

Department of Precision and Microsystems Engineering

Investigation of the nonlinear dynamics of a diamagnetically levitating plate

T.de Lint

Report no : 2021.052
Coach : Prof. Dr. P. G. Steeneken
Professor : Prof. Dr. P. G. Steeneken
Specialisation : Dynamics of Micro and Nanosystems
Type of report : Master Thesis
Date : 20 August 2021

Investigation of the nonlinear dy- namics of a dia- magnetically lev- itating plate

Tjebbe de Lint

Investigation of the nonlinear dynamics of a diamagnetically levitating plate

by

Tjebbe de Lint

to obtain the degree of Master of Science
at the Delft University of Technology,
to be defended publicly on Friday August 20, 2021 at 10:00 AM.

Student number:	4387538
Project duration:	April 23 rd , 2020 – August 20 th , 2021
Thesis committee:	Prof. dr. P.G. Steeneken, TU Delft, senior supervisor
	Dr. F. Alijani, TU Delft, senior supervisor
	X. Chen, TU Delft, daily supervisor
	Dr. T. Manzaneque García TU Delft, external committee member

This thesis is confidential and cannot be made public until August 20 , 2022.

An electronic version of this thesis is available at <http://repository.tudelft.nl/>.

Abstract

Over the past decades the diamagnetic effect has been used for a variety of applications. This ranges from levitating a living frog to stabilizing a force sensor, designing an acceleration sensor and determining the power of a laser. Diamagnetic levitation is the only method in which a passive stable levitation of an object can be achieved. Such a stable levitation has the benefits that the levitated object is isolated from its environment, thus eliminating mechanical damping and friction. Furthermore, no energy or control mechanisms are needed to sustain the levitation and to keep the object in its place. With the decrease in the damping, due to the elimination of the mechanical damping component, potential nonlinearities in the movement of the plate can present themselves. Such nonlinearities can be exploited in, for example, energy harvesters. In such energy harvesters, the nonlinearities can be used to increase the range of frequencies at which energy can be harvested. As both diamagnetism and nonlinearities have their benefits, this study aims at investigating the nonlinear behaviour of a diamagnetically levitating plate in the vertical direction. An analytical model has been constructed which aimed at predicting the (non)linear behaviour of a pyrolytic graphite plate, levitating over a 2x2 array of permanent magnets, which is actuated electrostatically. The model is verified numerically using COMSOL and has been used to predict both the static - and dynamic behaviour of the plate. Experiments have been conducted to assess the validity of the analytical predictions. It is found that the static behaviour and the linear dynamic behaviour of the plate can be predicted analytically within certain margins. The analytical model indicates that all combinations of the magnetic force and electrostatic forces investigated have a softening effect on the levitating plate. A softening response is also observed in experiments. However, the analytical model cannot perfectly predict the experimental frequency response functions. A possible reasons for this discrepancy might be mode coupling. Overall, the study of the nonlinear dynamics for a diamagnetically levitating plate will help to gain more understanding on its fundamental mechanism and thus pave a way for its wider applications.

Keywords: Diamagnetic Levitation | Nonlinear Dynamics | Stable levitation | Softening behaviour | Rigid body modes | Experimental frequency response function

Preface

Dear reader,

At the start of my thesis in April 2020 I looked upon the coming year with a mix of enthusiasm and determination as before me lay one of the biggest challenges I had to face thus far. Before me lay my master thesis, the crown jewel of my time as a student, in which all my obtained knowledge of the past 7 years needed to culminate in one single research.

Now, more than a year later, I can say that it has been a journey with lots of ups and downs. It has been a year with, most of all, a lot of learning. Next to a lot of technological knowledge I have learned how to structure a year long-project and divide it into intermediate milestones. These have been valuable learnings that I will definitely use in my future career.

During this thesis I have had help from many people. First of all, I want to extend my gratitude to Peter, Farbod and Xianfeng for their guidance and counsel during this thesis. The interesting discussions that we had guided me further in the research and lay as the foundation for several interesting ideas. Also I want to thank all my friends who have supported me during this thesis. A special thanks goes to my roommates, who have been an enormous help. The late night walks, conversations about the structure of the thesis and critical questions have helped me in producing the current research. Furthermore I want to thank my girlfriend. During each breakthrough and setback you have always been there for me to discuss it.

Lastly, and most importantly, I want to thank my parents and brothers. The continuous support and live lessons you have taught me have enabled me to do this thesis in the first place. For that, I am very grateful.

I have put my heart and soul into this research. The result lays before you. I hope you enjoy it.

*Tjebbe de Lint
Delft, August 2021*

Contents

Abstract	iii
1 Introduction	1
1.1 Thesis outline	2
1.2 The diamagnetic effect and its applications	3
1.2.1 Diamagnetism: how does it work?	3
1.2.2 Applications of the diamagnetic effect	4
1.3 Investigation of the dynamics of a diamagnetically levitating plate	8
1.4 Research goal	10
2 Theoretical background	11
2.1 The setup	11
2.2 The magnetic force on a diamagnetically levitating plate	12
2.2.1 Basic equations	12
2.2.2 Magnetic field strength around a 2x2 array of magnets	13
2.2.3 The magnetic force on a diamagnetically levitating plate	15
2.3 Electrostatic force on a diamagnetically levitating plate	16
2.4 Dynamical responses	17
2.4.1 Duffing oscillator	17
2.4.2 Systems with quadratic - and cubic nonlinearity	18
3 COMSOL models	21
3.1 Magnetic force on a diamagnetically levitating plate	21
3.2 Electrostatic force on a diamagnetically levitating plate	24
4 Numerical verification of the analytical forces	27
4.1 The magnetic force on a diamagnetically levitating plate	27
4.2 The electrostatic force on a diamagnetically levitating plate	28
5 Experimental setups	29
5.1 Setup of the magnets, the plate and the electrodes	29
5.2 Setup for the stable levitation height measurements	30
5.3 Setups for investigating the (non)linear dynamics of a diamagnetically levitating plate	32
6 Characterisation of the static behaviour of the diamagnetically levitating plate	37
6.1 Construction of the equation of motion for the diamagnetically levitating plate	37
6.2 Analytical calculations	38
6.3 Experimental procedure	38
6.4 Results of the stable levitation height for various DC-voltages	39
6.5 Discussion of the results	40
6.6 Concluding remarks	40
7 Characterising the linear dynamic behaviour of the diamagnetically levitating plate	41
7.1 The eigenmodes	41
7.2 Analytical calculations	43
7.3 Experimental details	44
7.4 Results of the linear eigenfrequency for various DC-voltages	44
7.5 Discussion of the results	45
7.6 Concluding remarks	45

8	Investigating the nonlinear dynamic behaviour of the diamagnetically levitating plate	47
8.1	Analytical calculations	47
8.2	Results for the analytically calculated stiffnesses and expected softening or hardening from the analytical model.	48
8.3	Discussion of the analytically calculated stiffnesses and expected softening - or hardening behaviour	50
8.4	Experimental measurements	51
8.5	Frequency response functions obtained from experiments.	51
8.6	Discussion of the experimental results	54
8.7	Results of comparing the analytical - and the experimental frequency response functions.	54
8.8	Discussion of the comparison of the analytical - and experimental frequency response functions	56
8.9	Concluding remarks	56
9	Conclusions	57
10	Recommendations	59
10.1	Investigation of the eigenmodes at driving frequencies close to 15.5 Hz	59
10.2	Extend the model such that it can calculate both the translational - and rotational stiffnesses	59
10.3	Investigate the eigenmode in x' -direction analytically and experimentally	59
10.4	Investigate the rotational eigenmode around the y' -axis analytically and experimentally	60
10.5	Put the full electrostatic force formula into the equation of motion and solve it to obtain the analytical frequency response function	60
A	Nondimensionalisation of the nonlinear equation of motion	61
B	High Voltage Amplifier and its low voltage connection scheme	63
C	Matlab code for finding the analytical frequency response function	65
D	Code of the analytical (mathematica) model	69

List of Figures

1.1	Image of a flying frog above a very strong magnet [10].	3
1.2	Force sensor setup designed by Abadie et al. [4].	5
1.3	Acceleration sensor from Barrot et al. [5].	5
1.4	Laser sensor setup from Pinot et al. [6].	6
1.5	Energy harvester setup by Palagummi et al. using diamagnetic plates [13].	6
1.6	The vertical energy harvester investigated by Wang et al. [19].	7
1.7	Measurement setup by Chen et al. to investigate the rigid body dynamics of a diamagnetically levitating plate [2].	8
1.8	Setup to investigate the frequency response of a diamagnetically levitating plate by DePasquale et al. [27].	9
1.9	Resulting frequency responses found by DePasquale et al. for a diamagnetically levitating plate [27].	9
2.1	The top - and 45 degree side view of the setup that is investigated. The coordinate system is indicated in both figures. The origin of the plate is at the exact center of the top planes of the array of magnets.	12
2.2	Force displacement curves for a linear -, hardening - and softening spring.	17
2.3	Frequency response curves for a Duffing equation of motion. A linear -, hardening - and softening behaviour are shown.	18
3.1	Isometric view of the setup used to calculate the magnetic force with COMSOL.	22
3.2	The meshed setup used to calculate the magnetic force with COMSOL.	23
3.3	The magnetic force as a function of the levitation height found numerically.	23
3.4	The meshed setup for calculating the electrostatic force using COMSOL.	25
3.5	The COMSOL electrostatic force as a function of the voltage applied and the levitation height.	25
4.1	Comparison of the analytical- and numerical magnetic force on the diamagnetically levitating plate.	27
4.2	Comparison of the analytical - and numerical electrostatic force on the diamagnetically levitating plate.	28
5.1	The schematic top - and 45 degree side view of the setup used for the investigation.	29
5.2	The real life top - and side view of the setup used for the investigation.	30
5.3	The schematic setup of the static experiment.	31
5.4	The static experimental setup.	31
5.5	The schematic general setup of the respective dynamic experiments.	32
5.6	Input - and output devices for finding the eigenmodes.	33
5.7	Input - and output devices for finding the eigenmodes in reality.	33
5.8	Input - and output devices for finding the shift in the linear eigenfrequency.	34
5.9	Input - and output devices for finding the shift in the linear eigenfrequency in real life.	35
6.1	Measurement areas for the static experiments.	38
6.2	Comparison of the levitation heights calculated analytically and found experimentally.	39
6.3	2^{nd} comparison of the levitation heights calculated analytically and found experimentally.	39
7.1	The measurement points used for identifying the eigenmodes.	42
7.2	The primed coordinate system.	42
7.3	Two eigenmodes of the levitating plate.	43

7.4	Linear eigenfrequency comparison between the results from the analytical model and the results from the experiments.	44
8.1	Linear stiffness values for varying DC-voltages as calculated by the analytical model. . .	49
8.2	Quadratic stiffness values for varying DC-voltages as calculated by the analytical model. . .	49
8.3	Cubic stiffness values for varying DC-voltages as calculated by the analytical model. . .	49
8.4	The values of the left- and right side of the analytical condition shown in Equation 2.39. . .	50
8.5	Points measured to investigate whether other modes are excited at frequencies between 15 and 16 Hz.	51
8.6	Experimental frequency response function found for a DC-voltage of - 200 V and varying AC-voltages.	52
8.7	Experimental frequency response function found for a DC-voltage of - 500 V and varying AC-voltages.	52
8.8	Experimental frequency response function found for a DC-voltage of - 700 V and varying AC-voltages.	52
8.9	Experimental frequency response function found for an AC-voltage of 800 V and varying DC-voltages.	53
8.10	Experimental frequency response functions found for multiple points on the plate.	53
8.11	The comparison between the analytical - and experimental frequency response function for a DC-voltage of -700 V and an AC-voltage of 800 V.	55
8.12	The comparison between the analytical - and experimental frequency response function for a DC-voltage of -700 V and an AC-voltage of 850 V.	55
B.1	The used High Voltage Amplifier (500x).	63
B.2	Connection diagram of the High Voltage Amplifier.	64

Introduction

Levitation is something worth striving for. At a large scale we are now able to levitate or fly by means of an airplane, helicopter, hot air balloon or maglev train due to enormous technological advancements over the past 100 years. At this scale, levitation is mainly used for transporting goods and people. Where levitation is beneficial at a large scale it also has its benefits on the small scale. At such a scale, levitation is used to eliminate the mechanical damping, resulting in a higher Q-factor. Therefore, it can be used to increase the sensitivity of sensors by offering a higher signal-to-noise ratio.

Although levitation can be achieved through various principles at the small scale, the only way in which a stable passive levitation can be achieved is through magnetic levitation [1]. More specifically, such a stable passive levitation can be achieved using diamagnetism [2]. Diamagnetic materials are materials which are repelled by any magnetic field [3]. If a proper configuration of the magnetic field is chosen, a diamagnetic material can achieve a stable passive levitation. For such a stable passive levitation no energy is needed to sustain the levitation and no control mechanism are necessary to keep the levitating object in its place.

The diamagnetic effect, which causes the repelling of a diamagnetic object by a magnetic field, has already been exploited for several sensing applications. It has been used in stabilizing a force sensor [4], levitating an acceleration sensor [5] and determining the power of a laser [6].

As stated earlier, by means of levitation the mechanical damping between an object and its environment can be eliminated. As a result, potential nonlinearities in the movement of the levitating object will be less damped and might present themselves. This is especially true if the levitating object is influenced by a magnetic field, which is inherently nonlinear. Such potential nonlinearities can be exploited for several purposes. One of these purposes is the extraction of energy from the environment [7]. Here, the nonlinearities could be exploited to increase the bandwidth over which energy can be harvested from its environment [7].

Within this framework of diamagnetism and nonlinear dynamics, the aim of this research is to investigate the nonlinear dynamics of a diamagnetically levitating plate in the vertical direction. To do so, an analytical model has been constructed to make predictions for both the static - and (non)linear dynamic behaviour of the plate. Furthermore, experiments have been conducted to test the predictions made by the analytical model.

1.1. Thesis outline

This report starts with an introduction to the concept of diamagnetism and the applications of the diamagnetic effect. Furthermore, the existing literature on the dynamics of diamagnetically levitating plates is presented. From the literature presented here, a knowledge gap has been identified. Subsequently, a research objective has been set to fill this knowledge gap.

The second chapter discusses the theoretical background of the forces that are present in the setup which is investigated. First, the magnetic force, due to the magnetic field of a 2x2 array of permanent magnets, acting on a diamagnetically levitating plate will be derived analytically. Secondly, the electrostatic force is explained as this is the actuation method in this setup. Lastly, the dynamic behaviour of an equation of motion for a translational rigid body mode will be explained. These steps are all taken to make analytical predictions for the experimental results.

The third chapter explains the numerical models that have been constructed. These numerical models, which are finite element models, are constructed using COMSOL. The numerical models are constructed to investigate the validity of the analytical models to represent the forces that are present.

After the analytical formulas and numerical models are explained, the results of these models are compared in chapter 4.

Next, chapter 5 describes the measurement setups that have been used to conduct the experiments. Here, a distinction has been made between the experimental setup used to investigate the static behaviour of the plate, a setup used to characterise the linear dynamic behaviour of the plate in the z-direction and the setup used to investigate the nonlinear dynamic behaviour in the z-direction.

The results from the analytical model, constructed in Appendix D, are compared with experimental results in chapters six, seven and eight. In chapter six the results for the stable levitation heights of the plate will be discussed. Subsequently, chapter seven discusses the linear dynamic behaviour of the plate in the z-direction. In chapter eight, the nonlinear dynamic behaviour of the levitating plate in the z-direction will be discussed.

The report finalizes with the overall conclusions of the research and the recommendations for future research.

1.2. The diamagnetic effect and its applications

Diamagnetism was discovered in 1778 by Dutch scientist Anton Brugman, who observed during his experiments that bismuth, unlike other materials, was repelled by either pole of a magnet [8]. Although there have been independent observations of the repelling force of a magnet on specific materials in the first half of the 19th century, a systematic investigation of diamagnetism was firstly done by Michael Faraday in 1846 [8]. He investigated a large variety of substances and eventually divided them in two groups. The first group consisted of substances that are magnetic. These substances are attracted by a magnet and are nowadays called paramagnetic. The second group consisted of substances that are diamagnetic, which means that they are repelled by a magnet [8, 9]. Shortly after in 1847 William Thomson, also known as Lord Kelvin, predicted that diamagnetic materials were able to levitate magnetically [8]. It was also predicted that a stable levitation could be achieved using diamagnetic materials [9, 10]. Even though these prediction were already made in the 19th century it took almost a century before the first stable diamagnetic levitation was achieved. This was done in 1939 by German physicist Werner Braunbek [8]. From the 1950's onward multiple applications of diamagnetic levitation have been shown to be feasible [8]. However, the most notable example of a diamagnetically levitating object is the 'flying frog' by Andrey Geim and his fellow authors. In their experiments, Andrey Geim and his fellow authors managed to levitate a living frog, seen in figure Figure 1.1, in a magnetic field of 16 T [11].

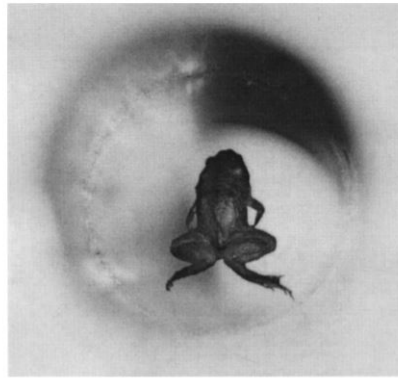


Figure 1.1: Image of a flying frog above a very strong magnet [10].

Nowadays diamagnetic levitation and stabilization is used in sensors [4–6, 12], energy harvesters [13–19] and other applications [20–24]. In subsection 1.2.2 these applications will be discussed.

1.2.1. Diamagnetism: how does it work?

Materials can be divided into three groups depending on their reaction to an external magnetic field. These three groups are: ferromagnetic materials, paramagnetic material and diamagnetic materials [3]. Ferromagnetic and paramagnetic materials are materials that increase the magnetic field when subjected to an external magnetic field. In contrast to ferro- and paramagnetic materials, diamagnetic materials are materials that weaken a magnetic field to which it is subjected [3].

Ferromagnetic - and paramagnetic materials are attracted towards a magnetic field whilst diamagnetic materials are repelled by a magnetic field [3]. This difference is illustrated by the magnetic susceptibilities of the respective materials. To explain how the magnetic susceptibilities influence the magnetic field, Equation 1.1 [3] is used. In this equation, $\vec{B}$ is the magnetic field inside a material, $\vec{B}_{app}$ is the applied magnetic field and χ_m is the magnetic susceptibility of the respective material [3].

$$\vec{B} = \vec{B}_{app}(1 + \chi_m) \quad (1.1)$$

For ferromagnetic materials χ_m is positive and very large, thus yielding a significant strengthening of the external magnetic field. For paramagnetic materials, the values of χ_m are positive and small. For diamagnetic materials, the values of χ_m are negative and small, with as an outlier a superconductor which has a magnetic susceptibility of -1 [3]. When inserting a magnetic susceptibility of -1 into Equa-

tion 1.1 it can be observed that the magnetic field is equal to zero. This shows that there is no resulting magnetic field within a superconductor [3].

The underlying physics of all the magnetic effects of materials can only correctly be explained using quantum-mechanics [25]. Using classical physics, an idea can be obtained on the basic principles, even though it is eventually incorrect. Using classical physics, diamagnetism can be explained as follows: each electron within an atom has an angular momentum $\mathbf{J}$ which is linked to its magnetic moment μ as shown in Equation 1.2 [25]. When subjected to a magnetic field, the angular momentum faces a change which is proportional to the applied magnetic field. As a result, the magnetic moment changes inversely proportional to the magnetic field. The extra magnetic moment induced therefore opposes the applied magnetic field. That is the diamagnetic effect. Equation 1.2 shows this relationship. In this formula μ is the magnetic moment, $\mathbf{J}$ is the angular momentum of an electron, q_e is the absolute charge of an electron and m is the mass of an electron [25].

$$\mu = -\frac{q_e}{2m} \mathbf{J} \quad (1.2)$$

The diamagnetic effect is, according to classical physics, a result of the change in angular momentum of the electrons [25]. Therefore, this effect occurs in all materials. However, it is usually overruled by the paramagnetic - or ferromagnetic effect. Therefore, the diamagnetic effect is usually seen by materials that are neither paramagnetic nor ferromagnetic [3, 25].

1.2.2. Applications of the diamagnetic effect

The diamagnetic effect can be used to achieve a stable levitation without any energy input [10]. This poses a lot of possibilities for its application in different fields, such as sensing applications [4–6, 12] and energy harvesting [13–19]. Here, different applications of diamagnetic levitation are shown and explained. First the use of the diamagnetic effect within sensors is discussed. Next, its use for energy harvesters will be discussed. Lastly, other applications of the diamagnetic effect are discussed.

Sensing

One of the fields in which the diamagnetic effect is already applied is the field of sensing. The diamagnetic effect has already been investigated for its use in force sensors [4, 12], an acceleration sensor [5] and laser power sensor [6].

A force sensor based on the use of a seismic mass was proposed by Abadie, Piat, Oster and Boukallel in 2012 [4]. The benefit of using a seismic mass over a microcantilever, which are more developed, is that a levitating sensor using a seismic mass can be constructed with standard equipment. Its downside is that the bandwidth is low compared to microstructure force sensors [4].

The design proposed by Abadie et al. is shown graphically in Figure 1.2 [4]. In this setup, the pink plates represent the diamagnetic material whilst the other components, which are M1, M1' and M2, are permanent magnets. This is seen at the front of Figure 1.2. As can be seen, the exact same setup is used at the back of the figure. The two levitating magnets of the front - and back configuration are joined through a tube, depicted in green. In this case, the diamagnetic effect is used to create a stability in the y-direction. The permanent magnets M1 and M1' create a stability of the levitating magnet in the xz-plane [4].

This setup can measure a force in the x-direction. A sensor measures the displacement of the tip of the tube that connects the levitating magnets. These displacements are then converted to a force. The forces that can be measured range from the nanonewton scale to the micronewton scale, up to 100 micronewtons. Regarding the resolution of the obtained force, this is dependent on the sensor that is used and on the disturbances from the environment. As an example, one can reach, under specific circumstances, a force resolution of up to 5 nN using a STIL confocal chromatic sensor [4].

A similar force sensor was already built in 2003 by Boukallel, Abadie and Piat [12]. The difference is that not a tube but an epoxy resin is used to connect the levitating magnets. The force is measured in the same way, measuring the displacement first and converting these displacements to a force. This research also looked solely at the force in one horizontal direction but it stated that a force measurement in the z-direction was also possible [12].

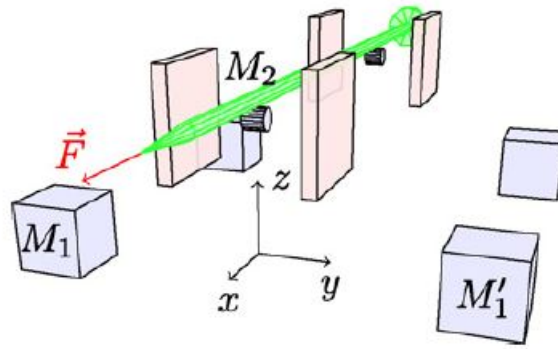


Figure 1.2: The setup designed by Abadie et al. to sense forces in the x-direction [4].

Another type of sensor which can benefit from the diamagnetic effect is an acceleration sensor [5]. Here, the diamagnetic effect is used to isolate the seismic mass from contact with the environment. With the use of both the diamagnetic effect and electrostatic actuators one is able to design acceleration sensors which are highly sensitive. The proposed design for such an actuator is given in Figure 1.3 [5].

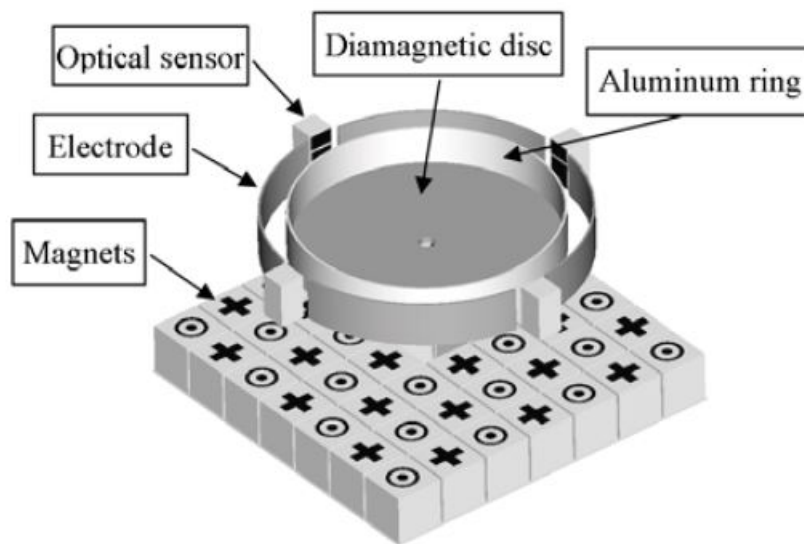


Figure 1.3: The proposed acceleration sensor from Barrot et al. [5].

The electrodes are used to keep the seismic mass at a certain position with respect to the base. The voltage applied is used to keep the seismic mass in this same relative position when there are disturbances. The voltage necessary to keep the seismic mass in the relative position is directly related to the acceleration of said seismic mass [5]. The sensor as shown in Figure 1.3 is able to measure accelerations in two directions in the horizontal plane. The accelerations can be measured with a resolution in the μg range and a maximum acceleration of a $0.5 g$ can be measured. They also propose an alteration with which the acceleration in the vertical direction can be measured as well.

Research has also been done into the possibility to use diamagnetic levitation to sense the power of a laser [6]. This research shows that due to irradiation power the levitating height of a pyrolytic graphite plate can change. This happens for an irradiation power between 20 mW and 1 W. It is shown that this relationship can be used to act as a laser power sensor within this irradiation power range. In this research a special property of carbon graphite is exploited. As a result of heating, the magnetic susceptibility of pyrolytic carbon increases. This in turn causes a larger repelling force to act on the diamagnetic plate. As a result the levitation height of a pyrolytic graphite plate changes when a laser

is pointed at the plate. The difference in levitation height, measured by a laser sensor, is then used to determine the power of the laser. The setup that was used in this research is depicted in Figure 1.4 [6].

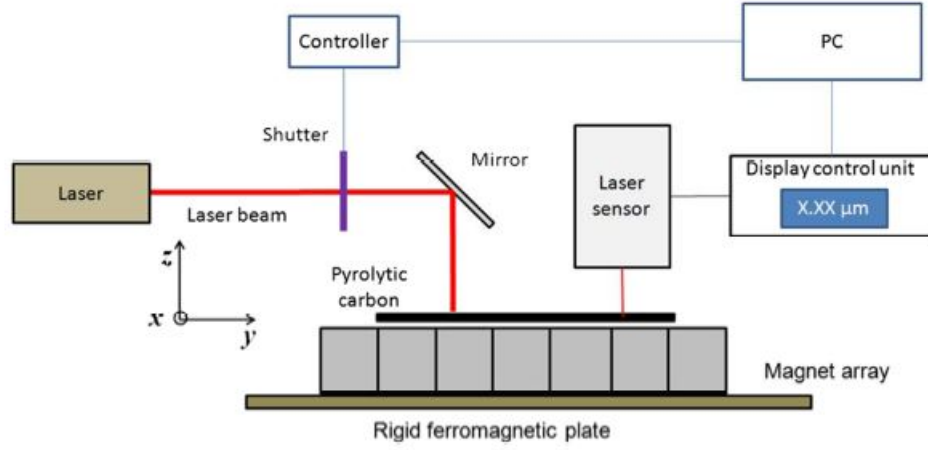


Figure 1.4: The setup used by Pinot et al. The goal is to measure the power of a laser by measuring a change in the levitation height of the levitating plate [6].

Energy harvesting

Another field in which the use of the diamagnetic effect can be exploited to make better designs is the field of energy harvesters [13–19]. Energy harvesters are used to extract energy from the environment, such as vibrations [13–19, 26]. A benefit of using the diamagnetic effect for energy harvesters is that the levitating object can be isolated from the environment. This eliminates the presence of friction and mechanical damping [17].

