# MEMORY EFFECTS IN TURBULENT FLOWS

1144 1186

P.J.H. BUILTJES



# MEMORY EFFECTS IN TURBULENT FLOWS

C10026 82605

# PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE TECHNISCHE WETENSCHAPPEN AAN DE TECHNISCHE HOGESCHOOL DELFT, OP GEZAG VAN DE RECTOR MAGNIFICUS PROF. IR. L. HUISMAN, VOOR EEN COMMISSIE AANGEWEZEN DOOR HET COLLEGE VAN DEKANEN, TE VERDEDIGEN OP WOENSDAG 30 MAART 1977 TE 14.00 UUR

# DOOR

PETER JOHANN HENRY BUILTJES

natuurkundig ingenieur

geboren te Rotterdam





Dit proefschrift is goedgekeurd door de promotor PROF. IR. J. O. HINZE

# SUMMARY.

The hypothesis of Boussinesq concerning the relation between the Reynolds' stresses and the local mean-velocity gradients, assumes the turbulent transport to be of the gradient type. It considers the turbulence as a hypothetical Newtonian fluid and the relationship between the turbulence and the meanvelocity is strictly local. The turbulence, however, behaves more as a hypothetical non-Newtonian fluid, the turbulent transport being a combination of gradient-type and convective-type transport. When there is a sudden change either in time or in space in the mean-velocity profile the turbulence can, depending on its relaxation time (for which, for example, the Lagrangian integral time scale may be taken), exhibit a delayed reaction to this new situation. In such cases so-called extra memory effects are present and the above relationship can no longer be described by a strictly local formula.

By an extension of the hypothesis of Boussinesq it is possible to deduce a relaxation equation which takes account of the extra memory effects. It appears that this relaxation equation, which contains the eddy viscosity and a relaxation time, can be derived from the complete transport equation of the turbulence which should, in principle, be able to describe the extra memory effects.

In order to investigate extra memory effects, experiments have been done (a) in a turbulent boundary-layer disturbed by a half sphere, (b) in the region just downstream of a cylinder and (c) in the region just behind a grid. Measurements have been made of the mean velocity  $\overline{U}_1$ , the turbulence intensities  $u_1'$  and  $u_2'$ , the turbulent shear stress  $-u_2u_1$ , the energy spectra of  $\overline{u_1^2}$ , the space-time correlations of  $u_1$ ,  $u_1^2$ ,  $u_2$ ,  $u_2^2$  and  $-u_2u_1$  and of some probability distributions.

In order to be able to apply the relaxation equation to these experiments it is necessary to know the value of the relaxation time and the eddy viscosity. For the relaxation time, the time scale deduced from the envelope of the spacetime correlations of  $-u_2u_1$  can be taken. This time scale turned out to be roughly equal to the Lagrangian longitudinal integral time scale of  $u_1$ , so that the latter may be taken instead. In the case of the turbulent boundary-layer one can use the value in the undisturbed situation for the eddy viscosity, for the wake flow one can use the value of the eddy viscosity in the self-preserving part of the wake.

From the investigations it is concluded that in the disturbed boundarylayer and in the region just downstream of the cylinder there is an extra memory effect on  $-u_2u_1$ . Even in the self-preserving part of the wake there proved to be an extra memory effect. The values of  $-u_2u_1$  calculated with the relaxation equation agree reasonably well with the measured values, the values of  $-u_2u_1$  according to the hypothesis of Boussinesq being quite different from the measured values. In the region just behind the grid the extra memory effect on  $-u_2u_1$  proved to be small. However, there is a distinct extra memory effect on  $u_1^2 - u_2^2$ , and consequently on the maximum shear stress in planes making an angle of 45 degrees with the main flow direction, which can in principle also be described by a relaxation equation.

#### SAMENVATTING.

De hypothese van Boussinesq die de relatie tussen de Reynoldse spanningen en de lokale gemiddelde snelheidsgradienten beschrijft, beschouwt het turbulente transport als een gradienttype transport. Deze hypothese beschouwt de turbulentie als een hypothetische newtonse vloeistof en de relatie tussen de turbulentie en de gemiddelde snelheid als zuiver lokaal. De turbulentie echter gedraagt zich veel meer als een hypothetische niet-newtonse vloeistof; het turbulente transport is een combinatie van gradienttype en convectief transport. Wanneer er een plotselinge verandering van het gemiddelde snelheidsprofiel is, in de tijd of in de ruimte, kan de turbulentie, afhankelijk van zijn relaxatietijd (waarvoor bijvoorbeeld de Lagrange integrale tijdschaal genomen kan worden), met een zekere tijdvertraging reageren op deze nieuwe situatie. In zulke gevallen treden zogenaamde extra geheugeneffecten op en de bovengenoemde relatie kan niet langer worden beschreven door een zuiver lokale formule.

Het is mogelijk om via een uitbreiding van de hypothese van Boussinesq een relaxatievergelijking af te leiden die rekening houdt met de extra geheugeneffecten. Het blijkt dat deze relaxatievergelijking die de turbulentie viscositeit en een relaxatietijd bevat, kan worden afgeleid uit de volledige transportvergelijking voor de turbulentie, welke vergelijking in principe in staat is extra geheugeneffecten te beschrijven.

Met het oog op het onderzoek aan extra geheugeneffecten zijn de volgende experimenten uitgevoerd: (a) in een turbulente grenslaag verstoord door een halve bol, (b) in het gebied dicht achter een cylinder en (c) in het gebied dicht achter een rooster. Er zijn metingen verricht van de gemiddelde snelheid  $\overline{U}_1$ , de turbulentie intensiteiten u<sub>1</sub>' en u<sub>2</sub>', de turbulente schuifspanning  $-\overline{u_2u_1}$ , de energiespectra van  $\overline{u_1^2}$ , de ruimte-tijd correlaties van u<sub>1</sub>, u<sub>1</sub><sup>2</sup>, u<sub>2</sub>,  $u_2^2$  en  $-u_2u_1$  en van enkele waarschijnlijkheidsverdelingen.

Om in staat te zijn de relaxatievergelijking op deze experimenten toe te passen is het noodzakelijk de waarde van de relaxatietijd en van de turbulentie viscositeit te kennen. Voor de relaxatietijd kan de tijdschaal afgeleid uit de omhullende van de ruimte-tijd correlaties van -u<sub>2</sub>u<sub>1</sub> worden genomen. Deze tijdschaal bleek ongeveer gelijk te zijn aan de Lagrange longitudinale integrale tijdschaal van u<sub>1</sub>, zodat deze laatste tijdschaal ook gebruikt kan worden in plaats van de tijdschaal van de ruimte-tijd correlatie. In het geval van de turbulente grenslaag kan men voor de turbulentie viscositeit de waarde van de ongestoorde situatie gebruiken, voor de zogstroming kan men de waarde van de turbulentie viscositeit in het gelijkvormigheidsgebied gebruiken.

Uit de onderzoekingen kan men concluderen dat er in de gestoorde grens-

laag en in het gebied dicht achter de cylinder een extra geheugeneffect van  $-\overline{u_2u_1}$  optreedt. Zelfs in het gelijkvormigheidsgebied van het zog bleek er een extra geheugeneffect te zijn. De waarden van  $-\overline{u_2u_1}$ , die berekend worden via de relaxatievergelijking sluiten redelijk aan bij de gemeten waarden, de waarden van  $-\overline{u_2u_1}$  volgens de hypothese van Boussinesq verschillen aanzienlijk van de gemeten waarden. In het gebied dicht achter het rooster bleek het extra geheugeneffect van  $-\overline{u_2u_1}$  klein te zijn. Er is echter een duidelijk extra geheugeneffect van  $\overline{u_1^2} - \overline{u_2^2}$ , en dientengevolge van de maximale schuifspanning in vlakken die een hoek van  $45^{\circ}$  maken met de hoofdstroomrichting. Dit extra geheugenheugeneffect kan in principe ook beschreven worden door een relaxatievergelijking.

# TABLE OF CONTENTS.

I	INTRODUCTION	page	2
	A. Memory effects B. Turbulence as a hypothetical non-Newtonian fluid C. Earlier experiments on disturbed velocity fields		2 3 5
II	THEORETICAL BACKGROUND OF A RELAXATION EQUATION WITH THE HYPOTHESIS OF BOUSSINESQ AS A STARTING POINT		9
	A. The hypothesis of Boussinesq B. The limitations of the hypothesis of Boussinesq C. A modification of the hypothesis of Boussinesq		9 11 12
III	THE CONNECTION BETWEEN THE RELAXATION EQUATION AND THE COMPLETE TRANSPORT EQUATION		17
	<ul><li>A. The turbulence models</li><li>B. The relation between the relaxation equation and the complete transport equation</li></ul>		17 18
 IV	THE RELAXATION EQUATION		24
	A. Introduction B. The memory time C. Extra memory effects in a self-preserving flow		24 24 25
	D. The connection between the relaxation equation and some other theories		27
V	THE EXPERIMENTAL EQUIPMENT AND MEASURING METHODS		34
VI	THE TURBULENT BOUNDARY-LAYER		38
	A. The undisturbed boundary-layer B. The disturbed boundary-layer		38 45
VII	THE WAKE OF A CIRCULAR CYLINDER		53
	A. Introduction B. The wake far behind the cylinder C. The region close to the cylinder	• 1 2	53 53 55
VIII	THE TURBULENT FLOW FIELD BEHIND A GRID		61
	A. A theoretical remark about the turbulence stresses B. The region far behind the grid C. The region close to the grid		61 62 63
IX	RECAPITULATION AND CONCLUSION		72
	ACKNOWLEDGEMENT		74
	NOMENCLATURE		75
	REFERENCES		78
	FIGURES		

### I. INTRODUCTION

### A. Memory effects

Memory effects will occur whenever an event that has happened in the past has a noticeable influence on the situation at a later moment. Stated in this way it is clear that in a turbulent velocity field memory effects exist, which show up for example in the existence of the so-called Lagrangian auto-correlation. A Lagrangian time correlation exists when there is a connection between the structure of the turbulence in the past and at a later moment.

This is one kind of a memory effect; it is inherent to the turbulence itself and present even if the turbulence is stationary and homogeneous. From this memory effect of the turbulence one can deduce a time scale of the turbulence, a kind of inner time scale that is solely determined by the structure of the turbulence.

One can also look at another memory effect, namely the reaction of the turbulence to a change in "external" conditions, that is to say to a change in the mean-velocity field. This change can be a change with respect to time as well as with respect to position. In the case of a change with respect to time (non-stationarity) there will be a memory effect if the time scale of the change in the mean-velocity field becomes of the same order as the inner time scale of the turbulence. When there is a change in conditions with respect to position (non-homogeneity) there will be a memory effect if the Lagrangian time scale of the change in the mean-velocity field becomes of the same order as the inner time scale of the turbulence. In both cases the local turbulence can not be expected to be determined by the local meanvelocity field.

We will consider only changes with respect to position in this investigation. The memory effects caused by these changes can occur in the longitudinal direction, that is the direction of the mean velocity, as well as in the transverse direction. In the following we will consider only the effects in the longitudinal direction. In order to distinguish these memory effects from the previously mentioned memory effects inherent to the turbulence itself, we shall call these effects extra memory effects.

Of course it is also possible that the mean-velocity field itself shows a memory effect. That is to say that it will take a certain time before the mean velocity has adjusted itself to a change in its "external" constraints, for example a sudden change in the position of surrounding fixed walls. To which extent the mean velocity will show such a memory effect depends in the case of laminar flow on the molecular structure of the fluid. In the theory of rheology this kind of memory effect is common. In a turbulent Newtonian flow the extra memory effect depends on the turbulence through interaction between the turbulence and the mean flow.

Turbulence is a characteristic of flows. In the above view we have more or less talked about turbulence as if turbulence were a special kind of fluid which can be more or less distinguished from the "mean-velocity" fluid Although this is in principle not correct it has been shown that in a lot of situations it is possible to talk about a turbulent fluid instead of a turbulent flow (<u>1</u>). If we suppose that the turbulence, or more correctly the turbulent fluid, behaves as a Newtonian fluid whose effective viscosity at a certain instant is determined only by the velocity field at that instant, it is impossible to describe the extra memory effects we stated above. If turbulence indeed behaves as if it had extra memory effects it is thus necessary to look at turbulence as a hypothetical non-Newtonian fluid instead of a hypothetical Newtonian fluid.

# B. Turbulence as a hypothetical non-Newtonian fluid.

Rivlin  $(\underline{2})$  was one of the first who paid attention to the fact that there is an analogy between the turbulent mean flow of a Newtonian fluid and the laminar flow of a non-Newtonian fluid. He, and also Liepmann  $(\underline{3})$ , suggested that the secondary currents which occur when a Newtonian fluid flows turbulently through a duct of non-circular cross-section could be explained by supposing that the turbulence behaves as a non-Newtonian fluid. Namely the laminar flow of a non-Newtonian fluid through a non-circular duct shows secondary currents, in contrast with the laminar flow of a Newtonian fluid.

A non-Newtonian fluid has a constitutive relation in which the stress component at a point in the flow can be expressed as a polynomial in the function of gradient of the mean velocity, the acceleration, etc. This is equivalent under rather general conditions to a constitutive relation where the stress component at a point in the flow at a certain instant is expressed in terms of the velocity gradient at that instant and, in a Lagrangian description, at all previous instants (2). This gives the possibility of including memory effects.

Especially during the last years several workers have paid attention to the non-Newtonian behaviour of the turbulence. We mention the work of Crow  $(\underline{4})$ , Townsend  $(\underline{5})$ , Lumley  $(\underline{6})$ , Proudman  $(\underline{7})$  and Dowden  $(\underline{8}, \underline{9})$ . Crow  $(\underline{4})$ 

-3-

. considers the smaller-scale motions of the turbulence as being ruled by a visco-elastic behaviour. Townsend (5) suggested, by studying the turbulent/ non-turbulent interface, that a visco-elastic behaviour could explain several observed phenomena. The work of Lumley (6) is of an other character. He tries to give the conditions for the existence of a turbulent visco-elastic constitutive relation and considers with these ideas the process of reaching an equilibrium in a turbulent flow. Proudman (7) investigates the possibility of using a special constitutive relation to describe the turbulence. He finds that the most suited relation is the constitutive relation of a thirdorder v-fluid. A third-order v-fluid is a non-Newtonian fluid whose properties are determined by only one dimensional constant v (with the dimension of a kinematic viscosity), and with time derivatives of the stress up to the third order. Dowden (8, 9) enlarges this idea and applies this constitutive relation, among other things, to the decay of turbulence. From these investigations it is clear that several properties of the turbulence can be well described if the turbulence is supposed to behave as a non-Newtonian fluid.

However, as Lumley  $(\underline{6})$  pointed out, one has to be careful when applying the theory of rheology to the turbulence. In the rheology there are two conditions that have to be satisfied in constructing constitutive relations for materials (see, for example, Truesdell (10)). The first condition concerns the principle of determinism. This principle consists of two parts. The first part says that the stress is determined by the past only. The second part says that the stress at a point is determined only by a small surrounding of that point (principle of local action). The first part of the principle of determinism is also correct in the case of turbulence. The principle of local action however is not correct. The turbulent stress at a point in a turbulent field is determined by a surrounding that has a radius of the order of the Lagrangian integral length scale. This length scale is by no means small with respect to the length scales of the mean-velocity field. Consequently the principle of local action does not hold in the case of turbulence. The second condition that has to be satisfied in rheology concerns the principle of material objectivity. This principle says that a constitutive relation should be invariant under uniform translation (Galilean-invariance) and under uniform rotation of the coordinate system. The Galilean-invariance holds also in the case of turbulence. The invariance under rotation gives difficulties. This invariance says that the structure of the turbulence should be invariant under rotation. In the case of a non-Newtonian fluid the molecular structure has a time scale of the order of the mean free path divided by the velocity of the molecules. This is a very small time scale with respect to the meanvelocity time scales. So one has to rotate very fast before there is even a small influence of the rotation on fluid properties of a molecular nature. Consequently, the constitutive relation of the non-Newtonian fluid obeys the invariance under rotation. The time scale of the turbulence however is large with respect to the time scale of the molecules. So even at a moderate rotation-velocity the turbulence structure can be influenced through Coriolis and centrifugal forces. Consequently, the turbulence constitutive relation need not obey the principle of rotation-invariance. These differences between the non-Newtonian fluids and the turbulence mean that, although there is a great resemblance, one has to be careful in applying theories pertaining to rheology to theories describing the behaviour of the turbulent flow of a Newtonian fluid.

# C. Earlier experiments on disturbed velocity fields.

The purpose of this investigation is to consider extra memory effects. So we are interested in situations where the mean-velocity profile is changing in a not too gradual way. A lot of experiments have been done in the past that fit this requirement. We will give a small survey of experiments where sudden spatial changes in the mean-velocity field occur. Most of the experiments on perturbation of turbulent flows that are known in the literature are done on perturbated boundary layers. We will first mention the experiments concerned with a sudden change in surface roughness.

The first experiments of this kind was done by Jacobs  $(\underline{11})$  in a pipe flow. He investigated the change from rough to smooth as well as from smooth to rough. He measured only mean-velocity profiles. This kind of investigation about a change from smooth to rough in a pipe flow was also done by Logan and Jones  $(\underline{12})$ . They measured both mean-velocity profiles and turbulence intensities. They found that the mean velocity and the turbulence close to the wall reaches an equilibrium state sooner than in the part of the flow more removed from the wall. The same has been found by Makita  $(\underline{13})$  in an investigation involving a change from rough to smooth in a rectangular channel.

Taylor  $(\underline{14})$  performed an investigation along a wall involving the change from rough to smooth. He measured only mean-velocity profiles. Antonia and Luxton  $(\underline{15})$  also did experiments along a wall with a change from smooth to rough. They measured the mean velocity, the turbulence intensities, the turbulence shear stress and turbulence length scales. They investigated carefully the growth of the inner, second boundary layer which occurs after the change. This boundary layer grows until it covers the whole layer. Schofield  $(\underline{16})$  investigated a change in surface roughness in an adverse pressure gra-

-5-

dient boundary layer. He measured only the mean-velocity distributions.

Experiments have also been done which are concerned with an atmospheric situation. We mention the work of Panofsky and Townsend  $(\underline{17})$  concerning the change of mown to unmown grass, and of Plate and Hidy  $(\underline{18})$  concerning the change in a wind tunnel of flow along a wall to flow along a water surface In both experiments only mean-velocity profiles are measured.

In all these experiments it is found that the turbulent velocity profile close to the wall is sooner adapted to the change in surface roughness than in the part of the boundary layer more removed from the wall. One gets the idea that the small eddies close to the wall react more quickly to changes than the bigger eddies more removed from the wall. Unfortunately very few measurements have been done in the region very close behind the change in surface roughness. Moreover it appears that the strongest change in the meanvelocity profile is very close to the wall, which is just the region that reacts the soonest. Because of these two facts it turned out that with the results of these experiments we are not able to investigate extra memory effects.

A second class of experiments on perturbed boundary layers are boundary layers with a sudden change in the imposed pressure gradient. Moses (19) did experiments on a boundary layer with a negative pressure gradient followed by a boundary layer without a pressure gradient. He found that the meanvelocity profile reaches an equilibrium state sooner than does the turbulence profile. Goldberg (20) has extended this investigation by considering the response of a boundary layer to five different pressure gradients. Bradshaw and Ferris (21) investigated the change of a boundary layer that is in an equilibrium state under a positive, i.e. adverse, pressure gradient to a boundary layer without a pressure gradient. Bradshaw (22) considered the same case but with a negative, i.e. favourable, pressure gradient. Also from these experiments it is clear that the inner region of the boundary layer reaches an equilibrium state sooner than the outer region. The changes in adverse pressure gradient had to be relatively moderate, because otherwise separation of the boundary layer would take place. This is one reason that no distinct extra memory effects can be found in these experiments.

A third class of perturbed boundary layers are layers with an obstacle attached to the wall. Mueller (23) and Mueller and Robertson (24) did experiments behind three different kind of wedges. They measured both the mean velocity and the turbulence. Plate and Lin (25) measured the disturbed boundary layer behind a sine-like obstacle and behind a wedge. Bearman (26) investigated the flow behind a two-dimensional obstacle. All the experiments show nearly the same behaviour. The boundary layer separates along the obstacle and reattaches the wall after a distance of about 7 to 15 times the height of the obstacle, depending on the shape of the obstacle. Then the relaxation to the undisturbed boundary layer starts. Of course the magnitude of the disturbance depends on the relative height of the obstacle with respect to the thickness of the boundary layer.

About the same situation occurs in the case of a step in the boundary layer. We mention the work of Tani (27) and Tani, Suchi and Komoda (28) in the case of a two-dimensional boundary layer and of Johnston (29) in the case of a three-dimensional boundary layer. For a review of this kind of experiment concerned with an obstacle in a turbulent boundary layer we refer to the article by Bradshaw and Wong (30).

An experiment that should also be mentioned in this survey is the classical work of Clauser  $(\underline{31})$ . Clauser performed experiments on the disturbance of a turbulent boundary layer by a small cylinder placed spanwise perpendicular to the main flow direction. The disturbance caused by the cylinder placed close to the wall fades away more quickly than the disturbance caused by the cylinder placed further away from the wall. This result is in agreement with the foregoing experiments. Another indication about the idea that smaller eddies return quicker to their original state than bigger eddies is given by the work of Lissenburg ( $\underline{32}$ ). The turbulent flow through a round pipe is disturbed by a sudden short contraction of the pipe. From the energy spectra measured behind the contraction it can be concluded that the part of the spectrum containing the lower frequencies (bigger eddies) returns much more slowly to the undisturbed situation than the part of the spectrum containing the higher frequencies (smaller eddies).

For further details about investigations of disturbed boundary layers we refer to the article by Tani  $(\underline{33})$  and the work of Plate  $(\underline{34})$ .

There are only a few experiments on disturbances in free turbulent flows. We mention the work by Narasimha and Prabu (35, 36) in a wake flow behind a twin-plate. The behaviour of the wake under a sudden change in the outer pressure gradient was examined. We should also mention the work done on the distortion of practically isotropic grid turbulence with the intention of testing the rapid-distortion-theory by Townsend. The work of Maréchal (37), Tucker and Reynolds (38) and Reynolds and Tucker (39) are investigations on the distortion of grid turbulence under irrotational strain. The work of Rose (40) and Champagne, Harris and Corrsin (41) are investigations of the response of grid turbulence to an uniform shear. We will not go into these

-7-

# investigations any further.

In most of these earlier experiments that are concerned with the disturbance of turbulent flows the greatest accent has been given to the behaviour of the mean-velocity field and not to the behaviour of the turbulence. The disturbances were in most cases either relatively slow or small. Consequently, these experiments appear not to be suited to a careful and accurate investigation of extra memory effects in turbulent flows.

The most important fact that comes out of the investigations mentioned above is that the response of the turbulence to a disturbance depends on the size of the turbulent eddies. A small eddy, with a small time scale will show a smaller extra memory effect than a bigger eddy with a bigger time scale. Whether a disturbance of the mean-velocity field will have a marked effect depends on the ratio between the time scale of the change of the mean-velocity field to the time scale of the turbulence that is disturbed. This dependence of the turbulent behaviour on the eddy size has a support in the wave theory of Landahl ( $\underline{42}$ ). In this wave theory Landahl shows that the only suitable length scale that determines the behaviour of the eddy is the eddy size itself. Consequently the time scale that determines the behaviour of an eddy to a disturbance is the time scale of the eddy itself.

Because the above mentioned experimental results proved to be unsuitable to investigate extra memory effects we decided to do experiments of our own. In the first place the flow field behind a hemispherical cap attached to the fixed wall in a turbulent boundary layer has been investigated. The experiment with the hemisphere has one special complication. The mean flow behind the hemisphere is essentially three-dimensional. In order to create a twodimensional situation we have tried to use a half cylinder as disturbance of the boundary layer. However, the half cylinder caused a much more weak disturbance than the hemisphere. Consequently it was decided to use, in addition to the three-dimensional experiment on the hemisphere, the wake of a circular cylinder in order to investigate extra memory effects in a two-dimensional situation. Finally the turbulent flow field produced by a grid has been investigated. This investigation of grid turbulence has the advantage that, after a certain distance behind the grid, the mean-velocity is constant through the whole field. Consequently there is no mean-velocity gradient anymore and, as is known from investigations described in the literature, the turbulence shear stress is also zero in an axial plane. So, the final state of the turbulent field is quite different from the final state in the case of the boundary layer and the wake flow. The three different experiments are described in chapter VI, VII and VIII.

-8-

# II. THEORETICAL BACKGROUND OF A RELAXATION EQUATION WITH THE HYPOTHESIS OF BOUSSINESQ AS STARTING POINT.

In the following we will give a theoretical background for a possible description of extra memory effects in turbulent flows.

Starting from the hypothesis of Boussinesq we will show that it is possible to create a formula which is able to describe extra memory effects.

# A. The hypothesis of Boussinesq.

By definition we divide the total velocity  $U_i$  in the  $x_i$ -direction into a mean velocity  $\overline{U}_i$  and a turbulent velocity  $u_i$ 

$$U_{i} = \overline{U}_{i} + u_{i}$$
(2.1)

For the turbulent velocity field the normal conservation laws hold. For the conservation of mass we find for an incompressible fluid

$$\frac{\partial \overline{U}_{i}}{\partial x_{i}} = 0 \qquad \frac{\partial u_{i}}{\partial x_{i}} = 0 \qquad (2.2)$$

Here and in the following the summation convention is used.

The conservation of momentum leads to the so-called Reynolds equation:

$$\frac{\partial \overline{U}_{i}}{\partial t} + \overline{U}_{j} \frac{\partial}{\partial x_{j}} \overline{U}_{i} = -\frac{1}{\rho} \frac{\partial P}{\partial x_{i}} + \nu \frac{\partial^{2} \overline{U}_{i}}{\partial x_{j} \partial x_{j}} - \frac{\partial}{\partial x_{j}} \overline{u_{j} u_{i}}$$
(2.3)

The first term on the left hand side is the change of the mean velocity with time, the second term is the convection term. The first term on the right hand side gives the influence of a force caused by the pressure P, the second term is the viscosity term with v the kinematic viscosity, and the last term contains the turbulent velocity correlations. Because it is common to think of these turbulent correlations as turbulence stresses we shall call the correlation the turbulence normal stress if i=j and the turbulence shear stress if i≠j.

The turbulence stress term is an extra unknown in the eqs. (2.2) and (2.3). So it is impossible to solve this set of equations without further information about this term. Consequently we must either know the value of this term or we have to express this term in known quantities. The first suggestion for relating the turbulence stress to known quantities was made by Boussinesq in 1877 (<u>43</u>). Boussinesq found that the wall shear stress in a laminar flow is raised when the flow becomes turbulent. The wall shear stress also rises when the viscosity of the flow increases. Boussinesq con-

-9-

cludes that the turbulence rises the effective viscosity of the flow. Consequently, he wrote in analogy with the expression for the molecular stress known from the kinetic theory of gases:

$$-\overline{u_{i}u_{j}} = \epsilon_{m} \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}}\right)$$
(2.4)

This expression is a constitutive relation which relates the turbulence to the mean-velocity field. The factor  $\epsilon_m$  is called the eddy viscosity. Boussinesq stated that  $\epsilon_m$  is a scalar. In general the eddy viscosity is much bigger than the molecular viscosity.

