Finding order in the design landscape of simple optical systems

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ABSTRACT

Contrary to the frequent tacit assumption that the local minima of a merit function are points scattered more or less randomly over the design landscape, we have found that, at least for simple imaging systems (doublets with three and triplets with five variables) all design shapes we have observed thus far form a strictly ordered set of points, the "fundamental network". The design shapes obtained for practical specifications with global optimization algorithms are a subset of the set of local minima in the fundamental network and are organized in a way that can be understood on the basis of the fundamental network.

Keywords: saddle point; global optimization; optical system design

1. INTRODUCTION

It is well known that, in typical design problems, the result obtained with local optimization is critically dependent on the choice of the starting point. Therefore, the presence of multiple local minima in the merit function landscape is one of the main difficulties in optical system design. Significant progress in global optimization of optical systems over the past two decades alleviates this difficulty for many design problems.[1-6] However, a better understanding at a fundamental level of the complexity of the design landscape could lead to further improvement of optical design techniques. The goal of this paper is to analyse the design landscape for simple imaging systems, and to make an inventory of the types of local minima that can be encountered in global optimization runs in these cases.

In general global optimization problems it is tacitly assumed that the local minima of the merit function are points scattered more or less randomly over the merit function landscape. In contrast, we have found structure in the optical merit function landscape. In this paper we discuss this structure in the case of simple optimization problems for doublets and triplets. As well known, for these systems good starting configurations for subsequent local optimization can be found with aberration theory (e.g. by annulling third-order spherical aberration, coma and axial color for designing achromatic doublets, and by annulling all primary monochromatic and chromatic aberrations to design a Cooke triplet). However, an analysis of the entire design landscape reveals unexpected features that might survive generalization to more complex systems for which finding a good starting configuration is much more difficult.

For simplicity, in this paper we consider global optimization problems with a number of variables $N=3$ (doublets) and especially $N=5$ (triplets). The image defects in the merit function are transverse ray aberrations (computed with respect to the chief ray) and our merit function is in this paper the default error function of the software we have used, CODE V. Practical experience shows that lens curvatures are more important variables than distances between surfaces. Therefore, in order to keep the number of variables low while preserving the essential structure that will be described in what follows, the variables are the lens curvatures. The glass and air thicknesses are kept constant for all systems of a given global optimization run. A linear constraint is always used, either a "solve" on the last surface to keep the effective focal length constant when the object is at infinity, or a "total track" constraint otherwise. Therefore, the dimensionality of the optimization problem is decreased by one.

We show that, for these simple imaging systems, the possible design shapes form an ordered set of points in the optical merit function landscape. This structure is observed when we consider not only local minima, but saddle points as well. In Section 2 we will discuss the concept of saddle point and we will show how saddle points organize the set of local

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minima into a network. In Section 3 we will discuss the concept of "fundamental network". This network contains as nodes all design shapes we have observed thus far in several sets of global optimization runs with different settings. Finally, in Section 4 we discuss triplet networks and we show how the set of local minima obtained with global optimization algorithms for practical settings is organized in a way that can be understood on the basis of the fundamental network.

2. SADDLE POINTS AND NETWORKS

Each set of optimization variables defines a point in an *N*-dimensional solution space. A point in this space for which the gradient of the merit function vanishes is called a critical point. As in the case of local minima and maxima, saddle points are critical points. As an intuitive analogy, in a two-dimensional mountain landscape the saddle points are the mountain passes, and two large balls placed on opposite sides of the "saddle" roll down to different valleys. Saddle points with similar properties exist in design spaces with arbitrary dimensionality and, as shown below, they play an essential role in structuring the merit function landscape.

For the mathematically interested reader we mention that an important characteristic of critical points is the so-called Morse index. We consider the matrix of the second-order derivatives of the merit function with respect to the optimization variables (i.e. the Hessian of the merit function). If the Hessian has a nonzero determinant, the number of negative eigenvalues of the Hessian gives the Morse index of the critical point. A negative eigenvalue means that along the direction defined by the corresponding eigenvector of the Hessian the critical point is a maximum. Therefore, minima have Morse index 0, maxima have Morse index N, and for saddle points the Morse index has values between 1 and $N - 1$. If, for instance, $N = 2$, then every saddle point has a Morse index of one.

The high-order equivalent of the "mountain pass" from the analogy above is a saddle point that is a maximum in one direction (called the downward direction) and a minimum in all other directions perpendicular on the downward one (i.e. a saddle point with a Morse index of one). Intuitively, the downward direction of a saddle point with a Morse index of one is similar to the downward direction of a familiar two-dimensional saddle point. If we choose for local optimization two starting points close to each other, but on opposite sides of the saddle, then after optimization they will lead to two distinct minima. All saddle points shown in this paper have the Morse index 1. Therefore, for simplicity, in what follows "saddle point" will mean "saddle point with a Morse index of one", unless stated otherwise explicitly.

