Traffic Flow Theory
An introduction with exercises

Knoop, V.L.

DOI
10.5074/t.2021.002

Publication date
2021

Document Version
Final published version

Citation (APA)
Knoop, V. L. (2021). *Traffic Flow Theory: An introduction with exercises*. TU Delft OPEN Publishing. https://doi.org/10.5074/t.2021.002

Traffic Flow Theory: An introduction with exercises

Victor L. Knoop

TU Delft open

Traffic Flow Theory: an Introduction with Exercises

Victor L. Knoop

TU Delft
Delft University of Technology

Traffic Flow Theory: an Introduction with Exercises

Victor L. Knoop

First publication: August 2018

Third Edition: November 2020

Cover image: Traffic flow at the A4 near Leiden; notice the queue spillback and a offramp with spare capacity. Picture by Victor L. Knoop.

ISBN Paperback / softback: 978-94-6366-377-9 Ebook : 978-94-6366-378-6

DOI: https://doi.org/10.5074/t.2021.002

Contents

PREFACE

Traffic processes cause several problems in the world. Traffic delay, pollution are some of it. They can be solved with the right road design or traffic management (control) measure. Before implementing these designs of measures, though, their effect could be tested. To this end, knowledge of traffic flow theory is needed.

This book is meant as learning book for students. To learn an engineering discipline, practising is essential. One of the core qualities of this book, is that more than 250 practice questions (and answers) are available. Therefore, this book can be used as material for courses.

The historical perspective is that the book is an end product of developing, using and continuously improving the lecture notes at Delft University of Technology. I am grateful for the comments by students, and the help of colleagues to this book; particularly for chapter 15, I have been following up from material written by Marie-Jette Wierbos.

I believe that sharing this work can help students, lecturers and possibly practitioners. An online version of the course Traffic Flow Theory and Simulation, including lectures given at Delft University of Technology is freely and openly available via Open Courseware at TU Delft (`https://ocw.tudelft.nl/`, search for traffic and you will find the course on `https://ocw.tudelft.nl/courses/traffic-flow-theory-simulation/`). I believe in open access, and a community where knowledge is shared. The book is hence free to use, and free to distribute to students.

The book is, like science, not finished. By now, it has reached a state of maturity that students highly value the book as it is. Therefore, this is for me the time to share this work. I plan to have updates to the book. If you have remarks – errors, additional request, things which are unclear – please let me know at `v.l.knoop@tudelft.nl`.

The book is meant as introduction to the field of traffic flow theory. Only basic calculus is assumed as base knowledge. For more in-depth knowledge, the reader can continue in other books, including:

- May, A.D. Traffic flow fundamentals. 1990.
- Leutzbach, W. Introduction to the theory of traffic flow. Vol. 47. Berlin: Springer-Verlag, 1988.
- Daganzo, C.F. Fundamentals of transportation and traffic operations. Vol. 30. Oxford: Pergamon, 1997.
- Treiber, M., and A. Kesting. "Traffic flow dynamics." Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg (2013).
- Elefteriadou, L. An introduction to traffic flow theory. Vol. 84. New York, NY, USA: Springer, 2014.
- Ni, D. Traffic Flow Theory: Characteristics, Experimental Methods, and Numerical Techniques. Butterworth-Heinemann, 2015.

Of course, there also is a vast, and ever expanding, body of scientific literature which the reader can use as follow-up material.

1

Variables

After this chapter, the student is able to:

- Use the right terms for level of descriptions, and stationarity and homogeneity
- Give an interpretation to variables $\{x, n, t, v, q, k\}$
- Analyse and explain the differences between observation methods and averaging methods (time mean vs space mean)
- Compute and explain Edie's generalized variables of traffic

This chapter describes the main variables which are used in traffic flow theory. Section 1.1 will show the different levels (microscopic, macroscopic and other levels) at which traffic is generally described. Section 1.2 will describe different principles (local, instantaneous and spatio-temporal) to measure the traffic flow. The last section (1.3) describes traffic flow characteristics.

1.1. Levels of description

This section will show the different levels at which traffic is generally described. Sections 1.1.1 and 1.1.2 will discuss the variables in the microscopic and macroscopic descriptions in more detail.

In a microscopic traffic description, every vehicle-driver combination is described. The smallest element in the description is the vehicle-driver combination. The other often used level of traffic flow description is the macroscopic traffic description. Different from the microscopic description, this level does not consider individual vehicles. Instead, the traffic variables are aggregated over several vehicles or, most commonly, a road stretch. Typical characteristics of the traffic flow on a road stretch are the average speed, vehicle density or flow (see section 1.1.2).Other levels of description can also be used, these are described in the last section(see section 1.1.3).

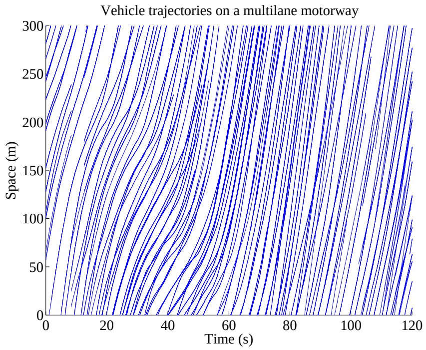


Figure 1.1: Vehicle trajectories on a multilane motorway

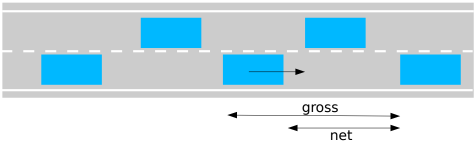


Figure 1.2: The difference between gross and net spacing (or headway)

1.1.1. MICROSCOPIC

In a microscopic traffic description, the vehicle-driver combinations (often referred to as "vehicles", which we will do from now on) are described individually. Full information of a vehicle is given in its trajectory, i.e. the specification of the position of the vehicle at all times. To have full information on these, the positions of all vehicles at all times have to be specified. A graphical representation of vehicle trajectories is given in figure 1.1

The trajectories are drawn in a space time plot, with time on the horizontal axis. Note that vehicle trajectories can never go back in time. Trajectories might move back in space if the vehicles are going in the opposite direction, for instance on a two-lane bidirectional rural road. This is not expected on motorways. The slope of the line is the speed of the vehicles. Therefore, the trajectories cannot be vertical – that would mean an infinite speed. Horizontal trajectories are possible at speed zero.

Basic variables in the microscopic representation are speed, headway, and space headway. The speed is the amount of distance a vehicle covers in a unit of time, which is indicated by v. Sometimes, the inverse of speed is a useful measure, the amount of time a vehicle needs to cover a unit of space; this is called the pace p. Furthermore, there is the space headway or spacing (s) of the vehicle. The net space headway is the distance between the vehicle and its leader. This is also called the gap. The gross space headway of a (following) vehicle is the distance *including* the length of the vehicle, so the distance from the rear bumper of the leading vehicle to the rear bumper of the following vehicle. Similarly, we can define the time it takes for a follower to get to reach (with its front bumper) the position of its leader's rear bumper. This is called the net time headway. If we also add the time it costs to cover the distance of a vehicle length, we get the gross time headway. See also figure 1.2. The symbol used to indicate the headway is h.

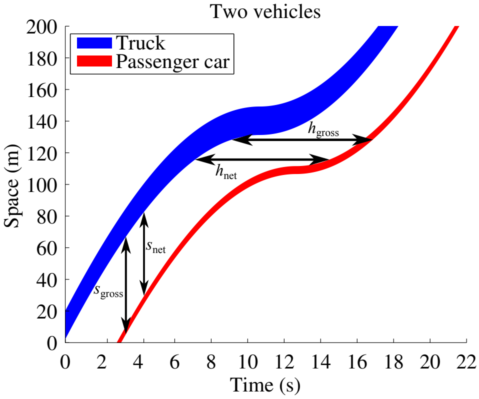


Figure 1.3: The microscopic variables explained based on two vehicles

From now on, in this reader we will use the following conventions:

- Unless specified otherwise, headway means time headway
- Unless specified otherwise, headways and spacing are given as gross values

Figure 1.3 shows the variables graphically. The figure shows two vehicles, a longer vehicle and a shorter vehicle. Note that the length of the vehicles remains unchanged, so the difference between the gross and net spacing is the same, namely the vehicle length. The lines hence have the same thickness (i.e., vertical extension), being the vehicle length. They seem thicker as the line is more horizontal; this is a perception error since the reduce in thickness orthogonal to the direction of the line (which has no physical meaning). Whereas the difference between gross and net space headway is constant (namely the vehicle length), the difference between the gross and net time headway changes based on the vehicle speed.

In a trajectory plot, the slope of the line is the speed. If this slope changes, the vehicle accelerates or decelerates. So, the curvature of the lines in a trajectory plot shows the acceleration or deceleration of the vehicle. If the slope increases, the vehicle accelerates, if it decreases, it decelerates.

1.1.2. MARCOSCOPIC

In a macroscopic traffic description, one does not describe individual vehicles. Rather, one describes for each road section the aggregated variables. That is, one can specify the density k, i.e. how close in space vehicles are together. Furthermore, one can specify the flow q i.e. the number of vehicles passing a reference point per unit of time. Finally, one can describe the average speed u of the vehicles on a road section. Other words for flow are throughput, volume or intensity; we will strictly adhere to the term flow to indicate this concept.

All of the mentioned macroscopic variables have their microscopic counterpart. This is summarized in table 1.1. The density is calculated as one divided by the average spacing, and is calculated over a certain road stretch. For instance, if vehicles have a spacing

1

Table 1.1: Overview of the microscopic and macroscopic variables and their relationship; the pointy brackets indicate the mean.

