DELFT UNIVERSITY OF TECHNOLOGY
Faculty of electrical engineering
Telecommunication and Traffic-control Systems Group

Title: Comparative study of hybrid direct sequence / slow frequency hopping, direct sequence and slow frequency hopping wireless communication systems with DPSK modulation in an indoor Rician fading environment for one resolvable path.

Author: R.G.A. Rooimans

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Mentors : Dr. L. Vandendorpe and prof. dr. R. Prasad

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A comparative study of direct sequence, slow frequency hopping and hybrid direct sequence/slow frequency hopping spread spectrum systems with DPSK modulation and diversity in an indoor wireless environment is presented in this report. Performances will be assessed by means of bit error probability and outage probability computation with the restriction of one resolvable path. The computation is done for a star-connected multiple access radio network. Furthermore, the influence of three types of forward error correction codes, namely the (7,4 Hamming code), the (15,7) BCH code and the (23,12) Golay code, on the performance is studied.

PREFACE

I have investigated the performance of an wireless communication system with help of a mathematical model within the framework of my "task". The results of this performance analysis are given in this report. Although the derivation of the complete model has already been done, this report doesn't contain this derivation, because a simplified version of the model has been studied. I will continue the study of the complete model in my graduation work.

Since this study is based on previous studies, parts of the theory in the first chapters can be found in other reports as well. Besides, parts of the results with respect to the performance of direct sequence are obtained from the work of H.S. Misser.

Most of all this report is written for people doing research in the field of "Code Division Multiple Access" techniques. For people who are not familiar with wireless communications, it would be advisable to read some literature in this field in order to get acquainted with some specific terms. In that respect I would like to refer to the book "Digital communications" of J.G. Proakis.

October 23, 1992

R.G.A. Rooimans

SUMMARY

Wireless communication systems have regained a lot of interest recently. Especially the applications of such systems in the indoor environment gives the advantages of flexibility and costs saving with respect to cabling systems.

From previous studies it was observed that direct sequence (DS) suffers severely of the near-far effect: the phenomenon that a transmitter close to the receiver gives more power there than a transmitter further away. A hybrid direct sequence/slow frequency hopping (DS/SFH) system overcomes this problem.

The performance of a hybrid DS/SFH system is investigated for two types of diversity, viz. selection diversity and maximal ratio combining with help of a theoretical model. With the same model the performances of a DS and a SFH system separately are investigated and compared with the performance of the hybrid system. Next the influence of three types of forward error correcting codes (FEC), viz. the (15,7) BCH code, the (7,4) Hamming code and the (23,12) Golay code, on the performance of the hybrid system is investigated.

The performances of the systems are expressed in terms of bit error probability or outage probability. In this model only one resolvable path is considered, which simplifies the calculations.

In general the hybrid DS/SFH yields the best performance and the DS system is superior over a SFH system. Maximal ratio combining is a better diversity technique than selection diversity. The use of FEC codes in a nondiversity hybrid and nondiversity direct sequence system leads to a saving of antennas. A SFH system will only give an acceptable bit error rate when a number of frequencies of 1024 are used in combination with the (23,12) Golay code.

Furthermore a trade off can be made among the number of antennas, number of frequencies in the hopping pattern, the length of the spreading codes and the frequency range. So it is highly dependent of the application (which determines the bit error rate) and the available frequency range, what system configuration can be used.

LIST OF SYMBOLS

$\alpha(t)$	attenuation factor
β	path gain
β_{max}	maximal path gain
γ_b	bit signal-to-noise ratio
ξ	decision variable
μ	covariance of the current and the previous filter output
μ_o	variance of the current matched filter output
μ_{-1}	variance of the previous matched filter output
σ_r^2	power received over specular paths
σ_n^2	variance of the noise samples
τ	path delay
$(\Delta f)_c$	coherence bandwidth
$(\Delta t)_c$	coherence time
$a_k(t)$	direct sequence code waveform of user k
$b_k(t)$	data waveform of user k
$b_k^o(t)$	current data bit of user k
$b_k^{-1}(t)$	previous data bit of user k
B_d	Doppler spread
C_{1k}	discrete aperiodic correlation
d_1	random variable of value 1 or 0 which describes the interference of the previous bit with the bit under consideration
d_2	random variable of value 1 or 0 which describes the interference of the current bit with the bit under consideration
E_b	bit energy
G_p	processing gain
K	number of simultaneously transmitting users
L	number of resolvable paths
m	mean of the matched filter outputs
M	order of diversity
N	number of chips
N_b	number of bits per hop

N_o	single sided spectral density of the white Gaussian noise
P	received signal power
Q	number of frequencies in the hopping pattern
r	distance between transmitter and receiver
R	Rice factor
R_c	channel bitrate
R_{1k}	partial correlation for user k
S	peak value of the total received signal
T_b	data bit duration
T_c	chip time
T_h	hop duration
T_m	time delay spread
U_m	matched filter output
X_o	current optimum demodulator output
X_{-1}	previous optimum demodulator output
W	transmission bandwidth

LIST OF ABBREVIATIONS

AWGN	Additive White Gaussian Noise
CDF	Cumulative Density Function
CDMA	Code Division Multiple Access
DPSK	Differential Phase Shift Keying
DS	Direct Sequence
FDMA	Frequency Division Multiple Access
FEC	Forward Error Correction
FFH	Fast Frequency Hopping
LOS	Line Of Sight
MRC	Maximal Ratio Combining
PDF	Probability Density Function
PN	Pseudo Noise
RF	Radio Frequency
SFH	Slow Frequency Hopping
SD	Selection Diversity
SS	Spread Spectrum
TDMA	Time Division Multiple Access
TH	Time Hopping

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1. INTRODUCTION

Wireless communication is a part of telecommunication, that has regained a lot of interest recently. This appears from the increasing demand for wireless mobile services such as vehicular telephony, radio paging systems and cordless telephones.

Except applications of this wireless services in an outdoor environment, there's an increasing development of wireless systems in the indoor environment. Especially in the office environment the application of this systems, such as wireless computer networks and cordless telephones, mainly offers two advantages.

First this system can provide flexibility in the placement of computer terminals. Since reorganizations in companies have almost become a fashion nowadays and almost always go to with changes in office arrangements and physical moves, this can be done with minimal disruption of work.

Secondly, this concept could provide economic advantages as well, because there is no need to invest a lot of money in cable systems.

In the last few years, a lot of attention has been paid to the performances of direct sequence spread spectrum (DS SS) communication systems over fading channels in presence of multi-user and multipath interference. However, it has been observed that DS SS technique suffers severely from the near-far effect. This effect means that a transmitter nearer to the receiver yields a bigger signal power at the receiver than a transmitter which is geographically further away. A combination of DS and slow frequency hopping (SFH), i.e. hybrid DS/SFH spread spectrum technique overcomes this problem.

In this report we investigate the performance of hybrid DS/SFH systems working with selection diversity (SD) and maximal ratio combining (MRC) as diversity techniques. As particular cases, the performances of DS and SFH systems are derived from the hybrid model and compared with the performance of the hybrid system.

The performance of the system will be expressed in terms of bit error ratio (BER) or outage probability, both as function of the signal to noise ratio. The BER and outage probability will be calculated on the basis of a theoretical mathematical model, with the restriction that only one resolvable path is used. This simplifies the calculations and actually gives the lower bound of the performance, because of the absence of the multipath interference in this case.

Furthermore, the influence of three types of forward error correcting (FEC) codes on the performance is investigated. The codes are: the (15,7) BCH code, the (7,4) Hamming code and the (23,12) Golay code.

The system we will use for the performance analysis is a star connected CDMA network with K users as shown in figure 1.

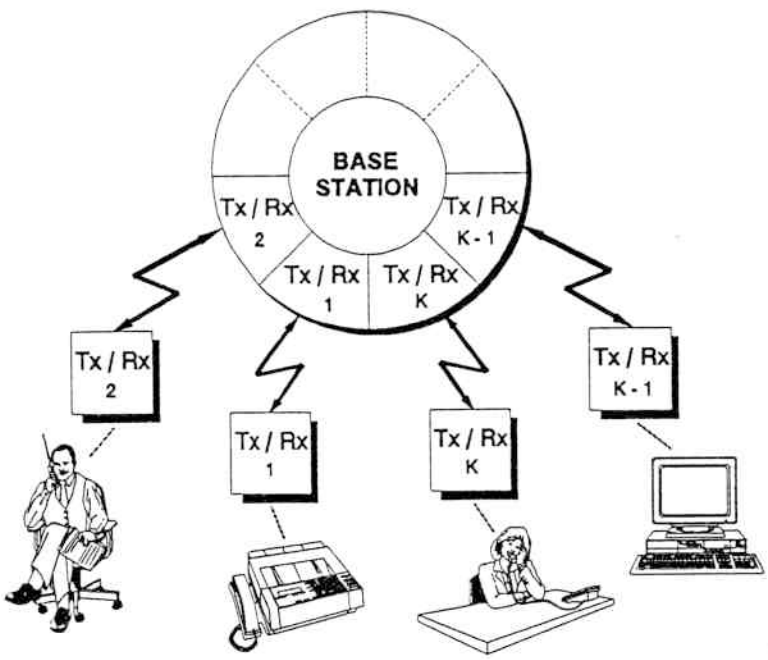

***Figure 1:** Star connected indoor wireless local area network with CDMA*

The base station consists of a bank of spread spectrum transmitters/receivers, one for each active user. We assume that each user has an unique spread spectrum code sequence.

Because of the presence of multipath fading on the radio path, which results in a loss of phase coherence in the received signal, coherent detection of the signal is almost impossible. In order to avoid this problem differential phase shift keying (DPSK) is used as the modulation technique.

In chapter two there will be a discussion about multiple access techniques, which are used in the model. Chapter three will consider the propagation aspects of indoor wireless communications, such as pathloss and multipath interference. In chapter four a description of the theoretical model is presented. Chapter five is a detailed analysis of the hybrid model based on both SFH and DS in case of selection diversity. Chapter six is a detailed study of the hybrid model but now in case of maximal ratio combining. In chapter seven the numerical results and the discussion of the results are presented. Chapter 8 gives the conclusions and recommendations for further research.

2. MULTIPLE ACCESS TECHNIQUES IN WIRELESS COMMUNICATIONS

In a wireless communication system all users must have the possibility to have communication simultaneously, which means that they must have multiple access capability. There are mainly three multiple access techniques: frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA).

In FDMA all users are able to transmit simultaneously but use disjoint frequency bands. Every single user has his own channel in the frequency domain, across which he can transmit his information.

In TDMA every single user has the possibility to transmit his information during a short period of time, the latter also referred to as a time slot. So all users occupy the same bandwidth, but sequentially in time.

In CDMA channels are created by using mutually different code sequences. Such a code sequence serves as a selection means and as a carrier of information. By using CDMA, the transmitted signal spectrum will be spread over a frequency range much greater than the message bandwidth. That is why this technique is often referred to as spread spectrum technique. In figure 2 the three multiple access methods are shown.

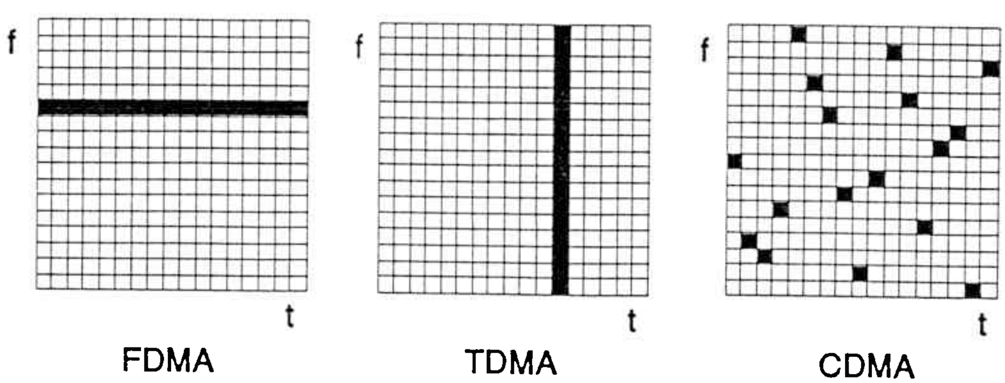

Figure 2: Multiple access techniques

Since CDMA is the most promising of the three techniques, this one will be considered from now on. The two most important ways to implement CDMA, viz. direct sequence (DS) and frequency hopping (FH) will be discussed in the next two sections.

2.1 Direct Sequence

The principle of direct sequence is the modulation of the data signal with a pseudo random code sequences, which consist of wideband digital signals. These three signals are shown in figure 3.

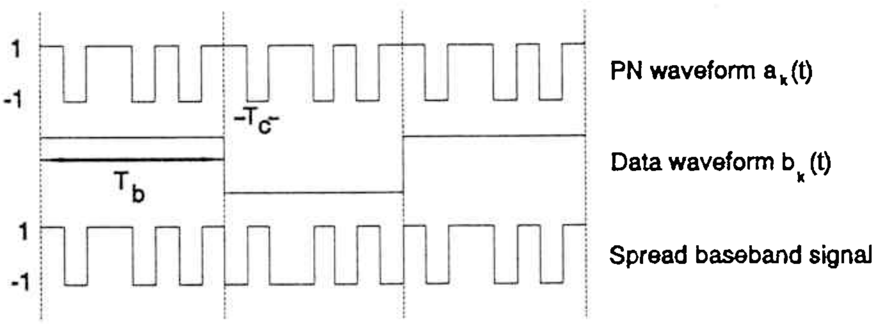

***Figure 3:** Principle of direct sequence*

Each user obtains such a sequence that distinguishes itself sufficiently from the other code sequences. Therefore, it is necessary to have enough sequences of certain length with mutually low crosscorrelation coefficients. Interesting in this respect are the maximal-length sequences , such as Goldcodes. The generation process of these sequences is often referred to as *pseudo-noise (PN) process*, due to the correlation properties, which are noise-like. Such a proces looks random, but can be replicated by authorized users.

