RELIABILITY EVALUATION OF A STRUCTURE AT SEA

HANS F. BURCHARTH

Department of Civil Engineering
Aalborg University, Denmark

Contents

1 Introduction

Conventional design practice for coastal structures is deterministic in nature and is based on the concept of a design load, which should not exceed the resistance (carrying capacity) of the structure. The design load is usually defined on a probabilistic basis as a characteristic value of the load, e.g. the expectation (mean) value of the 100-year return period event, however, often without consideration of the involved uncertainties. The resistance is in most cases defined in terms of the load which causes a certain design impact or damage to the structure and is not given as an ultimate force or deformation. This is because most of the available design formulae only give the relationship between wave characteristics and structural response, e.g. in terms of run-up, overtopping, armour layer damage etc. An example is the Hudson formula for armour layer stability. Almost all such design formulae are semi-empirical being based mainly on central fitting to model test results. The often considerable scatter in test results is not considered in general because the formulae normally express only the mean values. Consequently, the applied characteristic value of the resistance is then the mean value and not a lower fractile as is usually the case in other civil engineering fields. The only contribution to a safety margin in the design is then the one inherent in the choice of the return period for the design load. It is now more common to choose the return period with due consideration of the encounter probability, i.e. the probability that the design load value is exceeded during the structure lifetime. This is an important step towards a consistent probabilistic approach.

A safety factor or a conventional partial coefficient (as given in some national standards) might be applied too, in which cases the methods are classified as Level I (deterministic/quasi probabilistic) methods. However, such approaches do not allow the determination of the reliability (or the failure probability) of the design, and consequently it is neither possible to optimize, nor to avoid over-design of a structure. In order to overcome this problem more advanced probabilistic methods must be applied where the uncertainties (the stochastic properties) of the involved loading and strength variables are considered. Methods where the actual distribution functions for the variables are taken into account are denoted Level III methods. Level II methods comprise a number of methods in which a transformation of the generally correlated and non-normally distributed variables into uncorrelated and standard normal distributed variables is performed and reliability indices are used as measures of the structural reliability. Both Level II and III methods are discussed in the following. Described is also an advanced partial coefficient system which takes into account the stochastic properties of the variables and makes it possible to design to a specific failure probability level.

2 Failure modes and failure functions

Evaluation of structural safety is always related to the structural response as defined by the failure modes. Neglect of an important failure mode will bias the estimation of the safety of the structure.

Fig. 1 illustrates the failure modes for a conventional rubble mound breakwater with a capping wall.

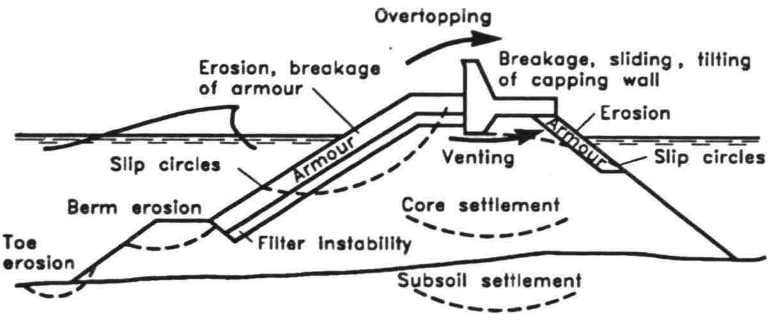

FIG. 1. Failure modes for a rubble mound breakwater.

Each failure mode must be described by a formula and the interaction (correlation) between the failure modes must be known. As an illustrative example let us consider only one failure mode, "hydraulic stability of the main armour layer", described by the Hudson formula

$$D_n^3 = \frac{H_s^3}{K_D\,\Delta^3\,cot\alpha} \tag{1}$$

where D_n is the nominal block diameter, $\Delta = \frac{\rho_s}{\rho_w} - 1$, where $\frac{\rho_s}{\rho_w}$ is the ratio of the block and water densities, α is the slope angle, H_S is the significant wave height and K_D is the coefficient signifying the degree of damage (movements of the blocks).

The formula can be split into load variables X_i^{load} and resistance variables, X_i^{res}. Whether a parameter is a load or a resistance parameter can be seen from the failure function. If a larger value results in a safer structure it is a resistance parameter and if a larger value results in a less safe structure it is a load parameter.

According to this definition one specific parameter can in one formula act as a load parameter while in another it can act as a resistance parameter. An example is the wave steepness in the van der Meer formulae for rock, which is a load parameter in the case of surging waves but a resistance parameter in the case of plunging waves. The only load variable in eq. (1) is H_s while the others are resistance variables.

Eq. (1) is formulated as a *failure function* (performance function)

$$g = A \cdot \Delta \cdot D_n\,(K_D\,cot\alpha)^{1/3} - H_s \begin{cases} < 0 & \text{failure} \\ = 0 & \text{limit state (failure)} \\ > 0 & \text{no failure (safe region)} \end{cases} \tag{2}$$

All the involved parameters are regarded as stochastic variables, X_i, except K_D, which signifies the "failure", i.e. a specific damage level chosen by the designer. The factor A in eq. (2) is also a stochastic variable signifying the uncertainty of the formula. In this case the mean value of A is 1.0.

In general eq. (2) is formulated as

$$g = R - S \tag{3}$$

where R stands for resistance and S for loading. Usually R and S are functions of many random variables, i.e.

$$R = R(X_1^{res}, X_2^{res}, \ldots, X_m^{res}) \quad \text{and} \quad S = S\left(X_{m+1}^{load}, \ldots\ldots, X_n^{load}\right) \quad \text{or} \quad g = g\left(\bar{X}\right)$$

The limit state is given by

$$g = 0 \tag{4}$$

which is denoted the *limit state equation* and defines the so-called *failure surface* which separates the safe region from the failure region.

In principle R is a variable representing the variations in resistance between nominally identical structures, whereas S represents the maximum load effects within a period of time, say successive T years. The distributions of R and S are both assumed independent of time. The *probability of failure* P_f during any reference period of duration T years is then given by

$$P_f = Prob\,[g \leq 0] \tag{5}$$

The *reliability* $\mathcal{R}$ is defined as

$$\mathcal{R} = 1 - P_f \tag{6}$$

3 Single failure mode probability analysis

3.1 Level III methods

A simple method – in principle – of estimation of P_f is the Monte Carlo method where a very large number of realisations x of the variables X are simulated. P_f is then approximated by the proportion of the simulations where $g \leq 0$.

The reliability of the method depends of course on a realistic assessment of the distribution functions for the variables X and their correlations.

Given $f_{\bar{X}}$ as the joint probability density function (jpdf) of the vector $\bar{X} = (X_1, X_2, \ldots, X_n)$ then eq. (5) can be expressed by

$$P_f = \int_{R \leq S} f_{\bar{X}}(\bar{x})\, d\bar{x} \tag{7}$$

Note that the symbol x is used for values of the random variable X.

If only two variables R and S are considered then eq. (7) reduces to

$$P_f = \int_{R \leq S} f_{(R,S)}(r,s) dr\, ds \tag{8}$$

which can be illustrated as shown in Fig. 2. If more than two variables are involved it is not possible to describe the jpdf as a surface but requires an imaginary multi-dimensional description.

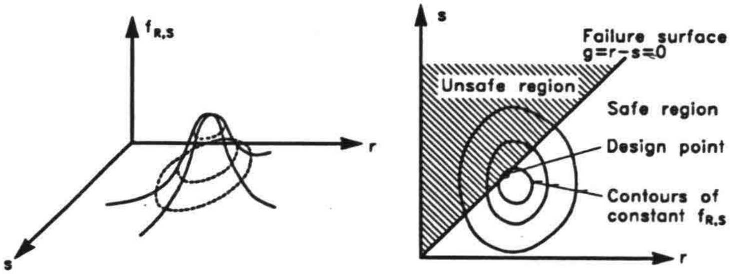

FIG. 2. Illustration of the two-dimensional joint probability density function for loading and strength.

Fig. 2 also shows the so-called *design point* which is the design point on failure surface where the joint probability density function attains the maximum value, i.e. the most probable point of failure.

Unfortunately, the jpdf is seldom known. However, the variables can often be assumed independent (non-correlated) in which case eq. (7) is given by the n-fold integral

$$P_f = \int\int_{R \leq S}\int \cdots \int f_{X_1}(x_1)\ldots f_{X_n}(x_n)\, dx_1 \ldots dx_n \tag{9}$$

where f_{X_i} are the marginal probability density function of the variables X_i. The amount of calculations involved in the multi-dimensional integration eq. (9) is enormous if the number of variables, n, is larger than say 5.

If only two variables are considered, say R and S, then eq. (9) simplifies to

$$P_f = \int\int_{R \leq S} f_R(r)\, f_S(s)\, dr\, ds \tag{10}$$

which by partial integration can be reduced to a single integral

$$P_f = \int_0^{\infty} F_R(x)\, f_S(x)\, dx \tag{11}$$

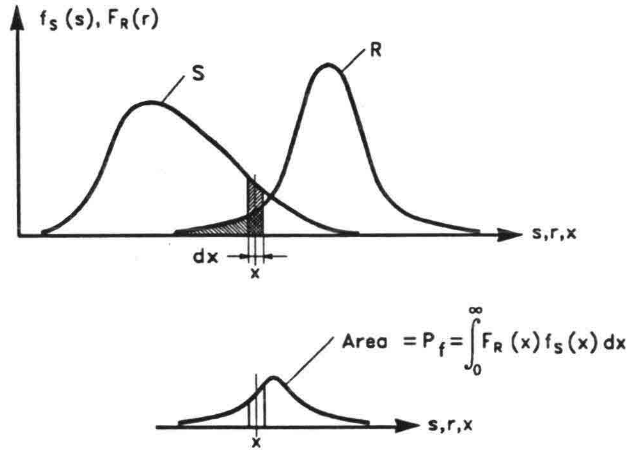

FIG. 3. **Illustration of failure probability in case of two independent variables, S and R.**

where F_R is the cumulative distribution function for R. Formally the lower integration limit should be $-\infty$ but is replaced by 0 since, in general, negative strength is not meaningful.

Eq. (11) can be explained as the product of the probabilities of two independent events, namely the probability that S lies in the range x, $x+dx$ (i.e. $f_S(x)dx$) and the probability that $R \leq x$ (i.e. $F_R(x)$), cf. Fig. 3.

3.2 Level II methods

3.2.1 Linear failure functions of normal-distributed random variables

In the following is given a short introduction to calculations at level II. For a more detailed description see Hallam et al. (1977) and Thoft-Christensen and Baker (1982). Only the so-called *first-order reliability method* (FORM) where the failure surface is approximated by a tangent hyberplane at some point will be discussed. A more accurate method is the *second-order reliability method* (SORM) which uses a quadratic approximation to the failure surface.

Assume the loading $S(x)$ and the resistance $R(x)$ for a single failure mode to be statistically independent and with density functions as illustrated in Fig. 3. The failure function is given by eq. (3) and the probability of failure by eq. (10) or eq. (11).

However, these functions are in many cases not known but might be estimated only by their mean values and standard deviations. If we assume S and R to be independent normally distributed variables with known means and standard deviations, then the linear failure function $g = R - S$ is normally distributed with mean value,

$$\mu_g = \mu_R - \mu_S \tag{12}$$

and

$$\text{standard deviation,} \quad \sigma_g = \left(\sigma_R^2 + \sigma_S^2\right)^{0.5} \tag{13}$$

The quantity $(g - \mu_g)/\sigma_g$ will be unit standard normal and consequently

$$P_f = prob[g \leq 0] = \int_{-\infty}^{0} f_g(x)\,dx = \Phi\left(\frac{0-\mu_g}{\sigma_g}\right) = \Phi(-\beta) \tag{14}$$

where

$$\beta = \frac{\mu_g}{\sigma_g} \tag{15}$$

is a measure of the probability of failure and is denoted the *reliability index* (Cornell 1969), cf. Fig. 4 for illustration of β. Note that β is the inverse of the coefficient of variation and is the distance in terms of number of standard deviations from the most probable value of g (in this case the mean) to the failure surface, $g = 0$.

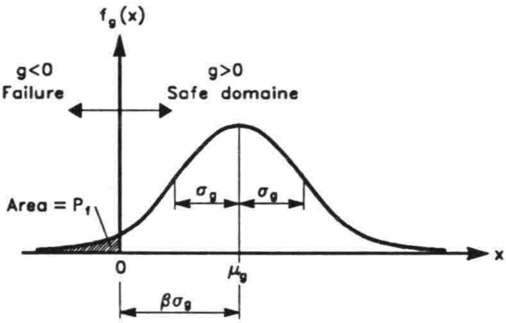

FIG. 4. Illustration of the reliability index.

