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DOI
10.1109/MTITS.2017.8005598

Publication date
2017

Document Version
Final published version

Published in
5th IEEE International Conference on Models and Technologies for Intelligent Transportation Systems, MT-ITS 2017 - Proceedings

Citation (APA)
Drabicki, A., Kucharski, R., Cats, O., & Fonzone, A. (2017). Simulating the effects of real-time crowding information in public transport networks. In *5th IEEE International Conference on Models and Technologies for Intelligent Transportation Systems, MT-ITS 2017 - Proceedings* (pp. 675-680). Article 8005598 IEEE. https://doi.org/10.1109/MTITS.2017.8005598

Simulating the effects of real-time crowding information in public transport networks

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***Abstract*—The objective of this paper is to understand the consequences of providing real-time information on crowding levels (RTI-CL) in public transport networks. We propose to extend the mesoscopic, simulation-based assignment model with the passengers' knowledge of instantaneous crowding levels on-board the public transport vehicles. We illustrate the results on a sample transit network, where we investigate the arising changes in network performance and journey experience as a result of RTI-CL provision. We demonstrate that effects of providing the crowding information in real-time on en-route path choices is strongly related to number of assumptions that we address in the paper. The resulting changes are dependent on: network congestion level (the effects are mostly pronounced in moderately congested networks), passengers' behaviour (increased sensitivity to crowding induces higher variability in vehicle loads), the RTI-CL penetration rate (network performance becomes worse with ubiquitous access to information) and information provision type (smoothed information over recent vehicle runs leads to higher accuracy than instantaneous information based on the single latest run only). Based on these, we formulate conclusions for further studies, larger-scale applications and practical implementation of real-time passenger crowding information systems.**

Keywords—public transport; transit assignment; passenger congestion; real-time information; crowding information

I. INTRODUCTION

Exploiting the possibilities of providing real-time information on current travel conditions to passengers has gained momentum in public transport research. This has been facilitated by rapid advancements in modern-day, ATIS-driven public transport systems, which often incorporate a wide range of technological solutions used to collect the data on the on-going performance of public transport system components. Aside from monitoring and maintenance purposes, the same ITS-fed data can be simultaneously handled to provide real-time updates on travel conditions to passengers. For instance, data collected from the AVL can be provided to public transport users in form of real-time information on travel times (RTI) – i.e. as instantaneous information about current departure times, estimated waiting and riding times etc. RTI is nowadays broadcast to passengers and available through a wide range of journey planning devices (electronic display signs, on-line apps etc.).

In the longer run, access to full travel information in real-time can potentially improve public transport journey experience and perception of service quality. Crucially, RTI availability is likely to induce changes in passengers' travelling strategies, as these would allow more adaptive behaviour based on different decision-making objectives [1]. Simulation studies [2], [3] have shown that access to RTI can bring some improvements in journey times – and additionally more profound impact on path choices (and resulting line loads). These distinct decision-making patterns need to be included in state-of-the-art assignment algorithms, to reproduce the actual passenger behaviour in the realm of "ATIS-equipped" public transport systems.

Another interesting yet much less explored possibility is to utilize the data collected from passenger counting systems (such as increasingly popular APC systems, ticket-gate and smart-card systems) and provide it as real-time information on current crowding levels (RTI-CL) in public transport network. The inclusion of RTI-CL in passenger route choice and assignment models remains a relatively much less exploited notion, both in theoretical and applied studies. Nuzzolo et al. [4] designed an assignment algorithm which accounts for real-time crowding prediction distributed to passengers, evaluated in a day-to-day simulation procedure. They conclude that in the longer run this leads to minor changes in route choices or overall journey times, but significantly higher shifts in temporal choices (i.e. departure time choices). As such, the benefits of crowding prediction are increased in higher congestion scenarios, with noticeable reductions in fail-to-board incidents. Other empirical studies revealed relevant (though rather limited) impact on distribution of pass. loads inside individual carriages of underground trains [5]. Furthermore, stated preference studies carried out by [6] revealed passengers' willingness to wait for later, less-crowded

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train services – which is strongly influenced by trip purpose and propensity to arrive on-time.

