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# **Investigation Study of the Influence of Pole Numbers on Torque Density and Flux-Weakening Ability of Fractional Slot Concentrated Winding Wheel-Hub Machines**

ZICHONG ZHU<sup>101</sup>, (Student Member, IEEE), YUNKAI HUANG<sup>101</sup>, JIANNING DONG 2, (Member, IEEE), AND FEI PENG<sup>1</sup>, (Member, IEEE)

School of Electrical Engineering, Southeast University, Nanjing 210096, China

Faculty of Electrical Engineering, Mathematics, and Computer Science, Delft University of Technology, 2628CD Delft, The Netherlands

Corresponding author: Yunkai Huang (huangyk@seu.edu.cn)

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**ABSTRACT** Fractional slot concentrated winding machines (FSCWMs) with the low operation speed and large diameter usually have a large number of poles and slots; thus, numerous pole/slot combinations are feasible. The common practice to choose pole/slot combinations by multiplying the basic combinations may neglect some competitive candidates. Taking 36-slot FSCWMs as examples, this paper investigates the influence of pole numbers on torque density and flux-weakening ability, the two most vital performances of wheel-hub machine. It is shown that the machines with pole number slightly less than the slot number have the highest torque densities. Each component of synchronous inductance is separately analyzed, and its variation against pole numbers shows obvious regularity. Machines with pole numbers larger than the slot numbers have an excellent flux-weakening ability due to the large inductance and small permanent magnet flux linkage. The measurements together with the finite element analysis results confirm that the stator leakage inductance contributes the most to the superior flux-weakening ability. The identical analysis is also performed on 54-slot and 81-slot FSCWMs, with similar regularities observed.

**INDEX TERMS** Flux weakening, fractional slot concentrated winding, pole number, torque density, wheel-hub machine.

## I. INTRODUCTION

Fractional slot concentrated winding machine (FSCWM) is a good candidate for wheel-hub propulsion due to short end winding, high torque density and superior flux-weakening ability, especially when equipped with tooth coil winding [1]-[3].

For FSCWMs used in heavy vehicles with extremely low operation speed and large diameter, the number of poles and slots are usually large to increase torque output and improve heat dissipation in slot center [4]. As a result, hundreds of pole/slot combinations are feasible [5]. For examples, 36-slot FSCWM investigated in this paper has 12 viable pole number candidates, on the other hand, 56-pole FSCWM can

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match with 14 feasible slot numbers. When the slot number increases to 54 and 81, feasible pole number candidates even reach to 18 and 28 respectively. Machines with these pole/slot combinations have close fundamental frequencies, but different performance characteristics. It is difficult to determine the most suitable one simply based on the range of electrical frequency, which is a common practice when selecting pole number for integer slot distributed winding machines.

Many researches investigate the influence of pole/slot combinations on performances of FSCWM, including torque density [6], fault-tolerant ability [7], [8], cogging torque [9], rotor eddy current loss [10], unbalanced magnetic pull and torque ripple [11], [12]. These researches focus mainly on basic pole/slot combinations, e.g., 6/9 (6-pole 9-slot FSCWM with tooth coil winding, hereinafter), 8/9, 8/12, 10/9, 10/12, 14/12 and 14/15. According to the conclusions from these



comparative studies, multiplying basic pole/slot combination is a common practice when selecting pole and slot numbers for FSCWMs with low speed and/or large diameter. For instance, 40/48 in [13] and 32/36 in [4] have four 10/12 and 8/9 basic combinations (unit motor) respectively. Nevertheless, these combinations derived from multiplying basic pole/slot combinations are not necessarily the best candidates. Some combinations, such as 28/36 [14], are rarely researched yet have better torque overload ability than common 32/36, as shown in Section III.

In addition, comparative studies of these basic pole/slot combinations are usually not fair enough, because the model machines are designed with various current densities, winding parameters, magnetic saturation and stator dimensions. Therefore, it is difficult to reasonably compare the flux-weakening ability of these combinations. There are few researches involved in this aspect [15], [16], especially regarding the influence of pole numbers.

Taking 36-slot FSCWMs as examples, this paper investigates the influence of pole numbers on torque density and flux-weakening ability, the two most vital performances of wheel-hub machine. FSCWMs with double layer winding and surface-mounted permanent magnet (SPM) rotor are focused due to shorter end winding, less magneto-motive force (MMF) harmonics and more sinusoidal back electromotive force (EMF) than alternatives with single layer winding or interior permanent magnet rotor [17].

The fundamental winding factor and open-circuit air-gap flux density are highly related to pole number. Consequently, the torque output which can be well predicted by the product of these two parameters shows obvious regularity against pole number. In this paper, apart from the analysis on winding factor and air-gap flux density, decomposition and calculation of synchronous inductance with finite element analysis (FEA) are undertaken to explore the effect of pole numbers on each inductance component and flux-weakening ability. In addition, the experimental measurement of inductances are used to validate the FEA results.

The main contribution of this paper is revealing the regularities that show the effect of pole numbers on torque density and flux-weakening ability. These regularities are valid for FSCWMs with identical slot number but different pole numbers. These regularities help to select the most suitable pole/slot combination or narrow down the searching space from hundreds of candidates in the early design stage, instead of simply multiplying the basic combinations. Moreover, by decomposing the synchronous inductance into different components, it found that the synchronous inductance is dominated by the stator leakage inductance, which improves the flux-weakening ability of FSCWM with SPM rotor, instead of the harmonic leakage inductance insisted in previous literatures [18]-[20]. In addition, calculation of the self- and mutual components of the stator leakage inductance by FEA concludes that the mutual component is nonnegligible, which is usually ignored in most of analytical methods proposed by previous researches.

### **II. POLE NUMBER SELECTION**

### A. POLE NUMBER DETERMINATION

For a certain number of slots z, viable number of pole pairs p is first determined by torque production capacity, which can be measured by back EMF. Phase EMF  $E_{ph}$  and output torque T can be expressed as

$$E_{ph} = \sqrt{2}\pi f k_{wp} T_{ph} \Phi_{m1} = \sqrt{2}\pi f k_{wp} T_{ph} \frac{2}{\pi} B_{m1} \frac{2\pi r_{si}}{2p} l_{stk}, \quad (1)$$

$$T = \frac{3E_{ph} I_m}{\omega} = \frac{3I_m}{2\pi f/p} \sqrt{2}\pi f k_{wp} T_{ph} \frac{2}{\pi} B_{m1} \frac{2\pi r_{si}}{2p} l_{stk}$$

$$= 3\sqrt{2} r_{si} l_{stk} T_{ph} k_{wp} B_{m1} I_m, \quad (2)$$

where  $I_m$ , f,  $k_{wp}$ ,  $T_{ph}$ ,  $\Phi_{m1}$ ,  $B_{m1}$ ,  $r_{si}$ ,  $l_{stk}$  and  $\omega$  denote effective value of phase current, fundamental frequency, fundamental winding factor, total turns in series in each phase winding, fundamental flux per rotor pole, amplitude of fundamental open-circuit air-gap flux density, inner radius of stator core, axial length of stator lamination and rotor mechanical angular frequency respectively.

