Hydroplaning

Lubrication theory

Research question

Can we model hydroplaning fast and accurate using lubrication theory?

Research question

Can we model hydroplaning fast and accurate using lubrication theory?

TU Delft

Research question

Can we model hydroplaning fast and accurate using lubrication theory?

TU Delft

Fast?

Fast?

Fast?

Content

Content

I. Hydroplaning

Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling

Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling

IV. Fluid Structure interaction

Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling

IV. Fluid Structure interaction

V. Results

Content

I. Hydroplaning

II. Fluid Mechanics: Lubrication Theory

III. Solid Mechanics: Tire modelling

IV. Fluid Structure interaction

V. Results

Hydroplaning

Hydroplaning

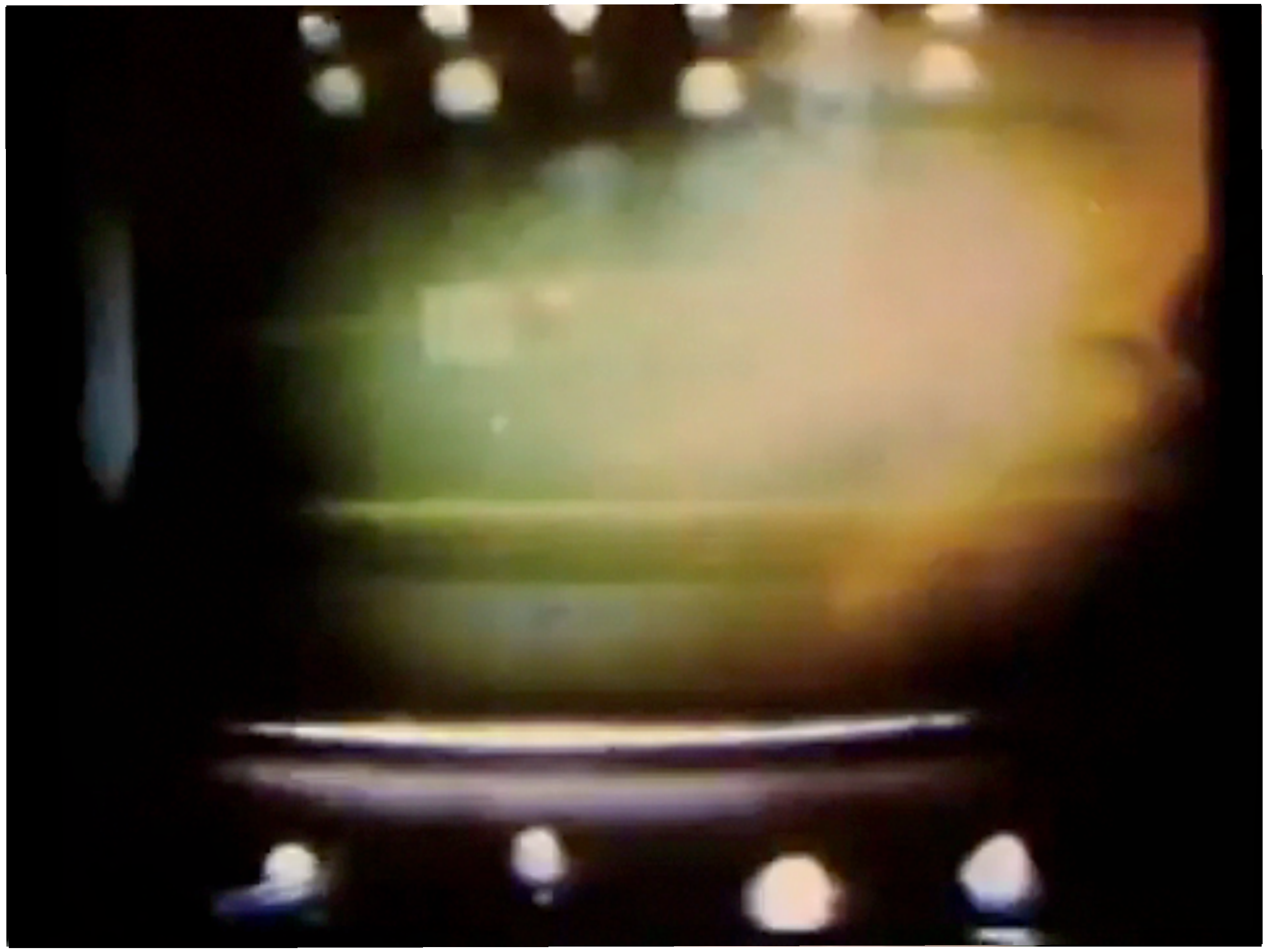
Observations

Observations

- Footprint

Observations

- Footprint

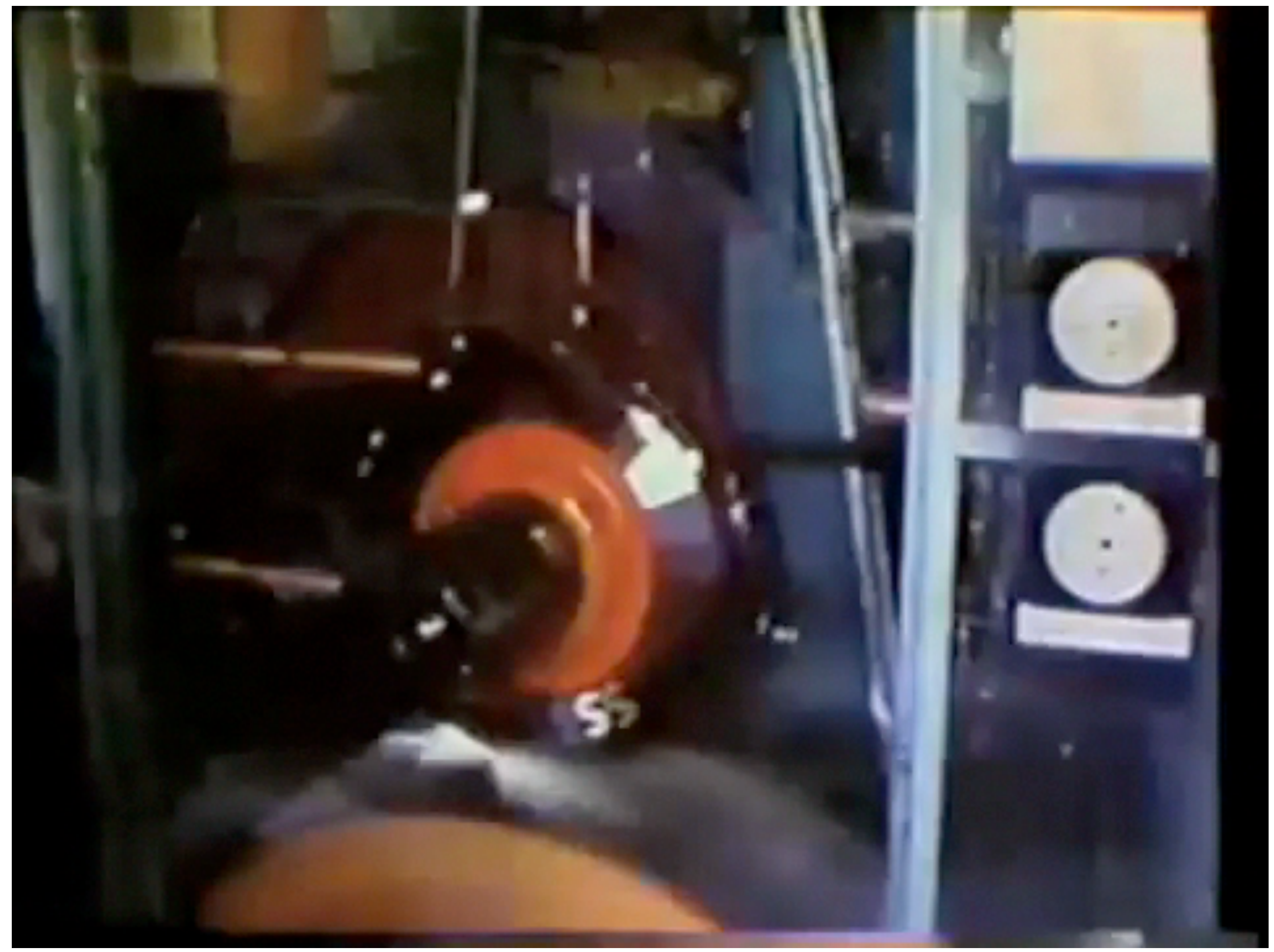


Observations

- Footprint
- Bow wave

Observations

- Footprint
- Bow wave
- Spin down



Observations

- Footprint
- Bow wave
- Spin down
- Loss of:

Observations

- Footprint
- Bow wave
- Spin down
- Loss of:
  - Traction

Observations

- Footprint
- Bow wave
- Spin down
- Loss of:
    - Traction
    - Directional control

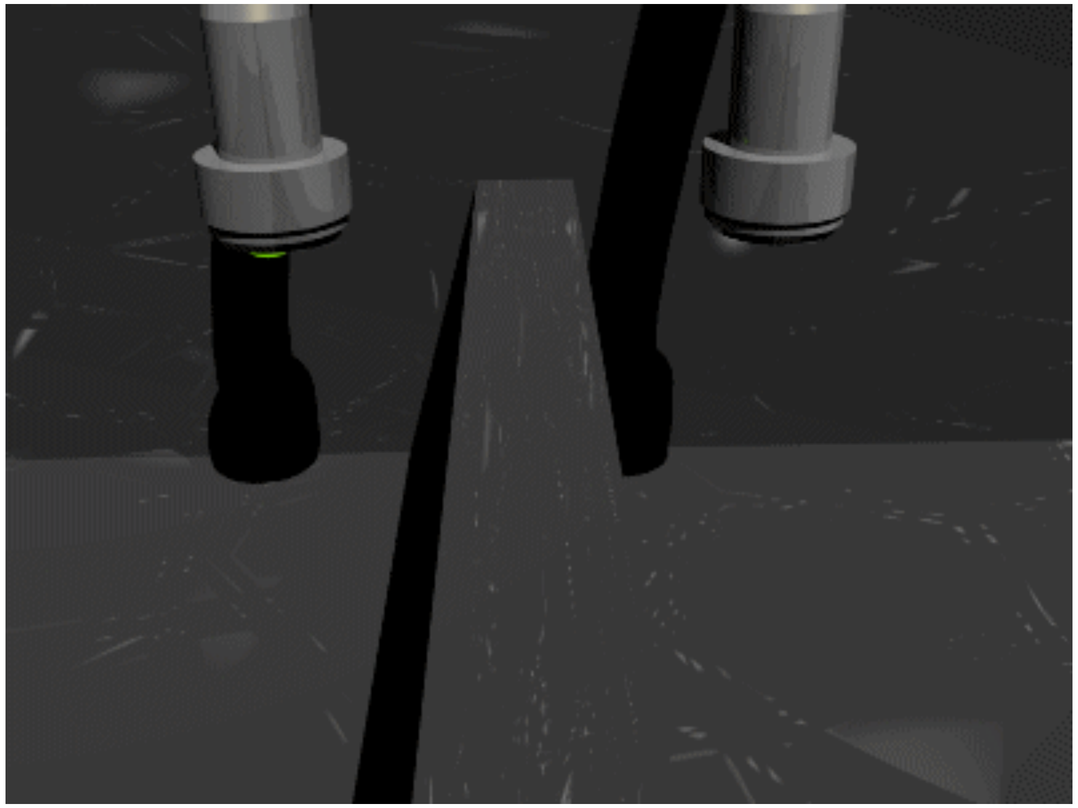
Parameters

Parameters

- Fluid:

Parameters

- Fluid:
  - Viscosity



Parameters

- Fluid:
  - Viscosity
  - Inertia

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture

Parameters

- Fluid:
    - Viscosity
    - Inertia
- Tire:
    - Tread design
    - Width
- Surface:
    - Texture

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
- Vehicle:

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
- Vehicle:
  - Weight

Parameters

- Fluid:
  - Viscosity
  - Inertia
- Tire:
  - Tread design
  - Width
- Surface:
  - Texture
  - Pavement crown
- Vehicle:
  - Weight

Operating parameters

Operating parameters

- Inflation pressure

Operating parameters

- Inflation pressure
- Vehicle velocity

Hydroplaning formula

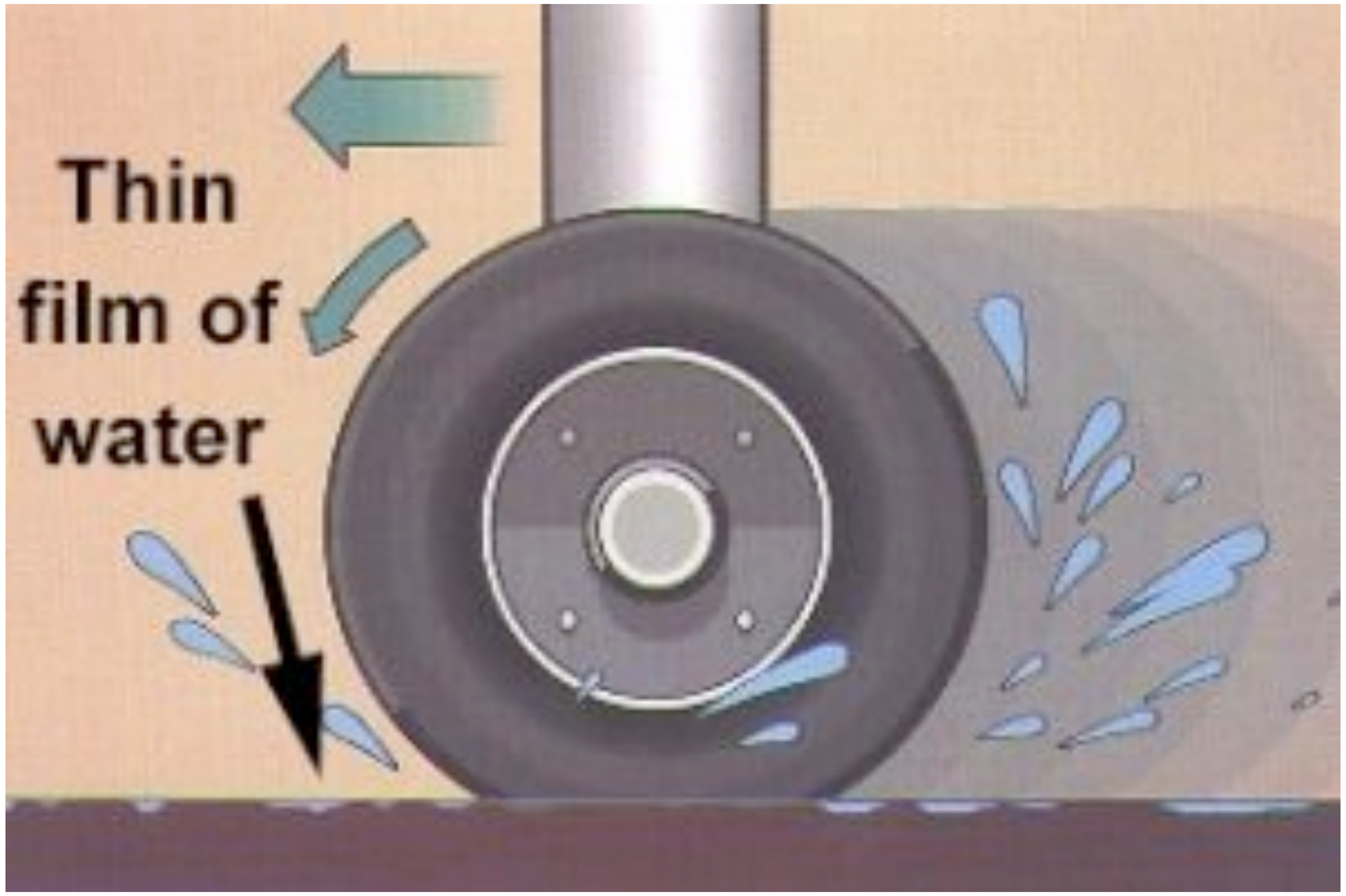
$$v = 6.36\sqrt{p}$$

Vehicle velocity: v in $\dfrac{km}{hour}$

Tire pressure: p in kPa

Dominant fluid effects

- Viscosity
- Inertia



Dominant fluid effects

- Viscosity
- Inertia

Modelling

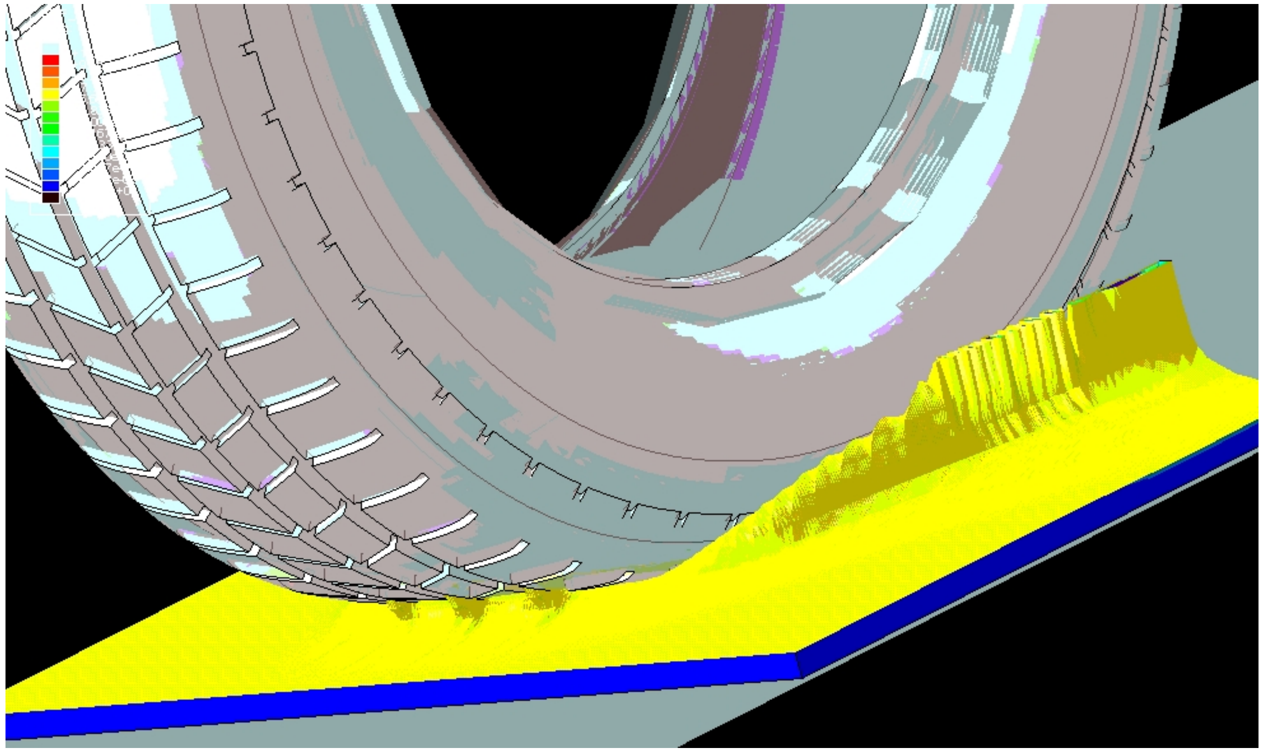
Modelling

- Analytical

$$F = \frac{\rho b V^2 R \lambda}{3}$$

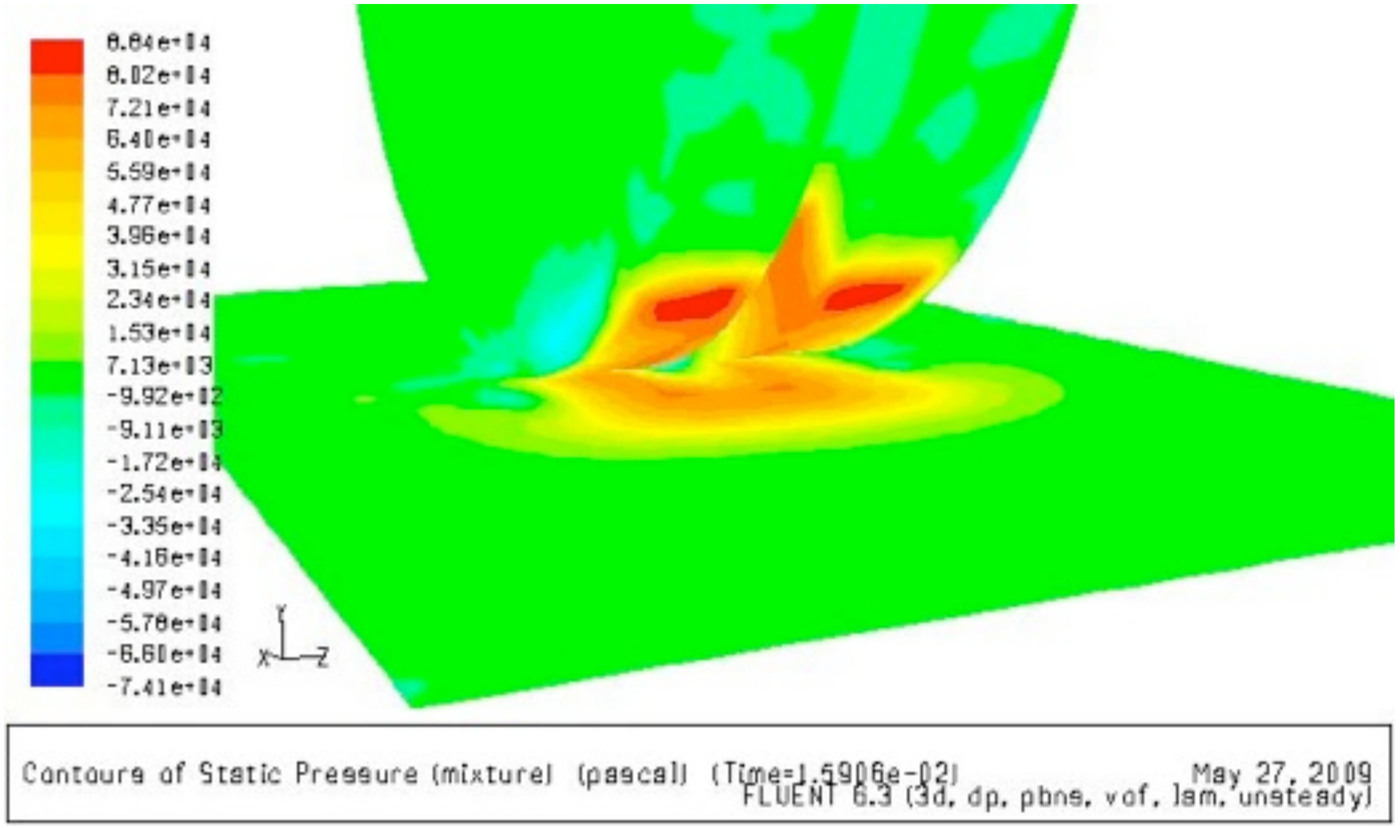
Modelling

- Analytical
- FEM & FVM



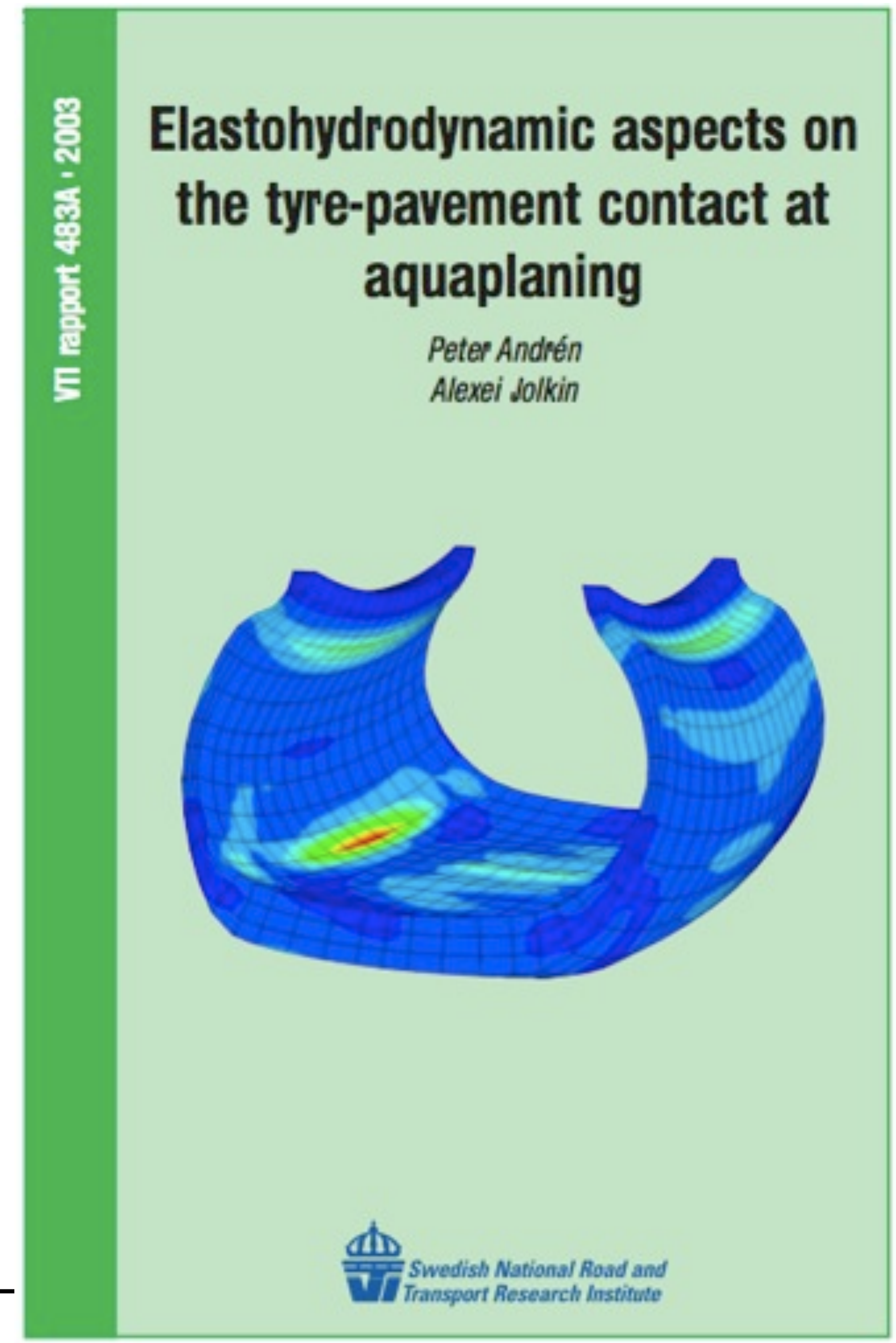
Modelling

- Analytical
- FEM & FVM
- CFD



Modelling

- Analytical
- FEM & FVM
- CFD
- Lubrication theory



Lubrication theory

Lubrication theory

Why?

Full 3D simulation

Pressure

Velocity: x-,y-, z-direction

Why?

Full 3D simulation

Pressure

Velocity: x-,y-, z-direction

Why?

Full 3D simulation

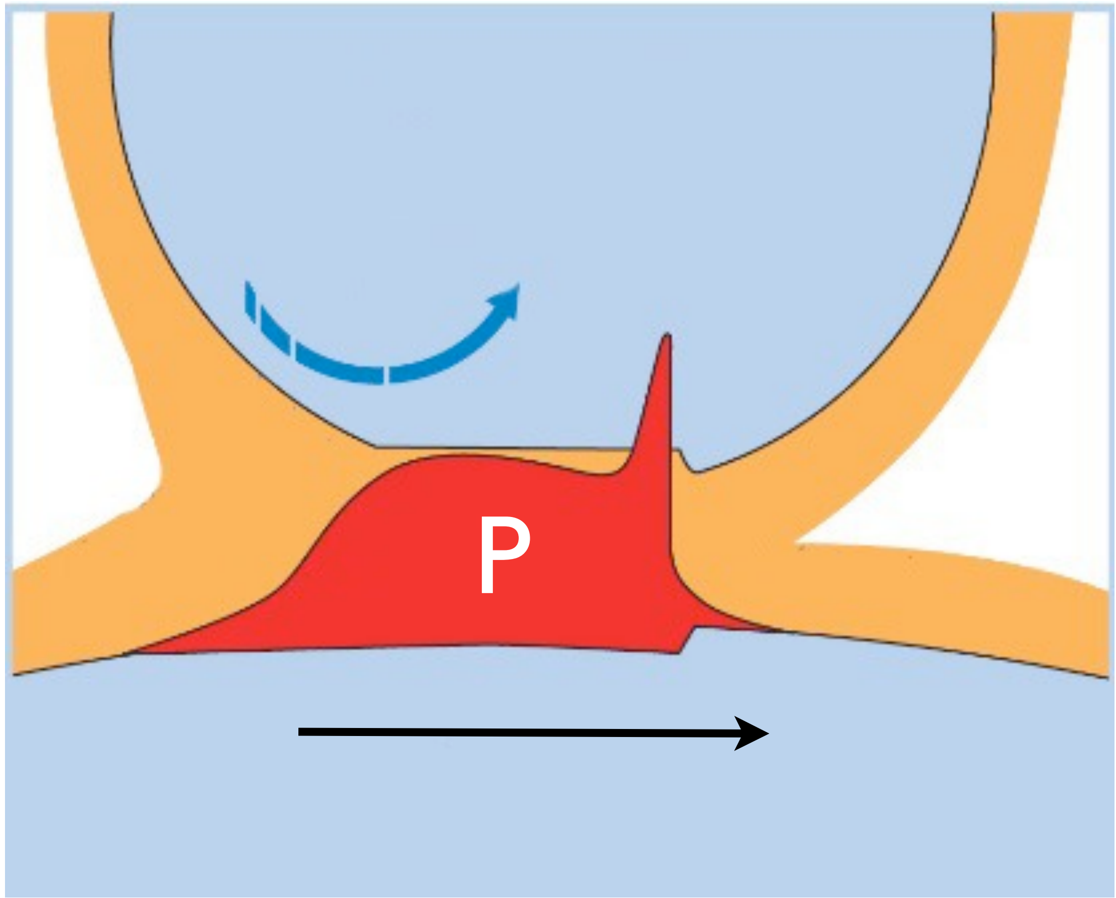
4

Why?

Full 3D simulation

4

Simplified Reynolds model



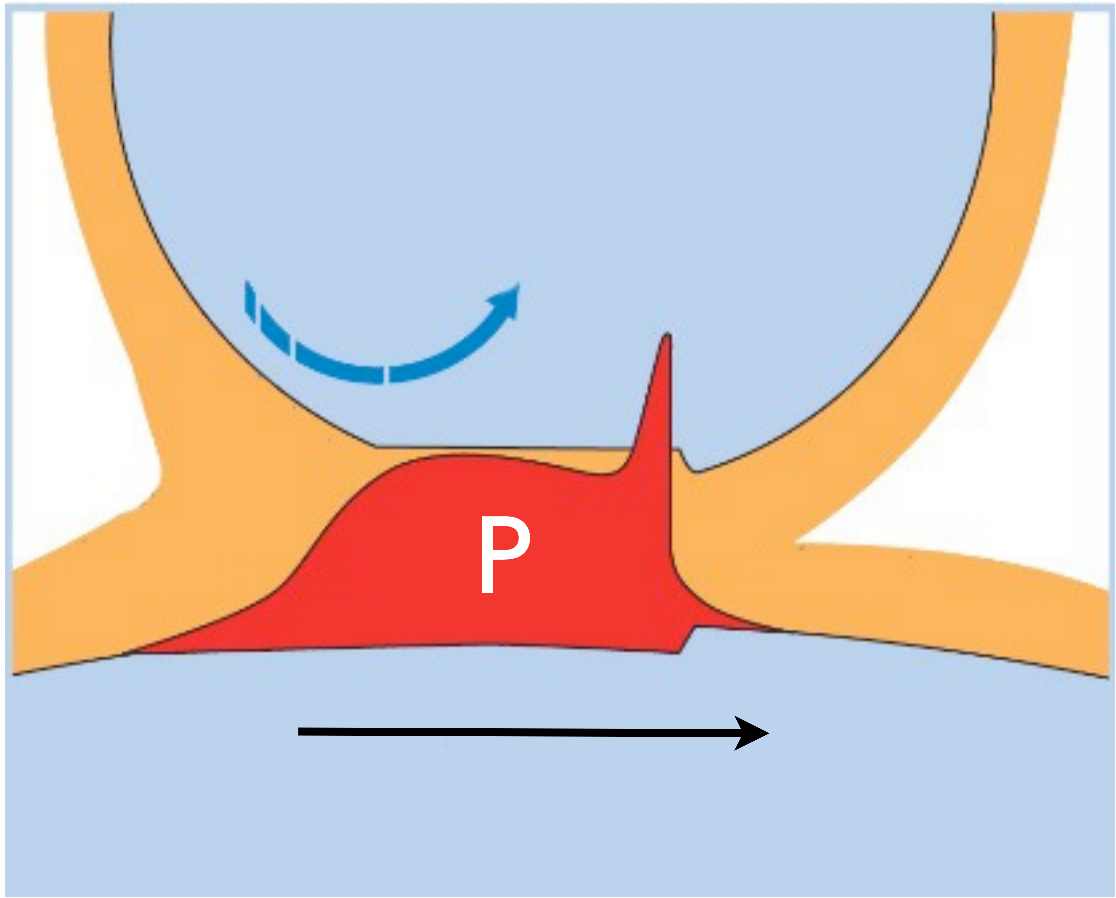
Pressure

Why?

Full 3D simulation

4

Simplified Reynolds model



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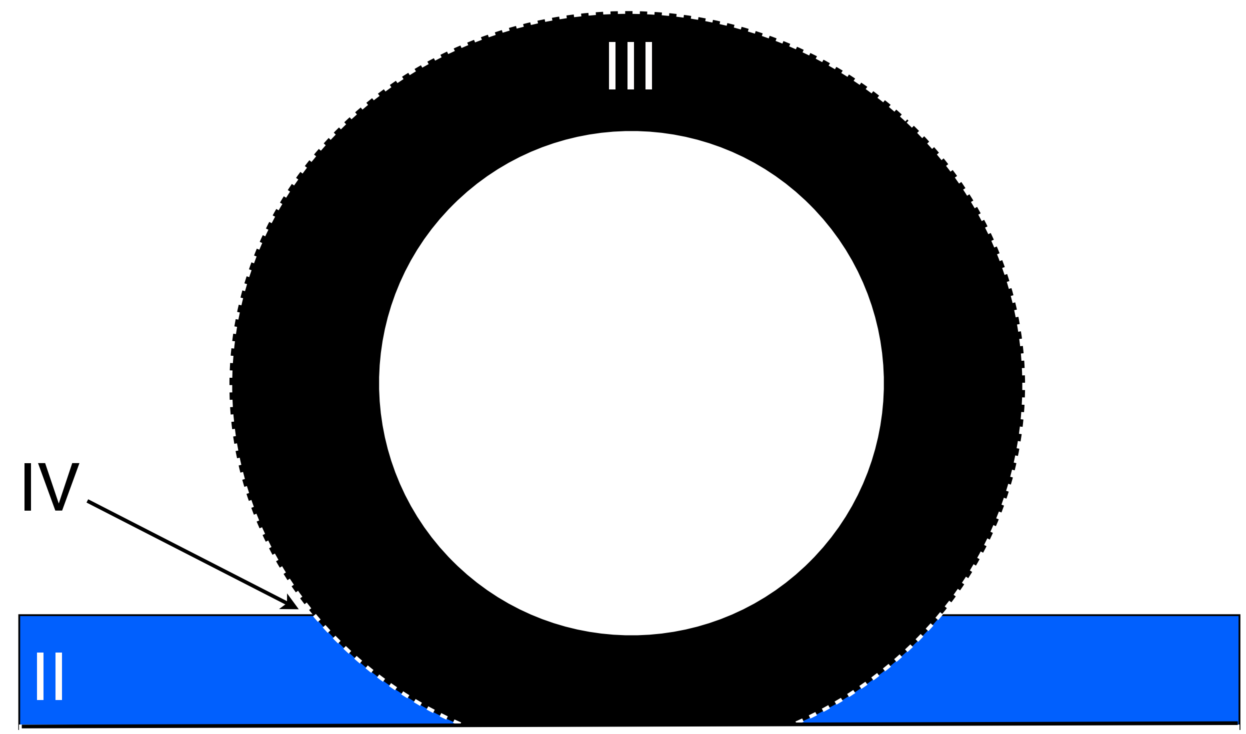
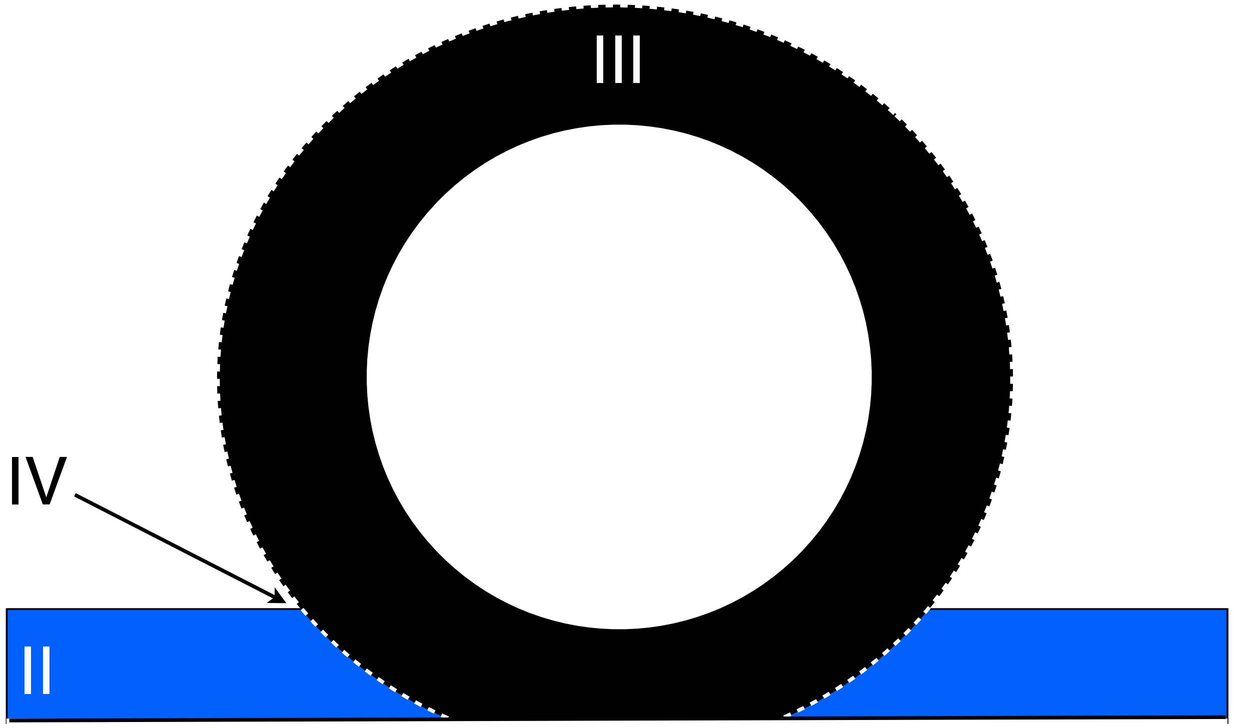
II: Lubrication theory



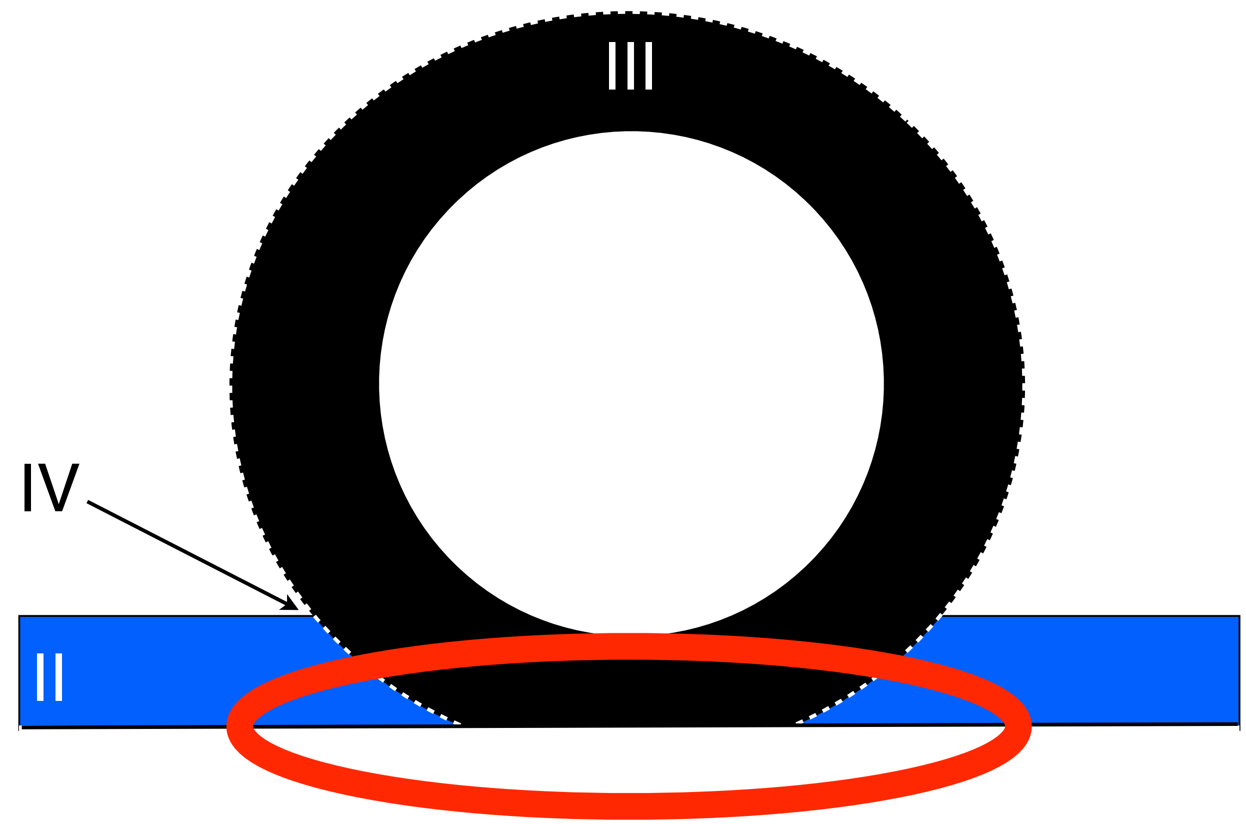
II: Lubrication theory

- Reynolds equation



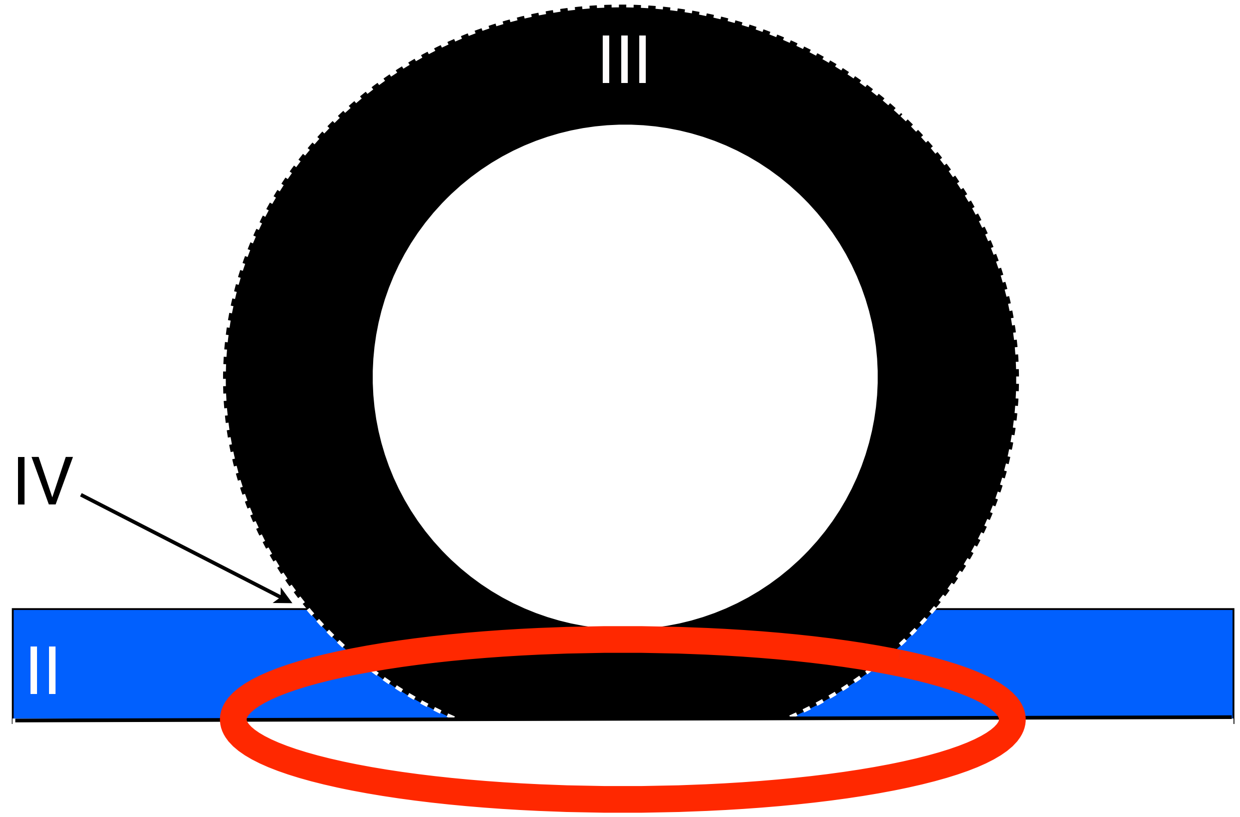
II: Lubrication theory

- Reynolds equation



II: Lubrication theory

- Reynolds equation
- Inertia correction



II: Lubrication theory

- Reynolds equation
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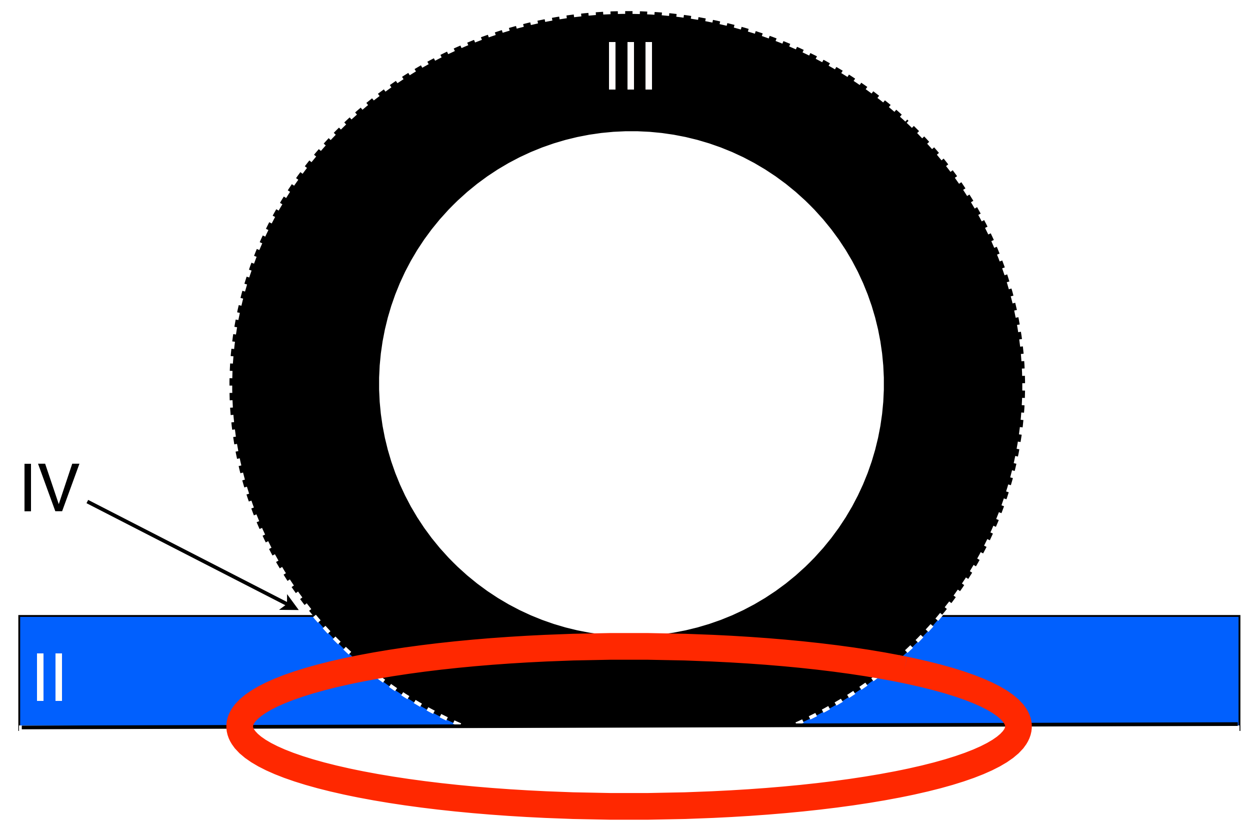
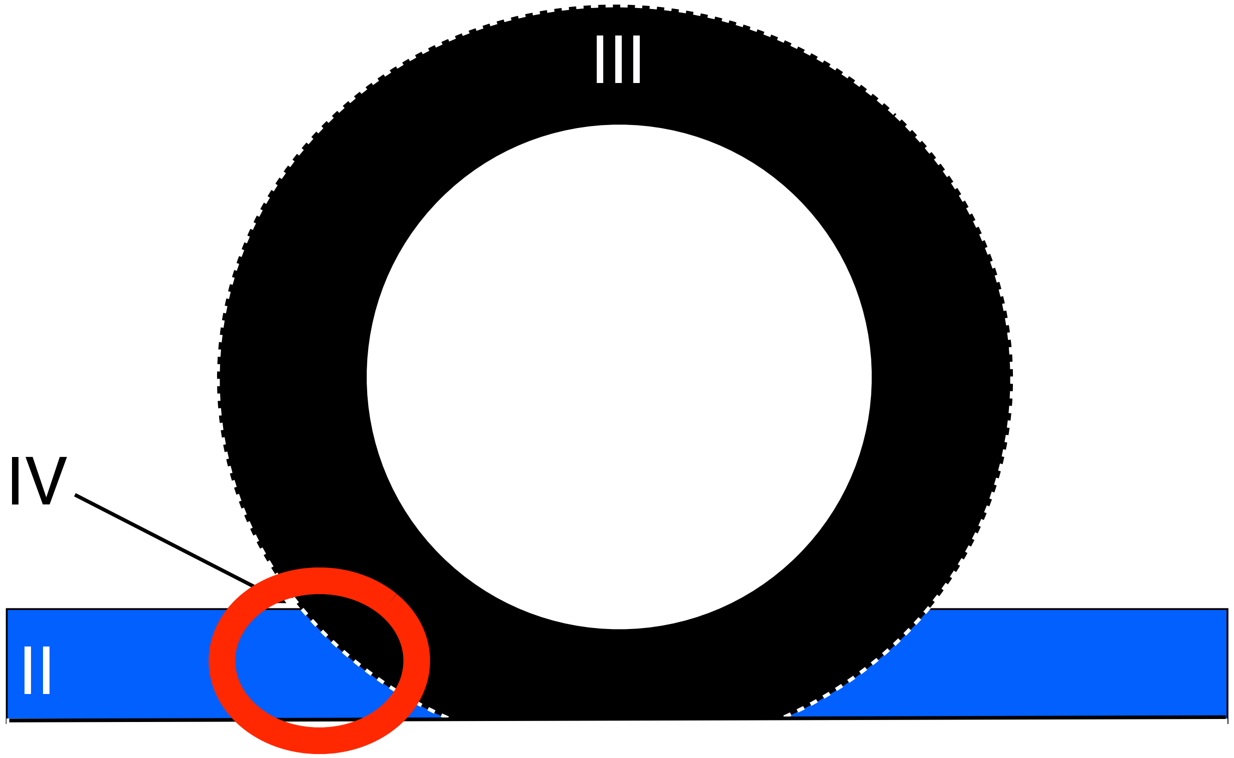
- Reynolds equation
- Inertia correction



II: Lubrication theory

- Reynolds equation
- Inertia correction
- Inlet condition



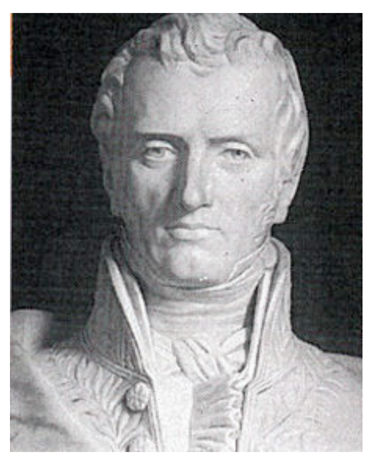
II: Lubrication theory

- Reynolds equation
- Inertia correction
- Inlet condition

Navier-Stokes equations

Incompressible Newtonian fluid

$$\overbrace{\rho\left(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}\right)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}}$$



Navier & Stokes (1822)

Assume: thin film

$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\substack{\text{Unsteady}\\\text{acceleration}}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\substack{\text{Convective}\\\text{acceleration}}}\Big)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\substack{\text{Pressure}\\\text{gradient}}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}}$$

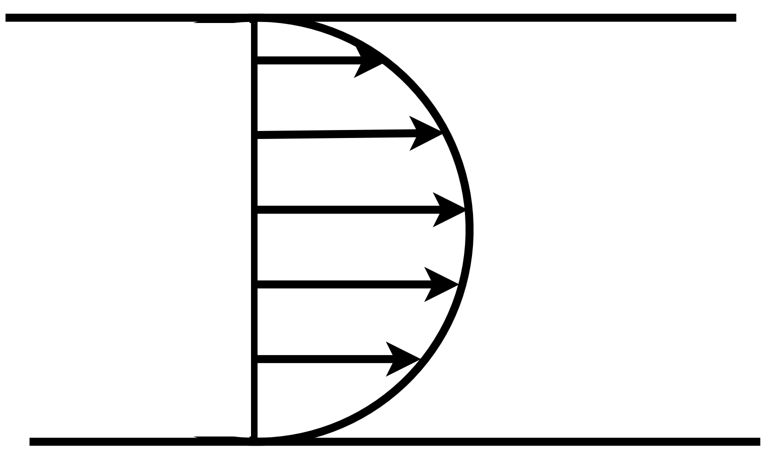
Assume: thin film

$$0 = \overbrace{\underbrace{-\nabla p}_{\substack{\text{Pressure} \\ \text{gradient}}} + \underbrace{\mu \nabla^2 \mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}}$$

