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Optimal matching for ridesharing systems with endogenous and flexible user participation

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ABSTRACT

The performance of ridesharing systems is intricately entwined with user participation. To characterize such interplay, we adopt a repeated multi-player, non-cooperative game approach to model a ridesharing platform and its users' decision-making. Users reveal to the platform their participation preferences over being only riders, only drivers, flexible users, and opt-out based on the expected utilities of each mode. The platform optimally matches users with different itineraries and participation preferences to maximize social welfare. We analytically establish the existence and uniqueness of equilibria and design an iterative algorithm for the solution, for which convergence is guaranteed under mild conditions. A case study is conducted with real travel demand data in Chicago. The results highlight the effect of users' flexibility regarding mode preferences on system performance (i.e., the average utility of users and the percentage of successful matches). A sensitivity analysis on the level of subsidy and the distribution of utility between matched riders and drivers shows that uneven distributions of utility may lead to a higher percentage of successful matches. Additional insights are provided on the effect of a user's origin and destination locations on their role choice and likelihood to be matched.

1. Introduction

The concept of ridesharing dates back to World War II and saw renewed interest during the energy crisis of the 1970s. After a period of decline, ridesharing has experienced a renaissance in the 2010s, driven by the advancement of mobile communication technologies (Chan and Shaheen, 2012). Despite its societal benefits — such as reducing pollution and traffic congestion — ridesharing has seen only moderate market penetration. This is partly because the effectiveness of ridesharing systems depends on attracting a critical mass of users, a condition that remains largely unmet. For instance, in Switzerland — a country known for its efficient public transportation system (Michael Page, 2023) — 78% of households own a car, while only 53% possess a public transport season ticket. In 2021, Swiss residents traveled an average of 30 km per person per day, 69% of which was by car (Swiss Federal Statistical Office, 2021). Although ridesharing holds the potential to reduce car ownership over time, current car occupancy rates remain low: only 1.1 persons per vehicle during peak hours in Switzerland (Swissinfo, 2017), and 1.2 in the United States, where public transportation is even less developed (USDOE, 2022). Given this context, increasing user participation in ridesharing systems has become a central objective for both policymakers and platform providers.

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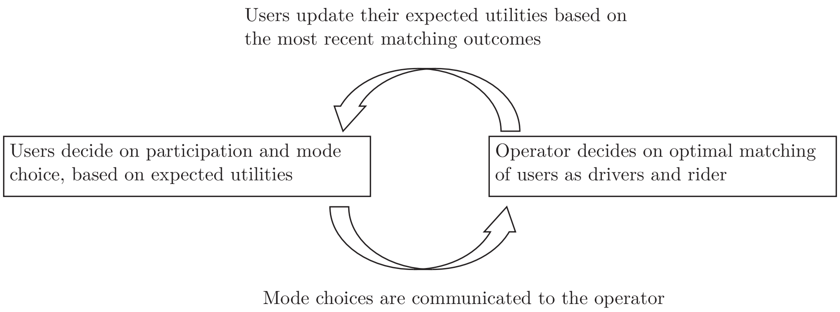

Fig. 1. Diagram of the decision-making framework.

The participation of users in ridesharing systems is intricately linked to the service level of these platforms, forming a bidirectional relationship: user participation determines the system's performance, which in turn also shapes users' willingness to participate. A substantial body of literature has explored various factors that influence users' willingness to participate in ridesharing, and the way to promote it. However, simply increasing overall participation is insufficient in practice, as ridesharing systems often suffer from an imbalance between riders and drivers. To address this challenge, recent studies have increasingly emphasized the role-specific nature of participation, recognizing that the motivations for engaging in ridesharing differ depending on whether a user participates as a driver or a passenger (Julagasigorn et al., 2021). While some motivations — such as environmental concern or a desire for social interaction — are shared across roles, others are distinctly role-specific. For example, empathy has been identified as a key motivator for drivers, whereas convenience (in terms of time and location), comfort, perceived usefulness, ease of use, and social status are more prominent among passengers.

The effectiveness of ridesharing systems can be significantly enhanced by incorporating participant flexibility—that is, users' ability to adapt their behavior or roles in response to changing system conditions. Flexibility can manifest in various forms, such as tolerance for delays (Ma and Koutsopoulos, 2022) or flexibility in preferred departure times (Stiglic et al., 2016). In this study, we focus on role flexibility—the possibility for users to be matched as either drivers or riders. Prior research has shown that enabling such flexibility helps alleviate imbalances between supply (drivers) and demand (riders), thereby improving matching efficiency. However, most existing models — such as Agatz et al. (2011), Masoud and Jayakrishnan (2017b), Liu et al. (2020), and Peng et al. (2024) — treat the proportion of users with role flexibility as exogenous. In these frameworks, each announced trip includes a pre-specified role: the user either intends to drive, to ride, or is open to either role. The platform then assigns roles based on system-level optimization goals. For instance, Masoud and Jayakrishnan (2017b) shows that when all users own vehicles and are willing to participate as both riders and drivers, system efficiency is maximized.

A prevailing assumption is that role flexibility in ridesharing systems depends solely on car ownership (Amerehi and Healy, 2025). In this view, only users who own vehicles are considered capable of assuming either driver or rider roles, while non-owners are limited to being riders. However, such a static interpretation underestimates the central role that flexibility plays in shaping system performance. Role flexibility is not simply a binary condition determined by ownership—it is a dynamic behavioral attribute that interacts deeply with platform operations, including real-time demand patterns, matching algorithms, and incentive structures. This form of flexibility has the potential to significantly enhance system adaptability, resilience, and matching efficiency, especially in imbalanced markets. Yet, its emergence and impact remain poorly understood in existing models. To fully realize the benefits of ridesharing, it is essential to move beyond the ownership-based view and treat flexibility as an endogenous, operationally relevant factor in system design.

In this paper, we consider a platform-based, peer-to-peer ridesharing system in which a recurring group of users participates on a daily basis. Each day, users report a preferred *participation mode*—choosing to join the system as a rider, a driver, or a flexible participant who is open to either role. These choices are made based on users' expectations of the utility associated with each *participation mode*. In particular, we model their mode selection behavior using a multinomial cross-nested logit model, which allows for correlated utilities among different *participation mode*. Given users' declared preferences, a central platform determines the final matching of riders and drivers in a way that maximizes the total utility of all matched participants. Importantly, selecting a *participation mode* does not guarantee assignment, as the platform may choose not to match certain users based on overall system optimization. The platform facilitates registration and coordination, and connects matched riders and drivers to arrange pickups. A schematic representation of the interaction between user decisions and platform matching is provided in Fig. 1, which is essentially a repeated multi-player non-cooperative game.

Our contribution is threefold. First, we propose a game-theoretic framework that endogenously models ridesharing users' flexibility in matching roles. We formulate a repeated interaction between a central platform — responsible for matching riders and drivers — and users, who can choose to participate as riders, drivers, or flexibly in either role. This framework can also be used to analyze the impact of policy design on users' willingness to be flexible. Second, we establish key mathematical properties of the system, including conditions for optimal matching and the existence of equilibrium. In particular, we show that when the platform's matching problem is formulated as a linear program (LP), an equilibrium outcome always exists. Building on this result, we develop an iterative algorithm to compute the equilibrium solutions in a numerical case study. Third, we offer managerial insights: enabling users to express flexible role preferences can significantly improve matching success rates. Moreover, targeted subsidy schemes that encourage flexibility — especially under demand imbalances — can further enhance overall system performance.

The remainder of this paper is organized as follows. An overview of the relevant literature is given in Section 2. The methods used to formulate and solve this problem are provided in Section 3. Here, we also describe the existence and uniqueness of the equilibrium solution. Numerical results for a case study of the city of Chicago are provided in Section 4. We conclude in Section 5.

2. Literature review

Our study sits at the intersection of ridesharing participation analysis and the optimal matching and pricing of ridesharing systems. The most relevant related studies are briefly summarized below.

2.1. Ridesharing participation analysis

Understanding participation behavior is crucial, as the efficiency and viability of ridesharing systems heavily depend on achieving a critical mass of users. A significant body of literature has focused on identifying the factors that affect individuals' willingness to participate in ridesharing. Early studies using survey data and behavioral theories have categorized these determinants into contextual factors — such as travel time savings, monetary incentives, and availability of parking — and personal or psychological factors, including trust, safety concerns, social preferences, and demographic attributes (Teal, 1987; Canning et al., 2010; Correia and Viegas, 2011; Neoh et al., 2017; Amirkiaee and Evangelopoulos, 2018; Bulteau et al., 2019).

Many works have emphasized the role of cost as a central motivator, particularly for passengers, while highlighting the variation in sensitivities between potential drivers and riders. Using TNC data from San Francisco (Shaheen et al., 2016), Hangzhou (Chen et al., 2017), and Chicago (Wang and Noland, 2021), researchers have confirmed the importance of travel time and fare in ride-splitting decisions, and the impact of cost-related factors may vary with roles (Park et al., 2018; Tahmasseby et al., 2016). These insights form the behavioral foundation for ridesharing system design, especially when participation is voluntary and users self-select into roles.

Concerning the cost-related factors, a relatively small body of research that adopts a utility-based approach to model ridesharing participation decisions. Equilibrium models have been widely used to analyze this multimodal competition. Early work by Daganzo (1982) examined route and mode choice under different occupancy rates, showing how carpools and solo drivers compete under congestion and tolling. Xu et al. (2015) developed a dynamic model simultaneously addressing role choice (driver/rider) and route selection, later applied to HOT lane design by Di et al. (2018). Ma et al. (2020) introduced stochastic equilibrium with elastic demand and ride-matching constraints, while Yao and Bekhor (2023) integrated stable matching with route choice for multi-passenger systems. These equilibrium-based approaches effectively capture strategic interactions among travel modes in complex multimodal networks.

Most related studies assume deterministic and precise user valuations. For instance, Tafreshian and Masoud (2020b) and Yao and Bekhor (2023) adopt deterministic utility-maximizing frameworks, where users choose to act as either drivers or riders, and their utilities are determined by the platform's operational decisions. Our study is most similar to Correia and Viegas (2011) in terms of the choice modeling approach, as they use a binary logit model to analyze the decision to participate in carpooling. However, their model does not account for role flexibility. In contrast, we use a cross-nested logit model, which accommodates an additional option—being flexible in matching roles. This extension introduces interdependencies among the utilities of different participation modes, making it better suited for modeling endogenous flexibility.

2.2. Optimal matching with flexible participants

The ability to efficiently match drivers with riders is essential to the success of a ridesharing program. We focus on the peer-to-peer ridesharing model that does not rely on dedicated drivers, which is distinct from the ride-splitting services offered by ride-hailing companies like Uber Many studies focused on optimizing such systems using mathematical programming approaches (Baldacci et al., 2004; Masoud and Jayakrishnan, 2017a; Xia et al., 2019; Wang et al., 2021; Liu et al., 2020) or reinforcement learning techniques (Qin et al., 2021; Haliem et al., 2021). Readers are referred to Tafreshian et al. (2020) for a comprehensive review.