From this hypothesis we see that, because of the fact that the turbulence stress is supposed to be proportional to the mean-velocity gradient, the turbulence is looked at as a hypothetical Newtonian fluid (see chapter I).

Prandtl  $(\underline{44})$  has extended the analogy between the kinetic theory of gases and the behaviour of the turbulence in his so-called mixing length theory. In the kinetic theory of gases it is known that the kinematic viscosity is proportional to the root-mean-square value of the molecular velocity times the mean free path. In analogy with this Prandtl wrote for the eddy viscosity  $\epsilon_m$ :

$$\epsilon_{\rm m} = v.\ell_{\rm t}$$
 (2.5)

For the velocity v he chooses the turbulence intensity  $u_j' = \sqrt{u_j^2}$  in the same, transverse direction of  $\ell_t$  and he called the length scale  $\ell_t$  the turbulence mixing length. This mixing length is a length scale that depends on the eddy size. It is a measure of the length after which an eddy looses its identity.

The hypothesis of Boussinesq and the mixing length theory of Prandtl have been widely used with a lot of success, in physical and in industrial respect. Although for a long time the objections against these theories have been increasing, nowadays a lot of people still use these very valuable theories.

We can give some generalisations of the hypothesis of Boussinesq. The expression (2.4) is not correct in the case of a contraction with respect to i and j. The right hand side of the formula then becomes zero. It is common to separate the normal stress term from the shear stress term to overcome this difficulty and write (see for example, Hinze (45) p. 23, 24):

$$-\overline{u_{i}u_{j}} = -\frac{1}{3} \overline{u_{k}u_{k}} \delta_{ij} + \epsilon_{m} \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}}\right)$$
(2.6)

We can view the isotropic part of  $\overline{u_i u_j}$ , i.e.  $\frac{1}{3} \overline{u_k u_k}$ , as a turbulent pressure  $P_t$ .

Up till now we have supposed that the stress  $\overline{u_i u_j}$  is only dependent on the mean-velocity gradient in the same direction i-j. We can generalize the formulation, so that in principle  $\overline{u_i u_j}$  depends on the mean-velocity gradient in every direction. We have also supposed that  $\epsilon_m$  is a scalar. Formally it is possible to write a fourth order tensor for  $\epsilon_m$ , so  $\epsilon_m$  can have a different value in each direction. These two generalisations leads to the following expression:

$$-\overline{u_{i}u_{j}} = -P_{t} \delta_{ij} + (\epsilon_{m})_{ijk\ell} \left(\frac{\partial \overline{v}_{k}}{\partial x_{\ell}} + \frac{\partial \overline{v}_{\ell}}{\partial x_{k}}\right)$$
(2.7)

It is clear that in practice eq. (2.7) is a very intractable and complicated expression.

Because, as is already stated in the first chapter, we can consider the turbulence as a hypothetical non-Newtonian fluid we can write a more general expression still:

$$-\overline{u_{i}u_{j}} = F_{ij}(x_{k}, t, \overline{U}_{k}, P_{t}, \frac{\partial U_{k}}{\partial x_{l}})$$
(2.8)

Under the conditions of material objectivity this expression becomes (see Lumley (6)):

$$-\overline{u_{i}u_{j}} = \epsilon_{o} \delta_{ij} + \epsilon_{m} \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}}\right) + \epsilon_{o} \left(\frac{\partial \overline{U}_{j}}{\partial x_{k}} + \frac{\partial \overline{U}_{k}}{\partial x_{j}}\right) \left(\frac{\partial \overline{U}_{k}}{\partial x_{i}} + \frac{\partial \overline{U}_{i}}{\partial x_{k}}\right)$$
(2.9)

The factors  $\boldsymbol{\epsilon}_{o}, \boldsymbol{\epsilon}_{m}$  and  $\boldsymbol{\epsilon}_{c}$  can be functions of the invariants of  $\partial \overline{U}_{i} / \partial x_{j} + \partial \overline{U}_{j} / \partial x_{i}$  and of scalar quantities, such as  $P_{t}$ . The expression (2.9) has the form of a constitutive relation of a Reiner-Rivlin fluid.

The expressions (2.7) and (2.9) are generalisations of the hypothesis of Boussinesq, but they are still "local" expressions in the sense that they contain only the local mean-velocity gradients. Consequently they cannot describe extra memory effects. Although an expression like eq. (2.9) is valuable we have to be careful in applying rheological theories to turbulence. As we have already stated in the first chapter, the turbulence does not obey the principle of material objectivity and the principle of local action.

# B. The limitations of the hypothesis of Boussinesq.

In the following we will use the hypothesis of Boussinesq in the more

general form of expression (2.6). This expression states that the turbulence stress is locally determined by the mean-velocity gradient, in other words the turbulence stress at a certain place at a certain instant is, not to mention  $\boldsymbol{\epsilon}_{\rm m}$ , wholly determined by the mean-velocity gradient at that place at that instant. The question is what will happen when there is a sudden change in the mean-velocity gradient. It is clear that expression (2.6) holds only if the time scale in which  $\partial \bar{U}_i/\partial x_j$  changes is large with respect to the time scale of the turbulence, for example the Lagrangian integral time scale. When these time scales become of the same order the turbulence will react with a certain timelag to the changes of the mean-velocity gradient. So there will be an extra memory effect and  $-u_i u_j$  will no longer be determined by the local mean-velocity gradient alone. This is the Lagrangian way of describing the extra memory effect. In fact there is an extra memory effect when the length scale characterizing changes in  $\partial \bar{U}_i/\partial x_j$  is of the same order as the "inner" length scale of the turbulence.

There are a number of experiments that clearly show that expression (2.6) cannot describe the behaviour of the turbulence in certain situations. In experiments in asymmetric flows it appears that the place where the mean velocity reaches its maximum value does not necessarily coincide with the place where the turbulence stress is zero. This is in contradiction with expression (2.6). We will come back to this fact in chapter IV. In an experiment of a fast spatially changing and consequently short boundary layer under a severely changing outer pressure gradient Deissler  $(\underline{47})$  found that the shear stress maintains the same value along a streamline, the value of the shear stress is frozen. So also in this case expression (2.6) does not hold.

From these remarks and the investigations it will be clear that in general the turbulence stress is not a locally determined quantity described by a simple gradient-type transport model.

# C. A modification of the hypothesis of Boussinesq.

The question arises as to whether it is possible to modify expression (2.6) in order to take account of the fact that the turbulence stress is not locally determined by the mean-velocity gradient. To answer this question we present the following reasoning. In this reasoning we will use the same assumptions which are usually made in considering turbulent diffusion (see <u>45</u>, p. 384-392). By talking about a mean value at a certain instant we mean an ensemble-averaged value.

For the turbulent transport of u. in the x\_-direction one can write:

$$\overline{u_{j}u_{i}} = \frac{1}{T} \int_{0}^{T} dt_{0} u_{j} (t_{0}) U_{i} (t_{0})$$
(2.10)

The integration over T means an averaging procedure. The velocity  $U_i(t_o)$  is determined by the history of a turbulent fluid particle at all timest  $\leq t_o$ . Let  $y_k(t_o;t)$  be the distance travelled during a time t in the  $x_k$ -direction by a fluid particle that crosses the control plane at time  $t_o$ . We suppose now that we can expand  $U_i(t_o)$  in a series around  $t_o$ :

$$U_{i}(t_{o};t) = (\overline{U}_{i})_{t_{o}} - y_{k}(t_{o};t)(\frac{\partial U_{i}}{\partial x_{k}})_{t_{o}} + \frac{1}{2} y_{k}y_{\ell}(t_{o};t).$$

$$(\frac{\partial^{2}\overline{U}_{i}}{\partial x_{k}\partial x_{\ell}})_{t_{o}} - \frac{1}{6} y_{k}y_{\ell}y_{m}(t_{o};t)(\frac{\partial^{3}\overline{U}_{i}}{\partial x_{k}\partial x_{\ell}\partial x_{m}})_{t_{o}} + \dots$$

$$(2.11)$$

For the distance  $y_k(t_o;t)$  we can write

$$y_{k}(t_{0};t) = \int_{0}^{t} U_{k}(t_{0}-t')dt'$$
 (2.12)

If we suppose, as usual, that the mean velocity has no influence on the transport and is uncorrelated with the turbulence and we neglect terms of a higher order than the second we find from eqs. (2.10), (2.11) and (2.12):

$$\frac{\overline{u_{j}u_{i}}}{u_{j}u_{i}} = -\left(\frac{\partial\overline{U}_{i}}{\partial x_{k}}\right)_{t_{0}} \int_{0}^{t} dt' \frac{\overline{u_{j}(t_{0})} u_{k}(t_{0}-t')}{u_{j}(t_{0}) u_{k}(t_{0}-t')} + \left(\frac{\partial^{2}\overline{U}_{i}}{\partial x_{k}\partial x_{k}}\right)_{t_{0}} \int_{0}^{t} dt' \frac{\overline{u_{j}(t_{0})} u_{k}(t_{0}-t') u_{k}(t_{0}-t')}{2} \qquad (2.13)$$

The averaging procedure over T is indicated by an overscore. The correlation  $u_j(t_0) u_k(t_0-t') = [Q_{jk}]_L$  is the Lagrangian auto-correlation. We can write for the integral containing  $[Q_{jk}]_L$  an eddy viscosity  $\epsilon_{jk}$ . If we assume that it is possible to write for the integral of the triple correlation an eddy viscosity  $\epsilon_{jk}^{o}$  times a length scale  $L_l$  we find:

$$-\overline{u_{j}u_{i}} = \epsilon_{jk}(t) \frac{\partial \overline{U}_{i}}{\partial x_{k}} - \epsilon_{jk}^{o}(t) \frac{L_{\ell}}{2} \frac{\partial^{2}\overline{U}_{i}}{\partial x_{k}\partial x_{\ell}}$$
(2.14)

When we consider the influence on the transport of all times preceding t<sub>o</sub> we can take t equal to infinity and consequently  $\epsilon_{jk}$  and  $\epsilon_{jk}^{o}$  are no longer a function of time. In general  $\epsilon_{jk}$  and  $\epsilon_{jk}^{o}$  have different values for different

directions. In the following we assume that a scalar value for  $\epsilon_{jk}$  and  $\epsilon_{jk}^{o}$  suffices. The expression (2.14) is not symmetric in i and j. If we further assume that the expression should also hold for i=j and that  $\overline{u_{ju}}_{ji}$  is only determined by velocity gradients in the i-j direction we find:

$$-\overline{u_{j}u_{i}} = -P_{t} \delta_{ij} + \epsilon_{m} \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}}\right) - \epsilon_{m} \circ \frac{L_{\ell}}{2} \frac{\partial}{\partial x_{\ell}} \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}}\right)$$
(2.15)

Although we now have an expression that is still more or less local, we have enlarged the surroundings by which  $-u_{j}u_{i}$  is determined. We can now, by the way of the higher order derivates, calculate the influence of a change in the mean-velocity gradient. How big the influence is of the higher order derivatives depends on the value of the "memory length"  $L_{\ell}$  and on  $\epsilon_{m}^{\circ}$ .

Expression (2.15) is one way to describe an extra memory effect. By trying to describe an extra memory effect we can also say that the stress  $\overline{u_{j}u_{i}}$  at a certain instant t will depend on the whole history of the meanvelocity field up to the moment t. We can write for this, taking the hypothesis of Boussinesq as a basis

$$-\overline{u_{j}u_{i}}(t) = -P_{t}(t) \delta_{ij} + \int_{-\infty}^{t} dt' \left( \epsilon_{m} \left\{ \left( \frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}} \right)(t') \right\} M(t-t') \right)$$
(2.16)

The function M(t-t') is a kind of weighting function, a memory function, which satisfies the condition

$$\int dt'' M(t'') = 1$$
(2.17)

Expression (2.16) is the same expression as used by Crow  $(\underline{4})$  to describe the visco-elastic behaviour of the small-scale turbulence. So, expression (2.16) is connected with a visco-elastic behaviour.

The weighting function must take care of the fact that the mean-velocity gradient close to t has a bigger influence on  $-u_ju_i$  than the mean-velocity gradient at an earlier instant. It is logical to connect this memory function with the Lagrangian auto-correlation. Although the Lagrangian auto-correlation is not exactly a simple exponential function, we will take for the memory function an exponential function. When we further write  $x_k$  instead of t we find

$$-\overline{u_{j}u_{i}}(x_{k}) = -P_{t}(x_{k}) \delta_{ij} + \int_{-\infty}^{x_{k}} dx_{k}' \left( \epsilon_{m} \{ (\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}})(x_{k}') \} \right).$$

$$\cdot \frac{1}{L_{k}} e^{-\frac{(x_{k}-x_{k}')}{L_{k}}} \right) \qquad (2.18)$$

There is a close connection between this expression (2.18) and the expression (2.15). Namely, one can find eq. (2.15) from eq. (2.18) by expanding the mean-velocity gradient in eq. (2.18) in a series around  $x_{\mu}$  (see also (89)).

Expression (2.18) is a convolution integral. A convolution integral, in general, can be a solution of an inhomogeneous first order differential equation. In this situation this differential equation reads as follows

$$L_{k} \frac{\partial}{\partial x_{k}} \left( -\overline{u_{j}u_{i}} + P_{t} \delta_{ij} \right) + \left( -\overline{u_{j}u_{i}} + P_{t} \delta_{ij} \right) = \\ \varepsilon_{m} \left( \frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}} \right)$$

$$(2.19)$$

The convolution integral (2.18) is a solution of eq. (2.19) only when  $L_k$  is a constant. When  $L_k$  is a function of  $x_k$  the solution of eq. (2.19) becomes:

$$(-\overline{u_{j}u_{i}} + P_{t} \delta_{ij})(x = x_{A}) = (-\overline{u_{j}u_{i}} + P_{t} \delta_{ij})(x = x_{o}) .$$
  

$$\cdot \exp \left(-\int_{x_{o}}^{x_{A}} \frac{dx'}{L_{k}(x')}\right) + \int_{x_{o}}^{x_{A}} dx' (\epsilon_{m} (\frac{\partial \overline{u}_{i}}{\partial x_{j}} + \frac{\partial \overline{u}_{j}}{\partial x_{i}}) (x')$$
  

$$\cdot \frac{1}{L_{k}(x')} \exp \left(-\int_{x'}^{x_{A}} \frac{dx''}{L_{k}(x'')}\right), \qquad (2.20)$$

where x is written for  $x_{t}$ .

The differential equation (2.19) is a more general expression than expression (2.15) and (2.16). This differential equation can in principle describe extra memory effects in turbulent flows. The term  $L_k \frac{\partial}{\partial x_k} (-u_j u_i + P_t \delta_{ij})$  can be seen as the term that takes account of the deviation from the hypothesis of Boussinesq. When this term is small with respect to the other terms there will be no marked extra memory effect. It is clear that the value of this term, next to the derivative of the stress, depends strongly on the value of the memory length  $L_k$ .

The experiments we performed in the course of this investigation are concerned mostly with two-dimensional flow situations where the boundary layer approximation holds. We simplify expression (2.15) to take this into account, assume that  $\epsilon_m = \epsilon_m^{o}$  and use  $L_l/2$  instead of  $L_l$  for the memory length. For the coordinate system we take  $x_1$  in the direction of the main flow and  $x_2$  transverse to the  $x_1$ -direction. We find:

$$-\overline{u_2 u_1} = \epsilon_m \left( \frac{\partial \overline{U}_1}{\partial x_2} - L_1 \frac{\partial}{\partial x_1} \left( \frac{\partial \overline{U}_1}{\partial x_2} \right) - L_2 \frac{\partial}{\partial x_2} \left( \frac{\partial \overline{U}_1}{\partial x_2} \right) \right)$$
(2.21)

We will call this expression the relaxation expression. The memory length  $L_1$  will be greater than the memory length  $L_2$ , according to the boundary layer approximation.

For the differential equation (2.19) we find with the same simplification:

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + L_{2} \frac{\partial}{\partial x_{2}} \left( -\overline{u_{2}u_{1}} \right) + \left( -\overline{u_{2}u_{1}} \right) = \varepsilon_{m} \frac{\partial U_{1}}{\partial x_{2}}$$
(2.22)

We will call this equation the relaxation equation.

# III. THE CONNECTION BETWEEN THE RELAXATION EQUATION AND THE COMPLETE TRANSPORT EQUATION.

In this chapter we will consider the complete transport equations of the turbulence stresses. If the relaxation equation (2.22) considered in Chapter II does describe the long distance behaviour of the turbulence shear stress, then there must be a connection with the transport equation. In the following it will be shown that eq. (2.22) can be derived from the transport equation under certain simplifying assumptions.

# A. The turbulence models.

The hypothesis of Boussinesq and the further generalisations of it have at their heart the idea of relating the turbulence stress to the gradient of the mean velocity by means of a semi-empirical formula. This is not the only possibility to describe the turbulence stress. The best way to determine the turbulence stress would be to solve the complete transport equation of the turbulence stress. From the Reynolds equation it is easy to deduce the complete transport equation for the turbulence stress  $\overline{u_i u_j}$  (see Hinze (<u>45</u>), p. 68-75). We get:

$$\frac{\partial}{\partial t} \overline{u_{i}u_{j}} + \overline{u}_{j} \frac{\partial}{\partial x_{j}} \overline{u_{i}u_{j}} = - \{\overline{u_{i}u_{k}} \frac{\partial\overline{u}_{j}}{\partial x_{k}} + \overline{u_{j}u_{k}} \frac{\partial\overline{u}_{i}}{\partial x_{k}}\}$$

$$- 2\nu \frac{\partial\overline{u_{i}}}{\partial x_{k}} \frac{\partial\overline{u}_{j}}{\partial x_{k}} + \frac{p}{\rho} (\frac{\partial\overline{u}_{i}}{\partial x_{j}} + \frac{\partial\overline{u}_{j}}{\partial x_{i}}) - \frac{\partial}{\partial x_{k}} (\overline{u_{i}u_{j}u_{k}} + \frac{\delta_{jk}}{3} \frac{\overline{u_{i}p}}{\rho}$$

$$+ \frac{\delta_{ik}}{3} \frac{\overline{u_{j}p}}{\rho} - \nu \frac{\partial}{\partial x_{k}} \overline{u_{i}u_{j}}) \qquad (3.1)$$

The first term on the left hand side is the change with time of  $\overline{u.u.j}$ , the second term is the convective transport.

The first term on the right hand side is the production term, then follows the dissipation term and the turbulence pressure-velocitygradient correlation. The last term is the turbulent and viscous diffusion term. By solving this equation we would find the correct value of  $\overline{u_i u_j}$ . Unfortunately there are a lot of terms unknown in this equation. If it were possible to express the unknown terms in known quantities, we would be able to solve the equation. In recent years a lot of work has been done in trying to express the unknown terms in the transport equation in known quantities. The growth of the capacity of computers has added greatly to the possibility of calculating  $\overline{u_i u_j}$ .

-17-

because it is now possible to solve several coupled differential equations within an acceptable amount of time.

At present there are several so-called turbulence models, some of them starting from other transport equations than the eq. (3.1) for  $u_iu_j$ . We will mention the model by Launder  $(\underline{48}, \underline{49}, \underline{50})$ , by Lumley  $(\underline{51}, \underline{52}, \underline{53})$ , by Rotta  $(\underline{54})$  and by Bradshaw  $(\underline{55}, \underline{56}, \underline{57}, \underline{58})$ . For information about these models one is referred to the above mentioned articles and the surveys that can be found in the book of Launder and Spalding  $(\underline{59})$ , the course of the Von Kármán institute  $(\underline{60})$  and the articles by Rotta  $(\underline{61})$ , Launder  $(\underline{62})$  and Mellor  $(\underline{63})$ . We will restrict ourselves here to making only some remarks about these models.

Although some of these models give remarkably good results in calculating turbulent flow fields there are several difficulties. In the models a certain number of constants had to be introduced which are often assumed to be universal. The universal character, however, has not been proven, so that it might well be that a constant, determined in the case of the decay of isotropic turbulence can have a different value for turbulence with a different structure, for example in a boundary layer. So the first difficulty is that these constants in general are not universal. Another difficulty is the fact that although the equation for the unknown terms are formally correct the physical idea behind the equations is by no means clear. For example, it is very difficult to get a physical idea of terms like the dissipation of the dissipation occurring in the transport equation for the dissipation used in some models. We must remark that Bradshaw has tried to overcome these difficulties.

However, all the models have one basic restriction. The models have as a basis the condition that the turbulent flow field should be in a nearly equilibrium and a nearly isotropic state. Consequently these models will be insufficient to describe strongly disturbed turbulent flow fields (see also (101), (102)).

# B. <u>The relation between the relaxation equation and the complete transport</u> equation.

We will look at a situation where the boundary layer approximation holds and where the mean-velocity field is two-dimensional. Again we take  $x_1$  in the main flow direction and  $x_2$  transverse to it. We then find for the normal and the shear stress component originating from equation (3.1)

$$\frac{\partial}{\partial t} \overline{u_1}^2 + \overline{U}_1 \frac{\partial}{\partial x_1} \overline{u_1}^2 + \overline{U}_2 \frac{\partial}{\partial x_2} \overline{u_2}^2 = -2 \overline{u_2 u_1} \frac{\partial \overline{U}_1}{\partial x_2} - 2\nu \frac{\partial u_1}{\partial x_\ell} \frac{\partial u_1}{\partial x_\ell} + \frac{2p}{\rho} \frac{\partial u_1}{\partial x_1} - \frac{\partial}{\partial x_2} \overline{u_1}^2 \frac{\partial 2u}{\partial x_2} + \nu \frac{\partial^2 \overline{u_1}^2}{\partial x_2^2}$$
(3.2)

$$\frac{\partial}{\partial t} \overline{u_2}^2 + \overline{U}_1 \frac{\partial}{\partial x_1} \overline{u_2}^2 + \overline{U}_2 \frac{\partial}{\partial x_2} \overline{u_2}^2 = -2\nu \frac{\partial u_2}{\partial x_k} \frac{\partial u_2}{\partial x_k} + \frac{2p}{\rho} \frac{\partial u_2}{\partial x_2}$$
$$- \frac{\partial}{\partial x_2} \overline{u_2}^3 - \frac{2}{\rho} \frac{\partial}{\partial x_2} \overline{u_2}^p + \nu \frac{\partial^2 \overline{u_2}^2}{\partial x_2^2}$$
(3.3)

$$\frac{\partial}{\partial t} \overline{u_3}^2 + \overline{U}_1 \frac{\partial}{\partial x_1} \overline{u_3}^2 + \overline{U}_2 \frac{\partial}{\partial x_2} \overline{u_3}^2 = -2\nu \frac{\partial u_3}{\partial x_k} \frac{\partial u_3}{\partial x_k} + \frac{2p}{\rho} \frac{\partial u_3}{\partial x_3}$$
$$- \frac{\partial}{\partial x_2} \overline{u_3}^2 \overline{u_2}^2 + \nu \frac{\partial^2 \overline{u_3}^2}{\partial x_2^2}$$
(3.4)

$$\frac{\partial}{\partial t} \overline{u_2 u_1} + \overline{u}_1 \frac{\partial}{\partial x_1} \overline{u_2 u_1} + \overline{u}_2 \frac{\partial}{\partial x_2} \overline{u_2 u_1} = -\overline{u_2^2} \frac{\partial U_1}{\partial x_2} - 2\nu \frac{\partial u_1}{\partial x_\ell} \frac{\partial u_2}{\partial x_\ell}$$
$$+ \frac{\overline{p}}{\rho} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) - \frac{\partial}{\partial x_2} \overline{u_1 u_2^2} - \frac{1}{\rho} \frac{\partial}{\partial x_2} \overline{u_1 p} + \nu \frac{\partial^2 \overline{u_2 u_1}}{\partial x_2^2}$$
(3.5)

For the unknown terms in these equations we will now use the expressions that are commonly used in the turbulence models, notwithstanding their limitations.

At first we will assume that the turbulence is stationary. Next we want to describe the dissipation. For the dissipation term in eq. (3.1) we use the expression that has first been suggested by Rotta (54):

$$2\nu \frac{\partial u_{i}}{\partial x_{\ell}} \frac{\partial u_{j}}{\partial x_{\ell}} = \frac{2}{3} \delta_{ij} \epsilon$$
(3.6)

Here  $\varepsilon$  is the dissipation of the total turbulent kinetic energy given by

$$\varepsilon = v \frac{\partial u_i}{\partial x_{\ell}} \frac{\partial u_i}{\partial x_{\ell}}$$
(3.7)

under the condition that the turbulence is homogeneous, so that

$$\frac{\partial u_{i}}{\partial x_{l}} \frac{\partial u_{l}}{\partial x_{i}} = 0$$

By using expression (3.6) we find that the dissipation for all the normal components has the same value namely  $\frac{2}{3} \epsilon$ . For  $\overline{u_2 u_1}$  we find that the dissipa-

tion is zero.

For the description of the turbulent pressure-velocity correlation it is common to divide this correlation in two parts:

$$\frac{\overline{p}}{\rho} \left( \frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right) = \frac{\overline{p_{M}}}{\rho} \left( \frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right) + \frac{\overline{p_{T}}}{\rho} \left( \frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right)$$
(3.8)

The pressure  $p_{M}$  is connected with the mean-velocity gradient, the pressure  $p_{T}$  is connected with the turbulence (for further information, see for example  $(\underline{60})$ ). The correlations can be described by:

$$\frac{\overline{P_{M}}}{\rho} \left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}}\right) = c_{1} \overline{q^{2}} \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}}\right) \qquad \text{product} \qquad (3.9)$$

$$\frac{\overline{P_{T}}}{\rho} \left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}}\right) = -c_{2} \frac{\varepsilon}{q^{2}} \left(\overline{u_{i}u_{j}} - \frac{\delta_{ij}}{3} \overline{q^{2}}\right) \qquad \text{return} \qquad (3.10)$$

Here  $c_1$  and  $c_2$  are constants, which may be different depending on i and j,  $\overline{q^2}$  is equal to  $\overline{u_k u_k}$  (see note at page 23). For the normal components we find from eq. (3.9) that the correlation connected ted with  $p_M$  is negligibly small. For the shear stress we find for the  $p_M^$ correlation  $c_1 \ \overline{q^2} \ \partial \overline{u}_1 / \partial x_2$ . For the part of the correlation connected with  $p_T$ we find from eq. (3.10) for  $\overline{u_1^2}$ :  $-c_2 \ \varepsilon/\overline{q^2} \ (\overline{u_1^2} - \overline{q^2}/3)$ , for  $\overline{u_2^2}$  and  $\overline{u_3^2}$  we find an analogous expression. For the shear stress we find for the  $p_T$ -correlation  $-c_2 \ \varepsilon/\overline{q^2} \ (\overline{u_2 u_1})$ .

