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Prestressed Concrete Girder Bridge

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Publication date
2020

Document Version
Accepted author manuscript

Published in
Examples for the Design of Reinforced and Prestressed Concrete Members Under Torsion

Citation (APA)
Benítez C., K. S., & Lantsoght, E. O. L. (2020). Torsion Design Example: Prestressed Concrete Girder Bridge. In E. Lantsoght, G. Greene, & A. Belarbi (Eds.), *Examples for the Design of Reinforced and Prestressed Concrete Members Under Torsion* (Vol. SP-344, pp. 149-167). (ACI Special Publication). American Concrete Institute.
https://www.concrete.org/publications/internationalconcreteabstractsportal.aspx?m=details&id=51728295

SP-XXXX—XX

TORSION DESIGN EXAMPLE: PRESTRESSED CONCRETE GIRDER BRIDGE

Kevin S. Benítez C. and Eva O. L. Lantsoght

Synopsis: The design of a cast-in-place, post-tensioned concrete, multi-cell box girder bridge under combined torsion, shear, and flexure is presented in this example. The bridge covers three spans of different lengths, supported by two abutments and two bents; its cross-section consists of three 12 ft (3.7 m) lanes, two 10 ft (3.0 m) shoulders, and two concrete barriers. The detailed procedure for the design based on ACI 318-14 is presented, and a comparison is done with the design results for: AASHTO LRFD 2017, EN 1992-1-1:2004, and MC-2010. With this example, the authors illustrate the differences between provisions of the aforementioned codes for design of torsional effects, outlining the different theories and approaches used for each of these.

Keywords: box girder bridges; prestressed concrete; shear; torsion.

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DESCRIPTION OF THE DESIGN TASK

Geometry and loads

This example concerns the design for torsion of a three-span bridge, based on the geometry of an example from the California Department of Transportation[1]. Figure 1 shows the elevation view of the bridge used for this example. The total length of the three spans is 412' (125.6 m): the first span is 126' (38.4 m), the center span is 168' (51.2 m), and the third span is 118' (36.0 m).

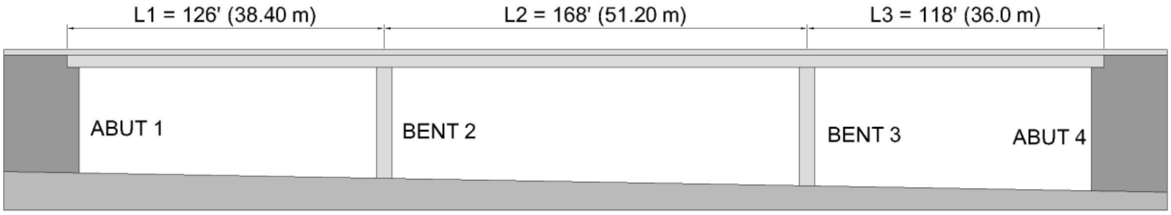


Figure 1—Elevation view of the bridge.

Figure 2 presents the cross-section of the bridge. The total width of the bridge is 58'-10'' (17.93 m). The bridge carries three 12' (3.7 m) traffic lanes, two 10' (3.0 m) shoulders and two 1'-5'' (0.45 m) concrete edge barriers.

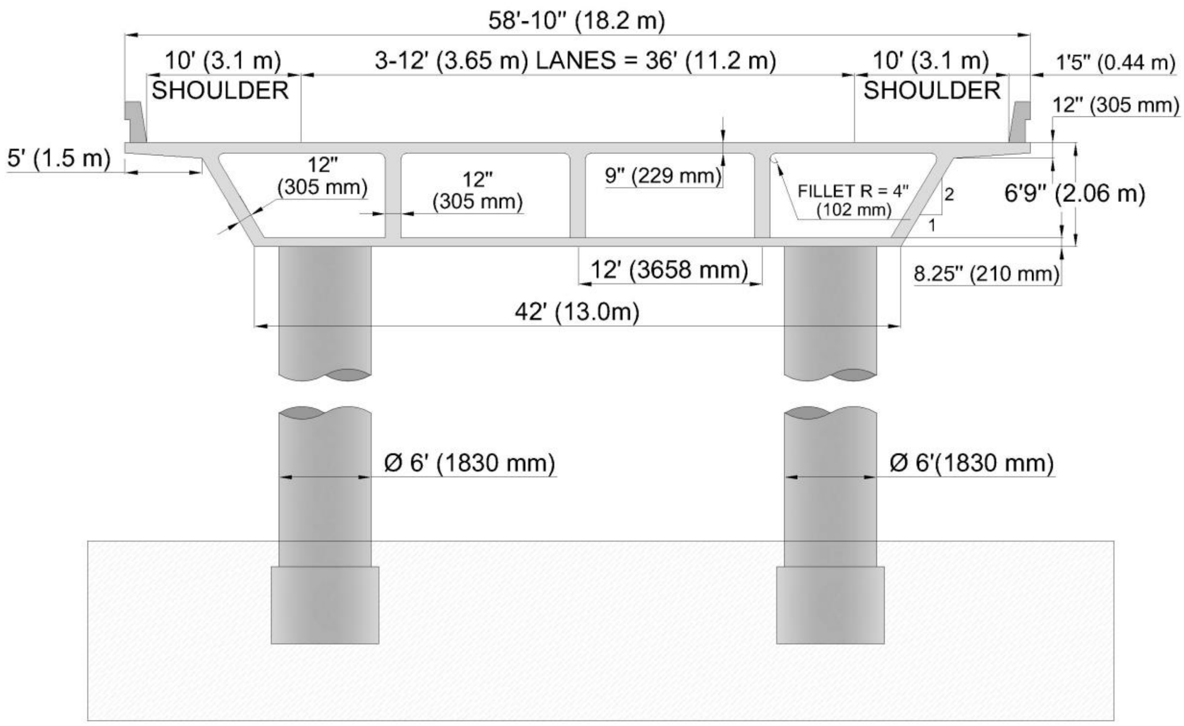


Figure 2—Cross-section of the bridge**.**

The loads that act on the bridge include the self-weight, load of the asphalt concrete (A.C.) wearing surface with a thickness of 3 in (75 mm), and the live load in accordance with AASHTO LRFD 2017[3] HL-93 (design truck plus design lane load).

Figure 3, Figure 4, and Figure 5 show the bending moments, shear forces, and torsional moments respectively resulting from the loading on the structure (self-weight, wearing surface, and live load). As expected, only the live loads result in torsional moments. CSiBridge[6] was used for the determination of the load effects.

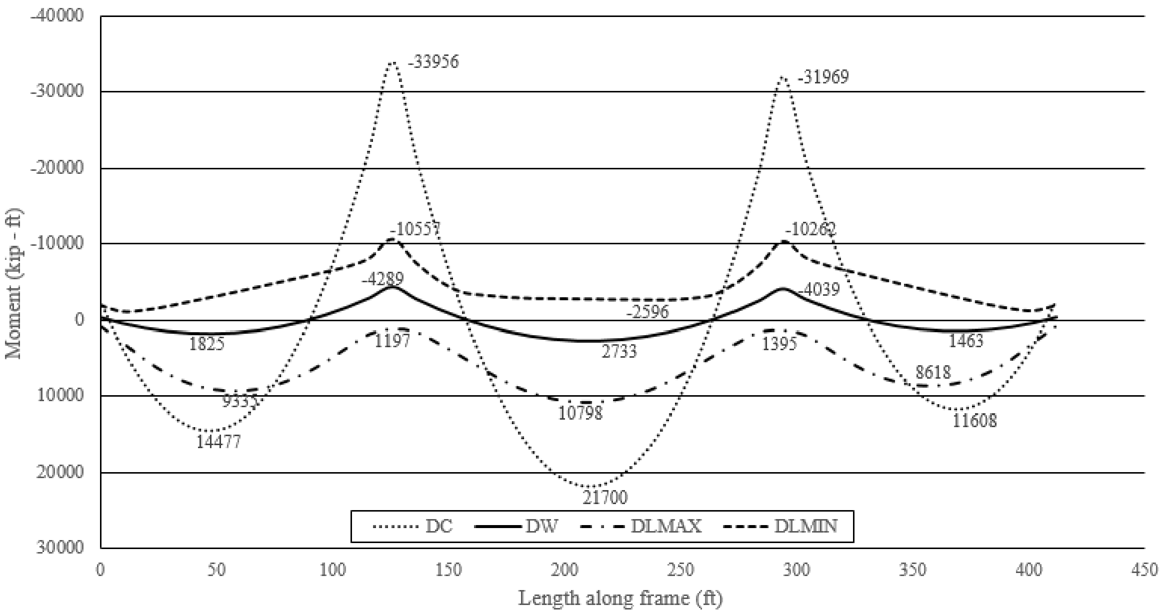


Figure 3—Bending moment diagrams for separate load cases. Conversion: 1 kip-ft = 1.356 kNm, 1 ft = 0.31 m.