In designing an energy harvester using the diamagnetic effect, one needs to choose whether to use the diamagnetic effect in the vertical - or horizontal direction. In 2015, an energy harvester which uses diamagnetism in the horizontal direction was investigated [13]. The benefit of the use of diamagnetism in the horizontal direction compared to the use in the vertical direction is that the limit of the amplitude of oscillation of the levitating magnet is mitigated. Another benefit was that the natural frequency was easily altered by varying the size of the gap between the two lifting magnets. The focus of this device was to extract energy for vibrations at low frequencies. The setup of this device can be seen in Figure 1.5 [13].

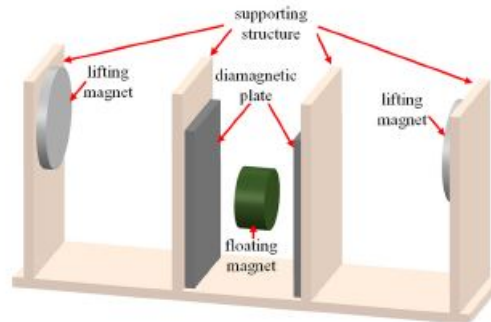


Figure 1.5: The setup used by Palagummi et al. of an energy harvester which is stabilized in a horizontal axis with the use of diamagnetic plates [13].

In this setup, the gravitational force of the levitating magnet is compensated by a set of permanent magnets. The diamagnetic plates, which are placed around the levitating magnet, ensure an equilibrium

in the horizontal direction. A coil, which is integrated in the diamagnetic plate, is used to extract the energy of the system. Eventually it was found that the average system efficiency for the experimental setup was equal to 1.93% [13]. In a subsequent paper written by Palagummi and Yuan in 2017, in which a similar setup was used, a system efficiency of 25.8 % was found [14].

The diamagnetic effect can also be used to achieve a stable equilibrium in the vertical direction. This is applied for an energy harvester, described by Wang et al., in 2013 [19]. This energy harvester also focused on extracting energy from the system for low frequencies. The energy is extracted by coils that are integrated in the diamagnetic plates. The full setup, consisting of a levitating magnet, a lifting magnet and two diamagnetic plates into which copper coils are integrated, can be seen in Figure 1.6 [19]. Here, the lifting magnet ensures the lifting of the levitating magnet whilst the repelling forces of the diamagnetic plates are used to achieve stability in the vertical direction. The experiments done showed a system efficiency of 0.72 %.

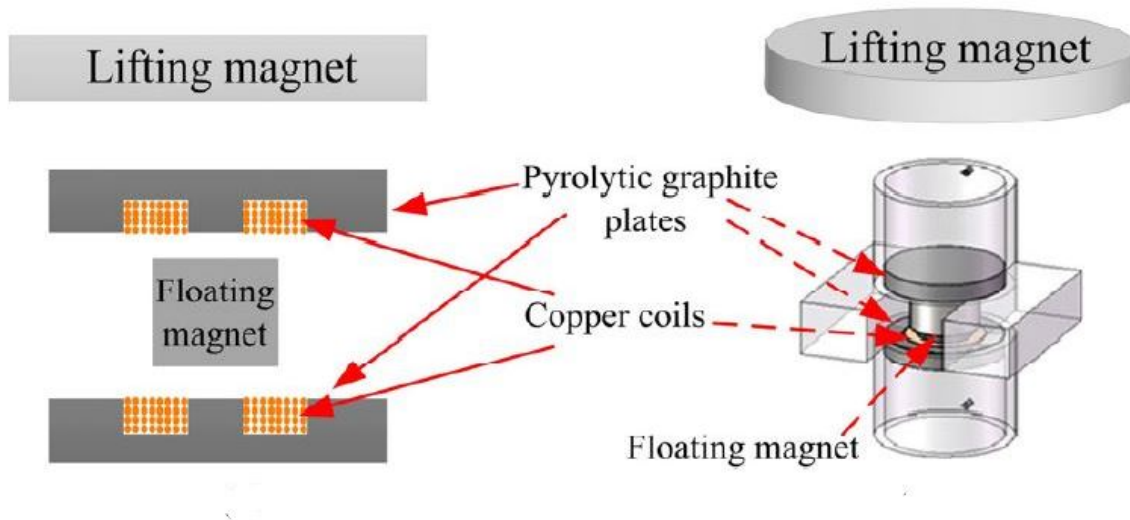


Figure 1.6: The vertical energy harvester investigated by Wang et al. [19].

In another paper, written by Palagummi and Yuan in 2014 [15], another vertical diamagnetic levitation system is analysed. Here, multiple parameters have been optimized. A similar setup as shown in Figure 1.6 is used to acquire the results. A system efficiency of 1.54% was reached.

Other applications

Apart from sensing and energy harvesting, the diamagnetic effect can also be exploited in other fields. One of these fields is biological processes [20]. In one paper, titled 'Diamagnetic Levitation Changes Growth, Cell Cycle, and Gene Expression of *Saccharomyces cerevisiae*' [20], it was concluded that the magnetic field, as well as temperature and gravity, has an effect on growth of *Saccharomyces cerevisiae*, which is a yeast. Other fields in which the diamagnetic effect can be applied are: bearings [21, 22], calibration [23] and microrobots [24].

1.3. Investigation of the dynamics of a diamagnetically levitating plate

A levitating plate has six degrees of freedom (DOF): three translations and three rotations. Since such a levitating plate is free of its surroundings it has the potential for very high Q-factors, in the order of millions for very small diamagnetic resonators [2]. As such, the rigid body dynamic movements of the plate offer great potential for resonant sensors and energy harvesters [2, 27]. Before these movements can be exploited for potential sensors it is important to know the dynamic behaviour of the rigid body modes of the levitating plate. Such a research has been done by Chen et al. in 2020 [2]. They investigated a setup in which a pyrolytic graphite plate was levitating, diamagnetically, above a 2x2 array of permanent magnets. Here, adjacent magnets had an opposite magnetization. The setup that was used is shown, both in real life and the setup used for the analysis, in Figure 1.7.

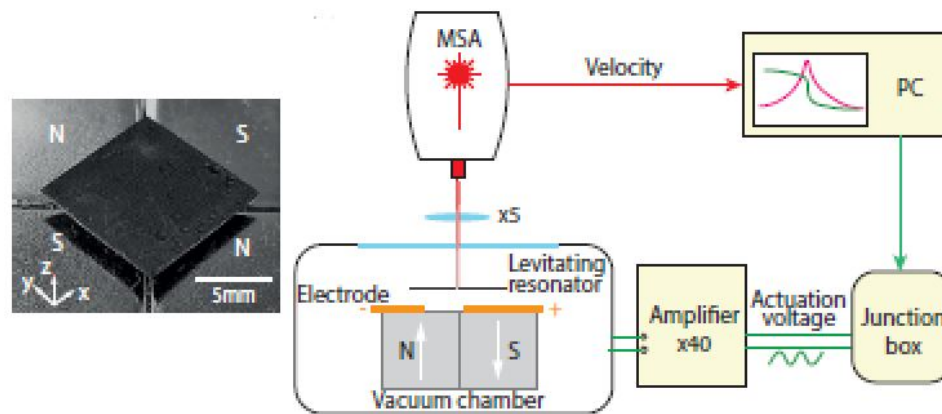


Figure 1.7: The measurement setup used by Chen et al. to investigate the rigid body dynamics of a diamagnetically levitating plate. This diamagnetic plate levitated above a 2x2 array of permanent magnets in which adjacent magnets had an opposite magnetization [2].

They found five of the six eigenmodes of a diamagnetically levitating plate along with their eigenfrequencies. Furthermore, they investigated the Q-factors due to eddy current damping as a function of plate size, experimentally and numerically, for two of the eigenmodes. Here it was found that the Q-factor of the vertical mode increases significantly with a reduction in levitating plate size. The simulated Q-factor also corresponded well with the experimental results [2].

For a similar configuration as used by Chen et al., another research was done on the dynamic behaviour of the diamagnetically levitating plate. This research has been performed by De Pasquale and his fellow authors [27]. In their research, they levitated a pyrolytic graphite plate over an 2x2 array of magnets with an 'opposite' configuration. By exciting the base, at which the magnets are attached, with a shaker and measuring the center of the plate with a laser, the oscillation amplitude of the plate was found. The total experimental setup that they used is shown in Figure 1.8.

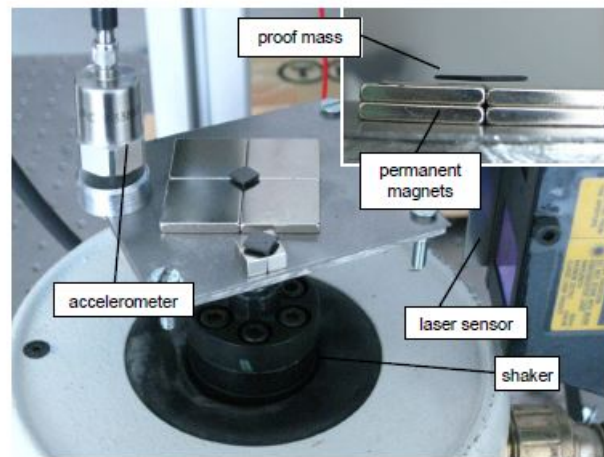


Figure 1.8: The setup used by DePasquale et al. to investigate the frequency response of a diamagnetically levitating plate [27].

By doing such measurements, the frequency response of the plate was measured. In this research they performed measurements with different side lengths of the plate and with a different number of magnetic layers. Doing so, the frequency responses that were found varied. The frequency response that were found are depicted in Figure 1.9 [27].

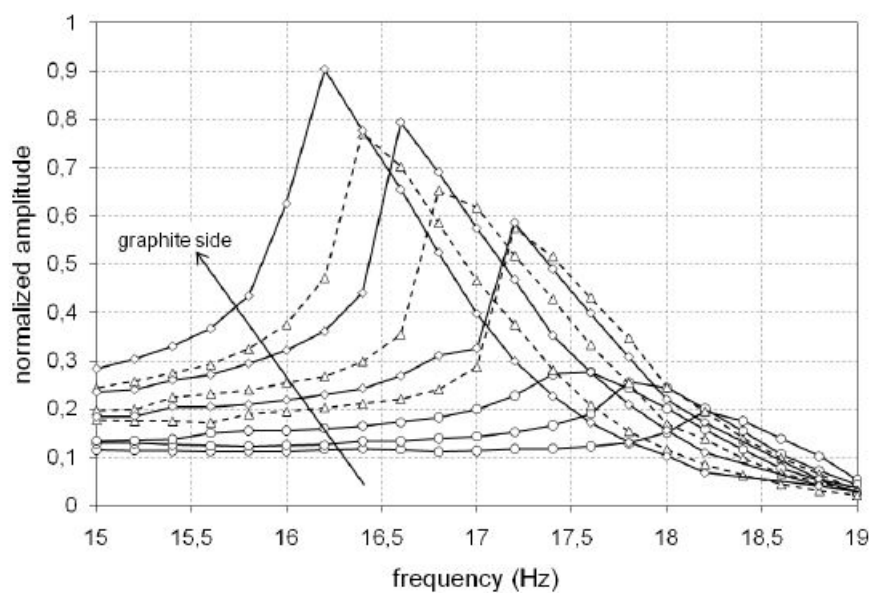


Figure 1.9: The results from the experiments done by DePasquale et al. The frequency response has been found for varying side lengths of the levitating plate as well as for a varying number of permanent magnet layers for the base. The results with a single layer of permanent magnets are shown with the circles. For two layers of magnets, the results are depicted with triangles whereas for three layers of magnets the results are depicted with squares. Furthermore, these frequency response are found for three levitating plates with respective side lengths of 9, 10 and 11 mm [27].

1.4. Research goal

In the previous sections the diamagnetic effect and its applications within sensors and energy harvesters have been explained. Furthermore, initial steps in the characterisation of the dynamic behaviour of a diamagnetically levitating plate have been shown. The eigenmodes of a diamagnetically levitating plate can be found [2], as well as the frequency response when the magnets were shaken with a shaker [27]. Although there have been initial studies to characterise the dynamic behaviour of the plate there is no literature found which extends the characterisation of the movement of the plate into the nonlinear regime. As diamagnetic levitation allows for potentially very high Q-factors [2] the appearance of nonlinearities in the dynamic behaviour is possible for an electrostatically actuated diamagnetically levitating resonator. Such nonlinearities are potentially very useful. Therefore, the aim of this thesis has been to fill this knowledge gap. More specifically, the research objective has been set as follows:

To model and experimentally test the dynamic movements of a diamagnetically levitating plate made of pyrolytic graphite, levitating above a 2x2 array of NdFeB magnets where adjacent magnets have an opposite magnetization, in the nonlinear regime. The levitating plate is actuated with an electrostatic force. This research focuses on the rigid body mode perpendicular to the top plane of the magnets.

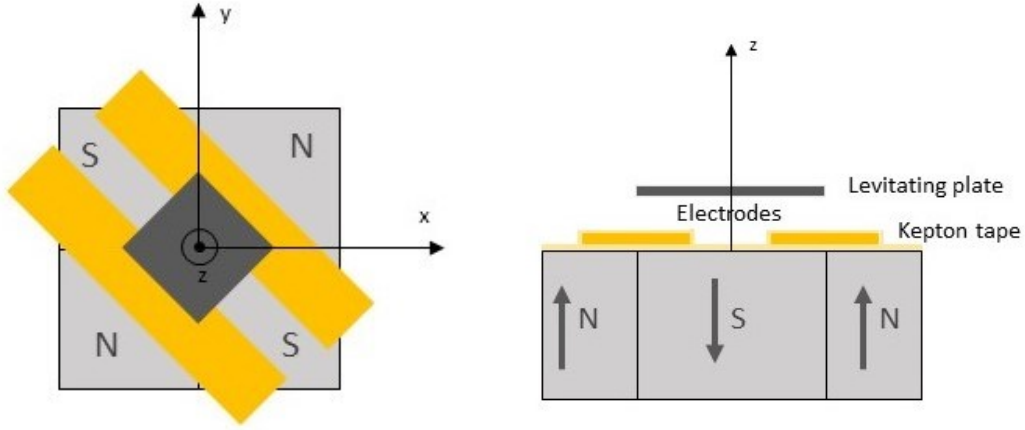
2

Theoretical background

As a first step in investigating the nonlinear dynamics of a diamagnetically levitating plate the theoretical background of the present forces, which are of magnetic and electrostatic nature, will be given. First, the setup will be explained briefly to get the reader acquainted with the setup which will be investigated. Secondly, the derivation of the magnetic field strength due to a 2x2 array of permanent magnets is given. Furthermore, the magnetic force acting on a diamagnetically levitating plate is derived. Thirdly, the method of actuation, which is an electrostatic actuation, will be explained. Here, the electrostatic force on a levitating plate will be shown. Lastly, the potential dynamic responses of the diamagnetically levitating plate will be discussed.

2.1. The setup

First of all, the reader needs to be acquainted with the setup for which the dynamic behaviour will be investigated. The setup consists of 4 NdFeB magnets, 2 electrodes and a levitating plate as is shown in Figure 2.1. The 4 magnets are arranged in a 2x2 array and are orientated in such a way that adjacent magnets have an opposite magnetization. The electrodes are placed on the magnets under a 45 degree angle. Each electrode is placed in such a way that its centerline is at the exact same position as the side of the levitating plate when observed from a top view. However, it should be noted that the levitating plate and the electrodes do not touch since they reside at a different height. In order to isolate the electrodes and the magnets a layer of kepton tape is placed between them. This kepton tape is also used to cover the electrodes. Lastly, there is the levitating plate. This levitating plate is floating at an angle of 45 around the z-axis. As the plate is floating it is therefore free from any contact with its environment. A top - and side view of the setup can be observed in Figure 2.1. In Figure 2.1 the coordinate system is also indicated. The origin, in x- and y-direction, is situated at the exact center of array of magnets. The z-value of the origin is equal to 0 at the top planes of the magnets. The z-axis is perpendicular to the top plane of the magnets whilst the x- and y-axis span the plane in which the top planes of the magnets lie.



(a) The top view of the setup consisting of the magnets, (b) The 45 degree side view of the setup of the magnets, the electrodes and the levitating plate.

Figure 2.1: The top - and 45 degree side view of the setup. The setup consists of the magnets (light grey), the levitating plate (dark grey), the electrodes (gold) and the kepton tape (light yellow.) The kepton tape insulates the electrodes from its surroundings. The centerlines of electrodes are directed in the same direction as the sides of the levitating plate. The coordinate system is also indicated, with its origin at the point at which the top surfaces of all magnets come together.

2.2. The magnetic force on a diamagnetically levitating plate

The magnetic field investigated in this section is the magnetic field induced by the 2x2 array of permanent magnets. The magnets are assembled and adjacent magnets have an alternating magnetization. The presence of the electrodes is neglected when analytically calculating the magnetic force on the levitating plate.

2.2.1. Basic equations

The magnetic field strength within a material can be calculated by Equation 2.1 [28]. Equation 2.1 shows the constitutive relationship between the magnetic flux density $\mathbf{B}$, the magnetic field strength $\mathbf{H}$ and the magnetization $\mathbf{M}$ of an object.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad (2.1)$$

All materials exhibit some sort of magnetic behaviour. They exhibit either a ferro- para- or diamagnetic behaviour. This results in either a strengthening or weakening of the magnetic field [3]. The magnetization of a material is dependent on its magnetic susceptibility, χ_m , and the magnetic field strength to which the material is subjected. This relationship is shown in Equation 2.2 [3].

$$\mathbf{M} = \chi_m \mathbf{H} \quad (2.2)$$

Combining Equation 2.1 and Equation 2.2 the following formula for the the magnetic flux density can be constructed:

$$\mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} \quad (2.3)$$

For permanent magnets the formula is slightly different due to its remanent magnetization, $\mathbf{M}_r$. The formula for the magnetic flux density of a magnet is shown in Equation 2.4.

$$\mathbf{B} = \mu_0((1 + \chi_m)\mathbf{H} + \mathbf{M}_r) \quad (2.4)$$

For the magnets used in this setup, which are NdFeB magnets, it is assumed that the magnetic susceptibility is small. Therefore the magnetic susceptibility is neglected in the calculation of the magnetic field around a permanent magnet. When neglecting the magnetic susceptibility of the magnet, Equation 2.4 can be rewritten as shown in Equation 2.5.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}_r) \quad (2.5)$$

2.2.2. Magnetic field strength around a 2x2 array of magnets

As a next step, the magnetic field strength around a permanent magnet will be calculated. In order to do so, the charge model is used [28]. The starting point of this model are the two magnetostatic field equations from Maxwell's equations. These are Gauss' law for magnetism, depicted in Equation 2.6, and Ampère's law, depicted in Equation 2.7 [28]. It is assumed that there is no current density within the magnet and that there is no change in the electric field over time. Therefore, the right side of Equation 2.7 is equal to 0. As a result Equation 2.7 is reduced to Equation 2.8. Furthermore, the magnetic scalar potential, Ψ , is introduced. The magnetic scalar potential is related to the magnetic field strength as shown in Equation 2.9 [28].

$$\nabla \cdot \mathbf{B} = 0 \quad (2.6)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad (2.7)$$

$$\nabla \times \mathbf{H} = 0 \quad (2.8)$$

$$\mathbf{H} = -\nabla\Psi \quad (2.9)$$

Substituting Equation 2.5 into Equation 2.6 yields the following result:

$$\nabla \cdot \mathbf{B} = \mu_0 \nabla \cdot (\mathbf{H} + \mathbf{M}_r) = 0 \quad (2.10)$$

Combining Equation 2.9 and Equation 2.10 gives a relationship between the magnetic scalar potential and the remanent magnetization of a permanent magnet as depicted in Equation 2.11 [28].

$$\nabla^2\Psi = -\nabla \cdot \mathbf{H} = \nabla \cdot \mathbf{M}_r \quad (2.11)$$

For a magnet bound by its volume V the magnetic scalar potential can be calculated with the use of the Green's function [28]. This results in Equation 2.12 as the function which describes the magnetic scalar potential at a point in space represented by $\mathbf{r}$ [28].

$$\Psi(\mathbf{M}_r, \mathbf{r}, \mathbf{r}_0) = -\frac{1}{4\pi} \int_V \frac{\nabla \cdot \mathbf{M}_r(\mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|} dV + \frac{1}{4\pi} \oint_S \frac{\mathbf{M}_r(\mathbf{r}_0) \cdot \hat{\mathbf{n}}}{|\mathbf{r} - \mathbf{r}_0|} dS \quad (2.12)$$

Equation 2.12 is constructed of two separate parts. The first part is a volume integral of the volume charge density ρ_m , depicted in Equation 2.13 [28], divided by the absolute distance between the point in space at which the magnetic potential needs to be known, $\mathbf{r}$, and the position of the magnetic source, $\mathbf{r}_0$. Note that the volume integral is done over the coordinates of $\mathbf{r}_0$, which are the coordinates of the magnetic source. The second part is equal to the surface integral of the surface charge density σ_m , depicted in Equation 2.14 [28], divided by the absolute distance between the magnetic source and a point in space. This fracture is integrated over all surfaces of the magnet [28].

$$\rho_m = -\nabla \cdot \mathbf{M}_r \quad (2.13)$$

$$\sigma_m = \mathbf{M}_r \cdot \hat{\mathbf{n}} \quad (2.14)$$

In the present analysis it is assumed that the remanent magnetization of the magnet is pointing purely in the z-direction. Therefore, the vector of $\mathbf{M}_r$ is as shown in Equation 2.15.

$$\mathbf{M}_r = M_0 \cdot \hat{k} \quad (2.15)$$

Inserting Equation 2.15 into Equation 2.13 tells us that the volume charge density is equal to 0. Therefore, Equation 2.12 can be reduced to:

$$\Psi(\mathbf{r}) = \frac{1}{4\pi} \oint_S \frac{\mathbf{M}_r(\mathbf{r}_0) \cdot \hat{\mathbf{n}}}{|\mathbf{r} - \mathbf{r}_0|} dS \quad (2.16)$$

When evaluating the surface charge density using Equation 2.15, it is found that this surface charge density solely has a value at the top - and bottom surface of the magnet. The reason for this is that these are the only surfaces at which the magnetization vector and the outward normal vector $\hat{\mathbf{n}}$ are aligned. At all the other surfaces, the magnetization vector and normal vector are perpendicular to each other. Therefore its inner product at these surfaces is equal to 0. As a result, the magnetic scalar potential can be represented by two sheets of the surface charge density. These sheets cover the top -and bottom surface of the magnet and have a thickness which can be neglected.

It should be noted that the outward normal vectors of the top - and bottom surface of the magnets point exactly in the opposite direction. Therefore, their surface charge density is also different. For the top plane the surface charge density is equal to M_0 whilst the surface charge density for the bottom surface is equal to $-M_0$.

Using these results for the surface charge densities and realising that these surface charge densities reside on the top - and bottom surface of the magnet respectively, the magnetic scalar potential around a permanent magnet is found as:

$$\Psi(\mathbf{r}, \mathbf{r}_{top}, \mathbf{r}_{bottom}) = \frac{1}{4\pi} \oint_S \frac{M_0}{|\mathbf{r} - \mathbf{r}_{top}|} dS + \frac{1}{4\pi} \oint_S \frac{-M_0}{|\mathbf{r} - \mathbf{r}_{bottom}|} dS \quad (2.17)$$

These surface integrals are done over the surfaces of the top - and bottom plane. Note that these surfaces lie in the x-y plane at the respective heights of z_{top} and z_{bottom} . In this case, a magnet is analysed which has its x-value of the bottom - and top surface bounded by $x = 0$ and $x = L_x$. For the y-direction, the surface has a lower bound at $y = 0$ and an upper bound at $y = L_y$. Combining this knowledge with Equation 2.17 gives the magnetic scalar potential as shown in Equation 2.18.

$$\begin{aligned} \Psi(\mathbf{r}, (0, L_x), (0, L_y), \mathbf{r}_{top}, \mathbf{r}_{bottom}) = & \frac{1}{4\pi} \int_0^{L_y} \int_0^{L_x} \frac{M_0}{|\mathbf{r} - \mathbf{r}_{top}|} dx_0 dy_0 \\ & + \frac{1}{4\pi} \int_0^{L_y} \int_0^{L_x} \frac{-M_0}{|\mathbf{r} - \mathbf{r}_{bottom}|} dx_0 dy_0 \end{aligned} \quad (2.18)$$

It should be noted that the two parts of Equation 2.18 are very similar. The only differences between these two parts are the sign before the value of M_0 and the z-value of $\mathbf{r}_{top}$ and $\mathbf{r}_{bottom}$. Since these two parts are so similar the following derivation will only be done for one part. For this part M_0 will be an input, as well as the coordinates of the point at which the scalar potential should be known (x,y,z), which are combined in the vector $\mathbf{r}$. The z-height, z_0 of the magnetic sheet is also an input. The boundaries of the integral are the boundaries of x_0 and y_0 . These boundaries are also an input to the equation. The position of the magnetic source are combined in the vector $\mathbf{r}_0$ (x_0, y_0, z_0). For future calculations Equation 2.19 is used.

$$\Psi(M_0, \mathbf{r}(x, y, z), \mathbf{r}_0(x_0, y_0, z_0), (0, L_x), (0, L_y)) = \frac{1}{4\pi} \int_0^{L_y} \int_0^{L_x} \frac{M_0}{|\mathbf{r} - \mathbf{r}_0|} dx_0 dy_0 \quad (2.19)$$

Note here that the numerator of the fracture is just the absolute distance between the point at which the scalar potential should be known, $\mathbf{r}$, and the point of the magnetic source, $\mathbf{r}_0$. This distance is shown in Equation 2.20.

$$r = |\mathbf{r} - \mathbf{r}_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \quad (2.20)$$

Inserting Equation 2.20 into Equation 2.19 gives the following result for the magnetic scalar potential:

$$\Psi(M_0, x, y, z, (0, L_x), (0, L_y), z_0) = \int_0^{L_x} \int_0^{L_y} \frac{M_0}{4\pi \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} dx_0 dy_0 \quad (2.21)$$

This can also be written as follows:

$$\Psi(M_0, r(x_0, y_0, z_0, x, y, z), (0, L_x), (0, L_y), z_0)) = \int_0^{L_x} \int_0^{L_y} \frac{M_0}{4\pi r} dx_0 dy_0 \quad (2.22)$$

When integrating Equation 2.21 over x_0 and by using Equation 2.20, Equation 2.23 is derived using Mathematica.

$$\Psi(M_0, x, y, z, (0, L_x), (0, L_y), z_0) = \frac{M_0}{8} \int_0^{L_y} [\ln(-(x - x_0) + r) - \ln((x - x_0) + r)]_0^{L_x} dy_0 \quad (2.23)$$

The next step in calculating the magnetic scalar potential is to integrate the integral of Equation 2.23 over y_0 . When doing so, Equation 2.24 is found.

$$\begin{aligned} \Psi(M_0, x, y, z, (0, L_x), (0, L_y), z_0) = \frac{M_0}{8\pi} \left[\left[-2(z - z_0) \arctan \frac{(x - x_0)(y - y_0)}{r(z - z_0)} \right. \right. \\ \left. \left. + (y - y_0) \ln \left[\frac{x - x_0 + r}{x_0 - x + r} \right] + 2(x - x_0) \ln(y - y_0 + r) \right]_0^{L_x} \right]_0^{L_y} \end{aligned} \quad (2.24)$$

As stated before, Equation 2.24 expresses the magnetic scalar potential only as a function of one surface of a single magnet. For an accurate description of the magnetic scalar potential due to a single magnet both top - and bottom surface need to be taken into account. The top surface resides at a z-height equal to z_{top} and has a magnetization of M_0 . This bottom surface resides at a z-height equal to z_{bottom} and has a magnetization of $-M_0$. Therefore, the total magnetic scalar potential for a magnet, which spans the cubical volume between $(0, L_x), (0, L_y), (z_{bottom}, z_{top})$, is as depicted in Equation 2.25. Note that the scalar potential is calculated from Equation 2.24.

$$\Psi_{singlemagnet} = \Psi(M_0, x, y, z, (0, L_x), (0, L_y), z_{top}) + \Psi(-M_0, x, y, z, (0, L_x), (0, L_y), z_{bottom}) \quad (2.25)$$

In the setup depicted in Figure 2.1 the magnetic scalar potential is a function of all four magnets. The magnets are identical in shape but adjacent magnets have an opposite magnetization. The opposite magnetization should be taken into account in the calculation of the total magnetic scalar potential. In the investigated setup the sizes of the magnets are all the same and their top - and bottom surfaces lie at the same z-height. Taking all these considerations into account, the formula for the total magnetic scalar potential for four magnets has been constructed using Equation 2.24. By filling in Equation 2.24 the total magnetic scalar potential results in Equation 2.26.

$$\begin{aligned} \Psi_{total}(x, y, z) = & (\Psi(M_0, x, y, z, (0, L_x), (0, L_y), z_{top}) + \Psi(-M_0, x, y, z, (0, L_x), (0, L_y), z_{bottom})) \\ & + (\Psi(-M_0, x, y, z, (0, -L_x), (0, L_y), z_{top}) + \Psi(M_0, x, y, z, (0, -L_x), (0, L_y), z_{bottom})) \\ & + (\Psi(-M_0, x, y, z, (0, L_x), (0, -L_y), z_{top}) + \Psi(M_0, x, y, z, (0, L_x), (0, -L_y), z_{bottom})) \\ & + (\Psi(M_0, x, y, z, (0, -L_x), (0, -L_y), z_{top}) + \Psi(-M_0, x, y, z, (0, -L_x), (0, -L_y), z_{bottom})) \end{aligned} \quad (2.26)$$

From Equation 2.26 the strength of the magnetic field can be calculated by using Equation 2.9.

