The influence of posture and stature on bicycle handling qualities

Master thesis

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Gazelle

The influence of posture and stature on bicycle handling qualities

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Abstract

The second most used way of transport in the Netherlands is the bicycle. There is a great variety of city bicycles in which each person has their preference. One of the most critical aspects of personal preference is the cycling experience or feeling, better defined as Handling Qualities. Improving the Handling Qualities can lead to more comfort and safety in cycling. The goal is to determine what could improve Handling Qualities which will help within the design stage of bicycles. Handling Qualities are different for each bicycle and depend on many various aspects of the bicycle's geometry. In addition, there are differences in riders' posture and stature that have an influence on the bicycle's control.

To find the most critical parameters that influence the handling qualities, three different bicycles are analysed. Furthermore, three different postures or sitting positions (forward lean) are compared to find the influence of the rider on the Handling Qualities of the total human-bicycle system. In addition, three different statures combined with the right side of the bicycle are compared. A database for human measurements is used to calculate the rigid body of the rider based on an anthropometric model [1]. The bicycle parameters are gathered from Solidworks bicycle models, given by Gazelle. A dynamical model is created based on the Whipple bicycle model [2] with an arm model extension [3]. (Steering is the most essential control input). The eigenvalues are calculated to analyse the stability of each combination. Furthermore, the Handling Quality Metric (HQM) [4] is calculated to compare Handling Qualities.

As a result of using an arm model, there is no self-stability within any of the rider-bicycle systems. The system can still keep the bicycle upright due to the human arm control of the bicycle. An analysis of eigenvalues shows that the dynamic behaviour of each combination is greatly influenced by the arm model. The different sitting positions and statures also gave a great variety of dynamic behaviour.

The HQM values are greatly influenced by the added arm model, some rider bicycle combinations gave extremely high values. However within high values, there were varying results; for every combination, the HQM values were best for the most upright sitting positions. Furthermore, the best HQM values found are for the tallest person and the worst for a very small person.

Both dynamic behaviour and HQM values are greatly influenced by the added arm model. However, the differences between postures and statures have become clear within this model. An increase in stature has a positive influence on the HQM values as well as a more upright position. The model can be used in the design stage of the bicycle to predict handling qualities according to the HQM standard. Influential parameters for the desired handling qualities can be found. Different sitting positions and people can be evaluated. Therefore, bicycles can be designed for specific target groups.

Contents

1 Introduction

In a typical Dutch street, you will undoubtedly notice one thing; there are so many bicycles. This amount of bicycles is not coincidental when cycling is the Netherlands' second most popular mode of transport [5]. Another notable thing in the Netherlands is the many different bicycles which vary in size and appearance. All kinds of bicycles are present, for example, racing, mountain, and of course the most common: the city "bicycle". The city bicycle already has a great variety in its shape, such as bikes for females and males, differences in frame structure and the use of different materials. City bicycles can also be in the form of electric bicycles. These bicycles need a battery and a motor located somewhere on the bicycle, which affects the bicycle's properties compared to non-electric bicycles. The electric bicycle is becoming more popular than ever to make life even easier [6], especially among older adults. Interestingly, within this great range of bicycles, everyone has their own preference [6]; their additional features and gadgets lead to their preferred cycling experiences. The common factor in all the examples mentioned above is that everyone wants to go from A to B as easily and safely as possible.

The bicycle industry uses aesthetics and usability to design bicycles to the customers' needs [7]. However, the cyclist does not always have the desired cycling experience in terms of ease and effort. This might be caused by many factors like the features described before or by the rider's interaction with the bicycle. To that end, the underlying interaction mechanisms between the rider and the bicycle should be determined and standardised to improve the bicycle design according to the rider's expectations. The underlying systems of riding a bicycle need to be explained in order to find what influences the riding experience. Another way of describing the riding experience is defining and evaluating the 'handling qualities' of a bicycle [8], [4].

1.1 Handling Qualities

Rider-bicycle interaction means all the factors from the human closed-loop control system and the bicycle dynamics that can influence each other and create an optimal rider(user) experience. An optimal experience requires minimum mental effort and maximum precision in operating the machine and achieving the highest performance [9]. In all vehicles driven by a human, this optimal experience can be evaluated by the term 'handling quality', first introduced by Harper and Cooper in aircraft-pilots systems performance [9]. According to their definition, 'Handling quality' is the *ease* and *precision* at which a pilot can *perform* their task".

Adopting this principle from aerospace engineering, Handling qualities could be defined for cycling and what are the most important aspects influencing the handling of a bicycle. It is desired to evaluate the handling qualities by modelling the human-bicycle system. Therefore it is necessary to understand the underlying systems of the bicycle and rider. For the bicycle, the most influential factors are within its dynamics. Since humans are the greatest portion of the mass of the total system, it is important to analyse the riders' dynamics. Therefore the mechanisms of controlling a bicycle need to be understood. In riding a bicycle there are two main aspects that influence the handling of a bicycle; the rider and the bicycle itself.

1.2 Bicycle geometry & dynamics

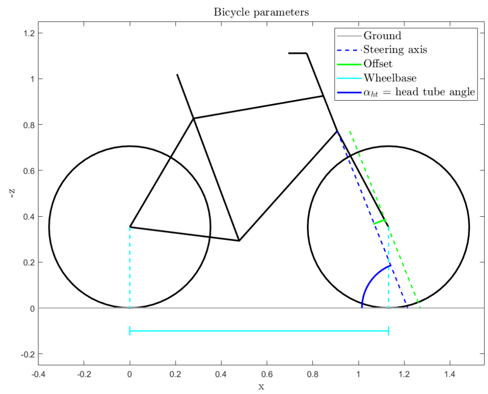

Figure 1: Bicycle Geometry with essential parameters

Much research has been done to understand the dynamics of the bicycle. The geometric properties that can influence the dynamics and therefore the handling qualities are evaluated. A brief description of the bicycle geometry will be given and its influence on the dynamics will be given later. A bicycle consists of a front frame with a steer, a rear frame and two wheels [10]. The dimensions of the front frame that have an effect on steering are; the head tube length and angle, the fork offset and the wheel radius [11],[2]. These last three front frame parameters create the trail as indicated in figure 1. The trail mainly has an influence on the steering of the bicycle [12], especially the response from the steer to the human [13]. If a bicycle has a large trail, the steering gives a lagged response (or is indirect), meaning it takes longer to rotate around the steering axis.

If the trail is small, the steering gives a more direct response [14]. Consequently, this has an impact on the input of the human on the steer. According to Patterson [15], the trail of the bicycle contributes to the control feedback of steering. The important parameters of the rear frame are; the seat tube angle, seat tube length and the wheelbase indicated in figure 1. The seat tube angle and length mainly influence the energy consumption, and power output of the rider [11]. These two parameters can also influence the sitting position of the rider [16]. The wheelbase influences the steering of the bicycle. A large wheelbase causes a greater curvature when turning the bicycle.

A standard set of equations has been created which allows analysis of the bicycle on its dynamic behaviour [2]. These equations can be extended with an arm model [3].

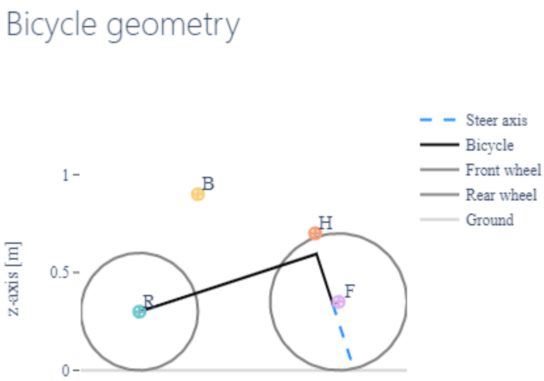

Figure 2: Benchmark bicycle with COM's[17],[2]

Another aspect is the mass properties of the bicycle and human which are important to the dynamic behaviour of the system [1]. Both front and rear frames, the wheels and the rider have a mass, a centre of mass (COM) and mass moment of inertia, see figure 2. A simple method of measuring rather than calculating moments of inertia and COMs of the bicycle was described by Moore et al. [18]. When the COMs, mass moments of inertia and masses are determined, they can be used in the dynamical model described in the next section.

There are many theories about how it is possible to ride a bicycle and how a bicycle can stay upright [14]. The question is: what movements are made whilst riding a bicycle forward and whilst keeping the balance on the bicycle [19]. Of course, the bicycle can move forward, but the front wheel turning allows lateral (sideway) movement of the total system. The lateral movement of steering is accompanied by the leaning of the whole system. *Steering* is the rotation about the steering axis, see figure **??**. *Leaning* is the side-way movements of the total human-bicycle system and not the rider's leaning.

The system's forward movement, leaning and steering are the most important movements involved in riding a bicycle [14].

Lateral movement is important for controlling the bicycle and has been analysed by many experts ([20], [14], [2], [3]). The dynamic explanation of riding a bicycle is as follows; when the bicycle starts to lean to one side, the COM shifts to the same side. Steering in that same direction causes the bicycle's COM to shift under the human's COM again. Whether this is to turn a corner or the human moves, there is a disturbance that causes the steering to turn, and in combination with lean, the bicycle can stay upright. Bicycles can have self-stability, which means the bicycle, on its own, when given a certain speed, will keep the bicycle upright. The combination of steer and lean also explains this, and thus the COM shift [12].

1.2.1 Dynamic models

A simple model can be created with a rear frame, a front frame including the steer, and a passive rider fixed on the bicycle, [20]. The equations of motion can be derived as provided by Åström [21] by using classical mechanics. Åström also states that the effect of the trail complicates a bicycle model.

These equations and models have been altered and refined by many researchers. A linearized bicycle model was determined by Meijaard, and Schwab [2]. A bicycle was simplified to create a benchmark for a linearised bicycle model. The bicycle was transformed into a model containing four rigid bodies, two wheels, a front frame and a rear frame, including a rider with accompanying parameters; mass, mass moments of inertia and geometric parameters. The total system has two degrees of freedom; the steering angle and the lean angle. The lean and steer angle were transformed into a set of linearised equations of motion. With this linearised set of equations, the eigenvalues of the rider-bicycle system can be calculated. The eigenvalues can be used to analyse the mode of the system. By analysing the modes of the system, one can explain whether the lean or steering motions accommodate each other, or if either is dominant. The linearized model was experimentally validated [22]. It was concluded that the model is correct for low speed and low frequencies and can be used to analyse bicycles within a speed range of 0 to 6 m/s. In a renewed version of the linearised bicycle model, the rider model was given more degrees of freedom [3]. The upper body of the rider was given two positions; one with a forward lean of the body with stretched arms and one with flexed arms and a more upright position. This model uses a software package called SPACAR [23], which can be used in Matlab, to calculate the model parameters. A metric of *modal controllability* was used to determine how the extra degrees of freedom influence the control of the bicycle. It was concluded that even with the possibility of the rider's upper body lean, the bicycle is still mainly controlled by steering actions.

1.3 Human influence

1.3.1 Human modelling

Since mass properties are of great influence on the dynamic system, the mass and inertia properties of the human have a great influence on the bicycle handling qualities. Secondly, the control of the human has a great influence on the total system. Control theory can be used to model the human control of vehicles or machines [24]. Human control inputs can be modelled with a neuromuscular model [25].

Using these principles led to the creation of the Handling Quality Metric (HQM) which will be used to evaluate the performance of the human bicycle system [4]. The proprioceptive feedback is essential when handling the steer of a bicycle and feeling the forces that are acting on the steer while cycling [15],[14],[26]. Proprioceptive feedback is one of the control mechanisms of humans that help to alter the steering input of the bicycle [27].

1.3.2 Posture

The human body has been modelled in many ways, for instance, as a point mass combined with the rear frame of the bicycle [19]. Or as a rigid body composed of all body parts [28]. However, it seems that there has not been a lot of research on the influence of different statures and postures. In most of the available literature the model with anthropometric data, data is based on the Ceasar database (American). This study contains bicycles of Gazelle, which are mostly sold in the Netherlands a Dutch database of anthropometric data will be used. Furthermore, there is a lot of difference in human stature therefore different statures will be analysed. The difference in sitting position has been analysed in some research [3], [29], however, this has not been done for the same bicycles. Therefore it will be investigated what influence these differences have on the dynamic behaviour and handling qualities.

1.4 Problem definition

In the previous section, the definition of handling qualities is defined. As well as what influences handling qualities and how they can be calculated. The weight distribution has a great influence on the dynamic human-bicycle system. Therefore a closer look has been taken at what influences weight distribution. First of all, in electric bicycles, the motor and batteries contribute to the weight distribution. Secondly, the added features of the user are changing how the weight is distributed. However, this does not explain the different experiences of different users with the same features on their bicycles. An explanation of this difference could be the sitting position of the human on the bicycle. Since the human is the greatest portion of the system's total weight. Therefore this research will examine different sitting positions as well as different statures of riders. The main research question is:

"What is the influence of different postures and statures on handling qualities?"

The different statures have an influence on the size of the bicycle, so there also will be looked at different sizes of bicycles. Dynamic behaviour will be analysed to evaluate the differences in postures and statures on the total system. Then the handling qualities can be evaluated.

2 Methods

The influences of sitting position and bicycle parameters on the dynamical behaviour and thus the handling qualities need to be determined. In order to do so a mathematical dynamic model will be used and examined. To examine the model the Handling Qualities metric will be used. The elements that are needed for modelling and examining:

1. Human parameters
2. Bicycle parameters
3. Equations of motion
4. SPACAR: calculations of EOMs
5. HQM model

A human-bicycle model is made in Matlab with an extension called SPACAR. The model is based on the linearized bicycle model [26] and [3]. First, an explanation of the used human and bicycle data will be given. The human and bicycle parameter databases are explained in section 2.1. After sorting the data the equations of motion can be created in section 2.2.1. The extension of Matlab SPACAR will calculate the elements of the EOMs, which is explained in section 2.2.4. When the EOMs are complete the human bicycle model can be examined. This will be explained last in section 2.3.

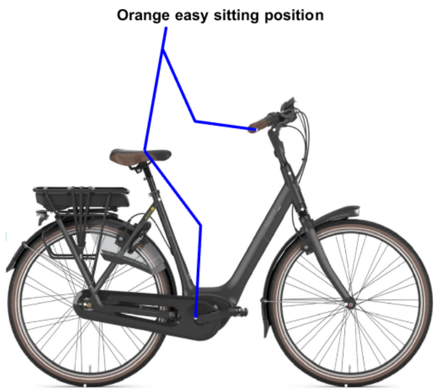

Figure 3: Orange (Easy) [30]

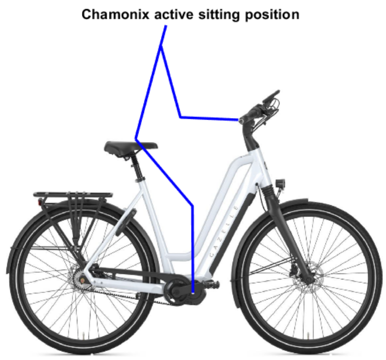

Figure 4: Chamonix (Active) [31]

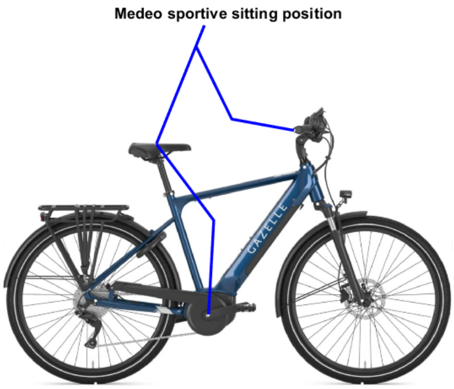

Figure 5: Medeo (Sportive) [32]

2.1 Data selection

It was chosen to use three different statures and types of bicycles, so only nine human-bicycle combinations remain. To retain an extensive range of statures the selected statures are; 1.60, 1.79, and 1.96. These differences in length represent a tiny person, an average Dutch person and a tall person. Research conducted by Jan Siksma concluded that within the different Gazelle bicycles, there are three types of sitting positions [29]. This results in categorizing the bicycle into three types; Easy, active and sportive. The easy bicycles are mostly ridden in an upright manner. The active in a more forward lean position and the sportive type in the most forward position. Therefore it was chosen to analyse an easy, active and sportive type of bicycle. The chosen bicycles are the; Orange, Chamonix and Medeo see figures 3, 4 and 5.

The first part of the model finds the human data with the right bicycle size, figure 6. The selection of parameters begins with two inputs:

1. The stature height of the human
2. The type of bicycle

The parameters of the rider and bicycle are described next in sections 2.1.1 and 2.1.2 accordingly.

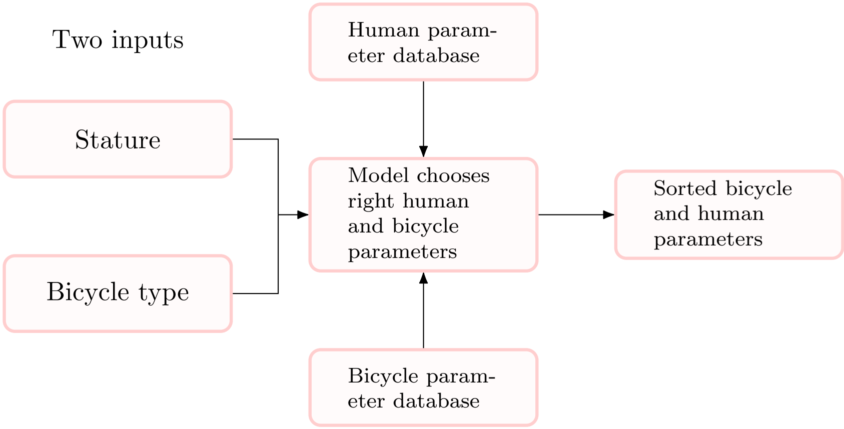

Figure 6: First steps of the human-bicycle model

2.1.1 Human measures data

The human measures data is gathered from the Dutch DINED database 2004 [33]. With the help of this anthropometric database, each human measure within the stature lengths of 1.60 to 2.10 meters was identified. The human measures are used to create a rigid rider for the human-bicycle model. Most measures that are needed to create this model are directly taken from the database, others have been calculated based on the gathered data. In table 7 the calculated and acquired measures are given.

Measure	symbol
Length stature	$L_{stature}$
Chest circumference	c_{ch}
Width shoulders	l_{ss}
Length (or height) of head	l_h
Length of neck	l_{ne}
Length torso	$l_{to} = L_{stature} - h_{fo} - l_{ul} - l_{ll} - l_h - l_{ne}$
Head circumference	c_h
Length of the upper arm	l_{ua}
Length of the lower arm	l_{la}
Upper arm circumference	c_{ua}
Lower arm circumference	c_{la}
Length hip to hip	l_{hh}

Table 1: Human measures [33]

Measure	symbol
Length of upper leg	l_{ul}
Length of lower leg	l_{ll}
Upper leg circumference	c_{ul}
Lower leg circumference	c_{ll}
Height of foot	h_{fo}

Table 2: Human measures [33]

These measures are used to calculate the moments of inertia of each body segment. For each body part, a relative mass will be taken from the total mass of the rider. The total mass of the human is based on a normal female and male BMI of 21.75. The relative masses are based on an anthropometric model [18]. The relative masses are:

Body part	Symbol	Relative weight
Rider's mass	m_{Br}	$L_{stature}^2 BMI$
Head	m_h	$0.068 m_{Br}$
Torso	m_{to}	$0.510 m_{Br}$
Lower leg	m_{ll}	$0.061 m_{Br}$
Upper leg	m_{ul}	$0.100 m_{Br}$
Lower arm	m_{la}	$0.022 m_{Br}$
Upper arm	m_{ua}	$0.028 m_{Br}$

Table 3: Relative masses of rider body and parts [1]

Section 2.2.2 explains how the arms' relative masses define the arm model.

