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# Adaptive Disturbance-Based High-Order Sliding-Mode Control for Hypersonic-Entry Vehicles

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In this paper, an adaptive, disturbance-based sliding-mode controller for hypersonic-entry vehicles is proposed. The scheme is based on high-order sliding-mode theory, and is coupled to an extended sliding-mode observer, able to reconstruct online the disturbances. The result is a numerically stable control scheme, able to adapt online to reduce the error in the presence of multiple uncertainties. The transformation of a high-order sliding-mode technique into an adaptive law by using the extended sliding-mode observer is, together with the multi-input/multi-output formulation for hypersonic-entry vehicles, the main contribution of this paper. The robustness is verified with respect to perturbations in terms of initial conditions, atmospheric density variations, as well as mass and aerodynamic uncertainties. Results show that the approach is valid, leading to an accurate disturbance reconstruction, to a better transient, and to good tracking performance, improved of about 50% in terms of altitude and range errors with respect to the corresponding standard sliding-mode-control approach.

Nomenclature		Т	=	temperature, K	
$a_h$	=	third time derivative of h not dependent on control,	$T_h$	=	temperature derivative with respect to the altitude,
		m/s <sup>3</sup>		_	K/M feedback angle of attack rate, rad/s
$a_V$	=	second time derivative of $V$ not dependent on	$u_{\alpha}$	_	feedback hank-angle rate, rad/s
		control, m/s <sup>3</sup>	$V^{\mu}$	_	velocity modulus m/s
$a(\mathbf{x}), b(\mathbf{x})$	=	generic functions	$\dot{\mathbf{x}}(t)$	=	state vector
$b_{h,\alpha}$	=	third time derivative of h dependent on $u_{\alpha}$ , m/s <sup>2</sup>	α	=	angle of attack, rad
$D_{h,\mu}$	=	third time derivative of <i>n</i> dependent on $u_{\mu}$ , m/s <sup>2</sup>	γ	=	flight-path angle, rad
$b_{V,\alpha}$	_	second time derivative of V dependent on $u_{\alpha}$ , m/s	$\gamma_{gas}$	=	specific-heat ratio
$C_{V,\mu}$	_	drag coefficient	$\theta$	=	longitude, rad
$C_D$	=	lift coefficient	$\kappa_i^m$	=	<i>i</i> th nonlinear sliding-mode gains of the <i>k</i> th variable
$D^{-L}$	=	drag acceleration, $m/s^2$	$\lambda_i^m$	=	<i>i</i> th linear sliding-mode gains of the <i>k</i> th variable
d	=	generic unknown function	μ	=	bank angle, rad
$d_h$	=	altitude-related disturbance/uncertainty, m/s <sup>3</sup>	ρ	=	atmospheric density, kg/m <sup>3</sup>
$d_V$	=	velocity-related disturbance/uncertainty, m/s <sup>3</sup>	σ	=	generic sliding variable, standard deviation
8	=	gravitational acceleration, m/s <sup>2</sup>	$\phi$	=	latitude, rad
h	=	altitude, m	Ψ	=	
k <sub>i</sub>	=	nonlinear sliding-mode gains of the <i>i</i> th variable	$(\cdot)$	=	first time derivative, $(\cdot)/s$
L	=	lift acceleration, m/s <sup>2</sup>	$(\cdot)$	=	second time derivative, $(\cdot)/s^2$
Μ	=	Mach number	ö	=	third time derivative. $(\cdot)/s^3$
$n_z$	=	load factor	õ	=	estimate of $(\cdot)$ $(\cdot)$
$\frac{Q}{2}$	=	dynamia prossum. N/m <sup>2</sup>	õ	_	residual of $(\cdot)$ , $(\cdot)$
q P	=	ayinamic pressure, $N/m^2$	$(\cdot)$	_	reference variable
r gas	_	radial position m	()	_	generic subscript
r	_	relative degree of the system	$O(\cdot)$	—	Senerie subscript
'D sat	_	saturation function			I Index landform
Jul		summer of the second			I. Introduction

sgn sign function

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**E** NTRY guidance of an unpowered vehicle is a difficult task, as the problem is governed by nonlinear equations of motion, and multiple constraints acting on the vehicle must be taken into account. For this reason, decades of research have provided several methods for dealing with this problem. Among these, the so-called Apollo entry guidance [1] has gained popularity in terms of reliability, and has been used since the beginning of the Apollo program itself until the last NASA missions (e.g., the Mars Science Laboratory [2,3]). This method is based on the design of one or more reference dragvelocity (or, alternatively, drag-energy) profiles that satisfy the requirements of the mission. In the hypothesis of having a nominal angle of attack, it is possible to extract the longitudinal states, that is, the altitude, velocity, and flight-path angle, as well as the bank-angle

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command from the drag-energy model. Lateral motion is usually controlled with so-called bank reversals, consisting of a rapid change of the sign of the bank angle, but preserving its modulus. With this approach, it is possible to keep the heading-alignment error under control, while minimizing the impact of the lateral guidance on the longitudinal performance of the system.

Over the last years, an alternative to the class of drag-energy methods has arisen, based on the use of optimal-control theory, and several tools have been developed over the years, such as DIDO [4,5] and SPARTAN [6,7]. The problem is described in terms of a cost function to be minimized (or maximized) and the differential equations representing the motion of the vehicle. Moreover, other constraints, such as the load factor and the heat flux, can be included in the optimization problem can be transcribed and solved with one of many off-the-shelf available software (e.g., SNOPT [8] or IPOPT [9]).

These two families of methods rely on several assumptions, though. For instance, they use analytical or semi-analytical models for the gravity field and the atmospheric density. Moreover, dispersions on the initial states, the mass of the vehicle, and other external disturbances affect the performance of the system. Therefore, a feedback scheme, able to track the desired trajectory and to reject these disturbances, is required. On this subject, several alternatives have been proposed over the years. In [10,11], linear and nonlinear feedback laws with a proportional–integral derivative structure for the longitudinal tracking were proposed. The lateral error was in both cases kept under control with the bank-reversal management logic.

Alternatively, in [12,13], the use of different tracking laws based on a tradeoff between longitudinal and lateral guidance performance was suggested, whereas in [14] a pre-terminal area for energy management ground-track control to limit the heading error was adopted. Other possible solutions foresaw the use of a recedinghorizon scheme based on the linearized time-varying dynamics to be controlled [15], and a unified predictor–corrector algorithm [16], which covers all the possible entry mission profiles. The problem was also approached by using different gain-scheduling controllers [17,18], or by tracking altitude and velocity via model reference adaptive control [19,20]. A possible alternative to the state-tracking schemes is a generalized constraint-tracking guidance, with a particular emphasis on the tracking of the heat flux [21,22].

A different way to approach the tracking problem comes from the field of attitude-control techniques, more specifically sliding-mode control (SMC) [23], which shows excellent robustness against perturbations with known upper bounds. This technique can be applied to nonlinear systems; therefore, no large amount of information has to be stored, as is the case for  $H_{\infty}$  controllers, which suffer from the rapid increase of the number of states needed to represent the uncertainties, and the need to apply a gain-scheduling technique, because they are conceived for linear systems.

Another advantage associated with the use of SMC techniques is its robustness with respect to uncertainties and disturbances, because of its nonlinear nature. Interesting results for the entry problem have been obtained by using terminal-guidance high-order sliding-mode (HOSM) controllers, in both the time [24] and range [25] domains, respectively. They are based on the definition of sliding surfaces associated with predefined terminal conditions to be achieved. More recently, a new class of adaptive HOSM controllers was proposed [26–28]. These methods generalize the possibility to apply virtually chattering-free sliding-mode controllers to systems with relative degree larger than 1 [29]. The controller is made adaptive by using a double-layer strategy to estimate online the minimum gain required to dominate unknown, but bounded disturbances acting on the system. The adaptation is obtained by using the concept of equivalent control, filtered out from the current control signal, and fed back into the double-layer algorithm. However, a drawback of this technique is the small step size required to obtain a stable numerical scheme for the gains. Moreover, it requires the design of a dedicated differentiator, which represents a parasitic dynamics [30], and needs to be included in the loop.

In this work, an alternative adaptive disturbance-based high-order SMC (ADHOSMC) scheme, based on an extended sliding-mode observer (SMO), is proposed. Instead of estimating the equivalent control via low-pass filtering, we propose to use a multi-input/multi-output (MIMO) sliding-mode disturbance observer, able to reconstruct online the disturbances acting on the system, and at the same time, to observe the  $\sigma$  dynamics, that is, the sliding variables representing the state errors, which can be fed in the loop. The advantage of this approach is twofold. First, it relaxes the requirements for the step size needed for the scheme. Second, at the same time, it provides the derivatives of the sliding variables needed to compute the tracking law.

