Journal of Sound and Vibration 626 (2026) 119624

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Corrigendum

Corrigendum to "Nonlinear Model Reduction to Random Spectral Submanifolds in Random Vibrations" [J. Sound Vib. 600 (2025), 118923]

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This corrigendum corrects a typo appearing in eq. (8) in Section 2.2, eq. (12) of the main theorem and eq. (13) in Remark 2 under the main theorem in [6]. Specifically, the typo involves the inadvertent omission of the linear term in all these equations. With the inclusion of the linear term, eq. (8) correctly reads as

$$\dot{\xi} = \mathbf{A}_E\xi + \mathbf{f}_{0\xi}(\xi, \mathbf{h}_0(\xi)),$$

whereas eq. (12) reads as

$$\begin{aligned}\dot{\xi} &= \mathbf{A}_E\xi + \mathbf{f}_{0\xi}(\xi, \mathbf{h}_\varepsilon(\xi;\theta^t(\nu))) + \varepsilon\mathbf{g}_\xi(\xi, \mathbf{h}_\varepsilon(\xi;\theta^t(\nu)), \theta^t(\nu)) \\ &= \mathbf{A}_E\xi + \mathbf{f}_{0\xi}(\xi, \mathbf{h}_0(\xi)) + \varepsilon\Big[D_\eta\mathbf{f}_{0\xi}(\xi, \mathbf{h}_0(\xi))\mathbf{h}_1(\xi;\theta^t(\nu), 0) + \mathbf{g}_\xi(\xi, \mathbf{h}_0(\xi), \theta^t(\nu))\Big] + \mathscr{O}(\varepsilon^2),\end{aligned}$$

and eq. (13) reads as

$$\dot{\xi} = \mathbf{A}_E\xi + \mathbf{f}_{0\xi}(\xi, \mathbf{h}_0(\xi)) + \varepsilon\mathbf{g}_\xi(\theta^t(\nu)).$$

In all later examples, computations and derivations, of [6], the linear term was included. This omission, however. affected the derivations appearing in "Appendix B: Deriving the leading order random SSM parametrization", which we correct below for completeness. The main results of [6] remain unchanged.

Appendix B. Deriving the leading order random SSM parametrization

We assume the leading-order random SSM parametrization $\mathbf{h}_1(\boldsymbol{\xi};\, \boldsymbol{\theta}^t(\boldsymbol{\nu}),\, \varepsilon)$ to be C^1 in time. We define $(\mathbf{V}_E)_R \in \mathbb{R}^{n\times d}$ and $(\mathbf{V}_E)_L \in \mathbb{R}^{n\times d}$ containing column vectors comprising the right and left eigenvectors associated with the slow subspace E. We also define $(\mathbf{V}_F)_R \in \mathbb{R}^{n\times(n-d)}$ and $(\mathbf{V}_F)_L \in \mathbb{R}^{n\times(n-d)}$ which comprise the right and left eigenvectors associated with the remaining (n−d)-dimensional fast subspace F. We normalize these eigenvectors to satisfy

$$((\mathbf{V}_E)_L)^{\mathrm{T}}(\mathbf{V}_E)_R = \mathbb{I}_d, \quad ((\mathbf{V}_F)_L)^{\mathrm{T}}(\mathbf{V}_F)_R = \mathbb{I}_{n-d},$$

DOI of original article: https://doi.org/10.1016/j.jsv.2024.118923.
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https://doi.org/10.1016/j.jsv.2025.119624

Available online 27 December 2025
0022-460X/

$$((\mathbf{V}_F)_L)^{\mathrm{T}}(\mathbf{V}_E)_R = 0, \quad ((\mathbf{V}_E)_L)^{\mathrm{T}}(\mathbf{V}_F)_R = 0. \tag{B1}$$

These choices are consistent with those used by the code *SSMTool* (see [4]). Given these choices, the linear transformation that block-diagonalizes $\mathbf{A}$ is

$$\mathbf{A}_E = ((\mathbf{V}_E)_L)^{\mathrm{T}}\mathbf{A}(\mathbf{V}_E)_R, \tag{B2}$$

$$\mathbf{B} = ((\mathbf{V}_F)_L)^{\mathrm{T}}\mathbf{A}(\mathbf{V}_F)_R.$$

The linear coordinate change between the phase space variable $\mathbf{x}$ and the modal coordinates is given by