DS involves spread spectrum and as the name suggests, the transmitted signal spectrum is much greater than the message bandwidth. The ratio between the message bandwidth and the transmitted signal bandwidth is called *processing gain.*

$$G_p = \frac{T_b}{T_c} \quad (1)$$

DS systems have several advantages and disadvantages over the other CDMA techniques.

Advantages:

- best noise and anti-jam performance;
- most difficult to detect
- best discrimination against multi-path fading.

Disadvantages:

- long acquisition time;
- fast code generator needed;
- near-far problem; if each user transmits with equal power, the signal of a user, closer to the receiver, yield a larger power at the receiver than the signal of a user further away.

2.2 Frequency hopping

In a frequency hopping system, the carrier frequency hops "randomly" from one value to another under control of an PN-proces. Since each user has his own hopping pattern, they are able to transmit their information simultaneously. Frequency hopping systems can be divided in two categories: *fast frequency hopping* (FFH) and *slow frequency hopping* (SFH).

In a fast frequency hopping system, a data bit is transmitted during multiple hops. If we consider a "snapshot" of the spectrum of this transmitted signal, we are not able to make a distinction between the different hops, because of the high hopping speed. The frequency band, allocated for this purpose will be fully occupied and spread spectrum is also involved.

A slow frequency hopping system transmits multiple data bits per hop and here no spread spectrum is involved, because of the slow hopping speed. If we consider a "snapshot" of the spectrum again, we only see the message bandwidth, slowly hopping from one carrier frequency to another. So slow frequency hopping is only used as a selection means for the different users rather than as spread spectrum means.

When several frequency hopping signals occupy a common RF channel, it might occur that these signals interfere. Events of this type are called *hits*. These hits become more and more of a problem when the number users hopping over a fixed bandwidth increases.

Frequency hopping systems have several advantages and disadvantages over other CDMA techniques.

Advantages:

- FFH yields the greatest amount of spreading;
- programmable hopping pattern so portions of the spectrum can be avoided;
- short acquisition time;
- less affected by the near-far problem.

Disadvantages:

- complex frequency synthesizer required;
- error correction needed.

3 PROPAGATION ASPECTS IN A INDOOR ENVIRONMENT

In order to design an indoor wireless communication system we need to have some knowledge of the indoor radio propagation characteristics. Two phenomena, namely path loss and multi path multipath interference, determine the characteristics of the radio channel in a large measure.

In general the spatially averaged value of the multipath power gain in a point at a distance r from the transmitter is represented by a distance-power law of the following form:

$$p \propto r^{-\alpha} \qquad (2)$$

In the free space $\alpha = 2$, but results from literature [4] shows that whitin buildings α can be smaller or greater than 2, dependent on the position of the transmitter with regard to the receiver. If the transmitter and receiver are placed in the same hallway of an office building for instance, α usually is smaller than 2. This gain over the free space situation is likely due to the wave guiding effects in the hallway. In rooms which are located off the hallway, α can exceed values of 3,4 or even 5, dependent on the position of the room with regard to the transmitter. In this performance analysis we assume that is case of a direct sequence system, the base station provides perfect average power control.

Multipath fading is caused by multiple reflections of the transmitted signal from the building structure and surrounding inventory. The resulting received waveform is a sum of time and frequency shifted versions of the original transmission. Besides, the received signal may be severely distorted, dependent on parameters of the signal and the radio channel. Figure 4 shows a typical pattern of a faded signal.

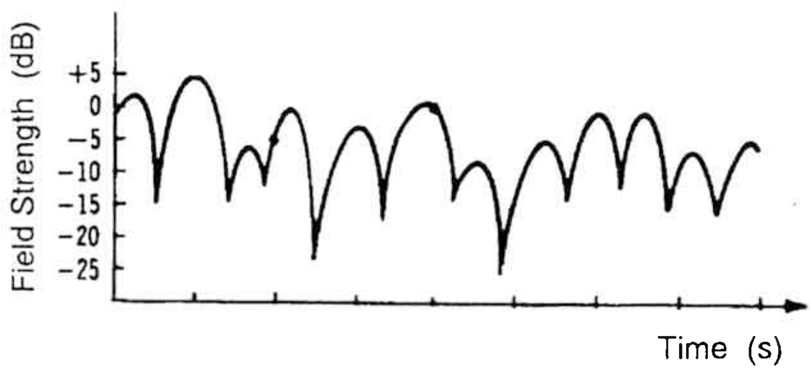

Figure 4: *Typical pattern of a signal subjected to multipath fading*

Since the channel parameters play an important role with respect to multipath fading, they will be discussed in the next sections.

3.1 Parameters of the radio channel

Starting point for the investigation of the properties of a channel is the impulse response of the channel. From this impulse response two parameters, viz. coherence bandwidth and coherence time, can be derived. The time-variant impulse response of the equivalent low-pass channel, which contains discrete multipath components, is described by:

$$c(\tau;t)=\sum_{n}\alpha_{n}(t).e^{-j2\pi f_{c}\tau_{n}(t)}.\delta(\tau-\tau_{n}(t)) \tag{3}$$

Here, $\alpha_n(t)$ is the attenuation factor for the received signal on the n*th* path and $\tau_n(t)$ is the propagation delay for the n*th* path. From such an impulse response several separated responses can be distinguished, due to the fact that the transmitted impulse reaches the receiver along different paths of different length. The *coherence bandwidth* and the *coherence time*, two important parameters of the channel will be discussed in the next two sections.

3.1.1 Coherence bandwidth of the channel

The range of values of τ over which the average output power of the channel as a function of τ (denoted by: $\phi_c(\tau)$), is essentially nonzero is called *multipath spread* or *time delay spread* of the channel and is denoted by T_m. The Fourier transform of $\phi_c(\tau)$ yields an autocorrelation function in the frequency variable, which is denoted by $\phi_c(\Delta f)$. This latter quantity is a measure of the frequency coherence of the channel. In figure 5 the relationship between these latter two quantities is shown.

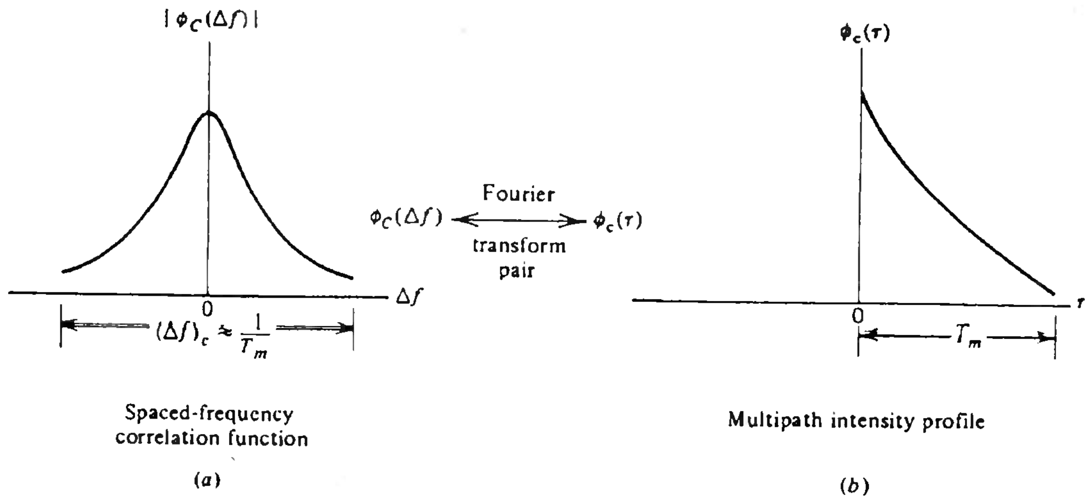

***Figure 5:** Relationship between the spaced-frequency correlation function and the multipath intensity profile*

As a result of the Fourier transformation relationship between $\phi_c(\tau)$ and $\phi_c(\Delta f)$, the reciprocal of the multipath delay spread is called the *coherence bandwidth* of the channel. That is:

$$(\Delta f)_c \approx \frac{1}{T_m} \tag{4}$$

Where $(\Delta f)_c$ denotes the coherence bandwidth. The channel is said to be frequency selective when the signal bandwidth is greater than the coherence bandwidth. In that case the signal is subjected to different gains and phase shifts across the band and is severely distorted by the channel.

On the other hand if the signal bandwidth is much smaller than the coherence bandwidth the channel is said to be frequency nonselective. All the frequency components in the transmitted signal undergo the same attenuation and phase shift.

3.1.2 Coherence time of the channel

The signal intensity as a function of the Doppler frequency f is given by the Doppler power spectrum of the channel, denoted by $S_c(f)$. The Doppler spread B_d is the range of values over which the Doppler power spectrum is nonzero. The Fourier transform of $S_c(f)$ yields an autocorrelation function in the time variable, denoted by $\phi_c(\Delta t)$. This latter quantity is a measure for the time coherence of the channel. This is shown in figure 6.

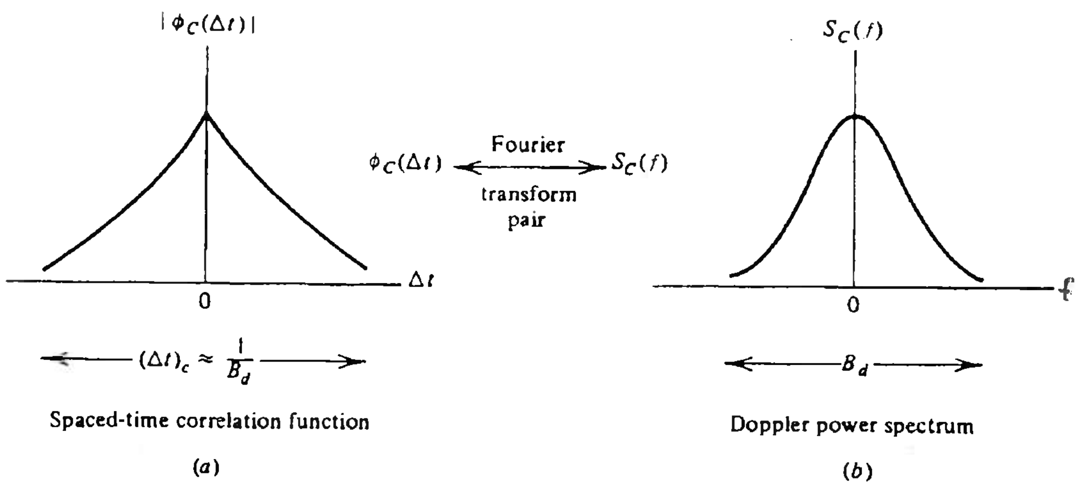

Figure 6: *Relationship between the spaced-time correlation function and the Doppler power spectrum*

Through the relationship between $S_c(f)$ and $\phi_c(\Delta t)$ the reciprocal of B_d is called the *coherence time* of the channel. That is:

$$(\Delta t)_c \approx \frac{1}{B_d} \tag{5}$$

Where $(\Delta t)_c$ denotes the coherence time. It is obvious that a slowly changing channel has a large coherence time or, equivalently a small Doppler spread.

The rapidity of the fading in a frequency nonselective channel is determined either from the correlation function $\phi_c(\Delta t)$ or from the Doppler power spectrum $S_c(f)$. Suppose that the periodical time duration T of the transmitted signal is smaller than the coherence time of the channel. Then the channel attenuation and phase shift are essentially fixed for the duration of at least one signalling interval. When this condition holds, the channel is said to be a *slow fading* channel.

In the next section we will discuss the influence of the channel parameters on the transmitted signal.

3.2 Influence of the channel parameters on the radio signal

In the indoor environment, many reflections of the signal arrive at the receiver at different times. All these reflections of the signal add up and cause signal peaks and dips, dependent on the phases of the different reflections.

The reflections can be grouped in clusters. These clusters contain the signals with a time difference between the arrival of the first and the last ray, which is less than the periodical time T of the transmitted signal. These clusters can be resolved by a receiver and are therefore called *resolvable paths*. The principle in shown in figure 7.

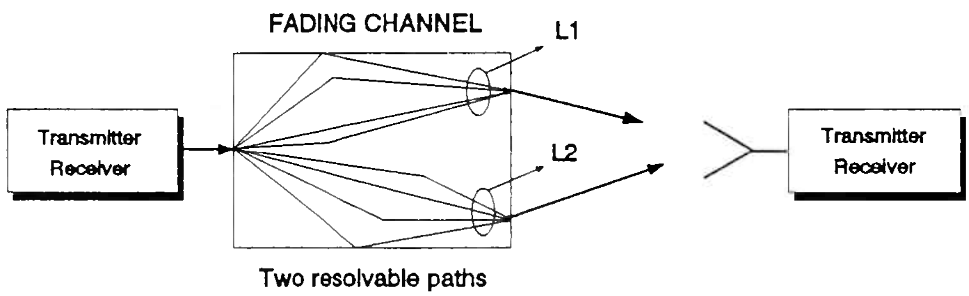

***Figure** 7: Multiple resolvable paths*

The criterion for the existence of multiple resolvable paths is that the signal bandwidth W is much larger than the coherence bandwidth $(\Delta f)_c$.

In case of direct sequence, each data bit is multiplied by a unique user code. Such a code consists of N pulses (chips) of duration $T_c = T_b/N$, where T_b is the duration of one data bit. The number of multiple resolvable paths L in this case is given by:

$$L=\frac{T_m}{T_c}+1 \tag{6}$$

Where T_m denotes the delay spread of the channel. So spread spectrum provides independently fading multiple signal paths, because the transmission bandwidth is much greater than the coherence bandwidth of the channel.

The major problem involving the reception of radio signals is when the transmitted signal is in a deep fade. A solution of this problem is to use the replicas of the signal, transmitted over independently fading channels. This latter phenomenon is called *diversity.* Direct sequence spread spectrum provides multiple resolvable paths if the delay spread is larger than the duration of a chip in a user code. This is called *inherent spread spectrum diversity.* In the next section we will discuss two ways receivers use diversity for signal reception.

3.3 Diversity techniques at the receiver

In wireless communications receivers mainly use two types of diversity, viz. *selection diversity* and *maximal ratio combining.*

Selection diversity is based on selecting the largest of a group of signals carrying the same information. The multiple resolvable paths can be used for selection diversity by selecting the path with the largest output of the matched filter. In figure 8 the schematic receiver configuration is shown.