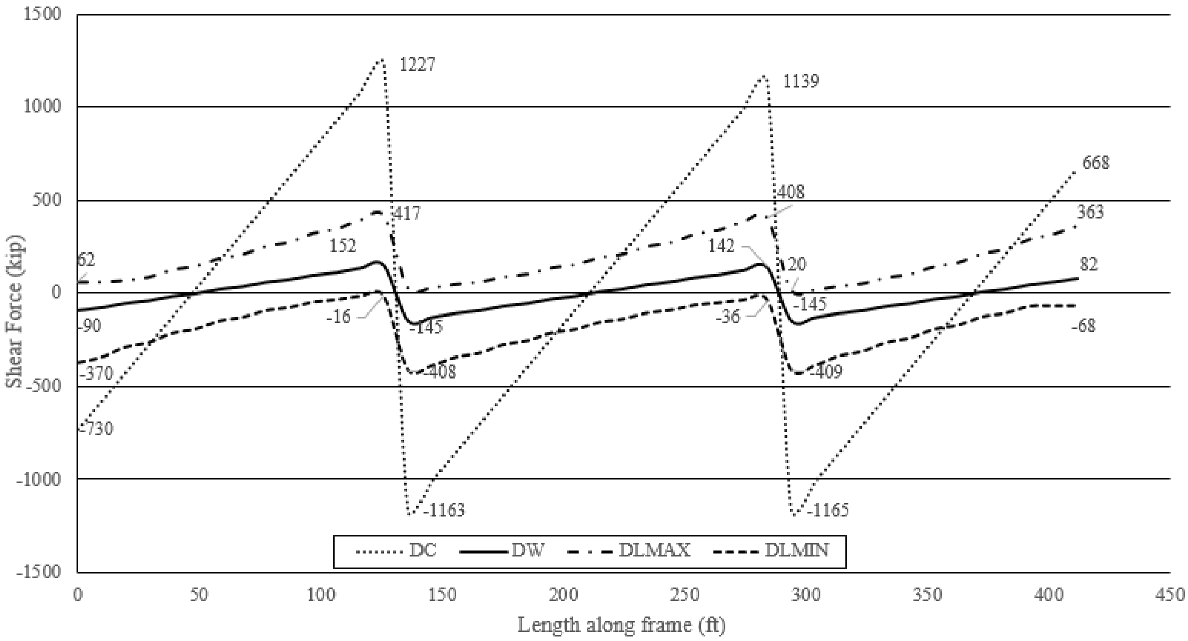


Figure 4—Shear diagrams for separate load cases. Conversion: 1 kip = 4.45 kN, 1 ft = 0.31 m.

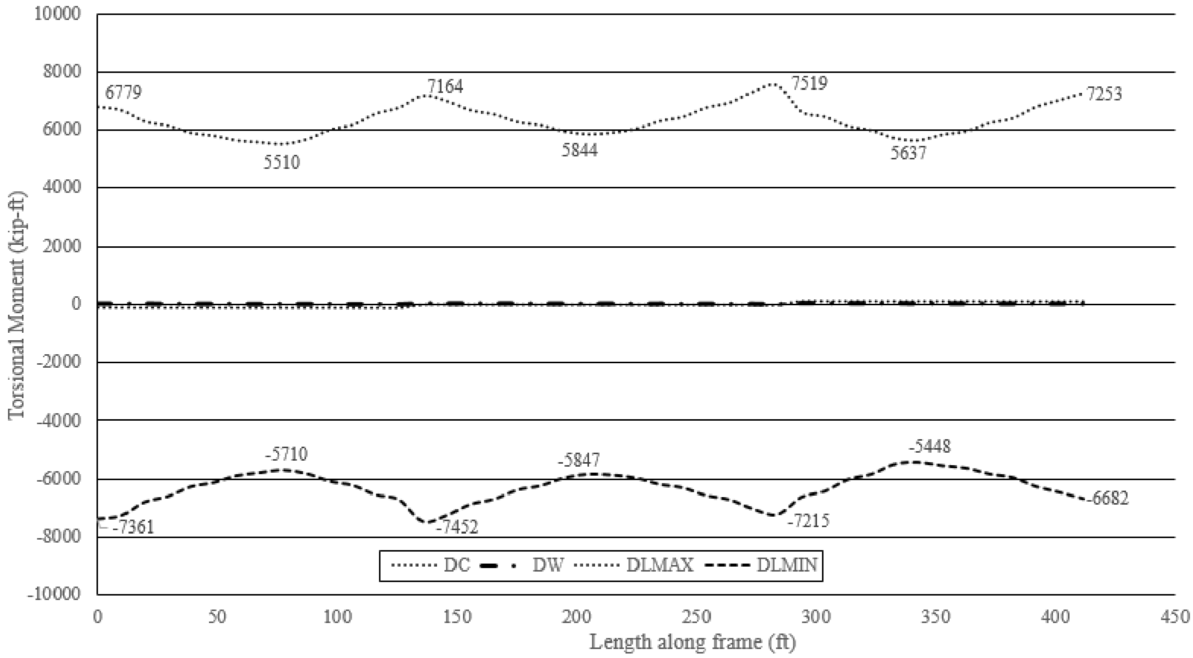


Figure 5—Torsional moment diagrams for separate load cases. Conversion: 1 kip-ft = 1.356 kNm, 1 ft = 0.31 m.

Materials

Concrete

Initial and 28-day concrete strength that will be used for the design of the bridge:

$f_{ci}' = 3500$ psi (24.1 MPa)
$f_c' = 5000$ psi (34.5 MPa)

γ_c = 0.15 kcf (23.6 kN/m^3)

Modulus of Elasticity of concrete

$$E_c = 33,000 \gamma_c^{1.5} \sqrt{f_c'}$$

$E_c = 4287$ ksi (29.6 GPa)

Reinforcing steel

$f_y = 60$ ksi (414 MPa)
$E_s = 29,000$ ksi (200 GPa)

Prestressing steel

$f_{pu} = 270$ ksi (1862 MPa)
$f_{py} = 243$ ksi (1675 MPa)
$f_{pj} = 202.5$ ksi (1396 MPa)

Statement of the design problem

The cross-section of this example bridge is subjected to a combination of flexure, shear, and torsion, hence the design for each of these limit states will be developed. Special attention is given to the design for torsion. The design of the prestressing steel is done in accordance with the requirements of AASHTO LRFD 2017[3]. Then, the design of all mild steel required to resist the effects of flexure, shear and torsion is done following the requirements of: AASHTO LRFD 2017[3], ACI 318-14[2], EN 1992-1-1:2004[4], and MC-2010[5]. The detailed procedure for the design with ACI318-14[2] is presented. In practice, the design engineer would only use the AASHTO LRFD 2017 provisions to determine the required reinforcement (or the relevant local code, if the bridge is not to be built in North America), but this example serves for the comparison between the approaches of the different codes. The design results following the other codes are included, so that a comparison between the requirements and design results of the studied design codes is possible.

