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# A practical approach to obtain the soil freezing characteristic curve and the freezing/melting point of a soil-water system

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## ABSTRACT

Knowing that extensive field tests and laboratory tests are time-consuming and expensive, this paper describes a practical approach to obtain crucial properties of frozen soil such as the soil freezing characteristic curve (SFCC), the freezing/melting point of a soil-water system and its hydraulic conductivity by means of limited input data. Different models and empirical equations are combined to provide a closed formulation which can be used in computer simulations to account for moisture migration in partially frozen soils. Input data such as grain size distribution and dry bulk density suffice to obtain the aforementioned properties. Further consideration of the pressure dependence of the freezing/melting temperature of water/ice even allows accounting for the phase change point depression and thus the phenomena of pressure melting.

The model is appropriate not just to represent qualitatively the SFCC of different soil types, but also to provide conformity between the model prediction and measured data of many soil types having a log-normal grain size distribution. This user-friendly approach is used as default setting of a newly implemented user-defined soil model for frozen and unfrozen soil in the geotechnical finite element code PLAXIS 2D.

## RÉSUMÉ

Conscient que les tests en laboratoire et sur le terrain sont chronophages et chers, cet article décrit une approche pratique pour obtenir des propriétés cruciales du sol, telles que la courbe caractéristique de gèlement des sols (CCGS), le point de gel/dégel d'un système eau/sol et sa conductivité hydraulique au moyen de données d'entrée limitées. Différents modèles et équations empiriques sont combinés pour fournir une formulation close qui peut être utilisée dans des simulations numériques pour le transfert hydrique dans des sols partiellement saturés. Les données d'entrée telles que la granulométrie et la masse volumique sèche suffisent pour obtenir les propriétés citées en amont. La considération ultérieure de la dépendance de la pression vis-à-vis de la température de gel/fusion de l'eau/glace permet même de prendre en compte le déplacement du point de changement de phase et donc la pression de fusion.

Ce modèle est approprié non seulement pour représenter qualitativement la CCGS de différents types de sols, mais aussi pour fournir de la conformité entre les données mesurées et prédites pour de nombreux sols, tant qu'ils possèdent une granulométrie suivant une distribution log-normale. Cette approche est utilisée par défaut dans le cadre d'un nouveau modèle d'utilisateur pour le gel et le dégel des sols dans le code aux éléments finis PLAXIS 2D.

## 1 INTRODUCTION

Over the past years, geotechnical engineers are more and more faced to deal with projects involving frozen ground due to the increase of engineering activities in cold regions, the use of artificial ground freezing and consequences of global warming. However, the analysis of frozen soil and the transition from frozen to unfrozen behaviour, vice versa, requires specific properties to be taken into account, which are not determined in standard site investigation and soil lab testing campaigns. This brings the need for a simplified method to determine such properties based on data that are commonly available, such as particle size distribution. The idea is, as an initial estimate, to correlate these data to the soil freezing characteristic curve and the hydraulic properties of partially frozen soils.

## 2 METHODOLOGY

### 2.1 Particle size distribution

The particle size distribution (PSD) is the physical data that is mostly available for any soil of interest. PSD information can be of value in making qualitative judgements of a number of physical properties and in providing initial rough estimates of the engineering properties of the soil such as permeability and strength. It is always useful to quantify the size and distribution of grains in a type of soil. This paper shows how PSD information can be used to estimate the soil freezing characteristic curve (SFCC) and the hydraulic conductivity of frozen soil. Both properties are relevant in frozen ground engineering.

Based on the U.S.D.A. classification scheme of soil, where equivalent diameters are given in Table 1, Shirazi & Boersma (1984) produced a texture diagram. The diagram is based on the assumption that the PSD in soil

is approximately log-normal. This assumption allows us to represent any combination of clay, silt and sand by a geometric (or log) mean particle diameter  $d_g$  and a geometric standard deviation  $\sigma_g$ .

