THE INFLUENCE OF SUBGRID-SCALE MODELLING ON THE PERFORMANCE OF A NEW NON-EQUILIBRIUM WALL-MODEL FOR LARGE-EDDY SIMULATION

William Sidebottom[1], Olivier Cabrit[1], Ivan Marusic[1], Charles Meneveau[2], Andrew Ooi[1] & David Jones[3]
[1]*Department of Mechanical Engineering, The University of Melbourne, Parkville, Victoria, 3010, Australia*
[2]*Department of Mechanical Engineering, Johns Hopkins University, Baltimore, Maryland, 21218, USA*
[3]*Maritime Division, Defence Science and Technology Organisation, Fishermans Bend, Victoria, 3207 Australia*

Abstract The computational cost of wall-resolved large-eddy simulations (LES) rapidly becomes prohibitive with increasing Reynolds number. Wall-modelled LES attempts to significantly reduce the computational cost of simulating wall-bounded turbulent flows by modelling the effect of the near-wall small-scale motions, rather than fully or partially resolving them. The present study concentrates on a new wall-model that is able to predict fluctuating wall-shear stress given a large-scale velocity input. The velocity input for the model is affected by the choice of subgrid-scale (SGS) model. Therefore, this study also focusses on the impact of the SGS-model on the distribution of quantities at the wall. Results show that the new wall-model is able to resolve more of the wall shear-stress variance than a standard wall-model; and that the SGS-model affects the distribution of fluctuations of both wall-shear stress and wall-pressure.

INTRODUCTION

Subgrid-scale (SGS) models are required in large-eddy simulations (LES) since the grid spacing is typically far greater than the viscous length scale at which kinetic energy is dissipated [2]. The scales of motion that are not resolved by the grid are responsible for mixing and energy transfer, typically from larger- to smaller-scales in the turbulent energy cascade. SGS-models therefore account for the scales of motion that are not resolved by the grid by introducing a 'viscosity' to reproduce the expected energy transfer. This viscosity must be chosen carefully to avoid overdamping the resolved scales. Two SGS-models are used in this study—the Smagorinsky model [12] and the σ-model [9]. Both are eddy-viscosity models with the general formulation: $\mu_{\text{sgs}} = \rho \left(C_m \Delta\right)^2 D_m$, where μ_{sgs} is the SGS viscosity, ρ is the density, C_m is a model constant, Δ is the SGS length scale, and D_m is the frequency operator based on the resolved velocity field. For the Smagorinsky model, $C_m = 0.18$ and $D_m = \sqrt{2\langle S_{ij}\rangle\langle S_{ij}\rangle}$, where $\langle\cdot\rangle$ denotes a large-scale quantity. For the σ-model, $C_m = 0.15$ and $D_m = \sigma_3(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)/\sigma_1^2$, where $\sigma_1 > \sigma_2 > \sigma_3 > 0$ are the singular values of the velocity gradient tensor. The Smagorinsky model is known to not behave well in wall-bounded flows, but is used here as a baseline for comparison. The σ-model has many interesting characteristics and behaves well near solid-boundaries.

In *wall-resolved* LES, over 90% of grid points are used to resolve the inner-layer, which covers only 10% of the boundary layer thickness [11]. In *wall-modelled* LES, however, turbulence away from the wall is computed with LES and flow near the wall is computed using a reduced-order model, typically based on the law-of-the-wall and RANS equations [10]. These models are generally only able to predict mean quantities at the wall. Indeed, there are very few models that are able to predict fluctuating quantities at the wall. The wall-model used in this study is able to predict the fluctuating component of the wall-shear stress, and is based on the coupling that exists between the small-scale structure of turbulence in the vicinity of the wall and the large-scale motions of the logarithmic layer. This interaction can be well described by a superposition mechanism [3] and an amplitude modulation effect [6]. Based on these phenomena, Marusic *et al.* [5, 7] proposed a model capable of predicting time-series of the streamwise velocity fluctuations near the wall given a large-scale single-point input taken away from the wall. Recently, Mathis *et al.* [8] extended the original model to predict the wall-shear stress fluctuations. The input requirement for large-scale information taken away from the wall makes this model well suited to LES. The model is defined as:

$$\underbrace{\tau'^{+}_{wp,x}}_{\substack{\text{predicted near-}\\\text{wall signature}}} = \underbrace{\tau'^{*}_{w,x}\{1+\alpha u'^{+}_{OL}\}}_{\substack{\text{amplitude}\\\text{modulation}}} + \underbrace{\alpha u'^{+}_{OL}}_{\text{superposition}}, \tag{1}$$

where $\tau'^{+}_{wp,x}$ is the predicted streamwise fluctuating wall shear-stress normalised by wall variables; $\tau'^{*}_{w,x}$ is the 'universal' streamwise wall shear-stress signal that would exist in the absence of any inner-outer interactions [8]; α is the superposition/modulation coefficient ($\alpha = 0.1$ [8]); and u'^{+}_{OL} is the fluctuating streamwise velocity normalised by wall variables. The O-subscript denotes a variable taken in the outer- or log-region and the L-subscript denotes a variable that has been filtered to only contain large-scales.

The wall-pressure is calculated with a similar model to that shown in eq. (1). However, the model for the pressure is used *a posteriori*. That is, the wall-pressure is not calculated during the simulation, but is found during post-processing using the large-scale pressure input from the logarithmic region. The model for the wall-pressure is: $\langle p_{wp}\rangle = \langle\overline{p_{wp}}\rangle + \mathcal{C}\alpha\langle p'_{\text{mid-log}}\rangle$, where $\langle p_{wp}\rangle$ is the predicted instantaneous wall-stress, $\langle\overline{p_{wp}}\rangle$ is equal to the free-stream mean pressure, $\mathcal{C}$ is a correction factor to correct the location of the input to the model, and $\langle p'_{\text{mid-log}}\rangle$ is the fluctuating pressure in the middle of the log-region.