Some corresponding values of β and P_f are given in Table 1.

Table 1. Corresponding values of β and P_f.

β	$P_f = \Phi(-\beta)$
0.0	0.50
0.5	0.31
1.0	0.16
1.5	0.067
2.0	0.023
3.0	0.0013
4.0	$0.32 \cdot 10^{-4}$
5.0	$0.29 \cdot 10^{-6}$

If R and S are normally distributed and "correlated" then eq. (14) still holds but σ_g is given by

$$\sigma_g = \left(\sigma_R^2 + \sigma_S^2 + 2\rho_{RS}\,\sigma_R\,\sigma_S\right)^{0.5} \tag{16}$$

where ρ_{RS} is the *correlation coefficient*

$$\rho_{RS} = \frac{C_{ov}[R,\ S]}{\sigma_R\,\sigma_S} = \frac{E\left[(R-\mu_R)\,(S-\mu_S)\right]}{\sigma_R\,\sigma_S} \tag{17}$$

R and S are said to be *uncorrelated* if $\rho_{RS} = 0$.

In general, if the failure function $g = g\left(\bar{X}\right)$ is a *linear* function of the normally distributed basic variables $X_1,\ X_2,\ \ldots\ X_n$, i.e.

$$g = a_o + a_1X_1 + a_2X_2 + \ldots + a_nX_n \tag{18}$$

then $\beta = \frac{\mu_g}{\sigma_g}$ and P_f can be found from eq. (14) using

$$\mu_g = a_o + a_1\mu_1 + a_2\mu_2 + \ldots + a_n\mu_n \tag{19}$$

and

$$\sigma_g^2 = a_1^2\sigma_1^2 + \ldots + a_n^2\sigma_n^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \rho_{ij}a_ia_j\sigma_i\sigma_j \tag{20}$$

where ρ_{ij} expresses the correlation coefficient between any pair of variables, cf. eq. (17).

Besides the illustration of β in Fig. 4 a simple geometrical interpretation of β can be given in case of a linear failure function $g = R - S$ of the independent variables R and S by a transformation into a *normalized coordinate system* of the random variables $R' = (R - \mu_R)\,/\sigma_R$ and $S' = (S - \mu_S)\,/\sigma_S$, cf. Fig. 5.

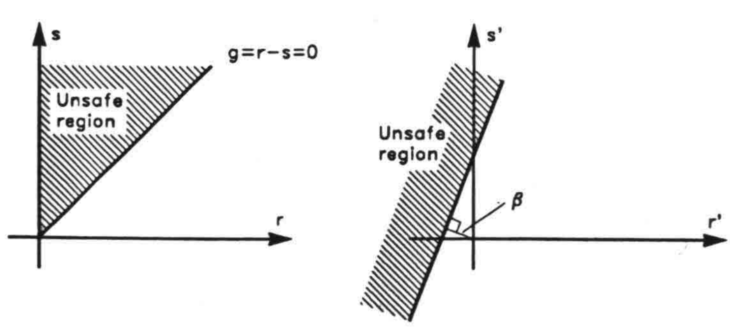

FIG. 5. Illustration of β in normalized coordinate system.

With these variables the failure surface $g = 0$ is linear and given by

$$R'\sigma_R - S'\sigma_S + \mu_R - \mu_S = 0 \tag{21}$$

By geometrical considerations it can be shown that the shortest distance from the origin to this linear failure surface is equal to

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{(\sigma_R^2 + \sigma_S^2)^{0.5}}$$

in which eqs. (12) and (13) are used.

3.2.2 Non-linear failure functions of normal-distributed random variables

If the failure function $g = g\left(\bar{X}\right)$ is *non-linear* then approximate values for μ_g and σ_g can be obtained by using a *linearized failure function.*

Linearization is generally performed by Taylor-series expansion about some point retaining only the linear terms. If the expansion is performed around the mean values $(X_1, \ldots, X_n) = \mu_1, \ldots, \mu_n$ then

$$g \simeq g(\mu_1, \ldots, \mu_n) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(X_i - \mu_i) \ , \tag{22}$$

where $\partial g/\partial X_i$ is evaluated at $(\mu_1, \ldots, \mu_n)$. The approximate values of μ_g and σ_g are then

$$\mu_g \simeq g(\mu_1, \ldots, \mu_n) \tag{23}$$

$$\sigma_g^2 \simeq \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} Cov\left[X_i,\ X_j\right] \tag{24}$$

If the random variables $\bar{X}$ are "uncorrelated", i.e. $\rho_{X_iX_j} = 0$, then e.g. (24) reduces to

$$\sigma_g^2 \simeq \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\sigma_{X_i}\right)^2 \tag{25}$$

because $Cov\left[X_i,\ X_i\right] = \sigma_{X_i}^2$ and $Cov\left[X_i,\ X_j\right] = 0$ for all i and j, $i \neq j$.

When linearization is performed around the expected mean values the method is often called a *first-order mean value approach* (FMA).

The values of μ_g and σ_g, and thereby also the value of β, depend on the choice of linearization point. Moreover, the value of β defined by eq. (15) will change when different but equivalent non-linear failure functions are used. For example an equivalent failure function to eq. (2) would be

$$g = A^3\,\Delta^3\,D_n^3\,K_D\cot\alpha - H_s^3 \tag{26}$$

which expresses the Hudson formula as does eq. (2), but will result in different β-values.

In order to overcome these problems a transformation of the basic variables $\bar{X} = (X_1, X_2, \ldots, X_n)$ into a new set of normalized variables $\bar{Z} = (Z_1, Z_2, \ldots, Z_n)$ is performed. For *uncorrelated* normal distributed basic variables $\bar{X}$ the transformation is

$$Z_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{27}$$

in which case $\mu_{Z_i} = 0$ and $\sigma_{Z_i} = 1$. By this linear transformation the *failure surface* $g = 0$ in the x-coordinate system is mapped into a failure surface in the z-coordinate system which also divides the space into a safe region and a failure region, cf. Fig. 6.

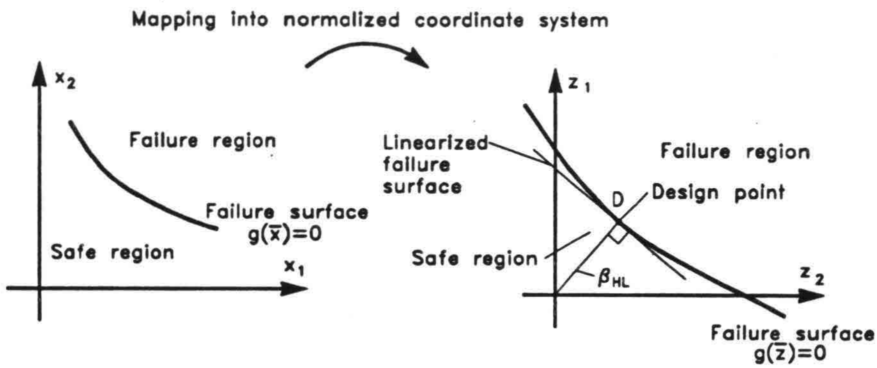

FIG. 6. Definition of the Hasofer and Lind reliability index, β_{HL}.

Fig. 6 introduces the Hasofer and Lind reliability index β_{HL} which is defined as the distance from origo to the nearest point, D, of the *failure surface* in the z-coordinate system. This point is called the *design point.* The coordinates of the design point in the original x-coordinate system are the most probable values of the variables $\bar{X}$ at failure. β_{HL} can be formulated as

$$\beta_{HL} = \min_{g(\bar{z})=0} \left(\sum_{i=1}^{n} z_i^2 \right)^{0.5} \tag{28}$$

The special feature of β_{HL} as opposed to β is that β_{HL} is related to the failure "surface" $g(\bar{z}) = 0$ which is invariant to the failure function because equivalent failure functions result in the same failure surface.

The two reliability indices β and β_{HL} will coincide when the failure surfaces are linear, cf. Figs. 5 and 6. Obviously, this will also be the case if non-linear failure functions are linearized by Taylor Series expansion around the design point.

Linearization around the design point instead of mean values is therefore very much to be preferred, also because the design point is the most probable point of failure, cf. Fig. 2. Linearization around mean values can lead to quite erroneous results but due to the simplicity of the method it might be used to get a first order-of-magnitude impression of the failure probability.

The method where linearization is performed around the design point is often called a *first-order design point approach* (FDA).

The calculation of β_{HL} and the design point coordinates can be undertaken in a number of different ways. An iterative method must be used when the failure surface is non-linear. In the following a simple method is introduced.

Let θ denote the distance from the origin to any point at the failure surface given in the normalized coordinate system

$$\begin{cases} \theta = \left[\sum_{i=1}^{n} z_i^2\right]^{\frac{1}{2}} \\ g(z_1, z_2, \ldots, z_n) = 0 \end{cases} \tag{29}$$

Construct the multiple function (Lagrange function)

$$\begin{aligned} F &= \theta + K_1 g \\ &= \left[z_1^2 + z_2^2 + \ldots + z_n^2\right]^{\frac{1}{2}} + K_1 g(z_1, z_2, \ldots, z_n) \end{aligned} \tag{30}$$

where K_1 is an unknown constant (multiplier).

Maximum or minimum of θ occurs when

$$\begin{cases} \dfrac{\partial F}{\partial z_i} = [z_1^2 + z_2^2 + \ldots + z_n^2]^{-\frac{1}{2}} \cdot z_i + K_1 \dfrac{\partial g}{\partial z_i} = 0 \quad i = 1, 2, \ldots, n \\ g(z_1, z_2, \ldots, z_n) = 0 \end{cases} \tag{31}$$

Assume that only one minimum exists and the coordinates of the design point D are given by

$$\left(z_1^d, z_2^d, \ldots, z_n^d\right) = (\beta_{HL}\alpha_1, \beta_{HL}\alpha_2, \ldots, \beta_{HL}\alpha_n) \tag{32}$$

Then

$$\theta_{min} = \beta_{HL} = \left[\sum_{i=1}^{n} (\beta_{HL}\alpha_i)^2\right]^{\frac{1}{2}} \quad \text{and consequently}$$

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \tag{33}$$

Eq. (31) becomes

$$\begin{cases} \beta_{HL}^{-\frac{1}{2}} \cdot (\beta_{HL}\alpha_i) + K_1 \dfrac{\partial g}{\partial z_i} = 0 \quad i = 1, 2, \ldots, n \\ g(\beta_{HL}\alpha_1, \beta_{HL}\alpha_2, \ldots, \beta_{HL}\alpha_n) = 0 \end{cases} \tag{34}$$

or

$$\begin{cases} \alpha_i = \dfrac{-\frac{\partial g}{\partial z_i}}{\frac{\beta_{HL}^{-\frac{1}{2}}}{K_1}} = \dfrac{-\frac{\partial g}{\partial z_i}}{K} \\ g\,(\beta_{HL}\alpha_1,\ \beta_{HL}\alpha_2,\ \dots\ ,\beta_{HL}\alpha_n) = 0 \end{cases} \tag{35}$$

Inserting eq. (35) into eq. (33) gives

$$K = \left[\sum_{i=1}^{n}\left(\frac{\partial g}{\partial z_i}\right)^2\right]^{\frac{1}{2}} \tag{36}$$

The α-values defined by (32) are often called *sensitivity factors* (or influence factors) because α_i^2 provides an indication of the relative importance on the reliability index β_{HL} of the random variable X_i. If α_i^2 is small it might be considered to model X_i as a deterministic quantity equal to the median value of X_i. In such case the relative change in the reliability index by assuming X_i deterministic can be approximated by

$$\frac{\beta_{HL}(X_i\ :\ \text{deterministic})}{\beta_{HL}(X_i\ :\ \text{random})} \simeq \frac{1}{\sqrt{1-\alpha_i^2}} \tag{37}$$

The corresponding change in failure probability can be found from eq. (14) or from Table 1. Eq. (37) is used for the evaluation of a simplification of a failure function by reducing the number of random variables.

The sensitivity of β_{HL} to change in the value of a deterministic parameter b_i can be expressed by

$$\frac{d\beta_{HL}}{d\,b_i} = \frac{1}{K}\frac{\partial g}{\partial b_i} \tag{38}$$

where K is given by eq. (36) and the partial derivative of g with respect to b_i is taken in the design point.

Eq. (38) is useful when it is considered to change a deterministic parameter (e.g. the height of wave wall) into a stochastic variable.

