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Hydroelasticity of Marine Structures

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CHINA

HYDROELASTICITY OF MARINE STRUCTURES

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Hydroelasticity is that branch of science which is concerned with the motion of deformable bodies through liquids. A floating or fixed marine structure is a flexible body which deforms due to the actions of fluid loadings or other externally applied forces. This paper discusses a general three dimensional hydroelastic theory applicable to marine structures which may be moving or fixed in regular sinusoidal waves. Applications of the theory are illustrated with reference to ships, multi-hulls (SWATHS, catamaran, semi-submersible), jack-up rig, fixed plate, etc.

INTRODUCTION

Consider a naval architect who is about to design a marine structure e.g. a conventional or unconventional ship, hovercraft, hydrofoil, offshore floati fixed platform etc. that is not some close relative of an existing structur creation must meet specific requirements i.e. an initial cost, safety, relia maintainability, performance etc. There is no possibility of prototype tes and misjudgement could produce horrendous consequences. It is against this background that one of the naval architect's problems is to be discussed. T *the response of the marine structure to waves.*

Historically, the naval architect has been involved mainly in the design of ships though in recent years interest has shifted to the design of offshore structures. He has the responsibility of ensuring that the designed ship has good seakeeping qualities. In brief, this seakeeping or seaworthiness may be defined as the ability of the ship to travel safely in the roughest seas and proceed on course with the minimum delay. The problem has changed with time - from sailing vessels which followed the prevailing winds to the advent of moderately powered steam ships capable of travelling directly to windward. Though structural damage occurred to the early steam ships, full engine power could be used in almost any weather condition even though ship speed was reduced by the actions of wind and sea.

For modern fast ships travelling in rough seas, the engine power available to any ship's captain is excessive and must be reduced voluntarily to avoid severe structural damage to the hull. Thus the decision of a ship's captain to maintain a particular schedule now depends on the behaviour of the ship's response in the seaway as well as the amount of available power at his command. Thus if bodily motions become too great because of the wave excitations, ship operations are greatly reduced because of excessive motions, deck wetting, maintaining course, speed reductions, slamming with consequential high stresses, etc. Alternatively, since the ship's structure is flexible, distortions may become excessive with resultant cracking or buckling of plates, pipes, etc. If all these possibilities in modern ship design and operations are untoward, they are not immediately catastrophic. But catastrophes do happen as a result of excessive wave excited hull responses. For example, tankers have split in waves, hatch covers have been dislodged and washed overboard, some types of hulls twist and crack, and smaller vessels, such as trawlers, capsize for reasons that are not fully understood.

In recent years, with the ever increasing demand for petroleum products, offshore exploration has become common place in deeper and deeper waters. This has demanded the building and development of large specialised offshore structures which may be fixed or floating. Again it is of the utmost importance that the naval architect ensures that the design can withstand the roughest of seas without impairing the safety of personnel and the structural integrity of the offshore structure.

In contrast to ships, these offshore structures can be either floating (slow moving or stationary) or fixed to the sea bed with very large geometrical dimensions and shapes which are far from slender. However the naval architect is faced with a plethora of similar design problems whether he is dealing with ship like bodies or offshore structures but the emphasis and importance of a particular aspect i.e. forward speed, arbitrary geometry, operations etc. may be different.

In order to assess the wave excited responses of a flexible marine structure, the naval architect has to be able to describe and identify the form of the structure and the fluid actions applied to it. He needs techniques which are the products of structural analysis and naval hydrodynamics (see, for example, figure 1).

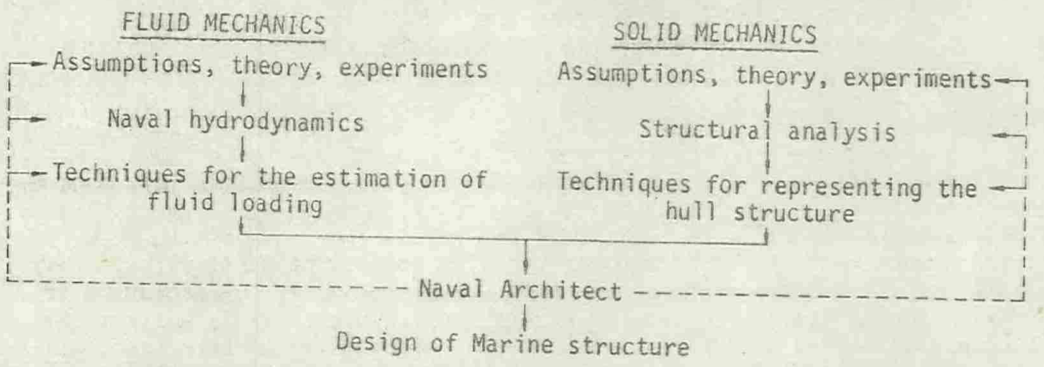

Figure. 1. Flow chart illustrating information transfer in the design process.

These have their origins in the theories of solid mechanics and fluid mechanics respectively. This does not mean that it is customary for designers to employ methods they receive ready made from classical physicists and applied mathematicians who may specialise in the relevant branches of applied mechanics and hydromechanics. Far from it, the techniques adopted are fashioned by naval architects since there is a strong feed back from actual designs. In addition naval architects tend to specialise either in structures or in naval hydrodynamics and seldom in both.

When the structural and hydrodynamic theories are available, the naval architect has a very difficult task in reconciling the two theories to describe the overall behaviour of the marine structure in rough seas since the theories are of necessity idealised. However, if one accepts the predictions of fluid actions offered by naval hydrodynamics and the representation of the marine structure as some form of elastic structure proposed by the structural analyst, the estimation of the responses associated with the marine structure becomes in effect a refined vibration problem. However, because of the physical nature of the sea environment, in general the vibrations excited in the structure are random and the oscillating marine structure - wave system constitutes a non-conservative system.

There are no new ideas introduced in discussing the marine structure as a more or less elastic body excited by waves but it is not common to estimate the responses of the structure in any unified way using the estaiblished techniques of dynamics. Therefore the aim of this paper and lecture is to show that by generalising the approach of Bishop and Price (1979) a suitable unifying hydroelasticity theory involving both the techniques of structural analysis and naval hydrodynamics may be developed and this describes in a realistic manner *the responses of an arbitrary shaped, flexible marine structure travelling with forward speed in the free surface of a seaway.*

In fact, the theoretical model describing the responses associated with a fixed marine structure i.e. offshore platform or a moving, totally submerged structure are special cases of the general problem just posed. For example, a deflection associated with a floating, flexible marine structure will involve a combination of bodily motions and distortions of the structure whilst for a fixed body only distortions of the structure are admitted into the theory. However, because of the imposed geometric conditions relating to the arbitrary shape of the marine structure, it becomes necessary to adopt and develop suitable three dimensional structural and hydrodynamic theories. Thus from the same basic theory and set o assumptions the responses of a rigid body, flexible ship-like body, semi-submersible/SWATHS/catamaran hull, fixed or moving jack-up rig, fixed offshore platform etc. may be determined and discussed.

GENERALISED EQUATIONS OF MOTION

In the hydroelasticity theory of ships developed by Bishop and Price (1979) the hu is assumed beamlike and its vibration characteristics in vacuo are determined in absence of damping and external forces. By adopting a suitable beam theory a set principal modes and natural frequencies of the dry ship may be determined. When allowance is made for the water in which the dry hull is placed all forces are tr as external actions applied to the hull. From the relevant mathematical model, resonance frequencies and responses (i.e. displacement, bending moment, shearing force, twisting moment, etc.) at any arbitrary position in the hull may be determ

Whether symmetric and/or antisymmetric responses of the hull are under investigation, it has been shown that the generalised linear equations of motion describing the responses of the flexible dry hull may be expressed in the form

$$\underset{\sim}{a}\ddot{\underset{\sim}{p}}(t) + \underset{\sim}{b}\dot{\underset{\sim}{p}}(t) + \underset{\sim}{c}\underset{\sim}{p}(t) = \underset{\sim}{Z}(t)$$

where

$\underset{\sim}{a}$ is the real inertia matrix of the dry hull, which is diagonal apart from off diagonal antisymmetric rigid body contributions,

$\underset{\sim}{b}$ is the structural damping matrix of the dry hull,

$\underset{\sim}{c}$ is the diagonal stiffness matrix of the dry hull with elements $c_{ss}=\omega_s^2 a_{ss}$.

The column matrices $\underset{\sim}{p}(t)$ and $\underset{\sim}{Z}(t)$ represent the responses of the hull and input loading respectively. In fact, it can be shown that this input loading may be represented in the form

$$\underset{\sim}{Z}(t) = -\underset{\sim}{A}\ddot{\underset{\sim}{p}}(t) - \underset{\sim}{B}\dot{\underset{\sim}{p}}(t) - \underset{\sim}{C}\underset{\sim}{p}(t) + \underset{\sim}{\Xi}(t)$$

where $\underset{\sim}{A}$, $\underset{\sim}{B}$ and $\underset{\sim}{C}$ are square matrices associated with the motions of the flexible body in the fluid and $\underset{\sim}{\Xi}(t)$ is a column matrix representing the wave excitation.

To describe non-beamlike structures which may be floating or fixed a more general mathematical model must be developed. One alternative approach to the two dimensional beamlike model is to assume a finite element discretisation of the three dimensional structure.

By adopting the techniques developed in finite element theory (see, for example, Zienkiewitz (1977)), it has been shown by Bishop, Price and Wu (1984) that if the continuous dry structure is discretised with m degrees of freedom then the (mx1) column nodal displacement vector $\underset{\sim}{U}=\{U_1,U_2,...,U_m\}$ is a solution of the set of equations describing the responses of the dry structure. Namely,

$$\underset{\sim}{M}\ddot{\underset{\sim}{U}} + \underset{\sim}{C}\dot{\underset{\sim}{U}} + \underset{\sim}{K}\underset{\sim}{U} = \underset{\sim}{P} \qquad (1)$$

where the (mxm) square matrices M, C, K describe the system mass, damping, stiffness properties respectively and the (mx1) column matrix P represents a general external loading applied to the structure. The matrices M and K are semi-definite or definite depending on the boundary restraints imposed on the structure.

For free motion P=0=C, a solution of the reduced matrix equation is sought in the general form

$$\underset{\sim}{U} = \underset{\sim}{\Delta} e^{i\omega t}$$

where Δ denotes a principal eigenvector matrix and the eigenvalue ω represents a natural frequency of the dry structure. At each natural frequency $\omega = \omega_r (r=1,2,..,m)$ there exists solutions for U and Δ. These may be represented as

$$\underset{\sim}{U}_r = \underset{\sim}{D}_r e^{i\omega_r t}$$

for each r=1,2,...,m with the column matrices $\underset{\sim}{U}_r = \{U_1, U_2, .., U_m\}_r$ and the characteristic vector of the rth principal mode $\underset{\sim}{D}_r = \{D_1, D_2, ..., D_m\}_r$ where the element

$$\underset{\sim}{D}_{rj} = \{u_r, v_r, w_r, \theta_{xr}, \theta_{yr}, \theta_{zr}\}_j$$

represents the displacement vector of the rth modal shape at the jth node (i.e. u,v,w translations and $\theta_x, \theta_y, \theta_z$ rotations about the equilibrium coordinate axes Ox, Oy, Oz respectively, see fig. 2.)

If a submatrix of D_r corresponding to the rth modal shape of all the nodes within one particular finite element is denoted by d_r then, as shown by Bishop, Price and Wu (1984), the rth principal modal shape at any arbitrary point within that finite element may be expressed as

$$\underset{\sim}{u}_r = \underset{\sim}{\ell}^T \underset{\sim}{N}\underset{\sim}{L}\underset{\sim}{d}_r = \{u_r, v_r, w_r\} \quad . \qquad (2)$$

Here $\underset{\sim}{\ell}^T \underset{\sim}{N}\underset{\sim}{L}$ denotes a transformation matrix between the local coordinate system defined within the finite element and the equilibrium axes system used to define the overall mode shape of the structure and as will be discussed later, also the external fluid loading.

For forced motion P≠0≠C, according to a theorem by Rayleigh (1894) any distortion of the structure may be expressed as an aggregate of distortions in the principal modes. That is to say, the nodal displacement in the forced solution may be expressed in the form

$$\underset{\sim}{U} = \sum_{r=1}^{m} \underset{\sim}{D}_r p_r(t) = \underset{\sim}{D}\underset{\sim}{p}(t) \qquad (3)$$

where the (mx1) column vector p represents the m principal coordinates associated with the principal modes of the dry structure and the (mxm) square matrix $\underset{\sim}{D} = \{\underset{\sim}{D}_1, \underset{\sim}{D}_2, ..., \underset{\sim}{D}_m\}$ is the principal mode matrix.

Thus the displacement at any arbitrary point within the structure may be written in the form

$$\underset{\sim}{u} = \{u,v,w\} = \sum_{r=1}^{m} \underset{\sim}{u}_r p_r(t) = \sum_{r=1}^{m} \{u_r, v_r, w_r\} p_r(t)$$

where $\underset{\sim}{u}_r$ denotes the rth principal mode shape vector associated with the dry structure.