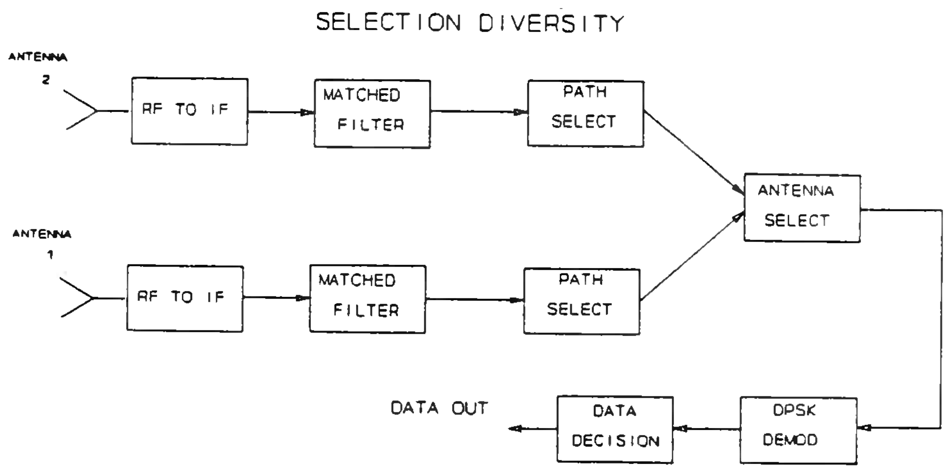

Figure 8: *Selection diversity receiver configuration*

The order of diversity M that can be achieved with selection diversity is defined as the number of resolvable paths multiplied by the number of antennas.

If there are too few resolvable paths one could use multiple antennas in order to increase the order of diversity. These antennas must be spaced sufficiently far apart so the multi-pathcomponents in the signal have significantly different propagation delays at the antennas. In figure 9 the selection diversity principle is shown.

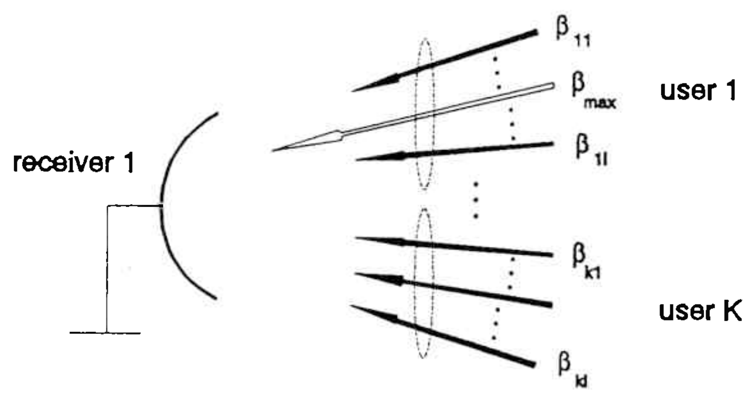

Figure 9: Selection diversity principle

Maximal ratio combining is based on summing the demodulation results of a group of signals carrying the same information. The result of this summation is used as a decision variable in the data decision process. The maximal ratio combining receiver configuration is shown in figure 10.

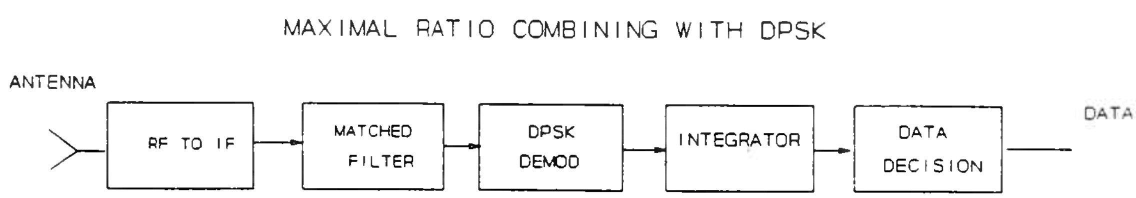

Figure 10: Maximal ratio combining receiver configuration

In general the order of diversity M that can be achieved with maximal ratio combining is equal to the number of resolvable paths L. However in case of one resolvable path a higher order of diversity can be obtained by using more than one maximal ratio combining receiver. Although this would be very expensive and even impracticable in practice, it is still of interest to consider this aspect in a theoretical model. This type of diversity is very attractive for spread spectrum with DPSK modulation, because it's very easy to implement and it gives a significant improvement in performance.

4. DESCRIPTION OF THE THEORETICAL MODEL

The derivation of the model has already been done in a previous study. The aim of this study is to adapt the results to the situation where there is only a single path, i.e. L=1 and compare the performances for SFH, DS and the hybrid system.

By considering a signature sequence which is equal to 1 anywhere, we get the results valid for SFH. On the other hand, the results of DS alone can also be obtained by considering that a single frequency is used in the hybrid system (Q=1).

Normally the assumption of multiple resolvable paths (L > 1) is valid in a hybrid system or a DS system, but not for a SFH system, because the bandwidth of the hopped signal is not large in comparison with the input signal bandwidth. Selection diversity is still of interest, because the order of diversity is equal to the number of resolvable paths multiplied by the number of antennas. In the following sections the descriptions of the transmitted signal, the channel and the receiver are given.

4.1 Description of the transmitter model

In figure 11 the transmitter model of a hybrid DS/SFH system is shown.

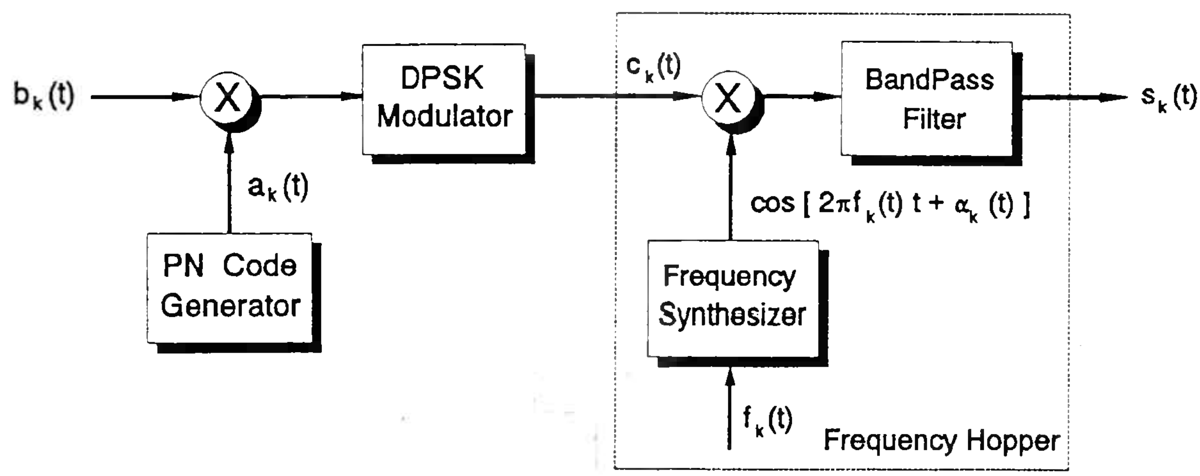

Figure 11: *Transmitter model of a hybrid DS/SFH system*

Each user produces a data waveform given by

$$b_k(t)=\sum_j b_k^j P_T(t-jT) \tag{7}$$

where b^j_k belongs to the set of {0,1} and is the *jth* data bit of user k. Furthermore P_T is a rectangular pulse of unit height and duration T. The signal $b_k(t)$ is first multiplied by a spreading sequence $a_k(t)$.

$$a_k(t)=\sum_j a_k^j P_{T_c}(t-jT_c) \tag{8}$$

where T_c is the chip duration and P_{Tc} a rectangular pulse of unit height and duration T_c. This sequence is periodical with period T. This signal $a_k(t)b_k(t)$ is DPSK modulated and can be written as:

$$c_k(t)=a_k(t)b_k(t)\cos(\omega_c t+\theta_k) \tag{9}$$

Where it is now assumed that b^j_k belongs to the set {-1,1}. θ_k is a random phase associated with user k.

This modulated signal is then frequency hopped according to the hopping pattern associated with user k. This produces a signal given by:

$$d_k(t)=a_k(t)b_k(t)\cos(\omega_c t+\theta_k)\cos[\omega_k(t)t+\alpha_k(t)] \tag{10}$$

The frequency $f_k(t)$ and the phase $\alpha_k(t)$ are constant over a time interval of duration T_h = $N_b T_b$, where T_b is the bit duration. The bandpass filter in the frequency hopper, shown in figure 11, removes unwanted frequency components present at the output of the multiplier. The hopped signal then becomes:

$$s_k(t)=\sqrt{2P}a_k(t)b_k(t)\cos(\omega_c t+\omega_k(t)t+\theta_k+\alpha_k(t)) \tag{11}$$

where P is the transmitted power of the *kth* signal. The quantity $\alpha_k(t)$ represents the phase shift introduced by the frequency hopper when it switches from one frequency to another. We assume $\alpha_k(t)$ to be constant during the time intervals that $f_k(t)$ is constant.

4.2 Description of the channel model

The link between the kth user and the base station is characterized by a lowpass equivalent transfer function given by:

$$h_k(t) = \beta_k \delta(t-\tau_k)\exp(j\gamma_k) \tag{12}$$

where k refers to user k. We assume that the path gains β are Rician distributed. They are not made dependent on the bit number, because the fading is assumed to be slow here. In this study, we assume to deal with random hopping patterns. The phase factor γ is uniformly distributed over $[0,2\pi]$. The random variables β, γ and τ are independent for different values of k.

The Rician PDF describes the envelope of the sum of two statistically independent non-zeromean Gaussian random variables with identical variances σ^2. This distribution is applicable when a significant part of the received signal envelope is due to a constant path (such as a line-of-sight component or fixed signal reflectors). Since in an indoor environment there are a lot of fixed signal reflectors the Rician would be appropriate.

The Rice probability density function is defined as:

$$p_\beta(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+s^2}{2\sigma^2}\right)I_o\left(\frac{sr}{\sigma^2}\right) \qquad r \geq 0,\ s \geq 0 \tag{13}$$

where $I_0(\ )$ is the modified of the first kind and zero order. The noncentrality parameter s is the peak value of the specular radio signal. This signal due to superposition of the dominant LOS signal and the scattered signals reflected from walls, ceiling and stationary inventory. The parameter s is defined as:

$$s^2 = m_1^2 + m_2^2 \tag{14}$$

where m_1 and m_2 are the mean values of the Gaussian random variables.

The Rician distribution is characterized by the parameter R, which is the ratio of the peak power and the power received over specular paths. R is defined as:

$$R = \frac{s^2}{2\sigma^2} \tag{15}$$

From propagation measurements, done in an indoor environment [4] some values of R are determined. We know that R = 6,8 dB corresponds to a 30-year-old brick building with reinforced concrete and plaster, as well as some ceramic block interior partitions. R=11 dB corresponds to a building that has the same construction, but has an open-office interior floor plan and non-metallic ceiling tiles.

4.3 Description of the receiver model

The receiver of a hybrid SFH/DS system consists mainly of two parts: the frequency dehopper and the DPSK demodulator. We will choose user 1 as the reference user and assume that the receiver is capable of acquiring the frequency hopping pattern and the time synchronization with the signal of user 1.

We assume that a received signal is composed of the contributions of the different users paths and additive white gaussian noise. Besides, it is obvious that the received signal for a certain user is the time convolution of the transmitted signal and the channel impulse response. Therefore, we have:

$$r(t)=\sqrt{2P}\sum_{k}\beta_k a_k(t-\tau_k)b_k(t-\tau_k)\cos[\omega_c t+\omega_k(t-\tau_k)t+\phi_k]+n(t) \qquad (16)$$

with

$$\phi_k=[\omega_c+\omega_k(t-\tau_k)](-\tau_k)+\alpha_k(t-\tau_k)+\theta_k+\gamma_k \qquad (17)$$

and $n(t)$ is white Gaussian noise with single-sided power spectral density N_o [W/Hz].

We consider user 1 as a reference user. Therefore, the dehopping sequence is the one associated with user 1. The dehopper produces a signal (after filtering):

$$r_d(t)=\sqrt{P/2}\sum_{k}\beta_k a_k(t-\tau_k)b_k(t-\tau_k)\delta[f_1(t),f_k(t-\tau_k)]\cos[\omega_c t+\phi_k+\beta_1(t)]+n_d(t) \qquad (18)$$

The phase $\beta_1(t)$ is constant over the time interval T_h and is introduced by the frequency dehopper. The δ function accounts for the possible hits with the frequency used by reference user 1 and the frequency used in delayed version of signals k.

We can express $r_d(t)$ by means of its Rice components:

$$r_d(t) = x(t)\cos(\omega_c t) + y(t)\sin(\omega_c t) \tag{19}$$

with

$$x(t)=\sqrt{P/2}\sum_k \beta_k a_k(t-\tau_k)b_k(t-\tau_k)\delta[f_1(t),f_k(t-\tau_k)]\cos[\phi_k+\beta_1(t)]+n_c(t) \tag{20}$$

$$=\sqrt{P/2}\sum_k \beta_k a_k(t-\tau_k)b_k(t-\tau_k)\delta[f_1(t),f_k(t-\tau_k)]\cos[\psi_k]+n_c(t) \tag{21}$$

$$y(t)=\sqrt{P/2}\sum_k \beta_k a_k(t-\tau_k)b_k(t-\tau_k)\delta[f_1(t),f_k(t-\tau_k)]\sin[\phi_k+\beta_1(t)]+n_s(t) \tag{22}$$

$$=\sqrt{P/2}\sum_k \beta_k a_k(t-\tau_k)b_k(t-\tau_k)\delta[f_1(t),f_k(t-\tau_k)]\sin[\psi_k]+n_s(t) \tag{23}$$

The optimal DPSK demodulator is given in figure 14.