EXAMPLE 1

Consider the hydraulic stability of a rock armour layer given by the Hudson equation formulated as the failure function, cf. eqs. (1) and (2)

$$g = A\,\Delta\,D_n\,(K_D\,cot\alpha)^{\frac{1}{3}} - H_s \tag{39}$$

all the parameters are regarded uncorrelated random variables X_i, except K_D which signifies the *failure criterion*, i.e. a certain damage level here chosen as 5% displacement corresponding to $K_D \simeq 4$. The factor A is also a random variable signifying the uncertainty of the formula.

All random variables are assumed normal distributed with known mean values and standard deviations, cf. Table 2. The normal distribution can be a bad approximation for H_s which is usually much better approximated by an extreme distribution, e.g. a Weibull or Gumbel distribution as will be discussed later. The normal distribution of H_S is used here due to the simplicity involved but might be reasonable in case of depth limited wave conditions.

Table 2. Basic variables.

i	X_i	μ_{X_i}	σ_{X_i}	coefficient of variation σ_{X_i}/μ_{X_i}
1	A	1	0.18	18%
2	D_n	1.5 m	0.10 m	6.7%
3	H_s	4.4 m	0.70 m	16%
4	Δ	1.6	0.06	3.8%
5	$cot\alpha$	2	0.10	5.0%

The failure surface corresponding to the failure function (39) reads for $K_D = 4$

$$A\,\Delta\,D_n\,(cot\alpha)^{\frac{1}{3}}\,1.59 - H_s = 0$$

or

$$X_1\,X_4\,X_2\,X_5^{\frac{1}{3}}\,1.59 - X_3 = 0 \tag{40}$$

By use of the transformation eq. (27) the failure surface in the normalized coordinate system is given by

$$(1 + 0.18\ z_1)\ (1.6 + 0.06\ z_4)\ (1.5 + 0.10\ z_2)\,(2 + 0.10\ z_5)^{\frac{1}{3}}\ 1.59 - (4.4 + 0.70\ z_3) = 0$$

In order to make the calculations in this illustrative example more simple we neglect the small variational coefficients of Δ and $cot\alpha$ and obtain

$$(1 + 0.18\ z_1) \cdot 1.6 \cdot (1.5 + 0.10\ z_2) \cdot 2^{\frac{1}{3}} \cdot 1.59 - (4.4 + 0.70\ z_3) = 0 \tag{41}$$

or

$$0.864\ z_1 + 0.32\ z_2 + 0.058\ z_1 z_2 - 0.70\ z_3 + 0.40 = 0 \tag{42}$$

$$0.864\ \beta_{HL}\alpha_1 + 0.32\ \beta_{HL}\alpha_2 + 0.058\ \beta_{HL}^2\alpha_1\alpha_2 - 0.70\ \beta_{HL}\alpha_3 + 0.40 = 0$$

$$\beta_{HL} = \frac{-0.40}{0.864\,\alpha_1 + 0.32\,\alpha_2 + 0.058\,\alpha_1\alpha_2\beta_{HL} - 0.70\,\alpha_3}$$

By use of eq. (35)

$$\alpha_1 = -\frac{1}{K}(0.864 + 0.058\,\beta_{HL}\alpha_2)$$

$$\alpha_2 = -\frac{1}{K}(0.32 + 0.058\,\beta_{HL}\alpha_1)$$

$$\alpha_3 = \frac{0.7}{K}$$

By eq. (36)

$$K = \sqrt{(0.864 + 0.058\,\beta_{HL}\alpha_2)^2 + (0.32 + 0.058\,\beta_{HL}\alpha_1)^2 + (0.7)^2}$$

The iteration is now performed by choosing starting values for β_{HL}, α_1, α_2 and α_3 and calculating new values until small modifications are obtained. This is shown in Table 3. The convergence is faster if a positive sign is used for α-values related to loading variables and a negative sign is used for α-values related to resistance variables.

Table 3.

	Iteration No.			
	start	1	2	3
β_{HL}	3.0	0.438	0.342	0.341
K		1.144	1.149	1.149
α_1	−0.50	−0.744	−0.747	−0.747
α_2	−0.50	−0.263	−0.266	−0.266
α_3	0.50	0.612	0.609	0.609

The probability of failure is then

$$P_f = \Phi(-\beta_{HL}) = \Phi(-0.341) = 0.367$$

cf. Table 1 for some corresponding values of β and P_f.

The design point coordinates in the normalized z coordinate system are

$$\left(z_1^d,\ z_2^d,\ z_3^d\right) = (\beta_{HL}\alpha_1,\ \beta_{HL}\alpha_2,\ \beta_{HL}\alpha_3)$$

$$= (-0.255,\ -0.091,\ 0.208)$$

Expression (33) $\beta_{HL} = \left(\sum_{i=1}^{3}\left(z_i^d\right)^2\right)^{\frac{1}{2}}$ provides a check on the design point coordinates.

Using the transformation

$$X_i^d = \mu_{X_i} + \sigma_{X_i} z_i^d$$

and the values of μ_{X_i}, σ_{X_i} given in Table 2 the design point coordinates in the original x coordinate system are found to be

$$\left(x_1^d,\ x_2^d,\ x_3^d\right) = (0.954,\ 1.491,\ 4.546)$$

The relative importance of the random variables to the failure probability is evaluated through the α^2-values. Table 4 shows that the uncertainty related to D_n is of minor importance compared to the uncertainties on A and H_s.

Table 4.

i	X_i	α_i	α_i^2 (%)	$\frac{\beta_{HL}(X_i\ :\ \text{deterministic})}{\beta_{HL}(X_i\ :\ \text{random})} \simeq \frac{1}{\sqrt{1-\alpha_i^2}}$	$\frac{P_f(X_i\ :\ \text{deterministic})}{P_f(X_i\ :\ \text{random})}$
1	A	−0.747	55.8	1.50 *)	0.831 *)
2	D_n	−0.266	7.1	1.04	0.989
3	H_s	0.609	37.1	1.26 *)	0.899 *)
			100.0		

*) The assumption of validity only for small α-values is not fulfilled

If all 5 parameters in the Hudson formula was kept as random variables with mean values and standard deviations as given in Table 2 then the corresponding values would be as shown in Table 5.

Table 5.

i	X_i	α_i	α_i^2 (%)	$\frac{\beta_{HL}(X_i\ :\ \text{deterministic})}{\beta_{HL}(X_i\ :\ \text{random})} \simeq \frac{1}{\sqrt{1-\alpha_i^2}}$	$\frac{P_f(X_i\ :\ \text{deterministic})}{P_f(X_i\ :\ \text{random})}$
1	A	−0.705	49.7	1.41 *)	0.857 *)
2	D_n	−0.275	7.6	1.04	0.986
3	H_s	0.631	39.8	1.29 *)	0.896 *)
4	Δ	−0.154	2.3	1.01	0.999
5	$cot\alpha$	−0.068	0.5	1.00	1.000
			100.0		

*) The assumption of validity only for small α-values is not fulfilled

It is clearly seen why Δ and $cot\alpha$ can be regarded as constants.

If the normally distributed basic variables $\bar{X}$ are *correlated* the procedure given above can be used if a transformation into non-correlated variables $\bar{Y}$ is performed before normalizing the variables.

The correlation between any pair of the random variables $\bar{X}$ is expressed by the covariance matrix

$$\bar{\bar{C}}_{\bar{X}} = \begin{bmatrix} Var[X_1] & Cov[X_1, X_2] & \cdots & Cov[X_1, X_n] \\ Cov[X_2, X_1] & Var[X_2] & & \vdots \\ \vdots & & \ddots & \vdots \\ Cov[X_n, X_1] & & & Var[X_n] \end{bmatrix} \tag{43}$$

If $\bar{\bar{C}}_{\bar{Y}}$ is a diagonal matrix

$$\bar{\bar{C}}_{\bar{Y}} = \begin{bmatrix} Var[Y_1] & \cdots & \cdots & 0 \\ \vdots & Var[Y_2] & & \vdots \\ \vdots & & \ddots & \\ 0 & & & Var[Y_n] \end{bmatrix} = \begin{bmatrix} \sigma^2_{Y_1} & & & 0 \\ \vdots & \sigma^2_{Y_2} & & \vdots \\ \vdots & & \ddots & \\ 0 & & & \sigma^2_{Y_n} \end{bmatrix} \tag{44}$$

then no correlation between any pair og random variables $\bar{Y}$ exists.

A set of uncorrelated variabels $\bar{Y}$ can be obtained by the transformation

$$\bar{Y} = \bar{\bar{A}}^T \bar{X} \tag{45}$$

where $\bar{\bar{A}}$ is an orthogonal matrix with column vectors equal to the orthonormal eigenvalues of $\bar{\bar{C}}_{\bar{X}}$.

The diagonal elements of $\bar{\bar{C}}_{\bar{Y}}$, i.e. $\sigma^2_{Y_1} \cdots \sigma^2_{Y_n}$, are equal to the eigenvalues of $\bar{\bar{C}}_{\bar{X}}$.

After determination of $\bar{Y}$ and $\sigma_{\bar{Y}}$ the following transformation, analog to (27), into uncorrelated and normalized variables $\bar{z}$ is performed

$$z_i = \frac{Y_i - \mu_{Y_i}}{\sigma_{Y_i}} \tag{46}$$

The reliability index β_{HL}, defined in the z-coordinate system as given in Fig. 6 and eq. (28), can be determined by the described iterative procedure of eqs. (35) and (36).

3.2.3 Non-linear failure functions containing non-normal distributed random variables

It is not always a reasonable assumption to consider the random variables normally distributed. This is for example the case for parameters such as H_s characterizing the sea state in long-term wave statistics. H_s will in general follow extreme distributions (e.g. Gumbel and Weibull) quite different from the normal distribution, and cannot be described only by the mean value and the standard deviation.

For such cases it is still possible to use the reliability index β_{HL} but an extra transformation of the non-normal basic variables into normal basic variables must be performed before β_{HL} can be determined as described above.

A commonly used transformation is based on the substitution of the non-normal distribution of the basic variable X_i by a normal distribution in such a way that the density and distribution functions f_{X_i} and F_{X_i} are unchanged at the design point.

If the design point is given by $x_1^d, x_2^d, \ldots, x_n^d$ then the transformation reads

$$\begin{aligned} F_{X_i}\left(x_i^d\right) &= \Phi\left(\frac{x_i^d - \mu'_{X_i}}{\sigma'_{X_i}}\right) \\ f_{X_i}\left(x_i^d\right) &= \frac{1}{\sigma'_{X_i}}\varphi\left(\frac{x_i^d - \mu'_{X_i}}{\sigma'_{X_i}}\right) \end{aligned} \tag{47}$$

where μ'_{X_i} and σ'_{X_i} are the mean and standard deviation of the approximate (fitted) normal distribution.

From eq. (47) is obtained

$$\begin{aligned} \sigma'_{X_i} &= \frac{\varphi\left(\Phi^{-1}\left(F_{X_i}\left(x_i^d\right)\right)\right)}{f_{X_i}\left(x_i^d\right)} \\ \mu'_{X_i} &= x_i^d - \Phi^{-1}\left(F_{X_i}\left(x_i^d\right)\right)\sigma'_{X_i} \end{aligned} \tag{48}$$

Eq. (47) can also be written

$$F_{X_i}\left(x_i^d\right) = \Phi\left(\frac{x_i^d - \mu'_{X_i}}{\sigma'_{X_i}}\right) = \Phi\left(z_i^d\right) = \Phi\left(\beta_{HL}\alpha_i\right)$$

Solving with respect to x_i^d gives

$$x_i^d = F_{X_i}^{-1}\left[\Phi\left(\beta_{HL}\alpha_i\right)\right] \tag{49}$$

The iterative method presented above for calculation of β_{HL} can still be used if for each step of iteration the values of σ'_{X_i} and μ'_{X_i} given by eq. (48) are calculated for those variables where the transformation (47) has been used.

For correlated random variables the transformation given by eq. (45) is used before normalization.

EXAMPLE 2

The same failure function and non-correlated normal-distributed variable as in Example 1 are considered except that H_s now follows a Gumbel distribution but with the same average and standard deviation as given in Table 2.

The Gumbel distribution function and density function are

$$F_G(x_3) = e^{-e^{-A(x_3-B)}}$$
$$f_G(x_3) = \frac{d\,F_G(x_3)}{dx_3} = A\,e^{\left[-e^{-A(x_3-B)}-A(x_3-B)\right]} \tag{50}$$

The distribution parameters A and B can be determined by the following expressions for the mean and the standard deviation

$$\mu_{x_3} = B + \frac{0.57722}{A}$$
$$\sigma_{x_3} = \frac{\pi}{\sqrt{6}}\frac{1}{A} \tag{51}$$

Using the Table 2 values $\mu_{x_3} = 4.4$ m amd $\sigma_{x_3} = 0.7$ m gives $A = 1.83\ m^{-1}$ and $B = 4.08$ m.

