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Value of Multiple Production Measurements and Water Front Tracking in Closed-Loop Reservoir Management

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Abstract

This paper extends previous work on value of information (VOI) assessment in closed-loop reservoir management (CLRM) to estimate the added value of performing multiple measurements along the producing life of the reservoir. The new procedure is based on the workflow from our previous paper which allows to quantify the VOI of a single observation under geological uncertainty. Here we show that, by modifying that workflow slightly, it is possible to assess the value of a series of measurements without a prohibitive increase in computational costs. The approach is illustrated with two cases based on a simple water flooding problem in a two-dimensional five-spot reservoir: the first one, in which we assess the value of a series of production measurements, and the second one, in which we estimate the additional value of water front positions tracked by an interpreted time-lapse seismic survey. We believe that our proposed workflow is a complete methodology to estimate the VOI in a CLRM context because we take into account that the production strategy is updated periodically after new information has been assimilated in the models. However, future work will be required to reduce the computational load to allow for the application of the workflow to real field cases.

Introduction

Over the past decades, new sensing technologies have been boosting the flow of detailed reservoir information from oil and gas fields. Different types of well-based sensors and field-wide data have become available, which, in combination with the development of numerical techniques for reservoir model-based optimization and history matching, increase the possibilities to monitor and improve reservoir management operations. Because many of these sensing technologies come at significant costs, the design of reservoir surveillance strategies has been receiving more attention. The solution of this design problem requires the ability of assessing the value of future measurements during the field development planning (FDP) phase of an oil field. In this context, techniques to quantify the value of information (VOI) under geological uncertainty become increasingly important.

An additional complexity arises when it is attempted to quantify the VOI associated with the deployment of a sensor that is expected to provide multiple measurements in its lifespan. Recently, we

have proposed a methodology to make use of the closed-loop reservoir management (CLRM) framework (i.e., under the assumption that frequent life-cycle optimization will be performed using frequently updated reservoir models) to assess the VOI of a single measurement (Barros et al., 2015). The first contribution of the present paper is to show how that methodology can be extended to also assess the value of a series of measurements at the same location.

Another challenge consists of quantifying the value of field-wide sensing methods, such as time-lapse seismic surveys. In Barros et al. (2015) we illustrated the VOI assessment methodology with an example considering only production data from the wells. Here we propose to repeat the workflow in combination with the work of Leeuwenburgh and Arts (2014) so that we are also able to analyze the value of water front tracking measurements, which can be an alternative to assess the VOI of a time-lapse seismic survey.

In the Background section we introduce the most relevant concepts that will be explored throughout the paper and recap what has been done in terms of VOI assessment of a single measurement. Next, in the Methodology section, we explain how to extend our workflow for a series of measurements and thereafter, in the Examples section, we illustrate it with some case studies and we analyze the results. Finally, in the Conclusions section, we comment on the main issues regarding the application of our methodology to real-field cases, and we suggest directions for future work.

Background

Closed-loop reservoir management (CLRM)

CLRM is a combination of frequent life-cycle production optimization and data assimilation (also known as computer-assisted history matching); see Fig. 1. Life-cycle optimization aims at maximizing a financial measure, typically net present value (NPV), over the producing life of the reservoir by optimizing the production strategy. This may involve well location optimization, or, in a more restricted setting, optimization of well rates and pressures for a given configuration of wells, on the basis of one or more numerical reservoir models. Data assimilation involves modifying the parameters of one or more reservoir models, with the aim to improve their predictive capacity, using measured data from a potentially wide variety of sources such as production data or time-lapse seismic. For further information on CLRM see, e.g., Jansen et al. (2005, 2008, 2009), Naevdal et al. (2006), Sarma et al. (2008); Chen et al. (2009) and Wang et al. (2009).

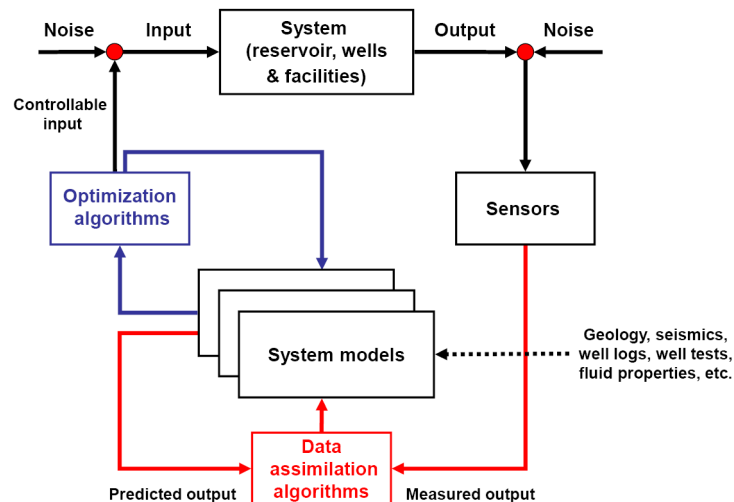


Figure 1—Closed-loop reservoir management as a combination of life-cycle optimization and data assimilation.

