Feasibility of the Polders

When can the Dutch polderconcept become economically unviable?

Additional Thesis

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Cover: Photograph of a neighbourhood in Zeeland after the 1953 floods

TU Delft

Abstract

Adequate flood protection is important to many countries, but especially so to the Netherlands. With a large share of its population centers located below sea level, in so-called polders, the need for flood protection systems quickly becomes apparent. This need is even more pressing with the rise of sea levels and the increase in river discharge variability due to the onset of climate change. To future proof themselves, the Netherlands needs to maintain and strengthen their flood defences. Especially precarious is the situation for polders, which are low-laying areas protected by one or more dikes. From a technical perspective, the feasibility of the polder system has been proven to withstand the expected water level rise as result of climate change for at least 2 to 3 meters sea level rise, shown by Kok et al. (2008). However, research into the economic perspective on the feasibility of the polder concept has been less extensive. This leads to the main question of this research: *When can the Dutch polder concept become economically unviable?*

Research on the topic of economic viability over time has been performed by Eijgenraam (2006), who based his work on the original work of van Dantzig (1956). However, this research still contains some knowledge gaps. The work by Eijgenraam is concerned with the optimization of the Cost-Benefit Analysis (CBA), without regard for any (financial) constraints that might be imposed on the optimization. Furthermore, the research conducted was set in a deterministic way, not including the uncertainty in many of the parameters that are used in the framework such as the sea level rise or population growth. The final knowledge gap is on the use of the discount rate. The discount rate is prescribed to be constant for investments being made in the current financial environment by The Ministry of Finance in the Netherlands. However, as the discounting of future costs and benefits to the present value is a non-linear process, the variability in the discount rate will result in non-linear results as well. As such this research argues for a variable discount rate, determined by various models.

To more accurately determine the economic viability of the Dutch polder concept, additions to the research of Eijgenraam are proposed that fill in the current knowledge gaps. The result of this research is a mathematical framework for the optimization of dike reinforcements in two dimensions, the lifetime of the structure and the crest height increase of the dike. The framework consist of a discounted Cost-Benefit Analysis with a financial constraint and a constraint on the maximum allowable time before reinforcement is needed. This framework contains stochastic elements in it's parameters and a stochastic model for the discount rate.

The derived framework was subsequently used to analyse two case studies based on regions in the Netherlands. The two case studies were based on the dikering of *IJsselmonde* (dikering 17) and the dikering of *Walcheren* (dikering 29). The results were determined for the two climate scenario's posed by the IPCC and KNMI by means of a Monte Carlo simulation. In the base case, both dikerings were deemed economically viable for both climate scenario's. A sensitivity analysis was performed on several of the input parameters of the model. This analysis showed the the costs of increasing the crest height of a dike is the most influential parameter in the assessment of the economical viability of the polder concept.

The case studies led to the conclusion that *the polderconcept is deemed economically viable based on the derived framework in this research as long as the cost per meter crest height increase per kilometer dike do not exceed a certain threshold*. When the costs exceed a factor 2.5 for *Walcheren* of and a factor of 1.2 for *IJsselmonde*, the economic viability, as judged in this framework, is lost. For the simplified cases this leads to thresholds of €11.3 mln. for *Walcheren* and €5.5 mln. for *IJsselmonde*. The difference in this threshold is mainly attributed to the difference in population and asset density between regions, as well as the difference in initial risk level. Economic viability in this research means the sign of the adjusted CBA. Whenever this sign turns from positive to negative, economic viability is lost. However, this does not mean that reinforcement is not the (economic) most optimal solution.

The marginal costs of reinforcement only exceed the marginal costs of flood risk for *IJsselmonde* after a 3.5-fold increase in costs from considered base case. For *Walcheren*, there is no turning point and reinforcement appears to be the most economically feasible solution for all reinforcement costs considered. As such, dike reinforcement under sea level rise remains the economically more feasible solution judged on the derived framework in this research for the simplified case studies of *Walcheren* and *IJsselmonde*.

This research has opened up the possibility to compare alternatives over different time periods and reinforcement measures with different constraints and stochastic parameters, adding to the work done by Eijgenraam. As such, a more risk-informed discussion on the general viability of the polderconcept can be had, ultimately resulting in a more informed decision on the future of the polders in the Netherlands.

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Introduction

This chapter will introduce the reader to some background information on the topic of flood protection in The Netherlands. After the background introduction, the most recent research on the topic of economic optimization of flood defences will be showed. The knowledge gaps in the presented research will be presented and discussed, being the driver for this research. After the knowledge gaps have been adressed, the objective of this research is presented, including the main research question and corresponding sub-questions. This section will also include the scope of the research. This chapter continues with the research approach, laying down the steps that will be undertaken to come to an answer to the research question. The chapter concludes with an outline of the rest of the report.

1.1. Background

Flood protection is not a new topic in the Netherlands, as early as the 14^{th} the concept of a polder was introduced and around the 15^{th} century the polder with a pumping station as it is currently known was conceived (Nederlands Openluchtmuseum, 2022). Although not new, it currently is a hotly debated one. In the past year Europe saw some very destructive floods. It should come as no surprise that the main driver behind these extreme weather events is climate change. According to the IPCC the ”Extreme weather events causing highly impactful floods and droughts have become more likely and (or) more severe due to anthropogenic climate change” (Pörtner et al., 2022).

It should therefore require no explanation that adaptation is necessary for the continuation of life in the Delta regions such as the Netherlands. How and when these adaptations are done in an economically most optimal way has been a long-asked question by economists and engineers alike. Several approaches such as van Dantzig (1956) and the later improved version by Eijgenraam (2006) have been proposed to solve this optimization problem.

The discussion on how and when to improve flood defences has recently been revived as the Deltacommissioner of the Netherlands, Peter Glas, gave a second round of advice to the government on the topic of housing and climate adaptation. In his advice the Deltacomissioner called upon a more critical view on where and how we should build houses in the Netherlands Glas (2021). As such, the question on when and especially where to reinforce flood defences became relevant again. Should new housing be created in urban areas that lie relatively lower than their less urban, but higher laying, counterparts? Some, such as Rijcken (2022), argue that the Netherlands has the technological capacity to keep the water out of polders for the expected water level rise in the future. Others argue that it is best to prepare for the worst and ”climate proof” the future houses by moving them to higher laying areas. Whatever side is chosen, the main focus appears to be on the technical side of the discussion, rather than the economical side. This research attempts to quantify the economical viability of the polderconcept in the Netherlands to add an additional perspective to the discussion and help come to a better conlusion on the future of flood protection in the Netherlands.

1.2. Knowledge gaps

Although the technical feasibility of polders has been researched extensively, such as by Kok et al. (2008), the economical assessment and feasibility of the polders has been underexposed. After the flood disaster of 1953, Van Dantzig has concerned himself with the question on how to optimally heighten dikes from an economical point of view. Many cost benefit analysis today still rely on his original work. After nearly 50 years, Eijgenraam (2006) has made a re-assessments on the optimal heightening of dikes when economic growth is factored in, as well as determining the optimal timing of the reinforcement. However, this research is concerned only with the end result of the cost benefit analysis, assessing the internal rate of return for the investments. Neglecting any possible constraints caused by the financing of the projects undertaken. In reality resources are scarce and not every decision, even if they have a positive net result, can be carried out. As such this research proposes to add constraints to the optimization problem to better reflect the use of scarce resources. Furthermore, the derivation given by Eijgenraam (2006) is deterministic in several components such as growth rates for population and asset value. Although the taken rates reflect the average value of the quantity very well, they do not capture the natural variability in the quantities. Especially if this variability is very large, the outcome of economical assessments might differ greatly from realization to realization as the assessment often contains non-linear functions. As a final knowledge gap, the discount rate used in the determination of the net present value is reconsidered. Discounting costs over time is a very non-linear process. The true discounted costs may therefore vary largely from realization to realization. In recent years the stability of the economic system has been tested several times, where interest raises have been significantly raised and lowered. Taking a deterministic rate for projects with durations of over 50 to 100 years does not reflect the true variability in the economic and political system.

1.3. Objective

This research will build upon the research of Eijgenraam, adding constraints on his posed optimization. Next to the added constraints, the optimization will also be set-up in a stochastic way to include the uncertainty of certain parameter estimates. The final addition to his model will be the inclusion of a variable discount rate, to be able to reflect changes in economic trends. The objective of this research is to advance the understanding in the economic workings of the Dutch polder concept and produce new insights on how to judge the economic feasibility of the Dutch polder concept. To this end the following research questions is posed:

When can the Dutch polder concept become economically unviable?

In order to answer the main question, three sub-questions are posed.

- Which frameworks for judging the economic feasibility of the polder concept are possible?
- How does the assessment of economic feasibility change over time?
- Which variable(s) has/have the most influential contribution to the the viability assessment?

The time-element in this context concerns the feasibility assessment of a polder throughout the life-time of the flood defences protecting it, with the (possible) inclusion of constraints on the investment moments to reflect the use of scarce resources.

1.3.1. Scope

This research limits itself to the economic viability of a polder. The technical feasibility of the polder concept and political will to keep investing in the concept are left out of this research. Kok et al. (2008) argue and show that the technical feasibility is not the limiting factor for the future of the polderconcept. In terms of designing, constructing and maintaining the flood defenses in the Netherlands, the feasibility criteria is therefore assumed to be met.

1.4. Research approach

The main question of this research will be addressed by formulating an answer to each of the sub-questions. The first of which will be answered by performing a literature study on the existing policies of the Netherlands, as well as producing new criteria to assess the economic feasibility of the polder concept. These criteria will be tested on their relative stringency and developed into multiple mathematical models. The models will be tested on two case studies. The second question will be answered by means of the two case studies. The answer to the third question comes from a sensitivity analysis on the two case studies. In this sensitivity analysis the parameters will be varied to determine the most influential one.

1.5. Report outline

Chapter 2 will provide a literature study on a few key concepts that are needed to perform an economic analysis of a polder. Chapter 3 will introduce the methodology used in this research, leading to a model for analysing the economic feasibility of a region. Chapter 4 will introduce the data that will be used for a case study based on the proposed model. Next to the introduction of the data, some key statistics ont he two case studies are presented. In chapter 5 the model will be applied on two cases. One for a polder that is primarily influenced by river discharge and one for a polder that is primarily influenced by sea level rise. The results will be presented in chapter 5. Chapter 6 will consist of a discussion on the model and the results. Finally, chapter 7 will present an answer to the original research question and draw a conclusion. Next to a conclusion, several recommendations for further research will be proposed in this chapter.

2 Literature Study

The aim of this literature study is to gain insights into the workings of the current state of flood protection and the financing of the polder model. By discussing a variety of topics that are needed in an economic and financial analysis of a water safety system, the base for a model will be created.

The section will start with the motive behind the main research question by introducing the polderconcept and the National Deltaprogramme. In particular, it will take a closer look at the most recent advice Deltacommissioner Peter Glas gave on the need for the relocation of housing. Which in turn gave rise to the question this research aims to answer.

Hereafter, the current state of affair in the Netherlands will be assessed, as well as the possible future states due to climate change. The focus will be on sea level rise and discharge variability, with a brief mention of other consequences.

Next, the general policies regarding water and safety in the Netherlands will be discussed. A brief history will be given, after which the Dutch approach to safety standards will be introduced. The section ends with the current safety standards.

Later, the technical side of safety will briefly be discussed. Although this research looks at economic feasibility of the polderconcept, some basic concepts will be introduced to understand the failure mechanisms of common flood defences. The section ends with a derivation of the costs of increasing a flood defence to the required safety level.

After this, some general economic theories will be explained to gain insight into the considerations one should make when choosing a particular safety level. The concept of costs and benefits will be discussed in detail, as well as the different approaches one can take based on risk tolerance. The aim of this section is to gain insight into the thought process a government has when choosing to what level it will protect an area.

Finally, this section will look at the financing of projects in general, which will assist in determining the costs for the polder model. To this end, a detailed explanation of the determining factor in finance, the interest rate, will be discussed. The study will zoom into three common models to determine interest rates over time.

2.1. The polderconcept

As mentioned in the introduction, polders have a rich history in the Netherlands. Before proceeding with the analysis, it is important to define what a polder and the Dutch polderconcept is, as well as why it is needed.

2.1.1. Definition

The Dutch dictionary defines a polder as "*A section of land between dikes on a place where water used to be*". Although in a general sense this is what is meant by a polder, the technical definition for engineering practices is a bit more nuanced. According to Hoes and van de Giesen, 2015 a polder is defined as "*as a level area which has originally been subject to a high groundwater or surface water, and is separated from the surrounding hydrological regime, to be able to control the water levels in the polder*" and should show the following characteristics:

- A polder does not receive any foreign water from a water course, but only from rain, seepage, or by irrigation intake;
- A polder has an outlet structure (sluice or pump) that controls the discharge;
- The ground water and surface water level are independent from the water level in the adjacent land. These water levels are artificially maintained in order to optimize the objectives of the polder.

To put it simply, a polder is relatively low-laying area that is closed off from foreign water by a system of flood defences. If water does enter, it is swiftly pumped out to maintain the integrity of the polder. This approach allows areas that would normally be flooded, to be used for other purposes that yield utility. This reclaimed land is often used for agriculture, but can also be used for housing. A schematic overview of a polder is given in figure 2.1.

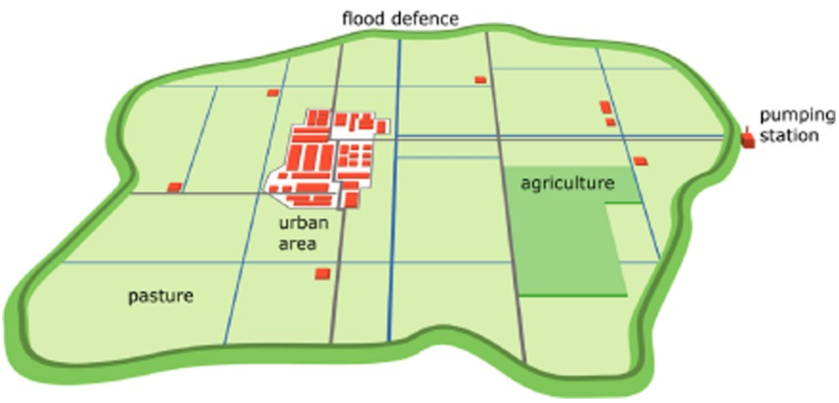
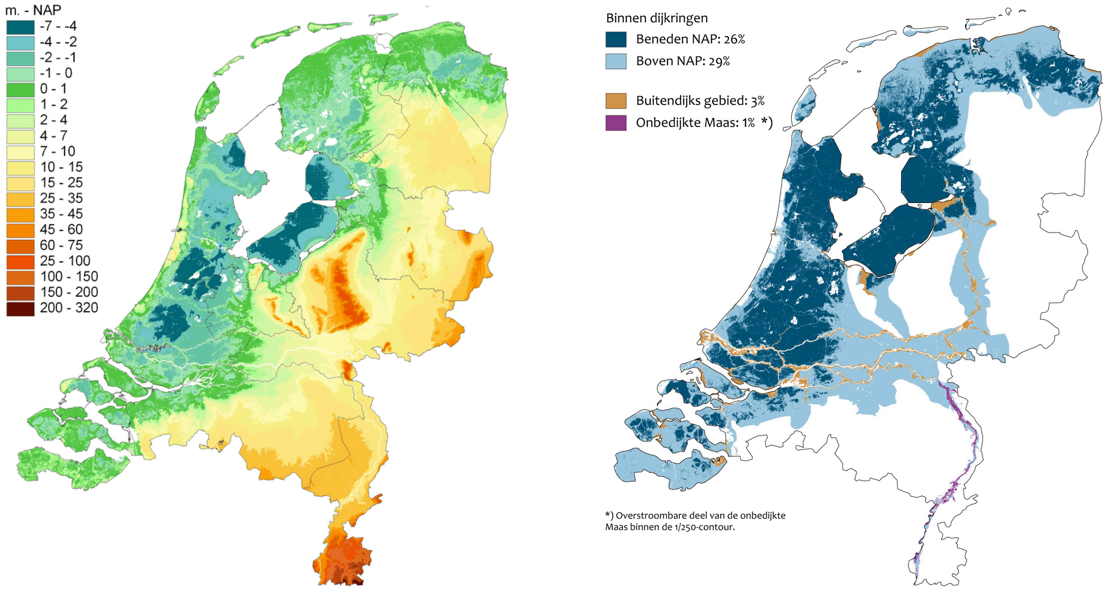


Figure 2.1: A schematic overview of a polder (Stijnen et al., 2012)

Note that the Dutch polderconcept should not be confused with the Dutch poldermodel, which is a method of consensus decision-making based on the policymaking in the 1980's and 1990's.

2.1.2. The need for polders

As can be seen in figure 2.2a, a large part of the Netherlands is at or below the current sea level. The relative depth of the Netherlands makes it more susceptible to flooding.



(a) Contour map of the Netherlands **(b)** Regions under and above NAP

Figure 2.2: Two views on elevations maps of the Netherlands (Blom-Zandstra et al., 2009)

Although below sea level, this land was considered valuable enough to protect from the water. "To protect" in this sense means that limited (or preferably, no) water from the sea or river should be able to enter the system, possibly damaging and/or flooding the low-laying areas. To this extent a "polder" was developed. It can be seen as a large area that lies beneath sea and is protected from the outside water via flood defences.

Some noteable polders in the Netherlands are: The Flevopolder, which is the result of the construction of the Afsluitdijk, is currently the world's largest artificial island. The Zuidplaspolder which is the lowest point of the Netherlands and shares this title with Lammefjord for the entire European Union. The Dutch polderconcept is also being exported around the global. A noteable project is that of Pulau Tekong in Singapore, the first polder in Southeast-Asia. The polder will create a new area northeast of mainland Singapore, being approximately 810ha in size. The design was made by dutch Engineering firm Royal HaskoningDHV, Boskalis will be responsible for making the ten kilometer long dike system. The project is expected to be finished this year (NOS, 2018).

2.2. National Deltaprogramme

The Netherlands has always had a precarious relationship with water. However, it wasn't until the devastating North Sea flood of 1953 that measures on a national level were taken by the government to protect it's citizens against a flood. To fulfill it's promise of protection, the first Deltacommission was formed within one month of the 1953 floods. This first iteration of the commission was tasked with advising the government on which measures to implement to guarantee water safety and prevent a possible next flooding. This ultimately led to the creation of the Delta Works in Zeeland. A collection of five stormsurge barriers, two sluices and six dams, of which the final construction was finished in 1997 with the completion of the Maeslantbarrier. The Delta works are, next to the Afsluitdijk, regarded as one of the largest water safety projects to ever be undertaken. The final report of this iteration of the Deltacommission was published near the end of 1960.

After nearly 50 years, in September 2007, the second iteration of the Deltacommission was installed by the minister of Transport and Water management and tasked with advising the government on strategies for dealing with the impacts of climate change on the Dutch water system. The result of this commission was a report with twelve recommendations. One of these recommendations was ”the strengthening of the political-administrative organization for our water safety” as well as ”securing financial resources for water safety” (Deltacommissie, 2008). This resulted in the Delta law (the so called *Deltawet*) being implemented in 2011, which mandated a yearly Deltaprogramme to be drafted.

2.2.1. Deltaprogramme 2022

By law, the Deltacommissioner has to present the Deltaprogramme to the House of Representatives on the third Tuesday of September (*Prinsjesdag*). This has happened each year since the inception of the Deltaprogramme in 2011. The programme contains a detailed plan for the next six years, and an outline of the plans for the six years after that. On the 21st of September the Deltaprogramme for 2022 was presented.

The report contains three substantive sections, *Water safety*, *Fresh water* and *Spatial adaption*. For this research, the first and latter sections are of most interest to this report. Some of the most important points from these section were:

Water safety: All primary flood defences should adhere to the ”new” norms by 2050. The water authorities and Rijkswaterstaat are currently mapping all dikes that need reinforcements. This mapping should be finished by 2022. The deadline of 2050 means that, on average, 50km of dike should be reinforced each year. This criteria is currently not being met.

Spatial adaptation: Some stress tests on certain areas were performed to see how increased precipitation and threats from rising sea levels would impact these areas. This is part of the *Deltaplan Spatial Adaptation*, which contains all projects and mitigations that are needed to develop the Netherlands in a climate robust manner. The plan is worked out in detail for the coming six years.

2.2.2. Additional advice Deltacommissioner

On the 13th of July 2021, a joint request for advice from the Ministry of the Interior and Kingdom Relations and the Ministry of Infrastructure and Water Management was made to the Deltacommissioner. The consult concerned the inclusion of the developments of climate adaptation when planning for the , both on the short- and long-term. Before 2030, a total of 900.000 new houses need to be build, of which 220.000 need to be build in large-scale housing areas. The request was divided into two tracks.

Track 1: Advice on the short term based on insights with the emphasis on costs

The first track was a request for advice on how to proceed with the required housing need in the next two to three years, from a cost-effective perspective on climate-adaptive design. The response from the Deltacommissioner on track one was given per letter on the first of September 2021, but is not considered relevant for this research and hence is not presented.

Track 2: Advice on the longer term

The second track was a request for advice on how to proceed with climate adaptive urbanisation in the Netherlands with a time horizon of 2050 to 2100.

The response from the Deltacommissioner on the second track was given per letter on the third of December 2021. The advice rested on two leading principles. The first of which was that the consequences of climate change should not be carried over to future generations. To achieve this, the water- and soil-system should be leading when considering the spatial planning of houses in the Netherlands. This concerns the construction methods as well as the construction location. The second principle concerned the inclusion of the effects of climate change in the investment plans for housing. An emphasis is put on the flexibility that is needed to prevent a lock-in, e.g. a large investment in an area that makes it nearly financially impossible to abandon an area without suffering great losses. The most important advice from the Deltacommissioner for this research is: ”(...) Next to that I would like to advice to start an investigation on how urbanisation and the investments that are associated with the urbanisation can be redistributed spatially over the Netherlands. Next to that I advice to start a migration to locations that are the least vulnerable from a climate change perspective" Glas (2021).

2.3. Climate change

The climate is changing. The rate and reason might be under debate, the effects are real and measurable. In the recent report of the IPCC The complete list of these effects extends far beyond the scope of this research. Therefore this section will be limited to the two main threats to water safety: Increased river discharge variability and the trend in sea level rise. Some other threats outside of the scope of this research will briefly be mentioned at the end.

2.3.1. River discharge

The effects of climate change on river discharge are two-fold as it is mainly the variability that is effected. An increase in variability means more extreme discharges, causing both low and high water levels. In the Netherlands, the main rivers of interest are the Rhine and the Meuse. These will be the rivers taken into consideration for this research. The predicted discharge in 2050 and 2100 haven been taken from Klijn et al., 2015, which based the discharge of the KNMI climate scenario's of 2014 (van den Hurk et al., 2014). These scenario's are based on IPCC reports and contain four scenario's. These four scenario's are the combination of two variables: The global temperature increase and the change in airflow patterns, which can both take low and high values. For this research the main interest is in the change in temperature, such that the two scenario's that are looked at are that of a high and low change in temperature with a large change in airflow patterns. These scenario's corresponds to G_H and W_H in the KNMI report. The distinction is made by the indication of a "+ (-)" next t to the year, indicating a high (low) change in temperature. Next, it is important to find the corresponding water level for a given discharge per river, this is done via Q-H relationships. To determine these water levels, the normative discharge with an exceedance probability of 1/1.250 is chosen. The results are shown in table 2.1.

	Current	2050-	2050+	2100-	2100+
Discharge Rhine [m^3/s]	14400	15300	15200	14900	17100
Increase in water level Rhine [m]	-	0.31	0.28	0.18	0.94
Discharge Meuse [m^3/s]	3900	4250	4250	4100	4750
Increase in water level Meuse [m]	-	0.27	0.27	0.16	0.57

Table 2.1: Discharge and water level of Rhine and Meuse in 2050 and 2100 Klijn et al., 2015

The Q-H relationship is assumed identical to the relationship used by Kok et al., 2008, which relates a 10% river discharge increase in the Rhine with approximately 0.50 meters increase in water level. For the Meuse, a 10% increase corresponds to slightly more than 30 centimeters of water height increase. .

It should be noted that a river has spatial variability and that this water level is not the exact water level along the entire Rhine and Meuse. Nonetheless the numbers give a ballpark idea about the costs that will be incurred for dikes along these rivers.

2.3.2. Sea level rise

The effects of climate change on the sea level rise (SLR) largely depend on the chosen emission scenario and corresponding increase in temperature. Although the author of this research is an optimist by nature, and would therefore prefer to believe in a scenario where global emissions will decrease, the truth is that this is most likely not true. When it comes to collaboration on a global scale, a neutral or even pessimistic view seems to be more warranted. Therefore the upper-bound of the estimates from the IPCC report Oppenheimer et al., 2019 will be assumed in this research. Table 2.2 shows the rate of global mean sea level rise as well as the expected value in 2050 and 2100 with the confidence interval in brackets, these values are also shown graphically in figure 2.3.
For this research, the high-end estimates of the RCP4.5 and RCP8.5 are used. This corresponds to an expected sea level rise of 0.34 and 0.40 meters for 2050 and 0.72 and 1.10 meters for 2100 for RCP 4.5 and RCP8.5 respectively.

	RCP2.6	RCP4.5	RCP8.5
Annual SLR [mm/y]	4 [2 - 6]	7 [4 - 9]	15 [10-20]
Expected SLR 2050 [m]	0.24 [0.17 - 0.32]	0.26 [0.19 - 0.34]	0.32 [0.23- 0.40]
Expected SLR 2100 [m]	0.49 [0.29 - 0.59]	0.55 [0.39 - 0.72]	0.84 [0.61 - 1.10]

Table 2.2: Expected sea level rise by 2050 ad 2100 by the IPCC

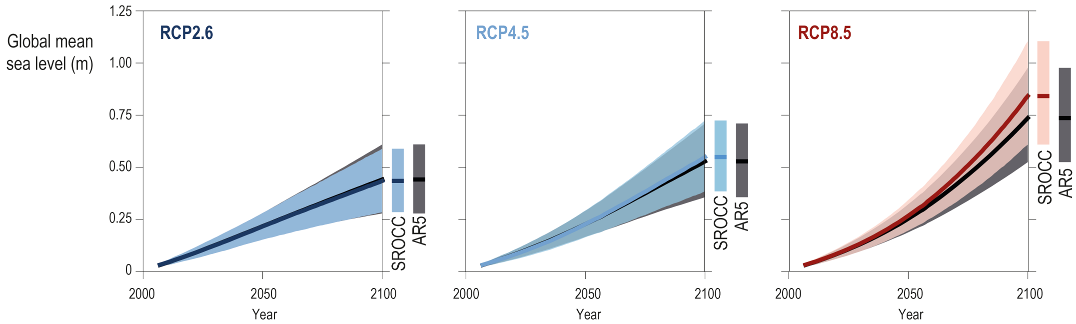


Figure 2.3: IPCC global sea level rise estimates for different scenario's Oppenheimer et al., 2019

A recent study of researchers at the TU Delft found evidence of an increase in the yearly SLR rate after 1990. Steffelbauer et al. (2022) showed that the average measured SLR rate over eight stations in the Netherlands before the break-point in the 1990's is 1.7 ± 0.3 mm yr^{-1}, and increased to 2.7 ± 0.4 mm yr^{-1} after the break-point. This increasing rate can also be seen in the adjusted predictions by the KNMI. Since 2014 the predictions have been adjusted by as much as 20%. This supports the argument for choosing the upper-bound of the proposed confidence interval by the IPCC scenario's.

2.3.3. Other consequences

Although river discharge variability and sea level rise constitute the main effects that are considered in this research, there are other effects that can have an impact on the boundary conditions for water safety. The consequences will be briefly described, as well as their impact on the (direct) environment. They will not, however, be taken into account during the financial analysis.

Drought

With a changing climate come periods with increased length where there is no precipitation. This can lead to drought, which in turn causes low ground water levels. If the ground water levels are low for prolonged period of time, the head in dikes will drop. When, eventually, the water in rivers rises again, the head difference over the dike will be greater than it was before the drought. This increases the risk for several failure mechanisms. Next to the increase of flood defences, there is the societal risk of a decreased water supply.

Salt intrusion

When discharges in the rivers are low and/or the sea level is high, salt water can intrude estuaries and river mounds. If the discharge is lowered for a prolonged period of time, the brackish water will intrude further upstream and can intrude into the groundwater. This has the potential of decreasing the size of the current ecological habitat of many species, as well as contaminate aquifers filled with drinking water.

2.4. Policies on water safety

This section discusses the historic context of water safety in the Netherlands. After a brief look at the history, the current state of affairs will be discussed. Finally, the structure of water safety in the Netherlands will be explained.

2.4.1. The history of safety

Water safety standards in the Netherlands have existed for an extensive period of time but were only formally enshrined in law since 1995. Since then, safety standards have been incorporated in two laws and the legal definition has been changed.

Wet op de Waterkering

The 'Wet op de Waterkering', which roughly translates to law of the Flood Defences, is a law that was introduced in 1995 to give a legal anchor for safety standards against flooding. This law denotes a safety standard for each area within a dike ring in the Netherlands. The norm is defined as "*The average exceedence probability - per annum - of the highest of the highest high water level for which the primary water defence intended to directly protect the outside water must be calculated, taking into account other factors that determine the water-defence capacity*" De Nederlandse Overheid, 1995. These safety standard were derived by the first Deltacommission in 1953, and differed per area. Most dike sections in the western part of the Netherlands had a safety standard of 1/10.000, while most river dike sections inland had a safety standard around 1/1.250. By law, the safety standards had to be reviewed every five years. The allowed exceedence probability per dike section can be seen in figure 2.4.

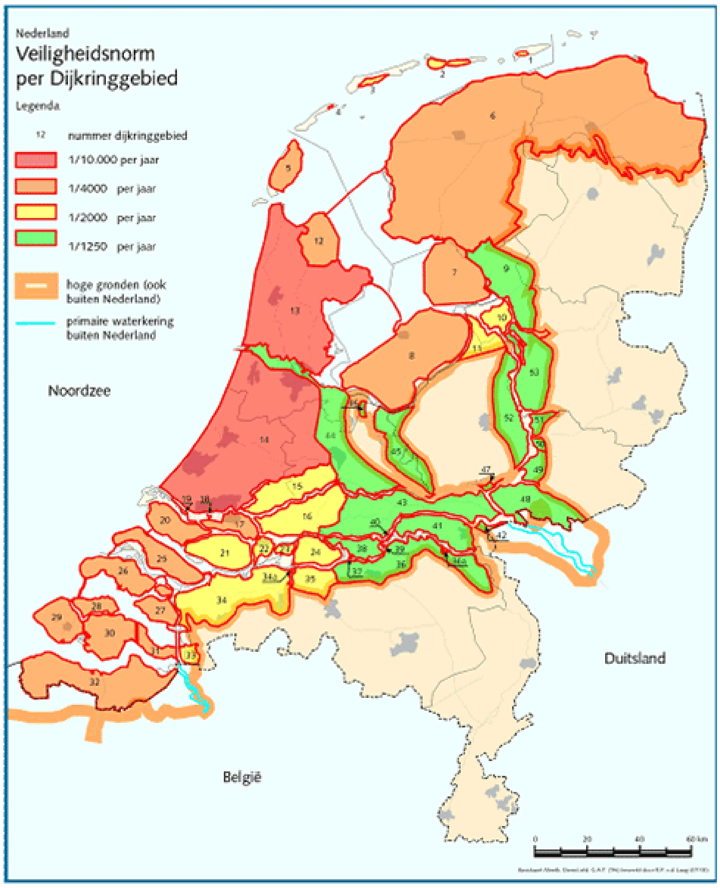


Figure 2.4: Safety standard per dike section

Waterwet

The *Wet op de Waterkeringen* was incorporated, together with seven other water-related laws, into the Waterwet in 2009. This law still incorporates the safety standards of the Wet op de Waterkeringen, but adds several other water-related requirements on paper. The total list of requirements contains subjects on water quality and quantity as well as on the usage of water.

The new standard

On the first of January 2017 a new safety standard was introduced. This new standard changed the question from: *What is the failure probability of the flood defence given a hydraulic load?* to *What is the probability of a fatal flood?*. Meaning that it is not the strength of the individual flood defences (often dikes) that is assessed, but rather the system and probability of flooding of an area. The goal is that in 2050, each inhabitant of the Netherlands has a probability of death due to flooding of 1:100.00. Whether this change was for the better, is up for debate. An independent Dutch advice committee on water safety (ENW) has posed some questions around the approach and given advice on how to improve it (Kok et al., 2020).

2.4.2. Philosophy of safety

The dutch philosophy on water safety after 2017 can be deduced back to one key concept, that of *Risk*. Risk is defined as the probability of damage due to an event, such as a flood, multiplied by the expected damages as a consequence of said event. Summing over all n possible floods and their respective damages yields the total risk for an area, as given per:

$$\text{Risk} = \sum_{i=1}^{n} \mathbb{P}[\text{Event}_i] \cdot \mathbb{E}[\text{Damage}_i]$$

Where:
Risk: The expected costs incurred, often expressed in monetary amounts per year.
$\mathbb{P}[\text{Event}_i]$: The probability of an event occurring per year, in Hydraulic Engineering this often concerns a flood)
$\mathbb{E}[\text{Damage}_i]$: The damage, often expressed in monetary amounts, as a result of an event i occurring

To give an illustration of the new standard introduced in 2017, two examples will be shown. Before the introduction of the new standard, dike sections were given an allowable failure probability. Assume there is a dike section that protects an area with a river running through it, with a value of €125 on the left side of the river and €250 on the right side of the river. Assuming the frequency of exceedence for this particular dike section is 1/1,250, it is found that the risk for each side R_l, R_r equals:

$$R_l = \frac{1}{1,250} \cdot 125 = 0.1 \qquad R_r = \frac{1}{1,250} \cdot 250 = 0.2$$

When the two sections are assumed to be independent, a total risk R_1 can be calculated as the sum of the two individual risks, given by:

$$R_1 = \sum_{i=l}^{r} R_i = 0.1 + 0.2 = 0.3$$

Doing a similar exercise, but requiring the risk to remain constant for both areas and equal the lower risk of the left side, it is found that the allowable failure probability $\mathbb{P}[\text{Flooding}]$ for both sections now equals:

$$\mathbb{P}[\text{Flooding}_l] = \frac{0.1}{125} = \frac{1}{1,250} \text{ and } \mathbb{P}[\text{Flooding}_r] = \frac{0.1}{250} = \frac{1}{2,500}$$

Hence, the required safety level has increased. As dikes are usually made symmetric, this means that a safety standard of 1/2.500 will be required for both areas, reducing the total risk for the two areas to:

$$R_2 = \frac{1}{2,500} \cdot (125 + 250) = 0.15$$

Although fictional, this example illustrates the current state of affairs of legal safety in the Netherlands. Due to the introduction of the new safety standards, a lot of dikes changed from being considered "safe" before 2017 to being considered "unsafe" afterwards.

2.4.3. Safety standards

The current safety standards are derived from the new standard that was introduced in 2017. As it is fairly young, not all new safety standards have been met. The current safety standard is an acceptable risk of dying due to flooding that has to be less than 1/100,000 or equivalently $\mathbb{P}(Death|Flood) = 10^{-5}$.

Primary Flood defences

The primary flood defences of the Netherlands offer protection against floods from the North sea, the Waddensea as well as the large rivers (Rhine, Meuse) and a selection of estuaries, most of which are in the province of Zeeland. The primary flood defences are maintained by both the water authorities and Rijkswaterstaat as per the Waterwet.

Regional Flood defences

Regional flood defences protect the Netherlands against water from within, often from the many lakes, smaller rivers and canals. Roughly speaking there are three types of regional flood defences: Those that prevent against water from large rivers and seas but are not primary flood defences. Those that prevent against flooding from water that comes from the smaller rivers, lakes or canals. And finally those that only function once a primary flood defence fails. Those are normally in a ”dry” state. Water authorities, regional sections of Rijkswaterstaat en the provinces each have a responsibility for the maintenance of the regional flood defences.

Hoogwaterbeschermingsprogramma

To prevent flooding in the Netherlands, the *Hoogwaterbeschermingsprogramma* (HWBP), roughly translating to ”Floodprotection programm”, will cary out reinforcements on flood defences between now and 2050. This includes around 1,500 kilometers of dikes and 500 slocks and pumping stations. This programme is a joint effort by Rijkswaterstaat and the water authorities. Yearly, around €800 million is invested in reinforcement, of which 50% comes from Rijkswaterstaat, 40% from the joint water authorities and the final 10% from the water authority in which the reinforcement takes place (Staf deltacommissaris, 2022).

2.5. The technical side of safety

Although the focus of this research is not on the technical aspects of water safety, it is important to understand the technical principals of several common flood defences. The assumption is made that the the technical feasibility is met for different hydraulic boundary conditions. However, the costs per flood defence do depend on the hydraulic boundary conditions. A higher water level for example, will lead to increased costs for dike reinforcement. This sections will introduce some common flood defences, discuss their failure mechanisms and technical adaptations that can be made to prevent these failure mechanisms. These technical adaptations will be quantified to the extent that is relevant for this research.

2.5.1. Flood Defences

Per definition, flood defences are hydraulic structures with the primary objective of providing protection against flooding along the coast, rivers, lakes and other areas where water and land (or other water bodies) interact. There are several types of flood defences, some common ones are: Dike, dams, dunes, storm surge barriers and water retaining structures like walls and hydraulic structures (Jonkman et al., 2021). As the focus of this research is on inlets near the sea and rivers, dikes and storm surge barriers will be looked at in more detail.

Storm surge barriers

These are structures that are partly move-able to allow for the passage of ships and river discharge under normal circumstances. During periods of high water, these structures can close the waterway off to protect the hinterland against flooding. Famous examples in the Netherlands are the Eastern Scheldt storm surge barrier and the Maeslantkering.

Dike

A dike is a flood defence that is (supposed to be) immovable. They protect the hinterland against water by simply having a higher crest elevation than the waterlevel can reach during periods of high water such as storms or high discharge in rivers. The main focus of this research will be on dikes, as they are the most prevalent form of flood defences in the Netherlands.

Note that next to their water retaining function, flood defences often have a secondary function like recreational, agricultural or transportation. Although secondary, these functions often (partly) determine of the safety requirements.

2.5.2. Failure mechanisms

When designing flood defences, the designing and constructing party has to show that the structure is sufficiently safe. This is done by assessing the possible failure mechanism and making sure the strength of the design exceeds the load provided by the boundary conditions, which is generally done by adhering to the so-called NEN-normen. Often a limit state function Z is defined for the safety, which is the difference between the resistance (strength) R and the load S. The criteria that is often set is that:

$$Z = R - S > 0$$

Where the probability of failure for a certain failure mechanisms can be expressed as:

$$P_f = \mathbb{P}(Z < 0)$$

Where the probability of failure must meet a certain requirement, often based on set regulations and/or expected damages. The assessment of the failure probability can be done on several different stochastic levels, ranging from completely deterministic to completely stochastic. A short overview, based on Jonkman et al., 2017, will be provided below.

Level 0 - Deterministic approach
The level 0 method is a deterministic approach to design. There is no variation in the load and resistance parameters and hence the concept of failure probability can not be applied here. This method is hardly ever used in designing (hydraulic) constructions and will not be discussed further.

Level I - Semi-probabilistic approach
The level I method is a semi-probabilistic approach to design. The parameters that are uncertain are modelled by the so-called characteristic values for both the load and resistance. This is done by choosing a low-percentile (often 5%) in the case of the strength parameter, and a high-percentile (often 95%) in the case of load parameters.

Level II - Approximation of full-probabilistic approach
The level II method is an approximation of a full probabilistic approach to design. In this method the mean of the variables (both resistance and load) and their covariance matrices are taken into account to determine P_f via the limit-state function. Note that the limit-state function in this case has the mean, standard deviation and the covariance matrix included. By equating the limit-state function to zero, a reliability index β can be derived, which is equal to the shortest distance from the origin to the surface that is described by equating the limit-state function to zero. In case of non-linear limit state functions, the equations first have to be linearized. The reliability index β can be transformed to a failure probability by means of the standard normal distribution. Note that this approach often requires iterating.

Level III - Full probabilistic approach
The level III method is a full probabilistic approach. The biggest difference with the previously mentioned method is that the formulation for P_f is exact. If possible with analytical formulations, although this is strongly limited by the number of variables. More often numerical integration or Monte Carlo simulations are used.

Level IV - Risk-based approach
The level IV method is a probabilistic approach where the consequences (the costs) are also taken into account. Here, the risk is used as a measure of the reliability. Different designs can then be compared

on an economic basis, by taking the uncertainty, costs and benefits into account.

For standard engineering practices, the level I method is most often used (Jonkman et al., 2021). Specific failure mechanisms for dikes and storm surge barriers will be discussed further.

Storm surge barriers
There are three large risks associated with Storm surge barriers: Failure to close, structural failure and failure to defend against flood. The first one regards a malfunction either the operating system of the structure, or the structural elements (the physical closing) of the barrier. The second is related to the loads on the structure. Once these loads exceed the strength of the structure, failure might occur. The final one occurs when the structure was not designed for the water levels it is supposed to defend against. This last one might become the case if sea level rise or discharge variability increases more than anticipated when the structures were designed and constructed.

Dikes
For dikes, there is a large plethora of failure mechanism. A selection of the most important failure mechanisms is displayed in figure 2.5.

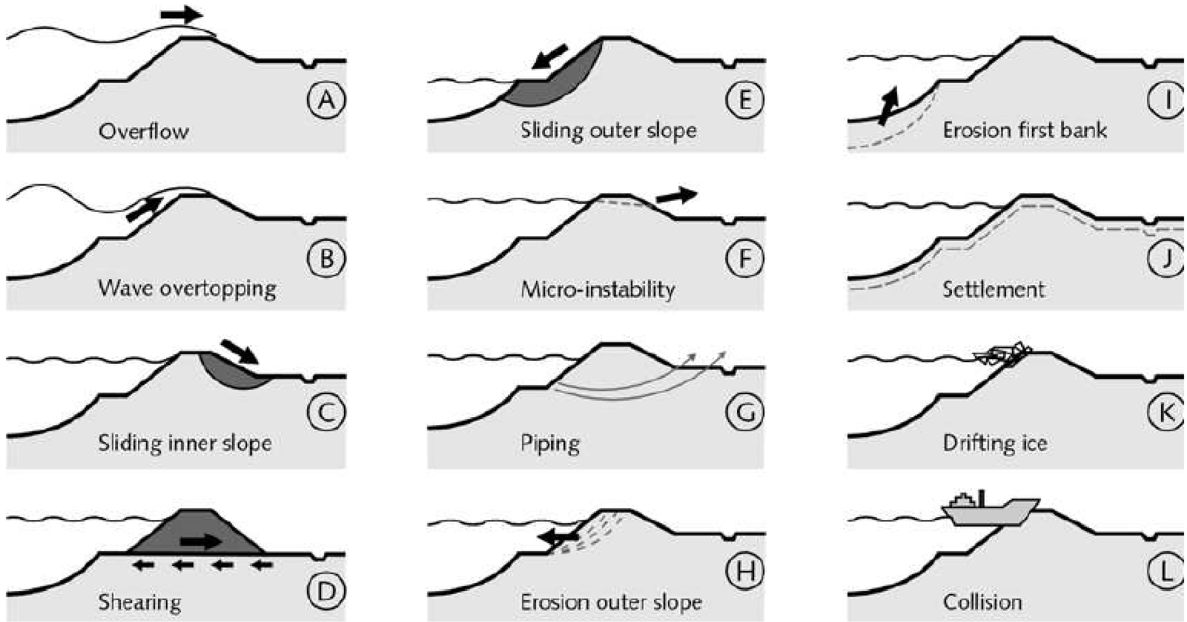


Figure 2.5: List of the most important failure mechanisms (TAW, 1999)

In this section the focus will be on the three main failure mechanism: Overtopping, macro-stability and internal erosion.

Overtopping
Overtopping occurs when a wave that approaches the outer slope of a dike will run up the slope and reach over the crest. This happens when the design level of the crest is below the highest point of the waves. The waves will top over the crest and a flow of water will flow over the inner slope of the dike. The failure mechanisms that is associated with overtopping is the stability of the inner slope. The reason behind this is that the inner slope is eroded, which can lead to progressive damage and potential collapse (Jonkman et al., 2021). The required height and slope for different wave conditions can be determined via the *EuroTop manual* (van der Meer et al., 2016).

Macro-stability
Macro-stability can be divided into two main topics: Stability of the outer slope and stability of the inner slope. The first being less common than the second one. Often, slope instabilities will be triggered by hydrological events, such as high water of a sea or river, which cause the pore water pressure to

increase. This increase in pore water pressure decreases the effective strength. With a decrease in effective strength comes the risk that a structure will fail, as the soil is unable to bear the loads. Interestingly, failure of the inner slope occurs when the water on the outer slope is high for a prolonged period of time. This increases water infiltration and lowers the effective stress. However, if the water level suddenly drops, called a *sudden drawdown* but the soil is saturated, the pressures exerted by the outward slope might be to high for the soil to bear. This is because the river or sea provides a force against the dike, which it keeps in balance with pressures in the soil, once the force is removed the dike exerts this force on thin air and might, as a result, fail.

Internal erosion
Internal erosion is not a single mechanisms but consist of three sub-mechanisms that all need to happen for internal erosion to occur. These sub-mechanisms are uplift, heave and piping. Uplift occurs when the pore pressures in the aquifer underneath the dike increase due to the large differential in hydraulic head between the inner and outer side of the dike. The exact specifications can be calculated via TAW, 1999. When the upward pressure and the inner side exceeds the weight of a top layer, this layer can be moved upward and rupture. This rupture allows water to seep through. When the pressure exceeds a certain critical gradient not only water is transported, but sand as well. During this stage, sand boils may form. When sand is transported a ”pipe” can form under the dike. This pipe will grow in length and diameter until the structural integrity of the dike is compromised and it collapses. The exact criteria were derived by Sellmeijer, 1988. As noted before, for structural failure to occur, all three sub-mechanism have to occur.

This section is concluded with the remark that most of the failure mechanisms described above can be solved by increasing the crest height and width of a dike section. This not only makes the dike higher, giving it more resistance against overtopping, it also makes the dike more stable and increases the piping length. The next section will demonstrate this principle.

2.5.3. Adaptation

When boundary conditions such as river discharges or sea levels change, there is a need for adapting existing flood defences to meet the safety requirements. In this section a relationship between an increase in the water level and the costs for adaptation will be made. Again, the distinction between dikes and storm surge barriers is made.

Storm surge barrier
Storm surge barriers have a large variability in appearance and structural design, so drawing a single conclusion is rather difficult. One thing that they do often have in common is the inability to adept to changing boundary conditions. If the water level rises above the structure, there is no ”quick fix” to increase the height of the structure, as often is possible with dikes. However, it should be noted that these structures are often designed with a long lifetime in mind. Next to the long lifetime they are often also prepared for worst case scenarios, making them likely to withstand even the worst climate predictions.