Using derived orthogonality relationships, the symmetric principal mass matrix, $\underset{\sim}{a}$ and stiffness matrix, $\underset{\sim}{c}$, may be expressed as

$$\underset{\sim}{a} = \underset{\sim}{D}^T \underset{\sim}{M}\underset{\sim}{D} \ , \qquad \underset{\sim}{c} = \underset{\sim}{D}^T \underset{\sim}{K}\underset{\sim}{D}$$

where the superscript T denotes a transpose matrix and the element $c_{ss} = \omega_s^2 a_{ss}$. After substituting for U from equation (3), premultiplying by D^T and using these orthogonality conditions, the equation of motion (1) may be caste into the familiar form,

$$a\ddot{p}(t) + b\dot{p}(t) + cp(t) = Z(t) \tag{4}$$

where the (mxl) column matrix

$$Z(t) = D^T P(t) = \{Z_1(t), Z_2(t), \ldots, Z_m(t)\} \tag{5}$$

represents the generalised external force arising from the waves, mooring lines, propeller, gravity, etc., and

$$b = D^T C D$$

is the generalised structural damping matrix which is usually assumed diagonal. This simplification is introduced because of the great scarcity of information on the distribution of damping throughout the structure.

Under these assumptions, the generalised linear equations of motion may be expressed as

$$a_{rr}\ddot{p}_r(t) + b_{rr}\dot{p}_r(t) + c_{rr}p_r(t) = Z_r(t) \tag{6}$$

for r=1,2,..,m.

THE FLUID-STRUCTURE PROBLEM

A flexible, floating structure travelling with constant forward speed $\bar{U}$ at an arbitrary heading angle χ (=180°, head waves) in deep water, regular sinusoidal waves of amplitude a, frequency ω, wave number k or wavelength $\lambda(=2\pi/k)$ experiences waves of encounter frequency

$$\omega_e = \omega - \bar{U}k\cos\chi = \omega - (\bar{U}\omega^2\cos\chi)/g.$$

The freely floating structure, excited by the waves, responds in both rigid body and distortion modes. The rigid body responses are associated with the first six principal modes of the dry structure which form a subset of the infinite number of modes describing the dynamic characteristics of the dry flexible structure. These rigid body responses are discussed extensively in seakeeping analyses and only a resumé of the underlying theory is presented here before embarking on a general hydrodynamic - structures theory associated with the flexible structure.

For a fixed flexible structure, rigid body modes do not exist and only the distortion responses of the structure need be considered.

AXES AND GENERAL FORMULATION

Figure 2 illustrates the three right hand axes systems used to define the fluid actions. $Ax_0y_0z_0$ is a fixed spatial set of axes, Oxyz is an equilibrium set of axes moving with forward speed $\bar{U}$ and remaining parallel to $Ax_0y_0z_0$ and O'x'y'z' is an axis system fixed in the arbitrary shaped structure and, prior to any disturbance, coincides with Oxyz.

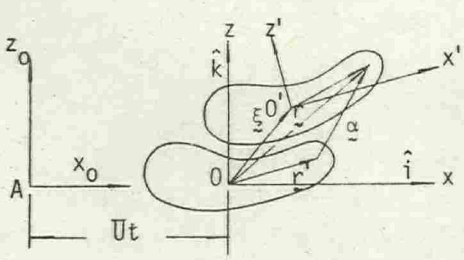

Figure 2. Axes systems.

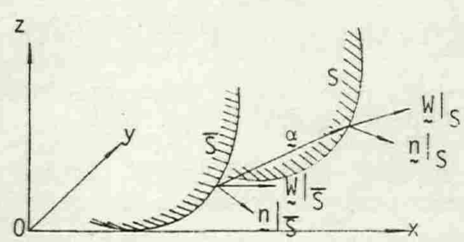

Figure 3. Instantaneous positions of unit normal vectors and steady flow velocity vectors on the wetted surfaces.

Assuming the fluid to be incompressible, inviscid and irrotational, there exists a potential function $\Phi(x_0,y_0,z_0,t)$ such that the fluid velocity $\underset{\sim}{V}(x_0,y_0,z_0)$ is

$$\underset{\sim}{V} = \text{grad}\ \Phi = \nabla\Phi$$

and throughout the fluid, the Laplace condition $\nabla^2\Phi = 0$ is valid.

Newman (1978) in his many extensive studies of rigid ship motions has shown that this potential satisfies the following boundary conditions:

(i) On the free surface, $z_0=\zeta$,

$$\Phi_{tt} - 2\nabla\Phi.\nabla\Phi_t + \tfrac{1}{2}\nabla\Phi.\nabla(\nabla\Phi.\nabla\Phi) + g\Phi_{z_0} = 0 \quad , \tag{7}$$

where $\Phi_{tt} = \partial^2\Phi/\partial t^2$ etc. and ζ is the elevation of the wave disturbance.

(ii) On the sea bed, $z_0=-d$,

$$\Phi_{z_0} = 0 \quad . \tag{8}$$

(iii) A suitable far field boundary condition.

(iv) On the instantaneous total wetted surface area S of the structure

$$\partial\Phi/\partial n = \Phi_n = \underset{\sim}{V}^S.\underset{\sim}{n} \tag{9}$$

where $\underset{\sim}{V}^S$ denotes the local velocity on the wetted surface S and $\underset{\sim}{n}$ is the outward drawn unit normal vector into the fluid.

From figure 2 it is seen that if $\underset{\sim}{\xi}$ represents a translation of 0' from 0 and $\underset{\sim}{\Omega}$ denotes a rotation of the rigid structure, the displacement of the point $\underset{\sim}{r}'(=x',y',z')$ relative to 0 is

$$\underset{\sim}{\alpha} = \underset{\sim}{\xi} + \underset{\sim}{\Omega}\mathrm{x}\underset{\sim}{r}' \tag{10}$$

and the velocity of this point is

$$\underset{\sim}{V}^S = \bar{U}\hat{i} + \dot{\underset{\sim}{\alpha}} = \bar{U}\hat{i} + \dot{\underset{\sim}{\xi}} + \dot{\underset{\sim}{\Omega}}\mathrm{x}\underset{\sim}{r}' \quad . \tag{11}$$

Figure 3 illustrates the effect of the body disturbance on the change in the direction of the unit vector $\underset{\sim}{n}$ relative to the equilibrium axes. This direction is unaffected by a pure translation but rotations of the structure must be accounted for. Thus if $\underset{\sim}{n}|_S$, $\underset{\sim}{n}|_{\bar{S}}$ denote the unit vectors relating to the disturbed and steady state condition respectively it follows that to a first approximation,

$$\underset{\sim}{n}|_S = \underset{\sim}{n}|_{\bar{S}} + \underset{\sim}{\Omega}\mathrm{x}\underset{\sim}{n}|_{\bar{S}} \quad . \tag{12}$$

The total potential Φ may be represented in the equilibrium frame of axes as

$$\Phi(x_0,y_0,z_0,t) \equiv \bar{U}\bar{\phi}(x,y,z) + \phi(x,y,z,t) \tag{13}$$

where $\bar{\phi}$, ϕ denote velocity potential components due to the steady motion of the structure in calm water and the unsteady motion in waves respectively.

When only the steady motion is considered, the velocity of the steady flow relative to the moving equilibrium frame of axes is

$$\underset{\sim}{W} = \bar{U}\text{grad}(\bar{\phi} - x) \tag{14}$$

and the boundary condition (iv) takes the form $\underset{\sim}{W}.\underset{\sim}{n}=0$ on the mean wetted surface $\bar{S}$.

However, when the structure is disturbed, the flow velocity associated with the instantaneous wetted surface S i.e. $\underset{\sim}{W}|_S$ may be expressed in terms of the steady flow velocity i.e. $\underset{\sim}{W}|_{\bar{S}}$ and a contribution due to the disturbed displacement of the structure. That is, to a first approximation the variation of $\underset{\sim}{W}|_S$ about $\underset{\sim}{W}|_{\bar{S}}$ due to a parasitic disturbance $\underset{\sim}{\alpha}$ may be written as

$$\underset{\sim}{W}|_S = [1 + (\underset{\sim}{\alpha}.\nabla)]\underset{\sim}{W}|_{\bar{S}} = [1 + (\underset{\sim}{\alpha}.\nabla)]\{\bar{U}\nabla(\bar{\phi}-x)\}|_{\bar{S}} = \bar{U}\nabla(\bar{\phi}-x)|_S \tag{15}$$

since for any vector e.g. $\nabla\bar{\phi}$, it follows from the above that

$$\nabla\bar{\phi}|_S = [1 + (\underset{\sim}{\alpha}.\nabla)]\nabla\bar{\phi}|_{\bar{S}} \qquad \text{etc.} \tag{16}$$

Using these previous relationships, i.e. equations (9, 11, 13, 14) the boundary condition (iv) on S may be expressed as

$$\partial\Phi/\partial n = \bar{U}\partial\bar{\phi}/\partial n + \partial\phi/\partial n = (\bar{U}\nabla\bar{\phi} + \nabla\phi).\underset{\sim}{n} = (\bar{U}\hat{\underset{\sim}{i}} + \dot{\underset{\sim}{\alpha}}).\underset{\sim}{n}$$

or

$$\underset{\sim}{W}.\underset{\sim}{n} + \nabla\phi.\underset{\sim}{n} = \dot{\underset{\sim}{\alpha}}.\underset{\sim}{n} \quad \text{on } S.$$

After the necessary substitutions for $\underset{\sim}{W}|_S$, $\underset{\sim}{n}|_S$ etc. it follows that to a first order approximation

$$\partial\phi/\partial n = [\dot{\underset{\sim}{\alpha}} + \underset{\sim}{\Omega}\times\underset{\sim}{W} - (\underset{\sim}{\alpha}.\nabla)\underset{\sim}{W}].\underset{\sim}{n} \quad \text{on } \bar{S}\,. \tag{17}$$

This is the familiar relationship derived by Timman and Newman (1962) in the development of the mathematical model to describe the rigid body motions of a ship moving in waves.

RIGID BODY MOTIONS - FLUID ACTIONS

When investigating the behaviour of the six bodily motions of a floating rigid ship-like structure, Salvesen, Tuck and Faltinsen (1970) expressed the unsteady oscillatory potential ϕ as

$$\phi = (\phi_0 + \phi_D + \sum_{r=1}^{6} \eta_r\phi_r)e^{i\omega_e t} \tag{18}$$

where η_r represents the amplitude of motion in surge, r=1; sway, r=2; heave, r=3; roll, r=4; pitch, r=5 and yaw, r=6. The incident wave potential amplitude

$$\phi_0(x,y,z) = \frac{iga}{\omega}\exp\{kz - ik(x\cos\chi - y\sin\chi)\}\,, \tag{19}$$

ϕ_D is the diffracted wave potential amplitude and ϕ_r is the radiation potential amplitude due to a motion of unit amplitude in each of the six body modes of motion in calm water. According to Newman (1978), these linear velocity potentials satisfy the following boundary conditions:

(i) On the free surface all the potentials ϕ_0, ϕ_D or ϕ_r (r=1,2,...,6) satisfy the linearised boundary condition

$$\bar{U}\partial^2\phi/\partial x^2 - 2i\omega_e\bar{U}\partial\phi/\partial x - \omega_e^2\phi + g\partial\phi/\partial z = 0 \quad \text{on } z=0 \tag{20}$$

where ϕ represents either ϕ_0, ϕ_D or ϕ_r.

(ii) Suitable bottom and radiation conditions at infinite distance from the oscillating, translating structure.

(iii) The incident and diffraction potentials satisfy the relationship

$$\partial\phi_D/\partial n = -\partial\phi_0/\partial n \quad \text{on } \bar{S}\,. \tag{21}$$

(iv) The radiation potentials are governed by the body boundary condition derived from equation (17), i.e.

$$\partial\phi_r/\partial n = i\omega_e n_r + \bar{U}m_r \qquad \text{on } \bar{S} \tag{22}$$

for r=1,2,...,6 and the components

$$\underset{\sim}{n} = (n_1,n_2,n_3),\ \underset{\sim}{r}\times\underset{\sim}{n} = (n_4,n_5,n_6),\ \underset{\sim}{r} = (x,y,z),\ (n.\nabla)\underset{\sim}{W} = -\bar{U}(m_1,m_2,m_3)$$

$$(\underset{\sim}{n}.\nabla)(\underset{\sim}{r}\times\underset{\sim}{W}) = -\bar{U}(m_4,m_5,m_6).$$

This formulation implies that the steady motion problem in calm water must be initially solved before these boundary conditions can be properly defined. However, as shown by Inglis and Price (1980), this complication greatly increases the complexity of the solution for the linear velocity potentials. As a simplification, if it is assumed that the perturbation of the steady flow due to the presence of the body is negligible then

$$\underset{\sim}{W} = -(\bar{U},0,0) = -\underset{\sim}{U} = -\bar{U}\hat{i} \tag{23}$$

and this approximation allows the unsteady motion problem to be derived independent of the description of the steady motion problem in calm water. In this situation, the boundary condition in equation (22) greatly simplifies since now

$$m_1=0=m_2=m_3=m_4 \quad , \quad m_5=n_3, \quad m_6=-n_2 \ . \tag{24}$$

It has been shown by Inglis and Price (1981, 1982a,b), Price and Wu (1983a,b) that the velocity potential of a pulsating or, and translating source satisfies the free surface boundary condition and together with the other boundary conditions, the fluid actions associated with the structure oscillating and moving in the free surface may be determined. In fact, the total oscillatory hydrodynamic force amplitude, Z_r on the structure may be obtained by integrating over the mean wetted surface $\bar{S}$ the unsteady pressure component defined by Bernoulli's equation. That is

$$Z_r(t)=e^{i\omega_e t}Z_r=e^{i\omega_e t}\rho\iint_{\bar{S}} n_r\{i\omega_e-\underset{\sim}{U}.\nabla\}[(\phi_o+\phi_D)+\sum_{k=1}^{6} n_k\phi_k]dS= e^{i\omega_e t}(\Xi_r + H_r) \tag{25}$$

where Z_r (r=1,2,3) represent the total hydrodynamic components of force in the Ox, Oy, Oz directions respectively and the remaining terms r=4,5,6 represent the moments about these axes. The exciting force and moment due to the waves is

$$\Xi_r = \rho\iint_{\bar{S}} n_r\{i\omega_e - \underset{\sim}{U}.\nabla\}(\phi_o + \phi_D)dS = \Xi_{or} + \Xi_{Dr} \tag{26}$$

where Ξ_{or} represents the Froude-Krylov component of the excitation and reflects that the presence of the structure does not influence the pressure distribution in the incident wave. The diffraction force, Ξ_{Dr}, accounts for the scattering of the incident waves by the structure. For $\underset{\sim}{U} = (\bar{U},0,0)$, the Froude-Krylov contribution reduces to

$$\Xi_{or} = \rho\iint_{\bar{S}} n_r\{i\omega_e - \underset{\sim}{U}.\nabla\}\phi_o dS = -i\rho\iint_{\bar{S}} n_r\omega\phi_o dS \tag{27}$$

and is independent of forward speed. Thus in the wave exciting force all speed dependence arises from the diffraction component.