From (2), the torque constant  $k_T$  can be written as (3). When keeping  $l_{stk}$ ,  $r_{si}$  and  $T_{ph}$  constant,  $k_T$  is proportional to  $k_{wp}$  and  $B_{m1}$ . Ideally,  $B_{m1}$  depends simply on the thickness and magnetic properties of permanent magnet (PM) and the effective air gap length. Actually, it decreases continuously when increasing pole number, as shown in the next section.

$$k_T = \frac{T}{I_m} = 3\sqrt{2}r_{si}l_{stk}T_{ph}\overbrace{k_{wp}B_{m1}}^{f(p)}$$
(3)

It can be seen that a large  $k_{wp}$  is a prerequisite to acquire large torque constant. Small  $k_{wp}$  value means winding current is not effectively used, and larger current or more coil turns are needed to compensate.  $k_{wp}$  is usually split into the distribution factor  $k_{dp}$  and the pitch factor  $k_{pp}$  when rotor and stator skew are not used, as in (4).  $k_{dp}$  depends not only on pole/slot combination but coil connection pattern (coil distribution for each phase winding). Normally, more than one connection patterns exist for a specific pole/slot combination [21]. It is hard to express  $k_{dp}$  as a uniform function of p and p. For certain coil connection pattern,  $k_{dp}$  is usually calculated through the star of slots diagram [22].

$$k_{wp} = k_{dp}k_{pp} \tag{4}$$

Fortunately, pitch factor  $k_{pp}$  is readily derived and can be written as a general function of p and z, as in (7), where  $h_c$  is coil pitch and equals to 1 for FSCWMs with tooth coil winding.  $\tau$  and  $\alpha_s$  denote the electrical pole pitch angle and slot pitch angle, given by (5) and (6). Thus,  $k_{pp}$  can be used to exclude some inappropriate pole number candidates. According to (7),  $24 \le 2p \le 48$  is a prerequisite for keeping  $k_{pp}$  thus  $k_{wp}$  larger than 0.85 for 36-slot FSCWM (z = 36). Feasible pole numbers are 24, 26, 28, 30, 32, 34, 38, 40, 42, 44, 46 and 48.

$$\tau = \pi \tag{5}$$



$$\alpha_s = \frac{2p\pi}{z}$$

$$k_{pp} = \cos\left(\frac{\tau - h_c\alpha_s}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{p\pi}{z}\right) = \sin\left(\frac{p\pi}{z}\right)$$
(6)

The method shown in [21] is used to determine the coil connection pattern that has largest  $k_{wp}$  for each pole/slot combination, while other patterns that produce smaller  $k_{wp}$  are not discussed in this paper. For example, coil connection patterns for 28/36 and 44/36 are illustrated in Fig. 1. Coil connection patterns of other combinations are not presented due to the space limit.

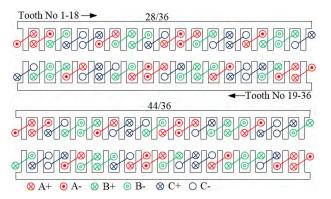


FIGURE 1. Coil connection patterns of 28/36 and 44/36.

#### **B. INVESTIGATED MACHINE**

To evaluate the influence of pole numbers on torque density and flux-weakening ability, 36-slot FSCWMs with 12 feasible pole numbers derived above are comparatively analyzed. Design specifications are listed in Table 1, and the rated torque-speed curve is shown in Fig. 2.

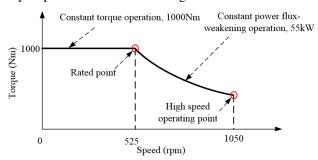


FIGURE 2. Torque-speed envelop of investigated wheel-hub machine.

 TABLE 1. Specifications of 36-slot FSCWM for wheel-hub propulsion.

Rated speed n (rpm)	525	Maximal speed (rpm)	1050
Power $P(kW)$	55	DC bus voltage $U_{dc}(V)$	600
Rated torque (Nm)	1000	Overload torque (Nm)	2500
Rated current (A)	140	Overload current (A)	400
Rated current density (A/mm <sup>2</sup> )	9.56	Cooling mode	Liquid

For fair comparisons, these 12 candidates use same stator core and are designed with identical rotor hub outer diameter and PM thickness. Main geometry parameters are

schematically defined in Fig. 3. Pole-arc coefficient  $\alpha_p$  of all candidates is fixed at 0.8, i.e., PM usages are identical. Stator yoke and rotor hub have relatively large thickness to avoid magnetic saturation. A 2mm thick nonmagnetic retaining sleeve is used to fix PMs onto the rotor hub and protect them from being destroyed by centrifugal force. The physical air gap  $\delta_p$  is 1mm, selected according to machining and assembly requirements.

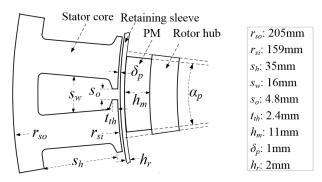


FIGURE 3. Definition of each dimension and its value.

Winding parameters are listed in Table 2. The number of turns per coil  $T_s$  is also identical for all candidates. To keep serial turns in each phase winding  $T_{ph}$  unchanged when altering pole number, parallel paths in each phase winding a is fixed at 1. Therefore, winding of different candidates can be obtained simply by reconnecting tooth coils to match various pole numbers.

TABLE 2. Design parameters of 36-slot FSCWMs with different poles.

2 <i>p</i>	24	26	28	30	32	34	38	40	42	44	46	48
Z	36											
$T_s$	8											
a		1										
$T_{ph}$	96 (12 coils in series)											
$\overline{q}$	1/2	6/13	3/7	2/5	3/8	6/17	6/19	3/10	2/7	3/11	6/23	1/4

It can be seen from Table 2 that feasible pole numbers distribute symmetrically on both sides of the slot number. The two candidates with pole numbers distributing symmetrically in the table have exactly identical winding connection pattern, as shown in Fig. 1, while only phase sequence is changed. Fig. 4 gives the spectra of 3-phase MMF of 36-slot FSCWMs with different poles. It can be observed that each pole/slot combination has a counterpart that shares the same MMF spectrum. For example, 28/36 and 44/36 have identical coil connection pattern and MMF waveform, thus MMF spectrum, but they use the 14th and 22nd space harmonics (in mechanical reference) as the fundamental harmonic respectively. Since the 14th and 22nd harmonics have different rotation directions, phase sequences of these two candidates are opposite. This is also the case for other machines. Machines that use higher order harmonics (in the dotted ellipse) have different torque density and flux-weakening



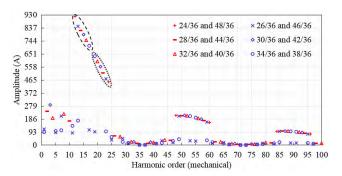


FIGURE 4. Spectra of the MMF of 36-slot FSCWMs when excited by the rated current (up to the 100th space harmonic).

ability compared with that using lower order harmonics (in the dashed ellipse), as discussed in the following sections.

