A MARKOV CHAIN BASED STUDY ON THE AVAILABILITY OF SHIP SYSTEMS OF A NAVAL VESSEL

Insights for Early Stage Ship Design requirements

TU Delft

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A MARKOV CHAIN BASED STUDY ON THE AVAILABILITY OF SHIP SYSTEMS OF A NAVAL VESSEL

INSIGHTS FOR EARLY STAGE SHIP DESIGN REQUIREMENTS

By

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ABSTRACT

During the early stages of ship design exploration, design freedom is abundant but problem knowledge is scarce and most costs are already committed. This amounts to an incentive to look at the influences of the requirements on a design, to better understand how a concept is defined. This thesis investigates the operational architecture concerning how systems are used (often in a temporal fashion). A way to quantify the operational architecture is with operability, which describes the ability of a ship to perform its mission. This thesis identified the system performance consisting of availability and vulnerability part of operability as its main topic. Markov theory was selected to assess the availability of systems in order to maximize the operability of a naval vessel. Transition rates of systems are a key element of Markov theory but are hard to obtain during Early Stage Ship Design. A network-theory-based approach allocated nonquantifiable focus points to each system to determine their importance within a network. These were then linked to transition rates, which are required for a Markov chain to calculate the availability of a system. Four case studies were performed to test the model: several configurations were compared, two different focus points approaches were used, different ranges of rates were applied, and systems were added or subtracted. The model proved to behave according to educated guesses beforehand but also gave new insights in connections between nodes. The eigenvector centrality approach used to allocate focus points was found to be better than the combined connectivity approach. The scale of the ranges of rates should not result in very high values for system availability since they do not provide data which makes comparisons between network configurations possible. Very low values of system availability were deemed too far off from real-life values. This thesis paves the road towards a total assessment of operability. It has focused on the availability of systems, and delivered a method to make preliminary decisions for systems design in the early stages.

PREFACE

After a considerable amount of time studying, I am proud to finally present my thesis for the master's degree in Marine Technology and call myself an engineer, like my father and brother before me. I had a rough start, but when I eventually started with my master, grades got better and time passed quickly. The sixth months I spent in Norway were the best experience I have had so far and I learned a lot about myself and life. Writing my thesis went quicker than I could have imagined a couple of years ago, when getting a master's degree was still miles away. For the time being, I am curious what the next year, 2019, has in store for me.

I would like to thank professor Hans Hopman for his sharp questions considering the big picture and the small details, as well as Austin Kana who I find very supportive with his constructive critique, suggestions and grateful grammar checks.

A special thanks goes out to Agnieta Habben Jansen, who has been the best supervisor I can imagine. She always provided good feedback on my work, was very supportive when my hopes were down, and always found the time to read through every piece of documentation I sent her way.

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CONTENTS

List of figures IX
List of tables X
Glossary XI
1 Introduction 1
2 Background 2
2.1 Current Research 2
2.1.1 Early stage ship design 2
2.1.2 Architectural framework 4
2.1.3 Similarities in research on vulnerability 5
2.2 Operability 7
2.3 Dependability methods 8
2.3.1 Fault tree analysis 8
2.3.2 Bayesian networks 10
2.3.3 Petri nets 11
2.3.4 Markov theory 11
2.3.5 Power-hierarchy 12
2.3.6 Choosing a method 13
2.4 Research Goal 13
3 Markov Theory 16
3.1 Markov Chains 16
3.2 Continuous-Time Markov Chains 16
3.3 Defining States 18
3.4 Transition Rates Problem 20
4 Network Theory 21
4.1 Network Configuration 21
4.2 Focus Points 22
4.2.1 Node importance 22
4.2.2 Nonquantifiable values 22
4.3 Determining Node Importance 23
4.3.1 Optimization methods 23
4.3.2 Network properties 24
4.4 Transition Rates 25
5 Model Definition 28
5.1 The Model 28
5.2 The Calculation 29
6 Case Studies 30
6.1 Change in configuration 30

6.2 Application of two focus point approaches ... 32
6.2.1 Comparison between approaches ... 32
6.2.2 Comparison to the baseline ... 34
6.3 Change in range of rates ... 36
6.4 Addition or removal of a system ... 38
7 Discussion ... 41
7.1 Discussion on The Case Studies ... 41
7.1.1 Change in configuration ... 41
7.1.2 Change in transition rates ... 41
7.1.3 Change in range of rates ... 41
7.1.4 Addition and subtraction of systems ... 42
8 Conclusions and Recommendations ... 43
8.1 Conclusions ... 43
8.1.1 Use of Markov theory ... 43
8.1.2 Regarding operability ... 43
8.1.3 Reflection on the model ... 44
8.2 Recommendations ... 45
8.2.1 Adaptations to the model ... 45
8.2.2 Adaptations for the contribution to operability ... 45
9 Bibliography ... 47
A. Appendix 1: Transition matrix as used in 5.2 ... 49
B. Appendix 2: Focus points calculation ... 50
C. Appendix 3: Results of nine rate ranges ... 51
D. Appendix 4: MATLAB® code ... 52

LIST OF FIGURES

Figure 1: Concept exploration and definition process (van Oers *et al.*, 2018) 2

Figure 2: Example of a multiplex network of an OPV (Duchateau, de Vos and van Leeuwen, 2018) 3

Figure 3: A generic design timeline, (Duchateau, 2016) 3

Figure 4: Visual representation of the architectural framework for naval distributed systems for a specific scenario (Brefort *et al.*, 2017). 4

Figure 5: Operability tree, own composition 8

Figure 6: A simple fault tree for the top event that system X fails, own composition. 9

Figure 7: Bayesian network representation of Figure 5, own composition... 10

Figure 8: Petri net representation, own composition 11

Figure 9: Discrete-time Markov chain, own composition.. 12

Figure 10: Power-hierarchy among dependability-model types (Malhotra and Trivedi, 1994)......................... 13

Figure 11: Schematic overview of the next steps.. 14

Figure 12: Transition rates between states (Rausand, 2011) 17

Figure 13: Network of a simple system of systems 18

Figure 14: Network configuration of a simple system of systems 22

Figure 15: Schematic overview of the model 28

Figure 16: Configuration 1 31

Figure 17: Configuration 2 31

Figure 18: Configuration 3 31

Figure 19: Configuration 4 31

Figure 20: Average availability of weapon system 1 33

Figure 21: Average availability of weapon system 2 33

Figure 22: Average availability of weapon system 1 in configuration 4, various ranges of rates...................... 37

Figure 23: Configuration 5 38

Figure 24: Configuration 6 39

Figure 25: Configuration 7 39

Figure 26: Operability tree 46

LIST OF TABLES

Table 1: Overview of existing methods for early stage vulnerability assessment of naval ships by (Habben Jansen, Kana and Hopman, 2017). 5

Table 2: Conditional occurrence table of the Bayesian network for the immediate events 10

Table 3: Conditional occurrence table of the Bayesian network for the top event 10

Table 4: System states of a two-pump system 16

Table 5: Overview of state transition rates from state X to state X' 17

Table 6: Importance analysis of each node in Figure 14. 24

Table 7: Two approaches for the importance analysis of each node in Figure 14. 25

Table 8: Lowest and highest value of each approach 25

Table 9: Range of rates 26

Table 10: Scaling of the range for all values of both approaches 26

Table 11: Two approaches for the importance analysis of each node in Figure 14. 27

Table 12: Rates of systems for both approaches of node importance 27

Table 13: Average availability of configuration 1-4, with a transition rate of $1.0e^{-5}$ and recover rate of $1.0e^{-3}$. 30

Table 14: Average availability of configuration 1-4, with a transition rate of $1.0e^{-4}$ and recover rate of $1.0e^{-3}$. 32

Table 15: Average system availability for each configuration and both approaches 32

Table 16: Difference in average availability between the baseline case and the eigenvector centrality approach 34

Table 17: Difference in average availability between the baseline case and the combined connectivity approach 34

Table 18: Number of focus points per system per approach 35

Table 19: Various ranges of rates 36

Table 20: Ranked ranges from best to worst performing. 37

Table 21: Average availability of weapon system 1 for different ranges and configurations. 38

Table 22: Average availability of the new configurations, with and without the connection between the switchboards. 39

Table 23: Focus points for each system in the new configurations. 39

GLOSSARY

Term	Definition
Focus point approach	A method of determining the importance of a node within a network.
Availability	The ability of a functional unit to be in a state to perform a required function under given conditions at a given instant of time or over a given time interval, assuming that the required external resources are provided. (NATO, 1997)
Combined connectivity	The sum of the downstream- and upstream connectivity, minus one due to duplication issues.
Configuration	The arrangement of nodes and edges within a network.
Degree centrality	The degree of a node in a network is the number of edges attached to it.
Downstream connectivity	The number of nodes connected to a node that depend on its output.
Edge	A connection between two nodes. Can either be directed or undirected.
Eigenvector centrality	The sum of the degree centrality of the nodes attached to a node.
Focus point	An abstract value used to determine the amount of resources allocated to the design of a system and its setup, relative to other systems.
Mode	The condition a system is in, i.e. working, failed or in error.
Network	A set of connected nodes.
Node	The representation of a system with a network.
Operability	The ability to perform the ship's mission in a safe and reliable functioning condition, according to pre-defined operational requirements. (Adapted from Wikipedia, 2008)
Range	The minimum and maximum value of both transition- and repair rates, used in a MATLAB® simulation.
Recover rate	A rate describing the frequency of transitioning back to a system's initial mode.
State	A set of modes each system of a network is in.
System	A piece of machinery or equipment used to perform a function within a vessel.
System of systems	A group of connected systems with a common goal.
Transition matrix	A square, stochastic matrix describing the transition process of a Markov chain.
Transition rate	General term for a rate describing the frequency of transitioning between two states. Also used to indicate the transition of a system from a working to a failed mode.
Upstream connectivity	The number of nodes connected to a node it depends on.
Vulnerability	The characteristics of a system that cause it to suffer a definite degradation (incapability to perform the designated mission) as a result of having been subjected to a certain level of effects in an unnatural (man-made) hostile environment. (Gortney, 2010)

1 INTRODUCTION

Early Stage Ship Design (ESSD) is the field in the maritime technology sector that describes the progress of exploring the customer's wishes by investigating different solutions for these wishes, which are then used to focus towards a solution that is worked out into more detail (Duchateau, 2016). The goal of ESSD is to examine the influences of the requirements on the design, in order to better understand what one wants to design. Ships are becoming more complex, and naval ships—the focus of this thesis—are currently even amongst the most complex vessels. Detailed design information is often lacking in the early stages, but the design freedom is high. This requires other design methods than conventional design methods, such as the design-spiral.

Over the past years, lots of research has gone into developing new ways to get a better understanding of design requirements, prior to the definition of a design concept. Most research on ship systems can be categorized as falling under a certain architectural framework, consisting of the physical, logical and operational architecture (Brefort *et al.*, 2017). The physical architecture describes where systems are located, whilst the logical architecture defines how systems are connected and how systems are used (in time) is the core of the operational architecture. The physical and logical architecture have been researched extensively, as for instance in system vulnerability analyses. The operational architecture requires more exploration though. The vulnerability studies by Habben Jansen, Kana and Hopman (2017) assess the operational architecture to a certain degree, by explaining how the vulnerability of systems influences its availability. Markov theory is used for this because it is able to handle uncertainty and captures the dynamic (time) effects of complex systems well.

Operability is a measure to assess the effectiveness of the operational architecture. Operability is a complex idea, consisting of elements related to how the ship performs (seakeeping, maneuverability), how systems perform (availability, vulnerability) and how crew performs (maintainability, recoverability). In this thesis, focus is given to the design of systems. Vulnerability has already been looked at by Habben Jansen, Kana, and Hopman (2017), so availability will be its main feature. The following research question is thus proposed:

How can Markov theory be used in Early Stage Ship Design regarding systems design to analyze the availability of systems in order to maximize the operability of a naval vessel?

The abovementioned design problem during the early stages of design is further discussed in the background section in chapter 2. Reviewed earlier research, as well as the architectural framework, the operability idea, dependability methods, and the research question, will also be explained in greater detail throughout this chapter. Mathematical methods, such as the aforementioned Markov theory, and network theory used in this thesis are explained in chapters 3 and 4. Chapter 5 explains what the developed model looks like. Four case studies test the applicability of the model in chapter 6. This report will conclude with results and recommendations in chapters 7 and 8.

2 BACKGROUND

This chapter explains the background leading to the initialization of this thesis. It describes the field of Early Stage Ship Design and what kinds of research this encompasses. A section explaining operability in detail follows. Dependability methods are discussed next and the research goal will conclude the last section of this chapter.

2.1 CURRENT RESEARCH

2.1.1 Early stage ship design

During Early Stage Ship Design, the customer's wishes are explored. This exploration is meant to quickly evaluate different concepts at a limited level of detail in order to understand the requirements and trade-offs of a design. This process is illustrated in Figure 1. The most desirable concepts are then selected and the insights are then used for the concept definition where the details can be finalized (van Oers *et al.*, 2018). A design is comprised of for instance a general layout of the vessel (where is what on the ship), a structural analysis (how strong the structure is), hydrodynamic calculations (how does it perform in the water), preliminary routing of piping and cables (how energy is transported) and a visualization of what it will look like (what everyone actually sees).

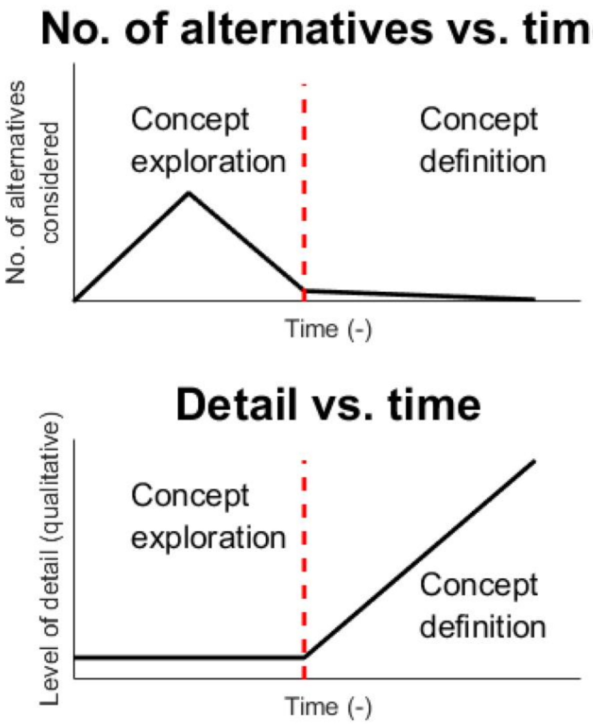

Figure 1: Concept exploration and definition process (van Oers *et al.*, 2018)

But as all of this is conceptual design, it is always prone to changes. Often, ship design is performed with the help of a so-called design-spiral. The design begins with big choices, but as more and more elements and details are added during the design, one must go back to the beginning where the first steps were made in order to check if these are still the right choices. This is an exhaustive and iterative process, not suitable anymore for today's complex naval vessels (Pawling, Percival and Andrews, 2017). In the early stages of design, there aren't a lot of choices set in stone and thus the freedom in design is quite high. Detailed information and hard facts are often lacking on the other hand, which leaves some choices in a fuzzy state. As the design progresses and moves beyond the early stages, it is not uncommon that some of the early design choices were wrong and thus an effort has to be made in order to correct this, with all its consequences.

Ships are becoming more and more complex. Naval ships are moving to a new model where integrated full electric propulsion (IFEP) and power generation is the norm (Ames, 2017). This is different from the conventional separate propulsion often provided by a combined diesel-electric and gas turbine propulsion system, as it doesn't need a mechanical connection between the engines and propulsors. This allows for more flexibility in the placement of the system. With this growing complexity, interdependency between distributed systems on board a vessel becomes even stronger (Dougal and Langland, 2016). In rare occasions, systems function entirely on their own, but in general, systems work together to enable or support the operational scenario. Systems not only need power to operate but often also require cooling or chilled water, fuel and even data (for e.g. weapon systems). An example of such a 'multiplex network' is given in Figure 2. Improvement of ship design and its techniques is thus warranted.

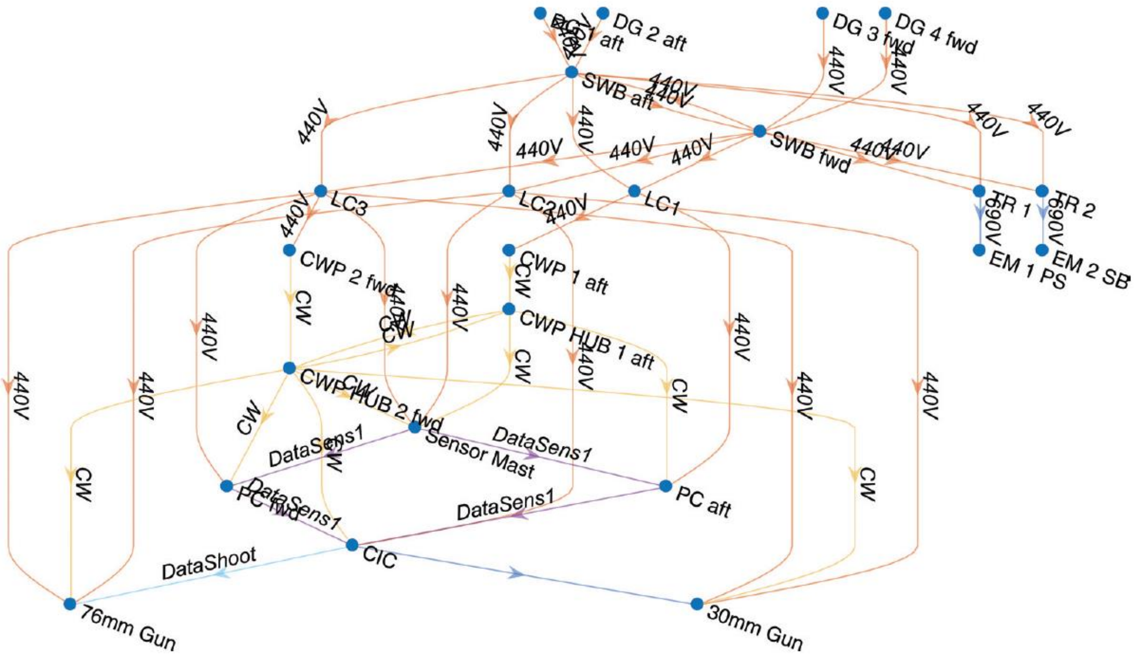

Figure 2: Example of a multiplex network of an OPV (Duchateau, de Vos and van Leeuwen, 2018)

Decreasing the number of errors made in the early stages, and thus increasing the effectiveness of the early stage design phase is currently the focus point of research in Early Stage Ship Design. A better design method would have more freedom in the later stages, knowledge of problems far earlier in the conceptualization and fewer costs committed in the early stages of the project (Kana, 2017). Duchateau (2016) illustrates this with an adaptation of the original figure by Mavris and DeLaurentis (2000), as shown in Figure 3, where early stage ship design is portraited by the preliminary design stage.

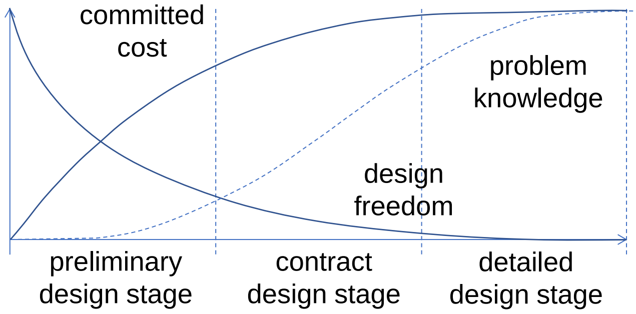

Figure 3: A generic design timeline, (Duchateau, 2016)

In the past two decades or so, major improvements in ship design have already been made regarding more efficient ship design, more detailed ship design and computer-aided ship design, among others. The recent study by Duchateau (2016) heavily investigated the way ships are designed, especially in these early stages. The study explains the importance of asking questions such as "what-if" scenarios and "What is it we are actually looking for?", in order to understand why a ship has to be designed in a certain way.