The force and moment due solely to the rigid body motions of the structure in calm water are given by

$$H_r = \rho\iint_{\bar{S}} n_r\{i\omega_e - \underset{\sim}{U}.\nabla\}\sum_{k=1}^{6} n_k\phi_k dS = \sum_{k=1}^{6} T_{rk}n_k \tag{28}$$

where

$$T_{rk} = \omega_e^2 A_{rk} - i\omega_e B_{rk} \tag{29}$$

denotes the hydrodynamic force and moment in the rth direction per unit oscillatory displacement in the kth mode. The terms A_{rk} and B_{rk} (r,k=1,2,...,6) represent added mass or inertia and damping coefficients respectively associated with these displacements.

FLEXIBLE BODY THEORY

For a free floating flexible body, the rigid body modes discussed previously are only the first six principal modes of the infinite number of modes of the dry structure. The fluid actions associated with the structure depend on the distortions of the structure as well as the rigid body motions. Therefore a general and rational velocity potential theory must be developed to include the effects of distortion and rigid body motions, forward speed and account for arbitrary shaped three dimensional structures. That is the conventional potential theory adopted in seakeeping theory is only a small portion of the general linear theory for flexible structures.

PRINCIPAL COORDINATES AND DISPLACEMENTS

According to a theorem due initially to Rayleigh (1894), see also Timoshenko (1955), any distortion of the structure may be expressed as an aggregate of distortions in its principal modes. That is the components of the deflection defined in the equilibrium coordinate axis system Oxyz may be expressed as

$$u(x,y,z,t)=\sum_{r=1}^{\infty}p_r(t)u_r(x,y,z) \;,\quad v(x,y,z,t)=\sum_{r=1}^{\infty}p_r(t)v_r(x,y,$$

$$w(x,y,z,t)=\sum_{r=1}^{\infty}p_r(t)w_r(x,y,z) \tag{30}$$

where $p_r(t)$ is the rth principal coordinate and u_r, v_r, w_r are the rth principal modes of the dry hull. These latter functions are defined with respect to the mean equilibrium position of the floating structure in which initially the axes systems Oxyz and O'x'y'z' are coincident. By adopting a suitable transformation, these mode shapes may be expressed as functions of (x',y',z'), and by representing the principal mode in the vector form

$$\underset{\sim}{u}_r(x',y',z')=u_r\hat{i} + v_r\hat{j} + w_r\hat{k} = \{u_r,v_r,w_r\}$$

such that the displacement

$$\underset{\sim}{u}(x',y',z',t) = u\hat{i} + v\hat{j} + w\hat{k} = \sum_{r=1}^{\infty}p_r(t)\underset{\sim}{u}_r \quad ,$$

the velocity of any point (x',y',z') on the surface of the structure travelling with forward velocity $\overline{U}$ can be expressed as

$$\underset{\sim}{V}^S(x',y',z',t) = \overline{U}\hat{i} + \dot{\underset{\sim}{u}} = \overline{U}\hat{i} + \sum_{r=1}^{\infty}\dot{p}_r(t)\underset{\sim}{u}_r \;. \tag{31}$$

In a similar manner the rotation vector at any point (x',y',z') is given by

$$\underset{\sim}{\theta}(x',y',z',t) = \sum_{r=1}^{\infty}p_r(t)\underset{\sim}{\theta}_r \tag{32}$$

where

$$\underset{\sim}{\theta}_r(x',y',z',t)=\{\theta_{xr},\theta_{yr},\theta_{zr}\}=\tfrac{1}{2}\mathrm{curl}\,\underset{\sim}{u}_r=\tfrac{1}{2}[(\frac{\partial w_r}{\partial y'} - \frac{\partial v_r}{\partial z'})\hat{i}+(\frac{\partial u_r}{\partial z'} - \frac{\partial w_r}{\partial x'})\hat{j}+(\frac{\partial v_r}{\partial x'} - \frac{\partial u_r}{\partial y'})\hat{k}].$$

By repeating and extending the arguments used in the rigid body theory, the unit normal vector on the instantaneous wetted surface area S may be written as

$$\underset{\sim}{n}|_S = \underset{\sim}{n}|_{\overline{S}} + \underset{\sim}{\theta}\times\underset{\sim}{n}|_{\overline{S}} \tag{33}$$

whilst the velocity of the steady flow $\underset{\sim}{W}$ due to the deflection of the structure is

$$\underset{\sim}{W}|_S = [1 + (\underset{\sim}{u}.\nabla)]\underset{\sim}{W}|_{\overline{S}} \;. \tag{34}$$

Now in the body fixed axis system O'x'y'z' the rigid body modes of the dry structure may be expressed as

$$\underset{\sim}{u}_1=(1,0,0) \;,\; \underset{\sim}{u}_2=(0,1,0) \;,\; \underset{\sim}{u}_3=(0,0,1) \;,\; \underset{\sim}{u}_4=(0,-z',y'),\; \underset{\sim}{u}_5=(z',0,-x'),\; \underset{\sim}{u}_6=(-y',x',0) \tag{35}$$

and these correspond to the principal coordinates $p_1(t)$, $p_2(t)$, ..., $p_6(t)$. If we denote

$$\underset{\sim}{\eta} = \{p_1(t),p_2(t),p_3(t)\},\quad \underset{\sim}{\Omega} = \{p_4(t),p_5(t),p_6(t)\} \quad\text{and}\quad \underset{\sim}{\alpha} = \underset{\sim}{\eta} + \underset{\sim}{\Omega}\times\underset{\sim}{r}'$$

then it follows that the deflection

$$\underset{\sim}{u} = \underset{\sim}{\alpha} + \sum_{r=7}^{\infty}p_r(t)\underset{\sim}{u}_r \;,$$

rotation

$$\underset{\sim}{\theta} = \underset{\sim}{\Omega} + \sum_{r=7}^{\infty}p_r(t)\underset{\sim}{\theta}_r \;,$$

velocity

$$\underset{\sim}{V}^S = \overline{U}\hat{i} + \dot{\underset{\sim}{\alpha}} + \sum_{r=7}^{\infty}\dot{p}_r(t)\underset{\sim}{u}_r \quad ,$$

unit normal

$$\underset{\sim}{n}|_S = \underset{\sim}{n}|_{\overline{S}} + \underset{\sim}{\Omega}\times\underset{\sim}{n}|_{\overline{S}} + \sum_{r=7}^{\infty}p_r(t)\underset{\sim}{\theta}_r\times\underset{\sim}{n}|_{\overline{S}} \;,$$

and the velocity of the steady flow

$$\underset{\sim}{W}|_S = [1 + (\underset{\sim}{\alpha}.\nabla)]\underset{\sim}{W}|_{\overline{S}} + \sum_{r=7}^{\infty}p_r(t)(\underset{\sim}{u}_r.\nabla)\underset{\sim}{W}|_{\overline{S}}$$

It is immediately clear that if the body has no distortion i.e. $p_r(t)=0$ for $r\geq7$ then the rigid body theory is obtained.

VELOCITY POTENTIAL

The unsteady component of the velocity potential function must include contributions accounting for the distortions of the structure in the fluid. Thus as a generalisation of the formulation adopted previously in the rigid body theory, the total velocity potential is again of the form

$$\Phi(x_0,y_0,z_0,t) \equiv \overline{U}\,\overline{\phi}(x,y,z) + \phi(x,y,z,t) \tag{36}$$

where now, for the flexible body, the unsteady component

$$\phi(x,y,z,t) = \phi_0(x,y,z,t) + \phi_D(x,y,z,t) + \sum_{r=1}^{\infty}\phi_r(x,y,z,t) \quad . \tag{37}$$

In a linear structural dynamics theory the deflection of the structure may be expressed by a series of distortions in the principal modes and a similar interpretation may be adopted for the unsteady velocity potential. That is, the unsteady potential may be described by a series of potentials $\phi_1, \phi_2, \ldots, \phi_6, \phi_7, \phi_8, \ldots$ each component corresponding to each of the principal modes of the dry structure. The subscript numbers 1 to 6 relate to rigid body modes and 7 onwards to the distortions of the dry structure. Therefore no distinction need be made between rigid and flexible body radiation potentials $\phi_r(r=1,2,\ldots,\infty)$, each being treated equally in the mathematical model and satisfying the same set of boundary conditions (i.e. free surface, bottom and radiation conditions, body etc.) .

By analogy with the rigid body theory, each of these radiation potentials may be written in the form

$$\phi_r(x,y,z,t) = \phi_r(x,y,z)p_r(t) \quad \text{for } r=1,2,\ldots,\infty \, . \tag{38}$$

Thus the principal coordinate $p_1(t)$ relates to surge motion; $p_2(t)$, sway; $p_3(t)$, heave; $p_4(t)$, roll; $p_5(t)$, pitch; $p_6(t)$, yaw and $p_7(t)$, $p_8(t)$,... to the distortions of the structure. Further in a linear theory for a sinusoidal excitation of frequency ω_e the principal coordinates are given by

$$p_r(t) = p_r e^{i\omega_e t} \tag{39}$$

where the amplitude p_r of the principal coordinate may be complex in form.

GENERALISED TIMMAN-NEWMAN RELATIONSHIPS

The boundary condition applicable on the instantaneous wetted surface area S of the flexible body is again of the form

$$\partial\Phi/\partial n = \underset{\sim}{V}^S.\underset{\sim}{n} \quad .$$

After substituting from equations (14), (31) and (36) this reduces to

$$(\overline{U}\nabla\overline{\phi} + \nabla\phi).\underset{\sim}{n} = (\overline{U}\hat{i} + \dot{\underset{\sim}{u}}).\underset{\sim}{n} \quad \text{or} \quad \partial\phi/\partial n = (\dot{\underset{\sim}{u}} - \underset{\sim}{W}).\underset{\sim}{n} \;\text{ on } S \; .$$

However, since

$$\underset{\sim}{W}|_S = [1 + (\underset{\sim}{u}.\nabla)]\underset{\sim}{W}|_{\overline{S}}$$

etc. it follows, after neglecting the second order terms in ϕ, $\underset{\sim}{u}$ and $\underset{\sim}{\theta}$, that the linearised boundary condition may be reduced to the form

$$\partial\phi/\partial n = [\dot{\underset{\sim}{u}} + \underset{\sim}{\theta}x\underset{\sim}{W} - (\underset{\sim}{u}.\nabla)\underset{\sim}{W}].\underset{\sim}{n} \;\text{ on } \overline{S} \quad . \tag{40}$$

Further substitution of these quantities in series form gives

$$\sum_{r=1}^{\infty}[\partial\phi_r/\partial n - i\omega_e\underset{\sim}{u}_r.\underset{\sim}{n} - \underset{\sim}{\theta}_r x\underset{\sim}{W}.\underset{\sim}{n} + (\underset{\sim}{u}_r.\nabla)\underset{\sim}{W}.\underset{\sim}{n}]p_r e^{i\omega_e t} = 0 \;\text{ on } \overline{S}$$

and this boundary condition must be satisfied for any arbitrary combination of p_r. This is always true if the condition is satisfied for each p_r. That is

$$\partial\phi_r/\partial n = [i\omega_e\underset{\sim}{u}_r + \underset{\sim}{\theta}_r x\underset{\sim}{W} - (\underset{\sim}{u}_r.\nabla)\underset{\sim}{W}].\underset{\sim}{n} \;\text{ on } \overline{S} \tag{41}$$

for each $r=1,2,\ldots,\infty$. This expression is a generalisation of the Timman-Newman relationships derived previously for the rigid body modes. This is immediately seen since for modes $r=1,2,\ldots,6$ the variables $\underset{\sim}{\alpha}$ and $\underset{\sim}{\Omega}$ used in the rigid body theory have their equivalents in $\underset{\sim}{u}$ and $\underset{\sim}{\theta}$. Thus this body surface boundary condition is valid for all modes - rigid body modes or flexible modes.

