# Optimal Channel Reuse in Cellular Radio Systems with Multiple Correlated Log-Normal Interferers

Aysel Şafak

*Abstract*— **The probability of co-channel interference (PCI) due to multiple correlated log-normal signals is calculated for cellular radio systems operating in Rayleigh fading and log-normal shadowing environment. The effects on the PCI of the correlation between the signals, the standard deviation due to shadowing, the number of interferers, the co-channel protection ratio, and the traffic load is investigated. The results are used for analyzing the optimal channel reuse, the cluster size, and the spectrum efficiency in terms of these parameters.**

## I. Introduction

IN cellular radio systems, the principal design constraint is the limited available frequency spectrum. Frequency reuse is one of the fundamental approaches for achieving efficiency of spectrum usage. However, if the system is not properly designed, serious interference may occur. Reusing the same frequency channels in different cells is limited by the co-channel interference between the cells [1]. The minimum distance required between two nearby co-channel cells is based on specifying a maximum tolerable PCI allowed in the system. Therefore, a realistic evaluation of the PCI helps establishing the optimal frequency reuse distance between the co-channel cells for a given choice of a cellular radio system.

Evaluation of the PCI requires, amongst others, a realistic model of the wave propagation in a fading and shadowing environment. A mobile radio signal envelope is modeled as a Rayleigh fading signal superimposed on a shadowing signal. Rayleigh fading is caused by multipath propagation while the terrain configuration is responsible for shadowing. The random fluctuations in the signal amplitude can be described approximately by a log-normal probability density function (pdf) over long periods, and by a Rayleigh pdf for much shorter periods [2]. Rayleigh fading of different signals is uncorrelated; however, in a shadowed environment, the mean levels of the desired and the interfering signals might be correlated [3].

In cellular radio systems, the total co-channel interference power in a receiver results from the sum of the powers of individual random co-channel interfering signals. Therefore, one needs to determine the statistics of the sum of the powers of random interfering signals, each with a log-normal pdf. A number of approximations to this problem have been developed to supplement the widespread use of Monte Carlo simulation [4]–[6]. These approximate methods have limitations while complete Monte Carlo simulation, though more accurate, requires extensive computer time.

Similarly, the statistics of the sum of the powers of uncorrelated interferers undergoing Rayleigh fading needs to be determined. Incoherent sum of the powers of these signals is described by an $n$-fold convolution of their pdf's, which results in a gamma distribution if they all have the same mean. However, the coherent addition of the powers of Rayleigh distributed phasors is described by an exponential pdf.

The calculation of the PCI for a system with Rayleigh fading and uncorrelated log-normal shadowing is reported in numerous publications [7]–[13]. In this context, one can refer to [14] and [15] for an exact analysis of outage probability with multiple uncorrelated interferers in an environment with fading and shadowing. This analysis is based on an observation that the exponential distribution of the desired signal power, due to Rayleigh fading, can be interpreted as a kernel for the Laplace transform of the pdf of the joint interference power of multiple interferers undergoing Rayleigh fading and log-normal shadowing. The effect of individual interferers can thus be accounted for by the product of their Laplace transforms instead of the more difficult convolution operation. Unfortunately, this technique is not suitable for studying the case of correlated interferers, since, in this case, the composite pdf's of the desired and individual interferers are not separable.

Though the importance of correlation between the signals has already been recognized [16], [17], the effect of multiple correlated log-normal interferers on the PCI in cellular radio systems has not been given due consideration, except for the cases of one and two correlated log-normal interferers [7], [18]–[20]. This paper addresses the effect of correlation between multiple log-normal signals on the performance of cellular radio systems. The PCI due to multiple correlated log-normal signals has been calculated for the mobile radio systems operating in an environment with Rayleigh fading and log-normal shadowing. The results obtained are used to calculate the normalized frequency reuse distance, the cluster size, and the spectrum efficiency. The results are based on an extension of the method of Schwartz and Yeh [6] for determining the mean and variance of the sum of the powers of multiple correlated log-normal signals [21].

The PCI due to multiple correlated log-normal signals is derived in Section II for mobile radio systems operating in an environment with Rayleigh fading and log-normal shadowing.

Manuscript received June 24, 1992; revised September 15, 1992 and August 10, 1993.

The author was with the Telecommunications and Traffic Control Systems Group, Delft University of Technology, 2600 GA Delft, The Netherlands. She is now with the Department of Electrical and Electronic Engineering, Eastern Mediterranean University, Gazi Mağusa, North Cyprus, Mersin 10, Turkey.

IEEE Log Number 9214720.

Section III describes the calculation of the PCI as a function of the normalized frequency reuse distance while Section IV relates the total PCI to the number of cells per cluster. Section V presents the effects of traffic load, the number of channels per cell, bandwidth per channel, the unit cell area, and the protection ratio on the spectrum efficiency. Section VI is devoted to the conclusions.

## II. Probability of Co-Channel Interference

The fundamental parameters in the design of a cellular radio system are the transmitter power, the cell radius, the normalized frequency reuse distance, the cluster size, and the spectrum efficiency. In order to determine these parameters, it is necessary to relate them to the PCI caused by a number of co-channel interferers [7]–[13]. The total PCI, due to $N$ interferers, is defined by

$$F(CI) \triangleq \sum_{n=1}^{N} F(CI|n)F_n(n). \tag{1}$$

Here, $F_n(n)$ is the probability of $n$ co-channel interferers being active. Assuming that the blocking probability, $B$, and the number of channels per cell, $N_s$, are the same in all cells, and considering the interfering signals only from the nearest neighboring $N = 6$ co-channel cells, $F_n(n)$ may be written as [9]

$$F_n(n) = \binom{N}{n} B^{n/N_s}(1 - B^{1/N_s})^{N-n} \tag{2}$$

where $B^{1/N_s}$ denotes the probability that a particular co-channel interferer is active. Note that, in some versions of (2) given in the literature, $B^{1/N_s}$ is replaced by $A_c/N_s$ where $A_c$ denotes the carried traffic per cell [10].