In the set of local minima resulting from a global optimization run, saddle points define relationships between the various minima. Two minima are said to be linked with each other if there exists a saddle point between them that leads after optimization on its two sides to the two given minima. It has been shown that in optical system design the local minima form a connected network in which each link contains a saddle point.[7]

Figure 1 shows a network that corresponds to a simple monochromatic doublet run. The five systems denoted by 'm" are local minima. Note that minimum m1 (the "hub") is linked to all other four minima via the four saddle points denoted by "s". (In several figures in this paper, the two downward paths of local optimization started on both sides of the saddle at a saddle point $s_{i\text{-}j}$ lead to minima m_i and m_j, as indicated by the continuous lines between systems.) In Ref. 8, crown and flint glasses are used as usual for the two lenses in order to achieve color correction and the two networks in the case of flint-crown and crown-flint glass order are shown. If we compare them with the one shown in Fig. 1, it turns out that the number of solutions and the network structure remains in both cases the same as in Fig. 1, despite of the fact that in the two glass order situations the merit function values of the minima differs significantly (the hub remains a poor minimum in both cases, in one case two second-row minima from Fig. 1 are good solutions, in the other case the other two). In many situations we have observed the same behavior when we change specifications (e.g. aperture, field, transverse magnification, distance between surfaces, glass types). Then, the total set of minima and its organization in a network is much less affected than the merit function values of individual systems. In fact, the type of the best solution changes more frequently than the network structure when specifications are changed. (Examples of changes in the network structure in such situations are given in Sec. 4.) Since we are interested in this paper in results that are more generally valid, the emphasis is here on the total set of minima and its organization in a network, rather than on finding the best solution.

In principle, if in a global search all saddle points are known, then all local minima result automatically from local optimization on both sides of the saddle points. The good solutions can then be selected from the total set of minima. We use two methods to find saddle points.

The network shown in Fig. 1 has been found with our own program NETMIN that is based on a method called saddle-point detection.[9,10] Without any a-priori knowledge about saddle-point properties, in the vicinity of known local minima all saddle points connected to them are sought. Then, on the other side of these saddle points new minima are found, until the complete network is detected. Detecting saddle points in this way is computationally expensive, but for systems for which the complexity is not too large, NETMIN gives good results. In these cases, comparison with the results of Global Synthesis of CODE V shows that usually all minima found by Global Synthesis are also found by NETMIN.

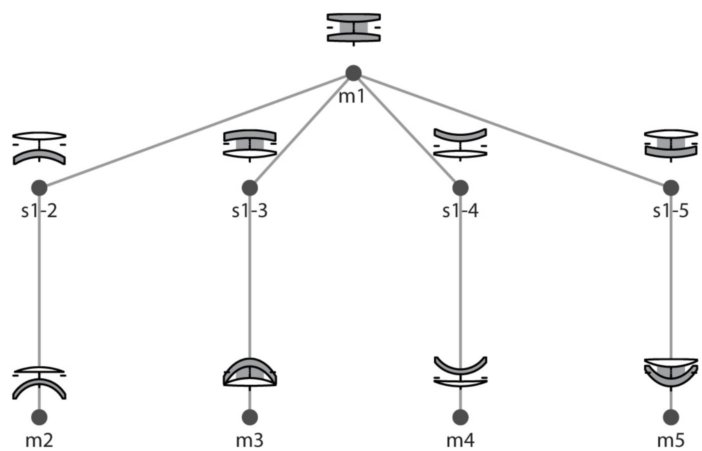

Fig. 1: Network corresponding to a monochromatic doublet global search

Above we have discussed some general mathematical properties of saddle points of smooth mathematical functions with *N* variables. Remarkably, the vast majority of the saddle points detected for simple imaging systems have an additional property that, to our knowledge, was first described in our papers on optical system design.[8,11,12] Most of these saddle points can be obtained by a simple technique, saddle-point construction (SPC), from local minima of optimization problems with a lower dimensionality, e.g. doublet saddle points can be obtained from singlet local minima and triplet saddle points can be obtained from doublet local minima. As a practical design method, SPC is much more efficient than saddle point detection and can be used successfully also for very complex systems with many variables and constraints (e.g., in designing lithographic objectives[13,14]) because it enables the generation of new system shapes with only a small number of local optimizations. In lens design, SPC is achieved by inserting a lens into an optimized system. Any optical merit function can be used, e.g. one based on transverse or wavefront aberrations. In the optimized system, we insert a meniscus–shaped lens (or airspace within an existing lens) of zero thickness and equal curvatures. Since such a glass or air meniscus disappears physically and affects neither the light path nor the system's merit function, we call it a 'null element'. However, if the new system is slightly modified, the new lens enables the merit function to decrease. With the new lens, there are two new variables, i.e., the two new surface curvatures. We have shown that for some specific curvature values of the 'null element', the local minimum is transformed into a 'saddle point' in the variable space of increased dimensionality. For more details see Refs. 8, 11, 12 and our website.[15] However, in addition to the practical aspect, simply the fact that saddle points are *obtainable* in this way from local minima with lower dimensionality is a fundamental property. (We encounter here, methodologically, a new sort of reductionism whose importance will be discussed in detail elsewhere).