In the special (though usually taken) case, when the steady flow is approximated to $\underset{\sim}{W} = -\bar{U}i$, the body boundary condition reduces to

$$\partial\phi_r/\partial n = i\omega_e(u_r n_1 + v_r n_2 + w_r n_3) + \bar{U}/2[n_3(\partial u_r/\partial z' - \partial w_r/\partial x') - n_2(\partial v_r/\partial x' - \partial u_r/\partial y')] \text{ on } \bar{S} \quad (42)$$

for each $r=1,2,\ldots,\infty$. After substituting for the dry hull body mode shapes $r=1,2,\ldots,6$ in equation (35) the body surface boundary condition derived in the rigid body theory section is again obtained.

LINEARISED FLEXIBLE BOUNDARY CONDITIONS

The linear velocity potentials associated with the flow around the flexible body satisfy the following boundary conditions:

(i) On the free surface the incident, diffracted and radiation potentials i.e. ϕ_O, ϕ_D and ϕ_r ($r=1,2,\ldots,6,7,.,\infty$) respectively satisfy the linearised boundary condition

$$\bar{U}^2\partial^2\phi/\partial x^2 - 2i\omega_e\bar{U}\partial\phi/\partial x - \omega_e^2\phi + g\partial\phi/\partial z = 0 \quad \text{on } z = 0 \quad (43)$$

where ϕ represents either ϕ_O, ϕ_D or ϕ_r.

(ii) Suitable bottom and radiation conditions at infinite distance from the oscillating, translating structure.

(iii) The incident and diffracted potentials satisfy the relationship

$$\partial\phi_D/\partial n = -\partial\phi_o/\partial n \qquad \text{on } \bar{S} \ . \quad (44)$$

(iv) The radiation potentials are governed by the body boundary condition

$$\partial\phi_r/\partial n = [i\omega_e\underset{\sim}{u}_r + \underset{\sim}{\theta}_r \times \underset{\sim}{W} - (\underset{\sim}{u}_r.\nabla)\underset{\sim}{W}].\underset{\sim}{n} \qquad \text{on } \bar{S} \quad ,$$

or when $\underset{\sim}{W}=-\bar{U}\hat{i}$,

$$\partial\phi_r/\partial n = i\omega_e(u_r n_1 + v_r n_2 + w_r n_3) + \bar{U}/2[n_3(\partial u_r/\partial z' - \partial w_r/\partial x') - n_2(\partial v_r/\partial x' - \partial u_r/\partial y')] \text{ on } \bar{S} \quad (45)$$

for each $r=1,2,\ldots,6,7\ldots,\infty$.

PRESSURE DISTRIBUTION

The fluid pressure acting on the instantaneous wetted body surface S during the oscillatory motion of the flexible structure is according to Bernoulli's formulation given by

$$p = -\rho[\partial\phi/\partial t + \underset{\sim}{W}.\nabla\phi + \tfrac{1}{2}(W^2 - \bar{U}^2) + \tfrac{1}{2}\nabla\phi.\nabla\phi + gz] . \quad (46)$$

Unfortunately to determine this expression a knowledge of the position of S is necessary and as discussed by Newman (1978) this difficulty may be satisfactorily overcome by relating the pressure on the surface S to the pressure on the body mean surface $\bar{S}$ by means of a Taylor series expansion. Thus, for the flexible structure, it follows that

$$p|_S = [1 + (\underset{\sim}{u}.\nabla) + \tfrac{1}{2}(\underset{\sim}{u}.\nabla)^2 + \ldots\ ]p|_{\bar{S}} \quad . \quad (47)$$

If it is assumed that the oscillatory motion of the body and parasitic flow are small i.e. neglecting the second order terms of the unsteady component, then the pressure on the wetted surface S becomes

$$p|_S = -\rho[\partial\phi/\partial t + \underset{\sim}{W}.\nabla\phi + \{\tfrac{1}{2}(W^2 - \bar{U}^2) + gz\} + \{gw + \tfrac{1}{2}(\underset{\sim}{u}.\nabla\ W^2)\}]_{\bar{S}} \ . \quad (48)$$

This implies that the oscillatory flow and the motion of the structure are linearised but the steady flow due to the steady forward motion remains non-linear.

Table 1 illustrates a comparison of the orders of magnitude of the terms involved in this formulation i.e. equation (48) based on differing body geometry assumptions. For example, in the three dimensional case in column 1, the geometry of the structure is such that the three main dimensions are of the same order. Thus it follows that and $(x,y,z)=\{o(1),o(1),o(1)\}$, $\underset{\sim}{n}=(n_1,n_2,n_3)=\{o(1),o(1),o(1)\}$

$$\bar{\phi} = o(L) = o(1) \ , \qquad \underset{\sim}{W} = \bar{U}\{o(1),o(1),o(1)\} \text{on } S \text{ and } \bar{S} \ .$$

For the unsteady potential ϕ, the incident wave potential ϕ_0 is independent of body geometry and its order of magnitude depends on the wave amplitude $\bar{a}=a/L$ and wave frequency ω. Because of the fluid boundary conditions on the body, the diffraction potential is of the same order as ϕ_0 and the radiation potential

$$\phi_R = \sum_{r=1}^{\infty}\phi_r = \phi - (\phi_0 + \phi_D) = o(|\underset{\sim}{u}|L\omega_e) \ .$$

The order of magnitude of each term is shown in Table 1, column 1 and the terms are non-dimensionalised with respect to the four parameters $\bar{a}=a/L$, $\bar{u}=|\underset{\sim}{u}|/L$, $\delta=\omega_e\sqrt{(L/g)}$ and Froude number $Fn=\bar{U}/\sqrt{(gL)}$.

If it is assumed that $Fn=o(1)$ and $\bar{a}=o(1)$ then the leading order of terms in the pressure equation (48) is of $o(\bar{u})$ and no term can be ignored. If $\delta=o(\varepsilon^{-\frac{1}{2}})$ the last terms in the pressure equation are of $o(\bar{u})$ and the remaining terms are of $o(\bar{u}\varepsilon^{-\frac{1}{2}})$ or $o(1)$. According to Newman (1978), this is the resonant frequency region for the rigid body motions (i.e. heave, pitch and roll) so that $\bar{u}=o(1)$ and all the terms in the pressure equation must be retained. However in the high frequency region i.e. $\delta \geq o(\varepsilon^{-1})$ the resonant frequencies of the structure are to be found - though this naturally depends on the type and flexibility of the structure - and the magnitude of the distortions are unlikely to be of $o(1)$. In this case the leading order of terms in the pressure equation is of $o(1)$ and reduces to

$$p = -\rho[\partial\phi/\partial t + \underset{\sim}{W}.\nabla\phi + \tfrac{1}{2}(W^2-\bar{U}^2) + gz]_{\bar{S}} \quad . \tag{49}$$

Table 1. Comparison of the orders of magnitude of the terms involved in the expression for the pressure on the body surface in equation (48).

Item	General 3D case	Slender body	Thin body	Flat body
Length of body L	α	α	α	α
Beam of body B	α	β	β	α
Draught T	α	β	α	β
$\underset{\sim}{r}=(x,y,z)$	(α,α,α)	(α,β,β)	(α,β,α)	(α,α,β)
$\underset{\sim}{n}=(n_1,n_2,n_3)$	(α,α,α)	(β,α,α)	(β,α,β)	(β,β,α)
$\underset{\sim}{r}\times\underset{\sim}{n}=(n_4,n_5,n_6)$	(α,α,α)	(β,α,α)	(α,β,α)	(α,α,β)
$\bar{\phi}$	$o(L)$	$o(\varepsilon^2L)$	$o(\varepsilon^2L)$	$o(\varepsilon^2L)$
$\underset{\sim}{W}=\bar{U}(\partial\bar{\phi}/\partial x-1,\partial\bar{\phi}/\partial y,\partial\bar{\phi}/\partial z)$	$\bar{U}(\alpha,\alpha,\alpha)$	$\bar{U}(\alpha,\beta,\beta)$	$\bar{U}(\alpha,\beta,\gamma)$	$\bar{U}(\alpha,\gamma,\beta)$
$\phi_R=\sum_{r=1}^{\infty}\phi_r$	$o(\|\underset{\sim}{u}\|L\omega_e)$	$o(\|\underset{\sim}{u}\|L\varepsilon\omega_e)$	$o(\|\underset{\sim}{u}\|L\varepsilon\omega_e)$	$o(\|\underset{\sim}{u}\|L\varepsilon\omega_e)$
$\partial/\partial t(\phi_0+\phi_D)/gL$	$o(\bar{a})$	$o(\bar{a})$	$o(\bar{a})$	$o(\bar{a})$
$\underset{\sim}{W}.\nabla(\phi_0+\phi_D)/gL$	$o(\bar{a}Fn\delta)$	$o(\bar{a}Fn\delta)$	$o(\bar{a}Fn\delta)$	$o(\bar{a}Fn\delta)$
$\partial/\partial t\phi_R/gL$	$o(\bar{u}\delta^2)$	$o(\bar{u}\varepsilon\delta^2)$	$o(\bar{u}\varepsilon\delta^2)$	$o(\bar{u}\varepsilon\delta^2)$
$\underset{\sim}{W}.\nabla\phi_R/gL$	$o(\bar{u}Fn\delta)$	$o(\bar{u}\varepsilon Fn\delta)$	$o(\bar{u}\varepsilon Fn\delta)$	$o(\bar{u}\varepsilon Fn\delta)$
$(W^2-\bar{U}^2)/gL$	$o(Fn^2)$	$o(Fn^2\varepsilon^2)$	$o(Fn^2\varepsilon^2)$	$o(Fn^2\varepsilon^2)$
z'/L	α	β	α	β
w/L	$o(\bar{u})$	$o(\bar{u})$	$o(\bar{u})$	$o(\bar{u})$
$(\underset{\sim}{u}.\nabla)W^2/gL$	$o(\bar{u}Fn^2)$	$o(\bar{u}\varepsilon Fn^2)$	$o(\bar{u}\varepsilon Fn^2)$	$o(\bar{u}\varepsilon Fn^2)$

$[\alpha \equiv o(1), \quad \beta \equiv o(\varepsilon), \quad \gamma \equiv o(\varepsilon^2)]$

When the body is slender, thin or flat, the leading order of terms in the pressure equation (48) is of $o(\bar{u}\varepsilon)$ provided that $\bar{a}=o(1)$ and the Froude number $Fn=o(1)$. In this case, Table 1 indicates that the pressure equation reduces to

$$p|_S = -\rho[\partial\phi/\partial t + \underset{\sim}{W}.\nabla\phi + \tfrac{1}{2}(W^2-\bar{U}^2) + gz' + gw]_{\bar{S}} \ . \qquad (50)$$

The third and fourth terms describe contributions from steady state and hydrostatic components respectively. They modify the generalised still water forces and in turn the equilibrium position of the body.

Based on the approximation that the steady flow $\underset{\sim}{W}=-\bar{U}\hat{i}$, the previous pressure equation on S reduces to

$$p = -\rho[\partial\phi/\partial t - \bar{U}\partial\phi/\partial x]_{\bar{S}} - \rho g[z' + w]_{\bar{S}} \ . \qquad (51)$$

GENERALISED FLUID FORCES

The rth component of the generalised external force $\underset{\sim}{Z}$ acting on the flexible structure due to the fluid only may be expressed in the form

$$Z_r(t) = -\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r p dS \qquad (52)$$

where $\underset{\sim}{n}^T$ is the transpose of the unit normal vector pointing out of the body surface into the fluid and $\underset{\sim}{u}_r$ is the rth principal mode vector associated with the dry structure. This integration extends over the instantaneous body wetted surface S.

Expression (52) may be shown to be equivalent to the rth generalised fluid force defined previously in equation (5). That is

$$Z_r = \underset{\sim}{D}_r^T\underset{\sim}{P} = \sum_e \underset{\sim}{d}_r^T\underset{\sim}{P}_e$$

where the summation $\sum_e$ includes contributions from all the elements within the instantaneous wetted surface S and the vector P_e denotes the distribution of the externally applied fluid force over the element and is defined with respect to the equilibrium axes.

By use of the principle of virtual work it has been shown by Bishop, Price and Wu (1984) that the generalised force related to the hydrodynamic pressure p can be expressed as

$$\bar{P}_e = -\iint_{S_e} p\underset{\sim}{N}^T.\underset{\sim}{\bar{n}} dS$$

where $\bar{n}$ is the outward drawn unit normal vector on the wetted finite element surface S_e measured in the local element coordinate system. $\bar{P}_e$ is associated with the nodal displacements and is equivalent to a set of concentrated forces or moments acting at the nodes of the elements doing an equivalent amount of work to the pressure p during the motion of the flexible structure. In addition transformations $\underset{\sim}{L}^T$ and $\underset{\sim}{\ell}$ may be defined such that

$$\underset{\sim}{P}_e = \underset{\sim}{L}^T\underset{\sim}{\bar{P}}_e \ , \qquad \underset{\sim}{\bar{n}} = \underset{\sim}{\ell}\underset{\sim}{n} \ ,$$

and these relate quatities between the local element coordinate system and the global or equilibrium axes systems.

