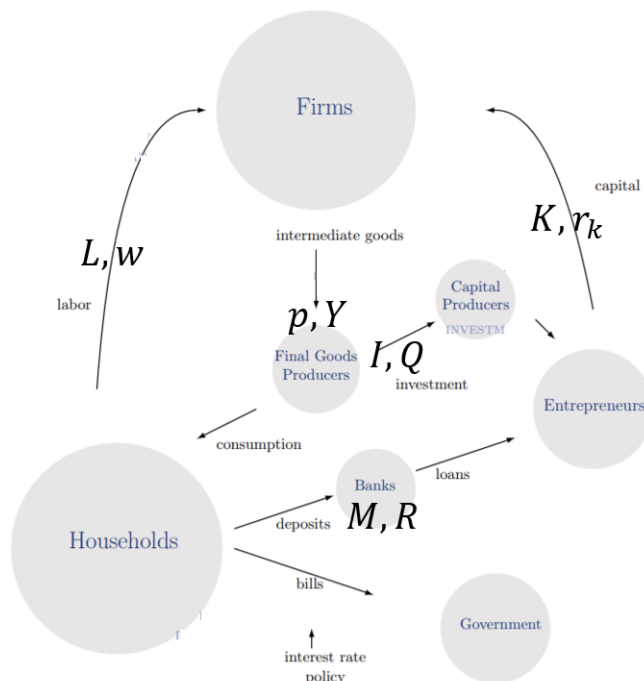


Figure 5-2: The same model structure from a DSGE model [1] as Figure 2-1, added in black the variable couples defined in this section matched with the corresponding variables and agents from the DSGE model.

In order to structure these variables, couples are formed based on a conjugate relation. This is done according to the following principle: A consumer good has a price, labor has a price, capital has a price, a stock has a price and money has a price. Therefore, labor L is coupled to wage w , capital K is coupled to rental-rate of capital r^k , cash balance M to interest rate R , investment I is coupled to return on equity Q , final goods price p is coupled to final goods production Y . Notice how labor, capital, and cash balance are interpreted as a generalized position which couple to wage, the rental rate of capital and interest rate, interpreted as generalized effort. Investment and production of final goods are interpreted as a generalized flow which couple to final goods price and return on equity, interpreted as generalized momentum. The couples are presented in an overview, see Figure 5-2.

The following variables are not coupled: Consumption, depreciation rate (of capital), intermediate goods production and governmental spending. To simplify the model, governmental spending is eliminated. The government could be interpreted as a separate controller as they control tax policy. In this model tax and government spending are not taken into account. Similarly, intermediate goods production is eliminated. To simplify one type of goods is taken instead of the distinction between final and intermediate goods. Consumption and depreciation rates are dissipating variables. The difference between labor or capital is that it dissipates. Therefore, these two variables are related to a damper - i.e. an $\mathcal{R}$ element.

The couples from Figure 5-2 are related through either a $\mathcal{I}$ -element, a $\mathcal{C}$ -element or a $\mathcal{R}$ -element. This depends on if the relationship is either capturing demand, factor demand or a type of consumption. Factor demand, a $\mathcal{C}$ -element, describes the relationship between labor and wage. Mechanically, this is the relation between a position - i.e. labor in economics - and a force - i.e. wage in economics. Hence, the relation is described by a spring - i.e. a $\mathcal{C}$ -element. Similarly, a $\mathcal{C}$ -element, describes the relationship between capital and rent of capital. Capital is also mechanically interpreted as a position, and rent of capital in a force. Demand, a $\mathcal{I}$ -element, describes the relationship between price and production of goods (or output). Mechanically, this is the relation between velocity - i.e. production of goods in economics - and momentum - i.e. the price of goods in economics. Hence, the relation is described by a mass - i.e. a $\mathcal{I}$ -element. Similarly, a $\mathcal{R}$ -element describes the relation between investment and return on equity. Investment is also mechanically interpreted as velocity, and return on equity as momentum.



Figure 5-3: The four identified elements that describe the relationship between eight system variables. From left to right: The relation between labor and wage, the relation between the price of goods and output (production), the relation between rental of capital and capital and the relation between the price of stocks and investment.

Figure 5-3 presents the four bond graph elements that capture the four relations of variable pairs. Each element is connected to an arrow. The arrow contains an effort, a flow, and a storage variable. As is explained in Section 2-3-1. $\mathcal{C}_1$ describes the relationship between labor and the wage. Similar to a spring in mechanics storing a position, the $\mathcal{C}_1$ element

stores labor. The corresponding flow variable is the labor supply. $\mathcal{C}_3$ describes the relation between capital and the rental rate of capital - i.e. rent. $\mathcal{C}_3$ stores capital. $\mathcal{I}_2$ describes the relationship between the price of goods and output - i.e. production of goods. Similar to a mass in mechanics storing momentum, the $\mathcal{I}_2$ element stores the price of goods. The corresponding effort variable is inflation. $\mathcal{I}_4$ describes the relationship between the price of stocks and investment. $\mathcal{I}_4$ stores the price of stocks.

5-2-2 Model configuration

Method

The four identified elements are connected to form a model. To do this, a trial-and-error process of model configurations is executed. The bond-graph-modeling approach offers a simple solution to try different possibilities. Because of the neutral confirmation, representing a model with momentum-position or action-angle as the initial framework, it results in the same bond-graph representation. The translation to a real mechanical - i.e. physical - model is difficult. This is difficult because the action-angle initial coordinate framework has a different translation than the momentum-position initial coordinate framework.

The variables will be connected to the economic agents. For example, consumption is connected to the household. In this way, each agent will be matched with corresponding variables, and hence, corresponding elements.

The actual model is build using a bond graph method. The elements are connected at a junction - i.e. the multi-port bond graph element - that will represent the transactions for one agent. The different agents - i.e. subsystems - are connected and together this forms the model. The model is presented in both a bond graph and differential equations.

Results

A possible interaction of the elements from Figure 5-3 is presented in Figure 5-4. A $\mathcal{C}$ element is always connected to a 0-junction. A $\mathcal{I}$ element is always connected to a 1-junction. Goods are added to the three generalized position variables: Labor, capital and money. Due to the action-angle transformation, these four variables can be expressed in terms of money \$. Without this transformation, labor, goods, and capital would be expressed in terms of a quantity #. In order to exchange one quantity to another, a 2-port element must transform the quantity of labor to a quantity of money.

The configuration of the four variables, goods, labor, capital, and money, interacting according to Figure 5-4 does not render results. This example failed because two springs were attached to two masses. In this way, the two springs and the two masses would behave as if they were one spring and one mass. This results in the behavior of a mass-spring system. The bond-graph modeling approach enables experimenting with different model configurations because of the conservation of the symplectic space, the mechanical translation needs to make sense. The lesson learned in this attempt is that in order to capture the behavior of different $\mathcal{C}$ and $\mathcal{I}$ elements they need to be attached in a specific order.

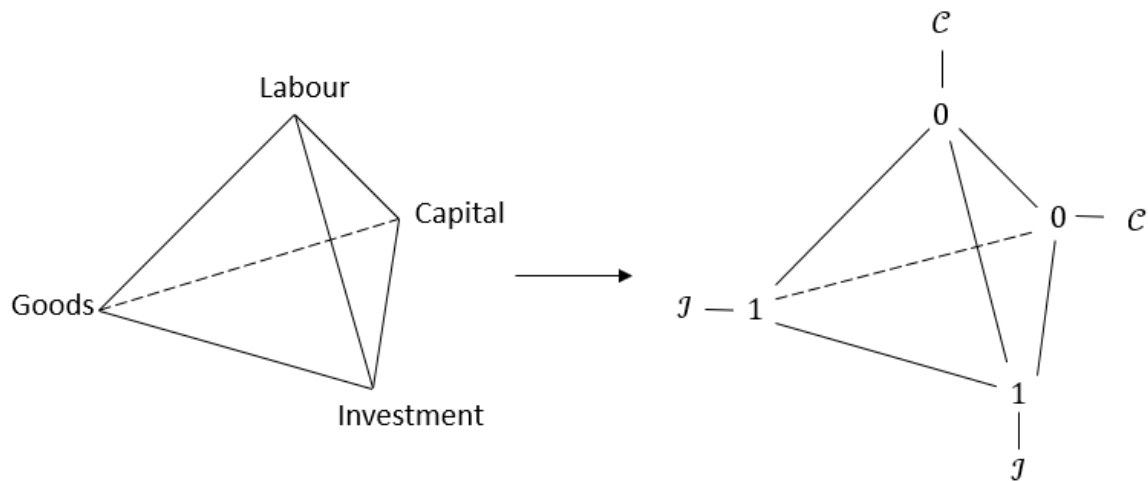


Figure 5-4: Possible model configuration of the four elements from Figure 5-3 attached in a tetrahedron (3D). On the left the four 'generalized position' variables that can transact into one another, on the right the translation into a bond graph model.

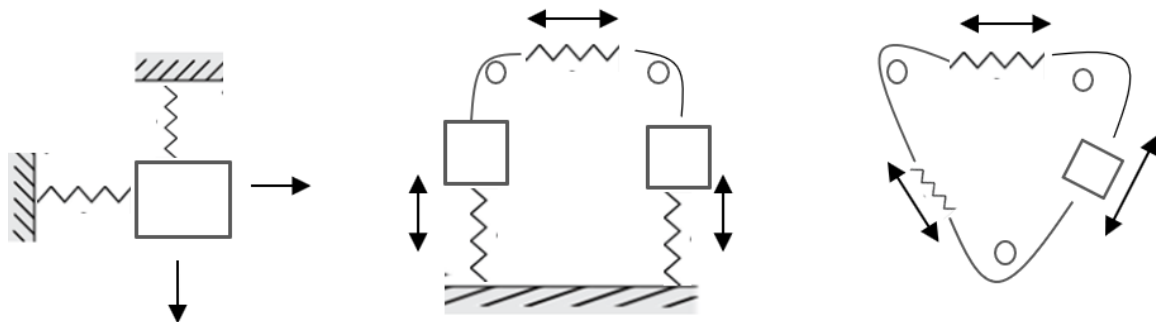


Figure 5-5: Possible mechanical systems containing of masses and springs. The dynamical behavior is modelled and matched with possible behavior of economic variables.

Instead of working directly with the bond-graph representation, also mechanical systems are tried. Figure 5-5 presents three possible mechanical confirmations. On the left, one mass attached to two springs presents the effect of more degrees of freedom. From this confirmation, it is learned that more degrees of freedom quickly increase in complex dynamics. This is not preferable since the results from Smets-Wouters do not show high complexity. In the middle, two springs are attached to a wall, both attached to a mass, these masses are attached to a spring including two rotations. The rotation enables the measurement of a position in different directions. This makes it possible to express the different generalized positions in one model. From this confirmation, it is learned that a decision is to be made of which generalized position is attached to a wall. This decision is not trivial since the behavior of the corresponding generalized position variable is affected. Is labor attached to a wall and

capital not, or the other way around? To avoid this, the right confirmation is not attached to a wall at all. This corresponds to what is often seen in economics: A circular flow. The work of Cantillon and Quesnay [32] is used as a reference. Unfortunately, this did not result in a working model. Somewhere in the model, the interaction between two generalized positions should be eliminated.

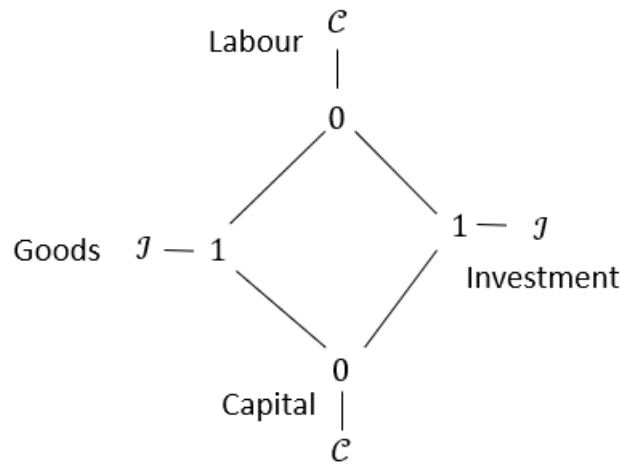


Figure 5-6: Possible bond graph configuration of the four identified elements from Figure 5-3.

The variables and corresponding elements are related to an economic agent. From the 7 economic agents presented in Figure 5-2, 2 are eliminated. The government and the final goods producer. Because of the simplification mentioned in the previous paragraph. Furthermore, for now, the bank is also eliminated. This is because the bank will be represented by the controller, which is modeled separately from the system. Table 5-2 presents the variables connected to the economic agents.

Household	Firm	Entrepreneur	Investor
Consumption	Production of final goods	Rental rate of capital	Investment
Labour	Final goods price	Capital	Return on equity
Labour supply		Depreciation Rate	
Wage			

Table 5-2: Overview of the four agents chosen to be in the first-principles model taken from the DSGE model, and the corresponding economic variables.

The variables are matched with the economic agents. The household demands consumption and supplies labor. Therefore, the variables: consumption, labor, labor supply, and wage are connected to the household. The firm supplies (final) goods, corresponding with the production of final goods and the price variables. The entrepreneur supplies capital, but this capital depreciates. Therefore, the variables: rental rate of capital, capital and depreciation rate are connected to the household. Finally, the investor 'produces' investment, corresponding with the investment and return on equity variable.

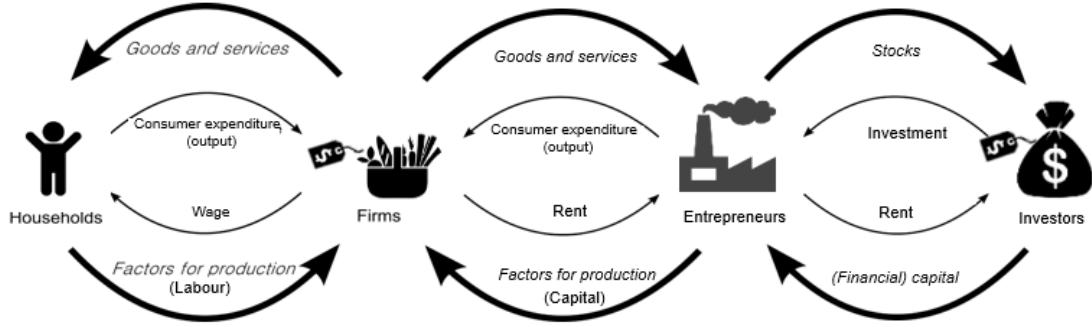


Figure 5-7: The circular flow of transactions between the four identified economic agents. The balloons from 5-2 convert to the agents in this figure and the corresponding economic variables that express the transactions between the agents.

