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On the Heaving Motion of Two Circular Cylinders on the Surface of a Fluid

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By

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Reprinted from Reports of Research Institute
for Applied Mechanics, Kyushu University
Vol. XVII, No. 58, 1969

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ON THE HEAVING MOTION OF TWO CIRCULAR CYLINDERS ON THE SURFACE OF A FLUID

By Makoto OHKUSU*

We investigate the hydrodynamic force upon two circular cylinders when they are given a forced heaving motion.

The wave amplitude at a distance from the cylinders and the increase or decrease in the inertia of the cylinders due to the fluid motion can be theoretically obtained by a procedure which is similar in principle to Ursell's one for one cylinder.

In addition we measure the wave amplitude at infinity and show that theoretical and measured wave amplitudes are in good agreement. We find that the theoretical added mass of the two cylinders has a negative value in some cases. It is desirable to confirm these results by experiment.

1. Introduction

We calculate the hydrodynamic force acting on two circular cylinders connected with each other, when they are immersed in a fluid of infinite depth with their axes in the free surface and given a forced heaving motion, and compare calculation with experiment. In the aspect of theoretical calculation Ursell's solution[1] for one circular cylinder and Tasai's solution[2] for a cylinder with Lewis form cross-section are well known. In addition Tasai[3] and Porter[4] compared these results with the measurements. An investigation of similar problems for the case of two cylinders also seems to be necessary for understanding, for example, the behaviour of catamaram ships or floating station in waves. In treating such problems it is a method most commonly used that we determine the singularity distribution on the body surface to satisfy the boundery condition on the surface by solving numerically integral equation.

In this paper, however, we adopt the method of series expansion by wave free potentials which Ursell proposed in his first paper[1] on one circular cylinder.

Measurements are made mainly about the wave amplitude at a distance from the cylinders. But in order to compare directly calculation with experiment it seems to be indispensable to measure pressure on the surface of the cylinders.

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2. Formulation of problem and calculation

We consider the fluid motion which arises when two infinitely long circular cylinders, which are connected with each other and immersed in a fluid with their axes lying in the mean free surface, oscillate vertically about their mean position. The motion is assumed to be two dimensional and in the stationary state with a period of $2\pi/\omega$. We deduce the amplitude of the waves which travel away from the cylinders and the added mass of the cylinders due to the fluid motion under the following assumptions.

(i) Surface tension and viscosity of a fluid can be neglected.

(ii) The fluid motion is irrotational and a velocity potential and a conjugate stream function exist.

(iii) Compared with the dimension of the cylinder cross-section the amplitude of the cylinder oscillation and the fluid motion is small, and the length of the wave which arises is large. After all to the first order we can linearize the boundary conditions on both the free surface and the body surface.

Take the origin of the coordinate at the center of the line joining the axis A and B of two circular cylinders as shown Fig. 1, where the axes of the cylinders are in the mean free surface in their mean position. The χ-axis is to the right and the y-axis is vertically downward. The coordinate of the center A, B of the cylinders is respectively $(-p, 0)$, $(p, 0)$ and then the distance of between A and B is $2p$.

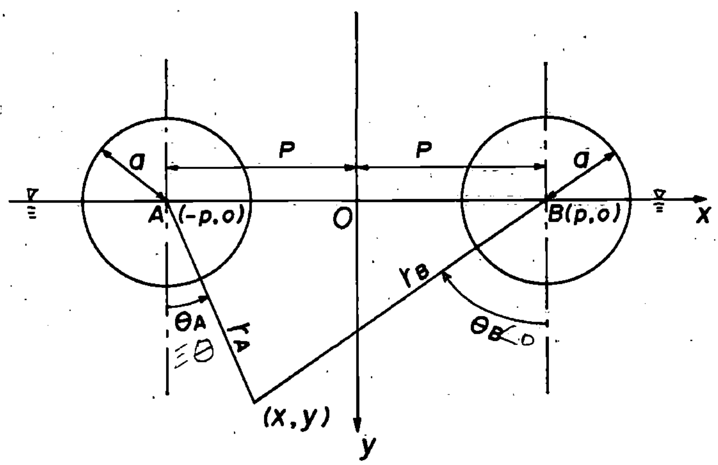

Fig. 1 Coordinate system.

Suppose that the cylinders oscillate about their mean position like

$$y = Re[le^{-i\omega t}], \tag{1}$$

where l is the amplitude of the motion.

Then the velocity potential Φ can be expressed as follows

$$\Phi=Re[\phi e^{-i\omega t}], \tag{2}$$

and it is required to find a velocity potential ϕ which satisfies the boundary condition on the cylinder surface and the free surface, and gives a progressing wave train at infinity.

The velocity potential ϕ satisfies

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0, \quad \text{for } y\leqq 0 \tag{3}$$

On the free surface the pressure is constant, then to the first order

$$K\phi+\frac{\partial\phi}{\partial y}=0, \tag{4}$$

where K is ω^2/g.

The boundery condition on the cylinders is that the velocity component normal to the boundary surface just inside the fluid is equal to the corresponding component of the velocity of the cylinders.

$$\frac{\partial\phi}{\partial n}=-i\omega l\cos\theta, \tag{5}$$

where n is outward normal to the cylinder surface and θ is the angle which the normal makes with a vertical line through the center of the cylinder (counterclockwise is positive).

In addition the so-called radiation condition at infinity is necessary. This is

$$\phi\sim Y(y)e^{+iK|x|} \ as\ |x|\to\infty \tag{6}$$

It is of course that ϕ must satisfy the condition at $y\to\infty$.

The following condition is imposed on a conjugate stream function ψ as a boundary condition on the cylinder surface instead of (5).

$$\frac{\partial\psi}{a\partial\theta}=i\omega l\cos\theta,$$

$$\psi=i\omega l\ a\sin\theta\pm C, \tag{7}$$

where C is constant and positive sign is taken on the cylinder A and negative sign on the cylinder B.

The velocity potential ϕ must be, of course, symmetrical and the stream function ψ must be antisymmetrical with respect to the y-axis.

Applying an idea[1] by which Ursell constructed the solution for similar problem on one cylinder, we can comparatively easily obtain the solution for two cylinders which satisfies the foregoing conditions. Suppose the velocity potential ϕ can be expressed in the form

$$\phi=\phi_A{}^0+\phi_B{}^0+\phi_A{}^1+\phi_B{}^1+\phi_A{}^2+\phi_B{}^2+ \cdots\cdots\cdots \tag{8}$$

where $\phi_A{}^0$, $\phi_B{}^0$ is respectively the velocity potential that represents a fluid motion when a cylinder A or B oscillates individually, that is, Ursell's solution, and ϕ_A^n, ϕ_B^n is obtained one by one as a diffraction potential of ϕ_B^{n-1}, ϕ_A^{n-1} as follows. $\phi_A{}^1$ is a diffraction potential of $\phi_B{}^0$ by the cylinder A under the condition of $\frac{\partial}{\partial n}(\phi_A{}^1+\phi_B{}^0)=0$ on the surface of the cylinder A, $\phi_B{}^1$ is a diffraction of $\phi_A{}^0$ by the cylinder B and once more $\phi_A{}^2$ a diffraction of $\phi_B{}^1$ etc.

