Endogenous Technological Change, Dynamic Discounting and Uncertainty in Climate Policy Models

Sustainable Energy Technology

M.Sc. Thesis

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Delft, 24-11-2016

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Executive Summary

Policy-makers are increasingly reliant on models for climate change policy. Due to growing environmental awareness, the long-term nature of climate change and the complexity of climate science, these models have painstakingly gotten more thorough and intricate over the years. Earlier climate policy models are just a fraction of what these models are today. However, as with any modeling effort, the models are just as good as its assumptions. Climate policy models are by no means an exception to this. This thesis treats several of its assumptions in order to provide a deeper understanding of climate policy models and ultimately be able to give better policy advice through the use of these.

Some of the most important assumptions with current climate policy models are those regarding *technological change*, *discount rates* and *uncertainty*. Technological change is usually regarded as an autonomous process which occurs outside the model, that is, exogenous. Current empirical evidence and economic theory points to the fact that technological change is an endogenous process which reacts to different factors such as prices and incentives. On the other hand, the debate on discount rates is one of the most discussed issues among economists. Climate policy models deal with long-term benefits and damages which are discounted to negligible values due to the power of compounding. Non-constant discounting has been proposed by economists as an alternative. Finally, uncertainty is present in climate models along every step. The sources of uncertainties are varied and span climate science, economics, politics, etc. As models tend to get bigger, the number of absolute uncertainties grows as well. Hence, proper treatment of uncertainty is crucial for the success of climate policy models. This thesis makes use of the DICE Integrated Assessment Model to tackle the three previously described issues. The programming languages GAMS and Python are used to model an extended version of DICE.

Endogenous technological change is modeled in this thesis through the inclusion of an R&D market and an energy sector. The energy sector is then divided into two energies: fossil fuels and backstop fuels (sustainable). Overall, adding endogenous technological change results in a greater understanding of the dynamics of an energy market on the whole economy. The biggest effect comes from adding backstop fuels to the model. This addition lowers total emissions and increases total welfare by 9%. However, one of the important insights is that backstop fuels are not enough to limit dangerous atmospheric temperature rise without the help of other policy measures such as command-and-control policies to limit the total amount of emissions.

In the original DICE model, the discount rate is kept constant throughout the whole timespan of the model. In this thesis, the discount rate is made non-constant by modeling it dynamically in time. The discount rate is modeled according to two formulations which link it to an economic and an environmental variable. The result is that the discount rate is decreasing in time with both formulations. This is in line with what many economists recommend for long-term environmental models. As damages and benefits are valued higher in the dynamic case in opposite to the constant case, more stringent emissions reductions in the short-term are recommended by the model which in turn increase the social cost of carbon.

Finally, an Exploratory Modeling and Analysis is done on the original DICE model to assess the effect of (deep) uncertainty on climate policy advice. Three uncertain parameters play a key role: the climate sensitivity, the elasticity of marginal utility and the exponent of the damage function. Because the exponent is the most sensitive parameter, it is only in cases of high valuation of climate damages when the 2°C target can be achieved. Likewise, the social cost of carbon only reaches conventional levels in less than 10% of the simulations. The analysis done in this thesis shows the importance of including uncertainty when using climate policy models for decision-making.

In conclusion, this thesis demonstrates that the three assumptions of technological change, discount rates and uncertainty are key in the results of the model and thus of primordial importance. It is recommended that future instances of climate policy models deal with these assumptions in a similar way: modeling technological change with R&D markets, linking the discount rate to an environmental/economic variable and treating uncertainty systematically with Exploratory Modeling and Analysis. Ultimately, the objective is to provide sound policy advice aided by better understood models.

Acknowledgements

First of all, I would like to thank the CONACYT for being my sponsor during my time at TU Delft. I am at a loss of words when trying to describe how grateful I am for getting a full scholarship to study my Master's degree. Just like climate change mitigation, the impacts of my academic preparation will have long-term benefits for Mexico. I am proud and blessed to be part of the upcoming generation of Mexicans whose future we hold in our hands.

Dr. Servaas Storm, I am extremely happy that I had the opportunity to do my Master thesis with you. You've supported me since the beginning and gave me an opportunity to do my thesis in a topic which I have no background on. That tells me how much you believed in me. I am grateful for your guidance and always pushing me forward with my thesis when I felt I was stuck. Thank you.

Dr. Jan Kwakkel, everytime I went into your office I came out learning something new. I am very grateful that you gave me an opportunity to work in your field which, even though it came quite late with the thesis, is extremely interesting and I hope that this work has helped you further develop it. Thank you for your patience and support.

Next, I would like to thank my whole family as there is nothing more important than it. Thanks for having my back throughout all these years and all my life. I will forever be blessed with having all of you by my side with huge amount of love. Another big one goes to all my friends here in Delft, I think this has been one exciting ride and without all of you this would mean nothing. Special mention to my roommates at THU for always being awesome, great support and sharing all those dinners together! I shalt never forget our time! Finally, a very important mention to my eternal source of inspiration, the universe. Without you I'd *probably* be nothing. Thank you for giving me this space in time to research about a pale blue dot!

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List of Abbreviations

AEEI	Autonomous Energy Efficiency Improvement
C	Carbon
CCS	Carbon capture and storage
CO_2	Carbon dioxide
COP21	21st Conference of Parties
CPI	Consumer Price Index
CTE	Carbon ton equivalent
DICE	Dynamic Integrated Climate-Economy
DICE2013x	Alternative DICE model
DICE-ED	DICE Endogenous & Discounting
EMA	Exploratory Modeling and Analysis
GAMS	General Algebraic Modeling System
GDP	Gross domestic product
GHG	Greenhouse gas
Gt	Gigaton
IAM	Integrated Assessment Model
IPCC	Intergovernmental Panel on Climate Change
kWh	Kilowatt-hour
LHS	Latin Hypercube Sampling
NOx	Nitrogen oxide
OECD	Organization for Economic Co-operation and Development
PRIM	Patient Rule Induction Method
R&D	Research and Development
RDM	Robust Decision Making
SCC	Social Cost of Carbon
TFP	Total factor productivity
USD	U.S. dollars

Knowledge is the only instrument of production that is not subject to diminishing returns.

John Clarke, economist.

Chapter 1

Introduction

Extinction is the rule. Survival is the exception.

Carl Sagan, astronomer.

Carl Sagan masterfully described Earth as a "Pale Blue Dot" in a picture from the satellite *Voyager 1* taken in 1990 and reproduced in Figure 1.1. This dot can be found halfway through the rightmost light stream. Here, in this very pixel, we can find everything that has ever happened in human history. And quite the history that has happened until now.

The very first humans who roamed this planet on a hunter-gatherer basis were using around 2 kWh per person per day [2]. Fire was eventually discovered and used for cooking and weapons; energy consumption rose to 6 kWh per person per day [2]. And as time moves on, humans were able to get more energy for their service. Early agricultural societies provided around 14 kWh and by 1400 AD the demand in medieval Europe was around 30 kWh per person per day [2]. In modern times, energy consumption calculated for the average person in the world is around 61 kWh per day with a maximum of around 233 kWh for people in the United States [3, 4]. What enabled this leap in energy use?

1.1 Context

Fossil fuels. Fossil fuels are the remnants of dead plants and organisms stored under the Earth's surface over millions of years. Fossil fuels contain energy which is released upon burning. As fossil fuels are mostly made from carbon and hydrogen, one of its byproducts while burning is carbon dioxide (CO_2). CO_2, when released in the atmosphere, acts as a greenhouse gas (GHG) which absorbs radiation and subsequently heats the Earth. This

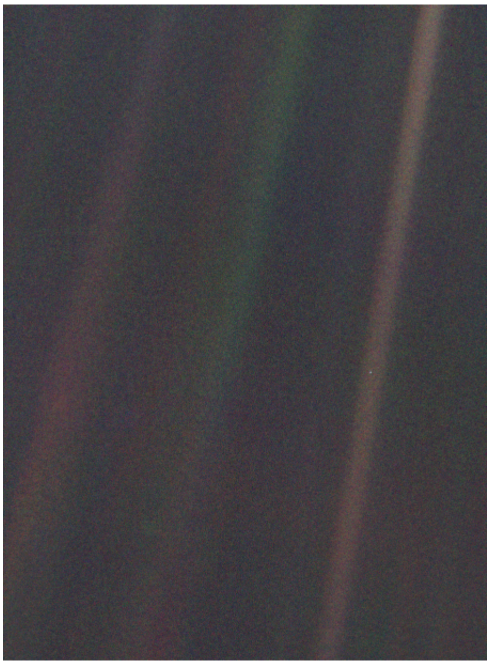

Figure 1.1: The Earth as a pale blue dot seen from the Voyager 1 in 1990 [1].

has profound impacts on the climate.

The discussion around the effect of GHGs on the climate is vast and extensive.[1] The Intergovernmental Panel on Climate Change (IPCC) has stated that human influence on the climate is clear. This influence can be seen primarily in the GHGs concentration in the atmosphere which has risen almost exponentially since 1750 [5]. Our understanding of the climate is limited to our current cognitive capabilities; however, year after year the effects of anthropogenic emissions on the climate are clearer. The link between concentrations of CO_2 and temperature change is visualized in Figure 1.2.

What is the effect of anthropogenic emissions of CO_2 and temperature change on the environment? Quite enough that it has warranted a new name for it: *the Anthropocene* [7]. The Anthropocene is a geological epoch starting from the 1800s where for the first time ever human influence on the environment and the climate system can no longer be denied. Our insatiable hunger for energy has modified the Earth's environment almost irreversibly. The climate system could take 50,000 years to recover from this human punch [8]! The consequences of rising temperatures are vast: negative impacts on crop

[1]Refer to [5] for a comprehensive review.

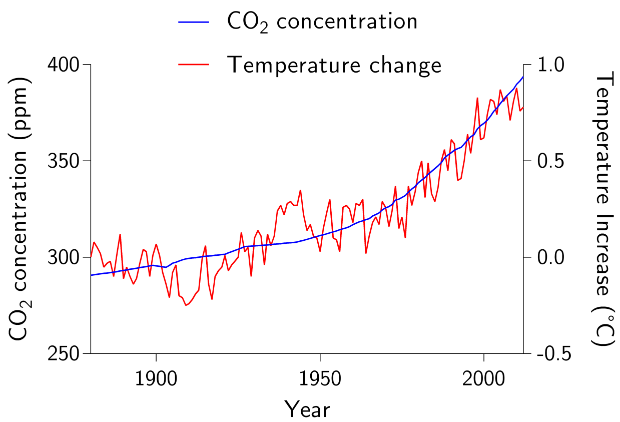

Figure 1.2: Relationship between CO_2 and temperature [6].

yields, species extinctions, more pronounced and extreme climate events, more intense and stronger hurricanes, sea level rise, urban displacement, biodiversity loss, ocean acidity and more [9]. Figure 1.3 shows the risk of increasing temperatures on five major Earth systems. An increase in the global mean temperature induces higher risks in every Earth system.

Due to climate change, the transition to a green economy is of the utmost importance. To enable this, the United Nations Framework Convention on Climate Change (UNFCCC) gathered together official representatives from 196 countries in Paris for the 21st Conference of Parties (COP21) during the month of December 2015. During this COP21, nations signed an agreement in which they promise to reduce their emissions in the coming years in order to limit global climatic change to no more than 2°C over pre-industrial temperatures [10].

The importance of climate change in the global policy discussion opened the way for new climate policy models to aid in the policy-making process. This thesis aims to improve the understanding of climate policy models in order have better tools which can aid the global policy-making process in stopping dangerous climatic change.

1.2 Scientific relevance

Climate change policy and modeling has thus been central and the object of much debate during the last decades. The literature has recognized three main problems with the

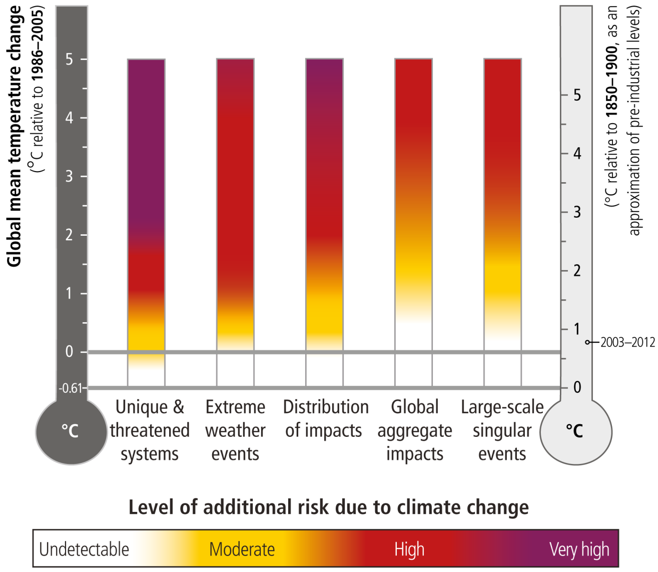

Figure 1.3: Impacts of climate change [9].

modeling of climate change: the choice of the discount rate, the uncertainties of climate change and the choice of technological change [11]. *Discount rates* are much debated in the literature [12], as the selection of the discount rate has an overwhelming influence on the modeling outcomes—and hence on decision-making. For example, a \$1 million benefit 200 years from now has a discounted (present) value of just \$1.33 with a discount rate of 7% but a discounted value of \$19053 with a discount rate of 2%. It is then easy to understand why some models which feature a high discount rate do not recommend climate change action [13]. It does not make sense when governments urge action on climate change but economic models say otherwise.

In particular, there are two main categories of reasons under which social discount rates[2] fall on, prescriptive and descriptive [14]. The *descriptive approach* states that the discount rate should equal current monetary interest rates on the belief that this is how society thinks about the future. William Nordhaus' work falls under this category. The other approach is the *prescriptive approach* which has fundamental ethical views as its base to choose the discount rate. The issue in this approach is to know which ethics to derive the discount rate from. Nicholas Stern's work is one of the most prominent

[2]The social discount rate is the discount rate applied to projects which are social in nature. Climate change mitigation is a prime example of this.

examples of this category. The discussion on these two is diverse in the literature, even with Seth Baum stating that all descriptive and prescriptive approaches are by nature both equally descriptive and prescriptive [15].

In William Nordhaus' seminal work on economic modeling of climate change, the Dynamic Integrated Climate-Economy (DICE) model, the optimal policy recommendation with a high social discount rate (3%) is inaction on climate change [13]. However, with a lower social discount rate (0.1%), Stern's work shows that immediate action on climate change is critical [16]. This poses a conundrum: what is the right social discount rate to use?

Alongside this, there are also ethical implications because most of the economic models apply discounted utilitarianism (DU) [17]. DU is used to maximize the weighted sum of generational utilities into the distant future, where the weight of utility of each next generation declines geometrically [18]. It is only concerned with the absolute utility, having as a backbone John Rawl's work of utilitarianism. It does not treat all generations equally, putting more importance on present generations. One way to deal with this is by using a social discount rate of zero as this would mean that every generation is valued equally [16] or by what is called sustainable discounted utilitarianism [19].

It could possibly be that the issue of discounting is something which will never be agreed upon by economists. In this thesis I do not presuppose to be the truth-bearer and solve the issue of discounting once and for all. I will explore different ways to treat the social discount rate which have not been applied to climate policy models in order to add to the scientific literature on the subject and hopefully bring light into this complex topic. Most climate policy models assume an exogenous social discount rate that is constant throughout the whole timespan of the model. This thesis will involve an exploration of dynamic social discount rates which are functions of either economic or environmental variables.

Besides this, technological change also plays a major role. As economies start decarbonizing, GHGs concentration will stop rising and energy efficiency will improve. One can only state that early technological change is beneficial for future generations. However, there is a big discussion on how technological change should be modeled [11]. It can be modeled as exogenous, semi-endogenous or completely endogenous. And because technological change affects future and present gains, there is then a nexus between technological change and the discount rate [20].

Many climate models work on the presumption of exogenous technological change. This is done by working with constant improvements in energy efficiency over time [11]. Another way to model technological change is by including a 'backstop technology' which is a carbon-free technology not currently developed and which costs decrease at an exogenous rate; this method is considered semi-endogenous and is used in Nordhaus' work [21]. And finally, there are several ways to model endogenous technological change: price-induced, directed technical change, learning-by-doing and by R&D investments [11]. It is the challenge of current modelers to include endogenous technological change in the models. In this thesis I will add endogenous technological change to a climate policy model in order to asses its impact on the results. This way I will be adding to the existing literature on the subject.

Additionally, there is a high degree of uncertainty when dealing with models with technological change [22]. In essence, there are three sources of uncertainty: uncertainty about future inventions, about usefulness of an infant technology and about the pace of technological progress towards market maturity [23]. This brings in *new growth theory* with knowledge spillovers and *Schumpeterian rent*.[3] Uncertainty about technological change is thus important to explicitly include in the modeling process.

And it is not only uncertainty about technological change, but there is a great deal of uncertainty surrounding climate science, economic indicators and the discount rate. And what separates these uncertainties are the nonlinearities exhibitied, the irreversibilities on the system and the very long time horizons [24].

There are many ways to tackle uncertainty in climate policy models: sensitivity analysis, expert assessment, model emulation and other variations [25]. In this thesis, a novel approach called *scenario discovery* with Exploratory Modeling and Analysis will be used to study the effect of uncertainty in climate policy models. Scenario discovery is a computer-assisted methodology in which a model is run over an ensemble of scenarios and analyzed with data-mining algorithms [26].

In summary, this thesis addresses three main themes in the climate policy modeling literature: discount rates, technological change and uncertainty. These topics are usually

[3]Economic rent earned between introduction and diffusion of an innovation.

treated separately in the literature. This thesis hopes to bring the three of them together in order to improve climate policy modeling.

1.3 Research question

The main research question proposed for this thesis is:

How sensitive are the climate policy conclusions from a standard climate policy model to the introduction of endogenous technological change, dynamic discounting and uncertainty?

1.3.1 Secondary Research Questions

The following questions will help structure the thesis:

- What is the most appropriate method to endogenize technological change?
- What is the effect of endogenous technological change on emissions abatement?
- What are the different ways of formulating a dynamic social discount rate?
- Are these formulations time-consistent?
- Is there a significant difference in policy advice between the different formulations?
- What are some of the key uncertainties in the model?
- To what extent and in which direction do the policy conclusions change with the introduction of (separately and in combination) endogenous technological change, dynamic discounting and uncertainty?

These questions will be answered throughout the thesis in order to assist in the resolution of the main research question.

1.4 Methodology

For the realization of this thesis, an Integrated Assessment Model (IAM) will be used, particularly, the DICE model developed by William Nordhaus [21]. The following subsections will elaborate further on this.

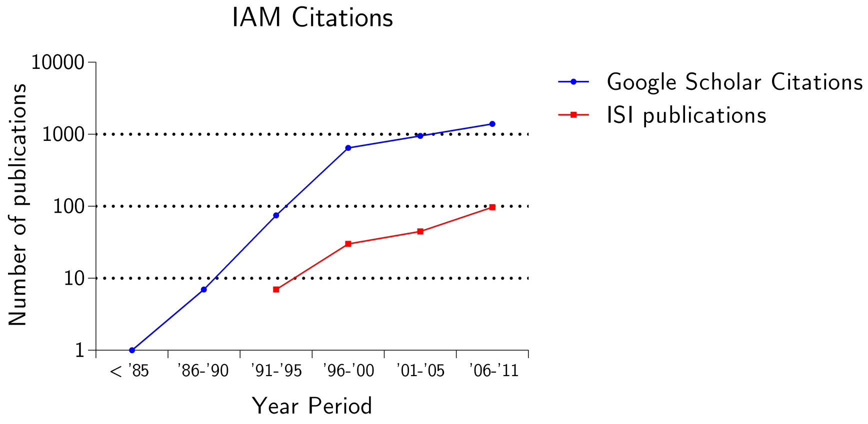

Figure 1.4: Time trend of IAMs publications. Adapted from [13].

1.4.1 Integrated Assessment Models

IAMs were initially developed in the 1970s as a way to deal with the complexities of environmental problems, which usually involve knowledge between two or more different domains (natural sciences, chemistry, economics, political science, etc). So therein lies the strength of IAMs, in which it is able to structure under a single framework the current knowledge of more than one academic domain.

The recent development of IAMs can be seen in Figure 1.4. The number of publications is growing with each year.

The necessity to use IAMs in order to study the climate problem is due to the nature of the problem itself. Climate change is a problem which spans several dimensions within society and can be characterized as a "wicked problem". A "wicked problem" is a problem which has several boundaries (climate system, policy, markets, society) and the solution to it transcends all of them, to the point that different stakeholders have different solutions [27]. For example, ecologists may view climate change as a threat to ecosystems while oil companies may think of it as a hazard to their business and politicians see climate change as a certain political stance.

This is where IAMs can be useful and relevant, as they can handle different domains within a single framework. Thus, the issue is simple: a problem that has several dimensions such as climate change can be worked out with a tool that includes several

Table 1.1: Example of prominent IAMs by type. Adapted from [28].

IAM type	Model
Welfare optimization	DICE, RICE, DEMETER, FUND, MERGE
General equilibrium	JAM, IGEM, SMG, WORLDSCAN, AIM
Partial equilibrium	MiniCAM, GIM
Simulation	PAGE, ICAM, E3MG
Cost minimization	GET-LFL, MIND, DNE21+

dimensions (IAMs).

In practice, there are several types of IAMs for climate change economics, namely: welfare optimization, general equilibrium, partial equilibrium, simulation and cost minimization [28]. Welfare optimization models tend to be simple, transparent and commonly use DU to give policy advice. General equilibrium models are more complex as they model different economic sectors independently and the objective is to find prices that will clear these markets. Partial equilibrium models use only a subset of the sectors of a general equilibrium model. Simulation models tend to use external predictions to calculate costs of different policy paths. Finally, cost minimization models are technology rich and designed to obtain a cost-effective solution for a particular objective. Table 1.1 shows the most prominent models from each IAM type.

For the topics treated in this thesis, welfare optimization models are the most appropriate choice. First of all, these models are the most simple and adaptable to change. Secondly, they tend to be the most transparent as the code is publicly available. Finally, the explicit representation of discounting and technological change through DU makes it accountable, easy to validate and comparable to other similar models. Out of the welfare optimization models, the DICE model is chosen as it is continuously updated, the code is freely available, its results have implications on policy [29] and because it is considered the base for discussion around climate policy advice.

1.4.2 DICE model

The DICE model was developed by William Nordhaus during the 1990s. It has been evolving ever since, with the latest update in 2013 [21]. It is a globally aggregated model which has a basis on economic foundations and geophysical equations.

The main workings of the models are explained through traditional *neoclassical growth theory*. In this logic, the key question to be answered is: is it welfare-enhancing to reduce consumption today in order to increase consumption in the future by the way of emissions reductions which prevents harmful climate change? The model describes how firms invest in capital, education, technologies as well as "natural capital" which acts as the bridge between economics and climate change. Concentrations of GHGs in the atmosphere are considered as negative natural capital while reductions in them reduce this negative capital.

The DICE model is a non-linear, inter-temporal, *policy optimization* model where the objective is to maximize an objective function (utility). This is in contrast with models that act as *policy evaluation* where there is no optimization of a function, but are rather an equilibrium model that generates paths with different policies. As a result, policy optimization models require much higher computational power to solve.

The model is written in the software General Algebraic Modeling System (GAMS) [30] and solved with CONOPT3 [31].

1.5 The Structure of the Report

Chapter 2 presents the DICE model, explains the full set of equations and describes some of the modifications that were done upon it as motivated by the research questions of this thesis. It finishes with a description of the two different scenarios which will be used for analysis in this thesis. **Chapter 3** covers the issue of endogenous technological change. A literature review on the subject is presented followed by a description of the new equations for the model with its corresponding calibration. An economic analysis follows after this. **Chapter 4** addresses the issue of social discounting. A literature review is first performed. Then, as in the previous chapter, the new equations for the model are presented and an economic analysis finishes the chapter. **Chapter 5** deals with uncertainty. A literature review is presented followed by the uncertainty analysis. Finally, **Chapter 6** concludes the report, elaborates on recommendations for future research and finishes on a technical and personal reflection.

Chapter 2

Dynamic Integrated Climate-Economy

This chapter presents all the relevant information about the DICE model. A literature review is first presented with all relevant equations and a comprehensive description of the model. Afterwards, the next section shows the modifications made to the DICE model as well as the scenarios which will be used throughout this thesis. Concluding remarks finish the chapter.

2.1 Literature Review

The DICE model stands for **D**ynamic **I**ntegrated model of **C**limate and the **E**conomy. It is part of a subset of models which economists, policy makers and scientists use to make decisions, evaluations and predictions called Integrated Assessment Models (IAMs). DICE was developed in the early 1990s by Yale economist William Nordhaus [21]. The latest version of the model was updated in 2013 (**DICE-2013R**) and it is the one which will be used for this thesis.

2.1.1 Model Description

This section draws heavily from Nordhaus' own detailed description of his model [21].

The DICE model is a globally aggregated model which has a basis on economic foundations and geophysical equations. The main workings of the models are explained

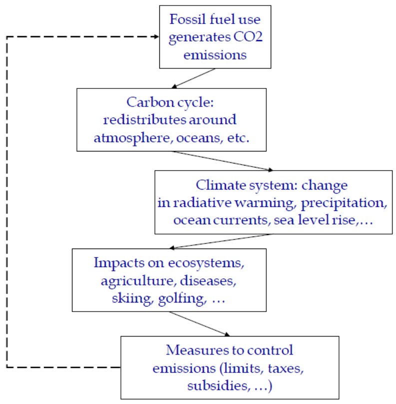

Figure 2.1: A schematic representation of the key connections in the DICE model [21].

through traditional *neoclassical growth theory* with a firm basis in the *Ramsey-Cass-Koopmans* growth model. In this sense, economies reduce consumption today in order to increase consumption in the future by the way of emissions reductions which prevents harmful climate change.

A simple flowchart of the inner workings of the DICE model and other IAMs can be found on Figure 2.1.

2.1.1.1 Objective Function

The most important definition in an IAM is the objective function; the whole model revolves around optimizing this function. In the case of DICE, this is represented by a social welfare function which measures utility. The world then has well-defined preferences. The social welfare function increases with increasing number of people and with the per capita consumption of each generation; however, it also demonstrates *diminishing marginal utility of consumption.*

The social welfare function, W, is:

$$W = \sum_{t=1}^{T_{max}} U[c(t), L(t)]R(t) \tag{2.1}$$

where $U[c(t), L(t)]$ (utility) is a function that involves per capita consumption $(c(t))$ and population $(L(t))$ which is discounted with the factor $(R(t))$. And with the sum notation, W is then a discounted sum of population-weighted utility of per capita consumption.

What is the utility function? It is a function where consumption is "generalized" and includes not only traditional goods like food and shelter but also non-market items such as leisure activities, environmental services and health services. The DICE model assumes a constant elasticity utility function given by:

$$U[c(t), L(t)] = L(t)[\frac{c(t)^{1-\alpha}}{(1-\alpha)}] \tag{2.2}$$

where α is the elasticity of the marginal utility of consumption. The parameter is thought of as the aversion to generational inequality. If the value is close to zero then consumptions between different generations are valued in almost the same way. If the value is high, then there is high inequality between generations. In the limiting case where the value is one, then the function acquires a logarithmic form due to *l'Hôpital's* rule.

Finally, the discount factor $R(t)$ is give by:

$$R(t) = \frac{1}{(1+\rho)^t} \tag{2.3}$$

where ρ is the social rate of pure time preference. In essence it gives different weights to the utilities of different generations.

2.1.1.2 Economic Equations

The economic module of the DICE model is founded on standard growth literature. Due to the very long time frames necessary for climate change modeling, many of the assumptions and predictions in this module are not to be taken at face value. Most macroeconomic models run for only a few years and in some exceptions to a few decades. However, in this particular case the model runs for 300 years.

The DICE model is simpler compared to other climate models because there is only one unit of consumption which is used for everything: consumption, investment and/or abatement. Additionally, many of the parameters like population and emissions variables are taken from national and international databases. All the parameters will be updated in this thesis to account for the newest and most recent numbers.

The production function is of a Cobb-Douglas form with constant-returns-to-scale[1] and with all the Inada conditions met. The function is given by:

$$P(t) = A(t)K(t)^{\gamma}L(t)^{1-\gamma} \tag{2.4}$$

where $A(t)$ is total factor productivity, $K(t)$ is capital stock and services and γ is the capital elasticity to output. $L(t)$ is population and the labor force, which is an exogenous variable, and given by:

$$L(t) = L(t-1)[1 + g_L(t)] \tag{2.5}$$

where $g_L(t)$ is the growth rate and given by:

$$g_L(t) = \frac{g_L(t-1)}{(1+\delta_L)} \tag{2.6}$$

where δ_L is the decline rate of $L(t)$.

Likewise, for $A(t)$, growth is given by:

$$A(t) = A(t-1)[1 + g_A(t)] \tag{2.7}$$

where $g_A(t)$ is the growth rate and given by:

$$g_A(t) = \frac{g_A(t-1)}{(1+\delta_A)} \tag{2.8}$$

where δ_A is the decline rate of $A(t)$.

The net output to society after damages and abatement $Q(t)$ is:

$$Q(t) = \frac{[1 - \mathit{\Lambda}(t)]P(t)}{[1 + \mathit{\Omega}(t)]} \tag{2.9}$$

where $\mathit{\Lambda}(t)$ is abatement costs and $\mathit{\Omega}(t)$ represents climate damages. These damages are, in turn, given by the following damage function:

$$\mathit{\Omega}(t) = \varphi_1 T_{AT}(t) + \varphi_2 T_{AT}(t)^{\varphi_3} \tag{2.10}$$

Equation (2.10) could be seen as the "thorniest issue in climate-change economics" [21]. T_{AT} is atmospheric temperature while φ_1, φ_2 and φ_3 are parameters of the equation. Estimating, calculating or approximating the damages of something of the magnitude of

[1] This is exhibited when output increases by the same proportional change as the change in the inputs.

climate change has proven to be very difficult [28, 32, 33]. However, Equation (2.10) is consistent with a recent survey on the matter [34]. In addition, Nordhaus applies a 25% increase in damages to take into account non-monetized impacts and damages such as sea-level rise, changes in ocean circulation, long-term warming and more. This represents a a value judgment from Nordhaus. Finally, the damage function is calibrated in theory for damages equal or below 3°C. Above 3°C, the damage function might prove erratic due to important tipping points.

The abatement costs, $\Lambda(t)$, is given by:

$$\Lambda(t) = \theta_1(t)\mu(t)^{\theta_2} \tag{2.11}$$

where $\mu(t)$ is the emissions reduction rate and θ_1 and θ_2 are calibration parameters for the abatement technology. This cost function is highly convex, which means that the marginal cost of reductions rises from zero in a non-linear fashion with the reductions rate.

The abatement technology in the DICE model is known as a backstop technology[2] which initially is very high in costs but decreasing in time. The backstop technology is included in the model by setting the parameters of Equation (2.11) to equal the marginal cost of abatement at a control rate of 100% (the power of the backstop technology) to the price of the backstop technology.

Besides these equations, there are a few standard economic equations necessary for the balancing of the model:

$$Q(t) = C(t) + I(t) \tag{2.12}$$

$$c(t) = \frac{C(t)}{L(t)} \tag{2.13}$$

where $C(t)$ is consumption and $I(t)$ is gross investment. The capital stock dynamics is given by:

$$K(t) = I(t) - \delta_K K(t-1) \tag{2.14}$$

where δ_K is the depreciation rate of capital and thus the equation states that capital at period t is equal to the new investment minus the depreciated capital from last period

[2] Refer to Section 3.1.1 for more information on this.

(t-1). $I(t)$ is determined in the model by the savings rate ($S(t)$). The savings rate is determined endogenously by the model. The key assumption in this case is that all savings go towards investment. This particular assumption is characteristic of the neoclassical Ramsey-Cass-Koopmans model which is derived from the Solow-Swan model. This is reminiscent of the classical proposition in economics known as *Say's law* which states a balance where supply creates its own demand. This would be in direct contrast with Keynesian economics where all savings do not necessarily go towards investments. There is no full utilization of resources in a Keynesian economy. As this thesis follows the neoclassical approach, all savings will get converted towards investment. Additional research is suggested in a climate model without Say's law. Nonetheless, as seen in Equation (2.14), the investment is added to the capital stock which ultimately drives production and consumption. An increase in consumption maximizes welfare in the model. The following equation shows the relationship between $I(t)$ and $S(t)$:

$$I(t) = S(t)Q(t) \tag{2.15}$$

How to calculate the emissions generated from production? This in done by the following equation:

$$E_{Ind}(t) = \sigma(t)[1 - \mu(t)]P(t) \tag{2.16}$$

where $E_{Ind}(t)$ are the total emissions generated by industrial activity and $\sigma(t)$ is the carbon intensity of the economy (Emissions/Output). The carbon intensity takes on a similar form as population and total factor productivity:

$$\sigma(t) = \sigma(t-1)[1 + g_\sigma(t)] \tag{2.17}$$

where $g_\sigma(t)$ is the growth rate and given by:

$$g_\sigma(t) = \frac{g_\sigma(t-1)}{(1 + \delta_\sigma)} \tag{2.18}$$

where δ_σ is the decline rate of carbon intensity.

Finally, there is a formula which limits the total amount of fossil fuels which can be extracted, in this case to **6000 tons of carbon content**. An important assumption is that incremental extractions costs are zero (it does not become more expensive to extract more) and that the fossil fuels are allocated efficiently throughout the market. The equation is given by:

$$CCum(t) \geq \sum_{t=1}^{T_{max}} E_{Ind}(t) \tag{2.19}$$

where $CCum(t)$ is the total amount of fossil fuels, with a maximum limit of 6000 tons of carbon content.

Table 2.1 presents all the variables and where they are found in the economic module.

2.1.1.3 Geophysical Equations

This section deals with the equations relating to the physical aspect of climate change. There is a nexus between the economic module and the geophysical module which will link both of them. For the realization of this thesis, this module will be left unchanged.

In the DICE model, the only GHG that is subject to any type of control is CO_2 from industrial sources. Other types of GHGs are modeled exogenously in radiative forcing as well as other sources of CO_2 emissions such as land-use changes.

Equation (2.16) calculates the total number of emissions from the industrial side. However, this is only half the picture. The other source is land-use changes and thus, total emissions are:

$$E(t) = E_{Ind}(t) + E_{Land}(t) \tag{2.20}$$

where $E_{Land}(t)$ is modeled exogenously by taking the latest results from the IPCC where the estimate is that land-use emissions are about 3 $GtCO_2$ per year [21].

With regard to the carbon cycle, it is assumed that it is a three reservoir model: atmospheric level, upper oceans/biosphere level and deep oceans level. Carbon can freely move between adjacent reservoirs. The carbon mixing between deep oceans level and the rest is extremely slow. Deep oceans act as a long-term sink for carbon. Every level is well mixed in the short-term. The following equations describe the dynamics:

$$M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1) \tag{2.21}$$

$$M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) \tag{2.22}$$

$$M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1) \tag{2.23}$$

Table 2.1: Table of variables for economic module.

Variable	**Description**	**Found on**
W	Social welfare	(2.1)
U	Utility	(2.1) (2.2)
t	Time	All
$c(t)$	Per capita consumption	(2.1) (2.2) (2.13)
$L(t)$	Population/Labor	(2.1) (2.2) (2.4) (2.5) (2.13)
$R(t)$	Discount factor	(2.1) (2.3)
α	Elasticity of marginal utility	(2.2)
ρ	Social rate of pure time preference	(2.3)
$P(t)$	Gross output	(2.4) (2.9) (2.16)
$A(t)$	Total Factor Productivity	(2.4) (2.7)
$K(t)$	Capital stock	(2.4) (2.14)
γ	Output elasticity	(2.4)
$g_L(t)$	Growth rate of population	(2.5) (2.6)
δ_L	Decline rate for population	(2.6)
$g_A(t)$	Growth rate of total factor productivity	(2.7) (2.8)
δ_A	Decline rate for total factor productivity	(2.8)
$Q(t)$	Net output	(2.9) (2.12)
$\mathit{\Lambda}(t)$	Abatement costs	(2.9) (2.11)
$\mathit{\Omega}(t)$	Climate damages	(2.9) (2.10)
φ_1	Parameter 1 for damage	(2.10)
φ_2	Parameter 2 for damage	(2.10)
T_{AT}	Atmospheric temperature	(2.10)
θ_1	Parameter 1 for abatement	(2.11)
θ_2	Parameter 2 for abatement	(2.11)
$\mu(t)$	Emissions reduction rate	(2.11) (2.16)
$C(t)$	Total consumption	(2.12) (2.13)
$I(t)$	Total investment	(2.12) (2.14)
δ_K	Capital depreciation rate	(2.14)
$S(t)$	Savings rate	(2.15)
E_{Ind}	Industrial emissions	(2.16) (2.19)
$\sigma(t)$	Carbon intensity	(2.16) (2.17)
g_σ	Growth rate of carbon intensity	(2.17) (2.18)
δ_σ	Decline rate of carbon intensity	(2.18)
$CCum(t)$	Total fossil fuels	(2.19)

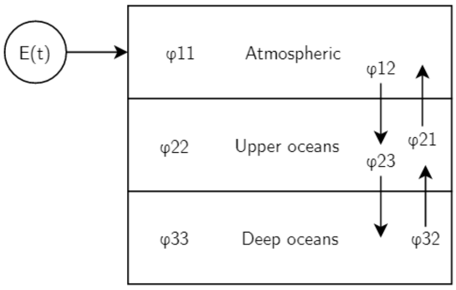

Figure 2.2: Dynamics between carbon cycle in DICE.

where ϕij is the flow parameter between level i and level j. M_{AT}, M_{UP} and M_{LO} represent carbon in the atmosphere, upper oceans and lower oceans respectively. Figure 2.2 shows in a schematic way the different dynamics in the carbon cycle of DICE.

As more GHGs are released into the atmosphere, the Earth's surface warms up due to radiative forcing.[3] This relationship between radiative forcing and GHGs is given by:

$$F(t) = \eta log_2[{}^{M_{AT}(t)}/{}_{M_{AT}(1750)}] + F_{EX}(t) \tag{2.24}$$

where $F(t)$ is the change in total radiative forcing since 1750 ($M_{AT}(1750)$) caused by CO_2, $F_{EX}(t)$ is exogenous forcings besides CO_2 and η is a parameter for equilibrium for every CO_2 doubling. The year 1750 is used as the reference level because it reflects emissions before the start of the Industrial Revolution. Most of the forcing is due to CO_2 and the rest of the GHGs are taken as exogenous due to the fact that their control is exogenous or they are poorly understood [21].

In theory, higher radiative forcing will eventually warm the atmospheric level which in turn will warm the upper ocean and finally the deep ocean. There is a lag between the warmings of the different levels due to diffusive inertia between levels. The equations for the temperatures of both the atmosphere and lower oceans are given by:

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\} \tag{2.25}$$

$$T_{LO}(t) = T_{LO}(t-1) + \xi_4\{T_{AT}(t-1) - T_{LO}(t-1)\} \tag{2.26}$$

[3]Radiative forcing is the difference between incoming energy and outgoing energy into the Earth. Positive radiative forcing warms the planet as it decreases outgoing energy.

Table 2.2: Variables for geophysical module.

Variable	Description	Found on
$E(t)$	Total emissions	(2.20) (2.21)
$E_{Ind}(t)$	Industrial Emissions	(2.16) (2.19) (2.20)
$E_{Land}(t)$	Land-use emissions	(2.20)
t	Time	All
$M_{AT}(t)$	Atmospheric carbon	(2.21) (2.22) (2.24)
$M_{UP}(t)$	Upper ocean carbon	(2.21) (2.22) (2.23)
$M_{LO}(t)$	Lower ocean carbon	(2.22) (2.23)
ϕ_{ij}	Flow parameter from i to j	(2.21) (2.22) (2.23)
$F(t)$	Radiative forcing	(2.24) (2.25)
$F_{EX}(t)$	Exogenous forcing	(2.24)
η	Equilibrium parameter for CO_2 doubling	(2.24)
$T_{AT}(t)$	Atmospheric temperature	(2.10) (2.25) (2.26)
$T_{LO}(t)$	Lower ocean temperature	(2.25) (2.26)
ξ_i	Climate sensitivity parameters	(2.25) (2.26)

where $T_{AT}(t)$ and $T_{LO}(t)$ are the mean temperature of the atmosphere and the deep oceans respectively. ξ_i are different parameters regarding climate sensitivity.[4] The climate sensitivity in the DICE model is estimated at 2.9°C for every doubling of CO_2 [21].