Table 1. U.S.D.A. classification scheme

Clay			$d$	$<$	0.002	mm
Silt	0.002	$\leq$	$d$	$<$	0.05	mm
Sand	0.05	$\leq$	$d$	$<$	2.0	mm

The geometric mean particle diameter  $d_g$  and the geometric standard deviation  $\sigma_g$  are defined as follows:

$$d_g = \exp\left(\sum_{i=1}^3 m_i \ln d_i\right) \quad [1]$$

$$\sigma_g = \exp\left(\sqrt{\sum_{i=1}^3 m_i (\ln d_i)^2 - \left(\sum_{i=1}^3 m_i \ln d_i\right)^2}\right) \quad [2]$$

In Eq. [1] and [2]  $m_i$  is the mass fraction of textural class  $i$ , and  $d_i$  is the arithmetic mean diameter of class  $i$ . The three textural classes are provided in Table 1 and are namely clay, silt and sand. The arithmetic means are given by  $d_{\text{clay}} = 0.001$  mm,  $d_{\text{silt}} = 0.026$  mm and  $d_{\text{sand}} = 1.025$  mm.

## 2.2 Soil freezing characteristic curve

Not all free pore water in a soil-water system freezes at the same temperature. According to Rempel et al. (2004), Wettlaufer & Worster (2006), Zhou (2014), two main mechanisms allow water to remain in its unfrozen state at temperatures below the bulk freezing point. These two mechanisms are namely the curvature-induced premelting and the interfacial premelting mechanism (Figure 1). The former one is a result of the existence of surface tension of the water meniscus formed between soil particles and is very similar to the capillary suction by bonding grains together. On the contrary, the latter one is a result of repulsion forces between ice and solid grains. These forces act as disjoining pressure tending to widen the gap by sucking in more water.

The amount of unfrozen water remaining in frozen soil with respect to freezing temperature can be seen as a soil property and the soil freezing characteristic curve (SFCC) is used to describe this relationship. Due to the analogy of the freezing characteristics and water retention characteristics in unsaturated soils (Black & Tice, 1989; Spaans & Baker, 1996; Coussy, 2005; Ma et al., 2015), models like van Genuchten (1980) and Fredlund & Xing (1994) have been employed to represent the freezing characteristic function (e.g. in Nishimura et al., 2009; Azmatch et al., 2012). Some attempts have also been conducted to find an empirical equation to compute the

unfrozen water content (e.g. Tice et al., 1976). In this paper we choose to relate the volumetric unfrozen water content  $\theta_u$  to the temperature using an empirical formulation based on test results of Anderson & Tice (1972). The specific surface area (SSA), the density of water ( $\rho_w$ ), the one of unfrozen soil ( $\rho_b$ ), and the temperature ( $T$ ) are the only input parameters. Eq. [3] gives the empirical relationship.

$$\theta_{uw} = \frac{\rho_w}{\rho_b} \exp(0.2618 + 0.5519 \ln(\text{SSA}) + -1.4495(\text{SSA})^{-0.2640} \ln |T|) \quad [3]$$

with

$$|T| = T_{f,\text{bulk}} - T \quad [4]$$

$T_{f,\text{bulk}}$  refers to the bulk freezing point (= 273.16 K) and  $T$  to the actual temperature in Kelvin. Eq. [3] is only valid for  $T < T_{f,\text{bulk}}$ .

To use Eq. [3] properly,  $\theta_u$  may not exceed the volumetric water content of a fully saturated soil, which is equal with the porosity of the soil. This cut-off value is needed because the above equation is empirical and provides values of  $\theta_u$  bigger than  $\theta_{\text{sat}}$  at temperatures close to  $T_{f,\text{bulk}}$ . An important assumption is that the cut-off point can be seen as the freezing/thawing temperature  $T_f$  of a soil-water system where pore water pressure equals zero. In other words,  $T_f$  obtained from the equation proposed by Anderson & Tice (1972) is the soil-type dependent freezing/thawing temperature. In section 3.1 this statement is validated.