Dike
In principal, dikes have three failure mechanisms as described in a previous section, all three of which would in principal require a different adaptation. However, there is an argument to make for only taking the water level rise into account. When a dike has to be redesigned for higher water levels, the crest height has to be increased. Assuming that the slope has to be maintained, the dike has to be scaled proportionally to the height increase. If we assume a constant berm width and crest width, it is possible to derive simple expressions for the increase in width and area. This principle is illustrated in figure 2.6. Note that the factor n is equal to the fraction of the increase in height Δh over the original height h. This allows to derive some expressions for the difference in width $\Delta w = w_{heightened} - w_{original}$ and in area

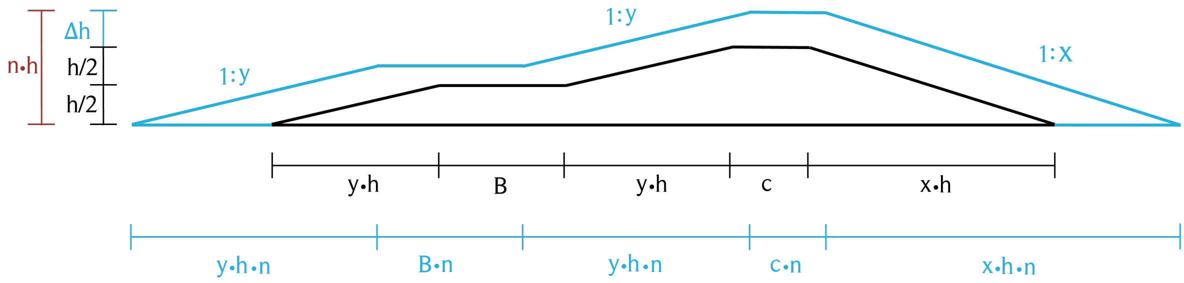


Figure 2.6: A common dike scaled by a factor n

ΔA.

$$\begin{aligned} \Delta h &= (n-1)h \sim n \\ \Delta w &= (n-1)h(2y+x) \sim n \\ \Delta \mathrm{A} &= (n^2-1)(yh^2+\frac{1}{2}xh)+(n-1)(Bh+\frac{1}{2}dh) \sim n^2+n \end{aligned} \tag{2.1}$$

As can be seen in equation 2.1, the width and height are scaled by the same factor n, while the area increases approximately with n^2. These results are in line with the derivations made by Jonkman et al. (2013). It should quickly become apparent that this means that with increasing water heights, the reinforcement costs will increase with a power that is larger than that of the water height. Note that for relatively small increases in crest height, a linear assumption is justified. However, as the crest height increase relative to the initial dike of the height starts approaching larger figures, the exact calculations have to be made.

2.6. Economics

This section will discuss some basic economic concepts, and puts these concepts into the context of this research. It is split up into a section on costs and benefits and how to properly count them and a section on utility theory. The (social) science of economics is concerned with the study on the division of scarce goods that have alternatives use among a group of individuals with limited resources. The premise of economic analysis is that individuals strive to maximize their own utility. From this premise, a lot of deductions can be made.

2.6.1. Costs and Benefits

To properly assess a polder, all costs that it incurs should be tallied up, and compared to the sum of all benefits it provides. This section takes a closer look at the definition of the costs and benefits in economics. After defining the concepts, some Finally, a special case of costs will be discussed, that of opportunity costs. These are often overlooked costs that are crucial for the economic and financial comparison of two alternatives.

Benefits

The Oxford dictionary defines Economic Benefits as: "*Benefits that can be expressed in financial terms as the result of an improvement in facilities provided by a government, local authority, etc.*". As an example the construction of a road is given, which brings on a plethora of benefits for road users. Lower vehicle operating costs, time saving due to the improvement and lower accidents costs as a result of the reduction in accidents.

A list containing the benefits of a polder could be drawn up in a similar fashion, however one would quickly run into a few challenges. Some of the benefits are easily quantified, added space for industry or housing for example. However, other benefits are more intangible and are not so easily quantified, an improved business climate due to perceived safety for example.

Costs
In a similar fashion as the benefits, the list of costs also contains a division based on the quantification of the costs. When drawing up a list of total costs, only the so-called *Prospective costs* and *Opportunity costs* should be included.

Prospective costs
Prospective costs are costs that are incurred when action is taken, and are therefore within control of the decider, they are often seen as the opposite of *Sunk costs*.

Opportunity costs
An often overlooked aspect of the total costs is the opportunity costs, as they are per definition not observed. This is also what differentiates economic costs from accounting costs. In simple terms, Opportunity costs are the .This concept is closely linked to the that of the *Time value of money*, which is discussed in section 2.7.

Sunk costs
A sunk cost is a cost that has already been incurred and can't be recovered. In classical economics, these costs should not be taken into account when making a decision, only the prospective costs should have be considered when making a decision. Often, in personal decision but also business , these costs are mistakenly taken into

To make an informed decision, all of costs should be weighed against all of the benefits. When deciding whether to "abandon" a polder, the Dutch government has to weigh both the the quantifiable as well as the intangible costs and benefits.

2.6.2. Theory of utility

Utility refers to the measurement of the effectiveness of an applied good or service, which is a way in economics to indirectly measure unobserved variables such as satisfaction, happiness or feeling. Utility is a useful concept as it places the added benefit, often described in a monetary amount, into perspective. As an illustration two individuals are compared, which both get a raise of €10 per day. Without thinking of the context via utility, it appears that both individuals would improve their financial situation by an equal amount, after all they are now both €10 per day richer. However, this train of thought does not take their initial wealth into account. Individual one might already make €10.000 per day, making the raise only a 0.1% increase in his or her earnings. On the contrary, individual two might make €10 per day, making the raise a 100% increase in earnings. Obviously, a way to differentiate between these situations is needed. This is where the theory of utility enters the picture, it places the added benefit into context.

There are two main utility theories, that of Cardinal Utility and that of Ordinal Utility. Although they are in many points alike, the main difference is in the quantification of utility. Cardinal Utility assumes that agents (individuals or institutions) can quantify their utility (often done in so-called "Utils" as the unit of choice) and rank them accordingly. Meaning that it is possible to state by how much an agent prefers a combinations of goods and/or services above another combination. Ordinal Utility theory assumes that agents are able to rank their preference, but not quantify them. Hence, in Ordinal Utility theory it is not possible to state how much a combination of goods and/or services is preferred above another combinations, only that it is preferred.

For the purpose of this research, the Ordinal Utility theory is assumed valid. The reason behind this is that this research is only concerned with ordinal preferences of combinations of goods and/or services, not their actual value. To apply the Ordinal Utility theory, four assumptions have to be satisfied. These are:

1. **Rationality of the consumer**: Given the budget constraint of a rational individual or institution, they will always strive to maximize utility.
2. **Ordinal Utility**: Agents can only tell their ordinal preference for goods and services.

3. **Transitivity**: If there are three goods A,B and C, and the choice of the individual or institution is A ≻ B and B ≻ C, then it follows that A ≻ C.
4. **Consistency**: If A ≻ B, then there should not exist a logical reasoning that leads to B ≻ A.

Note that for a utility function $U(x)$, the same properties hold, e.g. if $A \succ B$, then $U(A) \succ U(B)$.

An important concept that can be derived from the Theory of Utility is the concept of *Risk Aversion*. These three preferences can be graphically shown by means of the Utility function.

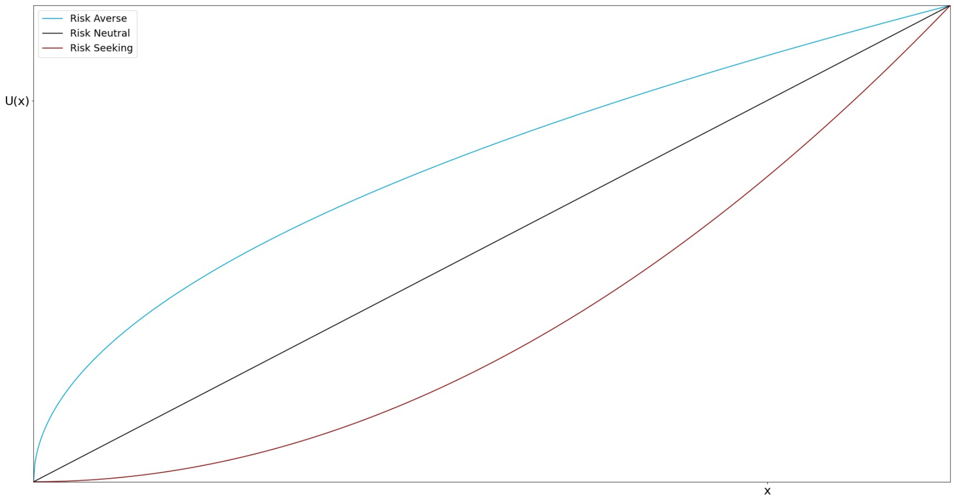


Figure 2.7: Risk approaches in Utility functions

As can be seen in figure 2.7, risk-aversion has a concave utility function whereas risk-seeking behaviour displays a convex utility function. When a person or institution is risk-neutral, this is displayed by a straight line, neither concave nor convex.

2.7. Financing water safety

Although economics and finance are closely linked, they are not the same. Where economics is concerned with the distribution and optimization of scarce resources, finance is concerned with the optimal use of funds with the idea of wealth maximization in mind. This section will dive deeper into the financial side of water safety. To this end, the current structure of a Cost-Benefit analysis in the Netherlands will be discussed and some weak points will be pointed out. After this discussion, the concept of *Time value of money* will be introduced. Certain aspects of the concept will be closer examined and three possible ways of determining a variable discount rate will be introduced.

2.7.1. Current structure

The discount rate is used in the Netherlands to discount the expected costs and benefits of a public or governmental investment project back to the present (Ministerie van Financien, 2020). It can be seen as the minimum required return on investment for a project or policy, required by the Dutch state. If the expected return is higher than the discount rate, the social prosperity rises, if the it lower, than it falls. A higher discount rate gives a lower weight to the costs and benefits in the future. Per 2020, the standard discount rate used in the Netherlands is set at 2.25%. This figure consist of -1% for the risk-free discount rate and of 3.25% for the risk premium. The rates were determined via three sources: The required return by households, companies and insights from the so-called Ramsey-rule. These rates are revised every five years and the discount rate for short- and long-term investments are assumed similar.

There are several things that raise concerns when considering discounting a process. The first is the stringent approach by Ministerie van Financien (2020) to quantify the discount rate as a single deterministic rate, where the interest rate (a close substitute for the discount rate) is naturally a highly variable rate. Setting an interest rate at a certain set percentage neglects all the inherent variability of the economy. The second point of concern is setting the rate for five years at a time. One does not have to look far to see that the years 2017 - 2022 have known inflation rates with monthly lows of 0.3% and highs of 12%. Setting the rate for five years at a time would completely misjudge the economic viability of a project in 2022 based on the rates determined in 2017.

For these reasons, this research argues in favour of changing the discount rate from a set rate to a variable rate to incorporate the natural fluctuations of the economy. This better reflects the true cost of a project in turbulent economic times.

2.7.2. Time value of money

To properly asses an investment, it should be noted that the value of an asset at a particular point in time C_t is different from the value of that same asset at a different point in time C_{t+n}, where n can attain both positive and negative values indicating a past or future value, respectively. To compare investments in different assets their future values should be compared to one another on a set point in time. A widely used metric to accomplish this goal is the Net Present Value of an asset, which discounts the future cash flows of the asset to the present value by taking the risk-free rate as the minimum base return, yielding the Present Value of all future cash flows. The principal investment is subtracted to achieve the Net Present value, as can be seen in equation 2.2.

$$NPV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} - C_0 \tag{2.2}$$

In which:
C_t = The value of an asset at time t
r = The discount (or interest) rate
C_0 = The principal investment at $t = 0$

The interest (or discount) rate r_t is an often misunderstood concept. This rate should be viewed as a rate of return on an investment that can always be achieved, without taking ”any” risk. No asset is completely risk-free, but governmental bonds (of selected countries) are often used as a proxy for the risk-free rate. To invest money in assets or projects, one takes on risk. The expected pay-out has to scale with the risk in order for an investor to justify their investment. After all, why would one take the risk of investing in a project that has a potential pay-out of 3% of the principal, but also has a 30% chance of failure when the risk-free rate offered by governmental bonds is also 3%. This would only constitute additional risk for no additional pay-out, something that rational humans would not consider as a viable option. Therefore, to compare investments a benchmark was established, the risk free rate. This report looks into three possibilities for determining the risk-free rate:

- Constant rate over the time, hence $r = r_t$
- A variable interest rate determined by an autoregressive type of model.
- A variable interest rate modelled as a random walk with a potential drift. Also known as a Brownian motion.

In traditional investment for projects a constant rate is often assumed, even when the investment period spans many decades. As will be demonstrated in this report, the assumption of a constant rate is a naive one. Therefore two additional methods, with variable interest rates, are researched.

2.7.3. Constant rate (with noise)

The assumption of a constant interest rate appears to be the easiest of the three cases, and in a modelling sense it indeed is, choosing the exact number for the interest rate is difficult. When noting that the lifetime of the considered structures often ranges from 50 to 100 years, setting an interest rate at one particular number seems almost foolish to do. To get an idea of the variability that risk-free rates exhibit, properties of the US 10-year treasury bonds are briefly discussed. To model the discount rate, one could think about taking the current rate, taken at the 25th of July 2022, which has a value of around 2.8%. This would, however be a multiple of well over 4 over the risk-free rate on this same asset a mere two years ago.

Although the last few years have seen high volatility due to the global pandemic, this phenomenon is not limited to only the last years. When looking at figure 2.8, it can be seen that the interest rates have varied from a recent low of around 0% to high of 15%+ in the 1980's.

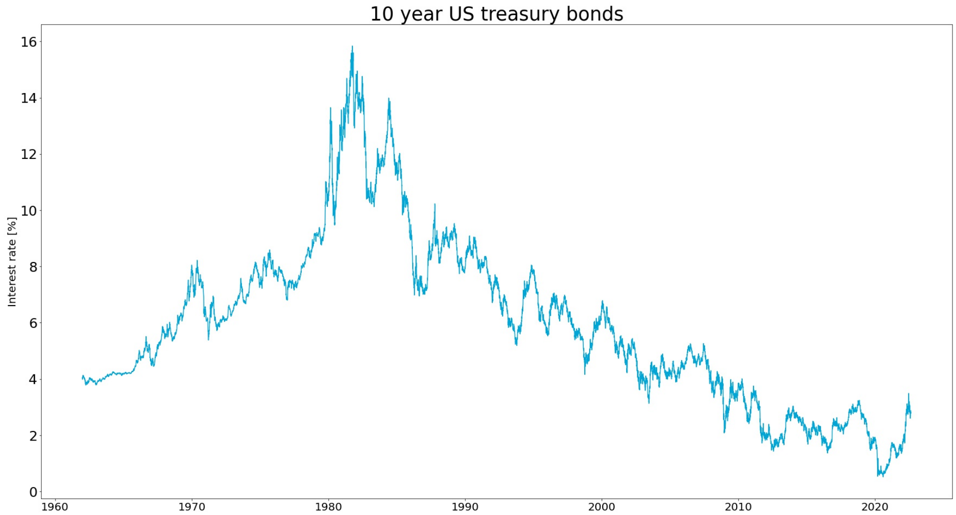


Figure 2.8: 10-year US treasury bonds

To conclude this section: Taking a single risk-free rate as the true value when financing a project neglects a lot of the inherent variability of this rate. Nonetheless, this is what the Dutch government recommends for large infrastructure project, often setting the rate at $\pm 2\%$. To extend this model a bit, some noise can be added as to simulate the random movements that the interest rate takes quarter to quarter. Often, this is done by modelling the rate as a Gaussian process, such that $r_t \sim \mathcal{N}(\mu, \sigma^2)$

2.7.4. Time series model

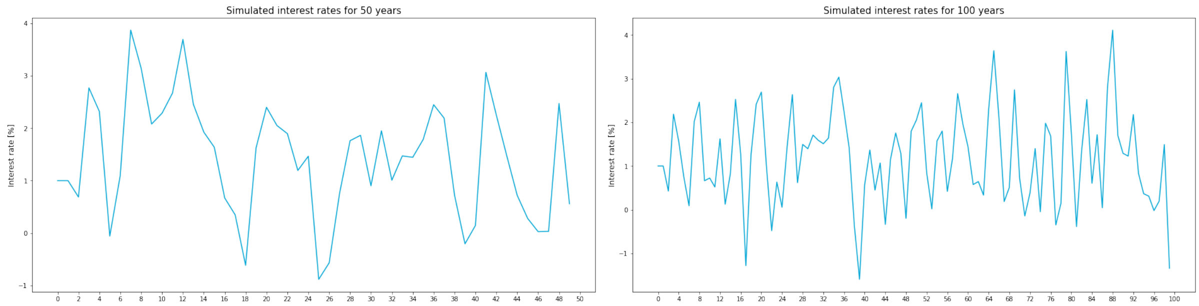
For statistical analysis of time series in economics and finance, often a autoregressive-moving-average (ARMA) models are used. As the name suggests, these models consist of two main parts: one term for the autoregression (AR) and another term for the moving average (MA). The AR part models the linear dependence of the variable with it's own past value(s). The MA part models the linear dependence of the variable with past value(s) of the stochastic errors that are inherent in time series. A standard formulation of the model can be seen in equation 2.3, containing p AR terms and q MA terms.

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2.3}$$

In which:
μ = The deterministic trend of the time series
ε_t = The white noise error terms at time t
φ_i = The i-th parameter coefficient of the AR part
θ_i = The i-th parameter coefficient of the MA part
X_{t-i} = The previous value of the variable of interest
ε_{t-i} = The previous values of the white noise error terms

The use of an ARMA(p,q) model, as opposed to just an AR(p) or MA(q) model, is the most appropriate in this situation for two reasons. First off, as the development of interest rates in the Netherlands, or any country for that matter, often depends on the interest rate from a previous period. This is reflected in the AR part of the model. The MA section is justified as it is used to characterise "shock" information to a series. The financial health of a country is judged not solely on its finances, but also by events that might impact it. Think of wars, trade relations that turn sour or change of political course. These events are hard to model, as they are per definition unforeseen. To add an element of randomness to the model, the MA section is added.

Two examples of a the development of a 50 and 100 year risk free interest rate are shown in figure 2.9a and 2.9b. In this case an ARMA(2,1) model was used with parameters:
μ = 1
φ_1 = 0.3
φ_2 = -0.05
θ_1 = 0.22



(a) Prediction for 50 years

(b) Prediction for 100 years

Figure 2.9: Prediction of the risk free interest rate r_t over two different periods, based on an ARMA(p,q) model

2.7.5. Interest rate as a random walk

Brownian motions, also known as random walks, find their origins in the description of the motion of small particles. It describes random motion in a three-dimensional plane with equal probability of moving one step in either of the three axis (up-down, left-right, forward-backward). In the case of a one-dimensional Brownian motion, it can be modelled as the scaled limit of a random walk. If we denote R to be a Rademacher variable, meaning it has a discrete probability distribution with the following probability mass function:

$$f(x) = \begin{cases} \frac{1}{2} \text{ if } x = -1 \\ \frac{1}{2} \text{ if } x = 1 \\ 0 \text{ otherwise} \end{cases}$$

and take the sum of n i.i.d. Rademacher variables, and divide by the square root of n, we obtain the following scaled continuous time process.

$$W_n(t) = \frac{1}{\sqrt{n}} \sum_{i=1}^{[nt]} R_i \qquad t \in [0, 1]$$

Taking the limit as $n \longrightarrow \infty$, yields the so-called one-dimensional Wiener process W_t. The Wiener process follows a normal distribution with mean 0 and variance t, such that at $t = s$:

$$f_{W_s}(x) = \frac{1}{\sqrt{2\pi s}} e^{-\frac{x^2}{2s}} \qquad \mathbb{E}[W(s)] = 0 \qquad \mathbb{V}[W(s)] = s$$

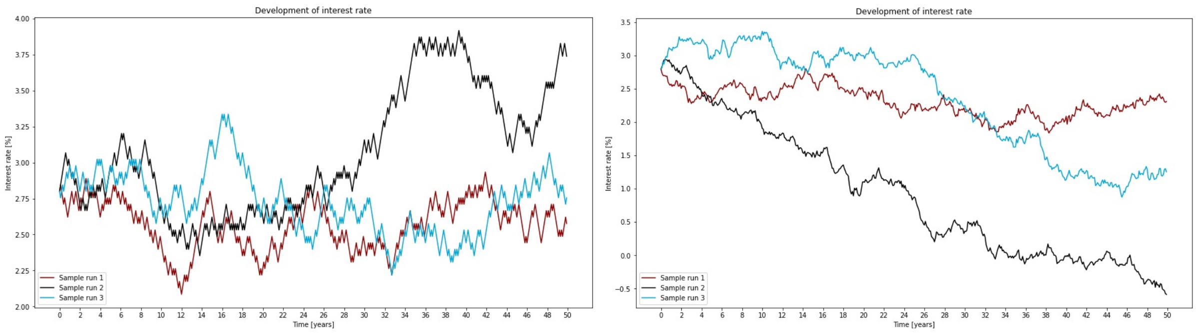
This process is defined by four properties.

1. $W(0) = 0$
2. $W(t)$ is continuous in t.
3. $W(t)$ has independent increments.
4. (t) has Gaussian increments. Meaning $W(t) - W(s) \sim \mathcal{N}(0, t - s)$ for $0 \leq s \leq t$

The Wiener process is used in a variety of financial models, albeit with modifications. For the value of bonds and stocks, which can not be negative, a Geometric Brownian motion is often used. For the modelling of interest rates, which can be negative but often exhibit a certain trend, the so-called Wiener process with drift is used. This stochastic process, $X(t)$ incorporates a trend with the stochastic nature of the Brownian motion. Defining the drift as μ and the variance of process as σ^2, the process can be written as per equation 2.4.

$$X(t) = \mu t + \sigma W(t) \tag{2.4}$$

Determining the parameters μ and σ can be done in a variety of ways. An example for a "regular" Wiener process is given in figure 2.10a. An example of a Wiener process with drift ($\mu = -0.05$ & $\sigma = 0.9$) is given in figure 2.10b. Note that in both cases the starting value was 2.8.



(a) Brownian motion

(b) Brownian motion with drift

Figure 2.10: Brownian motion with and without drift with a starting value of 2.8

Often, the Brownian motion is used for Monte Carlo simulation to determine the probability of exceedance of a certain boundary by the Brownian motion.Doob, 1949 showed that there exists a closed-form expression for the exceedance probability of constant boundary crossing for a Brownian motion with negative drift for some $t \leq 0$.

$$\begin{aligned}
\mathbb{P}(sup_{t\geq 0}\ X(t) > y) &= \mathbb{P}(X(t^*) > y \text{ for some } t^* > 0) \\
&= \mathbb{P}(W(t^*) + \mu t^* > y \text{ for some } t^* > 0) \\
&= \mathbb{P}(W(t^*) > y(1 - (\mu/y)t^*) \text{ for some } t^* > 0) \\
&= \mathbb{P}(\frac{W(t^*)}{1 - (\mu/y)t^*} > y \text{ for some } t^* > 0) \\
&= \mathbb{P}(sup_{t\geq 0}\ \frac{W(t)}{1 - (\mu/y)t} > y) = e^{2\mu y}
\end{aligned}$$

Which finally yields equation 2.5

$$\mathbb{P}(sup_{t\geq 0}\ X(t) > y) = e^{2\mu y} \tag{2.5}$$

This relatively simple expression allows for the calculation of an exceedance probability of the interest rate during the time period of interest. Conversely, it is possible to calculate the probability of non-exceedance by use of the complement rule.

$$\mathbb{P}(sup_{t\geq 0}\ X(t) < y) = 1 - \mathbb{P}(sup_{t\geq 0}\ X(t) > y) = 1 - e^{2\mu y}$$

This is a useful result when determining the maximum allowable interest rate for financial feasibility of a polder system, and it's corresponding probability.

3 Methodology

This chapter contains the methodology of the research and consists of three main sections with several subsections. The chapter starts with the CBA analysis which will include temporal elements, after which it continues to the proposed added constraints. The final section will combine all insights to propose a model for the analysis.

The first section comprises the Cost-Benefit Analysis (CBA) used in this research. Section 3.1 starts with the general definition of a CBA and moves into the costs that are taken into account in this research. Next,considering which costs have to be considered in this research, the subsection also quantifies these costs by putting them in a mathematical framework. After the costs, the same is done for the benefits. With both these quantities expressed, the section moves on to the inclusion of temporal elements. This means that the costs and benefits are adjusted to reflect their growth or decline over time, as derived by Eijgenraam (2006). Furthermore, expressions for the optimal and maximal reinforcement times will be (re)-derived. This leads to a framework for the number of reinforcements needed. The section will continue with the inclusion of the discounting principle on both the costs and benefits. Combining these derivations and frameworks leads to an optimization algorithm that will be presented at the end of the section, further referenced as the adjusted CBA.

Section 3.2 comprises the additional constraints proposed to the adjusted CBA analysis. Four constraints are derived and mathematically formulated. After the mathematical formulation, a small numerical example is given. This numerical example will be used in the next section to determine which of the four constraints will be further developed and incorporated in the adjusted CBA.

The final section, section 3.3, is concerned with the description of the proposed model for the case studies and analysis. It starts with choosing one of the four constraints based on several criteria. After a constraint is chosen, three models for the discount rate will be presented. The final part of this section consists of the description of the full mathematical framework by which the economical viability will be judged. It combines the insights of the adjusted CBA with the added constraints.

3.1. Cost-Benefit Analysis

A Cost-Benefit Analysis, often abbreviated as CBA, is a systematic process that can be used by individuals, businesses or governmental agencies to analyze which decisions to make and which decisions to forgo. As the name suggests, it consist of two quantities, the costs and the benefits, that are analyzed by some criteria. Often this criteria consists of the (discounted) benefits needing to exceed the (discounted) costs. In the field of Hydraulic Engineering a CBA is often used to evaluate the economically most optimal solution for a flood defence to protect an area. Whichever proposed defence has the largest positive value is often chosen. The costs and benefits that are taken into account when performing a CBA can consist of material as well as immaterial costs. One can think of the physical costs of strengthening a flood defence as well as adverse effects of a proposed flood defence on the ecosystem or even go as abstract as the utility individuals take i when tallying up the costs. Similarly, the benefits can consist of material as well as immaterial benefits. For this research the costs and benefits are limited to the material costs and benefits as they are more easily quantifiable and are (often) the majority share of the total costs and benefits.

3.1.1. Costs

For the purpose of this research two types of costs are included: The incurred costs as the flood risk increases or decreases and the costs of physically strengthening the flood defences. Each of these two types of costs will be explained in detail and a mathematical model will be provided.

Incurred costs as a result of flood risk

For the purpose of this research there are two types of costs associated with the flood risk, that of the expected costs incurred for individuals and for assets. In general the risk, which is the expected costs that are incurred as a result of an event, can be expressed as the product of the probability of loss of value ($\mathbb{P}$) and the value of the object or person (V).

$$Risk = \mathbb{P} \cdot V$$

Expanding this expression to include the difference in the value of lives and that of assets, but keeping the probability of loss of value the same for both quantities, yields that the expected costs as a result of flood risk can be expressed as:

$$C_{risk} = \mathbb{P} \cdot (V_{asset} + V_{live})$$

Where:
$\mathbb{P}$: The probability of a flood occurring
V_{asset}: The value of assets lost as a result of a flood
V_{live}: The value of lives lost as a result of a flood

The value of anything is per definition a subjective matter, especially when talking about something as precarious as a human life or a personal home. Luckily, generally accepted numbers do exist for both these quantities. The value of a human life for hydraulic engineering purposes is generally accepted to be €6.7 mln. (Slootjes & Wagenaar, 2016). The loss of asset value in this research is limited to the loss of housing. To this extent, the median house value in an area is assumed to be the value of the assets. Extension can be made by including the monetary value of infrastructure and the value of assets can be adjusted to include different types of buildings such as office space for companies.

To relate the probability of a flood occurring to the crest height and increase in crest height Δh, an exponential model is proposed.

$$\mathbb{P} = e^{\alpha(H_{crest} - H_{HWL} + \Delta h)} = e^{\alpha \cdot (H + \Delta h)} \tag{3.1}$$

Where:
α: A coefficient relating freeboard height to failure probability
H_{crest}: The crest height of a dike
H_{HWL}: The high water level
H: The freeboard height, defined as $H_{crest} - H_{SWL}$.

The reason for choosing an exponential model is two-fold. The first reason is that it captures the physical reality rather well, yielding exponential smaller failure probabilities for increasing crest heights. The second reason is that the natural exponential function has some attractive mathematical properties when making derivations.

Combining all derived expressions yields equation 3.2 for the expected costs as a result of flood risk.

$$C_{risk} = e^{\alpha \cdot (H+\Delta h)} \cdot (V_{asset} + V_{live}) \tag{3.2}$$

Note that it is possible to differentiate between the risk of loss of live and assets by including a so-called *evacuation factor*, which captures the ability of people to escape floods by going to higher grounds or leaving the area, whereas assets are unable to. To keep calculations simple in this research, this factor is assumed to be equal to zero.

Incurred costs due to physical strengthening of flood defence
The costs that are incurred when physically strengthening a dike are dependent on a number of parameters. These parameters include the geometry of the dike, the proposed increase in crest height, the length of the reinforcement and the unit cost per increase of meter crest height. Their relationship is summarized in equation 3.3.

$$C_{dike} = L_{dike} \cdot K_{dike} \cdot \Delta h \tag{3.3}$$

Where:
L_{dike}: The length of the dike section that required reinforcement [km]
K_{dike}: The costs of increasing the crest height by one meter per kilometer dike [€m/km]
Δh: The increase in crest height [m]

Out of these parameters, the cost per kilometer dike when the crest height is increased by one meter (K_{dike}) has the largest uncertainty. Jonkman et al. (2013) summarized the results of several research papers on these costs. The estimates range from €4.5 mln. in rural areas to €22.4 mln. for urbanised areas. Both these figures were for the Netherlands, between different countries there appears to be a large variance in the costs as well.

Note that throughout this section a linear relationship was assumed between the costs of strengthening a dike and the increase in crest height. For small Δh with respect to the initial crest level this relationship holds well, however if the increase is relatively large, the geometry of the dike plays a roll as well. In this case the generalized results from Jonkman et al. (2013) do not hold and the geometry of the dike should be taken into account. This can be done by deriving a unit cost of the area of a dike and combining this with the results derived in equation 2.1.

Cost minimization

To provide the lowest cost, the minimum of the two above mentioned costs should be found. Note that while they flood risk decreases for a larger crest height increase, the costs of strengthening a dike increases for increasing crest height additions. This effect is shown in figure 3.1, which displays the minimization of the combination of flood risk and the costs of strengthening a dike.

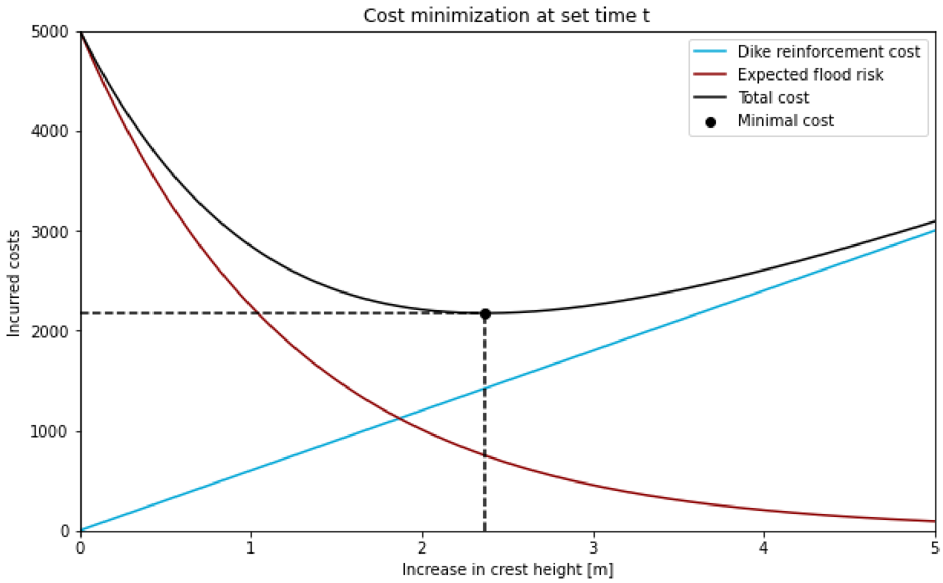


Figure 3.1: Typical Cost minimization for a set t

It becomes apparent that the point of lowest costs lies slightly to the right of the crossing of the two cost lines. To be more specific, it lies at the exact point where the slope of the two lines is equal, but with opposite signs. This can be mathematically explained as a minimization of the derivatives with respect to Δh. For the chosen two costs this yields:

$$\frac{\partial C_t}{\partial(\Delta h)} = \frac{\partial(e^{\alpha\cdot(H+\Delta h)}\cdot(V_{asset}+V_{live})+L_{dike}\cdot K_{dike}\cdot\Delta h}{\partial(\Delta h)} = 0$$

Which has a solution when the numerator equals zero, corresponding to the derivative of the two cost components equalling one another. This mathematical explanation gives an intuitive explanation for the need of the slope of the two cost components needing to equal each other with opposite sign. The physical strengthening is linear in Δh whereas the flood risk is (negative) exponential in Δh, explaining the right shift of the minimization point.

The two costs considered have an analytical solution for Δh as there are few terms and the terms are mathematically "well-behaved", meaning they have attractive properties. However, as one can imagine that as the cost component is expanded with more terms, this minimization can become mathematically cumbersome. To generalize the minimization task, the general framework in equation 3.4 is proposed.

$$\min_{\forall\Delta h} \quad C_t \tag{3.4}$$

Equation 3.4 displays the minimization process of the total costs for all possible crest height increases Δh. The equation can be expanded by taken the previously derived costs and plugging them into the equation, the result of which becomes:

$$\min_{\forall\Delta h} \quad e^{\alpha\cdot(H+\Delta h)}\cdot(V_{asset}+V_{live})+L_{dike}\cdot K_{dike}\cdot\Delta h$$

Which has the elegant solution where the partial derivative of this expression with respect to Δh was set equal to zero.

3.1.2. Benefits

The benefits considered in this research consist of the change in flood risk with respect to the unaltered situation. Unaltered in this context means the flood risk without any intervention, such as the increase of crest height level for a dike, after a period T. This unaltered situation can then be compared to the altered situation, one in which an intervention has taken place, to calculate the expected benefits from the intervention.

To illustrate this principle, imagine a dike that currently has a flood risk of 10^5, and a probability of flooding of $\mathbb{P}(Flooding) = 10^{-4}$. If growth rates of assets and population are momentarily put aside and the growth in polder risk is only affected by sea level rise, the gradual increase in flooding probability is the only factor contributing to the increased risk. If after a period T a reinforcement is made to the dikes surrounding the polder, such that the probability of flooding becomes $\mathbb{P}(Flooding) = 10^{-5}$
The results of the small example are illustrated in figure 3.2.

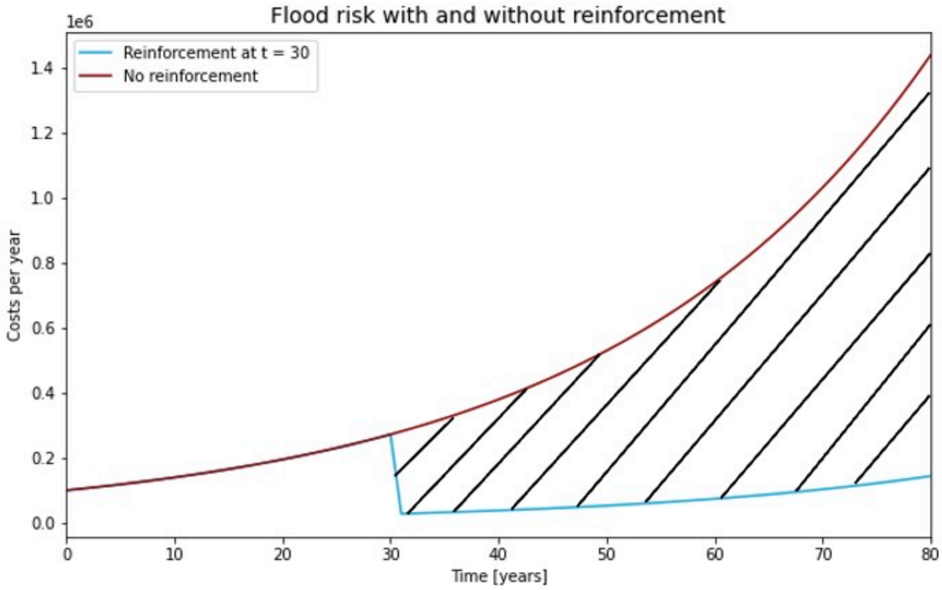


Figure 3.2: Comparison of total costs with and without reinforcement

The benefits in this case would be the difference in cost per year for all the years that there is a difference. Hence the difference between the two integrals of the costs (or summation for discrete time) over the time should be taken. This is marked by the shaded area in figure 3.2. In mathematical notation this means that:

$$B = \int_0^T C_{\text{no reinforcement}}(t)dt - \int_0^T C_{\text{reinforcement}}(t)dt$$

Or in the case of discrete time:

$$\begin{aligned} B &= \sum_{t_i=0}^{T} C_{\text{no reinforcement}} \cdot t_i - \sum_{i=0}^{t} C_{\text{reinforcement}} \cdot t_i \\ &= \sum_{t_i=0}^{T} (C_{\text{no reinforcement}} - C_{\text{reinforcement}}) \cdot t_i \end{aligned} \tag{3.5}$$

To maximize the cost-benefit analysis, the costs subtracted from the benefits should be maximized. Such that the minimization problem given in 3.4 becomes the maximization problem given in 3.6.

$$\max_{\forall \Delta h} \quad B_t - C_t \tag{3.6}$$

In reality the growth in population and asset value also contributes to the increase in risk over time. These factors will be included in the following section on the inclusion of temporal elements.

3.1.3. Inclusion of temporal elements

The derived expressions for the costs and benefits so far have been for stationary pints in time. Although time was included in the benefits, the analysis was still only performed on a set point in time. This section introduces the temporal domain into the analysis of costs and benefits. It will start with the increase of risk over time due to an increase in flooding probability as a result of sea level rise as well as an increase in damage due to the increase in value within an area. After this the increase in costs for physically strengthening a dike will be mathematically formulated. Next, a framework for minimizing these costs over time will be presented. After this the net present value method for a periodically changing discount rate will be discussed. Finally, a mathematical formulation for the benefits over time, including increase in asset value and population growth, will be provided.

Increased risk over time

If the expected costs are to be kept constant and it is given that the value within an area (and thereby the expected damage) increases, the failure probability must decrease. Hence a dike reinforcement is needed as more people move into an area or the value of assets increases, regardless of any change in hydraulic boundary conditions. This results in the need for periodic strengthening of a dike. Preferably the dike would be strengthened nearly continuously, as people are continuously migrating to an area and assets are increasing in value. In practice this is unfeasible, and investments are made periodic. This principle is demonstrated in figure 3.3, first derived by Eijgenraam (2006).

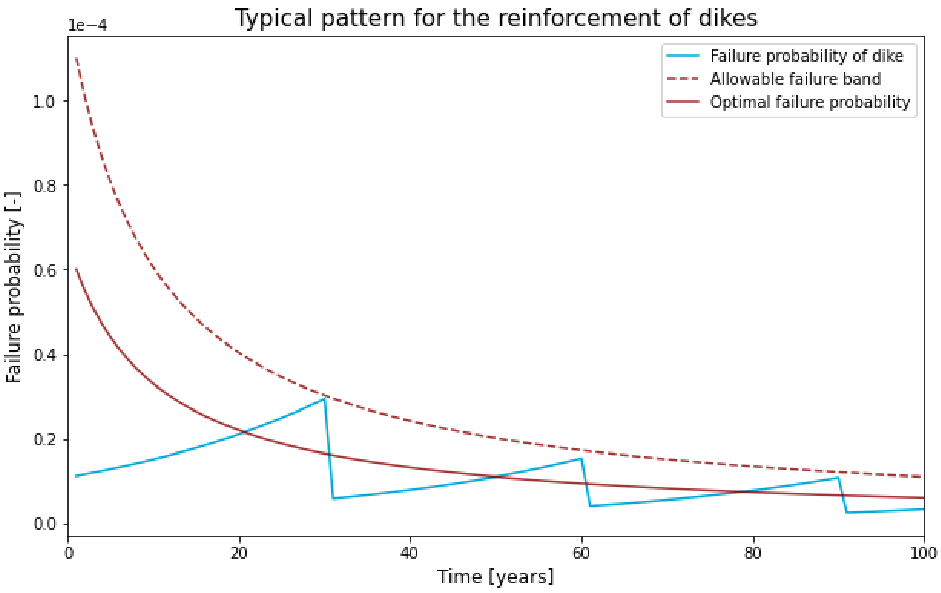


Figure 3.3: Typical "sawtooth" pattern for reinforcing dikes, as derived by Eijgenraam (2006)

To quantify the exact point of needed reinforcement, formulas are needed that capture the extent of increasing value in an area over time, as well as the increasing sea level rise over time. Eijgenraam, 2006 has derived a multitude of these expressions, which will be generalized and presented below.

The expected cost as a result of flood risk contains two elements that are temporally dependent. The first one is the increased probability of flooding, the second one is the increase is value. This can be captured in the definition of risk as:

$$Risk = \mathbb{P}(t) \cdot V(t)$$

Starting with the increase in flooding probability over time, the assumption is made that this is caused solely by sea (or river) level rise. For sea (river) level rise over time, a general expression for the increase or decrease of flooding probability over time can be derived from the proposed exponential model, given in equation 3.1. As the freeboard decreases when the sea (river) level rises, this term ($SLR(t)$) can be added in the exponent to simulate the effect of an increase in flood probability. This yields equation 3.7.

$$P(t) = e^{-\alpha(H_{crest}-H_0-SLR(t))} = e^{-\alpha(H-SLR(t))} \tag{3.7}$$

One could further expand this expression by noting that $SLR(t)$ can be expressed as:

$$SLR(t) = \beta \cdot t$$

Where β is a coefficient indicating the sea level rise year over year. Further noting that at time $t = 0$ the sea level rise is equal to zero allows for rewriting equation 3.7 into a more general form with the failure probability of the dike at time $t = 0$, denoted as P_0. This results in:

$$P(t) = e^{-\alpha \cdot H} \cdot e^{\alpha \cdot SLR(t)} = P_0 \cdot e^{\alpha \cdot \beta \cdot t}$$

If a dike reinforcement is introduced, were H is increased with a quantity Δh, the equation can be rewritten as:

$$P(t) = e^{-\alpha(H+\Delta h)} \cdot e^{\alpha \cdot SLR(t)} = P_0 \cdot e^{\alpha(\beta \cdot t-\Delta h)} \tag{3.8}$$

Equation 3.8 captures the increase or decrease in the probability of flooding over time for an increase in crest height Δh.

The next element to quantify is the increase in expected damage as a function over time. For this end, it is assumed that the increase in expected damage is solely a result of the population increase and increase in asset value. This allows for an elegant formula to be derived, displayed in equation 3.9.

$$V(t) = V_{0,asset} \cdot e^{t \cdot \theta} + V_{0,life} \cdot e^{t \cdot \gamma} \tag{3.9}$$

Where:
θ: The growth rate for assets
γ: The growth rate for the population
V_0: The value of life and assets at time t = 0

If the population growth rate (θ) and asset growth (γ) are assumed equal, and set to θ, the expression can be simplified to:

$$V(t) = (V_{0,asset} + V_{0,life}) \cdot e^{t \cdot \theta}$$

Combining these simplified expression for the development of value over time with that of the development of the flooding probability over time, yields equation 3.10.

$$C_{risk}(t) = \mathbb{E}(t) = V(t) \cdot P(t) = (V_{0,asset} + V_{0,life}) \cdot P_0 \cdot e^{(t(\alpha \cdot \beta+\theta)-\alpha \cdot \Delta h)} \tag{3.10}$$

Ideally, one would make sure that the risk remains nearly identical over time. In a perfect world this would mean that each millimeter of sea level rise and each euro increase in value due to both asset and population growth is compensated by a corresponding increase in crest height. In mathematical terms this would mean that the expression in the exponent is equal to zero. It then quickly becomes apparent that the verbal explanation of a matching safety increase matches the mathematical one, as a constant risk would imply that $t(\alpha \cdot \beta + \theta) - \alpha \cdot \Delta h = 0$. Hence, Δh as a function of t should be equal to:

$$\Delta h(t) = \frac{t(\alpha \cdot \beta + \theta)}{\alpha}$$

However, this is often infeasible when faced with reality. In practice a periodic investment is more common. This means that the dike is often designed safer than economically optimal and allowed to slowly becomes "less safe" over time until a reinforcement is needed again. The latest time t^* at which a reinforcement should be made can be found by equating the risk as a function t to the risk that is demanded for that particular polder E_{demand}. Equating these two quantities and solving for t^*, yields the following expression:

$$t^* = \frac{ln(\frac{E_{\text{demand}}}{(V_{0,asset}+V_{0,life})\cdot P_0}) + \alpha \cdot \Delta h}{\alpha \cdot \beta + \theta} \quad \text{for} \quad E_{\text{demand}} \geq (V_{0,asset} + V_{0,life}) \cdot P_0 \tag{3.11}$$

This equation captures both the increase in flooding probability as a result of sea (river) level rise over time, as well as the increase in value (and thereby expected damages when a flood occurs) over time.

If the rates θ and γ are significantly different from one another, equation 3.10 changes to:

$$\mathbb{E}(t) = V(t) \cdot P(t) = P_0 \cdot V_{0,asset} \cdot e^{(t(\alpha\cdot\beta+\theta)-\alpha\cdot\Delta h)} + V_{0,life} \cdot P_0 \cdot e^{(t(\alpha\cdot\beta+\gamma)-\alpha\cdot\Delta h)}$$

From which it quickly becomes apparent that closed form expression for $\Delta h(t)$ and t^* are not possible. However, as will be later demonstrated during the case study these will be solved by means of an iterative solve approach.

Increased cost of reinforcement over time
Next to the costs that are incurred over time as a result of increased flood risk, the cost of dike reinforcement are also time dependent. To this end, the functions of the dike reinforcement costs and flood risk should be related to time. For the costs of increasing the dike height, the assumption of a standard exponential growth with rate η is made, displayed in equation 3.12.

$$C_{dike}(t) = L_{dike} \cdot K_{dike,0} \cdot \Delta h \cdot e^{\eta\cdot t} \tag{3.12}$$

Where:
η: Growth rate for costs of dike reinforcement
$K_{dike,0}$: Costs of dike reinforcement at $t = 0$
L_{dike} : Length of the dike

Such that the total costs incurred as a function of time become:

$$C_t = C_{dike}(t) + C_{risk}(t) = L_{dike} \cdot K_{dike,0} \cdot \Delta h \cdot e^{\eta\cdot t} + (V_{0,asset} + V_{0,life}) \cdot P_0 \cdot e^{(t(\alpha\cdot\beta+\theta)-\alpha\cdot\Delta h)} \tag{3.13}$$

Solving the optimal point of minimal costs for each timestep t, yields an array of possible choices for Δh and C_t. The optimal choice of which will be discussed in the following section.