In NETMIN runs where all lenses have nonzero thickness, we also observe saddle points that contain meniscus lenses. An important property of many of them is that, if we reduce the meniscus glass (or meniscus airspace) thickness to zero and repeat the NETMIN run, we will obtain a saddle point with a null element that has curvatures that are very close to those of the original saddle point with nonzero thickness. It turns out that these (zero-thickness) saddle points can be obtained with SPC as well (much faster than with NETMIN). Even if the meniscus has nonzero thickness, we will call any such saddle point a 'null element saddle point' (NESP). For instance, all four saddle points in Fig. 1 are NESP's that can be obtained from an optimized singlet (that has a shape similar to that of any white lens in Fig. 1). For the saddle points, the lens or airspace that becomes a null element for zero thickness is marked in grey. The same grey marking is used in the doublet minima on both sides of each saddle point to indicate the lens or airspace resulting from the null element. Note that some doublet NESP's have one null element (glass in this case), other NESP's have a pair (air-glass) of null elements. For systems with more lenses the single null element can be airspace as well. NESP's play an essential role in the ordered structure described in the following section.

3. FUNDAMENTAL NETWORK

The networks we have detected with NETMIN for different types of simple and more complex systems confirm the intuitive image of many designers about the design landscape as consisting of a number of main valleys, separated by higher merit function barriers, e.g. the valleys A and B shown in Fig. 1. Superimposed on this main structure, we find occasionally smaller bumps and depressions,[16] e.g. the minima A1 and A2 in valley A. In our networks, the height of a merit function barrier is given by the difference between the merit function of a saddle point and that of a local minimum linked to it. The emphasis in this paper is on the main valleys, which will be called "system shapes".

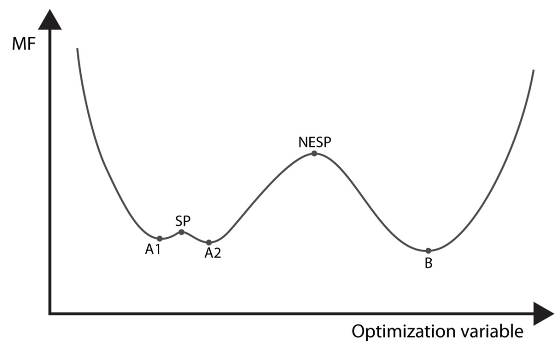

Fig. 2: Two main valleys, A and B. Two local minima, A1 and A2, with a low-barrier saddle point (SP) between them, form valley A.

In this section we introduce the concept of a "fundamental network", an idealization that we use to make the actual network observed in any specific case easier to understand and we show that this "fundamental network" is a perfectly ordered structure. For doublets and triplets, this ideal network is constructed on the basis of essential features (the sort of design shapes we can expect to encounter in a global search and the way they are linked in the network) that tend to be preserved when specifications are changed. In a loose analogy we can think about the regularity observed in ideal crystal lattices. Physical crystals have ideal lattices and real crystals feature defects in these ideal arrangements. Similarly, in lens design we have found that, at least for doublets and triplets, the observed network in a specific global search can be understood in terms of a perfectly ordered fundamental (i.e. "ideal") network in which "network defects" are present.

The network shown in Fig. 1 is in fact the fundamental network for the corresponding class of 3-dimensional doublet global searches and, as noted earlier, we have performed achromatic doublet global searches with practical settings that turn out to have the same network structure. Figure 3 shows the fundamental network for the 5-dimensional triplet global search class. All doublet and triplet numerical experiments we have performed thus far can be understood on the basis to Figs. 1 and 3, as shown in the next section. Moreover, the minima in these figures include all design shapes we have obtained in the global searches we have performed thus far, by using our own program NETMIN and "Global synthesis" of CODE V.

In Fig. 3, the systems denoted by "S" in the second and fourth rows are saddle points, and the systems denoted by "M" (drawn larger) in the first, third and fifth rows are the local minima that correspond to different system shapes. In the first four rows a perfectly regular arrangement can be easily observed. The uppermost minimum M1 has six links, each containing a saddle point, to the six minima drawn in the third row (M2-M7). The regular arrangement includes in fact the fifth row as well, but in a two-dimensional drawing such as Fig. 3 this is less obvious than for the first four rows. For instance, the blue links indicate how the well-known Cooke triplet design shape, minimum M15 in the fifth row, is linked (via two saddle points S3-15 and S6-15) to the third-row minima M3 and M6.

