The Cost of Risk Aversion

Eliciting risk preferences and re-evaluating flood protection standards in the Netherlands

M. Prevaes

Delft University of Technology

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by

M. Prevaes

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Student number:	4587049	
Thesis committee:	Prof. dr. ir. M. Kok,	TU Delft, HKV Lijn in Water
	Prof. dr. ir. S.N. Jonkman,	TU Delft
	ir. B. Strijker,	TU Delft, HKV Lijn in Water
	ir. G.W.F. Rongen,	TU Delft

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Voorwoord

Beste lezer,

Voor u ligt het resultaat van mijn studententijd in Delft, mijn master thesis. Een studententijd die, al zeg ik het zelf, rijkelijk gevuld was met zowel sociale als vakinhoudelijke momenten. Een aantal jaar geleden kwam ik in Delft aan met het idee om een bruggenbouwer te worden. Dat idee is gedurende die jaren verandert door een aantal uitstapjes. Uitstapjes zoals een jaar even niet studeren en erachterkomen wat er naast civiele techniek allemaal mogelijk is voor een 'civieler'. Of uitstapjes naar, achteraf gezien, een verkeerde masterkeuze die dan wel weer heeft geleid tot het ontdekken van mijn passie binnen de master, *Flood Risk*. Zelfs uitstapjes naar andere universiteiten en studierichtingen. Al deze uitstapjes tellen uiteindelijk op tot een grote stap naar de persoon die ik nu ben.

Het zijn dan ook dit soort uitstapjes waar je het meeste van leert. Sterker nog, het uitstapje naar econometrie heeft uiteindelijk ook geleid tot dit afstudeeronderwerp. In een zekere zin is dit afstuderen dan ook een mooie weerspiegeling van mij als persoon, een intersectie tussen waterbouwkunde en (gedrags)econometrie. Ik heb veel geleerd, nagedacht maar me vooral ook erg vermaakt. Dit verslag is daar een mooie reflectie van.

Ik wil dit voorwoord ook gebruiken om een aantal mensen te bedanken. Allereerst mijn afstudeercommissie: Bart, bedankt voor je bereidheid om je in een nieuw onderwerp te storten, je waardevolle inzichten tijdens de casus en natuurlijk de (soms afdwalende) inhoudelijke overleggen en gesprekken. Guus, bedankt voor het idee van het onderwerp (al zijn daar ook wat uitstapjes vanaf gemaakt), je onuitputtelijke technische kennis en de hulp die je gaandeweg hebt gegeven. Matthijs, bedankt voor je vakinhoudelijke kennis, kritische blik waar nodig en natuurlijk je eeuwig enthousiasme. Bovenal wil ik jullie alledrie bedanken voor het vertrouwen om nieuwe dingen uit te proberen binnen deze relatief onbekende intersectie van vakgebieden.

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Ten slotte wil ik mijn familie en vrienden bedanken voor hun onvoorwaardelijke liefde, vriendschap en steun, tijdens het afstuderen maar ook vooral daarbuiten.

Matthijs Prevaes
Delft, juli 2023

Preface

Dear reader,

Before you lies the result of my student life in Delft, my master's thesis. A student life that, if I may say so myself, was abundantly filled with both social and academic moments. A few years ago, I arrived in Delft with the intention of becoming a bridge builder. That idea has changed over the years due to a number of detours. Detours such as taking a year off from studying to explore what else is possible for a civil engineer. Or detours to, in hindsight, a wrong choice of master's degree that nevertheless led me to discover my passion within the field of Flood Risk. I even took detours to other universities and study disciplines. All these detours ultimately add up to a significant step towards the person I am today.

It is precisely these kinds of detours that teach you the most. In fact, the detour to econometrics eventually led to the topic of this thesis. In a way, this thesis is a reflection of myself as a person, an intersection between hydraulic engineering and (behavioural) econometrics. I have learned a lot, pondered deeply, and, above all, enjoyed myself. This report is a testament to that.

I also want to use this preface to express my gratitude to several people. First and foremost, my thesis committee: Bart, thank you for your willingness to dive into a new topic, your valuable insights during the case study, and of course, the (sometimes drifting) academic discussions and conversations. Guus, thank you for the idea of the topic (although there were some detours along the way here too), your inexhaustible technical knowledge, and the guidance you provided throughout. Matthijs, thank you for your subject knowledge, critical eye where needed, and, of course, your eternal enthusiasm. I want to thank all three of you for trusting me to try new things within this relatively unknown intersection of disciplines.

Additionally, I would like to thank HKV for their hospitality and the stimulating conversations during group meetings, student cafes, or simply by the coffee machine. The questions, discussions, and insights that arose from those interactions undoubtedly improved my thesis.

Finally, I want to express my gratitude to my family and friends for their unconditional love, friendship, and support, not only during but also beyond my studies.

Matthijs Prevaes
Delft, July 2023

Abstract

Where water flows, prosperity follows.

Many prosperous areas around the world are situated near large bodies of water such as oceans, seas, and rivers. These bodies of water play a vital role in enhancing the prosperity of societies through various means. They provide convenient access to transportation, food, recreation, and many other benefits. However it is not only prosperity that follows, but also *risk*. This risk, defined as the product of probabilities and outcomes, is an inherent price that has to be paid to enjoy the social and economic fruits that are provided by these oceans, seas or rivers. As whenever one is close to a body of water, there exists a probability of a devastating outcome: A flood.

In order to effectively minimize and handle risks, the field of Flood Risk Management has naturally evolved. Within this field numerous decisions need to be made, such as determining an acceptable probability of flooding for a particular area or deciding when to evacuate if a flood is imminent. These decisions involve individuals who process information and make judgments under risk. However, it has been observed that the decision-making process is vulnerable to the influence of risk preferences. The *St. Petersburg Paradox* provides an illustrative example of how risk preferences can affect behavior, as individuals are only willing to pay a limited amount of money for an expected outcome of an infinite sum. By considering risk preferences, researchers were able to explain the observed mismatch between the expected value and the willingness-to-pay. This example raises the question of whether similar discrepancies between willingness to pay and expected outcomes can also arise in flood risk management, and whether the inclusion of these risk preferences can aid in the decision-making process under risk.

This research looks at ways of incorporating risk preferences into the field of Flood Risk Management. A stated preferences method is used to uncover the risk preferences of individuals for flood risk related problems, consisting of a series of flood related choice problems. The subsequent results are fitted to several commonly used risk preference models, which consist of a utility- and probability weighting function. A modified version of *Prospect Theory* by Tversky and Kahneman (1992) is found to best describe the risk preferences of individuals towards flood risk related problems. This research shows that individuals have similar risk preferences for flood risk related choice problems as for general (behavioural) economic choice problems. The found utility function shows that individuals have a diminishing sensitivity for outcomes that are larger in magnitude, with a greater diminishing effect for positive outcomes than for negative outcomes. Additionally, it became apparent that individuals experience flood-related losses around 1.5 times more intensely than equal gains, aligning with the current understanding of behavioural economics. The identified probability weighting function indicates that individuals overestimate small probabilities, perceiving them as greater than their actual value, while simultaneously underestimating large probabilities. The point where overestimation switches to underestimation occurs around probabilities of 0.3, in line with the results found in behavioural economics. Special attention was given to probabilities between one in a hundred thousand and one in a hundred where, instead of assuming a functional form for the probability weighting function, an additional elicitation was performed. This elicitation revealed that individuals perceive probabilities below one in a hundred as largely the same. Suggesting that people are unable to distinguish between very small probabilities, such as one in a thousand and one in ten thousand. This finding carries significant implications for the perceived level of safety in flood risk management, considering that flood safety standards typically fall below one in a hundred.

The found risk preferences are subsequently used to reevaluate the Statistical Value of Life (VOSL), previously found to be around €6.7 mln. by de Blaeij (2003a) and Bockarjova et al. (2009). Based on the found risk preferences and answers to the choice problems a new value of €11.8 mln. is found. Adjusted for inflation this value is around 1.18 to 1.33 higher than the previous found values. This

research further looked at the influence of the additional risk premium in the Dutch discount rate for infrastructure projects and its effect on the safety standard in flood protections. Including a positive risk premium, which increases the overall discount rate, leads to a decrease in safety standards. This is the logical result of discounting future benefits in the form of reduced risk, while the incurred costs are borne in the present and are therefore not discounted. This decrease in safety standard is indicative of a risk seeking approach, which contradicts the risk-averse nature of the risk premium in the discount rate. To resolve this contradiction and adopt a risk-averse approach, several options for incorporating the risk-averse premium in the discount rate are proposed.

All the previous insights are used in a case study, which consists of the reevaluation of Dutch safety standards for all dike sections along rivers and coasts. The evaluation of optimal flooding probabilities for the Local Individual Risk (LIR) and the Social Cost-Benefit Analysis (SCBA) is modified to include the found risk preferences, along with the adjusted values for the discount rate and VOSL. The resulting criteria are then used to reassess the optimal classified flooding probabilities for various dike sections in the Netherlands. This adjusted evaluation shows that the ratio between the leading principles in the flood safety standards remains largely the same, but the optimal flood probabilities associated with these principles do change. Generally, these probabilities are reduced by approximately one order of magnitude when the best estimates found in this research are included, leading to a stricter safety standard for flooding. An analysis is conducted to examine the sensitivity of the different changes. This analysis demonstrates that the inclusion of risk preferences has the most significant impact on the optimal flooding probability for both the LIR and SCBA, with the probability weighting function exerting the most influence. When the best estimates for the adjusted evaluation are used, the additional costs for each resident of the Netherlands that are the result of risk aversion amount to €37 per year.

This research is concluded with the remark that the insights found in behavioural economics are useable and of value in the field of flood risk management. People tend to show the same degree of risk aversion when presented with an uncertain choice about flooding as they do when they are presented with an uncertain choice about economics, such as the problem posed in the St. Petersburg Paradox. Taking these risk preferences into account can help to better distribute the scarce resources such as time and money to where they are of most utility to society.

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Introduction

1.1. Background & Need for this study

Deltas throughout the world are attractive regions for communities to build cities, infrastructure and an overall prosperous society. Unfortunately, they are also highly vulnerable to floods. As the threat of sea level rise looms and continues to increase, delta regions have a growing need for flood protection. The technology to provide this protection is readily available, but up to what level should an area be protected? Often, there are laws or regulations set by governmental bodies that define the acceptable *risk* that an area can be exposed to. This risk, in the context of flooding, is calculated as the damage as a result of a flood multiplied by the probability of a flood occurring, summed over all probabilities and consequences. To reduce the risk, the probability of occurrence is often lowered through reinforcement of the existing protections or by (over)designing flood defences that are able to withstand higher loads than anticipated. However, increasing reinforcements or design changes also result in higher costs. Therefore, the 'art' of flood risk mitigation measures lies in finding the optimum between safety and costs.

Choosing the right mitigation measures requires adequate knowledge of both the risk and the cost associated with the measures. When examining the risk aspect, it becomes clear that a significant portion of the acceptable risk is determined by the allowable probability of flooding. There is an extensive body of literature describing several methods to determine the required norms, an example of which is the *(Social) Cost-Benefit Analysis*. These norms are determined by taking several factors into account such as the individual, societal and economic tolerable risks. At the most fundamental level these tolerable risks have been set to a certain standard by a (group of) individuals. Take for example the *Local Individual Risk* (LIR), which ensures a minimum level of flood protection for each inhabitant of the Netherlands. In 2017, it was determined that the LIR should not exceed a probability of death by flooding of 1/100,000 per year, as specified in the *Waterwet*. The choice of these safety standards is not based on physical constants like the speed of light or gravitational pull, but rather on political decisions influenced by a philosophy of fairness. Thus, the exceedance probability is not a physical quantity that can be estimated or solved by equations, but rather a value determined by knowledgeable individuals. No matter how knowledgeable the individual, humans decision making will be involved. This decision making might be susceptible to the influence of individual or societal risk preferences. The extent to which these preferences impact decision-making in the field of Flood Risk Management is currently unknown.

Zooming in on the cost part of the mitigation measures it becomes apparent that a factor that has a great influence on the total costs is the chosen interest (or discount) rate. This interest rate consists of two parts: a fixed rate and an added risk premium (Ministerie van Financien, 2020). Increasing the variable risk premium raises the overall interest rate, leading to a stronger discounting of future benefits to the present. Since (most of) the costs are often assumed to be incurred at the present moment, and therefore not discounted, it is only the future benefits that are reduced with a larger interest rate. When performing a social cost-benefit analysis, this will inevitably lead to a lower safety standard as less costs can be incurred due to the reduction in benefits. This results in an undesirable situation: An increase in the risk premium leads to a decrease in the flood safety standard. The risk premium serves as a financial mitigation strategy for risk and can therefore be viewed as a form of risk aversion. However, the decrease in safety standards would paradoxically lead to an increase in risk and could therefore be seen as risk seeking behaviour. The opposite of the observed risk aversion in the premium. The interplay between these two components is currently not well-studied. Another significant factor contributing to total costs is the value assigned to a statistical life in the event of death caused by flooding. The current commonly used figure for the value of a statistical life is either €6.7 mln. found by de Blaeij (2003a) or €6.8 mln. by Bockarjova et al. (2009). These figures were determined in 2003 and 2009, respectively, using choice-based methods for assessing traffic mortality and flood risk mortality. It is currently unknown whether these figures remain up-to-date after one or two decades, particularly in light of recent flood events.

The decision making process under risk and uncertainty plays a significant role in the assessment of the required flood protection. Since flood protection projects, particularly reinforcement efforts, often involve substantial investments, misjudging the associated risks can be a costly mistake. These costs are not limited to financial losses, as an inadequate flood protection can lead to unnecessary loss of life. For instance, consider the Hoogwaterbeschermingsprogramma (HWBP) in the Netherlands, which focuses on reinforcing approximately 1,500 km of dikes over the next thirty years (HWBP, 2023). The reinforcement of dikes in urban areas in the Netherlands have a cost that is in the order of €5 to 10 mln. per meter crest height increase per kilometer dike. Assuming that the dikes need an average of 1 to 2 meters of crest height increase under this project, an overweighing of the required failure probability of the dikes by only 5-10% can lead to an estimated €750 to 1,500 mln. in additional costs over these thirty years. However, it is not solely the financial costs that are of importance. Research has shown that probabilities are often subjectively weighed rather than objectively, with smaller probabilities receiving a disproportionately larger weight than larger probabilities (Tversky & Kahneman, 1992). This perception of (flood) safety can be an important factor when an individual decides where to reside.

It is therefore of interest to examine the influence of risk preferences on the current flood risk management standards. The influence of risk preferences in this context means both the inclusion of utility- and probability weighting functions, as well as a reevaluation of the value of a statistical life. Next to this, the effect of the risk premium part of the interest rate and its paradoxical risk preferences are of interest to better evaluate flood safety standards. These three influences are of interest both from a financial but also a social standpoint. This research aims to provide a better insight in the intersection of flood risk management and behavioural economics.

1.2. Objective

The objective of this research is to gain a better understanding of the intersection of behavioural economics/econometrics and flood risk management. Specifically, this research aims to identify and solve the current knowledge gaps that exist in the intersection. These knowledge gaps consist of unknown research fields as well as known research fields of which the it is currently not known whether the derived values are still valid at the present time.

The former, the unknown research fields, refer to the intersection of flood risk management and behavioural economics. In addition to the general elicitation of risk preferences, three specific topics of interest are: 1) the role of the risk premium in the interest rate, 2) the subjective weighting of small probabilities, and 3) the valuation of a statistical life. These objectives will be further refined as research questions in the following section.

The latter refers to the value of a statistical life, which was last elicited in relation to flood risk management in 2009, with the last general elicitation conducted in 2003. Since these elicitations, various social and economic events have occurred that may have influenced the valuation of a statistical life, such as inflation-driven increases in the cost of living or the occurrence of extreme flood events in the Netherlands, like those in Limburg.

1.2.1. Research questions

When faced with flood risk related problems, individuals have to make decisions under risk. This decision making process is largely dependent on the risk preferences of individuals. This insight, combined with the found knowledge gaps lead to the following main research question:

To what extent do individuals facing flood risk demonstrate risk preferences consistent with traditional behavioural economics? What is the impact of incorporating risk preferences on the acceptable flood probabilities of primary flood defences?

This main research question can be further divided into two distinct two sub-questions.

- *What models best describes individual risk preferences with regards to flood risk related decision problems?*
- *How do societal risk preferences alter the current safety standards in the Netherlands with regards to the allowable probability of flooding, the current leading principle, and costs of reinforcement?*

In this particular context, societal risk preferences refer to the combined risk preferences held by a group of individuals. Additionally, they encompass the influence of these preferences on various widely employed variables within flood risk management. Specifically, these key variables include the value of a statistical life, the risk premium incorporated into the discount rate, and the subjective weighting given to small probabilities.

These two sub-questions form the basis for a formulation of an answer to the main research question. The following section, *approach*, explains how an answer to each of these sub-questions is formulated.

1.3. Approach

To answer the research questions, several steps are undertaken. The first step involves reviewing the current literature on behavioural economics and its intersections with flood risk management. This review explores decision-making under risk in traditional economic choice problems and examines promising theories.

The second step entails eliciting risk preferences from individuals for both traditional economic choice problems as well as flood risk-related choice problems. The objective is to identify models that best fit the observed behavior and to determine whether individuals make similar decisions in flood risk related scenarios as they do in economic problems. If similarities exist, it suggests that standard behavioural economic theory is applicable across different domains in flood risk management. Additionally, this step allows for the elicitation of other variables of interest, such as the value of a statistical life and the weighting of subjective probabilities when probabilities are relatively small.

The third step involves testing the obtained results through a case study. This study examines the current flood safety standards established by Dutch law and incorporates risk preferences and subjective probability weighting. It includes modifications to the standards by reevaluating the value of a statistical life, adjusting the interest rate, and modifying related parameters. Each of these three elements is examined in isolation, before a best estimate for each is given and their joint effect is examined.

The combined insights of these three steps provide the necessary results to answer the main research question. Consequently, the final step consists of the synthesis of the previously found results into answers to the two sub-questions, which in turn can be used to answer the main research question.

1.4. Report outline

The remainder of this report consists of six chapters, next to the introduction.

Chapter 2 serves as the main literature review and is divided into several sections. The chapter starts by expanding on and summarizing several behavioural models commonly found in economics. Next, the current practices used in Flood Risk management in the Netherlands are then explained, after which the final section discusses several intersections of the field of behavioural sciences with Flood Risk management.

Chapter 3 presents the methodology used to elicit risk preferences, the value of a statistical life, and the subjective weighting of small probabilities through a survey. It explains the chosen method, details how the results are processed, and outlines the selection of a risk preference model.

Chapter 4 showcases the results of the survey and the subsequent processing of the results. It starts by summarizing the survey outcomes. After this, the results of the risk preferences, the value of a statistical life and the subjective probability weighting of small probabilities are shown.

Chapter 5 is comprised of a case study. In this case study the findings of chapter 4 are used in a reevaluation of the existing flood safety standards in the Netherlands. To perform this reevaluation, a novel derivation for the flood safety standards that includes risk preferences will be derived. The results of the adjusted flood safety standards are calculated for the three variables of interest: The interest rate, the value of a statistical life and the inclusion of risk preferences. This is done separately for each of the variables as well as jointly.

Chapter 6 consists of the discussion of the found results. It evaluates the findings from chapter 4 and 5, before giving an interpretation of the results. The implications of the findings are presented, followed by a section addressing the limitations of the research and the proposed model.

The final part of this research, chapter 7, contains the conclusion and the recommendation. It begins by answering the research questions posed in the introduction and placing the answers in a broader societal context. A general conclusion is then drawn, followed by several recommendations for further research.

2

Literature Study

2.1. Decision making under Risk

This section will discuss decision making under risk. It will do so by discussing several utility models, the axioms and assumptions behind them, as well provide some examples. The section will start with some general notation used throughout this research.

2.1.1. Axioms and definitions

In *Risk* the possible outcomes and probabilities are known a priori. Under *Uncertainty* the possible outcomes can be know, but the probabilities remain unknown. This sections looks at decision making under risk, and thus at problems with known probabilities about outcomes. In general, the (exceedance, failure or other) probabilities that are used in flood risk management are known, or at the very least estimated. As such, a logical starting point for this research is the existing literature on decision making where probabilities are known.

Before the different models describing behaviour under risk are discussed, several definitions and axioms commonly used throughout all models will be presented. The definitions will lay the groundwork for understanding the models while the axioms provide the reasoning behind and limitations of the discussed utility models.

Definitions

Prospect: A list of consequences with associated probabilities. Denoted by $P = (p_1 : x_1; p_2 : x_2; ...; p_n : x_n)$. Where, p_n is the probability of an outcome x_n. A prospect P is the combination of all possible outcomes and associated probabilities.

Preference: The order given by an individual to alternatives. There are three types of preferences that can be distinguished. Let p and q be two prospects, then the following notation is used to indicate a preference or indifference between the two prospects.

$p \succ q$: p is strictly preferred to q
$p \succeq q$: p is weakly preferred to q
$p \sim q$: Indifference between p and q

Note that indifference does not indicate that a person has no preference, it implies that both $p \succeq q$ and $q \succeq p$.

Monotonicity: The assumption that, independent of other outcomes or events, if an outcome A has a higher pay-off than outcome B with equal probability, than A will always be preferred over B. This statement is assumed to hold true, regardless of which framework or model describing behaviour is used.

Choice list: A list that consists of a prospect with two outcomes, shown as options to choose between. This list is used to elicit risk preferences. One of the options is kept fixed while the other is varied. Finding the point where an individual is indifferent between the two options (often taken as at the point where they switch options), allows for the extraction of their risk attitude. An example of a choice list is given in table 2.1.

Table 2.1: Example of a *Choice list*

Option A		Option B	
Probability	**Pay-out in €**	**Probability**	**Pay-out in €**
0.1	5	1	1
0.1	6	1	1
0.1	7	1	1
0.1	...	1	1

Note that the rows of the table are asked one by one, where the questions stop whenever the subject switches from one option to the other option.

Axioms
Axioms are statements or propositions that are regarded as being accepted, established or self-evidently true. To describe behaviour several of these axioms are needed, as they form the starting point for the models used. The most important axioms are discussed in this section. von Neumann and Morgenstern (1947) showed that, under the following four axioms of rational behaviour, individuals faced with risky outcomes will behave as if they are maximizing the expected value of a later to be determined function over the entire outcome space. Functions that adhere to these axioms are known as *von Neumann-Morgenstern utility functions*. These four axioms will be discussed below as they form the basis for near all (utility) models.

Completeness: When two prospects are compared, an individual must prefer one prospect to the other *or* the individual must be indifferent between the two. This means that for all prospects q and r, it must hold that either $q \geq r$, or $r \geq q$, or both.

Transitivity: If a prospect q is (weakly) preferred over a prospect r and r is subsequently (weakly) preferred over a prospect s, then q must be (weakly) preferred to s. Formally this means that if $q \geq r$ and $r \geq s$ it must hold that $q \geq s$.

Continuity: Consider three prospects q, r and s, where $q \geq r$ and $r \geq s$. Then there must exist a probability p such that the following certainty equivalent holds $r \sim (p : q; 1 - p : s)$.

Independence: Consider again the three prospects q, r and s, and all possible probabilities p, where $q \geq r$ and $r \geq s$. Then it must hold that $(p : q; 1 - p : s) \geq (p : r; 1 - p : s)$.

2.1.2. Expected Outcome

Consider a prospect $L = (p_1 : x_1; p_2 : x_2; ...; p_n : x_n)$. According to the Expected Outcome Theory, the expected value of prospect L is given as:

$$EV(L) = \sum_{i=1}^{n} p_i x_i \tag{2.1}$$

and it can be considered as a weighted average of the outcomes by means of their probabilities. In (Hydraulic) Engineering this is often equated with the concept of *Risk*. Expected Outcome theory is unable to explain some of the basic choices individual actors make. To illustrate a shortcoming of Expected Outcome theory, consider the following example based on the *St. Petersburg Paradox* (Bernoulli, 1954).

Example
Suppose an individual is asked to play a game where a fair coin is flipped repeatedly until a single flip of tails is obtained. When a flip of tails comes up at throw number n, the individual playing the game receives a pay-out of €2^n. Before playing this game, the individual is asked how much he or she is willing to pay to play this game. According to Expected Outcome theory, the individual should be willing to shell out an infinite amount of money to play this game. To see why this is true, first note that this game can be schematized as the following prospect: $L = (0.5 : 2; 0.25 : 4 : 0.125 : 8; ...)$. As such, the expected value of this prospect, as per equation 2.1, should be:

$$EV(L) = \sum_{n=1}^{\infty} \frac{1}{2^n} \cdot 2^n = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 4 + \frac{1}{8} \cdot 8 + ... = \infty$$

and it logically follows that the individual should be willing to pay any price (in fact it should be infinite) to play this game. However, one can sense from their own judgement that paying an infinite amount for this lottery would not be sensible. Many experiments have been carried out, such as by Hayden and Platt (2009), confirming that people are at most willing to spend around €2, with the median being €1.50 in a hypothetical version of the game and €1.75 in an actual version of the game. This paradox illustrates that there is more to decision making under risk then the expected outcome theory.

2.1.3. Expected Utility

To solve the problem of the infinite expected value posed in the St. Petersburg Paradox, Daniel Bernoulli (1738) proposed to measure the value of a lottery not by its expected outcome in isolation but by the expected outcome of the lottery relative to the initial wealth of the individual playing the game. Relating the outcome of the lottery to the initial wealth positions creates a difference in perception of the received amount. Bernoulli theorized that this difference in perception is captured in a function of the wealth increase that is specific to the individual. The result is the theory of *expected utility*.

$$EU(L) = \sum_{i=1}^{n} p_i u(x_i) \tag{2.2}$$

Where $u(x)$ is the individual wealth function as described by Bernoulli. This is the so-called *utility function* of an individual. As the concept of utility is instrumental to this research, the concept will be explained in detail below.

Utility function

Utility is a construct devised to measure preferences of alternatives. The concept is often used as a substitute for happiness, welfare or enjoyment. All of which are hard, if not impossible, to measure. To quantify these variables, mathematical functions were derived that capture the perception of individuals towards increases or decreases with respect to a base level of "happiness". These functions give rise to preferences of alternatives. As each individual is assumed unique, these utility functions often contain more than one parameter to fit the preferences of alternatives of an individual. The general shape and properties however, are similar among individuals. Note that parameters of utility functions are latent variable, meaning that they are variables that are not directly observable in experiments.

More formally a utility function, denoted by $u(\cdot)$, is a function that is able to represent the ordering of alternatives by an individual or group. It assigns a real number to each alternative, such that the evaluation of the utility function for alternative 1 is higher than for alternative 2 if alternative 1 is preferred to alternative 2 as it has a higher utility. Consider the following example of the evaluation of two alternatives using utility functions.

Example

Suppose an individual has a utility function $u(x, y) = x\sqrt{y}$, where x and y are two goods. The first alternative contains a combination of $x = 3$ and $y = 9$, the second alternative contains a combination of $x = 2$ and $y = 16$. Then the individual has a utility of $u_1(3, 9) = 9$ and $u_2(2, 16) = 8$ for alternative 1 and 2, respectively. As such, the individual will prefer alternative 1 to alternative 2.

Utility functions can generally be classified into two classes, *cardinal* and *ordinal*. Both of which have a different approach to the same concept of utility, differing in the degree of quantification possible.

Cardinal utility

Cardinal utilities can be independently ranked and given an explicit value, often expressed in *utils*. As the utilities can be explicitly expressed, it is possible to rank them based on absolute strength of preference. Note that the previous given example is an example of cardinal utility as the two alternatives were quantified in terms of utils. In this example, it could be stated that alternative 1 has a utility that is $1/8^{th}$ higher than that of alternative 2.

Ordinal utility

Ordinal utilities can not reveal the absolute strength of preferences, they can be solely used to judge the *relative* strength of preferences between options. For the given example that would mean that it is only possible to state that alternative 1 is preferred to alternative 2, but it is not possible to quantify by how much.

A consequence of this relative strength is that utility functions that are raised to an odd power can not be distinguished from one another. As the ranking for the functions $u(x)$ and $u^n(x)$, for $n = 3, 5, 7, ...$. are identical. The reason for this is that the transformation of function to an odd power is a monotonic transformation, meaning that ordering is maintained.

For many applications of utility functions it is not necessary to write them explicitly and exact, e.g. use cardinal utility. Often, it suffices to known which alternative is preferred. As such, it is ordinal utility that is most often used to reveal preferences of individuals.

2.1.4. Defining risk preferences

With the theories of expected outcome and expected utility known, it is possible to give a definition to the concept of *risk preferences*. Risk preferences denote the attitude an individual has towards risk. This attitude is shown by the shape of their utility function. There are three types of preferences that will be discussed.

Risk neutral
When an individual has a risk neutral approach, he or she will act according to the expected outcome theory. An increase or decrease in their wealth position is weighed equally, regardless of the position of their wealth. Mathematically, their utility function can be expressed as $u(x) = c \cdot x$ where c is some constant, corresponding to a straight line.

Risk averse
When an individual has a risk averse approach, he or she will deviate from the expected outcome theory. An increase or decrease in their wealth position is now dependent on the current position of wealth. Specifically, the individual will weigh smaller gains with a higher probability more heavily than larger gains with smaller probabilities, even if the expected outcome is equal. Mathematically, their utility function can be expressed as any type of concave function such as $u(x) = c \cdot x^n$. Where c is again some constant and $0 < n < 1$. Note that other concave functions are also possible, such as the logarithmic function.

Risk seeking
When an individual has a risk seeking approach, he or she will again deviate from the expected outcome theory. An increase or decrease in their wealth position is now dependent on the current position of wealth. Specifically, the individual will weigh smaller gains with a higher probability less heavily than larger gains with smaller probabilities, even if the expected outcome is equal. Mathematically, their utility function can be expressed as any type of convex function such as $u(x) = c \cdot x^n$. Where c is again some constant and $n > 1$. Note that other convex functions are also possible, such as the exponential function.

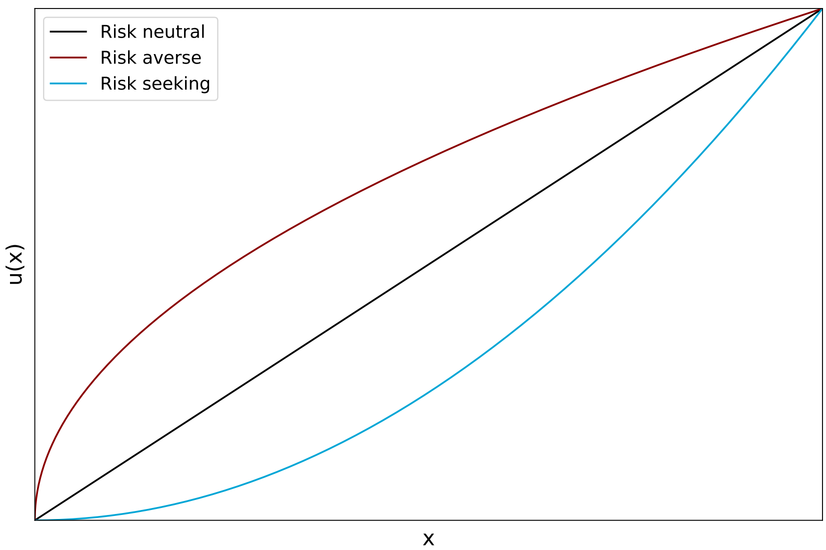


Figure 2.1: Three different types of risk preferences that can be observed in typical utility functions

The three types of risk preferences are shown in figure 2.1. Note that the function is capped at the point where the three functions align, at the top right corner. In reality, the functions will continue after this point with their respective shapes. It can be seen that the risk neutral approach corresponds to a straight line, e.g. a line where all outcomes are weighted equally regardless of the position along the x-axis. Conversely, it quickly becomes apparent that the evaluation of the utility for both the risk averse and risk seeking approach is dependent on the position along the x-axis of the graph.

Quantifying risk preferences
Under expected utility, the degree of risk aversion can be expressed by the coefficient of absolute risk aversion. This coefficient is given in equation 2.3.

$$r(x) = -\frac{u"(x)}{u(x)} \tag{2.3}$$

Where $u'(x)$ and $u"(x)$ indicate the first- and second derivative of the utility function $u(x)$ with respect to x. The sign of the coefficient of absolute risk aversion indicates the risk preference of the individual. The sign can be directly associated with the shape of the utility curves under expected utility. The second derivative of a concave function will be negative whereas the first derivative will be positive, resulting in a positive coefficient of absolute risk aversion. In the case of a convex utility curve both the first and second derivative will be positive, resulting in a negative coefficient of absolute risk aversion.

Linking these concepts to the coefficient of absolute risk aversion, it can be deduced that a positive coefficient corresponds to risk aversion, while a negative coefficient indicates risk seeking behaviour. If the coefficient is exactly zero, the risk preference of the individual is neutral. The higher the coefficient of absolute risk aversion is, the more risk averse an individual is.

Certainty Equivalent and the Risk Premium
Consider a prospect q that is offered to someone. One might want to know when (and if) an individual is indifferent between receiving the prospect q or a given amount with guaranteed certainty. The outcome that makes a person indifferent between receiving the prospect or receiving the guaranteed amount is called the *Certainty Equivalent (CE)*. This CE can be used as a measure for risk aversion. Using the standard notation for preferences, it is denoted that the CE is equivalent to:

$$(1 : CE) \sim (p_1 : x_1; p_2 : x_2; ...; p_n : x_n)$$

Retrieving the CE can be done with choice lists and via two main methods, which will be discussed further in the section on *Measuring utility*.

The *Risk Premium* is the difference between the expected value of a prospect and the certainty equivalent of an individual.

$$\text{Risk Premium} = EV(q) - CE$$

As such, the risk premium can be seen as the price an individual has to ”pay” for not pursuing a risk neutral approach to the prospect. This premium can be both positive or negative, depending on the risk preferences of the individual. A positive risk premium indicates risk aversion, whereas a negative risk premium indicates risk seeking preferences.
The risk premium can also be approximated as:

$$\text{Risk Premium} \approx \frac{1}{2}\mathbb{V}(q)r(x)$$

when the variance of the prospect q is small (e.g. $\sigma_q \to 0$), as shown by Pratt (1964).

These two definitions, the certainty equivalent and the risk premium, are independent of the type of utility model being used and are therefore applicable on a wide range of problems related to choice under risk.

Measuring utility
There are several ways to measure utility, two of the most common techniques which will used in this research are discussed in this section. These two methods bare a lot of similarities, apart from that they draw inference on two different parameters: the probability and the outcome of alternatives.

Certainty equivalent method
Two outcomes m and M are fixed, where $m < M$. The utility of m is fixed to zero and the utility of M is fixed to one. This does not lead to a loss of generality as a result of *uniqueness of utility*. After the fixation of the utilities, the following steps are undertaken.

1. Fix the probability p
2. Vary c, the certainty equivalent, up to the point where the individual is indifferent, e.g. $c \sim (p : M; 1 - p : m)$

If expected utility is assumed to hold, once can express the utility of the certainty equivalent c as $u(c) = p \cdot u(M) + (1 - p) \cdot u(m) = p$. If the steps 1 and 2 are repeated for several values of the probability p, the certainty equivalents can be found. This leads to a set of certainty equivalents for various probabilities, from which a utility curve can be constructed.

Probability equivalent method
Again, two outcomes m and M are fixed, where $m < M$. The utilities of m and M are set to zero and one respectively. The difference with the Certainty equivalent method is that the variation in the steps of the method are different. For the probability equivalent method, the following steps should be undertaken.

1. Fix the certainty equivalent c
2. Vary p, the probability of an outcome, up to the point where the individual is indifferent, e.g. $c \sim (p : M; 1 - p : m)$

If expected utility is again assumed to hold, the same expression for the utility of c can be used to find the various probability equivalents. These equivalents can then be used to construct the utility curves of the individual.

Violations of Expected Utility
One of the best known counter-examples to Expected Utility theory was shown by Allais (1953). The counter example leans on the exploitation of the certainty effect. In the so-called common consequence task that Allais presents, individuals are asked twice to to choose between two alternatives to elicit their (risk) preferences. The first set concerned the following alternatives.

$$A = \begin{cases} \text{1 million francs} & \text{with an 11\% probability} \\ 0 & \text{with an 89\% probability} \end{cases} \quad \text{or} \quad B = \begin{cases} \text{5 million francs} & \text{with a 10\% probability} \\ 0 & \text{with a 90\% probability} \end{cases}$$

The second set consisted of the following two alternatives.

$$A' = \text{1 million francs with a 100\% probability} \quad \text{or} \quad B' = \begin{cases} \text{5 million francs} & \text{with an 10\% probability} \\ \text{1 million francs} & \text{with an 89\% probability} \\ 0 & \text{with a 1\% probability} \end{cases}$$

Note that the second set of choices can be obtained from the first set of choices by adding 0.89 of the probability mass to 1 million francs and consequently subtracting 0.89 of the probability mass from the zero outcome.
According to Expected Utility Theory, if one prefers alternative A over B, it should logically follow that

he or she also prefers alternative A'over B' in the second set. However, as Allais observed in his experiments the majority of people choosing B in the first set opts for A' in the second set, violating Expected Utility Theory. The violation can be formally proved by noting that in the first set of choices the preference of B over A suggests that:

$$0.10 \cdot u(\text{5 million}) + 0.90 \cdot u(0) > 0.11 \cdot u(\text{1 million}) + 0.89 \cdot u(0)$$

While in the second experiment, a preference of A' over B' suggests that:

$$1.00 \cdot u(\text{1 million}) > 0.10 \cdot u(\text{5 million}) + 0.89 \cdot u(\text{1 million}) + 0.01 \cdot u(0)$$

Now, if $0.89 \cdot u(\text{1 million})$ is added to the first equation and $0.89 \cdot u(0)$ is subtracted from the first equation and re-arranging terms, it is found that:

$$1.00 \cdot u(\text{1 million}) < 0.10 \cdot u(\text{5 million}) + 0.89 \cdot u(\text{1 million}) + 0.01 \cdot u(0)$$

Which is a direct contradiction with the result obtained in the second set of choices, the equality sign switches relative to the second equation. As such, this is a violation of the independence condition that is necessary for Expected Utility theory to hold.

2.1.5. Rank-dependent probability weighing

In the previous section a violation of Expected Utility Theory was shown by means of the Allais paradox. A solution to the inconsistencies presented by Allais' experiments could be to weigh the value of probabilities different dependent on their values. Low probabilities, corresponding to a greater uncertainty, could receive a larger weight while higher probabilities, corresponding to a lower uncertainty, could receive a smaller weight. This idea of weighing probabilities led to the theory on *Subjective Expected Value*, which in turns led to the theory on *Rank-dependent Utility*. Both of these theories will be discussed in this section.

Subjective Expected Value

In Expected Utility Theory individuals take the objective value of probabilities and the subjective value of the outcome. In Subjective Expected Value Theory, the individuals take the subjective value of probabilities and the objective value of the outcome. According to this theory, individuals should weigh the prospects via equation 2.4.

$$SEV(p) = \sum_{i}^{n} \pi(p_i) x_i \tag{2.4}$$

Where $\pi(\cdot)$ is the *Probability weighting function*. This function holds many of the same properties as the utility function. For example, $\pi(\cdot)$ is an increasing function with $\pi(0) = 0$ and $\pi(1) = 1$. However, unlike the utility function the probability weighing function is bounded on [0,1].

This theory has a problem with the axioms of behavioural economics, presented in introduction of this chapter. *Monotonicity* is violated as soon as the weighing function $\pi(\cdot)$ is non-linear. To illustrate the violation of monotonicity an example will be provided. Before presenting this example, a generalization of the Subjective Expected Value model is given, where a utility weighing function is used instead of the value of the outcome to generalize the result of this violation. This results in the following function for a prospect $q = (p_1 : x_1; p_2 : x_2; ...; p_n : x_n)$.

$$V(q) = \sum_{i=1}^{n} \pi(p_i) u(x_i)$$

Note that if $u(x) = x$ is taken, this model simplifies to the Subjective Expected Value model presented in equation 2.4. To show the shortcomings of this model, consider the following example.

Example
Assume the following specification of the subjective expected value model: $u(x) = x$ and $\pi(p) = p^2$. As such, we have that:

$$V(p) = \sum_{i=1}^{n} p_i^2 \cdot x_i$$

Now consider the following two prospects $q = (0.5 : 100; 0.5 : 101)$ and $r = (1 : 99)$. It follows that:

$$V(q) = 0.25 \cdot 100 + 0.25 \cdot 101 < 0.99 = V(r)$$

Which is a clear violation of the axiom of monotonicity, as prospect q always has a higher pay-off than prospect r.

Several attempts at solving these types of violations were made. One of which, *Prospect Theory*, presented by Daniel Kahneman and Amos Tversky which will be looked at in depth later on. Their original version did not solve the violations completely. However, a solution to this problem was presented by John Quiggin in two papers. He suggested to weigh the probability of getting *at least* x, rather than to weigh the probability of getting *exactly* x. This resulted in the theory of *Rank-dependent Utility*.

Rank-dependent Utility
Rank-dependent Utility (RDU) can be seen as a combination of Expected Utility theory and Subjective Expected Value theory, where the outcomes are ranked. A weight is attached to a state of nature that depends on both its probability and its ranking relative to the other states.

To compute the RDU, the outcomes should first be ordered by absolute value, such that $x_1 \leq x_2 \leq ... \leq x_n$. After this ordering, a prospect of the form $q = (p_1 : x_1; p_2 : x_2; ...; p_n : x_n)$ can be considered. Evaluating the prospect can then be done by means of equation 2.5.

$$RDU(q) = \sum_{i-1}^{n} \pi_i u(x_i) \tag{2.5}$$

Where π_i are the *decision weights*, defined as:

$$\begin{aligned} \pi_i &= w(p_i + ... + p_n) - w(p_{i+1} + ... + p_n) && \text{for all } i < n \\ \pi_n &= w(p_n) && \text{if } i = n \end{aligned}$$

with $w(\cdot)$ the probability weighing function, as seen in the previous section where it was denoted by $\pi(\cdot)$. The decision weight can be viewed as the "marginal contribution" of event i to the probability weight $w(p_i + ... + p_n)$.

Note that if a linear probability weighting function is chosen, e.g. $w(p) = p$, RDU simplifies to the Expected utility model, since:

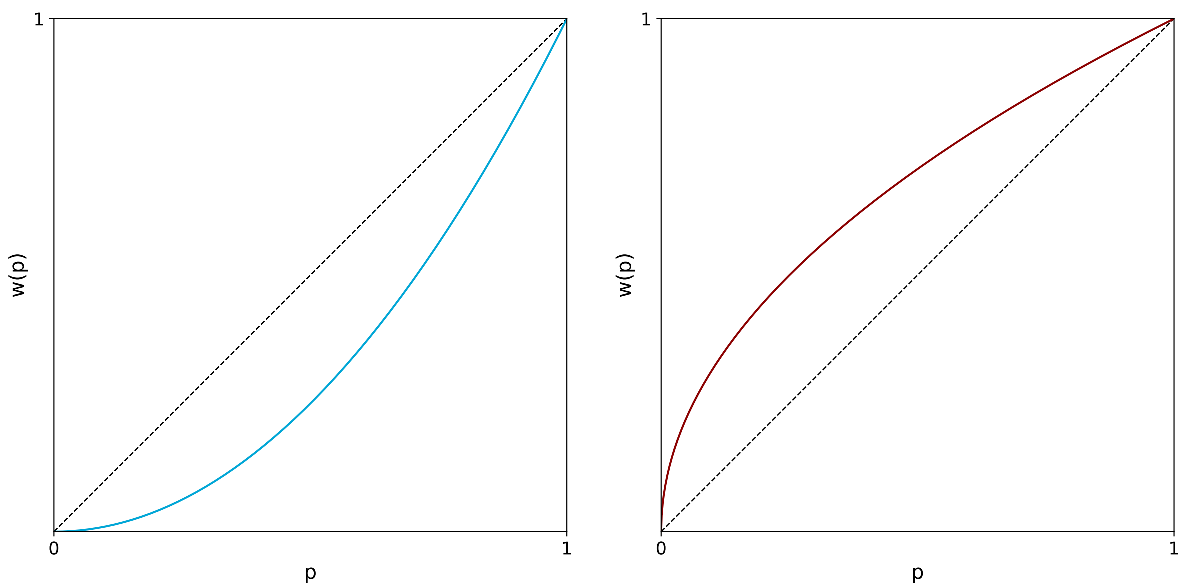
$$\begin{aligned} \pi_i &= w(p_i + ... + p_n) - w(p_{i+1} + ... + p_n) \\ &= p_i + ... + p_n - (p_{i+1} + ... + p_n) \\ &= p_i \end{aligned}$$

Pessimism and Optimism in Rank-dependent Utility
In the general sense *pessimism* and *optimism* refer to the expected turn-out of a situation. Pessimism refers to the expectation that a situation will turn out for the worse, while optimism refers to the expectation that a situation will turn out for the better. The definition in general sense bear some relation to the definitions used in (behavioural) economics, but with some slight alterations. In the general economic sense, optimism and pessimism mean the same as in the commonly used definitions. However, when looking at the technical definitions in relation to risk preferences, there are significant differences.

Pessimism
Worsening the rank of an outcome, by increasing the.probability of getting a better outcome, increases its decision weight $w(\cdot)$. The result is that the worse the outcome is relative to the other outcomes the more decision weight it receives. Linking the concept of *pessimism* to the concept of the probability weighting function, it is found that a concave probability weighting function enhances optimism and thereby risk-aversion for gains.

Optimism
Worsening the rank of an outcome, by increasing the.probability of getting a better outcome, decreases its decision weight $w(\cdot)$. The result is that the better the outcome is relative to the other outcomes the more decision weight it receives. Linking the concept of *optimism* to the concept of the probability weighting function, it is found that a concave probability weighting function enhances optimism and thereby risk-seeking behaviour for gains.

Pessimism can be graphically explained as a probability weighting function that monotonically increases and runs below the 45 degree line on a unit square with the probability weight $w(p)$ on the y-axis and the objective probability p on the x-axis. Optimism can be graphically shown as a probability weighting function that monotonically increases and runs above the 45 degree line on the same figure. Note that these shapes correspond to a concave and convex shape of the probability weighting functions, respectively. The results are shown in figure 2.2a and 2.2b.



(a) Pessimistic probability weighting function

(b) Optimistic probability weighting function

Figure 2.2: Pessimistic and Optimistic probability weighting functions

The concepts of risk aversion and risk seeking behaviour can also be linked to Pessimism and Optimism. To illustrate this, consider the following numerical example.

Example
Take the following prospect $q = (0.25 : 20; 0.25 : 40; 0.25 : 60; 0.25 : 80)$ and for simplicity assume a linear utility function $u(x) = x$. For the probability weighting functions both a pessimistic (concave) and optimistic (convex) probability weighting function will be chosen. Let $w_p(p) = p^2$ be the pessimistic weighting function and $w_o(p) = p^{\frac{1}{2}}$ be the optimistic probability weighting function. Recall that to determine an individuals risk preference, the difference between the expected value and the certainty equivalent (CE) should be calculated. Furthermore, recall that the following relation between the CE and any utility function (which is the RDU in this case) holds for any prospect L: $u(CE(L)) = RDU(L)$, e.g. the utility of the certainty equivalent of a prospect L should be equal to the rank-dependent utility of said prospect L.

Using a linear utility function, the evaluation of the utility of the certainty equivalent simplifies to $CE = RDU(L)$. As such, the Certainty Equivalent is simply the computed Rank-dependent Utility. The results of the probability weighting function for both the pessimistic and optimistic case are shown in table 2.2.

Table 2.2: Probability weights for pessimistic and optimistic probability weighting functions

Probability weight	Pessimistic	Optimistic
$\pi_{80} = w(0.25) - w(0)$	0.0625	0.5000
$\pi_{60} = w(0.50) - w(0.25)$	0.1875	0.2071
$\pi_{40} = w(0.75) - w(0.50)$	0.3125	0.1589
$\pi_{20} = w(1.0) - w(0.75)$	0.4375	0.1340

The RDU, and thereby the CE, for prospect L can be calculated via equation 2.5 in both the pessimistic and optimistic case to be:

$$RDU_p = 0.0625 \cdot 80 + 0.1875 \cdot 60 + 0.3125 \cdot 40 + 0.4375 \cdot 20 = 37.5 = CE_p$$

$$RDU_o = 0.5000 \cdot 80 + 0.2071 \cdot 60 + 0.1589 \cdot 40 + 0.1340 \cdot 20 = 61.5 = CE_o$$

The expected value for this prospect is 50, and as such it can be confirmed that the pessimistic probability weighting function corresponds to a risk averse preference as $37.5 < 50 \longrightarrow CE_p < EV$. Similarly it can be confirmed that the optimistic probability weighting function corresponds to a risk seeking preference as $61.5 > 50 \longrightarrow CE_o > EV$.

2.1.6. Prospect Theory

Prospect Theory is a theory in behavioural economics that was developed by Daniel Kahneman and Amos Tversky. Their work resulted in the 2002 Nobel Memorial Prize in Economics being awarded to the researchers. Prospect Theory was proposed as a solution to the violations that occurred in empirical validations of the Expected Utility Theory for risky prospects. In their paper Kahneman and Tversky (1979) address three main critiques of Expected Utility theory, namely *the Certainty effect*, *the Reflection effect* and *the Isolation effect*. The former was described in previous sections by the paper of Allais (1953). The other two effects will be explained below.

The Reflection effect
The Certainty effect describes preferences between positive prospects, i.e. only prospects involving gains and not losses. In their 1979 paper, Kahneman and Tversky replaced the positive prospects in their posed problems with negative prospects. The results were a reversion of the preference order around zero for the problems posed in the paper. This pattern was labeled as the *Reflection effect*, and implies that risk aversion in the positive domain is accompanied by risk seeking in the negative domain. Their experiments further demonstrated that the preferences between the corresponding negative prospects are also subject to the *Certainty effect*. The choices by individuals in the negative domain of problems tend to favour certainty over uncertainty, which contributes to risk seeking preference for a loss that is merely probable over a smaller loss that is certain. Recall that this effect was reversed for problems in the positive domain, e.g. risk aversion led to the preference of smaller, but near certain gains over larger, but merely probable gains. Finally, the experiments demonstrated that the reflection effect eliminates aversion for uncertainty as an explanation of the certainty effect. They

demonstrated that certainty increases the aversion of losses as well as the desirability of gains.

The Isolation effect
When individuals are faced with a choice, they often disregard components that the alternatives share and focus on the components that distinguish them to simplify the choice process. This can lead to inconsistent preferences as a pair of prospects can often be decomposed in more than one way, which can result in a difference of preferences among the same prospect. This phenomenon is referred to as *The Isolation effect*, and is the second violation of Expected Utility theory noted by Kahneman and Tversky.

The four-fold pattern
One of the important findings of the study by Kahneman and Tversky (1979) was the so-called *Four-fold pattern*. This pattern describes risk preferences for losses and gains with small and large probabilities, resulting in two preferences across four possibilities. The four-fold pattern can be seen in table 2.3.

Table 2.3: The fourfold patterns that describes the different combinations of probabilities and gains/losses that are typically associated with risk averse or -seeking preferences as derived by Tversky and Kahneman (1992)

	Gains	Losses
High probability	Risk averse	Risk seeking
Low probability	Risk seeking	Risk averse

Kahneman and Tversky found in their study that individuals have reversed risk preferences in the positive (gains) and negative (losses) domains. In general, people exhibit risk seeking behaviour for gains with small probabilities, while for losses they are generally risk averse. Two clear examples of the reversion of risk preferences are the tendency of individuals to purchase insurance for large losses with small probabilities and the purchase of lottery tickets, which having a (very) small probability of winning a large amount. The cost of insurance is often greater than the expected loss, resulting in a positive risk premium, indicating risk aversion. The cost of a lottery ticket is often greater than the expected gain resulting in a negative risk premium, indicating risk seeking preferences.

The theory
The theory developed by Kahneman and Tversky consists of two phases in the choice process, the *editing phase* and the *evaluation phase*. The former occurs before the latter in chronological order. Both of these phases will briefly be explained below.

Editing phase
The editing phase of the evaluation contains a preliminary analysis of the offered prospects, which often yields a simplified representation of the stated prospects. The editing phase consists of the operation of several operations that transform the probabilities and outcomes that are associated with the prospects being offered. There are four major operations, *coding*, *Combination*, *Segregation* and *Cancellation*, each of which is discussed below.

Coding
The empirical results from the paper of Kahneman and Tversky suggests that individuals perceive outcomes as gains and losses, rather than as final states of their personal wealth or welfare. Those gains and losses are defined relative to some reference point that might differ per individual. This reference point can be affected by the formulation of the offered prospects, which can alter the location of the reference point and the consequent coding of outcomes as gains or losses.

Combination
Combination refers to the possibility of combining probabilities of identical outcomes to simplify the offered prospects. For example, the prospect $(0.10 : 500; 0.55 : 400; 0.35 : 500)$ can be reduced to the form $(0.45 : 500; 0.55 : 400)$ and subsequently evaluated in this form.

Segregation
Segregation concerns the elimination of a riskless component in a single prospect in the editing phase. For example, when considering the prospect $(100 : 0.30; 300 : 0.70)$ an individual might recognize that there is a certainty of receiving 100 and the prospect can therefore be segregated into a sure gain of 100 and a risky prospect of $(200 : 0.7)$. This operations also works on negative prospects. For example, when an individual considers the prospects $(-200 : 0.45; -500 : 0.55)$, it can be viewed as a sure loss of 200 and a risky prospect of $(-300 : 0.55)$.

Cancellation
Cancellation involves the discarding of common constituents between prospects. Consider the following example, where an individual is asked to give his or her preference between the following prospects $(400 : 0.1; 150 : 0.3; -200 : 0.6)$ and $(200 : 0.1; 150 : 0.3; -100 : 0.6)$. This evaluation can be simplified by noting the common constituents, such that the choice between the following two prospects will be evaluated: $(200 : 0.1; -200 : 0.6)$ and $(-100 : 0.6)$.

Next to these four major operations there are two smaller and less quantifiable operations that were observed in experiments that are important to mention. The first is that of *simplification*. This operations concerns the rounding of outcomes and/or probabilities by individuals. A prospect of $(149 : 0.51; -99 : 0.49)$ is likely to be recorded as an even chance of gaining 150 or losing 100. Kahneman and Tversky note that an important form of simplification occurs when an extremely unlikely event is incorporated in the prospects, as this unlikely event is often discarded. As the exceedance probabilities in hydraulic engineering are often in the "extremely unlikely" range, this is an important finding with far reaching consequences.

The second operation that they noted occurs was that of the scanning of offered prospects to detect *dominated alternatives*, which are subsequently rejected without further consideration or evaluation by the individual. Kahneman and Tversky present the example of the following two prospects: $(0.20 : 500; 0.49 : 101)$ and $(0.15 : 500; 0.51 : 99)$. If the act of simplification is carried out, the second constituent of both prospects will appears to be $(0.50 : 100)$ in which case the the former prospect will dominate the second one, leaving it out for consideration by the individual. From this example it quickly becomes apparent that the order in which the operations are carried out influences the results of the evaluation of the prospects. In large part, this comes down to the formulation of problems. In their paper, Kahneman and Tversky assume that the formulation of the prospects leaves no room for further editing.

Evaluation phase
After the Editing phase, the evaluation phase follows. Once the prospects are (mentally) adjusted by the individual presented with the choice problem, it is assumed that he or she evaluates the edited prospects and chooses the one with the highest value. Let V denote the overall value of the edited prospect to the individual, which can be expressed into two scales π and v.
π is again the weighting function of the probabilities p by an individual, denoted by $\pi(p)$. Note that π is not a probability measure and it is often the case that $\pi(p) + \pi(1 - p)$ does not sum to unity.
The second scale, v, assigns a subjective value to the outcome x , denoted by $v(x)$. It should be noted that this subjective value is relative to the chosen reference point of the individual, and not an absolute value.

Using these definitions it is possible to derive a framework for the judgement of prospects. The basic equation of the theory, describing the value of a prospect, is given in equation 2.6. Let $(p : x; q : y)$ be a *regular* prospect under consideration, where there are at most two non-zero outcomes. This prospect can be viewed as a lottery where there is a probability p of receiving x and a probability of q of receiving y, leaving a probability of $1 - p - q$ of receiving nothing. For the prospect to be considered *regular*, the following conditions must hold: $p + q < 1$ or $x \geq 0 \geq y$ or $x \leq 0 \leq y$. It then follows that the evaluation of the prospect is performed as per equation 2.6.

$$V(p : x; q : y) = \pi(p)v(x) + \pi(q)v(y) \tag{2.6}$$

Note that the zero outcome is not explicitly mentioned in the prospect, as it is assumed that $v(0) = 0$. Furthermore, outcomes that have a zero probability get no weight ($\pi(0) = 0$) and certainties get unity weight $\pi(1) = 1$.

The evaluation of strictly positive or negative prospects follows different rules than the prospect considered above. If $p + q = 1$ and either $x > y > 0$ or $x < y < 0$, equation 2.6 transforms into equation 2.7.

$$V(p : x; q : y) = v(y) + \pi(p)[v(x) - v(y)] \tag{2.7}$$

Where it can be seen that the riskless component is segregated in the term $v(y)$. Note that the right-hand side of equation 2.7 can be written as $\pi(p)v(x) + [1 - \pi(p)]v(y)$, which reduces to equation 2.6 if $\pi(p) + \pi(1 - p) = 1$.

Value function
The value function is a crucial aspect of the presented theory, explicitly modelling that changes in wealth or welfare are the drivers of changes in utility rather than the final states. Kahneman and Tversky make the compelling argument that this is compatible with the principles of judgement and perception of individuals. After all, one senses changes in temperature when entering a new space rather the absolute temperature of the room. Another example is the sudden perceived increase in brightness when driving out of a tunnel, the surroundings appear much brighter than they were before driving into the tunnel. Therefore, Kahneman and Tversky hypothesise that the evaluation of monetary changes follows the same concave function of the magnitude of physical change as is the case for sensory and perceptual dimensions.

The consequences of this hypothesis is that the value function for changes of wealth is normally concave above the reference point (e.g. $\frac{\partial^2 v(x)}{\partial x^2} < 0$ for $x > 0$) and normally convex below the reference point (e.g. $\frac{\partial^2 v(x)}{\partial x^2} > 0$ for $x < 0$). This property has been empirically validated by several experiments in their own paper.Take for example the following prospects, having the form $(0.50 : x; 0.50 : x)$ is often deemed unattractive by the majority of individuals. Furthermore, the aversiveness to such prospects generally increases for larger stakes, meaning that if $x > y \geq y$, then $(0.50 : y; 0.50 : -y)$ is preferred to $(0.50 : x; 0.50 : -x)$. Following equation 2.6, it must therefore hold that:

$$v(y) + v(-y) > v(x) + v(-x) \longleftrightarrow v(-y) - v(-x) > v(x) - v(y)$$

Two interesting results can be inferred from these relationships. If y is set equal to zero, it follows that $v(x) < -v(-x)$. This inequality shows that the value function for losses is larger than the value function for gains. Furthermore, if y approaches x it follows that $\frac{\partial v(x)}{\partial x} < \frac{\partial v(-x)}{\partial x}$, provided that the first derivative of v with respect to x exists.

This implies that the value function for losses is steeper than the value function for gains. In experiments it has often been found that ”losses hit about twice as hard as gains”. The derived results lead to a value function that is shaped like an asymmetrical S as the function is larger for losses than gains and generally steeper closer to the reference point. An example of a value function that adheres to the results of Kahneman and Tversky is given in figure 2.3.