### **III. INFLUENCE ON TORQUE DENSITY**

Since the number of slots per pole per phase q is small in FSCWMs, as in Table 2, distribution factor  $k_{dp}$  is large for all combinations ( $k_{dp} > 0.95$ ). Meanwhile, the difference of  $k_{dp}$  between various combinations is not as obvious as that of pitch factor  $k_{pp}$ . Therefore, fundamental winding factor  $k_{wp}$ shows nearly consistent variation trend with  $k_{pp}$ . Calculated  $k_{wp}$  is shown in Fig. 5 (a). It can be seen that combinations with 2p close to z have larger  $k_{wp}$  that may have larger torque constant, according to (3).

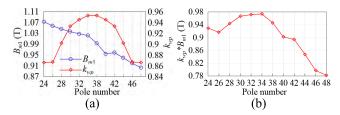


FIGURE 5. Variations of  $B_{m1}$  and  $k_{wp}$  when altering pole number. (a)  $B_{m1}$ and  $k_{wp}$ . (b)  $B_{m1} * k_{wp}$ .

Another factor that affects torque constant is the fundamental open-circuit air-gap flux density  $B_{m1}$ . As mentioned above, ideally,  $B_{m1}$  is independent of pole number. Nevertheless, in fact, it is significantly influenced by pole number due to inter-pole flux leakage. As shown in Fig. 6, more PM flux leakage is produced in 48/36 than 24/36, which inevitably results in a smaller  $B_{m1}$ . In order to consider the effect of flux leakage and local magnetic saturation, FEA is used to calculate  $B_{m1}$ , as shown in Fig. 5 (a). Clearly,  $B_{m1}$  tends to decrease when using more poles.

The product of  $k_{wp}$  and  $B_{m1}$  is shown in Fig. 5 (b), where  $B_{m1}$  is calculated by 2-D FEA. Since  $B_{m1}$  decreases nearly monotonously and  $k_{wp}$  has maximal values in the middle of pole array, the highest torque production does not appear in candidates with 2p closest to z, i.e., 34/36 and 38/36, but in candidates whose pole numbers are slightly less than those, e.g., 30/36, 32/36 and 34/36.

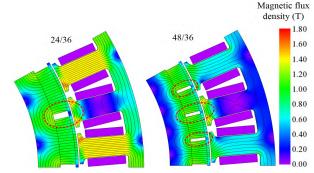


FIGURE 6. Inter-pole flux leakage at open circuit.

Fig. 7 shows semi-analytically and numerically calculated torques for all machines. Semi-analytical results are calculated using (2), where  $B_{m1}$  is obtained by 2-D FEA. Convex curves illustrate that under both rated and overload operating points, maximal torques appear in machines that have 2p slightly less than z. An exception is 26/36, its  $B_{m1}$  decreases significantly compared with 24/36, while  $k_{wp}$  does not have obvious increment. As a result, torque output of 26/36 is lower than that of 24/36. In addition, machines with 2p > zshow inferior torque production capability considering large inter-pole flux leakage.

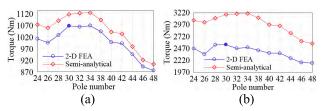


FIGURE 7. Torque output of 36-slot FSCWMs with different poles. (a) Rated point-140A. (b) Overload condition-400A.

It is noteworthy that the torque variation profile calculated from FEA shows excellent agreement with semi-analytical result at the rated point, as shown in Fig. 7 (a). This means (2) is effective to compare torque output of FSCWMs with different poles. However, under the overload condition, it is not as accurate as that at the rated point. As in Fig. 7 (b), the maximal torques appear in candidates with fewer poles, such as 28/36 and 30/36, which means that a certain degree of pole number reduction benefits overload ability.

## IV. SYNCHRONOUS INDUCTANCE CALCULATION

Accurate inductance calculation is essential for fluxweakening ability analysis, the synchronous inductance  $L_s$ determines characteristic current Ich and flux-weakening ratio  $\zeta$ , which are usually used to evaluate the flux-weakening ability, as in (8) and (9).

$$I_{ch} = \frac{\psi_m}{L_s}$$

$$\zeta = \frac{L_s i_d}{\psi_m}$$
(8)

$$\zeta = \frac{L_s i_d}{\psi_m} \tag{9}$$

where  $\psi_m$  is the PM flux linkage of phase winding.



Although the total inductance  $L_s$  can be calculated accurately by FEA, analytical calculation and decomposition of it are necessary to explore the effect of pole numbers and determine the contribution of each component to flux-weakening ability. This section calculates each component of  $L_s$  analytically, and FEA is used to validate these results. Larger errors are detected in these analytical methods that are widely used in literatures, which are not reported previously. The next section gives measured inductances to validate the calculated results.

Corresponding to each armature flux flow path,  $L_s$  is separated into several components as

$$L_s = L_g + L_\sigma = \overbrace{L_m + L_\delta}^{L_g} + \overbrace{L_{ti} + L_{sl} + L_{ew}}^{L_\sigma}, \quad (10)$$

where  $L_g$ ,  $L_\sigma$ ,  $L_m$ ,  $L_\delta$ ,  $L_{ti}$ ,  $L_{sl}$  and  $L_{ew}$  are air gap inductance, stator leakage inductance, magnetizing inductance, harmonic leakage inductance, tooth tip leakage inductance, slot leakage inductance and end winding inductance respectively.

## A. STATOR LEAKAGE INDUCTANCE

This part focuses on  $L_{ti}$ ,  $L_{sl}$  and  $L_{ew}$ . The sum of them, the stator leakage inductance  $L_{\sigma}$ , corresponds to the flux that traverses various parts of the stator core except the air gap.

Each component of  $L_{\sigma}$  is analytically calculated for FSCWM and integer slot distributed winding machine that have same pole number and fundamental flux linkage in [23], which assumes that  $L_{\sigma}$  depends only on the number of turns per coil  $T_s$  and slot dimensions, yet is independent of coil connection pattern. Other researches, e.g., [20] and [24], also consider  $L_{\sigma}$  related merely to  $T_s$  and stator geometry parameters, such as tooth tip height  $t_{th}$ , slot opening  $s_o$ , slot width  $s_w$ , slot depth  $s_h$ , coil span  $t_w$  and mean length of coil end  $l_{ew}$ , as shown in Fig. 3. Analytical formulas of  $L_{ti}$ ,  $L_{sl}$ and  $L_{ew}$  in these literatures base on corresponding permeance functions  $\lambda_{ti}$ ,  $\lambda_{sl}$  and  $\lambda_{ew}$  that are functions of above geometry parameters. Generally, they are written as (11), these permeance functions are derived from ideal slot shape and show poor accuracy in real machines, so they are not listed in detail here.

$$L_{\sigma} = L_{ti} + L_{sl} + L_{ew} = 12\mu_{0}T_{s}^{2} (\lambda_{ti} + \lambda_{sl} + \lambda_{ew})$$

$$= 12\mu_{0}T_{s}^{2} \begin{pmatrix} \lambda_{ti}(l_{stk}, s_{o}, t_{th}) \\ +\lambda_{sl}(l_{stk}, s_{h}, s_{w}, s_{o}, t_{th}) \\ +\lambda_{ew}(l_{ew}, t_{w}) \end{pmatrix}$$
(11)

In addition, (11) takes only self-inductance into account. Mutual coupling between each coil is assumed to be negligible in many literatures [20], [23]. Permeance function models that accounting for mutual coupling are rare in existing literatures [16]. In fact, the mutual component of  $L_{\sigma}$  is significant and should not be ignored. In addition, both self-inductance  $L_{\sigma s}$  and mutual inductance  $L_{\sigma m}$  have tight relationship with coil connection pattern, i.e., pole number.