The vehicle considered in this paper is the sharp edge flight experiment (SHEFEX)-3 prototype, a vehicle planned by the DLR, German Aerospace Center [31,32] for the demonstration of several entry technologies. The proposed tracking law can be used as feedback-control scheme together with onboard trajectorygeneration algorithms [33,34], as well as in conjunction with pure optimal trajectory-generation tools [6,7]. The work is organized as follows. In Sec. II, the vehicle and the scenario are briefly introduced, whereas in Sec. III the adaptive HOSM is described in detail, together with a series of simulations coming from a simplified example motivating the current work. In Sec. IV, the proposed technique is applied to the longitudinal equations of motion of an unpowered entry vehicle, whereas Sec. V focuses on the validation of the proposed algorithms, and compares the results with a traditional SMC algorithm. Finally, in Sec. VI, some conclusions on the work are drawn.

#### II. Vehicle and Scenario Characterization

SHEFEX is a program of technological development for atmospheric entry, conceived and led by the DLR, German Aerospace Center over the last 20 years [35]. The idea is to test technologies for atmospheric entry, such as structural and thermalprotection systems, with the focus to transform blunt areas into flat surfaces, to reduce costs without penalizing system performance.

SHEFEX-1 was successfully launched on 27 October 2005 from Andøya Rocket Range in Andenes, Norway. The experiments measured several aerodynamics parameters and their effect on the structure during atmospheric entry, and used passive control during entry. This mission also demonstrated that sounding rockets are suitable for atmospheric entry experiments. The SHEFEX project served as a starting point for SHEFEX-2, launched on 22 June 2012, also from Andøya Rocket Range. The goal of SHEFEX-2 was to validate analytical predictions and ground-test data, and to investigate technologies for hypersonic and space-transportation systems. To go on with the effort to increase the technological level for real space missions, the development of SHEFEX-3 began in 2012. The SHEFEX-3 vehicle has a faceted surface, which guarantees minor costs in terms of manufacturing. One of the proposed designs (the reference one for this work) is shown in Fig. 1.

The reference surface is equal to  $0.468 \text{ m}^2$ . The vehicle has its center of mass (c.m.) at 55%, starting from the nose, and is fully trimmable.

The entry interface has an initial altitude and velocity of 100 km and 6.5 km/s, respectively, and an initial flight-path angle equal to



Fig. 1 SHEFEX-3 entry vehicle.

Table 1 SHEFEX-3 vehicle parameters and initial state

Parameter	Value	Unit	State	Initial value	Unit
Mass	500	kg	Altitude h	100.0	km
Reference surface	0.468	$m^2$	Longitude $\theta$	2.1	deg
Length	1.85	m	Latitude $\phi$	68.5	deg
Width	1.85	m	Velocity modulus V	6500.0	m/s
Height	0.66	m	Flight-path angle $\gamma$	0.0	deg
c.m.	55%		Heading angle $\gamma$	-144.0	deg

0 deg. Details on the entry interface and the vehicle data are listed in Table 1, whereas the open-loop results are presented in Figs. 2 and 3. The open-loop commands are defined as follows: the angle of attack is initially equal to 45 deg, and slowly decreases until the value of 22 deg (corresponding to the maximum L/D) is achieved. This guarantees the correct exposure of the thermal-protection system at the beginning of the mission, when a large heat flux is experienced, whereas in the second part of the mission, the L/D ratio is maximized to extend the range capabilities of the vehicle. The bank-angle profile varies from an initial value of 60 deg to a value of 45 deg to ensure a sufficient margin of controllability.

The reference controls ensure proper final conditions in terms of altitude and velocity, which allow for the opening of the parachute system, while providing sufficient range, required for the onboard experiments. It is worth mentioning that angular-rate limits were included in the design of the control strategy. Specifically, these limits are equal to 5 deg /s for both the angle-of-attack rate and the bank-angle rate, and are compatible with the constraints coming from the flight-control system. In the frame of this work, no lateral control is included. However, the method can be fully coupled with bank-reversal management, or with other lateral control schemes.

Figure 2 shows the open-loop trajectory. Specifically, the plots in the top left and the center left, representing the altitude (Fig. 2a) and the flight-path angle (Fig. 2c), show one of the difficulties associated with this scenario [i.e., the phugoid oscillations (typical of flight at maximum L/D)], and a high variability of the states, which require an adequate reaction capability of the control scheme to be employed. In Fig. 2b, the velocity modulus is depicted, in which one can see that the velocity is almost constant during the first 200 s (as the drag is too low to reduce it), and then decreases once that the vehicle experiences a thicker atmospheric density, and therefore, a larger drag acceleration. In Fig. 2d plot, the nominal range is depicted. One can see that the spacecraft travels for about 3000 km. Plots in Figs. 2e and 2f present the corresponding open-loop profiles of angle of attack and bank angle.





Fig. 3 Open-loop trajectory: constraints.

Finally, in Fig. 3, the constraints acting on the vehicle [i.e., the dynamic pressure  $\bar{q}$  (Fig. 3a), the heat flux  $\dot{Q}$  (Fig. 3b), and the vertical load factor  $n_z$  (Fig. 3c)], together with their corresponding limits, are depicted. Specifically, they are computed as

$$\bar{q} = \frac{1}{2}\rho V^2, \quad \dot{Q} = k_q \sqrt{\rho} V^3, \quad n_z = \frac{|L\cos\alpha + D\sin\alpha|}{g_0}$$
 (1)

in which  $\rho$  is the atmospheric density expressed in kilogram per cubic meter;  $k_q$  is a constant depending on the material and the geometry of the thermal-protection system, for SHEFEX-3 equal to  $3.111 \cdot 10^{-4} \text{ kg}^{1/2}/\text{m}^3$ ; and  $g_0$  is the gravity acceleration at sea level ( $g_0 = 9.806 \text{ m/s}^2$ ). The structural limits of the vehicle and the active thermal-protection system dictate a limit for the preceding constraints. These limits are equal to  $\bar{q}_U = 2 \cdot 10^4 \text{ N/m}^2$ ,  $\dot{Q}_U = 2.5 \text{ MW/m}^2$ , and  $n_{z,U} = 5 \text{ g}$ , respectively.

#### III. Adaptive Disturbance-Based High-Order SMC

In this section, the ADHOSMC approach is explained. Sections III.A and III.B describe the HOSM theory and the extended sliding-mode disturbance observer, respectively. Their combination leads to the proposed ADHOSMC approach. In Sec. III.C, a practical example, motivating this work, is shown.

### A. HOSM Theory

The HOSM theory deals with the design of robust controllers for nonlinear systems. The name refers to the fact that the system to control is expressed in affine form with respect to the control signals. This is done by differentiating the equations of motion until the control appears linearly. We refer to the *n*th-order sliding-mode controller when the highest derivative of the state to track is of order *n*. It is therefore possible to link the state errors to the so-called sliding surfaces. The control will constrain the system to stay on this sliding surface, and this will ensure the correct tracking of the reference signals. As a practical example, suppose that we have the system dynamics described by

$$\sigma^{(n)}(t) = a(t) + b(t)u \tag{2}$$

in which  $\sigma(t) \in \mathbb{R}$  is the state error, or equivalently, the sliding variable associated with the state x(t), and defined as  $x(t) - x_{ref}(t)$ ; a(t) and b(t) are known functions  $\in \mathbb{R}$ ; and  $u(t) \in \mathbb{R}$  is the control. In real applications, the functions a(t) and b(t) do not perfectly match the models. Moreover, unmodeled terms may affect the results. We can therefore rewrite Eq. (2) as

$$\sigma^{(n)}(t) = a(t) + \Delta a(t) + [b(t) + \Delta b(t)]u = a(t) + b(t)u + d(t)$$
(3)

in which  $d(t) \in \mathbb{R}$  is an unknown, bounded function. Note that the function d(t) may contain combinations of several uncertainties and/ or disturbances. That is

$$d(t) = \Delta a(t) + \Delta b(t)u + d_u(t) \tag{4}$$

with  $\Delta a(t)$  and  $\Delta b(t)$  representing errors in the models of a(t) and b(t); and  $d_u(t)$  includes further, unknown terms. The operator ()<sup>(n)</sup> in Eqs. (2) and (3) represents the *n*th derivative with respect to the independent variable, in this case, the time *t*.