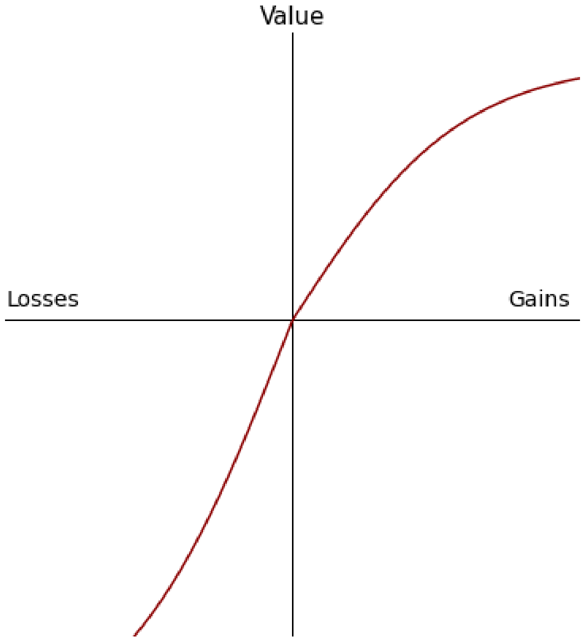


Figure 2.3: Example of a general utility function where losses are weighed more than gains as derived by Kahneman and Tversky (1979)

Weighting function
The weighing functions describes the weight that individuals attach to certain probabilities. It should not be confused with actual probabilities, as it does not adhere to the probability axioms. The values of the probability weighing function do not run from 0 to 1.

Based on the preferences that became apparent in experiments, Kahneman and Tversky prove that for small values of p, the probability weighting function $\pi(\cdot)$ is a sub-additive function of p, meaning that $\pi(rp) > r\pi(p)$. They demonstrated this by conducting a choice experiment between the following two prospects $(0.001 : 6000)$ and $0.002 : 3000)$. The first prospect was preferred over the second prospect by the majority of the participants. This means that:

$$\frac{\pi(0.001)}{\pi(0.002)} > \frac{v(3000)}{v(6000)} > \frac{1}{2}$$

Where the last equality must hold due to the concavity of v. Another choice problem that was posed, with larger values for p, shows that the sub-additive property does not hold for large p. Furthermore, they prove that very low probabilities are generally overweighed, e.g. $\pi(p) > p$. This was proved by the results of the following two choice problems, 1 and 1':

$$1 = \begin{cases} 5000 & \text{with a 0.1\% probability} \\ 5 & \text{with a 100\% probability} \end{cases} \quad \text{or} \quad 1' = \begin{cases} -5000 & \text{with a 0.1\% probability} \\ -5 & \text{with a 100\% probability} \end{cases}$$

The preferences of the two groups was reversed for the two choice problems. Individuals prefer a risky prospect when the outcomes are positive while they prefer a sure (small) loss when the outcomes are negative. Note that the preferences in problem 1 can be viewed as preferring a lottery ticket over a small payout while problem 1' can be viewed as insuring oneself for a small loss over the possibility of a larger loss. From the preferences of a gamble in the positive domain and an insurance in the negative domain it follows that:

$$\pi(0.001)v(5000) > v(5) \longleftrightarrow \pi(0.001) > \frac{v(5)}{v(5000)} > 0.001$$

where the last inequality holds when the value function for gains is concave. Note that problem 1' implies the exact same conclusion, given that the value function for losses is convex. These and other empirical findings provide strong evidence for the tendency of individuals to overweigh small probabilities, an important finding for risk perception in a broader sense and the ability to judge risks related to flooding in particular.

2.1.7. Cumulative Prospect Theory

Cumulative Prospect Theory (Tversky & Kahneman, 1992) is an extension of the original *Prospect Theory*. It employs cumulative rather than separable decision weights to extend the theory. It can be viewed as a combination of Rank-dependent Utility Theory and the original Prospect Theory. It solves two of the main critiques of the original prospect theory, namely the violations of monotonicity and the fact that Cumulative Prospect Theory (CPT) does not require the process of editing. Kahneman and Tversky solved these two main critiques by adjusting the probability weighing function to include different weights for the positive and negative domain. The evaluation function is given in equation 2.8.

$$
\begin{aligned}
PT(q) &= \sum_{i=1}^{k} \pi_i^- v(x_i) + \sum_{i=k+1}^{n} \pi_i^+ v(x_i) \\
\pi_i^- &= w^-(p_1 + p_2 + ... + p_i) - w^-(p_1 + p_2 + ... + p_{i-1}) \text{ for } 2 \leq i \leq k \\
\pi_i^+ &= w^+(p_i + p_{i+1} + ... + p_n) - w^+(p_{i+1} + p_{i+2} + ... + p_n) \text{ for } k+1 \leq i \leq n-1 \\
\text{Where } & \pi_1^- = w^-(p_1) \text{ and } \pi_n^+ = w^+(p_n)
\end{aligned}
\tag{2.8}
$$

CPT, in a similar way as Prospect Theory, uses a value function $v(\cdot)$. This value function, with an increased slope for the negative domain with respect to the positive domain, is of similar shape as the function shown in figure 2.3. Note that the value function is not split into two different functions, having one continuous function for both the positive and negative domain.

Unlike the value function, the probability weighing function $\pi(\cdot)$ is now split into two functions $\pi^+(\cdot)$ and $\pi^-(\cdot)$. The choices of individuals are weighed separately in the positive and negative domain. For gains (the positive domain) the decision weight $\pi^+(\cdot)$ of an outcomes equals the probability weight w^+ assigned to the probability of obtaining at least that outcome, minus the probability weight w^+ of the probability of getting more than that outcome. For losses (the negative domain) the decision weight $\pi^-(\cdot)$ of an outcomes equals the probability weight w^- assigned to the probability of obtaining at most that outcome, minus the probability weight w^- of the probability of less more than that outcome. After conducting several experiments to determine the general shape of such value functions $\pi(\cdot)$, Kahneman and Tversky found a shape similar to the one shown in figure 2.4.

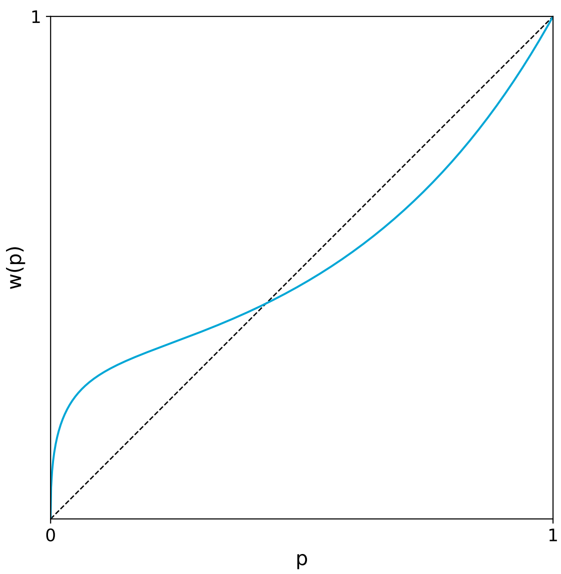


Figure 2.4: Example of the typical S-shape found in an empirical probability weighting function

From figure 2.4 it can be seen that small probabilities are given a larger weight $w(p)$ relative to the probability p that is used as an input. The black dotted line shows a neutral weighing of the probabilities, corresponding to the probability weighting function $w(p) = p$. The value of the probability weighing function (blue) is larger than that of the neutral probability weighing function (black) for probabilities up to around 0.4, after which the probability weighing function remains lower than the neutral function. Around probabilities of 0.3 it can be seen that the probability weighting functions has a slope that quickly decreases, being smaller than the neutral weighing of probability. The slope once again increases above the neutral function after probabilities around 0.7. This graphical representation shows the effect of *diminishing sensitivity*.

Diminishing sensitivity is the phenomena where individuals give large weights to small and large probabilities but are less sensitive to probabilities that lie between. To be more precise, the sensitivity is diminishing up to around the midpoint of the upper- and lowerbound of the probability overweighing. The tendency to overweigh small probabilities can partly explain why people tend to buy lottery tickets even though the expected earnings are lower then the ticket costs. If they estimate the probability of winning big higher than it actually is, their decisions to buy a ticket can be considered rational. The same logic goes for the purchase of insurance, where overweighing can (partly) explain the tendency of individuals to pay a fee that is often more than the expected damages.

2.2. Flood Risk management in the Netherlands

To determine risk preferences within flood risk management it is crucial to understand the current approach to flood risk management in the Netherlands. This requires the explanation of some key concepts such as *Economic risk*, *Societal risk* and *Individual risk*, which are explained in this section. Furthermore an overview of decision criteria used in the Netherlands is provided.

2.2.1. Defining flood risk

Risk is a well-known word with many different definitions. Within (Hydraulic) engineering, flood risk is defined as the product of the probability that a flooding will occur and the subsequent impact of said flooding. Risk in general is often expressed in monetary amounts, as the product of the probability of failure and the economic consequences of said failure. Within flood risk, the concept of risk is not limited to economic damage but can also be expressed in other measures such societal and individual risk (Kok et al., 2017). The Dutch approach to flood risk management considers three measures of risk: the economic risk, the individual risk and the societal risk. These three measures of risk are explained in more detail.

Economic risk

Economic risk is often equated with the annual expected damage, being equal to the expected value of a lottery as explained in section 2.1 on risk preferences. As such, economic risk is per definition a risk-neutral approach to flood risk management. This metric is often used as governmental agencies can efficiently spread the cost of any (flood related) damage among all residents in that area.

Societal risk

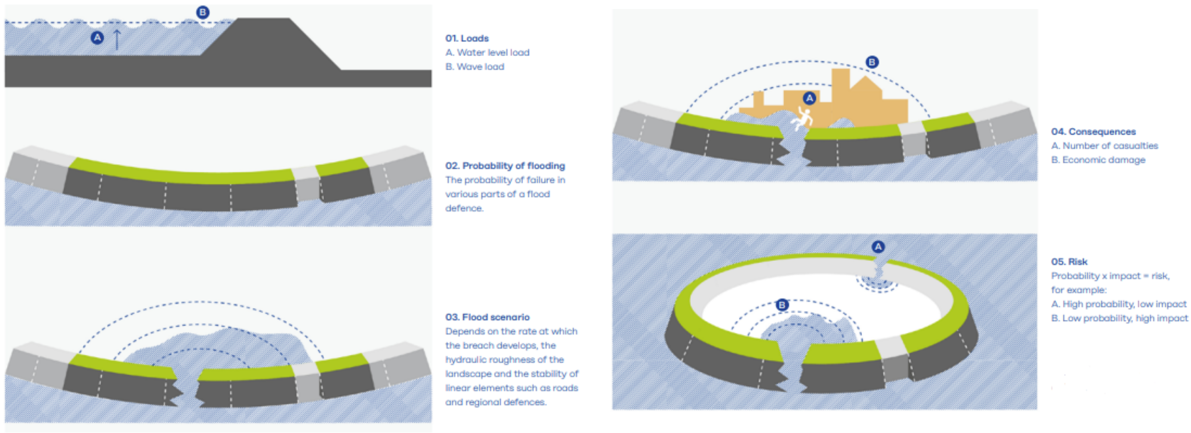
Societal risk is a measure of risk that provides an insight into the likelihood that there will be large number of casualties. The number of casualties displays non-linear behaviour for increasing numbers, meaning $\mathbb{E}[N \text{ casualties}] \neq N\mathbb{E}[1 \text{ casualty}]$. In particular, N casualties in one event is often considered worse than N times one casualty. When rephrasing the problem and assigning numerical values to it, one could say that twenty casualties in a single event is considered worse than twenty events where there is one casualty. The observed behaviour is concave, which is in agreement with risk averse preferences.

Individual risk

The previous two concepts relate to a population-wide risk. The local individual risk (LIR) is a measure of risk that captures the risk an individual faces. It represents the probability that an individual permanently present at a particular location will die as a result of flooding when potential evacuation is taken into account. This measure guarantees a certain basic level of protection for every resident in the Netherlands.

2.2.2. Quantifying flood risk

The process of quantifying flood risk consist of five steps. These steps are *Loads*, *Probability of flooding*, *Flood scenario*, *Consequences* and *Risk*. Each of the steps is shown in figure 2.5.



(a) Steps 1,2 and 3 **(b)** Steps 4 and 5

Figure 2.5: Visualization of steps to quantify flood risk (Kok et al., 2017)

An elaboration on each of the steps is given below.

Loads
In this step, the probability distributions of the loads to which different parts of the flood defence are subject should be determined. Several loads might occur simultaneously, which should be accounted for.

Probability of flooding
If the loads and their distributions are known, the probability of the flood defences failing should be evaluated as the next step. Failure in this sense means that the structure loses its water retaining capabilities at one or more places. Dependencies between sections of the flood defence should be taken into account at this step.

Flood scenario
In this step, the likely progress of any flood that might occur is determined, commonly referred to as a flood scenario. The scenario depends on the location of the breach, the rate at which it grows, the hydraulic roughness of the landscape and the stability of linear elements such as roads and regional flood defences. The above mentioned factors are uncertain. The uncertainty can be captured by assigning probabilities to the different scenarios. Summing all of the different flood scenarios results in the probability of flooding.

Consequences
In the fourth step, the consequences for each flood scenario are determined by combining the vulnerable assets and individuals with the characteristics of the flood, such as the water depths and flow rate of the water. The number of casualties as well as the damage will depend on how far in advance the threat of flooding is known and whether a timely decision to evacuate is taken and properly executed. Uncertainties can again be captured by assigning probabilities to the various outcomes.

Risk
The fifth and final step of the risk quantification regards the risk itself. With the probabilities of flooding and the consequences known, the two can be multiplied to obtain risk on an economic, individual and societal level. The first can be assessed by multiplying the expected damage with the probability of flooding. The second can be calculated by taking the probability into account that an individual is present in the area and is not properly evacuated in time. The third, societal risk, can be calculated by ranking the casualty numbers for each scenario from low to high before calculating the cumulative sum of scenario probabilities. This gives the cumulative probability for each number of casualties. These values can then be expressed as a societal risk curve, also known as a FN curve.

2.2.3. Relating flood risk to safety standards

The risk of flooding can never be reduced to zero as there will always remain some degree of uncertainty in both the strength and load parameters. As such, policy makers are faced with the question which level of risk is acceptable. A large factor in deciding the acceptable level of risk is the cost of risk reduction measures combined with the subsequent gain in reduction of risk.

Generally speaking, there are three ways in which flood risk can be managed.

1. Reduce the probability of flooding by reinforcing flood defences or reducing the load.
2. Reduce the scale of damage and/or the number of casualties by means of spatial planning: Avoid areas that are more vulnerable to floods.
3. Limit the consequences by ensuring the existence and execution of evacuation and crisis management.

The focus on flood risk management in the Netherlands has predominantly been by means of the first way: reducing the probability of flooding.

The Water Act

As of 2017, all laws regarding flood protection as well as water quality in the Netherlands are aggregated under the *Water Act*. This law prescribes the minimum required safety standard, expressed in an annual probability of flooding [1]. The standards in the Water Act is based upon acceptable flood risk for areas protected by the *primary flood defences*. The standards for these areas were set up based on two principles.

A Every individual should be able to rely on the same minimal level of protection. This basic level of protect is expressed as the local individual risk (LIR).

B If the impact of a flood is very high, a lower probability of flooding is deemed appropriate based on societal risk and a social cost-benefit analysis (SCBA).

The building blocks of the standards in the Water Act are graphically illustrated in figure 2.6.

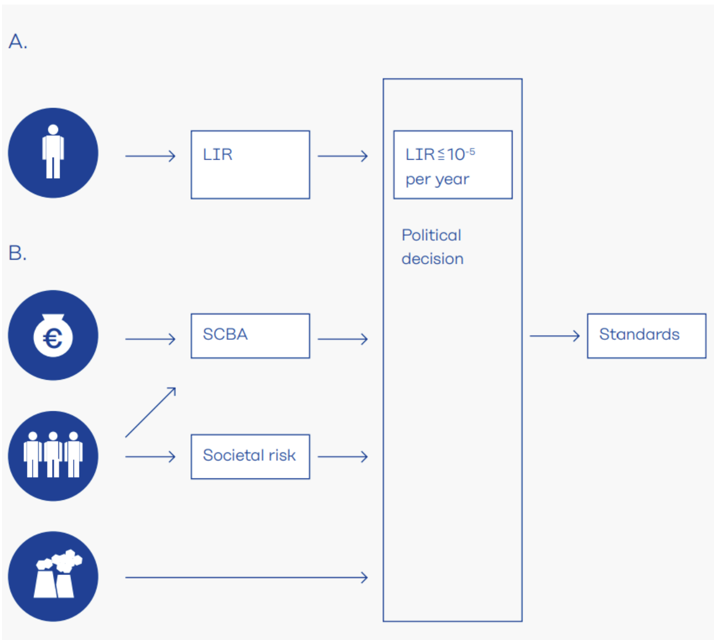


Figure 2.6: Principles underpinning the flood safety standards for the primary flood defences (Kok et al., 2017)

[1]There are exceptions in the way the minimum required safety standard is expressed. One of them is the Afsluitdijk, where the requirement is formulated in failure probabilities. Additional requirements might be formulated, as is the case for several barriers such as the Maeslantkering or levees like the Diefijk.

From item A. and B. it becomes apparent that the greater the potential consequences the more stringent the required standards must become. The size of impact of the consequences can roughly be caused by two parts: Large number of casualties or major economic damage.

The minimum required safety standard, expressed in a maximum allowable annual failure probability, results in the map given in figure 2.7.

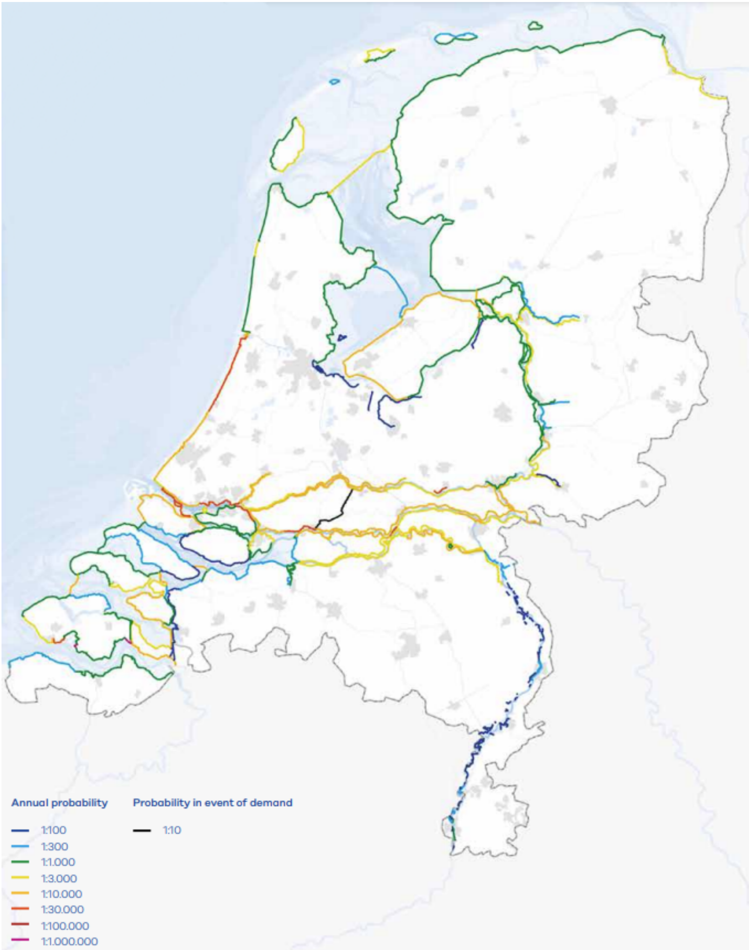


Figure 2.7: Flood safety standards as stipulated in the Water Act (Kok et al., 2017)

2.2.4. Decision criteria

As previously discussed, the Water Act relies on two building blocks consisting of three decision criteria. These criteria are discussed in further detail in this section.

Local Individual Risk

Every citizen of the Netherlands receives the same minimum level of protection. This minimum level of protection is guaranteed by the political decision that the local individual risk (LIR) must not exceed 1/100,000 per year. Defined as the that an individual in the Netherlands will die somewhere as the result of a flood, the LIR can be calculated as:

$$LIR = P(\text{Flood}) \cdot P(\text{Death}|\text{Flood}) \cdot (1 - f_{\text{evacuation}})$$

Where:
$P(\text{Flood})$ is the probability of a flood occurring
$P(\text{Death}|\text{Flood})$ is the probability of dying in the event of flood
$f_{\text{evacuation}}$ is the evacuation fraction, where $f_{\text{evacuation}} \in [0, 1]$

If the evacuation fraction $f_{\text{evacuation}}$ and the mortality $P(\text{Death|Flood})$ are known, it allows for the calculation of the maximum allowable probability of flooding given that the LIR has to adhere to the maximum value of 1/100,000. The mortality is determined in large part by the rate of water level rise and the maximum water depth. Jonkman et al. (2008) propose a method for estimating the loss of life due to a flood event based on three characteristics 1. information regarding the flood characteristics, 2. an analysis of the exposed population and evacuation, and 3. an estimate of the mortality amongst the exposed population. Mortality rates vary from 0.1 in small deep polders to 0.001 in shallow polders Deltares (2011). Next to geographical location, the type of flood is also of importance for the mortality fraction. Jonkman and Vrijling (2008) found mortality rates varying between 0.0001 and 0.1 for floods ranging from drainage floods to tsunamis. This is very dependent on the spatial geometry and geographical location and as such differs significantly per area. The evacuation fraction depends on time and amount of warning, the distance to safety and the available road capacity. The evacuation fraction can take on any value between zero and one. Kolen and van Alphen (2017) found that evacuation fractions in the Netherlands can range anywhere between 0.1 and 0.9, dependent on the available time and potential evacuation capacity scenario's the area specific evacuation fraction.

The choice for a mortality rate and an evacuation fraction are significant determinants in the optimal flood safety standard. Larger values

(Social) Cost-Benefit analysis
Before discussing the (social) cost-benefit analysis, some principles about cost optimization will be explained. Consider a flood defences, for example a dike section. In general, the more resources are invested in strengthening said flood defence, the lower the flood risk becomes. This reduction of flood risk has diminishing returns. At the same time, these investments costs resources at a (slightly) increasing rate per risk reduction. A such, there must come a point where the additional invested resources do not outweigh the risk reduction anymore, and the optimum is reached.

These are the basic principles used by van Dantzig (1956) when he discussed the problem of optimizing a hypothetical dike reinforcement. As he puts it in his 1956 paper, the problem that should be solved can be formulated as: *"Taking account of the cost of dike building, of the material losses when a dike-break occurs, and of the frequency distribution of different sea levels, determine the optimal height of the dikes."*. The solution of this problem lies in finding the point where the derivatives equal one another but have opposite signs, e.g. when the slopes are equal but opposite. Let $I(h)$ be the total cost of heightening the dike and let $L(h)$ be the flood risk[2], both functions of the increase of the effective crest height h. Then the optimal increase in crest height can be found by solving:

$$\frac{\partial I(h)}{\partial h} + \frac{\partial L(h)}{\partial h} = 0$$

The investment costs of the considered dike can be assumed approximately linear for small crest height increases, as the increase in the surface area of the dike is negligible. The solution for the differential equation posed by van Dantzig has a general shape as shown in figure 2.8.

[2]van Dantzig formulated the flood risk in terms of the required investment (with accumulated interest) that is needed to cover the expected values of all future losses.

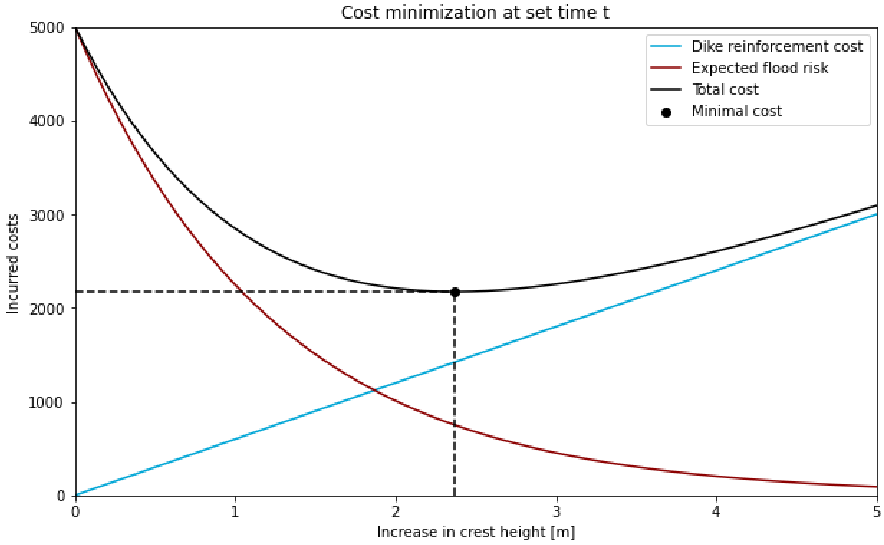


Figure 2.8: Typical Cost minimization for a set time t (Prevaes, 2022)

It can be seen that the reinforcement costs of a typical dike are approximately linear when the crest height is increased, represented by the blue line. The expected flood risk, shown in red, is an exponentially decaying function of the increase in crest height. The total cost, which is a summation of the expected flood risk $L(h)$ and the dike reinforcement costs $I(h_)$, are shown in black. This figure demonstrates the principle of the differential equation elegantly. It can be seen that the point of the minimal costs coincides with the point where the slope of dike reinforcement costs is equal to the slope of the expected flood risk , with the signs being opposite.

Note that it is also possible to find a more precise solution or a solution when the crest height increase is relatively large, although the mathematics involved are more difficult the principles remain the same. Further notes of Dantzig are that to make a proper analysis of the problem, the strength of the dike should be assumed to decrease over time due to land subsidence and degradation. Next to this, the flood risk will increase over time due to population growth and economic growth, independent of the probability of flooding. This principle is demonstrated in figure 2.9.

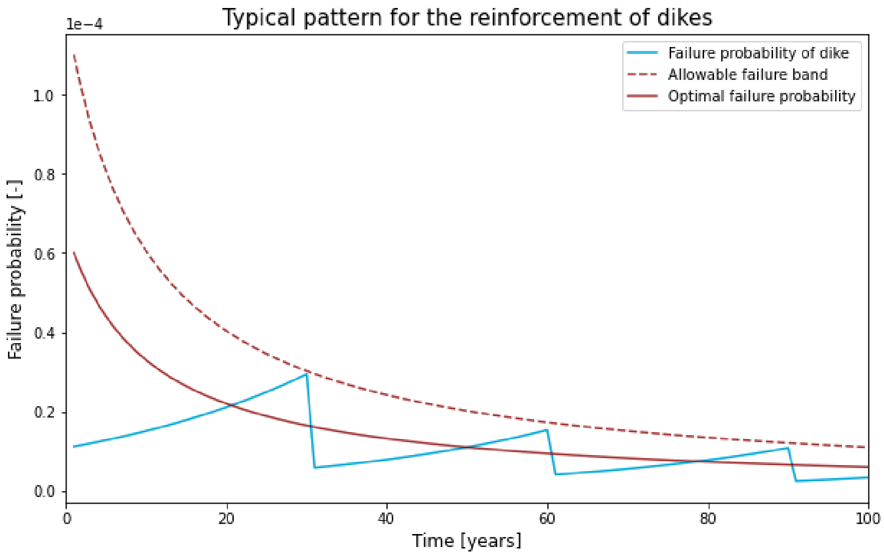


Figure 2.9: Typical "sawtooth" pattern for flood protection reinforcement, based on derivations by Eijgenraam (2006)

A downward shift of the failure probability over time can be seen in figure 2.9. This shift is the result of the increasing value that resides in the area as a result of population and economic growth. A cost-benefit analysis can be seen as a tally of all (social) benefits on one side and of all (social) costs on the other side. These two figures, be they discounted or not, can be compared and from this comparison a conclusion can be drawn.

Societal Risk
Societal risk is often equated with the probability of major loss of life. As stated by Kok et al. (2017): "*Assessment of the severity of societal risk is often based on a* risk-averse decision-making criterion, *which attached increasing weight to greater number of casualties*. The risk-aversion in the decision-making criteria can be seen when looking at so-called FN-curves. An example of such a curve is given in figure 2.10.

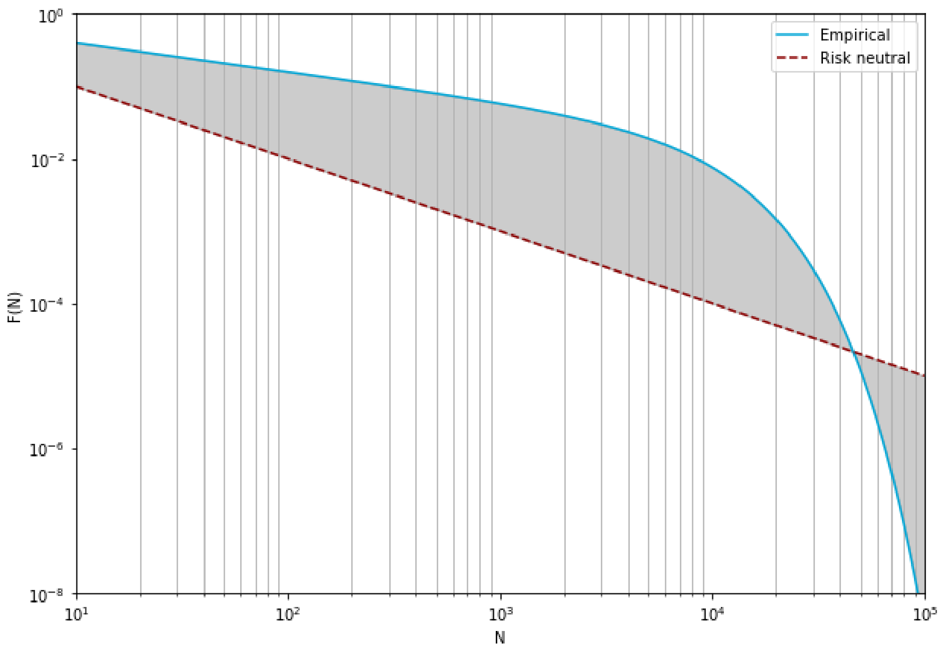


Figure 2.10: Example of a typical grouprisk or FN-curve

These curves display the exceedance probability of N casualties per year on the y-axis and the number of casualties on the x-axis. As such, the area under the curve represents the expected number of casualties. The risk aversion can be observed by the steep decrease in allowable failure probability for larger casualties. Risk aversion being defined as the difference between the expected value and the observed value, it is indicated as the shaded area in figure 2.10.

2.3. Risk preferences in Flood Risk management

This section highlights some examples of elicited risk preferences in Flood Risk management. It starts with the valuation of a human life, after which the risk premium that is used in the Dutch discount rate is discussed. After this, research on the inclusion of risk aversion in cost-benefit analysis will be discussed. Finally, some prior research on the group risk in FN-curves is shown.

2.3.1. Value of a Statistical Life

Often considered the most valuable possession an individual has, the valuation of a human life in the vernacular is set at an infinitely high valuation. As such, the question "*What is a human life worth?*" is sometimes considered unethical. However, for practical purposes it is often necessary to set a price on a human life. As the national means are a scarce good, basic economic theory dictates that the value can't simply be infinite but must be finite. After all, (a part of) the scarce national means have to be divided among possible investments related to safety. Whatever rational decision mechanism is chosen, it must be able to weigh the probability of profit against the probability of saving lives, thereby setting a finite value to the value of human life.

Vrijling and Gelder (2000) derive a valuation of a human life based on the ratio of the investment cost needed to provide an additional level of safety, divided by the additional number of lives saved. The full derivation is given in Appendix B.

The cost of investment to save an expected additional life, denoted by CSX, is expressed as the ratio of the total investment costs I and the discounted expected number of saved lives $(P_{f,0} - P_{f,opt}) \cdot N \cdot PV$ as:

$$CSX = \frac{I}{(P_{f,0} - P_{f,opt}) \cdot N \cdot PV}$$

It should quickly become apparent that this method of determining the CSX is a function of the value of a human life d, but it is certainly not equal to d. This theoretical framework is therefore not suitable in determining the value of a human life. Vrijling and Gelder therefore conclude their paper with an alternative for the valuation of a human life, based on the Net National Product (NNP) per head of the country. This NNP is the difference between the Gross National Product and the depreciation. In the year of publication (2000) this figure was around \$ 19,400 per year, equating to around \$ 450,000 to \$ 800,000 over a 70 year average lifespan, depending on the real rate of interest. In the period between 2000 and 2022 this figures has roughly doubled, yielding estimates of \$ 900,000 to \$ 1,600,000 over a 70 year average lifespan if measured today. The authors note that this alternative approach has as consequence that the value of a human life in a developing country is to be considerably lower.

Another approach to the valuation of a (statistical) human life was derived by de Blaeij (2003b), based applications in traffic management. The critique of de Blaeij (2003b) on the existing methods of valuation were that they only measure the impact of a death on the future Nation Product, but that these methods do not take the preferences of individuals into account. The leap forward in this research was the inclusion of the individual preferences To standardise the value of safety, the concept of the *Value of a Statistical Life* (VOSL) was introduced. The VOSL represents the value of a statistical human as opposed to any specific individual, removing the implicit assumption that an individual would accept any amount of cash for his or her life to immediately end as for most people there exist no figure that would be acceptable. The conducted research consists of two parts: A conjoint analysis (CA) and a contingent valuation (CV). For the CA-method, the stated-choice method (SC) was used. This method of eliciting preferences directly asks the respondents between two alternatives. In the second part, using the CV-method, a combination of a SC and CV questions were posed. Participants were first asked to choose among several options, after which an open question on their preferences were asked. Using the results of interviews, de Blaeij analysed the preferences of 1034 participants based on random utility maximisation by fitting the parameters via multinomial logit and mixed logit models. The random utility maximisation models were assumed linear. The VOSL estimate via the stated choice method resulted in a valuation of around €2,500,000. The VOSL estimate via the combined SC/CV-method resulted in a valuation of around €5,000,000.

2.3.2. Risk premium in Dutch discount rate

The discount rate is used in social cost-benefit analyses. To weigh the social benefits of government policies, investments or otherwise against the social costs of these policies, discounting is necessary, as the costs and benefits will not always occur at the same time. With the discount rate, costs or benefits that are at different points in time are brought under one denominator. Long-term effects have less weight than short-term effects when using a (positive) discount rate. In the Netherlands, the discount rate used in social cost-benefit analyses of governmental projects consists of a base rate and a risk premium. During the last revisit of the discount rate in 2020, the percentage of the base rate and risk premium were set at -1% and $3,25\%$ respectively (Ministerie van Financien, 2020). For projects that have large sunk costs, such as those in infrastructure, a rate of 1.6%, which consists of the same negative base rate but an adjusted risk premium with a factor 0.8. Furthermore it was advised to keep the*discount curve* flat, meaning that the discount rate for the short and long term is kept equal. This revisit happens every four years or if there are substantial shifts in the real market rate return.

The discount rate was determined using the so-called *Ramsey-rule*. The theoretical framework describes how, starting from a social welfare function, expected consumption growth and the risk profile of consumption growth over time, the government can make optimal choices regarding saving and investment. The key question is how much additional (expected) consumption is needed in the future to compensate for a loss of consumption in the present. In the optimum, the government cannot increase social welfare by distributing (expected) consumption differently over time. The model consists of two parts: A social welfare function and the assumed stochastic process of consumption.

The social welfare W is often expressed by means of the following function:

$$W = \sum_{t=0} e^{\delta t} \cdot \mathbb{E}[u(c_t)]$$

Where $\mathbb{E}[u(c_t)]$ is the expected utility from the consumption c at any time t. δ represents their time preference, which is similar to the time preference as seen in section 2.1.7. The utility function is often assumed as an iso-elastic utility function, corresponding to the *Constant Relative Risk Aversion* (CRRA) model, such that:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where γ measures the aversion against differences or change in consumption at different points in time. This can be interpreted as an indicator for risk aversion in the case of uncertainty. Without going into too much detail, the formula to determine the risk-adjusted discount rate r_r (the risk premium) is presented:

$$r_r = \delta + \gamma g - 0.5\gamma(\gamma+1)\sigma^2 + \gamma\sigma^2 = \delta - \gamma(g - 0.5\sigma^2) + 0.5\gamma\sigma^2$$

where g is the expected consumption growth per year and σ^2 the variance of said consumption growth. Note that the risk premium is directly dependent on the aversion against a change in consumption γ as well as the time preference following from the utility function δ. The utility function on which these are based, are a direct translation of the preferences of a (selection of) the Dutch population.

Now, consider the following train of thought on the workings of the risk premium in the discount rate. The risk premium increases the cost of borrowing and therefore the cost of construction. Applying the standard set of axioms of economic theory, this higher cost of borrowing should lead to a lower demand in the investments typically made with the borrowed funds. In the case of flood defences, this would inevitably lead to a lowered safety level. As such, the risk premium the discount rate incentivizes risk seeking behaviour in projects funded by these types of investments, which include flood defences. The question remains how sensitive the investing entities are to a shift in interest rates. As they are often governmental agencies who spent public funds, the sensitivity is most likely lower than in business practices.

2.3.3. Inclusion of risk aversion and equity weights in a (social) cost-benefit analysis

The objective of a cost-benefit analysis is to improve social welfare, which can be obtained in the form of maximizing utility under different alternatives. However, Kind et al. (2017) argues and shows that the current common practice of reducing the expected annual damages does not comply with this concept of social welfare. Since the common practice focuses on financial gain, rather than utility increase. As the marginal utility of money reduces for with increasing initial wealth, risk aversion and income differences should also be included in the cost-benefit analysis.

The risk premium as used by Kind et al. is the same concept that was introduced in section 2.1 of the literature research. They introduce a new metric that incorporates the risk premium, called the *risk-premium multiplier*, defined as:

$$R_{WTP/ED} = \frac{\text{willingness to pay}}{\text{expected damage}} = \frac{1 - [1 + P\{(1-z)^{1-\gamma} - 1]^{1/(1-\gamma)}}{Pz}$$

Where:
P: The probability of flooding

z: The fraction of consumption lost due to flooding
γ: The parameter that defines the elasticity of marginal utility of consumption in the assumed utility function $u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \gamma \geq 0, \gamma \neq 1$

This metric is heavily influenced by the share of consumption that is lost due to flooding z, which will be higher for lower-income households than higher-income household for equal floods. Hence it incorporates the effect of 'social vulnerability', where the risk premium can be interpreted as the individuals' valuation of risk dependent on income.

Social welfare functions show the social welfare W as a function of the utility of all n individuals in a society, such that:

$$W = W(U_1, U_2, ..., U_n)$$

To denote how much weight society assigns to an increase in well being for individual i, the partial derivative of social welfare with respect to the utility of said individual can be used. This metric $\partial W / \partial U_i$, is more commonly denoted as ω_{Ui}. The weights given to the additional well being of an individual is largely a subjective choice. Utilitarian societies give each individual a weight of $\omega_{Ui} = 1$, while egalitarian societies generally have weights that decrease with income.

The change in social welfare due to a shift in income for different individuals does not solely depend on the utility weights ω_{U_i} but also on the marginal utility of the shift in income, written as the partial derivative of utility to the income $\partial U_i / \partial Y_i$, often denoted as ω_{Y_i}.

The change in social welfare ∂W can therefore be written as:

$$\partial W = \frac{\partial W}{\partial U_1} \cdot \frac{\partial U_1}{\partial Y_1} \cdot \partial Y_1 + \frac{\partial W}{\partial U_2} \cdot \frac{\partial U_2}{\partial Y_2} \cdot \partial Y_2 + ... + \frac{\partial W}{\partial U_n} \cdot \frac{\partial U_n}{\partial Y_n} \cdot \partial Y_n$$

Where the partial derivatives can be replaced by their abbreviations ω_{U_i} and ω_{Y_i}, such that:

$$\partial W = \omega_{U_1} \cdot \omega_{Y_1} \cdot \partial Y_1 + \omega_{U_2} \cdot \omega_{Y_2} \cdot \partial Y_2 + \omega_{U_n} \cdot \omega_{Y_n} \cdot \partial Y_n$$

The equity weights ω_{Y_i} are often assumed equal for all, which implicitly introduces the assumption that the marginal utility of income is the same for all individuals. However, from empirical findings it has often been shown that the utility derived from €1 additional income is not equal for for example a student and a millionaire. As such, different equity weights are desirable to more accurately determine the true outcome of a CBA. Note that if the existing income distribution is considered fair, when flood damages are compensated or when the income is redistributed through other means, using equity weights in CBAs for flood risk management might not be necessary.

To incorporate these two effects, risk aversion and income difference, Kind et al. developed three extensions of the basic model of *Expected Value*. The first extension incorporates only the risk premium that is incurred due to risk aversion. The second extensions incorporates the equity weights given to individuals or groups with different initial wealth positions. The third and final extensions includes both the risk premium and the equity weights. The results of the four frameworks are given in table 2.4.

Table 2.4: Four frameworks to value the benefits of flood risk reduction (Kind et al., 2017)

	Expected Value			Certainty Equivalent		Equity Weighted Expected Value	Social Welfare
Policy concerns	Allocative efficiency			Allocative efficiency Social vulnerability		Allocative efficiency Equity	Allocative efficiency Social vulnerability Equity
Concepts used in the valuation	Expected damage			Expected damage Risk premium		Expected damage Equity weights	Expected damage Risk premium Equity weights
Monetary metric	Expected Annual Damage			Certainty Equivalent Annual Damage		Equity Weighted Expected Annual Damage	Equity Weighted Certainty Equivalent Annual Damage
When to apply:							
Damage compensation	Sufficient[1]	Insufficient		Insufficient		Insufficient	Insufficient
Damage as % of income		Low		High		Low	High
Income distribution		Fair	Unfair	Fair	Unfair	Unfair	Unfair
Other ways to redistribute income			Sufficient		Sufficient	Insufficient	Insufficient

Table 2.4 shows that different aspects are more or less suitable when using specific extensions of the expected value model. It becomes apparent that the inclusion of risk aversion is better suited to apply in the model if the damage as a percentage of income is relatively high. Furthermore it can be seen that not all methods are fair or sufficient, depending on which criteria is used.

2.3.4. Group risk in FN-curve

As previously mentioned, there are two acceptable levels of risk, the personal one and the societal one. This section will look at the societal risk.

The societal risk can be modelled by means of the TAW norm, which is inherently risk averse. Incidents involving a large number of casualties and small probability get more weight than incidents involving a small number of casualties but with a large probability. The criteria given by the TAW norm is given as:

$$\mathbb{E}[N_d] + k \cdot \sigma[N_d] < \beta \cdot 100$$

where k is the *risk aversion index* and N_d is the number of casualties. $\sigma[N_d]$ is the square root of the variance $\mathbb{V}[N_d]$. β is a policy factor which varies with the degree of 'voluntariness' with which an activity is undertaken and with the benefit perceived. It has higher values (order 10) for choices where the choice is completely free and lower values (order 0.01) for risks that are imposed and that have no perceived benefit.

Now, let P_f be the probability of a failure event f occurring. The expected value and variance can be further expressed in terms of conditional expectations and variances as:

$$\begin{aligned}
\mathbb{E}[N_d] &= P_f \cdot \mathbb{E}[N_d \mid f] \\
\mathbb{E}[N_d^2] &= P_f \cdot (\mathbb{E}^2[N_d \mid f] + \mathbb{V}[N_d \mid f]) \\
\mathbb{V}[N_d] &= \mathbb{E}[N_d^2] - (\mathbb{E}[N_d])^2 \\
&= P_f \cdot (\mathbb{E}^2[N_d \mid f] + \mathbb{V}[N_d \mid f]) - (P_f \cdot \mathbb{E}[N_d \mid f])^2 \\
&= P_f \cdot (\mathbb{V}[N_d \mid f] + (1 - P_f) \cdot \mathbb{E}^2[N_d \mid f])
\end{aligned}$$

Where the law of total expectation was used in the first two expressions and the definition of variance in the last expression. Possible simplifications of the expressions are possible if the exact distribution is known. This is illustrated in the following example.

Example

Assume that the number of deaths given an event f, denoted by $N_d \mid f$, follows an exponential distribution with parameter

$$\lambda = (P_{d\,|\,f} \cdot N_p)^{-1}$$

$$\text{s.t.} \quad \mathbb{E}[N_d \mid f] = \lambda^{-1} = P_{d\,|\,f} \cdot N_p \quad \text{and} \quad \sigma[N_d] = \sqrt{\mathbb{V}[N_d \mid f]} = \sqrt{\lambda^{-2}} = P_{d\,|\,f} \cdot N_p$$

Where $P_{d\,|\,f}$ is the probability of death given an event f occurred. The probability of not exceeding n deaths is therefore given by $P(N_d < n \mid f) = 1 - \exp(-\frac{n}{P_{d\,|\,f} \cdot N_p})$.

The bound of the acceptable level of risk then becomes:

$$\begin{aligned}
\mathbb{E}[N_d] + k \cdot \sigma[N_d] &= P_f \cdot \mathbb{E}[N_d \mid f] + k \cdot \sqrt{P_f \cdot (\mathbb{E}^2[N_d \mid f] + \mathbb{V}[N_d \mid f]) - (P_f \cdot \mathbb{E}[N_d \mid f])^2} \\
&= P_f \cdot P_{d\,|\,f} \cdot N_p + k \cdot \sqrt{P_f \cdot ((P_{d\,|\,f} \cdot N_p)^2 + (P_{d\,|\,f} \cdot N_p)^2) - (P_f \cdot P_{d\,|\,f} \cdot N_p)^2} \\
&= (P_{d\,|\,f} \cdot N_p) \cdot (P_f + k \cdot \sqrt{2P_f - P_f^2}) < \beta \cdot 100
\end{aligned}$$

From which it follows that the number of deaths given an event f must adhere to the following bound:

$$P_{d\,|\,f} \cdot N_p = \mathbb{E}[N_{d\,|\,f}] < \frac{\beta \cdot 100}{P_f + k \cdot \sqrt{2P_f - P_f^2}}$$

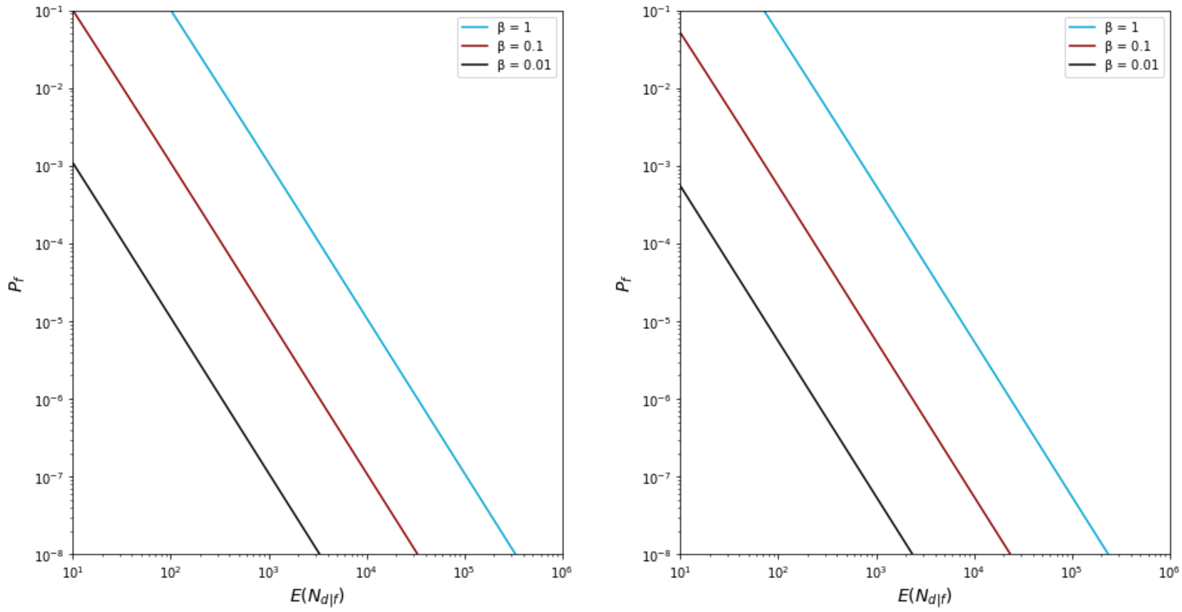
Comparing this to the deterministic case, where $\mathbb{E}[N_d] = P_f \cdot \mathbb{E}[N_d | f] = P_f \cdot P_{d\,|\,f} \cdot N_p$ and $\mathbb{V}[N_d | f] = 0$, where the bounds are given as:

$$\begin{aligned}
\mathbb{E}[N_d] + k \cdot \sigma[N_d] &= P_f \cdot \mathbb{E}[N_d \mid f] + k \cdot \sqrt{P_f \cdot (\mathbb{E}^2[N_d \mid f] + \mathbb{V}[N_d \mid f]) - (P_f \cdot \mathbb{E}[N_d \mid f])^2} \\
&= P_f \cdot \mathbb{E}[N_d \mid f] + k \cdot (\sqrt{P_f \cdot \mathbb{E}^2[N_d \mid f] + 0) - (P_f \cdot \mathbb{E}[N_d \mid f])^2} \\
&= \mathbb{E}[N_d \mid f] \cdot (P_f - k \cdot \sqrt{P_f - P_f^2}) \\
&= P_f \cdot P_{d\,|\,f} \cdot N_p \cdot (P_f - k \cdot \sqrt{P_f - P_f^2}) < \beta \cdot 100
\end{aligned}$$

From which it follows that the number of deaths given an event f must adhere to the following bound:

$$P_{d\,|\,f} \cdot N_p = \mathbb{E}[N_{d\,|\,f}] < \frac{\beta \cdot 100}{P_f + k \cdot \sqrt{P_f - P_f^2}}$$

Which is nearly the same results as in the exponential model, differing only by a factor of 2 in the square root. Some numerical results for several values of β are given in figures 2.11a and 2.11b.



(a) Allowable number of casualties as function of the failure probability as determined by the deterministic model

(b) Allowable number of casualties as function of the failure probability as determined by the exponential model

Figure 2.11: Bounds on the number of casualties for several values of the policy factor (β)

It can be seen that the difference between the exponential assumption and deterministic assumption is small, but not negligible. The deterministic model yields a slightly more conservative model as the expected number of deaths for the same probability is higher.

3

Elicitation & Methodology

3.1. Objective

The previous chapter provided a comprehensive overview of the different types of possible deviations from expected outcome theory. The question remains whether these deviations are present in the decision-making process in Flood Risk management, and if so, which model best represents this process. To address this, it is necessary to establish a baseline of individuals' risk preferences, followed by the elicitation of risk preferences specific to Flood Risk Management.

The objective of this method is to elicit risk preferences of individuals and groups. These elicitation efforts cover both general decision problems and those specific to Flood Risk Management. The resulting outcomes can be used to draw conclusions regarding the first two sub-questions. If the appropriate model is known, the third sub-question can be further investigated through a case study.

3.2. Data elicitation for variables of interest

This section explains the methodology used to gather the necessary data to address the research questions. It consists of two sections: the elicitation of risk preferences and the elicitation of valuation of a statistical life. Each of these sections is designed for both general risk preferences and flood risk-related risk preferences.

3.2.1. Elicitation of risk preferences

Risk preferences, whether general or specific to flood risk management, can be elicited using the certainty equivalent method. This method aims to determine the amount of certain income or return that an individual would consider equal in value to an uncertain income or return. It provides a way to quantify an individual's risk preferences by assessing their willingness to accept a lower (or higher) certain income rather than taking a chance on an uncertain outcome.

The process begins with presenting individuals with a choice between a guaranteed outcome, referred to as the *certain option* and a risky alternative with a probability distribution of possible outcomes. The risky alternative typically involves different levels of risk and potential returns. The individual is then asked to indicate the amount of certain income that they would find equally desirable or equivalent to the risky alternative. This value is recorded as the *Certainty Equivalent*.

General risk preferences

An example of one of the choice lists that is used in the elicitation of risk preferences in this research is given in table 3.1.

Table 3.1: Example of a *Choice list* used in this research

Certainty	**Gamble**	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
100% probability of €1	60% probability of €25	40% probability of €0
100% probability of €2	60% probability of €25	40% probability of €0
...	...	...
100% probability of €25	60% probability of €25	40% probability of €0

Where the dots represent every increment of €1 between €2 and €25. Note that individuals would be asked row by row whether they preferred the certainty or the gamble, starting at the top of the table.

The certainty equivalent (CE) is recorded as the point of indifference, in this case the mean value between the point at which the individual switches from the gamble to the certain pay-out (or vice versa) and the previous value. As an example, imagine an individual switches from the gamble to the certain pay-out when the certain pay-out reaches €7. The recorded CE in this case would be (7 + 6) / 2 = €6.5. Depending on the magnitude of the outcome, the increments can be adjusted to be either finer or coarser. To cover the range in probability and outcomes needed to construct the risk preferences, these parameter can be altered.

The elicitation of general risk preferences consists of the matrix given in table 3.2, overlapping partially with the choices presented in Gonzalez and Wu (1999). The **X** marks the combination of probability and outcomes associated with the example presented in table 3.1.

Table 3.2: Matrix containing the possible combinations of prospects that were used asked to participants in the questionnaire aimed at eliciting risk preferences for general outcomes

	Probability attached to the higher outcome										
Outcomes	0.01	0.05	0.1	0.25	0.4	0.5	0.6	0.75	0.9	0.95	0.99
(0; -25)						**o**					
(0; -100)			**o**								
(0; -400)	**o**										
(-100; -50)								**o**			
(-100; -200)										**o**	
(25; 0)							**x**				
(100; 0)				**o**							
(400; 0)		**o**									**o**
(100; 50)					**o**						
(200; 100)									**o**		

Note that not every entry of this matrix is asked to every participant, rather a selection consisting of various parts of the four-fold pattern is asked. The selection of posed prospects are indicated by an **o**. The result of the elicitation will be a (mean) CE in each entry of the matrix, which can be used to fit various models for both the probability weighting function and utility function of the group as a whole or on an individual basis.

Flood risk related risk preferences
In a similar manner as for the general risk preferences, flood risk related risk preferences can be elicited through choice lists. These choice lists will have the same probabilities and values as presented in table 3.2. The difference with the general choice problems lies in the context of the question. Participants will not simply be presented with two options, but rather they will be asked whether they are willing to purchase a certain insurance against flooding for the negative prospects and asked whether they would prefer a discount on next year's insurance, accompanied by a probability of occurrence, or a guaranteed pay-out for positive prospects. A complete list of posed questions can be found in Appendix C.

In addition to the overall interest in the shape of the probability weighting function from 0.01 to 1, there is a particular focus on probabilities that are close to zero within flood risk management. In particular, the range from 10^{-5} through 10^{-2} is of high interest as this encompasses the majority of exceedance probabilities for flooding. Therefore, there will be a division in the elicitation process of the Flood Risk related preferences. The region where $p_1 : p \in [10^{-5}, 10^{-2}]$ will be elicited through a different set of questions than the region where $p_2 : p \in [10^{-2}, 10^{0}]$. The elicited preferences allow for the determination of the value of a human life, next to the general shape of the utility- and probability weighting functions for both domains. Next to this general shape, a zoom-in of the probability weighting function for small values of p will be derived.

3.2.2. Elicitation of the valuation of a statistical life

The valuation of a statistical life will be obtained through two different approaches. The first approach involves using a set of general choice lists that focus on the risk of death while driving a car. The second approach utilizes a set of flood risk-related choice lists specifically addressing the risk of death due to flooding. Each of these elicitations will be explained in detail in this section.

Valuation of life under General Risk
For the valuation of life under general risk, two sets of choice lists were used to elicit the statistical value of a human life. The first elicitation concerns the willingness to pay for a toll road with a reduced travel time and risk of death due to car accidents. Participants were given the following information:

Suppose you have to drive from point A to point B and have the choice between two roads: Road 1 and Road 2. These roads differ in several aspects. Road 1 has a travel time of 1 hour, has a €5 toll and a person has an average chance of dying in traffic of 20 in 1,000,000 per year. Road 2 is quicker and safer, having a travel time of only 45 minutes and an average chance of dying in traffic of 10 in 1,000,000 per year. However, this comes at an increased cost of the toll of € 10.

Given the above information, which of the two roads do you prefer?

After revealing their preferences, participants would be asked the following question:

You selected Road i, how much would the toll of Road j need to be (in €) for you to prefer Road j above Road i?

Where i and j can be either 1 or 2, dependent on the answer they gave to the first question. After answering these questions, participants were presented with a similar question regarding two other roads with slightly different characteristics.

The second set of questions that participants were asked regarded the purchase of a car with different safety levels and price points. Participants were provided with the following information:

Suppose you have to buy a new car and have determined which brand model you would like. The only choice that you have to make is between three variants. All variants are identical apart from the safety standard that they adhere to. Assume that you intend to drive the car for five years after which you can sell it. The resell value will be the same for all three cars. The first variant, Variant A, means that in the coming year you have a probability of 10 in 100,000 that you will be in a deadly car crash. This variant bears no additional costs. If you pick the second variant, Variant B, your chances of being in a deadly car crash reduce to 5 in 100,000. This variant comes at an additional cost of €250 over the lifetime of the car (€50 per year). The final variant, Variant C, has an even lower chance of being in a deadly car crash, namely 1 in 100,000. However, this variant comes at an additional cost of €750 over the lifetime of the car (€150 per year).

Given the above information, which of the three variants do you prefer?

After revealing their preference, participants were asked the following question:

You chose Variant i. How much extra euro (per year) would you be willing to pay for Variant j? This reduces your probability of death due to a car crash from p_i to p_j per year.

Where i and j can be any of the three variants plus an additional variant D which has a lower probability of dying in a car accident than variant C. The probabilities p_i and p_j are the corresponding probabilities of dying due to a crash. These questions allow for the elicitation of several estimates, one of which being the valuation of a statistical human life. The method for the elicitation will be presented later in this chapter.

Valuation of life under Flood Risk
This choice list concern the valuation of live in two different locations with different probabilities of death due to flooding. Let p_{11} and p_{12} denote two probabilities chosen from the list p_1, where $p_1 = [10^{-5}; 10^{-4}; 10^{-3}; 10^{-2}]$. The probabilities are chosen in such a way that $p_{11} > p_{12}$. Participants would be given the following information:

Being outside the dike ring carries with it an increased risk in death due to flooding. This risk is estimated by experts to be around p_1 per year. Being inside a dike ring reduces this risk. The risk of death due to flooding inside a dike ring depends on several factors and has been estimated by experts to be around p_2 per year, making it p_{11}/p_{12} times less likely to be killed in a flood inside the dike ring as opposed to outside the dike ring. As there is a housing shortage, the municipality is offering additional yearly income to those willing to live outside of the dike ring.

Given the above information, which of the following two options would you prefer?

The individuals taking the surveys would be shown each row of table 3.3 successively.

Table 3.3: Choice list for the elicitation of mortality related risk in Flood Risk related risk preferences

Option A		Option B	
Additional income	*Probability of death*	*Additional income*	*Probability of death*
€50	p_{11}	€0	p_{12}
€100	p_{11}	€0	p_{12}
...	...	...	...
€1.950	p_{11}	€0	p_{12}
€2.000	p_{11}	€0	p_{12}

After going through the choice list, individuals would be directed to the next question where they are presented with the following information:

The previous question regarded a reduction of the probability of dying due to flooding of 1 in 1,000 to 1 in 10,000. Now imagine an identical situation as in the previous question but the initial and new probabilities vary. Please indicate below what the additional yearly income should be for you to consider moving from the inside of the dike to the seaside of the dike.