For instance, the packing design approach of van Oers (2011) and Duchateau (2016) is eagerly being used at the moment for general layout design and understanding, with various theses using it as a base for further research (Droste, 2016; Jaspers, 2017; Roth, 2017; Salvatori, 2018). Although this approach helps understanding why certain layouts seem to be worse than others, it does not help in selecting the best option from a set of designs. Different studies have rather focused on the routing of pipes and cables through a ship in early design stages, giving information about problems these might create given certain routings (Goodrum *et al.*, 2017; Duchateau, de Vos and van Leeuwen, 2018). Survivability and vulnerability of ships especially are researched as well. Habben Jansen (2017) looked at the survivability of naval machine systems by making use of Markov theory. Habben Jansen, Kana and Hopman (2017) also use Markov theory in order to assess the vulnerability of a naval vessel.

2.1.2 Architectural framework

Brefort *et al.* (2017) have developed an architectural framework to define the manner in which systems are organized and integrated, as shown in Figure 4. It consists of the physical architecture (i.e. spatial arrangements), the logical architecture (how different system components are connected) and the operational architecture (how a system and its components are used, often in a temporal fashion). This structural division is important because it shows the interdependencies of systems, which are important to know if one is to understand the working of these systems.

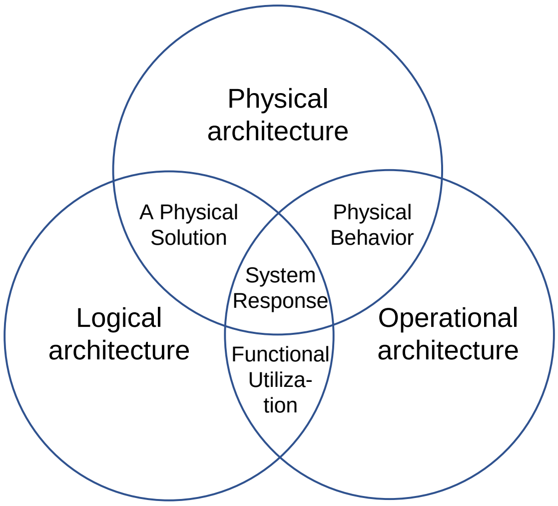

Figure 4: Visual representation of the architectural framework for naval distributed systems for a specific scenario (Brefort *et al.*, 2017).

The physical architecture describes the locations of systems, the dimensions of compartments and where equipment is situated in a ship. The packing approach mentioned earlier is an example of a tool for generating the physical architecture.

The logical architecture describes the functional characteristics of a system, and how systems are connected to or dependent on other systems (e.g. single-line diagrams). To understand the logical architecture, it is important that the relationships and dependencies between systems are clear, to which according to the paper, network theory based metrics lends itself well. An example of logical architecture can be found in the papers by Goodrum *et al.* (2017) and Duchateau, de Vos and van Leeuwen (2018).

The operational architecture (OA) defines the temporal behavior of a system, or as described by Brefort *et al.* (2017), *"Temporal behavior is intended to capture what needs to happen through time to accomplish a given mission scenario. It defines what systems are needed in which order, or what processes and input/outputs are needed through time (like personnel movement or the charging of a capacitor)."* Due to the temporal nature of the operational architecture, Brefort *et al.* (2017) mention the use of Markov theory in papers of Niese (2012) and Kana (2016) to analyze it. A different way of explaining the operational architecture may be given by Giraud Jr. (2014). He compares it to the floor plan when building a house, where operational architecture is explained as the operational view that defines operational processes and information requirements. It tells you what you are trying to accomplish. To quote Giraud Jr. (2014): *"OA is the art of taking unstructured problems and giving them enough structure to enable decision-makers to plan further useful action."* This is precisely what is needed in the early design stages, where much design freedom to tackle unstructured problems is present but not enough knowledge (structure) to make the right design choices. Both definitions are slightly different, however. The first one focusses more on the usability of systems, whilst the second one focusses more on decision-making regarding the use of systems.

The interrelations between the architectures are the physical solution, functional utilization, physical behavior, and system response. What system components look like in the physical space of a ship is called the physical solution, think of the physical layout and the way a radar and weapon system are logically connected. The physical behavior is the interrelation between the physical and operational architecture. The physical size of an engine is related to the amount of power it needs to generate, which is again related to the mission of interest of a ship. Thus it sets requirements for the characteristics of the physical layout. A relationship also exists between the logical and the operational architecture, called the functional utilization and *"characterizes the connection between system components and resources flows required in time to fulfill a mission of interest. It specifies which systems are used or can be used over time from a given logical arrangement, to fulfill a specified mission."* The center of the framework is called the system response, where all three architectures give a piece of information about a certain situation. It characterizes the performance of a chosen physical layout, a chosen set of interdependencies and a chosen mission. The system response may thus be defined as the primary measure of performance or effectiveness.

2.1.3 Similarities in research on vulnerability

Habben Jansen, Kana, and Hopman (2017) have used this architectural framework to classify some of the recent studies. In Table 1, one can see that the physical (P) and logical architecture (L) have been addressed aplenty, but research on the operational architecture is lacking. Though this very paper and another recent study by Habben Jansen (2017) can be categorized as research on the operational architecture. The paper concerns a methodology for an operational vulnerability assessment for naval ships. Its main concern is the availability of distributed systems after a vessel gets hit. They define vulnerability as a subdivision of survivability, together with susceptibility and recoverability. However, for ship designing purposes they leave out recoverability as part of the survivability of a vessel as it is more related to human interaction (repairs) than to how the ship is designed. The paper identifies the operational scenario, impact level, interdependencies and time as important factors for assessing the system vulnerability. Though it only deals with the interdependencies, meaning that a vulnerability study including the operational scenario, impact level, and temporal nature is to be investigated in further research. The paper acknowledges that current vulnerability tools are analysis tools instead of design aids, which makes them most useful during the detailed design stage but not during early stage design. As discussed earlier, it is very costly in both time and money to change a ship's design in the detailed design stage based on such analyses, thus such a tool is more desirable during early stage design.

Table 1: Overview of existing methods for early stage vulnerability assessment of naval ships by (Habben Jansen, Kana and Hopman, 2017).

Method	Theory	Arch.
Shields, 2016	Complexity theory	P, L
Kim, 2012	Various (probabilistic)	P, L
van Oers, 2012	Networks, optimization for physical parameters	P, L
Trapp, 2015	Networks, optimization for flow	L
de Vos, 2014	Networks, optimization for network parameters	L

Habben Jansen, Kana, and Hopman (2017) define a survivable system as *"one that is placed at a proper location,* and *connected adequately to other systems,* and *designed in a fashion that is in compliance with how it will be operated."* This can be linked to the architectural framework, where the location corresponds with the physical architecture, the connections with the logical architecture and the latter with the operational architecture. It is evident that most research has focused on the physical and logical architecture so far. But a situation-dependent and dynamic research such as presented in that paper lends itself more to the operational architecture and not only assesses whether a system is available after being hit but also how that affects the operational scenario. Habben Jansen, Kana and Hopman (2017) identify that different scenarios, interdependencies, time and a low level of detail are the requirements in order to implement this operational architecture. Markov theory is proposed as a possible good method for dealing with these aspects. Markov theory lends itself well for a time-based approach and is strong in dealing with uncertainties within a process. Though it can be used for any level of detail, Markov theory is useful when the model has a low level of detail, since that limits the number of uncertainties. Moreover, Markov theory is used to describe system states over a range of time, which can be thought of as the different scenarios.

This paper concerns vulnerability mostly, though some related aspects still need researching. For instance, recoverability is left out, even when this is very important for addressing the operational side. It is defined as *"the ability to restore (parts of) the ship or system functionality by means of active response".* Indeed, recoverability is linked to humans actually repairing broken systems and has limited effect on the ship design itself. But if the location, connection, and operation of systems can be designed in such a way that they fail less often, can that not be regarded as ship design as well?

The research by Habben Jansen (2017) takes a deeper look into Markov theory for assessing a similar vulnerability problem as Habben Jansen, Kana and Hopman (2017) did. Markov theory uses Markov chains, which can be analyzed in discrete-time or continuous-time, to study the probability of a system being available. A discrete-time Markov chain has to go through a number of iterations before it reaches a steady-state (if it even does), whilst a continuous-time Markov chain inherently provides the steady-state of a system over a certain range of time. The focus of the paper is on discrete-time Markov chains but recommends continuous-time Markov chains for long term system availability evaluations, especially when a larger system is considered with more components and connections. In addition, the paper recommends not only looking at absolute availability but also at partial availability (i.e. 50% or 75% availability as opposed to only 0% and 100% availability). Habben Jansen (2017) also mentions including other reliability issues, getting hit (the paper's focus) is different from components breaking down. In terms of the operational architecture, these same states (a system being unavailable) have different probabilities of occurrence and different operational consequences. This study does also take recoverability into account. This means that every damaged system or (component thereof) is able to return to its original state. It is also stated that *"recoverability is mainly an operational aspect and is usually not included comprehensively during the design,"* meaning that it would be better to include more often in future research.

Another study by van Oers, van Ingen and Stapersma (2012) looked at the vulnerability of a naval vessel as well but focusses heavily on how the location and routing of distributed systems affect the post-hit availability of a system. Network theory is used in conjunction with search algorithms to generate numerous ship service networks, of which the availability is then analyzed after the vessel has taken a certain number of hits. Though this research gives good insights in how the ship service network affects the availability of, for instance, a weapon system, due to systems being unavailable post-hit, it lacks the more dynamic approach taken by Habben Jansen, Kana, and Hopman (2017).

What these researches—as well as others such as Kim and Lee (2012)—don't take into account is the system availability due to other, non-hit related, circumstances. Think for instance of a pump failure or a leakage somewhere in the ship that causes a weapon system to overheat, making it unavailable to be used during a certain operational scenario. Other possibilities are crew not performing to their full potential or perilous weather conditions at sea that make it impossible to use certain radar or weapon systems.

This operational side to using a naval vessel is, to the author's knowledge, not researched sufficiently yet. To get a better picture of what mission a ship is actually capable of performing once at sea, multiple types of research have to be bundled together. Combining weather conditions, accompanying seakeeping, crew capability, ship, and system survivability, system reliability and probably even more could be categorized as the *operability* of a ship and could give a tremendous view of requirements for a designer in ESSD.

2.2 OPERABILITY

But what is the relationship between the operational architecture and operability and how exactly is operability defined? NATO does not have a definition of operability in their database, nor does the US Department of Defense (Gortney, 2010). According to Wikipedia (2008), *"Operability is the ability to keep an equipment, a system or a whole industrial installation in a safe and reliable functioning condition, according to pre-defined operational requirements."* This is however not entirely an interpretation useful in this thesis, as the whole ship and its mission are the key features for this research—as opposed to a single object—and therefore needs some adjustment. The adapted definition used in this thesis is: *"Operability is the ability to perform the ship's mission in a safe and reliable functioning condition, according to pre-defined operational requirements."* These operational requirements may be derived from the operational architecture, which describes how a system and its components are used. So, operability can be thought of as a way to measure the operational architecture, just as dimensions are a measure of physical architecture and the number of connections of logical architecture. The safe and reliable functioning condition mentioned in the definition make availability and vulnerability immediately spring to mind, though as stated earlier, more factors exist that influence the operability, as will be discussed below.

Availability can be regarded as the internal source of effects on the operability of ship service systems, while vulnerability as the external source. The first deals with all kinds of failures related to the performance of a system, while the latter deals with unforeseen consequences that are out of the ordinary (such as a hit to the ship by enemy fire). Availability is defined as *"The ability of a functional unit to be in a state to perform a required function under given conditions at a given instant of time or over a given time interval, assuming that the required external resources are provided*,*"* (NATO, 1997). And vulnerability (as used in Brefort *et al.* (2017)) is defined as *"The characteristics of a system that cause it to suffer a definite degradation (incapability to perform the designated mission) as a result of having been subjected to a certain level of effects in an unnatural (man-made) hostile environment*,*"* by Gortney (2010).

Seakeeping and maneuverability are also factors that influence the way the ship can be handled in say, rough weather. This, too, falls under "a safe and reliable functioning condition" and could make performing certain missions impossible. These factors can be categorized as a second external source influencing the operability, whilst crew performance is an internal source of operability.

Figure 5 schematically shows what the operability (as used in this thesis) of a ship consists of. Note that by no means this representation suggests this is a fully exhausted list of all factors that can possibly influence operability. It is rather an outline of the most relevant topics in regards to this thesis. Operability is divided into three main segments: ship performance, system performance, and crew performance:

1) The first includes seakeeping, maneuverability, and susceptibility (a subsection of survivability). These can be influenced by weather conditions for instance but are also linked to system and crew performance. A broken rudder has an influence on the maneuverability for instance.
2) The second segment deals with the availability of systems due to either internal or external causes. External causes can again be thought of as weather conditions (e.g. a frigate finding itself in arctic areas during wartime, while its weapons systems were not designed for those conditions) or as vulnerability. Internal causes are those actions relating to the systems themselves that make a system unavailable.
3) The last segment addresses the way in which human response influences the operability. This includes, for instance, the maintenance strategies used to keep systems running but also the time it takes crew to repair a system.

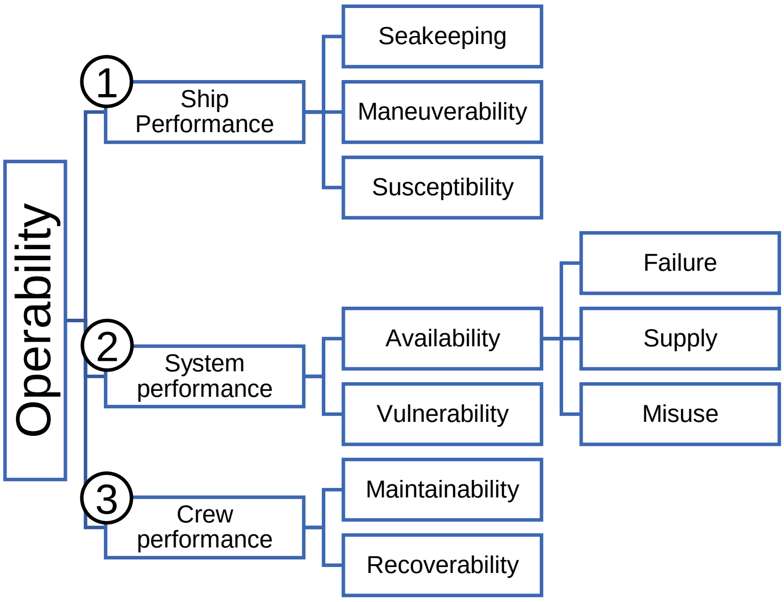

Figure 5: Operability tree, own composition

The focus of this thesis is on the design of ship systems however, thus only the system performance will be taken into consideration, which is in line with the research mentioned earlier in this chapter. Vulnerability, for instance, has been looked at thoroughly, though availability has not. Weather conditions, though they have an impact on the operability, will not be taken into consideration for systems design during ESSD, as the impact is fairly random and can only be impacted by the design of a system in a limited way.

The tree in Figure 5 shows that availability is considered to consist of three parts: failure, supply, and misuse. Failure entails all such manners in which a system can stop working, related to components of the system itself failing. Supply reflects the connectivity of a system, where any failure in a connected system causes the system itself to fail. Think of a power cut, a failing chilled water plant or the unavailability of important radar data. This is the functional utilization explained in the architectural framework of section 2.1.2 before. Lastly, misuse involves human errors leading to the failure of a system. This is obviously linked to crew performance (meaning that repairs have been carried out poorly for instance), but such connections are left out of the figure for the ease of reading. Human error is however not directly a design parameter (as also mentioned by Habben Jansen, Kana, and Hopman (2017)) and is left out in this research. The availability due to failure and supply is thus left to be researched.

2.3 DEPENDABILITY METHODS

Dependability methods are methods or models used to calculate reliability, availability, safety etc.. Rausand (2011) defines a couple of methods for causal and frequency analysis, relating to the availability of systems given certain failures. The most relevant to this study are fault tree analyses, Bayesian networks, Petri nets, and Markov theory. Each of them will be discussed in more detail below. Malhotra and Trivedi (1994) have defined a power-hierarchy of such dependability methods, which will also be discussed in this section.

2.3.1 FAULT TREE ANALYSIS

Fault tree analysis (FTA) is a well-known, widely used method for causal analysis of hazardous events (such as the availability of a system after certain events cause a failure to occur). It is a top-down logic diagram, showing the relationship between a critical event in a system and its causes. This analysis can either be qualitative, quantitative or both. FTA is used to identify all possible combinations of basic events that may result in a critical event in the system. The probability that a critical event will occur during a certain time interval or at a certain time *t* can be found, as well as the frequency with which the critical event happens. It is also used to identify elements of the system that need improving to reduce the probability of the critical event occurring.

In FTA a critical event called the top event is analyzed. Immediate causal events leading to the triggering of this top event (either by themselves or in combination with other events) are identified and connected to the top event by logic gates. Subsequently, potential causal events that may lead to these immediate causal events are identified and connected through a logic gate. This procedure goes on until the desired level of detail is reached. The events at this lowest level are called basic events. An example of a simple fault tree is given below.

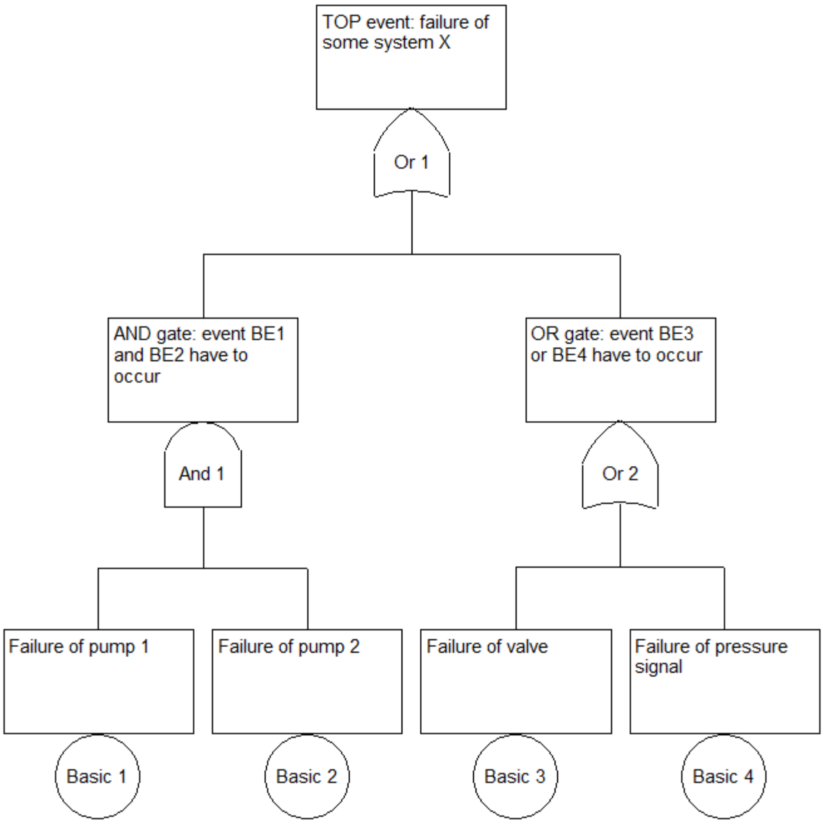

Figure 6: A simple fault tree for the top event that system X fails, own composition.

The top event of a system X failing in Figure 6 can be caused by either the left branch or right branch of the tree (OR logic gate). The left branch is defined by an AND logic gate, meaning that both basic events 1 and 2 need to occur at the same time before the event is triggered. The right branch is defined by an OR logic gate, meaning that if either basic event 3 or 4 occurs, the event is triggered. Thus system X fails if: basic events 1 and 2 occur, or if basic event 3 occurs, or if basic event 4 occurs. This is a binary analysis; an event either occurs or not. No intermediate states (partial failure) can be taken into account in an FTA. A fault tree such as the above is also *single event*-oriented, meaning that for a different kind of failure of system X a completely different tree must be constructed. The same goes for an entirely different system Y for example.