2.2.3. The magnetic force on a diamagnetically levitating plate

From the magnetic field strength one can calculate the magnetic force on the levitating plate through the magnetic energy. The magnetic energy, E_m , of the levitating plate can be calculated with the magnetization ($\mathbf{M}$) of the plate, the magnetic field strength ($\mathbf{H}$) and the magnetic permeability of free space (μ_0). The magnetic energy can be calculated as shown in Equation 2.27 [28].

$$E_m = -\frac{\mu_0}{2} \int_V \mathbf{M} \cdot \mathbf{H} dV \quad (2.27)$$

The magnetic force can be calculated from this magnetic energy by taking the negative derivative of the magnetic energy in each direction. Doing so and using Equation 2.2, the formula shown in Equation 2.28 is obtained for the magnetic force.

$$\mathbf{F}_m = \frac{\mu_0}{2} \iiint_V \nabla(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2) dV \quad (2.28)$$

When finding the magnetic force on the plate in each direction of the coordinate system Equation 2.29, Equation 2.30 and Equation 2.31 are found. In this research, only the magnetic force in the z-direction is considered.

$$F_x^m = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial x} dV \quad (2.29)$$

$$F_y^m = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial y} dV \quad (2.30)$$

$$F_z^m = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV \quad (2.31)$$

From the respective forces it is possible to extract the linear- and nonlinear stiffnesses in each direction. How this can be done is shown in subsection 2.4.2.

2.3. Electrostatic force on a diamagnetically levitating plate

To actuate the diamagnetically levitating plate, an electrostatic force is used. There are two reasons why an electrostatic force is used for the actuation. First of all, this actuation method allows for an actuation of the plate without any mechanical systems. Therefore, no mechanical damping is introduced and the benefits of the isolation of the plate from its environment are preserved. The second benefit of an electrostatic force is that two parameters can be tuned separately. These are the potential to induce a stable levitation height change, due to the DC-voltage, and the potential to oscillate the plate, due to the AC-voltage.

In order to calculate the electrostatic force acting on the plate, it is assumed that this force can be calculated using a parallel plate capacitor assumption. The standard formula for an electrostatic force between two electrodes is shown in Equation 2.32 [29]. Here, ϵ_0 is the permittivity of free space, ϵ_r is the relative permittivity of the medium between the electrodes (relative to ϵ_0), A is the overlap between the two electrodes, V is the applied voltage and z is the distance between the two electrodes. The electrostatic force is an attractive force between the electrodes [29]. Therefore, the electrostatic force is a negative force in the coordinate system that we use here.

$$F_{es} = -\frac{\epsilon_0 \epsilon_r A V^2}{2z^2} \quad (2.32)$$

For the setup shown in Figure 2.1 this formula can also be applied. For this setup ϵ_0 is equal to $8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ [29], ϵ_r is approximately 1, A is equal to $4.8897 \times 10^{-5} \text{ m}^2$ and V is varying. The value of z also varies with a change in the DC-voltage applied.

When a combination of the DC-voltage and AC-voltage, which is taken as a cosine, is applied to the electrodes, the electrostatic force is calculated as shown below.

$$F_{es} = -\frac{\epsilon_0 \epsilon_r A (V_{DC} + V_{AC} \cos(\omega t))^2}{2z^2} \quad (2.33)$$

After rewriting this equation and using the trigonometric rule $\cos(2\omega t) = \frac{1}{2} + \frac{\cos(2\omega t)}{2}$ it is found that the electrostatic force can be described as shown in Equation 2.34.

$$F_{es} = -\frac{\epsilon_0 \epsilon_r A (V_{DC}^2 + \frac{1}{2} V_{AC}^2 + 2V_{DC} V_{AC} \cos(\omega t) + \frac{1}{2} V_{AC}^2 \cos(2\omega t))}{2z^2} \quad (2.34)$$

2.4. Dynamical responses

The dynamical response of a moving object is characterised by its equation of motion. In this section the dynamical response of a Duffing oscillator will be given initially to explain the concept of hardening and softening. Afterwards the dynamical response of a system with quadratic - and cubic nonlinearities will be discussed.

2.4.1. Duffing oscillator

A duffing oscillator equation of motion is constructed when taking a spring which has both a linear - and a cubic component. The force necessary to stretch or compress such a spring is calculated as shown in Equation 2.35.

$$F_{duff} = \alpha x + \beta x^3 \quad (2.35)$$

For a constant α and varying β 's the force-deflection curves are shown in Figure 2.2. Whether a hardening - or softening behaviour is observed is dependent on the value of β . When β is equal to 0 Equation 2.35 reduces to a force equation for a simple linear spring. This is depicted in black in Figure 2.2. If the value of β is positive it is seen that the force necessary to stretch the spring increases more than linear with an increment in the displacement. This is called a hardening behaviour. In contrast, when β is negative the forces necessary to stretch the spring increases less than linear with an increment in the displacement. This is called a softening behaviour. In Figure 2.2 a linear curve (black), a softening curve (red) and a hardening curve (blue) are depicted. The linear curve has been constructed with the values $\alpha = 3.5$ and $\beta = 0$. The hardening curve has been constructed with the values $\alpha = 3.5$ and $\beta = 1$. Lastly, the softening curve has been constructed with the values $\alpha = 3.5$ and $\beta = -1$.

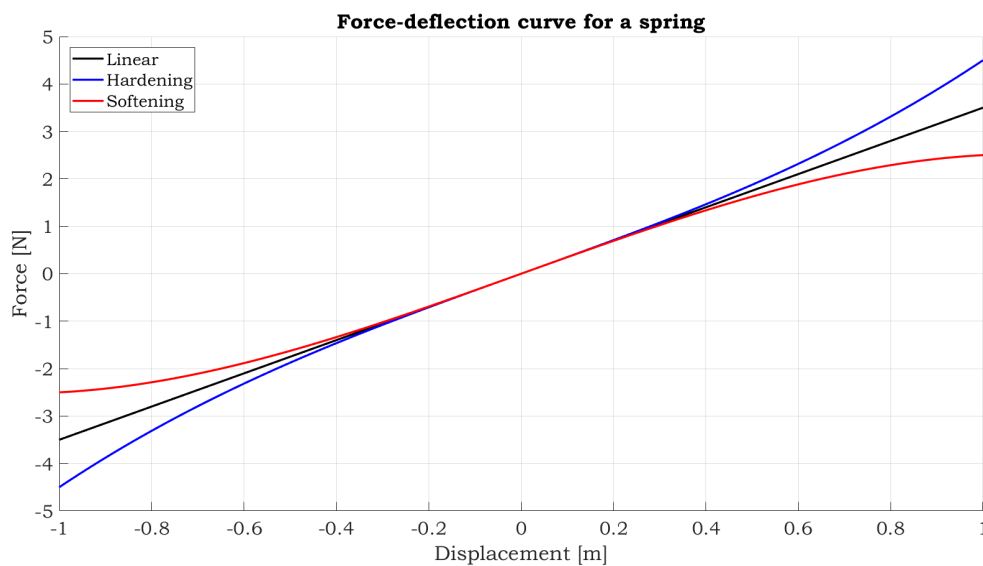


Figure 2.2: Force-displacement curves for stretching different springs, constructed with Equation 2.35. In black one can see a linear spring ($\alpha = 3.5$, $\beta = 0$). If the force necessary to stretch the spring increases more than linearly with an increment in the displacement a hardening response is found. This is depicted in blue ($\alpha = 3.5$, $\beta = 1$). In contrast, when the force necessary to stretch the spring increases less than linearly with an increment in the displacement a softening response is found. This is depicted in red ($\alpha = 3.5$, $\beta = -1$).

Another way to observe the hardening - or softening response is by looking at the frequency response function (FRF). A frequency response curve represents the amplitude of the motion of an object for varying driving frequencies. In such a frequency response a hardening behaviour is observed when the linear frequency response bends to the right. This indicates that the maximum amplitude is found at a frequency higher than the linear eigenfrequency. In contrast, a softening response is found if the linear frequency response bends to the left. This indicates that the maximum amplitude is found at a frequency which is lower than the linear eigenfrequency. To show what a hardening - or softening

response looks like in an FRF, Equation 2.36 is used [30]. This is a Duffing equation. Here, Γ is the damping coefficient, f is the driving force, ω_F is the driving frequency, t is the time and γ is the cubic nonlinearity. Furthermore, ϵ is used as the ordering parameter.

$$\frac{d^2x}{dt^2} + x + \epsilon(2\Gamma \frac{dx}{dt} - f \sin(\omega_F t + \gamma x^3)) = 0 \quad (2.36)$$

For this equation of motion an analytical solution can be derived with the method of averaging. This analytical solution is shown in Equation 2.37 [30]. In this formula, σ is the deviation from the linear eigenfrequency. Furthermore r is the amplitude of the motion. Lastly, γ is the cubic nonlinearity, Γ is the damping coefficient and f is the driving force.

$$\sigma = \frac{3}{8}\gamma r^2 \pm \sqrt{((\frac{f}{2r})^2 - \Gamma^2)} \quad (2.37)$$

A hardening - and softening response in an FRF are depicted in Figure 2.3. These are constructed by finding σ of Equation 2.37 for varying values of γ . Furthermore the following values are used in finding this figure: $\Gamma = 0.2$, $f = 1$ and r ranges between 0 and 5. When γ is set as 0 we see a linear response since no nonlinearities occur in the equation of motion. This is indicated in black in Figure 2.3. When γ is negative, in this case equal to -0.5, a softening response is observed. This is seen in red in Figure 2.2. A hardening response is in its turn observed when a positive value of γ , in this case equal to 0.5, is used as the cubic stiffness. This is depicted in blue in Figure 2.3.

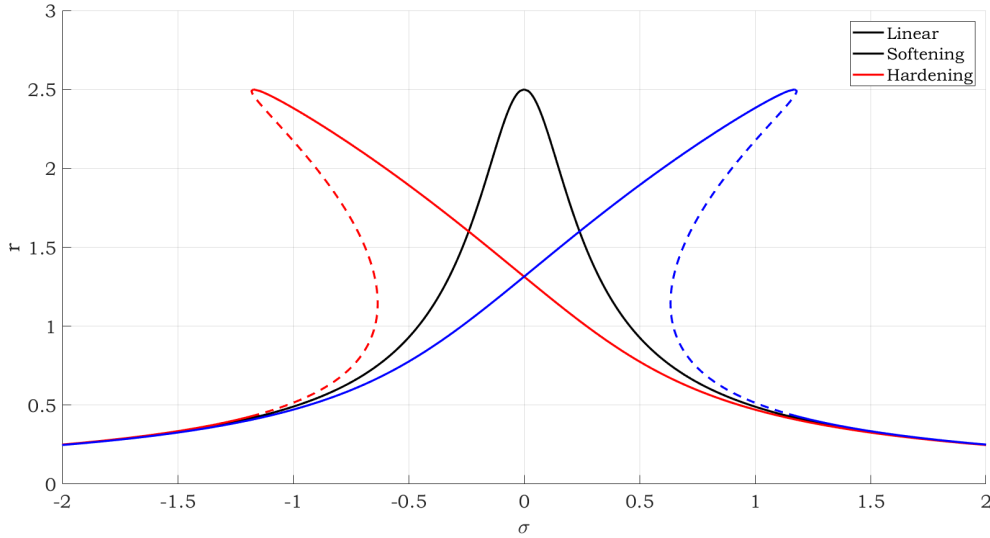


Figure 2.3: Different frequency response curves. In this case the amplitude of motion, r , is plotted against the deviation of the linear eigenfrequency, σ . The frequency response of a linear curve is depicted in black. When a negative cubic stiffness is set in the equation of motion one can observe a softening response. This is depicted in red. When a positive cubic stiffness is set in the equation of motion a hardening response is found. This is depicted in blue.

2.4.2. Systems with quadratic - and cubic nonlinearity

In this thesis the focus will be on an equation of motion in the z -direction. Here, the focus will be on an equation of motion with a linear -, quadratic - and a cubic stiffness. Furthermore it is assumed that the damping is solely linear. Under these assumptions the equation of motion (EoM) is as shown in Equation 2.38.

$$m \frac{d^2z}{dt^2} + c \frac{dz}{dt} + k_1 z + k_2 z^2 + k_3 z^3 = f \cos(\omega t) \quad (2.38)$$

Both the quadratic - and cubic stiffness influence the frequency response function. Here, these respective stiffnesses can both induce a hardening - or softening behaviour. Therefore it is not easily seen from the EoM whether a hardening or softening behaviour will occur in the frequency response.

To identify whether an equation of motion of a weakly nonlinear system with a linear, - quadratic and cubic stiffness exhibits a hardening or softening response Nayfeh & Mook have derived an analytical condition [31]. This condition has been derived for an equation which is very similar to Equation 2.38. When rewriting this analytical condition in terms of the parameters of Equation 2.38, Equation 2.39 is found. If Equation 2.39 is true a softening response is found for the equation of motion. When the right side of Equation 2.39 is smaller than the left side of Equation 2.39, a hardening response is found for the equation of motion [31].

$$\frac{k_3}{m} < \frac{9}{10} \frac{\left(\frac{k_2}{m}\right)^2}{\omega_0^2} \quad (2.39)$$

The analytical condition shown in Equation 2.39 shows that the hardening - or softening response depends on the values of the mass, the linear eigenfrequency and the quadratic- and cubic stiffnesses. Note that the linear eigenfrequency is a function of the linear stiffness. This relationship is shown in Equation 2.40.

$$\omega_0 = \sqrt{\frac{k_1}{m}} \quad (2.40)$$

In Equation 2.38, and therefore also in Equation 2.39, the respective stiffnesses are a summation of the stiffnesses due to the magnetic force and the stiffnesses due to the electrostatic force. These stiffnesses are calculated by taking the derivatives of the respective magnetic- and electrostatic force. For the calculations of these stiffnesses the origin of the coordinate system has been taken at the stable levitation height of the plate, instead of the top plane of the magnets. Within this new coordinate system the magnetic - and electrostatic force are calculated correctly by substituting $z=z_0+z$, where z_0 is the stable levitation height, into Equation 2.31 and Equation 2.34. As a result, the forces are as shown in Equation 2.41 and Equation 2.42.

$$F_m^{z=z+z_0} = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV \quad (2.41)$$

$$F_{es}^{z=z+z_0} = - \frac{\epsilon_0 \epsilon_r A (V_{DC}^2 + \frac{1}{2} V_{AC}^2 + 2V_{DC} V_{AC} \cos(\omega t) + \frac{1}{2} V_{AC}^2 \cos(2\omega t))}{2(z_0 + z)^2} \quad (2.42)$$

Within this new coordinate system the stiffnesses are evaluated at $z=0$. Therefore the respective stiffnesses are evaluated at the levitation height of the plate. The respective magnetic - and electrostatic stiffnesses are calculated as shown in Equation 2.43 till Equation 2.48.

$$k_{1mag} = - \frac{dF_m^{z=z_0+z}}{dz} \quad (2.43)$$

$$k_{2mag} = - \frac{1}{2} \frac{d^2 F_m^{z=z_0+z}}{dz^2} \quad (2.44)$$

$$k_{3mag} = - \frac{1}{6} \frac{d^3 F_m^{z=z_0+z}}{dz^3} \quad (2.45)$$

$$k_{1es} = - \frac{dF_{es}^{z=z_0+z}}{dz} \quad (2.46)$$

$$k_{2es} = - \frac{1}{2} \frac{d^2 F_{es}^{z=z_0+z}}{dz^2} \quad (2.47)$$

$$k_{3es} = - \frac{1}{6} \frac{d^3 F_{es}^{z=z_0+z}}{dz^3} \quad (2.48)$$

3

COMSOL models

Three forces, the magnetic -, electrostatic- and gravitational force, are acting on the levitating plate. Where in perfect conditions these forces can be represented accurately by analytical formulations, this is often not the case in real life setups where non-ideal conditions are present. As an example, the electrostatic force experienced by the plate may differ from a perfect parallel plate capacitor force due to fringe effects. To investigate the validity of the analytical formulations for calculating the forces in reality, multiple finite element models have been constructed. In setting up the finite element models it is assumed that the present forces are uncoupled. By assuming the forces are uncoupled the electrostatic - and magnetic force can be calculated in separate files, with different physics. This allows for faster calculation time of the models. COMSOL Multiphysics is used to make the finite element models. In both models, the 'AC/DC module' is used for the physics.

3.1. Magnetic force on a diamagnetically levitating plate

The first finite element model has been constructed to calculate the magnetic force acting on the levitating plate for various levitation heights. To calculate the magnetic force a stationary study was done using the interface 'Magnetic Fields, No Currents (mfnc)'. In this interface Gauss' Law, depicted in Equation 3.2, is solved. This is done by using the magnetic potential, the scalar V_m from Equation 3.1, as a variable [32].

$$\mathbf{H} = -\nabla V_m \quad (3.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3.2)$$

The constructed setup is the same as described in Figure 2.1 when neglecting the electrodes and the kepton tape. The setup constructed in COMSOL is shown in Figure 3.1. In COMSOL a sphere has been constructed which captures all pieces of the setup. To simulate the environment, a material (air) is assigned to the empty spaces within the sphere. For the calculations done in this model only the relative permeability of air is important. To simulate air, a blank material was assigned to the spherical domain with a relative permeability of 1. The magnets are all cubes with a side length of 12 mm. A blank material with a relative permeability of 1.02 was assigned to these magnets. The last piece of the geometry, the levitating plate, has the dimensions 10 mm x 10 mm x 0.25 mm. The initial levitation height of this plate was set at 1.425 mm above the top plane of the magnets. The levitating plate is made from pyrolytic graphite. For this material, the relative permeability differs in the x-, y-, and z-direction [2]. The relative permeability, μ_r , of the pyrolytic graphite is constructed using Equation 3.3. In the x- and y-direction the magnetic susceptibility (χ_x and χ_y) of pyrolytic graphite are equal to -85e-05. For the z-direction, its magnetic susceptibility (χ_z) is equal to -450 e-05 [2]. The full setup, excluding the air domain, can be seen in Figure 3.1. The parameters used are set out in Table 3.1.

$$\mu_r = 1 + \chi \quad (3.3)$$

Table 3.1: Parameter values to calculate the magnetic force as a function of the z-position of the plate.

Parameter	Value	Unit	Explanation
L_{plate}	0.01	m	Length of the plate.
W_{plate}	0.01	m	Width of the plate.
H_{plate}	0.00025	m	Height of the plate.
$L_{singlemagnet}$	0.012	m	Length of a single magnet.
$W_{singlemagnet}$	0.012	m	Width of a single magnet.
$H_{singlemagnet}$	0.012	m	Height of a single magnet.
$z_{initial}$	0.001425	m	Initial levitation height of the levitating plate.
χ_x, χ_y	-85e-5	-	Magnetic susceptibility of the plate in x-, and y-direction [2].
χ_z	-450e-5	-	Magnetic susceptibility of the plate in z-direction [2].
B_r	1.4	T	Remanent magnetization of a single magnet.
μ_0	1.257e-6	H/m	Magnetic permeability of free space.
μ_r^x	0.999915	-	Magnetic permeability of pyrolytic graphite in x-direction.
μ_r^y	0.999915	-	Magnetic permeability of pyrolytic graphite in y-direction.
μ_r^z	0.9955	-	Magnetic permeability of pyrolytic graphite in z-direction.

Next, the magnetic characteristics of each element in the setup have been set using the 'Magnetic Flux Conservation' domain. This domain adds Equation 3.4 to the set of equations consisting of Equation 3.1 and Equation 3.2 [32]. In this added equation μ_0 is the permeability of free space whilst μ_r is the relative permeability of a material with respect to μ_0 . The values for these respective permeabilities are found in Table 3.1.

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} \quad (3.4)$$

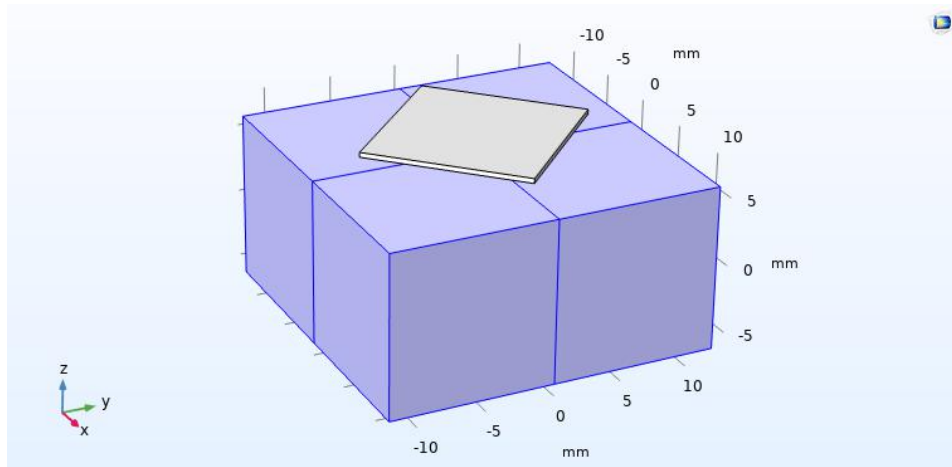


Figure 3.1: The setup for which the magnetic force on the levitating plate is calculated. The setup consist of 4 magnets (depicted in blue) and the levitating plate (depicted in grey).

After that, two 'Magnetic Flux Conservation' domains have been added to define the magnetic flux density due to the magnets. The magnetic flux density $\mathbf{B}$ due to these magnets is found by adding the remanent flux density of the magnets, $\mathbf{B}_r$, to equation Equation 3.4. For the configuration investigated in this model, the remanent flux density has an absolute value of 1.4 T and is directed into either the positive or the negative z-direction. Each magnets has its remanent flux density pointed in an opposite direction, in the z-axis, compared to its adjacent magnets.

The next step is to mesh the full setup. To obtain proper results the mesh sizes for the elements have been adjusted from the standard mesh size. For the levitating plate a maximum mesh size of 0.1 mm was chosen. The maximum mesh size for the magnets was set as 1 mm. The resulting mesh is found in Figure 3.2.

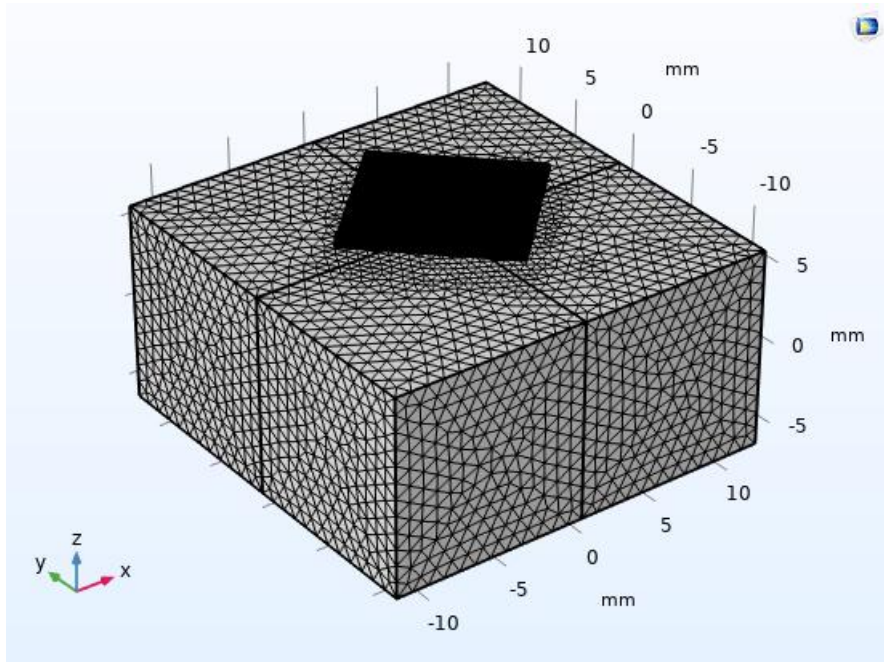


Figure 3.2: The meshed setup. The maximum mesh size of the magnets was set to 1 mm and the maximum mesh size of the plate was set to 0.1 mm.

To obtain the magnetic force for various levitation heights, a parametric sweep has been done. This is done by subtracting various values, ranging from -1.1 mm till 0.4 mm, from the initial levitation height of 1.425 mm. After the sweep has been done, a volume integration, using Equation 3.5 [2], over the volume of the levitating plate is done to calculate the total magnetic force acting on the plate.

$$F_z^m = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV \quad (3.5)$$

After post-processing the COMSOL output of the model, a graph containing the numerical magnetic force as a function of the levitation height is obtained. The results are shown in Figure 3.3.

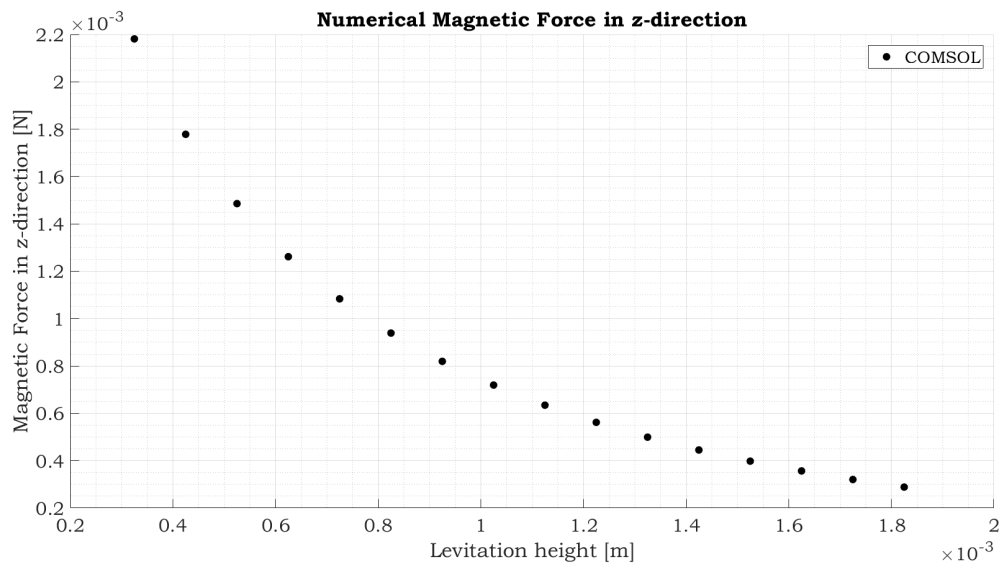


Figure 3.3: Magnetic force as a function of the levitation height calculated with the COMSOL model.

3.2. Electrostatic force on a diamagnetically levitating plate

The second finite element model has been built to calculate the electrostatic force acting on the levitating plate at various levitation heights. To calculate the electrostatic force a stationary study was done using the interface 'Electrostatic (es)'. In this interface Gauss' Law for electric fields, depicted in Equation 3.6, is solved. This is done by using the electric potential, the scalar V from Equation 3.7, as a variable [32].

$$\nabla \cdot \mathbf{D} = \rho_v \quad (3.6)$$

$$\mathbf{E} = -\nabla V \quad (3.7)$$

The setup of this model consists of a pair of electrodes and the levitating plate. This setup can be seen in Figure 3.4. A sphere which captures all pieces has been constructed to assign a material (air) to all the empty spaces within the sphere. A relative electrical permittivity of 1 was taken for air.

