Geotechnical Safety and Risk V
T. Schweckendiek et al. (Eds.)

doi:10.3233/978-1-61499-580-7-470

Comparison of Methods for Reliability Calibration of Partial Resistance Factors for Pile Foundations

Mahongo DITHINDE [a] , Johan RETIEF [b]
[a] *Department of Civil Engineering, University of Botswana, Botswana*
[b] *Department of Civil Engineering, University of Stellenbosch, South Africa*

Abstract. Uncertainties are a hall mark of engineering design leading to the development of risk and reliability informed basis for decision making. Accordingly three different but related levels of approach, namely a risk based, a reliability based and a semi-probabilistic approach have been proposed in ISO 2394. However, for code development the semi-probabilistic approach has been internationally embraced. The approach constitutes verification method in which allowance is made for the uncertainties and variability assigned to the basic variables by means of partial factors. A number of methods for reliability calibration of partial resistance factors have been reported in the literature. These include: Advanced First-Order Second Moment Approach (A-FOSM), Mean Value First-Order Second Moment Approach (MV-FOSM), Approximate First-Order Second Moment Approach and the Design Value method. These four methods are used to derive partial resistance factors. Published resistance statistics based on a comprehensive pile load tests from the geologic region of Southern Africa are used as input in the reliability calibration process. Comparison of results shows that resistance partial factors from a full scale reliability method are comparable to those obtained from approximate methods.

Keywords. Reliability calibration, partial factor, approximations, pile resistance

1. Introduction

Geotechnical design is a typical decision problem that is subject to a combination of inherent, modeling and statistical uncertainties. Accordingly the geotechnical fraternity is rigorously pursuing the concept of risk and reliability informed decision making. The philosophical and methodical treatment of the concepts of risk and reliability based decision making can be found in ISO 2394:2015, JCSS (2008), EN 1990:2002, TRB E-C079 (2005), etc. Three different but related levels of approach have been proposed, namely a risk based, a reliability based and a semi-probabilistic approach. However for code development the semi-probabilistic approach has been recommended due to its simplicity. This approach constitutes a verification method in which allowance is made for the uncertainties and variability assigned to the basic variables by means of representative values, partial factors and, if relevant, additive quantities.

The partial factor format is as a general rule utilized as basis for the definition of semi-probabilistic safety formats. Accordingly geotechnical design codes based on the partial factor format such as EN 1997-1 are in use.

To keep pace with international trends in geotechnical design, South Africa has also converted to the limit state design through the standard SANS 10160:2011. Within this suite of standards, Part 5 (SANS 10160-5) is devoted to *Basis for geotechnical design and actions*. Partial factors from BS EN 1997-1 were adopted as place holders pending the gathering of load and resistance statistics required for formal calibration exercises. However, for pile foundations, the local pile resistance statistics database reported by Dithinde and Retief (2013) is now available to enable reliability calibration of partial factors. The database represent typical soil types and pile construction methods for the region.

To set the stage for the impending calibration studies in South Africa, this paper compares and contrasts the numerical values and general characteristics of partial resistance factors obtained from different reliability calibration methods.

2. Calibration Process

In accordance with well-established norms (e.g. ISO 2394:2015, TRB E-C079 (2005)) the key calibrations steps followed in this paper are as follows:

- Formulation of a limit state function
- Establishment of the limit state design equation which include all parameters that describe the failure mechanism.
- Identification and quantification of uncertainties
- Setting of a target reliability index
- Determining partial factors corresponding to the set target reliability index

2.1. Limit state function

The limit state function is given by:

$$R - D - L = 0 \tag{1}$$

where R, D and L are the measured resistance, permanent and variable loads respectively, expressed as random variables.

In routine design practice, the measured load and resistance are presented in terms of their respective predicted values. To account for prediction uncertainty, the predicted values (X_P) are corrected by a model factor (M_X) and hence Eq. (1) becomes:

$$M_R R_p - M_G G_p - M_Q Q_p = 0 \tag{2}$$

where: M_R, M_G and M_Q are model factors for resistance, permanent and variable actions respectively, expressed as random variables following specific probability distributions.