FTA is easy to use and gives a clear and logic overview of what is being analyzed. It is suitable for complex systems, breaking them down step by step. This makes it also suitable for many different critical events. FTA gives a designer a better understanding of failure causes and thereby allowing him to rethink the design and operation of a system to eliminate potential hazards. However, FTA loses precision when more states than just working or failed are added and it handles sequence-sensitive scenarios poorly. For complex systems, fault tree analyses become complicated, time-consuming and quite difficult to follow. Lastly, the approach is static by nature and not suited for any dynamic systems, such as systems subject to complex maintenance strategies.

2.3.2 Bayesian networks

A Bayesian network is a directed acyclic graph illustrating the causal relationships between causes and outcomes. It consists of nodes and arcs, where a node is a state or condition and its arc indicates a direct influence, for instance, a statistical dependence. The network is used to identify all relevant factors that can significantly influence a critical event and the probability of a critical event occurring can be calculated. The most important contributors to this probability can also be identified.

Figure 7 shows what a Bayesian network looks like, based on the fault tree of the previous section. The basic events are the starting point of the network, leading to the immediate events and finally the top event.

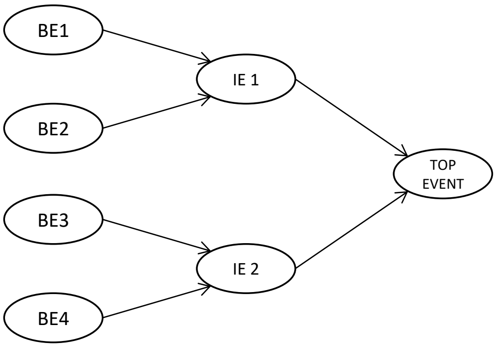

Figure 7: Bayesian network representation of Figure 5, own composition

Table 2 and Table 3 show how the AND and OR gates of the fault tree in section 2.3.1 are translated into a Bayesian conditional probability. Where:

$$\text{BE X} = \begin{cases} 1 & \text{if basic event X occurs} \\ 0 & \text{if basic event X does not occur} \end{cases} \tag{2.1}$$

Table 2: Conditional occurrence table of the Bayesian network for the immediate events

BE 1	BE 2	IE 1	BE 3	BE 4	IE 2
0	0	0	0	0	0
1	0	0	1	0	1
0	1	0	0	1	1
1	1	1	1	1	1

Table 3: Conditional occurrence table of the Bayesian network for the top event

IE 1	IE 2	Pr(TOP EVENT = 1)
0	0	0.00
1	0	1.00
0	1	1.00
1	1	1.00

Bayesian networks are a good alternative to fault tree analysis for risk analysis. Han, Marais, and DeLaurentis (2012) for instance use Bayesian networks to analyze interdependencies between systems of a US naval Littoral Combat Ship, where special focus is given to the failure propagation to other systems. Bayesian networks provide an intuitive graphical representation and are based on a mathematically rigorous theory. A Bayesian network is not limited to Boolean logic (working or failed) but can have any probability on its arcs (considering normal probability laws). A limitation is however, that the amount of work will increase almost exponentially with the number of nodes and will quickly require the aid of a computer.

2.3.3 Petri nets

A Petri net is a directed graph with arcs and two types of nodes, called places and transitions. Arcs always connect a place to a transition or the other way around. Tokens can be used to represent a resource flow and capacity through the network. Figure 8 shows what the fault tree in Figure 6 looks like in the form of a Petri net.

Petri nets are able to model and improve the aforementioned methods. They are based on universal technical modeling language (which you do need to know) and can even be simulated in order to illustrate and test system behavior. There is no limitation on the level of modeling detail and can capture both static and dynamic time-dependent behavior. The big downside to this method is that they can become very large, complex and confusing. Documentation on Petri nets used for reliability engineering is also still lacking.

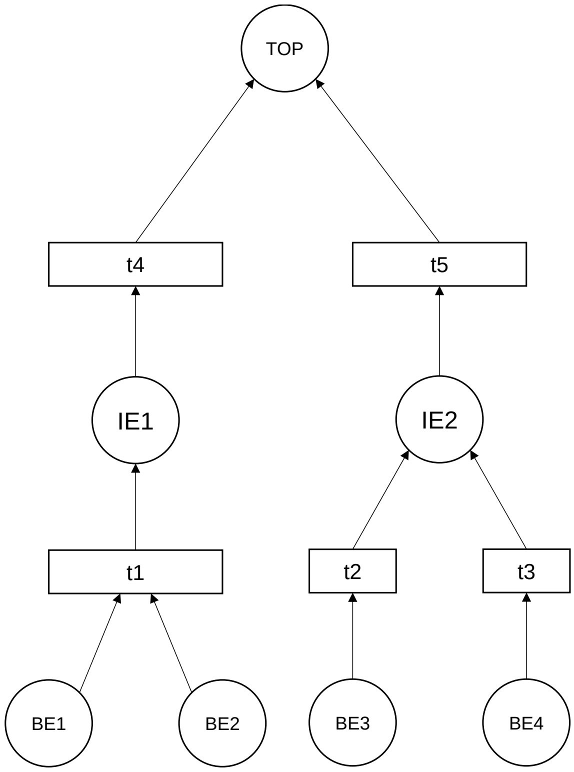

Figure 8: Petri net representation, own composition

2.3.4 Markov theory

A Markov process is a stochastic process describing a set of subsequent states, where the distribution of future states depends only on the current state and not on how the current state is reached; it is a memoryless process. It is often used to analyze systems with redundancy, interdependency, complex maintenance strategies, and/or sequence-dependent failures. Markov processes can be analyzed in discrete- and continuous-time with a finite number of states.

The discrete-time Markov chain in Figure 9 is an example of how the fault tree in Figure 6 can be replaced by a Markov chain. System X is working in state A but not in state C. In state B, one pump has failed but system X is still working. A ten percent chance exists that one pump fails, transitioning state A into state B. In state B there is a bigger chance of the second pump failing or the valve or the sensor can fail in the meantime, transitioning state B into state C, where system X has failed. There is a twenty percent chance that just the valve or the sensor fails, state A transitions then immediately to state B. Lastly, a seventy percent chance exists that no subsystem will fail and system X keeps working. Once a pump has failed, a thirty percent chance exists that no other subsystem will fail subsequently and system X keeps working. Note that this example does not take repairs into account, thus once state C is reached, it will always stay there (system X remains broken).

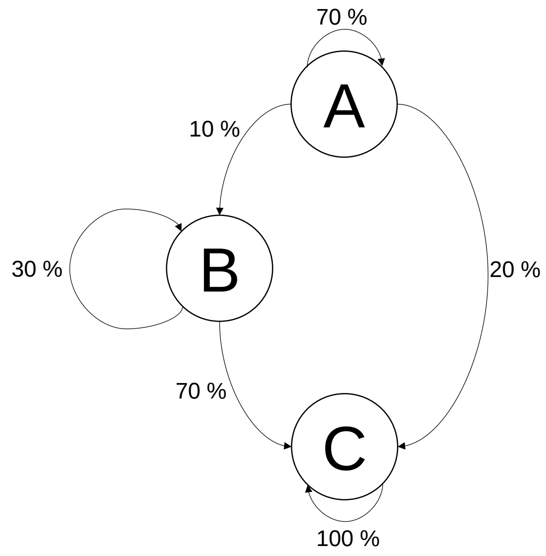

Figure 9: Discrete-time Markov chain, own composition

This model can be analyzed over a range of time, where eventually state C is always reached, thus after a certain number of steps, a steady state can (ideally) be reached and it can be seen how likely it is to end up in state C, given any starting state A or B.

A continuous-time Markov chain can be constructed in the same way, though instead of pure transition probabilities, transition rates are used, meaning that it will automatically calculate the steady state of a system, and thus its availability.

Markov theory is well-documented and has been applied and verified in many areas. It is suitable for small but complex systems with dynamic properties, which fault trees cannot adequately do. It provides a whole range of performance measures of a system—such as the availability, visit frequency of a state, the frequency and expected number of failures, overall mean uptime and downtime and mean time to the first system failure—which are difficult to obtain by other methods. Moreover, a state transition diagram with important information is easy for non-specialists to understand. However, the method is limited to small systems with a limited number of states, as it becomes quickly time-consuming. The continuous-time method is also limited by the requirements for constant transition rates.

2.3.5 Power-hierarchy

Malhotra and Trivedi (1994) have developed a power-hierarchy among dependability-model types, as can be seen in Figure 10. They include three of the four abovementioned methods, as well as some others that have not been discussed previously such as reliability block diagrams, fault trees with repeated events and reward models. The top methods are considered to be more powerful than the bottom methods in terms of calculating dependability. They divide the methods into two categories: combinatorial model types and Markov model types.

Reliability block diagrams (RBD), Reliability graphs (RG), fault trees (FT) and fault trees with repeated events (FTRE) fall into the combinatorial (also known as non-state space) model type category. These methods can model availability of repairable systems well, under the assumption that each system component has an *s*-independent repair person. From Figure 10 can be seen that RBD and FT are ranked at the same level, while RG and FTRE are each a step better in their performance.

Continuous-time Markov chains (CTMC), generalized stochastic Petri nets (GSPN), Markov reward models (MRM) and stochastic reward nets (SRN) are Markov-model types, according to this paper. All these types are based on basic CTMC theory or can either be converted to it. These can handle some dependencies in a system, which combinatorial models cannot. Figure 10 shows that all Markov model types are ranked higher than combinatorial-model types. Dugan *et al.* (1986) have proved that CTMC can replace FTRE and is thus more powerful. As the other methods are Markov related, they too are in the highest level of this hierarchy.

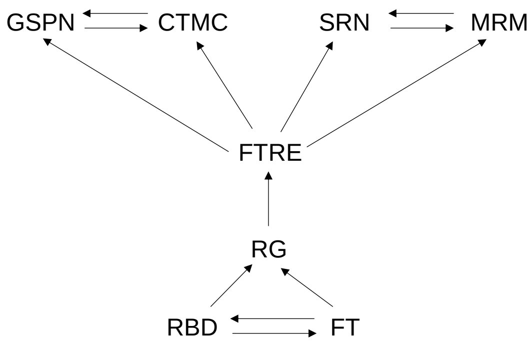

Figure 10: Power-hierarchy among dependability-model types (Malhotra and Trivedi, 1994)

2.3.6 CHOOSING A METHOD

This section has explained which methods exist that can possibly be used to calculate the availability of a system. The advantages and disadvantages have been addressed and Malhotra and Trivedi (1994) even ranked some and more of these methods in order of power. Interestingly enough, Bayesian networks were not considered in their research, but Rausand (2011) mentions that Markov models can replace Bayesian networks when dynamic effects need to be taken into account, so they can be imagined in between the top level and FTRE.

Fault tree analyses and reliability block diagrams are used frequently in the maritime sector, especially for assessing availability, reliability, and maintainability of systems and components. Due to limited documentation, the author is not sure whether or not these techniques are also used during early stage ship design for systems design.

As from Figure 10 can be seen, fault tree analysis is still a decent and very usable design tool, but as there are a lot of uncertainties in ESSD and naval systems especially are quite complex, a different method should perhaps be used. In line with most research mentioned in this chapter and due to the nature of how it works, Markov theory is proposed to be used in this thesis for the analysis of system availability, as many of the building bricks used in these papers can also be used for the calculation of system availability. To the author's knowledge, Markov theory has not been previously used for calculating the availability of systems with a ship design goal in mind, except for those studies that base the availability on compartment hits, e.g. vulnerability.

2.4 RESEARCH GOAL

The previous background section explained the state-of-the-art early stage design methods and dependability methods because it is important to know where improvements in systems design can be made in order to understand the effect on the operability of a naval vessel. A couple of things can be taken away from this. In the field of ESSD, improvements are ever present. This thesis shows that a lot of research is going on in different fields of ESSD and will do so in the future. But there is much to learn still about ship design, especially in the early stages where design freedom is still high, yet problem knowledge very little. Nevertheless is this the stage where many important decisions are made based on the knowledge at hand. Now that vulnerability has been thoroughly looked at, system performance related to machinery failures are looked at as the next step towards a bigger picture. A time-based approach such as Markov theory can show the behavior of systems over a longer amount of time, which leads to different results than a conventional, static approach does. It gives a time-based expectation and offers a certain form of availability to the researched systems.

This chapter concludes with the research question for this master thesis. The following question was proposed in the introduction: "*How can Markov theory be used in Early Stage Ship Design regarding systems design to analyze the availability of systems in order to maximize the operability of a naval vessel?*". This research question highlights the core of what this thesis is about. The goal is to find a different design method for systems during ESSD and use that in conjunction with current techniques. Here, *design* is meant to be the choice of a system itself or a network of dependent systems. Markov theory was identified as the most useful dependability method and previous research has also shown that it is a suitable method for analyzing similar problems. However, due to the way the operability tree is defined in section 2.2, the operability cannot be maximized. The many elements together form the operability of a naval vessel and not just the system performance alone. The scope of this thesis already only considered availability and not the other aspects of the tree. This means that the operability of a naval vessel cannot be maximized as was proposed in the initial research question. The research question proposed in the introduction of this master thesis is slightly altered accordingly:

How can Markov theory be used in Early Stage Ship Design regarding systems design to analyze the availability of systems in order to make design decisions?

In order to perform this research, the basic mathematical foundation is first explained. This consists not only of the Markov theory but also of network theory, which is needed for understanding the ship service networks. Figure 11 shows schematically what the steps will be to define a model that will be used in order to find analyzable results:

1) The first step is to explain in greater detail how Markov theory works and why Markov theory alone is not enough to solve the problem.
2) Subsequently, network theory is explained in step 2, which will complement Markov theory.
3) Step 3 is to apply both theories in practice, translating them into MATLAB® code. Any problems occurring will be addressed immediately during this step.
4) With the model now done, a case study can be performed in step 4 to generate results from combinations of different parameters.
5) Finally, in step 5, the results from the case study are analyzed and interpreted and the research question above can be answered.

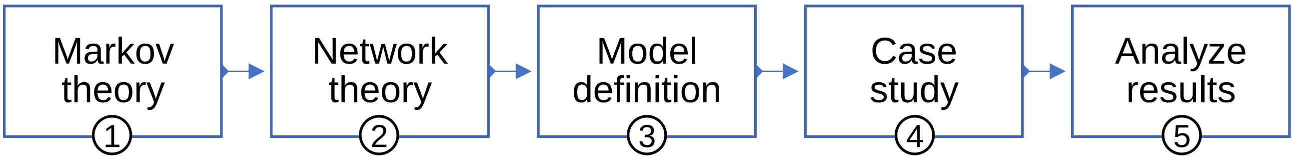

Figure 11: Schematic overview of the next steps

In this thesis, the following assumptions and limitations are used:

- This research is considered to be about naval ships specifically, though it can possibly also be useful for other complex vessels, such as drilling and pipelaying vessels.
- The customer’s wishes, and thus the fundamental functional systems and their interdependencies, are assumed to be known when using this method. A simplification of these systems will be used in this thesis.
- The operability of a naval vessel is only regarded from a systems design point of view. The emphasis is on the availability of systems.
- Vulnerability will not be assessed, as this has been looked at in other studies.
- Crew performance, i.e. human error, is left for further research.
- The impact of weather conditions is neglected.

- Only exponential distributions will be used for the transition rates used in Markov theory, as this will guarantee the memoryless Markov property.
- No maintenance strategies will be used to assess the availability, though the ability of a system to be repaired is taken into account.
- It is assumed that no system can fail at the same time step as another system is recovered.

3 MARKOV THEORY

This chapter describes what Markov chains are, and in particular, how continuous-time Markov chains work, as discrete-time Markov chains have already been addressed in section 2.3.4. They will be described to a basic level of understanding. The reader is referred to Rausand (2011) or similar books for a deeper understanding of the mathematics behind it.

3.1 MARKOV CHAINS

To study the availability of the system of systems of a naval vessel, Markov theory is used. Markov theory is based upon the principle that a certain state can transition into a different state, without influence of previous states; it is memoryless. A number of consecutive state transitions is called a Markov chain. Two types of Markov chains exist: discrete- and continuous-time Markov chains. A discrete-time Markov chain (DTMC) looks at the situation of a state at a certain time step. After the passing of enough time, the system will be in a steady state (if it is able to reach it, not all systems do), which is the most likely state to end up in after different transitions between all possible states of a system. A continuous-time Markov chain (CTMC) is based on the same principles but is not analyzed on a step by step base, yet rather on an *average* likelihood of ending up in a certain state. The difference between these methods is essential. Where a DTMC can be analyzed at any range of time (and this range impacts the result), a CTMC is always defined in the same 'time range'. A CTMC also does not have a certain starting point, it will always reach the same steady state given any starting state.

Another difference between DTMC and CTMC is the way a state transitions from one to the other. Since DTMC is step based, a state has a certain probability of changing to the next state (and step as well). This probability is always the same for this specific state transition but is able to change over time (the probability of transitioning from state A into state B for example). A CTMC however, bases its transition on a rate, which can be seen as the expected value over a range of time.

Due to this specific time aspect, continuous-time Markov chains are preferred over discrete-Markov chains, as the average availability of a system is what is most interesting to analyze.

3.2 CONTINUOUS-TIME MARKOV CHAINS

In order to explain how CTMC work and explain why they are relevant to this research, the system of two pumps in Figure 12 is considered, taken from the book by Rausand (2011). The system is always in a certain *state*, consisting of the *modes* that each subsystem is in. Modes are often also called states, but to avoid confusion modes is used from here on out instead. Each pump can either be in a *working mode* or a *failed mode*. Four possible *states* can be identified for this system and are given in Table 4. Each pump has a failure rate (λ_1, λ_2) and a repair rate (μ_1, μ_2). These are the transition rates from one state to the other. For instance, the failure rate of pump 2 (λ_2) is the rate that state 3 transitions into state 2 (The repair rate is the other way around). The failure rate is defined as 1 / MTTF (mean time to failure) and the repair rate is defined as 1 / MTTR (mean time to repair).

Table 4: System states of a two-pump system

State	Pump 1	Pump 2
0	Failed	Failed
1	Failed	Working
2	Working	Failed
3	Working	Working

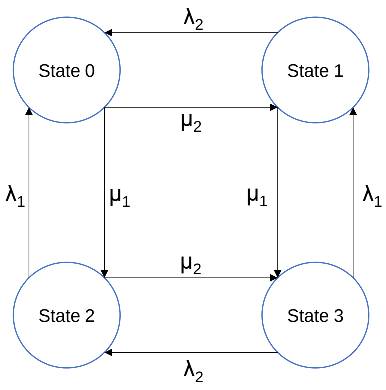

Figure 12: Transition rates between states (Rausand, 2011)

The transitions between states are organized in Table 5 below, where a non-primed state denotes the starting state and each primed state the resulting state after the transition. A zero indicates that no direct transition is possible between the states since it is assumed that both systems can't fail at the same time or be repaired at the same time in this example (see Figure 12). A Markov chain is a memoryless process, meaning that if a system is in a state *i* at time *s*, the probability of transitioning into state *j t* time later is independent of time *s*. The time a system spends in a state is thus exponentially distributed. Hence, following the Kolmogorov equation (refer to Rausand, 2011), every diagonal element is equal to the negative sum of every row, according to Equation (3.1), where *r* is the number of states.

