CRANFIELD
INSTITUTE OF TECHNOLOGY

THE PREDICTION OF EMPTY WEIGHT RATIO AND CRUISE PERFORMANCE
OF VERY LARGE SUBSONIC JET TRANSPORT AIRCRAFT

by

D. Howe

CRANFIELD INSTITUTE OF TECHNOLOGY

College of Aeronautics

Department of Aircraft Design

THE PREDICTION OF EMPTY WEIGHT RATIO AND CRUISE

PERFORMANCE OF VERY LARGE SUBSONIC JET TRANSPORT

AIRCRAFT

by   D. Howe

SUMMARY:

The empirical trend of the empty weight ratio of long
range subsonic jet transports is analysed in order to enable
the cruise performance of very large aircraft to be predicted.
Each of the main items which go to make up the empty weight
is dealt with individually in order that the effect of size
on each can be established. Consideration is given to the
effect of increased wing loading on take off runway
requirements.

It is concluded that it would be feasible to produce
an aircraft capable of carrying up to 1000 passengers over
the great majority of all transatlantic routes. Such an
aircraft would have an empty weight ratio of about 0.5 and
would weigh approximately twice as much as the largest subsonic
jet at present in existence.

CONTENTS

NOTATION

A   Aspect Ratio

b   Wing span   (ft)

C_1, C_2 Coefficients in wing weight relationship

D   Equivalent maximum fuselage diameter   (ft)

L   Fuselage overall length   (ft)

S   Wing area   (sq ft)

W   Gross, or take off weight   (lbs)

W_e Operating empty weight   (lbs)

W_f Fuselage structure weight   (lbs)

W_m $W_e - (W_s + W_p)$

W_p Weight of powerplant and associated structure   (lbs)

W_s Structure weight, excluding that associated with
        powerplant, $(W_f + W_w + W_t)$ (lbs)

W_t Tail unit structure weight   (lbs)

W_w Wing structure weight   (lbs)

λ Wing taper ratio, i.e. ratio of tip to root chords.

1.   INTRODUCTION

The past two decades have witnessed a spectacular increase in the size of transport aircraft. In the first instance the trend towards larger aircraft was a direct result of the need to fly non-stop over long intercontinental routes. More recently the dominant reason has been the improvement in operating economics which comes with large payload capacity. The Boeing B747 has already been in operation for long enough to demonstrate that this is indeed the case. Whilst these large aircraft introduce some ground handling and operational problems their ability to carry a large number of passengers on a single flight goes some way to alleviate air traffic congestion. It seems likely that these new large aircraft and derivatives of them will have sufficient capacity to cope with the intensity of traffic anticipated for the first half of the next decade. This is illustrated in Figure 1 which also indicates that aircraft with a seating capacity in excess of 600 may be desirable by 1990 and therefore suggests that there will be a definite requirement for even larger aircraft by that time. Whilst this potential situation could easily be changed by a technical breakthrough on the one hand or by political and economic considerations on the other, it is of interest to consider the technical prospects of very large subsonic transport aircraft. As an alternative to large capacity it might be that size could be economically useful in enabling aircraft to fly over much longer ranges than is currently the case. This would have the distinct advantage of reducing total journey time in those instances where the traffic intensity justified a long non stop flight.

The effect of scale on the economies of large aircraft has been investigated by Prof. Keith-Lucas (Ref.2). In order to evaluate the possible operational aspects of aircraft with capacities for up to 1000 passengers it was necessary to consider the effect of increased size on the design and weight of the aircraft. This was done in the context of the square-cube law and discussed the departures from it which are observed in practice. From the evidence examined it was concluded that the refinements in design associated with large aircraft

enabled the operating empty weight to be held as a constant
fraction of the gross weight. Using this assumption the
initial and operating costs were evaluated for aircraft operating
over a 3000 nautical mile stage length and having gross weights
of up to 1.6 million pounds. It was found that the trend
towards reduction of operating cost with size increase continued
to apply.

2.   ESTIMATION OF OPERATING EMPTY WEIGHT
      A major difficlty associated with the performance
prediction of aircraft which are outside existing experience
is the realistic evaluation of empty weight. Other important
factors such as powerplant and aerodynamic performance can be
reasonably evaluated within the framework of assumptions based
on a given state of the art. The real issue is whether improve-
ments and changes in design and operating characteristics can
continue to be employed to offset the fundamental effect of
the square-cube law, thereby maintaining the operating empty
weight as a more or less constant fraction of the gross weight.
In the first instance this question will be examined within
the context of the existing state of the art and subsequently
the effect of possible developments will be discussed. For
simplicity the investigation is limited solely to the case of
long range subsonic passenger aircraft.

      The structure weight of a transport aircraft accounts
for at least half of the empty weight and is the portion of it
which is most sensitive to changes of scale. The other parts
such as powerplant, systems and equipment are either much more
directly a function of gross weight or even virtually constant.

2.1   Wing structure weight
      The weight of the structure of a wing is influenced by
numerous parameters but the situation is considerably
simplified in the present instance by virtue of the restrictions
imposed upon the class of aircraft under consideration. Thus
such factors as design speed, normal acceleration, sweepback,
aerofoil section thickness, and taper ratio can be considered
as being approximately constant for a given design cruise Mach
number. The most important remaining variables are wing

loading and aspect ratio. Changes in the latter are not likely to be great.

Figure 1 of Ref. 2 presents the variation of wing loading with gross weight for a number of British and American jet transport aircraft. A mean line drawn through the presented data gives the relationship:-

$$\frac{W}{S} = 7W^{0.22} \qquad \qquad \ldots (1)$$

where W is the gross weight at take off (lbs) and S is the wing area (sq ft). This relationship is shown graphically in Figure 2. It is equivalent to:-

$$\frac{W}{S^{1.28}} = \text{constant} \qquad \qquad \ldots (1a)$$

rather than

$$\frac{W}{S^{1.5}} = \text{constant}$$

as is suggested by direct application of the square-cube law for similar geometry. Thus it can be concluded that designers have accepted the need to increase wing loading as gross weight has been increased to assist in the alleviation of increase of wing structure weight.