Assume: no slip

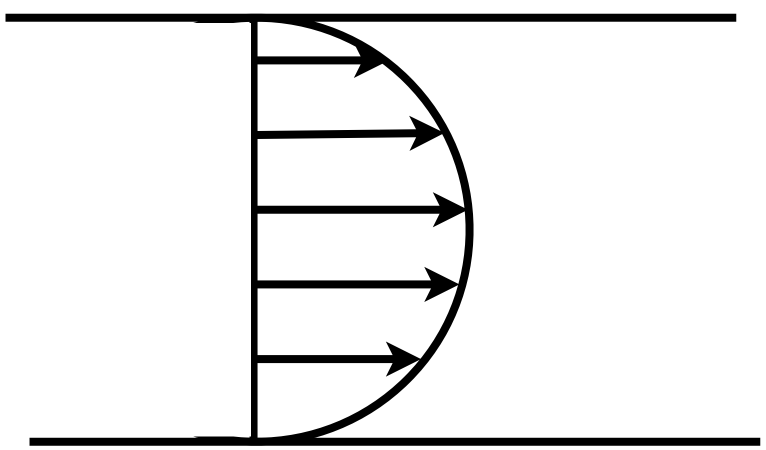
Assume: no slip

Poiseuille



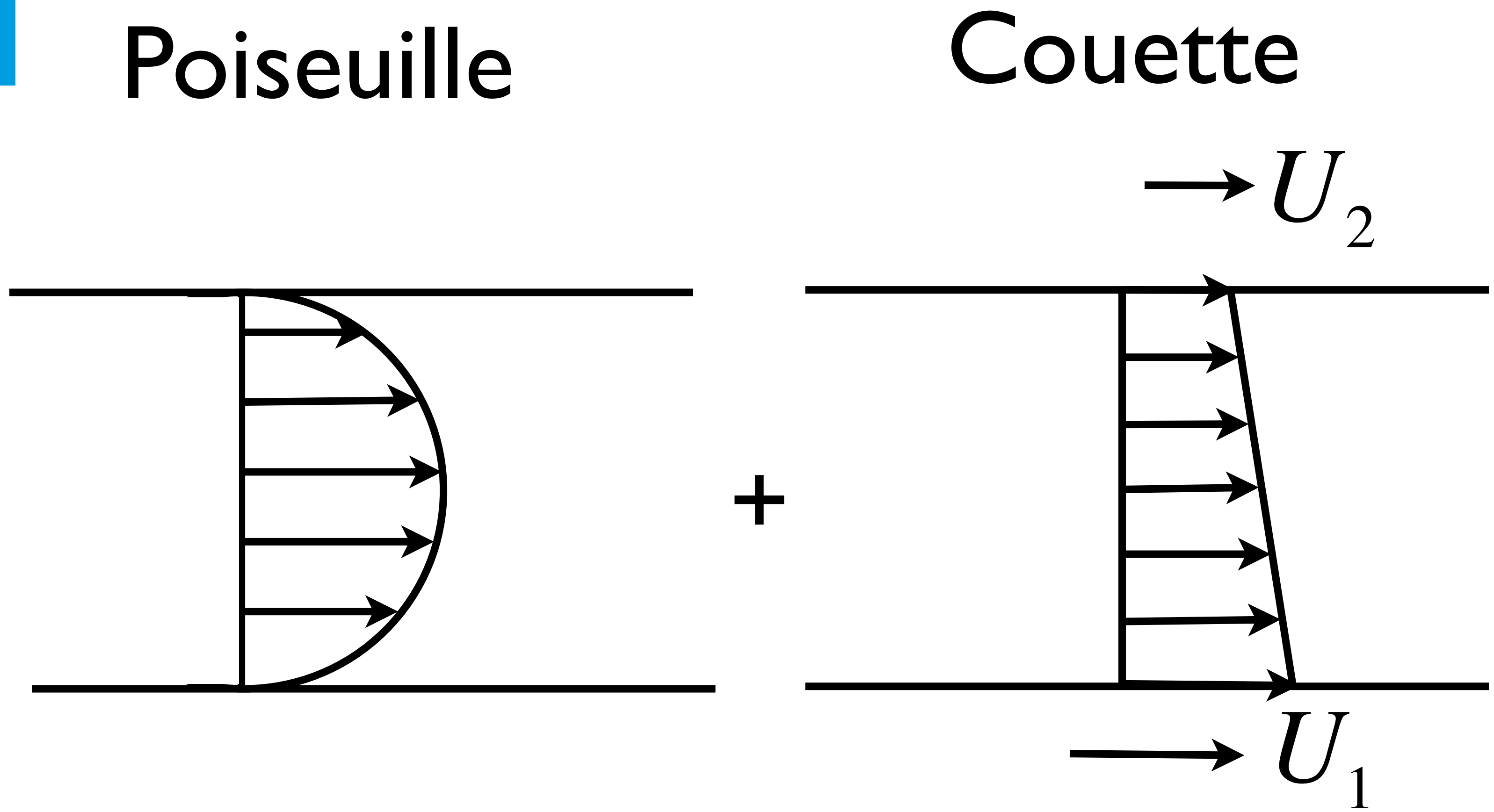
Assume: no slip

Poiseuille

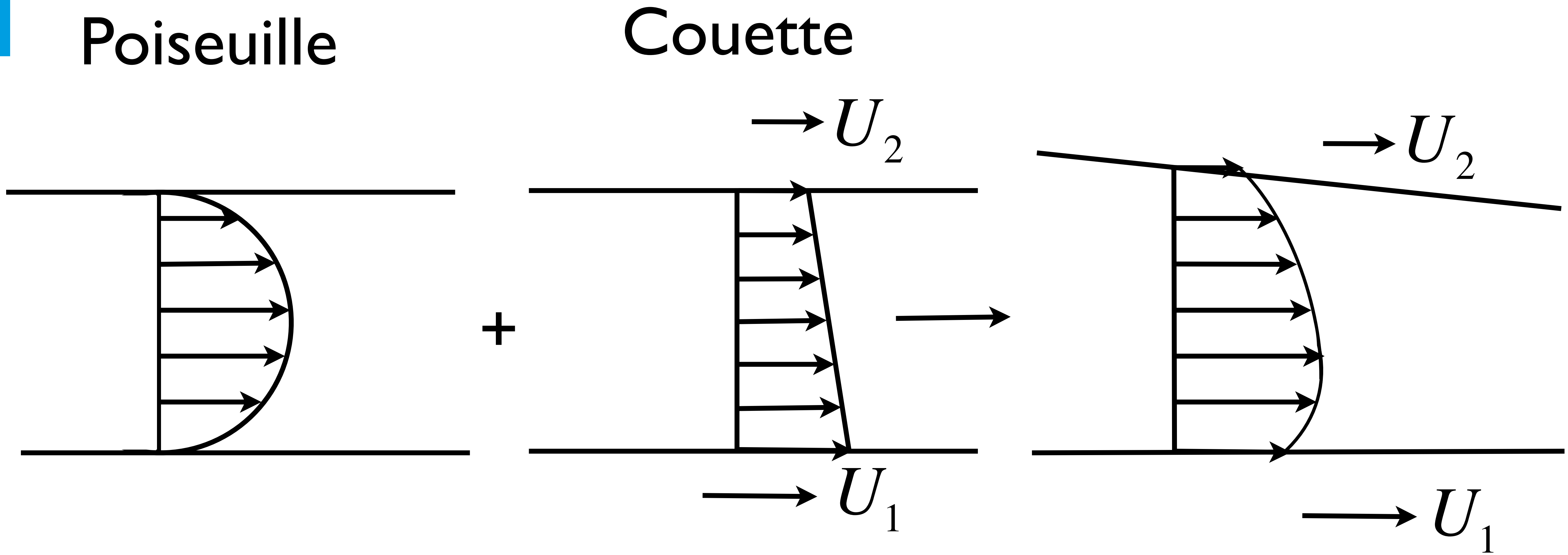


+

Assume: no slip

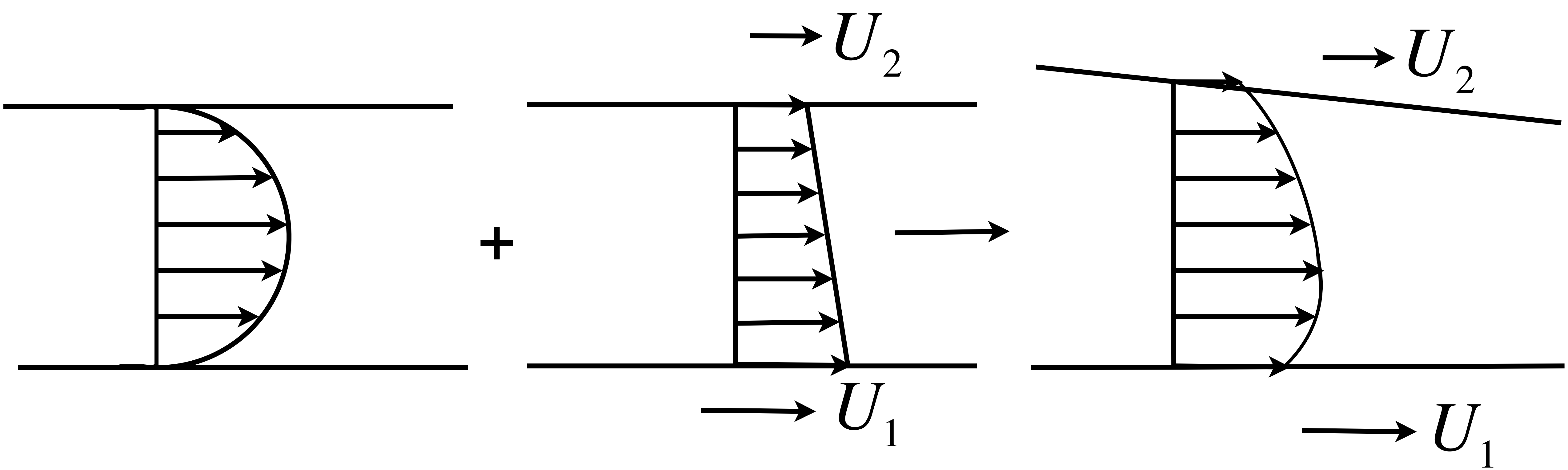


Assume: no slip



Assume: no slip

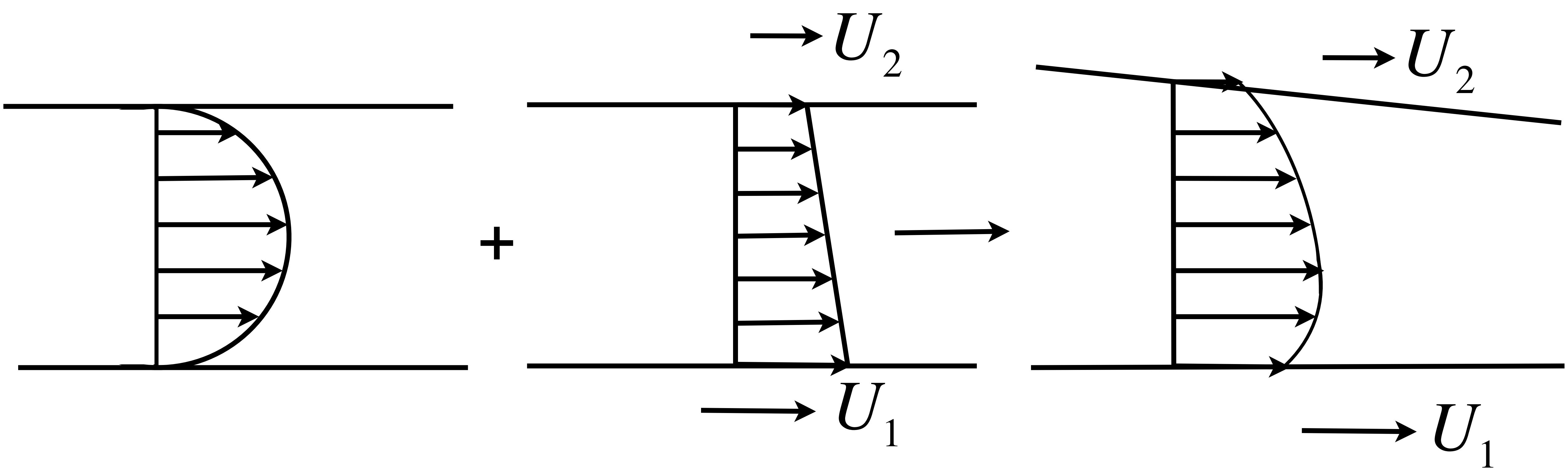
Poiseuille   Couette



$$\frac{-h^3}{12\mu}\nabla p$$

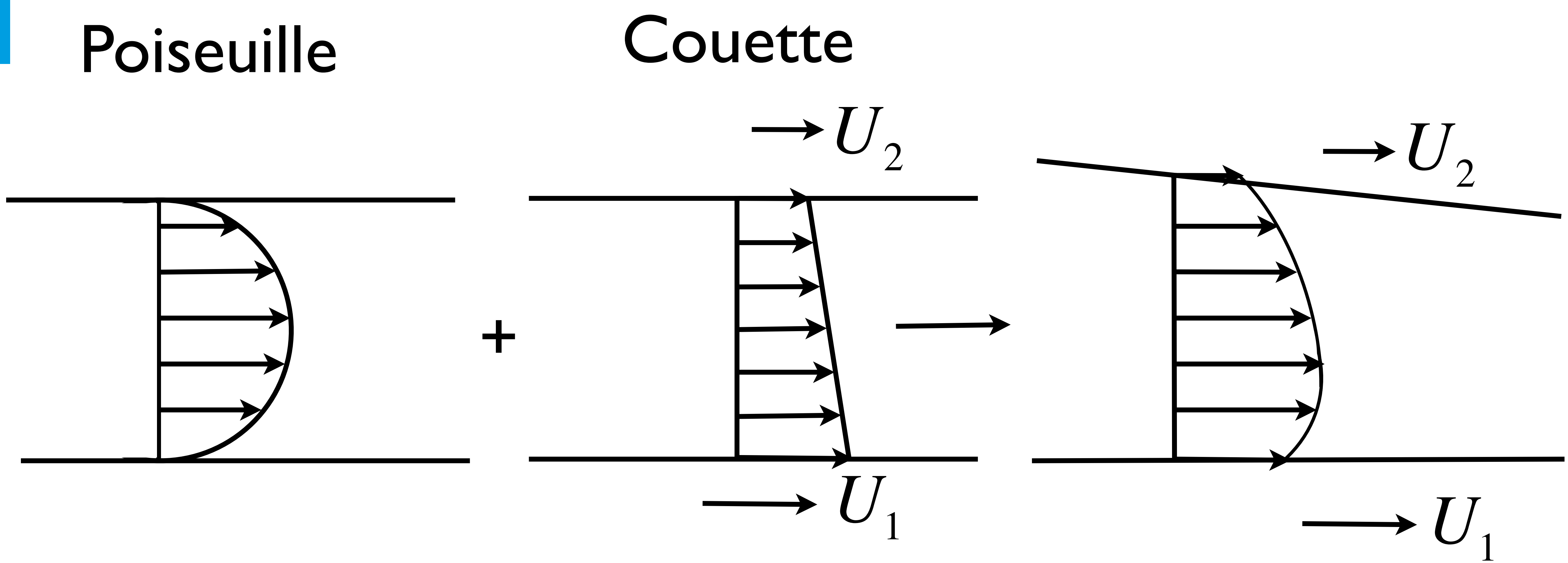
Assume: no slip

Poiseuille Couette



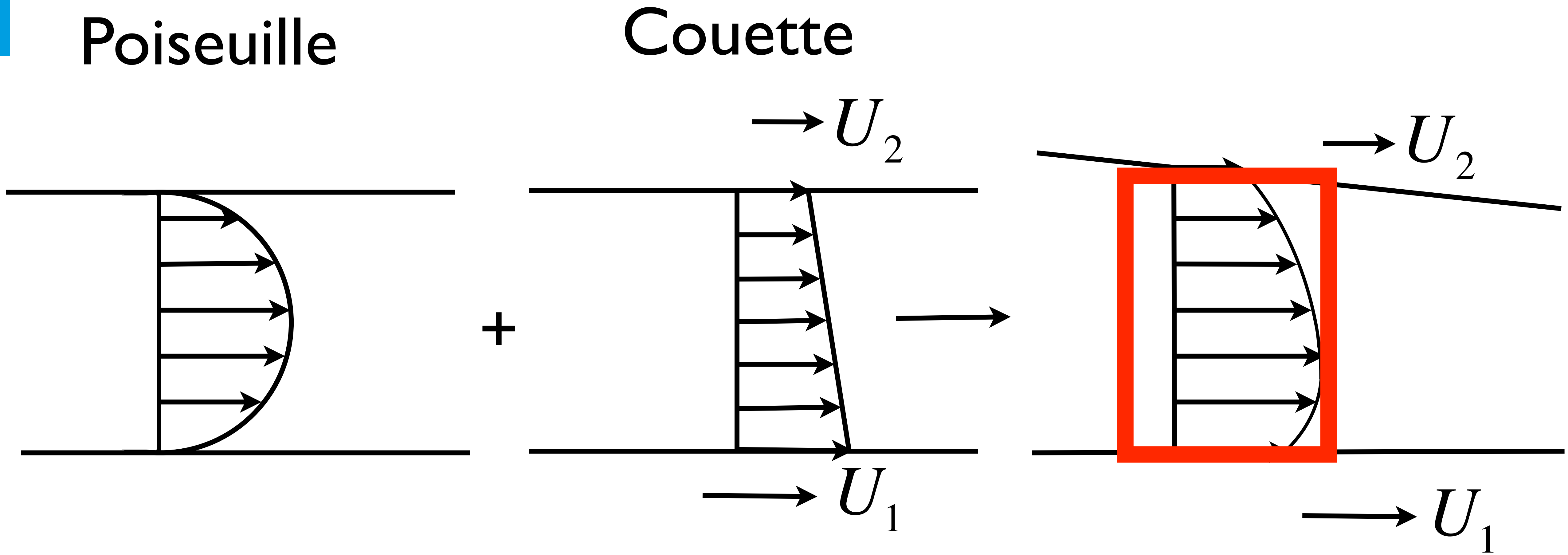
$$\frac{-h^3}{12\mu}\nabla p \quad +$$

Assume: no slip



$$\frac{-h^3}{12\mu}\nabla p \quad + \quad \left(\frac{U_1+U_2}{2}\right)h$$

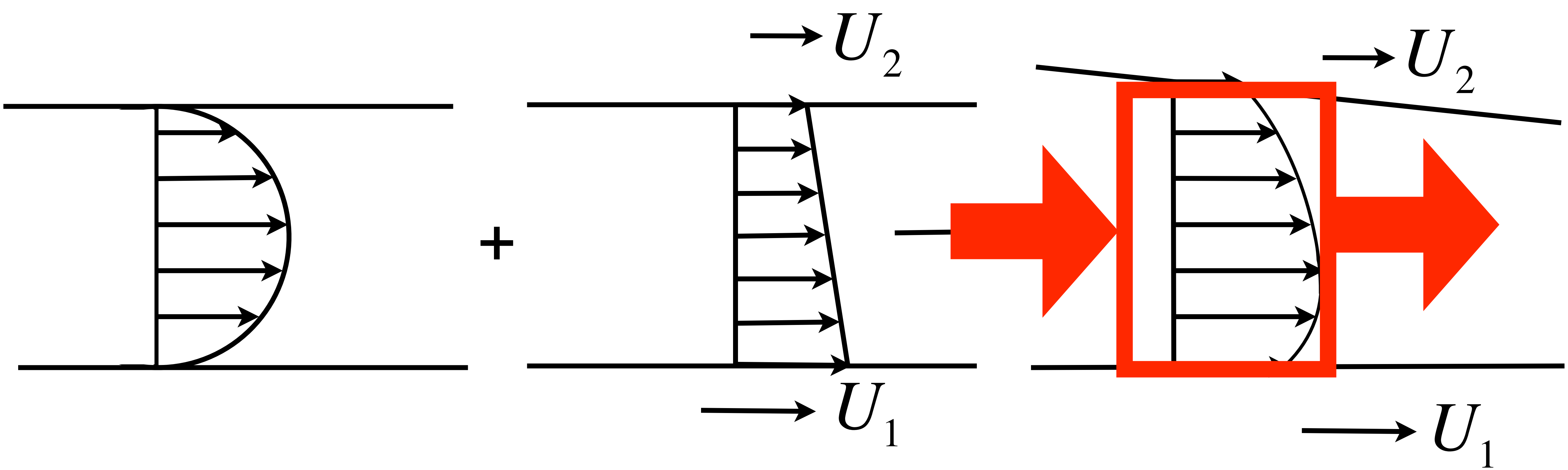
Assume: no slip



$$\frac{-h^3}{12\mu}\nabla p \;+\; \left(\frac{U_1+U_2}{2}\right)h$$

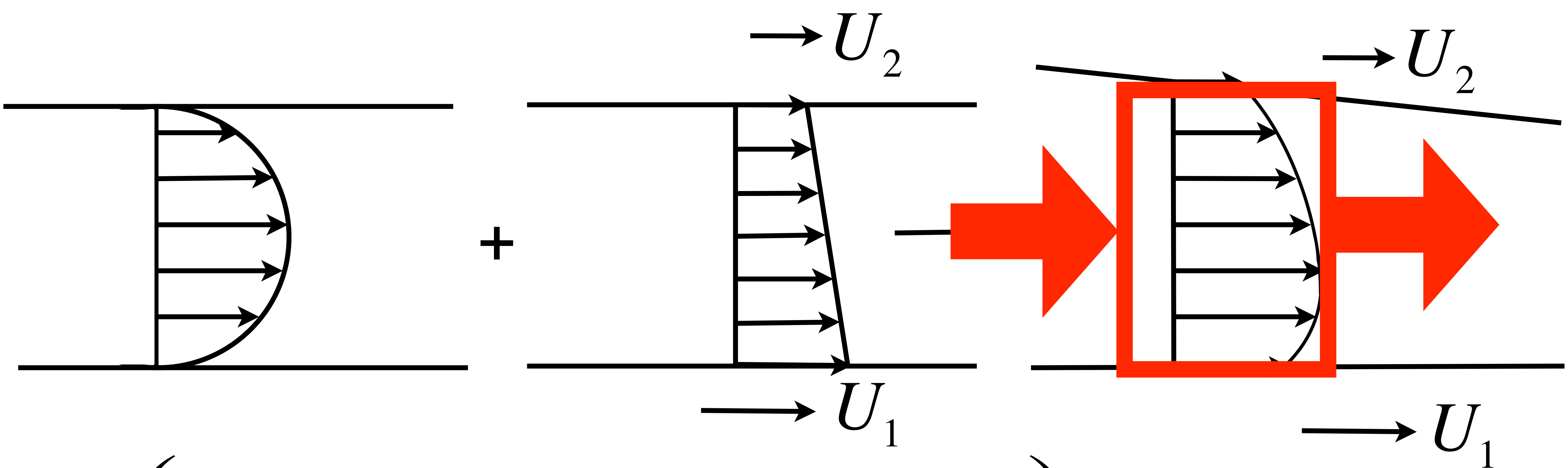
Assume: no slip

Poiseuille   Couette



$$\frac{-h^3}{12\mu}\nabla p \quad + \quad \left(\frac{U_1+U_2}{2}\right)h$$

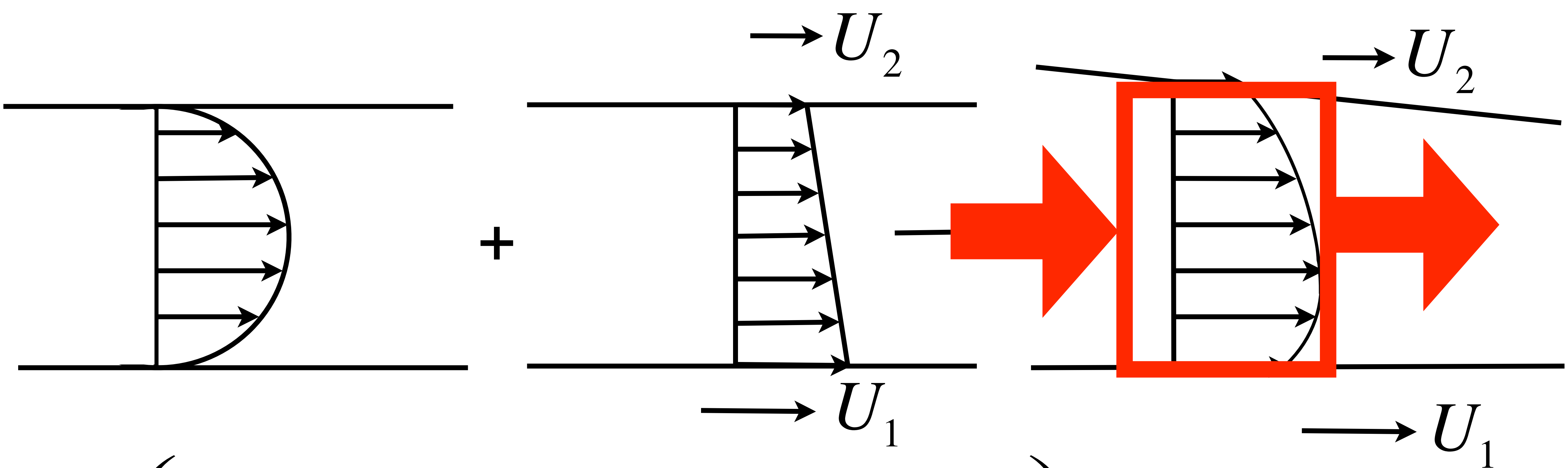
Assume: no slip



$$\nabla \cdot \left(\frac{-h^3}{12\mu} \nabla p + \left(\frac{U_1 + U_2}{2} \right) h \right)$$

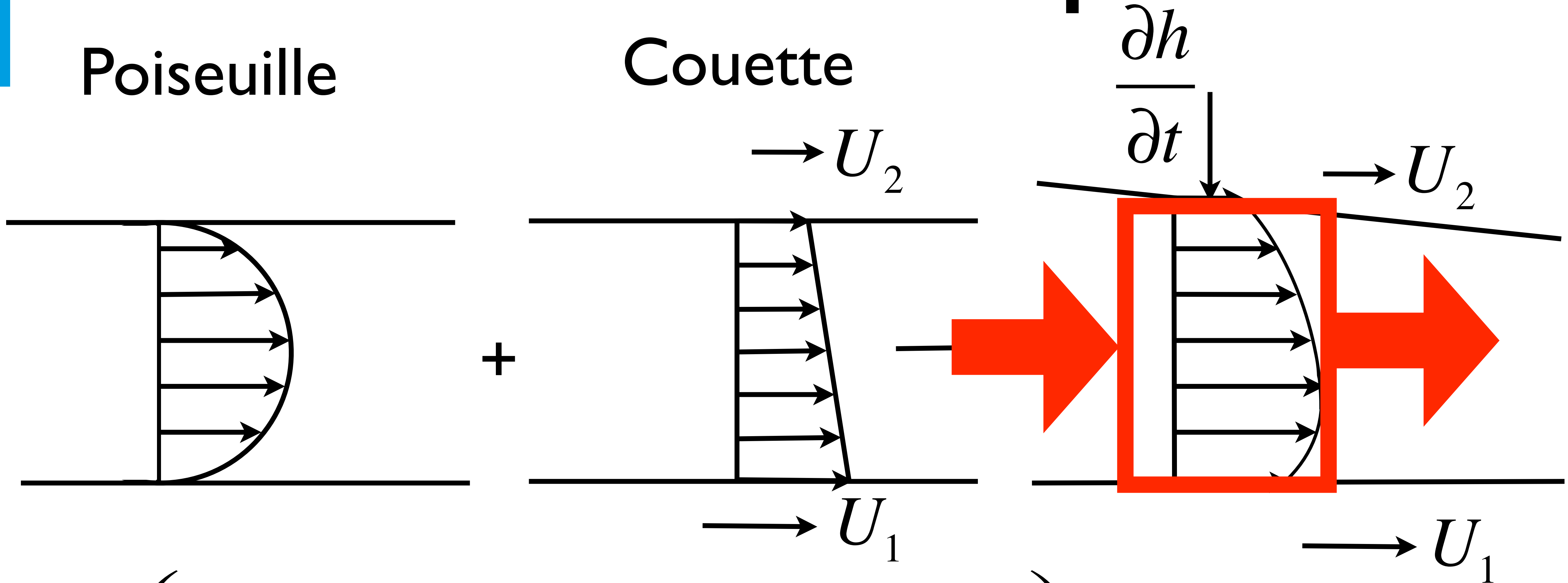
Assume: no slip

Poiseuille | Couette



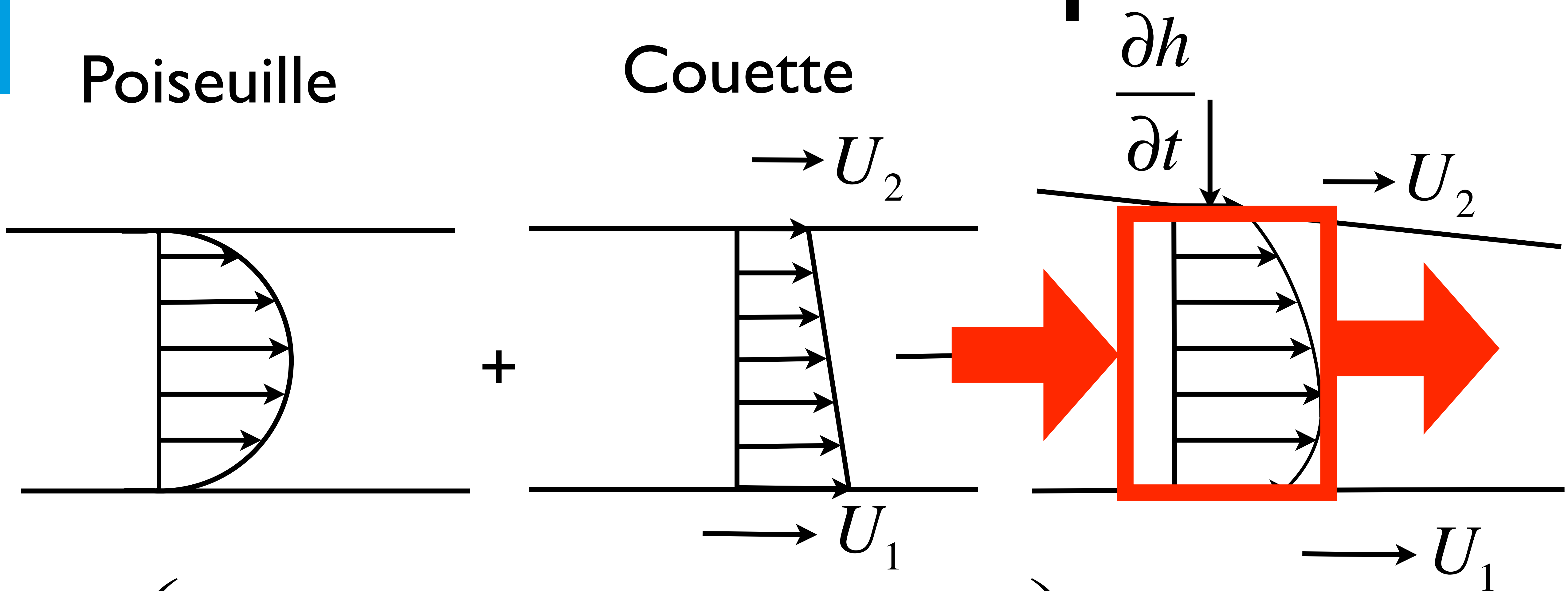
$$\nabla \cdot \left(\frac{-h^3}{12\mu}\nabla p + \left(\frac{U_1+U_2}{2}\right)h \right) =$$

Assume: no slip



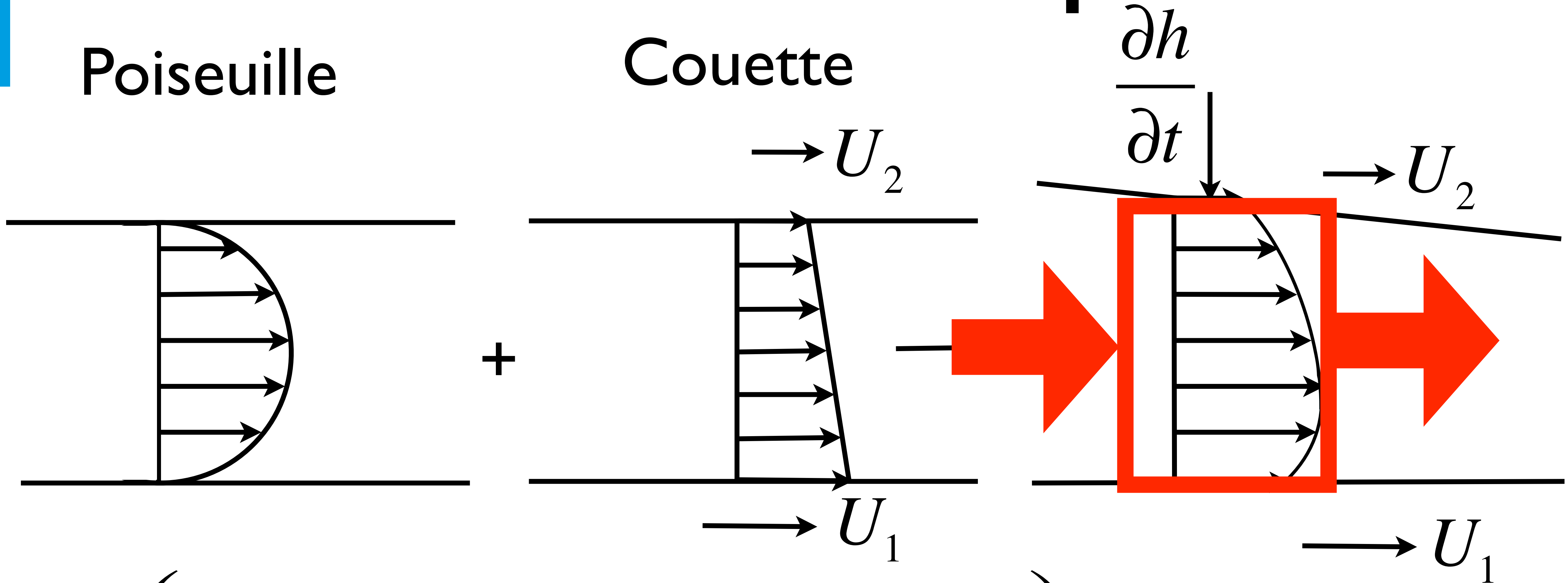
$$\nabla \cdot \left(\frac{-h^3}{12\mu} \nabla p + \left(\frac{U_1 + U_2}{2} \right) h \right) =$$

Assume: no slip

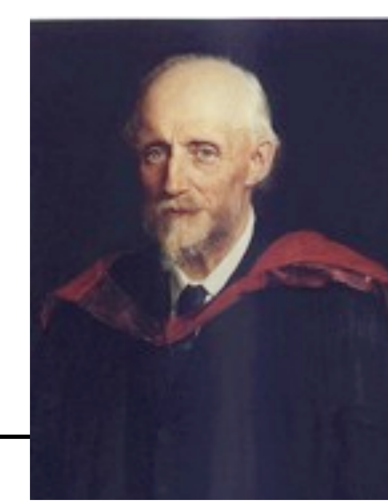


$$\nabla \cdot \left(\frac{-h^3}{12\mu} \nabla p + \left(\frac{U_1 + U_2}{2} \right) h \right) = \frac{\partial h}{\partial t}$$

Assume: no slip

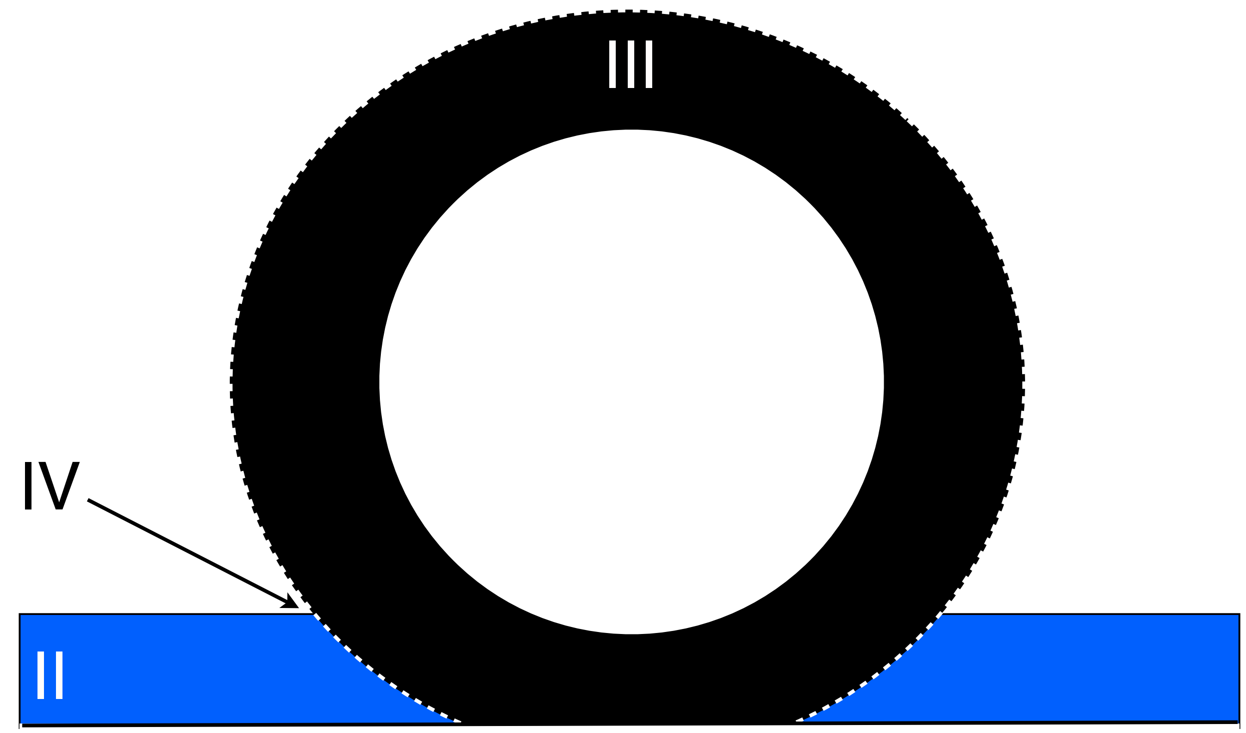


$$\nabla \cdot \left(\frac{-h^3}{12\mu} \nabla p + \left(\frac{U_1 + U_2}{2} \right) h \right) = \frac{\partial h}{\partial t}$$

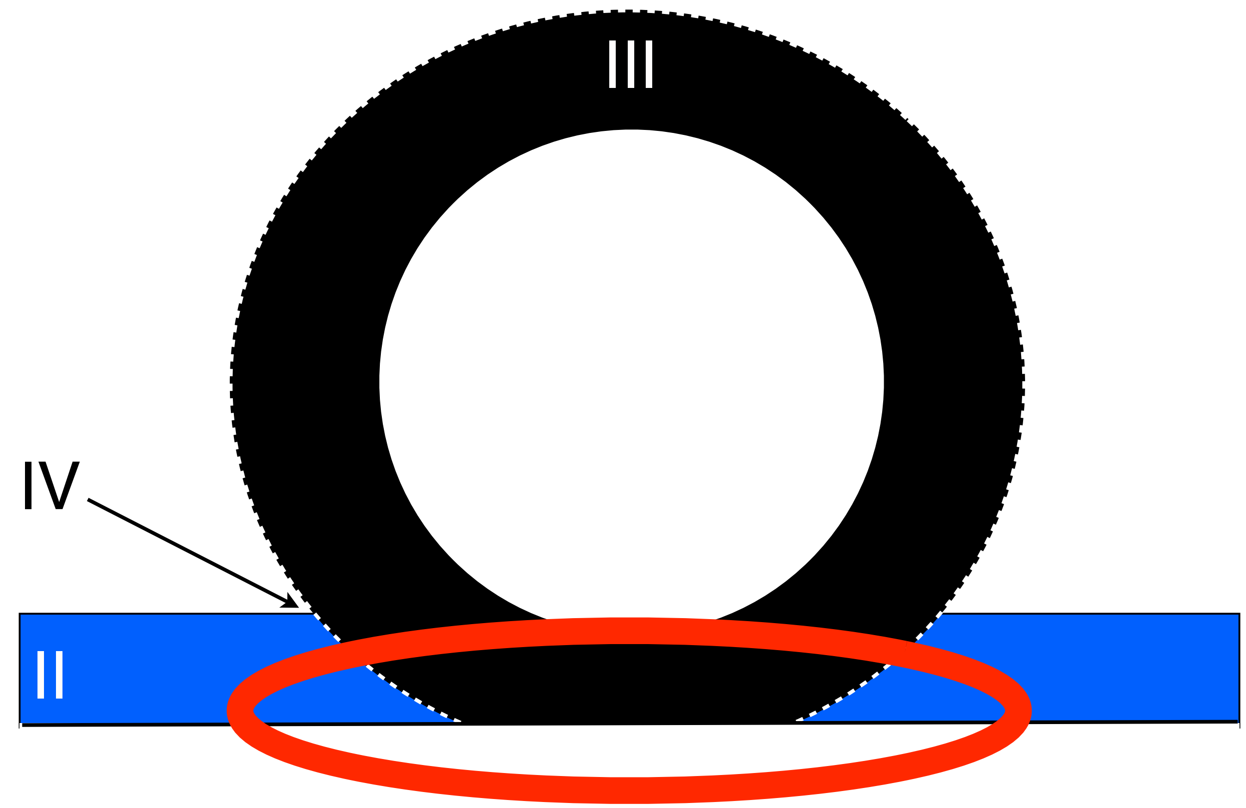


Reynolds (1886)

Inertia correction



Inertia correction

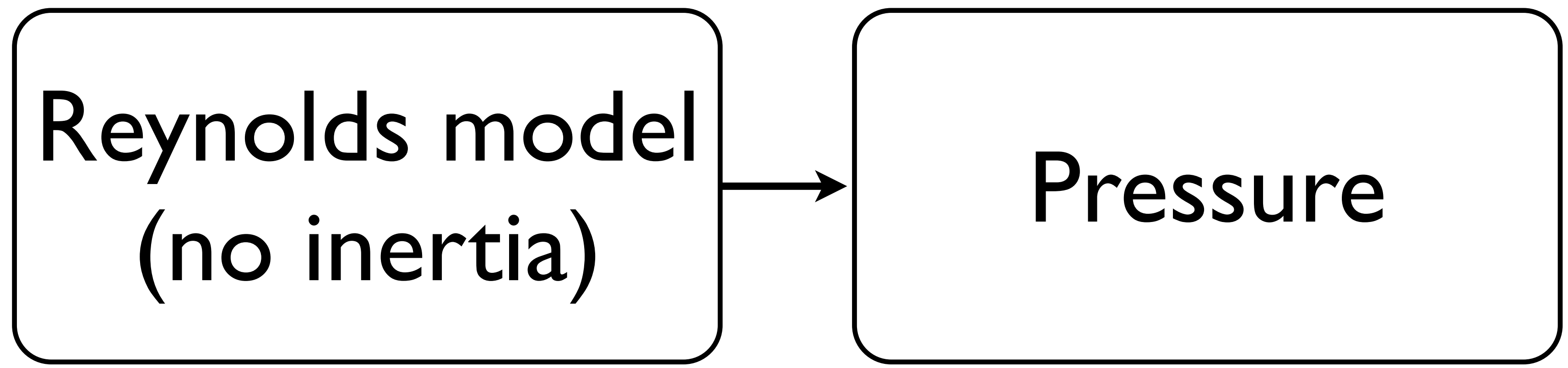


Inertia correction: 1D

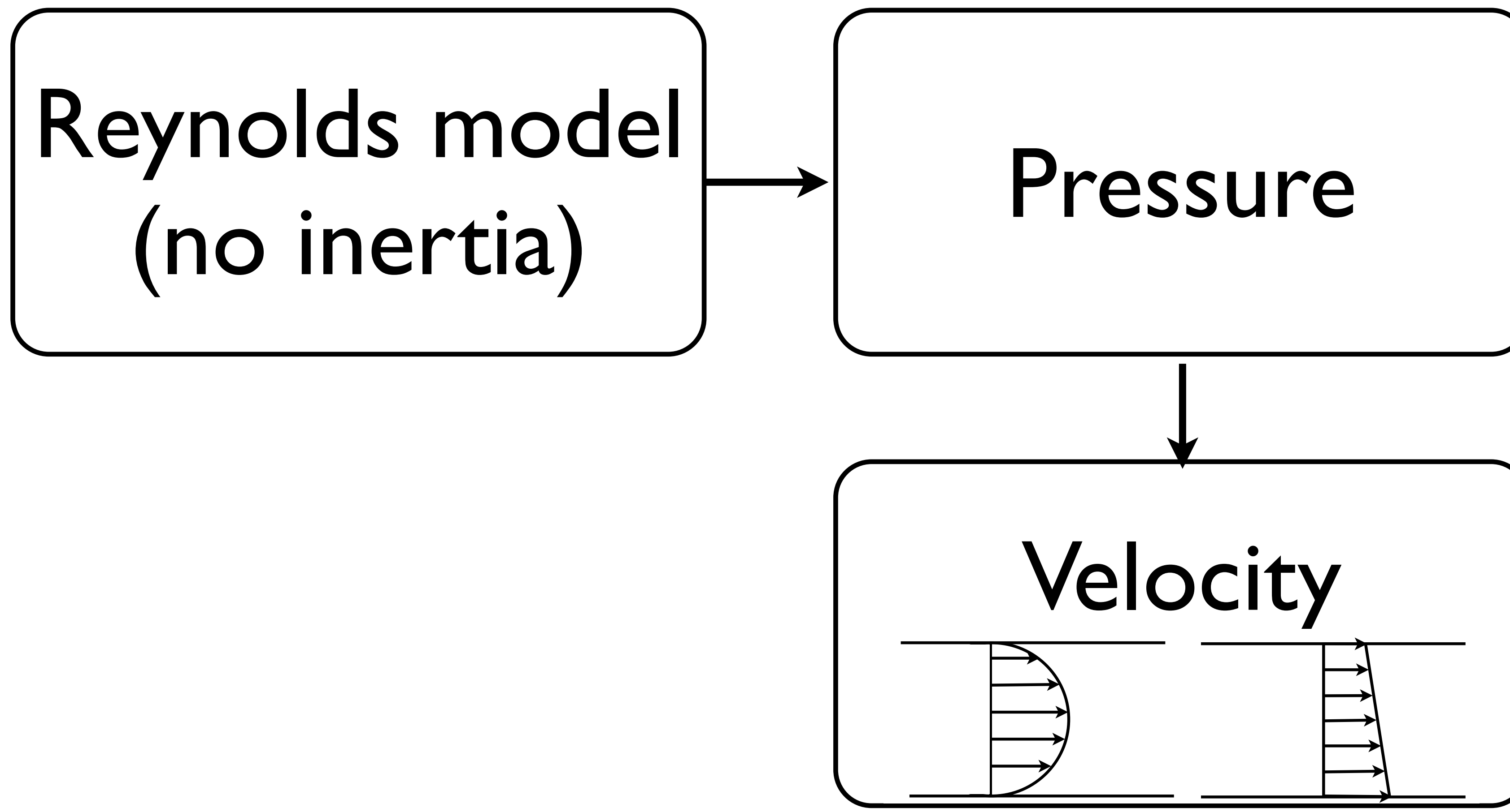
Inertia correction: 1D

Reynolds model
(no inertia)

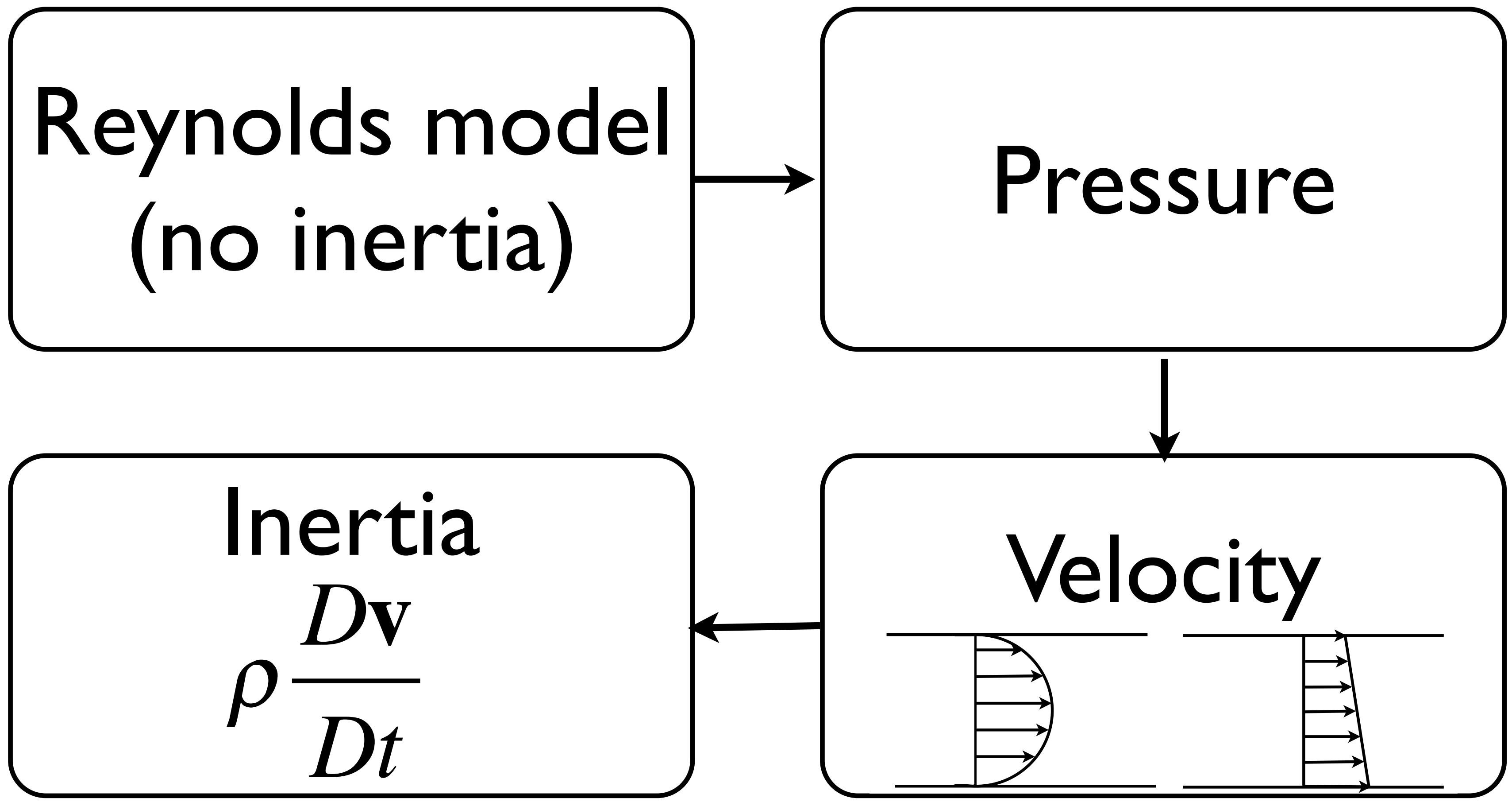
Inertia correction: 1D



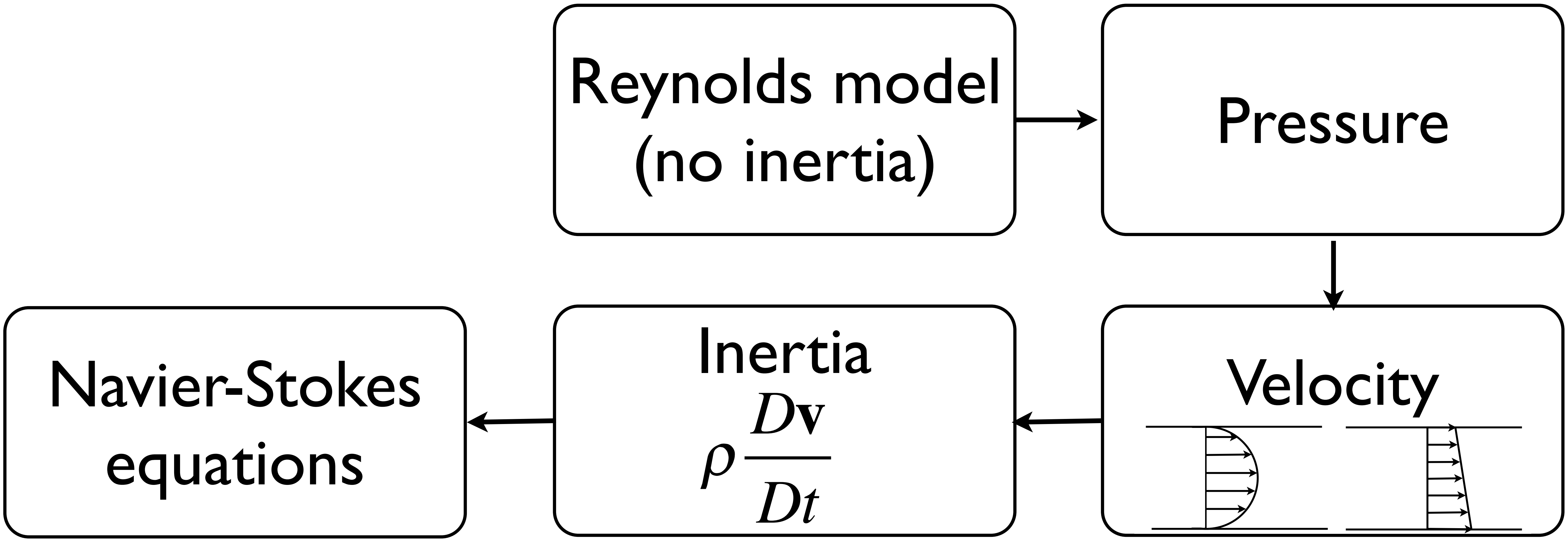
Inertia correction: 1D



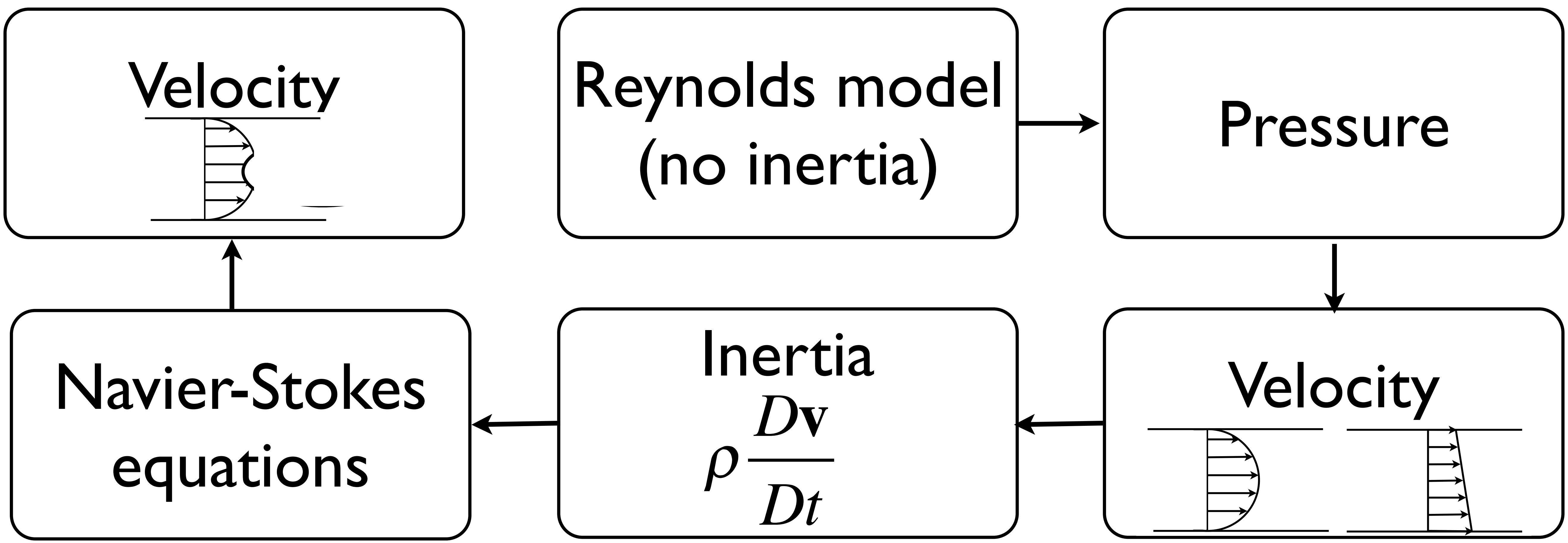
Inertia correction: 1D



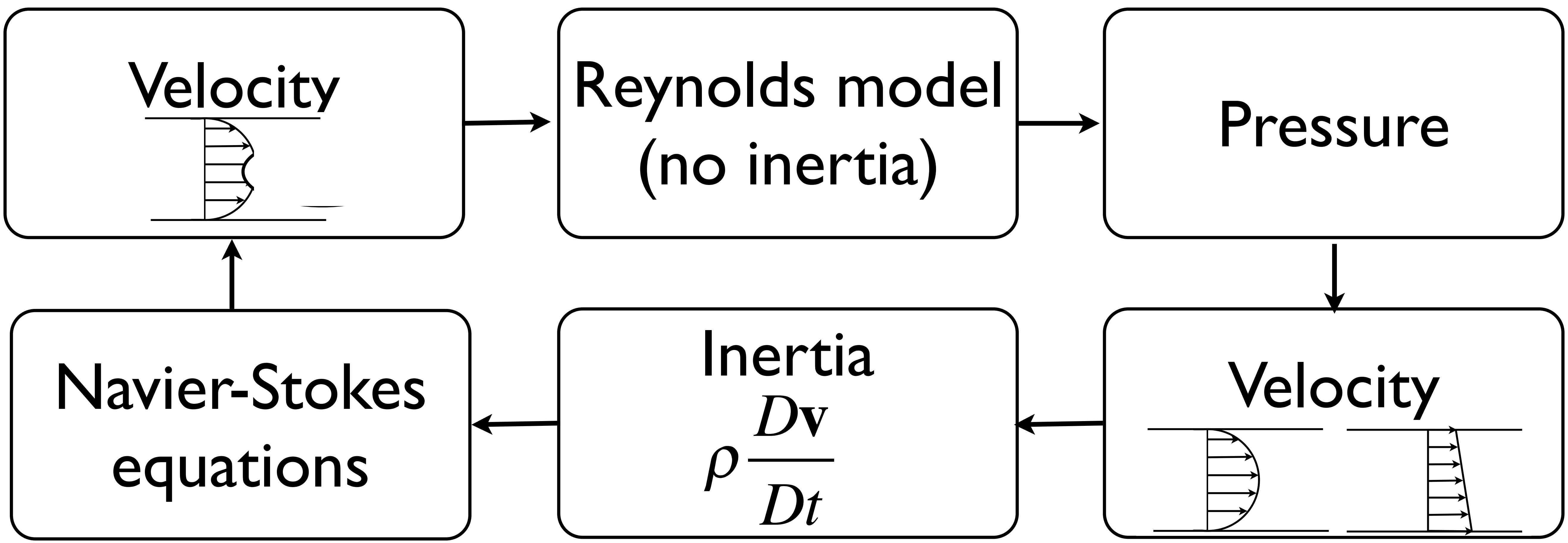
Inertia correction: 1D



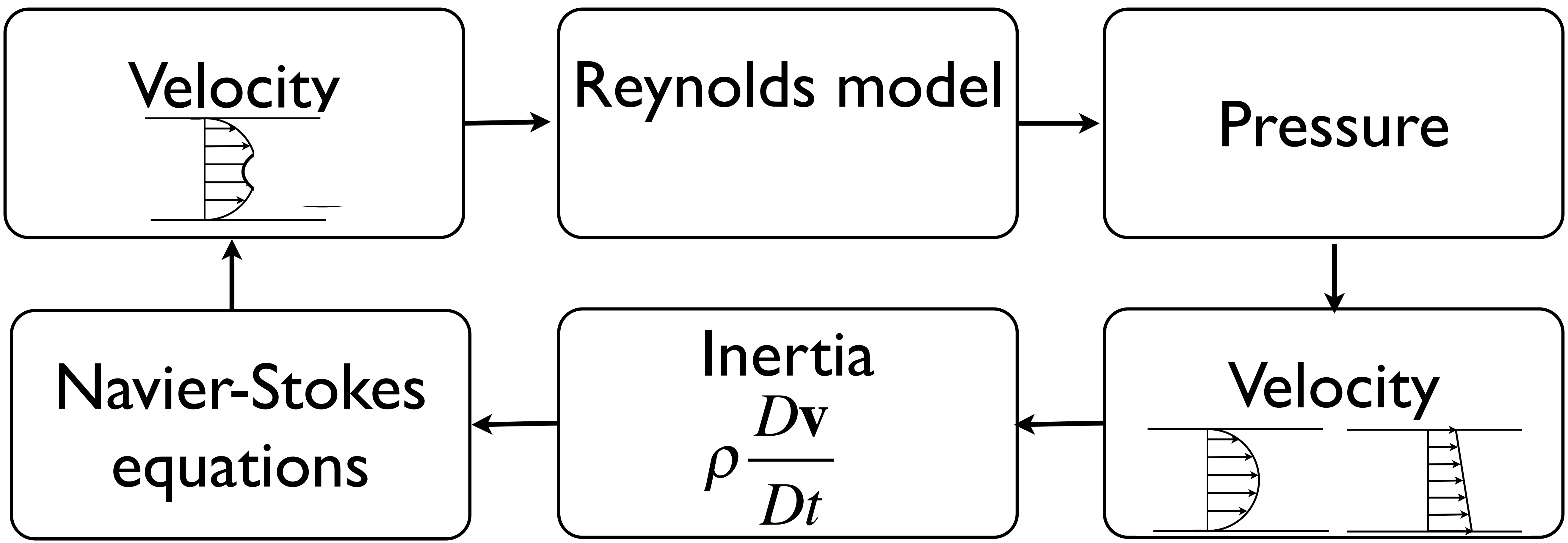
Inertia correction: 1D



Inertia correction: 1D

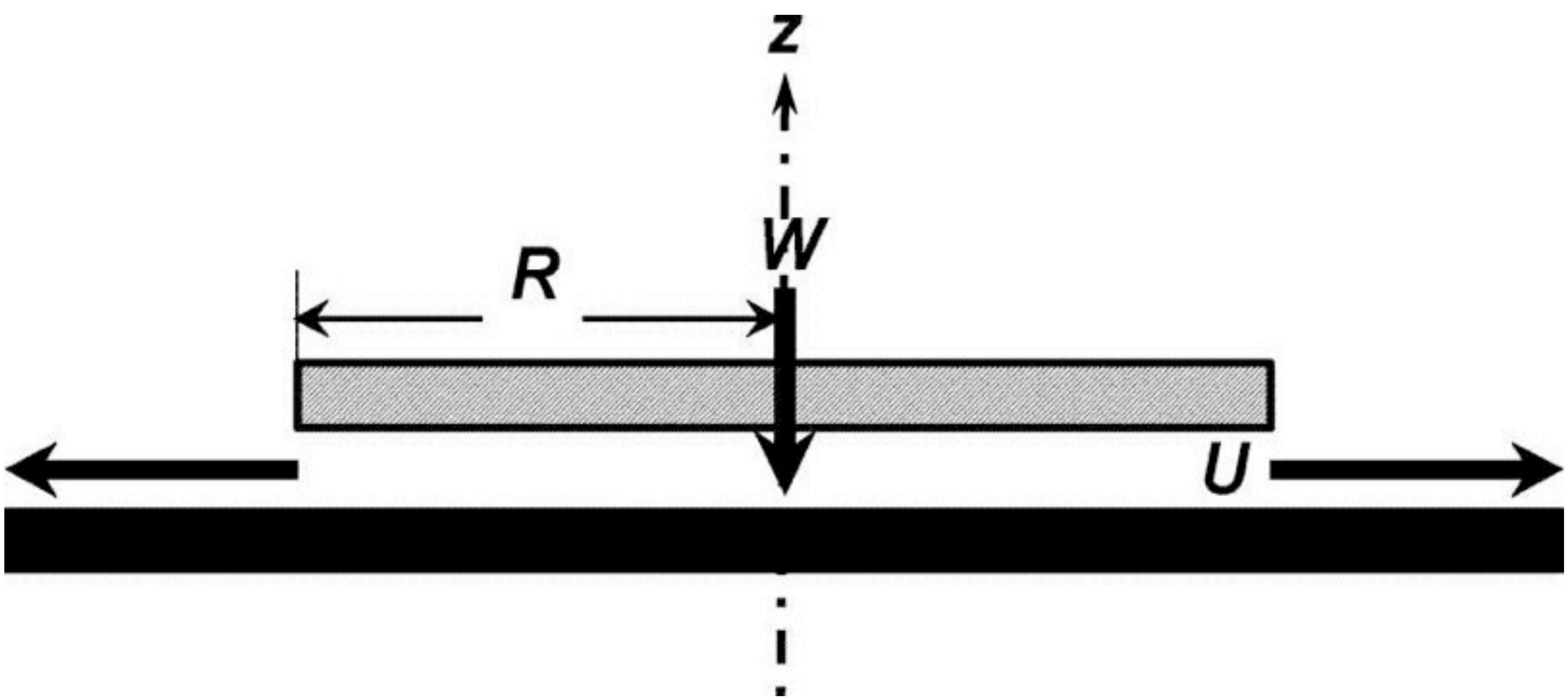


Inertia correction: 1D

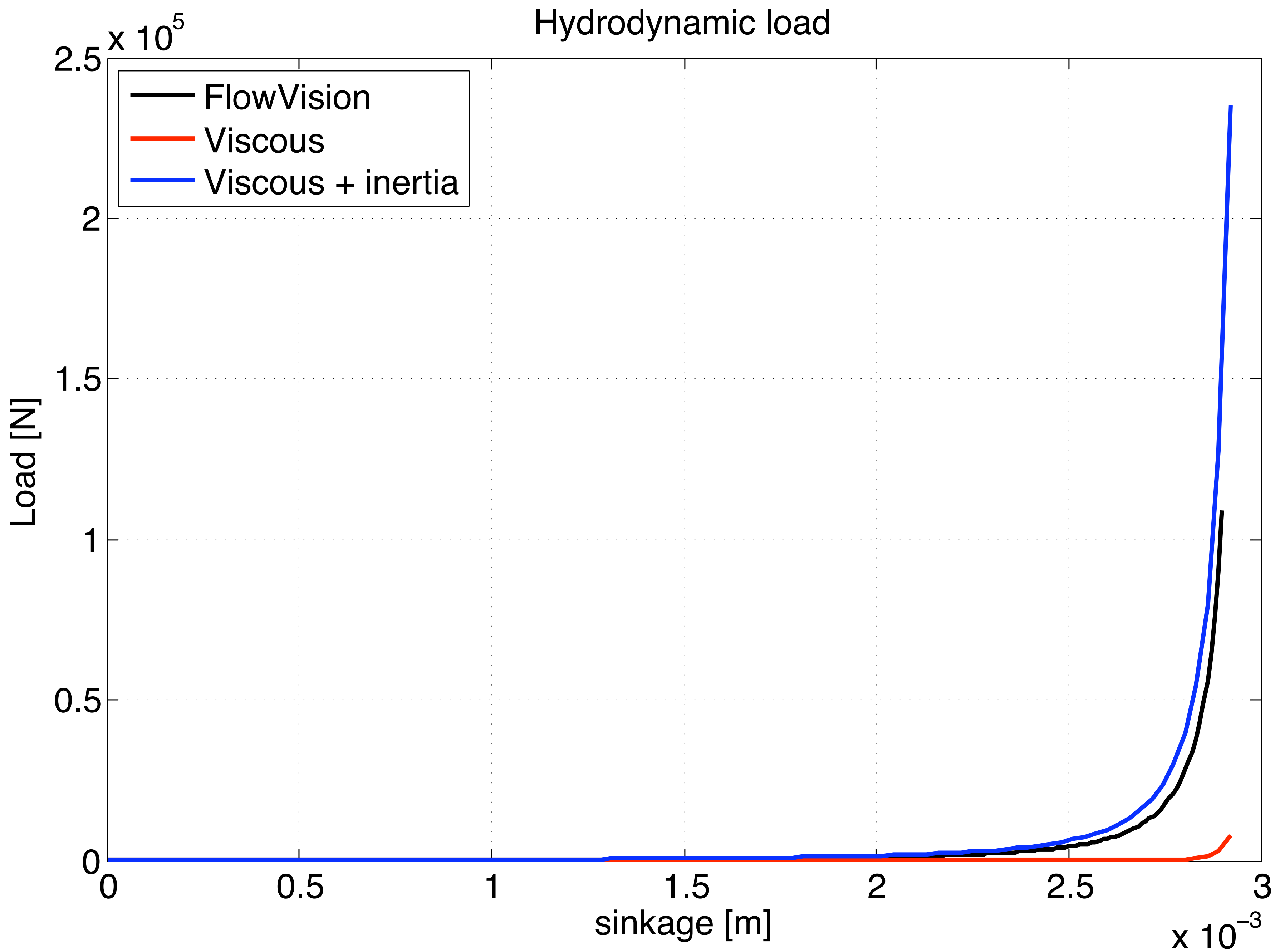


Squeeze

- Circular disc

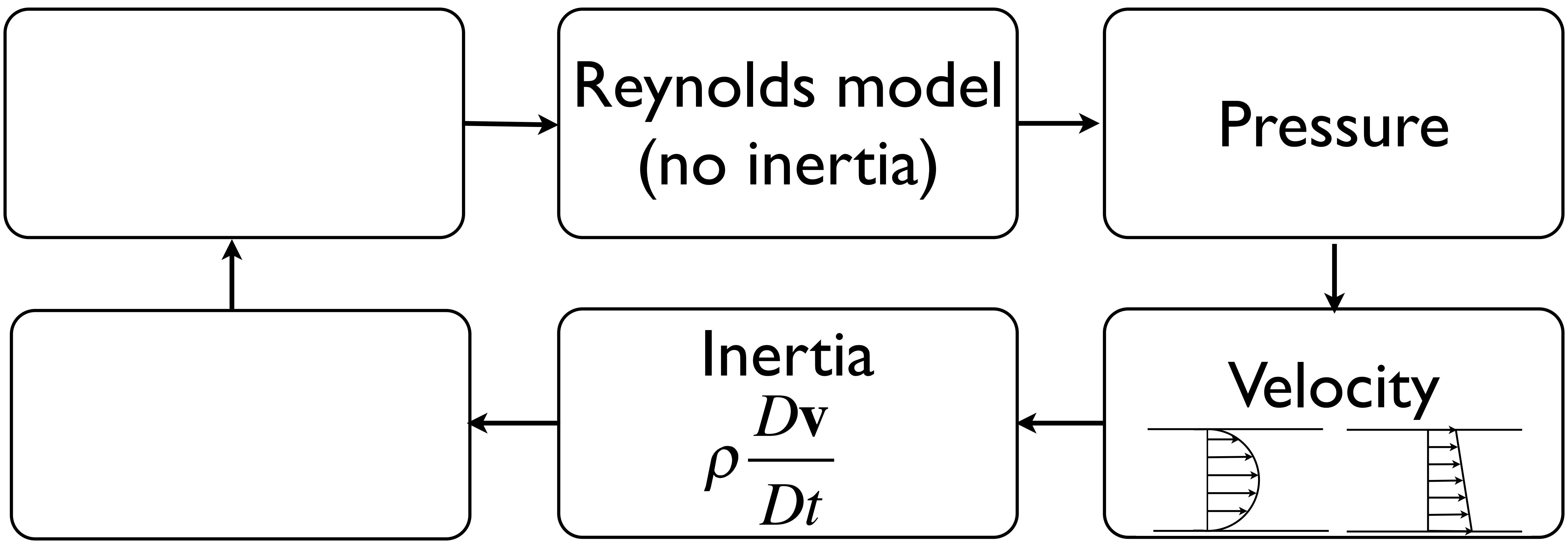


Squeeze

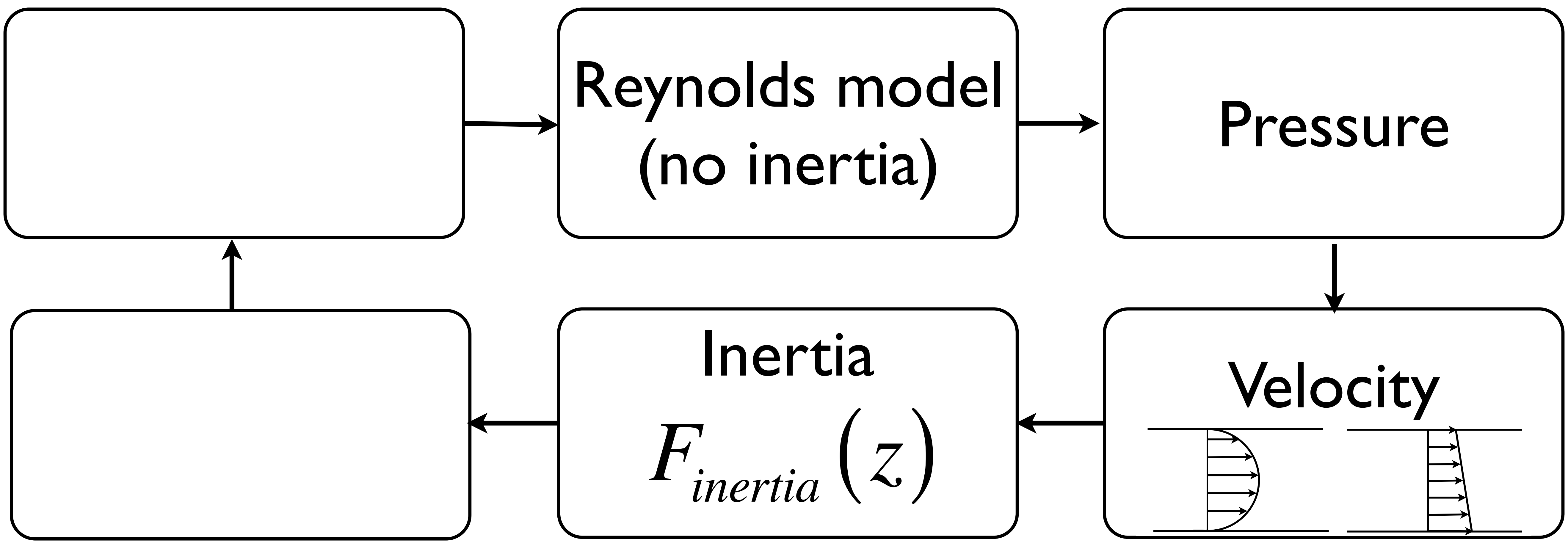


Hydrodynamic load
x 10^5
2.5
2
1.5
1
0.5
0
Load [N]
FlowVision
Viscous
Viscous + inertia
0
0.5
1
1.5
2
2.5
3
sinkage [m]
x 10^-3

