Modelling Rail Carrier Assignment and Relocation in Multimodal Pod System

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Abstract

A multimodal pod system features modular autonomous vehicles consisting of detachable transport units mounted on mode-specific carriers. Within this system, transport units arriving at the railway network need to be assigned to available rail carriers to continue their journey. For better alignment between carrier availability and transport demand, rail carriers are allowed to relocate empty within the network. This work presents a mathematical optimisation model for the associated rail carrier assignment and relocation problem. The model's performance is demonstrated through a small case study featuring five transport units, which highlights the impact of the number of rail carriers and their initial distribution in the railway network on the number of handled transport units and total relocation time. Specifically, increasing the number of rail carriers from two to three resulted in an average of 1.25 additional transport units to be assigned, with an average 55% increase in relocation time. Furthermore, varying the initial distribution of rail carriers results in a difference of up to two transport units handled, while relocation times varied by as much as 120% of the average relocation time. To enable a more general assessment, future work will apply the model to a larger and more realistic case study.

Keywords: modular vehicles, pods, matching, fleet circulation, optimisation

1. Introduction

In the multimodal pod system considered in the Pods4Rail[1] project, freight and/or passengers are transported by modular autonomous vehicles, i.e., pods. A pod consists of a transport unit mounted on a carrier, as shown in Figure 1. The transport unit is detachable from the carrier, with the carrier being specific to a transport mode, e.g., road or rail.

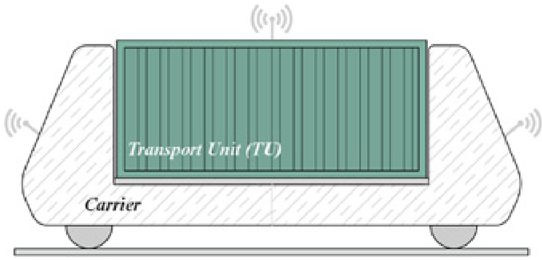

Figure 1: Illustration of a pod (3).

Transport units enter and exit the railway network at stations. To move the transport units from their pickup station to their delivery station, they must be assigned a rail carrier. The assignment of rail carriers to transport units is a challenge due to system constraints such as the limited number of rail carriers available, the pickup and delivery time windows of the transport units, and the railway network capacity. To accommodate the handling of transport units, rail carriers can be relocated, i.e., travel empty, between stations.

So far, the rail carrier assignment and relocation problem as such has been hardly considered in the railway-related literature. To the best of our knowledge, only one study directly addresses this problem. Liao et al. (3) consider a greedy heuristic approach in which assignments are made based on the earliest due date of the transport units and the earliest arrival time of the rail carriers. This approach, however, provides no guarantee in terms of the number of transport unit handled or the rail carrier relocation effort required.

In contrast, the related one-way car-sharing with relocation problem has been more extensively studied. In that context, vehicles are assigned to one-way trip requests while minimising relocation costs. For the modelling of this problem, research has explored (rolling horizon) optimisation, e.g. (4), (discrete-event) simulation, e.g. (1), or a combination of the two, e.g. (2). This car-sharing problem variant, however, differs significantly from the rail carrier problem, particularly due to the operational constraints inherent to railway systems, such as routing and safe separation on the network.

[1] `https://pods4rail.eu/`

In this work, we propose a mathematical optimisation model based on a multi-objective mixed integer linear programming (MILP) formulation to address the rail carrier assignment and relocation problem at the network level. Maximising the number of handled transport units with minimum relocation, the model assigns rail carriers to transport units, while accounting for constraints of the railway network. The model is demonstrated using a small case study, in which we briefly analyse the impact of the number of available rail carriers and their distribution in the network.

The paper is organised as follows. In Section 2.1, the rail carrier assignment and relocation problem is further specified, conceptually and mathematically. In Section 3, the mathematical model is demonstrated on a small case study. Section 4 concludes the paper.

2. Problem formulation

In this section, we further specify the rail carrier assignment and relocation problem. First, Section 2.1 provides a conceptual description of the problem and corresponding assumptions. Second, Section 2.2 presents a mathematical model formulation to formally describe the problem.

2.1. Problem description and assumptions

In the rail carrier assignment and relocation problem, transport units are assigned rail carriers for transportation within the railway network. To support effective and efficient service, rail carriers may be relocated within the network to better align carrier availability with the transport demand.