DESIGN PROCEDURE

Design of prestressing steel

For the bridge presented in this example, a parabolic tendon profile was used as shown in Fig. 6. The maximum eccentricities were chosen based on the points where maximum bending moments occur. Then the prestressing force was calculated so it could balance the total dead load plus a percentage of the total live load, and finally a stress check was done following the requirements of AASHTO LRFD 2017[3]. A prestressing loss of 25% of the initial prestressing force was calculated.
A total jacking force P_j = 9,300 kips (41,400 kN) after losses was calculated. Figure 7. shows the details of the position of the prestressing ducts over the cross-section of the bridge. The required tendon area is distributed evenly over the 5 webs of the box-girder to apply equal prestressing force in each one of the webs. The total area of prestressing steel to be used is calculated with the following equation:

$$A_{ps} = \frac{P_j}{f_{pj}} = \frac{9300kips}{202.5ksi} = 46in^2\left(29678mm^2\right)$$

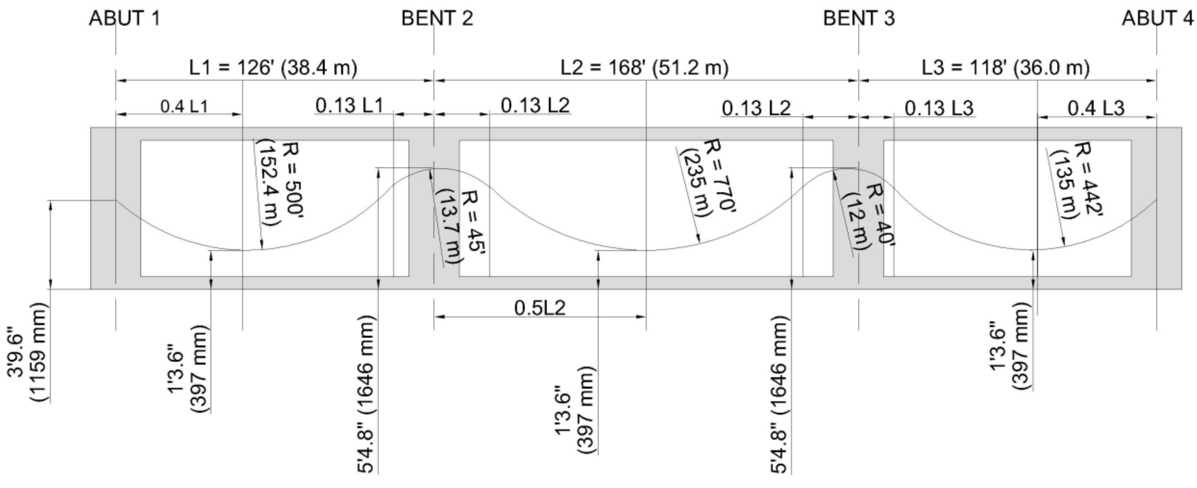
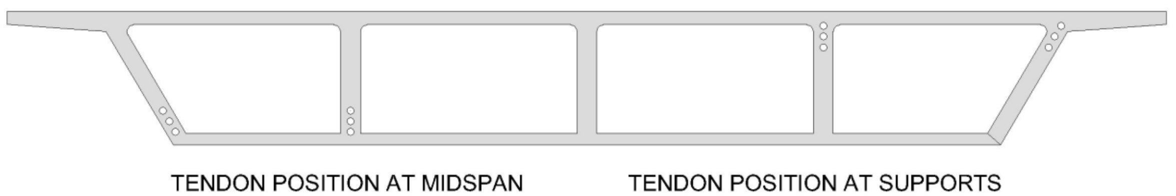
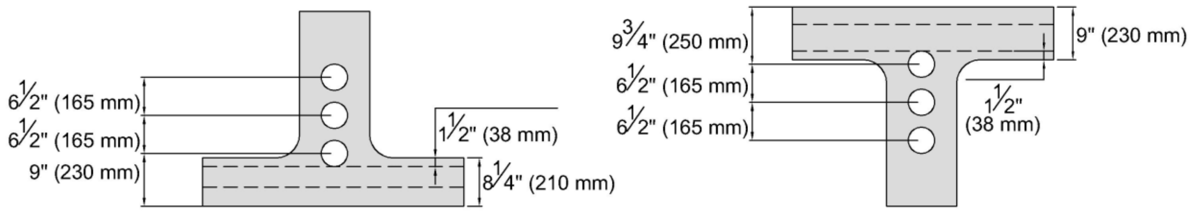


Figure 6—Prestressing tendon profile in longitudinal direction.



(a) general view



(b) detail with relevant dimensions

Figure 7—Prestressing tendons' position at midspan and at support (a) general view; (b) detail with all dimensions.

DESIGN PROCEDURE ACCORDING TO ACI 318-14

The example presented is a box-girder bridge. In practice, the design should be done according to AASHTO LRFD 2017[3], but for the purpose of the ACI SP, the reference design procedure that is selected to discuss in detail follows the provisions of ACI 318-14[2]. The entire design procedure for flexure, shear, and torsion will be shown for the ACI 318-14 provisions, but the prestresssing steel used will be the same as calculated with AASHTO LRFD 2017[3] as explained in the previous section. For the other design codes, the results will be summarized only. Detailed design procedures for all analyzed methods can be found in the background report[8].

The design for torsion is based on the principle of a thin-walled space truss in which the torsional resistance is provided by the outer skin of the cross-section that is confined by closed stirrups. The contribution of concrete in the core of a solid cross-sections to the torsional strength is neglected. Both solid and hollow cross-sections are idealized with the same thin-walled space truss. In the case of hollow sections, the area enclosed by the shear flow path A_o includes also the area of the void or voids enclosed by the outside perimeter. For this reason, cross-sections as the one presented in this example are efficient to resist torsional effects without increasing the self-weight of the element.

In structures like the one presented in this example, ACI 318-14[2] does not allow for reduction of torsional moments by redistribution of forces, since the torsional moment is required for the structure to be in equilibrium. Therefore, if the torsional moment is larger than the threshold torsion and torsion should be considered, the torsional reinforcement should be provided to resist the total factored design torsional moments.

The design for torsion is closely related to the design for shear. The design procedure consists of the following steps:

- Step 1: Determine the factored bending moment, shear force, and torsional moment at the face of Bent 1 of the box-girder based on the load combinations of Article 5.3.1.
- Step 2: Determine the section properties
- Step 3: Check the flexural resistance for the tendon layout shown in Fig. 6, based on the factored bending moment produced at the face of Bent 1.
- Step 4: Compute the additional mild steel required for flexure
- Step 5: Check if torsion can be neglected based on Article 22.7.4
- Step 6: Check if the dimensions of the cross-section are adequate based on Article 22.7.7.1.
- Step 7: Calculate the required area of transverse reinforcement for shear based on Article 22.5.10.5
- Step 8: Calculate the required area of transverse reinforcement for shear in exterior webs based on Article 22.5.10.
- Step 9: Calculate the required area of transverse reinforcement for torsion based on Article 22.7.6.
- Step 10: Calculate the required area of transverse reinforcement for exterior webs considering the effects of shear and torsion
- Step 11: Calculate the transverse reinforcement in flanges for torsion only based on Article 22.7.6.1
- Step 12: Check the maximum spacing for transverse reinforcement based on Articles 9.7.6.2.2 and 9.7.6.3.3
- Step 13: Calculate the required area of longitudinal reinforcement for torsion based on Article 22.7.6.1.
- Step 14: Check minimum required longitudinal and transverse reinforcement.