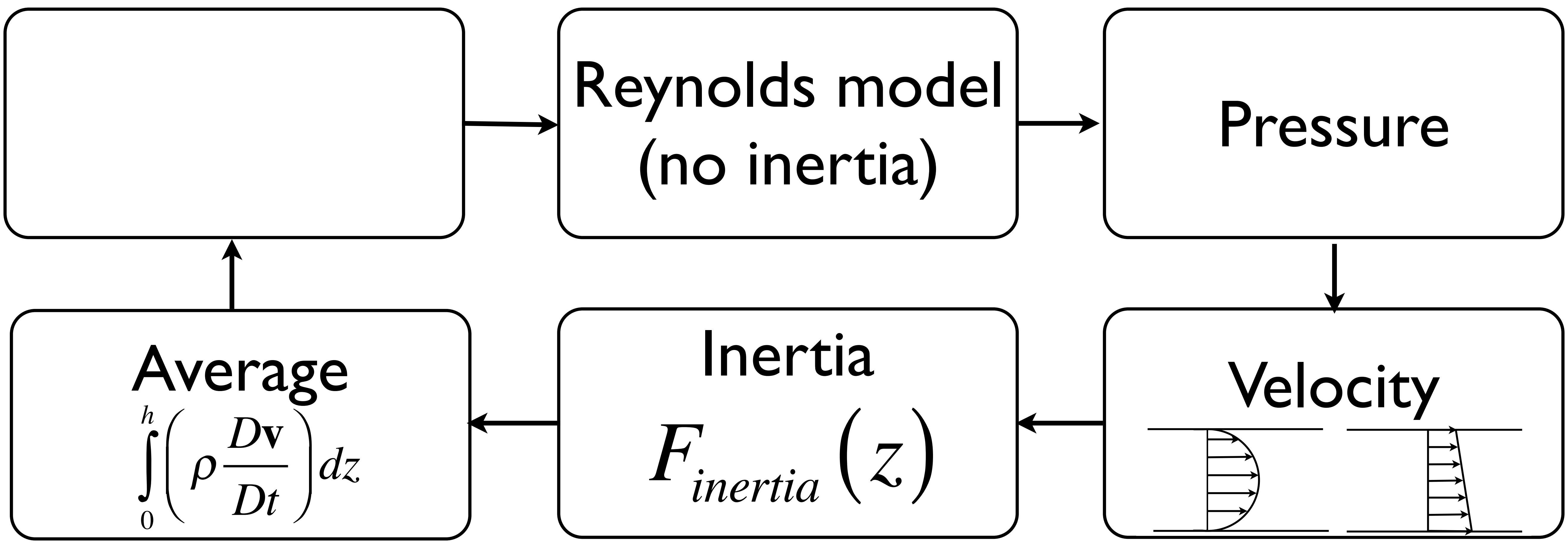
Inertia correction: 2D



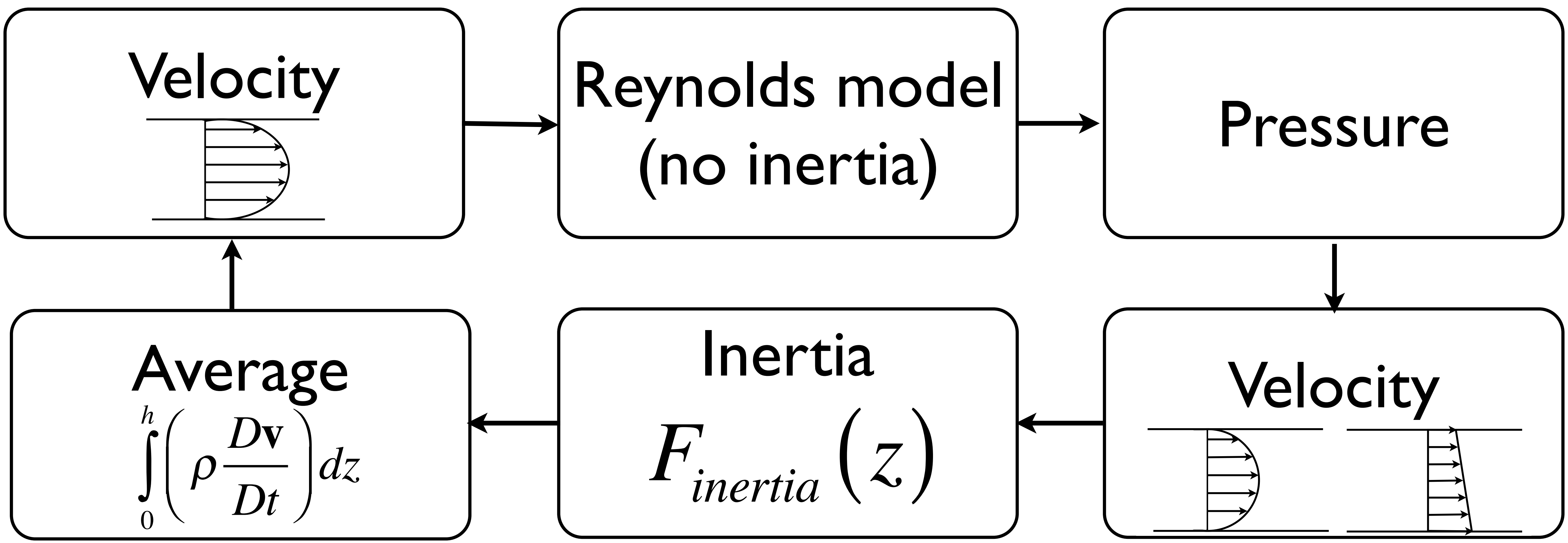
Inertia correction: 2D



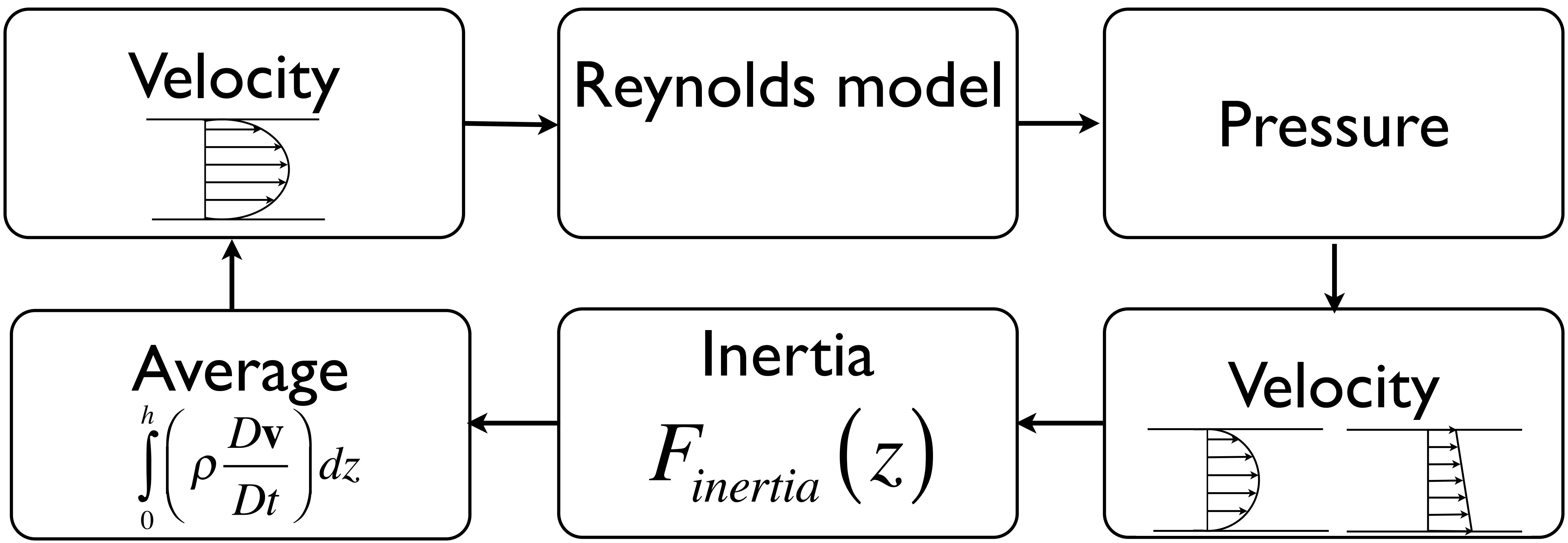
Inertia correction: 2D



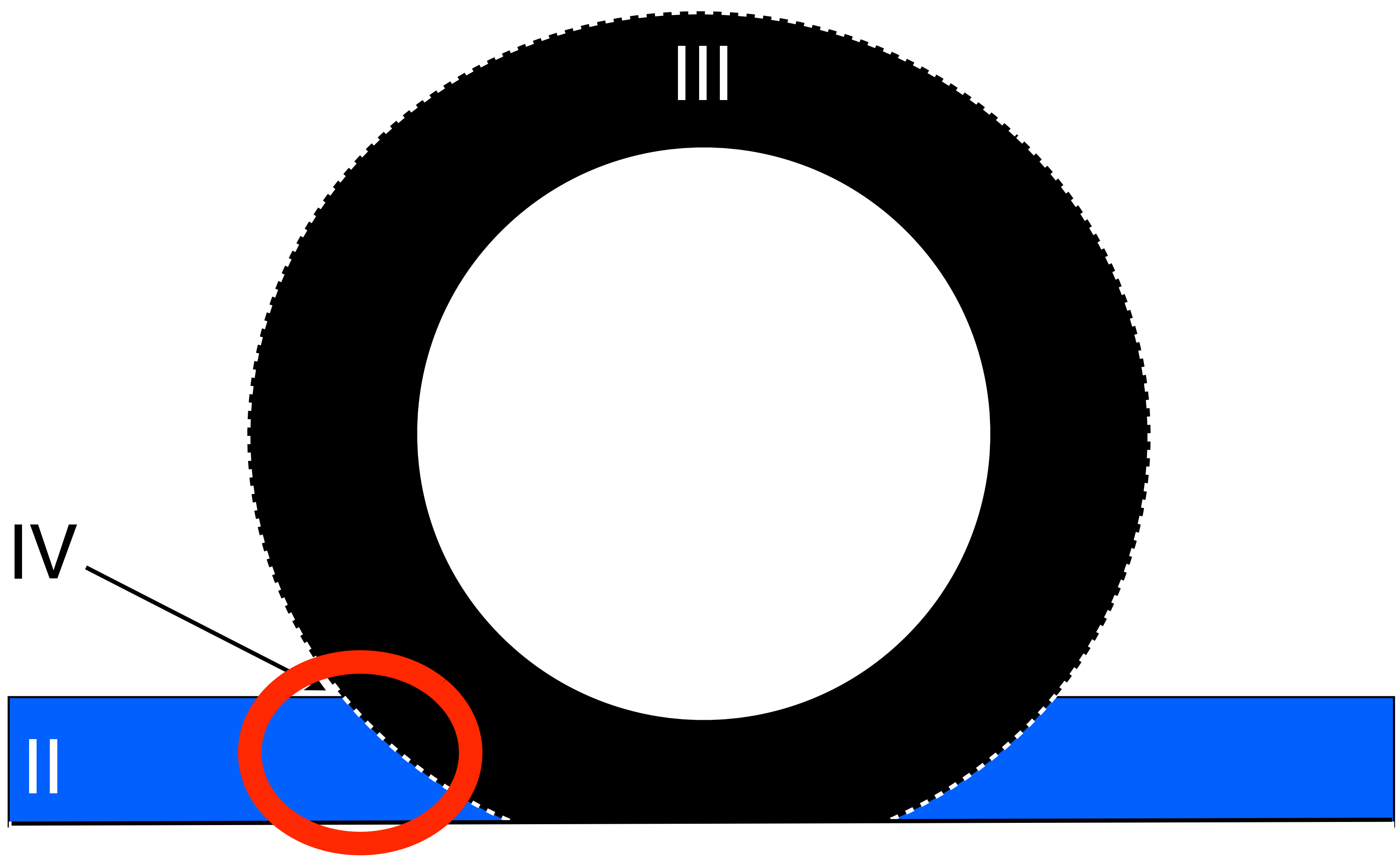
Inertia correction: 2D



Inertia correction: 2D

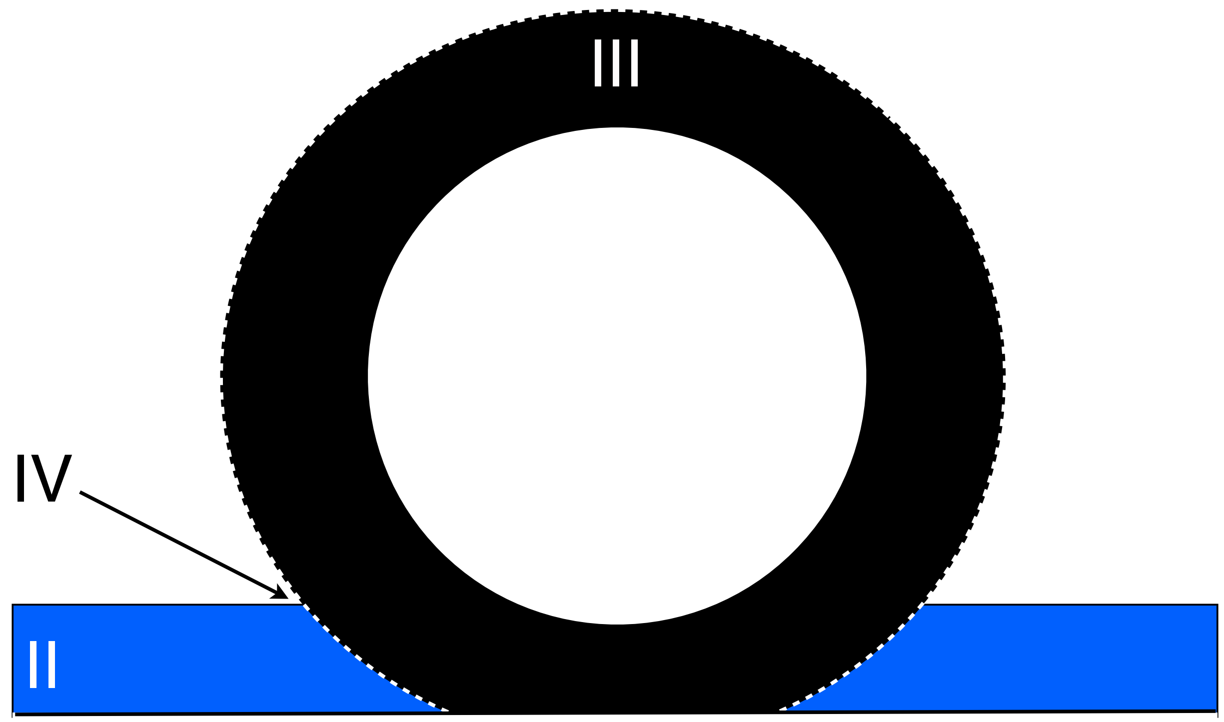


Inlet condition



Inlet condition

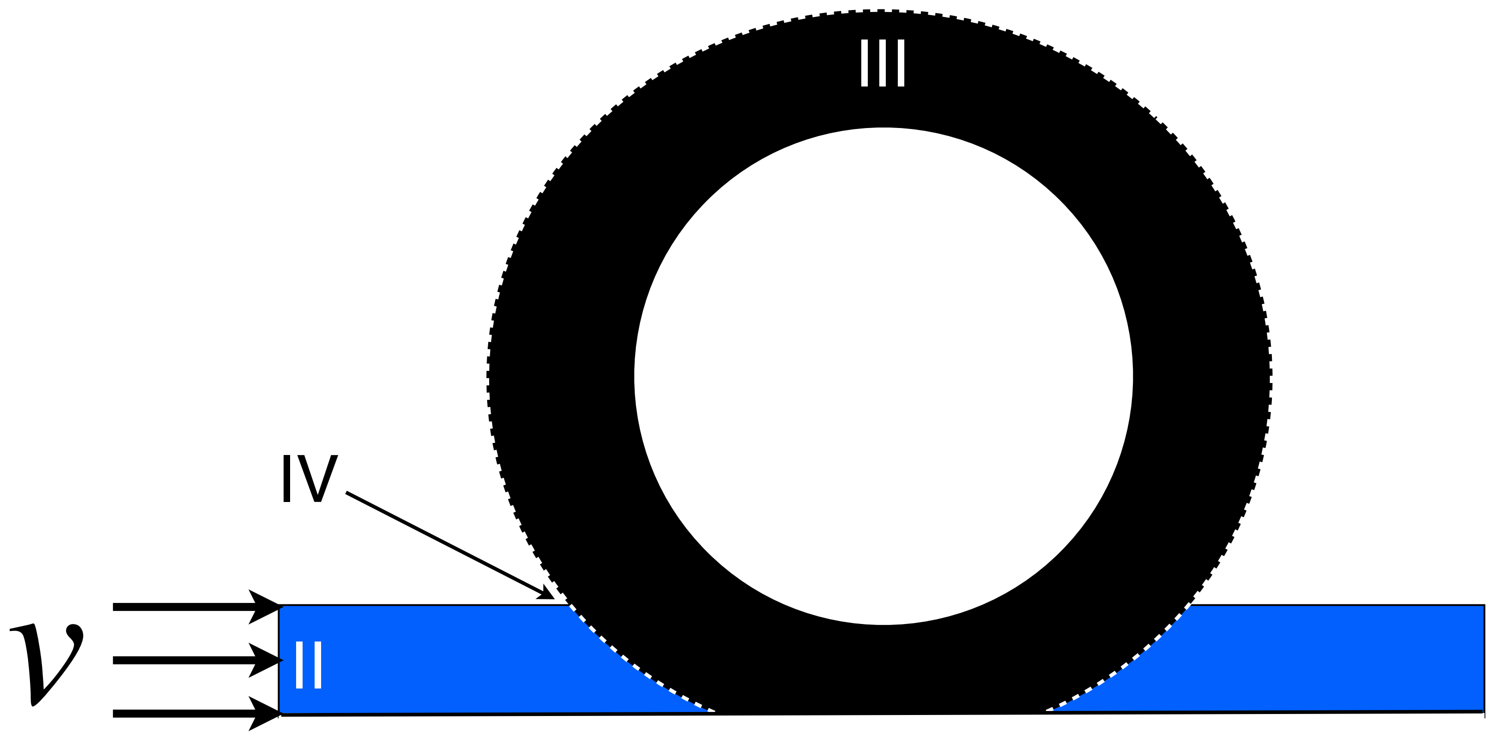
- Stagnation pressure




Bernoulli (1738)

Inlet condition

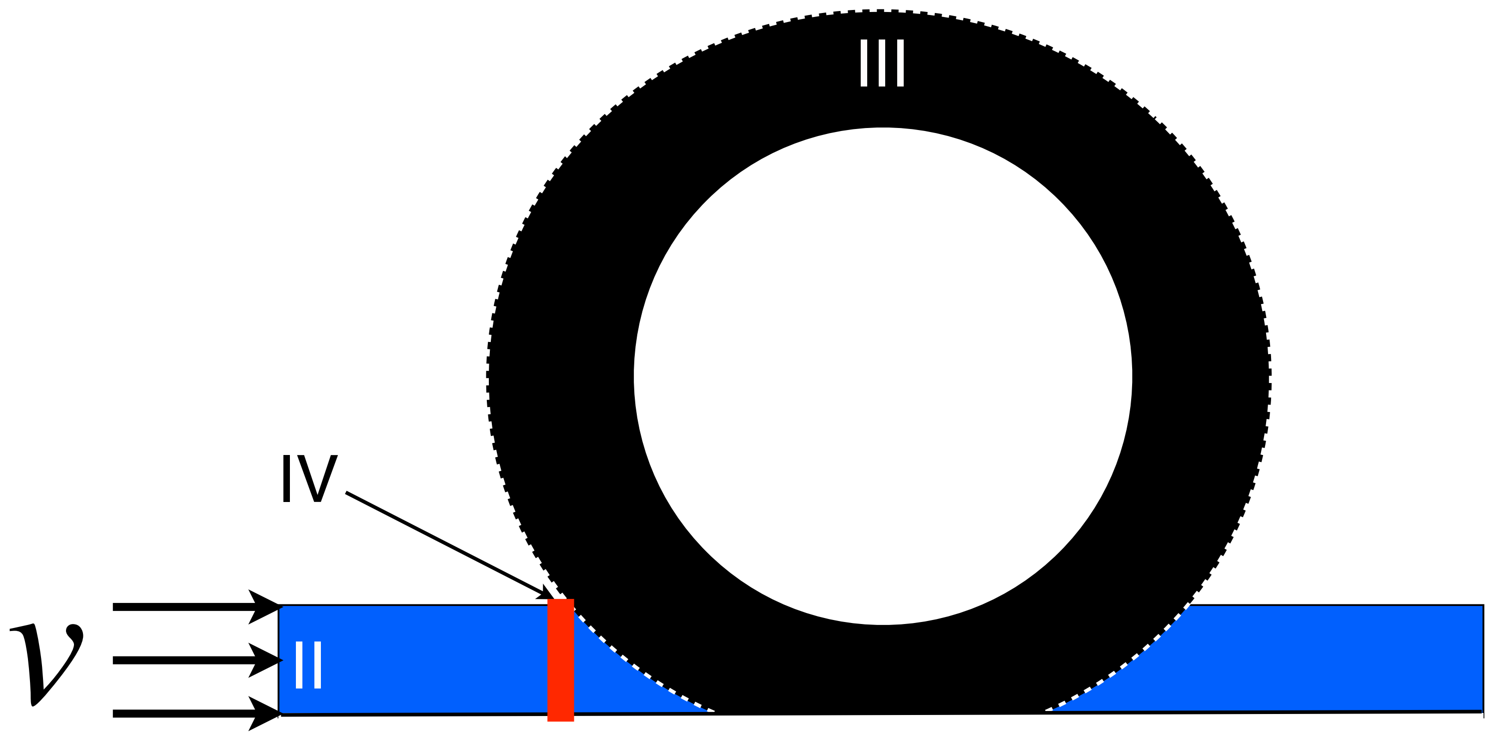
- Stagnation pressure



Bernoulli (1738)

Inlet condition

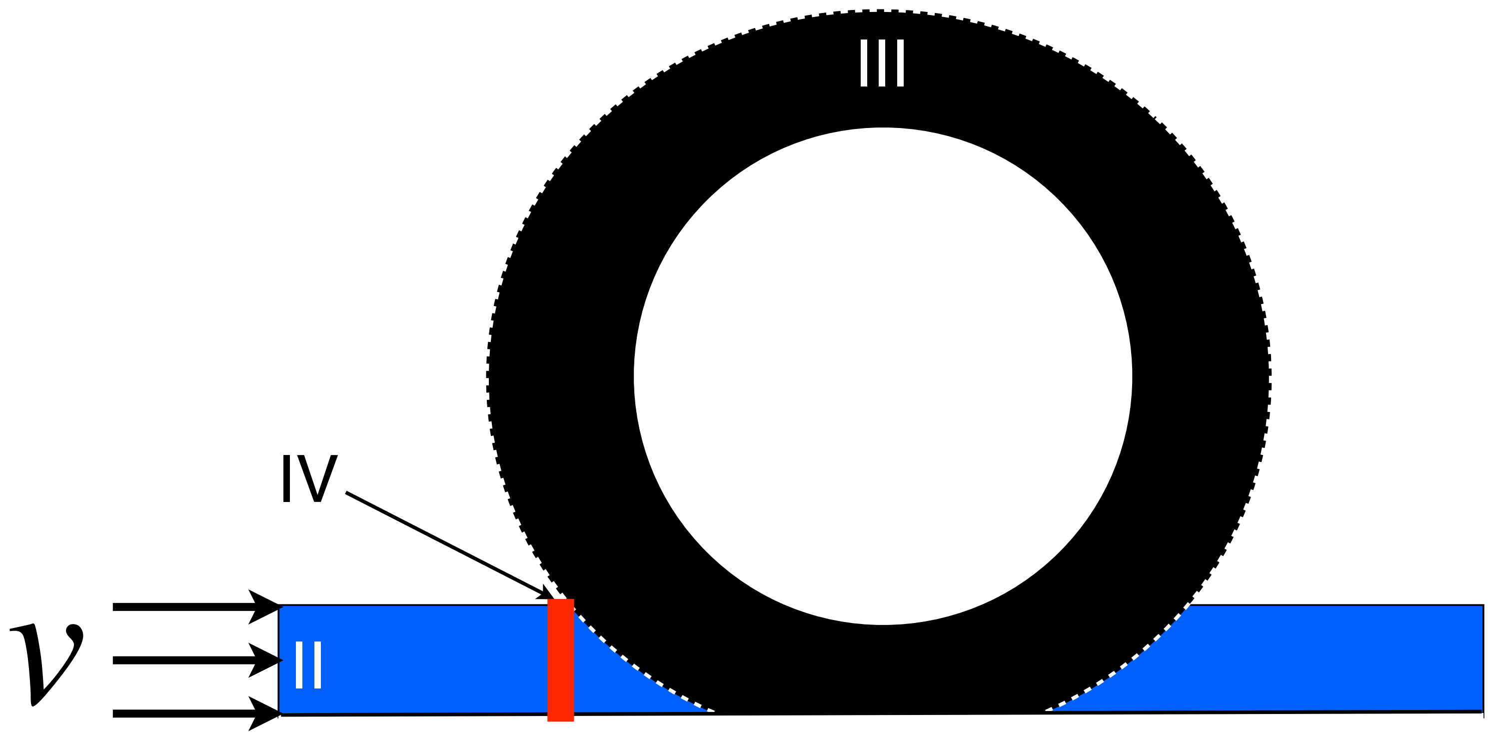
- Stagnation pressure



Bernoulli (1738)

Inlet condition

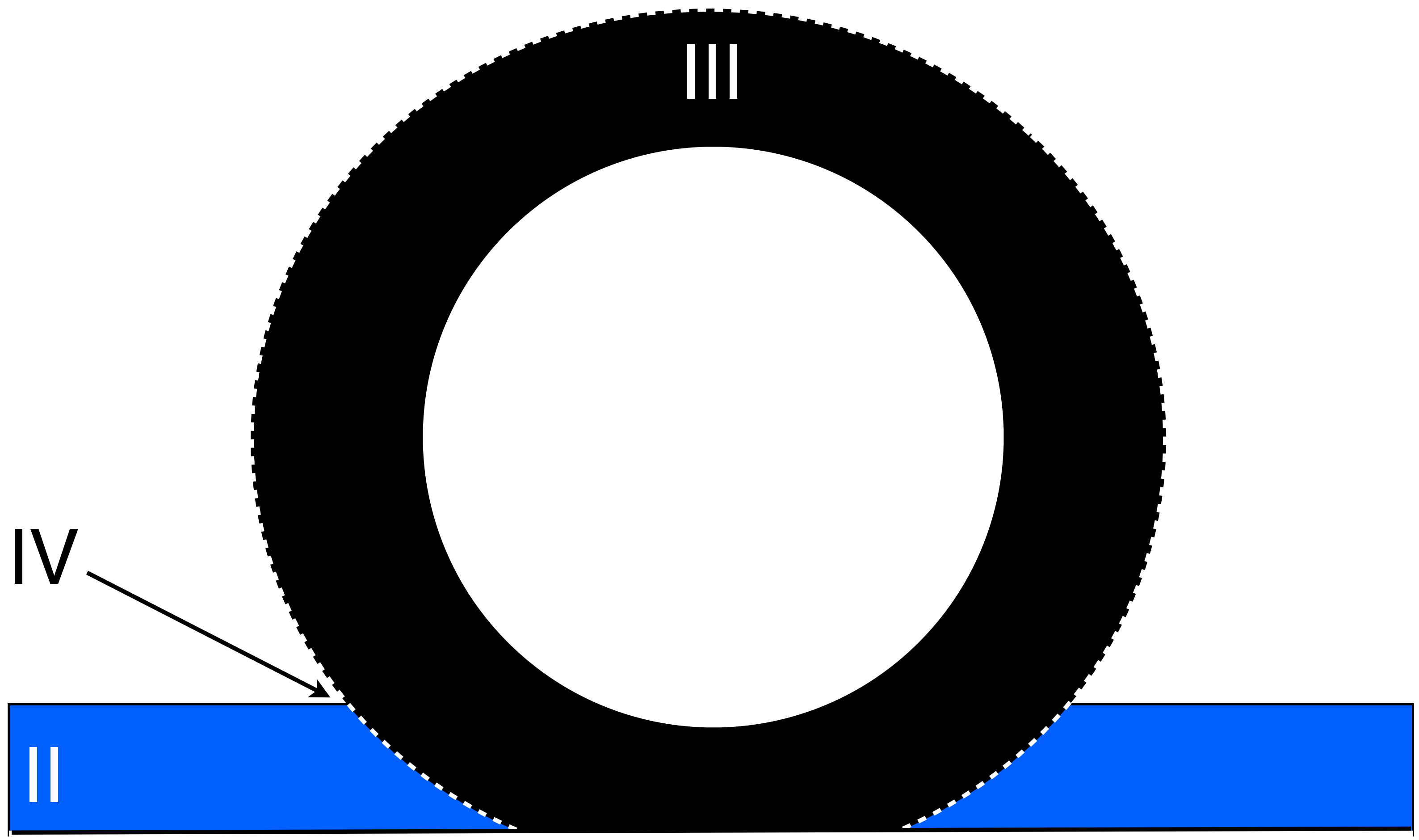
- Stagnation pressure



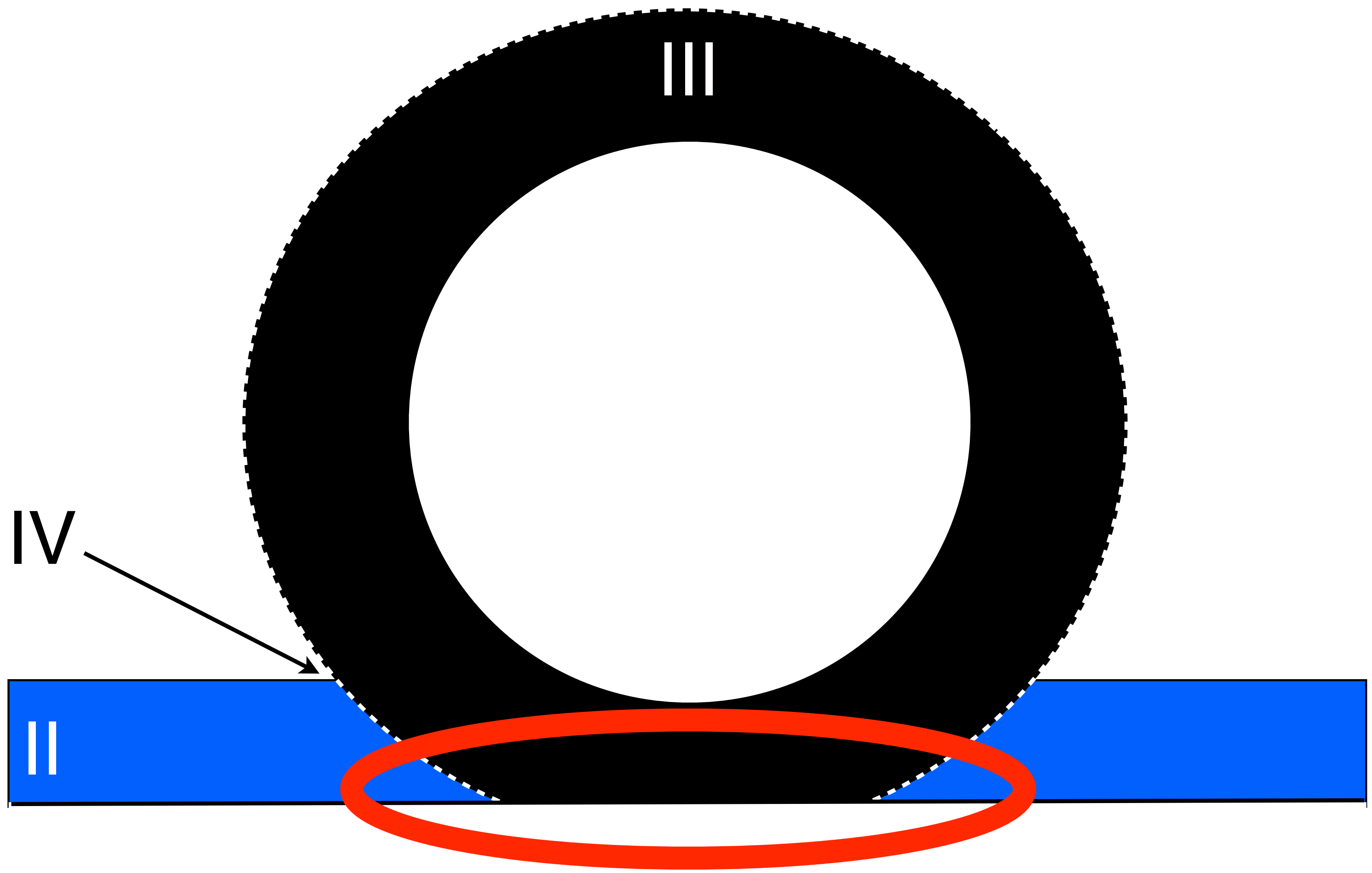
$$p = \frac{1}{2}\rho v^2$$

Bernoulli (1738)