2.1.2 Bicycle geometric data

Gazelle provided the bicycle parameters. The parameters are given for three types of bicycles with 5 to 7 sizes. An overview of the parameters can be found in figure 7 and in table 4.

Measure	symbol
Fork offset	f_0
Trail	$c = \frac{r_f \cos \alpha_{ht} - f_0}{\sin \alpha_{ht}}$
Wheelbase	w
Length handlebar	l_{hb}
Height steer	l_s
Length fork	l_f
Front hub width	w_{fh}
Front-wheel radius	r_F
Head tube angle	α_{ht}
Length head tube	l_{ht}
Width handlebar	w_{hb}
Rear hub width	w_{rh}
Rear wheel radius	r_R
Length seat tube	l_{st}
Length seat post or saddle height	l_{sp}
Seat tube angle	α_{st}
Bottom bracket drop	bb
Bottom bracket height	$h_{bb} = r_R - bb$
Chainstay	l_{cs}

Table 4: Geometry parameters

The height of the steer is chosen as average for statures to fit the rider onto the desired bicycle size and type. The same applies to the saddle height. The inner and outer widths of the tires are the same for every size of the chosen bicycle to compare each size equally. The selection of the size of the bicycle with stature is based on the crotch height when the feet are 20 cm apart [34]. The human size will be named l_{size} and is calculated with equation 1.

$$l_{size} = 0.68 * \sqrt{(h_{cr}^2 - 0.1^2)} \tag{1}$$

The seat tube length represents the frame size, which can be found in table 5. The variable l_{size} is subjected to the second column of the table and the right size is selected.

size	range of l_{st} [m]	Framesize [34]
S	0.44-0.5	44-50
M	0.5-0.55	50-55
L	0.55-0.6	55-60
XL	0.6-0.65	60-65
XL	0.65	65-66

Table 5: Frame sizes and corresponding seat tube lengths

After the data selection part of the model the chosen bicycle sizes are; small, large and extra large, corresponding to the chosen statures.

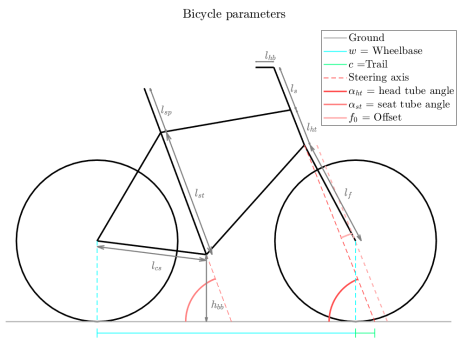

Figure 7: Bicycle with parameter indication

2.2 Dynamical Modelling

The mathematical bicycle model or equations of motion are described first. Then the necessary rigid bodies of the rider and the bicycle with their mass moments of inertia are fitted into the model. An arm model is added to the model to investigate the influence of the sitting positions more closely. Lastly, an explanation of the use of the HQM with the created human-bicycle model is given.

2.2.1 Equations of motion

The bicycle model is based on the linearised bicycle model [2]. First, a set of equations of motions is created, with $\mathbf{q} = [\phi, \delta]^T$ (lean and steer angle) as states and $\mathbf{f} = [T_\phi, T_\delta]^T$ (lean and steer torque) as inputs. These equations create the total system:

Linearised bicycle model

$$\mathbf{M}\ddot{\mathbf{q}} + v\mathbf{C}_1\dot{\mathbf{q}} + [g\mathbf{K}_0 + v^2\mathbf{K}_2]\mathbf{q} = \mathbf{f} \tag{2}$$

These equations of motion are a set of differential equations and are solved with a standard differential solution 2,3. It is possible to calculate the eigenvalues of the system for a specific range of speed. The use of eigenvalues gives inside into the dynamic behaviour of the system.

Standard solution differential equation: exponential solution

$$\mathbf{q} = \mathbf{q}_0 e^{\lambda t}$$

$$(\mathbf{M}\lambda^2 + v\mathbf{C}_1\lambda + [g\mathbf{K}_0 + v^2\mathbf{K}_2])\mathbf{q}_0 e^{\lambda t} = \mathbf{f}\mathbf{q}_0 e^{\lambda t} \quad (3)$$

By calculating the determinant of the system, the eigenvalues can be calculated, which will be one of the model's outputs. The use of the eigenvalues is explained in section 2.3.1.

$$det(\mathbf{M}\lambda^2 + v\mathbf{C}_1\lambda + [g\mathbf{K}_0 + v^2\mathbf{K}_2]) = 0$$

Now the variables for a certain combination of bicycle and human need to be determined to create the mass $\mathbf{M}$, damping $\mathbf{C}_1$ and stiffness $\mathbf{K}_0, \mathbf{K}_2$ matrices. The properties that influence these matrices are the masses, COMs and moments of inertia of the bicycle and the human body parts. The human inertia properties can be calculated using the human data [18]. The bicycle inertia properties can be measured, moreover, they can also be gathered from Solidworks models. The latter is done in this research and is described in section 2.2.3. The precise calculation of these matrices is done with the Matlab extension SPACAR which is explained in section 2.2.4. First, all the properties of each segment will be calculated or gathered from Solidworks, see 2.2.2 and 2.2.3.

2.2.2 Human inertia properties

The human parameters (see section 2.1.1) are used to create rigid bodies of all body parts [1]. The measures are also used to create the positions of human body parts on the bicycle. First, the coordinates of all the body parts are calculated (this is done at the same time as the bicycle parameter coordinates). Then the relative masses (see table 3) of all the body parts are calculated as well as the centres of mass and mass moments of inertia. The body parts are simplified to calculate the mass moments of inertia; the legs are cylinders, the torso is a block and the head is a sphere. The local moments of inertia of the upper leg are calculated with equation 4.

$$I_x, I_y = \frac{1}{12} m_{ul} \left(\frac{3c_{ul}^2}{4\pi^2} + l_{ul}^2 \right), I_z = \frac{m_{ul} c_{ul}^2}{10\pi^2} \quad (4)$$

The other local moments of inertia are calculated in the same manner [1].

The center of mass is taken at half the length of the cylinder. Then the cylinder is rotated into the global reference frame, see figure 8.

$$\mathbf{I}_i = \mathbf{R}_i \mathbf{I}_{local} \mathbf{R}_i^T \quad (5)$$

The total moment of inertia of the rider's body is composed of all the body parts and calculated with equations 6 and 7.

$$\mathbf{I}_{global} = \mathbf{I}_i + m_i + \begin{bmatrix} d_y^2 + d_z^2 & -d_x d_y & -d_x d_z \\ -d_x d_y & d_x^2 + d_z^2 & -d_y d_z \\ -d_x d_z & -d_y d_z & d_x^2 + d_y^2 \end{bmatrix} \quad (6)$$

$$\mathbf{I}_{br} = \sum \mathbf{I}_{global} \quad (7)$$

Note that the inertia properties of the arms are differently calculated, see section 2.2.4. Lastly, the moments of inertia of the human; I_{Br}; are combined with that of the rear frame of the bicycle; I_{Bf}; to create one moment of inertia; I_B. The total center of mass of the rider with the rear frame is calculated with the formula:

$$com_B = \frac{\sum m_i com_i}{\sum m_i} \quad (8)$$

Furthermore, three angles (or more if desired) are used for the upper body forward lean of the human. The sitting positions are chosen to accommodate the types of Gazelle bicycles (see figure 5, 4, 3), with the angles: 68°, 75° and 80°, measured from the x-axis to the torso midline.

	Sportive	Aactive	Easy
α_{fl}	1.868 rad or 68°	1.309 rad or 75°	1.396 rad or 80°

Table 6: Forward lean angles

2.2.3 Bicycle inertia properties

Solidworks models of the three chosen bicycles in three sizes are delivered by Gazelle. The inertia properties can easily be gathered from Solidworks using the mass properties function. The mass properties from Solidworks of the bicycles can be found in appendix A. Since the linearized bicycle model is used, the same manner is used to define the different bodies of the bicycle, see figure 2. The inertia properties are taken separately for the front frame, front wheel, rear frame and rear wheel. It is chosen to keep the position of the steer for each bicycle in the same position. This is due to the moment of inertia changing with the position of the steer and it is desired to compare the sitting positions equally.

2.2.4 SPACAR extension and Arm model

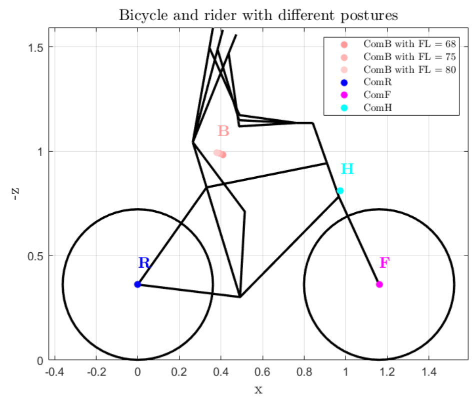

Figure 8: Bicycle with the rider in three different positions and centers of mass of rear and front wheels, front frame and the total COM of the rear frame and human body

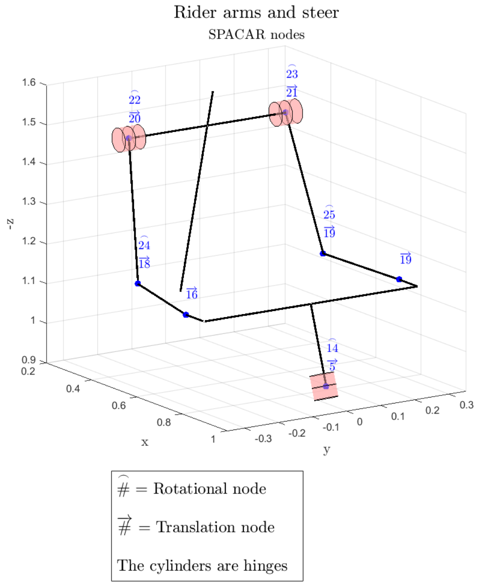

Figure 9: Arm model with SPACAR nodes (rotational and translational)

The model is based on models created by Arend Schwab [3]. The model is made with SPACAR, an extension in Matlab [23]. SPACAR calculates the matrices (mass, damping and stiffness) needed to use the linearized bicycle model [2]. SPACAR is based on the finite element method to calculate the properties of the system, [35]. By using the finite element method to define a dynamic system, the system is compiled out of elements, translational and rotational nodes. The elements are predefined and have a certain set of nodes. Within the nodes, the direction and rotation of the motions are defined. Then a set of fixation and predefined DOFs are needed for SPACAR to complete the system. When the kinematic part of the system is defined the dynamic properties are assigned to the right elements or nodes. Mass and inertia properties, gravity and applied velocities (for pre-described DOF) are prescribed to elements. Lastly, simulation specifications are made and the mode of SPACAR is chosen to be mode 4. Mode 4 calculates linearized equations of motion. Next a more precise description of the arm model is given, for evaluation of the fully defined system see appendix **??**. More information on the terminology of SPACAR can be found in the manual, [36]

The upper arms are modelled as beams and the lower arms as trusses. The shoulders are defined as hinges and allow the arm to rotate around one axis. The front frame (and steer) also contain a hinge which enables the rotation around the steer axis. The elbows consist of a connection that shares rotational and translational nodes. This means they are fixated together but can move as the shoulders rotate. The lower arms are also fixated on the steer, so whenever the steer rotates the lower arms and consequently the upper arms and shoulders move along. The same can be said for the upper arms, whenever these rotate relative to the upper body the lower arms and steer will also rotate. A figure of the connections can be seen in 9. The mass properties of the arms are separately defined from the rest of the body. A prescribed mass per unit length is assigned to each arm segment and SPACAR calculates the inertia properties for these segments accordingly to the elements they are assigned to be. These properties are calculated together with the rest of the system and combined into the output of SPACAR, which is the linearized equations of motion. The matrices: $\mathbf{M}$, $\mathbf{C}_1$, $\mathbf{K}_0$ and $\mathbf{K}_2$ are extracted and used for further calculation. After the matrices are calculated, the eigenvalues and HQM values can be calculated for the specific sitting positions.

2.2.5 HQM model

When the mathematical model of the bicycle with the rider is created the matrices will be transformed into state space form so they can be used in a human control model [4]. The human control model consists of a neuro-muscular model which give the inputs to the bicycle model. In figure 10 the total model is shown. The input to the total system is the lean angle of the rider bicycle system and the output is the state of the lean angle when controlled by the human ff checken!. The bicycle block computes four states: $\mathbf{x} = [\phi, \delta, \dot{\phi}, \dot{\delta}]^T$, which are used in the feedback loops. The inner loops receive the feedback of the lean rate, $\dot{\phi}$, and the steer angle, δ.

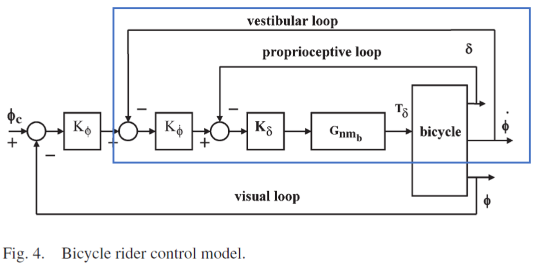

Figure 10: Human bicycle model

The states are fed back to the shown points in the control model, figure 10. The control gains of the inner loops are chosen so that the system behaves in the same manner as the neuromuscular model. The gains in the outer loop are chosen with the help of the crossover frequency theory [37], so they behave in the same open-loop manner as a pilot aeroplane control model.

The model estimates the needed control gains for the chosen bicycle and rider combination. The earlier mentioned HQM (see 1) is transformed to use on bicycles, see equations 9 and 10.

The HQM is a transfer function proportional to the control activity of the human (or the human feedback) over the input of the total system. The control activity is human feedback, which is one of the inner feedback loops in the model. The feedback is the rider's lean and steer rates divided by the steer gain.

$$HQM = \left|\frac{U_m}{C}(j\omega)\right| \cdot \frac{1}{|k_p|} \tag{9}$$

$$HQM = \left|\frac{\dot{\phi} + (\frac{\delta}{K_{\dot{\phi}}})}{\phi_c}\right| \cdot \frac{1}{|k_\phi|} \tag{10}$$

After the gains of the model are computed, the HQM values for three different postures, statures and bicycles are made.

2.3 Evaluation of model

2.3.1 Eigenvalues and Eigenvectors

The eigenvalues are calculated and plotted for each combination of a bicycle-rider system for three bicycles, statures and postures. They are calculated as described in section 2.2. Every combination has 4 sets of eigenvalues, which represent the 4 modes of the system. Usually, these are *capsize*, *castering* and two *weave* modes which are most often each other complex conjugates. The capsize mode is dominated by the roll angle of the system. The castering mode is dominated by the steer angle and decays fast. The weave modes are the oscillation between the steer and roll angles. The modes can be identified by considering the eigenvectors. When all the real parts of the eigenvalues are below zero, there is self-stability in the system. (Eigenvalues are stable when they are negative). To compare the results of this research a plot of the eigenvalues of a benchmark bicycle can be viewed in Figure 11.

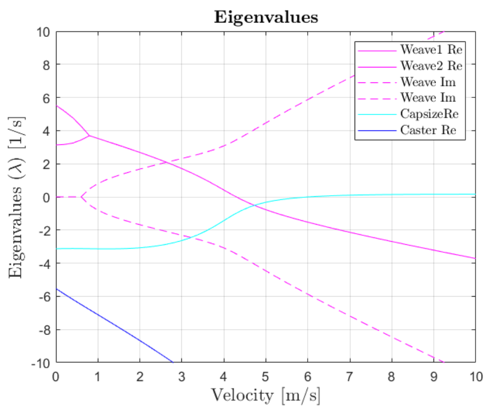

Figure 11: Eigenvalues of the benchmark bicycle

Plots and explanations of the eigenvectors can be found in appendix B.

2.3.2 HQM values

Once the human bicycle model is created and the HQM values are calculated they can be evaluated. The values will be calculated for three different velocities and for frequencies between 0 and 20 rad/s. These values will be plotted for each combination of rider and bicycle, for all different sitting positions.

3 Results

The results contain eigenvalues of each combination of bicycle, stature and posture. A plot with only the oscillating mode for three different postures is shown. The HQM plot is made for one combination of bicycle and stature with three different postures.

3.1 Eigenvalues

The changes in combinations within the bicycle-rider system influence the dynamic behaviour of the total system. Furthermore, the added arms model generates a different behaviour which can be well examined with the eigenvalues and eigenvectors. The arm model adds a stable pendulum-like oscillation to the steer. There is no longer self-stability in the system, though the arms stabilise the oscillating mode of the system. The classic eigenmodes of the bicycle-rider system are no longer present in the system. However, analysing the modes that are present gives inside into the behaviour of the system. All nine combinations are analysed, and divided into bicycle types, each starting with the smallest bicycle and so on. For each bicycle type and size, the three different sitting positions will be examined.

3.1.1 Chamonix bicycle

The Chamonix bicycle is a city bicycle designed for an easy to active sitting position. The eigenvalues and handling qualities are analysed for three sizes. As a result of the added arm model, it can be seen that the eigenvalues behave differently for all combinations. There is no longer a self-stability region and the eigenmodes cannot be identified as for the Benchmark bicycle. Note, that there the most forward lean is 68 degrees and the least forward lean is 80 degrees as it is measured from the x-axis to the upper-body position.

Small Chamonix

The results of the eigenvalues of the small Chamonix bicycle are plotted in figure 12.

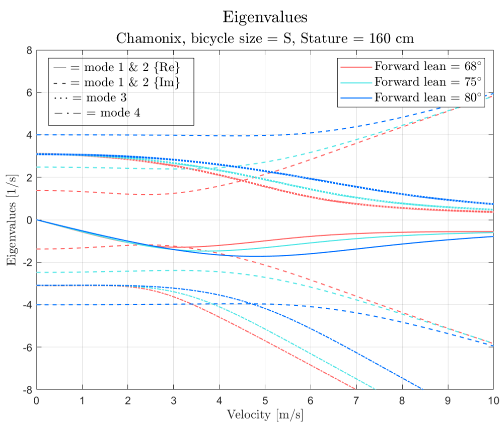

Figure 12: Eigenvalues chamonix Small bicycle with a person of 1.60 m tall with all forward leans

The first real eigenmode is always negative and is therefore always stable. This eigenmode barely decreases for low speeds and for the least forward lean (FL = 80 deg) it starts to decay at a higher speed. This eigenvalue starts to decay at a slightly lower velocity for a more forward lean. The other real eigenvalue mode does never become stable and stays positive for every velocity. In the oscillatory mode or the complex mode, a certain wave can be seen. First, the oscillation decreases slightly and for higher speeds, it becomes more expanded. For low velocities, there is an increase in the amplitude of the oscillatory motion with decreasing forward lean except for the most straight-up sitting position. For the forward lean of 80° the imaginary parts are a straight line. This might be due to the arms of the rider being almost straight in this position. The eigenvalues are plotted seperately in appendix C in figures: 41, 42, 43 and 44.