The Final Model

The following logic is applied to obtain the final model: In order to supply labor as a household, the household demands consumption goods. In order to supply these consumption goods, the firm demands both labor and capital to produce these goods. This completes the cycle of labor demand and supply. In order to supply capital, the entrepreneur demands both consumption goods and investment. This completes the cycle of consumption goods and capital demand and supply. In order to supply investment, the investor demands capital. This completes the cycle of investment demand and supply. These four cycles of labor, consumption goods, capital, and investment together form the logic behind the bond-graph model presented in Figure 5-8.

Each agent obtains an $\mathcal{R}$ element. For the household and the entrepreneur, the consumption and depreciation variable is captured in the corresponding $\mathcal{R}_1$ and $\mathcal{R}_3$ element. For the firm and the investor, $\mathcal{R}_2$ and $\mathcal{R}_4$ represent dissipation. Both the firm and the investor transact goods and stocks respectively. The $\mathcal{R}$ element captures the dissipation due to transaction costs, overhead, and other costs that are not directly connected to labor and capital.

The bond graph model can be converted into state space representation. With:

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (5-1)$$

Where x_1 is the labour variable, x_2 is the goods variable, x_3 is the capital variable and x_4 is the equity/stock variable. $\mathbf{u}$ is the input variable. This input variable is the sum of the control input - i.e. interest rate R - and disturbances - i.e. shocks ε and η .

$$\dot{x} = \begin{bmatrix} -\mathcal{R}_1\mathcal{C}_1 & \mathcal{I}_2 & 0 & 0 \\ -\mathcal{C}_1 & -\mathcal{R}_2\mathcal{I}_2 & \mathcal{C}_3 & 0 \\ 0 & -\mathcal{I}_2 & -\mathcal{R}_3\mathcal{C}_3 & \mathcal{I}_4 \\ 0 & 0 & -\mathcal{C}_3 & -\mathcal{R}_4\mathcal{I}_4 \end{bmatrix} x + Bu \quad (5-2)$$

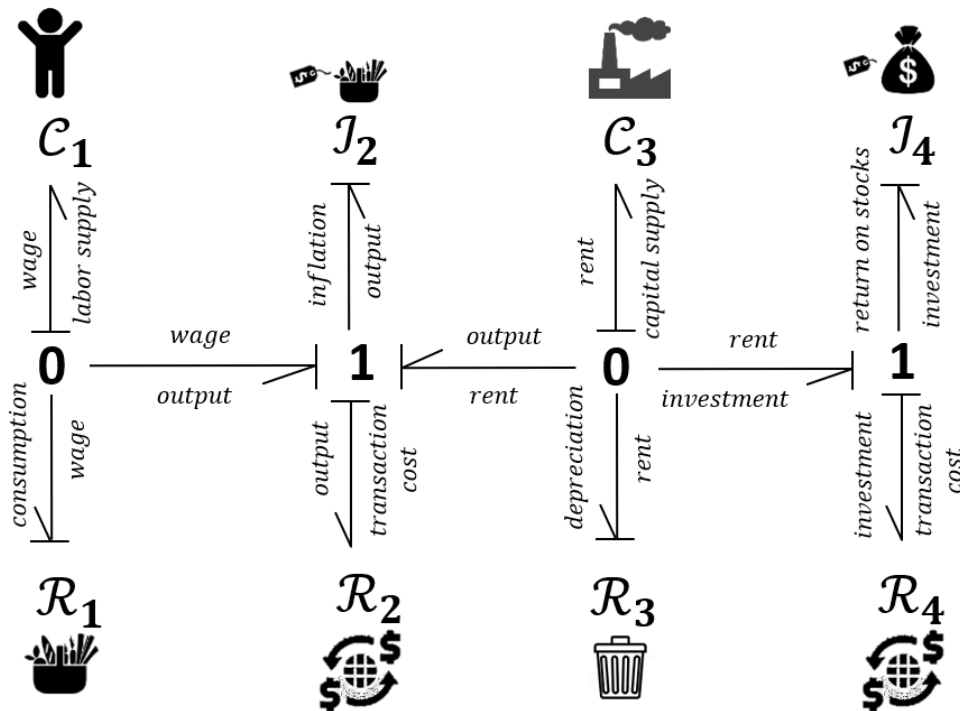


Figure 5-8: Complete Bond Graph macro economic model consisting of 4 separate subsystems. The transactions between agents expressed in Figure 5-7 and their corresponding variables translates to the bond-graph model. Each junction with two-elements attached represent an agent. The dissipation of each agent is represented by an individual $\mathcal{R}$ element.

5-2-3 Model Validation

Method

The first-principles model is compared to the DSGE model. The comparison is based on system variables. The variables included in the first-principles model are compared with the DSGE output variables - i.e. variables presented in their results.

In order to simulate results, parameter values need to be chosen. The parameters from the first-principles model are matched with the parameters presented in Smets-Wouters. A dimension analysis is done on the equations from Smets-Wouters to identify the dimensions of the parameters in the economic-engineering framework. With the obtained dimensions a match is done with the first-principles-model parameters. From the parameters, the eigenvalues of the first principles model can be deduced. Changing the parameters will influence the value of the eigenvalues. This is analyzed.

The value of the parameters also influences stability, controllability, and observability of the system. This is analyzed together with the economic interpretation of this effect.

Smets-Wouters describes the utility function, budget constraint, and concept of marginal cost. Their paper does describe capturing these economic concepts in their DSGE model.

The effectiveness of the first-principles model is tested to see if these economic concepts are captured and can be identified.

To compare the results of system responses, first, the shocks need to be identified as inputs on the system state variables. This will be presented in the next chapter.

Results

The first-principles model describes the dynamical behavior of sixteen variables. These sixteen variables are presented in Table 5-3. The second column presents the four state variables (x): Labour, price of goods, capital and price of stocks. Compared to the DSGE model, the price of stocks is not defined. The first column presents the derivatives of the four state variables ($\dot{x}$): Labour supply, inflation, capital supply and return on stocks. Compared to the DSGE model, labor supply is not defined separately from labor. Also, the price-of-stocks variable is not defined separately from the return on stocks. The third column presents the variables related to the state variables through the $\mathcal{C}$ and $\mathcal{I}$ elements: wage, output (i.e. production of goods), rent on capital and investment. These are all included in the DSGE model. The fourth column presents the dissipating variables related by the $\mathcal{R}$ element: consumption, transaction cost, and depreciation. The DSGE model does not include depreciation and transaction costs in their results.

$\dot{x}$	x	$\rightarrow \mathcal{I} \text{ or } \mathcal{C}$	$\rightarrow \mathcal{R}$
Labour supply $\dot{L}$	Labour L	Wage w	Consumption C
Inflation π	Price of goods p	Output Y	Transaction Costs t_g
Capital supply $\dot{K}$	Capital K	Rental on Capital r_k	Depreciation δ
Return on Stocks Q	Price of stocks p_s	Investment I	Transaction Costs t_g

Table 5-3: System variables of the first-principles model. x present the state variables and $\dot{x}$ the integral of the state variables, the other two columns are obtained by multiplication of the state variables with the system parameters.

Compared to the DSGE model, the first-principles model captures four more variables. Furthermore, it is able to plot all of these sixteen variables. Where the DSGE model plots twelve variables. The other way around, the DSGE model distinguishes between labor and employment, and capital and its utilization rate. This is not distinguished in the first-principles model.

The parameter values are chosen based on the parameters presented in the DSGE model. In the DSGE model, the wage equation (2-12) relates the wage variable by the labor variable through the parameter σ_L . Therefore, the parameter must have dimensions $\frac{\$yr}{\%}$ in the economic-engineering framework. Hence, it is identified as $\mathcal{C}_1$, relating wage and labor of the household in the first-principles model. The consumption equation (2-6) relates consumption and the nominal inflation rate (corrected by the interest rate) through the parameter σ_c . Therefore, the parameter must have dimensions $\frac{\$}{\%}$. Hence, it is identified as $\mathcal{R}_1$, relating inflation and consumption of the household in the first-principles model. Using this approach, $\mathcal{I}_2$, $\mathcal{C}_3$, $\mathcal{R}_3$ and $\mathcal{I}_4$ are identified as the parameters γ_p , ψ , τ and σ_m .

New Model	Dim.	DSGE	Value DSGE
$\mathcal{C}_1$	$\frac{\$yr}{\%}$	σ_L	0.4
$\mathcal{R}_1$	$\frac{\$}{\%}$	σ_c	0.6
$\mathcal{I}_2$	$\frac{\$}{\%yr}$	γ_p	1.4
$\mathcal{R}_2$	$\frac{\$}{\%}$	$? \rightarrow$	1
$\mathcal{C}_3$	$\frac{\$yr}{\%}$	ψ	0.93
$\mathcal{R}_3$	$\frac{\$}{\%}$	τ	0.1
$\mathcal{I}_4$	$\frac{\$}{\%yr}$	σ_m	0.5
$\mathcal{R}_4$	$\frac{\$}{\%}$	$? \rightarrow$	1

Table 5-4: System parameters of the first-principles model interpreted as parameters from the DSGE model, two parameters are not identified.

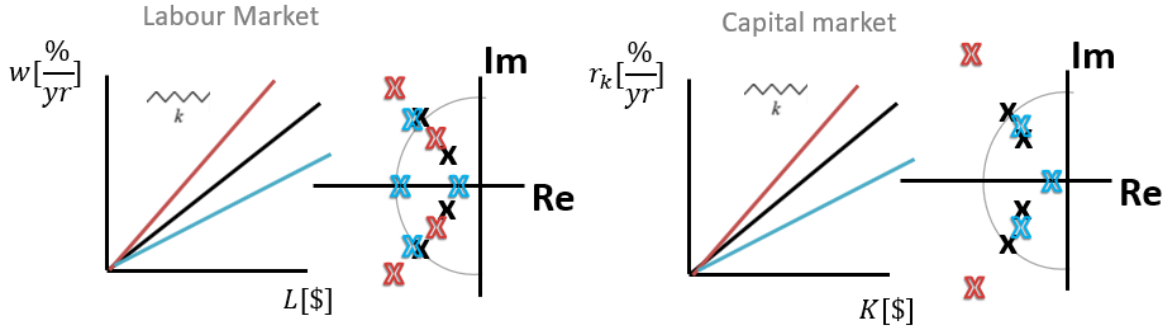


Figure 5-9: Changing eigenvalues when changing the parameters $\mathcal{C}_1$ and $\mathcal{C}_3$, expressing the relation between wage and labour, and rental on capital and capital respectively.

The $\mathcal{R}_2$ and $\mathcal{R}_4$ cannot be interpreted as a parameter from the DSGE model. Since the DSGE model does not present transaction costs in their model. Therefore, these parameters obtain a value of 1.

The parameter values from Table 5-4 result in the following, stable, eigenvalues:

$$\lambda_1 = -0.6821 \pm 1.3029i \quad \lambda_2 = -0.3967 \pm 0.2908i \quad (5-3)$$

Changing the parameter values results in different eigenvalues. Figure 5-9 presents the effect on the eigenvalues when changing $\mathcal{C}_1$ and $\mathcal{C}_3$. Increasing the $\mathcal{C}_1$ parameter results in an increase in the eigenvalue, both imaginary and real. When translating the imaginary part to business cycle behavior and the (negative) real part to declining trends, this means that a higher $\mathcal{C}_1$ will result in more volatile business cycle behavior. For the capital market, $\mathcal{C}_3$, this effect is even larger. Decreasing both the $\mathcal{C}_1$ and $\mathcal{C}_3$ results in a lower to zero imaginary part of the eigenvalue, which will result in a less declining trend.

Figure 5-10 presents the effects on the eigenvalues when changing $\mathcal{I}_2$ and $\mathcal{I}_4$. Increasing the $\mathcal{I}_2$ parameter results in zero imaginary part of the eigenvalue but a larger (negative) real part. This will result in a steeper trend downwards. Decreasing the $\mathcal{I}_2$ parameter results in a smaller real part. This will cause a less declining trend. Increasing the $\mathcal{I}_4$ parameter results

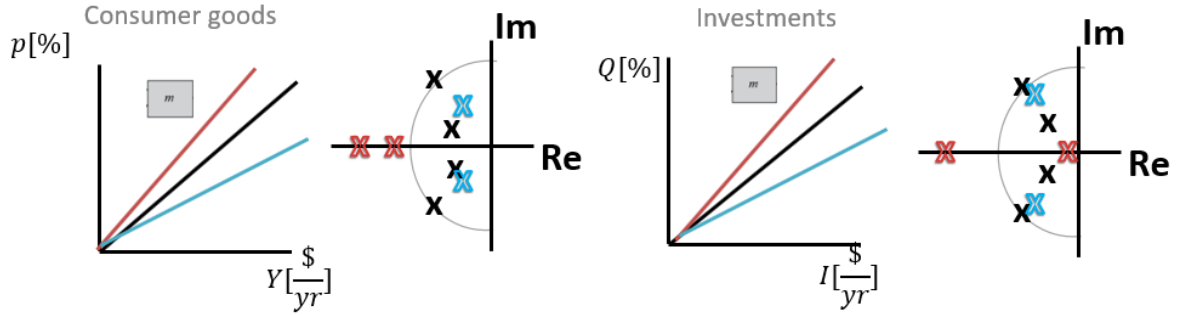


Figure 5-10: Changing eigenvalues when changing the parameters $\mathcal{I}_2$ and $\mathcal{I}_4$, expressing the relation between price and output (production), and price of stocks and investment respectively.

in zero imaginary part, so no business cycles, and one large real part and one close to zero. Decreasing $\mathcal{I}_4$ parameter results in a slight more decreasing trend.

Based on the results presented in Figure 5-9 and 5-10 the system is always stable. Also, controllability can be checked. In terms of controllability, first, the B matrix is defined in 5-4. Different types of input signals can be applied to the system, to each state variable. When taking all combinations of the variables d_1 , d_2 , d_3 and d_4 either 0 or 1, the system would always be controllable. When changing the eight parameters all to 0 the system would not always be controllable. If $\mathcal{I}_2$ is equal to 0, if $\mathcal{C}_3$ is equal to 0 or if $\mathcal{I}_4$ is equal to 0. Economically, this means that if consumer good prices, rental on capital or stock prices are fixed, the central bank cannot define an interest rate signal that will transfer any state of the system to any other state in finite time. Hence, the central bank does not have the tools to manipulate all state variables anymore.

$$B = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix} \quad (5-4)$$

The first-principles contains five economic concepts that are also described in the Smets-Wouters paper and included in the DSGE model.