Strictly speaking, it is necessary to prove the convergence of this series, but here we assume that the series converges and it gives the solution which we want to obtain, because on physical grounds it seems to be true and in addition we can do qualitatively such discussion as follows.

A diffraction potential ϕ_A^n is proportional to the amplitude of the diffracted wave and it is conceived[5] that the diffraction amplitude of a progressing two dimensional wave is necessarily smaller than that of the incident wave as far as the wave length is not so small compared with depthwise dimension of the cylinder.[1)] On the other hand the amplitude of the stationary wave which exist near around the cylinder rapidly decreases as a distance from the cylinder increases, and therefore a diffraction of the stationary wave becomes smaller as a diffraction occurs. That is $|\phi_A^n|=\varepsilon|\phi_A^{n-1}|$, $\varepsilon<1$. $\phi_A <$ progr. waves / standing waves.

Each of ϕ_A^n, ϕ_B^n is diffraction wave potential by the cylinder A and B, then they are expressed as a conbination of a series of symmetrical and antisymmetrical wave free potentials with respect to a vertical line through the center of each cylinder, and the potentials with a wave train diverging away at infinity, e. g. the functions describing a source and a dipole at A or B. It is proved by Ursell that such a series uniformly converges if a boundary condition on the cylinder is symmetric and the series constitutes of only symmetrical terms. By just the same procedure as Ursell we can easily prove the uniform convergence of the series when the boundary condition is antisymmetric and accordingly the series is constructed by only antisymmetrical terms. After all it follows that the series converges uniformly for such a case of more general boundary condition as this diffraction problem. Accordingly if we define the collection of diffraction potentials by A and B as

$$\phi_A=\phi_A{}^1+\phi^2{}_A+\phi_A{}^3+\cdots\cdots, \quad \phi_B=\phi_B{}^1+\phi_B{}^2+\phi_B{}^3+\cdots\cdots\cdots \tag{9}$$

where we have $\qquad \phi_A(x,\ y)=\phi_B(-x,y)$

then we immediately obtain the following expression for ϕ

$$\frac{\pi\omega}{gl}\phi=\frac{\pi\omega}{gl}(\phi_A{}^0+\phi_B{}^0+\phi_A+\phi_B)$$

$$=\frac{\pi\omega}{gl}(\phi_A{}^0+\phi_B{}^0)+$$

1) i.e. when the cylinder becomes the limit of a vertical wall and the wave is completely reflected without any reduction in amplitude of the incoming wave.

φ_s φ_s = Ursell's source potential.

$$
\begin{aligned}
&+\zeta_1/l\{\varphi_S(Kr_A,\ \theta_A)+\varphi_S(Kr_B,\ \theta_B)\\
&\quad+\sum_{m=1}^{\infty} P_m[f_m(Ka,\ r_A/a,\ \theta_A)+f_m(Ka,\ r_B/a,\ \theta_B)]\}\\
&+\zeta_2/l\{\varphi_D(Kr_A,\ \theta_A)-\varphi_D(Kr_B,\ \theta_B)\\
&\quad+\sum_{m=1}^{\infty} Q_m[g_m(Ka,\ r_A/a,\ \theta_A)-g_m(Ka,\ r_B/a,\ \theta_B)]\}
\end{aligned}
\tag{10}
$$

where $\varphi_S\ (Kr_A,\ \theta_A)$, $\varphi_S\ (Kr_B,\ \theta_B)$ are the velocity potentials due to a source placed at A and B, and $\varphi_D\ (Kr_A,\ \theta_A)$, $\varphi_D\ (Kr_B,\ \theta_B)$ the velocity potentials due to a dipole at A and B. r_A is a distance from A to a point$(x,\ y)$ and r_B a distance from B to this point. θ_A is the angle that the r_A makes with a vertical line through A and θ_B the angle that the r_B makes with the line through B (counterclockwise is positive) as shown Fig. 1, and they are given by

$$
\varphi_S(Kr,\ \theta)=-i\pi e^{-Kr\cos\theta}\ e^{iKr|\sin\theta|}
+\int_0^{\infty}\frac{\sin(Krt\cos\theta)-t\cos(Krt\cos\theta)}{1+t^2}e^{-Krt|\sin\theta|}\,dt
\tag{11}
$$

$$
\varphi_D(Kr,\ \theta)=\mp\pi e^{-Kr\cos\theta}e^{iKr|\sin\theta|}-\frac{Kr|\sin\theta|}{(Kr)^2}
\pm\int_0^{\infty}\frac{t\sin(Krt\cos\theta)+\cos(Krt\cos\theta)}{1+t^2}e^{-Krt|\sin\theta|}\,dt
\tag{12}
$$

$f_m\ (Ka,\ r_A/a,\ \theta_A)$, $f_m\ (Ka,\ r_B/a,\ \theta_B)$ are wave free potentials symmetrical about A or B, and $g_m\ (Ka,\ r_A/a,\ \theta_A)$, $g_m\ (Ka,\ r_B/a,\ \theta_B)$ are antisymmetrical about A or B and they are given by[6]

$$
f_m(Ka,\ r/a,\theta)=\frac{Ka\ \cos(2m-1)\theta}{(2m-1)\ (r/a)^{2m-1}}+\frac{\cos 2m\theta}{(r/a)^{2m}}
\tag{13}
$$

$$
g_m(Ka,\ r/a,\ \theta)=\frac{Ka\ \sin\ 2m\theta}{2m(r/a)^{2m}}+\frac{\sin(2m+1)\theta}{(r/a)^{2m+1}}
\tag{14}
$$

ζ_1/l, ζ_2/l, P_m, Q_m are complex numbers and the wave elevation ζ due to $\phi_A+\phi_B$ is at $x\to-\infty$

$$
\begin{aligned}
\zeta=Re[&-\zeta_1\ e^{-iK(\overline{x+p}+\omega t)}-\zeta_1 e^{-i(K\overline{x-p}+\omega t)}\\
&-i\zeta_2 e^{-i(K\overline{x+p}+\omega t)}+i\zeta_2 e^{-i(K\overline{x-p}+\omega t)}]
\end{aligned}
\tag{15}
$$

The conjugate stream function ψ can be easily derived from the velocity potential ϕ as follows

$$
\begin{aligned}
\frac{\pi\omega}{gl}\psi&=\frac{\pi\omega}{gl}(\psi_A{}^0+\psi_B{}^0)\\
&+\frac{\zeta_1}{l}\{\psi_S(Kr_A,\ \theta_A)+\psi_S(Kr_B,\ \theta_B)
\end{aligned}
$$

$$+\sum_{m=1}^{\infty} P_m[\xi_m(Ka,\ r_A/a,\ \theta_A)+\xi_m(Ka,r_B/a,\ \theta_B)]\Big\}$$

$$+\frac{\varsigma_2}{l}\Big\{\psi_D(Kr_A,\ \theta_A)-\psi_D(Kr_B,\theta_B)$$

$$+\sum_{m=1}^{\infty} Q_m[\eta_m(Ka,\ r_A/a,\ \theta_A)-\eta_m(Ka,\ r_B/a\ \ \theta_B)]\Big\} \tag{16}$$