The objective of this work is the development of the real-time transit assignment algorithm which accounts for passengers' access to real-time crowding information. This work contributes to the developing stream of research works on real-time crowding information (RTI-CL) systems for public transport users. We focus on the implications of instantaneous (actual) crowding information, which somehow complements work of [4] (which addressed principally the crowding prediction mechanisms). We apply a simulation-based assignment model (BusMezzo) and incorporate the RTI-CL in the generic path utility algorithm. We propose a simplified, "passenger-friendly" step-wise crowding information and two methods to derive this information from latest reported departures. We illustrate the model on a simple transit network, where we aim to observe the shifts in en-route passenger path choices. We show how the results change in various scenarios, namely: variable network congestion level, choice sensitivity parameter, and RTI-CL penetration rate. Finally, we aim to analyse the resultant information accuracy and how it correlates with the above mentioned demand conditions.

II. Model

A. Simulation model - BusMezzo

The proposed RTI-CL algorithm is implemented as an extension of the simulation-based, mesoscopic BusMezzo assignment model [7]. The BusMezzo uses a disaggregate representation of public transport demand and supply, i.e. individual users ("agents") and individual vehicles (with explicitly modelled physical capacity) – and models interactions between them by means of e.g. flow-dependent dwell times. Importantly, it is an event-based model, which implies that simulation process is triggered whenever an agent – passenger or vehicle – takes an action.

The dynamic passenger path choice model is designed as a sequential and event-based multinomial logit model, evaluated at each travel decision point (i.e. boarding, alighting and connection decisions). The action probability P_c is based on the action utility v_c, associated with choosing a given path from the current decision point to destination. The action probability is expressed with following equation:

$$P_c = \exp(\mu \cdot v_c) \Big/ \sum_{c' \in C} \exp(\mu \cdot v_{c'}) \qquad (1)$$

where μ is the sensitivity of choice model. High μ values yield deterministic choice process, while low values will result in probabilistic behaviour. The action utility is a logsum of the utility v_i of considered paths, expressed with following:

$$v_c = \ln \sum_{i \in A^c} e^{v_i} \qquad (2)$$

The path utility, in turn, is expressed as the sum of weighted trip cost components for each individual journey stage s. The utility usually consists of number of trip elements, which for sake of brevity we limit here to: in-vehicle times IVT_S, waiting times WT_S walk times WKT_S, and number of transfers NTR, along with a zero-mean random error term ε_i (3).

$$v_i = \sum_{s \in i} \beta_s^{IVT} \cdot IVT_s + \sum_{s \in i} \beta_s^{WT} \cdot WT_s + \sum_{s \in i} \beta_s^{WKT} \cdot WKT_s + \sum_{s \in i} \beta_s^{NTR} \cdot NTR_s + \varepsilon_i \qquad (3)$$

Crucially, the path choice results from sequentially updated decision-making process, being evaluated en-route. Whenever a choice process is triggered – that is, when passengers take decisions – the utilities computed with (eq. 3) are recalculated, and their corresponding choice probabilities are evaluated with (eq. 1) again. This implies that resultant O-D path is an aggregate output of the individual decisions taken en-route, and the final path is chosen not only based on expected pre-trip values, but is also influenced by their real-time en-route updates.

B. Modelling real-time crowding information in BusMezzo

We extend the BusMezzo model and propose following changes to account for passengers' access to real-time crowding information. We introduce a real-time crowding penalty, evaluated for each individual trip stage s. This penalty represents the disutility associated with the currently anticipated on-board crowding conditions when a passenger considers a given O-D path. We include it in the algorithm by means of the in-vehicle time multiplier in the utility formula, so that instead of (eq. 3) the utility is evaluated with:

$$v_i = \sum_{s \in i} \beta_{s,\tau}^{CL}(\tau) \cdot IVT_s + \sum_{s \in i} \beta_s^{WT} \cdot WT_s + \sum_{s \in i} \beta_s^{WKT} \cdot WKT_s + \sum_{s \in i} \beta_s^{NTR} \cdot NTR_s + \varepsilon_i \qquad (4)$$

where in-vehicle time is multiplied by the respective real-time crowding penalty factor $\beta_{s,\tau}^{CL}(\tau)$.

