Topological Stereotomic Design of System of Interlocking Stackable Modular Blocks for Constructing Multi-Storey Masonry Buildings

MSc.Thesis
Delft University of Technology
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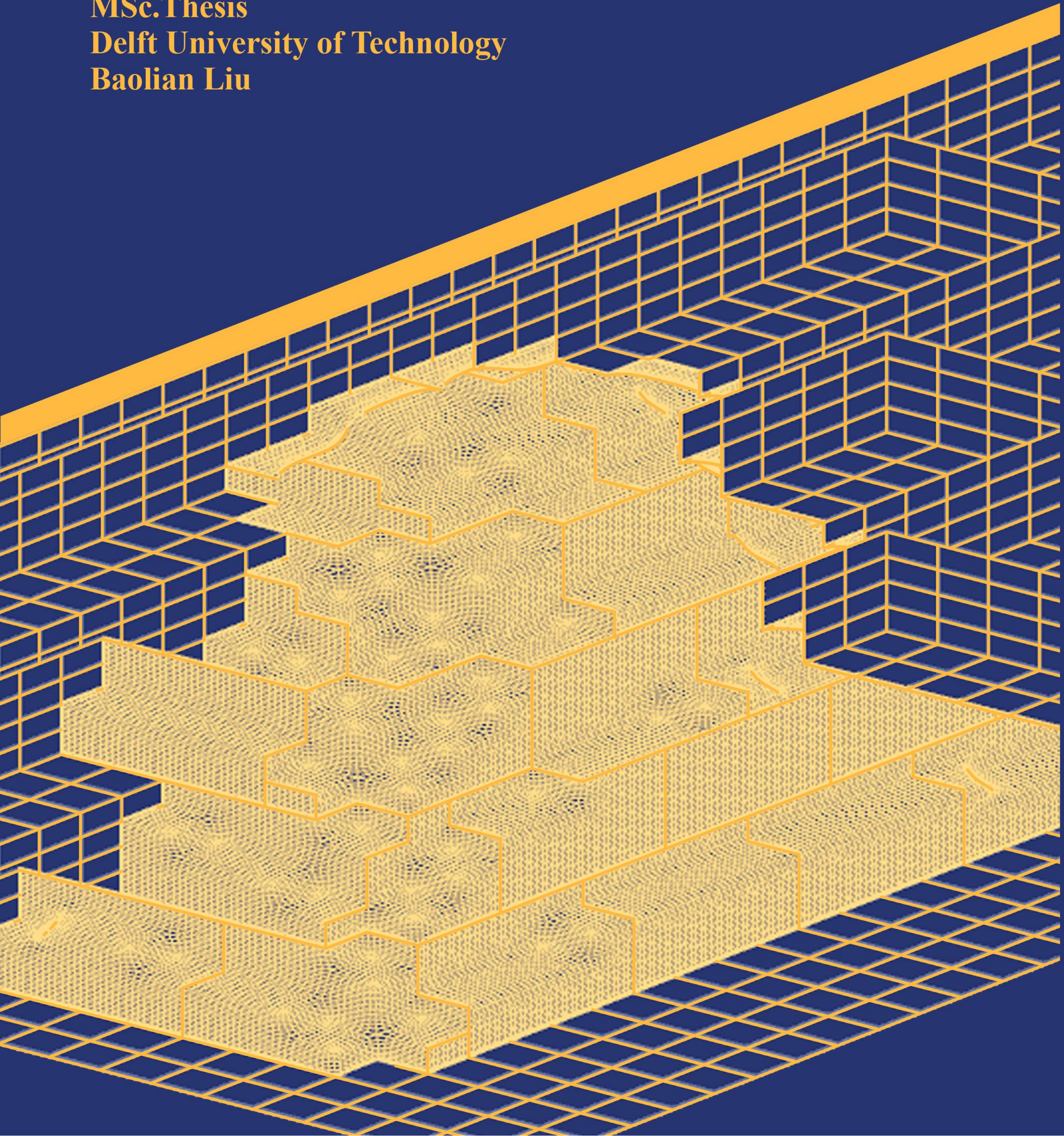

Master Thesis

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Preface

Learning the technology for practical construction is always the duty of architects when they design and helps them design better architecture, that's the motivation for me to choose Building Technology as my master track because I believe details determine the quality of a building.

Computer-Aided Design is what I found a powerful tool to help human being conducting their thoughts of design through a logical and detailed way during the two years' study. This method is undergoing rapid development in the past decades and brings innovations for generating complex but interesting shapes of architecture, leading people to a brand-new era. That's the reason I choose design informatics as the field of my graduation topic because I believe this method will somehow influence the traditional design methods in the future.

Shell/Vault structures always fascinate me due to their aesthetic beauty and their long history. During the course of Earthy at the beginning of the second semester, given by Pirouz Nourian and Shervin Azadi, their enthusiasm on shell structures influenced me and I found more advantages of these compression-only structures such as high structural efficiencies. Therefore, I decided to continue this topic as my graduation project to explore a more convenient methodology for construction of such structures aided by computational design to give a small contribution to the future development of shell structure design.

Baolian Liu
July 2022

Acknowledgements

I would like to thank my mentor Pirouz Nourian, who led me through the door of Computer-Aided Design and whose enthusiasm of this project has always encouraged me to overcome the difficulties. His ahead thoughts about the combination of shell structures design and computational method have always fascinated me and made me realize their significant influence in the future. I would like to thank my structural mentors, Simona Bianchi and Anjali Mehrotra, who would like to dedicate their help on this project and whose meticulous spirits on structural verifications have made this project more practical and realizable. As a beginner of python learner, I would also like to thank Shervin Azadi for his selfless help when I was facing technical problems of python. I would like to thank Geert Coumans, the delegate from board of examiners, who presented at each stage of my presentations and gave me really helpful feedbacks.

A big thank to my friend and project partner, Qinglu Chen, who designed the first part of this project with me and gave me spiritual supports during the design process. I also would like to thank my friends Lincheng Jiang, Wenyan Cai, Xiang Fang, and other friends who have ever helped me, their existence warmed my life in my two years' study in this foreign land. Many thanks to my boyfriend, Jinfeng Hang, who always gave me supports and accompany me to walk through the down moods.

In the end, I would like to thank my parents, without whose support I would not able to pursue and finish my master study.

Baolian Liu
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Abstract

With the advent of Computer-Aided Design, the design and fabrication of complex free-form shells have become easier to achieve. However, this results in extensive usage of custom-made formworks for the production of shell components and falseworks which provide support for the shell during the construction process. Therefore, a modular design method is proposed for generating form-active spatial structures out of stackable blocks of a few types, having in mind its potential applications such as housing. Instead of shells, spatial masonry structures are thus the main consideration in the design process considering building on top of a vaulted ceiling. By designing a 3D interlocking grid and introducing a four-step topological design that is coupled with structural verification processes based on finite element modelling and discrete element modelling simulations, the geometry of interlocking stackable modular blocks can be automatically generated for constructing such spatial masonry structures. The proposed method ensures that the designed vaults are modular, reconfigurable, and self-supporting during construction, thus increasing the efficiency of mass production while allowing for combinatorial mass customization in designs.

Keywords: 3D Tessellation, Stereotomy, Modular Vaulting, Funicular Shell, Form Finding, Masonry Construction, Structural behaviour, Assembly of interlocked element, Reconfigurable buildings

Contents

Chapter 1

Research Framework

1.1 Introduction

This section provides an overview of the research project. It first elaborates about the context of the research regarding masonry shell structures and its developments in recent years, then proposes the existing problems, design objective and research questions regarding the described context. Subsequently, its research scope (boundaries of the research) and research methodology are settled for the design of the project, then its social relevance, scientific relevance, planning and organization etc. are respectively discussed in the following subsections.

