

FractalAnalyzer: A MATLAB Application for Multifractal Seismicity Analysis

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Online Material: MATLAB code and associated figures and tables describing the multifractal seismicity analysis method.

INTRODUCTION

Earthquakes are seismic phenomena caused by the sudden release of energy in the Earth's crust. Their effects range from ground shaking to faulting. Geological and geophysical studies, especially in light of plate tectonic theory have been used to explain the occurrence of earthquakes. Thus from the point of view of statistical fractals, earthquakes cannot be interpreted as random independent events (i.e., having Poisson distribution). Rather, it is observed that the events of the same sequence are clustered in time and space (Shlien and Toksoz, 1970; Vere-Jones, 1970; Smalley *et al.*, 1987; De Natale *et al.*, 1988; Roy and Mondal, 2012a,b).

Several past approaches studied the clustering effect of the earthquakes. The generalized Poisson model is one of the most commonly used tools to understand the distribution of the seismic events (Ripley, 1988; Stoyan and Stoyan, 1994). This model assumes that the Poisson distribution can be used to describe the uncorrelated clusters of events, whereas the number of events in each cluster is defined by the power law distribution. Some researchers have studied these seismicity clusters by considering the occurrence of an earthquake to be a stochastic self-exciting process (Vere-Jones, 1970; Ogata *et al.*, 1982; Molchan and Kronrod, 2007). Another useful approach toward quantification of the clustering properties of an earthquake process is the multifractal analysis of the seismic events.

In the past, several authors have explained the regional seismicity clustering through fractal-like structures (Sadovskiy *et al.*, 1985; Smalley *et al.*, 1987), and these works suggest that multifractal analysis is an effective method for studying earthquake patterns. Moreover, this method provides a detailed explanation of the chaotic nature of distributions and geometry associated with the earthquake clustering phenomena (Bak and Tang, 1989; Huang and Turcotte, 1990; Chen *et al.*, 1991; Kagan and Jackson, 1991; Godano and Caruso, 1995; Guo and Ogata, 1997; Telesca *et al.*, 1999; Evison, 2001). The fractal dimensions needed for multifractal analysis can be calculated using different methods. The correlation integral approach calculates the correlation fractal dimension (Kagan and Knopoff, 1980; Grassberger and Procaccia,

1983a; Hirata, 1989). Generally, the fractal distributions or statistical scale invariance that exists in nature and in dynamic systems are found to be heterogeneous (Mandelbrot, 1989). Thus, a unique fractal dimension is not enough to understand the clustering of these events. In such cases, multifractal analysis is performed and the fractal dimensions are characterized by the general dimension D_q or the $f(\alpha)$ spectrum (Hentschel and Procaccia, 1983; Halsey et al., 1986; Neuman, 2010). Moreover, multidimensional fractal study provides a quantitative measure of the spatial clustering (i.e., whether cluster exists within clusters), thereby giving information about the crustal deformation in space and time nucleation of events. Therefore, multidimensional will explain the state from one level to other level of cluster existence, when it saturates at higher value, it indicates that no further cluster exists within cluster. This spatial clustering result can be used to understand the seismicity of a region (Aki, 1981, 1984; King, 1983; Legrand et al., 1996; Nakaya and Hashimoto, 2002; Oncel and Wilson, 2002, 2004, 2006; Roy and Ram, 2006; Li and Xu, 2012).

This paper presents the application of the correlation integral method for the determination of the correlation dimension and the multifractal or generalized fractal dimension. These results are further used to determine the clustering of seismicity (Grassberger and Procaccia, 1983b; Roy and Ram, 2006; Roy and Padhi, 2007). Multifractal analysis also helps to interpret the heterogeneous fractal nature of seismicity. This heterogeneity is an indication of the complex stress pattern of the region (Legrand et al., 1996; Oncel and Wilson, 2004). For easy implementation of this method, we present *FractalAnalyzer*, a MATLAB-based (http://in.mathworks .com/support/compilers/R2010a/win32.html; last accessed August 2015) application that uses this algorithm to interpret the spatiotemporal cluster of earthquake occurrence. FractalAnalyzer is an interactive graphical user interface (GUI) developed in MAT-LAB R2010a. It is capable of using up to a total of 22 fractal dimensions to define a cluster of earthquake events. The interactive feature of this application provides the flexibility of choosing the number of events to be considered within each subset. Additionally, this application includes a graph-plotting module that can be used to plot various graphs associated with this algorithm. To demonstrate the capability and effectiveness of this application, we used it to analyze the clustering of seismicity before the 08 October 2005 Kashmir earthquake ($M_{\rm w}$ 7.7).