Microscopic	symbol	unit	Macroscopic	symbol	unit	relation
Headway	h	s	Flow	q	vtg/h	$q = \frac{3600}{\langle h \rangle}$
Spacing	s	m	Density	k	vtg/km	$k = \frac{1000}{\langle s \rangle}$
Speed	v	m/s	Average speed	u	km/u	$u = 3.6 \langle v \rangle$

of 100 meters, there are 1/100 vehicles per meter, or 1000/100=10 veh/km. The flow is the number of vehicles that pass a point per unit of time. It can be directly calculated from the headways by dividing one over the average headway. For instance, if all vehicles have a headway of 4 seconds, there are 1/4 vehicles per second. That means there are 3600(s/h)/4(s/veh)= 900 veh/h. In table 1.1 units are provided and in the conversion from one quantity to the other, one needs to pay attention. Note that the provided units are not obligatory: one can present individual speed in km/h, or density in veh/hm. However, always pay attention to the units before converting or calculating.

RELATION TO THE MICROSCOPIC LEVEL

The average speed is calculated as an average of the speeds of vehicles at a certain road stretch. This speed differs from the average speed obtained by averaging speed of all vehicles passing a certain point. The next section explains the different measuring principles. The full explanation of the differences between the two speeds and how one can approximate the (space) average speed by speeds of vehicles passing a certain location is presented in section 3.4.

Another concept for a traffic flow, in particular in relation to a detector (see also section 1.2), is the occupancy o. This indicates which fraction of a time a detector embedded in the roadway is occupied, i.e. whether there is a vehicle on top of the detector. Suppose a detector has a length L_{det} and a vehicle a length of L_i. The occupancy is defined as the time the detector is occupied, τ_{occupied} divided by all time, i.e. the time it is occupied and time is is not occupied $\tau_{\text{not occupied}}$

$$o = \frac{\tau_{\text{occupied}}}{\tau_{\text{occupied}} + \tau_{\text{not occupied}}} \tag{1.1}$$

The occupation time can be derived from the distances and the speed. The distance the vehicle has to cover from the moment it starts occupying the detector up to the time it leaves the detector is its own length plus the length of the detector. Hence, the occupancy time is

$$\tau_{\text{occupied}} = \frac{L_i + L_{\text{det}}}{v} \tag{1.2}$$

Once the first vehicle drives off the detector, the distance for the following vehicle to reach the detector is the gap (i.e., the spacing minus the length of the vehicle) between the vehicles minus the length of the detector. The amount of time this takes is

$$\tau_{\text{not occupied}} = \frac{s - L_i - L_{\text{det}}}{v} \tag{1.3}$$

Substituting the expressions for the occupancy time and the non-occupancy time into equation 1.1 and rearranging the terms, we get

$$o = \frac{L_i + L_{\text{det}}}{s} \tag{1.4}$$

In practice, the detector length is known for a certain road configuration (usually, there are country specific standards). So assuming a vehicle length, one can calculate the spacing, and hence the density, from the occupancy.

1.1.3. OTHER LEVELS

Apart from the macroscopic and microscopic traffic descriptions, there are three other levels to describe traffic. They are are less common, and are therefore not discussed in detail. The levels mentioned here are mainly used in computer simulation models.

MESOSCOPIC

The term mesoscopic is used for any description of traffic flow which is in-between macroscopic and microscopic. It can also be a term for simulation models which calculate some elements macroscopically and some microscopically. For instance, Dynasmart (2003), uses such a mesoscopic description.

SUBMICROSCOPIC

In a submicroscopic description the total system state is determined by the sub levels of a vehicle and/or driver. Processes which influence the speed of a vehicle, like for instance mechanically throttle position and engine response, or psychologically speed perception, are explicitly modelled. This allows to explicitly model the (change in) reaction on inputs. For instance, what influence would cars with a stronger engine have on the traffic flow.

NETWORK LEVEL

A relatively new way of describing the traffic state is the network level. This has recently gained attention after the publication by Geroliminis and Daganzo (2008). Instead of describing a part of a road as smallest element, one can take an area (e.g. a city center) and consider this as one unit.

1.2. MEASURING PRINCIPLES

Whereas the previous sections described which variables are used to describe traffic flow, this section will introduce three principles of measuring the traffic flow. These principles are local, instantaneous and spatio-temporal.

1.2.1. LOCAL

With local measurements one observes traffic at one location. This can be for instance a position at the roadway. To measure motorway traffic, often inductive loops are used. These are coils embedded in the pavement in which a electrical current produces a (vertical) magnetic field. If a car enters or leaves this magnetic field, this can be measured in the current of the coil. Thus, one knows how long a loop is occupied. In the US, usually single loops are used, giving the occupancy of the loop. Using equation 1.4, this can be translated into density. The detectors also measure the flow. As will be explained later in section 3.1, this suffices to completely characterise the traffic flow.

This determination of density builds upon the assumption of the vehicle length being known. One can also measure the length of a vehicle for passing vehicles, using dual loop detectors. These are inductive loops which are placed a known short distance (order of 1 m) from each other. If one carefully measures the time between the moment the vehicle starts occupying the first loop and the moment it starts occupying the second loop, one can measure its speed. If its speed is known, as well as the time it occupies one loop, the length of the vehicle can also be determined.

1.2.2. INSTANTANEOUS

Contrary to local measurements, there are instantaneous measurements. These are measurements which are taken at one moment in time, most likely over a certain road stretch. An example of such a measurement is an areal photograph. In such a measurement, one can clearly distinguish spatial characteristics, as for instance the density. However, measuring the temporal component (flow) is not possible.

1.2.3. SPATIO-TEMPORAL MEASUREMENTS

Apart from local or instantaneous measurements, one can use measurements which stretch over a period of time *and* a stretch of road. For instance, the trajectories in figure 1.1 are an example thereof. This section will introduce Edie's definitions of flow, density and speed for an area in space and time.

A combination of instantaneous measurements and local measurements can be found in remote sensing observations. These are observations which stretch in both space and time. For instance, the trajectories presented in figure 1.1 can be observed using a camera mounted on a high point or a helicopter. One can see a road stretch, and observe it for a period of time.

Measuring average speed by definition requires an observation which stretches over time and space. At one location, one cannot determine speed, nor at one moment. One needs at least two locations close by (several meters) or two time instances close by. Ignoring these short distances one can calculate a local mean speed based on speeds of the vehicles passing by location. Ignoring the short times, one can calculate the time mean speed from the speed of the vehicles currently at the road. At this moment, we suffice by mentioning these average speeds are different. Section 3.4 will show how the space mean speed can be approximated from local measurements.

Figure 1.4 shows the same trajectories as figure 1.1, but in figure1.4 an area is selected. Trajectories within this area in space and time are coloured red. Note that an selected area is not necessarily square. It is even possible to have a convex area, or boundaries moving backwards and forwards in time. The definitions as introduced here will hold for all types of areas, regardless of their shape in space-time.

Let us consider the area X. We indicate its size by W_X, which is expressed in km-h, or any other unit of space times time. For all vehicles, we consider the distance they drive in area X, which we call $d_{X,i}$. Adding these for all vehicles i gives the total distance covered in area X, indicated by TD:

$$TD = \sum_{\text{all vehicles } i} d_{X,i} \tag{1.5}$$

For a rectangular area in space and time, the distance covered might be the distance from the upstream end to the downstream end, but the trajectory can also begin and/or end at the side of the area, at a certain time. In that case, the distance is less than the full distance.

Similarly, we can define the time a vehicle spends in area X, $t_{X,i}$, which we can sum for all vehicles i to get the total time spent in area X, indicated by TT.

$$TT = \sum_{\text{all vehicles } i} t_{X,i} \tag{1.6}$$

Obviously, both quantities grow in principle with the area size. Therefore, the traffic flow is best characterised by the quantities TD/W_X and TT/W_X. This gives the flow and the density respectively:

$$q = \frac{TD}{W_X} \tag{1.7}$$

$$k = \frac{TT}{W_X} \tag{1.8}$$

Intuitively, the relationship is best understood reasoning from the known relations of density and flow. Starting with a situation of 1000 veh/h at a cross section, and an area of 1 h and 2 km. In 1 hour, 1000 vehicles pass by, which all travel 2 kilometres in the area. (There the vehicles which cannot cover the 2 km because the time runs out, but there are just as many which are in the section when the time window starts). So the total distance is the flow times the size of the area: $TD = qW_X$. This can be simply rewritten to equation 1.8.

A similar relation is constructed for the density, considering again the rectangular area of 1 hour times 2 kilometres. Starting with a density of 10 veh/km, there are 20 vehicles in the area, which we all follow for one hour. The total time spent, is hence 10*2*1, or $TT = kW_X$. This can be rewritten to equation 1.8.

The average speed is defined as the total distance divided by the total time, so

$$u = \frac{TD}{TT} \tag{1.9}$$

The average travel time over a distance l can be found as the average of the time a vehicle travels over a distance l. In an equation, we find:

$$\langle tt \rangle = \left\langle \frac{l}{v} \right\rangle = l \left\langle \frac{1}{v} \right\rangle \tag{1.10}$$

In this equation, tt indicates the travel time and the pointy brackets indicate the mean. This can be measured for all vehicles passing a road stretch, for instance at a local detector. Note that the mean travel time is *not* equal to the distance divided by the mean speed:

$$\langle tt \rangle = l \left\langle \frac{1}{v} \right\rangle \neq l \frac{1}{\langle v \rangle} \tag{1.11}$$

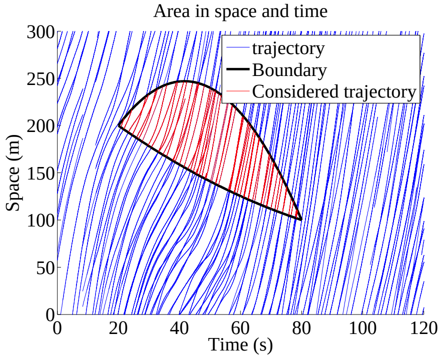


Figure 1.4: Vehicle trajectories and the selection of an area in space and time

In fact, it can be proven that in case speeds of vehicles are not the same, the average travel time is underestimated if the mean speed is used.

$$\langle tt \rangle = l \left\langle \frac{1}{v} \right\rangle \leq l \frac{1}{\langle v \rangle} \tag{1.12}$$

The harmonically averaged speed (i.e., 1 divided by the average of 1 divided by the speed) does provide a good basis for the travel time estimation. In an equation, we best first define the pace, p_i:

$$p_i = \frac{1}{v_i} \tag{1.13}$$

The harmonically averaged speed now is

$$\langle v \rangle_{\text{harmonically}} = \frac{1}{\langle p \rangle} = \frac{1}{\left\langle \frac{1}{v_i} \right\rangle} \tag{1.14}$$

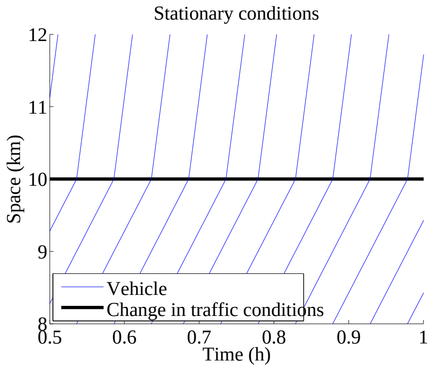
The same quantity is required to find the space mean speed. Section 3.4 shows the difference qualitatively. In short, differences can be several tens of percents.