1.2 Context

1.2.1 Masonry Shell Structure

Masonry is a traditional construction technique which could be dated back to ancient times and is of vital importance during the development of world's architectural heritage. A glorious fact is that many historical buildings are made in masonry and numerous greatest vaults and domes relied on these single units being laid and bound together to achieve grand space. Therefore, they have nonnegligible value not only in historical and cultural aspect, but also in economic aspects [1].

When it comes to shell structure, there is usually a vague definition between shell and vaults (or domes). *Shell Structures* have characteristic advantages in their aesthetic qualities as well as dealing with the problem in engineering (such as large span) due to their efficient load bearing behavior which is the result of a double curved geometry. From a narrow sense, contemporary shell structure emerged in 1920s [2], However, in a broad sense, shell structure are structures enclosing buildings with smooth continuous surface (such as vaults and domes) which could be traced back to its prosperity in Middle Ages [3] and that will be the case in this thesis being talking about.

Among the categories of shell structure, *masonry shell structure* is one of them. Despite the terminology, it cannot be separated from other categories such as concrete shell or vault shell structure because of the similar material use or geometry overlapping. Taking Pantheon in Rome as an example (Figure 1.1), it can be determined as masonry dome as well as concrete dome because it consists of material of concrete, but at the same time is a combination of different kinds of single units using masonry construction.

(a) (b)

Figure 1.1 *(a) Pantheon in Rome (b) Section of Pantheon [4]*

1.2.2 Development of Masonry Shell Structure

Although it is a fact that there were fading interests on masonry shells nowadays, there is a revived tendency in the last two decades. Due to the introduction of Computer-Aided Design and the development of modelling techniques, a new language emerged for these doubly-curved surfaces. Besides, techniques of digital fabrication methods (i.e. CNC wire cutting) are also in progress to enable more complex and flexible shapes becoming reality.

However, the issue of assembly has not raised widespread serious concern. Constructing such complex shapes brings the problem of extensive usage of custom-made formworks for production of shell components and falseworks to be used as support in the middle of construction process. Despite there were some ongoing researches regarding this topic by Block Research Group [5] and Gene T.C. KAO [6], these are far more enough. A growing body of literature has investigated this issue of assembly by adding chains [7], cable-net [8], or fabric formworks [9], but these approaches result in other difficulties of extra work on formwork and falsework structure verification and analysis.

Besides, the shell components and falseworks of monolithic structures made out of free-form pieces tend to be single-use as it is unlikely for the components to have any other use at the end of their life-cycle, being tailor-made for on particular shell structure. Consequently, the high costs of free-form shell structures have limited their application in special-purpose buildings such as stadiums or concert halls. Decreasing the construction cost of such form-active structures can unlock the potential of using them in ordinary multi-storey buildings, especially for reducing the embodied carbon of building floors by minimizing the use of tension-bearing materials (steel reinforcement) by adopting funicular geometries conducive to the natural flow of forces instead of relying on the bending strength of materials in uncomfortable settings.

1.3 Problem Statement

As mentioned above, it is a huge problem for the extensive use of falseworks (Figure 1.2(a)) which is labour intensive as well as time-consuming, therefore, this issue would be the first main concern in this project. Besides, it's a pity that the development of the second-floor vault architecture attracts little concern, hence its potential applications on such spatial masonry structures (Figure 1.2(b)) such as housing would be the second main concern during the design process. Therefore, the research problem can be represented in the following two parts:

1. To deal with the problem of extra usage of custom-made falsework during masonry shell construction.
2.To explore the potential for application of masonry shell structure on multi-storey buildings

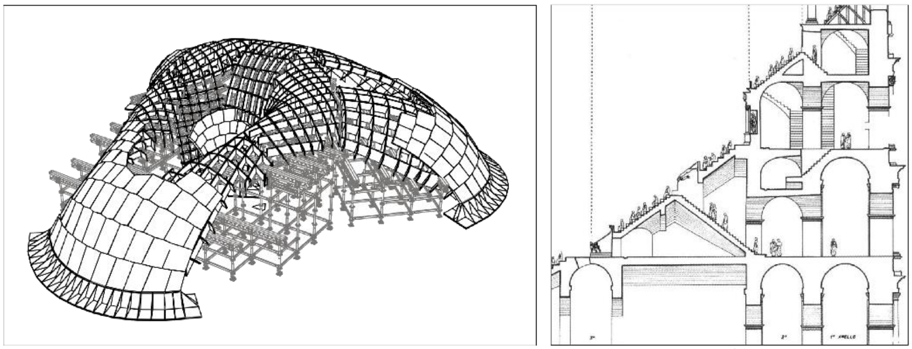

(a) (b)

***Figure 1.2** (a) Custom-made falseworks for constructing Armadilo Vault [10] (b)Example of multi-storey masonry vault structure (Section of Colosseum) [11]*

1.4 Objective

According to the two aspects of problem statement, the design objective can be thus stated as the following two parts:

1. To design a grid system consisting of limited stock of voussoir geometry of interlocking dry-stackable masonry blocks for mass-customization for form-active architectures, thus ensure equilibrium status during construction process.
2. To design a computational algorithm to approximate the interlocking grid system for the shape of form-active masonry vaulting structures.

1.5 Research Questions

Main research question: How to design a finite set of interlocking stackable blocks for generating a plethora of discrete form-active architectures?

Sub research questions may include:

1) How to generate new voussoir geometry to improve the stability during construction process?
2) How to ensure interlocking between masonry blocks to realize self-supporting during construction?
3) How to design the layering of masonry blocks to ensure a smooth surface for the second-floor construction?
4) How to remove the unnecessary part within masonry blocks to ensure a smooth intrados?
5) How to verify the feasibility of the new construction method?

1.6 Scope

This research relates to several fields of subjects within shell structure design, beginning with shape optimization, structural mechanics, geometry & topology, masonry construction and finally computer science. However, it will eventually jump out of the scope of this project (Figure 1.3).

The research focuses on prototyping a computational algorithm for users to generate 3-dimensional tessellations for masonry shell construction. Form-finding methods (DR, FDM, TNA) will be the preliminary research outside the scope of this algorithm. The research on tessellation, masonry construction, structural analysis and computer science will be a foundation for the generation of the algorithm. Eventually, the algorithm generated will be evaluated by application on the test cases.

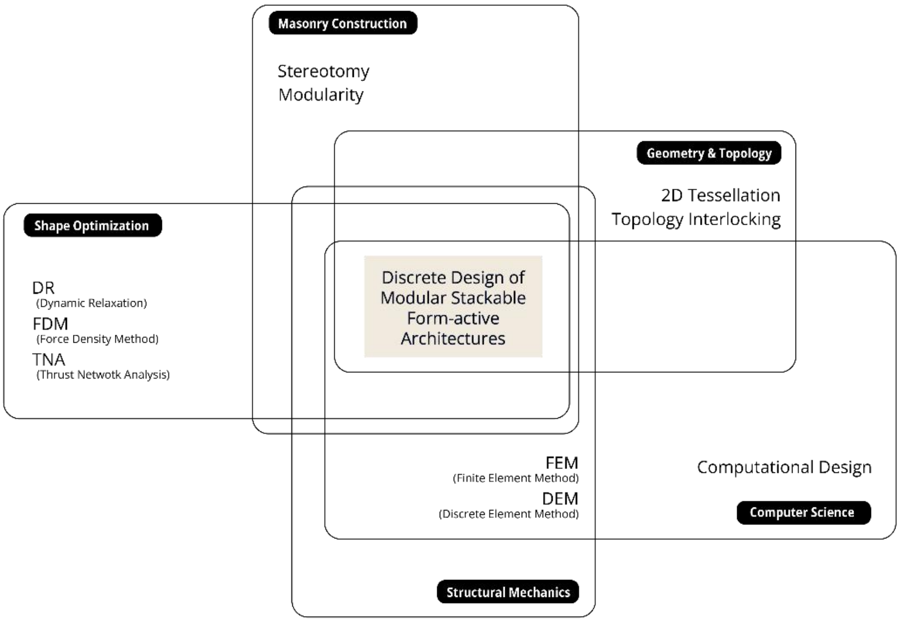


__Figure 1.3__ Position of this thesis within related fields of research

1.7 Social Relevance

As mentioned above, masonry shell structures are drawing far more attention than before due to its aesthetic feature as well as load-bearing efficiency. Therefore, the construction process appears to be particularly important because the traditional way of construction using customized falsework seems a waste of resources and labor intensive. Also, it becomes essential to explore the applications of shell structure for common multi-layer buildings. With the aim of reducing the potential formwork and falseworks during the construction of masonry shell structures and exploring application of shell structure for normal buildings such as housing, it would be in favor of accelerating construction process as well as improving structural efficiency for multi-storey architectures.

