Multi-player Multi-issue Negotiation with Complete Information[1]

Mengxiao Wu [a] Mathijs de Weerdt [b] Han La Poutré [a] Chetan Yadati [b]

Yingqian Zhang [b] Cees Witteveen [b]

[a] *Centre for Mathematics and Computer Science (CWI), Amsterdam*
[b] *Delft University of Technology, Delft*

1 Introduction

With the rapid development of multi-agent systems, automated negotiation has been widely used to solve coordination problems in complex systems. In this work, we propose a solution when multiple players allocate multiple resources amongst themselves through negotiation, which takes place round by round. In contrast to most previous work on two-player multi-issue negotiation or multi-player single-issue negotiation, we study *multi-player multi-issue* negotiation. Further, it is *multilateral* that all players involve a single negotiation, which is different from the multiple bilateral negotiation between more than two players.

Compared to the alternating-offer bargaining [2], in which one player proposes allocations for all players, we design a negotiation protocol that each player bids desired allocation only for himself sequentially in each round, which applies to many real negotiation scenarios directly. Moreover, the protocol lets all issues be bundled and negotiated concurrently. This way is optimal for multi-issue negotiation [1] as it increases the opportunities of making trade-offs between issues.

We set the negotiation under a *complete information* environment, in which all information is common knowledge, and develop equilibrium strategies of the players, which form a *subgame perfect equilibrium* (SPE). Given a negotiation deadline and a discount factor, an agreement is reached immediately at the end of the first round. When any player has multiple bids that have the same maximum utility, if he always chooses the one that is best for his opponents, the outcome is *Pareto-optimal*. Although we just analyze the negotiation with complete information in this paper, the proposed negotiation model is a fundamental result of automated negotiation studied. This paper is an important step towards incomplete information cases and provides a benchmark for multi-player multi-issue negotiation.

2 The Negotiation Model

Suppose $n \geq 2$ rational players negotiate to allocate a finite set of $m \geq 2$ divisible resources amongst themselves. Each player requires to get a combination of all types of resources and only a unanimous agreement can be accepted. We use the term issue to indicate the amount of a resource, which range is normalized to a continuous range $[0, 1]$. We let n players bid desired combinations of the m issues for themselves sequentially in consecutive rounds $r \in \mathbb{N}$ till a deadline γ, given the pre-specified bidding orders. A given player is represented by a different bidder in each of the rounds, provided that those bidders all share the same preference and information of the original player.

When it is bidder i's turn to bid in round $r \leq \gamma$, given the bids of previous bidders in round r, bidder i can either accept those bids and make his own bid $\mathbf{x}_i \in (0, 1)^m$, or reject those bids. If all bidders choose to bid in a round and the sum of every issue of all bids does not exceed 1, the bid profile $a = (\mathbf{x}_1, \dots, \mathbf{x}_n)$ is

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an allocation agreement and the negotiation stops; every bidder gets a utility $u_i(a, r) = \delta^{r-1} \cdot v_i(\mathbf{x}_i)$ where δ is a discount factor and $v_i(\mathbf{x}_i)$ is a general *monotonically increasing* function of any element in $\mathbf{x}_i$. Either some bidder choosing rejection or no agreement reached in the current round, the negotiation passes on to the next round. If the negotiation stops after the deadline without any agreement, every player gets zero utility. Quitting is not allowed in the negotiation.

3 The Negotiation Strategies

We investigate the equilibrium strategies to specify the optimal action of every bidder i in any round r, when it is his turn to bid, given the previous bids $h_i = (\mathbf{x}_1, \ldots, \mathbf{x}_{i-1})$ in round r. The equilibrium strategy is to try out all possible actions to find the optimal one, which introduces the maximum utility to the player, with consideration of his opponents' responses. All bidders' optimal actions in a round are best responses to each other; the action profile forms a *Nash equilibrium*. We let $-i$ denote the set of all bidders other than i in a round and let A denote the set of actions. Given any action $a_i \in A$, bidder i reasons his opponents' responses a_{-i} first, and then calculates the utility that he would get based on $a = (a_i, a_{-i})$. If a cannot form an agreement in round r, we define the utility of a for bidder i in round r to be equal to the utility that the player would get in round $r+1$. If any player chooses the rejection, the utility that every player would get in round r also equals to the utility that the player would get in round $r+1$. Thus, to calculate the utility of any bid $\mathbf{x}_i$, bidder i needs to reason the best response of each of the remaining bidders $j > i$ in round r, which is bidder j's optimal action. The reasoning also requires the information of the utilities that all players would get in round $r+1$. Eventually, bidder i in round r does *two-dimensional* reasoning from the current round to the last round and from the first bidder to the last bidder in each round, which is a *recursive* procedure with a base case that all players will get zero utilities after round γ, if no agreement has been reached.

We let H denote the set of all possible profiles of bids in the negotiation. We define the optimal action function $s_i : H \times \mathbb{N} \to A$, where $a_i^* = s_i(h_i, r)$ is bidder i's optimal action in round r, given previous bids h_i. We use a letter and the letter with a tilde to denote a bidder of the current round r and a bidder of the next round $r+1$ respectively, which represent the same original player. Formally, given the previous bids h_i, the optimal action function is defined by:

$$s_i(h_i, r) \in \operatorname*{argmax}_{a_i \in A} w_i(a_i, h_i, r)$$

where

$$w_i(a_i, h_i, r) = \begin{cases} 0 & \text{if } r > \gamma \\ u_i(\mathbf{x}, r) & \text{if } r \leq \gamma,\ a = \mathbf{x} \text{ is an agreement} \\ w_{\tilde{i}}(a_{\tilde{i}}^*, h_{\tilde{i}}, r+1) & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} &a = (h_i, a_i, a_{i+1}^*, \ldots, a_n^*),\ a_{i+1}^* = s_{i+1}(h_{i+1}, r),\ h_{i+1} = (h_i, a_i), \\ &\forall j \in \{i+2, \ldots, n\}\ \left\{a_j^* = s_j(h_j, r),\ h_j = (h_{j-1}, a_{j-1}^*)\right\}, \\ &\forall \tilde{j} \in N\ \left\{a_{\tilde{j}}^* = s_{\tilde{j}}(h_{\tilde{j}}, r+1), h_{\tilde{j}} = (h_{\tilde{j}-1}, a_{\tilde{j}-1}^*)\right\}. \end{aligned}$$

Proposition 1. *The equilibrium strategy of bidder i in round $r \leq \gamma$ is S_i^r: when it is his turn to bid, he uses the above optimal function to calculate the optimal bid/response, given the previous bids in round r. The equilibrium strategies of all players induce a subgame perfect equilibrium of the game. If an agreement exists in this game, it will be reached immediately at the end of the first round.*

Proposition 2. *The equilibrium outcome is a Pareto-optimal solution of the game if every player chooses his optimal action with the completely benevolent selection. That means, when bidder i has multiple bids that have the same maximum utility, he always chooses the one that is best for his opponents.*

References

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