In constructing the geometry both electrodes were constructed as is shown in Figure 3.4. Both electrodes have the dimensions 6 mm x 24 mm x 0.1 mm. One of the electrodes is orientated, under an angle of 45 degrees around the z-axis, with its center at the position $x=-5$ mm, $y=0$ mm and $z=0$ mm whilst the second electrode is also rotated 45 degrees and has its center at the position $x = -5$ mm, $y = 0$ mm and $z = 0$ mm. The electrodes are orientated like this to simulate the real life setup. A blank material with a relative permittivity of 1 has been assigned to these electrodes. Lastly the levitating plate has been constructed. This plate has the dimensions 10 mm x 10 mm x 0.25 mm. A blank material was assigned to this plate with a relative permittivity of 13. The levitation height is set as the variable G which initially has been set at 1 mm. All dimensions and relative permittivities used are summarised in Table 3.2

Table 3.2: Parameter values used in the electrostatic force COMSOL model. This model has been used to calculate the electrostatic force as a function of the z-position of the plate.

Parameter	Value	Unit	Explanation
L_{plate}	0.01	m	Length of the plate.
W_{plate}	0.01	m	Width of the plate.
H_{plate}	0.00025	m	Height of the plate.
$L_{electrode}$	0.024	m	Length of an electrode.
$W_{electrode}$	0.006	m	Width of an electrode.
$H_{electrode}$	0.00001	m	Height of an electrode.
$G_{initial}$	0.0001	m	Initial height of the levitating plate.
ϵ_0	8.85e-12	F/m	Electrical permittivity of free space.
$\epsilon_{r_{air}}$	1	-	Relative permittivity of air.
$\epsilon_{r_{copper}}$	1	-	Relative permittivity of copper.
$\epsilon_{r_{graphite}}$	13	-	Relative permittivity of pyrolytic graphite.

Next, the electrostatic characteristics of each element have been set using the 'Charge Conservation' domain. This domain adds the constitutive relationship shown in Equation 3.8 to the set of equations consisting of Equation 3.6 and Equation 3.7 [32]. This relationship shown in Equation 3.8 defines the relationship between the electric field ($\mathbf{E}$) and the electric field displacement ($\mathbf{D}$) through the electrical permittivity of free space (ϵ_0) and the relative permittivity ϵ_r of a material. The values for the permittivity of free space and the relative permittivities can be found in Table 3.2.

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} \quad (3.8)$$

Next, the ground and the terminal have been defined. Both have been defined as a boundary. The ground was assigned to the top plane of the first electrode. The voltage of this terminal was set to 0 V. The second electrode was set as the terminal with a voltage V . Initially this voltage was set to 800 V. Later, a parametric sweep has been done over the value of this voltage. Furthermore, the levitating plate was defined as a floating potential with an initial charge of 0 Coulomb and 0 Volt. Note that

this floating potential also is a boundary, just as the terminals. To calculate the electrostatic force the levitating plate was defined as a 'Force calculation' domain.

After the physics had been set a mesh was generated. In this case, a normal mesh size has been used for all the components. The meshed setup can be seen in Figure 3.4.

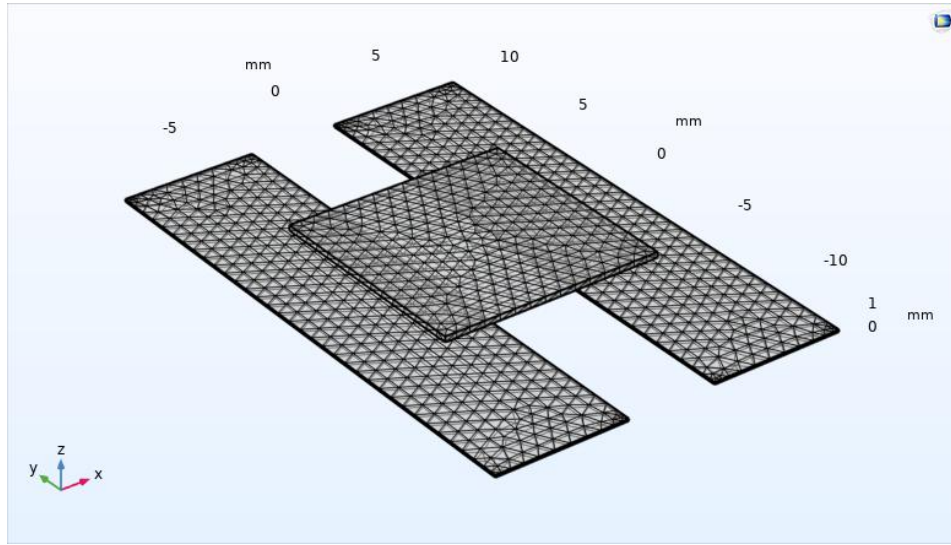


Figure 3.4: The meshed setup for calculating the electrostatic force using COMSOL.

Next, a parametric sweep has been done over the gap size G between the plate and the electrodes, as well as the voltage V that was set on the second electrode. The gap size has been varied between 0.4 mm and 1.8 mm with a step size of 0.05 mm. The voltage has been varied between 0 V and 1400 V with a step size of 100 V. The results were obtained from a global evaluation of the 'Force Calculation' domain. The results of the electrostatic force are shown visually in Figure 3.5.

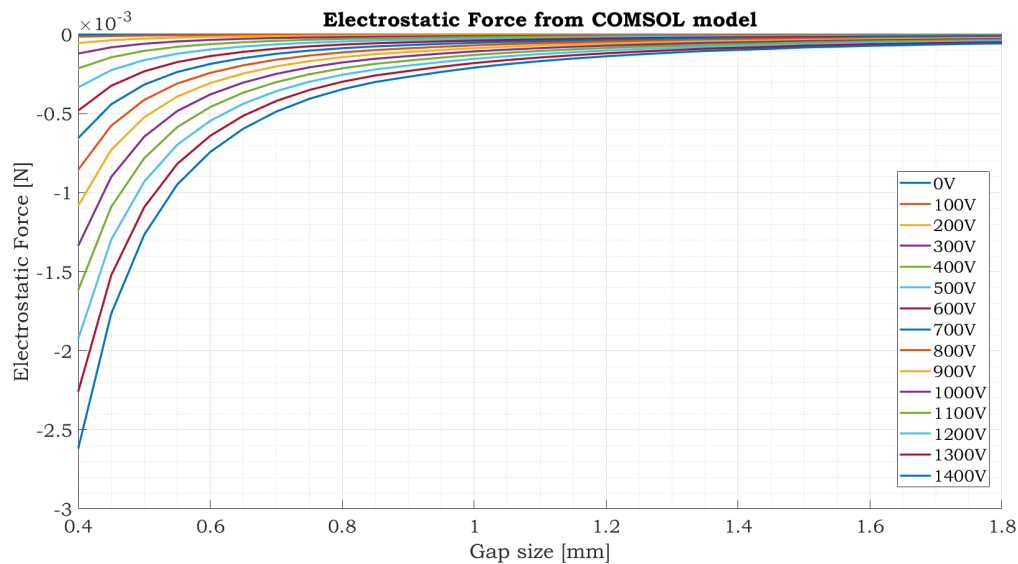


Figure 3.5: The electrostatic force obtained from the COMSOL model. Here, the electrostatic force is a function of both the voltage applied to the second electrode as well as the levitation height of the levitating plate.

Numerical verification of the analytical forces

In chapter 2 the analytical formulation of the magnetic- and electrostatic forces have been explained in detail. In chapter 3 the construction of the numerical models to obtain the numerical forces on the plate have been explained as well. In this chapter, the analytical - and numerical results will be compared. Here, both the magnetic - and electrostatic force acting on the levitating plate will be evaluated.

4.1. The magnetic force on a diamagnetically levitating plate

The derivation of the formula for the magnetic force acting on a diamagnetically levitating plate has been given in subsection 2.2.3. It has been shown that the magnetic force can be calculated analytically with Equation 4.1. In this formula, the magnetic field components are a function of all the magnets, the size and spatial orientation of these magnets and of all coordinates (x,y,z) of the coordinate system. The analytical force has been calculated by solving Equation 4.1 for the setup depicted in Figure 2.1. This is done by using the formulas presented in chapter 2. With these formulas a Matlab model has been constructed which could solve Equation 4.1 for various levitating heights. As such, the analytical force, depicted in red in Figure 4.1, is calculated. Equation 4.1 has also been evaluated in the numerical model that has been constructed. The numerical force is shown in black in Figure 4.1.

$$F_z^m = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV \quad (4.1)$$

In Figure 4.1 it can be seen that the analytically calculated magnetic force on the plate agrees very well with the numerical magnetic force. Therefore, the current analytical magnetic force has been used in the equation of motion constructed in section 6.1.

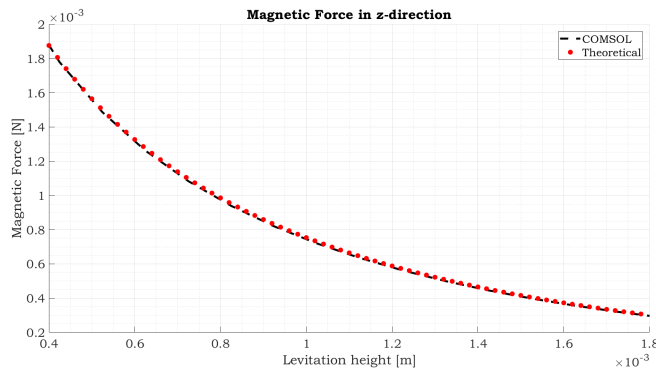


Figure 4.1: The comparison of the analytical- and numerical magnetic force on the levitating plate. The analytical force is represented by the red line whilst the COMSOL force is represented by the black line.

4.2. The electrostatic force on a diamagnetically levitating plate

As stated in section 2.3 it is assumed that the electrostatic force between the electrodes and the levitating plate can be modelled as the force from a parallel plate capacitor. For such a parallel plate capacitor Equation 4.2 can be used to calculate the electrostatic force [29]. Here, the area has been set equal to the overlap area between the electrodes and the levitating plate as found in the experiments. The value for the area was found to be equal to 48 percent of the total area of the plate. In the numerical model the force calculation tool of COMSOL has been used to evaluate the electrostatic force acting on the levitating plate. In COMSOL the area of overlap was exactly 60 percent of the levitating plate. When the analytical electrostatic force and the numerical electrostatic force are evaluated at different levitation heights, Figure 4.2 is found. These results are found for a voltage of 800 Volt. Furthermore, a corrected parallel plate capacitor force-displacement curve is also shown in Figure 4.2.

$$F_{es} = -\frac{\epsilon_0 \epsilon_r A V^2}{2z^2} \quad (4.2)$$

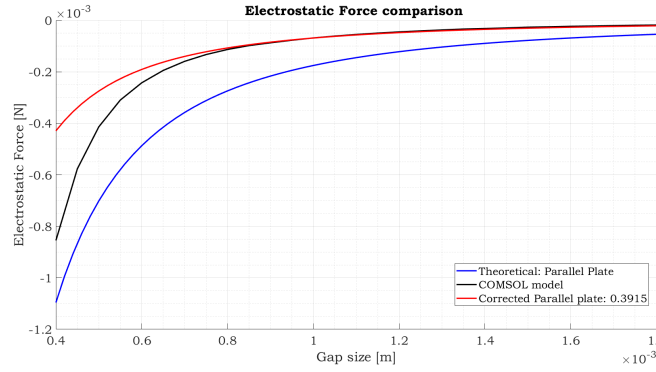


Figure 4.2: The comparison of the analytical - and numerical magnetic force on the levitating plate. The analytical force is represented by the blue line whilst the COMSOL force is represented by the black line. A corrected parallel plate capacitor force displacement curve, using a correction factor of 0.3915, is shown in red.

From Figure 4.2 it can be concluded that the parallel plate capacitor assumption for the electrostatic force does not represent the force on the plate well. The mismatch between the parallel plate capacitor electrostatic force and the numerical force can be mitigated by means of a correction factor. When using a correction factor of 0.3915 over the parallel plate capacitor force it was found that the resulting forces matched the numerical force well for the levitation height range between 0.8 mm and 1.8 mm. This is shown graphically in Figure 4.2. As the expected range of motion is within these boundaries, the corrected parallel plate force has been used for future calculations. The corrected parallel plate force formula, which accurately represents the electrostatic force acting on the plate, is shown in Equation 4.3.

$$F_{es} = -c_{cor} \frac{\epsilon_0 \epsilon_r A V^2}{2z^2} \quad (4.3)$$

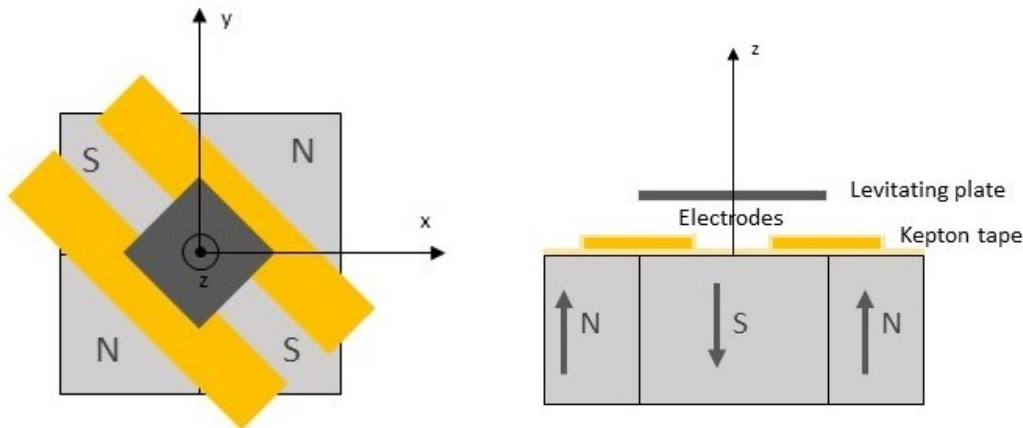
5

Experimental setups

To investigate the validity of the analytical model, experiments have been conducted. These experiments are set up in such a way that the results can be compared with the results of the analytical model. Doing so, the model can be either verified or debunked, depending on the extent to which the data of the experiments and the results from the models are comparable. In this chapter, the sets of experiments that have been conducted will be explained in detail. In section 5.2 the static experimental setup will be explained whereas section 5.3 focuses on the setups of the dynamic experiments.

5.1. Setup of the magnets, the plate and the electrodes

The setup used for the investigation is shown in Figure 2.1. For readability reasons it is shown below as well. In Figure 5.1 it can be seen that the magnets are in a 2x2 array with opposite magnetization. The magnets are covered by a layer of kepton tape before the electrodes are placed on it, with its centerline rotated 45 degrees around the z-axis such that the centerline are placed perfectly parallel with the sides of the levitating plate. The electrodes are again covered by a layer of kepton tape to ensure that the electrodes are insulated from its environment. Lastly, the plate is placed on the magnets. Note that the plate is levitating at a stable levitation height due to the repelling magnetic force. How this setup looks in real life is shown in Figure 5.2.



(a) The top view of the setup consisting of the magnets, (b) The 45 degree side view of the setup consisting of the the electrodes and the levitating plate. magnets, the electrodes and the levitating plate.

Figure 5.1: The top - and 45 degree side view of the setup consisting of the magnets (light grey), the levitating plate (dark grey), the electrodes (gold) and the insulating kepton tape (light yellow). The kepton tape insulates the electrodes from its surroundings. The centerline of the electrodes are directed in the same direction as the sides of the levitating plate. Note that the kepton tape is only included in 5.2(b).



(a) The top view of the setup consisting of the magnets, the electrodes, the kepton tape and the levitating plate. (b) The side view of the setup consisting of the magnets, the electrodes, the kepton tape and the levitating plate.

Figure 5.2: The real life top - and side view of the setup consisting of the magnets, the levitating plate, the electrodes and the insulating kepton tape.

5.2. Setup for the stable levitation height measurements

The first experiment focused on obtaining the stable levitation heights of the levitating plate. The goal of this experiment is to investigate how the levitation height of the plate changes due to an increase in the static electrostatic force. The static electrostatic force is induced by applying a DC-voltage over the electrodes. With an increase in the DC-voltage applied, the electrostatic force increases as well. Due to an increase in the electrostatic force, the equilibrium position is changed. To obtain a relatively significant change in the levitation height, in the order of tenths of millimeters, the applied electrostatic force needs to be of the same order as the combination of the magnetic and gravitational force. Using some calculations it was found that the voltages necessary to deliver such a force for this setup were in the range of hundreds of volts. To deliver these high voltages to the electrodes, a high voltage amplifier (HVA), which can amplify the voltage by a factor of 500, has been used. The attachment scheme of the amplifier is explained in detail in Appendix B.

The schematic of the setup used for the static experiments is shown in Figure 5.3. The voltage applied is set by the Delta Elektronika ES 030-5. This device gives an output between 0 V and 30 V to the High Voltage Amplifier (HVA). The voltage output of this device can be changed in steps of 0.1 Volt. The output of the Delta Elektronika ES 030-5 is amplified by a factor 500 by the HVA and then sent to the electrodes. Because of the voltage applied to the electrodes an electrostatic force is induced. This electrostatic force is exerted on the levitating plate and causes a change in its levitation height. A top view picture, taken by the Keyence Digital Microscope VHX-6000, is used to measure the levitation height of the plate. This picture is sent to the attached computer. With the software on the computer the levitation height of the plate can be extracted. The levitation height is found by averaging the measured levitation heights at several points along the plate. The real experimental setup is shown in Figure 5.4.

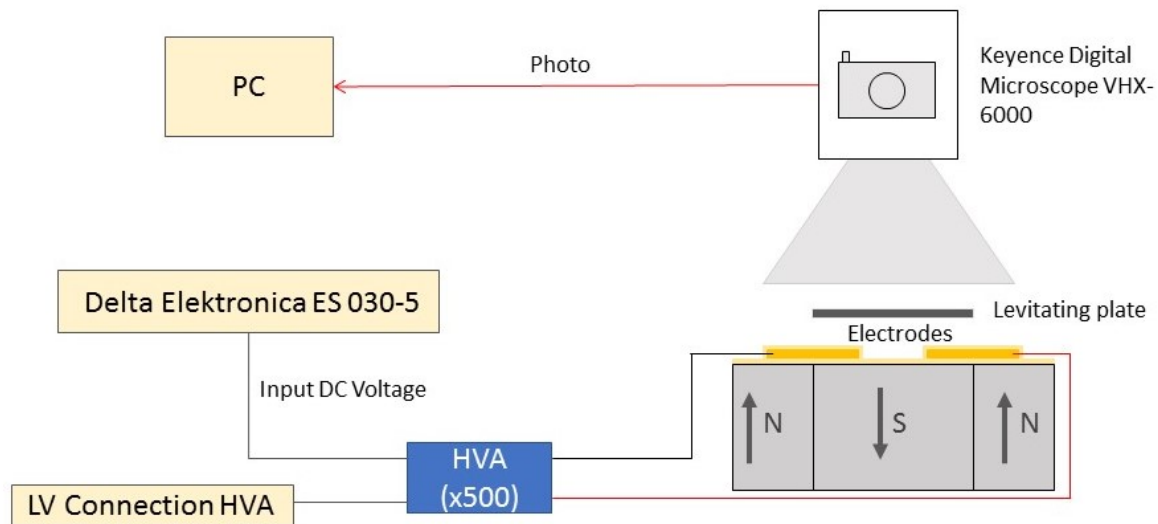


Figure 5.3: The schematic setup of the static experiment. The input voltage (DC-voltage) is generated by the Delta Elektronika ES 030-5. The HVA amplifies this signal and sends it towards the electrodes. Because of the voltage difference between the electrodes and the plate an electrostatic force is induced. This causes a change in the stable levitation height of the plate. A photo of the setup is then made by the Keyence Digital Microscope VHX-6000. The software on the computer is then used to determine the new levitation height of the levitating plate.

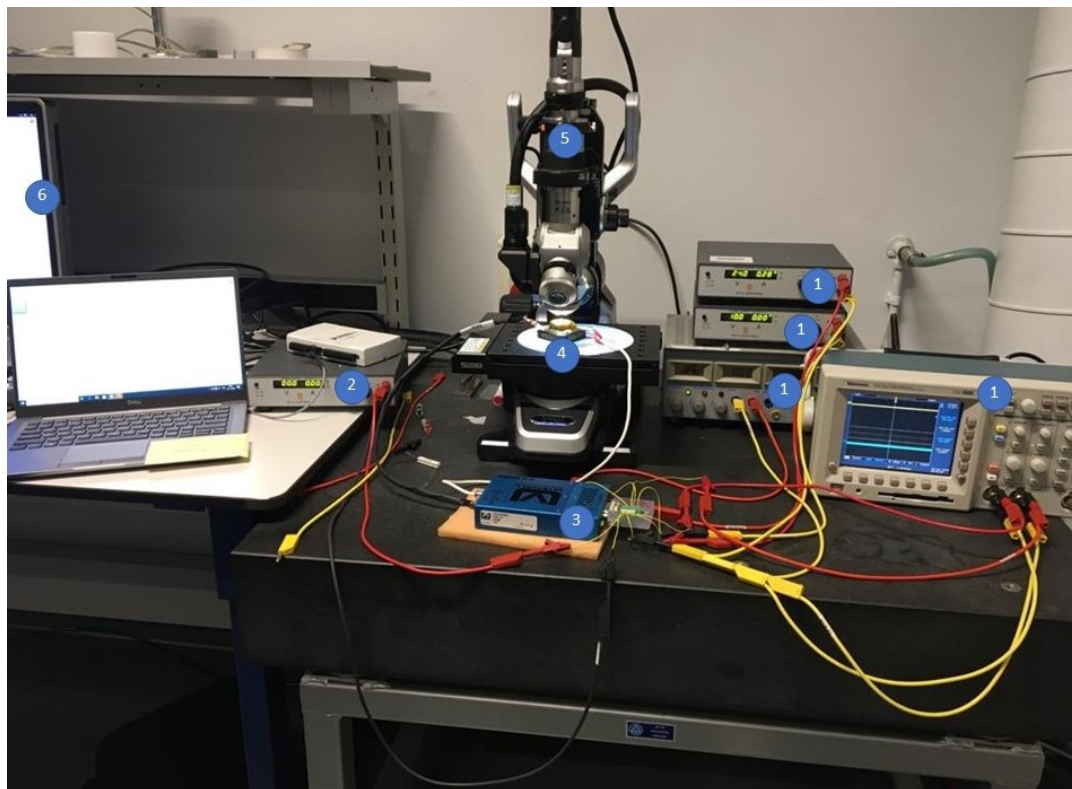


Figure 5.4: The static experimental setup. The different devices are listed by their numbers. 1: The low voltage connection devices for the HVA. 2: The input voltage generator, the Electronika ES 030-5. 3: The High Voltage Amplifier (HVA). 4: The magnets, the electrodes and the levitating plate. 5: The Keyence Digital Microscope VHX-6000. 6: The computer which processes the results.

5.3. Setups for investigating the (non)linear dynamics of a diamagnetically levitating plate

The second set of experiments focused on the characterisation of the dynamic movement of the levitating plate. To investigate this dynamic movement, the plate is excited using an electrostatic force. The goal of these experiments are to identify the eigenmodes of the plate and to measure the frequency response of the pyrolytic graphite plate for various electrostatic excitation forces. For these experiments the electrostatic force is a function of both the static voltage, the DC-voltage, and the dynamic voltage, the AC-voltage. In these experiments various combinations of these voltage inputs will be used to excite the levitating plate. Again, the high voltage amplifier is used to ensure that the electrostatic force is of the same order as the magnetic force.

The dynamic experiments focused on finding the eigenmodes of the levitating plate, investigating how the linear eigenfrequency changes with an increased applied DC-voltage and to investigate the nonlinearities in the movement of the levitating plate for various applied DC- and AC-voltages. For each of these investigations a different setup has been made. In this section the general setup for the dynamic experiments is discussed first. Secondly, the differences in the respective setups are discussed.

The general setup

The general setup that is used for the dynamic experiments has the same build up as the static experimental setup. First of all, a voltage input is given by the 'Input device'. For the dynamic experiments it is essential that the 'Input device' can deliver both a DC-voltage and an AC-voltage. The input voltage is amplified by the HVA. From the HVA the amplified voltage is sent to the electrodes. As a result an electrostatic force is induced. This electrostatic force causes the movement of the levitating plate. This movement is measured by the Polytec Scanning Head PSV-400. The measurement is done by measuring a single or multiple points on the plate by means of a laser. The output of the Polytec Scanning Head is a velocity which is sent to the output devices, which are different for the different dynamic experiments. The general setup is shown schematically in Figure 5.5.

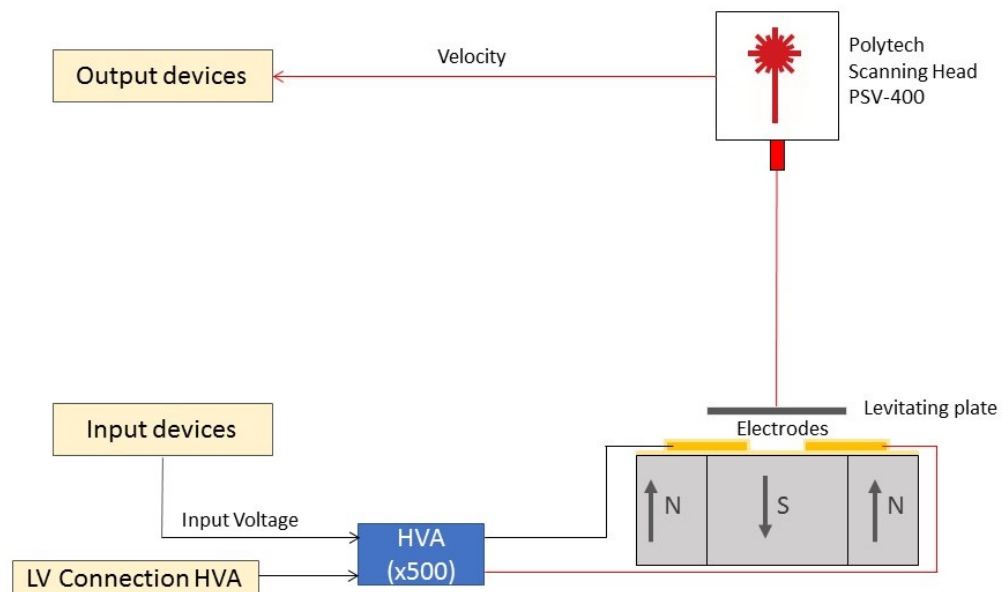


Figure 5.5: The general setup of the respective dynamic experiments. The constants over the dynamic experiments are the HVA, the setup of the electrodes, the magnets, the levitating plate and the use of the Polytec Scanning Head PSV-400. The differences are in the devices which supply the input voltage for the HVA and the devices used to process and display the movement of the plate.

Finding the eigenmodes

The input voltage of the experiments to identify the eigenmodes is set on a computer with the program 'PSV Acquisition'. Here it is possible to input both an AC- and a DC-voltage, or a combination of the both. In this case, both are used to identify the eigenmodes. This input voltage, in a Periodic Chirp form, is sent to the HVA passing through the Polytec PSV-400 Junction Box.

The movement of the plate is measured using the Polytec Scanning Head PSV-400. This device measures the velocity of the plate for the respective frequencies. The velocity of the plate is measured at several points on the plate. Doing so, the movement of the respective sides are measured for different driving frequencies. These velocities are sent to the computer, through the Polytec OFV-5000 Vibrometer Controller and the Polytec PSV-400 Junction Box. The results are post-processed using the software 'PSV 9.2'. Using this program, the eigenmodes could be identified. The respective input - and output devices used for identifying the eigenmodes are shown in Figure 5.6. The real life setup for finding the eigenmodes can be found in Figure 5.7.