Thus it follows from these previous results that the rth generalised fluid force can be written as

$$Z_r = -\sum_e \underset{\sim}{d}_r^T\underset{\sim}{L}^T\iint_{S_e}\underset{\sim}{N}^T\underset{\sim}{\ell}\underset{\sim}{n} p dS \ .$$

Taking the transpose of this expression, using the definition of the rth principal mode shape vector u_r for the overall structure and the fact that Z_r scalar, then it follows that

$$Z_r = -\sum_e\iint_{S_e}\underset{\sim}{n}^T(\underset{\sim}{\ell}^T\underset{\sim}{N}\underset{\sim}{L}\underset{\sim}{d}_r)p dS = -\iint_S\underset{\sim}{n}^T.\underset{\sim}{u}_r p dS$$

which agrees with the expression in equation (52).

Substituting the general expression i.e. equation (48) for the pressure p into equation (52), and including also the contribution from the generalised gravitational force, the rth generalised external force may be expressed in the component form

$$Z_r(t) = \Xi_r(t) + H_r(t) + R_r(t) + \bar{R}_r \qquad \text{for } r=1,2,..,m. \qquad (53)$$

and the rth generalised wave exciting force is defined as

$$\Xi_r(t) = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r[\partial/\partial t + \underset{\sim}{W}.\nabla](\phi_0 + \phi_D)dS \quad , \qquad (54)$$

the rth generalised radiation force

$$H_r(t) = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r[\partial/\partial t + \underset{\sim}{W}.\nabla]\sum_{k=1}^{m} p_k(t)\phi_k \, dS \quad , \qquad (55)$$

the rth generalised restoring force

$$R_r(t) = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r[gw + \tfrac{1}{2}(\underset{\sim}{u}.\nabla)W^2]dS \quad , \qquad (56)$$

and the rth generalised hydrostatic force

$$\bar{R}_r = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r[gz' + \tfrac{1}{2}(W^2-\bar{U}^2)]dS - \iiint_{V} \rho_b g w_r dV \quad . \qquad (57)$$

The first three components are associated with unsteady deflections of the structure whilst the hydrostatic contribution is independent of all unsteady motions. The latter contains components arising from the generalised hydrostatic fluid action, generalised forces due to the structure travelling with constant forward speed in calm water and the generalised gravitational force described by the volume integral.

GENERALISED WAVE FORCE

In regular sinusoidal waves the incident and diffraction potentials are both sinusoidal functions of encounter frequency ω_e. Hence the rth generalised wave exciting force may be written as

$$\Xi_r(t) = \Xi_r e^{i\omega_e t} = (\Xi_{or} + \Xi_{Dr})e^{i\omega_e t} \quad ,$$

where from equation (54) the rth generalised Froude-Krylov contribution is

$$\Xi_{or} = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r[i\omega_e + \underset{\sim}{W}.\nabla]\phi_0 dS \qquad (58)$$

and the rth generalised diffraction force accounting for the scattering of the incident wave due to presence of the flexible structure is

$$\Xi_{Dr} = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r[i\omega_e + \underset{\sim}{W}.\nabla]\phi_D dS \quad . \qquad (59)$$

When $\underset{\sim}{W}=-\bar{U}\hat{i}$, the rth generalised Froude-Krylov contribution reduces to

$$\Xi_{or} = \rho\iint_S \underset{\sim}{n}^T.\underset{\sim}{u}_r\omega\phi_0 dS \qquad (60)$$

and is independent of forward speed for each r=1,2,...,m.

GENERALISED RADIATION FORCE

Assuming a sinusoidal solution of the rth principal coordinate in the form

$$p_r(t) = p_r e^{i\omega_e t} \quad ;$$

it follows from equation (55) that the rth generalised radiation force may be written as

$$H_r(t) = \sum_{k=1}^{m} p_k T_{rk} e^{i\omega_e t} = \sum_{k=1}^{m} p_k (\omega_e^2 A_{rk} - i\omega_e B_{rk}) e^{i\omega_e t} \tag{61}$$

for r=1,2,...,m. The coefficients (r=1,2,...,m; k=1,2,...,m)

$$A_{rk} = (\rho/\omega_e^2)\ \mathrm{Re}\{\iint_S \underset{\sim}{n}^T . \underset{\sim}{u}_r [i\omega_e + \underset{\sim}{W}.\nabla]\phi_k dS\} \tag{62}$$

is in phase with the acceleration whilst

$$B_{rk} = (-\rho/\omega_e)\ \mathrm{Im}\{\iint_S \underset{\sim}{n}^T . \underset{\sim}{u}_r [i\omega_e + \underset{\sim}{W}.\nabla]\phi_k dS\ \} \tag{63}$$

is in phase with the velocity.

The terms A_{rk} and B_{rk} represent added mass or inertias and damping coefficients respectively associated with the rth mode per unit oscillatory distortion in the kth mode. The theory indicates that these coefficients may be determined experimentally by forced oscillation of the flexible structure in a prescribed dry hull mode shape at the arbitrary frequency ω_e as the structure travels with constant speed in calm water.

GENERALISED RESTORING FORCE

Since the displacement at any arbitrary point in the structure is given by

$$\underset{\sim}{u} = \{u,v,w\} = \sum_{k=1}^{m} \underset{\sim}{u}_k p_k e^{i\omega_e t} = \sum_{k=1}^{m} \{u_k, v_k, w_k\} p_k e^{i\omega_e t}\ ,$$

the rth generalised restoring force in equation (56) may be written as

$$R_r(t) = - \sum_{k=1}^{m} p_k C_{rk} e^{i\omega_e t} \tag{64}$$

where the generalised restoring force coefficient

$$C_{rk} = -\rho \iint_S \underset{\sim}{n}^T . \underset{\sim}{u}_r [g w_k + \tfrac{1}{2}(\underset{\sim}{u}_k.\nabla) W^2] dS$$

for r=1,2,...,m and k=1,2,...,m. The second term in this integral produces contributions to the rth generalised restoring force due to unit unsteady motions of the structure within the steady pressure field. For a general three dimensional body of arbitrary shape, this contribution is of the same order as obtained from the first term. However, for slender, thin or flat structures the second term is an order smaller than the first term and may be neglected by comparison. Thus for these types of structures the generalised restoring force coefficient reduces to the simpler form

$$C_{rk} = -\rho g \iint_S \underset{\sim}{n}^T . \underset{\sim}{u}_r w_k dS \quad \text{for } r=1,2,\ldots,m;\ k=1,2,\ldots,m. \tag{65}$$

It can be easily shown that this formulation includes a description of the restoring coefficients usually associated with the rigid body modes only.

GENERALISED EQUATIONS OF MOTION

From the theory presented, the generalised linear equations describing the responses of the floating flexible marine structure may be expressed in the general form

$$\omega_r^2 a_{rr} p_r(t) + \sum_{k=1}^{m} [a_{rk} \ddot{p}_k(t) + b_{rk} \dot{p}_k(t)] = Z_r(t) = \Xi_r(t) + H_r(t) + R_r(t) + \overline{R}_r \tag{66}$$

for r=1,2,...,m. In this formulation the generalised mass a_{rk} is symmetric since it must be remembered that there exists the possibility of off-diagonal contributions occurring in the rigid body modes.

STEADY EQUATIONS

For the structure in calm water, there exists a steady state solution

$$p_r(t) = \bar{p}_r$$

satisfying the equation

$$a_{rr}\omega_r^2\bar{p}_r = \sum_{k=1}^{m}\bar{p}_r C_{rk} + \bar{R}_r \qquad \text{for } r=1,2,\ldots, m. \tag{67}$$

FLOATING (FREE-FREE STRUCTURE)

For the *floating* structure moving or stationary in waves and in the absence of any steady state conditions, the generalised linear equation of motion may be written in the form

$$\omega_r^2 a_{rr}p_r(t) + \sum_{k=1}^{m}[(a_{rk}+A_{rk})\ddot{p}_k(t) + (b_{rk}+B_{rk})\dot{p}_k(t)+C_{rk}p_k(t)] = \Xi_r e^{i\omega_e t} \tag{68}$$

for r=1,2,...m. Alternatively in matrix form this set of equations becomes

$$(\underset{\sim}{a} + \underset{\sim}{A})\ddot{\underset{\sim}{p}}(t) + (\underset{\sim}{b} + \underset{\sim}{B})\dot{\underset{\sim}{p}}(t) + (\underset{\sim}{c} + \underset{\sim}{C})\underset{\sim}{p}(t) = \underset{\sim}{\Xi}(t) = \underset{\sim}{\Xi}e^{i\omega_e t} \tag{69}$$

where

$\underset{\sim}{a}$ is the inertia matrix of the dry structure and is diagonal apart from possible off-diagonal elements occurring in the rigid body modes (r=1,2,...,6).

$\underset{\sim}{A}$ is the hydrodynamic inertia matrix .

$\underset{\sim}{b}$ is the structural damping matrix of the dry structure and is usually assumed diagonal.

$\underset{\sim}{B}$ is the hydrodynamic damping matrix.

$\underset{\sim}{c}$ is the diagonal stiffness matrix with elements $c_{ss}=\omega_s^2 a_{ss}$. The natural frequency $\omega_s=0$ for the rigid body modes s=1,2,...,6.

$\underset{\sim}{C}$ is the hydrodynamic restoring or stiffness matrix.

Thus assuming a solution of the principal coordinate matrix in the form

$$\underset{\sim}{p}(t) = \underset{\sim}{p}e^{i\omega_e t} \quad ,$$

equation (69) reduces to

$$\underset{\sim}{I}\underset{\sim}{p} = \frac{\text{adj } \underset{\sim}{D}}{\det \underset{\sim}{D}}\ \underset{\sim}{\Xi} \tag{70}$$

where $\underset{\sim}{I}$ is a unit matrix,

$$\underset{\sim}{D} = -\omega_e^2(\underset{\sim}{a} + \underset{\sim}{A}) + i\omega_e(\underset{\sim}{b} + \underset{\sim}{B}) + (\underset{\sim}{c} + \underset{\sim}{C}) \ , \tag{71}$$

and the matrices $\underset{\sim}{A}$, $\underset{\sim}{B}$ and $\underset{\sim}{D}$ are functions of the frequency of encounter ω_e.

Hence from a knowledge of the principal mode shapes of the dry structure and the determined principal coordinates the displacement at any position in the structure is given by

$$\underset{\sim}{u}(x,y,z,t) = \sum_{r=1}^{m}\underset{\sim}{u}_r(x,y,z)p_r e^{i\omega_e t} \quad . \tag{72}$$

The bending moments, shearing forces, twisting moments and any other relevant response may be similarly determined using the appropriate principal mode shape of the dry structure.

FIXED (CLAMPED, PINNED ETC. STRUCTURES)

For a *fixed* flexible structure, rigid body modes no longer exist and only the distortion modes r=7,8,... are applicable and need be included in the theory. Naturally the shapes of these modes depend on the imposed end conditions i.e. clamped, pinned etc. In fact for a fixed cantilevered vertical structure (say). idealised end conditions may be imposed i.e. clamped-free and these mode shapes adopted in the theory. However, in reality such idealised end conditions may not be a true relfection of the actual end conditions due say to structure-soil interaction. The theory presented remains valid and by a suitable description of the externally applied generalised force a theoretical model based on the idealised clamped mode shapes may be developed which accounts for the more realistic physical end condition. From such a model the responses of the fixed structure may be determined once again. Thus for the *fixed* structure in waves of frequency ω the generalised linear equations describing the principal coordinate response now take the form

$$a_{rr}[\ddot{p}_r(t) + \omega_r^2 p_r(t)] + \sum_{k=7}^{m}[A_{rk}\ddot{p}_k(t) + (b_{rk}+B_{rk})\dot{p}_k(t)+C_{rk}p_k(t)] = \Xi_r e^{i\omega t} \qquad (73)$$

for r=7,8,...,m. By a simple process of redefining and renumbering (i.e. let r=1 denote the first distortion mode, etc.) this set of equations may be caste into the matrix form discussed previously.

MODIFICATIONS (MOORING LINES, NON-LINEARITIES ETC.)

Further modifications may be introduced into the existing theory. For example, if mooring lines are attached to the structure additional external concentrated applied loadings need be included in the description of the rth generalised external applied force. In a simple model, the mooring forces will provide an additional component to the restoring force term in the steady state condition and the general equation describing the unsteady motion may be written in the form

$$(\underset{\sim}{a} + \underset{\sim}{A})\ddot{\underset{\sim}{p}}(t) + (\underset{\sim}{b} + \underset{\sim}{B})\dot{\underset{\sim}{p}}(t) + (\underset{\sim}{c} + \underset{\sim}{C} + \Delta\underset{\sim}{C})\underset{\sim}{p}(t) = \underset{\sim}{\Xi}(t) \qquad (74)$$

where the generalised restoring matrix $\Delta\underset{\sim}{C}$ describes the linear contribution arising from the mooring lines.