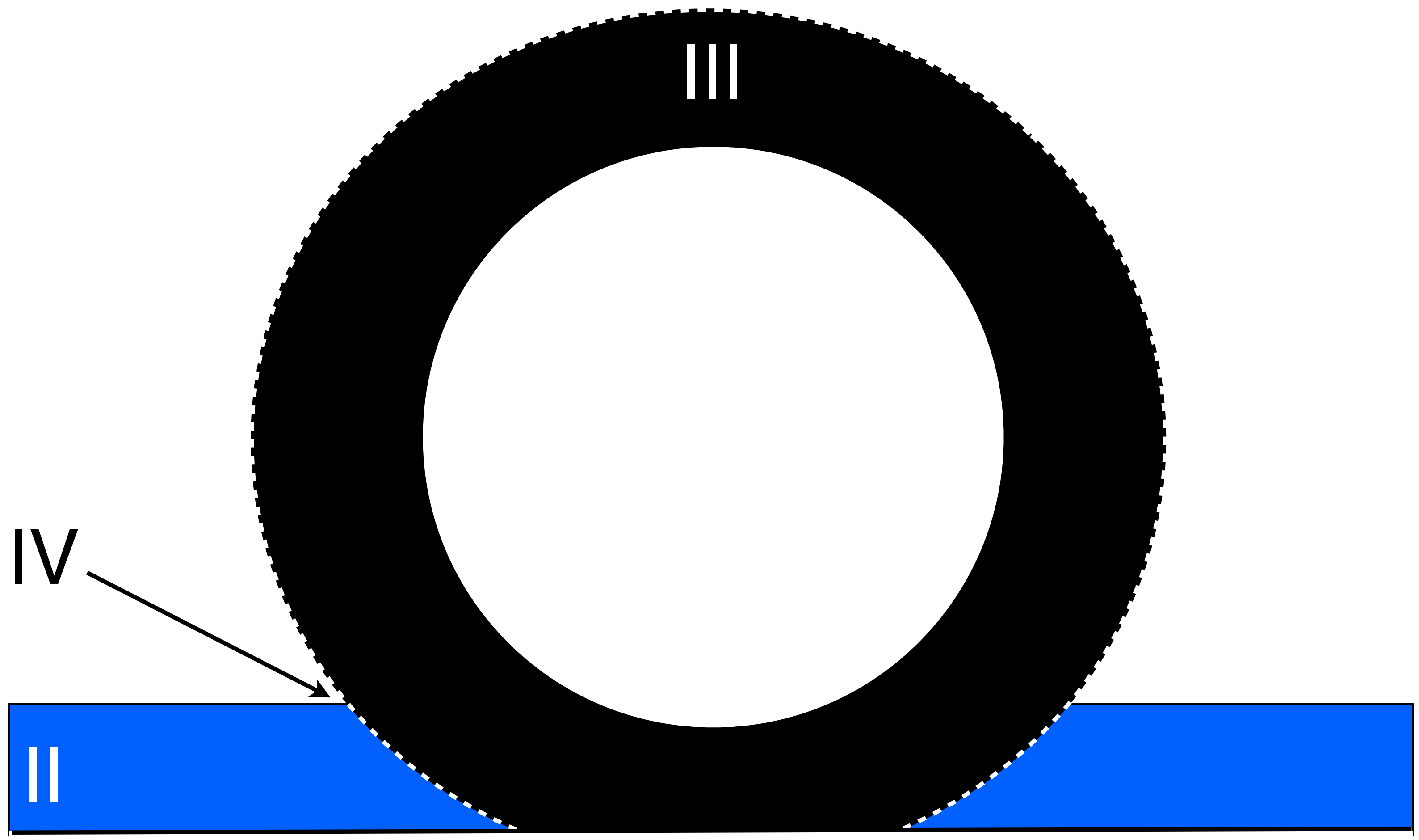
Summary



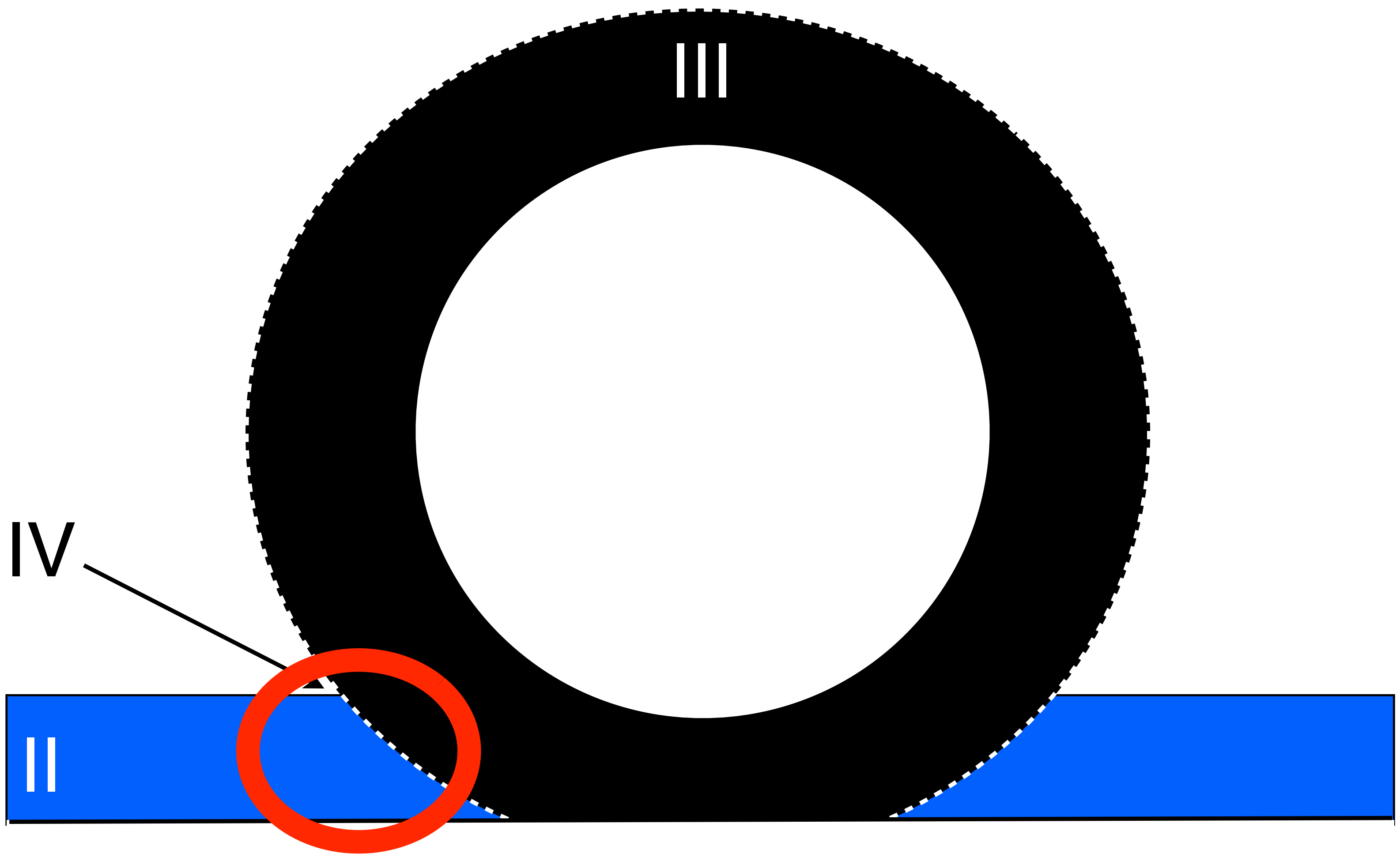
Summary



Summary



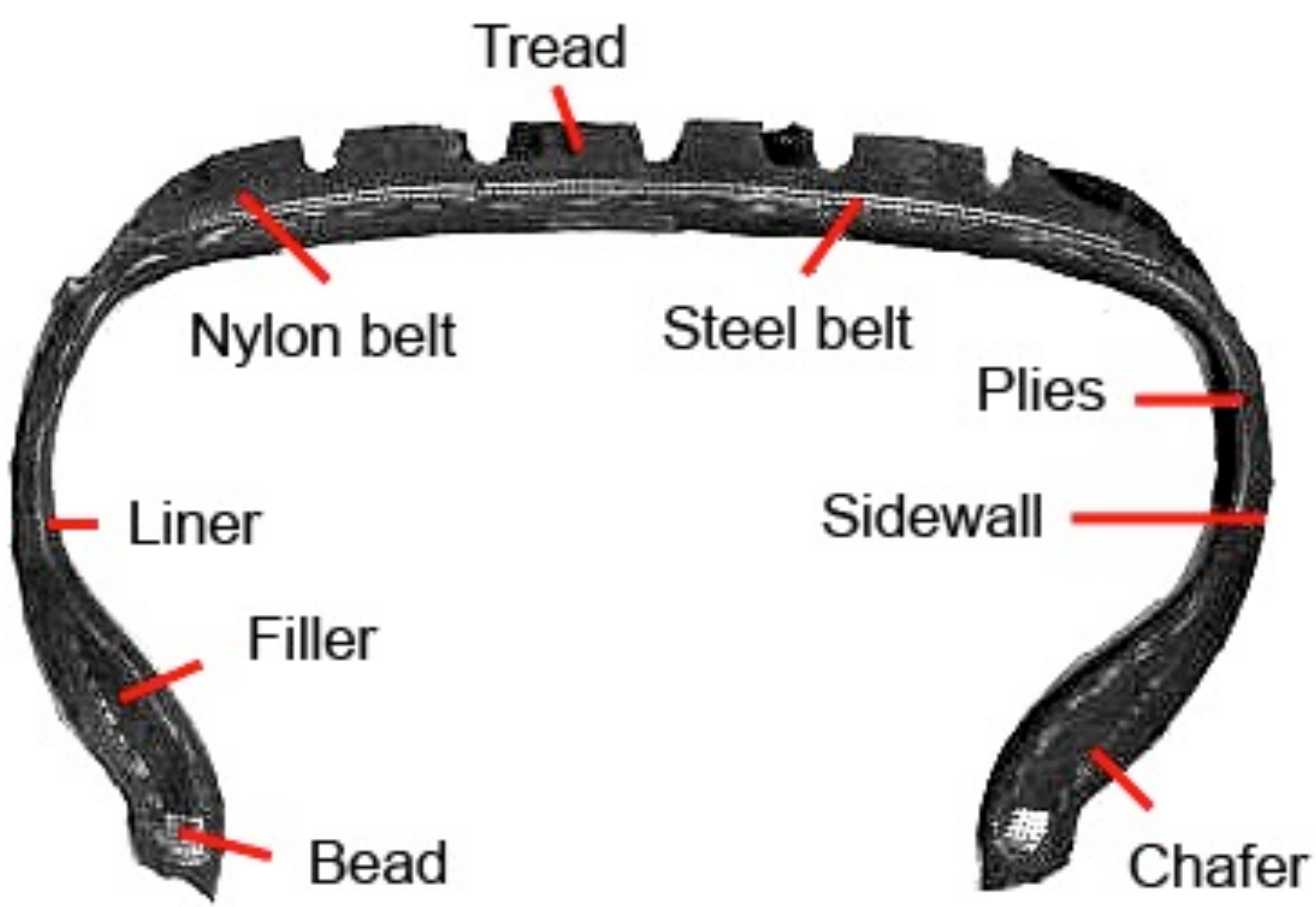
Summary



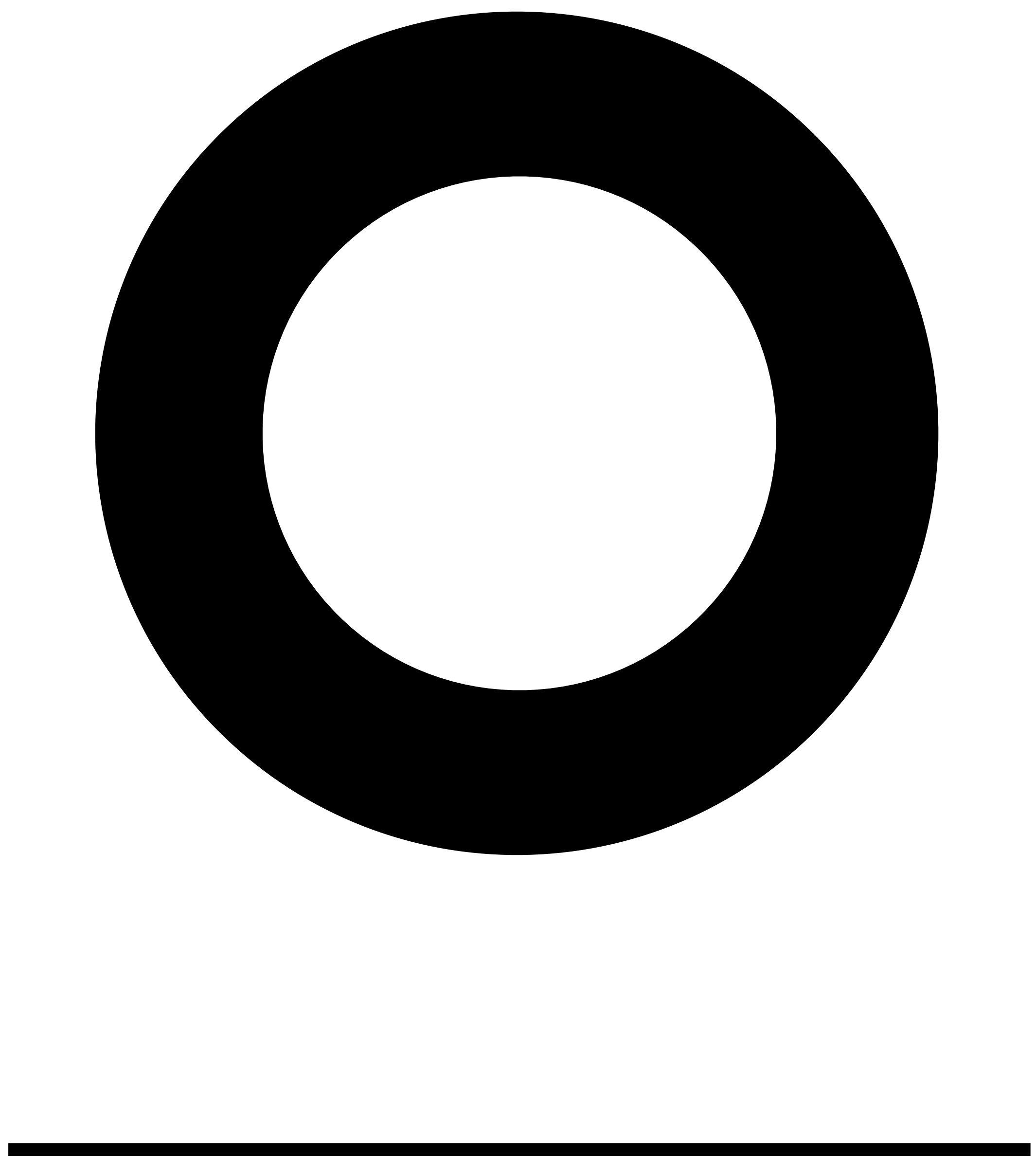
Tire modelling

Tire modelling

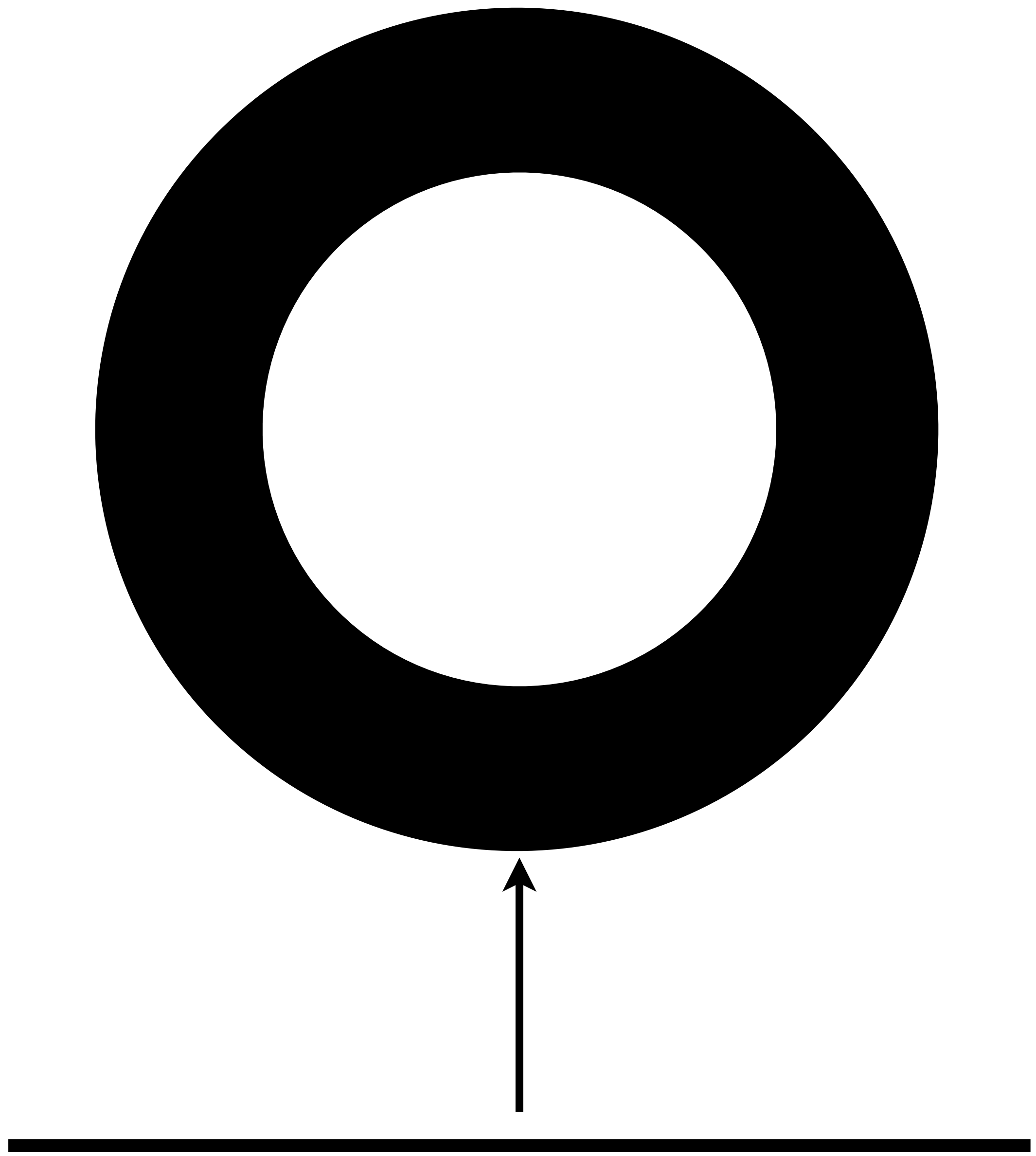
Tire construction



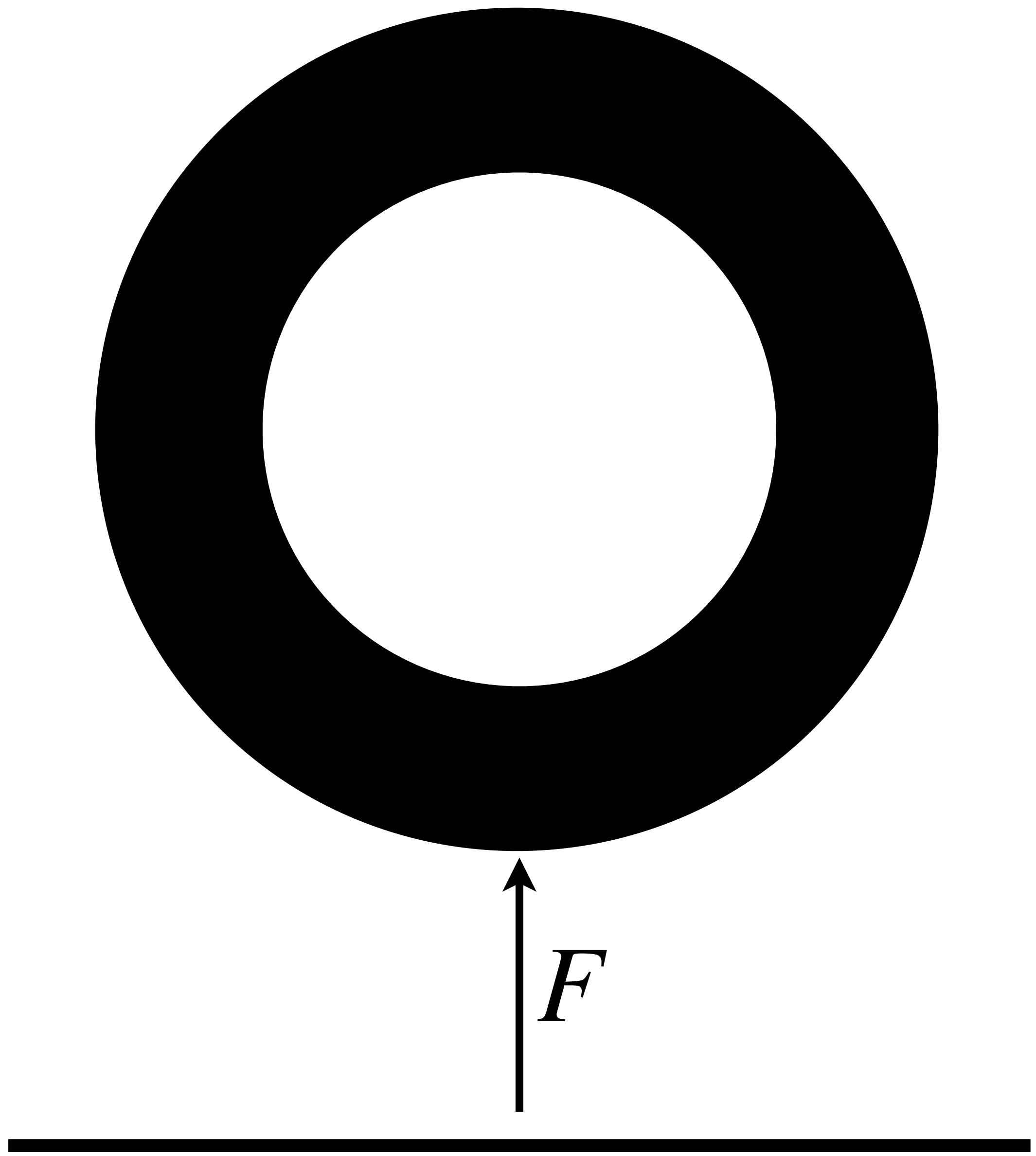
Linear elastic tire



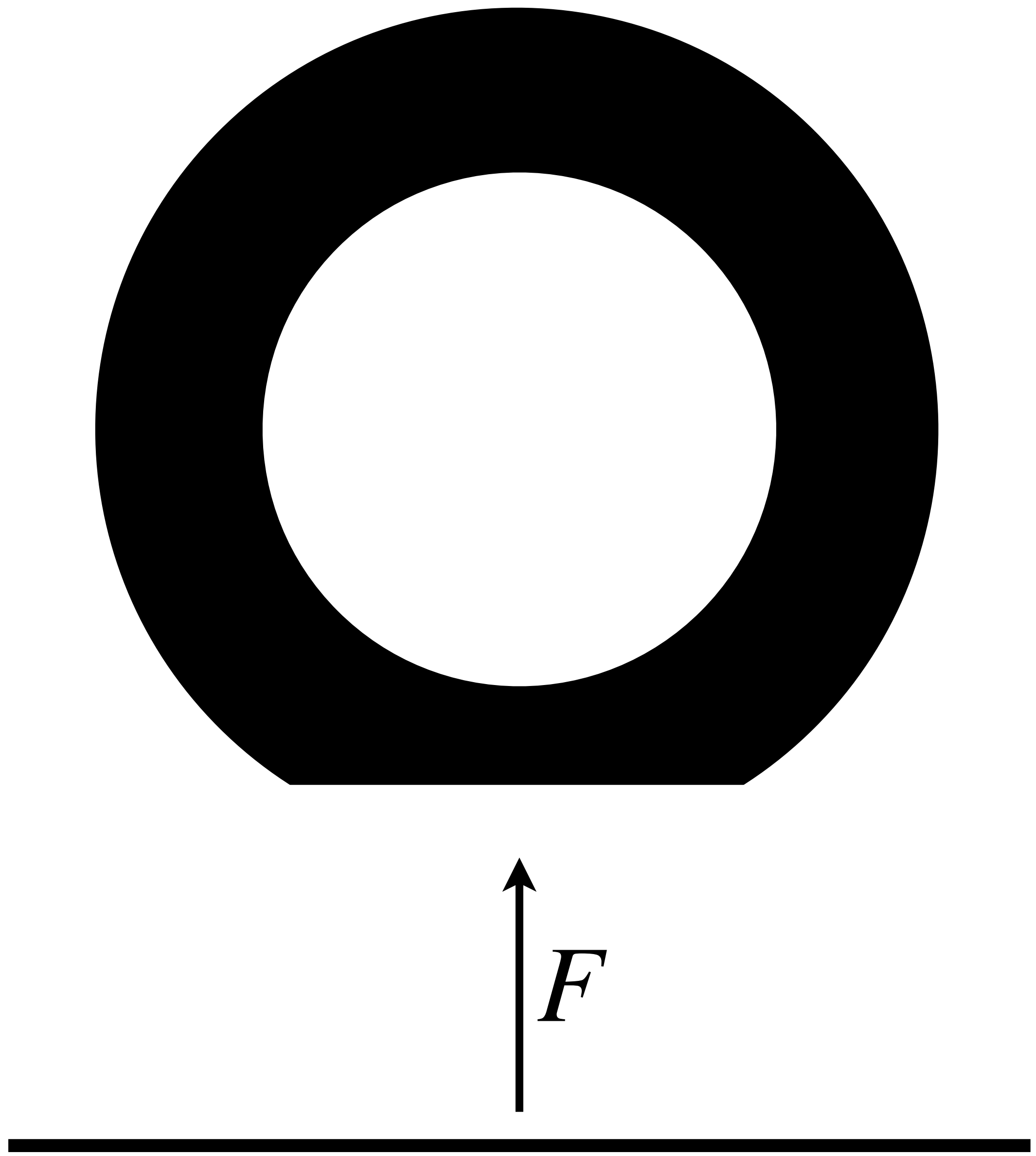
Linear elastic tire



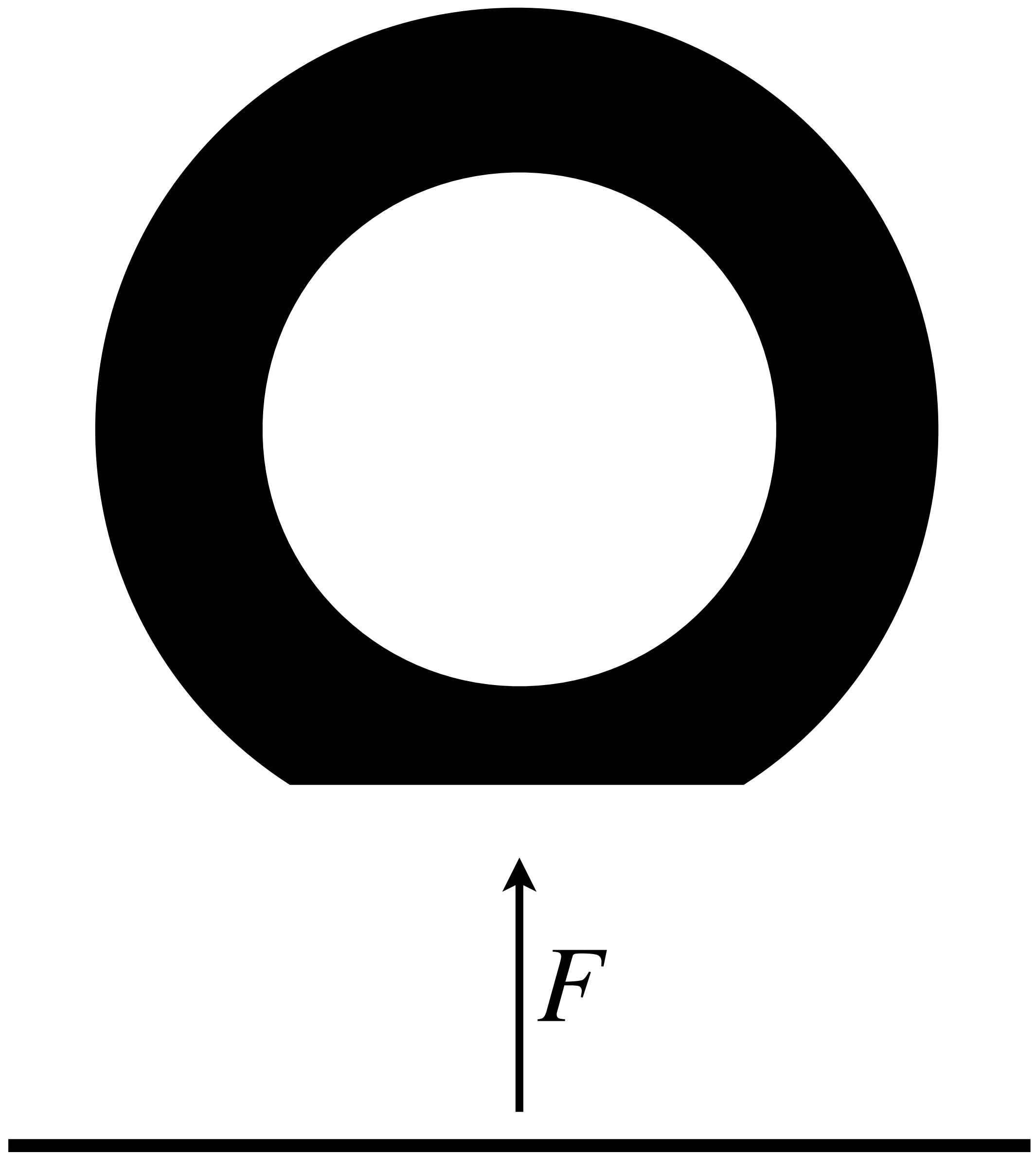
Linear elastic tire



Linear elastic tire

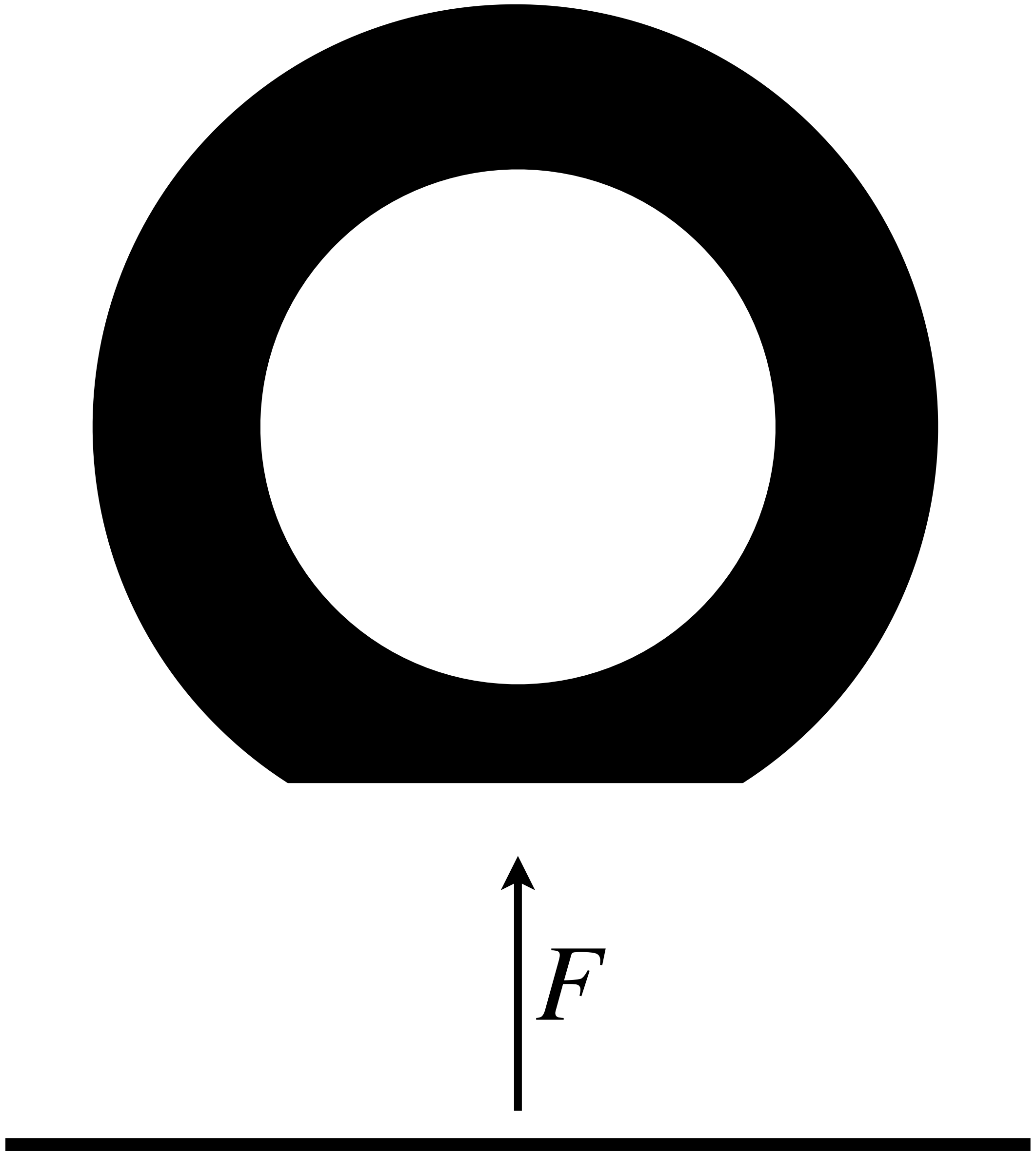


Linear elastic tire



$$Ku = F$$

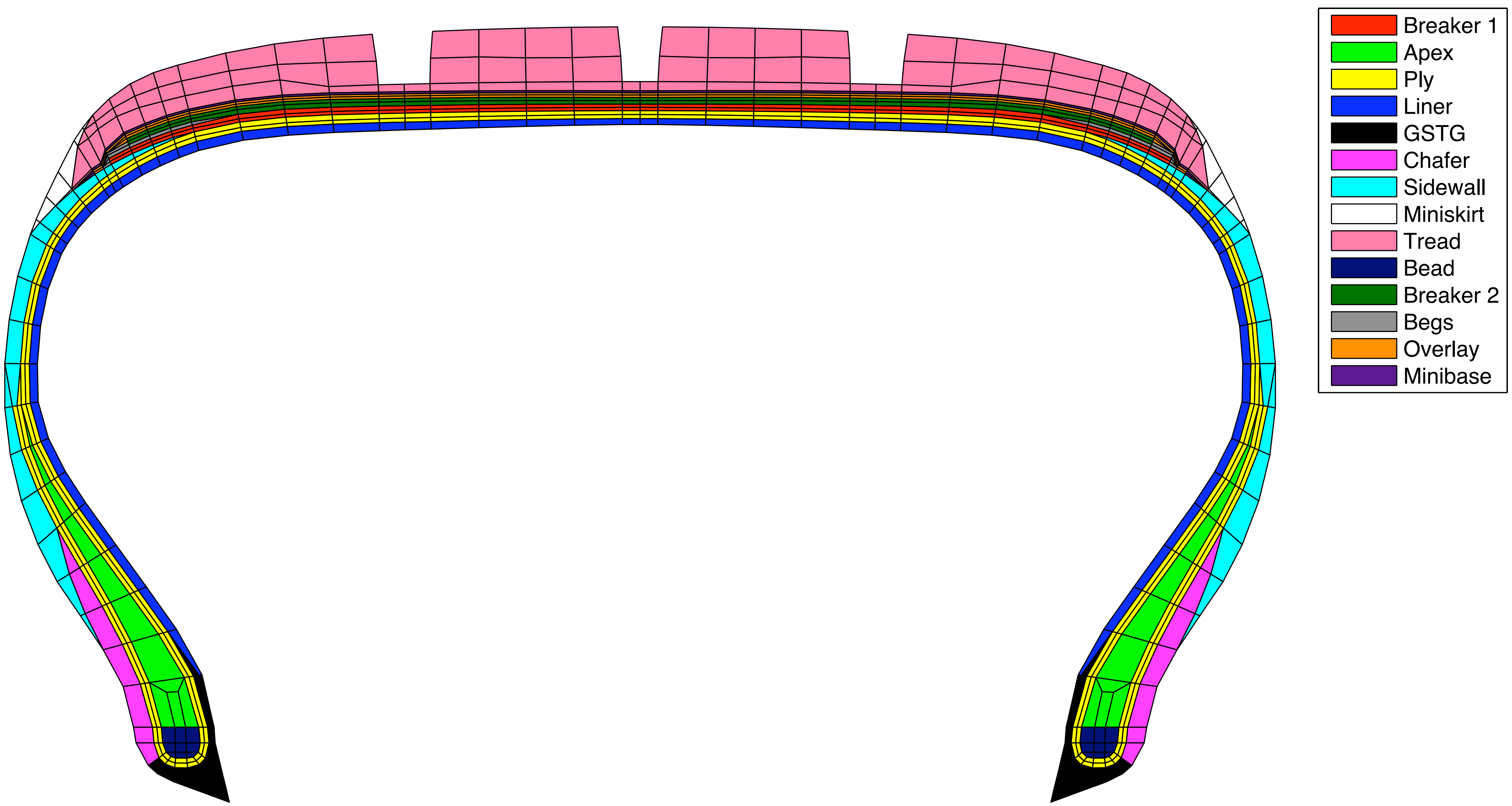
Linear elastic tire



$$Ku = F$$

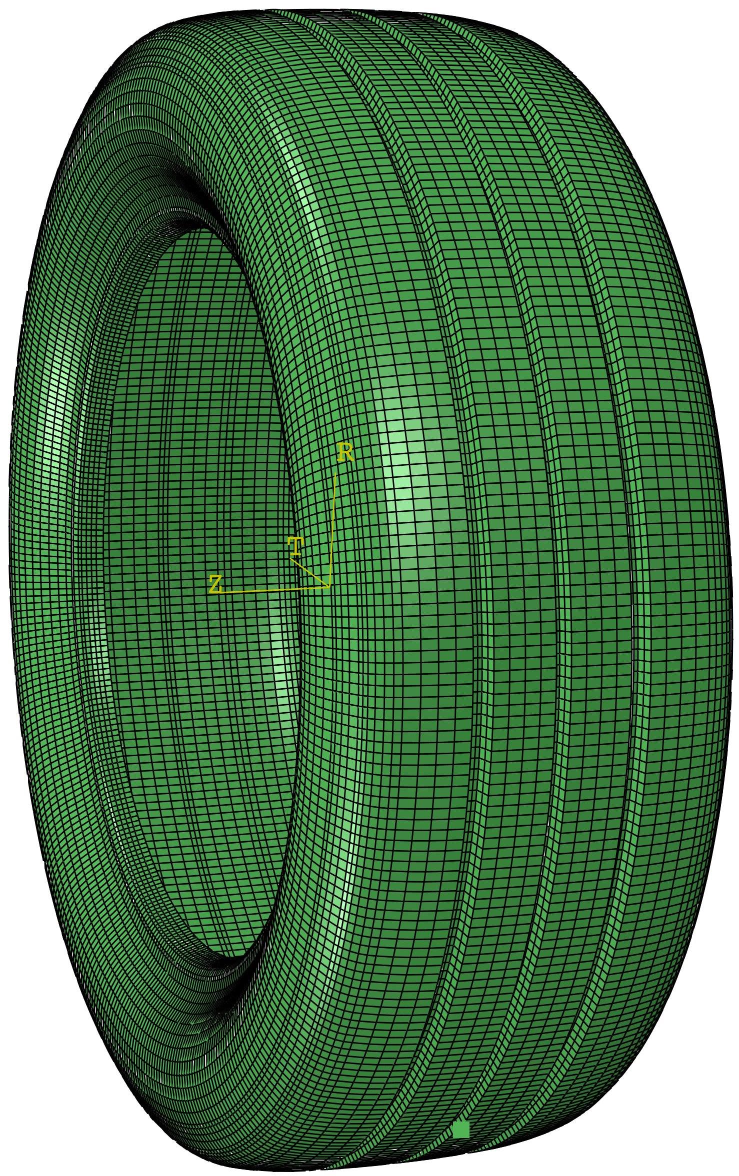
$$u = K^{-1}F$$

‘real’ tire

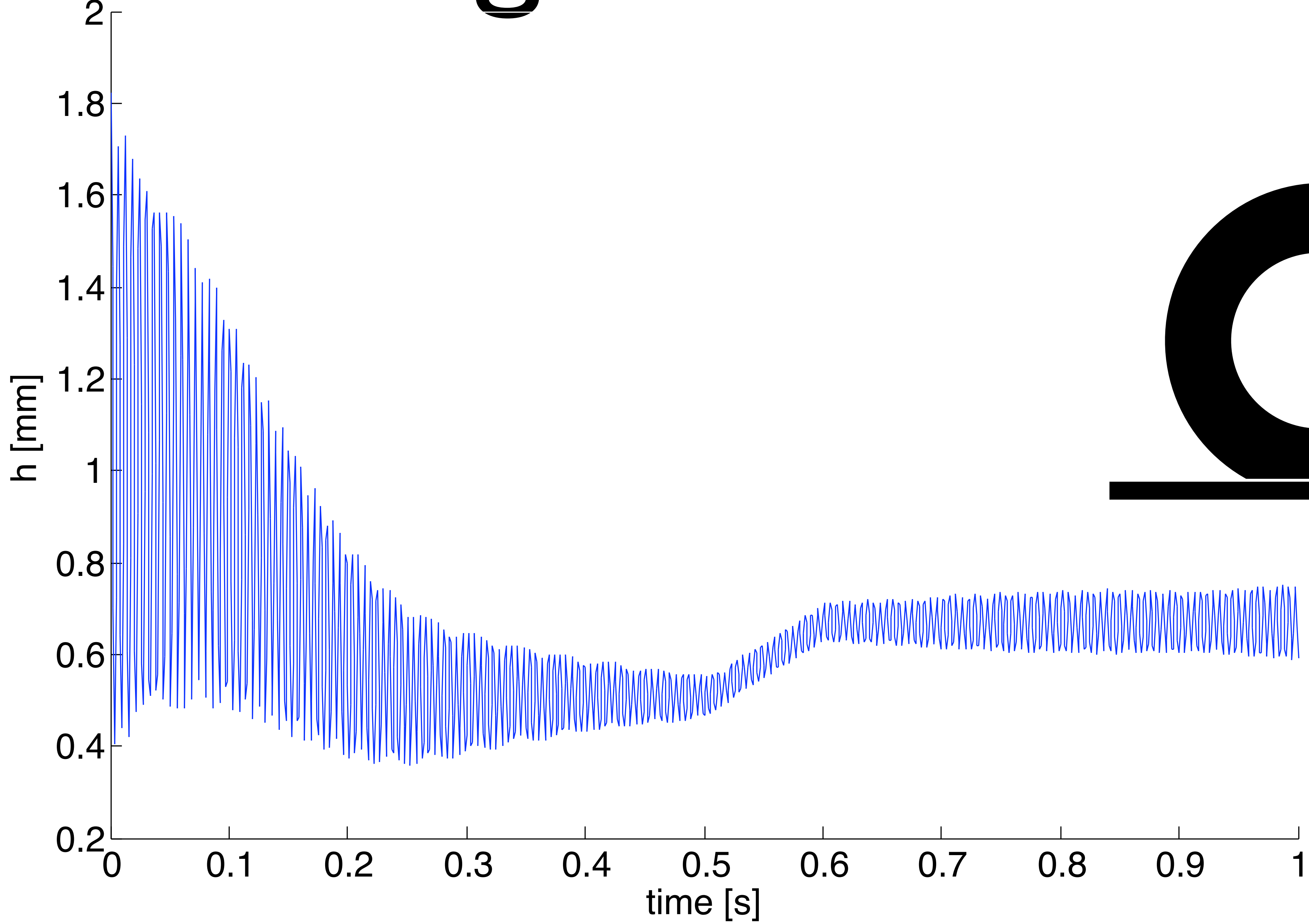


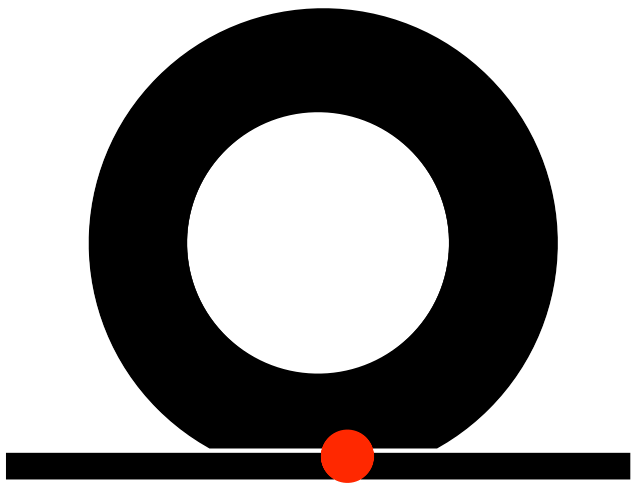
Breaker 1
Apex
Ply
Liner
GSTG
Chafer
Sidewall
Miniskirt
Tread
Bead
Breaker 2
Begs
Overlay
Minibase

‘real’ tire

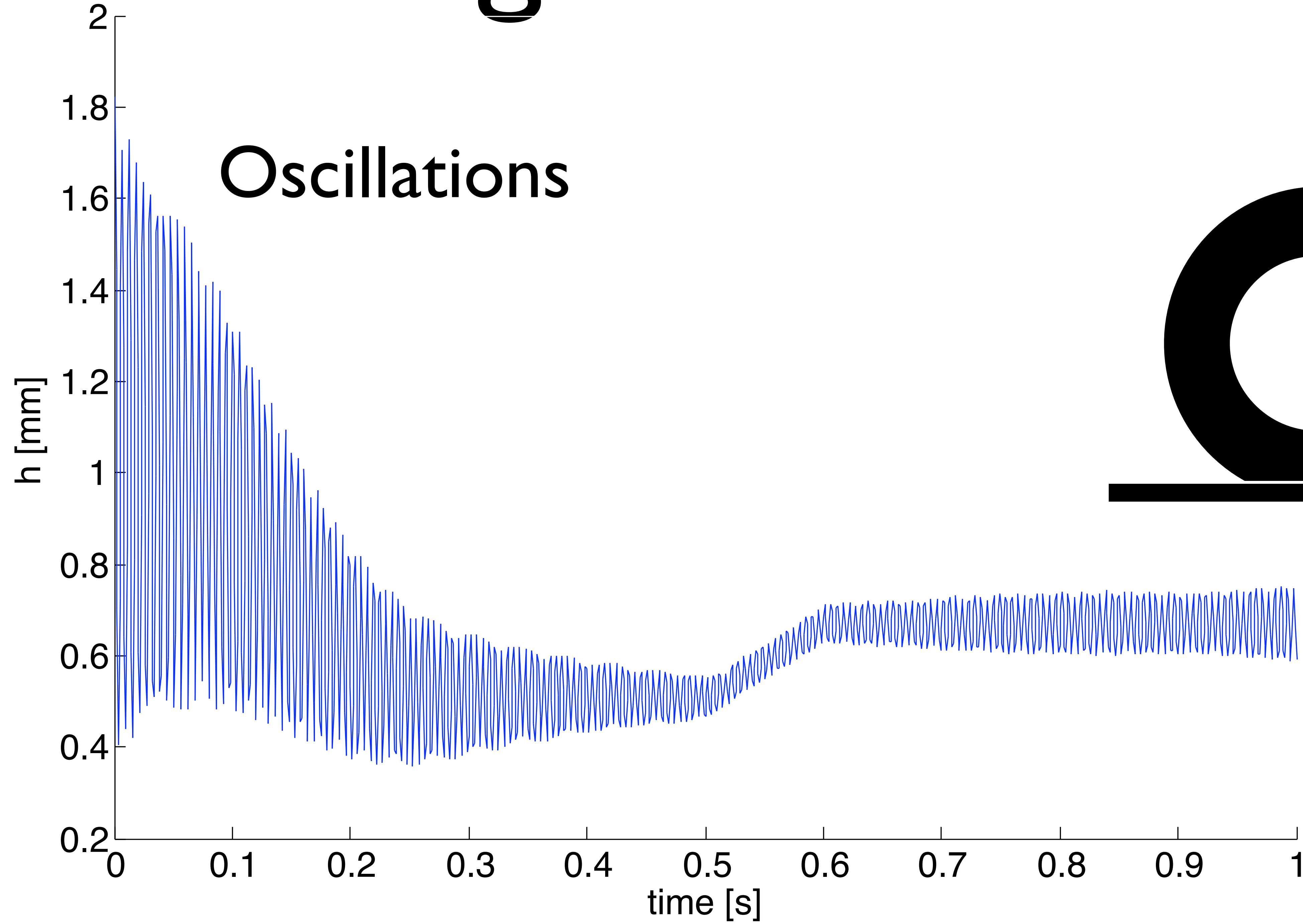


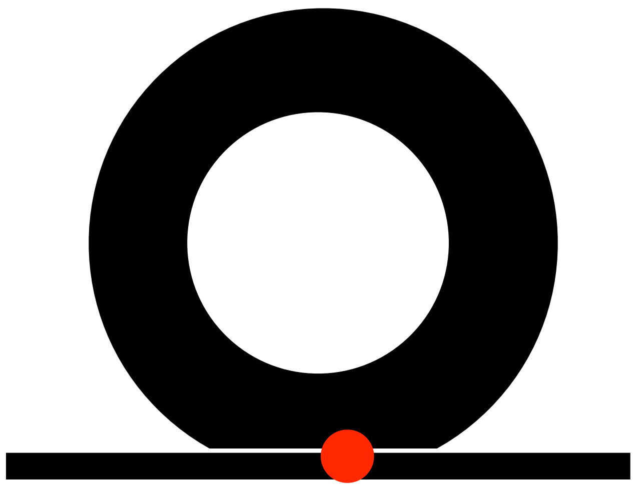
Rolling tire model



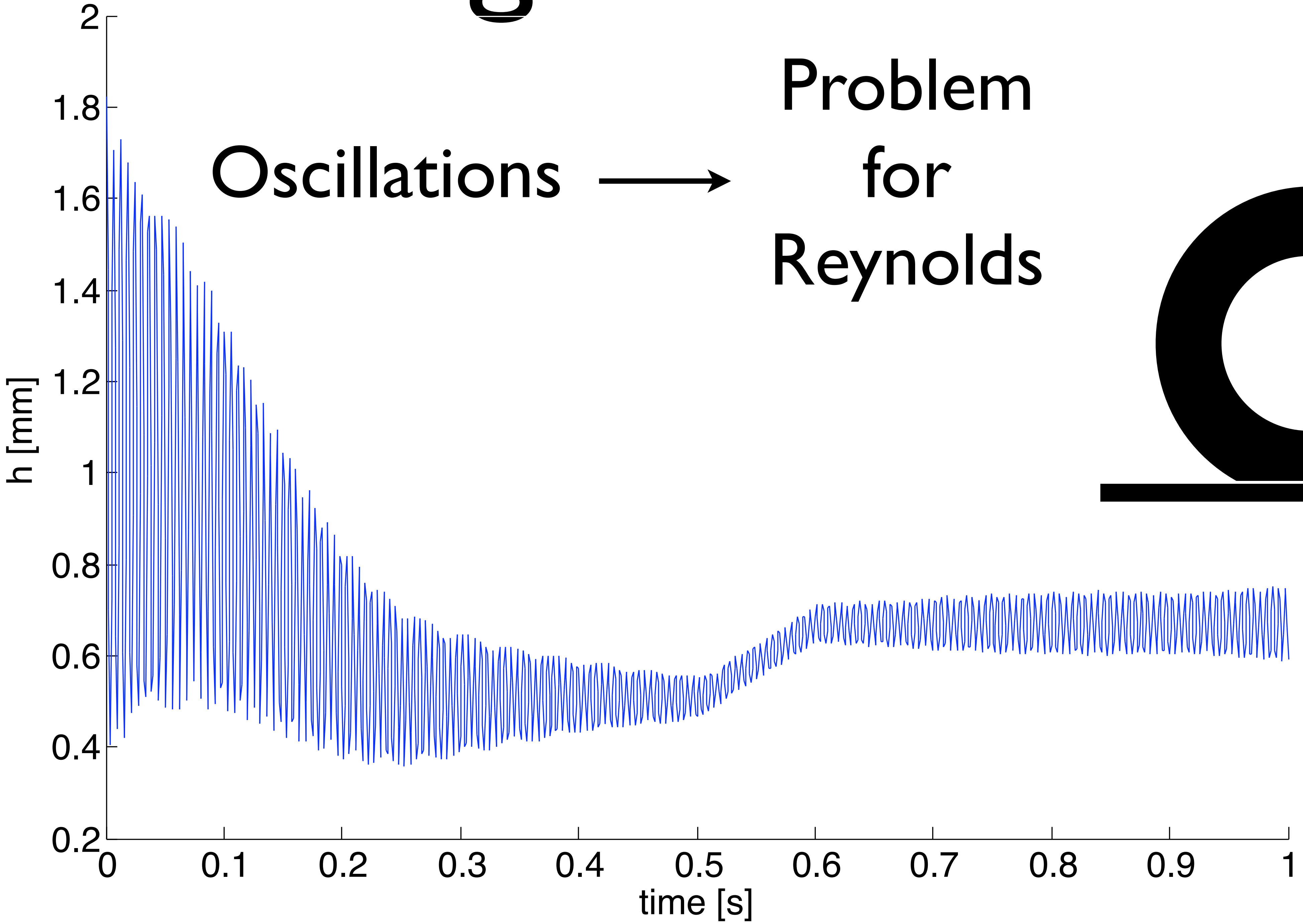


Rolling tire model

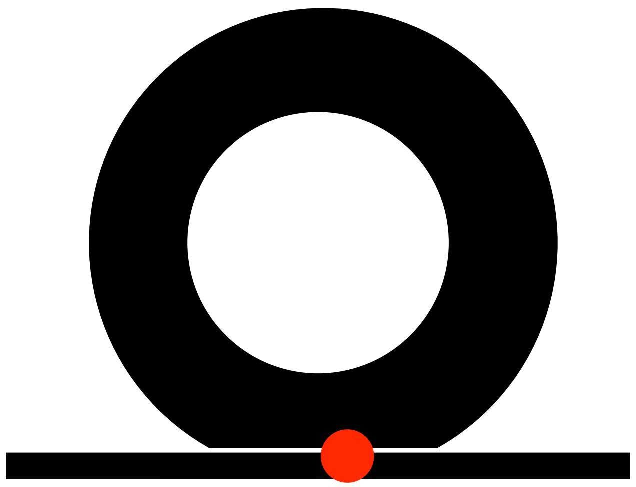




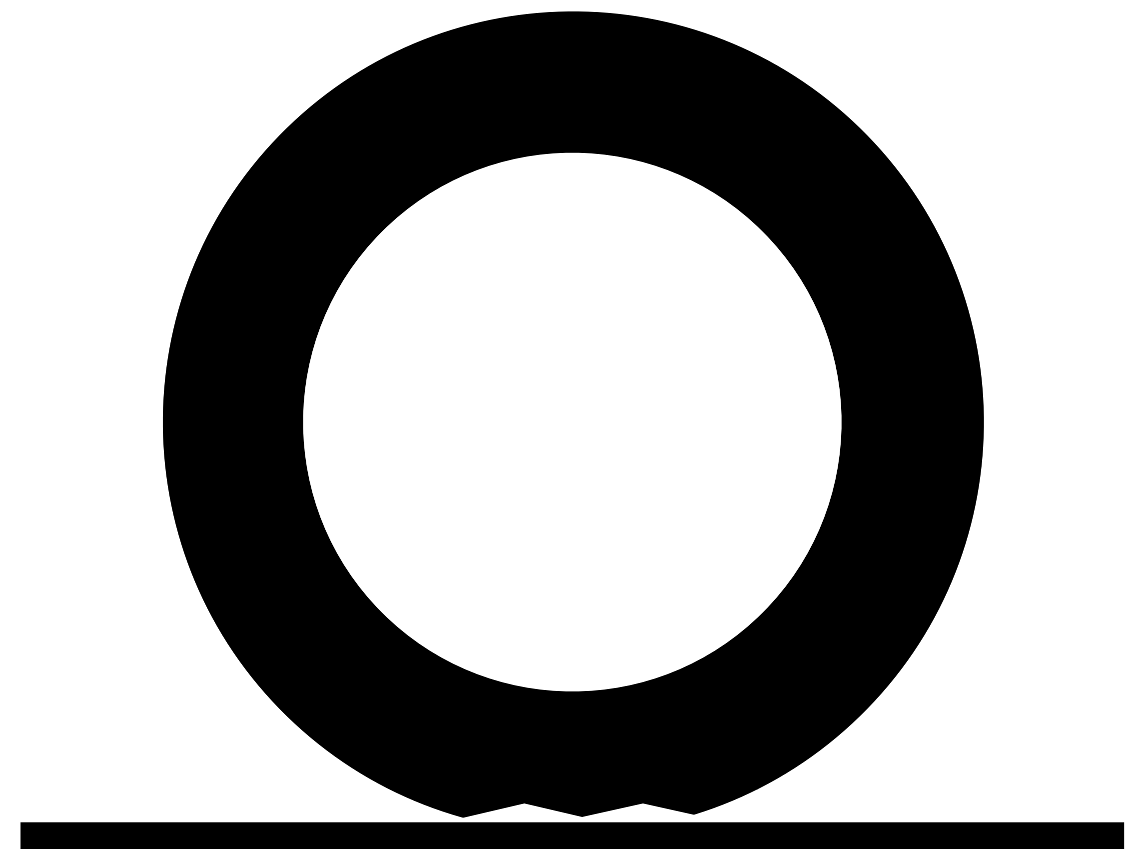
Rolling tire model



Oscillations ⟶ Problem for Reynolds

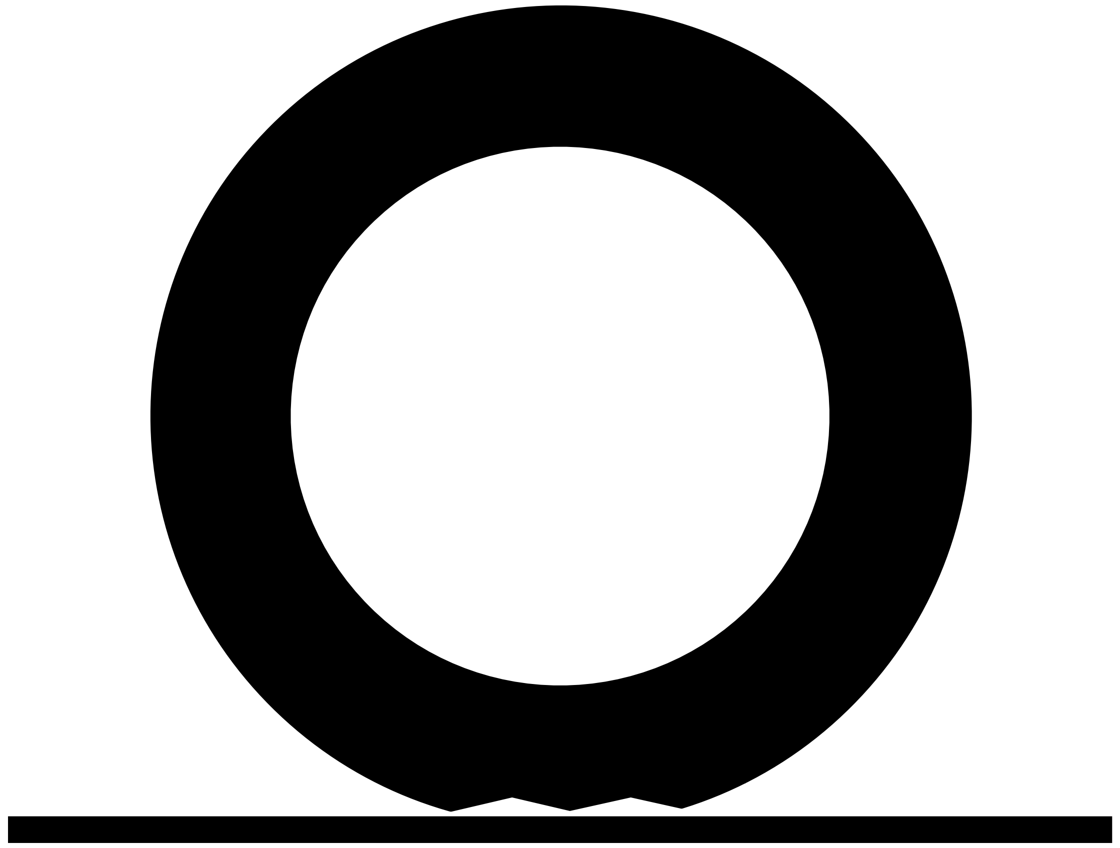


Solutions?



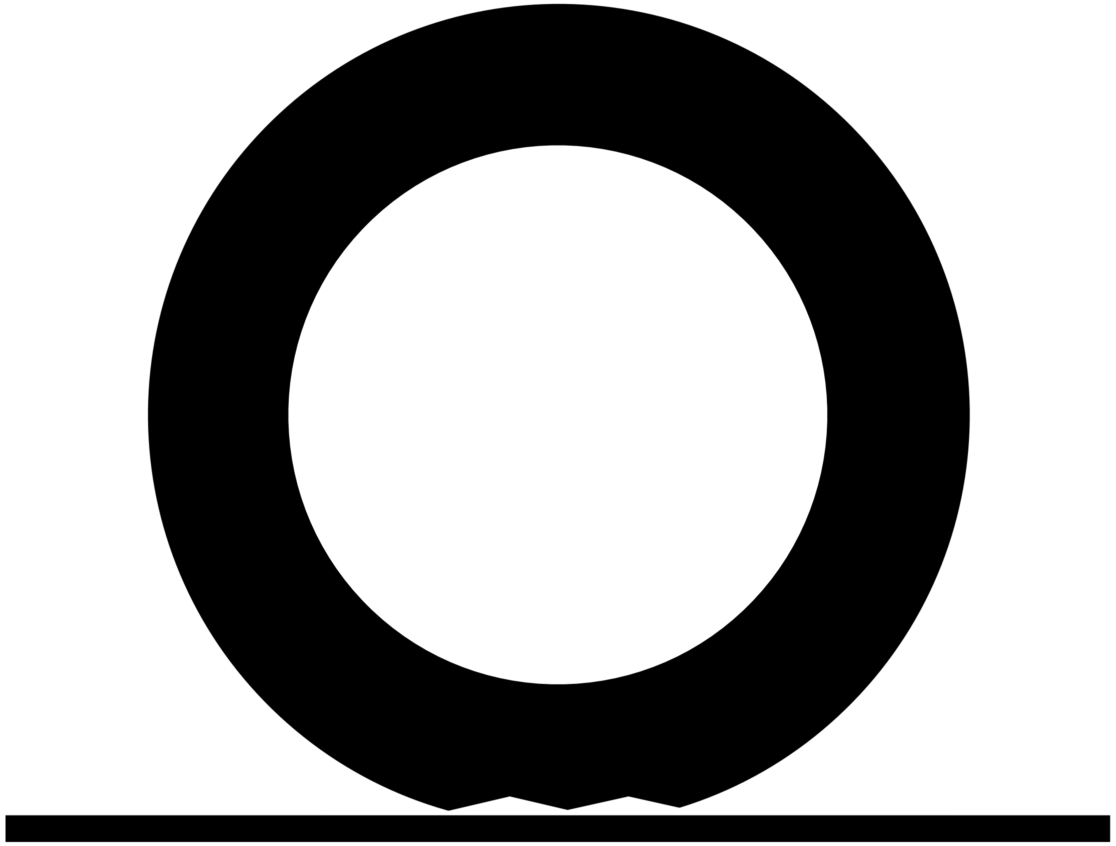
Solutions?

- Contact penalty



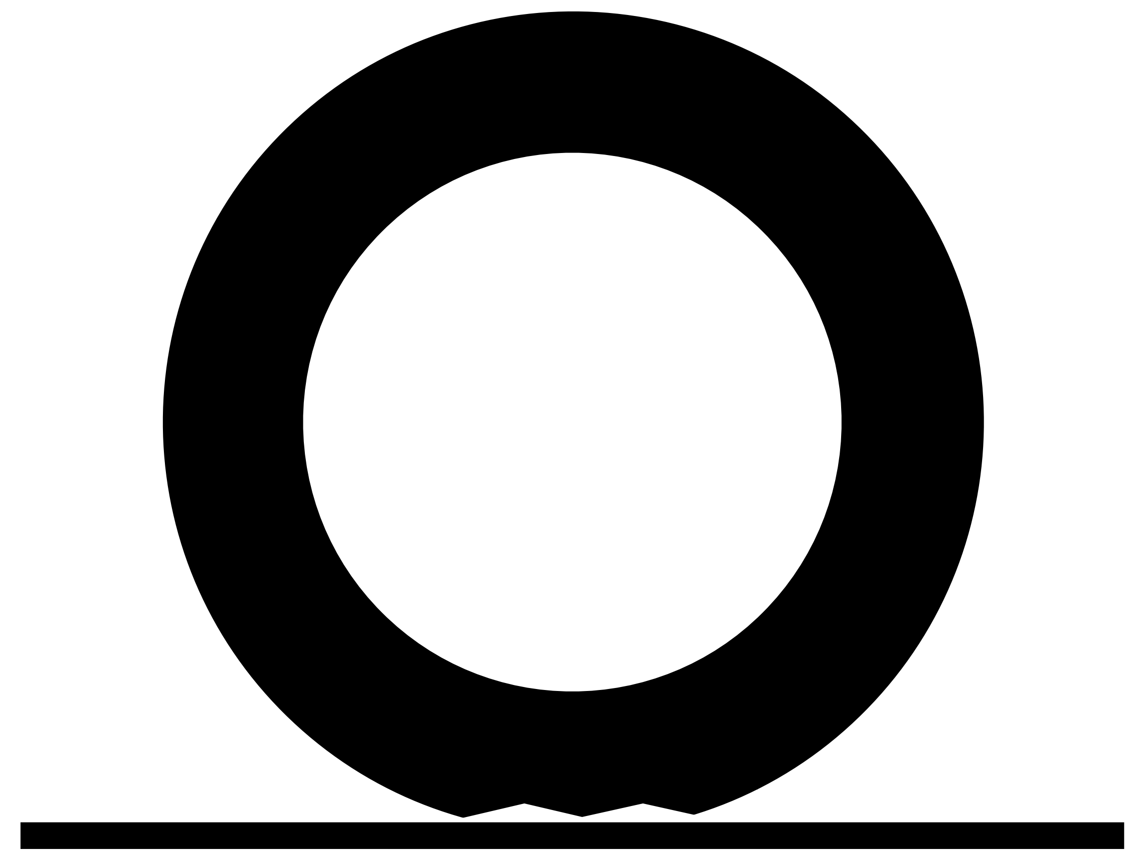
Solutions?

- Contact penalty
- Time step reduction



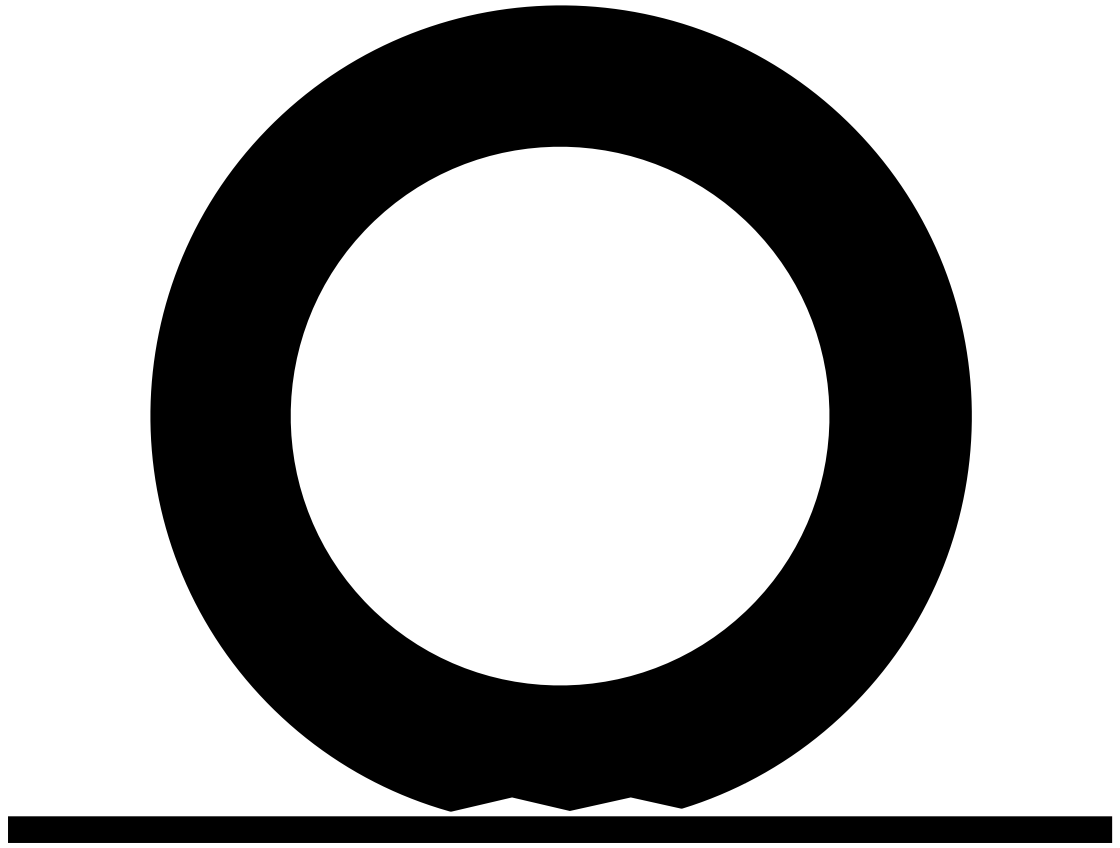
Solutions?

- Contact penalty
- Time step reduction
- Mesh refinement



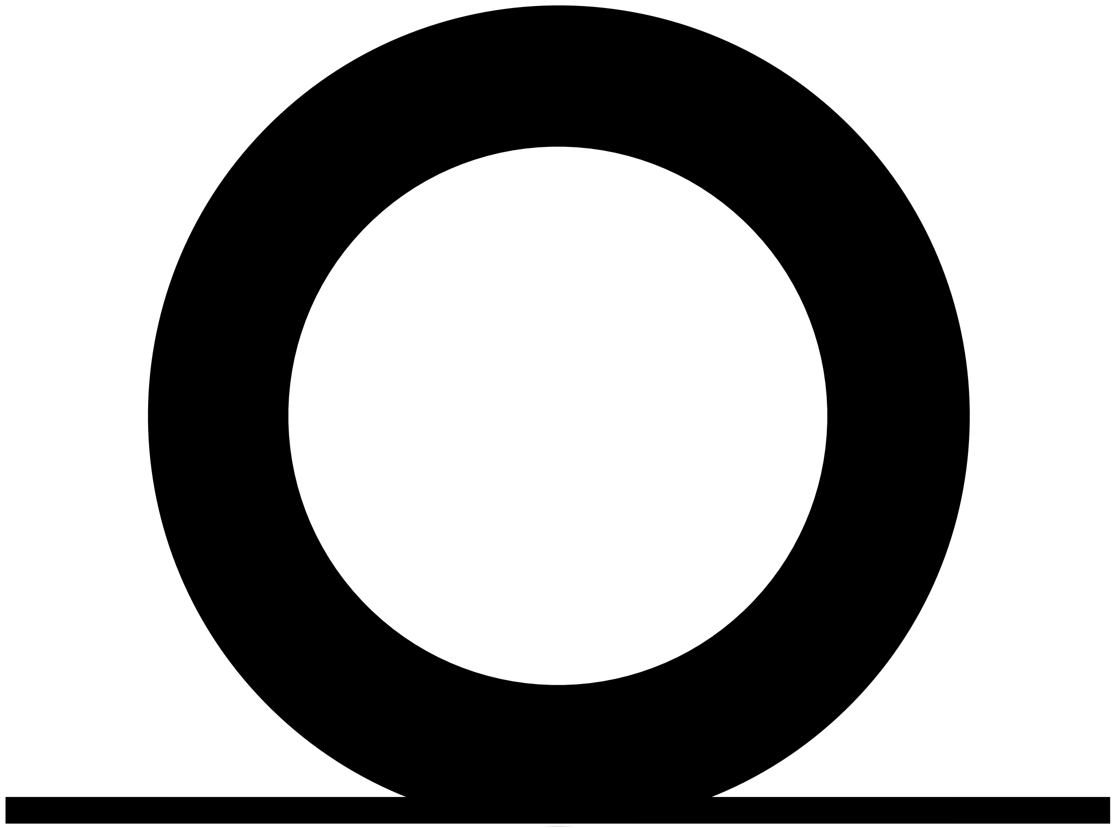
Solutions?

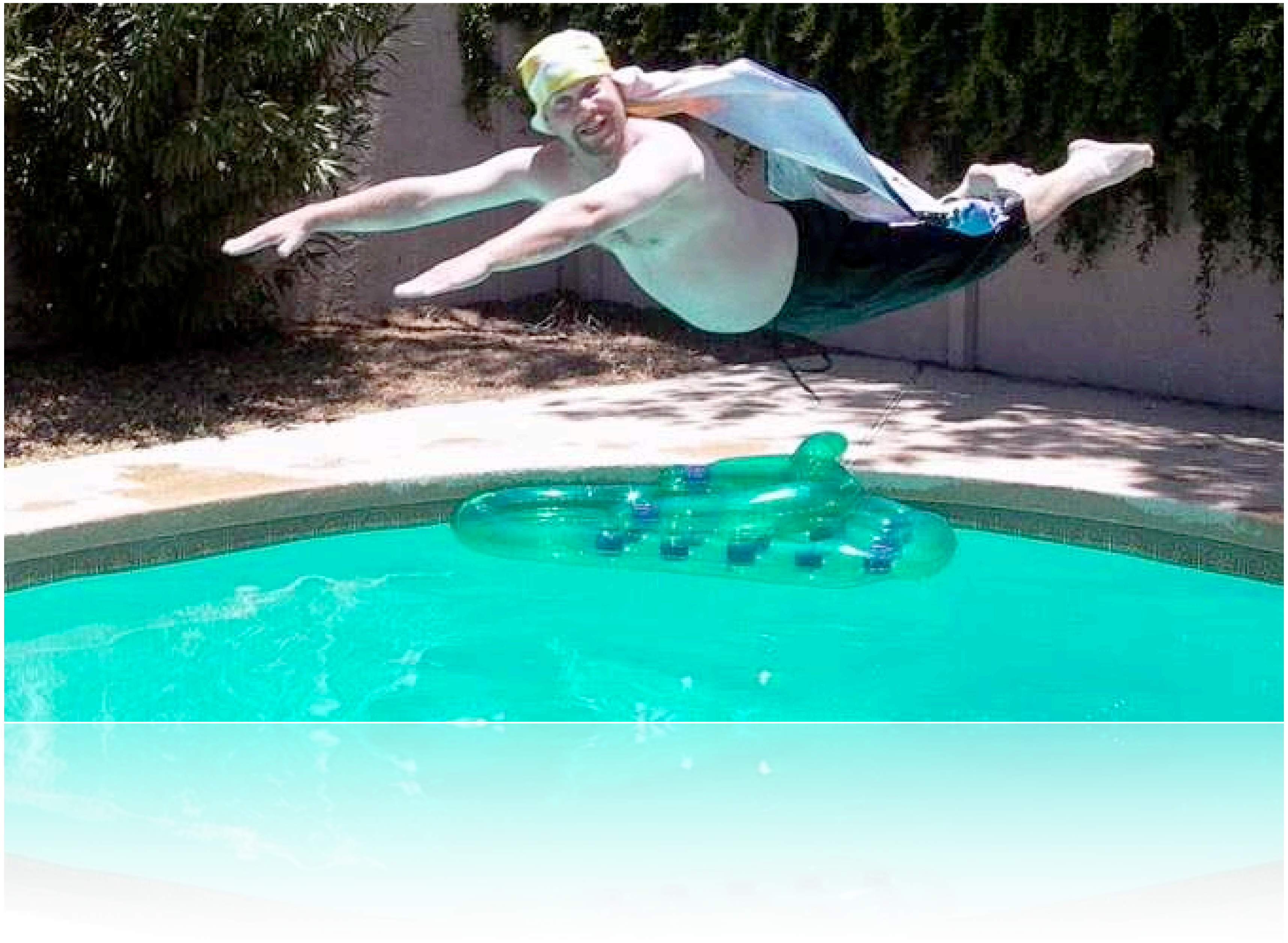
- Contact penalty
- Time step reduction
- Mesh refinement
- Move road (in FSI)



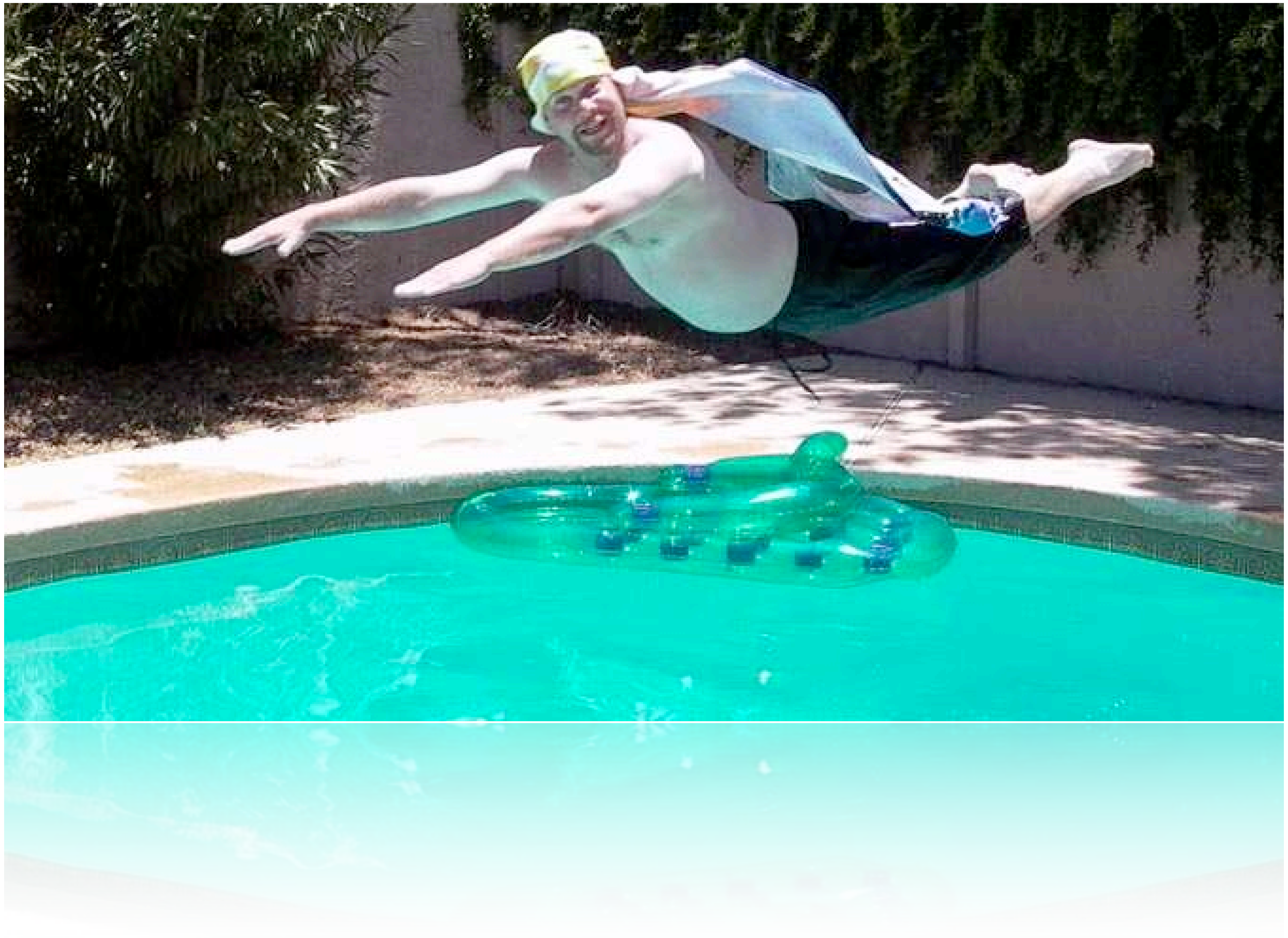
Solutions?