We consider the railway network as a set of stations connected by railway lines with limited capacity, modelled as the maximum number of rail carriers that may enter or exit a line per time unit. Accordingly, time is modelled as discrete, with time units corresponding to the minimum separation between rail carriers. We assume fixed travel times between stations, and a fixed maximum capacity in terms of rail carriers for each station.

We consider a fixed number of homogeneous rail carriers available in the network, which are initially located at a station. Each rail carrier can handle one transport unit at a time and may travel without transport unit, i.e., be relocated, when enabling the handling of a transport unit.

For the formulation of the rail carrier assignment and relocation problem, we define transport requests to represent transport units and their characteristics, i.e., pickup and delivery stations and associated time windows. To support operational flexibility, we allow transport requests to be rejected if they cannot be handled. If handled, a transport request is assigned a single rail carrier that transports the corresponding unit from its pickup to its delivery station. The loading and unloading of a transport unit takes a non-negligible amount of time.

In addition to transport requests, we include relocation requests to allow empty rail carriers to move between stations. Relocations helps to balance carrier availability with the demand and may occur either before the first assigned transport request or between consecutive transport requests. Relocation requests are derived from the combination of transport requests and the initial locations of the rail carriers.

2.2. Mathematical model

In this section, we propose a multi-objective MILP-based optimisation model for the rail carrier assignment and relocation problem. The three main components of the model are the decision variables, the objective function and the constraints.

2.2.1. Sets, parameters and variables

The sets, parameters and variables of the model are listed in Table 1. The sets represent collections of elements used in the model notation, the parameters define fixed input values, and the variables correspond to decisions made within the optimisation model.

The main sets are the set of requests R, the set of rail carriers C, the set of stations S, and the set of time units T. The set of requests is partitioned into two subsets: the set of transport requests $R^t \subset R$ and the set of relocation requests $R^l \subset R$.

Most parameters relate to the requests. $s_r^p \in S_r$ and $s_r^d \in S_r$ indicate the pickup and delivery station of request $r \in R$. $t_r^{p,\text{early}} \in T$ and $t_r^{p,\text{late}} \in T$ indicate the earliest and latest pickup time of request $r \in R$, and $t_r^{d,\text{early}} \in T$ and $t_r^{d,\text{late}} \in T$ indicate the earliest and latest delivery

Table 1: Sets, parameters and variables for the MILP model formulation.

Sets	
R	set of requests
$R^t \subset R$	set of transport requests
$R^l \subset R$	set of relocation requests
C	set of rail carriers
S	set of stations
T	set of time units
Parameters	
$s_r^p \in S$	pickup station of request $r \in R$
$s_r^d \in S$	delivery station of request $r \in R$
$t_r^{p,\text{early}} \in T$	earliest pickup time of request $r \in R$
$t_r^{p,\text{late}} \in T$	latest pickup time of transport request $r \in R^t$
$t_r^{d,\text{early}} \in T$	earliest delivery time of transport request $r \in R^t$
$t_r^{d,\text{late}} \in T$	latest delivery time of request $r \in R$
$\tau_r^{\text{load}} \in T$	loading time of request $r \in R$ ($= 0$ if $r \in R^l$)
$\tau_r^{\text{unload}} \in T$	unloading time of request $r \in R$ ($= 0$ if $r \in R^l$)
$\tau_r \in T$	minimum travel time of request $r \in R$ from pickup to delivery station
$\tau_{s,s'} \in T$	travel time from station $s \in S$ to adjacent station $s' \in S$
$l_{c,s}^{\text{init}} \in \{0,1\}$	$= 1$ if carrier $c \in C$ is initially located at station $s \in S$
$n_s \in \mathbb{N}$	carrier capacity at station $s \in S$
$M \in \mathbb{R}_+$	a large constant
$w_t, w_l \in \mathbb{R}_+$	weight for transport/relocation requests handling in objective function
$w_r \in \mathbb{R}_+$	handling priority weight of request $r \in R$
Variables	
$t_r^p \in T$	pickup time of request $r \in R$
$t_r^d \in T$	delivery time of request $r \in R$
$x_{r,c} \in \{0,1\}$	$= 1$ if request $r \in R$ is assigned to carrier $c \in C$
$y_r \in \{0,1\}$	$= 1$ if request $r \in R$ is handled
$z_{c,s,s',t}^c \in \{0,1\}$	$= 1$ if carrier $c \in C$ is departing from station $s \in S$ to station $s' \in S$ at time $t \in T$
$z_{r,s,s',t}^r \in \{0,1\}$	$= 1$ if request $r \in R$ is departing from station $s \in S$ to station $s' \in S$ at time $t \in T$
$o_{r,r',c} \in \{0,1\}$	$= 1$ if request $r \in R$ is handled before $r' \in R$ by carrier $c \in C$
$o_{r,r',c}^d \in \{0,1\}$	$= 1$ if request $r \in R$ is handled directly before $r' \in R$ by carrier $c \in C$
$b_{r,r',r'',c} \in \{0,1\}$	$= 1$ if request $r'' \in R$ is handled between requests $r, r' \in R$ by carrier $c \in C$
$l_{c,s} \in \{0,1\}$	$= 1$ if carrier $c \in C$ is located at station $s \in S$ after handling last request

