A multigrid method for matrix differential equations

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ABSTRACT

We consider the numerical solution of Sylvester and Lyapunov matrix differential equations with multigrid. Such equations have many applications, for example, in the field of numerical control, model reduction and for the computation of second moments (variance) in systems modeled by differential equations with stochastic coefficients.

The above applications are characterised by a low-rank structure of the right hand side in the differential equation. This enables a compression of the N^2 matrix element unknowns to only $O(N\log(\epsilon))$ significant entries, where ϵ is small number. When this low-rank compression is used throughout the multigrid process, a significant reduction of the solution time can be achieved. While a standard multigrid method would require $O(N^2)$ computations, here we need only $O(N\log(N))$.

The use of multigrid to solve the time-invariant Sylvester equations has been investigated before, in work by Grasedyck and Hackbusch [1]. Here we concentrate on the extension of the algorithm to the time-dependent case. We will show that the combination with time-stepping requires a special treatment for the compression of the unknowns. The performance of the algorithm will be illustrated by numerical experiments.

REFERENCES

[1] L. Grasedyck and W. Hackbusch. A Multigrid Method to Solve Large Scale Sylvester Equations. *Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig*, Preprint No. 48, 2004.