Large Chamonix

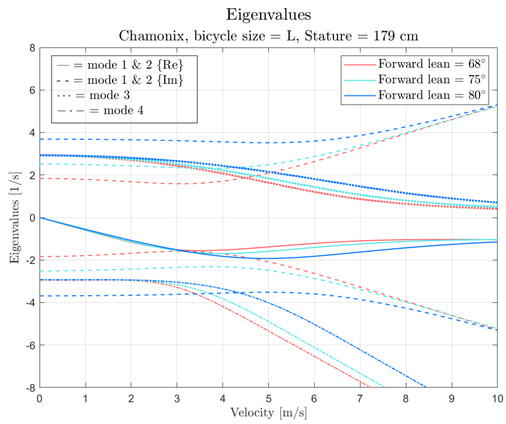

Figure 13: Eigenvalues chamonix large bicycle with a person of 1.79 m tall with all forward leans

The large Chamonix bicycle gives reasonably the same results as the small bicycle. The eigenvalues are plotted in figure 13 and in appendix C, figures: 45, 46, 47 and 48. In figures 13 and 48 it can be seen that the difference in amplitude of the oscillation is slightly less extensive than for the small bicycle.

Extra Large Chamonix

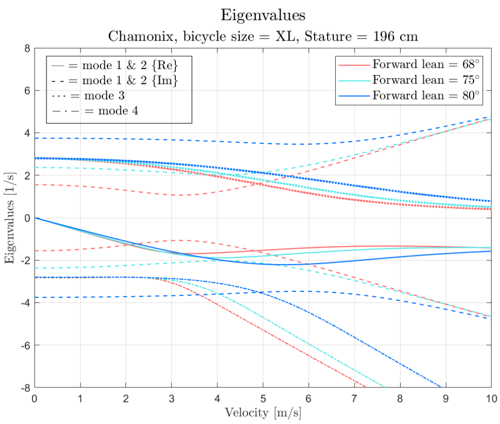

Figure 14: Eigenvalues chamonix extra large bicycle with a person of 1.96 m tall with all forward leans

The results of the extra-large bicycle do not differ too much from the small and large bicycles. Examining figure 14, the difference is most easily observed for the oscillatory mode or modes 1 and 2. For mode 4, there is a barely noticeable increase in values before it starts to decay. In mode 3 there is a smaller difference between the sitting positions. The different sitting positions can reviewed separately in appendix C, figures: 49, 50, 51 and 48.

3.1.2 Medeo

The Medeo bicycle is designed as a sportive bicycle, so a more forward lean is usually used for this bicycle.

Small Medeo

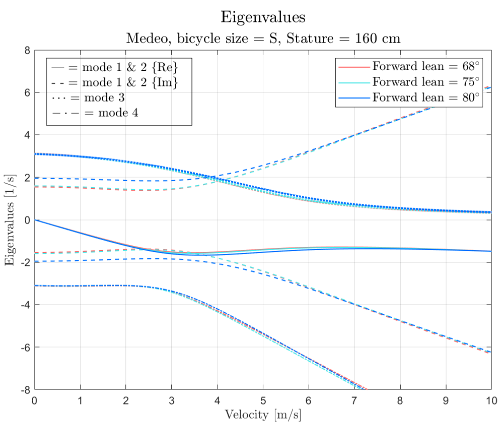

Figure 15: Eigenvalues Medeo small bicycle with a person of 1.60 m tall with all forward leans

Figure 15 shows the eigenvalues of the small Medeo bicycle. The oscillation mode has a smaller difference in amplitude between the different sitting positions than the small Chamonix bicycle. And for the two most forward lean there is barely a difference at all. For this bicycle, it can also be seen that mode 4 starts to decay around the same velocity with decreasing forward lean.

Large Medeo

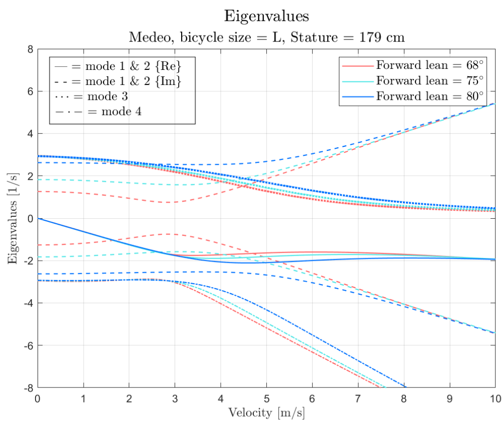

Figure 16: Eigenvalues Medeo large bicycle with a person of 1.79 m tall with all forward leans

The eigenvalues of the large Medeo can be seen in figures 57, 58, 59, and 60. The main difference between this combination and the small Medeo bicycle is that the differences in the oscillation mode are more pronounced. As seen in the extra large chamonix bicycle, the most negative real eigenmode slightly increases before it starts to decay for a forward lean of 68° and 75°.

Extra Large Medeo

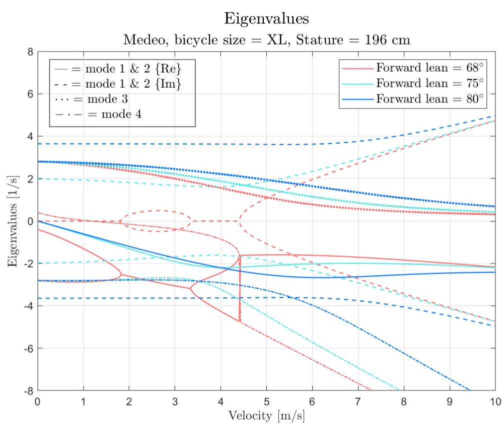

Figure 17: Eigenvalues Medeo extra large bicycle with a person of 1.96 m tall with all forward leans

Apart from the amplitude of the oscillation mode of the previously described bicycles, there are no exceptional differences in dynamic behaviour. This is the same for the extra large Medeo bicycle except for the combination where the forward lean is 68°, see figure 17 and more closely in figure 61.

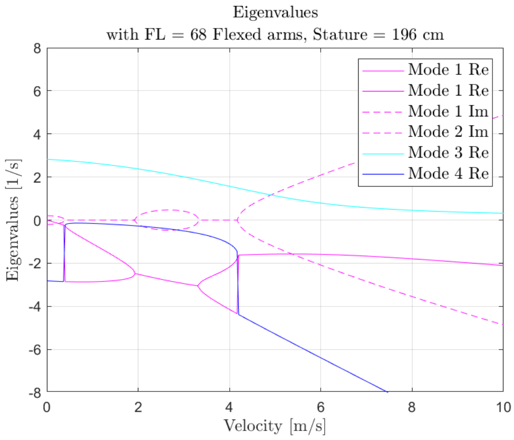

Figure 18: Eigenvalues Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 68 degrees

Velocities between 0.4 m/s and 1.9 m/s, 3.33 m/s and 4.167 m/s there are no imaginary eigenvalues. In these areas, it is hard to distinguish which eigenvalues belong to the oscillating mode and which to the always negative real eigenmode even when evaluating the eigenvectors, see figure 19. There appear to be two bifurcations in the oscillatory mode, which indicated a rapid change in dynamic behaviour. The changes in the behaviour of the system can be evaluated with the eigenvector plots in figures 19, 20 and 21. When the velocity is 1.4 m/s the steer angle is dominant for three negative eigenvalues and not out of phase of each other. After the first bifurcation at a velocity of 3 m/s the eigenvectors of the oscillating eigenvalues are about 100 degrees out of phase. After the second bifurcation at a velocity of 5 m/s, the phase difference is about 30 degrees. At higher speeds the distinction of 4 modes is possible.

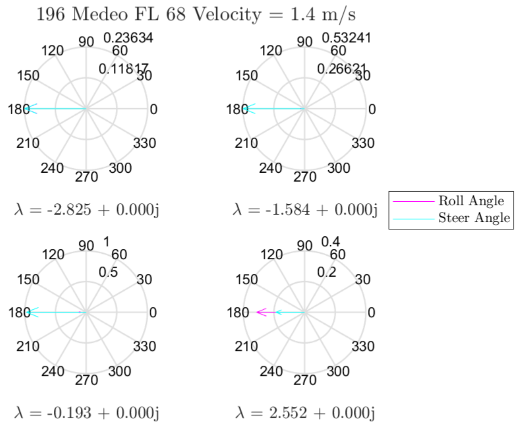

Figure 19: Eigenvectors Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 68 degrees at 1.4 m/s

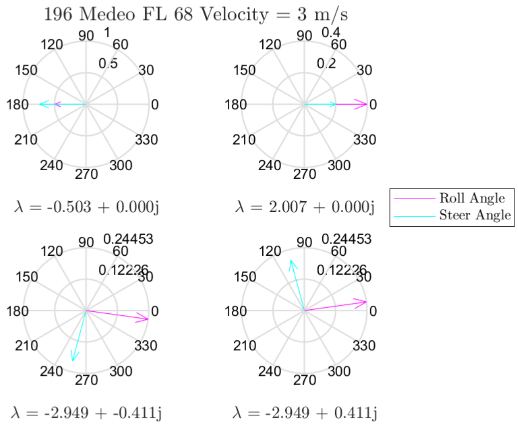

Figure 20: Eigenvectors Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 68 degrees at 3 m/s

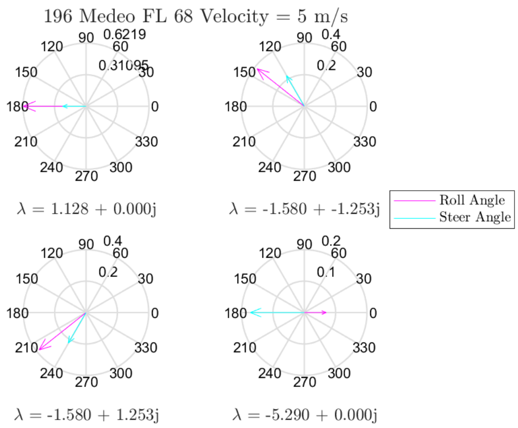

Figure 21: Eigenvectors Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 68 degrees at 5 m/s

The other two sitting positions show behaviour more like the extra large Chamonix bicycle, see figures 62 and 62. The difference in amplitudes of the oscillation mode is pronounced.

3.1.3 Orange

The Orange bicycle is a city bicycle designed for an easy sitting position. Therefore the most common sitting position is most straight up.

Small Orange

The results of the eigenvalues of the small Orange bicycle are shown in figure 22. These results are comparable to the results of the small Medeo bicycle. Again there is a very small difference in modes 1 and 2 of the smallest two forward lean angles.

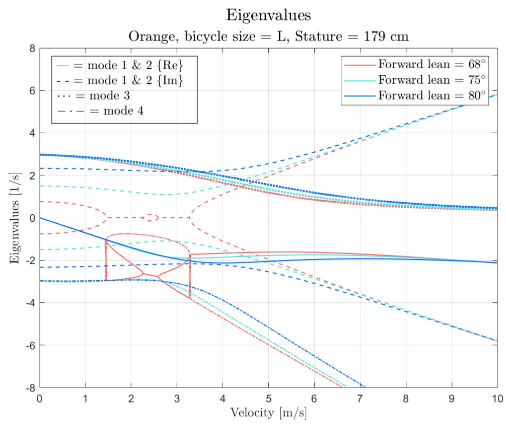

Figure 23: Eigenvalues Orange large bicycle with a person of 1.79 m tall with all forward leans

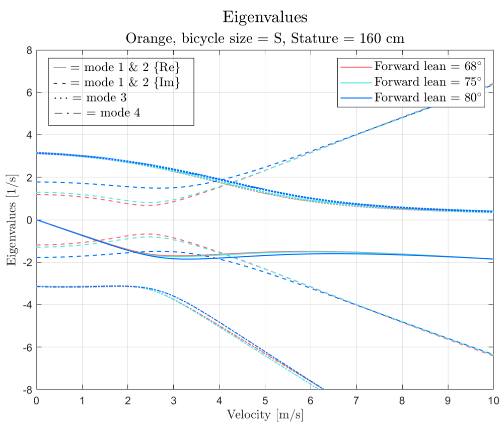

Figure 22: Eigenvalues Orange small bicycle with a person of 1.60 m tall with all forward leans

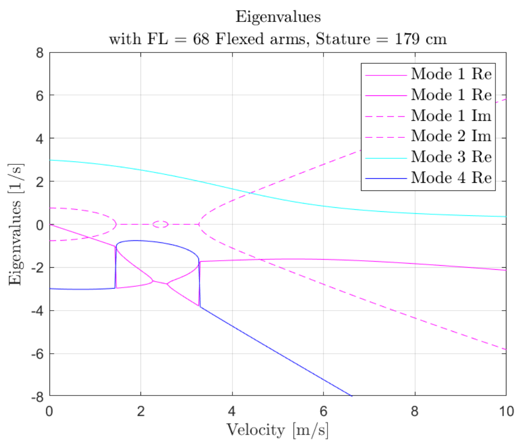

Figure 24: Eigenvalues Orange Small bicycle with a person of 1.79 m tall with a forward lean of 68 degrees

Large Orange

The eigenvalues of the forward lean of 75° and 80° show the same trend as seen in the small orange bicycle, but with a greater difference in amplitude of the oscillating mode, see figure 23. For the most forward lean, the imaginary eigenvalues are zero for certain velocities, see figures 24. This system shows the same behaviour as the extra-large Medeo bicycle but for different velocities. Plots of the eigenvectors of this rider-bicycle combination can be found in Appendix C.3.2.

Extra Large Orange

The same trend for the most forward lean eigenvalues plot continues in the extra-large Orange bicycle, see figure 25. The differences in the amplitude of the oscillation mode are noticeable. Moreover, the evolution of this mode toward expansion at higher speeds is different for each sitting position, figure 26.

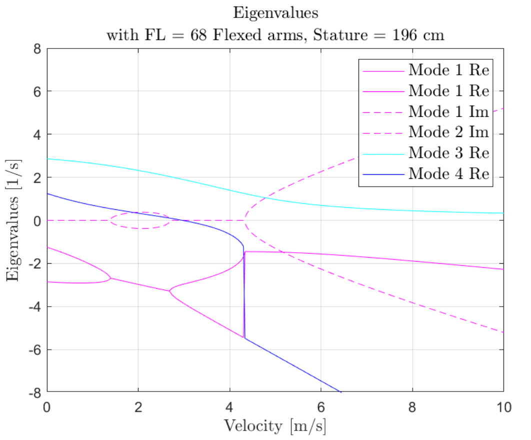

Figure 25: Eigenvalues Orange Small bicycle with a person of 1.96 m tall with a forward lean of 68 degrees

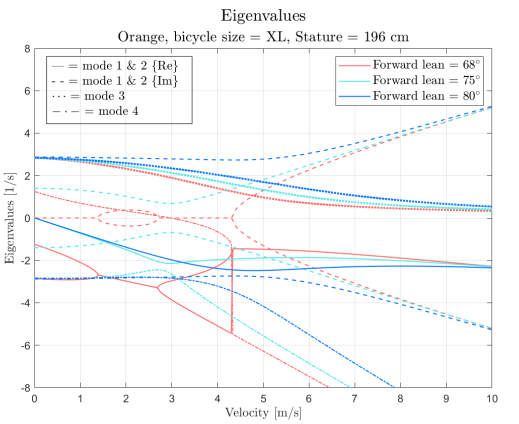

Figure 26: Eigenvalues Orange extra large bicycle with a person of 1.96 m tall with all forward leans

3.1.4 Conclusion of the eigenvalues

It has become clear that the dynamical system is heavily influenced by the arm model. Consequently, the posture has a great influence on the arm positions so therefore it can be said that the posture has an influence on the dynamic next to the change of center of mass of the system. However, this influence would have been less clear if the arm model was not used. (This statement can be used in discussion to clarify that it might be useful to conduct research to find answers to the question of how much the arm model influences the model and what the influence of the steer and steer position is.

3.2 Handling Quality Metric (HQM)

The handling qualities calculated for each rider-bicycle combination are plotted together for three forward leans and three different velocities; 4 m/s, 5 m/s and 7.5 m/s. The Levels 1,2 and 3 of the HQM values are based on the Cooper-Harper scale [9] and indicate the best handling qualities. Level 1 is the best and level 3 has the worst handling qualities. At the end of this section, there will be an overview plot of the maximum HQM values for each combination, as to compare all the different aspects that might influence the handling qualities. All the combinations are checked to have stable solutions for the chosen gains. All are stable except for the rider bicycle combination of stature 1.96m and a forward lean of 80° for the velocities 4 and 5 m/s. However, these can be considered marginally stable as they were checked on path tracking performance of which the results can be found in appendix D.

Chamonix bicycle

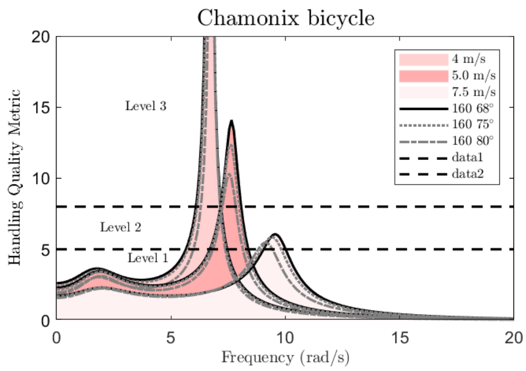

Figure 27: HQM Chamonix Small bicycle with a person of 1.60 m tall

In figure 27 the HQM values of the small Chamonix bicycle with the smallest person are given. The values are never below level 1 and indicate that this bicycle combination has poor handling qualities. At 4 m/s the values even reach a value of 82. Between the different sitting positions, there is a slight difference. The most upright position gives the best values and the most forward lean the worst.

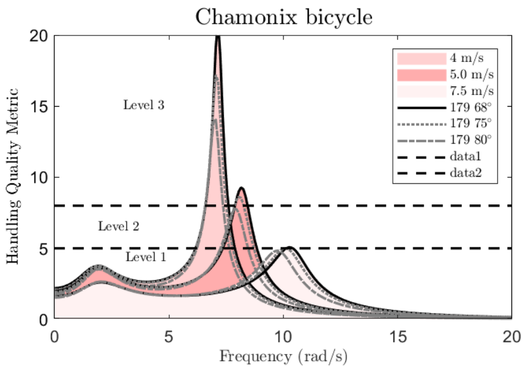

Figure 28: HQM Chamonix Large bicycle with a person of 1.79 m tall

The values for the large bicycle are better and are lower than for the small bicycle, see figure 28. Again the values are best for the most upright position. The peaks of HQM values are slightly shifted to lower frequencies with decreasing forward lean. At 7.5 m/s the handling qualities are best and stay below the level 1 line.

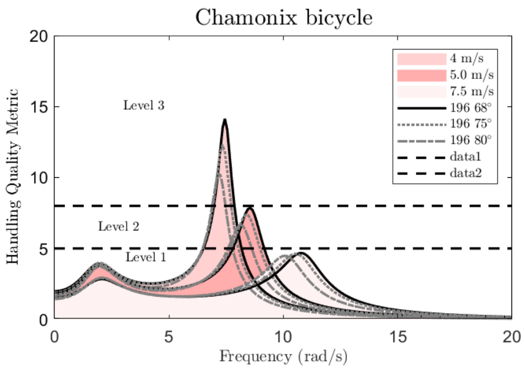

Figure 29: HQM Chamonix Extra large bicycle with a person of 1.96 m tall

The same observations on the differences between sitting positions are true for the extra-large bicycle, figure 29. This bicycle-rider combination gives the best HQM values for the Chamonix bicycle. At the highest velocity, the values stay well below level 1, and at 5 m/s below level 2.