One relationship which may be used to express the variation of wing structure weight, W_w, is:-

$$W_w = C_1 \left[bS\left(\frac{1+2\lambda}{3+3\lambda}\right) \right]^{C_2} \qquad \ldots (2)$$

where C_1 and C_2 are coefficients determined by the class of aircraft

b is the wing span (ft)

λ is the ratio of the nominal tip to root chord.

This weight relationship, like the others used in this report is essentially an interpretation of empirical information. That is the wing weights of actual aircraft are compared as a function of the parameter enclosed within the square brackets which has been chosen on the basis of a simple appraisal of wing bending material weight. It does not include an allowance for many important factors but nevertheless it is adequate for the present limited purpose in view of the remarks made previously. If a typical value of 0.33 is taken for λ then Eq.(2) may be rewritten in the form:-

$$W_W = C_1 \left[0.416A^{\frac{1}{2}} S^{\frac{3}{2}} \right]^{C_2} \quad \ldots (3)$$

where A is the aspect ratio.

Examination of known wing weights for aircraft of the type under consideration suggests that appropriate values for the coefficients are:-

$$C_1 = 0.42$$
$$C_2 = 0.94$$

whence:-

$$W_W = 0.18A^{0.47}S^{1.41} \quad \ldots (4)$$

A further simplification can be made by assuming a typical values of 7 for the aspect ratio:-

$$W_W = 0.45S^{1.41} \quad \ldots (4a)$$

This can be expressed in terms of the wing loading and the total weight in which case Eqs.(4) and (4a) become:-

$$W_W = 0.18A^{0.47}\left(\frac{W}{S}\right)^{-1.41} W^{1.41} \quad \ldots (5)$$

or for A = 7:-

$$W_W = 0.45\left(\frac{W}{S}\right)^{-1.41} W^{1.41} \quad \ldots (5a)$$

Further, if Eq.(1) is used to give the variation of wing loading with total weight then:-

$$W_W = 1.16A^{0.47} W^{1.1} \times 10^{-2} \quad \ldots (6)$$

or for A = 7:-

$$W_W = 2.8W^{1.1} \times 10^{-2} \quad \ldots (6a)$$

Thus clearly some increase of wing weight as a fraction of the total has been accepted in present designs as larger aircraft have been built, since aspect ratio has not been changed significantly. The increase in wing loading has not completely offset the effect of scale, nor has any structural design improvement which is implicit in the specification of C_1 and C_2. To eliminate scale effect on the structure weight of the wing Eq.(1) would have to be modified to:-

$$\frac{W}{S} = 7W^{0.29}$$

and Eq.(1a) would correspondingly become:-

$$\frac{W}{S^{1.41}} = \text{constant}$$

as is implied by Eq.(4).

The difference between this and the square-cube law relationship is the measure of the structural design improvements which have been achieved with increase in size.

2.2   Fuselage structure weight

It is known that one of the most important parameters which determines the fuselage structure weight is the gross surface area. Figure 2 of Ref. 2 compares the surface area per passenger with the total number of passengers which can be carried. It shows that there is little variation provided the passenger capacity exceeds about 100. For large aircraft the fuselage surface area is about 25 sq ft per passenger.

An alternative approach is given in Figures 3 and 4. The former of these shows the variation of the maximum passenger capacity with weight for transport aircraft in various nominal range groups. For the long range type the passenger capacity is seen to be approximately

$$PAX = 6.25W \times 10^{-4}$$

Figure 4 indicates the variation of the parameter (2LD) with weight, where L and D are the overall length (ft) and the equivalent maximum diameter (ft) of the fuselage, respectively. This parameter is somewhat easier to use than gross surface area but is obviously related closely to it. For the long range aircraft:-

$$2LD = 1.33W \times 10^{-2} \qquad \dots (7)$$

Combining this with the relationsip stated above gives a surface area of 26.5 sq ft per passenger if it is assumed that the gross surface area of the fuselage is 0.8 of that of the equivalent cylindrical surface defined by L and D.

Analysis of known fuselage structure weight, W_f, in terms of the parameter (2LD) suggests:-

$$W_f = 0.45(2LD)^{1.32} \qquad \ldots (8)$$

for a long range subsonic transport.

When use is made of Eq.(7) this becomes:-

$$W_f = 1.5W^{1.32} \times 10^{-3} \qquad \ldots (9)$$

A very limited approach of this type can only give an approximate indication of trends. In particular the power of W in Eq.(9) is derived directly from Eq.(8). Whilst it appears to be reasonably correct in respect of surface area direct comparison with known fuselage structure weights using Eq.(9) suggests it may be somewhat high for fuselages of up to about 12 ft diameter. The dominance of pressure loading on wide body designs, however, can be expected to cause a tendency to increase the magnitude of this power towards the value of 1.5 suggested by the square-cube law. Overall therefore it is considered that 1.32 is a reasonable mean value. Fortunately many of the other parameters which affect fuselage weight are nearly constant for the narrowly defined class of aircraft under consideration.

The powerful effect of size on fuselage weight as is shown by Eq.(9), is of considerable significance. It may indicate that designers have not been as successful in countering the effects of the square-cube law on fuselages as they have with wings.

2.3   Other structural items

The remaining structural items are the tail unit, undercarriage and engine nacelles and pylons. For convenience those structural components which are directly related to the power plant are best treated with it.

Although it has been found necessary to increase the relative tail size with increase of gross weight it has been observed that it is possible to express the tail unit weight as a function of the wing weight. For the long range subsonic class of aircraft the tail unit weight, W_t, can be defined as:-

$$W_t = 2.5W_w^{0.755} \qquad \ldots (10)$$

The wing weight has already been expressed in terms of the gross weight in Eqs.(5) and (6) and using these relationships:-

$$W_t = 0.685A^{0.35}(\tfrac{W}{S})^{-1.07} W^{1.07} \qquad \text{... (11a)}$$

$$\text{or} \qquad W_t = 8.7A^{0.35} W^{0.83} \times 10^{-2} \qquad \text{... (11b)}$$

where the wing loading is as defined by Eq.(1).