THEORY AND ALGORITHM

Fractals

Fractal dimension is a characteristic index of a fractal object or set (Hirabayashi *et al.*, 1992). As per Mandelbrot (1977), a

fractal is defined as a set for which Hausdorff–Besicovitch dimension strictly exceeds the topological dimension. The Hausdorff–Besicovitch dimension is not easy to calculate for fractals in the real world. There are several ways of defining the fractal dimensions (Takayasu, 1990). For example, the similarity dimension D_s is defined for an exactly self-similar or a smaller portion of distributions and has exactly the same kind of distribution in a larger portion set as

$$D_s = \frac{\log b}{\log a},\tag{1}$$

in which *a* is the linear size and *b* is the number of similar daughters. D_s has a more theoretical than practical base. Scale invariance distribution, which is not exactly self-similar in all scales, can be quantified with the capacity dimension D_1 , defined as

$$D_1 = \frac{\log N(r)}{\log(1/r)},\tag{2}$$

in which N(r) is the smallest number of coverings of the set with size r. D_1 has the practical advantage of extracting information about a system in a statistical sense. Further statistical extraction of the scale invariance distribution is done with the help of the information dimension D_I , which is based on probability distribution and defined as

$$D_{I} = \lim_{r \to 0} \sum_{i=0}^{N(r)} \frac{P_{i}(r) \log P_{i}(r)}{\log r}.$$
(3)

In equation (3), $P_i(r)$ is the probability for a point in the sample space to belong to the *i*th box with size *r*. Correlation dimension D_c depends on the correlation integral C(r) and the relation can be expressed as follows:

$$C(r) \approx r^{D_c - d},\tag{4}$$

in which d is the spatial dimension. C(r) is calculated from N using the following relation:

$$C(r) = \lim_{N \to \infty} \frac{1}{N^2},\tag{5}$$

in which N is the number of pairs of points in the sample space for which distance is less than r. In general, $D_1 \ge D_I \ge D_c$. The relationship holds true for homogeneous fractals, and the inequality is valid for heterogeneous multifractals (Hirabayashi *et al.*, 1992). They are applied to extract information about distributions of various theoretical and physical situations. A unique finger print for each of the dimension to be obtained will have its own significance as a multifractal object, which demands an infinite hierarchy of fractal dimensions, known as the generalized fractal dimensions D_q . This can be expressed as

$$D_q = \frac{1}{q-1} \lim_{r \to 0} \frac{\log\{\sum_i [P_i(r)]^q\}}{\log r},$$
(6)

in which D_q exhibits a nontrivial scaling behavior for different values of q = 1, 2, 3, ..., and $P_i(r)$ is the probability of an event lying within a square box of dimension r. The generalized dimension D_q is defined for all real values of q and is a monotonically decreasing function.

Mandelbrot (1989) showed that lower- and upperlimiting dimensions, $D_{-\infty}$ and D_{∞} respectively, exist that are related to the different regions of the set. $D_{-\infty}$ and D_{∞} correspond to the regions in which the measures are most dilute and most dense, respectively, and this phenomena is termed multifractality. Usually, the multiplicative cascades of the random processes generate multifractal structures, whereas the additive processes produce simple fractures (Bunde *et al.*, 1990). The correlation dimension D_c thus obtained is

$$D_c = \lim_{q \to 2} D_q = D_2. \tag{7}$$

In two dimensions, the values of D_q approaching a value of 2 signify a uniform coverage of the plane. We use the spherical triangular method to calculate the distance between two epicenters (Oncel and Wilson, 2002; Mandal *et al.*, 2005; Roy and Nath, 2007; Roy and Mondal, 2009).

Correlation Dimension

The fractal correlation dimension is derived from the correlation integral, which is a cumulative correlation function that measures the fraction of points in 2D space and is defined as

$$C(r) = \frac{2}{N(N-1)} \sum_{j=1}^{N} \sum_{i=j+1}^{N} H(r - r_{ij}),$$
(8)

in which N is the number of pairs that can be formed from a given cluster of seismic events (for 50 events, the N window will be ${}^{50}C_2$, which equals 1225), r is the length scale, r_{ij} is the distance between any two points of the cluster set obtained using the spherical triangle method, and H is the Heaviside step function. Grassberger and Procaccia (1983a) defined N as the total number of events that form the set. We consider it to be the set of all possible unique vector pairs, with two events taken each time. C(r) is proportional to the number of vector pairs of the fractal set having length less than r. Figure 1 shows a graph of $\log C(r)$ versus $\log r$ at different stages of an example fracture process. Based on regression analysis, we determine the equation of the line that fits the distribution. The slope of this line gives the fractal dimension (D_{c}) of the system. The large-scale deviations from linear dependence are associated with the finite size of samples, whereas the small-scale deviations are due to the boundary effects of data.