While most papers assume predefined and fixed roles for drivers and passengers, several recent studies have incorporated flexible role assignment into dynamic or large-scale ridesharing systems. Agatz et al. (2011) employed a rolling horizon framework to address the dynamic matching problem, allowing some participants to indicate flexibility in serving as either drivers or riders. Similarly, Masoud and Jayakrishnan (2017b) modeled a dynamic ridesharing system where unmatched riders may switch roles by driving their own vehicles and re-entering the system as drivers. Tafreshian and Masoud (2020a) proposed a graph partitioning approach for large-scale matching, under the assumption that participants do not specify their roles in advance. Extending this line of work, Peng et al. (2024) studied a stable matching problem that includes flexible users, explicitly accounting for both the outbound and inbound trips. Additionally, Tafreshian and Masoud (2020b) integrated role assignment and pricing decisions into a stable matching framework, further highlighting the operational importance of accommodating user flexibility in both roles and incentives.

However, many existing models treat role flexibility as something determined externally or assigned by the platform itself, which leaves important questions unanswered about how to incorporate user preferences directly into the system and give users more control over how roles are assigned. To address this gap, we make flexibility an endogenous choice that emerges from matching outcomes, and we capture the reciprocal interactions between platform operations and user behaviors.

3. Methodology

In this section, we first characterize the decisions of individual ridesharing users in Section 3.1. We delineate the mathematical formulation for modeling the optimal matching decision of the platform in Section 3.2. Then, we discuss the existence of equilibria and the conditions under which an equilibrium is unique in Section 3.3. Finally, we provide an iterative algorithm used to obtain solutions, and we discuss the conditions under which the convergence is guaranteed in Section 3.4.

3.1. Decisions of ridesharing users

Consider a road network where users with different itineraries reveal their travel preferences to be enrolled in the system. We consider a set J of classes among users registered in the ridesharing platform. Users in the same class, $j \in J$, have identical origin, destination, scheduling preference, and car ownership. Although classification of users may be difficult in practice, our setting can be generalized to cases with any other type of heterogeneity by adjusting the definition of set J. A notational glossary is provided in Table 3 in the Appendix. We note that some parameters that are only locally used, for example in proofs, are omitted from the table for readability.

We envisage a scenario where each registered user of a ridesharing platform can decide if she will participate in ridesharing or not at the beginning of each day. In particular, if she decides to participate, she can do it as a potential driver, a rider, or both. Formally, each user chooses a *participation mode* m from the set $M = \{\text{opt-out}, \text{rider}, \text{driver}, \text{flexible}\}$. Here, "flexible" refers to the case where users want to participate in ridesharing but are flexible in their mode as either riders or drivers. In class $j \in J$, we denote the number of users as n_j and the fractions of users who enrolled as opt-out, riders, drivers, and flexible users as $p_j^{\text{opt-out}}$, p_j^{rider}, p_j^{driver}, and p_j^{flexible}, respectively. Since a driver in the ridesharing program has to be a car owner, the corresponding proportions p_j^{driver} and p_j^{flexible} are forced to zero if users in class j do not own a car. We consider four types of matches between two users for a combination of user modes, $\mathcal{A} = \{\text{II}, \text{FI}, \text{IF}, \text{FF}\}$, where the first letter denotes the state of the rider and the second letter the state of the driver, inflexible (I) or flexible (F) for each one of them. Specifically, the four combinations are as follows: (1) an inflexible rider matches to an inflexible driver, (2) a flexible rider matches to an inflexible driver, (3) an inflexible rider matches to a flexible driver, (4) a flexible rider matches to a flexible driver.

Definition 1. A flexible user in the ridesharing scheme can be matched either as a rider or as a driver.

Definition 2. An opt-out user is a user who will not participate in the ridesharing system and use an alternative mode of transportation (driving alone or choosing another transportation mode, such as public transit).

The matching outcomes and the corresponding utilities depend on the *participation mode* choice, which might bring different utilities to users. Let $R = \{j\}_{j\in J} \cup \{ij\}_{i,j\in J}$ denote the set of possible matching outcomes. The element j stands for the case when a user from group j is not matched, and the element ij stands for the outcome when a rider from class i is matched with a driver from class j. For any class $j \in J$, given the *participation mode* $m \in M$ and the matching outcome $r \in R$, the corresponding utility is denoted by $u_j^m(r)$. We describe as below.

3.1.1. Opting-out users

Opting-out users will never be matched and thus leave the system. Then the corresponding utility of opting-out users from class $j \in J$ is denoted by $u_j^{\text{opt-out}}(j)$, which is exogenous and independent of the choices of other users. Obviously, in practice, $u_j^{\text{opt-out}}(j)$ depends on various exogenous factors, such as the road traffic conditions and the service level of public transportation. Then the expected utility of opting out would be

$$v_j^{\text{opt-out}} = u_j^{\text{opt-out}}(j). \tag{1}$$

3.1.2. Rider-only and driver-only users

Meanwhile, rider-only and driver-only users have stochastic matching outcomes and the corresponding utilities, depending on the matching decisions by the platform. We denote the number of matches of type $a \in \mathcal{A}$ with riders of class i and drivers of class j by x_{ij}^a. For any class $j \in J$ choosing *participation mode* $m \in M$, the corresponding probability of having matching outcome $r \in R$ is denoted by $\rho_j^m(r)$.

Now we characterize the probabilities and utilities of different matching outcomes. For any user in class i who chooses mode $m = \text{rider}$, there exist the following two types of matching outcomes:

(1) *Being matched with a driver in class* $j \in J$. Assuming that matching outcomes of users from the same group are independent, the probability $\rho_i^{\text{rider}}(ij)$ of an inflexible rider from class i being matched with a driver in class j is

$$\rho_i^{\text{rider}}(ij) = \frac{x_{ij}^{\text{II}} + x_{ij}^{\text{IF}}}{p_i^{\text{rider}} n_i}. \tag{2}$$

The numerator of the right-hand side of the above equation is the number of inflexible riders from group i actually matched with any users from group j, while the denominator is the number of inflexible riders from group i.

The utility of such an outcome for each user is determined by the gained utility from the match and the internal transfer scheme between drivers and riders. The gained utility for a match with the rider from group i and the driver from group j is defined as the discrepancy between the total utilities of both uses when they are matched and unmatched, respectively. Intuitively, it is the travel time and monetary savings from ridesharing. An internal transfer scheme will determine how the savings are distributed among the matched pair. For conciseness, we do not explicitly define the internal transfer scheme here. Roughly speaking, the internal transfer scheme is a pre-determined monetary transaction scheme between matched drivers and riders such that the utility gain for riders and drivers from the match is subject to a target ratio. Formally, for a match between a rider from $i \in J$ and a driver from $j \in J$, we assume that their joint utility is a constant u_{ij}, and if they opt out, the individual utilities for them would be $u_i^{\text{opt-out}}(i)$ and $u_j^{\text{opt-out}}(j)$, respectively. Then the gained utility g_{ij} is

$$g_{ij} = u_{ij} - u_i^{\text{opt-out}}(i) - u_j^{\text{opt-out}}(j). \tag{3}$$

We consider that the gained utility g_{ij} from matching will be split across riders and driver as fractions ϕ^{rider} and $\phi^{\text{driver}} = 1-\phi^{\text{rider}}$ (through the internal transfer scheme). Then the utilities of the rider and the driver, $u_i^{\text{rider}}(ij)$ and $u_j^{\text{driver}}(ij)$ are, respectively, given by

$$u_i^{\text{rider}}(ij) = u_i^{\text{opt-out}}(i) + \phi^{\text{rider}} g_{ij}, \tag{4}$$

$$u_j^{\text{driver}}(ij) = u_j^{\text{opt-out}}(j) + \phi^{\text{driver}} g_{ij}. \tag{5}$$

(2) *Being unmatched.* The number of unmatched enrolled riders from class i is $p_i^{\text{rider}} n_i - \sum_{j\in J}(x_{ij}^{\text{II}} + x_{ij}^{\text{IF}})$, such that the probability $\rho_i^{\text{rider}}(i)$ of an enrolled inflexible rider not being matched is

$$\rho_i^{\text{rider}}(i) = 1 - \sum_{j\in J} \frac{x_{ij}^{\text{II}} + x_{ij}^{\text{IF}}}{p_i^{\text{rider}} n_i}. \tag{6}$$

Like opting-out users, unmatched inflexible riders would leave the system and use an alternative mode of transportation. Particularly, we consider that users would be likely to prefer choosing opt-out in the beginning rather than being unmatched. This reason is that, users choosing to be inflexible riders have the expectation of being matched, and such expectation would lead to the aversion of being unmatched. Note that alternatively, one can choose some classical model like prospect theory to capture the impact of the loss aversion preference regarding the possibility of being unmatched (Kahneman and Tversky, 2013). However, the complexity of such kind of model, e.g., finding the reference point, would complicate our analysis. In this regard, we incorporate a fixed discomfort penalty δ of being unmatched such that the utility $u_i^{\text{rider}}(i)$ of an unmatched inflexible rider is

$$u_i^{\text{rider}}(i) = u_i^{\text{opt-out}}(i) - \delta. \tag{7}$$

Given the utilities and probabilities of being matched with different drivers and being unmatched, the expected utility for an inflexible rider from class i is

$$v_i^{\text{rider}} = \rho_i^{\text{rider}}(i)(u_i^{\text{opt-out}}(i) - \delta) + \sum_{j\in J} \rho_i^{\text{rider}}(ij)(u_i^{\text{opt-out}}(i) + \phi^{\text{rider}} g_{ij}). \tag{8}$$

By reorganizing the terms and using the fact that $\rho_i^{\text{rider}}(i) + \sum_{j\in J} \rho_i^{\text{rider}}(ij) = 1$, it can be written as

$$v_i^{\text{rider}} = u_i^{\text{opt-out}}(i) - \rho_i^{\text{rider}}(i)\delta + \sum_{j\in J} \rho_i^{\text{rider}}(ij)\phi^{\text{rider}} g_{ij}. \tag{9}$$

Next, we analyze the utilities of driver-only users. The expected utility for a driver-only user from class j is

$$v_j^{\text{driver}} = u_j^{\text{opt-out}}(i) - \rho_j^{\text{driver}}(j)\delta + \sum_{i\in J} \rho_j^{\text{driver}}(ij)\phi^{\text{driver}} g_{ij}. \tag{10}$$

where $\rho_j^{\text{driver}}(ij)$ is the probability of a drive-only user from class j being matched with a rider in class i, i.e.,

$$\rho_j^{\text{driver}}(ij) = \frac{x_{ij}^{\text{II}} + x_{ij}^{\text{FI}}}{p_j^{\text{driver}} n_j}. \tag{11}$$