An underlying assumption in all the theory presented relates to a linear description of the dry structural model so that mode shapes may be determined. However no such assumption need be imposed on the rth generalised external force although in this paper a linear description is only discussed. However with suitable modifications, a non-linear description of the fluid actions may be included in the theory and the equations describing the responses of the structure may be expressed in the form

$$(\underset{\sim}{a} + \underset{\sim}{A})\ddot{\underset{\sim}{p}}(t) + (\underset{\sim}{b} + \underset{\sim}{B})\dot{\underset{\sim}{p}}(t) + (\underset{\sim}{c} + \underset{\sim}{C})\underset{\sim}{p}(t) = \underset{\sim}{\Xi}(t) + \mathcal{F}(\underset{\sim}{p},\dot{\underset{\sim}{p}},\ldots) \qquad (75)$$

where the matrix $\mathcal{F}$ denotes a generalised contribution resulting from the non-linear external fluid actions and contains elements involving products of the coordinate $\underset{\sim}{p}(t)$. Such a mathematical model includes the Morison type formulation used to describe the non-linear fluid actions applied to fixed structures. Unfortunately when introducing such modifications, a more semi-empirical approach becomes necessary before the problem can be readily solved.

COMPUTATIONS AND EXAMPLES

It has been shown by Brard (1972) that when applying a singularity distribution method to a surface piercing structure with forward speed a line integral contribution must be included in the expression for the velocity potential at any point (x_1,y_1,z_1) in the fluid. That is

$$\phi(x_1,y_1,z_1)=\iint_S Q(x,y,z)G(x_1,y_1,z_1;x,y,z)dS+\frac{\bar{U}^2}{g}\oint_{\bar{C}} Q(x,y,0)G(x_1,y_1,z_1;x,y,0)n_1(x,y,0)dC$$

where (x,y,z) denotes a point on the structure, the contour $\bar{C}$ is the intersection of the structure's surface and the mean calm water surface, Q is the source density on the structure's surface and G is the appropriate Green's function.

For structures with port and starboard symmetry, Price and Wu (1983a,b) have developed a composite source distribution method to evaluate the unknown velocity potentials associated with the rigid body motions of a multi-hull body. Bishop, Price and Wu (1984) have extended this method to determine the required velocity potentials when the structure is flexible. This method is adopted to determine the hydrodynamic coefficients, responses etc. of the structures discussed in the following examples. The interested reader may refer to the cited references for more information about the numerical techniques used.

(i) UNIFORM SHIP

Figure 4 illustrates a typical set of calculated results for a flexible ship travelling with Froude number Fn=0.2 in unit amplitude sinusoidal head waves. In this case the monohull structure is idealised by a uniform beam of ship like proportions. The mean wetted surface area is discretised by 180 surface panel elements and a limited comparison is included between predictions based on the present three dimensional theory and the two dimensional strip-beam approach of Bishop and Price (1979). Figures 4(a,b) illustrate the variation with frequency of the generalised hydrodynamic coefficients and wave exciting modes r=3(heave), r=5(pitch) and the first two distortion modes r=7 and 9. The pitch principal coordinate $|p_5|$ in figure 4(c) shows the dominant response occurring in the low frequency range and the coupling existing between the principal coordinates due to the fluid-structure interaction. Figure 4(d) shows the affect of including different numbers of modal contributions in the summation of the vertical distortion calculated at position x=0.25L measured from the stern and figures 4(e,f) show the variation in the amplitudes of vertical deflection and bending moment respectively along the uniform beam at different wave encounter frequencies.

(ii) MULTI-HULL

Figure 5(a) illustrates a hypothetical multi-hull structure which in this case represents a small water-plane area twin hull ship (SWATHS), though with minor modifications it could easily be a semisubmersible rig, catamaran, etc. Figure 5(b) illustrates some calculated dry hull modal characteristics (i.e. mode shapes, bending moments, principal stresses) based on a beam-plate discretisation of the structure. Generalised hydrodynamic coefficients and wave forces are illustrated in figures 5(c,d) respectively for the SWATHS travelling with Froude number Fn=0.223 in regular sinusoidal head waves. When compared with figures 4(a,b) for the monohull, the fluid-structure interaction between the port and starboard pontoons is clearly visible in the results. For an imposed vertical oscillatory deflection, a more detailed analysis reveals the generation of a standing wave symmetric about the centreline of the rig with a maximum amplitude at the centreline and a frequency in the region where the curves show fluctuations. Figure 5(e) illustrates a typical principal coordinate response, and figures 5(f,g) show the variation in the amplitude of principal stress with frequency and position along the hull.

(iii) JACK-UP RIG

For a hypothetical jack-up rig travelling at Froude number Fn=0.1 in regular sinusoidal head waves, figure 6 illustrates a selection of results relating to leg A. Although only one dry principal mode shape is shown an extensive set of results was obtained from a discretisation of the structure involving beam and plate elements. The structure is now far from beamlike or slender, however by adopting the three dimensional approach generalised hydrodynamic coefficients, wave exciting forces etc. were determined for this floating structure before a dynamic analysis of the leg was performed. It is easy to visualise that a similar analysis may be undertaken to discuss the dynamics of a mast or superstructure fixed to a ship etc.

(iv) FIXED STRUCTURE

Figure 7 illustrates the first three symmetric dry principal modes (r=1,3,5) of a cantilevered square plate (10x10x0.1m). Also included are calculated values for the first five natural and resonance frequencies. For the plate fixed horizontally in water at a depth of 5m below the free surface, the generalised hydrodynamic properties may be evaluated as discussed previously and the principal coordinates determined for an imposed oscillatory generalised force. A typical principal coordinate i.e. $|p_3|$ is shown. Naturally the same approach may be utilised to discuss the influences of water depth, free surface etc. on the responses as well as considering different plate configurations (i.e. vertical partly immersed, geometry, etc.).

Although only a simple fixed structure is considered the analysis may be readily extended to more complex fixed structures (i.e. jack-up rig, platforms, single-point mooring) with little difficulty.

CONCLUSIONS

A general unified hydroelasticity theory has been developed for fixed or floating marine structures which may be either stationary or moving with constant forward speed in regular sinusoidal waves. Based on the techniques of structural dynamics a consistent appoach has been outlined for the evaluation of the responses (i.e. displacements, distortions, bending moments, shearing forces, twisting moments, stress, etc.) of a marine structure to waves. The coupling existing between the flexible structure and fluid has been highlighted in the development of the mathematical model to determine the fluid loadings arising from incident waves, diffracted waves, bodily motions and distortions of the structure etc.

In this paper it is shown that the theory has a wide range of application being able to analyse the responses of several marine structures each having a different geometric configuration. However, the techniques and analysis developed are not restricted to marine structure problems only but may be easily modified or extended to other fields of engineering in which the description of the interaction between fluid and structure is of great importance.

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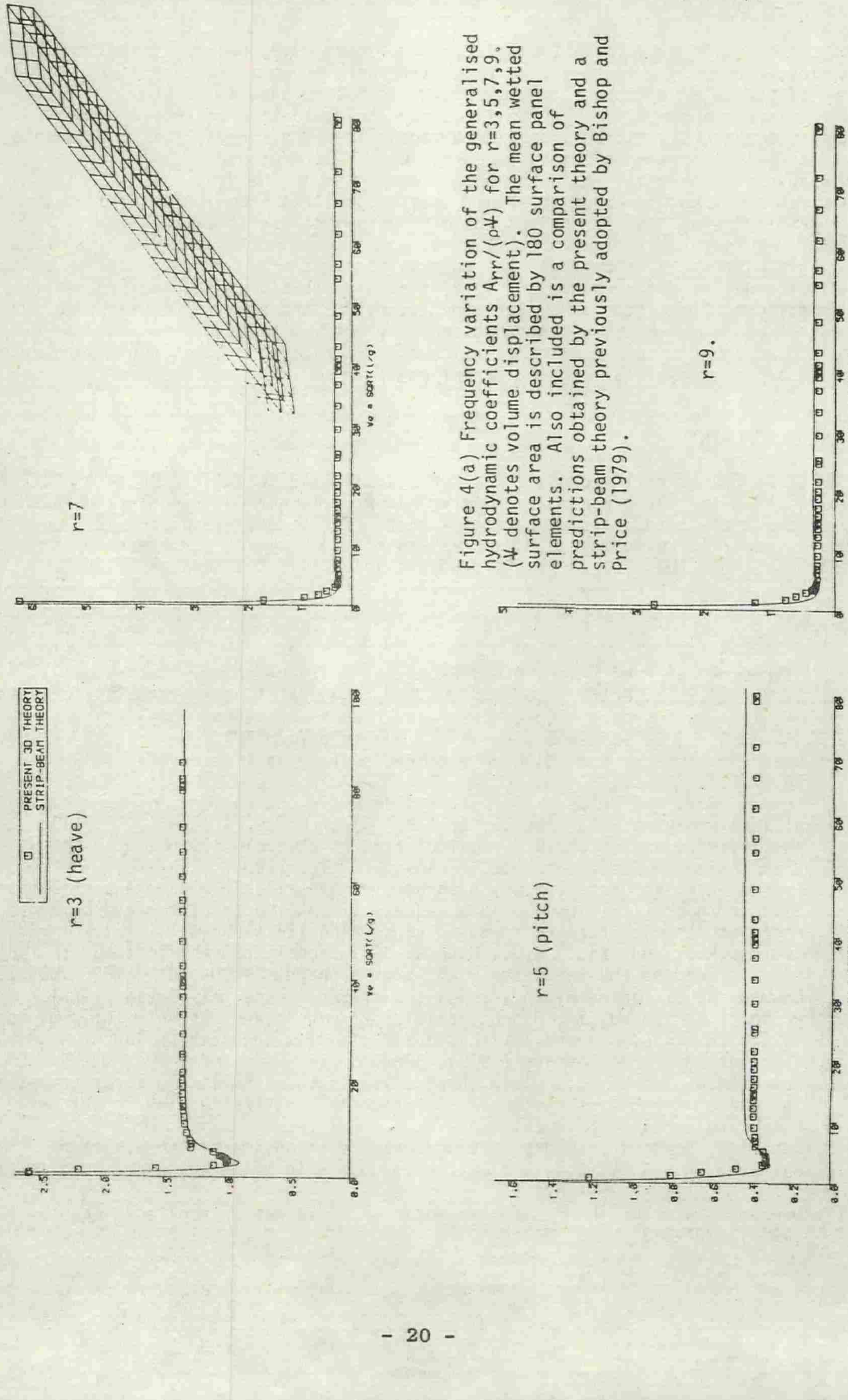


Figure 4(a) Frequency variation of the generalised hydrodynamic coefficients $A_{rr}/(\rho\forall)$ for r=3,5,7,9. ($\forall$ denotes volume displacement). The mean wetted surface area is described by 180 surface panel elements. Also included is a comparison of predictions obtained by the present theory and a strip-beam theory previously adopted by Bishop and Price (1979).

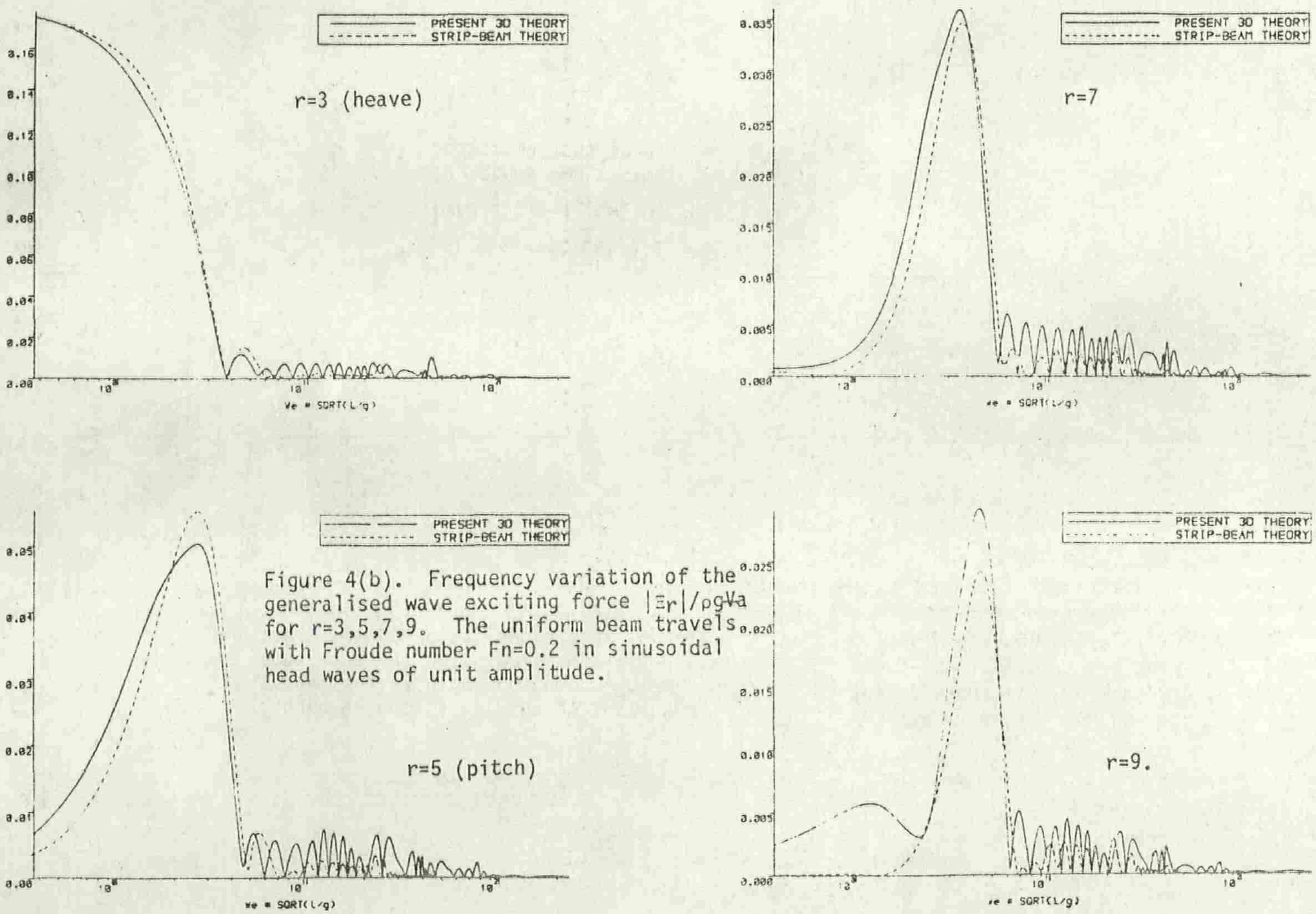


Figure 4(b). Frequency variation of the generalised wave exciting force $|\Xi_r|/\rho g V a$ for r=3,5,7,9. The uniform beam travels with Froude number Fn=0.2 in sinusoidal head waves of unit amplitude.