The last term to be described is the diffusion term. We will assume that the Reynolds number is not too small. In this case we can neglect the pressure-velocity correlations  $\overline{u_2p}$  and  $\overline{u_1p}$  and the viscosity terms. The terms left are the triple correlations. The triple correlation is described in most of the turbulence models by

$$\overline{u_{i}u_{j}u_{k}} = -c_{3} \frac{\overline{q^{2}/2}}{\varepsilon} \left( \overline{u_{i}u_{k}} \frac{\partial \overline{u_{j}u_{k}}}{\partial x_{k}} + \overline{u_{j}u_{k}} \frac{\partial \overline{u_{k}u_{i}}}{\partial x_{k}} + \overline{u_{k}u_{k}} \frac{\partial \overline{u_{i}u_{j}}}{\partial x_{k}} \right)$$
(3.11)

Here  $c_3$  is a constant. So we find for the triple correlations with eq. (3.11)

$$\frac{\partial}{\partial x_2} \overline{u_1^2 u_2} = \frac{\partial}{\partial x_2} \left[ -c_3 \frac{\overline{q^2/2}}{\epsilon} \left( 2 \overline{u_2 u_1} \frac{\overline{\partial u_2 u_1}}{\partial x_2} + \overline{u_2^2} \frac{\overline{\partial u_1^2}}{\partial x_2} \right) \right]$$
(3.12)

$$\frac{\partial}{\partial x_2} \overline{u_2^3} = \frac{\partial}{\partial x_2} \left( -c_3 \frac{\overline{q^2/2}}{\varepsilon} \cdot 3 \frac{\partial \overline{u_2^2}}{\partial x_2} \right)$$
(3.13)

$$\frac{\partial}{\partial x_2} \overline{u_3^2 u_2} = \frac{\partial}{\partial x_2} \left( -c_3 \frac{\overline{q^2/2}}{\varepsilon} \frac{\partial \overline{u_3^2}}{\partial x_2} \right)$$
(3.14)

$$\frac{\partial}{\partial x_2} \overline{u_1 u_2^2} = \frac{\partial}{\partial x_2} \left( -c_3 \frac{\overline{q^2/2}}{\epsilon} \left( \overline{u_2 u_1} \frac{\partial \overline{u_2^2}}{\partial x_2} + 2\overline{u_2^2} \frac{\partial \overline{u_2 u_1}}{\partial x_2} \right) \right)$$
(3.15)

In view of the fact that these terms describing the triple correlation contain only second order derivatives or a multiplication of two first order derivatives we will probably not make a very bigmistake by neglecting these terms. When, finally, we introduce a length scale L defined by:

$$L = c_4 \frac{(\overline{q^2}/2)^{3/2}}{\varepsilon} \qquad c_4 \frac{e^{\frac{2}{2}}}{\varepsilon} \quad turb. \ ton energie (3.16)$$

with  $c_{11}$  a constant, we find for the eqs. (3.2), (3.3), (3.4) and (3.5)

$$\bar{J}_{1} \frac{\partial}{\partial x_{1}} \overline{u_{1}^{2}} + \bar{U}_{2} \frac{\partial}{\partial x_{2}} \overline{u_{1}^{2}} = -2 \overline{u_{2}u_{1}} \frac{\partial \bar{U}_{1}}{\partial x_{2}} - \frac{2}{3} \varepsilon - \frac{c_{2}^{11}c_{4}}{2} \frac{(\bar{q}^{2}/2)^{1/2}}{L} .$$

$$\cdot (\overline{u_{1}^{2}} - \frac{1}{3} \bar{q}^{2}) \qquad (3.17)$$

$$\overline{U}_{1} \frac{\partial}{\partial x_{1}} \overline{u_{2}}^{2} + \overline{U}_{2} \frac{\partial}{\partial x_{2}} \overline{u_{2}}^{2} = -\frac{2}{3} \varepsilon - \frac{c_{2}^{22}c_{4}}{2} \frac{(\overline{q^{2}}/2)^{1/2}}{L} (\overline{u_{2}}^{2} - \frac{1}{3} \overline{q^{2}})$$
(3.18)

$$\overline{U}_{1} \frac{\partial}{\partial x_{1}} \overline{u_{3}^{2}} + \overline{U}_{2} \frac{\partial}{\partial x_{2}} \overline{u_{3}^{2}} = -\frac{2}{3} \varepsilon - \frac{c_{2}^{3^{3}c_{4}}}{2} \frac{(\overline{q^{2}/2})^{1/2}}{L} (\overline{u_{3}^{2}} - \frac{1}{3} \overline{q^{2}})$$
(3.19)

$$\overline{U}_{1} \frac{\partial}{\partial x_{1}} \overline{u_{2}u_{1}} + \overline{U}_{2} \frac{\partial}{\partial x_{2}} \overline{u_{2}u_{1}} = -\overline{u_{2}}^{2} \frac{\partial \overline{U}_{1}}{\partial x_{2}} + c_{1} \overline{q^{2}} \frac{\partial \overline{U}_{1}}{\partial x_{2}} - \frac{c_{2}^{21}c_{4}}{2} .$$

$$\frac{(\overline{q^{2}}/2)^{1/2}}{L} \overline{u_{2}u_{1}}$$

$$(3.20)$$

We first consider the equation for the shear stress. The factor  $(\overline{q^2}/2)^{1/2}/L$  has the dimension of 1/sec. We will put

$$T = \frac{2}{c_2^{21}c_4} \frac{L}{(\overline{q^2}/2)^{1/2}}$$
(3.21)

If we use this expression and multiply eq. (3.20) by T we get:

$$\overline{U}_{1} \ \mathbb{T} \ \frac{\partial}{\partial x_{1}} \ \overline{u_{2}u_{1}} + \overline{U}_{2} \ \mathbb{T} \ \frac{\partial}{\partial x_{2}} \ \overline{u_{2}u_{1}} = (c_{1} \ \overline{q^{2}} - \overline{u_{2}^{2}}) \ \mathbb{T} \ \frac{\partial U_{1}}{\partial x_{2}} - \overline{u_{2}u_{1}}$$
(3.22)

Because T is a turbulent time scale, the factors  $\overline{U}_1 T$  and  $\overline{U}_2 T$  can be considered as turbulence memory lengths. The factor  $(\overline{u_2}^2 - c_1 \overline{q^2})T$  has the dimen-

sion of a viscosity. We consider this factor as an eddy viscosity. If we introduce these suggestions we get:

$$L_1 \stackrel{*}{\sim} \frac{\partial}{\partial x_1} \left( -\overline{u_2 u_1} \right) + L_2 \stackrel{*}{\sim} \frac{\partial}{\partial x_2} \left( -\overline{u_2 u_1} \right) + \left( -\overline{u_2 u_1} \right) = \epsilon_m \stackrel{*}{\sim} \frac{\partial U_1}{\partial x_2}$$
(3.23)

This equation is the same as the relaxation equation (2.22). So we find that we can deduce the relaxation equation from the complete transport equation if we make the usual assumptions about the unknown terms. The relaxation equation is an approximate transport equation.

We will now consider the equations that describe the normal stresses. We assume that the constants  $c_2^{11}$ ,  $c_2^{22}$  and  $c_2^{33}$  will be equal. This seems to be a reasonable assumption because we find then a time scale  $T_q$  analogous to eq. (3.21) which is equal for the different components. This time scale  $T_q$  is then the time scale of the total turbulent kinetic energy. With  $T_q$  we find:

$$\overline{U}_{1} \quad \underline{T}_{q} \quad \frac{\partial}{\partial x_{1}} \quad \overline{u_{1}}^{2} + \overline{U}_{2} \quad \underline{T}_{q} \quad \frac{\partial}{\partial x_{2}} \quad \overline{u_{1}}^{2} = -2 \quad \overline{u_{2}u_{1}} \quad \underline{T}_{q} \quad \frac{\partial U_{1}}{\partial x_{2}} - \frac{2}{3} \quad \varepsilon \quad \underline{T}_{q} - (\overline{u_{1}}^{2} - \frac{1}{3} \quad \overline{q}^{2})$$

$$(3.24)$$

$$\overline{\overline{U}}_{1} \ \overline{\underline{T}}_{q} \ \frac{\partial}{\partial x_{1}} \ \overline{\underline{u}}_{2}^{2} + \overline{\overline{U}}_{2} \ \overline{\underline{T}}_{q} \ \frac{\partial}{\partial x_{2}} \ \overline{\underline{u}}_{2}^{2} = -\frac{2}{3} \varepsilon \ \overline{\underline{T}}_{q} - (\overline{\underline{u}}_{2}^{2} - \frac{1}{3} \ \overline{q}^{2})$$
(3.25)

$$\overline{U}_{1} \quad \underline{T}_{q} \quad \frac{\partial}{\partial x_{1}} \quad \overline{u_{3}}^{2} + \overline{U}_{2} \quad \underline{T}_{q} \quad \frac{\partial}{\partial x_{2}} \quad \overline{u_{3}}^{2} = -\frac{2}{3} \epsilon \quad \underline{T}_{q} - (\overline{u_{3}}^{2} - \frac{1}{3} \quad \overline{q}^{2})$$
(3.26)

Immediately it is clear that with all these simplifications we find  $\overline{u_2}^2 = \overline{u_3}^2$ . This result is a consequence of putting  $c_2^{22}$  equal to  $c_2^{33}$ . In general this will not be true and  $\overline{u_2}^2$  will be different from  $\overline{u_3}^2$ . In the following we shall only consider the equations for  $\overline{u_1}^2$  and  $\overline{u_2}^2$ . The equation for  $\overline{u_1}^2$  contains a production term;  $\overline{u_2}^2$  gets its energy through the pressure-velocity correlation (see also eqs. (3.2), (3.3)). When we subtract eq. (3.25) from eq. (3.24) we find:

$$\overline{U}_{1} \quad \underline{T}_{q} \quad \frac{\partial}{\partial x_{1}} \quad (\overline{u_{1}^{2}} - \overline{u_{2}^{2}}) + \overline{U}_{2} \quad \underline{T}_{q} \quad \frac{\partial}{\partial x_{2}} \quad (\overline{u_{1}^{2}} - \overline{u_{2}^{2}}) = -2 \quad \overline{u_{2}u_{1}} \quad \underline{T}_{q} \quad \frac{\partial \overline{U}_{1}}{\partial x_{2}} - (\overline{u_{1}^{2}} - \overline{u_{2}^{2}}) \quad (3.27)$$

With the same reasoning as for the  $-u_2u_1$  -equation we can put:

$$L_{11}^{\varkappa} \frac{\partial}{\partial x_1} \left( \overline{u_1^2} - \overline{u_2^2} \right) + L_{22}^{\varkappa} \frac{\partial}{\partial x_2} \left( \overline{u_1^2} - \overline{u_2^2} \right) + \left( \overline{u_1^2} - \overline{u_2^2} \right) = \varepsilon_q^{\varkappa} \frac{\partial \overline{u}_1}{\partial x_2}$$
(3.28)

We must remark that the memory lengths  $L_1^{\times}$  from eq. (3.23) and  $L_{11}^{\times}$  will be different from each other, according to the different time scales T and T<sub>q</sub> in the different equations. The same holds for  $L_2^{\times}$  and  $L_{22}^{\times}$ . The eq. (3.28) has the same form as the relaxation equation for  $-u_2u_1$ . Consequently, eq. (3.28) can also describe a kind of extra memory effect. So we find that the difference of the normal stresses,  $\overline{u_1^2} - \overline{u_2^2}$ , can in principle also show an extra memory effect with respect to changes in the mean-velocity gradient.

Finally we can make the following remark. From eq. (3.23) and eq. (3.28) we see that extra memory effects are small when:

$$\left| \frac{L_{1}^{\varkappa} \frac{\partial}{\partial x_{1}} (-\overline{u_{2}u_{1}})}{-\overline{u_{2}u_{1}}} \right| << 1 \text{ and } \left| \frac{L_{2}^{\varkappa} \frac{\partial}{\partial x_{2}} (-\overline{u_{2}u_{1}})}{-\overline{u_{2}u_{1}}} \right| << 1$$
(3.29)
$$\left| \frac{L_{11}^{\varkappa} \frac{\partial}{\partial x_{1}} (\overline{u_{1}^{2}} - \overline{u_{2}^{2}})}{\overline{u_{1}^{2}} - \overline{u_{2}^{2}}} \right| << 1 \text{ and } \left| \frac{L_{22}^{\varkappa} \frac{\partial}{\partial x_{2}} (\overline{u_{1}^{2}} - \overline{u_{2}^{2}})}{\overline{u_{1}^{2}} - \overline{u_{2}^{2}}} \right| << 1$$
(3.30)

In this case the relaxation equations transform into the hypothesis of Boussinesq for  $-\overline{u_2u_1}$  and for  $\overline{u_1^2} - \overline{u_2^2}$ 

$$-\overline{u_2 u_1} = \epsilon_m \frac{\partial \overline{U}_1}{\partial x_2}$$
(3.31)  
$$\overline{u_1^2} - \overline{u_2^2} = \epsilon_q \frac{\partial \overline{U}_1}{\partial x_2}$$
(3.32)

Note: The expression (3.10) implies that  $c_2$  is a constant, independent of i and j. In general it turned out to be necessary to assume that  $c_2$  is dependent of i and j. This shows in fact the inadequacy of expression (3.10).

### IV. THE RELAXATION EQUATION.

# A. Introduction.

In this chapter we will consider the relaxation equation more closely. Because in a situation where the boundary layer approximation holds the shear stress is the most important stress we will consider only the relaxation equation for  $-u_2u_1$ . So we will look at

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + L_{2} \frac{\partial}{\partial x_{2}} \left( -\overline{u_{2}u_{1}} \right) + \left( -\overline{u_{2}u_{1}} \right) = \epsilon_{m} \frac{\partial U_{1}}{\partial x_{2}}$$
(2.22)

We will summarize the advantage of this equation over the hypothesis of Boussinesq. Eq. (2.22) shows that it is not necessary for the shear stress to be locally determined by the mean-velocity gradient. The influence of the prehistory of  $-u_2u_1$  is represented to a much greater extent in eq. (2.22) than in the hypothesis of Boussinesq. The relaxation equation gives the possibility that the turbulence will behave as a hypothetical non-Newtonian fluid. We may also repeat the fact that the hypothesis of Boussinesq takes only account of a diffusive-like, i.e. gradient-type transport. Eq. (2.22) combines a diffusive-like transport and a convective-like transport. These remarks show that we can expect that the relaxation equation will have a much wider applicability than the hypothesis of Boussinesq.

# B. The memory time.

In chapter II we have said that both  $L_1$  and  $L_2$  are a kind of memory length. We will consider these memory lengths more closely. In accordance with chapter III we can write for the memory length the product of the mean velocity and a memory or relaxation time scale. So we write:

$$L_1 = \overline{U}_1 J_1 \tag{4.1}$$
$$L_2 = \overline{U}_2 J_2$$

The time scales  $J_1$  and  $J_2$  are relaxation time scales. The same can be said of the Lagrangian integral time scales  $(J_L)_1$  and  $(J_L)_2$ . So we can expect that  $J_1$  and  $J_2$  are at least proportional to  $(J_L)_1$  and  $(J_L)_2$ , and perhaps even equal to  $(J_L)_1$  and  $(J_L)_2$ . We assume in the following that these relaxation times are indeed equal to the Lagrangian integral time scales. So we get:

$$L_{1} = \overline{U}_{1} (J_{L})_{1}$$

$$L_{2} = \overline{U}_{2} (J_{L})_{2}$$

$$(4.2)$$

Formally we can also write for the memory lengths:

$$L_{1} = (\overline{U}_{1} + u_{1}')(J_{L})_{1} = \overline{U}_{1} (J_{L})_{1} + (\Lambda_{L})_{1} \simeq \overline{U}_{1} (J_{L})_{1}$$

$$L_{2} = (\overline{U}_{2} + u_{2}')(J_{L})_{2} = \overline{U}_{2} (J_{L})_{2} + (\Lambda_{L})_{2}$$
(4.3)

Here  $(\Lambda_{\rm L})_1$  and  $(\Lambda_{\rm L})_2$  are Lagrangian integral length scales. In a situation where the boundary layer approximation holds the memory length  $L_1$  will be much greater than the memory length  $L_2$ . In general  $(\Lambda_{\rm L})_1$  and  $(\Lambda_{\rm L})_2$  will be of the same order. Because  $L_1 >> L_2$  we can write  $L_1 \simeq \overline{U}_1 (\mathcal{I}_{\rm L})_1$  in expression (4.3).

The influence of the extra memory effect depends on the magnitude of the terms  $L_1 \frac{\partial}{\partial x_1} (-u_2 u_1)$  and  $L_2 \frac{\partial}{\partial x_2} (-u_2 u_1)$  with respect to the two other terms. So when the derivatives of  $-u_2 u_1$  in the  $x_1$ - and  $x_2$ -direction are very small, we expect a small extra memory effect. We expect also a small extra memory effect if  $L_1$  and  $L_2$ , or  $(J_L)_1$  and  $(J_L)_2$  are small. We know that small time scales are related to small eddies. So with the relaxation equation we find that in general small eddies will have a smaller extra memory effect than bigger eddies. This is in agreement with the experimental results described in chapter I.

# C. Extra memory effects in a self-preserving flow.

We want to consider the situation where the flow is in a self-preserving state. A self-preserving flow is a flow whose structure remains similar in the main-flow direction. The fully developed wake flow and free jet show such a self-preservation to a high degree. Now we call similarity complete when we need only one length scale and one velocity scale to let the reduced turbulent and mean-velocity profiles coincide. For incomplete similarity we need more than one scale.

If we want to consider the behaviour of the relaxation equation in a similarity situation we have to put:

$$\bar{U}_{1} = U \left(\frac{x_{1}}{L_{o}}\right)^{p} f(n)$$

$$\bar{U}_{2} = U \left(\frac{x_{1}}{L_{o}}\right)^{r} g(n)$$
(4.4)
(4.5)

-25-

$$- \overline{u_2 u_1} = U^2 \left(\frac{x_1}{L_0}\right)^s h(\eta)$$
(4.6)  
$$\eta = \frac{x_2/L_0}{(x_1/L_0)^q}$$
(4.7)

U and  $L_{o}$  are the velocity scale and the length scale. From these expressions we find, because the hypothesis of Boussinesq holds in such a situation:

We apply the expressions (4.4) - (4.8) to the relaxation equation (2.22). For this purpose we need also an expression for  $L_1$  and  $L_2$ . We put:

$$L_{1} = L_{0} \left(\frac{x_{1}}{L_{0}}\right)^{t_{1}} \ell(\eta)$$

$$L_{2} = L_{0} \left(\frac{x_{1}}{L_{0}}\right)^{t_{2}} m(\eta)$$
(4.9)
(4.10)

From the calculation we find that in the case of similarity  $t_1=1$  and  $t_2=q$ . So in all cases  $L_1$  is a linear function of  $x_1$ . For complete similarity we find q=1, so also  $L_2$  is in that case a linear function of  $x_1$ .

We will now look at the situation of a wake flow. Let the undisturbed outer flow have a velocity  $U_0$ . The mean velocity behind the cylinder is  $\overline{U}_1$ . For the two-dimensional wake we find from the Reynolds equation when there is no pressure gradient and the Reynolds number is large:

$$\overline{U}_{1} \frac{\partial \overline{U}_{1}}{\partial x_{1}} + \overline{U}_{2} \frac{\partial \overline{U}_{1}}{\partial x_{2}} = -\frac{\partial}{\partial x_{2}} \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$
(4.11)

Far behind the cylinder there is only a small difference between  $U_0$  and  $\overline{U}_1$ . The second term on the left hand side is small with respect to the first term. Consequently we can write:

$$U_{0} \frac{\partial U_{1}}{\partial x_{1}} = -\frac{\partial}{\partial x_{2}} \overline{u_{2}u_{1}}$$
(4.12)

With eq. (4.12) and by considering boundary conditions we find incomplete similarity with q = 1/2, p = -1/2, s = -1, so  $\epsilon_m$  is independent of  $x_1$  according to eq. (4.8).

For the relaxation equation in the wake flow we find:

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + \left( -\overline{u_{2}u_{1}} \right) = \varepsilon_{m} \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$
(4.13)

All the terms in this equation have in the case of similarity the same dependence upon  $x_1$ , namely  $x_1^{-1}$ . Consequently we can write:

$$L_1 \frac{\partial}{\partial x_1} (-\overline{u_2 u_1}) = A_1 (-\overline{u_2 u_1})$$
 (4.14)

or

$$\overline{u_2 u_1} = -\overline{u_2 u_1} (x_1 = 0) \exp \{A_1 \int_0^x \frac{1 \, dx_1'}{L_1(x_1')}\}$$
(4.14a)

The factor  $A_1$  is a negative constant, independent of  $x_1$ . We find from eq. (4.13) and eq. (4.14):

$$-\overline{u_2 u_1} = \frac{\epsilon_m}{A_1 + 1} \frac{\partial \overline{U}_1}{\partial x_2}$$
(4.15)

We are in a situation of similarity but it is not necessary that  $L_1 \frac{\partial}{\partial x_1} (-u_2 u_1)$  is small. From eq. (4.15) we conclude that notwithstanding a non-negligible extra memory effect the hypothesis of Boussinesq may still be used in a self-preserving flow.

However, this extra memory effect does not show up explicitly because  $\epsilon_m/(A_1+1)$  is independent of  $x_1$ .

From this result we see that the eddy viscosity according to the hypothesis of Boussinesq,  $(\boldsymbol{\epsilon}_{m})_{B}$ , is equal to  $\boldsymbol{\epsilon}_{m}/(A_{1}+1)$ . Consequently we find, because  $A_{1}$  is negative, that  $(\boldsymbol{\epsilon}_{m})_{B}$  is larger than the eddy viscosity  $\boldsymbol{\epsilon}_{m}$  according to the relaxation equation.

# D. The connection between the relaxation equation and some other theories.

In this part we will discuss the relaxation equation in the light of other theories that are concerned with the extension of the hypothesis of Boussinesq. We will also make some remarks that may contribute to a more general physical background of the relaxation equation.

We will first look at the work of Phillips  $(\underline{64})$ . Phillips assumes that instead of  $-u_2u_1$ ,  $\frac{\partial}{\partial x_2}$   $(-u_2u_1)$  is the locally determined property. So he writes for the case that the transport is mainly in the  $x_2$ direction:

$$\frac{\partial}{\partial x_2} \left( -\overline{u_2 u_1} \right) = A_2 \left( J_L \right)_2 \frac{\overline{u_2}^2}{u_2^2} \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.16)

In this expression  $A_2$  is a constant. Phillips applies this expression to a wake flow behind a cylinder and finds, in the self-preserving region, good agreement with the experiments by taking  $A_2 \approx 0.24$ . This factor is independent of  $x_2$ . Atesman (<u>65</u>) also uses this formula (4.16). He finds for a pipeflow  $A_2 \approx 0.33$  and for a homogeneous shearflow  $A_2 \approx 0.55$ .

There is a close connection between expression (4.16) and the relaxation equation. To show this we make use of the relaxation expression (2.21) and the relaxation equation (2.22).

We know that the relaxation expression is a first approximation of the relaxation equation. By using for  $-u_2u_1$  in eq. (2.22) the relaxation expression (2.21) we find a second approximation of the relaxation equation:

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + L_{2} \frac{\partial}{\partial x_{2}} \left( -\overline{u_{2}u_{1}} \right) = \epsilon_{m} \left[ L_{1} \frac{\partial}{\partial x_{1}} \left( \frac{\partial U_{1}}{\partial x_{2}} \right) + L_{2} \frac{\partial}{\partial x_{2}} \left( \frac{\partial \overline{U}_{1}}{\partial x_{2}} \right) \right]$$

$$L_{2} \frac{\partial}{\partial x_{2}} \left( \frac{\partial \overline{U}_{1}}{\partial x_{2}} \right) \right]$$

$$(4.17)$$

For the case considered by Phillips, namely that the terms with the  $x_2$ -derivative are much bigger than the terms with the  $x_1$ -derivative we get:

$$\frac{\partial}{\partial x_2} \left( -\overline{u_2 u_1} \right) = \epsilon_m \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.18)

So we find here the hypothesis of Phillips with  $e_m = A_2(J_L)_2 \overline{u_2^2}$ .

We can make a remark about  $\boldsymbol{\epsilon}_{m}$  and  $\boldsymbol{A}_{2} (\boldsymbol{J}_{L})_{2} \boldsymbol{u}_{2}^{2}$ . We know that, according to Prandtl's mixing-length theory, we can write  $\boldsymbol{\epsilon}_{m} = \boldsymbol{u}_{2}' \boldsymbol{\ell}_{t}$  with  $\boldsymbol{\ell}_{t}$  the mixing length. It is reasonable to assume that  $\boldsymbol{\ell}_{t}$  is proportional to the Lagrangian integral length scale  $(\boldsymbol{\Lambda}_{L})_{2}$  (see (<u>45</u>), p. 384, 385). For  $(\boldsymbol{\Lambda}_{L})_{2}$  we can write  $(\boldsymbol{\Lambda}_{L})_{2} = (\boldsymbol{J}_{L})_{2} \boldsymbol{u}_{2}'$ . So we find  $\boldsymbol{\epsilon}_{m} = \text{const.} (\boldsymbol{J}_{L})_{2} \boldsymbol{u}_{2}^{2}$ .

We will now look at an analogy between the relaxation equation and the equation describing a spring-mass system. We can look at  $-u_2u_1$  as a departure from an equilibrium situation  $-u_2u_1 = 0$ . When we do this we can interpret the factor  $\partial \overline{U}_1 / \partial x_2$  as an external force, a disturbance force. If we extend this idea we can find the following equation for a second order vibration of a spring-mass system in, for simplicity, one dimension

$$C_{1} \frac{\partial^{2}}{\partial x_{1}^{2}} (-u_{2}u_{1}) + C_{2} \frac{\partial}{\partial x_{1}} (-u_{2}u_{1}) + C_{3} (-u_{2}u_{1}) = \frac{\partial U_{1}}{\partial x_{2}}$$
(4.19)

The first term on the left hand side is a term that expresses the mass acceleration,  $C_1$  is a measure of the mass accelerated. The second term is a friction or quenching term,  $C_2$  is a measure of the friction. The third term is the elastic term, the factor  $C_3$  is a measure of the rigidity.

If we compare eq. (4.19) with the relaxation equation (4.13) we find the following. In the relaxation equation there is no term describing the mass acceleration. Consequently we can assume that the mass-effects will be small in the turbulence because there is also no equivalent term for the mass acceleration term in the complete transport equation of the turbulence. The other terms are present. The extra memory term is equivalent to the friction term, with the friction factor  $C_2$  equivalent to  $L_1/\varepsilon_m$ ; the shear stress is equivalent to the rigidity term, with  $C_3$  equivalent to  $1/\varepsilon_m$ . If we assume that the extra memory effect is small, then in the equivalent spring-mass system not only the acceleration term has to be neglected but also the friction term, and a relation equivalent to the hypothesis of Boussinesq is obtained. In the equivalent spring-mass system this hypothesis would describe an equilibrium between the elastic term and the external force.