Here ψ_S $(Kr,\ \theta)$, ψ_D $(Kr,\ \theta)$ are the conjugate harmonic function of φ_S $(Kr,\ \theta)$, φ_D $(Kr,\ \theta)$ and they are

$$\psi_S(Kr,\ \theta)=\mp\pi e^{-Kr\cos\theta}e^{iKr|\sin\theta|}$$

$$\pm\int_0^{\infty}\frac{t\sin(Krt\cos\theta)+\cos(Krt\cos\theta)}{1+t^2}e^{-Krt|\sin\theta|}dt \quad \text{for } \theta\gtrless 0 \tag{17}$$

$$\psi_D=i\pi e^{-Kr\cos\theta}e^{iKr|\sin\theta|}+\frac{Kr\cos\theta}{(Kr)^2}$$

$$-\int_0^{\infty}\frac{\sin(Kr\,t\cos\theta)-t\cos(Krt\cos\theta)}{1+t^2}e^{-Krt|\sin\theta|}dt \tag{18}$$

ξ_m $(Ka,\ r/a,\ \theta)$, η_m $(Ka,\ r/a,\ \theta)$ are also the conjugate of f_m $(Ka,\ r/a,\ \theta)$, $g_m(Ka,\ r/a,\ \theta)$

$$\xi_m(Ka,\ r/a,\ \theta)=\frac{Ka\sin(2m-1)\theta}{(2m-1)(r/a)^{2m-1}}+\frac{\sin 2m\theta}{(r/a)^{2m}} \tag{19}$$

$$\eta_m(Ka,\ r/a,\ \theta)=-\frac{Ka\cos 2m\theta}{2m(r/a)^{2m}}-\frac{\cos(2m+1)\theta}{(r/a)^{2m+1}} \tag{20}$$

If ψ satisfies the condition (7) on the surface of the cylinder A, then it satisfies the condition on the cylinder B because it is antisymmetrical about the y-axis. Since $\psi_A{}^0$ is the solution for the case of one cylinder, it satistfies the following condition on the cylinder A.

$$i\cdot\pi\cdot Ka\sin\theta=\frac{\pi\omega}{gl}\psi_A{}^0 \tag{21}$$

Therefore the condition for ψ on the cylinder A becomes as follows

$$C-\frac{\pi\omega}{gl}\psi_B{}^0=\frac{\varsigma_1}{l}\Big\{\psi_S(Ka,\ \theta)+\psi_S(Kr_B,\ \theta_B)$$

$$+\sum_{m=1}^{\infty} P_m[\xi_m(Ka,\ 1,\ \theta)+\xi_m(Ka,\ r_B/a,\ \theta_B)]\Big\}$$

$$+\frac{\varsigma_2}{l}\Big\{\psi_D(Ka,\ \theta)-\psi_D(Kr_B,\ \theta_B)$$

$$+\sum_{m=1}^{\infty} Q_m[\eta_m(Ka,\ 1,\ \theta)-\eta_m(Ka,\ r_B/a,\ \theta_B)]\Big\}. \tag{22}$$

where
$$\theta_B=\tan^{-1}\left(\frac{\sin\theta\ -2P/a}{\cos\theta}\right)\ , \tag{23}$$

$$(r_B/a)^2=1-2(2P/a)\sin\theta+(2P/a)^2\ . \tag{24}$$

If we determine the unknown coefficients P_m, Q_m, ς_1/l, ς_2/l and C satisfying this equation, we obtain the solution ϕ or ψ by inserting these coefficients into the equation (16) or (10) and the ratio $\overline{A}$ defined as

$$\overline{A}=\frac{\text{wave amplitude at infinity}}{\text{amplitude of forced heaving}}$$

$$=2\left|\left(-\varsigma_0/l-\varsigma_1/l\right)\cos KP+i\ (\varsigma_2/l)\ \sin KP\right|\ , \tag{25}$$

where ζ_0/l is the ratio when one cylinder oscillates, that is a contribution due to $\psi_A{}^0$ or $\psi_B{}^0$ (Appendix 1). The ratio is proportional to the square root of the damping force acting upon the cylinder which is out of phase with the displacement of the cylinder. In addition we can obtain a vertical component Py of a fluid dynamic force per unit length acting upon the cylinder from the expression for the velocity potential ϕ.

$$P_y=-\rho a e^{-i\omega t}\int_{-\pi/2}^{\pi/2} i\,\omega\,\phi\cos\theta\,d\theta\ , \tag{26}$$

where ρ is the fluid density.

Then the component of Py, which is in phase with the acceleration of the cylinder is

$$-\frac{\rho g l a}{\pi}\int_{-\pi/2}^{\pi/2} Im\left[\frac{\pi\omega}{gl}\phi\right]\cos\theta d\theta\ , \tag{27}$$

and the added mass coefficients $\overline{m}$ with the nondimensional quantity, which is defined as the added mass/the mass of the fluid displaced by unit length of the cylinder, is given by

$$\overline{m}=\frac{2\int_{-\pi/2}^{\pi/2} Im\left[\frac{\pi\omega}{gl}\phi\right]\cos\theta\,d\theta}{\pi^2\ Ka} \tag{28}$$

The coefficients P_m, Q_m, ς_1/l, ζ_2/l and C are the roots of infinite number of equations. Then we replaced this system of equations with a finite number of equations, where the coefficients are $P_m\ (Ka, M)$, $Q_m\ (Ka, M)$ $(m=1, 2, \ldots\ldots, M)$. We evaluated the known functions in the equations (22) at some chosen values of θ and determined P_m, Q_m, ς_1/l, ς_2/l and C so as to be fitted at these θ by least square method. 2(M+1) unknowns = 14. M=6.

Here we selected $M=6$ (P_m, Q_m, ς_1/l, ζ_2/l are complex numbers, then the number of unknown coefficients amouts to 28 because we can eliminate C), $\theta=-90°$, $-81°$, $-72°$, ..., 0°, 9°, 18°, ..., 90° and solved a set of 28 simultaneous

14 for real part
14 for imag. part.

i.e. actually 12, since both ξm and ηm occur!

linear equations for P_m, Q_m, ζ_1/l, ζ_2/l which the least square condition provided. Where the terms which are expressed by an integral in $\psi_S\ (Kr,\ \theta)$, $\psi_D\ (Kr,\ \theta)$, etc., were evaluated by the expansion as shown in Appendix 2.

The added mass coefficient was calculated by numerically integrating the velocity potential according to the equation (28).

The calculation was carried out by using a computer FACOM 230-60 of Kyushu University Computer Center.

Some examples of the calculated $\bar{A}$ and $\bar{m}$ for four cases $2P/a=3.0$, 4.0, 5.0 and 6.0 are shown in Table. 1 and Fig. 4, 5, 6, 7, and 8.

Finally we add that the procedure adopted here can be applied to the problem on the swaying or rolling motion of two cylinders with almost no modification.