There has been a general tendency for a decrease in the relative weight of transport aircraft undercarriages in recent years. However with increased size it has become necessary to resort to more complex undercarriage layouts and a reversal of the previous trend would appear to be inevitable. For the purpose of the present investigation therefore it will be assumed to be of adequate accuracy to use an undercarriage weight which is 0.035 of the gross weight.

2.4 Total structural weight

Excluding those structural units associated with the powerplant the total weight of the structure, W_s, is given by the appropriate sum of Eqs.(5) or (6), (9) (11), and the undercarriage allowance:-

$$W_s = 0.18A^{0.47}(\tfrac{W}{S})^{-1.41} W^{1.41} + 1.5W^{1.32} \times 10^{-3}$$

$$+ 0.685A^{0.35}(\tfrac{W}{S})^{-1.07} W^{1.07} + 0.035W \qquad \text{... (12a)}$$

or if the wing loading is taken from Eq.(1):-

$$W_s = 1.16A^{0.47}W^{1.1} \times 10^{-2} + 1.5W^{1.32} \times 10^{-3} + 8.7A^{0.35}W^{0.83}$$

$$\times 10^{-2} + 0.035W \qquad \text{... (12b)}$$

The ratio of $W_{s/W}$ for the special case of a constant wing loading of 120 lb sq ft and an aspect ratio of 7 is shown in Figure 5. Also shown is the corresponding ratio when use is made of Eq.(1) to define the wing loading. It is interesting to note that in this latter case a wing loading of 170 lb/sq ft is implied for a gross weight of 2 million pounds.

The third curve shown in Figure 5 represents the case of a constant wing loading of 200 lb/sq ft used in conjunction with an aspect ratio of 7 and an arbitrary reduction in W_s of 0.02W.

2.5   Powerplant weight

The weight of the powerplant is taken to include the installation and any structure directly associated with it. Known characteristics of long range jet transport aircraft suggest that the powerplant thus defined accounts for approximately 0.08W. There is a trend towards a reduction in this figure in the case of more recent designs of power plant and for the present purpose it will be assumed that the weight, W_p, is given by:-

$$W_p = 0.075W \qquad \dots (13)$$

A more accurate value could be achieved by introducing the thrust to weight ratio as an additional parameter. The effect is not likely to be large in view of the relatively small variations in this parameter which are observed in practice.

2.6   Systems, equipment, furnishing and miscellaneous items

Taken as a whole these items account for a substantial portion of the total weight. The individual contributions of the various separate items can vary considerably but for a given class of aircraft the total is much less variable. For this reason it is convenient to consider them together. An analysis of the differences between the operating empty weights and the sum of the weights of the structure and powerplant for long range transport types suggests that:-

$$W_m = 10,000 + 0.1W \qquad \dots (14)$$

where W_m (lbs) is defined as $\left[W_e - (W_s + W_p) \right]$

W_e (lbs) being the operating empty weight.

2.7   Operating empty weight

The operating empty weight, W_e, is by definition, given as the sum of Eqs.(12), (13) and (14).

Figure 6 shows the ratio of operating empty to gross weight for the three cases of structure weight given in Figure 5. The points indicated in this diagram are for actual subsonic jet transport aircraft.

The trends towards increase of empty weight ratio with increase in gross weight is apparent. Whilst the increase in wing loading discussed previously mitigates the effect to some extent it does not counteract it completely. The heaviest aircraft shown is the Boeing B-747B which has a wing loading of approximately 140 lb/sq ft at take off. This corresponds with Eq.(1) and the empty weight ratio lies close to the prediction curve although no detailed information for this aircraft, or any other type exceeding 350,000 lbs gross weight, was used in deriving Eqs.(12) to (14). It is thus reasonable to conclude that the curve appropriate to $W_{/S} = 7W^{0.22}$ is a good representation of current practice and forms an acceptable basis for extrapolation. As can be seen the empty weight ratio is of the order of 0.5 for aircraft which have gross weights approximately twice those in existence at the present time.

3.   FEASIBILITY OF VERY LARGE AIRCRAFT

The achievement of an empty weight ratio of the order of 0.5 for a very large aircraft implies a continuation of current design trends. One aspect of this is the refinement of structural design associated with increase of size which has been mentioned previously. It can be anticipated that this refinement will become a more difficult and expensive process as further progress is made since the margins available for improvement are bound to become less. On the other hand the solution may be achieved by means of a technical breakthrough such as might come by the extensive use of carbon fibre reinforced plastics in the structure.

A further requirement in the achievement of the predicted empty weight ratios is the acceptance of continually increasing wing loading. Up to the present time the effect of the accepted increase of wing loading has been offset in various ways. These include the use of more complex and improved high lift devices, higher installed thrust to weight ratio, and possibly most important of all, longer runways. There seems to be little further to be gained in the realm of high lift devices unless a completely new approach, such as boundary layer control, becomes practicable. Further increase of thrust to weight ratio implies some weight penalty, a relatively higher take off noise level, and possible poor thrust matching in cruising flight. Although runway lengths are being continually increased there must be some limit both for existing and new aerodromes. Figure 2 shows the approximate variation of required balanced field take off distance for two thrust to weight ratios as a function of wing loading. It can be seen that the additional length of runway required to cater for the increase of wing loading is not very great. As an alternative to increasing runway length the increase of thrust to weight necessary is not excessive either. A wing loading of 200 lb/sq ft would require approximately 16,700 ft of runway for take off if the installed thrust to weight ratio is 0.25.

Increase of aspect ratio is a further way by which the trend to longer runway lengths can be counteracted. However whilst this confers advantages in cruise conditions it does imply appreciable weight increase and the overall effect cannot be readily evaluated. It is perhaps not without significance that designers have chosen an aspect ratio of approximately 7 for many subsonic jet transport aircraft.