time of request $r \in R$. Note that $t_r^{p,\text{late}}$ and $t_r^{d,\text{early}}$ are only defined for transport requests $r \in R^t$. $\tau_r^{\text{load}} \in T$ and $\tau_r^{\text{unload}} \in T$ represent the loading and unloading time for transport request $r \in R^t$. $\tau_r \in T$ indicates the minimum time to travel between the request's pickup and delivery station.

Other parameters relate to various things. $\tau_{s,s'} \in T$ indicates the travel time from station $s \in S$ to adjacent station $s' \in S$. $l^{\text{init}}_{c,s} \in \{0,1\}$ indicates whether or not station $s \in S$ is the initial location of rail carrier $c \in C$. $n_s \in \mathbb{N}$ indicates the capacity of stations $s \in S$. Finally, we define $M \in \mathbb{R}_+$ to be a large constant, e.g., $|T|$, and weights w_t, w_l and w_r for the objective function. w_t and w_l are the weights for the two different objectives in the objective function, related to the handling of transport and relocation requests, respectively. w_r is the priority weight of transport request $r \in R^t$.

The model variables are mostly binary variables. The exception is formed by the timing variables $t^p_r \in T$ and $t^d_r \in T$ which determine the pickup and delivery time of request $r \in R$. The main binary variables are the assignment decision variables $x_{c,r},\ r \in R$ indicating whether request $r \in R$ is assigned to rail carrier $c \in C$, and the auxiliary handling variables $y_r,\ r \in R$ indicating whether request $r \in R$ is handled. Flow variables $z^c_{c,s,s',t} \in \{0,1\}$ and $z^r_{r,s,s',t} \in \{0,1\}$ indicate whether rail carrier $c \in C$ or request $r \in R$ starts travelling from station $s \in S$ to stations $s' \in S$ at time $t \in T$. Ordering variables $o_{r,r',c} \in \{0,1\}$ and $o^d_{r,r',c}$ are defined to indicate whether request $r \in R$ is handled (directly) before request $r' \in R$ by rail carrier $c \in C$. Auxiliary variables $b_{r,r',r'',c} \in \{0,1\}$ indicate whether request $r'' \in R$ is handled in between requests $r, r' \in R$ by rail carrier $c \in C$. Finally, $l_{c,s} \in \{0,1\}$ indicates whether rail carrier $c \in C$ is located at station $s \in S$ after handling its last request.

2.2.2. Objective function

The primary objective of the rail carrier assignment and relocation problem is to handle as many transport requests as possible. As a secondary objective, the empty travelling of rail carriers should be limited to reduce unnecessary use of network capacity. The two objectives are captured in the objective function (Equation (1)) as the maximisation of the number of handled transport requests and the minimisation of the travel time of relocation requests. The former is included as the weighted sum of non-handled transport requests $(1-y_r)$, in which the penalty weights w_r can, for example, be interpreted as priority factors for passenger over freight requests. The latter is included as a sum of handled relocation requests $(r \in R_l)$, weighted by their minimum travel time (τ_r). Weights w_t and w_l are included to represent the importance of the two respective objectives. To allow for relocation whenever it leads to the handling of at least one additional transport request, $w_t >> w_l$.

$$\text{minimise } w_t \sum_{r\in R^t} w_r(1-y_r) + w_l \sum_{r\in R^l} \tau_r y_r \tag{1}$$

2.2.3. Constraints

The constraints of the rail carrier assignment and relocation model are given by Equations (2) to (32). Constraints 2 ensures that each request is assigned to exactly one rail carrier, if the request is handled.

$$\sum_{c\in C} x_{r,c} = y_r \qquad \forall r \in R \tag{2}$$

The remaining constraints are categorised into the following groups: time window constraints, handling order constraints, traffic flow constraints, and network capacity constraints.