Smets-Wouters defines the economic concept of marginal cost (MC) as follows:

$$MC = r_k^\alpha w^{1-\alpha} \quad (5-5)$$

$$\alpha = \frac{w + r_k}{w} \quad (5-6)$$

First, the marginal cost is defined in Smets-Wouters. This marginal cost influences the profits of the firm. In the first-principles model, the concept of the α in the marginal cost equation can be found in the ratio between the total cost and the wage costs - i.e. the ratio between the total force on the mass of the firm and the force from the spring of the household. This

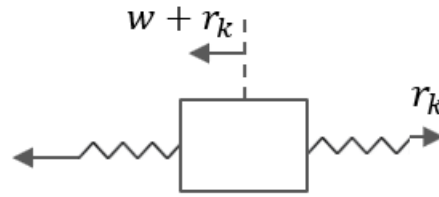


Figure 5-11: Mechanical representation of the economic concept of the marginal cost described by a mass, with two springs applying a force on this mass. The ratio between the force from one spring and the net force represents the marginal cost.

is presented in Figure 5-11. This value is always in between $0 < \alpha < 1$, similar to the theory of Cobb-Douglas and Solow-Swan [6].

Second, Smets-Wouters describes the budget constraint for the household. The 0-junction connects the elements for the household and the entrepreneur. The 0-junction sums the cash-flows while keeping the rates constant. This is interpreted as the budget constraint. A budget cannot be exceeded, similar to the rule of the sum of cash-flows. According to the bond-graph model, the budget constraint of the household is the income of labor minus the expenditures of consumption. The net of this equation flows into the $\mathcal{C}_1$ -element of the household. This describes the labor supply, but can also be interpreted as the financial wealth. This corresponds to the budget constraint equation of Smets-Wouters, except for the investment. This is due to the distinction of the household and the entrepreneur, in which the household consumes and the entrepreneur invests. According to the bond-graph model, the budget constraint of the entrepreneur is the income of capital (rent on capital), minus expenditures on investment and depreciation. The net of this equation flows into the $\mathcal{C}_1$ -element of the entrepreneur. This describes the capital supply, which is interpreted as an additional investment. Smets-Wouters doesn't describe a budget constraint for the entrepreneur. The research of the DNB [33] does describe a distinctive budget constraint for an entrepreneur. This budget constraint also contains labor and consumption.

Third, Smets-Wouters describes a profit function for the firm. The 1-junction connects the elements for the firm and the investor. The 1-junction sums the rates while keeping the cash-flows constant. These rates represent costs. Therefore, this is interpreted as the iso-cost line. The iso-cost line shows the costs which result in production output. According to the bond-graph model, the iso-cost line of the firm is the sum of rent, wages and transaction costs. The net of these costs results in inflation. The change in costs results in the change in prices of goods - i.e. inflation(or deflation). Similarly, for the investor, the iso-cost line includes rent and transaction costs.

Fourth, Smets-Wouters defines a utility function for the household. In Smets-Wouters this utility function is defined as a function of labor, money, and goods. According to the research of Ir. R. Smit [14] the utility function in economics can be interpreted as the Lagrangian in mechanics. Instead of forces, Lagrangian mechanics use the energies in the system. The central quantity of Lagrangian mechanics is the Lagrangian, a function that summarizes the dynamics of the entire system. Overall, the Lagrangian has units of energy, but no single expression for all physical systems. Any function which generates the correct equations of

motion, in agreement with physical laws, can be taken as a Lagrangian. A general definition of the Lagrangian is the kinetic energy minus the potential energy. According to the research of Ir. C. Hutter [15], a third term can be added in the Lagrangian expressing the dissipated energy. In economic engineering, this term is interpreted as consumption.

5-3 Discussion

With the first-principles model of the economy, all mathematical operations can be applied that can also be applied to a first-principles model of a physical system. This appears from the application of control characteristics such as the eigenvalue, stability, observability, and controllability. Also, the model can be represented in bond-graph, ODE's, state-space representation, whichever fits its purpose. This cannot be done with the DSGE model and their difference equations. Economists might not be interested in eigenvalues yet. But when the characteristics are interpreted in the economic domain, they might see the added value.

The first-principles model consists of four subsystems, each representing a different agent. This makes the understanding of the model more clear. Also, the subsystems can be used separately, and the system can be extended and dismantled by decoupling the subsystems. When doing this in the bond-graph formation, the corresponding ODE's will follow automatically. For example for the subsystem of the household, the ownership of a house can be added. Because of the bond-graph model, this is possible by decoupling this subsystem, researching and developing an extension, and attach the subsystem back to the original model. In this way, the first-principles model can become as complex as necessary, without the need for the development of a new model.

The logic behind the first-principles model, capturing the cycles of demand and supply, gives the first-principles model more ground than the DSGE model. The DSGE model presents their difference equations but does not present the logic behind these equations. When looking at the ten difference equations that together form the DSGE model, it is not possible to get a feeling of the working principles of the model. In contrast, the first-principles model offers this insight by the representation of the bond-graph model. Through this representation, the working principles of supply and demand of the system variables can be obtained.

The capturing of the economic concepts of marginal cost, the utility function, the budget constraint, the profit function, and the iso-cost line, indicates the correct modeling choices. It is not a proper validation method, but it argues for correctness.

The first-principles model can be presented by a 4-by-4 matrix that consists of 8 parameters. In comparison to the DSGE model, this is a very simple configuration. The DSGE model consists of 10 very long difference equations, containing more than 16 parameters. The first-principles model is built as simple as possible, while still capturing the most important variables and economic concepts.

The first-principles model presented in this chapter contains eight parameters. When modeling a 'real' mechanical system, the values of the parameters can be obtained through measurements. For example, a mass can be weight and the spring-coefficient can be estimated

through experiments. With this model of the economy, this is not possible. These findings suggest that the first-principles model is a grey box model. But instead of estimating the values of the parameters, the parameters of the DSGE [2] are taken, with the exception of $\mathcal{R}_2$ and $\mathcal{R}_4$.

The prior study to the DSGE model has noted the importance of defining all variables and parameters and their dimensions. It is found that in the DSGE model this is not the case. A possible explanation for this might be that the economic variables and parameters used in the model are common and their dimensions common knowledge. Another possible explanation for this is that it is not so important for the methods used in this model.

One interesting finding is that a cyclic model does not work. A note of caution is due here since there might be other possibilities. For this result, it means that there is a strict distinction between households and entrepreneurs. One person could be a household and an entrepreneur but their actions are modeled separately. As there are no other similar models to compare this model with, it is difficult to argue its validity.

The model could also be developed in discrete time. As economic data is mostly discrete, this would be a straightforward modeling choice. There are a couple of reasons against this. It is the first approach to modeling the economy as a mechanical model, and the central bank as a controller. Therefore, a continuous model appeared simpler. Furthermore, the dynamics of the economy are continuous, independent of the data. Since this model does not need any data, the model is not depending on it. The results presented in Smets-Wouters are also continuous and can be compared with the continuous results of the first-principles model.

5-4 Conclusions and Summary

The aim of this chapter was to develop a first-principles model of the economy that represents the system in a system and control formulation of the monetary policy problem. This first-principles model of the economy is modeled using the study resulting in the extended economic-engineering analog (presented in Chapter 3). This chapter presented the development of the first first-principles model of an economy. So far, it has not been done before. Therefore, it also serves as a proof-of-concept study.

The chosen subset, that represents the economy with all simplifications and assumptions, can be modeled as a 4th order system consisting of four 1st order subsystems each representing an agent that interacts with the other agents - i.e. subsystems. This is the first economic model that is build-up using this approach. Because of the first-principles approach, it is also the first economic model that consists of solvable differential equations as a result of capturing the economic relations of economic variables in formalisms. This contrasts the current approach, as used in the DSGE [2], that applies an auto-regression approach to estimate difference equations using data that are predefined by the authors. But the origin of these difference equations is not explained, therefore they cannot be understood by other academics except for the developers of the model and academics specialized in the field of monetary policy. The first-principles model presented in this chapter can be understood, replicated, extended by anyone with basic knowledge of calculus. Furthermore, the model can also be questioned

by anyone with a basic knowledge of calculus. The modeling decisions resulting in the differential equations that describe the model are explained and backed-up by 3 simple formalism's.

With this model all mathematical operations can be applied that can be applied to any physics or mechanics model. As the field of engineering is of a more advanced level, economics can also be brought to a more advanced level. It opens up the possibilities to model and control economic phenomena in the same way as engineers do. This would for one make economics more understandable, accessible by anyone who wants to, as it also influences everyone on a daily basis.

This study was limited by the absence of adequate validation. In spite of this limitation, the study certainly adds to our understanding of modeling of economics using engineering practices in general. The simplicity of the model makes the model more accessible to all sorts of academics to understand and to work with, not only economics in the field of monetary policy. Specifically, with the purpose to increase the understanding of monetary policy decisions and its risks. Furthermore, the model is built from mathematical principles that can be used by any academic that masters calculus. The model can be extended, or subsystems can be taken and analyzed. Using this same methodology economic models can more easily understand and compared with one another.

Applications of the Model

This chapter answers the final research question:

How can the developed model be used to support monetary policymakers with their decisions?

The purpose of this chapter is to apply system and control theory to the first-principles model, developed in the previous chapter. Since the purpose of the DSGE model is to support monetary policymakers, this same purpose is researched for the first-principles model.

The stochastic shock investigation presented in Smets-Wouters is interpreted as open-loop analysis. Section 6-1 presents an alternative to this investigation by using the transfer function. Section 6-2 presents the development of a PID-controller interpreted as policy actions from the central bank.

6-1 Insight in the Dynamics of the System

This section presents an alternative investigation tool to analyze the effects of shocks on the system. Chapter 2 showed how the DSGE model presents the results of ten different shocks to twelve different system variables in the form of a catalog. Their observations mention the initial response and the similarities between responses. From the engineering perspective, these not contribute to support policy decisions since they do not explain how to minimize undesired effects. Chapter 3 interpreted the shocks as disturbance inputs. This research introduces the Laplace domain transfer function as an alternative to investigate disturbance responses. It is not only an alternative but also extensive as a support to the decisions of monetary policymakers in contrast to the DSGE model.

6-1-1 Method

To analyze shock - i.e. disturbance - responses, system inputs, and outputs need to be identified. The system inputs are identified based on the shock variables presented in the

DSGE model. The following steps are executed:

1. Present shocks from DSGE in an overview
2. Interpret dimensions of the shock variables
3. Classify the shock variables according to the economic-engineering analog

The shocks, identified as disturbances with corresponding dimensions, need to be matched with the developed first-principles model, in order to be identified as a system input. The following steps are executed:

1. The equation from DSGE corresponding to each shock is analyzed
2. The economic variable from DSGE is matched with a variable in the first-principles model
3. The shock is identified as a disturbance input on a specific state variable

With the shocks identified as disturbance inputs acting on the four state variables of the first principles model, now the output variables can be chosen. This is done according to the results presented in Smets-Wouters. To compare the economic variables from these results and the possible output variables from the first-principles model, together they are presented in a table.

To reproduce the catalog of results presented in Smets-Wouters, the transfer function is used. The conversion of the state space representation to the transfer function offers a systematic way to reproduce each output variable to each input variable. A major advantage of using this method is that the transfer function can be written in terms of its parameters without the need to have its value. Also, the structure of the transfer function offers some logic. In this way, the differences and resemblances between all possible transfer functions can easily be compared without the need to plot a result. The conversion from state space representation to the transfer function is used to do this.

The observations from the DSGE model are interpreted as characteristics of the Laplace domain transfer function. Furthermore, more characteristics from system theory are interpreted as observations that could be helpful to monetary policymakers. The use of system theory is advantageous to economics, as more detailed characteristics can be described and analyzed.

The contribution of the system parameters is interpreted. The definition of parameters from Smets-Wouters is analyzed and interpreted in combination with their position in the transfer function. For example, the $\mathcal{C}_1$ parameter is the exact difference between the disturbance response for the wage and the labor variable. This is matched with the interpretation offered by economics and additional interpretation is given.

The disturbance responses are compared to the shock responses presented in the DSGE model.

6-1-2 Results

Stochastic Shocks

Table 6-1 presents an overview of the shocks presented in the DSGE model and their corresponding symbol in brackets. The corresponding dimensions are identified for every shock. The productivity shock acts on the α of the marginal cost equation. The marginal cost has dimensions $\%/yr$, hence the productivity shock has dimensions $\%/yr$. This is similar for the equity premium shock, the wage mark-up shock and the price mark-up shock that acts on the equity premium, wage, and inflation variable respectively. The preference shock acts on the consumption equation. The consumption equation has dimensions $\$/yr$, hence the preference shock has dimensions $\$/yr$. This is similar for the investment shock, the governmental spending shock, and labor supply shock. These shocks act on the investment, governmental spending and labor variable respectively. The monetary policy shock and the inflation objective shock act on the monetary policy reaction function, hence these two shocks have dimensions $\%/yr$.

Cost-Push Shocks	Demand-Pull Shocks	Build-in inflation
Productivity shock (ε^a)	Preference shock (ε^b)	Monetary policy shock (η^R)
Equity premium shock (η^Q)	Investment shock (ε^I)	Inflation objective shock ($\hat{\pi}$)
Wage mark-up shock (η^w)	Governmental spending shock (ε^G)	
Price mark-up shock (η^p)	Labour supply shock (ε^L)	

Table 6-1: Categorization of the ten different shocks presented by Smets-Wouters in the three defined groups: cost-push shocks (forces), demand-pull shocks (velocities), build-in inflation (control inputs)

The dimensions $\$/yr$ and $\%/yr$ correspond to a cash-flow and a rate. These are the derivatives of money and interest, from the action-angle analog. When transforming this back to the momentum-position analog, cash-flow and the rate are the analog of force and velocity. When a force or a velocity is applied to either a mass or a spring, this can be expressed by a *push* or a *pull*.

Interestingly, Robert J. Gordon used this expression in the categorization of economic shocks [34]. His so-called triangle model grouped the various shocks and used the terms cost-push and demand-pull. This grouping corresponds exactly with the results presented in Table 6-1. The third group in the triangle model was called built-in inflation. He defined this as follows: 'The built-in inflation is induced from adaptive expectations where agents adapt their behavior to experienced behavior in the past'. In the case of monetary policy, the agent that is mentioned is the central bank. Therefore, the built-in inflation is the control input. This results in the distinction between control inputs and disturbances. The control input is the built-in inflation and the disturbances are the cost-push shocks and the demand-pull shocks.

Disturbance Inputs

Each equation in the DSGE model contains one or more shock variables. The corresponding variables where these shocks act on are presented in the two left columns of Table 6-2. The identified system variable that corresponds to the variable from the DSGE model is presented in the right column of this table.

Three shocks from Table 6-1 are not taken into account. The governmental spending shock is not included because the government is not taken into account in the first-principles model. The monetary policy shock and the inflation objective shock are not included in this section because these are identified as control inputs. This section only describes disturbance inputs.