DESIGN CALCULATIONS

Step-by-step procedures

Step 1: Determine the factored bending moment, shear force, and torsional moment at the face of Bent 1 of the box-girder based on the load combinations of Article 5.3.1.

Using the bending moment, shear and torsional moment diagrams presented in Fig. 3, Fig. 4 and Fig. 5, the factored bending moment, shear force, and torsional moment are calculated using the following load combination: U = 1.2D + 1.6L. In this load combination D includes the values of DC and DW, and L consists of the design lane load and design truck based on the AASHTO LRFD 2017[4] HL-93 design vehicle. The results of the effects produced at the face of Bent 1 are presented below. For shear and torsion, the values are taken at a distance d from the face of Bent 1. These values will be used for all the calculations in this example based on ACI 318-14[2].

$M_u^- = 56804.2 kip - ft\ (77016 kN - m)$ at the face of Bent 1

$M_u^- = 44485.1 kip - ft\ (60314 kN - m)$ at a distance d of the face of Bent 1

$V_u = 3035 kip\ (13500.5 kN)$ at a distance d of the face of Bent 1

$T_u = 12108 kip - ft\ (16416.5 kN - m)$ at a distance d of the face of Bent 1

Step 2: Determine the section properties

Table 1—Section properties.

A_{cp}	44637 in^2	2.9×10^7 mm^2
p_{cp}	1275 in	32385 mm
A_g	13684 in^2	8.8×10^6 mm^2
d	64.8 in	1646 mm
b_w	60 in	1524 mm
A_{oh}	41710 in^2	2.7×10^{7}mm^2
p_h	1250 in	31750 mm

Step 3: Check the flexural resistance for the tendon layout shown in Fig. 6, based on the factored bending moment produced at the face of Bent 1.

The flexural resistance of the prestressing steel is calculated based on a rectangular concrete stress distribution as specified in §22.2.2. This assumption results in the following equation:

$$\frac{M_u}{\phi} = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\cdot\rho_p\cdot\frac{f_{pu}}{f_c'}\right) = 252.9 ksi$$

$$M_n = 46 in^2 \times 252.9 ksi\left(64.8 in - \frac{5.64 in}{2}\right) = 59982 kip - ft\ \left(81325 kN - m\right)$$

The stress in prestressing steel f_{ps} is calculated with the expression based on strain compatibility in §20.3.2.3.1 This verification is done in accordance with ACI 318-14 §9.5.1.1 using the following equation:

$$\phi M_n = M_u$$

The value of ø = 0.90 is chosen based on ACI 318-14 §21.2. The nominal moment including the reduction factor will be:

$$0.90 \times 59982 kip - ft = 53983 kip - ft\ (73193 kN - m)$$

The factored moment M_u at this point is $56804.2 kip - ft\ (77106 kN - m)$; therefore the flexural resistance provided by the prestressing steel is not enough.

Step 4: Compute the additional mild steel required for flexure

The next step is calculating the flexural resistance, taking into account the additional resistance provided by the mild steel. The addition of tension-resisting mild steel increases the depth of the equivalent rectangular stress block. For this purpose, the equation presented in step 1 is rewritten as follows, based on the rectangular concrete stress distribution as specified in §22.2.2.

$$\frac{M_u}{\phi} = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right)$$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left(\rho_p \cdot \frac{f_{pu}}{f_c'} + \frac{d_s}{d_p} \cdot \frac{f_y}{f_c'} \cdot \rho_l\right)\right) = 250ksi(1723.7MPa)$$

$$A_s = 30in^2 \ \left(19355mm^2\right)$$

$$\phi M_n = 62705.4kip - ft \ (85017.1kN - m)$$

This area of reinforcement corresponds to 39 #8 (25 mm) bars.

Step 5: Check if torsion can be neglected based on Article 22.7.4

The threshold torsion (T_{th}) shall be calculated according to the equations outlined in Table 22.7.4.1(b) (ACI 318-14) for hollow sections using Eq. 22.7.4.1(b) for prestressed members:

$$T_{th} = \lambda\sqrt{f_c'}\left(\frac{A_g^{\ 2}}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f_c'}}}$$

$$f_{pc} = \frac{P_j}{A}$$

$$T_{th} = 1 \times \sqrt{5ksi} \times \left(\frac{\left(13684in^2\right)^2}{1275in}\right) \times \sqrt{1 + \frac{9300ksi/13392in^2}{4 \times 1 \times \sqrt{5ksi}}} = 1609kip - ft \ \left(2182kN - m\right)$$

In this case $T_{th} < T_u$, so torsional effects should be considered for the design, and the factored torsional moment should be used.

Step 6: Check if the dimensions of the cross-section are adequate based on Article 22.7.7.1.

For checking the requirements of the code, the interaction equation 22.7.7.1(b) (for hollow sections) should be used. It is important to determine if the cross-section is large enough to avoid the excessive formation of cracks and to reduce the possibility of crushing of the surface concrete due to stresses induced by torsional effects. The interaction equation is:

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u}{1.7A_{oh}t}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$$

The present problem deals with a hollow cross-section. In such a cross-section, shear and torsion stresses are generated on the outer wall of the box. These should be summed, as shown in Eq. 22.7.7.1(b).

The concrete contribution to the nominal shear strength (V_c) is calculated using the approximate method provided in §22.5.8.2:

$$V_c = \left(0.6\lambda\sqrt{f_c'} + 700\right)b_w d = (0.6 \times 1 \times \sqrt{5000psi} + 700) \times 60in \times 64.8in = 2887kips \ (12842kN)$$

$$V_c = 5\lambda\sqrt{f_c'}b_w d = 5 \times 1 \times \sqrt{5000psi} \times 60in \times 64.8in = 1375kips \ (6116kN)$$

The results of the inequality from the interaction equation then become:

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u}{1.7A_{oh}t}\right)=\left(\frac{2991kip}{60in\times 64.8in}\right)+\left(\frac{10896kip-ft}{1.7\times 41710in\times 12in}\right)=0.866ksi\ (5.97MPa)$$

$$\phi\left(\frac{V_c}{b_w d}+8\sqrt{f_c'}\right)=0.75\left(\frac{1375kips}{60in\times 64.8in}+8\sqrt{5000psi}\right)=0.690ksi(4.76MPa)$$

Since $0.866ksi\ (5.97MPa) > 0.690ksi\ (4.76MPa)$, the size of the cross-section is not adequate and the dimensions should be increased. Using an iterative procedure, the new web width b_w = 83 in (2108 mm) is determined. All vertical webs will be flared. Exterior webs now have a thickness t = 19 in (483 mm) and interior webs t = 15 in (381 mm). With these new dimensions, the values are computed again to check if the interaction equation is satisfied, and the additional self-weight of the flared cross-section requires us to calculate again the values of V_u and M_u:

$$M_u^- = 44470kip-ft\ (60293.3kN-m) \text{ at a distance } d \text{ of the face of Bent 1}$$
$$V_u = 3035kip\ (13500.5kN) \text{ at a distance } d \text{ of the face of Bent 1}$$

$$V_c = \left(0.6\lambda\sqrt{f_c'}+700\right)b_w d = 4137.4kips\ (18404kN)$$
$$V_c = 5\lambda\sqrt{f_c'}b_w d = 1902kips\ (8461kN)$$
$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u}{1.7A_{oh}t}\right)=0.672ksi\ (4.63MPa)$$
$$\phi\left(\frac{V_c}{b_w d}+8\sqrt{f_c'}\right)=0.690ksi\ (4.76MPa)$$

Now $0.690ksi\ (4.76MPa) > 0.672ksi\ (4.63MPa)$, and the size of the cross-section is adequate.