We will now consider the connection between the relaxation equation and a consideration about transport processes that has been made by Corrsin (46).

By considering a one-dimensional transport by turbulence Corrsin starts from the following equation for the mean transport in the  $x_0$ -direction

$$\frac{\partial}{\partial t} \int_{-\infty}^{x_2} \overline{U}_1 (x_2', t) dx_2' = -\overline{u_2 u_1} (x_2, t)$$
(4.20)

Differentiating eq. (4.20) with respect to  $x_2$  gives:

$$\frac{\partial \overline{U}_{1}(x_{2},t)}{\partial t} = \frac{\partial}{\partial x_{2}} - \overline{u_{2}u_{1}} (x_{2},t)$$
(4.21)

Corrsin assumes that the gradient transport model of turbulence is true. By assuming gradient transport the hypothesis of Boussinesq holds:

$$-\overline{u_2 u_1} = \epsilon_m \frac{\partial U_1}{\partial x_2}$$
(4.22)

Because Corrsin considers only one-dimensional transport  $\epsilon_m$  in eq. (4.22) is independent of  $x_2$ .
By differentiating eq.(4.22) with respect to  $x_2$  and combining this result with eq. (4.21) one finds:

$$\frac{\partial \overline{U}_{1}}{\partial t} = \epsilon_{m} \frac{\partial^{2} \overline{U}_{1}}{\partial x_{2}^{2}}$$
(4.23)

This is the well known diffusion equation, and the eddy viscosity is now a kind of diffusion factor. So the hypothesis of Boussinesq is connected with a diffusion equation.

Corrsin now makes a link to the random-walk process. The complete theory of the random-walk equations is rather complex and not necessary in the following considerations. So Corrsin restricts himself to a one-dimensional process. He considers a one-dimensional frame with step size  $\delta Z$  and a time scale  $\delta t$ , necessary for taking this step. He calls p the probability that two consecutive steps are made in the same direction. He considers the behaviour of the property P.

When

$$p = 1/2$$
 and  $\lim_{\delta Z \to 0} \frac{(\delta Z)^2}{\delta t} = 2 D$ 

the limit of the random-walk process yields

$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial x_2^2} \tag{4.24}$$

On the other hand when:

p = 1 and  $\lim_{\delta Z \to O} \frac{\delta Z}{\delta t} = V$ 

the limit of the random-walk process yields

$$\frac{\partial^2 P}{\partial t^2} = V^2 \frac{\partial^2 P}{\partial x_2^2}$$
(4.25)

This equation is a wave-equation, V is the propagation velocity of the wave. Finally, when

 $\lim_{\delta Z \to 0} p = 1, \quad \lim_{\delta Z \to 0} \frac{1-p}{\delta t} = \frac{1}{2T} \text{ and } \lim_{\delta Z \to 0} \frac{\delta Z}{\delta t} = V$ 

the limit of the random-walk process gives the so-called telegraph equation:

$$\frac{\partial^2 P}{\partial t^2} + \frac{1}{T} \frac{\partial P}{\partial t} = V^2 \frac{\partial^2 P}{\partial x_2^2}$$
(4.26)

The factor T is a relaxation time. For T→∞ eq. (4.26) changes into eq. (4.25)

For  $\partial^2 P/\partial t^2 << \frac{1}{T} \partial P/\partial t$  eq. (4.26) changes into eq. (4.24) with D = V<sup>2</sup>T. Consequently we see that the diffusion equation (4.24) and the wave-equation (4.25) are limits of the telegraph equation (4.26).

Now we put  $P = \overline{U}_1$ . By using this in eq. (4.24) and combining eq. (4.24) with eq. (4.21) we find

$$\frac{\partial}{\partial x_2} \left( -\overline{u_2 u_1} \right) = \mathbb{D} \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.27)

Integrating this equation with respect to x<sub>2</sub> yields:

$$\overline{u_2 u_1} = \mathbb{D} \frac{\partial U_1}{\partial x_2}$$
(4.28)

We put  $P = \overline{U}_1$  into eq. (4.25). Next we differentiate eq. (4.21) with respect to t and we combine this result with eq. (4.25). We then find:

$$\frac{\partial}{\partial x_2} \frac{\partial}{\partial t} \left( -\overline{u_2 u_1} \right) = V^2 \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.29)

Again by integrating with respect to x<sub>o</sub> we get:

$$\frac{\partial}{\partial t} \left( -\overline{u_2 u_1} \right) = V^2 \frac{\partial \overline{U}_1}{\partial x_2}$$
(4.30)

At last we put  $P = \overline{U}_1$  into eq. (4.26) and we combine this result with eq. (4.21) and with eq. (4.21) differentiated with respect to t. Then we find:

$$\frac{\partial}{\partial x_2} \frac{\partial}{\partial t} \left( -\overline{u_2 u_1} \right) + \frac{1}{T} \frac{\partial}{\partial x_2} \left( -\overline{u_2 u_1} \right) = V^2 \frac{\partial^2 U_1}{\partial x_2^2}$$
(4.31)

By integrating eq. (4.31) with respect to  $x_{2}$  and multiplying by T we get:

$$\mathbb{T}\frac{\partial}{\partial t}\left(-\overline{u_{2}u_{1}}\right) + \left(-\overline{u_{2}u_{1}}\right) = \mathbb{V}^{2} \mathbb{T}\frac{\partial \overline{U}_{1}}{\partial x_{2}}$$

$$(4.32)$$

Finally we can write for eqs. (4.30) and (4.32) with the Taylor-hypothesis:

$$\overline{U}_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}}\overline{u_{1}} \right) = V^{2} \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$
(4.33)

and

$$\overline{U}_{1}T \frac{\partial}{\partial x_{1}} (-\overline{u_{2}u_{1}}) + (-\overline{u_{2}u_{1}}) = V^{2}T \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$
(4.34)

In the light of the foregoing the eqs. (4.28), (4.33) and (4.34) can be interpreted as follows. Eq. (4.28) is equal to the hypothesis of Boussinesq, - with  $\epsilon_{\rm m} = D$ . The "wave"-equation (4.33) describes convective transport. The "telegraph"-equation (4.34) combines the hypothesis of Boussinesq and the "wave"-equation, so this equation describes convective transport as well as diffusive transport. We see that the "telegraph"-equation (4.34) has the same form as the relaxation equation (4.13). From a comparison of eq. (4.34) with the relaxation equation (4.13) and eq. (4.2) one finds that the relaxation time T should be equal to the Lagrangian integral time scale  $(J_{\rm L})_1$ . Also  $\epsilon_{\rm m}$  should be equal to  $V^2T$ . We know that  $\epsilon_{\rm m} = {\rm const.} {\overline{u_2}}^2 (J_{\rm L})_2$ . If we assume that  $(J_{\rm L})_1 = {\rm const.} (J_{\rm L})_2$  we find that V is proportional to  $u_2$ '. So, the velocity of the turbulent "wave" is proportional to the turbulence intensity. Finally, the "wave"-equation (4.33) can only hold shortly after a severe change in  $\partial {\overline{u}_1} / \partial x_2$ .

As last part of this chapter we will consider a situation already mentioned earlier, namely that in asymmetric flows  $-u_2u_1$  can be zero at a place where  $\partial \bar{U}_1 / \partial x_2$  is not zero. This is clear from the experiments of Eskinazi and Yeh (<u>66</u>) and Kjellström and Hedberg (<u>67</u>) in a concentric annulus with a smooth and a rough wall, and the work of Hanjalic and Launder (<u>68</u>) in a channel with a smooth and a rough wall. They all find that the point of zero shear stress is situated closer to the smooth wall than the position of the maximum of the velocity. Other investigations where the point of zero shear stress does not coincide with the point where  $\partial \bar{U}_1 / \partial x_2$  is zero are the experiments of Tailland and Mathieu (<u>69</u>) in a wall jet and the work of Béquier (<u>70</u>) in a plane asymmetric jet.

The explanation that is given for this fact is that in such cases the big eddies play an important role in the transport process. From this idea Hinze (71) determines the following formula for such an asymmetric flow:

$$-\overline{u_2 u_1} = \overline{u_2^2} t^* \frac{\partial \overline{U}_1}{\partial x_2} + \frac{1}{\sqrt{2\pi}} \frac{d u_2'}{d x_2} u_2'^3 t^* \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.35)

The factor  $1/\sqrt{2\pi}$  is a consequence of the assumption that the distribution of the turbulence is normal. If the distribution is not normal this factor is a positive constant.

Launder  $(\underline{72})$  determines another formula for an asymmetric flow:

$$-\overline{u_2 u_1} = 0.4 \ (\overline{q^2})^{1/2} \ \ell \left(\frac{\partial \overline{U}_1}{\partial x_2} + \frac{\ell}{2} \frac{d\ell}{dx_2} \frac{\partial^2 \overline{U}_1}{\partial x_2^2}\right)$$
(4.36)

In eq. (4.35) t" is a traveling time of a turbulent fluid particle while in

eq. (4.36) l is a turbulence length scale. The formulas (4.35) and (4.36) are the same if the length scale l is proportional to  $u_2'$  t<sup>\*</sup>,  $\overline{q^2}$  is proportional to  $\overline{u_2}^2$  and the time t<sup>\*</sup> is independent of  $x_2$ . By assuming that in the normal situation the hypothesis of Boussinesq holds we can write for eqs. (4.35) and (4.36):

$$-\overline{u_2 u_1} = \epsilon_m \frac{\partial \overline{U}_1}{\partial x_2} + \frac{1}{2\sqrt{2\pi}} \frac{d \overline{u_2}^2}{d x_2} \epsilon_m \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.37)

$$-\overline{u_2 u_1} = \varepsilon_m \frac{\partial \overline{U}_1}{\partial x_2} + \frac{1}{4} \frac{d\ell^2}{dx_2} \varepsilon_m \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.38)

These expressions both give the possibility that  $-\overline{u_2u_1} = 0$  when  $\partial \overline{U}_1 / \partial x_2 \neq 0$ by introducing a transverse extra memory effect with the aid of  $\partial^2 \overline{U}_1 / \partial x_2^2$ .

A zero value of the shear stress when  $\partial \overline{U}_1 / \partial x_2 \neq 0$  can also be concluded from the relaxation equation (2.22). We get, written in the same way as eq. (4.37) and eq. (4.38):

$$-\overline{u_2 u_1} = \epsilon_m \frac{\partial U_1}{\partial x_2} - L_1 \frac{\partial}{\partial x_1} (-\overline{u_2 u_1}) - L_2 \frac{\partial}{\partial x_2} (-\overline{u_2 u_1})$$
(4.39)

We can use the relaxation expression (2.21) in this case also. We then get:

$$-\overline{u_2 u_1} = \varepsilon_m \frac{\partial \overline{U}_1}{\partial x_2} - L_1 \varepsilon_m \frac{\partial^2 \overline{U}_1}{\partial x_1 \partial x_2} - L_2 \varepsilon_m \frac{\partial^2 \overline{U}_1}{\partial x_2^2}$$
(4.40)

We see that eq. (4.39) and eq. (4.40) contain a term with a derivative in the  $x_1$ -direction, in contrast with eq. (4.37) and eq. (4.38) where such a term is absent. However, in general, for example in a fully developed turbulent boundary layer,  $L_1 \frac{\partial}{\partial x_1}$  will be of the same order as  $L_2 \frac{\partial}{\partial x_2}$ . It seems necessary in general when second order effects in the  $x_2$ -direction are considered also to take into account second order effects in the  $x_1$ -direction.

It is evident that also eq. (4.39) and eq. (4.40) are in principle able to describe the situation where  $-u_2u_1 = 0$  when  $\partial \overline{U}_1/\partial x_2 \neq 0$ . However, the effect is small and consequently the last two terms in the right hand side of eq. (4.39) and eq. (4.40) are small. On account of this fact, and the lack of sufficiently detailed experimental information we were, unfortunately, not able to apply eq. (4.39) and eq. (4.40) to the mentioned experiments.

#### V. THE EXPERIMENTAL EQUIPMENT AND MEASURING METHODS.

In order to investigate extra memory effects three different experiments have been carried out. Investigations have been done in a disturbed turbulent boundary-layer, in the wake just behind a cylinder and in the flow behind a grid.

The investigation of the turbulent boundary-layer has been carried out in a low-turbulence wind tunnel of the closed circuit type. The free stream turbulence is about 0.02 %. The working section is 4.5 m long and has a cross-section of 0.9 x 0.7 m<sup>2</sup>.

The boundary layer studied is on one side of a glass plate, put vertically in streamwise direction in the plane of symmetry of the working section. Transition from laminar to turbulent flow of the boundary layer is fixed at 0.6 m from the leading edge by means of a trip-wire placed spanwise at a short distance from the wall. A hemi spherical cap of 40 mm diameter is attached on to the glass plate in a centre position at 3.65 m from the leading edge. The free-stream velocity during most of the experiments was 10.5 m/s. The thickness of the turbulent boundary-layer at the location of the hemisphere was about 50 mm.

With the traversing system used it was possible to cover a distance in the longitudinal direction of more than 0.55 m behind the hemisphere, with an accuracy of about 1 mm. In the transverse direction along the plate a distance of about 0.15 m on both sides of the centre-line behind the hemisphere could be reached, also with an accuracy of about 1mm. The distance from the plate was adjusted with a micrometer with an accuracy of about 0.01 mm.

The cylinder is placed in a wind tunnel similar to the one used for the investigation of the disturbed boundary layer. Most of the experiments have been carried out with a cylinder of diameter 40mm. Occasionally also diameters of 20mm and of 1mm have been used. The cylinder is situated at about 0.9 m from the beginning of the working section. A free-stream velocity of 10.5 m/s is used during most of the experiments.

A traversing system is used with a range in the longitudinal direction of 0.70 m and an accuracy of about 1 mm. This traversing system could be placed at every location of the working section. The distance in the transverse direction through the wake was adjusted with a micrometer to an accuracy of about 0.01 mm.

The experiments with the grid flow have been carried out in a jet-type wind tunnel with a cross section at the beginning of the jet of  $0.28 \times 0.38 \text{ m}^2$ . The grid was placed at the end of the contraction section of the wind tunnel. The relative turbulence intensity of the airflow just upstream of the grid was about 0.6%.

The manufacture of the biplane, square mesh grid used is an important factor in producing a good and stable grid turbulence with a uniform meanvelocity field. In the first place the free-area factor is important. This factor gives the ratio between the free area through which the air can pass and the total area. From the literature it is known that the flow behind a grid with a free-area factor between about 0.25 and 0.60 is unstable, because the wakes of the bars can melt together randomly, caused by small changes in the pressure. With a free-area factor greater than 0.60 the flow becomes stable and one can get a stable turbulent flow field. In this investigation a grid is used with a free-area factor of 0.66. In order to get a uniform flow field behind the grid it is very important to fabricate the grid with the highest possible accuracy. The bars that have been used are cylindrical bars made of silversteel with a diameter of 1.880 mm and a maximum deviation of 0.002 mm. The mesh width of the grid is 10.000 mm, the maximum deviation in the distance between the bars is 0.001 mm. The position of the bars was adjustable. By means of screws at the end of the bars, a sufficiently high pretension could be given to the bars to avoid any vibration of an aeolic nature.

The design of a part of the grid is given in fig. 5.1. The total area of the grid is  $0.28 \times 0.38$  m<sup>2</sup>. During the experiment the velocity just before the grid was 10.5 m/s.

A traversing system is used with a range in the longitudinal direction of 0.9 m. The position can be adjusted with an accuracy of about 0.5 mm. The adjustment in the other directions is done with the aid of micrometers with an accuracy of about 0.01 mm.

The mean-velocity and turbulence measurements have been carried out with hot-wire anemometers operating at a constant temperature. The hot wire is placed in a wheatstone bridge with a feedback system. The output of the electronic system was at D.C.-level. The wires are platinum-coated tungsten of 5  $\mu$ m diameter. In most cases the length of the sensitive part was 1 mm. The distance between the prongs is 10 mm. Use is made of single wires perpendicular to the mean-flow direction and single wires in a swan-neck shaped holder in order to be able to measure the shear stress and the transverse component of the turbulence. For the measurement of the instantaneous shear stress and the instantaneous transverse turbulent component X-wires have been used. For the X-wires the distance between the prongs is also 10 mm. The distance between the wires had a diameter of 5  $\mu$ m and a length of 1 mm. The distance between the wires is about 0.5 mm. In the investigation normal pitot

-35-

#### tubes and a preston tube were also used.

The mean velocity is measured with a digital voltage meter. The turbulence is measured with a R.M.S. meter, Brüel and Kjaer type 2417. This R.M.S. meter has a flat response up to about 2.10<sup>4</sup> Hz and a low frequency -3dB point of 0.6 Hz. In order to determine energy spectra an ampex recorder model Fr-1900 and a band-pass filter Brüel and Kjaer type 1612 have been used. Sometimes the determination of the energy spectra was done on a P.D.P.-11 computer by means of a fast-fourier transformation. The analogue signal is then transformed into a digital signal. For the determination of peaks in the spectra, incidentally, use was made of a Brüel and Kjaer frequency analyzer type 2121. The correlation and probability-distribution measurements have been carried out with a 100-points Saicor-correlator type 5A1-42A. With a Disa-correlator type 55A06 some space-correlations have been measured.

In the following we will consider the measuring methods by which some turbulent properties are determined. First we look at the measurements made with the X-wire. The distance of 0.5 mm between the two wires is too small at a wire thickness of 5 µm when the wires are used in a position in which one wire is downstream of the other. However, when measuring the shear stress and the transverse turbulent component the wires are placed in planes in the mean-flow direction so that there is practically no aerodynamic interference between the wires. By measuring with the X-wire, the output of one of the wires is multiplied by a certain value in order to make the different sensitivities of the wires equal. This gives the possibility of determining -u\_u1 and uo directly. A difficulty is formed by the fact that the mean amplitude of the turbulent signal must exceed a certain value in order for the electronic equipment to work well. The turbulent signal is therefore multiplied by a certain factor. However, because the turbulent signal is very peaked, especially the signal of  $-u_0u_1$ , the peaks of the signal after multiplication are often too large for the electronic equipment. The multiplication factor must thus be chosen between these two boundaries. Because this is not always possible, errors in the output can occur.

Next we will discuss the measurements of the energy spectrum. Only spectra of  $\overline{u_1^2}$ , determined with a single wire, have been measured. In the first experiments the spectra were determined by leading the turbulent signal through a band-pass filter. During the experiments however it turned out to be necessary to determine the energy spectra for very low frequencies. Because the band-pass filter has a lowest band-pass with a centre frequency of 12.5 Hz it was decided to make use of a recorder. The turbulent signal was recorded on a tape over a period of 640 seconds. Then the tape was replayed

with a speed 32 times faster than the speed at which the signal was recorded. The resulting signal was then led through the band-pass filter. The averagingtime of 640/32 = 20 seconds appeared to be sufficient, the resulting lowest frequency is  $12.5/32 \simeq 0.39$  Hz.

Time-correlations with a single wire or a X-wire can be directly determined by means of the Saicor-correlator. More difficult is the determination of space-time correlations. Most of the space-time correlations have been measured with two X-wires. The first X-wire is fixed at a certain place. Then the second X-wire is placed behind the first wire. The position where the second wire is right behind the first wire is determined by measuring the space-time correlation. For, the space-time correlation must show a maximum value when the second wire is situated right behind the first wire. The values of the space-time correlation for small t are uncertain because the first wire influences the second wire, although not severely. However, due to the finite size of the X-wires it is impossible to determine the space-time correlation for very small t.

In the following chapters we will discuss the results of the experimental investigations. First we will consider the turbulent boundary-layer, then the wake flow and finally the grid turbulence.

## VI. THE TURBULENT BOUNDARY-LAYER.

## A. The undisturbed boundary-layer.

In the first place we will discuss the results of the investigation of the turbulent boundary-layer.

The coordinate system is taken with the origin in the centre of the base of the cap, with the  $x_1$ -coordinate in streamwise direction, the  $x_2$ -coordinate perpendicular to the plate and the  $x_3$ -coordinate in spanwise direction. The measurements have been made at a number of  $x_1$ -stations and in places with different values of  $x_3$ , including such a large value of  $x_3$  that the undisturbed layer at a given  $x_1$ -distance could be investigated.

We will first look at the results of the undisturbed boundary-layer. Measurements have been made of the mean velocity  $\overline{U}_1$ , the turbulence intensities  $u_1'$  and  $u_2'$  and the shear stress  $-\overline{u_2u_1}$ . Also measurements have been made of the wall shear stress, the energy spectrum of the  $u_1$ -component, the longitudinal integral scale and of some space-time correlations and probability distributions.

In fig. 6.1 the result is shown of the mean-velocity profile. The profile shows the normal behaviour. Namely near the wall a zone where the velocity is proportional to  $x_2$ , further away a logarithmic zone with a buffer region in between, for  $5 \le u^{\varkappa} x_2/\nu \le 25$ . The logarithmic zone is well described by:

$$\frac{U_1}{u^2} = A \ln \frac{u^2 x_2}{v} + B$$
(6.1)

with a value of A = 2.44 and B = 5.15. The factor  $u^{\times}$  is the wall shear stress velocity. The determination of  $u^{\times}$  will be discussed later. The value of B = 5.15 is almost equal to the value of 5 suggested by Huffman and Bradshaw (<u>73</u>). However, the spread in the value of B found in the literature is rather great. The factor A has the common value of 2.44 (see for further details Hinze (45) p. 626-630).

Fig. 6.2 gives the results of the relative turbulence intensities  $u_1'/U_0$  and  $u_2'/U_0$ . The values of the intensities are quite close to the results of Klebanoff (74). It should be noticed that for the boundary-layer thickness the distance from the wall is used where the mean velocity is equal to 0.995 U<sub>0</sub>. In fig. 6.3 the result is given of the value of the shear stress  $-u_2u_1$ , made dimensionless with the wall shear stress  $u^{\times 2}$ . The shear stress has been measured with a rotatable single wire and with a X-wire.

The results of the shear stress measurements agree satisfactorily with the measurements of Klebanoff  $(\underline{74})$ .

By measuring with the X-wire the R.M.S.-value of the shear stress, defined by  $(-u_2u_1)' = [(-u_2u_1)^2 - (-u_2u_1)^2]^{1/2}$ , we found a practically constant value of  $(-u_2u_1)'/-u_2u_1 \approx 3$  for  $0.3 < x_2/\delta_{995} < 0.9$ . This result is in agreement with the results of Antonia  $(\underline{75})$ .

In fig. 6.4 the result is shown of the eddy viscosity  $(\epsilon_m)_B$ , determined from  $(\epsilon_m)_B = -u_2u_1/\partial \overline{u}_1/\partial x_2$ . Though the results of  $(\epsilon_m)_B/u^*\delta_{995}$  show quite a scatter, the overall agreement with the results obtained by Klebanoff  $(\underline{74})$ and Townsend  $(\underline{76})$  is rather good. No correction is made for the intermittency factor.

#### The wall shear stress.

wit

We will make some remarks about the determination of the wall shear stress. Several methods have been used. First we mention the method of Preston ( $\underline{77}$ ). This method is refined by Patel ( $\underline{78}$ ). The determination of the wall shear stress by a preston tube is the most widely adopted method. One can also determine the wall shear stress from mean-velocity measurements very close to the wall in the viscous sublayer. When one accepts that in the viscous sublayer the velocity is proportional to  $x_2$ , one can determine u<sup>2</sup> from

$$u^{\kappa 2} = v \left(\frac{\partial \overline{U}_1}{\partial x_2}\right)_{x_2} = 0 \tag{6.2}$$

In an undisturbed boundary-layer one can also determine the wall shear stress from the logarithmic part of the velocity profile (Clauser-plot method). When there is no pressure gradient the following relation holds for  $x_2/\delta_{00} \leq 0.1$ 

$$-\overline{u_2 u_1} \simeq u^{*2} \tag{6.3}$$

Again a method to determine  $u^{\times 2}$ . It is also possible to combine the method of Patel and Preston and to apply their formulas not to a preston tube, but to a measured velocity profile with an accurately known distance from the wall. One can find  $u^{\times 2}$  in this way too. Finally it is possible to determine  $u^{\times 2}$  from the change of the momentum-loss thickness  $\delta_m$  by:

$$\frac{\partial}{\partial x_1} U_0^2 \delta_m = u^{\times 2}$$

$$(6.4)$$

$$th \delta_m = \int_0^{\infty} \frac{\overline{U}_1}{U} \left(1 - \frac{\overline{U}_1}{U}\right) dx_2$$

From all these different measurements we concluded that the most reliable and accurate methods of determining  $u^{*2}$  in a constant-pressure boundary layer are the preston tube method and the determination according to expression (6.3). From these methods a value of  $u^{*}/U_{o} \simeq 0.037$  has been obtained. The values according to the other methods could differ appreciably, by up to about 15% from this value.

# The integral length scale.

For a determination of the Eulerian longitudinal integral length scale  $\Lambda_{f,1}$  two methods have been used. The first, and in the literature most frequently used, method is by determining the longitudinal space-correlation and calculating the integral scale by:

$$\Lambda_{f,1} = \int_{0}^{\infty} f(\Delta x_1) \, d\Delta x_1$$
(6.5)

with  $f(\Delta x_1) = \overline{u_1(x_1) u_1(x_1 + \Delta x_1)/u_1^2}$ , the space-correlation in the  $x_1$ -direction between the turbulence velocity in two points separated by a distance  $\Delta x_1$ . Of course it is also possible, in order to determine  $\Lambda_{f,1}$ , to measure the Eulerian time-correlation and to calculate the space-correlation by using the Taylor hypothesis.

The calculation of  $\Lambda_{f,1}$  according to expression (6.5), however, can pose a problem. For a large interspace distance  $\Delta x_1$  it is possible that  $f(\Delta x_1)$  becomes negative. The value of  $\Lambda_{f,1}$  according to eq. (6.5) can then become so small that eq. (6.5) no longer gives a reliable value of  $\Lambda_{f,1}$ . It is common to define  $\Lambda_{f,1}$  in such a case by:

$$\Lambda_{f,1} = \int_{0}^{\Delta x_1} f(\Delta x_1) d\Delta x_1$$
(6.6)

where  $\Delta x_1^{\times}$  is the value of  $\Delta x_1$  at which  $f(\Delta x_1)$  becomes zero for the first time (see, for example  $(\underline{79})$ ).

However, an investigation showed that the value of  $\Delta x_1^*$  is dependent on the low cut-off frequency of the apparatus used (<u>98</u>). An example is given in fig. 6.5. Consequently the value of  $\Lambda_{f,1}$ , determined by expression (6.6), is dependent on the low cut-off frequency. The final result for this dependency is shown in fig. 6.6.