Table 2.2 shows a summary of all the variables in the geophysical module and where they can be found. $E_{Ind}(t)$ and $T_{AT(t)}$ represent linkages between the two modules, as they appear on equations on both of them.

[4]Climate sensitivity refers to how much change in temperature is caused by a double in CO_2.

2.2 Modifications and Scenarios

The 2013 DICE model is a very stable and reliable release. However, some modifications were done to this model in order to make it usable in terms of this thesis. For this, an alternative DICE model is used, aptly called *DICE2013x*.

The first modification concerns the emissions control rate, ($\mu(t)$), from the 2013 DICE model. As can be seen in Figure 2.3, the emissions profile of the DICE model behaves a bit erratically after year 2165 for the baseline run.[5] This is due to the fact that $\mu(t)$ is a free variable which can take any value. The upper limit on this variable is fixed at 1.2 which represents a modeling decision by Nordhaus. GAMS finds it optimal to have this control rate[6] for 23 periods out of 60 which is more than a third of the timespan of the simulation. This effect creates negative emissions which in turn lowers the total forcing and decreases the atmospheric temperature to below 1°C by the end of the model's timespan. This is by no means a wrong formulation or solution. This pathway is possible with the use of technologies which exhibit negative carbon emissions such as biomass with carbon capture and sequestration.[7] In order to make the model less erratic in terms of its emission profiles and make it more tractable, the upper limit on $\mu(t)$ will be set at 1.

$$\mu(t)_{UpperLimit} = 1$$

Another modification to the 2013 DICE model involves changes to the scenarios. The baseline scenario in the 2013 DICE model first calculates Hotelling rents for the carbon price with *no damages* in the model.[8] Then, it re-runs the model now with damages and with the carbon price fixed at these Hotelling rents. The optimal scenario is simply optimizing the model with *damages on*. For the DICE2013x model, the definition of these two scenarios is changed. The reason for this is that when including endogenous technological change and dynamic discounting, the baseline scenario setup of the 2013 DICE model complicates issues as $\mu(t)$ is no longer present.

The baseline scenario is defined as to how the economy would react if there was no

---

[5] Nordhaus shows only the emissions curve until 2100 for the optimal run [21]. This is convenient because it tells the convincing story that emissions have to peak mid-century. No explanation is given for the emissions curve after the year 2100.

[6] If the upper limit is relaxed, $\mu(t)$ during these periods can reach values up to 1.389.

[7] Future technology is quite uncertain. More on this on Section 5.2.

[8] This means that the price of carbon will increase hand in hand with the interest rate.

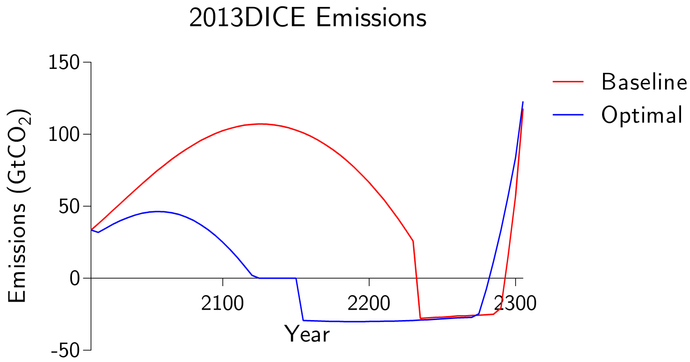

Figure 2.3: Emissions curve from the baseline and optimal runs in 2013 DICE.

externality. This means that firms and society do not see any damages from carbon emissions and thus the social cost of carbon is equal to the private cost of carbon. Although a highly unrealistic scenario, this specification gives an upper boundary on the economic activity. In this sense it is useful because it sets a reference point to which other scenarios can be compared. In the DICE2013x model, the baseline scenario is controlled by having zero damages in the model. This means:

$$\text{Baseline scenario} = \begin{cases} \varphi_1 = 0 \\ \varphi_2 = 0 \\ \varphi_3 = 2 \end{cases}$$

The optimal scenario is defined by adding the externality to the economic system. The economy would then react accordingly to carbon emissions while at the same time trying to maximize welfare. In this scenario, welfare is maximized while taking into account the damages that emissions cause on the economy. There is no limit on the atmospheric temperature or concentration of carbon. To run in optimal mode, the damage parameters are set to the calibrated values from Nordhaus' model [21]. This means:

$$\text{Optimal scenario} = \begin{cases} \varphi_1 = 0 \\ \varphi_2 = 0.00267 \\ \varphi_3 = 2 \end{cases}$$

Even though more scenarios could be run, for example, by limiting atmospheric temperature to 2°C, the objective of this thesis is to research the effect of endogenous technological change and dynamic discounting and not to look at several alternative scenarios. For this reason, only these two scenarios will be used (baseline and optimal)

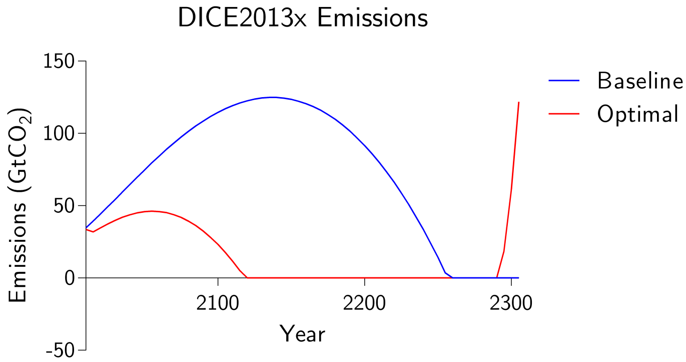

Figure 2.4: Emissions curve from the baseline and optimal runs in DICE2013x model.

for the rest of this thesis. The corresponding emissions curves for both scenarios with the new modifications are shown in Figure 2.4

2.3 Concluding Remarks

The DICE model has been presented in this section with all its appropriate equations and formulations. The model was described by its economic and environmental modules. Industrial emissions and atmospheric temperature are the two variables which link both of the modules.

Some small modifications were done on the DICE model in order to make it more tractable. This framework will be used throughout the thesis as its building block. In fact, a model will be built upon the DICE2013x model in order to include endogenous technological change coupled with a dynamic social discount rate.

Chapter 3

Endogenous Technological Change

After dealing with the DICE model and its equations, this chapter will deal with the issue of endogenous technological change. First, an extensive literature review is presented which delineates all the different methods to model endogenous technological change. After this, the extended model is presented. Following this, an economic analysis is realized and a concluding section closes the chapter.

3.1 Literature Review

Climate change is, among other issues, one of the first truly global problems. Solving climate change requires the delicate interplay between different actors and field of studies such as: economics, climate science, politics, ethics, cultural studies, etc. Managing this interplay has been one of the biggest challenges so far. However, one of the biggest hopes between all the different dimensions is that positive technological change will occur which will significantly reduce the problem's complexity. Technological change allows the introduction of new technologies/innovations into the economic system which can help ameliorate the environmental damages such as solar power and fuel improvements. Eventually, technological change could completely transform societies in ways in which climate change is no longer an issue. But first, what exactly is technological change in economics and how is it modeled?[1]

[1] Technological change is a synonym for a myriad of other similar forms of expressing the same thing such as technological development, technological achievement, technological progress, technical change, technical progress, etc. For the realization of this thesis, the term technological change will be mainly used.

Technological change, in economics, is an outcome of innovation. It can be defined using a production function. For example, from a neoclassical perspective, one of the most common production functions is one with a Cobb-Douglas form [35]:

$$Y = aL^{\alpha}K^{\beta} \tag{3.1}$$

where Y represents total output, K is the usage of capital and machines, L is labor input, α and β are constants describing output elasticity and finally a is total factor productivity, a change of which reflects neutral disembodied technological change.[2] Positive technological change, say by making processes more efficient with a new breakthrough, is represented by a. This can easily be visualized by making use of isoquant maps. An isoquant is a curve that graphs different combinations of labor and capital which give exactly the same output.

Figure 3.1 shows two production isoquants. All the points along each thick line represent the same amount of output. The initial isoquant is the thick line on the upper right side. The optimal combination of k and l (profit-maximization) for this isoquant is the point of tangency between it and the iso-cost line, which is the thin line that describes all the combination of production factors that have the same cost. This is visualized in point [l_o,k_o]. Technological change occurs when the curve is shifted inwards towards [0,0]. Because the slope of the iso-cost line remains the same, the new optimal point [l_1,k_1] has exactly the same k/l ratio as the previous one. This enables a reduction in capital and labor while keeping the output constant. This progress is called *neutral* as the labor-capital ratio is held constant; other types of progress are *labor-saving* and *capital-saving.*

Paul Romer formulated the foundation for endogenous technological change in the early 1990s [36]. He stresses the importance of technological change in three different premises. First, technological change lies at the very heart of economic growth and is responsible for increases in output/hour worked. Secondly, technological change is an intentional consequence of actors responding to diverse market incentives. This is where technological change adopts its endogenous form. And lastly, technological change is different from other economic goods; once technological change has been developed, the

[2] It can also be a measure of the economists' ignorance in understanding output production [35]. Because K and L are usually insufficient to explain output in a complex economy, this discrepancy is explained through a.

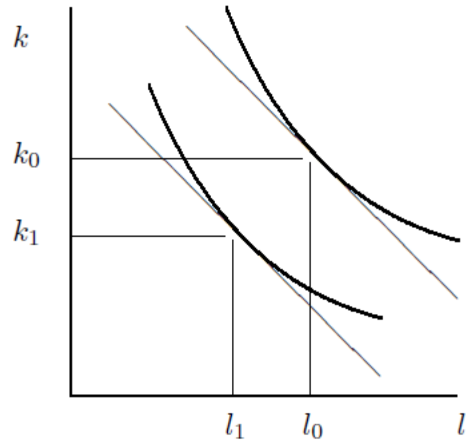

Figure 3.1: Isoquant curves [35].

cost to apply these "new instructions for the raw materials" is basically zero.

In the context of climate change modeling and energy, technological change is represented as an increase in the efficiency of the overall energy system, in particular, reducing the GHGs emissions per unit of output. Thereby, it makes the energy system "cleaner" as sustainable energies start dominating the market. This could be visualized in Figure 3.1 by assuming that the production factors are energy (e)[3] and labor (l). Energy intensity is then defined as e/y due to the fact that most of the emissions come from energy use. *Energy-saving* progress would then be the progress that reduces the use of e while keeping output y constant.

Hence, climate change modeling and technological change are highly intertwined matters which states the importance of their appropriate modeling [11]. Both are related through the externalities caused to the economic system.[4] Climate change is nothing more than the accumulation of all the negative externalities caused by the burning of fossil

[3]Instead of k.

[4]Externalities exist in the economy whenever a transaction incurs costs to particular parties which are not reflected on the market price. Externalities in nature could either be positive or negative. For example, educating oneself has a positive externality on society, as it can be argued that this is better overall for everybody. The market price of education does not include the price of all the indirect benefits of having an educated society. Unfortunately, the most important externalities are those that exhibit negative properties. One of the most common one is that of air pollution caused by the burning of fossil fuels for which health costs of breathing dirty air are not taken into account on the market price.

fuels, or as Nicholas Stern states *"..the greatest market failure the world has ever seen.."* [16]. On the other hand, technological change is caused by the positive externalities of the generation of knowledge. Knowledge is often described as a public good which means that some firms could potentially appropriate knowledge generated by other firms. Thus, knowledge is often under-provided. Adam's Smith invisible hand allows too much of the negative externalities while allowing too little of the positive externalities. This nexus is the reason why climate change modeling and technological change must go hand in hand.

However, most economic models of climate change initially developed in the 1990s were treating technological change as an exogenous variable [11]. This meant that the models assumed a constant technological change throughout time which did not depend or rely on any other factor; it was an autonomous variable. Now, models with endogenous technological change have been developed as the link with other factors (such as R&D) has become clearer.

In the following sub-sections, a review of all the ways that technological change is currently being modeled will be presented.

3.1.1 Exogeneous Technological Change

Exogenous comes from the modern Latin *exogena* denoting from outside (exo) the body (gena); it means anything that is growing or occurring outside an organism. In economic modeling, exogenous means that it occurs outside the model. In the case of climate change, and when dealing with (exogenous) technological change, this means that there are improvements of energy efficiency which occur outside the model. Technological change is not affected by changes in any of the variables of the model. As an example, Nordhaus uses exogenous technological change in the DICE model [21].

This way of modeling technological change is known as autonomous energy efficiency improvements (AEEIs) [11]. In its most straightforward way, modeling AEEIs is done by assuming a constant improvement of a in Equation (3.1) which directs the overall progress of the economy.[5] The pros of using AEEIs are its relatively simple use, easy integration with models and transparency. In fact, it can also facilitate sensitivity analysis as the parameter can be changed easily [37]. However, one of its biggest advantages is also its biggest drawback. Due to its simplicity, AEEIs can act as a "black box" character of technological change, which ignores policy decisions, price inducements and any other

[5] This is known as Hicks-neutral productivity.

innovation decision which affects technological change [11]. One other problem with AEEIs is that it completely ignores radical technological change (i.e. introducing a new shocking innovation) and relies only on the incremental kind of change. With uncertainty playing a major role in future technological change, incremental change is hardly an accurate portrayal of what to expect from the future. Studies criticize AEEIs as an improper way of modeling technological change due to the fact that it neglects the main causes of change and that it is also not consistent with empirical evidence [37].

Another common way to model exogenous technological change is by including a "backstop" technology in the model. This technology is something not yet feasible, but which will progressively lower its costs until it can fully enter the market. This type of technology is something which could potentially replace all fossil fuels such as nuclear fusion, carbon capture and sequestration (the emissions are the important issue) and/or solar energy. The approach to modeling is by having a cost curve and determining the date of introduction to the economy. One thing to note is that at several points in time, both the "old" and the "new" technology will be co-existing and it is essential to model this. The biggest drawback to this method is the inherent uncertainty about the "backstop" technology: uncertain negative environmental effects, material limitations, full availability, market penetration and so on. In a way, this type of modeling is sometimes called "semi-endogenous" as the cost and other factors are determined exogenously but its availability depends on the energy prices or other endogenous variables [37].

3.1.2 Endogenous Technological Change

There are several ways to endogenize technological change, namely: price-induced, directed technical change, learning-by-doing and by research and development (R&D). The reason to do this is that assuming exogenous technological change is an oversimplification as it is acknowledged that technological change is a very complex process involving prices, activities and time [38]. These methods usually involve incorporating a feedback mechanism in the model towards which policy can change the overall level of technological change [37]. Most commonly this is achieved through a "knowledge stock" which is accumulated with time and directs the level and direction of technological change. The challenge is to define how to accumulate this stock and how it affects emissions and energy usage.

One important consideration for all of the different ways of modeling endogenous technological change is whether to assume that the "base case" behavior of technological change is optimal.[6] One view shows that modeling with exogenous technological change is a constraint, which when taken off, will ultimately lower the costs of mitigation no matter what. The other view states that technological change is the "base case" and it is roughly optimal, so that any new effect with climate policy will lower the costs just marginally [38]. It is important to define well the "base case" to allow for proper comparison with other studies.

The following four subsections will present the four methods to endogenize technological change.

3.1.2.1 Price-induced technological change

Price-induced technological change is one of the most widely used approaches, first introduced by Hicks in his seminal work *The Theory of Wages* in 1932 [39]. It states that any change in the relative factor prices will cause firms to induce technological change as the more expensive factor will be subject to minimization. To illustrate this, imagine that the price of labor (L in Equation (3.1)) increases for a firm (due to a political decision of increasing minimum wages), this will force the management of the firm to look for ways to minimize the use of labor and substitute it with the use of automation or increased capital productivity.[7] The quest for this, caused by the change in the relative price factors, is what enables technological change.

In the case of climate change, one of the biggest inducers of technological change is the energy price. There is a growing literature on the subject that supports the claim that higher energy prices leads to positive technological change in the energy field [40] and in the renewables' [41]. For example, a study shows that the long-run elasticity of energy R&D with respect to energy prices is 0.35 [40]. The median lag of this is 4.9 years, which means that after 5 years of a change in the energy price, *half* the full effect of R&D will have already had happened. Other empirical studies working with energy patents show that innovation does indeed respond to incentives, that the social rate of return of environmental R&D is higher than the private one and that there are diminishing returns to research over time [42]. Some work has been done with price-induced technological change and shown positive results [11], nonetheless, most modelers prefer other ways of endogenizing technological change as price-induced technological change is just treated as a partial explanation to the whole issue where sometimes it is combined with an AEEI

[6] In the absence of climate policy.
[7] Assuming profit-maximizing firms.

parameter or in conjunction with learning-by-doing [37].

3.1.2.2 Directed technical change

Another way to endogenize technological change is by directed technical change, brought to light by Acemoglu in 2002 [43]. In regards to climate change modeling, his model includes two ways in which a good can be produced: with dirty or clean technology [44]. Any profit-maximizing firm will always choose the cheapest option, in this case dirty technology which already has an installed base, and thus reap the benefits of firms wanting to innovate the chosen option. This paints a gloomy picture: cleaner technology will not be used as firms prefer the dirtier technology and innovate on this. This is partly true unless the government decides to intervene.[8] This is where the *direction* comes into play. The government must be able to provide taxes and subsidies to correctly allocate the resources between clean and dirty technologies and be able to "direct" the technological change in a certain direction. Thus, supporting cleaner technologies in the short-run might slow growth down, but in the long-run it will provide a cleaner alternative than its counterpart [11].

Some models include some type of directed technical change with the inclusion of innovation possibility frontiers which are nothing more than all the expected isoquants from a specific sector.[9] Thus, technological change can be directed towards one of the isoquants. The difficulty lies in how to design the specific technology policy so as to direct the change towards the objective.

3.1.2.3 Learning-by-doing

Learning-by-doing in manufacturing was conceived from the fact that aeronautic engineers in the 1930s observed that the labor hours required to construct one product reduced as the manufactured products doubled in quantity [11]. It was then formalized by Kenneth Arrow in 1962 for economic purposes [45]. The basic understanding of learning-by-doing is that the more firms use or produce a certain technology, its costs start decreasing. Thus, several technologies can have initial high costs but these will decrease with usage and time. Learning-by-doing can be easily visualized with the help of experience curves which plot costs vs. cumulative production/use. Figure 3.2 shows an example of this type of curve. Each of the different phases present in Figure 3.2 represents a different

[8] As the exhaustible resources of the dirty technology start depleting, this makes the change to the cleaner one possible under *laissez-faire* [44].

[9] Refer to Figure 3.1 in Section 3.1.

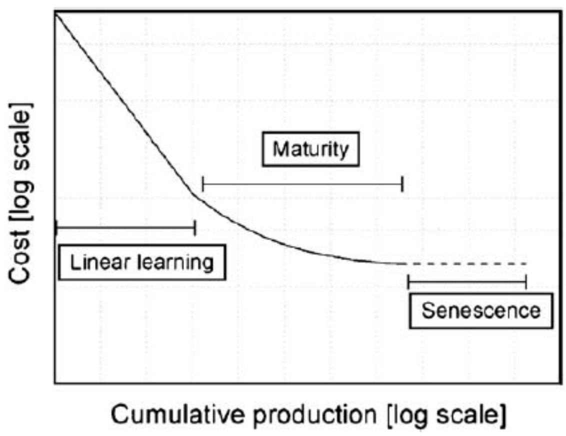

Figure 3.2: Qualitative description of a generic experience curve [46].

learning rate[10]: high in the beginning, lower in the maturity phase, and almost or even zero in the senescence phase.

One common way to mathematically express the experience curve is with a power law known as *Henderson's Law* [47]:

$$C_n = C_1 n^e \tag{3.2}$$

where C_n is the cost of the n-th unit of production, C_1 is the cost of the first unit of production, n is the cumulative production and e is the learning index which is equal to $e = \frac{log\alpha}{log2}$ with α being the learning rate.

This type of modeling is most common in models with high technology-specificity such as bottom-up models due to the disaggregation of the model and ease of implementation.[11] Regarding top-down models such as DICE, several studies have been including learning-by-doing technological change into them [48, 49]. Results show that learning-by-doing could greatly reduce abatement costs subject to a proper learning rate. Having the proper learning rate is thus the challenge where learning is usually seen as a specific phenomenon,

[10] A higher learning rate means that costs decrease more for a given amount of experience.

[11] Bottom-up models are often partial equilibrium models where the energy sector is detailed in a very technical way while using a large set of technologies. These models often try to optimize the sector on a cost basis. On the other hand, top-down models rely on a more economic description while minimizing the specificites of different technical issues of the energy sector. These models often try to optimize the economy by maximizing welfare according to different policy options.

not an aggregated one like in top-down models [50]. Another common result is that the carbon tax is usually lower in models with learning-by-doing than in models without it. This is due to the fact that, without R&D, adding more capacity to carbon-free technologies will lower the costs and increase the emissions reductions per dollar of investment [37].

There are some difficulties with regards to learning-by-doing. Spillover effects need to be taken into account, especially in bottom-up models which have a great deal of different technologies. Learning-by-doing is also seen as an *ad hoc* solution which lacks transparency [11]. For example, it is difficult to be confident about the causality of learning-by-doing. For all that matters, it could be the R&D factor that spurs more competitiveness in the marketplace and thus responsible for the increase in the production. One way to counter this is to create a "two-factor" model which has a cost function dependent on cumulative production and on R&D [51]. This could be done by adding $R\&D^e$ to Equation (3.2). Another issue is that learning is a self-reinforcing process and thus experience in one technology can "lock-in" unto a path dependency because the technology is becoming more competitive as more experience is gained until it overpowers the rest of the technologies [37]. Finally, carbon-free technologies usually enjoy great learning rates while carbon technologies enjoy almost no improvement, which means that the outcome is already pre-determined.

3.1.2.4 Research and development

R&D is an activity that firms undergo in order to lower their long-run production costs and just then it represents a market advantage. R&D is related to the generation of new knowledge, under which companies or government invest beforehand. R&D is an expensive activity, requiring investment of capital and wages of highly-specialized researchers.

R&D works under the knowledge market which in itself is full of imperfections. The most important one refers to the very nature of knowledge being a public good.[12] And due to this nature, the generation of knowledge creates spillovers, which can cover more than what it originally intended to. A quick example of this would be research done on aerodynamics of wind turbines which could spillover to the flight industry or the space

[12]A public good is a good that can benefit everybody, there is no exclusivity. It is characterized by two main aspects: non-rivalry and non-exclusivity. Non-rivalry refers to the fact that knowledge can be used by everybody at zero marginal cost (i.e. extra use does not lead to additional cost of knowledge production). Non-exclusivity refers to the fact that people (or firms) cannot be prevented from using the good; this leads to the free-rider problem where someone uses knowledge without actually paying for it.

industry. With this line of thought, there is then a positive externality to generating new knowledge. Society benefits as a whole when new research is generated, while the inventor does not [50]. Thus, the social rate of return of R&D is higher than the private rate of return. Nordhaus finds that social rates of return are around four times bigger than private rates of return in the U.S. [52]. Other similar studies show similar results [50, 53]. So, due to this very fact, the market underinvests in R&D as firms do not have the incentives to provide the socially optimal level.

On a related note, the effect of spillovers could cause high opportunity costs when increasing R&D (especially environmental). In a situation as this:

$$Y = C + I + R_E + R_O \tag{3.3}$$

where the output (Y) is composed of consumption (C), investment in physical capital (I), environmental research (R_E) and other research (R_O).[13] The point is that one cent spent on (R_E) will displace exactly one cent from both (C) and (I). The opportunity cost is one cent. But, if (R_O) has a social rate of return four times that of (I), then giving up that one cent of (R_O) has an opportunity cost of four cents [50]. So, the price of any R&D which displaces other R&D is 4 cents.

The above assumption on opportunity costs merits more analysis. This assumption is what has been traditionally been taken by neoclassical economists. Nonetheless, there are arguments to point to the fact that the opportunity costs of R&D could be considerably less or even zero. Investments are usually financed through bank credit which means that it is through completely new money, not from existing savings [54]. Climate mitigation investments (such as R&D) could practically be financed through bank credit which means that the opportunity cost of R&D would in turn be *zero* as there is no diversion of funds from other investment activities. Another option is that an increase in environmental R&D could be financed by spending less on marketing, military equipment or any other investment where the opportunity cost would be smaller. For the rest of this thesis, the opportunity cost of 4 cents assumption will be used to follow the steps of previous work on the topic [13, 50, 55]. Important to realize is that this reasoning could have important effects on the results. This is left and encouraged for future research.

There are even more considerations to take into account while modeling with R&D. One of them is making a distinction between private R&D and governmental R&D. As

[13] This presupposes full employment.

seen before, private R&D will always be sub-optimal due to a higher social rate of return. In order to avoid this, the government must be willing to finance extra research efforts by the way of tax breaks or subsidies in order to bring social and private rate to the same level. Public R&D is also of the more basic (fundamental) type[14], as great uncertainty and long-term planning is not fit for private firms. In this way, states can become a catalyst for innovation and growth [56]. Modeling both types could pose problematic [50], but should always be noted in the model assumptions. Another issue is the limited quantity of available money to spend on R&D, which will then favor certain technologies which could "crowd out" R&D investment in other distinct fields such as health [37]. As seen in the previous paragraphs, "crowding out" could have high opportunity costs.

How is R&D included in climate change models? The usual method is by declaring a new variable which represents R&D (or knowledge) which focuses on reducing GHGs emissions intensity and/or reducing abatement costs [11].[15] There have been several attempts to endogenize technological change in the DICE model by the way of R&D [52, 55, 57]. Nordhaus, for example, uses the following equation:

$$\frac{\dot{\sigma_t}}{\sigma} = \Psi_1 R_t^{\Psi_2} - \Psi_3 \tag{3.4}$$

where σ_t is the carbon energy/output ratio in year t, R_t is the level of R&D and Ψ_i are equation parameters.[16] Nordhaus makes use of the assumption that one cent of R&D will displace four cents of output [52]. The other two studies also focus on the effect of R&D on the level of emissions intensity (instead of the rate of change) and use a stock of accumulated knowledge (instead of a flow) [50].

One study shows that the effect of "crowding out" is very significant: without the effect the model shows that the total welfare gains are 45% more than in the base case, and with the full effect the gain is just a measly 2% [55]. Studies, as seen above, create a knowledge stock[17] which accumulates with time and has a negative effect on emissions intensity. Spillover effects are modeled by assuming that social and private rates are different [52, 57]. In general, these studies show that including technological change

[14] Instead of the more applied research.

[15] Another option is by productivity gains in specific sectoral production functions in multi-sectoral models. However, because DICE is not a multi-sectoral model, then this will not be furtherly pursued.

[16] The left side of the equation represents the rate of change of the ratio.

[17] A knowledge stock contains all the ideas, skills and experience which can affect the production function [37].

translates to higher welfare gains or smaller abatement costs.[18]

One common way to include the knowledge stock into the model is by modifying the Cobb-Douglas equation (3.1) as shown below:

$$Y = aK_r^{\beta} L^{\alpha} K^{1-\alpha} \tag{3.5}$$

where K_r is the new variable representing the knowledge stock and β is the knowledge elasticity to output.

Another approach involves using a continuum of intermediate goods and including it in a production function which has energy as an input:

$$Y = a \cdot \Phi(A_L L, A_E E) \tag{3.6}$$

where A_L and A_E are the endogenous augmentation levels for both labor (L) and energy (E). A_L and A_E then depend on the quality level of the intermediate good, the raw input, the type of service, the rate of change in the quality of the good (which depends on R&D) and more [37, 58].

3.1.3 Technology Diffusion

Technological change involves the introduction of either new products altogether (product innovation) or improvements upon the existing ones (process innovation). *Any* of these two options will most certainly have to go from research laboratory all the way into the market. This is a process that takes time and is the subject of great debate. One of the most common ways to represent this movement is through the use of a diffusion curve as shown in Figure 3.3. Following the shape of the curve, any new technology starts with a few early adopters which then follows a step of rapid adoption and eventually leveling off at a certain market penetration %. This theory works great with normal consumer goods with a big mass market. What about environmental technologies?

Environmental technologies, such as end-of-pipe technologies[19], must have a regulation (or incentive) in place or else there will be no adoption. Several studies have shown that

[18] Because the DICE model does not work with an objective cost function, then going into how R&D is modeled into these functions will not be presented here.

[19] Refers to technologies which reduce emissions after the product or process has already been developed such as tailpipe catalytic converters.

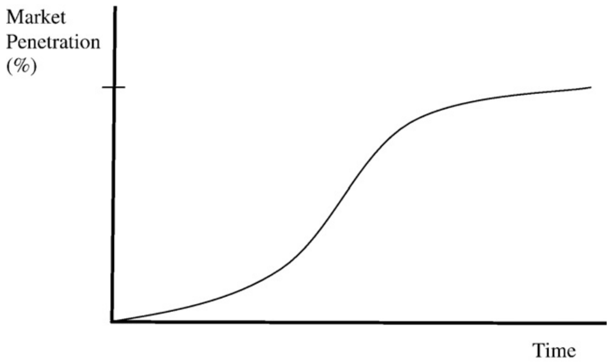

Figure 3.3: S-shaped diffusion curve [50].

environmental regulation is key to the adoption of end-of-pipe technologies [59, 60]. For example, stringent nitrogen oxide (NOx) regulations were the driving force behind the adoption of NOx pollution control technologies in the coal-fired electric power plants of the U.S. [59]. However, environmental technologies that focus on energy efficiency are adopted more slowly, as cost is the key driver which responds more to prices, not regulation [50]. One way or the other, policies can affect the shape of the diffusion curve and deems it necessary to include this in the modeling.

This is essential in bottom-up models with high technological specificity, however, top-down models with high levels of aggregation do not necessarily need this level of detail. This is due to the fact that these models are modeled with time periods of ten years which allow for the gradual diffusion of any new technology. This is supported by studies which show that the greatest impact of new energy patents take around four years and patents take around four years to respond to changing energy prices [40, 61]. With the DICE being a top-down model, technological change is only modeled as improvements to the existing technology instead of the introduction of new products.

3.1.4 Uncertainty

Most models with integrated endogenous technological change assume a deterministic world with known expectations [22]. However, there is a great deal of uncertainty when dealing with future technological change even though it is a vital aspect of long-term economic growth [23, 38, 62, 63]:

1. Uncertainty about future inventions.

2. Uncertainty about the usefulness of new infant technologies.

3. Uncertainty about the *pace* of technological progress towards market maturity.

4. Uncertainty about the effect of R&D on technological change and its pace.

Besides this, an even larger amount of uncertainties are also present such as those related to climate change science, the political situation or the economy as a whole. It is of no surprise that uncertainty can play a key role in endogenous technological change modeling.

There are two types of uncertainty, Arrovian (measurable) and Knightian (immeasurable) uncertainty [23]. It is the Knightian uncertainty which makes this phenomenon difficult to analyze and study. But it is not something that is out of reach. Understanding the dynamics of diffusion of new technologies and its impact on economic performance is the first step in characterizing uncertainty in technological change.

It is a fact that most innovations throughout history have failed [23]; these failures are correlated with the uncertainty and the inability to look into the future with clear eyes.[20] And as a whole, society is still doing the same mistakes. There is a gray area between *uncertainty* on one hand and *ignorance* on the other. Given that decision-makers do not always have an accurate probability distribution about the future, it is mostly ignorance that catapults failed innovations. However, no matter the term, it is incentives, policies and institutional rules that can help ease out the *ex ante uncertainties* in order to motivate firms to innovate.

R&D is a great tool to develop new technologies. However, most of the R&D is used for incremental product innovation (around 80% [23]). This means that R&D follows a path-dependency and does not seek for Schumpetarian profits. It is hard to say that technological change is exogenous when most of the R&D is used to improve upon existing technologies [23].

There are some studies that link the connection between uncertainty and endogenous technological change [62]. Most of these analyze either damage uncertainty, technological uncertainty or a combination of both. One approach is to assume that the backstop technology has a probability to be a bit "polluting" [64].[21] Another option is to have

[20] For an account of historical cases refer to [23].

[21] An example would be nuclear power whereas it is carbon-free but with other long-lived environmental problems.

uncertainty over the effectiveness of R&D, that is, how probable it is that the backstop technology will ultimately be successful [63]. Including uncertainty into models affects the technological policy in important ways by increasing the total amount of investment in R&D. However, different conclusions are sometimes reached due to the very nature of the modeling exercise and its appropriate assumptions. In every case, uncertainty plays a major role in the results, which indicates that it is something not to be ignored.

3.1.5 Concluding Remarks

An exhaustive review on the literature of technological change has been presented in this section. Technological change has an effect on the production function by lowering the use of the factors (namely labor and capital) in order to reach the same output. Energy models tended to use exogenous technological change in its nascent stages. New models and studies are showing that endogenizing technological change is beneficial and has important effects. The four main ways to endogenize technological change are price-induced technological change, directed technical change, learning-by-doing and by R&D. Additionally, the diffusion of technologies when there is positive technological change follow an S-shaped curve. Finally, there are many factors that contribute to a growing uncertainty about future technological change. This can play a key role in the modeling process.

For the following sections, endogenous technological change through R&D will be used as it is the most appropriate option for top-down models such as DICE. Nevertheless, the theory from the other types of endogenous technological change will still be reviewed to explain different phenomena. Technology diffusion will not be treated as much as it is more appropriate for bottom-up models and the fact that patents by R&D have the greatest impact within four years.

3.2 Model

This section will focus on the additions made to the DICE2013x model in order to endogenize technological change through R&D. David Popp's work on endogenous technological change provides the backdrop for this section [55, 65]. These additions are calibrated accordingly to 2010 values in order to achieve consistency throughout the modeling process. Section 3.2.2 goes through all the calibration steps.

The following sub-section describes in detail the equations of the model which is aptly called DICE-ED for Endogenous & Discounting (ED) which is the main focus of the thesis.

3.2.1 Equations

The first change made to the model is adding an energy sector which has two basic fuels: fossil energy and backstop energy. Fossil energy includes the trident of oil, gas and coal and release emissions to the atmosphere when used. On the other hand, backstop energy includes all renewable and clean sources like solar and wind which do not release emissions into the atmosphere when used. This is a key distinction to make between the two fuels in this energy sector. The energy sector is added to the Cobb-Douglas production function as $ES(t)$, which is a measure of effective energy units. The formula, adding on the production function equation (2.4), is:

$$P(t) = A(t)K(t)^{\gamma}L(t)^{1-\gamma-\beta}ES(t)^{\beta}$$

where β is the energy elasticity to output in the production function. The production function still exhibits constant returns to scale as the sum of the exponents is equal to one. However, the cost of both the fossil energy and the backstop energy still need to be subtracted from this production function. This is found in some other similar models [65–67]. The appropriate form is then:

$$P(t) = A(t)K(t)^{\gamma}L(t)^{1-\gamma-\beta}ES(t)^{\beta} - p_F(t)F(t) - p_B(t)B(t) \tag{3.7}$$

where $F(t)$ is fossil fuel usage measured in tons of carbon while $B(t)$ is backstop usage measured in carbon ton equivalent (CTE).[22] It follows then that $p_F(t)$ and $p_B(t)$ are the prices of fossil fuels and backstop fuels respectively measured in price per ton of

[22] 1 CTE has the equivalent energy to a ton of carbon of fossil fuel energy. Using these notations makes for a simpler analysis.

carbon and price per CTE.

The energy sector described with $ES(t)$ uses a combination of the two fuels and also a knowledge stock of energy efficiency which represents improvements in the energy field that do not relate to the use of any of the fuels. The formulation for $ES(t)$ uses a nested constant elasticity of substitution form to account for the three different sources. The form is:

$$ES(t) = \Bigg[(\alpha_H H_E(t))^{\rho_H} + \bigg(\Big(\frac{F(t)}{\alpha_\Phi \Phi(t)}\Big)^{\rho_B} + B(t)^{\rho_B}\bigg)^{\rho_H/\rho_B}\Bigg]^{1/\rho_H} \tag{3.8}$$

where $H_E(t)$ is the knowledge stock of energy efficiency described above and α_H is a scaling factor related to how much savings are generated per unit of knowledge. $\Phi(t)$ represents the ratio of emissions per unit of carbon used. This ratio declines as time passes and represents exogenous technological change in the model. These improvements in the ratio can be thought of as changing to cleaner fuels (e.g. from coal to oil to gas) and/or improving the energy efficiency of the current energy system (e.g. combined cycle power plants compared to single cycle ones). $\Phi(t)$ is calibrated so that the exogenous technological change is similar to the one found in the DICE2013x model. α_Φ is the factor that reduces this exogenous technological change when R&D is added to the model. Finally, ρ_H represents the ease of substitution between the two fuels and the energy efficiency stock and ρ_B between the fossil fuel and the backstop fuel. The elasticity of the substitution is $1/(1-\rho_i)$. The advantage of doing this is that the fossil fuel and the backstop fuel are modeled as imperfect substitutes and thus backstop fuel can still be used even when its price exceeds the one of fossil fuel [65, 68]. Modeling the energy sector without this would yield unrealistic results.

The price of fossil fuels follows the same methodology as the one explained by Nordhaus [66]. In the short term, it is the sum of the marginal cost of carbon extraction ($mcoe$), or how much 1 ton of carbon costs to extract independently of the supply, plus a markup which is the difference between consumer prices and the marginal cost of extraction. Basically, this markup contains costs such as distribution costs, transportation costs and taxes. The form of the equation, calibrated in the following section, is:

$$p_F(t) = mcoe + markup\left[\frac{CCum(t)}{CCum_{max}}\right]^4 \tag{3.9}$$

Note that the equation is highly convex due to the power of four and thus the fossil fuels are quite price-elastic in the short term.

The backstop fuel price has a different formulation overall. The price decreases as R&D increases the knowledge stock associated with backstop fuels. There are of course no limitations to the extraction or use of backstop fuels as the supply is in theory endless and renewable. The functional form for the backstop price is:

$$p_B(t) = \frac{p_{B0}}{H_B(t)^{\eta}} \tag{3.10}$$

where $H_B(t)$ is the stock of backstop knowledge, η is the factor between prices and knowledge and p_{B0} is the initial backstop fuel price. Technological change in this specific case comes through by a learning-by-doing framework. The above equation is similar in form to Equation (3.2) seen in Section 3.1.2.3. In this line of thought, a doubling of the knowledge stock would then reduce the costs by $1 - 2^{-\eta}$. This is commonly known as the progress ratio.

The different knowledge stocks ($H_E(t)$ and $H_B(t)$) are accumulated in a similar way as how the capital stock accumulates. New knowledge is created by R&D. The formulation is:

$$H_i(t) = f(R_i(t)) + (1 - \delta_H)H_i(t-1) \tag{3.11}$$

where δ_H refers to the decay of old knowledge and R_i is the R&D of either the energy efficiency stock (i=E) or the backstop fuel stock (i=B).

The equation linking new knowledge with R&D must first comply to several conditions due to empirical work which suggests that energy R&D exhibits diminishing returns [40]. The first derivative of $f(R_i(t))$ must be positive while the second derivative must be negative so that there are diminishing returns over time [42]. This is true of R&D within a specific field such as energy, however note that for a global R&D there might not be diminishing returns to R&D [40]. One form that meets these requirements is:

$$f(R_i(t)) = aR_i(t)^{b_i}H_i(t)^{\phi_i} \tag{3.12}$$

where as long as b_i and ϕ_i are between 0 and 1 then the conditions are met. This specification is also seen in other modern growth models [69] which are all inspired from the early work of Paul Romer [36]. When ϕ_i is greater than zero, prior R&D increases

R&D productivity.[23] When ϕ_i is lower than zero, prior R&D makes new R&D harder to discover by "fishing out" all the possibilities from the pool of knowledge [69].

As noted in the literature review, the social returns to R&D are higher than the private returns and thus firms will underinvest in R&D. It is important to take this factor into account when modeling R&D. Additionally, the opportunity cost of crowding out other R&D by energy R&D is also substantial and it is modeled here by subtracting 4 dollars of private investment for every R&D dollar crowded out. The formula for the capital stock for this model is:

$$K(t) = \{I(t) - 4 * crowdout * (R_E(t) + R_B(t))\} + (1 - \delta_K)K(t-1) \tag{3.13}$$

where *crowdout* is the % of R&D crowded out by energy R&D.