Table 5: Overview of state transition rates from state X to state X'

	State 0'	State 1'	State 2'	State 3'
State 0	$-(\mu_1 + \mu_2)$	μ_2	μ_1	0
State 1	λ_2	$-(\lambda_2 + \mu)$	0	μ_1
State 2	λ_1	0	$-(\lambda_1 + \mu_2)$	μ_2
State 3	0	λ_1	λ_2	$-(\lambda_1 + \lambda_2)$

$$a_{ii} = -\sum_{\substack{j=0 \\ j\neq 1}}^{r} a_{ij} \tag{3.1}$$

These same rates are used to construct the transition matrix Q accordingly, as is given by Equation (3.2):

$$Q = \begin{pmatrix} -(\mu_1+\mu_2) & \mu_2 & \mu_1 & 0 \\ \lambda_2 & -(\lambda_2+\mu_1) & 0 & \mu_1 \\ \lambda_1 & 0 & -(\lambda_1+\mu_2) & \mu_2 \\ 0 & \lambda_1 & \lambda_2 & -(\lambda_1+\lambda_2) \end{pmatrix} \tag{3.2}$$

The states are listed according to Table 5. I.e. the second element of the first column corresponds to the transition from state 1 to state 0', the third element the transition from state 2 to state 0' and so on.

$$\begin{aligned} -(\mu_1+\mu_2)P_0 + \lambda_2 P_1 + \lambda_1 P_2 &= 0 \\ \mu_2 P_0 - (\lambda_2+\mu_1)P_1 + \lambda_1 P_3 &= 0 \\ \mu_1 P_0 - (\lambda_1+\mu_2)P_2 + \lambda_2 P_3 &= 0 \\ P_0 + P_1 + P_2 + P_3 &= 1 \end{aligned} \tag{3.3}$$

To calculate the steady-state of this matrix, the steady-state equations in Equation (3.3) have to be solved, where *Pi* is the steady-state probability to be calculated. This can be done by solving them individually or easily by exchanging one of the columns of the transition matrix with ones (preferably with the least number of zeros). The steady-state can be calculated by Equation (3.4):

$$sQ' = [0\ 0\ 0\ 1] \tag{3.4}$$

Where Q' is the transition matrix adapted with ones and the vector [0 0 0 1] resembles an arbitrary starting state. Assuming a failure rate of $2.3e^{-4}$ and $1.7e^{-4}$ for pump 1 and 2 respectively, and a repair rate of $8.3e^{-2}$ and $4.2e^{-2}$ respectively, the steady-state vector *s*, follows from Equation (3.5):

$$s = [0.0012\ 0.0027\ 0.0041\ 0.9932] \tag{3.5}$$

Which means that on average, the system is fully functional for 99% of the time and down 0.12% of the time. Or in average hours per year, 8700.3h/y and 0.1h/y respectively.

3.3 DEFINING STATES

Now instead assume the system in Figure 13, consisting of two diesel engines, two switchboards, two radar systems, and two weapon systems; eight systems in total. The power supply is redundant if one engine fails the other can take over to provide the system of power. Likewise, weapon system 1 is provided by data (green lines) of both radar systems, such that if one fails, the other can still provide data. In order to use the Markov theory explained in section 3.1, a number of *states* must be identified for this network. Since there are more than just two systems, the number of states are not found as easily. As mentioned before, a Markov chain works with two types of *modes* that a system can be in. These are *working mode* and *failed mode*, indicating that a system is either working or not. Accordingly, every state consists of a combination of modes that the systems of a network are in.

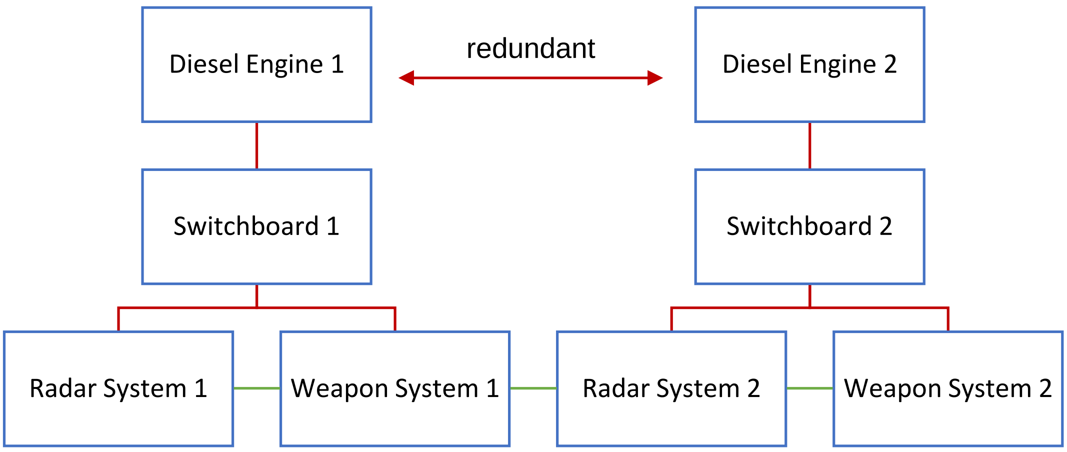

Figure 13: Network of a simple system of systems

Usually, in state 0, all systems are in a failed mode, or rather, the situation where all systems have failed. Opposingly, state X is the state where all systems are functioning, where X is the total numer of states. Whenever a system is recovered, state 0 transitions into the next state corresponding to the *transition rate* of one of the systems in the network. For instance, state 1 might be the state where system A has been recovered, and only system A. State 2 might be the state where system B has been recovered, and only system B. But state 9 might be the state where system A and B have both been recovered. Following the logic explained in section 3.1 only one system can ever change modes, for simplicity's sake. This means that state 9 can be reached from both state 1 or 2, with the subsequent restoring of both systems A and B (in any order). For every network, given two modes, this amounts to 2^n combinations of working and failed modes for a network consisting of *n* systems. Thus for the network in Figure 13, which consists of 8 systems, $2^8 = 256$ different states exist.

However, this thesis introduces a third mode: *error*. Looking back at the example in Figure 12, the two pumps are independent systems, so which system fails first does not matter. In Figure 13 to the contrast, the diesel engines provide power to all other systems, hence if these fail all *downstream systems* don't work anymore. This is what *supply* stands for in the operability tree in Figure 5 and is denoted by the *error mode*. The error mode means that a system is down even if that system itself did not fail. Mathematically speaking, this would give $3^8 = 6561$ different states. On closer inspection, that is not true, however. This application would imply that a system could transition to an error mode without any cause. I.e. there is a link missing between the mathematical application of three modes and the logical application of three modes. The above is perhaps better explained with an example.

Initially, state *A* is the state where all systems of the network in Figure 13 are working, this is denoted by a *1* in Equation (3.6). Note that the first two elements of the vector belong to the diesel engine systems, the third and fourth to the switchboards and so on.

$$State\ A = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \tag{3.6}$$

Then, when the first diesel engine fails, state *A* transitions into state *B*, where a failed system is denoted by a 0, as given in Equation (3.7).

$$State\ B = [0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \tag{3.7}$$

Alternatively, an arbitrary system is able to go into error mode, mathematically speaking. State *A* transitions into state *C*, where an error system is denoted by a 2, see Equation (3.8).

$$State\ C = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 2 \quad 1 \quad 1] \tag{3.8}$$

But Equation (3.8) is not possible in reality. A system can only be in the error state if it doesn't work because of *other* systems failing, according to the definition in this thesis. This means that if *both* diesel engines fail, no power is generated and all systems will go into error mode, as is visible in Equation (3.9).

$$State\ D = [0 \quad 0 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2] \tag{3.9}$$

This hints to the fact that there are actually still $2^8 = 256$ states for this 3-mode approach, with the exception that multiple systems can change modes in one state transition, as opposed to only a single system that was suggested at the beginning of this section. Though only *one* system will still change from either *working* to *failed* mode or vice versa. With the failing of the second diesel engine, for example, state *B* transitions into state *D* and the remaining *working* systems also transition but into *error* mode. The difference here is that state *D* occurs by the *transition rate* of a diesel engine and not by those of the systems going into *error* mode. The *transition rates* of all other systems have no place in that cell of the transition matrix, though ultimately their availability decreases due to their being unavailable in the error mode.

So, for a network consisting of eight systems and three types of modes, 256 different states exist. These are the exact same states as for a 2-mode system, with the exception that dependency is to be implemented, meaning that if certain predecessors of a system fail, the considered system itself goes into an *error* mode. In essence, a system that is in *error* mode has the same functionality as a system in *failed* mode, namely that it isn't able to perform its function.

In order to calculate the availability of an individual system, any *'2'* in the defined states above, is regarded as a *'0'*, according to Equations (3.10) and (3.11):

$$[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1] \rightarrow [0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2 \quad 2 \quad 2] \rightarrow [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{3.10}$$

and

$$[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1] \rightarrow [0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 0 \quad 0 \quad 2] \rightarrow [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{3.11}$$

Since both the diesel engines are not working, no other system can work and a *1* changes into a *2* (error mode). The steady-state vector (the probability of being in a certain state, see Equation (3.5)) will be multiplied by the value of the mode for each system in each state. This is why a *2* is changed into a *0*, as the system is regarded to be not-working, and would otherwise be *twice* as available in a given state. This results in multiple cases where a state consists purely of zeros. The information lost because of this transformation is however, still stored in the states that contain *2's* and will be used separately by the model.

3.4 TRANSITION RATES PROBLEM

The past three sections form the basis for the availability calculations in this thesis. The *average* availability is the key to performing the operability calculation later on. Four elements discussed in this chapter are needed for this calculation:

- A network of n nodes representing systems with known interdependencies.
- A number of defined modes m that a system can be in.
- A number of states s that prescribe all possible combinations of m modes for n nodes.
- A failure rate λ_n and repair rate μ_n for n systems.

The number of n nodes follows directly from the functional systems defined by the customer's wishes. They must, however, be put in a preliminary configuration, similar to that of the logical architecture. Depending on the level of detail of this calculation, the number of m modes is easily deduced, though each mode other than working and failure adds a new layer of complexity to the interdependencies of systems (like the prior discussed error mode). The number of states s is a result of the number of nodes n and modes m ($s = m^n$), though they must be carefully analyzed in order to understand if they portray the right situation. Lastly, the transition rates can, for instance, be obtained empirical or from a manufacturer of a system.

However, this study is theoretical in nature and during the early stages of design, data regarding failure rates and repair rates is not or limited available. Similar data for comparable systems on existing ships may be used for an indication, but naval ships often have new kinds of technology (railgun) installed or have a custom-made piece of equipment on board (like a special type of radar). This means that such data is usually not readily available.

As a result, in order to use Markov chains in ESSD, the way these transition rates can be acquired has to be looked at. The next chapter will, therefore, delve into network theory, which is used in an experimental approach for looking at how transition rates can be obtained from the network as a whole, instead of on an individual system basis, without knowing how the systems themselves perform.

4 NETWORK THEORY

This second theoretical chapter explains how network theory will be used to determine input for the Markov chain, including the network, its dependencies and a way to acquire transition rates. For more reading material on network theory, the reader is referred to Newman (2010).

4.1 NETWORK CONFIGURATION

A system of systems, such as the one in Figure 13, can easily be visualized by the use of network theory. Each system is therefore depicted by a 'node', whilst every connection between two systems is depicted by an 'edge'. There are several ways to illustrate the connections between systems. Edges, for instance, can be directed or undirected. Directed means that flow between two nodes can only occur in the indicated direction of the edge. An undirected edge can thus have flow in both directions.

For the system of systems analyzed in this thesis, directed edges are used most of the time because they indicate energy flow originating from an energy source (diesel motor) to other systems. In some cases, a system-type is redundant. Where this is the case, the edge connecting both systems is undirected, as energy can be redirected through the second system if the first one fails.

Assume the network in Figure 13 is a set of functional systems for a naval ship. Though the propulsion is also linked to two diesel engines, these themselves are left out of this network for this example. The emphasis of this example is on two weapon systems aboard a vessel. The diesel engines and weapon systems are not directly linked, but the power from the diesel engines is fed to a switchboard, from which it is then fed to a radar system and to the weapon system. The radar system provides the weapon system of important targeting data. For this simplified case, a chilled-water plant is left out for now. The weapon system thus needs both the power from the diesel engines and the data from the radar system to operate. Assume a similar network for the second weapon system but in addition, the second radar system can also provide data for the first weapon system. So the total network is comprised of eight systems, as can be seen in Figure 14, where red lines represent power flow and green lines data flow. According to Newman (2010), the connections between each system can also be visualized in an adjacency matrix, constructed according to Equation (4.1). From Equation (4.2), it can be seen that both directed and undirected edges are used, an undirected edge exists between node 2 and 6 for instance, allowing the second diesel engine to provide power to the entire network if the first one fails.

$$A_{ij} = \begin{cases} 1 & \text{if an edge exists between node i and j} \\ 0 & \text{if no edge exists between node i and j} \end{cases} \tag{4.1}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.2}$$

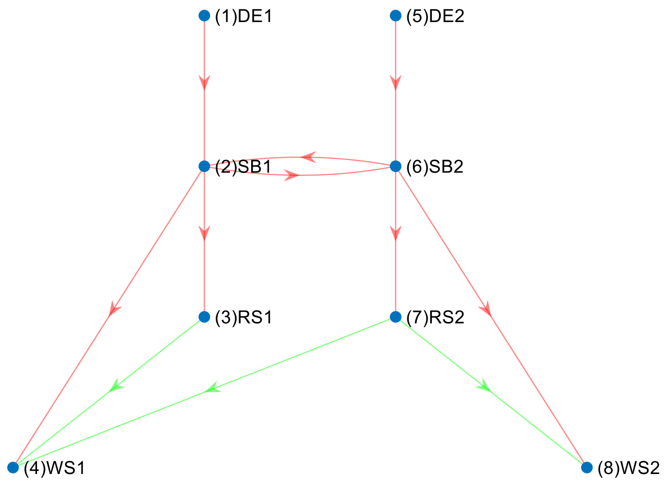

Figure 14: Network configuration of a simple system of systems

4.2 FOCUS POINTS

4.2.1 NODE IMPORTANCE

Now suppose that each node can be assigned a value, based on the amount of focus it is given to during the early ship design stages. This amount of focus, or *focus points*, should be seen as the amount of resources dedicated to the design of not only the system itself but also the connections it has to other systems, the way it is connected and ideally also the location of the system in the vessel. These resources can be thought of as for instance work-hours, product quality, and monetary value. In this thesis, the number of *focus points* awarded to a node is assumed to be related to the importance of a node in the entire network, where importance represents the consequences of the system's state in the network.

As explained in section 3.4, transition rates, usually referred to as *failure* and *repair* rates in this context, are needed for each system as input to the Markov chain calculation. These *focus points* form the basis for developing a method to acquire such transition rates. The simplest way to demonstrate this is via the failure and repair rate of a system, where more *focus points* could lead to a smaller failure rate and a bigger repair rate respectively. However, a system such as a diesel engine is fabricated by a manufacturer. The quality of the product determines the failure and repair rates, and not the way it is connected in a network. This means, that pure failure and repair rates cannot be used in the transition matrix of a continuous-time Markov chain discussed in section 3.2. Instead, this thesis assumes using rates in the transition matrix consisting of several items, including but not solely the failure and repair rates of a system. The location within the ship and the way a system is connected can also be factored in these rates. The following terms are thus proposed: **transition rate** and **recover rate**. The first denotes the *transition* from one state to another, the second denotes the transition back to its original, or *recovered*, state.

A node with a high number of focus points can now be seen as if for instance a better performing system was picked from a manufacturers product catalog, or it is better connected to important systems, i.e. more (monetary) resources are used in order to ensure the system works in the desired fashion within the network. Assume thus that the more attention a node gets during the design phase, the more reliable that corresponding system works. But this does not mean that for instance, a system with a low number is not reliable at all, it means that *relative* to the other considered systems, less attention needs to be given to this system than to others for the functioning of, for example, the weapon systems.

4.2.2 NONQUANTIFIABLE VALUES

Decision-making support tools often use nonquantifiable values. These are values that in itself do not mean anything and cannot be expressed in meaningful figures. They can, however, be compared to other nonquantifiable values within the same application. In shipbuilding, two of such decision-making support tools—AHP (analytical hierarchy process) and QFD (quality function deployment)—are widely applied.

AHP is a method to select a design from a group of fixed options. It is based on pairwise comparison between criteria and between options. A scale of nonquantifiable values, ranging from equally important to extremely more important, is used to determine how much a certain criteria or option is valued over the other. The most and least important criteria are thus identified. The fixed option are compared to each other per criteria. The method determines which option is the best per criteria, and ultimately, which is the best option in general. AHP is, however, very dependent on subjective human input regarding the scale, which is why this is often done in a team.

QFD is a method used to translate the wishes of a customer into engineering characteristics. A list of wishes is compared to a list of substitute quality characteristics in the part of the QFD that is called the House of Quality. Nonquantifiable values indicating a strong, medium or weak link between these are used to rank certain wishes to certain characteristics. The House of Quality subsequently identifies the rank of each wish of the customer and each substitute quality characteristic, giving the designer information about the most important aspects of a design.

The focus points described in the previous section are also nonquantifiable values and the essence of this method is, in fact, a decision-making support tool. The fact that other such tools exist (like AHP and QFD) and are widely used in shipbuilding, gives credibility to the application of nonquantifiable values in this method.

4.3 DETERMINING NODE IMPORTANCE

4.3.1 OPTIMIZATION METHODS

But how can these focus points be determined? The amount of resources is most probably not infinite, thus one can think at least of a limiting factor, a maximum capacity of sorts, that the number of focus points must adhere to. With this limit in place, a designer could be allocating focus points to each system in a way he sees fit. In the mathematical field of Operations Research, an optimization problem is often used for such cases. A *knapsack problem*, for instance, would suit this model well. The problem involves a ‘knapsack’ with a certain capacity and a number of unique items that can be either placed in it or not. Each item has a weight and a value. The total weight of all items may not exceed the capacity. The optimization problem is thus to get the most value inside the knapsack, provided that the total weight is lower than or equal to its capacity (Jensen and Bard, 2002). The problem with using this method is that a clear weight, value, and even capacity are missing for this particular application. Focus points could be weight, but what then would the gain (value) of the problem be? What resources determine the capacity? And how does one discriminate the weight of systems? This leads to a vicious circle, where one wants to know the importance of a node by assigning a weight of importance to it beforehand.

Another possibility could be by using a *linear programming relaxation* (Jensen and Bard, 2002). In this optimization method, the number of items in the knapsack problem is instead found by minimizing or maximizing a so-called *objective function*. The objective function consists of a number of i items each with a number of j copies, that together add up to a certain value. The objective function is commonly bound by several constraints, as opposed to the knapsack problem where an item is either in or out of it. Such constraints are for example that items A and B cannot be chosen at the same time, or that the sum of k items cannot be higher (or lower) than a certain value. The capacity would be an example of such a constraint. The maximum profit p of a factory with m different machines producing n number of goods with value v, is a simple example of an LP relaxation, as can be seen in Equation (4.3). Here, x_1 and x_2 are goods produced by 3 and 6 machines respectively of different types and are bound by a production constraint that the total number of produced goods cannot exceed 500 and no negative production can occur.

$$
\begin{aligned}
&\max\ 3x_1 + 6x_2 \\
&such\ that: \\
&x_1 + x_2 \leq 500 \\
&x_1 \geq 0, x_2 \geq 0
\end{aligned}
\tag{4.3}
$$

Although the LP relaxation has more flexibility, it still faces the same issues a knapsack problem has, if these methods were used to determine the number of focus points a node is given. The constraints, even a capacity constraint, are difficult to define and again the same vicious problem occurs. One cannot subjectively assign focus points to a node without already having a certain prejudice. A different, objective, method has to be found instead.

4.3.2 Network properties

Sometimes it is easier going back to the source of a problem, i.e. the network itself. A network has certain inherent properties that can always be found, and define the importance of a node within a network.

A number of ways exist to analyze this importance of a node in a network. First, there is the **degree centrality** (Newman, 2010). The degree centrality is a measure of the in- and outgoing edges of a node; the value is basically the number of connections of a node. The second is the **eigenvector centrality** (Newman, 2010). This value is basically the sum of the *degree centralities* of all nodes that a certain node is connected to. A third way is by looking at how many nodes are (in)directly dependent on a node and the fourth way is, vice versa, how many nodes are (in)directly connected to a certain node, giving an indication of how many systems are needed for a system to function. These ways are called **downstream connectivity** and **upstream connectivity** respectively, and—to the author's knowledge—are introduced for the first time in this thesis.