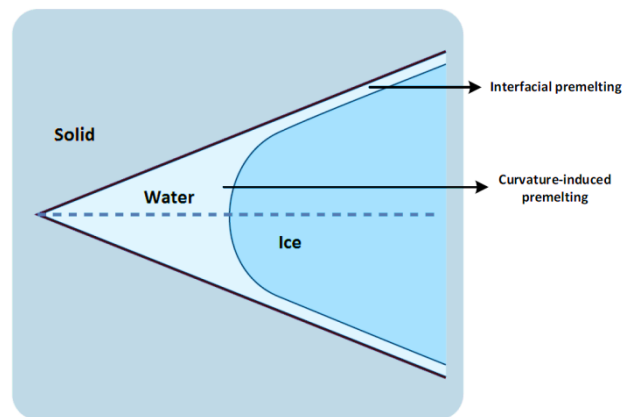


Figure 1. Curvature induced premelting and interfacial premelting during intrusion of ice into a wedge-shape wet preferential solid (after Wettlaufer & Worster, 2006 and Ghoreishian et al., 2016)

### 2.2.1 Specific surface area

The specific surface area (SSA) of a soil is defined as the sum of the surface area of soil particles per unit mass and is expressed in square meter per gram (m<sup>2</sup>/g). Many physical and chemical soil processes in soil are closely related to the SSA. Sepaskhah et al. (2010) uses a non-linear regression analysis to relate the geometric mean soil particle diameter  $d_g$  in mm ([1]) to the measured SSA in m<sup>2</sup>/g. This empirical power pedo-transfer function allows the estimation of the specific surface area as follows:

$$SSA = 3.89 \cdot d_g^{-0.905} \quad [5]$$

Petersen et al. (1996), show that the magnitude of the SSA of a soil depends largely on the amount of clay and type of clay minerals in the soil. The fact that the SSA differs largely between types of clay minerals cannot be taken into account using this textural information approach. Nevertheless, the proposed equation for SSA has also been examined in Fooladmand (2011) and was found to provide a good approximation of the specific surface area of soils.

### 2.3 Pressure dependence of the freezing/thawing temperature

Thermodynamic equilibrium of freezing soil can be described by the Clausius-Clapeyron equation (Henry, 2000). This equilibrium between liquid water and ice phases can be expressed as follows (Thomas et al., 2009):

$$\frac{p_{ice}}{\rho_{ice}} - \frac{p_w}{\rho_w} = -L \ln \frac{T}{T_f} \quad [6]$$

where  $p_w$  and  $p_{ice}$  indicate the pore water and ice pressure, respectively;  $\rho_w$  and  $\rho_{ice}$  the density of pore water and ice, respectively, and  $L$  is the latent heat of fusion of water.  $T$  represents the current temperature in Kelvin and  $T_f$  is the melting/freezing temperature of ice/water for a given soil and pressure. The process of water migration to the freezing zone due to a pressure gradient and temperature gradient is named cryogenic suction,  $s_c$ . This capillary action due to ice/water interface tension is derived in Eq. [7] (Thomas et al., 2009).

The phenomenon of pressure melting can already be described using the Clausius-Clapeyron equation; however, the freezing/melting temperature,  $T_f$ , by itself depends on pressure. The pressure dependence of the freezing point affects the amount of water kept unfrozen at negative temperatures. The relationship between unfrozen water content and pressure is important in studying the physical properties and mechanical behaviour of frozen soils under high pressure (Zhang et al., 1998).

$$\begin{aligned} s_c &= p_{ice} - p_w \\ &= \rho_{ice} \left( \frac{p_w}{\rho_w} - L \ln \frac{T}{T_f} \right) - p_w \\ &\approx -\rho_{ice} L \ln \frac{T}{T_f} \end{aligned} \quad [7]$$

Probably the most well-known and often quoted relation for the pressure dependence of the melting temperature is the empirical equation proposed by Simon & Glatzel (1929). This formulation cannot be used for falling melting curves or curves with maxima (Kechin, 1995). Thus, the application of this formulation to represent the pressure dependence of water freezing and/or ice melting is not appropriate. We therefore propose to use the melting-pressure equation for ice according to Wagner et al. (2011):

$$\frac{p_{melt}}{p_t} = 1 + \sum_{i=1}^3 a_i \left( 1 - \left( \frac{T}{T_t} \right)^{b_i} \right) \quad [8]$$

where  $T_t = 273.16$  K refers to the vapour-liquid-solid triple point temperature and  $p_t = 611.657$  Pa to the triple point pressure, respectively. The coefficients  $a_i$  and exponents  $b_i$  are given in Table 2.