Also, now with R&D and the energy sector, there are no abatement costs *per se* in this model. These are transferred as the total cost of energy. Abatement costs can be thought of as the cost it takes to include the backstop energy in the model. Thus, total output is regarded as:

$$Q(t) = \frac{P(t)}{1 + \Omega(t)} \tag{3.14}$$

with the balance equation now being:

$$Q(t) = C(t) + I(t) + R_E(t) + R_B(t) \tag{3.15}$$

Finally, some constraints are needed on the growth of the backstop fuel and the decline of the fossil fuel. Due to path dependency and technological lock-in, fossil fuels can only be decreased at a certain rate. It is difficult to get rid of fossil fuels from one period to the next. The following equation is proposed in order to model this:

$$F(t+1) \geq \xi * F(t) \tag{3.16}$$

where ξ refers to how much inertia can the energy system take on decarbonization.

[23]Similar to how knowledge builds upon knowledge until the most recent one is "standing on shoulders" of previous knowledge.

Likewise, backstop fuel cannot grow unconstrained from one period to the next due to the difficulties in implementing large-scale renewables in the energy system. There is then a limit to the growth of backstop fuel per period. This is modeled as:

$$B(t+1) \leq 0.005 + \zeta * B(t) \tag{3.17}$$

where ζ is the limit on growth of the backstop fuel.

3.2.2 Calibration

This section includes a description of all the calibration steps necessary to include endogenous technological change in the DICE-ED. This section can be skipped without any real lose in continuity.

The first step is to add an energy sector to the DICE2013x model. This is done by implementing the above equations without any type of R&D or knowledge stock improvement. The initial fossil fuel use ($F(0)$) is calibrated by seeing the DICE2013x first period emissions and converting them from CO_2 to C by dividing by 3.666.[24]

- $F(0) = \frac{33.553\ GtCO_2}{3.666\ CO_2/C} = 9.15248\ GtC$

β is calculated as the percentage of output spent on energy expenses. In this initial run, there is no backstop fuel so only the cost of the fossil fuel is important. It can be calculated with the following formula:

$$\beta = \frac{Cost_0}{Q_0 + Cost_0}$$

where $Cost_0$ is the initial energy expenses and Q_0 is the initial output gathered from the DICE2013x model equal to 63.473 trillion 2005 USD per year.[25] After this, appropriate values for A_0 and E_0 are chosen to equal the initial output Q_0.

$\Phi(t)$ is calibrated such that emissions from the DICE model are similar to the emissions from the DICE-ED model. The form of $\Phi(t)$ is:

$$\Phi(t) = \exp\left[\left(\frac{g_\phi}{\delta_\phi}\right)(1 - \exp(-\delta_\phi t))\right] \tag{3.18}$$

[24]This is because the atomic weight of carbon is 12 u while the atomic weight for carbon dioxide is 44 u. The conversion rate is then $\frac{44u}{12u}$ which equals 3.666.

[25]Q_0 changes slightly between the baseline and optimal run to account for damages.

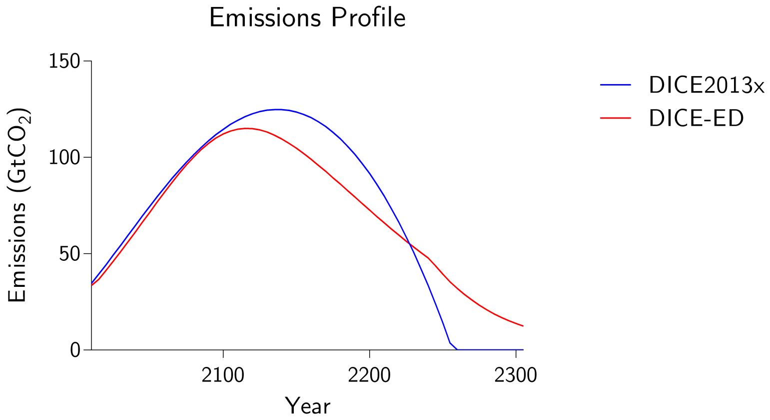

Figure 3.4: Comparison of Emissions between both models.

where g_ϕ is the growth rate and δ_ϕ is the decline rate of the growth rate. After calibration, these two values amount to:

- $g_\phi = -6.85\%$
- $\delta_\phi = 6.74\%$

Riemann sums were used to calculate the area under the curves and the result is that there is a minimum difference of 2.71% in both of the models. Figure 3.4 shows the result of this calibration.

The price of fossil fuels, $p_F(t)$, and backstop fuels, $p_B(t)$, are important for the ultimate allocation of these resources. Nordhaus calibrates his price function with a *mcoe* of 113 1990 USD/ton and a *markup* of 700 1990 USD/ton [66]. However, for a globally aggregated model, Popp calculates a weighted average of all the regional markups to come up with a value of 163.29 1990 USD/ton [55]. The form of the function is then:

$$p_F(t) = 113 + 163.29 + 700\left[\frac{CCum(t)}{CCum_{max}}\right]^4$$

There is only one small caveat that must be addressed. This is that the above function is calibrated to 1990 USD while the DICE-ED uses 2005 USD. Thus, it is important to change these values to appropriate ones. This is done by using the Consumer Price Index (CPI) of the United States [70]. To change from one year to the next, the following formula is used:

$$\text{Second Year Dollars} = \text{First Year Dollars} * \frac{CPI_{\text{Second Year}}}{CPI_{\text{First Year}}}$$

From Table 24 of the CPI report [70], $CPI_{\text{Second Year}}$ is 195.3 and $CPI_{\text{First Year}}$ is 130.7. With this information in hand, the final form of the price function for the fossil fuels is:

$$p_F(t) = 412.85 + 1045.98\left[\frac{CCum(t)}{CCum_{max}}\right]^4 \tag{3.19}$$

Likewise, for the price of backstop fuels, there are two important parameters that must first be defined. The initial price $p_B(0)$ is taken from Nordhaus' own calibration of the DICE model and is 344 2005 USD per ton of CO_2 [21]. Multiplying this value by 3.666 converts it to CTE. The final value chosen is 1200 2005 USD per CTE which is a common starting price for backstop fuels [65]. Finally, η is set at 0.4 which yields a progress ratio of 24%. This means that doubling $H_B(t)$ implies a 24% reduction in the cost. Although a highly arbitrary measure, the 24% progress ratio is considered appropriate for the long-run price of backstop fuels.[26] The final and calibrated form of the function is then:

$$p_B(t) = \frac{1200}{H_B(t)^{.4}} \tag{3.20}$$

The R&D sector is calibrated to initial historical levels in 2010. $R_E(0)$ is the initial level of energy R&D. This is calculated as 2% of the world's R&D expenditure in 2010.[27] The 2% comes from the fact that 2% of the U.S. total R&D is for energy purposes. Total R&D is 999136.3 2010 USD [71]. Converting to 2005 USD with the CPI and multiplying by 2% gives:

- $R_E(0) = 17.8974$ billion 2005 USD

The initial level of backstop R&D, $R_B(0)$, is approximated as 10% of the initial level of energy R&D [72].

- $R_B(0) = 1.78974$ billion 2005 USD

Coupled with $F(0)$, the initial backstop usage $B(0)$ is approximated as the amount of renewable energy being used in 2010. In 2010, around 9.6% of the total energy came

[26] There is currently no reliable measure of η for backstop fuels.

[27] This is proxied as OECD countries which make the bulk share of R&D expenditures in the world.

from renewable sources [73].[28] With $\frac{F(0)}{1-.096}$ being the total amount of energy, then $B(0)$ is:

- $B(0) = 0.972$ CTE

δ_H is the decay rate of knowledge which is a difficult parameter to calibrate as the literature is not specific enough. A study on clinical research found that the half-life of truth was of 45 years [75]. This would imply a decay rate of 1.4%.[29] Other decay rates used within the literature range from 0% (for convenience) up to 25% [76, 77]. Due to the difficulties of estimating a proper decay rate and the wide range found on the literature, this thesis will work with the assumption that there is no decay of knowledge in the energy and backstop fields. Therefore:

- $\delta_H = 0\%$

Both knowledge stocks, $H_E(t)$ and $H_B(t)$, are intermediate variables with no real physical meaning. This allows them to be modeled from any starting point, as long as it is logical. As $H_E(t)$ enters Equation (3.12) multiplicatively, it cannot have a starting value of zero. Also, a value is selected so that emissions in the case with no energy R&D remain unaffected with this initial energy knowledge stock. $H_B(t)$ is normalized to 1 so that in the runs without backstop R&D the price will remain unaffected due to the functional form of Equation (3.20). Thus, the starting values are:

- $H_E(0) = 0.0001$
- $H_B(0) = 1$

The ease of substitution parameters in Equation (3.8), ρ_H and ρ_B, are calibrated according to different criteria. ρ_H is changed so that the initial elasticity of energy R&D with respect to energy prices is equal to 0.35, which is an empirical value obtained from the literature [40]. ρ_B is obtained from the first-order conditions for energy demand. This substitution parameter makes use of the initial values of the model. The derived equation is: $\rho_B = {}^{\log(p_F(0)/p_B(0))}/_{\log(F(0)/B(0))} + 1$. The resulting values are:

- $\rho_H = 0.38$

[28] Not taking into account nuclear and traditional biomass as sources. Nuclear is not counted due to its controversial status while traditional biomass is not counted due to the fact that it is not considered sustainable [74].

[29] $0.5 = \exp^{-\delta_H * t}$ with $t = 50$ years then $\delta_H = 1.4\%$

- $\rho_B = 0.524$

This means that the elasticities of substitution are 1.61 and 2.1 respectively. A higher elasticity means that it is easier to substitute between alternatives. In this case, for an increase in energy prices, more backstop energy is induced than energy efficiency measures.

α_H and α_Φ are also parameters from Equation (3.8). α_H is calibrated so that each new dollar of R&D gives four dollars of energy savings. α_Φ is chosen at 80%. This is what remains from the exogenous technological change once endogenous technological change is added to the model. Their values are:

- $\alpha_H = 0.336$
- $\alpha_\Phi = 0.8$

ξ is the maximum decline of fossil fuels per period, a concept known as decarbonization. The IPCC notes that the decarbonization of the world energy system is comparatively slow, at an annual rate of -0.3% through the 20$^{\text{th}}$ century [78]. Some models use decarbonization rates around this area [79]. However, from 1990-2007, the world has been *carbonizing* at a rate of 0.03% per year [80]. However, with an increase in energy efficiency, the decarbonization of GDP has been between 1.2% to 2.5% [78]. From 2010 onwards, there is much uncertainty about future decarbonization although international agreements such as the Paris one promise some momentum forward. For this modeling practice, the rate of -3% will be chosen. This is to be the maximum decarbonization, as it is a constraint. More decarbonization than this is not possible in the model. In a period, the rate would be: $(1 - 3\%)^5 = -14\%$. Thus:

- $\xi = 0.86$

ζ is the maximum growth that the backstop fuel can have per period. From The Statistical Review of World Energy 2016 by British Petroleum, the combination of hydropower and other renewables has been growing at a rate of 3.4% per year from 1965 until 2010 [3]. On a five year period, this would mean a rate of: $(1 + 3.4\%)^5 = 18\%$. Being marginally more optimistic and rounding up, ζ is defined as:

- $\zeta = 1.2$

Finally, a_i, b_i and ϕ_i are chosen so that future elasticities fit the desired time path. Figure 3.5 and Figure 3.6 show the results of this calibration. The expected R&D with

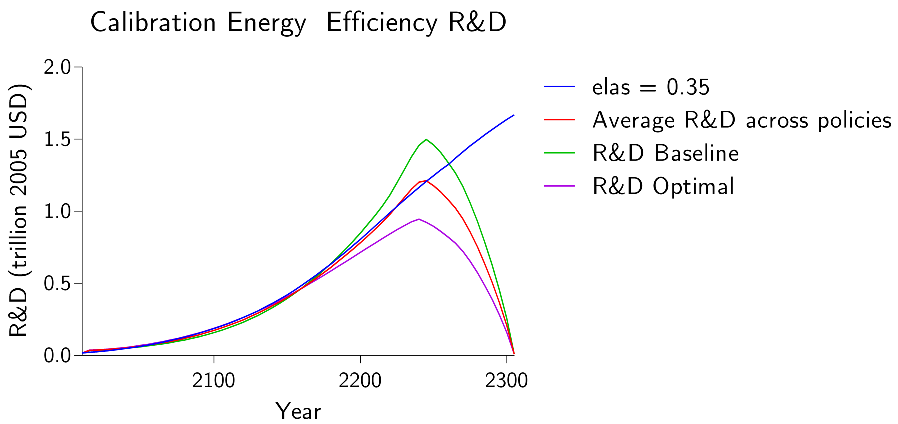

Figure 3.5: Calibration of Energy Efficiency R&D.

an elasticity of 0.35 is calculated and shown on the figures. Calibration requires having the average R&D path of both policies to be in line with this expected elasticity. The values of a_i, b_i and ϕ_i are chosen to match this path as closely as possible.[30] To take diminishing returns to R&D into account, the sum of b_i and ϕ_i has to be less than 1 [63].

- $a_E = 0.0262$
- $b_E = 0.29$
- $\phi_E = 0.60$

- $a_B = 0.01$
- $b_B = 0.067$
- $\phi_B = 0.60$

[30]The values for the energy efficiency R&D are changed slightly in the case without backstop R&D in order to simulate the same path.

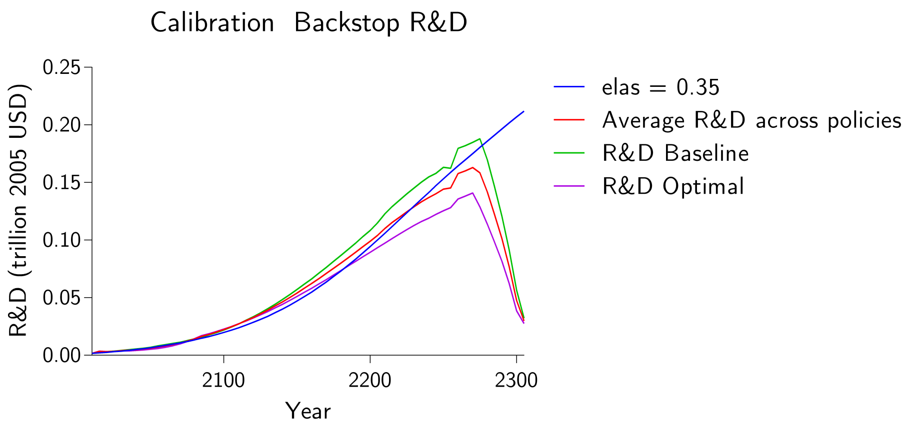

Figure 3.6: Calibration of Backstop R&D.

3.3 Economic Analysis

This section presents the results of the model runs and its economic analysis.

As discussed in Section 2.2, two different scenarios were chosen for the analysis. The baseline run involves having no damages in the economic system and thus represents an extreme case of business-as-usual. The optimal run contains the damages caused by emissions and thus fully appropriates the externality in the economic system. Each of these scenarios builds upon two cases:

1. Endogenous technological change with energy R&D but no backstop.
2. Endogenous technological change with energy R&D and backstop R&D.

The purpose of this is to also isolate the effect of the backstop technology.

One of the main sub-questions of this thesis is to analyze the effect that endogenous technological change has on the model. At this point, it is possible to isolate endogenous technological change as the main driver of change as there has been no modification with regards to discounting. Each scenario will analyze the major impact on the variables from the climate, economic and energy modules of DICE-ED.

3.3.1 Baseline Scenario

The baseline run presents an extreme case of business-as-usual due to the absence of damages from the carbon emissions. The model maximizes welfare without any regard

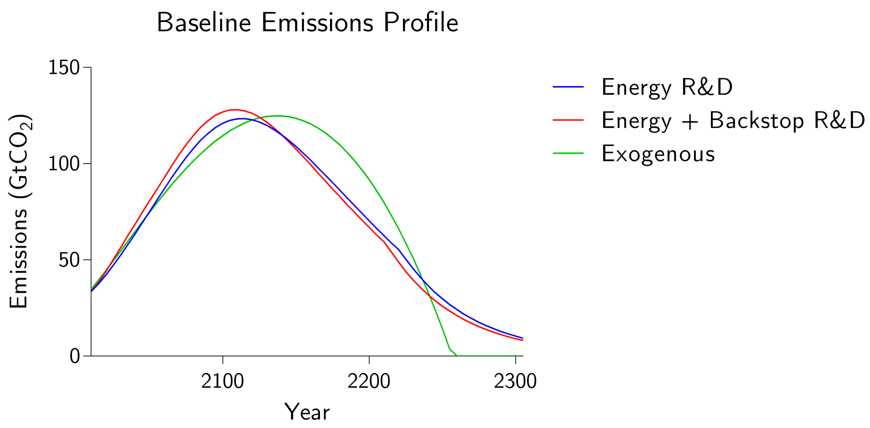

Figure 3.7: Emissions profile of the baseline run.

for the climate module. The climate module in this case plays a secondary role with the module only reacting to the changes from the economic and energy modules.

First, the climate module will be analyzed followed by the energy module. The economic module analysis finishes the analysis on the baseline scenario.

3.3.1.1 Climate module

One of the most important drivers of the carbon module are the carbon emissions. Carbon emissions are what ultimately policy-makers aim at while implementing new policies. What is the effect of endogenous technological change on emissions? Figure 3.7 shows the emissions profile of the exogenous case and both endogenous cases.

Endogenous technological change has a clear effect on the timing of emissions. The DICE2013x model emissions go to zero at the end because the limit of fossil fuel extraction reaches 6000 GtC which is the upper limit. In this simulation with exogenous technological change, absolutely *all* the fossil resources are used and this path is considered optimal by the solution algorithm. With endogenous technological change, the fossil fuel extraction does not reach its upper limit, although it gets close to it.[31] It follows that including endogenous technological change in the model reduces the amount of emissions to the atmosphere. This would mean that the maximum temperature would decrease by a small part when there is endogenous technological change. In fact, the maximum

[31] 5872.438 GtC in the Energy R&D case and 5916.486 GtC in the Energy + Backstop R&D case.

temperature increase in the exogenous case is **7.05°C** while the maximum temperature increase in the endogenous cases is **6.80°C**.

There is also a slight difference in the emissions path when comparing both endogenous cases. In the short-run, including the backstop fuel increases emissions but ultimately decreases the emissions in the long-run. This is due to the fact that in the short-run, the backstop fuel is considerably more expensive than the fossil fuel and thus it makes more sense to let emissions increase which generates greater cumulative discounted utility. When the price of the backstop has decreased enough, then fossil fuel becomes less useful and the emissions can ultimately be lower.

Following the logic from the previous paragraph, a change in the initial backstop fuel price would ultimately change the emissions curve to accommodate the new market price. A higher initial backstop fuel price would induce more emissions in the short-run while a lower initial backstop fuel price would have the counter-effect. A small sensitivity analysis is presented in Figure 3.8. A higher initial backstop fuel price (1600 2005 USD/GCTE) indeed induces a bit more emissions in the short-run compared to the base case of 1200 2005 USD/GCTE. The big difference comes with the lower price of 800 2005 USD/GCTE. As expected, while the price of the backstop fuel is still above the fossil fuel price, the emissions in the short-run are almost in line with the base case. After the year 2160, denoted by the green dotted line, the price of the backstop fuel is now lower than the price of the fossil fuel. Around this point, emissions start decreasing drastically as the model optimizes and prefers to use the cheaper and cleaner backstop fuel than the polluting fossil fuel. In comparison, for the high initial backstop fuel price, it is only after 55 years in 2215 that the price of the backstop fuel is lower than the price of the fossil fuel. With the base case, the backstop fuel is only cheaper after the year 2190. Assuming the lower backstop fuel price decreases total cumulative emissions by more than 200 GtC.

Assuming a lower backstop fuel price not only drastically reduces the emissions but also lowers the maximum atmospheric temperature. With the lower backstop price, the maximum increase in atmospheric temperature rise is **6.60°C**. This is a 0.20°C difference with the base case of **6.80°C** for a 3% change.

The above analysis explains some important dynamics of the R&D market and the assumptions. The assumption of the initial price of the backstop fuel is important. Practically there is no possible way to have an accurate initial price due to the variability

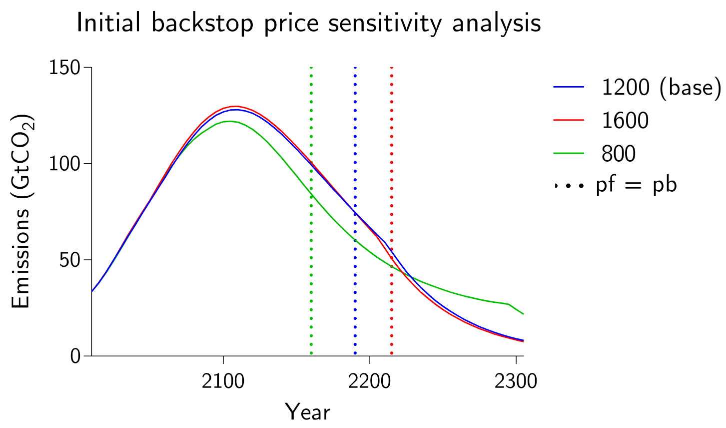

Figure 3.8: Sensitivity analysis due to the initial backstop price.

of backstop sources in terms of location and resources. But the real important point is that having a lower backstop fuel price will ultimately reduce emissions in the long-run when simple market forces will allocate resources to the more plentiful cheaper resource. For policy-making, focusing on R&D that purposefully lowers the backstop fuel price will yield fruitful emissions reductions in the long-term.

3.3.1.2 Energy Module

The energy module is introduced to the model in this thesis so it is not possible to do a comparison with the case of exogenous technological change. Nonetheless, a close look at its major variables gives further insights about the structure and limits of the model.

Figure 3.9 shows the evolution of the backstop fuel with time for the case of Energy + Backstop R&D. It is only after year 2130 that the backstop fuel represents 50% of the energy use in the base case. With a lower initial backstop fuel price, the year in which the backstop fuel reaches the 50% mark is preponed. In the long-run, all cases reach almost 100%. This graph clearly shows the energy transition which is bound to happen. The timing of the transition is what is really up to debate.

3.3.1.3 Economic Module

The economic module is the module which changes the most with endogenous technological change. For a first impression of the results, the welfare of the different cases must be compared to the one with exogenous technological change. Table 3.1 shows the welfare

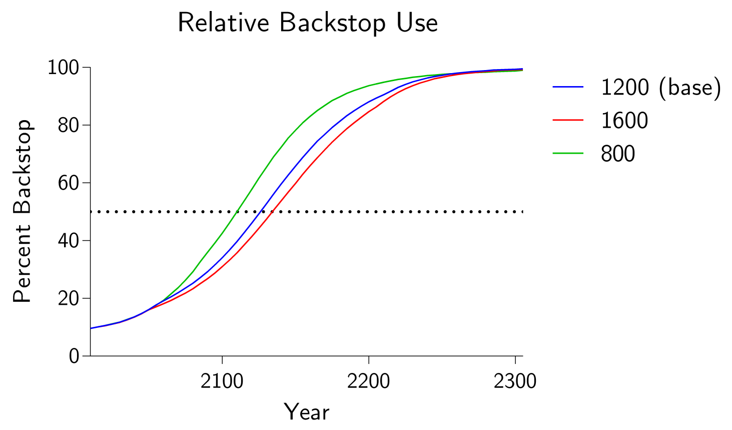

Figure 3.9: Relative usage of the backstop technology.

Table 3.1: Welfare comparison in baseline scenario.

	Welfare (utils)	Relative to exogenous
Exogenous	2741	0%
Energy R&D	2927	+6.8%
Energy + Backstop R&D	2986	+8.9%

comparison between the exogenous and the two endogenous cases. With endogenous technological change, the welfare of the economic system is increased up to 8.9%. This means that there is more "utility" to society as there is more discounted consumption per capita. The issue that consumption is better for society is a subjective issue and is beyond the scope of this thesis.

One key variable that changes significantly when endogenizing technological change is the savings rate. Figure 3.10 shows that the average savings rate increases with endogenous technological change. A higher savings rate implies that more is being invested, the capital stock is increasing, and future production can increase. An increase in the production can be seen in the long-run with endogenous technological change as portrayed in Figure 3.11. The output with the Backstop R&D increases by a bigger margin when the price of the backstop fuel has decreased enough to be lower than that of the fossil fuel, marked with the green dotted line. With the possibility to add cheap, unlimited energy to the production function, the overall output increases.[32] This is analogous to

[32]In an almost identical fashion, consumption per capita grows in the exact same way as output and is

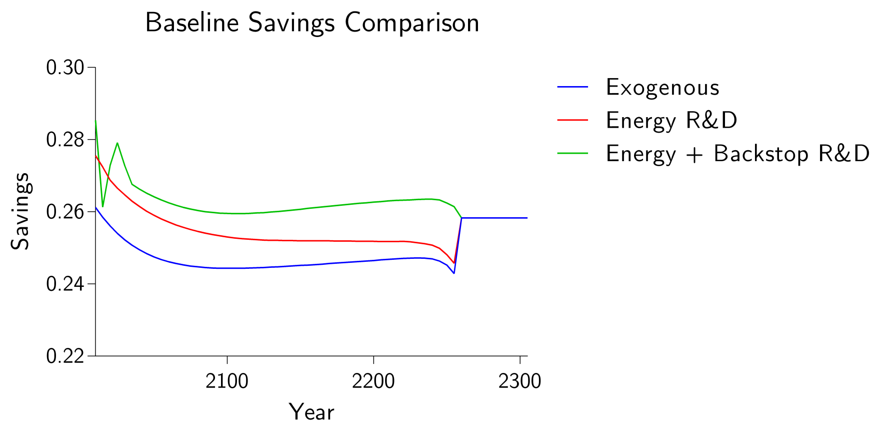

Figure 3.10: Baseline savings rate comparison.

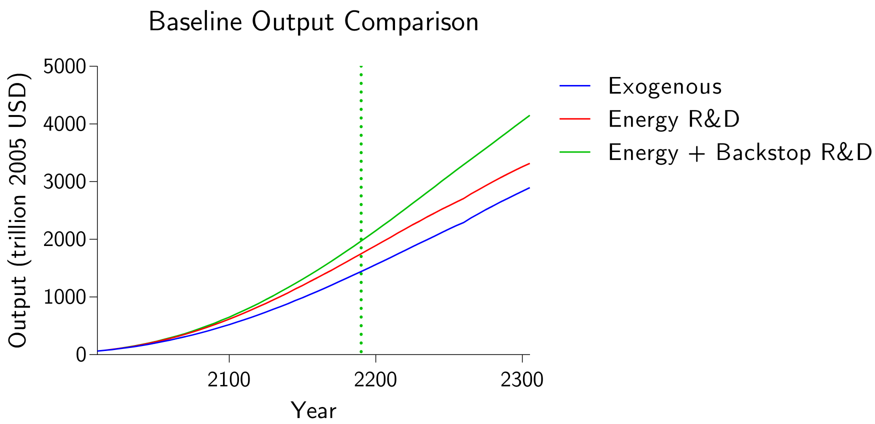

Figure 3.11: Baseline output comparison.

growth in the 20th century with cheap oil. Even though the differences in final output might differ in the three cases, the effect on total welfare is not as big as it initially looks due to the discount factor. Long-run output is discounted at almost negligible present values with the current discounting practice. The next chapter looks at this discounting issue in greater detail.

The inclusion of the R&D sector has to be accounted for. As seen in the literature review, the social rates of return of R&D are four times the private rates of return. This would mean that the actual rates of return have to be at least four times bigger than

thus not illustrated here to avoid redundancy.

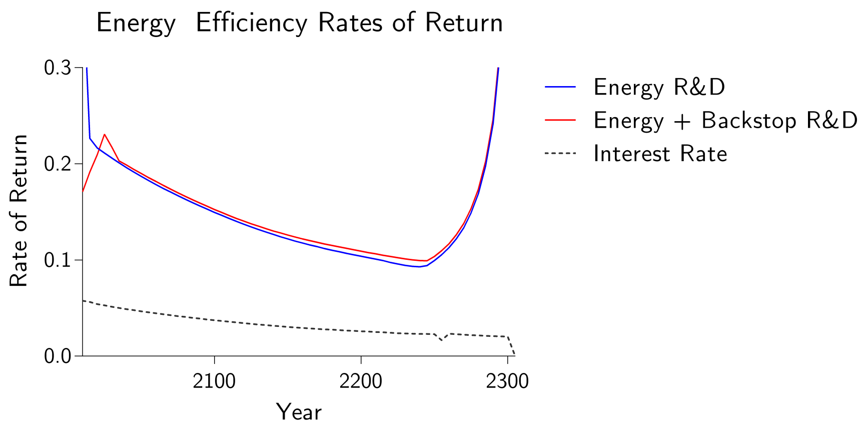

Figure 3.12: Baseline Energy Efficiency Rates of Return

the real interest rate. Figure 3.12 shows the real interest rate (dashed black line) and the rates of return in both endogenous cases. The rates of return are calculated as the change in output over a change in R&D, in mathematical notation:

$$\frac{\partial Q}{\partial R_i} = \frac{\partial Q}{\partial ES} \cdot \frac{\partial ES}{\partial p_i} \cdot \frac{\partial p_i}{\partial H_i} \cdot \frac{\partial H_i}{\partial R_i} \tag{3.21}$$

All the different partial derivatives are calculated within the model.[33] In most of the timespan of the modeling, the rates of return to R&D are exactly four times the real interest rate. It is only at the extremes that the rate of return is greater than the real interest rate. This is especially true at the end of the timespan because output increases significantly while R&D decreases as seen during the calibration section.

3.3.2 Optimal Scenario

The optimal scenario includes damages into the economic model. Here, the climate module plays a major role now as an increase in temperature causes monetary damages to the economy which must be taken into account. This scenario simulates what would happen if all the economy was left solely to the market forces. Will an invisible hand come to the rescue?

In this section, it is also important to see the difference between the baseline scenario and optimal scenario as this could help policy-makers understand the effects of the externality upon the system.

[33] Refer to Appendix B for the full code.

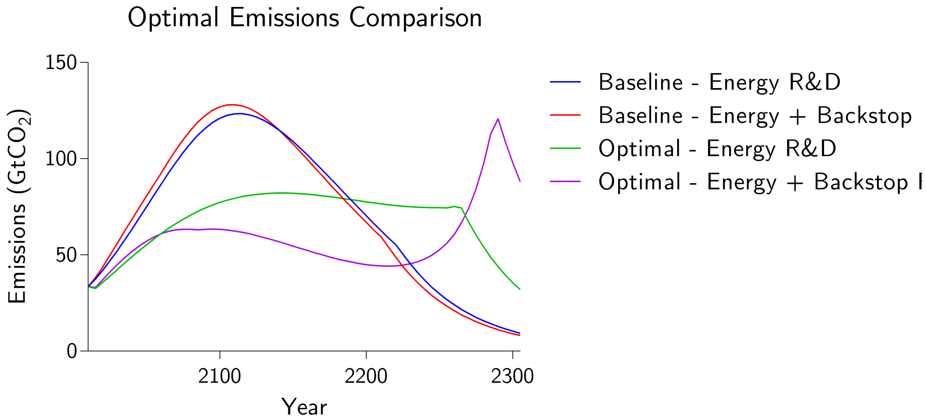

Figure 3.13: Optimal emissions comparison

3.3.2.1 Climate Module

As before, emissions are analyzed first. Figure 3.13 shows a comparison between the optimal and baseline scenarios. As expected, total emissions are decreased when the optimal scenario is run. This is due to the simple effect that for every $GtCO_2$ emitted, there is a corresponding damage. Figure 3.14 shows the level of cumulative emissions for each scenario and for both endogenous cases. Cumulative emissions for the optimal run with backstop R&D amount to 4798 GtC while its baseline counterpart amount to 5889 GtC which gives a difference of 18.5%! It is again the effect of the backstop fuel which decreases total emissions significantly.

For the optimal run with backstop R&D, emissions have to initially peak around the end of the 21st century. For this curve, a second and larger peak occurs at the end due to the effect commonly known as the *end-of-horizon effect*. The solution algorithm believes that the "end of the world" is near and thus optimizes a new increase in emissions which generates extra output to contribute for a higher consumption. This is one problem inherent with long-term modeling. One way to avoid this type of algorithmic inconsistencies is to present results only until the end of the 21st century which is what most politicians would consider acceptable. Figure 3.15 shows the same graph as before but only until the year 2100 which makes it potentially easier to explain and illustrate to the general public.

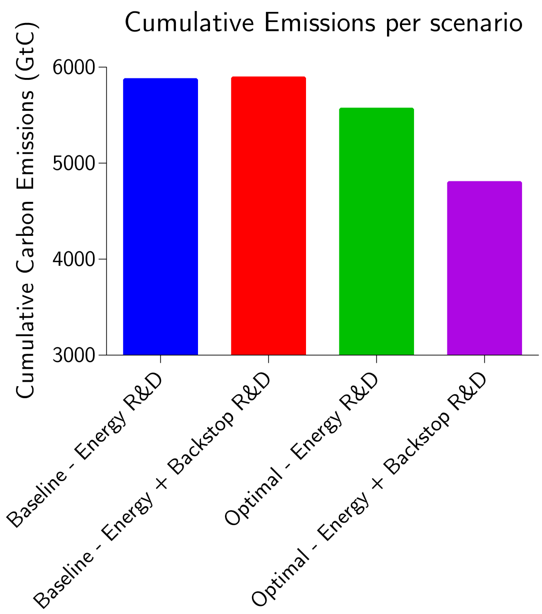

Figure 3.14: Cumulative emissions comparison.

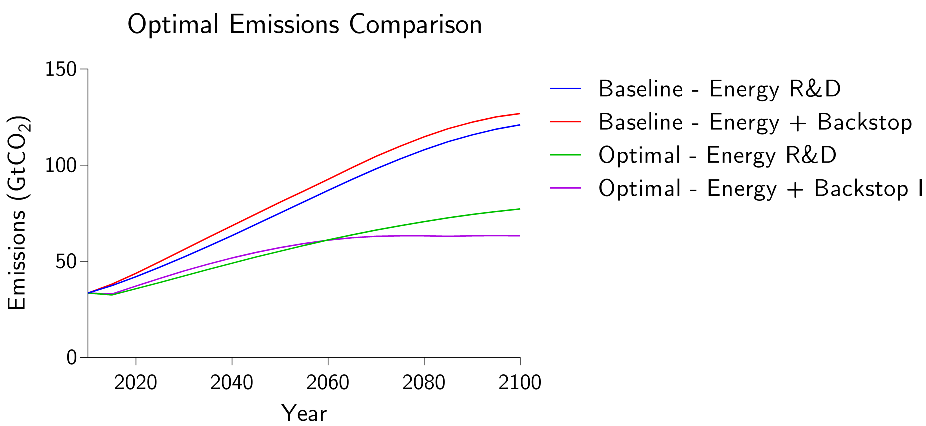

Figure 3.15: Emissions only until year 2100.

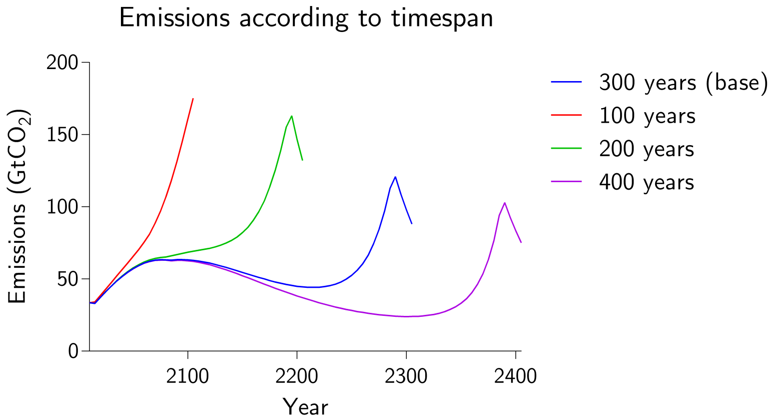

Figure 3.16: Sensitivity analysis on timespan.

Table 3.2: Area under curve analysis.

	Area	Relative to base
100 years	7892	0.45
200 years	15309	0.88
300 years (base)	17397	1.00
400 years	18019	1.04

In order to assess the impact that the timespan has on the emissions curve, a small sensitivity analysis was conducted that changed the timespan of the model. Figure 3.16 shows the results of this. Changing the timespan 100 years has a similar shape as the base case (300 years), although with a difference in the timing. The area under the curve gets progressively bigger as the timespan increases. Again, *Riemann Sums* were used to analyze the area under the curve. Table 3.2 show the results of this analysis. However, for a timespan of only 100 years, the emissions profile changes drastically. Zooming in until the year 2100 proves this. Figure 3.17 shows how the emissions profile gets dramatically bigger when the modeling timespan is only of 100 years. The analysis of the impact of stipulating timespans on the simulation outcomes is beyond the scope of this thesis. Figure 3.17 illustrates that the impact is not negligible and model outcomes are sensitive to the stipulated timespan; further investigations of this issue are left for future research.

With regard to atmospheric temperature, running the optimal scenario lowers the ultimate atmospheric temperature. The temperature increase at the end of the timespan

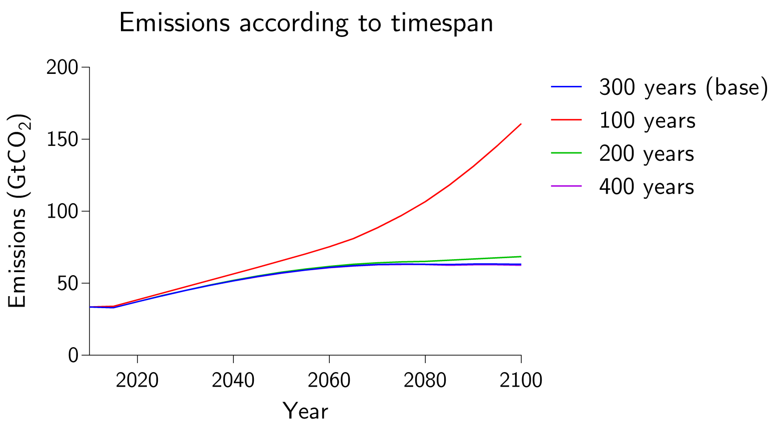

Figure 3.17: Sensitivity analysis on timespan until year 2100.

with the backstop R&D is **6.15°C** while in the baseline scenario it is **6.81°C**. This makes for a difference of 9.7%. The reason that this model gives very high temperature increases is three-fold:

1. The solution algorithm optimizes without any command-and-control policy in place such as limiting temperature to 2°C or concentrations below 550 ppm.

2. Damage functions are usually calibrated to lower temperatures. When working with high temperatures, there is no tipping point or extreme case modeled. There exists a lack of information into how the damages will look for high temperatures. Including extreme damages after a certain temperature could potentially improve the modeling results.

3. The numbers obtained from these simulations are not particularly important. What really matters is the effect of endogenous technological change. Any real estimates on the future state of the world after 300 years are sure to be standing on unstable ground.

Again, the initial backstop fuel price plays a major role in the results. This can be visualized in Figure 3.18. The low backstop fuel price shows a big reduction in atmospheric temperature increase from 6.17°C (base) to 4.73°C. This is a 23.34% difference with the base case! The reason for this is that having a lower initial backstop fuel price increases the elasticity of substitution between fossil fuel and backstop fuel. The base case of 1200 2005 USD/GCTE has an elasticity of substitution of 2.1 while with 800 2005 USD/GCTE the elasticity of substitution is 3.39. This makes it easier to substitute between fuels and

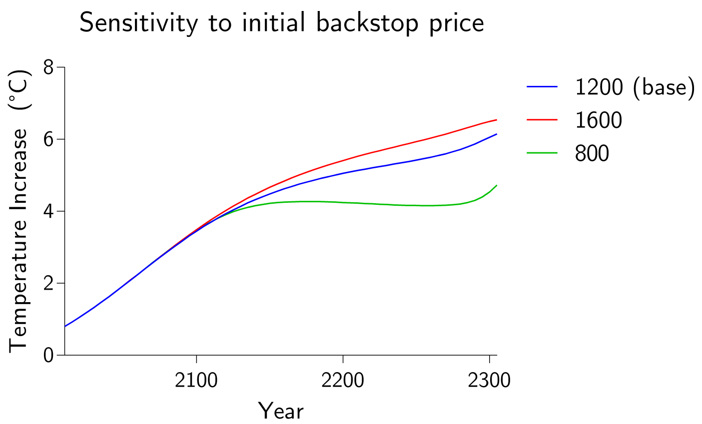

Figure 3.18: Temperature sensitivity analysis on initial backstop price.

accelerates the rate of backstop usage. However, even if the difference between these cases is relatively big, it is not enough as the temperatures by 2100 are around 4°C higher!