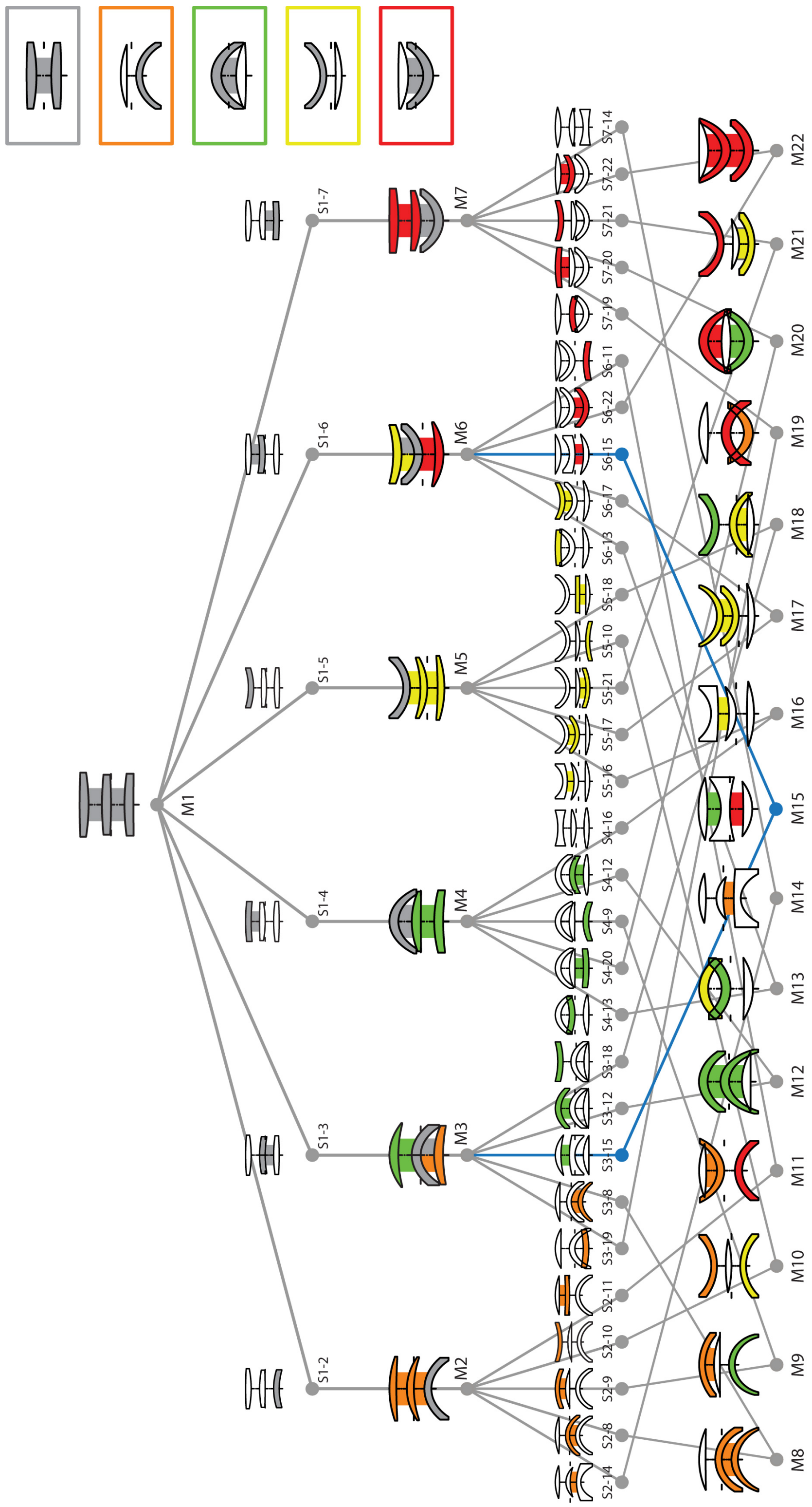

Fig. 3: Fundamental network for three lenses. In the upper part, the doublet design shapes in Fig.1 are reproduced in colored boxes (m1 grey, m2 orange, m3 green, m4 yellow, m5 red) in order to show how the triplet systems can be obtained from these five doublets with SPC. For the saddle points, the lens or airspace that corresponds to a null element is marked with the box color of the doublet from which it can be obtained. The same color is used in the resulting triplet minima on both sides of the saddle points.

In fact, any third-row minimum also has six links, as M1, one drawn upwards to M1 and five drawn downwards (like the blue link) that finally arrive at all other five third-row minima, with a minimum in the fifth row as an intermediate stage. The total number of 15 minima in the fifth row (M8-M22) is exactly the number necessary to link any third-row minimum with all other five in the same row. It can be thus seen that the possible design shapes in the doublet and triplet landscapes are not a random set of points, but a strictly ordered set. Our present results suggest that, when only curvatures are variable and when distances between surfaces are not too large (e.g. not larger than for typical Cooke triplet specifications), we can expect five types (i.e. design shapes) of doublets and 22 types of triplets, *not more*.