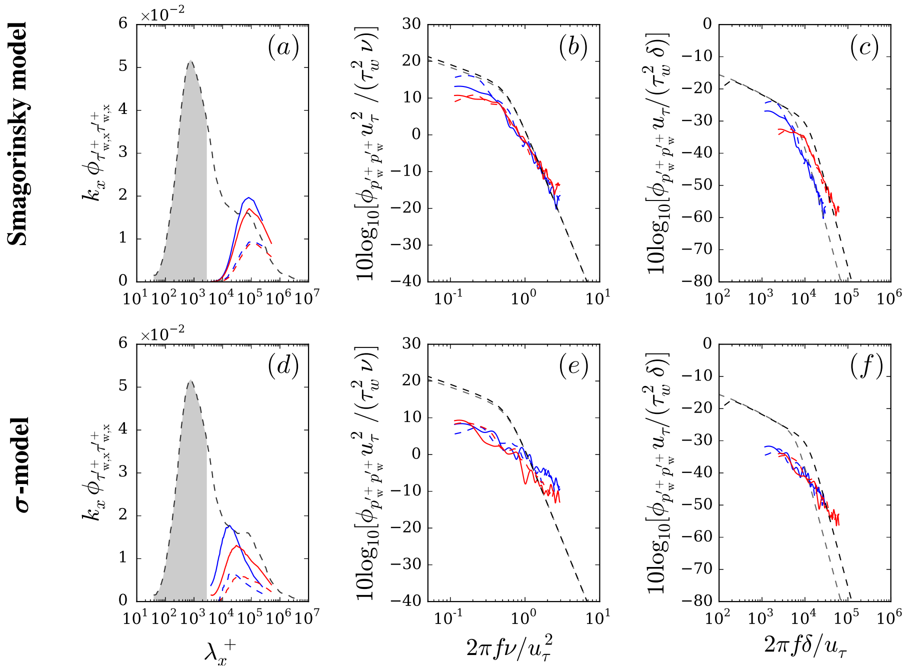

Figure 1. Effect of SGS-model and wall-model on variance of wall-shear stress—(a) & (d) (dashed black line indicates the reconstructed variance profile using the full inner-outer scale-interaction wall-model and experimental data [1], shaded area represents region that is relatively independent of large-scales)—and frequency power spectra of wall-pressure—(b), (c), (e) & (f) (dashed black lines show the empirical prediction of Goody's model [4]; (b) & (e) show scaling by inner variables (τ_w as the pressure scale and ν/u_τ^2 as the time scale), and (c) & (f) show scaling by outer variables (τ_w as the pressure scale and δ/U_∞ as the time scale)). (a)-(c) show results with the Smagorinsky SGS-model; and (d)-(f) show results with the σ-model. Results from channel flow simulations are shown at two Reynolds numbers: ——, $Re_\tau \approx 10,100$; ——, Re_τ $21,000$. Dashed colour lines indicated cases with the 'standard wall-model'; solid colour lines indicate results with the 'new' wall-model.

RESULTS AT THE WALL

The results in Fig. 1 (a) and (d) show that both the SGS-model and the wall-model significantly influence the predicted wall-shear stress variance profile. Results in Fig. 1 (b), (c), (e) and (f) show that the difference between the results of wall-pressure spectra from the 'standard' and 'new' wall-models is ostensibly indistinguishable given the same SGS-model. This suggests that the truncation error of the SGS-models dominate over the truncation error of the wall-models and that the wall-model has no significant influence on the flow away from the wall.

References

[1] O. Cabrit, R. Mathis, V. Kulandaivelu, and I. Marusic. Towards a statistically accurate wall-model for large-eddy simulation. In *Proc. 18th Australasian Fluid Mechanics Conference*, Launceston, Australia, 2012.

[2] C. Higgins, M. B. Parlange, and C. Meneveau. Energy dissipation in large-eddy simulation: dependence on flow structure and effects of eigenvector alignments. In E. Federovich, R. Rotunno, and B. Stevens, editors, *Atmospheric turbulence and mesoscale meteorology, tribute to Doug Lilly*. Cambridge University Press, Cambridge, 2004.

[3] N. Hutchins and I. Marusic. Large-scale influences in near-wall turbulence. *Phil. Trans. R. Soc. A*, **365**:647–664, 2007.

[4] Y. F. Hwang, W. K. Bonness, and S. A. Hambric. Comparison of semi-empirical models for turbulent boundary layer wall pressure spectra. *J. Sound Vib.*, **319**:199–217, 2009.

[5] I. Marusic, R. Mathis, and N. Hutchins. Predictive model for wall-bounded turbulent flow. *Science*, **239**(5988):193–196, 2010.

[6] R. Mathis, N. Hutchins, and I. Marusic. Large-scale amplitude modulation of the small-scale structures in turbulent boundary layers. *J. Fluid Mech.*, **628**:311–337, 2009.

[7] R. Mathis, N. Hutchins, and I. Marusic. A predictive inner–outer model for streamwise turbulence statistics in wall-bounded flows. *J. Fluid Mech.*, **681**:537–566, 2011.

[8] R. Mathis, I. Marusic, S. I. Chernyshenko, and N. Hutchins. Estimating wall-shear-stress fluctuations given an outer region input. *J. Fluid Mech.*, **715**:163–180, 2013.

[9] F. Nicoud, H. Baya Toda, O. Cabrit, S. Bose, and J. Lee. Using singular values to build a subgrid-scale model for large eddy simulations. *Phys. Fluids*, **23**(8):085106, 2011.

[10] G. I. Park and P. Moin. An improved dynamic non-equilibrium wall-model for large eddy simulation. *Phys. Fluids*, **26**:015108, 2014.

[11] U. Piomelli and E. Balaras. Wall-layer models for large-eddy simulations. *Annu. Rev. Fluid Mech.*, **34**:349–374, 2002.

[12] J. Smagorinsky. General circulation experiments with the primitive equations: 1. the basic experiment. *Mon. Wea. Rev.*, **91**(3):99–164, 1963.