Probability of death due to flooding on the riverside of the dike: p_i *per year*
Probability of death due to flooding on the inside of the dike p_j *per year*

How much additional yearly income do you want before you would consider moving from the inside of the dike to the riverside of the dike?

Where $p_i, p_j \in [10^{-5}, 10^{-2}]$. This last question was posed a total of three times with varying values for p_i and p_j. This results in several certainty equivalent that can subsequently be used in the estimation of the statistical value of a human life.

A variation on the choice list and subsequent question will be presented to the participants, where there is no additional income but rather an additional taxation for living within the dike ring, covering the negative domain of the fourfold pattern.

3.3. Method of elicitation

This section outlines the methodology used to collect preferences and certainty equivalents, as discussed in the previous section. It further provides details about the collected metadata from the participants. Additionally, it discusses the distribution of the tool and any incentives provided to encourage participants to complete the survey.

Method of elicitation
To ensure a substantial amount of data for this research, an online survey was chosen as the recording method. The survey was administered through the digital platform *Qualtrics*. The survey remained open for a duration of two weeks (14 days). In addition to the initial invitations, reminders were sent after 7 days, 10 days, and 13 days to maximize participation.

Collection of meta data
The survey is conducted anonymously. However, certain participant characteristics are of interest for later analysis. These characteristics enable subgroup divisions and help determine if socio-economic status influences the provided responses. The following list outlines the metadata collected from individuals completing the questionnaire. Each characteristic includes the option "Prefer not to disclose" for individuals who choose not to provide the information:

- Age bracket: *Under 35*, *between 35 and 50* and *above 50*
- Yearly income bracket: *€0 - €20,000*, *€20,000 - €40,000*, *€40,000 - €60,000*, *€60,000 - €100,000*, *€100,000+*

- Expenditure: *Open answer*
- Profession in Flood Risk Management:*Yes* or *No*.
- Rent or possession of a house: *Own*, *Rent*, *Other*
- Elevation of house: *Above NAP*, *Below NAP*, *Do not know*
- Description of self assessed risk preference: *Risk Averse*, *Risk Neutral*, *Risk Seeking Do not know*

This meta data can be used to subdivide groups and see if there are significant differences between the utility- and probability weighting functions, as well as whether the flood risk related metrics have a significance variation between participants with certain characteristics. The income bracket and expenditure will play a significant role in the valuation of a human life, enabling to see if this valuation is dependent on these two parameters.

Distribution and incentives
The survey was conducted digitally, and participants were not monitored during the survey process. Participant recruitment was carried out through the author's network and the author's company, *HKV Lijn in Water*. Additionally, the survey link was shared digitally and physically within the Delft University of Technology community. Next to these streams of participants the link to the survey was spread around the Delft University of Technology both digitally and physically.

To encourage participation, a reward was offered at the end of the survey. This reward involved the option for participants to receive their personal utility and probability weighting functions presented in two graphs, along with a brief explanation of the graphs' content.

3.4. Processing of elicited data

This section explains how the collected data can be processed to address the research questions. It outlines a general procedure for estimating the utility- and probability weighting functions from the obtained data, presenting several commonly used functions and describing how to identify the optimal one. This procedure applies to both general risk preferences and flood risk-specific risk preferences. Additionally, a method for eliciting the valuation of a statistical human life is provided.

3.4.1. Eliciting risk preferences

The process of eliciting risk preferences is identical for the general preferences and the Flood Risk management related preferences. In general there are two components to nearly all theories, a *utility function* and a *probability weighing function*, that form the basis for evaluating different prospects. The number of parameters to estimate depends on the chosen combination of utility- and probability weighting functions, which together form the model.

Approach
As previously stated, the decision making process regarding risky prospects is presumed to consist of two functions: the utility function $v()$ and the probability weighting function $w()$. These two functions are assumed to describe the weight individuals attach to the combination of outcomes and probabilities. The method below describes prospects with two outcomes, but the theory can easily be expanded to prospects with more outcomes. The certainty equivalent is assumed to be a weighed combination of the probabilities and outcomes in the following form:

$$v(CE) = w(p)v(x) + [1 - w(p)]v(y)$$

Where p is the probability associated with outcome x and $1 - p$ is with y of the prospect. Estimating either $v()$ or $w()$ would not be difficult apart from the fact that in practice it is not $v(CE)$, the value of the certainty equivalent, that is measured but rather the monetary value CE. As such, the observations should be equated to:

$$CE = v^{-1}(w(p)v(x) + [1 - w(p)]v(y))$$

To solve this equality a functional form on either $w()$ or $v()$ has to be assumed after which the other function can be estimated. After estimation the initial functional form can be evaluated in iterative steps until convergence is reached. To estimate the parameter in this model, the following model is proposed:

$$\hat{CE} = v^{-1}(w(p)v(x) + [1 - w(p)]v(y)) + \varepsilon_{CE}$$

Where $\hat{CE}$ is the predicted value of the CE given the prospect and ε_{CE} is the residual of the estimation. To allow for flexibility of the optimization of the fit of the parameters, a flexible loss function is proposed. This loss function is defined as the sum of the (squared) difference between the predicted certainty equivalent $CE_{pred,i}$ and the observed certainty equivalent $CE_{obs,i}$ for all k prospects under consideration. The objective is to minimize this difference, which is the residual of the model ε_{CE}.

$$\varepsilon_{CE,k} = \sum_{i=1}^{k} |CE_{pred,i} - CE_{obs,i}|^n$$

Where $n = 1$ corresponds to a linear loss function and $n = 2$ corresponds to a quadratic loss function. The advantages and disadvantages of each method will be discussed in a later section. The algorithm can be summarized in the following steps:

1. Elicit the certainty equivalents via choice lists
2. Estimate $v()$ and $w()$ using initial parameter estimates
3. Evaluate the residuals. If the change in residuals is below a predetermined threshold terminate. Else use the estimated parameters as initial estimates and return to point 2.

A graphical overview of the algorithm can be found in figure 3.1.

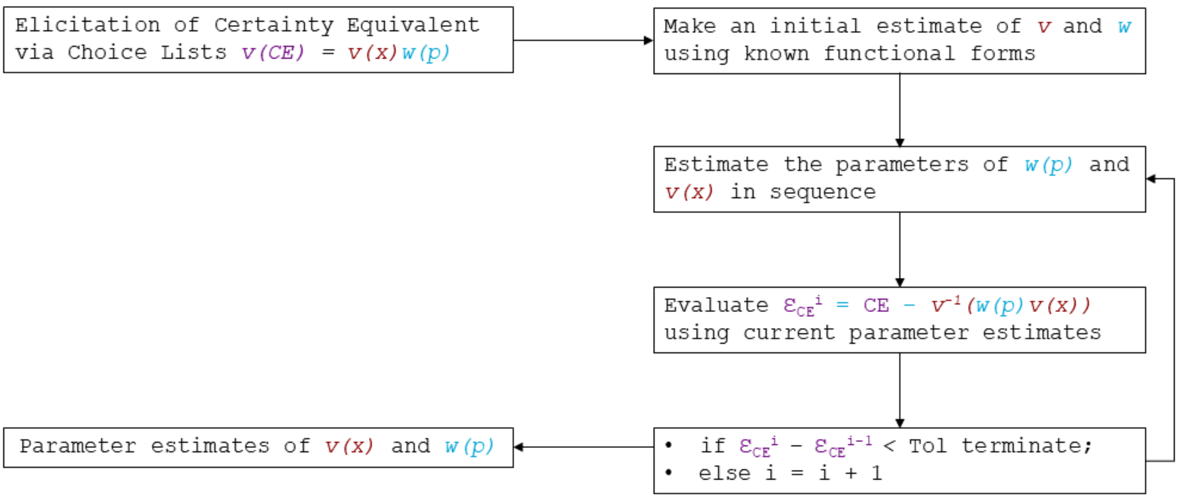


Figure 3.1: Graphical representation of the algorithm for finding the probability weighting function $w(\cdot)$ and utility function $v(\cdot)$ from the found certainty equivalent $u(CE)$ by means of a choice list

This algorithm imposes that $v(0) = 0$ and $0 < w(p) < 1$ for $0 < p < 1$. The optimum can be defined by setting a limit on the change in ε_{CE}.

This process is a general description for the estimation of the parameters of the utility- and probability weighting functions. The functional form of both components can be varied. The next section discusses several of the functional forms that are used in this research.

Utility function

Several functional forms of the utility function could explain the observed choices made by the participants. Therefore, a variety of functions will be evaluated. The different mathematical formulations under consideration are presented in table 3.4. The four formulations are functional forms that are used in different descriptive theories.

Table 3.4: Overview of several different types of utility functions with their descriptive theory and functional mathematical form

Type of function	Descriptive theory	Utility function $u(x)$
Linear	EO[1]	u(x) = $a \cdot x$
Logarithmic (von Neumann & Morgenstern, 1947)	EU[2]	u(x) = $\begin{cases} c \cdot \ln(x) & x > 0 \\ c \cdot \ln(-x) & x < 0 \end{cases}$
Iso-elastic (Yaari, 1987)	CRRA [3]	u(x) = $\begin{cases} \frac{x^{1-\eta}-1}{1-\eta} & \eta \geq 0, \eta \neq 1 \\ \ln(x) & \eta = 1 \end{cases}$
Power (Tversky & Kahneman, 1992)	CPT[4]	u(x) = $\begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda \cdot (-x)^{\beta} & x < 0 \end{cases}$

The linear function is based on the expected outcome theory, such as expected value theory. The logarithmic function by von Neumann and Morgenstern was one of the first functions to incorporate a utility function that accommodates risk-averse or risk-seeking preferences. The iso-elastic utility function by Yaari belongs to the family of constant relative risk aversion (CRRA) utility functions, where the degree of risk aversion remains constant regardless of an individual's level of wealth or consumption. This property simplifies the analysis and mathematical modeling of decision-making under uncertainty. The power function by Tversky and Kahneman allows for differentiation between positive and negative prospects and is a widely used model.

Except for the power model by Kahneman and Tversky, all models require the elicitation of one parameter. The power model necessitates the elicitation of three parameters: one for positive prospects and two for negative prospects.

Probability weighing function

In many empirical estimate, it was shown that the probability weighting function $w(p)$ initially has values where $w(p) > p$ before moving to values where $w(p) < p$. Furthermore, it starts concave and ends convex and finally has an intersection with the neutral line $w(p) = p$ at around 1/3. Meaning that any function under consideration should have the following three properties: It should be a regressive function, s-shaped and finally asymmetrical.

To this end, the *power* function (Tversky & Kahneman, 1992) is used as it fulfills all criteria. The mathematical formulation is given as:

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{-\gamma}}$$

Where $\gamma \in [0, \inf]$ determines the curvature of the probability weighting functions, with $\gamma = 1$ corresponding to a linear probability weighting function.

Elicitation of Flood Risk specific metrics

The procedure for eliciting the shape of the utility- and probability weighting function is the same for both general risk preferences and flood risk related risk preferences. Therefore, the method described in the previous section is applicable to flood risk related preferences as well. Once both choice lists have been utilized to estimate model parameters, the estimated models can be compared to determine if there are significant differences in the way individuals weigh probabilities and outcomes in general choice problems versus flood risk-specific choice problems. Additionally, it is possible to elicit probabilities within a narrower domain, specifically when probabilities range from orders of 10^{-5} to 10^{-2}.

[1] Expected Outcome

[2] Expected Utility

[3] Constant Relative Risk Aversion. Earlier iterations of this function were devised by Arrow (1965)

[4] Cumulative Prospect Theory

3.4.2. Valuation of a statistical life

The valuation of a human life is complex and abstract question if posed directly. While many may consider their own life to be invaluable, it is necessary to assign a finite value for practical and mathematical purposes. This last claim on an invaluable life is problematic, both mathematically and practically, as it would imply that no one would be willing to take any risk. However, we know that the valuation of a human life must be finite, as individuals undertake risky activities all the time such as smoking, driving or simply walking on the sidewalk. Directly asking an individual what they think their life is worth often leads to nonsensical answers, as it is near impossible to directly value ones most prized possession. Fortunately, there is a way of indirectly valuing one's life through choice lists. This section explains how the answers to these choice lists, combined with general risk preferences, can be used to determine the value individuals place on their own life. The process of elicitation differs between general choice problems and flood risk specific problems, and both methods are discussed below.

Elicitation via General choice problems

The elicitation of the statistical value of a human life via general choice problems was done by means of a choice lists and direct elicitation, posed in two different sets of questions. The first set regarded a preference indication between two roads with differing probabilities of death, toll costs and travel time. After this preference indication, participants were shown an alternative road with a higher level of safety (expressed as a lowered probability of dying in a car accident) and directly asked how much they were willing to pay in addition to the regular toll costs for this safer road. This leads to a similar equality as with the certainty equivalent, but with several additional terms.

Suppose that out of the two road choices (A and B), a participant chooses Road A. The participant would subsequently be asked how much he or she is willing to maximally pay to use road B, which is the safer option. This leads to the general equality shown in equation 3.1.

$$u(\mathrm{D}) \cdot w(p_1) + u(\mathrm{ND}) \cdot w(1-p_1) + u(M_1) + u(t_1) = u(\mathrm{D}) \cdot w(p_2) + u(\mathrm{ND}) \cdot w(1-p_2) + u(M_2) + u(t_2) \quad (3.1)$$

Where:
$u(\mathrm{D})$: The utility derived from dying
$u(\mathrm{ND})$: The utility derived from *not* dying
$w(p_n)$: The probability associated with dying for different scenarios
$u(M_n)$: The utility of the toll costs of road n
$u(t_n)$: The utility of the cost of time spent on the road n

Assuming that the derived utility- and probability functions reflect the true preferences of the participant, this equation contains three unknowns: $u(t_n)$, $u(D)$ and $u(ND)$. The participant was asked a near identical question regarding two different roads (C and D) which leads to a second equation with a similar form.

To solve for the third unknown, another equation is needed. This equation can be set-up by means of an additional set of choice problems. This second set of choice problems consisted of the preference indication of variants of a car with differing safety levels (again expressed as a lowered probability of dying in a car accident) and costs. After the preference indication individuals were directly asked how much they would be willing to spend on a safer variant of the chosen car. The exact questions posed to the participants can be found in the Appendix C.

Suppose that out of three variants (A,B and C) the participant chooses car B. The participant would subsequently be asked what the maximum is that he or she is willing to pay for the safer variant (in this case C). This again leads to an equality, of which the general formulation is given in equation 3.2.

$$u(\mathrm{D}) \cdot w(p_3) + u(\mathrm{ND}) \cdot w(1-p_3) + u(M_3) = u(\mathrm{D}) \cdot w(p_4) + u(\mathrm{ND}) \cdot w(1-p_4) + u(M_4) \quad (3.2)$$

Which has two unknowns $u(D)$ and $U(ND)$ which are the same unknowns as in the first set of equations. The combination of these two equations allows for all three unknowns to be solved under an important assumption. This assumption regards the utility of time, $u(t)$. For this elicitation it is assumed that if the difference in time points is relatively large with respect to the value of the two time points, the difference in utility for two sets of two time points can be assumed as constant. For practical applications of this research, this assumptions implies that $u(45$ min$) - u(30$ min$) = u(60$ min$) - u(45$ min$)$.

The result is a set of three equations with three unknowns, which can be solved by first choosing one variable in equation 3.2 and expressing the other variable in terms of the chosen variable. This expression can then be plugged into the two equations of 3.1 to be simultaneously solved for the two remaining unknowns. Both u(D) and u(ND) are expressed in *utils* and therefore have to be converted back to monetary amounts by taking the inverse of the utility function such that the monetary amount one attaches to not dying, e.g. the value one attaches to his or her own life, is given as: $V_{life} = u^{-1}(u(\text{ND}))$. Note that any utility function that captures the risk preferences of the individual in a proper manner can be used to derive the valuation. The same utility function for all participants can be used, or an individual utility function per participant can be used to derive this value. Both will be performed and compared.

Elicitation via Flood Risk related choice problems

The elicitation of the statistical value of a human life through flood risk management (FLM) related choice problems is more straightforward compared to the previous elicitation using general choice problems. The equation that arises from the FLM related choice problems can be expressed in a general form, as shown in equation 3.3. In these choice problems, participants are presented with the option of living inside or outside a dike ring, with both houses being identical except for their probabilities of death due to flooding. In one scenario, participants are offered additional income for living outside the dike ring, while in the other scenario, they face an additional tax for living inside the dike ring.

$$u(\text{D}) \cdot w(p_1) + u(\text{ND}) \cdot w(1 - p_1) = u(\text{D}) \cdot w(p_2) + u(\text{ND}) \cdot w(1 - p_2) \pm u(\text{CE}) \tag{3.3}$$

Where:
u(D): The utility derived from dying
u(ND): The utility derived from *not* dying
$w(p_n)$: The probability associated with dying for different scenarios, where $p \in [10^{-5}, 10^{-2}]$
u(CE): The utility of the additional received income or of the additional taxation.

This equation contains two unknowns, u(D) and u(ND), and therefore two of the elicited preferences are required, which can then be simultaneously solved to obtain both unknowns. Again, the value in *utils* has to be converted back to monetary amounts for a fair comparison.

To illustrate the working, consider the first and second posed question of the survey with regards to FLM related choice problems on the valuation of a human life. These two questions lead to the following set of equations to be solved.

$$u(\text{D}) \cdot w(\frac{1}{1,000}) + u(\text{ND}) \cdot w(\frac{999}{1,000}) = u(\text{D}) \cdot w(\frac{1}{10,000}) + u(\text{ND}) \cdot w(\frac{9,999}{10,000}) + u(CE_1)$$
$$u(\text{D}) \cdot w(\frac{1}{100}) + u(\text{ND}) \cdot w(\frac{99}{100}) = u(\text{D}) \cdot w(\frac{1}{10,000}) + u(\text{ND}) \cdot w(\frac{9,999}{10,000}) + u(CE_2)$$

In total, there were four variations of the probabilities for the additional income question, as well as four variations for the additional taxation question. Since the probabilities are the same for both types of questions, a direct comparison can be made between the certainty equivalents (CE) to quantitatively and qualitatively determine risk preferences. If the two CEs have equivalent magnitudes, the risk preference is considered risk neutral. If the CE from the additional income is higher (lower) than the CE from the additional taxation, the risk preference is classified as risk seeking (risk averse).

3.4.3. Evaluation of the probability weighting function for small probabilities

The evaluation of the probability weighting function for probabilities in the domain $p \in [10^{-5}, 10^{-2}]$ is done in a similar manner as the valuation of a statistical life. However, rather than having u(D) and u(ND) as parameters to be estimated, the values of $w(p_i)$, where $p_i \in [10^{-5}, 10^{-2}]$ are posed as unknowns. To this end, the values of u(D) and u(ND) have to be assumed as fixed. One could take the derived values from the general choice problems and use them in the FLM related choice problems as known values, or the other way around. The preference would be to use the found values in the FLM related equations as these have one less parameter to estimate/ assume as the utility of time drops out. As such, equation 3.1 will be used to elicit $u(\mathrm{D})$ and $u(\mathrm{ND})$ and the found values are substituted into equation 3.3.

3.5. Model selection and comparison between groups

This section starts off by discussing the selection criteria that are used to assess the fit of different utility- and probability weighting models to the observed data. Next to this, a selection criteria will be presented that quantifies the comparison between the general- and FLM related parameter estimates of the different models.

3.5.1. Comparison of different utility- and probability weighting models

To compare the performance of the different utility- and probability weighting models, several test statistics can be used. This research looks at two commonly used comparison by means of a goodness-of-fit: The sum of linear and squared errors and the coefficient of determination.

Sum of Squared Residuals and Sum of Linear Residuals

The Sum of Squared Residuals (SSR) is a measure of the difference between the observed and predicted values in a regression model. It is calculated as the sum of the squared differences between the observed and predicted values for each data point. The formula for SSR is given by:

$$SSR = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

where y_i is the observed value for the i^{th} data point, $\hat{y}_i$ is the predicted value for the i^{th} data point, and n is the number of data points.

The Sum of Linear Residuals (SLR) is another measure of the difference between the observed and predicted values in a regression model. It is calculated as the sum of the absolute differences between the observed and predicted values for each data point. The formula for SLR is given by:

$$SLR = \sum_{i=1}^{n} |y_i - \hat{y}_i|$$

where y_i is the observed value for the i^{th} data point, $\hat{y}_i$ is the predicted value for the i^{th} data point, and n is the number of data points. Like SSR, SLR is used to evaluate the goodness-of-fit of a regression model. However, unlike SSR, SLR places equal weight on all residuals, regardless of their magnitude. This means that outliers, which have a larger effect on SSR due to the squaring operation, are not given more weight in SLR. SLR may be preferred over SSR when the presence of outliers is suspected or when the absolute magnitude of the residuals is more important than their squared magnitude.

The prediction and observation in this case are the predicted and the observed CE value. Different combinations of utility- and probability weighting function will produce different estimates for the CE value. To avoid a bias in selection the CE values are normalized, such that utility functions that on average produce larger CE estimates are not disproportionally punished for these larger estimates.

Coefficient of Determination

The coefficient of determination, also known as R-squared (R^2), is a measure of the goodness of fit of a regression model. It is defined as the proportion of the variance in the dependent variable that is explained by the independent variables in the model. The R-squared value ranges from 0 to 1, with a value of 1 indicating a perfect fit and a value of 0 indicating no fit. The formula for R-squared is given by:

$$R^2 = 1 - \frac{SSR}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

where $\bar{y}$ is the mean of the observed values.

The goodness-of-fit of a regression model can be tested using R-squared and the SSR. A higher R-squared value and a lower SSR value indicate a better fit of the model to the data. However, it is important to note that a high R-squared value does not necessarily indicate that the model is the best fit for the data, as it may be overfitting the data or not considering important variables. In such cases it may be necessary to use other methods such as residual plots or statistical tests to assess the fit of the model.

3.5.2. Comparison of general- and FLM related parameter estimates

This section addresses the division of choice problems and their corresponding answers into two categories: general- and FLM related choice problems. In order to analyze the parameter estimates and test their significance, it is necessary to consider the distribution of choices and the subsequent distribution of the parameter estimates. In particular, it is of interest to know whether the means of the parameter estimates are statistically different between the general- and FLM related parameter estimates. Since the specific distribution is unknown, three different types of tests are proposed. These tests differ in terms of the strictness of the underlying assumptions. Each test will briefly be discussed along with the necessary assumptions. Which of these tests is best suited depends on the parameter estimates that are presented in the chapter on Results.

Equal variance between groups

The F-test, also known as the variance ratio test, is a statistical test used to compare the variances of two or more groups. It is commonly employed in analysis of variance (ANOVA) to assess whether the means of the groups are significantly different. The F-test calculates the ratio of the variance between groups to the variance within groups, and determines whether this ratio is significantly different from 1. The test statistic is given as :

$$F = \frac{\text{SSR}/(k-1)}{\text{SSE}/(n-k)}$$

where:
F: the F statistic
SSR: the sum of squares due to regression
SSE: the sum of squares of residuals
k: the number of predictors
n: the sample size

If the calculated F-value is greater than the critical value, it suggests that there are significant differences among the groups. Conversely, if the calculated F-value is smaller than the critical value, it implies that the differences among the groups are not statistically significant.

Unequal variance between groups

Welch's ANOVA test is a modified version of the traditional ANOVA that is used when the assumption of equal variances among the groups is violated. While the standard ANOVA assumes equal variances, Welch's ANOVA relaxes this assumption and provides a more robust test in such cases. The test statistic is similar as the test statistic for a regular F-test apart from the fact that the degrees of freedom are adjusted.

Nonparametric test

The Mann-Whitney U test, also known as the Wilcoxon rank-sum test, is a nonparametric test used to determine whether two independent samples come from the same population. Unlike parametric tests, such as the t-test, the Mann-Whitney U test does not require the assumption of normality. The test statistic is given as:

$$U = R - \frac{N_1(N_1+1)}{2}$$

where:
U: the Mann-Whitney U statistic
R: the sum of ranks in one of the groups
N_1: the sample size of one of the groups

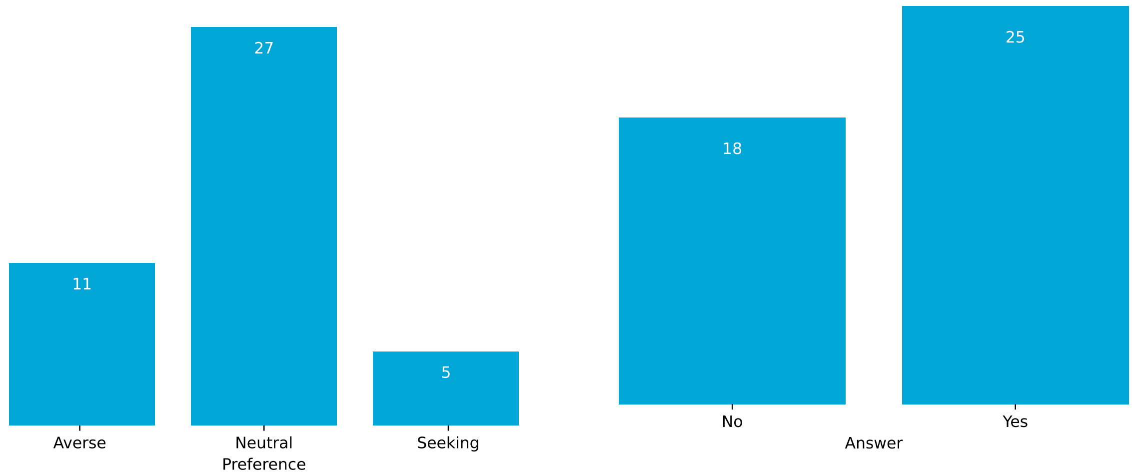
In the Mann-Whitney U test, the observations from both samples are combined, ranked, and compared to assess if one group tends to have larger values than the other. The test statistic U represents the sum of the ranks of the samples from the first group, and it follows a specific distribution under the null hypothesis of equal populations. The p-value associated with the test statistic is then used to determine the statistical significance of the results.

4

Results

4.1. Summary of survey outcome

A total of 43 individuals completed the entire survey. The appendix provides detailed information about the characteristics of the (sub)groups based on the collected metadata. Several important characteristics are highlighted here, starting with the self-assessed risk preference and whether the individuals are professionally employed in the flood risk management sector. These characteristics can be found in figure 4.1a and 4.1b.

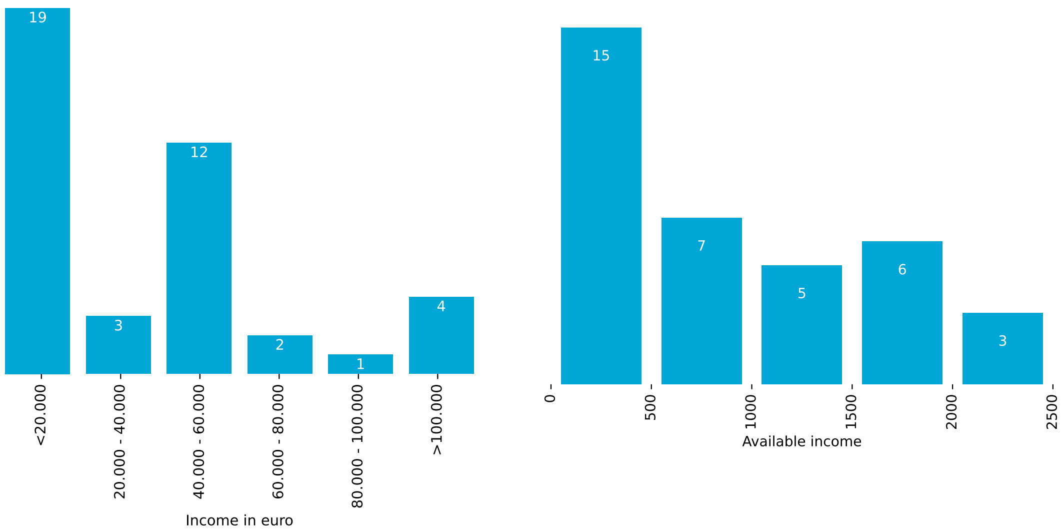


(a) Self-assessed risk preferences by the participants of the survey

(b) Professional employment in flood risk management of the participants of the survey

Figure 4.1: Histogram of the self-assessed risk preferences of the participants of the survey and whether they are employed in flood risk management

It can be observed that the majority of individuals perceive themselves as having a risk-neutral approach to gambles, while the remaining individuals are divided roughly two-to-one between risk-averse and risk-seeking preferences. Another significant factor that will impact both the utility- and probability weighting functions is the financial position of the participants. Figure 4.2 displays the distribution of income and monthly expendable income.



(a) Yearly gross income of participants of the survey

(b) Monthly expandable income of participants of the survey

Figure 4.2: Histogram of the yearly gross income of the participants of the survey and their monthly expendable income

It can be observed that the majority of participants have a yearly income below €60.000, although there is a noticeable number of individuals with incomes exceeding €100.000. This income disparity is reflected in the monthly expendable income, with the majority having less than €600 available per month, but with some outliers exceeding €1500. Expendable income is defined as the monthly amount individuals have left after paying their fixed expenses.

A summary of the found certainty equivalents of all participants from the posed choice problems is shown in the histogram figures in figure 4.3. Similar figures have been made for the other two subgroups, shown in figures E.1 and E.2 in the appendix.

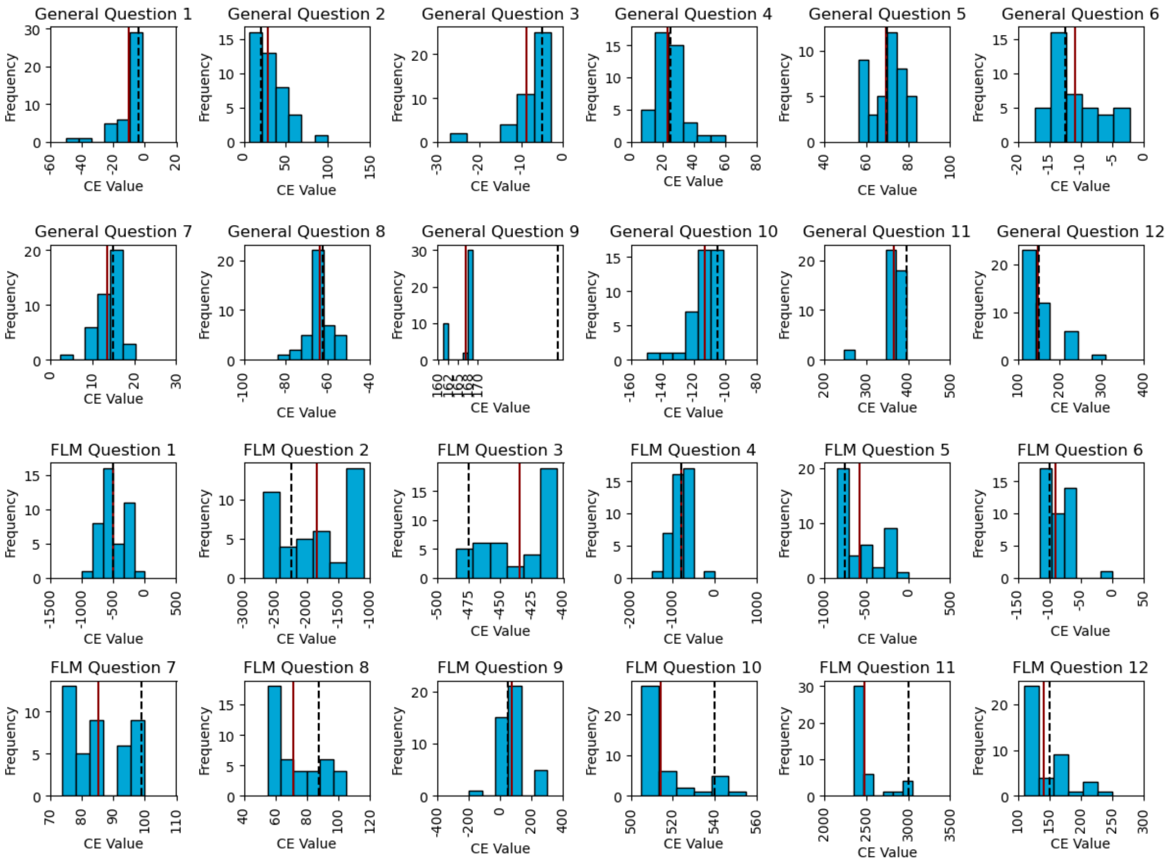


Figure 4.3: Histograms of the recorded certainty equivalents of the participants of the survey, collected per choice problem. The first twelve histograms correspond to the general choice problems, the second twelve histograms correspond to the flood risk related choice problems as described in Appendix C

In this figure, the red line represents the mean certainty equivalent of the group of participants. The black dotted line represents a risk neutral certainty equivalent. Note that some problems have positive certainty equivalents while others have negative certainty equivalents, depending on whether the problem posed was in the positive or negative domain. The relative position of the black line (the risk neutral CE) with respect to the red line (the mean CE of the participants) in combination with the knowledge of whether a the choice problem posed gives an indication of the (average) risk preference of the participants. Recall the definition of risk averse preference: being willing to settle for a lower pay-out than would be received according to the expected outcome. Risk seeking preferences are opposite: not being willing to settle for a lower pay-out than is expected on a risk neutral approach. These preferences can be distinguished based on the (average) certainty equivalent values from figure 4.3. If the red line lies left of the black dotted line, the participants are, on average, willing to accept a lower pay-out than would be expected based on a risk neutral approach. Subsequently if the red line lies to the right of the black dotted line, participants are willing to refuse the expected value of the gamble in search of higher pay-outs. Examples of risk averse approaches can be found in General Questions 1,3 and most pronounced 9 as well as in FLM Questions 7, 8 and 10. Examples of risk seeking approaches can be found in General Question 6 as well as in FLM questions 2,3 and 5.

For some problems the two lines coincide, indicating that the group of participants, on average, has a risk neutral approach to the problem. Examples of risk neutral approaches to the choice problems are General Question 4,5, 7 and 8 as well as FLM Question 1, 4 and 9. Note that individuals can still vary from the risk neutral approach; it is solely the group average that has a risk neutral approach.

4.2. Elicitation of risk preferences

This section presents the results of the elicitation of risk preferences. It begins by eliciting the parameters of the decision-making model for the general choice problems, followed by the FLM related choice problems. In both sections, various utility models are fitted and the results are presented. The section concludes with a comparison of the outcomes, and an analysis using both the general and FLM-related choice problems is presented. It is important to note that the utility models are varied and fitted, while the probability weighting function is kept constant as the *Power* function for all models.

4.2.1. General risk preferences

General risk preferences were elicited via the prospects laid out in Appendix C, resulting in the first 12 certainty equivalents (CE's) presented in figure 4.3. These 12 CE's are subsequently used in combination with the outcomes and probabilities to estimate the parameters for various utility models.

Table 4.1 shows several summary statistics of the parameter estimates for the various models. The table shows the mean, standard error and the minimum- and maximum value of the parameters estimates. The standard error of the curvature of the probability weighting function (given in γ) is shown in brackets next to the mean estimate of γ.

Table 4.1: Average estimates of the parameters for several utility functions for all participants based on the general choice problems, including several key statistics and the curvature of the probability weighting function with standard error in brackets

Model	Parameters	Mean	Standard error	Min - Max	Curvature (γ)
Linear	a	2.549	0.449	[0.973, 18.91]	0.868 (0.0961)
Logarithmic	c	1.496	0.0607	[0.838, 2.812]	1.108 (0.0254)
Iso-elastic	η	0.268	0.0269	[-0.273, 0.620]	0.627 (0.00515)
	α	0.694	0.0117	[0.425, 0.778]	
Power	β	0.834	0.0143	[0.514, 0.987]	0.631 (0.00748)
	λ	1.454	0.00207	[1.408, 1.479]	

To test which model best suits the observed CE values, several goodness-of-fit variables were examined. The first of which is the correlation coefficient, R^2, between the observed CE values and the predicted CE values by the different models. The second two statistics are quite similar, the *Sum of Linear Residuals* and *Sum of Squared Residuals*. The full lists of statistics are shown in table A.2 in Appendix A.

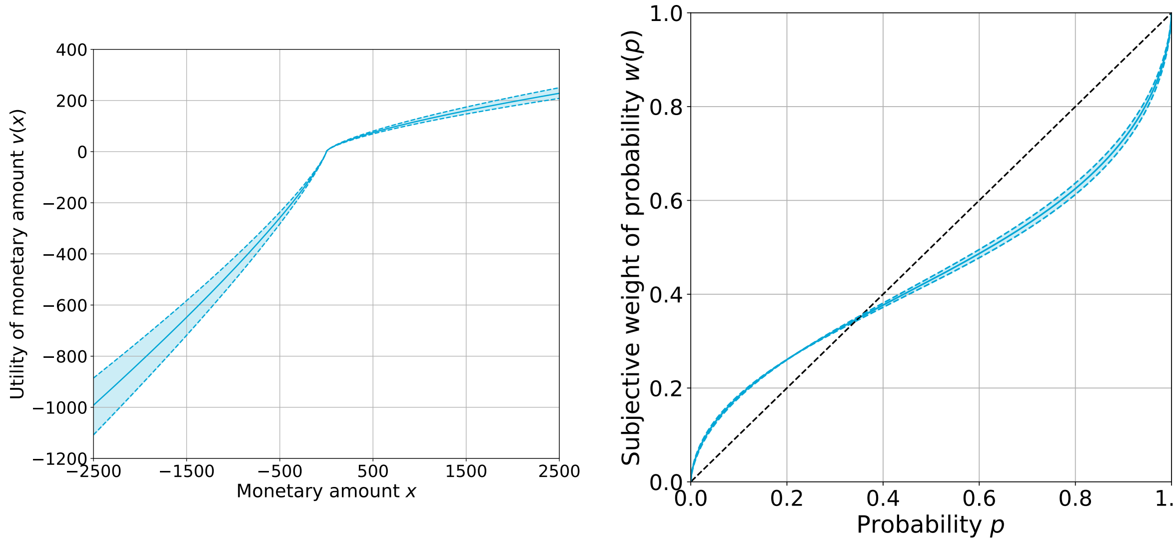
The R^2 indicates the percentage of the variance in the dependent variable that the independent variables collectively explain. In this context, a high correlation coefficient indicates that a large percentage of the variance of the found CE values is explained by the CE values as calculated by the models in table A.1. These CE values are calculated by the models with the given parameter estimates and the provided combination of outcomes (x, y) and probabilities (p). This correlation coefficient has been calculated per individual and presented as an aggregate statistic, as well as for the entire group by means of the average parameters. The R^2 for individuals therefore shows how much of the variance of the found CE values per individual is explained by the utility functions of said individual. The R^2 for the group shows how much of the variance of the found average CE values for the group is explained by the utility function of the entire group, e.g. the utility function with the average of the parameters. These two statistics indicate how well the model predicts individual as well as societal risk preferences.

The table also includes the *Sum of Squared Residuals* (SSR) values for the different models. This statistic gives an indication of how much of the observed CE values is not explained by the model. It measures the quadratic difference between the observed and predicted CE values, with higher values indicating less accuracy in predicting the correct CE values and a larger error. To account for differences in magnitude, the SSR is normalized.

The final metric, the *Sum of Linear Residuals* (SLR), works in a similar way to the SSR, except that the residuals are weighed linearly. Because of this linear weight, an estimate that is twice as far off the true value as another estimate is weighed twice as unfavorably. When using the SSR, this value would be weighed four (2^2) times as unfavorably compared to two times. Large differences between the SSR and SLR values suggest some predicted values have relatively large differences from the observed values. Ideally, both values should be close to zero. The SSR is more suitable for mean estimates, while the SLR is preferred for median estimates.

Based on the results of the parameter estimates and the goodness-of-fit statistics, the *Power* model was chosen as the best fitting utility function. The first reason is its relatively high score on both R^2 statistics, indicating that this utility function explains a large part of the variance between the predicted and observed CE values for both the individual function and the collective function. Although its SSR and SLR values are not the lowest, this is offset by its ability to differentiate between positive and negative outcomes within its utility function. This property is valuable for flood risk-related problems as they involve negative outcomes in the form of losses and positive outcomes in the form of avoided damage, which are considered as benefits in a cost-benefit analysis. Based on these three considerations, the *Power* model is considered the best model among those tested.

The *Power* model, which consists of the *Power* utility and probability weighting function, is plotted in Figure 4.4 using the best estimates for the group parameters.



(a) Estimate of the utility function for all participants including 95% confidence interval

(b) Estimate of probability weighting function for all participants including 95% confidence interval

Figure 4.4: Estimate of probability weighting- and utility function including uncertainty for all participants based on the general choice problems using the Power model

Figure 4.4a illustrates the utility functions of the power model. The x-axis represents the input, often expressed in monetary amounts, while the y-axis represents the utility $v(x)$ derived from the monetary input x. The shape of the utility function indicates that negative outcomes are assigned higher absolute weights (in terms of utility) compared to positive outcomes. This is a result of the combination of different exponents in the Power model (α and β), which cause varying rates of diminishing utility for positive and negative outcomes, as well as a relatively greater weight given to negative outcomes, indicated by a value of λ that is greater than one. The shaded area represents the uncertainty in parameter estimates of the participants, in this case represented by a 95% confidence interval around the mean. This is the uncertainty around the mean parameter estimate for each participants, not the uncertainty per participant. It can be seen that the uncertainty increases for higher values, with differences of approximately 200 utils (or about 20%) at the maximum plotted value of 2500 for x.

Figure 4.4b shows the probability weighting function that corresponds to the found utility function. The x-axis shows the input, a probability p between zero and one. The y-axis shows the weight $w(p)$ an individual attaches to the input probability, which also has values between zero and one. The typical S-pattern described in the theory can clearly be observed. Smaller probabilities are given a disproportionately high weight in relation to the larger probabilities. It can be seen that a probability of 0.1 has a weight of around 0.2 while a probability of 0.8 has a weight around 0.65. Noteable is that the uncertainty of the probability weighting function is relatively low. This can be attributed to two factors. The first being that all of participants had relatively similar estimates for the curvature of the probability weighting function (γ), resulting in a low variation. The other being that during the estimation procedure part of the uncertainty was 'absorbed' by the utility function. This can occur as the utility- and probability weighting function are estimated together in a bi-linear estimation problem. It is important to note that this phenomenon may not be observed when considering each function in isolation, but it is not a concern as long as both functions are used together to estimate the certainty equivalent.

4.2.2. Flood risk related risk preferences

FLM risk preferences were elicited via the prospects laid out in the previous section, resulting in the last 12 certainty equivalents (CE's) presented in figure 4.3.

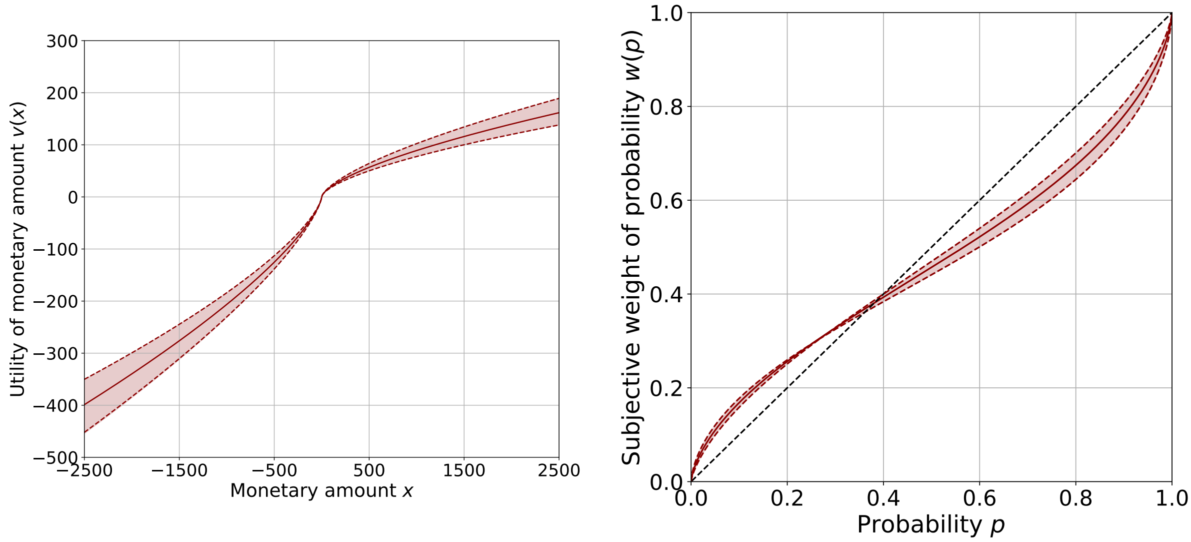
These 12 CE's are subsequently used in combination with the outcomes and probabilities to estimate the parameters for various utility models. The results of which are shown in table 4.2.

Table 4.2: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM choice problems

Model	Parameters	Mean	Standard error	Min - Max	Curvature (γ)
Linear	a	3.025	0.437	[-0.140, 14.28]	1.424 (0.109)
Logarithmic	c	0.485	0.0527	[0.161, 1.126]	0.892 (0.0282)
Iso-elastic	η	0.234	0.0298	[-0.114, 0.391]	0.887 (0.00926)
	α	0.650	0.0201	[0.424, 0.789]	
Power	β	0.719	0.0163	[0.505, 0.947]	0.698 (0.0212)
	λ	1.438	0.00359	[1.299, 1.460]	

A similar procedure for determining the best fitting model as for the general choice problems has been performed for the FLM related choice problems. The goodness-of-fit statistics are shown in table A.4 in Appendix A.

For the same reasons as were mentioned for the general risk preferences, the *Power* model is chosen to best represent the risk preferences for the FLM related choice problems. Its R^2 values are among the highest and the ability to differentiate between positive and negative outcomes is preferred. Using the same model for both the general- and FLM related choice problems further enables a direct comparison between parameter estimates to see if the risk preferences are consistent between general- and FLM related choice problems. The *Power* model for the FLM related choice problems is plotted in figure 4.5, again using the best estimates for the group.



(a) Estimate of the utility function for all participants including 95% confidence interval

(b) Estimate of probability weighting function for all participants including 95% confidence interval

Figure 4.5: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM related choice problems using the power model

The x- and y-axes in figure 4.5a and 4.5b have the same meaning as the axes in the general risk preferences. The same general characteristics observed in figure 4.4a can be seen in figure 4.5a. Negative outcomes have a lesser diminishing effect on outcomes with a greater magnitude compared to positive outcomes. Additionally, there is a key difference in the scale of the utility function for FLM-related choice problems, which has a lower absolute magnitude of $v(x)$. For the largest value of x, the difference is around 2 to 2.5. The positive outcomes appear to be of similar magnitude, as reflected in the values of α, which are closely grouped together. This could be an indication that individuals weigh the negative outcomes of flood risk related choice problems less severe than they weigh a similar outcome of a general economic choice problem. This could be due to the additional context around flood risk related choice problems, and possibly emotions that are paired to this context. The uncertainty around the utility function is of a similar relative magnitude as the uncertainty observed in the utility function found through the general choice problems.

Looking at figure 4.5b, the familiar S-shape of the probability weighting function can be recognized. The curvature of the probability weighting function is slightly lower compared to the curvature found in the general probability weighting function, as indicated by the parameter γ. Consequently, small probabilities are given a lower weight in this function, although the difference is small. Examining the uncertainty, a broader band of uncertainty around this probability weighting function can be observed. This suggests that individuals may have more difficulty estimating probabilities associated with flood risk-related choice problems compared to probabilities associated with general choice problems. Conversely, due to the smaller curvature, individuals, on average, would tend to estimate flood risk-related probabilities closer to their true probabilities.

Comparison of general and FLM-specific risk preferences

This section compares the parameter estimates of the various fitted utility and probability weighting functions between the general and FLM related choice problems. Starting with the utility functions, it can be seen that the scale in the negative domain differences significantly. Where an input of -2500 for x in the general risk preferences leads to a value of around -1000 utils, for the same input the FLM elicited risk preferences return around -400 utils which is a difference of around 2.5 magnitudes. The positive outcomes do not differ significantly. This difference for negative outcomes indicates that based on the survey, FLM related losses are weighed less drastically as those in general choice problems. A reason for this could be that the interpretation of negative outcomes is clouded by the context of an outcome related to flood risk, while for the general outcomes the monetary value was less clouded. Note that they still have a similar inclination to overweigh negative outcomes relative to positive outcomes. Moving to the probability weighting function, it can be seen that there is virtually no difference in either the shape or uncertainty of the general- and FLM related risk preferences. This suggests that the estimation of probabilities between the two preferences does not differ significantly and that, unlike outcomes, they might be influenced by different contexts.

To quantify these observed differences, three different tests are performed, based on assumptions of the residuals of the parameter fit. These tests are the Chow Test (more commonly known as the F-test), the Mann-Whitney Test (also known as the Wilcoxon rank-sum Test) and Welch's ANOVA Test. As explained in the methodology, these tests have different levels of restrictions with the F-test being the most restrictive and Welch's ANOVA Test being the least restrictive. The results of the comparison between the parameters are shown in table 4.3. The first value in each entry of the table is the test statistic, the value in between brackets is the corresponding p-value. If the corresponding p-value is marked as approximately zero, it's value is below 10^{-5}.

Table 4.3: Comparison of equality between the parameter estimates of the FLM and general utility- and probability weighting functions by means of several test-statistics with the corresponding p-value given in brackets

Model	Parameter	F Test	Mann-Whitney Test	Welch's ANOVA Test
Linear	a	0.563 (0.455)	733 (0.321)	0.563 (0.455)
	γ	14.13 (0.000322)	258 ($\approx$ 0)	14.14 (0.000325)
Logarithmic	c	0.436 (0.511)	933 (0.394)	0.736 (0.391)
	γ	279 ($\approx$ 0)	0.0 ($\approx$ 0)	60.8 ($\approx$ 0)
Iso-elastic	η	0.718 (0.399)	841 ($\approx$ 1)	0.718 (0.399)
	γ	589 ($\approx$ 0)	4.0 ($\approx$ 0)	588 ($\approx$ 0)
Power	α	3.428 (0.678)	883 (0.697)	3.428 (0.0687)
	β	27.53 (≈ 0)	1373 (≈ 0)	27.526 (≈ 0)
	λ	15.02 (0.000217)	1414(≈ 0)	15.02 (0.000253)
	γ	8.707 (0.00416)	691 (0.167)	8.707 (0.00482)

A p-value below 0.05 is a common threshold for rejecting the null hypothesis, which in this case is the hypothesis of equal means between the two groups. It can be observed that for nearly all functions, the utility-specific parameters between the two sets of choice problems have means that do not allow us to reject the null hypothesis of equal means based on the evidence. This holds true for all utility-specific parameters except the parameter λ in the *Power* model, where there is enough evidence to reject the null hypothesis of equal means between the groups of general and FLM related choice problems. The same conclusion can be drawn for the means of the curvature of the probability weighting function (γ) in all the models, as each corresponding p-value is below 0.05.

Therefore, it can be concluded that the means of the probability weighting function are significantly different between the general choice problems and the FLM related choice problems. This indicates that when conducting subsequent analyses using these models, FLM and general choice problems should have slightly different values for their parameters, although the model itself remains valid for use in both cases.

4.3. Valuation of a Statistical Life

The method for eliciting the statistical value of a human life has been explained in chapter 3. A brief recap will be provided in this section, along with the results of the survey and analysis. Participants were indirectly asked how much they valued their own live via two sets of questions: one related to road safety and one related to flood safety. In addition to presenting the results, the sensitivity of the elicitation of the model parameters is quantified.

4.3.1. Elicitation via General choice problems

The elicitation of the statistical value of a human life via general choice problems was done via two separate sets of questions. The first set involved comparing two roads with different probabilities of death, toll costs, and travel time. This resulted in two equations with three unknowns, which can be expressed in the general form shown in equation 3.1.

$$u(\text{D}) \cdot w(p_1) + u(\text{ND}) \cdot w(1-p_1) + u(M_1) + u(t_1) = u(\text{D}) \cdot w(p_2) + u(\text{ND}) \cdot w(1-p_2) + u(M_2) + u(t_2)$$

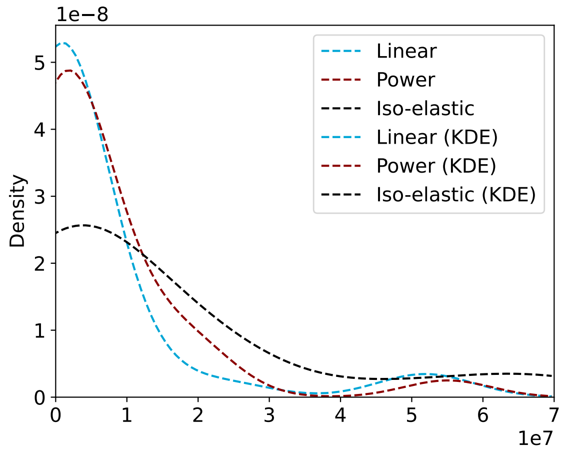
The second set of choice problems resulted in one equation with two unknowns, given below.

$$u(\text{D}) \cdot w(p_3) + u(\text{ND}) \cdot w(1-p_3) + u(M_3) = u(\text{D}) \cdot w(p_4) + u(\text{ND}) \cdot w(1-p_4) + u(M_4)$$

Where the parameter of interest in $u(ND)$, the utility derived of *not dying*.

These equations form a set of three equations with three unknowns. To solve them, one variable in equation 3.2 is chosen, and the other variable is expressed in terms of the chosen variable. This expression is then substituted into the two equations of 3.1 to be simultaneously solved for the remaining two unknowns. Both $u(\text{D})$ and $u(\text{ND})$ are expressed in *utils*, so they need to be converted back to monetary amounts by taking the inverse of the utility function. This yields the monetary value one assigns to not dying, known as the Value of Statistical Life (VOSL), given as: $VOSL = u^{-1}(u(\text{ND}))$. It's important to note that any utility function that accurately captures an individual's risk preferences can be used to derive the valuation.

The three most promising utility functions from the previous section were used to calculate the VOSL. The results of the analysis are summarized in figure 4.6a and table 4.6b.



(a) Histogram showcasing the distribution and kernel density estimate of the VOSL as elicited by general choice problems for three different utility models

Model	**Median** $VOSL$	**Median** $u(VOSL)$
Linear*	0.191 [€ mln.]	0.571 [mln. Utils]
Iso-elastic	12.82 [€ mln.]	0.206 [mln. Utils]
Power	4.397 [€ mln.]	0.0153 [mln. Utils]

(b) Summary statistics of different utility models used to estimate the (utility of the) valuation of a statistical life for general choice problems

Figure 4.6: Histogram and table showcasing the results of the valuation of a statistical life for general choice problems using three different utility models

* *The mean estimate for the linear model was €4.634 mln. corresponding to 9.031 mln. Utils.*

Figure 4.6a shows a kernel density estimate of the histogram representing the distribution of estimates of the valuation of a statistical human life for the three most promising models: the *linear*, *Iso-elastic* and *Power* model. The values in the histogram are capped at €70 mln. for readability. A non-capped version of this histogram is given in figure E.18 in Appendix E. The distribution of values appears to be asymmetric, with the majority of the mass concentrated between €0 and 20 million in valuation. The degree of asymmetry differs between the different utility models used and can be better understood by looking at table 4.6b.

Table 4.6b presents the median estimate for both the utility derived from *not dying* and the corresponding estimate for the VOSL. It is evident that the estimates differ significantly across models, ranging from around €191,000 to €12.82 million. The median was chosen as the representative estimate due to the presence of (severe) outliers in the individual responses and parameters. Given the relatively large weight of these outliers, a linear weighting of each data point, as achieved by the median estimate, is deemed more appropriate than a quadratic weighting as with the mean. The reasons behind the substantial discrepancy in estimates will be explained in a later section discussing the *Sensitivity of elicitation*.

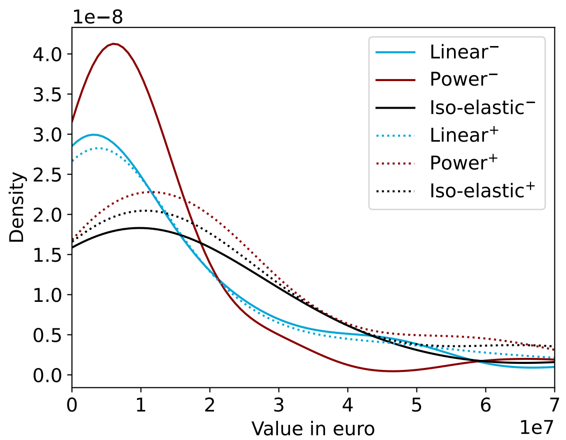
4.3.2. Elicitation via Flood Risk management related choice problems

The elicitation of the statistical value of a human life via FLM related choice problems is more straightforward compared to the previous elicitation via general choice problems. The general form of the equation that follows from the FLM related choice problems is given in equation 3.3. Recall that participants were presented a choice between living inside or outside of the dike ring, in identical houses that only differ in the probability of death due to flooding. In one scenario they were offered additional income for living outside of the dike ring, whereas in the other scenario they were taxed additionally for living inside the dike ring.

$$u(\text{D}) \cdot w(p_1) + u(\text{ND}) \cdot w(1 - p_1) = u(\text{D}) \cdot w(p_2) + u(\text{ND}) \cdot w(1 - p_2) \pm u(\text{CE})$$

Once again, the parameter of interest is $u(\text{ND})$. This equation involves two unknowns, and thus, two elicited preferences are necessary to solve for both unknowns simultaneously. The values in *utils* must be converted back to monetary amounts.

A total of four variations of probabilities were used in the additional income question, and four variations of probabilities were used in the question regarding additional taxation for living inside the dike ring. The same three most promising utility functions from the previous section were employed to calculate the VOSL. The results of the analysis are summarized in figure 4.6a and table 4.6b.



(a) Histogram showcasing the distribution by means of a kernel density estimate of the VOSL as elicited by flood risk related choice problems for three different utility models, for the positive and negative outcomes

Model	**Median** $VOSL$	**Median** $u(VOSL)$
Linear$^+$	18.71 [€ mln.]	37.19 [mln. Utils]
Linear$^-$	5.18 [€ mln.]	12.23 [mln. Utils]
Iso-elastic$^+$	2,571 [€ mln.]	15.11 [mln. Utils]
Iso-elastic$^-$	543.1 [€ mln.]	3.05 [mln. Utils]
Power$^+$	18.32 [€ mln.]	11.63 [mln. Utils]
Power$^-$	12.58 [€ mln.]	5.99 [mln. Utils]

(b) Summary statistics of different utility models used to estimate the (utility of the) valuation of a statistical life for flood risk related choice problems

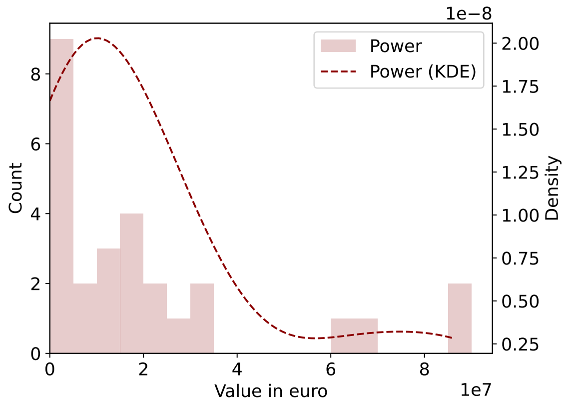
Figure 4.7: Histogram and table showcasing the results of the valuation of a statistical life for flood risk related choice problems using three different utility models

Again, figure 4.7a presents the kernel density estimate of a histogram displaying the distribution of estimates of the valuation of a statistical human life with the parameter estimates as obtained by the choice lists for the three best models. The same limits on the x-axis are used in this figure for comparison, a non-capped version can be found in figure E.18 in Appendix E. The distribution of FLM evaluations follows a similar pattern to the general problems, but with a significantly higher number of extremely high valuations. Consequently, the average value is even higher than that of the general elicitation, and the reported median values in the table are also higher. Notably, the *Power* model exhibits a distinct difference in densities between positive and negative outcomes, while this difference is less pronounced in the other models.