VOI assessment of a single measurement

Recently, we have proposed a new methodology to assess the VOI of future measurements by making use of the CLRM framework (Barros et al., 2015). Our approach presented there consists of ‘closing the loop’ in the design phase to simulate how information obtained during the producing life-time of the reservoir comes into play in the context of optimal reservoir management. By considering both data assimilation and optimization in their procedure, we are able “to not only quantify how information changes knowledge, but also how it influences the results of decision making”. This is possible because a new production strategy is obtained every time the models are updated with new information, and the strategies with and without additional information can be compared in terms of the value of the optimization objective function (typically NPV) obtained when applying these strategies to the virtual asset (synthetic truth); see Fig. 2. Despite being quite generic, that workflow does not address the case where a series of measurements (at multiple observation times) is available.

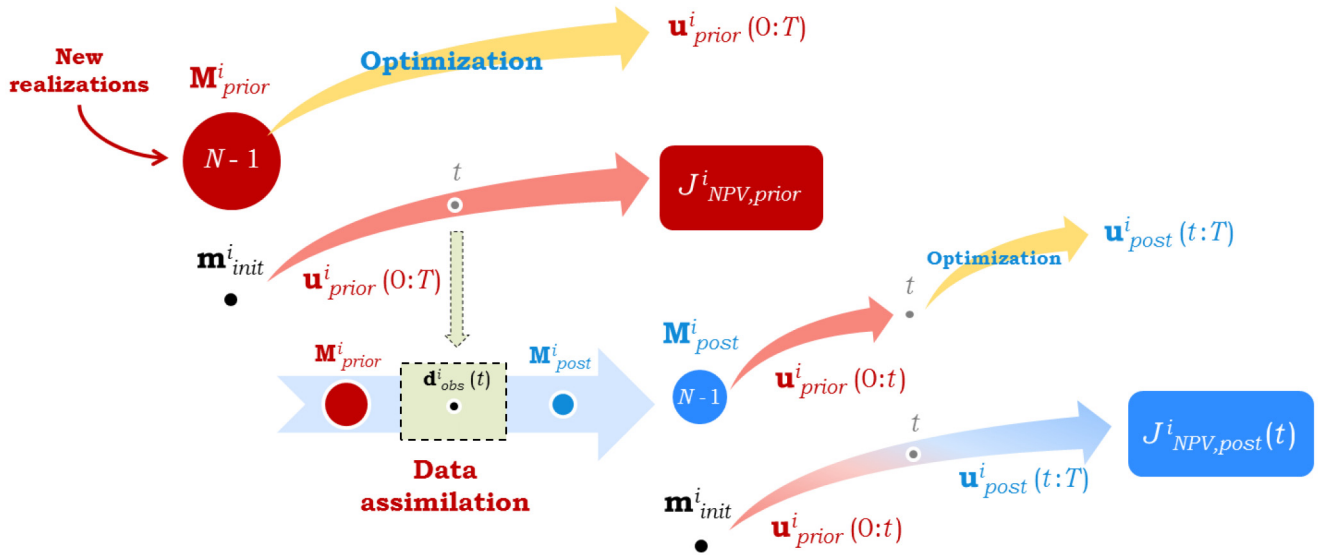


Figure 2—VOI assessment workflow proposed by Barros et al. (2015). (t indicates the observation time and T indicates the end time)

Water front tracking measurements

Water front tracking ‘measurements’ represent a good alternative to incorporate information from interpreted 4D (or time-lapse) seismics. Time-lapse seismic surveys fall under the category of field-wide sensing methods and have been showing great potential for reservoir surveillance. With a much higher volume of data, they provide a different type of information than the well-based measurements. A typical 4D seismic dataset gives good insight into the evolution of pressures and saturations in the whole extension of the reservoir. It may be a challenge to assimilate all these data to the reservoir models using classical history matching techniques such as the ensemble Kalman filter (EnKF). Recognizing that one of the main advantages of having 4D seismic data available lies on the possibility of interpreting saturation maps to track the water front in water flooding problems, Leeuwenburgh and Arts (2014) propose a way of parametrizing these data to facilitate the history matching. They use the fact that the water front can be detected in the seismic data, as a jump in the saturation values. Then they apply “a fast marching method (. . .) to calculate distances between observed and simulated fronts, which are used as innovations in the EnKF”. Fig. 3 depicts the general idea behind their work. This new parametrization of the data allows Leeuwenburgh and Arts (2014) to simplify the assimilation of the time-lapse seismic dataset, without having to use any localization or inflation schemes to ensure the good behavior of the history

matching ensemble techniques. For a detailed description of the fast marching method used to track the evolution of interfaces (water fronts in our setting), see [Sethian \(1996\)](#).