Step 7: Calculate the required area of transverse reinforcement for shear based on Article 22.5.10.5

To find the required area of transverse reinforcement, the part of the sectional shear force carried by the steel is determined first. In other words, the factored shear capacity of the concrete is subtracted from the ultimate sectional shear force to find the part of the shear force that needs to be carried by the stirrups. Eq. R22.5.10.5 is used:

$$\frac{A_v}{s}=\frac{V_u-\phi V_c}{\phi f_y d}$$

$$\frac{A_v}{s}=\frac{3035kip-0.75\times 1902kip}{0.75\times 60ksi\times 64.8in}=0.56\,{in^2}/{in}\ \left(14.23\,{mm^2}/{mm}\right)$$

The area of reinforcement required will be placed in the 5 webs of the cross-section. Therefore, using #5 (16 mm) two-legged stirrups for each of the 5 webs gives:

$$A_v = 2\times 5\times\frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 3.1in^2\left(2000mm^2\right)$$

$$s=\frac{3.1in^2}{0.56\,{in^2}/{in}}=5.54in\ (141mm)$$

For shear, the required transverse reinforcement is #5 (16 mm) stirrups at 5.50 in (140 mm) on center.

Step 8: Calculate the required area of transverse reinforcement for shear in the exterior webs based on Article 22.5.10.

The exterior webs of the box are skewed. The finite element model of the bridge provides the shear force per web. These values are then used to calculate the area of reinforcement for the exterior webs. The factored shear force for the left exterior web, considering the additional self-weight of the larger section is 782 kips (3479 kN). The concrete contribution to the shear capacity for the exterior web is:

$$V_c = 5\lambda\sqrt{f_c'}b_w d = 5\times1\times\sqrt{5000\,psi}\times19in\times64.8in = 435kips\ (1935kN)$$

Eq. 22.7.7.1 is used to check if the cross-section is large enough for the exterior webs:

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u}{1.7A_{oh}t}\right) = 0.689ksi\ (4.75MPa)$$

$$\phi\left(\frac{V_c}{b_w d}+8\sqrt{f_c'}\right) = 0.690ksi\ (4.76MPa)$$

Here, $0.690ksi\ (4.76MPa) > 0.682ksi\ (4.70MPa)$. As such, the size of the exterior webs is adequate.

The required amount of transverse reinforcement is:

$$\frac{A_v}{s} = \frac{782kip - 0.75\times435kip}{0.75\times60ksi\times64.8\times(\cos27^o)} = 0.16\,{in^2}/{in}\ \left(4.07\,{mm^2}/{mm}\right)$$

Using #5 (16 mm) two-legged stirrups per web, the required spacing becomes.

$$A_v = 2\times\frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.61in^2\left(2000mm^2\right)$$

$$s = \frac{0.61in^2}{0.16\,{in^2}/{in}} = 3.82in\ (97mm)$$

For the right exterior girder the same area reinforcement at the same spacing will be provided. To conclude, for shear in the exterior webs, the required transverse reinforcement is #5 (16 mm) stirrups at 3.75 in (95 mm) on center.

Step 9: Calculate the required area of transverse reinforcement for torsion based on Article 22.7.6.

The provisions of ACI 318-14 assume that all torsion is carried by the tranverse and longitudinal reinforcement. These provisions neglect the contribution of the concrete. The presence of torsional forces does not affect the shear strength provided by the concrete.

The area of transverse reinforcement is calculated with Eq 22.7.6.1(a). Since torsion is carried only in the exterior webs, the skew angle of the webs needs to be considered in the calculation:

$$\frac{A_t}{s} = \frac{T_u}{\phi2A_o f_y\cot\theta\cos\alpha} = \frac{12107.8kip-ft}{0.75\times2\times0.85\times41710in^2\times60000psi\times\cot(37.5^o)\times\cos(27^o)}$$

$$\frac{A_t}{s} = 0.04\,{in^2}/{in}\ \left(1.00\,{mm^2}/{mm}\right)$$

§22.7.6.1.1 permits to use A_o as $0.85A_{oh}$, being A_{oh} the area enclosed by the outermost layer of stirrups. §22.7.6.1.2 determines that the angle θ (angle of the compression diagonals) for the design of prestressed concrete members can be taken as 37.5º. This angle is based on the space truss analogy, in which torsional stresses are resisted by compression diagonals placed at an angle θ. It is assumed that concrete does not resist any tension, and that the reinforcement steel is yielding. If the angle θ is reduced, the amount of transverse reinforcement required will decrease, and the amount of longitudinal reinforcement required will increase.

Step 10: Calculate the required area of transverse reinforcement for exterior webs considering the effects of shear and torsion

According to § 22.7.7.1, for hollow sections the torsional and shear stresses should be combined on the exterior webs of the cross-section. The area required for shear on the exterior webs was previously calculated as $A_v = 0.16\,{in^2}/{in}\ \left(4.07\,{mm^2}/{mm}\right)$. The total transverse reinforcement should be calculated using § 9.5.4.3.

$$\left(\frac{A_v}{s}+\frac{A_t}{s}\right)=0.16\,{}^{in^2}\!/_{in}+0.04\,{}^{in^2}\!/_{in}=0.20\,{}^{in^2}\!/_{in}\ \left(5.08\,{}^{mm^2}\!/_{mm}\right)$$

The required transverse reinforcement is larger than the minimum reinforcement according to §9.6.4.2. Then, the spacing is computed with the total area of reinforcement for the exterior webs:

$$s=\frac{0.61in^2}{0.20\,{}^{in^2}\!/_{in}}=3.10in\ (79mm)$$

For the exterior (skewed) webs, the provided transverse reinforcement becomes #5 (16 mm) stirrups at 3 in (75 mm) on center. For the internal (vertical) webs, the required transverse reinforcement remains #5 (16 mm) stirrups at 5 in. (125 mm) on center.

<u>Step 11: Calculate the transverse reinforcement in the flanges for torsion only based on Article 22.7.6.1.</u>
The area of transverse reinforcement is calculated as follows:

$$\frac{A_t}{s}=\frac{T_u}{\phi 2A_o f_y\cot\theta}=\frac{12107.8kip-ft}{0.75\times2\times0.85\times41710in^2\times60000psi\times\cot(37.5)}$$

$$\frac{A_t}{s}=0.035\,{}^{in^2}\!/_{in}\ \left(0.89\,{}^{mm^2}\!/_{mm}\right)$$

In the flanges, only torsional stresses will act, so only the area calculated for torsion will be considered for the design. This transverse reinforcement will be provided along with the flexural reinforcement in the top and bottom flanges.

$$A_v=2\times\frac{\pi}{4}\left(\frac{3}{8}\right)^2=0.22in^2\left(143mm^2\right)$$

$$s=\frac{0.22in^2}{0.035\,{}^{in^2}\!/_{in}}=6.30in\ (160mm)$$

The provided transverse reinforcement for the flanges consists of #3 (10 mm) stirrups at 6.25 in. (160 mm) on center.

<u>Step 12: Check the maximum spacing for shear and torsion transverse reinforcement based on Articles 9.7.6.2.2 and 9.7.6.3.3</u>
§9.7.6.2.2 states that the spacing for transverse reinforcement provided for shear shall not be larger than the minimum between d = 64.8 in (1646 mm) or 12 in (305 mm). §9.7.6.3.3 limits the spacing for transverse torsional reinforcement to a value no larger than the minimum of p_h = 1250 in (31750 mm) or 12 in (305 mm).
For the interior webs, a spacing of 5.50 in (140 mm) was used, which fulfils this requirement. For the exterior webs, a spacing of 3 in (75 mm) was used, which fulfils this requirement. For the flanges, a spacing of 6.25 in (160 mm) was used, which fulfils this requirement.