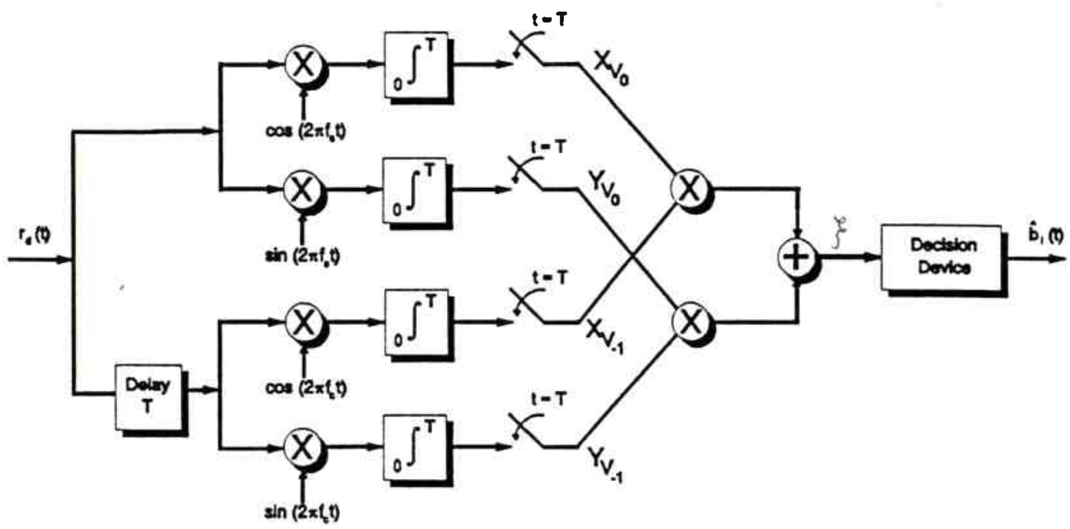

***Figure 12:** The optimal DPSK demodulator*

In DPSK the message bits are coded with two consecutive code bits. If a binary one is to be sent during a bit interval, it is send as a signal with the same phase as the previous bit. If a binary zero is sent, it is transmitted with the opposite phase of the previous bit.

Now the demodulator has to correlate two consecutive signal elements and create the decision variable ξ, which can be expressed as:

$$\xi = X_o X_{-1} + Y_0 Y_{-1} \tag{24}$$

The output of respectively the in-phase and the quadrature branches integrators are given by:

$$X_o = \sqrt{P/8} \sum_k \beta_k \cos[\psi_k] \int_{\lambda T}^{(\lambda+1)T} a_k(t-\tau_k) b_k(t-\tau_k) \delta[f_1(t), f_k(t-\tau_k)] dt + v \tag{25}$$

$$Y_o = \sqrt{P/8} \sum_k \beta_k \sin[\psi_k] \int_{\lambda T}^{(\lambda+1)T} a_k(t-\tau_k) b_k(t-\tau_k) \delta[f_1(t), f_k(t-\tau_k)] dt + \eta \tag{26}$$

$$\eta = \int_o^{T_b} a_1(s) n_c ds \quad ; \quad \nu = \int_o^{T_b} a_1(s) n_s ds \tag{27}$$

where υ and η are zero-mean gaussian random variables with equal variances given by $N_0T/16$. Bit under consideration is bit number $\lambda = jN_b + p$. Actually, the phase Ψ is time-dependent, but it has a fixed value for the time interval under consideration, because the δ function will only be equal to one if $f_k(t - \tau_k) = f_1(t)$, which means a fixed value for f and α.

Because of the limited interval for the delays, two bits will be under consideration: bits λ and λ-1 of the sequence k. We can assume without loss of generality that the jth path between transmitter 1 and receiver is the reference. Therefore, we assume $\tau_{1j} = 0$, $\Psi_{1j} = 0$. We define:

$$R_{1k}(\tau_k) = \int_0^{\tau_k} a_1(t) a_k(t-\tau_k) dt \tag{28}$$

$$\hat{R}_{1k}(\tau_k) = \int_{\tau_k}^{T} a_1(t) a_k(t-\tau_k) dt \tag{29}$$

In these definitions, the periodicity of the spreading sequence has been taken into account.

We can assume without loss of generality that $\tau_{1j} = 0$, $\Psi_{1j} = 0$. As a consequence, we can write:

$$X_o=\sqrt{P/8}\hat{R}_{11}(0)\beta_1 b_\lambda^1+\sqrt{P/8}\sum_{k\neq 1}\beta_k\cos[\psi_k][d_1(b_k^{-1})b_k^{-1}R_{1k}(\tau_k)+d_2(b_k^0)b_k^0\hat{R}_{1k}(\tau_k)]+\nu \tag{30}$$

$$Y_o=\sqrt{P/8}\sum_{k\neq 1}\beta_k\sin[\psi_k][d_1(b_k^{-1})b_k^{-1}R_{1k}(\tau_k)+d_2(b_k^0)b_k^0\hat{R}_{1k}(\tau_k)]+\eta \tag{31}$$

The DPSK demodulator computes the statistics $X_0X_{-1} + Y_0Y_{-1}$ and compares this value with a threshold being actually 0 in our case. X_{-1} and Y_{-1} are given by equations similar to 30 and 31 with λ becoming λ-1. We define:

$$X_{k\neq 1}=\beta_k\cos[\psi_k]R_{1k}(\tau_k) \tag{32}$$

$$X_k=0 \tag{33}$$

$$\hat{X}_{k\neq 1}=\beta_k\cos[\psi_k]\hat{R}_{1k}(\tau_k) \tag{34}$$

$$\hat{X}_{k\neq 1}=0 \tag{35}$$

and similar expressions for Y. Then,

$$X_o=\sqrt{P/8}\hat{R}_{11}(0)\beta_1 b_\lambda^1+\sqrt{P/8}\sum_{k\neq 1}[d_1(b_k^{-1})b_k^{-1}X_k+d_2(b_k^0)b_k^0\hat{X}_k]+\nu \tag{36}$$

$$Y_o=\sqrt{P/8}\sum_{k\neq 1}[d_1(b_k^{-1})b_k^{-1}Y_k+d_2(b_k^0)b_k^0\hat{Y}_k]+\eta \tag{37}$$

With help of all these expressions we are able to give the expressions for the bit error probability for both selection diversity and maximal ratio combining. This will be done in the next two chapters.

5. DETAILED STUDY OF HYBRID DIRECT SEQUENCE - SLOW FREQUENCY HOPPING WITH SELECTION DIVERSITY

Selection diversity means that the largest one of a group of M signals carrying the same information is selected. The order of diversity M is given by the number of resolvable paths times the number of antennas. In this study, where we considered L = 1, the order of diversity is equal to the number of antennas. Diversity means that the antenna giving the largest matched filter output is chosen. The decision variable is $\xi^{M}{}_{max}$ which is the largest among the set of M values.

The average bit error probability for the hybrid DS/SFH system with selection diversity is defined as the error probability due to multi-user interference only. In the next sections an expression for the bit error probability from literature will be altered in a form valid to describe the performance of the hybrid system.

5.1 Definition of the bit error probability in case of selection diversity

In order to be able to determine the performance of the hybrid system, it is necessary to have an expression for the bit error probability. We will take the expressions known from literature and adjust these in order to obtain a valid expression for this specific system.

Since the data bits are equiprobable, the bit error probability can be written as:

$$P[\xi^{M}_{max}<0\,|\,b_1^o b_1^{-1}=1] \tag{38}$$

If we assume multichannel reception in a time-invariant Rician fading channel and DPSK modulation for fixed delays, phase angles, bits and assuming the maximum of the envelope has been found (all other signals are seen as noise), we can use the bit error probability given in [1]:

$$P_e(\beta_{max}|\{\tau_k\})=Q(a,b)-\frac{1}{2}(1+\frac{\mu}{\sqrt{\mu_o\mu_{-1}}})\exp(-\frac{a^2+b^2}{2})I_o(ab) \tag{39}$$

where Q is the Marcum-Q function, which is defined as:

$$Q(a,b)=\int_{b}^{\infty} x.\exp(-\frac{a^2+x^2}{2}).I_o(ax) \tag{40}$$

and, where I_o is the modified Bessel function of the first kind and zero order, which is defined as:

$$I_o(a.b)=\frac{1}{2\pi}\int_{o}^{2\pi}\exp(a.b.\cos\theta)d\theta \tag{41}$$

The parameters a, b ,m ,μ ,μ_{-1} and μ_o are given below.

$$a=\frac{m}{\sqrt{2}}\left|\frac{1}{\sqrt{\mu_o}}-\frac{1}{\sqrt{\mu_{-1}}}\right| \tag{42}$$

$$b=\frac{m}{\sqrt{2}}\left|\frac{1}{\sqrt{\mu_o}}+\frac{1}{\sqrt{\mu_{-1}}}\right| \tag{43}$$

$$m=E[V_o|\beta_{max}] \tag{44}$$

$$\mu_o=var(V_o|\tau_k\ ,\ b) \tag{45}$$

$$\mu_{-1}=var(V_{-1}|\tau_k\ ,\ b) \tag{46}$$

$$\mu=E[(V_o-m)(V_{-1}-m)^*|\tau_k\ ,\ b] \tag{47}$$

where V = X + jY. We obtained the following expressions for these parameters:

$$m=\sqrt{P/8}\,\beta_1 b_k^o \hat{R}_{11}(o) \tag{48}$$

$$\begin{aligned}\mu_o=&\frac{P}{4}\sum_k d_1^2(b_k^{-1})E(X_k^2)+d_2^2(b_k^o)E(\hat{X}_k^2)+2d_1d_2b_k^ob_k^{-1}E(X_k\hat{X}_k)\\&+\frac{P}{4}\sum_k\sum_{k'\neq k}[d_1d_1b_k^{-1}b_{k'}^{-1}E(X_kX_{k'})\ +\ d_1d_2b_k^{-1}b_{k'}^oE(X_k\hat{X}_{k'})\\&\qquad+\ d_2d_1b_k^ob_{k'}^{-1}E(\hat{X}_kX_{k'})\ +\ d_2d_2b_k^ob_{k'}^oE(\hat{X}_k\hat{X}_{k'})]\end{aligned} \tag{49}$$

$$\begin{aligned}\mu_{-1}=&\frac{P}{4}\sum_{k} d_1^2(b_k^{-2})E(X_k^2)+d_2^2(b_k^{-1})E(\hat{X}_k^2)+2d_1d_2b_k^{-1}b_k^{-2}E(X_k\hat{X}_k)\\&+\frac{P}{4}\sum_{k}\sum_{k'\neq k}[d_1d_1b_k^{-2}b_{k'}^{-2}E(X_kX_{k'}) + d_1d_2b_k^{-2}b_{k'}^{-1}E(X_k\hat{X}_{k'})\\&\qquad + d_2d_1b_k^{-1}b_{k'}^{-2}E(\hat{X}_kX_{k'}) + d_2d_2b_k^{-1}b_{k'}^{-2}E(\hat{X}_k\hat{X}_{k'})]\end{aligned} \tag{50}$$

$$\mu=\frac{P}{4}\sum_{k=1}[d_1(b_k^{-1})b_k^{-1}X_k+d_2(b_k^{o})b_k^{o}\hat{X}_k]\sum_{k'}[d_1(b_{k'}^{-2})b_{k'}^{-2}X_{k'}+d_2(b_{k'}^{-1})b_{k'}^{-1}\hat{X}_{k'}] \tag{51}$$

We make the assumption that $\mu_0 = \mu_{-1}$. Therefore, the conditional bit error probability can be simplified and becomes:

$$P_e(\beta_{max}|\{\tau_k\})=\frac{1}{2}(1-\frac{\mu}{\mu_o})\exp(-\frac{b^2}{2})=\frac{1}{2}(1-\frac{\mu}{\mu_o})\exp(-\frac{m^2}{\mu_0}) \tag{52}$$

After that, the bit error probability given in equation 52 has to be averaged for all possible values of β_{max}. The conditioning on the delays has also to be removed. We now have to weight this result by considering the probability density function of the maximum amplitude. The derivation of this PDF is given in appendix A. For an order of diversity equal to M and assuming that all path gains are equally distributed independent Rician variables, this PDF becomes:

$$f_{\beta_{max}}(\beta_{max})=M[\int_0^{\beta_{max}}\frac{z}{\sigma_r^2}\exp(-\frac{S^2+z^2}{2\sigma_r^2})I_o(\frac{Sz}{\sigma_r^2})dz]^{(M-1)}.\frac{\beta_{max}}{\sigma_r^2}\exp(-\frac{S^2+\beta_{max}}{2\sigma_r^2})I_o(\frac{S\beta_{max}}{\sigma_r^2}) \tag{53}$$

In this case, let us remind the reader that the order of diversity is due to the number of antennas only. The order of diversity plays a role by means of the PDF of β_{max}. It is through this PDF that the influence of the order of diversity can be understood. The final bit error probability with selection diversity is now given by:

$$\int_{\beta_{max}}\int_{\tau_k}P_e(\beta_{max}|\{\tau_k\})f_{\beta_{max}}(\beta_{max})f_{\tau_k}(\tau_k)d\beta_{max}d\{\tau_k\} \tag{54}$$

It has been mentioned by many authors that integration over all path delays τ_k is very time consuming. Therefore an approximation has to be made. If the number of users is sufficiently large the central limit theorem can preferably used. As a result of this, the decision parameters μ_{-1} , μ_0 and μ are assumed to have a Gaussian distribution.