Proposition 1: Consider Eq. (2) in the nominal case (i.e., d(t) = 0). In the hypothesis of  $b(t) \neq 0$ , it is possible to state that the high-order SMC

$$u = -b(t)^{-1}(\bar{u} + a(t))$$
$$\tilde{u} = \sum_{i=0}^{n-1} \gamma_i |\sigma^{(i)}(t)|^{\alpha_i} \operatorname{sgn}(\sigma^{(i)}(t))$$
(5)

stabilizes the nonlinear system described by Eq. (2) if the terms  $\gamma_i$  are taken such that the polynomial

$$f(p) = p^n + \gamma_n p^{n-1} + \dots + \gamma_1 \tag{6}$$

is Hurwitz (that is, all its roots have negative real parts), and the terms  $\alpha_i$  are computed according to the formula

$$\alpha_{i-1} = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, \qquad i = 2, \dots, n \tag{7}$$

with  $\alpha_{n+1} = 1$ , and the seed  $\alpha_n$  is defined in the range  $[1 - \epsilon, 1)$ , with  $\epsilon << 1$ .

*Proof:* In [29], proposition 8.1, it is possible to find a rigorous proof of proposition 1 for the special case a(t) = 0, b(t) = 1. If we replace the affine mapping between control u and pseudocontrol  $\tilde{u}$  defined as

$$u = -b(t)^{-1}(\tilde{u} + a(t))$$
(8)

in Eq. (4), the system is reduced to

$$\sigma^{(n)}(t) + \sum_{i=0}^{n-1} \gamma_i |\sigma^{(i)}(t)|^{\alpha_i} \operatorname{sgn}(\sigma^{(i)}(t)) = 0$$
(9)

For the system described in Eq. (9), proposition 8.1 of [29] directly holds. The proof is complete, and is valid for the generic case  $a(t) \neq 0, b(t) \neq 0, 1$ .

*Remark 1*: Note that Eqs. (5) and (8) define a continuous controller. As a consequence, no chattering affects the system, and therefore, no saturation functions need to be selected to mitigate this effect at the expense of a robustness decrease. The nominal case represented by Eq. (2) is still a special case. In general, there will be disturbances and uncertainties, which will make the d(t) term different from zero. Edwards and Shtessel proposed an adaptive controller, based on a double-layer scheme, able to capture the derivative of the disturbance  $\dot{d}(t)$ , and to use it to estimate online the gain able to dominate the disturbance d(t) [28]. The scheme works very well for small step sizes, which are suitable for industrial applications. However, for entry-guidance schemes, this approach may be complicated to be used, as outer-loop control-system frequencies are usually lower (in the order of 1–10 Hz), and the aforementioned approach may lead to numerical instabilities. Moreover, the scheme still requires the design of a numerical differentiator, as not all the derivatives of the states [required in Eq. (5)], which are involved in the feedback loop, are directly measured.

Therefore, to guarantee the validity of the hypothesis of proposition 8.1 in [29] also in the presence of unknown disturbances, we propose an alternative scheme, able to cope with larger step sizes without reducing the accuracy of the results. The scheme is, at the same time, able to observe the  $\sigma$  dynamics by only using measurements of the states, and not their derivatives, and is based on the sliding-mode differentiation theory [36,37].

#### B. Extended SMO

Let us define an augmented  $\sigma$  state  $\sigma_a \in \mathbb{R}^{n+1}$ , defined as

$$\sigma_a = [\sigma_{a,1}, \dots, \sigma_{a,n}, \sigma_{a,n+1}]^T \tag{10}$$

in which  $\sigma_{a,n+1} = d(t)$ . The dynamics of Eq. (3) can be rewritten as

$$\dot{\sigma}_{a,1} = \sigma_{a,2}$$

$$\vdots$$

$$\dot{\sigma}_{a,n} = a + bu + d$$

$$\dot{\sigma}_{a,n+1} = \dot{d}$$
(11)

A system in the form of Eq. (11) can be estimated by using the sole measurement of the first state  $\sigma_1$ , supposed to be available [36,37]. (Here, the hypothesis of having a pure regulator problem, that is,  $x_{1,ref} = 0$ ,  $\sigma_1 = x_1 - x_{1,ref} = x_1$ , is implicitly assumed, whereas the general tracking problem is treated in Sec. IV.) The SMO can be written as

$$\hat{\sigma}_{a,1} = \hat{\sigma}_{a,2} + \lambda_1 \tilde{\sigma}_{a,1} + \kappa_1 \operatorname{sgn}(\tilde{\sigma}_{a,1})$$

$$\vdots$$

$$\dot{\hat{\sigma}}_{a,n} = a + bu + \lambda_n \tilde{\sigma}_{a,1} + \kappa_n \operatorname{sgn}(\tilde{\sigma}_{a,1})$$

$$\dot{\hat{\sigma}}_{a,n+1} = \lambda_{n+1} \tilde{\sigma}_{a,n+1} + \kappa_{n+1} \operatorname{sgn}(\tilde{\sigma}_{a,1})$$
(12)

in which  $\tilde{\sigma}_{a,1}$  is defined as  $\sigma_1 - \hat{\sigma}_{a,1}$ . The terms  $\lambda_i > 0$ , i = [1, n + 1] are linear gains, chosen such that the dynamics described by Eq. (12) is stable, and define a Luenberger observer. Nonlinear gains  $\kappa_i > 0$ , i = [1, n + 1] define a sliding-mode behavior, and enforce the variables  $\hat{\sigma}_i$  to converge exponentially to the true sliding states  $\sigma_i$ , i = [1, n + 1], within an accuracy defined by a constant  $\epsilon$  such that

$$\|\sigma_1(t) - \hat{\sigma}_1(t)\| \ll \epsilon \ll 1 \tag{13}$$

The consequence is that the estimated  $\sigma$  states converge to the true ones, whereas the (n + 1)th component converges to the disturbance d(t), which means that the disturbance is reconstructed in real time, and can be used to make the controller defined in Eq. (5) adaptive.

*Proposition 2:* The structure defined in Eq. (12) converges to the true  $\sigma$  state, provided that  $\kappa_1 > |\sigma_{a,2}(t) - \hat{\sigma}_{a,2}(t)|_{max}$ .

Proof: A formal proof of proposition 2 is described in [36].

*Remark 3:* A more general criterion for the selection of the linear and nonlinear gains, which appear in Eq. (12), is described in Sec. III.C.

If we indicate the estimate of the disturbance d(t) as  $\hat{d}(t)$ , we can modify Eq. (5), and define the disturbance-based high-order SMC law as

$$u = -\hat{b}^{-1}(\tilde{u} + \hat{a} + \hat{d}), \qquad \tilde{u} = \sum_{i=0}^{n-1} \gamma_i |\sigma^{(i)}|^{\alpha_i} \operatorname{sgn}(\sigma^{(i)})$$
(14)

in which  $\hat{a}$  and  $\hat{b}$  are the nominal functions a and b computed by using the state estimates and obtained by the SMO; this online estimator also provides the disturbance estimate  $\hat{d}$ .

Proposition 3: Suppose that the disturbance signal d(t) is bounded, together with its first and second derivatives, that is, there exist some values  $\delta_0$ ,  $\delta_1$ ,  $\delta_2$ , such that  $|d(t)| \le \delta_0$ ,  $|d(t)| \le \delta_1$ , and  $|\dot{d}(t)| \le \delta_2$ . Then, the adaptive law defined by Eq. (14) drives the dynamics of Eq. (5) toward the equilibrium point  $[\sigma_1, \ldots, \sigma_n]^T = [0, \ldots, 0]^T$  in finite time. *Proof:* Let us assume that the conditions for the SMO existence

*Proof:* Let us assume that the conditions for the SMO existence hold. Then, each of the variables in the SMO of Eq. (11) will converge to the true ones, as stated in proposition 2. This means that, once the observation sliding surface  $\sigma_{a,1}$  is reached and maintained, the dynamics is reduced to Eq. (9), and proposition 8.1 of [29] once again holds. The proof is complete.

*Remark 4:* Note that this is a theoretical result. In practice, what we obtain by using the adaptive law of Eq. (14) is a dramatic reduction of the disturbance acting on the system from the full unknown disturbance term *d* to a much smaller residual  $\epsilon_d = d - \hat{d}$ , which is bounded, by

$$|\epsilon_d| < \frac{\epsilon_0}{\mu} \tag{15}$$

in which  $\epsilon_0 \ll 1$  is a tuning parameter in the SMO, and  $\mu$  is the eigenvalue of the observer, properly defined in Sec. V.B, and taken  $\ge 1$  throughout this work. However, given the robustness of the HOSM framework, and the exponential stability of the nonlinear observer, the convergence is fast, and the proposed adaptive law makes the controller able to work in quasi-ideal conditions.