Generalized Dimension

Multifractal dimension parameter D_q represents the complicated fractal structure or multiscaling nature of the seismic



▲ **Figure 1.** Log C(r) versus log r is shown for one time window of the Kashmir earthquake region with latitude (32°–37° N) and longitude (70°–75° E); the slope gives D_{C} . The straight line demarcates the scaling region obeying the power law (i.e., scale invariance). R^2 represents correlation coefficients of the regression line.

events. In this work, we use D_q to analyze the seismicity clustering toward the multiscale notion. Some of the most common methods for calculating D_q are the fixed-mass method, the fixed radius method, and the box-counting method (Grassberger and Procaccia, 1983a; Halsey *et al.*, 1986; Mandelbrot, 1989). Pawelzik and Schuster (1987) extended the Grassberger–Procaccia method for the recovery of dimension from a time series. The related formulae are given as

$$\log C_a(r) = D_a \log r(r \to 0) \tag{9a}$$

and

$$C_q(r) = \left\{ \frac{1}{N} \sum_{j=1}^N \left[\frac{1}{N} \sum_{i=j}^N H(r - |X_i - X_j|) \right]^{q-1} \right\}^{1/(q-1)},$$
(9b)

in which $C_q(r)$ is the *q*th-order correlation integral. The above formulation is implemented in *FractalAnalyzer* to process the datasets. D_q is the slope of the straight line fitted on the data of log *r* versus log $C_q(r)$ graph using the linear regression method. For a different value of *q*, we get a different D_q . The curve of $q - D_q$ is termed the D_q spectrum.

FractalAnalyzer Application

FractalAnalyzer is a MATLAB-based interactive graphical application for multifractal analysis of the earthquake events and seismicity clusters. It consists of two modules: the multifractal

computation module (MCM) and *a* grapher module (GM). (E) Figures S1 and S2 (available in the electronic supplement to this article) show the graphical displays of *FractalAnalyzer* application in MCM mode and GM mode, respectively. MCM provides the functionalities to perform multifractal analysis for *q* (e.g., q = 2, 3, ..., 22). GM can be used to plot the various graphs that are associated with this method, such as $q - D_q$ plot, log $r - \log C(r)$ plot, and $D_q - t$.

The FractalAnalyzer application is built in MATLAB R2010a and requires a MATLAB Compiler Runtime (MCR) v.7.13 or higher. In addition, it is a Windows stand-alone application, and the GUI is built using the GUIDE tool of MAT-LAB. FractalAnalyzer is provided in both forms (a MATLABbased GUI application and a MATLAB package), thus it can be used as an end-user application or a developer's module. The MATLAB code is provided under the GNU license and can be freely edited and used for research purposes. The end-user package consists of the following files: install.bat, FA.exe, FA_ pkg.exe, and readme.txt. The text file provides details about the installation of the application. If the user has MCR (v.7.13 or higher) already installed on the system, then the FA.exe file should be run first to start the application; otherwise, for complete installation (including the required MCR), the install.bat file needs to be run first. To let the user be able to modify the algorithm (MATLAB script), the structure of the program is described in the following sections.

Multifractal Computation Module

MCM contains functionalities for performing correlationintegral-based multifractal analysis and saving the results in an organized format. (E) Figure S1 shows a screenshot of this module, providing information about the format of the input file. The input file required for MCM is a fixed seven-column format without any headers. The file extension is .dat. The seven columns are year (yyyy), month (mm), day (dd), hours, minutes, latitude, and longitude. Once the input file is supplied, this module provides the flexibility of choosing the number of data points to be kept in each cluster for D_q computation. Figure 2 shows a flowchart describing the process of computing the D_q values from the provided input. The main steps for multifractal analysis are described in the following sections.

Input Data

As mentioned above, *FractalAnalyzer* is very specific with the format of the input data. The data needs to be arranged in a fixed seven-column format without any headers and the file name should contain a .dat extension.

Choose the Window Size

The data need to be divided into several segments based on the length of the window. The multifractal dimension is then calculated for each of the segments independently. Further, to understand the stress accumulation, the fractal dimension values from all the segments are grouped together and on a temporal scale. The length of the window should neither be very large nor too small. The optimum length of the window is 50–75 events.



Figure 2. Flowchart describing the process of computation of the D_a values from the provided seismicity data input.

If the number of events within each segment exceeds 100, it is expected that the shorter patterns will be missed out (Roy and Mondal, 2012a). For windows with few events, it is difficult to obtain a reliable linear fit for the $\log r - \log C(r)$ graph; in this case, statistical invalidity would be prominent due to lack of data.