5.2 Gaussian approximation of the matched filter outputs

The parameters belonging to the Gaussian distribution are the mean and the variance of the stochastic variable under consideration. Furthermore we have to keep in mind that the variance can be found by:

$$VAR(x) = E[x^2] - (E[x])^2 \tag{55}$$

The means of the three μ-parameters can be written as:

$$E_{\tau,b}(\mu_o) = \frac{P}{4}\sum_k \frac{1}{Q}E_\tau[E(X_k^2)] + \frac{1}{Q}E_\tau[E(\hat{X}_k^2)] \tag{56}$$

$$E_{\tau,b}(\mu_{-1}) = E_{\tau,b}(\mu_o) \tag{57}$$

$$E_{\tau,b}(\mu) = \frac{P}{4}\sum_k \frac{1}{Q}E(X_k\hat{X}_k)] \tag{58}$$

We need to evaluate the square of several parameters. We have that:

$$[E(X_k^2)]^2 = (\sigma_r^2 + \frac{S^2}{2})^2 R_{1k}^4(\tau_k) \tag{59}$$

$$[E(\hat{X}_k^2)]^2 = (\sigma_r^2 + \frac{S^2}{2})^2 \hat{R}_{1k}^4(\tau_k) \tag{60}$$

$$E(X_k^2)E(\hat{X}_k^2) = (\sigma_r^2 + \frac{S^2}{2})^2 R_{1k}^2(\tau_k)\hat{R}_{1k}^2(\tau_k) \tag{61}$$

$$[E(X_k\hat{X}_k)]^2 = (\sigma_r^2 + \frac{S^2}{2})^2 R_{1k}^2(\tau_k)\hat{R}_{1k}^2(\tau_k) \tag{62}$$

Actually, we are interested in the expectation with respect to the path delays of these different squared parameters. When considering the expectation with respect to the path delays, various expressions are encountered. For the computation of various expectations of partial correlation functions, reference can partly be made to the work of Misser [2]. For the delay τ such that we have $0 \le nT_c \le \tau \le (n+1)T_c \le T$, we define the following terms:

$$R_{1k}(\tau)=A_{n1k}T_c+B_{n1k}(\tau-nT_c) \tag{63a}$$

$$\hat{R}_{1k}(\tau)=\hat{A}_{n1k}T_c+\hat{B}_{n1k}(\tau-nT_c) \tag{63b}$$

with

$$A_{n1k}=C_{1k}(n-N) \tag{64}$$

$$B_{n1k}=C_{1k}(n+1-N)-C_{1k}(n-N) \tag{65}$$

$$B_{n1k}=C_{1k}(n+1-N)-C_{1k}(n-N) \tag{66}$$

$$\hat{A}_{n1k}=C_{1k}(n) \tag{67}$$

$$\hat{B}_{n1k}=C_{1k}(n+1)-C_{1k}(n) \tag{68}$$

$$C_{1k}(n) = \begin{cases} \sum_{j=0}^{N-1-n} a_k^j a_1^{j+n} & \textit{for} \quad 0 \le n \le N-1 \\ \sum_{j=0}^{N-1+n} a_k^{j-n} a_1^j & \textit{for} \quad 1-N \le n \le 0 \end{cases} \tag{69}$$

We then get the following expectations:

$$\eta_{k1}=E[R_{1k}(\tau_k)] = \frac{T_c^2}{T}\sum_{n=0}^{N-1}[A_{n1k}+\frac{1}{2}B_{n1k}] \tag{70}$$

$$\eta_{k2}=E[R_{1k}^2(\tau_k)] = \frac{T_c^3}{T}\sum_{n=0}^{N-1}[A_{n1k}^2+A_{n1k}B_{n1k}+\frac{1}{3}B_{n1k}^2] \tag{71}$$

$$\eta_{k4}=E[R_{1k}^4(\tau_k)] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}[A_{n1k}^4+2A_{n1k}^3B_{n1k}+2A_{n1k}^2B_{n1k}^2+A_{n1k}B_{n1k}^3+\frac{1}{5}B_{n1k}^4] \tag{72}$$

$$\nu_{k1}=E[\hat{R}_{1k}(\tau_k)] = \frac{T_c^2}{T}\sum_{n=0}^{N-1}[\hat{A}_{n1k}+\frac{1}{2}\hat{B}_{n1k}] \tag{73}$$

$$\nu_{k2}=E[\hat{R}_{1k}^2(\tau_k)] = \frac{T_c^3}{T}\sum_{n=0}^{N-1}[\hat{A}_{n1k}^2+\hat{A}_{n1k}\hat{B}_{n1k}+\frac{1}{3}\hat{B}_{n1k}^2] \tag{74}$$

$$\nu_{k4}=E[\hat{R}_{1k}^4(\tau_k)] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}[\hat{A}_{n1k}^4+2\hat{A}_{n1k}^3\hat{B}_{n1k}+2\hat{A}_{n1k}^2\hat{B}_{n1k}^2+\hat{A}_{n1k}\hat{B}_{n1k}^3+\frac{1}{5}\hat{B}_{n1k}^4] \tag{75}$$

$$\alpha_{k11}=E[R_{1k}(\tau_k)\hat{R}_{1k}(\tau_k)] = \frac{T_c^3}{T}\sum_{n=0}^{N-1}[A_{n1k}\hat{A}_{n1k}+\frac{1}{2}A_{n1k}\hat{B}_{n1k}+\frac{1}{2}\hat{A}_{n1k}B_{n1k}+\frac{1}{3}B_{n1k}\hat{B}_{n1k}] \tag{76}$$

$$\begin{aligned}\alpha_{k22}=E[R_{1k}^2(\tau_k)\hat{R}_{1k}^2(\tau_k)]=\frac{T_c^5}{T}\sum_{n=0}^{N-1}[&A_{n1k}^2\hat{A}_{n1k}^2+A_{n1k}^2\hat{A}_{n1k}\hat{B}_{n1k}+\frac{1}{3}A_{n1k}^2\hat{B}_{n1k}^2+A_{n1k}\hat{A}_{n1k}^2B_{n1k}\\&+\frac{4}{3}A_{n1k}\hat{A}_{n1k}B_{n1k}\hat{B}_{n1k}+\frac{2}{3}A_{n1k}B_{n1k}\hat{B}_{n1k}^2+\frac{1}{2}\hat{A}_{n1k}B_{n1k}^2\hat{B}_{n1k}+\frac{1}{3}\hat{A}_{n1k}^2B_{n1k}^2+\frac{1}{5}B_{n1k}^2\hat{B}_{n1k}^2]\end{aligned} \tag{77}$$

We then find the following equations valid for $k \neq 1$.

$$E_\tau[E(X_k^2)]=(\sigma_r^2+\frac{S^2}{2})\eta_{k2} \tag{78}$$

$$E_\tau[E(\hat{X}_k^2)]=(\sigma_r^2+\frac{S^2}{2})\nu_{k2} \tag{79}$$

$$E_\tau[E(\hat{X}_k^2)]^2=(\sigma_r^2+\frac{S^2}{2})^2\nu_{k4} \tag{80}$$

$$E_\tau[E(X_k^2)E(\hat{X}_k^2)]=(\sigma_r^2+\frac{S^2}{2})^2\alpha_{k22} \tag{81}$$

$$E_\tau[E(X_k\hat{X}_k)]^2=(\sigma_r^2+\frac{S^2}{2})^2\alpha_{k22} \tag{82}$$

$$E_\tau[E(X_k\hat{X}_k)]^2=(\sigma_r^2+\frac{S^2}{2})^2\alpha_{k22} \tag{83}$$

The conditioning on the delays can now be removed by weighting the conditional bit error probability by the appropriate Gaussian probability density functions. Therefore, we finally get an expression for the average bit error probability in:

$$\int_{\beta_{max}}\int_{\mu_o}\int_{\mu} P_e(\beta_{max}|\{\tau_k\})f_{\beta_{max}}(\beta_{max})f_{\mu_o}(\mu_o)f_\mu(\mu)d\beta_{max}d\mu_o d\mu \tag{84}$$

This integral can be simplified by considering that μ appears explicitly in the function P_e (see eq. 39). We also know that:

$$\int_{-\infty}^{\infty}\mu f(\mu)d\mu \doteq E(\mu) \tag{85}$$

so eventually we get the following expression for the bit error probability in case of selection diversity:

$$\int_{\beta_{max}}\int_{\mu_o}\frac{1}{2}(1-\frac{E(\mu)}{\mu_o})\exp(-\frac{m^2}{\mu_o})f_{\beta_{max}}(\beta_{max})f_{\mu_o}(\mu_o)d\mu_o d\beta_{max} \tag{86}$$

in which $f_{\beta_{max}}(\beta_{max})$ is the probability density function given in eq.(53) and $f_{\mu_o}(\mu_o)$ is the Gaussian probability density function. The other way to describe the performance of a system is the outage probability, which will be discussed in the next section.

5.3 Definition of the outage probability in case of selection diversity

Outage probability is defined as the probability that the instantaneous bit error probability exceeds a preset threshold. We denote the threshold value as ber_o. The instantaneous value of the bit error probability can be obtained by using eq.(86). The averaging over β_{max} should be removed and a fixed value for β_{max}, β, should be substituted. Eq.(86) then alters in

$$P_e(\beta)=\int_{-\infty}^{\infty}\frac{1}{2}[1-\frac{E(\mu)}{\mu_o}]\exp(-\frac{A^2\beta^2T_b^2}{\mu_o})f_{\mu_o}(\mu_o)d\mu_o = ber(\beta) \tag{87}$$

The outage probability in case of selection diversity can then be calculated as follows:

$$P_{out}=P(0\leq\beta_{max}\leq\beta_o)=P(ber(\beta)\geq ber_o)=\int_0^{\beta_o}M[1-Q(S,r)]^{M-1}.r.\exp(-\frac{r^2+S^2}{2\sigma_r^2}).I_o(\frac{Sr}{\sigma_r^2})dr \tag{88}$$

in which β_0 is the value of β at which the instantaneous bit error probability is equal to ber_0. The integrant is just the PDF of β_{max}, where M is the order of diversity.

5.4 The use of forward error correction coding

Error correcting codes can improve the performance of the system in some cases. In this section three specific codes are considered viz., (15,7) BCH code, the (7,4) Hamming code and the (23,12) Golay code. The figures between brackets (n,k) means that k bits are transformed into a block of n bits by coding.

From coding theory we know that a code with a Hamming distance d_{min} is able to correct at least $t=(d_{min}-1)/2$ errors [1]. The Hamming distance for the three types of codes are:

- BCH code: $d_{min}=5$;
- Hamming code: $d_{min}=3$;
- Golay code: $d_{min}=7$.

This means that the codes can correct two, one and three errors respectively. The probability of having m errors in a block of n bits is:

$$P(m,n)=\binom{n}{m}P_e^m(1-P_e)^{n-m} \tag{89}$$

The probability of having more than t errors in a code block of n bits is:

$$P_{ec}=\sum_{m=t+1}^{n} P(m,n) \tag{90}$$

An approximation for the bit error probability after decoding is given in [6] as:

$$P_{ec1}=\frac{1}{n}\sum_{m=t+1}^{n} m.P(m,n) \tag{91}$$

Since the block codes can correct at least t errors, eqs. (90) and (91) are actually upperbounds on the block error and the bit error probability respectively.

Suppose we place a sphere of radius t around each of the possible transmitted code words in the code space. Codes of which all these spheres are disjoint and where every received code word falls in one of those spheres, are called *perfect codes*. They can correct $t=(d_{min}-1)/2$ errors. The Hamming code and the Golay code are examples of such codes.

Codes of which all spheres of radius t are disjoint and where every received code word is at most at distance t+1 from one of the possible transmitted code words, are called *quasi perfect codes*. This codes can sometimes correct t+1 errors.

The biterror probability can be derived using the block error probability given in [1], as:

$$P_{ec2}=\frac{1}{n}\sum_{m=t+1}^{n} m.P(m,n)+\frac{t+1}{n}\left[\binom{n}{t+1}-\beta_{t+1}\right]P_e^{t+1}(1-P_e)^{n-t-1} \tag{92}$$

$$\beta_{t+1}=2^{n-k}-\sum_{i=0}^{t}\binom{n}{i} \tag{93}$$

Like perfect codes, quasi perfect codes are optimum on the binary channel in the sense that they result in a minimum error probability among all codes having the same block length and the same number of information bits. Therefore P_{ec2} is a lowerbound for all perfect linear block codes.

Since the (15,7) BCH is neither a perfect code nor a quasi perfect code, P_{ec1} and P_{ec2} are the upperbound and the lowerbound for the bit error probability respectively for this code.

6. DETAILED STUDY OF HYBRID DIRECT SEQUENCE - SLOW FREQUENCY HOPPING WITH MAXIMAL RATIO COMBINING

Maximal ratio combining is accomplished by summing the demodulation results of a group of signals carrying the same information. The result of this will be used as decision variable. Adding the signal components together means that the noise terms are also added together. The noise is determined by the AWGN and the noise due to multi-user interference. In the next section an expression for the total noise power will be given.

6.1 Noise considerations in case of maximal ratio combining

Let us consider the decision variable again:

$$\xi = Re[\sum_{i=1}^{M} (AT\beta_{1i}b_1^0+N_{1i})(AT\beta_{1i}b_1^{-1}+N_{2i})^*] \tag{94}$$

Each term involved in the combination process is corrupted by AWGN and the multi-user interference. If we assume the multi-user interference to be Gaussian, then the sum of the AWGN and noise due to multi-user interference is represented by N_{1i} and N_{2i}.

We could also make the assumption that the covariance between N_{1i} and N_{2i}^* can be neglected and that the noise terms are independent for different values of the index i, but this not correct of course. The computation of the noise term can be obtained from the computation of μ_0 by taking the expectation over the pathdelays. It is given by:

$$E_\tau(\mu_o)(\frac{8}{P})=2\frac{1}{Q}(\sigma_r^2+\frac{S^2}{2})\sum_{k\neq1} E_\tau[R_{1k}^2(\tau_k)]+2\frac{1}{Q}(\sigma_r^2+\frac{S^2}{2})\sum_{k\neq1} E_\tau[\hat{R}_{1k}^2(\tau_k)]+2\sigma_N^2(\frac{8}{P}) \tag{95}$$

For path k, we have a signal power given by: $P\beta_k^2$ and a noise power given by NT = $E_\tau(\mu_0)$. Therefore, the signal to noise ratio of path k is given by:

$$\frac{P\beta_k^2}{E_\tau(\mu_o)} \tag{96}$$

With this information we are able to give an expression for the bit error probability in case of MRC. This will be done in the next section.