Minimizing the total costs over time
The simple framework for a set time t given in equation 3.4 can be expanded with the temporal elements derived above. However, before presenting the temporal framework the constraint on the reinforcement times is derived further into a mathematical model.

After the required reinforcement at time t^*, the reinforcement process can be seen as a reset back to t_0 with a new initial failure probability after reinforcement, denoted by $P_{0,1}$. The process of solving for a maximum allowable t_2^* is then identical to the previously mentioned steps. This process can be repeated as many times as is needed for lifetime of the structure, resulting in n reinforcement times, the final of which occurs at t_n^* and has a new initial failure probability of $P_{0,n}$. The derived expression gives a criteria for when to reinforce the dike at latest, earlier is however always a possibility. This leaves a domain of times in which the reinforcement could be performed, the optimal reinforcement time is then a function of the length of the time domain and the discount rates (current and future). The exact optimal point can be derived by numerical simulation.

To illustrate this principle, imagine a dike that needs a first reinforcement at a latest time of $t = t_1$, a second reinforcement at $t = t_2$ and so on until it is in need of its n^{th} reinforcement at $t = t_n$. This leaves the following domains for the reinforcements R_n.

$$
\begin{aligned}
&R_1 \in [t_0, t_1] \\
&R_2 \in [t_1, t_2] \cup [t_0, t_1] = [t_0, t_2] \\
&\quad \vdots \\
&R_n \in [t_{n-1}, t_n] \cup [t_{n-2}, t_{n-1}] \cup ... \cup [t_0, t_1] = [t_0, t_n]
\end{aligned}
$$

The feasible domains may overlap. In this case the reinforcement might *need* to happen in far spaced periods of time, but they *can* also happen earlier.

If it appears that more than one reinforcement will be needed within the lifetime of the dike, then the minimum of the sum of two (or more) reinforcements should be sought. More than one reinforcement will be needed if the sum of two (or more) subsequent choices for t^* is less than the lifetime of the structure. This principle is demonstrated in figure 3.4, where t_i^* demonstrates the latest possible reinforcement time and t_i the chosen optimal reinforcement time.

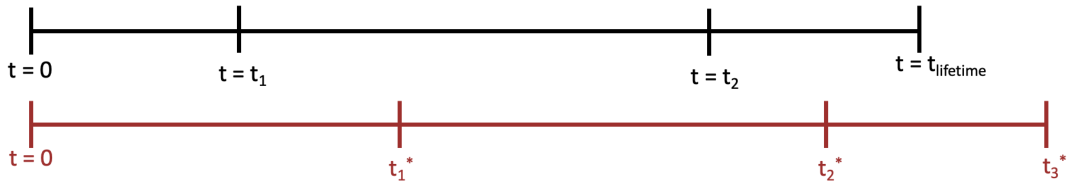


Figure 3.4: Example of multiple reinforcement times

It can be seen that there were three moments before which a reinforcement needs to occur, yielding the three possible reinforcement domains of $t \in [t_0^*, t_1^*], [t_0^*, t_2^*], [t_0^*, t_3^*]$. However, the lifetime lies within Hence two reinforcements at $t = [t_1, t_2]$ are needed in this example, as the range $[t_2^*, t_3^*]$ lies partially outside the lifetime of the dike.

These required reinforcement times can be added to the maximization problem of equation 3.6 as constraints. This extends the derived framework to the following formalized mathematical notation:

$$
\begin{aligned}
&\max_{\forall t \in [t_1, ..., t_n]} \quad B_t - C_t \\
&\text{s.t} \quad t_1, t_2, ..., t_n \in [t_0, t_1^*], [t_0, t_2^*], ..., [t_0, t_n^*]
\end{aligned}
$$

Benefits over time

As the benefits in this research are assumed to consist of a decrease in risk, the equations derived for the costs of increased risk over time can be repurposed to derive an expression for the benefits over time.

Combining the general formulations given in equation 3.5 and 3.2, an expression can be found for the difference in costs over time. Let $C_0(t)$ be the costs associated with flood risk when no reinforcement is made and $C_1(t)$ be the costs that are associated with flood risk when a reinforcement of Δh is made. The result is given in equation 3.14

$$
\begin{aligned}
B(t) &= \sum_{i=0}^{t} (C_{\text{no reinforcement}} - C_{\text{reinforcement}}) \cdot t_i \\
&= (V_{0,asset} + V_{0,life}) \cdot P_0 \cdot \sum_{t_i=0}^{T} e^{(t_i(\alpha\cdot\beta+\theta))} - e^{(t(\alpha\cdot\beta+\theta)-\alpha\cdot\Delta h)} \\
&= (V_{0,asset} + V_{0,life}) \cdot P_0 \cdot \sum_{t_i=0}^{T} e^{(t_i(\alpha\cdot\beta+\theta))} \cdot (1 - e^{-\alpha\cdot\Delta h})
\end{aligned} \tag{3.14}
$$

Equation 3.14 is logically increasing in benefits as Δh increases. In the limit case where $\Delta h \longrightarrow \infty$, the benefits are the costs of not reinforcing. This is to be expected, as when the crest height is heightened ever more, the expected risk decreases to zero. This results in only the ”original” expected flood risk (e.g. the situation in which there is no reinforcement) to be a contributing factor to the benefits. The negative exponential function displays the diminishing returns of increasing Δh.

Net present value of costs

As described in the literature section, the discount rate is vital part of the economical success of any project. Future expenses should be discounted towards the present to accurately compare costs. Furthermore, when looking only at the problem through an economical point of view, delaying costs will (almost) always lead to lower discounted total costs, excluding negative discount rates. For illustration purposes a small example is demonstrated.

Imagine a dike reinforcement that is needed to ensure the safety for the coming 50 years. The entire reinforcement can be done all at once at $t = 0$, with a cost $C_0 = 100$. Another option is to split the investment into two equal parts, one at $t = 0$ and one at $t = 25$. The final options is continuous reinforcement, investing 2% of C_0 every year to meet safety standards. For simplicity of the example, a constant discount rate of 2% is assumed. The results of the different strategies are shown in figure 3.5.

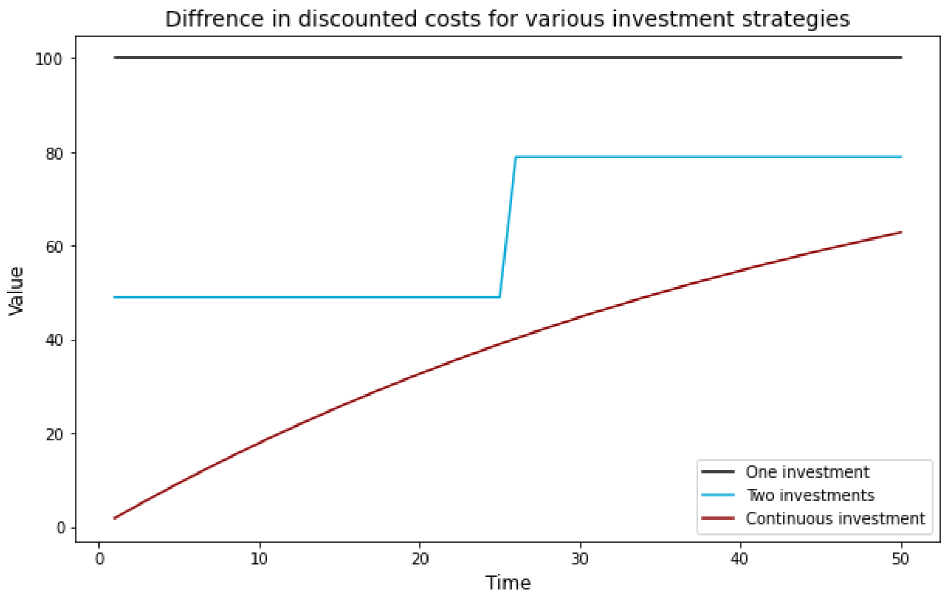
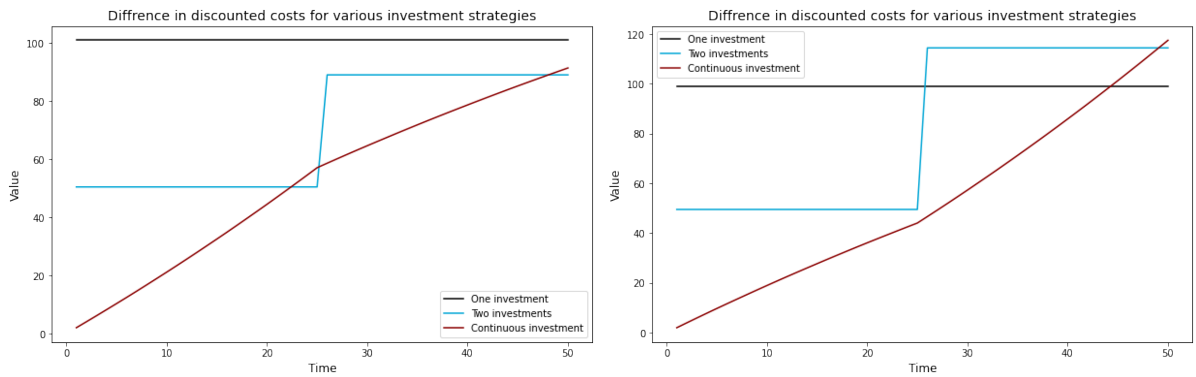


Figure 3.5: Discounted costs over time for various investment strategies

As expected, spreading the investment out over a larger time period results in a lower total present value for the costs. The result becomes even more pronounced over longer periods of time, a 100 year lifetime reduces the discounted costs for a two-step investment strategy to 67% and a continuous investment strategy to only 43% of the one-time investment strategy.

The statement that delaying costs to future points in time results in lower discounted costs has a big underlying assumption, that of a positive discount rate in the denominator of equation 2.2. However, a positive discount rate is not a given in the present economical climate. To demonstrate the effect a (partial) negative discount rate can have on a project, a different example is demonstrated. The same three investment strategies as the previous example are used, but now on an discount rate of $\pm 1\%$ for the first and last half of the project. Figure 3.6a displays the result for a negative discount rate in the first half of the investment period, a a positive one for the remainder. Figure 3.6b has an opposite approach, starting with a positive rate and ending with a negative rate.



(a) Starting negative rate, ending positive rate

(b) Starting negative rate, ending positive rate

Figure 3.6: Discounted costs for a two-state discount rate model

There are two things to note from this example. The first being that the discounting function obviously is not a symmetric one, costs made in the early stage of the constructions lifetime will have a far greater impact on the total discounted costs than later costs. The second one is that the presence of a negative rate on its own can not tell you about the best investment strategy, as can be seen in figure 3.6a the discounted costs are still lower for the two-investment and continuous investment strategy than for the ”lumpsum” approach. It should require no explanation that the results are more pronounced when the life time of the project is increased.

If the discount rate is allowed to vary for each timestep, the discounting process that was given in equation 2.2 has to be modified to allow for a variable r_t. This no longer produces an exponent of power t but a product of the denominator, such that the discounted costs up to time T can be given by equation 3.15.

$$C_{discounted} = \sum_{t=1}^{T} \frac{C_t}{\prod_{t=1}^{t}(1+r_t)} \tag{3.15}$$

Such that if there are two costs incurred at time $t = 1$ and $t = 3$, the total discounted costs become:

$$C_{discounted} = \frac{C_1}{1+r_1} + \frac{C_3}{(1+r_1)\cdot(1+r_2)\cdot(1+r_3)}$$

Similarly, the benefits can be discounted. As the benefits per equation 3.14 are solely a function of the increased risk, they follow the same framework for discounting as presented in this section. Both terms of the calculated costs can be discounted via equation 3.15. Such that:

$$B_{discounted}(t) = \sum_{t=1}^{T} \frac{B_t}{\prod_{t=1}^{t}(1+r_t)}$$

The discounting of costs and benefits can be applied to the derived expression for the total costs and benefits in equation 3.13 and 3.14. The difference between the two should be maximized to achieve an optimal economical reinforcement plan. The result of this maximization is a set of costs $C_1, C_2, ..., C_n$

and benefits $B_1, B_2, ..., B_n$ with a corresponding set of times $t_1, t_2, ..., t_n$ at which the sum of all net results is largest, given the combination of time dependent discount rates and dike reinforcement heights. Combining the principle of discounting costs and benefits to the previously derived framework yields the following mathematically generalized optimization problem:

$$\max_{\forall t \in [t_1,...,t_n]} \quad \sum_{t=1}^{T} \frac{B_t - C_t}{\prod_{t=1}^{t}(1+r_t)}$$
$$\text{s.t} \quad t_1, t_2, ..., t_n \in [t_0, t_1^*], [t_0, t_2^*], ..., [t_0, t_n^*]$$

The range of maximum allowable times t_1^*, ... , t_n^* are functions of the investments made in dike reinforcement C_{t_1}, ... , C_{t_n} as the investment is directly responsible for the new initial failure probabilities $P_{0,1}$, ... $P_{0,n}$. Making the subsequent allowable failure times dependent on one another. The interaction between the different components is visualized in a flow chart in figure 3.7.

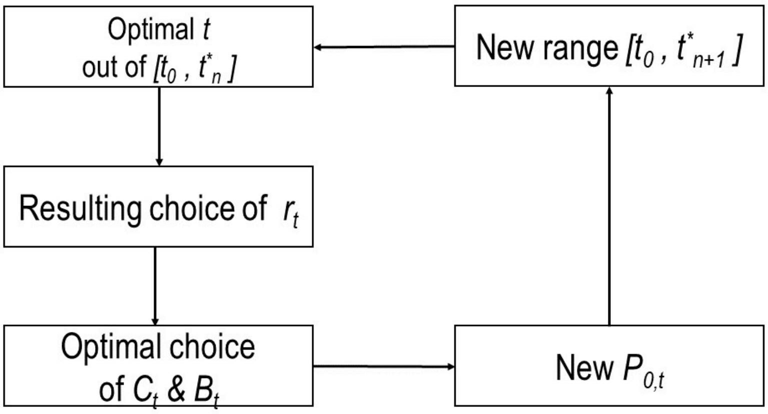


Figure 3.7: Flow chart of the interaction of different terms in discount rate model

This framework nearly completes the model that is needed to solve the research question. However, to ”future-proof” the polderconcept an additional constraint is needed, which will be derived in the next section.

3.2. Added constraints on the Cost-Benefit Analysis

Determining whether a polder becomes economically unfeasible first requires a better understanding of what exactly is meant by ”economic unfeasible”. The CBA gives a criterion for how much a certain decision would yield in costs and benefits, but does not tell the person in charge of making the decision if the financial resources for the decision are available. This is an often overlooked aspect of a CBA, and this research argues that the financial constraints should be added to a CBA to give the person in charge of making a decision the full picture of the investment. To stress the importance of taking the financing of a polder in consideration when performing a CBA, the following example is illustrated.

Imagine a closed system polder generating an annual revenue, consisting of revenue generated by taxation and by subsidy from the government, which is direct to flood defences and having annual expenditures related to flood defences. The polder has a positive outlook based on the CBA criteria, but it’s expenditure is larger than it’s revenue. Hence this polder will need resources to finance it’s flood defences, which should not be a problem as it’s ”business case” is positive, it generates more in benefits than that it generates in cost. The resources needed could come from a central government, that has a finite budget. If there is only one such polder, there is most likely no problem as the budget from the central government will be more than enough to finance this one polder. However, now imagine that there are more polders just like the one in our example. In particular, imagine there are so many polders that there need for resources exceed the available resources. One could argue that the polders with the "best" CBA should be given resources and that this solves the problem. However, as was demonstrated in the previous section the required safety level for a polder will increase year over year due to asset and population growth, even without any sea level rise. As the polders are an integral part of the the central government, their growth rates in both asset value and population will match those of the nation at large. Meaning that if there is a pie consisting of the total budget of the government, the share needed per polder remains constant over time. However, it is well established that there is some form of sea (river) level rise, which causes a relative growth of the required budget per polder to the available budget provided by the government. In other words, the proverbial needed piece of the pie grows larger over time. Taken to it's extreme, eventually the requires slice for a single polder will outgrow the size of the entire pie and no polder reinforcement can be financed, however much more benefits compared to costs it produces.

This example demonstrates the need for an additional constraint on the budget that a polder can exhaust per year, to keep it sustainable for the future. This sustainability directly relates to the main research question on the viability of the polder concept. In this section a selection of possible approaches to budget constraints will be discussed and compared.

1. **Marginal cost ≤ Marginal revenue**
   This approach likens most to the comparison of the Dutch flood protection system as a business. In finance and economics the comparison of marginal benefits and marginal costs is made to maximize profits. In particular, whenever the marginal revenue is equal to the marginal costs, a firm has maximised profit. This means that the available budget is not necessarily exhausted, rather the budget is used to the point where the marginal costs equal the marginal revenue. The marginal revenue in this case comes from the decrease in expected flood risk, as a result of the increase in crest height.
   In this scenario the polder has to sustain itself. This entails that the marginal "revenue" of a single polder has to exceed or at least be equal to the marginal costs associated with that particular polder.

2. **Budget for flood protection = Expenses flood protection**
   This approach assumes that the government appoints a fixed share of it's available yearly budget for flood protection, and in turn exhausts this budget to provide a maximised level of flood protection. Two methods of budget division are considered for this approach.

   (a) *Budget shared pro rata to economic risk among polders*
   The dedicated annual budget is shared among the polders based on the economic risk that each polder is exposed to. The economic risk is a combination of the loss of assets and life with their respective probability of As the government will strive to minimize economic impact in case of a flood, this approach will yield an equilibrium in economic risk amongst the different polders.

   (b) *Budget shared pro rata to the individual risk among polders*
   The dedicated annual budget is shared among the polders based only on the individual risk that one has when living in a polder. As the government will strive to minimize the individual risks among polders, this will yield an equilibrium where all polders will share the same individual risk.

3. **Reinforcement costs for flood protection≤ Expected damages due to flooding**
   Opposed to the popular mantra first attributed to Dutch philosopher Desiderius Erasmus "Prevention is better than cure", this approach looks at the costs after a flood has occurred. Particularly, it compares the expected costs that are incurred after an area floods and compares this to the total costs of reinforcing the dikes that are protecting that area. The level of reinforcement is thus not dependent on a law stating the required flood protection level, but solely on the value that sits within that area and the costs of reinforcement.

4. **Individual risk = constant**
   This approach likens most to the current approach taken by the Dutch government. In this sce-

nario, each inhabitant of the Netherlands receives a similar protection level, regardless of population density or asset value in that area. As the budget is limited, it is possible that the government can't provide the necessary protection level based on the available resources. If the budget is inadequate for the required level of protection, the assumption is made that the budget will be split such that all inhabitants get a similar protection level.

3.2.1. Mathematical formulation of added constraints

The four possible constraints have to be put into a mathematical framework to decide upon their stringency and which should be added as constraints to the maximization problem presented in section *3.1.3 Inclusion of temporal elements*. To derive their stringency, the polders are schematized as circular regions with a constant asset and population density. Their revenue comes from a local taxation as well as a national taxation. The costs considered are those of physically strengthening a dike through crest height increases as well as added risk as a result of population/asset growth and/or increased probability of flooding.

The full derivation of all four constraints can be found in section *A.3.1 Stringency of the financial constraints*. The final mathematical result is presented below for all four constraints.

Marginal costs ≤ Marginal revenue

The first possibility for an additional constraint concerns the derivative of the costs and revenue with respect to the increase of crest height Δh. The marginal revenue should be larger or equal than the marginal costs, resulting in a normative situation where they are equal to one another. In mathematical notation this means that:

$$MR = \frac{\partial TR}{\partial(\Delta h)} \text{ and } MK = \frac{\partial TK}{\partial(\Delta h)}$$

The expression for each was derived in the appendix, the final results are shown below.

$$MR = \begin{cases} 0 & \text{for SLR} < \Delta h \\ = -\frac{L_{dike,n}^2 \cdot (e^{SLR-\Delta h}-1)}{4\pi} \cdot (\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}) & \text{for SLR} \geq \Delta h \end{cases}$$

This equation demonstrates the marginal revenue with respect to the crest height increase Δh. There are two expressions, dependent on the relative size of the crest height against the expected sea level rise. As can be seen, the marginal revenue for a polder is zero when the crest height increase is larger than the expected sea level rise. The quantity turns negative when the expected sea level rise is larger than the crest height increase.

$$MK = \begin{cases} L_{dike,n} \cdot P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta h x}{h} + 2yh + x + b + \frac{1}{2}d) \\ -L_{dike,n}^2 \cdot (e^{SLR-\Delta h} - 1)[\frac{\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}}{4\pi}] & \text{for SLR} < \Delta h \\ L_{dike} \cdot P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta h x}{h} + 2yh + x + b + \frac{1}{2}d) & \text{for SLR} \geq \Delta h \end{cases}$$

This equation demonstrates the marginal costs with respect to the crest height increase Δh. If the crest height increase is greater than the expected sea level rise, the costs consist of (the derive of) the costs of reinforcement minus the expected increase in safety. If the crest height increase is smaller than the expected sea level rise, the costs consist solely of the (the derivative of) the costs of reinforcement.

Budget for flood protection = Expenses flood protection

The second possibility for an additional constraint concerns the costs and revenues themselves, which should equal one another. In mathematical notation this means that:

$$TR = TC$$

The expression for the total revenue and total costs has been derived in the appendix, the results are captured in the equations below.

$$TR = \begin{cases} B & \text{for SLR} \geq \Delta h \\ B + \frac{L^2_{dike,n}}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live}) & \text{for SLR} < \Delta h \end{cases}$$

where:

$$B = \frac{L^2_{dike,n}}{4\pi}(T_{fixed} + T_{variable} \cdot V_{\text{woz}}) + (\frac{L_{dike,n}}{\sum_{k=1}^{N} L_{dike,k}})^2 \cdot B_{subsidy} \quad \text{for k} \neq \text{n}$$

As can be seen, the total revenue consist of two parts: the generated tax and the expected decrease in costs due to a decrease in flood risk. If the sea level rise is greater than the increase in crest height, the revenue consists solely of the generated tax. If the crest height increase is larger than the sea level rise, it consists of both components.

The total costs can be expressed as:

$$TK = \begin{cases} \frac{L^2_{dike,n}}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live} + K_{dike} \cdot \frac{4\pi}{L_{dike}}) & \text{for SLR} \geq \Delta h \\ K_{dike} \cdot L_{dike} & \text{for SLR} < \Delta h \end{cases}$$

It can be seen that the total costs consist of two parts as well: The expected flood risk costs and the costs of strengthening the dike, e.g. increasing the crest height.

Reinforcement costs for flood protection ≤ Expected damage due to flooding

For the third constraint two quantities had to be assessed. The reinforcement costs for flood protection and the expected damage due to the flooding. The reinforcement costs can be expressed as:

$$\begin{aligned} C_{dike} &= L_{dike} \cdot \Delta A \cdot K_{dike} \\ &= L_{dike} \cdot K_{dike} \cdot (\Delta h^2 y + 2\Delta hyh + \frac{\Delta h^2 x}{2h} + \Delta hx + \Delta hB + \frac{\Delta hd}{2}) \end{aligned}$$

The expected damage due to flooding consist of the loss of life and the loss of assets. As such, it can be expressed as:

$$\mathbb{E}[damages|Flooding] = \frac{L^2_{dike} \cdot e^{SLR}}{4\pi} \cdot (\mathbb{P}(fatality, 0) \cdot P \cdot V_{live} + \mathbb{P}(asset, 0) \cdot H \cdot V_{assets})$$

Individual risk = constant

The fourth and final constraint has an equality between the expected sea level rise and the increased crest height, meaning $SLR = \Delta h$. As such, the total revenue and total costs can be reduced to the following expressions:

$$TR = B = \frac{L^2_{dike,n}}{4\pi}(T_{fixed} + T_{variable} \cdot V_{\text{woz}}) + (\frac{L_{dike,n}}{\sum_{k=1}^{N} L_{dike,k}})^2 \cdot B_{subsidy} \quad \text{for k} \neq \text{n}$$

The total revenue is similar as in the previous section, but now solely consists of the term B. This term indicates the generated tax revenue.

$$TK = \Delta A \cdot K_{dike} \cdot L_{dike} = ((\frac{\Delta h^2}{h^2} + \frac{2\Delta h}{h})(yh^2 + \frac{1}{2}xh) + (\frac{\Delta h}{h})(Bh + \frac{1}{2}dh)) \cdot K_{dike} \cdot L_{dike}$$

The total costs solely consists of the costs of physically strengthening the dike, as per definition of this criteria there is no increase or decrease in risk.

The expressions for the four criteria allow for the addition of the constraints in the mathematical framework of the maximization problem presented in the previous section. To determine which of the four constraints should be added to this framework, a numerical example will be worked out in the next section.

3.2.2. Numerical example

To get a sense of the stringency of these different interpretations, a numerical example is presented. This numerical example considers a fictitious island nation, which is only affected by sea level rise. The nation consist of two island, a larger and a smaller one. Each of these islands is protected by a single dike-ring enveloping the island, making it act like a singular large polder. A schematic overview of the example is given in figure, 3.8, including the key difference in statistics from table 3.1.

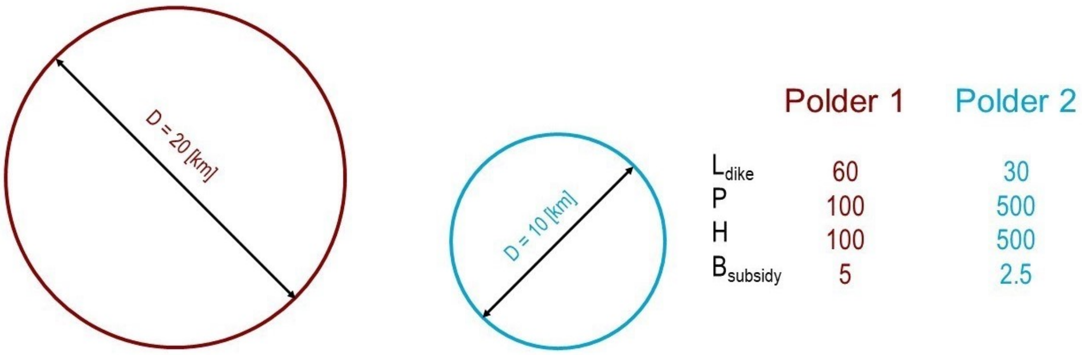


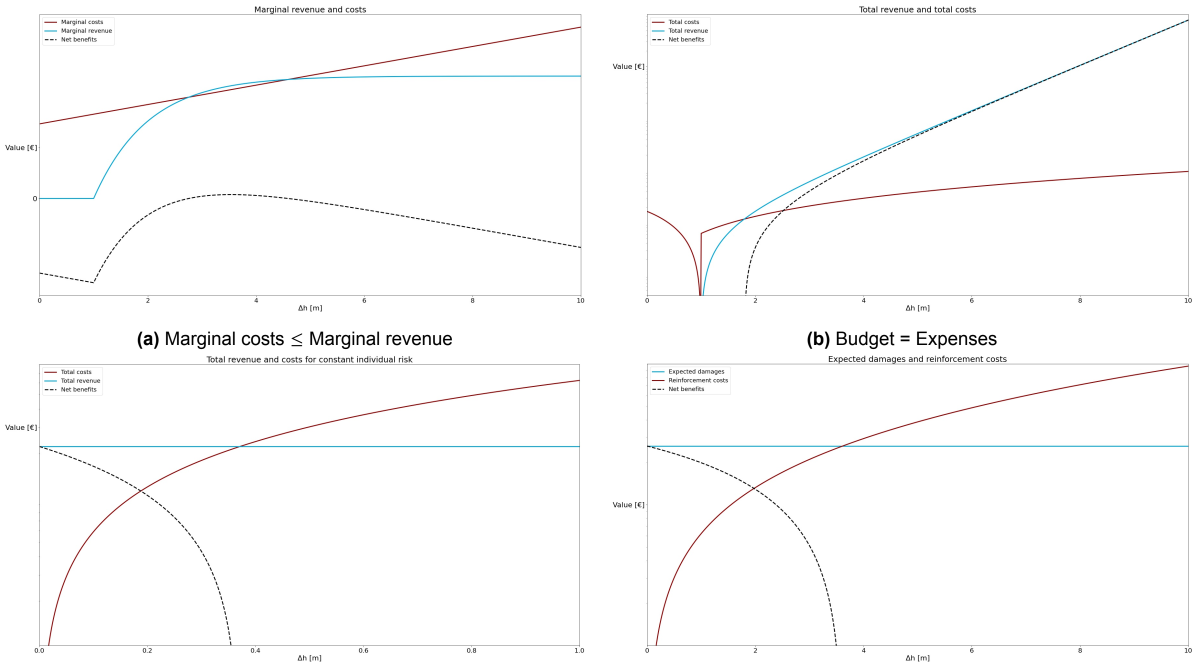
Figure 3.8: Schematic overview of stringency derivation example

The islands have a circular shape with a diameter just under 20 and 10 kilometers for island one and two respectively. Island 1 has a population of around 30.000 people and island 2 has a population of around 40.000. The monetized risk of the inhabitants and assets for island 1 is around €30. mln. per year. For island 2 this is around €16 mln. per year. The single dike-ring, at time $t = 0$, has a safety level that results in individual risks for persons and assets of $\mathbb{P}(fatality)$ and $\mathbb{P}(asset)$. The island nation has a certain GDP (revenue) and allocates a certain amount $B_{subsidy}$ to flood protection. Next to that, each local municipality island also raises a so called 'flood protection tax'. In most demographic regards, both islands resemble the Netherlands, with some small changes in each. Key statistics of each island is shown in table 3.1.

Variable	Unit	Description	Polder 1	Polder 2
P	[Individuals $\cdot$ km^{-2}]	Population density	100	500
H	[Assets$\cdot$km^{-2}]	Asset density	100	500
R	[Individuals]	Number of residents per household	3	4
V_{WOZ}	[€]	Valuation of a house	300.000	300.000
V_{live}	[€]	Economic value of a life	$6.7 \cdot 10^6$	$6.7 \cdot 10^6$
V_{asset}	[€]	Economic value of a house	$3 \cdot 10^5$	$3 \cdot 10^5$
$\mathbb{P}(fatality)$	[-]	Current individual probability of death	$5 \cdot 10^{-4}$	$5 \cdot 10^{-4}$
$\mathbb{P}(asset)$	[-]	Current probability of loss of value of an asset	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
$B_{subsidy}$	[€ per 50 years]	Budget provided by Central government	$5 \cdot 10^8$	$2.5 \cdot 10^8$
L_{dike}	[km]	Length of the dike section	60	30
SLR	[m]	Expected sea level rise over 50 years	1	1
K_{dike}	[€$\cdot m^{-1} \cdot km^{-1}$]	Costs per m per km of dike reinforcement	$4.5 \cdot 10^6$	$4.5 \cdot 10^6$

Table 3.1: Key statistics of fictitious island nation

The presented figures and statistics will be used on each of the four different constraints with Δh being the parameter to solve for. This will yield limits on the minimum (or maximum) increase in crest height Δh that the proposed constraint can tolerate, given an idea of the stringency of the additional constraints. The results of the analysis can be found for each constraint in figure 3.9. Each of the constraints will get a further explanation based on the figure below. The figures are displayed in full size in the appendix as well, in figures A.1 through A.4.



(a) Marginal costs ≤ Marginal revenue

(b) Budget = Expenses

(c) Reinforcement costs ≤ Expected damages

(d) Individual risk = constant

Figure 3.9: Results of numerical calculations for the four different constraints

The derivations made are done with the assumption of a 1 meter sea level rise over the given time period. hence, the increase in crest height Δh will always be with respect to the expected sea level rise of 1 meter. A description per figure is given in each of the different constraints presented below. In general the value is shown on the y-axis and the increase in crest height Δh can be seen on the x-axis. The revenue is shown by a blue line, the costs by the red line and the net result is shown in black.

Marginal costs ≤ Marginal revenue

This constraint is displayed in figure 3.9a. It can be seen that the marginal costs (red line) are a linear functions of the crest height increase given the expected sea level rise. This function is linear as we are considering the derivative of the increased costs for crest level height , which itself is a quadratic function in crest level height. The marginal benefits (blue line) are zero up until the point of the expected sea level rise. After which they increase up until they reach a stationary pint around 5 meter crest level height increase. This indicates that extending the reinforcements beyond 5 meter increase do not yield any marginal benefits. Furthermore it can be seen that there are two points where the marginal costs line (red) intersects with the marginal revenue line (blue). This means that there is a range of Δh for which the marginal revenue is larger than the marginal costs, satisfying the criteria. This puts both an upper- and a lower-bound on the possible values of Δh.

Budget flood protection = Expenses flood protection

This constraint is displayed in figure 3.9b. It can be seen that the costs decrease when approaching the 1 meter increase in crest height. This is to be expected as the costs consist of the added flood risk, which decrease with increasing crest height. Note that the sharp decline and subsequent increase around 1 meter are due to the mathematical formulation. In reality this line is expected to decrease and smoothly transition into the graph right of 1 meter. The benefits (blue line) only exists for crest level height increases above 1 meter. As the benefits considered in this research consist of the decrease in flood risk, the values are only defined for actual reduction, e.g. when the crest level increase is larger than the expected sea level rise. Furthermore it can be seen that there is a single point after which the revenue exceeds the costs. This puts a lower bound on the values that Δh can attain, but does not put an upper bound on the values it can attain.

Reinforcement costs for flood protection $\leq$ Expected damage due to flooding
This constraint is displayed in figure 3.9c. It can be seen that in this case the expected benefits are given as a constant with respect to the increase in crest level height. The expected costs are the result of the reinforcement costs. These costs grow quadratically with the crest level height increase. Furthermore it can be seen that there is a single point after which the costs exceed the revenue. Since the costs exceed the revenue after a certain point, the opposite situation of the previous constraint occurs. There is now an upper-bound that is placed on the value of Δh.

Individual risk = constant
The final constraint is displayed in figure 3.9d. In this case the individual risk should remain constant and hence, it is assumed that there are no benefits due to further increasing the crest height beyond the "required" height of the sea level rise (1 meter). Note that the x-axis only goes from 0 to 1 meter in this case, as this is the expected sea level rise. The expected costs are again a quadratic function of the crest height as they consist of the reinforcement costs. Furthermore it can be seen that there is again a single point where the costs exceed the revenues. This again leads to an upper-bound on the value of Δh. Hence we find a feasible range of values ranging from zero up to this intersection point.

Note that as the numbers used in these derivations come from a fictional example, it is their relative position toward one another (in terms of Δh) that is important rather than their absolute values.

3.3. Model description

This section will describe the model that will be implemented to solve the original research question. It will start of with the choice of two of the given constraints from the section on additional constraints. After the constraints are chosen, the three options for the discount rate model will be presented. Next, the temporal model will be introduced. This model will be used to analyze the second sub-question of this research. Finally, the spatial model will be introduced, which will be used to answer the third sub-question of this research.

3.3.1. Choice of constraints

A quantitative comparison of the different approaches can be made with the figures derived in the previous section. To this end the approaches are compared not by their total costs or benefits, but rather to what extent an increase in Δh is possible before the method ceases to be feasible according to it's own criteria. Table 3.2 shows the results of this comparison.

Method	**Feasible domain of Δh**	**Limit(s) on Δh**
Marginal cost ≤ Marginal utility	$\Delta h \in [2.8, 4.6]$	$2.8 \leq \Delta h \leq 4.6$ [m]
Budget safety = Expenses safety	$\Delta h \in [1.8, \infty)$	$\Delta h \geq$ 1.8 [m]
Reinforcement costs ≤ Expected Damages	$\Delta h \in (0, 3.6]$	$\Delta h \leq 3.6$ [m]
Individual risk = constant	$\Delta h \in (0, 0.38]$	$\Delta h \leq 0.38$ [m]

Table 3.2: Comparison of different methods based on maximum Δh for a given SLR of 1 [m]

Note that as this is a case with fictitious numbers, the exact values of Δh are not important. Rather, it is their relative relationship between different methods that are needed for the derivation of stringency.

From this table, three notable results emerge.

- The first approach yields two points of intersection between the marginal revenue and marginal costs, while the other methods yield one intersection point.
- The second approach yields a minimum requirement on Δh, while the third approach yields a maximum requirement on Δh.
- If the third scenario is excluded, there exists a range where the other three approaches have a feasible Δh, based on their own criteria e.g. have a domain that overlaps. This is given by $\Delta h \in [2.8, 3.6]$

Based on the limits posed on Δh, it can be observed that third approach imposes the strictest requirements, as a maximum increase of crest height of $\Delta h = 0.38$ [m] is allowed before the polder becomes financially infeasible. This is followed by the fourth approach, which has a maximum increase of $\Delta h = 3.6$ [m]. The first approach places the third strictest requirements on the maximum crest height increase with $\Delta h = 4.6$ [m], but also poses a more limited range for the start of financial feasibility. The second approach imposes a lower bound on the increase in crest height, but does not pose an upper bound on the increase in crest height. This would mean that any increase above $\Delta h = 1.8$ [m] would yield a financially stable system.

Choosing two of the four options was done based on two criteria:

1. The ability to sufficiently quantify the approach, and:
2. The previously derived stringency of the approach.

Next to these criteria the general idea of a novel approach to the assessment of the polder concept is kept in the back of the mind.

For the first criteria, the available data should be sufficient to quantify. Although the data processing comes in a later section, it will become apparent that the required data is provided for nearly all approaches. The marginal costs for increasing the safety level ten-fold are given by Slootjes and Wagenaar, 2016 in their research. The marginal utility is easily calculated by the increase or decrease in risk to people and assets. Figures for the extrapolated budget will be provided as well as for the expenses in this section as well. These figures can be re-used for the third method. However, the fourth method, that of reinforcement costs being equal to expected damages, leaves a rather large unknown in the data set. Namely that of the costs of re-building and moving (parts of) cities to a different location. This is hard to quantify as a city is not build in a single time frame, but over many decades if not centuries. Discounting all of these different values back to the current day or to a future date would be a task too large in scope for this research. For this reason, all approaches apart from the fourth are recommendable via this criteria.

For the second criteria, table 3.2 is used. Starting with the first method, it gives a rather discounting result: That of two points where in between the increase in crest height yields a feasible result, as this is the only method that yields such a result, it is an discounting one to develop further. The second method is unique as well in that it yields a lower limit for Δh, indicating that the crest height needs to be increased about 80% higher than the proposed 1 meter sea level rise. The third method is limited to one meter of sea level rise and is unable to reach this limitation in the proposed simplified model, making it a rather poor candidate for this criteria. The fourth method yields an upper limit that falls (partly) in the domain of the first and second method. As it is covered in between these two methods, those would suffice.

Combining the insights of the two criteria with a general indication of novelty from the approach, the first two models are chosen. As marginal costs and utilities are not a standard practice in hydraulic engineering (or engineering in general), this approach might deliver new insights. Furthermore, the marginal costs and benefits are possible to quantify, making it a feasibly method to study. The second approach is chosen as it gives a contrast to the first one based on the limitations on Δh. The other reason is that it is possible to quantify as well.

3.3.2. Discount rate model

The time dependent discount rate r_t has a large impact on the solution of the maximization framework of discounted costs. As such, determining the correct rate is crucial. Crucial as it may be, it is not easy. As introduced in the literature section, three models will be proposed for modelling the discount rate, two of which find their origins in economic and/or econometric origins. These are a constant discount rate with noise, an discount rate modelled by an ARMA(p,q) model and an discount rate modelled by a Brownian motion with drift.

The full derivation and reasoning behind the choices for the specific parameters in the models can be found in section A.2.

Constant discount rate with noise

The constant discount rate is determined to have a mean value of 1.9%, based on the targets set by the ECB as well as historical data. The analysis showed that the distribution of the discount rate is reasonably normally distributed. Hence the choice of of the distribution is Gaussian. The standard deviation was determined to be around a quarter of the discount rate value, so around 0.5%.
Hence the values of the constant discount rate with noise are drawn from $r_t \sim \mathcal{N}(1.9,\, 0.5)$, which has a 95% probability of drawing a value between [1.0 , 3.0].

ARMA(p,q) model

For the ARMA(p,q) model the discount rate of the full sample was examined and fitted according to the Akaike Information Criterion (AIC) and significance of the parameters. This yielded an ARMA(2,2) model as best fit for the data set, meaning that two lagged values of the discount rate are taken into account as well as two lagged errors. The discount rate r_t according to the chosen ARMA(2,2) model has the following form and parameters.

$$r_t = 0.53r_{t-1} - 0.88r_{t-2} + \varepsilon_t - 0.36\varepsilon_{t-1} + 0.85\varepsilon_{t-2}$$

Brownian motion with drift

The last proposed model is a Brownian motion with drift. A preliminary analysis of the historical data yielded two possibilities for the drift coefficient μ, one negative and one positive. The choice was made for the positive rate, as a negative rate would yield values well below -1%, which are not sustainable for prolonged periods of time. Hence the choice was made for a Brownian motion with drift coefficient $\mu = 0.087$ and standard deviation $\sigma = 2.872$, the starting value of the Brownian motion is the discount rate of 2020, set at $r_0 = -0.328$.

These three models represent options from which values of r_t can be drawn for each timestep in consideration. The choice for one (or more) of the models will be presented in the chapter *Case Study*.

3.3.3. Temporal model

For the temporal model, the second chosen budget constraint will be introduced to the previously derived framework for discounted costs. The reasoning for this is that if a polder were to be self sufficient and sustainable in the future, it should at the bare minimum be able to cover it's yearly (averaged) expenses with the budget it raises and the budget provided by the central government.

It follows from this criteria that the resources spent on dike reinforcement should be less or equal than the resources that are available at time t. As reinforcements are usually performed with the idea that the reinforcement will last a long time, the assumption is made that the budget over different years can be "saved" up to be spent at some time t in the future. Hence the required budget up to and including time t must not exceed the budget that has become available up to and including time t. Let P_t be the available budget at time t, defined as:

$$P_t = \sum_{i=1}^{t} P_i$$

The original maximization problem from equation 3.4 gets two transformations, one additional constraints and one additional condition. The transformation is the discounting process, displayed in the first line of equation 3.16. The additional constraint is given in the second line, displaying how the costs at each time instant t should be smaller than the sum of the available budget up to that instant, for all timesteps in considerations. The conditions is given in the third line, where all possible reinforcement moments leading to the costs C_t should be within the timeframe allowed before strengthening of the dike section is needed to adhere to the given failure probability demand.

$$\begin{aligned} &\max_{\forall t \in [t_1, \ldots, t_n]}, \quad \sum_{t_i=1}^{T} \frac{B_t(\Delta h) - C_t(\Delta h)}{\prod_{j=1}^{t}(1 + r_j)} \\ \text{s.t.} \quad & C_t(\Delta h) \leq \sum_{i=1}^{t} P_i(\Delta h) \;\; \exists t \in [t_1, \ldots, t_n], \; \exists \Delta h \in [0, \infty] \\ & t_1, t_2, \ldots, t_n \in [t_0, t_1^*], [t_0, t_2^*], \ldots, [t_0, t_n^*] \end{aligned} \tag{3.16}$$

Note that it is implicitly stated in equation 3.16 that the maximization over time also includes the maximization over Δh. The formal double maximization problem is then given as:

$$\begin{aligned} &\max_{\forall t \in [t_1, \ldots, t_n], \forall \Delta h \in [0, \infty]} \quad \sum_{t_i=1}^{T} \frac{B_t(\Delta h) - C_t(\Delta h)}{\prod_{j=1}^{t}(1 + r_j)} \\ \text{s.t.} \quad & C_t(\Delta h) \leq \sum_{i=1}^{t} P_i(\Delta h) \;\; \exists t \in [t_1, \ldots, t_n], \; \exists \Delta h \in [0, \infty] \\ & t_1, t_2, \ldots, t_n \in [t_0, t_1^*], [t_0, t_2^*], \ldots, [t_0, t_n^*] \\ \text{where:} \quad & C_t(\Delta h) = L_{dike} \cdot K_{dike,0} \cdot \Delta h \cdot e^{\eta \cdot t} + (V_{0,asset} + V_{0,life}) \cdot P_0 \cdot e^{(t(\alpha \cdot \beta + \theta) - \alpha \cdot \Delta h)} \\ & B_t = (V_{0,asset} + V_{0,life}) \cdot P_0 \cdot \sum_{t_i=0}^{T} e^{(t_i(\alpha \cdot \beta + \theta))} \cdot (1 - e^{-\alpha \cdot \Delta h}) \end{aligned}$$

The budget P_t, length of dike L_{dike} and costs per dike height increase $K_{dike,0}$ can be derived directly from the figures obtained in the subsequent data collection section, or be (partly) explicitly formulated in terms of parameters (in simplified scenario's) by equation A.8. The temporal dependent parameters β, θ, γ and η can be derived from historical data given in the case study section. After deriving the minimal total costs, the CBA can be performed. This entails the comparison of the discounted total benefits with the discounted total costs as per equation 3.16

4

Case study

This section consists of the data collection and extrapolation as well as the introduction of the two case studies. The section on data collection consists of specific regional data as well as the data that is required to model the discount rate. The case studies consider dikering 17 and dikering 29 in the Netherlands.

The section starts with the collection and processing of data that is required for the case studies. The available budget for the two cases in consideration will be disclosed first, as well as the extrapolation of the budget to 2050 and 2100. After this the loss of value for both cases will be derived and extrapolated as well. The final section considers collection and processing of data used to model the discount rate.

After the collection and processing of data, an overview of the two case studies with key parameters will be given. The cases consider dikering 17, primarily influenced by river discharge variability, and dikering 29, primarily influenced by sea level rise. The key statistics are divided into deterministic parameters and stochastic parameters that will be used in a later chapter for the Monte Carlo analysis.

4.1. Data collection and processing

This section contains the collection and processing of three sets of data. The first set regards the available budget by both the local and central governmental body of the regions in the Netherlands. The second set of data comprises the loss of value when a dike breaches and flooding occurs for both the dike rings that are considered in the case study. This loss of value comprises both the loss in life as well as the loss in assets. The final sets comprises data on Dutch governmental bonds with a 10 year duration, which is assumed to be the best proxy for the risk-free rate in the Netherlands. The two areas in consideration, dikering 17 and 29, are shown in relation to the Netherlands in figure 4.1. The mentioned dikerings are indicated with a black circle.