All of this stresses the importance of adding endogenous technological change and specifying a backstop technology within the model. One important insight is that renewable energies are not sufficient to limit the rise of temperature. Figure 3.18 shows that even in an optimist situation with low backstop fuel price the atmospheric temperature rise reaches levels above 4°C! The recent Paris agreement would fail catastrophically as it aims for a maximum temperature increase of 2°C [10]. For this, policy-makers must not only advocate for the growth of renewables, but also for command-and-control policies which can limit total emissions and limit atmospheric temperatures below dangerous levels. The key takeaway is that renewable energies are not a panacea, they need to be combined with other policy measures and strategies in order to tackle global climate change. This is of no surprise as climate change is a wicked problem: the solution is multi-dimensional and cannot be brought about by just one single policy instrument.

3.3.2.2 Energy Module

Counter-intuitively, running the optimal scenario has some important effects. Due to the fact that less fossil fuels are used in the optimal scenario, the price of the fossil fuels rises at an almost glacial pace. This then has an impact on total backstop usage because the price of fossil fuel is mostly below that of the backstop fuel. Figure 3.19 shows the

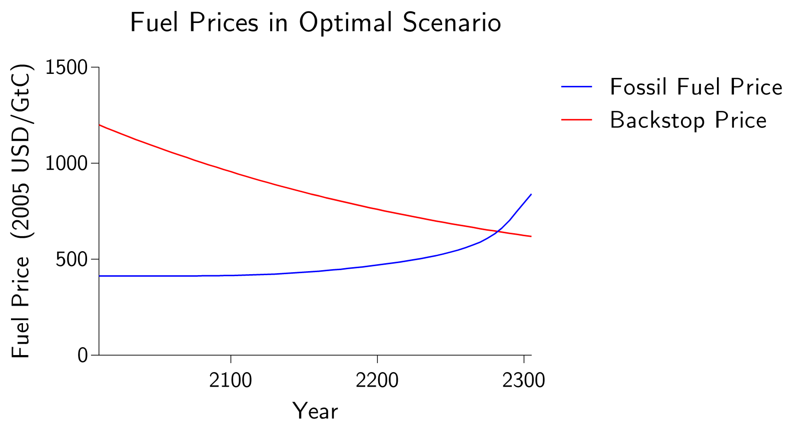

Figure 3.19: Fuel prices in optimal scenario.

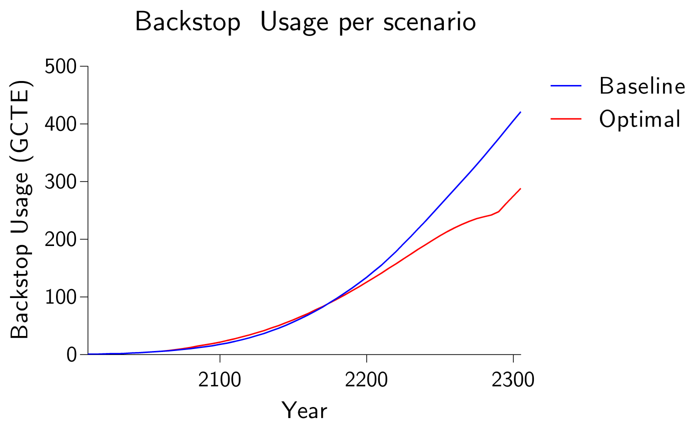

Figure 3.20: Backstop usage difference between the two scenarios.

dynamics between these two. The backstop fuel price only gets below the fossil fuel price in the year 2285 which proves to be quite late. Comparing the backstop usage from the optimal and the baseline scenario gives proof of this (Figure 3.20).

3.3.2.3 Economic Module

With the baseline scenario being an extreme case of business-as-usual and backstop usage being less as shown above, it can only be expected that the optimal scenario results in lower overall welfare. Table 3.3 shows the results on the welfare. Comparing it to the exogenous case, endogenous technological change has a similar effect in terms of the

Table 3.3: Welfare comparison for optimal scenario.

	Welfare (utils)	Relative to exogenous
Exogenous	2688	0%
Energy R&D	2871	+6.8%
Energy + Backstop R&D	2932	+9.1%

relative gains. In absolute values, the optimal scenario results in lower welfare due to the presence of the externality via the damage function.

An important topic to discuss with the optimal scenario is the development of the Social Cost of Carbon (SCC). In simple terms, the SCC is the economic cost caused by an additional ton of carbon emissions. In programming jargon, the SCC is the shadow price of emissions along the output path. In an ideal economy[34], the SCC would represent the carbon price. The SCC is dynamic in time, changing every year with each new state of the model. It is calculated as:

$$SCC(t) \equiv \frac{\dfrac{-\,\partial W}{\partial E(t)}}{\dfrac{\partial W}{\partial C(t)}} \tag{3.22}$$

The numerator and the denominator are easily calculated by GAMS as the marginal values of the variables $E(t)$ and $C(t)$. Figure 3.21 shows the SCC for the exogenous run and both endogenous runs. The three runs share a similar shape with the notable difference of the peaks. Including endogenous technological change with only energy R&D increases the SCC slightly compared to the exogenous case. However, when including backstop R&D, the SCC increases significantly compared to the exogenous case. The reason for this is that including a backstop fuel in the model makes the marginal utility of consumption smaller because in the endogenous cases the consumption is divided between investments and R&D while in the exogenous case there is no R&D sector. Opening up the R&D sector in the endogenous cases then reduces the value of consumption and its effect on welfare. Consumption is less valuable as dollars can now go to the R&D sector where they will also produce some value by reducing costs of the backstop fuel or increasing energy efficiency.

[34]Free of regulatory or tax distortions.

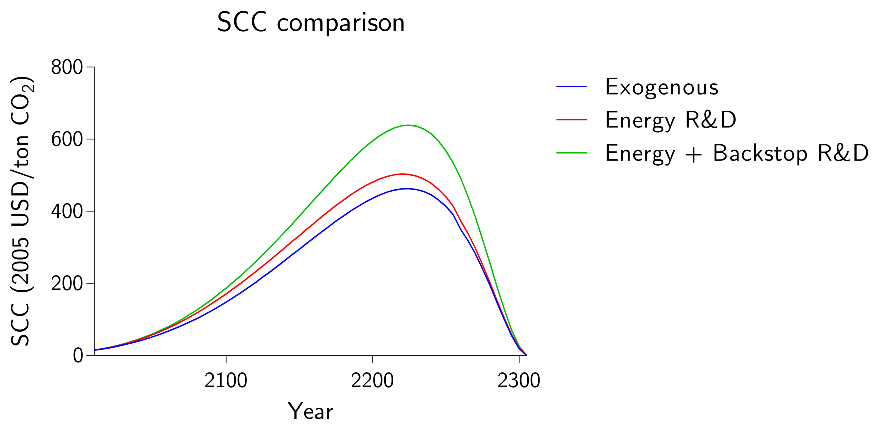

Figure 3.21: Social cost of carbon comparison between runs.

Table 3.4: SCC for different cases. Units in 2005 USD/ton CO_2.

	2010	**2020**	**2050**	**2100**	***Growth per year***
Exogenous	14.84	21.31	52.18	148.02	*2.6%*
Energy R&D	14.68	22.05	58.16	170.99	*2.8%*
Energy + Backstop R&D	14.92	22.42	60.62	187.09	*2.8%*

Table 3.4 and Figure 3.22 show the SCC until the end of the century. Now under close examination, the SCC of the endogenous case with backstop R&D is significantly bigger than the exogenous case as early as 2050. This difference in 2050 accounts to 16.17%. Annual growth rates among the three cases roam around the 2.8% mark through the end of the century. However, these rates change to around 1.7% when taken until the year 2225 (where the SCC is at its maximum). This again states the importance of the timespan in policy-making.

As for the total output, in this scenario the output gets decreased in comparison to the baseline scenario. The damage function is highly accountable for this. Figure 3.23 shows similar results such as those in the baseline scenario. Output with backstop R&D is increased due to the possibility of the backstop technology. The optimal output is lower due to the damage function. Figure 3.24 shows the difference between both endogenous cases. Including a backstop fuel ultimately lowers the fraction of lost output due to damages because of the emergence of the non-polluting fuel which does not increase atmospheric temperature. Long-run damages (around 10%) are incredibly big because of

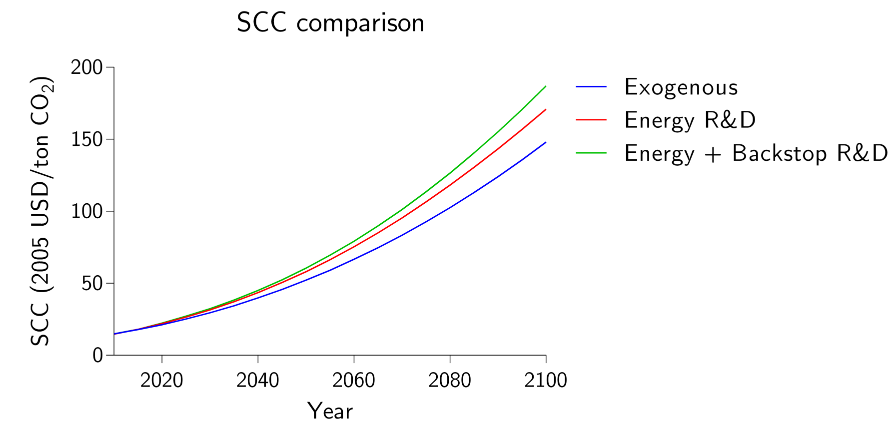

Figure 3.22: Social cost of carbon comparison until year 2100.

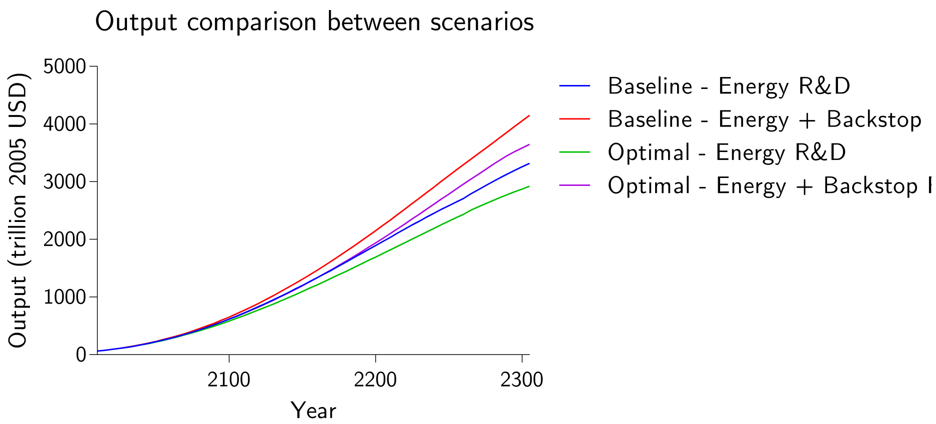

Figure 3.23: Comparison of output between different scenarios.

the high temperature increase (6.15°C). It is possible that in a world of 6°C damages might well be over 50%. There is much uncertainty again about damages in the high temperature range and it is considered an area of improvement for future modeling purposes.

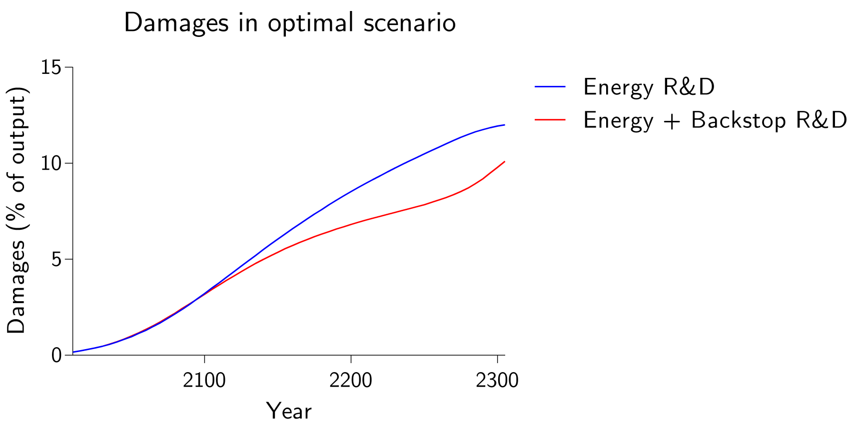

Figure 3.24: Damages across endogenous cases in optimal scenario.

3.4 Concluding Remarks

This chapter has dealt with endogenous technological change in a climate policy model. An extensive literature review was first presented. The DICE2013x model was reformulated to include endogenous technological change via R&D. Along with this, the calibration steps were explicitly described for transparency and to achieve consistency in the modeling. Finally, the model was run and its results were presented.

Including endogenous technological change has some important effects on the model results. First of all, total welfare is improved over the exogenous case by 8.9% with a backstop fuel. Total emissions are also significantly lower in the long run; in the early periods emissions are actually higher than the exogenous case as the price of the backstop fuel is higher than the price of the fossil fuel which incites extra emissions. Ultimately, lower temperatures are achieved with the inclusion of endogenous technological change. The bigger effect is seen when a backstop fuel is introduced into the model compared to just modeling energy R&D. In all modeling scenarios, the backstop fuel ultimately represents 100% of the economy's energy requirements by the end of the model's timespan.

In economic terms, the inclusion of endogenous technological change increases both net output through an increase in the general savings rate of the economy. This ultimately increases total welfare by having a higher consumption per capita. The SCC also increases compared to the exogenous case which for policy-makers means a higher carbon tax. Another important insight is that to tackle global rise of atmospheric temperatures, the inclusion of a backstop fuel is not enough. Extra policy measures have to be in place with a focus on limiting emissions in order to prevent dangerous climate change.

Many of the benefits accrued to a backstop fuel are seen in the far-distant future. These benefits are all brought back to the present time via the discount rate. With the current practice, many of these benefits are almost worth nothing due to the power of compounding. The discussion of discounting in climate policy models is primordial and with good reason. The next chapter focuses on this.

Chapter 4

Social Discounting

This chapter deals with the important issue of social discounting.[1] First, a literature review is presented to introduce the topic. Then, without claiming any right way of how to choose the discount rate, different methods of dynamic discounting will be presented and assessed. Each of them will be presented in terms of equations, calibration and reasoning. A comparative economic analysis follows this section. Concluding remarks are presented at the end.

4.1 Literature Review

Discount rates are used in economic analyses to account for the difference in time between varying economic effects. In essence, the discount rate links the future with the present so that all values can be compared on similar terms (i.e. the present). This is due to the fact that money has a time value, where money is worth more in the present than in the future due to potential earning capacity. Thus, when comparing different cash values in different points in time, it is necessary to account for the time value of money. One of the easiest ways to visualize the discount rate is with the following formula:

$$PV = \frac{FV}{(1+r)^t}$$

where PV is the present value of any transaction, FV is the future value, r is the discount rate and t is the time in years. The discount rate is mostly positive as it allows investments today to produce more in the future; this is what has been expected and

[1]As a reminder, social discounting is discounting applied to social projects such as climate change mitigation. In this chapter, the social part is implicit when mentioning discount rates.

what has become the norm.[2]

Due to the nature of climate change, models for policy analysis are usually confronted with long time spans. Discount rates are highly debated in the literature [12], as the selection of this particular value changes the whole decision-making. One of the most common examples is the difference in selection between William Nordhaus and Nicholas Stern. Nordhaus uses a discount rate of 4.3% on his DICE model and the result is inaction on climate change [66] while Stern employs a discount rate of 1.4% and the results are the opposite, urgent climate action [16]. How is it that a 2.9% difference is enough to drastically change the results of the DICE model?

The current generation has to basically make a decision between investing money on mitigation or in capital and education. If the real rate of return of mitigation is higher than the real rate of return of capital and education, then future generations will be better off if investment is made on mitigation. So, in a way, money is spent today to avoid future climate change damages in the future.

The question that arises is how to select the appropriate social discount rate. And throughout the literature, there are mostly two views on this issue: the *prescriptive approach* and the *descriptive approach*. The prescriptive approach is of a more normative behavior with ethical foundations as its base. It asks the question: "What should the discount rate be from an ethical point of view when considering future generations?". The descriptive approach, on the other hand, is of a positive nature with (mostly) market interest rates as its base. It asks the question: "How does the current generation value the future?".

Before going into each approach, it is necessary to analyze one of the most common approaches to social discounting, the Ramsey equation [82]:

$$r = \rho + \alpha g \tag{4.1}$$

where r is the social discount rate, ρ is the social rate of pure time preference, α is the value of the elasticity of marginal utility and g is the growth rate per capita.

[2] There are also other reasons why the discount rate should be positive: society's impatience to consume today and the justification that future generations will probably be richer than the present ones, thus their consumption should be valued less [81].

Table 4.1: Modeling values of Nordhaus and Stern

	ρ	γ	g	r
Nordhaus	3%	1	1.3%	4.3%
Stern	0.1%	1	1.3%	1.4%

ρ is the trade-off between the utilities of present generations and the utilities of future generations[3], assuming that they will exist. It is how the future is seen through today's telescope, as argued in [81]. If all generations are valued equally, then the value would be zero. If ρ is high then the value of consumption is highly differentiated between generations [21]. Some values close to zero are sometimes assigned due to human extinction possibilities in which humankind will not last forever [16].

α is the general trade-off between different consumption now versus consumption in the future, regardless of the date. It measures the relative effect of income on welfare. It is also a measure of society's aversion to interpersonal inequality[4] and risk in consumption [81]. In essence, a higher value means a more egalitarian society (intergenerational) which translates into a higher discount rate in which society would be less willing to act on climate change, as it will redistribute resources towards the future. This seems like a paradox, where the more egalitarian society is, the less it cares about climate change, as future generations will be thought off to be richer than current ones.

g is then the expected growth in the economy, mostly through growth in GDP. This can be extrapolated from past values or on an expected basis.

Both ρ and α require value judgments from the modelers and this is the crux of the literature's discussion on the social discount rate. As an example, Table 4.1 shows the values used in the modeling practices of Nordhaus and Stern.

As can be seen, the primary difference lies in the selection of ρ.[5] The difference in r might not seem really substantial, after all, it is just less than a 3% difference. However, it is significant when looking at long time horizons. For example, the present value of a

[3] Utility can be thought of as the enjoyment and happiness that one gets from consumption.

[4] Between the rich and the poor.

[5] The choice of α is also criticized in the literature, marking it as unethical to choose 1 as the value. A higher value (around 2-3) would be a preferred choice [81].

$100 damage 100 years from now is worth just $1.49 to Nordhaus and $24.9 to Stern! That is almost 17 times smaller in comparison!

4.1.1 Descriptive Approach

The descriptive approach, as previously said, is of a more positive nature, with facts and objectivity as its strength. Many climate change models are based on this, with Nordhaus' DICE as the most prominent one. There are three main arguments which support this view [14]:

1. Spending on mitigation crowds out other investment.[6] It is important to choose the option that maximizes total consumption.

2. If the rate of return (mitigation projects) < the rate of return (other investments), then current and future generation will be worse off.

3. There is no justification of using any other option other than society's actual choices with the current rates of return on savings.

Therefore, the descriptive approach is concerned with opportunity costs. Choosing the option with the highest return is the only viable option, both for future and present generations. If no money is spent on mitigation today and other alternatives are chosen, future generations will be richer and this will allow them to adapt to climate change. Market interest rates serve as the best proxy for "alternative investments" which could be made instead of mitigation actions.

One of the most important issues to solve is the difference between social and private returns. The descriptive approach must specify if it is using after-tax return (private return) or pre-tax return (social return) as the values differ and can have significant effects on project valuation [83]. Another parallel issue is whether to use government bonds, private stocks, treasury bills, etc.[7] And finally, it must also justify the selection of a country to base the worldwide phenomenon on.

4.1.1.1 Criticism

One of the strongest criticisms is that mitigation projects do not crowd out other investments on a dollar to dollar basis. Therefore, it is not fair to compare mitigation projects

[6] If everything is financed through bank credit then this would not hold as explained in Section 3.1.2.4.
[7] More on this in Section 4.1.3.

to other investment projects with the same lens.

Choosing the discount rate for climate change on the market interest rate is debatable. Different studies show that the choice of the discount rate differs between products, income, time framing and other factors [84]. For example, a study shows that discount rates for the adoption of air conditioners varies between 5% for high income households and 89% for low income households [85]. Also, most individuals only think about their own lifetime when doing savings and investment decisions, never in the long-term future that climate change requires. Why then is it justifiable to equate the market interest rate with the discount rate used for climate change when both things are of a completely different nature?

Besides this, another problem surfaces up when treating discount rates as market interest rates. Climate change modeling spans several centuries, however, market interest rates are usually only known until the next 30 years. Few interest rates, if any, have maturities above 30 years [60]. So, after 30 years, future interest rates are fundamentally uncertain and current modeling practices do not capture this. On top of this, decisions using more than a 30 year time frame not only affects present generations, but also future generations as well.

Uncertainty plays a major role in the selection of the discount rate. Studies show that in all cases where uncertainty is taken into account, the discount rate should equal the *lowest possible expected rate of return* [60, 86]. This is due to the fact that in the long-term, lower rates of return have more weight in the averaging process than the higher rates (if both have the same probability distribution) [83].

Nordhaus uses the descriptive approach and thus links the discount rate with market interest rates [66]. This simple decision leads him to conclude that at the moment of modeling, no action is needed to combat climate change. What he advocates is a *"climate policy ramp"* which includes ratcheting up the reduction of carbon emissions but only until a future date, not immediately.

4.1.2 Prescriptive Approach

The most important thing to know about the prescriptive approach is that it is based on ethical terms. This approach is usually associated with low discount rates. This is mainly due to the selection of the social rate of pure time preference. It is argued that ρ should be equal to zero on the basis that utility today and utility in the future holds exactly the same value. This is in line with *intergenerational neutrality*, which the prescriptive approach defends on a moral ground. However, Stern uses a low value of 0.1% due to the extreme case of human extinction[8] as it is improbable that humans will last forever and thus some minimal discounting is justified [16]. Another example of the prescriptive approach can be found in William Cline's work [87].

If the problem of climate change is seen under John Rawls's *"veil of ignorance"* [88], then all generations hold exactly the same value and subjecting future generations to possible harm is morally indefensible [83].

Prescriptive rates do not match the market interest rates because it is argued they they do not offer a good indicator of the marginal trade-offs to society. This is due to several reasons: market imperfection, suboptimal tax policy and difficulty in transferring to future generations [14].

In addition, the precautionary principle also plays a role. The precautionary principle, in its many definitions, is about reducing the risk and damages to the public health and the environment when scientific consensus is absent [89]. The consensus on the causes on climate change is clear, but the consensus on the damages is not. A lower discount rate would abide by the precautionary principle in order to minimize damages to the public health and the environment.

4.1.2.1 Criticism

There is much criticism towards the prescriptive approach. Critics state that the discount rate should not be used because the market rate of return is usually higher than the low numbers (≈2%) of the prescriptive approach, and this means that society is forgoing better opportunities elsewhere [14]. Counterarguments involve realizing that climate change investment is better than no investment at all and that society cannot set aside funds for future generations which will help them adapt to climate change.

Another argument that is pointed out is that low discount rates are not consistent with normal, expected behavior. That is, the government does not apply these discount rates in similar areas like education or research [14]. However, prescriptivists argue that

[8] Say by a meteorite hitting the Earth or an unsuspected alien invasion.

climate change is an ethical case with far-reaching impacts so it should not follow the same behavior as other areas.

Stern uses the prescriptive approach in his work, taking the values as seen in Table 4.1. The result of this is well known: urgent climate action in contrast with Nordhaus' recommendations.

4.1.3 Which one is the better one?

Is it that any of these two approaches is of a better nature for modeling? This is a difficult question to answer as the discussion on it is still continuing. It can be argued that both of them address different sides of the same coin. The prescriptive approach, on one hand, is concerned with the *distribution* of resources across generations. The descriptive approach, on the other hand, is more concerned with the *allocation* of resources across generations. Thus, the former is characterized by distribution and the latter with efficiency [83].

In one paper there is a strong argument that neither the descriptive approach nor the prescriptive approach are free from value-judgments and thus none of them can be defended on description alone [15]. Any discount rate that is assigned goes through a three step process: standing, measurement and aggregation. Both approaches go through value judgments at each step.

Standing refers to who will be described. In other words, who is going to be represented through this discount rate? The descriptive approach makes a huge value judgment in this step, just considering people who participate in the financial market in a certain country. By doing this, descriptivists are not considering many who are considerably affected more by climate change: the poor, animals and other countries. Additionally, even though future humans are taken into account, they have absolutely no power or influence on the discount rate. On the other hand, the prescriptive approach includes everybody into their discount rate but only the modelers/analysts have a say in this and it can be considered a form of elitism.

Measurement is the step where the approaches decide on what is the right thing to measure. One of the key issues is *dynamic inconsistency*, where society has different preferences at distinct points in time. Choosing anything to measure involves some type of value judgment. The descriptive approach places emphasis on monetary units as it bases the discount rate on it. It also favors the higher rates of return on equities [21, 60], which is in itself a value judgment, instead of choosing bond rates. If bond rates were chosen, both approaches could yield similar discount rates [15].[9] For the counterpart, the prescriptive approach can again be accused of elitism, where the few analysts decide on what is better to measure, which is their own particular value judgment. Another crucial question is how to define utility. Is it monetary consumption, happiness index, social connections? There is no clear answer on the subject.

Aggregation refers to the process of collecting the individual measurements of all the people standing, and combining them into a unique societal measurement. The descriptive approach works under the premise of "one dollar = one vote". This has its shortcomings, as only the few rich can actually be "voting" on the interest rate (which affects the discount rate). Therefore, not everyone has a say on the discount rate. The prescriptive approach works under the "one utility = one vote" which can be measured with α from Equation (4.1). Again, elitism is still prevalent.

In another paper, there is a strong argument that both approaches are fundamentally correct and none is inconsistent [83]. The prescriptive approach on intergenerational equality should not dismiss choosing the interest rate of return as the discount rate (when uncertainty is included). In particular, it is all about the choice of the particular projects. In order to leave the highest welfare for future generations, then it is necessary to compare different alternatives at the current interest rate. This tries to minimize generational inequality as resources are always maximized. For the prescriptive approach, the real issue is the savings and investment rates that matter, not the selection of projects or its corresponding discount rate. However, using only interest rates will not maximize possible future welfare if the right projects are not selected. Thus, the descriptive approach must incorporate climate change mitigation projects. So, both are correct: the prescriptive approach in that the current generation has to do its best to ensure intergenerational equality and the descriptive approach is correct in the method of doing this, by comparing it with the market interest rate [83].

4.1.4 Non-constant discounting

Both of the above approaches involve having a constant discount rate that does not change with time. The main reasons for constant discounting is economic efficiency and temporal consistency. For short-term projects, this is a very common practice and usually accepted by most economists [90]. For long-term projects, any future benefit or cost

[9]There are other methods used to measure how people discount: stated preference survey, brain imaging and using public policy as a proxy.

is discounted to almost zero due to the geometric decay of the discount factor. One alternative is to not discount at all. This, however, is in conflict with how most people value their life and thus makes it politically infeasible. The other alternative is the *juste milieu* between these two: non-constant discounting.[10]

First of all, there are several arguments as to why constant discounting is inappropriate:

1. There is an ever-changing intergenerational discount rate. The discount rate for a generation in 1900 is different from one in the 2000 which might be different for one in 2100. This means that different generations in time will have a different valuation of the future. This can be due to political, economical, cultural and environmental reasons. And given to the long times in climate change modeling, it is wise to capture this dynamic effect.

2. Policy and model results are very sensitive to the chosen discount rate.

3. As more technology is available to society which increases affluence and future consumption, the lower the implied discount rate. One example of this is linking the discount rate to the capital per capita. As it grows and society becomes richer, then the discount rate starts decreasing as richer societies tend to be more far-sighted [92]. More on this below.

4. As time passes by, climate damages are increasing and thus the issue of climate change becomes even more evident. It is difficult to see a situation in which society reverses previously done climate damages. Consequently, the discount rate will be lower the more imminent the threat is [93, 94].

5. Large future damages with moderate discount rates have almost no effect due to the power of compounding [82, 86, 91].

6. Interest rates are not fixed over hundreds of years, instead the rates are uncertain. Using this uncertainty can increase climate change valuation by up to 95% in comparison to a constant rate [60].

[10]Most of the attention in non-constant discounting goes to what is called *slow discounting.* This is when a discount rate starts at a positive value and tends to zero as time goes to infinity. For a mathematical explanation of this refer to [91].

4.1.4.1 Hyperbolic discounting

One stream of economists have been using *hyperbolic discounting* to model climate change models. What exactly is hyperbolic discounting? It derives from how humans discount in real-life without thinking about it. The most common example used to explain this situation is if an individual was offered €100 today or €102 tomorrow, what would the individual take? Many would prefer €100 as a single day is a significant delay in this short-time frame. But what if the same individual was offered €100 in 10 years time or €102 in 10 years and 1 day? Most would prefer waiting an extra day for the €102 even though in both cases the delay between the prizes is exactly the same, one day! This is because over a long-time frame, the delay of a single day is no longer very relevant. This type of inconsistency is captured in hyperbolic discounting. When applied to climate change, something similar happens. Society would expect to discount the welfare of the closer generations, as there is an emotional bond with them (children and grandchildren). However, when dealing with further generations, say the 11^{th} or 12^{th}, the difference between one or the other does not mean any emotional difference to the current society, so the future rate of time preference is lower and society discounts them at almost the same discount rate. [95].

Figure 4.1 shows a plot of the discount factor with time as a hyperbolic function. This type of plot can help explain some behavioral aspects of humans such as impulsive gratification in the short-term and preference reversal [84]. Hyperbolic discounting is easily applied to current models of climate change and it is preferred by some economists (especially prescriptivists) as it discounts future generations on a more ethical way.

One common form to express the hyperbolic discounting curve is [96]:

$$\text{Value} = \frac{\text{Value at no delay}}{\text{constant} + (\text{Impatience factor} \times \text{Delay})} \tag{4.2}$$

where the constant is a small number for when the delay is zero or small and the impatience factor can be modified according to the specific case: climate change, financial risks, consumption goods, etc.

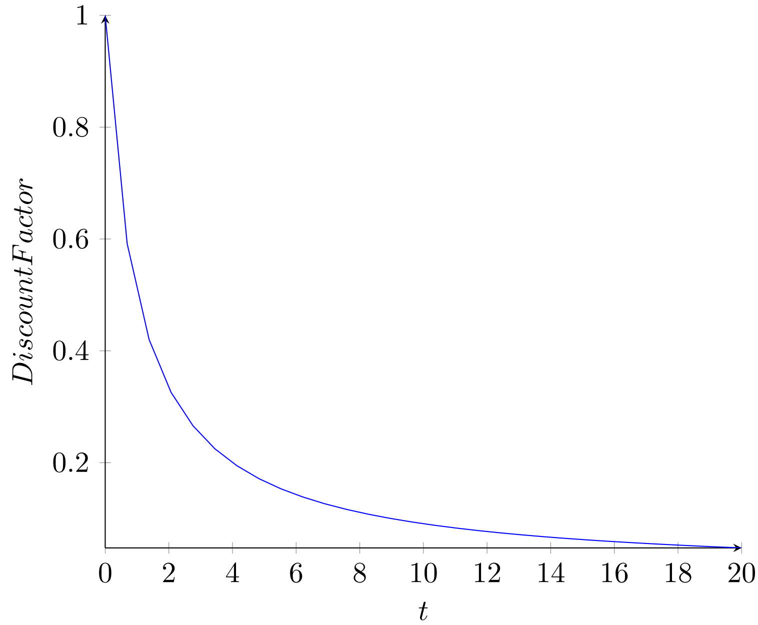

Figure 4.1: Hyperbolic Discounting

The case is that even though hyperbolic discounting can be seen in the behavior of consumers, it is still far from perfect to apply in cases of climate change, where the social and economical situations of distant generations are instrinsically different from one another.[11] Hyperbolic discounting is criticized mainly due to the *time inconsistency* problem. Time inconsistency refers to a situation in which preferences change over time such that an optimal choice made today is no longer optimal when re-evaluated tomorrow. Also, applying individual behavioral aspects to a social decision is conceptually fraught with irrationalities [84].

4.1.4.2 Coupled with Technological Change

Economist Martin Weitzman argues that the most important issue when dealing with deep-future (long-term) discount rates in climate change models is to know the underlying trend in the *real rate of interest* as this can be used as a savings program which can deal with climate damages in the deep-future [97]. So, what is it that drives the real rate of interest?

[11] Most individuals only face decisions based on a 20-40 year time frame (e.g. buying a house, retirement, savings fund, R&D research, business decisions, etc); considering the time frame of hundreds of years of climate change is unimaginable for the individual.

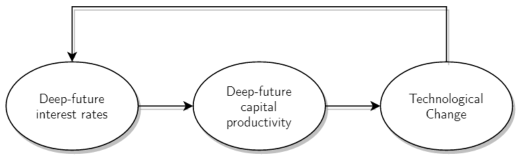

Figure 4.2: Link between interest rates, capital productivity and technological change.

In principle, the real rate of interest is dependent on the productivity of investment at the moment. And in turn, this depends on the productivity of capital, that is, how much output one can get from some inputs. The productivity of capital changes in time with technological change. This can be visualized in Figure 4.2.

In this sense, if deep-future interest rates depend on technological change then it is possible to link the discount rates with technological change. However, a question arises. How certain is it that technological change will continue indefinitely into the deep-future? Inspired by [97], it is possible to imagine a world where new ideas are nonexistent, just as an artist or inventor might run out of ideas. Also, there might be a finite number of ideas which can be explored. However, this seems implausible. Just as a deck of 52 cards can be combined in more ways than the total seconds in existence since the Big Bang, the total number of ways ideas can be combined to produce new ideas is insurmountable. Just from past observation, technological change has been growing at an ever-increasing rate. There is then no reason as to why the future productivity of capital should be lower than today's [97].

Weitzman argues about the nature of the problem itself, describing the long term effects of compounding as a concept which is highly counterintuitive. He suggest a declining discount rate for climate change modeling, eventually reaching to zero [97].

4.1.5 Concluding Remarks

A review of the literature and issues surrounding the discount rate has been made in this section. The Ramsey Equation (4.1) proved to be of critical importance for the understanding of the different concepts. Discounting has two main approaches: the descriptive approach and the prescriptive approach. The descriptive approach relies on a market principle, arguing that projects with the higher rates of return should always be chosen. This approach is criticized mainly for the inherent problem of choosing a current market rate for the long timespan of climate change. The prescriptive approach is based on an ethical foundation. The prescriptive approach lends itself to lower discount rates. These low rates are the main criticism against the prescriptive approach because it is not consistent with expected behavior. A study was reviewed which showed that both approaches are full of value-laden assumptions. Finally, a case was given for non-constant discounting. Hyperbolic discounting was presented as an initial approach of non-constant discounting. A coupling with technological change was presented as an alternative to the discussion.

One of the main points of this thesis is to see the effect of a dynamic discount rate (non-constant) in a climate policy model. This is done in order to try to steer away the conversation from the descriptive/prescriptive arguments. The following section describes the model and its implications.

4.2 Model

This section will explain the modifications done to the DICE-ED model in order to have a dynamic discount rate that depends on the previous work of endogenous technological change and environmental variables. Two different methods are explained below, the first sub-section for each method explains the reasoning behind the formulation. Then, the actual equations are explained followed by the calibration.

A key distinction to make while adding a dynamic discount rate is that the long-run steady-state savings rate will change. Initially, with a constant discount rate, the steady-state savings rate is given by:[12]

$$s^* = \left(\frac{\gamma(\delta_K + \nu)}{\delta_K + \rho + \nu\alpha} \right) \tag{4.3}$$

where s^* is the steady-state savings rate, γ is the capital elasticity to output, δ_K is the capital rate of depreciation, ν is the growth rate of labor-augmenting technological progress, ρ is the social rate of pure time preference and α is the elasticity of marginal utility of consumption.

Including a non-constant discount rate (ρ) has important quantitative effects on savings and growth [98]. A lower effective discount rate over the long-run would exhibit higher steady-state savings rate and capital accumulation [98]. Thus, an effective discount rate can replace ρ in Equation (4.3). The definition of the effective discount rate depends on the way it is modeled. For a "quasi-hyperbolic" form, the effective discount rate is calculated as ρ_{ss}/κ where κ is the discount factor and ρ_{ss} is the steady-state rate of time preference [98, 99]. In this case, the steady-state discount rate will be lower after a given period of time. To approximate the effective discount rate, the following formulation is proposed:

$$\lambda = \rho(T_{max} - 10) \tag{4.4}$$

where λ is the effective discount rate, $\rho(T_{max} - 10)$ is the rate of time preference at time period $(T_{max} - 10)$. Quantifying the effective discount rate depends on model parameters which can differ significantly with each strand of the literature. This will be an approximation as the effective discount rate in its most general case is a weighted average of future rates of time preference [98]. In theory, λ will be a bit higher than

[12]This comes from the steady-state solution for the Ramsey-Cass-Koopman model.

what it should be. However, the steady-state savings rate in Equation (4.3) is not very sensitive to the change of λ, so the above formulation is deemed appropriate to capture the effects of a higher savings rate with a non-constant discount rate. The reason why λ is evaluated at time period $(T_{max} - 10)$ is because the savings rate in the model is fixed to achieve steady-state. λ will replace ρ in Equation (4.3).

On a last note, one of the main issues with non-constant discounting is the time-inconsistency. The following methods practically avoid this issue by supplanting the decision to a formula instead of a subjective criteria. So, in every period, the formula can be re-evaluated by a new social planner and the result would be the same. Thus, the formulations are time-consistent. However, the only concern is the initial rate which is to be selected subjectively. This is a point of discussion outside the scope of this thesis.

4.2.1 Method 1: Decreasing Marginal Impatience

Initially, most of the early work with dynamic discount rates involved the assumption of *increasing marginal impatience* [100]. This means that as the levels of consumption of a country or agent get higher, then the future is discounted at a higher rate. This would imply that richer people are more impatient. They only want the "here-and-now" and do not think much about the future. From personal experience one can deduce that this is not generally the case. It is counter-intuitive to think that as consumption levels rise then the discount rate[13] also rises with it.

The alternative is called *decreasing marginal impatience*, which is the exact direct opposite. It basically states that poor people are more impatient, i.e., the lower the level of consumption then the higher the discount rate. One basic explanation is that investment in future-oriented capital will increase the "valuation and appreciation" of future utilities [101]. There is a link between these investments and future utilities. This means that richer countries will ultimately invest more in future-oriented capital and will ultimately end up being more patient.[14] It comes to no surprise that these investments can be thought of as R&D. R&D is mostly future-oriented with the promise of increasing future production (and consumption) and thus creating a valuation for future utilities. This is the basic premise for the justification of a dynamic discount rate that depends on technological change (R&D).

[13]In reality it is the rate of pure time preference, but as both are connected by the Ramsey formulation (Equation (4.1)), sometimes both are used interchangeably in the literature.

[14]Rich people have more assets and more money overall to invest. That is the big difference.

Figure 4.3: Discount rate by income level. Adapted from [102].

There is a vast literature on empirical findings that find evidence for the above theory.[15] Figure 4.3 shows how the discount rate declines with increased income level from a survey of consumer finances in 1992 [102]. Another study using panel data found that poorer households have a discount rate that is three to five points higher than in rich families [103].

4.2.1.1 Equations

The social rate of pure time preference (ρ) needs to be equated to a formula which relates to technological change. In this case, given that the original DICE2013x model does not have an R&D market, the investment profile will be used as a proxy for technological change.

$$\rho(t) = f(I(t))$$

In particular, it is interesting to see how the case of the original DICE2013x model compares with the DICE-ED model. One formulation is:

$$\rho(t) = z_1 \left(\frac{I(t) - I_{DICE}(t)}{I_{DICE}(t)} \right) + z_2$$

where z_1 and z_2 are calibration factors and $I_{DICE}(t)$ is the value of investment taken from the DICE2013x runs in order to create a reference point.

[15] Refer to Table 1 of [101] for more information.