6.2 Definition of the bit error probability in case of maximal ratio combining

If we assume that the values of the path gains are known, we can use the bit error probability given in [1] for maximal ratio combining and diversity of order M:

$$P_{e,mrc|\gamma_b}=\frac{1}{2^{(2M-1)}}\exp(-\gamma_b)\sum_{k=0}^{M-1}p_k\gamma_b^k \qquad (97)$$

with

$$p_k=\frac{1}{k!}\sum_{n=0}^{M-1-k}\binom{2M-1}{n} \qquad (98)$$

$$\gamma_b=\frac{E}{N}\sum_{k=1}^{M}\beta_k^2 \qquad (99)$$

where N is the power spectral density corresponding to the total noise power. Then we have to take the expectation of $P_{e,mrc|\gamma b}$ with respect to γ_b in order to find the averaged bit error probability. The probability density function of γ_b is given in [1] as:

$$p_{\gamma_b}(\gamma_b)=\frac{1}{2E/N}\left(\frac{\gamma_b}{(E/N)S_M^2}\right)^{\frac{(M-1)}{2}}\exp\left(-\frac{(S_M^2+\gamma_b N/E)}{2}\right)I_{M-1}\left(\frac{S_M}{\sigma^2}\sqrt{\frac{\gamma_b}{E/N}}\right) \qquad (100)$$

with $S_M^2 = MS^2$ and I_{M-1} is the (M-1)th-order modified Bessel function. We finally compute the bit error probability by:

$$\int_{\gamma_b}P_{e,mrc|\gamma_b}\,p_{\gamma_b}(\gamma_b)d\gamma_b \qquad (101)$$

6.3 Definition of the outage probability in case of maximal ratio combining

The as with selection diversity, the instantaneous value of the bit error probability can be obtained by using equation (101). The averaging over γ_b should be removed and a fixed value for γ_b, γ_o, should be substituted. Equation (101) then alters in

$$P_e(\gamma_{b_o})=\frac{1}{2^{(2M-1)}}\exp(-\gamma_{b_o})\sum_{k=0}^{M-1} p_k\gamma_{b_o}^k = ber_o \tag{102}$$

The outage probability in case of maximal ratio combining is given as:

$$P_{out}=P(ber(\gamma_b)\geq ber_o)=\int_0^{\gamma_{b_o}} p(\gamma_{b_o})d\gamma_{b_o} \tag{103}$$

where ber_o is the preset bit error threshold.

7. NUMERICAL RESULTS OF THE PERFORMANCE

The performance of the investigated systems is expressed in terms of bit error rate or outage probability, both as a function of the signal to noise ratio. The signal to noise ratio is given as the ratio of energy per bit E_b and the spectral density of the AWGN N_0. In the next sections we first evaluate the performance of the hybrid DS/SFH system, the SFH system and DS system seperately and then we make a comparison of the three systems.

7.1 The performance of the hybrid direct sequence-slow frequency hopping system

The performance analysis of the hybrid system involves an evaluation of the effect of the number of frequencies, the order of diversity and the effect of FEC codes on the performance. Figure 13 shows the effect of the number of frequencies in case of a system with selection diversity.

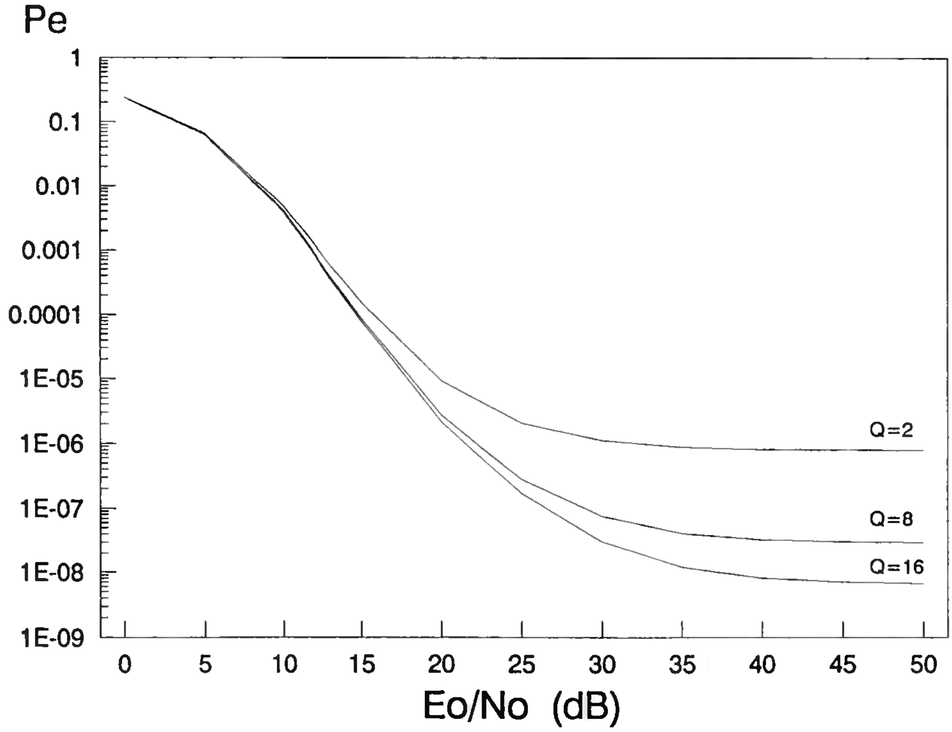

Figure 13: Effect of the number of frequencies on the performance of hybrid DS/SFH with selection diversity; L=1, M=2, R=6,8 dB, N=255, R_c=32 kbit/s.

From figure 13 we see that an increase of the number of frequencies in the hopping pattern enhances the performance of the hybrid system. This is due to a decrease of the multi-user interference, which decreases the probability of a hit. At low values of the signal to noise ratio the BER is independent of the number of frequencies, because the AWGN is dominating.
An increase of the number of frequencies requires an increasing frequency range in the frequency domain. A measure for the frequency range occupied by the hybrid DS/SFH system is given by N.Q.W., in which N is the period of the spreading codes, Q is the number of frequencies in the hopping pattern and W is the message bandwidth.

Figures 14 and 15 show the influence of the order of diversity in comparison with nondiversity with FEC coding for N=255 and N=127 in case of hybrid DS/SFH with selection diversity.

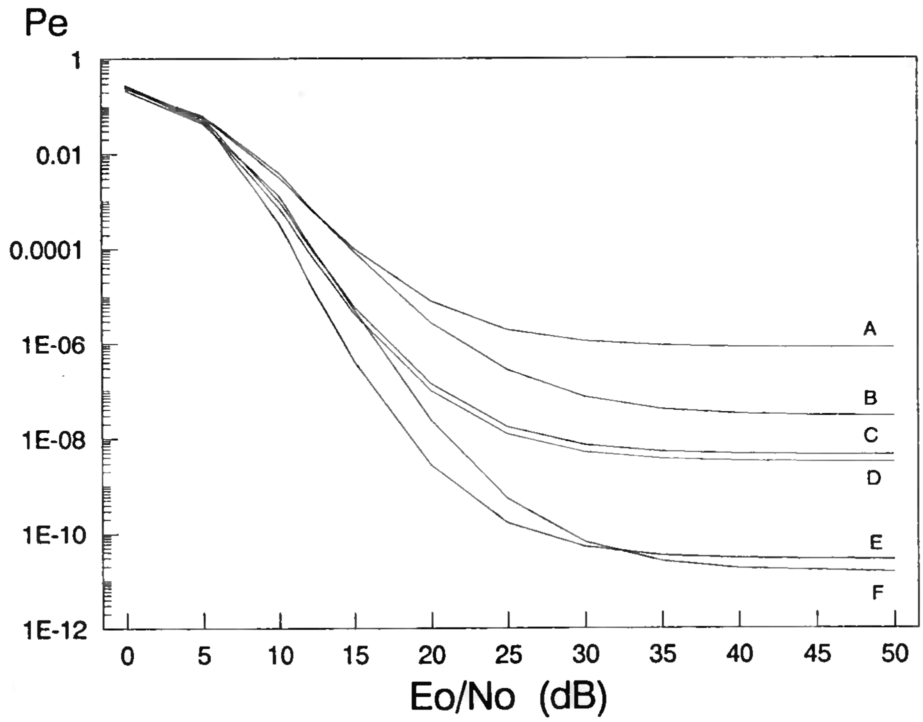

***Figure 14:** Comparison of a hybrid DS/SFH with selection diversity and a nondiversity hybrid system with Forward Error Correction coding; L=1, R=6,8 dB, Q=8, N=255, R_c=32 kbit/s*

A: M=1 with the (7,4) Hamming code;

B: M=2 without coding;

C: M=1 with the (15,7) BCH code (upper-bound);

D: M=1 with the (15,7) BCH code (lower-bound);

E: M=1 with the (23,12) Golay code;

F: M=3 without coding.

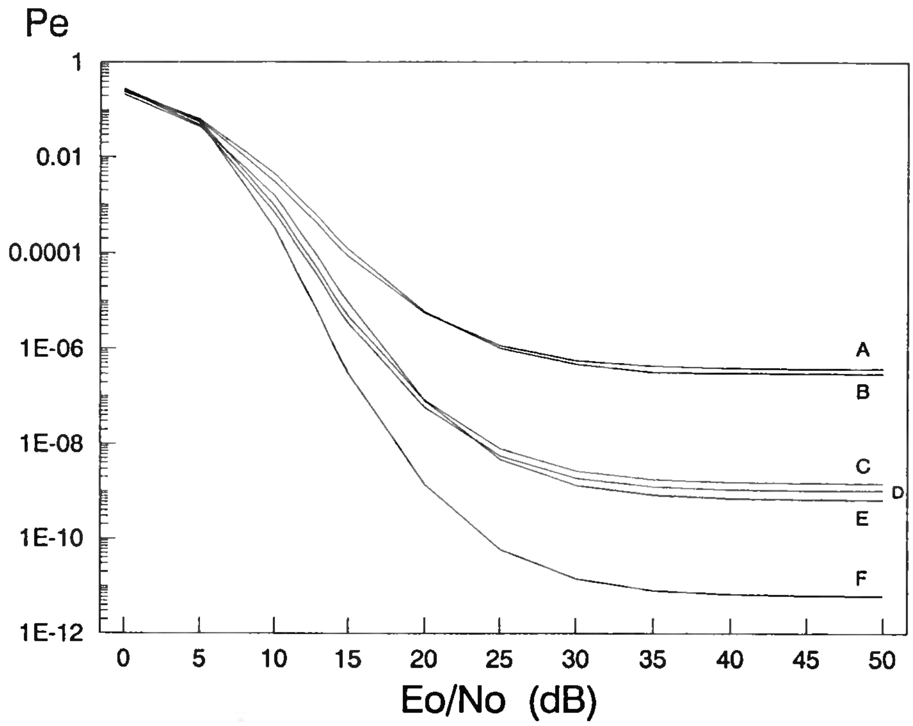

Figure 15: Comparison of a hybrid DS/SFH with selection diversity and a nondiversity hybrid system with Forward Error Correction coding; L=1, R=6,8 dB, Q=8, N=127, R_c=32 kbit/s

A: M=1 with the (7,4) Hamming code;

B: M=2 without coding;

C: M=1 with the (15,7) BCH code (upper-bound);

D: M=1 with the (15,7) BCH code (lower-bound);

E: M=3 without coding;

F: M=1 with the (23,12) Golay code;

From figure 14 we see that the BER of 10^{-8} of a noncoding hybrid system with M=2 is almost the same as the BER of a nondiversity hybrid system with BCH coding. Furthermore nondiversity with Golay coding gives an equivalent BER of 10^{-11} as in case of M=3 in a noncoding system. At first sight the application of the Golay code in a nondiversity hybrid system leads to a saving of the number of antennas. However the frequency range used by the system, which is proportional to N.Q.W, is still considerable.

Figure 15 shows the effect of a reduced number of spreading codes N. In comparison with figure 14 it is seen that in the performance of the noncoding system decreases for N=127. However the performance of the hybrid system with FEC coding increases for N=127 in comparison with the same system with N=255. A nondiversity hybrid system with N=127 and Golay coding can achieve a BER of 10^{-12} and the same system with N=255 can achieve a BER of 10^{-11}. Besides the hybrid system with N=127 only requires half of the frequency range compared with the hybrid system with N=255.

The outage probability of the hybrid DS/SFH system is investigated for selection diversity and maximal ratio combining, both with diversity order of 2 and 4. The threshold in this case is 0,01. Figure 16 shows the performance of the hybrid system in terms of outage probability.

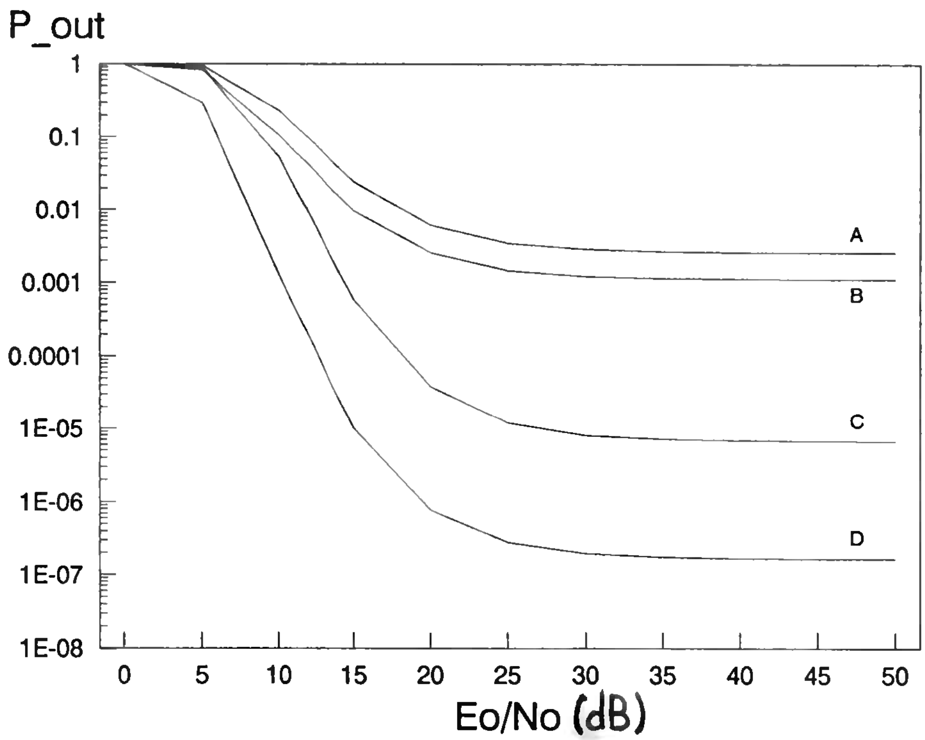

Figure 16: *Outage probability of hybrid DS/SFH with selection diversity and maximal ratio combining; L=1, R=6,8 dB, Q=8, N=255,* R_c*=144 kbit/s*

A: M=2 with selection diversity;

B: M=2 with maximal ratio combining;

C: M=4 with selection diversity;

D: M=4 with maximal ratio combining.