Shock	DSGE Variable	System variable
ε^α	Inflation	$\dot{x}_2$
η^w	Wage	$\mathcal{C}_1 x_1$
ε^b	Consumption	$\mathcal{C}_1 \mathcal{R}_1 x_1$
ε^L	Wage	$\dot{x}_1$
η^p	Inflation	x_2
ε^I	Investment	$\dot{x}_3$
η^Q	Return on Equity	$\dot{x}_4$

Table 6-2: Shock variables from the DSGE model [2] influencing corresponding variables from the DSGE model interpreted to which system variable from the first-principles model it is acting on

Four state variables are differentiated in the first-principles model. The four state variables are labour (x_1), price of goods (x_2), capital (x_3), and the price of stocks (x_4). Therefore, the wage mark-up shock, acting on the wage variable, acts indirectly on the labour variable expressed by $\mathcal{C}_1 x_1$.

$$\dot{x} = \begin{bmatrix} -\mathcal{R}_1 \mathcal{C}_1 & \mathcal{I}_2 & 0 & 0 \\ -\mathcal{C}_1 & -\mathcal{R}_2 \mathcal{I}_2 & \mathcal{C}_3 & 0 \\ 0 & -\mathcal{I}_2 & -\mathcal{R}_3 \mathcal{C}_3 & \mathcal{I}_4 \\ 0 & 0 & -\mathcal{C}_3 & -\mathcal{R}_4 \mathcal{I}_4 \end{bmatrix} x + \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix} u \quad (6-1)$$

Equation 6-1 presents the state-space representation of the first-principles model including the input matrix B. The parameters d_1 , d_2 , d_3 and d_4 enable the application of the shocks as disturbance inputs on the system. The parameter d_1 represents the labour supply shock, since this shock acts on $\dot{x}_1$. The parameter d_2 represents the productivity shock, since this shock acts on $\dot{x}_2$. The parameter d_3 represents the investment shock, since this shock acts on $\dot{x}_3$. Finally, the parameter d_4 represents the equity premium shock, since this shock acts on $\dot{x}_4$.

The system variables, presented in Table ??, can serve as an output variable. The first column presents the derivative of the four state variables in the second column. The third column presents the four variables obtained by multiplying the state variable with the corresponding $\mathcal{C}$ and $\mathcal{I}$ parameter. Finally, the fourth column presents the four variables obtained

by multiplying the variables from the third column with the corresponding $\mathcal{R}$ parameter.

The variables presented in blue in Table 5-3 correspond to the output variables presented in the DSGE model. Hence, the first-principles model increases the number of output variables that can be analyzed.

Transfer Functions

The effect of each shock to each system variable can be investigated. Smets-Wouters presented a large catalog of empirically obtained results with corresponding observations. In contrast to the DSGE model, the first-principles model can be expressed in a transfer function. The transfer function can be obtained from multiplying the state space matrices which is shown in Equation 6-2. This has the advantage that each shock, or disturbance input, to each system variable can be investigated in a systematic manner.

$$H = \frac{y}{u} = C(sI - A)^{-1}B = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \frac{\begin{bmatrix} 3^{rd} & 2^{nd} & 1^{st} & 0 \\ 2^{nd} & 3^{rd} & 2^{nd} & 1^{st} \\ 1^{st} & 2^{nd} & 3^{rd} & 2^{nd} \\ 0 & 1^{st} & 2^{nd} & 3^{rd} \end{bmatrix}}{\det(sI - A)} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (6-2)$$

Equation 6-3 presents the transfer function between the labour variable and the labour supply shock. The labour variable is obtained by taking C-matrix as $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$. The labour supply shock is obtained by $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$ taking the B-matrix as $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$. According to the numerator of $(sI - A)^{-1}$ from Equation 6-2, this will result in a 3th order numerator of the obtained transfer function.

$$H = \frac{s^3 + (\mathcal{C}_3\mathcal{R}_3 + \mathcal{I}_4\mathcal{R}_4 + \mathcal{I}_2\mathcal{R}_2)s^2 + (\mathcal{C}_3\mathcal{I}_2 - \mathcal{C}_3\mathcal{I}_4)s + \mathcal{C}_3}{\dots} \quad (6-3)$$

In order to obtain the transfer function between the wage variable and the labour supply shock, the C matrix is taken as $\begin{bmatrix} \mathcal{C}_1 & 0 & 0 & 0 \end{bmatrix}$. This will result in a similar transfer function as presented in Equation 6-3, multiplied by $\mathcal{C}_1$. This systematic way of obtaining the combinations between all input and output variables is presented in Table 6-3. The first row presents how to obtain the transfer function between all possible input variables and the wage, consumption and labor supply variable from the transfer function between the labor variable and all possible output variables. The second row presents this similarly to the price of goods variable.

Transfer functions between a shock and two variables can easily be compared. For example, the response of an investment shock to the price of goods and capital:

$$H(s) = \frac{\mathcal{C}_3s^2 + (\mathcal{C}_1\mathcal{R}_1\mathcal{C}_3 + \mathcal{C}_3\mathcal{I}_4\mathcal{R}_4)s + \mathcal{C}_1\mathcal{R}_1\mathcal{C}_3\mathcal{R}_3\mathcal{I}_4}{\dots} \quad (6-4)$$

Variable			Variable		Variable		Variable
Labour supply	$f \leftarrow$	(1,1)	Labour	$\rightarrow \mathcal{C}_1$	Wage	$\rightarrow \mathcal{R}_1$	Consumption
Inflation	$f \leftarrow$	(2,1)	Price of goods	$\rightarrow \mathcal{I}_2$	Output	$\rightarrow \mathcal{R}_2$	Transaction costs
Capital supply	$f \leftarrow$	(3,1)	Capital	$\rightarrow \mathcal{C}_3$	Rent	$\rightarrow \mathcal{R}_3$	Depreciation
Return on stocks	$f \leftarrow$	(4,1)	Price of stock	$\rightarrow \mathcal{I}_4$	Investment	$\rightarrow \mathcal{R}_4$	Transaction costs

Table 6-3: The numerator of the transfer function is multiplied with one parameter to obtain the corresponding transfer function of the defined variable

$$H(s) = \frac{(s + \mathcal{I}_4 \mathcal{R}_4)(s^2 + (\mathcal{C}_1 \mathcal{R}_1 + \mathcal{R}_2 \mathcal{I}_2)s + \mathcal{C}_1 \mathcal{I}_2 + \mathcal{C}_1 \mathcal{I}_2 \mathcal{R}_1 \mathcal{R}_2)}{\dots} \quad (6-5)$$

The observations of Smets-Wouters can be summarized as follows:

- Rise/Fall
- Similarities between shocks

The initial value theorem is used to analyze the initial shock response to a system variable (Equation 6-6). Therefore, this is can identify the rise or fall of a shock response. Similarly, the final value theorem is used to analyze the value the shock response converges to.

$$y(0) = \lim_{s \rightarrow \infty} sH(s) \quad (6-6)$$

$$y(\infty) = \lim_{s \rightarrow 0} sH(s) \quad (6-7)$$

The results of applying these two theorems to the transfer function between the labor supply shock and four variables are presented in Table 6-4.

Location	Variable	Initial	Final
(1,1)	Labour L	1	$\mathcal{C}_3$
(2,1)	Price of goods p_g	0	$-\mathcal{C}_1 \mathcal{C}_3 \mathcal{I}_4 \mathcal{R}_3 \mathcal{R}_4$
(3,1)	Capital K	0	$\mathcal{C}_1 \mathcal{I}_2 \mathcal{I}_4 \mathcal{R}_4$
(4,1)	Price of stocks p_s	0	$-\mathcal{C}_1 \mathcal{C}_3 \mathcal{I}_2$

Table 6-4: Initial value theorem and Final value theorem applied to the transfer functions between the disturbance input d_1 and the four state variables

Figure 6-1 presents the simulation results of applying an impulse response to the four variables. The initial and final value theorem results from Table 6-4 correspond to the results in the figure.

The expression of response characteristics in terms of parameters gives interpretation to each system parameter. With the 16 parameters of Smets-Wouters in mind, together with their estimation based on data: The influence of the parameters on the system response is very

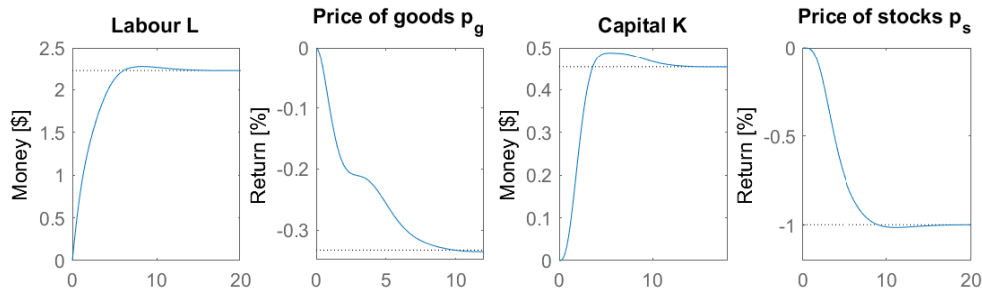


Figure 6-1: Result of a labor supply shock - i.e. a disturbance applied to the first state - to four system variables: labor, price of goods, capital and price of stocks.

large. Therefore, it is interesting to note that the parameters can be interpreted. According to the DSGE model of the Federal-Reserve [1] the elasticity of work effort with respect to the real wage the estimated parameter for the US is 0.7 where in Europe it is 0.4. Hence, the labor variable will converge to a higher value in the US compared to Europe. This is based on the final value theorem. It can be interesting to get some feeling with these parameters when for example comparing a labor market in different regions. This can also be done for all other parameters.

6-1-3 Discussion

It is somewhat surprising that the terminology used by Gordon to classify the different shocks match with the mechanical interpretation that follows from the economic-engineering analog. This result may be explained by the fact that the effects of shocks have been studied for such a long time that the researchers, in this case, Gordon, felt the urge to compare it to more physical behavior like pushing and pulling. This finding provides some support for the conceptual premise that economic theory can be modeled as a mechanical system.

It is interesting to note that from the ten shocks four act as disturbances on the four state variables. This means that the six other shocks cannot be implemented as disturbances on this first-principles model. The preference and labor supply shock both act on the first state-variable, but in the first-principles model this distinction cannot be made. If this distinction is to be made, the consumption variable needs to be expressed as a state variable. This will result in a 5 by 5 state-space matrix - i.e. A-matrix. This will add a 0 eigenvalue to the previous eigenvalues. This can be done with the wage mark-up shock, adding the wage variable as a state variable and the price mark-up shock with the price variable. As this is the first first-principles model of the economy, it is chosen to further analyze only the four shocks acting as disturbance inputs on the four state variables.

The transfer function is a very common tool used in control engineering, but not in economics (yet). The possibilities of the transfer function presented in this chapter are just a small subset of what is possible. The overshoot, the rise time, the settling time are for example other characteristics that can be extracted from the transfer function. As applying these techniques is very straightforward to control engineers, it is just to show the enumerate possibilities of applying system and control to monetary policy. The development of the

first-principles model opens the possibilities to apply all sorts of tested and validated mathematical techniques. Now the logic becomes clear and insight in the effects of shocks, and the underlying dynamics behind these effects enlarge knowledge to monetary policymakers. For example, they can analyze which shocks could potentially cause risks. Also, they can now analyze how to mitigate these effects and how policy can influence this.

6-1-4 Conclusions and Summary

The development of the first-principles model opens the possibilities to apply all types of system theory to monetary policy. Because all states are known and all the states are controllable. The DSGE shock identification is interpreted as an open-loop analysis, therefore an open-loop analysis is done through using the Laplace domain transfer function.

The interpretation of dividing shocks into demand-push and cost-pull corresponds with the results in the first-principles model. This confirms the correct modeling choices of the first-principles model. In addition, it indicates that economists have seen resemblances with a mechanical interpretation or at least find contribution in expressing results using mechanical terminology to better understand the behavior.

The transfer function offers a systematic way to extract and analyze all possible disturbance inputs with all system variables as outputs. As a large catalog of system variables is of interest by monetary policymakers, all different transfer functions can easily be overseen and compared. The first, second and third order numerators can be grouped into similar response behavior. It enables policymakers to deal with all the effects of different shocks on the system variables at once when making policy decisions.

The logic becomes clear why certain responses are obtained. The transient can be deduced either from the transfer function and corresponding poles and zeros, but also from the mechanical equivalent mass-spring-damper behavior. This contrasts the DSGE model, where they present observations of the transient without explaining the underlying reasoning of certain behavior. Furthermore, when looking at results from different DSGE models for the same shock response difference responses are presented but this cannot be deduced from either the model or logic in underlying dynamics. Using the first-principles method developing models, these differences can easily be spotted and explained. It enlarges the insight into cause-effect relationships.

The transfer function can be expressed using parameters without knowledge of the exact value of the parameter. Therefore, no data is necessary to express shock responses and to understand the reason for the specific response. According to Smets-Wouters, data is not always available. Hence, the combination of the first-principles model and the use of the transfer function solves this problem. Also, data-independency creates uniformity in the model. Meaning that both Europe and the US can model their economy with the exact same model. This has advantages in the general understanding of the model and it is more efficient. The difference between the US and Europe is made in terms of parameter values. The higher parameter for the labor market in the US (σ_L) indicates a higher increase in wages when the demand for labor increases compared to Europe.

The introduction of the transfer function to analyze shock responses - i.e. open loop disturbance responses - is an addition to existing analysis tools of monetary policymakers. The analysis of different response characteristics can be extended to not only observe the rise and falling behavior but the exact initial response and the exact convergence. The underlying dynamics of shock responses can now be understood and explained which increases the risk analysis. The risks of monetary policy actions influence the whole economy. Therefore, more transparency and insight are preferable.

6-2 Monetary Policy Simulations

This section presents a policy design tool to analyze the effects of policy decisions on the system. To design policy is not yet possible with the DSGE model because they do not distinguish the central bank as a controller, but approach the central bank and the economy as one.

The PID-controller is applied to the first-principles model. This will show the effectiveness of applying control theory to monetary policy. The working principles of the PID controller will result in a policy function that offers insight into the mitigating effects of policy decisions.

6-2-1 Monetary Policy Reaction Function

To understand the current monetary policy reaction function, it is converted to a differential equation. This monetary policy reaction function is taken from the DSGE model. The function represents the interest rate equation. The Laplace domain transfer function is taken from the obtained differential equation. The Laplace domain characteristics of the monetary policy reaction function on the inflation rate are discussed. Dr. R. Wouters is consulted on the results.

The P-action of a PID controller is interpreted in economic terms. Instead of translating an economic function to a control engineering one, now this is performed the other way around. The same is done for I and D action. The results are compared to what is found in a specific paper that presents the application of a PID controller to monetary policy models.