Determine r-Values

The distance (r_{ij}) between any two points of a set is calculated as stated above and operated with r, defining the space containing a fraction of points. The result is characterized with the Heaviside function and summed over all the possible pairs of points that lie within a defined window. The MATLAB code for determination of r-values is contained within the distance.m file, and it forms the integral part of the *FractalAnalyzer* package.

Calculating the Correlation Integrals

FractalAnalyzer uses a correlation integral approach to determine the multifractal correlation dimension for a given sequence of seismic events. The cumulative correlation function is used to measure the distribution as shown by equation (8) (Grassberger and Procaccia, 1983a; Roy and Ram, 2006). Here, N is used as $({}^{N}C_{2})$ for each window, as stated above in the Correlation Dimension section. Using the *r*-values and the Heaviside function value, the corresponding correlation integrals are calculated.

Regression Analysis for D_q

The correlation integral (Cq(r)) values are obtained, these data values are plotted with respect to r. Both, Cq(r) and r are plotted on a logarithmic scale. D_q is obtained from a log-log plot corresponding to the linear portion as stated previously in the Generalized Dimension section. A minimum of 51% continuous points should be chosen to perform proper multifractal analysis, otherwise we would have unreliable estimate (Oncel and Wilson, 2002).

FractalAnalyzer uses the plotting functions of MATLAB to fit the points on a line, such that the fitness function or regression coefficient has a value > 0.99. In the region of active seismicity, the minimum values of 0.98 are also acceptable (Oncel and Wilson, 2002). If the misfit is more, then the results obtained from multifractal analysis will be ambiguous. For further development purposes, the subroutine distance.m can be used.

Grapher Module

GM is the secondary module of *FractalAnalyzer*, which provides the flexibility to view the several plots associated with the multifractal analysis procedure. This helps in immediate analysis of the results without the need to switch to any additional software. The module allows viewing the following plots: $\log r - \log C(r)$, $D_q - q$, and $D_q - t$ (Fig. 3b).

$\log r - \log C(r)$ Plot

This is a plot of the cumulative correlation function C(r) versus the maximum allowable separation between two points r. C(r) is defined as described in the Correlation Dimension section, mainly obtained with the controlling parameter r. As discussed in the Correlation Dimension section, when C(r) and r are plotted on logarithmic scales to obtain D_c , they show a linear relationship with slight deviations. Significant deviation from the above said value in the Regression Analysis for D_q section will lead to ambiguity, which is shown in \mathbb{E} Tables S1 and S2.

Once MCM is executed, the entire study dataset is divided into several sets with *n* number of events contained in each set. MCM allows the user to change the value of *n* (default 100) as per the requirements, but it should not be < 30. The data (log *r* and log C(r)) are saved in the folder corresponding to each cluster. For example, the output for the third set of 50 events for $D_q = 2$ is saved in log_files_50_3\correlation/Cqr_2.dat. This data file can be loaded in GM to view the plot. (E) Fig-



▲ **Figure 3.** Using the *FractalAnalyzer* application, the multifractal dimensions D_q as a function of time are plotted for the spatial distribution of seismic events ($m_b \ge 3.5$) (a) before and (b) after the Kashmir earthquake 8 October 2005 (M_w 7.7).

ure S3 shows the graphical display of the *FractalAnalyzer* application in GM mode using the plot of $\log r$ versus $\log C(r)$. The MATLAB code for this subroutine is located in Fractal-Analyzer.m.

$D_q - q$ Plot

In general, D_q is defined for all real values of *q*. FractalAnalyzer calculates D_q for q = 2, 3, 4, ..., 22. For values >20, D_q is found to approach zero and is thus restricted to 22. Thus, the validation of a fractal system can be done using the $D_q - q$ plot to see the trend of monotonically decreasing D_q with q, as stated in the Fractals section.

Again, in the case in which the MCM is executed, D_q value is computed for q = 2, 3, 4, ..., 22 and is saved in a file with .dqq extension in the folder corresponding to each cluster. For example, the output for the third cluster of 50 events is saved inlog_files_50_3\DQVsQ.dqq. Further, this data file can be loaded directly in GM mode to view the plot. E Figure S4 shows *FractalAnalyzer* in GM mode. An interesting point is observed: the value of D_q decreases monotonically with increase in the value of q and approaches 0 for higher q values. To use this function as a subroutine for further development purpose, we suggest using the computeDQVsQ.m function. This file contains the MATLAB implementation for plotting the D_q values with respect toq.