2.2. Design Equation

According to SANS 10160:2011 the design equation for pile foundations is given by Eq. (3):

$$\frac{R_k}{\gamma_M} = \gamma_G G_k + \gamma_Q Q_k \tag{3}$$

where R_k = characteristic predicted pile capacity; γ_M = partial factor accounting for both material property and resistance model uncertainty; G_k = characteristic permanent action; γ_G = partial factor for permanent action; Q_k = characteristic variable action; and γ_Q = partial factor for variable action.

In this study the main task is determine γ_M while γ_G and γ_Q are given in SANS 10160:2011. For simplicity the calculation was done in the load space which entails expressing Q_k in terms of G_k. When Q_k is expressed in terms of G_k, Eq. (3) becomes:

$$\frac{R_k}{\gamma_M} = G_k \left[1 + \frac{Q_k}{G_k}\right] \tag{4}$$

From Eq. (4) the expressions for G_k and Q_k are as follows:

$$G_k = \frac{R_k}{\gamma_M \left[1 + \frac{Q_k}{G_k}\right]} \tag{5}$$

$$Q_k = \frac{R_k}{\gamma_M} - G_k \tag{6}$$

2.3. Load and resistance statistics

The statistics (mean, standard deviation and coefficient of variation) as well as the probability distribution of each random variable considered in the limit state function serve as input data. It is apparent from the limit state function (i.e. Eq.(2)) that the random variables can be classified into resistance and load related variables. These statistics express the uncertainties in the calculations of the pile resistance and loads.

The resistance statistics also referred to as model factor statistics (M) are based on a pile load test database from the Southern African geological setting and pile design practice compiled by Dithinde (2007) and also reported in Dithinde et al (2011) and Dithinde and Retief (2013). The measured resistances from the respective load-settlement curves were interpreted on the basis of Davisson's offset criterion (Davison 1972). However, for working piles, Chin's extrapolation (Chin 1970) was carried out prior to the application of the Davisson's offset criterion.

The statistics as reported in Dithinde and Retief (2013) are shown in Table 1. It should be noted that all the piles in the database were compression piles, The database was classified into various practical pile design classes as follows: (i) driven piles in non-cohesive soil (D-NC) with 29 cases, (ii) bored pile in non-cohesive soil (B-NC) with 33 cases; (iii) driven piles in cohesive soils (D-C) with 59 cases, (iv) bored pile in cohesive soils (B-C) with 53 cases; and the combinations (v) all driven piles (D) with 87 cases, (vi) all bored piles (B) with 83 cases (vii) all piles in non-cohesive soil (NC) with 58 cases, (viii) all piles in cohesive soil (C) with 112 cases and all piles (ALL) with 174 cases.

Table 1: Model factor (*M*) statistics (After Dithinde and Retief, 2013)

Pile class	*N*	*Mean mM*	*Std.Dev. sM;*	*COV*
D-NC	28	1.11	0.36	0.33
B-NC	30	0.98	0.23	0.24
D-C	59	1.17	0.3	0.26
B-C	53	1.15	0.28	0.25
D	87	1.15	0.32	0.28
B	83	1.09	0.28	0.25
NC	58	1.04	0.30	0.29
C	112	1.16	0.29	0.25
ALL	170	1.12	0.30	0.27

Regarding the probability distribution for *M*, a detailed analysis reported in Dithinde and Retief (2013) indicate that although at the customary 5% confidence level, the chi-square goodness-of-fit test results indicate that both the Normal and Lognormal distributions are valid, the Lognormal distribution has a slight edge, particularly towards the lower tail. Accordingly a lognormal distribution is adopted for *M*.

With regard to load statistics, values given in SANS 10160:2011 have been adopted.

2.4. Target reliability index

In accordance with the South African loading code SANS 10160: 2011, target reliability index of $\beta_T = 3.0$ has been prescribed for the reference reliability class *RC2* for which reliability procedures are specified in the code. The background for the target level of reliability for South African construction within the scope SANS 10160: 2011 has been discussed by Retief and Dunaiski (2009).