The corresponding conditional PCI, $F(CI|n)$, due to $n$ active interferers, is defined by

$$F(CI|n) \triangleq \text{Prob}(v < \alpha) = \int_0^{\alpha} f_v(v)\,dv. \tag{3}$$

Here, $f_v(v)$ denotes the pdf of $v = P_s/P_n$, and can be found from the joint pdf of $P_s$ and $P_n$ as follows

$$f_v(v) = \int_0^{\infty} f_{P_s,P_n}(P_s, P_n)\left|\frac{\partial(P_s, P_n)}{\partial(v, w)}\right| dw \tag{4}$$

with $w = P_n$ and the corresponding Jacobian $|\partial(P_s, P_n)/\partial(v, w)| = w$. Here, $P_s$ is the short-term power of the desired signal, $P_n$ denotes the cumulative interference power from $n$ active co-channels, and $\alpha$ stands for the specified minimum co-channel protection ratio.

In a shadowed environment, the power levels of the desired signal and interferers might be correlated, since the same obstacles may cast overlapping shadows to signals transmitted from different base stations. Assuming that the desired and interfering signals are correlated, the joint pdf of the local mean powers of the desired, $P_{od}$, and the sum of interfering signals, $P_{ou}$, may be written as [22], [23]

$$f_{P_{od},P_{ou}}(P_{od}, P_{ou}) = \frac{\exp\left(-\dfrac{(\tau_d^2 + \tau_u^2 - 2\rho_{d,u}\tau_d\tau_u)}{2(1-\rho_{d,u}^2)}\right)}{2\pi\sigma_d\sigma_u P_{od}P_{ou}\sqrt{1-\rho_{d,u}^2}}. \tag{5}$$

Here, $\tau_d = (1/\sigma_d)\ln(P_{od}/\xi_d)$ with $\sigma_d, P_{od}$, and $\xi_d$ defined as, respectively, the standard deviation, the local mean power, and the area mean power of the desired signal. Similarly, $\tau_u = (1/\sigma_u)\ln(P_{ou}/\xi_u)$ with $\sigma_u, P_{ou}$, and $\xi_u$ being, respectively, the standard deviation, the local mean power, and the area mean power of the sum of interfering signals. The sum of the powers of a finite number of log-normally distributed signals can be approximated, at least as a first order, by another log-normal pdf [4]–[6]. Therefore, the local mean power, $P_{ou}$, of the sum of the powers of $n$ interfering signals is also described by a log-normal pdf with $\sigma_u$ and $\xi_u$. Both $\sigma_u$ and $\xi_u$ can be obtained by using the extension of the method of Schwartz and Yeh [6] for $n$ correlated signals as described in [21].

Note that $\rho_{d,u}$ is the correlation coefficient between the desired and the sum of interfering signals and is calculated using the method presented in [21] as a function of the area mean power levels, standard deviation, and the correlation coefficients between the individual signals. For realistic scenarios in cellular radio systems, the correlation coefficients between the individual signals may vary between 0.4 and 0.6. Nevertheless, these values depend on the angular separation between their directions of arrival and are expected to decrease with increasing angular separation between them. Therefore, the assumption of a single correlation coefficient between the signals received by a user terminal is not necessarily very realistic but may still provide useful information about the effects of correlation.

The pdf of $v, f_v(v)$, can be determined by inserting (5) into (4) where $P_s$ is replaced by $P_{od}$ and $P_n$ by $P_{ou}$. The conditional PCI for shadowing, $F(CI|n)$, is found from (3)–(5) as follows

$$F(CI|n) = \frac{1}{\sqrt{2\pi}}\int_{p/\sigma_{\text{eff}}}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt. \tag{6}$$

Here

$$p = \ln\left(\frac{\xi_d}{\xi_u\alpha}\right) \tag{7}$$

and $\sigma_{\text{eff}}$ is the effective standard deviation given by

$$\sigma_{\text{eff}}^2 = \sigma_d^2 + \sigma_u^2 - 2\rho_{d,u}\sigma_d\sigma_u. \tag{8}$$

In a mobile radio environment, the received signal consists of a Rayleigh fading signal superimposed on a shadowing signal. Fast amplitude fluctuations of mobile radio signals can be represented by the Rayleigh pdf. The corresponding short-term power (average power measured over a number of RF cycles) of the desired signal, $P_d$, is exponentially distributed around the local mean power, $P_{od}$

$$f_{P_d}(P_d|P_{od}) = \frac{1}{P_{od}}\exp\left(-\frac{P_d}{P_{od}}\right). \tag{9}$$

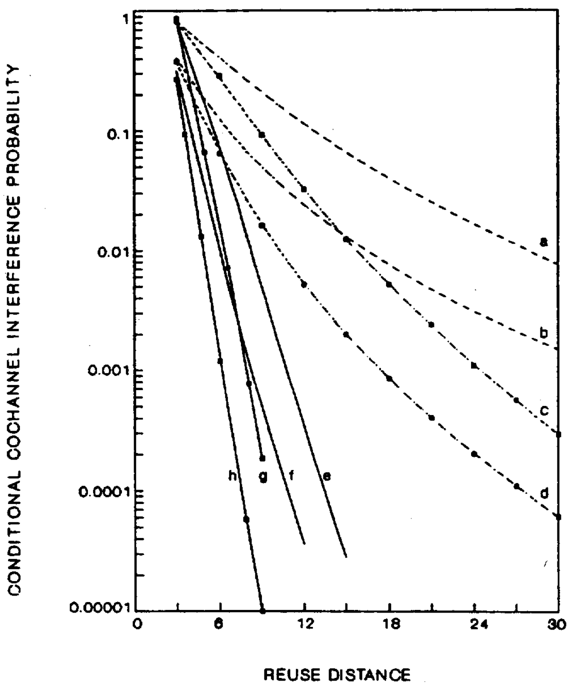



Fig. 1. Conditional PCI with pure shadowing for $\alpha = 8$ dB as a function of normalized frequency reuse distance with standard deviation $\sigma$, correlation coefficient $\rho$, and the number of interferers $n$ as parameters. $\sigma = 12$ dB: a) $\rho = 0, n = 6$; b) $\rho = 0, n = 1$; c) $\rho = 0.4, n = 6$; d) $\rho = 0.4, n = 1$. $\sigma = 6$ dB: e) $\rho = 0, n = 6$; f) $\rho = 0, n = 1$; g) $\rho = 0.4, n = 6$; h) $\rho = 0.4, n = 1$.