Table 1. The calculated values of $\bar{A}$, $\bar{m}$

$2p/a$ \ ka	3.0		4.0		5.0		6.0	
	$\bar{A}$	$\bar{m}$	$\bar{A}$	$\bar{m}$	$\bar{A}$	$\bar{m}$	$\bar{A}$	$\bar{m}$
0.05	0.170	3.104	0.172	2.865	0.173	2.665	0.174	2.488
0.10	0.313	2.280	0.319	2.029	0.323	1.782	0.326	1.559
0.15	0.441	1.915	0.456	1.646	0.467	1.255	0.473	1.028
0.20	0.560	1.734	0.590	1.435	0.613	1.047	0.624	0.585
0.25	0.674	1.630	0.730	1.305	0.774	0.727	0.779	-0.156
0.30	0.787	1.600	0.888	1.208	0.963	0.168	0.853	-1.327
0.35	0.905	1.624	1.091	1.071	1.120	-1.134	0.572	-1.662
0.40	1.033	1.697	1.376	0.592	0.855	-2.207	0.137	-0.680
0.45	1.187	1.829	1.659	-1.369	0.290	-1.198	0.143	-0.586
0.50	1.391	2.041	1.126	-2.983	0.062	-0.407	0.312	0.227
0.55	1.708	2.358	0.356	-1.587	0.265	-0.026	0.426	0.371
0.60	2.312	2.600	0.040	-0.712	0.395	0.017	0.509	0.452
0.70	1.828	-5.174	0.394	-0.042	0.557	0.369	0.627	0.544
0.80	0.039	-1.365	0.562	0.215	0.658	0.468	0.707	0.599
0.90	0.383	-0.426	0.664	0.352	0.728	0.532	0.766	0.643
1.00	0.571	-0.040	0.734	0.442	0.779	0.581	0.811	0.682

3. Experiment

The experiment was carried out at the small water tank at Tsuyazaki (60 M $\times$ 1.5 M $\times$ 1.5 M), Research Institute for Applied Mechanics, Kyushu University. We layed two cylinders between the water tank walls at the right angles to the axes of the cylinders in the free surface, where the velocity component parallel to the axes of the cylinders vanishes and the fluid motion is expected to be two dimensional, and we measured the height of a single regular wave train travelling away from the cylinders which was generated by giving to the cylinders the forced motion with a period $2\pi/\omega$.

In Fig. 2 are shown the forced heaving apparatus used and the position of a wave height meter at the water tank which is of ultra-sonic type and used for the measurement of the wave height. In Fig. 3 are illustrated the details of this apparatus and the dimension of the cylinders which are made of wood.

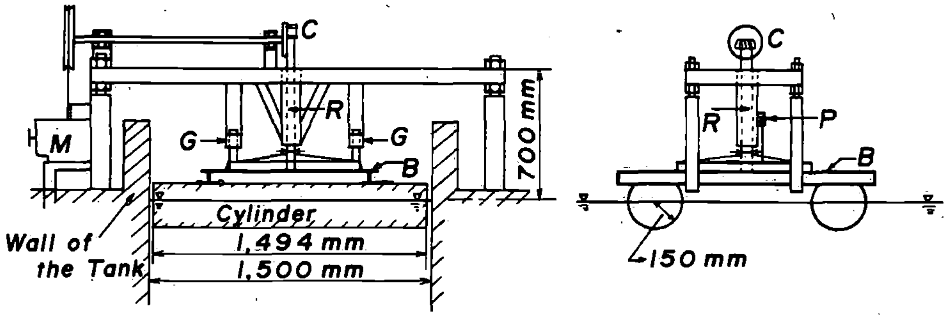

Fig. 2 Forced heaving apparatus.

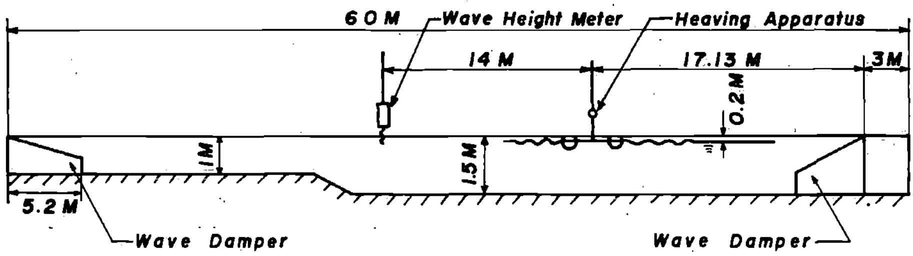

Fig. 3 Water tank and arrangement of experimental apparatus

The forced heaving apparatus is driven by a AC motor M through a crank C as shown in Fig. 2. A guide equipment G is installed to the apparatus and a beam B is sufficiently stiffened so that we may make as small as possible the vibration of a driving rod R or a beam B which is likely to disturb the wave form generated by the heaving motion of the cylinders. The heaving displacement of this apparatus is measrued by a potentio meter P and we confirmed that it made almost perfect sinusoidal motion.

About the diffraction wave from the ends of the water tank, Tasai[3] investigated when he carried out his experiment in this same tank and we confirmed his result by making the similler experiment again. That is, the amplitude of diffracted waves from the wave damper placed at the ends of this tank is a few or 10 percents of that of incident wave and it may be said that the wave system propagates at 1/2 of the phase velocity (group velocity) corresponding to the heaving period. In our experiment we adopted the records of the wave height meter as a data when all the water surface at the right side of the apparatus was filled with the waves.

The radius a of the cylinders is 150 mm and the amplitude of their forced heaving motion is selected to be $a/10$, that is 15 mm. If this amplitude is too small, the accuracy of the measurement of the wave height or pressure on the body surface decreases. On the other hand the larger the amplitude, the larger the influence of nonlinearity. Then this value was carefully adopted after we examined Tasai's results for one cylinder and made some measurements with several kinds of the amplitude.

Since the measured amplitude of motion by the potentio meter was a little changed due to its period (a shorter period made smaller the amplitude about 0.5 mm) we measured the amplitude every time we changed the heaving period T_w $(=2\pi/\omega)$. As shown later only a little variation of T_w some times results in a sudden and large change of the wave height. Accordingly we made the experiment varying T_w by as small step as possible and in the range of T_w where a sudden change of the wave heights occured we endevoured to find out a period at which the wave height reached to the maximum or minimum by continuosly varying the number of revolution of the motor.

Photo 1, 2 are some examples of the records of the minimum and maximum wave height.

The experiment were carried out for four cases $2p/a=3.0$, 4.0, 5.0 and 6.0 ($2p$ is the distance between the centers of two cylinders). The results are shown in Fig. 4, 5, 6, 7 in the form of $\overline{A}$ (wave amplitude/motion amplitude)$\sim Ka$ $(=\omega^2 \cdot a/g)$. These figures show that the wave height becomes almost zero (not perfectly zero because there is a little space between the ends of the cylinders and the tank wall whether we like it or not and consequently a small three dimensional wave is generated around this space) at some Ka which depends upon the value of $2p/a$. When Ka deviates a little to smaller side from Ka of the minimum wave height, the wave height become maximum. The difference between both values of Ka is, for example, only 0.15 in case of $2p/a=4$, which corresponds to 0.13 sec in T_w. Especially for the case of $2p/a=3.0$ this difference