The other method, which is thought to be a better one, is to determine  $\Lambda_{f,1}$  by means of the one-dimensional energy spectrum. The following relation holds

$$\lim_{k_{1} \to 0} \frac{E_{1}(k_{1})}{u_{1}^{2}} = \frac{2\Lambda_{f,1}}{\pi}$$
(6.7)

Here  $k_1$  is the wave number, defined by  $k_1 = 2\pi n/\overline{U}_1$ , with n the frequency.  $E_1(k_1)$  is the one-dimensional energy spectrum of  $u_1$ . The one-dimensional spectrum has the following shape. When put in a double-logarithmic form there is a distinct part of the spectrum for low frequencies that is practically constant. For sufficient large Reynolds numbers there is a part in the higher wave number range where  $E_1(k_1)$  is proportional to  $k_1^{-5/3}$ . This part is called the inertial subrange. For still higher frequencies the spectrum shows a falloff with a dependence of at least  $k_1^{-7}$  (for further details, see (45)). We see that for low frequencies  $E_1(k_1)$  is practically constant. Consequently it is easy to determine  $\Lambda_{f,1}$  accurately. However, it must be remarked that because most of the turbulence measurements are made with the aid of a.c.coupled circuits and since the measurements take place in a restricted amount of time, in principle  $E_1(k_1)$  can go to zero when  $k_1$  goes to zero. So also the energy spectrum is influenced by the low cut-off frequency. In most cases however the constant part of the spectrum is large enough. One can easily extrapolate the constant part of the spectrum to zero Herz. In this way one can find an accurate value for  $\Lambda_{f,1}$  because one finds a value of  $\Lambda_{f,1}$  that is not directly dependent on the low cut-off frequency. So we decided to determine  $\Lambda_{f,1}$  from the results of the energy spectrum (see for a more detailed discussion (80)).

In fig. 6.7 some measured energy spectra are given. One can see that especially the energy spectrum at  $x_2/\delta_{99} \approx 0.64$  shows a part where the (-5/3)law holds. The energy spectrum close to the wall  $(x_2/\delta_{99} \approx 0.015)$  shows a part where a (-1)-law is very closely followed. At that place there is a high value of  $\partial \bar{u}_1/\partial x_2$  and consequently there is a strong interaction possible between mean and turbulent flow. According to a theory given by Tchen (<u>99</u>) the mean-velocity gradient interacts with a part of the energy spectrum and causes a dependence with  $k_1^{-1}$  of  $E_1(k_1)$  when the interaction is strong enough. The spectrum at  $x_2/\delta_{99} \approx 0.64$  shows for  $k_1 < 2$  an increase of  $E_1(k_1)$ . This is probably caused by the intermittent character of the flow at that place. By determining  $\Lambda_{f,1}$  according to eq. (6.7) this part for  $k_1 < 2$  has been neglected.

In fig. 6.8 the final result of  $\Lambda_{f,1}$  obtained from the energy spectra, is shown. We see that for  $x_2/\delta_{99} \ge 0.2$  a value of  $\Lambda_{f,1}/\delta_{99} \simeq 0.6$  is found. This value of 0.6 seems to agree with the value of  $\Lambda_{f,1}$  found from the correlation curves when we extrapolate the results to a cut-off frequency of zero Herz (see fig. 6.6). It must be remarked that this value of 0.6 is higher than the value of 0.4 that is normally found in the literature  $(\underline{81}, \underline{82}, \underline{83})$ .

## The space-time correlations.

Measurements have been made of space-time correlations of  $u_1$ ,  $u_1^2$ ,  $u_2$ ,  $u_2^2$  and  $-u_2u_1$ . The correlation of  $u_1$  has first been determined with two single wires. Those of  $u_2$  and  $-u_2u_1$  has been made with two X-wires. With the two X-wires the correlation of  $u_1$  is also determined. These correlations of  $u_1$  agreed well with the correlations found with the two single wires.

We will first discuss the results of the space-time correlation of  $u_1$ , defined by

$$R_{u_{1}}(\Delta x_{1},0,0;\Delta t) = \frac{\overline{u_{1}(x_{1};t) \ u_{1}(x_{1}+\Delta x_{1};t+\Delta t)}}{u_{1}'(x_{1};t) \ u_{1}'(x_{1}+\Delta x_{1};t+\Delta t)}$$
(6.8)

In fig. 6.9 an example of  $R_{u_1}(\Delta x_1, o, o; \Delta t)$  is shown. The different correlations show a maximum at a time  $\Delta t$  with  $\Delta t = \Delta x_1/\bar{U}_1$ . Here  $\Delta x_1$  is the distance between the two wires and  $\bar{U}_1$  is the convective velocity of the turbulence which is equal to the mean-velocity at that place, within the accuracy of the measurements. The correlation that can be formed with the different maxima will be called the envelope-correlation. This envelope-correlation is equal to the Eulerian time-correlation that can be measured in a frame that is moving with the mean velocity  $\bar{U}_1$ .

From the envelope-correlation we can determine a time scale. This time scale is, just like the Lagrangian integral time scale, a kind of relaxation time.

There are three different methods of determining a time scale from the envelope-correlation. In the first place one can define a time scale by integrating the envelope-correlation from t is zero to t is infinity. This method is used by Sabot et.al. $(\underline{84})$ . A difficulty of this method is that one has to know the envelope-correlation up to very high values of t in order to be able to determine the time scale accurately. Blackwelder et.al.  $(\underline{85})$  suggested another way to determine a time scale. He stated that for not too small t the envelope-correlation can be described by a simple exponential curve. The third method is to assume that it is possible to describe the whole envelope-correlation by a single exponential curve. However, Favre  $(\underline{100})$  has found from experiments behind a grid that in general neither the first part, nor the whole envelope-correlation curve can be described by an exponential function. Notwithstanding this fact we will assume that it is possible to

describe the envelope-correlation approximately by one single exponential curve. In the following we will determine a time scale according to this third method.

It proved that the time scales obtained by the first and the second methods are almost equal and are proportional to the time scale  $\Lambda_{f,1}/u_1'$  (<u>84</u>). The time scale according to the third method will in general be somewhat smaller than the time scales of the other methods. In fig. 6.9 one can see that the envelope-correlation is fairly well described by a single exponential curve. The differences between different determinations of the spacetime correlation are shown by an approximate error-area.

The result of  $J_u$  in a boundary layer is given in fig. 6.10. We see that for  $x_2/\delta_{99} \gtrsim 0.6^{1}$  the time scale decreases. This is probably caused by the intermittency of the turbulence there. For  $x_2/\delta_{00} < 0.6$  we find:

$$J_{u_1} \simeq 0.9 \Lambda_{f,1}/u_1'$$
 (6.9)

Sabot (<u>84</u>) and Blackwelder (<u>85</u>) found in respectively a pipe flow and a boundary layer  $J_{u_1} \simeq \Lambda_{f,1}/u_1'$ .

Measurements have also been made of the correlation of  $u_1^2$  defined by:

$$R_{u_1^2}(\Delta x_1, 0, 0; \Delta t) = \frac{\overline{u_1^2(x_1; t) \ u_1^2(x_1 + \Delta x_1; t + \Delta t)}}{\overline{u_1^2(x_1; t) \ u_1^2(x_1 + \Delta x_1; t + \Delta t)}}$$
(6.10)

An example is given in fig. 6.11. In order to avoid difficulties concerning the exact value of  $R_{u,2}$  for  $\Delta x_1 = 0$  and  $\Delta t = 0$ , we have chosen the normalized value of 1 for  $R_{u,2}$  in that case.

From the measured auto-correlations the following value of  $J_{u_2}$  is obtained

$$J_{u_1^2} \simeq 0.37 \Lambda_{f,1}/u_1'$$
 (6.11)

It must be remarked that the determination of the correlation of  $u_1^2$  and consequently of  $J_{u_1^2}$  is rather difficult because of the high, peaked values of  $u_1^2$  that occur in the signal. From expression (6.9) and (6.11) we find

$$J_{u_1} \simeq 2.4 J_{u_1^2}$$
 (6.12)

One can ask whether there is a simple relation between the correlation of  $u_1$  and  $u_1^2$ . When the probability distribution of  $u_1$  is Gaussian, the following expression holds (see also (86))

$$R_{u_1^2} = 1 + 2(R_{u_1})^2$$
(6.13)

The correlation of  $R_u$  has a value between one and zero. The maximum value of  $R_{u_1^2}$  is  $\overline{u_1^4}/(\overline{u_1^2})^2$  and this is three when the probability distribution of  $u_1$  is Gaussian. So  $R_{u_1^2}$  has a value between three and one. If we use the assumption that the envelope-correlation can be described by an exponential function we can write

$$\begin{array}{c} -t/J_{u_{1}} \\ R_{u_{1}} \simeq e \\ -t/J_{u_{1}}^{2} \\ R_{u_{1}}^{2} \simeq 2 e \\ \end{array}$$
(6.14)

When we put expression (6.14) and (6.15) in (6.13) we find:

$$J_{u_1} \simeq 2 J_{u_1^2}$$
 (6.16)

This can only be true when the probability distribution of u<sub>1</sub> is Gaussian. The value of 2 is roughly of the same magnitude as the experimentally determined value of 2.4. In the part concerning the disturbed boundary-layer we will make some more remarks about this result.

The space-time correlation of u<sub>o</sub> is defined by:

$$R_{u_{2}}(\Delta x_{1}, 0, 0; \Delta t) = \frac{\overline{u_{2}(x_{1}; t) \ u_{2}(x_{1} + \Delta x_{1}; t + \Delta t)}}{u_{2}'(x_{1}; t) \ u_{2}'(x_{1} + \Delta x_{1}; t + \Delta t)}$$
(6.17)

An example is given in fig. 6.12. From the experiments we found:

$$J_{u_2} \simeq 0.35 \Lambda_{f,1}/u_1'$$
 (6.18)

Consequently we find the following relation between the time scale of  ${\rm J}_{{\rm u}_1}$  and  ${\rm J}_{{\rm u}_2}$ 

$$J_{u_1} \simeq 2.6 J_{u_2}$$
 (6.19)

The correlation of  $u_2^2$  is defined in the same way as expression (6.10). An example of this correlation is given in fig. 6.13. From the results we find

$$J_{u_2^{2^{\sim}}} = 0.3 \Lambda_{f,1}^{/u_1'}$$
 (6.20)

Consequently we find

$$J_{u_2} \simeq 1.2 J_{u_2^2}$$
 (6.21)

This value differs from the value of 2 in expression (6.16). It must be ad-

mitted, however, that the value of  $J_{11,2}$  is rather uncertain.

Finally we measured the space-time correlation of -u2u1 defined by

$$R_{-u_{2}u_{1}}(\Delta x_{1},0,0;\Delta t) = \frac{-u_{2}u_{1}(x_{1};t) - u_{2}u_{1}(x_{1}+\Delta x_{1};t+\Delta t)}{-u_{2}u_{1}'(x_{1};t) - u_{2}u_{1}'(x_{1}+\Delta x_{1};t+\Delta t)}$$
(6.22)

An example is given in fig. 6.14. From the experiments we find:

$$J_{-u_2u_1} \simeq 0.3 \Lambda_{f,1}/u_1'$$
(6.23)

This leads to:

$$J_{u_1} \simeq 3 J_{-u_2 u_1}$$
 (6.24)

By comparing the results of  $J_{u_2}$  and  $J_{-u_2u_1}$  with the results of Blackwelder (85) and Sabot (84) one finds a rather large difference. Instead of the value of 2.6 in eq. (6.19) they found a value of 3.5; instead of 3 in eq. (6.24) they found 1.5. However, it must be remarked that their definition of the correlations is different from the definition used in this investigation. Blackwelder (85) defined:

$$R_{u_{i}u_{j}}(\Delta x_{1},0,0;\Delta t) = \frac{\overline{u_{i}(x_{1};t) \ u_{j}(x_{1}+\Delta x_{1};t+\Delta t)}}{u_{i}'(x_{1};t) \ u_{j}'(x_{1}+\Delta x_{1};t+\Delta t)}$$
(6.25)

(no summation convention). Sabot defined:

$$R_{u_{2}u_{1}}(\Delta x_{1},0,0;\Delta t) = \frac{[u_{1}+u_{2}](x_{1};t)u_{1}(x_{1}+\Delta x_{1};t+\Delta t) - [u_{1}-u_{2}](x_{1};t)u_{1}(x_{1}+\Delta x_{1};t+\Delta t)}{2u_{2}'(x_{1};t)u_{1}'(x_{1}+\Delta x_{1};t+\Delta t)}$$

(6.26)

These space-time correlations do not describe the memory behaviour of the shear correlation  $-u_0u_1$ .

From the results we find that in a turbulent boundary-layer the following approximate relation between the time scales holds:

$$J_{u_1^2} \simeq J_{u_2^2} \simeq J_{u_2^{u_1}}$$
 (6.27)

When considering the values of the time scales it should be noticed that the experimentally determined values of these time scales must be looked at as rough ones and certainly not as accurate values.

#### B. The disturbed boundary-layer.

From the measurements of the mean velocity the picture of the mean flow

pattern as shown in fig. 6.15 has been obtained. The point S on the wall marks the region of reattachment. S is situated at a value of  $x_1$  of about 60 mm. This means that the reattachment takes place already at  $x_1/h \approx 3$  behind the object with height h. This very quick return to the wall is caused by the coanda-effect of the spherical surface of the cap. Because of this quick return the disturbance of the boundary layer is rather sharp and severe. The two streamwise trailing vortices in fig. 6.15 originate from the corner eddies present in the corner formed by the cap and the wall. Though the velocities induced by these trailing vortices are rather weak, the maximum values being about two percent of the free-stream velocity, they have a noticeable effect. The centres of the trailing vortices move slowly outward in both spanwise and traverse direction. In the region between  $x_1 = 0.35$  m and  $x_1 = 0.50$  m the  $x_3$ -coordinate of these centres varies roughly from about 23 mm to about 27 mm, the  $x_2$ -coordinate varies from about 10 mm to about 20 mm.

In fig. 6.16 some examples of measured velocity profiles are given in the centre plane of the cap, i.e. at  $x_3 = 0$  m. Just downstream of the cap the mean velocity is strongly retarded. It proved that at  $x_1 = 0.50$  m the mean velocity profile has surpassed the undisturbed profile and consequently the undisturbed profile is not yet reached at that station. From fig. 6.16 we note that there is a big change of the mean-velocity gradient in the  $x_1$ direction, so it is possible that extra memory effects will occur in the main flow direction.

In fig. 6.17, 6.18 and 6.19 some results of  $\overline{u_1^2}$ ,  $\overline{u_2^2}$  and  $-\overline{u_2u_1}$  in the disturbed layer at  $x_3 = 0$  m are given. The values of the turbulence intensities and the shear stress are divided by the values that are measured in the undisturbed layer. In fig. 6.17 one can see that the biggest change in  $\overline{u_1^2}$  takes place at  $x_2 = 20$  mm (20 mm is just the height of the cap). As may be expected, more removed from the wall the return to the undisturbed layer is slower than more close to the wall. From fig. 6.18 it is clear that the value of  $\overline{u_2^2}$  is also very high close to the wall. This is caused by the wake-like character that the flow shows behind the cap. Finally in fig. 6.19 the results are shown of  $-\overline{u_2u_1}$ . Here too the biggest change takes place at  $x_2 = 20$  mm. From the results it is clear that the behaviour of  $\overline{u_1^2}$ ,  $\overline{u_2^2}$  and  $-\overline{u_2u_1}$  at  $x_2 = 20$  mm is quite similar.

In fig. 6.20 the results are given of the wall shear stress. These results have been obtained with the preston tube and with the calculation method based on the formulas of Preston and Patel. Especially close to the half-sphere the calculation method has been used. The return to the undisturbed value is rather rapid. After about four times the boundary layer thickness behind the cap the deviation from the undisturbed value is only 5%.

## The extra memory effects.

The experiments show a definite region of severe axial changes of the mean-velocity gradient where it is expected that the simple, local, Boussinesq relationship will fail. In order to show this, the value of  $(\epsilon_m)_B$  according to the hypothesis of Boussinesq has been calculated using the measured values of  $-u_2u_1$  and the graphically determined values of the mean-velocity gradient. In fig. 6.21 the results are given at  $x_1 = 0.25$  m and 0.50 m,  $x_3 = 20$  mm, together with the results of the undisturbed layer. These results have not been corrected for intermittency. From fig. 6.21 it is clear that in the disturbed situation the value of  $(\epsilon_m)_B$  is far removed from the undisturbed value.

We now assume that in this case we have a situation where the shear stress is not locally determined by the mean-velocity gradient. In chapter II we stated that the simplest way to describe an extra memory effect is by means of the relaxation expression (2.21). According to the fact that  $L_2 << L_1$ , and that we have a strong disturbance in the  $x_1$ -direction, we can simplify eq. (2.21) with  $L_1 \frac{\partial}{\partial x_1} >> L_2 \frac{\partial}{\partial x_2}$  to:

$$-\overline{u_2 u_1} = \varepsilon_m \left( \frac{\partial \overline{U}_1}{\partial x_2} - L_1 \frac{\partial}{\partial x_1} \left( \frac{\partial \overline{U}_1}{\partial x_2} \right) \right)$$
(6.28)

We can apply this formula to the experiments if we know what we have to take for  $L_1$ . We already stated that this memory length is equal to  $\overline{U}_1(J_L)_1$ . Because it is nearly impossible to measure the longitudinal Lagrangian time scale we use the following expression for  $(J_L)_1$  in order to relate this time scale to Eulerian quantities:

$$(\mathbf{J}_{\rm L})_{1} = 0.4 \Lambda_{\rm f,1}/u_{1}' \tag{6.29}$$

So  $(J_L)_1$  is proportional to the Eulerian integral length scale in the  $x_1$ direction. The value 0.4 is a mean value of the value of  $\beta = \Lambda_L/\Lambda_f$ , with  $\Lambda_L$  the Lagrangian integral length scale, that can be found from several transverse experiments known in the literature (see Hinze (<u>45</u>) p. 416-427). The value of 0.4 is also in agreement with the theoretical values found by Saffman (<u>87</u>) and Philip (<u>88</u>). This relation of Saffman (6.29) is one way to determine the memory length  $L_1$ . In the course of this investigation the relation of Saffman has been used first in order to calculate  $L_1$ . However, one can make an objection against the use of the relation of Saffman. With this relation we determine a time scale of  $u_1'$  which time scale we use in describing an extra memory effect of  $-u_2u_1$ . So it is perhaps better to use a time scale of  $-u_2u_1$  itself, instead of a time scale of  $u_1'$ . After a correlator was available, we were able to measure the space-time correlation of  $-u_2u_1$  and we could determine a time scale  $J_{-u_2u_1}$  from this correlation, as we have described in the previous section. This time scale will be a better time scale than the time scale according to Saffman. Namely the space-time correlation of  $-u_2u_1$ , which is an Eulerian correlation in a convected frame, describes the real memory behaviour of  $-u_2u_1$ , if we assume that the transverse diffusion is small. From the result (6.23) we see that  $J_{-u_2u_1}$  in this case is 25% smaller than the time scale of Saffman according to eq. (6.29). However, caused by experimental difficulties, the value of  $J_{-u_2u_1}$  is rather uncertain. Consequently, although  $J_{-u_2u_1}$  is a better time scale we will use in the following nearly always the time scale according to Saffman.

If we take in the expression (6.29) for  $\Lambda_{f,1}$  the values of fig. 6.8, then we can calculate from eq. (6.28) the value of  $\varepsilon_m$ , if we also determine the second derivative term. We must remark that several difficulties occur. In the first place it proved that  $L_1$  is not constant in the  $x_1$ -direction, so we have to take a mean value. Also the graphical determination of  $\frac{\partial}{\partial x_1} (\partial \overline{U}_1 / \partial x_2)$ is very inaccurate, especially for  $x_2/\delta_{99} > 0.6$ . Notwithstanding these uncertainties we give in fig. 6.22 the results of eq.(6.28) applied to the experiment of fig. 6.21. In this figure 6.22 we made  $\varepsilon_m$  dimensionless with the local value of u<sup>x</sup> and the boundary-layer thickness. We also divided  $\varepsilon_m$  by the intermittency factor  $\Omega$ , as determined by Klebanoff (<u>74</u>). From fig. 6.22 we see that the value of  $\varepsilon_m/u^x \delta_{99}$  for the disturbed boundary-layer calculated from the relaxation expression (6.28) is very close to the value of the undisturbed boundary-layer (for further details see Hinze et.al. (<u>89</u>)).

We must remark that this result is rather uncertain and rough. But we see that in principle it is possible to describe the observed extra memory effects by the relaxation expression (6.28). As an extra result we find with eq. (6.28) a value of  $\epsilon_m/u^{x}\delta_{99}$  that is independent of  $x_1$  and has the value of the undisturbed layer. So we have found that, when corrected for extra memory effects in the above way, the dimensionless eddy viscosity has characteristics as if it were a kind of constant fluid property of a hypothetical visco-elastic fluid, representative for the turbulence.

In chapter II we stated that the relaxation equation is the best way to describe extra memory effects, especially when the value of  $L_1$  is a function of  $x_1$ . Consequently for the disturbed layer we can try to use the relaxation equation. With the same simplification as used by expression (6.28) we find:

$$L_1 \frac{\partial}{\partial x_1} (-\overline{u_2 u_1}) + (-\overline{u_2 u_1}) = \epsilon_m \frac{\partial U_1}{\partial x_2}$$

The question arises as to what we have to take for  $L_1$  and  $\epsilon_m$ . In the foregoing we have found that  $\epsilon_m/u^{x_i}\delta_{99}$  is a constant in the disturbed layer. The value of  $\delta_{99}$  changes from about 50 mm at  $x_1 = 0.10 \text{ mup}$  to about 55 mm at  $x_1 = 0.50 \text{ m}$ . In fig. 6.20 we see that  $u^{x_2}$  changes only little in the region of  $x_1$  between 0.15 and 0.50 m. Consequently it seems that we do not make a very big error if we take a mean, constant value for  $\epsilon_m$  along the  $x_1$ -direction. For the memory length  $L_1$  we use, as stated above, the expression according to the relation of Saffman.

## The memory length L, and the energy spectra.

For the determination of  $(J_L)_1$  according to expression (6.29) we need the value of  $\Lambda_{f,1}$ . It is clear that it is not necessary that the values of  $\Lambda_{f,1}$  of the undisturbed layer will be correct in such a strongly disturbed layer. Consequently we measured the energy spectra at several places behind the cap in order to determine  $\Lambda_{f,1}$ . In fig. 6.23 some energy spectra are given. For  $x_1 \ge 0.1$  m the part of the energy spectrum for  $k_1 \ge 100 \text{ m}^{-1}$  has already returned to the undisturbed situation. At  $x_1 = 0.50$  m the whole spectrum has nearly the value of the undisturbed layer (see fig. 6.7). So again we find that the larger eddies ( $k_1 \le 30 \text{ m}^{-1}$ ) return much slower to the situation of the undisturbed boundary-layer than the smaller eddies ( $k_1 \ge 60 \text{ m}^{-1}$ ).

The energy spectra taken closely behind the hemisphere at  $x_1 = 0.075$  m and 0.15 m show a distinct peak at  $k_1 = 60 \text{ m}^{-1}$  and  $k_1 = 50 \text{ m}^{-1}$  respectively. This agrees with a frequency of about 60 Hz. The Strouhal number S that determines the shedding frequency of the so-called von Kármán-vortices is defined by:

$$S = \frac{n.d}{U}$$
(6.31)

Here n is the frequency of the vortices, d is the diameter of the object and U is the velocity before the object. When we take for U a mean value of about 8 m/s we find with d = 40 mm a Strouhal number of about 0.3. This is a higher value than the Strouhal number of a cylinder, which is 0.19 at the corresponding Reynolds number.

From the energy spectra we have determined the values of  $\Lambda_{f,1}$  according to expression (6.7). In fig. 6.24 the result is shown as a function of  $x_1$  at three different distances from the wall. Also here we find that close to the

-49-

wall the return to the undisturbed situation is quicker than more removed from the wall. The value of  $\Lambda_{f,1}$  in the disturbed layer is smaller than in the undisturbed layer. This is evident because the turbulence in the disturbed layer is changing very fast and consequently the connection between the turbulence at different places is weak and so the length scale is small.

The most severe disturbance takes place at  $x_3 = 0$  m and  $x_2 = 20$ mm (see fig. 6.19). So it seems useful to apply the relaxation equation (6.30) to this situation. From the results of  $\Lambda_{f,1}$ ,  $\overline{U}_1$  and  $u_1$ ' we can calculate the value of  $L_1$  with expression (6.29). The result is shown in fig. 6.25. With the exception of the point at  $x_1 = 0.075$  m the results of  $L_1$  for the disturbed boundary-layer are well described by:

$$L_1 \simeq 0.31 x_1$$
 (6.32)

In fig. 6.25 the values of  $L_1$  in the undisturbed layer are also given. In the undisturbed layer it seems likely that  $L_1$  is also a linear function of  $x_1$ .

# The relaxation equation.

With  $L_1 \simeq 0.31 x_1$  we can solve the relaxation equation (6.30). From the measured velocity profiles,  $\partial \overline{U}_1 / \partial x_2$  is determined graphically. For the eddy viscosity we have taken a mean value of  $\epsilon_m = 14.2.10^{-4} m^2/s$ , i.e. the value of the undisturbed boundary-layer. The solution of the relaxation equation with  $L_1 = ax_1$  reads:

$$-\overline{u_{2}u_{1}} (x_{1} = x_{1A}) = -\overline{u_{2}u_{1}} (x_{1} = x_{10})(\frac{x_{1A}}{x_{10}})^{-1/a} + \frac{x_{1A}}{\frac{-1/a}{a}} \int_{-1/a}^{x_{1A}} dx_{1}' \in \frac{\partial \overline{u}_{1}}{\partial x_{2}} (x_{1}') x_{1}'^{\frac{1}{a}} - 1$$
(6.33)

with  $-u_2u_1$  (x<sub>1</sub> = x<sub>10</sub>) the value of the shear stress at the starting point in the undisturbed layer. We can calculate  $-u_2u_1$  as a function of x<sub>1</sub> with expression (6.33). In fig. 6.26 the result of this calculation is shown. In the same figure we have also given the results of the measured values of  $-u_2u_1$  and the results of  $-u_2u_1$  calculated with the hypothesis of Boussinesq:

$$-\overline{u_2 u_1} = \varepsilon_m \frac{\partial U_1}{\partial x_2}$$
(3.31)

It is clear that with this local formula we find a big discrepancy between the measured and the calculated values of  $-u_2u_1$ . Although the results of  $-u_2u_1$  according to the relaxation equation are somewhat lower than the measured values it is clear that the relaxation equation gives values that are much closer to the measured shear stress than the values according to the hypothesis of Boussinesq. This result has been obtained with a value of  $L_1 = 0.31 x_1$ . With a value of  $L_1 \simeq 0.43 x_1$  the calculated values of  $-u_2u_1$  are equal to the measured values. This corresponds with a value of the relaxation time of  $(J_L)_1 \simeq 0.55 \Lambda_{f,1}/u_1'$ . According to the uncertainty in this factor the difference between 0.4 and

0.55 is not very serious.

From these results we may conclude that the relaxation equation gives a reasonable description of the extra memory effects considered. Also the idea of taking the Lagrangian integral time scale for the relaxation time seems to be a good guess. It also seems to be correct to take for the eddy viscosity the value of the undisturbed situation.