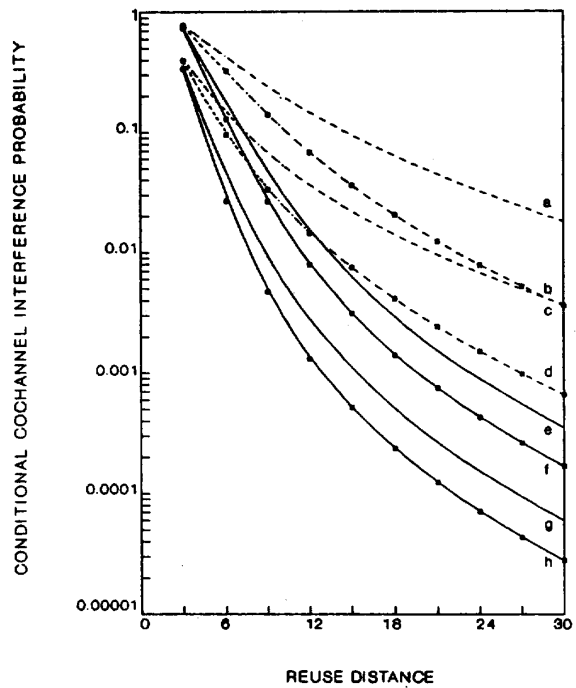



Fig. 2. Conditional PCI with fading and shadowing for $\alpha = 8$ dB as a function of normalized frequency reuse distance with standard deviation $\sigma$, correlation coefficient $\rho$, and the number of interferers $n$ as parameters. $\sigma = 12$ dB: a) $\rho = 0, n = 6$; b) $\rho = 0.4, n = 6$; c) $\rho = 0, n = 1$; d) $\rho = 0.4, n = 1$. $\sigma = 6$ dB: e) $\rho = 0, n = 6$; f) $\rho = 0.4, n = 6$; g) $\rho = 0, n = 1$; h) $\rho = 0.4, n = 1$.

The pdf of the incoherent sum of the powers of $n$ exponentially distributed interfering signals can described by an $n$-fold convolution of their pdf's; if these signals all have the same local mean powers their sum would have a gamma distribution [24]. The short-term power, $P_u$, of the cumulative interference is assumed to be described by an exponential pdf of the form (9), with a local mean power, $P_{ou}$. This is justifiable if the addition of $n$ Rayleigh distributed phasors can be assumed to be coherent over some sufficiently long time interval. Coherent addition of the Rayleigh phasors was reported to yield more optimistic results (lower PCI) compared to incoherent addition. However, the difference between the two predictions is less than 10% [10]. Here, we assumed coherent addition of the Rayleigh phasors.

In case of combined Rayleigh fading and log-normal shadowing, the joint pdf of the instantaneous powers of the desired and the sum of interfering signals may be written as

$$f_{P_d,P_u}(P_d, P_u) = \int_0^\infty \int_0^\infty f_{P_d}(P_d|P_{od}) f_{P_{od},P_{ou}}(P_{od}, P_{ou}) \cdot f_{P_u}(P_u|P_{ou})\, dP_{od}\, dP_{ou}. \tag{10}$$

Insertion of (10) into (4) with $v = P_d/P_u$ and $w = P_u$ and the use of (3) lead to the following expression for the conditional PCI with combined fading and shadowing

$$F(CI|n) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\exp(-t^2/2)}{[1 + \exp(p + \sigma_{\text{eff}} t)]}\, dt. \tag{11}$$

## III. Normalized Frequency Reuse Distance

The frequency reuse distance normalized by cell radius, $R_u$, is defined as the ratio of the distance between the centers of two nearby co-channel cells, $D$, and the cell radius of the desired station, $R$

$$R_u = \frac{D}{R}. \tag{12}$$

A reduction in the normalized frequency reuse distance would increase the system capacity and the efficiency of spectrum usage. Therefore, it is reasonable to try to minimize the normalized frequency reuse distance, while still meeting the specified system performance requirements. The minimum normalized frequency reuse distance required between two nearby co-channel cells is based on specifying a tolerable co-channel interference, which depends on the signal-to-interference ratio. Assuming the shortest distance, ($D$-$R$), for all interferers, as a worst case [25], [26], and a $R^{-\gamma}$ type dependence of the area mean power on distance, the signal-to-interference ratio may be written as the ratio of the area mean powers of the desired and the sum of interfering signals (see [21], eq. (4))

$$\frac{\xi_d}{\xi_u} = (R_u - 1)^\gamma \exp\left(-\sum_{k=2}^{n} G_1(\sigma_{w_k}, m_{w_k})\right) \tag{13}$$

where $n$ denotes the number of active co-channel interferers. $G_1(\sigma_{wk}, m_{wk})$ is a function used to find the area mean power

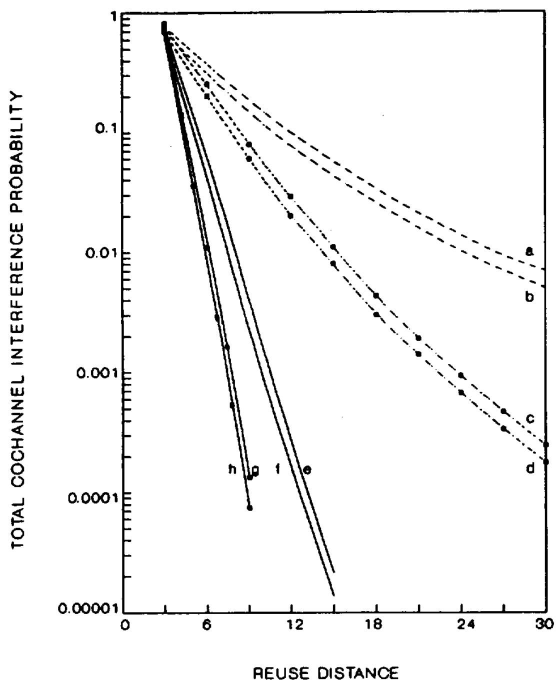



Fig. 3. Total PCI with pure shadowing for $N = 6, \alpha = 8$ dB, and $N_s = 10$ as a function of normalized frequency reuse distance with standard deviation $\sigma$, correlation coefficient $\rho$, and the blocking probability $B$ as parameters. $\sigma = 12$ dB: a) $\rho = 0, B = 0.2$; b) $\rho = 0, B = 0.01$; c) $\rho = 0.4, B = 0.2$; d) $\rho = 0.4, B = 0.01. \sigma = 6$ dB: e) $\rho = 0, B = 0.2$; f) $\rho = 0, B = 0.01$; g) $\rho = 0.4, B = 0.2$; h) $\rho = 0.4, B = 0.01$.