- Contact penalty
- Time step reduction
- Mesh refinement
- Move road (in FSI)





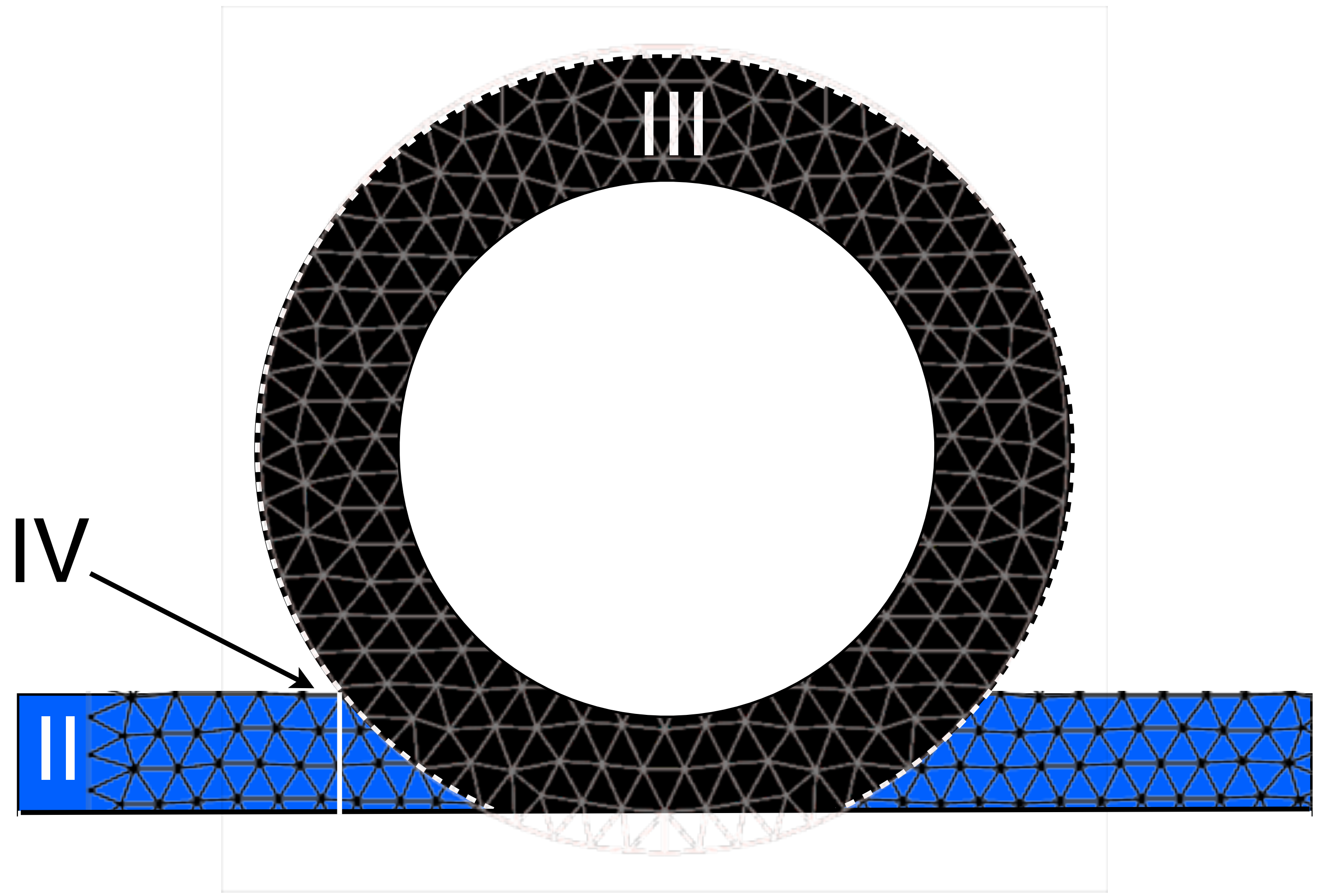
Fluid Structure Interaction



Fluid Structure Interaction

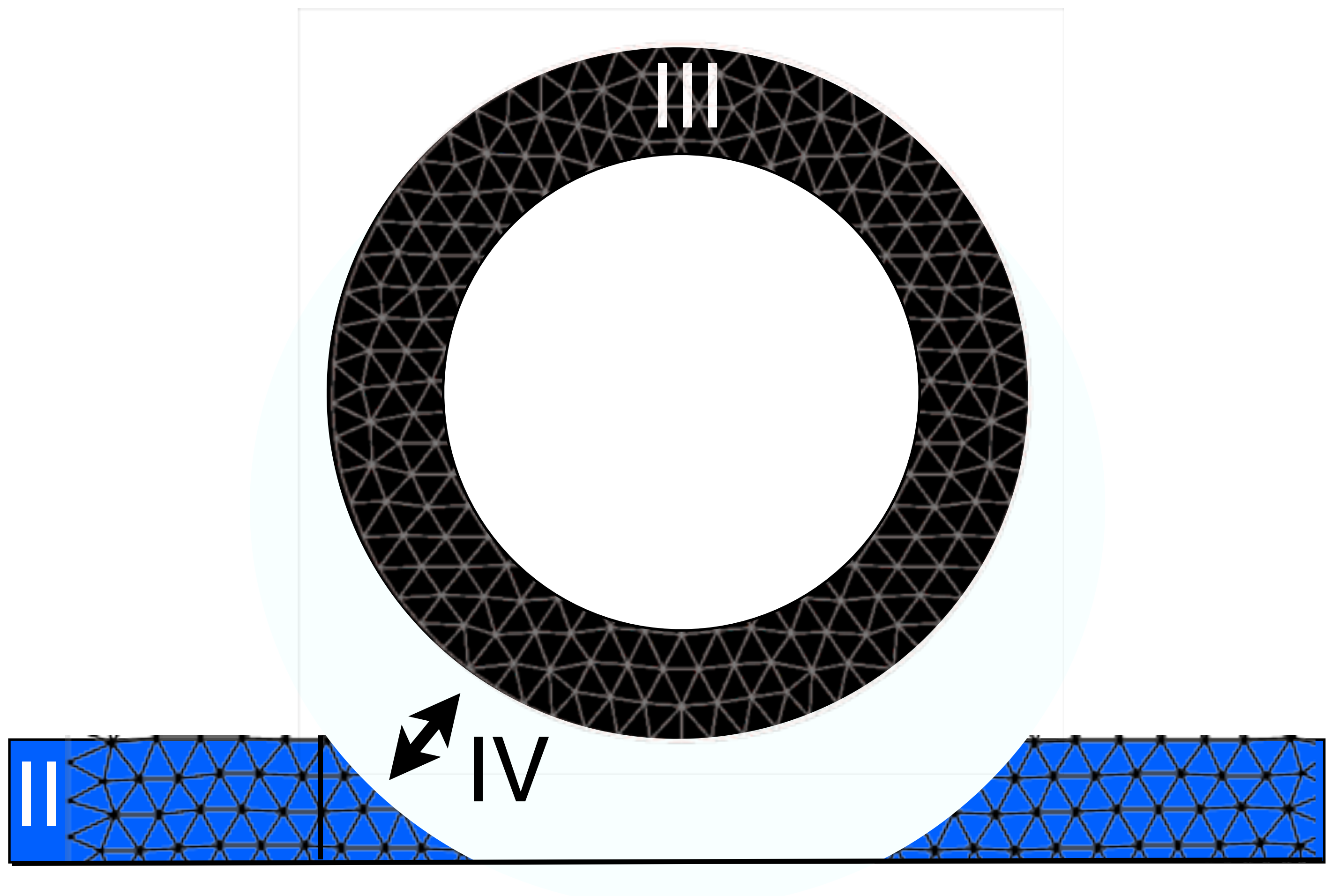
Fluid structure interaction

- Monolithic vs. Partitioned



Fluid structure interaction

- Monolithic vs. Partitioned

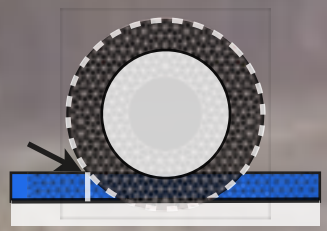


Monolithic vs. Partitioned

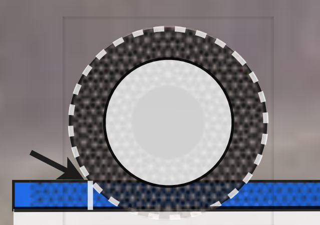
Monolithic vs. Partitioned

Monolithic vs. Partitioned

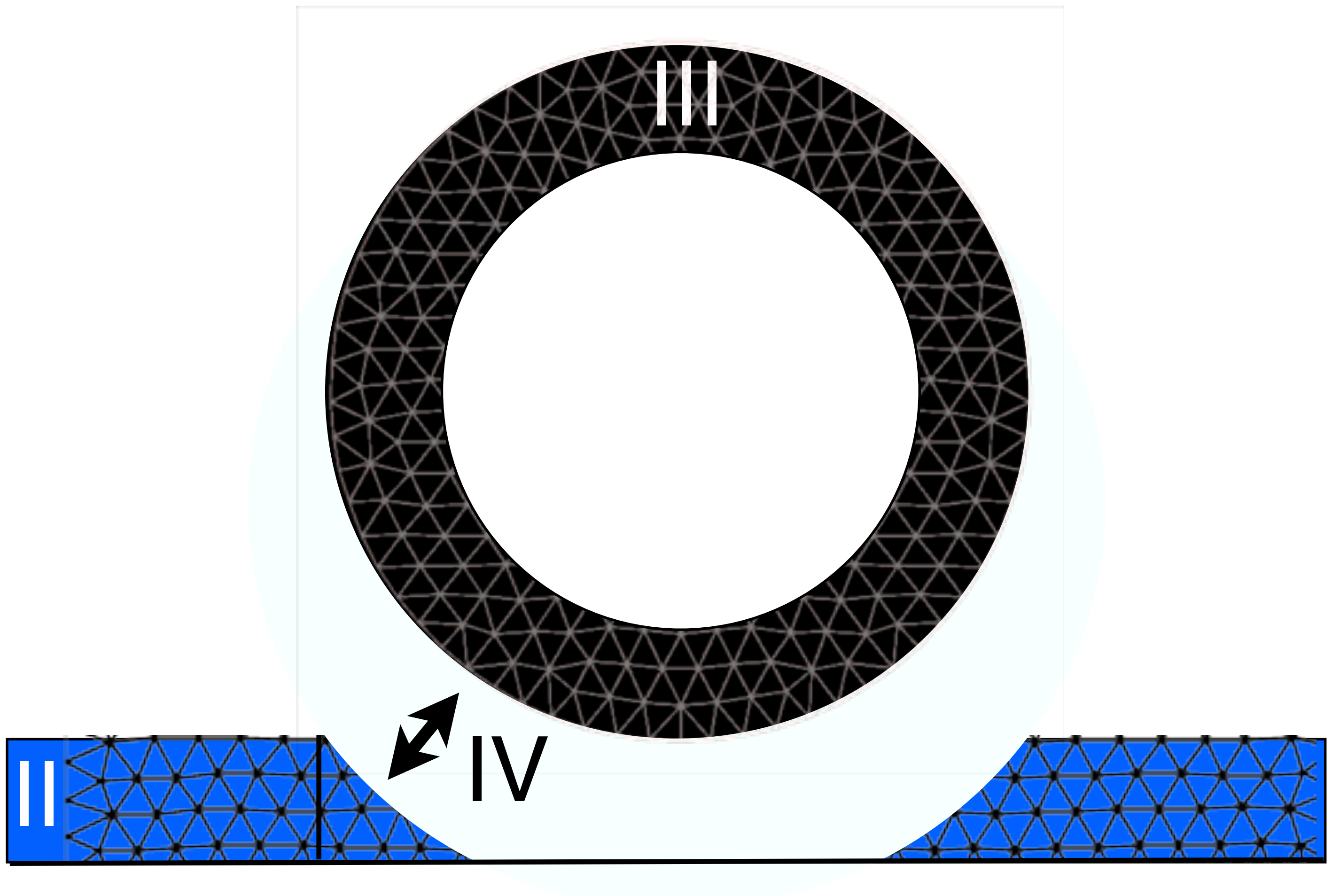
Monolithic vs. Partitioned



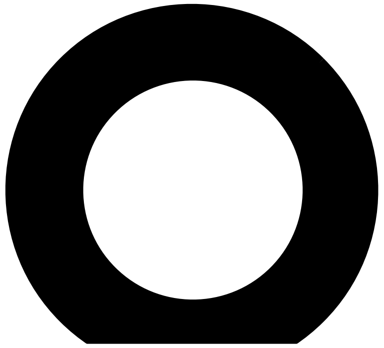
Monolithic vs. Partitioned



Classical staggering



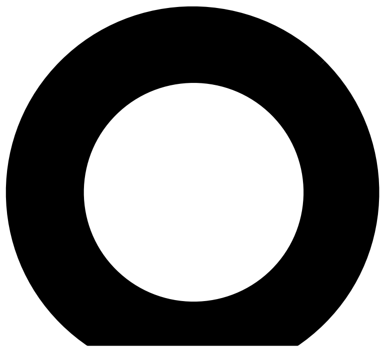
Classical staggering



Elastic tire

Reynolds model

Classical staggering

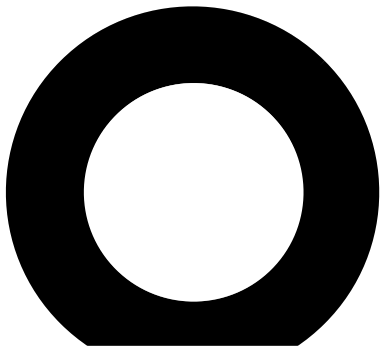


Elastic tire

h, U ↓

Reynolds model

Classical staggering

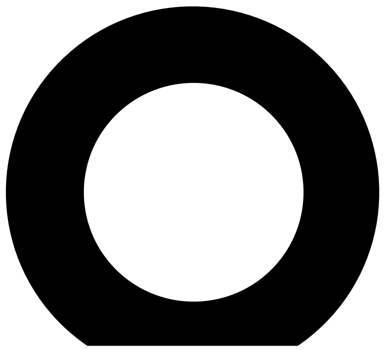


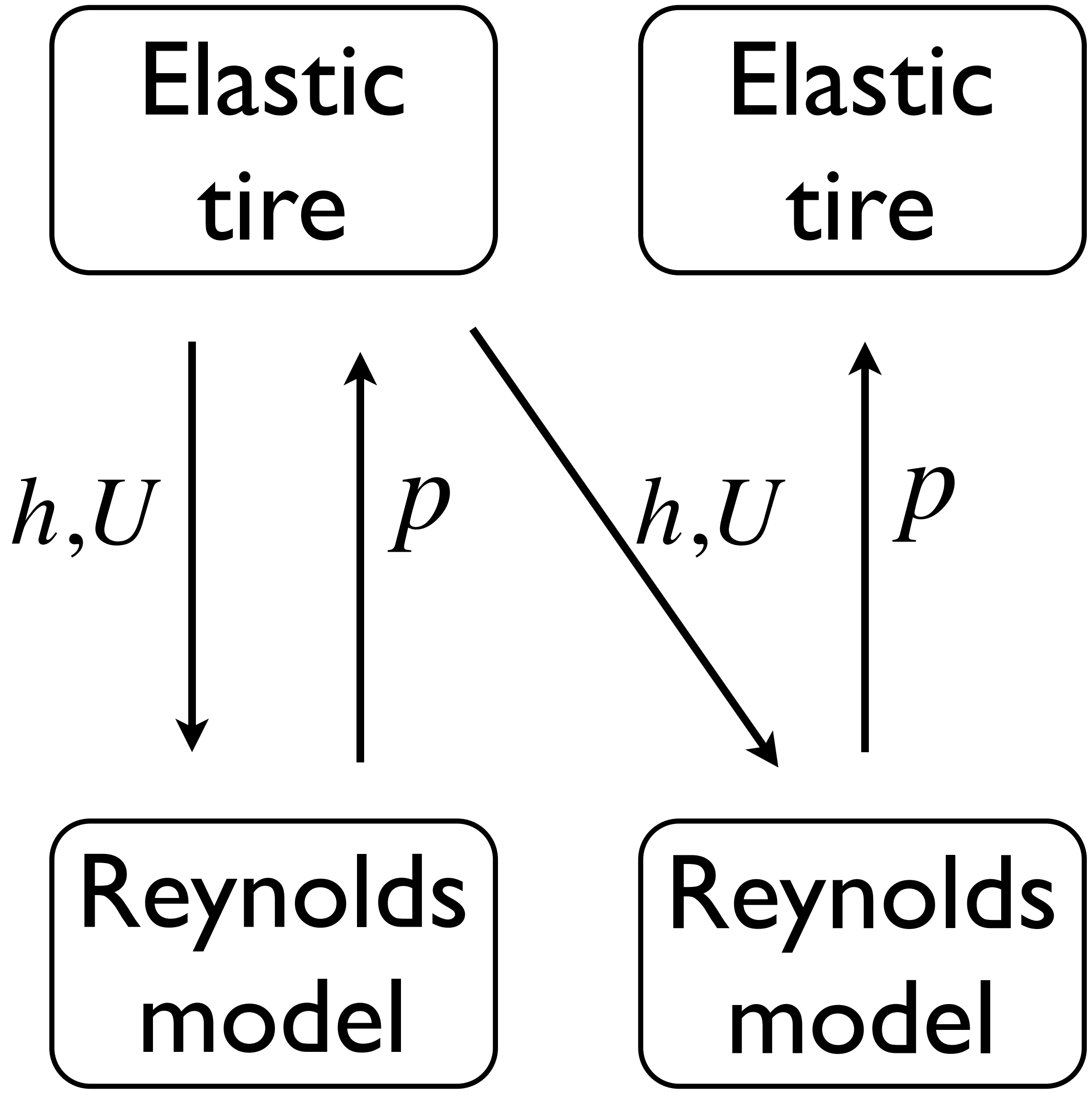
Elastic tire

h,U ↓ ↑ p

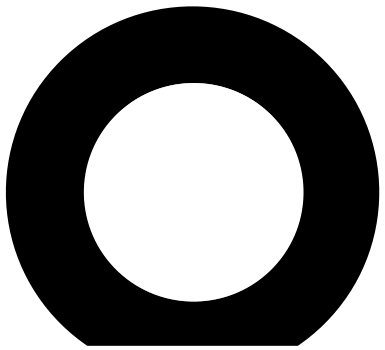
Reynolds model

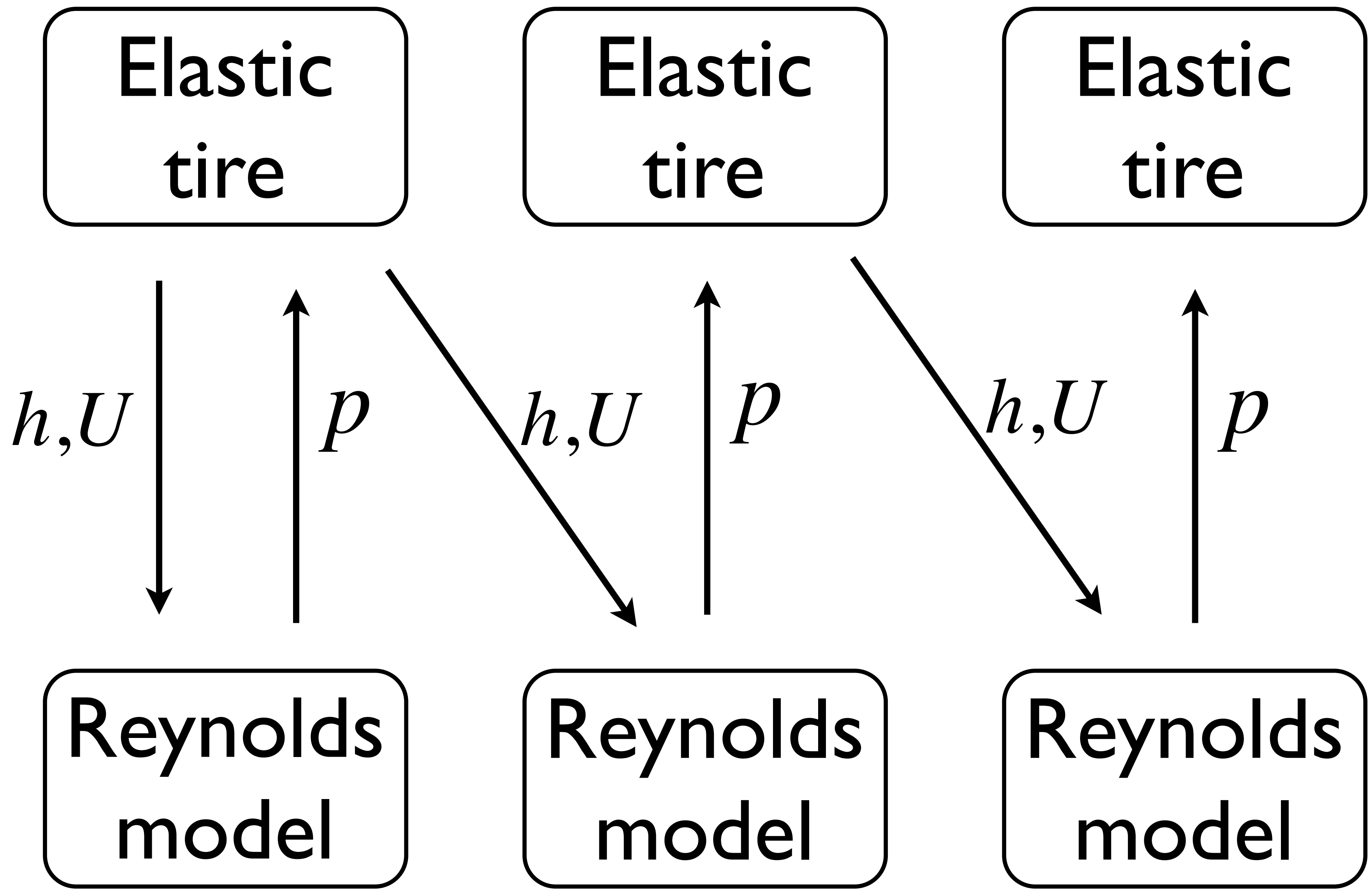
Classical staggering



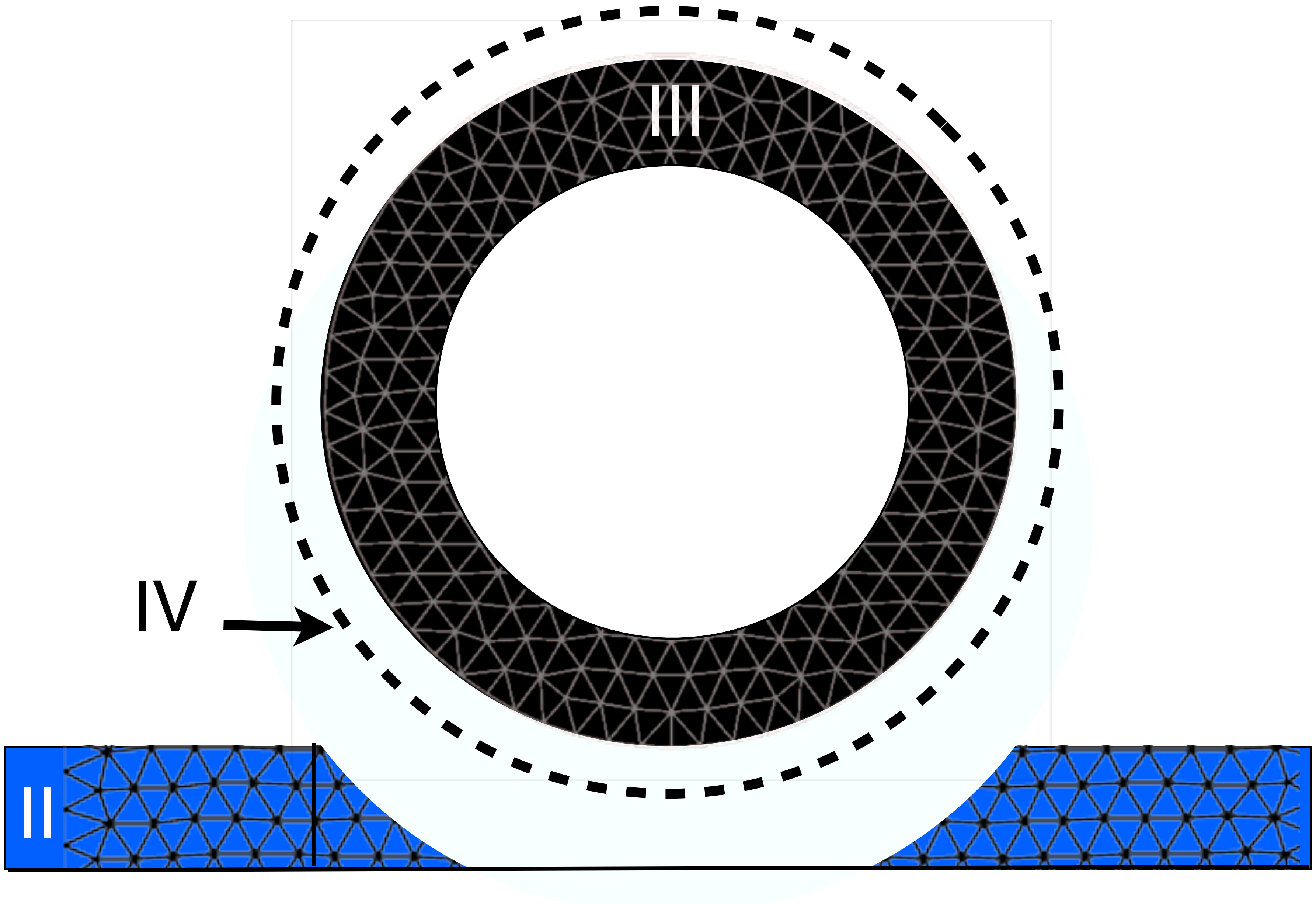


Classical staggering

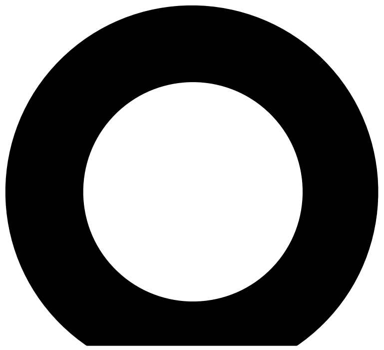




Interface Quasi Newton



Interface Quasi Newton



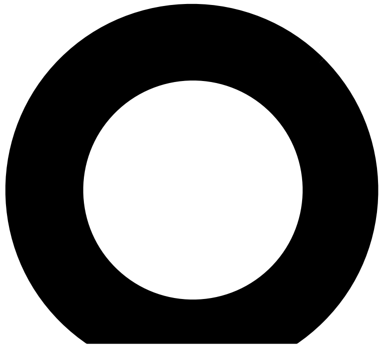
Elastic tire

Elastic tire

Reynolds model

Reynolds model

Interface Quasi Newton



Elastic tire

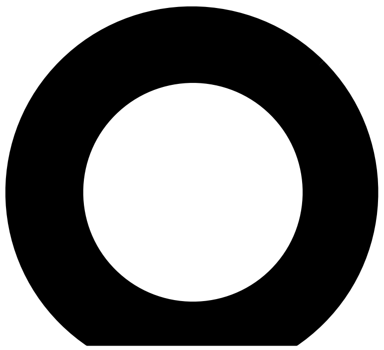
Elastic tire

h^k.....