The inflation decomposition as presented in Smets-Wouters (see Figure 6-2) is imitated by applying a stochastic signal to the developed first-principles model. A stochastic Gaussian Distributed signal is chosen, as it looks similar to the contribution of the shocks in Figure 6-2. The disturbance signal is applied to the fourth state, the equity premium variable. A PID controller is designed to mitigate the volatile effect of the inflation rate. This PID controller imitates monetary policy. This monetary policy consists of only the interest rate which acts on the fourth state, the equity premium variable.

The values of the parameters of the first-principles model are chosen according to the matches with the DSGE model (See Table 5-4). The values for $\mathcal{R}_2$ and $\mathcal{R}_4$ are 1 as they are unknown. A PID controller is designed with the goal to observe the effects of the different P, I and D actions and to minimize volatility in the inflation variable. The different effects of P, I and D action to the model are discussed.

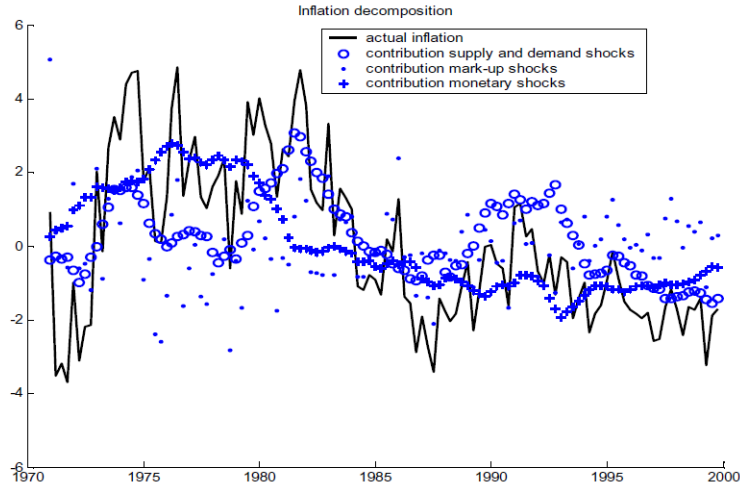


Figure 6-2: Inflation decomposition as presented in Smets-Wouters. The contribution of three different types of shocks are presented in blue. The y-axis presents a rate in $\frac{\%}{yr}$ and the x-axis presents the years from 1970 to 2000. The amplitude of the shocks are expressed in $\frac{\%}{yr}$.

6-2-2 Price Stability

The monetary policy reaction function can be written as follows:

$$R = \rho \int R dt + (1 - \rho)\pi + r_{\delta\pi}(\pi - \int \pi dt) + r_{\delta Y}(Y - \int Y dt) \quad (6-8)$$

The GDP term $r_{\delta Y}(Y - \int Y dt)$ is eliminated in accordance with Dr. R. Wouters. The Laplace domain transfer function between the input inflation rate π and the output interest rate R becomes then:

$$\frac{R}{\Pi} = \frac{(1 - \rho)}{s - 1} \quad (6-9)$$

This is the transfer function of the controller. If the DSGE model was put in the control framework as presented in chapter 3. The monetary policy reaction function would result in an unstable pole of 1.

A PID-controller can be designed alternative to the monetary policy reaction function. Only P-action would result in the following differential equation:

$$R = K_p(\pi^* - \pi) \quad (6-10)$$

This corresponds to the Taylor rule [35]. This finding was also reported by Hawkins [18]. Adding the I and D action will result in the following differential equation:

$$R = K_p(\pi^* - \pi) + K_i(p^* - p) + K_d(\dot{\pi}^* - \dot{\pi}) \quad (6-11)$$

In order to simulate the different effects of the P, I and D action, a stochastic (Gaussian) distributed signal are applied to the system. This signal is presented in Figure 6-3.

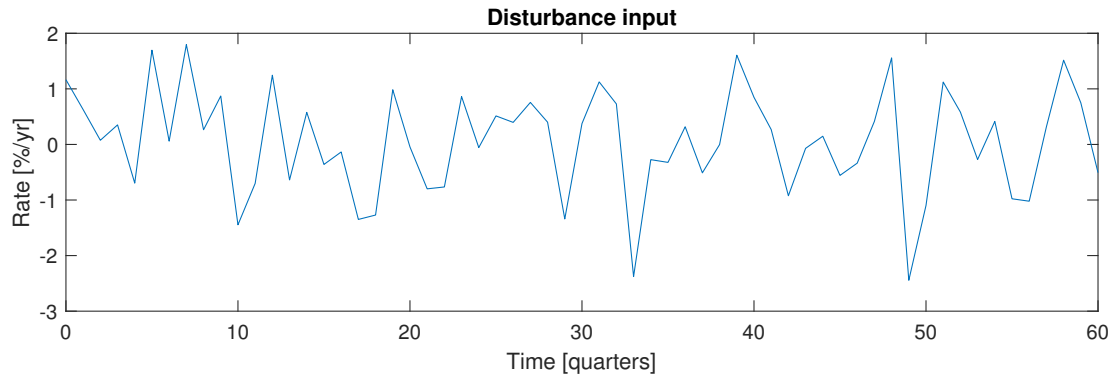


Figure 6-3: Stochastic disturbance (Gaussian) distributed random signal on the investor market, simulating an equity premium (cost-push) shock

Figure 6-4 presents the results of designing the PID controller. The upper plot shows the inflation rate. The lower plot shows the interest rate. The blue line presents the inflation rate when no policy was applied - i.e. no control. The range of the inflation rate is in between $\pm 5 \frac{\%}{yr}$. This is similar to the result of the inflation decomposition of Smets-Wouters.

P-Control

The P-action was tuned to a value of $K_p = 5$. For the inflation rate, this resulted in a large decrease in the volatility. The range of inflation rate with the P-controller policy resulted in between $\pm 2 \frac{\%}{yr}$. The corresponding interest rate moved in between $\pm 1 \frac{\%}{yr}$.

PI-Control

The I-action was also tuned to a value of $K_i = 5$, K_p was kept similar. The I-action caused a faster response to the inflation rate but also a slightly larger volatile behavior. The corresponding interest rate resulted in a slightly larger range and larger peak behavior.

PID-Control

The D-action was also tuned to a value of $K_d = 5$. The D-action resulted in a slower response of the inflation rate and also in a lower volatile behavior. The corresponding interest rate resulted in a flattened peak behavior. A smaller range of the interest rate was obtained in comparison to the P and PI controller.

6-2-3 Discussion

Contrary to expectations, the monetary policy reaction function results in instability. This is somewhat unexpected because this is not what would feel natural in practice. Nevertheless,

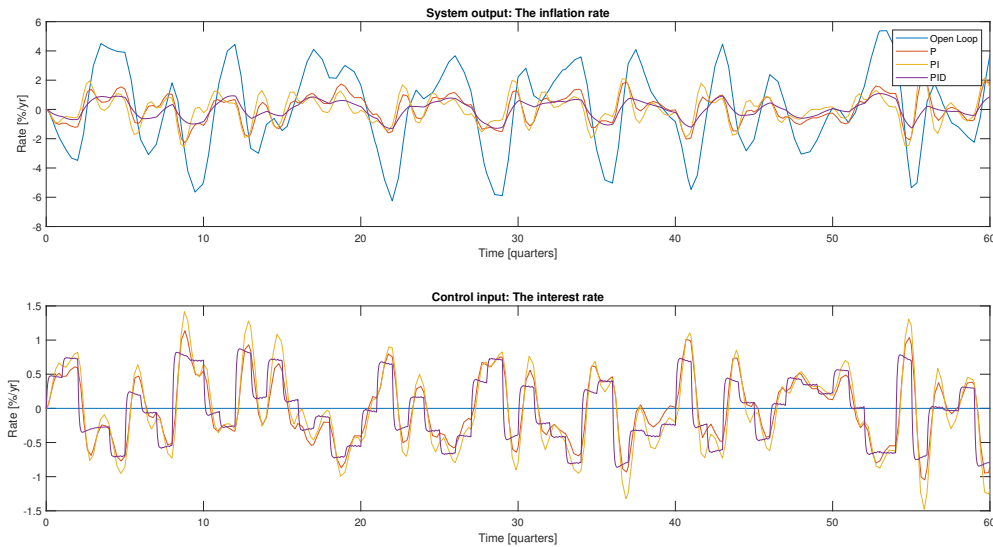


Figure 6-4: P, PI and PID control applied to the inflation variable, presented with the corresponding interest rate

when analyzing data, the long term central banks have had the urge to expand the money supply and the balance of the Federal Reserve, the ECB, the Peoples Bank of China and the bank of Japan, show unstable behavior. In system and control, it is more convenient to stabilize systems, but in economics, this does not always seem preferable. In economics, growth is preferable, which in control-engineering can translate to unstable behavior. Nevertheless, economics prefer stable growth with minimum volatility.

The use of a PID controller might seem like a rather easy choice. But this decision is made because it offers insight into the differences between P, I and D action.

In reviewing the literature, the Taylor rule was found as a monetary policy rule. This Taylor rule can be compared to a P-controller. The parameter α of the Taylor rule is then the K_p parameter of the P-controller. The P-controller showed the best result tuning to a value of $K_p = 5$. This finding is contrary to the Taylor rule study which has suggested that $\alpha = 0.5$. Also, the Taylor rule is bounded by $0 < \alpha < 1$. It seems possible that these results are due to the fact that the economists are not known with control theory and the PID-controller. Increasing the K_p parameter from 0.5 to 5 resulted in decreasing volatility. The corresponding interest rate did not change that much. It could, therefore, be interesting to experiment more with increasing the α of the monetary policy reaction function that is used in practice.

The PI-controller did not add to decreased volatility. But it could be useful when the ECB changes its target inflation. I-action is known to result in a zero steady-state error. The PID-controller decreased the volatility in comparison to the P-controller. Therefore, it is interesting to look at the possibilities for the monetary policy reaction function to steer with the derivative of the inflation rate.

The decision to build a continuous time model can be questioned because in economics it is common to work with discrete time. But as it is the first model build, it is chosen to work with a continuous time model. If the model is going to be compared to the results when working with data and system identification techniques, it could be better to develop a discrete time equivalent of the presented model.

Prior study to the correspondence of a PID controller to monetary policy [18] has shown an overview of different DSGE models and the monetary policy reaction function. They proposed the correspondence to the PID controller but did not show any simulations or results. Therefore, the differences between P, PI and PID could not be compared to their analysis.

6-2-4 Conclusions and Summary

The PID-control is the most convenient first application of a controller simulating monetary policy actions. The results seem meaning-full. This could be the first step into a modeling technique that can be of added value to monetary policymakers.

6-3 Conclusions and Summary

The decisions of monetary policymakers can be supported by the developed first-principles model. This is done by applying system and control theory. The transfer function offers insight into the shock responses and corresponding risks. The PID-controller offers insight into the mitigation of undesired shock responses and extends the toolbox of policy actions.

The application of system and control theory adds to the understanding of risks in monetary policy. Risk analysis is generally the main driving factor behind policy decisions. The effects of policy decisions have a large influence on the economy, and thereby the people that are part of this economy. The understanding of risks in policy actions is preferable for both policymakers and the people influenced by the policy actions.

The distinction between policy actions and shocks makes it possible to analyze the effects of policy actions on shocks - i.e. control input on disturbances. This was not possible with the DSGE model that only presented the effects of shocks, but not the effects of policy actions responding to these shock effects. The distinction in the model better simulates monetary policy practices. Also, it makes it possible to analyze different types of policy actions to shock, which is also not possible with the DSGE model.

Conclusions and Recommendations

In this thesis, system and control theory was applied to the monetary policy objective of a central bank. This chapter summarizes the conclusions of this thesis, discusses the ethical side of the research and lists recommendations for future research.

7-1 Conclusions

The overall goal of this thesis was to apply system and control theory to monetary policy and model it as if it were a mechanical system. Therefore, the current model used by the European Central Bank (ECB) together with the existing economic-engineering framework from Mendel (Chapter 3), were combined to express monetary policy as a control problem (Chapter 4). In order to develop a model for this control problem, the economic-engineering framework is extended to enable the mechanical interpretation of the economic concepts of money and interest (Chapter 5). Then the actual model was developed (Chapter 6) and two applications were presented to show its effectiveness (Chapter 7). According to the main research question, it is possible to combine the field of system and control engineering and economics to apply it to monetary policy. The most important conclusions are included below, divided into their contribution to the understanding of the working principles and accessibility of the model.

In terms of understanding:

- *The DSGE model does not offer insight to understand the working principles of the model:*

From a control-engineering perspective, the equations from the DSGE model (Smets-Wouters) are predefined and estimated using regression techniques. Therefore, they do not explain the underlying principles of economic phenomena. The result, in order to support the decisions of policymakers, does not explain the obtained effects. This limits the policymaker in terms of understanding, but also the public that is affected by the decisions of the policymakers.

- *The control formulation of the monetary policy objective approaches practices better than the existing models:*

A distinction is made between shocks - i.e. disturbances and policy actions - i.e. control input. This contrasts the DSGE model that approaches shocks that they can and cannot manipulate similarly. In practice, the central bank responds to certain behavior by changing the interest rate. Therefore, approaching this process as a feedback mechanism resulting in a control input as a response to a disturbance better matches with monetary policy practices. This can improve the understanding of monetary policy as results of shocks can be simulated with and without policy actions - i.e. control. Also, monetary policy actions can be designed by simulation the effect of policy action to certain disturbances. This result is independent of the use of can also be applied to a DSGE model if they become usable by engineers.

- *A 4th order model describes the behavior the transactions between four types of agents representing the dynamics of the economy:*

This is the first model describing economic behavior that is developed using a first-principles approach. Therefore, it is also the first model consisting of differential equations. So far, dynamics of economies could not be expressed as differential equations with analytic solutions [18], [36], [37]. The model is the simplest representation of economic behavior while still capturing the most important economic variables and concepts. It is based on three basic linear formalism's that together form 4th order dynamical behavior. With this model all operations can be applied that can also be applied to a model describing a physical system. The application of these operations can be of added value to describe certain behavior in monetary policy models.

- *The first-principles model offers insight in underlying dynamics of the system and adds to the understanding of the system parameters:*

The Laplace domain transfer function can be taken from any disturbance - i.e. shock - input to any economic variable as an output. The first-principles model enables the expression of these transfer functions in terms of their parameters. The parameters obtain meaning in terms of the characteristics of the dynamic response of the system. These characteristics can be extracted from these transfer functions, such as the initial response and the convergence value. These characteristics can offer their insight into the effects of possible economic phenomena and thereby support monetary policymakers in their decisions. Furthermore, different disturbance input effects can be compared systematically to the same economic variable by comparing the transfer functions. This systematic comparison can offer insight in risk-analysis of possible shock effects.