1.8 Scientific Relevance

This graduation project is within the topic of Design Informatics under Genesis Lab within Building Technology track. The goal of this research is aiming at creating a computational method to minimize the gap between digital masonry shell modeling and its assembly process in practice. This tool would provide convenience for shell designers to generate assembly-aware masonry shell structures.

1.9 Research Methodology

This section explains a proposed methodology that will be used during the design process. The design process is based on two main steps to realize the fulfillment of the algorithm as is shown in Figure 1.4.

A. Literature Study

In this step, a broad range of study of literature is conducted for the following steps and all knowledge required for the thesis is obtained. The literatures studied are with scientific background, but are also with articles and publications from commercial sources. The website frequently used are Researchgate, Scopus, Google Scholar, TU Delft Repository and ETH Block Research Group Repository. The key words searched on the website are based on the design concept proposed, which are words related to “topology interlocking”, “segmented shell”, “dry-fit block”, “stackable modular blocks”, “masonry structures” etc. as well as the key words of research for preliminary foundation such as “form-finding” and “thrust network analysis” etc. The results are evaluated based on its relevance and reliability, year of publication and information about author. A reference management software Zotero is used for organization of all the literatures.

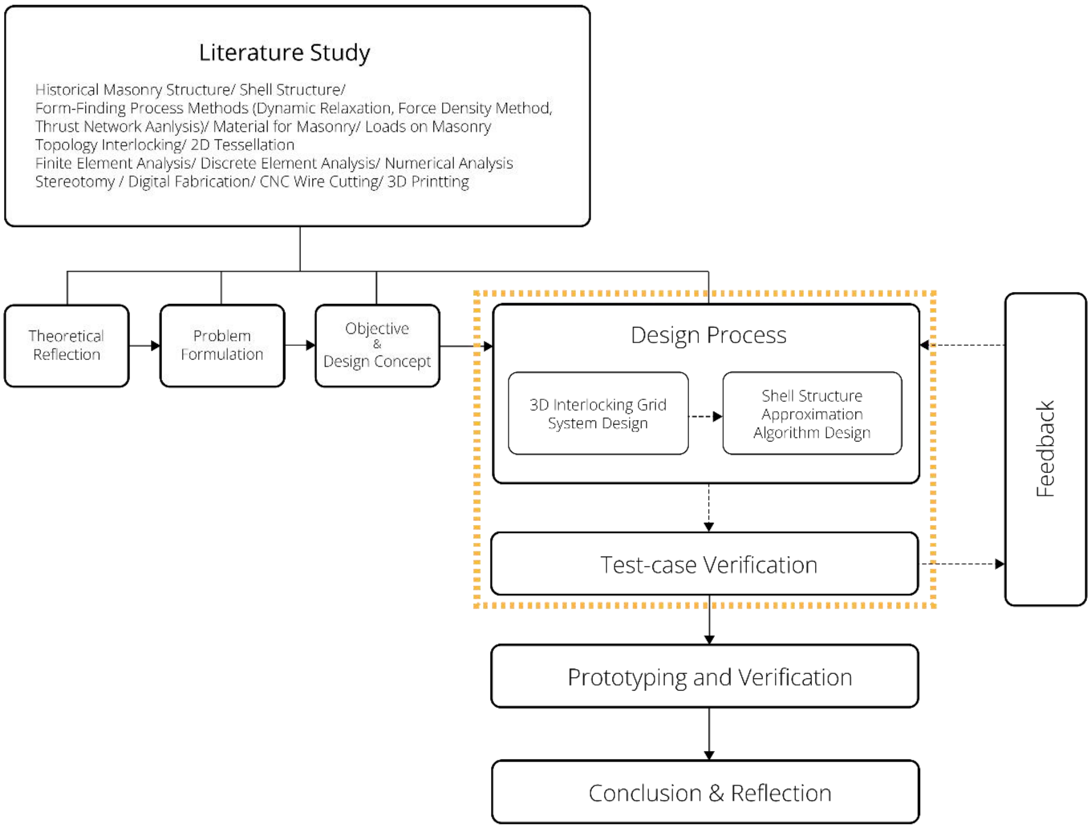


***Figure 1.4** Research Methodology*

B. Problem Formulation and Objective

Problem statement and research questions are proposed in this step based on the literature background in the first step. Sub problems and sub questions are also raised up. Next, design objective and design concept are developed as solutions to solve the proposed problems.

C. Design Process

In this step, the design of grid system and algorithms for approximating shell structures are developed as is illustrated in the orange rectangle in Figure 1.5. the development of the feasible concept in previous step are represented in virtual models and algorithms. Then prototyping the algorithm is conducted in python for the realization of design concept.

In this research, the design process is divided into two parts. The first part is to develop a 3D interlocking grid system and the second part is to approximate the interlocking system into form-active shell structures. It is worth noting that necessary structural verifications are in the first part to test the stability of the designed grid. The second part is developed within COMPAS [12] framework - an open-source framework that provides geometry processing independent of commercial CAD software – to conduct the algorithm design.

D. Test-case Verification

After the two parts of design process, several test-case of shell structures will be implemented to test the feasibility of the prototyping component. If it doesn't work smoothly and successfully, feedback will be transferred back to the phase of design method to be improved.

E. Prototyping and Verification

When the whole design process is settled, prototyping and fabrication would be conducted in representation of physical models to see the stability of its structural behavior.

F. Conclusion & Reflection

At last, conclusion and reflection of the methodology are discussed, as well as the limitations regarding this methodology. Summarization of the aspects needed to be improved are elaborated in the end.

1.10 Planning and Organization

The process of this research is conducted sequentially according to the planning and organization (*Figure 1.5*). The process is divided into 5 phases for finishing this project and they are aligning with the research process:

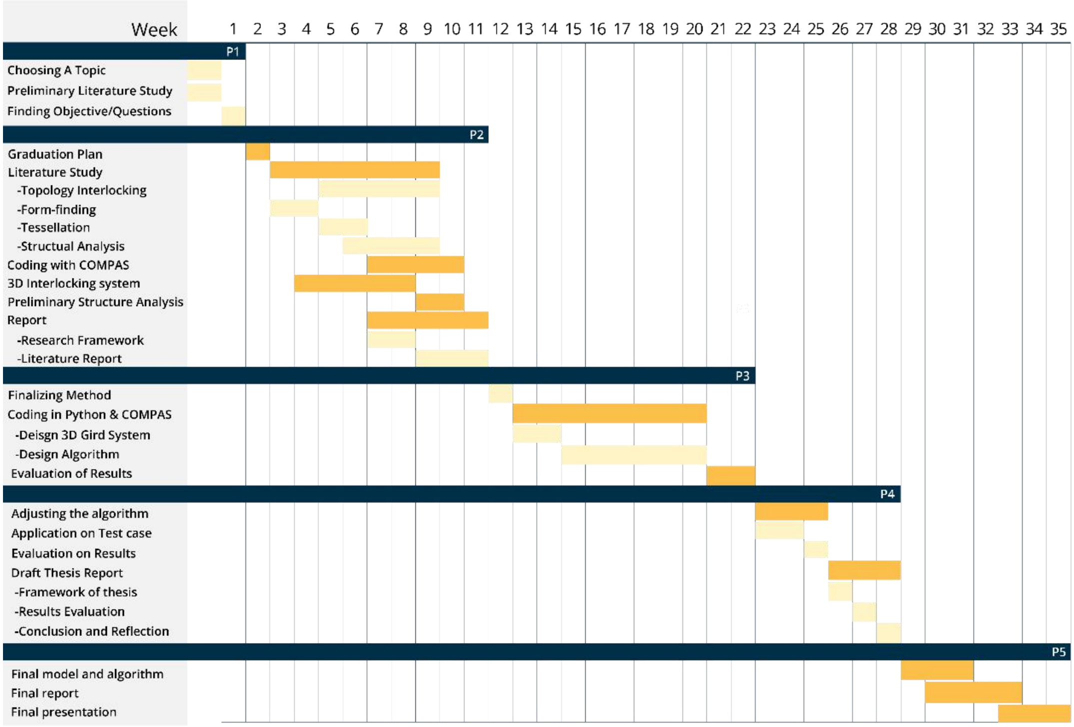


Figure 1.5 *Planning and Organization for the project*

- **Phase 1** includes preliminary literature study and context study, in which research questions and problem statement would be proposed and objective would be defined.
- **Phase 2** conducts detailed literature study based on the objective in phase 1 and develops detailed draft design concept for the realization of design objective. Then after design methodology is settled, preliminary design 3D interlocking system tessellation would be conducted together with Q. Chen.
- **Phase 3** continues developing design details individually. Then after 3D interlocking system tessellation is finished, the algorithm of 3D tessellation shell structure transformation will be designed in python and COMPAS.
- **Phase 4** conducts test case applications and results evaluation.
- **Phase 5** shows the final result and final report.

Chapter 2

Literature Study

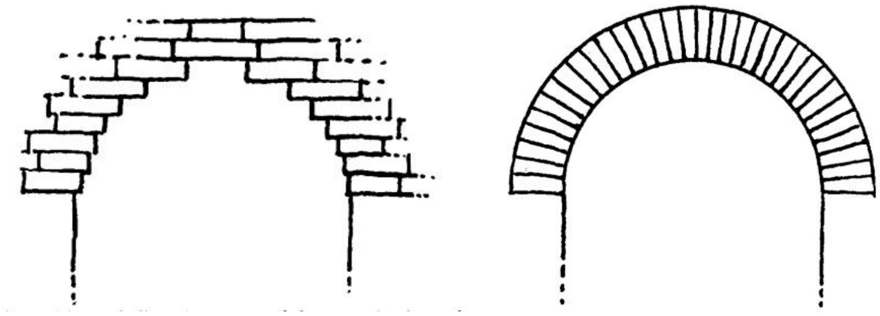
2.1 Masonry Construction

2.1.1 Masonry Material Property

Masonry constructions, whose history could be dated back to 8000 years ago, developed at the beginnings of the earliest urban civilization. By replacing the ancient material use of wood and straw, masonry enabled stronger construction and longer-standing structures. The common peculiarity of masonry material is that their strength in tension is much lower than compression [4]. Therefore, masonry shell structures would maximize their advantages when the structure is compression-only.

2.1.2 Masonry Layering Method

The most common layering of masonry vault in history is called Corbelled Dome and True Dome (*Figure 2.1*), of which true arches, vaults, domes developed from the use of corbels [13]. In the layering method of Corbelled Dome, the stones are laid in horizontal course, with the first layer extending a short distance beyond the wall and each layer a little more the one below until the gap is closed. The projecting stone acts as a cantilever and the stone above are subject in tension. The strength of the dome will be increased if the joints are at right angles of the line of thrust (*Figure 2.1(b)*). However, additional problem will be caused during construction process because the stones are no longer lined horizontally, wherever corbelled domes only need simple techniques to finish construction.



(a) **(b)**

Figure 2.1 *(a) Corbelled Dome-the joints are horizontal (b)True Dome-the joints are at right angles of the line of thrust [13]*

2.1.3 Corbelled Dome

Due to its advantage of more convenient method during construction, corbelled dome is prevailed from ancient history, especially well-known in pre-Hellenic Minoan civilization of Mycenae [14]. Many famous cases of corbelled domes could be found during history process, as is stated in time line [15]. Despite some drawbacks, on obvious advantage is that it has the capacity of transmitting compressive force along parallels when the construction of the ring is not completed [16]. This can be proved by a partially collapsed dome (Figure 2.2), where the remaining structure is still able to stand without the collaboration of collapsed surrounded structure.

Figure 2.2 *Example of partial collapse of Trullo Vault [16]*

2.2 Topological Interlocking

Topological interlocking is a design concept in which simple elements are arranged in a way that an entire structure is held together by kinematic constraints inflicted through the form and through the mutual arrangement of the elements [17]. The idea of using topological interlocking blocks can be dated back to ancient Inca structures which allowed assembling stable and self-aligning structures without any additional joints. Currently, the most prevailed topology interlocking is based on 3 basic types (Figure 2.3).

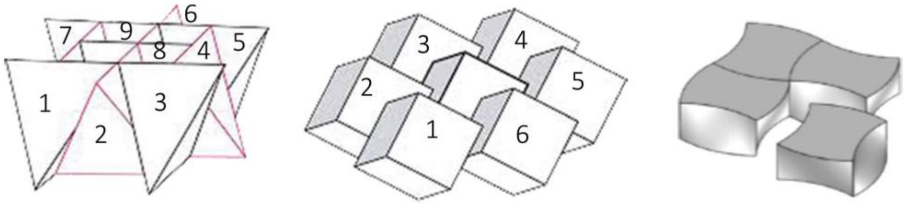


Figure 2.3 *Three types of topological interlocking (a) Repetitive of tetrahedrons (b) Interlocking assemblies based on a hexagonal grid (c) Assembly of Osteomorphic blocks [18]*

2.2.1 Type 1: Interlocking Assemblies of Repetitive Tetrahedron

Early researches have brought the idea of reciprocal structural systems, studied by several scientists, including Leonardo da Vinci (Figure 2.4). This reciprocal structure is made of wooden beams with one element leaning on the other two to support the whole structure. Later development of this principal by Sebastian Truchet has generated the first type of topology interlocking (Figure 2.5). This type of interlocking based on the opposite direction of edges to ensure the constraints of movement. However, this type of interlocking brings the problem of porosity in between the structure when they are assembled together (Figure 2.6).