In the normalized coordinate system the failure surface is then (compared with eq. (41))

$$(1+0.18z_1)\cdot 1.6\cdot(1.5+0.1z_2)\cdot 2^{\frac{1}{3}}\cdot 1.59 - \left(\mu'_{x_3} + \sigma'_{x_3}z_3\right) = 0$$

$$0.864z_1 + 0.32z_2 + 0.058z_1z_2 - \sigma'_{x_3}z_3 + \left(4.8 - \mu'_{x_3}\right) = 0$$

$$\beta_{HL} = \frac{-\left(4.8-\mu'_{x_3}\right)}{0.864\alpha_1 + 0.32\alpha_2 + 0.058\alpha_1\alpha_2\beta_{HL} - \sigma'_{x_3}\alpha_3}$$

By eq. (35)

$$\alpha_1 = -\frac{1}{K}\left(0.864 + 0.058\beta_{HL}\alpha_2\right)$$
$$\alpha_2 = -\frac{1}{K}\left(0.32 + 0.058\beta_{HL}\alpha_1\right)$$
$$\alpha_3 = \frac{\sigma'_{x_3}}{K}$$

By eq. (36)

$$K = \sqrt{\left(0.864 + 0.058\beta_{HL}\alpha_2\right)^2 + \left(0.32 + 0.058\beta_{HL}\alpha_1\right)^2 + \left(\sigma'_{x_3}\right)^2}$$

By eq. (49)

$$x_3^d = F_G^{-1}\left[\Phi\left(\beta_{HL}\alpha_3\right)\right]$$

By eq. (48)

$$\sigma'_{x_3} = \frac{\varphi\left(\Phi^{-1}\left(F_G\left(x_3^d\right)\right)\right)}{f_G\left(x_3^d\right)}$$

$$\mu'_{x_3} = x_3^d - \Phi^{-1}\left(F_G\left(x_3^d\right)\right)\cdot\sigma'_{x_3}$$

The results from each step of iteration are shown in Table 6.

Table 6.

	Iteration No.							
	start	1	2	3	4	5	6	7
β_{HL}	3.0	1.717	0.553	0.569	0.463	0.461	0.457	0.457
K		1.295	1.363	1.165	1.155	1.144	1.143	1.143
α_1	−0.5	−0.629	−0.629	−0.735	−0.742	−0.749	−0.749	−0.750
α_2	−0.5	−0.199	−0.220	−0.254	−0.260	−0.262	−0.262	−0.263
α_3	0.5	0.772	0.754	0.627	0.619	0.609	0.608	0.607
x_3^d		5.359	4.568	4.525	4.475	4.471	4.469	4.469
σ'_{x_3}	1.0	1.027	0.731	0.715	0.697	0.695	0.694	0.694
μ'_{x_3}	3.0	4.033	4.139	4.264	4.270	4.275	4.276	4.276

The probability of failure is then

$$P_f = \Phi(-\beta_{HL}) = \Phi(-0.457) = 0.324$$

The coordinates of the design point D in the normalized z-coordinate system are

$$\begin{aligned}\left(z_1^d\ ,\ z_2^d\ ,\ z_3^d\right) &= (\beta_{HL}\alpha_1\ ,\ \beta_{HL}\alpha_2\ ,\ \beta_{HL}\alpha_3) \\ &= (-0.342\ ,\ -0.12\ ,\ 0.277)\end{aligned}$$

Note that $\beta_{HL} = \left(\sum_{z=1}^{3}\left(z_i^d\right)^2\right)^{\frac{1}{2}}$.

The coordinates of the design point D in the original x-coordinate system are calculated by the transformation

$$x_i^d = \mu_{x_i} + \sigma_{x_i} z_i^d \qquad i = 1,\ 2 \qquad \text{(cf. Table 2 for } \mu_{x_i} \text{ and } \sigma_{x_i}\text{)}$$

$$x_3^d = \mu'_{x_3} + \sigma'_{x_3} z_3^d \qquad \text{(cf. Table 6 for } \mu'_{x_3} \text{ and } \sigma'_{x_3}\text{)}$$

to be

$$\left(x_1^d\ ,\ x_2^d\ ,\ x_3^d\right) = (0.934\ ,\ 1.474\ ,\ 4.468)$$

The reliability index is now $\beta_{HL} = 0.457$ which is larger than $\beta_{HL} = 0.341$ from Example 1. However, the failure probability does not change so much (from 36.7% in Example 1 to 32.4% in this example).

A more widely used method of calculating β_{HL} is

1. Select some trial coordinates of the design point in the z-coordinate system

$$\bar{z}^d = \left(z_1^d\ , z_2^d\ ,\ \dots\ ,\ z_n^d\right)$$

2. Calculate a_i $\quad i = 1, 2, \ldots, n$ by

$$a_i = \frac{\partial g}{\partial z_i}\Big|_{\bar{z}=\bar{z}^d}$$

3. Determine a better estimate of $\bar{z}^d$ by

$$z_i^d = a_i \frac{\sum_{i=1}^{n} \left(a_i\, z_i^d\right) - g|_{\bar{z}=\bar{z}^d}}{\sum_{i=1}^{n} (a_i)^2}$$

4. Repeat 2) and 3) to achieve convergence

5. Evaluate β_{HL} by

$$\beta_{HL} = \left[\sum_{i=1}^{n} \left(z_i^d\right)^2\right]^{\frac{1}{2}}$$

The method is based on the assumption of the existence of only one minimum. However, several "local" minima might exist. In order to avoid convergence against such local minima (and thereby overestimation of β_{HL} and the reliability) several different sets of trial coordinates might be tried.

3.2.4 Time-variant random variables

The failure functions within breakwater engineering are generally of the form

$$g = f_1(\bar{r}) - f_2(H_s\,,\ W\,,\ T_m) \tag{52}$$

where $\bar{R}$ represents the resistance variables and H_s, W and T_m are the load variables signifying the wave height, the water level and the wave period. The random variables are in general time-variant.

Discussion of Load Variables:

The most important load parameter in breakwater engineering is the *wave height.* It is a time-varying quantity which is best modelled as a stochastic process. Distinction is made between *short-tem* and *long-term* statistics of the wave heights. The first one deals with the distribution of the wave height H during a stationary sequence of a storm, i.e. during a period of constant H_s (or any other characteristic wave height). The short term wave height distribution follows the Rayleigh distribution in case of deep-water waves and some truncated distribution in case of shallow water waves.

The long term statistics deals with the distribution of the storms which are then characterized by the max value of H_s occurring in each storm. The storm history is given as the sample (H_{s1}, H_{s2}, ... , H_{sn}) covering a period of observation Y. Extreme value distributions like the Gumbel and Weibull distributions are then fitted to the sample. For strongly depth limited wave conditions a normal distribution with mean value as a function of water depth might be considered.

The distribution of H_s can be substituted by the distribution of the maximum value within T years, i.e. the distribution of H_s^T. The calculated failure probability then refers to the period T (which in practice might be the lifetime of the structure) if distribution functions of the other variables in (52) are assumed unchanged during the period T.

As an example consider a sample of n independent storms, i.e. H_{s1}, H_{s2}, ... , H_{sn}, obtained within Y years of observation. Assume that H_s follows a Gumbel distribution

$$F(H_s) = exp\left[-exp(-\alpha(H_s - \beta))\right] \tag{53}$$

i.e. the distribution of H_s within a period of average length between the observations Y/n.

The distribution parameters α and β can be estimated e.g. by the maximum likelihood method or the methods of moments. Moreover, the standard deviations of α and β signifying the statistical uncertainty due to limited sample size can be estimated too.

The sampling intensity is $\lambda = n/Y$. Within a T-years reference period the number of data will be λT. The probability of the maximum value of H_s within the period T is then

$$F\left(H_s^T\right) = \left(F(H_s)\right)^{\lambda T} = \left[exp\left[-exp(-\alpha(H_s - \beta))\right]\right]^{\lambda T} \tag{54}$$

The expectation (mean) value of H_s^T is given by

$$\mu_{H_s^T} = \beta - \frac{1}{\alpha} ln\left[-ln\left(1 - \frac{1}{\lambda T}\right)\right] \tag{55}$$

and the standard deviation of H_s^T – in case of maximum likelihood estimates – is

$$\sigma_{H_s^T} = \left(\frac{1}{n\alpha^2}\left[1.109 + 0.514\left(-ln\left(-ln\left(1 - \frac{1}{\lambda T}\right)\right)\right)\right.\right.$$
$$\left.\left. +0.608\left(-ln\left(-ln\left(1 - \frac{1}{\lambda T}\right)\right)\right)^2\right]\right)^{0.5} \tag{56}$$

This expression includes the statistical uncertainty due to limited sample size. Some uncertainty is related to the estimation of the sample values H_{s1}, H_{s2}, ... , H_{sn} due to measurement errors, errors in hindcast models etc. This uncertainty corresponds to a coefficient of variation $\frac{\sigma_{H_s}}{\mu_{H_s}}$ in the order of 5 - 20%. The effect of this might be implemented in the calculations by considering a total standard deviation of

$$\sigma = \left(\sigma^2_{H_s^T} + \sigma^2_{H_s}\right)^{0.5} \tag{57}$$

In the level II calculation eq. (54) is normalized around the design point and eqs. (55) and (56) or (57) are used for the mean and the standard deviation, cf. the procedure given in Example 2.

Instead of substituting H_s in eq. (52) with H_s^T the following procedure might be used: Consider T in eqs. (54) to (56) to be 1 year.

The outcome of the calculations will then be the probability of failure in a 1 year period, $P_f\,(1\ year)$. If the failure events of each year are assumed independent for all variables then the failure probability in T years is

$$P_f(T\ years) = 1 - [1 - P_f(1\ year)]^T \qquad (58)$$

However, for typical resistance variables such as concrete strength it is not realistic to assume the events of each year to be independent. The calculated values of the failure probability in T-years using $H_s^{1\ year}$ and H_s^T will be different. The difference will be very small if the variability of H_s is much larger than the variability of other variables.

The *water level* W is also an important parameter as it influences the structural *freeboard* and limits the wave heights in shallow water situations. Consequently, for the general case it is necessary to consider the joint distribution of H_s, W and T_m. However, in case of deep-water waves W is often almost independent (except for barometric effects) of H_s and T_m and might therefore be taken as a non-correlated variable and might be approximated by a normal distribution with a certain standard deviation. The distribution of W is assumed independent of the length of the reference period T.

The "wave period" T_m is correlated to H_s. As a minimum the mean value and the standard deviation of T_z and the correlation of T_z with H_s should be known in order to perform a level II analysis. However, the linear correlation coefficient is not very meaningful as it gives an insufficient description when the parameters are non-normal distributed. Alternatively the following approach might be used: From a scatter diagram of H_s and T_m a relationship of the form $T_m = Af\,(H_s)$ is established in which the parameter A is normal distributed (or some other distribution) with mean value $\mu_A = 1$ and a standard deviation σ_A which signifies the scatter. T_m can then be substituted by the variable A in (52). A is assumed non-correlated to all other parameters. Generally, the best procedure to cope with the correlations between H_s, W and T_m is to work on the conditional distributions. Assume the distribution of the maximum value of H_s within the period T given as $F_1\left(H_s^T\right)$. Further, assume the conditional distributions $F_2\left(W|H_s^T\right)$ and $F_3\left(T_m|H_s^T\right)$ to be known. Let Z_1, Z_2 and Z_3 be independent standard normal variables and

$$\Phi(z_1) = F_1\left(H_s^T\right)$$

$$\Phi(z_2) = F_2\left(W|H_s^T\right)$$

$$\Phi(z_3) = F_3\left(T_m|H_s^T\right)$$

The inverse relationships are given by

$$H_s^T = F_1^{-1}\left[\Phi(z_1)\right]$$

$$W = F_2^{-1}\left[\Phi(z_2)\,|H_s^T\right]$$

$$T_m = F_3^{-1}\left[\Phi(z_3)\,|H_s^T\right]$$

Let the resistance variables $\bar{R}$ converted into standard normal variable $\bar{z}_o$. The resistance term is written $f_1(\bar{r}) = f_3(\bar{z}_o)$. Then the failure function eq. (52) becomes

$$g = f_3(\bar{z}_o) - f_2\left(F_1^{-1}[\Phi(z_1)],\ F_2^{-1}\left[\Phi(z_2)\,\|H_s^T\right],\ F_3^{-1}\left[\Phi(z_3)\,|H_s^T\right]\right) = 0$$

because g now comprises only independent standard normal variables the usual iteration methods for calculating β_{HL} can be applied.