From figure 16 it is seen that there's almost no difference between the performance with selection diversity and the performance with maximal ratio combining for M=2. But for M=4 it is obvious that maximal ratio combining yields a better performance than selection diversity.

7.2 The performance of the slow frequency hopping system

By taking the spreading sequences equal to one everywhere in the hybrid DS/SFH system and by taking the chip time T_c equal to the bit duration T_b , we get a slow frequency hopping system. It is obvious that because of these changes in spreading sequences and chip time, the cross correlation functions have to be recalculated.

We investigated the effect of the number of frequencies, the order of diversity and the influence of FEC coding on the performance of the SFH system with maximal ratio combining. Figure 17 shows the influence of the number frequencies in the hopping pattern.

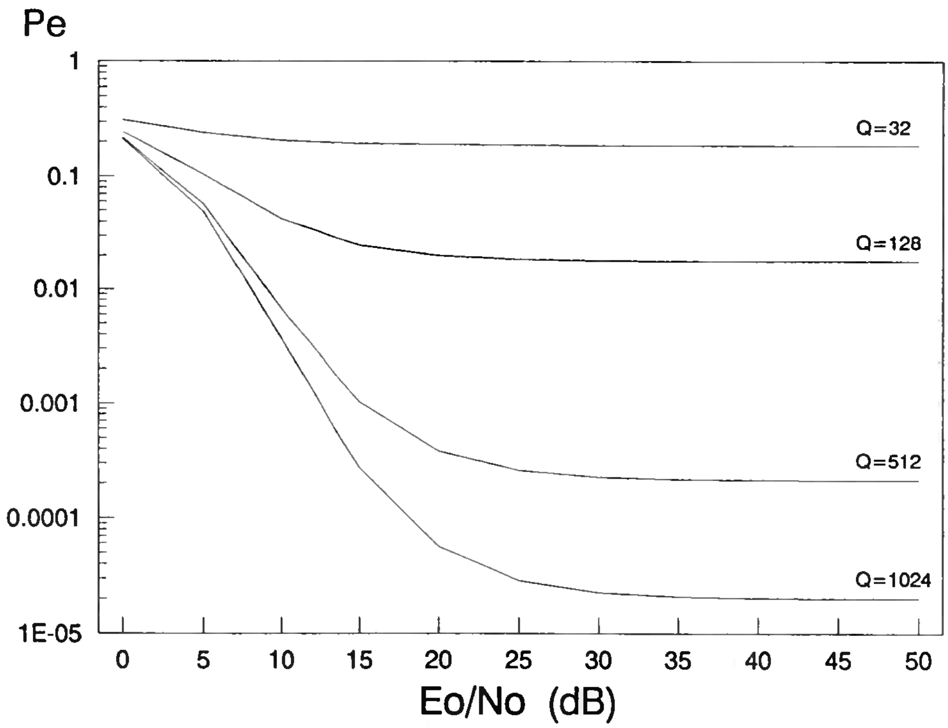

Figure 17: Influence of the number of frequencies on the performance of slow frequency hopping with maximal ratio combining; L=1, M=2, R=6,8 dB, R_c=32 kbit/s

From figure 17 we see that a low number of frequencies in a SFH system leads to an unacceptable BER for indoor radio. Only with a number of frequencies exceeding 1024 leads to a BER of 10^{-5} or lower. It is obvious that an increase of the number of frequencies in order to obtain a better performance requires an increase of the frequency band occupied by this SFH system. A mesure for the frequency range in case of slow frequency hopping is Q.W, in which W is the message bandwidth.

Figure 18 shows a comparison of nondiversity slow frequency hopping with FEC coding and slow frequency hopping with maximal ratio combining

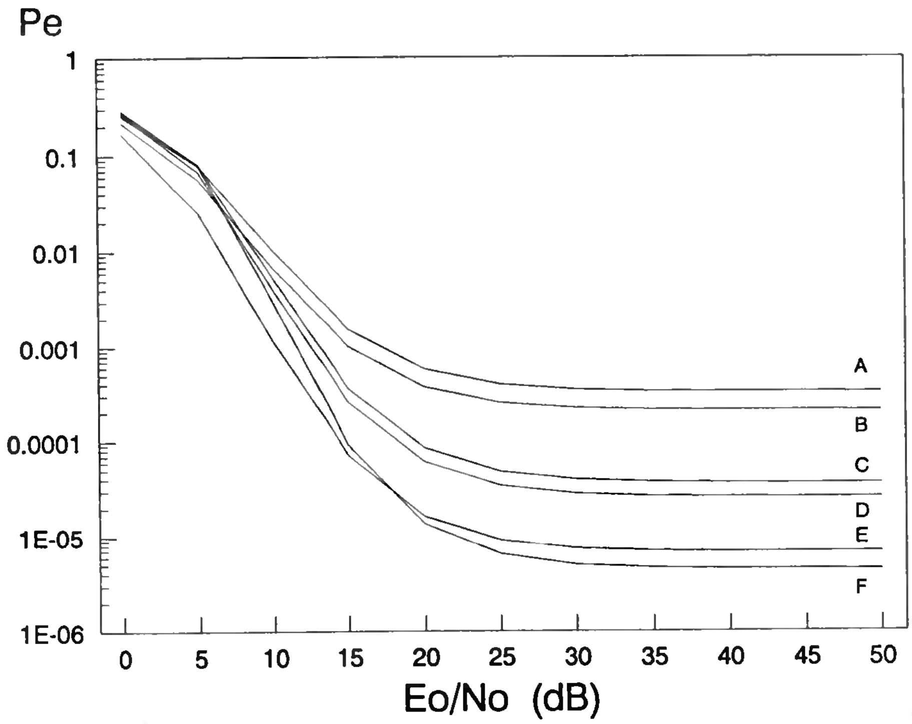

Figure 18: Comparison of nondiversity slow frequency hopping with Forward Error Correction codes and slow frequency hopping with maximal ratio combining; L=1, Q=512, R=6,8 dB, R_c*=32 kbit/s*

A: M=1 with the (7,4) Hamming code;

B: M=2 without coding;

C: M=1 with the (15,7) BCH code (upper-bound);

D: M=1 with the (15,7) BCH code (lower-bound);

E: M=3 without coding;

F: M=1 with the (23,12) Golay code.

From figure 18 we see that a nondiversity SFH-system with Hamming coding yields almost the same BER of 10^{-4} as a SFH-system with order of diversity of two. A BER of 10^{-5} can be achieved by nondiversity with the BCH code. The best performance (a BER of 10^{-6}) can be achieved by nondiversity SFH with the Golay code. This is the same result obtained by a noncoding SFH-system with order of diversity equal to three. Compared with figure 17 we see that the application of FEC coding in a nondiversity SFH-system with 512 frequencies leads to almost the same performance as a SFH-system with 1024 frequencies and order of diversity of two. This means a reduction of the necessary frequency range by 50%. Besides nondiversity means that only one maximal ratio combining receiver combination is required.

7.3 The performance of the direct sequence system

Systems based on direct sequence have been investigated in previous studies [2]. In order to compare the performance of these systems with the performance of slow frequency hopping and hybrid DS/SFH systems, part of the results of the work of H.S. Misser [2] have been used in this section. In figure 19 the performance of a nondiversity direct sequence system with FEC coding is compared with noncoding direct sequence with selection diversity.

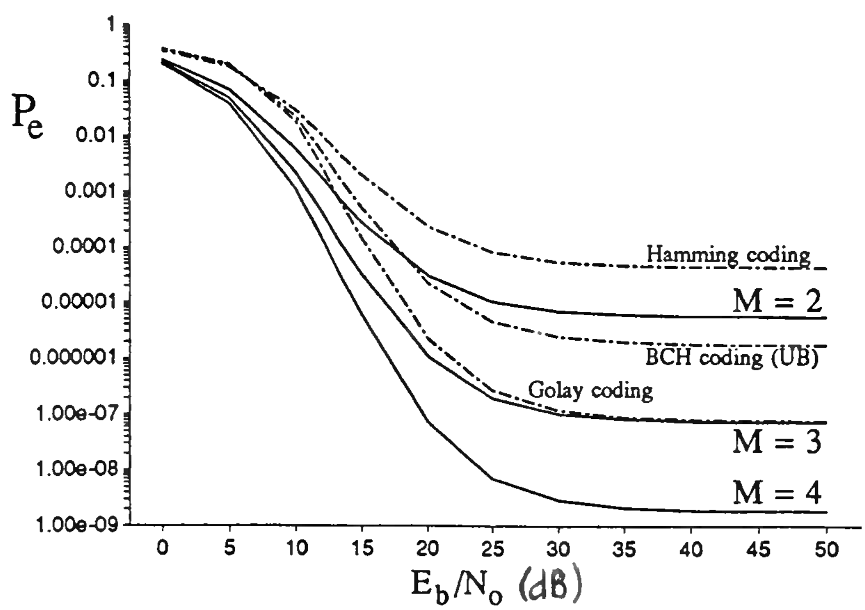

Figure 19: Comparison of nondiversity direct sequence with Forward Error Correction coding and direct sequence with selection diversity; L=1, N=255, R=6,8 dB, R_c=32 kbit/s.

From figure 19 it is seen that nondiversity direct sequence with BCH code yields almost the same BER of 10^{-6} as in case of selection diversity with M=2. However nondiversity direct sequence with Golay coding gives a BER of approximately 10^{-8}, which is the same BER as direct sequence with M=3. In order to obtain an even better performance of let's say 10^{-9}, an order of diversity of four is needed. As we have seen before, FEC coding can lead to a saving with respect to the number of antennas.

Figure 20 shows what happens if the number of resolvable paths is increased from one to five.

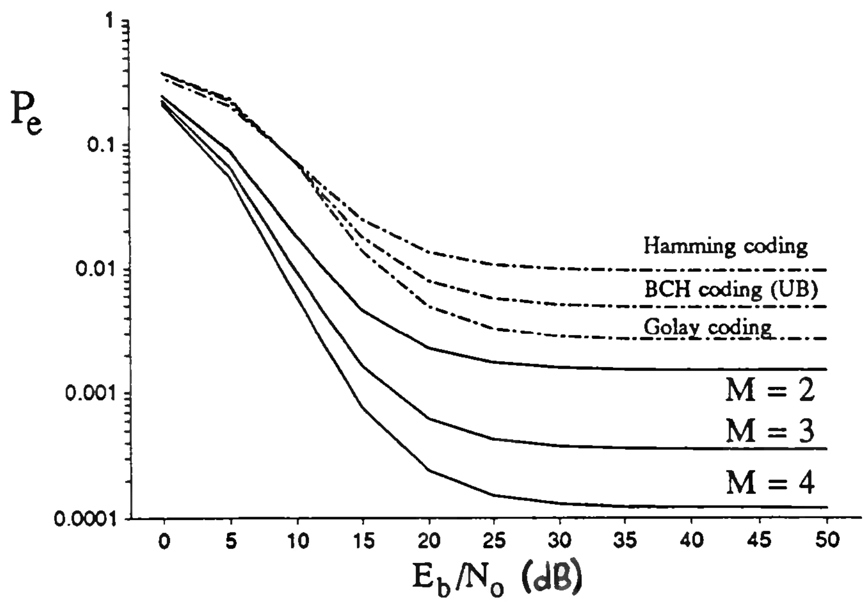

Figure 20: *Comparison of nondiversity direct sequence with Forward Error Correction coding and direct sequence with selection diversity; L=5, N=255, R=6,8 dB, R_c=32 kbit/s.*

It is seen from figure 19 and figure 20 that for low values of L (such as L=1) the use of FEC coding without diversity can lead to acceptable bit error probabilities. However, for larger values of L (L=5) the use of FEC coding only is not sufficient. In that case diversity is necessary either in combination with FEC coding or not.

7.4 Comparison of the performance of the three systems

In this section a comparison is made of a hybrid DS/SFH system, a direct sequence system and a slow frequency hopping system with maximal ratio combining. Figure 21 shows the influence of the number of frequencies on the performance hybrid DS/SFH and slow frequency hopping both with order of diversity two.