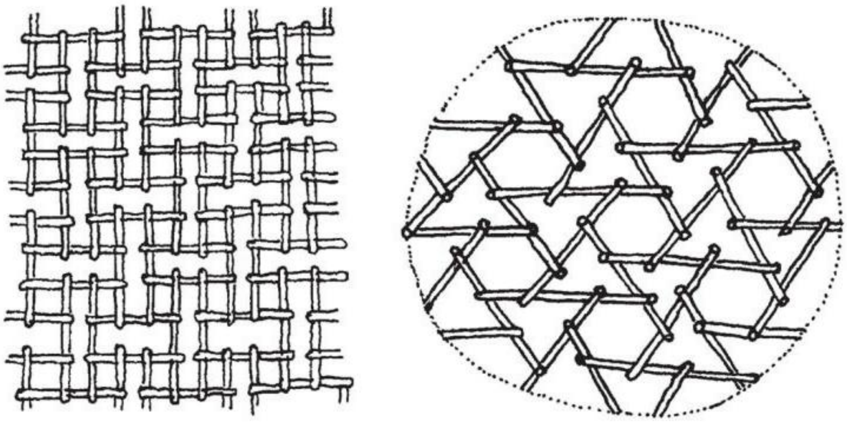


Figure 2.4 *Reciprocal Frame Structures by Leonardo da Vinci (source: Larsen, 2008)*

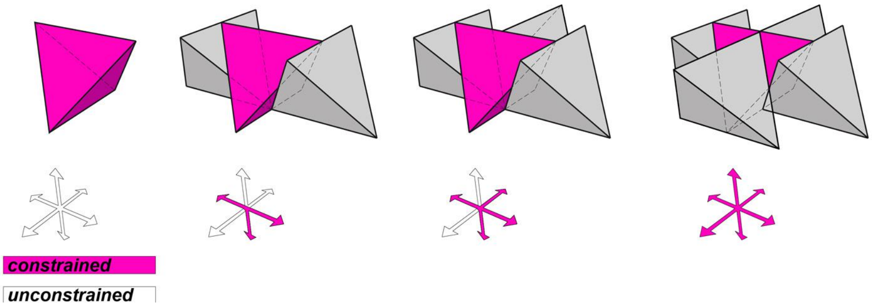


Figure 2.5 *Tetrahedron configuration of interlocking properties. From left to right, the middle tetrahedron is constrained increasingly by its neighbors.[19]*

Figure 2.6 *The problem of porosity of Type 1 when they are assembled together (Source: Philipp Mecke)*

2.2.2 Type 2: Interlocking Assemblies based on a Hexagonal Grid

Another tiling consisting of hexagon (honeycomb) forms the second of topology interlocking by A.J.Kanel-Blov [20]. The arrows on every edge of hexagon are assigned arrows in a way that any hexagon within the tiling the arrows heading inwards and outwards alternatively. By considering each hexagonal shape as the mid-section of a polyhedron, the inclined faces can be extended into an interlocking arrangement of cubes, octahedra, or dodecahedra (Figure 2.7 and 2.8). In this type of interlocking system, any element is constrained within all six directions: up, down, left, right, front and back direction. The middle cannot be taken away unless one of the neighbor six elements is missing.

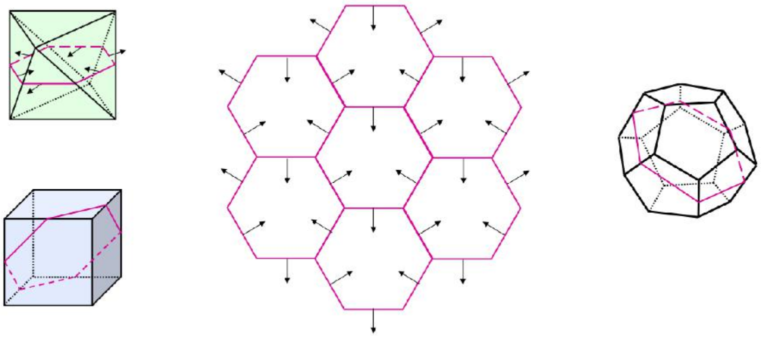


***Figure 2.7** Hexagonal tiling of the plane and the associated platonic solids with their respective hexagonal middle sections [20]*

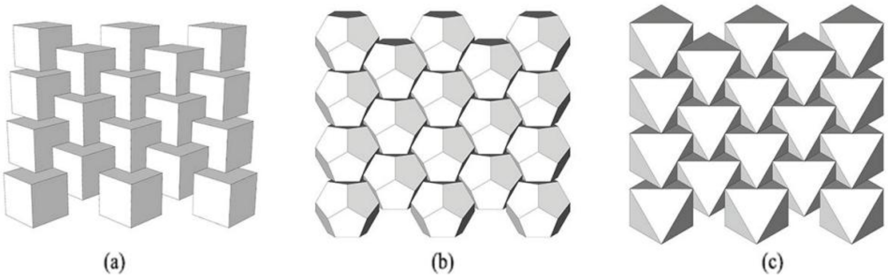
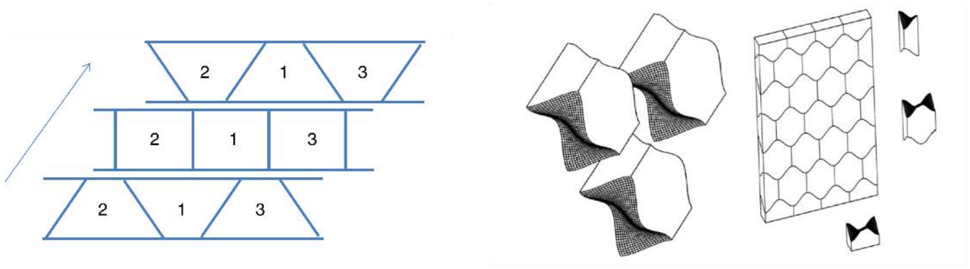


***Figure 2.8** Interlocking assemblies based on a hexagonal grid. a) cubes; b) dodecahedra; c) octahedral (Source: Kanel-Belov et al., 2008)*

2.2.3 Type 3: Interlocking Assemblies of Osteomorphic Blocks

The previous two types realize interlocking by their geometry and relative position. The third type is different in logic. This type generates topologically interlocking structures based on considering sections normal to assembly plane. The first type could be extended with a continuous transformation of a section as it moves through the assembly (Figure 2.9(a)). The simplest implementation of these shapes is Osteomorphic blocks (Figure 2.9 (b)).



(a) (b)

Figure 2.9 *(a) Transformation of a normal section within a single element; b) Osteomorphic blocks and their interlocking within a planar structure [17]*

2.2.4 Comparison between the Three Types

Comparing the three types of topological interlocking methods, it's not hard to tell that despite the exquisite surface for generating interlocking for Type 3, this type has the difficulty in manufacturing due to this doubly-curved surface. For Type 1, the obvious problem would be the porosity generated when the structure is assembled together, and this problem are not solvable at this stage. For Type 2, it seems to have a huge potential to be developed into a spatial grid system because of its regularity. However, this type of interlocking results in a gap when the second layer of polyhedral is added, thus preventing it from forming a space-filling system. Nevertheless, Type 2 still has the biggest potential to be developed into a space-filling system by redesign based on this original grid, thus this type is the foundation for the first design part.

2.3 Funicular Form-finding

The shape of form-active/funicular structures are formed by a hanging chain or a string loaded with any number of weights. In structural terms, these structures are in pure tension/compression and no portion is subjected to bending [21]." Therefore, a form-finding process is required to generate this kind of structure and research into computational form finding can be traced back to early 1960's. By definition, "form-finding is a process to find an (optimal) shape of a form-active structure that is in (or approximates) a state of static equilibrium [21]." Early examples of form finding by using physical models include the hanging chains of Antoni Gaudi, and the earliest example of structural form-finding for an arch was published by an English Engineer and Scientist Robert Hooke (1635-1703) (Figure 2.10(a)). Other examples include the parallelogram of forces illustrated by Simon Stevin (1548-1620), who was one of the first to develop the mathematical representation of forces as vectors. He showed several examples of suspended weights creating funicular shapes in both two and three dimensions (Figure 2.10(b)).

Currently, existing form finding methods can be divided into three categories: Stiffness Matrix Method (SM), Geometric Stiffness Methods, and Dynamic Equilibrium Methods (Figure 2.11) [22]. In this Section, three representative methods are discussed: Force Density Method (FDM), Dynamic Relaxation (DR), and Thrust Network Analysis (TNA). The concept of Force Density Method was presented by Schek in 1973 and Dynamic Relaxation was first brought up by R. Barns in his PhD thesis in 1977, not mention the most precedent of stiffness matrix method in 1964. The newly development method of Thrust Network Analysis is developed by P.Block in his PhD thesis in MIT (2009).