4.2.1.2 Calibration

z_1 is calculated from empirical data. One study shows that when controlling for age, education and race, the time preference of the lowest fifth percentile of households is 3.5% while the highest fifth percentile is 0.8% [103].[16] The implicit slope of these two points is -0.1421. This means that for every subsequent fifth percentile, the time preference is 0.14% smaller. z_1 takes the value of this slope. z_2 is calibrated in such a way that the initial period is equal to the initial rate (0.015). This value differs from the baseline and optimal scenarios. $I_{DICE}(t)$ is simply taken as the investment path when solving the DICE2013x model for both the baseline and optimal scenarios.

- $z_1 = -0.1421$
- $z_2 = 0.017522$ (baseline)
- $z_2 = 0.016813$ (optimal)

4.2.2 Method 2: Atmospheric Temperature Increase

This subsection involves linking the discount rate with an environmental variable which in this case is atmospheric temperature rise. The basic premise of this is that as imminent climate change becomes more obvious (proxied by an increase in the atmospheric temperature), then concerns for long-term sustainability become evident and in this particular case the discount rate will be minimized. It would be expected that utilities would not be discounted at almost negligible values in a dangerous world of high temperatures. This reasoning has also been applied in several papers working with environmental quality and discount rates [93, 94].

The IPCC estimates that in a business-as-usual scenario, the atmospheric temperature increase could reach the ranges of 7-8°C by 2300 [9]. This is considered to be the upper limit and will be taken as 7.5°C. As atmospheric temperatures start rising towards this, the discount rate starts to decline with it. The following sub-section elaborates further on this.

[16] The lowest fifth percentile refers to the richer households while the highest fifth percentile refers to the poorest households. Other studies find similar results between the discount rate and the wealth level [85, 104].

4.2.2.1 Equations

As in the previous cases, the changing parameter will be the social rate of pure time preference (ρ) in order to account for the differences in utilities in distinct generations. An equation which links the dynamic ρ with the atmospheric temperature increase is needed:

$$\rho(t) = f(T_{AT}(t))$$

One simple but functional form for the above equation is:

$$\rho(t) = x_1(T_{AT}(t) - T_{MAX})^{x_2} \tag{4.5}$$

where x_1 and x_2 are calibration factors and T_{MAX} is the aforementioned temperature of 7.5°C.

4.2.2.2 Calibration

x_2 will be set to 1 in order to achieve linearity in the formulation. With an initial T_{AT} (0.80°C) and an initial social rate of pure time preference (0.015), x_1 can be calculated accordingly. The calibration factors then take the values of:

- $x_1 = -0.002239$ °C^{-1}
- $x_2 = 1$

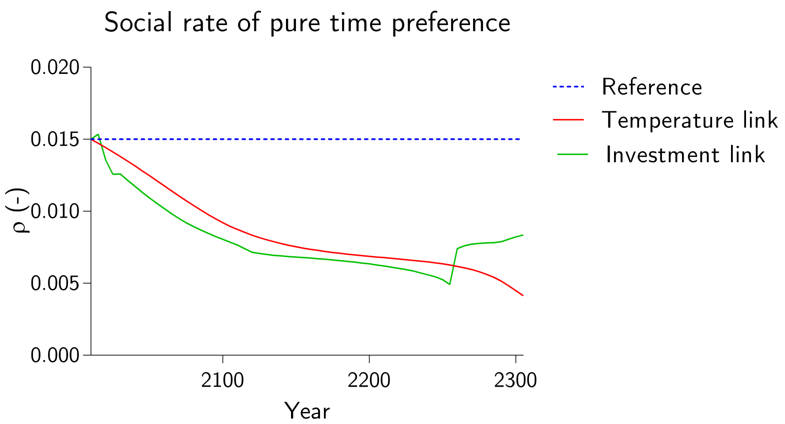

Figure 4.4: Social rate of pure time preference by method.

4.3 Economic Analysis

The previous subsections showed the different methods to have a dynamic discount rate. This does not make any distinction of which one is correct or if having a dynamic discount rate is correct at all. The point in this exercise is to show the policy and economic implications of having a dynamic discount rate. This section will elaborate further on this.

The optimal scenario will be used throughout this section to make the analysis. Method 1 from above will be described as "Investment link" and method 2 will be described as "Temperature link".

4.3.1 Rates and Factors

Before delving into the different aspects of the modules it is vital to see the impact of these formulations *on* the discount rate. After the formulations above and running the model for every method, the social rate of pure time preference is visualized in Figure 4.4. The reference and initial point is 0.015 as this was the one used by Nordhaus throughout his modeling. It is true that any initial point could have been chosen instead. This is a discussion outside the scope of this thesis.

From Figure 4.4 stands out the difference in the results of the different methods. Each formulation gives different although similar trending rates. This is to be expected. If all formulations were to give out the same rate across the model runs, then the discounting dilemma could potentially have a solution. Starting with the investment link, the rate drops from the reference level with time. This is due to the fact that including endogenous technological change increases production and investment capabilities in comparison to the DICE2013x model and thus the formulation will always lower the rate. There is a small jump in the last periods. This is due to how the structure is modeled with regard to the savings rate where it is fixed for the last 10 periods to achieve a steady-state. Afterwards, the temperature link is straight-forward. As temperatures increase gradually with the release of emissions, the rate decreases. It is almost obvious that the rate will never be higher than the initial rate, unless temperatures can be decreased drastically. Both these declining rates are in line with what many economists advise to do for long-term environmental models [60, 86, 91, 97, 105]. Finally, it is important to note that many other methods could have been chosen with very different profiles, maybe even higher rates than the initial one. This would probably mean a sign of weakness, but that is not necessarily the case. The point of it is to demonstrate that different formulations can have quite distinct results. It is a common practice for economists to look for concave, smooth lines to explain some of the basic workings of the economic system. It is still considered important to explore distinct links to capture a better understanding of the effect of a dynamic rate on climate change policy. Further work should be pursued which explores more discounting possibilities.

Ultimately, what really matters is not the discount rate *per se* but the discount factor. Figure 4.5 shows the discount factors for the different methods. During the early periods of the model, the discount factors decrease at an almost equal pace. However, it is in the long-term when the results differ. On the reference case, the discount factor is already below 0.1 for the last 100 years and almost reaches zero at the end. Including a dynamic discount rate which decreases the rate will have higher discount factors. This is important for policy-making as long-term damages are accounted more heavily in the model. With the current numbers, a one million USD damage 300 years from now would be valued around **seven times higher** with dynamic discounting compared to the reference case. Table 4.2 shows the normalized net present value of a one million USD damage with the different methods. In 100 years (short-term), the damages are valued almost in a similar way across the different methods. It is in the long-term when they start differing significantly. However, it is good not to be blinded just by the number, after all, *the double of a really small number is still a really small number*. Damages from 300 years from now will be discounted at almost negligible present values no matter the discounting method. The key insight is that this could play a big difference if the damages are out of proportion (i.e. non-quantifiable as the severity is not calculable). If a minimax criterion

Table 4.2: Normalized net present value of a one million USD damage by discounting method.

	100 years	200 years	300 years
Reference	1	1	1
Temperature link	1.35	2.82	6.55
Investment link	1.52	3.43	7.47

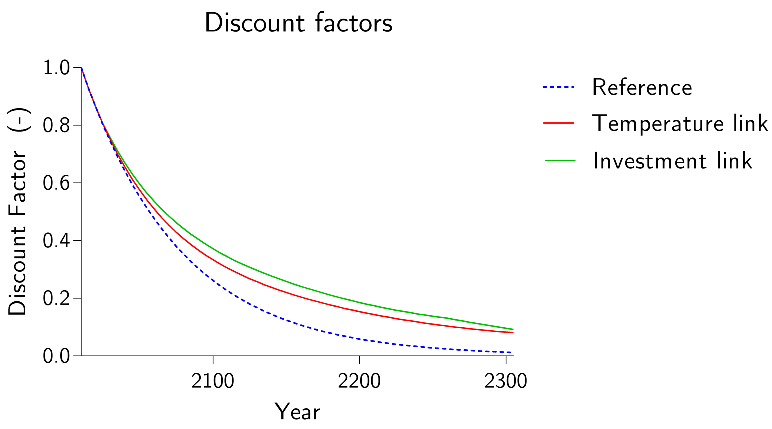

Figure 4.5: Discount factors by method.

is applied, the choice of a discounting method could prove to be the difference.[17]

With this in mind, the following subsections will treat the different modules of the model and its effect on climate change policy.

4.3.2 Climate Module

Like previous analyses, one of the main subjects of interest in the climate module is the emissions profile. Figure 4.6 shows the emissions path for the discounting methods in the optimal scenario. The emissions overall are lower; it is just at the end-of-horizon where emissions are above the reference case. *Riemann sums* were used to calculate the area under the curve and it shows that dynamic discounting methods would lower total emissions into the atmosphere. Compared to the reference case, the area under the dynamic discounting methods was on average 12% lower. Climate change policy usually

[17]The minimax criterion comes from decision theory and the basic idea is minimizing the loss for the worst case scenario. In this case, extreme temperatures with huge or non-quantifiable damages are the worst case scenario.

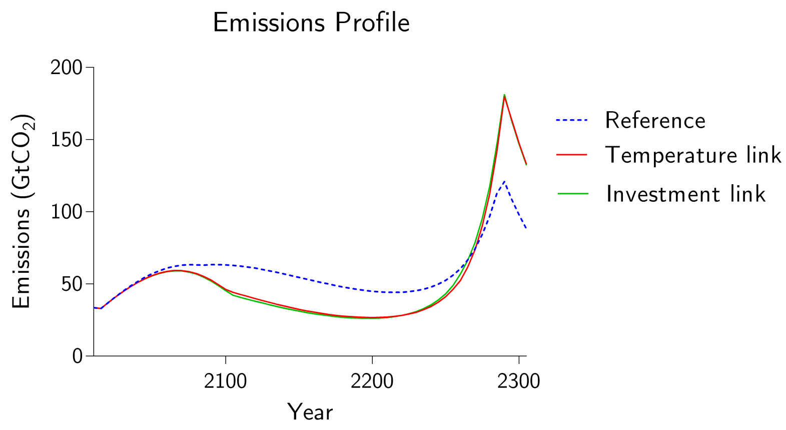

Figure 4.6: Emission path by discounting method.

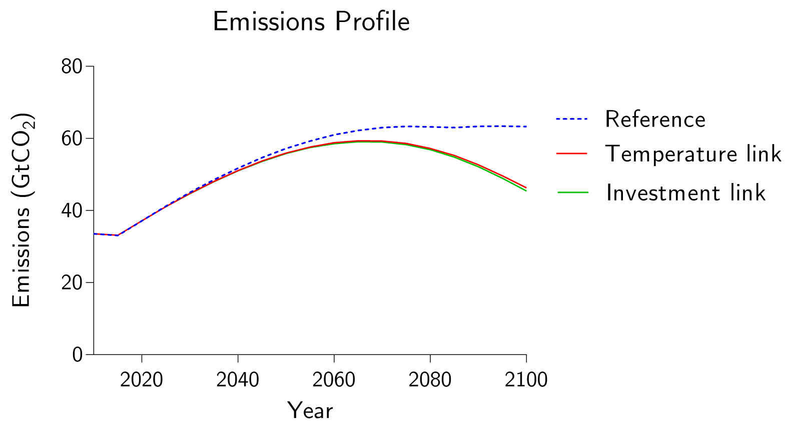

Figure 4.7: Emission path by discounting method until year 2100.

focuses until the year 2100. Zooming in on this section also paints a new picture with the emissions profile. Figure 4.7 shows how using a discounting method lowers the total emissions by the end of the century.[18]

Lower emissions with the discounting methods gives way to lower atmospheric temperatures increases. Figure 4.8 shows the temperature profile with the different methods. There is not much to read here; lower temperatures increases are achieved with dynamic discounting due to a lower emission profile. This bodes well news for the environment

[18]Refer again to Section 3.3.2.1 for a discussion on the dangers of only showing data until the year 2100.

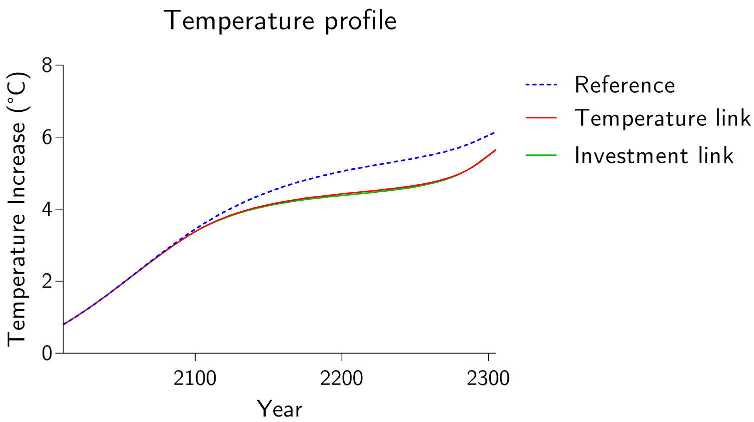

Figure 4.8: Temperature increase profile by discounting method.

as temperatures are lower and by consequence damages are minimized. However, one important policy conclusion is that leaving just the market without any intervention is not enough to limit climate change. An emissions policy or any other type of control policy is necessary in order to limit global temperature change.

4.3.3 Energy Module

The energy module is not severely affected by the method of discounting. Figure 4.9 shows the evolution of the fossil fuel price. In the reference case, more fossil fuel is used which depletes the reserves and therefore increases the overall price.[19] With any of the discounting methods, the fossil fuel is depleted at a lower pace and this results in a lower price throughout the model.

On the other hand, the backstop fuel price is not affected by any discounting method. What does change is the % usage of the backstop fuel. Figure 4.10 shows the change in the backstop relative usage in comparison to the reference case. Because the reference case employs more fossil fuel usage, the % usage of backstop fuel increases by a small margin with a dynamic discount rate. The impact of this particular detail is dependent on how the energy module is modeled. For example, a bottom-up model with high technology specificity would have a richer interaction between energy technologies which cannot be captured in a top-down model such as this.

[19] Fossil fuel price is modeled according to the exhaustion of the reserves. For clarity, the equation is $p_F(t) = mcoe + markup\left[\frac{CCum(t)}{CCum_{max}}\right]^4$. Refer to Section 3.2.1 for more information.

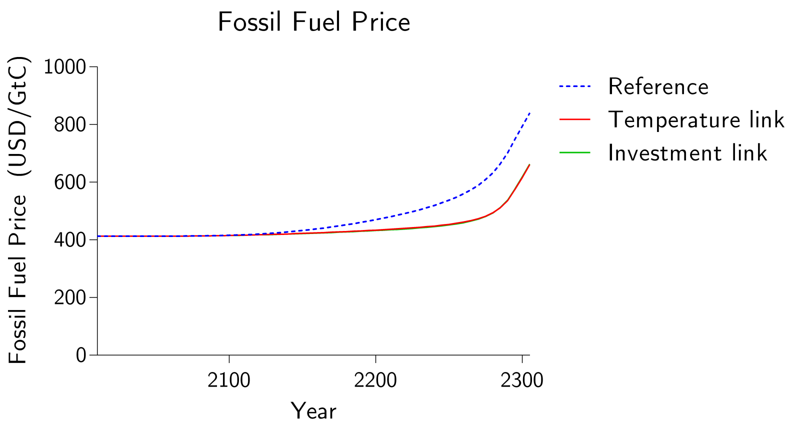

Figure 4.9: Fossil fuel price by discounting method.

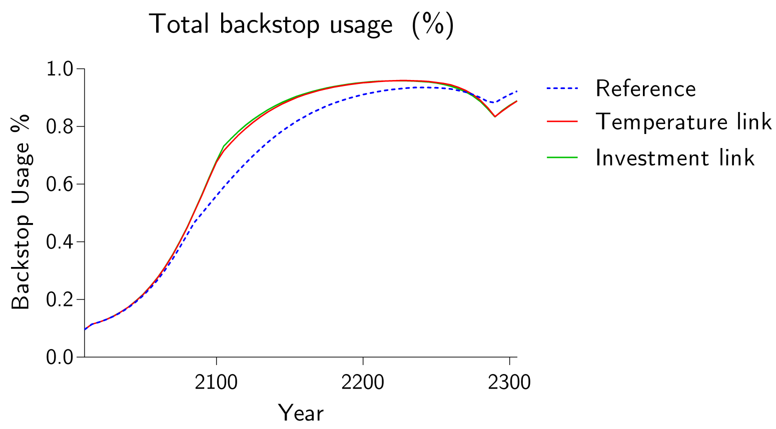

Figure 4.10: Backstop usage % by discounting method.

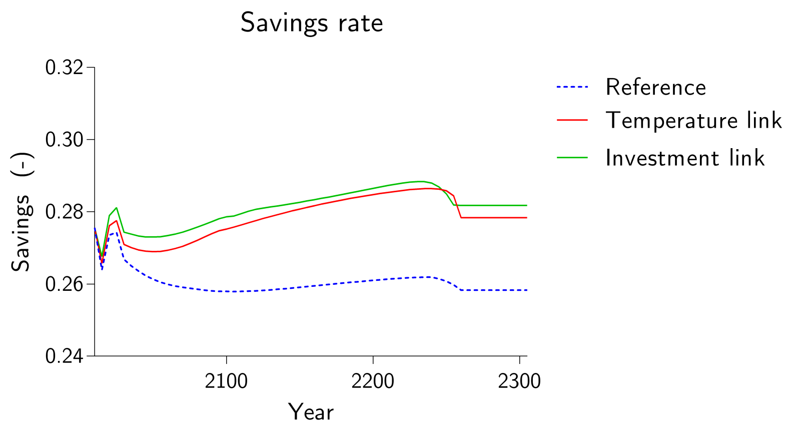

Figure 4.11: Savings rate by discounting method.

4.3.4 Economic Module

The biggest impact with adding a dynamic discount rate is on the savings rate. Figure 4.11 shows this in full effect. Both dynamic links have savings rates which are around two points higher than the reference case for most of timespan. In the Ramsey-Cass-Koopmans framework which is used in this model, the savings rate is determined endogenously. These rates are the optimum rates to maximize welfare according to the constraints. For policy purposes, this result should factor in the importance of increasing overall savings rates in the economy. This can be done through several mechanisms such as an increase in the consumption tax or a decrease in the capital gains tax. These savings rates are analogous to the *Golden Rule Savings Rate* from the Solow model. However, the opposite argument is also true: in order to allow for true dynamic discounting in real-life, the savings rate must be higher. *A lower discount rate implies a higher savings rate.*

In a similar manner, the real interest rate is different when adding a dynamic discount rate. Figure 4.12 shows the evolution of the real interest rate depending on the discounting method. As can be seen, the decline of the real interest rate is due to the lower social rate of pure time preference.

Another critical effect of a dynamic discount rate is on the social cost of carbon (SCC).[20] Figure 4.13 shows the difference in the SCC with the reference case. As can be seen, the SCC rises considerably with a dynamic discount rate. In fact, the peak for

[20]Refer to Equation (3.22) in Section 3.3.2.3.

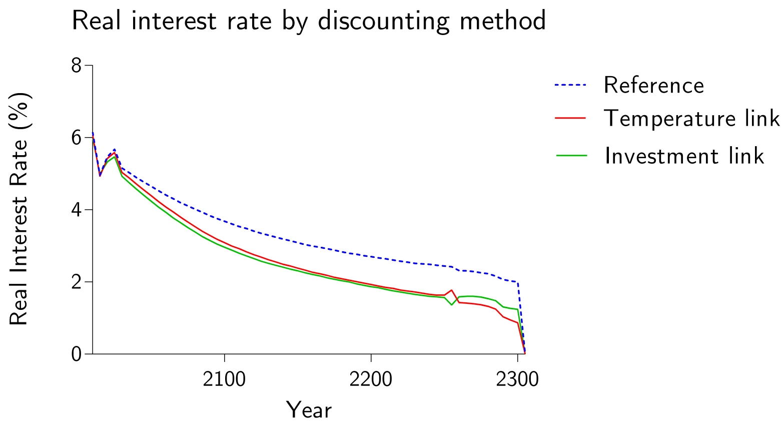

Figure 4.12: Real interest rate by discounting method.

the reference case is at 639.25 2005 USD/tonCO_2 while for the temperature link it is 985.62 2005 USD/tonCO_2 and for the investment link it is 995.23 2005 USD/tonCO_2. Comparing to the reference case, this is a difference of 54% for the temperature link scenario and 56% for the investment link scenario! For policy purposes this means that a higher carbon tax or emissions trading prices must increase in comparison to the reference. Again, this is more evident if the results are zoomed in until the year 2100. Figure 4.14 shows this important insight. One of the main issues here with comparing it to values from the literature is that these values always involve a fixed discount rate [29]. In any case, as the discount rate decreases the SCC will increase as this implies that future utilities are given greater value. This is why it is of no surprise that the SCC with the discounting methods is greater than the one with a fixed rate. Further research is encouraged to calculate the SCC with dynamic discount rates.

With dynamic (and declining) discounting, output in the long-run increases. This is linked with the discount factor being higher compared to the reference case. Figure 4.15 shows the slight increase. Additionally, the damage to the economic system is minimized with these discounting methods. Figure 4.16 shows how damages differ between the different methods. The uptick at the end is due to the end-of-horizon effects caused by the emissions (Figure 4.6). Either way, the damages with a discounting method are around 1% smaller throughout the whole timespan of the model. It might not seem like much, but this is just relative. In the last period, 1% of the total output in the reference case is 364.5 trillion 2005 USD. This is **5.7 times** the initial output, which is an approximation of the gross world product in 2010!

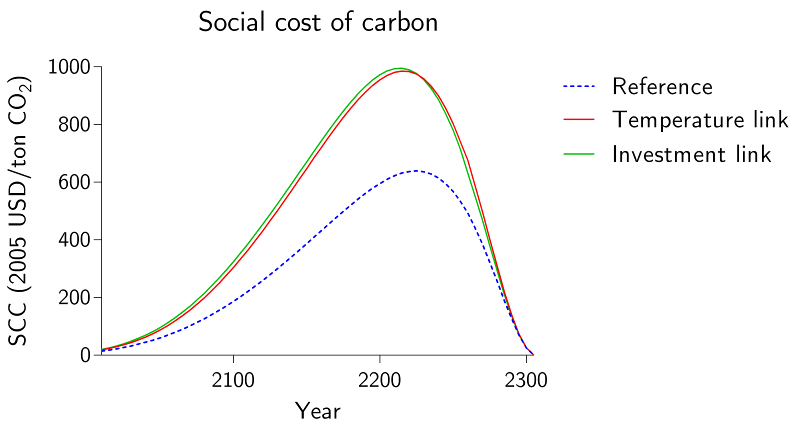

Figure 4.13: Social cost of carbon by discounting method.

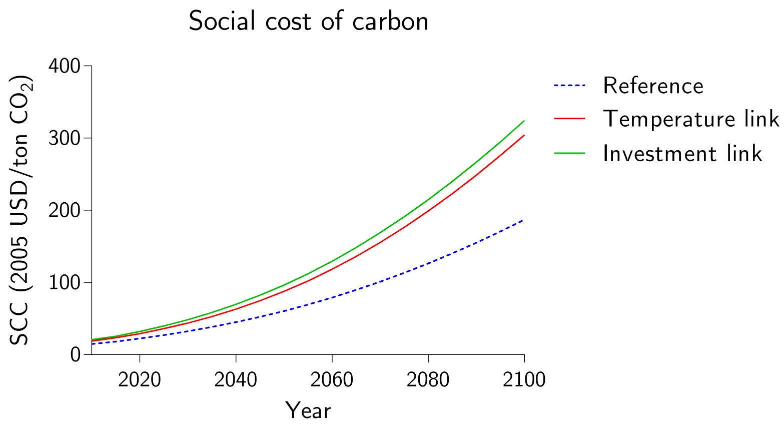

Figure 4.14: Social cost of carbon until year 2100 by discounting method.

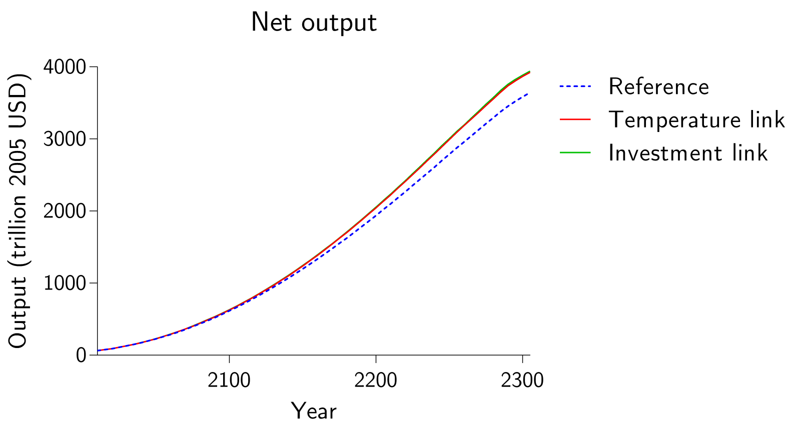

Figure 4.15: Net output by discounting method.

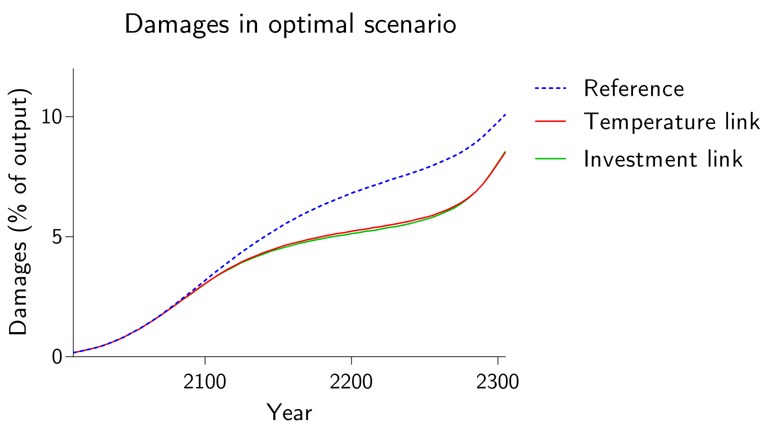

Figure 4.16: Damages as % of output by discounting method.

4.4 Concluding Remarks

This chapter focused solely on the controversial issue of social discounting. First, a literature review was presented to explain all the different views on discount rates. Non-constant discounting was presented as an alternative to the discussion. Afterwards, the modifications to the DICE-ED model and its equations were presented which captured the link between discount rates and economic and environmental variables. Afterwards, the model was run and the results were presented.

Both formulations (economic and environmental link) decrease gradually with time. This is in line with what many economists advise to do for this type of models. This has the effect of increasing the overall discount factors so that future damages and benefits have higher weight. Using a dynamic discount rate values damages and benefits *seven* times higher than the case of a constant discount rate.

Environmentally, the effect of a decreasing discount rate is that emissions and atmospheric temperature rise are lower. This is due to the fact that future damages and benefits are valued higher and future big damages are avoided as much as possible.

However, the biggest impact of a dynamic discount rate is on the savings rate. Higher savings rates are achieved with the formulations for the discount rate. This is important for policy-making overall as savings rate need to be increased in an economy with non-constant discounting. Finally, the SCC and both the output increase significantly with a dynamic discount rate.

This bears the consideration of using non-constant discount rates in long-term environmental models. Policy-makers should always use precaution and review the assumptions of models designed for policy-making because a constant discount rate has important ramifications.

This bodes the question, is this enough? Are we completely certain of these results? Dealing with climate change and long-term environmental modeling, a vast amount of uncertainties are always present. The following chapter opens the discussion of uncertainty in climate policy models.

Chapter 5

Uncertainty

In the previous chapters, all the models have been treated in a deterministic way; everything is known beforehand and there is no treatment of uncertainty. However, this can be an incomplete view as there is a great deal of uncertainty over all the inputs to the model. This chapter deals with the issue of uncertainty in climate policy models. The first section presents a literature review on the subject. Afterwards, the experimental design for the uncertainty analysis is presented. The uncertainty analysis follows after this. Concluding remarks close the chapter.

5.1 Literature Review

Uncertainty, in its most colloquial terms, is saying *"I really do not know."*. In a more political jargon, uncertainty is referred to as the *unknown unknowns* as put forth by American politician Donald Rumsfeld [106].[1] In scientific terms, the exact definition of uncertainty differs between fields. For economics, one of the earliest definitions of uncertainty came from Frank Knight who distinguished between risk and uncertainty where the latter is immeasurable and not possible to calculate [107]. In model-based decision support, uncertainty is defined as *"any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system."* [108]. It is this definition that will be used throughout this chapter.

Uncertainty is not a unidimensional concept; it can be characterized with three dimensions: *location, level* and *nature* [108, 109].[2] The location of the uncertainty is where in

[1] Risk, on the other hand, is called the *known unknowns*.
[2] For an alternative classification of uncertainty refer to [25].

Table 5.1: Levels of uncertainty.

Level of uncertainty	Enumeration/Ranking	Example
1 (shallow uncertainty)	Yes/Yes (Probabilities)	20%: High discount rates 50%: Low discount rates 30%: Mid discount rates
2 (medium uncertainty)	Yes/Yes (Likelihood)	Not likely: High discount rates More likely: Low discount rates Equally likely: Mid discount rates
3 (deep uncertainty)	Yes/No	High, low and mid discount rates. No way of knowing any distribution.
4 (recognized ignorance)	No/No	Being open to any new surprise.

the model does the uncertainty occur. For example, the location of the uncertainty could be in the boundary of the model, in the inputs of the model, in the solution algorithm, etc. The level dimension refers to where the uncertainty lies in a line which on one end is complete determinism and on the other end is total ignorance [108]. This dimension has four levels [109]. Level 1 (shallow uncertainty) is when the uncertainty's outcomes can be enumerated and exact probabilities can be assigned. Level 2 (medium uncertainty) is when the uncertainty's outcomes can be enumerated but exact probabilities cannot be assigned, rather classifying them as perceived likelihood. Level 3 (deep uncertainty) is when the uncertainty's outcomes can be enumerated but there is absolutely no way of ranking them in any order. Level 4 (recognized ignorance) is when not even the outcomes can be enumerated. Table 5.1 shows a summary of these levels with examples. Finally, the nature of uncertainty is about the basic essence of uncertainty and contains three categories: epistemic, ontic and ambiguity. Epistemic uncertainty is due to imperfect knowledge and can be reduced by more research [108]. Ontic uncertainty is related to the inherent variability of the phenomena. And ambiguity is when there are multiple stakeholders with different worldviews who can interpret things in different ways [109].

Uncertainty plays a major role in climate policy models. Uncertainty in climate policy models differ significantly from other policy models due to three reasons: damage non-linearity, environmental irreversibilities and extremely long time horizons [24]. The IPCC, one of the most important scientific bodies on climate science, tackles uncertainty as a level 2 phenomenon (likely, very likely, extremely likely, etc) [9]. However, another issue is the misunderstanding between modelers and policy makers. Modelers' aim is to minimize uncertainty to as much as possible and increase the knowledge surrounding it; on the other hand, policy makers only want to minimize the chances for making a political mistake [110]. Models are sometimes used in a predictive manner, that is, they predict the behavior of the system with known information. The presence of (deep) uncertainty in climate models poses a gargantuan obstacle for doing predictive modeling. Selecting one set of parameter from an almost infinite set is not a prediction [111]. What are some ways to deal with uncertainty?

One common way to tackle uncertainty in models is through a sensitivity analysis where a probability distribution is set to different parameters and the model is run and the results are compared.[3] With climate change, however, the uncertainties are so diverse and so large that it is not possible to state confidently the different probability distributions of the uncertain parameters. This situation is the previously mentioned Level 3 deep uncertainty [112]. It is not possible to agree between the consequences, probabilities or even effects of these parameters into the model.

A method to tackle deep uncertainty and aid in modeling practices is called *scenario discovery* [26, 111]. This method works by applying statistical tools and algorithms to results of simulation models. This differs from the traditional scenario method (commonly known as scenario-axes) where a limited quantity of the future states of the world is characterized, and many of these times in accordance to the modeler's worldview [112]. Limiting the scenarios to a few (usually four) when in reality there are an extensive amount is one of the main weaknesses of the scenario-axes approach [113].

The scenarios from scenario discovery are usually presented as a mere possibility but not as a truth prediction. This helps when dealing with multiple stakeholders which possess different worldviews and makes it easier to be accepted by all [26]. Scenarios are then considered as future states of the world where policies may not be able to meet its goal or they deviate from the optimum. Input parameters to a model are changed accordingly to produce many *combinations* of future states and compare against a single criterion such as cost or an environmental target. With the appropriate algorithms, all these input parameters can be characterized and the relevant ones are defined as the

[3]Some other ways: stochastic programming, expert assessment, model emulation, spatial or temporal variability, multiple models and data-based approaches. For a more detailed explanation refer to [25].

key drivers of the scenarios. This allows for a quantitative analysis of the model that addresses the weaknesses of scenario-axes approach and at the same time offer *ad hoc* recommendations for all the distinct stakeholders.

Scenario discovery is a process that supports *Robust Decision Making* (RDM). RDM is a method that uses current information to better prepare strategies in the face of current uncertainties instead of trying to predict the highly and deeply uncertain future world [26, 113]. It is a rigorous approach that soothes out the shortcomings of other uncertainty analyses by using a quantitative decision-analytic approach [112]. At its most basic underpinning, RDM is a method to choose among different strategies and lower the risks inherent with the uncertainties. These strategies that RDM provides are used by scenario discovery. Ultimately, the objective of RDM is to identify some strategies that perform well and are insensitive to most or as much uncertainties as possible [109]. Together, RDM and scenario discovery offer a new way that exploits the power of high-tech computers and information technologies in order to assist humans in their decisions.

How does scenario discovery work? It works in a three step process. First, a computer simulation model is run over all the uncertain inputs while holding constant a policy action. The results are compared against a performance level. Secondly, a set of tools are applied to look for the combinations of uncertain values that give results comparable to the performance level. These combinations are recommended to be as simple as it can be, represented by a few key driving forces [26]. Special indicators are employed in this step in order to assess the interpretability and reliability of the aforementioned combinations. Thirdly, the combinations are assessed with diagnostic tools (e.g. quasi-p-value and resampling test) to check for weaknesses and strengths.

One specific technique which is used in model-based decision support is the Exploratory Modeling and Analysis (EMA) developed by the RAND Corporation to design robust strategies for deep uncertainties [114, 115]. EMA is designed to work with deep uncertainties by using them in computer experiments such as scenario discovery. One particular model has almost an infinite set of possibilities from which to run from (i.e. parameters and equations can be defined in different ways) which EMA aims to explore by running a vast amount of computational experiments. An analysis follows from this which aims to answer specific questions about where the model fails or succeeds in order to provide adaptive policy advice and "out of the box" thinking [114].

In terms of climate change policy, scenario discovery along with EMA can help quench the problems usually confronted with the uncertainties of the climate system or the economic model.

5.2 Experimental Design

It is imperative to not just report single measurement results but know that they work within a range (the uncertainty range). To make scientific sense, sensitivity analysis and robustness analyses must be done so that policy makers can have a better working understanding of the model [68].

For the experiment, the EMA Workbench will be used to conduct the uncertainty analysis. The workbench was developed by Dr.ir. Jan Kwakkel in the programming language Python [116]. The EMA workbench has been connected to GAMS through a connector developed in this thesis.[4] EMA is a methodology to analyze complex and uncertain systems with a vast amount of computational experiments. Each experiment represents a different set of input parameters to the model. EMA is useful when the available information is insufficient to represent just one single deterministic model.

For this chapter, the model that will be used is the original DICE model and not the DICE-ED model (or the DICE2013x). There are several reasons for this. First of all, the original DICE model is already uncertain enough that warrants its own analysis. The DICE-ED model, which is almost triple in size, is left for future treatment. Because this is the first time that EMA will be applied to the DICE model, it is more useful if it is applied to the original one with far-reaching implications. Another reason is that computation times vary significantly. The DICE model takes a couple of seconds to solve while the DICE-ED model takes more than a minute. This difference is exacerbated when the model has to be run thousands of times.

The following sub-section enumerates the distinct uncertainties which will be used for the analysis.

5.2.1 Uncertainties

As seen in Chapter 4, one of the most important parameters is the social discount rate. For this reason, the social rate of time preference (ρ) will have an uncertainty range from

[4]See Appendix C for the code.

0.0001 to 0.015. 0.0001 corresponds to the non-zero value Stern uses due to the inclusion of a possibility of human extinction in the near-future [16]. 0.015 is the deterministic value used in DICE and considered by many to be in the upper range [82]. In the face of uncertainty, Weitzman argues that the far-distant future should be discounted at the lowest possible rate [86]. This is achieved in this analysis through the sampling method which ensures that low rates are chosen from the uncertainty space.

Another related parameter is the elasticity of marginal utility (α). This parameter can be thought of as the relative social valuation of consumption of different generations. Thus, a low value means that consumptions between different generations are close substitutes (low aversion to inequality) while a higher number means that these consumptions are highly differentiated (high inequality aversion). Economists usually set this around 1 in climate change modeling [81] (1.45 in the DICE model), however, in reality it could span a bigger range. For this analysis, the parameter will be in the range of 1 to 3 to follow the related literature [32, 81].

The depreciation rate of physical capital (δ_K) is also uncertain, especially when increasing temperatures and climate change can lead to a higher rate as physical capital has shorter lifespan due to extreme weather, storms, rising sea levels, etc [117]. In theory, the depreciation rate rises with atmospheric temperature [118]. The uncertainty range is defined from 0.1 (standard value) to 0.2.

Technological change comes through the DICE model by the growth rate of TFP. This growth rate ($g_A(t)$) will be varied from 0.07 to 0.09 to represent worlds with low and high technological change. Because it enters the production function multiplicatively, the actual results will be very sensitive to this parameter. The range used here is to study the effects of a lower or higher growth in technological change.

Decarbonization rates are highly uncertain and varied in the literature [78]. This will be explored by changing $g_\sigma(t)$ from -0.011 to -0.008 which are common values for annual decarbonization rates.

On the climate side, three key parameters will be assessed. The first one is the climate sensitivity (ξ_i) which in the current version is equal to 2.9°C [21]. The climate sensitivity is very likely to be in the range of 2°C to 4.5°C [119]. The second parameter is the coefficient for the damage function (φ_2). The parameter will take the range of 0.002 to

Table 5.2: Uncertainty ranges and the deterministic values.

Uncertainty	Symbol	Uncertainty Range	Deterministic Value
Pure rate of time preference	ρ	0.0001 – 0.015	0.015
Elasticity of marginal utility	α	1 – 3	1.45
Depreciation rate on capital	δ_K	0.1 – 0.2	0.1
TFP growth rate	$g_A(t)$	0.07 – 0.09	0.079
Decarbonization growth rate	$g_\sigma(t)$	-0.011 – -0.008	-0.01
Climate sensitivity	ξ_i	2 – 4.5	2.9
Damage function coefficient	φ_2	0.002 – 0.004	0.00267
Damage function exponent	φ_3	2 – 4	2

0.004 to account for varying impacts to climate. The third parameter is the exponent of the damage function (φ_3) currently set at 2. This means that the damage function has a quadratic behavior with temperature. This is highly uncertain as the parameter could take any value and has big impacts on the results [120, 121]. The uncertainty range for this parameter is set from 2 to 4. With these uncertainty ranges, the damages with an atmospheric temperature rise of 2°C would be 0.8% in the minimal case to 6.4% in the most extreme case.[5] It is with these uncertainty ranges that possible catastrophes can be simulated, in line with a fat-tail distribution of catastrophic risks and probability [122].

A summary of the uncertainties and the ranges is shown in Table 5.2. This is not, however, the full list of uncertainties in the DICE model. Almost every parameter in the model is subject to some type of uncertainty. The ones presented here are just a small but important sample of all of them.

After the uncertainties are recollected and accounted for, the different experiments are generated with a Latin Hypercube Sampling (LHS). LHS is a statistical method to generate samples from a multidimensional space (such as the uncertainty space in this case) in an almost random fashion. It is based on the *Latin square* and is usually used for computer experiments or the Monte Carlo method.

[5] Recall Equation (2.10): $\Omega(t) = \varphi_1 T_{AT}(t) + \varphi_2 [T_{AT}(t)]^{\varphi_3}$ where φ_1 is 0.