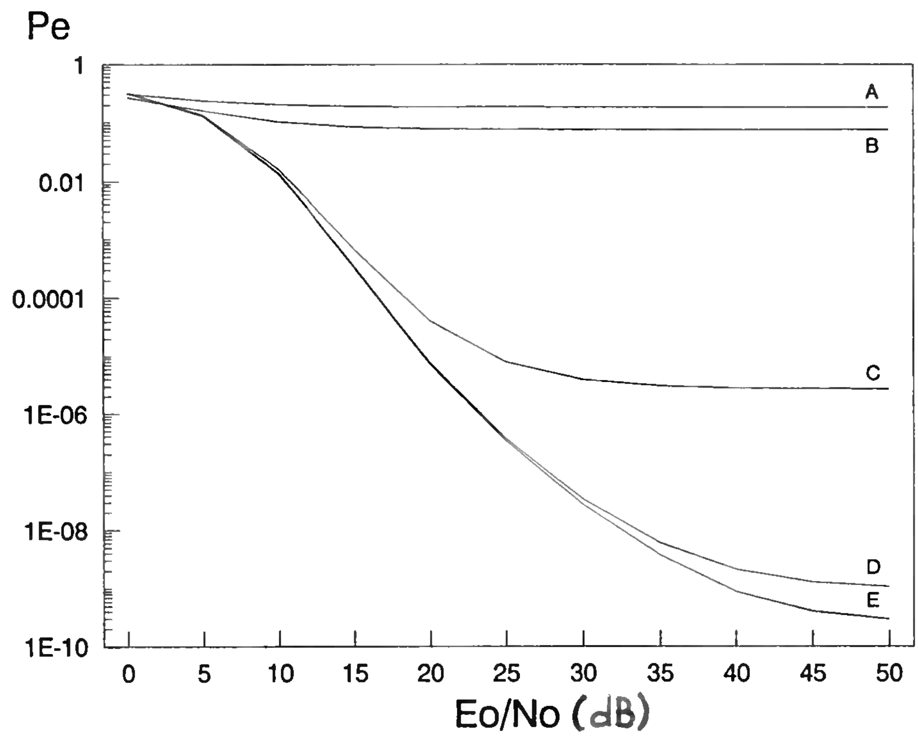

Figure 21: Comparison of the performance of hybrid DS/SFH, direct sequence and slow frequency hopping with maximal ratio combining; $L=1$, $M=2$, $N=255$, $R=6,8$ dB, $R_c=32$ kbit/s
A: slow frequency hopping with $Q=32$;
B: slow frequency hopping with $Q=64$;
C: direct sequence;
D: hybrid DS/SFH with $Q=32$;
E: hybrid DS/SFH with $Q=64$.

It is seen again from figure 21 that slow frequency hopping with a relative low number of frequencies (such as Q=32 or Q=64) yields a relative high bit error probability. Only with a number of frequencies larger than 1024 a slow frequency hopping system can give a BER almost equivalent with direct sequence, but as told before, this goes at expence of an increased frequency band. Hybrid DS/SFH yields the best performance of all three systems. With a number of frequencies of 32 and 64 a BER of 10^{-9} respectively 10^{-10} can be obtained. However, we have to keep in mind that in order to obtain a diversity order of two with maximal ratio combining, two MRC receiver combinations are required! If we want to obtain a diversity order of two with only one MRC receiver, we have to consider two resolvable paths. But this measure will introduce multipath interference which will certainly decrease the performance in case of M=2.

In figure 22 the influence of the order of diversity in all three systems with maximal ratio combining is shown.

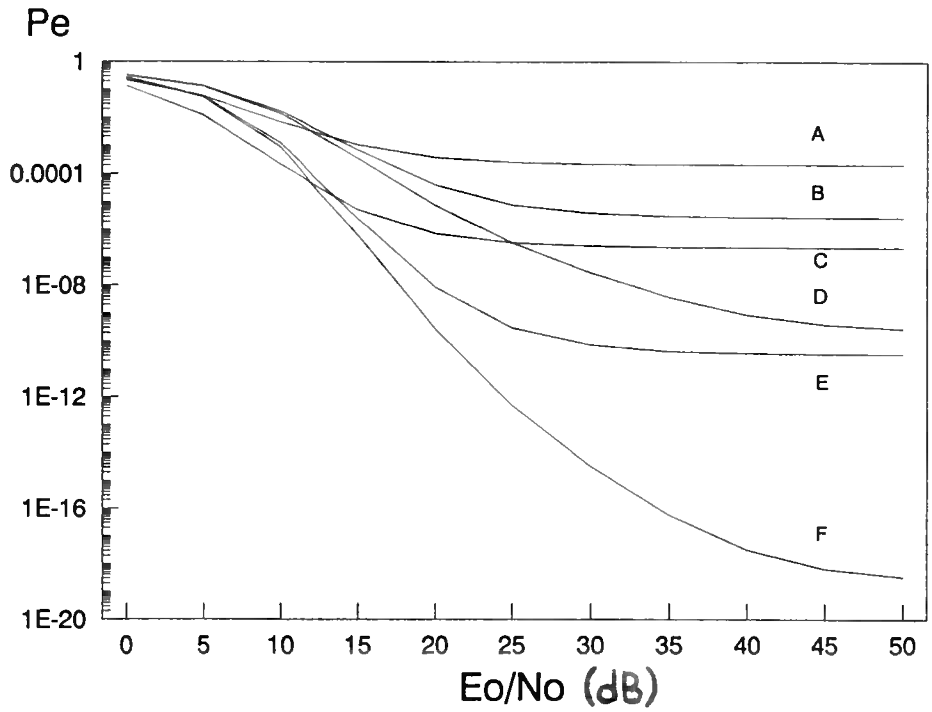

***Figure** 22: Comparison of the performance of hybrid DS/SFH, direct sequence and slow frequency hopping with maximal ratio combining; L=1, N=255, R=6,8 dB, R_c=32 kbit/s*

A: slow frequency hopping with M=2 and Q=128;

B: direct sequence with M=2;

C: slow frequency hopping with M=4 and Q=128;

D: hybrid DS/SFH with M=2 and Q=64;

E: direct sequence with M=4

F: hybrid DS/SFH with M=4 and Q=64.

Figure 22 shows that unless the number of frequencies of 128 in the slow frequency hopping system, the performance can be improved by an increase in diversity order. But this goes at expence of four MRC receiver combinations, which is almost impracticable!

Hybrid DS/SFH with M=2 yields almost the same BER as direct sequence with M=4. These orders of diversity can only be obtained by using more than one MRC receiver or considering more than two respectively four resolvable paths. As we discussed before, this measure will introduce multipath interference, which will decrease the performance of both the hybrid as the direct sequence system.

In figure 23 a comparison of hybrid DS/SFH and direct sequence both with selection diversity is shown. We have considered the number of frequencies in the hybrid system and the order of diversity (which is equal to the number of antennas because L=1) for both systems.

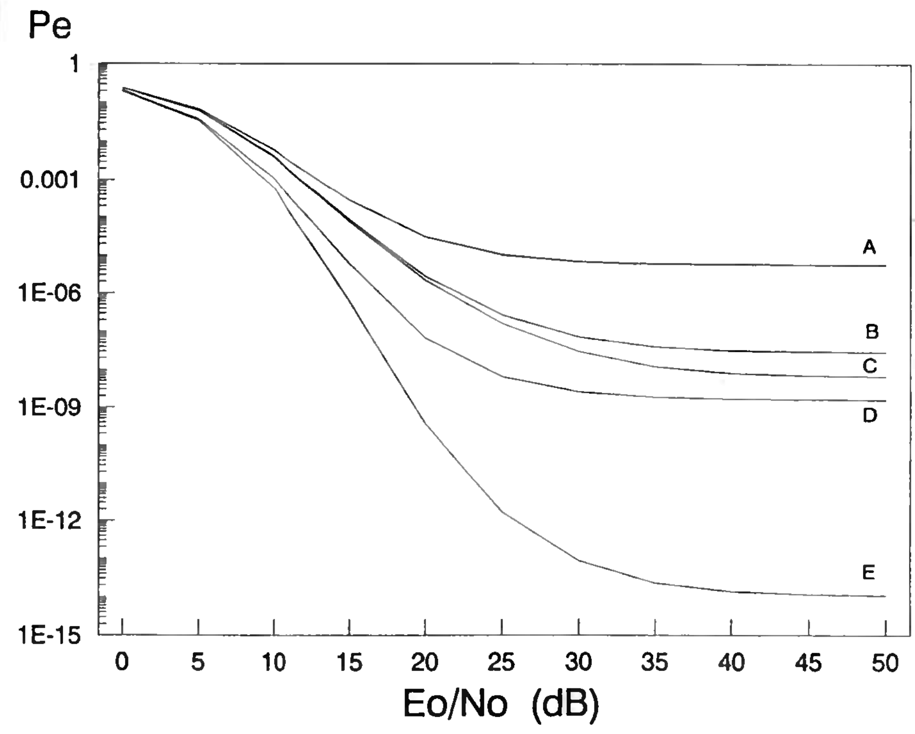

Figure 23: Comparison of the performance of hybrid DS/SFH and direct sequence with selection diversity; L=1, N=255, R=6,8 dB, R_c=32 kbit/s

A: direct sequence with M=2;

B: hybrid DS/SFH with M=2 and Q=8;

C: hybrid DS/SFH with M=2 and Q=16;

D: direct sequence with M=4;

E: hybrid DS/SFH with M=4 and Q=8.

We see from figure 23 that direct sequence with M=4 yields almost the same BER of 10^{-9} as hybrid DS/SFH with M=2 and Q=16. The hybrid system requires a larger frequency range than direct sequence, because Q.N.W is a measure for the occupied frequency range. So here it is obvious that an exchange can be made between the number of antennas and frequency range. If the frequency range is limited for one reason one could use a direct sequence system with four antennas instead of hybrid DS/SFH with two antennas, which yield the same performance.

If extreme low bit error probabilities are demanded, one could use hybrid DS/SFH with four antennas.

8. CONCLUSIONS AND RECOMMENDATIONS

From the numerical results we can draw the following conlusions:

1. In general receivers using maximal ratio combining yield better performance than receivers using selection diversity. Especially when the order of diversity increases the advantage of maximal ratio combining is obvious. However In case of one resolvable path this higher diversity order can only be obtained by using an equivalent number of MRC-receiver combinations. Although this is an interesting aspect in a theoretical model, in practice it's too expensive and impracticable.

   Another way of having a higher order of diversity is considering more than one resolvable path, but this will introduce multipath interference and so a decrease in performance. So actually we have determined the lower bounds of the bit error rates in case of maximal ratio combining with diversity order larger than one.

2. An increase of the number of frequencies in the slow frequency system enhances the performance. But this goes at expence of an increasing frequency range. Besides a slow frequency hopping system needs a considerable number of frequencies to have an acceptable performance, however this number can be decreased by using FEC coding. Especially the use of the (23,12) Golay code in a nondiversity slow frequency hopping system yields even a better performance than a slow frequency hopping system with maximal ratio combining and diversity order of three.

3. The performance of direct sequence increases considerably with an increase of the diversity order. In case of one resolvable path we see that the application of FEC coding in a nondiversity system leads to the same performance as direct sequence with selection diversity of order $M=2$ or $M=3$. This means that the use of FEC codes leads to a saving of the number of antennas. However if the number of resolvable paths increases the use of FEC coding only is not sufficient. In that case diversity is necessary.

4. An increase of the number of frequencies in the hybrid DS/SFH system also enhances the performance. Even with a small number of frequencies and selection diversity with

order two leads to a relatively low BER (approximately 10^{-8}). Nondiversity hybrid DS/SFH with FEC coding and lead to a considerable saving with respect to the number of antennas, especially the use of the (23,12) Golay code in combination with N=127 gives even a better performance than a noncoding hybrid system with selection diversity and with order of diversity of three.

Comparison of hybrid DS/SFH with direct sequence shows that bandwidth can be exchanged with the number of antennas.

5. Comparison of all three systems shows that a hybrid DS/SFH system is in general superior to a direct sequence system and a slow frequency hopping system. However it is highly depentdent on the available frequency range and the application (which determines the allowed bit error rate), which system will be used. If the available frequency range is too limited one could use multiple antennas or FEC codes to obtain a required bit error rate.

Recommendations for further research:

- Adjustment of the computer software in order to calculate the performance of the hybrid DS/SFH system for more than one multiple resolvable path (L>1);

- Calculation of the outage probability, throughput and the delay of the hybrid system with L>1;

- Simulation of the hybrid system with help of computer software;

- A total comparison of the theoretical results, experimental results and the results obtained from the simulation and see if the model has to be adjusted.

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APPENDIX: DERIVATION OF THE PDF OF THE MAXIMAL PATH GAIN

Selection diversity is based on selecting the strongest signal from several statistically independent signals carrying the same data. These signals are usually the multiple resolvable paths due to inherent diversity of spread spectrum, but these signals can also arrive from different antennas, in order to increase the order of diversity.

Suppose we have M identically distributed random variables $\{\beta_1,...,\beta_M\}$. For the largest random variable β_{max}, the following inequality holds:

$$\beta_{max} > \beta_i \qquad i \in \{1,2,...,M\} \tag{A.1}$$

The probability that $\beta_{max} < y$, denoted as $P_{\beta max}(y)$ is now given by:

$$P_{\beta_{max}}(y) = Pr\{\beta_1 < y\}. \;...\; .Pr\{\beta_M < y\} = P_{\beta_1}(y). \;...\; .P_{\beta_M} \tag{A.2}$$

This means that the CDF of β_{max} is the product of the CDFs of the M path gains.

Since β is Rician distributed, β^2 has a non-central chi-square probability distribution with two degrees of freedom. With the PDF given in equation 13 we can derive the CDF just by integrating the PDF.

$$P_{\beta_i}(\beta_i) = \int_0^{\beta_i} \frac{r}{\sigma^2} \exp\left(-\frac{S^2+r^2}{2\sigma^2}\right).I_o\left(\frac{S.r}{\sigma^2}\right) \tag{A.3}$$

The CDF of β_{max} is now given by:

$$P_{\beta_{max}}(\beta_{max}) = [P_{\beta_i}(\beta_i)]^M \tag{A.4}$$

The PDF can now be obtained by calculating the derivative with respect to β_{max}.

$$f_{\beta_{max}}(\beta_{max}) = M.[P_{\beta_{max}}(\beta_{max})]^{M-1}.\frac{dP_{\beta_{max}}(\beta_{max})}{d\beta_{max}} \tag{A.5}$$

By calculating this derivative, we get the following PDF:

$$f_{\beta_{max}}(\beta_{max})=M[\int_0^{\beta_{max}} \frac{z}{\sigma_r^2}\exp(-\frac{S^2+z^2}{2\sigma_r^2})I_o(\frac{Sz}{\sigma_r^2})dz]^{(M-1)}.\frac{\beta_{max}}{\sigma_r^2}\exp(-\frac{S^2+\beta_{max}}{2\sigma_r^2})I_o(\frac{S\beta_{max}}{\sigma_r^2}) \qquad (A.6)$$