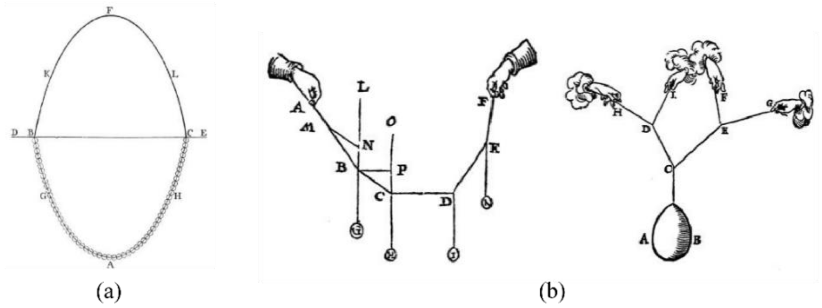

(a) (b)

Figure 2.10 *(a)Hooke's hanging chain and the inverted rigid catenary arch (b)Diagram of funicular shapes by Stevin [21]*

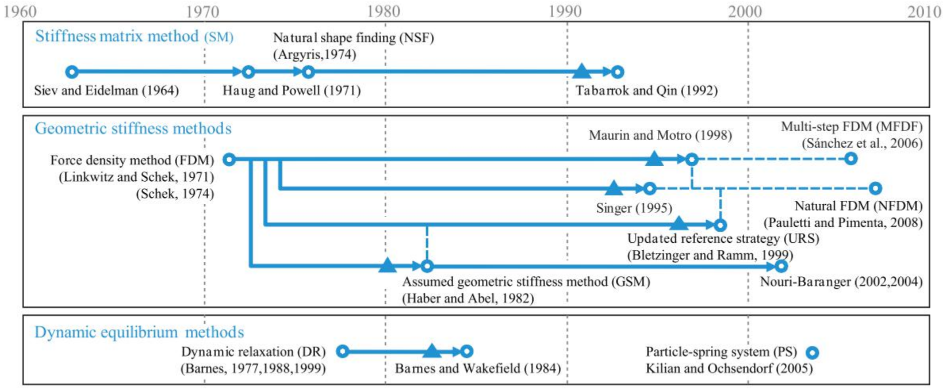


Figure 2.11 *Development and categorization of form finding methods with key references [22]*

2.3.1 Force Density Method

Force Density Methods (FDM), whose key concept is by using "force-length ratio" or "force densities" in the branches as network description parameter (degree of freedom). In other words, by prescribing one single quantity for each branch, namely force density, the appropriate state of equilibrium by solving one system of linear equations is obtained [23]. This result in an advantage of no necessity of information of material input. With the second step of materialization, the shape generated will not change. Also, FDM is especially a good choice for free-form bending structures as solutions are generated from simple linear solutions [21].

2.3.2 Dynamic Relaxation

As for Dynamic Relaxation (DR), invented in 1965 by Alistair Day, is a numerical procedure that solves a set of nonlinear equations. The basis of this method is to trace step by step for small time increments, Δt, the motion of each interconnected node of the grid until the structure achieve static equilibrium [21]. Due to the traced motion of nodes, DR is extremely suitable for grid shells either made from initially straight elements or prefabricated curved elements.

2.3.3 Thrust Network Analysis

Thrust Network Analysis (TNA) as a newly developed form finding method, is appropriate for compressive funicular shells, thus particularly for any type of vaulted system in unreinforced masonry [21]. The concept of TNA is originated from graphic statics, a 2-dimensional method for producing reciprocal form and force diagrams (Figure 2.12), so called thrust lines. Then TNA developed this principal into 3D being able to define specific force densities for FDM [24]. It allows a flexible and controlled process for funicular structures under gravitational loading and through the use of intuitive graphical methods, it is able to gain control over the exploration of form [25].

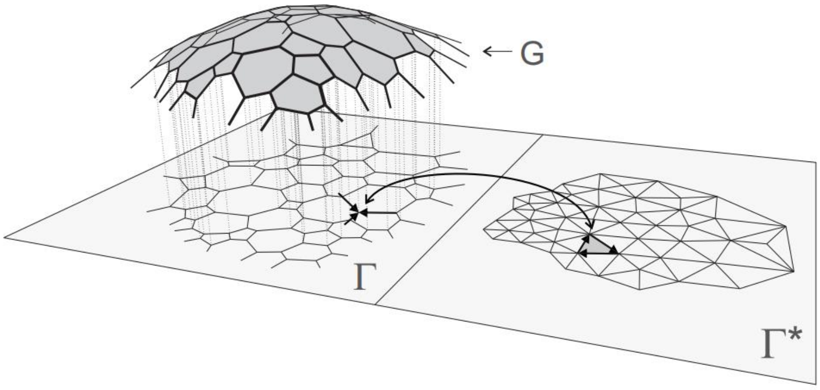


***Figure 2.12** The base theory of TNA (P. BLOCK 2009)*

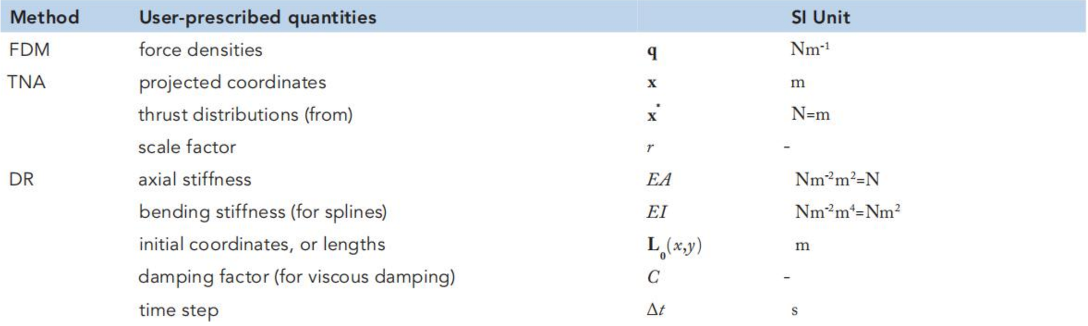
2.3.4 General Comparison

In this sub section, a general comparison is conducted between the above three methods, and to choose the most suitable one for the algorithm design. Every form-finding procedure consists of at least four parts: 1) a discretization for describing the initial geometry of the shell; 2) a data structure that stores the information of the shell structure; 3) Equilibrium equations that define the relationship between internal and external forces; 4) a solver, which describes how the equilibrium equations are solved [21]. The comparison is thus conducted between the last three aspects described above, namely prescribed user input, equilibrium equations, and solving strategy per method.

Considering the user-prescribed quantities, the input for FDM and TNA is reduced to a bare minimum, while the drawback of DR is the much larger number of parameters to control the process (Figure 2.13). It is an advantage for less prescribed inputs, though force densities are not physically meaningful and therefore difficult to control.

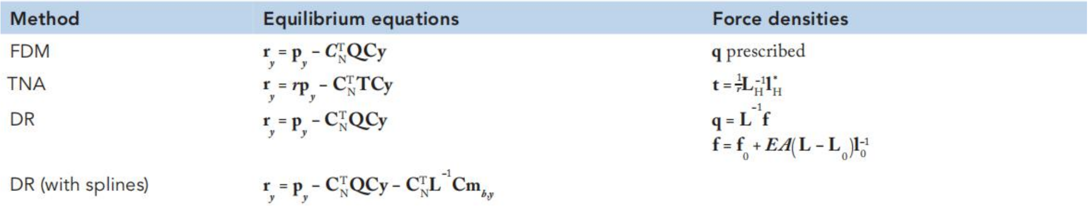
For equilibrium equations, if the force densities in each method are defined in the same way, then they should also yield identical shapes (Figure 2.14). This also means that whether the form-finding result was obtained through FDM, DR, or other method, the material properties or the initial length can be varied without disturbing shape as well as and static equilibrium as long as forces **f** and actual length **l** are kept the same.