It is important to note that all doublet saddle points in Fig. 1, as well as the vast majority of the saddle points in Fig. 3 (34 out of 36) are NESP's. In fact, the two remaining saddle points S4-16 and S7-14 are also part of a general pattern that will be discussed in more detail in a subsequent publication. It will be shown there that all fourth-row saddle points, including these two, are closely related to Morse Index 2 saddle points that are NESP's. Therefore, all minima in Figs. 1 and 3 (the design shapes) can be obtained with SPC from minima of simpler optimization problems. Because so many saddle points are NESP's, Figs. 1 and 3 can be obtained almost entirely (excepting the links containing S4-16 and S7-14) with SPC, and can be verified independently of NETMIN. For reproducing the network with SPC, see the specifications in the next section and Refs. 8 and 12 for technical details on SPC. When saddle points are constructed with SPC, they differ slightly from those shown in Fig. 3 because zero-thickness null elements are present. However, after the two minima on each side of a NESP are obtained, these two minima can be easily reoptimized with the same thickness as in Fig. 3, and then the minima seen in Fig. 3 are obtained.

4. REAL NETWORKS

The relationship between the fundamental network and real networks is discussed in this section in the triplet case. In the real network (obtained with NETMIN) shown in Fig. 4 the system specifications have been chosen not according to practical requirements, but such that we obtain all system shapes that we have observed thus far in all our 5-dimensional triplet runs. In fact, the entire fundamental network in Fig. 3 is included in Fig. 4. In addition to that we also observe network elements that correspond to the smaller depressions in the main valleys of the design landscape, similar to local minima A1 and A2 in Fig. 2. Several minima in the fifth row of Fig. 3 are replaced in Fig. 4 with boxes (drawn dashed) in which all systems -the minima (denoted by a,b,c… after the system shape number) and saddle points between them-resemble the minima they replace in Fig. 3. Because of this resemblance, we say that all systems within such a box (we call it a "cell") have the same system shape. For each system in Fig. 4, below "M" and "S" the merit function value (i.e. the default CODE V error function) is given. Note that the merit function barriers between the minima corresponding to the same system shape within a cell are typically significantly lower than the barriers between the same mimima and the neighboring ones that correspond to different system shapes. The low-barrier saddle points within a cell ("SP" in Fig. 2) seem, at the present stage of the research, not to be related to NESP's.

Each system in Fig. 4 has three lenses with the same glass SK16 (Schott) and glass thickness of 1 mm, placed at two equal distances of 1.5 mm from each other. The first five curvatures are variable and the last one is used to enforce an effective focal length of 30 mm. The object is at infinity, the image is at the paraxial position, the stop is at the 3rd surface, there is no vignetting, the F number is 3.3333, there are three fields (0, 7.14 and 10 degrees) and three wavelengths corresponding to the standard F, d, and C visible spectral lines. (Three wavelengths are not strictly necessary because chromatic aberration correction is not possible with only one glass type, but they are used in order to have specifications that do not differ unnecessarily from those used in the next example, which is a realistic one).

The systems in Fig. 4 look very similar to those in Fig. 3, but are not identical to them since the system drawings in Fig. 3 are extracted from a slightly different run with transverse magnification -1 in which the sixth curvature is also variable and the condition of constant effective focal length is replaced with a constraint of total track equal to 120 mm. In all other respects the two runs containing the system drawings in Figs. 3 and 4 have the same specifications, given above.

In the network shown in Fig. 4, the only type of departure from the fundamental network structure is the presence of cells, as described earlier. In general however, additional types of differences are present. Typically, as shown in Fig. 5, we observe only a subset of the set of fundamental design shapes found in Fig. 3. (Up to now, we did not observe in any run other system shapes that cannot be related to those in Fig. 3.)