$D_q - t$ Plot

The plot of D_q versus time (t) is studied to identify the region that leads to low generalized fractal dimension (D_q) values. Intense clustering leads to low D_q values. As the events get clustered, their limit tends toward a point, that is, they are approaching dimension 0. Hirata *et al.* (1987) demonstrated this with rock sample microfracturing in a laboratory experiment. Thus, the D_q versus *t* plot helps analyze the stress state of the regional crust. This can help in better hazard mitigation of the study region and its adjoining areas.

When *FractalAnalyzer* is run in MCM mode, the D_q value is computed for each cluster of seismic events belonging to the study region and is grouped in a single file in the DQ folder with. dqv extension. For example, for q = 2, the output will be saved in DQ/DQ_2.dqv. The MATLAB code for this functionality is saved in computeDC.m.

APPLICATION

To test the capability and effectiveness of the *FractalAnalyzer* application and to test efficiency of the correlation integral method, we use of two sets of seismic events, one before the strong 8 October 2005 Kashmir earthquake (M_w 7.7), and the other immediately after it.

Data

For study purpose, we use the earthquake datasets for the region bounded by 32° and 36° N latitudes and 71° and 75° E longitudes. A total dataset of ~1300 events (1974–2012) is obtained from the U.S. Geological Survey (USGS) Preliminary Determined Epicenter database (body-wave magnitude $m_b \ge 3.5$). The whole data are further divided into two sections: the events before the 2005 Kashmir earthquake and the events after it. The two subsets are arranged in a seven-column format so as to match the input file criteria of *FractalAnalyzer*.

Results

FractalAnalyzer is used to study the multifractal distribution of all events with $m_b \ge 3.5$ that occurred in the northwest Himalayan region during 1974–2012. From the $D_q - t$ plot, it is observed that the D_q value fluctuates with time (Fig. 3a,b). D_q values are plotted against mean time of each 50-event window for consecutive periods to study the variation of spatial-generalized fractal dimension with time. The correlation integral approach is used for the entire analysis as stated above in earlier section.

Generalized Fractal Dimension

The nature of slopes of the linear fits for the $\log C_q(r) - \log r$ plots obtained for all the 20 consecutive time windows prior to the 8 October 2005 earthquake of M_w 7.7 appears to be of similar nature. Hence, we show only one such window for demonstration purpose (© Fig. S3). The range of r for which the corresponding plot of $\log C_q(r)$ versus $\log r$ is a straight line is an indication of the range over which a fractal model holds true. D_q can be obtained from this linear portion of the plot. The D_q values obtained for different values of q are shown in (E) Figure S4, and the curve is known as the D_q spectrum. It characterizes the multifractal or heterogeneous fractal for the spatial distribution of events. The multifractal nature suggests the events form subclusters within a cluster of the fractal structure.

The decrease and then increase in the value of D_q with respect to the spatial distribution of seismic events indicates clustering and dispersion in multifractal structure. In (\bigcirc Figures S5 and 3b, the variation of D_q with respect to time has a nature similar to those of the results published in the past (Hirata *et al.*, 1987; Nakaya and Hashimoto, 2002; Roy and Nath, 2007; Roy and Padhi, 2007). The fluctuation is due to strain accumulation and liberation around the stressed zone.

Correlation Fractal Dimension

 D_c can be obtained from the linear portion of the log C(r)versus log *r* plot (Fig. 1). 🗊 Figure S6 shows events prior to the 8 October 2005 Kashmir earthquake lying within the study area. Low D_c values of 1.28 and 0.92 are observed for windows with mean time 2003.25 and 2005.083 years, respectively. The clustering of the aftershocks of the 2005 Kashmir earthquake is analyzed for the study area, and the low D_c values can be observed in (E) Figure S7. The technique of using 50 events for each window provides a precursor for the 2005 Kashmir earthquake (Roy and Nath, 2007; Roy and Padhi, 2007). Here, we define precursor as relative low D_c value for intermediate size events set, which excludes aftershocks. This precursor is an indicator of strain accumulation (Hirata et al., 1987; Nakaya and Hashimoto, 2002). Again we may state that clustering of intermediate size seismicity helps to identify crustal deformation on a regional scale (Oncel and Wilson, 2006), which possibly contributes to large earthquake preparation in selforganized mode (Bak and Tang, 1989; Bak et al., 1994; Al-Kindy and Main, 2003). Even Jaume and Sykes (1999) state that combined observational and simulation evidence indicates the period of increased moment release in moderate earthquake signals establishes long-wavelength correlation in the regional stress field. However, we would like to add that the above precursors cannot be classified as classical, because an exhaustive search for such precursors was not carried out and we do not know how common such an extreme D_c value is or how good D_c would be at predicting individual earthquakes.