Discussion of Resistance Parameters

The service life of coastal structures is in most cases a span of years, say 20 to 100 years. During periods of that length a decrease in the structural resistance is to be expected due to various types of material deterioration. Chemical reaction, thermal effect, and repeated loads (fatigue load) can cause deterioration of concrete and natural stone leading to disintegration and rounding of elements. Also the resistance against displacements of armour layers made of randomly placed armour units will decrease with the number of waves (i.e. with time) due to the stochastic nature of the resistance. Consequently, for armour layers it means a reduction of D_n and K_D with time, cf. the Hudson equation.

Although of great importance in some cases, it is not easy to account for the material effects in reliability calculations. The main problem is the assessment of the variation with time which depends a lot on the intrinsic characteristics of the applied rock and concrete. However, only fairly primitive methods are available for assessment of the relevant characteristics. Moreover, the variation with time depends very much on the load-history which can be difficult to estimate for the relevant period of structural life.

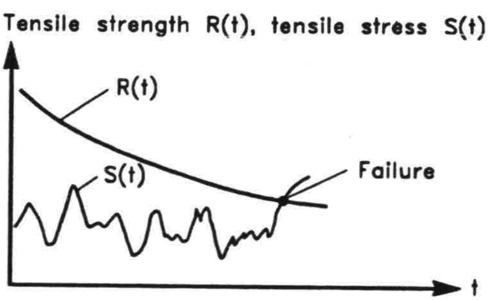

FIG. 7. Illustration of a first-passage problem.

Fig. 7 illustrates a situation where a resistance parameter $R(t)$, e.g. signifying the tensile strength of concrete armour units, decreases with time t. $R(t)$ is assumed to be a deterministic function. The load $S(t)$, e.g. the tensile stress caused by wave action, is assumed to be a stationary process. The probability of failure, i.e. $P(S > R)$, within a period T is

$$P_f(T) \simeq 1 - exp\left[-\int_0^T \nu^+ (R(t))\, dt\right] \tag{59}$$

where $\nu^+ (R(t))$ is the mean-upcrossing rate (number of up-crossings per unit time) of the level $R(t)$ by the process $S(t)$ at time t. ν^+ can be computed by Rice's formula

$$\nu^+ (R(t)) = \int_{\dot{R}}^{\infty} \left(\dot{S} - \dot{R}\right) f_{S\dot{S}} \left(R(t), \dot{S}\right) d\dot{S}$$

in which $f_{S\dot{S}}$ is the joint density function for $S(t)$ and $\dot{S}(t)$.

Implementation of time-variant variables into level II analyses is rather complicated. For explanation reference is given to Wen and Chen, 1987.

4 Failure probability analysis of failure mode systems

It is clear from Fig. 1 that a breakwater can be regarded as a system of components which can either fail or function. Due to interactions between the components, failure of one component may impose failure of another component and even lead to failure of the system. A so-called *fault tree* is often used to clarify the relations between the failure modes.

A fault tree describes the relations between the failure of the system (e.g. excessive wave transmission over a breakwater protecting a harbour) and the events leading to this failure. Fig. 8 shows a simplified example based on some of the failure modes indicated in Fig. 1.

A fault tree is a simplification and a systematization of the more complete so-called cause-consequence diagram which indicates the causes of partial failures as well as the interactions between the failure modes. An example is shown in Fig. 9.

The failure probability of the system, e.g. the probability of excessive wave transmission in Fig. 8, depends on the failure probability of the single failure modes and on the correlation and linking of the failure modes.

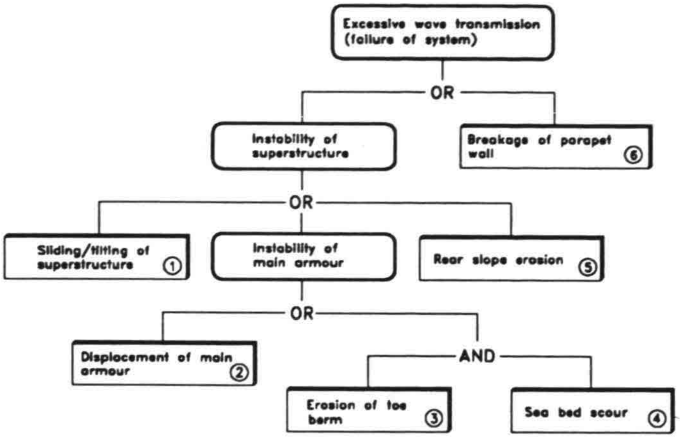

FIG. 8. Example of simplified fault tree for a breakwater.

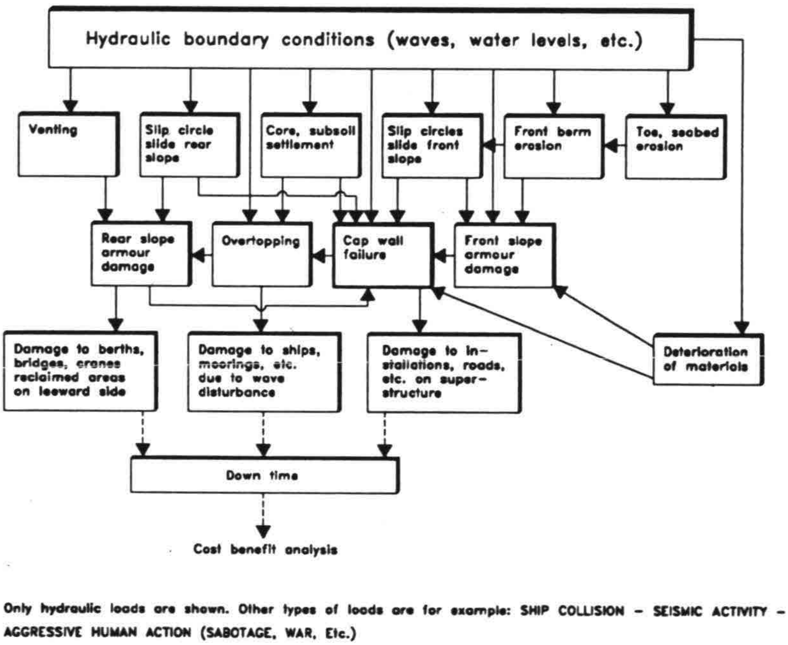

FIG. 9. Example of cause-consequence diagram for a rubble mound breakwater.

The failure probability of a single failure mode can be estimated by the methods described in chapter 3. Two factors contribute to the correlation, namely *physical interaction,* such as sliding of main armour caused by erosion of a supporting toe berm, and *correlation through common parameters* like H_s. The correlations caused by physical interactions are not yet quantified. Consequently, only the common-parameter-correlation can be dealt with in a quantitative way. However, it is possible to calculate upper and lower bounds for the failure probability of the system.

A system can be split into two types of fundamental systems, namely series systems and parallel systems, Fig. 10.

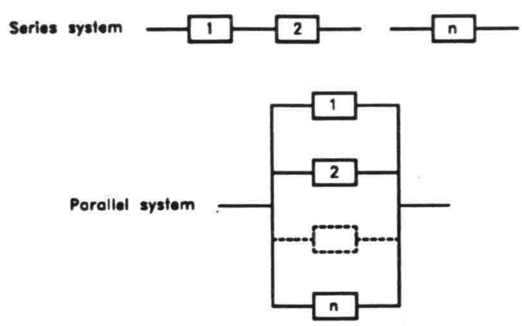

FIG. 10. Series and parallel systems.

Series systems

In a series system failure occurs if any of the elements $i = 1,\ 2,\ \dots\ ,\ n$ fails. The upper and lower bounds of the failure probability of the system, P_{fS} are

$$\text{Upper bound} \quad P_{fS}^U = 1 - (1 - P_{f1})(1 - P_{f2})\ \dots\ (1 - P_{fn}) \tag{60}$$

$$\text{Lower bound} \quad P_{fS}^L = \max P_{fi} \tag{61}$$

where max P_{fi} is the largest failure probability among all elements. The upper bound correponds to no correlation between the failure modes and the lower bound to full correlation. Eq. (60) is sometimes approximated by $P_{fS}^U = \sum_{i=1}^{n} P_{fi}$ which is applicable only for small P_{fi} because P_{fS}^U should not be larger than one.

The OR-*gates* in a fault tree corresponds to series components. Series components are dominating in breakwater fault trees. Really, the AND-gate in Fig. 8 is included for illustration purpose and is better substituted by an OR-gate.

Parallel systems

A parallel system fails only if all the elements fail.

Upper bound $$P_{fS}^{U} = \min P_{fi} \tag{62}$$

Lower bound $$P_{fS}^{L} = P_{f1} \cdot P_{f2} \ldots P_{fn} \tag{63}$$

The upper bound corresponds to full correlation between the failure modes and the lower bound to no correlation.

The AND-*gates* in a fault tree correspond to parallel components.

In order to calculate upper and lower failure probability bounds for a system it is convenient to decompose it into series and parallel systems. Fig. 11 shows a decomposition of the fault tree, Fig. 8.

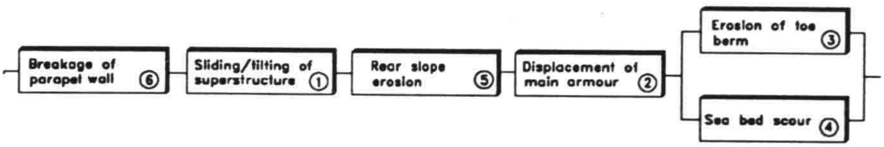

FIG. 11. Decomposition of the fault tree Fig. 8 into series and parallel systems.

EXAMPLE 3

The level II analysis of the single failure modes for a specific breakwater schematized in Figs. 8 and 11 revealed the following probabilities of failure in a 1-year period

i	1	2	3	4	5	6
P_{fi} %	3	6	4	3	0.5	1

Note that these P_{fi}-values cannot be used in general because they relate to a specific structure. However, they are typical for conventionally designed breakwaters with respect to order of magnitude and large variations.

The simple failure probability bounds for the system are, cf. eqs. (60), (61), (62) and (63):

Upper bound (no correlation):

$$P^U_{fS} = 1-(1-P_{f6})(1-P_{f1})(1-P_{f5})(1-P_{f2})(1-\text{min. of } [P_{f3},\ P_{f4}]) = 12.9\%$$

or for small values of P_{fi}

$$P^U_{fS} = P_{f6}+P_{f1}+P_{f5}+P_{f2}+\text{min. of } [P_{f3},\ P_{f4}] = 13.5\%$$

Lower bound (full correlation):

$$P^L_{fS} = \text{max of } [P_{f6},\ P_{f1},\ P_{f5},\ P_{f2},\ P_{f3}\cdot P_{f4}] = 6\%$$

The simple bounds corresponding to T-years structural life might be approximated by the use of eq. (58) *)

	Structure life in years 20	50	100
$P^U_{fS}\%$	94	100	100
*) $P^L_{fS}\%$	71	95	100

*) It is very important to notice that the use of eq. (58), which assumes independent failure events from one year to another, can be misleading. This will be the case if some of the parameters which contribute significantly to the failure probability are time-invariant, i.e. are not changed from year to year. An example would be the parameter signifying a large uncertainty of a failure mode formula, e.g. A in eq. (2). If all parameters were time-invariant then the correct lower bound would be $P^L_{fS} = \max\limits_{i=1-n} [P_{fi}]$ independent of T, i.e. 6% for all T in the example. It follows that use of eq. (58) leads to too large values of P^L_{fS} for $T > 1$ year.

In order to obtain correct P_{fS}-values it is very important that the fault tree represents precisely the real physics of the failure development. This is illustrated by Example 4 where a fault tree alternative to Fig. 8 is analysed, however, containing the same failure mode probabilities as given in Example 3.

EXAMPLE 4

Fig. 12 shows the fault tree which differs from the fault tree in Fig. 8 in that in Fig. 12 only failure mode 6 can directly cause system failure, while in Fig. 8 each of the failure modes 6, 5, 1, 2 and (3+4) can cause system failure.