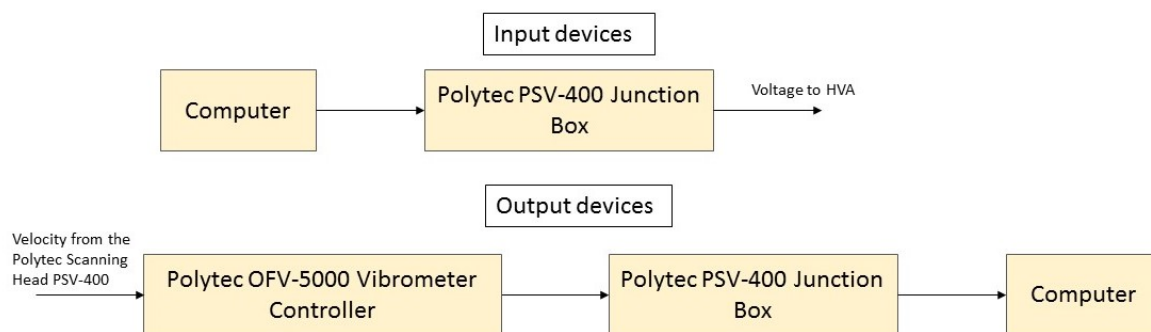


Figure 5.6: Input - and output devices for finding the eigenmodes.

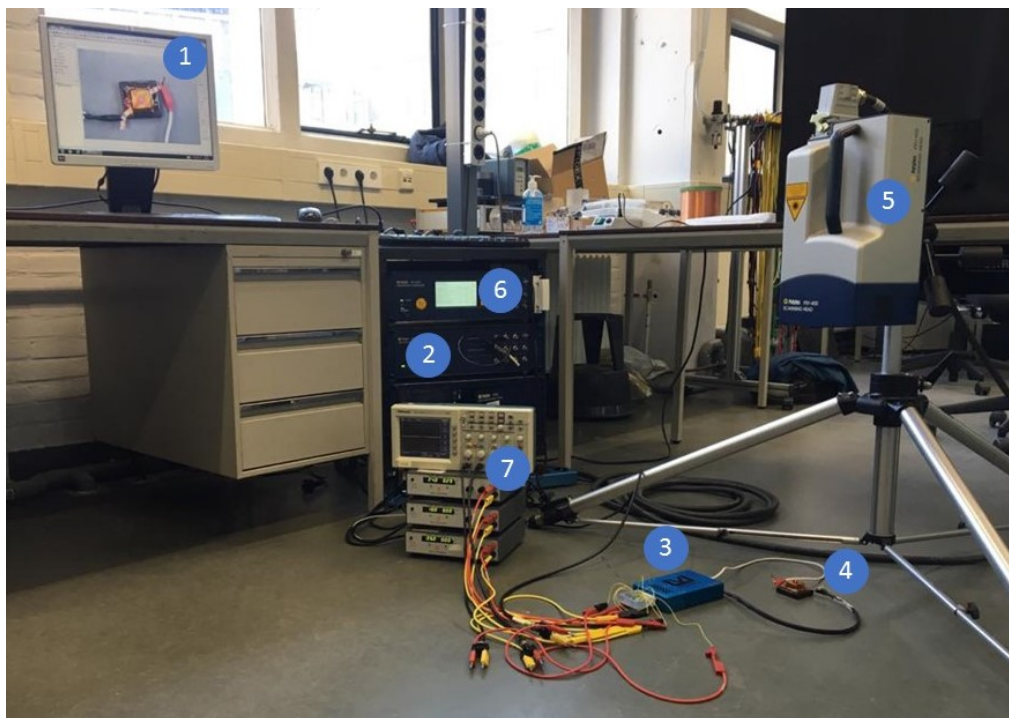


Figure 5.7: Input - and output devices for finding the eigenmodes in reality. The different devices are listed by their numbers. 1: The computer 2: The Polytec PSV-400 Junction Box. 3: The High Voltage Amplifier (HVA). 4: The magnets, the electrodes and the levitating plate. 5: The Polytec Scanning Head PSV-400 6: The Polytec OFV-5000 Vibrometer Controller 7: The low voltage connection of the HVA.

Finding the shift in the linear eigenfrequency for various DC-voltages

The input voltage of the experiments to identify the shifting of the linear eigenfrequency is supplied by the HF2LI Lock-in Amplifier from Zurich Instruments. This lock-in amplifier can output both a DC- and AC-voltage to the HVA. The total voltage output of this device is 10 V and thus sufficient for the experiments done here, where the maximum DC-voltage is below 3 V and the maximum AC voltage is 0.1 V. In the interface of the lock-in amplifier one can indicate that a frequency sweep needs to be done, along with the range of frequencies which need to be investigated. Doing so, the frequency response curve is found.

In this case, the movement of the plate is also measured with the Polytec Scanning Head PSV-400. This device measures the velocity of the plate for the respective frequencies. In this case only one point is measured. This point is at the exact center of the plate to ensure that only the translational mode in the z-direction is measured. The output of the Polytec Scanning Head is then sent, through the Polytec PSV-400 Controller, to the Zurich Instruments Lock-in Amplifier. Between the Controller and the Lock-in Amplifier there is an attenuator which dampens the voltage amplitude of the signal by a factor 2. The frequency response curve of the levitating plate is eventually displayed in the software of the lock-in amplifier.

Using this setup the shift of the linear eigenfrequency for ascending DC-voltages could be investigated. The respective input- and output device used for this investigation are shown in Figure 5.8. The setup in reality is seen in Figure 5.9.

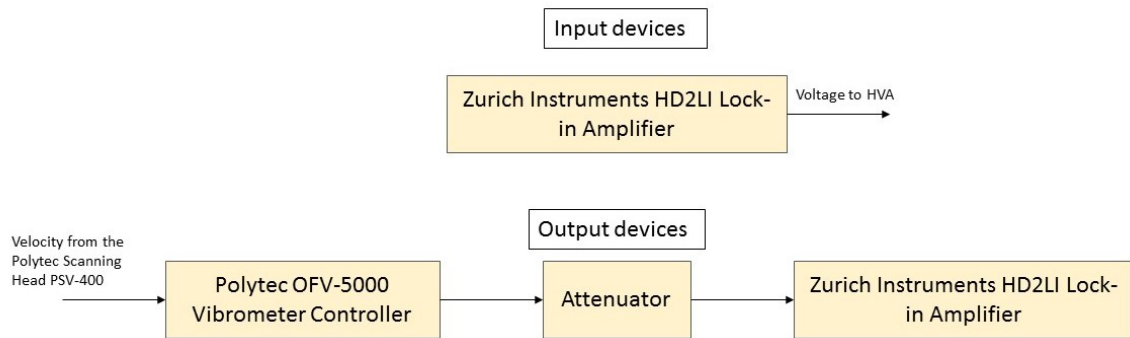


Figure 5.8: Input - and output devices for finding the shift in the linear eigenfrequency with ascending DC-voltages.

Investigating the nonlinearities in the movement of the levitating plate for various applied DC- and AC-voltages

The input- and output devices used for investigating the nonlinearities in the movement of the levitating plate for various applied DC- and AC-voltages are exactly the same as indicated in Figure 5.8. The difference when comparing these experiments to the experiments described for finding the shifting of the linear eigenfrequency are the AC-voltages applied. In these experiments the maximum AC-voltage, supplied to the HVA, is 1.8 V instead of the 0.1 V. Again, a frequency sweep is performed to obtain the frequency response function for the exact center of the plate. By applying a larger AC-voltage combined with a big DC-voltage the nonlinear behaviour of the levitating plate can be investigated in the obtained frequency response functions.

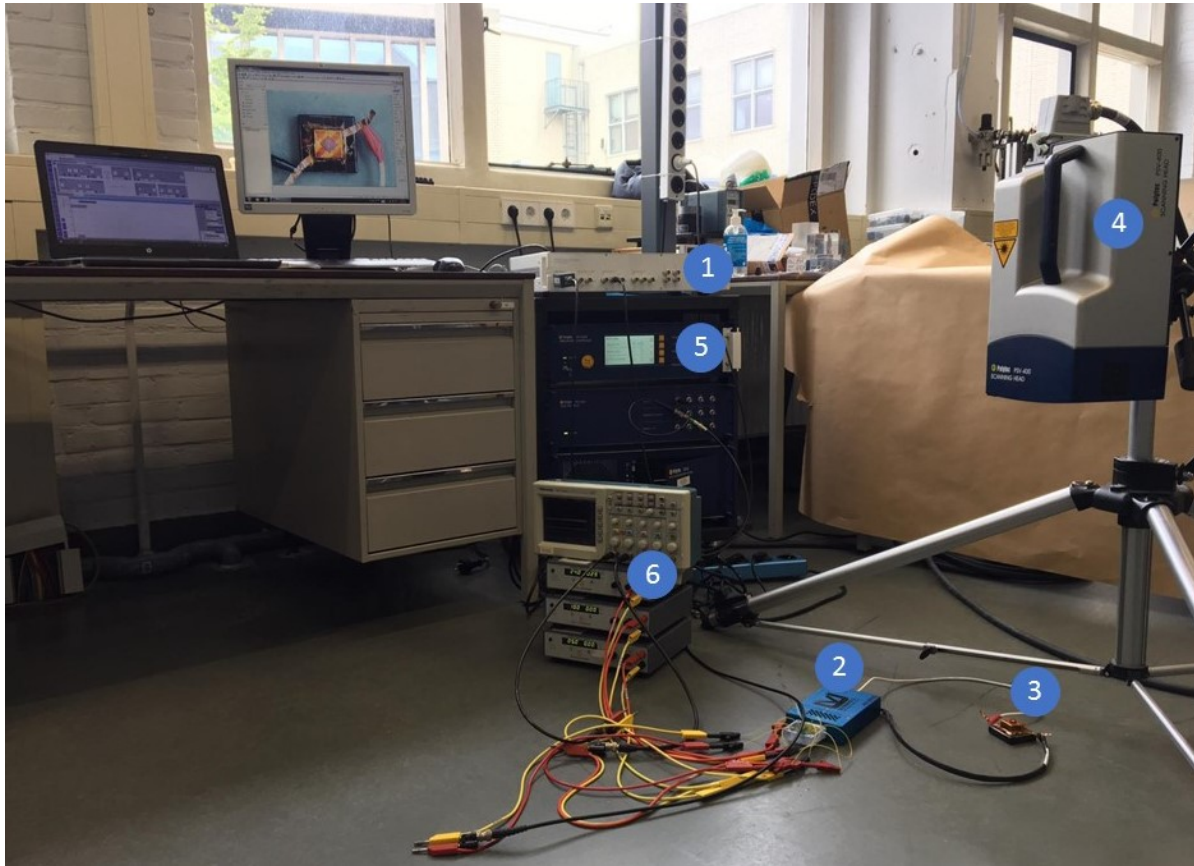


Figure 5.9: Input - and output devices for finding the shift in the linear eigenfrequency in real life. The different devices are listed by their numbers. 1: The Zurich Instruments HF2LI Lock-in Amplifier. 2: The High Voltage Amplifier (HVA). 3: The magnets, the electrodes and the levitating plate. 4: The Polytec Scanning Head PSV-400 5: The Polytec OFV-5000 Vibrometer Controller 6: The low voltage connection of the HVA.

Characterisation of the static behaviour of the diamagnetically levitating plate

In this chapter, the equation of motion of the movement of the levitating plate in the vertical direction will be constructed. From this equation of motion, an analytical model is constructed which can calculate the stable levitation height of the plate. This is done for various DC-voltages. First it will be shown how this stable levitation height is calculated analytically. Secondly, the experimental procedure for measuring the stable levitation height of the plate will be presented. Lastly, a comparison between the analytical predictions and experimental measurements will be given.

6.1. Construction of the equation of motion for the diamagnetically levitating plate

With the information of chapter 4 the analytical equation of motion for the translational mode in the z-direction of the levitating plate can be constructed. The initial equation of motion is depicted in Equation 6.1.

$$m \frac{d^2 z}{dt^2} + c \frac{dz}{dt} = F_m^{z=z+z_0} + F_{es}^{z=z+z_0} + F_g \quad (6.1)$$

When inserting Equation 2.41, Equation 2.42 (with a correction factor) and the formula for the gravitational force into Equation 6.1, the following equation of motion is found.

$$m \frac{d^2 z}{dt^2} + c \frac{dz}{dt} = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV - c_{cor} \frac{\epsilon_0 \epsilon_r A V^2}{2(z + z_0)^2} - mg \quad (6.2)$$

Note that the electrostatic voltage is a function of both the static DC-voltage and the dynamic AC-voltage. Therefore, Equation 6.2 can also be written as Equation 6.3. As both of these voltages can be set individually it is possible to investigate both the static behaviour of the plate as well as its dynamic behaviour.

$$m \frac{d^2 z}{dt^2} + c \frac{dz}{dt} = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV - c_{cor} \frac{\epsilon_0 \epsilon_r A (V_{DC} + V_{AC} \cos(\omega t))^2}{2(z + z_0)^2} - mg \quad (6.3)$$

In Equation 6.2 it should be noted how the magnetic force opposes both the electrostatic - and gravitational force. With Equation 2.41 and Equation 2.42 (with correction factor), combined with the gravitational force, an analytical model has been constructed which is able to calculate the stable levitation height for various voltages applied. Furthermore, this model is able to calculate the respective magnetic - and electrostatic stiffnesses for various voltages. This model has been constructed in the program Mathematica. The code used in this model is shown in detail in Appendix D. The results from this model are evaluated in this chapter, chapter 7 and chapter 8.

6.2. Analytical calculations

When the static equilibrium is analysed the acceleration - and velocity terms are equal to 0. Therefore, Equation 6.2 is reduced to Equation 6.4, which is nothing more than a balance of forces.

$$0 = \frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV - c_{cor} \frac{\epsilon_0 \epsilon_r A V^2}{2(z + z_0)^2} - mg \quad (6.4)$$

Equation 6.4 can also be rewritten as Equation 6.5

$$\frac{\mu_0}{2} \iiint_V \frac{\partial(\chi_x H_x^2 + \chi_y H_y^2 + \chi_z H_z^2)}{\partial z} dV = c_{cor} \frac{\epsilon_0 \epsilon_r A V^2}{2(z + z_0)^2} + mg \quad (6.5)$$

Both the magnetic - and electrostatic force are a function of the levitation height, z_0 , of the plate. Furthermore, the electrostatic force is a function of the voltage applied to the electrodes. If a static voltage is applied to the electrodes, the electrostatic force increases. As a result, the levitating plate is drawn towards the electrodes until a levitation height is obtained in which a balance of forces is achieved again. Therefore, the stable levitation height decreases with an increase in the static voltage applied. As the electrostatic force changes with the voltage squared, it is predicted that the stable levitation height is symmetrical around a voltage of 0 V. The analytical results for the stable levitation height for various DC-voltages are found in section 6.4.

6.3. Experimental procedure

The stable levitation heights, for various voltages applied, have also been found experimentally. To do so, the setup as described in section 5.2 has been used. The basic idea of these experiments is to apply a static voltage to the electrodes to induce a change in the stable levitation height. This stable levitation height is then measured by taking a picture of the full setup and measuring the distance between the electrodes and the levitating plate at all four sides of the plate. The measurement areas for these experiments are depicted in Figure 6.1. At each of these areas multiple measurements are taken. In the end, an average is taken over all the measurements to end up with the stable levitation height of the plate. The results from the experiments are shown in section 6.4.

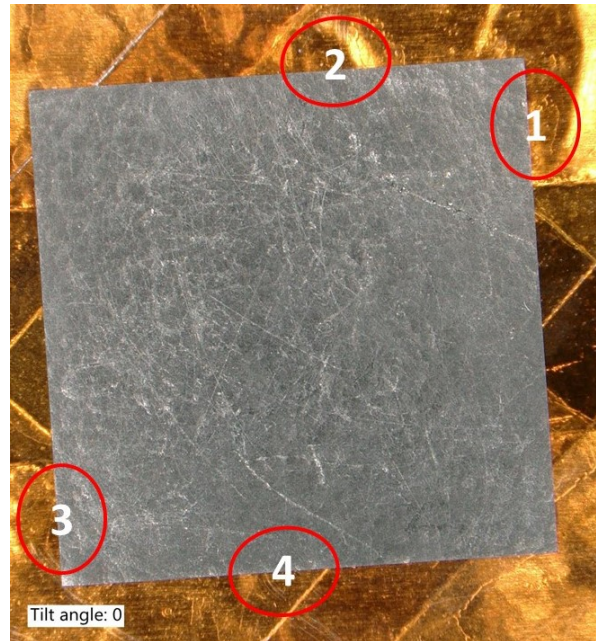


Figure 6.1: Measurement areas at which the levitation heights have been measured in the static experiments.

6.4. Results of the stable levitation height for various DC-voltages

Here, the results from the analytically calculated stable levitation heights are compared with the experimental results. The evaluation of the stable levitation heights have been done analytically for DC-voltages between -1250 V and 1250 Volt with steps of 50 V. Additionally two sets of experiments have been conducted. In the first set of experiments high DC-voltages are used, ranging between -1500 Volt and -700 Volt and between 700 Volt and 1400 Volt. The analytical results and the results of the first set of experiments are shown in Figure 6.2.

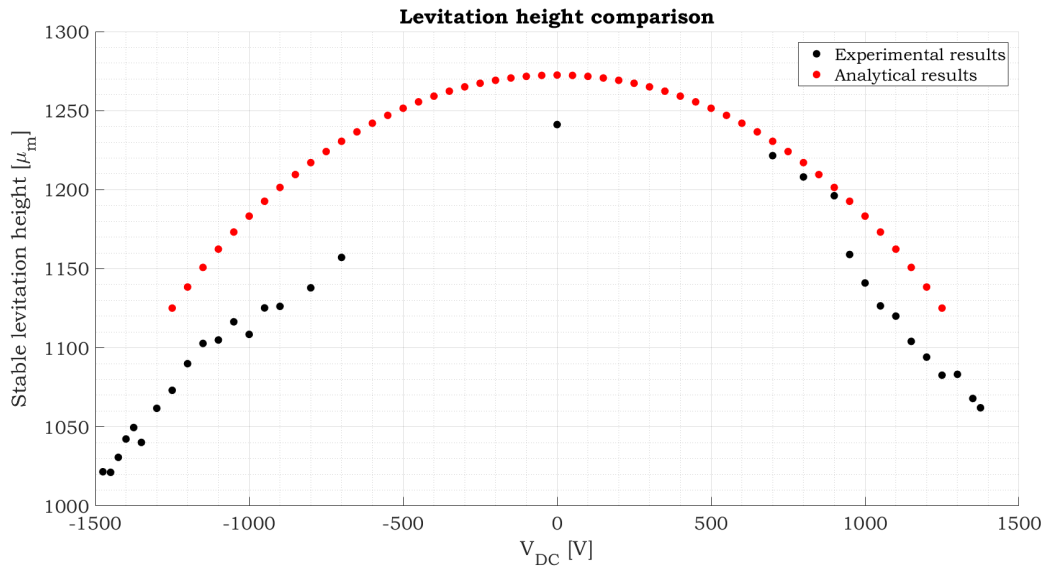


Figure 6.2: Comparison of the levitation heights calculated from the analytical model with the levitation heights found experimentally. The DC-voltage applied ranged between -1500 V till -700 V and 700 V till 1400 V for the experiments.

Second set of experiments

In the second set of experiments the applied voltage ranged between -1200 Volt and 1200 V. In this case the step size has been taken as 100 V. The results from the analytical calculations and the second set of experiments are found in Figure 6.3.

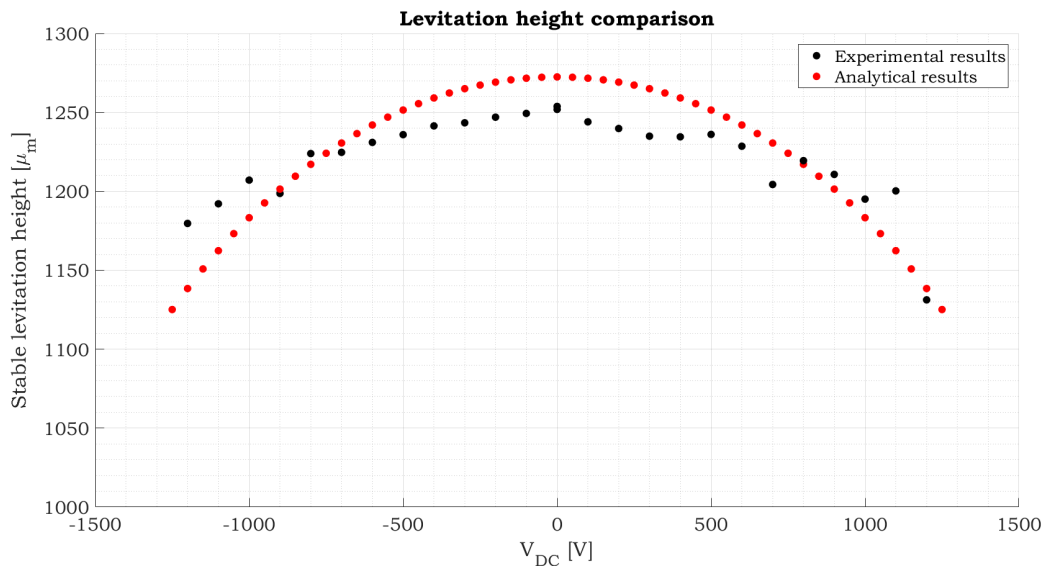


Figure 6.3: 2nd comparison of the levitation height calculated from the analytical model with the levitation height found experimentally. The DC-voltage applied ranged between -1200 Volt and 1200 Volt for the experiments.

6.5. Discussion of the results

Figure 6.2 and Figure 6.3 show some interesting results. The observations that could be made from these figures are listed below:

- The stable levitation height decreases with an increase in the DC-voltage applied

As expected, the stable levitation height decreases with an increase in the DC-voltage applied. This is observed in both the analytical - and the experimental curves. Furthermore, it is observed that the experimental results do not show a symmetry around a voltage of 0 V, which was expected. This is potentially due to the fact that the real life setup is not constructed as optimal as is assumed in the analytical model. For example, the layer of kepton tape over the electrodes has some pleats. If a measurement is done at such a pleat, this can influence the results. Another reason could be that the levitating plate is not perfectly flat or that the electrostatic force is not applied perfectly symmetrical. These factors might all influence the experimental results. Even so, Figure 6.2 still shows that the analytical curve and experimental curve both show a similar decrease in the levitation height for increasing DC-voltages.

- There is a discrepancy between the analytical - and experimental results

It is observed that even without an electrostatic force, a discrepancy between the analytical - and experimental result is found. This indicates that the analytical magnetic force does not fully represent the magnetic force in real life. One potential reason for this discrepancy could be that the magnets in the real life setup have fillets, which are not taken into account in the analytical model. The analytical model assumes that the magnets are all perfect cubes. As a result the analytical model could overestimate the magnetic force and therefore also the stable levitation height. Although there is a small overestimation without the application of an electrostatic force, this is only a small discrepancy. At this specific point the deviation is less than 3 %. When analysing all measurements, of both figures, it was found that the maximum deviation of the analytical results with respect to the experimental results is less than 8 %.

- At a DC-voltage of 0 V, two separate measurements deviate slightly from each other

Just as found when comparing the first set of experiments with the analytical model, Figure 6.3 shows a discrepancy between the experimental results and analytical results at a DC-voltage of 0 V. In Figure 6.3 it is found that between the two measurements at a DC-voltage of 0 V a small difference can be observed. This is due to experimental errors. Even though these two measurements differ slightly, this difference is negligible.

6.6. Concluding remarks

The aim of this chapter was to investigate whether an analytical model, constructed using formulas from chapter 2 and the correction factor found in chapter 4, was able to predict the stable levitation height of the levitating plate. In order to do so, the analytically calculated stable levitation heights, for various DC-voltages, are compared to the experimental results obtained for these stable levitation heights. From that comparison it has been found that the stable levitation height decreases with an increase in the DC-voltage applied, both analytically and experimentally. Furthermore, a discrepancy between the analytical - and experimental results was found. The maximum deviation was less than 8 %. It was found that the analytical model overestimates the stable levitation height slightly. This is potentially due to the fact that fillets of the magnets are not taken into account in the analytical model.

Characterising the linear dynamic behaviour of the diamagnetically levitating plate

The linear dynamic behaviour of the levitating plate is the movement that the plate makes when actuated with a small force. To investigate the linear dynamic behaviour of the eigenmode in the z-direction it is first of all important to know at which frequency this eigenmode is excited. Therefore, the very first step is to investigate at which frequencies the respective eigenmodes of the plate are excited. Next, the analytical calculation for the linear eigenfrequency of the eigenmode in the z-direction is explained. Here, the expectations for the shift in the eigenfrequency due to a change in the applied DC-voltage is explained as well. This shift in the eigenfrequency has also been tested experimentally. The experimental details for these measurements are given in section 7.3. Lastly, the analytical linear eigenfrequencies will be compared with the experimentally obtained eigenfrequencies.

7.1. The eigenmodes

The very first step in the identification of the linear dynamic behaviour of the eigenmode in the z-direction is to ensure that the right frequencies are investigated. Therefore the eigenmodes of the levitating plate, together with their associated eigenfrequencies, have been investigated experimentally. This is done by using the setup as described in section 5.3. The basic idea is that for 9 points on the plate the frequency response is found. The measurement points used for identifying the eigenmodes are shown in Figure 7.1. From the respective frequency responses the eigenfrequencies are found. After the eigenfrequencies have been found, a presentation has been made with the software 'PSV Acquisition 9.2'. From this presentation the corresponding eigenmodes have been extracted.

A levitating plate has 6 degrees of freedom. These are the translations in x-, y-, and z-direction as well as the rotations around these respective axes. With the measurement setup as described in section 5.3 modes within the x-y plane cannot be observed as they happen perpendicular to the laser direction. The modes that cannot be observed are the x-translational mode, the y-translational mode, and the rotational mode around the z-axis. Therefore, this setup would only be able to capture the rotations around the x-axis, the y-axis and the translational mode in the z-direction.

To find the eigenmodes, a DC voltage of -500 V in combination with an AC voltage of 5 V was applied to the electrodes. A frequency sweep was done between 5 and 35 Hz. From the obtained frequency responses, two eigenmodes have been found. The first eigenmode, which has an eigenfrequency at 13.63 Hz, was a rotational mode around an axis which is not described by the initial coordinate system. This axis is found when rotating the y-axis of the original coordinate system 45 degrees counterclockwise around the z-axis. This has been named the y'-axis. The y'-axis is shown schematically in Figure 7.2. Note that the x'-axis has also been shown in this figure. The second eigenmode, with an eigenfrequency of 16.75 Hz, was the translational mode in the z-direction. This is the mode that we are interested in here. For this eigenmode, a Q-factor of 23.05 has been found. The shapes of these two eigenmodes are shown visually in Figure 7.3

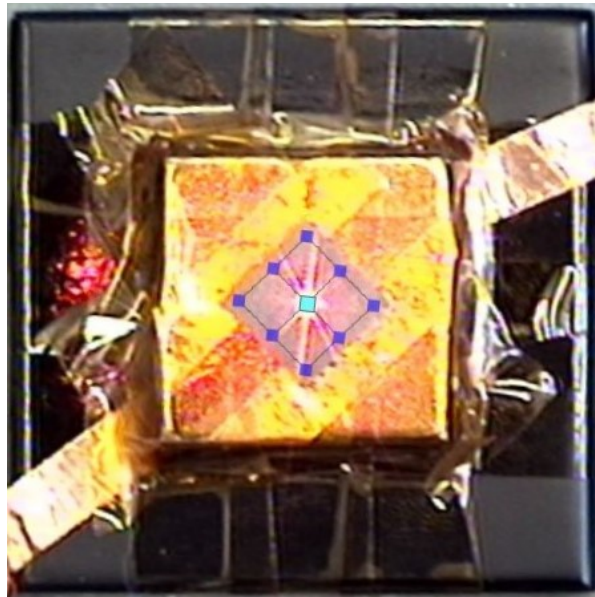


Figure 7.1: The measurement points used for identifying the eigenmodes.

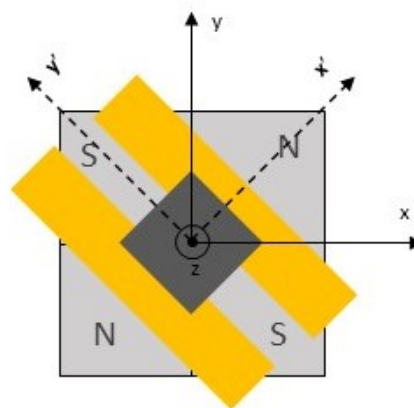


Figure 7.2: The coordinate system of the plate. The primed axes are found when rotating the original coordinate system with 45 degrees counterclockwise around the z-axis.