Table 2. Coefficients  $a_i$  and  $b_i$  of the melting-pressure equation Wagner et al. (2011)

$i$	$a_i$	$b_i$
1	$0.119539337 * 10^7$	$0.300000 * 10^1$
2	$0.808183159 * 10^5$	$0.257500 * 10^2$
3	$0.333826860 * 10^4$	$0.103750 * 10^3$

The contribution of cryogenic suction and pore water pressure results in the ice pressure:

$$p_{ice} = s_c + p_w \quad [9]$$

By substituting Eq. [9] into [8] we obtain Eq. [10]:

$$\frac{s_c + p_w}{611.657 \text{ Pa}} = 1 + \sum_{i=1}^3 a_i \left( 1 - \left( \frac{T_f}{273.16 \text{ K}} \right)^{b_i} \right) \quad [10]$$

Eq. [10] relates the freezing/melting temperature to the cryogenic suction and the pore water pressure. Keeping in mind that the Clausius-Clapeyron equation [6] and the

melting-pressure equation [10] have to be in equilibrium for a given pore water pressure  $p_w$  and temperature  $T$ , the cryogenic suction  $s_c$  and the freezing/melting temperature  $T_f$  can be obtained.

$T_f$  obtained from Eq. [10] has now to be compared with the soil-dependent  $T_f$  (cut-off point of Eq. [3]). The following simple distinctions are made:

- $T_{f,soil} < T_{f,pressure}$ , the SFCC remains unchanged;
- $T_{f,soil} > T_{f,pressure}$ , the SFCC is updated.

Updating the SFCC occurs in a way such that the cut-off point equals the  $T_{f,pressure}$ . This results in a shift of the SFCC to a more negative temperature, as it is the pressure change that is now the main cause of the water not being frozen. This statement is also validated in section 3.1.

## 2.4 Hydraulic conductivity of frozen soil

Azmatch et al. (2012) asserts that the most used approach to determine the hydraulic conductivity of partially frozen soils is probably the use of the soil water retention curve (SWRC) in combination with different hydraulic conductivity estimation methods (e.g. van Genuchten (1980); Fredlund et al. (1994)). However, this method assumes that the SWRC is known. Relating the SWRC to the SFCC and determining the parameters needed to fit the two curves complicate the use of this approach. It is not daily engineering practice. The costs for direct measurements, the lack of data, as well as pressure of time, require a quick and reliable estimation of hydraulic properties for frozen soil. Tarnawski & Wagner (1996) suggest calculating the hydraulic conductivity for partially frozen soils by using the hydraulic conductivity function of the same unsaturated soil but unfrozen. This is based on the assumption that partly frozen pores have a similar effect on water flow as air filled pores, i.e., hindering moisture flow and that moisture flow takes place only through the smaller pores filled with water. Taking these assumptions into account, Campbell's model (1985) is used to calculate the hydraulic conductivity,  $k$  [m/s], for partially frozen soils as such:

$$k = k_{sat} \left( \frac{\theta_{uw}}{\theta_{sat}} \right)^{2b+3} = k_{sat} (S_{uw})^{2b+3} = k_{sat} k_r \quad [11]$$

where  $\theta_{sat}$  is the volumetric water content of a saturated soil and therefore assumed to be equal to the porosity.  $\theta_{uw}$  is the current volumetric unfrozen water content, which can be obtained from the empirical Eq. [3] and  $b$  is an empirical parameter based on the geometric mean particle diameter ( $d_g$ ) and the geometric standard deviation ( $\sigma_g$ ) according to Campbell (1985). The  $b$ -parameter is defined as the slope of the water potential versus the volumetric water content on a log-log scale plot and is given in Eq. [12].  $k_{sat}$  is the hydraulic conductivity of the saturated soil in unfrozen state. The ratio  $\theta_{uw}$  over  $\theta_{sat}$  is

the so-called unfrozen water saturation  $S_{uw}$ , whereas the relative permeability  $k_r$  is defined in Eq. [13].