These four ways can be better described by looking at Figure 14. Node 1, a diesel engine, is only directly connected to node 2 (a switchboard), thus its *degree centrality* is 1. Node 2 is connected to node 1, node 3, node 4 and node 6; its *degree centrality* is 4. Since node 1 is only connected to node 2, its *eigenvector centrality* is 4 (the *degree centrality* of node 2). Downstream, node 1 is only disconnected from node 5 (flow can only be traced down or sideways, thus no flow can occur between these points), thus its *downstream connectivity* is 7. This includes node 1 itself since nodes without any *downstream connectivity* will have a value of 0, which is not desired for mathematical purposes later on. In the same fashion, the *upstream connectivity* of node 1 is 1, as it is only dependent on itself. The other way around, node 4 has a *downstream connectivity* of 1 and an *upstream connectivity* of 7 (disconnected from node 8). Table 6 gives an overview of all the nodes.

Table 6: Importance analysis of each node in Figure 14.

	Dgr. Cen.	Eigv. Cen.	Dwn. Con.	Ups. Con.
Node 1	1	4	7	1
Node 2	4	10	6	4
Node 3	2	7	2	5
Node 4	3	9	1	7
Node 5	1	4	7	1
Node 6	4	10	6	4
Node 7	3	9	3	5
Node 8	2	7	1	6

From Table 6 it is clear that the four methods give very different results. *Degree centrality*, in this case, doesn't say a lot about the location of a node in the network at large, whilst *eigenvector centrality* does; *degree centrality* is besides already incorporated in *eigenvector centrality*. *Degree centrality* is therefore left out. Both *downstream* and *upstream connectivity* have a fairly large spread between the values and only give useful information about their respective connectivity. Combining these two would give a better overall importance for both connectivities, though a correction has to be made for counting the considered node twice. This combination, logically also introduced for the first time in this thesis, is called **combined connectivity**. The *combined connectivity* and the *eigenvector centrality* approaches are the only two ways used in this thesis to analyze the importance of a node in a network. From Table 7 it is evident that the spread of values is now much closer to each other than in Table 6.

Table 7: Two approaches for the importance analysis of each node in Figure 14.

	Eigv. C.	Comb. Con.	System
Node 1	4	7	Diesel engine 1
Node 2	10	9	Switchboard 1
Node 3	7	6	Radar system 1
Node 4	9	7	Weapon system 1
Node 5	4	7	Diesel engine 2
Node 6	10	9	Switchboard 2
Node 7	9	7	Radar system 2
Node 8	7	6	Weapon system 2

Eigenvector centrality assigns a lower value to the diesel engines than *combined connectivity* does, indicating that this approach considers that the diesel engines are less important in the network. To a certain degree that is true, the diesel engines are only there to provide the power for all the systems. But other systems are needed to transfer this power to the remaining systems and eventually to let the weapon systems work. The *combined connectivity*, on the other hand, values the diesel engines higher, considering that once the power is cut off, everything stops working.

This network-based approach clearly gives different results than an optimization method would. The advantage of these methods is that they do not rely on prior knowledge that can bias the way an optimization problem would be defined. A disadvantage may be that the spread of focus points might not be 'optimized' with this approach. However, any optimization naturally leans towards the 'best' solution and is based on a higher level of detail than is available during the early stages of ship design, which is exactly the opposite of the goal of ESSD.

For the remainder of this thesis, the *eigenvector* centrality and *combined* connectivity are used to determine the number of focus points a node gets. Even though the values of both approaches are fairly similar for this simple case, they may not be for more complex networks, since both approaches work in different ways. Thus they are both used in this analysis, and the results will be compared to understand what the difference in approach might mean.

4.4 TRANSITION RATES

The last step in the process is to convert the focus points found in the previous section into a usable *transition* rate and *recover* rate. But how can this be represented? Table 8 shows the lowest and highest value for each approach, as shown earlier. However, these values cannot be used right away, as this would create scaling problems in the Markov chain. A value of *10*, is usually not identified as a rate, which would look more like *1 / 10*, meaning once every ten times for example.

Table 8: Lowest and highest value of each approach

	Lowest value	Highest value
Eigenvector centrality	4	10
Combined connectivity	6	9

Still, a rate of 1 / 10 is not quite common if one were to compare this to real-life transition rates. In the Markov chain example in section 3.2 for instance (Rausand, 2011), failure rates in the order of magnitude of $1.00e^{-4}$ and repair rates of $1.00e^{-2}$ were used for pumps. Thus for correct scaling, the values above must be adapted into such orders of magnitude, to get at least in the same order of magnitude of values used as in a real-life problem. There are two ways how this scaling can be achieved.

The first would be to scale all values in Table 7 on an absolute linear scale, where 10 would correspond to the 'best' rate and a 4 to the 'worst' rate, as illustrated in Figure 15. But this would result in very different results between both approaches and within each approach. For instance, a diesel engine (4 focus points) would get a significantly lower rate than a switchboard (10 focus points). Such extreme variances are not ideal for this kind of approach, where the arithmetical value itself is not supposed to have this much of an impact.

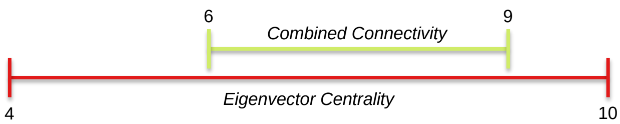

Figure 15: Absolute scaling

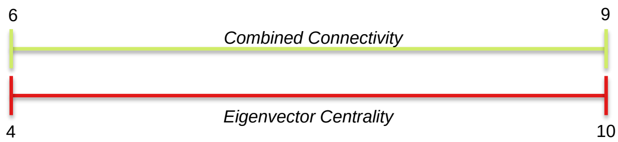

Figure 16: Relative scaling

The second way is thus to scale the values *relative* to each other, as shown in Figure 16. This scaling is achieved by setting the highest and lowest value of both approaches to the same respective maximum and minimum rate. This means that a 4 and 10 focus points in the eigenvector centrality approach don't have a stronger difference than 6 and 9 focus points would have in the combined connectivity approach. This also means that the focus points are all related to one another, meaning that if you would double all values, the same linear relationship applies between nodes.

The number of points can thus be ranked over a range, where the highest and lowest number of focus points in both approaches always correspond to the same transition rate. The values between the extremes vary instead.

The maxima and minima of this scalable range need to be assumed. In accordance with the orders of magnitude mentioned earlier, an arbitrary maximum of $1.0e^{-5}$ and a minimum of $1.0e^{-4}$ are used to transition from a "working" state to a "failed" state. A maximum of $1.0e^{-2}$ and minimum of $1.0e^{-3}$ are used to recover to a "working" state, as per Table 9. These values are somewhat similar to likewise systems found in a generic set of component data by Bouwer Utne (2017)—a failure rate of $6.66e^{-5}$ for a diesel engine for instance.

Table 9: Range of rates

	Minimum	Maximum
Transition Rate	$1.00e^{-4}$	$1.00e^{-5}$
Recover Rate	$1.00e^{-3}$	$1.00e^{-2}$

As demonstrated in Table 10, the highest number of focus points in any of the two approaches now corresponds with the maximum of the range, and the lowest number of focus points corresponds with the minimum of the range. The other amounts are linearly scaled between these minima and maxima.

Table 10: Scaling of the range for all values of both approaches

	Lowest value	Values in between	Highest value
Eigenvector centrality	4	7 and 9	10
Transition rate	$1.00e^{-4}$	$5.50e^{-5}$ and $2.50e^{-5}$	$1.00e^{-5}$
Recover rate	$1.00e^{-3}$	$5.50e^{-3}$ and $8.50e^{-3}$	$1.00e^{-2}$
Combined connectivity	6	7	9
Transition rate	$1.00e^{-4}$	$7.00e^{-5}$	$1.00e^{-5}$
Recover rate	$1.00e^{-3}$	$4.00e^{-3}$	$1.00e^{-2}$

For the ease of understanding, Table 7 from page 25 is repeated as Table 11. The result of this scaling is summarized in Table 12.

Table 11: Two approaches for the importance analysis of each node in Figure 14.

		Eigv. C.	Comb. Con.
Node 1	Eng 1	4	7
Node 2	Swb 1	10	9
Node 3	RdS 1	7	6
Node 4	WpS 1	9	7
Node 5	Eng 2	4	7
Node 6	Swb 2	10	9
Node 7	RdS 2	9	7
Node 8	WpS 2	7	6

Table 12: Rates of systems for both approaches of node importance

		Transition rate		Recover rate	
		Eigv. C.	Comb. Con.	Eigv. C.	Comb. Con.
Node 1	Eng 1	$1.0e^{-4}$	$7.0e^{-5}$	$1.0e^{-3}$	$4.0e^{-3}$
Node 2	Swb 1	$1.0e^{-5}$	$1.0e^{-5}$	$1.0e^{-2}$	$1.0e^{-2}$
Node 3	RdS 1	$5.5e^{-5}$	$1.0e^{-4}$	$5.5e^{-3}$	$1.0e^{-3}$
Node 4	WpS 1	$2.5e^{-5}$	$7.0e^{-5}$	$8.5e^{-3}$	$4.0e^{-3}$
Node 5	Eng 2	$1.0e^{-4}$	$7.0e^{-5}$	$1.0e^{-3}$	$4.0e^{-3}$
Node 6	Swb 2	$1.0e^{-5}$	$1.0e^{-5}$	$1.0e^{-2}$	$1.0e^{-2}$
Node 7	RdS 2	$2.5e^{-5}$	$7.0e^{-5}$	$8.5e^{-3}$	$4.0e^{-3}$
Node 8	WpS 2	$5.5e^{-5}$	$1.0e^{-4}$	$5.5e^{-3}$	$1.0e^{-3}$

5 MODEL DEFINITION

This chapter combines the mathematics introduced in chapters 3 and 4 and describes how the model works that can calculate the average availability of each system in any given network. The first section shows an overview of the steps the model goes through. The second section explains how the Markov chain can calculate the availability of a system.

5.1 THE MODEL

The discussed creation of a network, the defining of the number of corresponding states, and the creation of the Markov chain have all been programmed in a script using MATLAB® version 2017b. This script reads pre-defined data, such as node connections and interdependencies, from an Excel® file. This file also includes the calculations of the eigenvector centrality and combined connectivity. The appropriate code can be found in Appendix 4. Figure 17 shows schematically how it works and is described as follows:

1) As soon as the customer approaches a company with his wishes for a new vessel, the mission statement can be drafted, from which the operations and subsequent functions of the vessel can be derived.
2) This provides an overview of what kind of functional systems are needed to be installed aboard the ship. For instance, for a fast ship with Anti-Air Warfare as main purpose, functional systems for this mission would not just be what speed engine or type of radar systems are required but also what kind of anti-air weaponry can be installed aboard the ship. These systems are the input to the MATLAB® model, as well as their interdependencies.

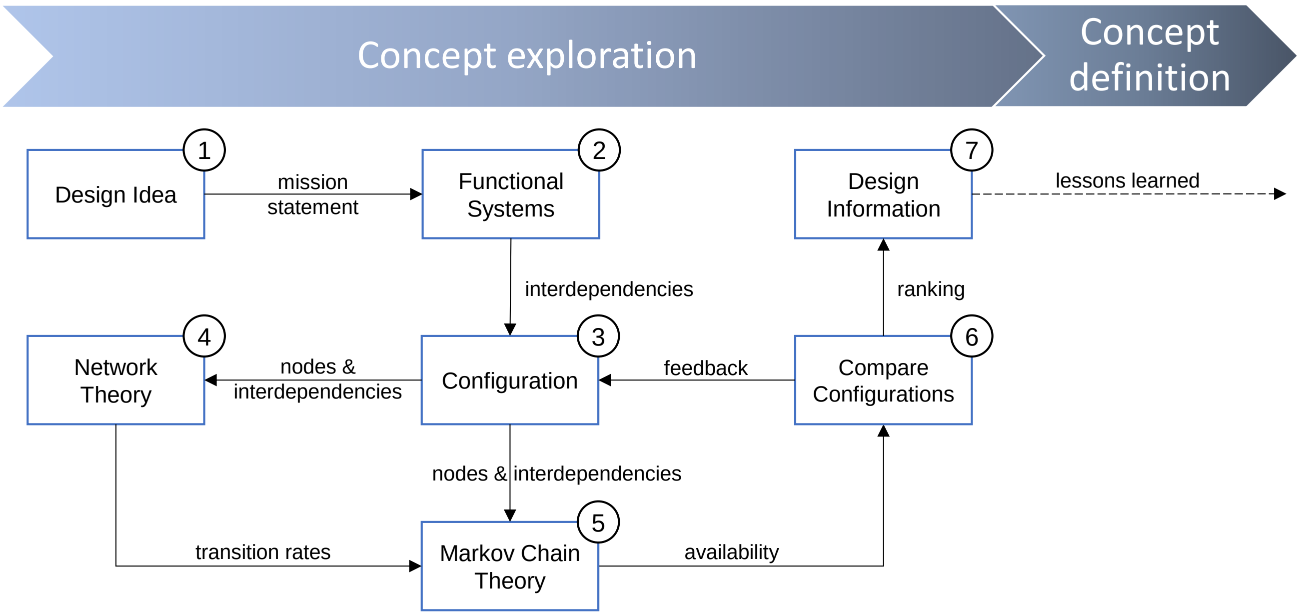

Figure 17: Schematic overview of the model

3) The script then generates a network based upon the user's input. Note that it does not generate a number of random network possibilities such as researched by van Oers, van Ingen and Stapersma (2012), though this could be an add-on.
4) The nodes and their interdependencies are subsequently analyzed to find the *eigenvector centrality* and *combined connectivity*, with which the transition rates will be determined.
5) Meanwhile, the number of states of the Markov chain is being established and corrected for the error mode. The Markov chain will use the transition rates from step 4 to calculate the likelihood of being in

each state, which if multiplied by the mode of a system, reveals the average availability of each individual system. This will be discussed in the next section.

6) The result can be compared with a different configuration in order to see where interdependencies between systems are much more important and how certain systems affect the availability of the network as a whole. With this feedback, a new configuration can be drafted, run and compared. The case studies in chapter 6 looks into this.
7) These results can then be relayed to the customer and provide him with different (not necessarily better) alternatives to his initial wishes, in order to proceed to the first real concept design. Although this step is not actually part of the thesis, it represents what the next step in the process would be, were this method to be applied to an actual case.

5.2 THE CALCULATION

Now that the number of states is defined and the rates have been assessed in sections 3.3 and 4.4 respectively, the transition matrix (as described in section 3.1) can be generated. For the network under consideration (Figure 14), this means the matrix has 256 columns and 256 rows. Each state transition mirrored around the diagonal with the transition rate on one side and the recover rate on the other side. A visual of what this matrix looks like is given in Appendix 1. It is a sparse matrix where blue dots indicate cells where a transition is possible, the white spaces indicate cells with values of zero. In the first 64 states, for example, both diesel engines are defined as having failed. This means that all six other systems cannot be working and are thus in error mode. Because of how this error mode is implemented (all 1's become 2's, become 0's), the first 64 states are zero-states (as previously mentioned in section 3.3) and thus no transition between these states occurs. This can be seen from the absence of blue dots in this area, save for the diagonal. From the 64th state on, one of the diesel engines is working and a number of transitions to different states are now possible, as can be seen from the dotted blue line starting just beyond state 50.

This matrix is then used to calculate the steady-state of the entire network as described in section 3.1 on Markov Theory. This yields a vector *s* that is 1x256 long, as given in Equation (5.1). For ease of reading, only the first two and last two elements are shown. This means that around 81% of the time all systems are functioning, which is depicted by the last element of steady-state vector *s* and the entire network is down 1.54% of the time. This value is not the same as the first element in vector *s* but rather the sum of the first 64 elements of vector *s*, where all other systems are in *error* mode due to the failing of both diesel engines.

$$s = [9.3565e^{-6}\ 1.0073e^{-5}\ \ldots\ 1.1337\ 80.9775] \tag{5.1}$$

The average availability can be determined for each system. The steady-state vector as given Equation (3.5) is multiplied by the value of the mode for each system in each state. This value is *0* if a system is down in that state and *1* if a system is up in that state. The summation product of all states is given in Equation (5.2). On average, the diesel engines work 91.7% of the time, whilst weapon system 2 works 98.2% of the time.

$$\begin{matrix} & Eng\,1 & Eng\,2 & Swb\,1 & Swb\,2 & RdS\,1 & RdS\,2 & WpS\,1 & WpS\,2 \\ Average\ availability = [& 0.917 & 0.917 & 0.999 & 0.999 & 0.987 & 0.996 & 0.995 & 0.982] \end{matrix} \tag{5.2}$$

But do these numbers in Equations (5.1) and (5.2) actually represent availability? And if they do, what do they mean? The simple answer would be *no* since no real system is being analyzed in this calculation. Furthermore, the transition rates, as defined in this thesis, have no meaning in a real-life situation. Finding real-life tangible values has never been the purpose of this thesis, instead, an abstract approximation giving design insights is sought after. The next chapter will dive deeper into how sense can be made from otherwise seemingly meaningless values.

6 CASE STUDIES

Now that the method has been described and the model been explained, the actual process of running tests to get results is possible. The network in Figure 14 is the basic configuration that was analyzed. In this chapter, this configuration will be altered. The initial case will consist of four configurations of the same eight nodes but connected in slightly different ways, in order to analyze the effect of connections and key systems. In addition to this, the two focus point approaches (eigenvector centrality and combined connectivity) are run for each configuration to understand how different views of importance change the way systems are given focus. The range of rates will also be varied to see how scaling affects the credibility of the results. Lastly, the influence of adding or subtracting a system to or from the configuration is examined. This amounts to four cases in total, aiming at evolving the method in such a way that it can be applied to a more complex case with confidence.

Before any results are discussed, the interpretation of these must be addressed. How significant is the difference for instance between an availability of 98.97% and 98.01%? Due to the abstract nature of this method, such a difference might not be entirely meaningful and should not be seen as a major improvement. Though, to indicate the changes within and between cases, they are still addressed. Were a system to change from 88% to 98%, it would give a far better indication of an actual improvement. Values are considered the same, whenever there is a difference of 1% or less between the two. It is also worth mentioning that these values do not mean anything outside of the boundaries of this thesis; one cannot claim that a system is actually 98.97% available. The goal here is to find relationships between the case studies based on the difference between them and to help define the method better.

6.1 CHANGE IN CONFIGURATION

In the first case, called the baseline case, the basic configuration is changed to analyze the effect of the way systems are connected. It stands to reason that a system with more redundancy has a higher availability. For instance, a system connected to only one power source has a lower availability than if it were connected to two power sources. With three it will be even higher, but with more, the added benefit will soon diminish and a limit will be reached. This test case will check if each extra layer of redundancy adds the desired result. What's more, the number of connections also impacts the way eigenvector centrality and combined connectivity are calculated. The four different configurations can be seen in Figure 18 to Figure 21. Figure 18 is the basic configuration as introduced in chapter 4. Figure 19 has two additional connections, each from a switchboard to the radar system it was not yet connected to. The connection between the two switchboards is removed in Figure 20 and an additional connection between radar system 1 and weapon system 2 is added. Figure 21 restores the connection between the switchboards. The extra connections from the switchboards to the radar systems have also been replaced with direct connections to the weapon systems.

For this baseline case, focus points are not yet considered to find transition rates. Instead, two fixed rates have been used for all systems: a transition rate of $1.0e^{-5}$ and recover rate of $1.0e^{-3}$, in order to demonstrate the difference between the configurations first. The result can be seen in Table 13.