Time window constraints. Constraints (3), (4), (5) and (6) ensure that the pickup and delivery times of handled requests are in line with their time windows. For transport requests (Constraints (3) and (4)), this means between earliest and latest pickup/delivery times. For relocation requests (5) and (6), this means after earliest pickup and before latest delivery time.

$$t_r^{p,\text{early}} y_r \leq t_r^p \leq t_r^{p,\text{late}} y_r \qquad \forall r \in R^t \tag{3}$$

$$t_r^{d,\text{early}} y_r \leq t_r^d \leq t_r^{d,\text{late}} y_r \qquad \forall r \in R^t \tag{4}$$

$$t_r^{p,\text{early}} y_r \leq t_r^p \qquad \forall r \in R^l \tag{5}$$

$$t_r^d \leq t_r^{d,\text{late}} y_r \qquad \forall r \in R^l \tag{6}$$

Handling order constraints. Through Constraints (7), (8) and (9), it is ensured that requests that are handled by the same rail carrier are ordered. That is, the pickup time of the request handled later is after the delivery time of the request handled earlier.

$$t_r^d < t_{r'}^p + M\left(1 - o_{r,r',c}\right) \qquad \forall r,r' \in R,\ c \in C \tag{7}$$

$$t_{r'}^d < t_r^p + M\left(3 - (1 - o_{r,r',c}) - x_{r,c} - x_{r',c}\right) \qquad \forall r,r' \in R,\ c \in C \tag{8}$$

$$o_{r,r',c} \leq 0.5 x_{r,c} + 0.5 x_{r',c} \qquad \forall r,r' \in R,\ c \in C \tag{9}$$

With the general order per pair of requests handled by the same rail carrier established, the direct order of requests per rail carrier can be derived. As Constraints (10) to (16) describe, requests are directly ordered if and only if they are ordered without any requests in between.

$$t^p_{r''} > t^d_r - M\,(1 - b_{r,r',r'',c}) \qquad \forall r, r', r'' \in R,\ c \in C \qquad (10)$$

$$t^d_{r''} < t^p_{r'} + M\,(1 - b_{r,r',r'',c}) \qquad \forall r, r', r'' \in R,\ c \in C \qquad (11)$$

$$o^d_{r,r',c} + b_{r,r',r'',c} \leq 1 \qquad \forall r, r', r'' \in R,\ c \in C \qquad (12)$$

$$o^d_{r,r',c} \leq o_{r,r',c} \qquad \forall r, r' \in R,\ c \in C \qquad (13)$$

$$o^d_{r,r',c} \geq o_{r,r',c} - \sum_{r'' \in R} b_{r,r',r'',c} \qquad \forall r, r' \in R,\ c \in C \qquad (14)$$

$$\sum_{r' \in R} o^d_{r,r',c} \leq 1 \qquad \forall r \in R,\ c \in C \qquad (15)$$

$$\sum_{r' \in R} o^d_{r',r,c} \leq 1 \qquad \forall r \in R,\ c \in C \qquad (16)$$

Traffic flow constraints. Constraints (17) to (22) ensure the alignment of pickup and delivery stations along a rail carrier route. For requests that are directly ordered on a rail carrier, Constraints (17) and (18) ensure that the delivery station of the first request is the pickup station of the second one. Constraints (19) and (20) ensure that the pickup station of the first request handled by a rail carrier is its initial location. Similarly, Constraints (21) and (22) ensure that the delivery station of a rail carrier's last request is its final location.

$$s^p_{r'} \geq s^d_r - M\,(1 - o^d_{r,r',c}) \qquad \forall r, r' \in R,\ c \in C \qquad (17)$$

$$s^p_{r'} \leq s^d_r + M\,(1 - o^d_{r,r',c}) \qquad \forall r, r' \in R,\ c \in C \qquad (18)$$

$$s^p_r \geq \sum_{s \in S} l^{\text{init}}_{c,s} - M\,(1 - x_{r,c} + \sum_{r' \in R} o^d_{r',r,c}) \qquad \forall r \in R,\ c \in C \qquad (19)$$

$$s^p_r \leq \sum_{s \in S} l^{\text{init}}_{c,s} + M\,(1 - x_{r,c} + \sum_{r' \in R} o^d_{r',r,c}) \qquad \forall r \in R,\ c \in C \qquad (20)$$

$$s^d_r \geq \sum_{s \in S} l_{c,s} - M\,(1 - x_{r,c} + \sum_{r' \in R} o^d_{r,r',c}) \qquad \forall r \in R,\ c \in C \qquad (21)$$

$$s^d_r \leq \sum_{s \in S} l_{c,s} - M\,(1 - x_{r,c} + \sum_{r' \in R} o^d_{r,r',c}) \qquad \forall r \in R,\ c \in C \qquad (22)$$