3. Reliability Calibration Methods Considered

Conceptually, the determination of partial factors in the reliability framework is the reverse of the process for computing the reliability index. Therefore in principle the various methods used to compute β can also be used to derive partial factors for a given β. For this study, the following four reliability calibration methods have been used:

a) Advanced first-order second moment approach (A-FOSM)
b) Design value method
c) Mean value first-order second moment approach (MV-FOSM)
d) Approximate mean value first-order second moment approach (Approx-MVFOSM)

3.1. Advanced First Order Second Moment Method (A-FOSM)

The A-FOSM method entails linearising the performance function at some point on the failure surface referred to as the design point. In general the design location is the point on the limit state surface at the shortest distance from the origin in the standard normal space. It is also the point which maximises the joint probability function on the failure surface for a given problem. The approach entails finding the design point of design values of the basic random variables corresponding to the target β. However, the design point is generally not known in advance and therefore an iteration technique is used to solve for the reliability index. To facilitate the iterations, simple and practical computational procedures have been developed by exploiting the nonlinear optimisation function in spreadsheets such as EXCEL. Accordingly in this paper, the spreadsheet optimisation method developed by Low and Tang (1997) was employed for the analysis.

3.2. Design Value Method

To enable practical application of the theory of reliability and their effective application in codes, various simplifications are accepted. In EN 1990, one such simplification to the A-FOSM approach is the Design Value method for obtaining partial factors. In accordance with EN 1990, the design value X_d is given by:

$$X_d = \frac{X_k}{\gamma} \tag{7}$$

in which γ and X_k are partial factor and characteristic value of a given basic random variable respectively.

In the context of the resistance the partial factors (γ_R) is then given by:

$$\gamma_R = \frac{R_k}{R_d} \tag{8}$$

where the R_k is the characteristic resistance and R_d is the design value of the resistance.

If the resistance follows a normal distribution, the design value (R_d) corresponding to a desired β-value is given by:

$$R_d = \mu_R(1 - \alpha_R \beta V) \tag{9}$$

in which α_R = sensitivity factor taken as -0.8 (*EN* 1990) and V = coefficient of variation of the resistance.

Since geotechnical performance is governed by the average resistance, the characteristic resistance (R_k) is the lower bound of the 95% confidence interval of the mean value. The characteristic value as a mean value at 95% confidence level is given by:

$$X_k = \bar{X}\left(1 - t_{n-1}^{0.95} V \sqrt{\frac{1}{n}}\right) \tag{10}$$

where X_k is the characteristic value, $\bar{X}$ is the arithmetic mean, V is the coefficient of variation of the desired property, n is the number of test results, and t is the value of the Student distribution corresponding to a confidence level of 95% and a degree of freedom of n-1.

From Eq.(9) and Eq. (10);

$$\gamma_R = \frac{\mu_R\left(1 - t_{n-1}^{0.95} V \sqrt{\frac{1}{n}}\right)}{\mu_{R(1-\alpha_R \beta V)}} \tag{11}$$

In reliability analysis, resistance is generally modeled as a lognormal variable and hence Eq. (11) is expressed as:

$$\gamma_R = \frac{\frac{\mu_R}{\sqrt{1+v^2}} exp\left(t_{n-1}^{0.95}\sqrt{\frac{1}{n}}\sqrt{ln(1+v^2)}\right)}{\frac{\mu_R}{\sqrt{1+v^2}} exp\left(\alpha\beta\sqrt{ln(1+v^2)}\right)} \tag{12}$$

3.3. Mean Value First-Order Second Moment Method (MV-FOSM)

This method derives its name from the fact that it is based on a first order Taylor series approximation of the limit state function linearised at the mean values of the random variables and it uses only second moment statistics (mean and variance) of the random variables. The MV-FOSM provides a closed form solution for reliability index. Likewise, there exists a closed form solution for derivation of partial resistance factors. The principal equation for determining resistance factor found in FHWA- HI-98-032 (2001) is given by:

$$\phi = \frac{\lambda_R\left(\gamma_{QD} + \gamma_{QL}\frac{Q_L}{Q_D}\right)\sqrt{\frac{\left(1+V_{QD}^2+V_{QL}^2\right)}{\left(1+V_R^2\right)}}}{\left(\lambda_{QD} + \lambda_{QL}\frac{Q_L}{Q_D}\right) exp\left\{\beta_T\sqrt{ln\left[\left(1+V_R^2\right)\left(1+V_{QD}^2+V_{QL}^2\right)\right]}\right\}} \tag{13}$$

in which γ_{QD} and γ_{QL} are the load factors for permanent (Q_D) and variable (Q_L) loads, λ_{QD} and λ_{QL} are the model factors for permanent and variable loads respectively.