<u>Step 13: Calculate the required area of longitudinal reinforcement for torsion based on Article 22.7.6.1.</u>
The relationship between the areas of transverse reinforcement and longitudinal reinforcement for torsion is used for calculating the required area of longitudinal reinforcement for torsion. In step 6, the value of θ = 37.5º for prestressed concrete was selected.

$$A_l=\frac{A_t}{s}p_h\left(\frac{f_{yv}}{f_{yl}}\right)\cot^2\theta=0.04\,{}^{in^2}\!/_{in}\times1250in\times1\times\cot^2 37.5=83.3in^2\ \left(53742mm^2\right)$$

As longitudinal reinforcement for torsion, #8 (25 mm) bars are used. The required number of bars required:

$$\#bars=\frac{A_l}{Arebar}=\frac{83.3in^2}{0.79in^2}\approx106bars$$

A total of 145 bars will be required for longitudinal reinforcement including flexure and torsion. The steel required for flexure will have to be placed near the interior face of the flange in tension. In the case of hogging moment (as in this example) these bars should be distributed over the upper flange. The result is a design with 39 #8 (25 mm) bars placed at 15 in (380 mm) on center.

The required longitudinal reinforcement for torsion will be distributed over the exterior faces of the two flanges and two exterior webs of the box girder, which are the elements that resist torsion. In total, 35% of the required reinforcement (36 bars) will be placed in the two exterior webs at a vertical spacing of 4.75 in (120 mm) and the other 65% (70 bars) will be placed over the two flanges at a horizontal spacing of 15 in (380 mm). At the corners of the flanges, the horizontal spacing is reduced to 10 in (250 mm) over a distance of 12 ft (3.7 m).

<u>Step 14: Check minimum required longitudinal and transverse reinforcement.</u>
§9.6.4.2. specifies that if transverse reinforcement for torsion is required, the minimum reinforcement is:

$$\left(\frac{A_v}{s}+\frac{A_t}{s}\right)_{\min} = 0.75\sqrt{f_c'}\,\frac{b_w}{f_{yt}} = 0.75\times\sqrt{5000\,psi}\times\frac{86in}{60000\,psi} = 0.076\,{in^2}/{in}\ \left(1.93\,{mm^2}/{mm}\right)$$

For transverse reinforcement 0.20 in^2/in was provided which is larger than the minimum required.
The minimum requirements for the longitudinal reinforcement for torsion are given in §9.6.4.3 Eq. 9.6.4.3(a) and (b):

$$A_{l,\min} = \frac{5\sqrt{f_c'}A_g}{f_y} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yt}}{f_y} = \frac{5\times\sqrt{5000\,psi}\times 9822in^2}{60000\,psi} - 0.0391\,{in^2}/{in}\times 1250in\times 1 = 13.8in^2\ \left(8903.2mm^2\right)$$

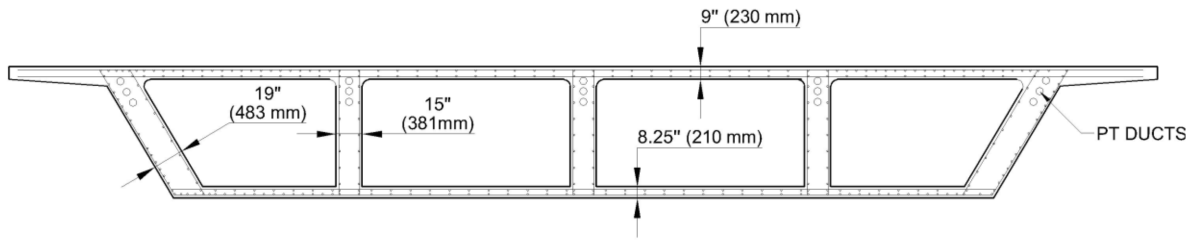
$$A_{l,\min} = \frac{5\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{25b_w}{f_{yt}}\right)p_h\frac{f_{yt}}{f_y} = \frac{5\times\sqrt{5000\,psi}\times 9822in^2}{60000\,psi} - \left(\frac{25\times 86in}{60000\,psi}\right)\times 1250in\times 1 = 13.1in^2\ \left(8451.6mm^2\right)$$

As longitudinal reinfocement for torsion 83.3 in^2 was provided, which is larger than the minimum required.

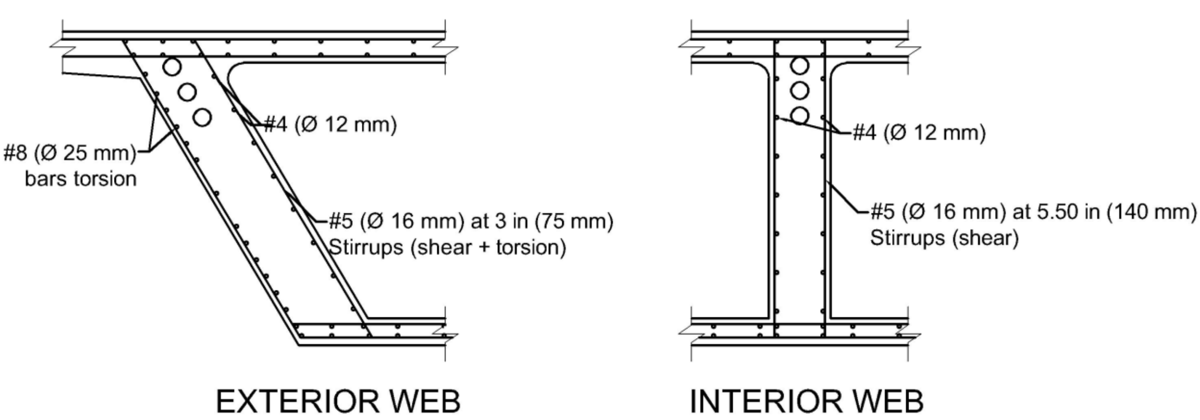
<u>Design results for torsion</u>

(c) detail of reinforcement in corners

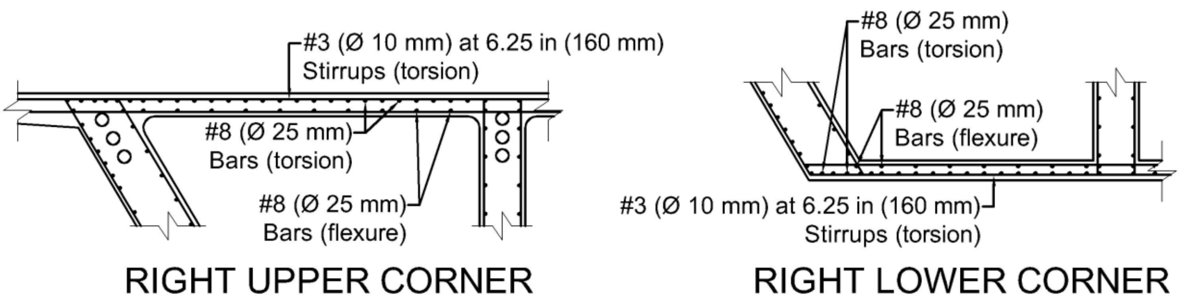
gives an overview of the resulting reinforcement for the section at the face of Bent 1 following the ACI 318-14 provisions. The design for shear was checked at a distance *d* from the face of the support. Detail (c) of Fig. 9 shows the placement of reinforcement around corners; longitudinal reinforcement for torsion should be placed at a smaller spacing around the corners, where the effects of warping torsion produce larger stresses.



(a) general view with dimensions



(b) detail of reinforcement in girders



(c) detail of reinforcement in corners

Figure 8—Overview of resulting reinforcement according to ACI 318-14. (a) general view with dimensions; (b) detail of reinforcement on girders; (c) detail of reinforcement in corners.