Complementary to the carrier flow in terms of stations, Constraints (23) to (26) address the flow of requests in terms of times. Constraints (23) ensure that the departure times at subsequent stations along the route of a request are at least the travelling time between the stations apart. Constraints (25) and (26) ensure the alignment of the pickup time with the departure time of the pickup station and the alignment of the delivery time with the departure towards the delivery station.

$$\sum_{t\in T}\sum_{s'\in S}(t+\tau_{s',s})z^r_{r,s',s,t} \le \sum_{t\in T}(t\ z^r_{r,s',s,t}) \qquad \forall r\in R,\ s\in S\backslash\{s^p_r,s^d_r\} \qquad (23)$$

$$\sum_{t\in T}\sum_{s'\in S} z^r_{r,s,s',t} = \sum_{t\in T}\sum_{s'\in S} z^r_{r,s,s',t} \qquad \forall r\in R,\ s\in S\backslash\{s^p_r,s^d_r\} \qquad (24)$$

$$t^p_r + \tau^{\text{load}}_r y_r \le \sum_{t\in T}\sum_{s\in S}(t\ z_{r,s^p_r,s,t}) \qquad \forall r\in R \qquad (25)$$

$$t^d_r - \tau^{\text{unload}}_r y_r \ge \sum_{t\in T}\sum_{s\in S}(t\ z_{r,s,s^d_r,t}) \qquad \forall r\in R \qquad (26)$$

As the models features timing variables for both rail carriers and requests, these are aligned by Constraints (27) and (28).

$$z^c_{c,s,s',t} \ge z^r_{r,s,s',t} - (1-x_{r,c}) \qquad \forall r\in R,\ c\in C,\ s,s'\in S,\ t\in T \qquad (27)$$

$$z^c_{c,s,s',t} \le z^r_{r,s,s',t} + (1-x_{r,c}) \qquad \forall r\in R,\ c\in C,\ s,s'\in S,\ t\in T \qquad (28)$$

Constraints (29) ensure that the number of rail carriers starting from or travelling to a certain station equals the number of rail carriers travelling from or ending at that station.

$$\sum_{c\in C} l^{\text{init}}_{c,s} + \sum_{c\in C}\sum_{s',s\in S}\sum_{t\in T} z^c_{c,s',s,t} = \sum_{c\in C}\sum_{s,s'\in S}\sum_{t\in T} z^c_{c,s,s't} + \sum_{c\in C} l_{c,s} \qquad \forall s\in S \qquad (29)$$

Network capacity constraints. Line capacity is addressed by Constraints (30) and (31): at most one rail carrier can leave or enter the same station within a time unit. Station capacity is addressed by Constraints (32): the number of rail carriers that started at or have entered but not left a station does not exceed that station's capacity at any time unit.

$$\sum_{c\in C}\sum_{s'\in S} z^c_{c,s,s',t} \le 1 \qquad \forall s\in S,\ t\in T \qquad (30)$$

$$\sum_{c\in C}\sum_{s'\in S} z^c_{c,s',s,t} \le 1 \qquad \forall s\in S,\ t\in T \qquad (31)$$

$$\sum_{c\in C} l^{\text{init}}_{c,s} + \sum_{r\in R}\sum_{s'\in S}\sum_{t'\le t} z^c_{c,s',s,t} - \sum_{r\in R}\sum_{s''\in S}\sum_{t'\le t} z^c_{c,s,s'',t} \le n_s \qquad \forall s\in S,\ t\in T \qquad (32)$$

3. Application to a small-scale case study

To illustrate the proposed model for the rail carrier assignment and relocation problem, we analyse its performance using a small-scale case study. We consider a railway network consisting of four stations which are interconnected with five double track lines with fixed travel times as shown in Figure 2. We assume a station capacity of three rail carriers for each station. In our analysis later on, we vary the number of rail carriers available in the network, along with their initial locations.