$$b = d_g^{-0.5} + 0.2\sigma_g \quad [12]$$

$$k_r = (S_{uw})^{2b+3} \quad [13]$$

Campbell (1985) describes that the saturated hydraulic conductivity is dependent on the size and the distribution of pores. In his book he gives an overview of a number of equations which have been derived for predicting this hydraulic property from soil texture. The same book describes an equation for  $k_{sat}$  considering clay and silt mass fractions and the dry bulk density of the soil, where clay weighs heavier than silt. Tarnawski & Wagner (1996) slightly modified the equation given by Campbell (1985) and proposed the following empirical equation to provide a default value for  $k_{sat}$  [m/s]:

$$k_{sat} = 4 \cdot 10^{-5} \left( \frac{0.5}{1 - \theta_{sat}} \right)^{1.3b} \cdot \exp(-6.88m_{clay} - 3.63m_{silt} - 0.025) \quad [14]$$

## 3 VALIDATION AND RESULTS

The empirical approaches described in the previous sections to obtain the SFCC as well as the hydraulic properties are verified and validated.

### 3.1 Soil freezing characteristic curve

The determination of the SFCC by means of the PSD and void ratio seems a user-friendly method. The verification and validation of this approach using limited input data are conducted in the following sections. Soil type and pressure dependence of the SFCC are investigated by comparing estimated volumetric unfrozen water contents over temperature and measured SFCC. The measured data is obtained from Smith & Tice (1988) and Zhang et al. (1998), respectively.

#### 3.1.1 Soil type dependence

Smith & Tice (1988) performed measurements of unfrozen water content on a variety of soils. Their selection covers a representative range in grain size distribution as well as specific surface area (SSA). The soil samples were fully saturated with distilled water. Initially the soil samples were cooled to between -10 °C and -15 °C and progressively warmed to 0 °C. The unfrozen water content at 0 °C equals to the porosity of the soil sample. The method of warming the sample might provide slightly different results of the unfrozen water content than when freezing the sample. One of the

reasons is that the pore water has to overcome the supercooling effect when freezing (Kozłowski, 2009). The test results of Oliphant et al. (1983) on Morin Clay and the ones from Williams (1963) show this effect. Smith & Tice (1988) didn't provide the grain size distribution curve of the different soils; therefore they are estimated by taking the limiting values of the U.S.D.A. soil triangle into account. The assumed particle size mass fractions are given in Table 3 and are visualised in Figure 2.

Table 3. Assumed particle size mass fractions for soils tested in Smith & Tice (1983)

Soil	$m_{\text{clay}}$	$m_{\text{silt}}$	$m_{\text{sand}}$
Castor Sandy Loam	0.06	0.22	0.72
Athena Silt Loam	0.15	0.58	0.27
Niagara Silt	0.08	0.87	0.05
Suffield Silty Clay	0.41	0.41	0.18
Regina Clay	0.52	0.25	0.23

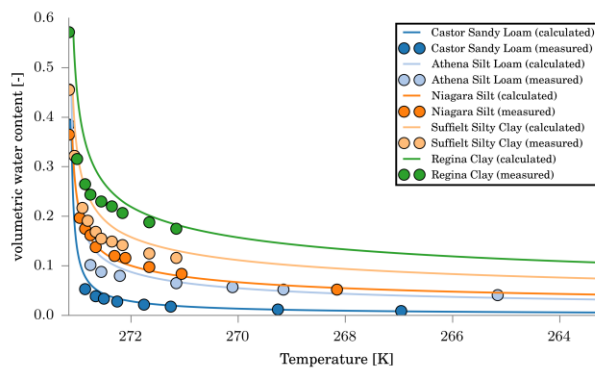


Figure 2. Comparison of measured and calculated SFCC of different soils

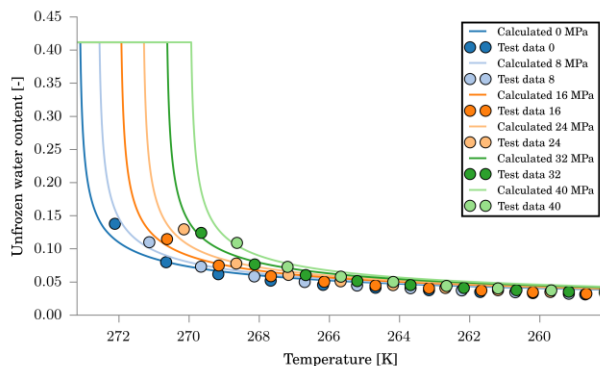


Figure 3. Comparison of measured and calculated SFCC of Lanzhou Loess at high pressures