It may be concluded that the trend of increasing wing loading is likely to continue and it may be associated with some increase of thrust to weight ratio. On this basis it does appear to be feasible to consider the building of aircraft which are approximately twice as heavy as current types. As far as can be foreseen any substantial increase

of wing loading beyond that indicated by current trends would
result in a need for unduly long runways.

A more profitable approach would be to change the
overall configuration to enable structure weight to be
reduced. One possible way of doing this could be to return
to the use of flying boats. In this case the elimination of
the undercarriage could be expected to more than offset the
penalty associated with a planing bottom and outrigger floats
so that a saving of around 0.02W is feasible. In this
particular case an increase of wing loading may also be
acceptable so that it might well be possible to achieve an
empty weight ratio of 0.46 with large aircraft, as was assumed
in Ref.2.

4. OPERATING PERFORMANCE CHARACTERISTICS

The empty weight ratios deduced can be used in conjunction
with assumed propulsion and aerodynamic characteristics to
evaluate the likely range and payload performance of large
subsonic jet transport aircraft. Typical results are given
in Figure 7. For simplicity these have been based on a
constant cruise lift to drag ratio of 18 and a constant
powerplant specific fuel consumption appropriate to an engine
bypass ratio of 5 and cruise at $M = 0.85$. The fuel used during
take off and climb was assumed to be equivalent to 0.05W in
each case and the only reserves allowed for were 0.02W as a
landing contingency. The design points shown on the curves
indicate the gross weight which corresponds to a fuselage size
equivalent to the specified payload. Gross weights below the
design point imply volume limitation as far as passenger
payload is concerned, whilst weights above have surplus
capacity and hence excessive allowance for fuselage weight.

The important deductions which may be made from these
curves are:-
1) When very long range with small payload is required the
   optimum aircraft size if small even by present day
   standards.

2) An aircraft with a nominal capacity for 1000 passengers
   would weigh approximately 1.6 million pounds. If the
   trend of wing loading increase with size continued it
   will require runways of approximately 14000 ft length
   and have an equivalent still air range of about 6000
   statute miles. If the empty weight ratio could be
   reduced from 0.5 to 0.45, the range would be increased
   by some 1000 statute miles.

The significance of these results can be understood
by reference to Figure 8 (Ref.1). This shows the equivalent
range required for various classes of transatlantic
operations in terms of city pairs served. An equivalent
still air range of 6500 statute miles enables virtually
complete coverage of all routes including Europe to the west
coast of North America.

5.   CONCLUSIONS
It can be concluded from the foregoing that the
extrapolation of present design trends indicates the feasibility
of producing an. aircraft capable of carrying as many as 1000
passengers over transatlantic route systems. Ideally the
aircraft would weigh about twice that of the largest types
at present flying. The operating empty weight ratio is likely
to be somewhat greater than is currently the case. In spite
of the range reduction implied by this, some 95 per cent of
all transatlantic routes could be flown with full payload.

Undoubtedly the capital investment required to produce
such an aircraft would present problems but it would almost
certainly be justified in the context of traffic densities
predicted for the last two decades of the twentieth century.

Although the operating costs estimated in Ref.2 are
based on the assumption of a somewhat lower empty weight ratio
than predicted in Figure 5 the difference is not so great as
to invalidate the trends shown.

The results also show that at the present time the best
way of obtaining very long non stop range with a subsonic
jet transport is to carry small payloads in relatively small
aircraft. The optimum size has a takeoff weight of around
200,000 lbs.

REFERENCES

1. HANNAN   A new generation of jet transport - The
    MAGRUDER AND   L-1011 Tristar.
    BEACH   Lockhhed Horizons No.8, January 1970

2. KEITH-LUCAS, D. The prospects of 1000 passenger aircraft.
                     British Association for the Advancement
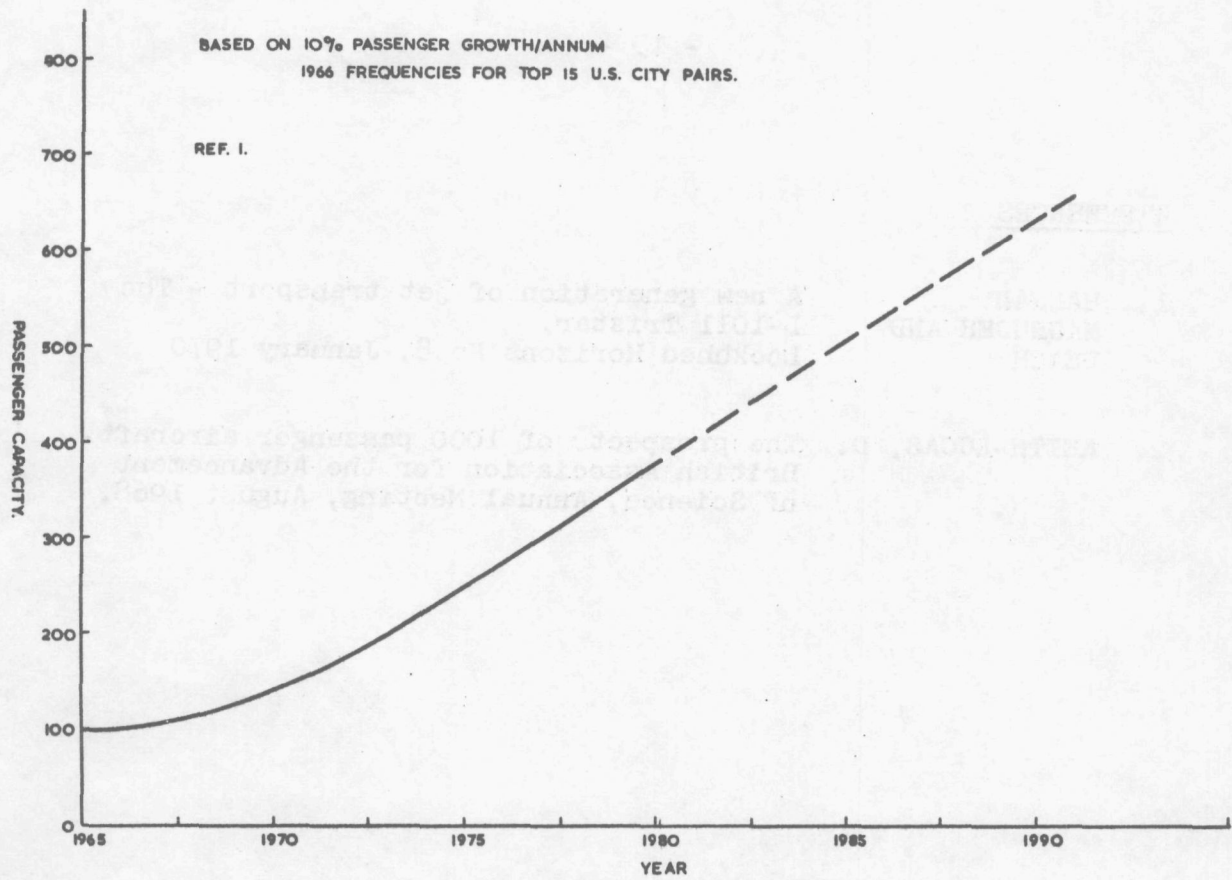   of Science, Annual Meeting, August 1968.