In-vehicle congestion level	Vehicle load factor $Vol_s(\tau)/Cap_s$	Real-time crowding factor $\beta_s^{CL}(\tau)$
●○○○	< 0.30	**0.90**
●●○○	0.30 – 0.60	**1.10**
●●●○	0.60 – 0.80	**1.70**
●●●●	> 0.80	**2.20**

Fig. 1. The proposed RTI-CL scale and corresponding crowding penalties.

The open issue that we address in the paper is that the crowding levels on downstream trip segments are unknown, since these refer to future system states. To this end, we distinguish between the actual in-vehicle crowding rate, denoted $\hat{\beta}^{CL}$ - which is reported for each departed vehicle in real-time based on the data reported by the APC system, and the anticipated crowding penalty, denoted β^{CL} - which is calculated from the system-fed data and is then provided to passengers. We assume that the crowding information is communicated to passengers by means of a simplified, user-friendly crowding scale (fig. 1). For the purposes of this study, we propose a 1-to-4 "crowding" scale, with crowding penalty $\beta_{s,\tau}^{CL}(\tau)$ values of 0.9, 1.1, 1.7 and 2.2, being dependent on the vehicle load-to-capacity ratio. This should reflect the non-linear character of crowding disutility which becomes more acute for the "upper-crowding" conditions, when all seats are taken or the density of standees exceeds a relative comfort threshold (which is in line with common research findings [8,9]).

We assume that the $\beta_{s,\tau}^{CL}(\tau)$ in (eq. 4) is obtained for the most recently departed vehicle runs r along respective segments s. We discuss the two following cases. In the first case (instantaneous), we use crowding factor of the latest vehicle run only, formally expressed with:

$$\beta_{s,\tau}^{CL}(\tau)=\hat{\beta}_{r',s}^{CL} \quad (5)$$

, where r' denotes the most recent vehicle run recorded along the line segment s. Alternatively, we propose to smooth the above by using crowding factor of several recent runs. We obtained the best sensitivity to both long-term trends and non-systematic variations by using the exponential smoothing formula with the smoothing parameter α equal to 0.50, which assigns higher relevance to the most recent crowding rates:

$$\beta_{s,\tau}^{CL}(\tau)=\alpha\cdot\hat{\beta}_{r',s}^{CL}+(1-\alpha)\cdot\beta_{r'-1,s}^{CL} \quad (6)$$

III. Results

To illustrate the concept, the proposed modelling framework has been applied to an extended version of the classical Spiess-Florian transit network [2, 10]. The network has been modified to better demonstrate the resultant phenomena (fig. 2). Passengers travel from two origin points (*O1, O2*) to the single destination point (*D*). Transit system is composed of 5 lines, each with their distinct run times and constant dispatching service frequencies from the origin stops. Transit vehicles are modelled with explicit capacity limits of max. 100 pax. (per vehicle), which is additionally increased to 200 pax. for the L3 line. It should be noted here that we introduce a number of modifications in comparison to the classical Spiess-Florian network topology: the L4 line is extended backwards to stop B; a second origin is added at the stop B; the L5 line is introduced on a parallel route from stop E to G, with direct transfer links between stops A-E and D-G and a 3-minute transfer link between the intermediate stops B-F.

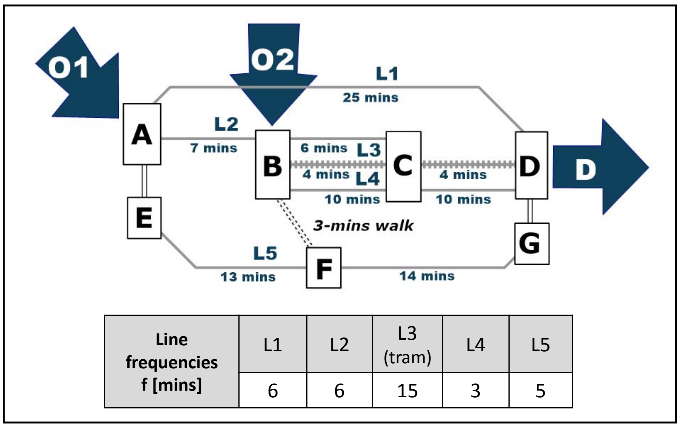

Fig. 2. Schematic layout of the sample transit network (modified Spiess-Florian) used in simulations.