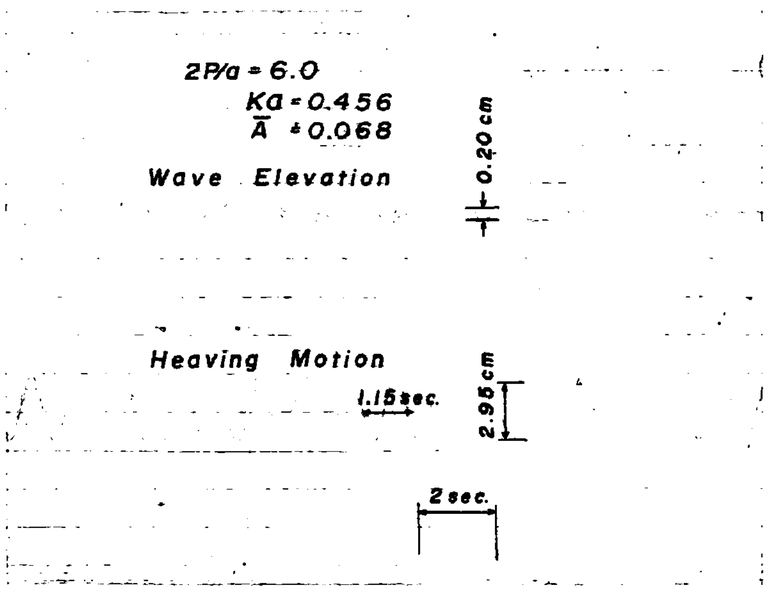

Photo. 1

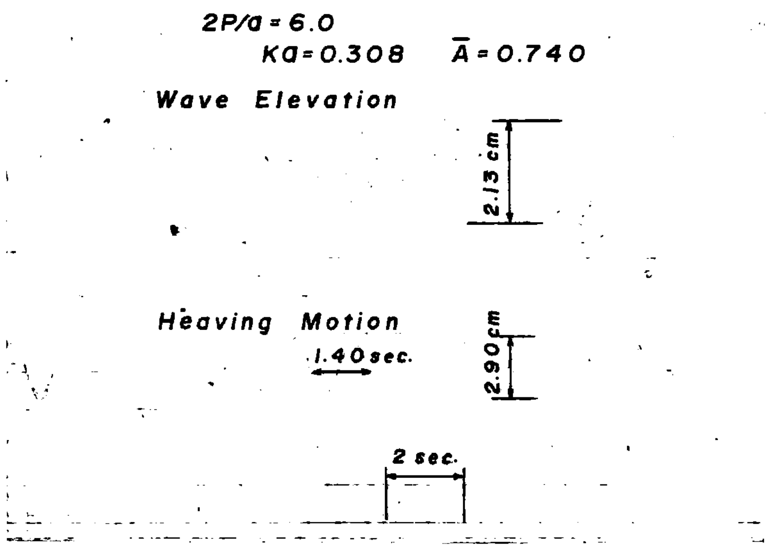

Photo. 2

is too small and we can not change ω by smaller step.

The noteworthy fact observed in our experiment is that the standing wave at the water surface between two cylinders becomes unusually high in the range of Ka where a abrupt change of travelling wave height occurs, that is, between the maximum and minimum wave height. And such a phenomenon was observed that there was almost no waves outside the cylinders while inside the cylinders the wave height was so large that the water got over the cylinder.

4. Discussion of results

In comparison of theoretical and measured $\bar{A}$ the agreement is very good in the frequency $Ka=0\sim1.2$ as shown in Fig. 4, 5, 6 and 7. We can find from these figures that maximum $\bar{A}$ becomes unusually large for small distance-radius ratio $2P/a$. There seems to be a deviation between theoretical calculation and measurements in the range of the frequency from Ka of maximum wave amplitude to Ka of minimum (zero) amplitude. It is perhaps due to the fact that the wave is so high in this range, but here is almost no measured value because the forced heaving apparatus used was not complete, then in future we should make sure of this results.

We tried to calculate the amplitude of the wave progressing to the left ($-x$ direction) from the cylinder B as shown in Fig. 9. We can immediately find from this figure that this amplitude reaches to its peak when Ka is in the range between maximum and minimum $\bar{A}_1$ and moreover its peak value is unusually

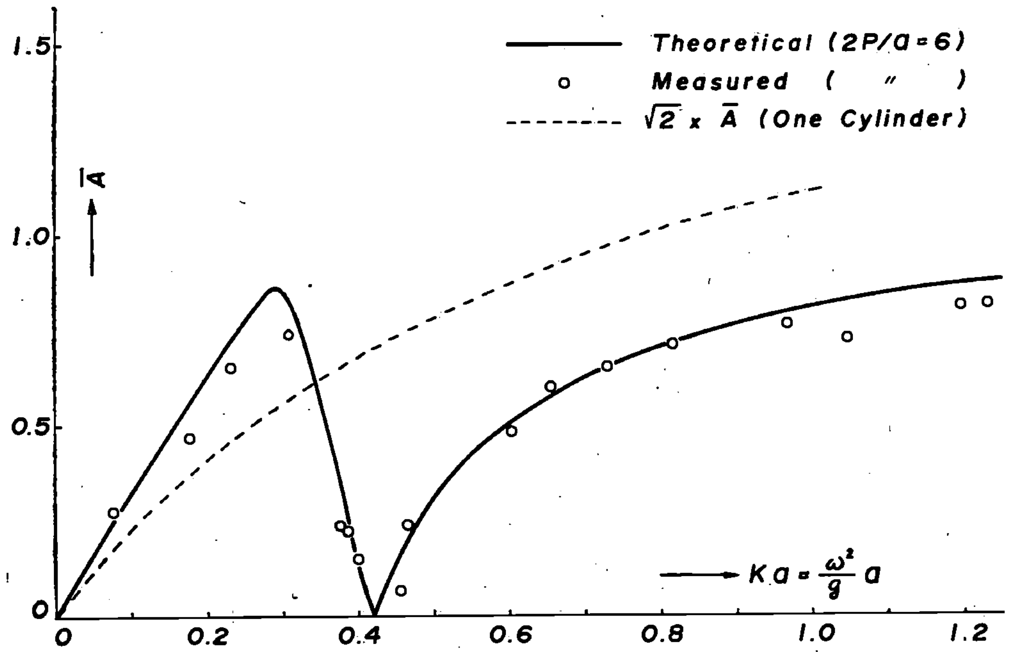

Fig. 4 The amplitude ratio (wave amplitude/heaving amplitude)