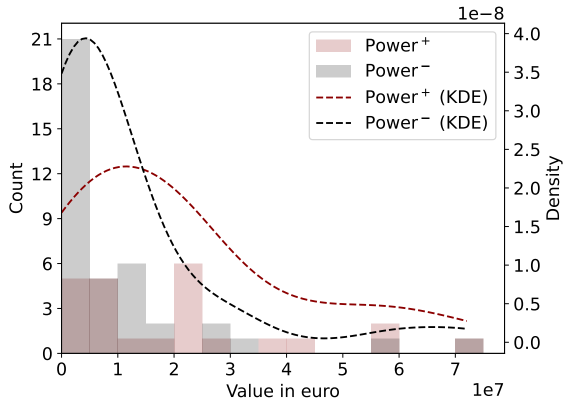
Table 4.7b includes the same two statistics as in the general section: the valuation of life and the utility of living. However, due to the two modes of elicitation (positive and negative), the estimates for both evaluation methods are shown for each model. It can be observed that the linear and *Power* model estimates are relatively close to each other, despite employing different utility measures. On the other hand, the Iso-elastic model displays a significantly higher valuation, even though the utility estimates are of a similar order of magnitude as the other two models. Additionally, it is noteworthy that the positive evaluation is 1.5 to 5 times higher than the negative evaluation.

Conducting a Welch's ANOVA test reveals that the null hypothesis of equal means between the estimates for the general and FLM elicitation can be rejected at a 5% significance level. For this purpose, the average of the positive and negative components of the FLM elicited valuations was taken.

To conclude this section, a closer look at the model of choice, the *Power* model, with its VOSL estimates is taken. The general- and FLM related choice problems are shown in figure 4.8.



(a) Histogram with KDE density estimate showcasing the distribution of the Power model estimates for the VOSL as elicited by general choice problems



(b) Histogram with KDE density estimate showcasing the distribution of the Power model estimates for the VOSL as elicited by flood risk related choice problems

Figure 4.8: Histogram with KDE density estimate showcasing the distribution of the Power model estimates for the VOSL for both the general- and flood risk related chocie problems

Figure 4.8a displays the estimate for the general choice problems. It can be seen that most of the mass is concentrated up to €20 mln. Figure 4.8b displays the estimates for the FLM related choice problems. This elicitation contains two estimates for the VOSL, one based on the positive outcomes and one based on negative outcomes. It can clearly be seen that the estimates for the mean

4.3.3. Sensitivity of elicitation

To illustrate the sensitivity of the valuation of a statistical life to the parameters of different models, a series of analysis was carried out for a range of parameter values. This was done for the elicitation via the general choice problems as well as the flood risk management related choice problems. To this end, each individual was given the same parameter for each evaluation of the value of a statistical life, rather than each individual having their personal parameter estimate. The results per evaluated value of the parameter were aggregated and the mean of all individuals was used as the point estimate for that parameter evaluation.

The results are presented in graphs with varying parameters for each of the three used models. Each set of graphs has a similar lay-out and corresponds with a parameter variation. The left graph represents the outcome of the general choice problems, the right graph represents the outcome of the flood risk management related choice problems. For all utility model specific parameters, the parameter of the probability weighting function (γ) was fixed to 0.75. The outcomes are also qualitatively described in table 4.4. The sign indicates whether the valuation of a statistical life increases or decreases if the parameter value is increased.

Table 4.4: Qualitative description of the influence varying the model parameters on the valuation of a statistical life parameters on the valuation of a statistical life

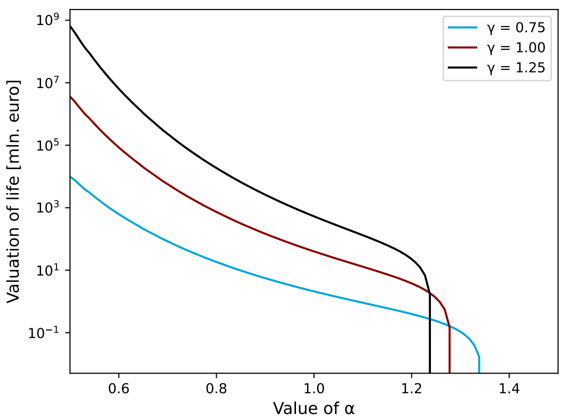
General			FLM		
Linear	**Iso-Elastic**	**Power**	**Linear**	**Iso-Elastic**	**Power**
c (0)	η (+)	α (-)	c (0)	η (+)	α (-)
γ (+)	γ (+)	γ (+)	γ (+-)*	γ (+ -)	γ (+ -)

* *The values of γ increase up to values of one after which they decrease.*

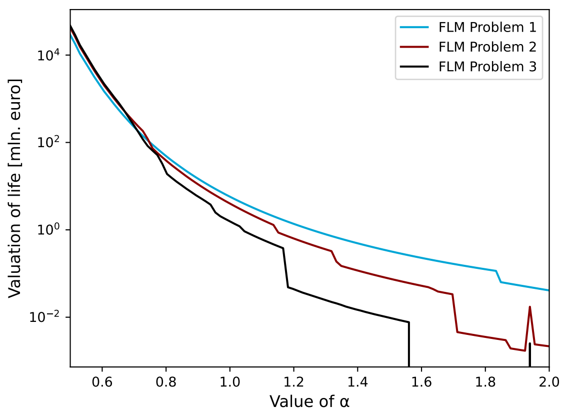
It can be seen from table 4.4 that influence of increasing or decreasing a utility model parameter is the same across the general- and FLM elicitation. The parameter of the probability weighting function γ has the same effect within the general or FLM elicitation, but varies between them. In particular, the FLM estimates start with increasing the valuation up to values of one, after which they decrease for increasing values of γ. In the general elicitation an increase in γ solely reflects an increase in the valuation. Note that the *Power* model is missing the parameters β and λ. These parameters were varied but were found to have no effect on the valuation. This is logical as the β parameter is responsible for the utility in the negative domain and as the valuation of a statistical life has a positive value, this parameter does not influence the outcome. A similar argument can be made for the λ parameter which quantifies the relative weight of a negative outcome with respect to a positive outcome. The sensitivity of the chosen model, the *Power* model is discussed in the main text. The sensitivity of the other two models is discussed in section A.4 in Appendix A.

Power model

For the *Power* model only two out of the four parameters were varied. The parameter dictating the diminishing sensitivity of positive outcomes (α) was varied from 0.5 to 2.0. The parameter dictating the curvature of the probability weighting function (γ) was varied from 0.75 to 1.5. The results for a varying α parameter are shown in figure 4.9.



(a) Results of the sensitivity analysis for the general choice problems for different values of the parameter that dictates the diminishing sensitivity of positive outcomes (α)

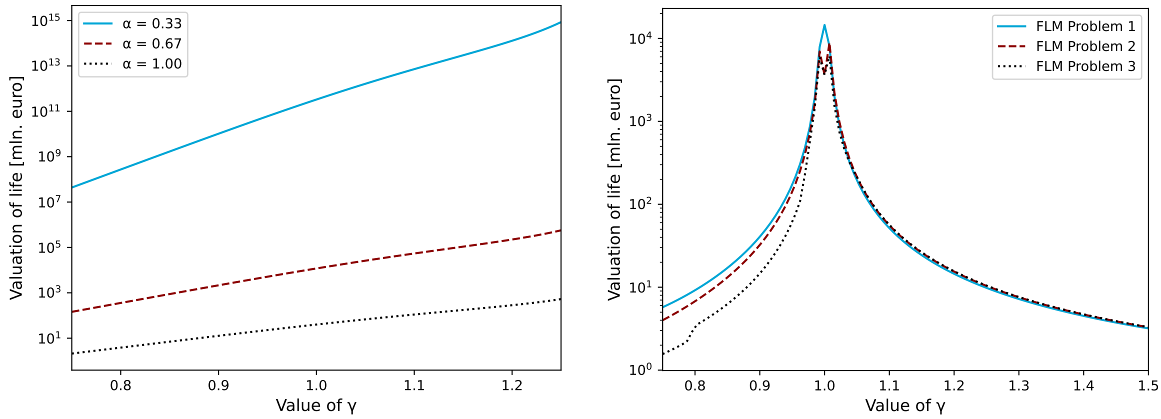


(b) Results of the sensitivity analysis for all three flood risk management related choice problems for different values of the parameter that dictates the diminishing sensitivity of positive outcomes (α)

Figure 4.9: Sensitivity analysis of the valuation of a statistical life for the Power model with a varying parameter that dictates the diminishing sensitivity of positive outcomes (α)

It can be seen in figure 4.9a that the valuation decreases rapidly when the value of α increases, which is to be expected based on the functional form of the utility function. This phenomenon holds for all values of the curvature of the probability weighting function γ with higher values starting with higher valuations and dropping more rapidly. A similar, but less extreme version of the decrease can be seen in figure 4.9, for the three different versions of the elicitation.

Figure A.8 shows the results when the curvature of the probability weighting function (γ) is varied.



(a) Results of the sensitivity analysis for the general choice problems for different values of the parameter that dictates the curvature of the probability weighting function (γ)

(b) Results of the sensitivity analysis for all three flood risk management related choice problems for different values of the parameter that dictates the curvature of the probability weighting function (γ)

Figure 4.10: Sensitivity analysis of the valuation of a statistical life for the Power model with varying curvature of the probability weighting function (γ)

It can be seen in figure 4.10a that for increasing values of γ the valuation increases for all three values of the parameter α. Figure 4.10b shows that all FLM valuations again lie close together. Increasing rapidly for values of γ up to one before rapidly descending after crossing the threshold of one.

To conclude this section, it is worth noting that the *linear* utility model yields the most stable valuation of a statistical human life. However, this model has drawbacks as it tends to produce relatively low estimates for the VOSL compared to previous research. On the other hand, the *Power* model provides stable results as long as the parameters responsible for the diminishing sensitivity of positive outcomes do not exceed one. If these parameters exceed one, it implies an increasing sensitivity to positive outcomes rather than diminishing sensitivity.

For the general valuation, the specific parameters do not have a significant impact as most values fall within the expected range. However, when considering the FLM valuation, special attention should be given to cases where the curvature approaches zero and the probability weighting function reduces to a linear weighting of probabilities (e.g., $w(p) = p$). This occurs when the parameter γ approaches one. In such cases, the risk preferences become neutral, and the elicitation method may not be effective.

Cross validation

A final method for testing the sensitivity of the elicited parameters was done by means of a cross validation. In this approach, the risk preference parameters obtained from the FLM elicitation were used to predict the general CE values for each individual, and vice versa. The conclusion of the analysis is discussed in this section, and the full analysis can be found in Appendix A.2.

For clarity, the reader is reminded that a certainty equivalent is the product of the weight given to probability of outcomes and the utility given to said outcomes. he weight given to the probability is determined by the probability weighting function, while the utility of the outcomes is calculated using the utility function. Both of these functions have parameters in their functional forms that were elicited in this study. In this analysis, the parameters obtained from the FLM elicitation were used to calculate the weight and utility of the choice problems (which involve probabilities and outcomes) in the general choice problems. Similarly, the parameters obtained from the general elicitation were used to calculate the weight and utility of the FLM-related choice problems. The estimated values were then compared to the actual observed CE value of the original problem.

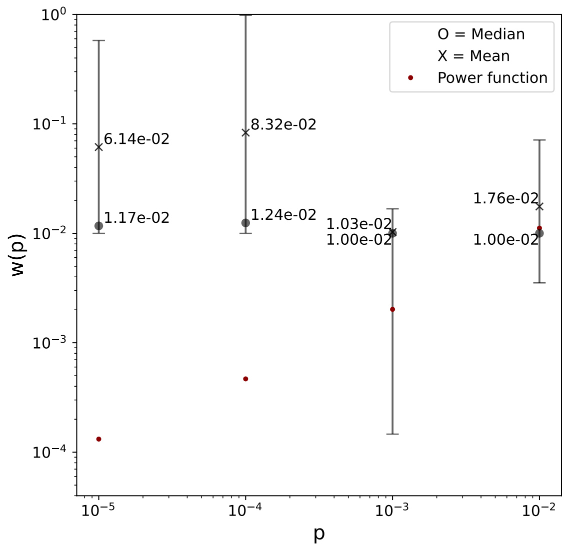
The analysis reveals that the estimated CE values obtained through cross-validation align with the observed CE values within the same order of magnitude. Generally, the differences were around 20 to 30%, often indicating a slight underestimation. This, along with the previous validation, suggests that the elicitation method is robust and provides further confidence in the elicited risk preferences and the models used. It is important to note that several cases were identified where the predicted CE value deviated significantly (either 50% above or below) from the observed value. Suitable explanations were found for these differences, such as the lack of fit near zero or the relatively large difference in the outcomes of the probability weighting function for small probabilities when the curvature of the function (γ) is altered.

4.4. Probability weighting function for small probabilities

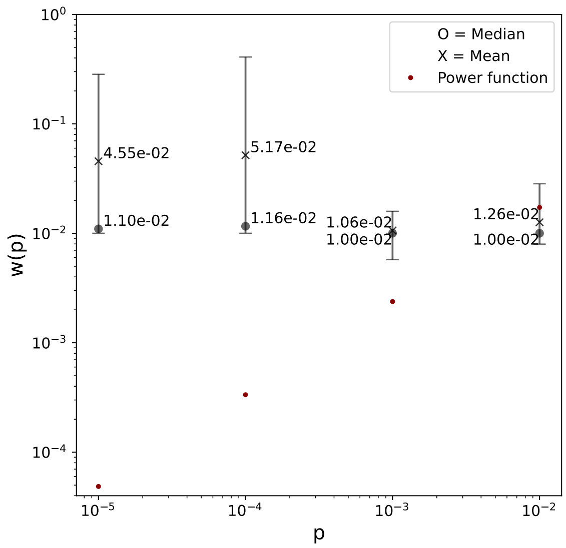
This section showcases the results of the evaluation of the probability weighting function for 'small' values of p, meaning where $p \in [10^{-5}, 10^{-2}]$. This is done by using the found results of VOSL by means of the general choice problems presented in table 4.6b. These results can be used as values for $u(\text{D})$ and $u(\text{ND})$ in the following equation:

$$u(\text{D}) \cdot w(p_1) + u(\text{ND}) \cdot w(1 - p_1) = u(\text{D}) \cdot w(p_2) + u(\text{ND}) \cdot w(1 - p_2) \pm u(\text{CE})$$

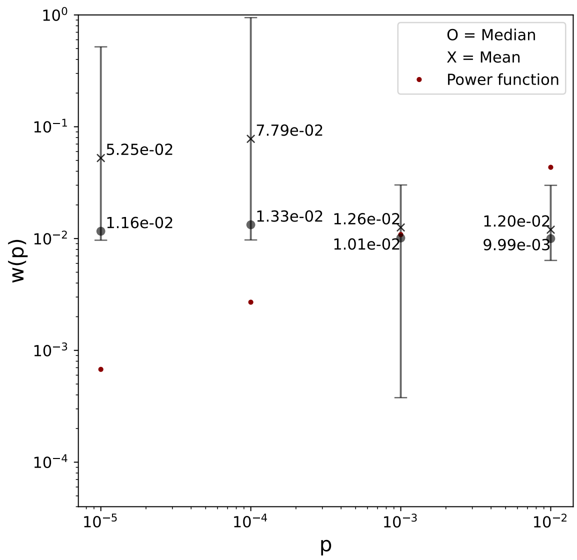
Note that the utility values of D and ND are used, not the monetary amounts. Furthermore the utility values per individual are used rather than an aggregate, corresponding to the elicited parameters per individual per utility function. This results in four arrays containing the estimates for $w(p)$ of all individuals. Summary statistics can be calculated using these arrays of values. In this case, the mean and median values are chosen, as well as the 95% confidence intervals of the estimates. The results for the three different utility models are shown in figure 4.12.



(a) Mean, median and 95% confidence interval for the estimates of small probabilities using the Linear model



(b) Mean, median and 95% confidence interval for the estimates of small probabilities using the Iso-elastic model



(c) Mean, median and 95% confidence interval for the estimates of small probabilities using the Power model

Table 4.5: Numerical results of the mean, median and 95% confidence interval for the estimates of small probabilities for all three utility models

Value of p	**Model**	**Median**	**95% Confidence Interval**
10^{-5}	Linear	$1.17 \cdot 10^{-2}$	[0.0100 0.578]
	Iso-elastic	$1.10 \cdot 10^{-2}$	[0.0100 0.284]
	Power	$1.16 \cdot 10^{-2}$	[0.00967 0.518]
10^{-4}	Linear	$1.24 \cdot 10^{-2}$	[0.0100 0.981]
	Iso-elastic	$1.16 \cdot 10^{-2}$	[0.0100 0.408]
	Power	$1.33 \cdot 10^{-2}$	[0.009718 0.947]
10^{-3}	Linear	$1.00 \cdot 10^{-2}$	[0.000146 0.0167]
	Iso-elastic	$1.00 \cdot 10^{-2}$	[0.00574 0.0159]
	Power	$1.01 \cdot 10^{-2}$	[0.000377 0.0302]
10^{-2}	Linear	$1.00 \cdot 10^{-2}$	[0.00353 0.0712]
	Iso-elastic	$1.00 \cdot 10^{-2}$	[0.00797 0.0284]
	Power	$9.99 \cdot 10^{-3}$	[0.00637 0.0299]

Figure 4.12: The mean and median estimates of the probability weighting function for small probabilities, that is for $p \in [10^{-5}, 10^{-2}]$, elicited by treating their values as parameters. Three different results including a 95% confidence interval are presented, produced by three different utility models.

The median valuation is given by an **o**, the mean valuation is given by an **x**. This figure also contains the average valuation of $w(p)$ of the participants using the probability weighting function as described by (Tversky & Kahneman, 1992), shown as red dots. The x- and y-axis of all three figures has been limited to the same values, such that the estimates and uncertainties can be directly compared.

Across all three figures, 4.11a, 4.11b and 4.11c, it can be seen that the evaluation of $w(p)$ for $p = 10^{-5}, 10^{-4}$ is significantly lower than the evaluation given by the *Power* function by Tversky and Kahneman. This *power* function weighs the probabilities more heavily than would be expected based on a neutral approach (e.g. $w(p) = p$), as such the evaluations using this method are significantly higher than the neutral approach. It can be seen that most evaluations have a median value around 10^{-2}, while the mean fluctuates between 10^{-2} and 10^{-1}, dependent on the model and probability evaluated.

The uncertainty surrounding the estimates differ vastly between models. For example, it can be seen that the uncertainty around the estimate for $w(p)$ for $p = 10^{-3}$ is relatively narrow when using the iso-elastic model while it is relatively wide when using the other two models.

This section concludes with the remark that in the elicitation of the valuation of a statistical life the *Power* form of the probability weighting functions was assumed. As such there is an implicit bias towards the values of $w(p)$ produced by the *Power* function. One could work around this by making the evaluation between the two types of questions, general and FLM, iterative. This would mean that the found value of $w(p)$ in this section is re-used in the valuation of a statistical life by means of the general choice problems, after which that value can be used in the FLM related problems to come up with an estimate for $w(p)$ and so one, until convergence is reached.

4.5. Interpretation of the found results

This section discusses, summarizes and interprets the results that were found in this chapter. The section is split in three parts: *interpretation of risk preferences*, *Valuation of a statistical life* and *Interpretation of the estimation of small probabilities*.

4.5.1. Interpretation of risk preferences

Several functional forms for the modelling of risk preferences have been shown in the chapter on Methodology and Results. This section discusses what the numerical results of the parameter estimates mean in a broader context. The chosen functional form of the utility function was the *power* model. This model contains three parameters α, β and λ. The first parameter α indicates how much weight (or utility) is given to increasing *positive* outcomes and typically has values between zero and one for risk averse individuals and values above one for risk seeking individuals. A value that is closer to one indicates that an individual does not give diminishing weights to increasing outcomes. A value closer to zero indicates the opposite, individuals tend to weigh larger outcomes relatively less than they do smaller outcomes. If the value is above one, an individual values increasing outcomes relatively more. The second parameter β indicates how much weight is given to decreasing *negative* outcomes and also typically has values between zero and one for risk averse individuals and above one for risk seeking individuals. The meaning behind the values is the same as for the parameter α but then in the negative domain. The final parameter λ shows the relative weight that negative outcomes get when compared to positive outcomes with the same magnitude. Due to loss aversion, these values are almost always above one with typical values ranging between 1.5 and 2.5.

The research findings show that the estimated values for α and β fall within the range of risk aversion, with slightly higher values for β than α. This suggests that the weight that is given to negative outcomes decreases at a slower rate than the weight that is given to positive outcomes, for outcomes that are greater in absolute magnitude. This indicates risk aversion, as the equal but opposite outcomes are not weighted in the same manner. The risk aversion is further supported by the fact that the value of λ is well above one. A final important thing to note is that the three parameters were elicited for general- as well as FLM related choice problems. The latter giving smaller values for all parameters. This indicates that, based on this research, individuals tend to be slightly more risk averse for choice problems in Flood Risk management as opposed to those posed in general economics.

In addition to the utility function, there is the probability weighting function, which is characterized by the parameter γ. This parameter determines the degree of curvature in the typical ”S-shape” of the function and usually ranges between zero and one. Values closer to zero give a greater degree of curvature, meaning that smaller probabilities are given a relatively larger weight. This effect diminishes as values grow larger, with values of one giving a complete linear weighting of probabilities. The larger weight that is being given to small probabilities is part of what causes individuals to participate in lotteries or buy insurances. This research found values of γ that indicate that small probabilities are overestimated. Specifically, the general choice problems yielded a lower average value for γ compared to the FLM-related choice problems. This indicates that, in FLM-related problems, individuals tend to weigh probabilities closer to their objective value than in general economic problems.

4.5.2. Valuation of a statistical life

A recalculation of the Value of a Statistical Life (VOSL) has been conducted to compare the findings with the often-cited €6.7 million reported by de Blaeij (2003a). Two methods were used to elicit the VOSL, both based on a combination of stated preference and willingness-to-pay. However, they differed in context and the number of values elicited. The first method, resembling the original study by de Blaeij, focused on a traffic problem. The second method was set in a flood risk context, where participants were asked if they would be willing to reside in the higher risk area outside a dike ring in exchange for additional income. A variation of this method involved participants paying an additional tax to live in the lower risk area inside the dike ring, mirroring the first problem but only allowing for losses.

Using the most suitable utility- and probability weighting function, the median VOSL was found to be €4.4 mln. for the first method of elicitation and €18.3 million (positive) and €12.6 million (negative) for the second method. Notably, the positive and negative elicitation differed by a factor of 1.46, aligning with the concept that 'losses hit about 1.5 to 2 times harder than gains'. There was a significant difference in valuation between the two elicitation methods. One possible explanation is the presence of an additional element in the first elicitation, namely the valuation of time. When each of the three answers is given equal weight, the average estimate for the VOSL in this research is €11.8 million. This is 1.7 to 1.8 times higher than the values reported by de Blaeij (2003a) and Bockarjova et al. (2009). However, when adjusted for inflation, these differences decrease to a factor of 1.18 and 1.33, respectively. The sensitivity of these estimates was further analyzed, revealing a strong dependence on the chosen value of the curvature of the probability weighting function (γ). Increasing values of γ led to higher estimates of the VOSL in the first method, while the second method yielded similar results for values up to one, after which the estimates started to decrease.

It is important to note that the reported value of around €12 million is the median result of an analysis that indirectly elicited a self-estimated worth of life in the face of uncertainty. This figure should not be confused with other variations such as Quality-adjusted life years (QALY) or economic valuations, as performed by Vrijling and Gelder (2000).

4.5.3. Interpretation of the weighting of small probabilities

This research examined two approaches to understand how individuals weigh (very) small probabilities, ranging from 10^{-5} to 10^{-2}. The first approach was by means of the probability weighting function (denoted as $w(\cdot)$) with the elicited parameter γ from either general or FLM related choice problems. The second approach was by means of an alternative method that parameterized the weight attached to a probability (denoted as $w()$), making use of the elicited value of a statistical life in the previous section. The two results were compared in one figure for each utility model, shown in figure 4.12.

The first method aligned with current theory, demonstrating that small probabilities tend to be systematically overestimated. Depending on the value of γ, this research observed substantial overestimations of the smallest probabilities, ranging from approximately 10 to 100, i.e., $w(p) \approx 10 - 100 \cdot p$ for $p \in [10^{-5}, 10^{-4}]$. As the magnitude of the probabilities increases, the relative discrepancy between the objective value p and the subjective weighting $w()$ decreases. When p is around 10^{-3}, the subjective weight attached ranges anywhere from a factor 1 to 10. If p goes towards 10^{-2} (e.g. a chance of one in a hundred), the discrepancy tends to disappear. This suggests that as probabilities become smaller, individuals find it more challenging to judge them objectively, tending to perceive smaller probabilities as larger and assigning disproportionately greater weights to them.

The second method produced results that differed from established theory. While the overall conclusion remains the same, indicating that small probabilities receive disproportionately large weights, the nuances for different magnitudes of probabilities varied. Unlike the previous method, which exhibited diminishing returns for relatively larger probabilities, this method showed relatively consistent estimates for the median weight attached to objective probabilities. Although there was some variation in chosen utility model and difference in individual weights, the median estimates for the weight were relatively consistent around a magnitude of 10^{-2}. These findings suggest that beyond a certain magnitude, in this case $p \approx 10^{-2}$, individuals become insensitive to further decreases in the objective probability. This could be attributed to various factors, with the most likely explanation being that individuals are unable to differentiate between probabilities beyond a certain threshold. In other words, for the average individual, a probability of, for example, 10^{-4} feels more or less the same as a probability of 10^{-5}.

In conclusion, regardless of the method or utility model used, the findings of this research indicate that objective probabilities ranging from 10^{-5} to 10^{-2} are consistently misjudged. The degree of misjudgment depends on the chosen method of elicitation and/or parameters, but overall, probabilities are generally overestimated. The first method of elicitation revealed a larger discrepancy between the objective value of p and the subjectively attached weight w as probabilities became smaller. These results align with Cumulative Prospect Theory Tversky and Kahneman (1992) and subsequent studies. The second method demonstrated that individuals become insensitive to further decreases in the magnitude of probabilities beyond $p \approx 10^{-2}$, or approximately a one in a hundred chance.

5

Case study

5.1. Introduction

In the literature review, an introduction to the *Water Act* was given. The chapter explains that the standards laid out in this law are based upon two principles: A minimum level of protection, guaranteed by the *Local Individual Risk* (LIR) and an economic and/or social criteria. If the impact of a flood is very high, a lower probability of flooding might be deemed appropriate based on either the *Group Risk* or a *Social cost-Benefit Analysis* (SCBA). This results in a maximum allowable flood probability for each of the different dike sections defined in the *Water Act*. The maximum allowable flood probability might be governed by either of the three principles. An overview of the current leading principles for the flood protection standards for different dike sections is given in figure 5.1.

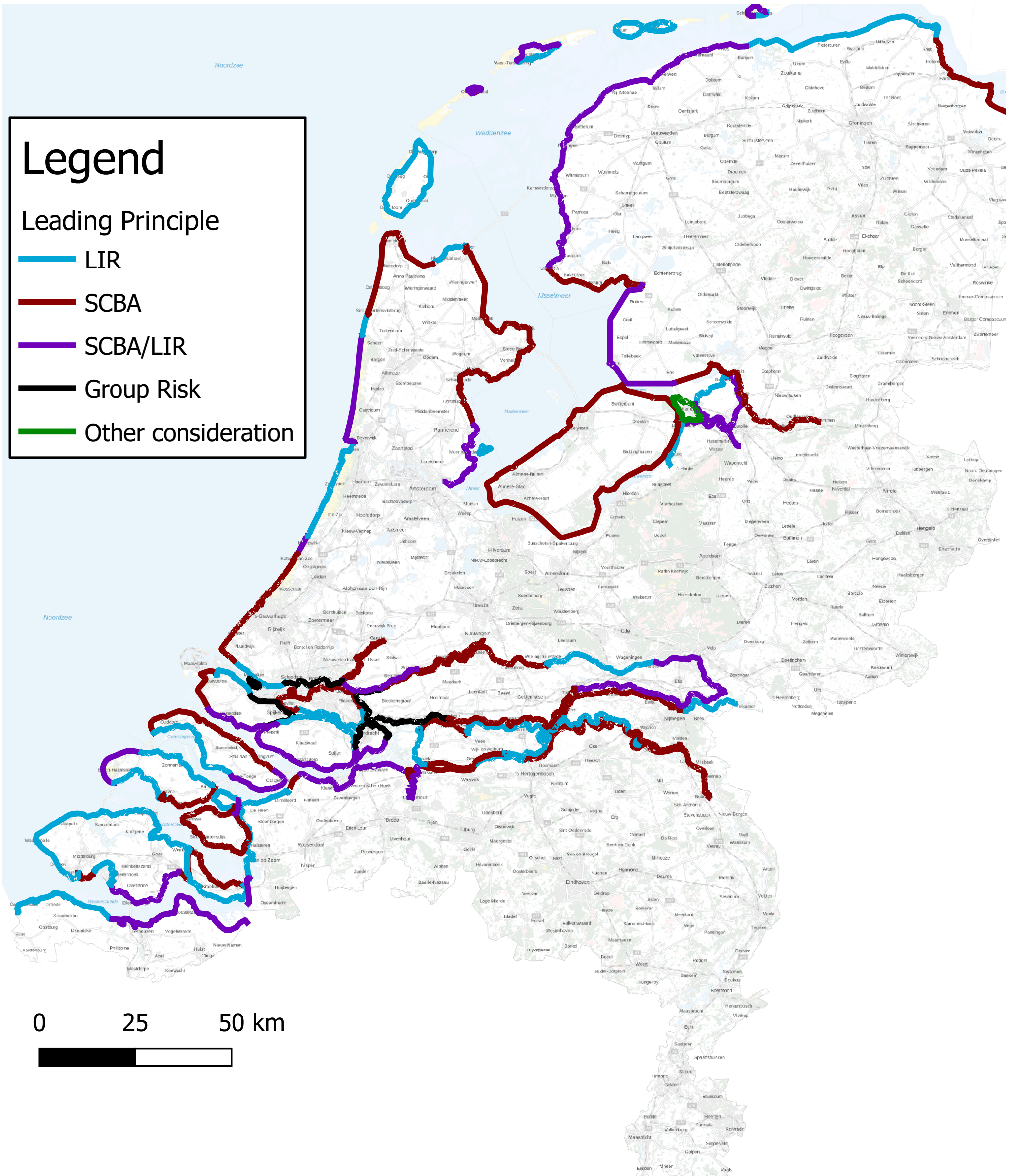


Figure 5.1: Overview of the current leading principle for the flood safety standards in the Netherlands placed on their respective dike sections, based on Slootjes and Wagenaar (2016)

From figure 5.1 it becomes apparent that near large population centers, such as the Rotterdam area, the group risk as well as the SCBA become the dominant safety standard. In less populous areas, such as the southern part of Zeeland or the Wadden isles, the LIR becomes the dominant safety standard.

The strictness of each of the three flood protection standards is determined in a risk neutral approach, using standard values for the Value of a Statistical Life (VOSL) and the interest rate. However, as was demonstrated in chapter 4 a risk neutral approach to choice problems is the exception rather than the safety standard. Furthermore the VOSL in a risk preference adjusted elicitation manner was shown to have a deviating value from the commonly used €6.7 mln, which in turn has an effect on the strictness of the flood protection standards. Each of the three principles is calculated in a different manner, the aforementioned variables may have an affect on a selection of the principles rather than on all principles. This leads to the final variable that will be investigated in this chapter, the interest rate. The interest rate governs the discounting of future benefits and in turn influences the optimal flooding probability as calculated by the SCBA. This case study examines the effects of varying the three aforementioned variables and their (joint) effect on the governing principles as laid out in figure 5.1.

5.1.1. Standards for primary flood defences

In 2016 a factsheet was published containing the expected results of a flooding per dike section as part of updating the flood protection standards of primary flood defences. This factsheet, devised by Slootjes and Wagenaar (2016), provides information per dike section on location, water depths, damages, casualties and needed investment costs for 10 times higher level of protection as well as which principle is leading for that dike section. This factsheet is the primary source of data for the case study and as such, a brief section will be devoted on how the figures in the factsheet were derived and how the results are presented.

The determination of the leading principle for the safety safety standard is relatively straightforward, it is the strictest safety standard out of three element: the LIR, the SCBA or the Group Risk. Such that the governing probability per year becomes:

$$p_{req} = \min\left(p_{LIR}, p_{SCBA}, p_{grouprisk}\right)$$

To allow for some flexibility when designing flood defences for long term use, it is often not one number that engineers have to design for but rather a range or 'class' of probabilities. An overview of the commonly used classes is given in table F.2 in Appendix F. The governing safety standard is chosen to be the strictest of the resulting classifications, as such:

$$p_{req} = \max\left(Class(p_{LIR}), Class(p_{SCBA}), Class(p_{grouprisk})\right)$$

Where the $\max()$ operator denotes the choice for the 'strictest' safety standard, i.e. the safety standard that results in the lowest allowable probability of flooding. The challenge lies in determining this optimal flooding probability, expressed per annum, for each of the three principles. How the optimal flooding probability is calculated, is briefly explained for each principle below.

Group Risk

The most simple of the three is the probability assigned to the *group risk*. If a dike section is marked as a 'Group Risk Hotspot Trajectory', this value is set to one class more strict than the strictest of either p_{LIR} or p_{SCBA}, such that:

$$Class(p_{grouprisk}|Hotspot) = Class(\min(p_{LIR}, p_{SCBA}) + 1$$

Automatically setting it as the strictest class and making it the governing principle. If the section is not a Group Risk Hotspot Trajectory, the class is governed entirely by the minimum of p_{LIR} and p_{SCBA} (the maximum of their respective classes).

Local Individual Risk

The maximum allowable probability of flooding as derived based on the LIR is dependent on a number of factors. The most important of which are the evacuation fraction of an area, the mortality fraction per area and the worst case scenario. The exact formulation for calculating the LIR, dependent on the neighbourhood B, is given in equation 5.1.

$$LIR(B) = p_f \cdot (1 - f_{evacuation}) \cdot (0.4 m_{worst\ case}(B) + 0.6 \sum_i P_{cond,i} m_i(B)) \tag{5.1}$$

Where:
$f_{\text{evacuation}}$: the average evacuation fraction for the corresponding area
p_f: the flood probability of the corresponding dike section (equal to the current legal safety standard frequency for dike rings)
$p_{\text{cond},i}$: the conditional probability of scenario i
$m_{worst\ case}(B)$: the worst case mortality in neighborhood B
$m_i(B)$: the mortality in neighborhood B for scenario i

To calculate the allowable flood probability p_f, equation 5.1 can simply be rewritten as:

$$p_f = \frac{LIR}{(1 - f_{evacuation}) \cdot (0.4 m_{worst\ case}(B) + 0.6 \sum_i P_{cond,i} m_i(B))}$$

This optimal flooding probability can subsequently be classified by means of table F.2 for a comparison with the other principles.

The results of Slootjes and Wagenaar (2016) were derived using a LIR value of $5 \cdot 10^{-6}$. The evacuation fraction $f_{evacuation}$ is derived from Maaskant et al. (2009), the combined mortality fraction of the conditional probabilities and worst case scenario's were derived by Deltares (2011).

Social Cost-Benefit Analysis
The SCBA is based on the principle of economic optimization. The total costs, consisting of the incurred risk and the costs for risk reduction, should be minimized. In essence one is looking for the point where the additional risk reduction, expressed in monetary amounts, is equal to the additional costs incurred for said risk reduction. Mathematically, one is looking for a point where the derivatives are equal in absolute magnitude. Let I be the required investment cost for increasing the flood safety level and C the incurred costs as a result of flooding. If p is an annual probability of flooding after a flood defence is build, and the probability of flooding without defence is assumed to be equal to one, the following equality must hold:

$$I = p \cdot \sum_{i=1}^{n} C_i$$

Showcasing that the investment in flood safety should equal the expected losses over the lifetime n of the measures. Discounting the costs C_i per year results in equation 5.2:

$$I = p \cdot n \cdot \sum_{i=1}^{n} \frac{C_i}{(1+r)^i} \tag{5.2}$$

Where r is the discount rate. Note that if one wants to incorporate a growth in either value or population at risk, displayed in the costs C, the term $\sum_{i=1}^{n}(1+g)^i$ can be added where g is the assumed annual growth rate. A peculiar case arises when $g = r$, as the equality simplifies back to $I = p \cdot \sum_{i=1}^{n} C_i$.

This simplified model can be expanded further by incorporating more detailed terms, such as the dynamic effects of economic growth and climate change incorporated by Eijgenraam (2005, 2006, 2008). Incorporating these additional terms and optimising for economic efficiently was performed in a algorithmic computer program *OptimiseRing* (Brekelmans et al., 2009, Duits, 2011). A simplification will be used here, which is a linear regression model based on the ratio of total damages over the additional costs of a tenfold decrease in flooding probability as derived by Kind (2012). It was found that the following linear relation holds for the 73 dike sections that were evaluated:

$$\frac{1}{p} = 38 \cdot \frac{C_{total}}{I_{10}} \tag{5.3}$$

Where C_{total} is the total damage as a result of flooding and I_{10} is the investment costs required for a tenfold increase in safety. The correlation coefficient was thus found to be 38. This derivation assumes a discount rate of 5.5%. Equation 5.3 can be solved for p (p_f) to find the economically optimal level of flood safety, given as:

$$p_f = \frac{I_{10}}{38 \cdot C_{total}}$$

Again, this flooding probability can subsequently be classed by means of table F.2 for comparison among principles.

The costs as presented by Slootjes and Wagenaar (2016) consist of three parts: The injury to individuals, the loss of life and the loss of economic value. If a flood occurs and there are k injured individuals, m individuals who died and €E in economical damage, the total costs can be expressed as:

$$C_{total} = k \cdot V_{injury} + m \cdot V_{life} + E \tag{5.4}$$

Where:
V_{injury}: The monetary amount associated with an injury
V_{VSOL}: The monetary amount associated with loss of life, i.e. the Value of a statistical life
E: The economic damage as a result of a flood

Results
The results of the evaluation by Slootjes and Wagenaar (2016) of the three above mentioned principles is summarized in a table, containing all the information per dike section. An example of the results of one such dike section is given in table 5.1. In total, there are 234 safety standard trajectories, of which 208 are dike trajectories and 26 are primary flood defenses. Table F.1 in Appendix F contains an explanation for each of the terms used in table 5.1.

Table 5.1: Example of the data per dike section (Section 2-1, Ameland Duin) as presented in Slootjes and Wagenaar (2016)

safety standard	
Lower limit value in the law (annual probability)	$1/300$
Signaling value in the law (annual probability)	$1/1,000$
Determined by	*LIR*
Requirements from LIR, MKBA, and Group Risk	
LIR requirement - lower limit class (annual probability)	$1/300$
LIR requirement - signaling class (annual probability)	$1/1,000$
LIR requirement - lower limit (annual probability)	$1/500$
LIR requirement - signaling value (annual probability)	$1/1,000$
MKBA requirement - lower limit class (annual probability)	$1/100$
MKBA requirement - signaling class (annual probability)	$1/300$
MKBA requirement - signaling value (annual probability)	$1/200$
Group Risk Hotspot Trajectory	*No*
safety standard Trajectory	
Length (km)	20.5
Type	Dune
Composition (percentage) per km	
Soft flood defense	20.5 (100%)
Hard flood defense	0.0 (0%)
Costs	
Cost of 10x higher protection level per km [mln. €/km]	*1.9*
Cost of 10x higher protection level for safety standard trajectory [mln. €/km]	39.0
Consequences of Floods	
Evacuation fraction (lower bound)	*0*
Mortality from flooding from this trajectory (%)	*0.2*
Mortality from flooding from other trajectories (overlap) (%)	0.5
Affected individuals (year 2011)	*870*
Casualties (year 2011)	*1*
Economic damage (year 2011) [million euro]	*63*
Monetized affected individuals (year 2050) [million euro]	20
Monetized casualties (year 2050) [million euro]	20
Economic damage (year 2050) [million euro]	130
Total damage (year 2050) [million euro]	170

To the extent of this research, not all rows of the table are relevant. The primary interest lies in the rows that are in *italic*. These results will be used in the subsequent re-evaluation of this chapter, of which the objective is explained in the next section.

5.1.2. Objective of the case study

This case study will examine the difference in 'ranking' between the local individual risk, the group risk and social cost-benefit analysis. Ranking in this context means how stringent the leading decision criteria is relative to the other criteria. In particular, the potential difference in ranking will be examined when the three aforementioned variables are changed This leads to the following questions:

1. What is the influence of the discount rate on the ranking of the decision criteria? In particular, what is the effect of a variation in the risk premium part of the discount rate?
2. What is the influence of a reevaluation of a statistical human life on the ranking of the decision criteria?
3. What is the influence of the inclusion of risk preferences on the ranking of the decision criteria? In particular, how does the ranking change if the elicited risk preferences derived in this research are used?

These three factors will be examined in isolation at first, evaluating their effect on the ranking, before evaluating all factors jointly with the best estimates for each of the factors. This will result in a map similar to figure 5.1 with the potentially adjusted leading decision factors. Next to this, an analysis of the cost of the risk preferences will be made. This analysis is based on the new derived optimal flooding probabilities.

5.2. Method of evaluation and application

This section outlines the theoretical derivations of the proposed method. It starts by discussing the necessary modifications to the SCBA to include variable values for the VOSL and interest rates. After this, the derivations for the inclusion of risk preferences in both the SCBA and LIR are discussed.

5.2.1. The inclusion adjustable VOSL and interest rates

The inclusion of an adjustable VOSL and an adjustable interest rate are grouped together as they have a similar effect on the SCBA. Note that the LIR is not influenced by a change in either of the two parameters and is therefore not discussed in this section.

Value of a statistical life

Referring back to equation 5.4, it can be seen that the VOSL is explicitly stated in the total costs used in the SCBA. As such it is relatively straightforward to modify the VOSL, given in the parameter V_{life}. The total costs consist of two other elements, the economic damage E and the value assigned to injury as a result of flooding V_{injury}. The former is assumed to remain constant while the latter is assumed to vary with the same relative magnitude as the VOSL does. As such the new total costs can be expressed as:

$$C_{new} = n \cdot (k \cdot V_{injury} + m \cdot V_{life}) + E$$

Where the n denotes the multiple of the standard value of the VOSL that is used in the analysis.

Variable interest rate

To include a variable interest rate, the starting point is equation 5.2. When the interest rate is changed, the reduction in risk in future years $t > 0$ changes as the fraction $\frac{1}{(1+r)}$ becomes has an inverse relation with the interest rate r. To generalize the found expression, it is preferable to express it in terms of the original probability and the additional interest rate.

The desired new optimal flooding probability p_1 can be found by equation 5.5. The full derivation of the expression can be found in section B.2 in Appendix B.

$$p_1 = p_0 \cdot \frac{(1 - (1 + r_0)^{-n} \cdot (r_0 + r_{add})}{(1 - (1 + r_0 + r_{add})^{-n}) \cdot r_0} \tag{5.5}$$

Where:
p_0: The original optimal flooding probability before modifications in the interest rate
r_0: The interest rate that was used in the original derivation
r_{add}: The added part of the interest rate that leads to a new optimal safety standard
n: The number of years that should be discounted

This expression can be seen as a multiplication factor, which can be either smaller or greater than one dependent on the added part of the interest rate.

5.2.2. The inclusion of Risk Preferences

The inclusion of risk preferences requires two steps. The first is a logical derivation of how and where the risk preferences should be applied in both the LIR and the SCBA. The second is the mathematical derivation of the inclusion of the risk preferences. These two steps will be discussed for both the LIR and the SCBA in this section.

Local Individual Risk
The LIR is defined as the annual probability for a hypothetical person present at a specific location to die as a result of a flood, taking into account the possibility of preventive evacuation (de Bruijn, 2009). The LIR is therefore designed to guarantee everyone a 'safe' level of protection from flooding. The choice for 10^{-5} is partly based on economic considerations and party based on subjective. Although the exact origins are unclear, Cornwell and Meyer (1969) argued that individuals perceive involuntary deadly events with a probability higher than 10^{-5} as unacceptable, while events that have a lower probability are generally accepted. As discussed in the literature study, the interpretation of probabilities is often subjective. It is therefore not a far stretch to assume that individuals subjectively weigh the safety standard as dictated by the LIR. The dictated demand that is in the order of 10^{-5} must therefore be subjective, not objective. This would imply that, based on the found risk averse preference, the required probability must be less than 10^{-5} because the LIR is a subjective probability that overweighed the true underlying objective probability. Hence it is expected that the required probability of flooding for an individual to 'feel' safe is lower than the one that is derived on the basis of expected value. After all, the degree of risk aversion differs per individual, and as such the weight that they attach to the probability of flooding.

If it is not the objective probability p_f on the righthandside of equation 5.1 that should equal to subjectively weighed LIR, it should be that the subjective weight given to the probability of flooding $w(p_f)$. Introducing this slight modification to equation 5.1 the following expression for the subjective weight of the LIR can be derived:

$$LIR = w(p_f) \cdot (1 - f_{evacuation}) \cdot (0.4 m_{worst\ case}(B) + 0.6 \sum_i P_{cond,i} m_i(B))$$

To regain the objective flooding probability requires a conversion through the inverse of the probability weighting function w^{-1}. Isolating p_f by means of the inverse function and rearranging terms results in an expression for a flooding probability governed by subjective probability weights as:

$$p_f = w^{-1}\left(\frac{LIR}{(1 - f_{evacuation}) \cdot (0.4 m_{worst\ case}(B) + 0.6 \sum_i P_{cond,i} m_i(B))}\right)$$

This probability of flooding can now be used to determine the class that corresponds to the given dike section and can subsequently be compared to p_{SCBA}.

Social Cost-Benefit Analysis
One can view the possible occurrence of a flood as a prospect in the traditional economic sense. If a flood occurs with a probability p and has expected incurred cost C when said flood occurs and zero costs when it does not occur, the prospect F can be written as $F = (p : C; 1 - p : 0)$. This prospect must have a certainty equivalent (CE): A value for which, when exceeded, an individual or society would be willing to accept the risk of the flood.

In a risk neutral approach, one would use expected value theory to see that the CE of this prospect must, by equation 2.1, be equal to:

$$u(CE) = w(p) \cdot u(C) + (1 - w(p)) \cdot u(0) = p \cdot C + (1 - p) \cdot 0 = p \cdot C = CE$$

This is the same result as was found in the current approach of the SCBA, as is to be expected.

However, as is demonstrated in this thesis, a risk neutral approach is not the only preference an individual or society can have towards prospects. Rather, one could have a risk averse or -seeking approach with varying degrees of seeking and evasive behaviour. If one does not assume a linear utility- and probability weighting function Expected Outcome theory, given in equation 2.1, ceases to be valid and the following equality must be used:

$$u(CE) = w(p) \cdot u(x) + (1 - w(p)) \cdot u(y)$$

Using this general form and substituting $x = C$ and $y = 0$, the certainty equivalent can be derived by taking the inverse of the utility function, such that:

$$CE = u^{-1}(w(p) \cdot u(C)) \tag{5.6}$$

This allows for the expression of a yearly expenditure that a society with a given risk preference, expressed in a suitable utility- and probability weighting function, would be willing to spent for flood protection that guarantees an annual exceedence probability of p if the expected damage of said flood is C.

Conversely if an annual budget of B is available, which is then assumed to be the CE value of society, the societal desired probability of flood occurrence must be equal to:

$$p = w^{-1}(\frac{u(B)}{u(C)}) \tag{5.7}$$

Note that the implicit assumption in equation 5.7 is that the utility of the annual budget $u(B)$ for flood protection does not exceed the utility of the expected cost in case of a flood $u(C)$, i.e. $\frac{u(B)}{u(C)} \leq 1$. This assumption seems justified from a logical point of view as the derived risk preferences were shown to be risk averse, making the utility function concave. Furthermore it is unlikely that B would exceed C from an economic perspective: If the annual budget exceeds the incurred cost (which has a return period greater than one year as $p \leq 1$) one would logically opt out of spending the annual budget.

Any functional form for $u(\cdot)$ and $w(\cdot)$ can be used in the derived expressions above, provided that they reflect the societal risk preferences.

5.3. Results

This section presents the numerical results of the case study. The discussion and conclusion of the results can be found in chapter 7. The results are divided into four sections, one for each of the variables and a final one that combines the different variables with best estimates into one combined effect.

For the interest rate a standard value of 2.25% was assumed, in line with the current interest rates used in infrastructure projects. For the risk preferences, the *power* model was chosen. The parameters that were used in the model were those derived in chapter 4, as the general and FLM related parameters were found to be significantly different, both are used in this evaluation. The Value of a Statistical Life (VOSL) has an initial value of €12.82 mln. and €12.06 mln. The first value corresponds to the VOSL found by the general risk preferences, the second as found by the FLM specific risk preferences, both found in chapter 4.

Table 5.2 gives a qualitative overview of the individual effect of the variation of the three variables on either the LIR or the SCBA. Where a + indicates that the safety standard has become more strict and a – indicates that the safety standard has become less strict for increasing values of the parameter, a 0 indicates that the variable has no effect on the safety standard.

Table 5.2: Qualitative assessment of the influence of the interest rate, VOSL and risk preference on the optimal flooding probabilities as derived by the LIR and SCBA

	Interest rate	VOSL	Risk preference			
			α	β	λ	γ
LIR	0	0	0	0	0	-
SCBA	-	+	0	+	0	-

Table 5.2 shows that, looking at the LIR, there is only one variable that influences the outcome. This is the curvature of the probability weighting function (γ), which for increasing values creates a less strict safety standard. For values between zero and one, increasing values of γ cause a decrease in the curvature of the probability weighting function, making it more risk neutral. When γ crosses one, the convex and concave parts of the probability weighting curve flip around, creating an underweighting of small probabilities and overweighting of the larger probabilities. The more an individual overweighs small probabilities, expressed in a greater curvature of their probability weighting function, the stricter the resulting optimal flooding probability according to the LIR criteria becomes.

Looking at the SCBA, nearly all variables have an influence on the strictness of the safety standard. Most notable is that the curvature of the probability weighting function has the same effect on the SCBA as it has on the LIR, decreasing the strictness for decreasing curvature of the probability weighting function, caused by increasing values of γ. In the utility function, it can be seen that the parameter that is responsible for the diminishing sensitivity to larger positive outcomes (α) and the parameter that is responsible for the weight that negative outcomes are given relative to positive outcomes (λ) do not influence the outcome. The parameter that is responsible for the diminishing sensitivity for absolute larger negative outcomes (β), has an increasing effect on the strictness when its value increases. For values between zero and one a larger value of β decreases the diminishing weight given to larger outcomes. If the value exceeds one, larger outcomes are given a relatively larger weight. Furthermore it can be seen that increasing interest rates decreases the strictness of the safety standard. This is to be expected as the future benefits of the flood protection are discounted more heavily, leaving less resources for the investment in protection at the current time. An increase in the value of a statistical life (VOSL) increases the strictness of the safety standard. This result is also expected as an increase in the VOSL yields a higher protected value by the flood protection and thus higher (future) benefits. With an increase in benefits comes an increase in needed flood protection and thus a stricter flood protection standard.

The rest of this section discusses the influence of each of the mentioned variables on the optimal flooding probability in more detail.

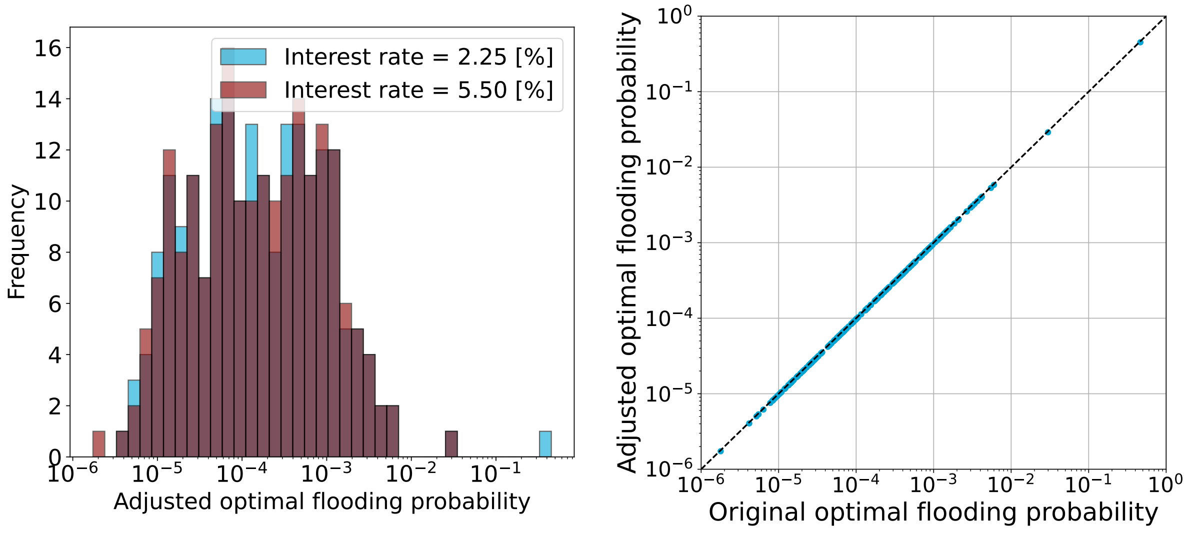
5.3.1. Influence of interest rate

Modifications in the interest rate only affect the outcome of the Social Cost-Benefit Analysis. As such, this section looks at the influence of the interest rate on the SCBA. The initial interest rate as used by Slootjes and Wagenaar (2016) was 5.5%, the most recent change in 2020 has reduced this rate to 2.25% (Ministerie van Financien, 2020).

Social Cost-Benefit Analysis

Figure 5.2 contains two figures that compare the outcome as a result of the modification of the interest rate. Similar types of figures are used in each of the three principles, as such more attention to what they contain will be given in this section.

Figure 5.2a shows a histogram of the results of both the original situation (an interest rate of 5.5%) and the adjusted situation (an interest rate of 2.25%). The x-axis shows the value of p on a logarithmic scale, the y-axis shows the frequency. This histogram is used to detect any shifts in the distribution of probabilities. Figure 5.2b shows the same data as figure 5.2a but with a different representation. The x-axis shows the optimal probabilities as derived in the expected outcome framework by Slootjes and Wagenaar (2016). The y-axis shows the optimal probabilities as found by adjusting the interest rate to the newly found figure of 2.25%. This figure shows a one-to-one mapping of the original values to the new values. This figure provides additional information on the transformation of the individual probabilities, i.e. whether all probabilities shift equally or there is a difference dependent on the magnitude.

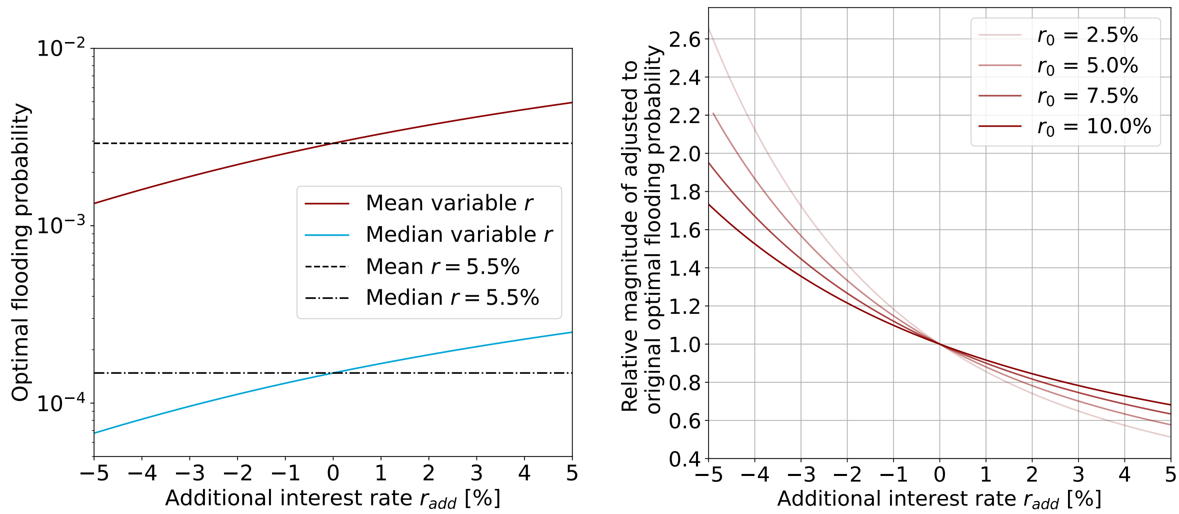


(a) Histogram of original and adjusted optimal flooding probabilities for the SCBA as a result of an adjustment in the interest rate

(b) Original optimal flooding probability plotted against the adjusted optimal flooding probability with an adjusted interest rate

Figure 5.2: Comparison between the original optimal flooding probabilities as derived in 2011 by means of the SCBA and the adjusted optimal flooding probabilities as a result of an adjustment in the interest rate

It can be seen from figure 5.2a that on a logarithmic scale there is no large shift in the distribution of the p-values as whole, indicating that the effect of the change in interest rate is limited. Looking further into the individual probabilities, figure 5.2b shows the same trend, the shift is near non existent. The change in optimal flooding probabilities p when the interest rate is changed from 5.5% to 2.25% is therefore limited. Next to the best estimates for the interest rate, the variability of the optimal flooding probability with respect to the interest rate is examined. Figure 5.3 shows the influence of different starting conditions and adjustments on the outcome of the optimal flooding probability. In figure 5.3a the additional part of the interest rate r_{add} is varied while the base-rate r_0 is kept constant at 5.5%. In figure 5.3b the base-rate of 5.5% is varied alongside the additional part of the interest rate.



(a) Mean and median optimal flooding probability as derived by the SCBA for varying risk premium additions to the interest rate with a base rate of r = 5.5%

(b) Mean relative magnitude of the optimal flooding probability as derived by the SCBA for varying risk premium additions to the interest rate for various base rates r

Figure 5.3: Sensitivity of the mean and median optimal flooding probability as derived by the SCBA to changes in the interest rate

Figure 5.3a shows the additional part of the interest rate on the x-axis and the resulting value of p on the y-axis. For reference, the mean and median outcomes of the original situation are shown in black dotted lines. It can be seen that the economic optimal flooding probability p as a result of the SCBA increases for increasing values of r_{add}, leading to a lower safety level. Note that if the additional interest rate goes below zero, the value of p becomes smaller than in the original situation, with the original situation and the modified situation having the same values for $r_{add} = 0$.

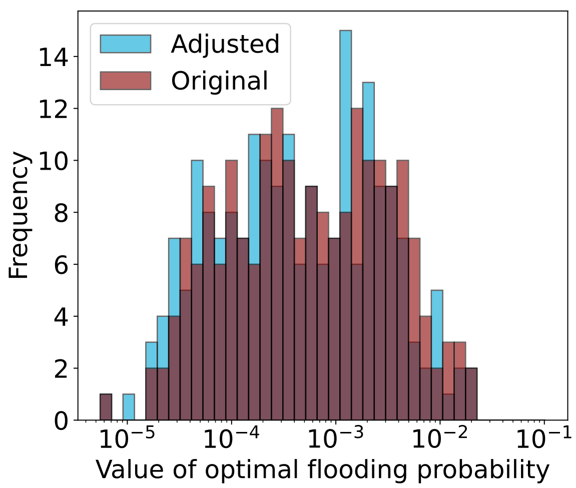
Figure 5.3b again shows the additional part of the interest rate on the x-axis and the resulting values of p on the y-axis. It can be seen that for increasing values of the base rate r_0, the effects of the additional part r_{add} are reduced. Indicating that an increase in the interest rate must be viewed in relation to the base rate to gauge its absolute effect. As expected, this relation between the base rate and additional interest rate can be traced back to equation 5.5.