Reynolds model

Reynolds model

Interface Quasi Newton



Elastic tire

Elastic tire

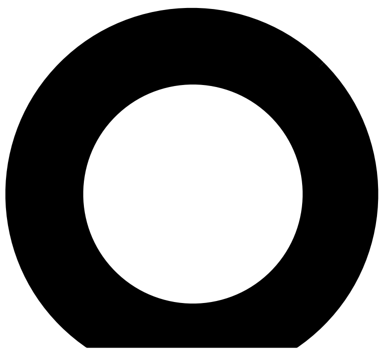
h^k - - - - -

Reynolds model

Reynolds model

TU Delft

Interface Quasi Newton

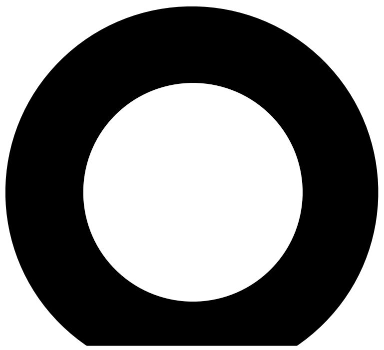


Elastic tire | Elastic tire

h^k ↓   ↑ p^k

Reynolds model | Reynolds model

Interface Quasi Newton



Elastic tire | Elastic tire

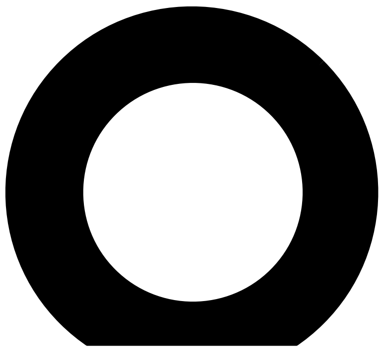
h^k → Reynolds model

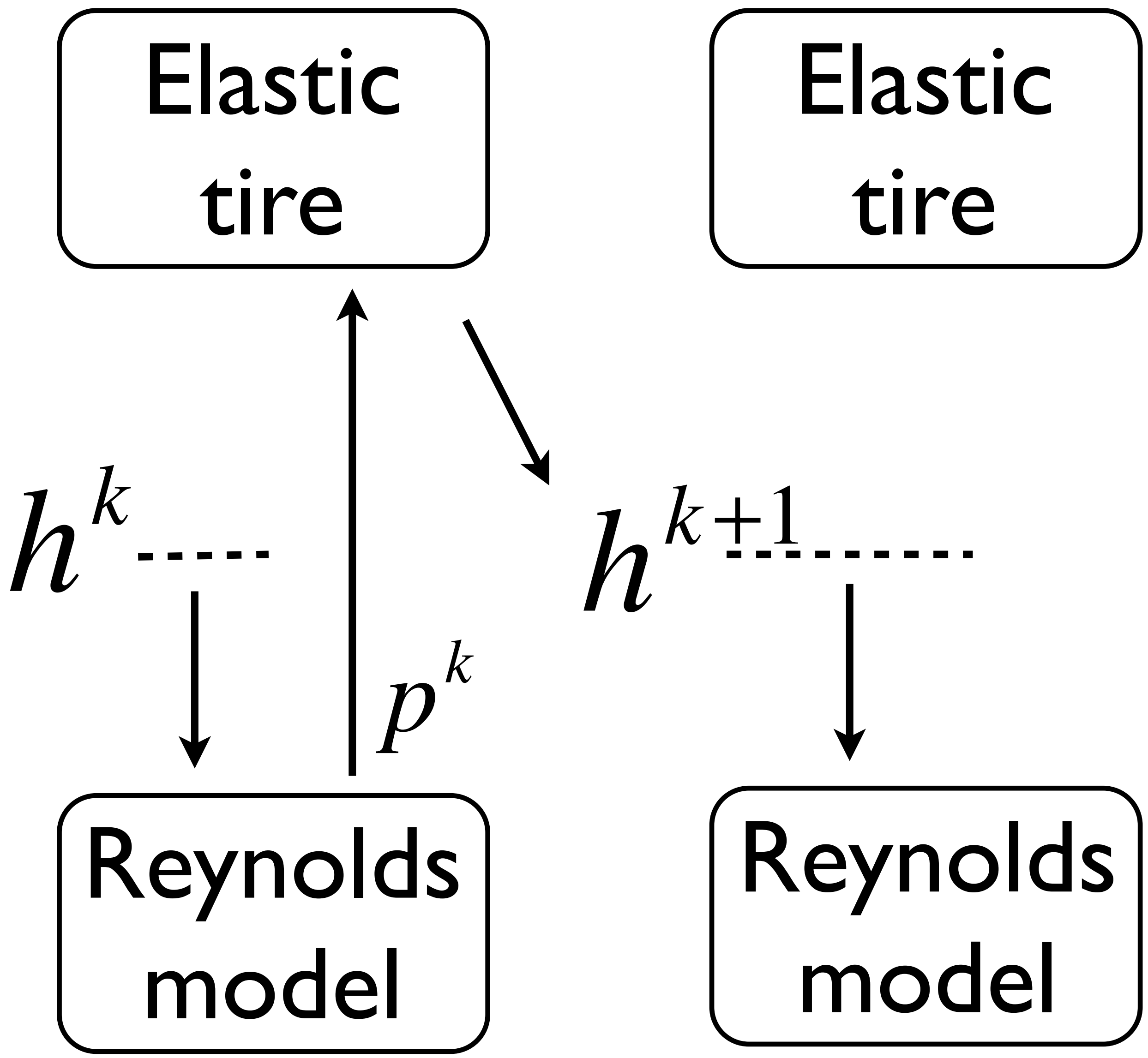
Reynolds model → p^k → Elastic tire

Elastic tire → h^{k+1}

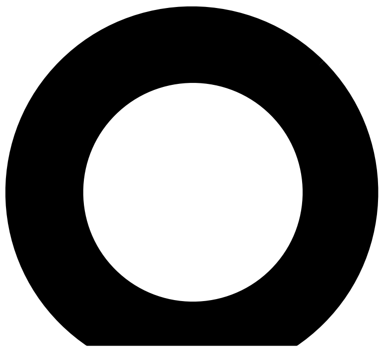
Reynolds model | Reynolds model

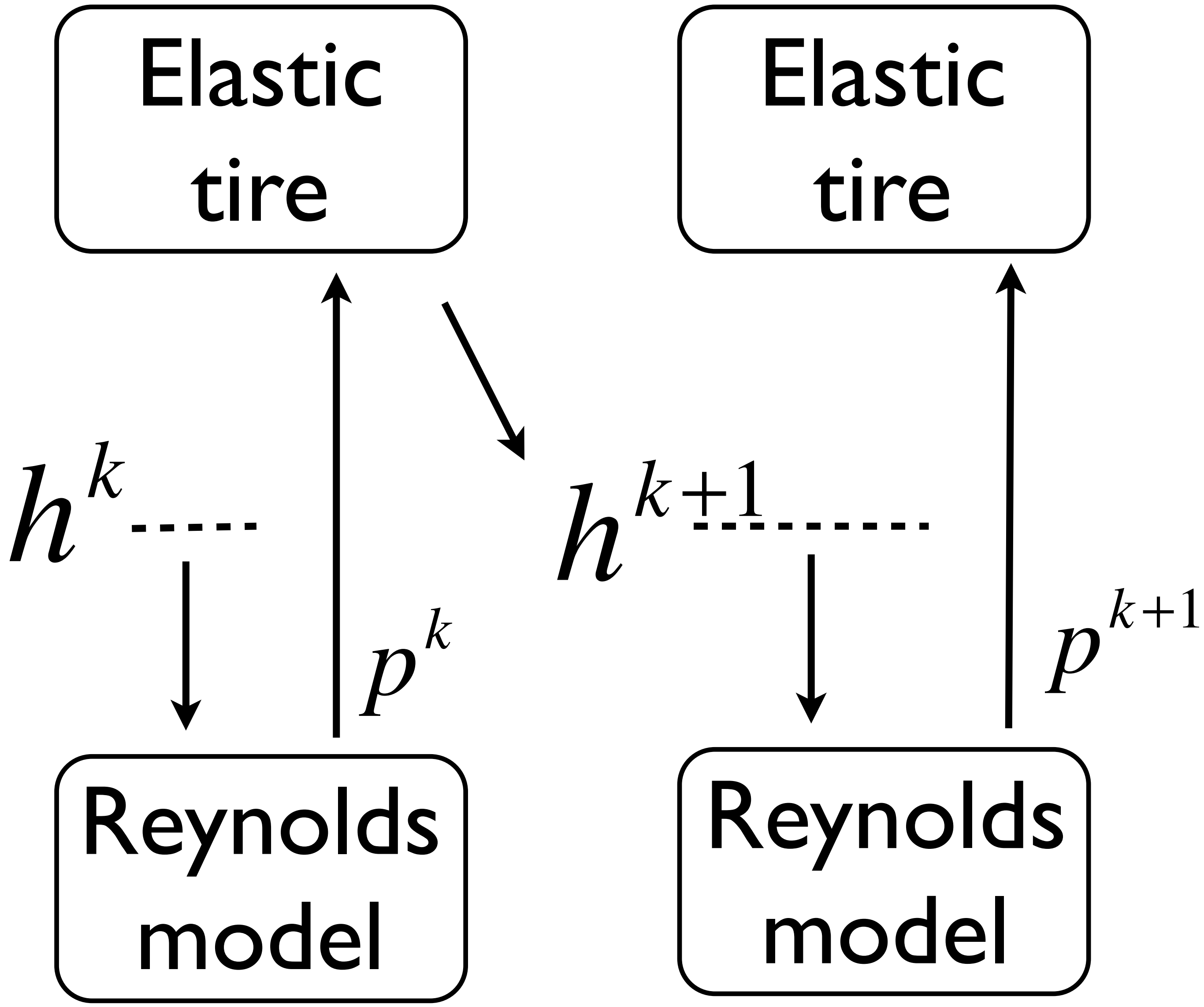
Interface Quasi Newton



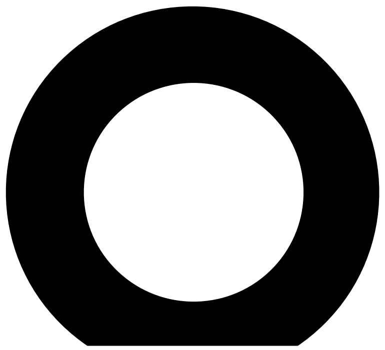


Interface Quasi Newton





Interface Quasi Newton



Elastic tire

Elastic tire

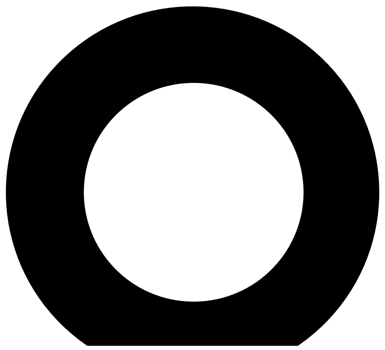
h^k ----- h^{k+1} ---------

p^k p^{k+1}

Reynolds model

Reynolds model

Interface Quasi Newton



Elastic tire

Elastic tire

h^k ..... h^{k+1} .... $\longrightarrow \Delta\mathbf{d}$

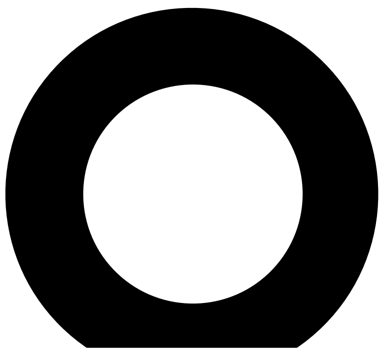
p^k p^{k+1} $\longrightarrow \Delta p$

Reynolds model

Reynolds model

TU Delft

Interface Quasi Newton



Elastic tire

Elastic tire

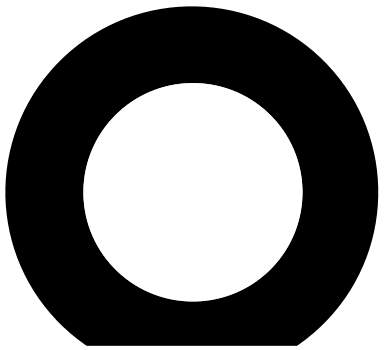
h^k ----- h^{k+1} ---------

p^k p^{k+1}

Reynolds model

Reynolds model

Interface Quasi Newton



Elastic tire

Elastic tire

h^{k} h^{k+1} h^{k+2}

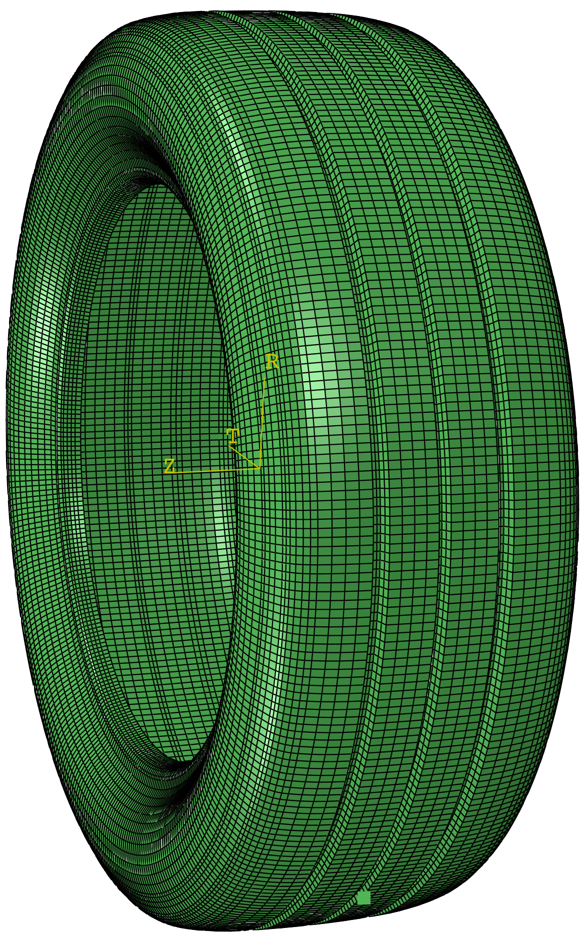
p^{k} p^{k+1}

Reynolds model

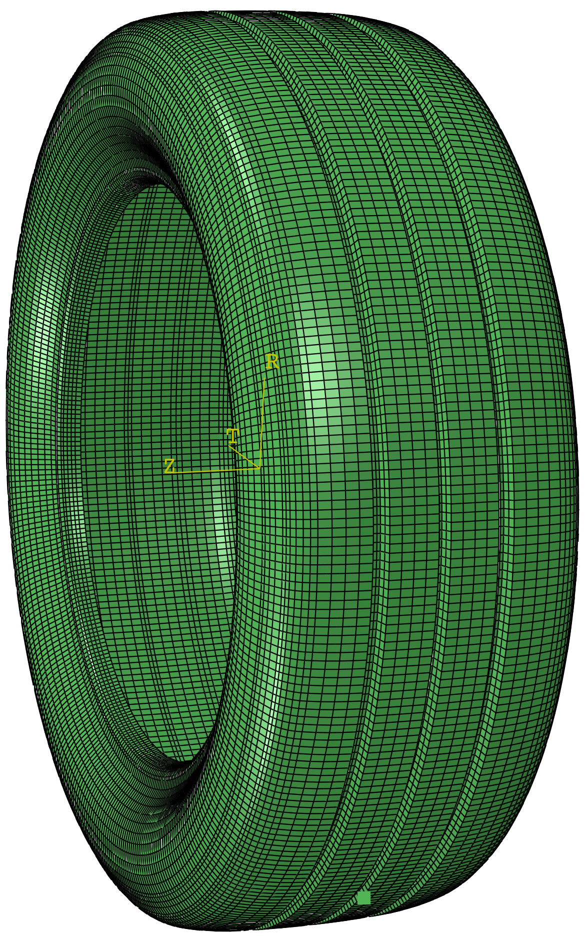
Reynolds model

Reynolds model

Abaqus - Matlab



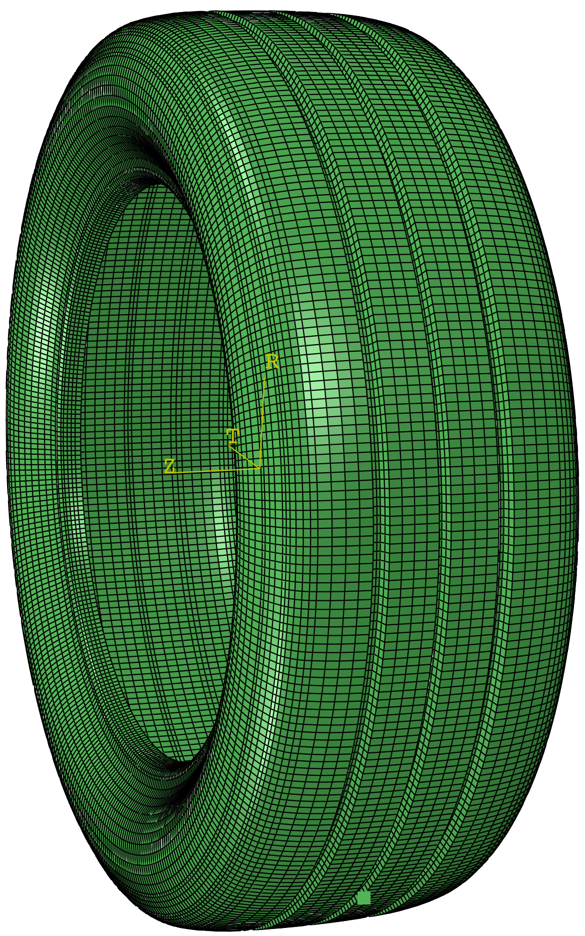
Abaqus - Matlab



→ Subroutine VUFIELD

← p Subroutine DSLOAD

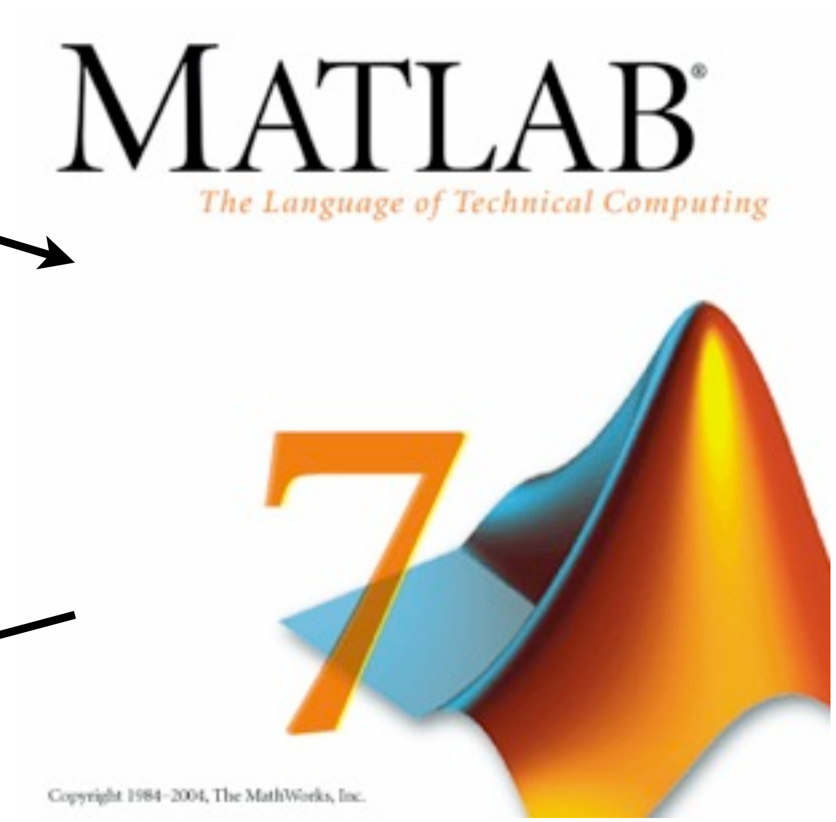
Abaqus - Matlab



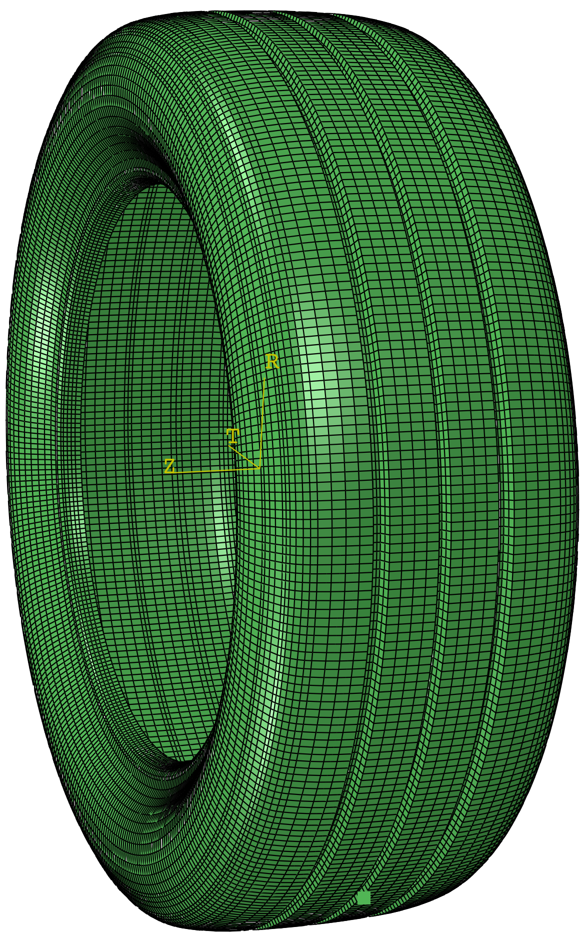
Subroutine VUFIELD

Subroutine DSLOAD

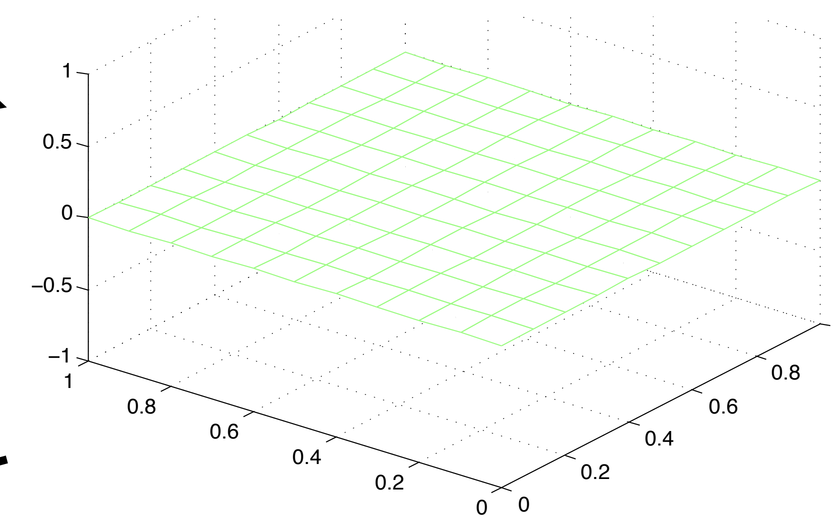
p



Abaqus - Matlab



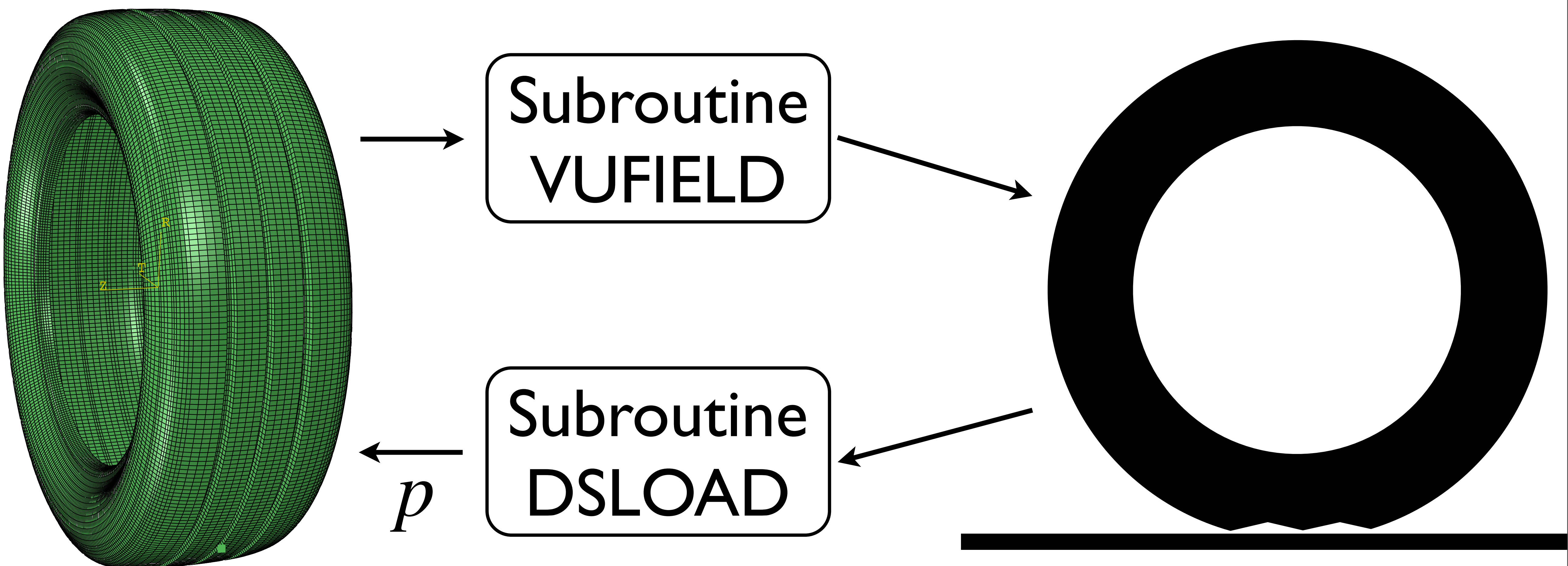
Subroutine VUFIELD



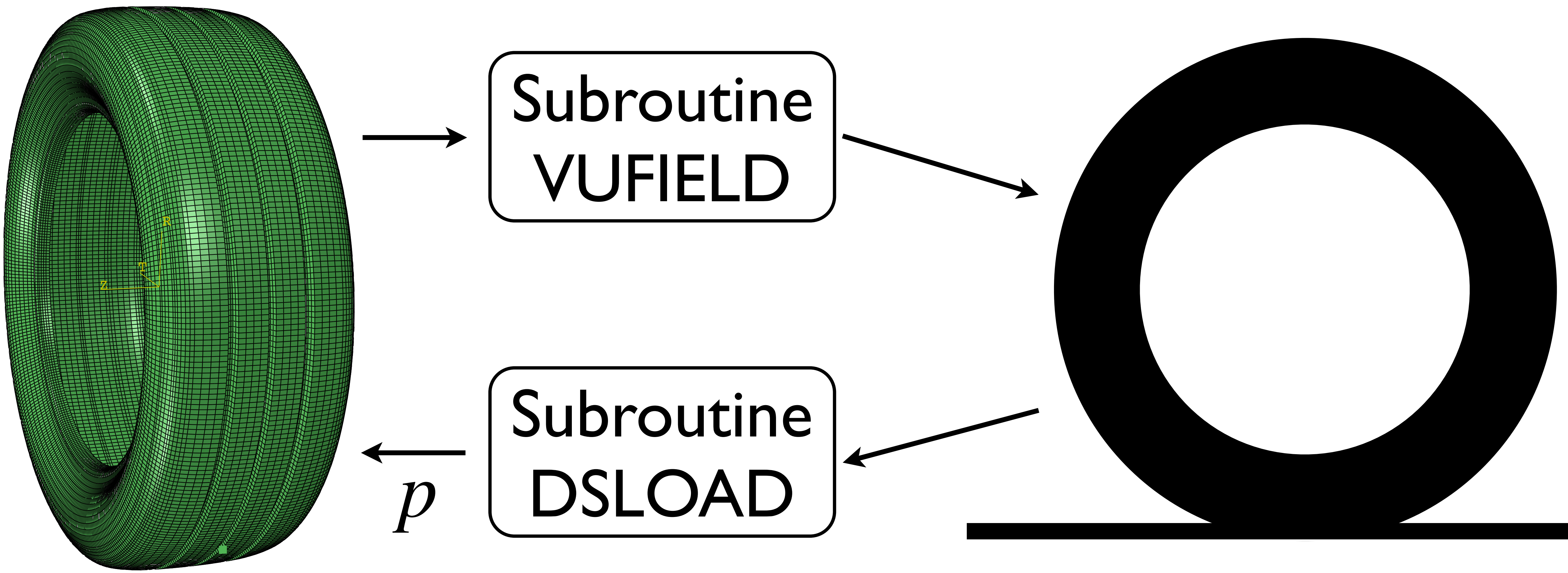
Subroutine DSLOAD

p

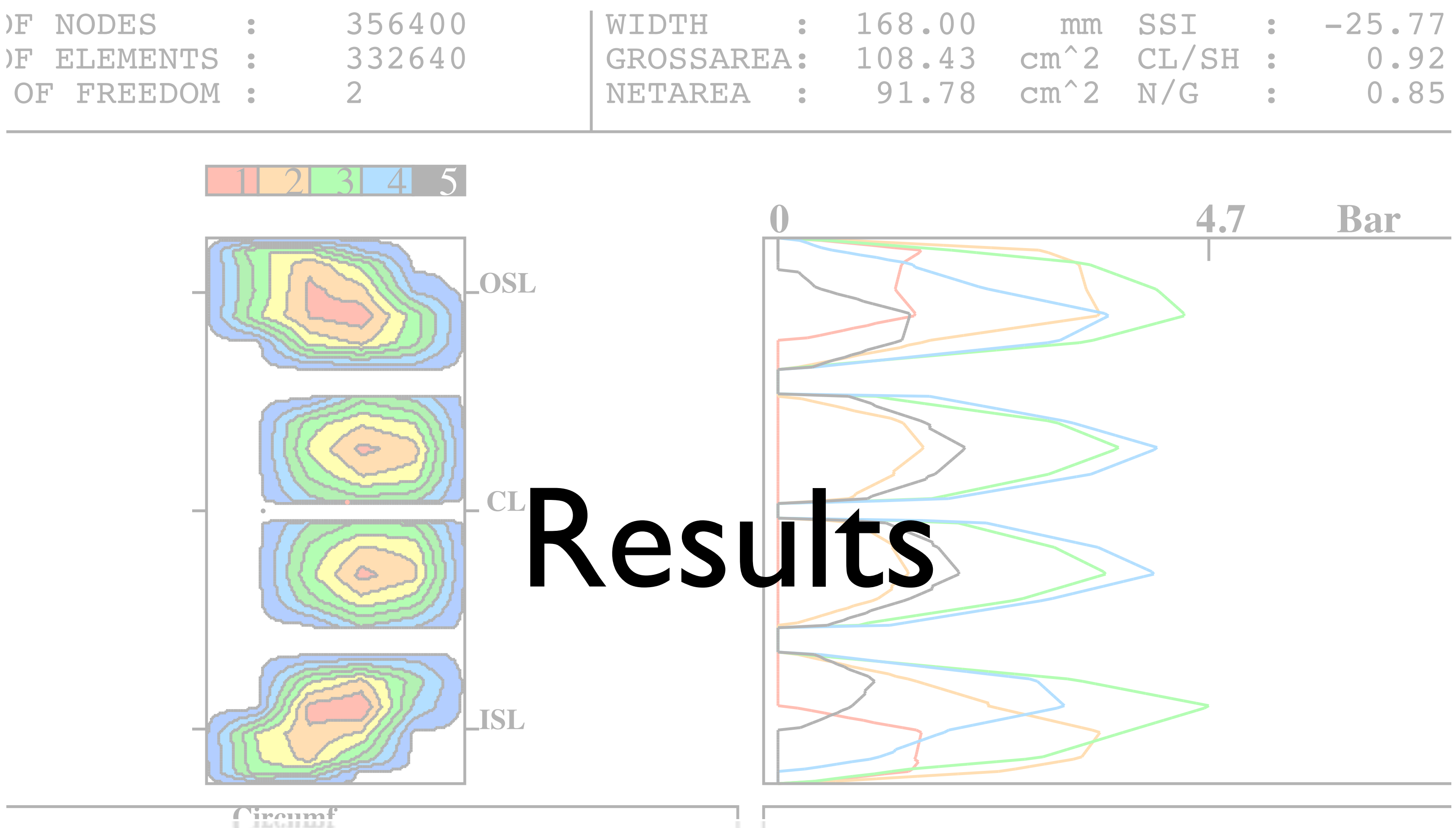
Abaqus - Matlab



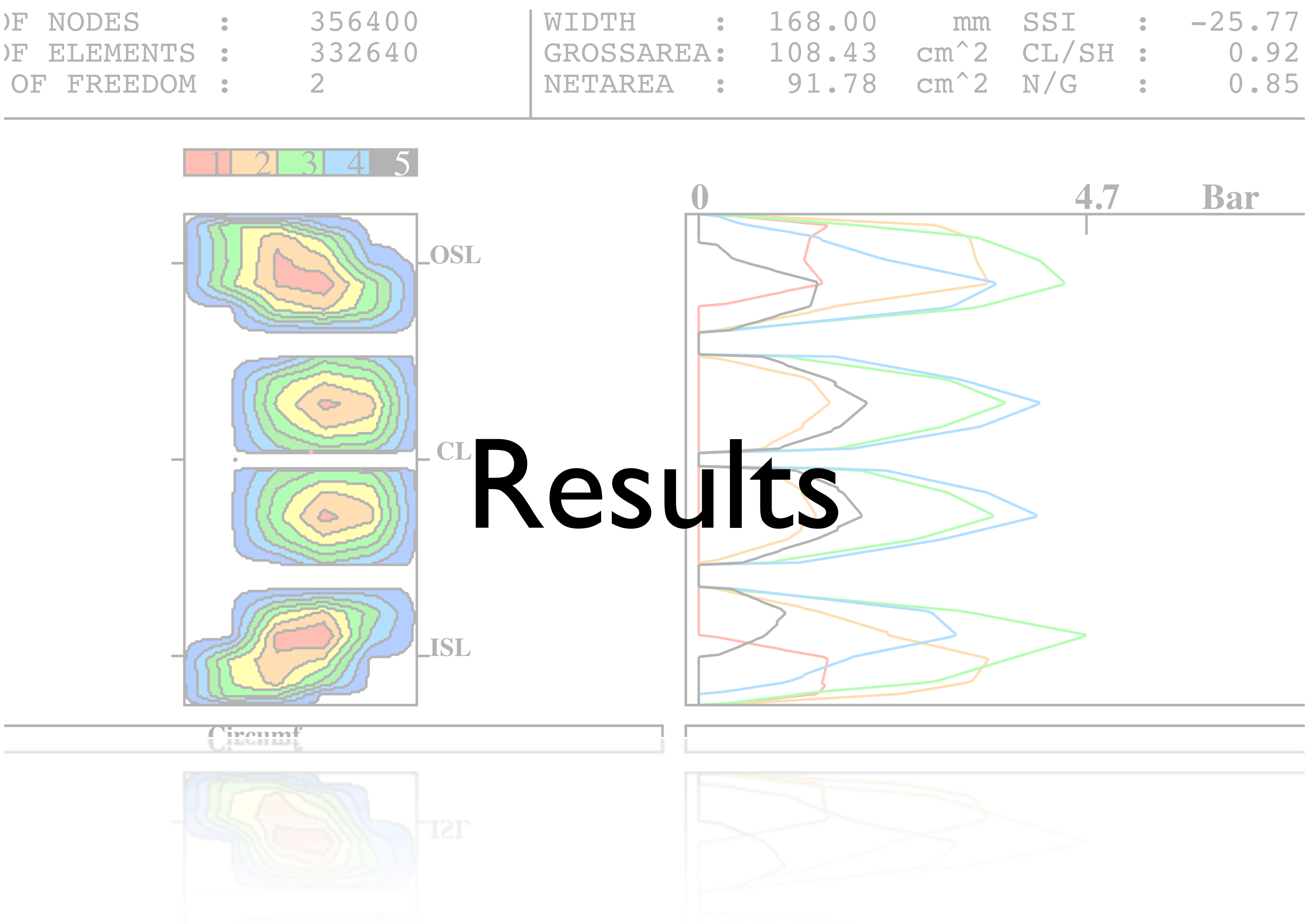
Abaqus - Matlab



Results



)F NODES : 356400
)F ELEMENTS : 332640
OF FREEDOM : 2
WIDTH : 168.00 mm
GROSSAREA: 108.43 cm^2
NETAREA : 91.78 cm^2
SSI : -25.77
CL/SH : 0.92
N/G : 0.85
1 2 3 4 5
OSL
CL
ISL
0
4.7
Bar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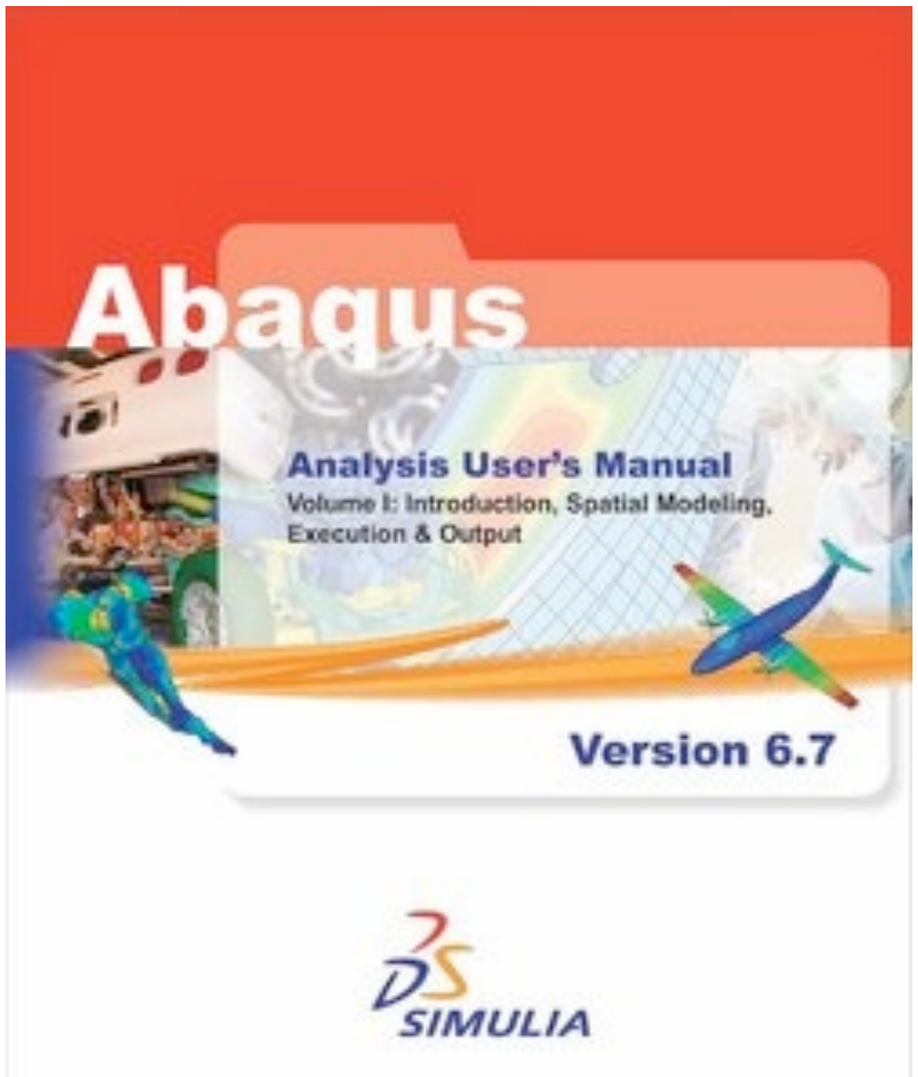
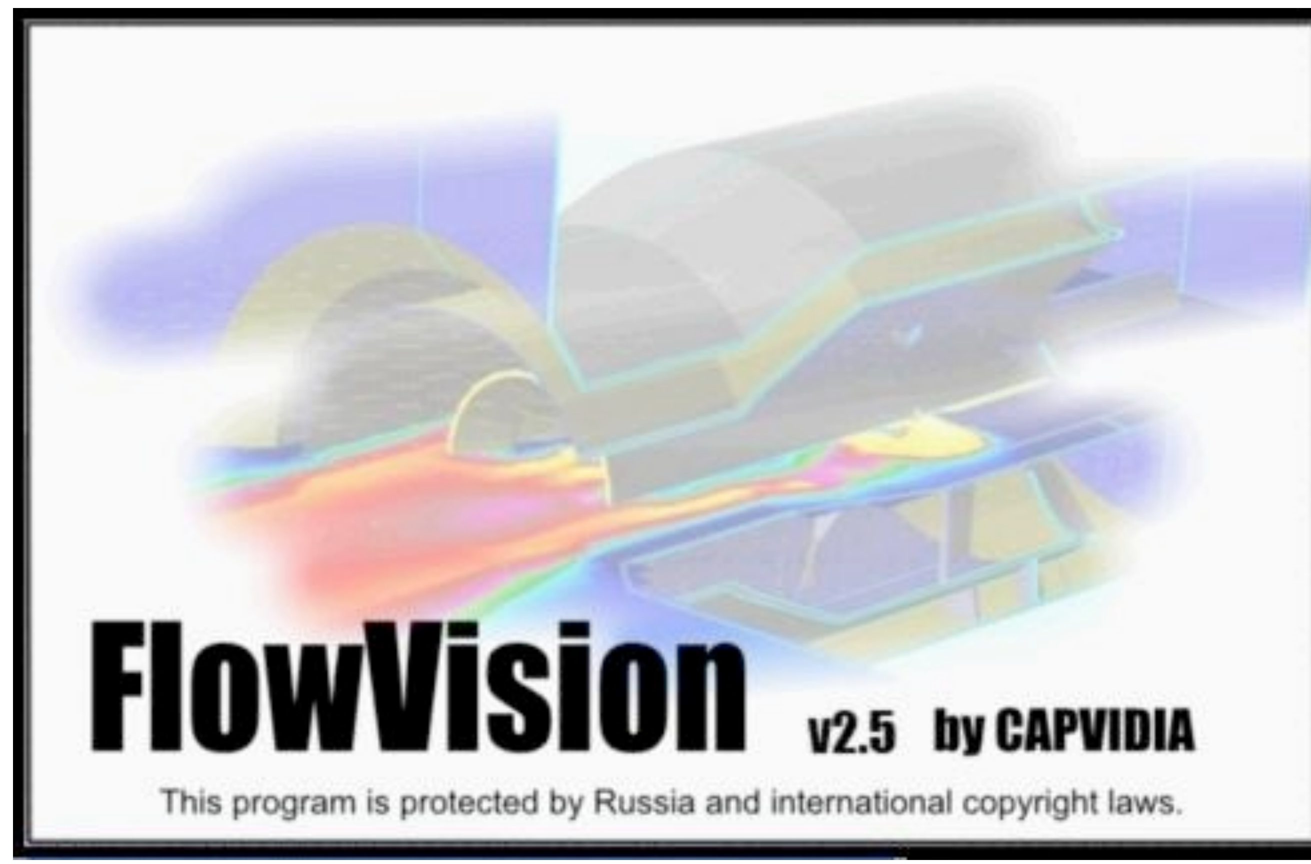
SSI : -25.77
CL/SH : 0.92
N/G : 0.85
1 2 3 4 5
OSL
CL
ISL
0
4.7
Bar

Results

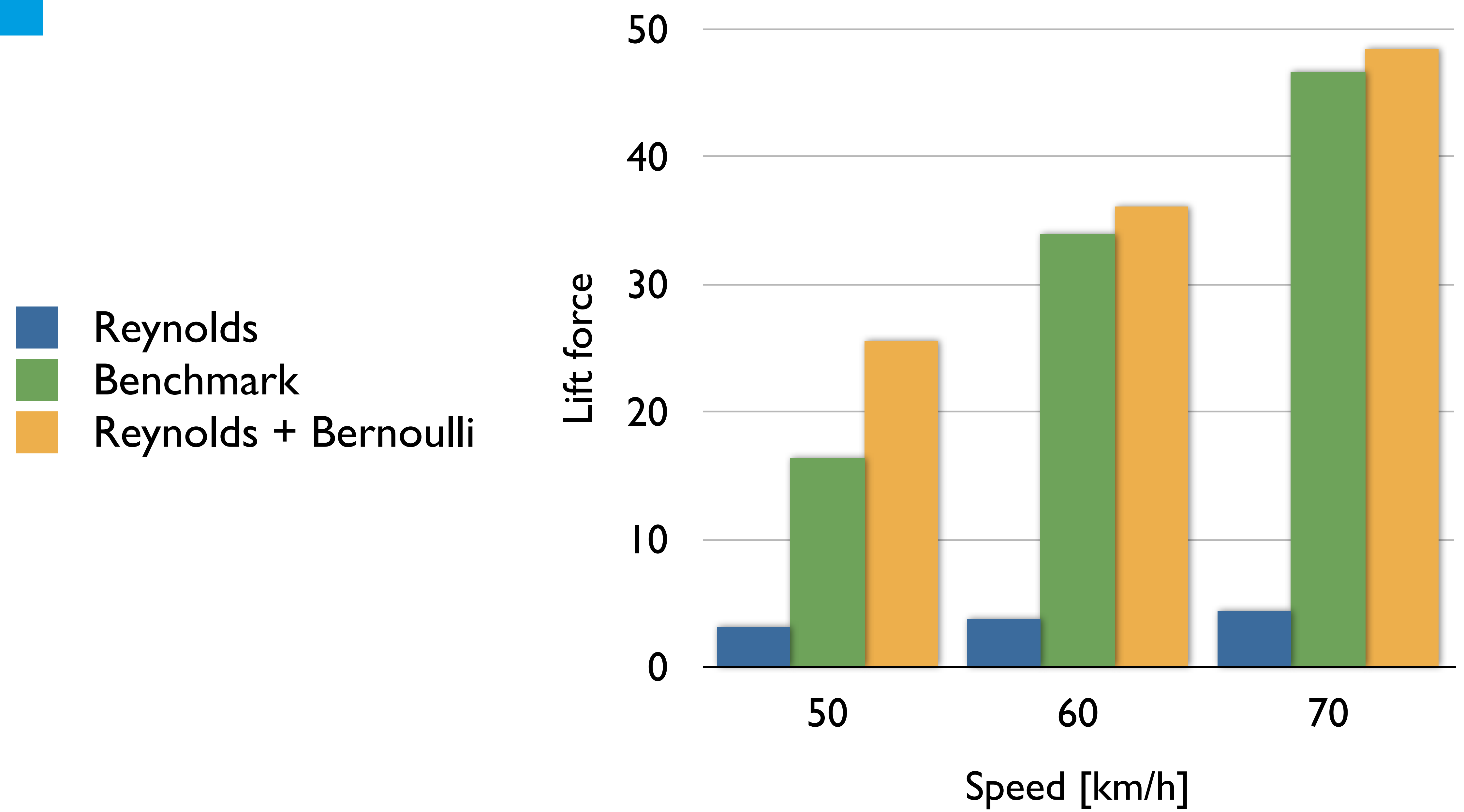
Results: Benchmark



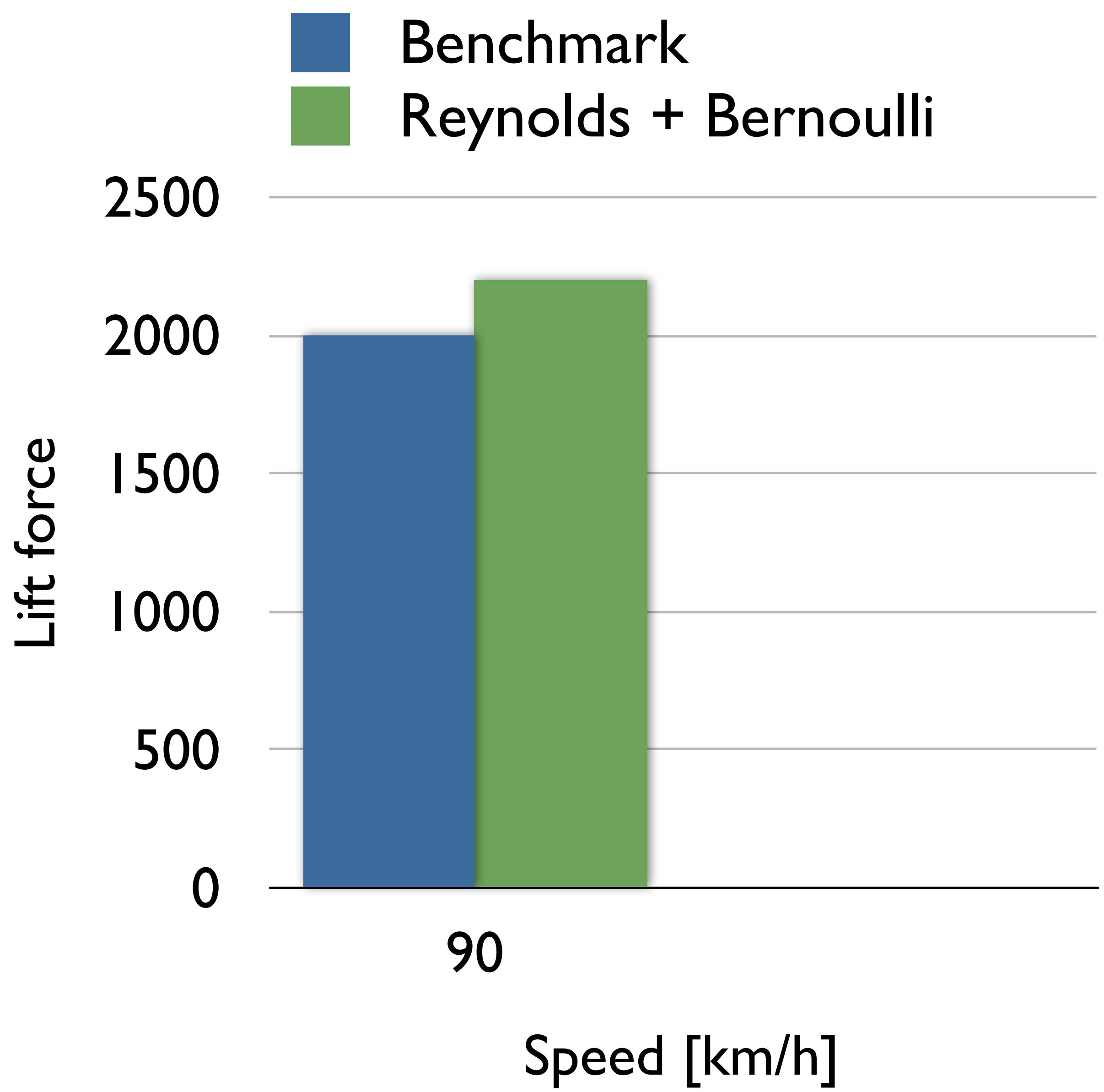
↔



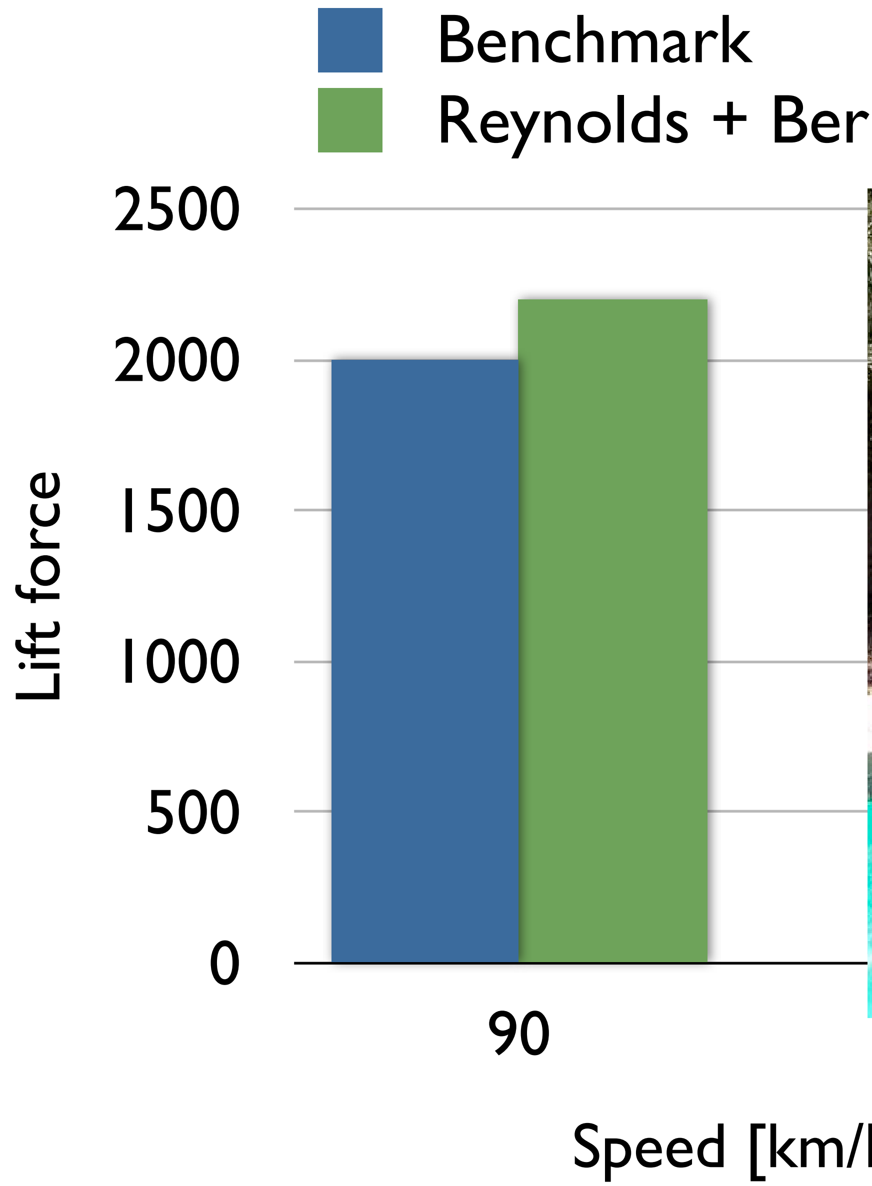
Results: Linear elastic tire



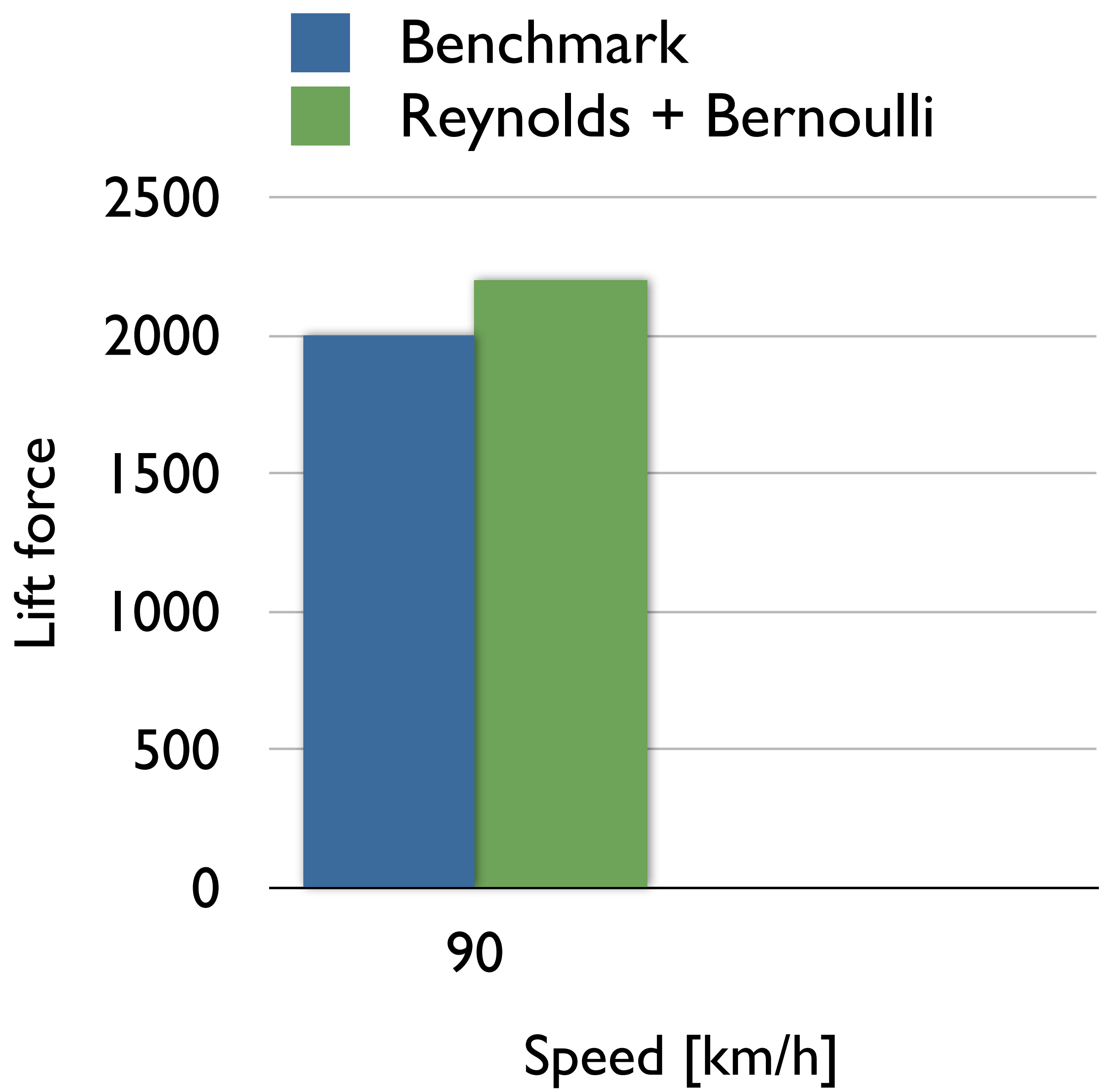
Results: 'real' tire



Results: ‘real’ tire

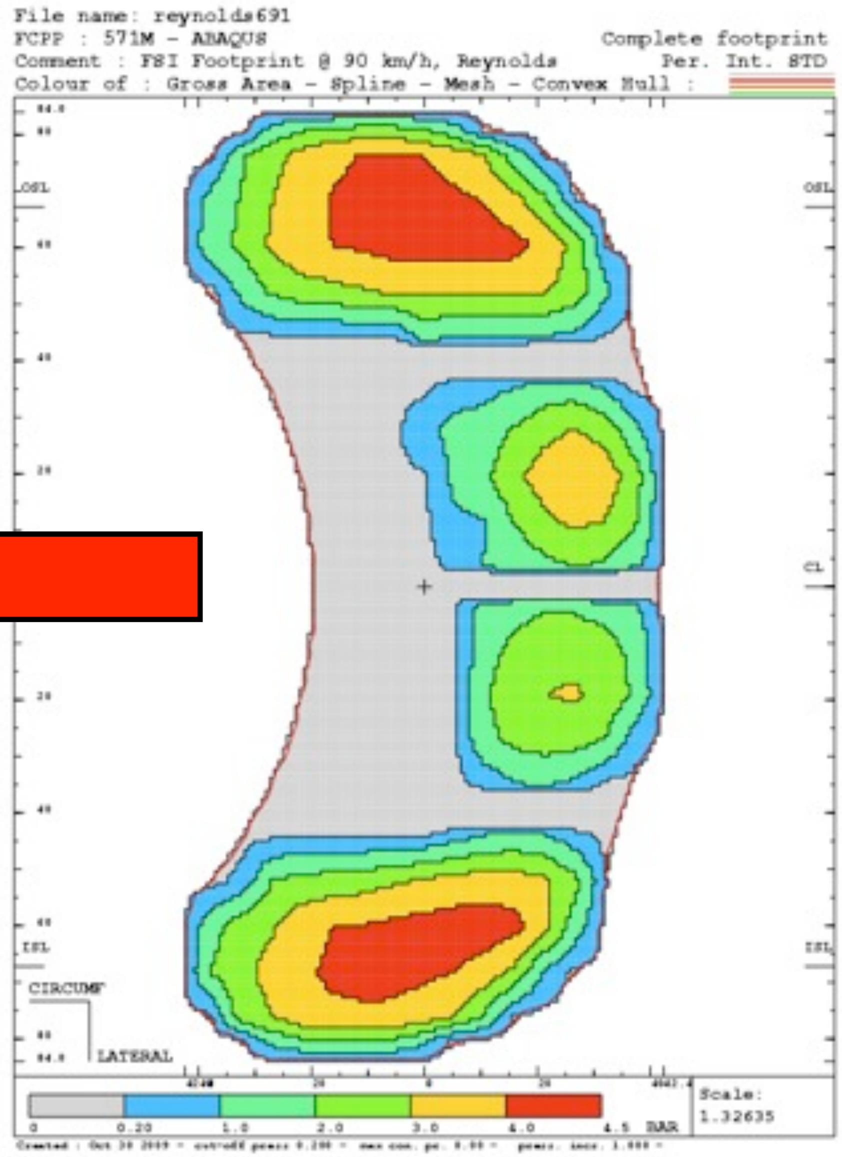


Results: ‘real’ tire

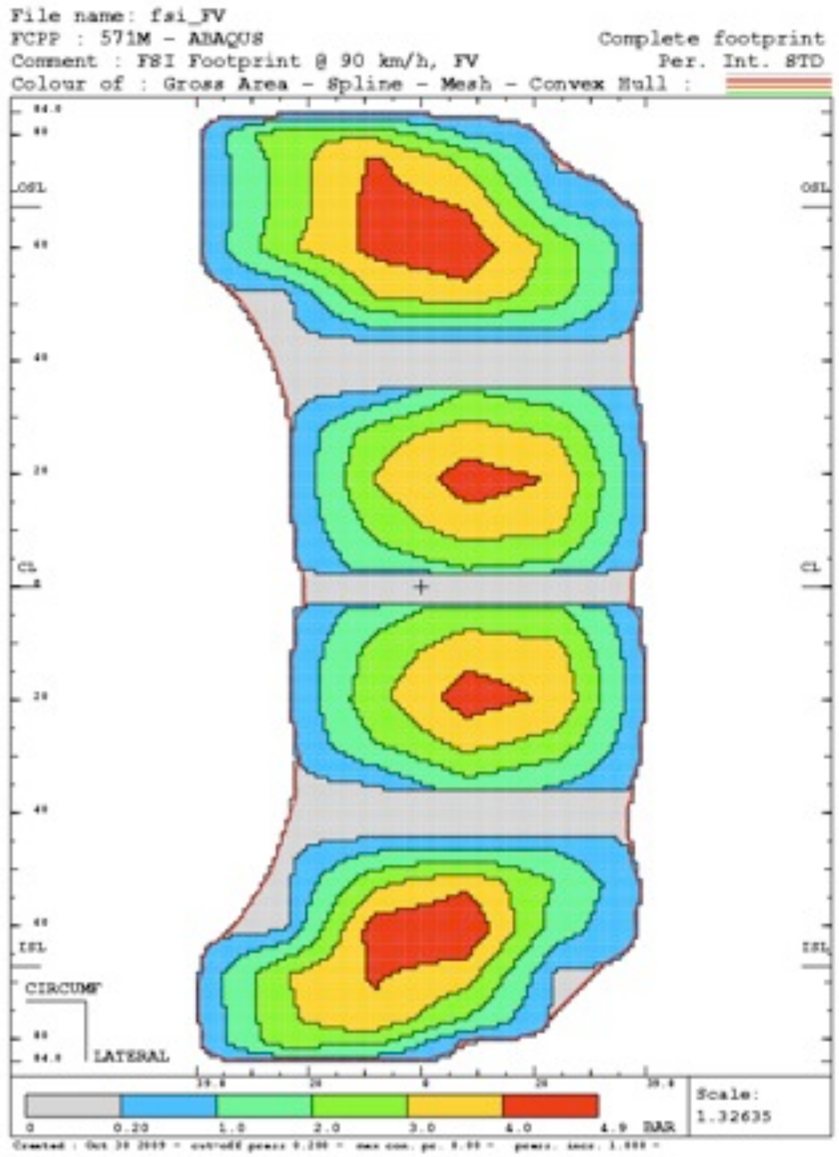


Footprint

Reynolds + Bernoulli

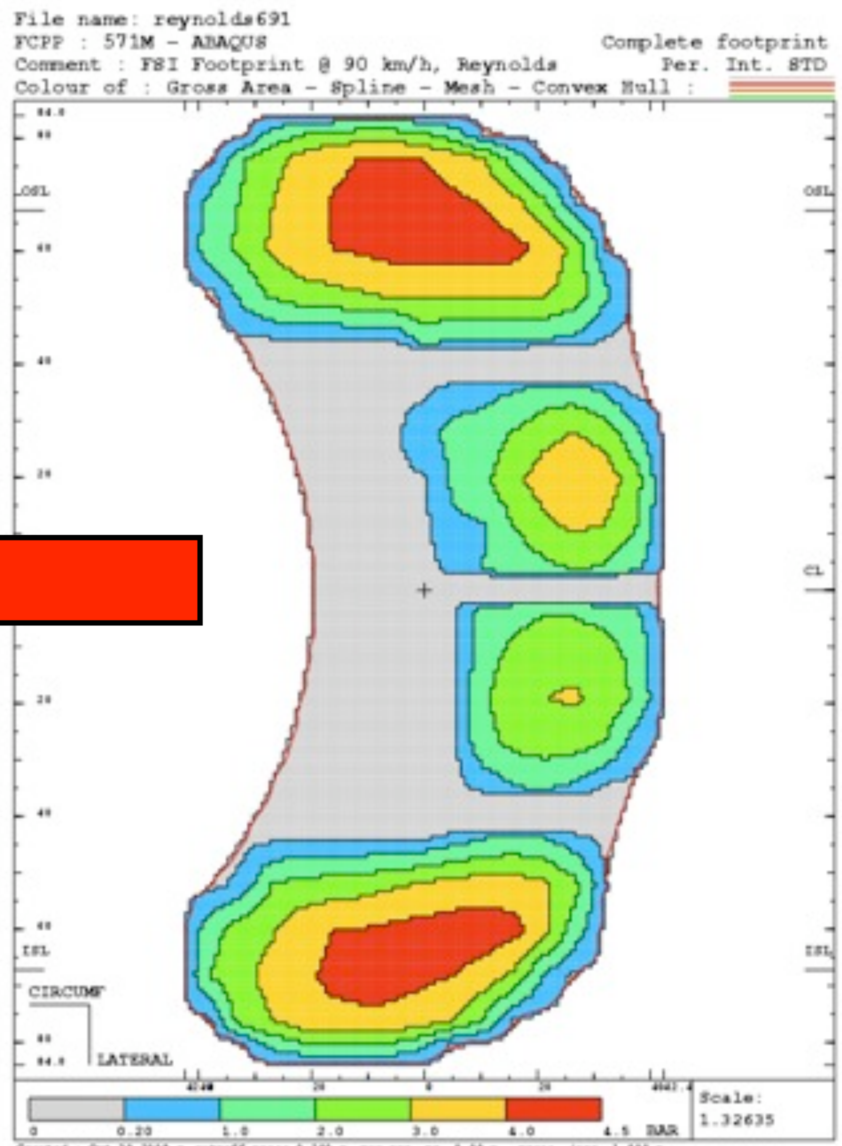


Benchmark

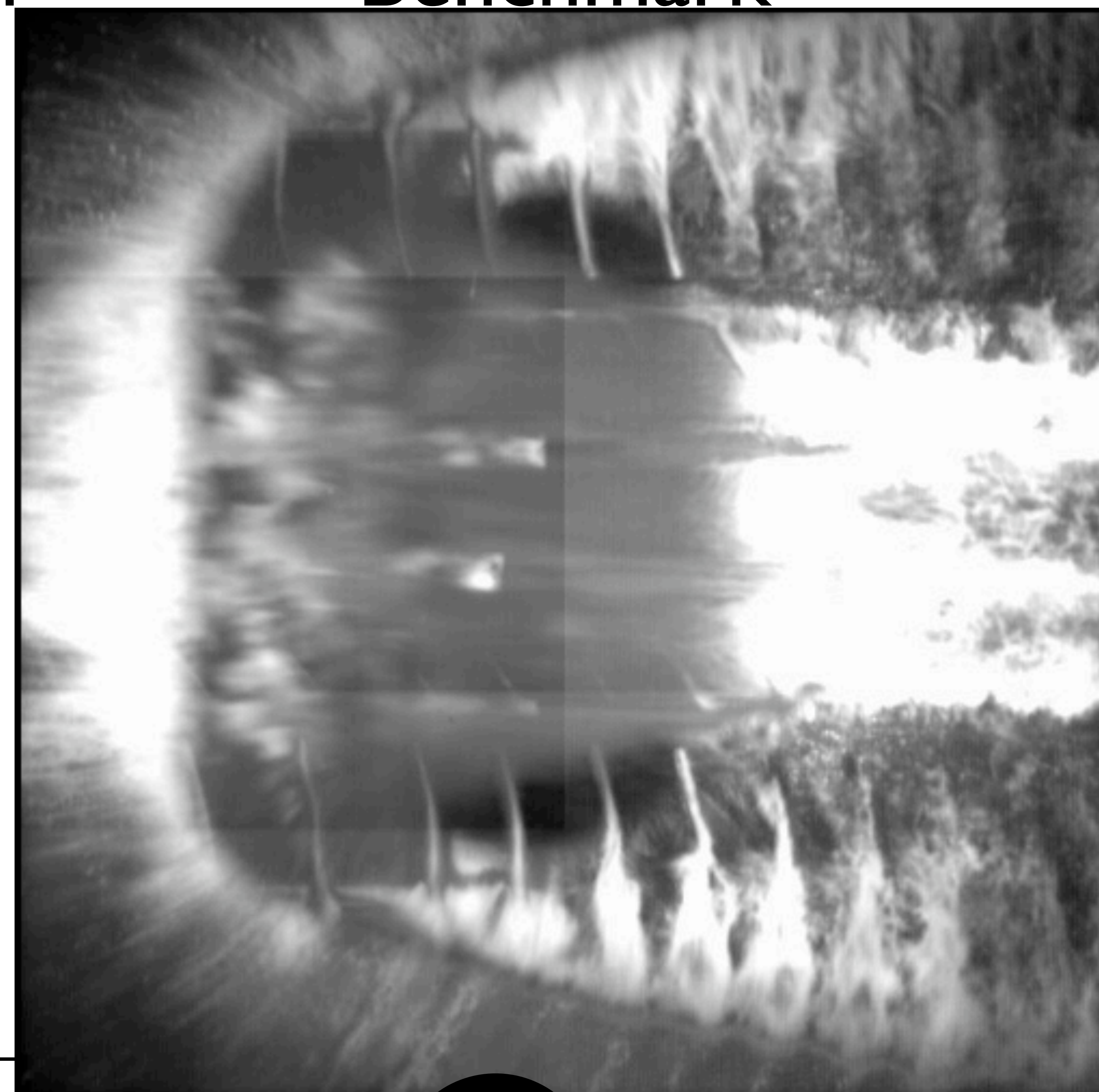


Footprint

Reynolds + Bernoulli

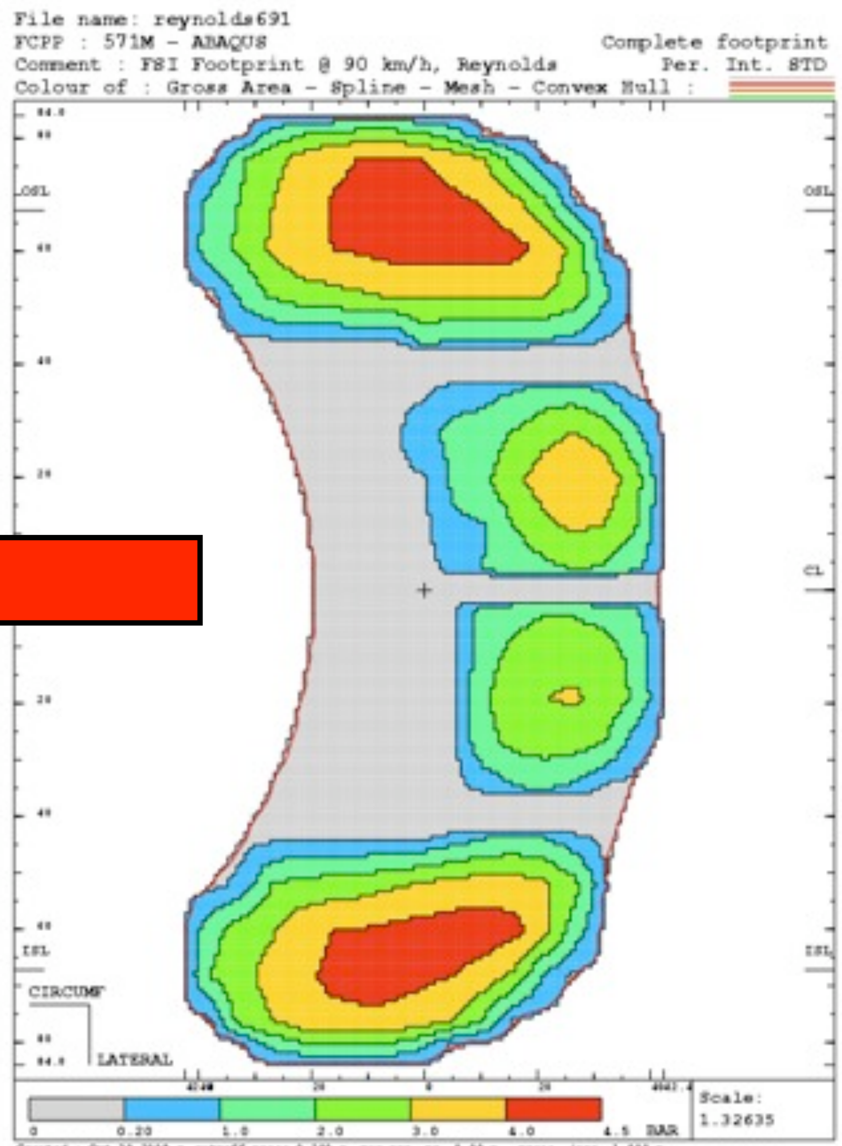


Benchmark

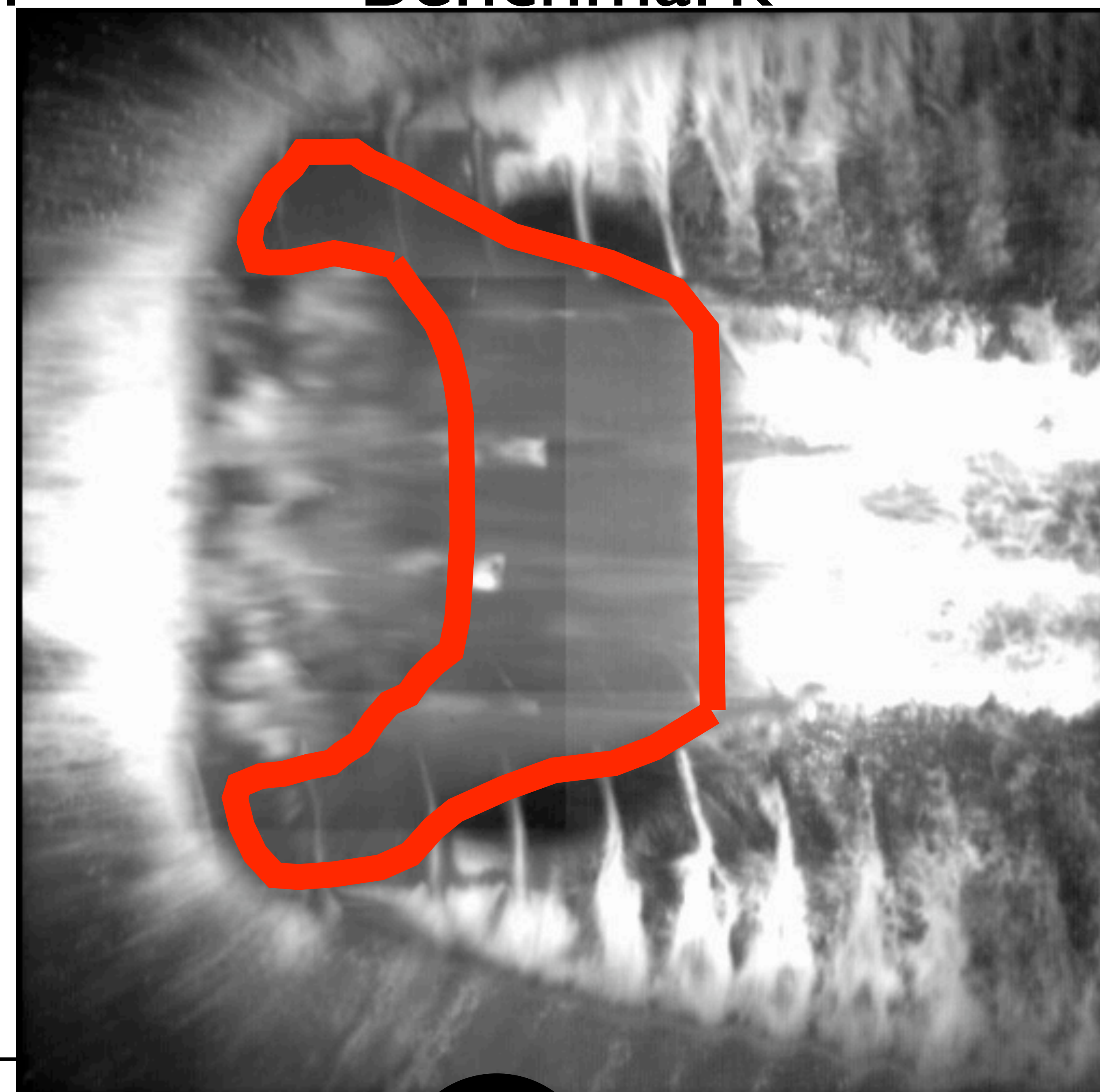


Footprint

Reynolds + Bernoulli



Benchmark



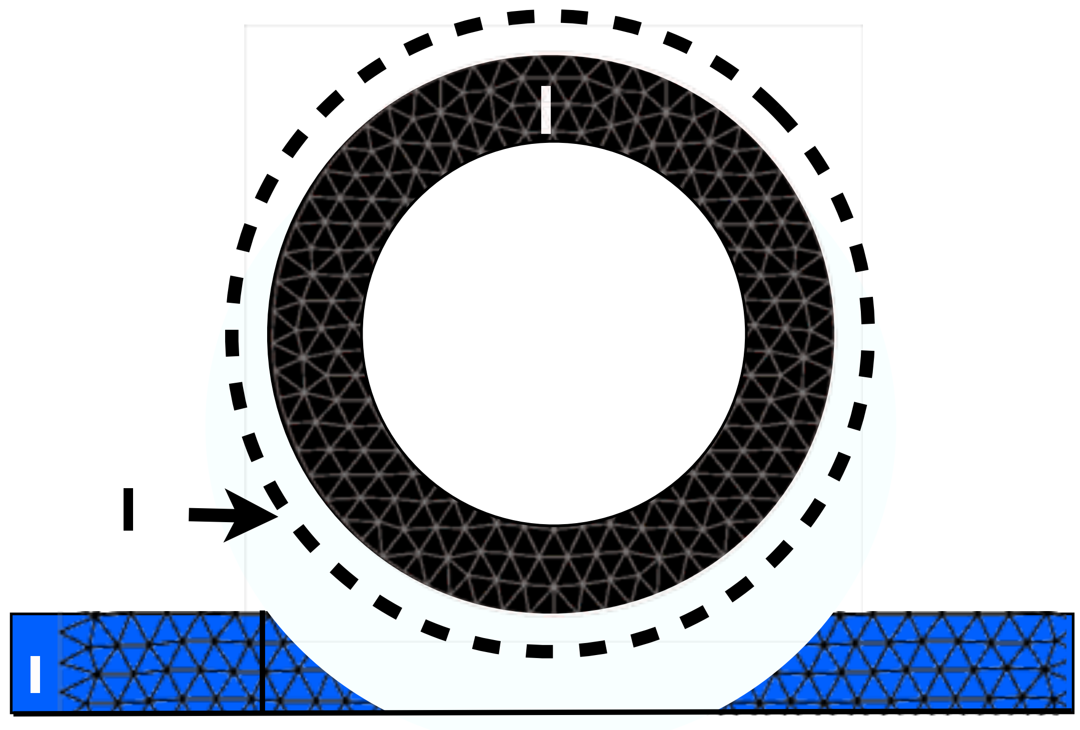
Fast?

Fast?

- Benchmark: 24 - 48 hours / 16 CPU's

Fast?

- Benchmark: 24 - 48 hours / 16 CPU's
- Interface method promising



Research question

Can we model hydroplaning fast and accurate using lubrication theory?

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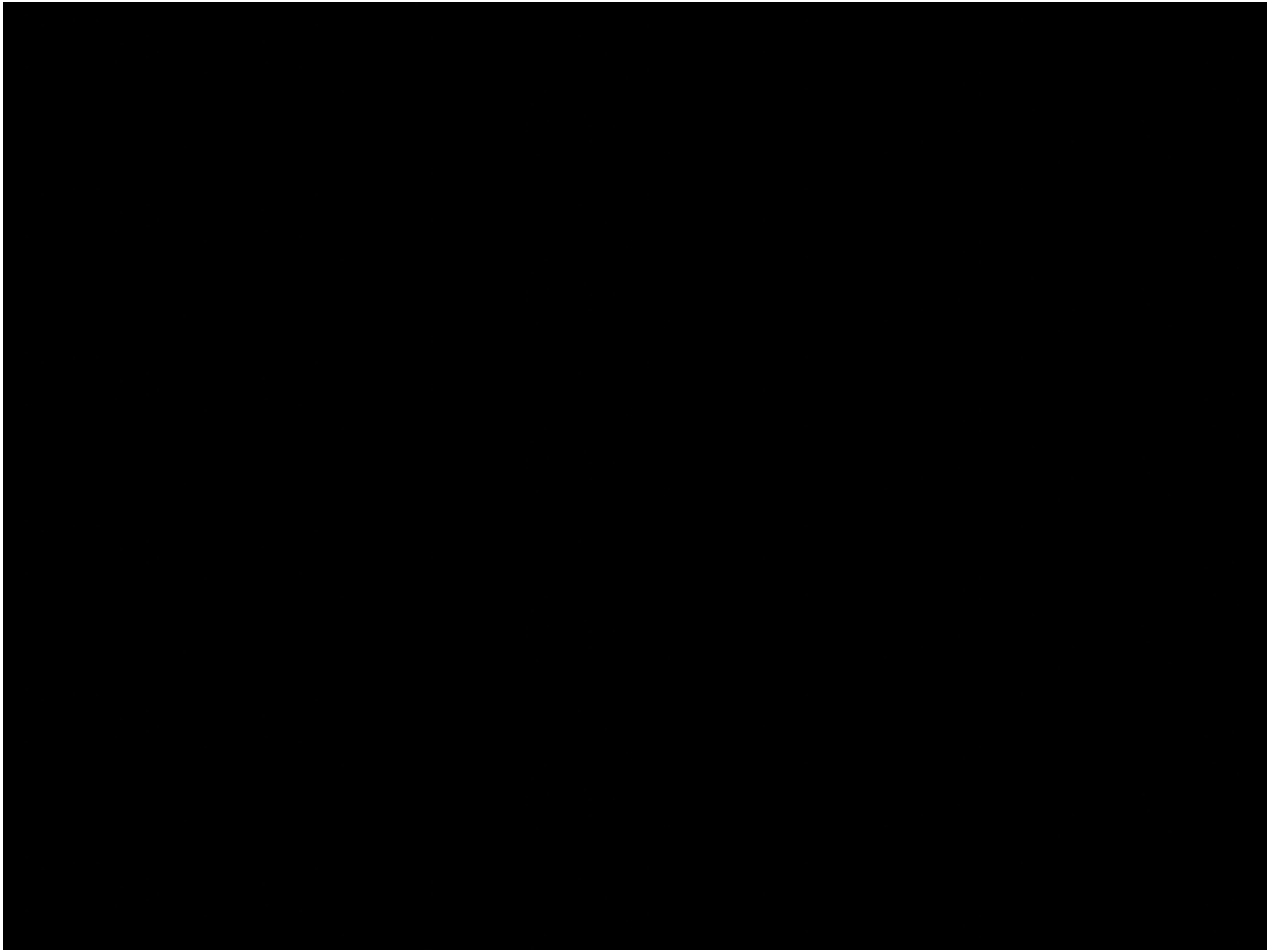
Questions?

Hydroplaning

Lubrication theory

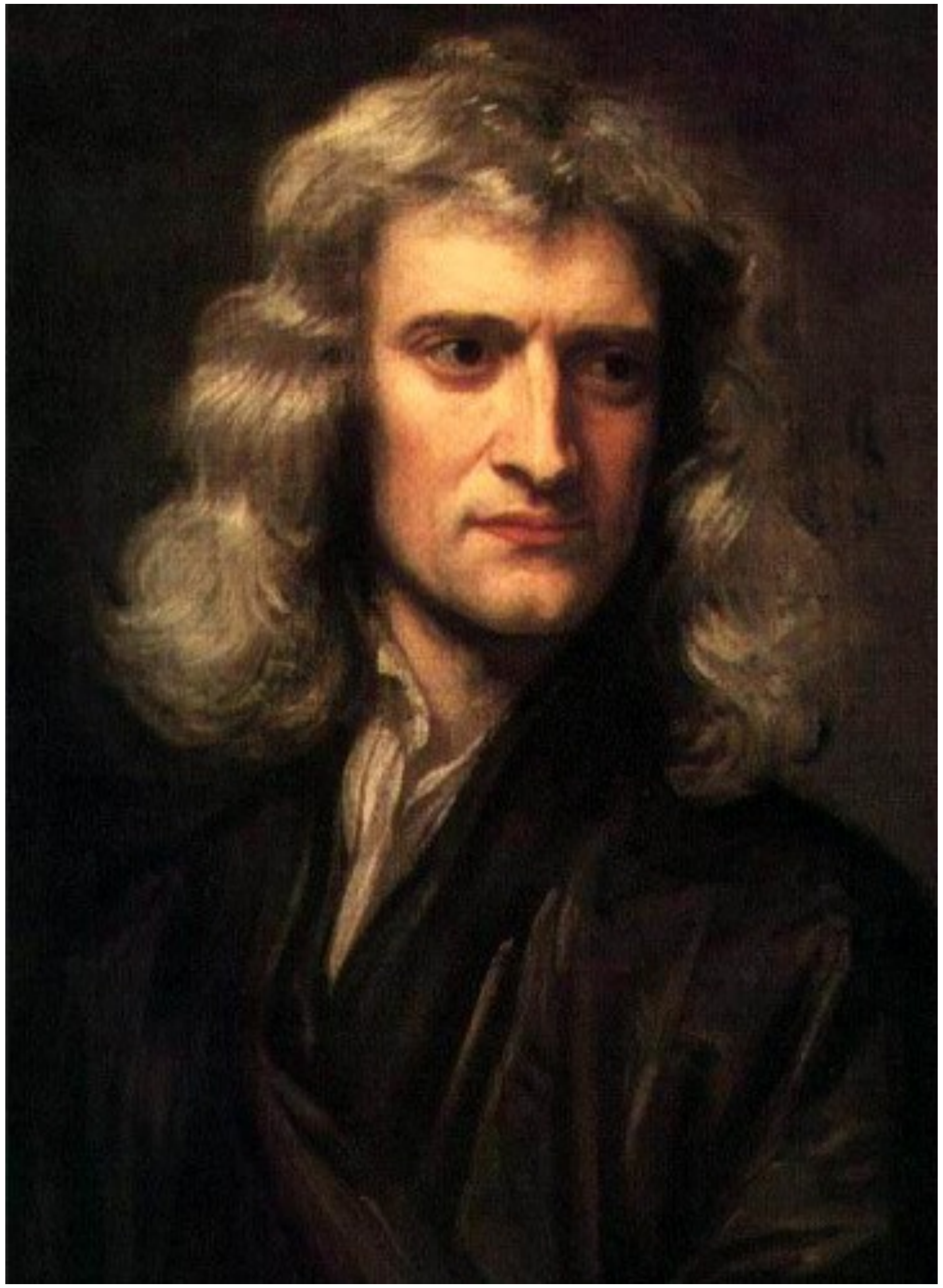


Problem description

Problem description

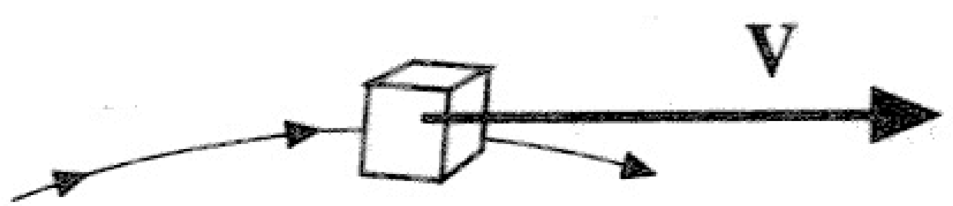
Newton's 2nd Law

$$F = m \cdot a$$



Moving control volume

$$\rho \frac{d}{dt}(\mathbf{v}(x, y, z, t)) = \mathbf{b}$$

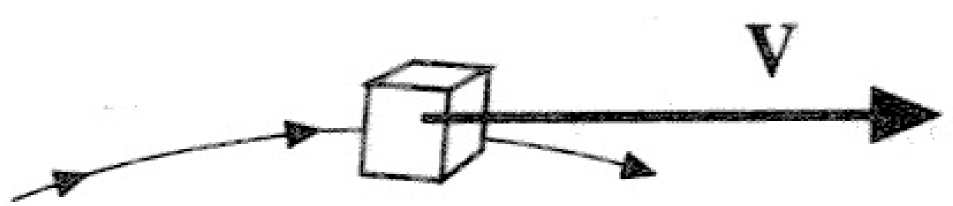


Infinitesimally small fluid element
of fixed mass moving with the flow

Moving control volume

$$\rho \frac{d}{dt}(\mathbf{v}(x, y, z, t)) = \mathbf{b}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{v}}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{v}}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{v}}{\partial z}\frac{dz}{dt} \right) = \mathbf{b}$$



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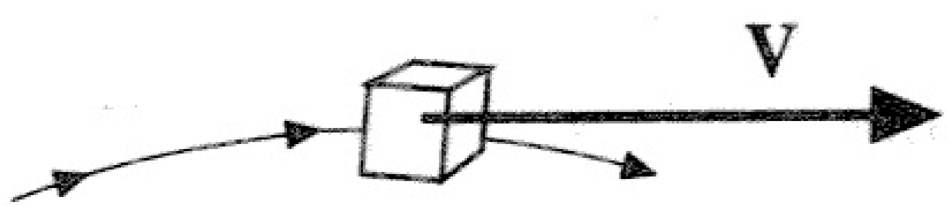
$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{b}$$



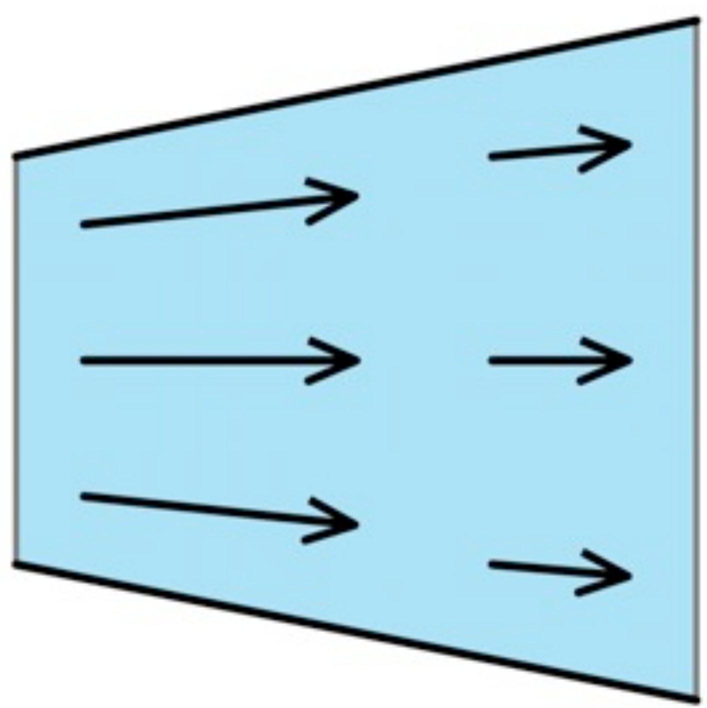
Infinitesimally small fluid element
of fixed mass moving with the flow

Moving control volume

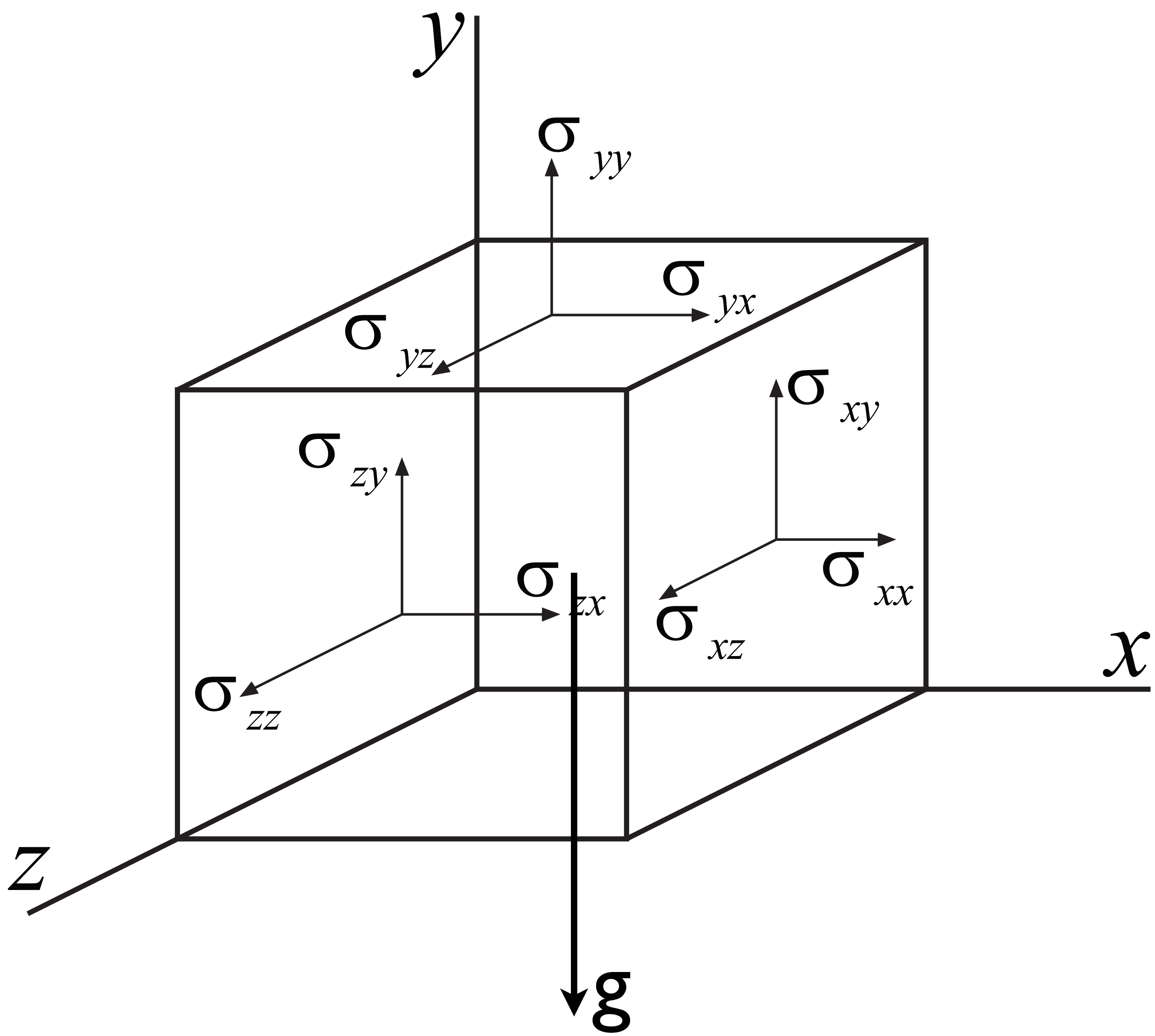
$$\rho \frac{d}{dt}(\mathbf{v}(x,y,z,t)) = \mathbf{b}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{v}}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{v}}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{v}}{\partial z}\frac{dz}{dt}\right) = \mathbf{b}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = \mathbf{b}$$



Body forces



Body forces

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}$$

Body forces

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} = -\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + \begin{pmatrix} \sigma_{xx}+p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}+p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}+p \end{pmatrix} = -pI + \mathbb{T}$$

Body forces

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} = - \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + \begin{pmatrix} \sigma_{xx} + p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} + p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} + p \end{pmatrix} = -pI + \mathbb{T}$$

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

Bored?