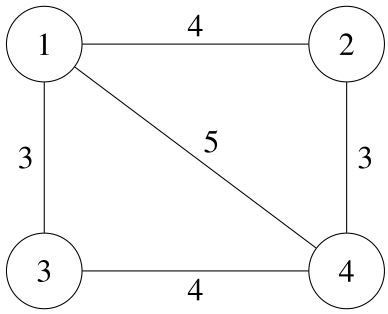

Figure 2: Schematic representation of small case study network.

A period of 30 time units is considered, during which five transport requests of equal priority are scheduled. The transport requests and their pickup and delivery characteristics are listed in Table 2. Given the transport requests, a first set of relocation requests is generated, i.e. relocation requests connecting transport requests that could be handled subsequently. Based on the time windows and stations, relocation requests are generated to connect transport requests 4 and 5 with transport requests 2 and 3. These four 'intermediate' relocation requests and their characteristics are added to Table 2 as fixed requests. Additional 'initial' relocation requests are defined per analysis scenario as they depend on the varying initial carrier locations.

As said, we define a set of scenarios by varying the number of rail carriers (two or three), and their initial locations at a station. With two rail carriers, there are ten possibilities to distribute them over the four station, including the four options to have the same initial location. In the same way, there are twenty possibilities to distribute three rail carriers over the four stations. An overview of the possible scenarios is given in Table A1, together with the results in terms of handled transport requests and total empty travel times.

Table 2: Fixed requests for test case study.

Request	Pickup station	Delivery station	Pickup time window	Delivery time window
Transport requests				
1	4	2	[11, 18]	[14, 21]
2	1	3	[12, 18]	[15, 21]
3	3	2	[14, 19]	[21, 26]
4	1	4	[1, 5]	[8, 13]
5	3	4	[1, 7]	[5, 11]
Intermediate relocation requests				
6	4	1	[8, →)	(←, 18]
7	4	3	[8, →)	(←, 19]
8	4	1	[5, →)	(←, 18]
9	4	3	[5, →)	(←, 19]

With two rail carriers, two to four transport requests (3.4 on average) can be handled with three to 11 relocation time units (6.5 on average). The exact number as well as which of the requests that are handled depends on the initial carrier locations, though request 1 is always handled. In five scenarios, four requests (1, 3, 4 and 5) are handled, with three to 11 time units of relocation. Requests 4 and 5 are handled in parallel, after which one rail carrier can directly pick up request 1. For the second rail carrier, request 3 is preferred over request 2 due to its relative proximity to station 4 and its later time windows. In four scenarios, three requests are handled, always including requests 1 and 2, with four or eight relocation time units. Request 4 or 5 is handled before request 1 depending on the initial carrier location. Only if there is a rail carrier at station 1, request 4 is handled. In the last scenario, in which both rail carriers are initially located at station 2, only two requests are handled. This is because there is no transport request with pickup station 2, in combination with the limited time. We note that for all scenarios with two rail carriers, a computation time of around two seconds can be reported.

With three rail carriers, three to five transport requests (4.65 on average) are handled with three to 15 relocation time units (10.05 on average). Requests 1, 2, and 3 are handled in all scenarios. In 14 out of 20 scenarios, all transport requests can be handled - with relocation time units varying between three and 15. Of the remaining six scenarios, five involve handling four requests and one involves handling three requests. In the scenarios where four requests are handled, either request 4 or 5 is rejected, depending on whether a rail carrier is initially located at station 1. The scenario in which only three requests are handled is the one with all rail carrier initially located at station 2, where no transport requests are to be picked up. On that note, when all rail carriers are initially located at either station 1 or station 3, where two transport requests are to be picked up at different times, all requests are handled. With all rail carriers initial located at station 4 with one scheduled pickup at a later time, four request are handled. It is worth noting that for some scenarios the computation time is significant. While typical computation times lie around five seconds, for several scenarios require between 15 and 20 seconds, and one scenario even takes around 40 seconds.

Based on the results of these scenarios, we conclude that both the number of rail carriers and the initial location of the rail carriers are key factors for the model objectives of, first, maximising the number of handled transport requests and, second, minimising the total relocation time. As shown in Figure 3, there is no clear relation between the two objectives. For the two-carrier scenarios, the relation is slightly positive, while for the three-carrier scenarios, the trend is slightly negative.