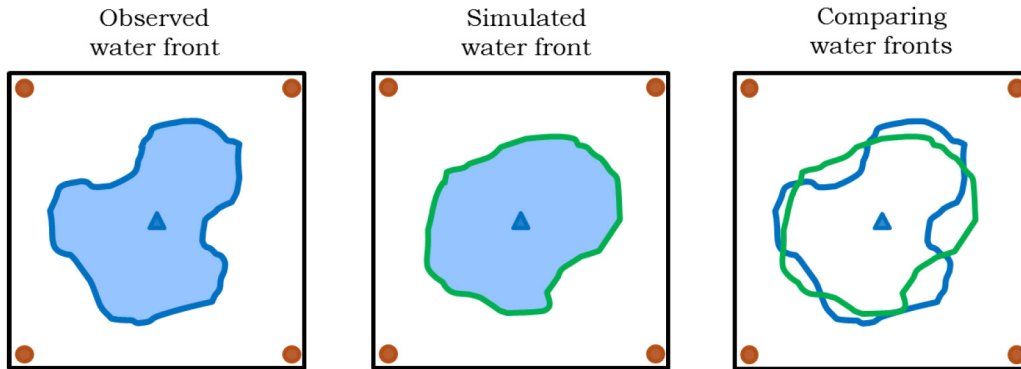


Figure 3—General idea behind the work of [Leeuwenburgh and Arts \(2014\)](#): detecting water fronts and comparing them in terms of a certain distance/metric to perform history matching.

Methodology

The new procedure is based on the workflow from our previous paper ([Barros et al., 2015](#)) which allows to quantify the VOI of a single observation under geological uncertainty. Throughout the description of the method and the illustrative examples presented there, we proposed to ‘close the loop’ once, considering new information to become available at one observation time. Then, in the examples, we repeated the procedure for several observation times to show how the VOI changes in time and to determine which moment in time would be best to collect data (if it were to be collected only once). We also showed there the importance of repeating the workflow for different plausible realizations of the truth, which made our procedure very expensive in terms of computational costs.

In order to assess the VOI of a series of future measurements, we propose here to ‘close the loop’ multiple times, repeating the original procedure from [Barros et al. \(2015\)](#) while gradually progressing over the producing life of the reservoir in time steps equal to the specified control time intervals. Every time new data become available, the models are history matched and the production strategy for the remaining control intervals are updated. [Fig. 4](#) depicts the new workflow for VOI assessment of a series of measurements, showing a simple case with 3 control time intervals only. Note that the repetition of the procedure for an ensemble of plausible truths also holds here.

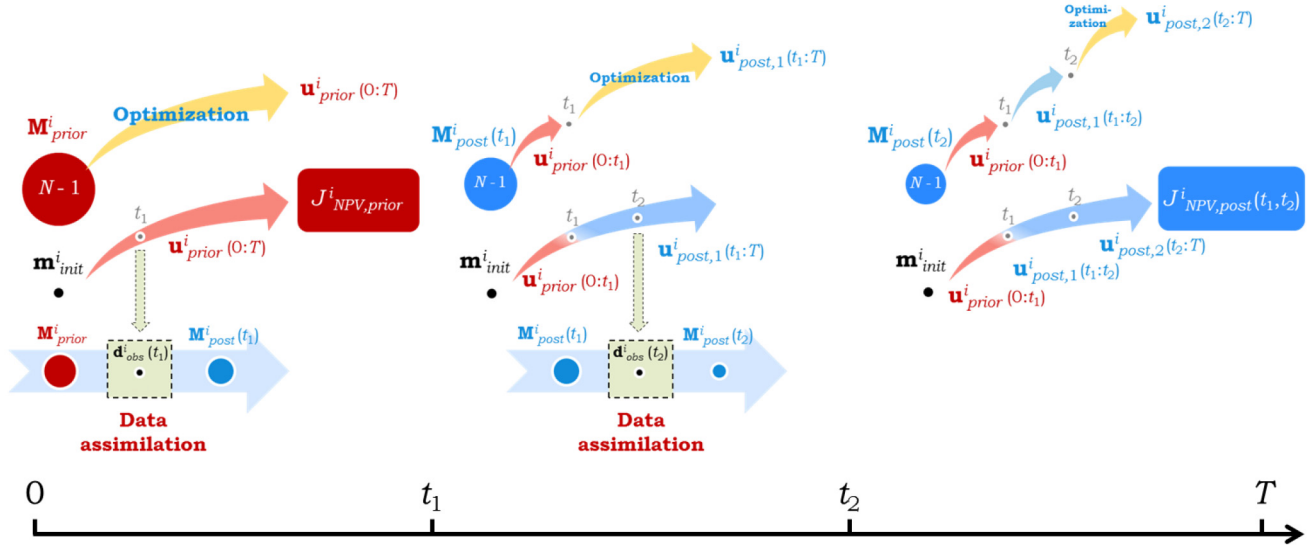


Figure 4—VOI assessment workflow proposed. (t_i indicates the observation times and T indicates the end time)

Initially, one might think that this new workflow would be much more computationally demanding than the previous one (for a single observation), but, in practice, it is not. Indeed, the assessment of the value of a series of future measurements implies repeating all the steps of the methodology through more observation times. However, we consider only one series of measurements. Thus, the repetition of history matching and optimization procedures while progressing over the producing life of the reservoir requires just as many reservoir simulations as repeating the workflow for a single observation for the different observation times, like we did in Barros et al. (2015). Regarding the repetition of the procedure for different realizations of the truth, the computational cost does not increase either: once a realization m^i_{init} is selected to be the synthetic truth, it will play the role of truth throughout the whole producing life of the reservoir in our setting. There is no need to consider new plausible truths as we progress over the control time intervals; therefore, the complexity of the procedure does not grow.