Table 13: Average availability of configuration 1-4, with a transition rate of $1.0e^{-5}$ and recover rate of $1.0e^{-3}$

	Eng 1	Eng 2	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2
Configuration 1	99.01	99.01	98.99	98.99	98.01	98.01	97.99	97.04
Configuration 2	99.01	99.01	98.99	98.99	98.97	98.97	98.00	97.04
Configuration 3	99.01	99.01	98.03	98.03	98.97	98.97	97.05	97.05
Configuration 4	99.01	99.01	98.99	98.99	98.01	98.01	98.95	98.95

The engines have the same availability in every configuration as expected since no focus points are used to change their rate value and no change in connection between diesel engine 1 and diesel engine 2 occurred. The connection between both switchboards is removed in configuration 3, which can be seen from their respective availabilities, though the difference is minimal. Once the engine directly connected to a switchboard fails, the possibility to use the other engine is lost. Interestingly enough, the availabilities of the radar systems do not decrease because of this, which is due to the cross connections (2-7 & 6-3) still connecting the remaining engine to these systems. This points to the fact that for the radar systems, a single connection is redundant enough. I.e. 2-7 and 6-3, without 2-6.

The drop in availability of weapon system 1 is still apparent due to the lower availability of the switchboards. Weapon system 2's availability 'increased' slightly, because it is also connected to radar system 1 and should have a higher availability than in configuration 2, however, the drop in availability of the switchboards overshadows this. The added power redundancy for the weapon systems in configuration 4 increases the availability of the weapon systems a lot as well.

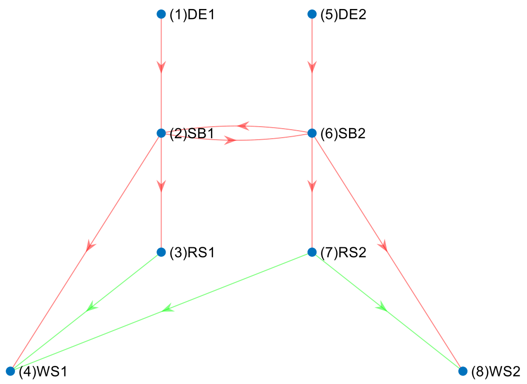

Figure 18: Configuration 1

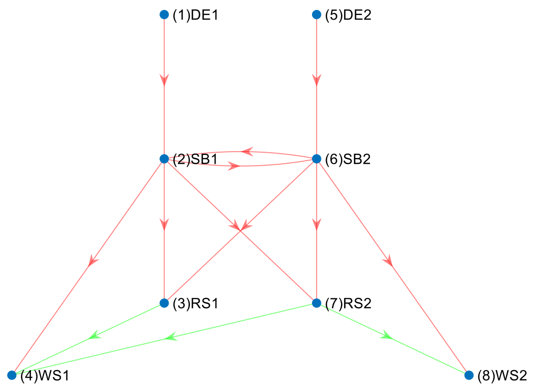

Figure 19: Configuration 2

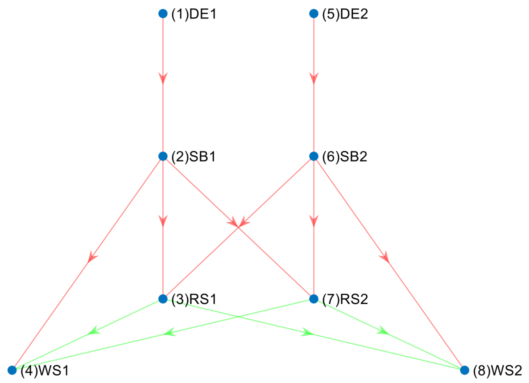

Figure 20: Configuration 3

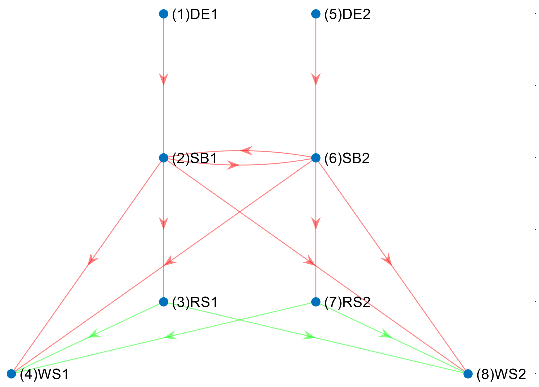

Figure 21: Configuration 4

Though this first case study might not show any unexpected results, considering a more redundant configuration inherently increases availability, but as stated before, these subtle changes might not be picked up once the node-specific transition rates will be used. These subtle changes can be identified clearer by changing one of the rates. A change of the transition rate into $1.0e^{-4}$ results in Table 14. The drop in availability of the switchboards in configuration 3 is now much more apparent. Looking especially at the weapon systems, the impact of adding a direct connection between the other switchboard and a weapon system is now also greatly exemplified, considering these rates.

Table 14: Average availability of configuration 1-4, with a transition rate of 1.0e^{-4} and recover rate of 1.0e^{-3}

	Eng 1	Eng 2	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2
Configuration 1	90.91	90.91	89.47	89.47	81.34	81.34	80.10	73.95
Configuration 2	90.91	90.91	89.47	89.47	88.17	88.17	80.67	73.95
Configuration 3	90.91	90.91	82.64	82.64	88.17	88.17	74.51	74.51
Configuration 4	90.91	90.91	89.47	89.47	81.34	81.34	86.93	86.93

This case has shown how different configurations, and thus connections, can improve the availability of a system. Some changes do not give a better result. The added redundancy for radar systems in configuration 2 is for instance not necessary, though the cross connection (2-7 & 6-3) itself is better than the horizontal connection (2-6). It must be noted however, that the removal of connection (2-6) does impact the availability of the weapon systems, thus it is also important to add redundancy to the weapon systems like in configuration 4. Lastly, the two different values for the transition rates give a significant difference in performance.

6.2 APPLICATION OF TWO FOCUS POINT APPROACHES

6.2.1 COMPARISON BETWEEN APPROACHES

In the second case, the same four configurations as in the previous section will be analyzed, but now the two approaches for assigning focus points to nodes are used instead of a predetermined transition and recover rate. With the added effect of changed rates, the differences between configurations may become more evident. In addition, this case examines if it is better to analyze these systems with diverse rates per system, or if one rate for all will be sufficient to get useful results. The total overview of the allocation of focus points is given in Appendix 2. The result of this allocation is also given Table 18 on page 35.

The first approach—as discussed in section 4.3.2—is the eigenvector centrality approach, where a node is assigned focus points determined by the number of nodes it is connected to and by the number those are individually connected to. The second approach is the combined connectivity approach, where a node will be assigned focus points relative to how many nodes dependent on it, and how many nodes it dependents on. The focus points are subsequently scaled on the same range of rates. This initial range will have a maximum transition rate of 1.0e^{-5} and minimum of 1.0e^{-4}, with 1.0e^{-2} the maximum of the recover rate and 1.0e^{-3} the minimum. These minimum rates are chosen in such a way that they correspond with the rates in Table 14, which gave a clearer picture of differences between configurations than Table 13.

Table 15: Average system availability for each configuration and both approaches

Config.	Approach	Eng 1	Eng 2	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2
1	Eigv. Cen.	90.91	90.91	99.07	99.07	98.09	98.78	98.77	97.80
	Comb. Con.	98.28	98.28	99.87	99.87	90.79	98.15	97.99	89.23
2	Eigv. Cen.	90.91	90.91	99.06	98.97	98.70	98.95	98.43	97.24
	Comb. Con.	98.28	98.28	99.87	99.87	90.88	98.25	98.00	89.23
3	Eigv. Cen.	90.91	90.91	90.60	90.60	99.02	99.02	90.29	90.29
	Comb. Con.	90.91	90.91	82.64	82.64	96.89	96.89	82.56	82.56
4	Eigv. Cen.	90.91	90.91	99.07	99.07	98.50	98.50	98.97	98.97
	Comb. Con.	90.91	90.91	99.07	99.07	90.06	90.06	90.13	90.13

Looking at Table 15—showing the results of respectively the eigenvector centrality approach and the combined connectivity approach—similarities between both approaches can be seen. The dark green cells have no difference between the approaches and the lighter green cells have values close enough to each other to be treated equally as well. An argument could be made for the similarities between the radar systems in configuration 3, but as stated at the beginning of this chapter, the values differ more than 1% and are not treated as equal. To understand the relationships between the green cells, one must look at the focus points and how they are distributed.

In configuration 1, both switchboards get the maximum number of focus points in each approach (10 and 9 respectively), as is also the case in configuration 4 (17 and 9). In configuration 2, eigenvector centrality allocates switchboard 1 16 points and switchboard 2 15 points, due to the fact that radar system 2 is connected to *one* more node than radar system 1.This increases the degree centrality of the radar system, and so also the eigenvector centrality of the switchboard. Because of the linear scaling between the minimum and maximum, the rates for a 15 and 16 points node do not differ a lot, which is clear from Table 15. This illustrates indeed the assumption that these focus points should not be interpreted as hard numbers.

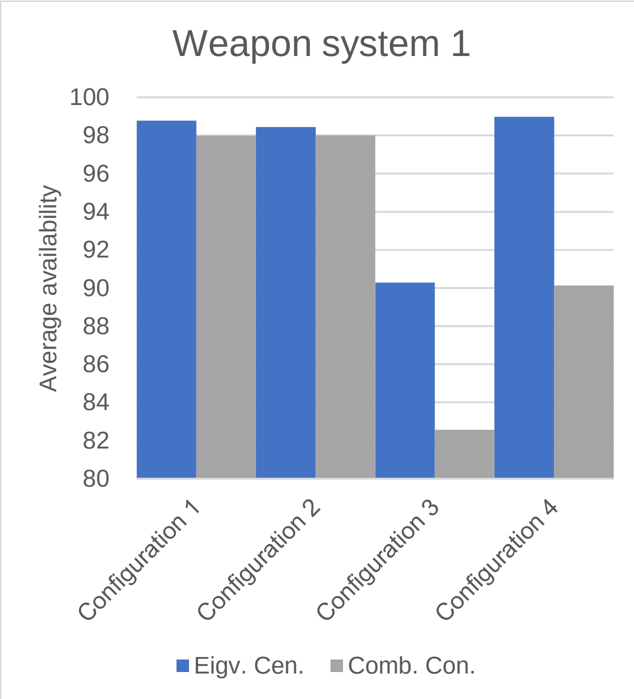

Figure 22: Average availability of weapon system 1

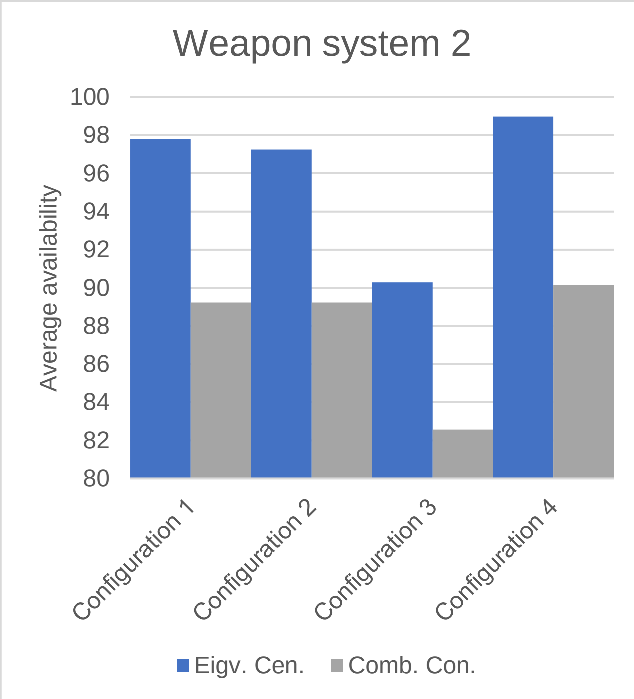

Figure 23: Average availability of weapon system 2

The findings of the previous section can be identified quickly, as can be seen in Figure 22 and Figure 23. The blue bars represent the eigenvector centrality approach, the gray bars combined connectivity. The dip in availability of the switchboards in configuration 3 is quite prominent, especially for the combined connectivity. In case of the eigenvector centrality, it makes sense since the loss of a connection means a decrease in its degree centrality. The combined connectivity depends on the upstream connection, which decreases due to the loss of the connection between the switchboards.

The values for the availabilities of the weapon systems are most interesting to look at. Given what was explained in the last section, configuration 4 should have the highest availability, as it has the most connections. This is indeed slightly visible for the eigenvector centrality, though the impact is small. This relationship is, however, different for the combined connectivity approach, where the overall availability of weapon system 1 has decreased in comparison to configuration 1. From inspecting its predecessors, one can see that all systems have a lower availability, and thus weapon system 1 also. This can be explained from the number of focus points. Configuration 1 has a 6-7-9 scale, where the engines and weapon system 1 get 7 focus points. Configuration 4 has a 7-9 scale, so the engines and weapon system 1 get 7 focus points as well. However, these are relative numbers, meaning that both 7’s get different rates due to the scaling.

The availability of the engines in configurations 1 and 2 is higher for the combined connectivity approach than for the eigenvector centrality. This is strange at first glance, but it makes sense once the focus points are investigated. The way configuration 3 and 4 are connected causes the combined connectivity to exist of a small scale, 6-7 and 7-9 respectively. Hence, only the minimum and maximum range values are allocated to nodes and no scaling takes places in between. This is not the case for configuration 1 and 2, where the engines are ranked in between the minimum and maximum.

6.2.2 Comparison to the baseline

Now that the individual differences between the approaches have been discussed, both approaches can be compared to the baseline case. In the end, only one of the approaches should be used to generate transition rates for this model. Table 16 shows the difference between the values of the baseline case and the case that uses the eigenvector centrality approach. The last column depicts the average difference for each configuration. The last row shows the average difference for each system individually. The intersection of these averages is a factor that can be used to compare the effectiveness of both approaches.

Table 16: Difference in average availability between the baseline case and the eigenvector centrality approach

	Eng 1	Eng 2	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2	Average
Configuration 1	0.00	0.00	+9.59	+9.59	+16.74	+17.44	+18.67	+23.85	11.99
Configuration 2	0.00	0.00	+9.58	+9.49	+10.53	+10.78	+17.76	+23.30	10.18
Configuration 3	0.00	0.00	+7.96	+7.96	+10.85	+10.85	+15.78	+15.78	8.65
Configuration 4	0.00	0.00	+9.59	+9.59	+17.16	+17.16	+12.04	+12.04	9.70
Average	0.00	0.00	9.18	9.16	13.82	14.06	16.06	18.74	10.13

The eigenvector centrality approach does grant higher availabilities, except for the diesel engines. This is easily explained by the fact that the baseline case has a transition rate of $1.0e^{-4}$, which only the diesel engines match, as they have the least number of focus points. All the other systems have thus a better rate and so a higher availability. It is worth mentioning though, that for the weapon systems, this difference seems to diminish with the improvement of each configuration, though they have the “biggest” improvement.

The same comparison can be made for the combined connectivity approach, as is shown in Table 17, where can also be seen that overall, the difference between configurations diminishes with the improvement of each configuration. The differences are in general smaller than for the eigenvector centrality. This is perhaps best explained by the fact that there are fewer steps in between the minimum and maximum for the combined connectivity, as was also mentioned in section 6.2.1. In addition, weapon system 2 has less improvement than weapon system 1, which was the other way around for the eigenvector centrality approach. The availability of weapon system 2 should even increase due to the added data connection from radar system 1.

Table 17: Difference in average availability between the baseline case and the combined connectivity approach

	Eng 1	Eng 2	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2	Average
Configuration 1	+7.37	+7.37	+10.39	+10.39	+9.45	+16.81	+17.89	+15.28	11.87
Configuration 2	+7.37	+7.37	+10.39	+10.39	+2.71	+10.08	+17.33	+15.28	10.12
Configuration 3	0.00	0.00	0.00	0.00	+8.72	+8.72	+8.05	+8.05	4.19
Configuration 4	0.00	0.00	+9.59	+9.59	+8.72	+8.72	+3.20	+3.20	5.38
Average	3.69	3.69	7.59	7.59	7.40	11.08	11.62	10.45	7.89

Table 18: Number of focus points per system per approach

		Eigenvector centrality	Combined Connectivity
Configuration 1	Eng 1	4	7
	Eng 2	4	7
	Swb 1	10	9
	Swb 2	10	9
	RdS 1	7	6
	RdS 2	9	7
	WpS 1	9	7
	WpS 2	7	6
Configuration 2	Eng 1	5	7
	Eng 2	5	7
	Swb 1	16	9
	Swb 2	15	9
	RdS 1	13	6
	RdS 2	15	7
	WpS 1	12	7
	WpS 2	9	6
Configuration 3	Eng 1	4	6
	Eng 2	4	6
	Swb 1	12	6
	Swb 2	12	6
	RdS 1	14	7
	RdS 2	14	7
	WpS 1	12	7
	WpS 2	12	7
Configuration 4	Eng 1	5	7
	Eng 2	5	7
	Swb 1	17	9
	Swb 2	17	9
	RdS 1	13	7
	RdS 2	13	7
	WpS 1	16	7
	WpS 2	16	7

Most interesting though is the intersection value of the averages. For the eigenvector centrality, this is 10.13, for the combined connectivity this is 7.89. But what do these values represent? This average can be interpreted as a measure of effectiveness of the approach, in comparison to the simple baseline. Meaning that the eigenvector centrality approach gives more optimistic results than combined connectivity does. However, are these approaches actually a more optimistic representation than the baseline? After all, with reasonable transition rates in the baseline, one can also get high values for the availability, as was previously demonstrated in Table 13. Yet, this research is not about finding the highest values. To that end, a transitions rate of for instance $1.0e^{-7}$ and a recover rate of $1.0e^{-2}$ could have been used, but how would that help design? An actual system of systems will never contain systems with equal rates (unless all systems are equal). The goal of this case study especially is finding a way that can mimic reality in order to say something about the different options. The eigenvector centrality approach can probably mimic a real-life situation better because the spread between focus points is larger and thus the contrast between systems is better taken into account.

6.3 CHANGE IN RANGE OF RATES

In the third case, the assumed minimum and maximum rates are varied. The goal of this case is to find out how much the space between these extremes influences the scaling, and with it, the results and what can be learned from them. As established in the previous section, the combined connectivity approach has a limited scale, where for configuration 3 and 4, only the minimum and maximum rates are actually used, and nothing in between. Meaning, that it does not make a lot of sense to apply different rates to the combined connectivity approach. Therefore, this case will only consider the eigenvector centrality approach. The following new ranges will be used:

Table 19: Various ranges of rates

	Transition rate		Recover rate	
#	Minimum	Maximum	Minimum	Maximum
0	1.00E-04	1.00E-05	1.00E-03	1.00E-02
1	1.00E-03	1.00E-05	1.00E-02	1.00E-01
2	1.00E-03	1.00E-05	1.00E-03	1.00E-02
3	1.00E-03	1.00E-05	1.00E-04	1.00E-03
4	1.00E-04	1.00E-06	1.00E-02	1.00E-01
5	1.00E-04	1.00E-06	1.00E-03	1.00E-02
6	1.00E-04	1.00E-06	1.00E-04	1.00E-03
7	1.00E-05	1.00E-07	1.00E-02	1.00E-01
8	1.00E-05	1.00E-07	1.00E-03	1.00E-02
9	1.00E-05	1.00E-07	1.00E-04	1.00E-03

Row 0 in Table 19 is the range that was previously used in section 6.2, based on the rates defined in chapter 4. Rows 1 to 9 are variations of these rates. Some have deliberately been chosen worse—such as a recover rate of $1.00e^{-4}$—in order to show how much the influence of different starting parameters is. Based on these values, a preliminary estimation can already be made as to which ranges should give the 'better' results. Ranges 7-9 have the lowest range of transition rates, which makes the failure of a system the least likely. Pair this with the 'best' recover rate and the result is range 7. The highest average availability is thus expected when range 7 is used. Range 3 is opposingly expected to generate the worst results, especially because of the low recover rates.

Appendix 3 gives the results from the subsequent MATLAB® run. The first column states which range has been used, the second for which configuration the calculation is done. Columns 3 to 10 represent every system in the network. The table is color graded, to quickly show the high and low values.