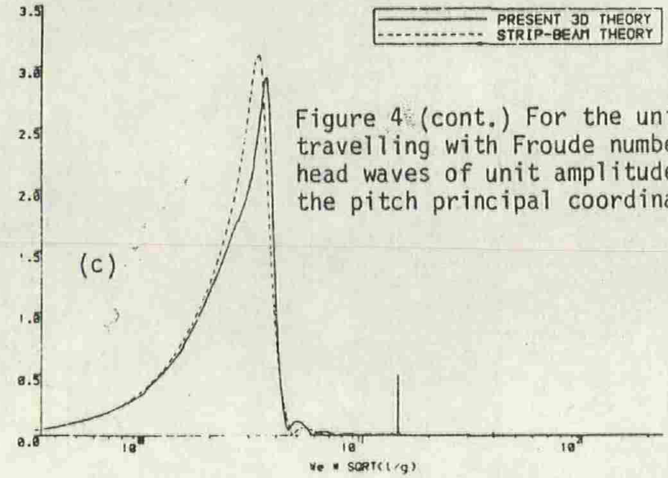
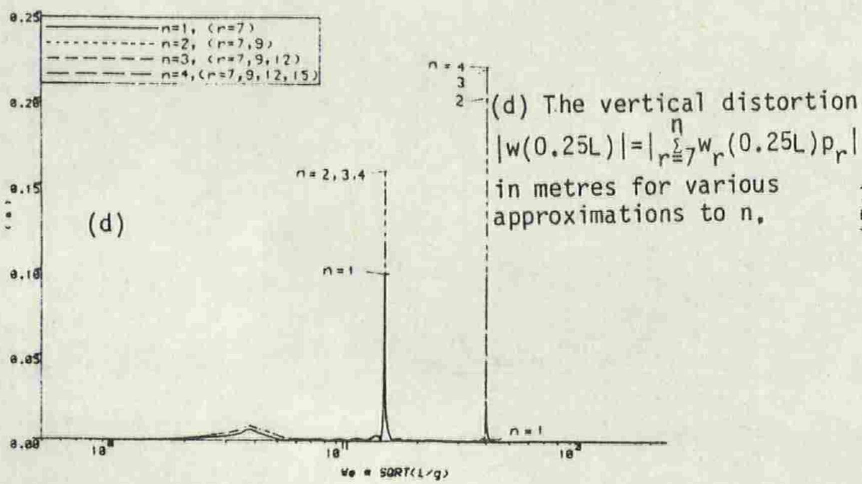
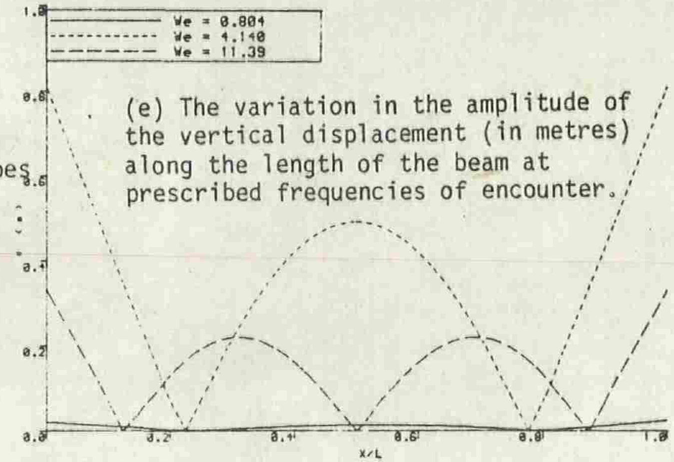


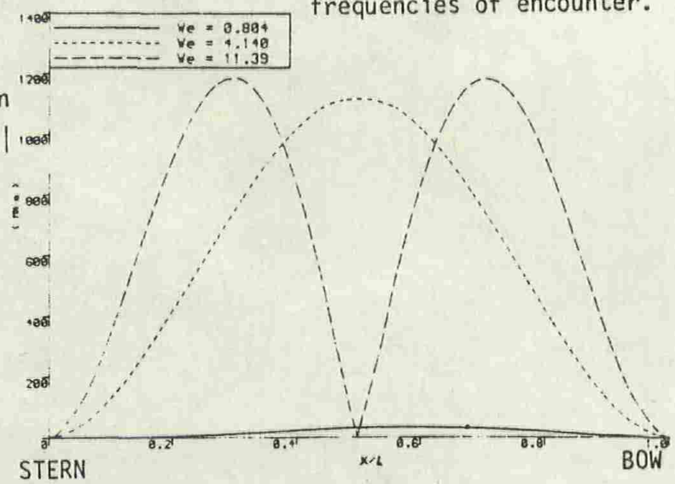
Figure 4 (cont.) For the uniform beam travelling with Froude number Fn=0.2 in head waves of unit amplitude, (c) describes the pitch principal coordinate $|p_5|/a$,



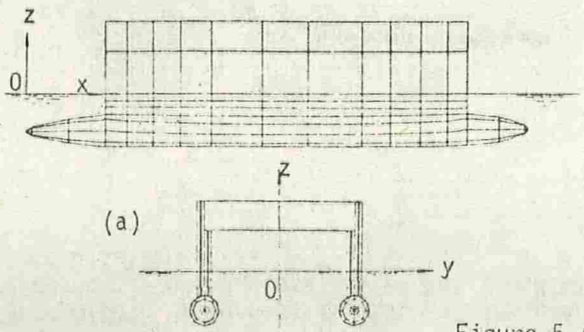
(d) The vertical distortion $|w(0.25L)| = |\sum_{r=7}^{n} w_r(0.25L)p_r|$ in metres for various approximations to n,



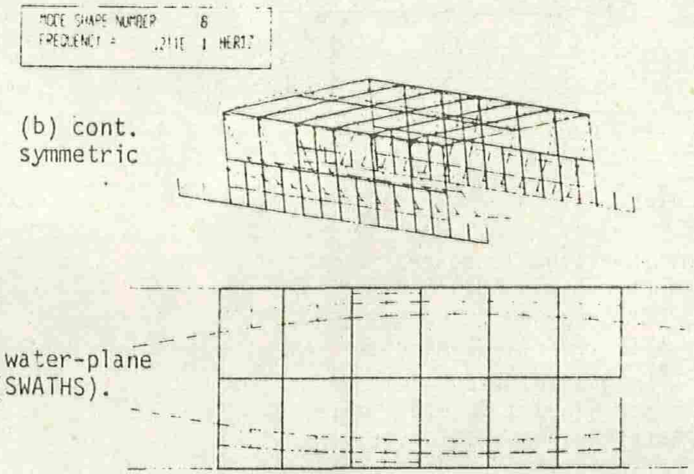
(e) The variation in the amplitude of the vertical displacement (in metres) along the length of the beam at prescribed frequencies of encounter.



(f) The vertical bending moment (MNm) along the length of the beam at prescribed frequencies of encounter.

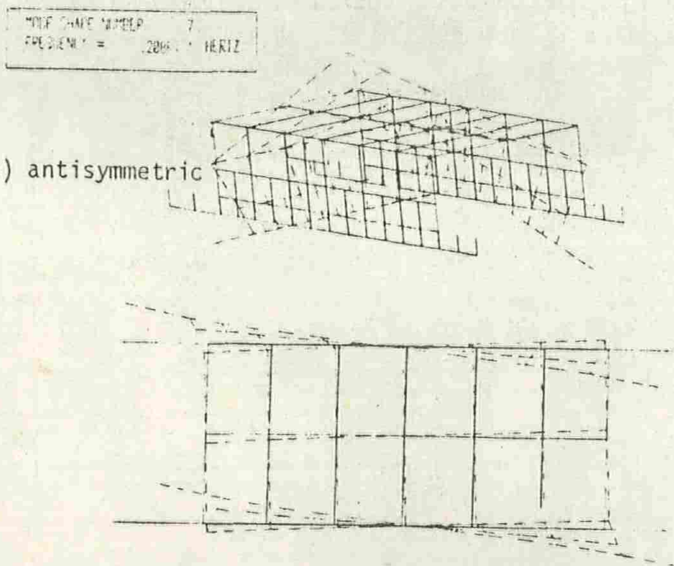


(a)

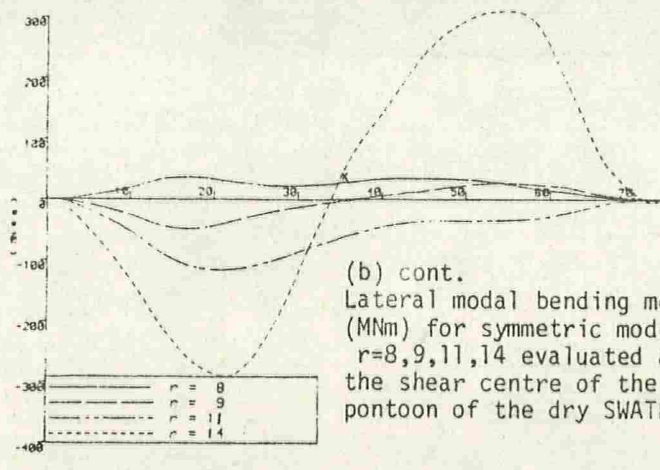


(b) cont.
symmetric

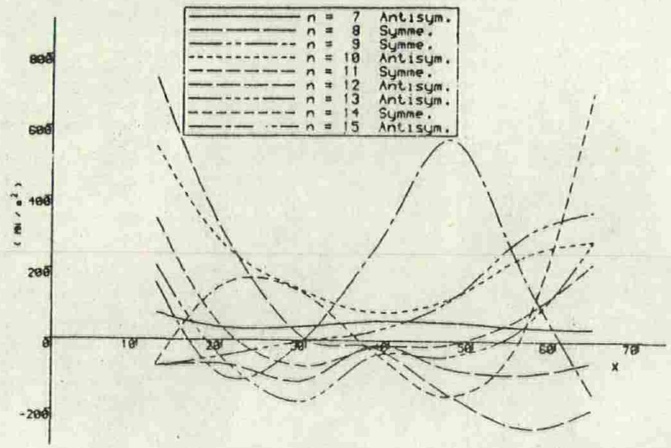
Figure 5 (a) A small water-plane area twin hull ship (SWATHS).
(b) Dry hull dynamic characteristics.



(b) antisymmetric



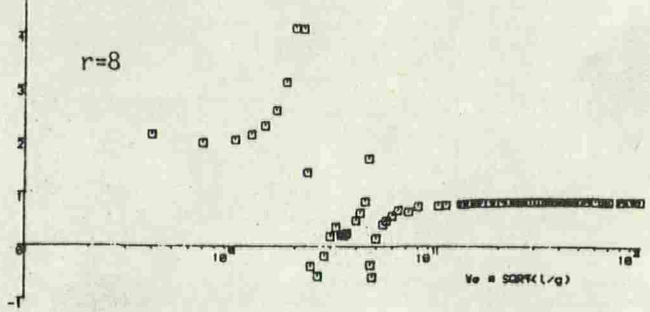
(b) cont.
Lateral modal bending moment (MNm) for symmetric modes r=8,9,11,14 evaluated along the shear centre of the port pontoon of the dry SWATHS.



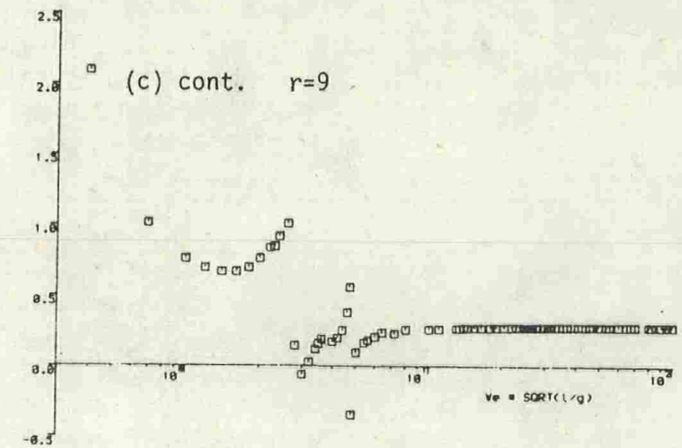
(b) cont. Amplitudes of the dry principal stress mode shapes (MN/m^2) on the outer surface of the port strut along the join with the bridge structure.