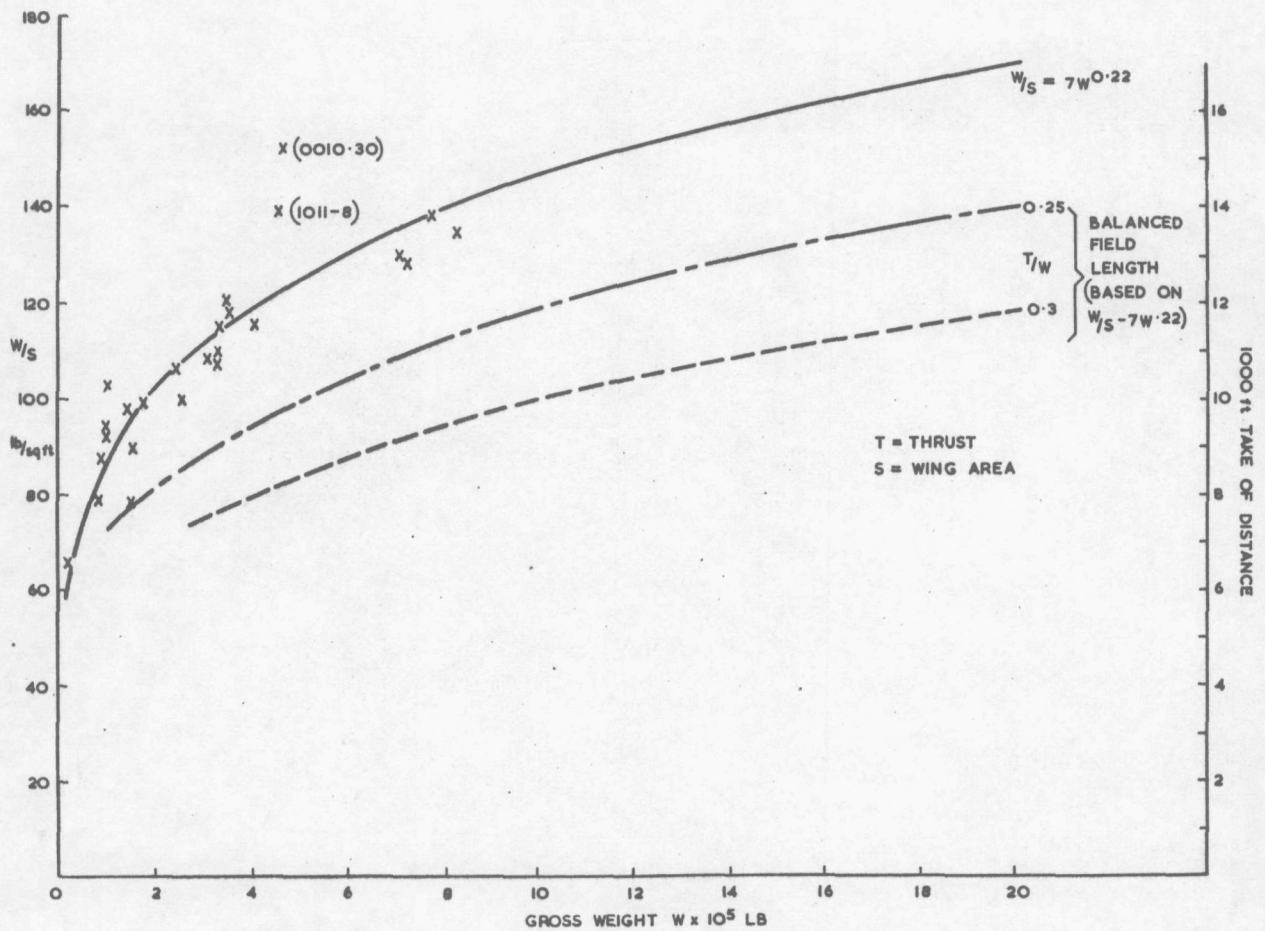FIG.I. GROWTH IN PASSENGER CAPACITY—CONSTANT FLIGHT FREQUENCY.

FIG.2. TREND OF WING LOADING CHANGE WITH SIZE INCREASE AND TAKE OFF LENGTH REQUIREMENT.