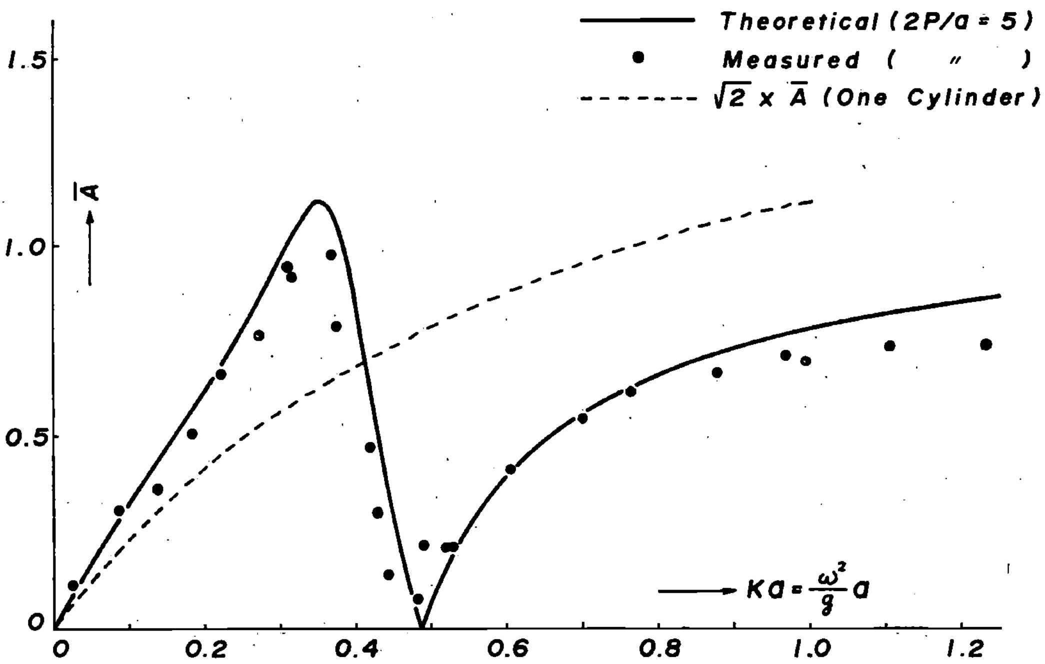

Fig. 5 The amplitude ratio (wave amplitude/heaving amplitude)

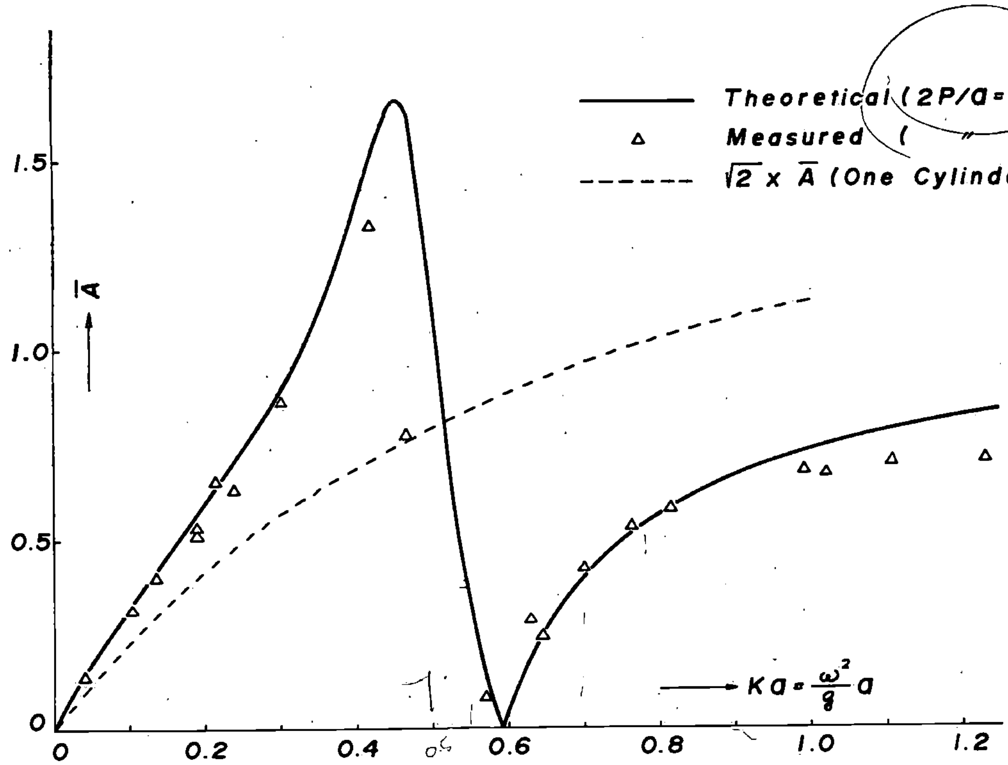

Fig. 6 The amplitude ratio (wave amplitude/heaving amplitude)

large, especially for small $2P/a$, compared with the wave amplitude which one cylinder produces without interference with another cylinder.

At the water surface between two cylinders there is no progressing wave but standing wave. If we neglect the stationary wave near around each of the cylinders, the amplitude of the standing wave between the cylinder is considered to be twice of the amplitude of the waves progressing to the left from the cylinder B because the wave with the same amplitude and the same phase is progressing to the right (x direction) from the cylinder A. For example, the amplitude of this standing wave for $2p/a=3.0$ amounts to $16 \times$ heaving amplitude. Since in such a case, as stated in Section 3 the water got over the cylinder, we could not continue the measurement.

As Ka tends to zero, the amplitude ratio $\overline{A}$ tends to $4Ka$ assuming $2p/a$ to be large enough to be able to neglect $(a/2p)^2$. It may be shown as follows. We know from Ursell's solution for one cylinder

$$\frac{\pi\omega}{gl}\varphi_B{}^0 \sim 2Ka(\varphi_S + \sum P_m f_m) \tag{29}$$

Then we can calculate the velocity V of a fluid near the cylinder A due to this

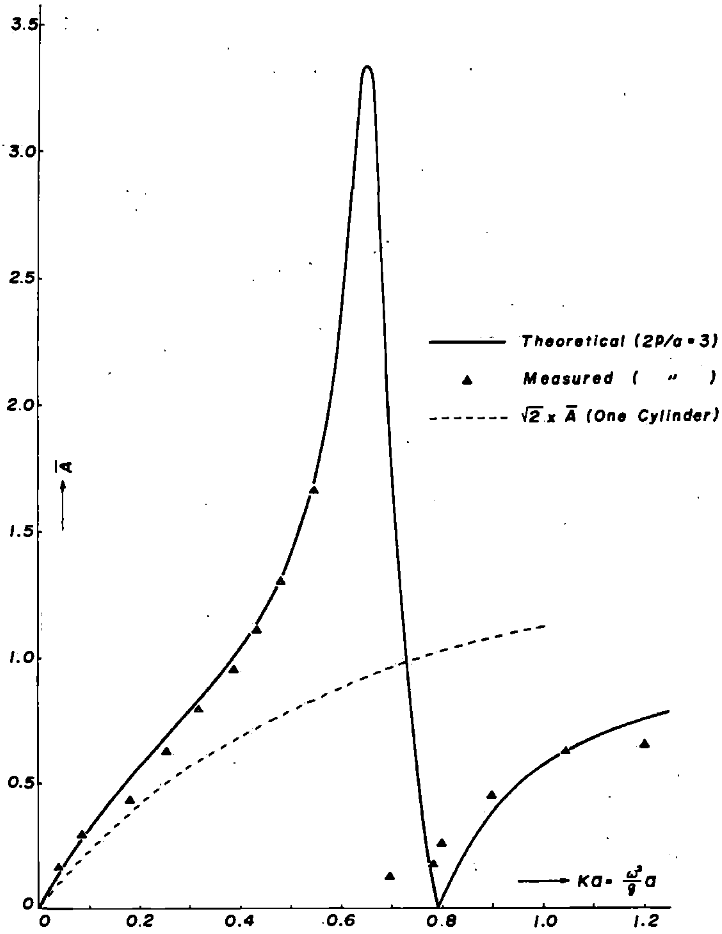

Fig. 7 The amplitude ratio (wave amplitude/heaving amplitude)

velocity potential. That is

$$\frac{\pi\omega}{gl}V \sim O(\overline{Ka}^2) + O\left(\frac{Ka}{(2P/a)^2}\right) \tag{30}$$