The SFCC obtained by the time domain reflectometry (TDR) method of the five different soils and the calculated comparison graphs are presented in Figure 2. The graph

clearly shows that the suggested approach, using the mineralogy of soils, is appropriate as a first and default approach to obtain the SFCC for most soil types. The higher the amount of fines, the higher the specific surface area which allows a higher capability to hold a certain amount of unfrozen water and, hence, causes a freezing point depression (Petersen et al., 1996; Andersland & Ladanyi, 2004; Watanabe & Flury, 2008). The calculated SFCC in Figure 2 do not just represent the correct qualitative behaviour but also show a good quantitative agreement.

### 3.1.2 Pressure dependence

The pressure dependence of the freezing point affects the amount of water kept unfrozen at negative temperatures. The relationship between unfrozen water content and pressure is important in studying the physical properties and mechanical behaviours of frozen soils under high pressure (Zhang et al., 1998). To validate this relationship experimental data from Zhang et al. (1998) is chosen. The soil used is a Lanzhou Loess. Its particle size mass fractions are  $m_{\text{clay}} = 0.12$ ,  $m_{\text{silt}} = 0.80$  and  $m_{\text{sand}} = 0.08$ . The applied pressure on the sample in the test tube is 0, 8, 16, 24, 32 and 40 MPa, respectively. The pressure is kept constant at every stage while determining the unfrozen water content of the frozen soil at different temperatures below the freezing point. The pore water pressure is set equal to the applied pressure on the sample. The initial void ratio is assumed to be 0.7. Figure 3 provides the comparison between measured data and calculated SFCC at the six different pressure levels.

Figure 3 reproduces accurately the ability of the empirical formulation to compute the volumetric unfrozen water content after Anderson & Tice (1972) by considering the pressure dependence of the freezing point (Eq. [10]).

## 3.2 Hydraulic properties

### 3.2.1 Saturated hydraulic conductivity

In order to validate the empirical approach of Eq.[14], soil textural classes and related saturated hydraulic conductivity classes provided by the U.S.D.A. are chosen to be comparative values. The calculated  $k_{\text{sat}}$  values are obtained by using the default grain size distribution of the U.S.D.A. soil textural classes and appropriate ranges of their void ratio (Table 4). The comparison of calculated and provided ranges for the saturated hydraulic conductivity can be seen in Figure 4.

The U.S.D.A. ranges shown for the saturated hydraulic conductivity in relation to texture are only a general guide and differences in bulk density may alter the rate. This dependence on the initial void ratio of the soil is demonstrated in Figure 4 when looking at the calculated ranges where a minimum void ratio (loose state) and a maximum void ratio (dense state) are considered. Two soil types, namely sandy clay and sandy clay loam show significant deviation between the two illustrated ranges. However, all other U.S.D.A. soil types show conformity with the estimated values. One has to keep in mind that



estimating the saturated hydraulic conductivity using this approach is only suggested when there is no other available information on the actual  $k_{sat}$ .

Table 4. Particle size mass fractions according to the U.S.D.A. soil textural classes and assumed void ratio ranges

Soil	$m_{clay}$	$m_{silt}$	$m_{sand}$	$e_{min}$	$e_{max}$
Sand	0.04	0.04	0.92	0.30	0.75
Loamy Sand	0.06	0.11	0.83	0.30	0.90
Sandy Loam	0.11	0.26	0.63	0.30	1.00
Loam	0.20	0.40	0.40	0.30	1.00
Silt	0.06	0.87	0.07	0.40	1.10
Silty Loam	0.14	0.14	0.21	0.40	1.10
Sandy Clay Loam	0.28	0.12	0.60	0.30	0.90
Clayey Loam	0.34	0.34	0.32	0.50	1.20
Silty Clay Loam	0.34	0.55	0.11	0.40	1.10
Sandy Clay	0.42	0.05	0.53	0.30	1.80
Silty Clay	0.48	0.45	0.07	0.30	1.80
Clay	0.70	0.13	0.17	0.50	1.80