Figure 4.1: Spatial position of dikering 17 and 29 in relation to the Netherlands

4.1.1. Available budget

Determining the available budget first requires a determination of the different revenue streams. As was stated in the literature study, the Dutch polder concept relies on two streams of revenue. The first one being the budget provided by the central governmental body. This is often a set amount per year, dependent on the GDP of the Netherlands. The second revenue stream comes from the taxes raised by the local water authorities. The revenue of the local water authorities is dependent on a number of parameters, but roughly contains two parts. A fixed part to be paid by everyone and a variable part that is dependent on the value of your home. Both parts will be evaluated. The full evaluation, including extrapolations and derivations, can be found in the appendix in section A.3.1.

The budget provided by the central government is found to be a share of 3.6% (2.3%) for *Hollandse Delta* (*Scheldestromen*) of the annual budget of the Deltafonds. As the Deltafonds is expected to grow over time at a steady rate, the budget was extrapolated to be €3.95 bln. in 2100. These figures can be combined to find a proxy for the budget provided by the central government for both water authorities.

The raised tax generated by the water authorities themselves consists of a fixed (C) and variable part (f), taking the form displayed in equation 4.1.

$$B_{tax} = C + f \cdot V_{house} \tag{4.1}$$

The generated budget is dependent on the parameters in the above mentioned expression, as well as the number of people that live within each polder. The future values for C, f and V_{house} were determined by means of extrapolation for the specific provinces. The value for the growth rate of the population was derived from figures of the CBS, presented in figure A.13.

Combining the found results for two factors contributing to the yearly budget, they can each be calculated and tallied up. The result is shown in table A.6.

	Dikering 29		**Dikering 17**	
	2050	*2100*	*2050*	*2100*
Central government	52.0	90.9	81.4	142.2
Local water authority	14.0	30.6	35.8	87.9
Total budget [mln. €]	66.0	121.5	117.2	230.1

Table 4.1: Final results of available budget in 2050 and 2100 for dikering 17 and 29

4.1.2. Loss of value

Determining the expected loss of life and economic damage has for a large part been done by the Ministry of Infrastructure and Water Management back in 2016 (Slootjes & Wagenaar, 2016). This was done by simulating a breach in the flood defence (often a dike), which resulted in an expected number of affected people, casualties and economic damage. These numbers were calculated for 2011 and 2050. Table A.7 provides some key figures about the two selected dikerings, including all the dike sections corresponding to the ring. These figures contain the expected economic damage, the loss of live and the total damage in 2011. All figures that are expressed in monetary units are in millions of euro's. Note that for the loss of live a value of €6.7 million per casualty was assumed. Within the column *Loss of live*, a cost of €12.500 was assigned to each affected, but not deceased, individual on top of the number of lost lives.

Dike section	Length [km]	Costs of increasing safety level [€$\cdot km^{-1}$]	Economic damage [€]	Loss of live [€]	Total damage [€]
17-1	27.0	4.4	780	433	1.213
17-2	26.5	5.8	2.600	1.279	3.879
17-3	9.5	2.5	11.000	8.575	19.575
Total	*63*	*4.7**	*14.380*	*10.287*	*24.667*
29-1	22.0	5.8	2.100	359	2.459
29-2	17.0	6.5	3.300	2.355	5.655
29-3	7.0	6.3	5.300	14.770	20.070
29-4	12.5	1.0	69	6	75
Total	*58.5*	*5.0**	*10.700*	*17.484*	*28.184*

Table 4.2: Expected damages in 2011, based on (Slootjes & Wagenaar, 2016)

* *These values are the average cost of increasing the safety level per kilometre in millions of euro's over the entire length of the dikering.*

The analysis by Slootjes and Wagenaar (2016) provides estimates for the monetary value of those affected and for the casualties in 2050, as well as an estimate for the economic damage in 2050, shown in table D.1. These values were derived by multiplying the figures from 2011 by an assumed annual growth of 1.9% over 39 years. This growth is used for both the economical figures as well as the population growth and by extent to the fatalities. The calculations were made for 2011 and extrapolated to 2050, however no estimates for 2100 were provided. To this extent, similar growth figures are used to extrapolate the values in the year 2100. These figures are presented in table D.2.

Before using these figures, a quick check on the 1.9% figures was performed. Starting with the expected economic growth, the BBP changes for the past 25 years are plotted and averaged, shown in figure A.16. The year-over-year change in BBP appears to average out around the value of 1.9%. Hence this value is assumed to be a plausible figure for the growth rate of asset value θ. However, as has been explained in the section *Available budget*, the population growth per year is around 59.000 people per year. Solving an exponential growth model for the growth rate exactly, it is found that the population growth rate (γ) is around 0.30% (0.2991). As this value is significantly different from the proposed value by Slootjes and Wagenaar, 2016, the values for population growth are adjusted. It is assumed that the value of the loss of live scales linearly with the growth or decline in population. The results for 2050 and 2100 are shown in table D.4.

4.1.3. Interest rate

The available data for the long-term Dutch interest rates are reported by *The Organisation for Economic Co-operation and Development* (OECD, 2022). The rates are given per year, starting in 1959 up to 2021. The development of the interest rate over time is shown in figure C.16 in the appendix. Some key statistical properties of the time series are displayed in table 4.3.

Statistic	**Value**
Datapoints	60
Minimum	-0.38
Maximum	11.55
$\mathbb{E}[X]$ (Mean)	5.51
$\mathbb{E}[X^2]$ (Standard deviation)	2.87
$\mathbb{E}[X^3]$ (Skewness)	-0.31
$\mathbb{E}[X^4]$ (Kurtosis)	-0.48

Table 4.3: Summary statistics of historical interest rate

It becomes apparent by looking at the minimum and maximum that all data point fall neatly between the mean and $\pm 2\sigma$. Looking at the skewness the data appears to be centrally distributed. The kurtosis provided is corrected by a factor 3 for the normal distribution, meaning that a value close to 0 represents a kurtosis that is near that of a normal distribution. As the kurtosis is slightly negative, there is a bit more scattering of the data points than would be expected based on a normal distribution. However, based on these statistics the conclusion can be drawn that the data is reasonably normally distributed.

4.2. Dikering 17

Dikering 17 can be considered a dikering that is primarily under the influence of rivers, specifically the Meuse. Hence the discharge variability is of the most importance for this analysis. As dikering 17 is part of the municipality of *Hollands Delta*, these figures will be used for the budget. The estimated damage due to loss of life and loss of assets are directly derived for dikering 29.

4.2.1. Key parameters

The key parameters used in the case study are shown in table 4.4.
The parameters $V_{0,life}$ and $V_{0.asset}$ represent the total expected damage in loss of life and assets when a flood occurs. There are two values for β as the analysis is concerned with the 2100+ ($\beta = 0.06$) and 2100- ($\beta = 0.02$) scenario presented by the KNMI, as per table 2.1. Both these values will be used in the calculations and simulations. Finally there is one parameter that is not listed here, the allowable risk level E_{demand}, as this variable is varied from 0.25 times the initial risk level ($P_0 \cdot (V_{asset} + v_{life})$) to 2.5 times the initial risk level.

Parameter	Distribution	Mean	Coefficient of variation
Deterministic parameters			
α	Deterministic	0.8	-
P_0	Deterministic	1/4000	-
L_{dike}	Deterministic	63.0	-
K_{dike}	Deterministic	$4.7 \cdot 10^6$	-
$V_{0,life}$	Deterministic	$14.380 \cdot 10^6$	-
$V_{0,asset}$	Deterministic	$10.287 \cdot 10^6$	-
E_{demand}	Deterministic	$1.5 \cdot P_0$	-
Stochastic parameters			
β	Normal	[0.002, 0.006]	[0.10, 0.05]
γ	Normal	0.003	0.05
θ	Normal	0.019	0.05
η	Normal	0.019	0.05
Lifetime	Exponential	80	0.0625
r_t	Normal	0.019	0.25

Table 4.4: Key parameters for case study of Dikering 17

4.3. Dikering 29

Dikering 29 can be considered a dikering that is primarily under the influence of the sea. Hence the sea level rise will be the most important parameter for this analysis. The local water authority that is associated with dikering 29 is that of *Scheldestromen*. As such, the values for local tax revenue as well as their share of the national deltaprogramme budget are assumed to (partly) be a proxy for the values that the dikering generates. The estimated damage due to loss of life and loss of assets are directly derived for dikering 29.

4.3.1. Key parameters

The key parameters used in the case study are shown in table 4.5.

Parameter	Distribution	Mean	Coefficient of variation
Deterministic parameters			
α	Deterministic	0.8	-
P_0	Deterministic	1/2000	-
L_{dike}	Deterministic	58.5	-
K_{dike}	Deterministic	$4.7 \cdot 10^6$	-
$V_{0,life}$	Deterministic	$17.484 \cdot 10^6$	-
$V_{0,asset}$	Deterministic	$10.700 \cdot 10^6$	-
E_{demand}	Deterministic	$1.5 \cdot P_0$	-
Stochastic parameters			
β	Normal	[0.005, 0.009]	[0.10, 0.12]
γ	Normal	0.003	0.05
θ	Normal	0.019	0.05
η	Normal	0.019	0.05
Lifetime	Exponential	80	0.0625
r_t	Normal	0.019	0.25

Table 4.5: Key parameters for case study of Dikering 29

There are two values for β as the analysis is concerned with the RCP4.5 scenario from (median estimate SLR of 0.55 [m]) and the RCP8.5 scenario (high-end estimate SLR of 1.10 [m]) by the IPCC for 2100, displayed in table 2.2. Both these values will be used in the calculations and simulations. Again, the allowable risk level E_{demand},is ranged fro 0.25 to 2.5 times the initial risk level.

5

Results

This chapter will present the found results from the case studies introduced in the previous chapter. To determine the results of the adjusted CBA with the constraints presented in the chapter on methodology, a Monte Carlo simulation was run. The simulation consisted of 10^5 calculations per run. The variables in each calculation were drawn from the distributions given in table 4.4 and 4.5.

Two notable parameters used in the optimization were the crest height increase Δh and the lifetime of the dike. The increase in crest height was varied from 0 to 5 meters for each calculation. As the maximal expected water level increase is around 1 to 1.5 meters, a crest height increase of 5 meters is well above the logical maximum value, guaranteeing convergence to an optimal increase. The lifetime of the polderdike is a stochastic quantity with an expected value of 80 years and standard deviation of 5 years. Hence each calculation has a different realized lifetime to simulate the uncertainty in lifetime assessment of dikes.

To determine the most influential parameters in the optimization framework, a sensitivity analysis was performed. To this extent various parameters are varied within range and the Monte Carlo analysis was performed again, with the varying parameters. The parameters chosen for the sensitivity analysis are both deterministic and stochastic of nature, requiring either simply substituting or redrawing from the distribution.

5.1. Dikering 17

This section presents the results for dikering 17, referred to as *Hollandse Delta* in previous sections.

5.1.1. Results optimization of CBA

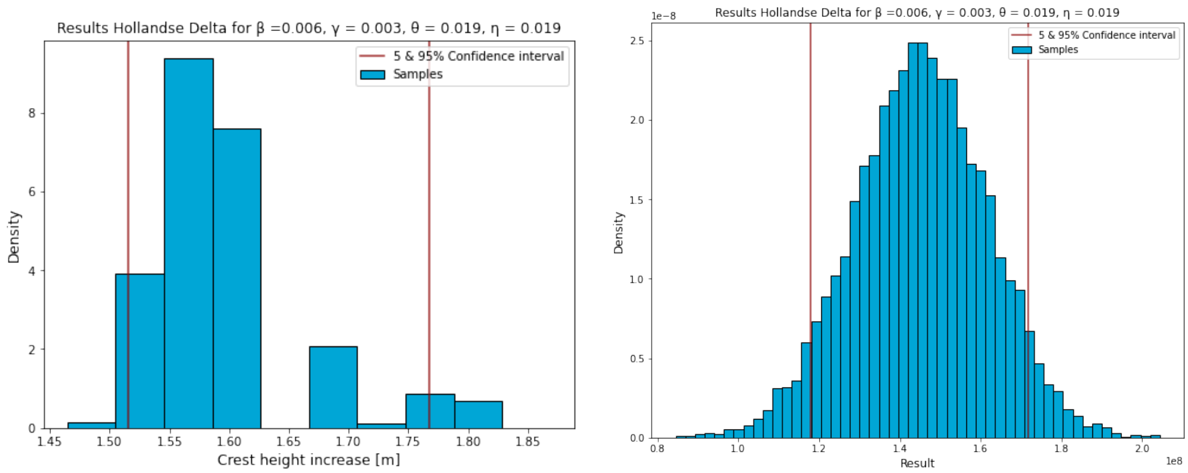
The results for the optimization of the CBA for dikering 17 can be seen in table 5.1 for two values of β, 0.002 and 0.006. The first value represents the mean, the values in between brackets represent the 5 and 95% confidence interval of the chosen quantity. All monetary values are in €mln.

Mean value of β	**0.002**	**0.006**
Chosen Δh [m]	1.0	1.6
Chosen reinforcement year [-] *	2020	2020
Benefits [mln. €]	709 (687, 725)	841 (811, 884)
Costs [mln. €]	684 (666, 698)	696 (684, 762)
Net result [mln. €]	25 (11, 59)	145 (118, 172)

Table 5.1: Monte Carlo results for Dikering 17 for different values of β

* *The chosen reinforcement year is taken as the mode of all possible reinforcement years, this holds for all tables presented in this chapter.*

The net results of both values of β with the optimal chosen value for Δh is positive, within a 95% confidence interval. Some additional results of the simulation performed to obtain the results in table 5.1 are shown in figure 5.1. This figure displays a histogram of the chosen crest height increases and a histogram of the net result of the CBA for $\beta = 0.006$. The intermediate results for the benefits and costs can be found in figure C.17 in the appendix. The same set of figures for $\beta = 0.002$ can be found in the appendix in figure C.18.



(a) Crest height increase

(b) Net result of CBA

Figure 5.1: Monte Carlo results for Hollandse Delta with β = 0.006

Figure 5.1a displays a normalized histogram from the optimal determined crest height increases for dikering 17. The 95% confidence interval for the 10^5 samples is shown in red. The mean is around 1.6 meter with a minimal (maximal) value around 1.45 (1.85) meter. The tight interval and high density around 1.55-1.60 meter display a proper convergence of the optimization task. The results of the figure indicate that, based on the current risk, population and other characteristics of the *Hollandse Delta* and the assumed annual growth rates for sea level rise, economic growth and population growth, the most optimal dike reinforcement height is around 1.60 meters. Due to the stochastic nature of the simulations, some results were a bit lower (around 1.50 meters) and some were a bit higher (1.85 meters).

Figure 5.1b displays a normalized histogram for the net results of the discounted CBA for dikering 17. It can be seen that the results are rather normally distributed with a mean around 145 €mln. and a minimal (maximal) value around 80 (210) €mln.

The intermediate results for the costs and benefits, displayed in figure C.17a and C.17b appear to display a bimodal distribution, where the costs have a more pronounced second mode than the benefits. The larger mass appears to be on the lower mode for both the benefits and the costs. The optimization problem is dependent on both the costs and benefits, which have opposite optimal investment years. To benefit the most of the reinforcement, it should be done as early as possible. To discount the costs as much as possible, it should be done as late as possible. This opposition of optimal timing creates a constant "tug" between the two components, resulting in the bimodal distribution observed.

5.1.2. Sensitivity Analysis

The sensitivity analysis will be performed for three quantities: the tolerable risk, the costs of dike reinforcement and the lifetime of the dike. To make the computations feasible, the value of β is chosen to be 0.006 and the number of drawn samples is reduced to 10^4. The 5- 95% confidence interval is shown as the shaded area in each of the graphs, where the solid line displays the mean value of the analysis. Next to the benefits, costs and net result, the chosen crest height increase Δh is displayed.

Tolerable risk E_{demand}

The tolerable risk E_{demand} is expressed as a fraction of the original risk level E_0. The fraction ranges from 0.25 to 2.5, in 10 steps. Each step consists of a Monte Carlo analysis. The results are graphically displayed in figure 5.2.

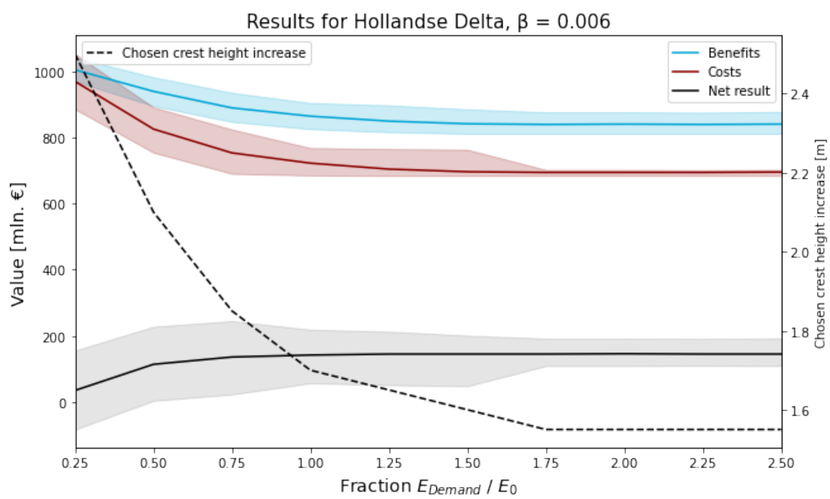


Figure 5.2: Sensitivity analysis for allowable risk for Hollandse Delta

This double-axed graph displays the adjusted CBA on the left axis and the chosen crest height increase Δh on the right axis. All fractions produce a positive CBA for dikering 17, given the other parameters. The costs and benefits remain relatively stable around a fraction of 0.50, slightly decreasing as the fraction increases. This is despite the decrease in chosen crest height as the fraction increases. However, at a fraction of 1.75 it appears that the crest height remains stable around a value of 1.55 [m]. It is also at this point that the variability in the costs greatly diminishes, which can be seen as the confidence interval shrinks. The mean value of the benefits as well as the confidence interval remain relatively stable after a fraction of 1.5.

Cost of dike reinforcement K_{dike}

The costs of dike reinforcement is ranged from €$4.7 \cdot 10^6$ to €$22.4 \cdot 10^6$ as per the minimum and maximum costs for reinforcement derived by Jonkman et al. (2013). Note that these figures are for the starting year 2020 and that the costs increase exponentially with a factor η throughout the lifetime of the dike. The results of the sensitivity analysis are graphically displayed in figure 5.3. The x-axis is expressed as a fraction of the chosen costs of the range (K_{dike}) divided by the original costs used in the analysis (K_0), which has a value of €$4.7 \cdot 10^6$.

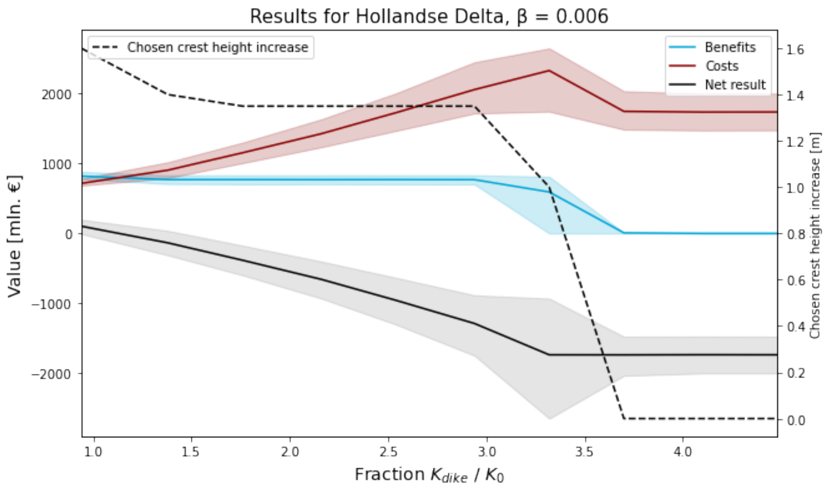


Figure 5.3: Sensitivity analysis for costs associated with crest height increase for Hollandse Delta

It becomes apparent from figure 5.3 that there is very particular range of K_{dike} where the heightening of a dike actually makes sense from an economical perspective. Whenever the fraction K_{dike}/K_0 exceeds 3.5, the choice is made to increase the crest height with 0 meters. This can have two causes:

1. The financial constraint is not met, which has the result that an increase in dike height is not possible. Note that the there are still costs incurred, namely those of the flood risk for a crest height increase of 0 [m]. This is the asymptotic part of cost part of the graph.
2. The costs of increasing the dike height is significantly higher than the incurred flood risk, causing the optimal CBA to be found in not increasing crest height.

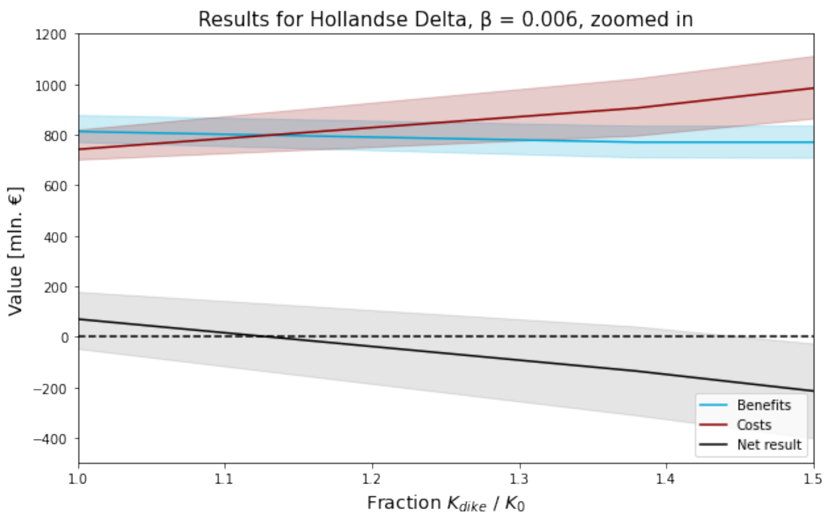


Figure 5.4: Zoomed in results for the sensitivity analysis for K_{dike}

After further investigation it appears to be a combination of both effects. This can be confirmed when looking at figure C.19a, where it can be seen that the first mentioned effect dominates after a Δh of around 2.2 [m]. Nonetheless there are still low points around 1.4 [m], however this is where the second effect appears to make make it's entrance. Even with the reduced flood risk the total costs are not lower than the option of not increasing the crest height. Taking the first mentioned effect to it's extreme, figure C.19b displays the results when the fraction attains it's largest value. For this value of K_{dike} there is no viable solution after a Δh of around 0.3 [m]. The costs are steeply increasing and before the proposed increased crest height reaches the expected sea level rise, the costs are exceeding the budget. The result is a situation in which large costs are incurred and additional safety is not provided.

The exact point of negative net results is hard to determine, as there are confidence bands in which the simulation results fall. Figure 5.4 shows the results for fractions of 1.0 to 1.5. It becomes clear that the mean costs are larger than the mean benefits for a fraction of around 1.15, while the 5 and 95% percentile are crossing for a fraction of around 1.45. These fractions correspond to costs of increasing the crest height of a dike one meter per kilometer (K_{dike}) of €5.5 and 6.8 mln. respectively.

It should be noted that although the net result of the adjusted CBA turns negative after fractions of around 1.2, there is still a crest height increase. This indicates that protecting the dike section, even though it presents more marginal costs than marginal benefits, is still a preferred option to refraining from increasing the crest height. It is only after around 3.7 times the original dike costs that the costs of marginally increasing the dike height exceed the marginal costs of not increasing the dike, which are the costs incurred due to additional flood risk.

Lifetime of the dike
The mean lifetime of the dikes is ranged from 60 to 100 years, with the same coefficient of variability as per table 4.4. The results of the analysis are graphically displayed in figure 5.5.

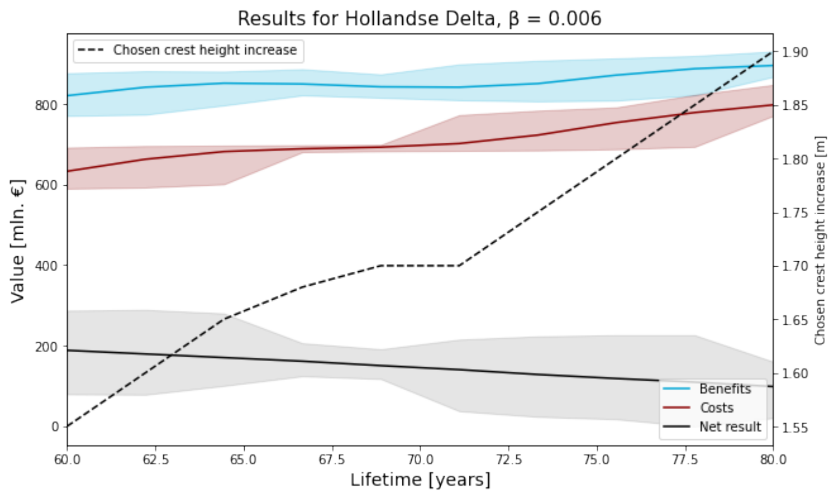


Figure 5.5: Sensitivity analysis for the lifetime variability for Hollandse Delta

From figure 5.5 it becomes apparent that the the chosen crest height is ever increasing over larger lifespans. This makes intuitive sense, as a longer lifespan means a greater potential for an increased water level. Furthermore, the benefits remain relatively constant, only increasing ever so slightly. Indicating that the crest level is chosen in such a manner that yields a similar risk level for all lifetime possibilities. This indication is confirmed when looking at the costs, which increase linearly with increasing crest height. Again indicating that the increased costs are solely the result of an increase in costs due to physical strengthening of the dike rather than an increase in flood risk. Using this set of parameters there is a feasible solution each lifetime choice, that yields a positive CBA for all choices. However, the CBA is decreasing over longer lifespans, despite the additional years to discount the costs over.

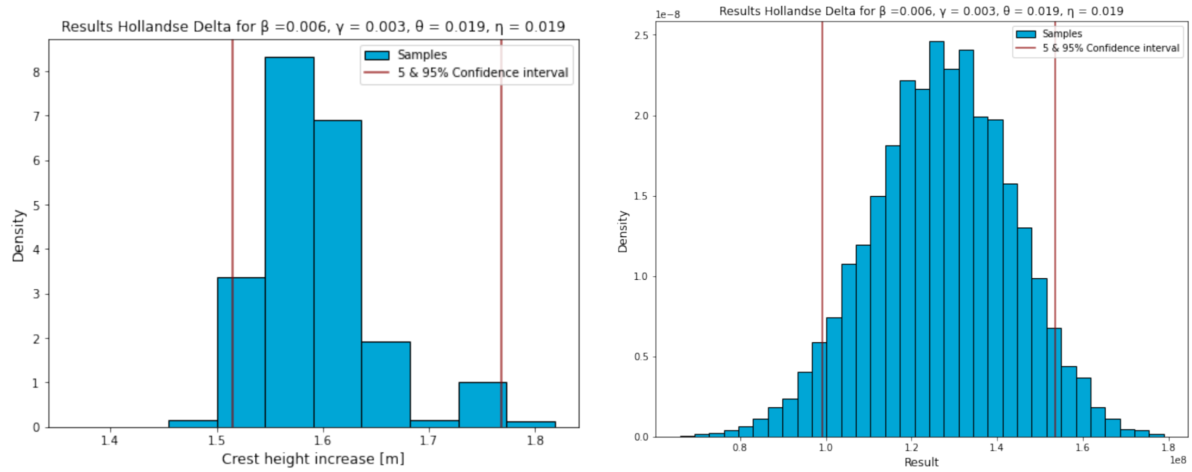
Change of interest rate model
The final sensitivity analysis that is performed is not directly linked to one of the input parameters, but rather to modelling of the discount rate. For previous sections a constant discount rate with noise was assumed. For this last analysis this was changed to a Brownian Motion with drift, with the parameters that were derived in the section *3.3.2 Interest rate model*. For this analysis the parameters of the sensitivity analysis are reset to the same values that were used in section *5.1.1 Results optimization of CBA*. The results of the optimization via a Monte Carlo analysis are shown in table 5.2.

Mean value of β	**0.002**	**0.006**
Chosen Δh [m]	1.0	1.3
Chosen reinforcement year [-]	2020*	2020*
Benefits [mln. €]	705 (665, 724)	837 (807, 876)
Costs [mln. €]	696 (624, 715)	710 (695, 769)
Net result [mln. €]	9 (-10, 27)	126 (99, 153)

Table 5.2: Monte Carlo results for Dikering 17 for different values of β using a Brownian Motion for the discount rate

* *As the reinforcement years were clearly split between the start and end of the chosen reinforcement period, the mode is displayed. However, the chosen reinforcement years had a larger variability in the case of a lower value for β.*

The results of the analysis with the discount rate modelled by a Brownian motion are for a large part in agreement with the results found in table 5.1 where a constant rate with noise was used. The net result appears to be slightly lower than when a constant rate with noise is used. The detailed results are shown in figures 5.6a and 5.6b.



(a) Crest height increase

(b) Net result of CBA

Figure 5.6: Monte Carlo results for Hollandse Delta with β = 0.006 using a Brownian Motion for the discount rate

The crest height distribution displayed in figure 5.6a is nearly identical to the distribution found when using a constant rate with noise. The shape and variability of the net result in figure 5.1b also has nearly identical characteristics, albeit that the net results is shifted slightly the left, which is the result of slightly lower benefits and higher costs. The results of the benefits and costs for the analysis using a Brownian Motion for the discount rate for $\beta = 0.006$ can be found in figures C.20a and C.20b in the appendix. The results for the case when $\beta = 0.002$ in combination with a Brownian Motion for the interest rate have the same characteristics as that of $\beta = 0.006$, being nearly identical to that of the results for $\beta = 0.002$ modelled by a constant rate with noise. The graphical results of this analysis can be found in figures C.21.

5.2. Dikering 29

This section presents the results for dikering 29, referred to as *Scheldestromen* in previous sections.

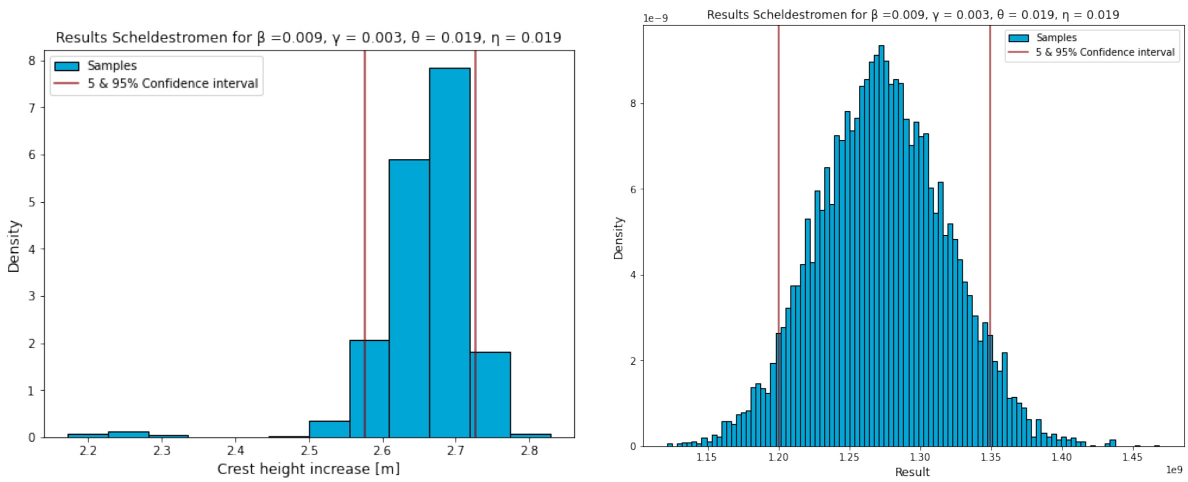
5.2.1. Results optimization of CBA

The results for the optimization of the CBA for dikering 29 can be seen in table 5.3 for two values of β, 0.005 and 0.009. The first value represents the mean, the values in between brackets represent the 5 and 95% confidence interval of the chosen quantity. All monetary values are again in €mln.

Mean value of β	**0.005**	**0.009**
Chosen Δh [m]	2.5	2.7
Chosen reinforcement year [-]	2045	2020
Benefits [mln. €]	1937 (1847, 1996)	2325 (2245 , 2408)
Costs [mln. €]	1011 (923, 1054)	1052 (1040, 1064)
Net result [mln. €]	926 (887, 966)	1273 (1199, 1349)

Table 5.3: Monte Carlo results for Dikering 29 for different values of β

The results for dikering 29 differ significantly from the results obtain for dikering 17. It can be seen in table 5.3 that the benefits are well over twice as large and the costs around 1.5 times as large as in table 5.1. This results in a net results of the CBA for dikering 29 that is significantly positive. Some additional results for $\beta = 0.009$ can be found in figure 5.7. Similar figures are generated for $\beta = 0.005$, given in the appendix in figure C.23.



(a) Crest height increase

(b) Net result of CBA

Figure 5.7: Monte Carlo results for Hollandse Delta with β = 0.006

The two figures in the main text, figure 5.7a and 5.7b, again display the distribution of optimal crest height increases and the distribution of the net results of the CBA, but then for dikering 29. From figure 5.7a it becomes apparent that there is a smaller bimodal peak for the crest heights around 2.2 - 2.3 [m] besides the more prominent chosen crest height of around 2.7 [m]. The individual distributions of the costs and benefits are displayed in figure C.22 in the appendix. In the cost function in figure C.22a the bimodal distribution that was observed in the crest height increases can be observed as well. The lower discounted costs are a logical consequence of having a lower crest height.

5.2.2. Sensitivity Analysis

The sensitivity analysis will again be performed for three quantities: The tolerable risk, the costs of dike reinforcement and the lifetime of the dike. To make the computations feasible, the value of β is chosen to be 0.009 and the number of drawn samples is reduced to 10^4. Again, the 5- 95% confidence interval is shown as the shaded area in each of the graphs, where the solid line displays the mean value.

Tolerable risk $E_{\textbf{demand}}$
The same fraction for the tolerable risk is chosen as in the section on dikering 17. The results are graphically displayed in figure 5.8.

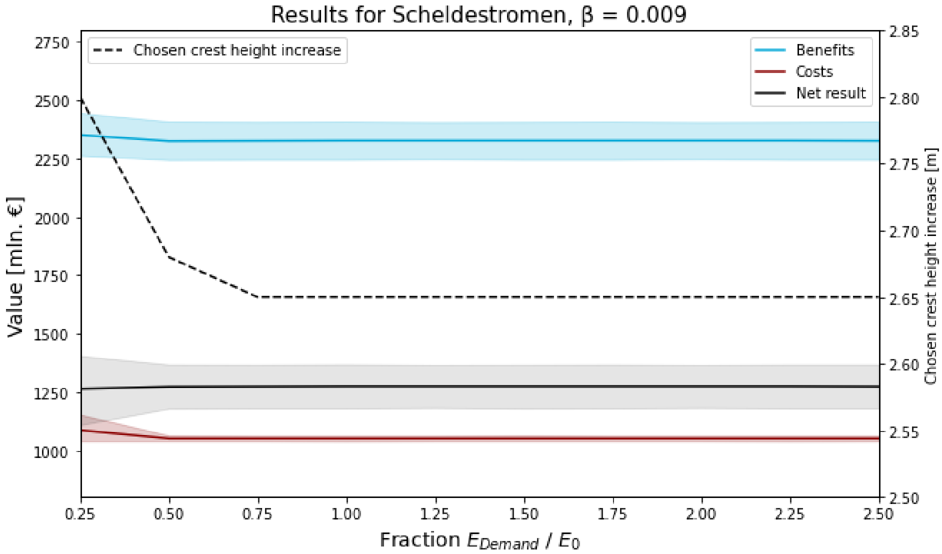


Figure 5.8: Sensitivity analysis for allowable risk for Scheldestromen

The same lay-out as in the section on dikering 17 is chosen to represent the choices. It appears that the CBA has a net-positive result for all fractions of the risk for dikering 29, constituting of benefits well over twice as large ast he costs. Furthermore it can be observed that as the tolerable risk level increases, the crest height is correspondingly lowered from 2.8 [m] at its high to around 2.65 [m] at its low. After the fractions reaches around 0.75 in value, the chosen crest height increase as well as the benefits and costs remain constant.

Costs of dike reinforcement $K_{\textbf{dike}}$
Again, the range of the costs of dike reinforcement is ranged from €$4.7 \cdot 10^6$ to €$22.4 \cdot 10^6$. The results of the sensitivity analysis are displayed graphically in figure 5.9.

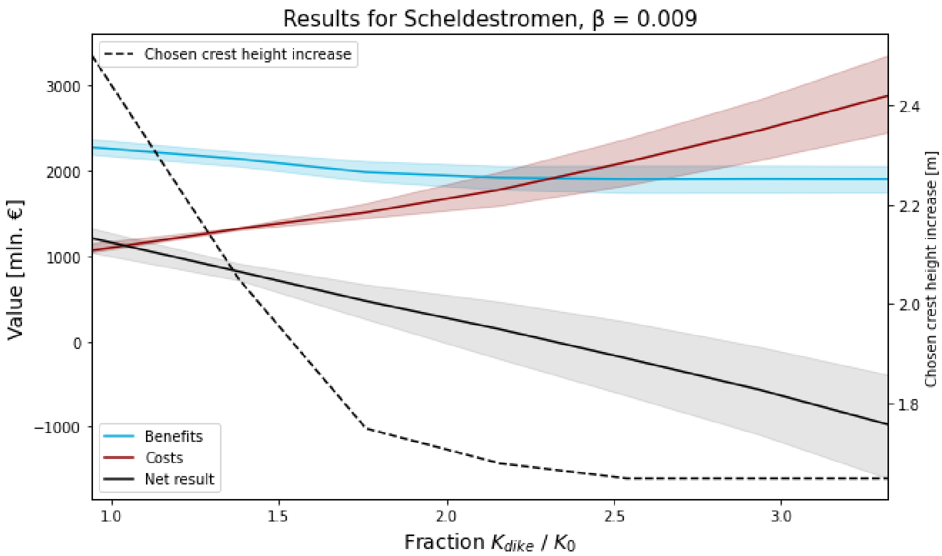


Figure 5.9: Sensitivity analysis for costs associated with crest height increase for Scheldestromen

An interesting result emerges from figure 5.9. The first thing to note is that the graph stops after the fraction reaches just below 3.5, after which the constraint could not be met anymore. This resulted in an unfeasible solution and hence no results. As such the costs for increasing the dike height per meter per kilometer in Scheldestromen can, according to this framework with constraints, not be increased beyond K_{dike} = €16 mln. Furthermore it can be observed that the crest height increase reaches a stable value around a fraction of 2.5, with $\Delta h = 1.65$. The mean costs reach beyond the mean benefits after a fraction of around 2.4, indicating a maximum on the costs of K_{dike} = €11.3 mln. for a net positive result. Note that this does not mean that increasing the crest height of the dike does not yield marginal benefits, it means that the marginal costs of increasing the dike height are higher than the marginal benefits from said increase. The 5 and 95% confidence intervals cross each other around a value for the fraction of 2.8, corresponding to K_{dike} = €13.2 mln.

Lifetime of the dike

The mean lifetime is again varied from 60 to 100 years. The results are shown in figure 5.10.

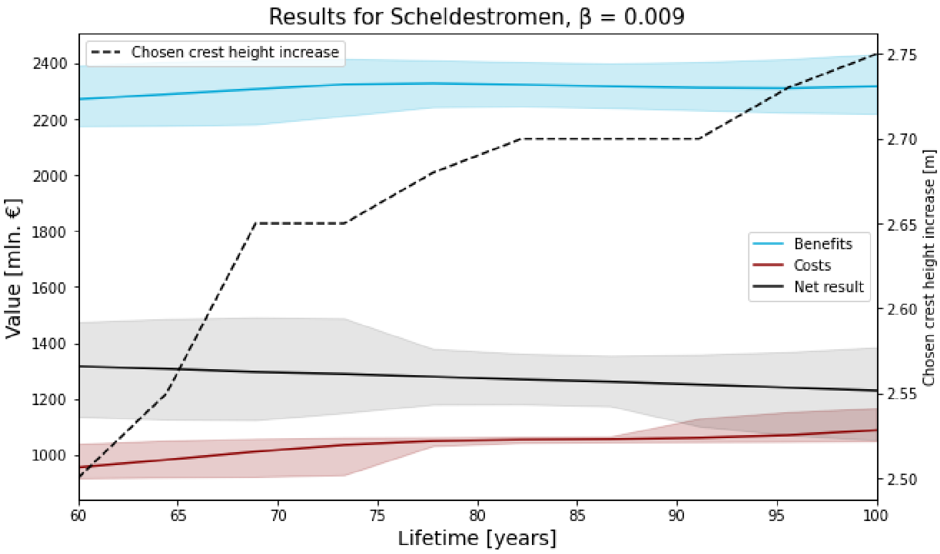


Figure 5.10: Sensitivity analysis for lifetime variability for Scheldestromen

From figure 5.10 it again becomes apparent that the crest height is increasing for increasing values of the mean lifetime, albeit at a slower rate than for dikering 17. A feasible and net positive result can be found for all considered lifespans of the structure. The benefits remain relatively constant and the costs increase linearly with the increase in crest height, as is to be expected.

Change of interest rate model

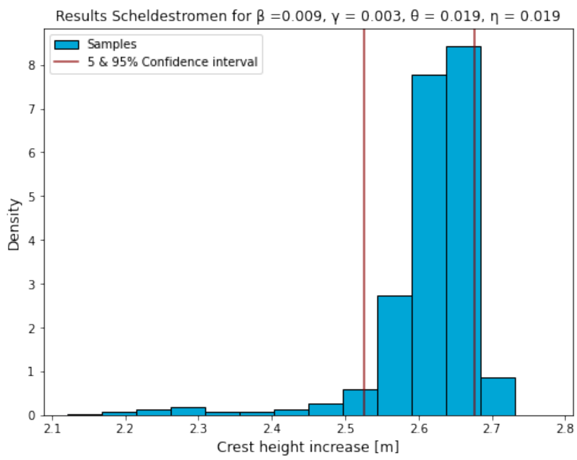
A final sensitivity analysis on the discounted rate is again performed. The constant rate with noise is replaced in favour of the Brownian motion with drift. The results of the analysis are summarized in table 5.4.

Mean value of β	**0.005**	**0.009**
Chosen Δh [m]	2.5	2.7
Chosen reinforcement year [-]	2020*	2020*
Benefits [mln. €]	1941 (1829, 2001)	2398 (2228 , 2944)
Costs [mln. €]	1042 (937, 1083)	1153 (1053, 1525)
Net result [mln. €]	898 (856, 941)	1245 (1164, 1332)

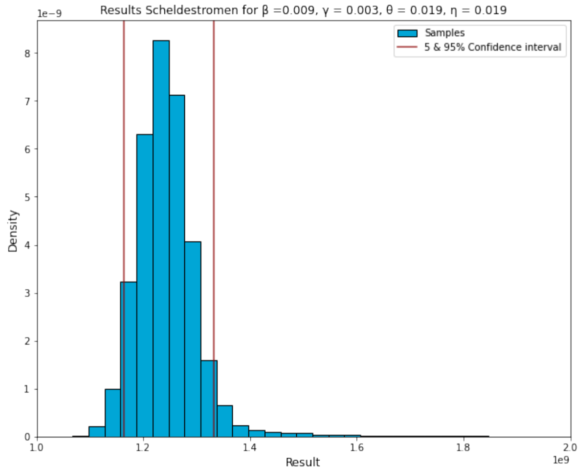
Table 5.4: Monte Carlo results for Dikering 29 for different values of β, using a Brownian motion to model the discount rate

* *As the reinforcement years were clearly split between the start and end of the chosen reinforcement period, the mode is displayed. However, the chosen reinforcement years had a larger variability in the case of a lower value for β.*

Again, the results of the analysis of the CBA with a Brownian Motion as the model for the discount rate are largely in agreement with the results where a constant rate with noise was used to model the discount rate, displayed in table 5.3. The net result again appears to be slightly lower than when a constant rate with noise is used, mainly due to the increased costs. The detailed results are shown in figures 5.11a and 5.11b.



(a) Crest height increase



(b) Net result of CBA

Figure 5.11: Monte Carlo results for Scheldestromen with β = 0.009 using a Brownian Motion for the discount rate

The crest height distribution displayed in figure 5.11a is again nearly identical to the distribution found when using a constant rate with noise. However, the net result displayed in figure 5.11b has a fatter right tail than the net result in figure 5.7b. This indicates that either the costs or the benefits have more probability mass in their extremes (left-tailed and right-tailed, respectively). This fatter right tails is ”compensated” by a slight shift of the entire distribution to the left, which causes a nearly identical mean for the constant rate with noise model and the Brownian Motion model. The results of the benefits and costs for the analysis using a Brownian Motion for the discount rate for $\beta = 0.005$ can be found in figures C.24a and C.24b in the appendix. Looking at figure C.24b, the source of the rght fat tail can be retrieved. The benefits are apparently slightly skewed towards larger values. This could be the result of realisation of the Brownian Motion that run far into the positive for later time periods, constituting a better investment moment than when the rate is positive.

The results for the case when $\beta = 0.005$ in combination with a Brownian Motion for the interest rate have the same characteristics as that of $\beta = 0.009$, being nearly identical to that of the results for $\beta = 0.005$ modelled by a constant rate with noise. The graphical results of this analysis can be found in figure C.25.

6
Discussion

This chapter contains the discussion of the research. The chapter is divided in four sections which discuss the obtained results and the proposed model.

The chapter starts with an evaluation of the proposed model. This will be done by comparing the outcomes of the model with previously established outcomes on the expected costs for flood protections under the given climate scenarios.

The second section considers the analysis of the results that were found. The results are discussed along the three sub-questions that were posed in the introduction of this research. Starting with a discussion on the results of the obtain economical framework. Next, the second sub-question on the economic viability of the polderconcept in the temporal domain will be discussed. Hereto, the results of the two case studies are discussed. The third section takes a closer look at the sensitivity of the different parameters used in the analysis of the case studies, and analyzes the most influential parameter. Finally, the fourth section will summarize the analysis of the three sub-questions to provide the results to the main question posed in this research.

The third section contains the implications of the analysed results from the second section. These implications are discussed with the advice of the Deltacommissioner in mind, providing an advice based on the outcomes of this research with respect to the economical viability of the polderconcept.

The fourth and final section discusses several limitations of the model. Three main limitations are pointed out and discussed in the presented framework. These limitations concern the use of the discount rate, the number of cost and benefit functions within the framework and the inclusion of non-quantifiable costs and benefits into the model.

6.1. Model evaluation

This section will examine the results of the research and compare them to research carried out in the past to evaluate the results of the model. Kok et al. (2008) estimate the costs of keeping the Netherlands at a similar risk level at around €0.5 to 1.0 bln per year. The analysis performed in this research was for an 80 year lifetime, hence the costs estimated by Kok et al. (2008) would be anywhere in between €40 and 80 bln. for all flood defences in the Netherlands in this period. In chapter A.3.1 it became apparent that the local water authorities that dikering 17 and 29 lie in have an annual budget of 3.6% and 2.3% of the assumed annual budget of the Netherlands, respectively. This results in an annual budget of 18 - 36 mln. €for dikering 17 and €11.5 - 23 mln. for dikering 29. Over the 80-year lifespan this would be anywhere between €1.44 - 2.88 bln. and €0.92 - 1.84 bln. for dikering 17 and 29 respectively.