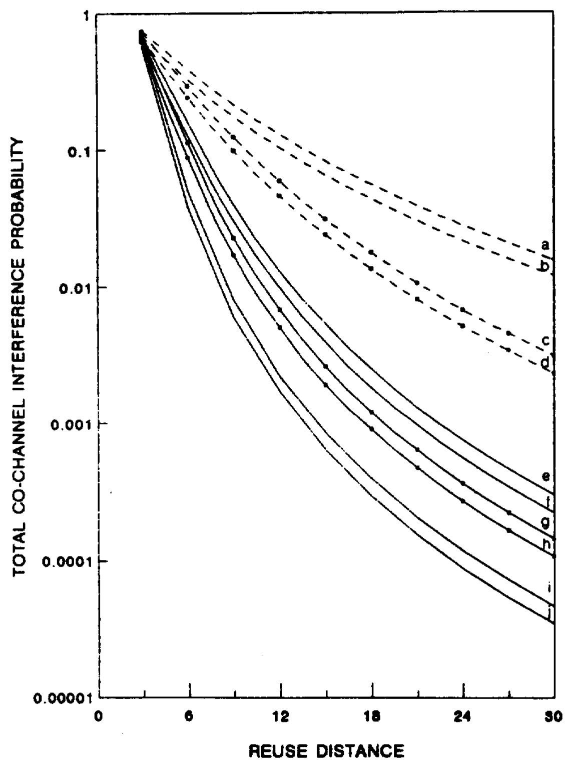



Fig. 4. Total PCI with fading and shadowing for $N = 6, \alpha = 8$ dB, and $N_s = 10$ as a function of normalized frequency reuse distance with standard deviation $\sigma$, correlation coefficient $\rho$, and the blocking probability $B$ as parameters. $\sigma = 12$ dB: a) $\rho = 0, B = 0.2$; b) $\rho = 0, B = 0.01$; c) $\rho = 0.4, B = 0.2$; d) $\rho = 0.4, B = 0.01. \sigma = 6$ dB: e) $\rho = 0, B = 0.2$, f) $\rho = 0, B = 0.01$; g) $\rho = 0.4, B = 0.2$; h) $\rho = 0.4, B = 0.01. \sigma = 0$ dB (pure fading): i) $\rho = 0, B = 0.2$; j) $\rho = 0, B = 0.01$.

of the sum of $n$ correlated log-normal interferers of the form $\exp(y_i)$ where $y_i$ are normally distributed. The parameters $\sigma_{wk}$ and $m_{wk}$ denote, respectively, the standard deviation and the mean of $w_k$ [21]

$$w_k = y_k - \ln\left(\sum_{i=1}^{k-1} e^{y_i}\right). \tag{14}$$

For more details, the reader is referred to [21]. Here, $\gamma$ is the propagation path loss exponent ($2 \leq \gamma \leq 5$) and is typically $\gamma = 4$ for UHF propagation in cellular radio.

By using (6), (7), and (13), the conditional PCI due to one and six correlated log-normal interferers is calculated and plotted in Fig. 1 as a function of the normalized frequency reuse distance for $\alpha = 8$ dB, $\sigma = 6$ dB, and 12 dB. The standard deviation of the desired signal and individual interferers due to shadowing are all assumed equal and denoted by $\sigma$. The area mean powers of the desired, $\xi_d$, and the interfering signals, $\xi_i$, are assumed equal to unity, i.e., zero dB. Similarly, the correlation coefficients between individual interferers and the desired signal are all the same and denoted by $\rho$. Fig. 1 confirms that the conditional PCI reduces with increasing normalized frequency reuse distance and increased correlation between the signals. Fig. 1 also shows that the high levels of the PCI occur with increasing standard deviation and the increasing number of interferers where the correlation between the signals reduces the PCI significantly compared to the uncorrelated case.

The conditional PCI for combined fading and shadowing, given by (11), is plotted in Fig. 2 as a function of the normalized frequency reuse distance with standard deviation, correlation coefficient, and the number of interferers as parameter. The results show that correlation between the signals helps reducing the conditional PCI. Presence of fading causes significant increase in the PCI. This effect becomes more significant at higher values of the normalized frequency reuse distance and for $\sigma = 6$ dB (see Figs. 1 and 2).

Fig. 3 shows the variation of the total PCI, due to $N = 6$ correlated interferers, versus normalized frequency reuse distance for pure shadowing with the blocking probability, the standard deviation, and the correlation coefficient as parameter. Fig. 3 depicts that the total PCI reduces with the increasing reuse distance and increasing correlation between the signals. Results also show that a higher level of co-channel interference occurs with increasing standard deviation. However, the effect of blocking probability is not very significant.

Total PCI for combined fading and shadowing is plotted in Fig. 4 as a function of the normalized frequency reuse distance with standard deviation, correlation coefficient, and blocking probability as parameters. Results for pure Rayleigh fading are

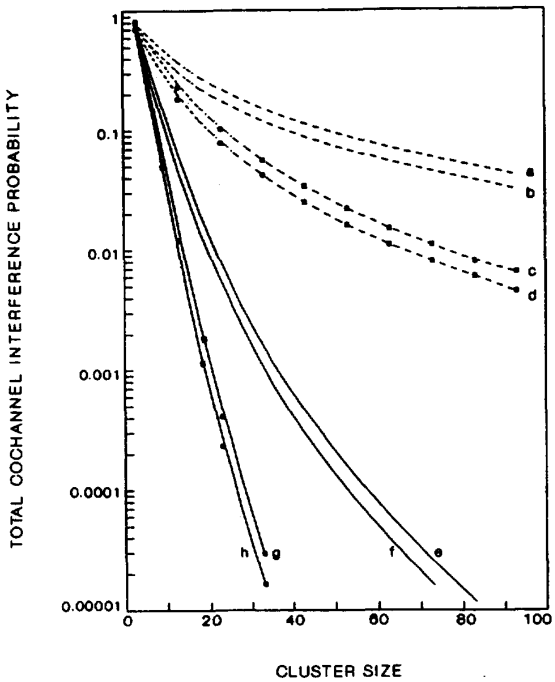



Fig. 5. Total PCI with pure shadowing for $N = 6, \alpha = 8$ dB, and $N_s = 10$ as a function of cluster size with standard deviation $\sigma$, correlation coefficient $\rho$, and the blocking probability $B$ as parameters as in Fig. 3.

also shown for the sake of comparison. Note that the total PCI is directly related to the total interference power, which is in turn proportional to the probability that a particular interfering base station is transmitting, i.e., $B^{1/N_s}$. Therefore, the ratio of total PCI's for $B = 0.01$ and 0.2, is approximately the same as the ratio of $(0.01/0.2)^{1/N_s}$. Consequently, the total PCI for $B = 0.01$ and 0.2 does not differ significantly.