Figure 24 gives a graphic representation, with a comparison to the baseline (case 1) and the initial eigenvector centrality approach with range 0 (case 2). Ranges 4, 7, and 8 score the highest results, followed by ranges 1, 5, and 9. Their results are a little higher than range 0, followed by the baseline case. Ranges 2 and 6 are far worse and range 3 does not even come close to the starting value of this graph. This graph corresponds thus to the expectations above.

They are also visible in Table 20, in which the ranges are ranked based on their performance according to Appendix 3. Range 3 is indeed by far the worst range that can be chosen to analyze this problem, as its results do not come close to representing real-life values. It is easy to explain why range 6 has lower results, as it has the worst recover rates, but range 2 differs only in the minimum transition rate from the base range 0, though its results are far worse, as is visible in Figure 24.

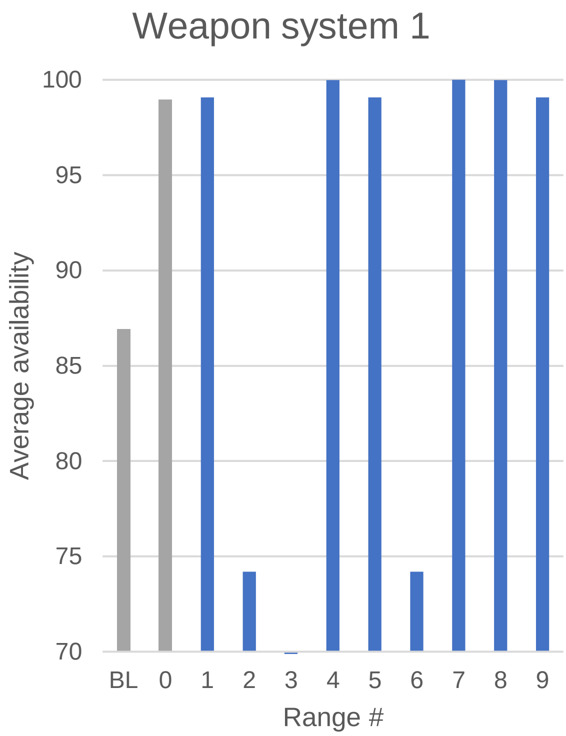

Figure 24: Average availability of weapon system 1 in configuration 4, various ranges of rates.

Table 20: Ranked ranges from best to worst performing.

	Transition rate		Recover rate	
#	Minimum	Maximum	Minimum	Maximum
7	1.00E-05	1.00E-07	1.00E-02	1.00E-01
4	1.00E-04	1.00E-06	1.00E-02	1.00E-01
8	1.00E-05	1.00E-07	1.00E-03	1.00E-02
1	1.00E-03	1.00E-05	1.00E-02	1.00E-01
5	1.00E-04	1.00E-06	1.00E-03	1.00E-02
9	1.00E-05	1.00E-07	1.00E-04	1.00E-03
0	1.00E-04	1.00E-05	1.00E-03	1.00E-02
2	1.00E-03	1.00E-05	1.00E-03	1.00E-02
6	1.00E-04	1.00E-06	1.00E-04	1.00E-03
3	1.00E-03	1.00E-05	1.00E-04	1.00E-03

But the goal here is no to find the best results. If that were the case, range 7 would be 'best' option and range 3 the 'worst'. This method investigates rather why there are differences between the results. Consider the values for the availability of weapon system 1 in Table 21, taken from Appendix 3. To understand the differences between configurations, the respective values have to be compared to one another. Previously it was assumed that a difference of 1% or less would be sufficient to treat values as equal, considering the way this method calculates transition rates. To that end, range 7 gives absolutely no information. Ranges 4 and 8 fall in the same category, even though the values might look better comparable. Range 1, 5, and 9 still have values within the 1% equality range, but the drop in configuration 3 finally becomes more apparent. Ranges 2 and 6 give actually a stronger correlation of the differences between the configurations, however, as stated before, the values do not present desired availabilities in real-life.

Table 21: Average availability of weapon system 1 for different ranges and configurations.

Config.	Range 7	Range 4/8	Range 1/5/9	Range 2/6
1	100.00	99.97	98.96	73.26
2	99.99	99.93	97.61	70.74
3	99.90	98.96	90.45	47.56
4	100.00	99.98	99.07	74.20

This case highlights thus why the range of these rates is important to get correct. With correct is meant that the generated results should not be so low that they don't properly mimic a real-life situation (ranges 2/3/6). Then again they should not be so high that a proper assessment of differences between configurations and approaches cannot be made (range 7). A seemingly small change in these rates can have a large effect on the values as is illustrated by comparing range 2 with range 0. It may thus be better to always analyze at least a couple of ranges before conclusions can be drawn.

6.4 ADDITION OR REMOVAL OF A SYSTEM

Lastly, systems can also be added to or removed from the configuration. Three diesel engines can be used instead of two, or a switchboard can be removed. The impact of such changes is studied in this last case. Two expectations can be made accordingly: the addition of a diesel engine increases the overall availability, whilst the removal of a switchboard decreases the availability of all systems except the diesel engines. Instead of using any of the four configurations previously defined in section 6.1, a fifth one is introduced. Configuration 5 in Figure 25 below is a configuration based on lessons learned from the first case. Basically, it is a combination of configuration 3 and 4. This configuration is also analyzed without the connection between the switchboards (henceforth denoted as *2-6*) as this was seen as a redundant connection in regards to the radar systems specifically. The results are shown in Table 22.

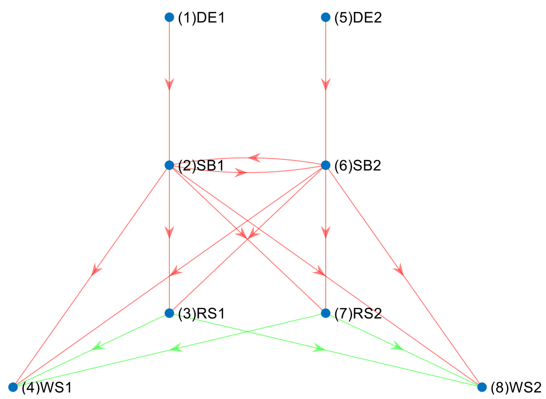

Figure 25: Configuration 5

Table 22: Average availability of the new configurations, with and without the connection between the switchboards.

		Eng 1	Eng 2	Eng 3	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2
Config 5	with 2-6	90.91	90.91	-	99.07	99.07	98.85	98.85	98.85	98.85
	without 2-6	90.91	90.91	-	90.74	90.74	99.04	99.04	99.04	99.04
Config 6		90.91	90.91	-	99.07	-	98.66	98.66	98.66	98.66
Config 7	with 2-6	90.91	98.47	90.91	99.89	99.89	99.63	99.63	99.63	99.63
	without 2-6	90.91	98.75	90.91	99.71	99.71	99.89	99.89	99.89	99.89

Without connection 2-6, the availability of the switchboards drops, as is expected. However, the availabilities of the radar and weapon systems have increased instead. This can be explained by looking at the focus points in Table 23. Without the connection, the switchboards get fewer focus points (17) than the radar and weapon systems (18). With the connection, the switchboards have the highest number of focus points (23) in the configuration. This might be an indication that it is more useful to give the radar and weapon systems the highest number of focus points, (i.e. better transition rates) as this has a greater influence on the availability than having higher performing switchboards. Then again, the difference between 98.85% and 99.04% availability is very small.

Table 23: Focus points for each system in the new configurations.

		Eng 1	Eng 2	Eng 3	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2
Config 5	with 2-6	6	6	-	23	23	20	20	20	20
	without 2-6	5	5	-	17	17	18	18	18	18
Config 6		6	6	-	14	-	12	12	12	12
Config 7	with 2-6	7	14	7	26	26	22	22	22	22
	without 2-6	6	12	6	19	19	20	20	20	20

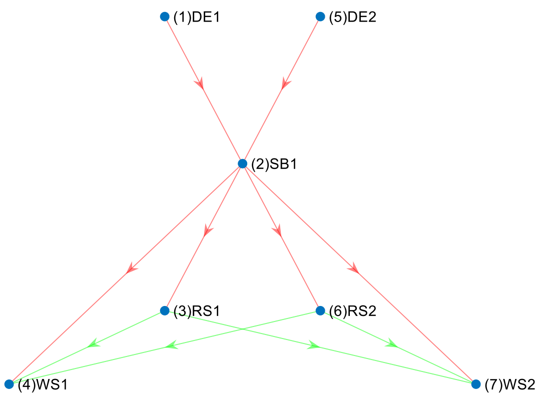

Figure 26: Configuration 6

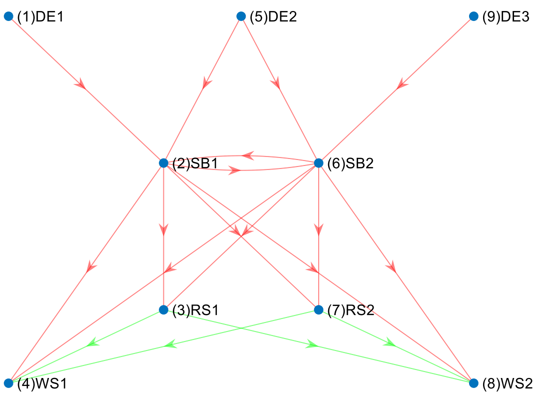

Figure 27: Configuration 7

Figure 26 is the same configuration, but one switchboard is removed and Figure 27 the configuration where one diesel engine is added. It was previously observed that the switchboards have a great impact on the overall performance of the model, the impact of the removal of one is thus interesting to assess. Configuration 6 represents this change. Only one switchboard means that it is automatically connected to all systems. Therefore, the redundancy analysis of the previous cases is not applicable. Thus it is expected that the availabilities of the radar and weapon systems decrease. Looking at Table 22, this is indeed the case.

Adding a third diesel engine should, logically speaking, increase the average availability of all systems other than the diesel engines. Mathematically speaking, i.e. with the focus point allocation, it is indeed perceived as well, as can be seen in Table 22. The middle diesel engine (Eng 2/node 5) is connected to two switchboards and therefore becomes more important than the other two diesel engines according to this method. If the connection between the switchboards (2-6) is removed, the same shift in focus points is perceived as for configuration 5. Though, due to the addition of the third diesel engine, this decrease in availability of the switchboards is not as big (99.07 → 90.74 versus 99.89 → 99.71).

7 DISCUSSION

What can be learned from these case studies and this research in general? Four slightly different configurations have been looked at. It was explained that more connections usually lead to a better performance, to a certain limit. But these connections also impact the way both focus points approaches work. Thus by changing the configurations, the way these approaches are applied to different networks could be inspected. The credibility of the initial rates used in this inspection also needs to be verified. Lastly, the influence of more or fewer systems is looked at. The next sections will give a short recap of each separate case, and how these helped to develop the model further.

7.1 DISCUSSION ON THE CASE STUDIES

7.1.1 CHANGE IN CONFIGURATION

Four configurations were introduced in section 6.1 and the differences between each identified. For most of these changes to the basic configuration, the outcome could have been guessed. Such as that adding an extra power connection to the weapon systems would result in a higher availability, from a logical perspective. The credibility of this method is therefore increased, due to the fact that an educated guess corresponds to the results of the model. One form of unnecessary redundancy, that of how the radar systems are connected to switchboards, was however also identified for the radar systems.

7.1.2 CHANGE IN TRANSITION RATES

However, the above is not entirely true, depending on what focus point approach is used. Considering once more the added power connection for weapon systems, the combined connectivity approach actually generates lower results in configuration 4 than in configuration 1, which is peculiar from a logical perspective. The eigen-vector centrality approach does generate higher values, as expected. The difference between these approaches is significant, and it has to do with the range of focus points in this case.

As explained in section 6.2.1, a problem exists between the number of different focus points in the combined connectivity approach. Since the network has the same eight nodes in every configuration, but just connected in a slightly different way, the upstream and downstream connectivity of most modes does not change a lot. The lowest value is always 6 and the highest 9, with only the value 7 as a substitute or addition to these values. This leads to the situation where usually half of the nodes have one value and the remaining half have the other value, generating a case that is almost similar to the baseline case with fixed rates.

For this reason, the combined connectivity approach should not be used to assess the availability of systems in the early stages of design, unless it will be clear that a larger network results in a far wider spread of focus points and thus transition rates. The more suitable option would probably be using the eigenvector centrality approach, which inherently generates a wider spread of focus points, where each different connection results in a different number of focus points.

But why do the transition rates even need to be system-dependent, in such an abstract case? In reality, all different systems have different types of failure rates and repair rates. In addition, the way each system is connected to other systems, and the position in the ship also impact the availability of a system. In this research, these three things are at least considered within the so-called transition and recover rates. This distinction makes it clear that the baseline case is a passable starting point for a basic estimation, but can definitely be improved by using the eigenvector centrality approach to get results that mimic reality in a better way.

7.1.3 CHANGE IN RANGE OF RATES

The next step in the further development of the model is the input that is used, which in this case means the range of rates used to calculate the availability. Nine different rates have been identified in section 6.3, some worse than others. This section has pinpointed to a proper definition of these rates, as a range that would have uncommonly low recover rates and uncommonly high transition rates resulted in useless information. A range with too-good sounding rates is also not preferred, as the internal differences between configurations (and previously between approaches) are hard to identify. The goal of this research, after all, is not to find the best results and conclude that a certain system actually works 98% of the time. The goal is to understand why the system works 98% of the time and not 99% or 80% for that matter. A range in between the extremes is thus preferred, as was shown in this research.

7.1.4 Addition and subtraction of systems

Lastly, the knowledge of previous cases can be combined and examined in a new configuration where the number of systems is changed for that configuration. Adding or subtracting a system has proven to show results that were to be expected. An additional diesel engine does indeed increase the availabilities of the systems, whilst the removal of a switchboard decreases the availability of the radar and weapon systems. The connection between the switchboards is an interesting design parameter. Removing it puts more emphasis on the performance of the radar and weapon systems, whilst keeping it puts more emphasis on the switchboards.

8 CONCLUSIONS AND RECOMMENDATIONS

This chapter concludes the final outcome and future implementation of this thesis. What is the outcome of this research? How can it be used? Is the model complete or does it lack anything still? Such questions will be answered in the next two sections, where the first deals with the conclusions of and reflections on this work. The second deals with recommendations for further research.

8.1 CONCLUSIONS

So what is the outcome of this research? What can be learned from it? What can immediately be taken away from this study, is that a gap exists in research on the operational architecture. Though this thesis does not completely fill in this gap, it helps define which other fields still need attention in order to find more pieces of the puzzle. The operability tree discussed in section 2.2 showed how operability—that is, the ability to perform the ship's mission in a safe and reliable functioning condition, according to pre-defined operational requirements—can be linked to the operational architecture. Availability was considered to be the part of system performance that needed looking at. From several dependability methods usable to calculate availability, Markov theory was the most promising, as it is a well-documented method (as opposed to Petri nets) with a wider applicability than fault trees and Bayesian networks and is used in similar studies as well. The research question for this thesis is stated below:

How can Markov theory be used in Early Stage Ship Design regarding systems design to analyze the availability of systems in order to make design decisions?

8.1.1 USE OF MARKOV THEORY

Markov theory has, indeed, proved to be useful when calculating the availabilities of the eight systems in the often-considered network in this thesis. Though, the result was not what was to be expected at first since data on rates was lacking. Basic Markov theory was, therefore, not enough to answer the research question, which is why network theory was introduced to generate appropriate input. Two approaches were introduced to find nonquantifiable focus points; the eigenvector centrality and the combined connectivity approach. These had to be used to define a scale resulting in a set of rates for the systems in the network. The eigenvector centrality approach was deemed more useful than the combined connectivity approach since it had a broader spectrum of focus points, leading to a more optimistic approximation of different transition rates for different systems throughout the network. Due to these nonquantifiable values, the method does not actually calculate availability, that is, not in a sense of real-life availability. Instead, it finds a nonquantifiable availability that can be compared to different configurations of a network and learn from the outcome. The result is, therefore, a model consisting of Markov and network theory, that looks into the availability of systems but does not generate direct answers to a real-life problem. The strength of the model is, however, the insights gained from comparing different systems and different configurations during the early stages of design, in order to get a better understanding of how these aspects can be used to define the first concept in the concept definition phase of Early Stage Ship Design.

8.1.2 REGARDING OPERABILITY

Section 2.4 explained that the total operability of a naval vessel cannot be assed by solely looking at availability of ship systems, as it was found to consist of more elements. This method does contribute to the calculation of the total operability, albeit in a small part. The actual availability of systems is not calculated with this method though, but rather an indication of what it could be, as just mentioned. Nevertheless has this thesis shown how operability—and thus a measure of operational architecture—can be addressed.

8.1.3 Reflection on the model

From a practical point of view, the most important question to ask after having performed this research is: 'How can this model be used by a ship designer?'. The model works strictly one way, i.e. it is limited to the assumption that transition rates are unknown at this design stage. If a certain availability is desired for a system or the overall configuration, then the model does not provide answers to what the transition rate should be directly. The input to the model can alternatively be changed in such a way, that the outcome may be the sought-after answer. It is in that case required that the configuration mimics the configuration that was in mind when the desired availability was established.

When configurations are compared to one another, it is not always correct to say that one configuration is 'better' than the other. In this analysis, such statements were usually made according to the value of availability for a certain system. But some configurations had surprising results, which was mainly due to the way focus points were allocated. This means that some configurations might indeed have 'better' results in regards to availability, but they may be worse in other regards. Cost can, for instance, have a big influence on what configuration is picked in the end. This method presents a way to compare configurations purely within the scope of this thesis and does not always provide a clear answer as to what configuration is 'better'. External aspects can be factored into this comparison in order to make such proper decisions.

It is however, difficult to compare the focus points within a configuration. If the switchboards in a configuration get 23 focus points, the radar and weapon systems 20, and the diesel engines only 5, what does this say about their respective importance? Given the nature of the eigenvector centrality, the diesel engines will always have a lower number of focus points compared to other, more central systems. For this reason alone, one cannot simply state that the weapon systems are four times more important than the diesel engines, for instance. Caution must thus be taken in regards to interpreting the focus points. It can be stated though, that more focus should be given to the switchboards, if this indeed leads to a transition rate close to what the method prescribes. This corresponds to the assumption made in section 4.2 that more focus points (i.e. more resources) lead to a better performing system. It is also worth restating, that the number of focus points are nonquantifiable and so cannot be directly converted into actual resources. Twenty focus points do not mean $20,000 or 2000 man-hours of designing, for example.

Essentially, when transition rates are unknown, the model provides a way to analyze the availability of systems in a broad sense. With the eigenvector centrality and this focus point approach, it is easy to see which system has a bigger influence on the availability than others. However, this influence might shift according to its connectivity. Depending on this connectivity, the outcome may be very different and provides an estimation of within what range a certain transition rate for a system should be. When the choice is made to use a certain system in the ship, and its transition rates are known at that moment, a comparison can be made with the transition rates used in this method. If it is worse than the value used in this approach, it may be wise to look for a different system. If the transition rate is better than the value used in this approach, it may be assumed that the system will function according to the model. It must be noted though, that transition rates were defined in section 4.2 to include the failure and repair rate of a system but are not strictly the same as the transition rates used here. This means that a designer cannot compare transition rate X from this method to the real failure rate Y of a system under consideration, without taking the way it is connected into account. The behavior of the systems and the configuration can hence be understood.

So how can this model be applied as a tool? The method described in the earlier chapters of this thesis has been used to develop a model, which after some testing has been developed into a tool. As design knowledge is limited during early stage ship design, it is important to have tools available that, with minimum data, can predict preliminary outcomes. With the proper input, a designer can quickly analyze the operational value (expressed in availability) of certain configurations given a set of functional systems. The following design data is undoubtedly required in order to use this tool:

- A set of systems contributing to the same goal.
- A general idea of interdependencies between these systems.

- An arbitrary set of transition rate ranges, in the order of magnitude of alike systems (see section 4.4).