Kok et al. (2008) estimate the costs of increasing the crest height of a dike in the Delta region by 1 meter per kilometer dike at around €14 mln. Hence, to make a fair comparison between the costs proposed by this model and the costs derived by Kok et al. the corresponding crest height increases for the different fractions of K_{dike} in the sensitivity analysis should be compared.

For dikering 17, a river dike with a length of 63 kilometers, the crest level increase is around Δh = 1.35 [m] for $K_{dike}/K_0 = 3.0$. This yields a total expenditure for dike reinforcement for dikering 17 with the figures provided above of $C = 1.35 \cdot 14 \cdot 63$ = €1.2 bln. This is beneath the lower end estimate provided in the research of Kok et al. although the difference is in the same order of magnitude.

For dikering 29, a sea dike with a length of 58.5 kilometers, the crest level increase is around Δh = 1.65 [m] for $K_{dike}/K_0 = 3.0$. This yields a total expenditure for dike reinforcement for dikering 29 of $C = 1.65 \cdot 14 \cdot 58.5$ = €1.35 bln. This estimate sits right in between the minimum and maximum of the derived lifetime budget for this dikering.

This section has demonstrated that for dikering 29 the results lie within the expected range of results provided by figures derived from Kok et al. (2008). For dikering 17 the costs were below the minimum costs suggested by Kok et al. However, as this research searched for an optimization it is not unthinkable that the total costs are slightly lower than in other research. As such, the conclusion is drawn that the model performs within the range that is expected.

6.2. Analysis of results

This research set out to answer the research question:

When can the Dutch polder concept become economically unviable?

In order to answer the main question, three sub-questions are posed.

- Which frameworks for judging the economic viability of the polder concept are possible?
- How does the assessment of economic viability change over time?
- Which variable(s) has/have the most influential contribution to the the viability assessment?

The results for each of the sub-questions posed will be presented below. After the presentation of each sub-question, the results will be generalized and summarized in the final section.

6.2.1. Framework for economical viability

This research argues that a cost benefit analysis (CBA) with two added components is beneficial to the judgement of the economical viability of the polderconcept. The first component comes in the form of an addition, namely that of temporal elements to the standard CBA. To this end the maximization is performed over the crest height as well as time. Furthermore, the interest rate r_t used in the discounting process is given additional degrees of freedom by being able to vary over each time period. The second component consist of a constraint on the possible reinforcement periods as well as the possible crest height increases. To this end, four constraints were determined and mathematically derived.

These constraints were quantitatively assessed on their stringency and in the end, one constraint was chosen for the framework and subsequent case study. This constraint stated that the cumulative yearly budget of a polder should exceed the costs of reinforcement at the given reinforcement time and crest height increase, to be a viable solution for the maximization problem.

The result of this sub-question is a mathematical framework that consists of three lines, shown below.

$$\max_{\forall t \in [t_1,\dots,t_n],\, \forall \Delta h \in [0,\infty]} \quad \sum_{t_i=1}^{T} \frac{B_t(\Delta h) - C_t(\Delta h)}{\prod_{j=1}^{t}(1+r_j)}$$

$$\text{s.t.} \quad C_t(\Delta h) \leq \sum_{i=1}^{t} P_i(\Delta h) \quad \exists t \in [t_1,\dots,t_n],\ \exists \Delta h \in [0,\infty]$$

$$t_1, t_2, \dots, t_n \in [t_0, t_1^*], [t_0, t_2^*], \dots, [t_0, t_n^*]$$

The first line displays the original maximization problem of net result of the CBA. Where the net result consist of the benefits at any over the crest height increase of a dike Δh, usually performed in any ordinary CBA. However, as can be seen in the subscript of the maximization, it is maximized in Δh for all the crest height increases as well as maximizedin t for all the time periods. This is the first addition that this research proposes. To minimize over time, the discounting process was modified to include a variable interest rate per time period, rather than the constant interest rate commonly assumed. The second and third line display two constraints on the maximization problem. Starting with the second, this is where the additional budget constraint comes into play. It can be seen that there should exists a combination of t and Δh such that the costs incurred as a result of that combination should be smaller than the cumulative budget up to that point. The third line introduces restriction for the possible reinforcement moments t_i, derived in equation 3.11 as a parameter that is based on the allowable risk and the growth rates of sea level rise and assets.

6.2.2. Viability in the temporal domain

To examine the viability in the temporal domain, two case studies were performed based on the derived framework for economic viability. These case studies consisted of dikering 17 and 29 in the Netherlands. The figures used in the analysis were derived from Slootjes and Wagenaar (2016) and Centraal Bureau voor de Statistiek (2022a) and adjusted to better fit current predictions. Both of the case studies were performed for two climate scenario's. In the case of dikering 17, this consisted of the 2100+ and 2100- scenario presented by the KNMI. For dikering 29 this consisted of the RCP4.5 and RCP8.5 scenarios presented by the IPCC.

The net result of the adjusted Cost-Benefit Analysis performed on the case studies was higher for an increased rise in annual water level, rather than a lower increase in annual water levels. This result holds for both the case studies. The outcome came as a surprise as it is generally accepted that a higher water level requires an increase in crest height, which in turn would lead to higher expected costs. However, for the two cases in consideration the increase in crest height was early twice as high as one would expect when looking only at the expected sea or river level rise. Where dikering 17 expected 0.25 and 0.50 meter for the two scenarios respectively, the crest height increases were 1.0 and 1.3 meter. Similarly, for dikering 29 the expected sea level rise was around 0.6 and 1.1 meter, while the maximization problem suggested optimal crest level increases of 2.5 and 2.7 meters respectively. This indicates that the reduction in flood risk from an increase in crest height weights more heavily in these cases than the costs of the physical strengthening does. As such, the costs between the two scenarios only slightly increased as the crest height only marginally increased. The main driver of the difference in net result therefore came from the difference in benefits. There was a larger difference in benefits as the case with higher expected water level rise (for both case studies) commenced the crest level increase earlier than the case with a lower expected water level rise. Therefore the reduction in flood risk had more time to add to the benefits in the case with a higher water level rise, resulting in high benefits.

To conclude this section, the viability in the temporal domain seems to be easily reachable as the main driver of the net result of the CBA is the reduction in flood risk over time. The parameters associated with the two cases and the chosen parameters were such that the reduction in flood risk dominates the increased costs of strengthening the dikes, making a higher crest level increase more favourable. A different set of parameters might give a different outcome however, therefore the third sub-question of this research was posed.

6.2.3. Sensitivity of parameters on the outcome of the assessment

To determine the parameters with the greatest influence on the economic viability of the polder in the temporal domain, several sensitivity analyses were carried out on the performed case studies. To distinguish between the several factors that contribute to the economic viability, the effect of each varying parameters is displayed for each factor. The results of which are summarized in table 6.1.

	β	E_{demand}	K_{dike}	Lifetime	r_t
Δh	++	- -	0	++	0
Chosen reinforcement year	-	0	-	-	±*
Costs	+	-	++	+	+
Benefits	++	-	0	+	0
Net Result	+	-	- -	0	-

Table 6.1: Results of sensitivity analysis

* *The change of the interest rate r_t from a constant rate with noise to a Brownian motion with drift can have either a positive or negative effect on the chosen reinforcement year*

In this table, the parameters that were varied throughout the simulations are given in the columns, whereas the parameters that were influenced are shown in the rows. The effects are qualitatively assessed on a scale from $--$ to $++$, where 0 means no effect. Note that the table refers to the effects of [column] on [row] when the value in the columns is increased. The results found in table 6.1 hold for both case studies.

The most notable results from the sensitivity analysis are summarized below.

- The cost of increasing a dike per meter crest height per kilometer is the most significant parameter determining the economic viability of the polder.
- With a higher rate of sea level rate β comes a larger need for crest height increase Δh over time. However, this increase in crest height comes with costs as well as added benefits due to decreased risk of flooding. The net result for both case studies became more positive for larger values for β.
- The initial risk with respect to the demanded risk is influential for the required increase in crest height, but does not provide a significant difference in the net result of the CBA. In particular, the results remain relatively constant after the fraction E_{demand}/E_0 hits 1.0.
- The lifetime of a dike predictably requires a larger increase in crest height for longer lifetimes. Both the benefits and the costs increase with the same relative rate, yielding a (nearly) constant net result for the CBA.
- Modelling the discount rate as a constant interest rate with noise produces nearly identical results for the CBA, crest height increase and chosen reinforcement year as when the discount rate is modelled as a Brownian Motion with drift.

6.2.4. Summary of results

Each of the three sub-questions provides a partial answer to the main question posed in this research. This section collects the partial answers and summarizes them to give an answer to the main question.

Several ways of judging the economic viability of the polderconcept were deemed executable in this research. All of the methods were concerned with an adjusted Cost-Benfit analysis that included a constraint and the addition of temporal as well as parameter optimization. The considered parameter was the crest level increase that is needed for a dike to provide flood protection. Four constraints were derived to judge the economic viability, of which one was chosen after a stringency derivation. The constraint concerned the ability of the polder to financially sustain the needed reinforcements over it's lifetime, to be considered viable.

The result of this derivation was a mathematical framework that was used on two case studies, which yielded an answer to the second sub-question on the temporal domain of economic viability. To this end, simulations with the figures provided by the case study were performed. The result was a positive outlook on the economic viability of polders over time, even seeing an increase in the net result of the adjusted CBA for larger expected water level rises.

To generalize the results and make them less parameter dependent, a sensitivity analysis was performed on three parameters and the modelling of the discount rate. The result was a qualitative assessment of the most influential parameters on five key outcomes: The needed crest level increase Δh, the chosen reinforcement year, the discounted costs, the discounted benefits and the net result of the adjusted CBA. It became apparent that the most influential parameters on the chosen crest height increase were the expected rate of sea level rise β and the lifetime of the flood protection. The chosen reinforcement year was relatively neutral throughout the parameters as it was often chosen to be reinforced immediately, going against conventional wisdom of delaying costs as much as possible to profit on the discounting principle. The costs, benefits and net results are interwoven and will be assessed by the latter component. The net result was greatly influenced by the cost of dike reinforcement K_{dike}. This parameter influenced the costs in such a way that if taken too large, no feasible solution could be found beyond certain crest height increases. The optimization even went as far as suggesting that for values of K_{dike} above €17.4 mln.for Hollandse Delta the "best" (e.g. economically the least expensive) would be to not increase the crest height and accept the increased likelihood of a flood and all costs that are associated with that.

This section is concluded with the answer to the original research question that: *The polderconcept is deemed economically viable based on the derived framework and performed case studies in this research as long as the cost of per meter crest height increase per kilometer dike do not exceed a certain threshold.*

This threshold was found to be €11.3 mln. for dikering 29 and €5.5 mln. for dikering 17 in case of the highest water level rise predictions. Note that these thresholds are set up according to a net positive result of the adjusted CBA analysis and do not tell whether a (partial) migration is economically more justified or not. To this end further research is needed.

6.3. Implications

The results of this research suggest that, taken on its own, the economic viability of the polderconcept is lost for the two case studies considered when the costs of reinforcement exceeds a certain threshold. If a policy or decision maker is solely interested in a net-positive result of costs and benefits, these thresholds provide a very clear line at which to stay below. If it is not possible to reinforce the flood defences below this price, the polderconcept can be considered economically unviable.

However, it may very well be possible that the question does not revolve around a net positive CBA but rather against a better CBA result than the alternative. In this case, policy and decision makers should question themselves to what extent they are willing to provide additional budget for flood protection beyond the zero crossing of the net result of a Cost-Benefit Analysis. Alternatives, such as the (partial) migration to other regions, should be fully analyzed in a similar fashion as was done in this research to determine their net CBA results. It might very well turn out that a (partial) migration becomes economically feasible only after a water level rise well above what is expected with current models. As such the polderconcept remains the better option for years to come. However, it might also turn out that (partial) migration will be the most economically feasible option within the expected water level rise. The proverbial numbers should be crunched to determine the outcome of such an analysis.

6.4. Limitations

The role of a model should be to assist in the decision making process. A model should therefore reflect the aspects of reality that are important for the decision making process as best as possible. Simultaneously, a model should be computationally feasible and perhaps more import: it should be understandable. As such, a model will by definition be limited in it's capabilities. The extend and scope of these limitations is what distinguishes the quality of a model.

The proposed model in this research is computationally rather expensive and as such, limitations on the scope of the model had to be put in place. This section will explain the limitations of the proposed model.

6.4.1. Discount rate modelling

The discount rate in this research is modelled by a time-variable rate. The time variability allows for a more flexible approach to model events such as downturns and uptakes in the economy and allow for stochastic simulations of future scenario's. The take in this research on the discount rate is a pragmatic one, considering it as a variable that is to be estimated by data.

However, Lee and Ellingwood (2015) argue that using standardized figures for the discount rates for time horizons beyond several decades do not capture the true costs for future generations. As choices in Civil and Hydraulic engineering are often made for decades to come, the decision makers determine the course of the infrastructure within a nation for a long time to come. Sometimes a choice might seem (economically) right at the moment, but puts a large burden on future generations. To reduce the effect of choosing optimally for the present but taking future burdens into consideration, Lee and Ellingwood (2015) suggests carefully choosing the discount rate. This research illustrates that the discount rates can be used for more than a quantitative analysis of future values, but can also be used to influence policy and decision making. The model presented in this research does not incorporate such attributes to the discount rate. However, as the interest rate modelling is made to be flexible in this research, it is possible to manipulate it in such a way that smaller or larger rates are given to later dates, which either discourages or encourages investment at a later moment. In this way the political aspect of the discount rate can be implemented in the framework derived in this research.

6.4.2. Number of cost and benefit functions

This research took several cost and benefit functions into account for the CBA analysis. There are however, many more aspects that one could think about that can be incorporated into the analysis. Examples of costs that could be considered is the added costs of introducing a large number of pumping stations with increasing relative depth of the poldersystem with respect to the sea or river level or the costs incurred as a result of the required sand nourishment due to additional coast erosion. On the benefit side of the analysis one could think of a better perceived business climate when flood risk is reduced. This business climate could in turn reduce interest rates on international loans or attract more investors to the Netherlands.

The question one must ask him- or herself is whether the addition of more terms into either the cost or benefit functions is worth the additional (computational) effort in relation to the added accuracy it provides. The purpose of a model should always be to aid in the decision making process, not dominate the decision making process.

6.4.3. Non-quantifiable costs and benefits

The previous section mentioned the addition of cost and benefit functions to the proposed model to better capture some of the nuances in the assessment of the economic viability. This research focused primarily on the most influential cost and benefit factors, which also needed to be quantifiable. There is however an entire section of costs and benefits that are not so easily quantifiable but can play a large part in the decision process on the viability of a poldersystem.

On the cost-side of the analysis this can consist of, but is not limited to, the perceived risk. On the benefit-side of the analysis one could think of the increased utility of inhabitants of an area when the flood risk is reduced. People might feel less stressed and have an overall better quality of life when the flood defences are strengthened, which contributes to a better quality of life. This improved quality of life can be counted as a benefit, however the question then becomes how one can quantify such a benefit in monetary terms for a fair value comparison.

7

Conclusion & Recommendation

This final chapter consists of two parts, a conclusion of the report and recommendation on further research. The conclusion summarizes the most important findings of this report and condenses them into an answer to the original research question. The section on recommendations consists of three recommendations. On the inclusion of terms used in the assessment of the economic viability, one on the extension of the model to incorporate risk aversion and finally one recommendation to incorporate the spatial domain in the analysis as well.

7.1. Conclusion

This research concerned itself with the assessment of the economic viability of the Dutch polder concept in light of the changing climate. As such, the main research question was posed as:

When can the Dutch polder model become economically unviable?

To answer this question a framework for economic viability was constructed based on a Cost Benefit Analysis (CBA) derived by Eijgenraam (2006), but with the addition of constraints, stochastic parameters and the inclusion of a variable discount rate. The posed constraint concerned the ability of the polder to finance itself throughout the reinforcements within its lifetime. The addition of a variable discount rate and the addition of stochastic parameters over the lifetime allowed for the inclusion of temporal elements into the CBA.

The derived framework was subsequently used to analyse two case studies based on regions in the Netherlands. The two case studies were based on the dikering of *IJsselmonde* (dikering 17) and the dikering of *Walcheren* (dikering 29). The results were determined for the two climate scenario's posed by the IPCC and KNMI. The results were determined for the two climate scenario's posed by the IPCC and KNMI by means of a Monte Carlo simulation. In the base case, both dikerings were deemed economically viable for both climate scenario's.

To quantify the effects that individual parameters had on the analysis of economic viability, a sensitivity analysis was performed. To this end, four parameters were varied and the analysis was run again to assess the difference. The result was a qualitative assessment of the most influential parameters on the economic viability of the two polders in consideration. The costs of raising a dike per meter per kilometer K_{dike} was found to be most influential on the results of the CBA.

The case studies led to the conclusion that *the polderconcept is deemed economically viable based on the derived framework in this research as long as the cost of per meter crest height increase per kilometer dike do not exceed a certain threshold*. When the costs exceed a factor 2.5 for *Walcheren* of and a factor of 1.2 for *IJsselmonde*, the economic viability, as judged in this framework, is lost. For the simplified cases this leads to thresholds of €11.3 mln. for *Walcheren* and €5.5 mln, for *IJsselmonde*.

Note that economically viable in this context means that the net results of the simplified case studies remains positive. However, even if the results turns negative the choice for increasing the crest height might remain the most economically justified choice. This is because the marginal costs of increasing the crest height is still lower than the marginal flood risk that is added as a result of the water level rise. For *Scheldestromen*, the net results turns negative but for costs considered in this research there is an optimal crest level height increase above zero. This indicates that heightening the dikes will always yield an economic better result than refraining from crest height increase. In this sense, the polderconcept can be seen as viable for this simplified case. For *Hollandse Delta* there is a point where there is no longer an optimal crest level increase above 0 meters. The crest height increase turns to zero around 3.7 times the original marginal dike reinforcement costs.

This research has opened up the possibility to compare alternatives over different time periods and reinforcement measures with different constraints and stochastic parameters, adding to the work done by Eijgenraam.

7.2. Recommendation

This section provides three recommendations for the advancement of research on the topic of economic viability of the polderconcept. The first recommendation is about including more elements in the costs and benefits functions of the presented framework. The second recommendation concerns the addition of risk aversion into the model.The final recommendation is about introducing a spatial aspect to either this or a completely new model.

7.2.1. Model extension

It has been mentioned as a limitation of the model in the previous chapter: The inclusion of more costs and/or benefit functions into the analysis. Currently two terms are quantified for the costs, the costs incurred when the crest level is elevated and the costs as a result of an increase in flood risk. On the benefit side one term is included in this research, namely that of the decrease in expected costs as a result of the decreased probability of flooding.

Although the included costs and benefits constitute the majority share of costs and benefits, they are by no means the definitive list. Extending the list of costs and benefits will increase the accuracy of the model evaluation. A balance should be sought between adding components and keeping the model computationally feasible and comprehensible. The section is concluded with the suggestion to include the costs of coastal erosion and added pumping capacity first, as these costs would most likely contribute the largest share.

7.2.2. Risk aversion

The assessment of the viability of a polder as on a whole is not limited to the economic viability. In fact, it is not even complete when the technical viability is factored in. A large part of the viability of a polder, or any for that matter, is dependent on the number of people that deem it a place that they would like to reside. Determining what a suitable place is to live, is partly determined by the risk perception that the individuals have of said area. This risk perception does not necessarily have to reflect the true risk. With small probabilities, people are notoriously bad at estimating their true frequency of occurrence. Examples of this are insurance against unlikely events or the irrational fear of dying in a plane crash, while happily going to work by car each day. The same can occur with the probability of flooding. People can feel unsafer than they truly are, resulting in a low residence rate for an area that is deemed ”unsafe” although it might well be perfectly safe.

The presented model results in a rather ”sterile” analysis of the polderconcept, looking at cold figures on the viability. The reality however is that if people do not feel safe in a given area, they will not reside there, no matter how safe it might really be. This phenomenon can be considered as an expression of *Risk aversion*. The model would benefit from the inclusion of risk aversion in the required crest level increase. A framework for such risk perception was presented by van Erp, 2017. The inclusion of risk aversion would make the model more suitable for the adaptation by policy makers, who are often driven by more than economic or technical motives.

7.2.3. The spatial domain

The analysis in this report was primarily focused on the temporal aspects of the viability of the polderconcept. However, once can also think about the feasibility in the spatial domain. Although less used for existing polders, during the design phase of a new area it is useful to know the limitations of the size the polder can be made to keep it (economically) viable. To this end a spatial model with a similar framework as created for the temporal domain could give a first indication of the limitations.

To encourage research into this area, the author has taken the liberty to set-up the beginnings of a model that is likened to the temporal model derived in this research, but then for the spatial domain. The set-up is presented in chapter B.

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A

Mathematical derivations, proofs and extrapolations

A.1. Stringency of the financial constraints

To derive the stringency of each of the five approaches, the variables of the first polder will be taken for the numerical example. These numerical examples use the previously derived formulas, with modifications for each approach. The approaches are judged based on the point where the increase in crest height Δh, is no longer economically feasible by the criteria provided. Before the derivation of the constraints, some general notation is introduced.

Costs

For the analysis in this report, the following costs were considered:

- $K_{fatalities}$: The costs associated with the loss of live as a result of a flood
- K_{assets}: The costs associated with the loss of assets as a result of a flood
- $K_{SLR}(\Delta h)$: The costs that are incurred when increasing the crest height of a dike by Δh meters.

The total number of fatalities N can be calculated as the integral of the product of the individual risk and the population density over the area:

$$\mathbb{E}[N] = \iint_A \mathbb{P}(fatality)(x,y)P(x,y)dxdy$$

When making the assumption that there is no spatial variation in the individual risk (IR) nor the population density P, and assuming a circular polder shape, the expression above can be rewritten as:

$$\mathbb{E}[N] = A \cdot IR \cdot P = \frac{L_{dike}^2}{4\pi} \cdot \mathbb{P}(fatality) \cdot P$$

The total cost of the loss of live is the expected number of lost lives multiplied by the valuation of a life. This leads to the expression for the total costs of the loss of live shown in equation A.1.

$$K_{fatalities} = \mathbb{E}[N] \cdot V_{live} = \frac{L_{dike}^2}{4\pi} \cdot \mathbb{P}(fatality) \cdot P \cdot V_{live} \tag{A.1}$$

The expected loss of economic value is calculated via a similar integration of the individual risk of assets over the area and asset density. Again making the assumption that the last two mentioned quantities are constant over the area, the following expression for the expected loss of economic value S can be derived:

$$\mathbb{E}[S] = \iint_A \mathbb{P}(asset)(x,y)H(x,y)dxdy = \frac{L_{dike}^2}{4\pi} \cdot \mathbb{P}(asset) \cdot H$$

Such that the total costs due to the loss of assets is given by equation A.2.

$$K_{assets} = \mathbb{E}[S] \cdot V_{asset} = \frac{L_{dike}^2}{4\pi} \cdot \mathbb{P}(asset) \cdot H \cdot V_{asset} \tag{A.2}$$

As this analysis is concerned with the increase or decrease in safety and the associated costs as a result of an increase in river discharge or sea level rise, it is not the absolute costs that are taken into account. The change in costs ΔK over a period T are considered, such that:

$$\Delta K_{fatalities} = \Delta K_{fatalities,T} - \Delta K_{fatalities,0}$$

$$\Delta K_{assets} = \Delta K_{assets,T} - \Delta K_{assets,0}$$

The increase (or decrease) in safety is expressed solely in the IR-terms. Ceteris paribus, the change in cost ΔK via equation A.2 and A.1 can be written as

$$\Delta K_{assets} = \Delta\mathbb{P}(asset) \cdot \frac{L_{dike}^2}{4\pi} \cdot H \cdot V_{asset}$$

$$\Delta K_{fatalities} = \Delta\mathbb{P}(fatality) \cdot \frac{L_{dike}^2}{4\pi} \cdot P \cdot V_{live}$$

Next to expected costs due to loss of lives or assets, there is a cost to physically strengthening the dikes to provide the required safety level. These costs are dependent on both the length of the dike section and the cost of strengthening the dike per unit of length. These quantities are summarized in equation A.3.

$$\begin{aligned} K_{SLR} &= L_{dike} \cdot K_{dike} \text{ where} \\ K_{dike} &= f(\Delta h_{crest}, h_0, w_0) \text{ as per equation 2.1} \end{aligned} \tag{A.3}$$

The costs for increasing the dike with a height Δh, is dependent on the geometry of the dike. For this section it is assumed that the dike has the same general geometry as shown in figure 2.6.

Benefits
For this analysis, the followings revenue streams are considered:

- B_{tax}: The tax revenue generated by a local authority
- $B_{subsidy}$: The subsidy provided to a polder by the overarching government
- $\Delta K_{fatality}$: The benefits in expected loss of human life as a result from an increased safety level
- ΔK_{assets}: The benefits in expected loss of assets as a result from an increased safety level

The local authorities are assumed to raise taxes in a similar fashion as the Dutch local authorities, with a fixed part and a variable part based on the value of your house. The tax-burden for an individual thus becomes:

$$B_{tax,i} = T_{fixed} + \text{V}_{\text{woz,i}} \cdot T_{variable}$$

Such that the total tax revenue B_{tax}, which is the sum of all individual tax contributions $B_{tax,i}$ over all M houses in a polder, becomes:

$$B_{tax} = \sum_{i=1}^{M} B_{tax,i} = M \cdot T_{fixed} + T_{variable} \cdot \sum_{i=1}^{M} \text{V}_{\text{woz},i} \tag{A.4}$$

Next to the taxes raised by the local authority, a subsidy provided by the overarching governmental body is provided. Such that for a system with N polders, the i^{th} polder receives a share of the total subsidy of:

$$B_{subsidy,j} = B_{subsidy} \cdot p_j \qquad 1 \leq j \leq N \tag{A.5}$$

Where p_j is the fraction allocated to polder j. Such that:

$$\sum_{j=1}^{N} p_j = 1 \longrightarrow B_{subsidy} = \sum_{j=1}^{N} B_{subsidy,j}$$

This fraction can be determined in a plethora of ways. One such way is taking the ratio of the economic value in area j over the total economic value. Another possibility would be to take the ratio of the number of people in area j over the total number of people in the system. This latter approach was chosen for the numerical example later in this section.

Lastly, there is a not so obvious benefit to be gained in the polder concept. When the safety level increases relative to the sea level rise, the polder will have a lower failure probability and thus individual risk, both for assets and individuals. This decrease in turn leads to a lower expected loss of life $\mathbb{E}[N]$ and a lower expected loss of economic value $\mathbb{E}[S]$, which constitutes a benefit. Conversely if the safety level decreases, the expected loss in live and economic value increases. This leads to positive values of ΔK, constituting a cost.

Summing all benefits yields the total benefit B, as demonstrated in equation A.6.

$$\begin{aligned} B &= B_{tax} + B_{subsidy,j} + \Delta K_{fatalities} + \Delta K_{assets} \\ B &= M \cdot T_{fixed} + T_{variable} \cdot \sum_{i=1}^{M} V_{woz,i} + B_{subsidy,i} \cdot p_i + \Delta K_{fatalities} + \Delta K_{assets} \end{aligned} \tag{A.6}$$

In a similar fashion the total costs K can be calculated, as demonstrated in equation A.7.

$$\begin{aligned} K &= \Delta K_{fatalities} + \Delta K_{assets} + K_{SLR} \\ K &= \frac{L_{dike}^2}{4\pi} \cdot (\Delta\mathbb{P}(fatality) \cdot P \cdot V_{live} + \Delta\mathbb{P}(asset) \cdot H \cdot V_{assets} + K_{dike} \cdot \frac{4\pi}{L_{dike}}) \end{aligned} \tag{A.7}$$

A.1.1. Marginal cost ≤ Marginal utility

To asses this approach, the marginal cost MK and marginal benefit (revenue) MR functions should first be defined. Assuming that the number of individuals, and therefore the number of houses, scales with the size of the area, it is possible to rewrite equation A.6 with Δh as the only variable. Furthermore the assumption is made that the government subsidy is divided by ratio of individuals in a polder over the total number of individuals in the nation. Before deriving the marginals costs and benefits, the total cost and benefit functions should be derived.

$$TR = \begin{cases} B & \text{if } \Delta K_{fatalities}, \Delta K_{assets} \geq 0 \\ B + \Delta K_{fatalities} + \Delta K_{assets} & \text{if } \Delta K_{fatalities}, \Delta K_{assets} < 0 \end{cases} \tag{A.8}$$

$$TK = \begin{cases} K_{SLR} + \Delta K_{fatalities} + \Delta K_{assets} & \text{if } \Delta K_{fatalities}, \Delta K_{assets} \geq 0 \\ K_{SLR} & \text{if } \Delta K_{fatalities}, \Delta K_{assets} < 0 \end{cases} \tag{A.9}$$

Note that the total revenue and costs following a dike strengthening are dependent on the loss or gain of the expected number of fatalities or damaged assets. In particular, if no strengthening of the dike is made, then the additional loss of life and damages due to the decrease in safety should be added to the costs. Conversely, if it is decided that a dike is strengthened such that the probability of failure decreases, than the additional safety can be calculated as revenue. This leads to two expressions for the total costs and two expressions for the total revenue, dependent on the sign of ΔK. It follows that equation A.8 for polder n out of N can then be expressed as:

$$TR = \begin{cases} B & \text{if } \Delta K_{fatalities}, \Delta K_{assets} \geq 0 \\ B + \frac{L_{dike,n}^2}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live}) & \text{if } \Delta K_{fatalities}, \Delta K_{assets} < 0 \end{cases}$$

where:

$$B = \frac{L_{dike,n}^2}{4\pi}(T_{fixed} + T_{variable} \cdot V_{\text{woz}}) + (\frac{L_{dike,n}}{\sum_{k=1}^{N} L_{dike,k}})^2 \cdot B_{subsidy} \quad \text{for k} \neq \text{n}$$

Furthermore, it follows that equation A.9 can be expressed as:

$$TK = \begin{cases} \frac{L_{dike,n}^2}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live} + K_{dike} \cdot \frac{4\pi}{L_{dike}}) & \text{if } \Delta K_{fatalities}, \Delta K_{assets} \geq 0 \\ K_{dike} \cdot L_{dike} & \text{if } \Delta K_{fatalities}, \Delta K_{assets} < 0 \end{cases}$$

To get the marginal benefits (revenue) and marginal costs, the derivative with respect to the change in dike height is taken, such that:

$$MR = \frac{\partial TR}{\partial(\Delta h)} \text{ and } MK = \frac{\partial TK}{\partial(\Delta h)} \tag{A.10}$$

The dependence in the change in crest height for both TK and TR is in the term K_{dike}, as well as in the increase (or decrease) in safety level $\Delta\mathbb{P}(fatality)$ and $\Delta\mathbb{P}(asset)$. To this end, it is important to specify the precise functions of both the safety levels as well as the costs to increase the dike height.

Assuming that the prices of reinforcing a dike are linearly proportional to the amount of material used, the costs can be calculated using equation 2.1.

$$\begin{aligned} K_{dike} &= \Delta A \cdot P_{reinforcement} \\ &= (n^2 - 1)(yh^2 + \frac{1}{2}xh) + (n-1)(Bh + \frac{1}{2}dh) \cdot P_{reinforcement} \end{aligned} \tag{A.11}$$

where $n = \frac{h+\Delta h}{h} = 1 + \frac{\Delta h}{h}$.

For the individual risks, both to buildings and to people, it is assumed that an increase of Δh is needed to maintain the same risk. For the sake of this example, any meter below or above the required height for maintaining the current individual risk $h+\Delta h$, reduces or increases the risks by a factor 10. Furthermore it is assumed that the expected sea level rise (SLR) has to balance the increase in crest height Δh, and that no additional height is needed. The increase in ΔIR after some time T then becomes:

$$\begin{aligned} \Delta\mathbb{P} = \Delta\mathbb{P}(fatality) = \Delta\mathbb{P}(asset) &= \mathbb{P}_T - \mathbb{P}_0 \\ &= \mathbb{P}_0 \cdot 10^{SLR-\Delta h} - \mathbb{P}_0 = \mathbb{P}_0 \cdot (10^{SLR-\Delta h} - 1) \end{aligned}$$

or, as this example is about order of magnitudes, a simple natural power function can be assumed instead of a power function with base 10, such that:

$$\Delta\mathbb{P}(fatality) = \Delta\mathbb{P}(asset) = \mathbb{P}_0 \cdot (e^{SLR-\Delta h} - 1)$$

Defining the change in cost in this manner allows for the manipulation of the conditions in equation A.8 and A.9. Instead of ΔK, one can write the domain of the function as two disjoint intervals based on the increase in crest height and the sea level rise, such that:

$$\text{if } \Delta K_{fatalities}, \Delta K_{assets} \geq 0 \longrightarrow \text{SLR} \geq \Delta h$$
$$\text{if } \Delta K_{fatalities}, \Delta K_{assets} < 0 \longrightarrow \text{SLR} < \Delta h$$

Hence equation A.8 and A.9 now become:

$$TR = \begin{cases} B & \text{for SLR} < \Delta h \\ B + \frac{L_{dike,n}^2}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live}) & \text{for SLR} \geq \Delta h \end{cases}$$

$$TK = \begin{cases} \frac{L^2_{dike,n}}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live} + K_{dike} \cdot \frac{4\pi}{L_{dike}}) & \text{for SLR} < \Delta h \\ K_{dike} \cdot L_{dike} & \text{for SLR} \geq \Delta h \end{cases}$$

The marginal revenue for the case where $SLR \geq \Delta h$, yields the following derivation:

$$\begin{aligned} MR &= \frac{\partial(B + \frac{L^2_{dike,n}}{4\pi} \cdot (e^{SLR-\Delta h} - 1) \cdot (\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}))}{\partial(\Delta h)} \\ &= -\frac{L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1)}{4\pi} \cdot (\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}) \end{aligned}$$

When the sea level rise is larger than the increase in crest height, the marginal revenue only consist of B. As B is independent of Δh, it's derivative is equal to zero. Combining this yields equation A.12.

$$MR = \begin{cases} 0 & \text{for SLR} < \Delta h \\ = -\frac{L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1)}{4\pi} \cdot (\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}) & \text{for SLR} \geq \Delta h \end{cases} \quad \text{(A.12)}$$

For the marginal costs, the derivative of the dike reinforcement costs are shown first.

$$\begin{aligned} \frac{\partial K_{dike}}{\partial(\Delta h)} &= \frac{((n^2-1)(yh^2 + \frac{1}{2}xh) + (n-1)(Bh + \frac{1}{2}dh)) \cdot P_{reinforcement}}{\partial(\Delta h)} \\ &= \frac{((\frac{\Delta h^2}{h^2} + \frac{2\Delta h}{h})(yh^2 + \frac{1}{2}xh) + (\frac{\Delta h}{h})(Bh + \frac{1}{2}dh)) \cdot P_{reinforcement}}{\partial(\Delta h)} \\ &= P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta hx}{h} + 2yh + x + b + \frac{1}{2}d) \end{aligned}$$

The marginal costs are always non-zero, regardless of the value of Δh in relation to SLR. Starting with the case where SLR $\geq \Delta h$, the following expression is found:

$$\begin{aligned} MK &= \frac{\partial K_{dike} \cdot L_{dike}}{\partial(\Delta h)} \\ &= L_{dike} \cdot P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta hx}{h} + 2yh + x + b + \frac{1}{2}d) \end{aligned}$$

In the case that SLR $< \Delta h$, the following expression is found:

$$\begin{aligned} MK &= \frac{\partial(L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1) \cdot (\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live} + K_{dike} \cdot \frac{4\pi}{L_{dike}}))}{4\pi \cdot \partial(\Delta h)} \\ &= L_{dike,n} \cdot P_{reinforcement} \cdot [\Delta h 2y + \frac{\Delta hx}{h} + 2yh + x + b + \frac{1}{2}d] \\ &- L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1)[\frac{\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}}{4\pi}] \end{aligned}$$

Combining these two expressions with their corresponding domain yields an equation that shows the marginal costs. This is displayed in equation A.13.

$$MK = \begin{cases} L_{dike,n} \cdot P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta hx}{h} + 2yh + x + b + \frac{1}{2}d) & \\ -L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1)[\frac{\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}}{4\pi}] & \text{for SLR} < \Delta h \\ L_{dike} \cdot P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta hx}{h} + 2yh + x + b + \frac{1}{2}d) & \text{for SLR} \geq \Delta h \end{cases} \quad \text{(A.13)}$$

To maximize profit, the two quantities MR and MK, should be set equal to one another. Two cases are distinguished, based on the domain of the functions:

Sea level rise smaller than crest height increase (SLR < Δh)
In this scenario the marginal revenue is equal to zero. This leads to the following equality:

$$L_{dike,n} \cdot P_{reinforcement} \cdot [\Delta h 2y + \frac{\Delta h x}{h} + 2yh + x + b + \frac{1}{2}d]$$
$$- L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1)[\frac{\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live}}{4\pi}] = 0$$

Sea level rise larger than crest height increase (SLR ≥ Δh)
This case yields the following equality:

$$L_{dike} \cdot P_{reinforcement} \cdot (\Delta h 2y + \frac{\Delta h x}{h} + 2yh + x + b + \frac{1}{2}d) =$$
$$- \frac{L^2_{dike,n} \cdot (e^{SLR-\Delta h} - 1)}{4\pi} \cdot (\mathbb{P}(asset,0) \cdot H \cdot V_{asset} + \mathbb{P}(fatality,0) \cdot P \cdot V_{live})$$

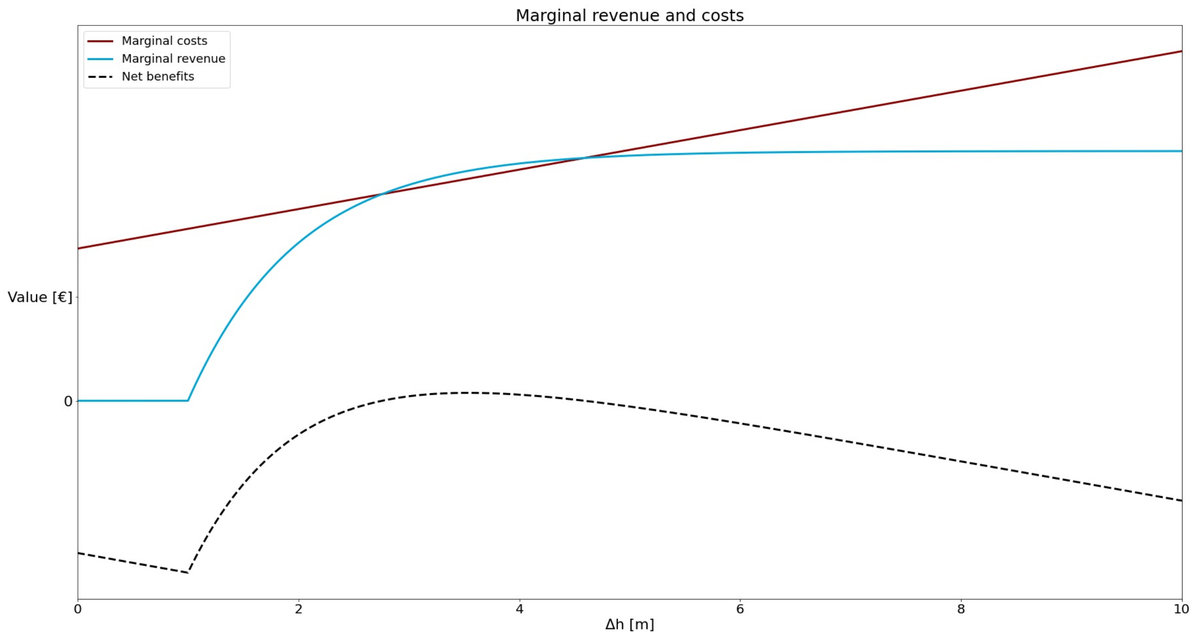


Figure A.1: Marginal revenue and costs as function of Δh

Both of these cases do not have a closed form expression for Δh. However, a graphical interpretation can help in understanding the numerical solution. A graphical interpretation of the solution is shown in figure A.1. There are two points where the marginals are equal, which are approximately at $\Delta h = 2.8$ [m] and $\Delta h = 4.6$ [m]

A.1.2. Budget for flood protection = Expenses flood protection

Where the previous approach used the marginals of the costs and benefits, this approach looks at the totals of the costs and benefits. The goal is not maximizing the marginal utility of an additional unit Δh, but rather maximizing the total utility. Hence, expression for the total costs TK and the total benefits (revenue) TR are needed. For this purpose, equation A.8 and A.9 can be used. Setting the total costs and benefits equal to one another leads to the following expressions, again distinguishing two cases:

Sea level rise smaller than crest height increase (SLR < Δh)

$$\frac{L_{dike,n}^2}{4\pi}(T_{fixed} + T_{variable} \cdot V_{\text{WOZ}}) + (\frac{L_{dike,n}}{\sum_{k=1}^{N} L_{dike,k}})^2 \cdot B_{subsidy}$$
$$= \frac{L_{dike,n}^2}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live} + K_{dike} \cdot \frac{4\pi}{L_{dike}})$$

Sea level rise larger than crest height increase (SLR ≥ Δh)
In case of a larger sea level rise than increase in crest height, the following expression is found:

$$B + \frac{L_{dike,n}^2}{4\pi}(\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live}) = K_{dike} \cdot L_{dike}$$

Again, in neither case a closed form solution for Δh can be found. However, it is possible to derive a numerical solution and graphical interpretation. This graphical interpretation is given in figure A.2.

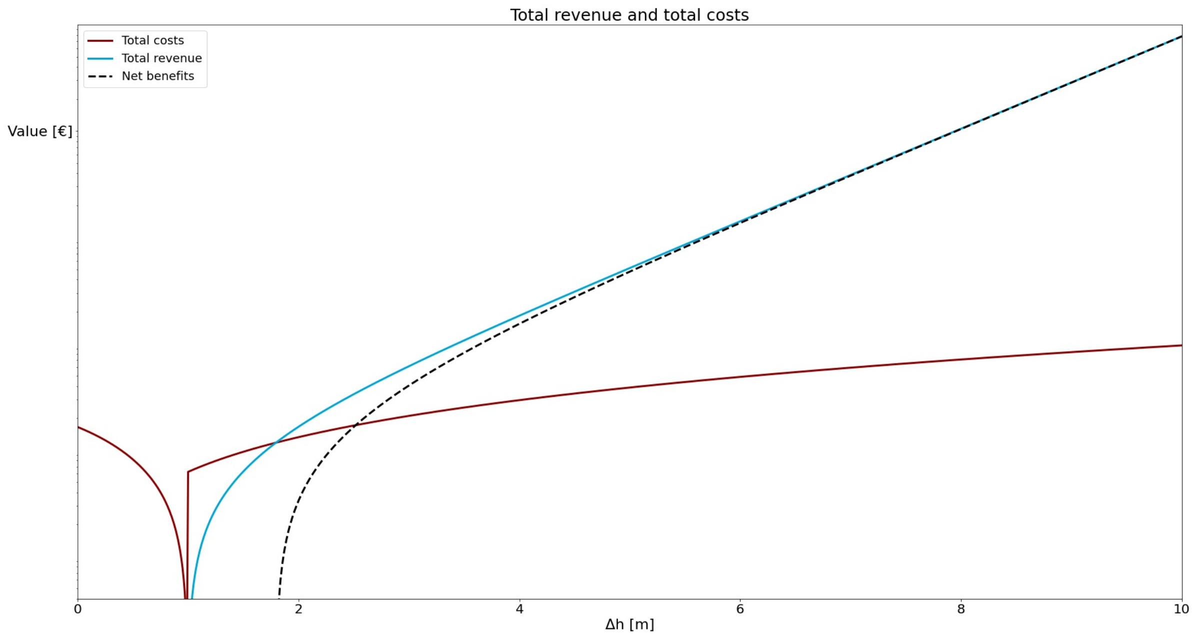


Figure A.2: Total revenue and costs as function of Δh

Interestingly, in this case there is no point where the total revenue exceeds the total costs in the first part domain ($SLR < \Delta h$). There is however a point in the second part of the domain ($SLR \geq \Delta h$) where the total revenue first exceeds the total costs. This point is around $\Delta h = 1.8$ [m], after which the total revenue will increase faster than the total costs for increasing Δh. This would mean that, for this particular polder and parameters, increasing the dike height always appears to be a financially smart decision. It should however be noted that physically this can not be true, as there must come a point of diminishing returns for the increase in safety against the cost of heightening a dike.

A.1.3. Individual risk = constant

In this approach the government is assumed to maintain a constant individual risk for each inhabitant. For the stringency analysis that means that the increase in crest level Δh should equal the increase (or decrease) in sea level rise. Then the total costs can be compared to the total revenue from an area, after which an economic assessment can be made. This approach only has one scenario, namely that Δh = SLR. The total revenue by equation A.8 thus reduces to:

$$TR = B = \frac{L_{dike,n}^2}{4\pi}(T_{fixed} + T_{variable} \cdot V_{\text{woz}}) + (\frac{L_{dike,n}}{\sum_{k=1}^{N} L_{dike,k}})^2 \cdot B_{subsidy} \quad \text{for k} \neq \text{n}$$

The total cost, by equation A.9, reduces to:

$$TK = K_{dike} \cdot L_{dike} = ((\frac{\Delta h^2}{h^2} + \frac{2\Delta h}{h})(yh^2 + \frac{1}{2}xh) + (\frac{\Delta h}{h})(Bh + \frac{1}{2}dh)) \cdot P_{reinforcement} \cdot L_{dike}$$

Equating these two quantities yields a closed-form solution for the increase in crest height. As the combined equation is quadratic in form, only the positive solution for Δh is shown here.

$$\Delta h = \frac{h \cdot (\sqrt{(\frac{2}{h} \cdot (yh^2 + \frac{1}{2}xh) + Bh + \frac{1}{2}dh)^2 + \frac{4 \cdot (yh^2 + \frac{1}{2}xh) \cdot (\frac{TR \cdot h}{L_{dike} \cdot P_{reinforcement}})}{h}} - \frac{2}{h} \cdot (yh^2 + \frac{1}{2}xh) - (Bh + \frac{1}{2}dh)}{2 \cdot (yh^2 + \frac{1}{2}xh)}$$

Where TR is the expression for the total revenue. Solving this equality with the values from table 3.1 yields that the increase in crest height is approximately $\Delta h = 0.38$ [m]. This intersection point of total costs and total revenue can also be derived graphically, via figure A.3.