3.1.3. Flexible user

Now we turn to analyze the utilities of flexible users, who can be matched as either riders or drivers. The probabilities $\rho_i^{\text{flexible}}(ij)$ and $\rho_i^{\text{flexible}}(ji)$ of a flexible user from class i being matched with a driver and a rider in class j are respectively given by

$$\rho_i^{\text{flexible}}(ij) = \frac{x_{ij}^{\text{FI}} + x_{ij}^{\text{FF}}}{p_i^{\text{flexible}} n_i}. \tag{12}$$

$$\rho_i^{\text{flexible}}(ji) = \frac{x_{ji}^{\text{IF}} + x_{ji}^{\text{FF}}}{p_i^{\text{flexible}} n_i}. \tag{13}$$

The probability of a flexible user from class i not being matched is

$$\rho_i^{\text{flexible}}(i) = 1 - \sum_{j\in J} \frac{x_{ij}^{\text{FI}} + x_{ij}^{\text{FF}} + x_{ji}^{\text{FF}} + x_{ji}^{\text{IF}}}{p_i^{\text{flexible}} n_i}. \tag{14}$$

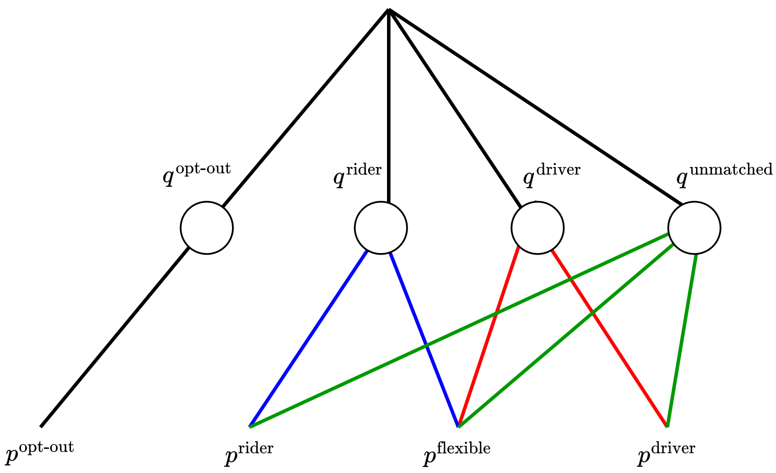

Fig. 2. Graphical depiction of the cross-nested logit model where p denotes the probability of choosing a mode and q denotes the probability of choosing a nest.

Then, the expected utility of flexible users is given by:

$$v_i^{\text{flexible}} = u_i^{\text{opt-out}}(i) + \sum_{j\in J} \left[\phi^{\text{rider}} g_{ij} \rho_i^{\text{flexible}}(ij) + \phi^{\text{driver}} g_{ji} \rho_i^{\text{flexible}}(ji)\right] - \delta \rho_i^{\text{flexible}}(i). \tag{15}$$

Given the expected utilities of four *participation modes* defined in Eqs. (1)–(15), the choices among them are characterized in a cross-nested logit manner (Bierlaire, 2006). The cross-nested logit model is commonly used in discrete choice analysis, which allows for the possibility that choices can belong to multiple overlapping nests. A nest is a group of alternatives that share unobserved similarities and thus have correlated utilities. In our setting, we consider four nests, each of which includes at least one *participation modes* $m \in M$. The first nest C_{driver} consists of two modes: being drivers and being flexible users, because both modes can yield the outcome of being matched as drivers. The second nest C_{rider} consists of two modes: being riders and being flexible users, because both modes can yield the outcome of being matched as drivers. The third nest $C_{opt-out}$ only includes opt-out. The last nest $C_{unmatched}$ includes all participation modes that may yield the outcome of not being matched, i.e., $C_{unmatched} = \{\text{rider}, \text{driver}, \text{flexible}\}$. Let C denote the set of all nests, i.e., $C = \{C_{driver}, C_{rider}, C_{opt-out}, C_{unmatched}\}$. A graphical representation of the cross-nested logit model, including the notation defined below, is given in Fig. 2.

By the cross-nested logit model, the probability of each user from any group $j \in J$ choosing mode $m \in M$ is given by

$$p_j^m = \sum_{c\in C} q_j^c r_j^c(m), \forall j \in J, m \in M. \tag{16}$$

where q_j^c is the probability of choosing nest $c \in C$, and $r_j^c(m)$ is the conditional probability of choosing mode m when nest c is chosen.

$$q_j^c = \frac{(\sum_{m'\in M} \epsilon_{m'c}^{\eta_c/\eta} e^{\eta_c v_j^{m'}})^{\eta/\eta_c}}{\sum_{c'\in C}(\sum_{m'\in M} \epsilon_{m'c'}^{\eta_{c'}/\eta} e^{\eta_{c'} v_j^{m'}})^{\eta/\eta_{c'}}} \tag{17}$$

$$r_j^c(m) = \frac{\epsilon_{mc}^{\eta_c/\eta} e^{\eta_c v_j^m}}{\sum_{m'\in M} \epsilon_{m'c}^{\eta_c/\eta} e^{\eta_c v_j^{m'}}} \tag{18}$$

with $\eta_c \geq \eta \geq 0$, $\epsilon_{mc} \geq 0$, $\sum_{c\in C} \epsilon_{mc} = 1$ for all $m \in M$ and there exist some nest $c \in C$ such that $\epsilon_{mc} > 0$. Therefore, given the utility of each mode, the probability of choosing each mode can be expressed as $p_j^m = P_j^m(\boldsymbol{v}_j)$, where

$$P_j^m(\boldsymbol{v}_j) = \sum_{c\in C} \frac{(\sum_{m'\in c} \epsilon_{m'c}^{\eta_c/\eta} e^{\eta_c v_j^{m'}})^{\eta/\eta_c}}{\sum_{c'\in C}(\sum_{m'\in c'} \epsilon_{m'c'}^{\eta_{c'}/\eta} e^{\eta_{c'} v_j^{m'}})^{\eta/\eta_{c'}}} \frac{\epsilon_{mc}^{\eta_c/\eta} e^{\eta_c v_j^m}}{\sum_{m'\in M} \epsilon_{m'c}^{\eta_c/\eta} e^{\eta_c v_j^{m'}}} \tag{19}$$

3.2. Decisions of the operator

The operator matches potential drivers and riders in order to maximize the total welfare of users. We macroscopically model the operator's decisions as an LP. The operator knows a priori the possible utilities of users before matching. We model the problem using a continuous number of drivers and riders such that the problem can be modeled as an LP. We formulate the problem as follows:

$$\max_{\mathbf{x}} \quad U(\mathbf{x}) = \sum_{i\in J}\sum_{j\in J}\sum_{a\in \mathcal{A}} u_{ij} x_{ij}^a + \sum_{j\in J} u_j^{\text{opt-out}}(j)\Big(n_j - \sum_{i\in J}\sum_{a\in \mathcal{A}} x_{ij}^a - \sum_{i\in J}\sum_{a\in \mathcal{A}} x_{ji}^a\Big), \tag{20}$$

$$\text{s.t.} \quad \sum_{i\in J}(x^{\mathrm{FI}}_{ij} + x^{\mathrm{II}}_{ij}) \le p^{\mathrm{driver}}_{j} n_j, \forall j \in J, \tag{21}$$

$$\sum_{i\in J}(x^{\mathrm{IF}}_{ji} + x^{\mathrm{II}}_{ji}) \le p^{\mathrm{rider}}_{j} n_j, \forall j \in J, \tag{22}$$

$$\sum_{i\in J}(x^{\mathrm{IF}}_{ij} + x^{\mathrm{FF}}_{ij} + x^{\mathrm{FF}}_{ji} + x^{\mathrm{FI}}_{ji}) \le p^{\mathrm{flexible}}_{j} n_j, \forall j \in J, \tag{23}$$

$$x^{a}_{ij} \ge 0, \forall i, j \in J, a \in \mathcal{A}. \tag{24}$$

The objective (20) is the utility of matched and unmatched users. Constraints (21) and (22) ensure that the number of inflexible drivers and riders is not exceeded, respectively. Constraints (23) ensure that every flexible agent is used at most once, in any role. Decision variables $\mathbf{x}$ are modeled as continuous decision variables. The reason for this is that due to the interaction with the cross-nested logit model, we have opted to model all variables in a continuous space. Especially for larger user groups (i.e., when n_j is large), the solution of the LP problem will be close to the solution of the IP problem.

In our formulation, we intentionally define the social welfare objective based on the observable component of utility, rather than on the full random utility or its expectation (i.e., the logsum term). This modeling choice is motivated by three main considerations. First, we interpret the random component of utility as capturing *unobserved heterogeneity*, consistent with Manski's view that the randomness in discrete choice models arises from the analyst's limited information, not from uncertainty in individuals' actual preferences (Manski, 1977, 1999). From the platform's perspective, only the observable part of utility is relevant for decision-making. It is therefore conceptually appropriate to evaluate welfare based on the part of utility that is actually observable and actionable. Second, incorporating the logsum term would compromise the mathematical structure of our optimization model. In particular, for the cross-nested logit, the logsum introduces nonlinear and non-convex dependencies on choice probabilities, making the problem no longer amenable to linear programming. In contrast, focusing on the observable utility maintains tractability and enables efficient computation and interpretation.

Here, utilities depend on the travel time, scheduling displacement, and values of time for each travel mode. Let $dist(j)$ be the distance between the origin and destination of class j, measured in units of time. Let $detour(i, j)$ be the additional time needed for a driver to reach the pickup and dropoff region of the matched rider. The entire duration of the trip of rider i is spent together. On top of that, the driver may spend some time alone, which is computed as $(dist(j) + detour(i, j) - dist(i))$. Let $\alpha^{\text{opt-out}}$ and α^{pool} be the values of time when traveling alone and sharing rides, s is the subsidy for ridesharing. To allow for scheduling preferences, t^*_j is the desired arrival time of class j. The unit costs of early and late arrival are denoted by β and γ, respectively, with $\beta < \gamma$. Solo drivers arrive exactly at their desired arrival time. Users sharing a ride determine the departure time that maximizes their joint utility. By de Palma et al. (2022), in our setting, two ridesharing users i and j maximize their joint utility if they arrive at $\min(t^*_i, t^*_j)$. In this case, one user arrives exactly on time while the other arrives early. Alternative definitions of the scheduling preferences may be used depending on the application. For example, when deadlines for arrival at the destination are strict, some matches may be infeasible, for which the utility can be set infinitely large. Then the utilities are formulated as follows:

$$u^{\text{opt-out}}_j = -\alpha^{\text{opt-out}} dist(j), \tag{25}$$

$$u_{ij} = s - 2\alpha^{\text{pool}} dist(i) - \alpha^{\text{opt-out}}(dist(j) + detour(i, j) - dist(i)) - \beta|t^*_i - t^*_j|. \tag{26}$$

By solving the problem in (20)–(24) the operator obtains an optimal matching. Here, we define two concepts of optimality. A social optimal solution is obtained when the total utility of the population is maximized. This is the case when $p^{\text{flexible}}_j = 1$ for every class $j \in J$. We note that this is a sufficient condition for obtaining a social optimum, but not a necessary condition. Other solutions may exist for which the same objective value can be obtained. We refer to an optimal solution to the problem in (20)–(24), conditional on a set of participation strategies $\mathbf{p}_j$ for every class $j \in J$ as a conditional optimal solution.