1.3. STATIONARITY AND HOMOGENEITY

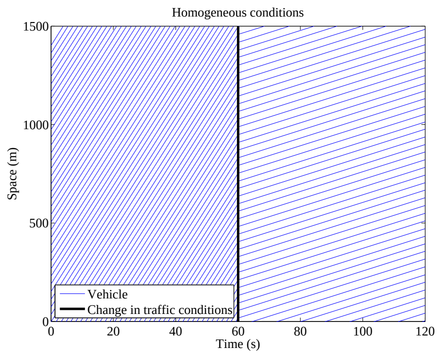
Traffic characteristics can vary over time and/or over space. There are dedicated names for traffic if the state does not change.

Traffic is called stationary if the traffic flow does not change over time (but it can change over space). An example can be for instance two different road sections with different characteristics. An example is given in figure 1.5a, where there first is a low speed, then the speed of the vehicles is high.

Traffic is called homogeneous if the traffic flow does not change over space (but it can change over time). An example is given in figure 1.5b, where at time 60 the speed decreases at the whole road section. This is much less common than the stationary conditions. For this type of situation to occur, the traffic regulations have to change externally. For instance, the speed limits might change at a certain moment in time (lower speeds at night).



(a) Stationary conditions



(b) Homogeneous conditions

SELECTED PROBLEMS

For this chapter, consider problems: 5, 6, 162, 172, 173, 174, 175, 196, 217, 232, A.10.3, A.11.3

2

CUMULATIVE CURVES

After this chapter, the student is able to:

- Construct (slanted/oblique) cumulative curves in practice or from theoretical problem
- Interpret these and calculate: delays, travel times, density, flow

This chapter discusses cumulative curves, also known as cumulative flow curves. The chapter first defines the cumulative curves (section 2.1), then it is shown how traffic characteristics can be derived from these (section 2.2). Section 2.4 shows the application of slanted cumulative curves.

2.1. DEFENITION

The function $N_x(t)$ is defined as the number of vehicles that have passed a point x at time t and is only used for traffic into one direction. Hence, this function only increases over time. Strictly speaking, this function is a step function increasing by one every time a vehicle passes. However, for larger time spans and higher flow rates, the function is often smoothed into a continuous differentiable function.

The increase rate of this function equals the flow:

$$\frac{dN}{dt} = q \tag{2.1}$$

Hence from the flow, we can construct the cumulative curve:

$$N = \int q\,dt \tag{2.2}$$

This gives one degree of freedom, the initial value. This can be chosen freely, or should be adapted to cumulative curves for other locations.

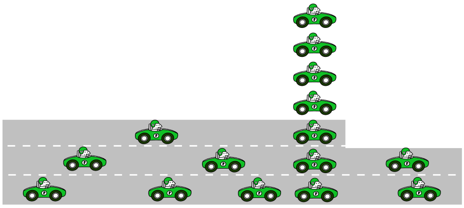


Figure 2.1: Illustration of a vertical queue

2.2. VERTICAL QUEUING MODEL

A vertical queuing model is a model which assumes an unlimited inflow and and an outflow which is restricted to capacity. The vehicles which cannot pass the bottleneck are stacked "vertically" and do not occupy any space. Figure 2.1 illustrates this principle.

Let us now study the dynamics of such a queue. We discretize time in steps of duration Δt, referred to by index t. The demand is externally given, and indicated by D. At time steps t we compute the flow into and out of the stack (the number of vehicles in the stack indicated as S). In between the time steps, indicated here as $t+1/2$, the number of vehicles in the stack is updated based on the flows q. Then, the stack provides the basis for the flows in the next time step.

The stack starts at zero. Then, for each time step first the inflow to the stack is computed.

$$q_{\text{in},t} = D \tag{2.3}$$

and the stack is updated accordingly, going to an intermediate state at time step t+1/2. This intermediate step is the number of vehicles in the queue if there was no outflow, so the original queue plus the inflow:

$$S_{t+1/2} = S_t + q_{\text{in}}\Delta t \tag{2.4}$$

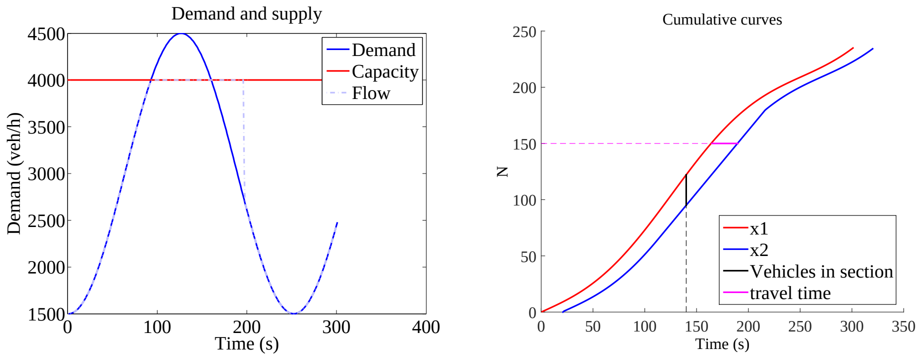
Then, the outflow out of the stack (q_{out}) is the minimum of the number of vehicles in this intermediate queue and the maximum outflow determined by the capacity C:

$$q_{\text{out}} = \min\{C\Delta t, S_{i+1/2}\} \tag{2.5}$$

The stack after the time step is then computed as follows

$$S_{i+1} = S_{i+1/2} - q_{\text{out}}\Delta t = S_i + \left(q_{\text{in},i} - q_{\text{out},i}\right)\Delta t \tag{2.6}$$

Let us consider a situation as depicted in figure 2.1, and we are interested in the delays due to the bottleneck with a constant capacity of 4000 veh/h. The demand curve is plotted in figure 2.2a. The flows are determined using the vertical queuing model. The flows are also shown in figure 2.2a. Note that the area between the flow and demand curve where the demand is higher than the flow (between approximately 90 to 160 seconds), is the same as the area between the curves where the flow is higher than the demand (between approximately 160 and 200 seconds). The reasoning is that the



(a) Demand and supply

(b) Cumulative curves

Figure 2.2: Demand and cumulative curves

area represents a number of vehicles (a flow times a time). From 90 to 160 seconds the demand is higher than the flow, i.e., the inflow is higher than the outflow. The area represents the number of vehicles that cannot pass the bottleneck, and hence the number of queued vehicles. From 160 seconds, the outflow of the queue is larger than the inflow. That area represents the number of vehicles that has left the queue, and cannot be larger than the number of vehicles queued. Moreover, the flow remains at capacity until the stack is empty, so both areas must be equal.

2.3. TRAVEL TIMES, DENSITIES AND DELAYS

This section explains how travel times and delays. Delay can be computed using cumulative curves. Note that this methodology does not take spillback effects into account. If one requires this to be accounted for, please refer to shockwave theory (chapter 4).

2.3.1. CONSTRUCTION OF CUMULATIVE CURVES

The cumulative curves for the above situation is shown in figure 2.2b. The curves show the flows as determined by the vertical queuing model. For the inflow we hence use equation 2.3 and for the outflow we use 2.5; for both, the cumulative curves are constructed using equation 2.2.

2.3.2. TRAVEL TIMES, NUMBER OF VEHICLES IN THE SECTION

A black line is drawn at $t = 140s$ in figure 2.2b. The figure shows by intersection of this line with the graphs how many vehicles have passed the upstream point $x1$ and how many vehicles have passed the downstream point $x2$. Consequently, it can be determined how many vehicles are in the section between $x1$ and $x2$. This number can also be found in the graph, by taking the difference between the inflow and the outflow at that moment. This is indicated in the graph by the bold vertical black line.

Similarly, we can take a horizontal line; consider for instance the line at $N = 150$. The intersection with the inflow line shows when the 150th vehicle enters the section, and the intersection with the outflow line shows when this vehicle leaves the section. So, the horizontal distance between the two lines is the travel time of the 150th vehicle. At times where the demand is lower than the capacity, the vehicles have a free flow travel time. So without congestion, the outflow curve is the inflow curve which is translated to the right by the free flow travel time.