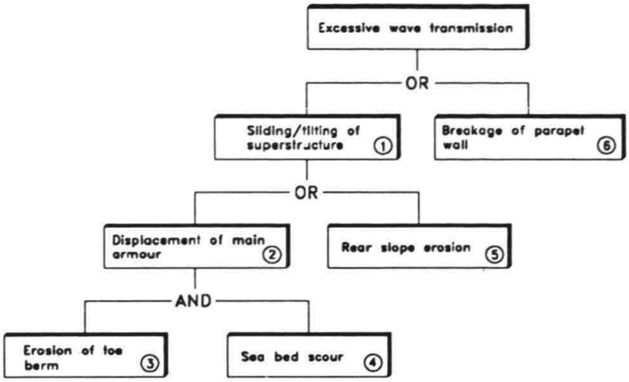

FIG. 12. Example of simplified fault tree for a breakwater.

The decomposition of the fault tree is shown in two steps in Fig. 13. Note that the same failure mode can appear more than once in the decomposed system.

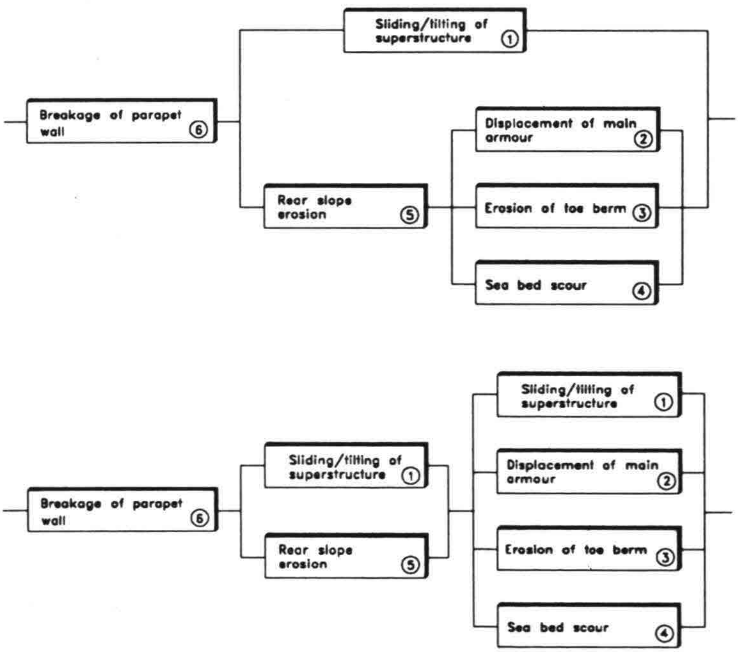

FIG. 13. Decomposition of the fault tree Fig. 12 into series and parallel systems.

The simpel bounds for the system are, cf. eqs. (60), (61), (62) and (63):

Upper bound:

$$P^U_{fS} = 1 - (1 - P_{f6})(1 - \text{ min. of } [P_{f1},\ P_{f5}])(1 - \text{ min. of } [P_{f1},\ P_{f2},\ P_{f3}, P_{f4}]) = 4.5\%$$

or for smaller values of P_{fi}

$$P^U_{fS} = P_{f6} + \text{ min. of } [P_{f1},\ P_{f5}] + \text{ min. of } [P_{f1},\ P_{f2},\ P_{f3},\ P_{f4}] = 4.5\%$$

Lower bound:

$$P^L_{fS} = \text{ max. of } [P_{f6},\ P_{f1} \cdot P_{f5},\ P_{f1} \cdot P_{f2} \cdot P_{f3} \cdot P_{f4}] = 1\%$$

Using the same P_{fi}-values and procedure as given in Example 3 the following system failure probabilities are obtained

	Structure life in years 20	50	100
$P^U_{fS}\%$	60	90	99
*) $P^L_{fS}\%$	18	39	63

These values are quite different from the values of Example 3 which underlines the importance of a correct fault tree. *) see note on page 28.

The real failure probability of the system P_{fS} will always be in between P^U_{fS} and P^L_{fS} because some correlation exists between the failure modes due to the common sea state parameters, e.g. H_s.

It would be possible to estimate P_{fS} if the physical interactions between the various failure modes were known and described by formulae and if the correlations between the involved parameters were known. However, the procedure for such correlations are very complicated and are in fact not yet fully developed for practical use.

The probability of failure cannot in itself be used as the basis for an optimization of a design. This is because an optimization must be related to a kind of measure (scale) which for most structures is the economy, but other measures such as loss of human life (without considering some cost of a life) are also used.

The so-called *risk*, defined as the product of the probability of failure and the economic consequences is used in optimization considerations. The economic consequences must cover all kind of expenses related to the failure in question, i.e. cost of replacement, down-time costs etc.

5 Uncertainties related to parameters determining the reliability of the structure

Calculation of reliability or failure probability of a structure is based on formulae describing its response to loads and on information about the uncertainties related to the formulae and the involved parameters.

Basically, uncertainty is best given by a probability distribution. Because the distribution is rarely known it is common to assume a normal distribution and a related coefficient of variation

$$\sigma' = \frac{\sigma}{\mu} = \frac{\text{standard deviation}}{\text{mean value}} \tag{64}$$

as the measure of the uncertainty.

The word uncertainty is here used as a general term referring both to errors, to randomness and to lack of knowledge.

5.1 Uncertainty related to failure mode formulae

The uncertainty of a formula can be considerable. This is clearly seen from many diagrams presenting the formula as a nice curve shrouded in a wide scattered cloud of data points (usually from experiments) which are the basis for the curve fitting. Coefficients of variation of 15-20% or even larger are quite normal.

The range of validity and the related coefficient of variation should always be considered when using a formula.

5.2 Uncertainty related to environmental parameters

The sources of uncertainty contributing to the total uncertainties in environmental design values are categorized as:

1. Errors related to instrument response (e.g. from accelerometer buoy and visual observations)

2. Variability and errors due to different and imperfect calculations methods (e.g. wave hindcast models, algorithms for timeseries analysis)

3. Statistical sampling uncertainties due to short-term randomness of the variables (variability within a stochastic process, e.g. two 20 min. records from a stationary storm will give two different values of the significant wave height)

4. Choise of theoretical distribution as a representative of the unknown long-term distribution (e.g. a Weibull and a Gumbel distribution might fit a data set equally well but can provide quite different values of a 200-year event).

5. Statistical uncertainties related to extrapolation from short samples of data sets to events of low probability of occurrence.

6. Statistical vagaries of the elements

Distinction is to be made between *short-term* sea state statistics and *long-term* (extreme) sea statistics. Short-term statistics is related to the stationary conditions during a sea state, e.g. wave height distribution within a storm of constant significant wave height, H_s. Long-term statistics deals with the extreme events, e.g. the distribution of H_s.

Related to the *short-term* sea state statistics the following aspects must be considered:

- The distribution for individual wave heights in a record in *deep water* and *shallow water* conditions, i.e Rayleigh distribution and some truncated distributions, respectively.

- Variability due to short samples of single peak spectra waves in deep and shallow water based on theory and physical simulations.

- Variability due to different spectral analysis techniques, i.e. different algorithms, smoothing and filter limits.

- Errors in instrument response and influence of location of measurement. Floating accelerometer buoys tend to underestimate the height of steep waves. Characteristics of shallow water waves can vary considerably in areas with complex sea bed topography. Wave recordings at positions with depth limited breaking waves cannot produce reliable estimates of the deep water waves.

- Imperfection of deep and shallow water numerical hindcast models and quality of wind input.

Estimates on overall uncertainties for short-term sea state parameters covering items 1 - 3 given above, are presented in Table 7 for use when no more precise site specific information is available.

Table 7. Typical variational coefficients $\sigma' = \sigma/\mu$ (standard deviation over mean value) for measured and calculated sea state parameters (Burcharth, 1989).

Parameter	Methods of determination	Estimated typical values σ'	Bias	Comments
Significant wave height, OFFSHORE	Accelerometer buoy, pressure cell, vertical radar	0.05-0.1	~ 0	
	Horizontal radar	0.15	~ 0	
	Hindcast, num. models	0.1-0.2	0-0.1	Very dependent on quality of weather maps.
	Hindcast, SMB method	- 0.15-0.2	?	Valid only for storm conditions in restricted sea basins.
	Visual observations from ships	0.2	0.05	
Significant wave height NEARSHORE determined from offshore significant waveheight taking into account typical shallow water effects (refraction, diffraction, shoaling, ...)	Numerical models	0.1-0.20	0.1	σ' can be much larger in some cases
	Manual calculations	0.15-0.35		
Mean wave period offshore on condition of fixed significant wave height	Accelerometer buoy records	0.02-0.05	~ 0	
	Estimates from amplitude spectra	0.15	~ 0	
	Hindcast, num. models	0.1-0.2	~ 0	
Duration of sea state with significant wave height exceeding a specific level	Direct measurements	0.02	~ 0	
	Hindcast, num. models	0.05-0.1	~ 0	
Spectral peak frequency offshore	Measurements	0.05-0.15	~ 0	
	Hindcast, num. models	0.1-0.2	~ 0	
Spectral peakedness offshore	Measurements and hindcast, num. models	0.4	~ 0	
		Degrees		
Mean direction of wave propagation offshore	Pitch - roll buoy	5°		
	Measurements η, u, v or p, u, v •)	10°		
	Hindcast, num. models	15 – 30°		
		σ'		
Astro tides	Prediction from constants	0.001-0.07	~ 0	
Storm surge	Numerical models	0.1-0.25	±0.1	

•) two horizontal velocity components and water level elevation or pressure.

Evaluation of the uncertainties related to the *long-term* sea state statistics and its use for design involves considerations of the following aspects:

- The encounter probability
- Estimation of the standard deviation of a return-period event for a given extreme distribution
- Estimation of extreme distributions by fitting to data sets consisting of uncorrelated values of H_s from
  - frequent measurements of H_s equally spaced in time
  - identification of the largest H_s in each year (annual series)
  - maximum values of H_s for a number of storms exceeding a certain threshold value of H_s (POT, peak over threshold, analysis)

The methods of fitting are the maximum likelihood method, the method of moments, the least square method and visual graphical fit.

- Uncertainty on extreme distribution parameters due to limited data sample size.
- Influence on the extreme value of H_s of the choise of threshold value in the POT analysis. (The threshold level should exclude all waves which do not belong to the statistical population of interest.)
- Errors due to lack of knowledge about the true extreme distribution. Different theoretical distributions might fit a data set equally well, but might provide quite different return period values of H_s. (The error can be estimated only empirically by comparing results from fits to different theoretical distributions.)
- Errors due to applied plotting formulae in case of graphical fitting. Dependent on the applied plotting formulae quite different extreme estimates can be obtained. The error can only be empirically estimated.
- Climatological changes.
- Physical limitations in extrapolation to events of low probability. The most important example might be limitations in wave heights due to limited water depths and fetch restrictions.
- The effect of measurement error on the uncertainty related to an extreme event.

It is beyond the scope of this contribution to discuss in more detail the mentioned uncertainty aspects related to the environmental parameters. Reference is given to Burcharth (1989).

5.3 Uncertainty related to structural parameters

The uncertainties related to material parameters (like density) and geometrical parameters (like slope angle and size of structural elements) are generally much smaller than the uncertainties related to the environmental parameters and to the design formulae.

6 Introduction of a partial coefficient system for implementation of a given reliability in the design

The following presentation explains in short the partial coefficient system developed and proposed by Subgroup-F under the PIANC PTC II Working Group 12 on Rubble Mound Breakwaters. For more details reference is made to Burcharth (1991).

6.1 Introduction to partial coefficients

The objective of the use of partial coefficients is to assure a certain reliability of the structures.

The partial coefficients, γ_i, are related to characteristic values of the stochastic variables, $X_{i,ch}$. In conventional civil engineering codes the characteristic values of loads and other action parameters are often chosen to be an upper fractile (e.g. 5%), while the characteristic values of material strength parameters are chosen to be the mean values. The values of the partial coefficients are uniquely related to the applied definition of the characteristic values.

The partial coefficients, γ_i are usually larger than or equal to one. Consequently, if we define the variables as either load variables X_i^{load} (as for example H_s) or resistance variables X_i^{res} (as for example the block volume) then the related partial coefficients should be applied as follows to obtain the design values

$$\begin{aligned} X_i^{design} &= \gamma_i^{load} \cdot X_{i,ch}^{load} \\ X_i^{design} &= \frac{X_{i,ch}^{res}}{\gamma_i^{res}} \end{aligned} \tag{65}$$

The magnitude of γ_i reflects both the uncertainty on the related parameter X_i and the relative importance of X_i in the failure function. A large value, e.g. $\gamma_{H_s} = 1.4$, indicates a relatively large sensitivity of the failure probability to the significant wave height, H_S. On the other hand, $\gamma_i \simeq 1$ indicates no or negligible sensitivity in which case the partial coefficient should be omitted. It is to be stressed that the magnitude of γ_i is not – in a mathematical sense – a stringent measure of the sensitivity of the failure probability to the parameter, X_i.