Accordingly at infinity the velocity potential ϕ is given by

$$\frac{\pi\omega}{gl}\varphi\bigg|_{n=\pm\infty} \sim \frac{\pi\omega}{gl}(\varphi_A{}^0 + \varphi_B{}^0) + O(\overline{Ka}^2) + O\left(\frac{Ka}{(2P/a^2)}\right) \tag{31}$$

Let $2p/a$ tends to infinity we obtain

$$\overline{A} \sim 4Ka \tag{32}$$

Added mass coefficient, due to a component of a fluid dynamic force upon one of the two cylinders which is in phase with the displacement of the cylinders, is given in Fig. 8 with that for one cylinder. Added mass coefficient for two cylinders is of the same order as for one cylinder in comparatively small Ka, but at some Ka dependent upon $2p/a$ the former begins to decrease and takes a negative value. And it gets to a minimum, then it begins to increase and seems to come back again to the level of one cylinder. Especially for small $2p/a$ the degree of the decrease is very steep and there can be two or more values of free heaving period of the two cylinders for $2p/a=3.0$ neglecting damping force.

According to Yoshiki and others[7], added mass coefficient of two cylinders when Ka tends to infinity ($\phi=0$ is the condition on the free surface) is $1+\frac{a^2}{2P^2}$, which is marked in Fig. 8. In our calculation also such a tendency as the added mass coefficient becomes over that of one cylinder is found in the case

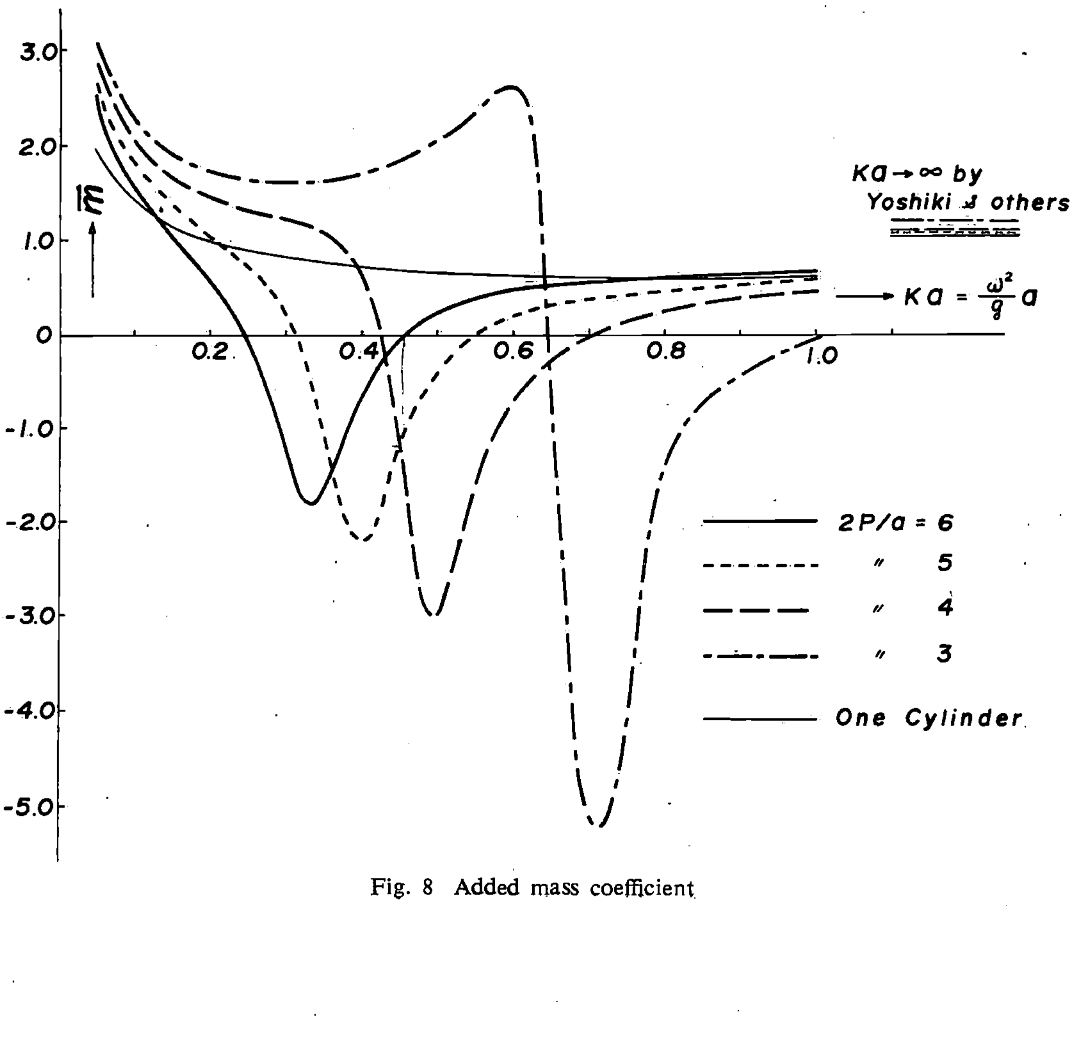

Fig. 8 Added mass coefficient

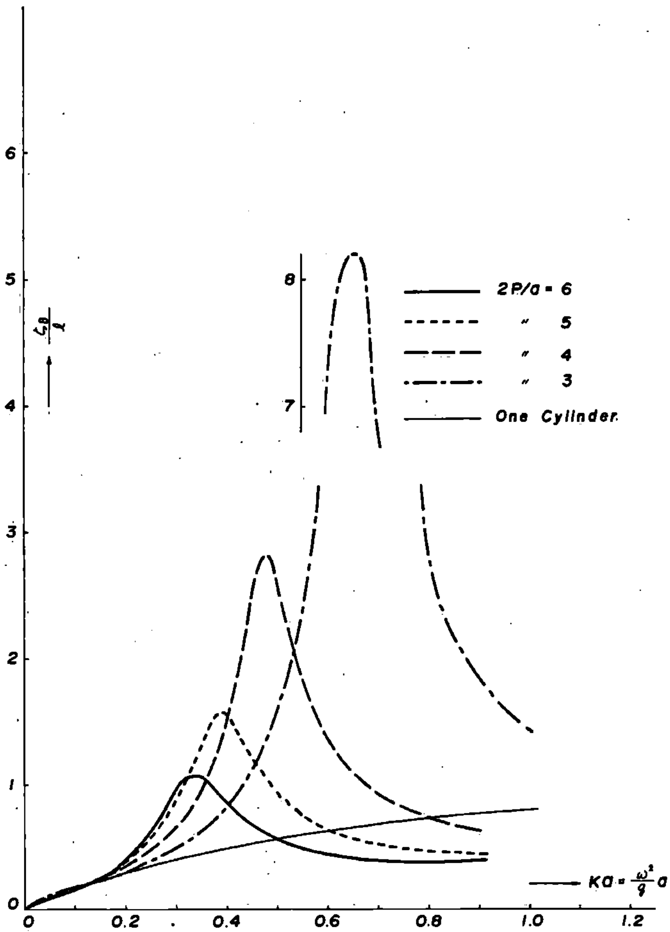

Fig. 9 Wave amplitude from cylinder B toward cylinder A.