5.3.2. Influence of the value of a statistical life

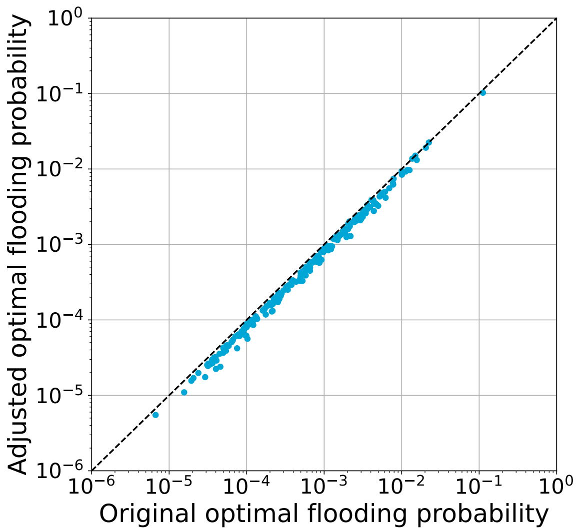
This section discusses the influence of the value of a statistical life on the resulting flooding probabilities. Note that the VOSL does not influence the LIR and as such only the SCBA is discusses in this section.

Social Cost-Benefit Analysis

Figure 5.4 has a similar lay-out as figure 5.2, consisting of both a histogram and a one-to-one mapping of the original and reevaluated flooding probabilities. The best estimate for the VOSL presented in figure 5.4 is the value found with the general risk preference, €12.82 mln. The results for the FLM risk preferences are shown in figure E.19 in Appendix E, showcasing similar results as the one in the main text.



(a) Histogram of original and adjusted optimal flooding probabilities for the SCBA as a result of an adjustment in the VOSL



(b) Original optimal flooding probability plotted against the adjusted optimal flooding probability with an adjusted in the VOSL

Figure 5.4: Comparison between the original optimal flooding probabilities as derived in 2011 by means of the SCBA and the adjusted optimal flooding probabilities as a result of an adjustment in the value of a statistical life

It can be seen that in figure 5.4a that the distribution of optimal flooding probabilities is relatively similar between the original and the adjusted p-values, with some minor adjustments in frequency of several bins. Looking at figure 5.4b, it can be seen that there is only marginal deviation from the original flooding probabilities. Notice that the entire group of points lies closer to the neutral axis but each individual point lies more scattered across an imaginary shifted neutral axis. This indicates that while changing the VOSL to €12.82 mln. does not drastically alter the outcomes of the evaluation as a whole, it does alter the outcomes of several specific dike sections. This can be a reflection of the deviation in number of casualties between the considered dike sections.

To investigate the sensitivity of the optimal probability of flooding to the absolute value of the VOSL, the relative magnitude is varied from 0.5 to 10. The results are shown in figure 5.5.

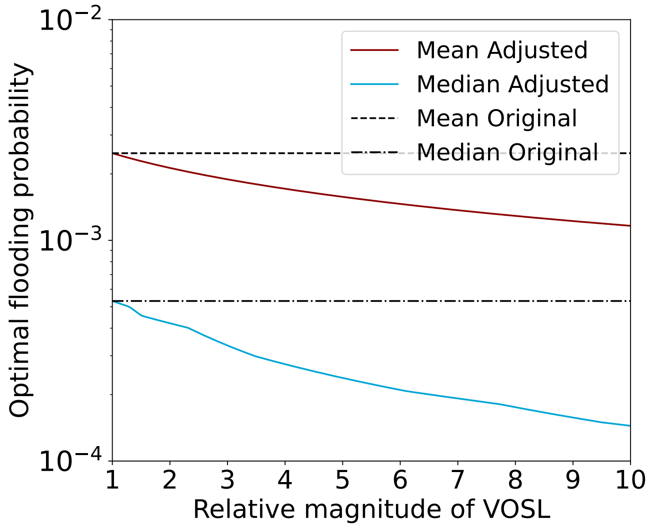


Figure 5.5: Mean and median of the adjusted optimal flooding probability as derived by the SCBA (p_{SCBA}) for varying relative magnitudes of the value of a statistical life of €6.7 mln.

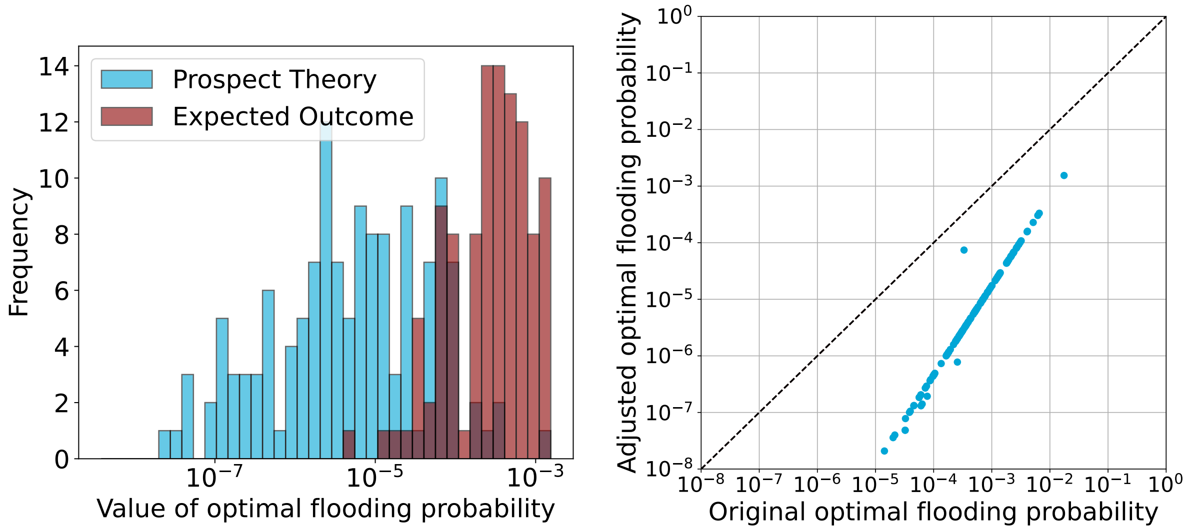
Figure 5.5 shows how the mean and median p-values of the evaluation changes with varying values for the VOSL. The x-axis shows the relative magnitude of the adjusted VOSL compared to the original VOSL. Meaning that if the original value of the VOSL was €6.7 mln. a relative magnitude of 2 corresponds to a new VOSL of €13.4 mln. It can be seen that for increasing magnitudes, mean and median p-values tend to decrease. From a theoretical perspective these results are expected, as a higher VOSL corresponds to a higher total loss expressed in €. If the risk were to kept constant, this results in the observed decrease in flooding probability. The decrease in flooding probability appear to be diminishing for larger values of the VOSL.

5.3.3. Influence of risk preferences

This section discusses the influence of including risk preferences into the analysis of the required flooding probability. To this end the *Power* model for the utility weighting function and the *Power* model for the probability weighting function are used. The combination of these two is denoted as *Prospect Theory* in this section. The parameters of the models are the mean estimates that were found in chapter 4, divided among FLM and general choice problems. As the introduction of risk preferences effects both the SCBA and the LIR, both will be discussed seperately in this section.

Local Individual Risk

The LIR is influenced by the probability weighting function $w()$, but not by the utility function. Figure 5.6 shows results of the reevaluation of the optimal flooding probabilities according for the LIR p_{LIR}, when the probabilities are weighted with a probability weighting function with parameter $\gamma = 0.631$. A similar figure for the FLM elicited parameter was made, using $\gamma = 0.698$. The results of which are shown in figure E.20 in Appendix E.



(a) Histogram of the original and adjusted optimal flooding probabilities for the LIR as a result of an adjustment in the risk preferences

(b) Original optimal flooding probability plotted against the adjusted optimal flooding probability as dictated by the LIR with an adjustment in risk preferences

Figure 5.6: Comparison between the original optimal flooding probabilities as derived in 2011 by means of the LIR and the adjusted optimal flooding probabilities as a result of an adjustment in the risk preferences

Figure 5.6a shows a histogram of the original and adjusted optimal flooding probabilities according to the LIR. The x-axis shows the value of p_{LIR} on a logarithmic scale, the y-axis shows the frequency. The first thing to notice is that the outcomes of Prospect Theory, on average, lead to a lower value for the optimal flooding probability p than the current approach. The second thing to notice is that the new distribution appears to have a higher variance with respect to the distribution of the current approach, translating into a more 'smeared out' distribution. This is confirmed when looking at figure 5.6b, which shows the one-to-one mapping of the optimal flooding probabilities. It can be seen that next to a general shift upward, the probabilities are also rotated on the logarithmic scale along the neutral axis. This indicates that relatively large probabilities are getting a disproportionately larger weight than relatively small probabilities. As the curvature of the probability weighting function (incorporated in the parameter γ) is the sole variable in the LIR analysis, it is of interest to see how variations in the value of γ affect the outcome. This variation is shown in figure 5.7.

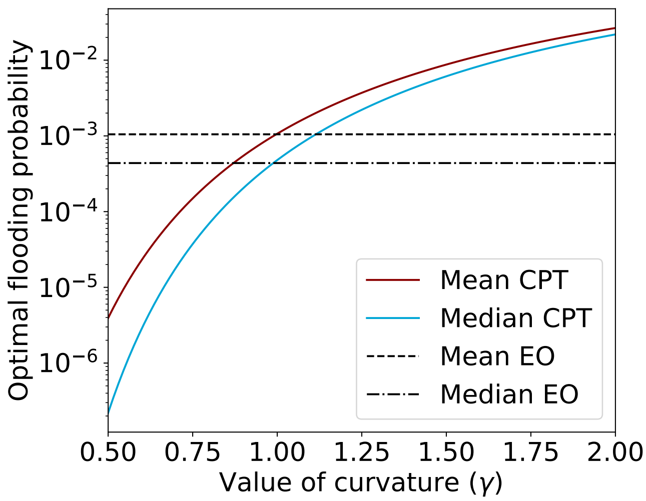
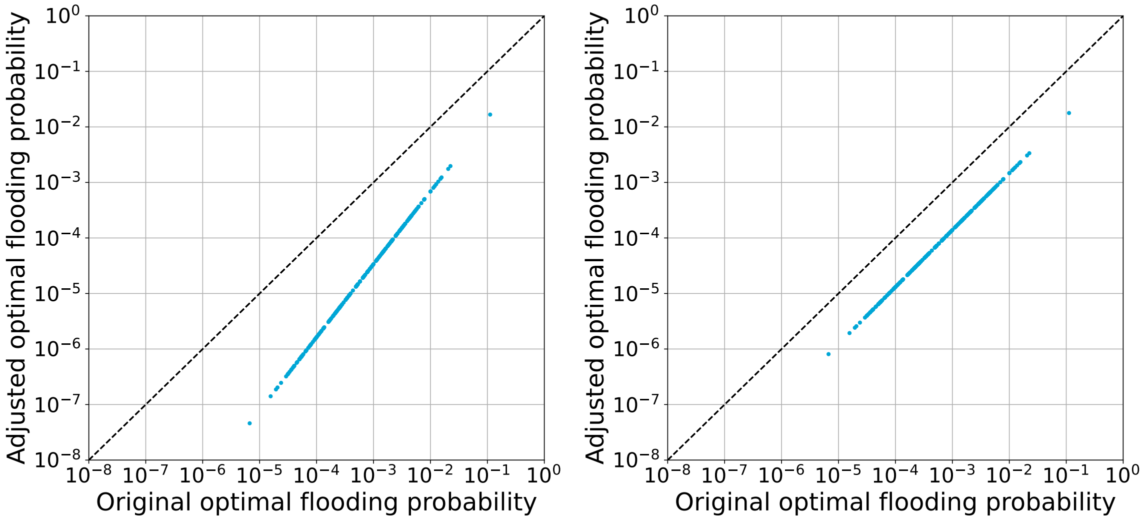


Figure 5.7: Mean and median of the adjusted optimal flooding probability as derived by the LIR (p_{LIR}) for varying values of the curvature of the probability weighting function (γ)

Figure 5.7 shows the effect on the mean and median estimate for p_{LIR} when the parameter dictating the curvature of the probability weighting function (γ) is varied. On the y-axis the value of the optimal flooding probability p is shown, on the x-axis the value of γ is shown. It can be seen that when γ the mean (median) of (Cumulative) Prospect Theory (CPT) coincides with the mean (median) of the Expected Outcome (EO), as is expected since a value of $\gamma = 1$ corresponds to a neutral probability weighting function and the estimate of the optimal flooding probability p is solely dependent on this function, not on the utility function, resulting in a risk neutral approach. Furthermore it can be seen that larger values of the curvature of the probability weighting function (γ) result in larger optimal flooding probabilities. These probabilities are decreasingly getting larger for increasing values of γ. Contrary, smaller values of γ lead to smaller optimal flooding probabilities.

Social Cost-Benefit Analysis
The SCBA is influenced by both the probability weighting function and the utility function. Using the estimates for the parameters α, β, λ and γ as derived in chapter 4, adjusted values for p_{SCBA} can be calculated. The results of this calculation are shown in figure 5.8.



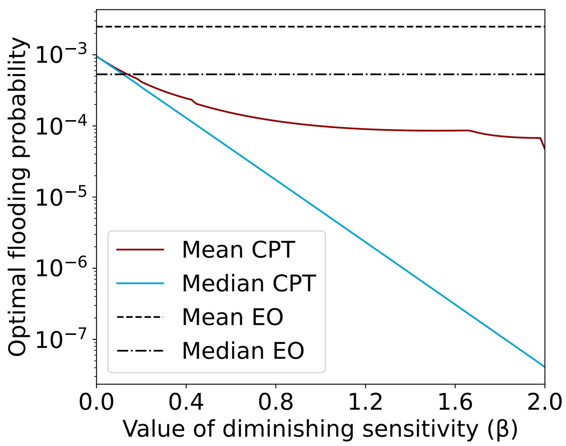
(a) Original optimal flooding probability plotted against the adjusted optimal flooding probability as dictated by the SCBA for the general elicited risk preferences

(b) Original optimal flooding probability plotted against the adjusted optimal flooding probability as dictated by the SCBA for the FLM elicited risk preferences

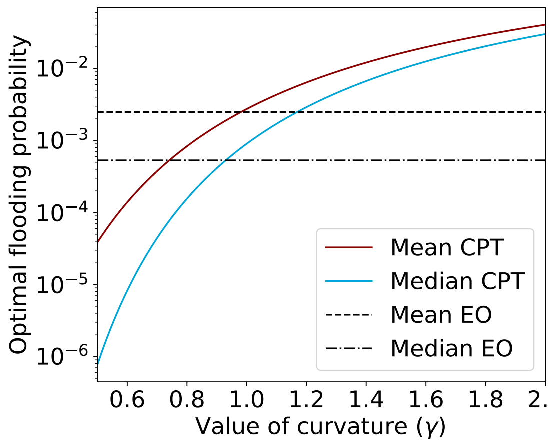
Figure 5.8: Comparison between the original optimal flooding probabilities as derived in 2011 by means of the SCBA and the adjusted optimal flooding probabilities as a result of an adjustment in the risk preferences

It can be seen in figure 5.8 that with both risk preferences, general and FLM, the adjusted probabilities are lower than the original probabilities. Interesting to note is that for the FLM risk preferences, shown in figure 5.8b, the probabilities appear to have shifted by an order of approximately 10, but stayed near parallel to the original probabilities. While if one looks at the results for the general risk preferences, shown in figure 5.8a, the probabilities have shifted by a different magnitude dependent on their original value. Higher probabilities have shifted relatively less than lower probabilities. This is a similar effect that was observed by the reevaluation of the LIR. A more detailed distribution of the probabilities is shown in the histogram in figure E.21 in Appendix E, displaying both the results of the FLM- and general risk preferences.

As the SCBA is influenced by both the utility- and probability weighting function, a total of four parameters can be varied. After evaluation only two of the parameters responsible for the diminishing sensitivity to negative outcomes (β) and the curvature of the probability weighting function (γ) appear to influence the optimal flooding probability. These two parameters are allowed to vary over a range of possible values while the other parameters α and λ are kept constant. The results of the performed analysis can be seen in figure 5.9.



(a) Mean and median optimal flooding probability as derived by the SCBA for varying values of the diminishing sensitivity to negative outcomes of the utility function (β)



(b) Mean and median optimal flooding probability as derived by the SCBA for varying values of the curvature of the probability weighting function (γ)

Figure 5.9: Sensitivity of the mean and median optimal flooding probability as dervied by the SCBA to changes in the shape of the utility- and probability weighting function (expressed in the parameters β and γ)

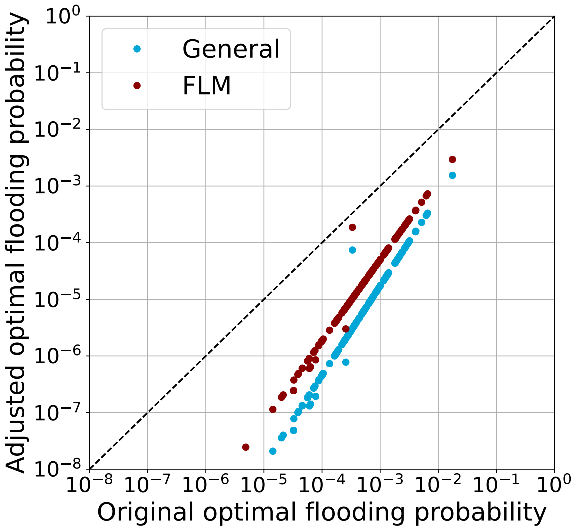
Figure 5.9a shows increasing values for β on the x-axis and the resulting flooding probabilities on the y-axis. It can be seen that for increasing values of β the median probabilities decreases rapidly. The mean probabilities appear to decrease at a much slower rate, indicating that the distribution of probabilities becomes rather skewed for larger values of β. Figure 5.9b shows increasing values for γ on the x-axis and the resulting flooding probabilities on the y-axis. It can be seen that for increasing values of γ the optimal flooding probabilities actually increase, in contrast to the result found in the analysis of the LIR. Larger values of γ have diminishing increases in the optimal flooding probability. Note that for both figure 5.9a and 5.9b the results are conditional on the general estimates of the other parameters.

5.3.4. Influence of aggregate effect

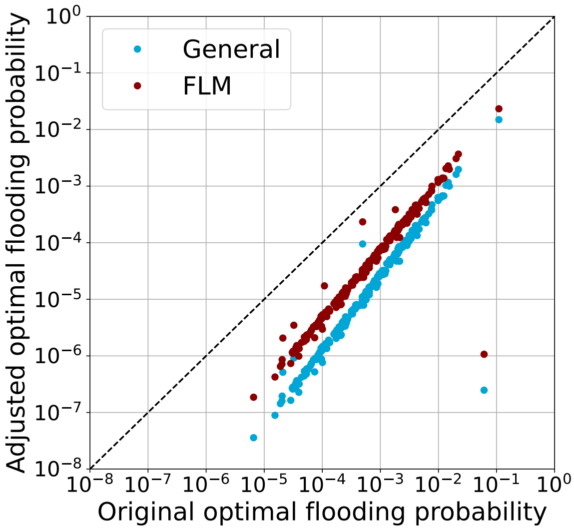
The aggregate effect consists of a best estimate for all three variables, combined in one evaluation. For clarity, the best estimates for the variables are repeated here. These values will be used to determine the influence of the aggregate effect for both the SCBA and the LIR.

Interest rate: The interest rate is set at the level determined in 2020, which is 2.25%.
Value Of a Statistical Life: The VOSL is set at €12.82 mln. when general risk preferences are used. The VOSL is set at €15.45 mln. (the average of 18.32 and 12.58) when FLM risk preferences are used. The value associated with injury is increased with the same relative magnitude. As such, it is increased to €23,918 for general- and €28,825 for FLM risk preferences.
Risk preferences: The power model for both the utility- and probability weighting function is used. The values for the general (FLM) parameters are: α = 0.694 (0.650), β = 0.834 (0.719), λ = 1.454 (1.438) and γ = 0.631 (0.698).

The aggregate effect on the LIR is identical to the sole effects of the risk preferences, as this is the only influential factor. As such, figures 5.6 remains valid as the analysis was performed with the best estimates for the risk preference parameters. The aggregate effect on the SCBA is more complicated as all three variables have an effect on the outcome. Furthermore, these effects can have opposite effects, resulting in an aggregate effect that is hard to predict. The results of the analysis for both the LIR and SCBA are shown in figure 5.10.



(a) Original optimal flooding probability plotted against the adjusted optimal flooding probability as dictated by the LIR for the best estimates of all three adjustments



(b) Original optimal flooding probability plotted against the adjusted optimal flooding probability as dictated by the SCBA for the best estimates of all three adjustments

Figure 5.10: Comparison between the original and adjusted optimal flooding probabilities as dictated by the LIR and the SCBA for both general- and FLM elicited risk preferences combined with the aggregate effect of all variables

Starting with the LIR, it can be seen in figure 5.10a that for both the general- and FLM risk preferences the combined effect of the VOSL, interest rate and risk preferences leads to stricter optimal flooding probability. These effects are disproportionally larger for smaller probabilities with respect to the larger probabilities, meaning that smaller probabilities get relatively smaller than the larger probabilities get smaller. The elicited optimal flooding probabilities elicited via general risk preferences are slightly smaller (i.e. stricter) than the probabilities elicited from the FLM risk preferences.
Moving to the SCBA, a very similar effect on the magnitude of the probabilities can be seen. The aggregate effect of the three variables on the SCBA results in a lowered optimal flooding probability. It can be seen in figure 5.10b that for both the general and FLM risk preferences the combined effect of the VOSL, interest rate and risk preferences on the SCBA leads to a stricter optimal flooding probability. With the General risk preferences, on average, being slightly more strict than the FLM risk preferences. Furthermore it can be seen that there is more variance among the reevaluated probabilities. This variance is most likely the result of an increase of the number of varying factors that play a role in the adjusted evaluation of the SCBA with respect to the evaluation of the LIR.

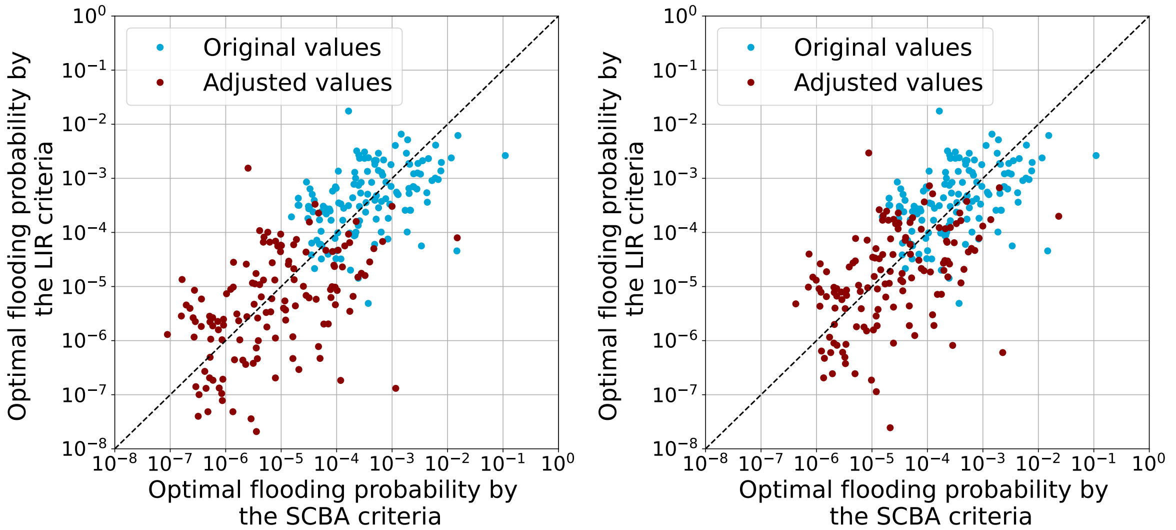
5.3.5. Changes in the leading principles

This section discusses which dike sections have their leading principles changed based on the best estimates given in section 5.3.4. As discussed in section 5.1.1, after an allowable flooding probability has been determined it is rounded to the nearest class. The class that either the LIR or SCBA (or group risk) falls into is subsequently compared to one another and the strictest is chosen. Table 5.3 shows an example of the dike sections that have a change in leading principle due to the proposed estimates for the three variables: the discount rate, the VOSL and the risk preferences. The full tables, for both the general- and FLM related risk preferences can be found in tables F.3 and F.4 in Appendix F.

Table 5.3: Example of changes in leading principles for the flood protection standards of dike sections

Dike section	Name	Old leading principle	New leading principle
1-1	Schiermonnikoog Duin	SCBA/LIR	LIR
3-1	Terschelling Duin	SCBA/LIR	LIR
...	...	...	...
41-3	Land van Maas en Waal - Maas	LIR	SCBA/LIR

It can be seen in this small example that the both the LIR and SCBA tend to switch to one another as a result of this reevaluation. To make a direct comparison between the LIR and SCBA, figure 5.11 was made. This figure shows every evaluated dike section with the found optimal LIR and SCBA flooding probability after the adjusted evaluation.



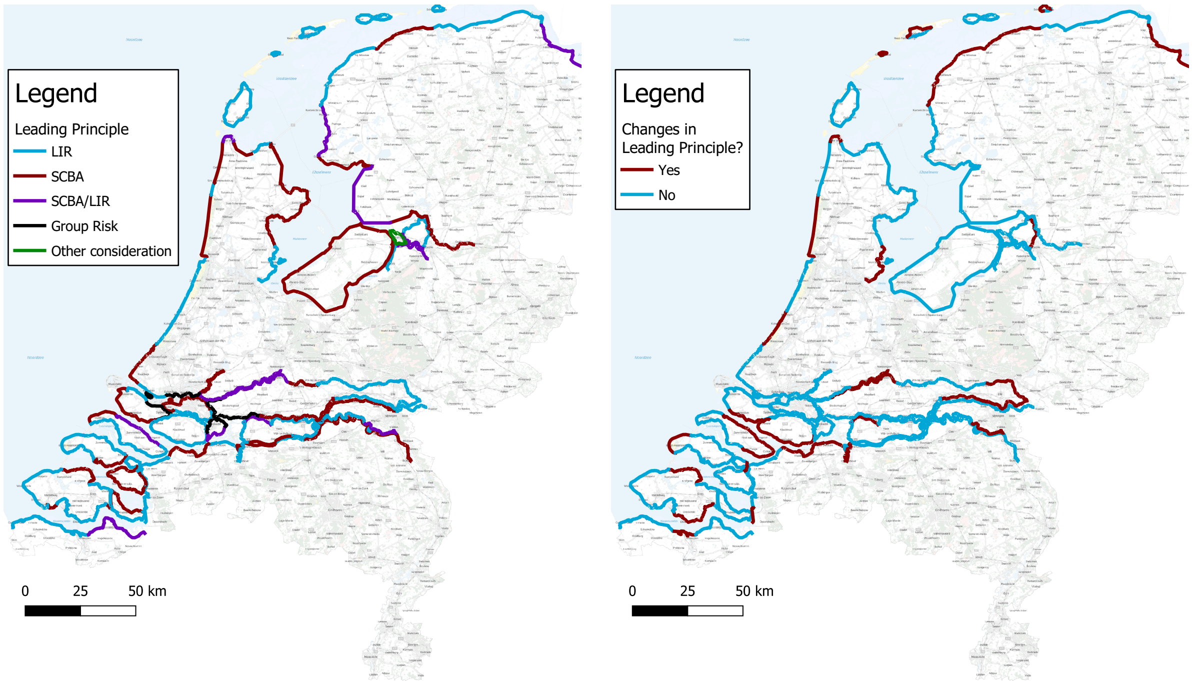
(a) Comparison between the original optimal flooding probabilities as derived in 2011 and the risk preference adjusted optimal flooding probabilities as derived in this research for both the LIR and SCBA, based on the risk preferences elicited by the general economic choice problems

(b) Comparison between the original optimal flooding probabilities as derived in 2011 and the risk preference adjusted optimal flooding probabilities as derived in this research for both the LIR and SCBA, based on the risk preferences elicited by the FLM related choice problems

Figure 5.11: Comparison between the original and adjusted optimal flooding probabilities as derived by the LIR and SCBA for both general- and FLM elicited risk preferences

Figure 5.11 shows a direct comparison between the optimal flooding probabilities as a result of the adjusted SCBA and as a result of the adjusted LIR. The x-axis shows the value of the optimal flooding probability after the adjusted SCBA analysis, the y-axis shows the value of the optimal flooding probability after the adjusted LIR analysis. Figure 5.11a shows the results for the general risk preferences, figure 5.11b shows the results for the FLM related risk preferences. As such, if a point lies above black dotted line, the SCBA presents a stricter criteria while if the point lies below the black dotted line the LIR presents a stricter criteria.

It can be seen that for both figures the division between the LIR and SCBA dominated dike sections remains largely the same. However, the optimal flooding probabilities are shifted by an order of 1 to 2 magnitudes to smaller values, creating stricter standards. The shift towards a stricter standard is slightly more pronounced for the general elicited risk preferences than for the FLM elicited risk preferences. After computing the shift, these probabilities are subsequently assigned to a class, after which it can be determined whether the leading principle has changed. The resulting changes in leading principles are shown in figure 5.12.



(a) Overview of the leading principle for the flood safety standards in the Netherlands placed on their respective dike sections after the inclusion of risk preferences

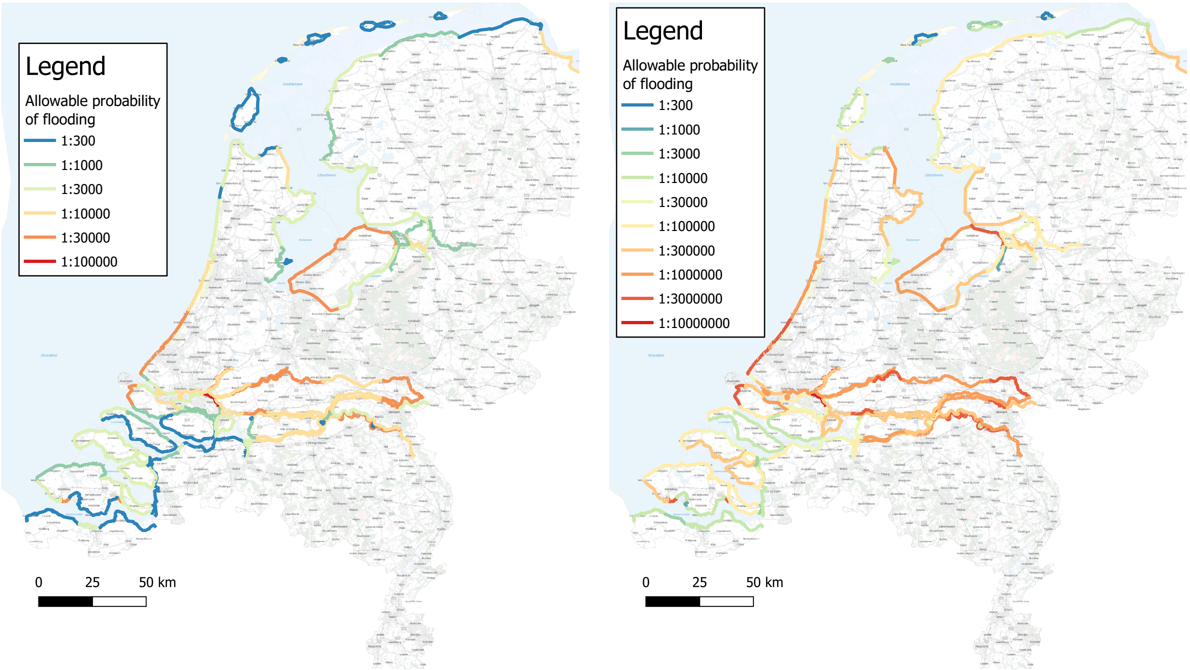
(b) Overview of the changes in leading principle for the flood safety standards in the Netherlands after the inclusion of risk preferences

Figure 5.12: Overview of the leading principle for the flood safety standards in the Netherlands based on general risk preferences

Figure 5.12a shows the leading principles for the different dike sections when the variables are adjusted and the general risk preferences are included. To better highlight the difference between this figure and the original leading principles as shown in figure 5.1, a map showing whether the dike sections have changed leading principle was created. This map is shown in figure 5.12b. It can be seen that there is not one single cause for a change in leading principle. Both sea dikes and river dikes are represented, as well as densely and less densely populated areas. The changes are also not limited or concentrated in one specific geographical area. The full list of dike sections that have changed their leading principles is given in Appendix F in table F.3 for the general risk preferences and table F.4 for the FLM risk preferences. Out of the 140 dike sections that were evaluated, around 80 had a change in leading principle. Before the application of risk preferences, 59, 41 and 40 dike sections were governed by the SCBA, LIR or a combination of the two respectively, excluding the non-evaluated sections, grouprisk hotspot sections and section with other considerations. After the risk preferences have been applied, these numbers changed to 51, 69 and 20. The main shift has been from a joint leading principle of the SCBA and LIR to a sole leading principle of the LIR. The LIR gained 28 dike sections while the SCBA lost 8. Note that the exact number of changes is highly dependent on the degree of risk aversion and as such on the parameters of the utility- and probability weighting function.

Changes in optimal safety standard

However, a shift in a leading principle does not tell the entire story. As became apparent from figure 5.11, not only do the leading principles switch they also shift towards smaller optimal flooding probabilities. This shift in probabilities resulted in a flood safety standard that was in the order of 10 to 100 more strict. Linking the shifted points in figure 5.11 to their respective dike sections and placing them on a map gives a clearer picture of where the largest increase in flood safety standards occur. These new safety standards are shown in figure 5.13.



(a) Overview of the optimal flooding probabilities for the flood safety standards in the Netherlands placed on their respective dike sections before the inclusion of risk preferences

(b) Overview of the optimal flooding probabilities for the flood safety standards in the Netherlands placed on their respective dike sections after the inclusion of risk preferences

Figure 5.13: Overview of the optimal flooding probabilities for the flood safety standards in the Netherlands based on a risk neutral approach and with the inclusion of general risk preferences

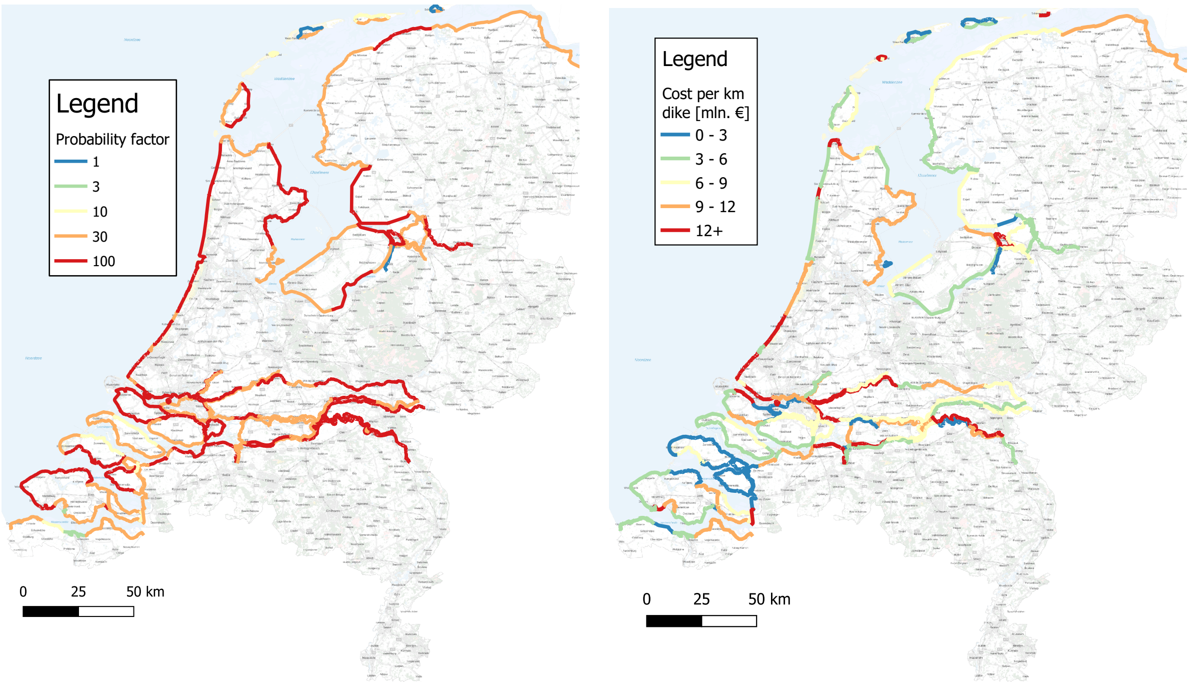
Figure 5.13a shows the optimal flooding probabilities before modifications by the risk preferences and adjustment of variables. This figure serves as a frame of reference for figure 5.13b, which contain the new optimal allowable probability of flooding after the inclusion of risk preferences and adjustment of variables. The first thing to note is that four new probability classes have been added, ranging from 1:300,000 to 1:10,000,000, reflecting the lowered optimal probabilities as found by the analysis presented in figure 5.11. The second thing to note is that, with a few exceptions, the probabilities all shift a factor 10 tot 100, also in line with the results found in figure5.11. Note that the biggest difference between the figures is that the probabilities in the map are classified whereas the probabilities in figure 5.11 are not.

The shift of probabilities is confirmed when looking at figure E.23 in Appendix E. These two histograms show the distribution of old and new optimal flooding probabilities, divided along the lines of whether the leading principle changes. It can be seen that the distribution of probabilities shifts with a factor 10 to 100 and the distribution of changes in leading principles remains relatively the same, although it is stretched out over more columns. The costs for increasing this safety standard also increase, again with proportionate shift as shown in figure E.24.

Associated costs with stricter safety standards

Figure 5.13 shows that a large portion of the dikes in the Netherlands will have a stricter safety standard under this method of evaluation. This stricter safety standard increases the costs that society has to bear for flood safety. Using the increased factors of safety together with the costs for a tenfold increase in safety standard from Slootjes and Wagenaar (2016), the costs of risk aversion per dike section and for the Netherlands can be calculated. To this end, the difference in optimal flooding probability between the original and the adjusted situation is first expressed in an increased safety factor.

Figure 5.14a shows the results for this safety factor. In this figure a factor of 10 means that the optimal flooding probability according to the new evaluation is 10 times more strict than the original optimal flooding probability. It can be seen that some dike sections remain at the same safety standard. More common is an increase in safety standard, with most of the dike sections seeing a multiplication of the safety standard between 30 and 100 times the original safety standard. The derived safety factor can be used in combination with the derived costs for a tenfold increase in safety level as derived by Slootjes and Wagenaar (2016). If the probability factor is 10, the costs for a tenfold increase in safety level is multiplied by 1. If the probability factor is 100, the costs are multiplied by 2, everything in between is scaled with a logarithm.



(a) Overview of the optimal flooding probabilities for the flood safety standards in the Netherlands placed on their respective dike sections before the inclusion of risk preferences

(b) Overview of the optimal flooding probabilities for the flood safety standards in the Netherlands placed on their respective dike sections after the inclusion of risk preferences

Figure 5.14: Overview of the optimal flooding probabilities for the flood safety standards in the Netherlands based on a risk neutral approach and with the inclusion of general risk preferences

The result is shown in figure 5.14b, which shows the costs per kilometer dike section to attain the desired flood safety standard when risk preferences are included. Prices above €12 mln. per kilometer are grouped together, below this threshold the costs are divided in €3 mln. intervals. Because the total costs are dependent on two factors, the costs per km dike section for a tenfold increase in safety standard *and* the increase in safety factor, it is hard to pinpoint the exact cause for the difference in costs per km dike in figure 5.14b. However, it can be seen that the in some of the highest costs are incurred by the sections that see the greatest increase in safety standard, i.e. have the largest probability factor.

To get a better idea of what drives the costs and changes in leading principles, several other correlations and causalities were examined. These include the examination of any relation between the leading principle or classification and the number of fatalities, economic damage, the evacuation fraction, original costs for increasing the safety standard and the costs associated with risk aversion per dike section. This was done by means of scatterplots as well as boxplots, given in figure E.25 through E.32 in Appendix E. Based on the performed analysis, the following conclusions can be drawn.

1. A higher safety standard generally corresponds to higher costs, although exceptions exist in densely populated areas.
2. The highest number of casualties and economic damage occur when the LIR and SCBA share the leading principle, both before and after the introduction of risk preferences.
3. The adjusted risk averse LIR is the leading principle associated with the highest costs per dike section, both on average as well as in the outliers.

Other observations that can be made from the figures are that: The evacuation fraction has a similar distribution among dike sections, both before and after changes in the leading principle. The SCBA sees the largest relative increase in dike sections that have a probability factor of 100, the LIR sees the relative largest increase in dike sections that have a probability factor of 30 or smaller.

The final step is to calculate the associated costs with risk aversion. This is done by multiplying the found costs per km dike section with the length of each section, resulting in a total costs per dike section. These total costs per dike section can then be summed to provide the total cost of of €19.3 bln. To put these costs in perspective, the Bureau of Economic Policy of the Netherlands estimates the GDP of the Netherlands to be €1,018 bln. at the end of 2023 (Planbureau, 2023), such that the proposed safety increase would be around 1.9% of the GDP. The Hoogwaterbeschermingsprogramma (HWBP), a Dutch dike reinforcement program, is active up to 2050. Assuming that these costs can be spread equally over the same period as the HWBP, ranging from 2020 to 2050, this would result in an annual costs of €642 mln. per year, or around 0.063% of GDP. An important note to make is that this analysis made the assumption that the increase in safety standard solely came from an increase in the strength of the dike sections. In reality there are different options for increasing the safety standard available such as enclosure dams or movable barriers, which could reduce the costs. Especially when considering that the cost per km dike section as presented in figure 5.14b are highest near the New Waterway Rotterdam and upstream of that. This safety standard of a large section of this waterway can be increased by increasing the safety standard of the Maeslant barrier, reducing costs.

Concluding this chapter it became apparent that the outcomes of the case study suggest that when the three mentioned variables are changed, the guiding principles for flood safety standards shift slightly in favour of the LIR. The most noteable change is the decrease of the optimal flooding probabilities by an order of approximately 10 to 100 for nearly all dike sections. Expressing this decrease in optimal flooding probabilities to safety standards, the costs are around €642 mln. per year. With a population slightly above 17.5 million people, the cost of risk aversion is approximately €37 per person per year, or about 19 utils.

6

Discussion

6.1. Evaluation

The evaluation of the results is divided into two parts: one for the outcomes of the survey and another for the subsequent use of the obtained results in a case study. The former focuses on comparing the utility and probability weighting functions found in this research to commonly observed values in (economic) choice problems. Additionally, a qualitative comparison will be provided between the elicited parameters for general risk preferences and risk preferences specific to Flood Risk Management (FLM). The latter part evaluates the changes in the leading principles of flood safety standards in the Netherlands based on the findings and examines whether they align with the derived theory.

6.1.1. Results of the survey

To the best of the author's knowledge, risk preferences in Flood Risk Management have not been directly elicited prior to this research. Therefore, a direct comparison with previously obtained results in Flood Risk Management is not feasible. However, extensive research has been conducted on general risk preferences of individuals, and the estimates from that body of research can serve as a proxy for the validity of the results obtained in this research. The estimates derived in this study for the parameters of the general utility function (FLM in parentheses), based on the the *power* model, are approximately 0.69 (0.65), 0.83 (0.72) and 1.45 (1.44) for α, β and γ respectively. The probability weighting function was found to have a parameter γ of 0.631 (0.698). These found results indicate a general tendency for diminishing sensitivity of larger outcomes, as both the parameters α and β are smaller than one. Positive outcomes exhibit a stronger diminishing sensitivity than negative outcomes, as the exponential term α is smaller than β. Furthermore negative outcomes are, on average, seen as 1.45 times as 'bad' as positive outcomes with the same magnitude, as reflected in the parameter λ. Subjectively, probabilities up to approximately 0.30 are overestimated, while probabilities greater than that are underestimated, as indicated by the curvature ($\gamma < 1$) of the probability weighting function.

In their original research, Tversky and Kahneman (1992) found values of 0.88 for α and β and 2.25 for λ. Tversky and Kahneman used a probability weighting function with the same functional form as the one presented in this research but with different parameters for losses and gains. The estimates for these parameters γ^+, γ^- were 0.61 and 0.69 respectively. Due to this differentiation in the parameters for the probability weighting function a direct one-to-one comparison between the values found by Tversky and Kahneman and this research is not possible as part of the risk aversion that is contained in the probability weighting function in their study is absorbed into the parameters β in this research. Nonetheless, the approximate order still gives an indication of the behaviour.

A more comparable and extensive overview of parameter estimates for CPT around the world is presented in Rieger et al. (2011). They employed the same functional form of the utility and probability weighting function as presented in this research, allowing for a more direct comparison of values, although the prospects of the choice problems differed. For reasons that will be explained in section 6.3, a comparison of parameters will be made with countries that are demographically and culturally similar to the Netherlands, which for the purposes of this research are identified as Denmark, Germany and the UK. This leads to ranges for α, β and γ of 0.45 to 0.50, 0.90 to 1.00 and 0.50 to 0.65, respectively. This study did not use mixed lotteries (e.g. lotteries with mixed positive and negative outcomes), such that there was no way of eliciting λ. Rather they reported θ, the ratio between a gain A and a loss B such that a fifty-fifty lottery between A and B is as attractive as an outcome of zero. This value ranged between 1.38 and 2.00, compared to the relative risk aversion of around 1.5 found in this study, suggesting alignment between the results.

In conclusion, the parameter estimates in this study reflect the general risk preferences observed in other studies. While the exact values of the parameters may differ across studies due to factors such as sample size, presented prospects, and heterogeneity in individual risk attitudes, the overall trend of an S-shaped probability weighting function and risk aversion in the utility function remains consistent. Small probabilities are generally overestimated, while large probabilities are underestimated. Relatively large outcomes are perceived as disproportionately less severe compared to relatively smaller outcomes. Therefore, it can be concluded that the identified risk preferences likely reflect commonly observed societal preferences.

6.1.2. Results of the case study

The incorporation of risk preferences into the Social Cost-Benefit Analysis (SCBA) and the Local Individual Risk (LIR) has, to the author's knowledge, not been implemented in the field of flood risk management. A direct comparison of the results found in this research with results found in previous literature is therefore difficult. However, the results can also be evaluated without a reference point. Looking at table F.3 and F.4 in Appendix F, 38 (39) out of the 140 checked dike sections have changed leading principles for the general (or FLM) elicited risk preferences. It should be noted that if the LIR and SCBA were initially jointly considered the leading principle and then shifted to either of the two, it is counted as a change in the leading principle. Such a shift occurred 23 times for LIR and 4 times for SCBA, resulting in approximately 11 (or 12 for FLM) dike sections transitioning from LIR to SCBA or vice versa.

Incorporating risk aversion into both the utility- and probability weighting functions leads to a significant decrease in the optimal flooding probability by one to two orders of magnitude. Dike sections that previously had an optimal exceedance probability of once every thousand years now have an optimal exceedance probability of once every ten to a hundred thousand years when risk aversion is considered. This reduction in optimal probabilities aligns with the findings regarding the estimation of small probabilities, revealing that individuals tend to perceive smaller probabilities as larger than their true values by approximately one to two orders of magnitude.

The final aspect of the case study focused on the economic analysis of the results. The costs for increasing the flood safety standards amounted to a total of around €19 bln. up to 2050, equivalent to an annual cost of around €642 mln. When comparing these costs to the current expenditure for the Delta program of €1.25 bln. per year (Government of the Netherlands, 2023), it can be seen that the incurred costs for risk aversion is about half of the costs for the current program. The HWBP (*High water protection program*), a program aimed at fortifying flood defences in the Netherlands, has an annual budget of around €400 mln. (Hoogwaterbeschermingsprogramma, 2019). The cost of risk aversion would therefore be around 1.5 times the current expenditure for this program. To decide if the higher costs are justified, it is important to explore alternative approaches that can improve safety standards, even outside the realm of flood risk. By considering different options and identifying the most efficient use of resources to enhance safety, a comprehensive evaluation can be conducted.

6.2. Implications

An important outcome of this research is the addition of a general framework that takes individual or societal preferences into account when performing a (social) cost-benefit analysis. By incorporating risk preferences, specifically in the form of utility- and probability weighting, into the existing analysis it becomes easier and more accurate to quantify risk-averse or risk-seeking behavior. Additionally, this framework enables the inclusion of externalities that were previously challenging to monetize. Traditionally, assessing the viability of a project faced difficulties in quantifying externalities that couldn't be easily expressed in monetary terms, such as the loss of biodiversity or the creation of a recreational lake near a city. This research proposes a solution by converting monetary outcomes into utility values, allowing for standard operations to be performed. Subsequently, the utilities can be converted back into monetary amounts, facilitating a comparison of externalities in the same units.

For instance, consider the noise disturbance experienced by residents due to nearby construction or the negative impact on biodiversity caused by a dam. Previously, these externalities were often considered as additional factors after financial or cost-benefit analyses had been completed. In the framework proposed by this research, the utility value of these externalities is determined through choice lists and can be directly integrated into the cost-benefit analysis. By conducting mathematical operations while values remain in utilities, the final result can be easily transformed back into monetary amounts using the inverse of the utility function, denoted as $u^{-1}(\cdot)$. This approach facilitates a more quantified inclusion of previously challenging-to-express externalities.

In addition to incorporating risk preferences into a general framework, his research also highlights the subjective weight individuals assign to small probabilities. It reveals that below a certain threshold, further lowering probabilities does not significantly impact individuals' subjective perception of those probabilities. In other words, individuals are relatively insensitive to differences between very small probabilities, such as the distinction between probabilities on the order of 10^{-4} and 10^{-5}. This finding carries significant implications for the current governance of the (LIR). Considering the subjective sense of safety, should the safety standard be as low as 10^{-5} if individuals do not perceive a substantial difference between a safety standard of 10^{-5} and 10^{-4}, or even 10^{-3}? This question gains particular relevance in light of the limited resources available for reinforcement projects, like the High Water Protection Program (HWBP).

6.3. Limitations

This section addresses the three key limitations of this research. It begins by discussing the challenge of converting individual utility functions to societal utility functions. Next, the spatial heterogeneity of risk preferences, which refers to changes in risk preferences across countries and regions, is examined. Finally, the temporal heterogeneity of risk preferences, which refers to the change in risk preferences over long periods, is explored.

6.3.1. Relating individual behaviour to group behaviour

This research has, for a large part, concerned itself with the elicitation of risk preferences of individuals in both the general economic sense as well as in the flood risk management sense. The application of the found risk preferences have to be put in a societal context rather than an individual context. To circumvent this problem, the incurred costs and benefits were assumed to be evenly distributed among the population. However, there are limitations to this assumption, which are discussed in this section.

The sum of individual utility evaluations based on a proportionate share of total expenditure on flood risk management does not necessarily equal the utility evaluation of the total expenditure. This is because an important aspect of a risk averse (or seeking) utility curve is that it is concave (or convex). When the input of a concave (or convex) function is multiplied by some constant c, the output is not multiplied by the same constant, i.e. $u(c \cdot x) \neq c \cdot u(x)$. In any case other than risk neutrality, dividing outcomes among individuals leads to a different cumulative result compared to when the outcome is assessed using a single utility function.

This has implications for weighing the costs and benefits of flood risk-related measures when dividing them among society.. To illustrate, suppose there are only two individuals in a society which have to divide an outcome $c \cdot x$ among them, where each individual takes a fraction c_1 and c_2 of x such that $c = c_1 + c_2$. If both individuals have risk-averse utility curves, the division of outcomes will result in a higher cumulative evaluation compared to $u_1 + u_2 > u$. Conversely, if the utility function is convex (risk-seeking), the inequality would be reversed. In general, if a utility function deviates from risk neutrality, the sum of evaluations of the two partitions will not be equal to the evaluation of the sum of the two partitions, i.e. $u(c_1 \cdot x) + u(c_2 \cdot x) > u(c \cdot x)$.

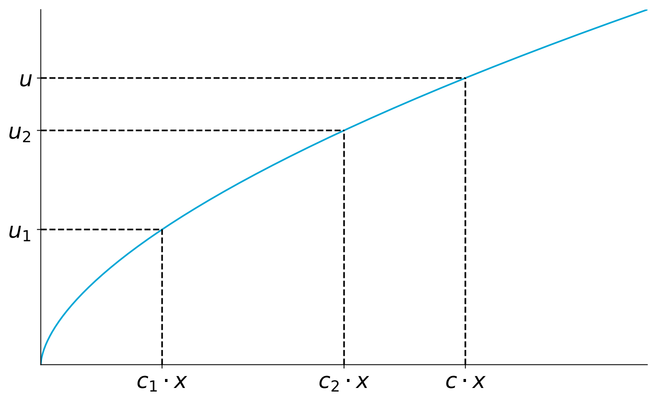


Figure 6.1: Illustration of the concave property of the utility function that results in a larger cumulative utility if the outcomes are split into multiple parts

This principle is demonstrated in figure 6.1, where it becomes evident that $u(c_1 \cdot x) + u(c_2 \cdot x) > u(c \cdot x)$.

Therefore, if each person weighs their individual share of the costs, the sum of all utilities will be higher than if the total costs are evaluated using a single utility function and subsequently divided among residents. The choice between these two methods depends on the specific context of the decision problem. However, it is of importance to recognize the difference in outcomes when summing individual utility functions versus using a single utility function in a societal context.

6.3.2. Spatial heterogeneity of risk preferences

The survey conducted for this research was focused on participants from the Netherlands, resulting in a nearly 100% Dutch response rate. However as shown by Weber and Hsee (1998), risk preferences are strongly influenced by culture and demographics. The Netherlands is country that, on average, has a relatively risk seeking preference compared to other countries. This limits the applicability of the found results, especially when areas are demographically and culturally different from the Netherlands. Therefore, it is recommended that if part of this research were to be replicated or used outside of (western) Europe, the risk preferences are adjusted or elicited again. The methodology can still be applied, but the specific values may need adjustment.

6.3.3. Temporal heterogeneity of risk preferences

The previous section discussed the social and cultural differences between individuals as a possible cause for difference in risk preferences. There is another cause that is influential in the risk preferences and especially in the degree of risk averse behaviour. As individuals age their risk preferences tend to change, although the specific patterns can vary among individuals. Research suggests that risk preferences tend to decrease with age, meaning that older individuals tend to become more risk-averse compared to younger individuals. This observation is supported by a study among German participants performed by Schurer (2015) concludes that ”tolerance to risk drops by 0.5 standard deviations across all socioeconomic groups from late adolescence up to age 45”.

Considering the distribution of participant ages shown in figure E.3, approximately half of the participants are below 25 years old. The other half is approximately split 2 to 1 between 25 to 50 and aged 50 or above. Looking at the age distribution in the Netherlands it becomes evident that the Dutch population has a proportionally larger share of people aged between 25 and 50 as well as people aged above 50 (Centraal Bureau voor de Statistiek, 2022). Therefore, the average Dutch citizen is expected to be older than the participants in this research. Combining this with the understanding that individuals tend to become more risk-averse as they age, it can be inferred that the identified risk preferences likely underestimate the degree of risk aversion in the general population.

7

Conclusion & Recommendation

7.1. Conclusion

Individuals constantly have to make decision under risk, the practice of Hydraulic Engineering and Flood Risk Management in particular is no exception. To model the decision making behaviour of individuals under risk, several models and theories have been proposed in the past. These models incorporate risk preferences as a means of quantifying the decision making process under risk and uncertainty. The majority of the published literature is based in behavioural economics, with few articles relating flood risk management with behavioural economics. This research aims to address this gap by examining the combination of flood risk management and behavioural economics. The main question guiding this research is whether risk preferences can be included in flood risk management and, if so, what effect they would have on current flood safety standards.

To answer this question, three sub-questions were posed. The first question focused on whether current theories on risk preferences and behavioural economics were applicable in flood risk management. To address this, the following question was formulated:

What models best describes individual risk preferences with regards to flood risk related decision problems?

Based on the conducted survey, it can be concluded that the risk preferences of individuals faced with flood risk related decision problems can be described by existing models in behavioural economics. The chosen model form consisted of a utility function combined with a probability weighting function. Several functional forms were tried for the utility function, where the *power function* as used by Tversky and Kahneman (1992) was found to be the model with the best explaining power and flexibility. The probability weighting function also consisted of a power function, similar to the one used by Tversky and Kahneman. The results of this model were consistent when applied to both traditional economic choice problems and flood risk management-related choice problems, indicating that individuals employ similar judgment systems in both contexts. This finding is significant as it enables the application of behavioural economic theory in flood risk management.

This finding is used to the derive societal risk preferences with regards to flood risk management. The elicited risk preferences were grouped and subsequently applied to several key variables used in flood risk management. The following three variables were elicited or recalibrated:

- *The value of a statistical life*
- *The subjective weight given to small probabilities*
- *The risk premium part of the interest rate*

Starting with the value of a statistical life (VOSL), this quantity was elicited via two sets of choice problems. The first set consisted of traffic related choice problems similar to the ones posed by de Blaeij (2003a). The second set consisted of flood risk related choice problems similar to the ones posed by Bockarjova et al. (2009). The latter set included both positive and negative choice lists for elicitation, wherein participants either received money or had to pay additional taxes. As a result, three estimates for the VOSL were obtained. Giving equal weight to each of the three answers, the average estimate for the VOSL in this research is €11.8 million. This is a factor 1.7 to 1.8 higher than the values found by de Blaeij and Bockarjova et al. The inflation adjusted values differ a factor 1.18 and 1.33, respectively. Therefore, it can be concluded that the found values in this research are in line with the previously found values, being around 20 to 30% higher. It should be noted that the elicitation was highly dependent on the estimation of the curvature of the probability weighting function, with values increasing sharply when greater curvature was introduced. This higher value has implications for the optimal flooding probability determined by the SCBA. If the value protected by a dike section is increased while maintaining a constant allowable risk, the probability of flooding must decrease. Consequently, the implication of this higher valuation is a decrease in the optimal flooding probability according to the SCBA.

The subjective weight given to small probabilities was elicited using the previously determined VOSL. This research revealed that regardless of the method or utility model employed, objective probabilities ranging from 10^{-5} to 10^{-2} are systematically misjudged. The degree to which probabilities are misjudged depends on the chosen method of elicitation and/or chosen parameters, but in general they tend to be overestimated. For probabilities below 10^{-3} the difference between the elicited subjective probabilities and the subjective probabilities by the probability weighting function approaches zero. This finding has implications for the use of small probabilities in subjective situations, such as the minimum safety standard stipulated by the LIR. If individuals have difficulty differentiating between a probability that is of the order 10^{-5} or 10^{-4}, the question can be asked to the governing institutions whether such stringent requirements as dictated by the LIR should be governing. Particularly if the SCBA dictates a less stringent, but economically more optimal requirement. Given the scarcity of resources, investments should be directed where they yield the greatest benefit (e.g., utility) for society as a whole.