COMPARISON OF DESIGN RESULTS FOLLOWING OTHER CODES

The design of the box girder presented on Fig. 2 was also done according to AASHTO LRFD 2017[3], EN 1992-1-1:2004[4], and MC-2010[5]. Table 2 presents the sectional dimensions of the girders following the requirements for each one of the design codes. Table 3 and Table 4 present the design results for each code in terms of longitudinal and transverse reinforcement, including the reinforcement ratios and approximate weight of reinforcing steel to be used.

The AASHTO LRFD 2017[3] provisions are based on the Modified Compression Field Theory[7] (MCFT). The Eurocode EN 1992-1-1:2004[4] uses a spatial truss model, and limits the angle of the compression field to a value between 22° and 45°. Therefore, in this example, the results will be computed for different angles of inclination between the allowable limits to analyze the difference in results assuming different angles.

The Model Code MC-2010[5] uses an approach based on Levels of Approximation. Four different levels are defined: Level 1 (LoA 1) uses a variable angle truss model, which is the same as EN 1992-1-1:2004[4], Level 2 (LoA 2) uses a generalized stress field approach, Level 3 (LoA 3) use the MCFT[7], and Level 4 (LoA 4) uses finite element modelling. The results are computed for LoA 1, considering different angles of inclination, LoA 2, and LoA 3.

Table 2—Dimension of girders for each design code. Conversion: 1 in = 25.4 mm and 1 in^2 = 645.16 mm^2

Design code	b_w of exterior webs (in)	b_w of interior webs (in)	A_g disregarding overhanging flanges (ft^2)
ACI 318-14	19	15	106.3
AASHTO LRFD 2017	12	12	95.1
EN 1992 1-1:2004 (θ = 45°)	12	12	95.1
EN 1992 1-1:2004 (θ = 35°)	13	12	95.6
EN 1992 1-1:2004 (θ = 22°)	18	16	106.5
MC-2010 (LoA 1) (θ = 45°)	12	12	95.1
MC-2010 (LoA 1) (θ = 37.5°)	12	12	95.1
MC-2010 (LoA 1) (θ = 25°)	16	14	101.8
MC-2010 (LoA 2)	13	13	97.4
MC-2010 (LoA 3)	13	13	97.4

For calculating the ρ_l the following equation is used:

$$\rho_l = \frac{A_l + A_s}{A_g}$$

The unit weight of steel used for calculations of total weight of reinforcement is γ_s = 0.283 lbs/in^3 (7850 kg/m^3).
Table 3—Longitudinal reinforcement required for flexure and torsion for each design code. Conversion: 1 in^2 = 645.15 mm^2, 1 in = 25.4 mm,1 lb = 0.45 kg.

Design Code	Required longitudinal reinforcement for flexure (in^2)	Required longitudinal reinforcement for torsion (in^2)	Longitudinal reinforcement provided for flexure	Longitudinal reinforcement provided for torsion	ρ_l(%)	Total weight of longitudinal reinforcement (lbs/in)
ACI 318-14	30	83.3	38 #8	106 #8	0.74%	32.3
AASHTO LRFD 2017	40	52.0	51 #8	67 #8	0.68%	26.3
EN 1992-2004 (θ = 45°)	40	46.1	51 #8	59 #8	0.63%	24.5
EN 1992-2004 (θ = 35°)	40	65.8	51 #8	84 #8	0.77%	30.1
EN 1992-2004 (θ = 22°)	40	114.0	51 #8	145 #8	1.00%	43.6
MODEL CODE 2010 –LoA 1 (θ = 45°)	40	27.3	51 #8	35 #8	0.50%	19.2
MODEL CODE 2010 –LoA 1 (θ = 37.5°)	40	35.6	51 #8	45 #8	0.56%	21.6
MODEL CODE 2010 –LoA 1 (θ = 22°)	40	58.5	51 #8	74 #8	0.68%	28.1
MODEL CODE 2010 –LoA 2	40	47.2	51 #8	60 #8	0.63%	25.0
MODEL CODE 2010 –LoA 3	40	53.7	51 #8	68 #8	0.67%	26.8

The calculation of the total weight of transverse reinforcement in Table 4 assumes that this spacing of stirrups will be distributed over 12.5 ft (3.8 m). For calculating the transverse reinforcement ratio ρ_t, the following equation is used:

$$\rho_t = \frac{A_v/s}{b_w} + \frac{A_t/s}{t_{flange}}$$

The codes that are followed for the design differ in their procedures. In AASHTO LRFD 2017[3] the angle of inclination of the compression field θ = 31.5° and is calculated according to the MCFT[7]. Since AASHTO LRFD 2017[3] does not require a check of the cross-sectional dimensions for the combined effects of shear and torsion combined, the dimensions are kept the same as shown in Fig. 2. For the shear design, the code takes into account the contribution of concrete and prestressing steel to the shear resistance. As a result, the required area of transverse reinforcement is smaller.

For Eurocode EN 1992 1-1:2004[4] three different angles of inclination of the compression strut were evaluated. This code requires to check the cross-sectional dimensions to resist the effects of shear and torsion. Using an angle of 45° will maximize the concrete strength components; therefore, there is no need to flare the webs. Reducing this angle reduces both the design torsional resistance moment and the design shear resistance, leading to a need to increase the cross-sectional dimensions. When using the Eurocode, the area of transverse reinforcement was calculated to resist the full factored shear force, leading to a larger required area of transverse reinforcement. If the angle θ decreases, the required area of transverse reinforcement for shear and torsion also decreases, but the area of longitudinal reinforcement for torsion increases.

In the Model Code MC-2010[5] Level of Approximation approach, the time and effort devoted to calculations increases for increasing Levels of Approximation. Level 1 is based on a variable angle truss model, so in this case the same results are found as for EN 1992 1-1:2004[4] when varying the angles of inclination of the compression field. Level 2 is based on a generalized stress field, and this approach determined that θ = 30°. Since here the concrete contribution to shear resistance is not taken into account, this angle of inclination of the compression field will require to flare all interior webs. A balanced quantity of steel and a increase in gross area of concrete of just 2% is required. Level 3 is based on the MCFT[7] and it resulted on an angle θ = 27°, and this approach contains the most time and effort-consuming method. Using this method also results in a balanced quantity of steel and an increase in gross area of concrete of 2%, similar to the results with Level 2. In this example, Level 2 and Level 3 have similar results of cross-sectional area of concrete and quantity of steel, so for design it could be a good option to either use the generalized stress field based LoA 2 or the MCFT-based[7] LoA 3.

Table 4—Transverse reinforcement required for shear and torsion for each design code. Conversion: 1 in^2 = 645.15 mm^2, 1 in = 25.4 mm,1 lb = 0.45 kg.

Design Code	Required transverse reinforce-ment for exterior web (shear and torsion) (in^2/in)	Required transverse reinforce-ment for interior web (shear) (in^2/in)	Required transverse reinforce-ment for flange (torsion) (in^2/in)	Transverse reinforce-ment provided in exterior web (shear and torsion)	Transverse reinforce-ment provided in interior webs (shear)	Transverse reinforce-ment provided in flanges (torsion)	ρ_t (%)	Total weight of transverse reinforcement (lbs/in)
ACI 318-14	0.195	0.111	0.035	#5 @3in	#5 @5.50in	#3 @6.25in	2.23%	15.0
AASHTO LRFD 2017	0.151	0.082	0.026	#5 @4in	#5 @7.25in	#3 @8.50in	2.29%	11.3
EN 1992-2004 (θ = 45°)	0.295	0.208	0.034	#5 @2in	#5 @2.75in	#3 @6.50in	4.81%	24.0
EN 1992-2004 (θ = 35°)	0.207	0.146	0.026	#5 @2.75in	#5 @4in	#3 @9.25in	3.27%	17.1
EN 1992-2004 (θ = 22°)	0.119	0.084	0.015	#5 @5in	#5 @7.25in	#3 @12in	1.86%	9.6
MODEL CODE 2010 –LoA 1 (θ = 45°)	0.271	0.190	0.037	#5 @2in	#5 @3in	#3 @5.75in	4.70%	23.4
MODEL CODE 2010 –LoA 1 (θ = 37.5°)	0.208	0.146	0.029	#5 @2.75in	#5 @4in	#3 @7.75in	3.47%	17.3
MODEL CODE 2010 –LoA 1 (θ = 25°)	0.127	0.089	0.017	#5 @4.75in	#5 @6.75in	#3 @12in	1.67%	10.2
MODEL CODE 2010 –LoA 2	0.157	0.110	0.023	#5 @3.75in	#5 @5.50in	#3 @10.25in	2.38%	12.6
MODEL CODE 2010 –LoA 3	0.133	0.097	0.019	#5 @4.5in	#5 @6.25in	#3 @11.75in	2.02%	10.8

Two design approaches based on the MCFT[7] were studied here: AASHTO LRFD 2017[3] and MC-2010[5] LoA III. The expressions for the angle θ differ across these two codes, so that for AASHTO LRFD 2017[3] θ = 31.5°, and for MC-2010[5] θ = 27.0°.