We simulate one hour of passenger demand inflow from 2 origin points, with the additional 30-minute warm-up and 30-minute cool-down periods on the supply side. A linear dwell-time function is assumed with a 2-second increase rate per each boarding or alighting passenger.

To assess the implications of the RTI-CL provision, we perform the simulations with specific focus on sensitivity to the following issues:

1. *Network crowding levels* (2 cases): the "MID cong." - where network is moderately saturated, and the "HIGH cong." - where network is oversaturated,

2. *RTI-CL evaluation algorithm* (2 cases): instantaneous – based on the single latest run (eq. 5), and smoothed – averaged for several recent runs (eq. 6),

3. *Passengers' behaviour* (2 cases): "probabilistic" when the sensitivity parameter μ in the path utility formula is low, and "quasi-deterministic" when sensitivity parameter μ is high,

4. *Demand penetration rate* (4 cases): 0% (reference scenario, i.e. no RTI-CL available), and penetration rates of 25%, 50% and 100% - i.e. share of passengers who utilise the RTI-CL.

The combination of all possible cases yields 28 scenarios. To attain statistically significant results from the stochastic simulation model, we run five simulations of each scenario and report averaged results. This number of simulation seems to be satisfactory in case of our relatively small-scale network, as the observed variation between simulations was sufficiently low. In the following subsections we report the simulation results and discuss their main implications.

A. Travel utility (welfare) changes

The passengers' objective is to maximize the overall travel utility (or the so-called welfare), expressed as a sum of weighted time trip components given by (eq. 4). If they become aware about actual crowding conditions (through RTI-CL system), they can revise the choices and, in turn, improve utility. Consequently, we expect the total welfare to increase due to RTI-CL. The highest benefits were observed in the "MID cong." scenarios with penetration rates of 25% and 50%, where access to RTI-CL leads up to a 4% increase in average welfare rate. On the other hand, in the 100% penetration rate scenarios the welfare was actually slightly lower as compared to the no RTI-CL scenario.

In general, the system becomes complex and multidimensional as passengers aim to minimize both travel times and crowding discomfort. The resulting welfare changes are strictly related to the real-time crowding factors as well as the choice sensitivity parameter μ. Thus, the output welfare rates do not vary much between individual scenarios, as for instance, benefits from reduced in-vehicle travel times (weighted by crowding factors $\beta_{s,\tau}^{CL}(\tau)$) are likely to be outweighed by a corresponding increase in (weighted) waiting times.

B. Flow shifts

Interestingly, provision of crowding information results with more noticeable shifts in segment volume loads in our sample network. The shifts were most evident in the "MID cong." scenarios with deterministic behaviour. Reductions in passenger volumes occur across the central part of the network (lines L1 to L4), with passengers migrating towards the hitherto less-popular line L5 – whose patronage rate increases substantially. (Fig. 3) depicts the relation between initial passenger flows without the RTI-CL (black) and additional flow shifts as a result of providing real-time crowding information (red, green).

Analogous flow shift patterns could be observed for the overcrowded network state (the "HIGH cong." scenario). However, these changes are relatively smaller since the network is already oversaturated, and all the line segments experience substantial congestion.

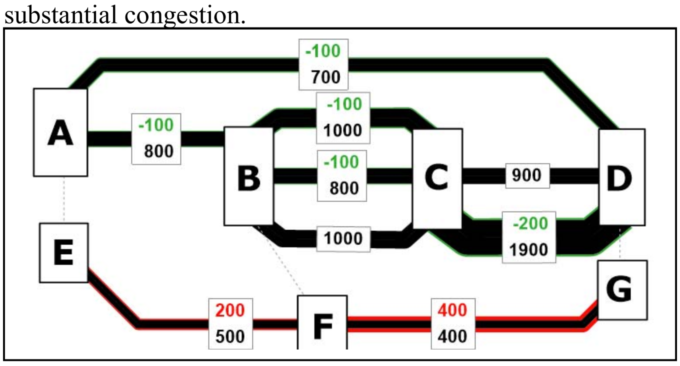

Fig. 3. Aggregate passenger flows along the line segments without RTI-CL (black) and arising flow shifts due to RTI-CL introduction (increase – red, decrease – green). Sample results for the "MID cong." scenario with 100% penetration rate.