Although the setup described in section 5.3 is not able to capture eigenmodes in the x-y plane it was still possible to observe a mode in the x' -direction in another way. This can be done setting the AC-voltage and the driving frequency of the AC-voltage manually and observing when the eigenmode in the x' -direction is excited. It was found that this eigenmode was excited at a frequency of 4.6 Hz. An overview of all the eigenmodes that were found, together with their associated eigenfrequencies, are shown in Table 7.1.

Table 7.1: All observed eigenmodes, with their associated eigenfrequencies, of the levitating plate.

Eigenfrequency [Hz]	Eigenmode [-]
4.6	Translational mode in the x' -direction
13.63	Rotational mode around the y' -axis
16.75	Translational mode in the z-direction

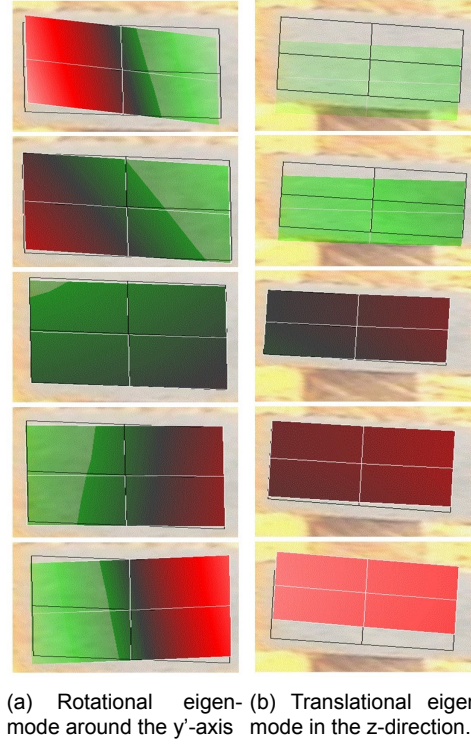


Figure 7.3: Two of the eigenmodes found for the levitating plate. The first eigenmode (7.3(a)), with an eigenfrequency of 13.63 Hz, was the rotational mode around the y' -axis. The second eigenmode (7.3(b)), with an eigenfrequency of 16.75 Hz, was the translational mode in the z -direction.

7.2. Analytical calculations

The linear eigenfrequency of the eigenmode in the z -direction, ω_0 , is calculated by taking the square root of the the total linear stiffness divided by the mass of the plate. The formula for the linear eigenfrequency is shown in Equation 2.40. Note that the total linear stiffness is calculated at the stable levitation height of the plate.

As described in subsection 2.4.2 the stiffness components are calculated in a coordinate system which has its origin at the center of the levitating plate. Within this coordinate system, the magnetic - and electrostatic force are calculated using Equation 2.41 and a corrected version of Equation 2.42. In analysing the shift of the linear eigenfrequency it is assumed that the AC- voltage is very small compared to the DC-voltage. Taking this, together with the correction factor found in section 4.2, into account Equation 2.42 is reduced to Equation 7.1.

$$F_{es}^{z=z_0+z} = -c_{cor} \frac{\epsilon_0 \epsilon_r A V_{DC}^2}{2(z_0 + z)^2} \quad (7.1)$$

The total linear stiffness used in calculating the eigenfrequency is the summation of the linear stiffness component due to the magnetic force and the linear stiffness component due to the electrostatic force. These respective stiffness components are found as is explained in subsection 2.4.2 and shown in Equation 2.43 and Equation 2.46. Note that the stiffnesses are calculated by taking the derivatives of Equation 2.41 and Equation 7.1. Note also that since the stable levitation height changes with an increase in the static voltage, the value of z_0 is dependent on the voltage applied. As a result, the values of the stiffnesses, and therefore the value for the linear eigenfrequency, change with an increase in the DC-voltage applied. As the electrostatic force scales with the voltage squared it is expected that the total linear stiffness values are symmetrical around a voltage of 0 V. Therefore, it is also expected that the linear eigenfrequency results are symmetrical around this voltage of 0 V.

From the force-displacement curves of the respective magnetic- and electrostatic forces acting on the plate, expectations on the linear stiffness components due to these respective forces can be made. In Figure 4.1 it is seen that as the distance between the plate and the magnets decreases, the steepness

of the slope increases. Therefore, an increase in the magnetic stiffness component is expected as the levitation height of the plate decreases. From Figure 4.1 it can also be seen that the slope is always negative. Since the linear stiffness component is negatively related to this slope value, as depicted in Equation 2.43, it is expected that the value of the magnetic stiffness is always positive.

From Figure 4.2 it is seen that as the distance between the plate and the magnets decreases, the steepness of the slope of the electrostatic force also increases. Therefore, it is also expected that the absolute value of the electrostatic stiffness increases with a decreasing levitation height. In contrast to the slope of the magnetic force-displacement curve, the slope of the electrostatic force-displacement curve is always positive. As the linear stiffness is equal to the negative of this slope, the value of the electrostatic linear stiffness is expected to be negative at all levitation heights.

As a result, the magnetic - and electrostatic linear stiffness will always oppose each other. Depending on which components increases the most, the total linear stiffness will decrease or increase. As a result, the linear eigenfrequency will change accordingly. The results for the analytical calculations of the linear eigenfrequency, for various DC-voltages, are shown in section 7.4.

7.3. Experimental details

The linear eigenfrequency has also been found experimentally for various applied DC-voltages. This is done by using the setup as described in section 5.3. The basic idea is that both a DC- and AC-voltage are applied to the electrodes. Here, the AC-voltage is very small compared to the DC-voltages applied. With these voltages applied a frequency sweep is done, with a laser pointing at the exact center of the plate, to obtain the frequency response function. From this frequency response function the frequency at which the maximum amplitude occurs is extracted. This is the eigenfrequency of the eigenmode. Since the z-mode is investigated, the range of frequencies close to 16.75 have been investigated. The experiments are done for various DC-voltages, whilst the AC-voltage has been taken as constant at 50 V. The results obtained are shown in section 7.4.

7.4. Results of the linear eigenfrequency for various DC-voltages

The analytical - and experimental results regarding the linear eigenfrequency of the eigenmode in the z-direction, for various DC-voltages, are shown graphically in Figure 7.4. The analytical calculations have been done for DC-voltages in the range of -1250 V and 1250 V with a step size of 50 V. for the experiments the DC-voltage has been changed between -1000 V and 1000 V with steps of 100 V.

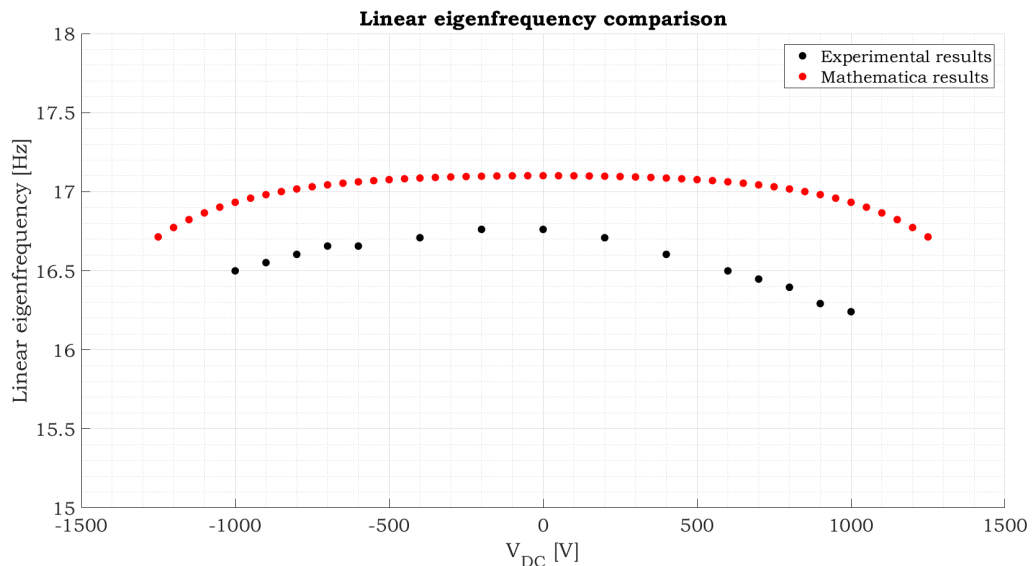


Figure 7.4: Linear eigenfrequency comparison between the results from the analytical model and the results from the experiments.

7.5. Discussion of the results

From the results shown in Figure 7.4 a number of interesting observations can be done. These observations are threefold and listed below.

- The analytical - and experimental results both show a decrease in the linear eigenfrequency with an increase of the DC-voltage

As expected in section 7.2 the linear eigenfrequency changes when applying a DC-voltage. Here, both the analytical model and experiments show a decrease in the linear eigenfrequency. As the mass of the plate is constant over the measurement it can be concluded that the summation of the total linear stiffness decreases with an increase in the DC-voltage. As a result, it can be concluded that the absolute value of the electrostatic linear stiffness component increases faster than the magnetic linear stiffness component with increasing DC-voltages.

- The analytical model overestimates the linear eigenfrequency for all DC-voltages

From the results in Figure 7.4 it is apparent that the analytical model overestimates the value of the linear eigenfrequency. Even when no DC-voltage is applied, the linear eigenfrequency is overestimated. This indicates that the magnetic linear stiffness is stronger than it is in the real life setup. This might be due to the earlier presented fact that the real life magnets have fillets, which are not taken into account in the analytical model. When comparing the analytical - and experimental results for all the voltages it is found that a maximum deviation of 5 percent is found. This is a small deviation.

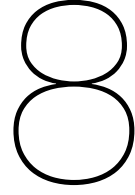
- The experimental measurement do not show symmetry around $DC = 0$ V

Lastly, it is found that the experimental measurements are not symmetric around $DC = 0$ V. As the voltage is squared in the electrostatic force it was expected that the linear eigenfrequency values show symmetry around the 0 V axis. As this is not the case, it can be concluded that there are experimental errors present in measurements. One reason for the experimental error might be that not solely the z-mode is captured in the experiments. Potentially, a slight rotation of the plate also influences the experimental results. This could be due to an asymmetrical electrostatic force acting on the plate.

7.6. Concluding remarks

The aim of this chapter was to investigate whether the analytical model was able to predict the linear eigenfrequency of the translational z-mode of the plate. In order to do so, first the eigenmodes have been identified. Three of the eigenmodes have been found for the levitating plate. These are the translational mode in the x'-direction, with an eigenfrequency of 4.6 Hz, the rotational mode around the y'-axis, with an associated eigenfrequency of 13.63 Hz and finally the translational mode in the z-direction, with an associated eigenfrequency of 16.75 Hz. For the translational mode in the z-direction a Q-factor of 23.05 was found.

Next, the analytical - and experimental results of the linear eigenfrequency of the translational eigenmode in the z-direction have been compared. It has been derived that the electrostatic - and magnetic linear stiffness components oppose each other from the respective magnetic - and electrostatic force-displacement curves. In comparing the analytical - and experimental results it was found that the linear eigenfrequency decreases with an increase in the DC-voltage applied. This indicates that the absolute value of the electrostatic linear stiffness component increases more than the absolute value of the magnetic linear stiffness components. Furthermore, it was found that the analytical model overestimates the value of the linear eigenfrequency. This is potentially related to the fact that the fillets of the magnets are not taken into account in the analytical model. Lastly, it was found that the analytical model can predict the experimental results within a margin of 5 percent. As such, it is concluded that the analytical model can predict the linear eigenfrequencies well.



Investigating the nonlinear dynamic behaviour of the diamagnetically levitating plate

Where chapters six and seven focused on the static - and linear dynamic behaviour of the plate, the present chapter will focus on the investigation of the nonlinear dynamic behaviour of the levitating plate. The focus is on the translational eigenmode in the z-direction. As a first step in the investigation, the analytical calculations of the nonlinear behaviour will be discussed. The analytical results are discussed in section 8.3. Secondly, the experimental measurements will be explained. Next, the experimental frequency response curves are shown and discussed. Lastly, the analytical - and experimental frequency responses are compared to each other. The chapter is closed with the concluding remarks.

8.1. Analytical calculations

The equation of motion investigated in this study contains a linear -, quadratic and cubic stiffness. This equation of motion is depicted in Equation 8.1.

$$m \frac{d^2 z}{dt^2} + c \frac{dz}{dt} + k_1 z + k_2 z^2 + k_3 z^3 = f \cos(\omega t) \quad (8.1)$$

As explained in subsection 2.4.2, Nayfeh & Mook have derived an analytical condition to check whether such a system displays a hardening - or softening behaviour. This analytical condition, rewritten in the parameters of Equation 8.1, is shown in Equation 2.39 [31]. To fill in the analytical equation of Equation 2.39 the linear -, quadratic - and cubic stiffness need to be known. As explained in subsection 2.4.2 the stiffnesses are calculated in a coordinate system with its origin at the center of the plate. The stiffnesses are calculated by evaluating Equation 2.43 till Equation 2.48 at $z=0$. Here, the magnetic stiffnesses are calculated by taking the respective derivatives of the magnetic force shown in Equation 2.41. To calculate the electrostatic stiffnesses Equation 2.42, in adjusted form, is used. First of all, the correction factor found in section 4.2 needs to be taken into account. Secondly, all time dependent force components in Equation 2.42 have been set to 0 when calculating the electrostatic stiffnesses,. As a result, the electrostatic force used for calculating the electrostatic stiffness values is reduced to Equation 8.2.

$$F_{z=z_0}^{es} = -c_{cor} \frac{\epsilon_0 \epsilon_r A (V_{DC}^2 + \frac{1}{2} V_{AC}^2)}{2(z + z_0)^2} \quad (8.2)$$

As derived in section 7.2, the magnetic linear stiffness is always positive. Furthermore, it has been found that the value of this magnetic linear stiffness increases with a decreasing levitation height. Doing a similar analysis over the steepness of the slope of the linear stiffness it is derived that the quadratic stiffness is always negative. Furthermore, its absolute value increases with a decrease in the levitation

height. Similarly, it is derived that the cubic stiffness is always positive and increasing with a decrease in the levitation height.

In section 7.2 it has also been derived that the electrostatic linear stiffness value is always negative and increases as the levitation height decreases. For the electrostatic force it can be derived that the quadratic stiffness values will always be positive and will increase with a decrease in the levitation height. For the cubic stiffness values due to the electrostatic force, it is derived that these will always be negative and its absolute values increases with a decrease in the levitation height.

From these derivations it is clear that the magnetic stiffness components will always oppose the electrostatic stiffness components. Depending on the values of the respective stiffnesses for different voltages, the total stiffnesses will decrease or increase in value for different voltages applied.

Next to the stiffness terms, Equation 8.1 also contains a force term. This force term is the result from the corrected time-dependent components of the electrostatic force shown in Equation 2.42. Note that only the cosine term which is dependent on the driving frequency is used in the equation of motion. The value of the force term is found when evaluating the time dependent component of the electrostatic force at $z=0$, just like the stiffnesses. Doing so, the force term is calculated as is shown in Equation 8.3.

$$f = -c_{cor} \frac{\epsilon_0 \epsilon_r V_{DC} V_{AC}}{z_0^2} \quad (8.3)$$

In order to compare the analytical frequency response function with the experimental frequency response function both are made dimensionless. Equation 8.1 has been made by dimensionless introducing a dimensionless length and a dimensionless time. The dimensionless length is constructed by dividing the displacement of the plate by the stable levitation height (h) found with the analytical model, for the DC- and AC-voltage applied. The dimensionless length (z^*) is calculated as shown in Equation 8.4. The dimensionless time is constructed by multiplying the real time with the linear eigenfrequency. The dimensionless time is constructed as shown in Equation 8.5.

$$z^* = \frac{z}{h} \quad (8.4)$$

$$t^* = \omega_0 t \quad (8.5)$$

With Equation 8.4 and Equation 8.5, Equation 8.1 has been made dimensionless. The full derivation of the nondimensionalisation of the equation of motion is explained in Appendix A. The nondimensional equation of motion that is found is shown in Equation 8.6.

$$\frac{d^2 z^*}{dt^{*2}} + \frac{c}{m\omega_0} \frac{dz^*}{dt^*} + \frac{k_3 h^2}{m\omega_0^2} z^{*3} + \frac{k_2 h}{m\omega_0^2} z^{*2} + \frac{k_1}{m\omega_0^2} z^* = \frac{f \cos(\frac{\omega t^*}{\omega_0})}{m\omega_0^2 h} \quad (8.6)$$

To construct the analytical frequency response, Equation 8.6 is solved for varying driving frequencies. The resulting maximum amplitudes are then extracted and plotted against the driving frequency. The code used for finding the analytical frequency response functions can be found in Appendix C.

8.2. Results for the analytically calculated stiffnesses and expected softening or hardening from the analytical model

The goal of this section is to show the analytically calculated stiffnesses. Furthermore, the analytical expectation for a softening - or hardening behaviour of the diamagnetically levitating plate will be shown.

In Figure 8.1, Figure 8.2 and Figure 8.3 the resulting stiffnesses, as calculated with Equation 2.43 till Equation 2.48, are displayed for varying DC-voltages. In calculating these results, the AC-voltage has been taken as 0 V.

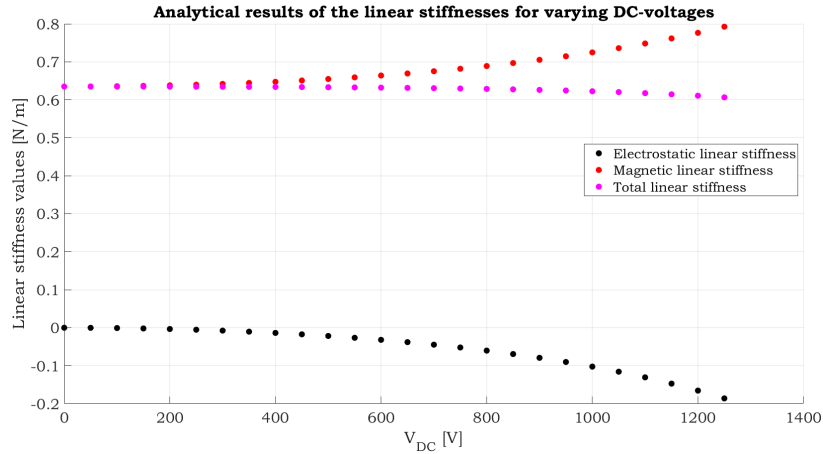


Figure 8.1: Linear stiffness values for varying DC-voltages as calculated by the analytical model.

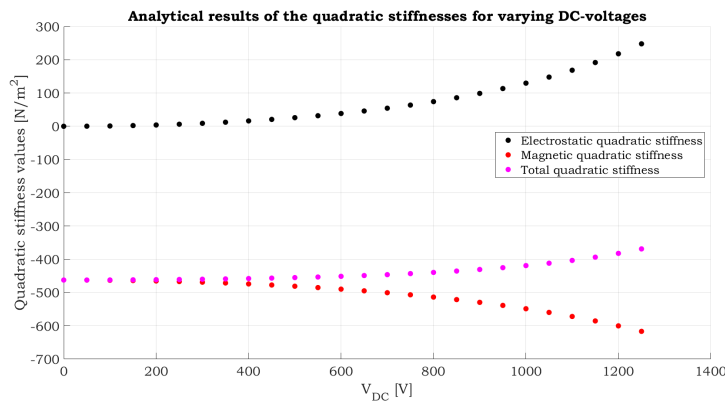


Figure 8.2: Quadratic stiffness values for varying DC-voltages as calculated by the analytical model.

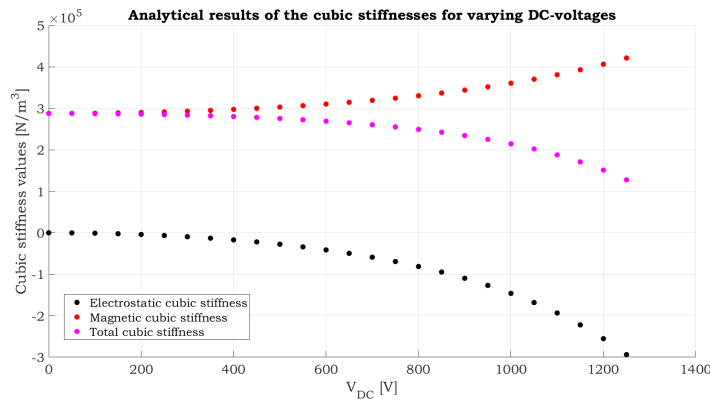


Figure 8.3: Cubic stiffness values for varying DC-voltages as calculated by the analytical model.

With the stiffnesses found in the previous figures, together with the mass of the plate, it is possible to check for each DC-voltage applied whether the movement of the plate will experience hardening- or softening. This is done by inserting the respective stiffness values into Equation 2.39. The values for the left - and right side of Equation 2.39 for various DC-voltages are depicted in Figure 8.4.

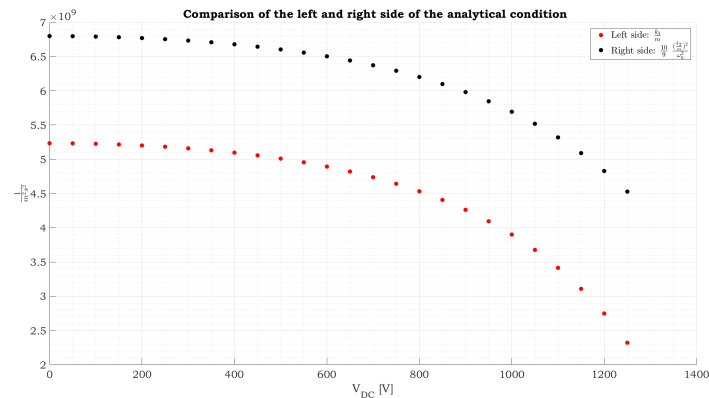


Figure 8.4: The values of the left- and right side of the analytical condition shown in Equation 2.39.

8.3. Discussion of the analytically calculated stiffnesses and expected softening - or hardening behaviour

From the analytical results, depicted in Figure 8.1 till Figure 8.4, a number of observations can be done. These are listed below:

- The total linear stiffness decreases with an increase in the DC-voltage

This conclusion is drawn when looking at Figure 8.1. Since the total linear stiffness is linked to the linear eigenfrequency by Equation 2.40 the eigenfrequency will thus also drop with an increase in the DC-voltage. This is also what is observed in Figure 7.4

- The magnetic - and electrostatic stiffness values always oppose each other

As has been expected in section 8.1 the magnetic- and electrostatic stiffnesses always oppose each other in sign. This is true for the linear -, quadratic - and cubic stiffnesses.

- The electrostatic stiffnesses increase more than the magnetic stiffnesses with an increase in the DC-voltage applied

Another interesting observation is that as the DC-voltage increases, the electrostatic stiffnesses increase more than the magnetic stiffnesses. This is most apparent at high DC-voltages. As a result, the respective total stiffness curves bend in the same direction as the electrostatic stiffness curves.

- The combination of stiffnesses can be tuned by increasing or decreasing the DC-voltage

As a result of the application of an electrostatic force the values of the linear-, quadratic and cubic stiffnesses are changed. Since the electrostatic force can be set manually a preferred set of stiffnesses can be chosen.

- For each DC-voltage applied a softening behaviour is expected for the movement of the plate

When the left side of the analytical condition shown in Equation 2.39 is smaller than the right side of that analytical condition, a softening behaviour is expected for the behaviour of the plate. In Figure 8.4 it can be seen that for each DC-voltage applied to the plate this is true. Therefore, a softening response is always expected for the plate. This is also true in the absence of an electrostatic force.

8.4. Experimental measurements

The dynamic response of the plate has also been found experimentally for varying DC- and AC-voltages. The DC-voltages applied ranged between - 100 V and -800 V whilst the AC-voltages ranged between 100 V and 850 V. The experiments are done by using the setup described in section 5.3. As a result of the AC-voltage, the plate will start to oscillate. The amplitude of this oscillation is measured at the exact center of the plate. A frequency sweep is performed to measure the amplitude of the oscillation for varying driving frequencies. As a result, an experimental frequency response function can be constructed. To measure the dynamic behaviour of the plate in the z-direction, the frequency sweep has been done for frequencies close to the eigenfrequency of the eigenmode in the z-direction. Therefore, a frequency range of 14.5 Hz - 22 Hz has been investigated. The results are shown in section 8.2.

In the first set of experiments, it has been observed that multiple modes of the plate are excited close to the linear eigenfrequency of the eigenmode in the z-direction. Therefore, a second set of experiments has been conducted to investigate whether other modes are excited around this eigenfrequency. To do so, an FFT analysis is done for a DC-voltage of -700 V and an AC-voltage of 850 V. In this case, the measurement setup to investigate the eigenmodes as described in section 5.3 is used. Unfortunately, this setup cannot detect the eigenmodes in the x-y plane as such movements are perpendicular to the laser. The FFT analysis has been done for several points on the plate. These points are arranged in a square and cover all corners of the plate. An image of the points analysed is shown in Figure 8.5. Here, the FFT analysis is done for different driving frequencies between 15 and 16 Hz. Unfortunately, from the FFT analyses done, no other modes could be identified.

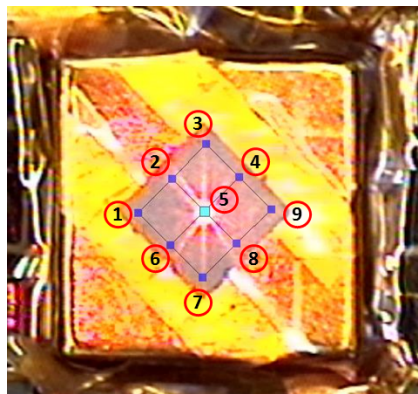


Figure 8.5: All of the points measured to investigate whether other modes are excited at frequencies between 15 and 16 Hz.

Additionally, a third set of experiments has been conducted to investigate whether any rotational modes are excited at a driving frequency of 15.5 Hz. In order to do so, the setup for measuring the nonlinear dynamics as described in section 5.3 is used. Here, the frequency response function for each point indicated in Figure 8.5 is found. The results of these experiments are found in Figure 8.10.

8.5. Frequency response functions obtained from experiments

In this part, the most important experimental results will be presented. Here, first three figures are presented which show the frequency response function of the plate for three different DC-voltages and varying AC-voltages. These frequency responses are made dimensionless in the y-axis by dividing the displacement of the plate by the stable levitation height associated with the applied DC-voltage. These stable levitation height are found in the static experiments and are shown in section 6.4. Next, the frequency response function for one AC-voltage and varying DC-voltages will be shown. This figure has been made dimensionless in the y-axis in the same manner. Lastly, a figure is shown which contains the frequency response functions for several points on the plate at one specific DC-voltage and one specific AC-voltage.

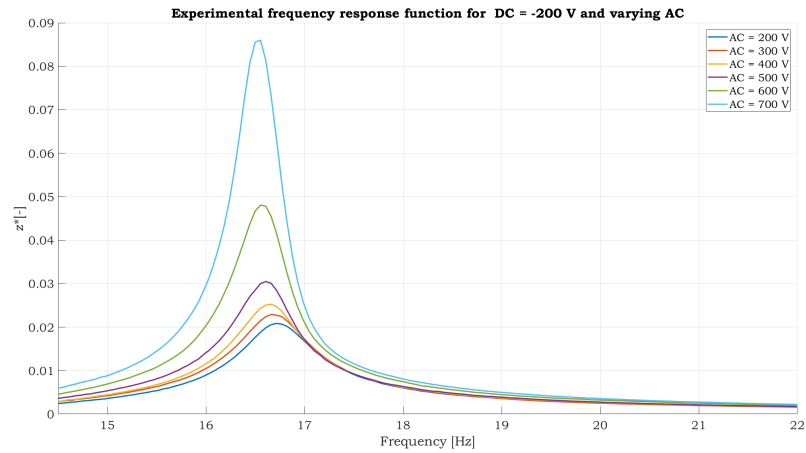


Figure 8.6: Experimental frequency response function found for a DC-voltage of - 200 V and varying AC-voltages.