3.4. Approximation to MV-FOSM Method

The approximation allows for separate determination of resistance and load factors. The full derivation of expression for determining resistance factors using this approach can be found in FHWA- HI-98-032 (2001) and is given *by:*

$$\phi = \lambda_R exp(-\propto \beta_T V_R) \tag{14}$$

Where ϕ = resistance partial factor, λ_R = resistance model uncertainty factor, β_T = target reliability index, α = fitting factor (0.7 to 1 but taken as 0.87 in accordance with FHWA- HI-98-032, 2001).

4. Calibrated Partial Factors

In line with the current South African code, only total resistance partial factors have been calibrated. For the A-FOSM and MV-FOSM methods, γ_R values are a function of the calibration points (i.e. L_n/D_n ratio). Nonetheless, for codified design a single γ_R value applicable to all the calibration points within a given pile class is required. The application of a single resistance factor to all the design situations will inevitably lead to some deviation from the target reliability index for some of the calibration range for L_n/D_n. To achieve consistent reliability within a range of calibration points for a given pile class, an optimum partial factor which best approximate the uniform target reliability is needed. In principle this can be obtained by minimizing the deviation from the target beta using an objective or penalty function penalising the deviation from the target reliability index.

A number of objective functions have been proposed in the literature (e.g. least square function, Lind's function, 1977). Investigation of the optimisation schemes by Dithinde (2007) showed that Lind's function gives results that are close to the least square function while still penalising under-designs more than over-designs, hence providing conservative results. Further analysis by Dithinde (2007) revealed that optimal partial resistance factors obtained by minimization of Lind's objective function were very close to partial factors corresponding to the calibration point represented by L_n/D_n of 0.5 (i.e. design situation with the highest partial factor). Therefore instead of performing the optimisation process, in this paper the partial factors corresponding to L_n/D_n ratio of 0.5 were taken as the optimal partial resistance factors.

The resistance partial factors obtained from the various calibration methods are presented in Table 2.

Table 2: γ_R values for various approximations

Pile class	*A-FOSM*	*Design Value*	*MV-FOSM*	*Approx MVFOSM*
D-NC	2.3	*2.5*	*2.9*	*2.1*
B-NC	2.0	*2.1*	*2.6*	*1.9*
D-C	1.8	*1.9*	*2.3*	*1.7*
B-C	1.8	*1.9*	*2.3*	*1. 7*
D	1.9	*2.1*	*2.5*	*1.8*
B	1.9	*2.0*	*2.4*	*1.8*
NC	2.2	*2.4*	*2.8*	*2.1*
C	1.8	*1.9*	*2.3*	*1.7*
ALL	2.0	*2.2*	*2.6*	*1.9*

5. Discussion of Results

- In comparing the results, the A-FOSM method is taken as the reference reliability calibration method with relatively more accurate results. In terms of general characteristics, partial factors from the four calibration methods depict common trends as follows: Piles in non-cohesive materials (D-NC, B-NC, NC) depict higher γ_R values. The scenario is attributed to the relatively higher variability exhibited by piles in non-cohesive materials as demonstrated by the coefficient of variations in Table 1.
- The influence of pile construction method is minimal in cohesive soils as the γ_R values for bored (D-C) and driven piles (B-C) are quite the same for all the calibration methods. In contrast, the influence of pile construction method in non-cohesive materials is appreciable. In this regard the γ_R value for driven piles is significantly higher than that of bored piles.
- A comparison of all driven (D) versus all bored (B) piles irrespective of soil type indicates very little difference in γ_R values, implying that in general pile installation method has little influence.
- The differences in γ_R values for piles in non-cohesive versus cohesive soils is quite distinctive when comparing values for all piles in cohesive materials (C) to all piles in non-cohesive (NC). Therefore it appears that

from design perspective, piles should be classified on the basis of soil type only. It follows that partial resistance factors should be differentiated on the basis of soil types.