- *The design of a PID-controller offers insight into different policy actions and their effects on minimizing undesired disturbances:*

The P-action is similar to conventional monetary policy rule - i.e. the Taylor rule. Increasing the value of the parameter above 1 will increase the volatile behavior of the inflation variable. Introducing I-action and D-action as steering on the price level and the derivative of the inflation rate will add to the toolbox of policymakers. I-action will result in fast steering towards a new target inflation rate. D-action will result in smooth steering and decreased volatile behavior of the inflation rate.

The possibility to simulate the effect of policy actions to certain shocks is a new ap-

proach, in comparison to the DSGE model. This will support policymakers in consideration of using different policy actions. Furthermore, it can specifically focus to minimize effects for a certain economic event.

In terms of accessibility:

- *The DSGE model cannot be used by an engineer:*
Although the DSGE model presented in the paper of Smets-Wouters uses terminology a control engineer is familiar with; e.g. "state space representation", "law of motion" and "linearization", the equations that together form the model are difficult to interpret. Not all parameters and variables are explained and it is unclear what the dimensions of these parameters and variables are. Also, for some of the parameters, no values are given. More importantly, there is no explicit definition of a controller. This implies that only economists, specialized in monetary policy, can understand and use the DSGE model. This limits the accessibility for academics to do further research on the model and build on it.
- *Action-angle coordinates express the economic variables money and interest in its mechanical analog:*
Instead of starting from a momentum-position coordinate framework, the coordinate change to action-angle enables the modeling of monetary policy as a mechanical system in terms of money and rates. The use of this initial coordinate framework results in an elegant and simple-to-use model. This simplicity adds in terms of accessibility, hence, academics can do further research to the model and potentially build on it.

The thesis presented a novel methodology to model monetary policy. The main contribution of this method is the distinction between (undesired) disturbances and (mitigating) policy actions. This contrasts the existing model used by the ECB, where they approach these disturbances and policy actions similarly - e.g. as shocks. As a result of the first principles model, the underlying dynamics of the system can be understood and because of this, the effect of the disturbances on system variables can be predicted using system theory characteristics. The influence of the system parameters get a meaning thanks to this method. Furthermore, the first-principles model is built on mathematical principles on which people can build on. The simplicity of the model also enables the division into subsystems and the extension of the model with more economic agents. Because of the distinction of the central bank and the economy in terms of a system and a controller, a policy can now be designed explicitly to reject undesired disturbances and can even be automated. Clearly, there are hurdles to be overcome. Therefore, the next section presents a point of view on the ethical side of the thesis work and recommendations for future research.

7-2 Ethics

From an engineering perspective, automating systems is often seen as a preferable solution. Nevertheless, this can also involve negative consequences. In an attempt to introduce control engineering techniques to monetary policy, it seems straightforward to introduce the possibilities of automating monetary policy. Similarly to the autopilot in a plane. But the ethical

question arises: *Is automatizing monetary policy right and is it desirable?*

Nowadays, a lot of news-items present the decisions of the ECB and of the Federal Reserve of the United States. This is an important news topic, as it has a large economic influence. Most countries are in debt with another country. Therefore, changing the interest rate influences this. Furthermore, the financial sector is also largely influenced by the interest rate. This consists of invested capital for private households, companies and also insurance and pension funds. With this large influence, it is important to get insight into the working principles of monetary policy. The central banks have the power and authority to make monetary policy decisions. But to the public, it is unclear what options the policymakers have had if they act consciously of risks and what the underlying mechanisms are that they base their decisions on. The introduction of system and control engineering to monetary policy could shed light on these unknowns. It could offer people more insight into the working mechanisms of monetary policy tools and their effects. Hence, it would become more transparent what kind of risks are taken and what that could mean to them. As it has such a large influence, it can be of added value to having a larger understanding of possible policy actions and the risks that are involved.

Similarly to self-driving cars, completely automatizing monetary policy brings a very large risk. But current-day cars have the technology to warn for potential hazards, to continuously analyze signals obtained by sensors, and, to drive almost autonomously with the driver always there to interfere. With that in mind, it could be interesting to research the possibilities to automatize a monetary policymaker in the form of a controller without actually applying the policy actions. These decisions can be compared to actual monetary policy actions and the effects of them. Also, parts of monetary policy can be automated. For example, decision-support consult can be automated, or different types of risk analyses. In this way, the final decisions will always be made by people.

It looks like there is potential to further explore the contributions of applying system and control theory to monetary policy. Nevertheless, the last question to discuss is: Is there a market for this economic-engineering approach? Maybe economists are not enthusiastic about this cooperation. Impediments could be; a lack of trust or suspicion that engineers are going to take over their jobs. Maybe, they would like to keep these models, mechanisms, and decisions in the dark. The willingness of both parties to work together can be seen as very ethical.

7-3 Future Research

With regard to the results, the following can be questioned: What is needed to apply the introduced methods in practice? As it is the first attempt to model monetary policy as an engineer, it is important to apply more effort in studying this approach. The modeling decisions presented in Chapter 5 can each be questioned and researched. Furthermore, it is very important to communicate with economists and monetary policymakers. In order to let them use engineering tools, they need to understand them and see the added value. Also, it is important to match with the demand of these economists and monetary policymakers. What kind of innovations are they looking for? It could be that they are not so much interested in the initial value of a shock response, but more in the overshoot of a shock response.

The data used to estimate the DSGE model can be used to validate the first-principles model. First, this data needs to be gathered. This can be done by reaching out to Dr. Mr. Wouters. Then system and identification techniques can be used to estimate a model. Also, the difference equations presented in the DSGE model can be re-used. This can be helpful in comparing the shock responses and validate the model. It can also be helpful in developing a different first-principles model or extending the first-principles model.

The PID-controller is designed to mitigate for one type of disturbance. This can be extended to disturbances applied to all four states. It can be researched what the influence is of P, I and D action to minimize the volatile response of the inflation variable. In order to apply the results in practice, the translation of P, I and D action in terms of the interest rate needs to be discussed with monetary policymakers.

Both the system and the controller can be extended to multi-input-multi-output. In addition to the inflation rate, economic growth and unemployment can be added as output signals. In order to do this, first economic growth and unemployment need to be defined in the first-principles model. It could be that the model needs an extension. But it is recommended to study the transformation of labor into employment using a canonical transformation. Furthermore, economic growth could be interpreted as the multiplication of inflation and output of the $\mathcal{I}_2$ element of the firm. In addition to the interest rate, the controller can be extended by money supply, open market operations, and quantitative easing. It is recommended to study the addition of money supply as a similar signal as the interest rate. Hence, the controller could also be modeled as a mechanical system. Furthermore, open market operations and quantitative easing could be interpreted as a transformer or a lever.

The application of more complex controllers can be studied. In order to reject the different stochastic disturbance signals, the application of robust control can be studied. The translation to the economic interpretation of robust control is important in order to make it applicable for monetary policymakers. Also, adaptive and predictive control could be researched, as economic shocks are unpredictable events.

Appendix A

Trial-and-Error

In order to find an interpretation of money in mechanics, a trial-and-error process was executed. After defining what variables belong to what economic agents, the interactions or transactions were sketched.

In Figure A-1 the government, the household (hh), the bank and the firm are transacting. Each agent acts according to its Hamiltonian and Lagrangian. In the sketch, the Legendre transform between these two are expressed in economical terminology. For example, the household has its savings and demand deposit account as the Hamiltonian, costs to consume as the Lagrangian and revenue as the labour multiplied by the hours worked.

Figure A-2 presents the sketches that describe the possibility of a mechanical model that represents the household and the firm. This sketch was made to find out how inflation works. It tries to express the mechanism of a household that consumes and what happens if it consumes more or less. The response of the firm that sells consumer goods is also captured in the model.

Figure A-3 expresses three of the monetary policy tools of a central bank. This sketch was to research if there are other possibilities for the controller the capture different tools than the interest rate. The open-market-operation was found analog to the Q-factor of a damped harmonic oscillator. It was decided to not further research this as this would cost a lot of research.

Figure A-4 expresses a flow diagram of the circular flow of income. The corresponding dynamics are drawn below. The symbols for quantity and price are used to find the proper analog. The dynamics of a harmonic oscillator are expressed and the corresponding Lagrangian and Hamiltonian.

Figure A-5 presents a sketch of a possible bond graph model. It is difficult to read, but it expresses the variables labour and wage, capital and investment, consumption and inflation, and in the dotted area money supply and interest rate. This sketch is to present the various options of expressing the transactions between agents in a mechanical model.

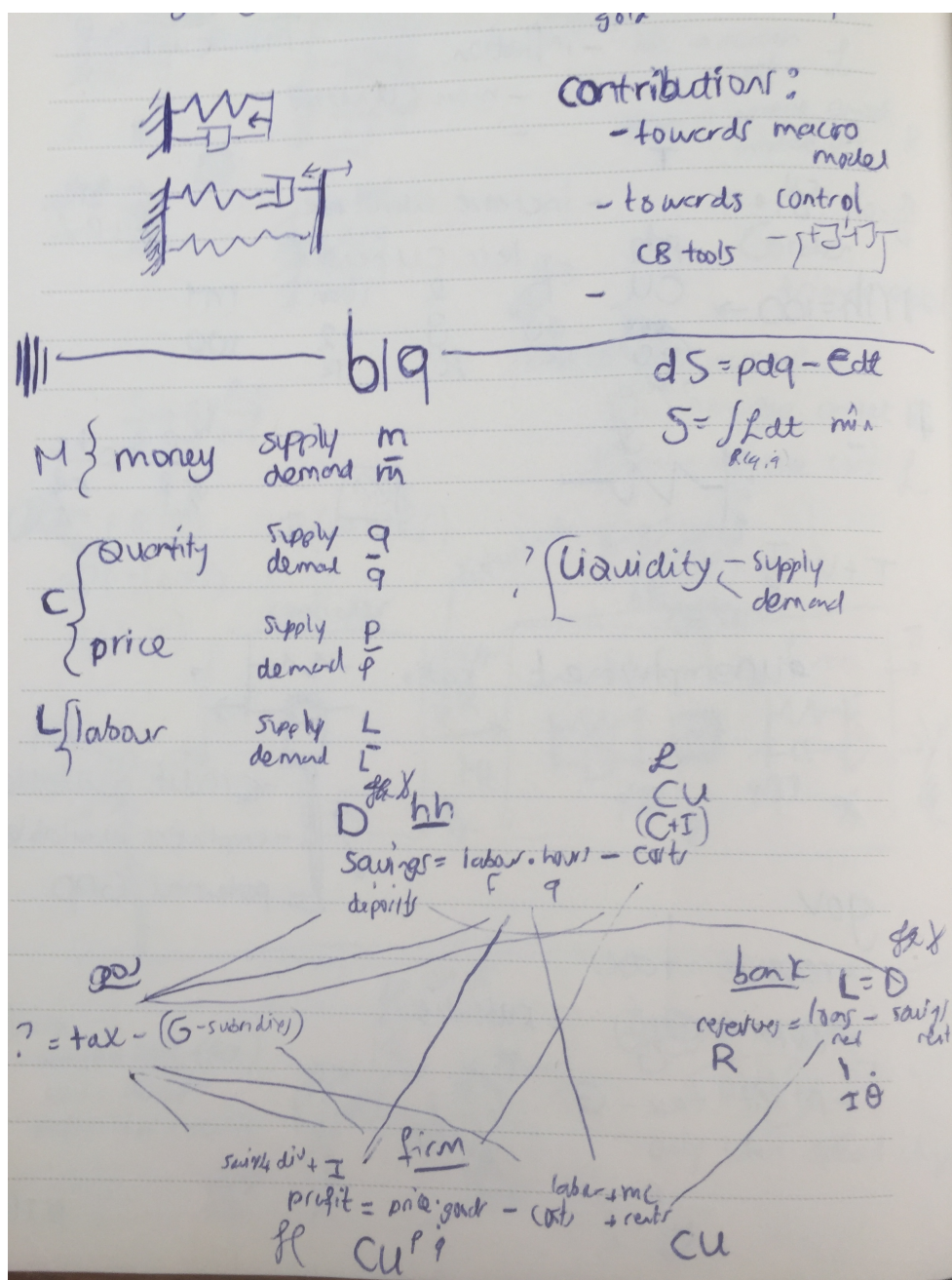


Figure A-1: Sketch of the government, the household, the bank and the firm interacting.

Figure A-6 presents a similar sketch as Figure A-5. The interactions between the economic agents household, firm, investor and bank are expressed in a sketch. This sketch includes if the dynamics of the variables can be expressed using a spring and a mass.

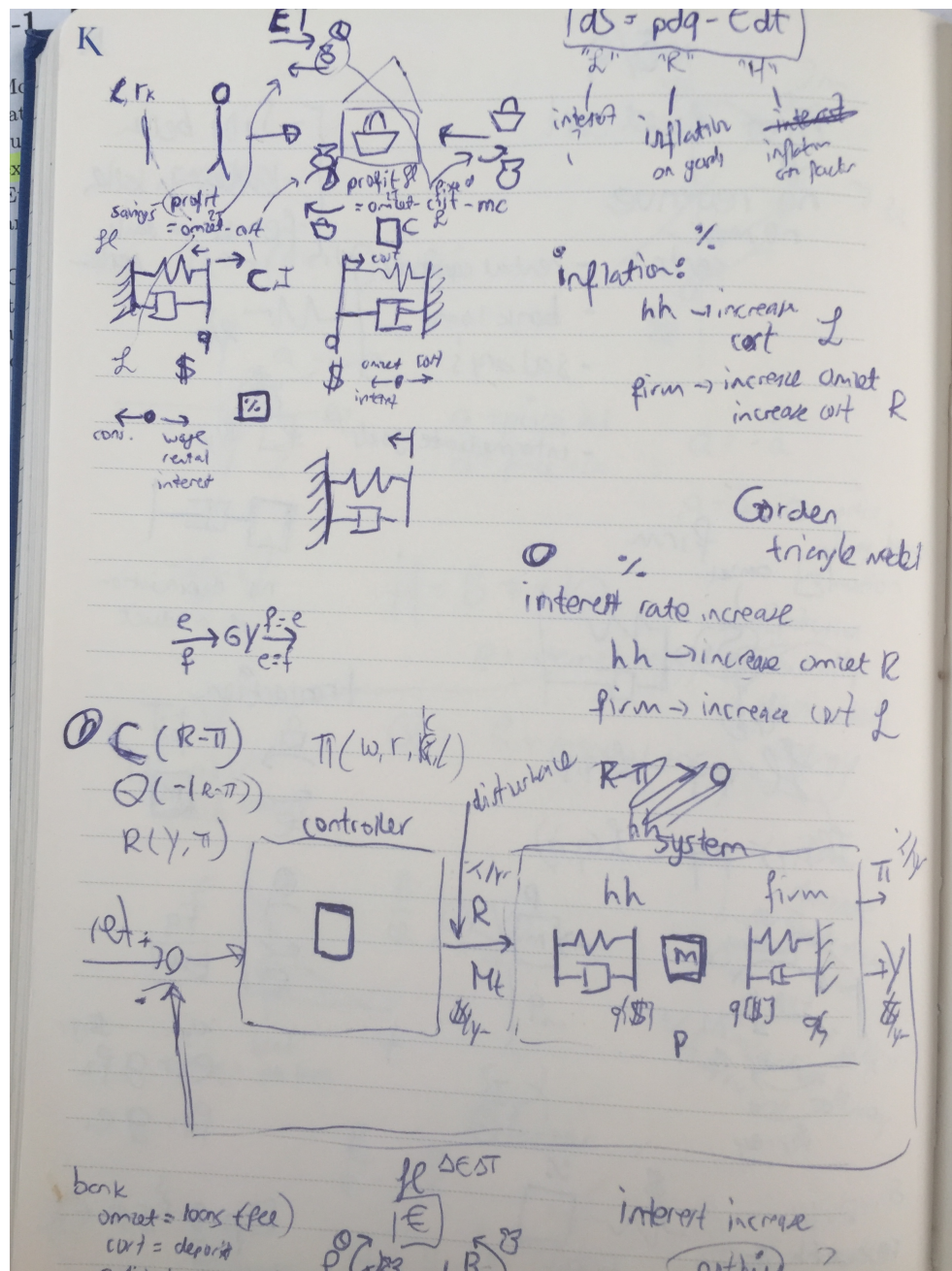


Figure A-2: Mechanism of the household and the firm that result in inflation.