The final variable examined was the risk premium component in the interest rate. Researched in the context of a case study, it was observed that an increase in the risk premium led to an increase in the optimal flooding probability calculated by the SCBA. This implies that if the interest rate is raised, the safety standards decrease. While this finding aligns with expectations, it contradicts the desired effect in flood risk management. Since the majority of the investments is traditionally made at present day, the interest rate mainly discounts the benefits gained by increasing the safety standard. Although the interest rate also discounts future reinforcement costs, these costs are often significantly smaller than the initial investment cost. The discounting of benefits in turn reduces the optimal safety standard as calculated by the SCBA, leading to a higher probability of flooding. In the context of this research, this can be interpreted as a preference for risk-seeking behavior. Remember that the investment costs can be seen as the certainty equivalent of a prospect that has a probability p of flooding and $1 - p$ of not flooding with the outcomes being the total damage as a result of a flood and zero. When the probability of flooding increases, the expected outcome also increases in magnitude, typically becoming more negative. Coupled with a decrease in investment costs (the certainty equivalent), this leads to a decrease in the risk premium and a preference for more risk-seeking behavior. This contrasts with the risk-averse preferences associated with the risk premium component of the interest rate, which is added to account for the inherent uncertainty in investments. These preferences appear to contradict one another. It is therefore suggested to reevaluate the discounting of flood defense benefits to align preferences. One possible mitigation strategy to achieve alignment is to discount costs and benefits in a different manner. For example, valuing each year without (expected) flood damages progressively could counteract the undesired effects of current discounting methods without completely disregarding the concept.

These three findings, together with the elicited risk preferences, were directly used in the answer to the second sub-question. This question concerned itself with the application of all previously derived results on the flood safety standards in the Netherlands. To this end, the following question was formulated:

How do societal risk preferences alter the current safety standards in the Netherlands with regards to the allowable probability of flooding, the current leading principle, and costs of reinforcement?

Based on the transformations applied to the calculation of the SCBA and the LIR, which includes the three variables as well as risk preferences, there occurs a shift in leading principles for a number of dike sections. The most significant change to the current evaluation method was the inclusion of risk preferences, changing optimal probabilities by a magnitude of 10 to 100 for both the LIR and the SCBA. The costs of the required reinforcements to the flood defences is of comparable order to the current expenditure of the Hoogwaterbeschermingsprogramma (HWBP). The curvature of the probability weighting function is found to have the most significant impact on the evaluation of the new optimal flooding probabilities and therefore the costs of reinforcement.

Combining the found answers to these sub-questions allows for the generation of an answer to the main research question. This main research question is repeated here for clarity, and is formulated as:

To what extent do individuals facing flood risk demonstrate risk preferences consistent with traditional behavioural economics? What is the impact of incorporating risk preferences on the acceptable flood probabilities of primary flood defences?

The first part of the question regards the applicability of risk preferences in traditional economic choice problems to the choice problems found in flood risk management. It became apparent that, based on the elicited risk preferences from the participants, the power utility function combined with the power probability weighting function provided the most flexibility combined with accurate results. These two functions adequately describe the risk preferences of individuals. Although there was a significant difference (at $p = 0.05$) between the parameter estimates for the model made for the general economic choice problems and the flood risk management related choice problems, the general trend of risk aversion became apparent for both choice problems and thus for both fields of research. Therefore, it can be concluded that the risk preferences observed in traditional economic problems are comparable to those observed in flood risk-related problems, which was supported by means of a cross validation. This is a significant finding as it allows for the utilization of a vast body of literature and research from (traditional) behavioural economics and econometrics in the field of flood risk management.

The second part of the question focuses on reevaluating the flood safety standards in the Netherlands by implementing the discovered risk preferences and their consequences. When the best estimates for the calculated and elicited variables, including the value of a statistical life, the interest rate, and the parameters of the utility and probability weighting function, are employed, the proportion of dike sections governed by the LIR slightly increases compared to the dike sections governed by the SCBA. However, the more influential factor is the required increase in flood safety standards, ranging from one to two orders of magnitude across all leading principles. Previously, a dike section might have had an optimal exceedance probability of once every thousand years, but with the revised evaluation, the optimal exceedance probability is now in the range of once every ten to a hundred thousand years.

In conclusion, this research has examined the intersection between flood risk management and behavioural economics and econometrics. It has demonstrated that the commonly used theories on risk preferences in behavioural economics are applicable to flood risk management and has determined the necessary values and parameters associated with the utility and probability weighting function for flood risk-related choice problems. Overall, individuals exhibit risk aversion in both general economic choice problems and flood risk-related choice problems. A framework has been developed to incorporate these functions into the current evaluation of flood safety standards in the Netherlands. The findings of this research have subsequently been used to reevaluate the flood safety standards, resulting in a slight shift towards the LIR in governing dike sections in the Netherlands. Furthermore, the inclusion of risk aversion has led to significantly higher required safety standards, ranging from ten to a hundredfold increase. These stricter safety standards incur an average cost of approximately €37 per year for the average Dutch citizen, reflecting the cost of risk aversion.

Although further research is necessary, this thesis has demonstrated the potential benefits of integrating behavioural economics into flood risk management and has paved the way for further exploration in this area.

7.2. Recommendation

This section presents three recommendations for advancing research on risk preferences in Hydraulic Engineering and, more specifically, in Flood Risk Management. The first recommendation suggests incorporating time-dependent risk preferences into the derived model. The second recommendation proposes including heuristics, biases, and fallacies in the model. The final recommendation advocates for an alternative approach to using the utility and probability weighting function by adopting a Bayesian approach to risk preferences. This approach removes the need for a probability weighting function, as all uncertainty is incorporated into the utility function.

To encourage further research in these three areas, a literature study on these topics is provided in Appendix D. Additionally, for the first two recommendations, various examples are included to demonstrate how choice lists can be used to elicit parameters and test relevant hypotheses.

7.2.1. Time-dependence of risk preferences

Time-dependent risk preferences, sometimes referred to as *intertemporal choice*, refers to the choice that individuals have to make when two or more of the options in the choice lists have a different point in time when the pay-out occurs. This research has primarily focused on eliciting risk preferences for choice lists that have outcomes that occur simultaneously. In reality, and especially with climate adaptive strategies, choices have to be made between outcomes that do not occur in the same point of time. For instance, consider the choice of when to reinforce a dike section under rising sea levels. As the rate of sea level rise may accelerate at an unknown rate, current strategies involve evaluating adaption paths. These paths present multiple options and choices over the expected lifetime of a structure, considering the uncertainty in sea level rise and resulting in different alternatives. The current practice is to discount the benefits and costs of the alternatives in different points of time to a single point in time, most often the present day. This discounting provides an objective way to measure the monetary value of the alternatives.

However, as this research has demonstrated, the monetary value of an outcome is not the only way in which individuals value those outcomes. Risk preferences can be adjusted to include a similar form of monetary discounting, but then of the utilities that individuals attach to outcomes. Appendix D showcases several functional forms of this discounting process for intertemporal choice. This discounting process is a linear operation on the utilities, allowing it to be incorporated into the findings of this research without having to elicit the risk preferences again. Integrating the intertemporal choice model into the derived risk preference model offers a valuable addition for decision-making in flood risk management.

7.2.2. Inclusion of heuristics, biases and fallacies

Although individuals are assumed to behave rationally when placed in the correct model for risk preferences, sometimes there choices can contradict one another. This often is the result of a violation of monotonicity in either the utility- or probability weighting function. However, another explanation for the contradiction of choice is that an unobserved heuristic, biases or fallacy is (partly) responsible. Several of the most common heuristics, biases and fallacies are given in Appendix D with explanation and examples. The three most common ones are *Framing*, the *Recency bias* and *Anchoring*.

These heuristics, biases, and fallacies can be elicited using a similar set of choice problems as proposed in this research, with some modifications. For example, the framing heuristic would present the survival probability in case of a flood rather than the probability of death due to flooding. Similarly, the recency bias can be elicited by reminding participants of recent flooding events. If it is found that individuals are susceptible to these heuristics, biases, or fallacies, the risk preference model can be adjusted to account for them. This again is an addition to the model that does not require the found risk preferences to be re-elicited.

7.2.3. Bayesian approaches to decision under risk

Up to this point, all models proposed in this research contained both a utility function and a probability weighting function with the possibility of extension. There is however another class of models that might have a strong explanatory power for the choice problems described in this research, those based on Bayesian analysis. The basis of Bayesian analysis is that one continuously updates their beliefs if new information arises. This method is attractive as it would mitigate one of the limitations found in the discussion, that of changing risk preferences with age.

One method in particular is of interest, that of Bayesian Decision theory by van Erp (2017). The theory uses a slight adjustment of *homo oeconomicus* and belongs to the class of neo-Bernoullian decision theories as it adopts Bernoulli's original utility function. It differs from the original function of Bernoulli in that it incorporates not only the most likely path (by means of expected value) but also the worst- and best-case scenarios. The valuable part of this theory is that it provides an intuitive way of quantifying uncertainty and does not make use of the probability weighting function, but solely of the utility function with the product and sum rules of Bayesian probability theory.

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A

Analysis of risk preference models

A.1. Estimation of the utility- and probability weighting function for different models

This section discusses various utility- and probability weighting functions that were fitted to the observed certainty equivalents. It discusses the fit of different utility models and analyses the residuals of the fit.

A.1.1. Utility- and probability weighting functions based on the general choice problems

Table A.1 shows several summary statistics of the parameter estimates for the various models. The table shows the mean, standard error and the minimum- and maximum value of the parameters estimates. The standard error of the curvature of the probability weighting function (given in γ) is shown in brackets next to the mean estimate of γ.

Table A.1: Average estimates of the parameters for several utility functions for all participants based on the general choice problems, including several key statistics and the curvature of the probability weighting function

Model	Parameters	Mean	Standard error	Min - Max	Curvature (γ)
Linear	a	2.549	0.449	[0.973, 18.91]	0.868 (0.0961)
Logarithmic	c	1.496	0.0607	[0.838, 2.812]	1.108 (0.0254)
Iso-elastic	η	0.268	0.0269	[-0.273, 0.620]	0.627 (0.00515)
	α	0.694	0.0117	[0.425, 0.778]	
Power	β	0.834	0.0143	[0.514, 0.987]	0.631 (0.00748)
	λ	1.454	0.00207	[1.408, 1.479]	

To test which model best suits the observed CE values, several goodness-of-fit variables were examined. The first of which is the correlation coefficient, R^2, between the observed CE values and the predicted CE values by the different models. The second two statistics are quite similar, the *Sum of Linear Residuals* and *Sum of Squared Residuals*. The statistics are shown in table A.2.

Table A.2: Goodness-of-fit statistics for different models using the general choice problems

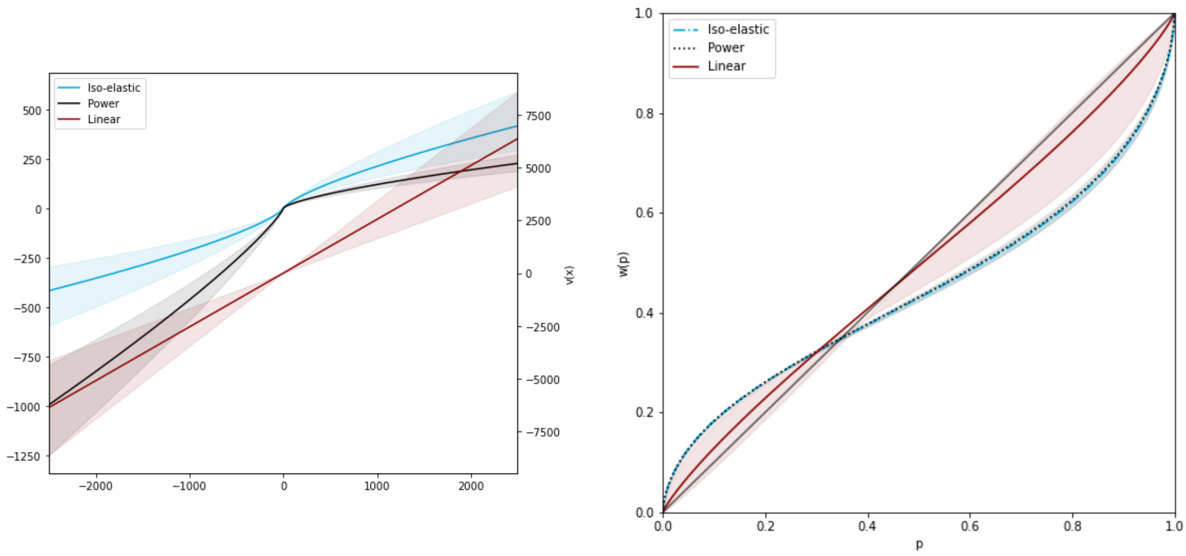
Model	R^2 for individuals	R^2 for group	SLR	SSR
Linear	0.9968 (0.00296)	0.9923	213.9	659.0
Logarithmic	0.9260 (0.03510)	0.8601	384.3	360.7
Iso-elastic	0.9970 (0.00410)	0.9897	144.4	251.6
Power	0.9992 (0.00233)	0.9894	190.5	438.5

The R^2 indicates the percentage of the variance in the dependent variable that the independent variables collectively explain. In this context, a high correlation coefficient indicates that a large percentage of the variance of the found CE values is explained by the CE values as calculated by the models in table A.1. These CE values are calculated by the models with the given parameter estimates and the provided combination of outcomes (x, y) and probabilities (p). This correlation coefficient has been calculated per individual and presented as an aggregate statistic, as well as for the entire group by means of the average parameters. The R^2 for individuals therefore shows how much of the variance of the found CE values per individual is explained by the utility functions of said individual. The R^2 for the group shows how much of the variance of the found average CE values for the group is explained by the utility function of the entire group, e.g. the utility function with the average of the parameters. These two statistics indicate how well the model predicts individual as well as societal risk preferences.

The highest R^2 values for the individuals is found when using the *power* model, while the lowest values are found when using the *logarithmic* model. Aggregating the parameters into an average changes the highest value to the *Linear* model, the *logarithmic* models has the lowest score. All values appear to decrease when the aggregate is chosen instead of the individual functions. This is to be expected as the choice process is assumed to be a process with a high degree of variability between individuals. The results indicate that personalized utility- and probability weighting functions yield a better explanatory power than an average function over all participants. Furthermore it shows that the *Power* model has the best explanatory power for the variance on an individual level, while the *Linear* model has the best explanatory power for the group.

Table A.2 further displays the *sum of squared residuals* (SSR) values for the different models. This statistic gives an indication of how much of the part of the observed CE values that are not explained by the model. The higher this number is, the less accurate the model is in predicting the correct CE values, as there is a larger error. Note that the residuals are weighed quadratically in this goodness-of-fit variable. It can be seen that the highest SSR values are recorded for the *Linear* model while the lowest SSR values are found when using the *Iso-elastic* model. Another metric, the *Sum of linear residuals* (SLR), gives a slightly different picture. This metric works in a similar way as the SSR, apart from the fact that the residuals are weighed linear. Here the *Logarithmic* model has the highest score with the *Iso-elastic* model again having the lowest score. Based on these tables, the *Linear*, *Iso-elastic* and *power* models appear to have the best scores on both the correlation coefficient as well as the SS-L/SSR values. To further test the goodness-of-fit, the residuals are examined for these three models. The analysis can be found in section A.3.1.

Using the estimated parameters for all models, the utility- and probability weighting functions can be plotted. These curves are shown in figure A.1. The curves of the individual models are given in figure E.4 through E.6 in Appendix E.



(a) Estimate of utility function for all participants

(b) Estimate of probability weighting function for all participants

Figure A.1: Estimate of probability weighting- and utility function including uncertainty for all participants based on the general choice problems

Figure A.1a shows the utility functions of the three models under consideration. The x-axis represents the input, often expressed in monetary amounts. The y-axis represents the utility $v(x)$ derived from the monetary input x. In this figure, the *power* and *iso-elastic* utility functions share the same y-axis on the left. Due to the large difference in estimate of utility the *Linear* model has it's own axis on the right. Note that the magnitude of the estimated utility for the different models can not be compared one-to-one. Often it is either the inverse of the utility function, such that one ends up with the input x, or the ordinal ranking of utility that is of interest, both of which are unique to the specific utility function. The shaded area represents the uncertainty in parameter estimates of the participants, in this case represented by two standard errors of difference from the mean. It can be seen that all utility functions have widening uncertainty for larger values of x. The linear and iso-elastic model appear to have a symmetry in the uncertainty while the power model shows a certain asymmetry between positive and negative values. This is to be expected as both these models contain one parameter while the power model contains three parameter. As such, the power model also has the largest relative uncertainty out of the three models.

Figure A.1b shows the probability weighting functions of the same three models. The x-axis shows the input, a probability p between zero and one. The y-axis shows the weight $w(p)$ an individual attaches to the input probability, also ranging between zero and one. The typical S-pattern described in the theory can clearly be seen in the iso-elastic and power model, but is less pronounced in the linear model. This can be explained by looking at the average estimate of all parameters, with the linear parameter being closest to one and the other two being significantly below one. All average estimates of γ were below one. Note that the iso-elastic and power models have estimates that lie close together. Looking at the uncertainty, it becomes apparent that the aforementioned two models also share a similar degree of uncertainty, being relatively narrow. The linear model has a larger degree of uncertainty than the other two models. If the same two standard errors of difference to the mean are applied to the estimate of the linear model, the parameter γ reaches values above one which flips the concave and convex parts of the curves.

A.1.2. Utility- and probability weighting functions based on the FLM related choice problems

This section discusses the results of the analysis for the choice problems that were related to flood risk. The results of the parameter estimates is given in table A.3.

Table A.3: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM choice problems

Model	**Parameters**	**Mean**	**Standard error**	**Min - Max**	**Curvature** (γ)
Linear	a	3.025	0.437	[-0.140, 14.28]	1.424 (0.109)
Logarithmic	c	0.485	0.0527	[0.161, 1.126]	0.892 (0.0282)
Iso-elastic	η	0.234	0.0298	[-0.114, 0.391]	0.887 (0.00926)
	α	0.650	0.0201	[0.424, 0.789]	
Power	β	0.719	0.0163	[0.505, 0.947]	0.698 (0.0212)
	λ	1.438	0.00359	[1.299, 1.460]	

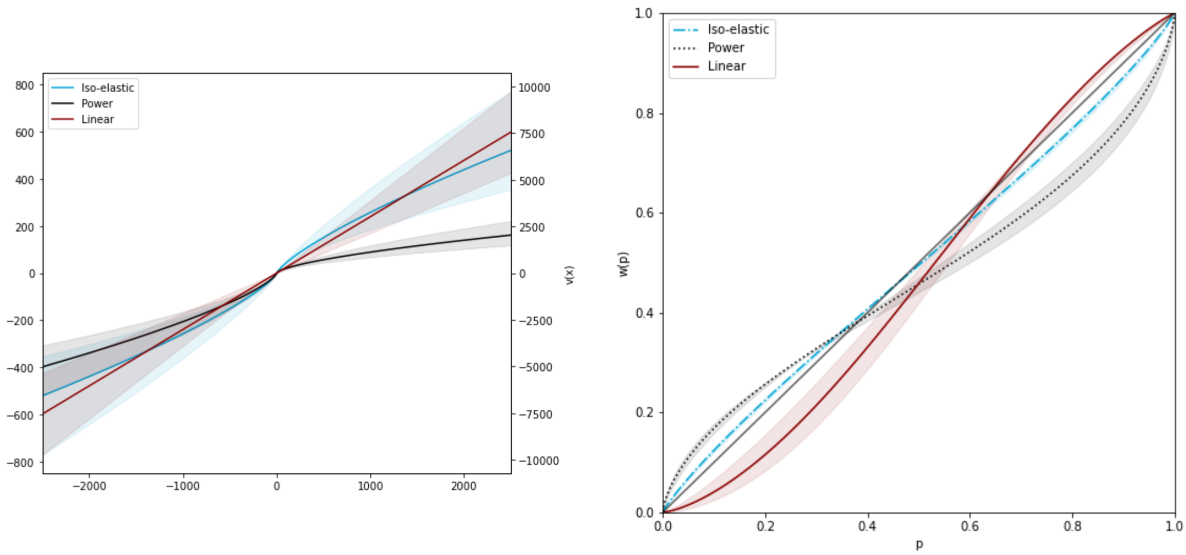
A similar procedure for determining the best fitting model as for the general choice problems has been performed for the FLM related choice problems. Table A.4 shows the individual and aggregated correlation coefficients as well as the SSR values for the FLM related choice problems.

Table A.4: Goodness-of-fit statistics for different models using the FLM related problems

Model	R^2 **for individuals**	R^2 **for group**	**SLR**	**SSR**
Linear	0.9743 (0.06165)	0.9741	158.9	149.90
Logarithmic	0.686 (0.0908)	0.418	415.6	394.0
Iso-elastic	0.967 (0.0102)	0.989	134.9	107.28
Power	0.9727 (0.0288)	0.9653	192.3	278.0

It can be seen in A.4 that the R^2 values for both the individuals and the group as a collective are near to one when using either the *linear*, *Iso-elastic* or *Power* model, while the *Logarithmic* model has a significantly lower R^2 value. Looking at the SSR and SLR scores, the lowest value is recorded using the *iso-elastic* model and the highest value is recorded using the *Logarithmic* model. All models show non-normality in the residuals, indicating that parts of the observed values are not explained by the model choice. This does not have to pose a problem however, as it known that it is not solely the risk preferences that determine the choice between options, but socio-economic factors play a part as well. The full examination can be found in section A.3.2.

Using the estimates for the parameters of the three models, the utility- and probability weighting functions are plotted. The result of which can be found in figure A.2.



(a) Estimate of utility function for all participants

(b) Estimate of probability weighting function for all participants

Figure A.2: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM choice problems

The lay-out of figure A.2 is the same as in the section on the general choice problems. Again, the linear model is given its own y-axis in figure A.2b due to the larger magnitude. The same degree of risk aversion in the Power functions can be seen as was observed in the general choice problems, with the positive outcomes given a smaller weight than the negative outcomes. The magnitude of the power function for higher values of x is smaller than in the general choice problems. This is also the case for the iso-elastic model. The linear model shows slightly larger output values.

Figure A.2b again shows the probability weighting functions of the three different models, with values ranging from zero to one. The curves have two notable features. The first being the the uncertainty around the Iso-elastic probability weighting function is relatively small. This might be an indication that the uncertainty in the certainty equivalents is 'absorbed' in the uncertainty of the utility function. This results in a curve that has the convex and concave parts reversed. If uncertainty is included the estimates for the curvature of the probability weighting function (γ) of the linear model remains above one.

A.2. Cross-validation of utility models

To check the robustness of the estimates for the chosen utility model, the power function, a cross validation of the parameters for general- and FLM elicited choice problems is performed. To this end, the general CE values per individual are predicted by means of the FLM elicited risk preference parameters and vice versa. This section discusses the results of the analysis.

For clarity, the reader is reminded that a certainty equivalent is the product of the weight given to probability of outcomes and the utility given to said outcomes. The weight given to the probability is determined by the probability weighting function, the utility of the outcomes is calculated by means of the utility function. Both these functions have certain parameters in their functional forms, which were elicited. This analysis takes the parameters of the FLM elicitation and uses them to calculate the weight and utility of the choice problems (which consist of probabilities and outcomes) of the *general* choice problems. Similarly, the parameters that were the result of the general elicitation were used to calculate the weight and utility of the FLM related choice problems. The estimation is consequently compared to the actual, observed CE value of the original problem.

A.2.1. General parameters for predicting FLM related CE-values

Using the parameters that were elicited by means of the general posed choice problems, the CE values of the choice problems that are related to flood risk management are calculated. The estimation has the same procedure as in the regular process, apart from the fact that the parameters are changed. A similar comparison as in figure E.14, which shows the predicted CE value against the observed CE value for all twelve posed choice problems, can be made using this different set of parameters. This is shown in figure E.17. Next to this comparison, it is of interest to see what the relative magnitude of these estimates is in comparison to the observed value, with corresponding 95% confidence interval, shown in figure A.3 was created.

Figure A.3 shows the observed (or measured) CE values on the x-axis. On the y-axis it shows the relative magnitude of the estimated CE value with respect to the observed CE values. These estimated values are the result of the utility- and probability weighting function that were found using the general choice problems, but applied to the FLM related choice problems. Each point in the graph represents the mean certainty equivalent of the respondents to one general choice problem. This problem consists of an outcome (in this case a monetary outcome) and a probability of obtaining that outcome. All the answers of the respondents are gathered and combined into one mean answer to each of the problems, which is what the dots represent. Using the individual utility- and probability weighting functions of the participants, an estimate for the CE of the posed problem is made. This is then normalized by dividing by the measured CE value. A range of estimates exist, which are shown in the error bars for each of the points. These error bars show the 95% confidence interval of the estimates of the participants for each of the choice problems posed.

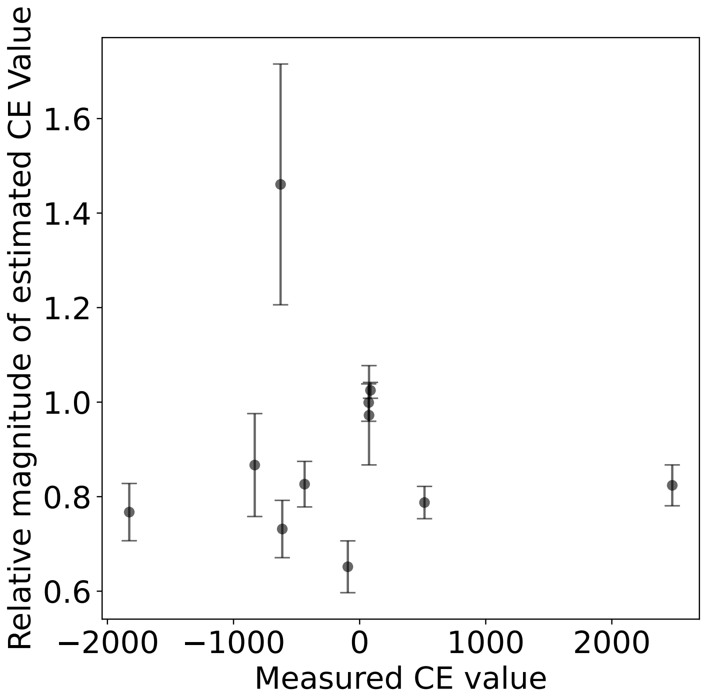


Figure A.3: Boxplot of the observed certainty equivalents (CE) of the general choice problems against the predicted CE values of the general choice problems that were predicted with the FLM elicited parameters for the utility- and probability weighting function

It can be seen that in general the predicted CE values are underestimated with respect to the average CE value. This ranges from around 10 to 30% underestimation. An underestimation in this context means that, based on the utility- and probability weighting function with the FLM parameters, the individuals would accept a lower certainty equivalent (so a lower amount of money) for the same choice problem when the parameters that were elicited for the general problems are used. Conversely, an overestimation means that individuals would like to receive more money for the same choice problem.

One noticeable example exists around -800, for which the estimated CE values is overestimated by a factor of 1.5. This most likely is the result of the prediction using a relatively small probability as input for the probability weighting function, in combination with an average estimate for the utility function. Although the part of the outcome might be given a comparable weight to the measured CE values, it is the probability weighting function that causes large changes. It is known that the probability weighting function is particularly sensitive to small probabilities, often overestimating them by a significant degree. If the parameters are changed for this function, the degree of overweighting changes relatively more in the domain of small probabilities, which most likely caused the shift around -800.

A.2.2. FLM related parameters for predicting general CE-values

Similar to the previous section, but in reversed order, the FLM related parameters can be used to predict the general observed CE values. For each of the different problems posed, figure E.16 shows the correlation between the predicted and observed values.

Figure A.4 again shows the relative magnitude of the estimates in comparison to the observed value, with corresponding 95% confidence interval.

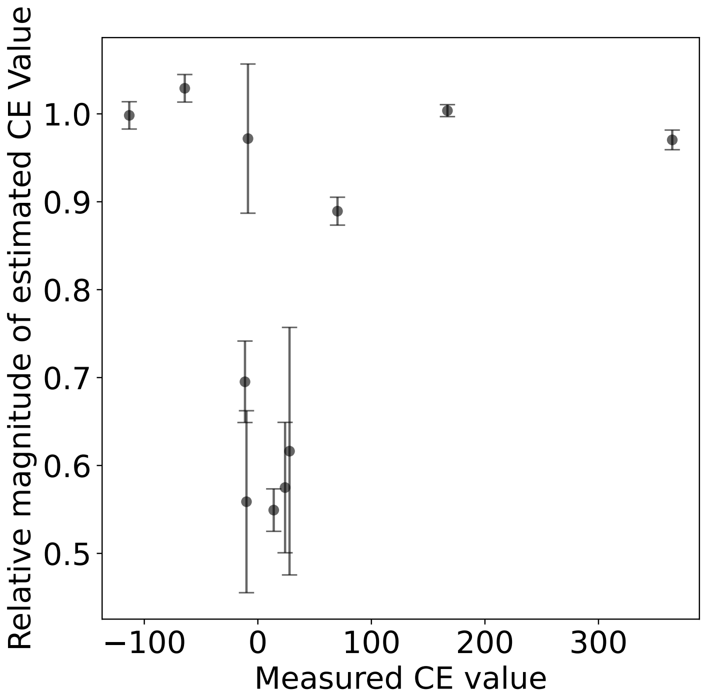


Figure A.4: Boxplot of the observed certainty equivalents (CE) of the FLM related choice problems against the predicted CE values of the FLM related problems that were predicted with the general elicited parameters for the utility- and probability weighting function

It can be seen that the endpoints are predicted with a higher degree of certainty and closer to the true values than those near zero. Values near zero tend to be underestimated anywhere from 30 to 50%. Again, this means that using the utility- and probability weighting function of the general elicited choice problems into the FLM posed choice problems, individuals tend to want less money for the same problem. For values further away from zero, the estimated CE values lie closer to the measured CE values, indicating that individuals are willing to pay nearly the same amount of money for the choice problems, regardless of whether their risk preferences are described by FLM or general parameters.

The noticeable exception occurs around zero, where the predicted CE value is underestimated. The most likely cause of this underestimation is the lack of available points that were used to fit the FLM risk preferences. As a reminder, the FLM risk preferences were elicited with choice problems that contained outcomes that were relatively larger than the outcomes used in the general choice problems. This results in a 'better' fit for smaller values for the general choice problems, and a 'worse' for smaller values when FLM related risk preferences are used. This is also the explanation for the relatively large uncertainty, and the smaller uncertainty for larger values. Conversely, this is why the general risk preferences showed a greater uncertainty for the FLM choice problems in figure A.3.

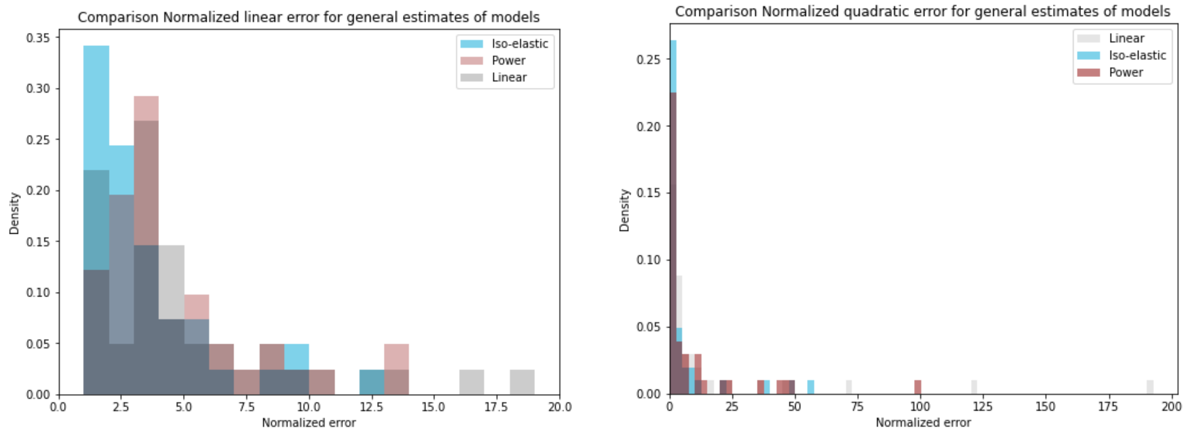
In conclusion, it can be seen that the cross validation yields estimations of the CE values that are of the same order as those observed. This indicates that the method of elicitation is robust and provides further confidence for the found risk preferences. Several outliers were detected, for which appropriate explanations were found.

A.3. Analysis of the residuals of the different risk preference models

This section gives a more detailed explanation of how the residuals of the model estimates were obtained and subsequently analysed.

A.3.1. Residuals of the general choice problem estimates

To examine and compare residuals, the values are normalized and calculated for each CE estimate per participant. Two metrics were investigated, the linear normalized residuals and the squared normalized residuals. The normalizing procedure consists of dividing the linear error term by the observed CE value for the first metric and dividing the squared error term by the square of the observed CE value. Note that a squared error term puts more weight to predicted values that are further off the observed value, while the linear error term weights the residuals linear. The results are presented in the two histograms shown in figure A.5.



(a) Histogram of linear residuals

(b) Histogram of quadratic residuals

Figure A.5: Histogram of linear and quadratic normalized residuals for the general choice problems

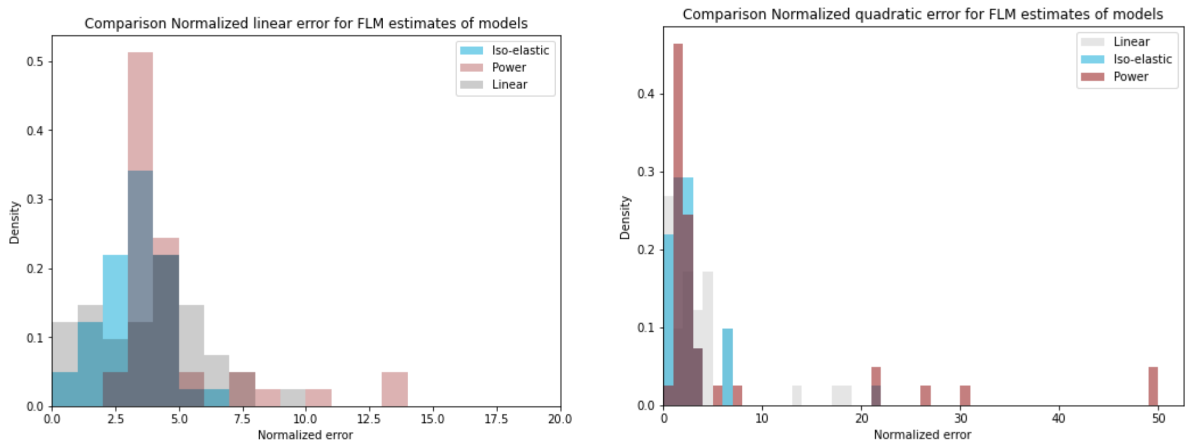
Figure A.5a shows the linear residuals with a binwidth of 1, figure A.5b shows the squared residuals with a binwidth of 5. The highest density for the linear residuals is located at or near zero for both figures, indicating a (near) perfect fit. There are however several outliers for all models. Most notably for the *linear* model, which causes the large SSL/SSR scores in comparison to the other models. Performing both a Shapiro and a Kolmogorov–Smirnov (KS) test shows that both residuals for all models are not normally distributed. To give an indication, the Shapiro test statistic for the *power* model is reported as 0.536 and 0.397 respectively, while the KS test statistic was 0.314 and 0.500. All of these test statistics correspond to p-values that are far below the threshold of 0.05. For the other two models the values of the test statistics were of similar magnitude. As such, the null hypothesis of normality can be rejected based on the observations for all three models.

A.3.2. Residuals of the FLM related choice problem estimates

The three models with the best estimates for R^2 are chosen to be investigated further, these are the Linear, Iso-elastic and Power model. A histogram of which can be seen in figure A.6.
In figure A.6 both histograms have a binwidth of one. The highest density for the linear residuals is located around three, the highest density for the squared residuals is located around two. Again, several outliers can be detected, in this case it is the *power* model that has the largest outliers relative to the other models. Again performing both a Shapiro and a Kolmogorov–Smirnov (KS) test shows that both residuals for all models have p-values that are far below the threshold of 0.05. As such, the null hypothesis of normality can be rejected based on the observations for all three models.

A.4. Sensitivity analysis for the value of a statistical life

This section discusses the sensitivity of the VOSL when using elicitations via the Linear and Iso-elastic model.



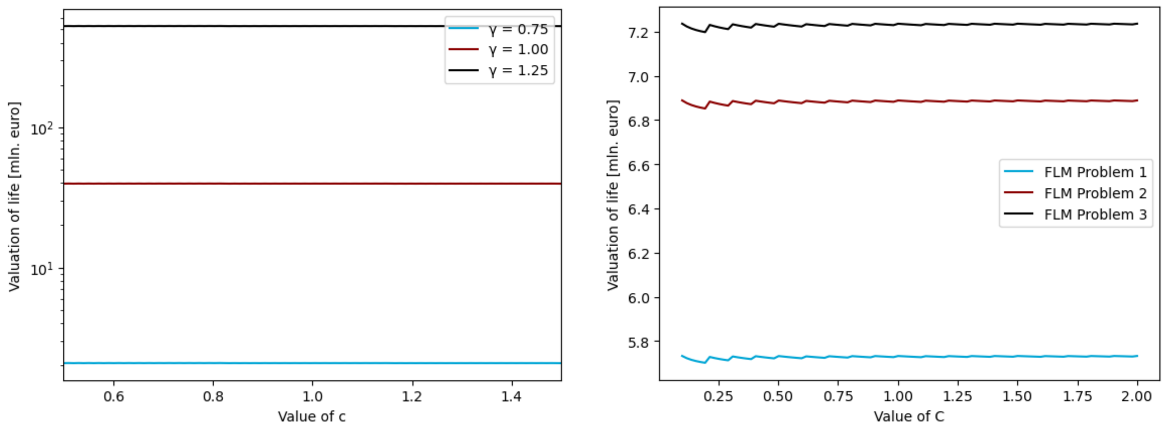
(a) Histogram of linear residuals

(b) Histogram of quadratic residuals

Figure A.6: Histogram of linear and quadratic normalized residuals for the FLM choice problem

A.4.1. Linear model

For the linear model the parameter dicating for the magnitude of the linear utility (c) was varied from 0.1 to 2.0. The parameter dictating the curvature of the probability weighting function (γ) was varied from 0.75 to 1.5. The results for a varying c parameter are shown in figure A.7.



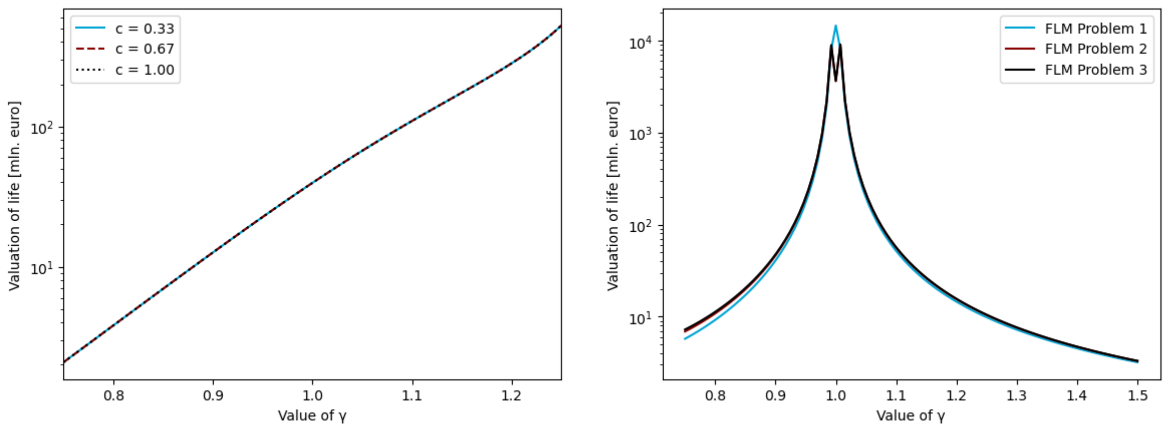
(a) Results of the sensitivity analysis for the general choice problems for different values of the parameter that dictates the relative magnitude of the utility function (c)

(b) Results of the sensitivity analysis for all three flood risk management related choice problems for different values of the parameter that dictates the relative magnitude of the utility function (c)

Figure A.7: Results of the valuation of a statistical life for the Linear model with a varying parameter that dictates the relative magnitude of the utility function (c)

It can be seen in figure A.7a that for the range of c values tested, the estimation remains constant for the general choice problems. Depending on the value of γ the estimates differ by approximately a factor 10. Figure A.7b shows that for the different FLM problems presented the estimates are also (nearly) constant for values of c. Interestingly enough the choice of the FLM problem has a

Figure A.8 shows the results when the parameter dictating the curvature of the probability weighting function (γ) is varied.



(a) Results of the sensitivity analysis for the general choice problems for different values of the parameter that dictates the curvature of the probability weighting function (γ)

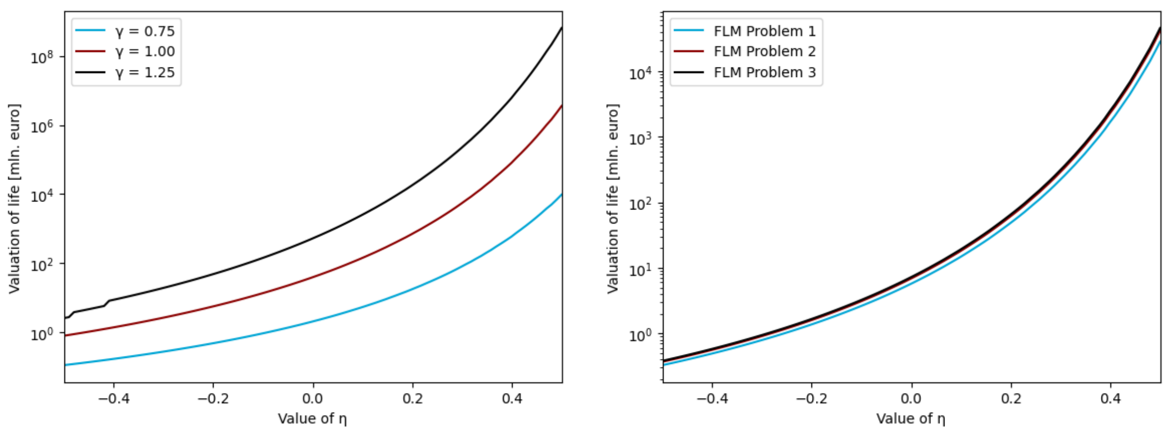
(b) Results of the sensitivity analysis for all three flood risk management related choice problems for different values of the parameter that dictates the curvature of the probability weighting function (γ)

Figure A.8: Results of the valuation of a statistical life for the Linear model for different values of the parameter that dictates the curvature of the probability weighting function (γ)

It can be seen in figure A.8a that with increasing values of γ the valuation increases for all tested values of c. Figure A.8b shows that the valuation for all three FLM problems is nearly identical. Increasing rapidly for values of γ up to one before rapidly descending after crossing the threshold of one.

A.4.2. Iso-elastic model

For the Iso-elastic model the parameter dictating the curvature for both the positive and negative outcomes (η) was varied from -0.50 to 0.50. The parameter dictating the curvature of the probability weighting function (γ) was varied from 0.75 to 1.50. The results for a varying parameter η are shown in figure A.9.



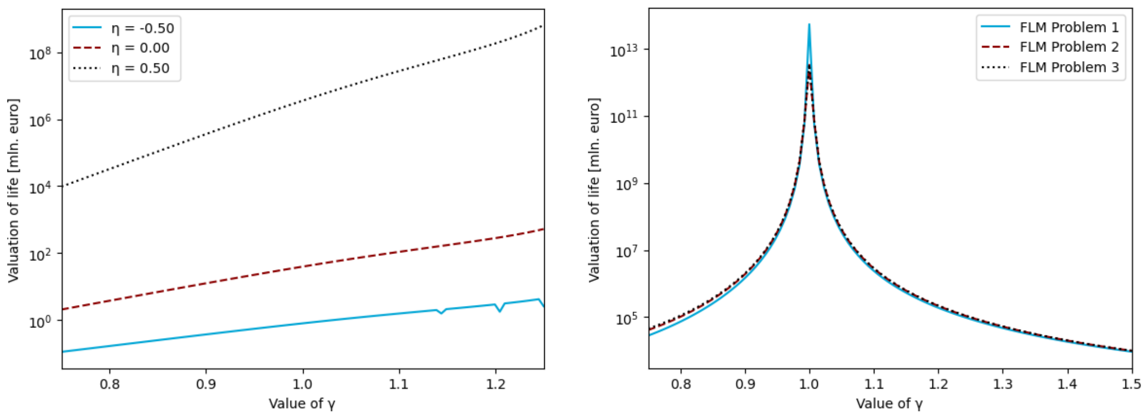
(a) Results of the sensitivity analysis for the general choice problems for different values of the parameter that dictates the curvature of the utility function (eta)

(b) Results of the sensitivity analysis for all three flood risk management related choice problems for different values of the parameter that dictates the curvature of the utility function (eta)

Figure A.9: Results of the valuation of a statistical life for the Iso-elastic model with a varying parameter that dictates the curvature of the utility function (η)

It can be seen in figure A.9a that for increasing values of η the valuation increases as well, regardless of the value of γ. The difference in estimation between the values of γ is around a factor 10. Figure A.9b has the same trend of increasing valuation for increasing values of η, with the different methods of elicitation showcasing similar results.

Figure A.10 shows the results when the parameter dictating the curvature of the probability weighting function (γ) is varied.



(a) Results of the sensitivity analysis for the general choice problems for different values of the parameter that dictates the curvature of the probability weighting function (γ)

(b) Results of the sensitivity analysis for all three flood risk management related choice problems for different values of the parameter that dictates the curvature of the probability weighting function (γ)

Figure A.10: Results of the valuation of a statistical life for the Iso-elastic model with a varying parameter that dictates the curvature of the probability weighting function (γ)

It can be seen in figure A.10a that for increasing values of γ the valuation increases for all evaluated values of η. Note that lower values of η produce lower valuations, in line with results from figure A.9. Figure A.10b shows that all FLM valuations again lie close together. Increasing rapidly for values of γ up to one before rapidly descending after crossing the threshold of one.

B

Derivations

B.1. Value of a statistical life

Assuming an exceedance probability than can be modelled in the tails as the exponential distribution with parameters A and B, allows for the expression of the exceedance probability as:

$$P_f = e^{-\frac{h-A}{B}}$$

Letting the expected total cost $\mathbb{E}(C)$ related to flooding and flood risk management be the sum of an initial investment I_0, a variable part I_1 and the expected cost of the number of casualties, this quantity can be expressed as:

$$\mathbb{E}(C) = I_0 + I_1 \cdot (h - h_0) \cdot N + P_f \cdot N \cdot d \cdot PV$$

Where N is the expected number of casualties and d the valuation of a human life. PV is the present value factor. Differentiating this expression with respect to the dike height h and equating the expression to zero results in the optimal level of protection as:

$$P_{f,opt} = \frac{I_1 \cdot B}{d \cdot PV}$$

This allows for the derivation of the optimal height of the dike h_{opt}. The total investment in safety, which is the expected total cost $\mathbb{E}(C)$ minus the expected costs of the number of casualties $P_f \cdot N \cdot d \cdot PV$, is therefore equal to:

$$\begin{aligned} I &= I_0 + I_1 \cdot (h_{opt} - h_0) \cdot N \\ &= I_0 + I_1 \cdot (A - B \cdot ln(\frac{I_1 \cdot B \cdot N}{d \cdot PV}) - h_0) \cdot N \end{aligned}$$

The cost of investment to save an expected extra life, denoted by CSX, can then be expressed as the ratio of the total investment costs I and the discounted expected number of saved lives $(P_{f,0} - P_{f,opt}) \cdot N \cdot PV$ as:

$$CSX = \frac{I}{(P_{f,0} - P_{f,opt}) \cdot N \cdot PV}$$

B.2. Variable interest rate

This section gives a derivation for the expression of a new optimal flooding probability as a result of a variable interest rate.

If one imagines two scenario's where the first scenario has an interest rate r_0 and the second one has a different interest rate r_1 such that $r_1 > r_0$, the difference in risk reduction can be expressed as a fraction of the original risk reduction. The total discounted risk reduction over the lifetime of n years of the measure can be expressed as:

$$TC_{risk} = \sum_{i=1}^{n} C_{risk} \cdot (1+r)^{-i}$$

Where the cost of the risk consists of the value of the assets under risk and the probability of the assets ceasing to hold value. If for simplicity the annual risk is assumed as the product of a probability of flooding p and a damage as a result of flooding V, then the annual risk can simply be expressed as: $C_{risk} = p \cdot V$. The ratio of the total costs with the initial interest rate over the total cost with the new interest rate can then be expressed as:

$$\frac{TC_0}{TC_1} = \frac{\sum_{i=1}^{n} C_{risk} \cdot (1+r_0)^{-i}}{\sum_{i=1}^{n} C_{risk} \cdot (1+r_1)^{-i}} = \frac{\sum_{i=1}^{n} p_0 \cdot (1+r_0)^{-i}}{\sum_{i=1}^{n} p_1 \cdot (1+r_1)^{-i}}$$

Note that it is assumed that p does not change over time to keep the mathematics manageable. This assumption is justified by the use of classes in the subsequent analysis. This usage of classes will overestimate the necessary flooding probability, ultimately making it resilient against future decreases of said probability. Since $r_1 > r_0$, it is possible to express r_1 in terms of r_0, such that $r_1 = r_0 + r_{add}$ where r_{add} is the increase in interest rate from the original situation. This expands the previous expression to:

$$\frac{TC_0}{TC_1} = \frac{p_0}{p_1} \frac{\sum_{i=1}^{n} (1+r_0)^{-i}}{\sum_{i=1}^{n} (1+r_0+r_{add})^{-i}}$$

Since this derivation is based on the SCBA principle, the total risk reduction over the lifetime of the measure should equal the investment costs at $t = 0$, from which the desired probability p_1 can be found. These required investment costs are expressed as the cost that are required for a 10x safety level and are assumed independent of the interest rate. If these costs are denoted by I, it follows that:

$$\frac{I_0}{I_1} = \frac{TC_0}{TC_1} = \frac{p_0}{p_1} \frac{\sum_{i=1}^{n} (1+r_0)^{-i}}{\sum_{i=1}^{n} (1+r_0+r_{add})^{-i}}$$

Setting I_0 equal to I_1, under the assumption of independence of the interest rate, reduces the lefthand-side to unity. The desired flooding probability can now be found by equation B.1:

$$p = p_1 = \frac{p_0 \cdot \sum_{i=1}^{n} (1+r_0)^{-i}}{\sum_{i=1}^{n} (1+r_0+r_{add})^{-i}} = \frac{p_0 \cdot \frac{1-(1+r_0)^{-n}}{r_0}}{\frac{1-(1+r_0+r_{add})^{-n}}{r_0+r_{add}}} = p_0 \cdot \frac{(1-(1+r_0)^{-n} \cdot (r_0+r_{add})}{(1-(1+r_0+r_{add})^{-n}) \cdot r_0} \tag{B.1}$$

The last equality follows from the expansion of a finite sum via the geometric series. Here, p_0 is known and constant, being equal to $\frac{I}{V}$. If a starting value for r_0 is assumed, the only variable is the additional interest rate r_{add}. This derivation allows for a quick comparison of the change in optimal flooding probability for increases or decreases in the interest rate via the term r_{add}.

C
Questionnaire

This chapter contains the questions that were asked to participants in the questionnaire that was used to elicit the risk preference, adjusted VOSL and weighting of small probabilities.

C.1. Risk preferences

This section shows the questions that were asked to the participants of the study that are related to general risk preferences and Flood Risk management related risk preferences.

C.1.1. General risk preferences

This section shows the questions related to general risk preferences that were posed to participants in the study. Each entry marked with an **o** in table C.1 corresponds to a choice list that was asked to the participants.

Table C.1: Matrix containing the possible combinations of prospects that were used asked to participants in the questionnaire aimed at eliciting risk preferences for general outcomes

	Probability attached to the higher outcome										
Outcomes	0.01	0.05	0.1	0.25	0.4	0.5	0.6	0.75	0.9	0.95	0.99
(0; -25)						**o**					
(0; -100)			**o**								
(0; -400)	**o**										
(-100; -50)								**o**			
(-100; -200)										**o**	
(25; 0)							**o**				
(100; 0)				**o**							
(400; 0)		**o**									**o**
(100; 50)					**o**						
(200; 100)									**o**		

Each of the entries in table C.1 corresponds to a choice problem posed to the participants. Participants were asked whether they preferred a sure outcome or the gamble (represented by outcomes and probabilities in table C.1). The general formulation of these questions is:

Do you prefer a certain pay-out C or a gamble where you have a $100 \cdot p$% chance of losing €x_1 and a $(1 - p) \cdot 100$ % chance of losing €x_2?

Where x_1 and x_2 are the outcomes shown in table C.1. The tables presented below contain all the 12 questions that were asked to the participants for these general choice problems. For each table presented, the first two and last two choices are shown, indicating the interval range and length of the outcome space. In between these rows, on the dots, would be all intermediate values within the specified interval. All certain pay-outs are constructed in such a way that the *gamble* must be preferred in the first choice. The rows are presented as a choice one by one, starting from the top and continuing up to the point where the individual switches. This point, where the individual switches between the *gamble* and the *certainty*, and the point before are recorded as the bounds of the certainty equivalent. The average of the two is used in calculations as the certainty equivalent. The tables are listed based on the order in which they appeared in the questionnaire.

Table C.2: Choice list for the outcome $(0; -400)$ used in the general Risk preferences

Certainty	**Gamble**	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €0	0.01 probability of €0	0.99 probability of €-400
Certainty of €-2	0.01 probability of €0	0.99 probability of €-400
...	...	...
Certainty of €-28	0.01 probability of €0	0.99 probability of €-400
Certainty of €-30	0.01 probability of €0	0.99 probability of €-400

The interval of table C.2 is €2 per row. Starting at €- and continuing to €-30.

Table C.3: Choice list for the outcome $(400; 0)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €4	0.05 probability of €400	0.95 probability of €0
Certainty of €8	0.05 probability of €400	0.95 probability of €0
...	...	...
Certainty of €56	0.05 probability of €400	0.95 probability of €0
Certainty of €60	0.05 probability of €400	0.95 probability of €0

The interval of table C.3 is €4 per row. Starting at €4 and continuing to €60.

Table C.4: Choice list for the outcome $(0; -100)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-2	0.05 probability of €-100	0.95 probability of €0
Certainty of €-4	0.05 probability of €-100	0.95 probability of €0
...	...	...
Certainty of €-26	0.05 probability of €-100	0.95 probability of €0
Certainty of €-30	0.05 probability of €-100	0.95 probability of €0

The interval of table C.4 is €2 per row. Starting at €-2 and continuing to €-30.

Table C.5: Choice list for the outcome $(100; 0)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €5	0.25 probability of €100	0.75 probability of €0
Certainty of €7.5	0.25 probability of €100	0.75 probability of €0
...	...	...
Certainty of €37.5	0.25 probability of €100	0.75 probability of €0
Certainty of €40	0.25 probability of €100	0.75 probability of €0

The interval of table C.5 is €2.5 per row. Starting at €5 and continuing to €40.

Table C.6: Choice list for the outcome $(100; 50)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €55	0.40 probability of €100	0.60 probability of €50
Certainty of €57.5	0.40 probability of €100	0.60 probability of €50
...	...	...
Certainty of €87.5	0.40 probability of €100	0.60 probability of €50
Certainty of €90	0.40 probability of €100	0.60 probability of €50

The interval of table C.6 is €2.5 per row. Starting at €55 and continuing to €90.

Table C.7: Choice list for the outcome $(0; -25)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-1.5	0.50 probability of €-25	0.50 probability of €0
Certainty of €-3	0.50 probability of €-25	0.50 probability of €0
...	...	...
Certainty of €-21	0.50 probability of €-25	0.50 probability of €0
Certainty of €-2.5	0.50 probability of €-25	0.50 probability of €0

The interval of table C.7 is €1.5 per row. Starting at €1.5 and continuing to €22.5.

Table C.8: Choice list for the outcome $(25; 0)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €1.5	0.60 probability of €25	0.40 probability of €0
Certainty of €3	0.60 probability of €25	0.40 probability of €0
...	...	...
Certainty of €21	0.60 probability of €25	0.40 probability of €0
Certainty of €22.5	0.60 probability of €25	0.40 probability of €0

The interval of table C.8 is €1.5 per row. Starting at €1.5 and continuing to €22.5.

Table C.9: Choice list for the outcome $(-50; -100)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-50	0.75 probability of €-50	0.25 probability of €-100
Certainty of €-52.5	0.75 probability of €-50	0.25 probability of €-100
...	...	...
Certainty of €-82.5	0.75 probability of €-50	0.25 probability of €-100
Certainty of €-85	0.75 probability of €-50	0.25 probability of €-100

The interval of table C.9 is €2.5 per row. Starting at €-50 and continuing to €-85.

Table C.10: Choice list for the outcome $(200; 100)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €160	0.90 probability of €200	0.10 probability of €100
Certainty of €162.5	0.90 probability of €200	0.10 probability of €100
...	...	...
Certainty of €192.5	0.90 probability of €200	0.10 probability of €100
Certainty of €195	0.90 probability of €200	0.10 probability of €100

The interval of table C.10 is €2.5 per row. Starting at €160 and continuing to €195.

Table C.11: Choice list for the outcome $(-100; -200)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-100	0.95 probability of €-100	0.05 probability of €-200
Certainty of €-102	0.95 probability of €-100	0.05 probability of €-200
...	...	...
Certainty of €-126	0.95 probability of €-100	0.05 probability of €-200
Certainty of €-128	0.95 probability of €-100	0.05 probability of €-200

The interval of table C.11 is €2 per row. Starting at €-100 and continuing to €-128.

Table C.12: Choice list for the outcome $(400; 0)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €360	0.99 probability of €400	0.01 probability of €0
Certainty of €362.5	0.99 probability of €400	0.01 probability of €0
...	...	...
Certainty of €392.5	0.99 probability of €400	0.01 probability of €0
Certainty of €395	0.99 probability of €400	0.01 probability of €0

The interval of table C.12 is €2.5 per row. Starting at €360 and continuing to €395.

Table C.13: Choice list for the outcome $(400; -100)$ used in the general Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €100	0.50 probability of €400	0.50 probability of €-100
Certainty of €120	0.50 probability of €400	0.50 probability of €-100
...	...	...
Certainty of €360	0.50 probability of €400	0.50 probability of €-100
Certainty of €380	0.50 probability of €400	0.50 probability of €-100

The interval of table C.13 is €20 per row. Starting at €100 and continuing to €380.

C.1.2. Flood Risk related risk preferences

This section shows the questions that were used in the FLM related choice problems used in the elicitation of risk preferences that were posed to participants in the study. Each entry marked with an **o** in table C.14 corresponds to a choice list that was asked to the participants.

Table C.14: Matrix containing the possible combinations of prospects that were used asked to participants in the questionnaire aimed at eliciting risk preferences for FLM related outcomes

	Probability attached to the higher outcome										
Outcomes	0.01	0.05	0.1	0.25	0.4	0.5	0.6	0.75	0.9	0.95	0.99
(0; -25)						**o**					
(0; -100)			**o**								
(0; -400)	**o**										
(-100; -50)								**o**			
(-100; -200)										**o**	
(25; 0)							**o**				
(100; 0)				**o**							
(400; 0)		**o**									**o**
(100; 50)					**o**						
(200; 100)									**o**		

Each of the entries in table C.14 corresponds to a choice problem posed to the participants. Participants were asked whether they preferred a sure outcome or the gamble (represented by outcomes and probabilities in table C.14). The general formulation of these questions is:

Do you prefer to buy insurance for €C or have a $100 \cdot p$% chance of losing €x_1 and a $(1 - p) \cdot 100$ % chance of losing €x_2 as a result of flood damages?