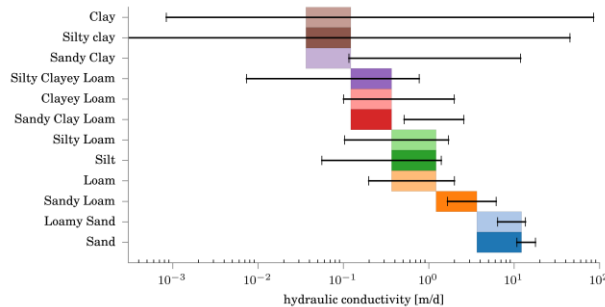


Figure 4. Comparison of saturated hydraulic conductivity ranges – U.S.D.A. ranges (coloured bars) vs. calculated  $k_{sat}$  ranges based on the PSD and void ratio (lines)

### 3.2.2 Hydraulic conductivity of frozen soil

One of the first direct measurements of hydraulic conductivity of partially frozen soil was conducted by Burt & Williams (1976). They found that the hydraulic conductivity coefficient depends on soil type and temperature and is related to the unfrozen water content. At temperatures within a few tenths of 0 °C, the coefficient ranges from  $10^{-5}$  to  $10^{-9}$  cm/s, and decreases only slowly below about -0.5 °C. Furthermore, the same study showed that soils known to be susceptible to frost heave have significant hydraulic conductivities well below 0°C.

The test data of Burt & Williams (1976) serve as a comparison basis. The hydraulic conductivity values are estimated using Eq. [11]. This equation makes use of the PSD, as well as Eq. [3] to obtain the volumetric unfrozen water content empirically and Eq. [14] to predict the saturated hydraulic conductivity. The comparison between

estimated and predicted values of hydraulic conductivities is illustrated in Figure 5. The PSD was given for most of the soil types. The initial void ratio, however, had to be estimated. In Table 5 the used values are shown.

Table 5. Particle size mass fractions and initial void ratio for soils tested in Burt & Williams (1976)

Soil	$m_{clay}$	$m_{silt}$	$m_{sand}$	$e_{init}$
Carleton Silt	0.03	0.40	0.57	0.60
Oneyda Clayey Silt	0.28	0.42	0.30	0.60
Leda Clay	0.40	0.45	0.15	0.50

A clear finding is the drop in hydraulic conductivity occurring within a very small temperature range of less than 0.50 °C. This drop can be reproduced by using the proposed approach. The zone of a freezing soil where this temperature range appears is the so called frozen fringe. Furthermore it can be observed that after this sudden steep decline in hydraulic conductivity, a threshold value is reached, meaning no further relevant decrease in  $k$  is expected. The minimum conductivity value is therefore related to the initial saturated hydraulic conductivity,  $k_{sat}$ , and chosen to be  $k_{sat} * 10^{-6}$ . This limiting value is also important regarding a numerical implementation of the moisture transfer equation in order to avoid numerical problems (condition of the global flow matrix).

## 4 DISCUSSION

Obviously there is no perfect conformity of the estimated hydraulic conductivities with the direct measurements. Facts which affect the accuracy of the results are the empirical estimation of the volumetric unfrozen water content as well as the use of Campbell's model which is again empirical and originally formulated for unsaturated soil. Other points are the quality of direct measurements and the influence of ice lenses in the soil samples (see the results of densely lensed Leda clay in Figure 5). They highly influence the hydraulic conductivity.