The vertical distance is the number of vehicles in the section (ΔN) at a moment t. In a time period dt this adds $\Delta N dt$ to the total travel time (each vehicle contributes dt). To get the total travel time, we integrate over all infinitesimal intervals dt:

$$tt = \int \Delta N dt \tag{2.7}$$

The horizontal distance between the two lines is the travel time for one vehicle, and vertically we find the number of vehicles. Adding up the travel times for all vehicles gives the total travel time:

$$tt = \sum_i tt_i \tag{2.8}$$

In a continuous approach, this changes into

$$tt = \int tt_i di \tag{2.9}$$

Both calculation methods lead to the same interpretation: *the total time spent can be determined by the area between the inflow and outflow curve.*

2.3.3. DELAYS

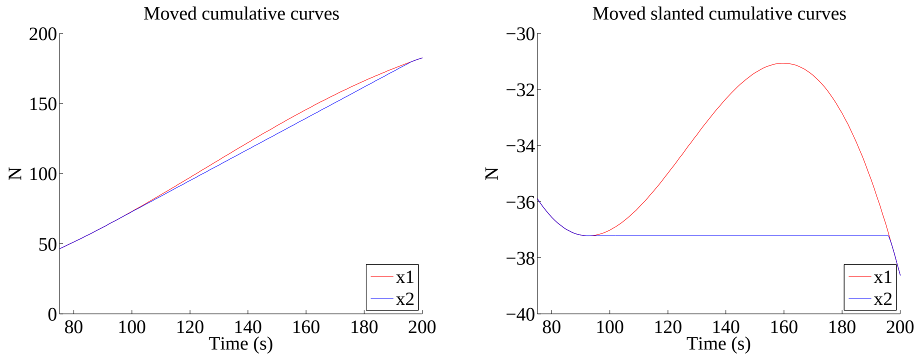
Delays for a vehicle are the extra time it needs compared to the free flow travel time; so, to calculate delay, one subtracts the free flow travel time from the actual travel time. To subtract the free flow travel time from the travel time, we can graphically move the outflow curve to the left, as is shown in figure 2.3a. For illustration purposes, the figure is zoomed at figure 2.3b. The figure shows that if the travel time equals the free flow travel time, both curves are the same, leading to 0 delay.

Similar to how the cumulative curves can be used to determine the travel time, the moved cumulative curves can be used to determine the delay. The delay for an individual vehicle can be found by the horizontal distance between the two lines. The vertical distance between the two lines can be interpreted as the number of vehicles queuing. The total delay is the area between the two lines:

$$\mathcal{D} = \int tt_i - tt_{\text{free flow}} di \tag{2.10}$$

This is the area between the two lines. If we define N_{queue} as the number of vehicles in the queue at moment t, we can also rewrite the total delay as

$$\mathcal{D} = \int N_{queue}(t) dt \tag{2.11}$$



(a) Moved cumulative curves

(b) Slanted cumulative curves

Figure 2.3: Determining the delay and the flows from cumulative curves

2.4. SLANTED CUMULATIVE CURVES

Slanted cumulative curves or oblique cumulative curves is a very powerful yet simple tool to analyse traffic streams. These are cumulative curves which are off set by a constant flow:

$$\tilde{N} = \int q - q_0 dt - \int q_0 dt = \int q dt - \int q_0 dt \tag{2.12}$$

This means that differences with the freely chosen reference flow q_0 are amplified: in fact, only the difference with the reference flow are counted. The best choice for the reference flow q_0 is a capacity flow.

Figure 2.3b shows the slanted cumulative curves for the same situation as in figure 2.3a. The figure is off set by $q_0 = 4000$ veh/h. Because the demand is initially lower than the capacity, $\tilde{N}$ reaches a negative value. From the moment outflow equals capacity, the slanted cumulative outflow curve is constant. Since the demand is higher than the capacity, this increases. At the moment both curves intersect again, the queue is dissolved.

The vertical distance between the two lines still shows the length of the queue, N_{queue}. That means that equation 2.10 still can be applied in the same way for the slanted cumulative curves, and the delay is the area between the two lines.

Slanted cumulative curves are also particularly useful to determine capacity, and to study changes of capacity, for instance the capacity drop (see section 6.2). In that case, for one detector the slanted cumulative curves are drawn. By a change of the slope of the line a change of capacity is detected. In appendix C a Matlab code is provided by which cumulative curves can be made, and which includes the computation of several key performance indices.

2.5. PRACTICAL LIMITATIONS

Cumulative curves are very useful for models where the blocking of traffic does not play a role. For calculating the delay in practise, the method is not very suitable due to failing detectors. Any error in the detection (a missed or double counted observation), will change one of the curves and will offset the cumulative flow, and this is never corrected;

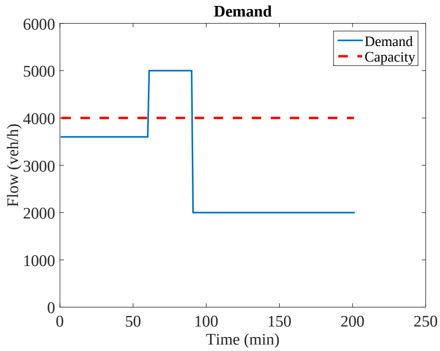


Figure 2.4: Demand and capacity

this is called cumulative drift. Recently, an algorithm has been proposed to check the offsets by cross checking the cumulative curves with observed travel times (Van Lint et al., 2014). This is work under development. Moreover, some types of detectors will systematically miscount vehicles, which makes the above-mentioned error larger.

Apart from their use in models, slanted cumulative curves are very powerful to show changes in capacity in practise.

2.6. EXAMPLE APPLICATION

Consider a road with a demand of:

$$q_{\text{in}} = \begin{cases} 3600\text{v/h} & \text{for } t < 1\text{h} \\ 5000\text{v/h} & \text{for } 1\text{h} < t < 1.5\text{h} \\ 2000\text{v/h} & \text{for t} > 1.5\text{h} \end{cases} \tag{2.13}$$

The capacity of the road is 4000 veh/h. A graph of the demand and capacity is shown in figure 2.4.

1. Construct the (translated=moved) cumulative curves
2. Calculate the first vehicle which encounters delay (N)
3. Calculate the time at which the delay is largest
4. Calculate the maximum number of vehicles in the queue
5. Calculate the vehicle number (N) with the largest delay
6. Calculate the delay this vehicle encounters (in h, or mins)
7. Calculate the time the queue is solved
8. Calculate the last vehicle (N) which encounters delay
9. Calculate the total delay (veh-h)

10. Calculate the average delay of the vehicles which are delayed (h)

This can be answered by the following:

1. For the cumulative curves, an inflow and an outflow curve needs to be constructed; both increase. For the inflow curve, the slope is equal to the demand. For the outflow curve, the slope is restricted to the capacity. During the first hour, the demand is lower than the capacity, hence *the outflow is equal to the demand.* From t=1h, the inflow exceeds the capacity and the outflow will be equal to the demand. The cumulative curve hence increases with a slope equal to the capacity. As long as there remains a queue, i.e. the cumulative inflow is higher than the outflow, the outflow remains at capacity. The outflow remains hence increasing with a slope equal to the capacity until it intersects with the cumulative inflow. Then, the outflow follows the inflow: see figure 2.5a and for a more detailed figure 2.5b.

2. The first vehicle which encounters delay (N) Delays as soon as q>C: so after 1h at 3600 v/h = 3600 vehicles.

3. The time at which the delay is largest: A queue builds up as long as q>C, so up to 1.5 h. At that moment, the delay is largest

4. The maximum number of vehicles in the queue: 0.5 h after the start of the queue, 0.5*5000=2500 veh entered the queue, and 0.5*4000=2000 left: so 500 vehicles are in the queue at t=0.5h

5. The vehicle number (N) with the largest delay: N(1.5h)=3600+0.5*5000 = 6100

6. The delay this vehicle encounters (in h, or mins): It is the 2500th vehicle after t=1h. The delay is the horizontal delay between the entry and exit curve. It takes at capacity 2500/4000 = 37.5 mins to serve 2500 vehicles. It entered 0.5 hours = 30 mins after t=1, so the delay is 7.5 mins

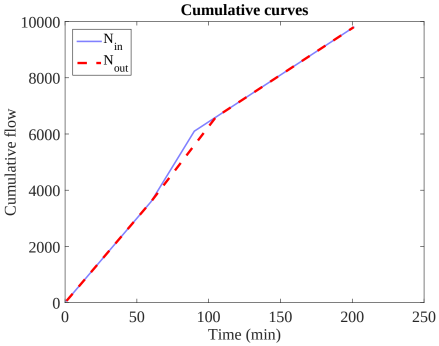
7. The time the queue is solved: This is the time point that the inflow and outflow curves intersect again. 500 vehicles is the maximum queue length, and it reduces with 4000-2000=2000veh/h. So 500/2000=15 minutes after the time that q<C the queue is solved, i.e. 1:45h after the start.

8. The last vehicle (N) which encounters delay This is the vehicle number at the moment the inflow and outflow curves meet again. 15 minutes after the vehicle number with the largest delay: 6100+0.25*2000 = 6600 veh

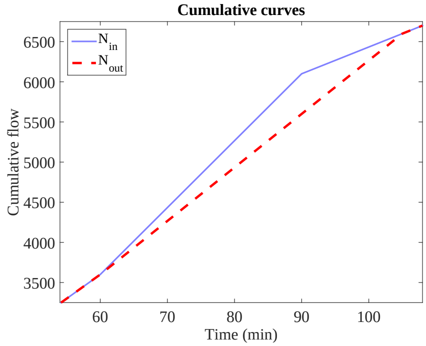
9. The total delay. This is the area of the triangle between inflow and outflow curve. This area is computed by 0.5 * height * base = 0.5 * 500 * (30+15)/60 = 187,5 veh-h. Note that here we use a generalised equation for the area of a triangle. Indeed, we transform the triangle to a triangle with a base that has the same width, and the hight which is the same for all times (i.e., we skew it). The hight of this triangle is 500 vehicles (the largest distance between the lines) and the width is 45 minutes.