It is noted from Figs. 1–4 that, for a given value of the PCI at the boundary of a cell, the correlation between the signals allows the use of smaller normalized frequency reuse distances than for the uncorrelated case. For example, for a total PCI = 0.01, $R_u$ is found as 8 and 6.24 for, respectively, $\rho = 0$ and 0.4 for pure shadowing ($\sigma = 6$ dB) and 13.5 and 11.4 for combined fading and shadowing ($\sigma = 6$ dB) (see Figs. 3 and 4). Similarly, fading and increased shadowing ($\sigma = 12$ dB) result in significant increase in the minimum required normalized frequency reuse distance, for a given value of the total PCI.

## IV. Cluster Size

The number of cells per cluster, referred to as cluster size as well, is a parameter of major interest, since in practice this number determines how many channel sets must be formed out of the total allocated spectrum. If the total number of channels available to the system is fixed, smaller cluster sizes provide more channels per cell and per cell site base station. Therefore, each cell site can carry more traffic, thereby reducing the total number of cell sites needed for a given total load.

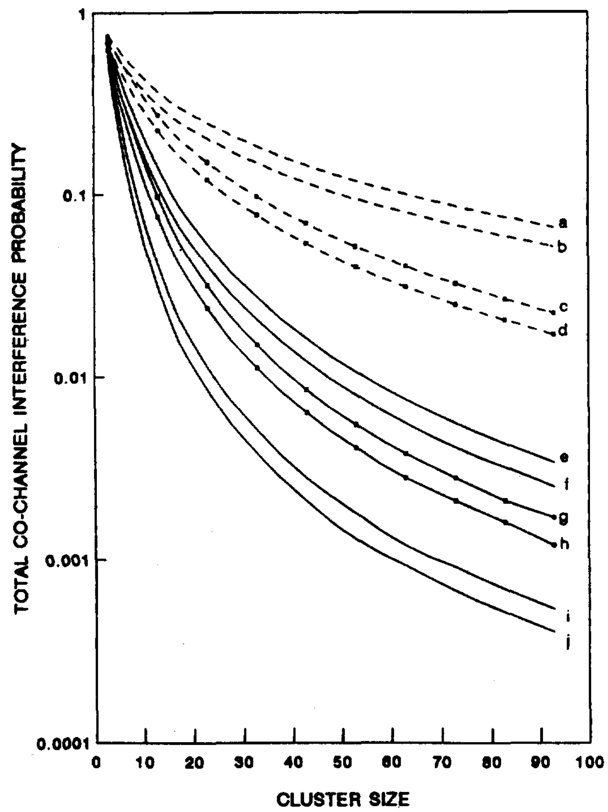



Fig. 6. Total PCI with fading and shadowing for $N = 6, \alpha = 8$ dB, and $N_s = 10$ as a function of cluster size with standard deviation $\sigma$, correlation coefficient $\rho$, and the blocking probability $B$ as parameters as in Fig. 4.

If the channels are divided into groups, then each cell provides its users with $N_s = N_T/C$ channels. $N_T$ is the total number of channels available for the cellular radio system, and the cluster size, $C$, is determined by

$$C = i^2 + ij + j^2 \qquad i, j \geq 0. \tag{15}$$

The fact that $i$ and $j$ are integers makes geometrically realizable only certain values of the number of cells per cluster, e.g., 3, 4, 7, 9, 12, 13 $\cdots$.

Service area of a high capacity cellular radio system is covered without gaps or overlaps with many clusters of cells. In hexagonal layout, each cluster is composed of several hexagonal cells. The relationship between the normalized frequency reuse distance, $R_u$, and the number of cells per cluster, $C$, required to completely cover any planar area with a fixed assignment plan is simply given by [27]

$$R_u = \sqrt{3C}. \tag{16}$$

By using (16), (13), (7), and (6), the total PCI, due to $N = 6$ correlated interferers, has been evaluated and plotted in Fig. 5 as a function of the cluster size for pure shadowing with standard deviation, the correlation coefficient, and the blocking probability as parameter. The total PCI was observed to increase significantly with an increase in shadowing from $\sigma = 6$ dB to $\sigma = 12$ dB. Here also the effect of the

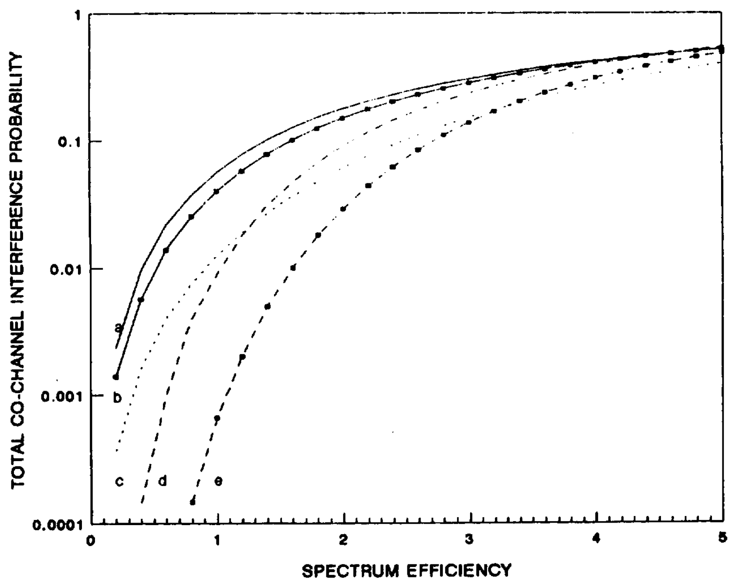



Fig. 7. Total PCI versus spectrum efficiency with $N = 6, \alpha = 8$ dB, $B = 0.02, A_c = 5$ erlang, $N_s = 10, S = 1\,\text{km}^2$, and $W = 25$ kHz for a) combined Rayleigh fading and shadowing ($\sigma = 6$ dB) $\rho = 0$; b) combined Rayleigh fading and shadowing ($\sigma = 6$ dB) $\rho = 0.4$; c) pure Rayleigh fading ($\sigma = 0$ dB) $\rho = 0$; d) pure shadowing ($\sigma = 6$ dB) $\rho = 0$; e) pure shadowing ($\sigma = 6$ dB) $\rho = 0.4$;