## The space-time correlations.

Just as in the undisturbed layer we have measured in the disturbed layer some space-time correlations. With a space-time correlation we can determine an Eulerian time scale in a convected frame. The time scale which we determine in this way is a mean value over the path along which the correlation is measured, and not a time scale at a certain point. This remark is especially important in the case of a fast changing turbulent flow field. In the undisturbed layer we found that the time scales are proportional to  $\Lambda_{f,1}/u_1'$ . In the disturbed layer we can expect that the time scales will be proportional to a mean value of  $\Lambda_{f,1}/u_1'$  along the covered distance. From the space-time correlations we have obtained the following results:

Ju	~ 0.6 <sup>A</sup> f,1 <sup>/u</sup> 1'
Jula	≃ 0.6 Å <sub>f,1</sub> /u <sub>1</sub> '
Juz	≃ 0.6 Å <sub>f,1</sub> /u <sub>1</sub> '
Ju2	≃ 0.351 <sub>f,1</sub> /u <sub>1</sub> '
Ju_2u_1	≃ 0.351 <sub>f,1</sub> /u <sub>1</sub> '

(6.34)

From these results we see that the time scale  $J_{u_1}$  is somewhat lower, with respect to  $\Lambda_{f,1}/u_1'$ , than in the undisturbed layer. The time scale  $J_{u_2}$  is higher, the time scale  $J_{-u_2u_4}$  somewhat higher than in the undisturbed layer.

There is one result that deserves some special attention. The time scales of  $J_u$  and  $J_{u,2}$  are almost equal. In the undisturbed layer the time scale of  $J_u^{-1}$  is larger than the time scale of  $J_{u,2}^{-2}$ , roughly by a factor 2.4 (see eq. (6.12)). We have already remarked that when the distribution of  $u_1$ 

is Gaussian  $J_{u_1} \simeq 2J_{u_1}^2$  (see eq. (6.16)). A reason for the observed discrepancy in the disturbed layer would be a deviation from the Gaussian distribution of  $u_1$ . So we decided to measure the probability distribution of  $u_1$  and  $u_2$  in the disturbed and the undisturbed layer. For completeness we also measured the distribution of  $-u_2u_1$ . The results are shown in fig. 6.27, 6.28 and 6.29. It must be remarked that a correction has been made for the nonlinear behaviour of the hot-wire.

In the undisturbed layer the distributions of  $u_1$  and  $u_2$  are rather close to the Gaussian distribution. In the disturbed layer there is a big departure from the Gaussian distribution. This big deviation from the Gaussian distribution can be the reason that the time scales of  $J_{u_1}$  and  $J_{u_1^2}$  are almost equal. However, we must be careful with this reasoning. Although the distribution of  $u_2$  is also far from the Gaussian distribution in the disturbed layer,  $J_{u_2}$  is about 1.7  $J_{u_2^2}$ . The very sharp distribution of  $-u_2u_1$  is caused by the intermittent character of  $-u_2u_1$ .

The most important result is the value of the time scale  $J_{-u_2u_1}$ . From eq. (6.34) we see that the time scale of  $-u_2u_1$  is only about 10% lower than the time scale of expression (6.29) which we have used in the relaxation equation. Consequently it will not make much difference whether we take in the relaxation equation the Eulerian time scale of  $-u_2u_1$  in a convected frame,  $J_{-u_2u_1}$ , or the Lagrangian integral time scale of  $u_1'$  according to Saffman.

Finally we will make a few remarks about the behaviour of the complete relaxation equation (2.22) in an undisturbed boundary-layer. In a normal boundary-layer  $L_1$  will be much greater than  $L_2$ , and the derivative in the  $x_1$ -direction will be much smaller than in the  $x_2$ -direction. Consequently in a boundary-layer approximation we have:

$$L_{1} \frac{\partial}{\partial x_{1}} \left(-\overline{u_{2}u_{1}}\right) \simeq L_{2} \frac{\partial}{\partial x_{2}} \left(-\overline{u_{2}u_{1}}\right)$$
(6.35)

There will be a noticeable memory effect in the undisturbed layer if the terms of eq. (6.35) are say about 10% of the value of  $-u_2u_1$ . It proved how-ever that in an undisturbed boundary-layer the following value holds:

$$\frac{L_1 \frac{\partial}{\partial x_1} (-\overline{u_2 u_1})}{-\overline{u_2 u_1}} \simeq \frac{L_2 \frac{\partial}{\partial x_2} (-\overline{u_2 u_1})}{-\overline{u_2 u_1}} \simeq 0 (0.01)$$
(6.36)

Consequently the relaxation equation reduces in the undisturbed layer to the normal hypothesis of Boussinesq with the normal value of the eddy viscosity.

#### A. Introduction.

We will now consider the experiments done in the wake of a circular cylinder. Close behind the cylinder the velocity in the centre of the wake is rather small. This velocity increases strongly in the  $x_1$ -direction and as a result there is a relatively rapid change of the lateral mean-velocity gradient in the  $x_1$ -direction. Consequently we may expect the extra memory effect considered, to be effective here.

The origin of the coordinate-system is chosen in the centre of the cylinder, at half-height in the wind tunnel. The  $x_1$ -direction is in the main flow direction, the  $x_2$ -direction is transverse to it and the  $x_3$ -direction is in the direction of the cylinder. The situation is shown in fig. 7.1. In fig. 7.1 also the definitions of the mean velocities considered in the following are given.  $U_0$  is the free-stream velocity,  $\overline{U}_1$  is the local velocity at a certain place behind the cylinder and  $\Delta \overline{U}_{1,m}$  is the maximum velocity difference, that is to say  $\Delta \overline{U}_{1,m} = U_0 - \overline{U}_1$  at  $x_2 = 0$  mm. We must remark that the mean flow is two-dimensional in the  $x_1-x_2$  plane.

Measurements have been made at several distances behind the cylinder. Very close to the cylinder, at  $x_1 \leq 10$  d (d is the diameter of the cylinder) the flow is very unstable. This is caused by the von-Kármán vortices. The influence of these vortices fades rapidly away in the  $x_1$ -direction. In the case of the cylinder of d = 40 mm it was not possible to use the whole working section of 4.5 m of the wind tunnel to investigate the development of the wake. After a  $x_1$ -distance of about 75d there was a small but notice-able effect of the boundary layers at the walls of the tunnel on the wake.

# B. The wake far behind a cylinder.

We will discuss some experimental results obtained behind the cylinders with d = 40 mm at  $x_1 = 70 \text{ d}$ , d = 20 mm at  $x_1 = 180 \text{ d}$  and d = 1 mm at  $x_1 = 500 \text{ d}$ . We will compare these results with the results found by Townsend behind a cylinder of d = 1.6 mm at 500 d (<u>90</u>).

Townsend has found that the mean-velocity profile behind a cylinder is in a self-preserving state at  $x_1 \ge 100$  d, and that the turbulent profile is in a self-preserving state at  $x_1 \ge 500$  d. Already from this result it is clear that there can be no simple local relation between the turbulence and the mean velocity.

In the following we will give some results of the measurements. The dis-

tances in the wake are made dimensionless with  $(x_2)_{99}$ . This is the value of  $x_2$  where  $U_0 - \bar{U}_1 = 0.01 \ \Delta \bar{U}_{1,m}$ . In the literature it is common to make the distances in the wake dimensionless with  $\sqrt{d(x_1+a)}$ , where a is the origin of similarity (see (<u>45</u>), p. 496). This is not a suitable scale in the present situation, because the measurements are not always done at such large values of  $x_1$  that the wake is in a self-preserving state. Namenly, we find behind the cylinder of  $d = 40 \ \text{mm}, (x_2)_{99}/d \simeq 2.5 + 0.06 \ (x_1/d)$  for the region  $5 < x_1/d < 70$ . So we find that  $(x_2)_{99}$  is about a linear function of  $x_1$ , instead of  $(x_2)_{99}$  proportional to  $\sqrt{x_1}$  as we expect in the self-preserving part of the wake.

In fig. 7.2 the results are given of the mean-velocity distribution. The different velocity-profiles coincide reasonably. Only the profile determined by Townsend (<u>90</u>) is somewhat narrower, but they all show the same behaviour. In fig. 7.3 the results of  $\overline{u_1^2}/\Delta \overline{U}_{1,m}^2$  are shown. The results of d = 1 mm at  $x_1 = 500 \text{ d}$  and of d = 40 mm at  $x_1 = 70 \text{ d}$  are nearly the same. However, our results of  $\overline{u_1^2}/\Delta \overline{U}_{1,m}^2$  are about 20% higher than the results of Townsend. A reason for this could be the fact that it is difficult to determine the small value of  $\Delta \overline{U}_{1,m}$  accurately. It must also be remarked that the values of  $\overline{u_1^2}/\Delta \overline{U}_{1,m}^2$  determined by Uberoi and Freymouth (<u>91</u>) in the wake of a cylinder are also about 10% higher than the results of Townsend. In fig. 7.4 the results of  $-\overline{u_2u_1}/\Delta \overline{U}_{1,m}^2$  are given. We see that there is a great discrepancy between the results of Townsend and our own measurements. In order to investigate this, consider the following reasoning.

If the mean-velocity profile and the turbulent profiles are in a selfpreserving state we can calculate  $-u_2u_1$  from the mean-velocity profile according to the following expression:

$$\frac{-\overline{u_2 u_1}}{U_0^2} = \frac{1}{2} \frac{x_2}{x_1 + a} \frac{U_0 - \overline{U_1}}{U_0} \approx \frac{1}{2} \frac{x_2}{x_1} \frac{U_0 - \overline{U_1}}{U_0}$$
(7.1)

(for further details, see Hinze (45) p. 499).

The results of the calculations according to this expression are also shown in fig. 7.4. The measured and computed values by Townsend agree nicely. Our measured and computed values for d = 1 mm agree only roughly. We see that the small differences between the mean-velocity profile of Townsend and the meanvelocity profile at d = 1 mm (see fig. 7.2) lead to considerable differences in the results of  $-u_2u_1$ . It proved that the calculated values at d = 40 mmare quite different from the measured values. This might be caused by the fact that the wake at  $x_1 = 70 \text{ d}$  will be rather far removed from the selfpreserving state.

Finally in fig. 7.5 the results of  $(\epsilon_m)_B/U_0$ d are given. The results of d = 1 mm and of d = 20 mm do not differ very much from the results of Townsend. The results of d = 40 mm at  $x_1 = 70 \text{ d}$  however are quite different. This might be another indication that at  $x_1 = 70 \text{ d}$  the self-preserving state has not yet been reached.

# C. The region close to the cylinder.

We will consider the wake of the cylinder of d = 40 mm in the region of about  $x_1 = 5 d$  up to  $x_1 = 70 d$ . The measurements are done at a free-stream velocity of 10.5 m/s.

In fig. 7.6 some velocity profiles are given at several distances behind the cylinder. It is clear that the wake close to the cylinder has a narrower profile and a smaller centre velocity with respect to the situation farther from the cylinder (except the velocity at  $x_1 = 5d$ ). Along a line  $x_{2}$ =constant in the  $x_{1}$ -direction, which is nearly a streamline, there is a distinct change of the mean-velocity gradient. In fig. 7.7, 7.8 and 7.9 the results are given of respectively  $\overline{u_1^2}/\Delta \overline{U}_{1,m}^2$ ,  $\overline{u_2^2}/\Delta \overline{U}_{1,m}^2$  and  $-\overline{u_2 u_1}/\Delta \overline{U}_{1,m}^2$  in the  $x_1$ -direction along a line  $x_2$  = constant, all relative to the values at  $x_1$ =70d. At  $x_2=30$  mm all the profiles show the same behaviour: a very high value just behind the cylinder followed by a rather sharp fall-off. At  $x_0=120$  mm the values of all the three factors grow gradually in the  $x_1$ -direction; close to the cylinder  $x_0 = 120$  mm corresponds approximately to the boundary of the wake. The results at x2=60 mm are between the above results. Close to the cylinder the values are about 2 to 2.5 times higher than the values at  $x_1=70$  d. This difference fades gradually away. It must be remarked that in the region 15d  $\lesssim$  x<sub>1</sub>  $\lesssim$  45d the value of the shear stress shows a maximum in the transverse profile at about this same distance  $(x_{2} = 60 \text{ mm})$ .

In order to investigate the extra memory effects more closely we have tried to use the relaxation equation in this case too. We can use the same equation as in the case of the disturbed boundary-layer, namely

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + \left( -\overline{u_{2}u_{1}} \right) = \epsilon_{m} \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$
(6.30)

If we want to calculate the shear stress as a function of  $x_1$  according to this equation, we must know the value of  $L_1$  and of  $\epsilon_m$ . For  $\epsilon_m$  we will take also in this case the value of the equilibrium situation, that is to say the value of the self-preserving part of the wake. For  $L_1$  we will use, as we have

stated above, the value according to the time scale of Saffman. In order to determine  $L_1$  from relation (4.2) and (6.29) we must know the value of the integral length scale  $\Lambda_{f,1}$ . Consequently measurements have been made of the energy spectra behind the cylinder.

## The energy spectra.

In fig. 7.10 results of the energy spectra are given at  $x_1 = 10$  d. From the figure we see that the value of  $\Lambda_{f,1}$  (calculated with eq. (6.7)) in the centre of the wake is higher than that further away from the centre. The spectrum at  $x_2/(x_2)_{99} \approx 0.18$  shows a peak at about 100 Hz, the spectra at  $x_2/(x_2)_{99} \approx 0.34$  and 0.73 show a peak at about 50 Hz. With a Strouhal number of 0.19 we find with expression (6.31) a Strouhal frequency  $(n_{str.})$  for this configuration of 50 Hz. So the peaks are caused by the von-Kármán vortices; at  $x_2/(x_2)_{99} \approx 0.18$  the double Strouhal frequency occurs, caused by the alternating vortices present in the centre of the wake (see fig. 7.1).

In fig. 7.11 results are shown of spectra at  $x_2/(x_2)_{99} \approx 0.5$  at different places behind the cylinder. The Strouhal peak fades away in the  $x_1^{-1}$  direction, but is still present at  $x_1 = 20$  d. The value of  $\Lambda_{f,1}$  grows in the  $x_1^{-1}$ -direction, caused by the broadening of the wake. Also here it is found that the bigger eddies ( $k_1 \leq 30 \text{ m}^{-1}$ ) return more slowly to their undisturbed state than the smaller eddies.

In order to investigate the peak in the energy spectra more closely we have used a P.D.P.-11 computer. The analogue turbulent signal, recorded on a tape, is digitalized and with the digitalized data the spectrum is calculated with the aid of a fast-fourier transformation. The results of the energy spectra determined in this way are the same as with the direct method. However, it is possible to determine the peak in the spectrum much more accurately because the effective band width of the computer is much smaller than that of the real filter. The result is shown in fig. 7.12. As we expected the peaks are very sharp. From this figure we see that, although there is a small increase in the energy for larger values of  $x_1$  for the region n <<  $n_{str}$  and n >> n str., it is difficult to conclude from these results that the energy of the Strouhal peak is transfered to the turbulence. It must be remarked that these results can also be obtained with a very accurate band-pass filter. The results obtained with the P.D.P.-11 and with the band-pass filter are the same within the accuracy of the measurements. From fig. 7.12 we find that the Strouhal frequency is about 51 Hz. With this value we get a Strouhal number of 0.194 which is close to the commonly used value of 0.19.

With the P.D.P.-11 computer we have also determined the very low frequency part of the energy spectrum. The result is shown in fig. 7.13. Down to a frequency of about 0.2 Hz the energy spectrum is nearly constant. For frequencies below 0.2 Hz a decrease of the spectrum begins caused by the A.C.-coupled circuits used. From fig. 7.13 we conclude that the complete electronic equipment has a low cut-off frequency, defined as the -3 dB-point, of about 0.1 Hz.

# The relaxation equation.

From the results of the energy spectra we have determined  $\Lambda_{f,1}$  according to expression (6.7). Some results are given in fig. 7.14. Close to the cylinder the value of  $\Lambda_{f,1}$  is small in the outer region of the wake. This is caused by the fact that the eddies there are relatively small. At  $x_1 = 70$  d the change of  $\Lambda_{f,1}$  across the wake is much smaller than at  $x_1 = 10$  d and 20 d. From this result we can conclude that in the self-preserving part of the wake,  $\Lambda_{f,1}/(x_2)_{99}$  will be equal to about 0.6 across the whole layer.

From the values of  $\Lambda_{f,1}$  we can determine the length scale  $L_1$ . The results are shown in fig. 7.15. For  $x_2 = 30$  mm we find approximately  $L_1 \approx 0.41 x_1$ , for  $x_2 = 60 \text{ mm } L_1 \approx 0.38 x_1$  and for  $x_2 = 120 \text{ mm we get } L_1 \approx 0.40 x_1^{-0.9}$ . We see that there is only a small variation of  $L_1$  across the wake. From the experiments of Townsend (<u>90</u>) one can calculate a value of  $L_1$  in the self-preserving wake of about 0.26  $x_1$ . So the  $L_1$ -curves of fig. 7.15 should show a part with a dependence on  $x_1$  with an exponent smaller than one in order to meet this slower lineair variation for large values of  $x_1$ .

In fig. 7.9 we saw that the biggest changes of  $-u_2u_1$  take place at  $x_2 = 30 \text{ mm}$  and 60 mm. So we decided to apply the relaxation equation to these two locations. For the value of the eddy viscosity we have used at both places the value of the self-preserving wake,  $(\epsilon_m)_B/U_0 d = 0.017$  (see fig. 7.5). With the above mentioned values of  $L_1$  we have calculated the shear stress according to the relaxation equation (6.30) with the solution (6.33). The results of the calculations are shown in fig. 7.16 and fig. 7.17 together with the measured values of  $-u_2u_1$  and the shear stress calculated according to the relaxation equation gives results much closer to the measured values than the results according to the hypothesis of Boussinesq to the hypothesis of Boussinesq. Again it appears that the relaxation equation with the values used for  $\epsilon_m$  and  $L_1$  gives a good description of the extra memory effects. It must be noticed that for  $x_1/d \leq 15$  the measured values of  $-u_2u_1$  are higher than the calculated values.

This is caused by the fact that the region close to the cylinder shows a very complicated and chaotic flow pattern; so it is likely that the rather simple relaxation equation will no longer hold in this region.

#### The space-time correlations.

In the wake of the cylinder also space-time correlations have been measured. In fig. 7.18-7.22 some examples are given with the fixed probe at  $x_1 = 14$  d. In the correlations of  $u_1$  and  $u_2$  the influence of the von-Kármán vortices can be easily seen. Although the von-Kármán vortices influence each correlation curve, for simplicity only the influence on the correlation for  $\Delta t = 0$  has been shown. The von-Kármán vortices have a smaller influence on the  $-u_2u_1$ -correlation. This is caused by the fact that the von-Kármán vortices have an influence on the  $u_1$ -component which is practically 90 degrees out of phase with the influence on the  $u_2$ -component.

From the correlation curves the time scales have been determined. We found that the time scales were almost independent of  $x_2$ . With the definitions used in chapter VI the following results are found for  $14 \le x_1/d \le 65$ 

(7.2)

 $J_{u_{1}} \simeq 0.4 \Lambda_{f,1}/u_{1}'$   $J_{u_{1}^{2}} \simeq 0.3 \Lambda_{f,1}/u_{1}'$   $J_{u_{2}} \simeq 0.4 \Lambda_{f,1}/u_{1}'$   $J_{u_{2}^{2}} \simeq 0.25\Lambda_{f,1}/u_{1}'$   $J_{-u_{2}^{u_{1}}} \simeq 0.35\Lambda_{f,1}/u_{1}'$ 

There is a small increase in the  $x_1$ -direction of the value of  $J_u$  and a small decrease of the value of  $J_{u_1^2}$ , both with respect to  $\Lambda_{f,1}/u_1$ '. As in the case of the disturbed boundary-layer we find in the near wake  $J_{u_1^2} \simeq J_{u_2^2}$  (see chapter VI). Although some time scales are smaller than in the case of the disturbed boundary-layer there is no big discrepancy between the two different situations. Just as in the case of the turbulent boundary-layer we find that the time scale of  $-u_2u_1$  is only about 10% lower than the time scale of Saffman, defined by expression (6.29), which time scale has been used in the relaxation equation. So also in this case it does not make much difference whether we use in the relaxation equation the time scale of Saffman or  $J_{-u_2u_1}$ .

Finally we must make some remarks about the behaviour of the complete relaxation equation in the self-preserving part of the wake. Far behind the

cylinder the velocity  $\overline{U}_1$  in the centre of the wake is nearly equal to the free-stream velocity  $U_0$ . Because of this fact and because of the fact that the wake is very wide there, the derivatives in the  $x_2$ -direction become of the same order as in the  $x_1$ -direction. The memory length  $L_1$  however is an order of magnitude greater than  $L_2$ . According to these approximations the relaxation equation in the self-preserving part of the wake becomes:

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + \left( -\overline{u_{2}u_{1}} \right) = \epsilon_{m} \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$
(6.30)

with  $L_1 \simeq U_0 (J_L)_1$ . The extra memory effects will be small if  $L_1 \frac{\partial}{\partial x_1} (-u_2 u_1)$  is small with respect to  $-u_2 u_1$ . From the results of Townsend (<u>90</u>) however one can determine

$$\frac{\left| \frac{L_{1}}{\partial x_{1}} - \frac{u_{2}u_{1}}{u_{2}u_{1}} \right| \simeq 0.2 - 0.3$$
(7.3)

So, also far behind the cylinder there must be a noticeable extra memory effect. However, according to the considerations given in chapter IV part C.it is clear that one can still use a kind of hypothesis of Boussinesq with an eddy viscosity  $(\boldsymbol{\epsilon}_{m})_{B}$  equal to  $\boldsymbol{\epsilon}_{m}/(A_{1}+1)$  with  $A_{1} \simeq -0.25$  (see eq. (4.15)). Consequently, the real equilibrium eddy viscosity  $\boldsymbol{\varepsilon}_{\mathrm{m}}$  is about 25% lower than the eddy viscosity  $\left(\boldsymbol{\varepsilon}_{m}\right)_{B}$  calculated with the normal hypothesis of Boussinesq. In the calculation of the shear stress according to the relaxation equation we have used the value of  $(\boldsymbol{\epsilon}_{m})_{B}$ . This value of  $(\boldsymbol{\epsilon}_{m})_{B}$  however belongs to the hypothesis of Boussinesq. In principle we must use in the relaxation equation the 25% lower value of  $\boldsymbol{\varepsilon}_{\rm m}.$  In that case the results of the hypothesis of Boussinesq with  $(\epsilon_m)_B$  and the results of the relaxation equation with  $\epsilon_m$  will coincide for large values of  $x_1$ . However, if we should use this value of  $\epsilon_m$ in the relaxation equation the calculated values of  $-u_2u_1$  close to the cylinder would decrease with 25%. Consequently, the agreement between the measured and the calculated values of  $-u_2u_1$  would become less. It must be remarked, however, that it is very difficult to determine the exact value of  $\boldsymbol{\varepsilon}_{m}$ . On the other hand, the relaxation equation will be always better, in principle, than the hypothesis of Boussinesq because this hypothesis neglects the existing extra memory effects.

The extra memory effects in the self-preserving part of the wake are caused by the fact that in the wake of the cylinder the turbulence always lags behind the development of the mean velocity by a constant time factor. This result is, as already stated in chapter IV, in agreement with the remarks by Tennekes and Lumley  $(\underline{1}, p.119-120)$ . They conclude from the fact that the time scale characteristic for the turbulence is equal to about 0.5 times the time scale connected with the development of the wake that the turbulence will always lag behind the development of the mean velocity. The structure of the turbulence can then only be in a self-preserving state when the turbulence ce lags a constant time factor behind the development of the mean velocity.

#### VIII. THE TURBULENT FLOW FIELD BEHIND A GRID.

The last experiment that we will discuss is the investigation of the flow field behind a grid.

The origin of the coordinate system is chosen in the grid-plane with the  $x_1$ -direction in the direction of the main flow, and the  $x_2$ - and  $x_3$ -direction along the grid. The coordinate system is shown in fig. 8.1. The experiments were carried out at a velocity measured just before the grid of 10.5 m/s. The flow at the end of the wind tunnel without a grid has a turbulence intensity of about  $u_1'/U_0 \approx 0.006$ , consequently all the turbulence behind the grid can be considered as being produced by the grid. Measurements have been done at several places behind the grid. The region of  $x_1/M \leq 15$  has been considered extensively in order to investigate the development of the grid turbulence.

# A. A theoretical remark about the turbulence stresses.

Before we consider the results of the measurements we must make some remarks about the turbulence stresses that are of importance in this case of the grid turbulence. It is possible to consider the turbulence stresses as the components of a tensor. For a two-dimensional mean-velocity field and for any axial plane in the case of an axi-symmetric flow, we can write such a tensor in the following way

$$\mathbf{T} = \begin{pmatrix} \overline{u_1^2} & -\overline{u_2 u_1} & 0\\ -\overline{u_2 u_1} & \overline{u_2^2} & 0\\ 0 & 0 & \overline{u_3^2} \end{pmatrix}$$
(8.1)

We can transform this tensor T to another coordinate system in such a way that in that special system the shear stress is zero. This special system is called the principal-axes system. We then get the following tensor

$$T_{p} = \begin{pmatrix} (\overline{u_{1}}^{2})_{p} & 0 & 0 \\ 0 & (\overline{u_{2}}^{2})_{p} & 0 \\ 0 & 0 & (\overline{u_{3}}^{2})_{p} \end{pmatrix}$$
(8.2)

The angle between the original system of eq. (8.1) and the principal-axes system of eq. (8.2) is called  $\alpha_n$ . This angle is given by:

$$\tan 2\alpha_{p} = \frac{-2u_{2}u_{1}}{u_{1}^{2} - u_{2}^{2}}$$

If  $-u_2u_1$  is zero,  $\alpha_p$  is zero and the original system coincides with the principal-axes system. With respect to the principal coordinate system there is a maximum shear stress in a plane that makes an angle of 45 degrees with the principal axes. The value of this maximum shear stress is given by:

$$(-\overline{u_2u_1})_{\max} = \frac{(\overline{u_1^2})_p - (\overline{u_2^2})_p}{2} = \{(\frac{\overline{u_1^2} - \overline{u_2^2}}{2})^2 + (-\overline{u_2u_1})^2\}^{1/2}$$
(8.4)

(8.3)

In the following we will consider the behaviour of  $\alpha_p$  and of  $(-u_2u_1)_{max}$  behind the grid.

# B. The region far behind the grid.

In the literature a lot of experiments are known about grid turbulence. They are all concerned with the region rather far behind the grid. We will only mention a few experiments. For more information one can use Hinze  $(\underline{45}, \text{chapter 3})$ . We mention the work of Uberoi  $(\underline{92}, \underline{93}, \underline{94})$ , of Comte-Bellot  $(\underline{95})$  and of Van Atta  $(\underline{96}, \underline{97})$ . The work of Van Atta is concerned with the behaviour of energy spectra and correlations. The article of Comte-Bellot gives a review of a number of grid experiments in order to investigate the anisotropy found in all the experiments. The mean value of the anisotropy factor  $u_1'/u_2'$  of all the experiments showed to be about 1.2, but also values of 1.35 occur, depending on the geometry of the grid, the Reynolds number and the distance  $x_1/M$ . Most of the investigators mention the fact that this anisotropy factor remains nearly constant in the  $x_1$ -direction, which is anyway so when  $u_1'$  and  $u_2'$  follow the same decay law.