Where x_1 and x_2 are the outcomes shown in table C.14. The tables presented below contain all the 12 questions that were asked to the participants for these FLM related choice problems.

Table C.15: Choice list for the outcome $(0; -50,000)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-200	0.01 probability of €-50,000	0.99 probability of €0
Certainty of €-250	0.01 probability of €-50,000	0.99 probability of €0
...	...	...
Certainty of €-600	0.01 probability of €-50,000	0.99 probability of €0
Certainty of €-650	0.01 probability of €-50,000	0.99 probability of €0

The interval of table C.15 is €50 per row. Starting at €-200 and continuing to €-650.

Table C.16: Choice list for the outcome $(0; -3,000)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-1,000	0.75 probability of €-3,000	0.25 probability of €0
Certainty of €-1,200	0.75 probability of €-3,000	0.25 probability of €0
...	...	...
Certainty of €-2,600	0.75 probability of €-3,000	0.25 probability of €0
Certainty of €-2,800	0.75 probability of €-3,000	0.25 probability of €0

The interval of table C.16 is €200 per row. Starting at €-1,000 and continuing to €-2,800.

Table C.17: Choice list for the outcome $(0; -500)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-400	0.95 probability of €-500	0.05 probability of €0
Certainty of €-410	0.95 probability of €-500	0.05 probability of €0
...	...	...
Certainty of €-480	0.95 probability of €-500	0.05 probability of €0
Certainty of €-490	0.95 probability of €-500	0.05 probability of €0

The interval of table C.17 is €10 per row. Starting at €-400 and continuing to €-490.

Table C.18: Choice list for the outcome $(0; -8,000)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-500	0.10 probability of €-8,000	0.90 probability of €0
Certainty of €-600	0.10 probability of €-8,000	0.90 probability of €0
...	...	...
Certainty of €-1,300	0.10 probability of €-8,000	0.90 probability of €0
Certainty of €-1,400	0.10 probability of €-8,000	0.90 probability of €0

The interval of table C.18 is €100 per row. Starting at €-500 and continuing to €-1,400.

Table C.19: Choice list for the outcome $(0; -1, 500)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-100	0.50 probability of €-1,500	0.50 probability of €0
Certainty of €-200	0.50 probability of €-1,500	0.50 probability of €0
...	...	...
Certainty of €-900	0.50 probability of €-1,500	0.50 probability of €0
Certainty of €-1,000	0.50 probability of €-1,500	0.50 probability of €0

The interval of table C.19 is €100 per row. Starting at €-100 and continuing to €-1,000.

Table C.20: Choice list for the outcome $(0; -250)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €-70	0.40 probability of €-250	0.60 probability of €0
Certainty of €-80	0.40 probability of €-250	0.60 probability of €0
...	...	...
Certainty of €-150	0.40 probability of €-250	0.60 probability of €0
Certainty of €-160	0.40 probability of €-250	0.60 probability of €0

The interval of table C.20 is €10 per row. Starting at €-70 and continuing to €-160.

After these first six questions, participants were presented with the following information: *Suppose now that you have bought a particular insurance for the previous year. There were no floods last year and as such your insurance company did not make any costs and offers you an option for a partial refund. The insurance company gives you the following two options:*

1. *You will get a certain discount on this year's insurance policy.*
2. *You will get a (higher) reimbursement on your bank account if there is no flooding of your home. Whether or not there will be a flooding is uncertain, but the insurance company gives you likelihoods in the form of chances.*

In the following questions you are asked to make state your preference between the two options, with varying monetary amounts and varying chances.

After which they are presented with the following choice problems, shown in table form.

Table C.21: Choice list for the outcome $(100; 0)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €75	0.99 probability of €100	0.01 probability of €0
Certainty of €77.5	0.99 probability of €100	0.01 probability of €0
...	...	...
Certainty of €95	0.99 probability of €100	0.01 probability of €0
Certainty of €97.5	0.99 probability of €100	0.01 probability of €0

The interval of table C.21 is €2.5 per row. Starting at €75 and continuing to €97.5.

Table C.22: Choice list for the outcome $(350; 0)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €50	0.25 probability of €350	0.75 probability of €0
Certainty of €60	0.25 probability of €350	0.75 probability of €0
...	...	...
Certainty of €130	0.25 probability of €350	0.75 probability of €0
Certainty of €140	0.25 probability of €350	0.75 probability of €0

The interval of table C.22 is €10 per row. Starting at €50 and continuing to €140.

Table C.23: Choice list for the outcome $(1,000; 0)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €10	0.05 probability of €1,000	0.95 probability of €0
Certainty of €20	0.05 probability of €1,000	0.95 probability of €0
...	...	...
Certainty of €90	0.05 probability of €1,000	0.95 probability of €0
Certainty of €100	0.05 probability of €1,000	0.95 probability of €0

The interval of table C.23 is €10 per row. Starting at €10 and continuing to €100.

Table C.24: Choice list for the outcome $(600; 0)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €500	0.90 probability of €600	0.10 probability of €0
Certainty of €510	0.90 probability of €600	0.10 probability of €0
...	...	...
Certainty of €580	0.90 probability of €600	0.10 probability of €0
Certainty of €590	0.90 probability of €600	0.10 probability of €0

The interval of table C.24 is €10 per row. Starting at €500 and continuing to €590.

Table C.25: Choice list for the outcome $(5,000; 0)$ used in the FLM Risk preferences

Certainty	Gamble	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €2,300	0.60 probability of €5,000	0.40 probability of €0
Certainty of €2,400	0.60 probability of €5,000	0.40 probability of €0
...	...	...
Certainty of €580	0.60 probability of €5,000	0.40 probability of €0
Certainty of €590	0.60 probability of €5,000	0.40 probability of €0

The interval of table C.25 is €100 per row. Starting at €2,300 and continuing to €3,200.

Table C.26: Choice list for the outcome $(400; -100)$ used in the FLM Risk preferences

Certainty	**Gamble**	
Pay-out	**Pay-out possibility 1**	**Pay-out possibility 2**
Certainty of €100	0.50 probability of €400	0.50 probability of €-100
Certainty of €120	0.50 probability of €400	0.50 probability of €-100
...	...	...
Certainty of €280	0.50 probability of €400	0.50 probability of €-100
Certainty of €300	0.50 probability of €400	0.50 probability of €-100

The interval of table C.26 is €20 per row. Starting at €100 and continuing to €300.

D

Additional recommendations

D.1. Time-dependent risk preferences

In this research, several models to explain human behaviour have been discussed. These models all relied on choice lists or prospects with the assumption of direct pay-outs. However, in reality the outcomes do not have to occur at the same point in time. The outcomes might very well be spread out over a certain period of time. In this case the monetary options should be discounted to present time to be able to make a fair comparison between alternatives. This section introduces several models that show how *intertemporal choices*, which are time-dependent risk preferences, can be made. Before starting with the models, several definitions are given that are used throughout this section.

Decision time: The time at which a decision is made.

Consumption time: The time at which the consequence of a decision is experienced.

Temporal distance: The time between the consumption time and the decision time.

Impatience: Future utilities get a lower weight than present utilities, even when properly discounted with any form of monetary discounting.

Discounted utility

The most common model for modeling intertemporal choice is *discounted utility*. This model discounts the utilities via a discount function, which is dependent on the time at which an event occurs. The model is shown in equation D.1.

$$u^t(c_t, ..., c_T) = D(0) \cdot u(c_t) + ... + D(T-t) \cdot u(c_T) \tag{D.1}$$

Where:
c_t: The consumption at time t
$D(t)$: The discount function at time t
$u(c_t)$: The utility of the consumption at time t.

Discounted utility has several types of independence that must be assumed for the theory to work:

1. *Utility independence*: The order of the discounted utilities is not relevant.
2. *Consumption independence*: The utility in period t depends solely on the consumption at time t.
3. The utility function is independent of time.
4. The discount function is independent of the type of consumption.

Discounted utility can be used with any type of discount function. The discount function $D(\cdot)$ can take several forms, of which a selection will be discussed.

Constant discounting
The simplest form of discounting is *constant discounting*, more commonly known as exponential discounting. This form of the discount function is used in most practical applications. It has the following functional form:

$$D(t) = \frac{1}{(1+r)^t} = \delta^t$$

where:
r: The discount rate, typically it is assumed that $r \in [0, \mathrm{inf})$
δ: The discount factor, defined as: $\delta = \frac{1}{1+r} \in (0,1)$

Constant discounting has some attractive properties, as it implies constant impatience which in turn has the result of stationary time preferences. Consider two pay-outs, x and y at times s and t, such that the choice is between the prospects $(s : x)$ and $(t : y)$. Constant discounting implies that if x is weakly preferred to y that:

$$(s : x) \geq (t : y) \rightarrow \delta^s u(x) \geq \delta^t u(y)$$
$$\delta^{s+\tau} u(x) \geq \delta^{t+\tau} u(y) \rightarrow (s + \tau : x) \geq (t + \tau : y)$$

Hence there is no change in preference if both options are moved a time step τ to the future, which is the definition of constant impatience. Individuals keep their preferences in the same order if the temporal distance between two prospects remains the same, even if they are shifted into time.

In reality, most individuals tend to exhibit decreasing impatience. This implies that as options lie further in the future, individuals will become less sensitive to the temporal distance between options and will opt for the higher pay-out. A consequence of decreasing impatience is that time inconsistency will arise. To incorporate this into intertemporal models of behavioural economics, *hyperbolic discounting* was introduced. This broad terms encompasses a type of discount function where decreasing impatience is taken into account. Four specific functions of hyperbolic discounting are discussed below.

Hyperbolic discounting
Another form of modeling the discount function is by means of *hyperbolic discounting*. This form of discounting is able to incorporate decreasing impatience, whereas constant discounting is not able to model this behaviour. There is an elegant link of human intuitive behaviour under uncertainty over time with Bayesian updating. Sozou (1998) shows that hyperbolic time-preferences can be explained by an uncertain underlying hazard rate, with an exponential prior distribution. There are four types of hyperbolic discounting that will be discussed in this section, starting with Quasi-hyperbolic discounting.

Quasi-hyperbolic discounting
This form of discounting is also known as the *beta-delta model*. It incorporates decreasing impatience only if time $t = 0$ is involved, but consists solely of constant impatience if only future periods are involved. The discount function has the following functional form:

$$D(t) = \begin{cases} 1 & \text{if } t = 0 \\ \beta\delta^t & \text{if } t > 0 \end{cases}$$

where:
β: The present-bias parameter, $\beta \in (0, 1)$
δ: Discount factor, $\delta \in (0, 1)$

The introduction of the present-bias parameter, coupled with the discount factor being equal to one at $t = 0$ implies that:
if $(0 : x) \sim (t : y)$ then $(\tau : x) \prec (t + \tau : y)$
if $s < t$ and $(s : x) \sim (t : y)$ then $(s + \tau : x) \sim (t + \tau : y)$

These are called *present-biased* preferences and they are inherent to this model.

Generalized hyperbolic discounting
This form of discounting solely incorporates decreasing impatience. The discount function is given as:

$$D(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}} \qquad \text{with } \alpha, \beta > 0$$

Which implies that:
if $(0 : x) \sim (t : y)$ then $(\tau : x) \prec (t + \tau : y)$
if $s < t$ and $(s : x) \sim (t : y)$ then $(s + \tau : x) \prec (t + \tau : y)$

Constant absolute decreasing impatience
This form of discounting can incorporate any degree of decreasing and also increasing impatience. The discounting function is given as:

$$D(t) = \begin{cases} ke^{re^{-ct}} & \text{for } c > 0 \\ ke^{-rt} & \text{for } c = 0 \\ ke^{-re^{-ct}} & \text{for } c < 0 \end{cases}$$

Where c is the degree of decreasing impatience.

Constant relative decreasing impatience
Similar to the previous model, this form of discounting can incorporate any degree of decreasing as well as increasing impatience. The discount function is given as:

$$D(t) = \begin{cases} ke^{rt^{1-d}} & \text{for } d > 1 \\ kt^{-r} & \text{for } d = 1 \\ ke^{-rt^{1-d}} & \text{for } d < 1 \end{cases}$$

Where d/t is the degree of decreasing impatience.

To illustrate the use of these models, consider the following example about eliciting time preferences for two of the four shown models.

Example
Suppose an individual is presented the following four choices:
A: Receive €50 in 1 month
B: Receive €60 in 2 month
C: Receive €50 in 2 month
D: Receive €60 in 3 month

And it is given that he or she is indifferent between option C and D. The question then becomes what the preferences of the individual between A and B are for different models.

If we assume that the behaviour can be modelled by Quasi-hyperbolic discounting, it can be deduced that, due to constant impatience, the preferences shifted over a time period τ should remain the same. Hence the individual should also be indifferent between option A and B.

If we assume that the behaviour can be modelled by Generalized hyperbolic discounting, the results change. As this type of discounting is known to have decreasing impatience, e.g. individuals are more willing to wait for a longer period to receive a higher pay-out. In this case, given that the individual is indifferent between C and D, it must follow that the decreased waiting time result in a preference of A over B.

Quantifying time preferences
To quantify the time preferences and estimate the parameters in the different discounting models, a similar approach as the elicitation of utility can be used. This entails the presentation of a certain type of choice list that contains two options. However, unlike the choice lists used in the elicitation of utilities, these lists have the data of pay-out rather than the probability of pay-out incorporated in the information to consider. Often two choices in different points in time are needed. For most models it is also

required that at least one of the options under consideration is an immediate pay-out, such that the time $t = 0$ is included. As most of these models have two (or more) parameters, at least two (or more) preferences have to be elicited. In general for k parameters at least k preferences have to be elicited, which requires the consideration of $2k$ options, as each preference has to have at least two options.

To illustrate how these parameters can be estimated, consider the following example based on the previous example.

Example
Assume that the previous example is still under consideration. To quantify the parameters of the Quasi-hyperbolic discount function, an additional data point containing the point $t = 0$ is needed. Let us discard the finding of A and B and assume that in a further round of questioning, it is found that the individual is indifferent between the following two options:
A': Receive €35 immediately
B': Receive €45 in 1 month

If we assume a linear utility function $u(x) = x$ for both models, then it follows that for the Quasi-hyperbolic discount function:

$$35 \cdot 1 = 45 \cdot \beta\delta^1$$

$$50 \cdot \beta\delta^2 = 60 \cdot \beta\delta^3$$

From which it is easily found that $\delta = \frac{5}{6}$ and therefore $\beta = \frac{35}{45} \cdot \frac{6}{5} = \frac{14}{15}$.

Note that if the utility function is also unknown or the parameters need to be estimated, additional degrees of freedom are added. As such, more data points need to be gathered to account for this lack of information. This is usually done by adding more questions in the form of choice lists to provide the additional data points.

D.1.1. Examples of choice problems

This section shows the elicitation procedure for time dependent risk preferences for both the general risk preferences and the risk preferences related to Flood Risk management.

General time dependent risk preferences

This section shows the questions related to general risk preferences that were posed to participants in the study. Each entry of table D.1 corresponds to a choice list that was asked to the participants.

Table D.1: Matrix containing the possible combinations of prospects asked to participants

	Outcomes (€)									
Time horizon (months)	-10	-25	-50	-100	-400	10	25	50	100	400
(0,1)										
(1,2)										
(0,6)										
(6,12)										

The outcomes that are listed are for the first time horizon. The time horizons are denoted as $(t_1, t_2) = [(0, 1); (1, 2); (0, 6); (6, 12)]$ to keep the length of the report limited. Again, for each table that is presented the first two and last two choices are shown to indicate the interval range and the length of the outcome space.

Table D.2 is for the negative prospects. For these prospects a logical endpoint is the zero point as otherwise waiting would result in a reversion of the outcomes, which is neither logical nor desirable for the estimation of parameters. The individuals would be asked the question:

Which of the following two options would you prefer?

Table D.2: Choice list for the outcome €-10 used in the time dependent risk preferences

Time horizon 1 (t_1)	**Time horizon 2** (t_2)
€-10	€-10
€-9	€-10
..	...
€-1	€-10
€0	€-10

The interval of table D.2 is €1 per row. Starting at €-10 and continuing to €0.
For the negative prospects, the later time point corresponds to the largest value, such that the direct pay-out is lower than the pay-out for which an individual must wait. The same interval for each outcome is used as for the negative prospects. To limit the length of the report, only the table for the outcome 10 is given in table D.3. The other outcomes are constructed in a similar manner.

Table D.3: Choice list for the outcome €10 used in the time dependent risk preferences

Time horizon 1 (t_1)	**Time horizon 2** (t_2)
€0	€10
€1	€10
..	...
€9	€10
€10	€10

Choice problems like the ones proposed here can be constructed for each entry of table D.1.

D.2. Inclusion of heuristics, biases and fallacies

Doing operations, such as estimating probabilities or judging outcomes, can be a hard or tedious procedure. Sometimes there is no time to evaluate each possible outcome, as a decision has to be made quickly. This was particularly the case during most human development. Threats needed to be analysed swiftly to ensure survival. As a result of the need for quick decisions, humans developed several shortcuts to evaluate situations and make decisions.

In their paper, Tversky and Kahneman (1974) show that people rely on a limited number of principles which reduce the task of evaluating and assessing probabilities and to simplify operations. Often, these so-called *heuristics* are useful and provide a short-cut to the right answer, however they can also cause severe systematic errors. These heuristics are the result of biases in judgements.

Kahneman and Tversky relate the subjective assessment of probability to the subjective assessment of physical quantities such as size or distance. Take for example the judgement of distance. If an object is blurry, it is often equated with being far away. Although this heuristic holds in general, heat, water or foggy windows can create a similar effect for objects that are relatively nearby, leading to a judgement that is incorrect.

This section will introduce three mental shortcuts that are both prevalent in the general population and are of interest to flood risk management. It starts by discussing the *representativeness* heuristic, before moving to the *availability* bias. Finally, the concepts of *Adjustment* and *Anchoring* are discussed to conclude the section.

D.2.1. Representativeness

The representativeness heuristic is a mental shortcut used by individuals to draw inference on the relation between two objects. Kahneman and Tversky identified several shortcuts that individuals make when making decisions that involve probabilities and as such, also risk. These shortcuts can be divided into three main categories regarding the *prior probability*, *sample size* and *the general misconception of chance*. Each of these three categories will be explained below.

Insensitivity to prior probability
Probabilities that need be judged are often comprised of two parts, the prior probability of an occurrence and some additional information that is specific for that instance of occurrence. This prior probability can be seen as a sort of population average of a certain trait or event happening. The specific information contains any additional information on the probability of the specific trait or event. However, when judging probabilities individuals often forget to take prior probabilities into account when assessing the likelihood of an event. The negligence of these prior probabilities can lead to over- or underestimation of the actual probabilities.

A famous example of such an insensitivity to prior probability is the *Linda problem*[1] (Tversky, 2002). As the second option presented in the Linda problem is a subset of the first problem, it can by definition never be more probable than the full set. This experiment demonstrates that individuals evaluate the likelihood that a description fits by the degree to which the description was representative of the stereotype that was painted in the example, even when there can be no mathematical ambiguity as to which of the two descriptions is more probable.

Insensitivity to sample size
This example has an elegant link to the notion of Bayesian belief updating, discussed in a later section of the literature research. Individuals tend to intuitively judge posterior odds almost exclusively by the sample proportion rather than by the sample size. In reality, the posterior odds should be a combination of prior believe and the observed data, in which the sample size determines the ”credibility” of the data with respect to the prior. To illustrate this fallacy, consider this example.

The following problem was posed to individuals: Consider an urn filled with balls, one third are a certain colour and two thirds are another colour. An individual has drawn five balls from the turn and found four red balls and one white ball. Another individual has drawn twenty balls and found and found twelve red balls and eight white balls. The question is which individual should be more confident that the turn contains one third white balls and two thirds red balls?

Most individuals feel that the first sample of five balls provides stronger evidence for the hypothesis that the turn is predominantly red rather than white, as the observed fraction of red balls is larger. In reality however, the correct posterior odds are eight to one for the draw of five balls and sixteen to one for the draw with twenty balls.

Misconception of chance
More commonly known as the *Gambler's fallacy*, this heuristic is based on the expectation by individuals that a sequence of events generated by a random process will represent the essential characteristics of that process. The main reason for this is that *chance* is commonly viewed as a self-correcting process. The deviations towards one direction are assumed to induce a deviation in the opposite direction, thus restoring the equilibrium. However, in reality there is no ”correction”, there is merely dilution of the deviation as the sample size increases.

An elegant example of this fallacy is seen by gamblers in a casino. If one is playing the roulette wheel and the game has a long streak of red, the tendency is to bet on black as *”it has to be due, given the red streak I just observed”*, while in reality each play at the wheel is independent of all the previous plays[2]. These examples are not just limited to outcomes but also to the perception of what is random. If a fair coin is flipped five times, the sequence H-H-T-H-T appears much more in line with what we think of as random than the sequence T-T-T-T-H despite both these sequences having equal probabilities of occurring.

[1] In this demonstration of the representativeness heuristic, individuals were told the following information:
Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
Subjects were subsequently asked which of the two is more probable: A. Linda is a bank teller. or B. Linda is a bank teller and is active in the feminist movement. Throughout multiple experiments, the majority opts for option B.

[2] In fact, the plays at the roulette wheel and the subsequent stochastic process can be considered a *Markov Process*.

D.2.2. Availability

The availability bias is a mental shortcut used by individuals to assess the frequency of the probability of occurrence by the ease with which instances of said occurrence come to mind. There are two separate aspects that contribute to this shortcut, the *availability heuristic* and the *recency bias*.

Availability heuristic

The first part is the availability heuristic operates on the notion that if an individual is able to recall an event easily, it will be given more weight than an event that can not easily be recalled. A consequence of this is that the probability with which events occur can be under- or overestimated. Kahneman and Tversky illustrate this heuristic in their 1973 paper with the following example: Participants were asked if, when a random word from the English language was taken, it is more likely for a word to start with a ’K’ or that the third letter of that word is a ’K’. The hypothesis was that words starting with a ’K’ could be more easily recalled and would therefore be favoured in frequency over the words that had ’K’ as a third letter. The results of the study supported this hypothesis as individuals tended to overestimate the frequency of words with ’K’ as a first letter with respect to words that have ’K’ as a third letter.

In the context of flood risk management, it is possible that experts let their judgement be clouded by the ease with which they can recall floods. As a large part of their work consists of thinking about and protecting against floods, the ease with which it can be recollected will on average almost certainly be higher than that of the average person. As such, there might be a tendency to overestimate the probability of flooding when either designing flood defences or when making decisions regarding evacuations of a looming flood.

Recency bias

The recency bias is a mental shortcut that favours recent events over historic ones. This entails that if a specific event has occurred more recent than another, it will be favoured as happening again, even if the two events have equal probabilities. This effect is most prominently present in the practice of investing. A recent streak of a particular asset, such as a stock or bond, rising in value will be viewed as more likely to rise in value in the future than another assets that has had a recent streak of decreasing in value. This tendency to think that streaks will continue due to the recency bias has cost many investors a fair amount of assets.

With regards to flood risk management, one could imagine that the judgement of the risk of a particular area flooding might be clouded by recent floods that were shown to the individual in charge of making a judgement. An example where the recency bias might come in to play is when the probability of a flood should be judged in the Netherlands as the floods of the province of Limburg in the summer of 2021 might invoke the recency bias.

D.2.3. Adjustment and Anchoring

When posing a choice or question to individuals, the expectation is that they formulate an answer independent of any previous questions or comments. To obtain a final answer to a question, individuals often make estimates by starting from an initial value that is subsequently adjusted to fit the situation. However, this initial value, baseline or starting can be influenced by the formulation of the problem or it may be the result of a partial computation. *Anchoring* often occurs as a cognitive bias whereby an individuals decisions are influenced by a particular reference point or ’anchor’, coupled with insufficient adjustment from the anchor. The result of this insufficient adjustment can be an estimation that is either too high or too low compared to the true value. There are two common instances of a lack of adjustment, that will be discussed below.

Incomplete computation

Anchoring can also occur when an individual bases his or her estimate on the result of a partially carried out calculation. A famous illustration of this heuristic is given by Kahneman and Tversky. Two groups of high-school students were asked to estimate, within a few seconds, one of the following two mathematical problems.

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \qquad or \qquad 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$$

As the students had to answer the question within a relatively short time frame, they often resorted to a partial computation of the sum to extrapolate the answer. The true answer to this sum is 40,320. The first group of students (corresponding to the left sum) that answered gave a median estimate of 2,250 while the second group of students (corresponding to the right sum) gave a median estimate of 512, resulting in an approximate fourfold difference in median estimate for the same problem.

The evaluation of conjunctive and disjunctive events
As a consequence of anchoring, individuals tend to overestimate the overall probability in conjunctive problems and tend to underestimate the overall probability in disjunctive problems. The consequences of this finding are two-fold. The first is that probabilities of successfully completing an undertaking, for example developing a new product, are often overestimated. This is because such undertakings typically have a conjunctive character: for the whole to succeed, each element in the series has to succeed. Conversely, the probabilities of evaluations of risks are often underestimated. This is because these types of undertakings typically have a disjunctive character: for the whole to succeed not all elements in the series has to succeed.

D.3. Bayesian approaches to decision under risk

This research has outlined some of the most prevalent and prominent models that behavioural economists use to explain human behaviour. These psychological models were devised to substitute the previous models that relied on the *homo oeconomicus* with models that better fit the observations of experiments. The field of behavioural sciences is in constant motion and new research is frequently published. A more recent model is that of *Bayesian Decision Theory*.

Developed by van Erp (2017), Bayesian decision theory is a successor of the psychological models that behavioural economists devised to better explain human behaviour. The theory uses a slight adjustment of *homo oeconomicus* and belongs to the class of neo-Bernoullian decision theories as it adopts Bernoulli's original utility function. It differs from the original function of Bernoulli in that it incorporates not only the most likely path (by means of expected value) but also the worst- and best-case scenarios. To understand the theory, Bernoulli's utility function and the algorithmic steps of Bayesian decision theory are introduced. After this, some remarks about the chosen position measure of the theory are presented.

D.3.1. Bernoulli's original utility function

Bernoulli started the derivation of his utility function by noting that an increment Δx in the initial wealth position x must correspond to some change in utility Δy. As such, the change in utility can be written as:

$$\Delta y = f(x = \Delta x) - f(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}\Delta x$$

Which in the limit leads to:

$$dy = \frac{f(x+dx) - f(x)}{dx}dx = f'(x)dx$$

Bernoulli then stated that the utilities of monetary increments Δx are invariant if the initial wealth position x is rescaled by some factor c. This invariance means that if someone has a fortune of €100,000 and a yearly income of €20,000, this person must have the same utility as someone who has a fortune of €50,000 and a yearly income of €10,000. The previous found expression should then become:

$$f(x+\Delta x) - f(x) = f(cx + c\Delta x) - f(cx) \longrightarrow \frac{f(cx+cdx) - f(cx)}{cdx}cdx = cf'(cx)dx$$

By substituting the found equations into one another, it can be found that:

$$f'(x) = cf'(cx)$$

This equality has the general solution

$$f'(x) = q\frac{1}{x} \longrightarrow dy = q\frac{dx}{x}$$

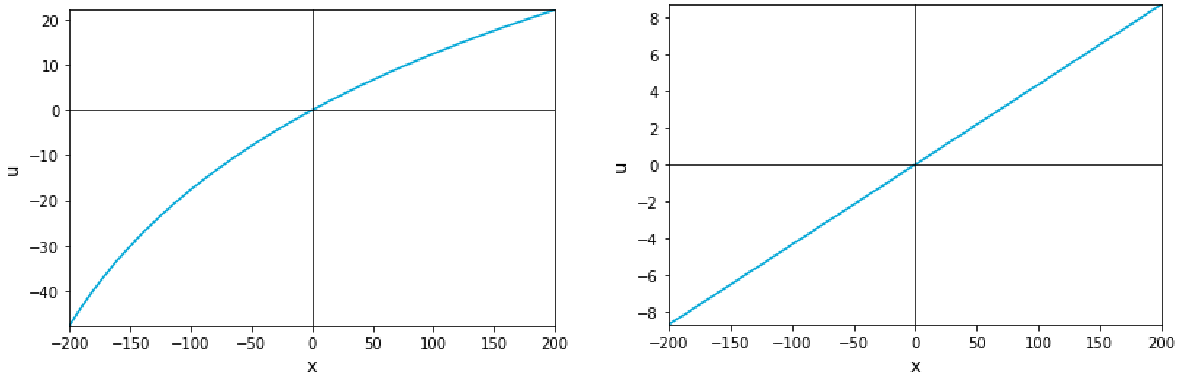
with q a constant, dy and dx the limit of utility and of monetary increments. This differential equation can be solved to find the following general solution:

$$\Delta y = q \log(\frac{x + \Delta x}{x})$$

Note that the equation above denotes the difference function as a utility function, rather than the utility function for a final asset position. This difference function Δy can also be written as a utility function $u(\cdot)$ that assigns a utility to the monetary increment Δx conditional on the current wealth position x. It follows that:

$$u(\Delta x \mid x) = q \log(\frac{x + \Delta x}{x})$$

Which can be used to map monetary increments given certain monetary positions to utilities. An example of this mapping for two different starting points of wealth can be seen in figure D.1.



(a) Initial wealth of $x = 300$

(b) Initial wealth of $x = 1,000,000$

Figure D.1: Bernoulli utility functions for two different starting positions of wealth

It can be seen from figure D.1a that if the gains or losses are relatively large in comparison to the initial wealth, the utility steeply declines or increases. Furthermore it can be seen that the property of loss aversion is preserved by the Bernoulli utility function, as the gradient of the utility is steeper in the negative domain of the graph, resulting in an assymetrical graph. For an initial wealth position that is significantly higher than the possible loss or gain, the utility function is symmetrical, as can be seen in figure D.1b. It can further be seen that the utility is near linear, indicating that a scaled version of loss or gain also results in a scaled loss or gain in utility, regardless of the size. This was clearly not the case in figure D.1a, where losses and gains near the initial wealth position create strong non-linear responses.

D.3.2. Algorithmic steps of Bayesian Decision theory

The algorithmic steps of the Bayesian decision theory as described by (van Erp et al., 2015) are threefold:

1. Construct outcome probability distributions by using the product and sum rules of Bayesian probability theory.
2. If the outcomes are of monetary nature, then by way of utility functions the utilities are mapped to the monetary outcomes of the outcome probability distributions.
3. The position of the resulting utility probability distributions is maximized.

With these three steps, the whole of Bayesian decision theory is described. Each of the three steps is explained in more detail below.

Elaborating on the firs step, let each problem of choice consist of a set of potential decisions $D_i = \{D_1, ..., D_n\}$. Furthermore, let each of these decisions D_i we might make give rise to a set of possible events $E_{j_i} = \{E_{1_i}, ..., E_{m_i}\}$. The events are then linked to the decisions by means of the conditional probabilities $P(E_{j_i} \mid D_i)$, which is the probability of observing E_{j_i} given that decision D_i was made. Furthermore, let each event allow for a set of potential outcomes, denoted by $O_{k_{j_i}} = \{O_{1_{j_i}}, ..., O_{p_{j_i}}\}$. Again, the outcomes can be associated to the events by means of the conditional probabilities $P(O_{k_{j_i}} \mid E_{j_i})$. This last expression can be used to obtain the conditional probability distribution of the outcomes on the decisions. This is done by marginalizing the bivariate probability distribution of an event and an outcome conditional on the decision taken.

$$\begin{aligned} P(O_{k_{j_i}} \mid D_i) &= \sum_{j_i=1}^{m_i} P(E_{j_i}, O_{k_{j_i}} \mid D_i) \\ &= \sum_{j_i=1}^{m_i} P(O_{k_{j_i}} \mid E_{j_i}) P(E_{j_i} \mid D_i) \end{aligned}$$

If the conditional probabilities of the outcomes on the decisions are known, the second step of the theory can be performed. The monetary outcomes can be mapped to utilities. For this step, Van Erp uses the Bernoulli utility functions such that we are left with the conditional probability distributions of the utilities, $P(u_i \mid D_i)$. Van Erp notes that the decision that results in outcomes being located most to the right are most advantageous to the decision maker, as they maximize utility.

This brings on the third step of the theory, maximizing the utility probability distribution. There is however a degree of freedom that has not been addressed. The position of a probability distribution is yet to be defined.

D.3.3. The position measure

After some iterations, such as taking the expected value as a position measure or the confidence bounds as a position measure, van Erp arrives at a combination of the sum of the confidence bounds plus the expected value as the best position measure. It follows that:

$$\frac{LB(u) + \mathbb{E}(u) + UB(u)}{3} = \begin{cases} \mathbb{E}(x), & LB(x) < a,\ UB(x) < b \\ \frac{a + 2\mathbb{E}(x) + k \cdot std(X)}{3} & LB(X) \leq a,\ UB(X) < b \\ \frac{2\mathbb{E}(x) - k \cdot std(x) + b}{3} & LB(x) > a,\ UB(x) \geq b \\ \frac{a + \mathbb{E}(x) + b}{3} & LB(x) \leq a,\ UB(x) \geq b \end{cases} \tag{D.2}$$

where a and b are the limits set on the lower- and upperbound respectively, defined as $a = \min(x_1, ..., x_n)$ and $b = \max(x_1, ..., x_n)$). $\mathbb{E}(x)$ denotes the expected value of x. The lower- and upperbound themselves are defined as:

$$LB(x) = \begin{cases} \mathbb{E}(x) - k \cdot std(x) & \text{if} \quad LB(x) > a \\ a & \text{if} \quad LB(x) \leq a \end{cases}$$

$$UB(x) = \begin{cases} \mathbb{E}(x) + k \cdot std(x) & \text{if} \quad UB(x) < b \\ b & \text{if} \quad LB(x) \geq b \end{cases}$$

where $std(x)$ is the standard deviation of x, given as:

$$std(x) = \sqrt{\sum_{i=1}^{n} p_i [x_i - \mathbb{E}(x)]^2}$$

and k is the chosen significance level of the bounds.

This position measure can be viewed as the average of the probabilistic worst, expected and best case scenario. This position measure makes a trade-off between either losses or gains in the probabilistic worst case scenario as well as the corresponding losses or gains in the probabilistic best case scenario.

E
Figures

E.1. Results

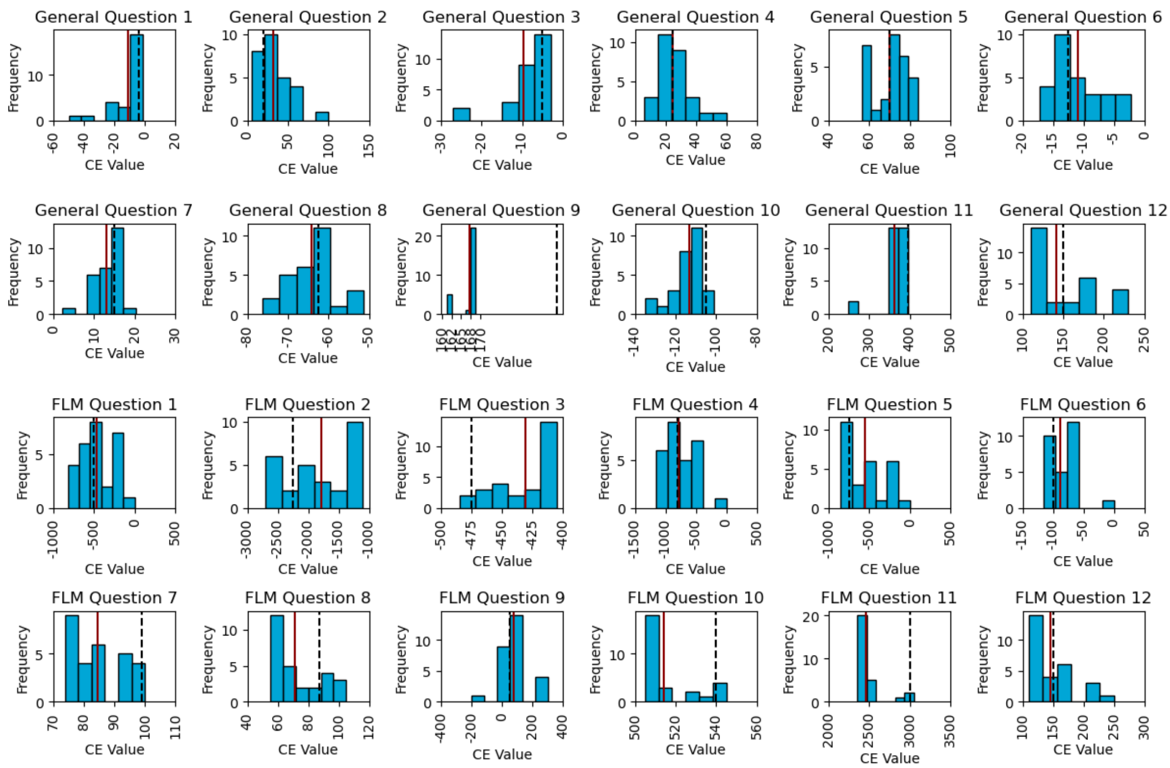


Figure E.1: Certainty equivalents of all Dutch participants, for all posed choice problems

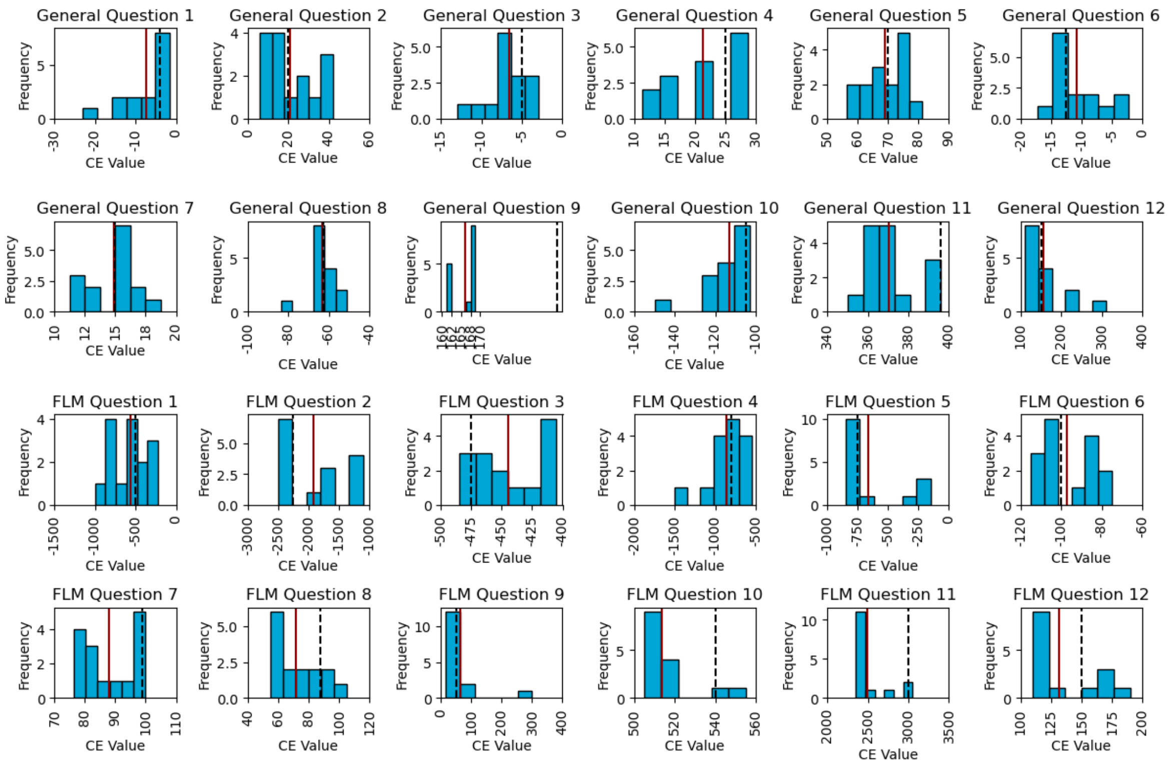


Figure E.2: Certainty equivalents of all participants from HKV, for all posed choice problems

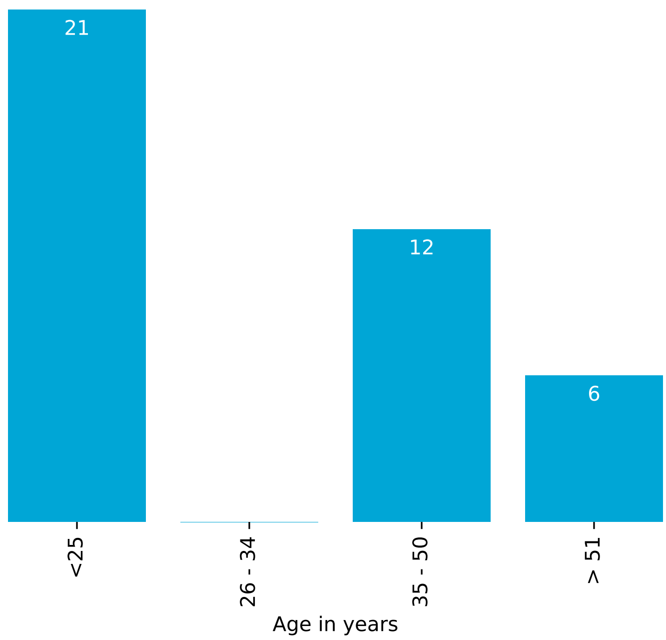
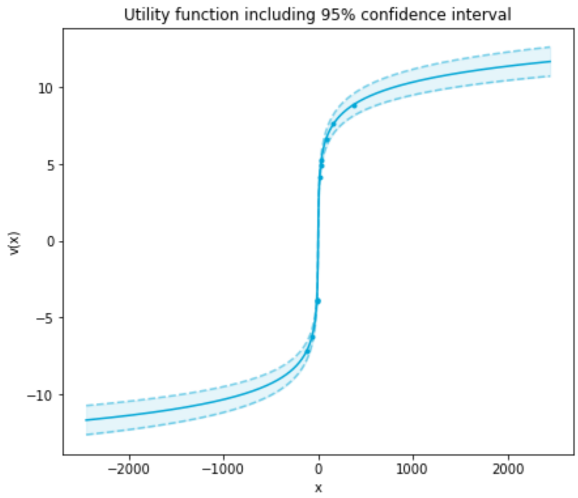
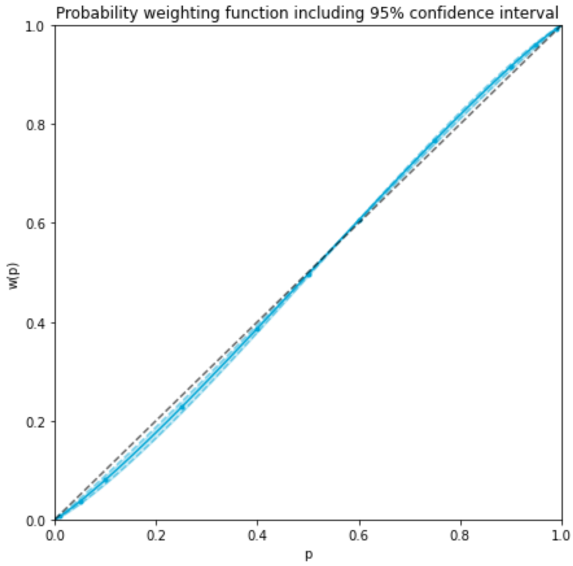


Figure E.3: Age distribution of participants

E.1.1. Utility- and Probability weighting functions

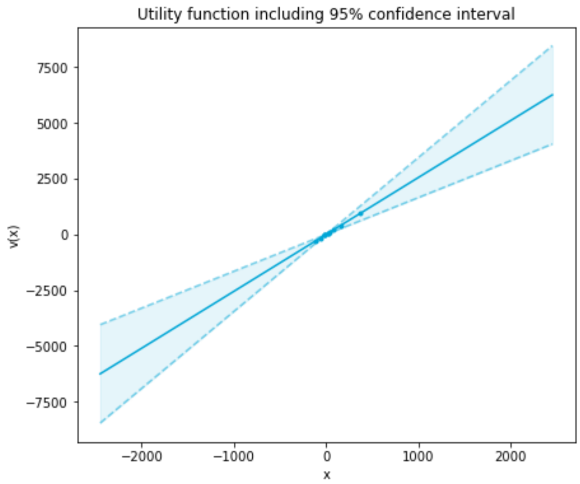


(a) Estimate of *logarithmic* utility function for all participant

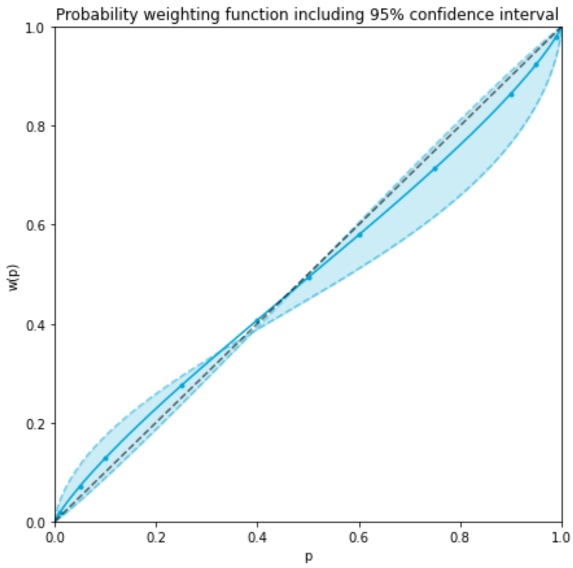


(b) Estimate of probability weighting function for all participants

Figure E.4: Estimate of probability weighting- and utility function including uncertainty for all participants based on the general choice problems using a logarithmic utility model

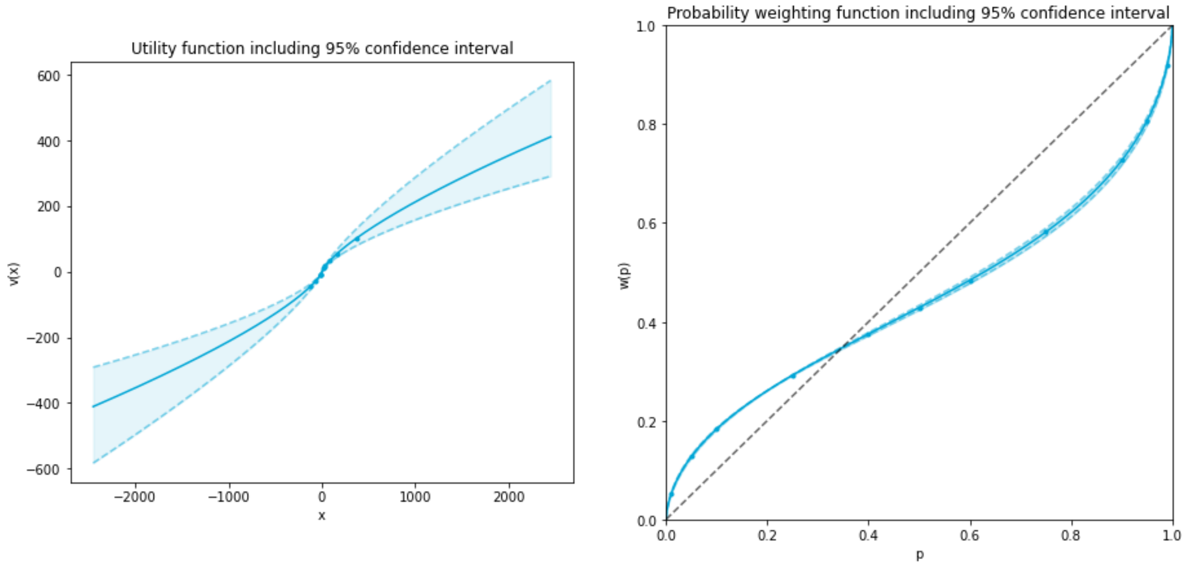


(a) Estimate of *linear* utility function for all participant



(b) Estimate of probability weighting function for all participants

Figure E.5: Estimate of probability weighting- and utility function including uncertainty for all participants based on the general choice problems using a linear utility model

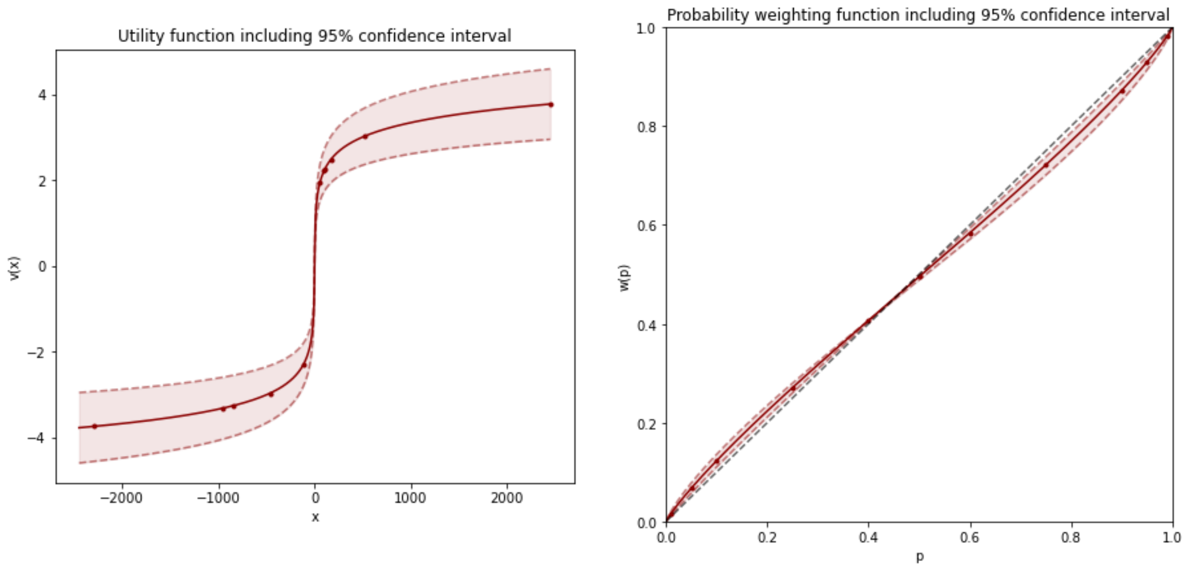


(a) Estimate of *iso-elastic* utility function for all participant

(b) Estimate of probability weighting function for all participants

Figure E.6: Estimate of probability weighting- and utility function including uncertainty for all participants based on the general choice problems using a iso-elastic utility model

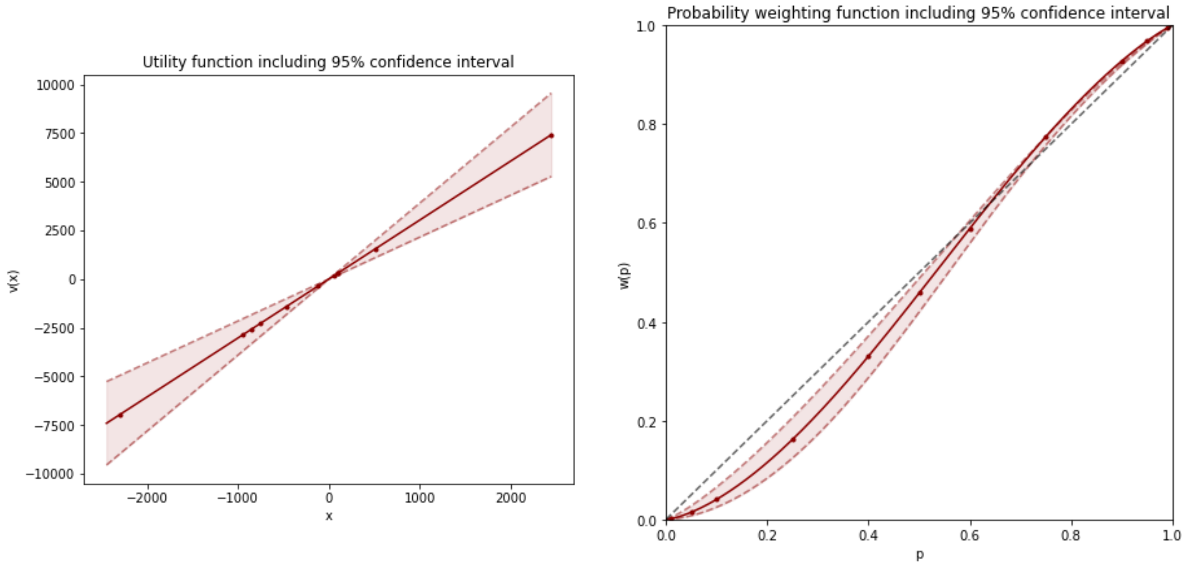
Results of Flood Risk Management related choice problems



(a) Estimate of *logarithmic* utility function for all participant

(b) Estimate of probability weighting function for all participants

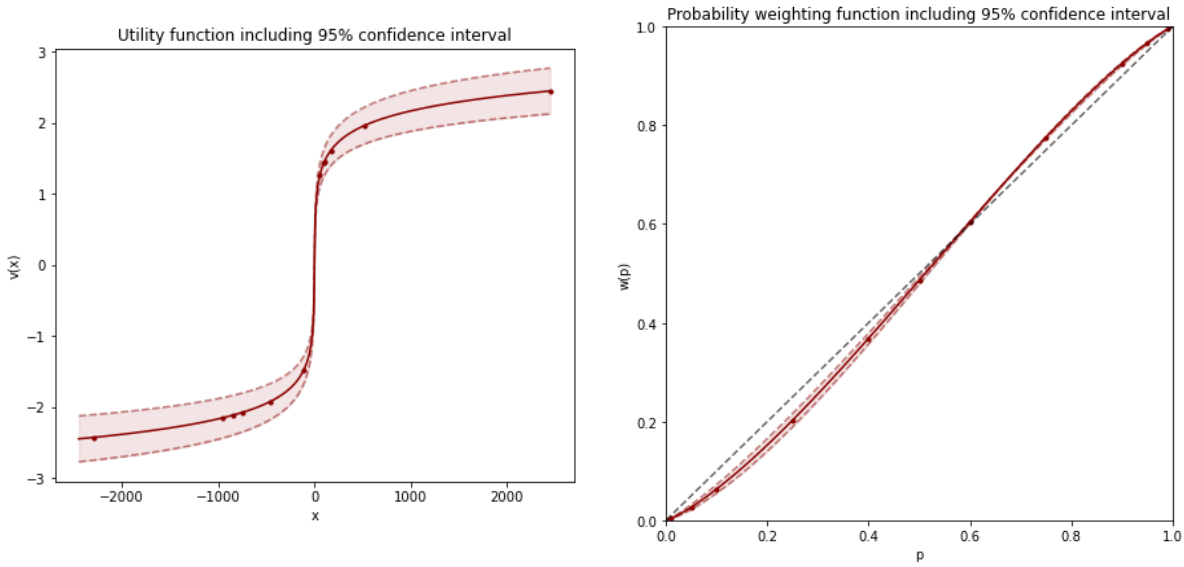
Figure E.7: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM choice problems using a logarithmic utility model



(a) Estimate of *linear* utility function for all participant

(b) Estimate of probability weighting function for all participants

Figure E.8: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM choice problems using a linear utility model

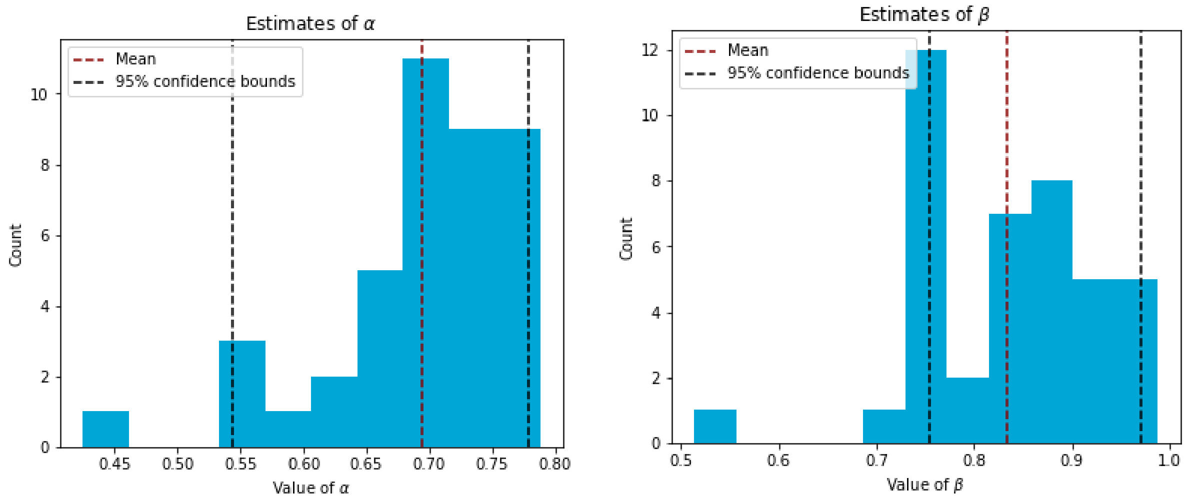


(a) Estimate of *iso-elastic* utility function for all participant

(b) Estimate of probability weighting function for all participants

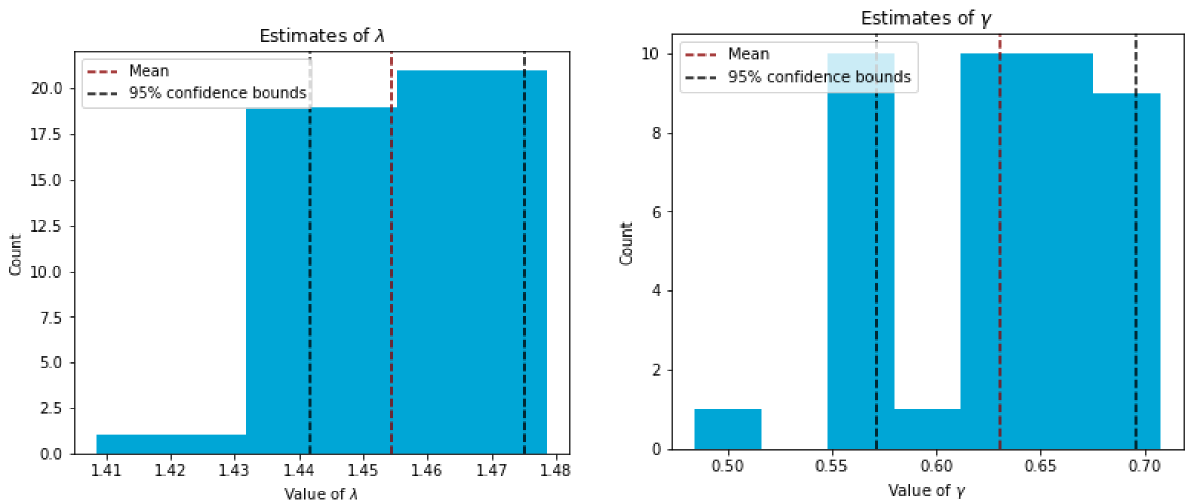
Figure E.9: Estimate of probability weighting- and utility function including uncertainty for all participants based on the FLM choice problems using a iso-elastic utility model