Discussions

The main Himalayan thrust behavior within the western syntaxis of the range is poorly understood. Some of the surface ruptures have been mapped. The exact locations of active faults are ambiguous, and a few paleoseismological studies have also been completed. Tectonic processes generally activate the fault system, in which strain accumulation yields highly stressed zone. The rupture may nucleate from those stressed zones, accounting for most of the high-frequency seismic energy radiation and eventually causing a large earthquake (© Fig. S8). The stressed zones control the distribution of earthquakes over a fault zone that triggers repeated earthquakes, as controlled by fault surface heterogeneities. Such a stress trigger has been reported for the adjoining Himalayan zone by, for example, the coloumb stress transfer approach (Gupta *et al.*, 2015). Interestingly, these zones possess different physical states and properties and hence can remain difficult to map by standard geophysical techniques. Imaging this intriguing nature of the subelements of the megathrusts is a challenge for geophysicists.

Analysis reveals significant variation in the multifractal properties of seismicity between the tectonic subdivisions of the area under study. Differences between D_2 and D_{22} (as shown in Fig. 3a,b) are related to the differences in the tendency for seismicity to be clustered or dispersed at different scales. Hence, the differences between the multifractal dimensions D_2 and D_{22} are interpreted to result from fractal heterogeneity between regional and local scales, respectively (Oncel and Wilson, 2006). Changes between the fractal dimension D_c and multifractal (q = 2-22) measures illustrate the sensitivity of the multifractal characterization changes in the local complexity. The large difference between D_2 and D_{22} implicates the presence of significant fractal heterogeneity within the hypocenter distribution of shallow seismicity. This is due to the differences in fault complexity at local scales (i.e., q = 15, 16, ..., 22). With the help of multifractal analysis, the fractal properties of complex fault systems can be more suitably characterized.

In addition to the precursor for the 2005 Kashmir earthquake, several other low- D_C -value zones exist. Some of these low D_c values can be explained by the occurrence of events of the order of magnitude of 6. The rest of these are the precursors to the 2005 Kashmir earthquake, as mentioned in (E) Table S1.

CONCLUSIONS

FractalAnalyzer is a simple, versatile tool for analyzing a wellconstrained catalog for seismically active regions to demonstrate a quantitative representation of seismicity. Here, we could have better spatiotemporal understanding of seismicity and its implication on physical understanding. FractalAnalyzer is also capable of identifying the seismicity clustering based on low D_c or D_q values. To demonstrate the effectiveness of this tool, we used it to study the 2005 Kashmir earthquake. The relative clustering is considered to be the set of all possible unique vector pairs formed by two elements each time from the fractal set. The modified conventional correlation integral approach has given added benefit in this multifractal analysis of the intermediatesize events prior to the major 8 October 2005 Kashmir earthquake. Consequently these patterns are totally in agreement, which may serve as a precursor for this major earthquake. Further, we use this tool to study the aftershocks. The results presented here demonstrate the capability of the multifractal method, as well as the *FractalAnalyzer* application. Moreover, the multifractal analysis approach gave a view of heterogeneity of the crust leading to such complex seismicity. The implementation of this method in a graphical interface for user-friendly interactive environment makes FractalAnalyzer a strong tool for studying the earthquake patterns. To be specific, FractalAnalyzer and its importance can be judged as a viable technique, given

quantitative spatiotemporal distributions for numerical warning rather than earthquake prediction. Thus, this reproducible numerical precursor prior to major earthquakes might help in improving hazard mitigation and, therefore, also in disaster management for other seismically active regions having past event episodes. Thus *FractalAnalyzer* may play a key role in analyzing strong earthquake preparation within a short span by using a well-constrained catalog of seismically active zones.

DATA AND RESOURCES

FractalAnalyzer and its recent updates are available to download from http://sourceforge.net/projects/fractalanalyzer/ (last accessed July 2015). **★**

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REFERENCES

- Aki, K. (1981). A probabilistic synthesis of precursory phenomena, in *Earthquake Prediction, Maurice Ewing Series*, D. W. Simpson and P. G. Richards (Editors) American Geophysical Union, Washington, D.C., 556–574.
- Aki, K. (1984). Asperities, barriers, characteristic earthquakes and strong motion prediction, J. Geophys. Res. 89, 5867–5872.
- Al-Kindy, F. H., and I. G. Main (2003). Testing self-organized criticality in the crust using entropy: A regionalized study of CMT global earthquake catalogue, *J. Geophys. Res.* 108, no. B11, 2521, ESE 5-1–ESE 5-9, doi: 10.1029/2002JB002230.
- Bak, P., and C. Tang (1989). Earthquakes as self-organized critical phenomena, J. Geophys. Res. 94, 15,635–15,637.
- Bak, P., K. Christensen, and Z. Olami (1994). Self-organized criticality: Consequences for statistics and predictability of earthquakes, in *Nonlinear Dynamics and Predictability of Geophysical Phenomena*, W. I. Newman, A. Gabrielov, and D. L. Turcotte (Editors), American Geophysical Monograph 83, 69–74.
- Bunde, A., S. Havlin, and H. E. Roman (1990). Multifractal features of random walks on random fractals, *Phys. Rev. A* 42, 6274-6277.
- Chen, K., P. Bak, and S. P. Obukhov (1991). Self-organized criticality in a crack-propagation model of earthquakes, *Phys. Rev. A* **43**, 625–630.
- De Natale, G., F. Musumeci, and A. Zollo (1988). A linear intensity model to investigate the causal relation between Calabrian and