We will first consider the homogeneity of the flow field far behind the grid. In fig. 8.2 results are given of two series of measurements of  $\overline{U}_1/U_0$  at  $x_1/M = 24$ , with M the mesh width of the grid. The bars are situated at  $x_2 = \pm 5$  mm,  $\pm 15$  mm, and so on. The two measurements show about the same behaviour. It seems that something is still to be seen of the influence of the bars. We see that at this distance differences of about 1% in the mean velocity occur. It must be remarked that the step size between the results, which is equal to about 0.002, corresponds to a difference of about 0.2  $^{\circ}/_{00}$  in the mean-voltage reading.

We now consider the development of the flow behind the grid. In fig. 8.3 the result of two series of measurements of  $\bar{U}_1/U_0$  is given as function of  $x_1$ .

We see that behind the grid  $\overline{U}_1$  is somewhat larger than U<sub>0</sub>. This is caused by the free-area reduction by the grid.

Of importance in the case of grid turbulence is the anisotropy factor. In fig. 8.4 the ratio of  $u_1'/u_2' \approx 1.17$ . Another important factor in grid turbulence is the decay of the turbulence intensities  $u_1'/\bar{U}_1$  and  $u_2'/\bar{U}_1$ . In fig. 8.5 this decay is given as a function of  $x_1/M$ . The decay of  $(\bar{U}_1/u_1')^2$  and  $(\bar{U}_1/u_2')^2$  is nearly linear with  $x_1$  for  $x_1/M \ge 10$ , with apparent origin at  $x_1/M \approx 7$ . For  $x_1/M > 45$  there is a deviation from the "linear" decay. A reason for this might be the influence of the boundaries of the flow field. However, if we were to assume the flow field to behave as a free jet the distance where the influence of the boundaries should occur would be much larger than  $x_1/M = 45$ . Of course the influence of the average boundaries themselves.

## C. The region close to the grid.

In the following we will consider the development of the grid turbulence in the region close to the grid. For this purpose we have chosen a two-dimensional situation,  $x_3 = 0$  mm, that is in a plane of symmetry where  $\partial \overline{U}_1 / \partial x_3$  is zero.

In fig. 8.6 the results of the mean-velocity profile at different distances behind the grid are given. The definitions of the velocities used in the following are also given in this figure. Note that  $\overline{U}_0$  is the average value of the velocity along the  $x_2$ -path considered at a given  $x_1$ -distance behind the grid, and  $U_0$  is the reference velocity just upstream of the grid. The centre of the bar is situated at  $x_2 = 5$ mm. The effect of the wake of the bar is clearly shown in the figure. After  $x_1/M = 8.25$  the mean velocity is practically constant.

The result of the transverse distribution of  $u_1'/\bar{U}_1$  is shown in fig. 8.7. In the free-jet part close to the grid the turbulence level is very low, in the wake the turbulence level is rather high. These differences are spread out rather quickly. The results of  $u_2'/\bar{U}_1$  (fig. 8.8) show roughly the same behaviour.

In fig. 8.9 the results of  $-u_2u_1/\overline{U}_1^2$  are shown. Close to the grid there is a small region around  $x_2 = 0$  where the shear stress is practically zero. Of course the shear stress is also zero in the centre of the wake. The shear stress should show a maximum at about the places where  $\partial \overline{U}_1/\partial x_2$  has a maximum value. That is to say at about  $x_2 = -2.5$  mm, 2.5 mm and 7.5 mm. The deviations shown in the figure are caused by the fact that the determination of the exact place where  $\partial \overline{U}_1 / \partial x_2$  has its maximum value, and the determination of the shear stress itself is rather inaccurate very close to the grid.

In order to investigate possible extra memory effects we consider the development of the turbulence in the  $x_1$ -direction at the place where  $-u_2u_1$  shows a maximum value, that is to say at  $x_2 = -2.5$  mm.

In fig. 8.10 the result of the velocity difference  $\Delta \overline{U}_{1,m}$  is shown. After about  $x_1/M = 13$  the velocity difference is practically zero.

The results of  $u_1'/\overline{U}_1$  and  $u_2'/\overline{U}_1$  are shown in fig. 8.11. The intensities  $u_1'$  and  $u_2'$  behave in the same way. In fig. 8.12 the result of  $-u_2u_1/\overline{U}_1^2$  is given. One can see that there is a very sharp fall off of the shear stress. For  $x_1/M \ge 20$  the shear stress is zero.

An important factor in the investigation of extra memory effects is the eddy viscosity  $\epsilon_m$ . So we decided to determine  $\epsilon_m$  according to the hypothesis of Boussinesq,  $(\epsilon_m)_B = -\overline{u_2u_1}/\partial \overline{u}_1/\partial x_2$ . The results are shown in fig. 8.13. For  $4 < x_1/M < 12$ ,  $(\epsilon_m)_B$  has an average value of about  $8.10^{-4} \text{ m}^2/\text{s}$ . It must be remarked that the determination of  $(\epsilon_m)_B$  for low and for higher values of  $x_1$  is rather inaccurate. However, it is of importance to know whether  $(\epsilon_m)_B$  has about a constant value in the  $x_1$ -direction also for large values of  $x_1$ . Unfortunately, we were not able to determine  $(\epsilon_m)_B$  for larger values than  $x_1/M \approx 12$ . But we can apply the following reasoning. We have already stated that  $(\epsilon_m)_B$  is proportional to  $u_2' \cdot \Lambda_{f,2}$ . From fig. 8.5 we see that for  $x_1/M > 10$ ,  $u_2'/\overline{U}_1$  is proportional to  $x_2'$  is proportional to  $x_1^{-0.5}$ . From results of the energy spectra that will be discussed later we can find  $\Lambda_{f,1} \propto x_1^{0.6}$ . If we suppose that  $\Lambda_{f,1} \propto \Lambda_{f,2}$  we find  $(\epsilon_m)_B \propto x_1^{0.1}$ . So according to this reasoning  $(\epsilon_m)_B$  will be almost constant, also for higher values of  $x_1$ .

# The value of $\alpha_p$ and of $(-u_2u_1)_{max}$ .

We are also interested in the behaviour of  $\alpha_p$  and of  $(-u_2u_1)_{max}$ . The results of  $\alpha_p$  are shown in fig. 8.14. The value of  $\alpha_p$  decreases from a value of about 35° down to a value of 0° at about  $x_1/M = 20$ . For  $x_1/M \ge 20$  the principal-axes system and the original system coincide. In fig. 8.15 the result of  $(-u_2u_1)_{max}$  is given. We see that  $\overline{U}_1^2/(-u_2u_1)_{max}$  increases linearly with  $x_1$ . According to expression (8.4)  $(-u_2u_1)_{max}$  is proportional to  $u_1^2-u_2^2$  for  $x_1/M \ge 20$ . In fig. 8.5 we have found that  $\overline{U}_1^2/\overline{u_1^2}$  and  $\overline{U}_1^2/\overline{u_2^2}$  are proportional to  $x_1$ . One can show that, because of the apparent origins of  $\overline{U}_1^2/\overline{u_1^2}$  and

 $\overline{U}_1^2/\overline{u_2^2}$  are equal,  $\overline{U}_1^2/(-\overline{u_2^2u_1})_{max}$  will be proportional to  $x_1$  also, with the same apparent origin.

To consider the difference in behaviour between  $-u_2u_1$  and  $(-u_2u_1)_{max}$  more closely, we will consider the behaviour of the correlation coefficients  $R_{21}$  and  $(R_{21})_{max}$  defined by

$$R_{21} = \frac{-u_2 u_1}{u_1' u_2'}$$
(8.5)

$$\left(R_{21}\right)_{\max} = \frac{\left(-u_2 u_1\right)_{\max}}{\left(u_1'\right)_{\max}\left(u_2'\right)_{\max}}$$
(8.6)

with

$$u_{1}')_{\max} = (u_{2}')_{\max} = \left\{ \frac{(\overline{u_{1}}^{2})_{p} + (\overline{u_{2}}^{2})_{p}}{2} \right\}^{1/2} = \left( \frac{\overline{u_{1}}^{2} + \overline{u_{2}}^{2}}{2} \right)^{1/2}$$
(8.7)

The intensities  $(u_1')_{max}$  and  $(u_2')_{max}$  are the normal stresses in the plane where the shear stress has its maximum value.

In fig. 8.16 the results of  $R_{21}$  and  $(R_{21})_{max}$  are given.  $R_{21}$  goes to zero at about  $x_1/M = 20$ .  $(R_{21})_{max}$  however goes to a constant value of about 0.16.

From these results we find that far behind the grid there is a shear stress indeed in a plane that makes an angle of 45° with the principal axes. We expected this already from the anisotropy of the turbulence. We will consider more closely the result of  $(-u_2u_1)_{max}$ . According to  $R_{21}$  one can see that there is a de-coupling between  $u_1$  and  $u_2$  beyond  $x_1/M \approx 20$ . However, in the plane that makes an angle of 45 degrees with the principal axes there is still a coupling between  $(u_1)_{max}$  and  $(u_2)_{max}$ , represented by the value of (R<sub>21</sub>)<sub>max</sub>. A de-coupling in this plane is only reached when (R<sub>21</sub>)<sub>max</sub> is zero, that is to say when there is an isotropic situation. One can make this acceptable by considering the complete transport equations for  $\overline{u_1^2}$  and  $\overline{u_2^2}$ , eqs. (3.2) and (3.3). The equation for  $\overline{u_1^2}$  has a production term,  $-u_2u_1 \partial \overline{U}_1/\partial x_2$ , the equation for  $\overline{u_2^2}$  does not contain such a term. The component  $\overline{u_2^2}$  gets its energy from the bigger component  $\overline{u_1^2}$  by means of the pressure-velocity correlation. This energy stream from  $\overline{u_1^2}$  to  $\overline{u_2^2}$  goes on, as long as  $\overline{u_1^2}$  is bigger than  $\overline{u_2^2}$ ; the pressure-velocity correlation is only zero when  $\overline{u_1^2} = \overline{u_2^2}$  (see also expression (3.10)). Consequently, when  $-u_2u_1$  is zero only the production term in the transport equation of  $\overline{u_1^2}$  is zero, but the energy stream from  $\overline{u_1^2}$  to  $\overline{u_2^2}$  goes on. Although there is a de-coupling between  $u_1$  and  $u_2$ , there is still a coupling between  $(u_1)_{max}$  and  $(u_2)_{max}$  up to the moment that the isotropic situation is reached. In the following we will see that this isotropic situation can only be reached asymptotically, that is to say this
isotropic situation will not be reached earlier than at the moment that the turbulence has disappeared and dissipated into heat.

## The relaxation equations.

We will consider the behaviour of  $\alpha_p$  and  $(-u_2u_1)_{max}$  by using the relaxation equations (2.22) and (3.28). The relaxation equation for  $-u_2u_1$  is in this case, with  $L_1 \frac{\partial}{\partial x_1} >> L_2 \frac{\partial}{\partial x_2}$ 

$$L_{1} \frac{\partial}{\partial x_{1}} \left( -\overline{u_{2}u_{1}} \right) + \left( -\overline{u_{2}u_{1}} \right) = \epsilon_{m} \frac{\partial \overline{U}_{1}}{\partial x_{2}}$$

$$(6.30)$$

The relaxation equation for  $\overline{u_1^2} - \overline{u_2^2}$  is, also with  $L_{11}^{\varkappa} \frac{\partial}{\partial x_1} >> L_{22}^{\varkappa} \frac{\partial}{\partial x_2}$ 

$$L_{11}^{\varkappa} \frac{\partial}{\partial x_1} \left( \overline{u_1^2} - \overline{u_2^2} \right) + \left( \overline{u_1^2} - \overline{u_2^2} \right) = \varepsilon_q \frac{\partial U_1}{\partial x_2}$$
(8.8)

We can make directly a remark about this equation for  $\overline{u_1^2} - \overline{u_2^2}$ . Far behind the grid the mean-velocity gradient is zero. If the term which takes account of the extra memory effect,  $L_{11}^{\varkappa} \frac{\partial}{\partial x_1} (\overline{u_1^2} - \overline{u_2^2})$ , were also zero we should find  $\overline{u_1^2} = \overline{u_2^2}$ , an isotropic situation. The experiments however show that no isotropic situation is reached. Consequently the term  $L_{11}^{\varkappa} \frac{\partial}{\partial x_1} (\overline{u_1^2} - \overline{u_2^2})$  can not be small far behind the grid and extra memory effects will play a part in this situation. We also see directly that the isotropic situation can only be reached asymptotically.

We will now consider the situation in which the extra memory effects for  $-\overline{u_2u_1}$  would be small. Then eq. (6.30) approaches the hypothesis of Boussinesq:

$$-\overline{u_2 u_1} = \epsilon_m \frac{\partial \overline{U}_1}{\partial x_2}$$
(3.31)

In analogy the relaxation equation of  $\overline{u_1^2} - \overline{u_2^2}$ , eq. (8.8), will approach the following expression in the case of negligible extra memory effects for  $\overline{u_1^2} - \overline{u_2^2}$ .

$$\overline{u_1^2} - \overline{u_2^2} = \varepsilon_q \frac{\partial \overline{U}_1}{\partial x_2}$$
(3.32)

So when the extra memory effects for  $\overline{u_1^2} - \overline{u_2^2}$  are small the difference between  $\overline{u_1^2}$  and  $\overline{u_2^2}$  is proportional to the mean-velocity gradient. However, eq. (3.32) clearly demonstrates that the first (memory) term in eq. (8.8) cannot be neglected, because eq. (3.32) would result in  $\epsilon_q \rightarrow \infty$  when  $\partial \overline{u_1} / \partial x_1 \rightarrow 0$  while  $\overline{u_1^2} - \overline{u_2^2} \neq 0$ , a physically unacceptable situation. The behaviour of  $\epsilon_q$  in the

case in which extra memory effects are negligible will be the same as the behaviour of  $\epsilon_{\rm m}$  in this case. We know that  $\epsilon_{\rm m}$  (or in the turbulent boundarylayer  $\epsilon_{\rm m}/{\rm u}^{\rm x}\delta_{99}$ ) is independent of  ${\rm x}_1$  in the case of negligible extra memory effects. Consequently we can write for that case

$$\epsilon_{\rm m} = A(x_2)\epsilon_{\rm q} \tag{8.9}$$

In the case of a turbulent boundary-layer we find  $A(x_2) \approx 0.4$ , almost across the whole layer. If we put the results of eqs. (3.31), (3.32) and (8.9) in the expression for  $\alpha_p$ , eq. (8.3), we find

$$\tan 2 \alpha_{p} = 2 A(x_{2})$$
 (8.10)

Consequently, in the case of an undisturbed boundary-layer and in the case of a self-preserving flow,  $\alpha_p$  should have a constant value in the  $x_1$ -direction. From fig. 8.14 it is clear that  $\alpha_p$  is not constant in the case of the grid turbulence for  $x_1/M \le 20$ . So there is a distinct extra memory effect of  $-u_2u_1$  or of  $\overline{u_1^2} - \overline{u_2^2}$  or of both.

We will now consider the behaviour of  $(-u_2u_1)_{max}$ . For  $x_1/M \ge 20$  we can write for eq. (8.8)

$$L_{11}^{*} \frac{\partial}{\partial x_{1}} \left( \overline{u_{1}^{2}} - \overline{u_{2}^{2}} \right) + \left( \overline{u_{1}^{2}} - \overline{u_{2}^{2}} \right) = 0$$
(8.11)

The value of  $(-u_2u_1)_{max}$  is given by expression (8.4). Because beyond  $x_1/M = 20$  the original system and the principal-axes system coincide we can write

$$L_{11}^{*} \frac{\partial}{\partial x_{1}} \left( -u_{2}u_{1} \right)_{\max} + \left( -u_{2}u_{1} \right)_{\max} = 0$$
(8.12)

This equation gives the behaviour of  $(-\overline{u_2u_1})_{max}$  for  $x_1/M \ge 20$ .

# The extra memory effect of -u2u1.

In order to be able to calculate the extra memory effects described by eq. (6.30) and eq. (8.8) we need the memory lengths  $L_1$  and  $L_{11}^{\times}$  and the "eddy viscosities"  $\epsilon_m$  and  $\epsilon_q$ . For the memory length  $L_1$  we have always taken  $L_1 = \overline{U}_1 (J_L)_1$ . The time scale  $(J_L)_1$  can be calculated according to the relation of Saffman, (6.29). In order to determine  $\Lambda_{f,1}$  the energy spectra were measured. In fig. 8.17 some results are given.

The energy spectra show a very great part that has a nearly constant value up to about  $k_1 = 100 \text{ m}^{-1}$ . There is no part in the spectra where a (-5/3)-law holds. This is caused by the fact that the Reynolds number in this case is rather low, there is no inertial subrange and consequently there is no -5/3

part in the spectra. We see that for  $k_1 < 100 \text{ m}^{-1}$  it takes a longer time to reach an equilibrium situation than for the higher values of  $k_1$ . So again we find that the bigger eddies need a longer time to reach an equilibrium situation than the smaller eddies.

With the value of  $\Lambda_{f,1}$  determined from the energy spectra according to relation (6.7) we can calculate  $L_1$ . The results are shown in fig. 8.18. In order to be able to calculate the shear stress from eq. (6.30) it is necessary to know the equilibrium value of  $\epsilon_m$ . The question arises whether the value shown in fig. 8.13 of  $\epsilon_m \simeq 8.10^{-4} \text{ m}^2/\text{s}$  is the real equilibrium value or not. If this value of  $\epsilon_m$  is the equilibrium value we see that the hypothesis of Boussinesq will describe the measured values of the shear stress rather well and consequently the extra memory effects will be small. However, it is also possible that there are still extra memory effects. As we have seen in the wake flow, this is only the case when

$$L_1 \frac{\partial}{\partial x_1} \left( -\overline{u_2 u_1} \right) = A_1 \left( -\overline{u_2 u_1} \right)$$

$$(4.14)$$

with  $A_1$  independent of  $x_1$ . From the experiments and the value of  $L_1$  we are able to calculate the factor  $A_1$ . It proved that  $A_1$  changes from about -0.12 at  $x_1/M = 2$  up to -0.03 at  $x_1/M = 9$ . Consequently as far as  $-u_2u_1$  is concerned, in grid turbulence the corresponding extra memory effects are negligibly small in contrast to the wake flow. We can conclude from this result and the earlier mentioned behaviour of  $\alpha_p$  that in consequence there must be an extra memory effect of  $\overline{u_1^2} - \overline{u_2^2}$ . So it proved that different turbulent quantities can behave in a different way with respect to the extra memory effects.

Just as in the case of the boundary layer and the wake flow we have tried to determine the space-time correlations of  $u_1$ ,  $u_1^2$ ,  $u_2$ ,  $u_2^2$  and  $-u_2u_1$ . Some results of the correlations are given in fig. 8.19-8.23. From the space-time correlations we have determined the corresponding time scales. In the region  $4.5 < x_1/M < 26.5$  we found the following results

$$J_{u_{1}} \approx 0.4 \Lambda_{f,1}/u_{1}'$$

$$J_{u_{1}}^{1} \approx 0.2 \Lambda_{f,1}/u_{1}'$$

$$J_{u_{2}}^{1} \approx 0.3 \Lambda_{f,1}/u_{1}'$$

$$J_{u_{2}}^{2} \approx 0.2 \Lambda_{f,1}/u_{1}'$$
(8.13)

The result of the time scale of  $-u_2u_1$  deserves special attention. At  $x_1/M = 4.5$  we obtained  $J_{-u_2u_1} \approx 0.25 \Lambda_{f,1}/u_1'$ , at  $x_1/M = 9$  (not shown) we obtained  $J_{-u_2u_1} \approx 0$ . For  $x_1/M = 0$  there is no turbulence, so we can expect that  $J_{-u_2u_1}$  is zero there.

For  $x_1/M \ge 10$  the value of J is indefinite. We suppose that J  $u_2u_1$  increases from zero at  $x_1/M = 0$  up to a maximum value of about 0.25  $\Lambda_{f,1}/u_1$ ' at  $x_1/M \cong 5$ , then decreases to zero at  $x_1/M \cong 10$ . From this result we see that J is not equal to the time scale of Saffman which we have used in the relaxation equation, namely  $J_{L,1} = 0.4 \Lambda_{f,1}/u_1$ ' for the whole  $x_1$ -area. Because  $J_{-u_2u_1} < J_{L,1}$  we find with the time scale J a still smaller extra memory effect than we had already found with  $J_{L,1}$ . So also with J  $u_2u_1$  we find that  $-u_2u_1$  shows a negligible extra memory effect behind the grid.

# The extra memory effect of $\overline{u_1^2} - \overline{u_2^2}$ .

We now consider the behaviour of  $\overline{u_1^2} - \overline{u_2^2}$ . We have already concluded that the extra memory effects of  $\overline{u_1^2} - \overline{u_2^2}$  can not be small. We consider these extra memory effects according to the relaxation equation of  $\overline{u_1^2} - \overline{u_2^2}$ eq. (8.8). In order to be able to calculate  $\overline{u_1^2} - \overline{u_2^2}$  according to eq. (8.8) we need the values of  $L_{11}^{\times}$  and  $\epsilon_q$ . We will first consider the memory length  $L_{11}^{\times}$ . For  $x_1/M > 15$ , eq. (8.8) changes into eq. (8.11). We can try to determine  $L_{11}^{\times}$  according to this equation. From fig. 8.5 we see that we can put for  $x_1/M \ge 10$ 

$$\frac{\overline{U}_{1}^{2}}{u_{1}^{2}} \simeq \frac{U_{0}^{2}}{u_{1}^{2}} = a_{1} (x_{1}-b_{1})$$

$$\frac{\overline{U}_{1}^{2}}{u_{2}^{2}} \simeq \frac{U_{0}^{2}}{u_{2}^{2}} = a_{2} (x_{1}-b_{2})$$
(8.14)
(8.14)
(8.14)

Again from fig.8.5 we see that  $b_1=b_2=b$ . If we put eq. (8.14) and (8.15) into eq. (8.11) we find

$$L_{11}^{\varkappa} = x_1 - b$$
 (8.16)

Consequently for  $x_1/M \ge 15$ ,  $L_{11}^{\times}$  is a linear function of  $x_1$ .

According to what we have done earlier there is also another method to determine  $L_{11}^{\varkappa}$ . The memory length  $L_{11}^{\varkappa}$  will be equal to

$$L_{11}^{*} = \bar{U}_{1} J_{u_{1}^{2} - u_{2}^{2}}$$
(8.17)

The time scale  $J_{u_1^2-u_2^2}$  can in principle be determined from the space-time correlation of  $u_1^{2^1} - u_2^{2^2}$ . So we decided to try to measure this correlation. An example of a correlation of  $u_1^2 - u_2^2$  is given in fig. 8.24. For the region 4.5 <  $x_1/M$  < 30 we find

$$J_{u_1^2 - u_2^2} \simeq 0.15 \Lambda_{f,1} / u_1'$$
(8.18)

One can ask if there is a connection between  $J_{u_1^2}, J_{u_2^2}$  and  $J_{u_1^2-u_2^2}$ . To investigate this we suppose that we can write as an approximation

$$\frac{-\tau/J_{u_1^2}}{u_1^2(t) u_1^2(t+\tau)} = \frac{-\tau/J_{u_1^2}}{u_1^4} e^{-\tau/J_{u_1^2}}$$
(8.19)

$$\overline{u_2^2(t) \ u_2^2(t+\tau)} = \overline{u_2^4} e^{-\frac{u_2^2}{2}}$$
(8.20)

For the correlation of  $\overline{u_1^2} - \overline{u_2^2}$  we can write

$$\frac{\langle u_{1}^{2} - u_{2}^{2} \rangle (t) \langle u_{1}^{2} - u_{2}^{2} \rangle (t+\tau)}{-u_{1}^{2}(t) u_{2}^{2}(t+\tau)} - \frac{\langle u_{1}^{2}(t+\tau) \rangle}{u_{2}^{2}(t+\tau)} = \frac{-\tau/J_{u_{1}}^{2}(t+\tau)}{-\tau/J_{u_{1}}^{2}} + \frac{-\tau/J_{u_{2}}^{2}(t+\tau)}{-\tau/J_{u_{2}}^{2}} + \frac{-\tau/J_{u_{2}}^{2}(t+\tau)}{-\tau/J_{u_{2}}^{2}} + \frac{-\tau/J_{u_{2}}^{2}(t+\tau)}{-\tau/J_{u_{2}}^{2}} = -2 \frac{\tau/J_{u_{1}}^{2} u_{2}^{2}}{u_{1}^{2} u_{2}^{2}} e$$

$$(8.21)$$

For the correlation of 
$$u_1^2 - u_2^2$$
 we can also write  
 $-\tau/J_{u_1^2-u_2^2}(t) < u_1^2 - u_2^2 > (t+\tau) = (u_1^2 - u_2^2)^2 e^{-\tau/J_{u_1^2-u_2^2}} = -\tau/J_{u_1^2-u_2^2}$ 

$$(\overline{u_1^4 + \overline{u_2^4}} - 2 \ \overline{u_1^2 u_2^2}) e^{(8.22)}$$

When we compare eq. (8.21) and eq. (8.22) we see that the expressions are only equal when

$$J_{u_1^2} = J_{u_2^2} = J_{u_1^2 u_2^2} = J_{u_1^2 - u_2^2}$$
(8.23)

From the results (8.13) and (8.18) we see that this is indeed roughly the case in this experiment.

With expression (8.18) we can determine  $L_{11}^{\times}$ . We find

$$L_{11}^{*} \simeq 0.16 x_{1}$$
 (8.24)

This value of  $L_{11}^{*}$  is much lower than the value of expression (8.16). Expression (8.16) is directly determined from the measurements of the turbulent intensities. Consequently, the latter value of  $L_{11}^{*}$  should be the correct value in the region  $x_1/M > 15$ . So we must conclude that expression (8.24) does not give the right value for  $x_1/M > 15$ , although there is no clear argument for this. For  $x_1/M < 15$ , expression (8.16) does not hold. The only value of  $L_{11}^{*}$  that we know in this region is expression (8.24). The results of the calculations with the different memory lengths will be shown. First however we must try to determine the equilibrium value of  $\epsilon_q$ . In fig. 8.25 the results of  $\epsilon_q$ , determined by the hypothesis of Boussinesq are given. From the results it is evident that we are not able to determine a reliable equilibrium value of  $\epsilon_c$ .