Medeo bicycle

As seen in the Chamonix bicycle the increasing stature has a positive effect on the handling qualities of the Medeo bicycle, figure 30, 31 and 32. In the small bicycle, there are less pronounced differences between the sitting positions than for the large and especially the extra-large bicycle.

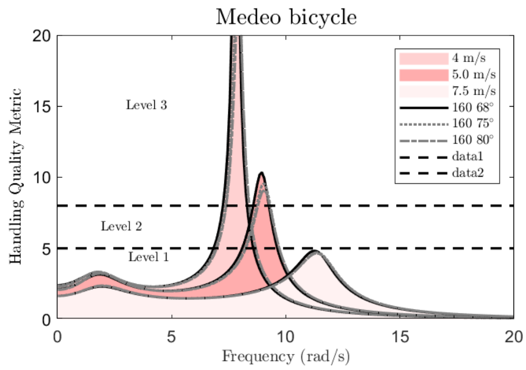

Figure 30: HQM Medeo Small bicycle with a person of 1.60 m tall

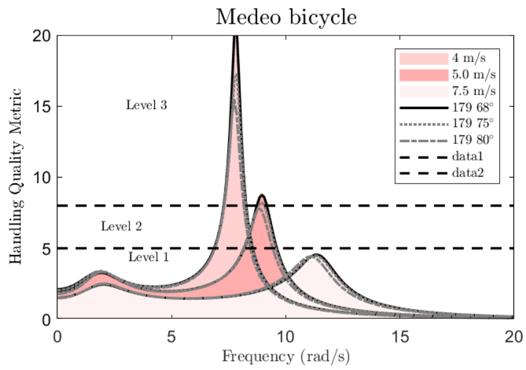

Figure 31: HQM Medeo Large bicycle with a person of 1.79 m tall

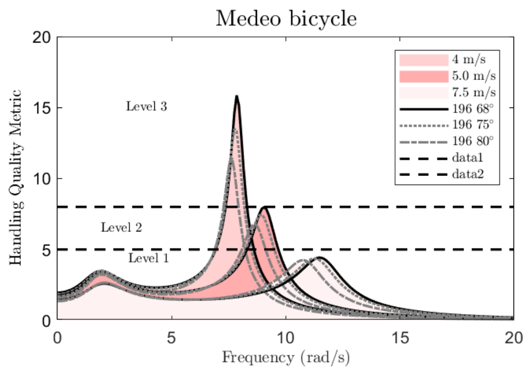

Figure 32: HQM Medeo Extra Large bicycle with a person of 1.96 m tall

Orange bicycle

Overall the Orange bicycle has the best values for the HQM compared to other bicycles, figures 33, 34, 35. Again the same trend between the statures and sitting positions applies to this bicycle. The differences between the sitting positions of the small bicycle are very slight. This difference is better seen in the extra-large bicycle.

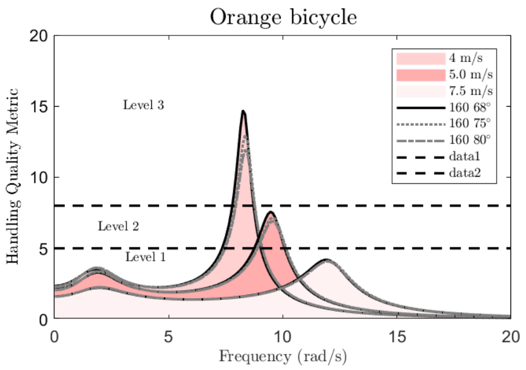

Figure 33: HQM Orange Small bicycle with a person of 1.60 m tall

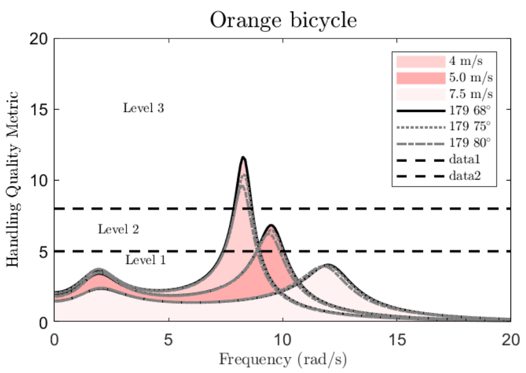

Figure 34: HQM Orange Large bicycle with a person of 1.79 m tall

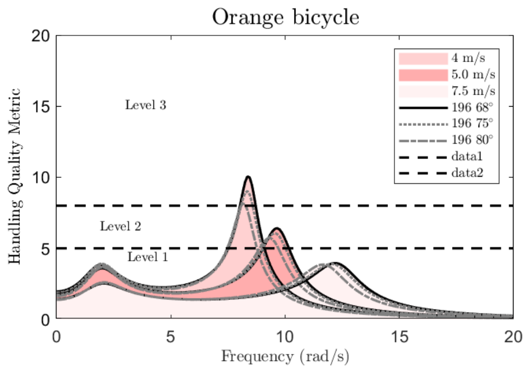

Figure 35: HQM Orange Extra Large bicycle with a person of 1.96 m tall

3.2.1 Comparison of all the rider-bicycle combinations & conclusion of the HQM values

To have an overview of all the rider-bicycle combinations, the maximum HQM values are plotted together in a bar chart, figure 36. Now it is clear that overall the Chamonix bicycle performs worst and the Orange the best. In all combinations, the stature seems to have the greatest improving effect. And overall the most upright position performs the best. It is unclear what parameter of the HQM is influenced most by the addition of the arm model. The control gains are changed because the open loop dynamics are drastically changed by the arm model. However, it is uncertain if the lean or steer rate is mostly affected by this addition. Furthermore, it can be concluded that the HQM values for some combinations are extremely large and reach over 20. This is most likely due to the added arm model. The results of the HQM values are also compared to rider bicycle combination without an arm model, see figure 37. The arm model has a negative effect on the handling qualities. By adding the arm model, the arms of the rider have their own inertia properties and influence the COM of the total system. It should be determined how much this influences the lean rate and how much the steer rate.

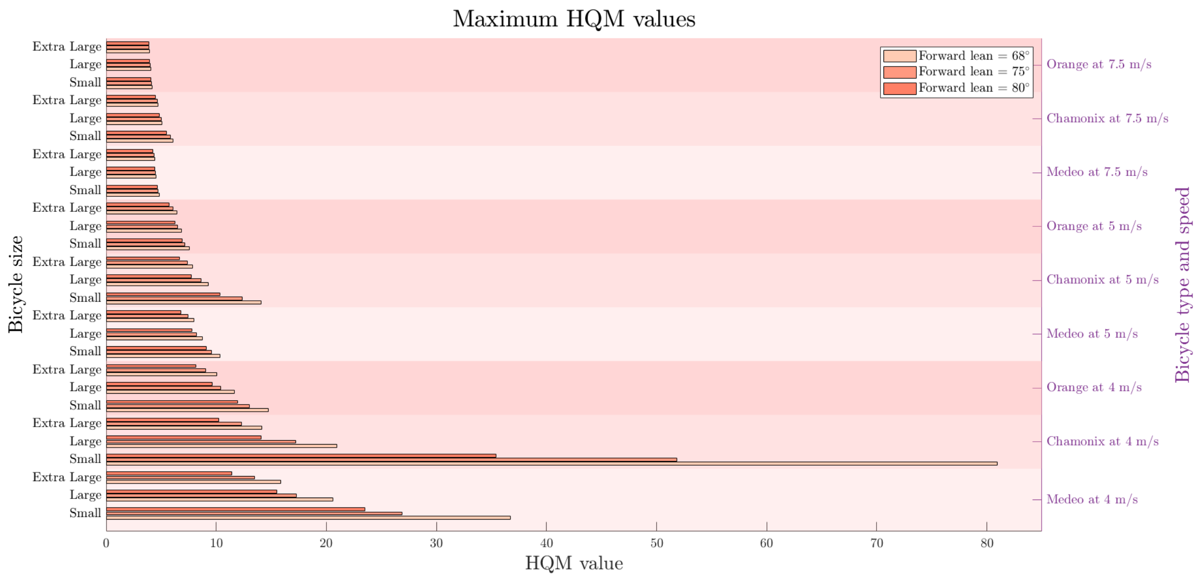

Figure 36: Maximum HQM values for three bicycles, velocities and postures

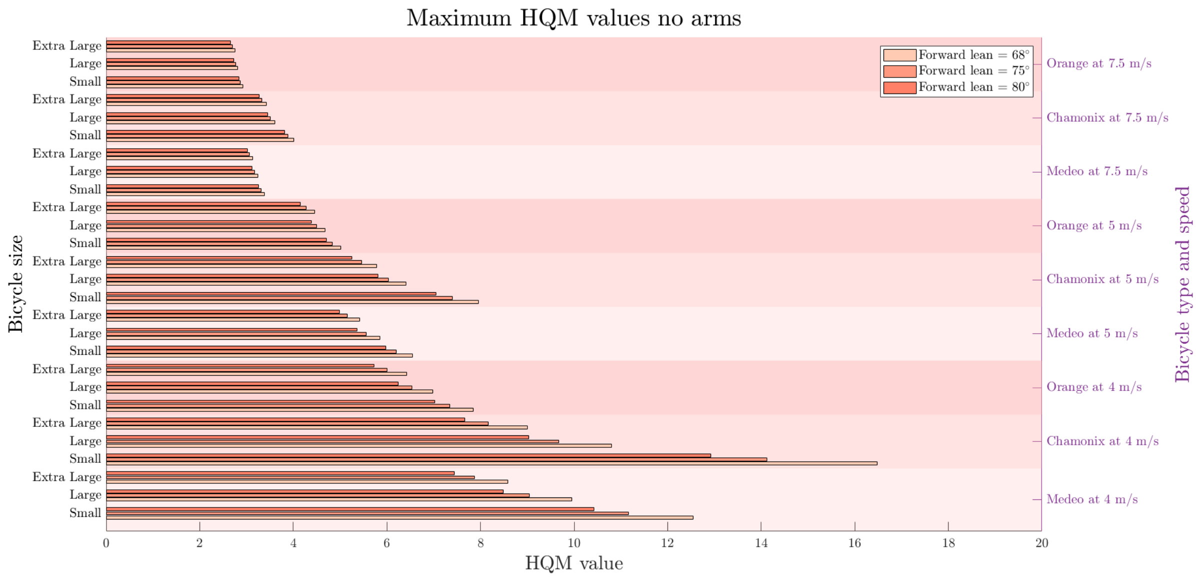

Figure 37: Maximum HQM values for three bicycles, velocities and postures without the arm model

4 Discussion

4.1 Results

The results for HQM values for different statures are mainly best for the tallest person and worst for the smallest person. This might be due to the location of the centers of mass of each of these combinations. With the COM of the smallest person being a lower location than of a taller person, it seems like that keeping the bicycle upright is easier with a high COM. The HQM model is designed to keep the bicycle upright so this result is not surprising. The arm model and thus the inertia properties of the arms do not seem to have a particularly great effect on the handling qualities between different sitting positions, which is remarkable because this does seem to greatly affect the oscillating behaviour of the system. The different sitting positions seem to have an influence on the handling qualities, although it is not as great as the influence of the different statures. It seems that the bicycle handles best in the most upright position. Which might also be caused by the location of the COM of the human body. The added arm model however has a great influence on the HQM values. This is probably due to the chosen gains that are needed for a stable system. The cause of these high HQM values could be explained by the system's chosen open loop gains. However, the results of the HQM values do not show this difference too much for different sitting positions.

4.2 Limitations

The arm model might not affect the HQM too much, however, it has a great effect on the eigenvalues and vectors. Therefore it can be seen that arms have a great influence on the dynamics system. When the arm model is used, the position of the steer has a great influence on the system as well. This is due to the arms being attached to the steer and whenever there is a change in the position of the steer it changes the location of the centers of mass of the arms. Consequently, if the position of the steer changes the COM of the front frame also changes.

4.3 Future research

It should be considered that the arm model influences the HQM values as well as the eigenvalues of the system. Since the steer input of the human is stated to be the most important, it should be validated with human experiments. As to see whether the level of the HQM values should be changed or if the added arm model is the cause of the extreme values.

5 Conclusion

It can be concluded that different statures and postures have a great influence on the handling qualities, however, this is minimal for higher speeds. The differences in dynamic behaviour that are found with the help of the eigenvalues are not clearly present when looking at the HQM model. The arm model gives insight into the differences in sitting positions and what input is dominant while cycling. It is evaluated that the most upright sitting position gives a greater oscillation than a more forward-leaning position. It should be noted that differences in this behaviour are strongly dependent on the steer's position. The use of the arm model with its oscillating behaviour gives inside into the steering behaviour and the changes there will be if the position of the steer (and consequently the moment of inertia) changes. This might seem obvious, but the differences are less clear when there is no use of the arm model. Overall stature has the greatest influence on handling qualities. For taller people, the HQM values are mainly better than for smaller people. The influence of the sitting position on the HQM becomes less important in higher velocities. When the maximum HQM values are plotted together the differences become most clear. Therefore the HQM can indicate the difference between bicycle-rider combinations as well as an eigenvalues analysis.
This model can evaluate the different postures and statures on their dynamic behaviour and handling qualities. The use of a Solidworks model allows the prediction of the handling qualities of a bicycle. Using human inertia modelling creates the possibility of evaluating a bicycle for a specific target group. Therefore this method of modelling can contribute to the design process of a bicycle.

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A Data tables

A.1 Human data

Measure	[m]	[m]	[m]
$L_{stature}$	1.60	1.79	1.96
l_{to}	0.3967	0.4600	0.5197
c_{ch}	0.8493	0.9484	1.0345
l_{ss}	0.4197	0.4594	0.4938
c_h	0.5808	0.6128	0.6406
c_{ua}	0.2985	0.2985	0.2985
l_{ua}	0.3126	0.3541	0.3902
c_{la}	0.1728	0.1728	0.1728
l_{la}	0.2414	0.2746	0.3034
l_{hh}	0.2450	0.2450	0.2450
c_{ul}	0.4555	0.4555	0.4555
l_{ul}	0.3846	0.4159	0.4431
c_{ll}	0.2985	0.2985	0.2985
l_{ll}	0.3461	0.4106	0.4667
m_{Br}	55.6800	69.6892	83.5548
m_h	3.7862	4.7389	5.6817
m_{la}	1.2250	1.5332	1.8382
m_{ll}	3.3965	4.2510	5.0968
m_{to}	28.3968	35.5415	42.6129
m_{ua}	1.5590	1.9513	2.3395
m_{ul}	5.5680	6.9689	8.3555

Table 7: Human measures

Forward lean angle	sportive	active	easy
α_{fl}	$1.868 rad$ or 68°	$1.3090 rad$ or 75°	$1.3963 rad$ or 80°

Table 8: Forward lean angles

A.2 Bicycle data

A.2.1 Bicycle geometry data

Size	Small	Large	Extra large
l_{hb}	0.0800	0.0800	0.0800
l_s	0.2050	0.2050	0.2050
h_{bb}	0.3010	0.3010	0.3010
l_{cs}	0.4950	0.4950	0.4950
l_f	0.4640	0.4640	0.4640
$w_f h$	0.1000	0.1000	0.1000
r_F	0.3610	0.3610	0.3610
α_{ht}	1.2305	1.2305	1.2305
l_{ht}	0.1600	0.1700	0.1850
w_{hb}	0.6250	0.6250	0.6250
w_{rh}	0.1300	0.1300	0.1300
r_R	0.3610	0.3610	0.3610
l_{st}	0.4500	0.5500	0.6000
l_{sp}	0.2307	0.2265	0.2598
α_{st}	1.2741	1.2741	1.2741
f_0	0.0420	0.0420	0.0420
c	0.0833	0.0833	0.0833
w	1.1250	1.1600	1.1770

Table 9: Geometry parameters Medeo bicycle

Size	Small	Large	Extra large
l_{hb}	0.0300	0.0300	0.0300
l_s	0.1800	0.1800	0.1800
h_{bb}	0.3010	0.3010	0.3010
l_{cs}	0.4710	0.4710	0.4710
l_f	0.4850	0.4850	0.4850
$w_f h$	0.1000	0.1000	0.1000
r_F	0.3610	0.3610	0.3610
α_{ht}	1.2043	1.2305	1.2305
l_{ht}	0.1450	0.1850	0.2200
w_{hb}	0.6650	0.6650	0.6650
w_{rh}	0.1300	0.1300	0.1300
r_R	0.3610	0.3610	0.3610
l_{st}	0.4900	0.5700	0.6100
l_{sp}	0.1907	0.2065	0.2498
α_{st}	1.2479	1.2479	1.2479
f_0	0.0500	0.0500	0.0500
c	0.0850	0.0748	0.0748
w	1.1320	1.1210	1.1260

Table 10: Geometry parameters Chamonix bicycle

Size	Small	Large	Extra large
l_{hb}	0.0800	0.0800	0.0800
l_s	0.2000	0.2000	0.2000
h_{bb}	0.2930	0.2930	0.2930
l_{cs}	0.4820	0.4820	0.4820
l_f	0.4750	0.4750	0.4750
$w_f h$	0.1000	0.1000	0.1000
r_F	0.3530	0.3530	0.3530
α_{ht}	1.1956	1.1956	1.1956
l_{ht}	0.1450	0.1650	0.1850
w_{hb}	0.5850	0.5850	0.5850
w_{rh}	0.1300	0.1300	0.1300
r_R	0.3530	0.3530	0.3530
l_{st}	0.4900	0.5700	0.6100
l_{sp}	0.1907	0.2065	0.2498
α_{st}	1.2130	1.2130	1.2130
f_0	0.0550	0.0550	0.0550
c	0.0799	0.0799	0.0799
w	1.1080	1.1310	1.1410

Table 11: Geometry parameters Orange bicycle

A.2.2 Bicycle data from solidworks

This section contains an example of the mass properties gathered from Solidworks. An overview of all the parameters of all combinations is given in the next section.