(PAX)

500

MAX PASSENGER CAPACITY

400

300

200

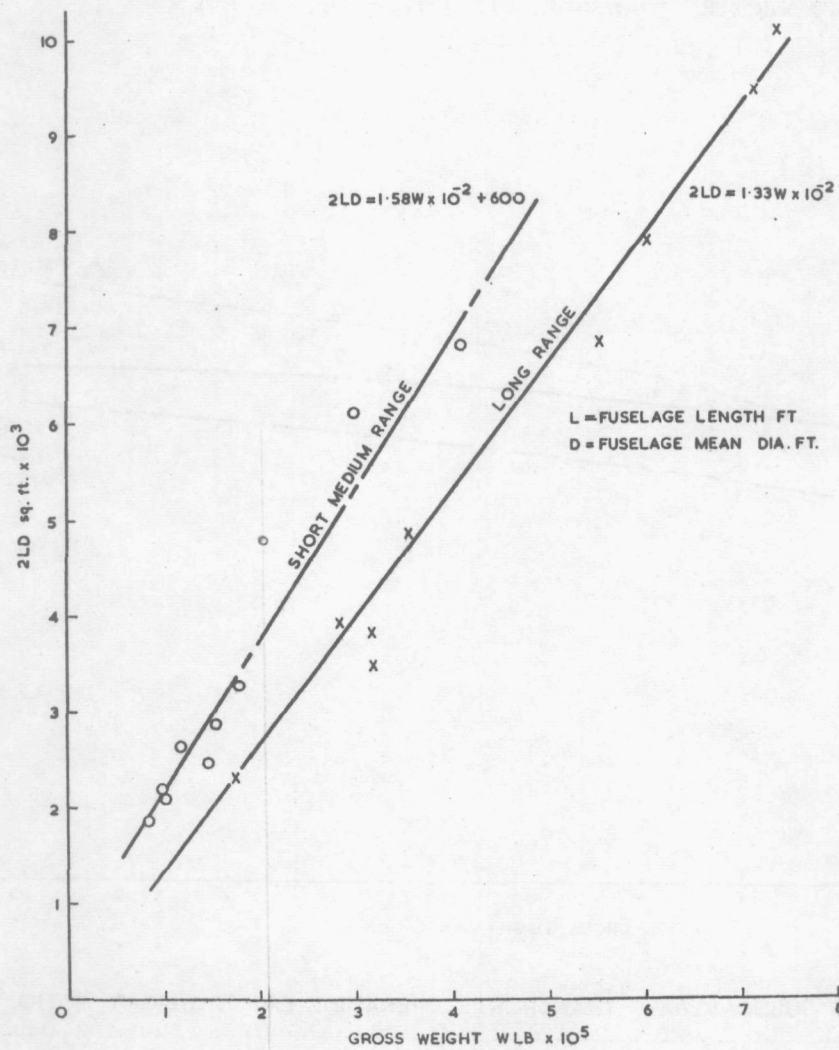100

PAX = 8·3 W x 10⁻⁴ + 20

PAX = 6·25 W x 10⁻⁴

SHORT-MEDIUM RANGE

LONG RANGE

0   1   2   3   4   5   6   7   8

GROSS WEIGHT W LB x 10⁵
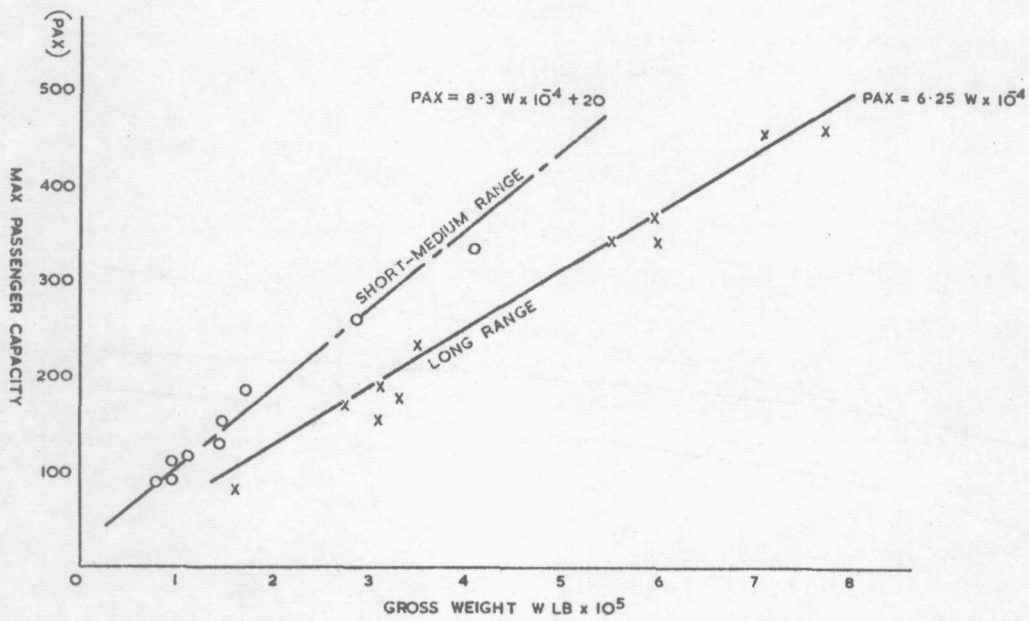
FIG.3.   VARIATION OF MAX. PASSENGER CAPACITY WITH SIZE.

10

9

8

7

6

2LD sq. ft. x 10³

5

4

3

2

1

2LD = 1·58W x 10⁻² + 600

2LD = 1·33W x 10⁻²

SHORT MEDIUM RANGE

LONG RANGE

L = FUSELAGE LENGTH FT
D = FUSELAGE MEAN DIA. FT.