When the partial coefficients are applied to the characteristic values of the parameters in eq. (2) we obtain the **design equation**, i.e. the definition of how to apply the coefficients.

The partial coefficients can be related either to each parameter or to combinations of the parameters (overall coefficients). In the first case we obtain the design equation

$$G = \frac{Z_{ch}}{\gamma_z} \frac{\Delta_{ch}}{\gamma_\Delta} \frac{D_{n,ch}}{\gamma_{Dn}} \left(K_D \frac{cot\alpha_{ch}}{\gamma_{cot\alpha}} \right)^{1/3} - \gamma_{H_s} H_{s,ch} \geq 0$$

or (66)

$$D_{n,ch} \geq \gamma_z \gamma_\Delta \gamma_{Dn} \gamma_{cot\alpha}^{1/3} \gamma_{H_s} \frac{H_s}{Z_{ch}\, \Delta_{ch} K_D\, cot\alpha_{ch}}$$

In the second case we could for example have only γ_{H_s} and an overall coefficient γ_z related to the first term on the right hand side of eq. (2). The design equation would then be

$$G = \frac{Z_{ch}}{\gamma_z} \Delta_{ch}\, D_{n,ch}\, (K_D cot\alpha)^{1/3} - \gamma_{H_s} H_s \geq 0$$

or (67)

$$D_{n,ch} \geq \gamma_z\, \gamma_{H_s} \frac{H_s}{Z_{ch}\, \Delta_{ch}\, K_D\, cot\alpha_{ch}}$$

Eqs. (66) and (67) express two different "code formats". By comparing the two equations it is seen that the product of the partial coefficients is independent of the chosen format, other things equal. It is desirable to have a system which is as simple as possible, i.e. as few partial coefficients as possible, but without invalidating the accuracy of the design equation beyond acceptable limits.

Fortunately, it is very often possible to use overall coefficients, like γ_z in eq. (67), without loosing significant accuracy within the realistic range of combinations of parameter values. This is the case for the system proposed in this paper where only two partial coefficients, γ_{H_s} and γ_z, are used in each design formula.

Usually several failure modes are relevant to a design. The relationship between the failure modes are characterized either as series systems or parallel systems. A fault tree can be used to illustrate the complete system. The partial coefficients for failure modes being in a system with failure probability, P_f are different from the partial coefficients for the single failure modes with the same failure probability, P_f. Therefore, partial coefficients for single failure modes and multi failure mode systems are treated separately.

6.2 Overall concept of the proposed partial coefficient system

In existing civil engineering codes of practise, e.g. for steel and concrete structures, it is a characteristic of them that

- partial coefficients are related to combinations of basic variables rather than to each of them in order to reduce the number of coefficients.
- the partial coefficients reflect the safety level inherent in a large number of well proven designs. Two sets of coefficients covering permanent and preliminary structures are usually given, but the related average probabilities of failure are not specified. In

other words, it is not possible by means of the normal structural codes to design a structure to a predetermined failure probability.

However, it is not advisable to copy this concept in safety recommendations for rubble mound breakwaters for the following reasons:

- For coastal structures and breakwaters there is no generally accepted tradition which reflects one or more levels of failure probability. On the contrary it is certain that the safety level of existing structures varies considerably and is often very low. Besides, it is very difficult to evaluate the safety level of existing coastal structures and breakwaters because of lack of information, especially on the environmental conditions, e.g. the water level variations and the wave climate. Consequently, it is not possible to produce sets of partial coefficients which, in a meaningful way, are calibrated against existing designs.

- Due to the very nature of coastal engineering where design optimization dictates considerable variations in the safety level of the various structures it is necessary (advisable) to have sets of partial coefficients which correspond to various failure probabilities. In other words the designer and the client decide on the basis of optimization and cost benefit analyses that the structure should be designed for a specific safety level (for example 20% probability of failure (P_f = 20%) within a structural lifetime of T = 80 years, where failure is defined as a certain degree damage). The code should then contain a set of partial coefficients corresponding to this failure probability.

- Because the quality of information about the long term wave climate (the dominating load) varies from very unreliable (uncertain) wave statistics based on few uncertain data sets to very reliable statistics based on many years of high quality wave recordings and hindcast values it is necessary that the partial coefficients must be a function of the quality of the available information on the wave climate. This means that the statistical uncertainty due to limited number of wave data and errors in the wave data should be implemented.

Extensive calculations, performed at University of Aalborg, of partial coefficients for armour layer stability formulae demonstrated that it was possible to develop a concept which satisfies these demands.

6.3 Method of determining the partial coefficient

The overall procedure for the development of a partial coefficient system was as follows

- Define the failure modes and the failure element structure
  (single element analysis and/or system analysis)

- Select the code format (design equations)

- Define intervals of the parameters, their statistical properties and combinations

- Select target probabilities of failure

- Calculate the partial coefficients
- Optimize and calibrate the system
- Verify the partial coefficient system against the observed behaviour of existing structures.

The partial coefficients γ_i are determined from a so-called level II reliability analysis. The applied computer programmes BWREL (Break Water RELiability programme) and BWCODE (Break Water CODE) were developed at the University of Aalborg by Dr. John Dalsgaard Sørensen especially for the reliability analysis of breakwaters. For further explanation reference is made to the Sub-group F report.

6.4 Breakwater Types and Failure Modes

The Working Group set out to study five different types of breakwaters and considered a wide range of failure modes. During the work it became clear that sufficiently well documented failure formulae were only available to justify recommendations of partial coefficients for conventional multi-layer rubble mound breakwaters with armour carried over the crest – and for the following Failure modes:

- Hydraulic instability of front face armour
- Instability of low crested rock breakwaters
- Hydraulic instability of rock toe berm
- Run-up on rock armoured slopes

The formulae for these failure modes are given in section 6.8 in the form of design equations, which shows how to apply the partial coefficients.

6.5 Partial Coefficient System Format for Single Failure Modes

For each failure mode only two partial coefficients γ_{H_s} and γ_z are used, cf. the example given by eq. (67). The partial coefficient are determined from formulae. Three different concepts for these formulae have been evaluated and the following were chosen as being acceptable with respect to deviations from the target probability of failure.

$$\gamma_{H_s} = \frac{\hat{H}_s^{T_{P_f}}}{\hat{H}_s^T} + {\sigma'_{FH_s}}^{\left(1+\left(\frac{\hat{H}_s^{3T}}{\hat{H}_s^T}-1\right)k_\beta P_f\right)} + \frac{k_s}{\sqrt{P_f N}} \tag{68}$$

$$\gamma_z = 1 - k_\alpha ln\, P_f \tag{69}$$

where

$\hat{H}_s^T$ is the central estimate of the T-year return period value of H_s, where T is the structural lifetime (T = 20, 50 and 100 years were used for the code calibration). γ_{H_s} is applied to $\hat{H}_s^T$ (the characteristic value of H_s, cf. the design equations).

$\hat{H}_s^{3T}$ is the central estimate of the $3T$-year return period value of H_s.

$\hat{H}_s^{T_{P_f}}$ is the central estimate of H_s corresponding to an equivalent return period T_{P_f} defined as the return period corresponding to a probability P_f that $\hat{H}_s^{T_{P_f}}$ will be exceeded during the structural lifetime T. T_{P_f} is calculated from the encounter probability formula $T_{P_f} = \left(1 - (1 - P_f)^{\frac{1}{T}}\right)^{-1}$, cf. Fig. 14.

$\sigma'_{F_{H_s}}$ is the variational coefficient of a function F_{H_s} modelled as a factor on H_s. F_{H_s} signifies the measurement errors and short term variability of H_s and has the mean value 1.0. $\sigma'_{F_{H_s}}$ is equal to σ' for H_s in Table 7. The statistical uncertainty on H_s is not included in F_{H_s}.

N is the number of H_s data, used for fitting the extreme distributions. The statistical uncertainty depends on this parameter.

k_α, k_β and k_s are coefficients which are determined by optimization. $k_s \simeq 0.05$ for all failure modes. The k_α and the k_β values are given in Tables 9-12.

The first term in eq. (68) gives the correct γ_{H_s} provided no statistical uncertainty and measurement errors related to H_s are present. The middle term in eq. (68) signifies the measurement errors and the short term variability related to the wave data. The last term in eq. (68) signifies the statistical uncertainty of the estimated extreme distribution of H_s. The statistical uncertainty depends on the total number of wave data, N, but not on the length of the period of observation, as might be expected. The 10 largest values of H_s over a 15 years period provides a much more reliable estimate of the extreme distribution than the 10 largest values of H_s over 1 year. However, in the statistical analysis it is assumed that the data samples are equally representative of the true distribution. In other words it is assumed that the data, besides being non-correlated, are sampled with a frequency and over a length of time which ensures that periodic variations (e.g. seasonal) are not biasing the sample. The designer must be aware of these restrictions.

If the extreme wave statistics is not based on N wave data, but for example on estimates of H_s from information about water level variations in shallow water, then the last term in eq. (68) disappears and instead the value of $\sigma'_{F_{H_s}}$ must account for the inherent uncertainty.

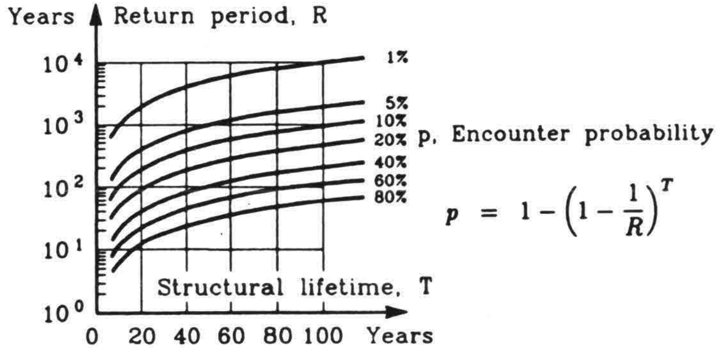

FIG. 14. Encounter probability, i.e. the probability p that the R-year return period event will be exceeded during a T-year structural life.

6.6 Format for Multi Failure Modes

A simple series system is considered, cf. chapter 4. The reliability of the system depends on the correlation between the failure modes. Two factors contribute to the correlation, namely the physical interaction, e.g. the erosion of a toe berm triggering a slide in the main armour layer, and the correlation through common parameters like H_s. The physical correlations are not yet generally known. Consequently, only the common parameter correlations have been implemented in the present work.

A simple system is to treat each failure mode, $i = 1, 2, 3, \ldots, n$ separately using the single failure mode models. The upper and lower bounds of probability of the system, P_f^s could then be estimated as

$$\text{Upper bound} \quad P_f^s = 1 - (1 - P_f^1)(1 - P_f^2)\ldots(1 - P_f^n) \tag{70}$$

$$\text{Lower bound} \quad P_f^s = \max P_f^i \tag{71}$$

Max P_f^i is the largest of the failure probabilities of the failure modes.

Eqs. (70) and (71) correspond to no correlation and full correlation, respectively. Due to the common parameters there will always be a correlation of some size. However, closer bounds must await further work on correlation between failure modes.

6.7 Investigated Ranges of Parameter Variations

The optimization of the partial coefficients is based on calculations where all combinations of realistic values of the failure formula parameters are considered.

The resistance parameters are modelled as normally distributed stochastic variables given by mean values and standard deviations. An example of these values and the related range of parameter variations is given below for the Hudson equation applied to rock armour

Hudson formula $\frac{H_s}{\Delta D_n} = (K_D cot\alpha)^{1/3}$

Design equation :$G = \frac{1}{\gamma_Z}\hat{Z}\hat{\Delta}\hat{D}_n(K_D cot\hat{\alpha})^{1/3} - \gamma_{H_S}\hat{H}_s^T \geq 0$

Notation

$D(p)$	indicates a deterministic value, p.
$N(x_1, x_2)$	indicates a normally distributed parameter with mean value x_1 and standard deviation, x_2.
z	is the design parameter
$\bar{p}$	defines the ranges of application of the code for this failure mode.
$\hat{X}$	expected value (mean value) of X.
F_{H_s}	error function on H_s.
$\hat{H}_s^T$	central estimate of the significant wave height which on average is exceeded once every T years.