Parameter estimation of Power model



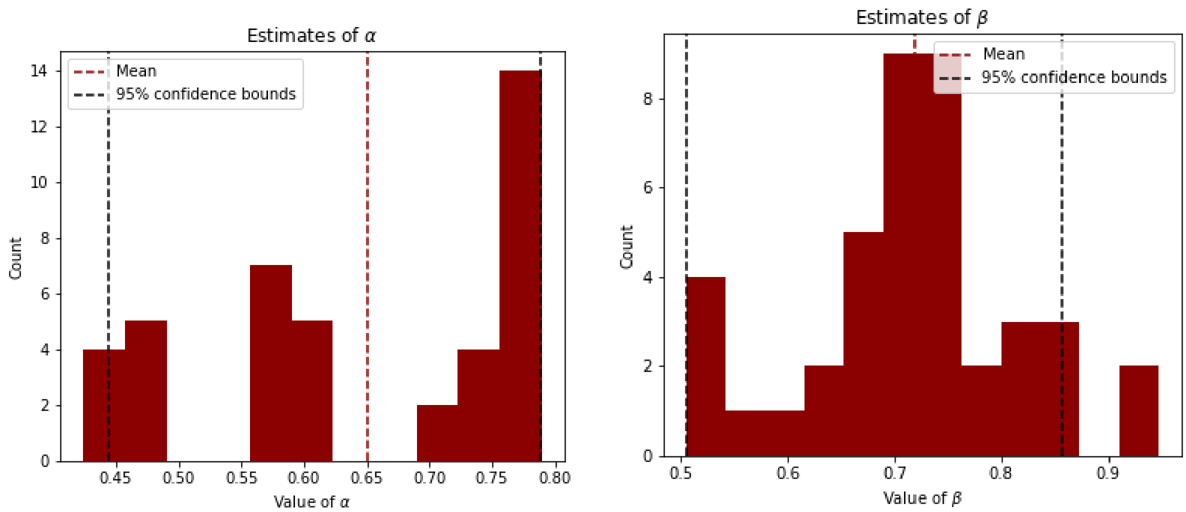
(a) Results of α estimates **(b)** Results of β estimates

Figure E.10: Histogram of parameter estimates (α & β) for all participants using the power model, based on the general choice problems



(a) Results of λ estimates **(b)** Results of γ estimates

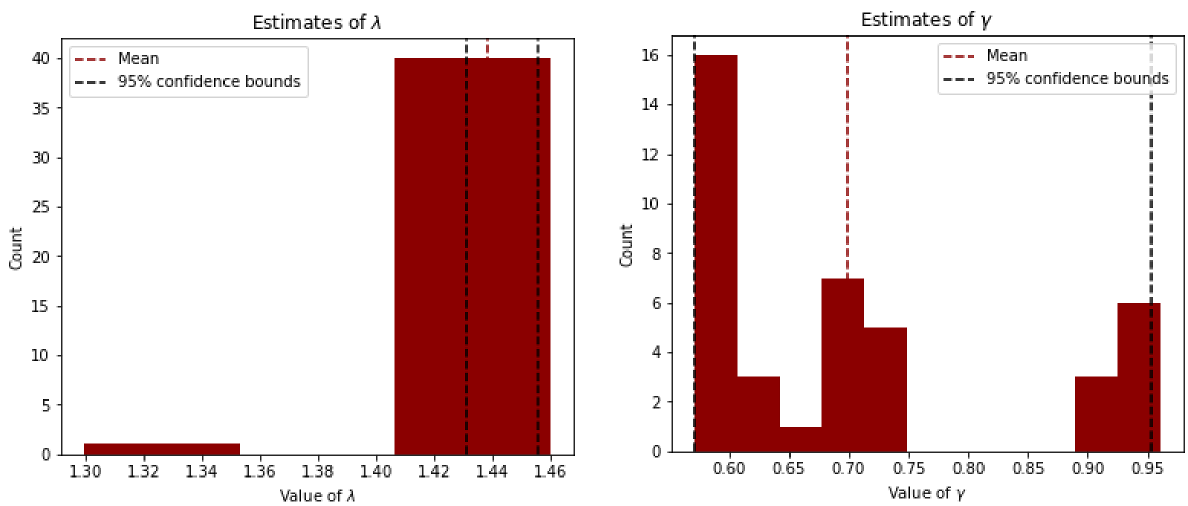
Figure E.11: Histogram of parameter estimates (λ & γ) for all participants using the power model, based on the general choice problems



(a) Results of α estimates

(b) Results of β estimates

Figure E.12: Histogram of parameter estimates (α & β) for all participants using the power model, based on the FLM related choice problems



(a) Results of λ estimates

(b) Results of γ estimates

Figure E.13: Histogram of parameter estimates (λ & γ) for all participants using the power model, based on the FLM related choice problems

E.1.2. Estimation of CE in comparison with actual CE

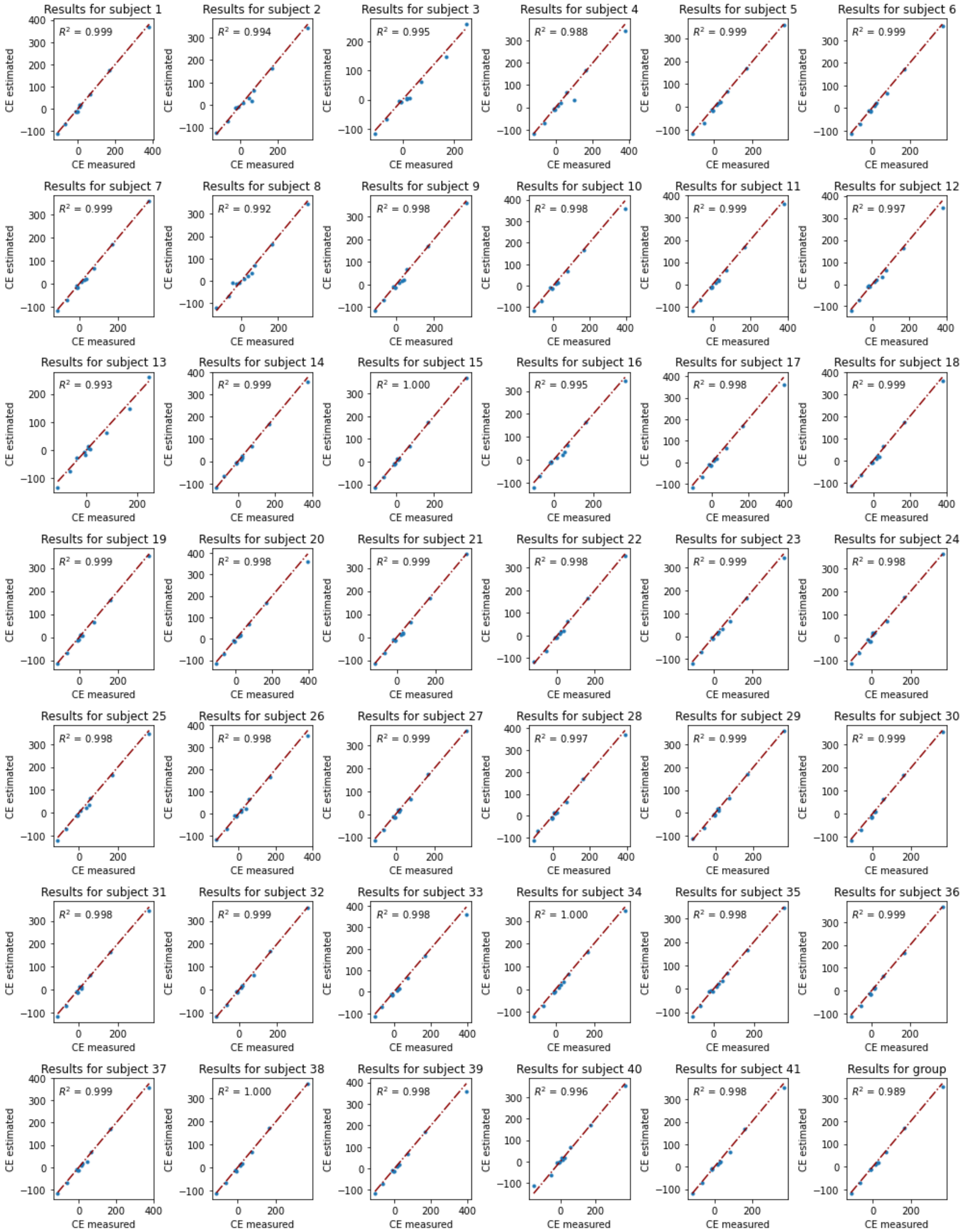


Figure E.14: Comparison of CE estimation with observed CE values for all participants using the *power* model, based on general choice problems

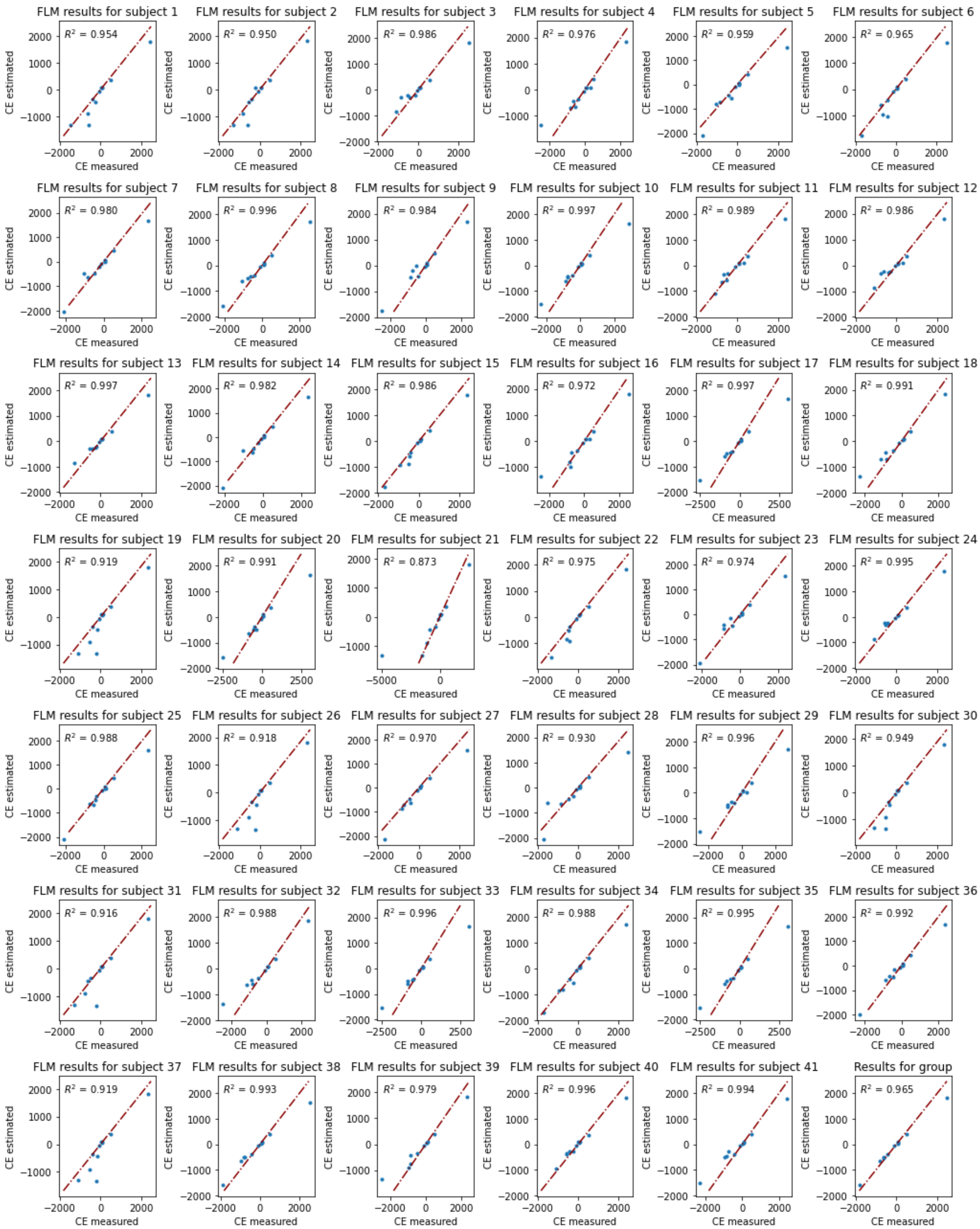


Figure E.15: Comparison of CE estimation with observed CE values for all participants using the *power* model, based on FLM related choice problems

Cross validated CE values

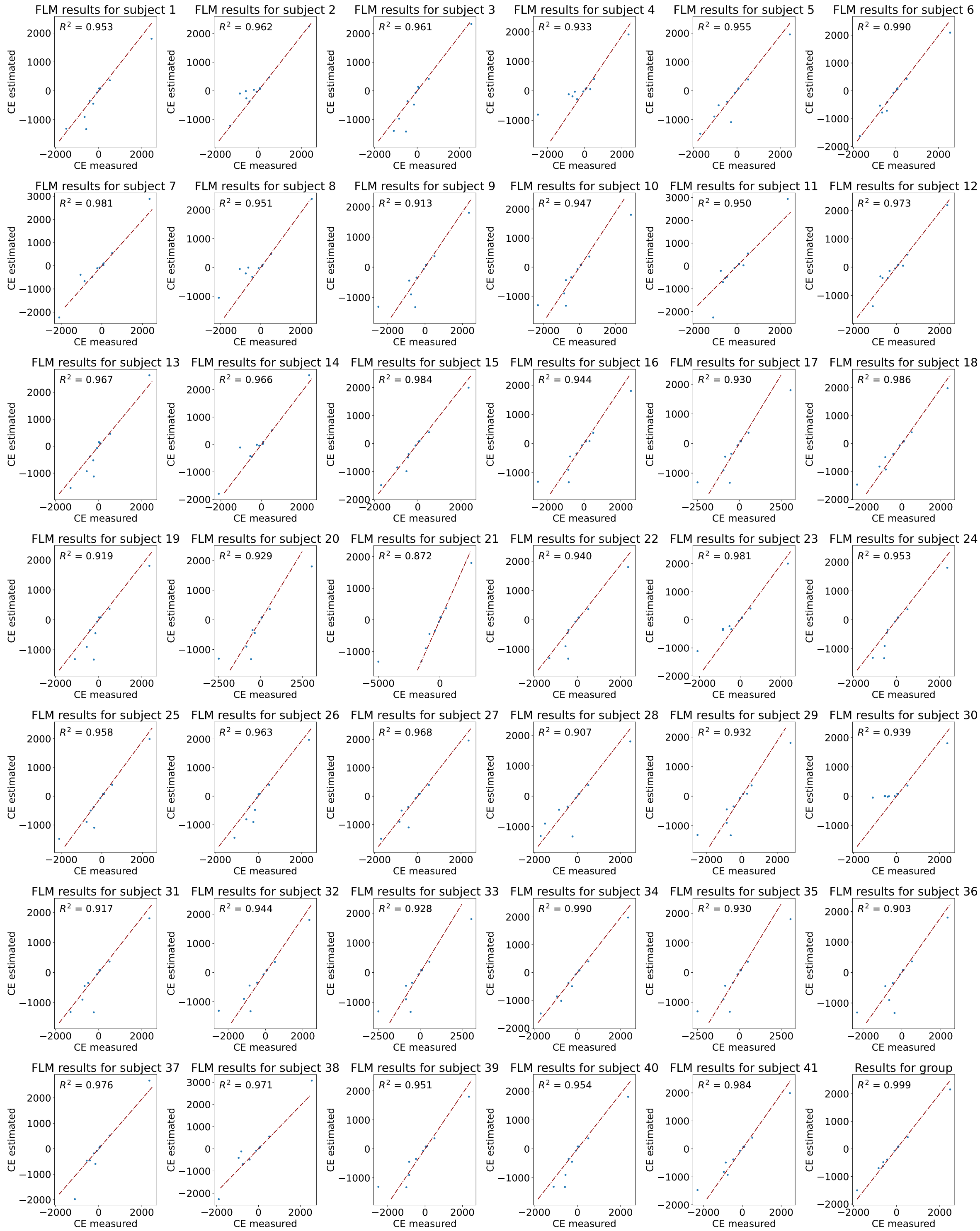


Figure E.16: Comparison of CE estimation via general risk preference parameters with observed FLM CE values for all participants using the *power* model

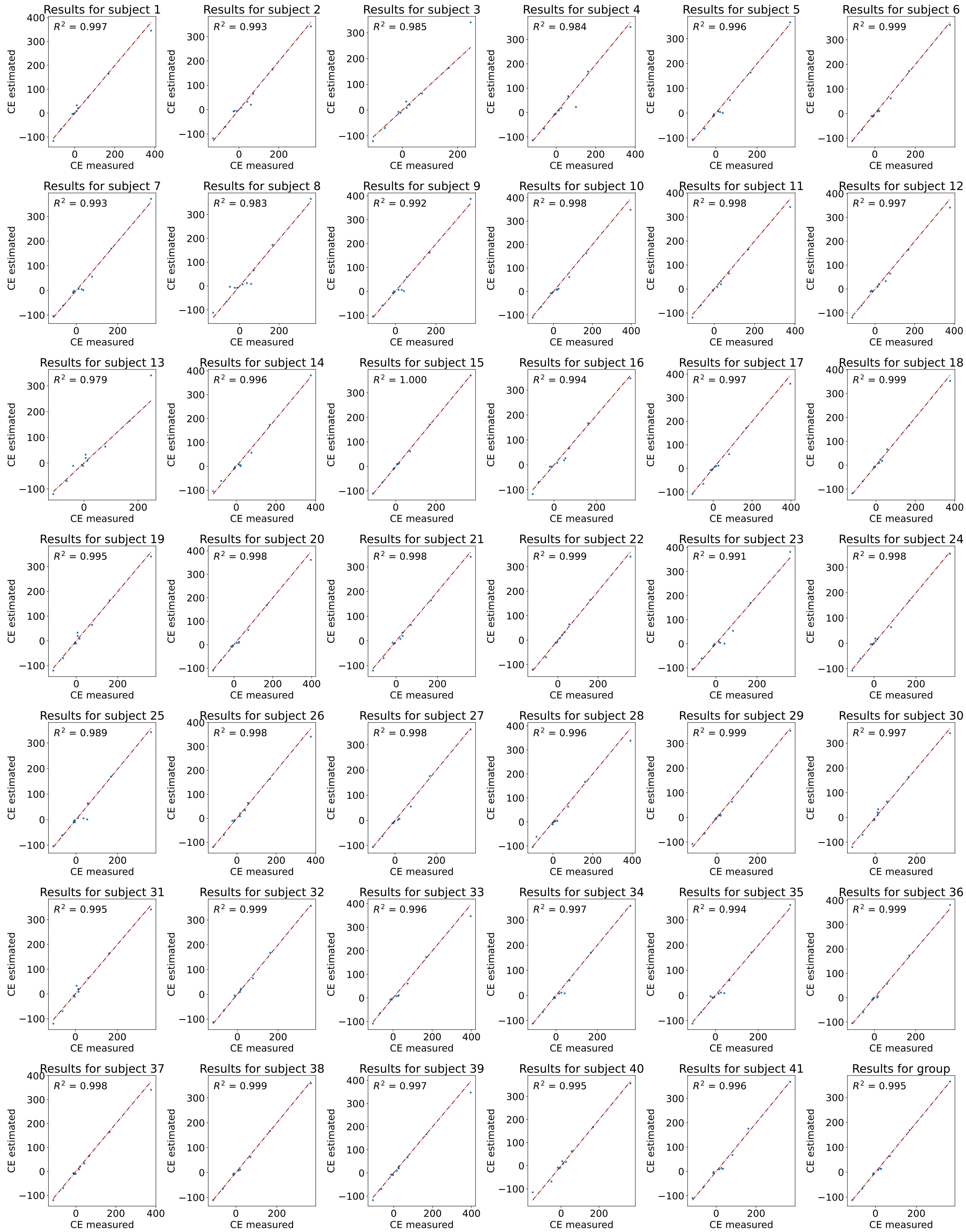
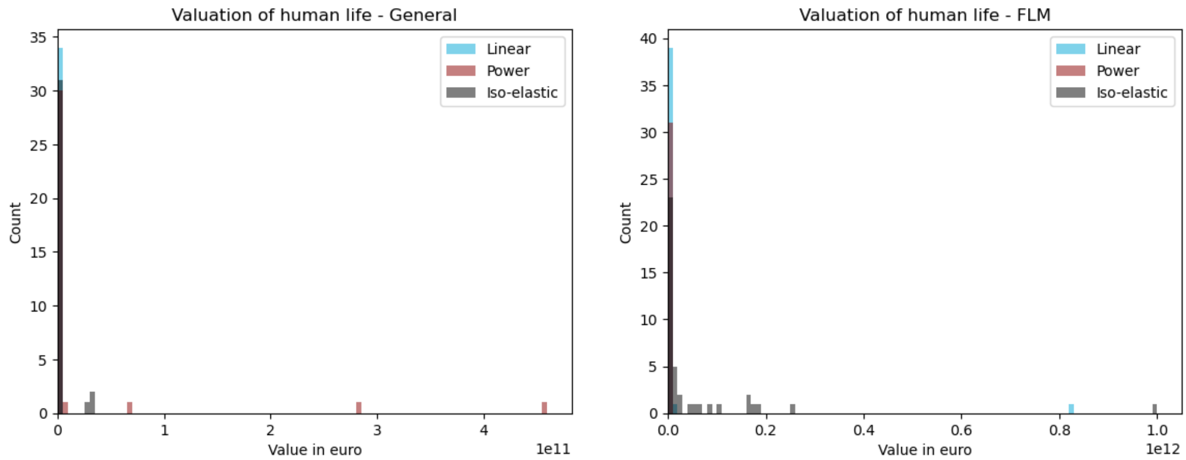


Figure E.17: Comparison of CE estimation via FLM risk preference parameters with observed general CE values for all participants using the *power* model

E.1.3. Valuation of statistical life



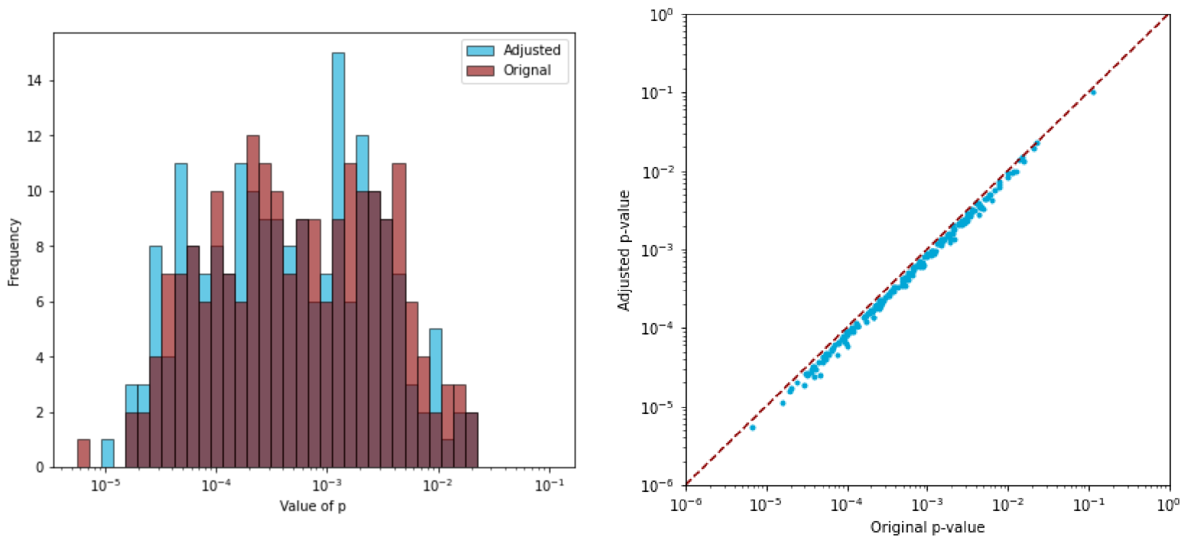
(a) Results for the general elicitation

(b) Results for the FLM elicitation

Figure E.18: Histogram of estimate for the valuation of a statistical human life without bounds for both general and FLM elicitations

E.2. Case Study

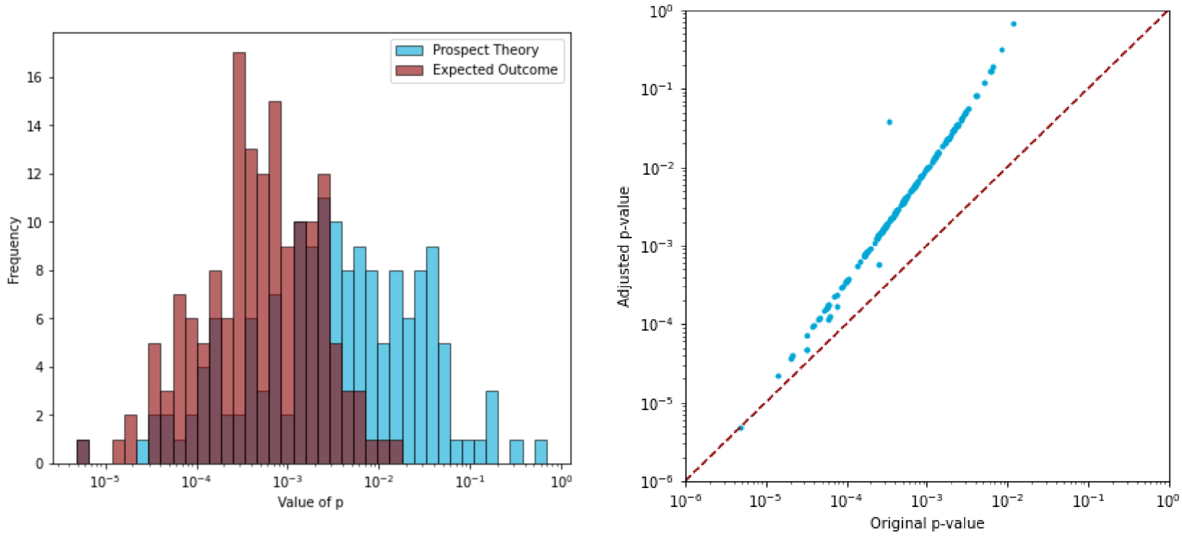
E.2.1. Comparison of adjusted flooding probability for FLM elicited parameters



(a) Histogram of flooding probabilities for the SCBA using adjusted VOSL values

(b) Standard flooding probability vs flooding probability using adjusted VOSL

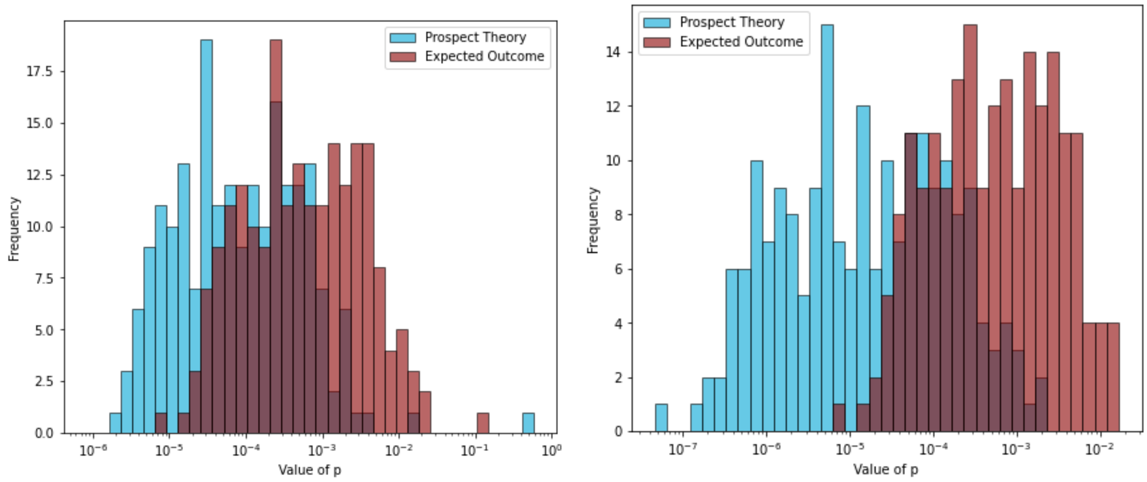
Figure E.19: Comparison between the results of the SCBA with original and adjusted VOSL values - FLM



(a) Histogram of flooding probabilities for the LIR using adjusted risk preferences

(b) Standard flooding probability vs flooding probability using adjusted risk preferences

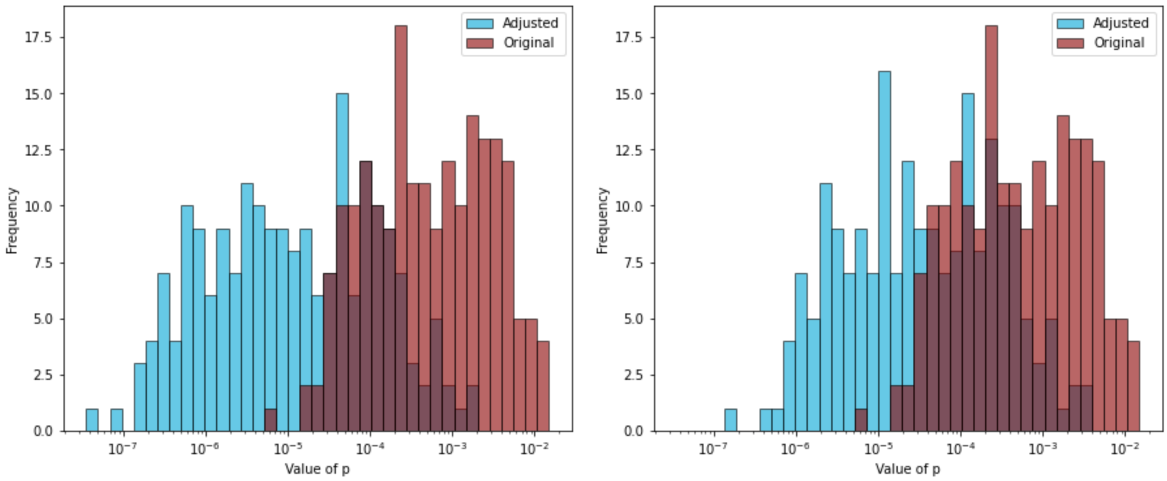
Figure E.20: Histogram of adjusted flooding probabilities for the LIR using FLM risk preferences



(a) Histogram of the results when using the parameters elicited through the FLM related choice problems

(b) Results when using the parameters elicited through the General choice problems

Figure E.21: Results of adjusted flooding probabilities for the SCBA using FLM risk preferences

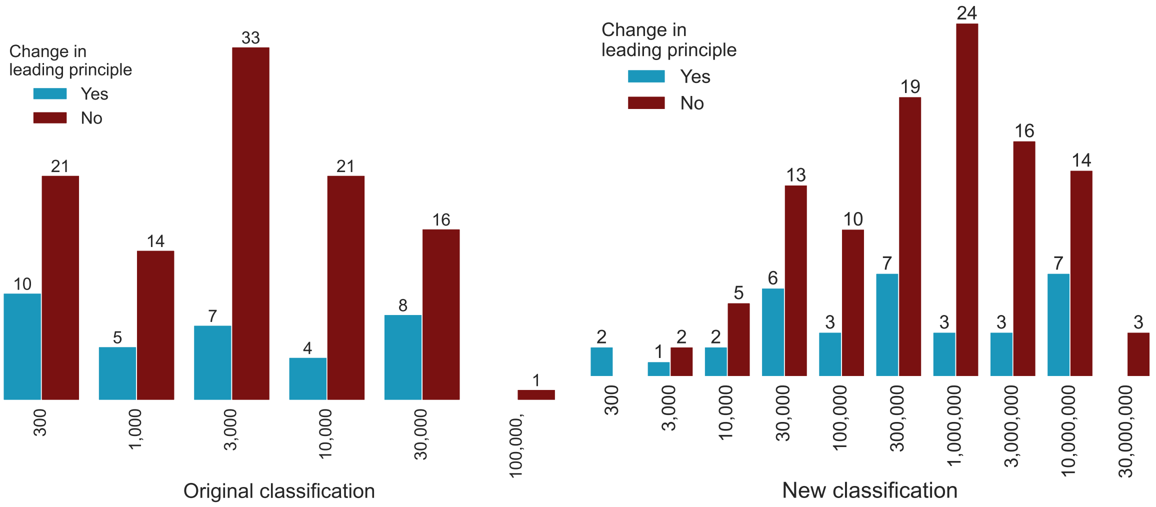


(a) Histogram of aggregated influence of the three variables using best estimates with general risk preferences

(b) Histogram of aggregated influence of the three variables using best estimates with FLM risk preferences

Figure E.22: Histogram of adjusted flooding probabilities for the aggregate effect

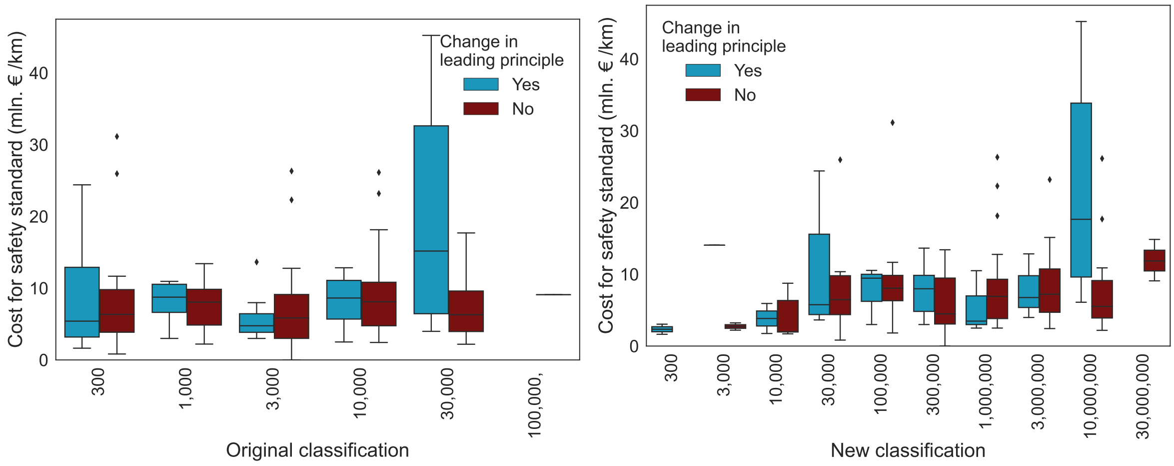
E.2.2. Evaluation of possible correlations between changes in leading principles and dike section properties



(a) Histogram showcasing the original classification of dike sections where a division is made based on whether dike sections will have a change in leading principles

(b) Histogram showcasing the new classification of dike sections where a division is made based on whether dike sections will have a change in leading principles

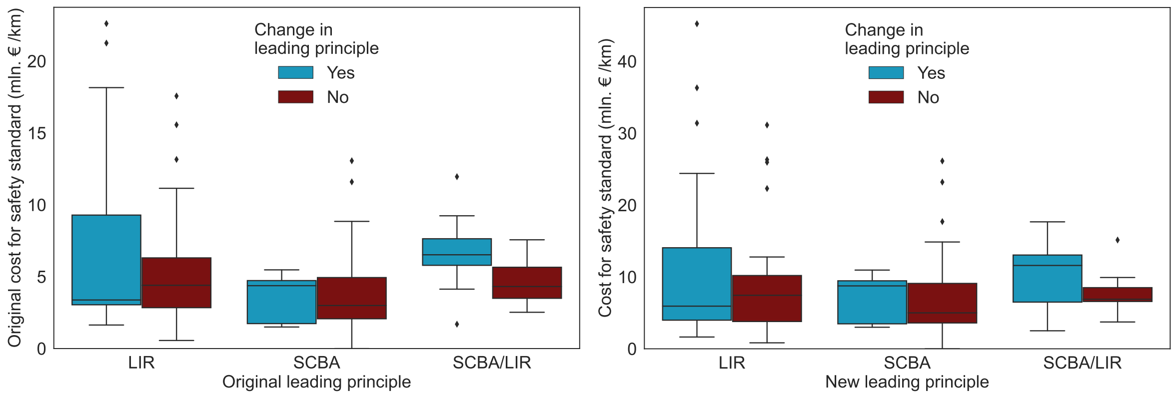
Figure E.23: Histogram showcasing the number of dike sections that have a certain classification before and after the inclusion of risk aversion, segregated based on whether a change in leading principle occurs



(a) Boxplot of the original classification of the allowable flooding probabilities and the costs for obtaining a 10x higher safety standard, divided on the lines of whether the leading principles changed

(b) Boxplot of the new classification of the allowable flooding probabilities and the costs for the safety standard for risk aversion, divided on the lines of whether the leading principles changed

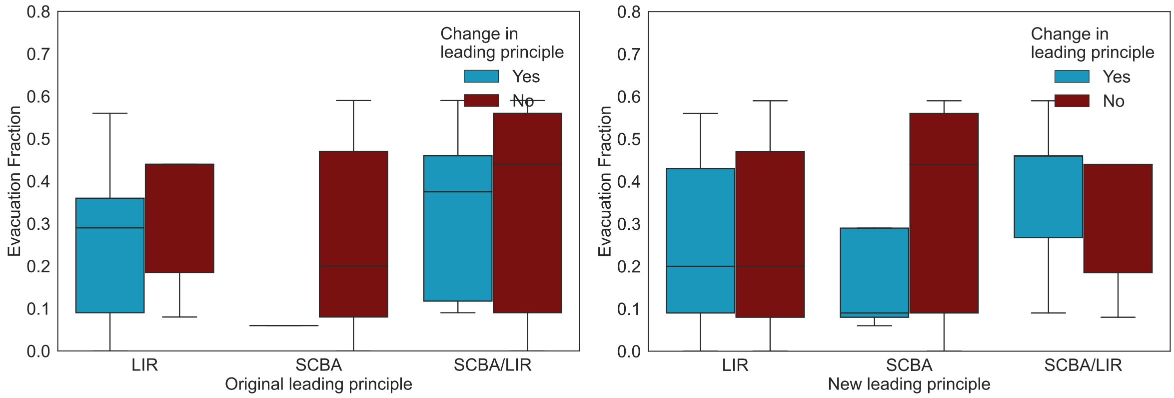
Figure E.24: Boxplot of the original and new classification of the allowable flooding probabilities with the costs for obtaining the safety standard, divided on the lines of whether the leading principles changed



(a) Boxplot of the original classification and the costs for obtaining a 10x higher safety standard, divided on the lines of whether the leading principles changed

(b) Boxplot of the new classification and the costs for the safety standard for risk aversion, divided on the lines of whether the leading principles changed

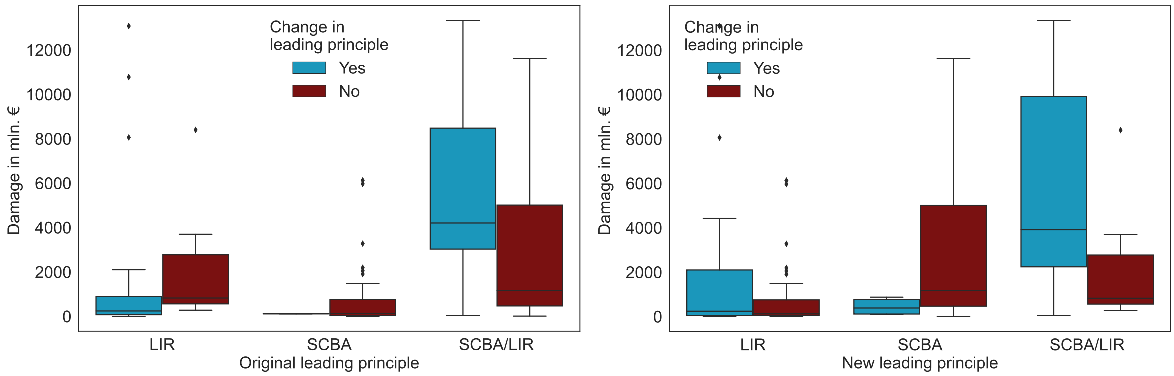
Figure E.25: Boxplot of the original and new classification with the costs for obtaining the safety standard, divided on the lines of whether the leading principles changed



(a) Boxplot of the original classification and the evacuation fraction, divided on the lines of whether the leading principles changed

(b) Boxplot of the new classification and the evacuation fraction, divided on the lines of whether the leading principles changed

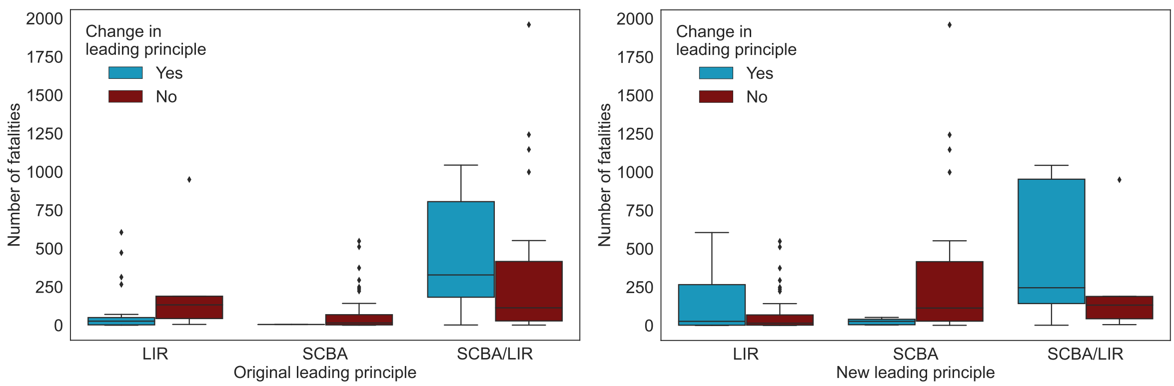
Figure E.26: Boxplot of the original and new classification with the evacuation fraction, divided on the lines of whether the leading principles changed



(a) Boxplot of the original classification and the expected damage as a result of a flood, divided on the lines of whether the leading principles changed

(b) Boxplot of the new classification and the expected damage as a result of a flood, divided on the lines of whether the leading principles changed

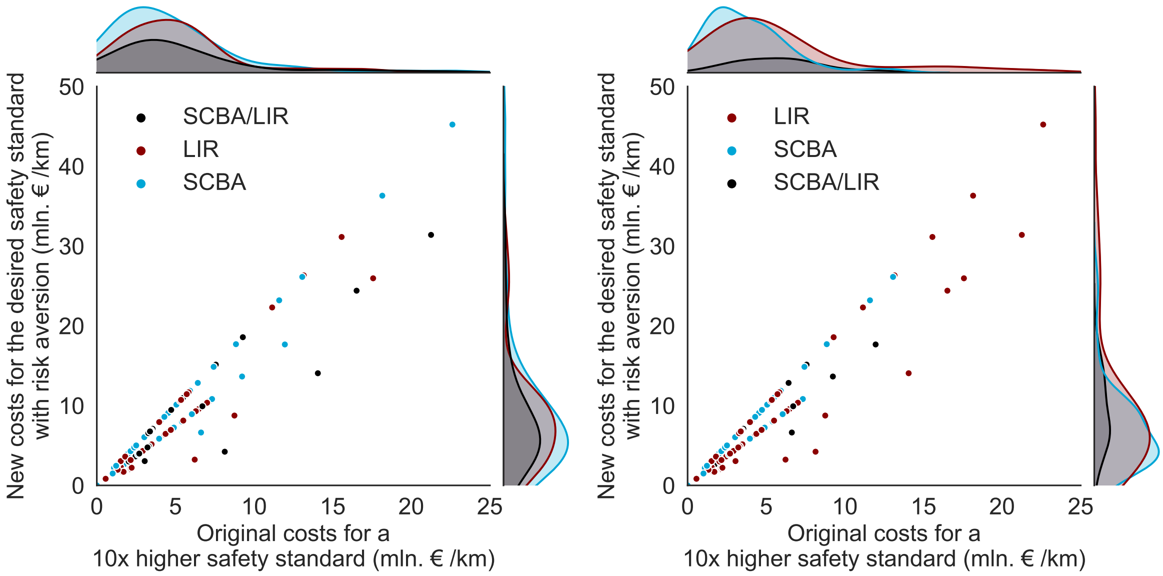
Figure E.27: Boxplot of the original and new classification with the expected damage as a result of a flood, divided on the lines of whether the leading principles changed



(a) Boxplot of the original classification and the expected number of fatalities, divided on the lines of whether the leading principles changed

(b) Boxplot of the new classification and the expected number of fatalities, divided on the lines of whether the leading principles changed

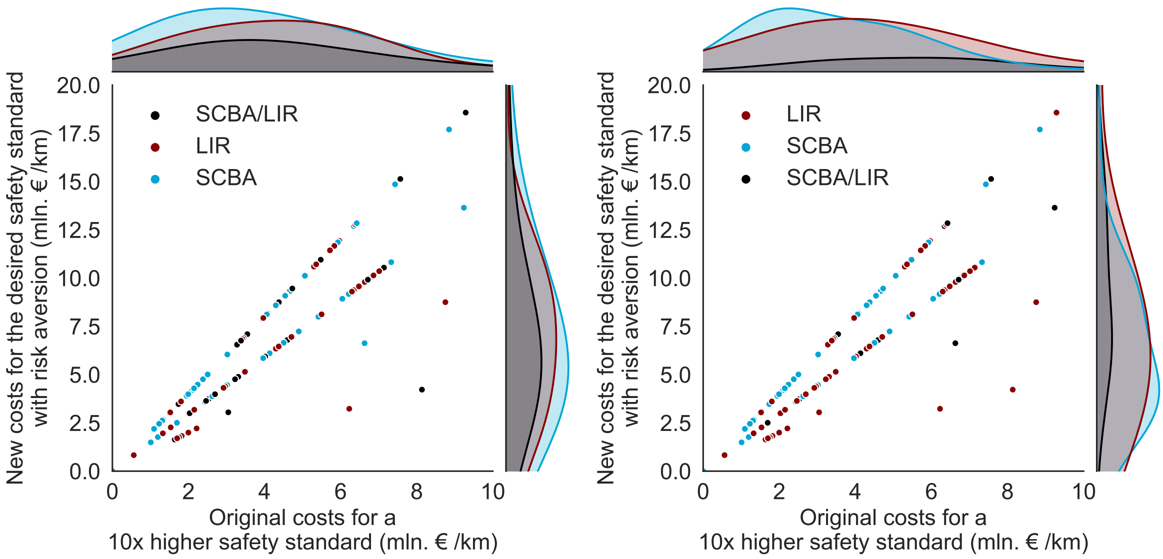
Figure E.28: Boxplot of the original and new classification with the expected number of fatalities as a result of a flood, divided on the lines of whether the leading principles changed



(a) Scatterplot including marginals, of the original costs for a 10x higher safety standard and the costs for the safety standard dictated by risk aversion, divided on the premise of the original leading principles

(b) Scatterplot including marginals, of the original costs for a 10x higher safety standard and the costs for the safety standard dictated by risk aversion, divided on the premise of the new leading principles

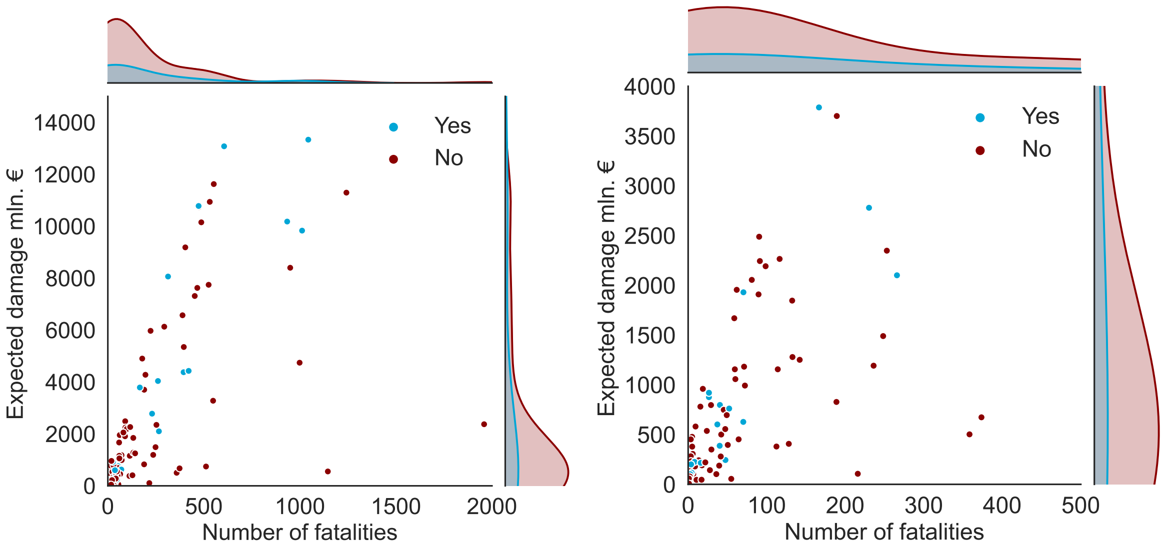
Figure E.29: Scatterplot including marginals, of the original costs for a 10x higher safety standard and the costs for the safety standard dictated by risk aversion, divided on the premise of leading principles



(a) Scatterplot including marginals, of expected number of fatalities and the expected economic damage, divided on the premise of the original leading principles - Zoomed in

(b) Scatterplot including marginals, of expected number of fatalities and the expected economic damage, divided on the premise of the new leading principles - Zoomed in

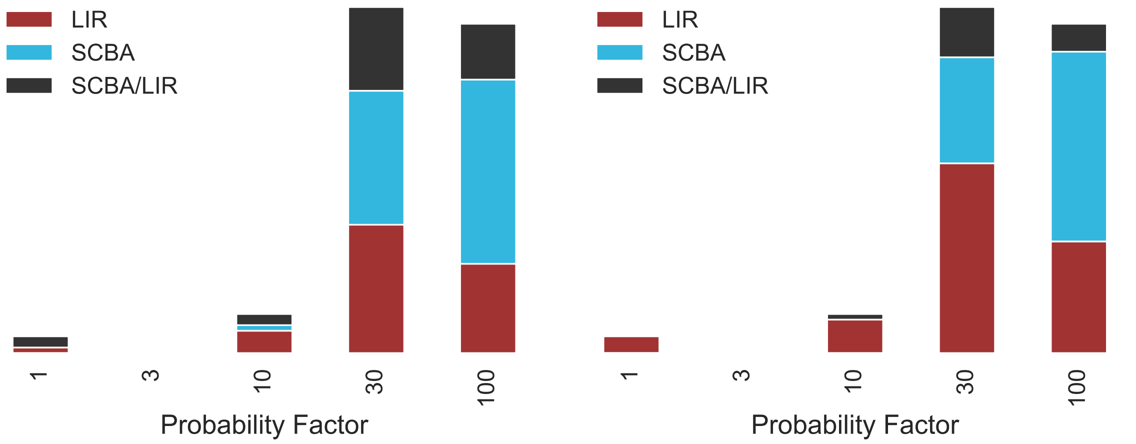
Figure E.30: Scatterplot including marginals, of expected number of fatalities and the expected economic damage, divided on the premise of leading principles - Zoomed in



(a) Scatterplot including marginals, of expected number of fatalities and the expected economic damage, divided on the premise of whether the leading principle changes

(b) Scatterplot including marginals, of expected number of fatalities and the expected economic damage, divided on the premise of whether the leading principle changes - Zoomed in

Figure E.31: Scatterplot including marginals, of expected number of fatalities and the expected economic damage, divided on the premise of whether the leading principle changes



(a) Stacked bar chart showing the distribution of probability factors, divided along the lines of the original leading principles

(b) Stacked bar chart showing the distribution of probability factors, divided along the lines of the new leading principles

Figure E.32: Stacked bar chart showing the distribution of probability factors divided along the lines of the leading principles

F

Tables

F.1. Case study

Table F.1: Description of the terminology used in Slootjes and Wagenaar (2016)

Name	Description
LIR requirement - lower limit class	Maximum allowable flood probability (lower limit) from LIR 10^{-5} base protection level per year assigned to a norm class.
LIR requirement - signaling class	Calculated requirement for the primary flood defense from LIR $5 \cdot 10^{-6}$ base protection level per year (signaling value) assigned to a norm class.
LIR requirement - lower limit	Maximum allowable flood probability (lower limit) from LIR 10^{-5} base protection level per year.
LIR requirement - signaling value	Calculated requirement for the primary flood defense from LIR $5 \cdot 10^{-6}$ base protection level per year (signaling value).
SCBA requirement - lower limit class	Maximum allowable flood probability calculated as one class more lenient than the 'median probability' assigned to a norm class.
SCBA requirement - signaling class	Economically optimal flood probability according to the 'median probability' concept (signaling value) assigned to a norm class.
SCBA requirement - signaling value	Economically optimal flood probability according to the 'median probability' concept (signaling value).
Group Risk Hotspot Trajectory	Yes: the respective norm trajectory is a so-called 'hotspot' trajectory from the group risk perspective. The signaling value and if necessary, the lower limit of the trajectory are assigned one class stricter than the strictest requirement of SCBA or LIR. No: the respective norm trajectory is not a 'hotspot' trajectory.

Table F.2: Lower- and upper-limits of the flooding probability classes

Interval Begin	Class
0	125
180	250
350	500
700	1000
1400	2000
2800	4000
6300	10000
14000	20000
28000	40000
63000	100000
Maximum	

Table F.3: Changes in leading principles for the norms of dike sections - General

Dike section	Name	Old leading principle	New leading principle
1-1	Schiermonnikoog Duin	SCBA/LIR	LIR
3-1	Terschelling Duin	SCBA/LIR	LIR
4-1	Vlieland Duin	SCBA/LIR	LIR
6-3	Friesland-Groningen - Friesland 3	SCBA/LIR	LIR
6-4	Friesland-Groningen - Friesland 4	SCBA/LIR	SCBA
6-7	Friesland-Groningen - Groningen 3	SCBA	SCBA/LIR
10-1	Mastenbroek 1	SCBA/LIR	LIR
13-1	Noord-Holland - Kust 1	SCBA/LIR	SCBA
13-4	Noord-Holland - Kust 4 stad	SCBA	SCBA/LIR
13-9	Noord-Holland - Markermeer 3	SCBA/LIR	LIR
14-7	Zuid - Holland - Kust 3	SCBA	LIR
14-8	Zuid-Holland - Kust 4	SCBA	LIR
14-9	Zuid-Holland - Kust 5	SCBA/LIR	LIR
15-1	Lopiker-en Krimpenerwaard - Oost	SCBA	SCBA/LIR
16-1	Alblasserwaard en de Vijfheerenlanden - Merwede	SCBA/LIR	LIR
16-2	Alblasserwaard en de Vijfheerenlanden - Merwede/Noord/Lek	SCBA/LIR	LIR
16-3	Alblasserwaard en de Vijfheerenlanden - Lek-West	SCBA	SCBA/LIR
16-4	Alblasserwaard en de Vijfheerenlanden - Lek-Oost	SCBA	SCBA/LIR
20-3	Voorne-Putten 2	SCBA/LIR	LIR
20-4	Voorne-Putten 3	SCBA/LIR	SCBA
21-2	Hoekse Waard 2	SCBA/LIR	LIR
22-2	Eiland van Dordrecht 2	SCBA/LIR	LIR
24-3	Land van Altena 3	SCBA	SCBA/LIR
25-2	Goeree-Overflakkee Haringvliet	SCBA	SCBA/LIR
25-3	Goeree-Overflakkee	SCBA/LIR	LIR
26-1	Schouwen Duiveland 1	SCBA/LIR	LIR
27-4	Tholen en St. Philipsland 4	SCBA/LIR	LIR
28-1	Noord-Beveland	LIR	SCBA
30-3	Zuid-Beveland West 3	SCBA/LIR	LIR
32-3	Zeeuwsch Vlaanderen 3	SCBA/LIR	LIR
33-1	Kreekrakpolder	SCBA/LIR	LIR
34-1	West-Brabant 1	SCBA/LIR	LIR
34-2	West-Brabant 2	SCBA/LIR	SCBA

Dike section	Name	Old leading principle	New leading principle
35-2	Donge 2	SCBA/LIR	LIR
41-4	Land van Maas en Waal - Maas	SCBA	SCBA/LIR
43-3	Betuwe, Tieler en Culemborgerwaarden 3	SCBA/LIR	LIR
43-4	Betuwe, Tieler en Culemborgerwaarden 4	SCBA/LIR	LIR
43-5	Betuwe, Tieler en Culemborgerwaarden 5	SCBA/LIR	LIR

Table F.4: Changes in leading principles for the norms of dike sections - FLM

Dike section	Name	Old leading principle	New leading principle
1-1	Schiermonnikoog Duin	SCBA/LIR	LIR
3-1	Terschelling Duin	SCBA/LIR	LIR
4-1	Vlieland Duin	SCBA/LIR	LIR
6-3	Friesland-Groningen - Friesland 3	SCBA/LIR	LIR
6-4	Friesland-Groningen - Friesland 4	SCBA/LIR	SCBA
6-7	Friesland-Groningen - Groningen 3	SCBA	SCBA/LIR
10-1	Mastenbroek 1	SCBA/LIR	LIR
10-3	Mastenbroek 3	SCBA/LIR	LIR
13-1	Noord-Holland - Kust 1	SCBA/LIR	SCBA
13-4	Noord-Holland - Kust 4 stad	SCBA	SCBA/LIR
13-9	Noord-Holland - Markermeer 3	SCBA/LIR	LIR
14-7	Zuid - Holland - Kust 3	SCBA	SCBA/LIR
14-8	Zuid-Holland - Kust 4	SCBA	SCBA/LIR
14-9	Zuid-Holland - Kust 5	SCBA/LIR	LIR
15-1	Lopiker-en Krimpenerwaard - Oost	SCBA	SCBA/LIR
15-3	Hollandse IJssel dkrg15	SCBA	SCBA/LIR
16-1	Alblasserwaard en de Vijfheerenlanden - Merwede	SCBA/LIR	LIR
16-2	Alblasserwaard en de Vijfheerenlanden - Merwede/Noord/Lek	SCBA/LIR	LIR
16-3	Alblasserwaard en de Vijfheerenlanden - Lek-West	SCBA	LIR
16-4	Alblasserwaard en de Vijfheerenlanden - Lek-Oost	SCBA	SCBA/LIR
20-3	Voorne-Putten 2	SCBA/LIR	LIR
20-4	Voorne-Putten 3	SCBA/LIR	SCBA
24-3	Land van Altena 3	SCBA	SCBA/LIR
25-2	Goeree-Overflakkee Haringvliet	SCBA	SCBA/LIR
25-3	Goeree-Overflakkee	SCBA/LIR	LIR
26-1	Schouwen Duiveland 1	SCBA/LIR	LIR
27-4	Tholen en St. Philipsland 4	SCBA/LIR	LIR
28-1	Noord-Beveland	LIR	SCBA
30-3	Zuid-Beveland West 3	SCBA/LIR	LIR
32-3	Zeeuwsch Vlaanderen 3	SCBA/LIR	LIR
32-4	Zeeuwsch Vlaanderen 4	SCBA/LIR	LIR
33-1	Kreekrakpolder	SCBA/LIR	LIR
34-2	West-Brabant 2	SCBA/LIR	SCBA
35-2	Donge 2	SCBA/LIR	LIR
41-4	Land van Maas en Waal - Maas	SCBA	SCBA/LIR
43-1	Betuwe, Tieler en Culemborgerwaarden 1	SCBA	SCBA/LIR
43-3	Betuwe, Tieler en Culemborgerwaarden 3	SCBA/LIR	LIR
43-4	Betuwe, Tieler en Culemborgerwaarden 4	SCBA/LIR	LIR

Dike section	Name	Old leading principle	New leading principle
43-5	Betuwe, Tieler en Culemborgerwaarden 5	SCBA/LIR	LIR