5.3 Uncertainty Analysis

The total number of uncertainties in Table 5.2 represents the total dimensions of the uncertainty space. In this case it is an *8-dimensional space*.[6] The key question while doing the uncertainty analysis is how many experiments should be run in order to have a balance between precision and computing time. To do this, the EMA workbench is run with Monte-Carlo sampling and 10,000 experiments. The total computing time was around 3 hours.

The criterion to determine when there have been enough experiments is when the fraction of interest stabilizes. The fraction of interest is defined as a proportion: how many experiments are successful in limiting atmospheric temperature rise to no more than 2.0°C by the year 2100 out of the total number of experiments. It can be defined as:

$$\omega(x) = \frac{\sum y}{\sum x} \tag{5.1}$$

where $\omega(x)$ is the fraction of interest, x is the amount of experiments and y is the amount of successful experiments. y is defined such that:

$$y = \begin{cases} 1 \text{ if } T_{ATM}(2100) \leq 2.0°\text{C} \\ 0 \text{ if } T_{ATM}(2100) > 2.0°\text{C} \end{cases}$$

Figure 5.1 shows the evolution of $\omega(x)$ with increasing number of experiments. When the code is run with 1000 or less experiments, $\omega(x)$ varies by a big margin. It is only after 3,000 experiments that $\omega(x)$ is stabilized around 0.245. Thus, for the uncertainty analysis, it is of utmost importance to run the code with at least 3,000 experiments.

The initial analysis consists of 3,000 experiments with the aforementioned uncertainties. The DICE model is run on the optimal scenario which optimizes welfare with the damage function. This optimal scenario is the same one as in the DICE-ED model.

One of the basic functions of an uncertainty analysis is to visualize the effect of different parameters on the model's outcomes. Figure 5.2 shows the effect of uncertainty

[6] If there was only 1 uncertain parameter then the uncertainty space could be visualized with just a line. 2 uncertain parameters could be visualized with a 2D plane. 3 uncertain parameters could be visualized with a 3D shape and so on for higher dimensions.

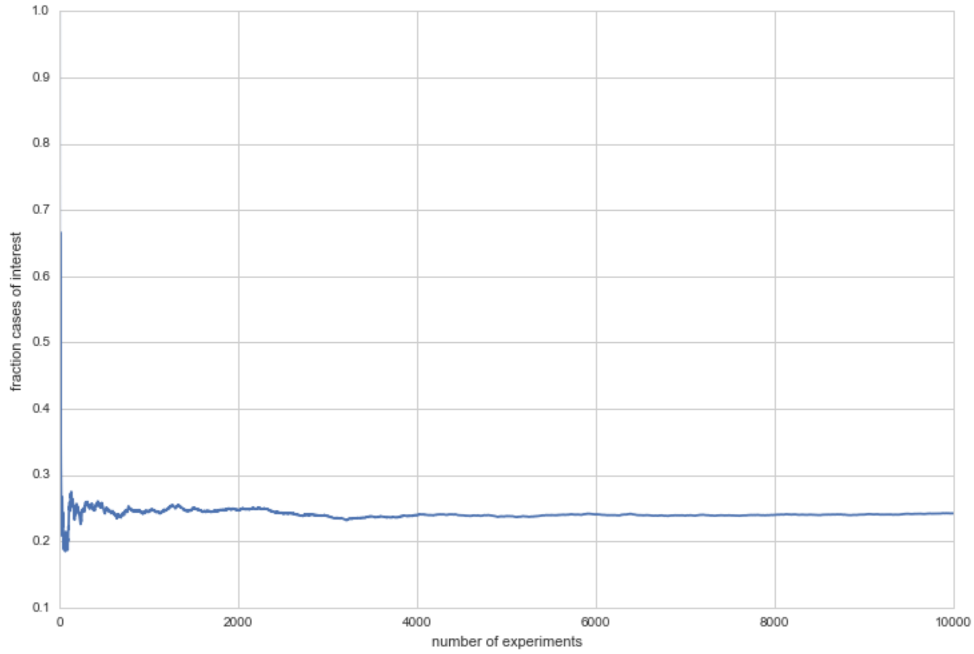

Figure 5.1: Fraction of interest evolution with increasing number of experiments.

on a key outcome of interest, the atmospheric temperature rise. As can be clearly seen, the effect of uncertainty creates a wide range of possibilities. However, due to the long nature of climate change modeling, sometimes it is only useful to present results until the end of the 21st century. This way, policy-makers can have a better grasp of what is immediate. Figure 5.3 shows the results of uncertainty on the atmospheric temperature rise until the year 2100. This particular figure is very relevant to today's discussion on the topic. In the recent COP21, an agreement was reached which aimed to limit atmospheric temperature rise to no more than 2°C [10]. It is this 2.0°C target which is of particular interest. The figure shows that in a particular set of experiments, the atmospheric temperature rise is equal or lower to 2.0°C. Under which assumptions of uncertainty does this particular set of experiments comply with the 2.0°C target? The solution lies within scenario discovery.

The EMA workbench analytics includes the Patient Rule Induction Method (PRIM). PRIM is a type of bump hunting algorithm of which the objective is to find regions in the input (uncertainty) space with relatively high (or low) values for a target variable [123]. It is a useful tool for scenario discovery, as it allows for the detection of certain scenarios which are of particular interest [26].

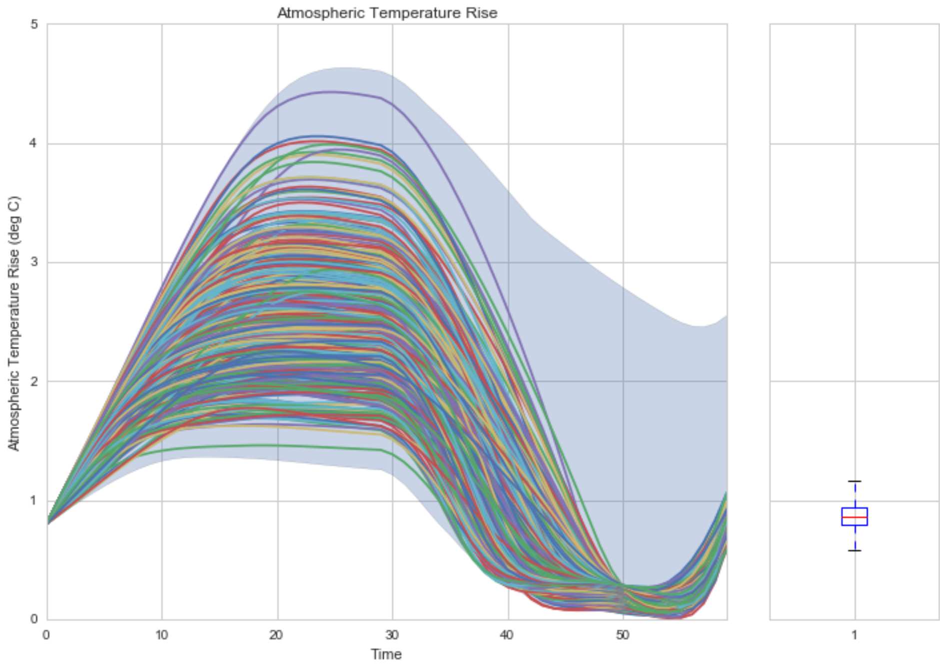

Figure 5.2: Effect of the uncertainty on the atmospheric temperature rise.

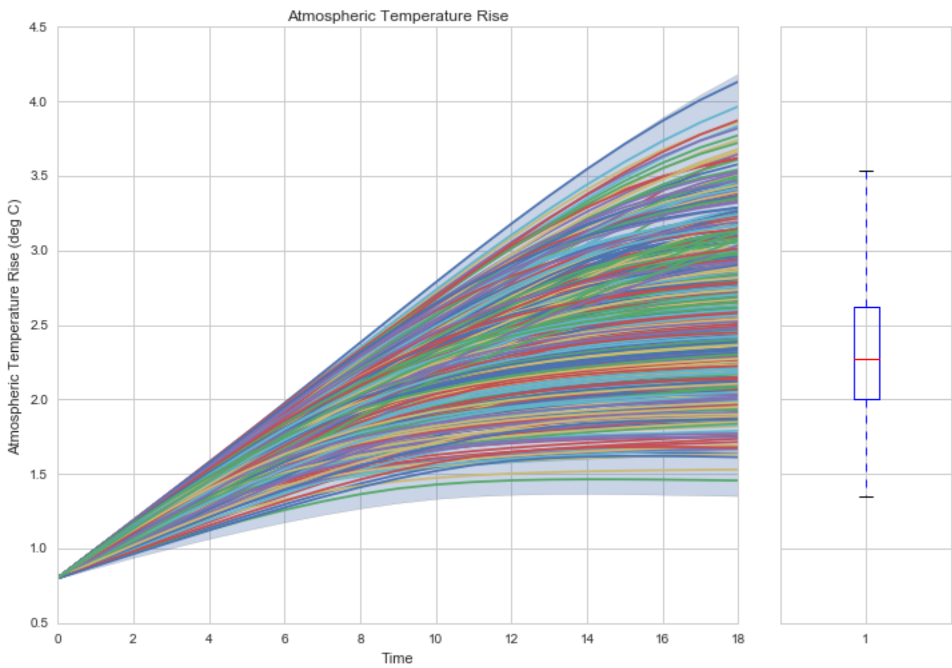

Figure 5.3: Effect of the uncertainty on the atmospheric temperature rise until the year 2100.

Out of the 3,000 experiments, 730 of them are of special interest, as the temperature rise by the end of the 21^{st} is equal or less than 2.0°C. This equals 24.4% which is in line with the 10,000 Monte-Carlo experiment from above.

The PRIM algorithm is run on the EMA Workbench in order to apply it for scenario discovery. The output of the PRIM algorithm is a series of boxes where each box represents a particular scenario. A series of boxes is called a *box set* which collectively describe a set of assumptions in the uncertainty space with the outcome of interest (in this case limiting temperature rise to no more than 2.0°C). Each box set is defined by its *coverage* and *density*. The coverage measures how many of the scenarios in the box effectively comply to the outcome of interest. In this case, the highest coverage possible is what is wanted. It is common practice to at least look for a coverage equal or above 0.8. Density is analogous to the fraction of interest, Equation (5.1). This means that it is the ratio of total experiments of interest in a particular scenario over the total number of experiments. Just like coverage, a high density is also of importance. Alongside these two metrics, the *interpretability* is also important. This refers to how easy the box set can be used to gain proper insight. This matter is highly subjective but experience shows that a box set should not have more than four boxes with three uncertain parameters [26].

Figure 5.4 shows the results of the PRIM algorithm. It is at a first glance that one can notice certain trade-offs. Increasing coverage comes at the expense of a reduction in the density. Interpretability is proxied as the number of restricted dimensions. A lower number of restricted dimensions corresponds to higher interpretability. Another trade-off here is that as interpretability increases, the coverage does as well but with a reduction in the density. Here lies the interactivity of the PRIM algorithm, where the modeler needs to decide on appropriate boxes based on these metrics.

The ideal box would have 100% density and 100% coverage with just a few restricted dimensions. However, such is not the case here. The enclosed area is of particular interest as it is the closest to the ideal point. Three boxes are particularly chosen in Figure 5.4 for their coverage, density and no more than three restricted dimensions. Table 5.3 shows the coverage, density and restricted dimensions for these boxes.

The three restricted dimensions in the three boxes are the same: the exponent of the damage function (φ_3), the elasticity of marginal utility (α) and the climate sensitivity

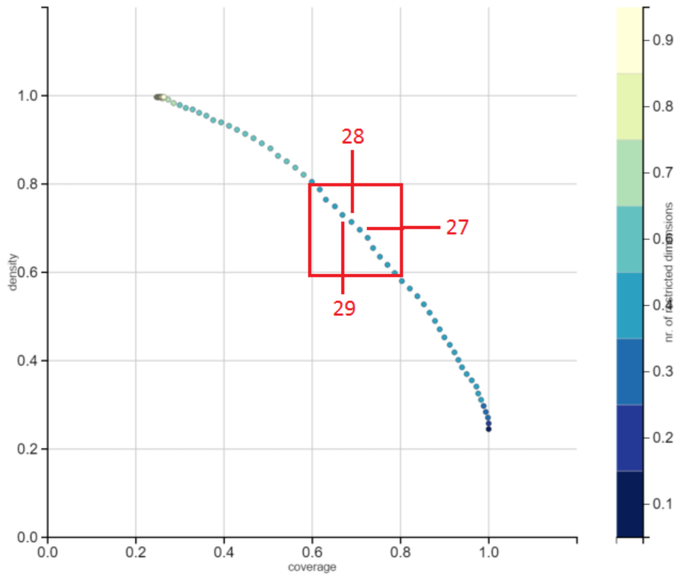

Figure 5.4: Efficiency frontier from PRIM algorithm with limit on temperature rise to 2.0°C.

Table 5.3: Coverage, density and restricted dimensions for selected boxes.

Box	Coverage	Density	Restricted dimensions
27	72.87%	71.60%	3
28	71.23%	73.75%	3
29	68.90%	75.19%	3

(ξ_i). Figure 5.6 shows the values of box 28 for these parameters and the quasi-p-values used to test significance.[7] Quasi-p values are found in parenthesis next to the parameters. The light gray area is the full range of the uncertainty while the blue line represents the uncertainty range in the sub-space found by the PRIM algorithm. The upper parameter *a3* is the exponent of the damage function which is extremely significant on the results (qp = 7.1E-54). The other two parameters, *elasmu* and *t2xco2*, are the elasticity of marginal utility and the climate sensitivity respectively. Both of these parameters are also significant.[8] Interpreting the graph, the atmospheric temperature rise will be lower or equal to 2.0°C by the end of the 21st century if the exponent of the damage function is between 2.9 and 4, the elasticity of marginal utility is between 1 and 2.1 and the climate sensitivity is between 2 and 4. This is somewhat unsettling as the climate sensitivity is a parameter which is inherent in nature, thus it is not possible to change it with policy-making. The best course of action is to try to understand it more and adapt to what the latest science tells. The exponent of the damage function, on the other hand, is mostly chosen subjectively by modelers. Many studies try to quantify the damage function [34], however, it is still mostly uncertain. Policy advice to increase valuation of the damages of climate change is not very conventional. Increased occurrences of large-scale hurricanes, thunderstorms and other climate events will tend to increase the damage function. However, this has its shortcomings as it is difficult to pinpoint the cause of these events with human-induced climate change. Figure 5.5 shows the effect of uncertainty on climate damages caused by the increase in atmospheric temperature. Damages range from a maximum of 10% to a minimum of 1%. Nonetheless, it is surprising to see that the elasticity of marginal utility is significant and not the pure rate of time preference (ρ). The reason is that the elasticity of marginal utility appears directly in the utility function and is thought of as aversion to generational inequality. A greater value of this elasticity means high inequality aversion and thus consumptions are highly differentiated. It is with higher values where climate change is given less weight and thus temperature rises above 2°C. Fortunately, most climate models use values between 1 and 2 and the uncertainty space in this case is from 1 to 2.1 (Figure 5.6). Therefore, current climate policy models are in line with achieving the 2°C target in this particular parameter. Ultimately, this is not to say that discounting with ρ is not important. It merely reflects the fact that uncertainty about the exponent of the damage function, elasticity of marginal utility and climate sensitivity are more important in determining the overall rise in atmospheric temperature.

[7] Box 27 and box 29 have similar results so they are avoided here for redundancy.

[8] Where quasi-p-values below 0.05 are considered significant.

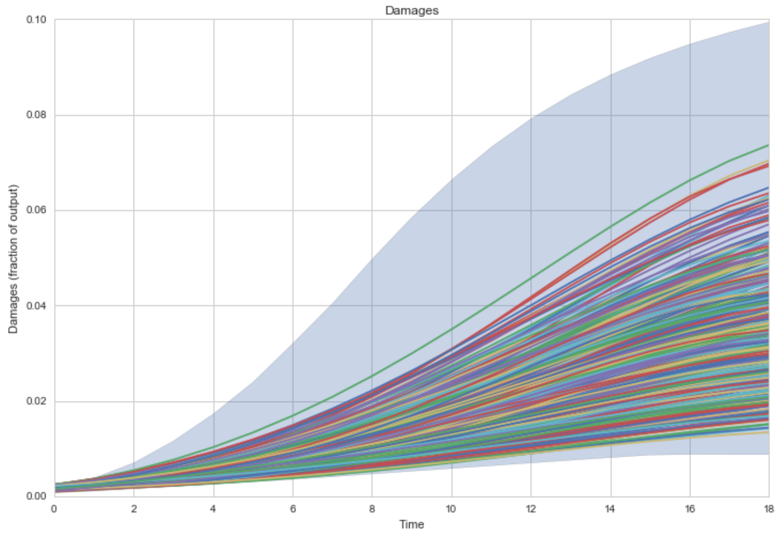

Figure 5.5: Effect of uncertainty on damages caused by the increase in atmospheric temperature.

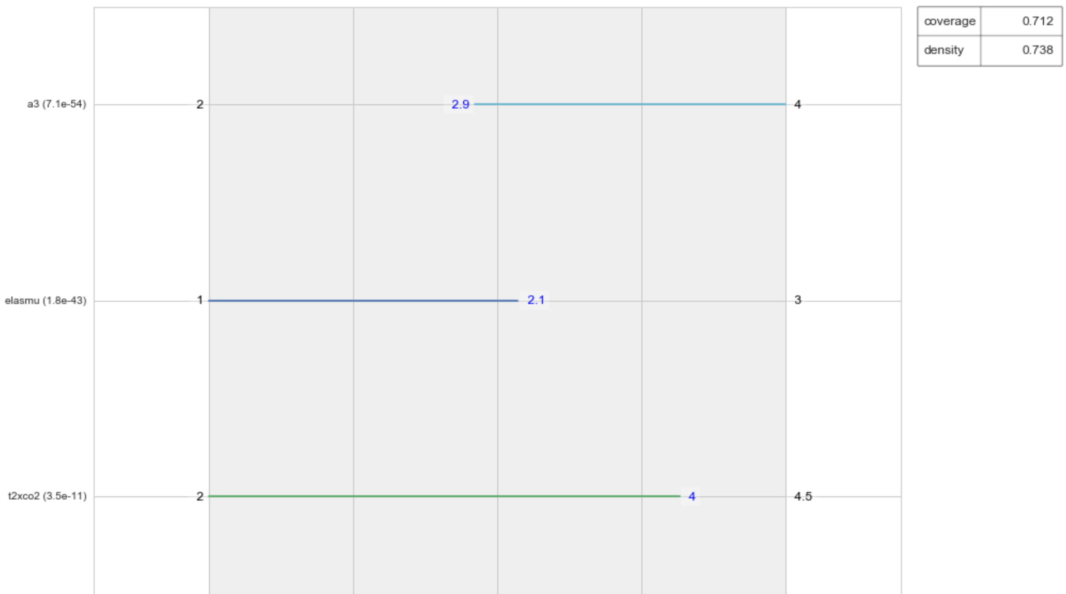

Figure 5.6: Range and quasi-p-values for restricted dimensions in box 28.

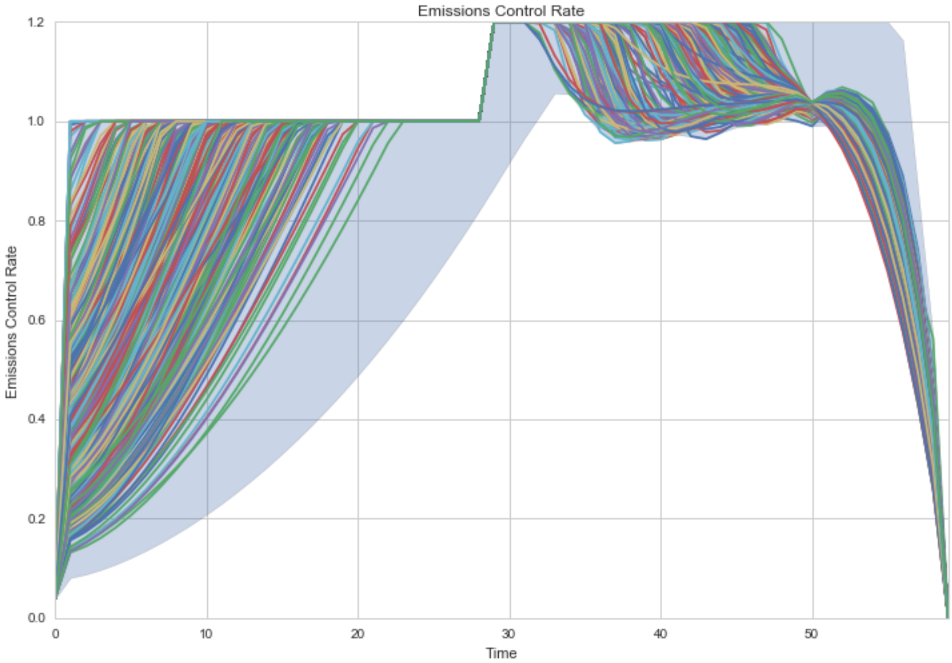

Figure 5.7: Effect of uncertainty on emissions control rate.

Reaching the temperature goal from COP21 means that a big amount of emissions need to be controlled to limit carbon emissions. Carbon emissions can be controlled by traditional sustainable energies or by carbon capture and storage (CCS). Figure 5.7 shows the effect of uncertainty on the emissions control rate. The control rates are limited to 1 in the first half of the timespan because it represents a modeling decision as carbon negative technologies are assumed not to be available. However, in the second half, carbon negative technologies are available and the limit on the emissions control rate is increased to 1.2. This represents a subjective assumption about the potential of carbon negative technologies by the modeler.

There have been proposals to decarbonize the economy by 2050 as a solution to climate change [124]. The EMA Workbench and scenario discovery can help find under which assumptions this is true. Figure 5.8 shows the emissions control rates until the year 2050. The PRIM algorithm is run on the workbench in order to perform scenario discovery. Out of the 3,000 experiments, 681 are cases of interest which represents 22.7%. This indicates that when taking into account uncertainty, in over one fifth of the times must emissions be 100% controlled by 2050.

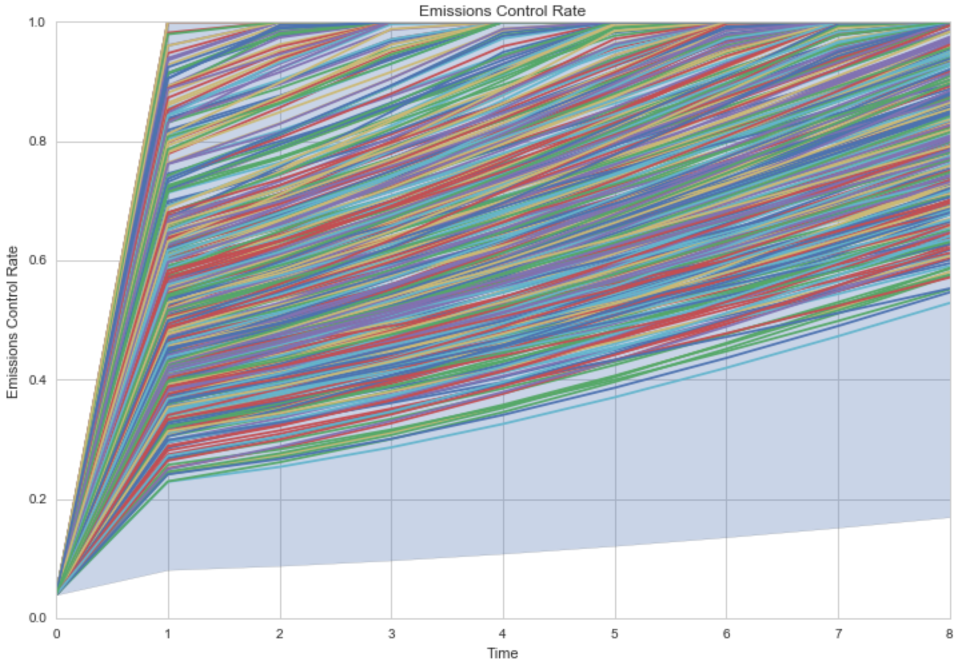

Figure 5.8: Effect of uncertainty on emissions control rate until year 2050.

Figure 5.9 shows the efficiency frontier for this case. The shape is similar as the previous case, along with the same exhibited trade-offs. In fact, the coverage, densities and the restricted dimensions are the same. This is to say that dealing with these deep uncertainties is of primordial importance to comply with environmental targets. Another important result of climate policy models is the social cost of carbon (SCC), which in an ideal economy would represent the harmonized global carbon tax. Figure 5.10 shows the minimum and maximum outline of the SCC when taking into account uncertainty. The distribution of the SCC is shown for the year 2050 (time period = 8). In the optimal run of the deterministic DICE model, the SCC in the year 2050 is of 45 2005 USD/tonCO_2. Out of the 3,000 experiments done here, only in 229 experiments (7.63%) is the SCC below 50 2005 USD/tonCO_2.[9] The SCC is very sensitive to changes in the uncertainty space and considering the importance in policy-making, it deserves the question: do current estimates of the SCC consider uncertainty? The "dismal theorem" proposed by Martin Weitzman states that in the face of uncertainty and fat-tailed distributions for catastrophic events with low but nonnegligible probabilities, the value of the SCC is theoretically infinite [122]. This is an area for future research.

[9] 50 is used instead of 45 to get numbers just above 45 too.

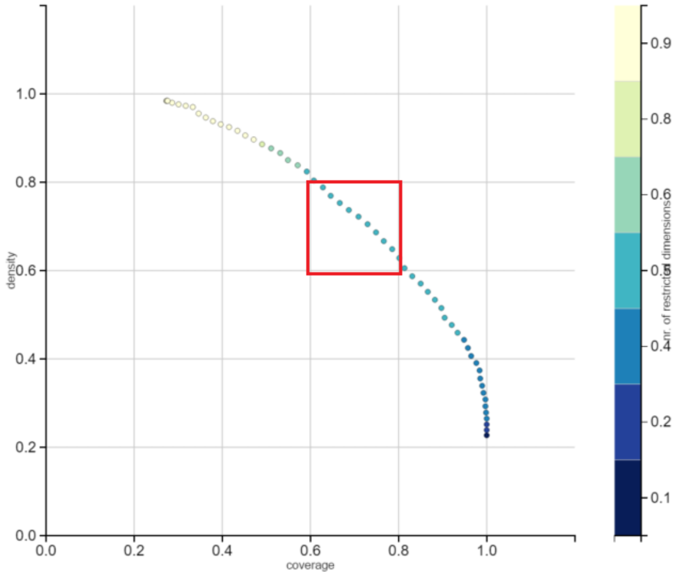

Figure 5.9: Efficiency frontier from PRIM with emissions control rate up to 1 until 2050.

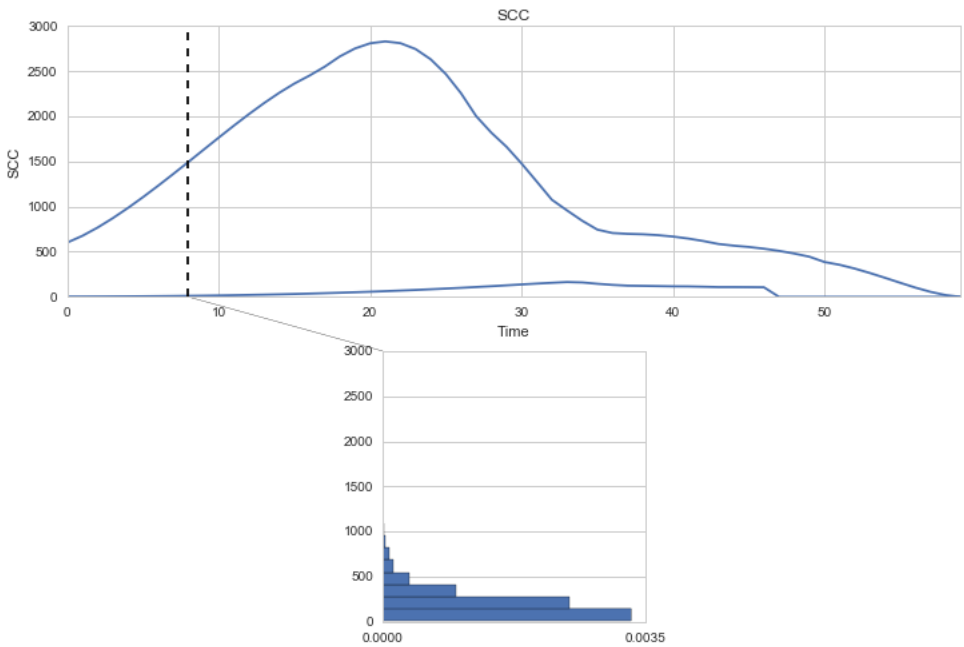

Figure 5.10: Uncertainty surrounding the SCC and the distribution in the year 2050.

Table 5.4: Feature selection

Rank	Uncertainty	Symbol	Value
0	Damage function exponent	φ_3	0.483197
1	Elasticity of marginal utility	α	0.254259
2	Climate sensitivity	ξ_i	0.157225
3	Pure rate of time preference	ρ	0.053009
4	Damage function coefficient	φ_2	0.036677
5	TFP growth rate	$g_A(t)$	0.006180
6	Decarbonization growth rate	$g_\sigma(t)$	0.004836
7	Depreciation rate on capital	δ_K	0.004618

This shows the importance of dealing with deep uncertainty in climate policy models. Nordhaus states that the 2°C target is ambitious as emissions control rates would need to reach zero by 2060 [21]. However, there is no mention of how to achieve this target or what could drive it. This is one of the benefits of the uncertainty analysis developed in this chapter with the EMA Workbench. The most important uncertain parameters were the exponent of the damage function, the elasticity of marginal utility and the climate sensitivity. The EMA Workbench includes analytics for feature selection. Feature selection is useful in big models as it allows to rank certain inputs by degree of importance with linear or logistic regressions. Table 5.4 shows the results of this and ranks the uncertainties by order of importance.

5.4 Concluding Remarks

This chapter dealt with the issue of uncertainty surrounding the DICE model. A literature review on the relevant topics was first introduced. Scenario discovery and Exploratory Modeling and Analysis were identified as key techniques for dealing with uncertainty. Afterwards, the experimental design was discussed along with the relevant uncertainties of the DICE model and their appropriate ranges. Finally, the uncertainty analysis was performed.

The initial analysis showed that a wide range of possible outcomes were obtained through the analysis. To segment and rationalize this, the 2°C target proposed in the COP21 was used to apply scenario discovery and the PRIM algorithm. Three key uncertainties were obtained from this: the climate sensitivity, the elasticity of marginal utility and the exponent of the damage function. These three uncertainties paint a gloomy picture for the environment. First, the climate sensitivity is an uncertain physical parameter, only more research can lead to a greater understanding of it. There is not much leeway in this parameter except taking the latest science as a basis. The elasticity of marginal utility is a subjective parameter chosen by the modeler. Nonetheless, current climate policy models usually take this parameter in the same range as the one proposed by scenario discovery. Finally, the exponent of the damage function is also another subjective parameter. The deterministic version of DICE uses a value of 2, however, the uncertainty space obtained through scenario discovery shows that the exponent should be between 2.9 and 4 in order to limit atmospheric temperature rise to no more than 2°C. This parameter can be increased either by an increase occurrences of natural disasters or through more research.

Additional to the atmospheric temperature rise, the emissions control rates were also subject to the same uncertain parameters. This connection is of no surprise as a higher emissions control rate limits atmospheric temperature rise. Finally, the SCC was also analyzed and the results showed that it is very sensitive to uncertainty. Nordhaus' SCC recommendation of 45 2005 USD/tonCO_2 by 2050 is only valid in 7.63% of the cases. This shows that it is highly unlikely that in an optimal scenario the SCC would be the one recommended by Nordhaus.

This concludes the chapter of uncertainty. The next chapter concludes and reflects on the thesis.

Chapter 6

Conclusions and Reflections

This chapter concludes the thesis. The first section elaborates on the main conclusions from the body of the thesis. Afterwards, recommendations for future research are presented. A reflection on the work and a personal reflection finish the chapter and the thesis.

6.1 Conclusions

The purpose of this thesis is to improve climate policy modeling with the introduction of endogenous technological change, dynamic discounting and uncertainty. Revisiting the main research question:

How sensitive are the climate policy conclusions from a standard climate policy model to the introduction of endogenous technological change, dynamic discounting and uncertainty?

The overarching conclusion is that policy conclusions differ greatly with the introduction of endogenous technological change, dynamic discounting and uncertainty.

Endogenous technological change

Including endogenous technological change (Chapter 3) has important effects on the DICE climate policy model (Chapter 2). Total welfare is improved by 9% compared to the exogenous technological change case. In addition, more stringent emissions reductions are recommended with the inclusion of endogenous technological change as the price of fossil fuels is now included and taken into account. In the optimal scenario (accounting for the externality), emissions peak by the end of the 21st century. All of this means that at the end of the timespan the atmospheric temperature rise will be lower. From the analysis, the difference in final temperature rise compared to the exogenous case amounted to 0.25°C.

However, the inclusion of a backstop (sustainable) fuel into the model has the biggest effect on the most important economic and environmental variables compared to the model without the backstop fuel. Higher emissions reductions and a lower atmospheric temperature rise are achieved with a backstop. Many climate policy models do not explicitly use a backstop fuel. Including a backstop fuel brings a wealth of information about the market dynamics that cannot be captured without it.

Nonetheless, the assumption of the initial backstop fuel price is a major determinant in the results. As would be expected, a lower backstop fuel price lowers atmospheric temperatures and reduces emissions. Even with the uncertainty surrounding the price of the backstop fuel, policy which reduces the backstop price such as R&D subsidies or tax breaks will have a net positive impact on the environment.

From the economic module, the social cost of carbon (SCC) with a backstop fuel is more than 16% higher than in the exogenous case. The SCC would represent the ideal carbon tax in the economy. This has implications for policy-making as current carbon taxes calculated under models with exogenous technological change are undervalued. This is akin to not capturing the full social cost of a ton of carbon. Additionally, a higher understanding of the dynamics is achieved with the inclusion of the backstop fuel. Economic output is increased significantly in the long-run as the backstop fuel represents a cheap, non-polluting way of generating output. In the short-term, economic output remains in line with the exogenous case due to the high price.

Even with the reduction in emissions and a higher SCC, this is not enough to prevent dangerous climate change. The recent Paris Agreement from the COP21 aims to limit atmospheric temperature rise to no more than 2°C by the end of this century. Temperatures of more than 4°C are achieved in this model even in the most optimistic situation with a low backstop price. Thus, additional policy measures such as command-and-control are necessary to comply with international agreements and bring further reduction in emissions.

What does all of this mean for policy-making? First of all, this thesis shows the importance of endogenous technological change as it explains the links and dynamics between the economic and climate module in a way that exogenous technological change cannot. Many models still include exogenous technological change and policy-makers should be aware of the limitations that this brings. This thesis has applied endogenous technological change to a common and ubiquitous model such as the DICE and shown how sensitive the policy recommendations are.

Dynamic discounting

Dynamic discounting was introduced into the model (Chapter 4) as a way to challenge the traditional notion of a constant social discount rate. This was achieved through two distinct formulations: one related to an economic variable and another to an environmental variable. Both formulations show declining social discount rates throughout the timespan of the model. This is in line with what many economists suggest for long-term environmental models.

Dynamic discounting values future damages and benefits more than the traditional method with a constant social discount rate. Damages are valued seven times higher by the end of the model with dynamic discounting due to declining social discount rates. This has important effects on both the economic and environmental variables. As with endogenous technological change, dynamic discounting has a positive impact on the environment with lower emissions and atmospheric temperatures. In fact, emissions peak around the year 2060 compared to the year 2100 with a constant social discount rate. Still, temperatures reach critical limits above the 2°C level by 2100 and thus more control policy is necessary and advocated.

Savings rate are increased in the model with dynamic discounting by two percentage points. Policy which aims to increase the overall savings rate of the economy such as an increase in the consumption tax or a decrease in the capital gains tax would be in line with declining social discount rates. On the other hand, the SCC also increases in the model with dynamic discounting. This is due to a higher valuation of future damages and benefits when the social discount rate is lower. Policy-makers should be aware of an undervaluation of the SCC in climate policy models with a constant social discount rate.

The importance of dynamic discounting has been shown in this thesis. Two exploratory formulations for defining a non-constant social discount rate were demonstrated. The justifications for a non-constant social discount rate which decreases with time has been advocated by numerous scientists. As this work represents an expansion of the knowledge frontier, more research is necessary to bring it into mainstream climate policy models. Nonetheless, it represents an important step towards a deeper understanding of the climate and the economy which can be reflected with appropriate policy advice.

Uncertainty

The treatment of (deep) uncertainty in this thesis (Chapter 5) was done through Exploratory and Modeling Analysis (EMA). Eight different uncertain parameters were considered. These parameters included discount rate parameters, climate sensitivity parameters, technological change, decarbonization, damage function parameters and the depreciation of physical capital. With an 8-dimensional uncertainty space, 3000 experiments were conducted with Latin Hypercube sampling.

Out of the 3000 experiments, only in 25% of them does the temperature rise stabilize below 2°C by the end of the 21st century. This is a major point of discussion with international agreements such as the one signed in the COP21 in Paris. The Patient Rule Induction Method (PRIM) was run to find the uncertainty space which complies to such target. After applying scenario discovery, three uncertain parameters were found to be the most sensitive to achieving the 2°C target: the exponent of the damage function, the elasticity of marginal utility and the climate sensitivity. Quasi-p values were obtained from the PRIM algorithm and the three parameters were significant. In the subset selected by the PRIM, the exponent of the damage function takes a value between 2.9 and 4 (the deterministic DICE model uses an exponent of 2). This means that the subjective valuation of climate damages needs to increase in order to be able to achieve the environmental targets. This is unfortunate, as this can be done through an increase in the occurrences of environmental disasters linked to climate change or by convincing modelers and policy-makers about this importance. The marginal elasticity of utility, on the other hand, is the aversion to generational inequality. This again is a subjective measure. A lower value means that consumption of different generations are close substitutes and thus more care is taken on the environment, ultimately lowering the atmospheric temperature. Lastly, the climate sensitivity is very uncertain even with all the research that goes into it. Unfortunately, there is nothing that can be done about the climate sensitivity as it is an inherent property of nature. It is only further research that can elucidate modelers with the right choice.

Same as the atmospheric temperature rise, another key variable obtained from climate policy models is the SCC. The SCC is very sensitive to the treatment of uncertainty. In his DICE model, Nordhaus recommends a SCC of 45 2005 USD/ton CO_2 by 2050 in the optimal scenario. When taking into account uncertainty in this thesis, the SCC is only below 50 2005 USD/ton CO_2 in 7.63% of the times. This means that it is more likely than not (over 90%) that the SCC is above 50 2005 USD/ton CO_2. This shows the importance of taking into account uncertainty in climate policy models because it gives a more accurate portrayal of what to expect in the uncertain future.

6.2 Future Research

Climate policy models such as DICE have a big and substantial influence over policy-makers. Many policy decisions rely on these models' results. Therefore, it is of utmost importance to have a clear understanding of the models and look for ways to improve it. Along the realization of this thesis, several strands of research were identified as lacking completely or underdeveloped. In no particular order of priority, future recommendations for research are:

- Study the effect of *end-of-horizon effects* and the timespan on model results. The sensitivity analysis made in this thesis showed that the timespan could potentially be critical in the results. Ideally, this study is made across models to generalize the concept.

- Relax the assumption of the opportunity costs of R&D as it is possible that there are no opportunity costs due to the availability of banking finance. This is easily achievable and could change the way these models are done.

- Another important assumption is that all the savings in the model go towards investment. This is due to the neoclassical framework of the model but it is not necessarily the case in real life. A suggested alternative is to study the climate model under a Keynesian framework.

- Include tipping points and extreme damages in the damage function in order to give higher temperature ranges more importance than current formulations do. Along with this study, including not only carbon but also the rest of the GHGs in the forcing equation would be relevant to have a better overall picture of the climate dynamics.

- Discounted utilitarianism has been the norm for climate policy models. An exploration of the DICE model under sustainable discounted utilitarianism can yield interesting and novel results with a bigger emphasis on sustainability.
- Perform a similar analysis with dynamic discount rates but on alternative climate policy models to compare results and have a better overview of the effect of non-constant rates.
- Completely endogenize the social discount rate within the neoclassical framework and make the social discount rate a decision variable for the solution algorithm. Compare these rates with the ones from this thesis to ensure consistency in policy advice.
- It is recommended to use the EMA Workbench developed partly in this thesis to other climate policy models. Additionally, it would be useful also to treat the extended DICE-ED model with the workbench.
- The SCC showed extreme sensitivity to the treatment of uncertainty. Given the importance of the SCC in policy-making, it is advised to analyze in more detail the effect of uncertainty in this case and what it means for policy-making.