10. The average delay of the vehicles which are delayed (h) 187,5 veh-h/ (6600-3600) veh = 0,0625 h = 3,75 min

2



(a) Overview of the cumulative curves



(b) More detailed view of the cumulative curves

Figure 2.5: Cumulative curves for the example

SELECTED PROBLEMS

For this chapter, consider problems: 3, 4, 68, 69, 70, 71, 100, 142, 143, 144, A.7.2, 205, 205, 234, 235, 260, A.11.2

3

RELATIONSHIPS OF TRAFFIC VARIABLES

After this chapter, the student is able to:

- Comment on the restrictions of the fundamental diagram
- Translate the fundamental diagram in three planes
- Interpret the shape of the fundamental diagram in terms of driving behavior

Chapter 1 defined the variables and their definition. This chapter will discuss the relationship between these variables. First of all the mathematically required relationships are shown (section 3.1), then typical properties of traffic in equilibrium are discussed (section 3.2). Section 3.3 discusses these relationships in the light of drivers, and expands this to non-equilibrium conditions. Finally, section 3.4 gives attention to the moving observer.

3.1. FUNDAMENTAL RELATIONSHIP

In microscopic view, it is obvious that the headway (h), the spacing (s) and the speed (v) are related. The headway times the speed will give the distance covered in this time, which is the spacing. It thus suffices to know two of the three basic variables to calculate the third one.

$$s = hv \tag{3.1}$$

Since headways and spacings have macroscopic counterparts, there is a macroscopic equivalent for this relationship. After reordering, equation 3.1 reads

$$\frac{1}{h} = \frac{1}{s}v \tag{3.2}$$

Table 3.1: The basic traffic variables and their relationship

Microscopic	Macroscopic
s	$k = \frac{1}{\langle s \rangle}$
h	$q = \frac{1}{\langle h \rangle}$
v	$u = \frac{1}{\langle v \rangle}$
$s = hv$	$q = ku$

The macroscopic equivalent of this relationship is the average of this equation. Remembering that $q = \frac{1}{\langle h \rangle}$ and $k = \frac{1}{\langle s \rangle}$, we get:

$$\boxed{q = ku} \tag{3.3}$$

This equation shows that the flow q is proportional with both the speed u and the density k. Intuitively, this makes sense because when the whole traffic stream moves twice as fast if the flow doubles. Similarly, if – at original speed – the density doubles, the flow doubles as well.

Table 3.1 summarizes the variables and their relationships.

3.2. FUNDAMENTAL DIAGRAM

If two of the three macroscopic traffic flow variables are known, the third one can be calculated. This section will show that there is another relationship. In fact, there is an equilibrium relationship between the speed and the density. First, a qualitative understanding will be given, after that the effect will be shown for various couples of variables. Also, different shapes of the supposed relationship will be shown (section 3.2.5).

3.2.1. QUALITATIVE UNDERSTANDING OF THE SHAPE

Let us, for the sake of argument, consider the relationship between density and flow. And let us furthermore start considering the most extreme cases. First, the case that there is no vehicle on the road. Since the density is 0, the flow is 0, according to equation 3.3. In the other extreme case the density on the road is very high, and the speed is 0. Using again equation 3.3 we find also for this case a flow of 0. In between, there are traffic states for which the traffic flow is larger than zero. Assuming a continuous relationship between the speed and the density (which is not necessarily true, as will be discussed section 6.2) there will be a curve relating the two points at flow 0. This is indicated in figure 3.1c.

This relationship is being observed in traffic. However, it is important to note that this is *not* a causal relationship. One might argue that due to the low speed, drivers will drive closer together. Alternatively, one might argue that due to the close spacing, drivers need to slow down.

(a) One extreme: an empty road

(b) One extreme: a jammed road

(c) Basic shape of the fundamental diagram

Figure 3.1: The extreme situations and an idea for the fundamental diagram

3.2.2. TRAFFIC STATE

We can define a traffic state by its density, flow and speed. Using equation 3.3, we only need to specify two of the variables. Furthermore, using the fundamental diagram, one can be sufficient. It is required that the specified variable then has a unique relationship to the others. For instance, judged by figure 3.1c, specifying the density will lead to a unique flow, and a unique speed (using equation 3.3, and thus a unique traffic state). However, specifying the flow (at any value between 0 and the capacity) will lead to two possible densities, two possible speeds, and hence two possible traffic states.

The speed of the traffic can be derived using the equation 3.3:

$$u = \frac{q}{k} \tag{3.4}$$

For a traffic state in the flow density plane, we can draw a line from the traffic state to the origin. The slope of this line is q/k. So the speed of the traffic can be found by the slope of a line connecting the origin to the traffic state in the flow density plane. The free flow speed can be found by the slope of the fundamental diagram at k=0, i.e. the derivative of the fundamental diagram in the origin.

3.2.3. IMPORTANT POINTS

The most important aspect of the fundamental diagram for practitioners is the capacity. This is the maximum flow which can be maintained for a while at a road. The same word is also used for the traffic state at which maximum flow is obtained. This point is found at the top of the fundamental diagram. Since we know that the flow can be determined from the headway, we can estimate a value for the capacity if we consider the minimum headway. For drivers on a motorway, the minimum headway is approximately 1.5 to 2 seconds, so we find a typical capacity value of $\frac{1}{2}$ to $\frac{1}{1.5}$ vehicles per second. If we convert this to vehicles per hour, we find (there are 3600 seconds in an hour) $\frac{3600}{2} = 1800$ to $\frac{3600}{1.5} = 2400$ vehicles per hour.

The density for this point is called the critical density, and the related speed the critical speed. The capacity is found when the average headway is shortest, which is when a large part of the vehicles is in car-following mode. This happens at speeds of typically 80 km/h; this then is the critical speed. From the capacity and the critical speed, the critical density can be calculated using equation 3.3. This varies from typically 20 veh/km/lane to 28 veh/km/lane.

For densities lower than the critical density, traffic is in an uncongested state; for higher densities, traffic is in a congested state. In the uncongested part, the traffic flow increases with an increase of density. In the congested branch, the traffic flow decreases with an increase of density. The part of the fundamental diagram of uncongested traffic states is called the uncongested branch of the diagram. Similarly, the congested branch gives the points for which the traffic state is congested.

The free flow speed is the speed of the vehicles at zero density. At the other end, we find the density at which the vehicles come to a complete stop, which is called the jam density. For the jam density, we can also make an estimation based on the length of the vehicles and the distance they keep at standstill. A vehicle is approximately 5 meters long, and they keep some distance even at standstill (2-3 meters), which means the jam density is $\frac{1}{5+3}$ to $\frac{1}{5+2}$ veh/m, or $\frac{1000}{5+3}$=125veh/km to $\frac{1000}{5+2}$=142 veh/km.

3.2.4. FUNDAMENTAL DIAGRAM IN DIFFERENT PLANES

So far, the fundamental diagram has only be presented in the flow density plane. However, since the fundamental equation (equation 3.3) relates the three variables to each other, any function relating two of the three variables to each other will have the same effect. Stated otherwise, the fundamental relationship can be presented as flow-density relationship, but also as speed-density relationship or speed-flow relationship. Figure 3.2 shows all three representations of the fundamental diagram for a variety of functional forms.

In the speed-density plane, one can observe the high speeds for low densities, and the speed gradually decreasing with increasing density. In the speed-flow diagram, one sees two branches: the congested branch with high speeds and high flows, and also a congested branch with a low speed and lower flows.

3.2.5. SHAPES OF THE FUNDAMENTAL DIAGRAM

There are many shapes proposed for the fundamental diagram. The data are quite scattered, so different approaches have been taken: very simple functions, functions with mathematically useful properties, or functions derived from a microscopic point of view. Even today, new shapes are proposed. In the remainder of this section, we will show some elementary shapes; the graphs are shown in figure 3.2.

GREENSHIELDS

Greenshields was the first to observe traffic flows and publish on this in 1934 (Greenshields, 1934). He observed a platoon of vehicles and checked the density of the platoon and their speed. He assumed this relationship to be linear:

$$v = v_0 - ck \tag{3.5}$$

Note that for $k = \frac{v_0}{c}$ the speed equals 0, hence the flow equals zero. Therefore, the the jam density equals $\frac{v_0}{c}$.

TRIANGULAR

The Greenshields diagram is not completely realistic since for a range of low densities, drivers keep the same speed, possibly limited by the current speed limit. The fundamental diagram which is often used in academia is the triangular fundamental diagram, reffering to the triangular shape in the flow-density plane. The equation is as follows:

$$q = \begin{cases} v_0 k & \text{if } k < k_c \\ q_c - \dfrac{k - k_c}{k_j - k_c} q_c & \text{if } k \geq k_c \end{cases} \tag{3.6}$$

TRUNCATED TRIANGULAR

Daganzo (1997) shows a truncated triangular fundamental diagram. That means that the flow is constant and maximized for a certain range of densities. The equation is as follows:

$$q = \begin{cases} v_0 k & \text{if } k < k_1 \\ v_0 k_1 & \text{if } k_1 < k < k_c \\ q_c - \dfrac{k - k_c}{k_j - k_c} q_c & \text{if } k \geq k_c \end{cases} \tag{3.7}$$

SMULDERS

Smulders (1989) proposed a fundamental diagram in which the speed decreases linearly with the density for the free flow branch. In the congested branch the flow decreases linearly with density.

DRAKE

Drake et al. (1967) proposes a continuous fundamental diagram where the speed is an exponentially decreasing function of the density:

$$v = v_0 \exp\left(-\frac{1}{2}\left(\frac{k}{k_c}\right)^2\right) \tag{3.8}$$

INVERSE LAMBDA

The capacity drop (see section 6.2) is not present in the fundamental diagrams presented above. Koshi et al. (1981) introduced an 'inverse lambda'fundamental diagram. This means the traffic has a free speed up to a capacity point. The congested branch however, does not start at capacity but connects a bit lower at the free flow branch. It is assumed that traffic remains in the free flow branch and after congestion has set in, will move to the congested branch. Only after the congestion has solved, passing a density lower than the density where the congested branch connects to the free flow branch, traffic flows can grow again to higher values. The description is as follows:

$$q = \begin{cases} v_0 k & \text{if } k < k_c \\ v_0 k_1 - \dfrac{k - k_1}{k_j - k_1} v_0 k_1 & \text{if } k \geq k_1 \text{and traffic is congested} \end{cases} \tag{3.9}$$

This shape of the fundamental diagram allows for two traffic states with similar densities but different flows. This can yield unrealistic solutions to the kinematic wave model (see section 4).