- Existence
- Smoothness

Bored?

- Existence
- Smoothness

Assume: no body force

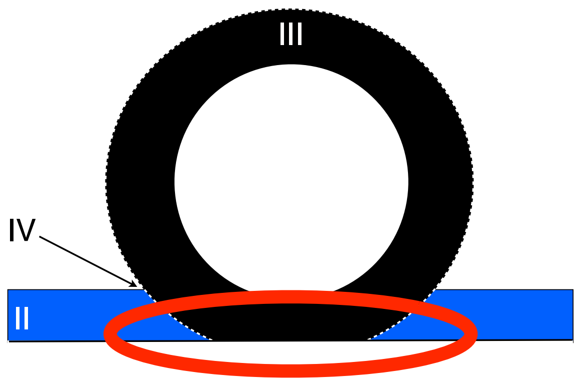
$$0 = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{\mathbf{f}}_{\substack{\text{Other}\\\text{body}\\\text{forces}}}.$$

TU Delft

Assume: no body force

$$0 = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu \nabla^2 \mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}}.$$

Assume: μ constant



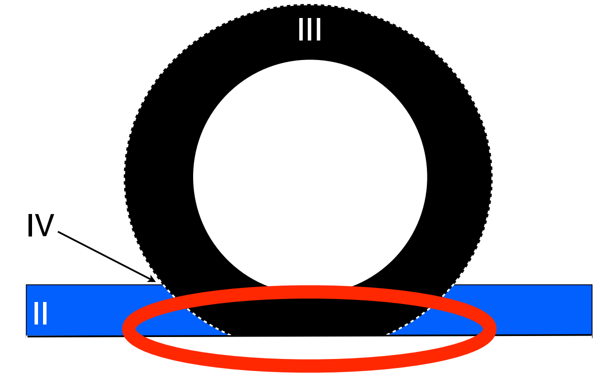
Conservation of momentum:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial^2 z}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial^2 z}$$

$$\frac{\partial p}{\partial z} = 0$$

Assume: μ constant



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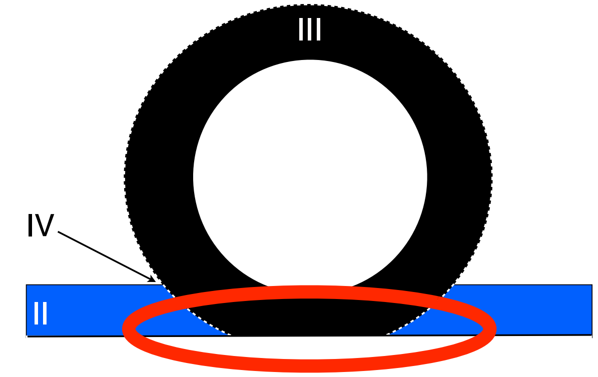
$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial^2 z}$$

$$\frac{\partial p}{\partial z} = 0$$

Conservation of mass:

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

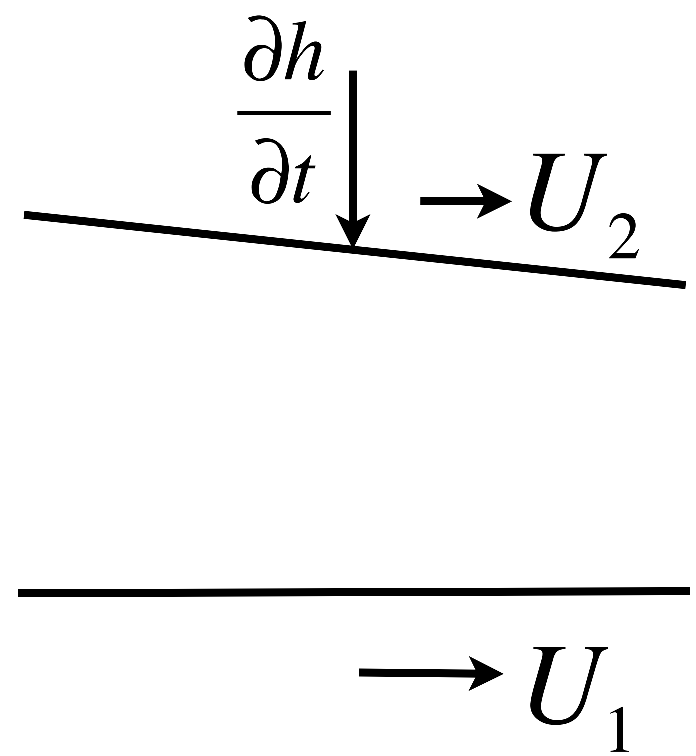
Continuity



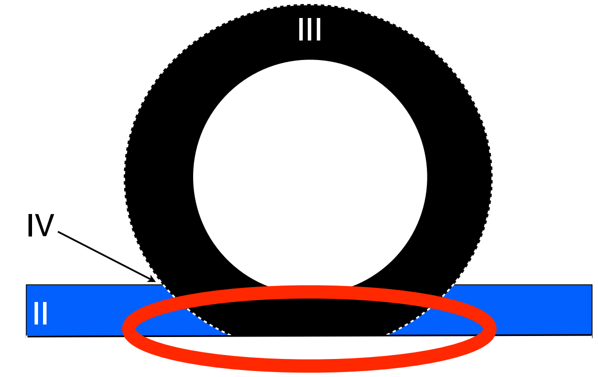
$$u\left(x,z\right)=\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(z^2-hz\right)+\frac{U_2-U_1}{h}z+U_1$$

$$v\left(x,z\right)=\frac{1}{2\mu}\frac{\partial p}{\partial y}\left(z^2-hz\right)+\frac{V_2-V_1}{h}z+V_1$$

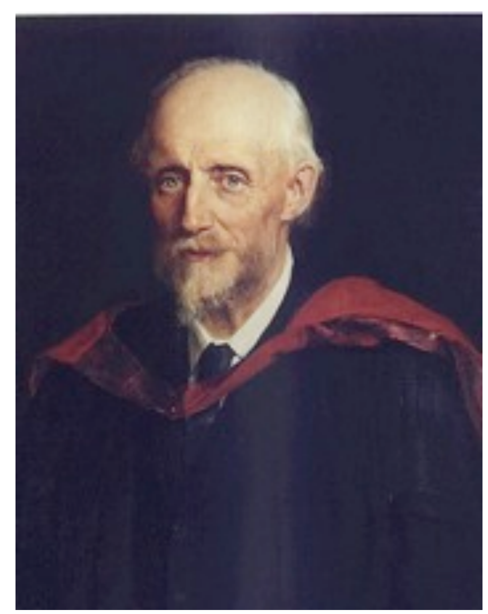
$$0=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$



Reynolds equation

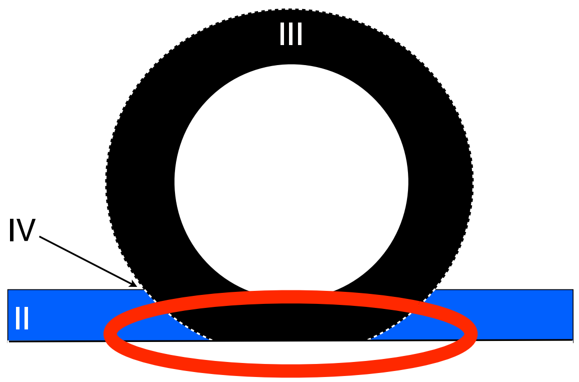


$$\nabla \cdot \left(\frac{h^3}{12\mu} \nabla p \right) = \nabla \cdot \left(\bar{U} h \right) + \frac{\partial h}{\partial t}$$



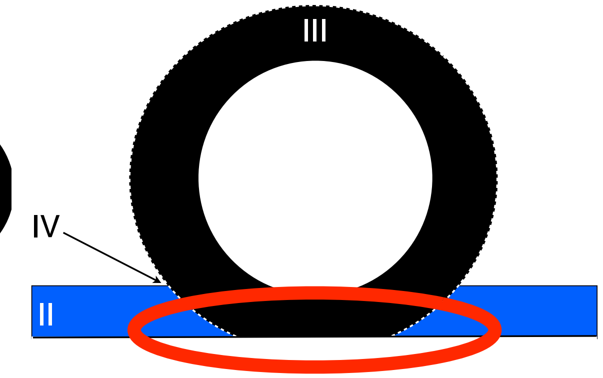
Reynolds (1886)

Squeeze



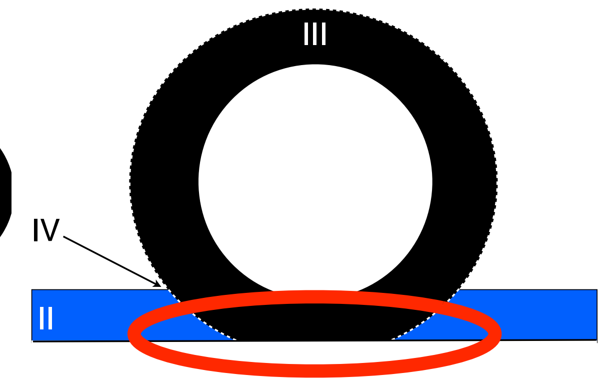
- Iterative scheme:
  - Solve the pressure
  - Determine velocity
  - Re-solve Reynolds, including inertia

Sliding: iterative 1D



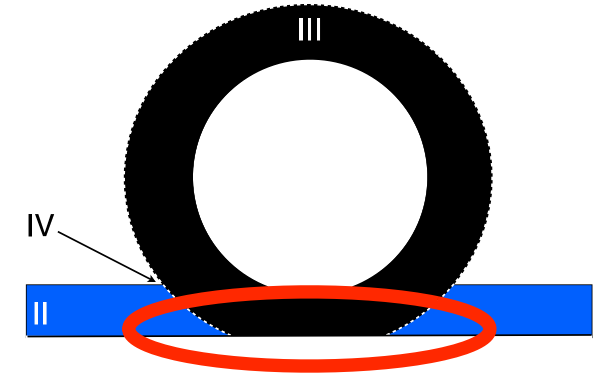
$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_c}{\partial z^2}$$

Sliding: iterative 1D



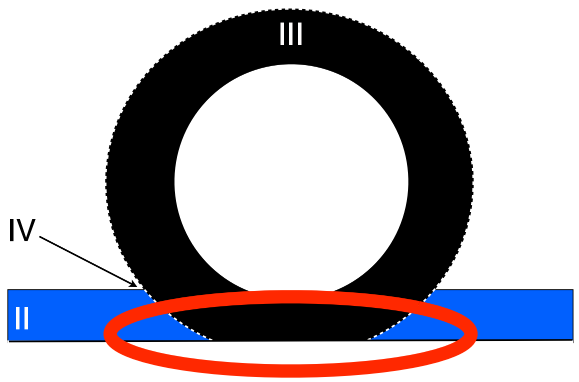
$$\rho\left(u_v \frac{\partial u_v}{\partial x} + w_v \frac{\partial u_v}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_c}{\partial z^2}$$

Sliding: Average 1D



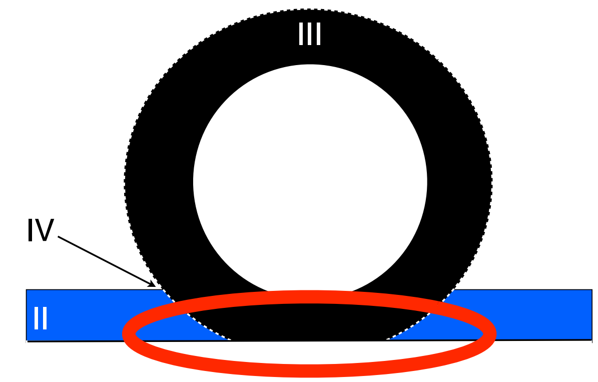
$$\overbrace{\rho\left(\frac{1}{h}\int_0^h\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)dz\right)}=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2}$$

Sliding: Average 1D



$$\overbrace{\rho\left(\frac{1}{h}\int_0^h\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)dz\right)}^{\text{Independent of } z}=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2}$$

Sliding: Iterative & Average 2D

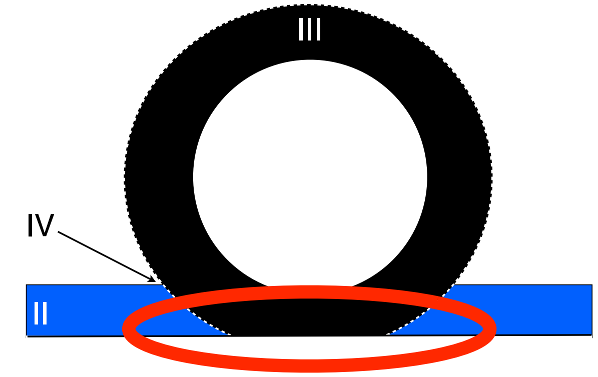


$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2}$$

$$0 = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial p}{\partial z} = 0$$

Sliding: Iterative & Average 2D

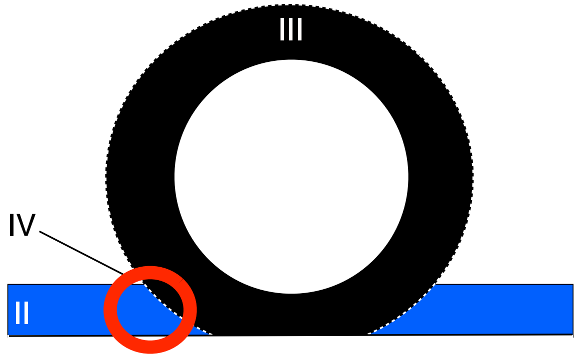


$$\rho\left(\frac{1}{h}\int_0^h \frac{Du}{Dt}dz\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial z^2}$$

$$\rho\left(\frac{1}{h}\int_0^h \frac{Dv}{Dt}dz\right) = -\frac{\partial p}{\partial y} + \mu\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial p}{\partial z} = 0$$

Inlet condition

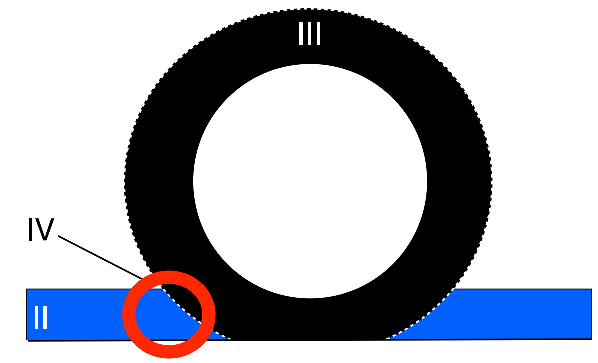


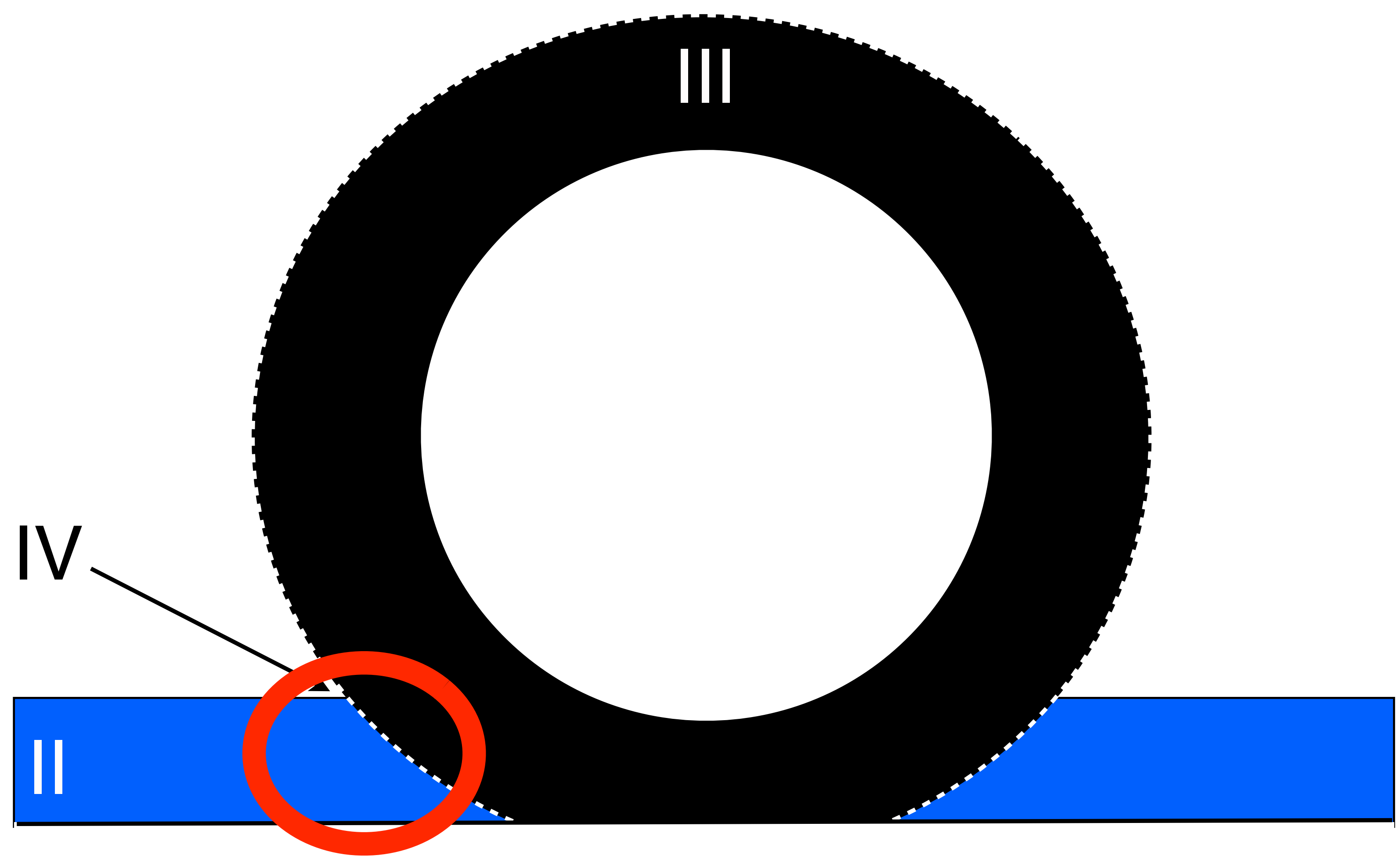
- Stagnation pressure

$$p = \frac{1}{2}\rho v^2$$

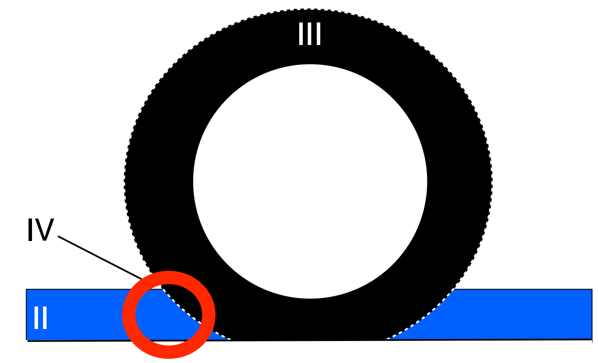
- Energy & Momentum correction
  - Converges to zero

Fill rate





Fill rate



$$\nabla \cdot \left(\frac{-h^3}{12\mu} f \nabla p + \bar{U} h f \right) = 0$$

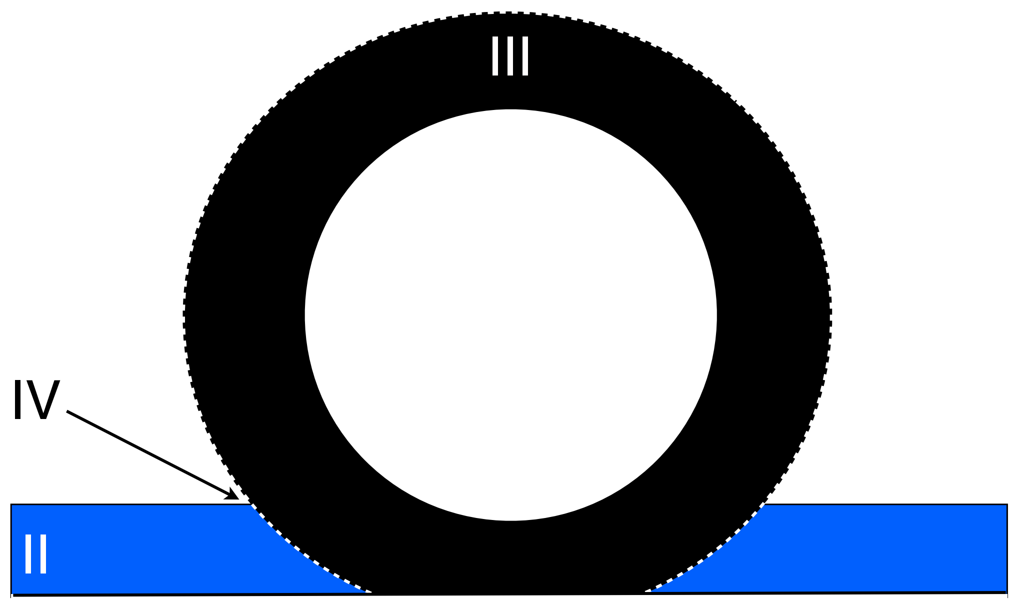
$$p = \xi \text{ for } \xi \geq 0$$

$$f = 1 \text{ for } \xi \geq 0$$

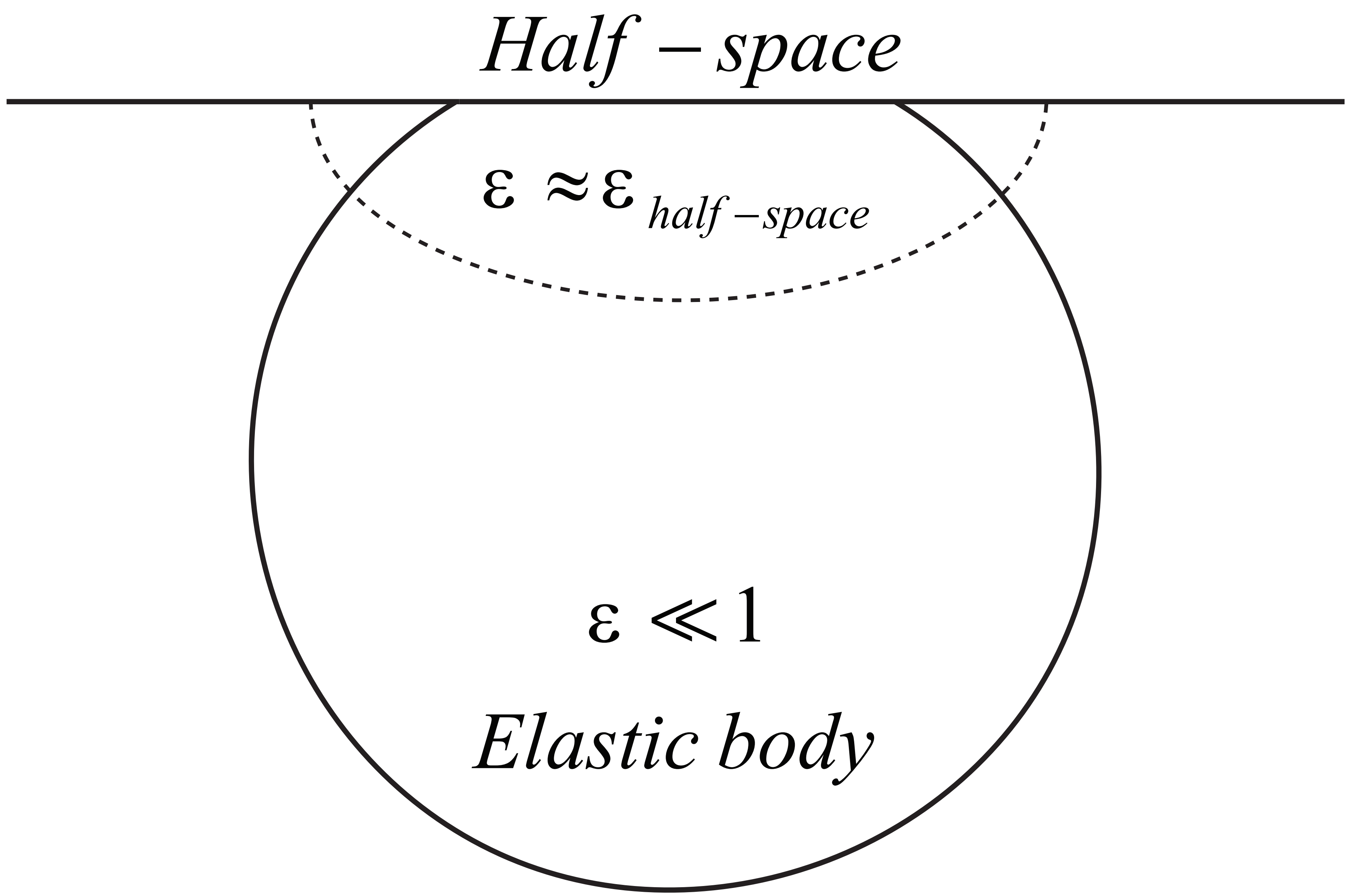
$$f = 1 + c_f \xi \text{ for } \xi < 0$$

Models

- Elastic half space
- Abaqus/Explicit model
  - ‘real’ tire



Elastic half space



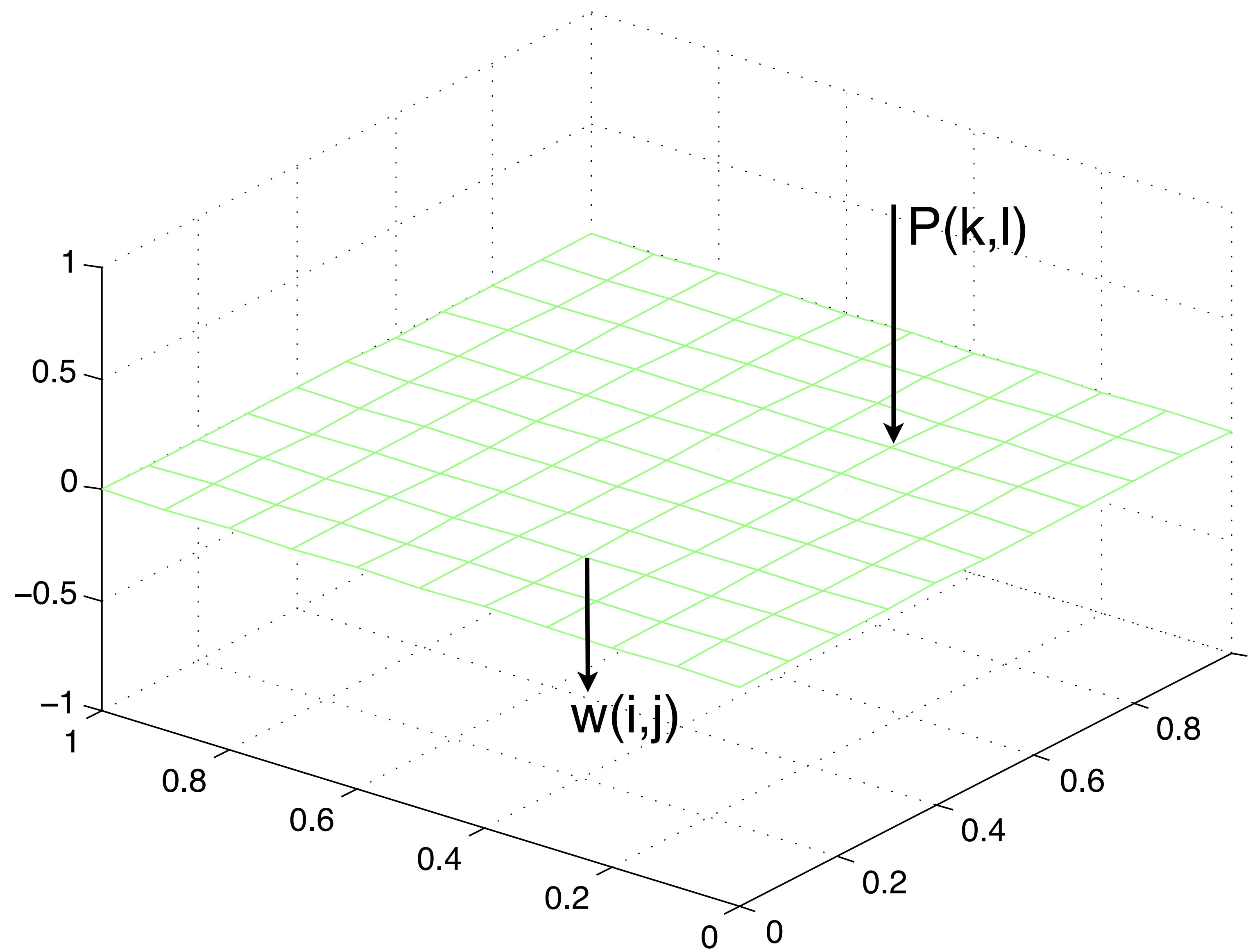
Elastic half space

- Influence matrix:

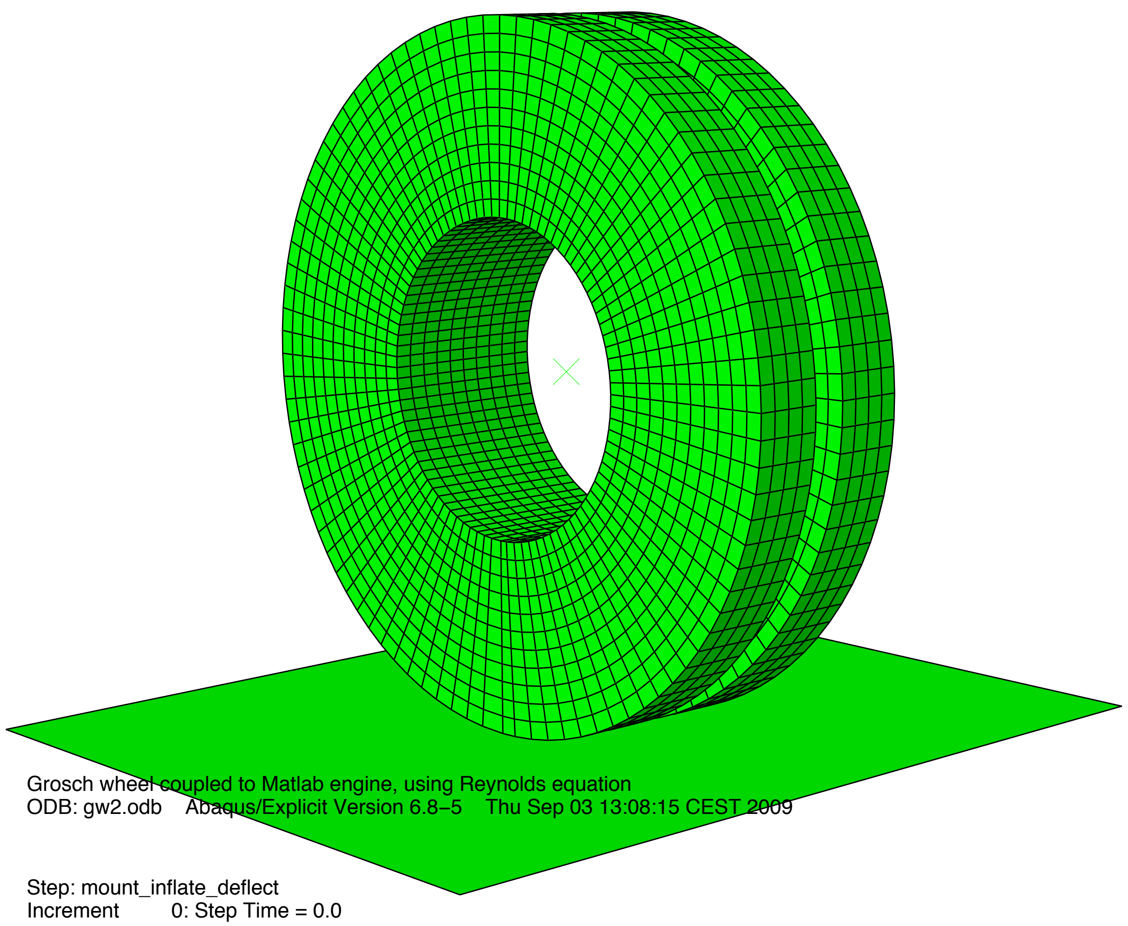
$$w\left(x_i, y_j\right) = w_{i,j} \approx \frac{2}{\pi E'} \sum_{k=1}^{n_x} \sum_{l=1}^{n_y} D_{ijkl} p_{kl}$$

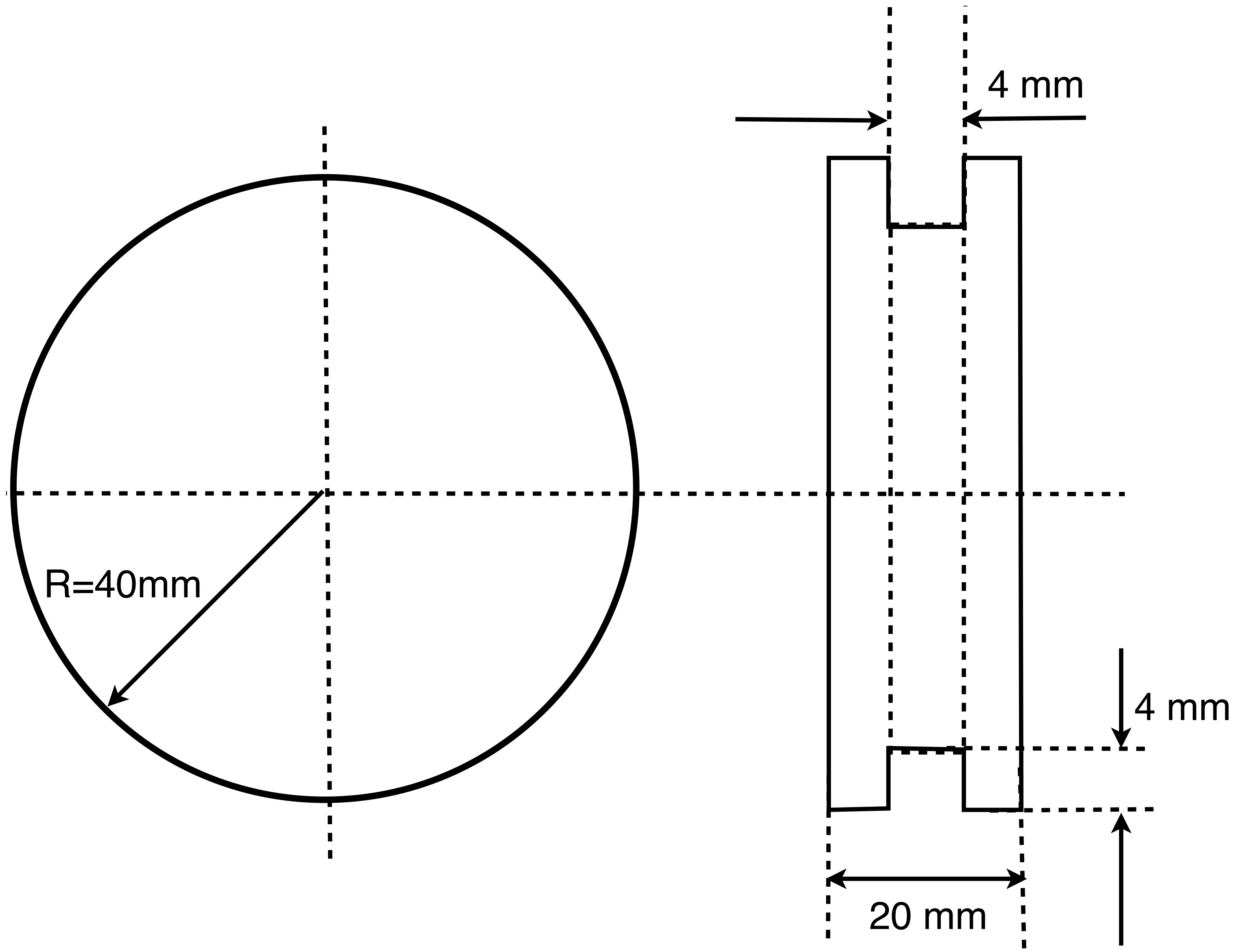
$$D_{ijkl} = \int\int \frac{1}{\sqrt{\left(x - x'\right)^2 + \left(y - y'\right)^2}} dx' dy'$$

Elastic half space

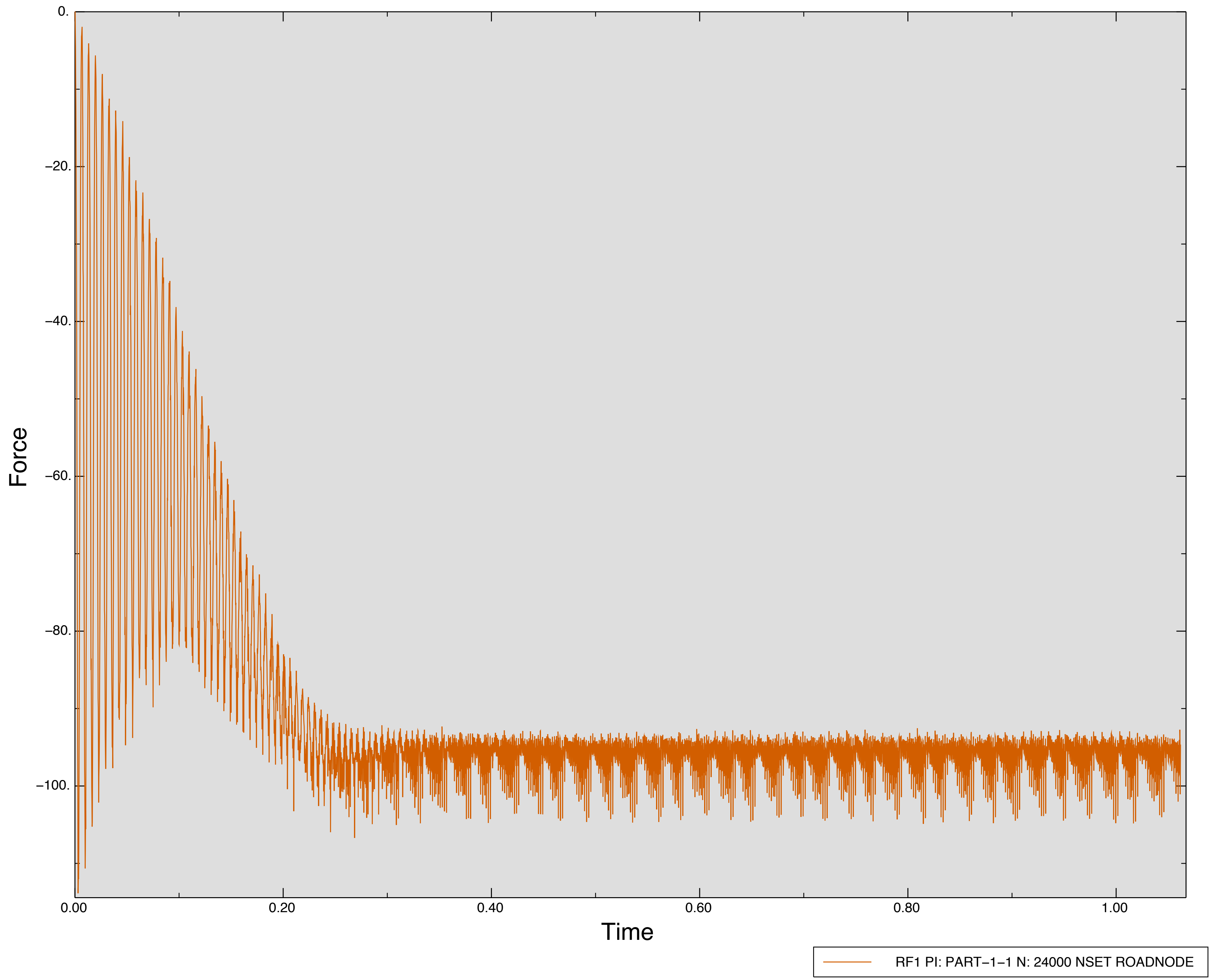


Grosch wheel





Problem: oscillations



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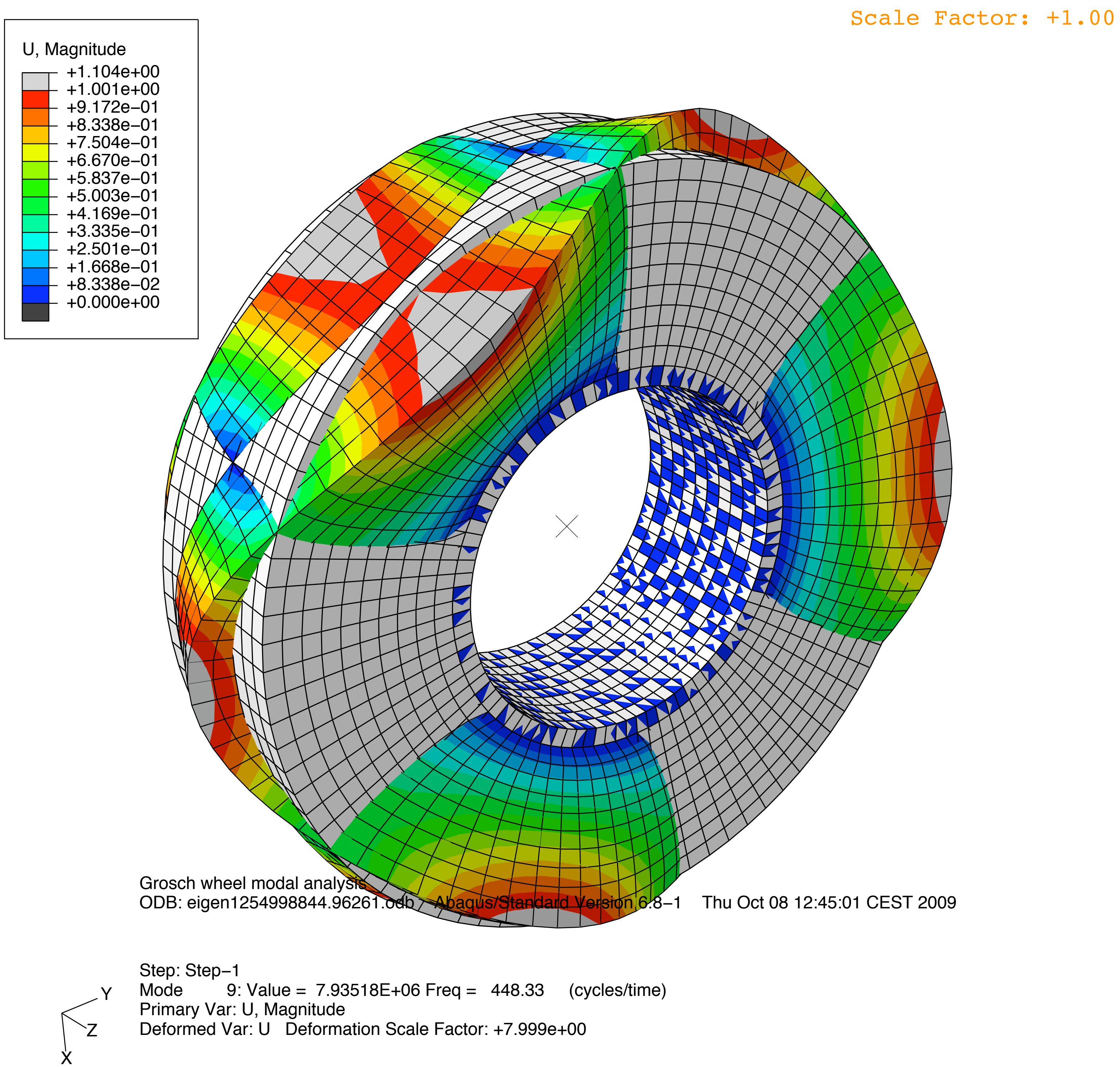
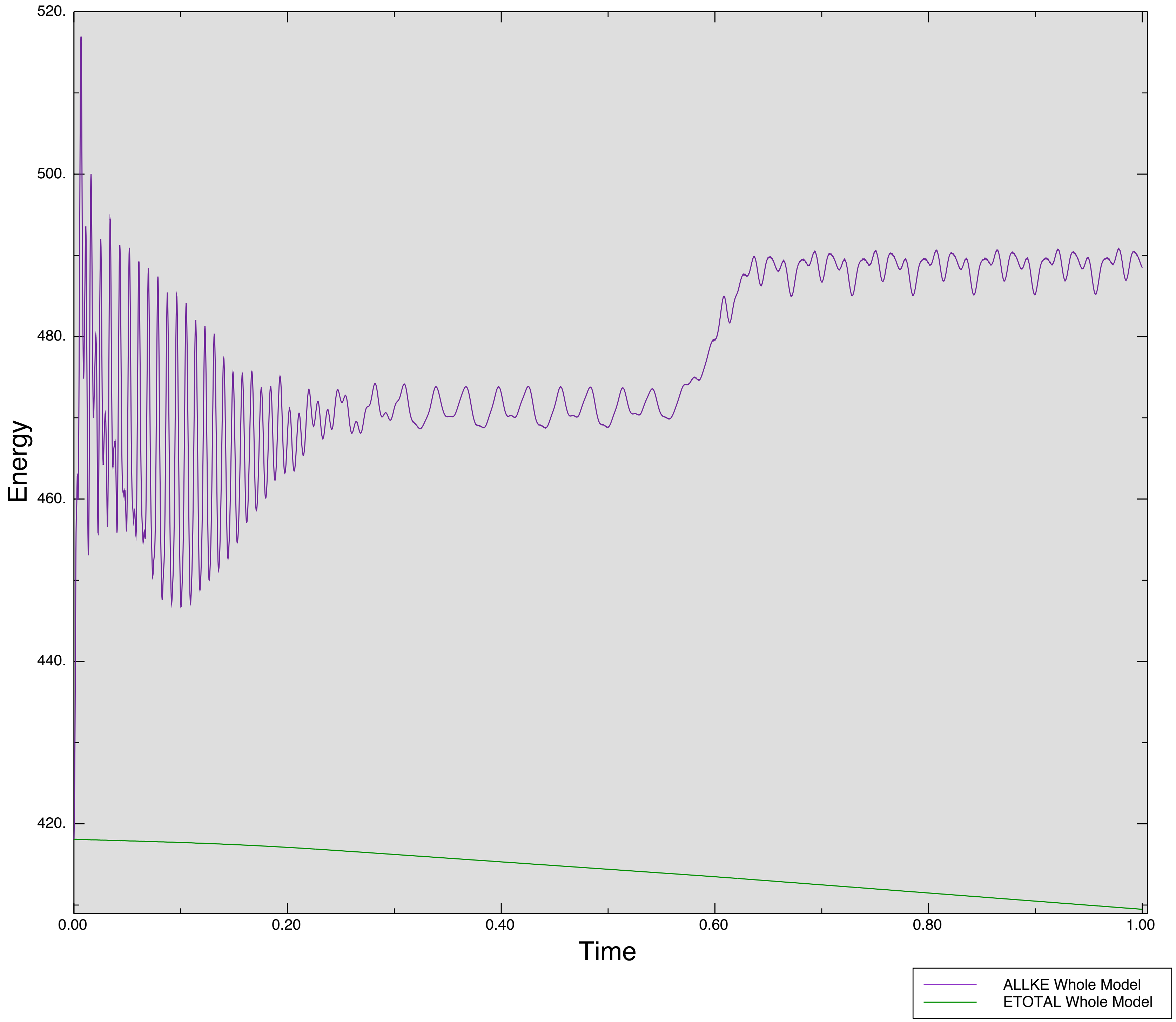
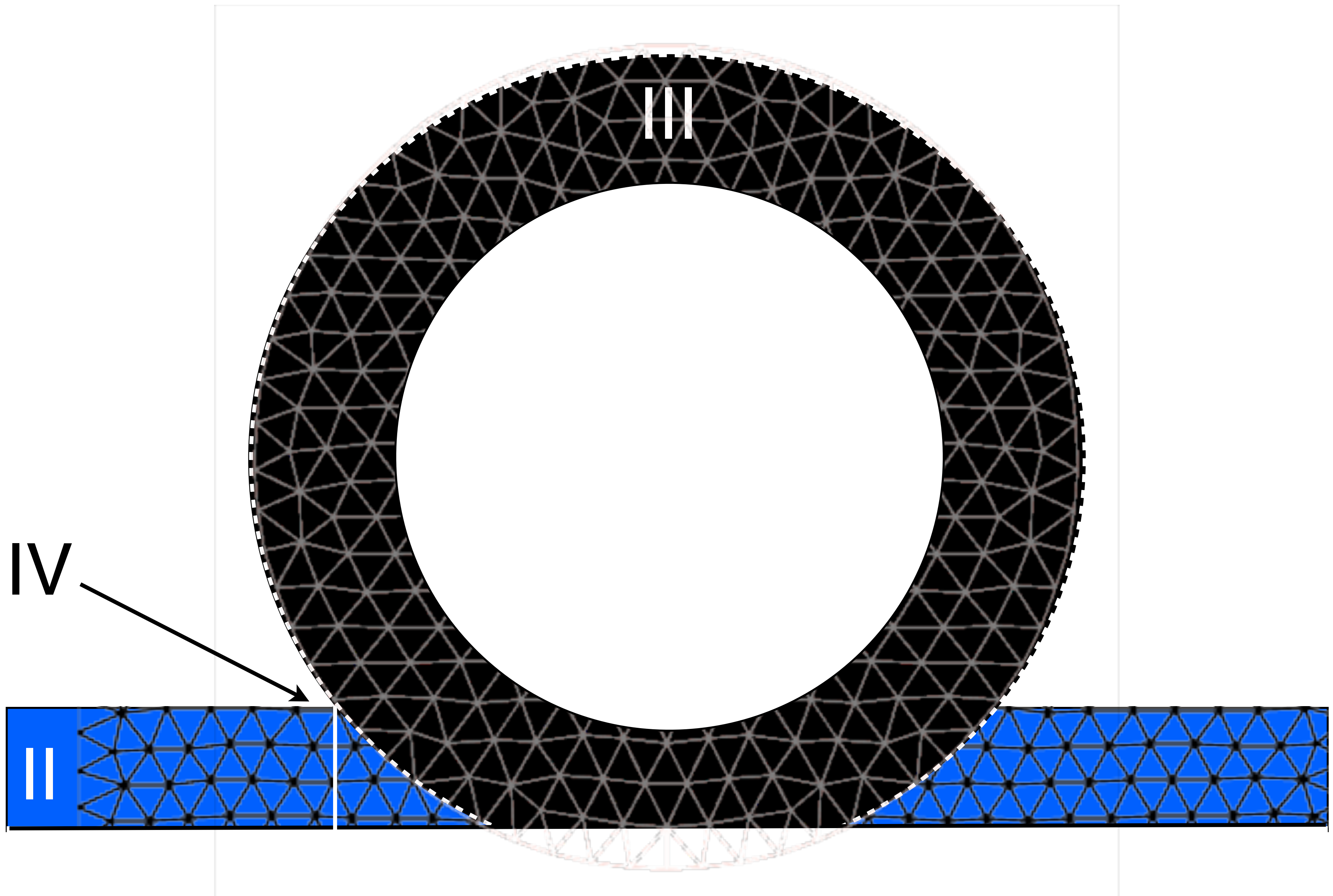
Eigenmodes?