To clearly present the relationships,  $L_{\sigma}$  is calculated by imposing symmetry boundary conditions on stator inner surface in 3-D FEA models [25], [26]. These conditions prohibit

armature flux from entering the air gap and constrain it flowing parallel to these faces, as in Fig. 8. Fig. 9 gives calculated flux linkages when feeding only sinusoidal A-phase current of 140A (results of 38/36, 40/36, 42/36, 44/36, 46/36 and 48/36 are not plotted because the flux linkage of them are identical to their counterparts that share same coil connection pattern).

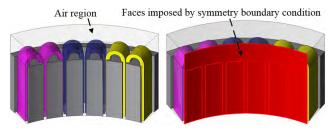


FIGURE 8. 3-D FEA partial model used to calculate stator leakage inductance (30/36).

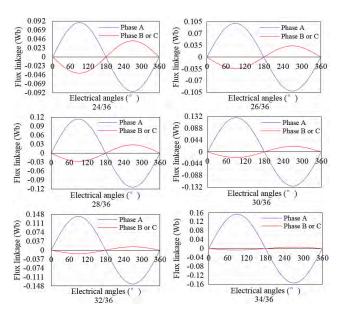


FIGURE 9. Flux linkages in each phase winding when feeding 140A current into phase A.

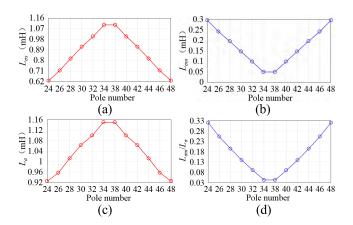
It can be seen that the mutual flux linkages are non-negligible and the ratio of B-phase and A-phase flux linkages changes dramatically in different machines. Fig. 10 (a) and (b) show resultant  $L_{\sigma s}$  and  $L_{\sigma m}$  calculated from flux linkages using (12) and (13), where  $\psi_a$ ,  $\psi_b$  and  $i_a$  denote A-phase flux linkage, B-phase flux linkage and A-phase current respectively. Fig. 10 (c) shows  $L_{\sigma}$ . Fig. 10 (d) gives the ratio of  $L_{\sigma m}$  and  $L_{\sigma}$ .

$$L_{\sigma s} = \psi_a / i_a \tag{12}$$

$$L_{\sigma m} = \psi_b / i_a \tag{13}$$

It can be seen from Fig. 10 (a) that  $L_{\sigma s}$  increases if more poles are used when 2p < z. Machines with 2p closest to z have the maximum  $L_{\sigma s}$  of 1.15mH, i.e., 34/36 and 38/36. However,  $L_{\sigma m}$  shows a totally different trend, as shown





**FIGURE 10.** Variations of  $L_{\sigma S}$ ,  $L_{\sigma m}$  and  $L_{\sigma}$  against pole numbers. (a)  $L_{\sigma S}$ . (b)  $L_{\sigma m}$ . (c)  $L_{\sigma}$ . (d)  $L_{\sigma m}/L_{\sigma}$ .

in Fig. 10 (b). It has minimum value 0.048mH in these two machines. This can be observed intuitively from mutual flux linkages in Fig. 9, where  $\psi_b$  decreases continuously when 2p increases from 24 to 34. In 24/36,  $\psi_b$  reaches half of  $\psi_a$ , but only 4.37% of  $\psi_a$  in 34/36. The ratio of  $L_{\sigma m}$  and  $L_{\sigma}$  decreases from 0.33 to 0.042 when 2p increases from 24 to 34. That is, machines with 2p close to z have larger  $L_{\sigma s}$  and smaller  $L_{\sigma m}$ . These features are attractive when considering fault-tolerant ability.

 $L_{\sigma}$  is dominated by  $L_{\sigma s}$  and has maximum value in machines with 2p closest to z, despite where  $L_{\sigma m}$  is minimum. More importantly,  $L_{\sigma s}$ ,  $L_{\sigma m}$  and  $L_{\sigma}$  are related to coil connection pattern. Analytical calculation using (11) that ignores these relations shows poor accuracy and is invalid for real machines.

#### B. AIR-GAP INDUCTANCE

Armature flux that traverses air gap is represented by  $L_g$ . It is usually split into two parts: magnetizing inductance  $L_m$  and harmonic leakage inductance  $L_{\delta}$ , as (14).

$$L_{\varrho} = L_m + L_{\delta} \tag{14}$$

Since the number of slots per pole per phase q is small in FSCWMs, MMFs produced by tooth-coil winding are far from sinusoidal and contain rich harmonics. The first term of  $L_g$  corresponds to the fundamental of armature field that interacts with rotor field to produce a constant torque.  $L_\delta$  corresponds to all harmonic components, and a large  $L_\delta$  may signify rotor eddy current loss due to these harmonics.

 $L_m$  and  $L_\delta$  are usually calculated using winding functions [27], [28], which intrinsically describe how winding coils are arranged [29]. Mutual inductance  $L_{ab}$  between phase A and B can be expressed as

$$L_{ab} = \frac{\mu_0 r_{si} l_{stk}}{\delta} \int_0^{2\pi} N_a(\theta) N_b(\theta) d\theta, \qquad (15)$$

where  $N_a(\theta)$  and  $N_b(\theta)$  are winding functions of phase A and B,  $\theta$  is the angular measure along the air gap (in mechanical reference),  $\delta$  is the length of effective air gap (the sum

of physical air gap, the thickness of retaining sleeve, and the thickness of PM).

Self-inductance L can be obtained using (15) as well if replace  $N_b(\theta)$  with  $N_a(\theta)$ . Winding functions are usually decomposed to Fourier series for harmonic analysis and written as

$$N_{a}(\theta) = \frac{2T_{ph}k_{w1}}{\pi}\cos(\theta + \varphi_{1}) + \frac{2T_{ph}k_{w2}}{2\pi}\cos(2\theta + \varphi_{2}) + \frac{2T_{ph}k_{w3}}{3\pi}\cos(3\theta + \varphi_{3}) + \dots$$

$$= \frac{2T_{ph}}{\pi}\sum_{\nu=1,2,3}^{\infty} \frac{k_{w\nu}}{\nu}\cos(\nu\theta + \varphi_{\nu}), \qquad (16)$$

where  $k_{wv}$  denotes the winding factor of the  $v^{\text{th}}$  harmonic.  $\varphi_v$  is the phase of the  $v^{\text{th}}$  harmonic, which can only be 0 or  $\pi$ , because  $N_a(\theta)$  is even symmetrical. Upon integrating, self-inductance L is solved as (17).