WU

An addition to the inverse-lambda fundamental diagram is made by Wu (2002). He assumes the speed in the free flow branch to decrease with increasing density. The shape of the free flow branch is determined by the overtaking opportunities, which in turn depend on the number of lanes, l. The equation for the speed is

$$q = \begin{cases} k\left(1-\left(\frac{k}{k_1}\right)^{l-1} * v_0 + \left(\frac{k}{k_1}\right)^{l-1} v_p\right) & \text{if } v > u_p \\ v_p k_1 - \frac{k-k_1}{k_j-k_1} v_0 k_1 & \text{if } k \geq k_1 \text{and traffic is congested} \end{cases} \tag{3.10}$$

KERNER

Kerner (2004) has proposed a different theory on traffic flow, the so-called three phase traffic flow theory. This will be described in more detail in section 6.4. For here, it is important to note that the congested branch in the three phase traffic flow theory is not a line, but an area.

3.3. MICROSCOPIC BEHAVIOUR

Section 3.2 showed the equilibrium relationships observed in traffic. This is a result of behavior, which can be described at the level of individual drivers as well. This section does so. First, the equilibrium behaviour is described in section 3.3.1. Then, section 3.3.2 discusses hysteresis, i.e. structural off-equilibrium behaviour under certain conditions.

3.3.1. EQUILIBRIUM BEHAVIOUR

The fundamental diagram describes traffic in equilibrium conditions. That can happen if all drivers are driving in equilibrium conditions, i.e. all drivers are driving at a headway which matches a speed. Using the relationships in table 3.1 one can change a fundamental diagram on an aggregated level to a fundamental diagram on an individual level. This way, one can relate individual headways to individual speeds.

The fundamental diagram gives the average distance drivers keep. However, there is a large variation in drivers' behaviour. Some keep a larger headway, and some drivers keep a smaller headway for the same speed. These effects average out in a fundamental diagram, since the average headway for a certain speed is used. On an individual basis, there is a much larger spread in behaviour.

3.3.2. HYSTERESIS

Apart from the variation between drivers, there is also a variation within a driver for a distance it keeps at a certain speed (which we assume in this section as representative of the fundamental diagram). These can be random variations, but there are also some structural variations. Usually the term Hysteresis is used to indicate that the driving behaviour (i.e. the distance) is different for drivers before they enter the congestion compared to after they come out of congestion. That is, the distance *at the same speed* is different in each of these conditions.

Two phenomena might play a role here:

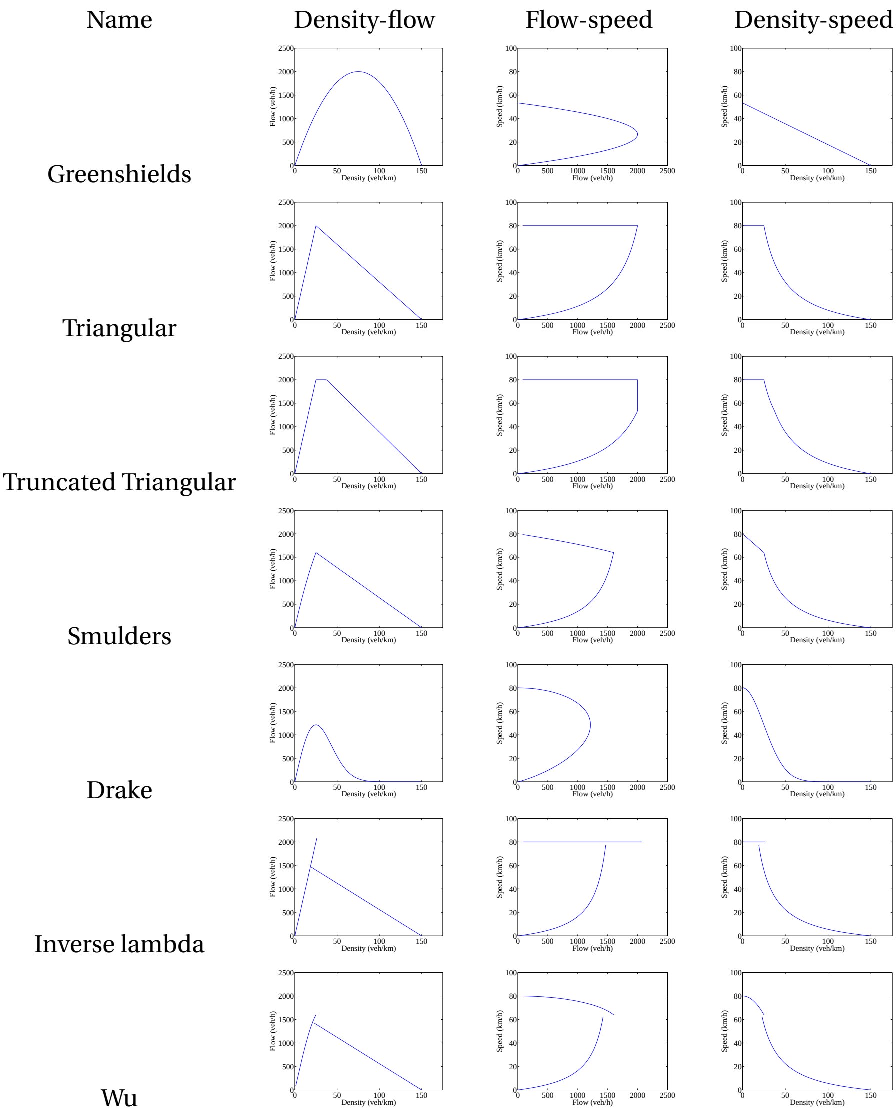
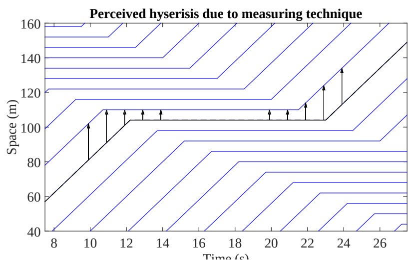
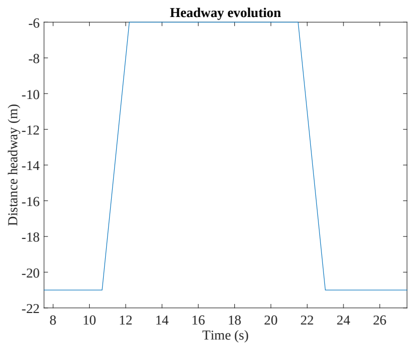


Figure 3.2: Different shapes of the fundamental diagram



(a) Trajectories



(b) Headways

Figure 3.3: Mistakes if making the headway analysis at one moment in time

1. Delayed reaction to a change of speed

2. Anticipation of a change in speed

Zhang (1999) provides an excellent introduction to hysteresis. The simplified reasoning is as follows.

Let's first discuss case 1, drivers have a delayed reaction to a change of speed. That means that when driving at a speed, first the speed of the leader reduces, then the distance reduces. So during the deceleration process, the headway is shorter than the equilibrium headway. When the congestion solves, first the leader will accelerate, and the driver will react late on that. That means that the leader will shy away from the considered car, and the distance will be larger than the equilibrium distance.

In case 2, if the driver anticipates the change in speed, the exact opposite happens. Before the deceleration actually happens, a driver will already decrease speed (by definition in anticipation), leading to a larger headway than the equilibrium headway for a certain speed. Under acceleration, the opposite happens, and a driver can already accelerate before that would be suitable in case of equilibrium conditions. Hence, in the acceleration phase, the driver has a shorter headway than in equilibrium conditions for the same speed.

In traffic, we expect drivers to have a reaction time. In fact, the reaction time can be derived from the fundamental diagram, as section 7.1 will show later on. It will also show that the best way to analyse car-following behaviour is not comparing the distance-speed relationship for one pair *at one moment in time*, as shown in figure 3.3. It shows that the drivers have no hysteresis – they copy the movement of the leader perfectly – but still the gap changes with a constant speed. Instead, one should make the analysis of car-following behavior *along the axis parallel to the wave speed*. Laval (2011) provides a very good insight in the differences one can obtain using this correct technique or using the (erroneous) comparison of instantaneous headways (as in the arrows in figure 3.3b).

3.4. MOVING OBSERVER

An observer will only observe what is in the observation range. Many observations are taken at a point in space . This point might move with time, for instance a driver might check the number of trucks he is overtaking. In this case, the driver is moving and observing, this is called a moving observer. This section discusses the effects of the speed of the moving observer, and also discusses the effect of observed subjects passing a stationary observer with different speeds.