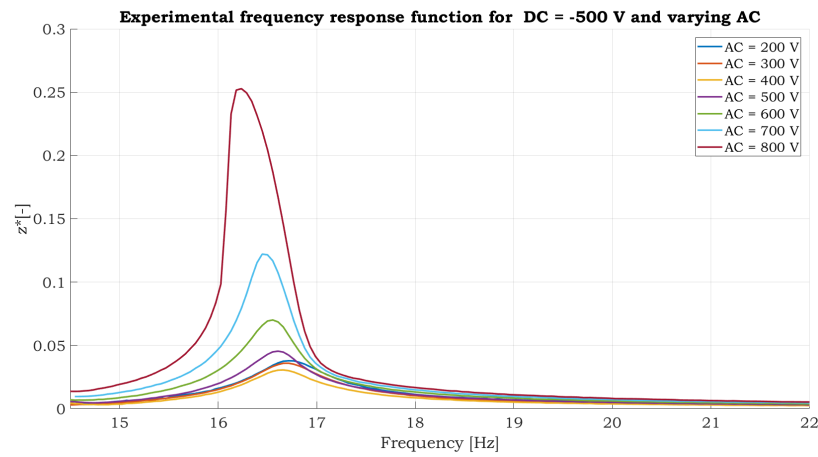


Figure 8.7: Experimental frequency response function found for a DC-voltage of - 500 V and varying AC-voltages.

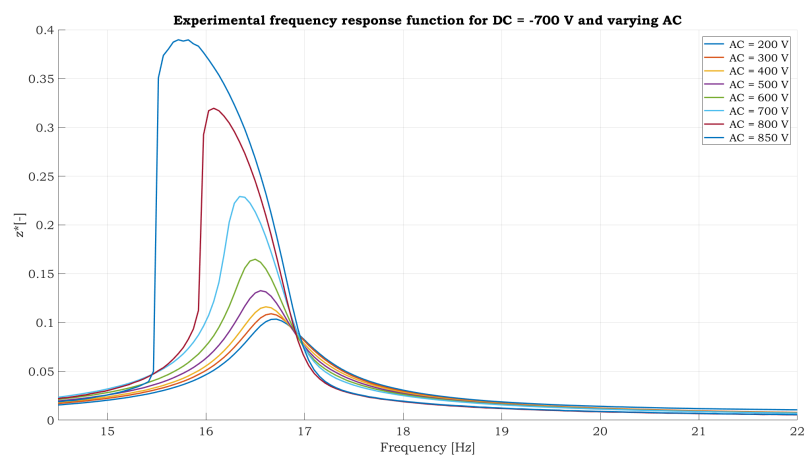


Figure 8.8: Experimental frequency response function found for a DC-voltage of - 700 V and varying AC-voltages.

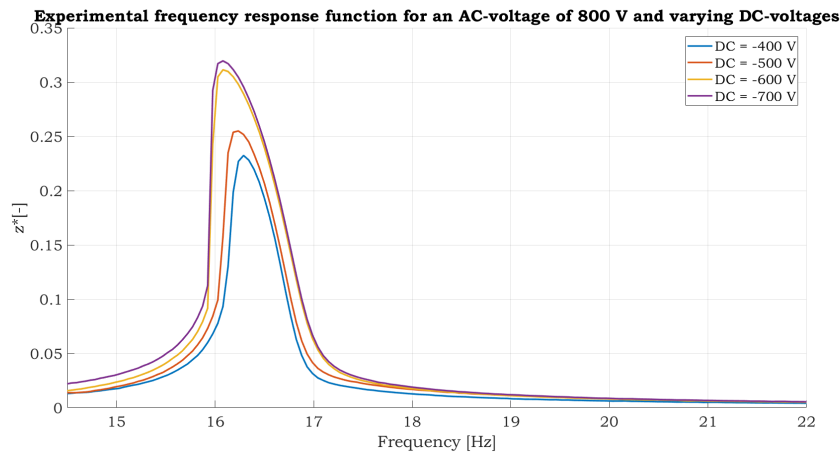


Figure 8.9: Experimental frequency response function found for an AC-voltage of 800 V and varying DC-voltages.

When visually investigating the behaviour of the plate close to the frequency at which the maximum amplitude occurs, for an applied DC-voltage of -700 V and an applied AC-voltage of 850 V, it is observed that multiple modes of the plate are excited. To investigate whether a rotational mode is observed at these frequencies, a figure has been constructed which contains the frequency response functions measured at multiple points on the plate. The measured points are indicated in Figure 8.5. The results that have been obtained are shown in Figure 8.10. Note that Figure 8.10 shows purely the raw data of the experiments because it serves only to investigate whether a rotational mode is observed.

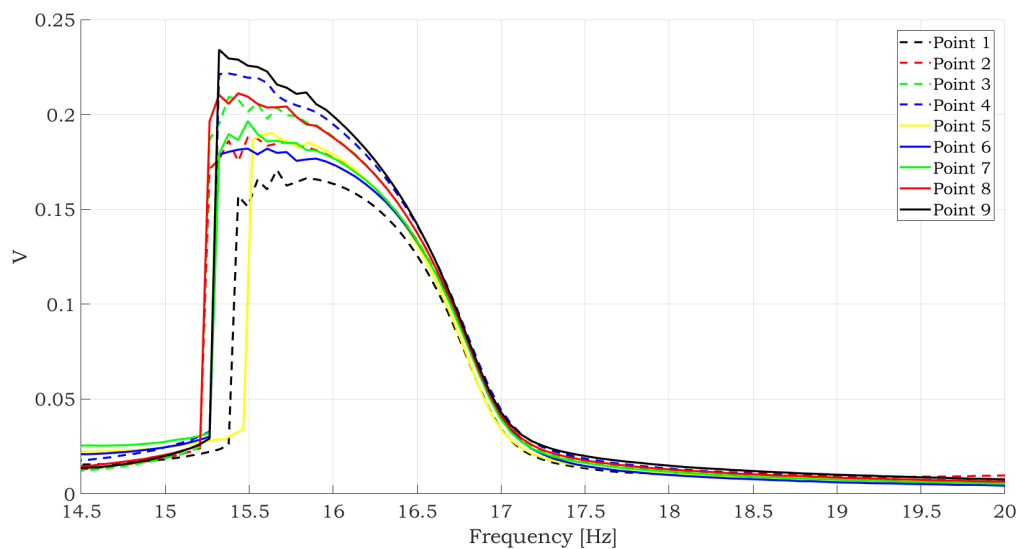


Figure 8.10: Experimental frequency response functions found for multiple points on the plate. The frequency response functions are found when applying a DC-voltage of -700 V and an AC-voltage of 850 V.

8.6. Discussion of the experimental results

From Figure 8.6, Figure 8.7, Figure 8.8, Figure 8.9 and Figure 8.10 some interesting conclusions can be drawn. These are listed below:

- A softening behaviour is found, but only at very high AC-voltages

From Figure 8.6 till Figure 8.9 it is clear the nonlinear behaviour that occurs is a softening behaviour. This is also what was expected from the analytical condition given in subsection 2.4.2. Even though a softening behaviour is indeed observed, this is only apparent at very high AC-voltages. At lower AC-voltages the frequency response shows a linear response.

- As the DC-voltage increases, the occurrence of nonlinear behaviour is observed at lower AC-voltages

In Figure 8.6 it is clear that all the frequency responses show a linear response, no matter how much AC-voltage there is applied. In contrast, Figure 8.7 and Figure 8.8 do show nonlinear behaviour when high AC-voltages are applied. When comparing these three figures it is also clear that as the DC-voltage increases, the nonlinearity is already observed at lower AC-voltages. This is best observed when looking at the cases where the AC-voltage is equal to 700 V. Where at a DC-voltage of -200 V and a DC-voltage of -500 V the frequency response at this AC-voltage do not show a nonlinearity, a slight softening behaviour can be observed in the frequency response when the DC-voltage is set to -700 V.

- As the DC-voltage increases the dimensionless amplitude increases as well

This conclusion is mainly drawn on the observations done in Figure 8.9. In this figure it can be seen that the dimensionless amplitude of motion, for a constant AC-voltage, increases with an increase in the DC-voltage applied. Another conclusion that can be drawn from this figure is that the nonlinear behaviour is better observed when a higher DC-voltage is applied.

- A rotational mode is observed at driving frequencies around 15.5 Hz

In Figure 8.10 it is seen that the amplitudes of the frequency responses differ for different points on the plate. It is seen that the difference in the maximum voltage output is largest between points 1 and 9. This indicates that there is indeed a rotational mode present at a driving frequency of 15.5 Hz, for a DC-voltage of -700 V and an AC-voltage of 850 V. This presumption is further strengthened by noticing that the output amplitude of point 4 is larger than point 6 and that of point 8 larger than point 2.

8.7. Results of comparing the analytical - and the experimental frequency response functions

Here, the analytical frequency response curves will be compared with the experimental frequency response curves. For this comparison, both frequency response functions have been made dimensionless in the y-axis. As stated before, the experimental frequency response function has been made dimensionless in the y-axis by dividing the amplitudes of the oscillations by the stable levitating height. This stable levitation height is found, experimentally, for the DC-voltage which is used to find the frequency response function. These stable levitation heights are presented in section 6.4. The analytical frequency response is dimensionless in the y-axis because the equation of motion has been made dimensionless, as is shown in section 8.1. As a result, the maximum amplitude found when solving this equation of motion is already the dimensionless amplitude of oscillation.

To investigate whether the analytical frequency response matches the experimental frequency response in that it shows softening, the comparisons of the respective frequency responses has been done for experimental curves which showed a clear softening behaviour. For the comparisons it has been chosen to compare the experimental curves which have been found for a DC of -700 V and respective AC-voltages of 800 V and 850 V. The obtained figures are shown in Figure 8.11 and Figure 8.12.

Analytically it was expected that the force used for oscillating the plate could be calculated by Equation 8.3. When solving the equation of motion with this force, with the model presented in Appendix C,

it showed that such a driving force gave unrealistic results. As a result, the force used for making the analytical frequency response functions has been taken as the force for which the peak amplitudes of the experimental - and analytical frequency responses matched.

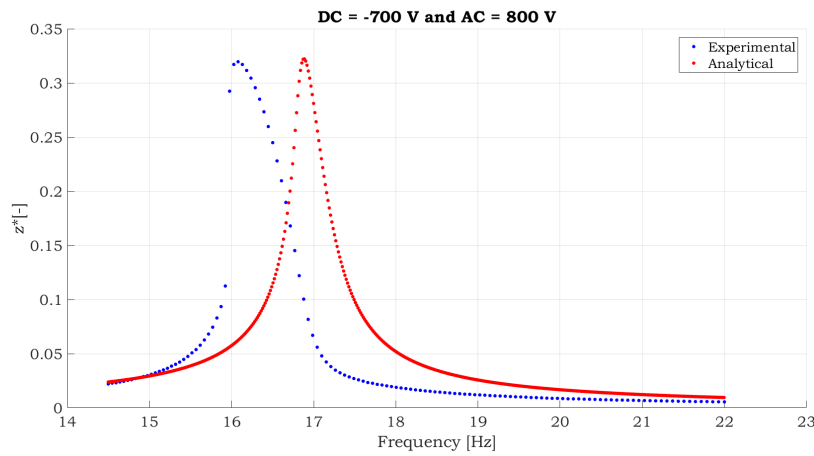


Figure 8.11: The comparison between the experimental frequency response function (blue) and the analytical frequency response function (red) for a DC-voltage of -700 V and an AC-voltage of 800 V.

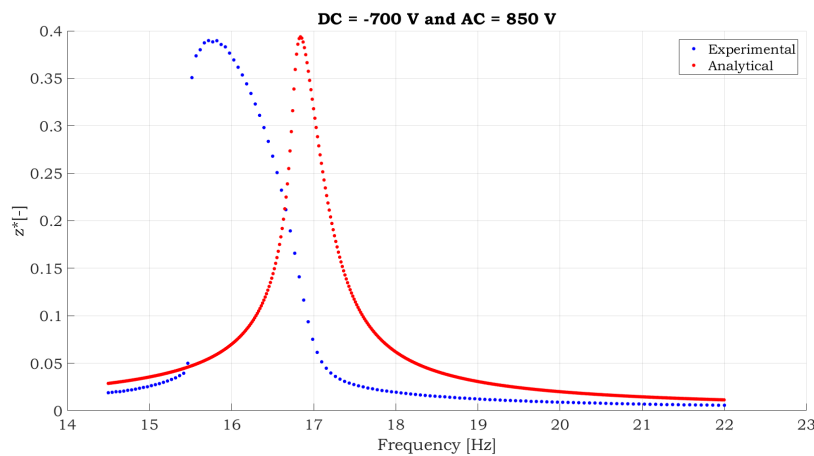


Figure 8.12: The comparison between the experimental frequency response function (blue) and the analytical frequency response function (red) for a DC-voltage of -700 V and an AC-voltage of 850 V.

8.8. Discussion of the comparison of the analytical - and experimental frequency response functions

In comparing the analytical - and experimental frequency response functions it can be concluded that the analytical frequency response cannot accurately predict the softening behaviour seen in the experiments. First of all, it is seen that the analytical frequency response curve is shifted right compared to the experimental frequency response curve. This could be due to a difference in the linear eigenfrequency, which has also been shown in section 7.4. As is explained in section 7.5, this might be due to the fact that the fillets of the magnets are not taken into account whilst calculating the magnetic force on the plate analytically. Furthermore it can be seen that the analytical frequency response functions do not show an explicit softening- or hardening behaviour. This is in sharp contrast with the experimental results, which do show a strong softening behaviour. One reason for this difference could be that in the experiments other modes are captured as well, which might affect the frequency response found. Another reason might be that higher order stiffnesses, which are not taken into account in the analytical model, also play a role in the dynamic movement of the levitating plate in the z-direction.

8.9. Concluding remarks

The aim of this chapter was to investigate the nonlinear dynamic behaviour of the diamagnetically levitating plate. More specifically, the movement of the plate in the z-direction is investigated. To do so, analytical predictions have been made on the nonlinear dynamic behaviour of the plate. Furthermore, experiments have been conducted to find the real life frequency response curves for various AC- and DC-voltages.

First of all, the results from the analytical model have been analysed. With the analytical model the respective linear -, quadratic - and cubic stiffnesses have been calculated for various DC-voltages. From these stiffnesses it was concluded that the magnetic - and electrostatic stiffnesses always oppose each other. Furthermore it was found that the absolute value of the electrostatic stiffnesses increases more than the absolute value of the magnetic stiffnesses with an increase in the DC-voltage applied. As a result, the respective total stiffness curves bend in the same direction as the electrostatic stiffness curves. A last conclusion drawn when analysing the stiffness results was that as the DC-voltage can be set, a preferred set of stiffnesses can be obtained. With the stiffnesses obtained it has been checked whether a softening - or hardening behaviour was expected for the system. This check was done for various DC-voltages. It was found that for all applied DC-voltages a softening behaviour was expected.

Secondly, experimental frequency response curves have been obtained for various DC- and AC-voltages. From these experiments it was concluded that, for high DC- and AC-voltages, indeed a softening behaviour was found in the frequency response curves. For lower DC- and AC-voltages solely a linear response was found. Furthermore it is found that a nonlinear behaviour is observed at lower AC-voltages if the DC-voltage is increased. Lastly it was found that with a decrease in the DC-voltage the dimensionless amplitude increases.

In checking the movement of the plate visually it was seen that multiple modes of the plate have been excited around a frequency of 15.5 Hz. From additional experiments it was found that indeed a rotational mode is excited for driving frequencies around 15.5 Hz.

At the end of this chapter, the analytical frequency response has been compared with the experimental frequency response to investigate if the softening behaviour found in experiments could be predicted analytically. It was found that the analytical frequency response could not predict the softening behaviour as seen in the experimental frequency responses. This might be because other modes are also excited in the experiments or that higher order stiffnesses, which are not taken into account in the model, do have an influence in the experiments.

Conclusions

The goal of this research was to model and experimentally test the dynamic movements of a diamagnetically levitating pyrolytic graphite plate within the nonlinear regime. In this research the plate is levitating over a 2x2 array of permanent magnets where adjacent magnets have an opposite magnetization. Furthermore, the levitating plate is actuated electrostatically. The focus of this research has been on the eigenmode perpendicular to the top plane of the magnets, which is the translational eigenmode in the z-direction.

The first step in this investigation was to construct analytical - and numerical models of the magnetic - and electrostatic force acting on the diamagnetically levitating plate. When comparing the analytical - and numerical models it was found that the magnetic force was very well predicted by the analytical model. For the electrostatic force it was found that it could be calculated well, within the expected range of motion of the plate, with a corrected parallel plate capacitor formula. Here, a correction factor of 0.3915 was found. With these forces, together with the gravitational force, an analytical model has been constructed which could calculate the stable levitation height and the linear - , quadratic - and cubic stiffness values in the z-direction.

To investigate the validity of the analytical predictions, several experiments have been conducted. First of all, experiments have been conducted to investigate how the stable levitation height changes with an increase in the DC-voltage applied to the electrodes. When comparing the analytically calculated stable levitation heights with the experimental results it has been found that both show a decrease in the levitation height as the DC-voltage increases. Furthermore it was found that the analytical results overestimated the experimental results slightly. This could be due to fillets of the magnets in the experimental setup, which are not taken into account in the analytical model. A maximum deviation of 8 percent between the analytical - and experimental results has been found.

Secondly, experiments have been conducted to investigate the shift in the linear eigenfrequency of the translational eigenmode in the z-direction for various DC-voltages applied to the electrodes. Doing so, first the eigenmodes and their associated eigenfrequencies have been found. Three eigenmodes were found. Their associated eigenfrequencies are at 4.6 Hz, 13.63 Hz and 16.75 Hz respectively. The investigated mode in this research, the translational mode in the z-direction, has its eigenfrequency at 16.75 Hz. Therefore, frequencies close to this eigenfrequency are investigated in finding the shift in the linear eigenfrequency for various applied DC-voltages. It is found that the experiments show a decrease in the linear eigenfrequency with an increase in the DC-voltage. This has also been expected from the analytical results. Additionally, from the analytical model it was found that the electrostatic - and magnetic linear stiffness oppose each other. As the linear eigenfrequency decreases with increasing DC-voltages it has been derived that the electrostatic stiffness component increases more, in its absolute values, than the magnetic stiffness component with increasing DC-voltages. This has also been expected from the analytical model. Lastly, it was found that the analytical model overestimates the experimental linear eigenfrequency slightly but that the maximum deviation between the analytical - and experimental results was only 5 percent.

Next, the nonlinear behaviour of the plate in the z-direction has been investigated. In doing so, analytical predictions have been made at first. The linear - , quadratic - and cubic stiffnesses due to the respective magnetic - and electrostatic force have been calculated for various DC-voltages applied.

It was found that the electrostatic stiffnesses increase more with an increase in the DC-voltage than the magnetic stiffnesses. The magnetic - and electrostatic stiffnesses oppose each other. As a result, the total stiffness curves, which are constructed by a summation of the magnetic - and electrostatic stiffnesses for various DC-voltages, bend in the same direction as the electrostatic stiffness curves. Furthermore it was found that a preferred set of stiffnesses could be chosen by increasing or decreasing the applied DC-voltage. With the obtained stiffnesses it was found that a softening behaviour is expected for all DC-voltages for which the stiffnesses were investigated.

When experimentally investigating these predictions it was found that indeed a softening response is found in the experimental frequency response functions. These experimental frequency response functions were obtained when applying high AC- and DC-voltages simultaneously to the electrodes. For relatively low DC- and AC-voltages applied solely a linear response is found. Furthermore it is found that a nonlinear behaviour is observed at lower AC-voltages if the DC-voltage is increased. Lastly it was found that with an increase in the DC-voltage applied, the dimensionless amplitude increases as well.

In checking the movement of the plate visually it was seen that multiple modes of the plate are excited at frequencies between 15.5 and 16 Hz. To investigate which modes were excited at these frequencies, first an FFT analysis was done on multiple points on the levitating plate. This analysis did not show any other excited modes. A second set of experiments has also been done to investigate if any rotational modes are excited for frequencies around 15.5 Hz. The results indeed indicated that a rotational mode is excited at frequencies close to 15.5 Hz.

Lastly, the analytical frequency response has been compared with the experimental frequency response to investigate if the softening behaviour found in experiments could be predicted analytically. It was found that the analytical frequency response could not predict the softening behaviour as seen in the experimental frequency responses. A potential reason for this discrepancy is an excitation of multiple modes within the experiments, which are not captured in the analytical model. Another potential reason is that higher order stiffnesses, not captured in the analytical model, also influence the dynamic movement of the plate in the z-direction.

10

Recommendations

Based on the research done in this study, a few recommendations have been made for future research.

10.1. Investigation of the eigenmodes at driving frequencies close to 15.5 Hz

As is described in section 8.5, in the experiments it is found that multiple modes are excited close to a driving frequency of 15.5 Hz. In this study it has already been shown that a rotational mode is excited at this driving frequency. However, when visually investigating the plate movement at this frequency it was seen that the plate also translates in the x' - and y' -direction (these axes are indicated in Figure 7.2). These translations, and a potential rotation around the z -axis, should be investigated as well. In order to do so, it is advised to use three cameras to investigate the movements. These cameras should capture the top - and the respective side views of the levitating plate. In the recordings all movements of the plate can be captured and extracted.

10.2. Extend the model such that it can calculate both the translational - and rotational stiffnesses

With the model code of Appendix D the stiffnesses in the respective x' -, y' - and z -direction can be calculated. Since this research only focuses on movements in the z -direction, only stiffnesses within this direction have been calculated. It is recommended to do the dynamic analysis in the x' - and y' -direction as well. Furthermore it is recommended to extend the analytical model to include rotational movements around all axes. As such, the rotational stiffnesses could be calculated as well. If this is done, the coupling coefficients between the respective rigid body modes can be calculated and analysed. Doing so, expectations for the dynamic movements in all directions can be derived, taking into account all the potential couplings with other modes.

10.3. Investigate the eigenmode in x' -direction analytically and experimentally

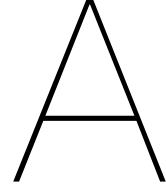
As discussed in the previous recommendation, the current analytical model already allows for calculating the stiffnesses in the x' - and y' -direction. In this research, the eigenfrequency of the x' -mode has also been found. It is recommended to do a dynamic analytical analysis for this mode and to experimentally test the predictions. For such experiments it is advised to extract the movement of the plate by means of a camera which focuses on the top view of the plate. Furthermore it is advised to record the movement from a side views as well to investigate whether other modes are also excited.

10.4. Investigate the rotational eigenmode around the y' -axis analytically and experimentally

If the model is extended to include rotational movements, as advised in section 10.2, the rotational stiffnesses can be calculated. In the present research the rotational eigenmode around the y' -axis has already been found. For future research it is recommended to investigate this mode both analytically and experimentally. For such experiments the setup for finding the eigenmodes of the plate, presented in this research, can be used.

10.5. Put the full electrostatic force formula into the equation of motion and solve it to obtain the analytical frequency response function

In this research the analytical equation of motion is constructed with the use of the linear -, quadratic - and cubic stiffnesses due to the respective magnetic- and electrostatic forces. Here, the stiffnesses are calculated by taking the derivatives of these respective forces. For future research it would be interesting to see if the analytical frequency response changes when inserting the full electrostatic force formula in the equation of motion, instead of the derived stiffnesses from this force formula. Potentially, the softening behaviour can be captured better if the electrostatic force formula is inserted fully into the equation of motion.



Nondimensionalisation of the nonlinear equation of motion

This appendix explains how a nonlinear equation of motion is nondimensionalised. With nondimensionalising the equation of motion each factor in the equation of motion will be dimensionless due to a perfect balance in the units of the parameters which make up the original equation of motion. In nondimensionalising the equation of motion the relative importance of the factors can be compared. The equation of motion which will be nondimensionalised is shown below.

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + k_3 x^3 + k_2 x^2 + k_1 x + k_4 = f \cos(\omega t) \quad (\text{A.1})$$

The first step in nondimensionalising this equation of motion (EoM) is to make the EoM dimensionless in time. This is done by introducing the dimensionless time, t^* , as a function of the linear eigenfrequency and the real time t as shown in Equation A.2.

$$t^* = \omega_0 t \quad (\text{A.2})$$

Using the dimensionless time, the first and second derivatives of x with respect to the real time can be rewritten. This is done as shown in Equation A.3 and Equation A.4

$$\frac{dx}{dt} = \frac{dt^*}{dt} \frac{dx}{dt^*} = \omega_0 \frac{dx}{dt^*} \quad (\text{A.3})$$

$$\frac{d^2 x}{dt^2} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d}{dt^*} \frac{dt^*}{dt} \left(\omega_0 \frac{dx}{dt^*} \right) = \omega_0^2 \frac{d^2 x}{dt^{*2}} \quad (\text{A.4})$$

Inserting Equation A.2, Equation A.3 and Equation A.4 into Equation A.1 gives the EoM which is dimensionless in time. This EoM is depicted in Equation A.5.

$$m \omega_0^2 \frac{d^2 x}{dt^{*2}} + c \omega_0 \frac{dx}{dt^*} + k_3 x^3 + k_2 x^2 + k_1 x + k_4 = f \cos\left(\frac{\omega t^*}{\omega_0}\right) \quad (\text{A.5})$$

The next step in making the EoM fully dimensionless is to nondimensionalize it with respect to a specific length. The dimensionless length x^* is introduced as a function of the length x and the specific length Q_0 . This is shown in Equation A.6.

$$x^* = \frac{x}{Q_0} \quad (\text{A.6})$$

Inserting Equation A.6 into Equation A.5 gives

$$m \omega_0^2 Q_0 \frac{d^2 x^*}{dt^{*2}} + c \omega_0 Q_0 \frac{dx^*}{dt^*} + k_3 Q_0^3 x^{*3} + k_2 Q_0^2 x^{*2} + k_1 Q_0 x^* + k_4 = f \cos\left(\frac{\omega t^*}{\omega_0}\right) \quad (\text{A.7})$$

Dividing all the respective factors of Equation A.7 by $m\omega_0^2 Q_0$ gives the nondimensionalised equation of motion as described in Equation A.8.

$$\frac{d^2 x^*}{dt^{*2}} + \frac{c}{m\omega_0} \frac{dx^*}{dt^*} + \frac{k_3 Q_0^2}{m\omega_0^2} x^{*3} + \frac{k_2 Q_0}{m\omega_0^2} x^{*2} + \frac{k_1}{m\omega_0^2} x^* + \frac{k_4}{m\omega_0^2 Q_0} = \frac{f \cos(\frac{\omega t^*}{\omega_0})}{m\omega_0^2 Q_0} \quad (\text{A.8})$$

Equation A.8 is fully dimensionless now. Note that the nondimensionalised linear stiffness should be equal to 1 since $\frac{k_1}{m} = \omega_0^2$.

B

High Voltage Amplifier and its low voltage connection scheme

The High Voltage Amplifier (HVA) is an integral part of this thesis. Without the amplifier, the necessary voltages to excite the levitating plate cannot be reached. Thus the HVA is absolutely necessary to test whether the experiments show the same results as the analytical model predicts. The High Voltage Amplifier itself is shown in Figure B.1.

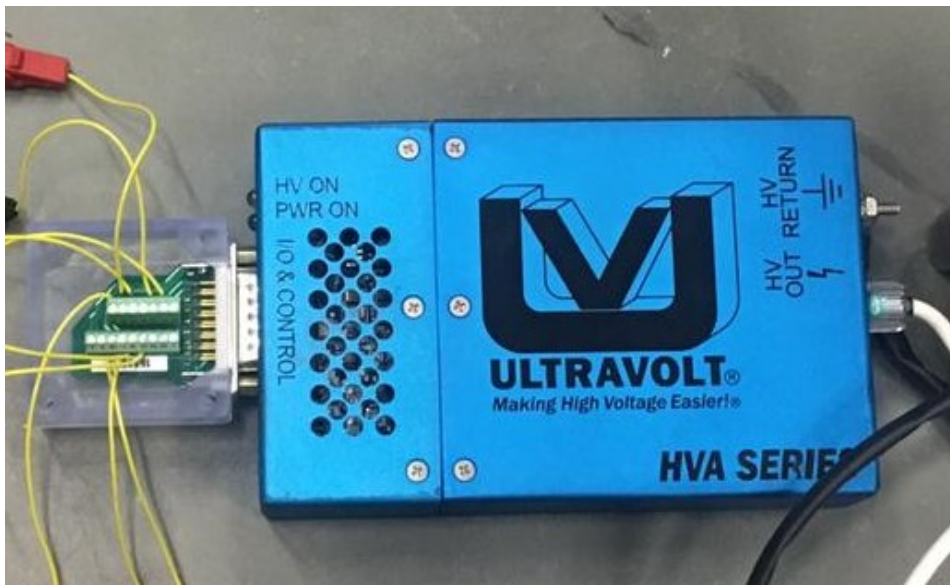


Figure B.1: The used HVA (500x). The low voltage connections on the left side are the input of the HVA whilst the right side shows the high voltage output of the HVA.