Notwithstanding this difficulty we decided to calculate  $\overline{u_1^2} - \overline{u_2^2}$  according to the relaxation equation (8.8). First we take for  $L_{11}^{\times}$  the value determined by the space-time correlations,  $L_{11}^{\times} \simeq 0.16 x_1$ . The unknown equilibrium value of  $\varepsilon_q$  has been determined by adjusting the calculated values to the measured values. The results are shown in fig. 8.26. We see that the measured values are well described by the calculated values according to the relaxation equation if we take  $\varepsilon_q = 7.10^{-4} \text{ m}^2/\text{s}$ ; so  $\varepsilon_q$  has about the same value as  $\varepsilon_m$ . The values of  $\overline{u_1^2} - \overline{u_2^2}$  determined by the hypothesis of Boussinesq with the same value of  $\varepsilon_q$  are clearly lower than the measured values. We can conclude that the extra memory effects of  $\overline{u_1^2} - \overline{u_2^2}$  are well described by the  $L_{11}^{\times} = 0.16 x_1$  at least in the region up to  $x_1/M \simeq 13$ .

Next we shall try to determine  $\overline{u_1^2} - \overline{u_2^2}$  according to the relaxation equation with  $L_{11}^{"} = x_1 - 0.07$  (from fig. 8.5 we find b  $\simeq 0.07$  m). This relation for  $L_{11}^{"}$  holds only for  $x_1/M > 15$ . In this region  $\partial \overline{U}_1/\partial x_2$  is practically zero, so we do not need a value of  $\varepsilon_1$ . We have performed the calculations by taking the measured value of  $\overline{u_1^2} - \overline{u_2^2}^2$  at  $x_1/M = 15$  as the starting point for the calculation. The results are shown in fig. 8.27. In the first place we see that the hypothesis of Boussinesq, but also the relaxation equation with  $L_{11}^{"} = 0.16 x_1$  and the same value of  $\varepsilon_q$  is not able to describe the measured values of  $\overline{u_1^2} - \overline{u_2^2}$  for  $x_1/M > 15$ . We also see that with  $L_{11}^{"} = x_1 - 0.07$  the relaxation equation can describe the measured values of  $\overline{u_1^2} - \overline{u_2^2}$  reasonably for  $x_1/M > 15$ . From these results we can conclude that the relaxation equation can in principle describe the extra memory effects of  $\overline{u_1^2} - \overline{u_2^2}$  behind a grid, although there is a great difference between the values of  $L_{11}^{"}$  in the region  $0 < x_1/M < 15$  and in the region without any production of turbulence,  $x_1/M > 15$ .

-71-

#### IX RECAPITULATION AND DISCUSSION.

From this investigation it is clear that extra memory effects can play an important role in turbulent flow fields, especially when there is a strong distortion of the mean-velocity gradient. Behind the half sphere in the turbulent boundary-layer and behind the cylinder the shear stress  $-u_2u_1$  exhibits distinct extra memory effects - in the case of the cylinder up to as far downstream as the self-preserving part of the wake flow. Behind the grid, the extra memory effect on  $-u_2u_1$  was small, but that on the difference between the normal stresses,  $u_1^2 - u_2^2$ , was clearly observable.

It proved that these extra memory effects can be described reasonably well by a so-called relaxation equation. This relaxation equation can be derived by an extension of the hypothesis of Boussinesq. The relaxation equation can also be obtained from the complete transport equations by substituting some commonly used expressions for the unknown terms. Physically the relaxation equation shows that the turbulent transport is a combination of gradient-type and convective transport and that the turbulence behaves as a non-Newtonian fluid.

Notwithstanding the success of the relaxation equation there remain certain difficulties and uncertainties, from a theoretical as well as from an experimental point of view.

From the theoretical considerations used to derive the relaxation equation it is clear that it is only a second order approximation (if we call the hypothesis of Boussinesq a first order approximation). Only the complete transport equations can describe the behaviour of the turbulence correctly, including the extra memory effects. This point leads immediately to the question of whether there is a relation between the relaxation equation and the turbulence models, which use the complete transport equations. From the derivation of the relaxation equation from the complete transport equations one can see that this equation can only be derived by using rather simple and in some cases even questionable approximations for the unknown terms. Some of the turbulence models use much more complex and, almost certainly, better approximations for the unknown terms. The question arises as to whether the theory using the relaxation equation may be considered as a kind of a very simple turbulence model. The answer is that in a way it can be so considered, though it has not been intended as a turbulence model. For the relaxation equation is far more an equation which tries to describe a certain aspect of the turbulence, namely the extra memory effect. Consequently there

will be situations that can be described by some turbulence models, but where the use of the relaxation equation will fail. On the other hand there remains the fact that the relaxation equation can describe very severely disturbed turbulent flow fields that can not be described by local, gradient-type, expressions. All the current turbulence models are based on the condition that the turbulent flow fields to be described should be in a nearly equilibrium and nearly isotropic state. So it is very likely that these turbulence models will not be able to describe very severely disturbed turbulent flow fields.

From these remarks we can conclude that if we want to extend the relaxation equation by including other terms it is probably not very fruitful to use only the existing turbulence models. Perhaps it would be better for such an extension to consider the ideas about constitutive relations for non-Newtonian fluids that have been developed in the theory of rheology. This, notwithstanding the difference between the theory of turbulence and rheology.

Another questionable point is the use of the eddy viscosity in the relaxation equation. If we accept the use of an eddy viscosity, there still remain at least two questions. One question is, is it always correct to use the value of the equilibrium (undisturbed) flow for the eddy viscosity, or is it possible that the value of the eddy viscosity to be used in the relaxation equation should not necessarily be the equilibrium value? Despite the fact that we find good results using the equilibrium value this seems an important question. The other question is concerned with the constant value of the eddy viscosity. In the case that the equilibrium value of the eddy viscosity is constant in the main flow direction, the question arises as to what the physical process underlying this constant behaviour is. The eddy viscosity in this case seems to be a kind of conservative property of the turbulence.

A problem of experimental nature is the one concerning the determination of the memory length  $L_1$  (or  $L_{11}^{"}$ ). We have seen that the use of the Lagrangian integral time scale of  $u_1$  as described by Saffman leads to good results. We have also seen that the time scale of  $-u_2u_1$ ,  $J_{-u_2u_1}$ , derived from the spacetime correlation of  $-u_2u_1$ , which should be a more suitable value for the relaxation time, has nearly the same value as the time scale according to Saffman. However, an accurate experimental determination of  $J_{-u_2u_1}$  is still not possible.

This brings us to the ideas concerning the continuation of this investigation. In addition to the above mentioned problems there is another problem that deserves special attention. The experiments did show that the larger eddies have a stronger extra memory effect than the smaller eddies. So it will be worthwhile investigating these extra memory effects experimentally in different narrow frequency bands, instead of the investigation of the complete turbulent signal containing all frequencies as in this investigation. This leads to the following problem. On the one hand there is the fact that the larger eddies adapt themselves more gradually to a new situation than the smaller eddies. The adaptation of the smaller eddies is almost instantaneous. On the other hand, however, there are a lot of indications that there exist a cascade process by which the larger eddies give their energy to smaller eddies and so on. Consequently, according to this cascade process the smaller eddies must also show an extra memory effect, which seems to be in conflict with their small relaxation times. Perhaps the smaller eddies adapt themselves only very fast to changes in their immediate surroundings but not to the changed mean-velocity field, which change has to be transported to the smaller eddies by a cascade process. However, the cascade process is still in discussion in the literature. It seems possible that the further investigations of the extra memory effects can also shed some light on this still questioned cascade process.

### Acknowledgement.

The author would like to thank all the members of the Laboratory for Aero- and Hydrodynamics of the Delft University of Technology for their assistence and advice during the investigations. The author is specially indebted to the former students J.A. Amini, J.G. Drenthen and J.J. Dirkmaat. NOMENCLATURE.

A, A<sub>1</sub>, A<sub>2</sub>, a<sub>1</sub>, a<sub>2</sub> constants origin of similarity B, b<sub>1</sub>, b<sub>2</sub> constants c1, c2, c3, c4 constants C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub> factors in an equation for a mechanical vibration Cd drag coefficient diameter of the object concerned d diffusion factor D  $\mathbb{E}_1(\overline{u_1^2})$ one-dimensional energy spectrum of  $\overline{u_1}^2$  $f(\Delta x_1)$ space-correlation k<sub>1</sub> wave number L, L, l length scales length scale in the x<sub>k</sub>-direction L<sub>k</sub>  $L_1, L_2, L_1^{*}, L_2^{*}, L_{11}^{*}, L_{22}^{*}$ memory lengths mixing length lt М mesh width of the grid M(t)weighting function frequency n Ρ static pressure; property  $P_{t} = \frac{1}{3} \overline{u_{k} u_{k}}$  $P_t$ turbulent pressure р  $P(u_1)$ probability distribution of u1 [Qjk]L Lagrangian autocorrelation a<sup>2</sup> total turbulent kinetic energy,  $\overline{q^2} = \overline{u_k u_k}$ R<sub>u1</sub>, R<sub>u2</sub>, R<sub>u1</sub><sup>2</sup>, R<sub>u2</sub><sup>2</sup>, R<sub>-u2</sub><sup>u1</sup> space-time correlations  $R_{u_1^2 - u_2^2}$ 

R <sub>21</sub> , (R <sub>21</sub> ) <sub>max</sub>	correlation coefficients
Re	Reynolds number
S	Strouhal number
t, t <sub>o</sub>	time
Т	time scale
t ×	traveling time
U.	velocity in the xdirection
Ū	mean velocity
u.	turbulent velocity
U_	outer velocity
U	velocity scale
u <sup>×</sup>	wall shear stress velocity
∆Ū <sub>1,m</sub>	velocity difference
Ψ	turbulent velocity scale
V	wave-velocity
xi	cartesian coordinate
yk	distance in the $x_k$ -direction
αp	angle between original system and principal- axes system
β	factor of proportionality
<sup>6</sup> 99, <sup>6</sup> 995	boundary layer thickness
δ <sub>m</sub>	momentum-loss thickness
δ.j	Kronecker delta
e <sub>o</sub> , e <sub>c</sub>	factors in a constitutive relation
$\epsilon_{jk}, \epsilon_{jk}, \epsilon_{m}, \epsilon_{q}$	eddy viscosities
ε	dissipation
η	distance factor
J <sub>1</sub> , J <sub>2</sub>	time scales

-76-

 $(J_{L})_{1}, (J_{L})_{2}$   $J_{u_{1}}, J_{u_{2}}, J_{u_{1}^{2}}, J_{u_{2}^{2}}, J_{-u_{2}^{u_{1}}},$   $J_{u_{1}^{2}-u_{2}^{2}}$   $(\Lambda_{L})_{1}, (\Lambda_{L})_{2}$   $\Lambda_{E}$   $\Lambda_{f,1}$   $\nu$   $\rho$ 

Lagrangian integral time scales time scales derived from space-time correlations

Lagrangian integral length scales Eulerian integral length scale Eulerian longitudinal integral length scale kinematic viscosity density

# REFERENCES.

[1]	Tennekes, H. and Lumley, J.L. "A first course in turbulence", MIT-press (1972).
[2]	Rivlin, R.S. Quart. Appl. Math. 15, 212 (1957).
[3]	Liepmann, H.W., Mécanique de la Turbulence, Paris, C.N.R.S. (1961).
[4]	Crow, S.C., J. Fluid Mech., 33, 1, 1 (1968).
[5]	Townsend, A.A., J. Fluid Mech. 26, 689 (1966).
[6]	Lumley, J.L., J. Fluid Mech. 41, 2, 413 (1970).
[7]	Proudman, I., J. Fluid Mech. 44, 3, 563 (1970).
[8]	Dowden, J.M., J. Fluid Mech. 56, 4, 641 (1972).
[9]	Dowden, J.M., J. Fluid Mech. 63, 1, 33 (1974).
[10]	Truesdell, C., "The principles of continuum mechanics". Coll. lectures in Pure and Appl. Science 5, Sacony Mobil Oil Co., Inc. (1961).
[11]	Jacobs, W., Z. Angew. Math. Mech. 19, 87 (1939).
[12]	Logan, E.L. and Jones, J.B., Trans. Amer. Soc. Mech. Engrs. 85, D, 35 (1963).
[13]	Makita, H., "Response of a turbulent boundary layer to a sudden change in surface roughness", M. Eng. Thesis Univ. of Tokyo (1968).
[14]	Taylor, R.J., J.Fluid Mech. 13, 529 (1962).
[15]	Antonia, R.A. and Luxton, R.E., J. Fluid Mech. 48, 4, 721 (1971).
[16]	Schofield, W.H., J. Fluid Mech. 70, 3, 573 (1975).
[17]	Panofsky, H.A. and Townsend, A.A., Quart. J. Roy. Met. Soc. 90, 147 (1964).
[18]	Plate, E.J. and Hidy, G.M., J. Geophys. Res. 72, 4627 (1967).
[19]	Moses, H.L., Gas Turbine Lab. Report 73, M.I.T. (1964).
[20]	Golberg, P., Gas Turbine Lab. Report 85, M.I.T. (1966)
[21]	Bradshaw, P. and Ferris, P.H., N.P.L. Aero Report 1145 (1965).
[22]	Bradshaw, P., N.P.L. Aero Report 1219 (1967).
[23]	Mueller, T.J., "On separation, reattachment and redevelopment of turbulent boundary layers" Doct. Diss. Univ. of Illinois (1961).
[24]	Mueller, T.J. and Robertson, J.M., Developments in Theor. and Appl. Mech. 1 (1963).
[25]	Plate, E.J. and Lin, C.W., Final Rep. on Grant no. DA AMC 36-039-63-67, Fl. Dyn. and Dif. Lab. Colorado State Univ. (1965).
[26]	Bearman, P.W., J. Fluid Mech. 21, 2, 241 (1965).
[27]	Tani, I., "Experimental investigation of flow separation over a step", Boundary Layer Research, Springer-Verlag-Berlin (1958).
[28]	Tani, I., Suchi, M., Komoda, H., Aeron. Res. Inst. Univ. of Tokyo rep. no. 364 (1961).
[29]	Johnston, J.P., J. Fluid Mech. 42, 2, 823 (1970).
[30]	Bradshaw, P. and Wong, F.Y., J. Fluid Mech. 52, 1, 113 (1972).

- [31] Clauser, F.H. Advances in Appl. Mech. 4, 1 (1956).
- [32] Lissenburg, R.C.D., Hinze, J.O. and Leijdens, H., Appl. Sci. Res. 31, 343, (1975).
- [33] Tani, I., Computation of Turb. Bound. Layers, AFOSR-IFP-STANFORD CONFERENCE, 483 (1968).
- [34] Plate, E.J., Aerodynamics of Characteristics of Atmospheric Boundary Layers, AEC Critical Review Series, chapter 4 (1971).
- [35] Narasimha, R. and Prabhu, A., J. Fluid Mech., <u>54</u>, 1, 1 (1972).
- [36] Prabhu, A. and Narasimha, R., J. Fluid Mech. <u>54</u>, 1, 19 (1972).
- [37] Maréchal, J., J. de Mécanique 11, 263 (1972).
- [38] Tucker, H.J. and Reynolds, A.J., J. Fluid Mech. 32, 657 (1968).
- [39] Reynolds, A.J. and Tucker, H.J., J. Fluid Mech. 68, 4, 673 (1975).
- [40] Rose, W.G., J. Fluid Mech. 44, 767 (1970).
- [41] Champagne, F.H., Harris, V.G. and Corrsin, S., J. Fluid Mech. 41, 81 (1970).
- [42] Landahl, M.T., J. Fluid Mech. 29, 3, 441 (1967).
- [43] Boussinesq, J. Mém. prés. par div. savant à l'acad. sci. Paris, 23, 46 (1877).
- [44] Prandtl, L., Z. angew. Math. u. Mech. 5, 136 (1925).
- [45] Hinze, J.O., "Turbulence" McGraw-Hill, Inc. (1975).
- [46] Corrsin, S., Advances in Geophysics 18A, 25 (1974).
- [47] Deissler, R.G., J. Fluid. Mech. 64, 763 (1974).
- [48] Hanjalić, K and Launder, B.E., J. Fluid Mech. 52, 609 (1972).
- [49] Launder, B.E., Reece, G.J. and Rodi, W., J. Fluid Mech. 68, 537 (1975).
- [50] Launder, B.E. and Spalding, D.B., Comp. Math. in Appl. Mech. and Eng. 3, 269 (1974).
- [51] Lumley, J.L. and Khajeh-Nouri, B., "Comp. modeling of turb. transport" Adv. in Geophys. vol. 18A. Acad. Press New York, 169 (1974).
- [52] Lumley, J.L., Phys. of Fluids 18, 6, 619 (1975).
- [53] Lumley, J.L., Phys. of Fluids 18, 6, 750 (1975).
- [54] Rotta, J.C. and Vollmer, H., DFVLR-AVA-Bericht IB, 251-74A, 45 (1974).
- [55] Bradshaw, P., Ferris, D.H. and Atwell, N.P., J. Fluid Mech. 28, 593 (1967).
- [56] Bradshaw, P., J. Fluid Mech. 46, 417 (1971).
- [57] Bradshaw, P., Aero- J. 76, 403 (1972).
- [58] Bradshaw, P. and Patel, V.C., Imp. College Aero. Rep. 73-05 (1973).
- [59] Launder, B.E. and Spalding, D.B., "Mathematical models in turbulence", Academic Press London (1972).
- [60] Von Kármán Institute, Lecture Serie 76, March 3-7, (1975).
- [61] Rotta, J.C., Fl. Dyn. of Int. Flow 80-109 Elsevier Publ. Co. A'dam (1967).
- [62] Launder, B.F., Imp. College of Science and Techn. R. L. 261 (1972).
- [63] Mellor, G.E. and Herring, H.J., AIAA-Journal 11, 5, 590 (1933).
- [64] Phillips, O.M., J. Fluid Mech. 27, 1, 131 (1967).

[65] Atesman, K.M. AICHE J. 17, 5, 1256 (1971).

[66] Eskinazi, S. and Yeh, H. J. Aeron. Sci. 23, 23 (1956).

[67] Kjellström, B. and Hedberg, S., Aktiebolaget Atomenergie, Stockholm, Rep. A. E-243 (1966).

[68] Hanjalic, K. and Launder, B.E., J. Fluid Mech. 51, 301 (1972).

- [69] Tailland, A. and Mathieu, J., J. Mécanique 6, 103 (1967).
- [70] Béquier, C. "Etude du jet plan dissymétrique en régime turbulent incompressible. Thesis Univ. d'Aix-Marseille (1971).
- [71] Hinze, J.O. Appl. Sci. Res. 22, 163 (1970).
- [72] Launder, B.E., Basic. Eng. 855 (1969).
- [73] Huffman, G.D. and Bradshaw, P., J. Fluid Mech. 53, 45 (1972).

[74] Klebanoff, P.S. Natl. Adv. Comm. Aeron. Tech. Notes 3178 (1954).

[75] Antonia, R.A., Phys. of Fluids 15, 9, 1669 Research Note (1972).

[76] Townsend, A.A., Proc. Cambridge Phil Soc. 47, 375 (1951).

[77] Preston, J.H., J. of the Royal Aer. Soc. 58, 109 (1954).

[78] Patel, V.C., J. Fluid Mech. 23, 1, 185 (1965).

- [79] Champagne, F.H., Harris, V.G. and Corrsin, S., J. Fluid Mech. 41, 8 (1970).
- [80] Comte-Bellot, G. and Corrsin, S., J. Fluid Mech. 48, 273 (1971).
- [81] Klebanoff, P.S. and Diehl, F.W., NACA T.N. 2475 (1951).
- [82] Bradshaw, P. and Ferris, D.H., N.P.L. Aero Rept. 1145 (1965).
- [83] Antonia, R.A. and Luxton, R.E., J. Fluid Mech. 48, 721 (1971).
- [84] Sabot, J., Renault, J. and Comte-Bellot, G., Phys. of Fluids 16, 9, 1403 (1973).
- [85] Blackwelder, R.F. and Kovasznay, L.S.G., Phys. of Fluids, 15, 9, 1545 (1972).
- [86] Frenkiel, F.N. and Klebanoff, P.S. "Statistical properties of velocity derivatives in a turbulent field", Dept. of Appl. Math. T.N. AML-45 70, Washington (1970).
- [87] Saffman, P.G., Appl. Sci. Res. 11A, 245 (1963).
- [88] Philip, J.R., Phys. of Fluids 10, suppl. 9, S69 (1967).
- [89] Hinze, J.O., Sonnenberg, R.E. and Builtjes, P.J.H., Appl. Sci. Res. 29,1 (1974).
- [90] Townsend, A.A., J. Research, 2A, 451 (1949).
- [91] Uberoi, M.S. and Freymuth, P., Phys. of Fluids 12, 1359 (1969).
- [92] Uberoi, M.S. and Wallis, S., J. Fluid Mech. 24, 3, 539 (1966).
- [93] Uberoi, M.S. and Wallis, S., Phys. of Fluids 10, 6, 1216 (1967).
- [94] Uberoi, M.S. and Wallis, S., Phys. of Fluids 12, 7, 1355 (1969).
- [95] Comte-Bellot, G. and Corrsin, S., J. Fluid Mech. 25, 4, 657 (1966).
- [96] Van Atta, C.W. and Chen, W.Y., J. Fluid Mech. 34, 497 (1968).
- [97] Van Atta, C.W. and Chen, W.Y., J. Fluid Mech. 38, 743 (1969).
- [98] Builtjes, P.J.H., Appl. Sci. Res. 31, 397 (1975).
- [99] Tchen, C.M., J. Res. Natl. Bur. Standards 50, 51 (1953).

- [100] Favre, A., J. Appl. Math. Trans. A.S.M.E., 32 E, 241 (1965).
- [101] Bradshaw, P., Agardograph no. 169 (1973).
- [102] Bradshaw, P., Sectional Lecture, IUTAM Congress, Delft (1976).





Fig. 5.1. Part of the design of the grid.



-83-



















Fig. 6.11. A space-time correlation of  $u_1^2$ 













Fig. 6.15. Mean flow pattern downstream of the hemi-spherical cap.





-91-





-92-









-95-







-98-







-99-












-105-



-106-



-107-



-108-









Fig. 8.1 One mesh at the centre of the grid with the used coordinate system

















Fig. 8.9  $-\overline{u_2}\overline{u_1}/\overline{U_1}^2$  close to the grid



-116-









-120-













Fig. 8.23 A space-time correlation of -u2u1









-125-

## STELLINGEN.

- 1. De criteria die gebruikt worden bij de meting van de intermitteringsfactor zijn min of meer subjectief. Hoewel er in de literatuur een vrij grote, nogal verwonderlijke overeenkomst bestaat aangaande de experimentele resultaten van de intermitteringsfactor is deze situatie voor de voortgang van het turbulentieonderzoek ongewenst. Er zal moeten worden gestreefd naar algemeen aanvaarde criteria.
- 2. Turbulentiemetingen met behulp van X-draden lijken over het algemeen aanleiding te geven tot grotere fouten dan metingen met roteerbare enkelvoudige gloeidraden. Het verdient daarom aanbeveling om, indien geen instantane waarden worden vereist, de turbulentiemetingen uit te voeren met een roteerbare enkelvoudige gloeidraad.
- 3. Voor de bepaling van de afstand tot de wand van een gloeidraad kan men gebruik maken van het met de gloeidraad te meten gemiddelde snelheidsprofiel in de visceuze sublaag. Extrapolatie van het, mits duidelijk aanwezige, lineaire snelheidsprofiel levert de afstand tot de wand. Afstandsbepalingen langs mechanische of optische weg zijn moeizamer, vergen extra voorzieningen en leiden in de meeste gevallen niet tot een nauwkeuriger afstandsbepaling.
- 4. Bij de beschrijving in het model van Launder van de produktie term in de transportvergelijking voor de dissipatie wordt een factor ingevoerd die bepaald wordt via aanpassing aan de metingen. Het blijkt dat een verandering van 3 % in de waarde van deze factor een onevenredig grote invloed heeft op de uitkomsten van het turbulentiemodel. Dit resultaat toont aan dat de juistheid van het fysisch model dat als basis dient voor het turbulentiemodel van Launder op zijn minst twijfelachtig is.

B.E. Launder, Lecture von Kármáninstitute, march 1975. B.E. Launder, J. of Fluid Mech., 68, 3, 537 (1975).

- 5. Jullien bepaalt uit diffusiemetingen de Lagrange integrale lengteschaal in een stroming tussen twee vlakke wanden. Hij vindt het fysisch onaanvaardbare resultaat dat de Lagrange integrale lengteschaal toeneemt met afnemende afstand tot de wand. Dit resultaat zal mede een gevolg zijn van de onjuiste onderstelling dat ook dicht bij de wand de verdeling van het warmtezog gaussisch is.
  - F. Jullien. Comptes Rendues Hebd. des Seances de l'Ac. de Sciences; Serie A, Sci., Math. 275, 16, 761 (1972).

6. Bradshaw en Wong kiezen bij hun bestudering van verstoorde turbulente grenslagen de Clauser parameter als de factor bepalend voor de mate van afwijking van de ongestoorde turbulente grenslaag. Deze parameter die uitsluitend een functie is van het snelheidsprofiel en de wandschuifspanning, beschrijft echter hoofdzakelijk het gedrag van de gemiddelde snelheid. Voor een volledige beschrijving van de mate van verstoring van een turbulente grenslaag is echter naast de Clauser parameter ook een turbulentieparameter noodzakelijk.

P. Bradshaw and F.Y. Wong, J. of Fluid Mech., 52, 1, 113 (1972).

- 7. Bradshaw stelt voor de beschrijving van de turbulente schuifspanning in sterk gekromde stromingen een Boussinesq-achtige formule voor. De invloed van de door de kromming veroorzaakte gradient van de gemiddelde snelheid op de turbulente schuifspanning is in deze formule een factor 10 groter dan de invloed van de snelheidsgradient van de gemiddelde snelheid van de stroming zonder kromming. Een fysische verklaring voor dit verschijnsel ontbreekt. Het lijkt waarschijnlijker dat in het geval van een sterk gekromde stroming Boussinesqachtige formules onvoldoende zijn om de turbulente schuifspanning te beschrijven.
  - P. Bradshaw, AGARDograph no. 169 (1973).
- 8. Het steeds meer uiteenvallen van het turbulentieonderzoek in een groep die zich uitsluitend bezighoudt met het berekenen van gemiddelde turbulente stromingen met behulp van mathematische turbulentiemodellen en een groep die zich uitsluitend bezighoudt met het experimenteel, op geconditioneerde wijze, bepalen van turbulente stromingen leidt tot een ongewenste kloof tussen beide groepen die het totale turbulentieonderzoek kan schaden.
- 9. Het verdient aanbeveling het vak magnetohydrodynamica in de studie van natuurkundig ingenieur op te nemen. Dit gezien het belang van dit vakgebied voor de toekomst en de didactische waarde ervan doordat een aantal uiteenlopende gebieden van de natuurkunde in dit vak geïntegreerd voorkomen.
- 10. Automatisering dient in het algemeen beperkt te blijven tot het vervangen van door de uitvoerenden als geestdodend ervaren arbeid. Verdergaande automatisering op grond van economische motieven die slechts betrekking hebben op het eigen bedrijf dient te worden afgewezen.