C. Sensitivity in passengers' choice behavior

The expected impact of RTI-CL access depends on the sensitivity rate of passengers' path choice model. As expected, the probabilistic scenario yields smaller changes than the deterministic one, since in the latter case passengers are more sensitive to utility changes.

For the "MID cong." scenario the relative changes in individual segment loads (expressed as a percentage rate of the "no-RTI-CL" volume rate) range between 90 – 120% in the probabilistic scenario, whereas in the deterministic scenario these fluctuations are more significant, ranging between 75% to 180%. A very similar trend can be traced for the "HIGH cong." scenario, with the highest shifts attributable on one hand to lines L1 and L4 which are extremely overcrowded - and on the other hand to line L5 which already experiences substantial passenger congestion, but individual vehicle runs indicate enough spare capacity for it to become attractive in case of a deterministic utility choice model.

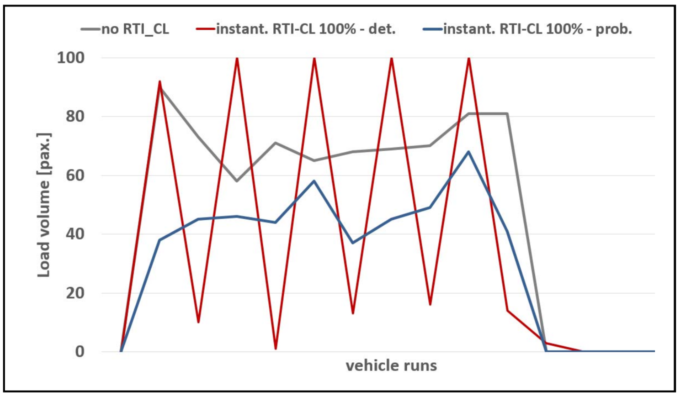

Fig. 4. Passenger loads on consecutive runs of line L1 without RTI-CL - compared to RTI-CL probabilistic and deterministic behavior ("MID cong." scenario).

Interestingly, more significant differences are exposed once we consider not just aggregate segment loads, but individual vehicle runs. Assuming that all the travellers respond to the RTI-CL (100% penetration rate), the deterministic scenario reveals high variability in passenger volumes over consecutive vehicle runs a "bouncing" pattern is produced, with alternating empty vehicles followed by overcrowded vehicles. The fluctuations are reduced in the probabilistic scenario, as individual vehicle loads follow a more uniform and consistent distribution pattern (fig. 4).

D. RTI-CL penetration rate

Furthermore, passenger shifts are strongly affected by the assumed RTI-CL penetration rates. A lower RTI-CL penetration rate leads to a substantial suppression of the aforementioned "bouncing" pattern of individual vehicle loads – which is a characteristic phenomenon in case of deterministic sensitivity scenarios with ubiquitous RTI-CL response rate. A closer investigation of both "MID cong." and "HIGH cong." scenarios reveals that network penetration rates of 50% or 25% yield more even distribution among consecutive departures for most line segments – though a certain share of vehicle runs might still remain overcrowded (fig. 5).

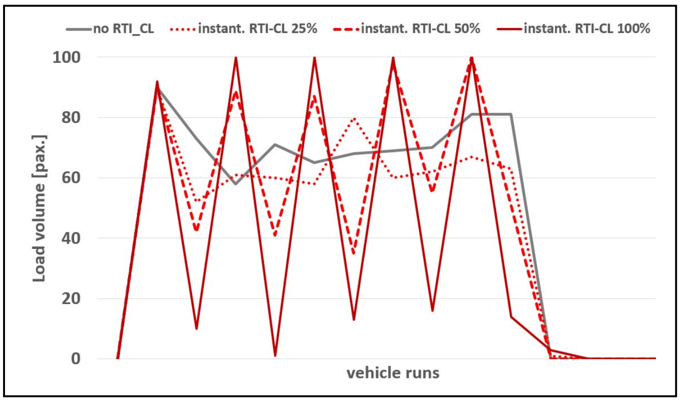

Fig. 5. Passenger loads on consecutive runs of line L1 without RTI-CL – compared to various RTI-CL penetration rates ("MID cong." deterministic scenario).