of $2p/a=6.0$. It is of course that negative added mass of the cylinder A or B is given rise to as an exciting force by the velocity potential, especially progressing wave potential, due to another cylinder B or A. Ka value at which the added mass becomes a minimum with a negative value coincides with the one at which the amplitude of progressing wave to the right (to the cylinder A) from the cylinder B (or to B from A) has the largest value shown in Fig. 9. And also this Ka coincides with Ka at which $\sqrt{2}\cdot\overline{A}_0$, where $\overline{A}_0$ is the amplitude ratio of one cylinder when there is no interference between two cylinders, is equal to $\overline{A}$ for the two cylinders as shown Fig. 4, 5, 6 and 7. Where $\sqrt{2}\cdot\overline{A}_0$ corresponds to the sum of damping forces acting upon the two cylinders when they oscillate without interference —— it is an imaginary case —— because the amplitude of the damping force for this case is twice of that for one cylinder and $\overline{A}^2$ is proportional to the amplitude. The forementioned fact means that all of the fluid dynamic force acting upon one cylinder as an exciting force due to the velocity potential of another cylinder contributes to the component in phase with the displacement that is the added mass of the former cylinder, when the exciting force is the largest.

We can calculate the behaviour of the added mass when the distance between the cylinder is sufficiently large and Ka tends to zero by using Haskind-Newman relation.[8]

Since $2p/a$ is large, the velocity potential $\phi_B{}^0+\phi_B$ which comes from the cylinder B to A constitutes of only the progressing wave train. The fluid dynamic force upon the cylinder A is interpreted as the sum of the forces due to $\phi_A{}^0$, the incident wave $\phi_B{}^0+\phi_B$ and its diffraction ϕ_A. Then the amplitude of heaving force F by the latter two velocity potentials can be obtained as follows by Haskind-Newman relation

$$F=\zeta_b\frac{\rho g^2}{\omega^2}\overline{A}_0 \tag{33}$$

where ζ_b is the wave amplitude of $\phi_B{}^0+\phi_B$ and $\overline{A}_0$ is the amplitude ratio of one cylinder.

We put the amplitudes of the forces out of phase and in phase with the displacement of the cylinder respectively as

$$\zeta_b\frac{\rho g^2}{\omega^2}\overline{A}_0\sin\delta \tag{34}$$

$$\zeta_b\frac{\rho g^2}{\omega^2}\overline{A}_0\cos\delta \tag{35}$$

After all the added mass coeffcient of the cylinder A due to $\phi_B{}^0+\phi_B$ and ϕ_A is given by

$$\overline{m}_0=\frac{\frac{\zeta_b}{l}\overline{A}_0\cos\delta}{\frac{\pi}{2}(Ka)^2} \tag{36}$$

Since the sum of the damping forces due to $\phi_B{}^0+\phi_B$, ϕ_A and $\phi_A{}^0$ is equal to half of the damping force of two cylinders, we get

$$\frac{\overline{A}^2}{2}=\frac{\zeta_b}{l}\overline{A}_0\sin\delta+\overline{A}^2{}_0 \tag{37}$$

$$\sin^2\delta=\frac{\frac{\overline{A}^2}{2}-\overline{A}^2{}_0}{\left(\frac{\zeta_b}{l}\right)\overline{A}_0} \tag{38}$$

In the calculation of $\overline{A}$ when $Ka\to 0$ we know $\left(\frac{\zeta_b}{l}\right)$, $\overline{A}_0$ tends to $2Ka$ and $\overline{A}$ tends to $4Ka$ when $Ka \to 0$, and therefore $\sin\delta$ tends to 1. Therefore $\overline{m}_0$ becomes zero for $Ka \to 0$, and the added mass coeffcient tends to that due to only ϕ_A^0, that is the value for one cylinder

$$\frac{2}{\pi}\left(\log\frac{1}{Ka} - 0.46\right) \tag{39}$$

Acknowledgement

I wish to express my gratitude to Prof. Fukuzo Tasai, Kyushu University, for his invaluable and constant guidance in the course of this work. Also I am indebted to Mr. Masahiro Kaname, Lecturer of Kitakyushu Technical College, for his cooperation in the experimemt and the members of our laboratory Mr. K. Watanabe, Mr. M. Yasunaga and Miss Y. Okazaki for their generous assistance.

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(Received October, 21 1969)

Appendix

A-1. Solution of one cylinder

According to Ursell ψ_A^0 or ψ_B^0 can be derived as follows. ψ_A^0 is expressed by

$$\frac{\pi\omega}{gl}\psi_A^0 = \frac{\zeta_0}{l}\left[\psi_S(Kr_A, \theta_A) + \sum_{m=1}^{\infty} r_m f_m(Ka, r_A/a, \theta_A)\right],$$

Left cylinder. $x_2 - P$

then the boundary condition on the cylinder A is given by

$$i\pi Ka\sin\theta = \frac{\zeta_0}{l}\left[\psi_S(Ka,\ \theta) + \sum_{m=1}^{\infty} r_m \psi_m(Ka,\ 1,\theta)\right]$$

If we take a finite number of this series, we can determine the coefficients ζ_0/l, r_m by least square method. The wave elevation at $x\to-\infty$ is

$$Re\left[-\zeta_0 e^{-i(K\overline{x+P}+\omega t)}\right]$$

positive down

A-2. Evaluation of integral

We put

$$I=\int_0^{\infty}\frac{t\sin(Kyt)+\cos(Kyt)}{1+t^2}e^{-Kt|x|}dt$$

$$J=\int_0^{\infty}\frac{\sin(Kyt)-t\cos(Kyt)}{1+t^2}e^{-Kt|x|}dt$$

then

$$iI+J=-\int_0^{\infty}\frac{e^{-K(|x|+iy)t}}{t+i}dt \qquad \text{(A-2, 1)}$$

If $K\sqrt{x^2+y^2}=Kr$ is comparatively small, we can use the following series instead of the equation (A-2, 1), which is derived from a well known expansion of the exponential integral.

$$iI+J=(A+iB)e^{-(Ky-iK|x|)}$$

$$A=\log Kr+\gamma+\sum_{m=1}^{\infty}\frac{(Kr)^n\cos n\theta}{n\cdot n!}$$

$$B=-\theta+\pi-\sum_{m=1}^{\infty}\frac{(Kr)^n\sin n\theta}{n\cdot n!}$$

(converge for all Kr)

where $\theta=\tan^{-1}|x|/y$ and γ is Euler's constant $(=0.5772\cdots\cdots)$.

If $K\sqrt{x^2+y^2}$ is large, the following asymptotic expansion is effective.

$$iI+J\sim\left[\frac{e^{i\theta}}{Kr}+\frac{e^{2i\theta}}{(Kr)^2}+\frac{2!\,e^{3i\theta}}{(Kr)^3}+\frac{3!\,e^{4i\theta}}{(Kr)^4}+\cdots\cdots\right]$$