North-Aegean earthquake sequences, *Geophys. J. Roy. Astron. Soc.* **95**, 285–293.

- Evison, F. F. (2001). Long-range synoptic earthquake forecasting: An aim for the millennium, *Tectonophysics* 338, 207–215.
- Godano, C., and V. Caruso (1995). Multifractal analysis of earthquake catalogues, *Geophys. J. Int.* **121**, 385–392.
- Guo, Z. Q., and Y. Ogata (1997). Statistical relations between the parameters of aftershocks in time, space, and magnitude, *J. Geophys. Res.* 102, 2857–2873.
- Gupta, D., D. Bhowmick, and P. N. S. Roy (2015). Himalayan hazard study on the basis of stress and strain state of 1991 Uttarkashi earthquake using coulomb stress transfer model, *J. Geomat. Nat. Hazards Risk* 6, no. 2, 131–148.
- Grassberger, P., and I. Procaccia (1983a). Measuring the strangeness of strange attractors, *Phys. Nonlinear Phenom.* 9, nos. 1/2, 189– 208, doi: 10.1016/0167-2789(83)90298-1.
- Grassberger, P., and I. Procaccia (1983b). Characterizations of stranger attractors, *Phys. Rev. Lett.* **50**, 346–349.
- Halsey, T. C., M. H. Jenson, L. P. Kadanoff, I. Procaccia, and B. I. Shraiman (1986). Fractal measure and their singularities: The characterization of strange sets, *Phys. Rev. A* 3, no. 2, 1141–1151.
- Hentschel, H. G. E., and I. Procaccia (1983). The infinite number of generalized dimensions of fractals and strange attractors, *Physica D* 8, no. 3, 435–444.
- Hirabayashi, T., K. Ito, and T. Yoshii (1992). Multifractal analysis of earthquakes, *Pure Appl. Geophys.* 138, no. 4, 591-610.
- Hirata, T. (1989). A correlation between the *b* value and the fractal dimension of earthquakes, *J. Geophys. Res.* 94, 7507–7514.
- Hirata, T., T. Satoh, and K. Ito (1987). Fractal structure of spatial distribution of microfracturing in rocks, *Geophys. J. Roy. Astron. Soc.* 90, 360–374.
- Huang, J., and D. L. Turcotte (1990). Are earthquakes an example of deterministic chaos? *Geophys. Res. Lett.* 17, 223–226.
- Jaume, S. C., and L. R. Sykes (1999). Evolving towards a critical point: A review of accelerating seismic moment/energy release prior to large and great earthquakes, *Pure Appl. Geophys.* 155, 279–306.
- Kagan, Y., and L. Knopoff (1980). Spatial distribution of earthquakes: The two point correlation function, *Geophys. J. Roy. Astron. Soc.* 62, 303–320.
- Kagan, Y., and D. Jackson (1991). Long-term earthquake clustering, Geophys. J. Int. 104, 117–133.
- King, G. (1983). The accommodation of large strains in the upper lithosphere of the Earth and other solids by self-similar fault system: The geometrical origin of *b*-value, *Pure Appl. Geophys.* **121**, 761–815.
- Legrand, D., A. Cisternas, and L. Dorbath (1996). Multifractal analysis of the 1992 Erzincan aftershock sequence, *Geophys. Res. Lett.* 23, no. 9, 933–936.
- Li, Q., and G. M. Xu (2012). Characteristic variation of local scaling property before Puer M 6. 4 earthquake in China: The presence of a new pattern of nonlinear behavior of seismicity, *Izvestiya Phys. Solid Earth* 48, no. 2, 155–161.
- Mandal, P., A. O. Mabawonku, and V. P. Dimri (2005). Self-organized fractal seismicity of reservoir triggered earthquakes in the Koyna-Warna seismic zone, western India, *Pure Appl. Geophys.* 162, 73–90.
- Mandelbrot, B. B. (1977). *Fractals: Form, Chance, and Dimension*, W.H. Freeman and Company, San Francisco, California.
- Mandelbrot, B. B. (1989). Multifractal measures, especially for the geophysicist, Pure Appl. Geophys. 131, 5–42.
- Nakaya, S., and T. Hashimoto (2002). Temporal variation of multifractal properties of seismicity in the region affected by the mainshock of the October 6, 2000 Western Tottori Prefecture, Japan, earthquake (M = 7.3), *Geophys. Res. Lett.* **29**, 133-1–133-4.
- Neuman, S. P. (2010). Apparent/spurious multifractality of data sampled from fractional Brownian/Lévy motions, *Hydrol. Process.* 24, no. 15, 2056–2067.
- Ogata, Y., H. Akaike, and K. Katsura (1982). The application of linear intensity model to the investigation of casual relation between a