This phenomenon can be further traced by looking at the network-wide distribution of vehicle load factors (fig. 6). For the "MID cong." scenario, the share of moderately loaded vehicles (40%–80%) goes up in the 25% and 50% RTI-CL penetration scenarios, but then falls down again once 100% of passengers respond to the RTI-CL. This does not happen in the "HIGH cong." scenario though, which could be possibly attributed to very little or no spare network capacity left. We note that in both scenarios the vast majority of passengers (as opposed to vehicle runs) experience line segments with high crowding levels.

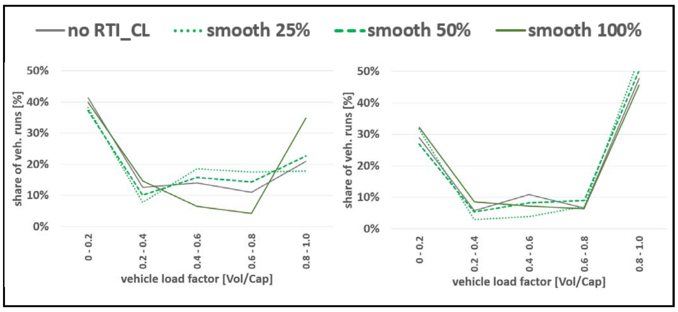

Fig. 6. Vehicle loads' distribution – in the "MID cong." (left) and the "HIGH cong." (right) deterministic scenarios.

E. *RTI-CL information accuracy*

Finally, we investigate the accuracy of the provided RTI-CL, which is a crucial measure for the system to be implemented and trusted by passengers. We propose to measure accuracy as the compliance rate between the provided RTI-CL (anticipated) and the actual RTI-CL (experienced) on-board the individual journey segments, evaluated as an aggregate sum (share) of all the passengers' travel decision instances. The accuracy measure also accounts for the share of demand flows which doesn't observe the RTI-CL in the path choice algorithm (i.e. the accuracy is measured for 100% of passengers regardless of the information penetration rate). We focus here specifically on two important indicators: share of accurate *RTI-CL* (i.e. no difference between expected and actual crowding rates), and share of strongly inaccurate *RTI-CL* (i.e. a difference of two levels or more on our 1-to-4 "crowding scale" - which implies a significant under- or overestimation) (fig. 7).

The resultant accuracy level seems to be highly dependent on penetration rate, the evaluation algorithm used for generating RTI-CL, as well as the choice sensitivity. In general, the smoothed RTI-CL (eq. 6) yields higher accuracy rates than instantaneous RTI-CL (eq. 5), with the difference becoming even more evident with higher demand sensitivity rates. For the lower-sensitivity probabilistic scenario, the accuracy rate tends to remain stable regardless of the penetration rate – but concurrently, in the deterministic scenario the resultant RTI-CL accuracy becomes much more variable. In the deterministic case, it actually exhibits two opposite correlation patterns with respect to penetration rate: decreasing accuracy rate for instantaneous RTI-CL, and concurrently – increasing accuracy rate for smoothed RTI-CL (fig. 4). Simultaneously, the risk of information inaccuracy becomes especially substantial for higher penetration rates of instantaneous RTI-CL, reaching as much as ca. 40% for the ubiquitous (100%) penetration rate.

Finally, we observe that the information is more accurate for the "HIGH cong." scenario, though it should be noted that it may become now "biased" by a high saturation degree of our small-scale network: the oversaturated network conditions imply that crowding information anyhow reaches the maximum rate for most of the vehicle runs.

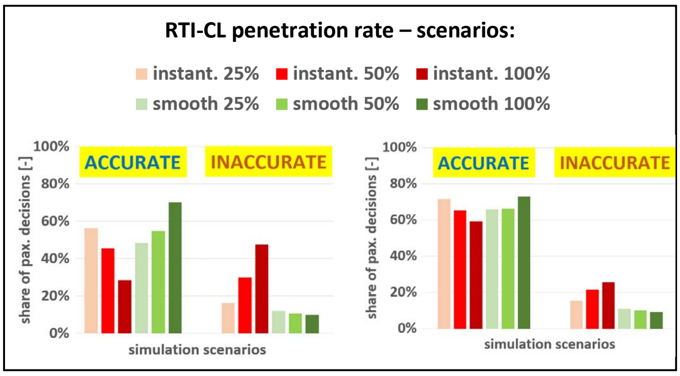

Fig. 7. Results: global accuracy measures of RTI-CL: percentage of pax. decisions with "accurate" and "very inaccurate" RTI-CL – with respect to different penetration rates and RTI-CL evaluation algorithms – for "MID cong." (left) and "HIGH cong." (right) deterministic scenarios.