When small values (< 30°) for the angle θ are assumed, the cross-sectional dimension need to be increased. This increase is necessary when using the provisions from EN 1992-1-1:2004[4] and MC-2010[5]. ACI 318-14[2] requires also to check the resistance of the cross-section, which resulted in an increase in the size of the cross-section of almost 12%. AASHTO LRFD 2017[3] does not require any check of the cross-section; only the combination of effects produced by shear and torsion has to be considered in the calculation of the longitudinal strain. So, using a lower angle of the compression field such as the minimum angle provided by EN 1992-1-1:2004[4] or MC-2010[5] results in an uneconomical solution in terms of required longitudinal reinforcement and required gross area of concrete.

For an angle of the compression strut of θ = 45°, large amounts of transverse reinforcement are found, and small amounts of longitudinal reinforcement result. On the other hand, for a smaller angle of the compression strut θ taken as 22° and 25°, the opposite results: small amounts of transverse reinforcement and large amounts of longitudinal reinforcement are found. This conclusion can also be drawn based on the resulting weight of reinforcement from Table 2 and Table 3. As such, for design it is recommended to balance the required areas of transverse and longitudinal reinforcement by choosing a mean angle between the allowable limits when using the provisions from EN 1992-1-1:2004[4] and MC-2010[5] Level of Approximation I. The recommendation from ACI 318-14[2] to use an angle θ = 37.5º is in line with these observations.

Using the procedures of AASHTO LRFD 2017[3] and MC-2010[5] LoA 3 will require more computational time and effort, since both use a MCFT[7]-based approach, but they will lead to a more economical and balanced design solution. In terms of resulting quantity of steel, both methods lead to a similar provided total area (sum of longitudinal and transverse steel) of steel reinforcement. There is some difference in the cross-sectional area, since the MC-2010[5] design solution requires an increase of 2% of A_g. resulting in an increase of 2.30 ft^2 (213677 mm^2) of concrete. As such, the design solution using AASHTO LRFD 2017[3] is the most economical design.

SUMMARY AND CONCLUSIONS

In this example, the detailed design of a cast-in-place, post-tensioned concrete, multi-cell box girder bridge under a combination of the effects of flexure, shear and torsion is presented. The starting point for the geometry was taken from a design example of the California Department of Transportation[1]. Then, using a linear finite elements software package for bridge design (CSI Bridge)[6], the bending moments, shear, and torsional moments where obtained. The design of the prestressing steel was done following AASHTO LRFD 2017[3] guidelines using a parabolic tendon profile. Consequently, the example includes the detailed procedure for the design of flexure, shear, and torsion following the requirements of ACI 318-14[2]. Finally the design results of AASHTO LRFD 2017[3], EN 1992-1-1:2004[4] and MC-2010[5] are presented, so a comparison between the design results is made.

Each of the analyzed design codes has its own design metodology and principles. ACI 318-14[2] uses a space truss analogy and a thin-walled tube analogy, AASHTO LRFD 2017[3] uses a simplification of the Modified Compression Field Theory[7], EN 1992-1-1:2004[4] uses a variable angle truss model based on an equivalent thin-walled tube, and MC-2010[5] uses different Levels of Approximation. For MC-2010 the theories used are a variable angle truss model, generalized stress field, or a simplification of the MCFT[7], depending on the Level of Approximation.

ACI 318-14[2] results in large amounts of required steel and concrete, as its provisions are more conservative in some aspects. When using a variable angle truss model as in EN 1992-1-1:2004[4] and MC-2010[5] LoA 1, the presented case study shows that it is recommended to use a mean angle between the minimum and maximum allowable limits, as this approach will result in a more balanced design solution of steel and concrete. Finally using more time and effort-consuming design solutions as for example MC-2010[5] LoA 2 and 3 or AASHTO LRFD 2017[3] will lead to the most economical and balanced of all solutions obtained.

LIST OF NOTATIONS

a = depth of equivalent rectangular stress block,
A_{cp} = area enclosed by the outside perimeter of concrete cross-section,
A_g = gross area of concrete cross-section,
A_l = total area of longitudinal reinforcement to resist torsion,
$A_{l,min}$ = minimum area of longitudinal reinforcement to resist torsion,
A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement,
A_o = gross area enclosed by torsional shear flow path,
A_{ps} = area of prestressed longitudinal tension reinforcement,
A_s = area of nonprestressed longitudinal tension reinforcement,
A_t = area of a closed stirrup, hoop or tie resisting torsion,
A_v = area of shear reinforcement,

b_w	=	web width,
d	=	distance from extreme compression fiber to centroid of longitudinal tension reinforcement,
d_p	=	distance from extreme compression fiber to centroid of prestressing reinforcement,
D	=	effect of service dead load,
DC	=	weight of supported structure,
DW	=	weight of wearing surface (superimposed dead load),
E_c	=	modulus of elasticity of concrete,
E_s	=	modulus of elasticity of steel,
f'_c	=	specified compressive strength of concrete,
f'_{ci}	=	specified compressive strength of concrete at time of initial prestress,
f_{pc}	=	compressive stress in concrete, at centroid of cross-section resisting externally applied loads,
f_{pj}	=	prestressing steel stress at effective jacking force,
f_{ps}	=	stress in prestressing reinforcement at nominal flexural strength,
f_{pu}	=	specified tensile strength of prestressing reinforcement,
f_{py}	=	specified yield strength of prestressing reinforcement,
f_y	=	specified yield strength for nonprestressed reinforcement,
f_{yv}	=	specified yield strength of transverse shear reinforcement,
f_{yt}	=	specified yield strength of transverse torsion reinforcement,
L	=	effect of service live load,
LL	=	vehicular live load,
M_n	=	nominal flexural strength at section,
M_u	=	factored moment at section,
p_{cp}	=	outside perimeter of concrete cross-section,
p_h	=	perimeter of centerline of outermost closed transverse torsional reinforcement,
P_j	=	effective jacking force (after losses),
s	=	center-to-center spacing of longitudinal and transverse reinforcement,
t	=	wall thickness of hollow section,
T_{th}	=	threshold torsional moment,
T_u	=	factored torsional moment,
U	=	strength of a member or cross-section required to resist factored loads,
V_c	=	nominal shear strength provided by concrete,
V_u	=	factored shear force at section,
α	=	angle defining the orientation of reinforcement for skewed girders,
ε_s	=	tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength,
θ	=	angle between axis of compression field and tension chord of a member,
λ	=	modification factor for lightweight concrete,
ρ_l	=	ratio of area of longitudinal reinforcement,
ρ_t	=	ratio of area of transverse reinforcement,
γ_c	=	unit weight of concrete,
γ_s	=	unit weight of steel,
ϕ	=	strength reduction factor,

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