point process and another stochastic process, *Ann. Inst. Stat. Math.* **34,** 373–387.

- Oncel, A. O., and T. H. Wilson (2002). Space-time correlations of seismotectonic parameters: Example from Japan and from Turkey preceding the İzmit earthquake, *Bull. Seismol. Soc. Am.* 92, 339–349.
- Oncel, A. O., and T. H. Wilson (2004). Correlation of seismotectonic variables and GPS strain measurements in western Turkey, J. Geophys. Res. 109, no. B11, 306.
- Oncel, A. O., and T. H. Wilson (2006). Evaluation of earthquake potential along the northern Anatolian fault zone in the Marmara Sea using comparisons of GPS strain and seismotectonics parameters, *Tectonophysics* 418, 205–218.
- Molchan, G., and T. Kronrod (2007). Seismic interevent time: A spatial scaling and multifractality, *Pure Appl. Geophys.* 164, 75–96, doi: 10.1007/s00024-006-0150-y.
- Pawelzik, K., and H. G. Schuster (1987). Generalized dimensions and entropies from a measured time series, *Phys. Rev.* 35, 481-484.
- Ripley, B. D. (1988). *Statistical Inference for Spatial Process*, Cambridge University Press, Cambridge, United Kingdom, 148 pp.
- Roy, P. N. S., and S. K. Mondal (2009). Fractal nature of earthquake occurrence in northwest Himalayan region, *J. Indian Geophys. Union* 13, no. 2, 63–68.
- Roy, P. N. S., and S. K. Mondal (2012a). Multifractal analysis of earthquakes in Kumaun Himalaya and its surrounding region, *J. Earth Syst. Sci.* 121, no. 4, 1033–1047.
- Roy, P. N. S., and S. K. Mondal (2012b). Identification of active seismicity by fractal analysis for understanding the recent geodynamics of central Himalaya, *J. Geol. Soc. India* 79, 353–360.
- Roy, P. N. S., and S. K. Nath (2007). Precursory correlation dimensions for three great earthquakes, *Current Science* 93, no. 11, 1522–1529.
- Roy, P. N. S., and A. Padhi (2007). Multifractal analysis of earthquakes in the southeastern Iran-Bam region, *Pure Appl. Geophys.* 167, 2271–2290.
- Roy, P. N. S., and A. Ram (2006). A correlation integral approach to the study of 26 January 2001 Bhuj earthquake, Gujarat, India, J. Geodynam. 41, 385–399.

- Sadovskiy, M. A., T. V. Golubeva, V. F. Pisarenko, and M. G. Shnirman (1985). Characteristic dimensions of rock and hierarchical properties of seismicity, *Phys. Solid Earth* 20, 87–95.
- Shlien, S., and M. N. Toksoz (1970). A clustering model for earthquakes occurrence, Bull. Seismol. Soc. Am. 67, 173–185.
- Smalley, R. F., J. L. Chatelain, D. L. Turcotte, and R. Prévot (1987). A fractal approach to the clustering of earthquakes: Application to the seismicity of the New Hebrides, *Bull. Seismol. Soc. Am.* 77, 1368–1381.
- Stoyan, D., and H. Stoyan (1994). Fractals, Random Shapes, and Point Fields: Method of Geometrical Statistics, Wiley, New York, New York, 389 pp.
- Takayasu, H. (1990). *Fractals in the Physical Sciences*, Manchester University Press, Manchester, United Kingdom.
- Telesca, L., V. Cuomo, M. Lanfredi, V. Lapenna, and M. Macchiato (1999). Investigating clustering structures in time-occurrence sequences of seismic events observed in the Irpinia-Basilicata region (southern Italy), *Fractals* 7, 221–234.
- Vere-Jones, D. (1970). Stochastic models for earthquakes occurrence, J. Roy. Stat. Soc. B32, 321–345.

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