Referring again to our previous paper (Barros et al., 2015), we also showed there that we can adapt the VOI workflow to compute the value of clairvoyance (VOC), which means that at some time of the reservoir life-cycle the true reservoir is suddenly revealed so we can perform optimization with perfect knowledge of the truth. Because clairvoyance implies perfect revelation of the truth, its value represents a ‘technical limit’ to the VOI. The same holds when dealing with a series of measurements, with a small difference: in order to make a fair comparison with the VOI, the VOC workflow has to assume that we obtain imperfect information while clairvoyance is not yet available. By considering this, the VOC workflow for a series of measurements requires data assimilation and robust optimization, which makes it somewhat more complex than the original VOC workflow from our previous paper.

Examples

Multiple production data (2D five-spot model)

As a first step to illustrate the proposed VOI workflow, we applied it to a simple reservoir simulation model representing a two-dimensional (2D) inverted five-spot water flooding configuration, the same example as in our previous paper from which we have taken the following description (Barros et al., 2015). In a 21×21 grid (700×700 m), with heterogeneous permeability and porosity fields, the model simulates the displacement of oil to the producers in the corners by water injected in the center; see Fig. 5. Table 1 lists the values of the physical parameters of the reservoir model. We used multiple ensembles of $N = 50$ realizations of the porosity and permeability fields, conditioned to hard data in the wells, to

model the geological uncertainties. The simulations were used to determine the set of well controls (bottom hole pressures) that maximizes the NPV. The economic parameters considered in this example are also indicated in Table 1. The optimization was run for a 1,500-day time horizon. With well controls updated every 150 days, i.e. $M = 10$ control intervals, and with five wells, the control vector \mathbf{u} has 50 elements. We applied bound constraints to the optimization variables ($200 \text{ bar} \leq p_{prod} \leq 300 \text{ bar}$ and $300 \text{ bar} \leq p_{inj} \leq 500 \text{ bar}$). The initial control values were chosen as mid in-between the upper and lower bounds. The whole exercise was performed in the open-source reservoir simulator MRST (Lie et al., 2012), by modifying the adjoint-based optimization module to allow for robust optimization and combining it with the EnKF module to create a CLRM environment for VOI analysis. The average NPV for the initial ensemble is \$ 53.5 million when using base line control (fixed mid in-between-bounds bottom hole pressures: 400 bar in the injector and 250 bar in the producers) and \$ 55.7 million when using robust optimization over the prior (i.e. without additional information). The workflow was applied for a series of observation times, $t_{data} = \{150, 300, \dots, 1350\}$ days. For this 2D model we assessed the VOI of the production data (total flow rates and water-cuts) with absolute measurement errors ($\varepsilon_{flux} = 5 \text{ m}^3/\text{day}$ and $\varepsilon_{wct} = 0.1$). The VOI, the VOC, and the spread of the ensemble in terms of NPV σ_{NPV} were computed for each of the nine observation times.

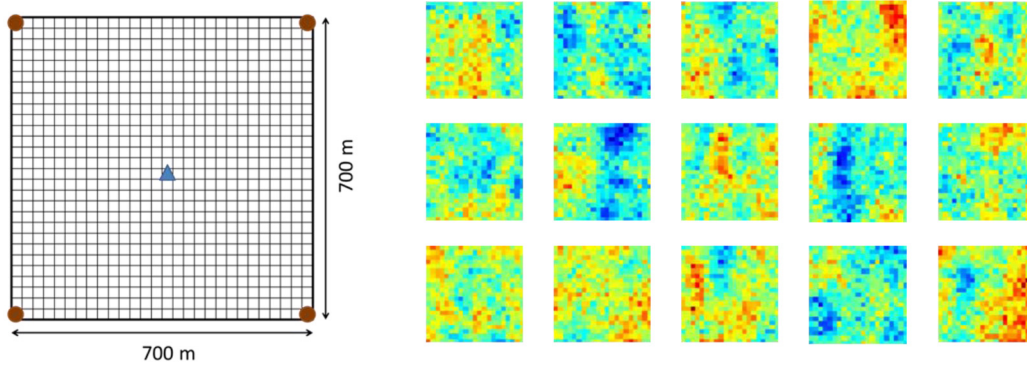


Figure 5—2D five-spot model (left); 15 randomly chosen realizations of the uncertain permeability field (right).

TABLE 1—PARAMETER VALUES FOR 2D FIVE-SPOT MODEL.