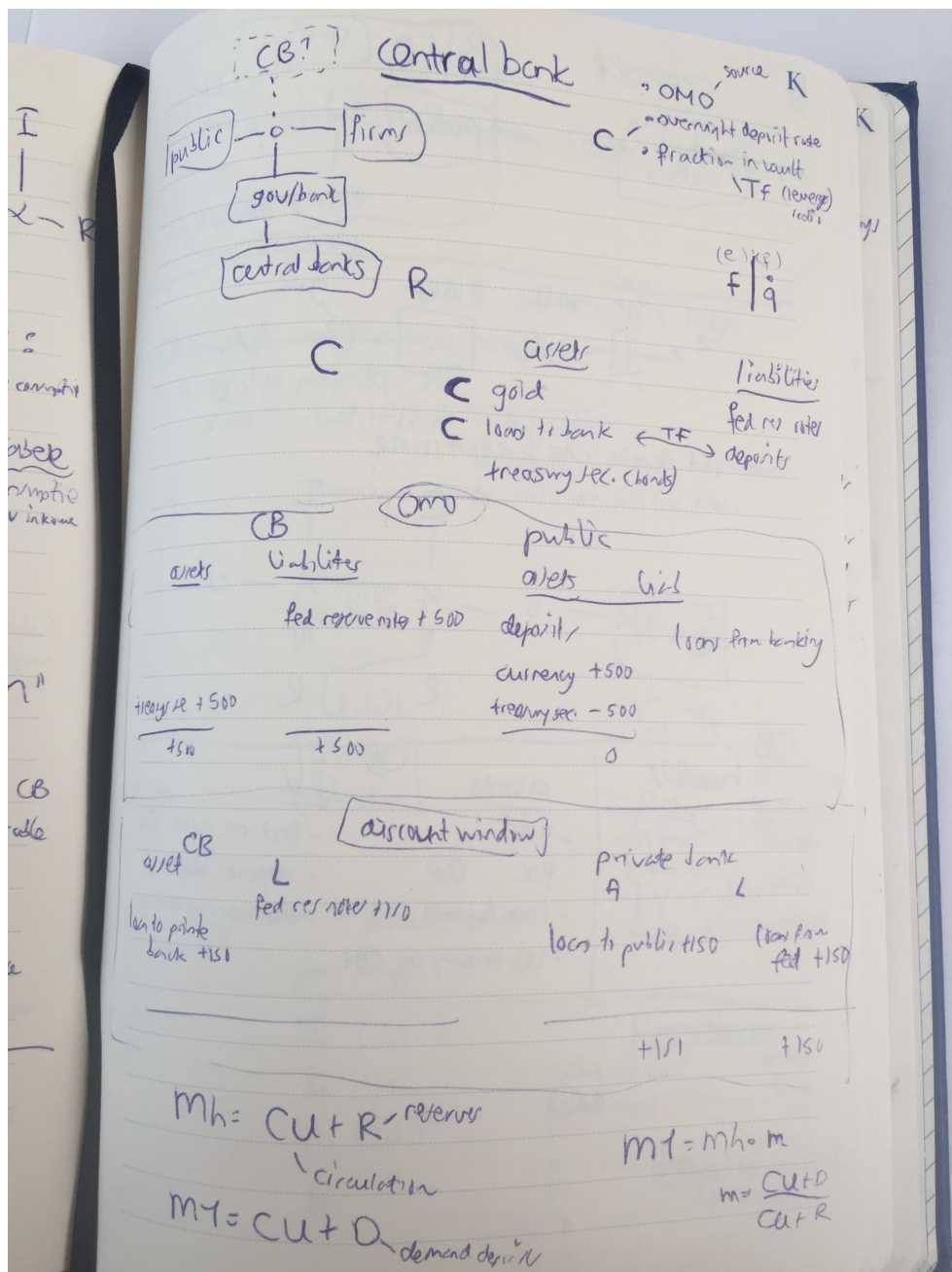


Figure A-3: The monetary policy tools of a central bank expressed in an overview. The open-market-operations, the discount window and the reserve ratio are expressed.

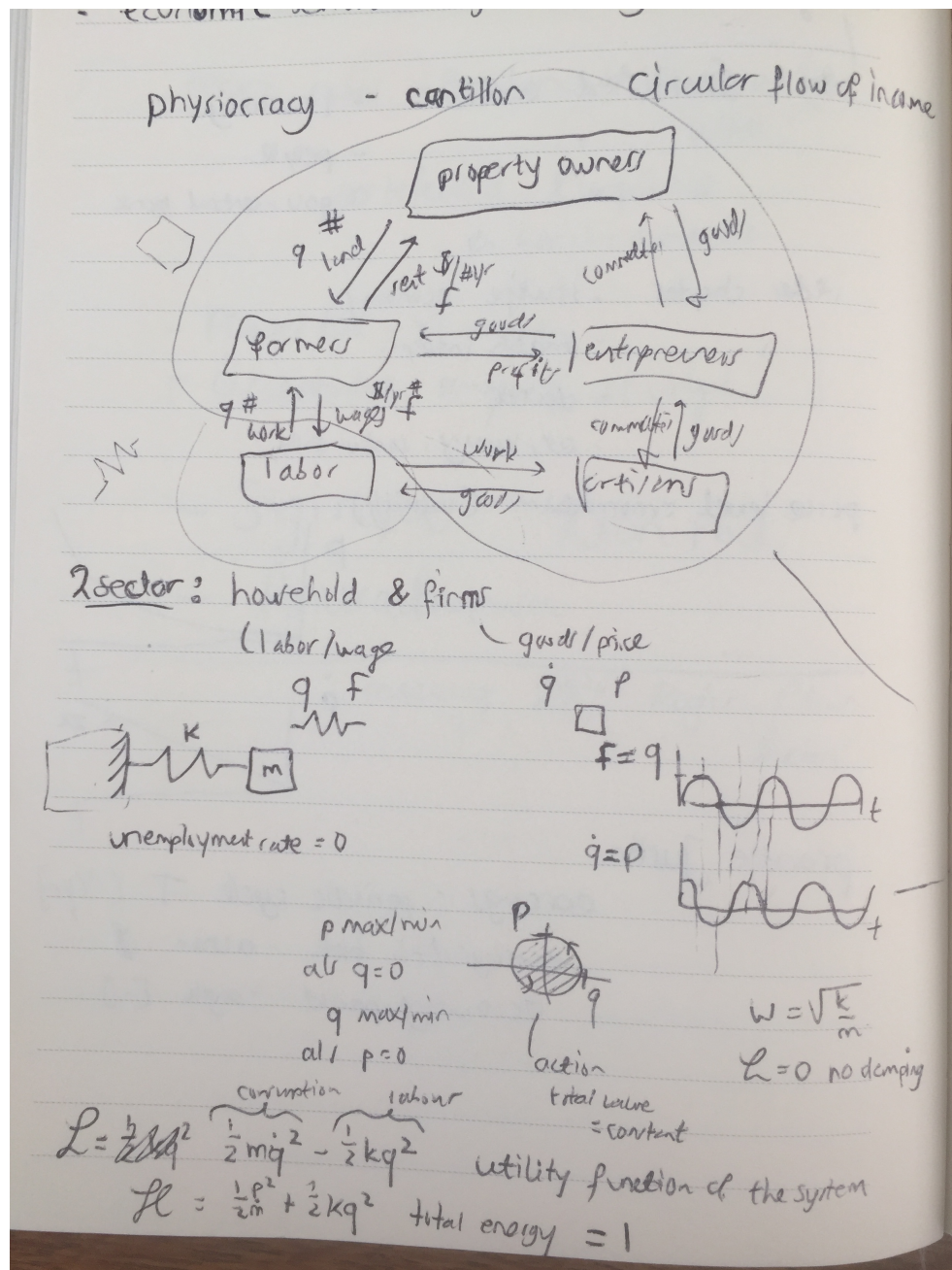


Figure A-4: Circular flow of income as expressed by Cantillon, interpreted as a dynamical system.

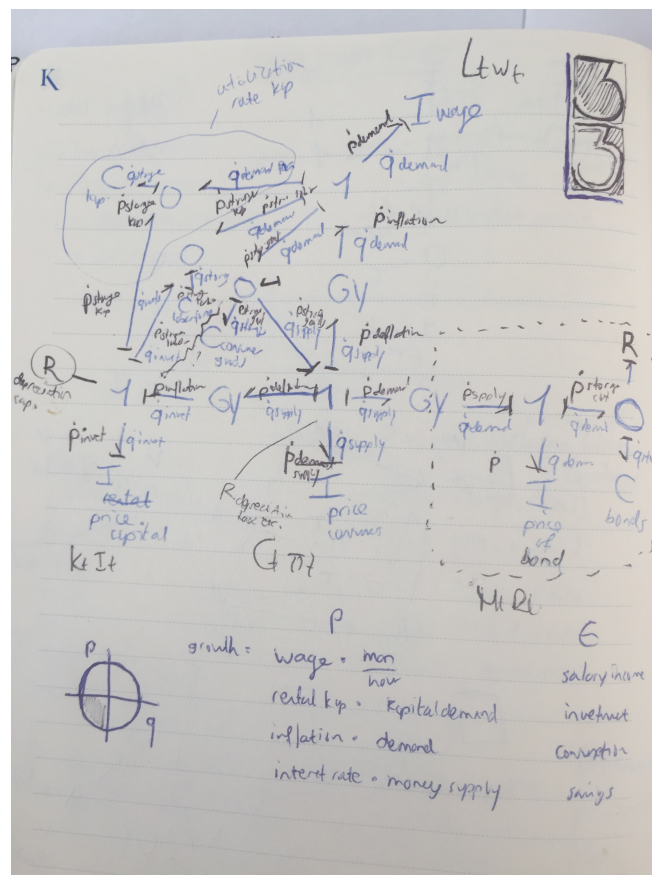


Figure A-5: Possible bond graph model of the economy. Interacting variables are expressed in black, the bond graph elements are expressed in blue.

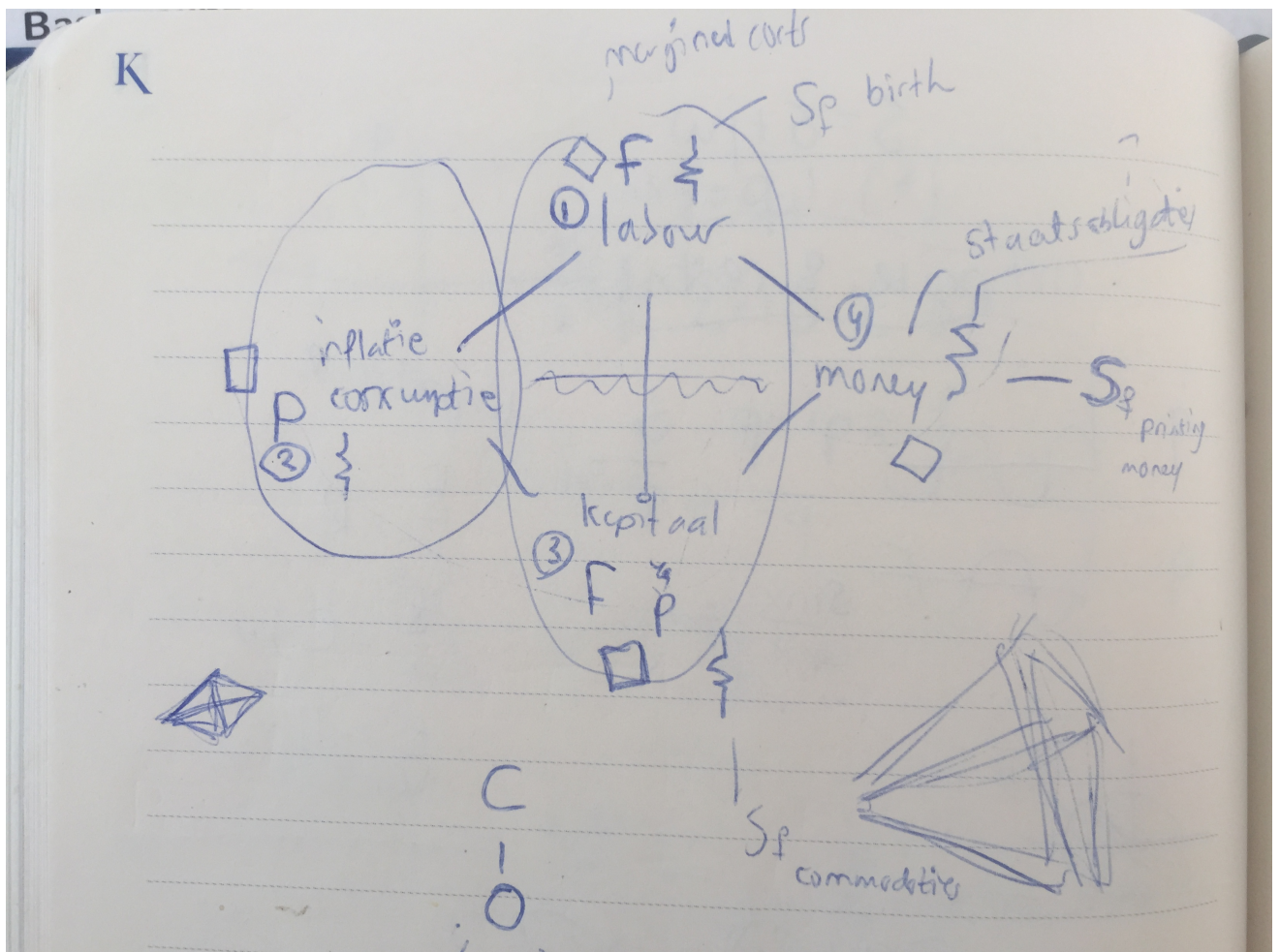


Figure A-6: Sketch of the interaction of the economic variables labour, consumption, capital and money.