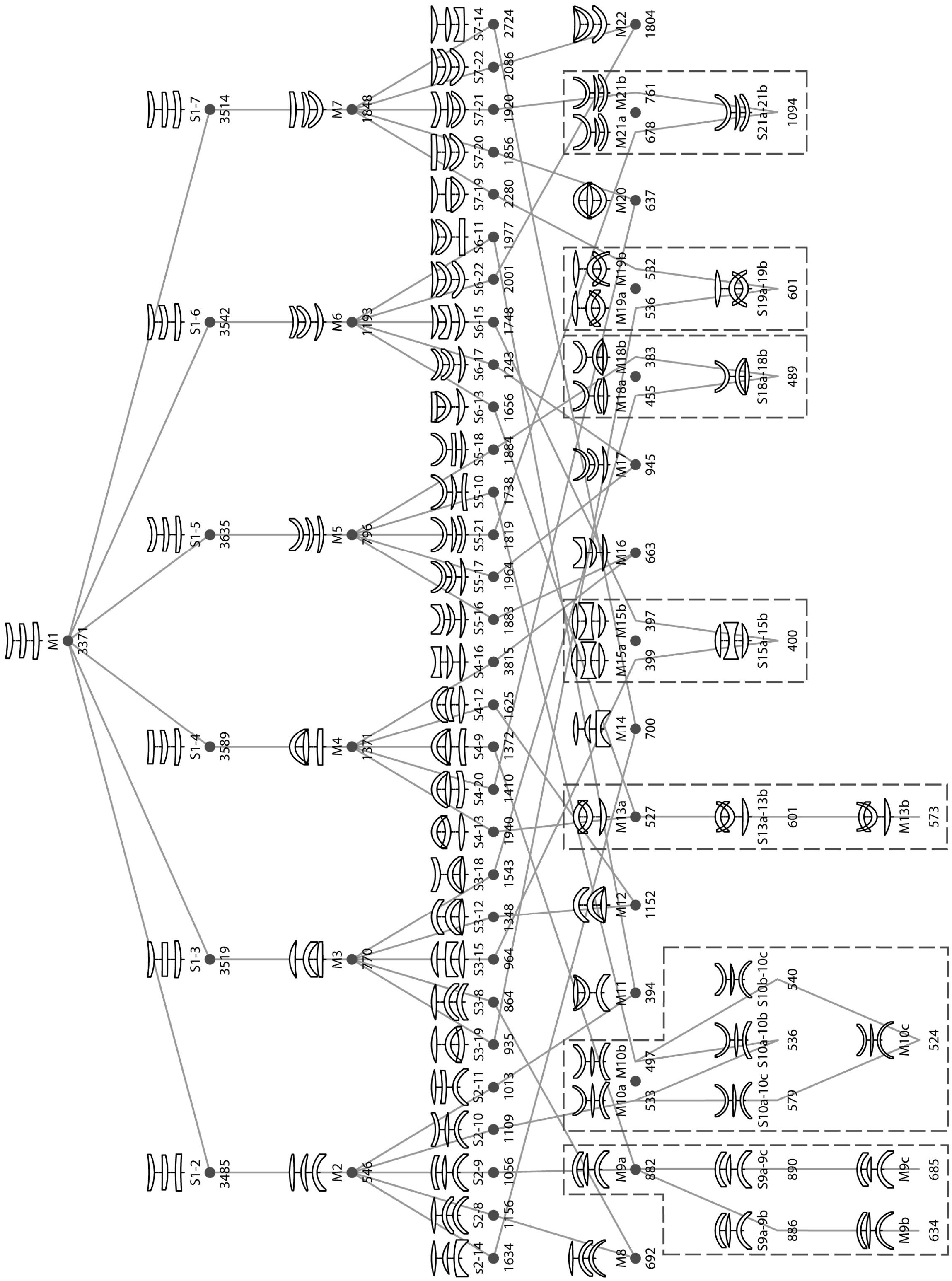

Fig. 4: Real network for three lenses that includes the fundamental one. Systems that have the same shape are drawn within dashed boxes. For simplicity, edge thickness control was disabled. The few edge thickness violations disappear when the distances between lenses increase (see Fig. 5).

In Fig. 1 of Ref. 8 we can find, for realistic specifications, the typical network corresponding to Cooke triplet global searches, for which the variables are the surface curvatures. Figure 5 shows essentially the same network (with two additional saddle points s18-(11v17) and s19-(10v15)), but, in order to facilitate comparison with the fundamental network, the arrangement is different and the skeleton of the network in Fig. 3 is used. (The system numbering is kept the same as in Ref. 8.) From Fig. 5 we obtain insight in the network behavior when specifications change in a given range. The system shapes in Fig. 5 are collected from a set of runs with different specifications (runs with object at infinity, runs for symmetric problems with transverse magnification -1, the field varies between 20 and 33 degrees). The aperture, the distances between surfaces and the glass types are the same for all runs included here and are typical for practical Cooke triplet settings. For a Cooke triplet network that corresponds to a single set of specifications see Ref. 7. The minima there are the same as in Fig. 5, but the saddle points s4-(6v3), s18-(5v7), s18-(11v7), s19-(6v3) and s19-(10v15) are absent.

As shown below, the network structure in Fig. 5 has only a small number of "network defects" when compared to the fundamental triplet network. First, the minimum M1 in Fig. 3 (the "main triplet hub") has disappeared (together with the six saddle points that connect it to the rest of the network). Then, as in Fig. 4, we observe several cells. One cell contains two Cooke triplet minima, m1 and m2, together with a saddle point s1-2 between them. A second cell contains the similarly looking systems m13, m16, and s13-16, and a third one contains m12, m14, and s12-14.