6.3 Reflections

At the end of the thesis, there are three different models which all aim towards a deeper understanding of the nexus between the climate and the economy. If I were to meet Nordhaus tomorrow, I would first consider explaining my work behind dynamic discounting. This is due to the fact that the treatment of technological change and uncertainty are widely recognized as limitations. The issue of discounting goes deeper into the economist's heart where everyone has their own particular opinion. I believe that dynamic discounting adds not too much complexity at the model while at the same time improving it in a significant way. From the two formulations of this thesis, I have a preference for the environmental link as this is a very common human behavior. Besides, it is time-consistent and easily applicable to many models.

At the time of this writing (October 29, 2016), Nordhaus just released a *beta* version of the 2016 version of DICE. In terms of this thesis, not much has changed. Discounting is still done in the exact same way as in the 2013 version, technological change is still exogenous and there is no implicit treatment of uncertainty. Climate sensitivity has changed from 2.9 to 3.1. The exponent of the damage function is still 2 and the elasticity of marginal utility is still 1.45. What seems to be the biggest difference is that the timespan of the model increased from 300 years to 500 years and the update of some climate parameters. Nonetheless, it seems that this new version is just an update on numbers and does not include any real structural transformation. My initial guess is that Nordhaus will use this new version to publish revised SCC values.

IAMs are usually used for calculation of the SCC. Are IAMs the best tool available for this? I cannot give a reasonable answer. But I would like to argue that they are one of the best available options we have. I believe that the SCC should not be one single deterministic value. With all the uncertainty surrounding the climate damages, the SCC will never be exactly calculated. I think it is better to think about the SCC as a possible range. Here, IAMs can be used to calculate it along with the appropriate uncertainty. Subsequently, the SCC in IAMs can be updated as new science comes along. So, IAMs are a useful tool, although not perfect. They give a guiding light in face of the darkness of uncertainty.

This uncertainty can show up as a fat-tailed distribution as in the "dismal theorem" from Weitzman. The exact numbers will probably never be quantified. But when dealing with these catastrophic risks, I think even the slightest chance of it happening is enough to warrant insurance. Why else do we insure our whole life even if the probability of damage is next to zero? This is in line with the minimax criterion for minimizing loss for a worst case scenario. This is particularly important with the "dismal theorem" as the expected value of damages could be infinite in theory. With a fat tail, the overall conclusions of the discounting chapter make even more logical sense. *Ceteris paribus*, discount rates should be lower in a world with these catastrophic risks. This is then tied to my initial recommendation I would make to Nordhaus.

In the uncertainty analysis, quasi-p-values were used to assess if the uncertainties in the boxes were there purely by chance. Why quasi-p-values and not regular p-values? This is because depending on the sampling method, the values might not be an accurate portrayal of the system. This particular type of test, mostly a binomial test, is used specifically in the scenario discovery literature. A binomial test is the appropriate one due the nature of the PRIM algorithm, where you either fail or succeed with the objective target.

As a final reflection, Robert Pindyck has criticized the use of IAMs for climate policy as the uncertainty regarding some parameters is too big to ignore.[1] This would then make IAMs useless for climate policy. I do agree with many of his arguments, especially the one where the damage function is completely arbitrary. However, his alternative of relying on expert opinion is no better. As the old joke says: if you put 10 economists in a room, you will get 11 opinions. He advocates for ranges with the expert opinion, which invariably will lead to ranges in the discount rate which will in turn lead to a big range for the SCC. I believe that the best option is to continue making strides with the IAMs while taking away subjective parametrization from the modelers as much as possible (e.g. with dynamic discount rates). Climate change is not surrendering, so we should keep fighting.

6.4 Personal Reflections

I found great satisfaction with this thesis. Why? I overcame a multitude of challenges which I put on myself. Before starting the thesis, I did not have any experience whatsoever with economics, with GAMS, with Python, with climate models, with modeling and even with writing a thesis. I was fortunate to have very supportive supervisors who guided me through this. At the end, I am very pleased with my work and the findings. One thing that I would like to improve for next time is working on my brevity and concise writing as I managed to write more than 120+ pages here. I abided by Dr. Storm's recommendation that it is always better to write more than not writing enough as text can always be deleted on-demand. In life, however, I believe that less is better in most cases.

This leads me to the basic question which followed and nagged me through these last nine months. **Are more complex and bigger models better?** I honestly do not know the answer. Looking rapidly through the code, the original DICE model contains 380 lines of code while my model contains 916 lines of code! This is without including several sub-routines which would add several hundred more. I have more than tripled the size of the model. At the same time, every new line of code adds uncertainty to the model. Each new parameter, each new assumption, each new variable is adding an extra layer of complexity and depth to the model. Is it worth it? I believe that the quest to fulfill research merits as much lines of codes as possible. It is after a careful examination

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of the model and the results that one can discriminate or not against extra lines of code. I hope that my work can incite future research with the purpose of telling us more with less lines of code. The final objective should be to have *the most simple complex* models!

This leads me to a another point of reflection. I had the chance to be in Paris the weeks before the COP21 where I interacted with thousands of young activists. At the moment I did not realize it, but many persons were collectively using models in a wrong way. There was absolutely no distinction between "scenarios" and "predictions". In fact, every "scenario" was taken as a "prediction". This of course eschewed the narrative and thus we were all doomed by the business-as-usual. There is a chasm between modelers and the general public in the reading of model results. This should be worked on from both sides. On one hand, modelers need to do a better job of explaining the models in terms that the users will understand. On the other hand, the users must be willing to dive deeper into the model, question the results and not accept things at face-value. I think there is a lack of synergy between modelers and general users. I hope that in the future I can help solve this issue.

At the end, this thesis has changed me. I think it has completely transformed how I approach challenges in life, how to do research, how to communicate and how to write. And the transformation has been for the best. I can finally consider myself a master student. This is where formal education has succeeded. Even if this thesis is read by 7.5 billion people or just 1, I think I can consider it a success. Not because of what is written or what was researched, but what is yet to come.

Bibliography

[1] NASA, “Solar System Portrait - Earth as Pale Blue Dot,” 1990.

[2] R. Kümmel, *The Second Law of Economics: Energy, Entropy, and the Origins of Wealth.* New York: Springer, 2011.

[3] BP, “BP Statistical Review of World Energy,” tech. rep., London, 2016.

[4] United Nations, “World Population Prospects: The 2015 Revision,” tech. rep., New York, 2015.

[5] IPCC, “Climate Change 2013: The Physical Science Basis. Summary for Policymakers,” tech. rep., Cambridge, 2013.

[6] Earth Policy Institute, “Eco-Economy Indicator: Temperature 2013,” tech. rep., Rutgers University, New Brunswick, NJ, 2013.

[7] P. J. Crutzen and E. F. Stoermer, “The "Anthropocene",” *IGPB Newsletter*, vol. 41, pp. 17–18, 2000.

[8] A. Berger and M. F. Loutre, “An Exceptionally Long Interglacial Ahead?,” *Science*, pp. 1287–1288, aug 2002.

[9] IPCC, “Climate Change 2014 Synthesis Report,” *Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, pp. 1–151, 2014.

[10] UNFCCC. Conference of the Parties (COP), “Adoption of the Paris Agreement.,” *Paris Climate Change Conference*, 2015.

[11] A. Löschel and M. Schymura, “Modeling Technological Change in Economic Models of Climate Change,” *Encyclopedia of Energy, Natural Resource, and Environmental Economics*, pp. 89–97, 2013.

[12] F. Ackerman and I. J. Finlayson, “The economics of inaction on climate change: a sensitivity analysis,” *Climate Policy*, vol. 6, no. 5, pp. 509–526, 2006.

[13] W. Nordhaus, “Integrated Economic and Climate Modeling,” in *Handbook of Computable General Equilibrium Modeling* (P. B. Dixon and D. W. Jorgenson, eds.), ch. 16, pp. 1069–1131, North Holland: Elsevier, 1st ed., 2013.

[14] K. Arrow, W. Cline, K.-G. Maler, M. Munasinghe, R. Squitieri, and J. Stiglitz, “Intertemporal Equity, Discounting, and Economic Efficiency,” in *Climate Change 1995: Economic and Social Dimensions of Climate Change* (J. P. Bruces, H.-S. Yi, and E. F. Haites, eds.), ch. 4, pp. 127–142, Cambridge University Press, 1996.

[15] S. D. Baum, “Description, prescription and the choice of discount rates,” *Ecological Economics*, vol. 69, no. 1, pp. 197–205, 2009.

[16] N. Stern, “The Economics of Climate Change,” *Stern Review*, p. 662, 2006.

[17] S. Dietz and G. B. Asheim, “Climate policy under sustainable discounted utilitarianism,” *Journal of Environmental Economics and Management*, vol. 63, no. 3, pp. 321–335, 2012.

[18] H. Llavador, J. E. Roemer, and J. Silvestre, *Sustainability for a Warming Planet*. Cambridge: Harvard University Press, 2015.

[19] G. B. Asheim and T. Mitra, “Sustainability and discounted utilitarianism in models of economic growth,” *Mathematical Social Sciences*, vol. 59, no. 2, pp. 148–169, 2010.

[20] G. Müller-Fürstenberger and G. Stephan, “What really matters: Discounting, technological change and sustainable climate,” *Ecological Economics*, vol. 70, no. 5, pp. 978–987, 2011.

[21] W. Nordhaus and P. Sztorc, “DICE 2013R: Introduction and User’s Manual,” tech. rep., 2013.

[22] C. Böhringer, T. P. Mennel, and T. F. Rutherford, “Technological change and uncertainty in environmental economics,” *Energy Economics*, vol. 31, no. SUPPL. 1, pp. 3–5, 2009.

[23] N. Rosenberg, “Uncertainty and Technological Change,” *The Mosaic of Economic Growth*, pp. 334–353, 1996.

[24] R. S. Pindyck, "Uncertainty in environmental economics," *Review of Environmental Economics and Policy*, vol. 1, no. 1, pp. 45–65, 2007.

[25] L. Uusitalo, A. Lehikoinen, I. Helle, and K. Myrberg, "An overview of methods to evaluate uncertainty of deterministic models in decision support," *Environmental Modelling and Software*, vol. 63, pp. 24–31, 2015.

[26] B. P. Bryant and R. J. Lempert, "Thinking inside the box: A participatory, computer-assisted approach to scenario discovery," *Technological Forecasting and Social Change*, vol. 77, no. 1, pp. 34–49, 2010.

[27] C. W. Churchman, "Free for All," *Management Science*, vol. 14, no. 4, pp. B141–B146, 1967.

[28] E. a. Stanton, F. Ackerman, and S. Kartha, "Inside the integrated assessment models: Four issues in climate economics," *Climate and Development*, vol. 1, no. 2, p. 166, 2009.

[29] IAWG, "Technical Support Document: Social Cost of Carbon for Regulatory Impact Analysis," tech. rep., Interagency Working Group, Washington, DC, 2010.

[30] GAMS Development Organization, "General Algebraic Modeling System (GAMS)," 2016.

[31] ARKI Consulting and Development A/S, "CONOPT3," 2016.

[32] R. S. Pindyck, "Climate Change Policy: What Do the Models Tell Us?," *Journal of Economic Literature*, vol. 51, no. 3, pp. 1–23, 2013.

[33] R. S. Pindyck, "The Use and Misuse of Models for Climate Policy," *NBER Working Paper*, 2015.

[34] R. S. J. Tol, "The Economic Effects of Climate Change," *Journal of Economic Perspectives*, vol. 23, no. 2, pp. 29–51, 2009.

[35] C. Naastepad, "Lectures on Technology and Economic Performance," tech. rep., TU Delft, 2002.

[36] P. M. Romer, "Endogenous technological change," *Journal of Political Economy*, vol. 98, no. 5, pp. S71–S102, 1990.

[37] K. Gillingham, R. G. Newell, and W. A. Pizer, “Modeling endogenous technological change for climate policy analysis,” *Energy Economics*, vol. 30, no. 6, pp. 2734–2753, 2008.

[38] D. Popp, R. G. Newell, and A. B. Jaffe, “Energy, the environment, and technological change,” in *Handbook of the Economics of Innovation*, vol. 2, ch. 21, pp. 873–937, Elsevier B.V., 1 ed., 2010.

[39] J. Hicks, *The Theory of Wages.* London: Macmillan, 2nd ed., 1963.

[40] D. Popp, “Induced Innovation and Energy Prices,” *American Economic Review*, vol. 92, no. 1, pp. 160–180, 2002.

[41] A. Diaz Arias and C. van Beers, “Energy subsidies, structure of electricity prices and technological change of energy use,” *Energy Economics*, vol. 40, pp. 495–502, 2013.

[42] D. Popp, “Lessons from patents: Using patents to measure technological change in environmental models,” *Ecological Economics*, vol. 54, no. 2-3, pp. 209–226, 2005.

[43] D. Acemoglu, “Directed Technical Change,” *The Review of Economic Studies*, vol. 69, no. 4, pp. 781–809, 2002.

[44] D. Acemoglu, P. Aghion, and D. Hemous, “The environment and directed technical change,” *The American Economic Review*, vol. 102, no. 1, pp. 131–166, 2012.

[45] K. Arrow, “The Economic Implications of Learning by Doing,” *The Review of Economic Studies*, vol. 29, no. 3, pp. 155–173, 1962.

[46] F. Ferioli, K. Schoots, and B. C. C. van der Zwaan, “Use and limitations of learning curves for energy technology policy: A component-learning hypothesis,” *Energy Policy*, vol. 37, no. 7, pp. 2525–2535, 2009.

[47] E. Tüzün and B. Tekinerdogan, “Analyzing Impact of Experience Curve on ROI in Software Product Line Engineering Adoption Process,” *Information and Software Technology*, vol. 59, pp. 136–148, 2015.

[48] S. Alberth and C. Hope, “Climate modelling with endogenous technical change: Stochastic learning and optimal greenhouse gas abatement in the PAGE2002 model,” *Energy Policy*, vol. 35, no. 3, pp. 1795–1807, 2007.

[49] A. Manne and R. Richels, "The impact of learning-by-doing on the timing and costs of CO2 abatement," *Energy Economics*, vol. 26, no. 4, pp. 603–619, 2004.

[50] W. A. Pizer and D. Popp, "Endogenizing technological change: Matching empirical evidence to modeling needs," *Energy Economics*, vol. 30, no. 6, pp. 2754–2770, 2008.

[51] P. Söderholm and T. Sundqvist, "Empirical challenges in the use of learning curves for assessing the economic prospects of renewable energy technologies," *Renewable Energy*, vol. 32, no. 15, pp. 2559–2578, 2007.

[52] W. Nordhaus, "Modeling Induced Innovation in Climate Change Policy," in *Modeling Induced Innovation in Climate Change Policy*, ch. 9, pp. 259–290, Resources for the Future, 2002.

[53] C. I. Jones and J. C. Williams, "Measuring the Social Return to R&D," *Quarterly Journal of Economics*, vol. 113, no. 4, pp. 119–135, 1999.

[54] K. Furey, "A Reading on Money and Money Creation," *Forum for Social Economics*, vol. 42, no. 1, pp. 38–58, 2013.

[55] D. Popp, "ENTICE: endogenous technological change in the DICE model of global warming," *Journal of Environmental Economics and Management*, vol. 48, pp. 742–768, 2004.

[56] M. Mazzucato, *The Entrepreneurial State*. London: Demos, 2011.

[57] P. Buonanno, C. Carraro, and M. Galeotti, "Endogenous induced technical change and the costs of Kyoto," *Resource and Energy Economics*, vol. 25, no. 1, pp. 11–34, 2003.

[58] S. Smulders and M. D. Nooij, "The impact of energy conservation on technology and economic growth," *Resources and Energy Economics*, vol. 25, pp. 59–79, 2003.

[59] D. Popp, "Exploring Links Between Innovation and Diffusion: Adoption of NOX Control Technologies at US Coal-fired Power Plants," *NBER Working Paper Series*, 2006.

[60] R. Newell and W. Pizer, "Discounting the benefits of climate change mitigation: How much do uncertain rates increase valuations?," tech. rep., Resources for the Future, Washington, DC, 2001.

[61] D. Popp, “The effect of new technology on energy consumption,” *Resource and Energy Economics*, vol. 23, no. 3, pp. 215–239, 2001.

[62] E. Baker and E. Shittu, “Uncertainty and endogenous technical change in climate policy models,” *Energy Economics*, vol. 30, no. 6, pp. 2817–2828, 2008.

[63] V. Bosetti and M. Tavoni, “Uncertain R&D, backstop technology and GHGs stabilization,” *Energy Economics*, vol. 31, pp. S18–S26, 2009.

[64] T. Goeschl and G. Perino, “On backstops and boomerangs: Environmental R&D under technological uncertainty,” *Energy Economics*, vol. 31, no. 5, pp. 800–809, 2009.

[65] D. Popp, “ENTICE-BR: The effects of backstop technology R&D on climate policy models,” *Energy Economics*, vol. 28, no. 2, pp. 188–222, 2006.

[66] W. D. Nordhaus and J. Boyer, *Warming the World.* London: The MIT Press, 2000.

[67] V. Bosetti and L. Drouet, “Accounting for Uncertainty Affecting Technical Change in an Economic-Climate Model,” *SSRN Electronic Journal*, pp. 1–26, 2005.

[68] B. van der Zwaan and A. Seebregts, “Endogenous learning in climate-energy-economic models: an inventory of key uncertainties,” in *Risk and Uncertainty in Environmental and Resource Economics*, (Wageningen, the Netherlands), pp. 1–11, ECN, 2002.

[69] M. E. Porter and S. Stern, “Measuring the "ideas" production function: evidence from international patent output,” *NBER working paper series*, 2000.

[70] M. Crawford, J. Church, and B. Akin, “CPI Detailed Report Data for February 2016,” Tech. Rep. April, Bureau of Labor Statistics, Washington, DC, 2016.

[71] OECD, “Gross domestic spending on R&D (indicator),” tech. rep., 2016.

[72] D. Anderson, “Renewable Energy Technology and Policy for Development,” *Annual Review of Energy and the Environment*, vol. 22, no. 1, pp. 187–215, 1997.

[73] W. Turkenburg, D. J. Arent, R. Bertani, A. Faaij, M. Hand, W. Krewitt, E. D. Larson, J. Lund, M. Mehos, T. Merrigan, C. Mitchell, J. R. Moreira, W. Sinke, V. Sonntag-O’Brien, B. Thresher, W. van Sark, and E. Usher, “Renewable Energy,” in *Global Energy Assessment - Toward a Sustainable Future* (J. Schmid, ed.), ch. 11, pp. 811–821, International Institute for Applied Systems Analysis, 2012.

[74] J. Goldemberg and S. Teixeira Coelho, “Renewable energy - Traditional biomass vs. modern biomass,” *Energy Policy*, vol. 32, no. 6, pp. 711–714, 2004.

[75] T. Poynards, M. Munteanu, V. Ratziu, Y. Benhamou, V. Di Martino, J. Taieb, and P. Opolon, “Truth Survival in Clinical Research: An Evidence-Based Requiem?,” *Annals of Internal Medicine*, vol. 136, pp. 888–895, 2002.

[76] F. Machlup, *Knowledge: Its Creation, Distribution and Economic Significance.* Princeton, New Jersey: Princeton University Press, 1984.

[77] A. B. Jaffe and M. Trajtenberg, *Patents, Citations & Innovations: a window on the knowledge economy.* Cambridge: The MIT Press, 2002.

[78] B. Fisher, N. Nakicenovic, K. Alfsen, J. Corfee Morlot, F. De La Chesnaye, J.-C. Hourcade, K. Jiang, M. Kainuma, E. La Rovere, A. Matysek, A. Rana, K. Riahi, R. Richels, S. Rose, D. P. van Vuuren, and R. Warren, “Issues related to mitigation in the long-term context,” in *Climate Change 2007: Working Group III: Mitigation of Climate Change* (J. Weyant and M. Babiker, eds.), ch. 3, pp. 219–222, Cambridge: Cambridge University Press, 207.

[79] R. Gerlagh and L. Wietze, “Induced Technological Change Under Carbon Taxes,” *CLIM – Climate Change Modelling and Policy*, pp. 1–26, 2003.

[80] J. Goldemberg and L. Tadeo Prado, “The "decarbonization" of the world’s energy matrix,” *Energy Policy*, vol. 38, no. 7, pp. 3274–3276, 2010.

[81] P. Dasgupta, “Discounting climate change,” *Journal of Risk and Uncertainty*, vol. 37, no. 2-3, pp. 141–169, 2008.

[82] L. Karp and C. Traeger, “Discounting,” in *Encyclopedia of Energy, Natural Resource, and Environmental Economics*, pp. 286–292, Elsevier Inc., 2013.

[83] C. R. Sunstein and D. A. Weisbach, “Program on Risk Regulation Climate Change and Discounting the Future: A Guide for the Perplexed,” *Reg-Markets Center Working Paper*, no. 08, 2008.

[84] J. Gowdy, J. B. Rosser, and L. Roy, “The evolution of hyperbolic discounting: Implications for truly social valuation of the future,” *Journal of Economic Behavior and Organization*, vol. 90, pp. 94–104, 2013.

[85] J. a. Hausman, "Individual Discount Rates and the Purchase and Utilization of Energy-Using Durables," *The Bell Journal of Economics*, vol. 10, no. 1, pp. 33–54, 1979.

[86] M. L. Weitzman, "Why the far-distant future should be discounted at its lowest possible rate," *Journal of Environmental Economics and Management*, vol. 36, no. 3, pp. 201–208, 1998.

[87] W. R. Cline, *The Economics of Global Warming.* New York: New York University Press, 1st ed., 1992.

[88] J. Rawls, *A Theory of Justice.* Cambridge: Harvard University Press, 1971.

[89] I. M. Goklany, "From precautionary principle to risk-risk analysis.," *Nature biotechnology*, vol. 20, no. 11, p. 1075, 2002.

[90] P. R. Portney and J. P. Weyant, "Introduction," in *Discounting and Intergenerational Equity* (P. R. Portney and J. P. Weyant, eds.), ch. 1, pp. 1–13, Washington, DC: Resources for the Future, 1999.

[91] C. M. Harvey, "The reasonableness of non-constant discounting," *Journal of Public Economics*, vol. 53, no. 1, pp. 31–51, 1994.

[92] B. Thierry, C. Camacho, and V. M. Veliov, "Adaptive Model-Predictive Climate Policies in a Multicountry Setting," in *The Oxford Handbook of the Macroeconomics of Global Warming* (L. Bernard and W. Semmler, eds.), ch. 6, pp. 114–141, New York: Oxford University Press, 2015.

[93] P. Sirianni, "Endogenous Environmental Discounting and Climate-Economy Modeling," Tech. Rep. August, Colgate University, 2011.

[94] A. Ayong Le Kama and K. Schubert, "A note on the Consequences of an Endogenous Discounting Depending on the Environmental Quality," *Macroeconomic Dynamics*, vol. 11, no. 02, pp. 272–289, 2007.

[95] L. Karp, "Global warming and hyperbolic discounting," *Journal of Public Economics*, vol. 89, no. 2-3, pp. 261–282, 2005.

[96] G. Ainslie, "Precis of Breakdown of Will," *Behavioral and Brain Sciences*, vol. 28, no. 5, pp. 635–673, 2005.

[97] M. L. Weitzman, ""Just Keep Discounting, But ..."," in *Discounting and Intergenerational Equity* (P. R. Portney and W. J. P., eds.), ch. 3, pp. 23–29, Washington, DC: Resources for the Future, 1999.

[98] R. J. Barro, "Ramsey Meets Laibson in the Neoclassical Growth," *The Quarterly Journal of Economics*, vol. 114, no. 4, pp. 1125–1152, 1999.

[99] R. J. Barro, "Myopia and Inconsistency in the Neoclassical Growth Model," *NBER Working Paper Series*, 1997.

[100] M. Das, "Optimal growth with decreasing marginal impatience," *Journal of Economic Dynamics and Control*, vol. 27, no. 10, pp. 1881–1898, 2003.

[101] G. Becker and C. Mulligan, "The endogenous determination of time preference," *Quarterly Journal of Economics*, vol. 112, pp. 729–758, 1997.

[102] A. A. Samwick, "Discount rate heterogeneity and social security reform," *Journal of Development Economics*, vol. 57, no. 1, pp. 117–146, 1998.

[103] E. C. Lawrance, "Poverty and the Rate of Time Preference: Evidence from Panel Data," *Journal of Political Economy*, vol. 99, no. 1, pp. 54–77, 1991.

[104] G. W. Harrison, M. I. Lau, and B. Melonie, "Estimating Individual Discount Rates ind Denmark : A Field Experiment," *American Economic Review*, vol. 92, no. 5, pp. 1606–1617, 2002.

[105] B. Groom, C. Hepburn, P. Koundouri, and D. Pearce, "Declining discount rates: The long and the short of it," *Environmental and Resource Economics*, vol. 32, no. 4, pp. 445–493, 2005.

[106] J. Ellenberg, "Miss More Planes!," in *How Not to Be Wrong: The Power of Mathematical Thinking*, ch. 12, pp. 232–233, New York: Penguin Books, 2014.

[107] F. H. Knight, "Enterprise and Profit," in *Risk, Uncertainty and Profit* (H. M. Co., ed.), ch. IX, 1st ed., 1921.

[108] W. Walker, P. Harremoës, J. Rotmans, J. van der Sluijs, M. van Asselt, P. Janssen, and M. Krayer von Krauss, "A Conceptual Basis for Uncertainty Management," *Integrated Assessment*, vol. 4, no. 1, pp. 5–17, 2003.

[109] J. H. Kwakkel, W. Walker, and V. A. Marchau, "Classifying and communicating uncertainties in model-based policy analysis," *Int. J. Technology, Policy and Management*, vol. 10, no. 4, pp. 299–315, 2010.

[110] B. Enserink, J. H. Kwakkel, and S. Veenman, “Coping with uncertainty in climate policy making: (Mis)understanding scenario studies,” *Futures*, 2013.

[111] R. A. Halim, J. H. Kwakkel, and L. A. Tavasszy, “A scenario discovery study of the impact of uncertainties in the global container transport system on European ports,” *Futures*, 2015.

[112] D. G. Groves and R. J. Lempert, “A new analytic method for finding policy-relevant scenarios,” *Global Environmental Change*, vol. 17, no. 1, pp. 73–85, 2007.

[113] S. W. Popper, R. J. Lempert, and S. C. Bankes, “Shaping the future,” *Scientific American*, no. April, pp. 66–71, 2005.

[114] J. H. Kwakkel and E. Pruyt, “Exploratory Modeling and Analysis, an approach for model-based foresight under deep uncertainty,” *Technological Forecasting and Social Change*, vol. 80, no. 3, pp. 419–431, 2013.

[115] S. Bankes, “Exploratory Modeling for Policy Analysis,” *Operations Research*, vol. 41, no. 3, pp. 435–449, 1993.

[116] J. H. Kwakkel, “EMA Workbench,” *http://simulation.tbm.tudelft.nl/ema-workbench/contents.html*, 2012.

[117] P. Tsigaris and J. Wood, “A Simple Climate-Solow Model for Introducing the Economics of Climate Change to Undergraduate Students Panagiotis,” *International Review of Economics Education*, vol. 23, pp. 65–81, 2016.

[118] S. Dietz and N. Stern, “Endogenous Growth, Convexity of Damage and Climate Risk: How Nordhaus’ Framework Supports Deep Cuts in Carbon Emissions,” *Econ. J.*, vol. 125, no. 583, pp. 574–620, 2015.

[119] M. L. Weitzman, “On Modeling and Interpreting the Economics of Catastrophic Climate Change,” *Review of Economics and Statistics*, vol. 91, no. 1, pp. 1–19, 2009.

[120] F. Ackerman, E. A. Stanton, and R. Bueno, “Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE,” *Ecological Economics*, vol. 69, no. 8, pp. 1657–1665, 2010.

[121] M. L. Weitzman, “GHG Targets as Insurance Against Catastrophic Climate Damages,” *Journal of Public Economic Theory*, vol. 14, no. 2, pp. 221–244, 2012.

[122] M. L. Weitzman, "Fat tails and the social cost of carbon," *American Economic Review*, vol. 104, no. 5, pp. 544–546, 2014.

[123] J. H. Friedman and N. I. Fisher, "Bump Hunting in High-Dimensional Data," *Statics and Computing*, vol. 9, pp. 123–143, 1999.

[124] C. Jägemann, M. Fürsch, S. Hagspiel, and S. Nagl, "Decarbonizing Europe's power sector by 2050 - Analyzing the implications of alternative decarbonization pathways," *Energy Economics*, vol. 40, no. 12, pp. 622–636, 2012.

[125] E. P. Lazear, "Entrepreneurship," *NBER Working Paper Series*, no. 9109, p. ALL, 2002.

[126] A. S. Kritikos, "Entrepreneurship and economic growth," in *International Encyclopedia of the Social & Behavioral Sciences* (J. D. Wright, ed.), pp. 675–680, Elsevier Ltd, second ed., 2015.

[127] R. G. Holcombe, "Entrepreneurship and economic growth," *The Quarterly Journal of Austrian Economics*, vol. 1, no. 2, pp. 45–62, 1998.

[128] M. Bianchi and M. Henrekson, "Is Neoclassical Economics still Entrepreneurless?," *KYKLOS*, vol. 58, no. 3, pp. 353–377, 2005.

[129] A. Colino, D. Benito-Osorio, and C. Rueda-Armengot, "Entrepreneurship culture, total factor productivity growth and technical progress: Patterns of convergence towards the technological frontier," *Technological Forecasting and Social Change*, vol. 88, pp. 349–359, 2014.

[130] M. A. Carree and A. R. Thurik, "The lag structure of the impact of business ownership on economic performance in OECD countries," *Small Business Economics*, vol. 30, no. 1, pp. 101–110, 2008.

[131] I. M. Kirzner, *Discover and the Capitalist Process.* Chicago: University of Chicago Press, 1973.

[132] F. A. Hayek, "The Use of Knowledge in Society," *American Economic Review*, vol. 35, no. 4, pp. 519–530, 1945.

[133] H. Barreto, *The Entrepreneur in Microeconomic Theory.* London: Routledge, 1989.

[134] R. E. Lucas Jr., "On the Size Distribution of Business Firms," *The Bell Journal of Economics*, vol. 9, no. 2, pp. 508–523, 1978.

[135] R. E. Kihlstrom and J.-J. Laffont, "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion," *Journal of Political Economy*, vol. 87, no. 3, pp. 719–748, 1979.

Appendices

Appendix A

Entrepreneurship and Technological Change

Supervisor: Dr. L.M. (Linda) Kamp

The purpose of this chapter is to explore the policy implications of entrepreneurship on technological change in climate policy models. This represents a gap in the literature as no climate model has included the effect of entrepreneurship on the environment. This is part of the requirements for the **Entrepreneurship Annotation** in TU Delft. First, a literature review is presented on the matter. This will cover the definition of entrepreneurship until its various ways of modeling. Afterwards, the DICE-ED model used throughout this thesis will be modified accordingly to include entrepreneurship in the equations. Next, an analysis will be performed. Finally, conclusions and recommendations will close the chapter.

A.1 Literature Review

The entrepreneur is the single most important player in a modern economy.

Edward Lazear, award-winning American labor economist [125].

A.1.1 Definition of entrepreneurship

The definitions for *entrepreneurship* or *entrepreneur* abound in sources and meanings. There is no single accepted definition for the term. It is believed to have been coined by the French economist Jean-Baptiste Say after studying Adam Smith's capstone work, *The Wealth of Nations*. The term originally comes from the Irish-French economist Richard Cantillon and the French word *enterprendre* which literally means *undertake*. An entrepreneur, in Cantillon's terms, is an *adventurer* or a *risk-taker*. For Say, the entrepreneur is more of a *planner* who can organize and manage a business to gain a profit at the end. More modern takes on the term are *businessman* or *innovator*.

From Frank Knight and the "Knightian uncertainty" as seen in Section 3.1.4, the definition of entrepreneurship goes more into the decision-making process of the individual [107]. Entrepreneurs follow investment decisions which are both risky and uncertain and they have to live up to these decisions. Risk is easily calculable with probability distributions while uncertainty is incalculable. Thus, an entrepreneur follows uncertain and unpredictable paths.

Additionally, Joseph Schumpeter, seen again in Section 3.1.4, has an alternative definition. He sees entrepreneurs as those who can do new combinations to initiate the process of creative destruction where the old is replaced with the new [126]. They are vital in capitalist economies.

There are more definitions and ideas about the term of entrepreneurship. For the rest of this chapter, an entrepreneur involves someone whose objective is a process of creative destruction, risk-taking, innovation and a search for profits.

There is much interest about the effect of entrepreneurship on technological change and thus on the economy. The following sub-section will review this interest.

A.1.2 Entrepreneurship and its effect on the economy

What is the cause for economic growth? In general, there are two sides [127]. On one side, Adam Smith regarded economic growth as a product of entrepreneurship and innovation which would lead to ever-increasing wealth and a greater division of labor as markets grew. On the other side, David Ricardo saw economic growth as a function of several inputs (land, capital, labor) and due to diminishing marginal factor productivity and fixed inputs such as land, economic growth was bound to be stagnated with time.

Both of these approaches are common economic knowledge. Nonetheless, the Ricardian view takes a bigger role in economic modeling due to its ease which with it can be parametrized where investment is the key to economic growth [127]. Another reason is that the Ricardian model was used constantly before 1989 within the context of centrally planned economies in Europe as it states that a central planner has the best position to increase investment and production [127]. The Smithian view, on the other hand, is harder to model due to the inherent difficult of modeling innovation and entrepreneurship [128].

The Ricardian view is fundamental in the production function, reminiscent of the one function treated in Section 3.1 and reproduced down below for convenience. Here, investment drives the stock of K (capital) which increases output. L (labor) is usually taken to be exogenous. The only other factor is a (total factor productivity) which has come now under close examination. There is currently literature studies on the effect of R&D on a so as to increase productivity over time [127]. The Smithian view, on the other hand, has grown more popular in the last decades due to the research work of Paul Romer on endogenous technological change [36]. Romer states that increasing investment in R&D could generate economic growth through the increase of human capital (which is the set of skills and knowledge appropriable by humans).

$$Y = aL^{\alpha}K^{\beta}$$

Say noted that entrepreneurs are those who are able to look for inefficient uses of resources and capital and move them into a more productive area; the objective of this being to generate higher yields. In other words, entrepreneurs are profit-seekers who can get the most out of every resource.

Just like an entrepreneur squeezes more output of the same resources, the total factor productivity (TFP), a in the equation above, has the same function. There is then a link between entrepreneurship and total factor productivity (mainly determined by technical change or technological change). TFP appears as the main component which describes different countries' economic performance over time [129].

Entrepreneurship has several positive effects on the economy. First and foremost, entrepreneurs are the creators of new jobs, and not only for themselves. Secondly, as stated above, they can increase the productivity of the economy by making resources more efficient on the basis of positive technological change. The introduction of new products and processes, creative destruction made able by entrepreneurs, is the third way in which they are able to affect the economy. They are one of the most important drivers of the economic system [126].

Job creation is of crucial importance to both an economy and its politicians. A bigger supply of labor will ultimately increase the overall production of the economy.[1] There is evidence of this fact with a study done on OECD countries where an increase in entrepreneurship leads a positive effect on both employment growth and GDP growth [130].

The economy is rewarded when there is an absolute bigger number of entrepreneurs as there is now more competition between the new firms and the incumbents. This pushes firms to improve their productivity, lower their costs and ultimately reward the consumers with more product variety plus the secondary effect of avoiding monopolies and/or oligopolies. New firms as well as the incumbents need to adapt to the new standard brought about by their coexistence. There is also evidence of an empirical relationship between entrepreneurial activity and productivity [126].

Lastly, the process of creative destruction brings about new innovations and can open up new markets. Examples of this are all the firms like Google, Apple, Google, Amazon which were founded upon entrepreneurial endeavors. As it currently stands, Elon Musk[2], is on the path to colonizing Mars and electrifying the mobility sector just with his decisions and dreams. Just these two objectives of him are enough to send ripples throughout the whole economy. Incumbent firms are generally more reluctant to

[1]This is akin to thinking about an increase in L in the Cobb-Douglas production function.

[2]Founder of Tesla Motors, Solar City, SpaceX and PayPal. Probably the world's most famous and exciting entrepreneur right now.

look for new areas and opportunities, either because of failure to adapt to a changing environment or organizational inertia [126].

However, with all this positive effects on the economy, research on entrepreneurship remained fairly vague during almost all of the twentieth century [126, 128]. It was until the information and communication technology revolution of the 1990s when the recognition of entrepreneurship started appearing in mainstream economic research. The link between the role of entrepreneurship and economic growth is still in its embryonic stages. It is part of this link that this chapter hopes to research.

A.1.2.1 Entrepreneurship and knowledge

In the neoclassical framework of this thesis, technological change comes through R&D. However, doing R&D is not sufficient for a positive effect on the economy. The opportunities opened by this new research must be exploited. This is where entrepreneurship comes in.

As profit-seekers[3], entrepreneurs act upon unseen opportunities. This falls in line with the model of American economist Israel Kirzner [131]. The ability to recognize these opportunities is part-knowledge part-acumen. Being aware of the current state of knowledge is something entrepreneurs need in order to identify opportunities. But there is a distinction. There is a need for distributed knowledge, as Friedrich Hayek puts it [132]. Distributed local knowledge refers to that which pertains to an individual at a certain point in time and space. Entrepreneurs with their own individual knowledge and their acumen are able to seek these profit opportunities. It is of no surprise that the microprocessor was invented by an electrical engineer for example.

As seen in Chapter 3, there are increasing returns to R&D and positive knowledge spillovers. This plays a major role in Kirzner's model of entrepreneurship. Kirzner states that entrepreneurship is running alongside R&D but they are not as interconnected as they seem [127]. R&D and knowledge does indeed augment some factors of production, however, they are not responsible for the introduction of new goods and services which further propel economic growth. Entrepreneurship is what enables what R&D and knowledge cannot. Knowledge externalities can be explained through entrepreneurship because the entrepreneurial opportunities of some produce opportunities for others. There

[3]There is also the *social entrepreneur* whose objective is not profit-seeking. However, these entrepreneurs will not be treated in this chapter for simplicity.

are also increasing returns to entrepreneurship because with more entrepreneurship comes more opportunities.

A.1.3 Modeling Entrepreneurship

There are some inherent difficulties with analyzing economic growth with the neoclassical framework. One is that the models are not fitted to account for new products derived from innovation, which might disrupt the market and thus open up new opportunities. This creation of market niches is a key link between entrepreneurship and economic growth [127]. In these models, growth is tended towards producing more of the old goods. Innovation is modeled as R&D which affects the production factors. No room is given for entrepreneurial discovery. R&D is not the cause for growth, it is merely the response to new growth opportunities [127].[4]

One of the cardinal reasons why entrepreneurship is not modeled is due to the fact that it is a phenomenon that is analytically not tractable. In fact, it has been stated that both the entrepreneur and the microeconomic theory of the firm cannot coexist and as such economists have chosen the firm over the entrepreneur [133]. It is difficult to capture in equations what the entrepreneur means for economic development. However, in recent years, several attempts have been made to include entrepreneurship in economic models [128].

The first method to model entrepreneurship is by assuming that entrepreneurs are *talented individuals* who organize the factors of production [134]. This relates to the role of "managers" given to entrepreneurs by Say. Different entrepreneurs will have different levels of talent to manage the factors of production. The following formulation shows the relationship between entrepreneurs and the production:

$$Y = x \cdot g[f(L, K)] \tag{A.1}$$

where $f(L, K)$ is a formula for production (Y) and x is an indication of the talent of the entrepreneur. This way to model entrepreneurship is analytically convenient as it enters the production function multiplicatively. $g[.]$ is a function to mitigate the effect of x on production by introducing diseconomies of scale in managing. One of

[4] This means that R&D is usually present where there are profit opportunities to be made. As an example, there is considerably more R&D directed towards the energy industry than the pasta-making industry.

the many limitations of this method is how to define, quantify and explain the causes for x.