Definition 3. A social optimal solution maximizes the total utility of the entire population (participating and non-participating). This solution is obtained when $p^{\text{flexible}}_j = 1$ for every class $j \in J$ for which the users own a car. For those classes $j \in J$ for which the users do not own a car, $p^{\text{rider}}_j = 1$.

Definition 4. A conditional optimal solution maximizes the total utility of the entire population (participating and non-participating), conditional on the participation strategies $\mathbf{p}_j$ for every class $j \in J$.

We also emphasize that due to the interaction of u_{ij} with $u^{\text{opt-out}}_i$ and $u^{\text{opt-out}}_j$, a pair of users (i, j) is only matched if they jointly benefit from the matching. As a result, unreasonable scenarios of extremely high detours and scheduling delays are avoided by definition.

3.3. Equilibrium conditions

Now we define the equilibrium of the interaction between the operator's decisions and the stochastic decision-making behavior of users. We define a *participation strategy* for each class $j \in J$ as a quadruple $\mathbf{p}_j = (p^m_j)_{m\in M}$. The *ridesharing strategy* $\mathbf{p}_j$ is determined through the utilities of all options via the defined multinomial logit model. The matching of drivers and riders is determined via the defined LP problem. As defined by Definition 5, an equilibrium solution refers to a solution where the operator matches all the users according to a conditional optimal solution of the LP, while all user classes choose a ridesharing strategy $\mathbf{p}_j$ for every class $j \in J$ for which they cannot improve their utility by only changing their own strategy. This problem is a fixed-point problem where the decisions are interrelated.

Definition 5. An equilibrium solution $(\mathbf{x}, \mathbf{p})$ refers to a solution where the operator matches the users according to a conditional optimal solution $\mathbf{x}$, and the ridesharing strategies $\mathbf{p}_j$ for every class $j \in J$ are such that no user can improve his/her utility by only changing his/her own strategy.

Given any ridesharing strategies $\mathbf{p}$ regarding users' mode choices, we define the set of possible matching outcomes $Q(\mathbf{p})$ as

$$Q(\mathbf{p}) = \{\mathbf{x} \in \mathbb{R}^{|J|^2} | (21)\text{–}(24), U(\mathbf{x}; \mathbf{p}) = U(\mathbf{x}^*; \mathbf{p})\}. \tag{27}$$

where $\mathbf{x}^*$ is an optimal solution to LP (20)–(24). For notational convenience, we define a special type of matching outcome as "Um", and denote the number of unmatched users from group j as x_j^{Um}. Then we can define $A^m(r) \in A \cup \text{Um}$ as the set of feasible match types for matching outcome r and *participation mode* m. For example, for users from group j choosing opt-out, i.e., m = opt-out, the matching outcome must be $r = j$, and the set $A^m(r)$ is $\{\text{Um}\}$. For rider-only users from group j and successfully matched as riders, i.e., m = rider, $r \in \{ji\}_{i \in J}$, the set $A^m(r)$ is $\{\text{IF}, \text{II}\}$. The equilibrium *participation strategy* $\mathbf{p} = (\mathbf{p}_j)_{j \in J}$ solves the following equation system:

$$p_j^m = P_j^m(\boldsymbol{v}_j), \qquad \forall j \in J, m \in M, \tag{28}$$

$$v_j^m = \sum_{r \in R_j} \rho_j^m(r) u_j^m(r), \qquad \forall j \in J, m \in M, \tag{29}$$

$$\rho_j^m(r) = \frac{\sum_{a \in A^m(r)} x_r^a}{p_j^m n_j}, \qquad \forall j \in J, m \in M, r \in R_j, \tag{30}$$

$$\boldsymbol{x} \in Q(\boldsymbol{p}), \tag{31}$$

$$p_j^m \in (0, 1], \qquad \forall j \in J, m \in M. \tag{32}$$

where R_j is the set of matching outcomes for which class j is unmatched, being matched as drivers or being matched as riders.

Now we define

$$G_j^m(\boldsymbol{p}) = P_j^m(H(\boldsymbol{p})), \quad \forall j \in J, m \in M, \tag{33}$$

where $H(\boldsymbol{p}) = (H_j^m(\boldsymbol{p}))_{j \in J, m \in M}$, and

$$H_j^m(\boldsymbol{p}) = \frac{1}{n_j p_j^m} \sum_{r \in R_j} \sum_{a \in A^m(r)} Q_r^a(\boldsymbol{p}) u_j^m(r). \tag{34}$$

Then the equations system above can be treated as a fixed-point problem $\boldsymbol{p} = G(\boldsymbol{p})$, where $\boldsymbol{p} = (p_j^m)_{j \in J, m \in M}$, $G(\boldsymbol{p}) = (G_j^m(\boldsymbol{p}))_{j \in J, m \in M}$. We can use the formulation of this problem as a fixed-point problem to prove that an equilibrium solution always exists, according to Theorem 1.

Theorem 1. *The equilibrium problem defined by (28)–(31) always admits a solution.*

Proof. We employ the Kakutani fixed-point theorem to prove the existence of a solution, which states that an upper hemicontinuous set-valued function $G : S \to 2^S$ from a nonempty convex compact subset S of Euclidean space to the corresponding power set 2^S has at least one fixed point.

First, we show that the function $G(\boldsymbol{p})$ can be defined on a compact and convex space of $\boldsymbol{p}$. By definition, we have $p_j^m \in (0, 1]$ for all $j \in J, m \in M$. We can further show that, p_j^m is lower bounded by a small constant $\underline{p}_j^m = \inf_{\boldsymbol{v}_j} P_j^m(\boldsymbol{v}_j)$. Thus, letting $\underline{p} = \min_{m,j} \underline{p}_j^m$, we can define a solution space as $S = [\underline{p}, 1]^{|M||J|}$, which is compact and convex.

Second, we proceed to show the upper hemicontinuity of $G(\boldsymbol{p})$ by proving that the set-valued function $Q(\boldsymbol{p})$ is continuous. Recall that $Q(\boldsymbol{p})$ is the set of optimal solutions for an LP with parameters $Q(\boldsymbol{p})$, as defined in Eq. (27). By Theorem 2 in Böhm (1975), we know that (1)$Q(\boldsymbol{p})$ has a closed graph, and (2) $Q(\boldsymbol{p})$ is continuous if $Q(\boldsymbol{p})$ is compact for any $\boldsymbol{p} \in S$. Here $Q(\boldsymbol{p})$ is bounded, apparently. Therefore, $Q(\boldsymbol{p})$ is continuous, and then $G(\boldsymbol{p})$ is continuous (which must be upper hemicontinuous). Thus, the proof is complete. □

Although an equilibrium solution always exists according to Theorem 1, it is not necessarily unique. Suppose that the platform always chooses exactly one solution from the optimal solutions to LP (20)–(24). To denote the chosen solution, we define a single-valued function $\bar{Q}(\boldsymbol{p}) \in Q(\boldsymbol{p})$. Then we can further define a counterpart of function $G_j^m(\boldsymbol{p})$ as

$$\bar{G}_j^m(\boldsymbol{p}) = P_j^m(\bar{H}(\boldsymbol{p})), \quad \forall j \in J, m \in M, \tag{35}$$

where $\bar{H}(\boldsymbol{p}) = (\bar{H}_j^m(\boldsymbol{p}))_{j \in J, m \in M}$, and

$$\bar{H}_j^m(\boldsymbol{p}) = \frac{1}{n_j p_j^m} \sum_{r \in R_j} \sum_{a \in A^m(r)} \bar{Q}_r^a(\boldsymbol{p}) u_j^m(r), \quad \forall j \in J, m \in M. \tag{36}$$

Under certain conditions, we can prove that the equilibrium solution for the fixed point problem $\bar{G}(\boldsymbol{p}) = \boldsymbol{p}$ is unique. We specify this result in Theorem 2 as below.

Theorem 2. *When η_c and $\eta_c/\eta - 1$ are sufficiently small, i.e., $\eta_c \to 0$ and $\eta_c/\eta \to 1$ for all $c \in C$, the function $\bar{G}(\cdot)$ is contractive, and the solution to the fixed point problem $\bar{G}(\boldsymbol{p}) = \boldsymbol{p}$ is unique.*

Proof. We prove the uniqueness of equilibrium by invoking the Banach fixed-point theorem (Border, 1985). By the Banach fixed-point theorem, the fixed-point problem $\boldsymbol{p} = \bar{G}(\boldsymbol{p})$ has a unique solution when the function $\bar{G}$ is contractive over a complete set. It has been shown in the proof of Theorem 1 that $\bar{G}(\boldsymbol{p})$ can be defined on a compact (and therefore complete) set.

Then, we proceed to examine when the function $\bar{G}(\cdot)$ would be contractive. Recall that we treat $\bar{G}(\boldsymbol{p})$ as composed by two functions $P(\cdot)$ and $\bar{H}(\cdot)$, i.e., $\bar{G}(\boldsymbol{p}) = P(\bar{H}(\boldsymbol{p}))$, where

$$\bar{H}_j^m(\boldsymbol{p}) = \frac{1}{n_j p_j^m} \sum_{r \in R_j} \sum_{a \in A^m(r)} \bar{Q}_r^a(\boldsymbol{p}) u_j^m(r), \tag{37}$$

and

$$P_j^m(\boldsymbol{v}_j) = \sum_{c \in C} \frac{(\sum_{m' \in c} \epsilon_{m'c}^{\eta_c/\eta} e^{\eta_c v_j^{m'}})^{\eta/\eta_c}}{\sum_{c' \in C} (\sum_{m' \in c'} \epsilon_{m'c'}^{\eta_{c'}/\eta} e^{\eta_{c'} v_j^{m'}})^{\eta/\eta_{c'}}} \frac{\epsilon_{mc}^{\eta_c/\eta} e^{\eta_c v_j^m}}{\sum_{m' \in M} \epsilon_{m'c}^{\eta_c/\eta} e^{\eta_c v_j^{m'}}} \tag{38}$$

First, recall that we have $P_j^m(\boldsymbol{v}_j) = \sum_{c \in C} q_j^c r_j^c(m)$ by Eq. (16). One can easily verify that

$$\lim_{\eta_c \to 0, \eta_c/\eta \to 1} \frac{\partial r_j^c(m)}{\partial v_j^n} = 0,$$

$$\lim_{\eta_c \to 0, \eta_c/\eta \to 1} \frac{\partial q_j^c}{\partial v_j^n} = 0.$$

Since q_j^c and $r_j^c(m)$ are upper bounded by 1, we are able to obtain that

$$\lim_{\eta_c \to 0, \eta_c/\eta \to 1} \frac{\partial P_j^m}{\partial v_j^n} = 0, \forall j \in J, m, n \in M.$$