A more general analysis of the network behavior shows that, when specifications change, neighbouring critical points can come close to each other (both in the variable space and as merit function values) and often "collide". In these cases, several scenarios are possible. (A rigorous mathematical analysis of most of these scenarios can be done within the framework of Catastrophe Theory.[17]) For instance, a pair of neighbouring critical points, a minimum and a saddle point, disappears after the "collision" and the link, or links, which used to reach the defunct minimum, will now continue until they reach the lower minimum on the other side of the defunct saddle point. According to this scenario, M12 and S4-12 in Fig. 3 are absent at the corresponding position in Fig. 5. The saddle point that corresponds to S3-12 and is linked in Fig. 3 to M12 is now linked to m18 (that corresponds in Fig. 3 to M4, i.e. the minimum on the other side of S4-12). In the same way, M22 and S7-22 from Fig. 3 are also absent.

Then, two neighbouring minima with a saddle point between them can be replaced by a single minimum. For instance, when the aperture is increased sufficiently, the two Cooke triplets m1 and m2 and the saddle point s1-2 between them merge into a single Cooke triplet minimum, and the network defect mentioned above disappears. Similarly, two saddle points with a minimum between them can be replaced by a single saddle point, a mechanism that causes several network defects in Fig. 5. There, the saddle point s19-18 replaces S7-20, M20 and S4-20 from Fig. 3 and has a shape similar to that of the absent local minimum M20. Also, s6-3 replaces S6-13, M6 and S6-17, while s5-7 replaces S3-19, M3 and S3-8. In all such cases, the replacement saddle point has as links the two external links from the fundamental network of the two defunct saddle points it replaces (i.e. the links that do *not* lead to the disappearing minimum).

In the set of runs shown in Fig. 5, most links in the network remain the same when specifications are changed within the range given above. However, we also observe eight unstable links (dashed in Fig. 5 and in Fig. 1 of Ref. 8) that may change in this range. Then, on one side of a saddle point denoted $s_{i-(jvk)}$ the downward path may lead either to minimum m_j or to m_k. ("v" denotes the Logical OR.) In two such cases, the explanation of this instability is simple. The downward path on one side from s19-(10v15) does not always lead to m15, as it should according to the fundamental network. When specifications vary, the systems m15 and s10-15 can come close to each other and then the merit function barrier between them becomes small. On the other hand, s19-(10v15) has a very large merit function. In this case, the local optimization algorithm, starting very high, can jump over this small barrier and can land in the lower m10, on the other side of s10-15. The same applies to s18-(11v17), when the expected minimum m17 and s11-17 come close to each other, and the resulting minimum can be the lower m11.