Talking about solving strategy, TNA can be a special case of FDM. FDM can be formulated as linear equations if the force density **q** doesn't change per iteration, while DR shows a different strategy in solving the equilibrium equation (Figure 2.15).



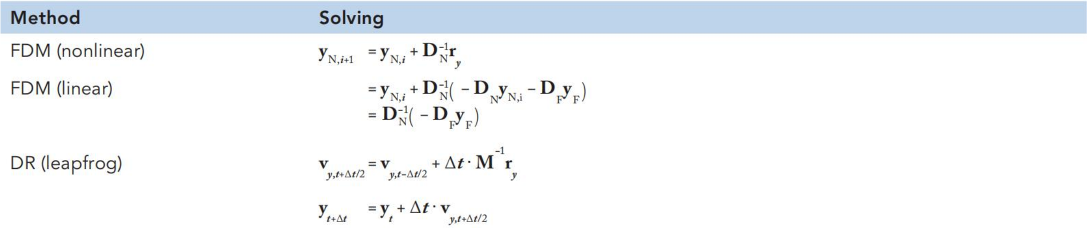
Method	User-prescribed quantities		SI Unit
FDM	force densities	$\mathbf{q}$	$\mathrm{Nm^{-1}}$
TNA	projected coordinates	$\mathbf{x}$	m
	thrust distributions (from)	$\mathbf{x}^*$	N=m
	scale factor	r	-
DR	axial stiffness	EA	$\mathrm{Nm^{-2}m^2{=}N}$
	bending stiffness (for splines)	EI	$\mathrm{Nm^{-2}m^4{=}Nm^2}$
	initial coordinates, or lengths	$\mathbf{L}_0(x,y)$	m
	damping factor (for viscous damping)	C	-
	time step	Δt	s

***Figure 2.13** Comparison for use input per method [21]*



Method	Equilibrium equations	Force densities
FDM	$\mathbf{r}_y = \mathbf{p}_y - C_\mathrm{N}^\mathrm{T}\mathbf{QCy}$	$\mathbf{q}$ prescribed
TNA	$\mathbf{r}_y = r\mathbf{p}_y - \mathbf{C}_\mathrm{N}^\mathrm{T}\mathbf{TCy}$	$\mathbf{t} = \frac{1}{r}\mathbf{L}_\mathrm{H}^{-1}\mathbf{l}_\mathrm{H}^*$
DR	$\mathbf{r}_y = \mathbf{p}_y - \mathbf{C}_\mathrm{N}^\mathrm{T}\mathbf{QCy}$	$\mathbf{q} = \mathbf{L}^{-1}\mathbf{f}$ $\mathbf{f} = \mathbf{f}_0 + EA(\mathbf{L} - \mathbf{L}_0)\mathbf{l}_0^{-1}$
DR (with splines)	$\mathbf{r}_y = \mathbf{p}_y - \mathbf{C}_\mathrm{N}^\mathrm{T}\mathbf{QCy} - \mathbf{C}_\mathrm{N}^\mathrm{T}\mathbf{L}^{-1}\mathbf{Cm}_{b,y}$	

***Figure 2.14** Equilibrium equations in vertical direction per method [21]*



Method	Solving
FDM (nonlinear)	$\mathbf{y}_{\mathrm{N},i+1} = \mathbf{y}_{\mathrm{N},i} + \mathbf{D}_\mathrm{N}^{-1}\mathbf{r}_y$
FDM (linear)	$= \mathbf{y}_{\mathrm{N},i} + \mathbf{D}_\mathrm{N}^{-1}(-\mathbf{D}_\mathrm{N}\mathbf{y}_{\mathrm{N,i}} - \mathbf{D}_\mathrm{F}\mathbf{y}_\mathrm{F})$ $= \mathbf{D}_\mathrm{N}^{-1}(-\mathbf{D}_\mathrm{F}\mathbf{y}_\mathrm{F})$
DR (leapfrog)	$\mathbf{v}_{y,t+\Delta t/2} = \mathbf{v}_{y,t-\Delta t/2} + \Delta t \cdot \mathbf{M}^{-1}\mathbf{r}_y$ $\mathbf{y}_{t+\Delta t} = \mathbf{y}_t + \Delta t \cdot \mathbf{v}_{y,t+\Delta t/2}$

***Figure 2.15** Solving strategy per method [21]*

To conclude, the methods differ in two aspects: how the internal forces are defined and how the resulting problem is numerically solved. Nevertheless, cases in which compression-only structures are more efficiently tackled by purely geometrical methods (i.e. FDM and TNA) [21]. And FDM is considered more suitable for this project due to its advantages of less required prescribed quantities and is easier to control.

2.4 Structural Mechanics

In this Sections, two representative methods for structural verifications are researched and elaborated, respectively Finite Element Modelling (FEM) [26] and Discrete Element Modelling (DEM) [27].

2.4.1 Finite Element Modelling (FEM)

FEM is originally developed as a computer-based simulation method for structural analysis in aerospace. Then it found its way in both design and analysis of complex structural systems, not only in aerospace but also in civil and mechanic engineering. In the late 1960s it also expanded its area into non-structural problems of fluids, thermo-mechanics and electromagnetics. As a branch within Computational Solids and Structures Mechanics (CSSM) classification, FEM is dominate the scene of linear problems and for nonlinear problems, its dominance is overwhelming [26].

The term FEM actually includes a wide spectrum of techniques that share common features with Direct Stiffness Method (DSM) (Figure 2.16), the most common implementation of FEM. Particularly, all major commercial FEM codes are based on DSM. Therefore, it is critical to understand its principal beforehand.

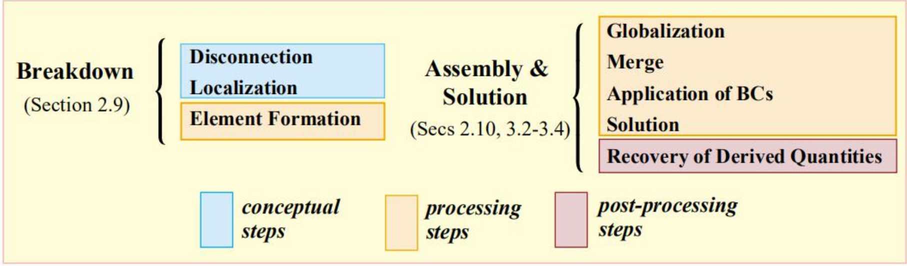


Figure 2.16 *The discrete stiffness method steps [26]*

The core of linear DSM is based on the linear relationship between **f** (joint force in each node) and **u** (displacement in each node). Linear static analysis deals with the static problems in which response in linear in the cause-and-effect sense [26]. In this case, a constant matrix K will be the linear relations between **f** and **u**, called master stiffness matrix as well as global stiffness matrix (Figure 2.17).

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{bmatrix} = \begin{bmatrix} K_{x1x1} & K_{x1y1} & K_{x1x2} & K_{x1y2} & K_{x1x3} & K_{x1y3} \\ K_{y1x1} & K_{y1y1} & K_{y1x2} & K_{y1y2} & K_{y1x3} & K_{y1y3} \\ K_{x2x1} & K_{x2y1} & K_{x2x2} & K_{x2y2} & K_{x2x3} & K_{x2y3} \\ K_{y2x1} & K_{y2y1} & K_{y2x2} & K_{y2y2} & K_{y2x3} & K_{y2y3} \\ K_{x3x1} & K_{x3y1} & K_{x3x2} & K_{x3y2} & K_{x3x3} & K_{x3y3} \\ K_{y3x1} & K_{y3y1} & K_{y3x2} & K_{y3y2} & K_{y3x3} & K_{y3y3} \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix}.$$

$$\boxed{\mathbf{f} = \mathbf{K}\,\mathbf{u}.}$$

Figure 2.17 *Linear relations between **f** and **u** [26]*

The process of DSM approach is mainly divided into two parts: breakdown process and assembly process (Figure 2.18). The first two of conceptual steps are not actually programmed, it actually begins at element-stiffness-equation forming step. Then in the assembly step, stiffness equations of each member will be merged into global stiffness equations. After applying boundary conditions, the modified equations will be submitted to a linear equation solver, which returns the unknown joint (node) displacements.