Therefore, when $\eta_c \to 0$ and $\eta_c/\eta \to 1$ for all $c \in C$, the ∞ matrix norm of the Jacobian $J_P(\boldsymbol{v})$ of $P(\boldsymbol{v})$ is

$$\lim_{\eta_c \to 0, \eta_c/\eta \to 1} \|J_P(\boldsymbol{v})\|_\infty = 0. \tag{39}$$

Then the function $P(\boldsymbol{v})$ is contractive when $\eta_c \to 0$ and $\eta_c/\eta \to 1$. By the property of contractive mapping, we can always find some constant $K_1 = \sup_{\boldsymbol{v}} \|J_P(\boldsymbol{v})\|_\infty$ such that

$$\lim_{\eta_c \to 0, \eta_c/\eta_1 \to 1} \|P(\boldsymbol{v}) - P(\boldsymbol{v}')\|_\infty \le K_1 \|\boldsymbol{v} - \boldsymbol{v}'\|_\infty, \forall \boldsymbol{v}, \boldsymbol{v}' > 0. \tag{40}$$

Second, by Mangasarian and Shiau (1987), optimal solutions of an LP are Lipschitz continuous in the right-hand-side parameters. Thus, the function $\bar{Q}(\boldsymbol{p})$ is Lipschitz continuous. Note that $\bar{H}(\boldsymbol{p})$ can be seen as the product of the function $\bar{Q}(\boldsymbol{p})$ and a linear function of $\boldsymbol{p}$, which are both bounded because p_j^m is lower bounded by $\underline{p}$. Then it is ready to prove that the function $H(\boldsymbol{p})$ is also Lipschitz continuous. Then we can find a real constant $K_2 \ge 0$ such that

$$\|\bar{H}(\boldsymbol{p}) - \bar{H}(\boldsymbol{p}')\|_\infty \le K_2 \|\boldsymbol{p} - \boldsymbol{p}'\|_\infty, \forall \boldsymbol{p}, \boldsymbol{p}' \in S. \tag{41}$$

Given that $\bar{G}(\boldsymbol{p}) = P(\bar{H}(\boldsymbol{q}))$, by (40) and (41), we have

$$\lim_{\eta_c \to 0, \eta_c/\eta_1 \to 1} \|\bar{G}(\boldsymbol{p}) - \bar{G}(\boldsymbol{p}')\|_\infty \le K_1 K_2 \|\boldsymbol{v} - \boldsymbol{v}'\|_\infty, \forall \boldsymbol{v}, \boldsymbol{v}' > 0.$$

where K_1 is approaching zero by (39), and K_2 is finite. Then it is obvious that $K_1 K_2 \le 1$, and $\bar{G}(\boldsymbol{p})$ is contractive. The proof is complete. □

We can thus conclude that an equilibrium solution to the problem always exists and, under certain conditions, is also unique. Under the limiting case where $\eta_c = 0$ and $\eta/\eta_c = 1$, Eq. (19) reduces to $P_{jm}(v_j) = 1/|\mathcal{M}|$, where $|\mathcal{M}|$ denotes the number of participation modes. This corresponds to a situation in which individuals are fully random in their choices and make decisions entirely independent of the observed utility values. While we acknowledge that the condition is not mild, this degenerate case offers useful intuition: when η is sufficiently small and η_c is of similar magnitude, the equilibrium tends to be unique and the fixed-point iteration algorithm described in Section 3.4 reliably converges. A more formal characterization of the parameter range that guarantees convergence can be challenging, as it is likely to be highly instance-dependent and sensitive to the structure of the underlying linear program. For this reason, we leave the rigorous analysis of such thresholds to future work.

Algorithm 1: Iterative algorithm

Input: A set of users J, with utility values $u_j^{\text{opt-out}}$ and joint utility functions u_{ij}
Initialize $\mathbf{p}_j$ as random variables on [0,1] such that $\sum_{m\in M} p_j^m = 1$ for all $j \in J$.
while *Optimal matching or mode choice changes* **do**
  Solve (20)–(24)
  Update the expected utility v_j^m for every $j \in J$ and $m \in M$
  Update every agent's mode choice decisions $\mathbf{p}_j$ according to the choice model
end
return Equilibrium matching decisions and mode choices

3.4. Iterative algorithm

To evaluate the practical convergence of our methods to an equilibrium, we design an algorithm that iteratively obtains and updates the decisions of the operator and the individual users. The general structure of the algorithm is described in Algorithm 1. This iterative process is repeated until no changes to the optimal matching $\mathbf{x}$ and the mode choice quadruple $\mathbf{p}_j$ are observed.

To foster convergence of the algorithm, we apply a moving average update rule for mode choice probabilities $\mathbf{p}_j$. Let $\mathbf{p}_j^{(k)}$ be the value for $\mathbf{p}_j$ in iteration k. The moving average is updated as follows:

$$\mathbf{p}_j^{(k)} = \lambda \mathbf{p}_j^{(k-1)} + (1-\lambda)\mathbf{p}_j \tag{42}$$

The parameter λ is tuned based on the specific characteristics of the system.

By the Banach fixed-point theorem, the property of $\boldsymbol{G}$ being a contractive mapping under certain conditions, as formulated in Theorem 2, ensures the convergence of the following iterative algorithm, as stated in the following corollary.

Corollary 1. *The solution $\boldsymbol{p}^*$ to the fixed point problem $\bar{G}(\boldsymbol{p}) = \boldsymbol{p}$ can be found as follows: start with an arbitrary $\boldsymbol{q}$ in S, and define a sequence $(\boldsymbol{q}^{(k)})_{k\in\mathbb{N}}$ by $\boldsymbol{q}^{(k)} = \bar{G}(\boldsymbol{q}^{(k-1)})$ for $k \geq 1$. Then $\lim_{k\to\infty} \boldsymbol{q}^{(k)} = \boldsymbol{p}^*$.*

The theoretical convergence of this iterative algorithm is confirmed on a set of experiments in Section 4.2. Additional properties on the expected utility values and the corresponding mode choice probabilities in equilibrium can be obtained. Obviously, p_j^{driver} and p_j^{flexible} are equal to 0 for classes $j \in C$ that do not own a car. In general, some properties on utilities can be obtained based on previously realized matchings, which are summarized in Remark 1.

Remark 1. For a class of users $j \in C$ with a recorded set of historic observations on matching decisions, the following equalities hold on the expected utility values in equilibrium:

1. If a class of users is never matched in the historic observations, for their expected utilities it holds that: $v_j^{\text{rider}} = v_j^{\text{driver}} = v_j^{\text{flexible}}$.
2. If a class of users is only matched as a rider in the historic observations, for their expected utilities it holds that: $v_j^{\text{rider}} = v_j^{\text{flexible}}$.
3. If a class of users is only matched as a driver in the historic observations, for their expected utilities it holds that: $v_j^{\text{driver}} = v_j^{\text{flexible}}$.

The properties in Remark 1 align with the intuition of the behavior of ride-sharing users as described in Section 3.1. If a user who indicates to be flexible is only matched as a rider, over time, their expectation will be that they are only matched as a rider. This will then also translate to the expected utility values. The same holds for being matched as a driver, or not matched at all.

4. Results

In this section, we evaluate the effect of role choice on the performance of a ridesharing system. We evaluate the results of a case study of the city of Chicago, of which the details are described in Section 4.1. In Section 4.2 we evaluate the convergence of the iterative algorithm. We identify the effect of geographical properties of agents on their role choice in Section 4.3. We explore the effect of allowing for flexibility for different levels of car ownership in Section 4.4, and we go deeper into the effect of car ownership on mode choice in Section 4.5. Finally, we evaluate the effect of subsidies and (un) even redistibution of gained utilities among riders and drivers in Section 4.6.

4.1. Case study

We evaluate the results of the developed problem, as well as our iterative algorithm, for a case study of the city of Chicago, USA. We use data provided by the City of Chicago (2010) to establish 77 nodes based on the communities in the cities. Origins and destinations of all commuters inside these communities are aggregated to reduce the number of nodes, for computational reasons. Using the communities as nodes, we construct a fully connected graph, where the distances between nodes are obtained as Euclidean distances between the geographical centers of the communities. The origin–destination data of commuters is based on the use of

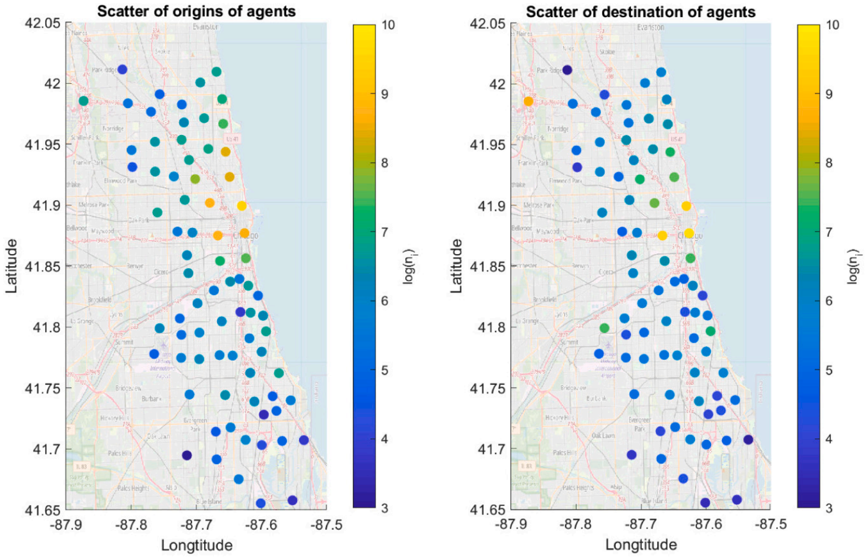

Fig. 3. Distribution of origins and destinations in Chicago (log scale).

ride-hailing vehicles, provided by City of Chicago (2022). The historical data have been used to construct demand rates of origin–destination pairs. In turn, this has been used to randomly generate instances of commuters. Commuters are generated according to a Poisson process with the obtained demand rate. The distribution of origins and destinations is displayed in Fig. 3.

We assume the car ownership rate among the generated commuters is equal to 75%, and commuters are divided into three groups by their desired arrival times (7:30 am, 8:00 am, and 8:30 am). The value of travel time for commuting solo (α^{solo}) is set to 6.8 [$/h], and the value of travel time for shared rides (α^{pool}) is set to 7.8 [$/h]. Earliness is penalized by β equal to 3.00 [$/h]. A base subsidy of 1.00$ is used per match. The gained utility is split evenly across rider and driver, which means that $\phi^{\text{rider}} = \phi^{\text{driver}} = 0.5$. The default value of $\eta_c = \eta = 1$, λ is set to 0.75, and δ is set to 0.2. The iterative algorithm has an iteration limit of 20. For the specific nested structure of our problem, as depicted in Fig. 2, we set the following values: $\epsilon_{opt-out,opt-out} = \epsilon_{rider,rider} = \epsilon_{driver,driver} = 1$, $\epsilon_{flexible,driver} = \epsilon_{flexible,rider} = 0.5$, and 0 otherwise.