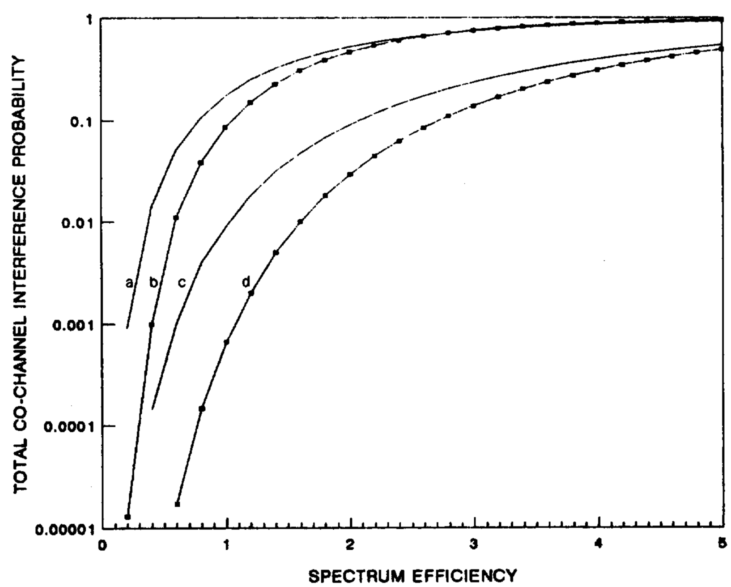



Fig. 8. Total PCI versus spectrum efficiency for pure shadowing with $\sigma = 6$ dB, $N = 6, B = 0.02, A_c = 5$ erlang, $N_s = 10, S = 1\,\text{km}^2$, and $W = 25$ kHz for $\alpha = 18$ dB: a) $\rho = 0$; b) $\rho = 0.4$ and $\alpha = 8$ dB: c) $\rho = 0$; d) $\rho = 0.4$.

blocking probability is insignificant. Similarly, total PCI for combined fading and shadowing is shown in Fig. 6 as a function of the cluster size; fading causes a sharp increase in PCI for $\sigma = 6$ dB. The correlation between the signals causes a reduction in the total PCI or, in other terms, the presence of correlation allows the use of smaller cluster sizes compared to the uncorrelated case, for a given value of the total PCI.

The cluster size and the number of base stations are limited by several factors including the economic constraints. In this context, it might be useful to quantify the effect of total PCI on the cluster size. If a maximum tolerable PCI of 0.01 is assumed at the boundary of a cell, then the cluster size should be larger than 21 and 13 for, respectively, $\rho = 0$ and 0.4 for pure shadowing with $\sigma = 6$ dB and $\alpha = 8$ dB (see Fig. 5). The corresponding cluster sizes for combined

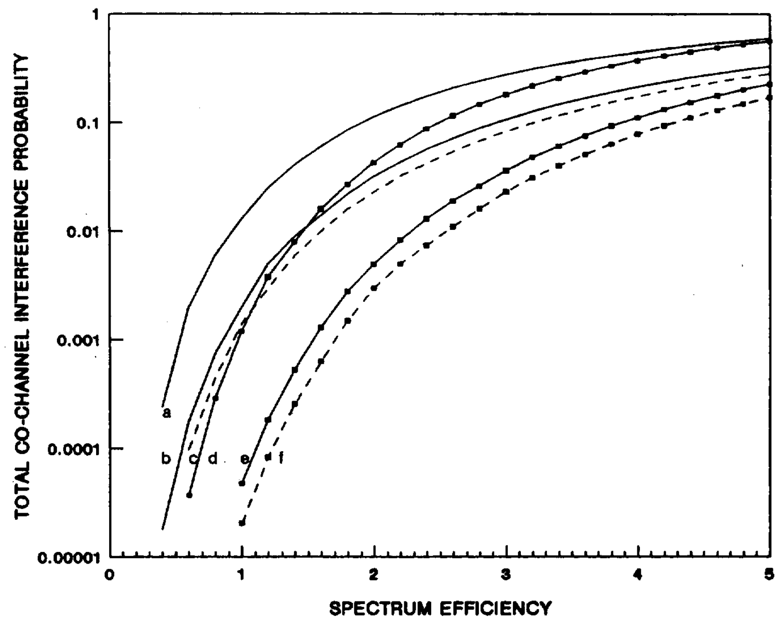



Fig. 9. Total PCI versus spectrum efficiency for pure shadowing with $\sigma = 6$ dB, $N = 6, \alpha = 8$ dB, $S = 1\,\text{km}^2$, and $W = 25$ kHz for $\rho = 0$: a) $B = 0.01, A_c = 4.42$ erlang, $N_s = 10$; b) $B = 0.2, A_c = 7.74$ erlang, $N_s = 10$; c) $B = 0.2, A_c = 22.16$ erlang, $N_s = 25$; $\rho = 0.4$: d) $B = 0.01, A_c = 4.42$ erlang, $N_s = 10$; e) $B = 0.2, A_c = 7.74$ erlang, $N_s = 10$; f) $B = 0.2, A_c = 22.16$ erlang, $N_s = 25$.

fading and shadowing are 61 and 43 (see Fig. 6). The cluster sizes for pure shadowing and combined fading and shadowing with $\sigma = 12$ dB are much larger. Similarly, a change in the blocking probability, as determined by quality considerations, from $B = 0.01$ to 0.2 leads to a slight increase in the cluster size. In view of (16), the normalized frequency reuse distances corresponding to pure shadowing with $\sigma = 6$ dB and $\alpha = 8$ dB are 8 and 6 for, respectively, $\rho = 0$ and 0.4; the corresponding values for combined fading and shadowing are found as 13.5 and 10.4.