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{bmatrix} ((\mathbf{V}_E)_L)^{\mathrm{T}} \\ ((\mathbf{V}_F)_L)^{\mathrm{T}} \end{bmatrix}\mathbf{x}, \ \mathbf{x} = \begin{bmatrix} (\mathbf{V}_E)_R & (\mathbf{V}_F)_R \end{bmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix}. \tag{B3}$$

Let us denote the random SSM parametrization and reduced dynamics as

$$\mathbf{x} = \mathbf{w}_\varepsilon(\xi; \theta^t(\nu)) = \mathbf{w}_0(\xi) + \varepsilon\mathbf{w}_1(\xi; \theta^t(\nu), \varepsilon), \tag{B4}$$

$$\dot{\xi} = \mathbf{r}_\varepsilon(\xi; \theta^t(\nu)) = \mathbf{r}_0(\xi) + \varepsilon\mathbf{r}_1(\xi; \theta^t(\nu), \varepsilon). \tag{B5}$$

Differentiating eq. (B4) with respect to time and expressing $\dot{\mathbf{x}}$ from the governing equations, we obtain the invariance equation for the random SSM in the form

$$\left(\mathbf{D}_\xi\mathbf{w}_\varepsilon(\xi; \theta_t(\nu))\right)\dot{\xi} + \dot{\mathbf{w}}_\varepsilon(\xi; \theta_t(\nu)) = \mathbf{A}\mathbf{w}_\varepsilon(\xi; \theta_t(\nu)) + \mathbf{f}_0(\mathbf{w}_\varepsilon(\xi; \theta_t(\nu))) + \varepsilon\mathbf{g}(\mathbf{w}_\varepsilon(\xi; \theta_t(\nu)), \theta_t(\nu)). \tag{B6}$$

Substituting eq. (B5) into eq. (B6), and expanding in ε followed by $\boldsymbol{\xi}$, we collect the $O(|\boldsymbol{\xi}|)$ terms to obtain the invariance equation

$$\mathbf{D}_\xi\mathbf{w}_0(0)\mathbf{D}_\xi\mathbf{r}_0(0) = \mathbf{A}\mathbf{D}_\xi\mathbf{w}_0(0). \tag{B7}$$

Choosing $\mathbf{D}_\xi\mathbf{w}_0(0) = (\mathbf{V}_E)_R$ and $\mathbf{D}_\xi\mathbf{r}_0(0) = \mathbf{A}_E$ solves eq. (B6).

Using these choices and collecting $O(\varepsilon)$ terms from eq. (B6), we obtain the leading order invariance equation for the random SSM parametrization and reduced dynamics as

$$\mathbf{D}_\xi\mathbf{w}_0(0)\mathbf{r}_1(0; \theta^t(\nu), 0) + \dot{\mathbf{w}}_1(0; \theta^t(\nu), 0) = \mathbf{A}\mathbf{w}_1(0; \theta^t(\nu), 0) + \mathbf{g}(0, \theta^t(\nu)),$$

$$(\mathbf{V}_E)_R\mathbf{r}_1(0; \theta^t(\nu), 0) + \dot{\mathbf{w}}_1(0; \theta^t(\nu), 0) = \mathbf{A}\mathbf{w}_1(0; \theta^t(\nu), 0) + \mathbf{g}(0, \theta^t(\nu)). \tag{B8}$$

The above invariance equation has two unknowns $\mathbf{r}_1(\mathbf{0}; \boldsymbol{\theta}^t(\boldsymbol{\nu}), 0)$ and $\mathbf{w}_1(\mathbf{0}; \boldsymbol{\theta}^t(\boldsymbol{\nu}), 0)$. We fix $\mathbf{r}_1(\mathbf{0}; \boldsymbol{\theta}^t(\boldsymbol{\nu}), 0)$ as

$$\mathbf{r}_1(0; \theta^t(\nu), 0) = ((\mathbf{V}_E)_L)^{\mathrm{T}}\mathbf{g}(0, \theta^t(\nu)), \tag{B9}$$

which is consistent with the choice for the reduced dynamics coefficients in [4] and [5]. Through this choice, the leading-order term in the reduced dynamics appears as the projection of the full random forcing onto E.