4.2. Convergence of iterative algorithm

We evaluate the convergence of the iterative algorithm considering the following values of $\eta_c = [1, 2, 3]$ to evaluate the effect of this behavioral parameter on convergence. We initialize $\mathbf{p}_j$ randomly for all $j \in J$. Each line represents the convergence following one of these random initializations. We compare the objective value $U(x)$ and the norm of the difference between the $\mathbf{p}$ vectors of the current and the previous iterations. The results are presented in Fig. 4.

We observe that the algorithm converges to a stationary solution within a couple of iterations. The algorithm takes more iterations to converge when the value for η_c becomes larger. Larger values of η_c indicate that users are more sensitive to utility and more likely to choose the option with the highest utility. This can, therefore, amplify the differences between iterations, making convergence more difficult. This also implies that for smaller values of η_c, larger values of λ combined with higher iteration limits can be used to smooth the progress and improve convergence. The results emphasize the importance of parameter tuning and can even suggest the need for user segmentation. To enhance convergence, user groups that are more sensitive to changes in utility can then be made more stable offers compared to other classes of users.

4.3. Network analysis

Next, we analyze the influence of geographical features on the mode choice and match of a user by looking at node-specific results. A set of ten randomly generated instances is used to evaluate the total number of riders and drivers in every region in the network. Fig. 5 displays the proportion of drivers and riders for users with an origin (left) and destination (right) at each node. Fig. 6 displays the percentage of users that are successfully matched for users with an origin (left) and destination (right) at each node. Car ownership has been set to 50% to properly highlight the differences across the network.

The results show that users with origins and destinations closer to the city center are more likely to be matched in general. From Fig. 5 it is clear that the majority of those are matched as riders, whereas drivers mainly have origins and destinations further in the suburbs. The reason for this is that in the system, a driver picks up the rider on their way. Therefore, it is usually favorable

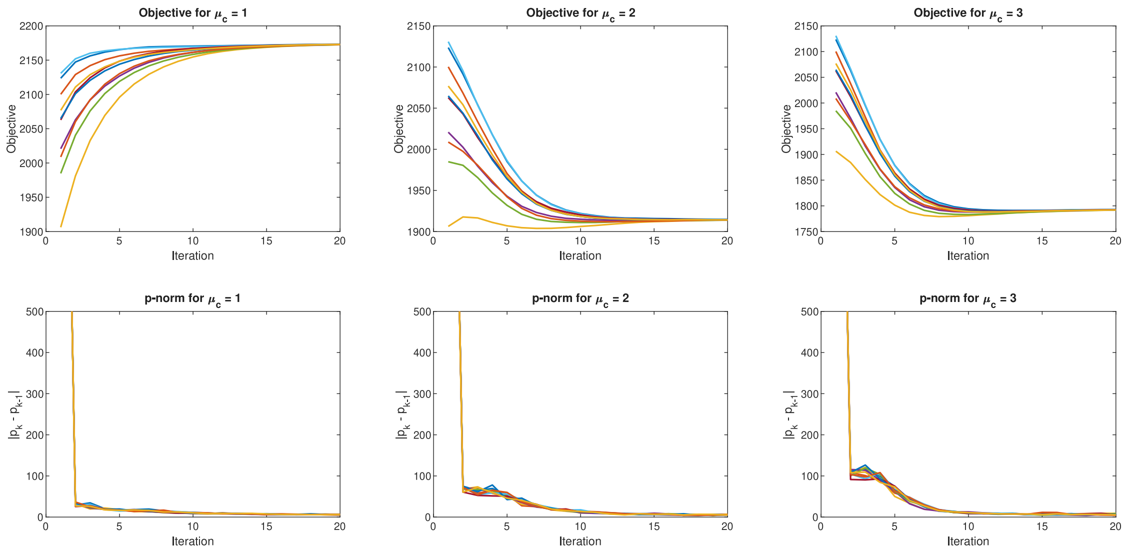

Fig. 4. Convergence of algorithm for varying values of μ_c. Each line represents a random $\mathbf{p}_j$ initialization.

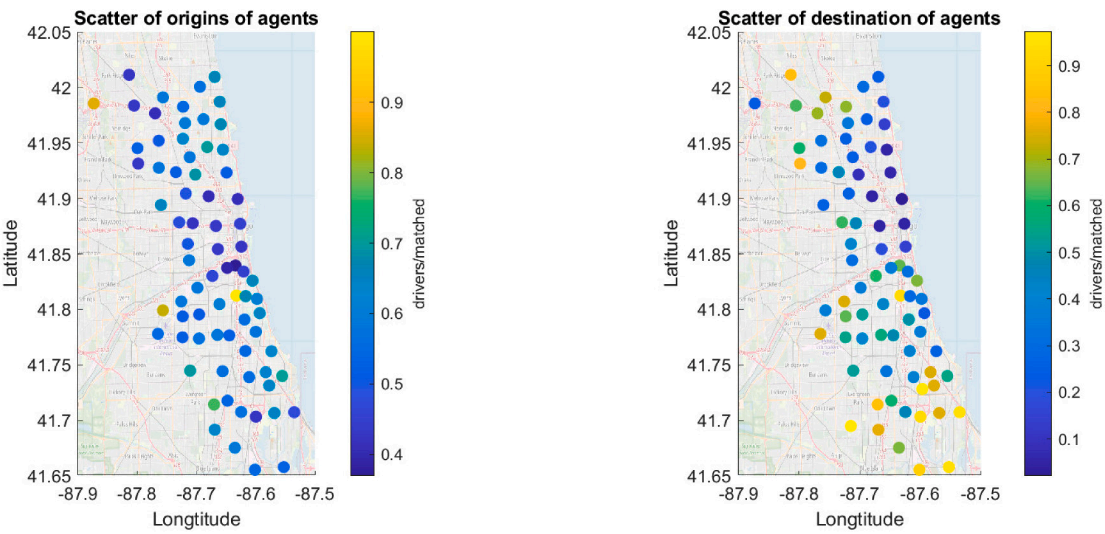

Fig. 5. Ratio of drivers over matched users in every region.

for the agent with the longest itinerary to perform the pickup. This effect is also seen in the utility these users assign to different modes, and with that, the probabilities with which they choose each mode. This is discussed in more detail in Section 4.5.

Our results imply that, in general, users with origins in the suburbs need to be subsidized more to reach a critical mass of ridesharing users. We can see that for those users, it is generally favorable to be a driver rather than a rider, both from the perspective of the user and the operator. Additionally, dynamic subsidy schemes that nudge suburban users to become drivers during peak hours or when local demand is low may help to enhance system performance. These results can help policymakers and operators of ride-sharing systems make more informed decisions.

4.4. Effect of flexibility

To further quantify the effect of allowing users to select a flexible role choice, we compare the equilibrium solutions to the social optimal solution (according to Definition 3) for four different levels of car ownership. The social optimal solution is obtained by solving the matching problem once for fixed probabilities p_j^m. For all classes that own a car, p_j^{flexible} are set to 1, and 0 for all other modes m. For all classes that do not own a car, p_j^{rider} are set to 1, and 0 for all other modes m. We emphasize that setting the probabilities in this way gives full freedom to the operator to determine the socially optimal matching. For every level, we

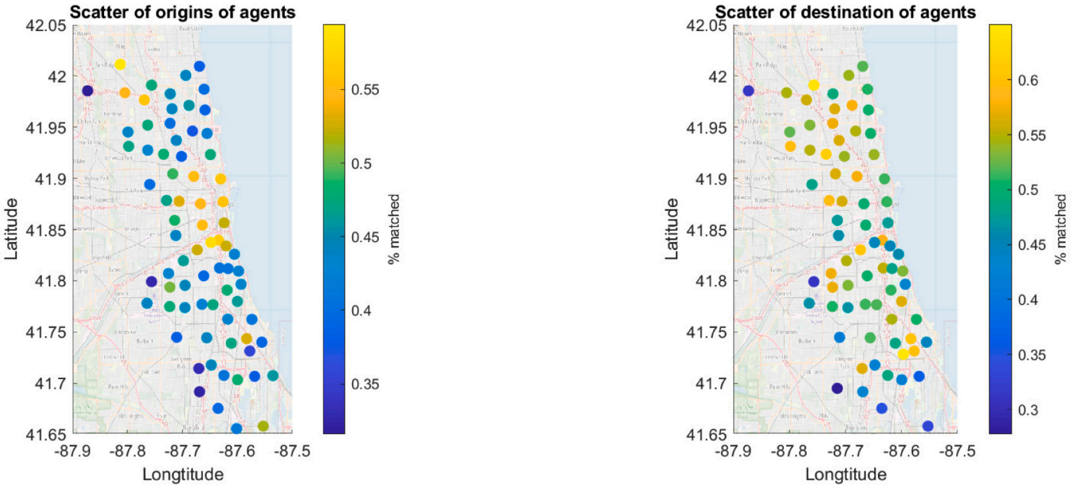

Fig. 6. Proportion of matched users in every region.

Table 1
Statistics on the effect of flexibility.

Car ownership	Soc. opt. utility	Flexible choice not allowed		Flexible choice allowed	
		Eq. utility (opt-gap %)	% matched	Eq. utility (opt-gap %)	% matched
25%	0.32	0.20 (−38.1)	22.2	0.21 (−32.3)	26.6
50%	0.47	0.28 (−40.5)	42.4	0.39 (−34.8)	49.2
75%	0.51	0.35 (−31.8)	59.8	0.43 (−28.2)	65.7
100%	0.53	0.37 (−30.5)	73.9	0.45 (−27.7)	74.4

The first column denotes the car ownership. The second column denotes the social optimal utility. The third and fourth denote the equilibrium utility and percentage of matched users for when users are not allowed to choose a flexible role. The fifth and sixth denote the equilibrium utility and percentage of matched users for when users are allowed to choose a flexible role. The number in parentheses behind the equilibrium utility denotes the optimality gap (i.e., the percentual difference between the equilibrium and social optimum). The percentage of users matched is the number of matched drivers and riders (both flexible and inflexible) as a percentage of the total number of users in the system.

randomly generate 10 instances of users. The results are displayed in Table 1, where the first column denotes the car ownership, the second column denotes the social optimal utility, the third and fourth denote the equilibrium utility and percentage of matched users for when users are not allowed to choose a flexible role, and the fifth and sixth denote the equilibrium utility and percentage of matched users for when users are allowed to choose a flexible role. The number in parentheses behind the equilibrium utility denotes the optimality gap (i.e., the difference between the equilibrium and social optimum).

The results indicate that the utility and the percentage of successful matches increase with the level of car ownership. When comparing the flexible to the inflexible scenario, we see that when users are allowed to be flexible, the utility in equilibrium is closer to the social optimal equilibrium. This is clear from the optimality gap, which is reduced due to an increase in the equilibrium utility. The percentage of successful matches is also increased. It is clear that when the percentage of car owners is high, flexibility mostly influences the utility, while the percentage of successful matches only marginally increases.