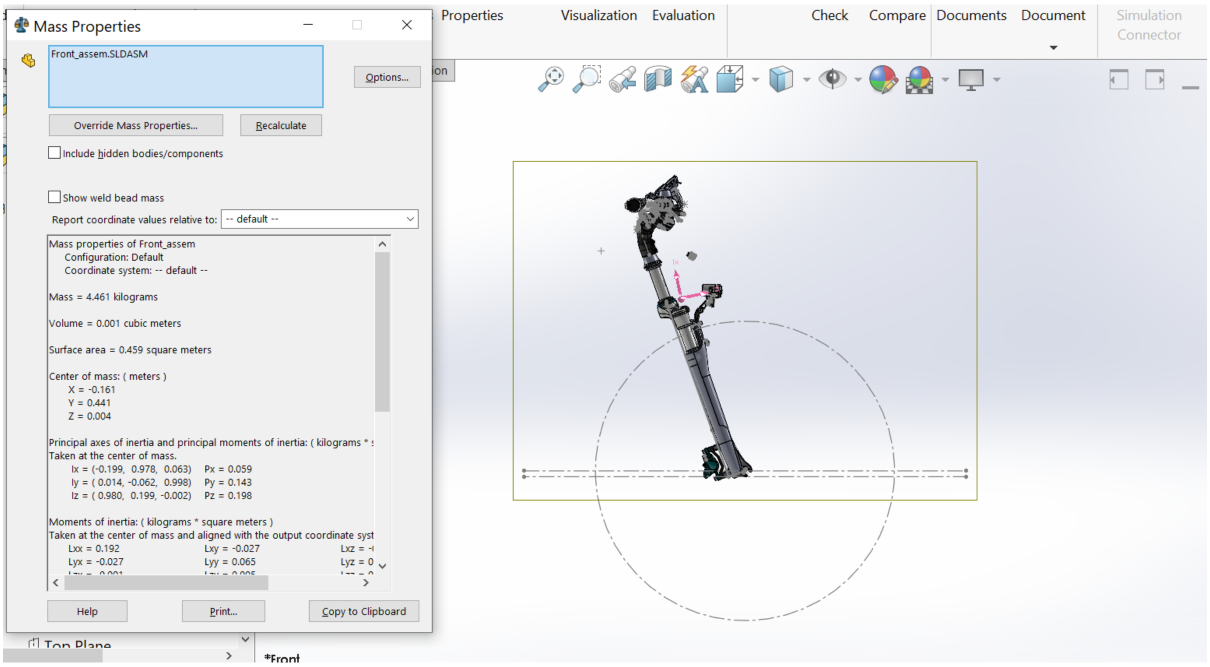


Figure 38: Screenshot of Solidworks with mass properties

```
Mass properties of Chamonix HMS MY21
     Configuration: G1890 - Chamonix T10 HMS L57
     Coordinate system: -- default --

Mass = 14.31979 kilograms

Volume = 0.00730 cubic meters

Surface area = 3.17916 square meters

Center of mass: ( meters )
    X = -0.00406
    Y = 0.25632
    Z = -0.00583

Principal axes of inertia and principal moments of inertia: ( kilograms * square ...
    meters )
Taken at the center of mass.
     Ix = ( 0.97452,  0.22203,  0.03166)    Px = 0.67886
     Iy = (-0.22170,  0.97502, -0.01366)    Py = 1.27794
     Iz = (-0.03390,  0.00629,  0.99941)    Pz = 1.86681

Moments of inertia: ( kilograms * square meters )
Taken at the center of mass and aligned with the output coordinate system. (Using ...
    positive tensor notation.)
    Lxx = 0.70967   Lxy = 0.12975   Lxz = 0.03844
    Lyx = 0.12975   Lyy = 1.24843   Lyz = 0.00051
    Lzx = 0.03844   Lzy = 0.00051   Lzz = 1.86551

Moments of inertia: ( kilograms * square meters )
Taken at the output coordinate system. (Using positive tensor notation.)
    Ixx = 1.65099   Ixy = 0.11487   Ixz = 0.03878
    Iyx = 0.11487   Iyy = 1.24915   Iyz = -0.02088
    Izx = 0.03878   Izy = -0.02088  Izz = 2.80658

One or more components have overridden mass properties:
```

```
    475409900@FRM^Chamonix HMS MY21
    463409000@FRM^Chamonix HMS MY21
    448337900@FRM^Chamonix HMS MY21
    470210800<Default>@ZDL^Chamonix HMS MY21<BBB BSD 292>
    380137401 (indicatief)<Default>@380137400 ...
        (indicatief)<Default-_flexible1>@ZDL^Chamonix HMS MY21<BBB BSD 292>
    AAN^Chamonix HMS MY21
    447190300<Default>@ADR^Chamonix HMS MY21<Derailleur>
    Snelbinders rechte flex assy - Chamonix HMS MY21<Default>@ADR^Chamonix HMS ...
        MY21<Derailleur>
    Horizontal Powertube Bosch<Default>
```

```
    Mass properties of frontframe
     Configuration: Default
     Coordinate system: -- default --

Mass = 3.64042 kilograms

Volume = 0.00164 cubic meters

Surface area = 0.70353 square meters

Center of mass: ( meters )
    X = -0.19840
    Y = 0.52009
    Z = 0.00343

Principal axes of inertia and principal moments of inertia: ( kilograms * square ...
    meters )
Taken at the center of mass.
     Ix = (-0.25724,  0.96559,  0.03830)    Px = 0.06622
     Iy = ( 0.01863, -0.03467,  0.99923)    Py = 0.32334
     Iz = ( 0.96617,  0.25776, -0.00907)    Pz = 0.37775

Moments of inertia: ( kilograms * square meters )
Taken at the center of mass and aligned with the output coordinate system. (Using ...
    positive tensor notation.)
    Lxx = 0.35712   Lxy = -0.07742  Lxz = -0.00206
    Lyx = -0.07742  Lyy = 0.08723   Lyz = 0.00964
    Lzx = -0.00206  Lzy = 0.00964   Lzz = 0.32297

Moments of inertia: ( kilograms * square meters )
Taken at the output coordinate system. (Using positive tensor notation.)
    Ixx = 1.34188   Ixy = -0.45305  Ixz = -0.00453
    Iyx = -0.45305  Iyy = 0.23057   Iyz = 0.01613
    Izx = -0.00453  Izy = 0.01613   Izz = 1.45099

One or more components have overridden mass properties:
    998504000@STR^Chamonix HMS MY21
    998504100<998504100>@STR^Chamonix HMS MY21<T10>
    540281900_LH<Default>@STR^Chamonix HMS MY21<T10>
    540281900_RH<Default>@STR^Chamonix HMS MY21<T10>
    BL_MT200<Links>@STR^Chamonix HMS MY21<T10>
    BL_MT200<Rechts>@STR^Chamonix HMS MY21<T10>
    705496400<Default>@STR^Chamonix HMS MY21<T10>
    310576400new<Default>@VVK^Chamonix HMS MY21<Default>
    Trelock Veo50 Bracket<Default>@444403800<Default-_flexible1>@VVK^Chamonix HMS ...
        MY21<Default>
    Trelock Veo50 Body<Default>@444403800<Default-_flexible1>@VVK^Chamonix HMS ...
        MY21<Default>
    448337900@VVK^Chamonix HMS MY21
```

A.3 Datasets to calculate HQM

The following data is in text form to load it into the HQM model.

A.3.1 Orange

Stature= 160, Forward lean = 68

```
IBxx = 7.788868e+00
IBxz = -4.581548e-01
IByy = 8.831749e+00
IBzz = 2.257897e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.108000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 6.585800e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 3.361076e-01
zB = -8.623917e-01
g = 9.810000e+00
M = 6.791155e+01 1.953378e+00; 1.953378e+00 6.489421e-01
K0 = -6.870944e+01 -2.289053e+00; -2.289053e+00 8.923746e-03
K2 =     0 5.840345e+01; -3.552714e-15 2.058711e+00
C1 = 2.884001e-15 2.659390e+01; -4.692151e-01 1.711667e+00
```

Stature= 160, Forward lean = 75

```
    IBxx = 7.997562e+00
IBxz = -7.675135e-01
IByy = 9.025492e+00
IBzz = 2.242947e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.108000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 6.585800e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 3.229480e-01
zB = -8.667949e-01
g = 9.810000e+00
M = 6.883634e+01 1.906694e+00; 1.906694e+00 6.363035e-01
K0 = -6.909631e+01 -2.224157e+00; -2.224157e+00 2.791739e-02
K2 =     0 5.872832e+01; -3.552714e-15 2.004216e+00
C1 = 2.864043e-15 2.574943e+01; -4.692151e-01 1.660140e+00
```

Stature= 160, Forward lean = 80

```
    IBxx = 8.100876e+00
IBxz = -1.002099e+00
IByy = 9.143013e+00
IBzz = 2.257153e+00
```

```
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.108000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 6.585800e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 3.132680e-01
zB = -8.689408e-01
g = 9.810000e+00
M = 6.933226e+01 1.873576e+00; 1.873576e+00 6.395601e-01
K0 = -6.930368e+01 -2.176799e+00; -2.176799e+00 1.214522e-01
K2 =     0 5.890245e+01; -3.552714e-15 1.964448e+00
C1 = 2.903667e-15 2.509582e+01; -4.692151e-01 1.627626e+00
```

Stature= 179, Forward lean = 68

```
    IBxx = 1.173708e+01
IBxz = -6.297403e-01
IByy = 1.268342e+01
IBzz = 2.677289e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.131000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 7.846626e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 3.240541e-01
zB = -9.606520e-01
g = 9.810000e+00
M = 9.867680e+01 2.203496e+00; 2.203496e+00 6.796686e-01
K0 = -8.947958e+01 -2.378997e+00; -2.378997e+00 -2.610643e-02
K2 =     0 7.430232e+01; -3.552714e-15 2.090837e+00
C1 = -1.551301e-15 3.241514e+01; -4.680669e-01 1.725686e+00
```

Stature= 179, Forward lean = 75

```
    IBxx = 1.206867e+01
IBxz = -1.123765e+00
IByy = 1.300612e+01
IBzz = 2.668393e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
```

```
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.131000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 7.846626e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 3.080937e-01
zB = -9.659923e-01
g = 9.810000e+00
M = 1.001400e+02 2.133530e+00; 2.133530e+00 6.819099e-01
K0 = -9.003178e+01 -2.289726e+00; -2.289726e+00 8.774501e-02
K2 = 1.136868e-13 7.475659e+01; -3.552714e-15 2.017398e+00
C1 = -1.486192e-15 3.109256e+01; -4.680669e-01 1.667930e+00
```

Stature= 179, Forward lean = 80

```
    IBxx = 1.223287e+01
IBxz = -1.497880e+00
IByy = 1.320449e+01
IBzz = 2.702564e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.131000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 7.846626e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 2.963537e-01
zB = -9.685950e-01
g = 9.810000e+00
M = 1.009351e+02 2.085647e+00; 2.085647e+00 7.055963e-01
K0 = -9.033247e+01 -2.224540e+00; -2.224540e+00 3.117475e-01
K2 =      0 7.500395e+01; -3.552714e-15 1.963773e+00
C1 = -2.280891e-15 3.007665e+01; -4.680669e-01 1.634603e+00
```

Stature= 196, Forward lean = 68

```
    IBxx = 1.638242e+01
IBxz = -8.381260e-01
IByy = 1.724156e+01
IBzz = 3.174395e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
```

```
w = 1.141000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 9.094532e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 3.125273e-01
zB = -1.049789e+00
g = 9.810000e+00
M = 1.351662e+02 2.523346e+00; 2.523346e+00 7.044658e-01
K0 = -1.119626e+02 -2.524975e+00; -2.524975e+00 -1.617301e-01
K2 =      0 9.198471e+01; -3.552714e-15 2.191549e+00
C1 = 6.006692e-15 3.862902e+01; -4.675822e-01 1.797273e+00
```

Stature= 196, Forward lean = 75

```
    IBxx = 1.686511e+01
IBxz = -1.558625e+00
IByy = 1.771943e+01
IBzz = 3.169581e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.141000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
mH = 4.660000e+00
mB = 9.094532e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 2.939312e-01
zB = -1.056011e+00
g = 9.810000e+00
M = 1.373068e+02 2.419003e+00; 2.419003e+00 7.283925e-01
K0 = -1.127053e+02 -2.407741e+00; -2.407741e+00 8.677158e-02
K2 =      0 9.259029e+01; -3.552714e-15 2.095951e+00
C1 = -2.644084e-14 3.670428e+01; -4.675822e-01 1.735777e+00
```

Stature= 196, Forward lean = 80

```
    IBxx = 1.710420e+01
IBxz = -2.104188e+00
IByy = 1.801548e+01
IBzz = 3.226533e+00
IRxx = 7.640000e-02
IRyy = 1.403000e-01
IFxx = 8.160000e-02
IFyy = 1.566000e-01
IHxx = 3.142000e-01
IHyy = 3.752000e-01
IHxz = -6.660000e-02
IHzz = 6.870000e-02
rF = 3.530000e-01
rR = 3.530000e-01
w = 1.141000e+00
c = 7.993715e-02
lam = 3.752458e-01
mR = 4.175000e+00
mF = 1.993000e+00
```

```
mH = 4.660000e+00
mB = 9.094532e+01
xH = 9.266000e-01
zH = -7.595000e-01
xB = 2.802523e-01
zB = -1.059044e+00
g = 9.810000e+00
M = 1.384987e+02 2.356154e+00; 2.356154e+00 7.932979e-01
K0 = -1.131209e+02 -2.322725e+00; -2.322725e+00 5.868577e-01
K2 =      0 9.292922e+01; -3.552714e-15 2.026626e+00
C1 = -2.414177e-14 3.524588e+01; -4.675822e-01 1.709301e+00
```

A.3.2 Chamonix

Stature= 160, Forward lean = 68

```
    IBxx = 7.234789e+00
IBxz = -9.817186e-01
IByy = 8.193424e+00
IBzz = 2.292143e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.132000e+00
c = 8.501767e-02
lam = 3.665191e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 6.422600e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.867849e-01
zB = -8.897040e-01
g = 9.810000e+00
M = 6.969940e+01 2.522255e+00; 2.522255e+00 8.897187e-01
K0 = -6.846996e+01 -2.598261e+00; -2.598261e+00 5.627313e-02
K2 = 1.136868e-13 5.719017e+01;      0 2.274294e+00
C1 = 8.763086e-16 2.882267e+01; -4.766219e-01 2.003077e+00
```

Stature= 160, Forward lean = 75

```
    IBxx = 7.436850e+00
IBxz = -1.292533e+00
IByy = 8.448481e+00
IBzz = 2.345139e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.132000e+00
c = 8.501767e-02
lam = 3.665191e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 6.422600e+01
```

```
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.732909e-01
zB = -8.942191e-01
g = 9.810000e+00
M = 7.071363e+01 2.493821e+00; 2.493821e+00 9.262419e-01
K0 = -6.889030e+01 -2.530917e+00; -2.530917e+00 4.327540e-01
K2 =      0 5.753683e+01;      0 2.218754e+00
C1 = 9.695030e-16 2.801149e+01; -4.766219e-01 1.957046e+00
```

Stature= 160, Forward lean = 80

```
    IBxx = 7.536908e+00
IBxz = -1.525274e+00
IByy = 8.612356e+00
IBzz = 2.408956e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.132000e+00
c = 8.501767e-02
lam = 3.665191e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 6.422600e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.633650e-01
zB = -8.964196e-01
g = 9.810000e+00
M = 7.130284e+01 2.495933e+00; 2.495933e+00 1.018226e+00
K0 = -6.913459e+01 -2.482138e+00; -2.482138e+00 1.429723e+00
K2 =      0 5.773830e+01;      0 2.178526e+00
C1 = -2.029193e-15 2.739273e+01; -4.766219e-01 1.935271e+00
```

Stature= 179, Forward lean = 68

```
    IBxx = 1.100544e+01
IBxz = -1.322527e+00
IByy = 1.197182e+01
IBzz = 2.816015e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.121000e+00
c = 7.479437e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 7.704005e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.696292e-01
zB = -9.899310e-01
g = 9.810000e+00
```

```
M = 1.016409e+02 2.683723e+00; 2.683723e+00 9.452002e-01
K0 = -8.980756e+01 -2.591574e+00; -2.591574e+00 2.260169e-01
K2 =      0 7.625452e+01;      0 2.304091e+00
C1 = 1.131960e-15 3.576233e+01; -4.743322e-01 2.063270e+00
```

Stature= 179, Forward lean = 75

```
    IBxx = 1.132674e+01
IBxz = -1.814148e+00
IByy = 1.236179e+01
IBzz = 2.884680e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.121000e+00
c = 7.479437e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 7.704005e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.533734e-01
zB = -9.953701e-01
g = 9.810000e+00
M = 1.032049e+02 2.636928e+00; 2.636928e+00 9.812689e-01
K0 = -9.039268e+01 -2.506141e+00; -2.506141e+00 5.187916e-01
K2 = -1.136868e-13 7.674655e+01;      0 2.232252e+00
C1 = -2.069881e-15 3.441408e+01; -4.743322e-01 2.009426e+00
```

Stature= 179, Forward lean = 80

```
    IBxx = 1.148589e+01
IBxz = -2.183154e+00
IByy = 1.261169e+01
IBzz = 2.975432e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.121000e+00
c = 7.479437e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 7.704005e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.414160e-01
zB = -9.980210e-01
g = 9.810000e+00
M = 1.040965e+02 2.625426e+00; 2.625426e+00 1.077858e+00
K0 = -9.072718e+01 -2.444194e+00; -2.444194e+00 1.335385e+00
K2 =      0 7.702782e+01;      0 2.180161e+00
C1 = 6.328031e-16 3.338931e+01; -4.743322e-01 1.987785e+00
```

Stature= 196, Forward lean = 68

```
    IBxx = 1.548755e+01
IBxz = -1.693128e+00
IByy = 1.646096e+01
IBzz = 3.407469e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.126000e+00
c = 7.479437e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 8.965437e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.554059e-01
zB = -1.079448e+00
g = 9.810000e+00
M = 1.392638e+02 3.061940e+00; 3.061940e+00 1.010708e+00
K0 = -1.127399e+02 -2.802098e+00; -2.802098e+00 1.631291e-01
K2 =      0 9.511394e+01;      0 2.470102e+00
C1 = -7.746700e-16 4.277555e+01; -4.740877e-01 2.214923e+00
```

Stature= 196, Forward lean = 75

```
    IBxx = 1.595701e+01
IBxz = -2.408318e+00
IByy = 1.701201e+01
IBzz = 3.489056e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.126000e+00
c = 7.479437e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 8.965437e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.365421e-01
zB = -1.085760e+00
g = 9.810000e+00
M = 1.415089e+02 2.984670e+00; 2.984670e+00 1.062258e+00
K0 = -1.135115e+02 -2.689201e+00; -2.689201e+00 5.070028e-01
K2 =      0 9.575992e+01;      0 2.375589e+00
C1 = -1.047643e-15 4.079219e+01; -4.740877e-01 2.154646e+00
```

Stature= 196, Forward lean = 80

```
    IBxx = 1.618963e+01
IBxz = -2.946241e+00
```

```
IByy = 1.736455e+01
IBzz = 3.608985e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 3.571200e-01
IHyy = 3.229700e-01
IHxz = 7.742000e-02
IHzz = 8.723000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.126000e+00
c = 7.479437e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 3.640420e+00
mB = 8.965437e+01
xH = 9.226300e-01
zH = -8.794100e-01
xB = 3.226663e-01
zB = -1.088836e+00
g = 9.810000e+00
M = 1.427955e+02 2.962299e+00; 2.962299e+00 1.206418e+00
K0 = -1.139561e+02 -2.607694e+00; -2.607694e+00 1.580931e+00
K2 = 4.547474e-13 9.613205e+01; -3.552714e-14 2.307355e+00
C1 = 1.620858e-15 3.929888e+01; -4.740877e-01 2.141406e+00
```

A.3.3 Medeo

Stature= 160, Forward lean = 68

```
    IBxx = 7.664210e+00
IBxz = -1.008428e+00
IByy = 8.453777e+00
IBzz = 2.103479e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.125000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 6.432874e+01
xH = 9.383000e-01
zH = -8.096200e-01
xB = 4.226142e-01
zB = -8.855879e-01
g = 9.810000e+00
M = 6.962388e+01 2.389482e+00; 2.389482e+00 8.011633e-01
K0 = -6.866274e+01 -2.649707e+00; -2.649707e+00 7.241472e-02
K2 = 1.136868e-13 5.826608e+01; 3.552714e-15 2.344609e+00
C1 = -2.676090e-16 3.112673e+01; -4.803600e-01 2.039124e+00
```