Figure 5(c). Frequency variation of the generalised hydrodynamic coefficients $A_{rr}/\rho\forall$ for r=8,9. ($\forall$ denotes volume displacement).

r=8



(c) cont. r=9



r=8

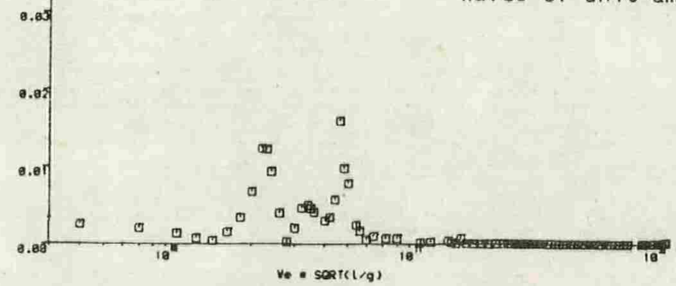
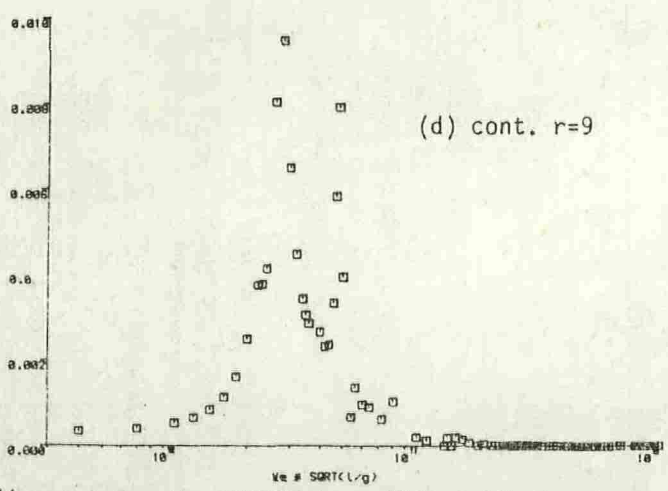


Figure 5(d) Frequency variation of the generalised wave exciting forces $|\Xi_r|/\rho g\forall a$ for r=8,9.

The SWATHS travel with Froude number Fn=0.223 in sinusoidal head wave waves of unit amplitude



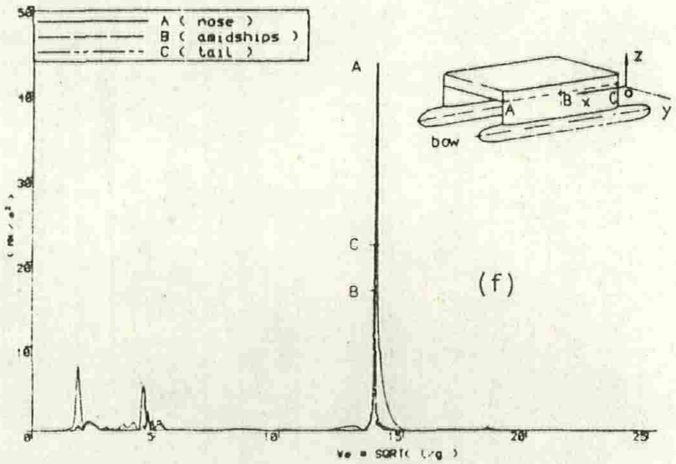
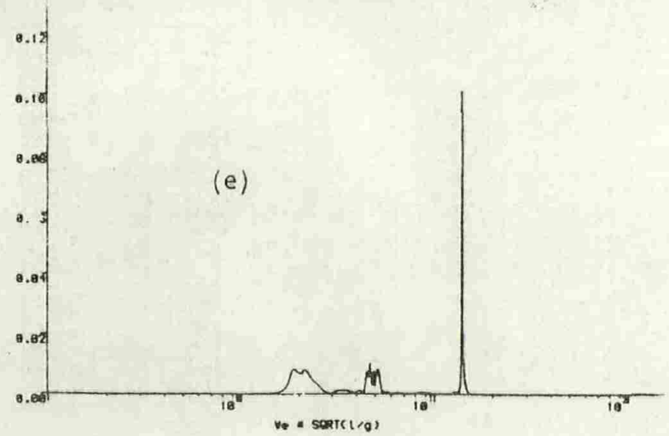
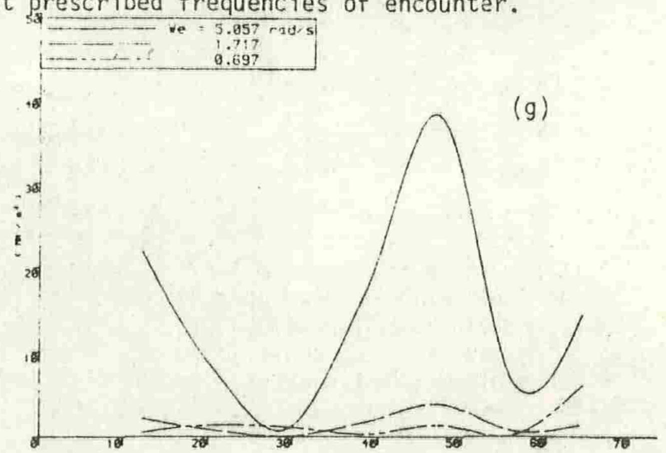


Figure 5(cont.) For the SWATHS travelling with Froude number Fn=0.223 in sinusoidal head waves of unit amplitude (e) describes the principal coordinate $|p_8|/a$; the variation in the principal stress amplitude (MN/m^2) (f) at points A, B, C on the outer surface as indicated and (g) along the line ABC at prescribed frequencies of encounter.





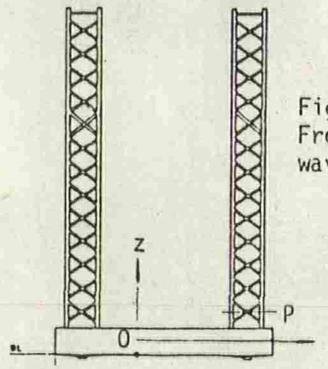
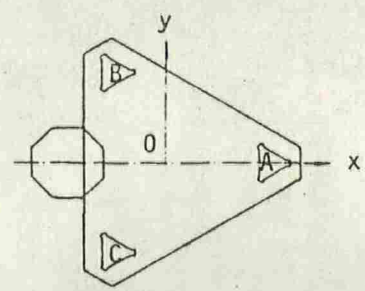
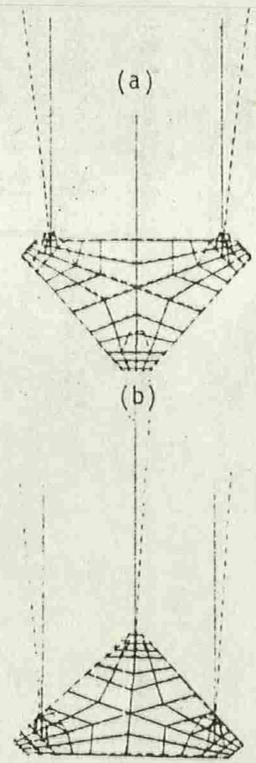
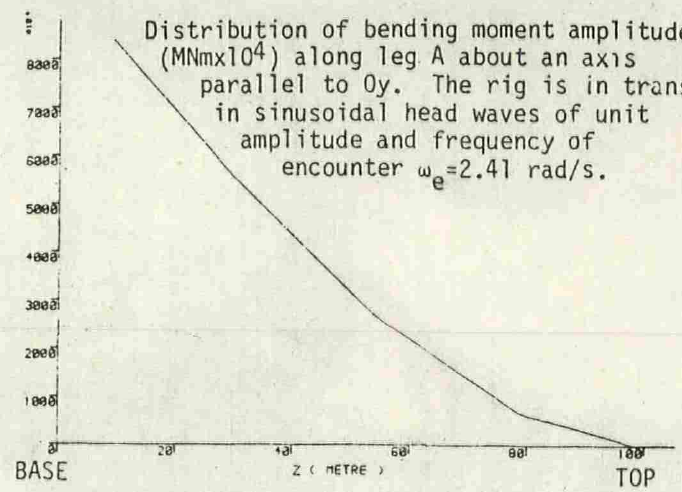


Figure 6. A jack-up rig in transit with Froude number Fn=0.1 in sinusoidal head waves of unit amplitude.

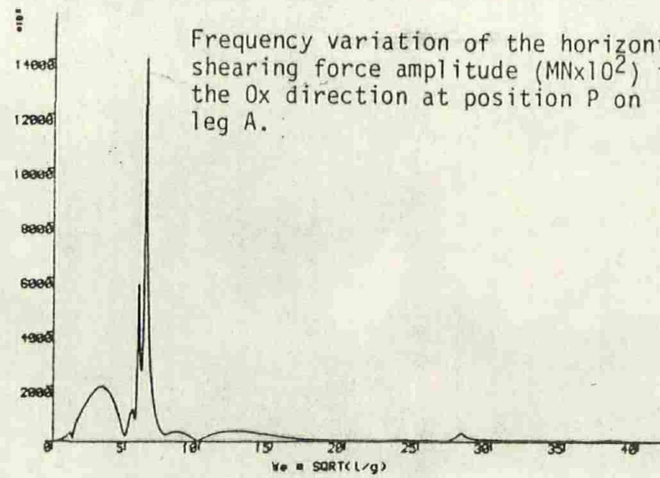




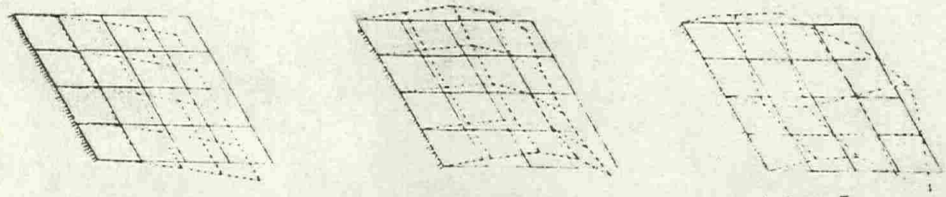
The principal dry modes of the rig in vacuo
(a) r=9 (symmetric), ω_r=2.47 rad/s.
(b) r=8 (antisymmetric), ω_r=2.29 rad/s.



Distribution of bending moment amplitude (MNmx10^4) along leg A about an axis parallel to Oy. The rig is in transit in sinusoidal head waves of unit amplitude and frequency of encounter ω_e=2.41 rad/s.



Frequency variation of the horizontal shearing force amplitude (MNx10^2) in the Ox direction at position P on leg A.



(a) r=1 (b) r=3 (c) r=5

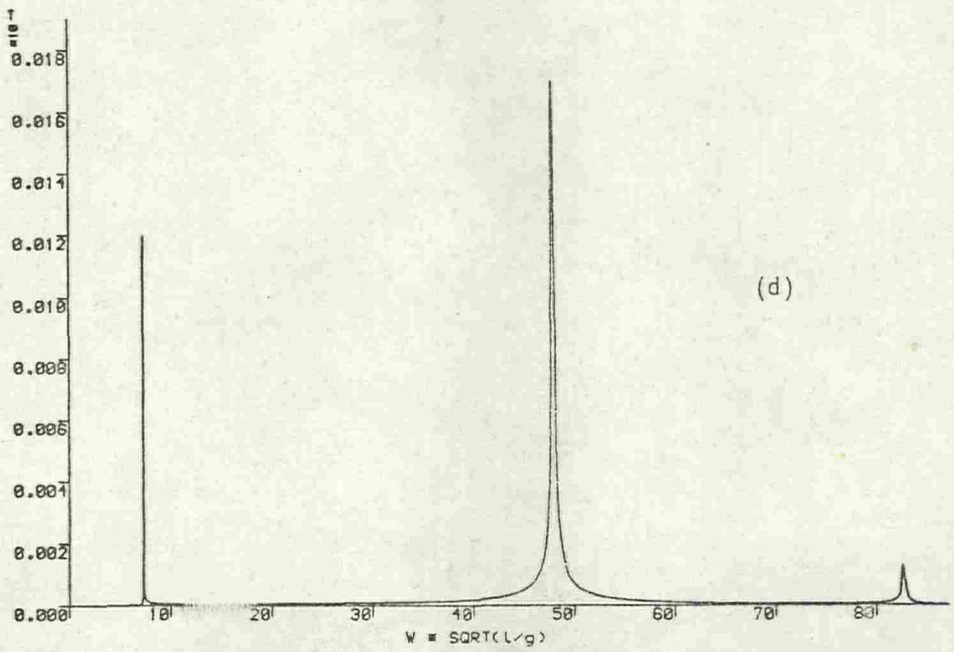


Figure 7. Horizontally cantilevered square plate (10x10x0.1m) submerged at 5m below the free surface. (a-c) denote the first three symmetric dry modes and (d) illustrates the frequency variation of the principal coordinate $|p_3|/a$.

Mode number	Distortion	Natural dry frequency (rad/s)	Resonance wet frequency (rad/s)
1	symmetric	12.94	7.27
2	antisymmetric	31.78	17.93
3	symmetric	80.43	47.4
4	antisymmetric	100.89	80.6
5	symmetric	115.74	81.8