Basically, the movement at speed v has no effect on density, but the relative speed of traffic changes. Therefore, applying equation 3.3, the relative flow changes. Written down explicitly, one obtains a relative flow q_{rel}:

$$q_{\text{rel}} = k(v - v) \tag{3.11}$$

If the observer moves with the speed of the traffic, the observed relative flow becomes zero. In practice if not all vehicles drive at the same speed, the flow needs to be divided into classes with the same speed and the partial flows for each of these classes should be calculated.

$$q_{\text{rel}} = \sum_{v} \left(k_{\text{Vehicles at speed } v}(v - v)\right) \tag{3.12}$$

LOCAL MEASUREMENTS

Suppose there is a local detector located at location x_{detector}. Now we reconstruct which vehicles will pass in the time of one aggregation period. For this to happen, the vehicle i must be closer to the detector than the distance it travels in the aggregation time:

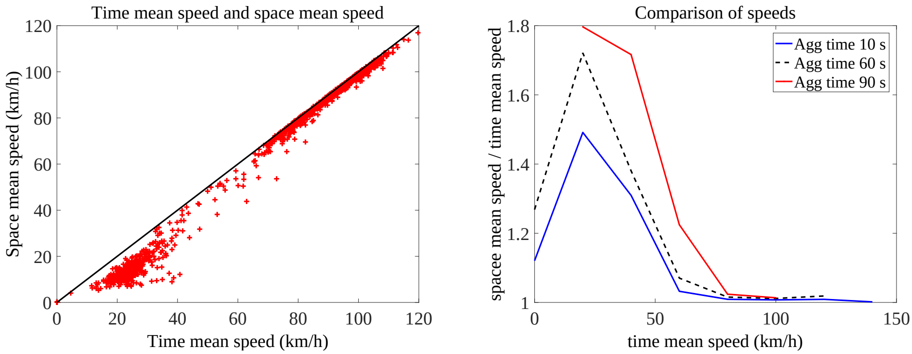
$$x_{\text{detector}} - x_i \leq t_{\text{agg}} v_i \tag{3.13}$$

In this formula, x is the position on the road and t_{agg} the aggregation time. For faster vehicles, this distance is larger. Therefore, if one takes the local (arithmetic) mean, one overestimates the influence of the faster vehicles. If the influence of the faster vehicles on speeds is overestimated, the average speed u_t is overestimated (compared to the space-mean speed u_s).

The discussion above might be conceived as academic. However, if we look at empirical data, then the differences between the time-mean speeds and space-mean speeds become apparent. Figure 3.4a shows an example where the time-mean speed and space-mean speed have been computed from motorway individual vehicle data collected on the A9 motorway near Amsterdam, The Netherlands. Figure 3.4b shows that the time mean speed can be twice as high as the space mean speed. Also note that the space-mean speeds are always lower than the time-mean speeds.

In countries where inductive loops are used to monitor traffic flow operations and arithmetic mean speeds are computed and stored, average speeds are overestimated, affecting travel time estimations. Namely, to estimate the average travel time, the average of $TT = L/v$ is needed, in which L is the length of the road stretch and v the travel speed. Since L is constant, the average travel time can be expressed as

$$\langle TT \rangle = L \left\langle \frac{1}{v} \right\rangle \tag{3.14}$$



(a) The arithmetic mean and the harmonic mean of the speeds of the vehicles passing a cross section of a motorway

(b) The arithmetic mean and the harmonic mean of the speeds of the vehicles passing a cross section of a motorway

Figure 3.4: The effect of inhomogeneities in speeds: the difference in arithmetic mean and harmonic mean speed; figures inspired by and based on the data used in Knoop et al. (2007)

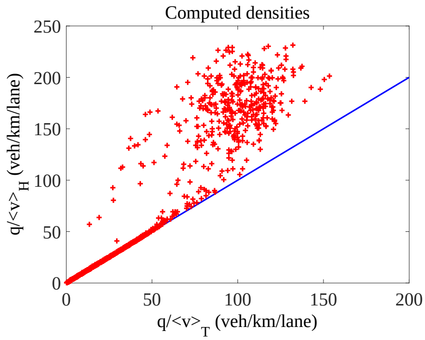


Figure 3.5: Densities computed using arithmic mean speeds or harmonic mean speeds; based on Knoop et al. (2007).

From this, it follows that the harmonically averaged speed is required for estimating the mean travel time.

Furthermore, since $q = ku$ (equation 3.3) can only be used for space-mean speeds, we cannot determine the density k from the local speed and flow measurements, complicating the use of the collected data for, e.g. traffic information and traffic management purposes. As figure 3.4b already shows, largest relative deviation is found at the lower speeds. In absolute terms, this is not too much, so one might argue this is not important. However, a low speed means a high density or a large travel time. For estimating the densities, this speed averaging has therefore a large effect, as can be seen in figure 3.5.

SELECTED PROBLEMS

For this chapter, consider problems: 1, 66, 90, A.5.3, 140, 141, 155, 156, 176, 177, 178, 188, 195, 194, 197, 215, 223, 224, 245, 255, A.10.5, 258, 285, 286, 290

4

Shock wave theory

After this chapter, the student is able to:

- Construct the traffic dynamics in space-time for a given demand profile and one (or more) stationary bottleneck(s), or give properties of traffic given the traffic in space-time

This chapter describes shock wave theory to capture queuing dynamics. This differs from cumulative curves in the way that the spatial extent of the queue is considered. In this section, fixed bottlenecks are discussed. At fixed bottleneck some lanes of a highway are (temporarily) blocked. Firstly the theory and derivation of equations will be discussed (section 4.1). Thereafter, two examples are given (Section 4.2 and 4.3). Then section 4.4 describes the stop-and-go waves in this modelling framework. This framework can also be applied to moving observers, which is presented in chapter 5.

4.1. Theory and derivation of equations

Let us consider a situation with two different states: state A downstream, with a matching q_A, k_A and v_A, and state B upstream (q_B, k_B and v_B). The states are plotted in the space-time diagram 4.1. We choose the axis in such a way that the shockwave moves through the point t=0 at x=0. We will now derive the equation to get the speed of this wave.

In the derivation, we base the reasoning on figure 4.1. The boundary between is called a shock wave. This wave indicates where the speed of the vehicles changes. It is important to note that there are no vehicles captured in the wave itself: the wave itself does not have a physical length. Thus the assumption is that vehicles change speed instantaneously.

Because there are no vehicles in the wave, the number of vehicles entering the wave must be equal to the number of vehicles exiting the wave, or we can state that rate of vehicles entering the wave must be equal to the rate of vehicles exiting the wave. This principle, in combination with the following equation (already presented in equation

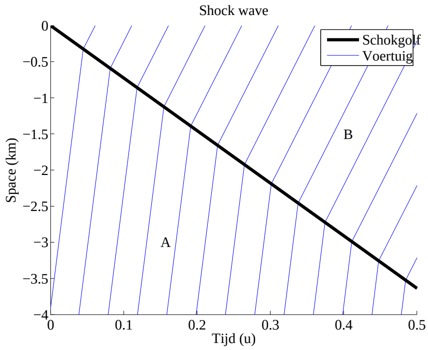


Figure 4.1: A shockwave where traffic speed changes from high to low.

4

3.3) is used to calculate the speed of the wave:

$$q = kv \tag{4.1}$$

The speed of the wave is indicated by w. Note that we can apply this equation to moving frames of reference as well. In that case, the flow changes, as does the speed. The density is invariant under a change of reference speed.

To determine the attachment and exit rate, we will move with the speed of the wave (in the frame of a moving observer). At the downstream end, the density is k_B. The speed in the moving frame of reference is $v_B - w$. The exit rate in the moving frame of reference is calculated using equation 4.1.

$$q_{\text{exit}} = k_B\,(v_B - w) \tag{4.2}$$

In the same way, the attachment rate can be determined. The upstream density is k_A. The speed of the vehicles in the frame of reference moving with the wave speed w is $v_A + w$. Using equation 4.1 again, we find the attachment rate in this moving frame of reference:

$$q_{\text{attachment}} = k_A\,(v_A - w) \tag{4.3}$$

Since these rates have to be equal, we find:

$$q_{\text{exit}} = q_{\text{attachment}} \tag{4.4}$$
$$k_A\,(v_A - w) = k_B\,(v_B - w) \tag{4.5}$$

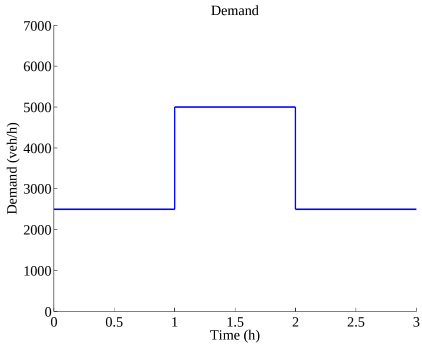
This can be rewritten as:

$$k_A v_A - k_A w = k_B v_B - k_B w \tag{4.6}$$
$$q_A - k_A w = q_B - k_B w \tag{4.7}$$

In the last step, the speeds have been substituted using equation 4.1. We can solve this equation for the shock wave speed w. We find

$$q_A - q_B = (-k_B + k_A)\,w \tag{4.8}$$

(a) The road



(b) Demand pattern

Figure 4.2: Situation

And isolating w gives the wave speed equation:

$$\boxed{w = \frac{q_A - q_B}{k_A - k_B} = \frac{\Delta q}{\Delta k}} \tag{4.9}$$

Note that in a space-time plot, the speed w is the slope of the shock wave between A and B. The right hand side is the ratio between the difference in flow and the difference in density of states A and B. This is also the slope of a line segment between A and B in the flow-density plot. This becomes very useful when constructing the traffic states.

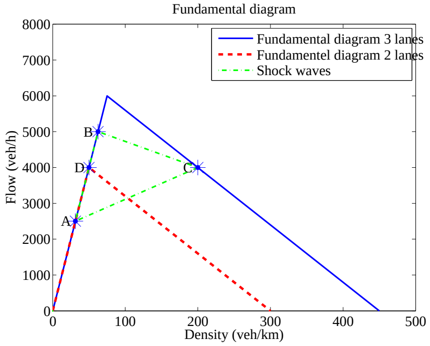
The above reasoning holds for the speed of any shock wave, moving backward or forward. The following section shows an example for both.

4.2. EXAMPLE: TEMPORAL INCREASE IN DEMAND AT A ROAD WITH A LANE DROP

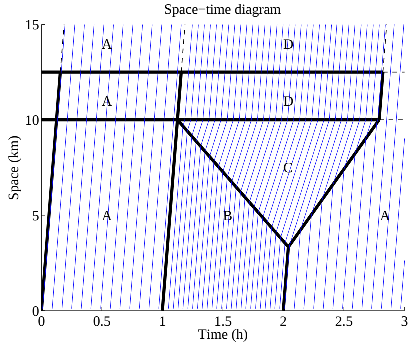
Let's consider a 3 lane road with a reduction to 2 lanes over a 1 km section between x=10 and x=12.5 (see figure 4.2b). For the road, we assume lanes with equal characteristics, described by a triangular fundamental diagram with a free speed of 80 km/h, a capacity of 2000 veh/h/lane and a jam density of 150 veh/km/lane. At the start of the road, there is a demand of 2500 veh/h which temporarily increases to 5000 veh/h between t=1h and t=2h (see the demand profile in figure 4.2b). The question we will answer in this example is: *What are the resulting traffic conditions?*

The final answer to the question are the traffic states which are shown in table 4.1, and shown on the fundamental diagram in figure 4.3a. The speed of the shock waves is given in table 4.2, and the resulting traffic situation is shown in figure 4.3b. We will now explain how this solution can be found.