A second method to model entrepreneurship is by viewing them as *risk takers* responsible for taking the risk in the firms [135]. Individuals are made either employees or entrepreneurs depending on their risk aversion. For example, there is a risk aversion cut-off level where anyone below this level is an entrepreneur and the rest will be employees. Each firm can then have one unit of entrepreneurship which is in charge of hiring employees and generate production. One of the shortcomings of this method is to differentiate between risk and uncertainty [107].

The third method to model entrepreneurship is as *innovative agents* who make "creative destruction" by implementing innovations which augment the economy or the firm [128]. This way entrepreneurs take advantage of profit opportunities created by the introduction of an innovation. This is defined by an ability factor (θ) which explains whether entrepreneurs would risk opening a new business, manage the existing one or trade for some middle ground. Alternatively, innovation comes from firms doing profit-seeking R&D. However, the limitation to this is that R&D is stripped from the entrepreneurial hands as it is firms who seek R&D with managers who do not have any real risk [128].

A.2 Model and Results

This section describes the modifications made to the DICE-ED model in order to include entrepreneurship in the model and be able to see its effect on economic growth and climate policy.

The DICE-ED model induces innovation through the inclusion of an R&D market which reacts dynamically to the prices of fossil fuels and backstop fuels[5]. The R&D market is subsequently divided into two: one for energy efficiency and another for backstop research.[6] For this section, a special focus will be made on the backstop research. The basic premise is that higher entrepreneurial activity will be able to introduce more products or innovations into the economy, thus lowering the price of the backstop technology. The first and third method to model entrepreneurship will be combined in this model.

[5]Backstop fuels or backstop technologies is a name given to all energy sources which emit no carbon emissions with usage. It is mostly referred to normal renewable energies but carbon capture and sequestration is also included as well as new technologies which could appear in the future.

[6]See Section 3.2.1 for a complete overview.

For one part, entrepreneurship will be given the role of talented individuals to account for the factors of production and also innovative agents by having an ability factor to quantify the level of entrepreneurship.

The modification to the model will be to the innovation possibility frontier of the backstop technology, Equation (3.12). A factor that describes entrepreneurial activity will be added to the model in order to describe future states of the world. The resulting equation modification is:

$$H_i(t) = \varepsilon f(R_i(t)) + (1 - \delta_H)H_i(t-1) \tag{A.2}$$

$$f(R_i(t)) = aR_i(t)^{b_i}H_i(t)^{\phi_i} \tag{A.3}$$

where ε is the total amount of entrepreneurial activity and the rest of the equation follows the same description as in the original model. The exact value of ε depends on the state of the world in regards to entrepreneurship. The future state of entrepreneurship is fairly uncertain. The value will be changed accordingly in the model and will take values such as:

$$\varepsilon = \begin{cases} 2 & \text{high entrepreneurship} \\ 1 & \text{base case (no entrepreneurship)} \\ 0.5 & \text{low entrepreneurship} \end{cases}$$

The exact point of these values is not to define any numerical precision on the level of entrepreneurial activity. The point is to see how the effect of a world supplied with more entrepreneurial activities changes in comparison to a base case. This will allow for the recommendation of generalized policy advice. The results of a model would indicate the effect of reducing or increasing entrepreneurship on both output and climate. A future reduction or increase of entrepreneurship can be done through policy making such as incentives, subsidies or tax breaks for new businesses.

With the new equation, the model is run on an optimal scenario to account for damages and the externality of carbon emissions.

When dealing with climate change, one of the most important variables to consider is the atmospheric temperature. Figure A.1 shows the effect of the entrepreneurial level on

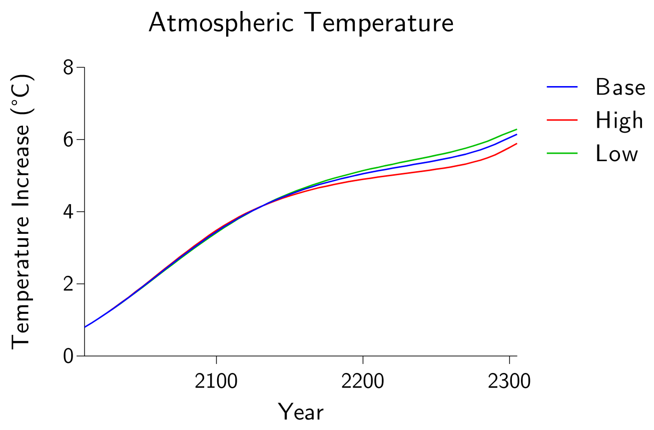

Figure A.1: Atmospheric temperature with entrepreneurship level.

the atmospheric temperature. Several key insights are gathered from this figure. First of all, the short-run temperature is not changed in any significant way with a different level of entrepreneurship. Almost until the year 2150 does the atmospheric temperature change levels. The key insight is that entrepreneurship is a long-term activity with benefits far into the future. This is represented in the model by an increase in the appropriable knowledge generated with the same amount of R&D. Another key insight is that a world with more entrepreneurial level is in relative terms "safer" than one without. This is seen at the end of the simulation where the atmospheric temperature is lower in the High case. Additionally, the highest gains are made from the first improvements to entrepreneurship. Figure A.2 shows the result of increasing entrepreneurship level on the atmospheric temperature by the end of the model's timespan. The curve flattens as more entrepreneurship is added to the model, this indicates diminishing marginal returns of entrepreneurship on the atmospheric temperature. From this it is clear that the biggest marginal gains will be achieved through the initial effort. For policy advice, inducing more entrepreneurship by subsidies, tax breaks or grants will have a positive effect on the environment. This effect, however, will only be seen in the long-term.

With the entrepreneurship specifically modeled for the backstop technology, the highest improvement is seen with the price of the backstop technology. Figure A.3 shows the marked improvement of the price on the backstop technology with increasing or decreasing entrepreneurship level. It becomes clear that an increase in the entrepreneurship level in the backstop field will result in a steeper price decrease as knowledge is generated at higher rates. With a decrease in the backstop price, fossil fuels are

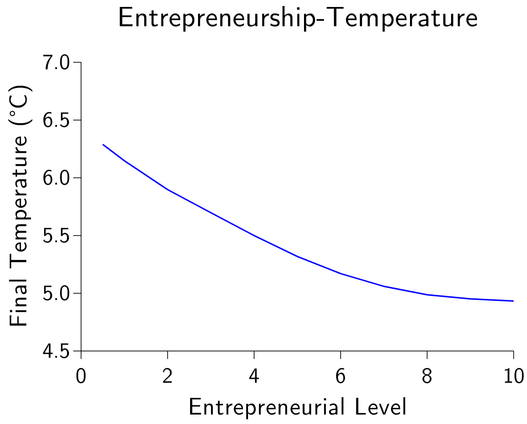

Figure A.2: Relationship between final atmospheric temperature and different entrepreneurship levels.

used less and thus their price is also affected as seen in Figure A.4. Less usage of fossil fuels means that the price is considerably less with the supply still intact. The key relationship is that a decrease in the price of the backstop technology will limit total atmospheric temperature rise as backstop technologies do not emit carbon emissions.

As for economic growth, the rise expected to see with increasing entrepreneurship level is not as marked as one would expect. Figure A.5 shows the results of this analysis. The increase in output is minimal and it is only seen in the long-term. With a minimal discount factor, the present value of this output is practically the same.[7] This may be due to the fact of modeling practices. This model is a climate policy model with an emphasis of entrepreneurship on the energy sector. It should then be of no surprise why the economic values do not change considerably with this. The social cost of carbon (i.e. the carbon tax in an ideal economy) is a variable which accounts for both economic and climate values. Equation (A.4) shows the definition of the social cost of carbon where the numerator is the marginal benefit on welfare from a rise in emissions (climate variable) while the denominator is the same marginal benefit but from a rise in consumption (economic variable). Figure A.6 shows the social cost of carbon with the three entrepreneurship levels. As with economic output, the change in social cost of carbon with differing entrepreneurship levels is negligible for the short-term. This has important effects because policy-makers usually only use the first 100 years of climate model simulations as these results are the appropriate ones for policy-making. In fact, if

[7] Refer to Section 4.3.

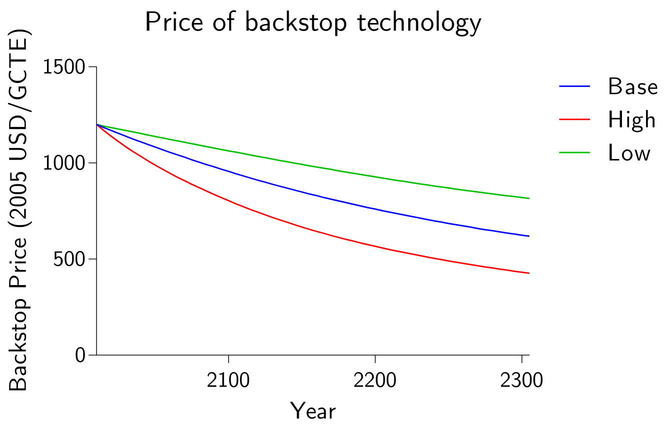

Figure A.3: Price of backstop technology with entrepreneurship level.

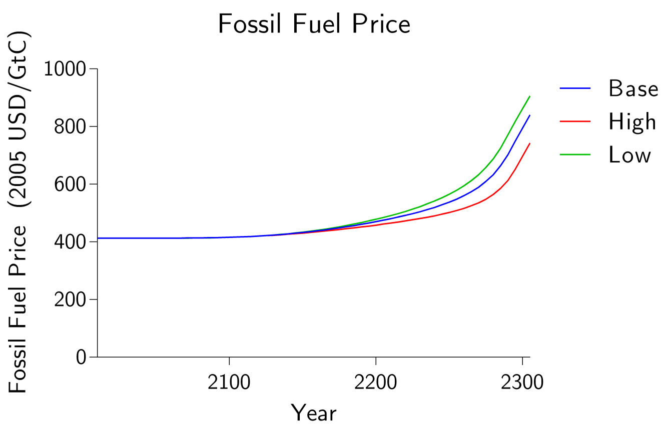

Figure A.4: Price of fossil fuels with entrepreneurship level.

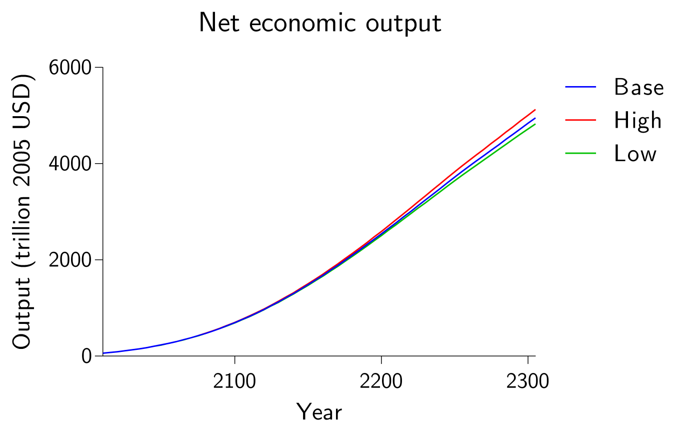

Figure A.5: Net economic output with entrepreneurship level.

only the results until the year 2100 were shown, a single line would represent the three cases as there is no significant difference.

$$SCC(t) \equiv \frac{\frac{-\,\partial W}{\partial E(t)}}{\frac{\partial W}{\partial C(t)}} \tag{A.4}$$

A.3 Conclusions

This chapter has dealt with entrepreneurship in regards to technological change in a climate policy model. A literature review was first presented in order to assimilate the lack of research done on the topic. The analysis presented an opportunity to study entrepreneurship in a climate policy model developed throughout the thesis. Technological change is modeled through an R&D market and entrepreneurship was linked to this R&D market.

Entrepreneurship is a vital component of economics. In the model, due to its precise modeling effort, entrepreneurship impacted the energy sector and ultimately the atmospheric temperature the most. **Entrepreneurship exhibited diminishing marginal returns on atmospheric temperature.** This means that early improvements to entrepreneurship are vital for long-term climate policy-making. The biggest improvements are to be gained from the first efforts. As entrepreneurship level increases, the atmo-

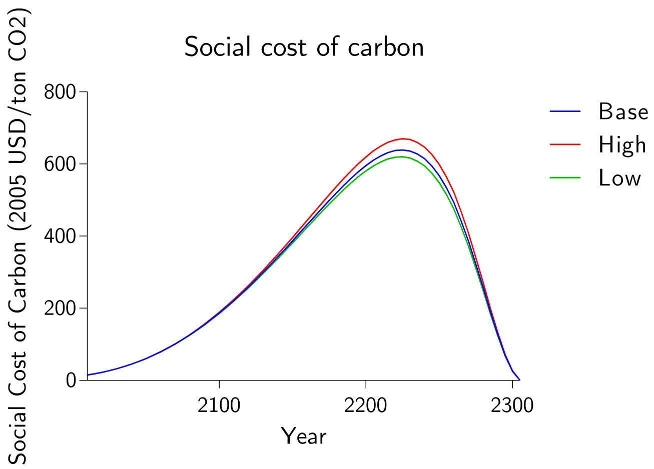

Figure A.6: Social cost of carbon with entrepreneurship level.

spheric temperature starts converging towards a steady-state temperature. This is more a reflection upon the economic concavity of the model, which is based upon the work of Ramsey-Cass-Koopmans.

In terms of prices, the backstop technology enjoyed a price decrease with a higher entrepreneurial level. On the other hand, the fossil fuel price did not increase in a more entrepreneurial level. This could be counter-intuitive but it is due to the fact that the price of the fossil fuel depends on the supply, so if less fossil fuel is used then the price does not increase. **Thus, entrepreneurship in the model increases the competitiveness of the backstop technology in comparison to the fossil fuel technology.** This is an important insight into policy-making as nurturing entrepreneurs via subsidies, grants, tax breaks, programs, etc. will keep fossil fuels on the ground and increase the use of sustainable energies.

In regards to economic growth, the inclusion of entrepreneurship in the model through technological change did not amount to much change. There are two reasons for this: most of the growth already comes from the original inclusion of technological change (R&D) and because entrepreneurship is modeled specifically in the energy sector. Other economic variables such as capital, investment and labor are the predominant factors in the net economic growth. **It is further recommended to include entrepreneurship in a model without a climate module, this way the exact effect on economic variables can be characterized.**

In a similar fashion, the social cost of carbon had no significant change when including entrepreneurship in the model. Most policy-makers are only interested in the short-run social cost of carbon, and with good reason. There was no change in the social cost of carbon when including entrepreneurship due to the long-term benefits of damages and emissions. Entrepreneurship only has an effect in the second half of the model when the new "inventions and innovations" become a prominent player in the market. **This is a reminder that investing in entrepreneurship is a long-term investment. Many times, the benefits are not immediate.** Policy-makers are urged to include long-term vision into their decisions even when the benefits will not be felt while they are still in office.

There is much work left to do. Entrepreneurship is very dynamic, its meaning has changed accordingly with time. The future is inherently uncertain and expectations about its potential are vast. The importance of including entrepreneurship while modeling technological change in climate models has been demonstrated in this analysis. Further research is encouraged on similar fronts: stochastic programming to account for uncertainty, entrepreneurship with different economic theories, positive shocks to the economy with a random probability distribution to simulate large-scale disruptive technologies and more. Only further research into how entrepreneurship deals with technological change and its effect on the economy and climate will yield better tools to give more appropriate policy-advice.

Appendix B

GAMS code

The following is the basic code for most of the thesis. For the full code please e-mail me at leoncio.montemayor@gmail.com.

```
$ontext
This is the DICE-ED model.
It is calibrated to 2010 initial values with 2005 USD.
By: Leoncio David Montemayor Rodriguez
email: leoncio.montemayor@gmail.com
$offtext

$title        DICED 2016
$offlisting
$offsymlist

set         t   Time periods (5 years per period)                   /1*60/ ;

parameters

**Time Step
tstep     Years per Period                                    /5/

** Preferences
elasmu    Elasticity of marginal utility of consumption     /  1.45 /
prstp     Initial rate of social time preference per year   / .015  /

** Population and technology
gama      Capital elasticity in production function        /.300    /
pop0      Initial world population (millions)              /6838    /
```

```
popadj   Growth rate to calibrate to 2050 pop projection  /0.134   /
popasym  Asymptotic population (millions)                 /10500   /
dk       Depreciation rate on capital (per year)          /.100    /
q0       Initial world gross output (trill 2005$)         /63.69   /
k0       Initial capital value (trill 2005$)              /135     /
a0       Initial level of total factor productivity (below)
ga0      Initial growth rate for TFP per 5 years          /0.079   /
dela     Decline rate of TFP per 5 years                  /0.006   /

** Emissions parameters
gsigma1  Initial growth of sigma (per year)                   /-0.01   /
dsig     Decline rate of decarbonization (per period)         /-0.001  /
eland0   Carbon emissions from land 2010 (GtCO2 per year)     / 3.3    /
deland   Decline rate of land emissions (per period)          / .2     /
e0       Industrial emissions 2010 (GtCO2 per year)           /33.61   /
miu0     Initial emissions control rate for base case 2010    /.039    /

** Carbon cycle
* Initial Conditions
mat0   Initial Concentration in atmosphere 2010 (GtC)        /830.4   /
mu0    Initial Concentration in upper strata 2010 (GtC)      /1527.   /
ml0    Initial Concentration in lower strata 2010 (GtC)      /10010.  /
mateq  Equilibrium concentration atmosphere  (GtC)           /588     /
mueq   Equilibrium concentration in upper strata (GtC)       /1350    /
mleq   Equilibrium concentration in lower strata (GtC)       /10000   /

* Flow paramaters
b12      Carbon cycle transition matrix                      /.088   /
b23      Carbon cycle transition matrix                      /0.00250/

* These are for declaration and are defined later
b11      Carbon cycle transition matrix
b21      Carbon cycle transition matrix
b22      Carbon cycle transition matrix
b32      Carbon cycle transition matrix
b33      Carbon cycle transition matrix
sig0     Carbon intensity 2010 (kgCO2 per output 2005$ 2010)

** Climate model parameters
t2xco2   Equilibrium temp impact (oC per doubling CO2)    / 2.9   /
fex0     2010 forcings of non-CO2 GHG (Wm-2)              / 0.25  /
fex1     2100 forcings of non-CO2 GHG (Wm-2)              / 0.70  /
tocean0  Initial lower stratum temp change (C from 1900)  /.0068  /
tatm0    Initial atmospheric temp change (C from 1900)    /0.80   /
```

```
c10       Initial climate equation coefficient for upper level /0.098  /
c1beta    Regression slope coefficient(SoA¬Equil TSC)          /0.01243/

c1        Climate equation coefficient for upper level     /0.098  /
c3        Transfer coefficient upper to lower stratum      /0.088  /
c4        Transfer coefficient for lower level             /0.025  /
fco22x    Forcings of equilibrium CO2 doubling (Wm-2)      /3.8    /

** Climate damage parameters
a10        Initial damage intercept                          /0       /
a20        Initial damage quadratic term                     /0.00267 /
a1         Damage intercept                                  /0       /
a2         Damage quadratic term                             /0.00267 /
a3         Damage exponent                                   /2.00    /

** Abatement cost
expcost2  Exponent of control cost function                / 2.8  /
pback     Cost of backstop 2005$ per tCO2 2010             / 344  /
gback     Initial cost decline backstop cost per period    / .025 /
limmiu    Upper limit on control rate after 2150           / 1.2  /
tnopol    Period before which no emissions controls base   / 45   /
cprice0   Initial base carbon price (2005$ per tCO2)       / 1.0  /
gcprice   Growth rate of base carbon price per year        /.02   /

** Availability of fossil fuels
fosslim        Maximum cumulative extraction fossil fuels (GtC) /6000/

** Scaling and inessential parameters
* Note that these are unnecessary for the calculations but are for
    convenience
scale1      Multiplicative scaling coefficient             /0.016408662 /
scale2      Additive scaling coefficient                   /-3855.106895/

**Endogenous Technological Change parameters
y0            Initial production net from DICE            /63.473   /
y0nodam       Initial production net from DICE no dam     /63.3647  /
i0            Initial investment (trillion 2005$)         /16.608   /
c0            Initial consumption (trillion 2005$)        /47.029   /
phigr         Growth rate phi (per period)                /-.0685   /
phigrgr       Decline rate of phigr (per period)          /.0674    /
alphah        Scaling factor energy efficiency            /0.336    /
alphac        Scaling factor carbon intensity             /.8       /
he0           Initial energy human capital                /0.0001  /
```

```
hb0          Initial backstop human capital              /1.0      /
ipfae1       IPF parameter a (energy eff)                /0.0312  /
ipfbe1       IPF parameter b (energy eff)(0<b<1)         /0.21    /
ipfphie1     IPF parameter phi (energy eff) (0<phi<1)    /0.52     /
ipfae        IPF parameter a (energy eff)                /0.0262  /
ipfbe        IPF parameter b (energy eff)(0<b<1)         /0.29     /
ipfphie      IPF parameter phi (energy eff) (0<phi<1)    /0.60     /
ipfab        IPF parameter a (backstop)                  /.01    /
ipfbb        IPF parameter b (backstop) (0<b<1)          /0.067    /
ipfphib      IPF parameter phi (backstop) (0<phi<1)      /0.60     /
subh         Subsititution parameter knowledge           /0.38     /
subb         Subsititution parameter backstop (0.524)
Δh       Decay knowledge                             /0.0      /
Δb       Decay backstop knowledge                    /0.0      /
crowd        Crowding out %                              /0.5      /
ccost0       Initial carbon cost (calc below)
beta         Beta production function (.0562)
pb0sub       Price of backstop for substitution          /1200     /
pb0          Initial price of backstop                   /1200     /
pf0          Initial price of fossil fuel                /412.85   /
eta          Parameter knowledge and price (bt)          /0.4      /
rde0         Initial energy RD (trillion 2005$)          /.0178974 /
rdb0         Initial RD backstop (trillion 2005$)        /.00178974/
bt0sub       Level of backstop for substitution          /0.972    /
bt0          Initial backstop use (G CTE)                /0.972    /
f0           Initial fossil fuel use (Gt C)              /9.15248  /
eh0          Initial energy units (calc below)
backlim      limit on growth of backstop (per period)   /1.2       /
inertf       Limit on decline of fossil use (per period)/0.86     /;

* Program control variables
sets     tfirst(t), tlast(t),tnotone(t),tmid(t),tnotlast(t);

*all of the parameters have an initial value of zero for all the time periods
PARAMETERS
l(t)         Level of population and labor
al(t)        Level of total factor productivity
sigma(t)     CO2-equivalent-emissions output ratio
rr(t)        Average utility social discount rate
ga(t)        Growth rate of productivity from
forcoth(t)   Exogenous forcing for other greenhouse gases
gl(t)        Growth rate of labor
gcost1       Growth of cost factor
gsig(t)      Change in sigma (cumulative improvement energy efficiency)
```

```
etree(t)       Emissions from deforestation
cost1(t)       Adjusted cost for backstop
lam            Climate model parameter
gfacpop(t)     Growth factor population
pbacktime(t)   Backstop price
optlrsav       Optimal long-run savings rate used for transversality
scc(t)         Social cost of carbon
cpricebase(t) Carbon price in base case
photel(t)      Carbon Price under no damages (Hotelling rent condition)
*Endogenous Technological Change new parameters
phi(t)         carbon emissions per carbon service
phicgr(t)      cumulative exponential growth rate of phi;

* Program control definitions, basically tfirst is 1 and tlast is 60
tfirst(t)   = yes$(t.val eq 1);
tlast(t)    = yes$(t.val eq card(t));
tnotone(t)  = yes$(ord(t) ge 2);
tmid(t)     = yes$(ord(t) ge 2 and t.val lt card(t));
tnotlast(t)= yes$(ord(t) le (card(t)-1));

* Parameters for long-run consistency of carbon cycle
b11 = 1 - b12;
b21 = b12*MATEQ/MUEQ;
b22 = 1 - b21 - b23;
b32 = b23*mueq/mleq;
b33 = 1 - b32 ;

* Further definitions of parameters
sig0 = e0/(q0*(1-miu0));
lam = fco22x/t2xco2;

l("1") = pop0;
loop(t, l(t+1)=l(t););
loop(t, l(t+1)=l(t)*(popasym/L(t))**popadj ;);

gsig("1")=gsigma1;
loop(t,gsig(t+1)=gsig(t)*((1+dsig)**tstep) ;);
sigma("1")=sig0;
loop(t,sigma(t+1)=(sigma(t)*exp(gsig(t)*tstep)););

pbacktime(t)=pback*(1-gback)**(t.val-1);
etree(t) = eland0*(1-deland)**(t.val-1);
rr(t) = 1/((1+prstp)**(tstep*(t.val-1)));
forcoth(t) = fex0+ (1/18)*(fex1-fex0)*(t.val-1)$(t.val lt 19)+
```

```
(fex1-fex0)$(t.val ge 19);
optlrsav = (dk + .004)/(dk + .004*elasmu + prstp)*gama;

*Transient TSC Correction ("Speed of Adjustment Parameter")
c1 =  c10 + c1beta*(t2xco2-2.9);

*Base Case       Carbon Price
cpricebase(t)= cprice0*(1+gcprice)**(5*(t.val-1));

*Endogenous Technological Change defintions

phicgr(t) = (phigr/phigrgr)*(1-exp(-(ord(t)-1)*phigrgr));
phi(t) = exp(phicgr(t));


VARIABLES
MIU(t)          Emission control rate GHGs
FORC(t)         Increase in radiative forcing (watts per m2 from 1900)
TATM(t)         Increase temperature of atmosphere (degrees C from 1900)
TOCEAN(t)     Increase temperatureof lower oceans (degrees C from 1900)
MAT(t)      Carbon concentration increase in atmosphere (GtC from 1750)
MU(t)    Carbon concentration increase in shallow oceans (GtC from 1750)
ML(t)      Carbon concentration increase in lower oceans (GtC from 1750)
E(t)            Total CO2 emissions (GtCO2 per year)
EIND(t)         Industrial emissions (GtCO2 per year)
C(t)            Consumption (trillions 2005$ per year)
K(t)            Capital stock (trillions 2005$)
CPC(t)          Per capita consumption (thousands 2005$ per year)
I(t)            Investment (trillions 2005$ per year)
S(t)            Gross savings rate as fraction of gross world product
RI(t)           Real interest rate (per annum)
Y(t)         GWP net of abatement and damages (trillions 2005$ per year)
YGROSS(t)  GWP GROSS of abatement and damages (trillions 2005$ per year)
YNET(t)        Output net of damages equation (trillions 2005$ per year)
DAMAGES(t)      Damages (trillions 2005$ per year)
DAMFRAC(t)      Damages as fraction of gross output
ABATECOST(t)    Cost of emissions reductions  (trillions 2005$ per year)
MCABATE(t)      Marginal cost of abatement (2005$ per ton CO2)
CCA(t)          Cumulative industrial carbon emissions (GTC)
PERIODU(t)      One period utility function
CPRICE(t)       Carbon price (2005$ per ton of CO2)
CEMUTOTPER(t)   Period utility
UTILITY         Welfare function
*Endogeneous Technological Change variables
```

```
HE(t)            Knowledge energy efficiency stock
HB(t)            Knowledge backstop stock
NEWHE(t)         New knowledge energy efficiency stock
NEWHB(t)         New knowledge backstop stock
RDE(t)           RD for energy efficiency (trillions 2005$ per year)
RDB(t)           RD for backstop (trillions 2005$ per year)
RETRDE(t)        Rate of return on energy R&D
RETRDB(t)        Rate of return on backstop R&D
GROWTHRD(t)      Rate of growth on R&D (% per year)
EH(t)            Energy units (eeu)
FOSSIL(t)        Level of fossil fuel used (GtC)
PRICEFOSSIL(t)   Price of fossil fuels  (2005$ per ton Carbon)
PRICEBT(t)       Price of backstop technology (2005$ per CTE)
CHANGEP(t)       Change price level PRICEBT  w.r.t. time (2005$ per CTE)
DEDP(t)          Derivative of EH w.r.t. PRICEBT
BT(t)            Level of backstop technology used (GCTE)
*Dynamic discounting variables
RHO(t)           Dynamic rate of pure time preference
FACTOR(t)        Discount factor
;

NONNEGATIVE VARIABLES  MIU, TATM, MAT, MU, ML, Y, YGROSS, C, K, I, H, HE, HB,
RDE, RDB, PRICEFOSSIL, PRICEBT, BT,S, NEWHE, NEWHB,EH, RHO, FACTOR;

EQUATIONS
*Emissions and Damages
EEQ(t)            Emissions equation
EINDEQ(t)         Industrial emissions
CCACCA(t)         Cumulative carbon emissions

FORCE(t)          Radiative forcing equation
DAMFRACEQ(t)      Equation for damage fraction
DAMEQ(t)          Damage equation

*Climate and carbon cycle
MMAT(t)           Atmospheric concentration equation
MMU(t)            Shallow ocean concentration
MML(t)            Lower ocean concentration
TATMEQ(t)         Temperature-climate equation for atmosphere
TOCEANEQ(t)       Temperature-climate equation for lower oceans

*Economic variables
YGROSSEQ(t)       Output gross equation
YNETEQ(t)         Output net of damages equation
```

```
YY(t)             Output net equation
CC(t)             Consumption equation
CPCE(t)           Per capita consumption definition
SEQ(t)            Savings rate equation
KK(t)             Capital balance equation
RIEQ(t)           Interest rate equation

* Utility
CEMUTOTPEREQ(t)  Period utility
PERIODUEQ(t)      Instantaneous utility function equation
UTIL              Objective function
*Endogeneous Technological Change equations
PFEQ(t)           Equation for the fossil price
PBEQ(t)           Equation for the backstop fuel price
CHANGEPBEQ(t)     Equation for the change of backstop fuel price
HEEQ(t)           Knowledge energy efficiency stock equation
HBEQ(t)           Knowledge backstop stock equation
NEWHEEQ0(t)       Equation for new knowledge energy in first run
NEWHBEQ0(t)       Equation for new backstop knowledge in first run
NEWHEEQ1(t)       Equation for new knowledge energy without backstop
NEWHEEQ(t)        Equation for new knowledge energy
NEWHBEQ(t)        Equation for new backstop knowledge
ENERGYEQ(t)       Energy equation
DEDPEQ(t)         Derivative energy w.r.t. price equation
DEDPEQ1(t)        Derivative energy w.r.t. price equation (optimal run)
FOSSILEQ(t)       Fossil fuel constraint equation
BTEQ(t)           Backstop constraint on growth
INERTEQ(t)        Fossil fuel limit decline equation
GROWTHRDEQ(t)     Equation for the rate of growth of R&D
CONSTRDEQ(t)      Constraint for rate of return of energy R&D
CONSTRDBEQ(t)     Constraint for rate of return of backstop RD
RETRDEEQ(t)       Equation for the rate of return on energy R&D
RETRDEEQ1(t)      Eq for the rate of return on energy R&D with backstop
RETRDBEQ(t)       Equation for the rate of return on backstop R&D

*Dynamic discounting equations
RHOEQ(t)          Equation for the dynamic discount rate
FACTOREQ(t)       Equation for discount factor
CEMUTOTPER1EQ(t) New period utility
RIEQ1(t)          New interest rate equation
;

** Equations of the model
*Emissions and Damages
```

```
eeq(t)..              E(t)            =E= EIND(t) + etree(t);
eindeq(t)..           EIND(t)         =E= FOSSIL(t)*3.666;
ccacca(t+1)..         CCA(t+1)        =E= CCA(t)+ tstep*EIND(t)/3.666;
force(t)..            FORC(t)         =E= fco22x * ((log((MAT(t)/588.000))/
log(2))) + forcoth(t);
damfraceq(t) ..       DAMFRAC(t)      =E= (a1*TATM(t))+(a2*TATM(t)**a3) ;
dameq(t)..            DAMAGES(t)      =E= YGROSS(t) * DAMFRAC(t);

*Climate and carbon cycle
mmat(t+1)..           MAT(t+1)        =E= MAT(t)*b11 + MU(t)*b21 + (E(t)*
(5/3.666));
mml(t+1)..            ML(t+1)         =E= ML(t)*b33  + MU(t)*b23;
mmu(t+1)..            MU(t+1)         =E= MAT(t)*b12 + MU(t)*b22 + ML(t)*b32;
tatmeq(t+1)..         TATM(t+1)       =E= TATM(t) + c1 * ((FORC(t+1)-
(fco22x/t2xco2)*TATM(t))-(c3*(TATM(t)-TOCEAN(t))));
toceaneq(t+1)..       TOCEAN(t+1)     =E= TOCEAN(t) + c4*(TATM(t)-TOCEAN(t));

*Economic variables
ygrosseq(t)..         YGROSS(t)       =E= (al(t)*(L(t)/1000)**(1-gama-beta))
 *(K(t)**gama)*(EH(t)**beta) ;
yneteq(t)..           YNET(t)         =E= YGROSS(t)*(1-damfrac(t));
yy(t)..               Y(t)            =E= YNET(t)- (PRICEFOSSIL(t)*FOSSIL(t)/
(phi(t)*1000)+BT(t)*PRICEBT(t)/1000);
cc(t)..               C(t)            =E= Y(t) - I(t) - RDE(t) - RDB(t) ;
cpce(t)..             CPC(t)          =E= 1000 * C(t) / L(t);
seq(t)..              I(t)            =E= S(t) * Y(t);
kk(t+1)..             K(t+1)          =L= (1-dk)**tstep * K(t) + tstep *
(I(t)-4*crowd*(RDE(t)+RDB(t)));
rieq(t+1)..           RI(t)           =E= (1+prstp) * (CPC(t+1)/CPC(t))**
(elasmu/tstep) - 1;

*Utility
cemutotpereq(t)..     CEMUTOTPER(t)   =E= PERIODU(t) * L(t) * rr(t);
periodueq(t)..        PERIODU(t)      =E= ((C(T)*1000/L(T))**(1-elasmu)-1)/
(1-elasmu)-1;
util..                UTILITY         =E= tstep * scale1 * sum(t,  CEMUTOTPER(t
    ))
+ scale2 ;

*Endogeneous Technological Change equations definition
pfeq(t)..             PRICEFOSSIL(t) =E= (412.85 + 1045.98*(CCA(t)/fosslim)
    **4) ;
pbeq(t)..             PRICEBT(t)      =E= pb0/(HB(t)**eta) ;
changepbeq(t)..       CHANGEP(t)      =E= PRICEBT(t) - pb0;
```

```
heeq(t+1)..          HE(t+1)        =L= tstep * NEWHE(t) + (1-Δh)**tstep
 *HE(t) ;
hbeq(t+1)..          HB(t+1)        =L= tstep * NEWHB(t) + (1-Δb)**tstep
 *HB(t) ;
newheeq0(t)..        NEWHE(t)       =E= 0 ;
newhbeq0(t)..        NEWHB(t)       =E= 0 ;
newheeq1(t)..        NEWHE(t)       =E= ipfae1 * RDE(t)**ipfbe1 * HE(t)**
ipfphie1 ;
newheeq(t)..         NEWHE(t)       =E= ipfae * RDE(t)**ipfbe * HE(t)**
    ipfphie;
newhbeq(t)..         NEWHB(t)       =E= ipfab * RDB(t)**ipfbb * HB(t)**
    ipfphib;
energyeq(t)..        EH(t)          =E= ((alphah*HE(t))**subh + ((FOSSIL(t)/
(phi(t)))**subb + BT(t)**subb)**(subh/subb))**(1/subh);
**dedpeq is defined below under scenario 3.
fossileq(t)..        FOSSIL(t)      =L= (0.1*(fosslim-CCA(t)))/tstep ;
bteq(t+1)..          BT(t+1)        =L= .005 + backlim*BT(t) ;
inerteq(t+1)..       FOSSIL(t+1)    =G= inertf*FOSSIL(t);
growthrdeq(t+1)..    GROWTHRD(t+1)  =E= ((RDE(t+1)+RDB(t+1))-(RDE(t)+RDB(t)))
    /
(RDE(t)+RDB(t))/tstep*100 ;
constrdeq(tmid)..    RETRDE(tmid)   =G= 4*RI(tmid) ;
constrdbeq(tmid)..   RETRDB(tmid)   =G= 4*RI(tmid) ;
retrdeeq1(t)..       RETRDE(t)      =E= ((BETA*Y(t)*alphah*((alphah*HE(t))**
(subh-1)))/((alphah*HE(t))*subh + (((FOSSIL(t)/phi(t))**subb +
BT(t)**subb)**(1/subb))**subh)) *
(ipfae1*ipfbe1*RDE(t)**(ipfbe1-1)*HE(t)**ipfphie1) ;
retrdeeq(t)..        RETRDE(t)      =E= ((BETA*Y(t)*alphah*((alphah*HE(t))**
(subh-1)))/((alphah*HE(t))*subh + (((FOSSIL(t)/phi(t))**subb +
BT(t)**subb)**(1/subb))**subh)) *
(ipfae*ipfbe*RDE(t)**(ipfbe-1)*HE(t)**ipfphie) ;
retrdbeq(t)..        RETRDB(t)      =E= ((BETA*Y(t)/EH(t))*DEDP(t)*((-eta*pb0
    )/
((HB(t))**(eta+1)))*(ipfab*ipfbb*(RDB(t)**(ipfbb-1))*HB(t)**ipfphib)) ;


*Resource limit
CCA.up(t)        = fosslim;

* Control rate limits
MIU.up(t)             = limmiu;
MIU.up(t)$(t.val<30)  = 1;

**  Upper and lower bounds for stability
```

```
K.LO(t)          = 1;
MAT.LO(t)        = 10;
MU.LO(t)         = 100;
ML.LO(t)         = 1000;
C.LO(t)          = 2;
TOCEAN.UP(t)     = 20;
TOCEAN.LO(t)     = -1;
TATM.UP(t)       = 40;
CPC.LO(t)        = .01;
S.LO(t)          = 0.001;
*Endogenous Technological Change bounds
FOSSIL.lo(t)     = 1;
FOSSIL.up(t)     = 100;
BT.lo(t)         = 0;
BT.up(t)         = 100;
RDE.lo(t)        = 0;
RDB.lo(t)        = 0;
RDE.up(t)        = 1;
RDB.up(t)        = 1;

* Control variables
* Set savings rate for steady state for last 10 periods
set lag10(t) ;
lag10(t) =  yes$(t.val gt card(t)-10);
S.FX(lag10(t)) = optlrsav;

* Initial conditions
CCA.FX(tfirst)     = 90;
K.FX(tfirst)       = k0;
MAT.FX(tfirst)     = mat0;
MU.FX(tfirst)      = mu0;
ML.FX(tfirst)      = ml0;
TATM.FX(tfirst)    = tatm0;
TOCEAN.FX(tfirst) = tocean0;
*Endogenous Technological Change initial conditions
HE.fx(tfirst)      = he0;
HB.fx(tfirst)      = hb0;
PRICEBT.FX(tfirst)= pb0;
BT.fx(tfirst)      = bt0;
RDE.FX(tfirst)     = rde0;
RDB.FX(tfirst)     = rdb0;
FOSSIL.FX(tfirst) = f0;

** Solution options
```

```
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
```

Appendix C

Python code

The following code shows the basic way of handling GAMS from Python. This was done with the API.

```
from gams import *
import os
import sys

#How to run a GamsJob from a file

ws = GamsWorkspace("D:\Dropbox\Dropbox\Thesis\workingdirectorypython")
t1 = ws.add_job_from_file("dice.gms") #.gms file must be in working directory
t1.run() #working directory has .lst file with results
t1.out_db.export() #to export results to .gdx file in working directory

#How to retrieve and print a solution from an output database
print "Solution with ifopt = 1:"
for rec in t1.out_db["TATM"]:
print "TATM" + str(rec.keys) + " = " + str(rec.level)

#to get a specific result from any variable
print t1.out_db.get_symbol("MIU")["20"]

#To change parameter and run the job
opt = ws.add_options() #add options command to change parameters
opt.defines["ifopt"] = "0" #changes parameter
t1.run(opt) #runs with new opt configuration
print "Solution with ifopt = 0:"
for rec in t1.out_db["TATM"]:
```

```
print "TATM" + str(rec.keys) + " = " + str(rec.level)

#to change a new one or more repeat process
opt.defines["prstp"] = "0.03"
opt.defines["ifopt"] = "1"
opt.defines["gama"] = "0.2"
t1.run(opt)
print "Solution changed parameters:"
for rec in t1.out_db["TATM"]:
print "TATM" + str(rec.keys) + " = " + str(rec.level)
```