The effect of overburden pressure and pressure in general is not investigated. In qualitative terms, an increase in pressure causes a freezing point depression. The result is a bigger availability of unfrozen water and considering double porosity networks a higher hydraulic conductivity at constant temperature. Reversely, as pressure (or temperature) decreases, the hydraulic conductivity of frozen soil sharply decreases. This is commonly observed in frozen soil (Stähli et al., 1999; McCauley et al., 2002). Summing up, also the pressure dependence can be achieved using the proposed empirical approach. Benson and Othman (1993) explain, however, that an increase in overburden pressure may also decrease the hydraulic conductivity of the frozen fringe, by compressing the pores and the cracks which consequently restricts the conduits for flow. This effect, causing a decrease in voids, cannot be taken into account using this model.

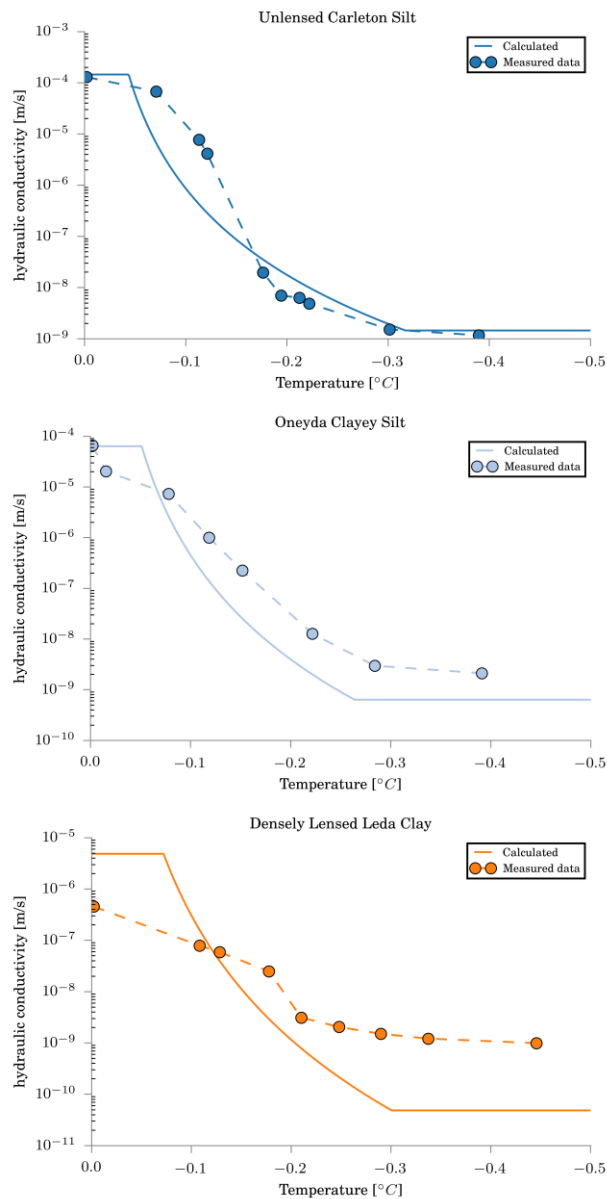


Figure 5. Comparison of measured and estimated hydraulic conductivities of different frozen soil types measured from Burt and Williams (1976)

## 5 CONCLUSION

This paper demonstrates how limited input data can be used to obtain some of the main properties of frozen soils. The described approach comprises empirical relationships developed over the last five decades and provides a closed formulation which can be used in numerical simulations to account for moisture migration in partially frozen soils.

The proposed approach is validated by comparing real test data with calculated values. Soil freezing

characteristic curves of different soils at varying confining pressure, as well as direct hydraulic conductivity measurements, serve as comparison basis. The soil type and the pressure-dependence of the SFCC for a handful different soils could be captured qualitatively and also in a quantitative respect. Furthermore, the important drop of the hydraulic conductivity occurring within the very small temperature range of less than 0.50 °C can be represented as well.

Keeping in mind the simplicity, the user-friendliness, the low expenditure of time and the obtained conformity between model predictions and measured data of many soil types, it is worth to consider this approach as the initial estimation of the SFCC, the freezing/melting point of a soil-water system and its hydraulic conductivity. Furthermore, first boundary value problems performed in a THM-FE environment considering unfrozen and frozen soil (Aukenthaler et al. 2016) have shown that the numerical implementation of this approach is stable and provides budding results.

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