Stature= 160, Forward lean = 75

```
    IBxx = 7.871812e+00
IBxz = -1.325979e+00
IByy = 8.705152e+00
IBzz = 2.147254e+00
```

```
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.125000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 6.432874e+01
xH = 9.383000e-01
zH = -8.096200e-01
xB = 4.091417e-01
zB = -8.900957e-01
g = 9.810000e+00
M = 7.056716e+01 2.341418e+00; 2.341418e+00 7.861340e-01
K0 = -6.905032e+01 -2.581899e+00; -2.581899e+00 8.682241e-02
K2 = 1.136868e-13 5.859083e+01;      0 2.287793e+00
C1 = -2.617105e-16 3.028077e+01; -4.803600e-01 1.978579e+00
```

Stature= 160, Forward lean = 80

```
    IBxx = 7.974572e+00
IBxz = -1.564064e+00
IByy = 8.864970e+00
IBzz = 2.204311e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.125000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 6.432874e+01
xH = 9.383000e-01
zH = -8.096200e-01
xB = 3.992317e-01
zB = -8.922927e-01
g = 9.810000e+00
M = 7.107246e+01 2.307450e+00; 2.307450e+00 7.889728e-01
K0 = -6.925779e+01 -2.532447e+00; -2.532447e+00 1.852976e-01
K2 =      0 5.876467e+01;      0 2.246357e+00
C1 = -3.389100e-16 2.962275e+01; -4.803600e-01 1.939729e+00
```

Stature= 179, Forward lean = 68

```
    IBxx = 1.176700e+01
IBxz = -1.330231e+00
IByy = 1.255756e+01
IBzz = 2.620518e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
```

```
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.160000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 7.712277e+01
xH = 9.733000e-01
zH = -8.096200e-01
xB = 4.080153e-01
zB = -9.848627e-01
g = 9.810000e+00
M = 1.015697e+02 2.819753e+00; 2.819753e+00 8.644492e-01
K0 = -8.986012e+01 -2.901115e+00; -2.901115e+00 3.410185e-02
K2 =      0 7.373351e+01;      0 2.478166e+00
C1 = -2.357790e-15 3.800833e+01; -4.785173e-01 2.169731e+00
```

Stature= 179, Forward lean = 75

```
    IBxx = 1.209775e+01
IBxz = -1.833805e+00
IByy = 1.294406e+01
IBzz = 2.676276e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.160000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 7.712277e+01
xH = 9.733000e-01
zH = -8.096200e-01
xB = 3.917770e-01
zB = -9.902960e-01
g = 9.810000e+00
M = 1.030735e+02 2.753317e+00; 2.753317e+00 8.729204e-01
K0 = -9.041839e+01 -2.809236e+00; -2.809236e+00 1.845738e-01
K2 = -1.136868e-13 7.418717e+01;      0 2.403503e+00
C1 = -2.136274e-15 3.670723e+01; -4.785173e-01 2.103257e+00
```

Stature= 179, Forward lean = 80

```
    IBxx = 1.226151e+01
IBxz = -2.212146e+00
IByy = 1.318911e+01
IBzz = 2.757561e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
```

```
w = 1.160000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 7.712277e+01
xH = 9.733000e-01
zH = -8.096200e-01
xB = 3.798324e-01
zB = -9.929441e-01
g = 9.810000e+00
M = 1.038970e+02 2.710725e+00; 2.710725e+00 9.100060e-01
K0 = -9.072479e+01 -2.742216e+00; -2.742216e+00 5.072666e-01
K2 =      0 7.443615e+01;      0 2.349041e+00
C1 = -2.699112e-15 3.570413e+01; -4.785173e-01 2.065211e+00
```

Stature= 196, Forward lean = 68

```
    IBxx = 1.642754e+01
IBxz = -1.648518e+00
IByy = 1.719568e+01
IBzz = 3.182489e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.177000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 8.972939e+01
xH = 9.903000e-01
zH = -8.096200e-01
xB = 3.955886e-01
zB = -1.076125e+00
g = 9.810000e+00
M = 1.394945e+02 3.300616e+00; 3.300616e+00 9.109921e-01
K0 = -1.128832e+02 -3.160557e+00; -3.160557e+00 -1.022278e-01
K2 =      0 9.110741e+01; 3.552714e-15 2.650156e+00
C1 = -1.383474e-15 4.555302e+01; -4.776618e-01 2.315645e+00
```

Stature= 196, Forward lean = 75

```
    IBxx = 1.690854e+01
IBxz = -2.377863e+00
IByy = 1.773989e+01
IBzz = 3.245696e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.177000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
```

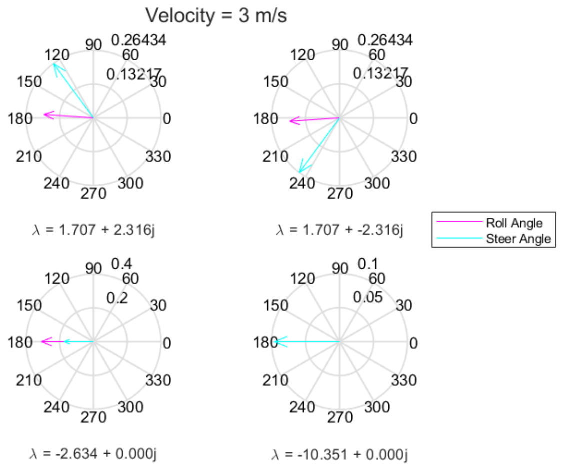
```
mH = 4.461000e+00
mB = 8.972939e+01
xH = 9.903000e-01
zH = -8.096200e-01
xB = 3.767406e-01
zB = -1.082431e+00
g = 9.810000e+00
M = 1.417110e+02 3.207350e+00; 3.207350e+00 9.574662e-01
K0 = -1.136408e+02 -3.041117e+00; -3.041117e+00 2.695612e-01
K2 =      0 9.171408e+01;      0 2.554498e+00
C1 = -1.806161e-15 4.368271e+01; -4.776618e-01 2.247598e+00
```

Stature= 196, Forward lean = 80

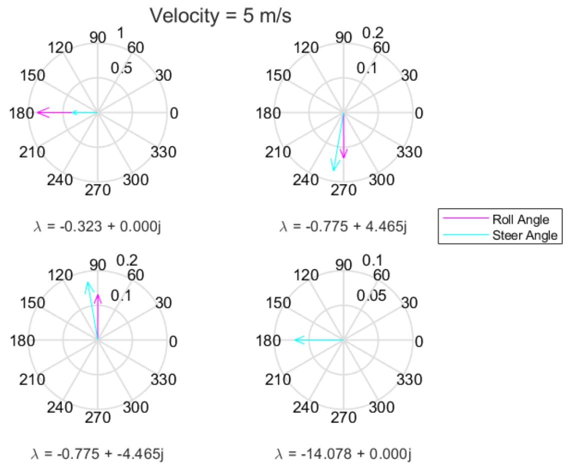
```
    IBxx = 1.714677e+01
IBxz = -2.926969e+00
IByy = 1.808457e+01
IBzz = 3.352140e+00
IRxx = 7.984578e-02
IRyy = 1.553692e-01
IFxx = 8.127634e-02
IFyy = 1.605732e-01
IHxx = 2.120000e-01
IHyy = 1.780000e-01
IHxz = 5.690000e-02
IHzz = 8.080000e-02
rF = 3.610000e-01
rR = 3.610000e-01
w = 1.177000e+00
c = 8.328116e-02
lam = 3.403392e-01
mR = 2.486700e+00
mF = 2.245310e+00
mH = 4.461000e+00
mB = 8.972939e+01
xH = 9.903000e-01
zH = -8.096200e-01
xB = 3.628763e-01
zB = -1.085505e+00
g = 9.810000e+00
M = 1.429755e+02 3.169066e+00; 3.169066e+00 1.088946e+00
K0 = -1.140757e+02 -2.955034e+00; -2.955034e+00 1.302518e+00
K2 =      0 9.206241e+01; 3.552714e-15 2.485556e+00
C1 = -2.127786e-15 4.227134e+01; -4.776618e-01 2.227837e+00
```

B Phasor plot explanation

To identify and for a closer look at the behaviour of the modes, the Eigenvectors can be plotted in a phasor plot. The phasor plots show the magnitude and phase of the steer and roll angle, in figures 39 and 40 phasor plots of the benchmark bicycle for 4 different velocities can be examined.

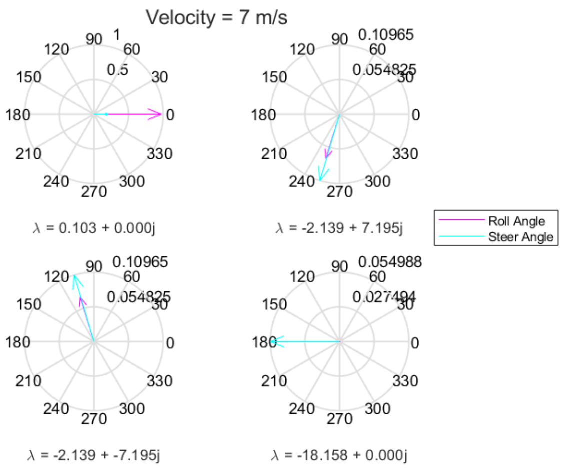


(a) 3 m/s

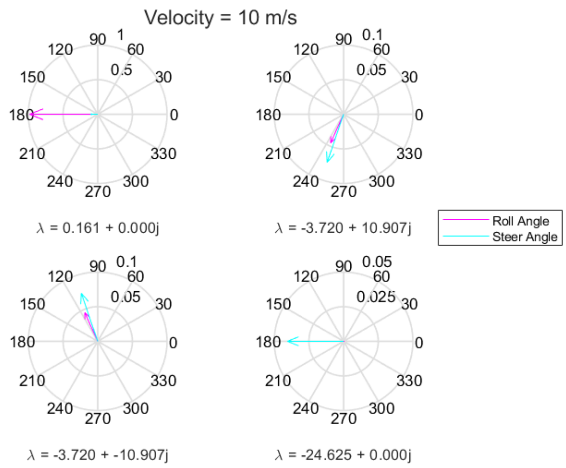


(b) 5 m/s

Figure 39: Phasor plot of the benchmark bicycle



(a) 7 m/s



(b) 10 m/s

Figure 40: Phasor plot of the benchmark bicycle

It can be seen that the steer angle component of the most negative real eigenvalue is dominant and that the magnitude rapidly decays with increasing velocity, which implies that this is the caster mode. The eigenvectors components of the other real eigenvalue mode become more dominated by roll angle with increasing speed, which explains that this is the capsize mode. The eigenvectors of the complex eigenvalues give inside into the weave mode's oscillating behaviour. With increasing speed the angle between the two components decreases which means that the angles are less out of phase with each other.

C Results – Eigenvalues plots seperately

C.1 Chamonix

C.1.1 Small

The results of the eigenvalues of the small Chamonix bicycle are plotted in figures 41, 42, 43 and 44.

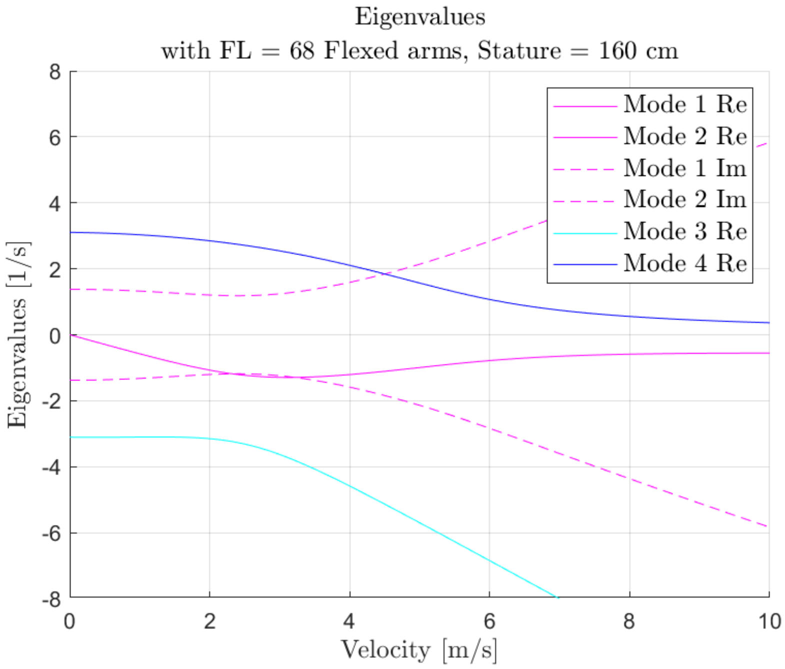


Figure 41: Eigenvalues Chamonix Small bicycle with a person of 1.60 m tall with a forward lean of 68 degrees

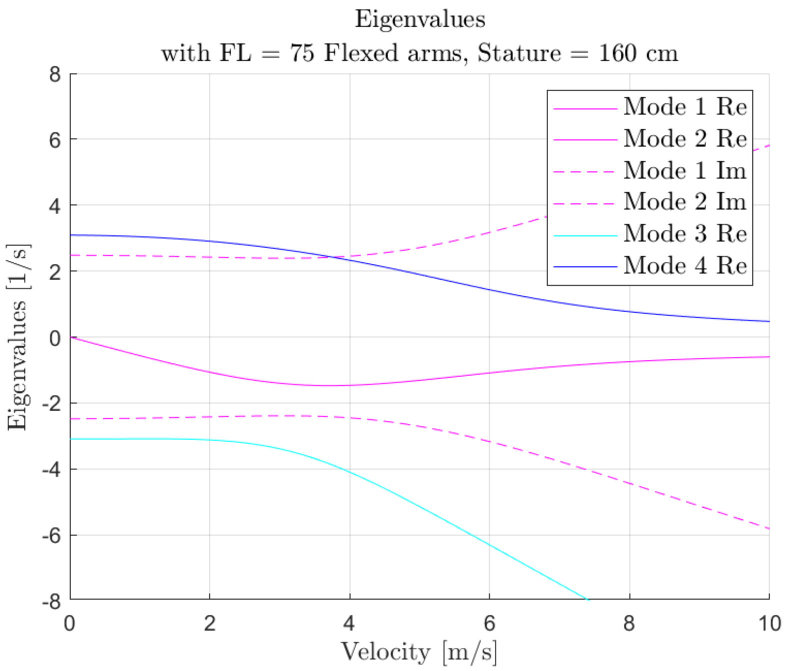


Figure 42: Eigenvalues Chamonix Small bicycle with a person of 1.60 m tall with a forward lean of 75 degrees

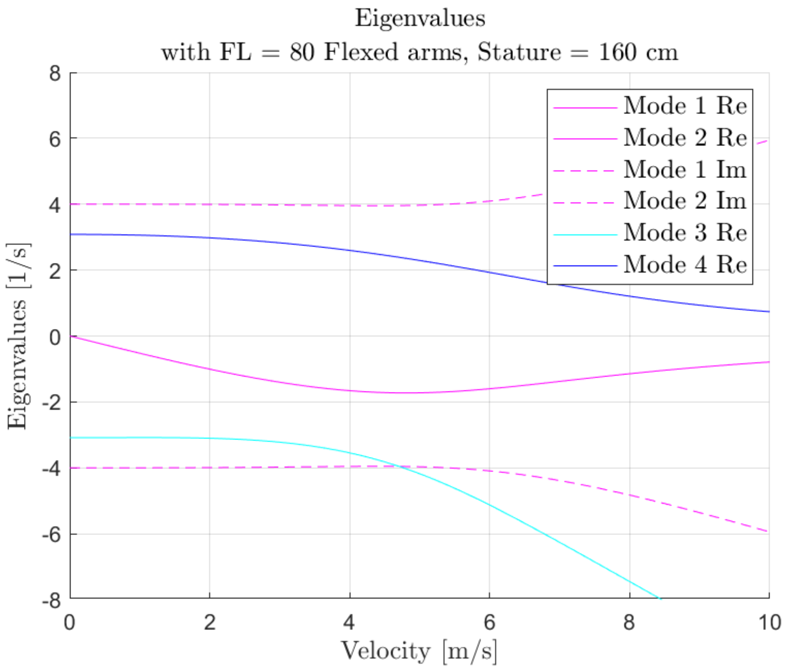


Figure 43: Eigenvalues Chamonix Small bicycle with a person of 1.60 m tall with a forward lean of 80 degrees

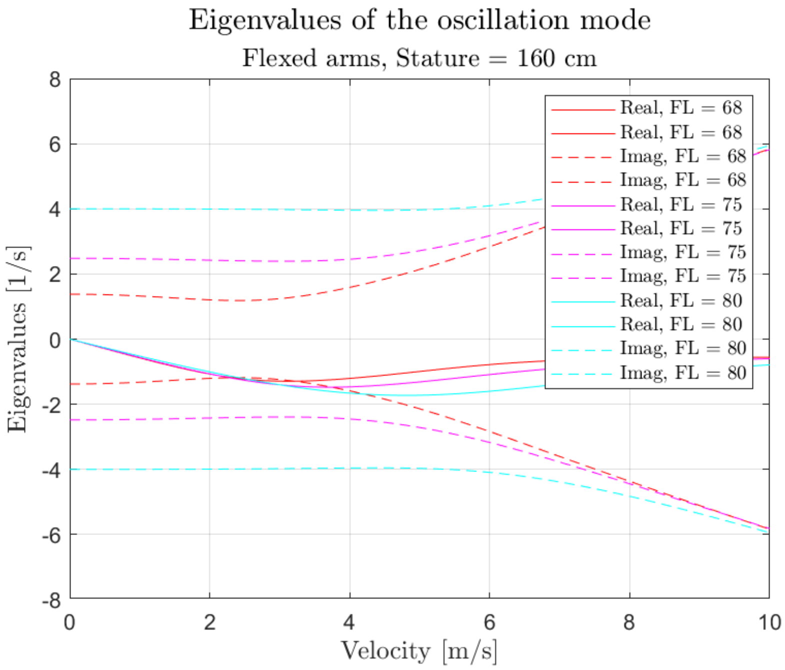


Figure 44: Eigenvalues Chamonix Small bicycle with a person of 1.60 m tall with all sitting positions