#### **IV.** Motivational Example

In this section, a simple example illustrating the motivation of the work is described. Suppose we have a system of third order, defined as

$$\sigma^{(3)}(t) = a(t) + b(t)u + d(t) \tag{16}$$

in which a and b are equal to 2 and 4, respectively, whereas d(t) is a time-dependent uncertainty acting on the system. Suppose that the initial state is defined as  $\sigma = \begin{bmatrix} 1 & 0.5 & 0 \end{bmatrix}^T$ , and that, in the absence of uncertainties (i.e., d(t) = 0), we apply the controller defined by Eq. (5). Results are omitted for brevity, but one can observe some of the interesting features of the HOSM theory: the error converges to the equilibrium point  $[0, 0, 0]^T$  in finite time, and the control signal does not show any trace of the chattering phenomenon. Let us consider the presence of the disturbance d(t). A possible approach to take the (unknown) disturbance d(t) into account is to define a double-layer adaptive scheme, based on the use of the socalled equivalent control [27,28,30]. The idea is to counteract the disturbance d(t) by means of two gains k(t) and  $\rho(t)$ , which become an upper bound for the first and second derivatives of the disturbance. It is possible to demonstrate that the corresponding candidate Lyapunov function converges to 0, which means that the gains themselves are bounded and converge to the unknown



Fig. 4 Application of double-layer adaptive HOSM: step size = 0.1 ms.

disturbance derivatives. An example of application of this technique (simulated with a step size equal to 0.1 ms) is represented in Figs. 4a-4d. There, the double-layer-based adaptive scheme is able to drive the states of the perturbed system, represented in Fig. 4a, by using the control signal (depicted in Fig. 4b) to its equilibrium point. Figures 4c and 4d show the two layers of gains k and  $\rho$ . They are integrated in the scheme to compensate for the disturbances. To guarantee convergence, the first gain has to be equal or greater than the absolute value of the first derivative of the disturbance. An example of this adaptive scheme is shown in Fig. 4c, in which the gain k tracks with some margin  $\epsilon_k$ (which is one of the tuning parameters) |d(t)|. The tracking of the disturbance is realized by using the second layer, defined by a further gain  $\rho$ , which is an upper bound for the second derivative of the disturbance d(t), and ensures the convergence of the scheme. However, although there is formal proof for the theoretical stability of the scheme, in practice, some numerical issues arise when larger step sizes are taken. For instance, if the step size is increased to 2.5 ms, while keeping all the other parameters constant, we get the results depicted in Figs. 5a–5d. One can see that numerical instabilities cause divergence of the states of the adaptive scheme, which directly causes the divergence of the states (Fig. 5a). Therefore, although this technique is an excellent choice for high-performance architectures, for machines with stricter CPU limitations, such as onboard computers, this algorithm may not be the best alternative.

To overcome this drawback, an alternative scheme based on the MIMO SMO is proposed. The advantage is twofold: first, it significantly relaxes the step-size requirements, while still bringing the state errors to 0. Second, it provides the variables needed for the feedback process, that is,  $\sigma$  and its derivatives. Figures 6a–6d and 7a–7d show the corresponding results obtained by using the proposed ADHOSMC scheme for step sizes equal to 0.1 and 2.5 ms, respectively. No qualitative differences can be observed in the states, which, for both cases, converge toward the equilibrium point of the system, and in the controls, which are chattering free. Moreover, for both cases, the disturbance observer converges in less than a second (about 690 ms) to the true d(t) with an accuracy of  $\pm 1\%$ .





### V. Longitudinal Controller Design

Let us now apply the approach discussed in the previous section to the longitudinal equations of motion of an unpowered reentry vehicle. First, we need to extract the input/output linearized model from these equations to have a system in the form of Eq. (2) to make the application of the ADHOSMC possible.

#### A. Input/Output Feedback Linearization

The longitudinal dynamics of the vehicle with respect to a nonrotating Earth is described by [10]

$$\dot{h} = V \sin \gamma$$

$$\dot{V} = -D - g \sin \gamma$$

$$\dot{\gamma} = \frac{1}{V} L \cos \mu + \left(\frac{V}{r} - \frac{g}{V}\right) \cos \gamma$$

$$\dot{\alpha} = u_{\alpha} + \dot{\alpha}_{ref}$$

$$\dot{\mu} = u_{\mu} + \dot{\mu}_{ref}$$
(17)

in which h, V, and  $\gamma$  are the altitude, velocity modulus, and flight-path angle, respectively, whereas r is the radial position; g is the gravity acceleration, whereas L and D are the lift and drag accelerations. The controls are, in this case, the angle-of-attack rate  $u_{\alpha}$  and the bankangle rate  $u_{\mu}$ . Indeed, they appear in affine form in the equations of motion. This is not the case for the angle of attack, which is "hidden" in the aerodynamic database, as for the vehicle analyzed here, the coefficients depend on angle of attack, Mach number, and altitude, and the bank angle, which appears as an argument of the cosine function. The objective is to derive the MIMO ADHOSMC that allows to track the reference altitude  $h_{ref}(t)$  and velocity  $V_{ref}(t)$ . An important difference with respect to [24,25,39] is that, in this case, the reference states are time dependent and not terminal, constant values; therefore, their derivatives are different from zero and need to be included in the controller design. To have the controls linearly appearing in the equations of motion, we differentiate the altitude three times and the velocity two times. The total relative degree of the system is 5, and is equal to the order of the system of Eq. (17).

If we differentiate the altitude three times with respect to time, we get

$$\dot{h}(t) = V \sin \gamma$$
  
$$\ddot{h}(t) = \dot{V} \sin \gamma + V \dot{\gamma} \cos \gamma$$
  
$$\ddot{h}(t) = \ddot{V} \sin \gamma + 2\dot{V} \dot{\gamma} \cos \gamma + V \ddot{\gamma} \cos \gamma - V \dot{\gamma}^2 \sin \gamma$$
(18)

From the preceding equation, it is clear that the expression for  $\ddot{\gamma}$  is needed. If we differentiate  $\gamma$  twice, we get

$$\dot{\gamma} = \frac{L}{V} \cos \mu + \left(\frac{V}{r} - \frac{g}{V}\right) \cos \gamma$$
$$\ddot{\gamma} = \frac{\dot{L}}{V} \cos \mu - \frac{L\dot{V}}{V^2} \cos \mu - \frac{L\sin \mu}{V} (\dot{\mu}_{ref} + u_{\mu}) + \dots$$
$$+ \left(\frac{\dot{V}}{r} - \frac{V\dot{r}}{r^2} - \frac{\dot{g}}{V} + \frac{g\dot{V}}{V^2}\right) \cos \gamma - \left(\frac{V}{r} - \frac{g}{V}\right) \dot{\gamma} \sin \gamma$$
(19)

Furthermore, differentiating the velocity twice with respect to time yields

$$\dot{V} = -D - g \sin \gamma$$
  
$$\ddot{V} = -\dot{D} - \dot{g} \sin \gamma - g \dot{\gamma} \cos \gamma$$
(20)

Atmospheric density and gravity-acceleration derivatives with respect to time can be easily computed either analytically or numerically. Assuming that the atmospheric density  $\rho$  and the gravity acceleration g depend only on the altitude, we can write

$$\dot{\rho} = \rho_h \dot{h} \quad \dot{g} = g_h \dot{h} \tag{21}$$

in which  $\rho_h$  and  $g_h$  are the derivatives of the atmospheric density and the gravity acceleration with respect to the altitude, respectively. From the analysis of Eqs. (18–20), it is clear that we need to extract differential information about the aerodynamic accelerations L and D from the model. Drag and lift acceleration derivatives with respect to time can be computed as

$$\dot{D} = D\left(\frac{\dot{\rho}}{\rho} + 2\frac{\dot{V}}{V} + \frac{\dot{C}_D}{C_D}\right), \quad \dot{L} = L\left(\frac{\dot{\rho}}{\rho} + 2\frac{\dot{V}}{V} + \frac{\dot{C}_L}{C_L}\right)$$
(22)