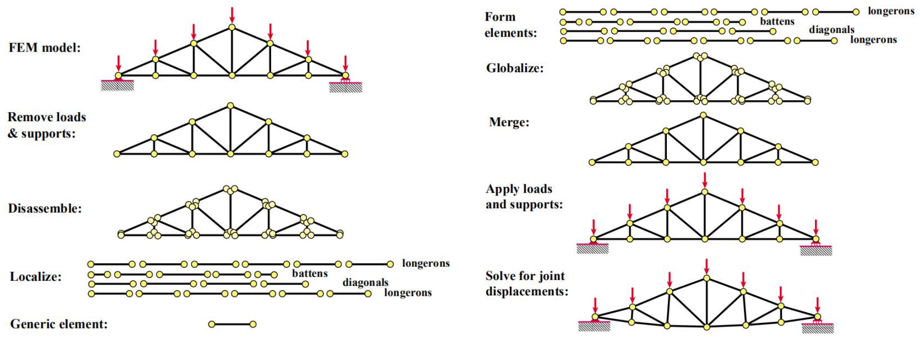


Figure 2.18 *Steps of DSM [26]*

2.4.2 Discrete Element Modelling (DEM)

Engineers often faces problems in which the mechanical behaviour of materials of structures consisting of separate components have to be predicted. Grains, stone blocks, bricks etc. are mechanical systems whose behaviour is fundamentally determined by the fact that they have a characteristic discrete internal structure which changes as a response to external effects (Figure 2.19). Compared with the continuum-based calculation techniques like FEM, DEM (developed by Cundall, 1971) drives from an inverse perspective: the material is viewed as an assembly of distinct bodies, the masonry units, acting along their boundaries (Figure

2.20). Therefore, it is able to reflect the phenomenon of a sliding failure of masonry vault or deformation of sandy soil.

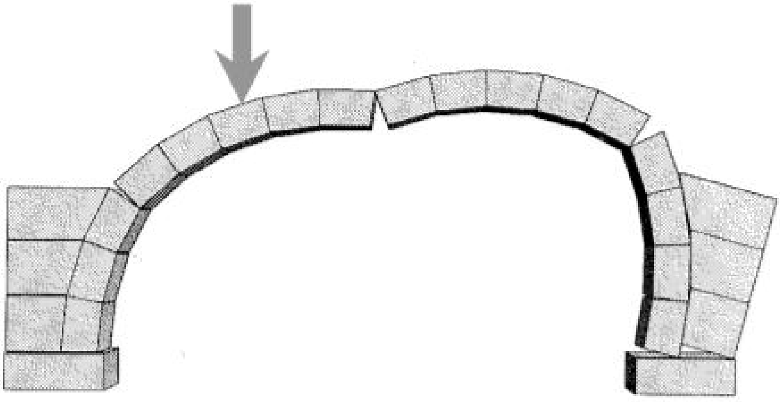


Figure 2.19 *Discrete element model of a masonry arch (Bicanic, 2003)*

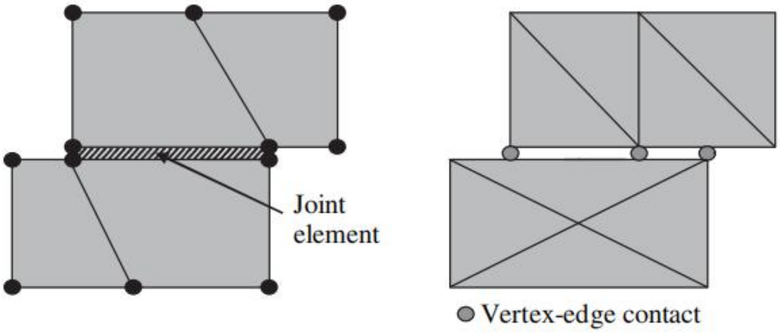
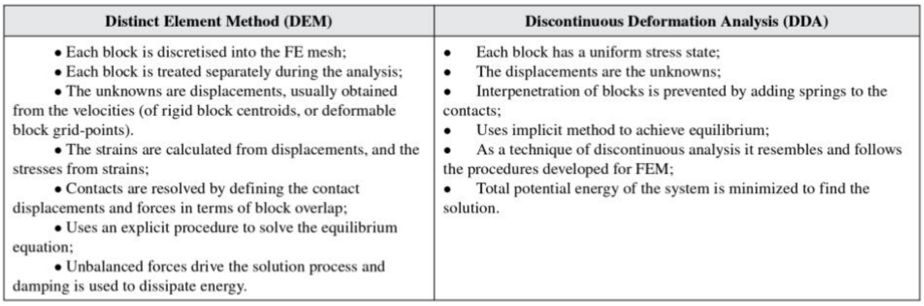


Figure 2.20 *Representation of contact between blocks (a) joint elements and (b) point (vertex-edge) contacts*

Any discrete element modelling consists of two basic components: the element, and the contacts between them. Then contact behaviour are represented in DEM code includes contact recognition and mechanical model based on geometry properties of the contact. It is worth mentioning that recent DEM codes used today in practice or in research are all based on time integration of the equations of motions within the system. Early versions of DEM based on stiffness matrix are not applied today but in a simplified manner (see Baraldi and Cecchi, 2017). In today's application the different time stepping method and the approximation of system is through a series of small but finite time intervals, expressed in the equation below [28].

$$\mathbf{M}(t) \cdot \mathbf{a}(t) = \mathbf{f}(t, \mathbf{u}(t), \mathbf{v}(t))$$

Another method called discontinuous distinct analysis (DDA) is developed by Shi and Goodman (1988) for solving stress-displacement problems for a jointed rock mass, in which the blocks are considered deformable but contacts are rigid which is opposed to DEM (Figure 2.21). Also, DDA used an implicit algorithm based on global stiffness matrix (unlike UDEC) [29].



Distinct Element Method (DEM)	Discontinuous Deformation Analysis (DDA)
• Each block is discretised into the FE mesh; • Each block is treated separately during the analysis; • The unknowns are displacements, usually obtained from the velocities (of rigid block centroids, or deformable block grid-points). • The strains are calculated from displacements, and the stresses from strains; • Contacts are resolved by defining the contact displacements and forces in terms of block overlap; • Uses an explicit procedure to solve the equilibrium equation; • Unbalanced forces drive the solution process and damping is used to dissipate energy.	• Each block has a uniform stress state; • The displacements are the unknowns; • Interpenetration of blocks is prevented by adding springs to the contacts; • Uses implicit method to achieve equilibrium; • As a technique of discontinuous analysis it resembles and follows the procedures developed for FEM; • Total potential energy of the system is minimized to find the solution.

Figure 2.21 *the major difference between DEM and DDA [25]*

2.4.3 General Comparison

Despite the core difference of FEM and DEM, the distinction between them has become somewhat blurred during the evolution of these methods as they borrowed features from each other [27], such as the development of Discrete-Finite Element Models.

- Some DE models with deformable blocks are based on physical assumptions essentially similar to some FE models with joint elements, and may thus be expected to lead to similar results.
- Methods derived from structural analysis, as well as some DE formulations that assemble stiffness matrices, resort to the solution techniques of nonlinear FE analysis.

Therefore, considering the fact that DEM is more suitable for analysing discrete masonry blocks created in this project, it becomes the primary tool for the structural verification process. However, due to its more mature development for FEM, it is considered as an auxiliary tool to verify the result of DEM and to deal with the cases DEM cannot tackle easily.

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