For the HVA to work correctly, several devices need to be connected to its low voltage side. These devices are listed below:

1. Function generator
2. DC-Source with a 24 V output (and a current < 400 mA)
3. DC-Source with a 10 V output
4. DC-Source with a tunable output between 0 and 10 V

The way in which the separate devices need to be connected to the HVA is indicated in Figure B.2. Note that in this connection scheme, the oscilloscope is also connected to the HVA. Although the

oscilloscope is not necessary for the HVA to work it is very useful to identify if something is off. It should also be noted that the device used as the function generator can be changed when different sorts of inputs are needed. The other devices are necessary to make sure for the device to work.

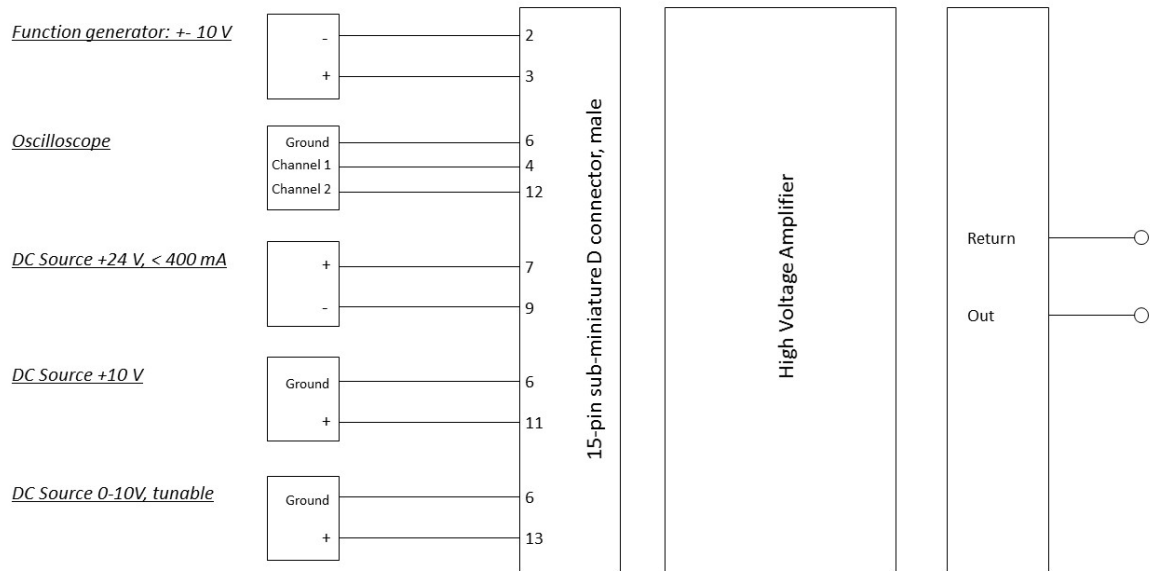
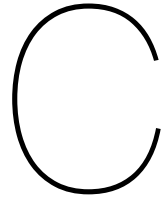


Figure B.2: Connection diagram of the High Voltage Amplifier. For the amplifier to work one needs to connect a function generator, a DC source with a 24 V output (and a current $< 400\text{ mA}$), a DC source of 10 V and an DC source with a tunable voltage between 0 and 10 V . Furthermore, an oscilloscope can also be connected to check if something is off regarding the voltage output.



Matlab code for finding the analytical frequency response function

```
1 %% This file serves to construct the analytical frequency response function for DC = -700 V
   and AC = 850 V
2 % Tjebbe de Lint
3 % Graduation project
4 clear all
5 close all
6 clc
7
8 %% Import the experimental data and post-process it
9 %% Convert ratio for the experimental data
10 V_to_v_125 = 125; % [mm/s/V], voltage to velocity conversion ratio
11 Attenuator = 2; % [-], The attenuator damped the signal by a factor two
12 h = 1.2043e+03/1000; % [mm], stable levitation height for DC = -700 V
13
14 %% DC= -700 V AC = 850 V
15 %% Importing the data
16 Freq_DC_min700_AC_850 = xlsread('D:\Documents\Zurich\Instruments\LabOne\WebServer\
   session_20210608_091656_00\Exp_24_000\dev1684_demods_0_sample_00000.csv',1,'E9:EX9'); %
   frequency [Hz]
17 Ampl_DC_min700_AC_850 = xlsread('D:\Documents\Zurich\Instruments\LabOne\WebServer\
   session_20210608_091656_00\Exp_24_000\dev1684_demods_0_sample_00000.csv',1,'E16:EX16');
   %amplitude [V]
18
19 %% Converting the amplitudes from Voltage to velocities
20 Ampl_DC_min700_AC_850 = Attenuator*V_to_v_125.*Ampl_DC_min700_AC_850; % [mm/s]
21
22 %% Converting the velocity to a displacement
23 Disp_DC_min700_AC_850 = Ampl_DC_min700_AC_850./(2*pi*Freq_DC_min700_AC_850); % [mm]
24
25 %% Make the displacement dimensionless in the y-axis
26 Disp_dimensionless_DC_min700_AC_850 = Disp_DC_min700_AC_850/h; % [-]
27
28 %% Plot the dimensionless figure
29 figure
30 hold on
31 plot(Freq_DC_min700_AC_850*2*pi, Disp_dimensionless_DC_min700_AC_850)
32 title('Amplitude [-] vs omega [rad/s] DC=-700V and AC=850V')
33 xlabel('omega [rad/s]')
34 ylabel('z [-]')
35
36
37
38
39 %% Calculating the frequency response function with ODE45, for DC = -700 V and AC = 850 V
40 %% Parameter values for the equation of motion
41 m = 5.5e-05; % [kg]
42 Q = 23.05;
```

```

43 c = 1/(2*23.05); % [Ns/m]
44 f = 0.0000058; % [N], force applied to the plate
45
46
47
48 %%% Calculate f
49 %% Side_length = 0.01;
50 %% A = 0.488*Side_length^2;
51 %% V_DC = 700;
52 %% V_AC = 850;
53 %% e_0 = 8.85e-12;
54 %% e_r = 1.0006;
55 %% f = e_0*e_r*A*V_DC*V_AC/((h1)^2);
56
57 %% Parameter values for the ODE45, obtained from the analytical mathematica model
58 h1 = 0.00119753; % [m], stable levitation height
59
60 alpha1_mag = 0.709439; % [N/m], magnetic linear stiffness
61 alpha2_mag = -533.436; % [N/m^2], magnetic quadratic stiffness
62 alpha3_mag = 347584; % [N/m^3], magnetic cubic stiffness
63
64 alpha1_el = -0.336106/4; % [N/m], electrostatic linear stiffness
65 alpha2_el = 421/4; % [N/m^2], electrostatic quadratic stiffness
66 alpha3_el = -468745/4; % [N/m^3], electrostatic cubic stiffness
67
68 alpha1 = abs(alpha1_mag+alpha1_el); % [N/m], absolute value of the total linear stiffness
69 alpha2 = abs(alpha2_mag+alpha2_el); % [N/m^2], absolute value of the total quadratic
    stiffness
70 alpha3 = abs(alpha3_mag+alpha3_el); % [N/m^3], absolute value of the total cubic stiffness
71
72
73 f_0 = sqrt(abs(alpha1)/m)/(2*pi); % [Hz], linear eigenfrequency
74 omega_0 = sqrt(abs(alpha1)/m); % [rad/s]
75
76 % Over which frequencies the frequency sweep is performed
77 omega = [14.5*2*pi:0.1:22*2*pi]; % range over which the frequency is swept [rad/s]
78
79 %% Checking the values of all the parameters in the nondimensionalised equation of motion
80 alpha_1_check = ((alpha1)/(m*omega_0^2));
81 alpha_2_check = -((alpha2*h1)/(m*omega_0^2));
82 alpha_3_check = ((alpha3*h1^2)/(m*omega_0^2));
83 f_check = (f*cos(omega(1)/omega_0)/(m*(omega_0^2)*h1));
84
85 %% Performing the frequency sweep and obtaining the frequency response
86
87 % Time span and initial values
88 y0 = [0; 0]; % initial values
89 tspan = [0:0.1:2000]; % [-], timespan which is the dimensionless time
90
91 tic
92 for i = 1:length(omega)
93     omega1 = omega(i)
94     [t_tot, y_tot] = ode45(@(t,y) equations1(t,y,m,alpha1,alpha2, alpha3,f,omega_0,omega1
        ,h1,c), tspan, y0);
95     y_k = y_tot(:,1); % [-], Dimensionless displacement
96     y_k1 = y_tot(:,2); % [-], Dimensionless velocity
97     t_k = t_tot; % [-], Dimensionless time
98     y_k([1:1:round(length(t_k)/2)]) = []; % Get rid of the transient behaviour
99     y_k1([1:1:round(length(t_k)/2)]) = []; % Get rid of the transient behaviour
100    t_k([1:1:round(length(t_k)/2)]) = [];
101
102    max_amp(i) = max(abs(y_k)); % [-] Maximum dimensionless displacement
103    max_amp1(i) = max(abs(y_k1)); % [-] Maximum dimensionless velocity
104 end
105 toc
106
107 %% Plot the analytical - and experimental frequency response functions
108 figure
109 hold on
110 grid on
111 set(gca, 'FontName', 'Bookman Old Style', 'FontSize', 18, 'Box', 'off')

```

```

112 plot(Freq_DC_min700_AC_850, Disp_dimensionless_DC_min700_AC_850, 'b.', 'Markersize', 14)
113 plot((omega/(2*pi)), max_amp, 'r.', 'Markersize', 14)
114 title('DC=-700V and AC=850V')
115 xlabel('Frequency [Hz]')
116 ylabel('z*[-]')
117 legend('Analytical', 'Theoretical')
118
119
120
121
122
123
124 %% Function, dimensionless EoM
125 function equationsv1 = equations1(t,y,m,alpha1,alpha2,alpha3,f,omega_0,omega,h,c)
126 equationsv1= [y(2); (-c)*y(2)-((alpha3*h^2)/(m*omega_0^2))*y(1)^3+((alpha2*h)/(m*omega_0^2))
               *y(1)^2-((alpha1)/(m*omega_0^2))*y(1)+(f*cos(omega*t/omega_0)/(m*(omega_0^2)*h))];
127 end

```


D

Code of the analytical (mathematica)
model

Analytical model mathematica code

```

In[ ]:= $Assumptions =
    x ∈ Reals && y ∈ Reals && z ∈ Reals && x0 ∈ Reals && y0 ∈ Reals && z0 ∈ Reals && z > z0;

Nf = 1;
Lx := 12 × 10-3; (* Size of magnets [m] *)
Ly := 12 × 10-3;
Lz := 12 × 10-3;
Xix := -85 × 10-6;
(*Magnetic susceptibility of a pyrolytic graphite plate in the x-direction [-] *)
Xiy := -85 × 10-6;
(*Magnetic susceptibility of a pyrolytic graphite plate in the y-direction [-] *)
Xiz := -450 × 10-6;
(*Magnetic susceptibility of a pyrolytic graphite plate in the z-direction [-] *)
Lp := 10 × 10-3; (*Side length of the levitating plate [m] *)
Lpdiaq := Sqrt[2] Lp / 2; (* Half of the diagonal of the plate [m] *)
tplate := 0.25 × 10-3; (*Thickness of the levitating plate [m] *)

mu0 := 4 Pi 10-7; (*Magnetic permeability of free space [H/m] *)
Br = 1.4; (* Remanent Magnetization of the permanent magnets [T] *)
densplate = 2200; (*Mass density of the pyrolytic graphite plate [kg/m3] *)
mplate = Lp2 tplate densplate; (*Mass of the levitating plate [kg] *)
ggrav = 9.81; (*Gravitational acceleration [m/s2] *)

mplate * ggrav; (*Gravitational force on the plate[N] *)

r := ((x - x0)2 + (y - y0)2 + (z - z0)2)(1/2); (*Distance between a
point in space (x,y,z) and the source of the magnetic field (x0,y0,z0).*)

In[ ]:= (* The function Psi is the double integral over x0 and y0 of
1/4PI 1/r over a rectangle along x and y axes. It was determined
using Mathematica and using some manual simplifications. *)

In[ ]:= Psi[x_, y_, z_, x0_, y0_, z0_, M0_] :=
    M0 / (8 Pi) (-2 (z - z0) ArcTan[(x - x0) (y - y0) / (r (z - z0))] +
        (y - y0) Log[(x - x0 + r) / (x0 - x + r)] + 2 (x - x0) Log[y - y0 + r])

In[ ]:= Psi[x_, y_, z_, x0_, y0_, z0_, M0_] :=

$$\frac{1}{8 \pi} M_0 \left( -2 (z - z_0) \operatorname{ArcTan} \left[ \frac{(x - x_0) (y - y_0)}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} (z - z_0)} \right] + \right.$$


$$(y - y_0) \operatorname{Log} \left[ \frac{x - x_0 + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}}{-x + x_0 + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} \right] +$$


$$\left. 2 (x - x_0) \operatorname{Log} [y - y_0 + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}] \right)$$


```

```
(*Double check integral, by double derivation,
to get the 'Green's function kernel' which gives the magnetic potential Psi due
to a magnetic charge at position (x0,y0,z0) evaluated at position (x,y,z).*)
Simplify[D[D[Psi[x, y, z, x0, y0, z0, M0], x0], y0]]
```

```
(*To determine the effect of the magnetic charges of a
surface of the magnet on the potential Psi at location (x,y,z),
evaluate the integral Psi over a surface of the magnet. This
surface is bounded by (Lxmin, Lxmax) and (Lymin, Lymax).*)
```

```
In[ ]:= Psiixint[x_, y_, z_, Lxmax_, Lxmin_, y0_, z0_, M0_] :=
Psi[x, y, z, Lxmax, y0, z0, M0] - Psi[x, y, z, Lxmin, y0, z0, M0]
Psiint[x_, y_, z_, Lxmax_, Lxmin_, Lymax_, Lymin_, z0_, M0_] :=
Psiixint[x, y, z, Lxmax, Lxmin, Lymax, z0, M0] -
Psiixint[x, y, z, Lxmax, Lxmin, Lymin, z0, M0]
```

Psiint: This represent the magnetic scalar potential at a location (x,y,z) with the magnetic source at the top surface of a magnet, with magnetization M0 along the positive z-axis, when this top surface is bounded by (Lxmax,Lymax) and (Lxmin,Lymin). For the bottom surface the same equation can be used but instead of M0 one should use -M0.

Note that the final magnetic scalar potential is a function only of the top and bottom surfaces of the magnet.

Now for 1 full magnet add top and bottom surface:

```
In[ ]:= Psi1M[x_, y_, z_, Lxmax_, Lxmin_, Lymax_, Lymin_, Lzmax_, Lzmin_, M0_] :=
Psiint[x, y, z, Lxmax, Lxmin, Lymax, Lymin, Lzmax, M0] +
Psiint[x, y, z, Lxmax, Lxmin, Lymax, Lymin, Lzmin, -M0]
```

Psi1Mtest-> 1 magnet

Psi4Mtest->2x2 array of magnets where adjacent magnets have an opposite magnetization.

```
Psi1Mtest = Psi1M[x, y, z, 1 Lx, 0, 1 Ly, 0, 0, -1 Lz, Mmag];
(*Magnetic scalar potential at a position (x,y,z) for 1 magnet
bounded by (Lxmax,Lymax), (Lxmin,Lymin) and (Lzmin, Lzmax).*)
Psi4Mtest = Psi1M[x, y, z, 1 Lx, 0, 1 Ly, 0, 0, -1 Lz, 1 Mmag] +
Psi1M[x, y, z, 0, -1 Lx, 1 Ly, 0, 0, -1 Lz, -1 Mmag] +
Psi1M[x, y, z, 0, -1 Lx, 0, -1 Ly, 0, -1 Lz, 1 Mmag] +
Psi1M[x, y, z, 1 Lx, 0, 0, -1 Ly, 0, -1 Lz, -1 Mmag];
(*Magnetic scalar potential at (x,y,z) due to a 2x2 array of magnets
where adjacent magnets have an opposite magnetic field.*)

Hx4Mtest = -D[Psi4Mtest, x]; (*Magnetic field strength in the x-direction [A/m].*)
Hy4Mtest = -D[Psi4Mtest, y]; (*Magnetic field strength in the y-direction [A/m].*)
Hz4Mtest = -D[Psi4Mtest, z]; (*Magnetic field strength in the z-direction [A/m].*)

Hz1Mtest = -D[Psi1Mtest, z];
(*Magnetic field strength in the z-direction due to only 1 magnet [A/m].*)
```

```
In[ ]:= (* Magnetic force equations *)
```

```
MagFDxtest = D[Xix Hx4Mtest^2 + Xiy Hy4Mtest^2 + Xiz Hz4Mtest^2, x];
(* Magnetic force in the x-direction. Note how the mu0/2
term has not been added here. This term will be added later.*)
```

```

MagFDytest = D[Xix Hx4Mtest^2 + Xiy Hy4Mtest^2 + Xiz Hz4Mtest^2, y];
(* Magnetic force in the y-direction. Note how the mu0/2
term has not been added here. This term will be added later.*)

MagFDztest = D[Xix Hx4Mtest^2 + Xiy Hy4Mtest^2 + Xiz Hz4Mtest^2, z];
(* Magnetic force in the z-direction. Note how the mu0/2
term has not been added here. This term will be added later.*)

MagFDztest45 =
D[Xix Hx4Mtest^2 + Xiy Hy4Mtest^2 + Xiz Hz4Mtest^2, z] /. x -> Sqrt[1/2] (X - Y) /.
y -> Sqrt[1/2] (Y + X); (*Magnetic force in the z-direction. Here,
the coordinate system is rotated 45 degrees counterclockwise around the z-
axis. This is done so that the integration over the levitating plate can be done
easier. Another benefit is that this decreases the calculation time of the model.*)

```

Calculating the stable levitation height and the respective stiffnesses in the z-direction

```

ccor = 0.3915; (*Correction factor for the electrostatic force [-]*)
eR = 1.000589; (*Relative permittivity of air [-]*)
Vdc = 700; (*Applied DC-voltage [V]*)
Vac = 800; (*Applied AC-voltage [V]*)
e0 = 8.85 * 10^(-12); (*Electrical permittivity of free space [F/m]*)
Aelectrostatic = 0.48897;
(*Relative area (for the electrostatic force) compared to the total area of the
plate [-]. This is the overlap area between the electrodes and the plate.*)

Pel[z_] := -ccor * Aelectrostatic * e0 * eR * (Vdc^2 + 0.5 * Vac^2) / (2 * z^2);
(*Electrostatic pressure [N/m^2]*)
Pg = -mplate ggrav / Lp^2; (*Gravitational pressure [N/m^2]*)
Pmag[z_] := (mu0 / 2) tplate MagFDztest45; (*Magnetic pressure [N/m^2]*)
Fg = -mplate * ggrav; (*Gravitational force [N]*)

Pztot = Pel[z] + Pg + Pmag[z]; (*Total pressure on the plate [N/m^2]*)

Fplatetot[z_] := 4 NIntegrate[Pztot /. z -> zh, {X, 0, Lp/2}, {Y, 0, Lp/2}];
(*Total force on the plate as a function of z [N]*)

Feltot[z_] := 4 NIntegrate[Pel[z] /. z -> zh, {Y, 0, Lp/2}, {Y, 0, Lp/2}];
(*Electrostatic force on the plate as a function of z [N]*)

Fmag[z_] := 4 NIntegrate[Pmag[z] /. z -> zh, {X, 0, Lp/2}, {Y, 0, Lp/2}];
(*Magnetic force on the plate as a function of z [N]*)

FindRoot[Fplatetot[z] - Feltot[z] - Fmag[z], {z, 0.001}] (*Finding the stable
levitation height. The search starts at a levitation height of 1 mm *)

hlev = z /. % (*Stable levitation height [m]*)

k1el = -4 D[Pel[z] Lp^2, {z, 1}] /.
z -> hlev (*Linear electrostatic stiffness [N/m]. This stiffness is
evaluated at the stable levitation height. Note that this stiffness is
overestimated by a factor 4. This is compensated for in the matlab files.*)

```

```

k2e1 = -1/2 × 4 D[Pe1[z] Lp^2, {z, 2}] /. z → hlev
(*Quadratic electrostatic stiffness [N/m^2]. This stiffness is
evaluated at the stable levitation height. Note that this stiffness is
overestimated by a factor 4. This is compensated for in the matlab files.*)

k3e1 = -1/6 × 4 D[Pe1[z] Lp^2, {z, 3}] /. z → hlev
(*Cubic electrostatic stiffness [N/m^3]. This stiffness is evaluated at
the stable levitation height. Note that this stiffness is overestimated
by a factor 4. This is compensated for in the matlab files.*)

(* k4=-1/24 4 NIntegrate[ D[Pztot,{z,4}]/. z→ hlev,{x,0,Lp},{y,0,Lp}]
k5=-1/120 4 NIntegrate[ D[Pztot,{z,5}]/. z→ hlev,{x,0,Lp},{y,0,Lp}] *)

k1mag = -4 NIntegrate[ D[Pmag[z], {z, 1}] /. z → hlev, {X, 0, Lp/2}, {Y, 0, Lp/2}]
(*Linear magnetic stiffness [N/m]. This stiffness
is evaluated at the stable levitation height.*)

k2mag = -1/2 × 4 NIntegrate[ D[Pmag[z], {z, 2}] /. z → hlev, {X, 0, Lp/2}, {Y, 0, Lp/2}]
(*Quadratic magnetic stiffness [N/m^2]. This
stiffness is evaluated at the stable levitation height.*)

k3mag = -1/6 × 4 NIntegrate[ D[Pmag[z], {z, 3}] /. z → hlev, {X, 0, Lp/2}, {Y, 0, Lp/2}]
(*Cubic magnetic stiffness [N/m^3]. This
stiffness is evaluated at the stable levitation height.*)

(* k4=-1/24 4 NIntegrate[ D[Pztot,{z,4}]/. z→ hlev,{x,0,Lp/2},{y,0,Lp/2}]
k5=-1/120 4 NIntegrate[ D[Pztot,{z,5}]/. z→ hlev,{x,0,Lp/2},{y,0,Lp/2}] *)

```


Bibliography

- [1] EH Brandt. "Levitation in physics". In: *Science* 243.4889 (1989), pp. 349–355.
- [2] Xianfeng Chen et al. "Rigid body dynamics of diamagnetically levitating graphite resonators". In: *Applied Physics Letters* 116.24 (2020), p. 243505.
- [3] Paul A Tipler and Gene Mosca. *Physics for scientists and engineers*. 6th. Macmillan, 2008.
- [4] Joël Abadie et al. "Modeling and experimentation of a passive low frequency nanoforce sensor based on diamagnetic levitation". In: *Sensors and Actuators A: Physical* 173.1 (2012), pp. 227–237.
- [5] François Barrot, Jan Sandtner, and Hannes Bleuler. "Acceleration sensor based on diamagnetic levitation". In: *IUTAM Symposium on Vibration Control of Nonlinear Mechanisms and Structures*. Springer. 2005, pp. 81–90.
- [6] P Pinot and Z Silvestri. "New laser power sensor using diamagnetic levitation". In: *Review of Scientific Instruments* 88.8 (2017), p. 085003.
- [7] BP Mann and ND Sims. "Energy harvesting from the nonlinear oscillations of magnetic levitation". In: *Journal of sound and vibration* 319.1-2 (2009), pp. 515–530.
- [8] G Kustler. "Diamagnetic levitation-historical milestones". In: *Revue Roumaine Des Sciences Techniques Serie Electrotechnique Et Energetique* 52.3 (2007), p. 265.
- [9] Roland Jackson. "John Tyndall and the Early History of Diamagnetism". In: *Annals of Science* 72.4 (2015), pp. 435–489.
- [10] MD Simon and AK Geim. "Diamagnetic levitation: flying frogs and floating magnets". In: *Journal of applied physics* 87.9 (2000), pp. 6200–6204.
- [11] Michael V Berry and Andre K Geim. "Of flying frogs and levitrons". In: *European Journal of Physics* 18.4 (1997), p. 307.
- [12] Mehdi Boukallel, Joël Abadie, and Emmanuel Piat. "Levitated micro-nano force sensor using diamagnetic materials". In: *2003 IEEE International Conference on Robotics and Automation (Cat. No. 03CH37422)*. Vol. 3. IEEE. 2003, pp. 3219–3224.
- [13] S Palagummi, J Zou, and FG Yuan. "A horizontal diamagnetic levitation based low frequency vibration energy harvester". In: *Journal of Vibration and Acoustics* 137.6 (2015).
- [14] Sri Vikram Palagummi and Fuh-Gwo Yuan. "An enhanced performance of a horizontal diamagnetic levitation mechanism-based vibration energy harvester for low frequency applications". In: *Journal of Intelligent Material Systems and Structures* 28.5 (2017), pp. 578–594.
- [15] S Palagummi and FG Yuan. "An optimal design of a mono-stable vertical diamagnetic levitation based electromagnetic vibration energy harvester". In: *Journal of Sound and Vibration* 342 (2015), pp. 330–345.
- [16] S Palagummi and FG Yuan. "A bi-stable horizontal diamagnetic levitation based low frequency vibration energy harvester". In: *Sensors and Actuators A: Physical* 279 (2018), pp. 743–752.
- [17] L Liu and FG Yuan. "Nonlinear vibration energy harvester using diamagnetic levitation". In: *Applied Physics Letters* 98.20 (2011), p. 203507.
- [18] S Palagummi and FG Yuan. "A vibration energy harvester using diamagnetic levitation". In: *Active and Passive Smart Structures and Integrated Systems 2013*. Vol. 8688. International Society for Optics and Photonics. 2013, 86880N.
- [19] XY Wang et al. "A magnetically levitated vibration energy harvester". In: *Smart Materials and Structures* 22.5 (2013), p. 055016.
- [20] Chasity B Coleman et al. "Diamagnetic levitation changes growth, cell cycle, and gene expression of *Saccharomyces cerevisiae*". In: *Biotechnology and bioengineering* 98.4 (2007), pp. 854–863.

- [21] Ahmet Cansiz. "Static and dynamic analysis of a diamagnetic bearing system". In: *journal of applied physics* 103.3 (2008), p. 034510.
- [22] Jie-Yu Chen, Jian-Bin Zhou, and Guang Meng. "Diamagnetic bearings for MEMS: Performance and stability analysis". In: *Mechanics Research Communications* 35.8 (2008), pp. 546–552.
- [23] Q Li, K-S Kim, and A Rydberg. "Lateral force calibration of an atomic force microscope with a diamagnetic levitation spring system". In: *Review of scientific instruments* 77.6 (2006), p. 065105.
- [24] Ron Pelrine et al. "Diamagnetically levitated robots: An approach to massively parallel robotic systems with unusual motion properties". In: *2012 IEEE International Conference on Robotics and Automation*. IEEE. 2012, pp. 739–744.
- [25] Michael A. Gottlieb and Rudolf Pfeiffer. *The magnetism of matter*. URL: https://www.feynmanlectures.caltech.edu/II_34.html. accessed: 10.08.2020.
- [26] Lei Liu and FG Yuan. "Diamagnetic levitation for nonlinear vibration energy harvesting: theoretical modeling and analysis". In: *Journal of sound and vibration* 332.2 (2013), pp. 455–464.
- [27] Giorgio De Pasquale et al. "Characterization of low-stiffness suspensions based on diamagnetic levitation for MEMS energy harvesters". In: *Proc. Power MEMS, Leuven, Belgium* (2010), pp. 77–80.
- [28] Edward P Furlani. *Permanent magnet and electromechanical devices: materials, analysis, and applications*. Academic press, 2001.
- [29] Mohammad I Younis. *MEMS linear and nonlinear statics and dynamics*. Vol. 20. Springer Science & Business Media, 2011.
- [30] Farbod Alijani. "ME46072: Nonlinear Dynamics, Lecture 7: Forced nonlinear oscillators". unpublished. 2019.
- [31] Ali Hasan Nayfeh and Dean T Mook. *Nonlinear Oscillations*. John Wiley & Sons INC., 1995.
- [32] COMSOL. "COMSOL Multiphysics AC/DC Module User's Guide Version 5.4". 2018.