C.1.2 Large

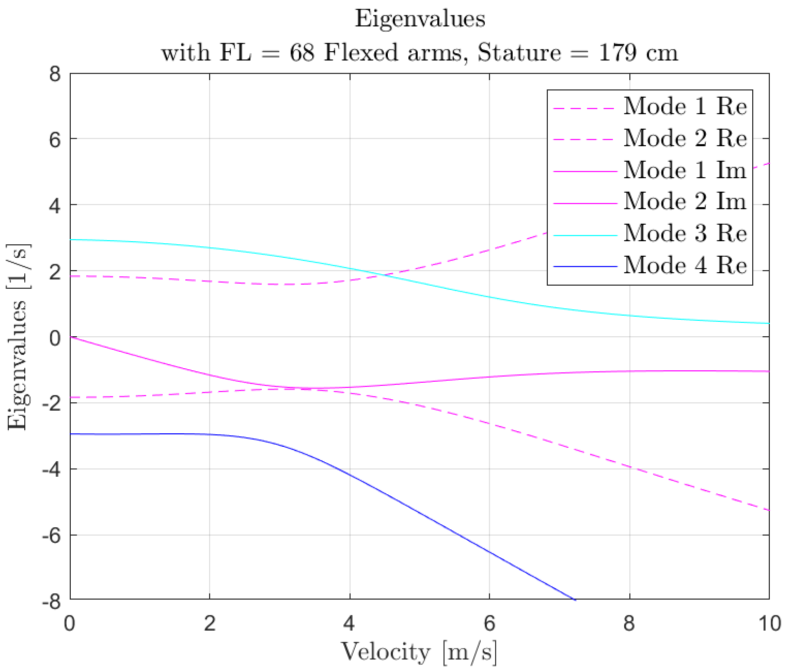


Figure 45: Eigenvalues chamonix Large bicycle with a person of 1.79 m tall with a forward lean of 68 degrees

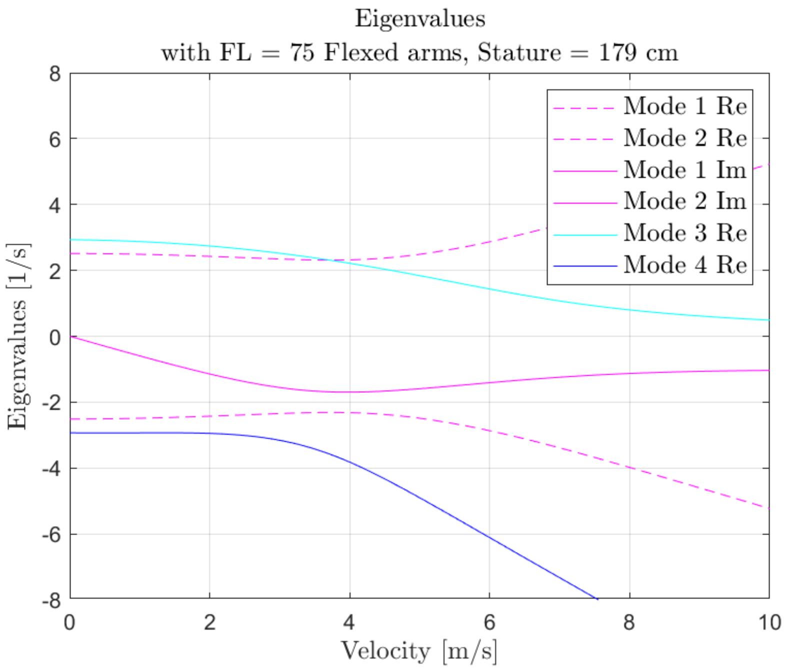


Figure 46: Eigenvalues chamonix Large bicycle with a person of 1.79 m tall with a forward lean of 75 degrees

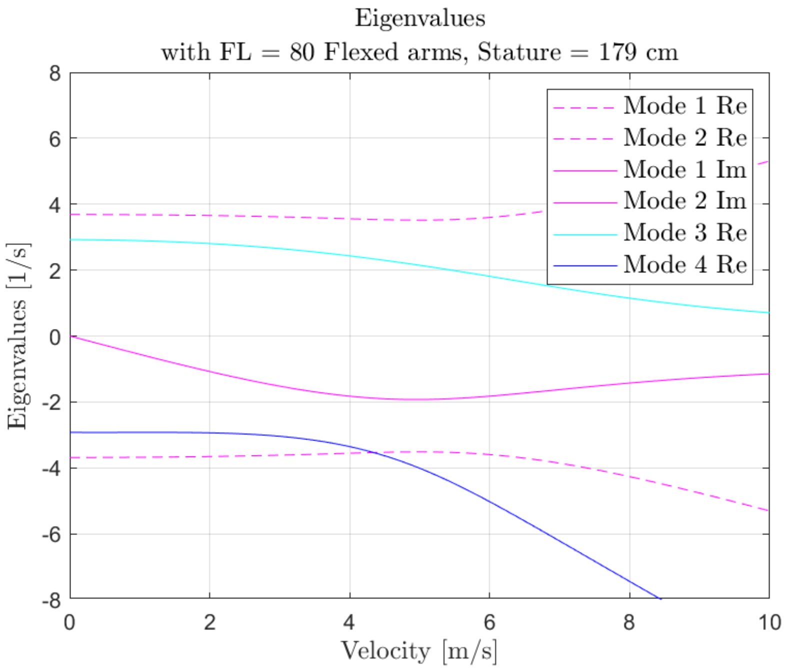


Figure 47: Eigenvalues Chamonix Large bicycle with a person of 1.79 m tall with forward lean of 80 degrees

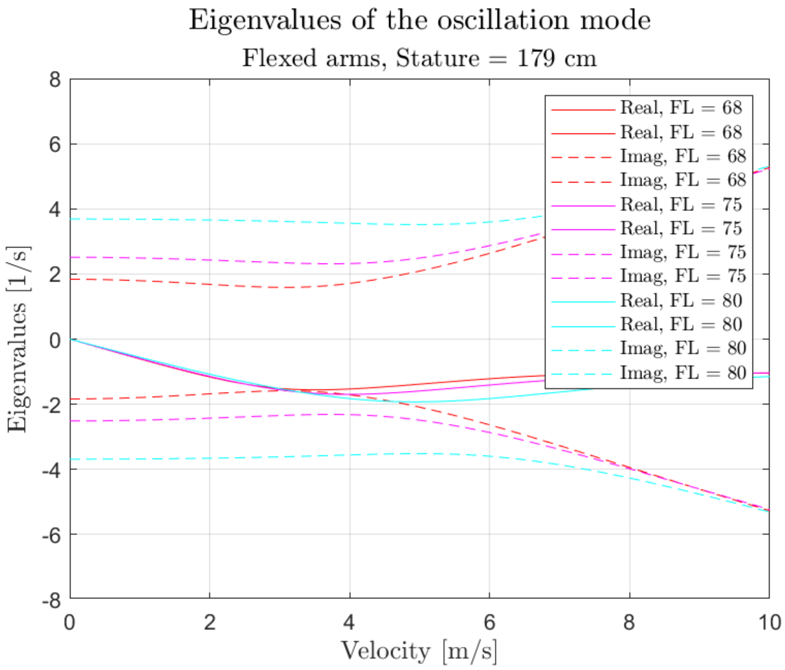


Figure 48: Eigenvalues Chamonix Large bicycle with a person of 1.79 m tall with all sitting positions

C.1.3 Extra Large

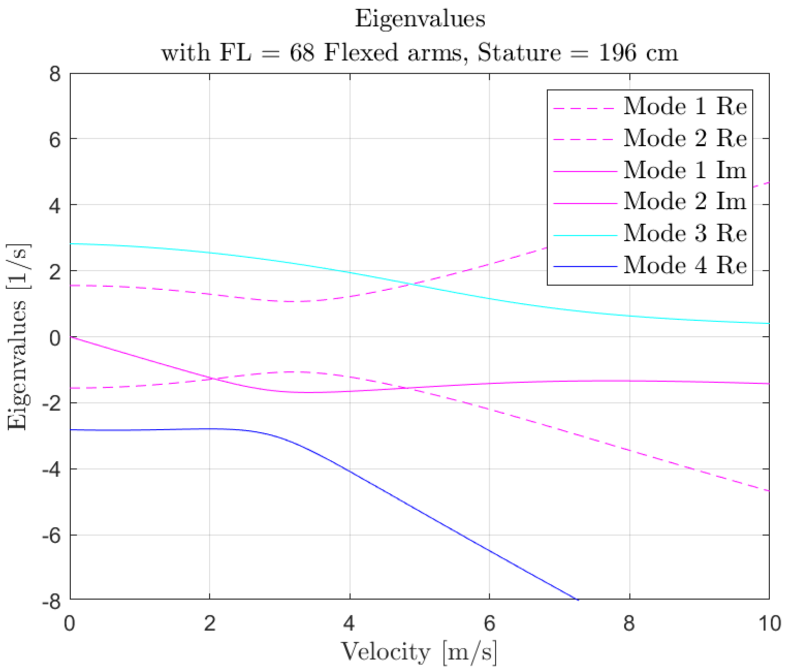


Figure 49: Eigenvalues Chamonix Extra Large bicycle with a person of 1.96 m tall with forward lean of 68 degrees

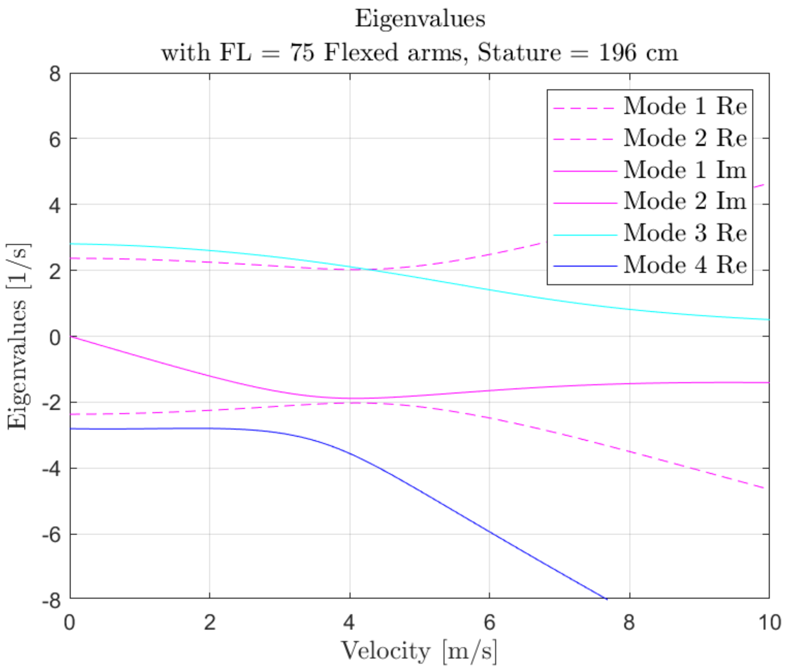


Figure 50: Eigenvalues Chamonix Extra Large bicycle with a person of 1.96 m tall with forward lean of 75 degrees

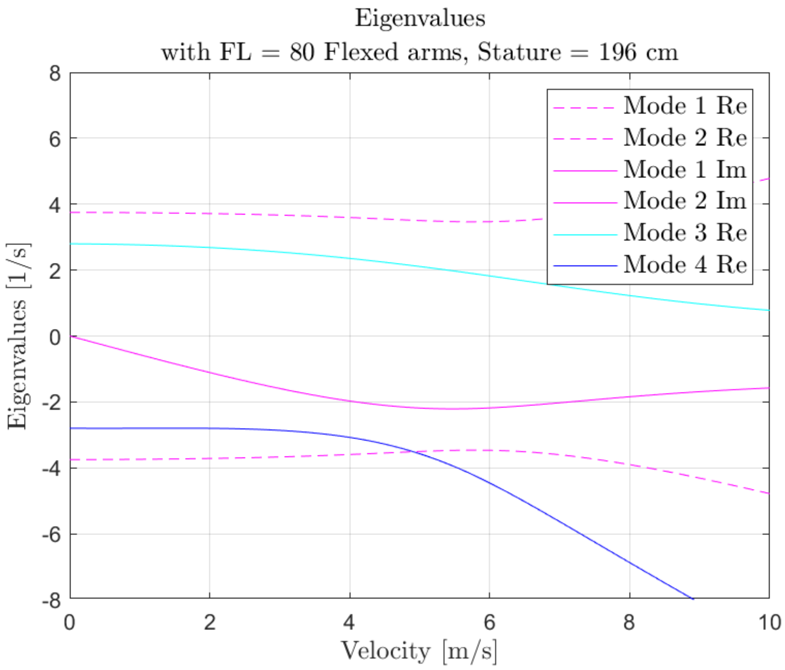


Figure 51: Eigenvalues Chamonix Extra Large bicycle with a person of 1.96 m tall with forward lean of 80 degrees

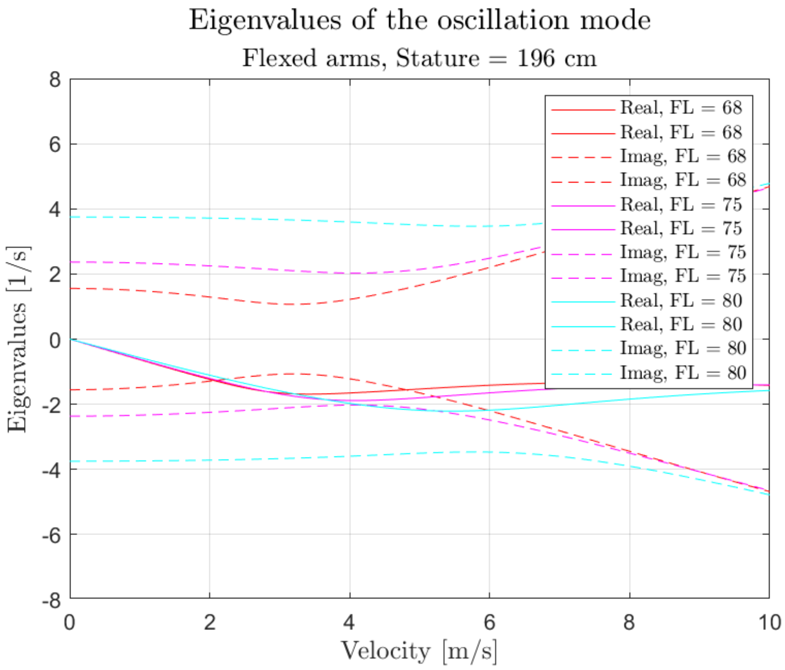


Figure 52: Eigenvalues Chamonix Extra Large bicycle with a person of 1.96 m tall with all sitting positions

C.2 Medeo

C.2.1 Small

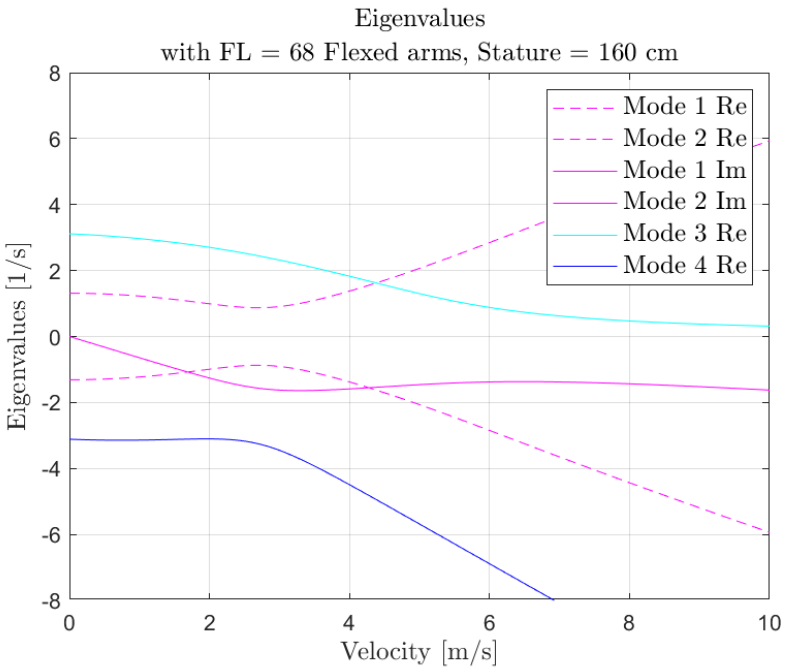


Figure 53: Eigenvalues Medeo Small bicycle with a person of 1.60 m tall with a forward lean of 68 degrees

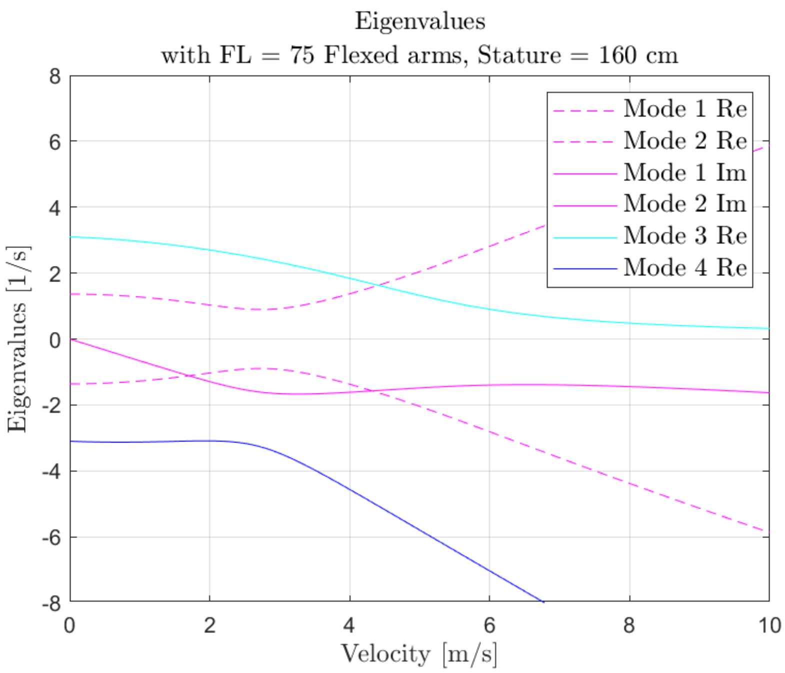


Figure 54: Eigenvalues Medeo Small bicycle with a person of 1.60 m tall with a forward lean of 75 degrees

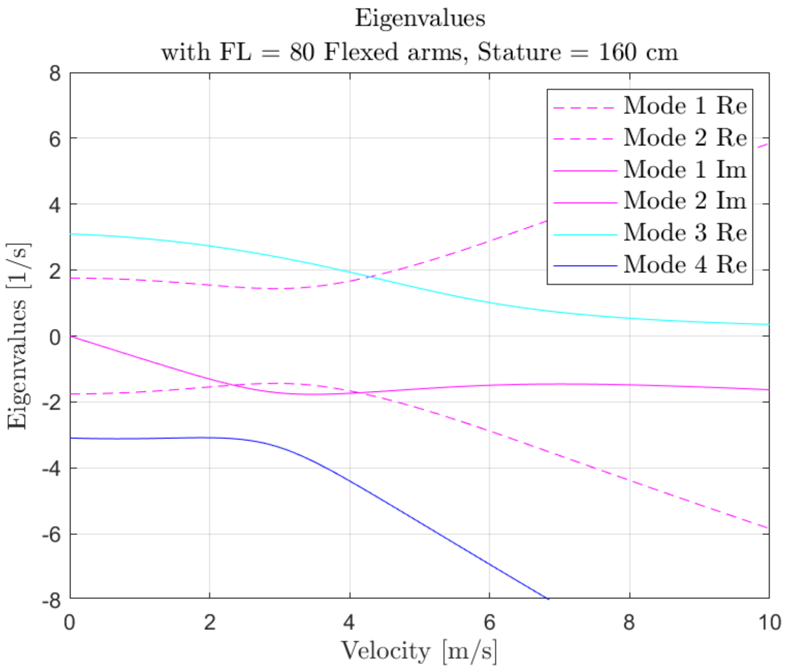


Figure 55: Eigenvalues Medeo Small bicycle with a person of 1.60 m tall with a forward lean of 80 degrees