The results indicate that by simply allowing users to be flexible in their mode choice when this is the best option for them, the average utility and percentage of successful matches can be improved substantially. This finding can be extremely useful for ridesharing systems as it can be implemented in practice without capital investments or the design of dedicated policies. It also suggests that platform architectures should support modular user roles, enabling individuals to switch seamlessly between being riders and drivers. Treating the “driver” and “rider” roles as fluid rather than fixed allows the system to dynamically allocate resources based on real-time demand, thereby improving overall efficiency and user satisfaction.

4.5. Effect of average car ownership on mode choice

The option of flexibility is only directly affecting car owners, as those individuals without a car are limited to the same set of role choices. In this section, we quantify the difference in the effect of flexibility on users with and without a car. Fig. 7 displays the probabilities of choosing a mode for varying values of car ownership, and Fig. 8 displays the proportion of users being matched for varying values of car ownership. It is clear that users with an origin and destination in the city center have a preference for being a rider. This preference decreases as car ownership increases, giving users who stay within the downtown area to match with other users inside the downtown area. It is mostly the users traveling from downtown to the suburbs who are benefiting from being flexible. The main reason for this is that this is a relatively smaller group of users.

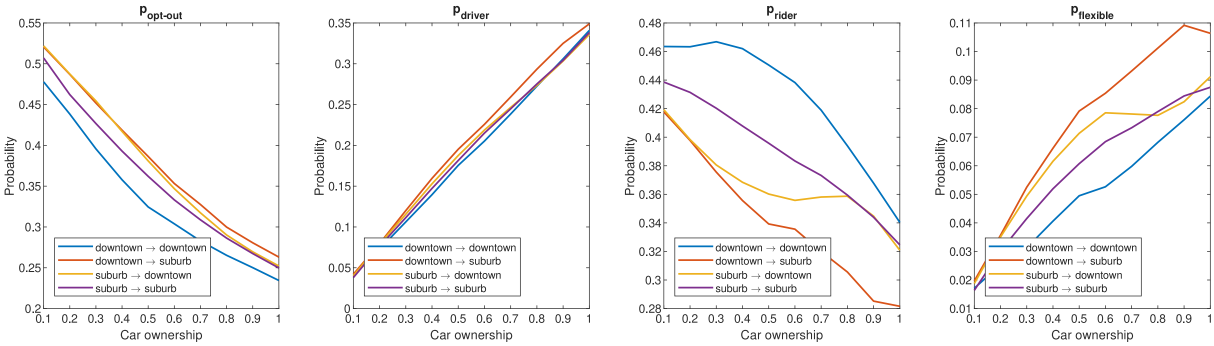

Fig. 7. Probabilities of choosing a mode for varying values of car ownership.

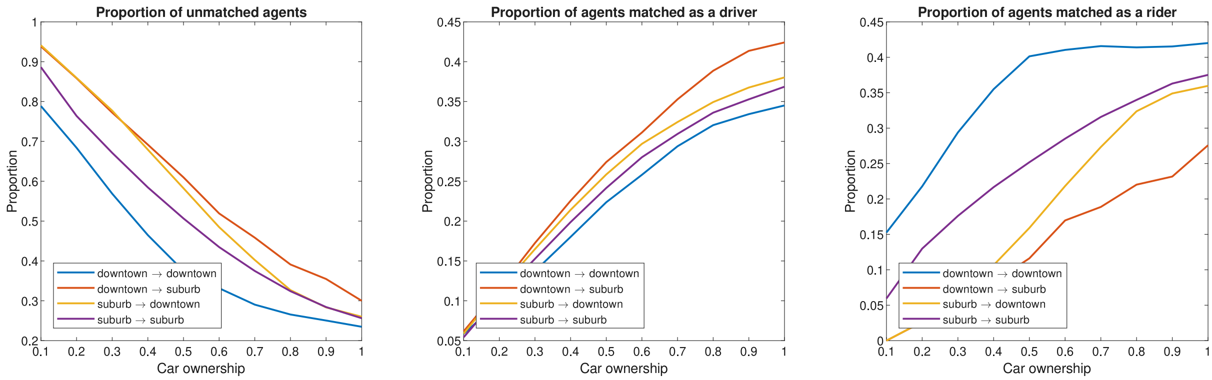

Fig. 8. Proportion of users being matched for varying values of car ownership.

When examining the effect of flexibility, this especially influences users with either an origin or a destination outside the downtown area. Giving these users the option to be flexible makes ride-sharing more appealing to them. Also, it increases the share of users that are potential drivers, which is generally the smaller share, given that users staying in the downtown area prefer to be riders.

The results of this paper, and specifically this subsection, signal both advantages and disadvantages of allowing car owners to choose a flexible role. On the one hand, it may increase the participation of car owners, and it may even lead to more car owners leaving their cars at home. On the other hand, it strengthens the position of car owners over non-car owners, which may, in turn, decrease matching efficiency because of imbalances between riders and drivers. In addition to this, it may lead to increased car ownership in the long term. Quantifying the advantages and disadvantages, therefore, marks an important direction for future research. Future research, through, for example, stated preference surveys, may investigate whether the advantages outweigh the disadvantages and, with that, come up with policy implications on whether offering a flexible option is always beneficial in practice. To avoid imbalances, platform operators should implement real-time monitoring tools capable of detecting such imbalances and dynamically adjusting incentives or recommendations. Selective limitations on flexibility, such as nudging users toward the driver role during periods of high rider demand or in underserved areas, may be necessary.

4.6. Effect of subsidies and distribution

In this section, we explore the influence of increasing subsidies and (un)evenly distributing the subsidies and gained utilities among riders and drivers in a pair. The results are displayed in Table 2. Clearly, the percentage of matches increases with an increase in the subsidies. This is straightforward, as it makes matching in any role more beneficial and therefore increases users' willingness to match. This is also clear from the decrease in p^{solo}, with users switching to other mode choices.

Interestingly, the effect of unevenly redistributing gained utility among the driver and rider in a pair is unequivocal across the scenarios. Redistributing more utility towards the driver naturally increases the probability of being a driver. The same holds for attributing more utility to the rider. Given the shortage of drivers, as noted in the previous sections, giving more utility to the rider is not beneficial from a system perspective. Giving more utility to a driver can be beneficial, but this depends on the level of subsidy as well. The reason for this is that the imbalance between riders and drivers is stronger when participation levels are lower, as noted in the previous sections. Therefore, choosing an optimal distribution is highly dependent on the other features of the system.

Table 2
Sensitivity analysis on subsidy and distribution of utility.

s	ϕ^{rider}	ϕ^{driver}	Matched %	p^{solo}	p^{driver}	p^{rider}	$p^{flexible}$
1	0.5	0.5	0.65	0.30	0.26	0.37	0.08
2	0.5	0.5	0.76	0.21	0.29	0.41	0.09
1	0.25	0.75	0.68	0.30	0.31	0.33	0.06
2	0.25	0.75	0.74	0.23	0.40	0.34	0.03
1	0.75	0.25	0.57	0.31	0.23	0.40	0.06
2	0.75	0.25	0.61	0.25	0.23	0.44	0.08

The first column denotes the level of subsidy. The second and third columns denote the distribution of gained utility among riders and drivers. The fourth column denotes the percentage of users who are matched. The final columns denote the average probability of users choosing a specific mode.

Giving different levels of subsidies to different classes of individuals, as well as redistributing utility based on features of the drivers and riders rather than using a single redistribution mechanism among the entire population, can further improve the performance of the ride-sharing system. However, this will lead to new issues in terms of fairness and truthfulness and is therefore outside the scope of this work. Nevertheless, the results in this section provide useful insights that can aid further developments in this field.

5. Conclusion

In this paper, we studied the participation of drivers and riders in a ridesharing system with flexible mode choices. We formulated the problem as a repeated multi-player non-cooperative game where the decisions of the operator were modeled as an LP, and the decisions of the users were modeled through a random utility model. The central operator decides on the optimal matching of drivers and riders, and users determine whether to participate in the ridesharing system and in what mode they wish to participate. To find the equilibrium solution to this multi-player problem, we developed an iterative approach that updates the expectations and decisions of users based on previous observations.

Our theoretical results show that an equilibrium solution to this problem exists and is unique under certain conditions. We developed an iterative algorithm, which is guaranteed to converge under the same mild conditions. Our numerical results are evaluated on a case study of the city of Chicago. Our iterative algorithm tends to converge to an equilibrium solution within ten iterations. We observe that users with an origin and destination in the downtown area have a lower probability of being unmatched and are more likely to be matched as a rider than as a driver. We have also shown that the considered ridesharing system benefits heavily from economies of scale. As the participation of users in the system increases, the percentage of matched users increases, and the user equilibrium solution gets closer to the system-optimal solution. The results also suggest that allowing users to be flexible in their mode choice improves the performance of the ridesharing system and the utility of the users. However, as it may improve the position of car owners compared to non-car owners, future research is needed to properly quantify the effect of such a policy in the long term. Naturally, subsidies can further improve the performance of the system by increasing the number of matches. By redistributing utility between a rider and a driver within a matched pair, users can be stimulated to opt for either one of the modes. This can be especially beneficial when a shortage of one of the two modes is observed. Other directions of future research include treating the problem as a Stackelberg-like game, where the operator is aware of how their decisions influence individual choices.

The results obtained in this paper can be useful in designing subsidy schemes that improve the performance of the ridesharing system and can be used to identify a critical mass for participation in the ridesharing service. These areas are marked as important directions for future research. In addition to this, future work can focus on incorporating socio-economic properties of users in their role-choice decisions. Park et al. (2018) studied data from Ohio State University's 2012 Campus Travel Pattern Survey. They found that people favoring the passenger role emphasized safety, flexibility, and parking cost-savings, whereas those favoring the driver role tended to find value in the convenience and opportunities for socializing through a carpool trip. People receptive to both roles emphasized flexibility and all types of cost reductions.

Besides the main conclusion that allowing for flexible role choices can significantly enhance the matching success rates, various managerial insights can be obtained from the results in this paper. Suburban participants, who tend to favor driving over riding, often require greater incentives to reach critical adoption thresholds—suggesting the value of targeted subsidies and dynamic role encouragement in low-density areas. More broadly, our analysis shows that allowing users, particularly car owners, to flexibly switch between rider and driver roles can significantly improve overall utility and match success rates, bringing system outcomes closer to socially optimal equilibria. However, this flexibility must be managed carefully. Real-time monitoring tools are essential to detect and address potential imbalances, especially when flexible users disproportionately choose to ride.

CRediT authorship contribution statement

Patrick Stokkink: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Zhenyu Yang:** Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Nikolas Geroliminis:** Writing – original draft, Validation, Supervision, Methodology, Investigation, Conceptualization.

Table 3
Notational glossary.