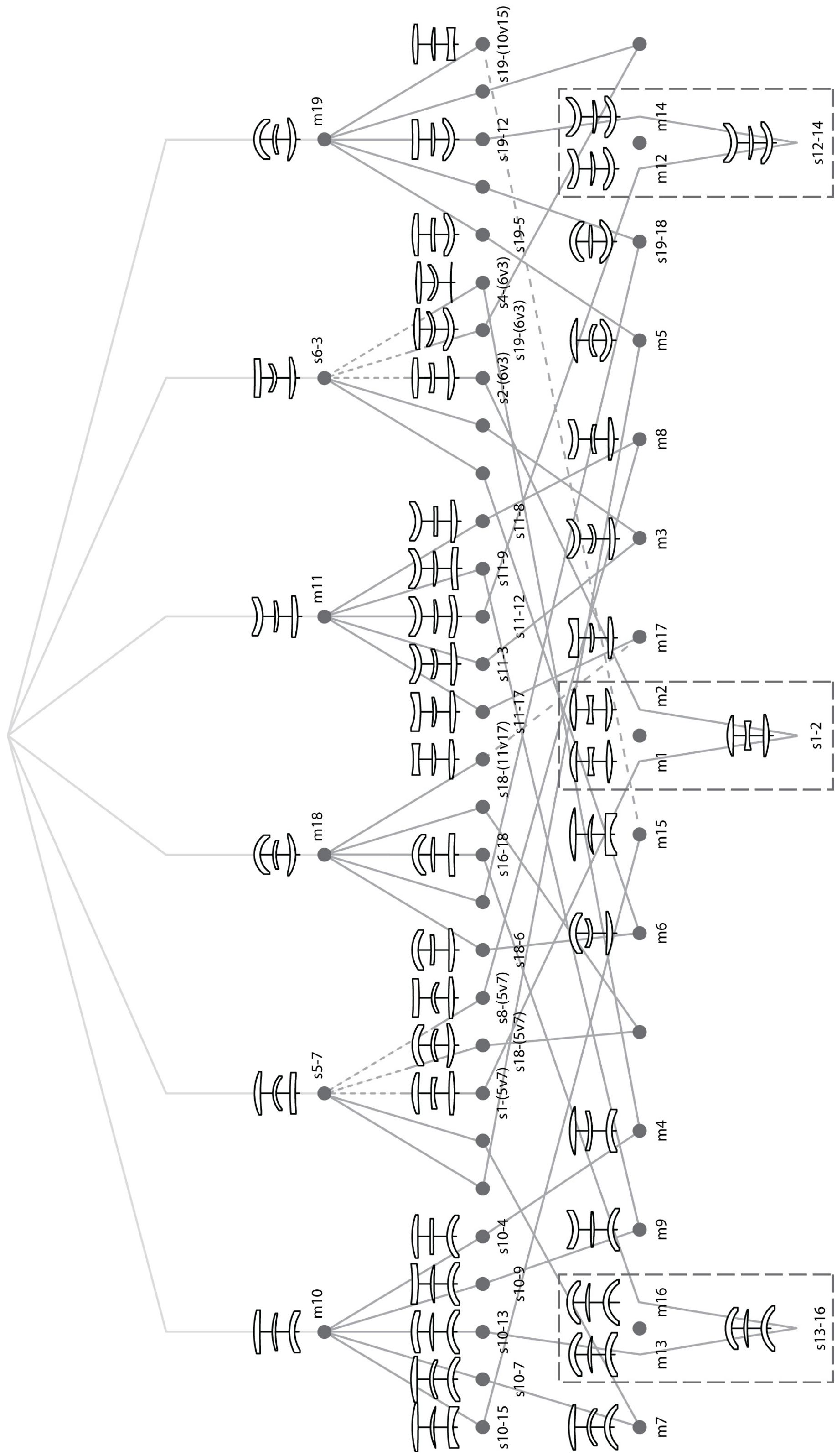

Fig. 5: Network for Cooke triplet runs. One saddle point, s4-(6v3), was found only when the thickness of one of the lenses was smaller than the fixed one that was used in this set of runs.

The other six unstable links in Fig. 5 are caused by the replacement of the second-line hubs M3 and M6 from Fig. 3 by the saddle points s5-7 and s6-3, as mentioned above. The three saddle points s1-(5v7), s8-(5v7), and s18-(5v7) having unstable links in Fig. 5 correspond in Fig. 3 to S3-15, S3-12 and S3-18, respectively. In Fig. 3, they lead on one side of their saddle to M3. In Fig. 5, the three corresponding optimization paths encounter a saddle point (s5-7) at (or close to) the old position of the now defunct hub M3. These paths are shown in Fig. 5 by three dashed saddle-saddle links to s5-7. For some specifications any of these three optimization paths continues downwards on one side of s5-7 and reaches m5, for other specifications it reaches m7 on the other side of s5-7. In the same way, the replacement of M6 by s6-3 produces three unstable links on one side of all three surviving saddle points s2-(6v3), s4-(6v3), and s19-(6v3) that were connected in the fundamental network to the now defunct hub M6. It can be observed that for these six saddle-saddle links, the instability has a fundamental nature that cannot be reduced to e.g. imperfections of the software used or to other accidental causes. Based on accidental causes, the downward path from the higher saddle point has to "choose" on which side of the lower saddle point it has to continue in order to reach a minimum, but a minor change in the specifications is sufficient to change the "choice".

CONCLUSIONS

The concepts of saddle point and of network of minima linked via saddle points make it possible, at least for doublets and triplets of the type considered here, to reveal a previously unsuspected order in the design landscape and to make predictions about what sort of local minima can be expected in new global optimization runs with changed specifications. The focus was here on the analysis of the triplet design landscape, because the triplet case is sufficiently complex to show that a non-trivial pattern is present, but sufficiently simple for reliable numerical analysis with our present tools.

The various local minima that can be found have been classified in system shapes, in a way that is, we believe, consistent with the intuitive image of many designers about the design landscape. The results of all numerical experiments we have performed up to now (as well as a theoretical model that will be discussed in a later publication) indicate that the set of all possible design shapes is organized in a perfectly ordered way in a so-called fundamental network that can be used as reference for the analysis of any actual global optimization run. The total number of local minima actually found can vary from run to run when the specifications are changed. Typically, not all system shapes in the fundamental network are observed, but several local minima can belong to the same system shape. The concept of fundamental network also makes is possible to explain differences in real networks with different specifications, e.g. the presence of unstable links.

The emphasis in this paper was on the analysis of numerical experiments revealing the newly discovered ordered structure, but these results do have practical importance as well. For instance, if we want to design a Cooke triplet, but choose a poor starting point and after local optimization the solution is unsatisfactory, then by using a combination of SPC and SPC in reverse order it is possible to navigate through the network and change the design shape in a systematic way, until the Cooke triplet shape is reached. Then, subsequent refinement with traditional tools leads to a design that meets practical requirements. The same approach was shown to lead to new system shapes in practical work on highly complex systems as well.[13-14] Such approaches can be better analyzed for simple systems, in order to improve their efficiency for designing systems of arbitrary complexity.

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