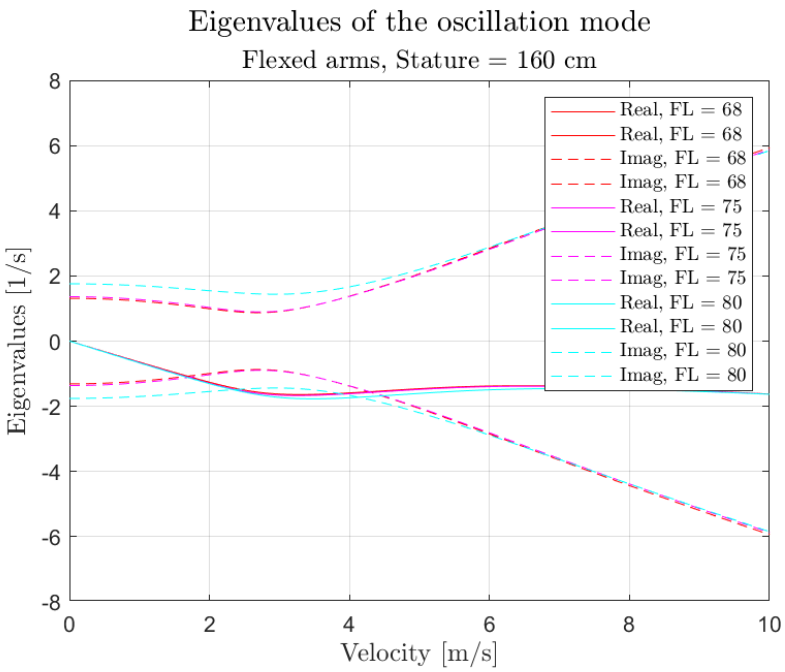


Figure 56: Eigenvalues Medeo Small bicycle with a person of 1.60 m tall with all sitting positions

C.2.2 Large

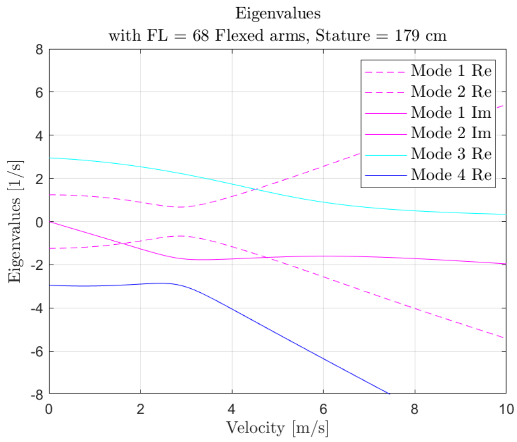


Figure 57: Eigenvalues Medeo Large bicycle with a person of 1.79 m tall with a forward lean of 68 degrees

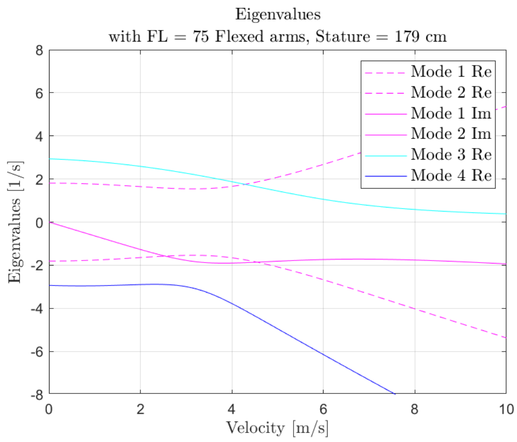


Figure 58: Eigenvalues Medeo Large bicycle with a person of 1.79 m tall with a forward lean of 75 degrees

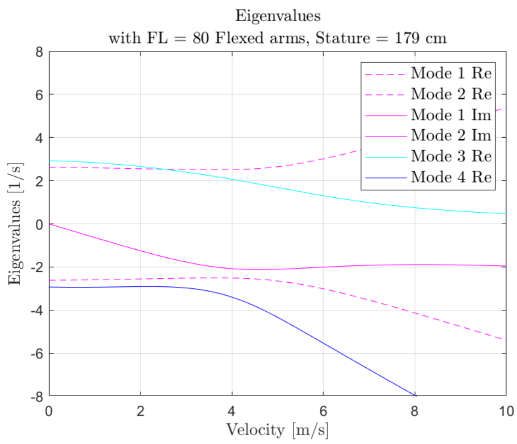


Figure 59: Eigenvalues Medeo Large bicycle with a person of 1.79 m tall with a forward lean of 80 degrees

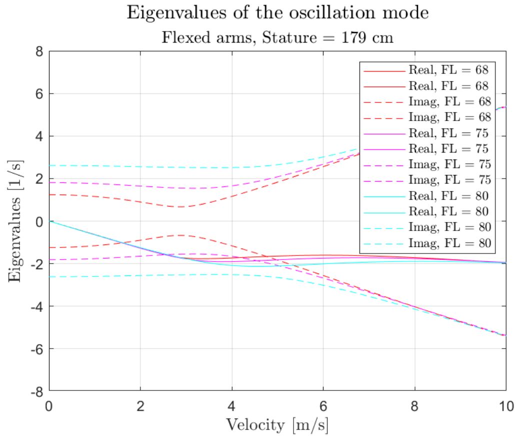


Figure 60: Eigenvalues Medeo Large bicycle with a person of 1.79 m tall with all sitting positions

C.2.3 Extra large

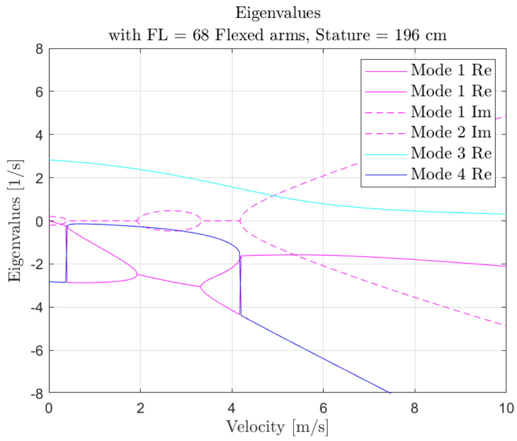


Figure 61: Eigenvalues Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 68 degrees

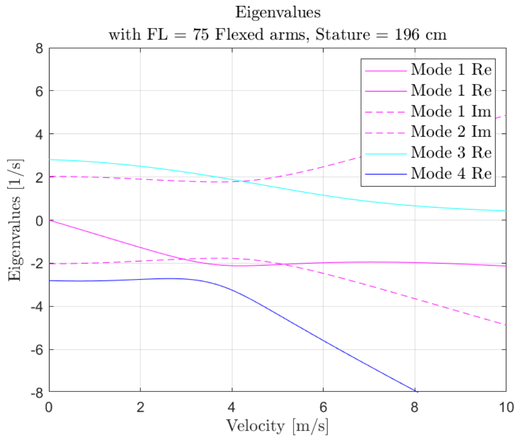


Figure 62: Eigenvalues Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 75 degrees

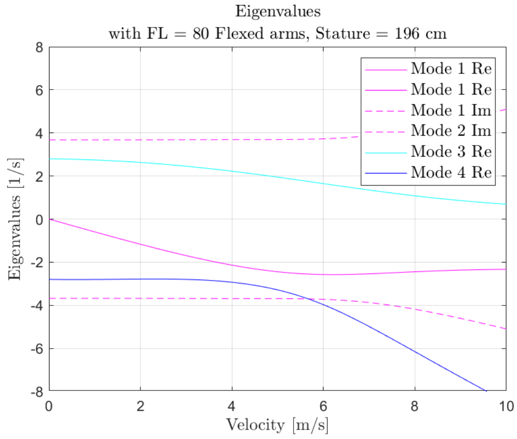


Figure 63: Eigenvalues Medeo Extra Large bicycle with a person of 1.96 m tall with a forward lean of 80 degrees

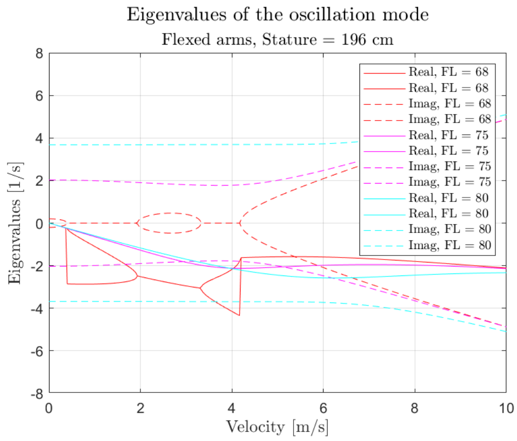


Figure 64: Eigenvalues Medeo Extra Large bicycle with a person of 1.96 m tall with all sitting positions

C.3 Orange

C.3.1 Small

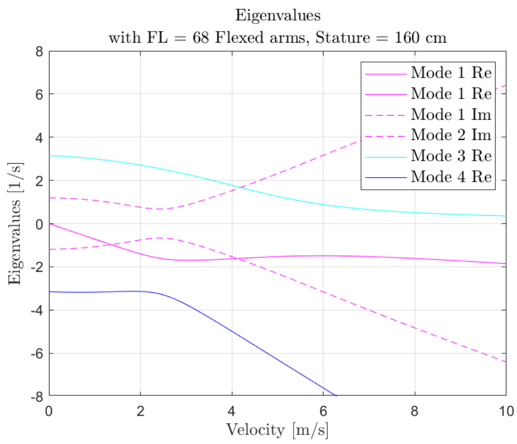


Figure 65: Eigenvalues Orange Small bicycle with a person of 1.60 m tall with a forward lean of 68 degrees

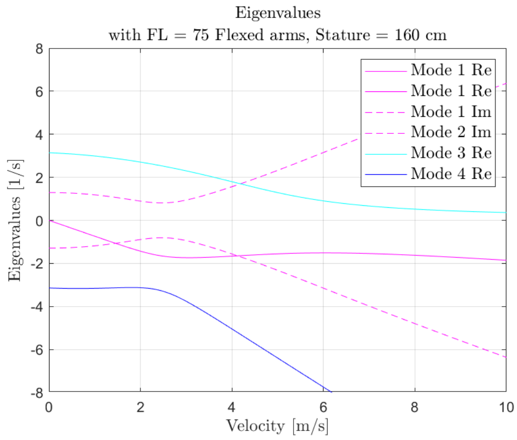


Figure 66: Eigenvalues Orange Small bicycle with a person of 1.60 m tall with a forward lean of 75 degrees

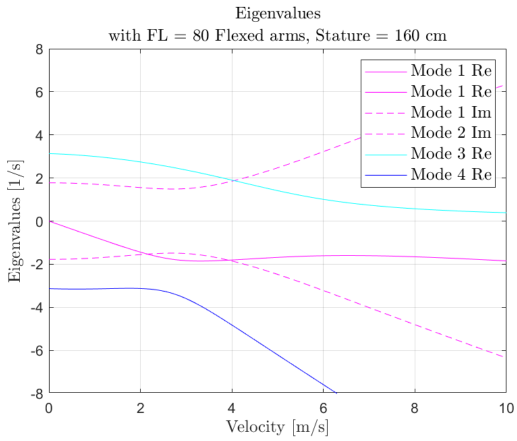


Figure 67: Eigenvalues Orange Small bicycle with a person of 1.60 m tall with a forward lean of 80 degrees

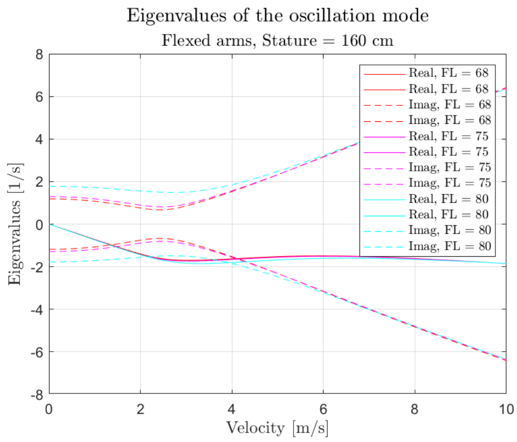


Figure 68: Eigenvalues Orange Small bicycle with a person of 1.60 m tall with a forward lean of 80 degrees

C.3.2 Large

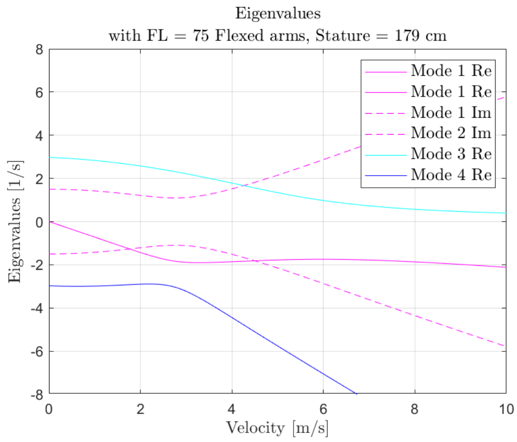


Figure 69: Eigenvalues Orange Small bicycle with a person of 1.79 m tall with a forward lean of 75 degrees

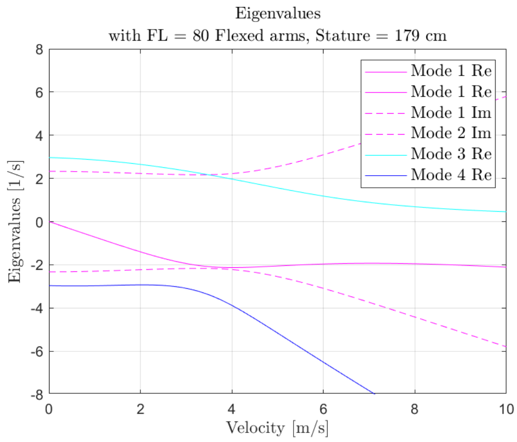


Figure 70: Eigenvalues Orange Small bicycle with a person of 1.79 m tall with a forward lean of 80 degrees

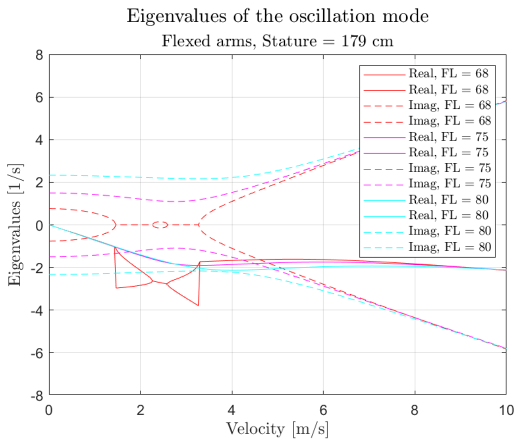


Figure 71: Eigenvalues Orange Small bicycle with a person of 1.60 m tall with a forward lean of 80 degrees

C.3.3 Extra large

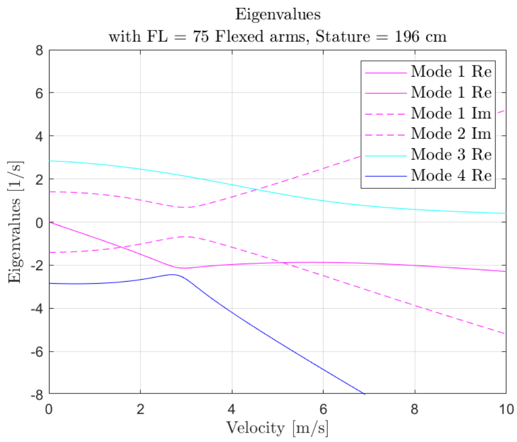


Figure 72: Eigenvalues Orange Small bicycle with a person of 1.96 m tall with a forward lean of 75 degrees

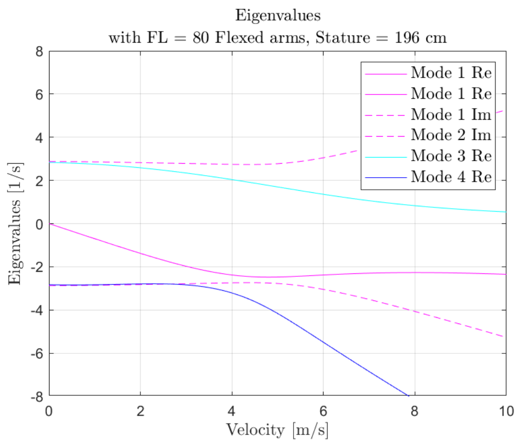


Figure 73: Eigenvalues Orange Small bicycle with a person of 1.60 m tall with a forward lean of 80 degrees

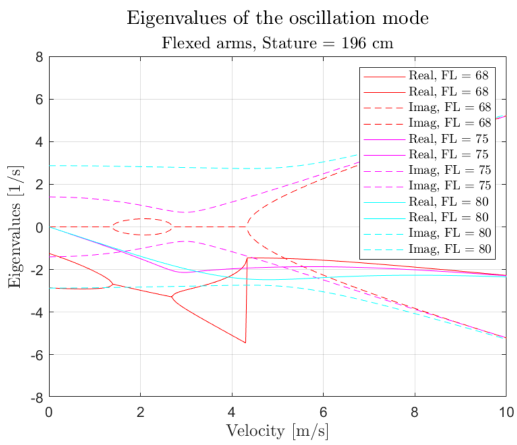


Figure 74: Eigenvalues Orange Small bicycle with a person of 1.96 m tall with a forward lean of 80 degrees

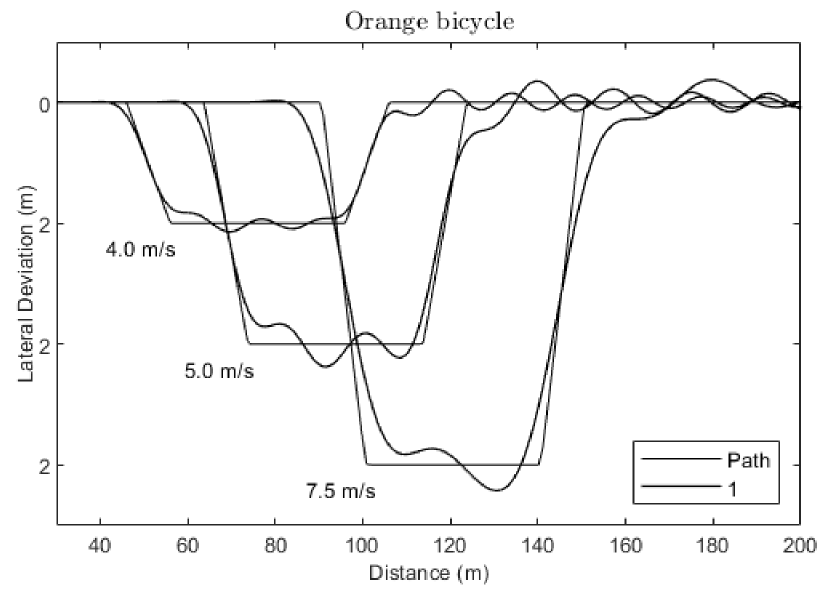


Figure 75: Path tracking performance of the combination L=1.96 m and a forward lean of 80°

D Additional check for marginally stable solutions of the HQM model

The rider-bicycle combination of the orange bicycle with a person of 1.96 m tall and a forward lean of 80° is checked on path tracking, see figure 75. As can be seen the