Parameters for the stochastic variables:

parameter	distribution	variation of $\bar{p}$
Δ	$N(p_1, p_2)$	$(p_1, p_2) = (1.4, 0.03), (1.6, 0.06)$
D_n	$N(z, pz)$	$p = 0.01,\ 0.05$
$cot\alpha$	$N(p, 0.1)$	$p = 1.5,\ 2,\ 3$
K_D	D	
F_{H_s}	N(1,p)	$p = \sigma_{FH_s} = (0, 0.10, 0.20)$
Z	N(1.0, 0.18)	
H_s	Extreme distribution fitted to local wave data	

The statistical model for the load parameteer H_s was described by three of the commonly used theoretical extreme distributions: Weibull, Gumbel and Exponential. The distributions are given below expressing the non-exceedence probability within T years. λ is the average number of H_s-data per year and N is the total number of data available for fitting the distribution.

The statistical uncertainty of the distributions is included through the parameters α and β which are modelled as stochastic normally distributed variables with variances based on the maximum likelihood estimates of α and β. It should be noted that no quality measure (correlation coefficient or X^2-test) of the fit of a distribution to a data sample, is included in the analyses that were carried out.

The considered distribution functions are listed below.
$N(x_1, x_2)$ indicates a normally distributed parameter with mean value x_1 and standard deviation, x_2.

Gumbel $$F_{H_s^T}(H_s) = [exp(-exp(-\alpha(H_s - \beta)))]^{\lambda T}$$

$$\alpha : N\left(\alpha, \alpha\sqrt{\frac{0.608}{N}}\right) \qquad \beta : N\left(\beta, \frac{1}{\alpha}\sqrt{\frac{1.109}{N}}\right)$$

Weibull $$F_{H_s^T}(H_s) = \left[1 - exp\left(-\left(\frac{H_s - H_s'}{\beta}\right)^{\alpha}\right)\right]^{\lambda T}$$

$$\alpha : N\left(\alpha, \sqrt{\frac{1}{N}}\right) \qquad \beta : N\left(\beta_1\left(\frac{\beta}{N^2}\left[\frac{\Gamma(1+2/\alpha)}{\Gamma^2(1+1/\alpha)} - 1\right]\right)^{\frac{1}{2}}\right)$$

$var[\alpha] = \frac{1}{N}$ is an assumption since it has not been possible to find an analytical expression.

Γ is the gamma function.

$$Exponential \quad F_{H_s^T}(H_s) = \left[1 - exp\left(-\frac{H_s - H_s'}{\alpha}\right)\right]^{\lambda T} \qquad \alpha : N\left(\alpha, \alpha\sqrt{\frac{1}{N}}\right)$$

The H_s data samples used in the analysis are real deep water and shallow water data set from the North Sea, the Atlantic Ocean, the Bay of Biscaya and the Mediterranean Sea. Table 8 shows the distribution parameters for the data sets.

Table 8. Distribution parameters for H_s-data samples.

			Gumbel		Weibull		Exp.	
	N	λ	α	β	α	β	H_s'	α
Bilbao	50	4.17	1.95	5.55	1.39	1.06	4.9	0.97
Sines	15	1.25	0.88	8.75	1.78	2.53	7.1	2.27
Tripoli	15	0.75	0.74	5.06	1.83	3.24	2.9	2.91
North Sea	30	1.88	1.30	6.65	1.28	1.48	5.7	1.39
Follonica	46	5.94	3.14	5.73	1.14	0.58	2.69	0.55
Pozzallo	22	6.94	3.62	4.68	1.05	0.48	2.20	0.47

The statistical uncertainty described through the variance of α and β does not include uncertainties due to

- lack of knowledge about the true extremal distribution
- climatological changes
- measurement errors
- variability due to imperfect calculations of H_s and short term randomness

The last two points are incorporated in the analysis by the multiplication term F_{H_s} on H_s. F_{H_s} is modelled as a normally distributed variable with a mean value of unity and a specified coefficient of variation, $\sigma'_{F_{H_s}}$, the size of which depends on the quality of the available information, cf. Table 7.

The first two points cannot be treated through F_{H_s}, but in a design situation the designer must try the different models for the extreme wave height and thereby select the most appropriate. A partial coefficient system cannot take these problems into account.

Moreover, it is assumed inherent in the analysis that the N values of H_s represent the statistical population to which H_s belongs. This sets limits to minimum length of the period of observation N/λ and N in order to prevent seasonal changes from biasing the results.

For the calibration of the system the following target values of $\sigma'_{F_{H_s}}$ and P_f were used:

$$\sigma_{F_{H_s}} = 0.00 \ , \ 0.10 \text{ and } 0.20$$

$$P_f = 0.01 \ , \ 0.05 \ , \ 0.10 \ , \ 0.20 \text{ and } 0.40.$$

6.8 Example of Design equations and Recommended Values of k_α and k_β

The values of k_α and k_β which have been obtained by carrying out optimization for each failure modes are presented as well as the related design equations in Tables 9 - 12. Note that limitations related to the equations are not given here.

Table 9. Main armour hydraulic stability.

Formula	Design equation	k_α	k_β
Hudson, rock	$\frac{1}{\gamma_z}\ \Delta D_{n50}\,(K_d cot\alpha)^{1/3} \geq \gamma_{H_s} H_s^T$	0.036	151
Van der Meer, rock			
Plunging waves	$\frac{1}{\gamma_z} 6.2 S^{0.2} P^{0.18} \Delta D_{n50} cot\alpha^{0.5} s_m^{0.25} N_z^{-0.1} \geq \gamma_{H_s} H_s^T$	0.027	38
Surging waves	$\frac{1}{\gamma_z} S^{0.2} P^{-0.13} \Delta D_{n50} cot\alpha^{0.5-P} s_m^{-0.5P} N_z^{-0.1} \geq \gamma_{H_s} H_s^T$	0.031	38
Van der Meer Tetrapods $cot\alpha = 1.5$	$\frac{1}{\gamma_z}\left(3.75 \frac{N_{od}^{0.5}}{N_z^{0.25}} + 0.85\right) s_m^{-0.2} \Delta D_n \geq \gamma_{H_s} H_s^T$	0.026	38
Van der Meer Cubes $cot\alpha = 1.5$	$\frac{1}{\gamma_z}\left(6.7 \frac{N_{od}^{0.4}}{N_z^{0.3}} + 1.0\right) s_m^{-0.1} \Delta D_n \geq \gamma_{H_s} H_s^T$	0.026	38

Table 10. Hydraulic stability of low crested rock breakwaters.

Formula	Design equation	k_α	k_β
Van der Meer, rock	As for main armour with factor $f_i = \left[1.25 - 4.8 \frac{R_c}{H_s^T} \left(\frac{S_m}{2\pi}\right)^{0.5}\right]^{-1}$ applied to D_{n50}	0.035	42

Table 11. Hydraulic stability of rock toe berm.

Formula	Design equation	k_α	k_β
Van der Meer, rock	$\frac{1}{\gamma_z} 8.7 \left(\frac{h_t}{h}\right)^{1.43} \Delta D_{n50} \geq \gamma_{H_s} H_s^T$	0.087	100

Table 12. Run-up on rock armoured slopes.

Formula		Design equation	k_α	k_β
Hunt	for	$(cot\alpha)^{-1} s_m^{-0.5} < 1.5$ $\frac{1}{\gamma_z} R_u a^{-1} cot\alpha\, s_m^{0.5} \geq \gamma_{H_s} H_s^T$ -	0.036	44
	for	$(cot\alpha)^{-1} s_m^{-0.5} > 1.5$ $\frac{1}{\gamma_z} R_u b^{-1} [cot\alpha\, s_m^{0.5}]^c \geq \gamma_{H_s} H_s^T$	0.018	36

6.9 Example of the use of the Partial Coefficient System

The following example will illustrate how the partial coefficient system is applied for design purpose.

Objective:

Determination of the average mass, or the nominal diameter D_{n50}, of quarry rock armour corresponding to the following design conditions:

Case 1. Moderate to severe damage with a probability $P_f = 0.2$ within a structural life of $T = 50$ years.

Case 2. Very severe damage (failure) with a probability $P_f = 0.2$ within a structural life of $T = 100$ years.

Case 3. Moderate to severe damage with a probability $P_f = 0.1$ within a structural life of $T = 100$ years.

The Van der Meer formulae for rock given in Table 9 are assumed valid.

Design parameters:

Densities: Rock 2.8 t/m^3, water 1.03 t/m^3, $\Delta = 1.72$

Slope: $cot\alpha = 1.5$, porosity $P = 0.4$

Wave climate: Weibull distribution of H_s with the site specific coefficients $(\alpha, \beta, H_s') = (1.39, 1.06, 0.44)$ determined by fitting to a hindcasted H_s-data set consisting of the $N = 50$ largest values within a 12 years period, i.e. $\lambda = 50/12 = 4.17$. σ'_{FH_s} is estimated to 0.2 for the hindcasted H_s values. Wave steepness $s_m = 0.04$, number of waves $N_z = 2500$.

Damage: Moderate to severe damage $S = 6$, very severe damage (failure) $S = 14$.

Procedure:

The procedure and the partial coefficent formulae described in section 6.5 are used.

Calculations:

In case of a Weibull distribution the central estimate of the significant wave height with an average return period of T years is given by

$$\begin{aligned}\hat{H}_s^T &= H_s' + \beta\,(exp\,[ln(ln(\lambda T))/\alpha]) \\ &= 0.44 + 1.06\,(exp\,[ln(ln(4.17T))/1.39])\end{aligned}$$

The equivalent return period is given by

$$T_{P_f} = \left(1 - (1 - P_f)^{\frac{1}{T}}\right)^{-1}$$

From this is obtained

Case	T (year)	P_f	T_{P_f} (year)	$\hat{H}_s^T$ (m)	$\hat{H}_s^{3T}$ (m)	$\hat{H}_s^{T_{P_f}}$ (m)
1	50	0.2	225	3.98	4.49	4.67
2	100	0.2	449	4.30	4.80	4.97
3	100	0.1	950	4.30	4.80	5.29

From Table 9 (for plunging waves)

$k_\alpha = 0.027$, $k_\beta = 38$

From the formulae

$$\gamma_{H_s} = \frac{\hat{H}_s^{T_{P_f}}}{\hat{H}_s^{T}} + \sigma'_{F_{H_s}}{}^{\left(1+\left(\frac{\hat{H}_s^{3T}}{\hat{H}_s^{T}}-1\right)k_\beta P_f\right)} + \frac{0.05}{\sqrt{P_f N}}$$

$$\gamma_Z = 1 - k_\alpha ln\, P_f$$

and the Van der Meer design equation is obtained

Case	γ_{H_s}	γ_Z	D_{n50} (m)	Average mass (t)
1	1.23	1.04	1.58	11.0
2	1.22	1.04	1.43	8.1
3	1.35	1.06	1.91	19.5

The example illustrates how easy it is to calculate the size of the armour for various design conditions. The system facilitates economical optimization of a design.

The system can be used also for the evaluation of the failure probability of existing structures.

6.10 Conclusions

A concept for the calculation of partial coefficients corresponding to given failure probability within given structure life is presented. Two partial coefficients γ_{H_s} and γ_z are applied to a design formula. Two partial coefficients are calculated from formulae (68) and (69) in which two failure mode specific coefficients, k_α and k_β, are used together with characteristic return period values of H_s, extracted from the site specific long term distribution of H_s.

So far the k_α , k_β coefficients have been calculated only for the failure modes which are described by existing uncertainty evaluated formulae . However, it is easy to expand the system as more failure mode formulae appear. It is important to notice that the reliability of the formulae must be documented, e.g. in terms of a standard deviation, in order to implement them in the partial coefficient system.

7 Acknowledgement

The useful comments of Dr. Zhou Liu and Dr. J. Dalsgaard Sørensen are greatly acknowledged.

8 References

BURCHARTH, H.F. (1989) Uncertainties related to environmental data and estimated extreme events. Report of Subgroup-B, PIANC PTCII Working Group 12 on Rubble Mound Breakwaters. (In print by PIANC 1992)

BURCHARTH, H.F. (1991) Introduction of partial coefficients in the design of rubble mound breakwaters. Proc. Conf. on Coastal Structures and Breakwaters. Inst. of Civil Eng., London.

BURCHARTH, H.F. (1991) Development of a partial coefficient system for the design of rubble mound breakwaters. Report of Subgroup-F. PIANC PTC II Working Group 12 on Rubble Mound Breakwaters. (In print by PIANC 1992)

HALLAM, M.G., HEAF, N.I. and WOOTOS, I.R. (1977). Rationalization of Safety and Serviceability factors in Structural Codes. CIRIA report No. 63, London.

THOFT-CHRISTENSEN, P. and BAKER, M.J. (1982). Structural Reliability Theory and its Application. Springer Verlag. Berlin.

WEN, Y.K. and CHEN, H.C. (1987). On fast integration for time variant structural reliability. *Probabilistic Engineering Mechanics,* Vol.2, 1987, pp 156-162.