## V. Spectrum Efficiency

Spectrum efficiency is measured by the carried traffic for the unit size of the frequency bandwidth and cell area [28], [29]. Thus, the system capacity is directly related to the spectrum efficiency. The spectrum efficiency, $E_s$, can be defined as

$$E_s = \frac{A_c}{N_s W C S} \qquad \text{erlang/MHz/km}^2. \tag{17}$$

Here, $N_s$ is the number of channels per cell, $W$ represents bandwidth per channel in MHz, $C$ the number of cells per cluster, and $S$ the area of a cell in $\text{km}^2$. The traffic carried by a cell, $A_c$, expressed in erlang/cell is given by

$$A_c = A(1 - B) \tag{18}$$

where $A$ denotes the offered traffic per cell in erlang. The blocking probability $B$ is determined using the Erlang-B formula [30]

$$B = \frac{A^{N_s}}{N_s! \sum_{n=0}^{N_s} \frac{A^n}{n!}}. \tag{19}$$

By expressing $p$ in (7) in terms of the spectrum efficiency through (17), (16), and (13), the total PCI was calculated as a function of the spectrum efficiency with pure shadowing, pure Rayleigh fading, and combined Rayleigh fading and shadowing. The results are shown in Fig. 7 for $W = 25$ kHz, $A_c = 5$ erlang, $B = 0.02, N_s = 10, S = 1\,\text{km}^2$, and $\alpha = 8$ dB. Note that, for a given value of the PCI, combined Rayleigh fading and shadowing reduces spectrum efficiency below that for pure shadowing and pure fading. At high values of the total PCI, pure fading can result in higher spectrum efficiency values compared to pure shadowing with $\sigma = 6$ dB.

Spectrum efficiency can be increased by accepting an increase in the total PCI (corresponding to lower service quality) and/or by tolerating a lower value of the protection ratio (corresponding to more robust receiver design). Correlation between the signals improves the spectrum efficiency. The effect of protection ratio on spectrum efficiency is shown in Fig. 8 where, for a given value of the total PCI, higher values of the protection ratio decrease the spectrum efficiency.

The effect of the carried traffic on spectrum efficiency is shown in Fig. 9. For a given value of the PCI, the spectrum efficiency can be improved by increasing the carried traffic, and the number of channels per cell. Again, increased correlation between the signals reduces the PCI for a given value of

TABLE I
EFFECT OF THE TOTAL PCI ON THE CLUSTER SIZE, THE NORMALIZED FREQUENCY REUSE DISTANCE, AND THE SPECTRUM EFFICIENCY.
($\sigma = 6$ dB, $\alpha = 8$ dB, $N_s = 10$, and $B = 0.2$)

| | | PCI = 0.01 | | PCI = 0.1 | |
|---|---|---|---|---|---|
| | | $\rho = 0$ | $\rho = 0.4$ | $\rho = 0$ | $\rho = 0.4$ |
| Cluster | pure shadowing | 21 | 13 | 12 | 9 |
| Size | fading & shadowing | 61 | 43 | 16 | 13 |
| Reuse | pure shadowing | 8 | 6.24 | 6 | 5.2 |
| Distance | fading & shadowing | 13.5 | 11.4 | 6.9 | 6.2 |
| Spectrum | pure shadowing | 1.42 | 2.3 | 3 | 3.95 |
| Efficiency | fading & shadowing | 0.6 | 0.75 | 1.9 | 2.2 |

the spectrum efficiency, or increases the spectrum efficiency for a given value of the PCI.

The constraints discussed in Section IV and the limitations on the available frequency spectrum by the regulations limit the spectrum efficiency that can be expected from a cellular radio system. In view of (17), the values of the spectrum efficiency corresponding to cluster sizes calculated in Section IV for a total PCI = 0.01 at the cell boundary are equal to 1.42 and 2.3, respectively, for $\rho = 0$ and 0.4 for pure shadowing with $\sigma = 6$ dB and $\alpha = 8$ dB. Corresponding values of the spectrum efficiency for combined fading and shadowing are 0.6 and 0.75.

Table I provides a summary of the effects of the total PCI on the cluster size, the normalized frequency reuse distance, and the spectrum efficiency with shadowing and combined fading and shadowing for the uncorrelated case and for a correlation coefficient of $\rho = 0.4$ between the signals. The presented values are valid for $\sigma = 6$ dB, $\alpha = 8$ dB, $N_s = 10$ and $B = 0.2$. The increased correlation between the signals yields smaller cluster sizes, shorter normalized frequency reuse distances, and improved spectrum efficiency, compared to the uncorrelated case. Combined fading and shadowing leads to larger cluster sizes, increased normalized frequency reuse distances, and lower spectrum efficiencies. Finally, an increase in the maximum tolerable total PCI at the cell boundary from 0.01 to 0.1 results in higher spectrum efficiency values and a reduction in $R_u$ and $C$, as expected.

## VI. CONCLUSIONS

The PCI due to multiple correlated log-normal interferers was calculated for cellular radio systems with Rayleigh fading and shadowing. The dependence of the total PCI on the correlation coefficients between the signals, the standard deviation due to shadowing, the number of interferers, the co-channel protection ratio, and the traffic load was investigated. The results were used for analyzing the optimal channel reuse, the cluster size, and the spectrum efficiency in terms of these parameters.

The presence of fading causes a significant increase in the conditional PCI compared to pure shadowing. This effect is more pronounced at higher values of the normalized frequency reuse distance and for $\sigma = 6$ dB (see Figs. 3 and 4). For a given value of the PCI at the cell boundary, the correlation between the signals permits the use of shorter normalized frequency reuse distances. Results also show that a higher level of co-channel interference occurs with an increase in the standard deviation, the blocking probability, and the number of channels per cell.

It is noted that high values of the standard deviation, the protection ratio, the number of channels per cell, and the blocking probability require larger cluster sizes in order to maintain the same level of PCI. Increased correlation between the signals was observed to result in a significant decrease in the cluster size for a given value of the PCI. The blocking probability was observed to be less effective while fading causes a sharp increase in the PCI compared to that for pure shadowing, especially for $\sigma = 6$ dB.

For a given value of the PCI, combined Rayleigh fading and shadowing yields lower spectrum efficiency compared to pure shadowing. Lower values of the protection ratio were observed to yield higher spectrum efficiencies for a given value of the PCI. Spectrum efficiency increases with the carried traffic, the number of channels per cell, and the blocking probability. It was also noted that the increased correlation between the signals improves the spectrum efficiency.

It can thus be concluded that the cellular radio systems perform better in a correlated shadowed environment than in an uncorrelated shadowed environment. The presented results can be used for designing macro- and micro-cellular systems in a correlated shadowed propagation environment.


## ACKNOWLEDGMENT

The author is grateful to Prof. Dr. J. C. Arnbak for his constant encouragement and careful review of the manuscript. She would also like to thank anonymous reviewers for their helpful suggestions.