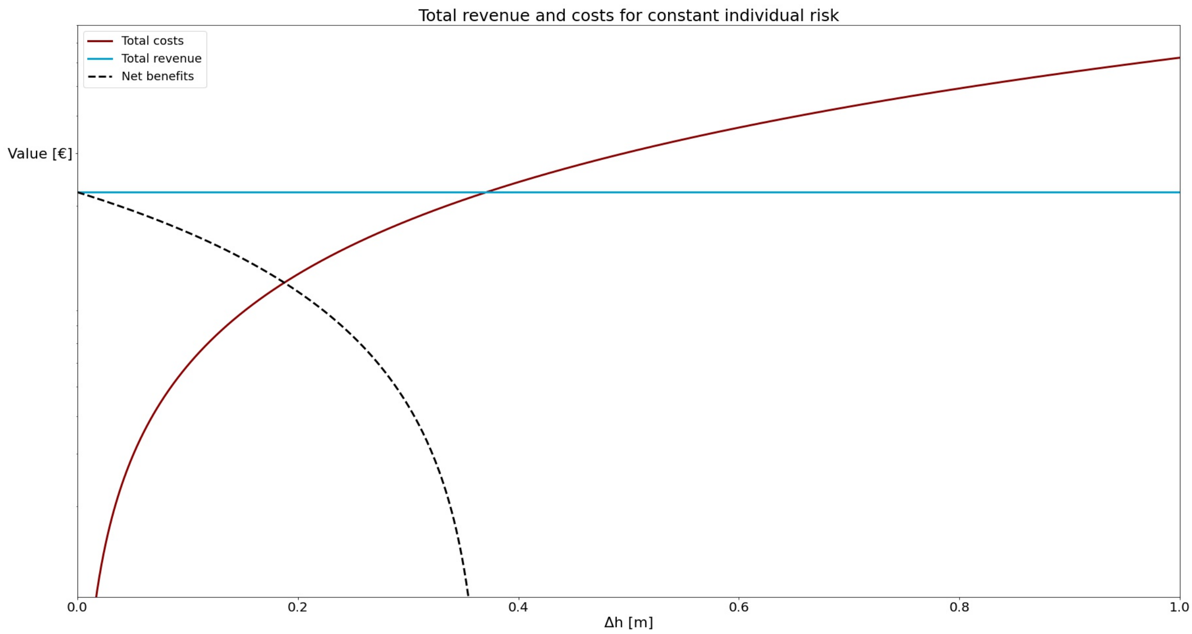


Figure A.3: Total costs and revenue for constant individual risk

Note that the x-axis on this figure only goes up to the assumed SLR of 1 [m], per definition of this approach. It becomes apparent that the required $\Delta h = SLR = 1$ [m] is not reached before the polder becomes financially unfeasible. Although the total revenue is independent of Δh, the assumption for the provided subsidy or raised tax is of great influence on the financial feasibility of the polder. If it is assumed that there just one polder that receives all of the subsidy, the point of feasibility jumps to $\Delta h = 0.8$ [m]. Similarly, if the subsidy provided increases, the line shifts upwards. Particularly, if the subsidy is multiplied by a factor of 2.8, the polder is on the edge of financial feasibility for an expected SLR of 1 meter.

A.1.4. Reinforcement costs for flood protection ≤ Expected damages due to flooding

For this approach the expected damage due to flooding should first be assessed. These expected damages can be expressed in the variables $K_{fatalities}$ and K_{assets}, which are given in equations A.1 and A.2 respectively. Combining the two expressions yields equation A.14.

$$\begin{aligned}\mathbb{E}[damages|Flooding] &= \mathbb{E}[N] + \mathbb{E}[S] \\ &= \frac{L_{dike}^2}{4\pi} \cdot (\mathbb{P}(fatality, T) \cdot P \cdot V_{live} + \mathbb{P}(asset, T) \cdot H \cdot V_{assets})\end{aligned} \tag{A.14}$$

Where $\mathbb{P}(asset, T)$ and $\mathbb{P}(fatality, T)$ are the values of the individual risk to assets and individuals at time T, with no intervention being made on the flood defences. Hence these quantities can be expressed as:

$$\mathbb{P}(asset, T) = \mathbb{P}(asset, 0) \cdot e^{SLR}$$
$$\mathbb{P}(fatality, T) = \mathbb{P}(fatality, 0) \cdot e^{SLR}$$

Such that equation A.14 can be rewritten as:

$$\mathbb{E}[damages|Flooding] = \frac{L_{dike}^2 \cdot e^{SLR}}{4\pi} \cdot (\mathbb{P}(fatality, 0) \cdot P \cdot V_{live} + \mathbb{P}(asset, 0) \cdot H \cdot V_{assets})$$

The costs for flood protection reinforcement are dependent on the required reinforcement height (and corresponding width), the length of the dike and the costs of reinforcement, as given in equation A.11. Replacing n with the expression for the change in crest height leads to equation A.15.

$$\begin{aligned}K_{dike} &= L_{dike} \cdot \Delta A \cdot P_{reinforcement} \\ &= L_{dike} \cdot P_{reinforcement} \cdot (\Delta h^2 y + 2\Delta hyh + \frac{\Delta h^2 x}{2h} + \Delta hx + \Delta hB + \frac{\Delta hd}{2})\end{aligned} \tag{A.15}$$

These two expressions allow for the derivation of an upper limit for increase in crest height. Assuming a similar structure for the individual risks as in the marginal costs and benefits section and setting A.14 equal to A.15, a numerical solution for Δh can be found. Using the values for all parameters but Δh from table 3.1 yields a graphical interpretation of this approach, shown in figure A.4. The expected damages have a constant value as they are taken as the expected costs that are incurred for a scenario where there is no dike reinforcement.

The reinforcement costs are lower than the expected damage for any Δh up to approximately $\Delta h = 3.6$ [m].

A.2. Proposed models for the interest rate

To determine the most suitable model for modelling the interest rate, three options are presented in this section. First a model for a constant interest rate will be shown, next a model for the interest rate modelled by an ARMA(p,q) model will be shown. Finally a model for the interest rate modelled by a Brownian motion will be discussed. These models will then be incorporated into the NPV expression to conclude the section.

A.2.1. Constant interest rate with noise

A first step in determining the constant interest rate would be to look at some summary statistics of the historical data. These summary statistics are shown in table 4.3. A possibility could be to take the mean value of the interest rate of the 60 year period as the constant interest rate. However, when looking at the standard deviation it becomes apparent that there is a large deviation from the mean throughout the years. Not only is there a large deviation, two distinct periods can be observed in figure C.16. In the first period, ranging from 1960 to 1980, the interest rate is steadily increasing from 4.5 to 12%. This constitutes a growth rate of around 0.38% per year. The second period, starting in 1980

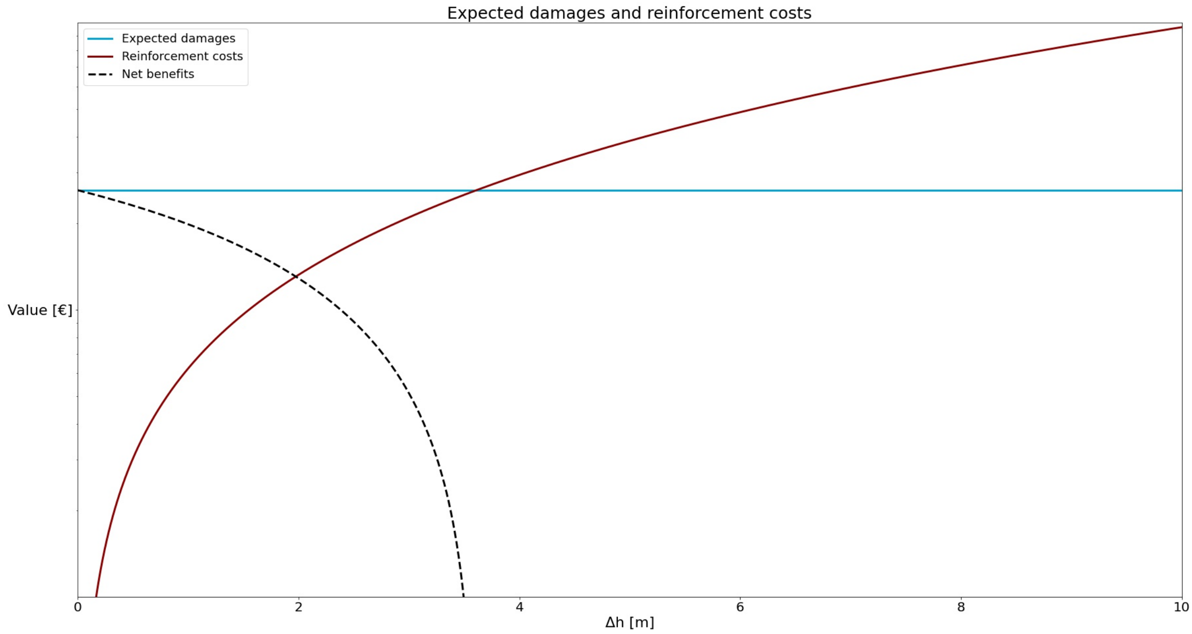


Figure A.4: Expected damages and reinforcements costs as function of Δh

and ending in 2020 is a period of steady decline, starting with in 1980 at 12% and ending in 2020 at a slightly negative value of -0.38%. This constitutes a growth rate of around -0.31%. Averaging both periods out, the constant interest rate in the first period would be 8.25% and in the second period 5.75%.

However, a notable event happened around the turn of millennium, where the Netherlands joined the European Union and with that the Economic and Monetary Union (EMU) as well. The European Central Bank has set four criteria for membership of the EMU, as per February 1992 according to the Maastricht Treaty. The fourth of which being that the governmental bonds should not exceed a 2% interest rate, in line with the inflation targets set by the aforementioned organisation. It would therefore make sense that the long-term interest rate should not exceed 2% in the case of a constant interest rate. The assumption is made that the same steady trend upwards as the first time period (1960 - 1980) occurs after 2020, and the interest rate is kept locked to 2% after it first hits the 2% mark. This means that, starting in 2020, the interest rate will hit 2% after six years, after which it is kept fixed for the remaining 24 or 74 years. The average interest rate for 2050 is taken as the weighted average of the preceding interest rates, making the interest rate $r_{2050} = 1.73\%$. The same exercise can be done for 2100, coming to an interest rate $r_{2100} = 1.89\%$. The noise that is added to this constant rate, to give it the appearance of a natural rate, is assumed to be normally distributed. The standard deviation is chosen to be a quarter of the value of the interest rate.

This section concludes with the observation that due to the large variability in interest rates it seems almost foolish to set the interest rate at a fixed value for projects that range from 30 to 80 years. The following sections will therefore research a variable interest rate.

A.2.2. ARMA(p,q) model

To determine the correct specification for an ARMA(p,q) model, the first step is to look at the autocorrelation and partial autocorrelation of the time series. These are shown in figure A.5
The partial autocorrelations display some information on the appropriate number of lags to include for the AR section of the model. The autocorrelation itself does not shown any useful information on the MA section. As this might be a sign of the presence of a unit root and therefore non-stationarity, a Dicky Fuller test is performed. The test statistic has a value of -0.0575 (p-value of 0.953), which is well below the threshold of the critical value to reject the H_0 of nonstationarity. To solve this issue, the first difference of the time series is taken. The result of taking the first difference is shown in figure C.7. The

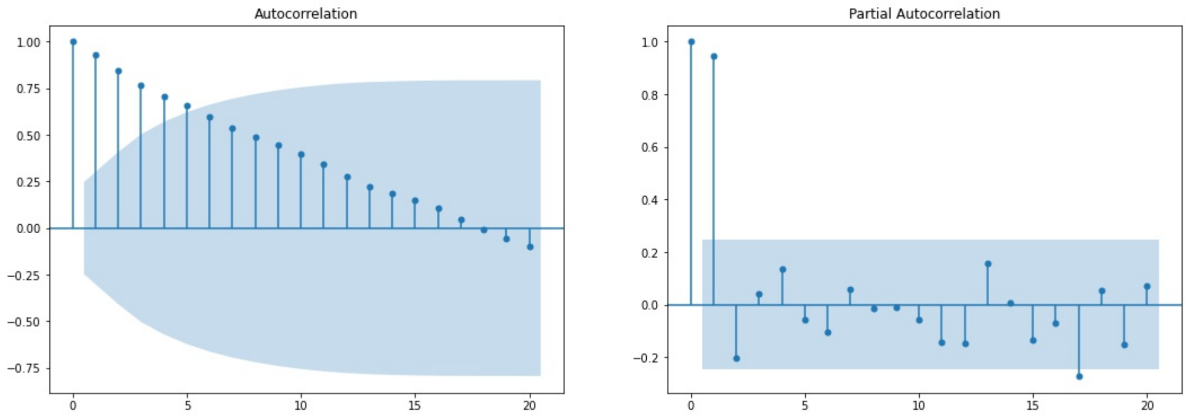


Figure A.5: (Partial) Autocorrelations of interest rates

Dickey-Fuller test is performed again and now a test statistic of -6.151 (p-value of $7.6 \cdot 10^{-8}$) is found, giving enough confidence to reject the H_0 of non-stationarity, and the conclusion is drawn that the time series is now stationary. This allows for the use of the (partial) autocorrelations for a first estimation of the order of the ARMA model. The (partial) autocorrelations are shown in figure A.6.

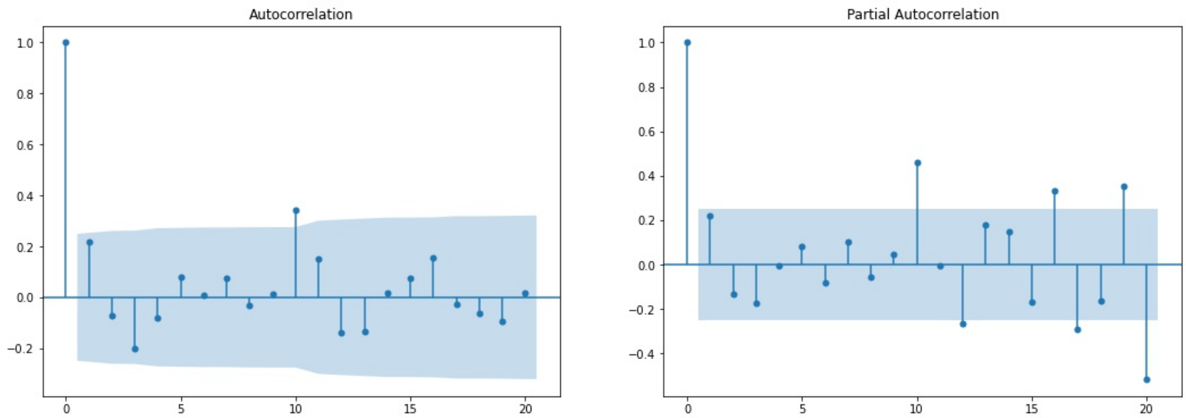


Figure A.6: (Partial) Autocorrelations of interest rates after taking first differences

Based on these autocorrelations an ARMA(1,1) appears to be a good first estimate. Next to the ARMA(1,1) model, 35 other models are compared via the Akaike Information Criteria (AIC). Six orders per term (both AR and MA) were compared. The five models with the lowest score are presented in table A.1.

Order of ARMA(p,q)	AIC
(2,3)	140.04
(2,4)	140.12
(4,2)	140.18
(4,3)	141.69
(3,4)	141.90

Table A.1: Akaike Information Criteria for five different ARMA models

It becomes apparent that an ARMA(2,3) model has the best AIC out of 36 combinations of models. The initial guess of an ARMA(1,1) model has an AIC of 144.05, ranking it at a 19th place. Deriving a model with the given time series and specifications above yields a model with parameters as displayed in table D.3, including the standard deviation and significance of the parameter. All of the MA parameters appear not to be of significance and hence, the order was reduced by one, from three to two. This yields the model in table A.2, of which all of the parameters are significant.

Parameter	Value	Standard deviation	p-value
φ_1	0.53	0.132	0.000
φ_2	-0.88	0.119	0.000
θ_1	-0.36	0.159	0.024
θ_2	0.85	0.159	0.00

Table A.2: Parameter estimation of ARMA(2,2) model

To examine whether the removal of one MA term did not result in a loss of information, the residuals are inspected on their properties. If the model captures the elements of the time series, the residuals are expected to behave as white noise. To verify this, the residuals are plotted in figure C.8 and the probability density function of the residuals are shown in figure C.9. These figures give a good indication that the residuals are standard normally distributed, adhering to the white noise properties. To verify this numerically a Ljung-Box test, as given per equation A.16 was performed on the first five lags.

$$LB = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho_k}^2}{n-k} \sim \chi^2_{1-\alpha}(h) \tag{A.16}$$

All of these yielded values below the criteria of $Q > \chi^2_{0.05}(5)$, resulting in p-values above 0.77. Hence the H_0 of independently distributed data is not rejected and it can be concluded that the data is indeed non-correlated. Further confirmation is obtained when looking at a QQ-plot of the residuals, shown in figure A.7, which display a (near) one-to-one alignment of the theoretical quantiles with the empirical quantiles. The (partial) autocorrelation, displayed in figure C.10, give final confirmation of the white noise properties.

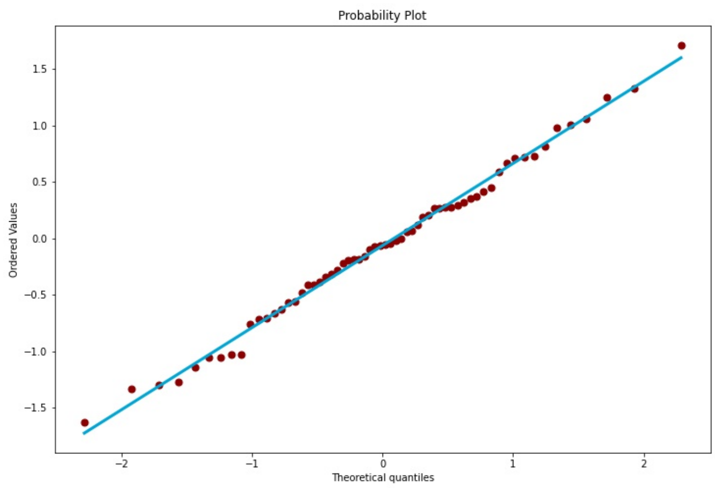


Figure A.7: QQ-plot of residuals of ARMA(2,2) model

The (partial) autocorrelation, displayed in figure C.10, give final confirmation of the white noise properties.

The time series model is now properly defined, which yields the following model that will be used to predict the risk free interest rate r_t for the desired investment period.

$$\Delta r_t = 0.53\Delta r_{t-1} - 0.88 r_{t-2} + \varepsilon_t - 0.36\varepsilon_{t-1} + 0.85\varepsilon_{t-2}$$

A.2.3. Brownian Motion

To construct the Brownian motion the following parameters need to be extracted from the data set:
μ: The drift coefficient
σ: The standard deviation

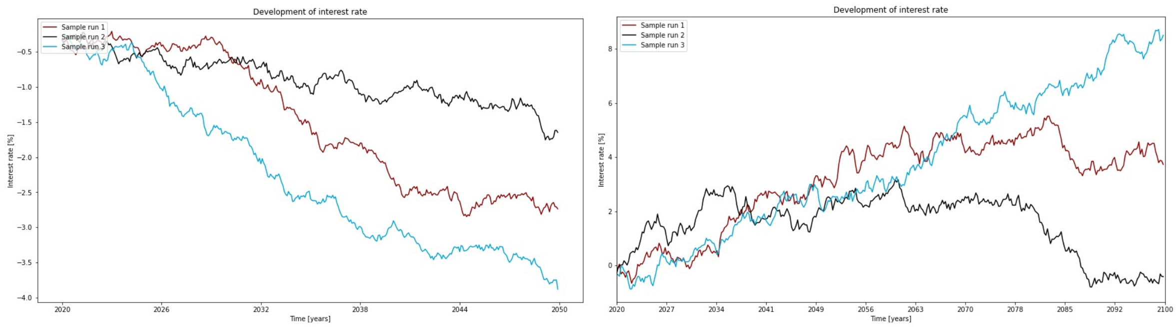
r_0: The starting value of the Brownian motion

These values can be derived from the data set in a variety of ways, two of which will be presented here. The first way is by taking the first differences of the data set. From these first differences, a mean can be computed to substitute for μ and the variance can be computed to calculate the σ. The second way of computing μ is by taking the mean of the interest rate value at each time step, and dividing by the length to get an increase per time step. The standard deviation σ is computed by taking the square root of the variance of the interest rates. The starting value r_0 is in both cases equal to the value of the interest rate in 2020, which is $r_0 = -0.328$. A summary of the parameters for the two different approaches is given in table A.3.

Parameter	Method 1	Method 2
μ	0.087	-0.076
σ	2.872	0.754
r_0	-0.328	-0.328

Table A.3: Parameters for two different BM approaches

These approaches yield some significant differences. An example of the two approaches is shown in figure A.8.



(a) Sample of BM with values of the first method **(b)** Sample of BM with values of the second method

Figure A.8: Three samples of two approaches for a 30-year sample of a Brownian Motion

Each figure consists of three runs for a 30-year period. The variety between runs is rather large, which is understandable given the variance of the approaches. More note able is that the first approach yields a negative drift, where the second approach yields a positive drift.
Negative interest rates means that the investor has to pay a premium for keeping their money invested in the asset, depreciating the investment each year. Keeping the rate negative is a temporary tool to prevent or reverse deflation, as the goal of many financial intuitions is to keep inflation around $\pm 2\%$. It is therefore rather unlikely that a negative interest rate persist for long, let alone that it reaches values well below -1%. For this reason, the second approach is chosen to determine the Brownian motion for interest rates.

Figure A.8 displays only three runs, however when the number of runs is increased, the average of the BM moves towards the value of μ. The approximation of the drift for 2050, together with confidence bands, is shown in figure A.9
The interest rate in 2050 is of interest. The distribution including the 5th and 95th percentile are shown in the histogram in figure A.10.
It becomes apparent that the mean value of the interest rate in the 2050 year is around 2.28%, with a 5th percentile of -2.48% and a 95th percentile of 6.96%. The same figures and data can be produced for a Brownian motion that extends eighty years into the future, to 2100. The development of the interest rate is shown in figure C.13, the histogram of the final year values is shown in figure C.14. The mean of the 100 year interest rate for the year 2100 is 6.68% with a 5th percentile of 1.81% and 95th percentile of 11.46%.

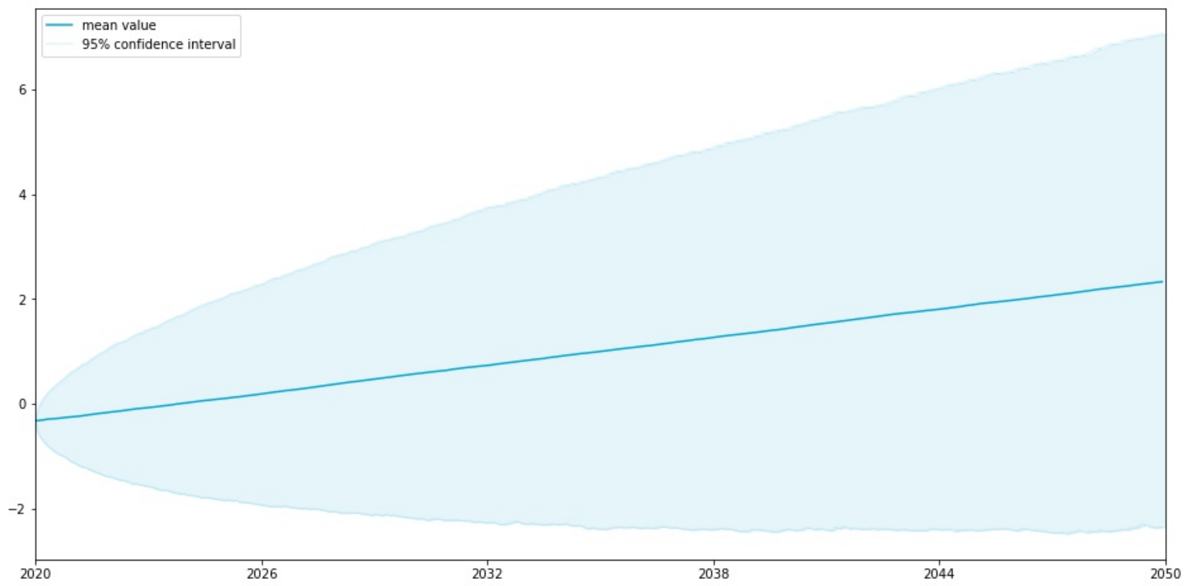


Figure A.9: Average of 10.000 sample BM for a thirty year period

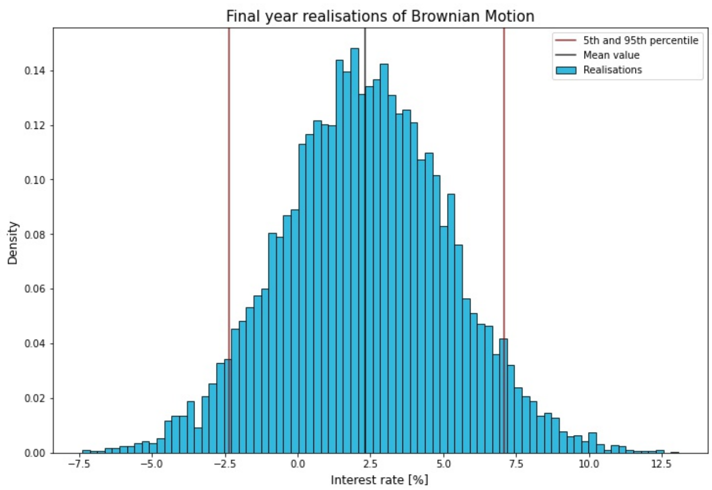


Figure A.10: Histogram of final year of 10.000 sample BM for a thirty year period

Using the result from Doob, 1949, presented in equation 2.5, probabilities of boundary crossings for the Brownian motion can be calculated. The result of Doob was derived for a Brownian motion with negative drift. As the drift is positive for the derived Brownian motion, a modification is needed. To circumvent this, it is noted that due to the *Reflection Principle* the exceedance probability of a positive boundary with negative drift is identical to the exceedance probability of a negative boundary with positive drift. Some examples of boundary crossing probabilities for the derived Brownian motion are shown in table A.4. The results are also presented graphically for interest rates ranging from 0.0 to 20% in figure C.15. It should be noted that the result of 2.5 was derived for all $t \in [0, \inf)$, whereas this research is interested in finite t. There exist expressions for finite time series, but these are cumbersome to calculate. For an indication of the order and the course of the probability over different boundaries, the result of Doob suffices.

To differentiate between the 2050 and 2100 rates,the "restart" property of Brownian motions was used. This property can best be described by viewing the Brownian motion at time t^* as a restart of the process, as if it were to restart from $t = 0$. However, now the exceedance boundary is not y but the difference between y and the level the Brownian motion is at, at the restart. Mathematically this means

Interest rate	Exceedance probability 2050	Exceedance probability 2100
2.5	0.647	0.962
4.5	0.457	0.676
6.5	0.322	0.480
8.5	0.277	0.339
10.5	0.161	0.240
12.5	0.113	0.169

Table A.4: Exceedance probabilities of interest rates for 2050 and 2100

that:

$$\mathbb{P}(sup_{t^* \geq b}\ X(t^*) > y) = \mathbb{P}(sup_{t \geq 0}\ X(t) > y - X(b)) = e^{2\mu(y - X(b))}$$

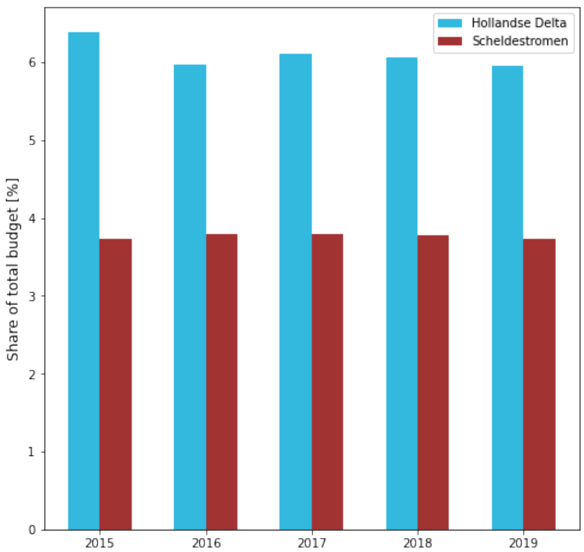
The value of $X(b)$ after 50 years is the mean of the Brownian motion at $t = 50$. Previously it was found to be around 2.28%. Which can be entered in the expression above. Intuitively this makes sense, a time period that stretches for a longer period of time most likely has a probability of exceeding a boundary that is larger, given the same drift coefficient. This can indeed be verified from table A.4, where the exceedance probabilities of a certain interest rate are higher for 2100 than for 2050.

A.3. Extrapolation of available budget and expected costs

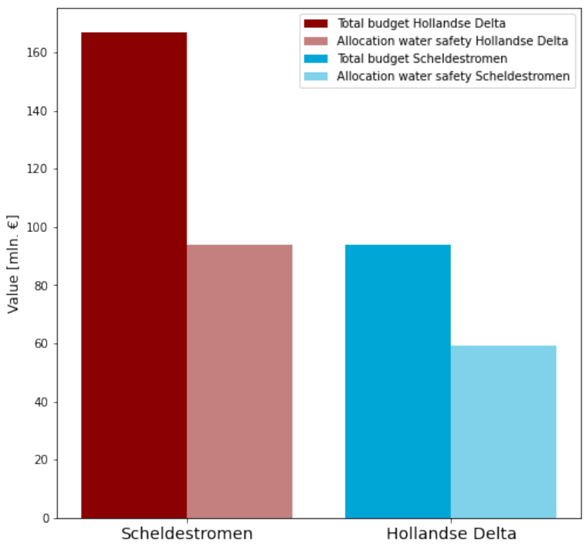
A.3.1. Derviation of available budget

Budget provided by central government

The first part to be evaluated is the budget provided by the central government. To this end, two estimates have to be made. The first estimate that has to be made is how the budget should be divided amongst the different water authorities and how an extrapolation to 2050 and 2100 can be made. The second estimate concerns the share of the GDP that is spend on flood protection. Although the central government does not directly hand-out a budget to the local water authorities, it is possible to imagine the budget being split up over projects that the water authorities undertake. To this end, the division is assumed to be made based on the share of total expenditure per water authority. The budgets from 2015 to 2019 of the water authorities are compared (Centraal Bureau voor de Statistiek, 2022d) to the total budget of all water authorities. The share of the two water authorities in consideration, *Hollandse Delta* and *Scheldestromen*, are shown in figure A.11a.



(a) Budget as share of the total budget



(b) Budget dedicated to flood protection as share of the total budget

Figure A.11: Budget for water authorities from 2015 to 2019

Hollandse Delta receives on average 6.1% of the total budget, while *Scheldestromen* receives around 3.8%. This share of the total water budget can be divided further into budget that is related to flood protection and budget that is related to other causes. Figure A.11b shows this division for the two water authorities considered in 2019. The share of budget dedicated to flood protection is around 60% for both water authorities over the period 2015-2019. The remaining 40% is split up between expenses related to the water distribution system and the water purification system.Combining the two figures yields a share of 3.6% and 2.3% of the total budget for water authorities dedicated to flood protection for the *Hollandse Delta* and *Scheldestromen* respectively.

The next figure to determine is the total expenditure to flood protection in the Netherlands in the year 2050 and 2100. To this end, the historic and expected expenditures of the Deltafonds were used and extrapolated. The historic figures come from 2013 to 2021 and the expected expenditure range from 2022 to 2025. The result are displayed in figure A.12.

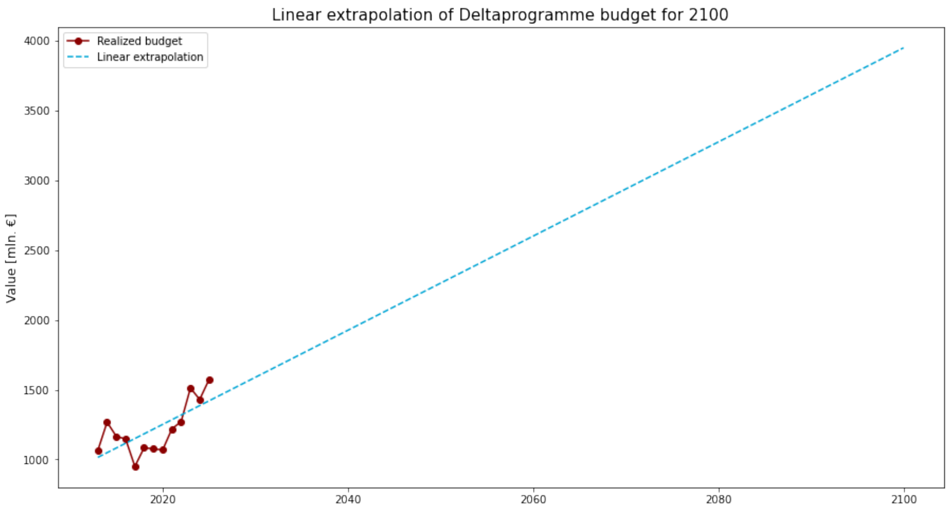


Figure A.12: Linear extrapolation for Deltafonds expenditure in 2050

The extrapolation yields a value of €2.26 bln. for 2050 and €3.95 bln. for 2100.

Combing the two derived figures, it is possible to estimate the GDP expenditure to flood protection for the two local water authorities in consideration. This yields €81.4 mln. for *Hollandse Delta* in 2050 and €142.2 mln. in 2100. For *Scheldestromen* this yields €52.0 mln. in 2050 and €90.9 mln. in 2100.

Budget provided by the local water authority

As previously stated, the local water authorities in the Netherlands collect their own taxes from the population in their jurisdiction. This taxation consist of many parts. For this research, the taxation is simplified into the two main components: a set tax and a variable part based on the value of your house. In general the tax can be written as:

$$B_{tax} = C + f \cdot V_{house}$$

Where:
f = The fraction of the value of the house that has to be paid
C = The constant part that has to be paid
V_{house} = The value of a house

To determine the budget that the local water authorities generate in future years, three main factors have to be taken into account: The population growth, the increase in the value of a house and the change in the constant C and variable part f of the taxation.

Starting with the estimated population growth, the statistics from Centraal Bureau voor de Statistiek, 2022b are used. These statistics are provided from 2020 to 2050, but no further. To get an estimate for 2050, a linear extrapolation was made using the least squares method. The result can be seen in figure A.13.

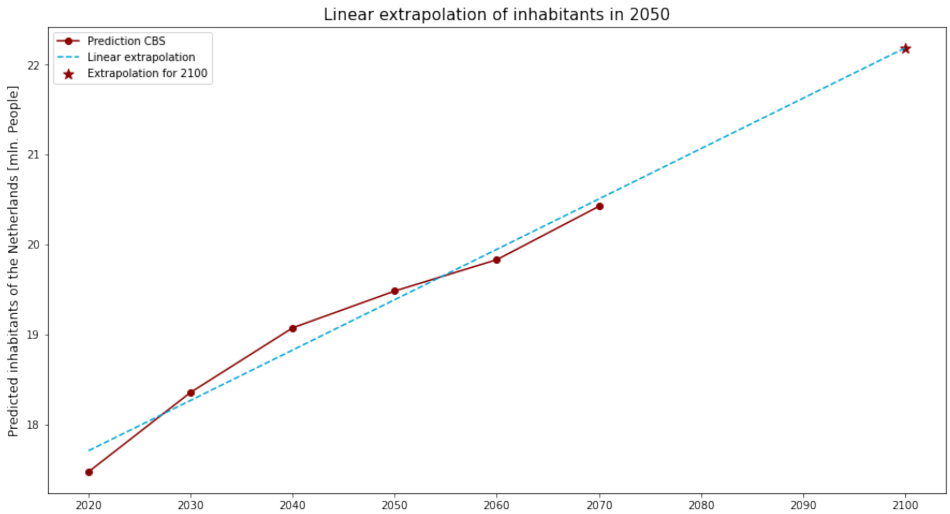


Figure A.13: Linear extrapolation for population size in 2100

As can be seen, the linear extrapolation yields an estimate of 22.2 million people living in the Netherlands in 2100. The estimate provided by the CBS for 2050 is around 19.5 million people. For dikering 17 the number of inhabitants in 2020 was 195.000, for dikering 29 this number was 115.000 (Centraal Bureau voor de Statistiek, 2022c). Using the same growth rate as for the general population in the Netherlands, the extrapolated values for 2050 and 2100 yield 217.650 and 247.800 for dikering 17 and 128.400 and 146.100 for dikering 29, respectively.

The next estimate to be made is the increase in the value of an average house. To this end, historical figures from Centraal Bureau voor de Statistiek, 2022a have been used, and an extrapolation to 2050 and 2100 has been made. The average value of a house in the Netherlands in 2020 was €334.500. A linear extrapolation was chosen, as higher orders gave unrealistic results. The extrapolation for the value of a house is shown in figure A.14. Next to the predicted value, an extrapolation was made for the index (2015 = 100) of the house values. The results can be found in figure C.2.
Using this extrapolation, it is found that the median house value in 2100 is around €927.000. The corresponding index is around 377. The average price of a house in dikering 17 was €218.000, for dikering 29 this was €208.000. Extrapolating these figures with the same growth rate to 2050 and 2100 yields €354.000 and €604.000 for dikering 17 and €337.500 and €576.000 for dikering 29 respectively.

Finally, the change in the constant C and variable f part of the taxation are considered. To determine the figures in 2050 and 2100, a linear extrapolation is made from the figures from the period 2011 to 2021. As a proxy for the taxes raised in a dikering, the taxes that are raised in a water authority are used. The historic figures for *Hollandse Delta* can be seen in figure A.15.
A similar graphical display for *Scheldestromen* was created and can be found in figure C.1 in the appendix.

Extrapolating these figures to 2050 and 2100 for both water authorities yields the results shown in table A.5. The full extrapolation can be seen in figures C.3 through C.6.
Combining the obtained results from the three main factors, an estimate for the raised tax per average household in 2050 and 2100 can be made. Combining this estimate with the estimate for the budget provided by the central government yields the total budget estimates. The results for these budget estimates are shown in table A.6, where it is used that there are on average three people per household.

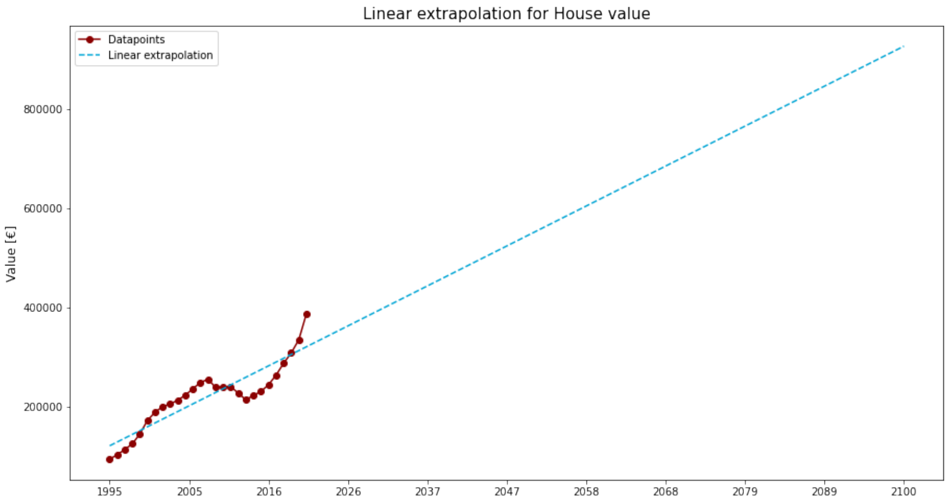


Figure A.14: Linear extrapolation for house values in 2100

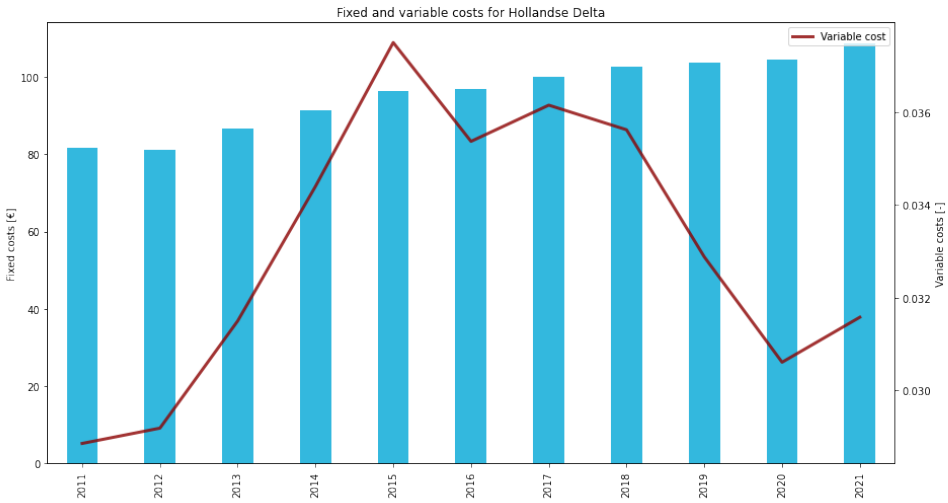


Figure A.15: Fixed and variable costs for Hollands Delta from the period 2011 - 2021

	Scheldestromen		**Hollandse Delta**	
	2050	*2100*	*2050*	*2100*
C [€]	235	442	190	329
f [%]	0.073	0.103	0.0406	0.0518

Table A.5: Results of local taxation factors in 2050 and 2100

	Dikering 29		**Dikering 17**	
	2050	*2100*	*2050*	*2100*
Central government	52.0	90.9	81.4	142.2
Local water authority	14.0	30.6	35.8	87.9
Total budget [mln. €]	66.0	121.5	117.2	230.1

Table A.6: Final results of available budget in 2050 and 2100 for dikering 17 and 29

A.3.2. Derivation of incurred costs as a result of losses

Determining the expected loss of life and economic damage has for a large part been done by the Ministry of Infrastructure and Water Management back in 2016 (Slootjes & Wagenaar, 2016). This was done by simulating a breach in the flood defence (often a dike), which resulted in an expected number of affected people, casualties and economic damage. These numbers were calculated for 2011 and 2050. Table A.7 provides some key figures about the two selected dikerings, including all the dike sections corresponding to the ring. These figures contain the expected economic damage, the loss of live and the total damage in 2011. All figures that are expressed in monetary units are in millions of euro's. Note that for the loss of live a value of €6.7 million per casualty was assumed. Within the column *Loss of live*, a cost of €12.500 was assigned to each affected individual.

Dike section	Length [km]	Costs of increasing safety level [€$\cdot km^{-1}$]	Economic damage [€]	Loss of live [€]	Total damage [€]
17-1	27.0	4.4	780	433	1.213
17-2	26.5	5.8	2.600	1.279	3.879
17-3	9.5	2.5	11.000	8.575	19.575
Total	*63*	*4.7**	*14.380*	*10.287*	*24.667*
29-1	22.0	5.8	2.100	359	2.459
29-2	17.0	6.5	3.300	2.355	5.655
29-3	7.0	6.3	5.300	14.770	20.070
29-4	12.5	1.0	69	6	75
Total	*58.5*	*5.0**	*10.700*	*17.484*	*28.184*

Table A.7: Expected damages in 2011

* *These values are the average cost of increasing the safety level per kilometre in millions of euro's over the entire length of the dikering.*

The analysis by Slootjes and Wagenaar (2016) provides estimates for the monetary value of those affected and for the casualties in 2050, as well as an estimate for the economic damage in 2050, shown in table D.1. These values were derived by multiplying the figures from 2011 by an assumed annual growth of 1.9% over 39 years. This growth is used for both the economical figures as well as the population growth and by extent to the fatalities. The calculations were made for 2011 and extrapolated to 2050, however no estimates for 2100 were provided. To this extent, similar growth figures are used to extrapolate the values in the year 2100. These figures are presented in table D.2.

Before using these figures, a quick check on the 1.9% figures was performed. Starting with the expected economic growth, the BBP changes for the past 25 years are plotted and averaged, shown in figure A.16.

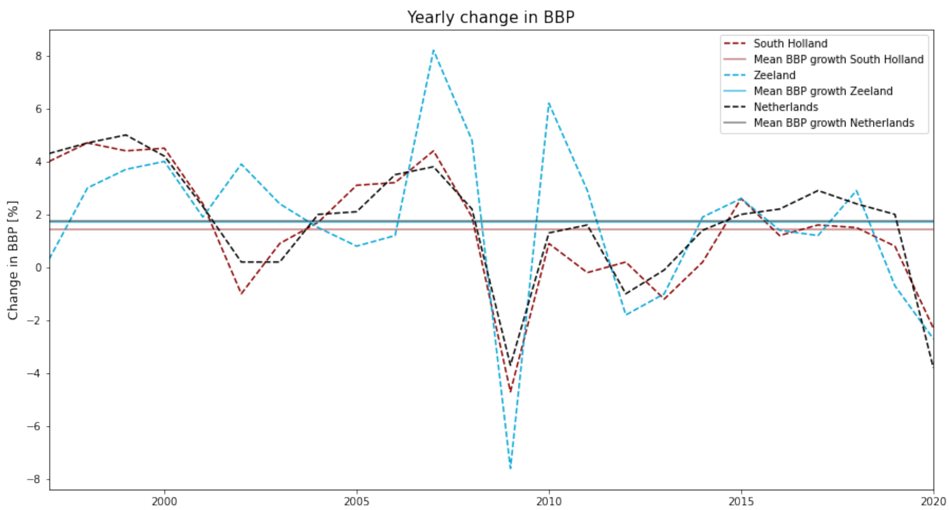


Figure A.16: Change in BBP for South Holland, Zeeland and the Netherlands

Which appear to average out around the value of 1.9%. Hence this value is assumed to be a plausible figure for the growth rate of asset value θ. However, as has been explained in the section *Available budget*, the population growth per year is around 59.000 people per year. Solving an exponential growth model for the growth rate exactly, it is found that the population growth rate (γ) is around 0.30% (0.2991). As this value is significantly different from the proposed value by Slootjes and Wagenaar, 2016, the values for population growth are adjusted. It is assumed that the value of the loss of live scales linearly with the growth or decline in population. The results for 2050 and 2100 are shown in table D.4.

B Extension of the model - Spatial Domain

For the spatial model, the first chosen budget constraint will be introduced to the framework derived in this report. However, the framework is now adjusted to minimize over the length of the dike section L_{dike}, rather than the temporal domain.