The time derivatives of the aerodynamic coefficients can be computed using the information contained in the aerodynamic database:

$$C_{D} = C_{D,\alpha}(\dot{a}_{ref} + u_{\alpha}) + C_{D,M}M_{V}V + (C_{D,M}M_{h} + C_{D,h})h$$
  
$$\dot{C}_{L} = C_{L,\alpha}(\dot{a}_{ref} + u_{\alpha}) + C_{L,M}M_{V}\dot{V} + (C_{L,M}M_{h} + C_{L,h})\dot{h}$$
(23)

in which  $C_{D,\alpha}$  and  $C_{L,\alpha}$  are the derivatives of the aerodynamic coefficients with respect to the angle of attack  $\alpha$ .  $C_{D,M}$  and  $C_{L,M}$  are the derivatives of the aerodynamic coefficients with respect to the Mach number M,  $C_{D,h}$  and  $C_{L,h}$  are the derivatives of the aerodynamic coefficients with respect to the altitude, whereas  $\dot{\alpha}_{ref}$ is the reference angle-of-attack rate. Finally, the terms  $M_V$  and  $M_h$ are the derivatives of the Mach number with respect to the velocity V and the altitude h, and are described in the Appendix. With all these relationships, the entire input/output model can be obtained. It has the following compact form:

$$h(t) = a_h + b_{h,a}u_a + b_{h,\mu}u_{\mu}, \quad \ddot{V}(t) = a_V + b_{V,a}u_a + b_{V,\mu}u_{\mu}$$
(24)

As for the single-input/single-output system of Eq. (5), uncertainties on aerodynamics, mass, atmosphere, and wind will cause variations of the functions  $a_i$ ,  $b_{i,j}$ , i = h, V,  $j = \alpha$ , and  $\mu$ . All these uncertainties can be combined into two extra terms to be added in Eq. (24), which become

$$h(t) = a_{h} + b_{h,a}u_{a} + b_{h,\mu}u_{\mu} + d_{h}$$
  
$$\ddot{V}(t) = a_{V} + b_{V,a}u_{a} + b_{V,\mu}u_{\mu} + d_{V}$$
 (25)

The expressions for the terms  $a_h, a_V, b_{h,a}, b_{h,\mu}, b_{V,a}$ , and  $b_{V,\mu}$  are given in the Appendix. They depend on the states and their derivatives, whereas  $u_{\alpha}$  and  $u_{\mu}$  are the control rates we need to determine.

*Remark 5:* Explicit expressions for the uncertainties  $d_h$  and  $d_V$  can be obtained by writing the perturbed version of Eq. (25), which can be obtained by replacing the nominal variables involved in Eqs. (18–24) with their perturbed version (i.e., replacing  $C_L$  with  $C_L + \Delta C_L$ , and so on). However, this development is omitted, as the scope of the adaptive-control scheme proposed here is to reconstruct uncertainties without any previous knowledge of them. Moreover, there may be other uncertainties not modeled by Eqs. (18–24), which will be dealt with the method here anyway.

It is now possible to design the adaptive high-order SMC scheme for the system of Eq. (25).

#### B. MIMO ADHOSMC

The objective of the feedback-control scheme is to track the given altitude and velocity profiles. In a similar fashion to what has been done in Sec. III, let us define two decoupled sliding surfaces  $\sigma_h$  and  $\sigma_V$ :

$$\sigma_h = h(t) - h_{\text{ref}}(t), \quad \sigma_V = V(t) - V_{\text{ref}}(t)$$
(26)

We can extend the approach developed in Sec. III, and specifically Eq. (14), to the MIMO system represented by Eq. (26). Let us define the following matrices and vectors:

$$\boldsymbol{A} = \begin{bmatrix} a_h \\ a_V \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} b_{h,\alpha} & b_{h,\mu} \\ b_{V,\alpha} & b_{V,\mu} \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} d_h \\ d_V \end{bmatrix}$$
(27)

$$\tilde{\boldsymbol{u}} = \begin{bmatrix} \sum_{i=0}^{2} \gamma_{h,i} |\sigma_{h}^{(i)}(t)|^{\alpha_{h,i}} \operatorname{sgn}(\sigma_{h}^{(i)}(t)) \\ \sum_{i=0}^{1} \gamma_{V,i} |\sigma_{V}^{(i)}(t)|^{\alpha_{V,i}} \operatorname{sgn}(\sigma_{V}^{(i)}(t)) \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} u_{\alpha} \\ u_{\mu} \end{bmatrix}$$
(28)

With these definitions, the MIMO control law can be written in matrix form as

$$\boldsymbol{u} = -\boldsymbol{B}^{-1}(\tilde{\boldsymbol{u}} + \boldsymbol{A} + \boldsymbol{D}) \tag{29}$$

All the terms in Eq. (29) are defined, except the vector D, which will be replaced by its estimated value  $\hat{D}$ , leading to the final form of the ADHOSMC law.

$$\boldsymbol{u} = -\hat{\boldsymbol{B}}^{-1}(\tilde{\boldsymbol{u}} + \hat{\boldsymbol{A}} + \hat{\boldsymbol{D}})$$
(30)

The quantities  $\hat{A}$ ,  $\hat{B}$  are computed by using the nominal expressions described in the Appendix, by using the derivatives estimated with the SMO, and  $\hat{D}$  is the vector containing the online estimates of the disturbances acting on the system. From the term u, the angle of attack and bank angle can be obtained as

$$\alpha(t) = \alpha_{\text{ref}}(t) + \Delta \alpha(t) = \alpha_{\text{ref}}(t) + \int_{t_0}^t u_\alpha \, \mathrm{d}\tau$$
$$\mu(t) = \mu_{\text{ref}}(t) + \Delta \mu(t) = \mu_{\text{ref}}(t) + \int_{t_0}^t u_\mu \, \mathrm{d}\tau \qquad (31)$$

in which  $u_{\alpha}$  and  $u_{\mu}$  are the feedback angle-of-attack rate and the bankangle rate, respectively. From the inspection of Eq. (30), one can see that the control can be synthesized only if the matrix **B** is nonsingular.

$$\det(\boldsymbol{B}) = b_{h,\alpha} b_{V,\mu} - b_{h,\mu} b_{V,\alpha} \neq 0 \tag{32}$$

If we look at the definitions shown in the Appendix, we can write

$$\det(\mathbf{B}) = 0 \Leftrightarrow b_{D,\alpha}L\cos\gamma\sin\mu = 0 \tag{33}$$

Because  $b_{D,\alpha}$  and L are always different from 0, from a physical point of view, Eq. (33) gives us the two only possibilities, in which the control synthesis cannot be applied. The former is related to the condition  $\gamma = \pm 90$  deg, which means that the controller cannot be applied in vertical motion. This condition is excluded during the trajectory planning in any case. The latter is related to the condition  $\mu = 0$  deg, which represents a well-known singularity for controllability of unpowered entry vehicles [40-42]. To exclude this possibility, a region around  $\mu = 0$  deg is avoided. Specifically, for all the simulations, the bank angle is limited in the interval [0.1, 89] deg. The angle of attack was limited to the range [-5, 7.5] deg with respect to the nominal angle-of-attack profile, in a similar fashion to what has been done for the space shuttle [10]. With the proposed approach, it is possible to track the altitude and velocity by modulating the bank angle and the angle of attack at the same time. The next step is the extension of the SMO to the system represented by Eq. (25).

#### C. Nonlinear Disturbance Observer

The control scheme synthesized in the previous section relies on several models (for instance, the atmospheric density and the aerodynamic database), which can be different with respect to the actual data. The missing information can be enclosed in  $\hat{D}$ , which will be estimated by a MIMO SMO. The technique is here extended to the longitudinal states involved in the atmospheric entry, that is, the altitude and the velocity. Moreover, because the flight-path angle and its derivatives appear in Eqs. (18–24), this state is also included in the observer, which will provide, together with the states and their derivatives, the estimates  $\hat{d}_h(t)$  and  $\hat{d}_V(t)$ . If we define the state vector  $\boldsymbol{x}$  as

$$\boldsymbol{x} = \{ \boldsymbol{h} \quad \dot{\boldsymbol{h}} \quad \ddot{\boldsymbol{h}} \quad \boldsymbol{v} \quad \dot{\boldsymbol{v}} \quad \boldsymbol{\gamma} \quad \dot{\boldsymbol{\gamma}} \}^T \tag{34}$$

the system of Eq. (17) can be rewritten in state-space form as

$$\dot{x}_{1} = x_{2} \quad \dot{x}_{2} = x_{3} \quad \dot{x}_{3} = a_{h} + b_{h,\alpha}u_{\alpha} + b_{h,\mu}u_{\mu} \quad \dot{x}_{4} = x_{5}$$
  
$$\dot{x}_{5} = a_{V} + b_{V,\alpha}u_{\alpha} + b_{V,\mu}u_{\mu} \quad \dot{x}_{6} = x_{7}$$
  
$$\dot{x}_{7} = a_{\gamma} + b_{\gamma,\alpha}u_{\alpha} + b_{\gamma,\mu}u_{\mu}$$
(35)