$$L = \frac{4\mu_0 r_{si} l_{stk} T_{ph}^2}{\pi \delta} \sum_{\nu=1,2,3,\dots}^{\infty} \frac{k_{w\nu}^2}{\nu^2}$$
 (17)

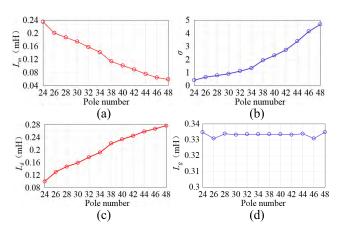
Self- and mutual components of  $L_g$  can be calculated together by replacing  $N_b(\theta)$  in (15) with synthesized 3-phase winding function  $N_{abc}(\theta)$ , as given in (18).

$$L_g = \frac{\mu_0 r_{si} l_{stk}}{\delta} \int_0^{2\pi} N_a(\theta) N_{abc}(\theta) d\theta$$
 (18)

In a three phase machine of basic pole/slot combination, windings are symmetrically arranged by  $2\pi/3$  radians. Therefore, triple harmonics are cancelled out from synthesized winding function  $N_{abc}(\theta)$ . Accordingly, for the machine whose pole/slot combination derived from multiplying the basic combination, the harmonics corresponding to these triple harmonics are eliminated. For example, the  $3^{rd}$  and  $6^{th}$  harmonics of  $N_a(\theta)$  of 8/9 are eliminated in  $N_{abc}(\theta)$ . Correspondingly, the  $12^{th}$  and  $24^{th}$  harmonics are excluded from  $N_{abc}(\theta)$  of 32/36 that contains four basic 8/9 combinations. These harmonics correspond to zero-sequence inductance, which has no effect on flux-weakening analysis, yet should be taken into account in fault analysis. Thus,  $L_g$  is given as (19), where t is the number of basic combinations contained in certain pole/slot combination.

$$L_{g} = \frac{3}{2} * \frac{4\mu_{0}r_{si}l_{stk}T_{ph}^{2}}{\pi \delta} \sum_{\substack{\nu = 1, 2, 3, \dots \\ \nu \neq t * 3k, k = 1, 2, 3, \dots}}^{\infty} \frac{k_{w\nu}^{2}}{\nu^{2}}$$
(19)

The multiplier 3/2 in (19) represents the contribution of mutual inductance, as the amplitude of the remained harmonics ( $\nu \neq t*3k$ ) are multiplied by 3/2 due to phase coupling. Therefore,  $L_m$ ,  $L_\delta$  and the ratio of them, referenced as the harmonic leakage factor  $\sigma$ , are given as (20)-(22). Since the term in the bracket is identical for all 12 candidates,  $L_m$  and  $L_\delta$ 



**FIGURE 11.** Variations of  $L_m$ ,  $\sigma$ ,  $L_\delta$  and  $L_g$  against pole numbers. (a)  $L_m$ . (b)  $\sigma$ . (c)  $L_\delta$ . (d)  $L_g$ .

vary with winding factors and harmonic spectra, which are tightly related to pole numbers.

$$L_{m} = \left(\frac{6\mu_{0}r_{si}l_{stk}T_{ph}^{2}}{\pi\delta}\right)\frac{k_{wp}^{2}}{p^{2}}$$

$$L_{\delta} = \left(\frac{6\mu_{0}r_{si}l_{stk}T_{ph}^{2}}{\pi\delta}\right)\sum_{\substack{\nu = 1, 2, 3, \dots, \nu \neq p, \\ \nu \neq t * 3k, k = 1, 2, 3, \dots}}^{\infty} \frac{k_{w\nu}^{2}}{\nu^{2}}$$
(20)

$$\sigma = \frac{L_{\delta}}{L_{m}} = \sum_{\substack{\nu = 1, 2, 3, \dots, \nu \neq p, \\ \nu \neq t * 3k, k = 1, 2, 3, \dots}}^{\infty} \frac{p^{2}k_{w\nu}^{2}}{k_{wp}^{2}\nu^{2}}$$
(22)

Fig. 11 shows analytically calculated  $L_m$ ,  $\sigma$ ,  $L_\delta$  and  $L_g$  using (20)-(22), where  $L_\delta$  and  $\sigma$  are counted up to the 200<sup>th</sup> space harmonic. From Fig. 11 (a),  $L_m$  decreases when more poles are used, with a drop from 0.235mH to 0.059mH when 2p doubles from 24 to 48.  $L_m$  of 48/36 is only a quarter of that of 24/36 since these two machines have identical MMF spectrum and harmonic winding factor, but different fundamental harmonic orders.

Opposite trends are observed in the variation of  $\sigma$  and  $L_{\delta}$ , shown in Fig. 11 (b) and (c). They both increase significantly when using more poles.  $\sigma$  increases from 0.42 to 4.69 while  $L_{\delta}$  increases from 0.099mH to 0.275mH. Starting from 32/36,  $L_{\delta}$  becomes larger than  $L_m$ . Approximately,  $L_g$  is dominated by  $L_m$  in machines with 2p < z. But when 2p > z,  $L_{\delta}$  takes a larger proportion. In addition, it is noteworthy that the sum of  $L_m$  and  $L_{\delta}$ , i.e.,  $L_g$ , is nearly constant regardless of pole numbers, as in Fig. 11 (d).

Analytical calculation using winding functions bases on some assumptions and hypotheses:

- 1. ignoring the slotting effect,
- 2. assuming stator core and rotor hub infinitely permeable, MMFs drop only on the effective air gap  $\delta$ ,

3.  $\delta$  is considered to be small and armature flux traverses it radially, then enters the rotor hub.

The first two assumptions may cause minor errors and can be modified by using more complicated models or correction factors [27]. Nevertheless, this is not the case for the third, particularly in machines with SPM rotor. Actually, not all armature flux enters the rotor hub. Part flux bypasses it and returns to the stator core directly, as in Fig. 12. As a result, magnetic circuit permeance and  $L_g$  are inevitably underestimated. For instance, armature field distribution on the stator inner surface of 28/36 and its spectrum are shown in Fig. 13. It can be seen that the amplitude of nearly all harmonics are increased due to a decreased effective air gap length, which means the armature reaction is enhanced and a larger  $L_g$  is produced, compared with winding function analysis.

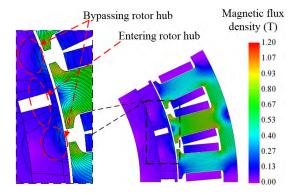


FIGURE 12. Armature field distribution of 24/36 with all three phases excited.

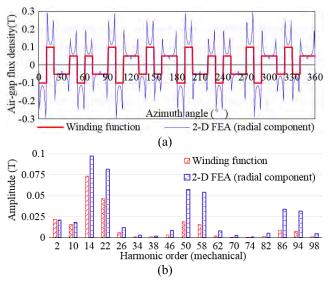


FIGURE 13. Armature field distribution (PMs are disabled) and its spectrum of 28/36. (a) Armature flux density in the air gap  $B_{ar}$ . (b) Spectrum of  $B_{ar}$ .