IV. CONCLUSIONS

The study proposes a novel path choice model whose objective is to account for passengers' access to instantaneous real-time information on crowding levels (RTI-CL) in public transport services. The modelling framework assumes that information on crowding is generated from system-fed updates on the most recently recorded crowding levels of vehicle runs which are currently progressing along the considered downstream O-D paths. We denote this type of information as "downstream-based RTI-CL", and illustrate the model with a sample transit network to demonstrate its effects. Main observations and conclusions are as follows:

1. Demand shifts are likely to be more pronounced in a moderately saturated network. As the most popular line segments become relatively more crowded, the RTI-CL increases the awareness of hitherto unconsidered O-D paths. Consequently, this may potentially lead to more even utilization of the available network O-D connections and increase the patronage rate of less-popular routes. However, in an oversaturated network, these benefits are lower since most available O-D paths would already experience similarly high congestion levels.

2. The impact of RTI-CL is highly dependent on the demand sensitivity assumptions. Lower sensitivity of route-choice yields limited changes in network flows, while an increased sensitivity produces more significant, yet less stable results. This stems from a crucial issue: although an increasingly widespread response to crowding information brings minor improvements in general (aggregate) measures at the first glance, it has significant implications for the arising network dynamics. As shown for the deterministic behaviour scenario, individual vehicle runs' loading rates become highly variable. Such uneven utilization of transit system capacity. can seriously deteriorate service quality and regularity, and potentially undermine the general effectiveness of the RTI-CL system.

3. The benefits of crowding information are strictly related to the penetration rate of RTI-CL, and may actually become lower in case of ubiquitous passengers' response. This is specifically the case for "high-sensitive" demand scenarios: for the 100% access rate, with fluctuations in individual vehicle, which at some stage become alternately underutilised or fully

crowded. A closer investigation of limited penetration rate scenarios (25%, 50%) indicates that a beneficial trade-off could be achieved – i.e. a somewhat improved network utilization rate without any deteriorations in output journey times.

4. The investigation of the RTI-CL accuracy shows that it is in general accurate in a moderate degree, i.e. for majority of passengers the crowding experienced on-board is in line with the provided crowding information. However, the share of inaccurate predictions may become noticeably high in some cases, with a general tendency to underestimate the actual network congestion. The results lead to conclusion that an increased sensitivity coupled with a high penetration rate seriously deteriorates the RTI-CL accuracy. In such cases, the real-time crowding information might no longer be valid, which would consequently seriously undermine the overall RTI-CL trustworthiness.

5. The processing of raw RTI-CL data and the corresponding RTI-CL evaluation algorithm (instantaneous or smoothed) has also relevant impact on network performance. Firstly, they lead to different flow shift patterns. Secondly, an important relation is observed in terms of RTI-CL accuracy in higher sensitivity scenarios: an increasing response rate to instantaneous RTI-CL deteriorates its accuracy rate – whereas an opposite (positive) pattern is observed when information is smoothed over several runs.

This paper shall be thus considered as a proof-of-concept for modelling transit systems real-time crowding information. The main contribution is the proposed transit assignment algorithm which covers real-time crowding information. The application on a sample transit network revealed a number of interesting phenomena, which should be taken into consideration in future implementation cases. However, the above findings shall be treated with care due to the limited-scale application. Changes in journey times were not discussed in more detail, and the welfare shifts might be "biased" owing to the small-scale transit network utilised in our simulation works. Further research with different network topology would be needed to derive more general conclusions.

Future follow-up works would include simulations on real-world networks, evaluation of optimal information provision strategies, empirical evidence (SP and RP data) concerning user preferences, and an iterative feedback between provided information vs. system performance. Additionally, we focused on arising spatial, en-route shifts - the possibility of temporal shifts shall be covered in more detail in the follow-up studies.

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