## REFERENCES


[1] V. H. MacDonald, "The cellular concept," *Bell Syst. Technical J.*, vol. 58, pp. 15–41, Jan. 1979.
[2] W. C. Y. Lee, "Elements of cellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. VT-35, pp. 48–56, May 1986.
[3] V. Graziano, "Propagation correlations at 900 MHz," *IEEE Trans. Veh. Technol.*, vol. VT-27, pp. 182–189, Nov. 1978.
[4] L. F. Fenton, "The sum of log-normal probability distributions in scatter transmission systems," *IRE Trans.*, vol. CS-8, pp. 57–67, Mar. 1960.
[5] M. A. Hamdan, "The logarithm of the sum of two correlated log-normal variates, *J. Amer. Statist. Assoc.*, vol. 66, pp. 105–106, Mar. 1971.
[6] S. C. Schwartz and Y. S. Yeh, "On the distribution function and moments of power sums with log-normal components," *Bell Syst. Tech. J.*, vol. 61, pp. 1441–1462, Sept. 1982.
[7] K. Daikoku and H. Ohdate, "Optimal channel reuse in cellular land mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. VT-32, pp. 217–224, Aug. 1983.
[8] R. C. French, "The effect of fading and shadowing on co-channel reuse in mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-28, pp. 171–181, Aug. 1979.
[9] R. Muammar and S. C. Gupta, "Co-channel interference in high capacity mobile radio systems," *IEEE Trans. Commun.*, vol. COM-30, pp. 1973–1978, Aug. 1982.
[10] R. Prasad and A. Kegel, "Improved assessment of interference limits in cellular radio performance," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 412–419, May 1991.

[11] D. C. Cox, "Co-channel interference considerations in frequency reuse small-coverage-area radio systems," *IEEE Trans. Commun.*, vol. COM-30, pp. 135–142, Jan. 1982.

[12] M. Hata, K. Kinoshita, and K. Hirade, "Radio link design of cellular land mobile communication systems," *IEEE Trans. Veh. Technol.*, vol. VT-31, pp. 25–31, Jan. 1982.

[13] Y. Nagata and Y. Akaiwa, "Analysis for spectrum efficiency in single cell trunked and cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-35, pp. 100–113, Aug. 1987.

[14] K. W. Sowerby and A. G. Williamson, "Outage probability calculations for mobile radio systems with multiple interferers," *Electron. Lett.*, vol. 24, pp. 1073–1075, Aug. 1988.

[15] J.-P. M. G. Linnartz, "Exact analysis of the outage probability in multiple-user mobile radio," *IEEE Trans. Commun.*, vol. 40, pp. 20–23, Jan. 1992.

[16] H. M. Sachs, "A realistic approach to defining the probability of meeting acceptable receiver performance criteria," *IEEE Trans. Electromagnet. Compat.*, vol. EMC-13, pp. 3–6, Nov. 1971. (see also A. S. Thompson, "Comments on a realistic approach to defining the probability of meeting acceptable receiver performance criteria," *IEEE Trans. Electromagnet. Compat.*, vol. EMC-14, pp. 74–78, May 1972.)

[17] J.-E. Stjernvall, "Calculation of capacity and cochannel interference in a cellular system," *Nordic Seminar Digital Land Mobile Radiocommun.*, 1985, pp. 209–217.

[18] A. Şafak, R. Prasad, and J. P. M. G. Linnartz, "Outage probability analysis for mobile radio systems due to correlated log-normal interferers," in *Proc. 1990 Bilkent Int. Conf. New Trends in Commun., Contr., Signal Processing,* Ankara, Turkey, July 1990, pp. 518–524.

[19] A. Şafak and R. Prasad, "Effects of correlated shadowing signals on channel reuse in mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 708–713, Nov. 1991.

[20] ——, "Multiple correlated log-normal interferers in mobile cellular radio systems," *Electron. Lett.*, vol. 28, pp. 1319–1320, 2nd July 1992.

[21] A. Şafak, "Statistical analysis of the power sum of multiple correlated log-normal components," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 58–61, Feb. 1993.

[22] A. Papoulis, *Probability, Random Variables and Stochastic Processes.* New York: McGraw Hill, 1965.

[23] P. Beckmann, *Probability in Communication Engineering.* New York: Harcourt, Brace & World, 1967.

[24] J. C. Arnbak and W. van Blitterswijk, "Capacity of slotted ALOHA in Rayleigh-fading channels," *IEEE J. Select. Areas Commun.*, vol. SAC-5, pp. 261–269, Feb. 1987.

[25] W. C. Y. Lee, *Mobile Cellular Telecommunications Systems.* New York: McGraw-Hill, 1989.

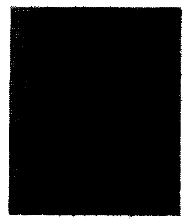
[26] J. Walker, *Mobile Information Systems.* New York: McGraw-Hill, 1989.

[27] W. R. Young, "Advanced mobile phone service; Introduction, background, and objectives," *Bell Syst. Tech. J.*, vol. 58, pp. 1–14, Jan. 1979.

[28] D. N. Hatfield, "Measures of spectral efficiency in land mobile radio," *IEEE Trans. Electromagnet. Compat.*, vol. EMC-19, pp. 266–268, Aug. 1977.

[29] H. Suzuki and K. Hirada, "Spectrum efficiency of M-ary PSK land mobile radio," *IEEE Trans. Commun.*, vol. COM-30, pp. 1803–1805, July 1982.

[30] W. C. Jakes, Ed., *Microwave Mobile Communications.* New York: Wiley, 1974.

**Aysel Şafak** was born in Uzunköprü, Turkey, in 1952. She received B.Sc. and M.Sc.(Eng.) degrees from Hacettepe University, Ankara, Turkey, in 1975 and 1977, respectively.

From 1977 to 1978, she worked as an assistant in Hacettepe University. Between 1979 and 1984, she was with the Satellite Communications Division of the Turkish PTT Administration in Ankara where she was involved with planning, evaluation, and procurement of analog and digital earth stations. She was with the Electrical Engineering Department of the Delft University of Technology, The Netherlands, from 1986 to 1992. Currently, she is with the Electrical and Electronics Engineering Department of the Eastern Mediterranean University, North Cyprus. Her research interests include mobile and indoor communications.