We want to estimate online the terms  $\hat{d}_h(t)$  and  $\hat{d}_V(t)$  defined in Eq. (25). If we define the new augmented state vector  $\mathbf{x}_a$ 

$$\boldsymbol{x}_a = \{ \boldsymbol{h} \quad \dot{\boldsymbol{h}} \quad \ddot{\boldsymbol{h}} \quad \boldsymbol{d}_h \quad \boldsymbol{v} \quad \dot{\boldsymbol{v}} \quad \boldsymbol{d}_V \quad \boldsymbol{\gamma} \quad \dot{\boldsymbol{\gamma}} \}^T$$
(36)

the perturbed equations of motion can be represented in state-space form as

$$\dot{x}_{a,1} = x_{a,2} \quad \dot{x}_{a,2} = x_{a,3} \quad \dot{x}_{a,3} = a_h + b_{h,\alpha}u_{\alpha} + b_{h,\mu}u_{\mu} + x_{a,4}$$
$$\dot{x}_{a,4} = \dot{d}_h \quad \dot{x}_{a,5} = x_{a,6} \quad \dot{x}_{a,6} = a_V + b_{V,\alpha}u_{\alpha} + b_{V,\mu}u_{\mu} + x_{a,7}$$
$$\dot{x}_{a,7} = \dot{d}_V \quad \dot{x}_{a,8} = x_{a,9} \quad \dot{x}_{a,9} = a_\gamma + b_{\gamma,\alpha}u_{\alpha} + b_{\gamma,\mu}u_{\mu}$$
(37)

For the scenario analyzed here, the measurements of attitude, position, and velocity are obtained with sufficient accuracy by the navigation subsystem [38], and are converted into altitude, velocity, and flight-path-angle measurements  $z_h$ ,  $z_V$ , and  $z_\gamma$ . These measurements can be integrated into the following MIMO nonlinear disturbance estimator:

$$\hat{x}_{a,1} = \hat{x}_{a,2} + \lambda_0^h \tilde{x}_h + \kappa_0^h \operatorname{sat}(\tilde{x}_h) \quad \hat{x}_{a,2} = \hat{x}_{a,3} + \lambda_1^h \tilde{x}_h + \kappa_1^h \operatorname{sat}(\tilde{x}_h) 
\dot{\hat{x}}_{a,3} = a_h + b_{h,a}u_a + b_{h,\mu}u_\mu + \hat{x}_{a,4} + \lambda_2^h \tilde{x}_h + \kappa_2^h \operatorname{sat}(\tilde{x}_h) 
\dot{\hat{x}}_{a,4} = \lambda_3^h \tilde{x}_h + \kappa_3^h \operatorname{sat}(\tilde{x}_h) \quad \dot{\hat{x}}_{a,5} = \hat{x}_{a,6} + \lambda_0^v \tilde{x}_h + \kappa_0^v \operatorname{sat}(\tilde{x}_V) 
\dot{\hat{x}}_{a,6} = a_V + b_{V,a}u_a + \hat{x}_{a,7} + \lambda_1^v \tilde{x}_V + \kappa_1^v \operatorname{sat}(\tilde{x}_V) 
\dot{\hat{x}}_{a,7} = \lambda_2^v \tilde{x}_V + \kappa_2^v \operatorname{sat}(\tilde{x}_V) \quad \dot{\hat{x}}_{a,8} = \hat{x}_{a,9} + \lambda_0^v \tilde{x}_\gamma + \kappa_0^v \operatorname{sat}(\tilde{x}_\gamma) 
\dot{\hat{x}}_{a,9} = a_\gamma + b_{\gamma,a}u_a + b_{\gamma,\mu}u_\mu + \lambda_1^\gamma \tilde{x}_\gamma + \kappa_1^v \operatorname{sat}(\tilde{x}_\gamma)$$
(38)

in which  $\lambda_i^h$ ,  $\lambda_i^v$ ,  $\lambda_i^\gamma$  are the linear gains, and  $\kappa_i^h$ ,  $\kappa_i^v$ ,  $\kappa_i^\gamma$  are the nonlinear gains of the observer, respectively; they are all positive, whereas the terms  $\tilde{x}_h$ ,  $\tilde{x}_V$ ,  $\tilde{x}_\gamma$  are the differences between the measurements and the observer estimates, computed as

$$\tilde{x}_h = z_h - \hat{x}_{a,1}$$
  $\tilde{x}_V = z_V - \hat{x}_{a,5}$   $\tilde{x}_\gamma = z_\gamma - \hat{x}_{a,8}$  (39)

The state vector  $\hat{x}_a$  is consequently defined as

$$\hat{\boldsymbol{x}}_{a} = \left\{ \hat{h} \quad \hat{h} \quad \hat{h} \quad \hat{d}_{h} \quad \hat{v} \quad \hat{v} \quad \hat{d}_{V} \quad \hat{\gamma} \quad \hat{\gamma} \right\}^{T}$$
(40)

To avoid observer's chattering, the sgn function is replaced by the saturation function, defined as

$$\operatorname{sat}(\tilde{x}_m) = \begin{cases} 1, & \tilde{x}_m \ge w_m \\ -1, & x \le -w_m \\ \frac{\tilde{x}_m}{w_m}, & |\tilde{x}_m| < w_m \end{cases} \qquad m = h, v, \gamma \qquad (41)$$

Equation (41) implies that, when the residuals defined in Eq. (39) are within the boundaries defined by  $w_h$ ,  $w_V$ , and  $w_\gamma$ , the SMO becomes a Luenberger observer with augmented linear gains:

$$\tilde{\lambda}_i^m = \lambda_i^m + \frac{1}{w_m} \qquad m = h, v, \gamma \tag{42}$$

The procedure to select the linear and nonlinear gains is directly taken from [37], and is based on the assumption that the disturbances and their derivatives can be unknown, but bounded, which is a realistic hypothesis given the scenario we are dealing with. Therefore, it is always possible to define some positive constants  $c_i^m$ , m = h, v, and i = 1, 2, such that

$$|d_h| \le c_1^h, \quad |d_h| \le c_2^h \quad |d_V| \le c_1^v, \quad |d_V| \le c_2^v$$
 (43)

holds. More rigorously, the disturbance derivatives are assumed Lipschitz continuous. With these premises, it is possible to realize the conditions for the SMO only if the nonlinear gains satisfy the following relationships [36]:

$$\kappa_1^h \ge |\tilde{x}_{a,2}| \quad \kappa_1^V \ge |\tilde{x}_{a,5}| \quad \kappa_1^\gamma \ge |\tilde{x}_{a,8}|$$
(44)

If we define the thresholds for the convergence of the observer  $\epsilon_h$ ,  $\epsilon_V$ , and  $\epsilon_\gamma$ , it is possible to compute the linear gains as

Table 2Monte Carlocampaign parameters

Parameter	Range $(3\sigma)$	Units
$\Delta h$	$\pm 1$	km
$\Delta V$	$\pm 100$	m/s
$\Delta \gamma$	$\pm 0.25$	deg
$\Delta \rho$	$\pm 20\%$	
$\Delta C_L$	$\pm 10\%$	
$\Delta C_D$	$\pm 10\%$	
$\Delta m$	$\pm 0.5\%$	



Fig. 8 Monte Carlo campaign (1000 runs): ADHOSMC vs SMC: altitude-velocity plane.

$$\lambda_{i}^{h} = C_{4}^{i} \mu_{h}^{i}, \qquad i = 1, \dots, 4$$
  

$$\lambda_{i}^{V} = C_{3}^{i} \mu_{V}^{i}, \qquad i = 1, \dots, 3$$
  

$$\lambda_{i}^{\gamma} = C_{1}^{i} \mu_{V}^{j}, \qquad i = 1, 2$$
(45)

in which the parameters  $\mu_h$ ,  $\mu_V$ , and  $\mu_\gamma$  are the poles of the Luenberger observer. These parameters have to satisfy the following inequality:

$$\mu_m \ge \sqrt{\frac{\mu_{\max}(\boldsymbol{P}_m)}{\mu_{\min}(\boldsymbol{P}_m)}} \frac{2\mu_{\max}(\boldsymbol{P}_m)c_2^m}{(1-\nu_m)\epsilon_m}, \qquad m = h, v, \gamma \qquad (46)$$







in which the terms  $\nu_h$ ,  $\nu_V$ , and  $\nu_\gamma$  are constant parameters defined in the range (0, 1). The coefficients  $C_i^i$  are computed as

$$C_j^i = \frac{j!}{i!(j-i)!}$$
  $i, j = 1, 2, 3$  (47)

and the parameters  $\mu_{\max}(\mathbf{P}_m)$  and  $\mu_{\min}(\mathbf{P}_m)$   $(m = h, v, \gamma)$  are the maximum and minimum eigenvalues of the matrices  $\mathbf{P}_m$ , which represent the solutions of the following Lyapunov equations:

$$P_h M_h + M_h^T P_h = -I_3$$

$$P_V M_V + M_V^T P_V = -I_2$$

$$P_\gamma M_\gamma + M_\gamma^T P_\gamma = -1$$
(48)

with

$$\boldsymbol{M}_{h} = \begin{bmatrix} -C_{3}^{1} & 1 & 0\\ -C_{3}^{2} & 0 & 1\\ -C_{3}^{3} & 0 & 0 \end{bmatrix}, \quad \boldsymbol{M}_{V} = \begin{bmatrix} -C_{2}^{1} & 1\\ -C_{2}^{2} & 0 \end{bmatrix}, \quad \boldsymbol{M}_{\gamma} = -C_{1}^{1} \quad (49)$$

and  $I_n$  is the identity matrix having dimensions  $n \times n$ . Once the linear gains are computed, the nonlinear gains can be obtained as

$$\kappa_1^m \ge \|\tilde{x}_m\|_{\max} \sqrt{\frac{\mu_{\max}(\boldsymbol{P}_m)}{\mu_{\min}(\boldsymbol{P}_m)}}, \qquad m = h, v, \gamma$$
(50)

$$\begin{aligned} \kappa_i^h &\geq \kappa_1^h C_3^{i-1} \mu_h^{i-1}, \qquad i=2,3,4 \quad \kappa_i^v \geq \kappa_1^v C_2^{i-1} \mu_V^{i-1}, \\ i=2,3 \quad \kappa_i^\gamma \geq \kappa_1^\gamma C_1^{i-1} \mu_\gamma^{i-1}, \qquad i=2 \end{aligned}$$
(51)

Rigorous mathematical proofs for these relationships can be found in [37].

With the estimates of the disturbances  $\hat{d}_h(t)$  and  $\hat{d}_V(t)$ , and the states' derivatives  $\dot{\hat{h}}(t)$ ,  $\ddot{\hat{h}}(t)$ ,  $\dot{\hat{h}}(t)$ ,  $\dot{\hat{V}}(t)$ ,  $\ddot{\hat{V}}(t)$ , the sliding variables can finally be computed as



c) ADHOSMC vs SMC: flight-path angle d) ADHOSMC vs SMC: range Fig. 9 Monte Carlo campaign (1000 runs): ADHOSMC vs SMC: states.





Fig. 11 Monte Carlo campaign (1000 runs): ADHOSMC vs SMC: constraints.

$$\hat{\sigma}_{h} = h(t) - h_{\text{ref}}(t) \quad \hat{\sigma}_{h} = \hat{h}(t) - \dot{h}_{\text{ref}}(t) \quad \hat{\sigma}_{h} = \hat{h}(t) - \ddot{h}_{\text{ref}}(t)$$
$$\hat{\sigma}_{V} = \hat{V}(t) - V_{\text{ref}}(t) \quad \dot{\sigma}_{V} = \hat{V}(t) - \dot{V}_{\text{ref}}(t) \tag{52}$$

With the use of the disturbance observer, we simultaneously estimate online the uncertainties acting on the system and the derivatives of the current states, needed for the design of the controller, by only using available measurements.

#### VI. Results

Simulation campaigns have been performed to assess the behavior of the proposed controller. The ADHOSMC is compared with a standard SMC, based on the traditional SMC theory [39]. The schemes are tuned to ensure similar control authority. More specifically, the two seeds  $\alpha_h$  and  $\alpha_V$  are both taken equal to 0.8, whereas the poles associated with Eq. (28) are placed at -0.4 and -0.6, respectively. For what regards the disturbance observer,

the linear gains are based on the poles  $\lambda_h = -6.18$ ,  $\lambda_V = -4.57$ , and  $\lambda_{\gamma} = -5.56$ , whereas the nonlinear gains are  $\kappa^{h} =$  $[2.82, 78.4, 484.9, 1664.7]^T$ ,  $\kappa^V = [0.51, 5.1, 11.7]$ , and  $\kappa^{\gamma} =$ [0.02, 0.17]. Finally, the saturation layers are chosen as  $w_h = 0.25, w_V = 0.1$ , and  $w_{\gamma} = 0.001$ . The nonlinear control strategy is therefore completely defined by a small number of constant parameters, with the advantage that no large data sets, coming from gain-scheduling techniques, are required. For what regards the triggering of the feedback scheme, because at the beginning of the entry the atmospheric density is thin, the aerodynamic accelerations are very small and cannot properly counteract gravity, and this condition may induce control saturation. To avoid it, the scheme is triggered once the aerodynamic accelerations become significant. A rule, which works well in practice, is to use the drag-to-gravity ratio as a measure of the effectiveness of the control. In this case, the trigger is associated with a drag acceleration equal to 0.5g, which happens in a time interval between 140 and 170 s after the beginning of the entry.



Fig. 13 Monte Carlo campaign (1000 runs): velocity and flight-path-angle reconstruction.

To assess the behavior of the developed control strategy, a full Monte Carlo campaign has been run. Dispersions on atmospheric density, altitude, velocity, and flight-path angle are considered. Moreover, aerodynamic dispersions and mass uncertainty have been included. The control sample rate for all the simulations is 0.1 s. All the uncertainties follow a normal distribution. Further details about the uncertainties are listed in Table 2.

From the previous table, one can see that, together with the initial errors and the atmospheric-density uncertainty, a variation of the  $C_L$ and  $C_D$  coefficients up to  $\pm 10\%$  is included. These limits are in line with the wind-tunnel tests performed by the DLR, German Aerospace Center, Institute of Aerodynamics and Flow Technology [43].

Moreover, uncertainties in the dry mass, together with the propellant residuals, were taken into account according to the margins suggested by the ESA [44]. A total of 1000 runs have been performed. For visibility purposes, only the results associated with the first 25 simulations are plotted, and are shown in Figs. 8-14, whereas in Figs. 15a and 15b, there is a comparison of the behavior of the sliding states for all the three SMC techniques considered here. Table 3 illustrates the benefits of using ADHOSMC compared to SMC and pure HOSM.

From Figs. 8 and 9, we can observe that both the ADHOSMC and SMC strategies correctly track the reference states. The dispersions reduce over time, and a first difference in the methods can be seen.



b) Velocity disturbance

Fig. 14 Monte Carlo campaign (1000 runs): disturbance reconstruction.

The ADHOSMC generates smoother results, and this is especially visible in the h - V plane, plotted in Fig. 8, and in the flight-path angle (Fig. 9c), in the interval between 140 and 170 s, which is exactly the moment at which the drag becomes large enough to counteract gravity. Note that the effective error that the control scheme has to deal with is much larger than what has been summarized in Table 2. Indeed, there is a lack of control authority during the first 2–3 min of mission, and during this phase the error may significantly increase. However, the control scheme can properly counteract the error once activated. The difference coming from the two strategies becomes more evident in Figs. 10a and 10b, in which one can see that, in general, the convergence to the equilibrium point is significantly faster (in the order of 80-100 s) than if we use the standard SMC. For the 1000 cases analyzed, the velocity errors go to 0 more slowly by using the ADHOSMC strategy than by using the SMC in less than 1% of the cases, whereas for the altitude it never happens. The reason is due to an angle-of-attack saturation. In that case, the other available control, that is, the bank angle, is used to keep tracking the reference altitude, and this causes a delay in the convergence of the velocity error.

Note that, in general, however, there is more control activity when the SMC is used than when the ADHOSMC is adopted, as is shown in Fig. 10. It is also interesting to see that, once the sliding surfaces are reached, the control profiles of the two schemes perfectly overlap.

This behavior is consistent with the fact that the two control schemes achieve the same sliding surfaces in different ways, and in different times, but when these are reached, the control activity to track them is the same, as the kinematic profiles involved in their definition are the same too.

An interesting difference between the two schemes can be observed in Fig. 11, which shows the constraints. Although both the control systems satisfy the limits in terms of dynamic pressure and heat flux, the delay in the convergence of the SMC with respect to the ADHOSMC causes several violations of the maximum value of vertical load factor, as it is visible from the bottom plot. This limit was violated in about 11% of the cases, whereas in total only six violations occur when the ADHOSMC is employed. The maximum violation in the two cases is in any case quite different. The worst cases in terms of load factor are associated with a value of 5.27g (ADHOSMC) and 7.06g (SMC). Moreover, the conventional SMC violates five times the maximum dynamic pressure, whereas this never happens with the ADHOSMC. Finally, both the systems satisfy the requirements in terms of heat flux.

In terms of observer's performance, we can see how the states and their derivatives are correctly reconstructed. Specifically, the altitude and its first and second derivatives are shown in Fig. 12, whereas in Fig. 13 the velocity modulus, the flight-path angle, and their derivatives are plotted. The online estimates (continuous black lines)



Fig. 15 Monte Carlo campaign (1000 runs): ADHOSMC vs SMC: state errors.