Substituting eq. (B9) in eq. (B8), we arrive at a unique linear inhomogeneous ODE for $\mathbf{w}_1(\mathbf{0}; \boldsymbol{\theta}^t(\boldsymbol{\nu}), 0)$ in the form

$$\dot{\mathbf{w}}_1(0; \theta^t(\nu), 0) = \mathbf{A}\mathbf{w}_1(0; \theta^t(\nu), 0) + \left(\mathbb{I}_n - (\mathbf{V}_E)_R((\mathbf{V}_E)_L)^{\mathrm{T}}\right)\mathbf{g}(0, \theta^t(\nu)). \tag{B10}$$

This equation has the general solution

$$\mathbf{w}_1(0; \theta^t(\nu), 0) = e^{\mathbf{A}(t-t_0)}\mathbf{w}_1(0; \theta^{t_0}(\nu), 0) + \int_{t_0}^{t} e^{\mathbf{A}(t-s)}\left(\mathbb{I}_n - (\mathbf{V}_E)_R((\mathbf{V}_E)_L)^{\mathrm{T}}\right)\mathbf{g}(0, \theta^s(\nu))ds.$$

If we further assume that the spectrum of $\mathbf{A}$ is real negative and the coefficients are uniformly bounded, the limit $t_0 \to -\infty$ gives us an unique globally bounded asymptotic expression for $\mathbf{w}_1$,

$$\mathbf{w}_1(0; \theta^t(\nu), 0) = \int_{-\infty}^{t} e^{\mathbf{A}(t-s)}\left(\mathbb{I}_n - (\mathbf{V}_E)_R((\mathbf{V}_E)_L)^{\mathrm{T}}\right)\mathbf{g}(0, \theta^s(\nu))ds. \tag{B11}$$

Here $(\mathbf{V}_E)_R((\mathbf{V}_E)_L)^{\mathrm{T}}$ is the oblique projection operator whose range is the slow subspace E.

Expressing this operator as $\mathbf{P}_E$ and using the linear change of coordinates in eq. (B3), we arrive at the final expression for the leading order SSM parametrization in modal coordinates,

$$\mathbf{h}_1(0; \theta^t(\nu), 0) = ((\mathbf{V}_F)_L)^{\mathrm{T}}\int_{-\infty}^{t} e^{\mathbf{A}(t-s)}(\mathbb{I}_n - \mathbf{P}_E)\mathbf{g}(0, \theta^s(\nu))ds. \tag{B12}$$

Note that eq. (B11) does not require the calculation of the fast eigenvectors and only depends on their slow counterparts. It also directly gives the result in the physical coordinates $\mathbf{x}$, unlike eq. (16) in [6]. In practice, we will use eq. (B11) to estimate the leading-order contribution of the random SSM parametrization. We also expect the convolution operation to be dominated by a small finite set of slowly decaying modes outside E when estimating the action of $e^{\mathbf{A}(t-s)}$ on the projected component of the forcing along the fast spectral subspace F.

Since we have chosen a graph-style parametrization for the SSM that projected the external forcing onto the slow dynamics, we expect the complementary projection of the forcing to appear in the SSM parametrization to take into account any shifts in the SSM away from the autonomous equilibrium. Note that $\mathbb{I}_n - \mathbf{P}_E$ is precisely the oblique projection operator that projects the forcing onto the remaining (n−d) fast linear modes. This suggests that if the forcing vector is nearly tangent to the slow subspace E and we are only interested in predictions for slow observables, we can ignore this leading-order contribution as well. In fact, in both data- and equation-driven SSM computations, this term is ignored due to these reasons without a significant loss of prediction accuracy (see [2,1, 4] and [3]).

The smoothness assumption in time for the parametrization is reasonable as we can always numerically view continuous random signals as smooth interpolations of the dataset representing them. For all the forcing vectors and forcing magnitudes in our examples (see Section 3 of [6]), we found the leading-order contribution to be negligible. This indicates that the forcing vectors were nearly tangent to the slow subspace E, in which case one can, in fact, ignore this contribution and still obtain accurate predictions.

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