In exception of MV-FOSM method, the γ_R values presented in Table 2 are generally lower than the overall values prescribed in SANS 10160-5. For example the overall partial factor for bored piles is 1.5 (model factor) x 1.6 (resistance partial factor) = 2.4. This is attributed to the fact that only model uncertainty was taken into account while other sources were neglected. Nonetheless, comparison of the γ_R values from the various methods lead to the following observations:

- The MV-FOSM method yields the highest γ_R values. Relative to the full scale reliability method, the approach gives γ_R values that are 26 to 31% higher. In contrast, the approximate MV-FOSM method produces γ_R values that are quite close to values from full scale reliability method. Therefore the approximate MV-FOSM method is better than the actual MV-FOSM method.
- The design value method produces γ_R values that are comparable with those for the A-FOSM method. Theoretically the two approaches should give similar results as they are both based on evaluating the performance function at the design point. The difference in results is attributed to the approximation of the sensitivity factor to -0.8 which might be different from the actual values.

6. Conclusions

The main conclusions of the analysis are as follows:

- For all the four methods, the variation of the γ_R values with different variables such as pile classes, soil type and installation methods exhibit the same characteristics. This suggests that although the methods appear radically different, they are based on a common and sound underlying theoretical basis.
- Results of the principal calibration method (i.e. A-FOSM) are comparable to results obtained from the Design Value and the approximate MV-FOSM methods. This implies that the approximate methods yield reasonable results, further suggesting that reliability calibration can as well be based on the simple approximation procedures.
- The γ_R values are influenced more by soil type than pile construction method. Therefore it appears that from design perspective, piles should be classified on the basis of soil type only.

References

Dithinde, M (2007). Characterisation of model uncertainty for reliability based design of pile foundations. PhD dissertation submitted to the University of Stellenbosch, Available at: http://www.hdl.handle.net/10019.1/12612 .

Dithinde, M.,Phoon, K. K., De Wet, M., and Retief, J. V. (2011). Characterisation of Model Uncertainty in the Static Pile Design Formula. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 137, No. 1.

Dithinde, M., and Retief, J.V. (2013). Pile design practice in southern Africa I: Resistance statistics. Journal of the South African Institution of Civil Engineering, Vol 55 Number 1 April , 2013.

EN 1990, (2002). Eurocode: Basis of structural design. Committee for Standardization (CEN): Brussels.

EN 1997:2004. Eurocode 7: geotechnical design – Part 1: General rules. European Committee for Standardization (CEN), Brussels.

FHWA. (2001). Load and resistance factor design (LRFD) for highway bridge substructures. Publication No.FHWA- HI-98-032.

ISO 2394 (2015). General principles on reliability for structures. International Organisation for Standardisation, Geneva.

JCSS (2008). Probabilistic Model Code. http://www.jcss.byg.dtu.dk/Publications/Probabilistic_Model_Code.aspx

Lind, N. C. (1977). Reliability-based structural codes-practical calibration. Proc. of the International Research Seminar on Safety of Structures Under Dynamic Loading, Trondheim, Norway, pp. 149-60.

Low, B. K. and Tang, W.H. (2007). Efficient spreadsheet Algorithm for FORM. Journal of Engineering Mechanics, ASCE, 133(12), 1378-1387.

Retief JV and Dunaiski PE (2009). The limit states basis of structural design for SANS 10160-1. Chapter 1-2 in Retief & Dunaiski (Ed) Background to SANS 10160. SunMedia Stellenbosch. ISBN 978-1-920338-10-7, 2009.

SANS 10160-5:2011. South African National Standard Part 5: Basis for geotechnical design and actions. SABS Standards Division, Pretoria

TRB E-C079 (2005), Calibration to Determine Load and Resistance Factors for Geotechnical and Structural Design, Transport and Research Board Circular.