0   1   2   3   4   5   6   7   8

GROSS WEIGHT W LB x 10⁵

FIG.4.   VARIATION OF EFFECTIVE FUSELAGE
SURFACE AREA WITH SIZE.
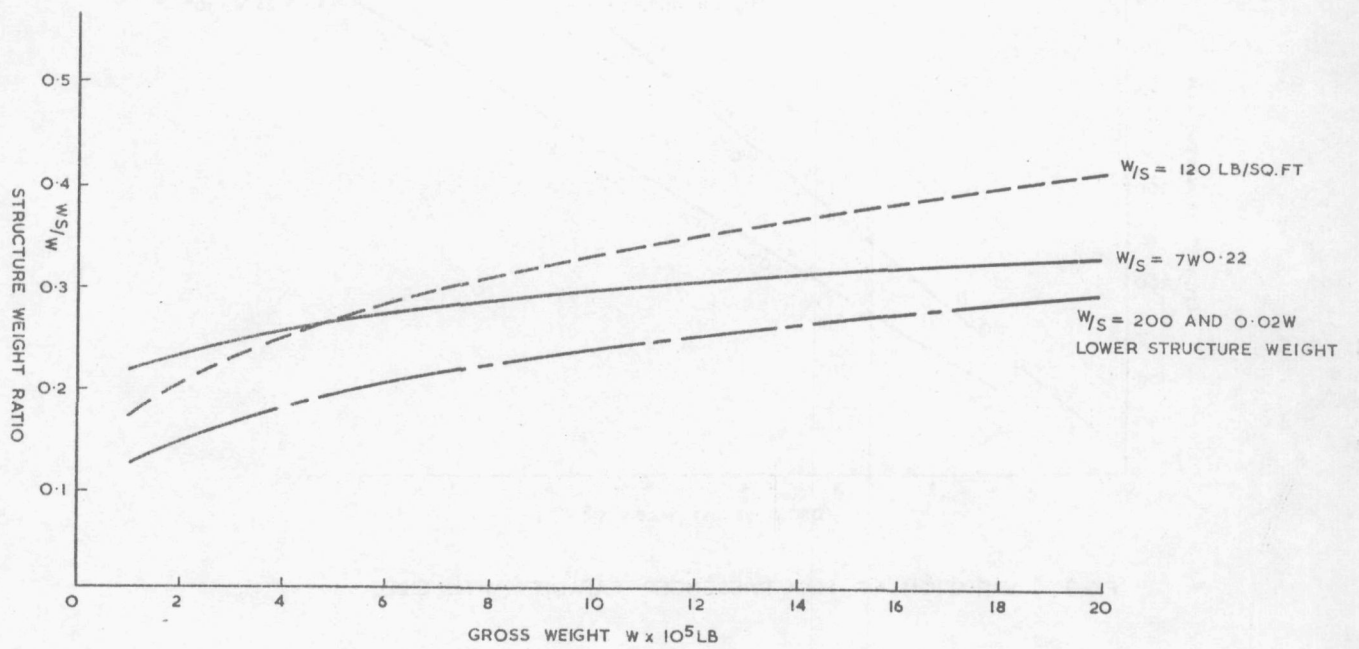
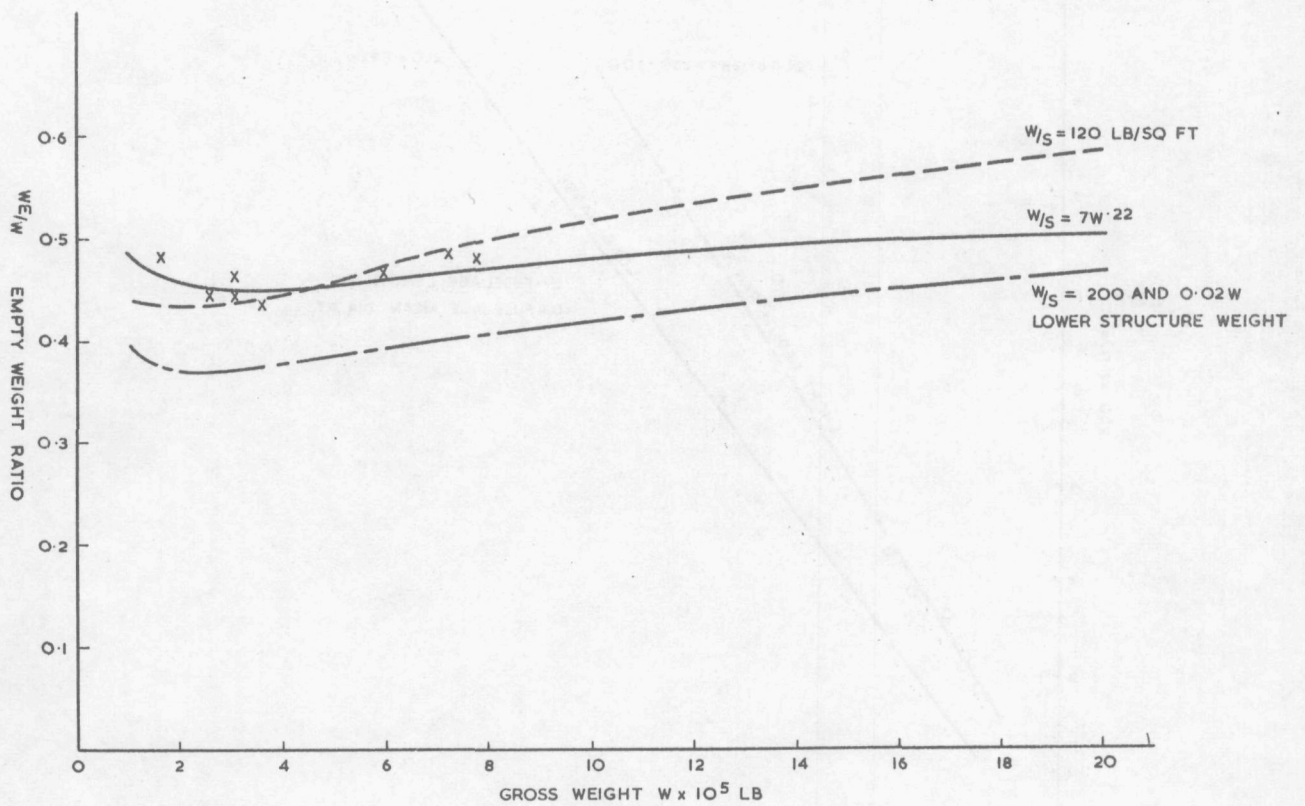FIG.5.   LONG RANGE SUBSONIC JET TRANSPORTS STRUCTURE WEIGHT RATIO.   A=7