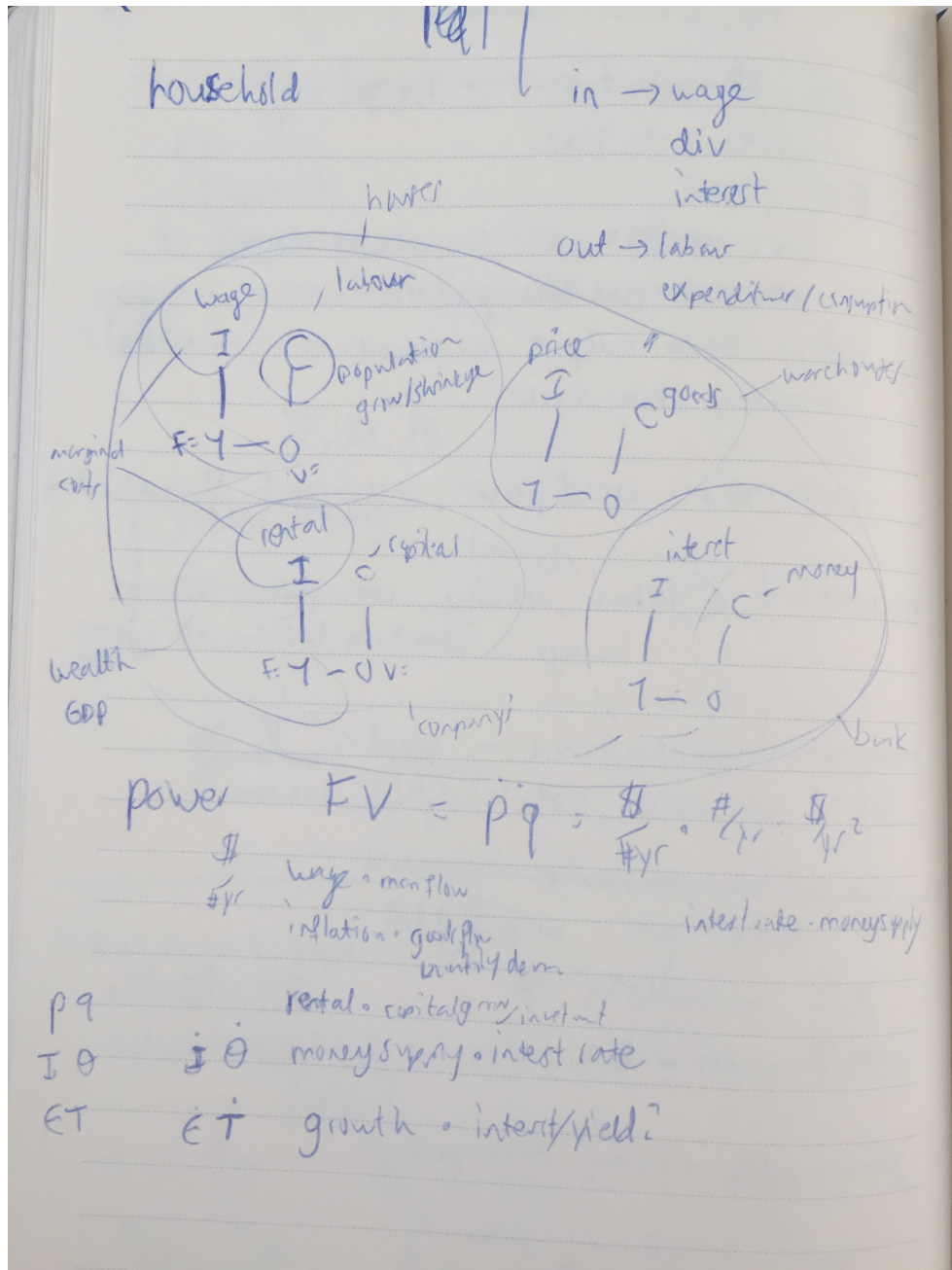


Figure A-7: Sketch of economic variables expressed in bond graph elements. Four separate bond graph models are presented, upper-left the household with variables labour and wage, upper-right the firm with variables price and goods, left-down the entrepreneur with variables rental of capital and capital and left-right the bank with variables interest and money.

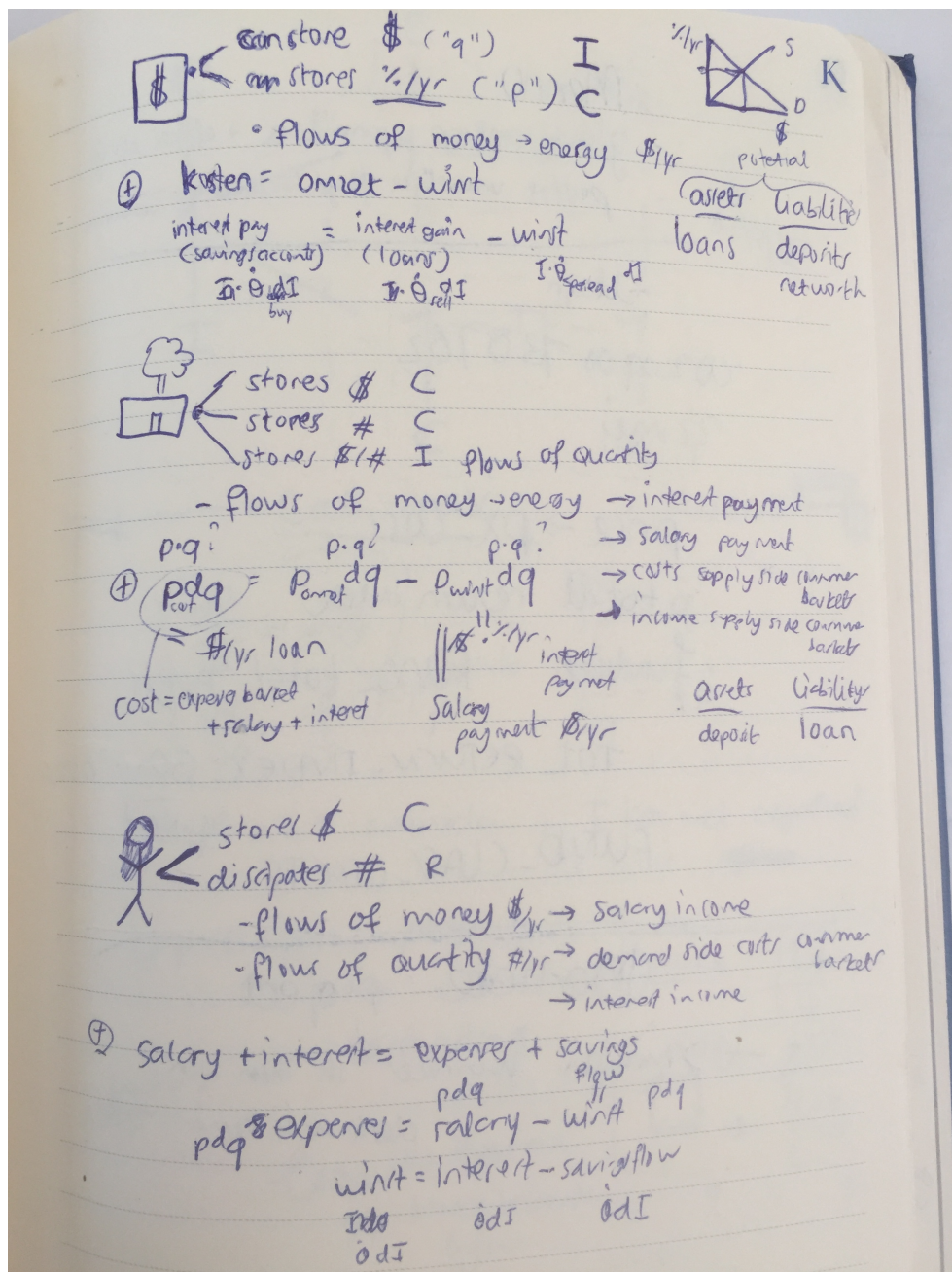


Figure A-8: Sketch of variables corresponding to the bank, the firm and the household. Corresponding bond graph elements are presented.

Bibliography

- [1] F. Smets and R. Wouters, “Shocks and frictions in us business cycles: A bayesian dsge approach,” *American Economic Review*, vol. 97, pp. 586–606, June 2007.
- [2] F. Smets and R. Wouters, “An estimated dynamic stochastic general equilibrium model of the euro area,” *Journal of the European Economic Association*, vol. 1, no. 5, pp. 1123–1175, 2003.
- [3] D. H. Kobe, “Canonical transformation to energy and tempus in classical mechanics,” *American Journal of Physics*, vol. 61, jan 1993.
- [4] P. Lafourcade and J. de Wind, “Taking trends seriously in dsge models: An application to the dutch economy,” 2012.
- [5] F. B. L. Jeffrey D. Sachs, *Macroeconomics in the Global Economy*. Prentice Hall, 1 ed., 1993.
- [6] D. Romer, *Advanced Macroeconomics*. McGraw-Hill Education, 4 ed., 2011.
- [7] W. D. N. Paul A. Samuelson, *Economics*. McGraw-Hill Education, 19 ed., 2009.
- [8] J. E. Stiglitz, *The Collected Scientific Papers of Paul A. Samuelson, Volume 1*. MIT-Press, 1966.
- [9] L. D. Landau and E. M. Lifshitz, *Mechanics, Vol. I of Course on Theoretical Physics*. Pergamon Press, 1972.
- [10] A. V.I., *Mathematical Methods of Classical Mechanics*. Springer-Verlag New York, 1989.
- [11] H.C. Corben and P. Stehle, *Classical Mechanics*. Dover Publications, 2 ed., 1994.
- [12] M. Born, *The Mechanics Of The Atom*. Bell, London, 1927.
- [13] D. M. B. Mendel, “Principles of economic engineering,” *Lecture Notes Delft University of Technology*, 2019.

- [14] R. Smit, “The interpretation and application of 300 years of optimal control in economics,” Master’s thesis, TU Delft, 2018.
- [15] C. Hutters, “The analytical mechanics of consumption,” Master’s thesis, TU Delft, 2019.
- [16] J. Black, N. Hashimzade, and G. Myles, *A Dictionary of Economics*. jan 2009.
- [17] G. Akerlof, W. T. Dickens, and G. Perry, “Near-rational wage and price setting and the long-run phillips curve,” *Brookings Papers on Economic Activity*, vol. 31, no. 1, pp. 1–60, 2000.
- [18] R. J. Hawkins, J. K. Speakes, and D. E. Hamilton, “Monetary policy and pid control,” *Journal of Economic Interaction and Coordination*, vol. 10, pp. 183–197, Apr 2015.
- [19] J. W. Brewer and P. P. Craig, “Bilinear, dynamic single-ports and bond graphs of economic systems,” *Journal of the Franklin Institute*, vol. 313, no. 4, pp. 185 – 196, 1982.
- [20] J. W. Brewer, “Progress in the bond graph representations of economics and population dynamics,” *Journal of the Franklin Institute*, vol. 328, no. 5, pp. 675 – 696, 1991.
- [21] A. Macchelli, “Port-hamiltonian formulation of simple macro-economic systems,” in *52nd IEEE Conference on Decision and Control*, pp. 3888–3893, Dec 2013.
- [22] J. Tenreiro Machado and M. Mata, “A fractional perspective to the bond graph modelling of world economies,” *Nonlinear Dynamics*, vol. Spring, 06 2014.
- [23] R. C. R. Dean C. Karnopp, Donald L. Margolis, *System Dynamics: Modeling, Simulation, and Control of Mechatronic Systems*. Wiley, 5 ed., 2012.
- [24] A. E.-N. Gene F. Franklin, J. David Powell, *Feedback Control of Dynamic Systems*. Pearson, 7 ed., 2014.
- [25] ECB, “European central bank - tasks - monetary policy,” 2019. Last accessed 15 April 2019.
- [26] D. H. Kobe, “Energy and tempus as canonical variables: application to a particle with a force quadratic in velocity,” *European Journal of Physics*, vol. 14, pp. 262–267, nov 1993.
- [27] D. H. Kobe and V. C. Aguilera-Navarro, “Derivation of the energy-time uncertainty relation,” *Physical Review A*, vol. 50, no. 2, 1994.
- [28] D. H. Kobe and K.-H. Yang, “Energy of a classical charged particle in an external electromagnetic field,” *European Journal of Physics*, vol. 8, pp. 236–244, oct 1987.
- [29] D. Sen, “The uncertainty relations in quantum mechanics,” *Current Science*, no. 2, 2014.
- [30] J. A. T. Machado and M. E. Mata, “A fractional perspective to the bond graph modelling of world economies,” *Nonlinear Dynamics*, vol. 80, pp. 1839–1852, Jun 2015.
- [31] P. M. V. den Hof, *System Identification Data driven Modeling of Dynamic Systems*. Lecture notes DISC Course SYSID, 2018.

-
- [32] R. E. Backhouse and Y. Giraud, *Famous Figures and Diagrams in Economics - Circular Flow Diagrams*, ch. 23, pp. 221–230. 2010.
 - [33] S. Pool, *Credit supply and macroeconomic fluctuations*. PhD thesis, University of Groningen, 2018.
 - [34] R. J. Gordon, *Macroeconomics: Theory and Policy, Chap 22.4 Modern theories of inflation*. McGraw-Hill Education, 2 ed., 1988.
 - [35] J. B. Taylor, “Discretion versus policy rules in practice,” *Carnegie-Rochester Conference Series on Public Policy*, vol. 39, pp. 195 – 214, 1993.
 - [36] M. R. Astrom KJ, *Feedback systems: an introduction for scientists and engineers*. Princeton University Press, 2008.
 - [37] B. S, *A history of control-engineering 1930-1955*. Peter Peregrinus Ltd., Stevenage, 1993.