 Table 3
 Monte Carlo campaign results (1000 cases)

Parameter	Units	ADHOSMC	SMC	HOSM
Mean $\Delta h_f$	m	2.44	3.66	-27.92
Standard deviation $\Delta h_f$	m	19.69	28.25	68.12
Mean $\Delta V_f$	m/s	-0.04	-0.01	-0.81
Standard deviation $\Delta V_f$	m/s	0.18	0.05	1.07
Mean $\Delta R_f$	km	0.50	0.99	0.90
Standard deviation $\Delta R_f$	km	7.43	7.71	7.82
Mean $\bar{q}_{\text{peak}}$	N/m <sup>2</sup>	15,226	15,391	15,295
Standard deviation $\bar{q}_{\text{peak}}$	$N/m^2$	1,695	1,831	1,556
Mean $\dot{Q}_{peak}$	$\dot{MW}/m^2$	1.744	1.753	1.748
Standard deviation $\dot{Q}_{\text{peak}}$	$MW/m^2$	0.089	0.124	0.079
Mean $n_{z, \text{peak}}$	g	4.091	4.309	4.100
Standard deviation $n_{z,peak}$	ģ	0.284	0.610	0.277

are overlapped with the corresponding true profiles (dotted gray lines). The estimates are very accurate over the entire mission timeline. Figures 14a and 14b show the comparison between true disturbances acting on the system  $[d_h(t) \text{ and } d_V(t), \text{ in dotted gray}]$ lines] and their estimates  $[\hat{d}_h(t)]$  and  $\hat{d}_V(t)$ , in continuous black lines]. Also, here, the approximation is very good with one exception. Errors with respect to the true disturbances can indeed be observed only at the moment of triggering the feedback-control scheme. The reason for this behavior resides in the fact that there is a discontinuity in the angular rates  $u_{\alpha}$  and  $u_{\mu}$  at the moment of triggering the control system or saturating the controls. Because they appear in the online SMO of Eq. (38), the condition of Lipschitz continuity invoked in Eq. (43) is locally violated, and this causes the presence of these errors. However, once the control activity is started, the hypothesis of Lipschitz continuity is valid again, and the estimated disturbances converge to the true ones immediately, and are bounded by the theoretical  $\epsilon$  (assumed equal to 0.01 m/s<sup>3</sup> for both the altitude and the velocity), as foreseen by the SMO theory. In any case, because the local discontinuity of the angular rates is a mathematical simplification, it does not limit the practical applicability for real systems, which will always have a finite angular acceleration, and, therefore, will not be affected by this local decrease of accuracy. Finally, the behavior of the altitude and velocity sliding states for the three SMC techniques are depicted in Figs. 15a and 15b. In both the plots, one can see that the use of the disturbance observer helps the ADHOSMC to reach the sliding surfaces, whereas without the control system does not reach the origin of the sliding state space. Note that the conventional SMC is also able to reach the origin, but it shows a worse transient, consistently with the results of Figs. 10a and 10b.

If we look at Table 3, we can see the benefits of using the ADHOSMC: the tracking error in terms of altitude is reduced, both with respect to the standard SMC, and more dramatically with respect to the application of the corresponding pure HOSM strategy. Also, the standard deviation is reduced of about 30% with respect to what is obtained by using the standard SMC. It is worth to recall that these results are obtained without any saturation function in the control law (as in the case of the SMC), which would cause a decrease of the robustness of the system, and no tradeoff between robustness and chattering reduction needs to be operated.

Also, in terms of final range, we can observe an improvement coming from the use of the proposed approach. The use of the ADHOSMC positively affects all the constraints. We can observe that all the peaks are equal or less to the ones obtained by using conventional SMC, and as previously stated, only six violations were observed on a total of 1000 cases (corresponding to the 0.6% of cases), against 112 violations observed when the SMC was used. Finally, the corresponding standard deviations are positively influenced as well when the ADHOSMC is employed. The reason for these results comes from the improvement in the transient behavior. Because all the peaks are experienced at about 180–200 s after the beginning of the entry, a better transient (i.e., a faster and smoother convergence to the reference states) automatically turns into a reduction of the peaks, which will become closer to the nominal ones.

#### VII. Conclusions

In this paper, a novel adaptive-control scheme for hypersonicentry vehicles has been proposed. The proposed method uses the chattering-free high-order SMC strategy, and is at the same time able to estimate the combination of known and unknown perturbations acting on the system. The disturbances  $d_h$  and  $d_V$ , coming from multiple uncertainties, are reconstructed online by only using the measurements provided by the navigation subsystem. Moreover, the scheme provides accurate estimates of the state derivatives, without the need to design a further differentiator.

The approach can be implemented by using a step size, which is in the range of the nowadays onboard computers, and, therefore, significantly relaxes the corresponding computational requirements. The results show the feasibility of the approach, together with a significant improvement in the response of the system, especially in terms of transient, with respect to standard SMC strategies. The transient-behavior improvement translates into a significantly smaller number of violations of the maximum value of constraints, in this specific case, the load factor, and in general, to an improvement of the final errors. Moreover, the estimates match very well with the true derivatives and disturbances, confirming the validity of the proposed adaptive approach.

#### **Appendix: Dynamic Inversion Terms**

The terms  $a_h$ ,  $a_V$ ,  $b_{h,\alpha}$ ,  $b_{h,\mu}$ ,  $b_{V,\alpha}$ , and  $b_{V,\mu}$  can be computed as follows:

$$a_h = 2\dot{V}\dot{\gamma}\cos\gamma - V\dot{\gamma}^2\sin\gamma + \sin\gamma a_V + V\cos\gamma a_\gamma \qquad (A1)$$

$$a_V = -a_D - g_h \sin \gamma \dot{h} - g \cos \gamma \dot{\gamma} \tag{A2}$$

$$b_{h,\alpha} = \sin \gamma b_{V,\alpha} + V \cos \gamma b_{\gamma,\alpha} \tag{A3}$$

$$b_{h,\mu} = V \cos \gamma b_{\gamma,\mu} \tag{A4}$$

$$b_{V,\alpha} = -b_{D,\alpha} \tag{A5}$$

$$b_{V,\mu} = 0 \tag{A6}$$

$$a_D = D\left(\frac{\rho_h}{\rho} + \frac{C_{D,h}}{C_D}\right)\dot{h} + D\left(\frac{2}{V} + \frac{C_{D,M}M_V}{C_D}\right)\dot{V} + D\left(\frac{C_{D,a}}{C_D}\right)\dot{\alpha}_{\text{ref}}$$
(A7)

$$a_{L} = L \left(\frac{\rho_{h}}{\rho} + \frac{C_{L,h}}{C_{L}}\right) \dot{h} + L \left(\frac{2}{V} + \frac{C_{L,M}M_{V}}{C_{L}}\right) \dot{V} + L \left(\frac{C_{L,\alpha}}{C_{L}}\right) \dot{\alpha}_{\text{ref}}$$
(A8)

$$b_{D,\alpha} = D \frac{C_{D,\alpha}}{C_D} \tag{A9}$$

$$b_{L,\alpha} = L \frac{C_{L,\alpha}}{C_L} \tag{A10}$$

$$a_{\gamma} = \left(\frac{1}{r} + \frac{g}{V^2}\right) \dot{V} - \left(\frac{V}{r^2} + \frac{g_h}{V}\right) \dot{h} \cos\gamma - \frac{L}{V^2} \dot{V} \cos\mu - \frac{L}{V^2} \sin\mu\dot{\mu}_{\text{ref}}$$
(A11)

$$-\dot{\gamma}\left(\frac{V}{r} - \frac{g}{V}\right)\sin\gamma + \frac{a_L}{V}\cos\mu \tag{A12}$$

$$b_{\gamma,\alpha} = \frac{b_L}{V} \cos \mu \tag{A13}$$

$$b_{\gamma,\mu} = -\frac{L}{V}\sin\mu \tag{A14}$$

Moreover, the partial derivatives of Mach number with respect to the altitude h and the velocity modulus V are computed as follows:

$$M_h = -\frac{M}{2} \frac{T_h}{T} \quad M_V = \frac{1}{\sqrt{\gamma_{\text{gas}} R_{\text{gas}} T}}$$
(A15)

with  $T_h$  representing the derivative of the atmospheric temperature T with respect to h, and computed numerically, whereas  $\gamma_{gas}$  is the air specific-heat ratio, equal to 1.4, and  $R_{gas}$  is the specific gas constant, assumed equal to 287.05 J/(kg · K)).

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