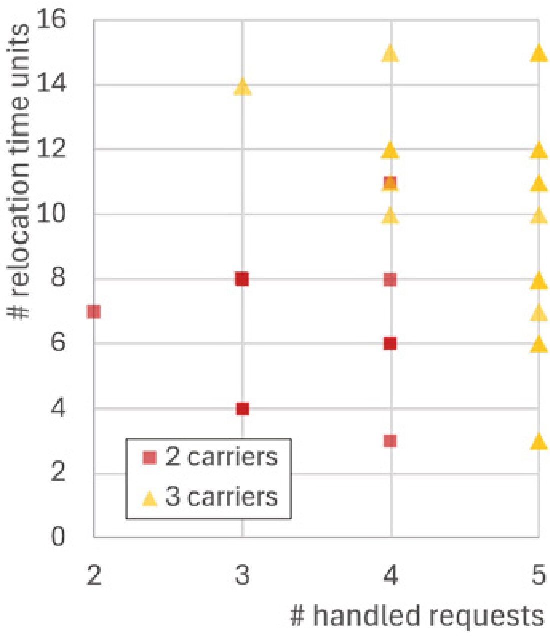

Figure 3: Total relocation time versus number of handled transport requests.

To conclude, we acknowledge the potentially strong effects of the considered case study, e.g., by the small network and short time period. To obtain more general results, application of the model to a variety of realistic case studies should be considered.

4. Conclusions

In this paper, we proposed a mathematical optimisation model for the rail carrier assignment and relocation problem in the context of a multimodal pod system. The model assigns rail carriers to transport units with specified pickup and delivery stations and time windows, while accounting for constraints of the railway network. The objective is to maximise the number of handled transport units while minimising the total relocation time of rail carriers.

The model is demonstrated using a small case study involving a four-station railway network and five transport units. A preliminary analysis showed the significant impact of the number of available rail carriers and their initial distribution within the network on both the (number of) handled transport units and the total relocation time. While these findings illustrate the model's capabilities and provide initial insights into its behaviour, they are limited by the simplified nature of the consider case study. To asses the model's practical applicability, it is essential to evaluate it on more realistic scenarios.

In future steps, we will explore the use of soft time windows, in combination with the possibility to form platoons of rail carriers. Furthermore, the model will be applied to a more practical case study in terms of both network size and time horizon. Given the already observed increase in computational effort, this will require the adoption of a more advanced solution method.

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Literature

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A. Detailed case study results

Table A1: Handled transport requests and total empty travel time per scenario.

Initial carrier locations (stations)	**Handled transport requests** (number: list)	**Total empty travel time** (time units)
2 carriers	*3.4*	*6.5*
1, 1	4: 1, 3, 4, 5	6
1, 2	3: 1, 2, 4	4
1, 3	4: 1, 3, 4, 5	3
1, 4	4: 1, 3, 4, 5	8
2, 2	2: 1, 2	7
2, 3	3: 1, 2, 5	4
2, 4	3: 1, 2, 5	8
3, 3	4: 1, 3, 4, 5	6
3, 4	4: 1, 3, 4, 5	11
4, 4	3: 1, 2, 5	8
3 carriers	*4.65*	*10.05*
1, 1, 1	5: 1, 2, 3, 4, 5	6
1, 1, 2	5: 1, 2, 3, 4, 5	11
1, 1, 3	5: 1, 2, 3, 4, 5	3
1, 1, 4	5: 1, 2, 3, 4, 5	8
1, 2, 2	5: 1, 2, 3, 4	11
1, 2, 3	5: 1, 2, 3, 4, 5	12
1, 2, 4	5: 1, 2, 3, 4, 5	3
1, 3, 3	5: 1, 2, 3, 4, 5	8
1, 3, 4	5: 1, 2, 3, 4, 5	12
1, 4, 4	5: 1, 2, 3, 4, 5	14
2, 2, 2	3: 1, 2, 3	10
2, 2, 3	4: 1, 2, 3, 5	15
2, 2, 4	4: 1, 2, 3, 5	10
2, 3, 4	5: 1, 2, 3, 4, 5	15
2, 4, 4	4: 1, 2, 3, 5	12
3, 3, 3	5: 1, 2, 3, 4, 5	6
3, 3, 4	5: 1, 2, 3, 4, 5	11
3, 4, 4	5: 1, 2, 3, 4, 5	15
4, 4, 4	4: 1, 2, 3, 5	12