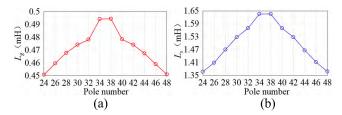
To improve accuracy, 3-D FEA is used to calculate  $L_g$ . Unlike what has been done for stator leakage inductance  $L_\sigma$  in Fig. 8, direct calculation of  $L_g$  by imposing symmetry

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(21)



boundary conditions to hinder stator leakage flux causes large error. In this paper,  $L_g$  is obtained by subtracting  $L_\sigma$  from  $L_s$ , according to (10). Fig. 14 shows  $L_s$  from 3-D FEA and resultant  $L_g$ . Comparing Fig. 14 (a) with Fig. 11 (d),  $L_g$  increases about 50% when considering true flow paths of armature flux with FEA. It can be seen from Fig. 10 (c) and Fig. 14 (a),  $L_g$  shows the same variation trend as  $L_\sigma$ , and machines with 2p close to z also have larger  $L_g$ .



**FIGURE 14.**  $L_q$  and  $L_s$  calculated by 3-D FEA. (a)  $L_q$ . (b)  $L_s$ .

Total inductance  $L_s$  is shown in Fig. 14 (b). It is generally concluded in many papers [18]–[20] that  $L_s$  is dominated by the harmonic leakage inductance  $L_\delta$  in FSCWM. However, comparing Fig. 10 (c) with Fig. 14 (a),  $L_s$  of FSCWM with SPM rotor is actually dominated by the stator leakage inductance  $L_\sigma$  due to small coil pitch and large effective air gap, instead of  $L_\delta$ .

## V. MEASUREMENT OF INDUCTANCE

It is impossible to constrain armature flux flow paths in the experimental measurements like that in FEA. In addition, the standard method recommended by IEEE 115-2009 to measure the stator leakage inductance  $L_{\sigma}$  with rotor removed test ignores the effect of harmonic leakage inductance. This method is applicable for integer slot distributed winding machines but shows poor accuracy when extending it to FSCWMs [20]. Thus far, it is difficult to distinguish each component from measured inductance.

In a compromise, indirect validations on calculated results are performed by measuring the total inductance  $L_s$  with and without permeable rotor hub. The 36-slot stator with tooth coils is fabricated to measure the inductance of machines with different pole numbers, as in Fig. 15. Each coil has two terminals for convenience of changing the coil connection patterns for various pole/slot combinations. Thus, all measurements use the same stator core and rotor hub, which helps to eliminate inherent machining errors and magnetic property difference.

Two measurements are carried out-one is performed when removing the rotor, while another with a permeable rotor hub. The calculated (3-D FEA) and measured synchronous inductances with LCR meter are listed in Table 3. The result for machines with 2p > z are not shown since they are identical to their counterparts.

The errors between measured and calculated results are about 4%, which may result from the difference of magnetic properties of the stator core in the FEA model and prototype. From measured results, the maximum  $L_s$  also appears



FIGURE 15. Stator with 36 tooth coils and the permeable rotor hub without PMs assembled.

**TABLE 3.** Synchronous inductances for 36-slot FSCWMs.

	$L_s$ (mH),	$L_s$ (mH),	$L_s$ (mH)	$L_s$ (mH)
Machine	Calc, no rotor	Meas, no	Calc, with rotor	Meas, with
	hub	rotor hub	hub	rotor hub
24/36	1.31	1.27	1.37	1.34
26/36	1.36	1.32	1.42	1.39
28/36	1.41	1.37	1.48	1.44
30/36	1.48	1.43	1.53	1.50
32/36	1.53	1.49	1.58	1.54
34/36	1.60	1.55	1.64	1.60

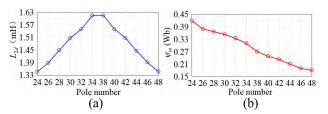


FIGURE 16. Synchronous inductance  $\mathcal{L}_{2d}$  and open-circuit phase flux linkage  $\psi_m$ .

in machines with 2p closest to z, which is consistent with the calculated result. When with the rotor hub, measured  $L_s$  increases from 1.34mH to 1.60mH as 2p increases from 24 to 34. This increment owes much to a larger stator leakage inductance  $L_\sigma$ , rather than air-gap component  $L_g$ , as presented in section IV. In addition,  $L_s$  measured with the rotor hub has little increment compared with that when rotor hub is removed. This implies that little armature flux enters the rotor hub, and inductance corresponding to this part of flux is relatively small compared with others.

## **VI. FLUX-WEAKENING ABILITY ANALYSIS**

The synchronous inductance calculated by 2-D FEA  $L_{2d}$  is shown in Fig. 16 (a). It contains all components of  $L_s$  except end winding inductance  $L_{ew}$ , as in (23). Therefore,  $L_{ew}$  can be indirectly calculated by differentiating  $L_{2d}$  and  $L_s$  from 3-D FEA, given as (24). Resultant  $L_{ew}$  is equal to 0.02mH for all 12 candidates. Clearly,  $L_{ew}$  is negligible compared with other components of  $L_s$  since FSCWMs have short end winding and non-overlapping coil arrangement. Therefore,  $L_{ew}$  is excluded when comparing flux-weakening ability of machines with different pole numbers, as in (25) and (26).

$$L_{2d} = L_m + L_\delta + L_{ti} + L_{sl} \tag{23}$$

$$L_{ew} = L_s - L_{2d} \tag{24}$$



$$I_{ch} = \frac{\psi_m}{L_{2d}}$$

$$\zeta = \frac{L_{2d}i_d}{\psi_m}$$
(25)

$$\zeta = \frac{L_{2d}i_d}{\psi_m} \tag{26}$$

Another factor that determines flux-weakening ability is PM flux linkage  $\psi_m$ , given by (27).  $\psi_m$  is inversely proportional to the number of pole pairs p when assuming open-circuit air-gap flux density  $B_{m1}$  is constant for all 12 machines. In fact,  $B_{m1}$  decreases continuously when using more poles, as shown in Fig. 5 (a). As a result,  $\psi_m$  decreases monotonously since serial turns  $T_{ph}$  is kept constant for all machines, as shown in Fig. 16 (b).

$$\psi_m = k_{wp} T_{ph} \Phi_{m1} = k_{wp} T_{ph} \frac{2}{\pi} \pi r_{si} l_{stk} \frac{B_{m1}}{p}$$
 (27)

Characteristic current  $I_{ch}$  and flux-weakening ratio  $\zeta$  calculated using (25) and (26) are shown in Fig. 17. It can be seen that machines with a larger number of poles have smaller  $I_{ch}$  due to a smaller  $\psi_m$ , although  $L_{2d}$  has maximum value in combinations with 2p close to z. Meanwhile,  $\zeta$  increases significantly when using more poles. For some machines,  $\zeta$ is even larger than 1, such as 46/36 and 48/36. This means that FSCWMs with 2p > z have excellent flux-weakening ability, while integer slot distributed winding machine with SPM rotor is usually considered inappropriate when emphasizing this performance.