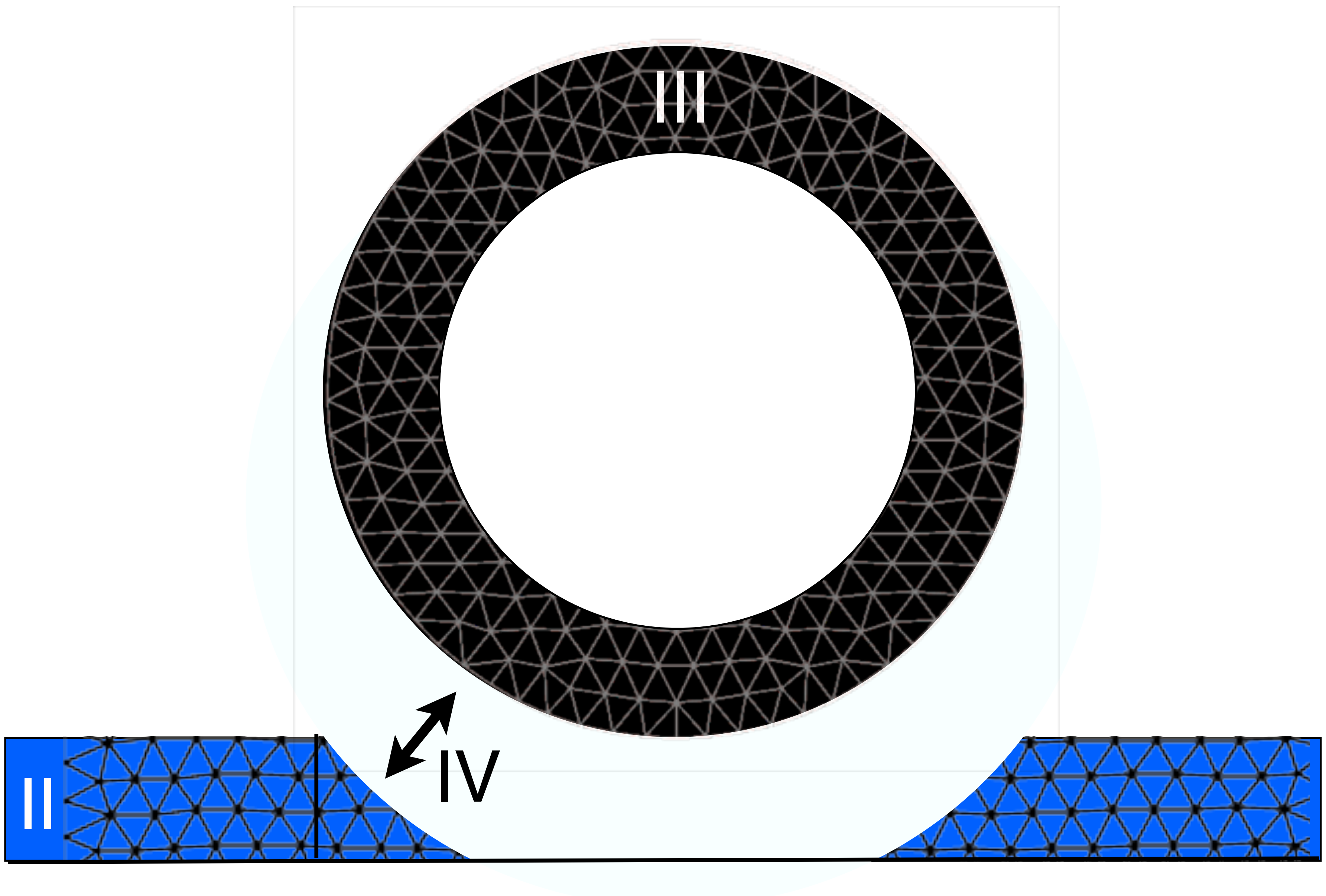


Scale Factor: +1.00
U, Magnitude
+1.104e+00
+1.001e+00
+9.172e−01
+8.338e−01
+7.504e−01
+6.670e−01
+5.837e−01
+5.003e−01
+4.169e−01
+3.335e−01
+2.501e−01
+1.668e−01
+8.338e−02
+0.000e+00
Grosch wheel modal analysis
ODB: eigen1254998844.96261.odb Abaqus/Standard Version 6.8−1 Thu Oct 08 12:45:01 CEST 2009
Step: Step−1
Mode 9: Value = 7.93518E+06 Freq = 448.33 (cycles/time)
Primary Var: U, Magnitude
Deformed Var: U Deformation Scale Factor: +7.999e+00
Y
Z
X

Problem: Energy




III
IV
II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II
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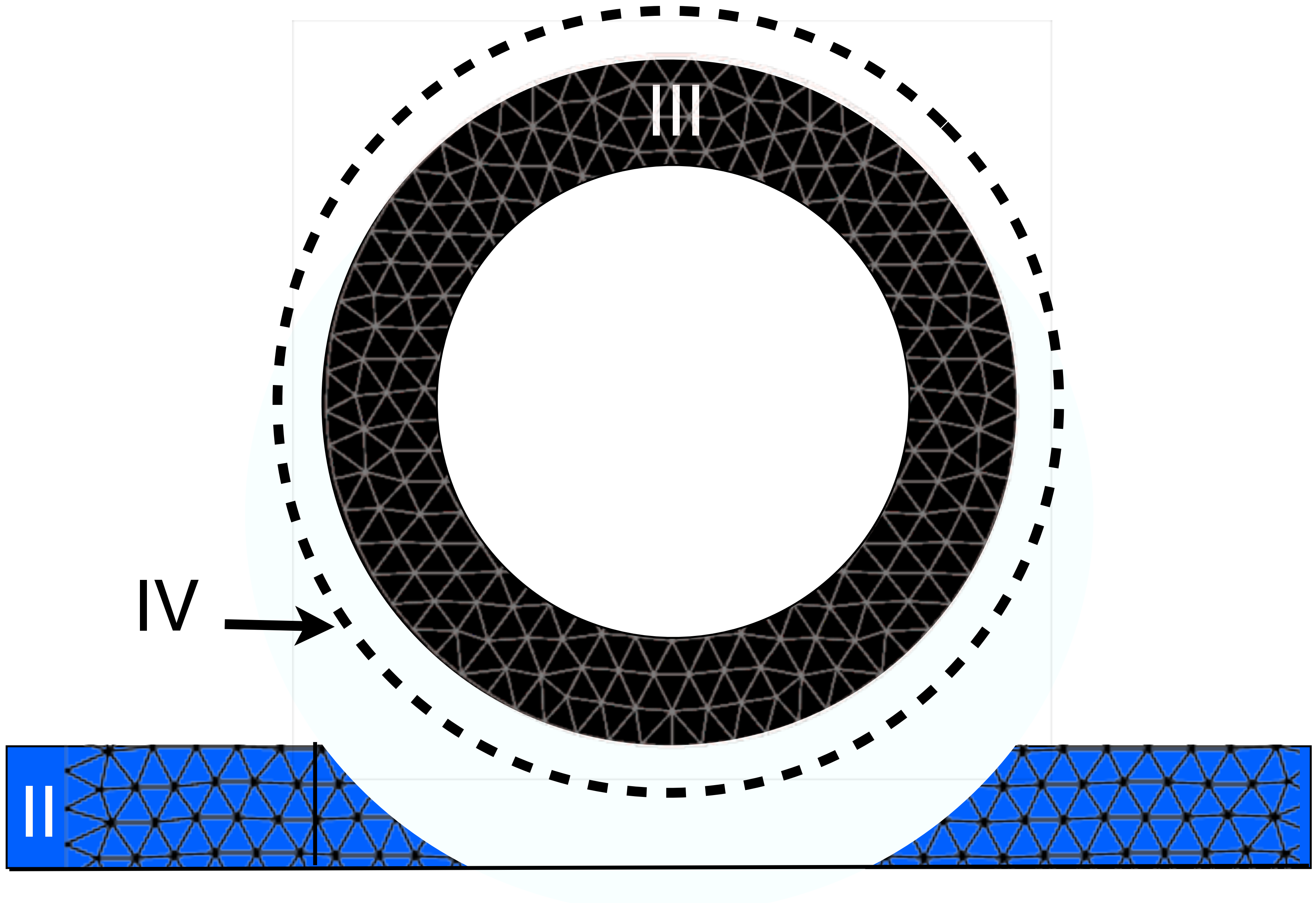
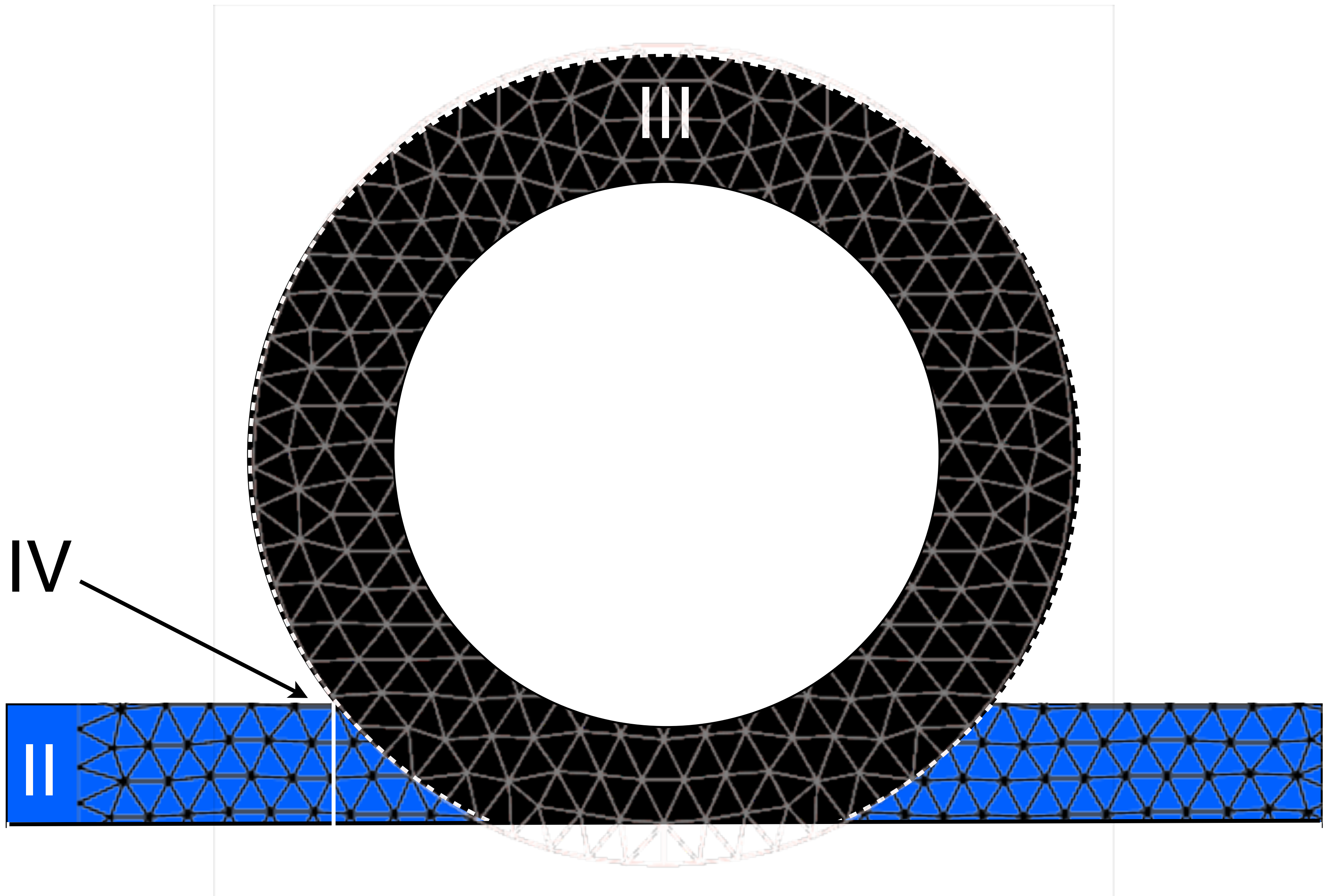
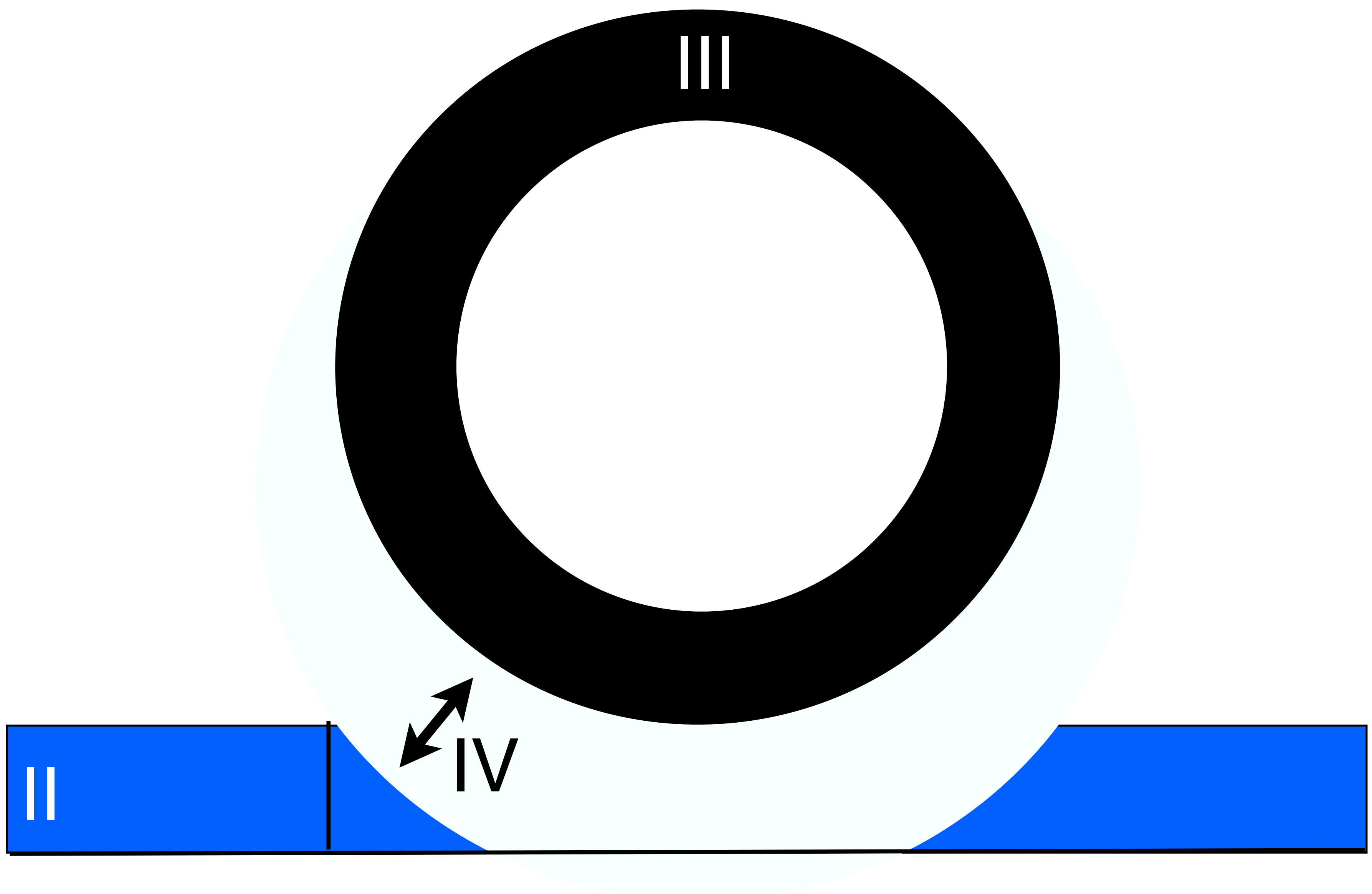
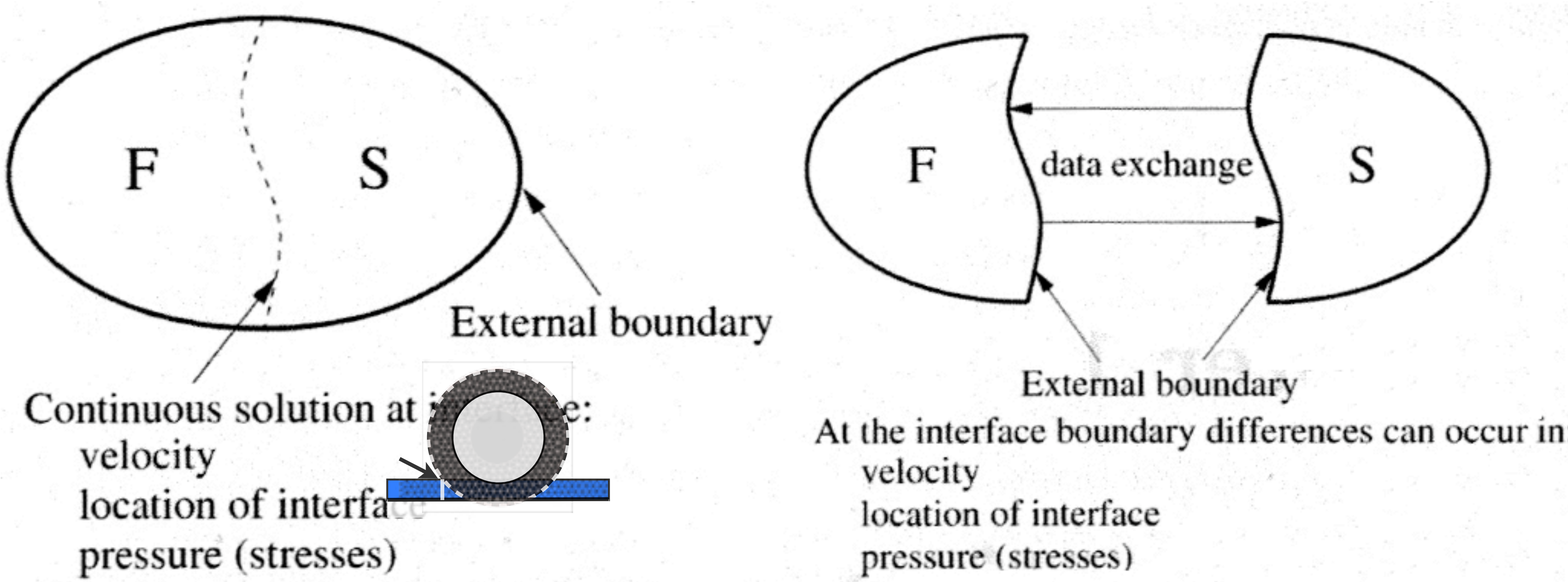

III
IV
II


III
IV
II


III
II
IV

Monolithic vs. Partitioned

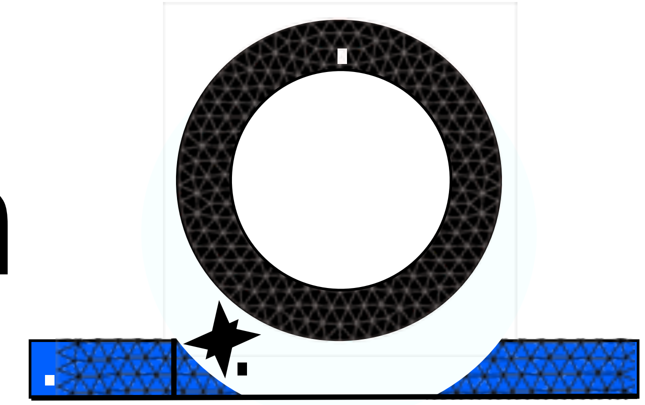


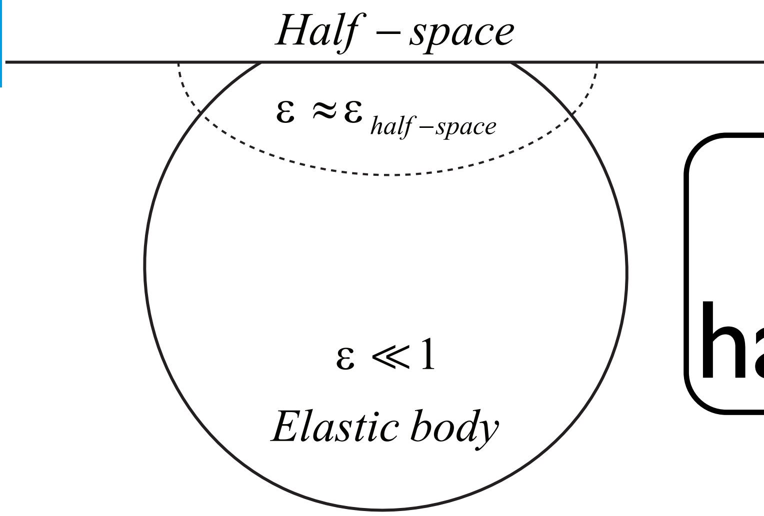
Monolithic vs. Partitioned

Monolithic vs. Partitioned

Thursday, January 21, 2010

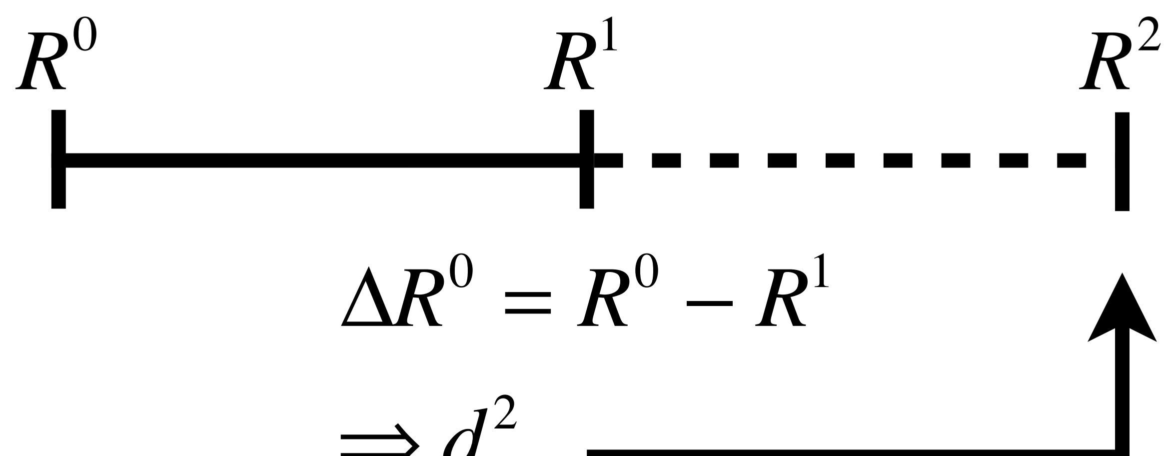
Interface Quasi Newton





Elastic half-space

$$R^k$$

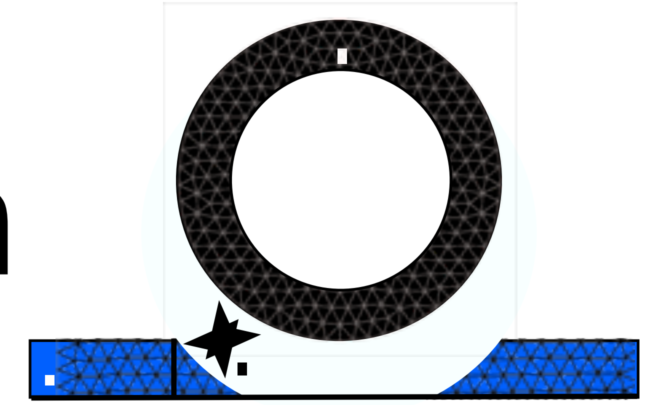


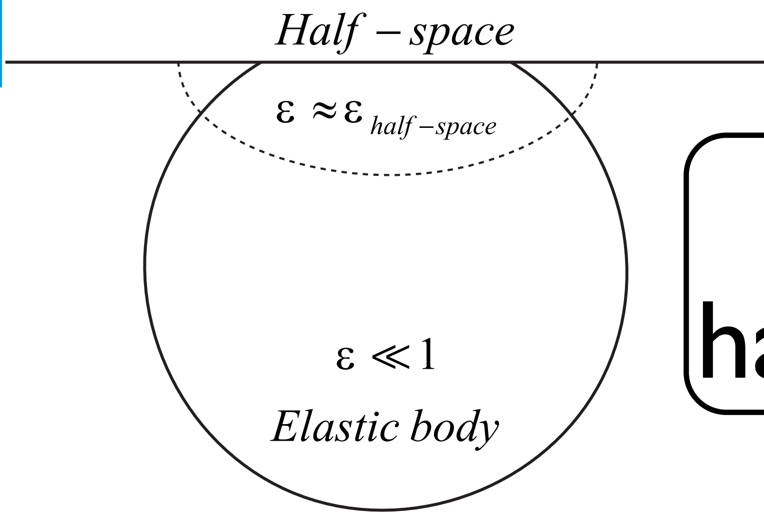
$$\Delta R^0 = R^0 - R^1$$

$$\Rightarrow d_s^2$$

Reynolds model

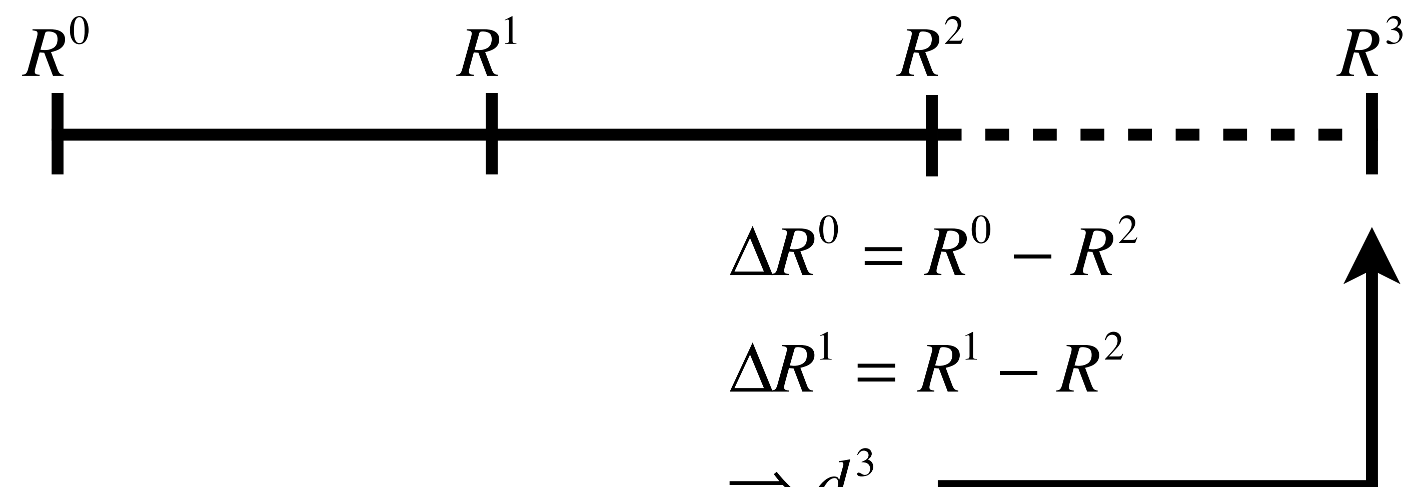
Interface Quasi Newton





Elastic half-space

$$R^k$$



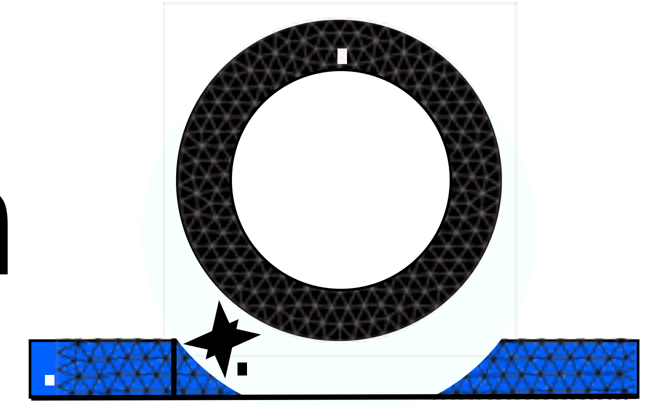
$$\Delta R^0 = R^0 - R^2$$

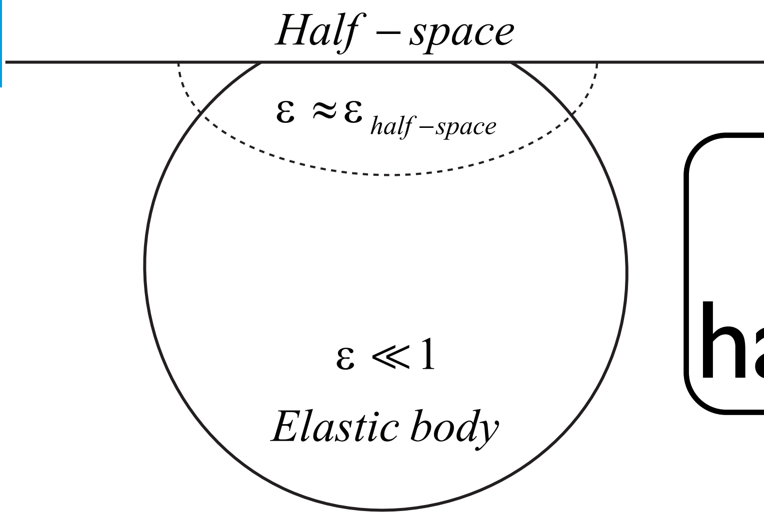
$$\Delta R^1 = R^1 - R^2$$

$$\Rightarrow d_s^3$$

Reynolds model

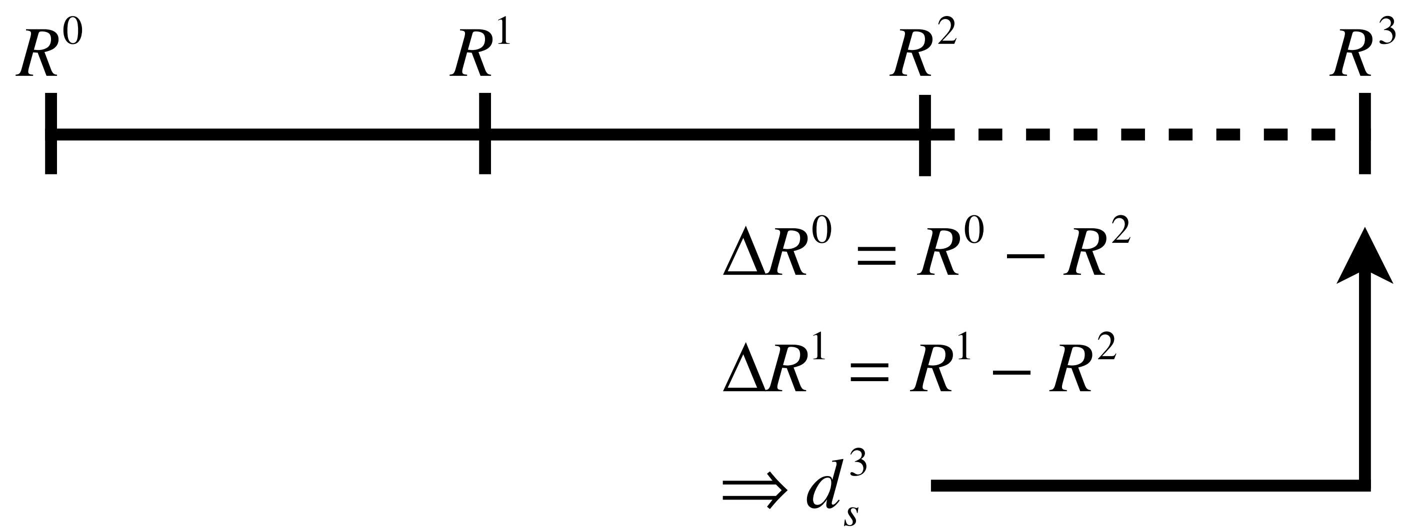
Interface Quasi Newton





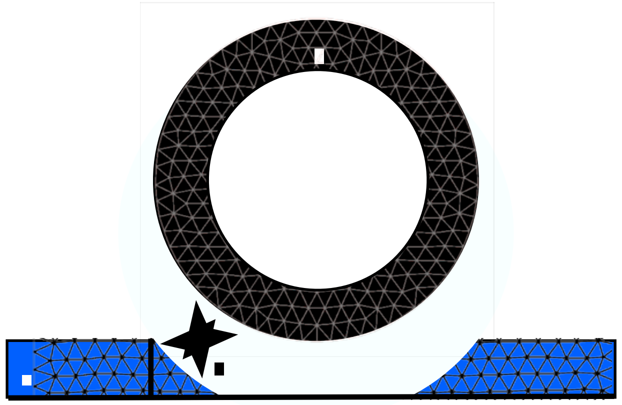
Elastic half-space

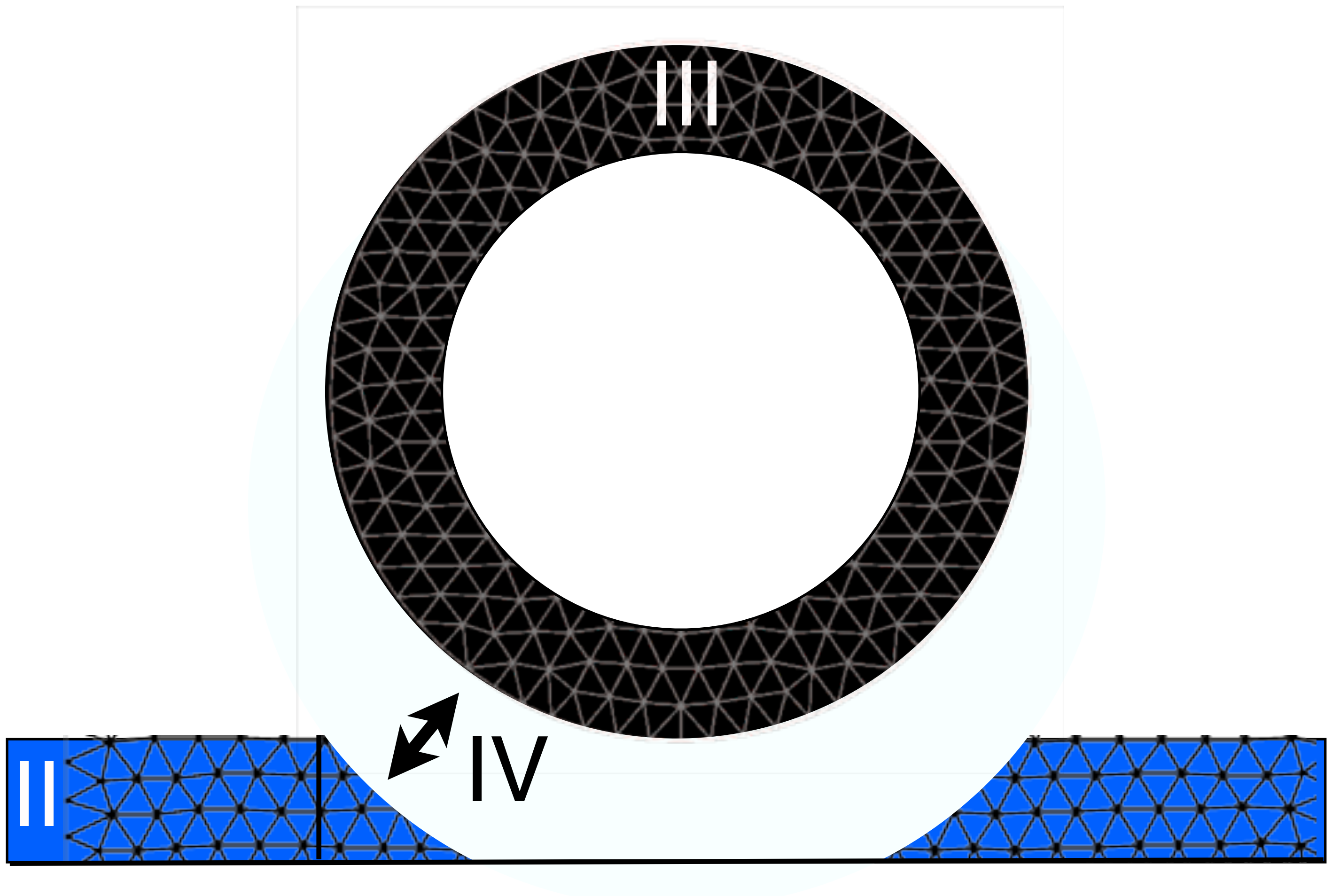
$$R^k$$



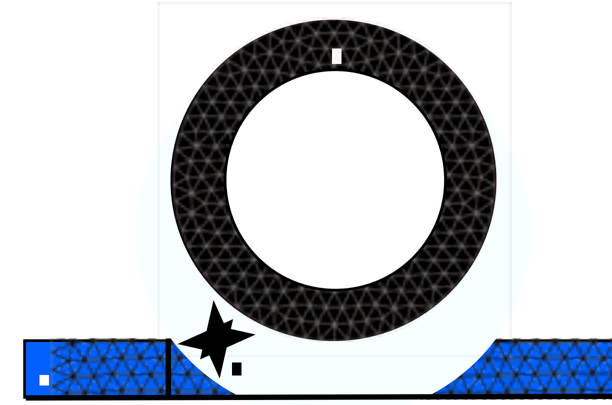
Reynolds model

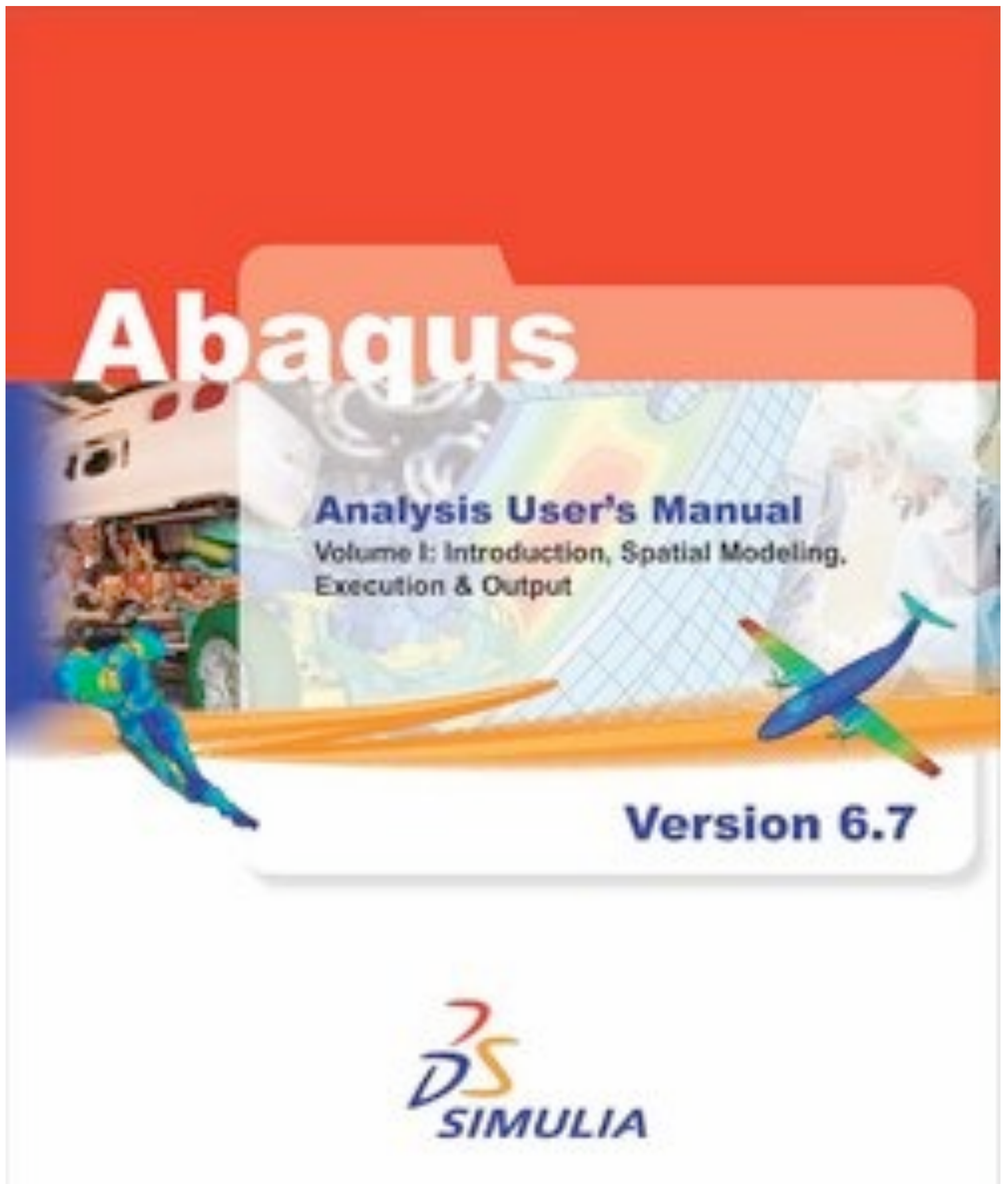
Abaqus - Matlab





Abaqus - Matlab





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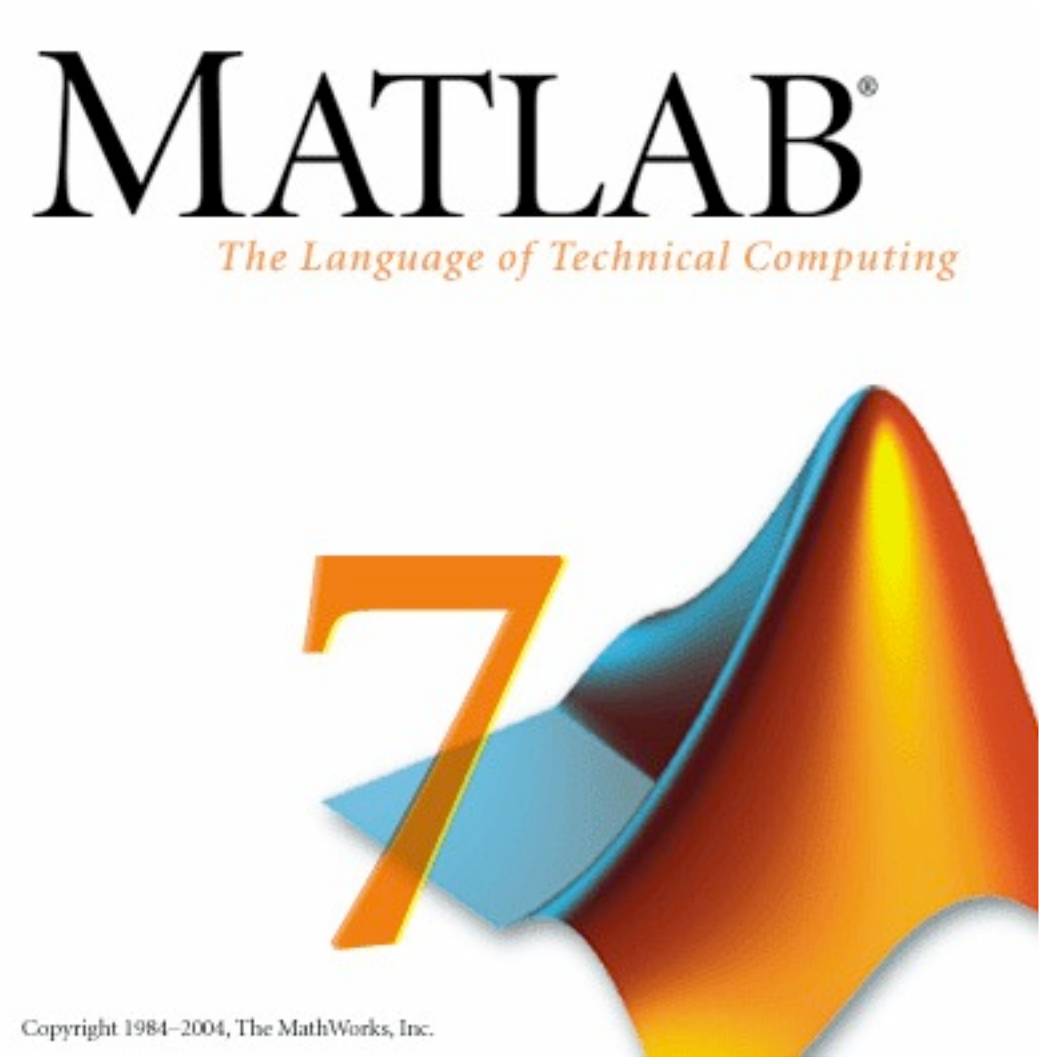
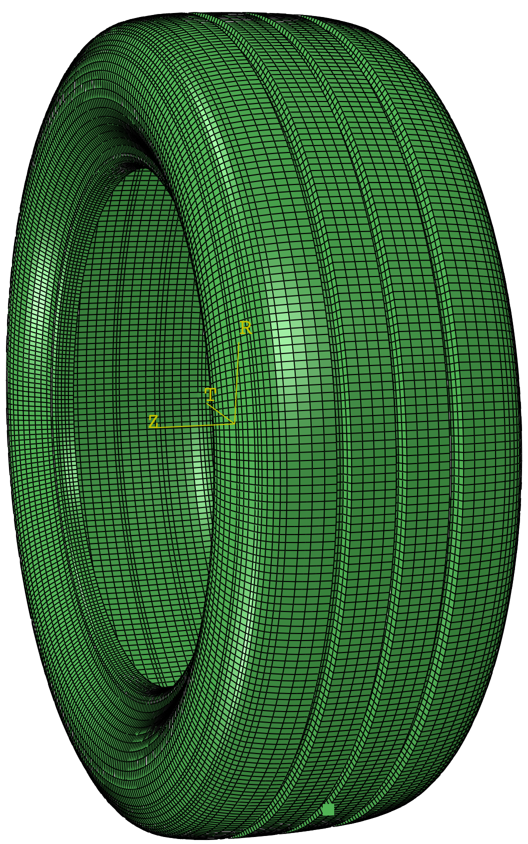
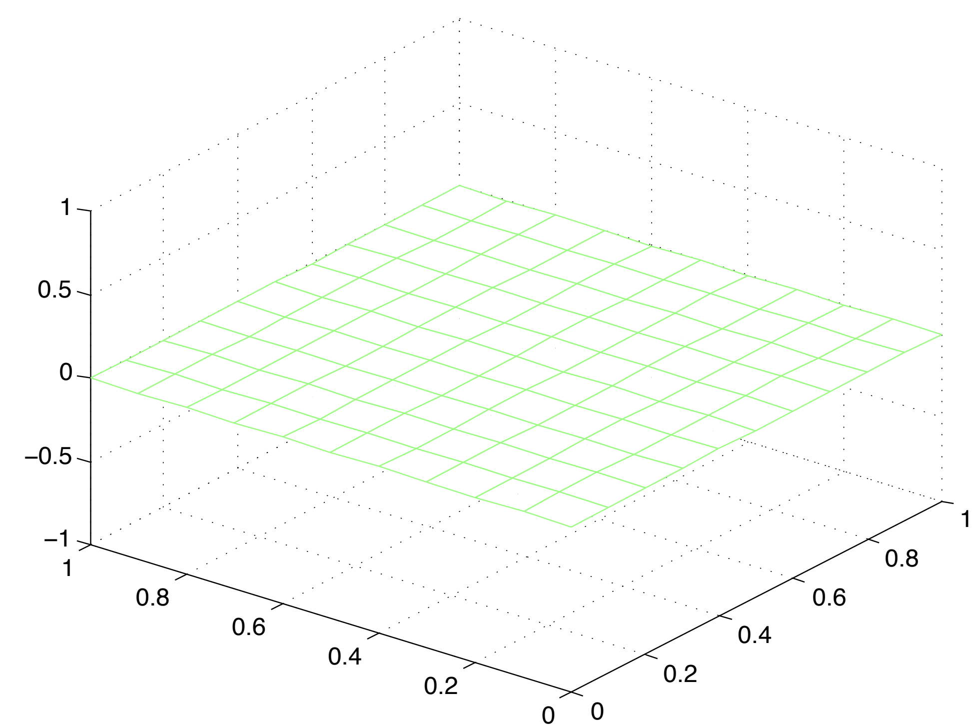


Abaqus - Matlab

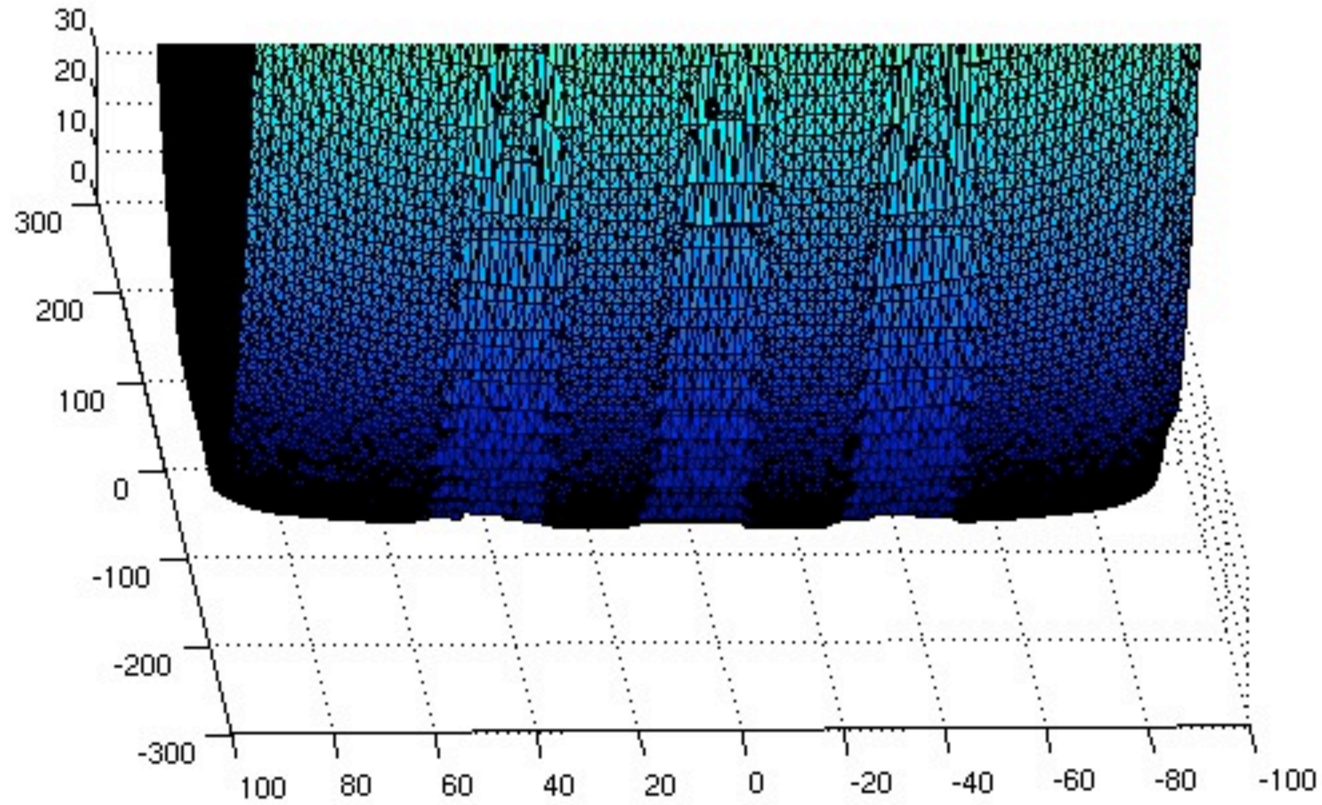
- Mesh mapping





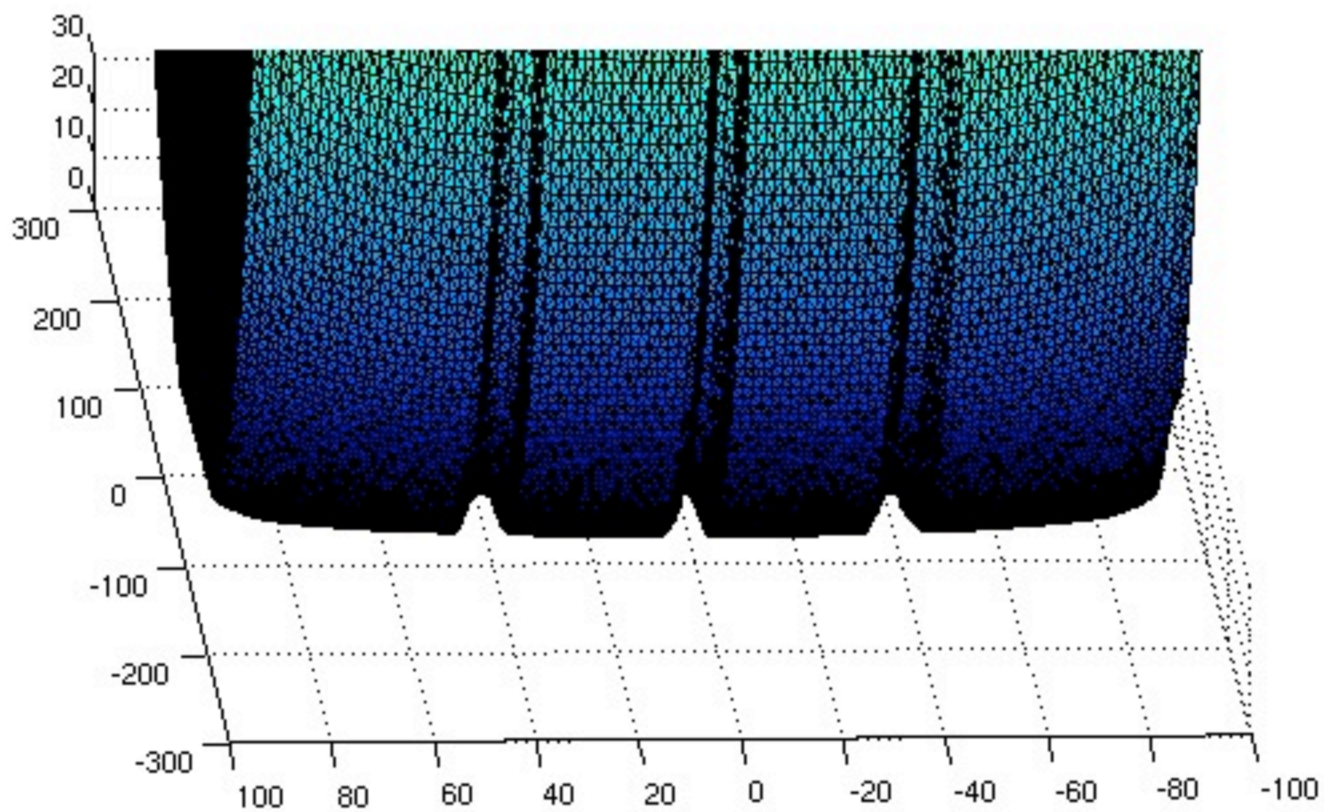
Abaqus - Matlab

- Mesh mapping



Abaqus - Matlab

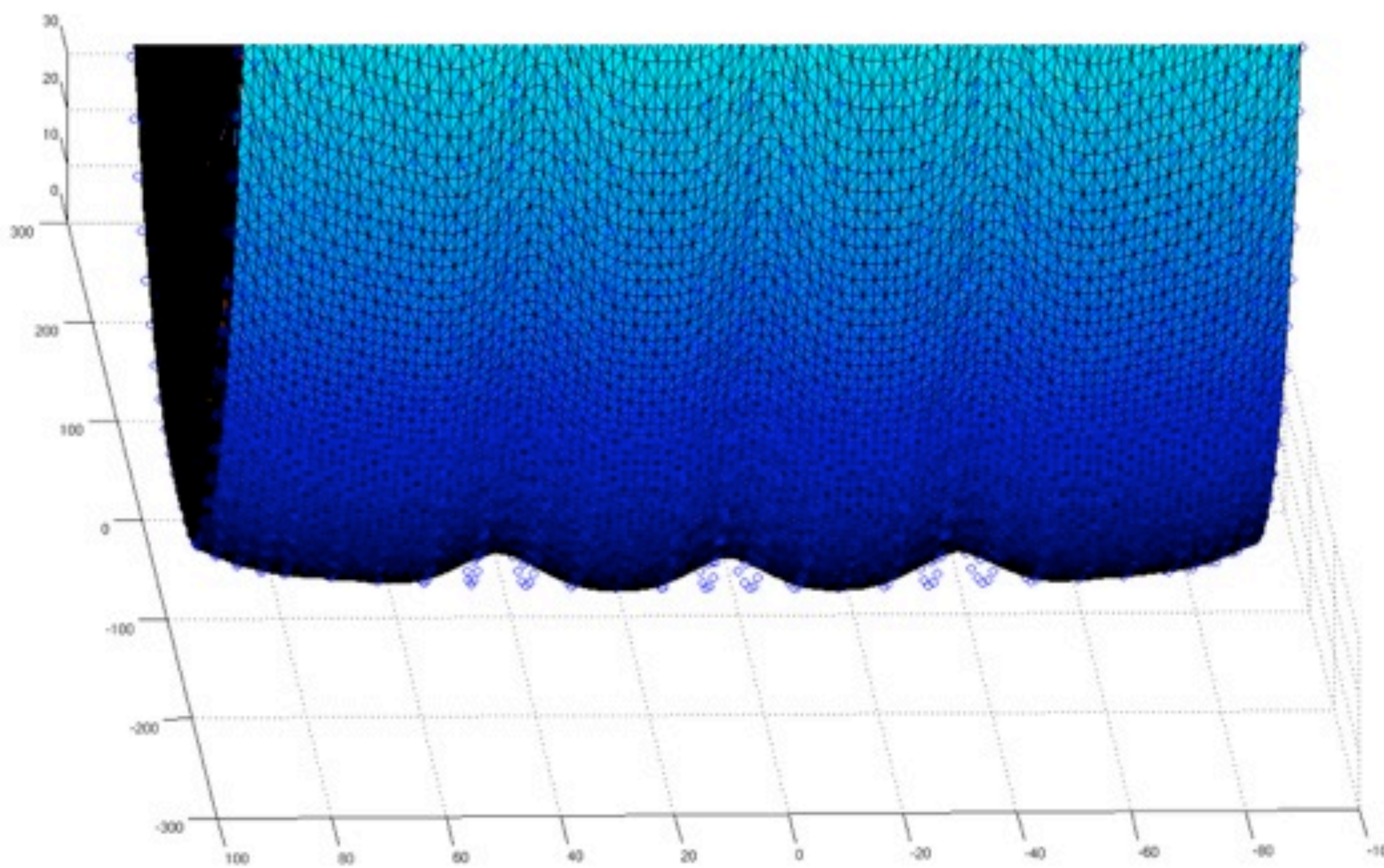
- Mesh mapping



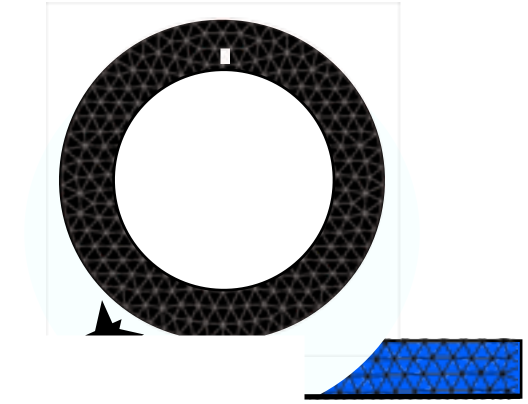
TU Delft

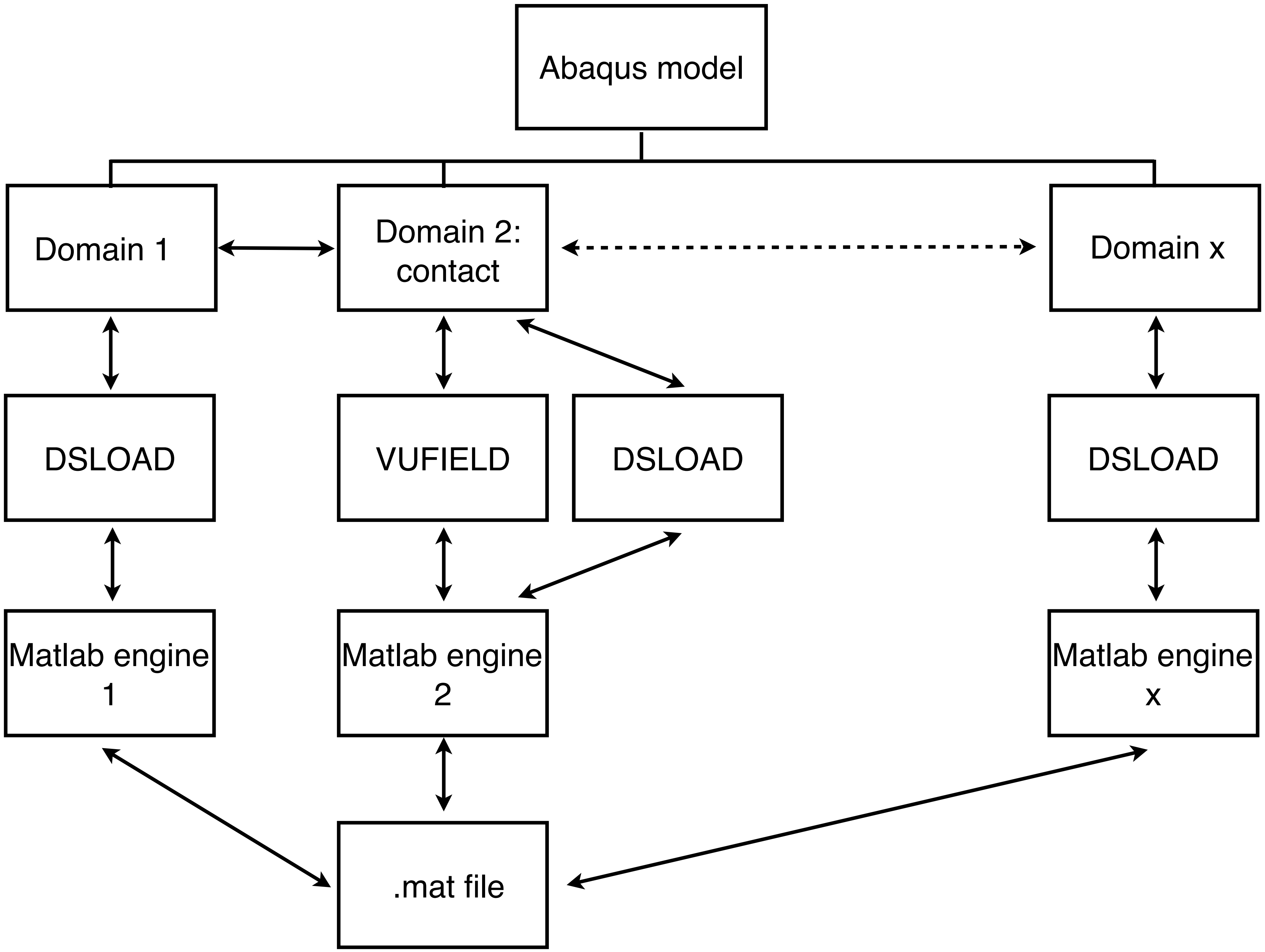
Abaqus - Matlab

- Mesh mapping



Abaqus - Matlab





Results

- Elastic half space:

Speed [km/h]	Reynolds	+ Bernoulli inlet	+ Energy correction	+ Momentum correction	Fill rate	CEL	FV
50	3,14	25,56	3,14	3,14	3,05	33	16,36
60	3,77	36,09	3,77	3,77	3,55	48	33,93
70	4,39	48,45	4,39	4,39	4,12	64	46,66

- Grosch wheel:

Speed [km/h]	Normal load [N]	Water layer [mm]	Reynolds + Bernoulli inlet
50	214	5	30,80
15	100	3	3,50

- ‘real’ tire:

Speed [km/h]	Normal load [N]	Water layer [mm]	Reynolds + Bernoulli inlet	FV
90	3924	3	2200	2000

Results: Grosch wheel

Results: Grosch wheel

Speed [km/h]
50

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Results: Grosch wheel

Speed [km/h]	Reynolds + Bernoulli
50	188%

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Recommendations

- Reynolds equation
  - Inlet condition
  - Fill rate
- Tire model
  - Contact algorithm
- Fluid structure interaction
  - Interface quasi Newton