Sets	
$\mathcal{A}$	Set of match types (indexed a)
C	Set of nests (indexed c)
J	Set of user classes (indexed i, j)
M	Set of participation modes (indexed m)
R	Set of possible matching outcomes (indexed r)
Parameters	
$dist(j)$	Distance between the origin and destination of users in class $j \in J$
$detour(i, j)$	Additional time needed for a driver of class j to pick up and drop off a rider of class i
g_{ij}	Gained utility of matched users from class i and j as rider and driver, respectively, compared to when both travel alone
n_j	Number of users in class $j \in J$
t_j^*	Desired arrival time of users in class $j \in J$
$u_j^m(r)$	Utility of users in class $j \in J$ choosing mode $m \in M$ under matching outcome $r \in R$.
u_{ij}	Joint utility of users in class $i \in J$ being matched as riders to users in class $j \in J$ as drivers
$U(\mathbf{x})$	Total system utility under matching $\mathbf{x}$
$\alpha^{\text{ opt-out}}$	Value of time when traveling alone
α^{pool}	Value of time when sharing rides
β	Cost of arriving early
γ	Cost of arriving late
δ	Discomfort penalty for users that choose to share a ride but remain unmatched
ϕ^{rider}	Share of the gained utility of a match claimed by the rider
ϕ^{driver}	Share of the gained utility of a match claimed by the driver
η_c	Nest-specified scale parameter in the cross-nested logit model, for nest $c \in C$
η	Global Scale parameter in the cross-nested logit model
ϵ_{mc}	Degree of membership in the cross-nested logit model for mode $m \in M$ and nest $c \in C$
Variables	
p_j^m	Fraction of users in class $j \in J$ that choose mode $m \in M$
$\rho_j^m(r)$	Probability of matching outcome r for users in class $j \in J$ that choose mode $m \in M$
v_j^m	Expected utility of users in class $j \in J$ for choosing *participation mode* $m \in M$
x_{ij}^a	Number of matches of type $a \in \mathcal{A}$ for riders of class $i \in J$ to drivers of class $j \in J$

Appendix

See Table 3.

References

Agatz, N., Erera, A.L., Savelsbergh, M.W., Wang, X., 2011. Dynamic ride-sharing: A simulation study in metro atlanta. Procedia Soc. Behav. Sci. 17, 532–550.
Amerehi, F., Healy, P., 2025. Transforming ridesharing: Harnessing role flexibility and HOV integration for enhanced mobility solutions. arXiv preprint arXiv:2504.05077.
Amirkiaee, S.Y., Evangelopoulos, N., 2018. Why do people rideshare? An experimental study. Transp. Res. Part F: Traffic Psychol. Behav. 55, 9–24.
Baldacci, R., Maniezzo, V., Mingozzi, A., 2004. An exact method for the car pooling problem based on lagrangean column generation. Oper. Res. 52 (3), 422–439.
Bierlaire, M., 2006. A theoretical analysis of the cross-nested logit model. Ann. Oper. Res. 144, 287–300.
Böhm, V., 1975. On the continuity of the optimal policy set for linear programs. SIAM J. Appl. Math. 28 (2), 303–306.
Border, K.C., 1985. Fixed Point Theorems with Applications to Economics and Game Theory. Cambridge University Press.
Bulteau, J., Feuillet, T., Dantan, S., 2019. Carpooling and carsharing for commuting in the Paris region: A comprehensive exploration of the individual and contextual correlates of their uses. Travel. Behav. Soc. 16, 77–87.
Canning, P., Hughes, S., Hellawell, E., Gatersleben, B., Fairhead, C., 2010. Reasons for participating in formal employer-led carpool schemes as perceived by their users. Transp. Plan. Technol. 33 (8), 733–745.
Chan, N.D., Shaheen, S.A., 2012. Ridesharing in North America: Past, present, and future. Transp. Rev. 32 (1), 93–112.
Chen, X.M., Zahiri, M., Zhang, S., 2017. Understanding ridesplitting behavior of on-demand ride services: An ensemble learning approach. Transp. Res. Part C: Emerg. Technol. 76, 51–70.
City of Chicago, 2010. Boundaries - census tracts - 2010. https://data.cityofchicago.org/Facilities-Geographic-Boundaries/Boundaries-Census-Tracts-2010/5jrd-6zik.
City of Chicago, 2022. Transportation network providers - trips (2018 - 2022). https://data.cityofchicago.org/Transportation/Transportation-Network-Providers-Trips-2018-2022-/m6dm-c72p.
Correia, G., Viegas, J.M., 2011. Carpooling and carpool clubs: Clarifying concepts and assessing value enhancement possibilities through a stated preference web survey in Lisbon, Portugal. Transp. Res. Part A: Policy Pr. 45 (2), 81–90.
Daganzo, C.F., 1982. Design of a park-and-ride carpool system. Transportation Research Part B: Methodological 16 (4), 315–327.
de Palma, A., Stokkink, P., Geroliminis, N., 2022. Influence of dynamic congestion with scheduling preferences on carpooling matching with heterogeneous users. Transp. Res. Part B: Methodol. 155, 479–498.
Di, X., Ma, R., Liu, H.X., Ban, X.J., 2018. A link-node reformulation of ridesharing user equilibrium with network design. Transportation Research Part B: Methodological 112, 230–255.
Haliem, M., Mani, G., Aggarwal, V., Bhargava, B., 2021. A distributed model-free ride-sharing approach for joint matching, pricing, and dispatching using deep reinforcement learning. IEEE Trans. Intell. Transp. Syst. 22 (12), 7931–7942.

Julagasigorn, P., Banomyong, R., Grant, D.B., Varadejsatitwong, P., 2021. What encourages people to carpool? A conceptual framework of carpooling psychological factors and research propositions. Transp. Res. Interdiscip. Perspect. 12, 100493.
Kahneman, D., Tversky, A., 2013. Prospect theory: An analysis of decision under risk. In: Handbook of the Fundamentals of Financial Decision Making: Part I. World Scientific, pp. 99–127.
Liu, X., Titheridge, H., Yan, X., Wang, R., Tan, W., Chen, D., Zhang, J., 2020. A passenger-to-driver matching model for commuter carpooling: Case study and sensitivity analysis. Transp. Res. Part C: Emerg. Technol. 117, 102702.
Ma, Z., Koutsopoulos, H.N., 2022. Near-on-demand mobility. The benefits of user flexibility for ride-pooling services. Transp. Res. Part C: Emerg. Technol. 135, 103530.
Ma, S., Wang, Y., Zhang, D., 2020. Ride-sharing and dynamic pricing with stochastic user equilibrium and elastic demand. Transportation Research Part C: Emerging Technologies 117, 102671.
Mangasarian, O.L., Shiau, T.-H., 1987. Lipschitz continuity of solutions of linear inequalities, programs and complementarity problems. SIAM J. Control Optim. 25 (3), 583–595.
Manski, C.F., 1977. The structure of random utility models. Theory and Decision 8 (3), 229–254.
Manski, C.F., 1999. Identification Problems in the Social Sciences. Harvard University Press.
Masoud, N., Jayakrishnan, R., 2017a. A decomposition algorithm to solve the multi-hop peer-to-peer ride-matching problem. Transp. Res. Part B: Methodol. 99, 1–29.
Masoud, N., Jayakrishnan, R., 2017b. A real-time algorithm to solve the peer-to-peer ride-matching problem in a flexible ridesharing system. Transp. Res. Part B: Methodol. 106, 218–236.
Michael Page, 2023. Switzerland: The train nation. Switzerland: The train nation - 94% of professionals rank public transportation system as efficient.
Neoh, J.G., Chipulu, M., Marshall, A., 2017. What encourages people to carpool? An evaluation of factors with meta-analysis. Transp. 44, 423–447.
Park, Y., Chen, N., Akar, G., 2018. Who is interested in carpooling and why: The importance of individual characteristics, role preferences and carpool markets. Transp. Res. Rec. 2672 (8), 708–718.
Peng, Z., Wang, W., Wang, C., Yu, B., 2024. Designing stable ride-sharing for commuting trip chain with role flexibility. Transp. 1–33.
Qin, G., Luo, Q., Yin, Y., Sun, J., Ye, J., 2021. Optimizing matching time intervals for ride-hailing services using reinforcement learning. Transp. Res. Part C: Emerg. Technol. 129, 103239.
Shaheen, S.A., Chan, N.D., Gaynor, T., 2016. Casual carpooling in the San Francisco Bay Area: Understanding user characteristics, behaviors, and motivations. Transp. Policy 51, 165–173.
Stiglic, M., Agatz, N., Savelsbergh, M., Gradisar, M., 2016. Making dynamic ride-sharing work: The impact of driver and rider flexibility. Transp. Res. Part E: Logist. Transp. Rev. 91, 190–207.
Swiss Federal Statistical Office, 2021. Population's mobility behaviour. https://www.bfs.admin.ch/bfs/en/home/statistics/mobility-transport/passenger-transport/travel-behaviour.html.
Swissinfo, 2017. Swiss government examines carpooling plans. https://www.swissinfo.ch/eng/society/share-and-work_government-tables-carpooling-plans/43355260.
Tafreshian, A., Masoud, N., 2020a. Trip-based graph partitioning in dynamic ridesharing. Transp. Res. Part C: Emerg. Technol. 114, 532–553.
Tafreshian, A., Masoud, N., 2020b. Using subsidies to stabilize peer-to-peer ridesharing markets with role assignment. Transp. Res. Part C: Emerg. Technol. 120, 102770.
Tafreshian, A., Masoud, N., Yin, Y., 2020. Frontiers in service science: Ride matching for peer-to-peer ride sharing: A review and future directions. Serv. Sci. 12 (2–3), 44–60.
Tahmasseby, S., Kattan, L., Barbour, B., 2016. Propensity to participate in a peer-to-peer social-network-based carpooling system. J. Adv. Transp. 50 (2), 240–254.
Teal, R.F., 1987. Carpooling: Who, how and why. Transp. Res. Part A: Gen. 21 (3), 203–214.
USDOE, 2022. Transportation energy data book. In: U.S. Department of Energy (DOE), Oak Ridge National Lab (2022) Transportation Energy Data Book Edition 40.
Wang, S., Noland, R.B., 2021. What is the elasticity of sharing a ridesourcing trip? Transp. Res. Part A: Policy Pr. 153, 284–305.
Wang, J., Wang, X., Yang, S., Yang, H., Zhang, X., Gao, Z., 2021. Predicting the matching probability and the expected ride/shared distance for each dynamic ridepooling order: A mathematical modeling approach. Transp. Res. Part B: Methodol. 154, 125–146.
Xia, J., Curtin, K.M., Huang, J., Wu, D., Xiu, W., Huang, Z., 2019. A carpool matching model with both social and route networks. Comput. Environ. Urban Syst. 75, 90–102.
Xu, H., Pang, J.-S., Ordóñez, F., Dessouky, M., 2015. Complementarity models for traffic equilibrium with ridesharing. Transportation Research Part B: Methodological 81, 161–182.
Yao, R., Bekhor, S., 2023. A general equilibrium model for multi-passenger ridesharing systems with stable matching. Transp. Res. Part B: Methodol. 175, 102775.