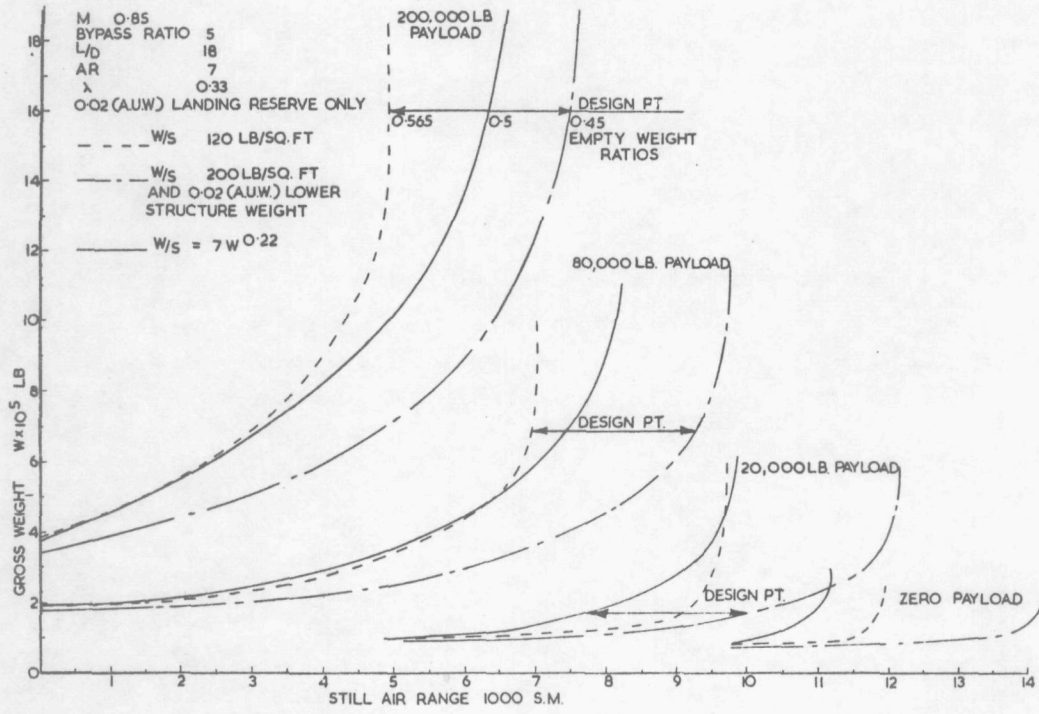FIG.6.   LONG RANGE SUBSONIC JET TRANSPORTS OPERATING EMPTY WEIGHT RATIO.   A=7

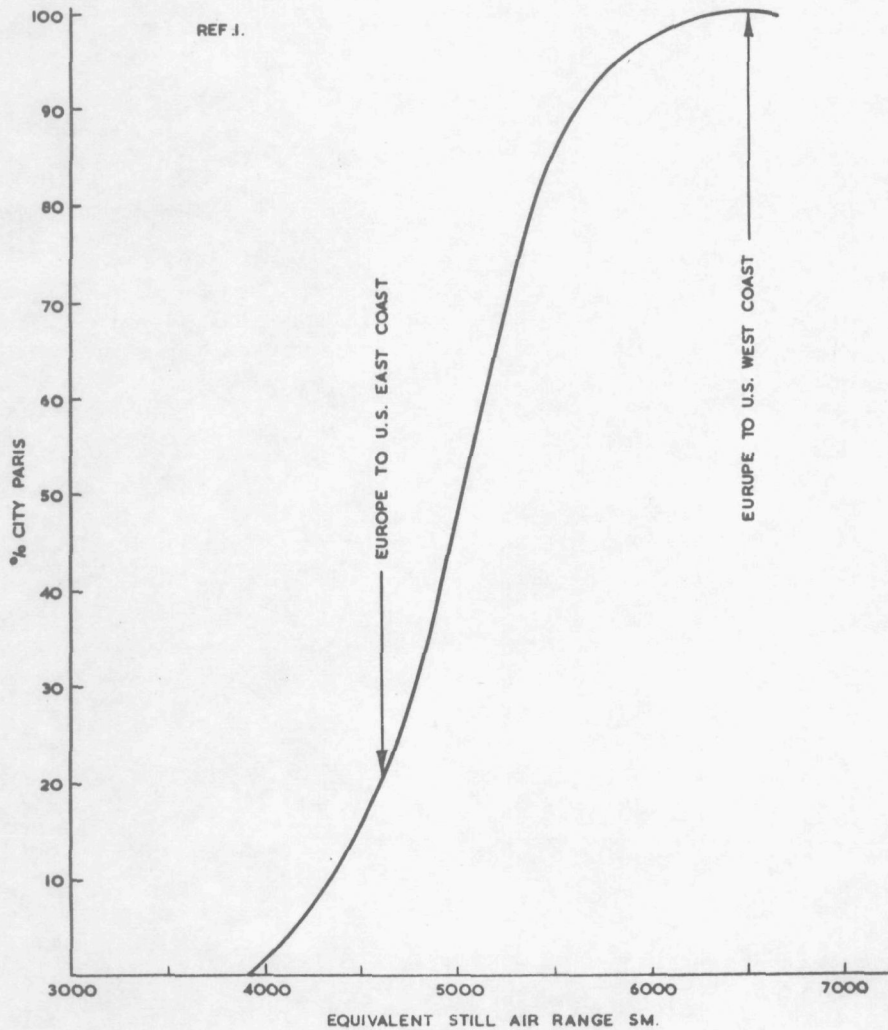FIG.7. LONG RANGE SUBSONIC AIRLINERS-VARIATION OF WEIGHT WITH PAYLOAD AND RANGE.

FIG.8. EQUIVALENT RANGE REQUIREMENTS
EUROPE TO NORTH AMERICA.