The following knowledge is subsequently acquired when using this tool:

- Elementary connectivity information, such as the influence of adding or removing certain connections on the availability of involved systems.
- A ballpark estimation of the analyzed system of systems' performance.
- An indication of individual system availability, given a certain network.
- If applied to the bigger picture, the contribution of system performance to the operability of a naval vessel.

In conclusion, a designer can use this method during the early stages of ship design to asses certain configurations. Without any prior knowledge of the quality of systems, he can come up with a configuration based on the required functional systems and get an indication of the performance. By comparing a set of different configurations—be it with a different number of systems or other connections—he can identify critical nodes and connections, redundant set-ups and the influence of each system on the whole system. The designer can then link this information to the amount of design focus that can be given to each system and apply it for early budgeting and planning.

8.2 RECOMMENDATIONS

Although this thesis answered several questions, it led to many more topics to be further investigated as well. The model itself can be improved with a more comprehensive algorithm, and the operability tree can be further explored. Recommendations will be given for both below.

8.2.1 ADAPTATIONS TO THE MODEL

The current model is based on the user putting in data for all nodes and for each configuration, leading to a tedious task where mistakes are easily made. A major improvement to this model can be achieved by using an add-on that automatically generates network configurations. This will greatly increase the envelope of possible reasonable routing options that can be analyzed based on their layout, connectivity, and operability. Doing this ties all three architectures (physical, logical and operational) together. An example of such automatic network generation has recently been exhaustively explored by de Vos and Stapersma (2018).

Furthermore, the model can be extended by adding capacities to the nodes and edges, which can determine if a system can actually get enough power, given the failure of a power supplier. The firing of a high energy weapon such as a railgun demands a lot of power, leading to a possible (temporary) shutdown of other systems. The question of whether a system is sufficiently powered can be derived from the network theory. The Markov chain analysis presented in this thesis can tackle this problem due to the addition of the error mode. Alternatively, partial availability (50% vs 0% or 100%) can also be introduced.

8.2.2 ADAPTATIONS FOR THE CONTRIBUTION TO OPERABILITY

This research is but a specific part of a broader study on operability, as can be seen in the operability tree in Figure 28. Only the failure and supply parts of the availability section have been discussed in this thesis. Combining this with the vulnerability study by Habben Jansen, Kana, and Hopman (2017) would lead to an already quite comprehensive model regarding system performance. The misuse part of availability is still lacking, though. As this is related to crew performance, a study on this section itself could cover that. Maintenance strategies, such as preventive maintenance or condition-based maintenance, can also be taken into account at this stage, even if it might be difficult to predict which will be used in the early stages of design. Ship performance itself is not part of systems design, but can contribute to it (propeller out of the water, systems being unusable due to unwanted tilting etc.). However, such research belongs to a different field of study than systems design, as indicated in the figure.

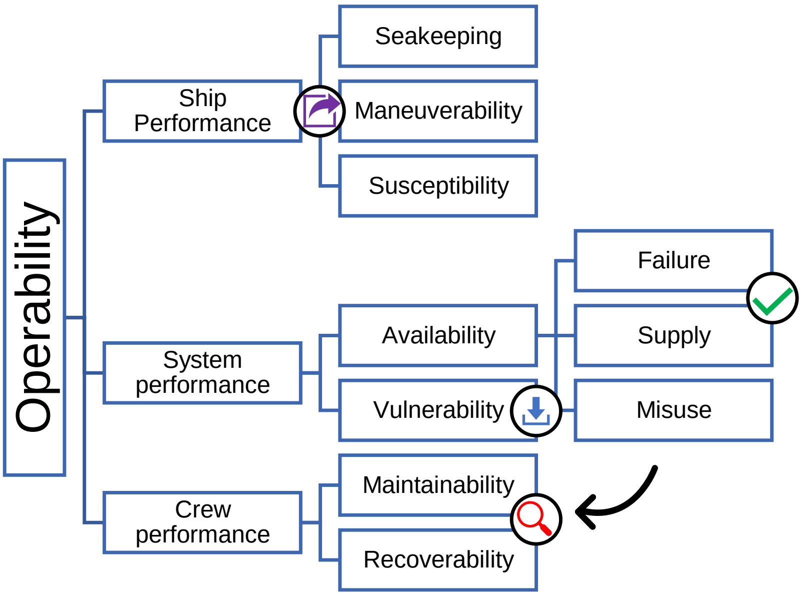

Figure 28: Operability tree

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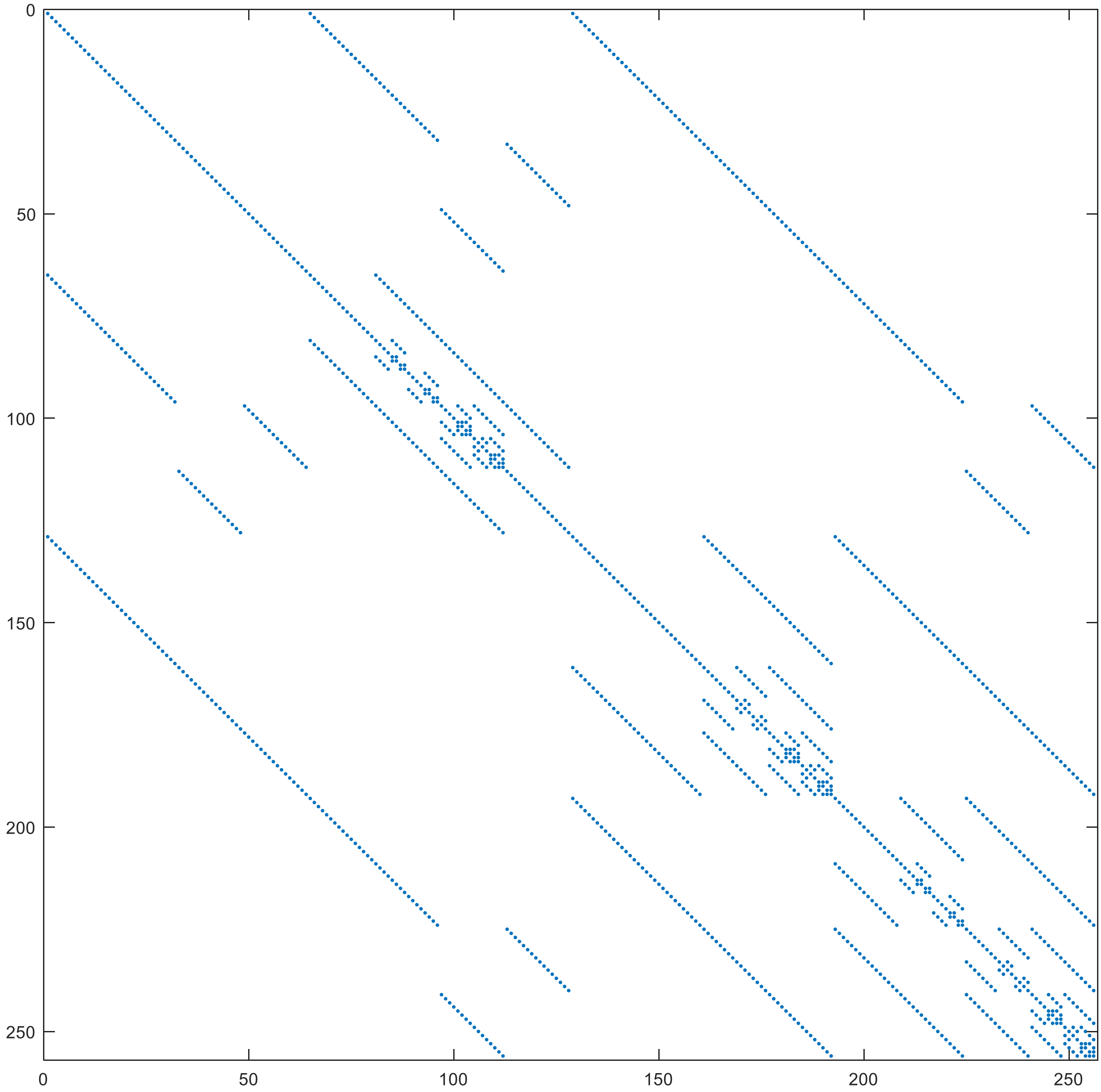
A. APPENDIX 1: TRANSITION MATRIX AS USED IN 5.2



This is a plot of what the transition matrix looks like based on the calculation in section 5.2. The states are on the x- and y-axis. All blue dots are cells with transition rates, or in case of the diagonal, the negative sum of a each row.

B. APPENDIX 2: FOCUS POINTS CALCULATION

		Degree centrality	Eigenvector centrality	Downstream connectivity	Upstream connectivity	Combined Connectivity
Configuration 1	Eng 1	1	4	7	1	7
	Eng 2	1	4	7	1	7
	Swb 1	4	10	6	4	9
	Swb 2	4	10	6	4	9
	RdS 1	2	7	2	5	6
	RdS 2	3	9	3	5	7
	WpS 1	3	9	1	7	7
	WpS 2	2	7	1	6	6
Configuration 2	Eng 1	1	5	7	1	7
	Eng 2	1	5	7	1	7
	Swb 1	5	16	6	4	9
	Swb 2	5	15	6	4	9
	RdS 1	3	13	2	5	6
	RdS 2	4	15	3	5	7
	WpS 1	3	12	1	7	7
	WpS 2	2	9	1	6	6
Configuration 3	Eng 1	1	4	6	1	6
	Eng 2	1	4	6	1	6
	Swb 1	4	12	5	2	6
	Swb 2	4	12	5	2	6
	RdS 1	4	14	3	5	7
	RdS 2	4	14	3	5	7
	WpS 1	3	12	1	7	7
	WpS 2	3	12	1	7	7
Configuration 4	Eng 1	1	5	7	1	7
	Eng 2	1	5	7	1	7
	Swb 1	5	17	6	4	9
	Swb 2	5	17	6	4	9
	RdS 1	3	13	3	5	7
	RdS 2	3	13	3	5	7
	WpS 1	4	16	1	7	7
	WpS 2	4	16	1	7	7

C. APPENDIX 3: RESULTS OF NINE RATE RANGES

Range #	Config.	Eng 1	Eng 2	Swb 1	Swb 2	RdS 1	RdS 2	WpS 1	WpS 2
1	1	90.91	90.91	99.16	99.16	98.26	98.96	98.96	98.06
	2	90.91	90.91	99.15	99.06	98.80	99.06	98.61	97.50
	3	90.91	90.91	90.68	90.68	99.12	99.12	90.45	90.45
	4	90.91	90.91	99.16	99.16	98.68	98.68	99.07	99.07
2	1	50.00	50.00	74.90	74.90	68.60	73.39	73.26	67.22
	2	50.00	50.00	74.66	74.17	72.03	73.90	70.74	63.81
	3	50.00	50.00	48.76	48.76	73.67	73.67	47.56	47.56
	4	50.00	50.00	74.90	74.90	71.43	71.43	74.20	74.20
3	1	9.09	9.09	17.10	17.10	8.92	14.18	12.99	7.39
	2	9.09	9.09	16.38	15.58	12.01	14.84	10.29	5.62
	3	9.09	9.09	7.25	7.25	13.84	13.84	5.78	5.78
	4	9.09	9.09	17.10	17.10	11.51	11.51	15.58	15.58
4	1	99.01	99.01	99.99	99.99	99.90	99.97	99.97	99.88
	2	99.01	99.01	99.99	99.98	99.95	99.98	99.93	99.82
	3	99.01	99.01	98.98	98.98	99.99	99.99	98.96	98.96
	4	99.01	99.01	99.99	99.99	99.94	99.94	99.98	99.98
5	1	90.91	90.91	99.16	99.16	98.26	98.96	98.96	98.06
	2	90.91	90.91	99.15	99.06	98.80	99.06	98.61	97.50
	3	90.91	90.91	90.68	90.68	99.12	99.12	90.45	90.45
	4	90.91	90.91	99.16	99.16	98.68	98.68	99.07	99.07
6	1	50.00	50.00	74.90	74.90	68.60	73.39	73.26	67.22
	2	50.00	50.00	74.66	74.17	72.03	73.90	70.74	63.81
	3	50.00	50.00	48.76	48.76	73.67	73.67	47.56	47.56
	4	50.00	50.00	74.90	74.90	71.43	71.43	74.20	74.20
7	1	99.90	99.90	100.00*	100.00*	99.99	100.00*	100.00*	99.99
	2	99.90	99.90	100.00*	100.00*	100.00*	100.00*	99.99	99.98
	3	99.90	99.90	99.90	99.90	100.00*	100.00*	99.90	99.90
	4	99.90	99.90	100.00*	100.00*	99.99	99.99	100.00*	100.00*
8	1	99.01	99.01	99.99	99.99	99.90	99.97	99.97	99.88
	2	99.01	99.01	99.99	99.98	99.95	99.98	99.93	99.82
	3	99.01	99.01	98.98	98.98	99.99	99.99	98.96	98.96
	4	99.01	99.01	99.99	99.99	99.94	99.94	99.98	99.98
9	1	90.91	90.91	99.16	99.16	98.26	98.96	98.96	98.06
	2	90.91	90.91	99.15	99.06	98.80	99.06	98.61	97.50
	3	90.91	90.91	90.68	90.68	99.12	99.12	90.45	90.45
	4	90.91	90.91	99.16	99.16	98.68	98.68	99.07	99.07

* Disclaimer: This value is not actually a 100%, which would be impossible. Due to rounding up of the values to two decimals, a 100.00 is shown instead.

D. APPENDIX 4: MATLAB® CODE

```
%% This is the MATLAB® code for the model as used in this thesis.%%
filename = 'Parameters.xlsx';                          % Imports the user data

for z = 1:9                                            % Varies the range
    for n = 1:4                                        % Varies the configuration
        for q = 1:2                                    % Varies the approach 1 =
combined connectivity, 2 = eigenvector centrality

%% Generate network graph
X = xlsread('Parameters',5,'A10:H10');
Y = xlsread('Parameters',5,'A11:H11');
names = {'(1)DE1' '(2)SB1' '(3)RS1' '(4)WS1' '(5)DE2' '(6)SB2' '(7)RS2'
'(8)WS2'};
D = xlsread('Parameters',n,'B3:I10');
s = xlsread('Parameters',n,'A14:N14');
t = xlsread('Parameters',n,'A16:N16');
edge_weights = xlsread('Parameters',n,'A18:N18');
node_weights = xlsread('Parameters',n,'B20:I20');
edge_color = xlsread('Parameters',n,'A24:N24');
Con = xlsread('Parameters',n,'B22:I22');
Nodetable = table(names',node_weights','VariableNames',{'Node','Value'});
G = digraph(s,t,edge_weights,Nodetable);
network = plot(G,'XData',X,'YData',Y,'NodeLabel',names);

for c = 1:length(edge_color)
    if edge_color(c) == 0
        highlight(network,[s(c),t(c)],'EdgeColor','r');
    else
        highlight(network,[s(c),t(c)],'EdgeColor','g');
    end
end

%% Generate all combinations to define the number of states
No_of_Systems = 8;                                                  % Define the
number of systems
S = allcomb([0 1],[0 1],[0 1],[0 1],[0 1],[0 1],[0 1],[0 1]);       % Manually in-
sert No_of_States pairs of [0 1]
No_of_States = size(S,1);                                           % Determines
the number of states

%% Adjust S matrix to include error state
K = S;                                                              % Adjustment
matrix of S, including error state (has values of 2)
DK = S;                                                             % Dummy matrix
of K, for correct conditional adjustment (does not have values of 2)

for r = 1 : No_of_States
    for k = 3 : No_of_Systems
        if DK(r,k) == 0
            %do nothing
        elseif DK(r,1) + DK(r,2) <= 0
            K(r,k) = 2;
            DK(r,k) = 0;
        elseif DK(r,3) + DK(r,4) <= 0
            K(r,k) = 2;
            DK(r,k) = 0;
        elseif sum(DK(r,:) .* D(k,:) .* node_weights) < Con(k)
            K(r,k) = 2;
```

```
            DK(r,k) = 0;
        end
    end
end

%% Generate failure and repair rate vectors
approach = ["Z3:Z10", "AC3:AC10";"AA3:AA10","AD3:AD10"];          % First two el-
ements are for combined connectivity, second two are eigenvector centrality

Rate_F = xlsread('Parameters',n,approach(q,1))';                   % Reads for
configuration n the failure rates of approach q from data file
Rate_R = xlsread('Parameters',n,approach(q,2))';                   % Reads for
configuration n the repair rates of approach q from data file
Range_rates = xlsread('Parameters',6,'M42:P49');

%% Generate transition matrix
UnQ = unique(K, 'rows');                                           % Sorts the
rows by uniqueness, possible not necessary anymore (as of 6/9/18)
No_of_States = size(UnQ,1);                                        % Determines
the number of unique states
Q_0 = zeros(No_of_States);                                         % Defines the
size of the transition matrix


%% Filling in of the transition matrix
for i=1:No_of_States
    for j=1:No_of_States
        count_higher = 0;                                          % This loop
checks what kind of transition is going on between two states
        count_lower = 0;
        count_error = 0;
        dummi = UnQ(i,:);
        dummj = UnQ(j,:);
        for k=1:8
            if (UnQ(i,k) == 2 && UnQ(j,k) == 0) || (UnQ(i,k) == 0 && UnQ(j,k) ==
2)
               count_error = count_error + 1;
            elseif UnQ(i,k) == 2 || UnQ(j,k) == 2
                dummi(k) = 0;
                dummj(k) = 0;
            elseif UnQ(i,k) > UnQ(j,k)
                count_higher = count_higher + 1;
            elseif UnQ(i,k) < UnQ(j,k)
                count_lower = count_lower + 1;
            end
        end
        if count_error >= 1
            Q_0(j,i) = 0;                                          % This loop
checks whether a transition between two states is possible and assigns a transi-
tion rate to it, if so.
        elseif count_higher == count_lower
            Q_0(j,i) = 0;
        elseif (count_higher == 0) && (count_lower == 1)
            Q_0(j,i) = Rate_F * (dummj - dummi)';
        elseif (count_higher == 1) && (count_lower == 0)
            Q_0(j,i) = Rate_R * (dummi - dummj)';
        else
            Q_0(j,i) = 0;
        end
    end
end
```

```
for i = 1:No_of_States
    Q_0(i,i) = -sum(Q_0(i,:));                                    % Fills in the
diagonal elements
end

UnQ_2 = UnQ;                                                      % Adapts the
state matrix to incorporate the error state
for i = 1:No_of_States
    for j = 1:No_of_Systems
        if UnQ_2(i,j) == 2
            UnQ_2(i,j) = 0;
        end
    end
end

%% Calculate the steady state vector
Q_1 = Q_0;                                                        % Amend transi-
tion matrix according to Markov
        for i = 1:No_of_States
            Q_1(i,No_of_States) = 1;
        end
p = zeros(1,No_of_States);
p(No_of_States) = 1;
P = 100*(p/Q_1);                                                  % Steady state
vector

%%Calculate the average availability of each system for independent and
%%dependent case

Avg_av_system = zeros(1,No_of_Systems);

for k=1:8
    Avg_av_system(:,k) = sum(UnQ_2(:,k).*P');                     % Average avail-
ability per system
end

if q == 1
    Result.cmb(n,:) = Avg_av_system;                              % Exports the
data to a structure
    Result.steady_state_cmb(n,:) = P;
else
    Result.egv(n,:) = Avg_av_system;
    Result.steady_state_egv(n,:) = P;
end


        end
    end

Result.ranges(z*4-3:z*4,:) = Result.egv;                          % Writes the re-
sults to a structure
FR_min = Range_rates(z+1,1);                                      % Switch to the
next range
FR_max = Range_rates(z+1,2);
RR_min = Range_rates(z+1,3);
RR_max = Range_rates(z+1,4);
xlswrite(filename,FR_max,6,'I30')                                 % Inserts the
next range
```

```
xlswrite(filename,FR_min,6,'H30')
xlswrite(filename,RR_max,6,'I34')
xlswrite(filename,RR_min,6,'H34')
end
```