The starting points for this derivation are equation A.8 and A.9. To find an expression for the marginal revenue and marginal costs, the partial derivative should be changed from $\partial(\Delta h)$ to $\partial L_{dike,n}$. Such that:

$$MR = \frac{\partial TR}{\partial L_{dike,n}} \quad \text{and} \quad MK = \frac{\partial TK}{\partial L_{dike,n}}$$

Plugging in the derivation and simplifying yields the marginal revenue with respect to the size of a polder as given below.

$$MR = \begin{cases} B' & \text{for SLR} \geq \Delta h \\ B' + \frac{L_{dike,n}}{2\pi}(\Delta \mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta \mathbb{P}(fatality) \cdot P \cdot V_{live}) & \text{for SLR} < \Delta h \end{cases}$$

where:

$$B' = \frac{L_{dike,n}}{2\pi}(T_{fixed} + T_{variable} \cdot V_{\text{woz}}) + \frac{2 \cdot L_{dike,n} \cdot (L_{total} - L_{dike,n})}{(L_{total})^3} \cdot B_{subsidy} \quad \text{for k} \neq \text{n}$$

A similar derivation can be made for the marginal costs, the result of which is shown below.

$$MK = \begin{cases} \frac{L_{dike,n}}{2\pi}(\Delta \mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta \mathbb{P}(fatality) \cdot P \cdot V_{live}) + K_{dike} & \text{for SLR} \geq \Delta h \\ K_{dike} & \text{for SLR} < \Delta h \end{cases}$$

Taking the limit case for the inequality between the marginal costs and benefits, the equality is chosen. Hence the two derived expressions should be set equal to one another to find the limit for $L_{dike,n}$, given the other parameters. This results in two expressions for the maximum length of a dike section L_{dike}.

$$\begin{aligned} & \min_{\forall L \in [0, L_{max}]} \quad C_L(\Delta h) \\ \text{s.t.} \quad & \frac{\partial C_L}{\partial(\Delta L_{dike,n})} \leq \frac{\partial TR_L}{\partial(\Delta L_{dike,n})} \quad \forall \Delta h \in [0, \Delta h_{max}] \\ \text{where} \quad & C_L(\Delta h) = e^{\alpha \cdot (H + \Delta h)} \cdot (V_{asset} + V_{live}) + L_{dike} \cdot K_{dike} \cdot \Delta h \end{aligned} \tag{B.1}$$

The first line in equation B.1 displays the minimization problem of the total costs with respect to the chosen crest height increase Δh. The second line displays the constraint that is being put on the minimization problem, namely that of the marginal revenue needing to equal the marginal costs. The marginals in this case are with respect to the length of the dike section n, $L_{dike,n}$. The final line displays the given equation for the total costs in consideration, dependent on the flood risk and physical strengthening of the dike section.

For the given expressions of the marginal revenue and marginal costs, it is possible to derive an explicit formulation for the minimal dike length $L_{dike,n}$. This is given as:

$$L_{dike,n} = \frac{\sqrt{(A + 2 \cdot B \cdot y - C)^2 - 8 \cdot B \cdot D} + A + 2 \cdot B \cdot y - C}{4 \cdot B}$$

Where:

$A = \frac{T_{fixed} + T_{variable} \cdot V_{\text{WOZ}}}{2\pi}$

$B = \frac{B_{subsidy}}{(L_{total})^3}$

$C = \frac{\Delta\mathbb{P}(asset) \cdot H \cdot V_{asset} + \Delta\mathbb{P}(fatality) \cdot P \cdot V_{live}}{2\pi}$

$D = K_{dike}$

$y = L_{total}$

C
Figures

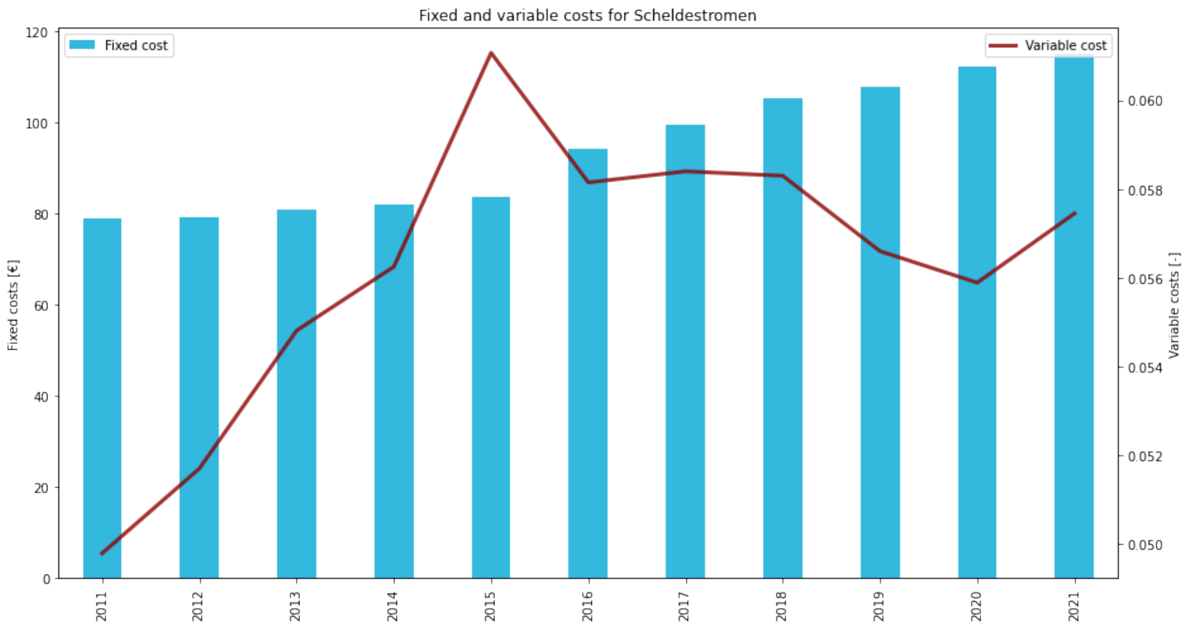


Figure C.1: Fixed and variable costs for Scheldestromen from the period 2011 - 2021

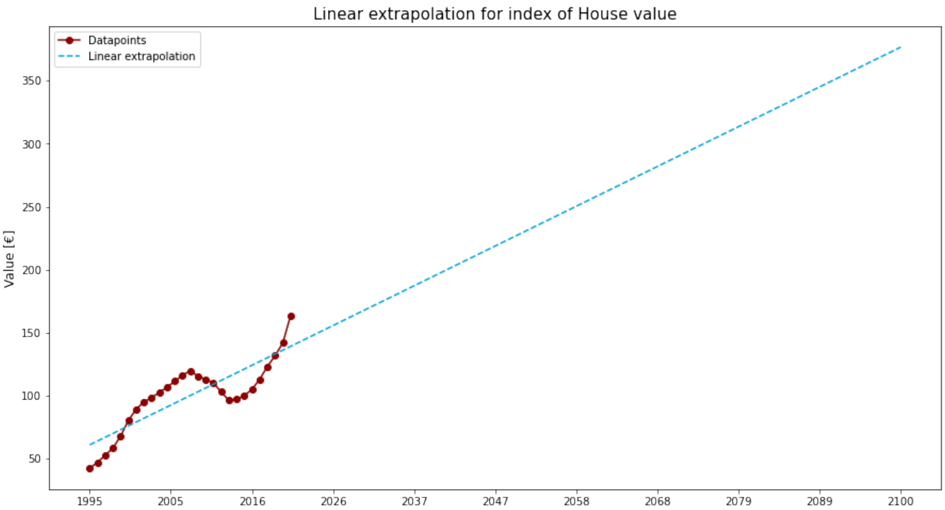


Figure C.2: Linear extrapolation for the index of house values in 2100

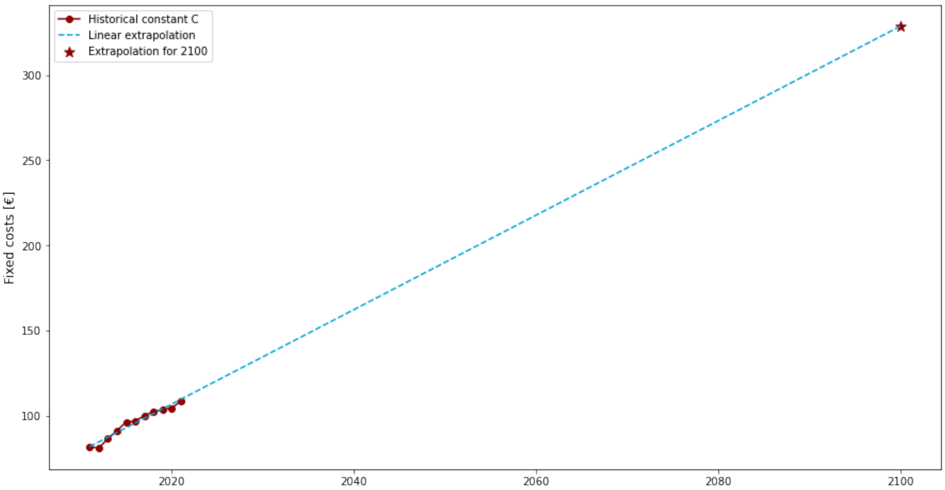


Figure C.3: Extrapolation for constant C in taxation for Hollandse Delta

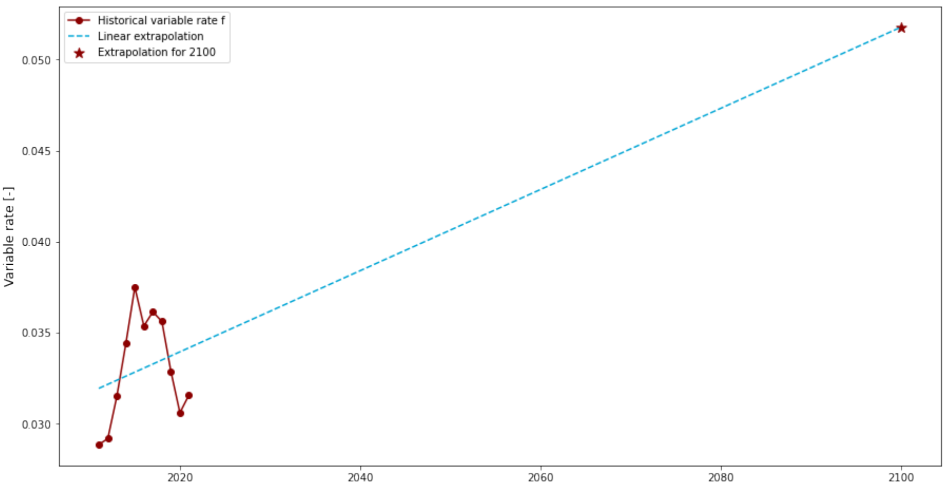


Figure C.4: Extrapolation for variable f in taxation for Hollandse Delta

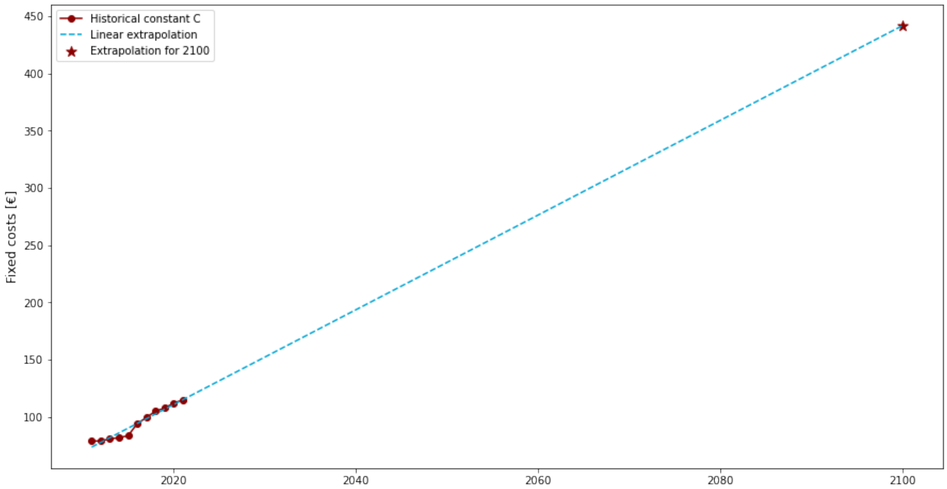


Figure C.5: Extrapolation for constant C in taxation for Scheldestromen

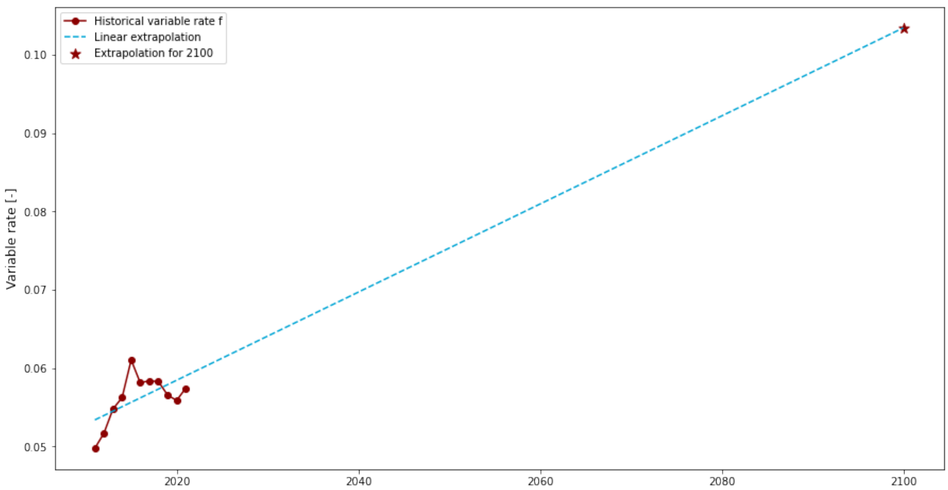


Figure C.6: Extrapolation for constant f in taxation for Scheldestromen

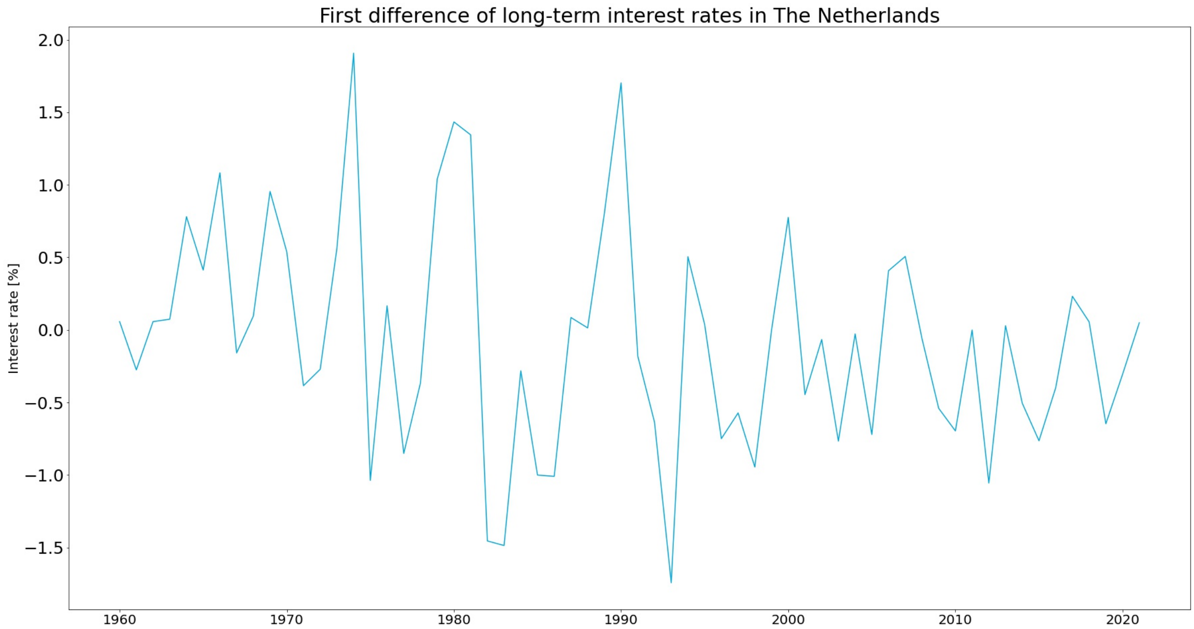


Figure C.7: First difference of long-term interest rate in the Netherlands

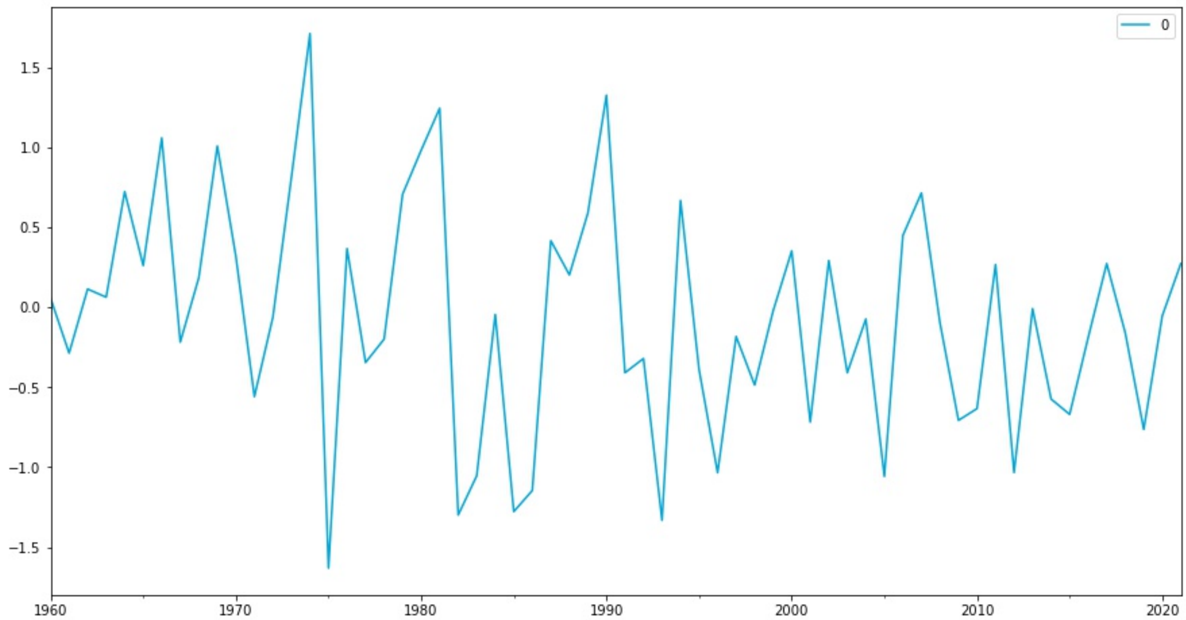


Figure C.8: Line graph of the residuals of the interest rates

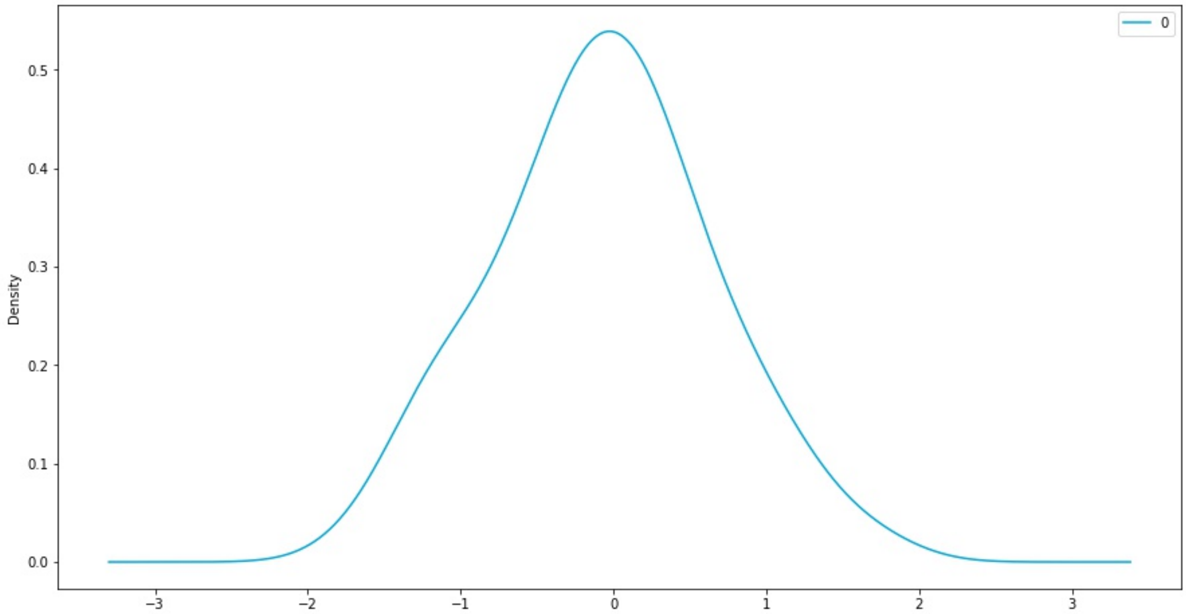


Figure C.9: Probability density function of the residuals of the interest rates

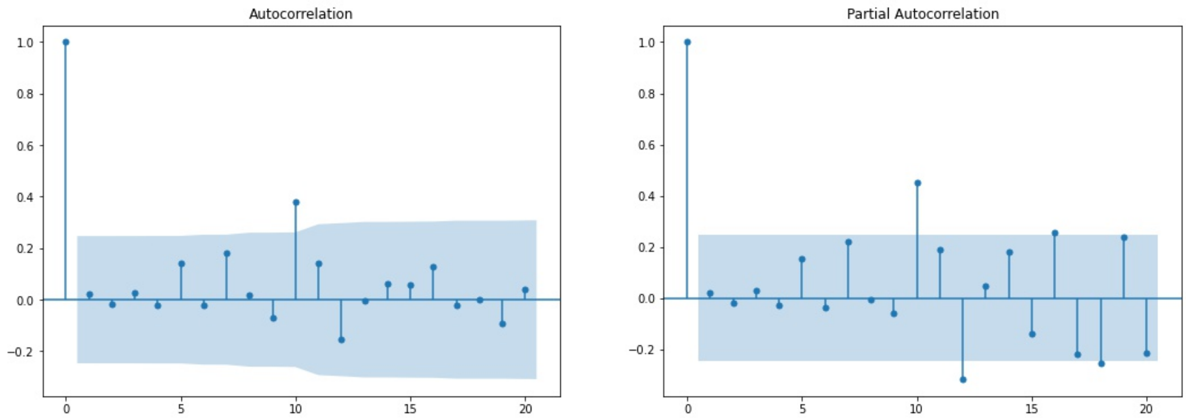


Figure C.10: (Partial) Autocorrelations of residuals of the interest rates

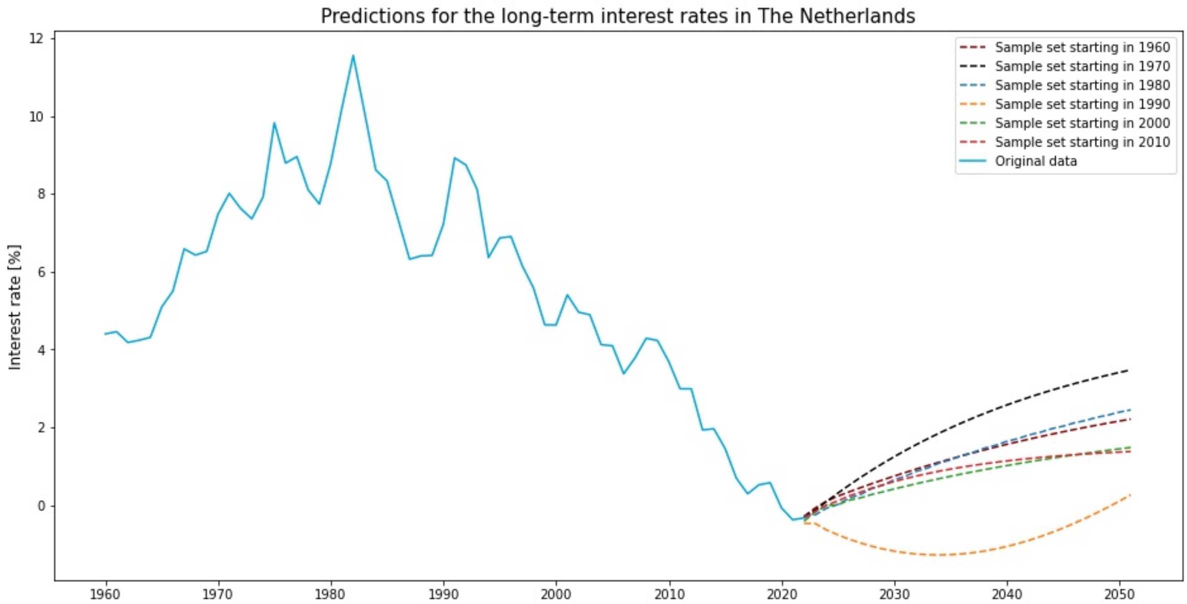


Figure C.11: ARMA(2,3) predictions for the interest rate in 2050 using multiple data-slices

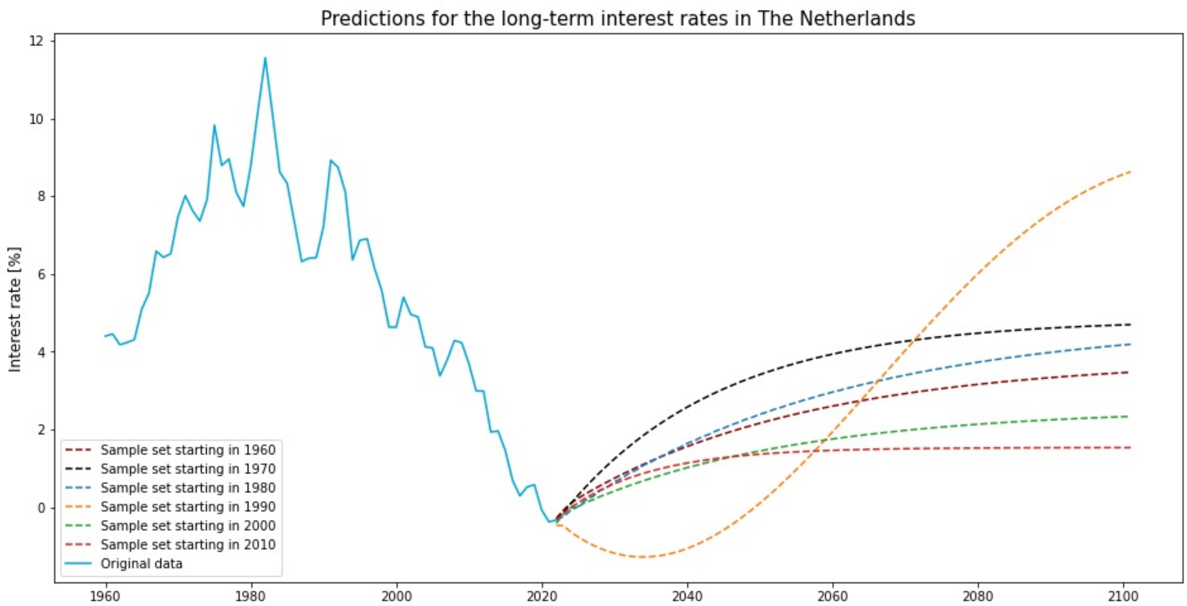


Figure C.12: ARMA(2,3) predictions for the interest rate in 2100 using multiple data slices

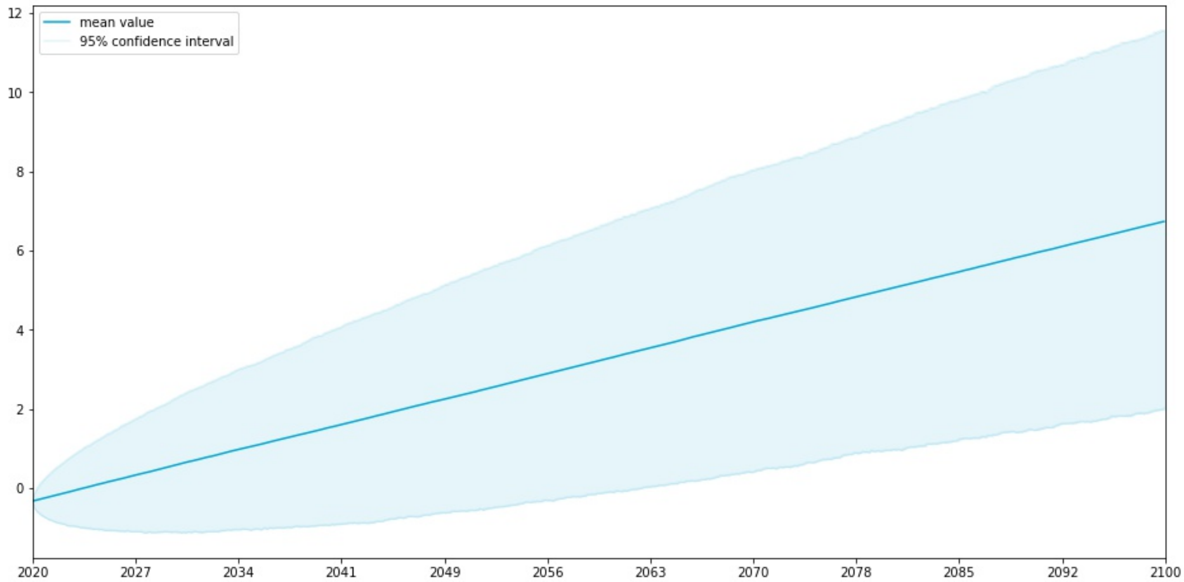


Figure C.13: Average of 10.000 sample BM for an eighty year period

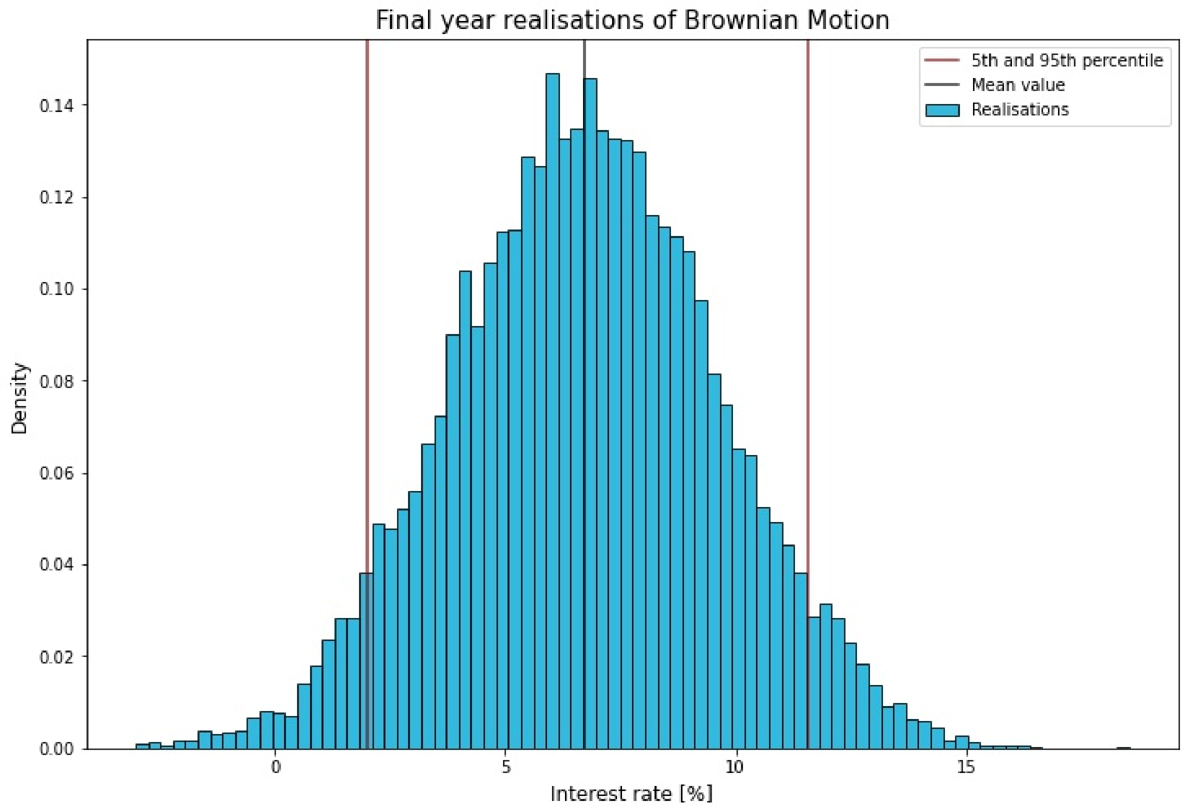


Figure C.14: Histogram of final year of 10.000 sample BM for an eighty year period

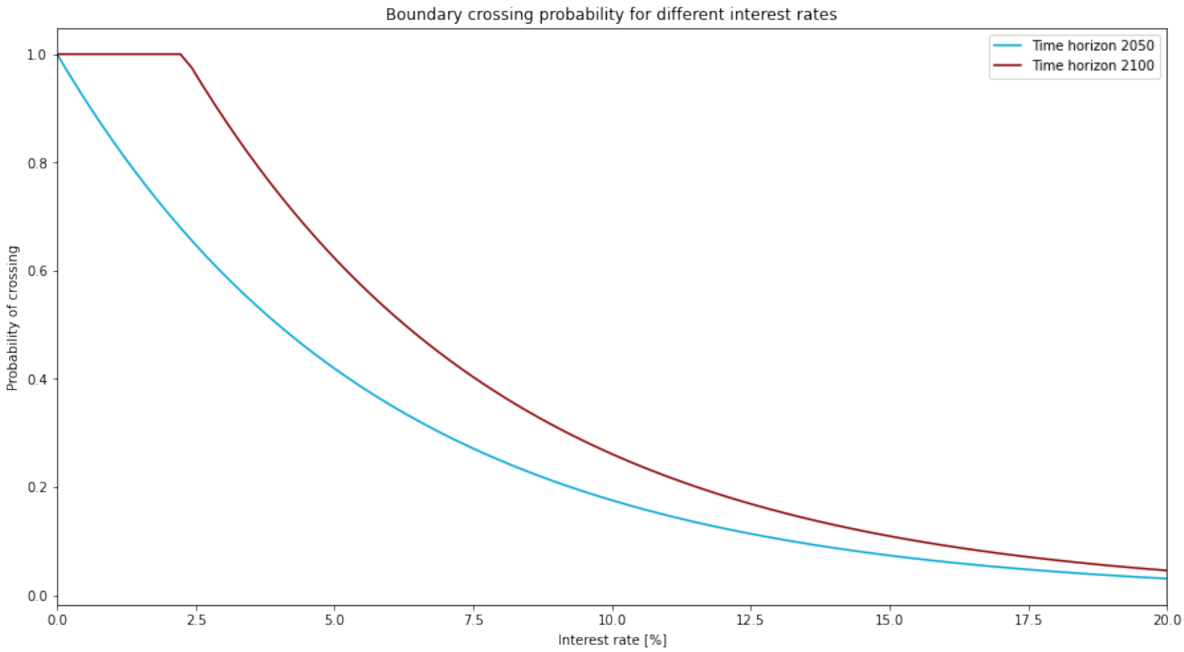
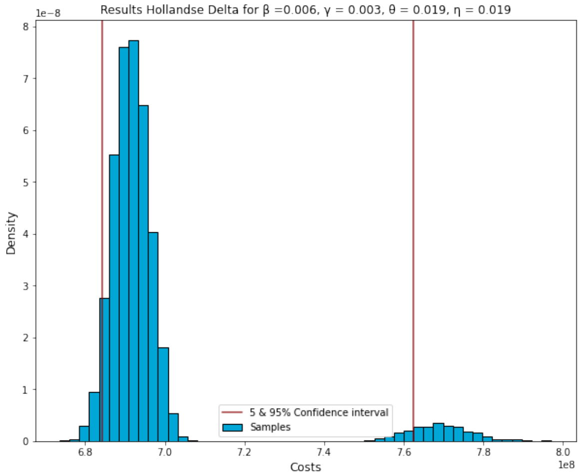
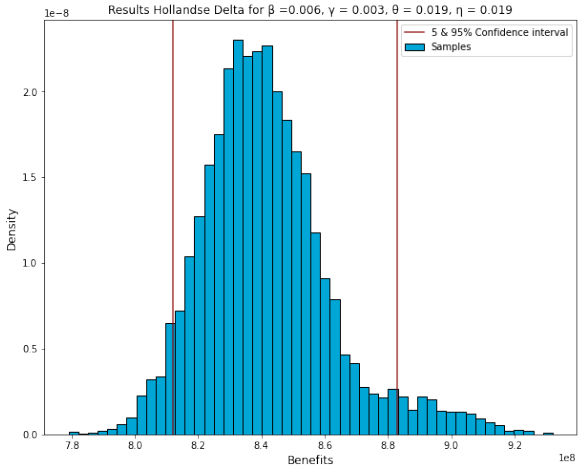


Figure C.15: Interest rate exceedance probabilities for 2050 and 2100

Figure C.16: Historical long-term interest rates in the Netherlands

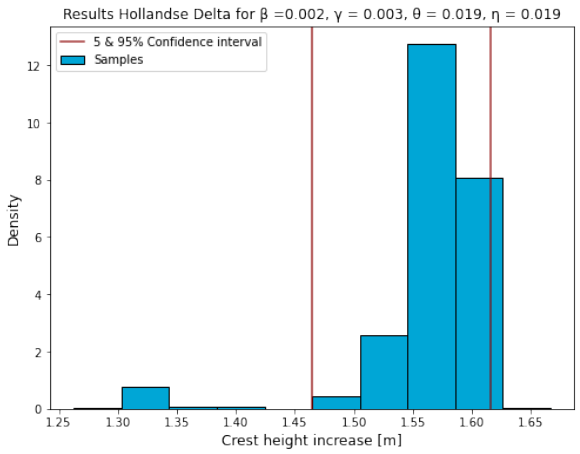


(a) Discounted costs

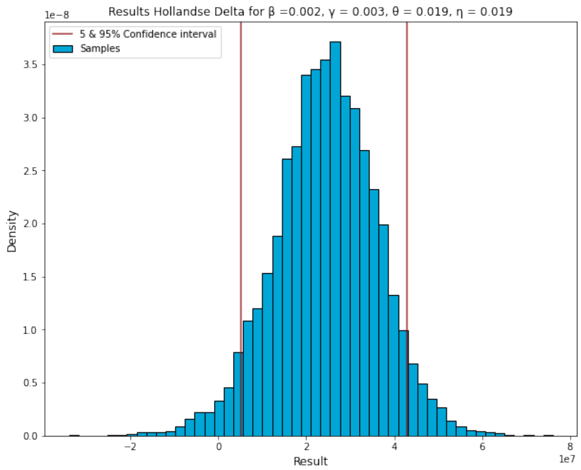


(b) Discounted benefits

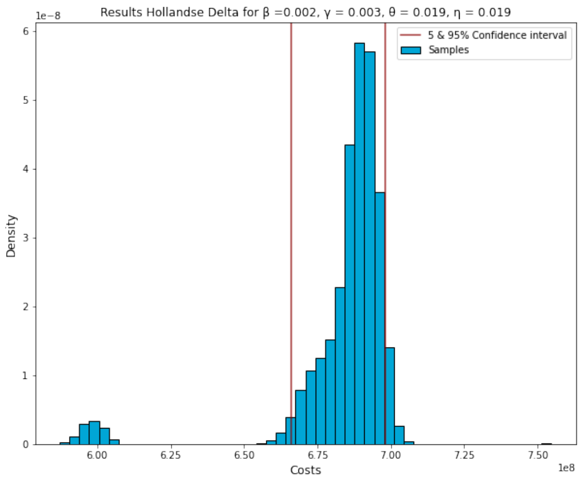
Figure C.17: Monte Carlo results for costs and benefits for Hollandse Delta with β = 0.006



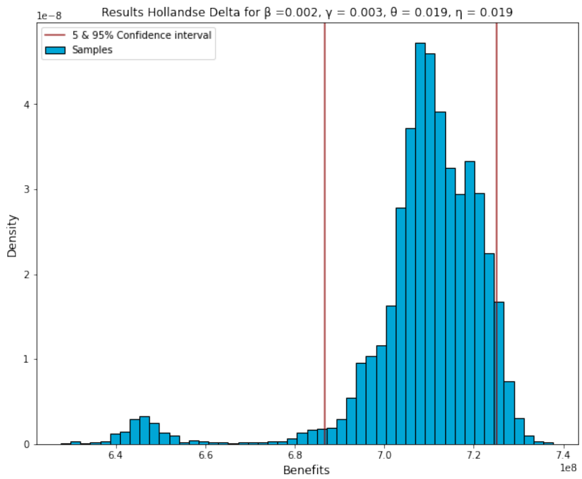
(a) Crest height increase



(b) Net result of CBA

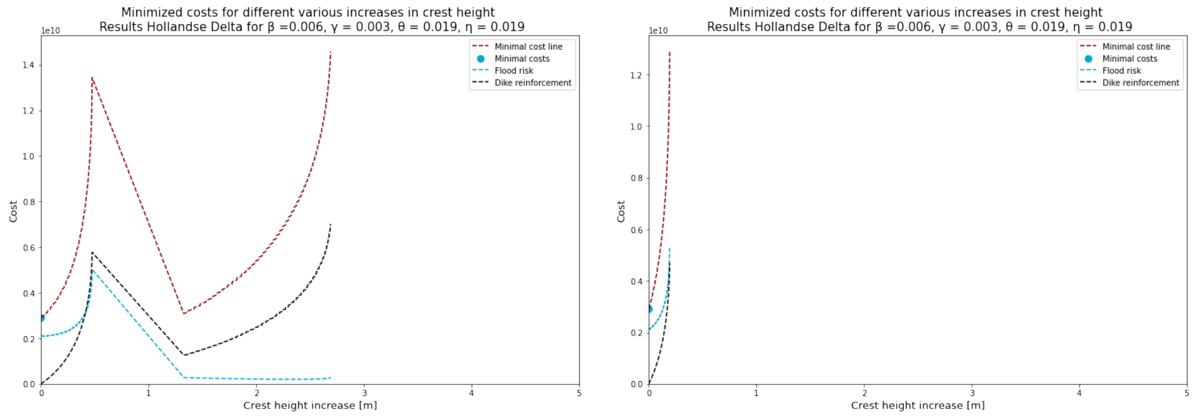


(c) Discounted costs



(d) Discounted benefits

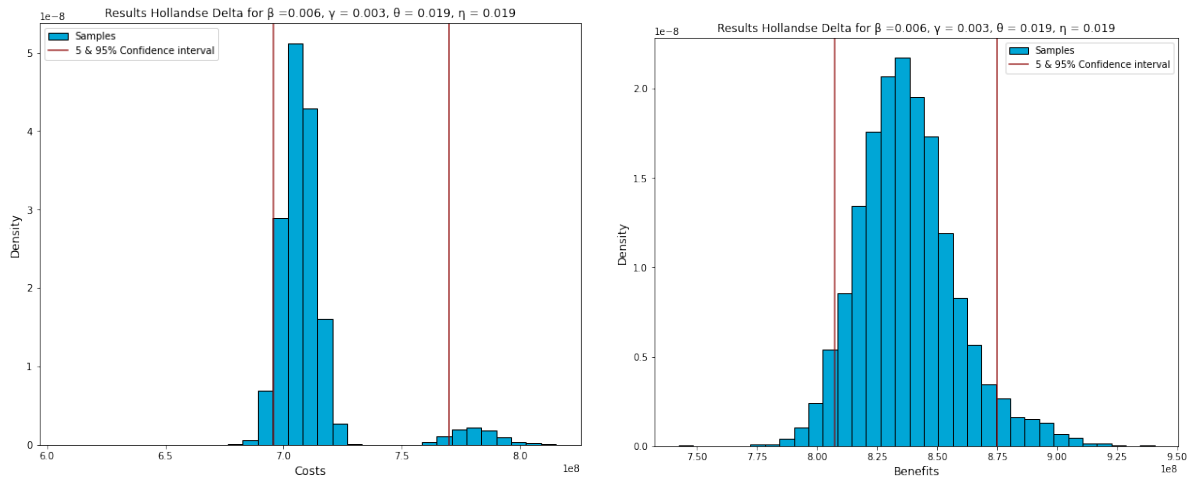
Figure C.18: Monte Carlo results for costs and benefits for Hollandse Delta with β = 0.002



(a) Cost minimization graph for lower K_{dike}

(b) Cost minimization graph for higher K_{dike}

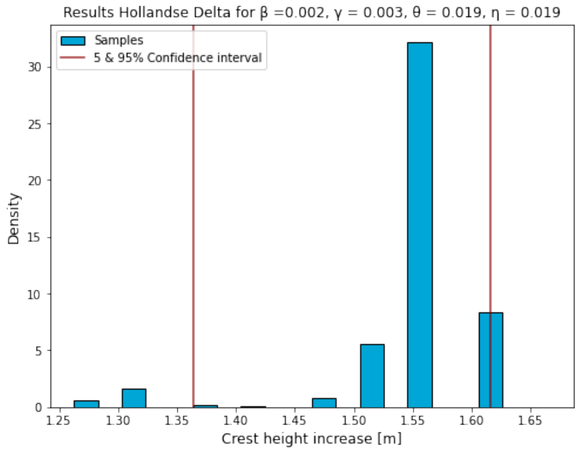
Figure C.19: Minimized costs for two cases where K_{dike} is varied



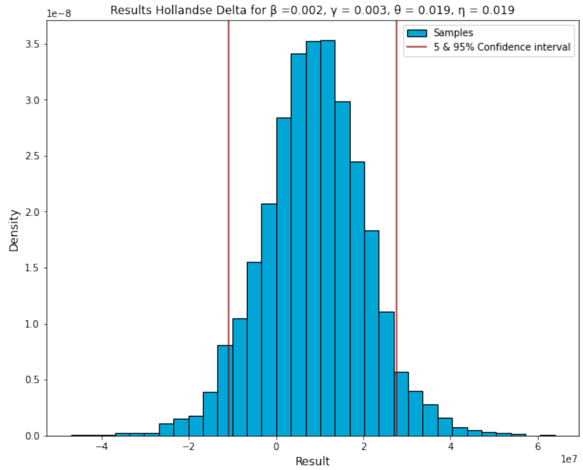
(a) Discounted costs

(b) Discounted benefits

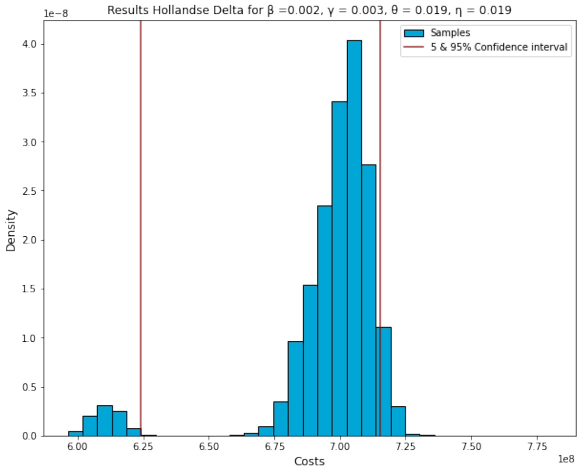
Figure C.20: Monte Carlo results for costs and benefits for Hollandse Delta with β = 0.006, using a Brownian Motion for the discount rate