Rock-fluid parameters		Initial conditions	
$\rho_o = 800$	kg/m ³	$p_o = 300$	bar
$\rho_w = 1,000$	kg/m ³	$S_{oi} = 0.8$	[-]
$\mu_o = 0.5$	cP	$S_{wi} = 0.2$	[-]
$\mu_w = 1$	cP	Economic parameters	
$n_o = 2$	[-]		
$S_{or} = 0.2$	[-]	$r_o = 80$	\$/bbl
$k_{ro, or} = 0.9$	[-]	$r_{wp} = 5$	\$/bbl
$n_w = 2$	[-]	$r_{wi} = 5$	\$/bbl
$S_{wc} = 0.2$	[-]	$b = 0.15$	[-]
$k_{rw, wc} = 0.6$	[-]		

Fig. 6 depicts the results of the analysis for production data. The dashed lines represent the expected values, i.e. the ensemble mean. The dark solid lines and the lighter solid lines correspond to the P_{50} and P_{10}/P_{90} percentiles respectively. Here, P_x is defined as the probability that x % of the outcomes exceeds this value. The markers correspond to the observation times at which the analysis was carried out. In Fig. 6 (top left) we can observe that clairvoyance loses value with observation time, following the stepwise

behavior that we also found in Barros et al. (2015). In addition, by observing the percentiles, we realize that, in this example, the VOC has a non-symmetric probability distribution. The high values of P_{10} indicate that, for some realizations of the truth, knowing the truth can be considerably more valuable than indicated by the expected VOC; however, the P_{50} values, which are always below those of the expected VOC, indicate what is more likely to occur. The same holds for the VOI, as can be observed in Fig. 6 (top right). The first observation (at $t_{data} = 150$ days) shows already a significant VOI, but, as expected, this value builds up in time as more observations are incorporated to the models. Note that in our example the earliest observation seems to be the most valuable one and that the incremental values for the following observations are relatively small, but that this may be case-specific.

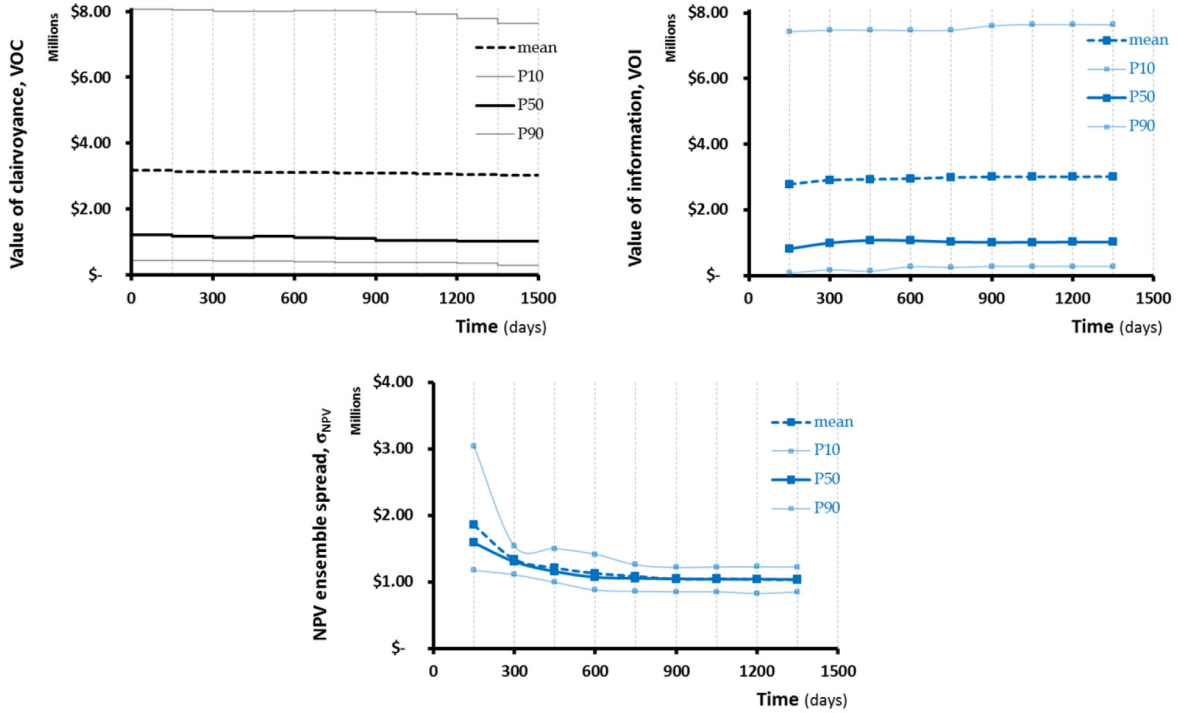


Figure 6—Results for the VOI analysis of production data in the 2D model: VOC (top left); VOI (top right); ensemble spread in terms of NPV (bottom).

Fig. 6 (bottom) shows how the spread of the ensembles of realizations changes as we progress ‘closing the loop’ over time. We express this spread as the standard deviation of the predicted NPV of the ensemble. Note that, because of the repetition of the procedure for different plausible truths, this spread is also a random variable. The initial uncertainty is $\sigma_{NPV, ini} = \$ 4.1$ million, computed as the average of the standard deviations in the NPV of the different prior ensembles. We observe that this spread reduces significantly as the first observations are assimilated, but that later on it reaches a point from which it does not change any more.