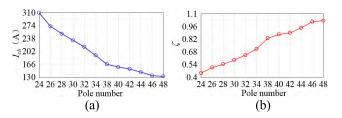
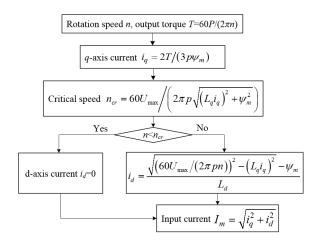


FIGURE 17. Characteristic current  $I_{ch}$  and flux-weakening ratio  $\zeta$ .

Most importantly, comparing Fig. 10 (c) with Fig. 14 (a), air-gap inductance  $L_g$  is only about half of the stator leakage inductance  $L_{\sigma}$ . In other words,  $L_{2d}$  is dominated by  $L_{\sigma}$  and the excellent flux-weakening ability owes to it, rather than the harmonic leakage inductance  $L_{\delta}$  suggested in [19], [20].

From above analysis, it seems that a machine with maximum pole number is preferred for flux-weakening operation. However, considering torque production inferiority, it may not be the best candidate. Since stator core loss is only about 1/12 of winding copper loss (e.g., these two kinds of loss are 268.3W and 3105.9W for 30/36 at rated operating point), machine that applies minimum current to reach the given torque and speed is preferred. Input current  $I_m$  is determined according to the procedures shown in Fig. 18, where  $U_{max}$ is the maximum output phase voltage of inverter with given dc bus voltage  $U_{dc}$ . With a 600V  $U_{dc}$ , normal 6-transistor inverter is able to produce 345V  $U_{max}$  with space vector pulse width modulation.

Q-axis current  $i_q$  required to obtain the rated torque during constant torque operation and corresponding critical speed



**FIGURE 18.** Procedures to calculate the minimum input current  $I_m$ .

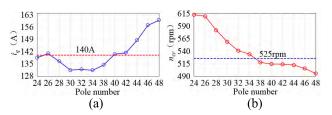


FIGURE 19. Q-axis current  $i_q$  and corresponding critical speed  $n_{cr}$ .

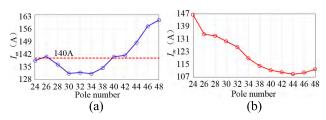


FIGURE 20. Input current  $I_m$ . (a) Rated operating point. (b) High-speed operating point.

 $n_{cr}$  are shown in Fig. 19. It can be seen that to produce rated torque, 30/36, 32/36 and 34/36 need the least currents. When 2p > z,  $I_m$  increases monotonously. 44/36, 46/36 and 48/36 need much more current than others due to relatively small  $\psi_m$ . Moreover, they have low critical speeds due to larger pole numbers, as in Fig. 19 (b). Machines with 2p > z have critical speeds lower than the rated value 525rpm, which means fluxweakening control and d-axis current  $i_d$  are necessary to meet the speed requirement. The existence of  $i_d$  further increases the amplitude of  $I_m$ , and thus produces more copper loss. Resultant  $I_m$  at rated operating point (1000Nm, 525rpm) and high-speed operating point (500Nm, 1050rpm) are shown in Fig. 20.

At the high-speed operating point, the minimum  $I_m$  appears in 44/36. For machines with fewer poles, larger  $i_d$  is necessary to weaken PM field, while for machines with more poles, larger  $i_q$  is used to produce the required torque due to smaller  $\psi_m$ . Therefore, for other operating points of fluxweakening operation, the minimum  $I_m$  always appears in



machine with  $34 \le 2p \le 44$ . Generally speaking, machines with 2p slightly larger than z are recommended when emphasizing flux-weakening ability.

It is noteworthy that the maximal torque density and optimal flux-weakening ability do not appear in the same design candidate, but in a range of  $30 \le 2p \le 44$ , the lower and upper limits correspond to the maximal torque density and the optimal flux-weakening ability respectively. To determine the combination that has the highest operation efficiency, driving cycle of the electrical vehicle and control strategies should be taken into account. This will be presented in future work.

#### **VII. CONCLUSION**

For FSCWMs with a large number of poles and slots, hundreds of pole/slot combinations are feasible. Simply multiplying the basic combinations may miss some competitive candidates. Taking 36-slot FSCWMs as examples, this paper investigates the influence of pole numbers on torque density and flux-weakening ability of wheel-hub machines.

By imposing constraints on the comparative studies, it is found that some conclusions and calculation methods in previous literatures are not valid or show poor accuracy. For example, stator leakage inductance contributes the most to the superior flux-weakening ability of FSCWM with SPM rotor, instead of previously thought harmonic leakage inductance. Experimental measurements of total inductance have confirmed this conclusion. More importantly, the influence of pole numbers show obvious regularities, which are also observed in 54-slot and 81-slot FSCWMs with various pole numbers. The analysis results obtained help to determine the preferred combination or narrow down the searching space in the early design stage.

The influence of pole numbers on other performances, such as rotor eddy current loss, iron loss, and the effect of slot numbers on performance of FSCWMs with identical pole number but different slot numbers will be presented in future work.

## **REFERENCES**

- J. Wang, X. Yuan, and K. Atallah, "Design optimization of a surfacemounted permanent-magnet motor with concentrated windings for electric vehicle applications," *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1053–1064, Mar. 2013.
- [2] G. Hong, T. Wei, and X. Ding, "Multi-objective optimal design of permanent magnet synchronous motor for high efficiency and high dynamic performance," *IEEE Access*, vol. 6, pp. 23568–23581, 2018.
- [3] A. M. El-Refaie, T. M. Jahns, P. J. McCleer, and J. W. McKeever, "Experimental verification of optimal flux weakening in surface PM Machines using concentrated windings," *IEEE Trans. Ind. Appl.*, vol. 42, no. 2, pp. 443–453, Mar. 2006.
- [4] Y. Huang, Z. Zhu, B. Guo, H. Lin, and S. Fang, "Design and thermal analysis on high torque low speed fractional-slot concentrated windings in-wheel traction motor," in *Proc. 22nd Int. Conf. Electr. Mach. (ICEM)*, 2016, pp. 1487–1492.
- [5] F. Libert and J. Soulard, "Investigation on pole-slot combinations for permanent-magnet machines with concentrated windings," in *Proc. Int. Conf. Elect. Mach. (ICEM)*, 2004, pp. 5–8.
- [6] E. Carraro, N. Bianchi, S. Zhang, and M. Koch, "Design and performance comparison of fractional slot concentrated winding spoke type synchronous motors with different slot-pole combinations," *IEEE Trans. Ind. Appl.*, vol. 54, no. 3, pp. 2276–2284, May/Jun. 2018.