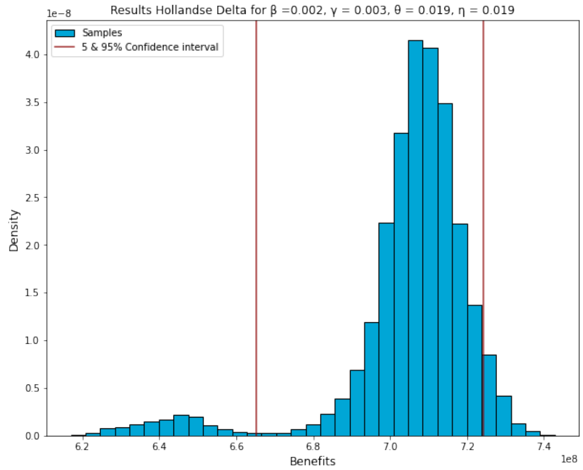
(a) Crest height increase



(b) Net result of CBA

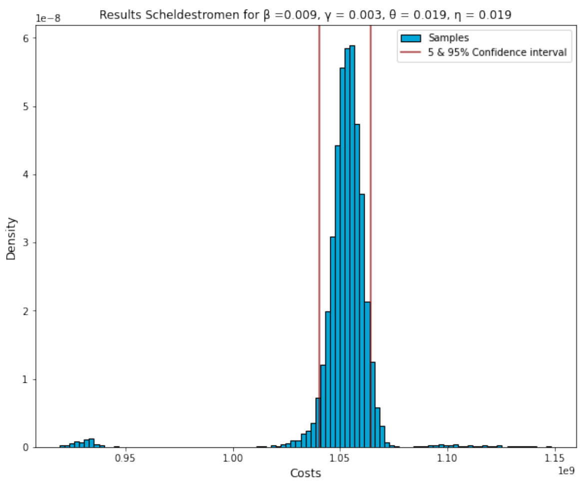


(c) Discounted costs

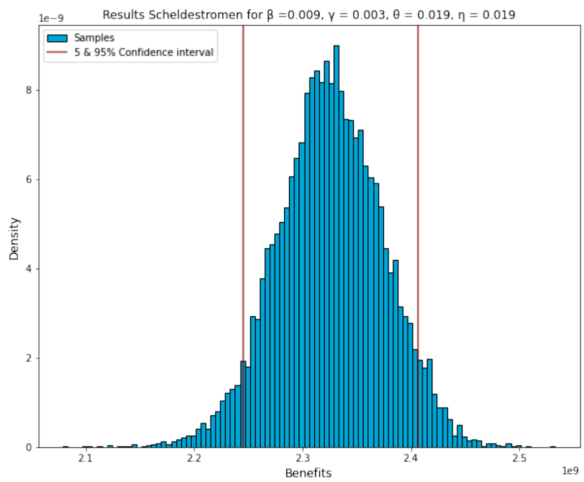


(d) Discounted benefits

Figure C.21: Monte Carlo results for costs and benefits for Hollandse Delta with β = 0.002, using a Brownian Motion for the discount rate

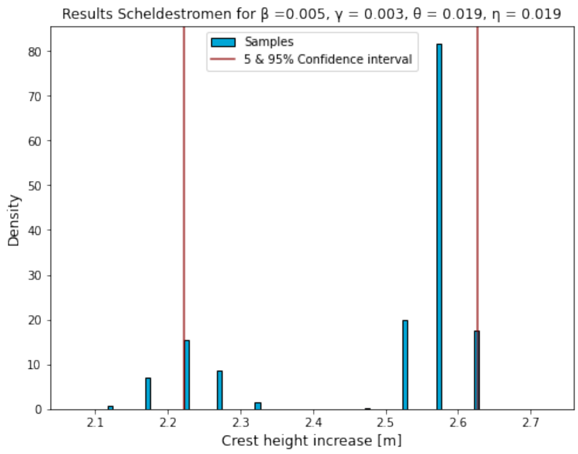


(a) Discounted costs

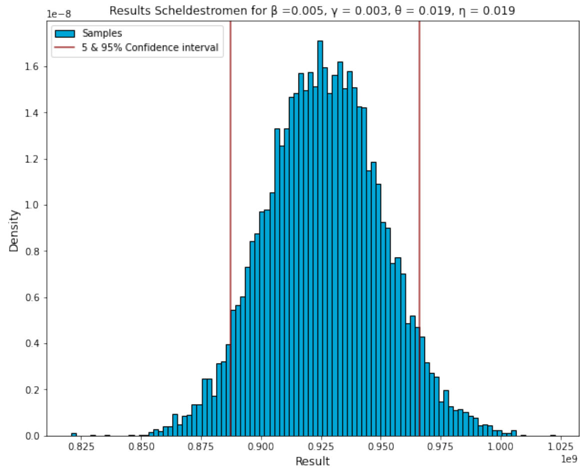


(b) Discounted benefits

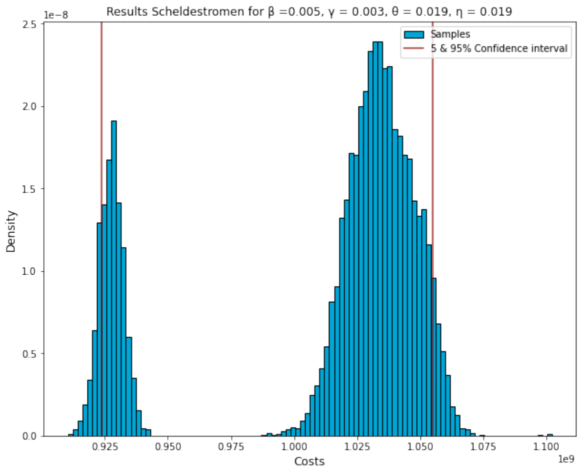
Figure C.22: Monte Carlo results for costs and benefits for Scheldestromen with β = 0.009



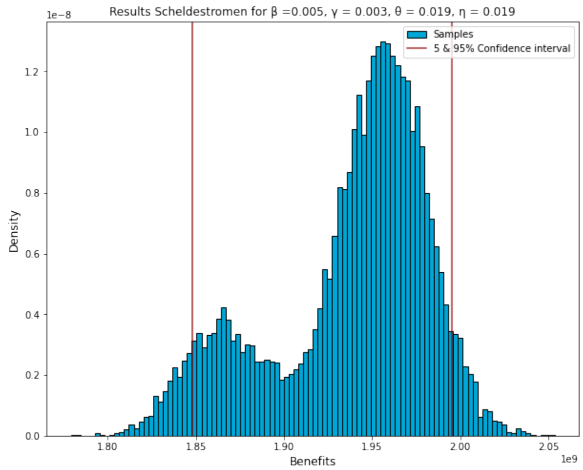
(a) Crest height increase



(b) Net result of CBA

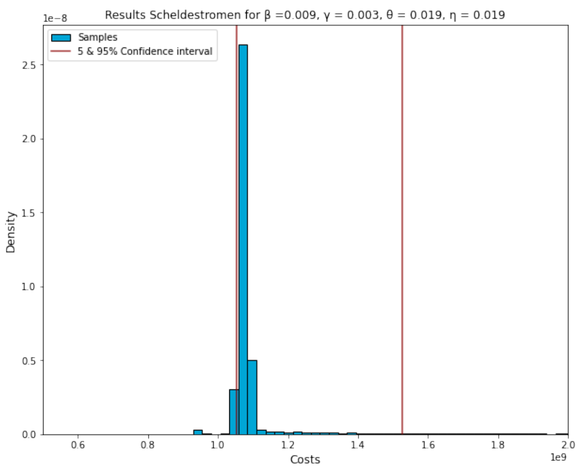


(c) Discounted costs

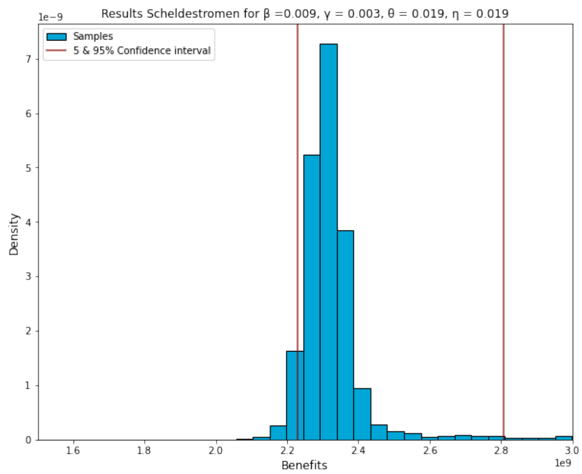


(d) Discounted benefits

Figure C.23: Monte Carlo results for costs and benefits for Scheldestromen with β = 0.005

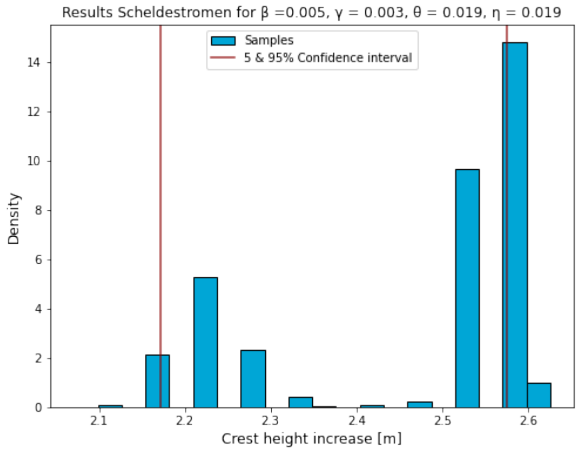


(a) Discounted costs

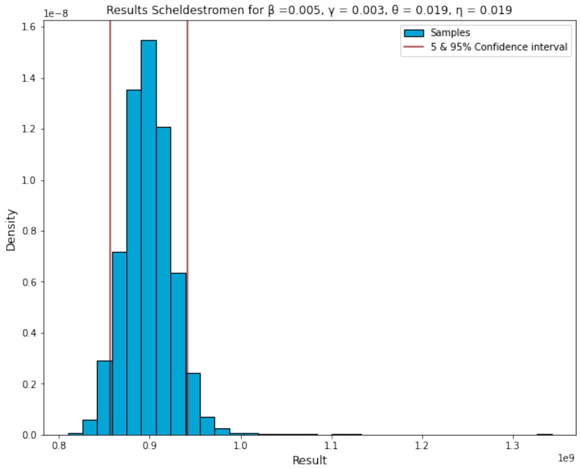


(b) Discounted benefits

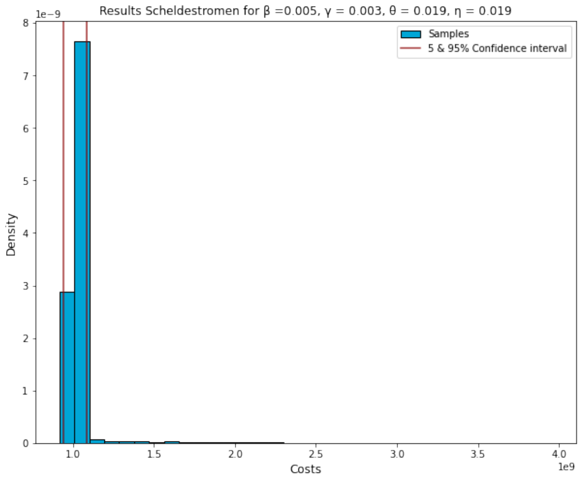
Figure C.24: Monte Carlo results for costs and benefits for Scheldestromen with β = 0.009, using a Brownian Motion for the discount rate



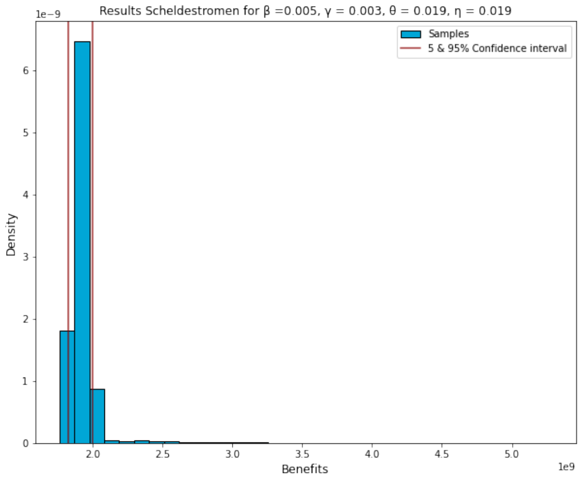
(a) Crest height increase



(b) Net result of CBA



(c) Discounted costs



(d) Discounted benefits

Figure C.25: Monte Carlo results for costs and benefits for Scheldestromen with β = 0.005, using a Brownian Motion for the discount rate

D
Tables

Dike section	Economic damage [€]	Loss of live [€]	Total damage [€]
17-1	1.600	900	2.500
17-2	5.400	2.700	8.100
17-3	22.000	19.000	41.000
Total	*29.000*	*22.600*	*51.600*
29-1	4.500	800	5.200
29-2	7.000	5.000	12.000
29-3	11.000	30.000	41.000
29-4	143	14	157
Total	*22.643*	*35.814*	*58.357*

Table D.1: Expected damages in 2050

Dike section	Economic damage [€]	Loss of live [€]	Total damage [€]
17-1	4.100	2.307	6.407
17-2	13.839	6.919	20.758
17-3	56.381	48.693	105.073
Total	*74.320*	*57.919*	*132.239*
29-1	11.532	2.050	13.326
29-2	17.939	12.814	30.573
29-3	28.190	76.883	105.073
29-4	367	36	402
Total	*58.029*	*91.783*	*149.555*

Table D.2: Expected damages in 2100

Parameter	Value	Standard deviation	p-value
φ_1	-1.19	0.17	0.000
φ_2	-0.73	0.13	0.000
θ_1	1.65	5.54	0.767
θ_2	1.54	11.91	0.897
θ_3	0.45	4.20	0.915

Table D.3: Parameter estimation of ARMA(2,3) model

			2050			2100
Dike section	**Economic damage [€]**	**Loss of live [€]**	**Total damage [€]**	**Economic damage [€]**	**Loss of live [€]**	**Total damage [€]**
17-1	1.600	486	2.086	4.100	565	4.665
17-2	5.400	1.437	6.837	13.839	1.668	15.507
17-3	22.000	9.634	31.634	56.831	11.185	68.016
Total	*29.000*	*11.558*	*40.558*	*74.320*	*13.419*	*87.739*
29-1	4.500	403	4.903	11.532	468	12.000
29-2	7.000	2.646	9.646	17.939	3.072	21.011
29-3	11.000	16.594	27.594	28.190	19.267	47.457
29-4	143	7	150	367	8	375
Total	*22.643*	*19.644*	*42.287*	*58.029*	*22.807*	*80.836*

Table D.4: Corrected expected damages in 2050 and 2100

E

Programming code

This chapter contains several of the written programming codes that were vital for the assesment. The codes listed contain: The derivation of the stringency, the generation of the discount rate via a Brownian Motion and the final optimization code as per the framework derived in this research.

The first code presented was used to generate the derived stringency plots.

```
#Scenario 1, marginal costs and marginal benefits

def MK(dh, k_values):
    "Equation"
    if k_values == "negative":
        K = -P * L**2 * ((((np.exp(-(dh -1)) * H * V_asset + np.exp(-(dh-1))*P*V_live) * (-(dh-1))) / 4* np.pi)
        + dh*2*y + (dh*x/h) + 2*y*h + x+ b + 0.5*d)
    if k_values == "positive":
        K = (P * L**2 *dh*2*y + (dh*x/h) + 2*y*h + x+ b + 0.5*d)

    return K

def MR(dh, k_values):
    if k_values == "positive":
        R = np.ones(len(dh))
    if k_values == "negative":
        R = P * L**2 * (((np.exp(-(dh -1)) * H * V_asset + np.exp(-(dh-1))*P*V_live) * (1-dh)) / 4* np.pi)
    return R

def Net(dh, K_values):
    return MR(dh, K_values) - MK(dh, K_values)


dh = np.linspace(0,10,10000)
k_values = "positive"

plt.figure(figsize=(30,16))

plt.plot(dh, MK(dh, k_values) ,label = "Marginal costs", color = TU_red, linewidth = 3) #Blauw
plt.plot(dh, MR(dh, k_values) ,label = "Marginal revenue",  color = TU_blue, linewidth = 3) #Rood
plt.plot(dh, Net(dh, k_values) ,label = "Net benefits", color = TU_black, ls = "--", linewidth = 3)
plt.title("Marginal revenue and costs for positive K", size = 25)
plt.xlabel("Δh [m]", size = 22)
plt.xlim(0,10)
#plt.yscale("log")
plt.legend(fontsize = 18)
plt.yticks(np.linspace(5*10**7, 0,2), labels = ["Value €[]", 0], size = 22)
plt.xticks(np.linspace(0,10,6), size = 20)
```

```
plt.savefig("scenario1_positive.jpg", bbox_inches = 'tight')
plt.show();

# Scenario 2, total revenue and total costs are equal

#Define the total revenue

def TR(dh, SLR):
    B = ((L**2 /(4*np.pi)) * (T_fixed + T_variable * V_asset)) + (L / L_total)**2 * B_subsidy
    K = np.zeros(len(dh))
    for i in range(len(dh)):
        if dh[i] < SLR:
            K[i] = B
        elif dh[i] >= SLR:
            K[i] = B + ((L**2)/(4*np.pi)) * (np.exp(dh[i] - SLR) -1)*(IR_0a * H * V_asset +
                IR_0f * P * V_live)
    return K

#Define total costs

def TK(dh, SLR):
    K = np.zeros(len(dh))
    for i in range(len(dh)):
        K_dike = Price * (((dh[i]/h)**2 + 2*dh[i]/h)*(y*h**2 + 0.5*x*h) +(dh[i]/h)*(b*h +
            0.5*d*h))
        if dh[i] < SLR:
            K[i] = (L**2 / (4*np.pi)) * (np.exp(SLR - dh[i])-1)*(IR_0a * H * V_asset + IR_0f
                * P * V_live + K_dike * (4*np.pi/L))
        elif dh[i] >= SLR:
            K[i] = K_dike * L
    return K

dh = np.linspace(0,10,1000)

plt.figure(figsize=(30,16))
plt.plot(dh, TK(dh, SLR) ,label = "Total costs", color = TU_red, linewidth = 3) #Blauw
plt.plot(dh, TR(dh, SLR) ,label = "Total revenue",  color = TU_blue, linewidth = 3) #Rood
plt.plot(dh, TR(dh,SLR) - TK(dh,SLR) ,label = "Net benefits", color = TU_black, ls = "--",
    linewidth = 3)
plt.title("Total revenue and total costs", size = 25)
plt.xlabel("Δh [m]", size = 22)
#plt.ylabel("Value €[]")
plt.yscale("log")
plt.legend(fontsize = 18)
plt.xlim(0,10)
plt.ylim(4*10**7, 10**13)
plt.yticks(np.linspace(10**12, 10**9,1), labels = ["Value €[]"], size = 22)
plt.xticks(np.linspace(0,10,6), size = 20)

#plt.savefig("scenario2_final.jpg", bbox_inches = 'tight')
plt.show();

#Scenario 3, expected damage and costs of increasing safety
def E(dh):
    result = ((L**2 * np.exp(SLR))/(4*np.pi)) * (IR_0f*P*V_live + IR_0a * H * V_asset) * np.
        ones(len(dh))
    return result

def K(dh):
    result = L * Price * (dh**2 * y + 2*dh*y*h + (dh**2 * x)/(2*h) +dh*x + dh*b + (dh*d/2) )
    return result

def total(dh):
    return E(dh) - K(dh)

dh = np.linspace(0,10,1000)
plt.figure(figsize=(30,16))

plt.plot(dh,E(dh), label = "Expected damages", color = TU_blue, linewidth = 3)
plt.plot(dh,K(dh), label = "Reinforcement costs", color = TU_red, linewidth = 3)
plt.plot(dh,total(dh), label = "Net benefits", color = TU_black, ls = "--", linewidth = 3)
```

```
plt.title("Expected damages and reinforcement costs", size = 25)
plt.xlabel("Δh [m]", size = 22)
#plt.ylabel("Value €[]")
plt.xlim(0,10)
#plt.ylim(10**8,1*10**10)
plt.yscale("log")
plt.legend(fontsize = 18)
plt.yticks(np.linspace(1*10**9, 10**10,1), labels = ["Value €[]"], size = 22)
plt.xticks(np.linspace(0,10,6), size = 20)
#plt.axvline(3.6)
#plt.savefig("scenario4_final.jpg", bbox_inches = 'tight')
plt.show();

#Scenario 4

T_fixed = 90
T_variable = 0.000343
L_total = 90
B_subsidy =  140* 10**7

#Define total revenue
def TR_4(dh, SLR):
    B = ((L**2 /(4*np.pi)) * (T_fixed + T_variable * V_asset)) + (L / L_total)**2 * B_subsidy
    K = B*np.ones(len(dh))
    return K

#Define total costs
def TK_4(dh, SLR):
    K_dike = Price * (((dh/h)**2 + 2*dh/h)*(y*h**2 + 0.5*x*h) +(dh/h)*(b*h + 0.5*d*h))
    K = K_dike * L
    return K

dh = np.linspace(0,SLR,1000)

plt.figure(figsize=(30,16))
plt.plot(dh, TK_4(dh, SLR) ,label = "Total costs", color = TU_red, linewidth = 3) #Blauw
plt.plot(dh, TR_4(dh, SLR) ,label = "Total revenue",  color = TU_blue, linewidth = 3) #Rood
plt.plot(dh, TR_4(dh,SLR) - TK_4(dh,SLR) ,label = "Net benefits", color = TU_black, ls = "--"
    , linewidth = 3)
plt.title("Total revenue and costs for constant individual risk", size = 25)
plt.xlabel("Δh [m]", size = 22)
#plt.ylabel("Value €[]")
plt.yscale("log")
plt.legend(fontsize = 18)
plt.xlim(0,1)
plt.ylim(10**7, 8* 10**8)
plt.yticks(np.linspace(3*10**8, 10**8,1), labels = ["Value €[]"], size = 22)
plt.xticks(np.linspace(0,1,6), size = 20)
plt.axvline(0.36)
plt.savefig("scenario10_final.jpg", bbox_inches = 'tight')
plt.show();
```

The code for generating a Brownian Motion is a modified version of the Brownian() Class by Sarkar, 2020. The modifications are shown in the code below.

```
#Function definitions
def interest_rate(
                    self,
                    r0 = 2.5,
                    mu=0.2,
                    sigma=0.68,
                    deltaT=52,
                    dt=0.1,
                    ):
        """
        Models the interest rate r(t) using the Wiener process W(t) as
        r(t) = mu*t +sigma.W(t)

        Arguments:
            mu: 'Drift' of the interest rate (upwards or downwards), default 1
            sigma: 'Volatility' of the interest rate, default 1
            deltaT: The time period for which the future prices are computed, default 52 (as
                in 52 weeks)
            dt (optional): The granularity of the time-period, default 0.1
            start: Starting value of the interest rate

        Returns:
            s: A NumPy array with the simulated interest rate prices over the time-period
                deltaT
        """
        n_step = int(deltaT/dt)
        time_vector = np.linspace(0,deltaT,num=n_step)
        # Forcefully set the initial value to r0
        self.x0 = 0
        # Weiner process (calls the `gen_normal` method)
        weiner_process = sigma*self.gen_normal(n_step)
        r =  r0+ mu*time_vector + sigma*self.gen_normal(n_step)

        return r

def plot_interest_rates(mu,sigma, r0, time):
    """ Plots interest rates for multiple scenarios  """
    x = np.linspace(2020,2050,6)
    label = []
    for i in range(len(x)):
        label.append(int(x[i]))

    values = []
    plt.figure(figsize=(15,8))
    for i in range(3):
        plt.title("Development of interest rate")#\n mu =" +str(mu), "\nsigma = " +str(sigma)
            )
        plt.ylabel("Interest rate [%]")
        plt.xlabel("Time [years]")
        b_interest = b.interest_rate(r0 = r0,
                                 mu=mu,
                               sigma=sigma,
                                 deltaT = time,
                               dt=1/12)
        values.append(b_interest)
        plt.plot(b_interest, color = color[i])
    plt.legend(['Sample run '+str(i) for i in range(1,6)],
               loc='upper left')
    plt.xticks(np.linspace(0,360,6), labels = label)
    textstr = '\n'.join((
    r' $Parameters$' ,
    r' $\mu = %.2f$' % (mu,),
    r' $\sigma = %.2f$' % (sigma, )))
    props = dict(boxstyle = ' round', facecolor = 'white', alpha = 0.5)

#Calculations
#2100 year prediction
n = 10000
```

```
time = 80
dt = 1/12

values = []
means = []
var = []
b_interest = 0
mean_final = []
for i in range(n):
    b_interest = b.interest_rate(r0 = -0.328083,
                                  mu=mean/length,
                                sigma=sigma,
                                  deltaT = time,
                                dt=dt)
    values.append(b_interest)
    means.append(np.mean(b_interest))
    var.append(np.var(b_interest))
    mean_final.append(b_interest[-1])

reshape = np.reshape(values, (n, int(time/dt)))

x = np.linspace(2020,2050,6)
label = []
for i in range(len(x)):
    label.append(int(x[i]))


y = np.linspace(0, int(time/dt), int(time/dt))
plt.xticks
avg = np.mean(reshape,axis = 0)
var = np.var(reshape, axis = 0)

lower = np.quantile(reshape , 0.05, axis = 0)
upper = np.quantile(reshape, 0.95, axis = 0)

plt.figure(figsize = (16,8))
plt.xlim(0, int(time/dt))
plt.xticks(np.linspace(0,int(time/dt),6), labels = label)
plt.plot(avg, color= TU_blue, label = "mean value")
plt.plot(lower, color= TU_blue, alpha = 0.1, label = "95% confidence interval")
plt.plot(upper, color= TU_blue, alpha = 0.1)
plt.fill_between(y, lower, upper, color=TU_blue, alpha=0.1)
plt.legend();

print("Average value of the mean after", n , "runs =", np.mean(means))
print("Average value of the variance after", n , "runs =", np.mean(var))
print("Average value of the 2050 value after", n , "runs =", np.mean(mean_final))
plt.figure(figsize = (12,8))
plt.hist(mean_final, color = TU_blue, bins = 80, alpha = 0.8, density = True, edgecolor = TU_black, label = "Realisations")
p5 = np.quantile(mean_final, 0.05)
p95 = np.quantile(mean_final, 0.95)
p50 = np.mean(mean_final)
plt.axvline(p5, color = TU_red, alpha = 0.8, label = "5th and 95th percentile")
plt.axvline(p95, color = TU_red,alpha = 0.8)
plt.axvline(p50, color = TU_black, alpha = 0.8, label = "Mean value")
plt.title("Final year realisations of Brownian Motion", size = 15)
plt.xlabel("Interest rate [%]", size = 12)
plt.ylabel("Density", size = 12)
plt.legend();


#Doob result
def Doob(mu, y):
    return np.exp(-2*mu*y)
doob1 = []
doob2 = []
boundary = np.linspace(0, 20, 100)
for i in range(len(boundary)):
    doob1.append(Doob(0.087,boundary[i]))
    doob2.append(Doob(0.087,boundary[i]-2.28))
```

```

doob2 = [1 if x > 1 else x for x in doob2]
plt.figure(figsize=(15,8))
plt.plot(boundary,doob1, color = TU_blue, label = "Time horizon 2050")
plt.plot(boundary,doob2, color = TU_red, label = "Time horizon 2100")
plt.title("Boundary crossing probability for different interest rates")
#plt.yscale("log")
plt.xlim(boundary[0], boundary [-1])
plt.xlabel("Interest rate [%]")
plt.ylabel("Probability of crossing")
plt.legend()
plt.savefig("Doob_results", bbox_inches = 'tight');
```

The optimization code for producing the minimum costs over both time and crest height is shown below. Note that the primary loop can be changed to produce different variations in variables.

```

#List to adjust the loop
var_lst = [0.005, 0.009]
var_name = "beta"

#Loop for varying various parameters
for z in range(len(var_lst)):
    var_value = var_lst[z]

    beta_mean = var_lst[z] #In this case, beta was the varied parameter
    samples = 10**4
    lifetime_mean = 80
    factor_eis = 1.5
    K_dike = 4.7*10**6

    #Deterministic analysis
    var_value = var_lst[z]
    n = len(cumbudget)
    t = np.linspace(1,n,n)

    #Parameters
    alpha_mean = 0.8   #Translationfactor
    #beta_mean = 0.002   #Sea level rise
    gamma_mean = 0.003  #Population increase
    theta_mean = 0.019  #Asset value increase
    eta_mean = 0.019    #Dike strengthening cost increase

    alpha = alpha_mean
    beta = beta_mean
    gamma = gamma_mean
    theta = theta_mean
    eta = eta_mean
    lifetime = 2020 + lifetime_mean

    P0 =  1/2000#10**-4
    L_dike = 63

    V0_asset = 14380*10**6
    V0_life = 10287*10**6
    N_people = 1
    N_asset = 1

    E_eis = factor_eis* P0 * (V0_asset * N_asset + V0_life * N_people) #* lifetime_mean
    r_mean = 1.9/100
    r_std = 0.25*r_mean

    dh = np.linspace(0,5,10000)
    r = np.random.normal(r_mean, r_std, size = n)


    lst1 = []
    lst = []

    t_max_lst = []
    dh_list = []
    C_lst = []
    CSafety_lst = []
    CDike_lst = []
    CSafety_discount_lst = []
    CDike_discount_lst = []

    Csts = []
    Budgt = []
    Budgt_check_list = []
    dh_unfeasible = []
    dh_feasible = []
    #Minimization task
    for i in range(len(dh)):
```

```
        C_total = 0
        CSafety = 0
        CDike = 0
        Budgt_check_list = []
        #t_max_lst.append(t_max(alpha, beta, gamma, theta, dh[i], P0,V0, E_eis))
        t_fin = t_max(alpha, beta, gamma, theta, dh[i], P0,V0_life, N_people,V0_asset, N_asset, E_eis)
        CSafety = C_safety(P0, N_people, N_asset, V0_life, V0_asset, t, alpha, beta, theta, gamma, dh[i], lifetime_mean)
        CDike = C_dike(L_dike, K_dike, t, eta,dh[i])
        C_total = np.add(CSafety, CDike)
        dh_new = dh
        if t_fin + 2020 < lifetime:
            dh_add = dh[i]
            P1 = P_t_SLR(alpha, beta, t_fin, dh[i], P0)
            V1_life = V_t(N_people, N_asset,V0_life, V0_asset, t_fin, theta, gamma)[1]
            V1_asset = V_t(N_people, N_asset,V0_life, V0_asset, t_fin, theta, gamma)[2]
            dh_new = np.add(dh_new,(np.ones(len(dh))*dh_add))
            CSafety += C_safety(P1, N_people, N_asset, V1_life, V1_asset, t+t_fin, alpha, beta, theta, gamma, dh_new[i], lifetime_mean)
            CDike += C_dike(L_dike, K_dike, t+t_fin, eta,dh_new[i])
            t_fin = t_max(alpha, beta, gamma, theta, dh_new[i], P1,V1_life, N_people,V1_asset, N_asset, E_eis)
        C_total += np.add(CSafety, CDike)
        #Criteria
        for j in range(len(C_total)):
            if C_total[j] < cumBudget[j]:
                Budgt_check_list.append(1)
            else:
                Budgt_check_list.append(0)

        #Storing and sorting data
        Budgt_check_list = np.array(Budgt_check_list)
        cumBudget = np.array(cumBudget)
        idx = np.where(Budgt_check_list > 0)[0]
        if len(idx) == 0:
            #print("There is no solution that meets the criteria for dh = "+str(dh[i])+"\r")
            dh_unfeasible.append(dh[i])
        elif len(idx) != 0:
            TC = discount(C_total[idx], r[idx],t[idx])
            CSafety_discount = discount(CSafety[idx], r[idx], t[idx])
            CDike_discount = discount(CDike[idx], r[idx], t[idx])

            t_max_lst.append(t_fin)
            C_lst.append(np.min(C_total[idx]))
            CSafety_lst.append(np.min(CSafety[idx]))
            CDike_lst.append(np.min(CDike[idx]))
            CSafety_discount_lst.append(CSafety_discount[np.argmin(TC)])
            CDike_discount_lst.append(CDike_discount[np.argmin(TC)])
            lst.append(np.min(TC))
            lst1.append(np.argmin(TC))
            #print(C_total[np.argmin(TC)]- cumBudget[np.argmin(TC)])
            dh_list.append(dh_new[i])
            Budgt.append(cumBudget[np.argmin(TC)])
            Csts.append(C_total[np.argmin(TC)])
            dh_feasible.append(dh[i])
            #print(len(Budgt) - len(Csts))


    fig, ax1 = plt.subplots(figsize = (12,8))
    ax1.plot(dh_feasible,lst, color = TU_black, label = "Discounted minimal cost line");
    #ax1.plot(dh_feasible,C_lst, color = TU_red, ls = "--", label = "Minimal cost line")
    ax1.scatter(dh_feasible[np.argmin(lst)],np.min(lst), color = TU_blue, marker = "*", s= 70, label = "Discounted minimal costs" )
    #ax1.scatter(dh_feasible[np.argmin(C_lst)],np.min(C_lst), color = TU_blue, marker = "o", s= 70, label = "Minimal costs" )
    ax1.set_title("Minimized discounted costs for different various increases in crest height \n Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean), size =15)
    ax1.set_xlabel("Crest height increase [m]", size = 13)
    ax1.set_ylabel("Cost", size = 13)
```

```
    #ax1.legend(loc= 'upper left')

    #ax2 = ax1.twinx()
    ax1.plot(dh_feasible, CSafety_discount_lst, color = TU_blue,  label = 'Discounted flood risk')
    ax1.plot(dh_feasible, CDike_discount_lst, color = TU_red, label = 'Discounted Dike reinforcement')
    ax1.legend(loc= 'upper right')
    plt.xlim(np.min(dh), np.max(dh));
    plt.ylim(0)


    print("Overall minimal costs" , np.min(lst), "for dh = ", dh_feasible[np.argmin(lst)], " needs reinforcement again in", t_max_lst[np.argmin(lst)]+2020)
    print("Rounded chosen dh = ", np.round(dh_feasible[np.argmin(lst)],1))
    print("Chosen reinforcement time = ", 2020+lst1[int(np.argmin(lst))] )
    plt.savefig("MinCostsDisc_S_beta_Var="+var_name+"="+str(var_value)+".png", bbox_inches = 'tight')

    fig, ax1 = plt.subplots(figsize = (12,8))
    #ax1.plot(dh_feasible,lst, color = TU_red, label = "Discounted minimal cost line");
    ax1.plot(dh_feasible,C_lst, color = TU_red, ls = "--", label = "Minimal cost line")
    #ax1.scatter(dh_feasible[np.argmin(lst)],np.min(lst), color = TU_blue, marker = "*", s= 70, label = "Discounted minimal costs" )
    ax1.scatter(dh_feasible[np.argmin(C_lst)],np.min(C_lst), color = TU_blue, marker = "o", s = 70, label = "Minimal costs" )
    ax1.set_title("Minimized costs for different various increases in crest height \n Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean), size =15)
    ax1.set_xlabel("Crest height increase [m]", size = 13)
    ax1.set_ylabel("Cost", size = 13)

    #ax2 = ax1.twinx()
    ax1.plot(dh_feasible, CSafety_lst, color = TU_blue, ls = "--",  label = 'Flood risk')
    ax1.plot(dh_feasible, CDike_lst, color = TU_black, ls= "--", label = 'Dike reinforcement')
    ax1.legend(loc= 'upper right')
    plt.xlim(np.min(dh), np.max(dh));
    plt.ylim(0)


    print("Overall minimal costs" , np.min(C_lst), "for dh = ", dh_feasible[np.argmin(C_lst)], "needs reinforcement again in", t_max_lst[np.argmin(lst)]+2020)
    print("Rounded chosen dh = ", np.round(dh_feasible[np.argmin(C_lst)],1))
    print("Chosen reinforcement time = ", 2020+lst1[int(np.argmin(C_lst))] )
    plt.savefig("MinCosts_S_beta_Var="+var_name+"="+str(var_value)+".png", bbox_inches = 'tight')


    # Monte Carlo Analysis

    #Deterministic parameters
    dh = np.linspace(0,5,100)
    zero_lst = np.zeros(len(dh))
    #Stochastic parameters
    lifetime_lst = np.add(np.ones(samples)*2020,np.random.normal(lifetime_mean,5,size = samples))

    #Stochastic parameters
    alpha_lst = np.random.normal(alpha_mean, 0.05*alpha_mean, size = samples)
    beta_lst = np.random.normal(beta_mean, 0.05*beta_mean, size = samples)
    gamma_lst = np.random.normal(gamma_mean, 0.05*gamma_mean, size = samples)
    theta_lst = np.random.normal(theta_mean, 0.05*theta_mean, size = samples)
    eta_lst = np.random.normal(eta_mean, 0.05*eta_mean, size = samples)

    #Lists to save output
    Final_Cost_List = []
    Final_Time_List = []
    Final_dh_List = []
    Final_Benefits_List = []
    TR_List = []
```

```

    #Minimization task
    for k in range(samples):
        lifetime = lifetime_lst[k]
        lftime = lifetime_lst[k] - 2020
        alpha = 0.8
        beta = beta_lst[k]
        gamma = gamma_lst[k]
        theta = theta_lst[k]
        eta = eta_lst[k]

        r = np.random.normal(r_mean, r_std, size = samples)
        lst1 = []
        lst = []
        t_max_lst = []
        dh_list = []
        C_lst = []
        Csts = []
        Budgt = []
        Budgt_check_list = []
        dh_unfeasible = []
        dh_feasible = []
        Benefits_lst = []
        Benefits_List = []
        Cost_List = []
        for i in range(len(dh)):
            Bresult = 0
            C_total = 0
            Budgt_check_list = []
            #t_max_lst.append(t_max(alpha, beta, gamma, theta, dh[i], P0,V0, E_eis))
            t_fin = t_max(alpha, beta, gamma, theta, dh[i], P0,V0_life, N_people,V0_asset,
                N_asset, E_eis)
            C_total = np.add(C_safety(P0, N_people, N_asset, V0_life, V0_asset, t, alpha,
                beta, theta, gamma, dh[i],lftime),C_dike(L_dike, K_dike, t, eta,dh[i]))
            for m in range(len(t)):
                Bresult += (P_t_SLR(alpha, beta, t[m], zero_lst[i], P0) - P_t_SLR(alpha, beta
                    , t[m], dh[i], P0)) * V_t(N_people, N_asset,V0_life, V0_asset, t[m], np.
                    mean(theta_lst), np.mean(gamma_lst))[0]
                Benefits_lst = np.ones(len(dh))*Bresult
            dh_new = dh
            if t_fin + 2020 < lifetime:
                dh_add = dh[i]
                P1 = P_t_SLR(alpha, beta, t_fin, dh[i], P0)
                V1_life = V_t(N_people, N_asset,V0_life, V0_asset, t_fin, theta, gamma)[1]
                V1_asset = V_t(N_people, N_asset,V0_life, V0_asset, t_fin, theta, gamma)[2]
                dh_new = np.add(dh_new,(np.ones(len(dh))*dh_add))
                C_total += np.add(C_safety(P1, N_people, N_asset, V1_life, V1_asset, t+t_fin,
                     alpha, beta, theta, gamma, dh_new[i], lftime),C_dike(L_dike, K_dike, t+
                    t_fin, eta,dh_new[i]))
                t_fin = t_max(alpha, beta, gamma, theta, dh_new[i], P1,V1_life, N_people,
                    V1_asset, N_asset, E_eis)
                #t_fin = t_max(alpha, beta, gamma, theta, dh_new[i], P1,V1, E_eis)

            #Criteria
            for j in range(len(C_total)):
                if C_total[j] < cumBudget[j]:
                    Budgt_check_list.append(1)
                else:
                    Budgt_check_list.append(0)

            #Storing and sorting data
            Budgt_check_list = np.array(Budgt_check_list)
            cumBudget = np.array(cumBudget)
            idx = np.where(Budgt_check_list > 0)[0]
            if len(idx) == 0:
                #print("There is no solution that meets the criteria for dh = ",dh[i], end =
                    "\r")
                dh_unfeasible.append(dh[i])
            elif len(idx) != 0:
                TC = discount(C_total[idx], r[idx],t[idx])
                TB = discount(Benefits_lst[idx], r[idx], t[idx])
```

```
                TR = np.subtract(TB,TC)


                Benefits_List.append(TB[np.argmax(TR)])
                t_max_lst.append(t_fin)
                C_lst.append(np.min(C_total[idx]))
                Cost_List.append(TC[np.argmax(TR)])
                lst.append(np.max(TR))
                lst1.append(np.argmax(TR))
                dh_list.append(dh_new[i])
                Budgt.append(cumBudget[np.argmax(TR)])
                Csts.append(C_total[np.argmax(TR)])
                dh_feasible.append(dh[i])



        Final_Time_List.append(lst1[np.argmax(lst)])
        Final_dh_List.append(dh_feasible[np.argmax(lst)])
        TR_List.append(lst[np.argmax(lst)])
        Final_Benefits_List.append(Benefits_List[np.argmax(lst)])
        Final_Cost_List.append(Cost_List[np.argmax(lst)])
        print(np.round(k/samples*100,2), "%", end="\r")


    #Plotting functionality

    plt.figure(figsize = (10,8))
    plt.hist(Final_Cost_List, bins = int(np.sqrt(samples)/6), density = True, color = TU_blue,edgecolor = TU_black,label = "Samples");
    plt.axvline(np.quantile(Final_Cost_List, 0.95), color = TU_red, alpha = 0.8, label = "5 & 95% Confidence interval")
    plt.axvline(np.quantile(Final_Cost_List, 0.05), color = TU_red, alpha = 0.8)
    plt.legend()
    plt.title("Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean))
    plt.xlabel("Costs", size = 12)
    plt.ylabel('Density', size = 12)
    plt.savefig("DiscountedCosts_Final_S_Var="+var_name+"="+str(var_value)+".png", bbox_inches = 'tight');

    plt.figure(figsize = (10,8))
    plt.hist(Final_Benefits_List, bins = int(np.sqrt(samples)/6), density = True, color = TU_blue,edgecolor = TU_black,label = "Samples");
    plt.axvline(np.quantile(Final_Benefits_List, 0.95), color = TU_red, alpha = 0.8, label = "5 & 95% Confidence interval")
    plt.axvline(np.quantile(Final_Benefits_List, 0.05), color = TU_red, alpha = 0.8)
    plt.legend()
    plt.title("Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean))
    plt.xlabel("Benefits", size = 12)
    plt.ylabel('Density', size = 12)
    plt.savefig("DiscountedBenefits_Final_S_Var="+var_name+"="+str(var_value)+".png", bbox_inches = 'tight');

    plt.figure(figsize = (10,8))
    plt.hist(TR_List, bins = int(np.sqrt(samples)/6), density = True, color = TU_blue, edgecolor = TU_black,label = "Samples");
    plt.axvline(np.quantile(TR_List, 0.95), color = TU_red, alpha = 0.8, label = "5 & 95% Confidence interval")
    plt.axvline(np.quantile(TR_List, 0.05), color = TU_red, alpha = 0.8)
    plt.legend()
    plt.title("Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean))
    plt.xlabel("Result", size = 12)
    plt.ylabel('Density', size = 12)
    plt.savefig("DiscountedResult_Final_S_Var="+var_name+"="+str(var_value)+".png", bbox_inches = 'tight');


    plt.figure(figsize = (8,6))
    plt.hist(Final_Time_List, bins = int(np.sqrt(samples)/10), density = True, color =
```

```
        TU_blue, edgecolor = TU_black,label = "Samples");
    plt.xlim(0,80)
    plt.title("Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(
        gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean))
    plt.xlabel("Chosen reinforcement year", size = 12)
    plt.ylabel('Density', size = 12);
    plt.savefig("ReinforcementYear_Final_S_Var="+var_name+"="+str(var_value)+".png",
        bbox_inches = 'tight')


    plt.figure(figsize = (8,6))
    plt.hist(Final_dh_List, bins = int(np.sqrt(samples)/15), density = True, color = TU_blue,
         edgecolor = TU_black,label = "Samples");
    plt.axvline(np.quantile(Final_dh_List, 0.95), color = TU_red, alpha = 0.8, label = "5 &
        95% Confidence interval")
    plt.axvline(np.quantile(Final_dh_List, 0.05), color = TU_red, alpha = 0.8)
    plt.legend()
    plt.title("Results Scheldestromen for \u03B2 ="+str(beta_mean)+", \u03B3 = "+str(
        gamma_mean)+", \u03B8 = "+str(theta_mean)+", \u03B7 = "+str(eta_mean))
    plt.xlabel("Crest height increase [m]", size = 12)
    plt.ylabel('Density', size = 12);
    plt.savefig("CrestHeightIncrease_Final_S_Var="+var_name+"="+str(var_value)+".png",
        bbox_inches = 'tight')


    B_mean_lst = []
    B_95_lst = []
    B_05_lst = []
    C_mean_lst = []
    C_95_lst = []
    C_05_lst = []
    R_mean_lst = []
    R_95_lst = []
    R_05_lst = []

    # Results in list
    B_mean = int(np.mean(Final_Benefits_List)/10**6)
    B_05 = int(np.quantile(Final_Benefits_List,0.05)/10**6)
    B_95 = int(np.quantile(Final_Benefits_List,0.955)/10**6)
    C_mean = int(np.mean(Final_Cost_List)/10**6)
    C_05= int(np.quantile(Final_Cost_List,0.05)/10**6)
    C_95 = int(np.quantile(Final_Cost_List,0.95)/10**6)
    R_mean = int(np.mean(TR_List)/10**6)
    R_05 = int(np.quantile(TR_List,0.05)/10**6)
    R_95 = int(np.quantile(TR_List,0.95)/10**6)
    print(B_mean, B_05, B_95)
    print(C_mean, C_05, C_95)
    print(R_mean, R_05, R_95)

    B_mean_lst.append(B_mean)
    B_95_lst.append(B_95)
    B_05_lst.append(B_05)
    C_mean_lst.append(C_mean)
    C_95_lst.append(C_95)
    C_05_lst.append(C_05)
    R_mean_lst.append(R_mean)
    R_95_lst.append(R_95)
    R_05_lst.append(R_05)

    str1 = str(B_mean_lst)
    str2 = str(B_05_lst)
    str3 = str(B_95_lst)
    str4 = str(C_mean_lst)
    str5 = str(C_05_lst)
    str6 = str(C_95_lst)
    str7 = str(R_mean_lst)
    str8 = str(R_05_lst)
    str9 = str(R_95_lst)
    with open("resultS_Var="+var_name+"="+str(var_value)+".txt", 'w', encoding='utf-8') as f:
        f.writelines([str1+"\n", str2+"\n", str3+"\n", str4+"\n", str5+"\n", str6+"\n", str7+
            "\n", str8+"\n", str9+"\n"])
```