Fig. 7 (left) depicts the expected values of VOI (blue dots) and VOC (black line), and Fig. 7 (right) shows the same results using a different scale of the vertical axis. The plots confirm that clairvoyance can be considered the technical limit for any information gathering strategy and that the expected VOC forms an upper-bound to the expected VOI. Here, we can see more clearly that the VOC indeed decreases and that the VOI increases in time. However, the decrease of VOC over time is less significant than what we observed in the previous paper, because here we history match the models with production data while clairvoyance is not yet available. We also note that the expected VOC and expected VOI converge to the

same value at the last observation time ($t_{data} = 1350$ days), because after this point there are no controls left to be re-optimized, which means it is too late to benefit from clairvoyance. Fig. 7 also illustrates that collecting production data over the producing life of this reservoir is worth (on average) approximately \$ 3.02 million, which represents a gain of 5.4% compared to relying on prior knowledge to operate the field.

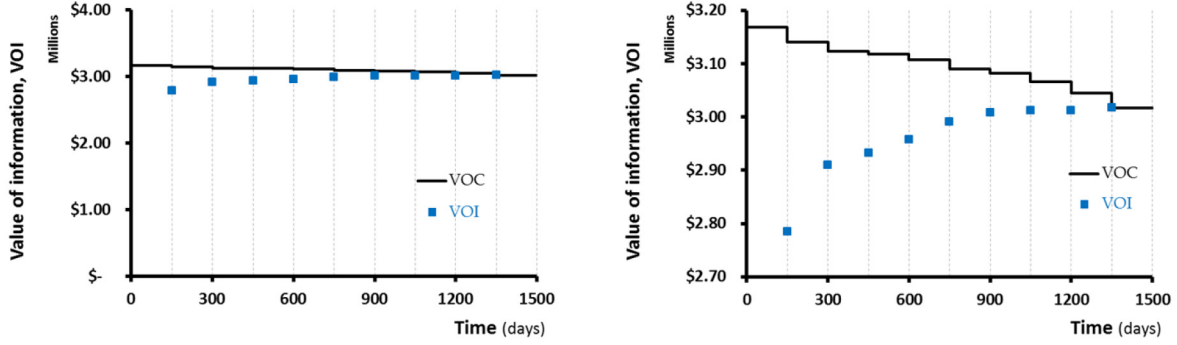


Figure 7—Results for the 2D model: the expected VOI is upper-bounded by expected VOC (left); same results plotted using a different scale of the vertical axis (right).

Multiple oil rates measurements

Next, we repeated the VOI analysis for the same case, but with more limited production data by considering oil rate measurements only (for the same 2D model and with an absolute measurement error $\varepsilon_{oil} = 5 \text{ m}^3/\text{day}$). By comparing the results with the previous example (total flow rate + water-cut measurements), we can have an idea of the additional value of also collecting accurate water production data.

Fig. 8 (left) depicts the expected values of the VOI for the measurements from the previous example (blue dots) and for oil rate measurements only (red dots), and Fig. 8 (right) shows the same results using a different scale of the vertical axis. We observe that measuring oil rates only is less valuable than collecting information of total rates and water-cuts, which is an expected result. We also note a difference compared to the previous example: when assimilating multiple oil rate measurements at different times, the value does not increase monotonically; see Fig. 8 (right). The decrease around $t_{data} = 300$ days can be attributed to the fact that, at low water-cuts (immediately after water breakthrough time in the producers), oil rate measurements are not capable of detecting the presence of water and, therefore, they are not as effective as the water-cut observations in revealing the uncertainties considered here. As a matter of fact, around 300 days, most of the realizations have just observed first water breakthrough. Finally, by comparing the values of the last points ($t_{data} = 1350$ days), we estimate that the additional value of collecting reasonably accurate water-cut measurements (10% error) rather than just oil rates is (on average), for this case, approximately \$ 300,000, which represents an increase of 1.1% in terms of NPV.

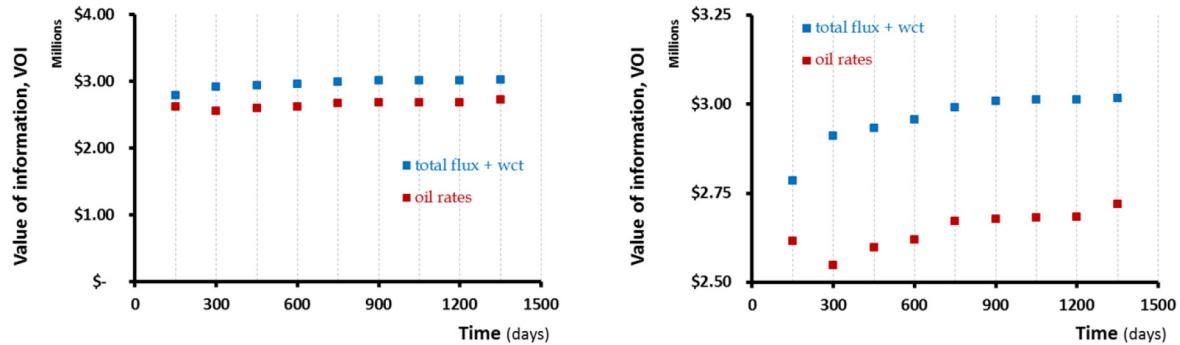


Figure 8—Results for the 2D model: the expected VOI of total flow rate and water-cut measurements (blue) and oil rate measurements only (red) (left); same results plotted using a different scale of the vertical axis (right).