- [7] H. Dogan, F. Wurtz, A. Foggia, and L. Garbuio, "Analysis of slot-pole combination of fractional-slots PMSM for embedded applications," in Proc. Int. Aegean Conf. Elect. Mach. Power Electron. Electromotion, Joint Conf., 2011, pp. 611–615.
- [8] X. Zhu and M. Cheng, "Design and analysis of 10 MW class HTS exciting double stator direct-drive wind generator with stationary seal," *IEEE Access*, vol. 7, pp. 51129–51139, 2019.
- [9] X. Ge and Z. Q. Zhu, "Sensitivity of manufacturing tolerances on cogging torque in interior permanent magnet machines with different slot/pole number combinations," *IEEE Trans. Ind. Appl.*, vol. 53, no. 4, pp. 3557–3567, Jul./Aug. 2017.
- [10] L. Wu, R. Qu, and D. Li, "Reduction of rotor eddy-current losses for surface pm machines with fractional slot concentrated windings and retaining sleeve," *IEEE Trans. Magn.*, vol. 50, no. 11, pp. 1–4, Nov. 2014.
- [11] M. Qiao, C. Jiang, Y. Zhu, and G. Li, "Research on design method and electromagnetic vibration of six-phase fractional-slot concentratedwinding PM motor suitable for ship propulsion," *IEEE Access*, vol. 4, pp. 8535–8543, 2016.
- [12] I. Petrov, P. Lindh, M. Niemelä, E. Scherman, and J. Pyrhönen, "High-torque-density IPMSM rotor pole geometry adjustment for smooth torque," *IEEE Access*, vol. 7, pp. 52650–52658, 2019.
- [13] S.-U. Chung, J.-M. Kim, D.-H. Koo, B.-C. Woo, D.-K. Hong, and J.-Y. Lee, "Fractional slot concentrated winding permanent magnet synchronous machine with consequent pole rotor for low speed direct drive," *IEEE Trans. Magn.*, vol. 48, no. 11, pp. 2965–2968, Nov. 2012.
- [14] B. Dotz, M. Ippisch, and D. Gerling, "Design considerations on a 36-slot 28-pole permanent magnet drive," in *Proc. IEEE 12th Int. Conf. Power Electron. Drive Syst. (PEDS)*, Dec. 2017, pp. 360–367.
- [15] A. M. El-Refaie and T. M. Jahns, "Scalability of surface PM Machines with concentrated windings designed to achieve wide speed ranges of constant-power operation," *IEEE Trans. Energy Convers.*, vol. 21, no. 2, pp. 362–369, Jun. 2006.
- [16] S. G. Min and B. Sarlioglu, "Analysis and comparative study of flux weakening capability in fractional-slot concentrated windings," *IEEE Trans. Energy Convers.*, vol. 33, no. 3, pp. 1025–1035, Sep. 2017.
- [17] D. Ishak, Z. Q. Zhu, and D. Howe, "Comparison of PM brushless motors, having either all teeth or alternate teeth wound," *IEEE Trans. Energy Convers.*, vol. 21, no. 1, pp. 95–103, Mar. 2006.
- [18] F. Wu, P. Zheng, L. Cheng, and C. Zhou, "Analysis and experimental evaluation of harmonic leakage inductance for polyphase PM machines having close slot and pole combinations," *IEEE Trans. Magn.*, vol. 51, no. 11, pp. 1–4, Nov. 2015.
- [19] P. Ponomarev, P. Lindh, and J. Pyrhonen, "Effect of slot-and-pole combination on the leakage inductance and the performance of tooth-coil permanent-magnet synchronous machines," *IEEE Trans. Ind. Electron.*, vol. 60, no. 10, pp. 4310–4317, Oct. 2013.
- [20] C. M. Donaghy-Spargo, B. C. Mecrow, and J. D. Widmer, "On the influence of increased stator leakage inductance in single-tooth wound synchronous reluctance motors," *IEEE Trans. Ind. Electron.*, vol. 65, no. 6, pp. 4475–4482, Jun. 2018.
- [21] J. Cros and P. Viarouge, "Synthesis of high performance PM motors with concentrated windings," *IEEE Trans. Energy Convers.*, vol. 17, no. 2, pp. 248–253, Jun. 2002.
- [22] N. Bianchi, S. Bolognani, M. D. Prè, and G. Grezzani, "Design considerations for fractional-slot winding configurations of synchronous machines," *IEEE Trans. Ind. Appl.*, vol. 42, no. 4, pp. 997–1006, Jul./Aug. 2006.
- [23] R. Krall, J. Krenn, and A. Schmid, "Comparison of leakage inductance between fractional slot winding and distributed winding," in *Proc.* 16th Int. Power Electron. Motion Control Conf. Expo., 2014, pp. 276–282.
- [24] P. Ponomarev, Y. Alexandrova, I. Petrov, P. Lindh, E. Lomonova, and J. Pyrhonen, "Inductance calculation of tooth-coil permanent-magnet synchronous machines," *IEEE Trans. Ind. Electron.*, vol. 61, no. 11, pp. 5966–5973, Nov. 2014.
- [25] A. Taieb Brahimi, A. Foggia, and G. Meunier, "End winding reactance computation using a 3D finite element program," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1411–1414, Mar. 1993.
- [26] O. Chiver, E. Micu, and C. Barz, "Stator winding leakage inductances determination using finite elements method," in *Proc. 11th Int. Conf. Optim. Elect. Electron. Equip.*, 2008, pp. 69–74.
- [27] T. A. Lipo, Analysis of Synchronous Machines. Boca Raton, FL, USA: CRC Press, 2012.
- [28] D. W. Novotny and T. A. Lipo, Vector Control and Dynamics of AC Drives. Oxford, U.K.: Clarendon, 1996.



[29] S. M. Raziee, O. Misir, and B. Ponick, "Winding function approach for winding analysis," *IEEE Trans. Magn.*, vol. 53, no. 10, pp. 1–9, Oct. 2017.



**ZICHONG ZHU** (S'15) received the B.S. degree in thermal energy and power engineering from the Nanjing Institute of Technology (NJIT) and the M.S. degree in electrical engineering from Southeast University (SEU), Nanjing, China, where he is currently pursuing the Ph.D. degree with the School of Electrical Engineering.

His research interests include electromagnetic analysis, thermal simulation and structural design of permanent magnet synchronous machines for

wheel-hub driving, wind power generation, and servo applications.



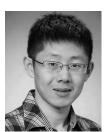
**YUNKAI HUANG** received the M.Sc. and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 2001 and 2007, respectively, where he is currently a Professor with the School of Electrical Engineering.

His research interests include design and control of permanent magnet (PM) machine and high-speed brushless machine, applications in domestic appliances, electric vehicles, and wind-power generation systems.



**JIANNING DONG** (S'10–M'17) received the B.S. and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 2010 and 2015, respectively. Since 2016, he has been an Assistant Professor with the Delft University of Technology (TU Delft), Delft, The Netherlands. Before joining TU Delft, he was a Postdoctoral Researcher with the McMaster Automotive Resource Centre (MARC), McMaster University, Hamilton, ON, Canada. His main

research interests include design, modeling, and control of electromechanical systems.



**FEI PENG** (S'15–M'16) received the B.S. and M.S. degrees in electrical engineering from Southeast University, Nanjing, Jiangsu, China, in 2010 and 2012, respectively, and the Ph.D. degree in electrical and computer engineering from McMaster University, Hamilton, ON, Canada, in 2016. He was a Postdoctoral Fellow with the McMaster Institute for Automotive Research and Technology (MacAUTO), McMaster University. In 2016, he joined the School of

Electrical Engineering, Southeast University, as an Assistant Professor. His research interests include optimal design and control of power converters, and modeling and digital control of motor drives.

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