Value of an interpreted time-lapse seismic survey (water front tracking ‘measurements’)

We have taken two approaches to assess the value of an interpreted time-lapse seismic survey. In the first one, we simply repeated the procedure from our previous paper (Barros et al., 2015) but using the water front tracking ‘measurements’ from Leeuwenburgh and Arts (2014) described in the Background section. The methodology for the distance reparametrization of detected water fronts was available in a modified version of the EnKF module for MRST. Once again, we used the same 2D reservoir model from the previous examples, and we adopted an absolute measurement error $\varepsilon_{dst} = 1$ [–], which refers to an error of 1 gridblock when detecting the water fronts. Following the workflow from Barros et al. (2015), we were able to estimate the VOI of these new ‘measurements’ at different observation times (if they were to be collected only once). Fig. 9 (top left) and Fig. 9 (bottom left) depict the results for this analysis, showing that these ‘measurements’ are valuable if available at early times, but not so much at later times. This is an expected result. However, by doing the analysis this way, we are actually assessing the VOI of the time-lapse seismic survey by comparing the value of acquiring a new seismic survey with the value of relying on prior knowledge only (not collecting any additional data throughout the producing life of the reservoir), which does not seem to be a realistic practice. Given the economic costs, an oil company will only consider paying to shoot a new seismic survey to monitor a reservoir if it has already decided to invest in sensor deployment to collect production data, which is a much cheaper option.

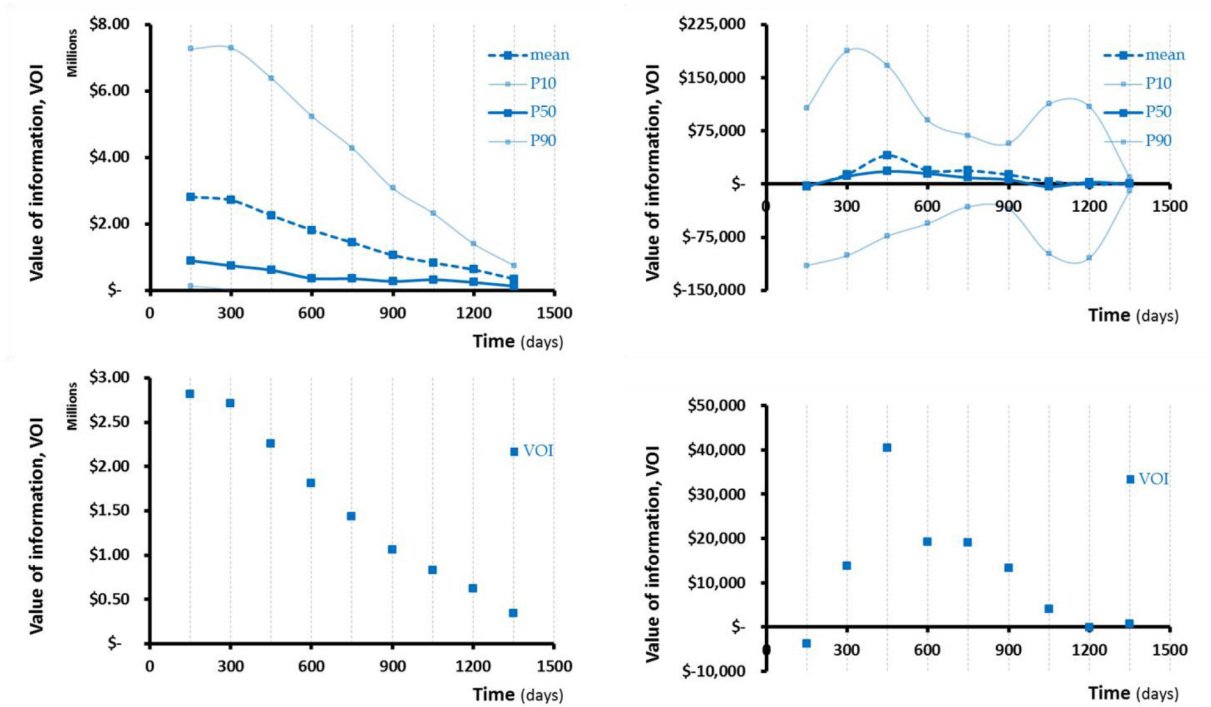


Figure 9—Results for the 2D model: VOI of interpreted 4D seismic data (at a single moment in time) without performing production measurements (top left: mean and percentiles; bottom left: mean only). Incremental VOI of interpreted 4D seismic data (at a single moment in time) in combination with performing production measurements (at multiple observation times) (top left: mean and percentiles; bottom left: mean only).

In order to make a more realistic analysis, in our second approach to assess the VOI of such ‘measurements’, we propose to assume that the time-lapse seismic survey is acquired in addition to the series of production data measurements considered in our first example. Thus, we applied the proposed workflow to analyze the VOI of assimilating production data at multiple observation times $t_{prod} = \{150, 300, \dots, 1350\}$ days together with a single water front tracking ‘measurement’ at time $t_{seismic}$. Then, we considered the VOI obtained after all the observations have been assimilated (at $t_{data} = 1350$ days) and compared it to the same value (at $t_{data} = 1350$ days) obtained in our first example (in which the models were history matched with the production data only). Finally, we repeated the analysis for different moments in time to shoot the seismic: $t_{seismic} = 150$ days, $t_{seismic} = 300$ days, \dots , $t_{seismic} = 1350$ days. The results obtained with this second approach are depicted in Fig. 9 (top right) and Fig. 9 (bottom right). For this example acquiring a single seismic survey, in addition to performing multiple production measurements, does only results in an incremental VOI of, at maximum, \$ 40,000 (for a survey shot at $t_{seismic} = 450$ days). (Note that, as in all examples, this is the VOI without accounting for the costs of acquiring the data.) Apparently the production data already provide sufficient information in this case.

Conclusion

We extended the work from our previous paper to create a new workflow that allows the VOI assessment of a series of future measurements. The method uses elements available in the CLRM framework, such as history matching and robust optimization. First, we identified the opportunity to combine these elements with concepts of information value theory to create a VOI analysis instrument. We then designed a generic procedure that can, in theory, be simply implemented in a variety of applications, including our optimal reservoir management problem. Next, the workflow was illustrated with two examples and the results were analyzed. In the third example, we illustrated the additional value of a time-lapse seismic

survey, which, however, in our example was limited because apparently the production data were already substantially informative. We believe that our proposed workflow is a complete methodology to estimate the VOI in a CLRM context because we take into account that the production strategy is updated periodically after new information has been assimilated in the models. However, the computational complexity of the method is, at present, prohibitively large. Future work will be required to reduce the computational load, e.g. through the use of proxy models or reduced-order modeling, to allow for the application to real field cases.

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References

- Barros, E.G.D., Jansen, J.D. and Van den Hof, P.M.J. 2015. Value of information in closed-loop reservoir management. *Accepted for publication in Computational Geosciences*.
- Chen, Y., Oliver, D.S. and Zhang, D. 2009. Efficient ensemble-based closed-loop production optimization. *SPEJ* **14** (4) 634–645. [http: 10.2118/112873-PA](http://10.2118/112873-PA).
- Jansen, J.D., Brouwer, D.R., Nævdal, G. and van Kruijsdijk, C.P.J.W. 2005. Closed-loop reservoir management. *First Break*, January, **23**, 43–48. [http: 10.3997/1365-2397.2005002](http://10.3997/1365-2397.2005002).
- Jansen, J.D., Bosgra, O.H. and van den Hof, P.M.J. 2008. Model-based control of multiphase flow in subsurface oil reservoirs. *Journal of Process Control* **18**, 846–855. [http: 10.1016/j.jprocont.2008.06.011](http://10.1016/j.jprocont.2008.06.011).
- Jansen, J.D., Douma, S.G., Brouwer, D.R., Van den Hof, P.M.J., Bosgra, O.H. and Heemink, A.W. 2009. Closed-loop reservoir management. Paper SPE 119098 presented at the SPE Reservoir Simulation Symposium, The Woodlands, USA, 2-4 February. [http: 10.2118/119098-MS](http://10.2118/119098-MS).
- Leeuwenburgh, O. and Arts, L.J. 2014. Distance parameterization for efficient seismic history matching with the ensemble Kalman Filter. *Computational Geosciences* **18**, 535–548. [http: 10.1007/s10596-014-9434-y](http://10.1007/s10596-014-9434-y).
- Lie, K.-A., Krogstad, S., Ligaarden, I. S., Natvig, J. R., Nilsen, H. M. and Skalestad, B. 2012. Open source MATLAB implementation of consistent discretisations on complex grids. *Computational Geosciences*, **16** (2) 297–322. [http: 10.1007/s10596-011-9244-4](http://10.1007/s10596-011-9244-4).
- Nævdal, G., Brouwer, D.R. and Jansen, J.D. 2006. Waterflooding using closed-loop control. *Computational Geosciences* **10** (1) 37–60. doi: 10.1007/s10596-005-9010-6
- Sarma, P., Durlofsky, L.J. and Aziz, K. 2008. Computational techniques for closed-loop reservoir modeling with application to a realistic reservoir. *Petroleum Science and Technology* **26** (10&11) 1120–1140. [http: 10.1080/10916460701829580](http://10.1080/10916460701829580).
- Sethian, J.A. 1996. A fast marching level set method for monotonically advancing fronts. *Proc. Nat. Acad. Sci.* **93** (4) 1591–1595. [http: 10.1073/pnas.93.4.1591](http://10.1073/pnas.93.4.1591).
- Wang, C., Li, G. and Reynolds, A.C. 2009. Production optimization in closed-loop reservoir management. *SPEJ* **14** (3) 506–523. [http: 10.2118/109805-PA](http://10.2118/109805-PA).