The inflow to the system is given, being 2500 veh/h (state A) and 5000 veh/h (state B) on a three lane road. The matching densities can be computed from the fundamental diagram for a three lane road. This gives densities of (equation 4.1) $k_A = \frac{q_A}{u_A} = \frac{2500}{80} =$ 31.25veh/km and $k_B = \frac{q_B}{u_B} = \frac{5000}{80} = 62.5$veh/km. The separation with the empty road



(a) Position of traffic states at the fundamental diagram



(b) Shock waves in space-time

Figure 4.3: The situation

4

Table 4.1: The states on the road

State	Flow (ven/h)	Density (veh/km)	Speed (km/h)
A	2500	31.25	80
B	5000	62.5	80
C	4000	200	20
D	4000	50	80

Table 4.2: The shock waves present on the road

State 1	State 2	shock wave speed w (km/h)
A	B	80
B	C	-7.3
A	C	8.9

moves forward with a speed of 80 km/h, i.e. the speed of the vehicles. This can also be found by the shock wave equation, equation 4.9. The difference in flow is 2500 veh/h and the difference in density is 31.25 veh/km. The shock wave then moves with:

$$w_{0A} = \frac{q_0 - q_A}{k_0 - k_A} = \frac{-2500}{-31.25} = 80 km/h \tag{4.10}$$

Note that it does not matter whether the speed of shock wave w_{0A} or shock wave w_{A0} is calculated. This remark holds for every combination of states.

Also the wave between state A and B moves forward with 80 km/h, calculated by equation 4.9:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 5000}{31.25 - 62.5} = 80 km/h \tag{4.11}$$

Note that this equals the free flow speed. This is because we use a triangular fundamental diagram. The speed w_{ab} is the direction of the line segment AB in figure 4.3a. This has the same slope as the slope of the free flow speed (i.e. the slope of the fundamental diagram at the origin) because its shape is triangular.

Now this wave hits the two lane section. The flow is higher then the capacity of the two lane section. That means that downstream, the road will operate at capacity, and upstream a queue will form (i.e. we have a congested state). The capacity of the downstream part is, according to the fundamental diagram, 4000 veh/h (state D). The speed follows from the (triangular) fundamental diagram, and is 80 km/h. The density hence is:

$$k_D = \frac{q_D}{u_D} = \frac{4000}{80} = 50 veh/km \tag{4.12}$$

If 4000 veh/h drive onto the two lane segment, 4000 veh/h have to drive off the three lane segment (no vehicles can be lost or created at the transition from three to two lanes). That means that upstream of the transition, we have a congested state with a flow of 4000 veh/h. The density is derived from the fundamental diagram:

$$q_{\text{cong}} = q_{\text{capacity}} - q_{\text{capacity}} \frac{k - k_c}{k_j - k_c} = 4000 \tag{4.13}$$

Substituting the parameters for the fundamental diagram, and realising that the capacity is found by $q_{\text{capacity}} = l u_{\text{capacity}} k_c$ (l is the number of lanes) we calculate the density at point C, 200 veh/km. Graphically, we can find point C on the fundamental diagram by the intersection of the congested branch and a line at a constant flow value of 4000 veh/h.

The speed at which the tail of the queue moves backwards, i.e., the speed of the boundary between B and C, is calculated by the shock wave equation (equation 4.9)

$$w_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{5000 - 4000}{62.5 - 200} = -7.3 \text{km/h} \tag{4.14}$$

Note that this speed can also be derived graphically from the fundamental diagram, i.e. the slope of the line segment BC. That the slope in figure 4.3a and figure 4.3b is graphically not the same, which is a consequence of different axis scales in the figures.

Table 4.3: The states on the road with a temporal bottleneck

Number	Flow	Density	Speed
A	2500	31.25	80
B	1000	387.5	
C	1000	12.5	12.5
D	6000	75	75

Then the demand reduces again and the inflow state returns to A. When lower demand hits the tail of the jam, the queue can solve from the tail. At the head of the queue, this change has no influence yet, since drivers are still waiting to get out onto the smaller roadway segment. The boundary between C and A moves with a speed of

$$w_{CA} = \frac{q_C - q_A}{k_C - k_A} = \frac{4000 - 2500}{200 - 31.25} = 8.9\text{km/h} \tag{4.15}$$

Here again, the speed could also be derived graphically, by the slope of the line segment AC.

Then, this wave arrives at the transition of the two lane road to a three lane road, and the congestion state C is dissolved. In the two lane part, the flow is now the same as the demand (the state in A, 2500 veh/h). The boundary between state D and A moves forward with a speed of

$$w_{DA} = \frac{q_D - q_A}{k_D - k_A} = \frac{4000 - 2500}{50 - 31.25} = 80\text{km/h} \tag{4.16}$$

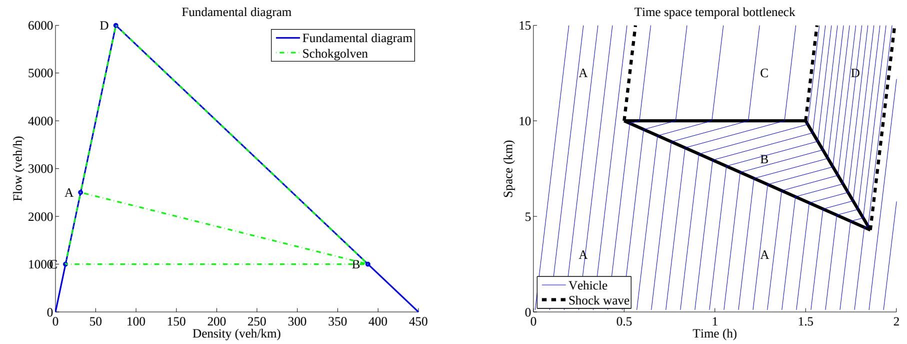
4.3. EXAMPLE: TEMPORAL CAPACITY REDUCTION

Another typical situation is a road with a temporal local reduction of capacity, for instance due to an accident. This section explains which traffic situation will result from that.

The case is as follows. Consider a three-lane freeway, with a triangular fundamental diagram. The free flow speed is 80 km/h, the capacity is 2000 veh/h/lane and the jam density is 150 veh/km/lane. The demand is constant at 2500 veh/h. From t=0.5h to t=1.5h, an incident occurs at x=10, limiting the capacity to 1000 veh/h. Calculate the traffic states and the shock waves, and draw them in the space-time diagram. Also draw several vehicle trajectories.

For referring to certain states, we will first show the resulting states, and then explain how these states are constructed. Figure 4.4a shows the fundamental diagram and the occurring states, 4.4b shows how the states move in space and time. The details of the states can be found in table 4.3, and the details of the shock waves can be found in table 4.4.

At the start, there are free flow conditions (state A) at an inflow of 2500 veh/h. For the assumed triangular fundamental diagram, the speed for uncongested conditions is equal to the free flow speed, so 80 km/h. The matching density can be found by applying equation 4.1: $k_A = \frac{q}{u} = \frac{2500}{80} = 31.25$ veh/km.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 4.4: The situation

Table 4.4: The shock waves present on the road with a temporal bottleneck

State 1	State 2	shock wave speed w (km/h)
A	C	80
B	C	0
A	B	-4.2
B	D	-16

From t=0.5h to t=1.5h, a flow limiting condition is introduced. We draw this in the space time diagram. The flow is too high to pass the bottleneck, so the moment the bottleneck occurs, a congested state (B) will form upstream. Downstream of the bottleneck we find uncongested conditions (once the vehicles have passed the bottleneck, there is no further restriction in their progress): state C. For state C, the flow equals the flow that can pass the bottleneck, which is given at 1000 veh/h. The speed is the free flow speed of 80 km/h, so the matching density can be found by applying equation 4.1: $k_C = \frac{q}{u} = \frac{1000}{80} = 12.5$ veh/km. The speed of the shock wave between state A and C can be calculated using the shock wave equation, 4.9:

$$w_{AC} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 1000}{31.25 - 12.5} = 80 km/h \tag{4.17}$$

This equals the free flow speed. Graphically, we understand this because both states can be found at the free flow branch of the fundamental diagram (figure 4.4a) and the shock wave speed is the slope of the line segment connecting these states. Because the fundamental diagram is triangular, this slope is equal to the slope at the origin (i.e., the free flow speed).

Upstream of the bottleneck a congested state forms (B). The flow in this area must be the same as the flow which can pass the bottleneck. This is because at the bottleneck no new vehicles can be formed. That means state B is a congested state with a flow of 1000 veh/h. From the fundamental diagram we find the matching density in the congested branch, 387.5 veh/km. The speed at which the shock between states A and B now moves, can be calculated using equation 4.9:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 1000}{31.25 - 387.5} = -4.2\text{km/h} \tag{4.18}$$

The minus sign means that the shock wave moves in the opposite direction of the traffic, upstream. We could also graphically derive the speed of the shock wave by the slope of the line between point A and B in the fundamental diagram.

Once the temporal bottleneck has been removed, the vehicles can drive out of the queue: state D. Because state C is congested, the outflow will be capacity (or the queue discharge rate in case there is a capacity drop). That is for this case a flow of 3x2000 veh/h = 6000 veh/h. Realising the vehicle speed equals the free flow speed of 80 km/h, the density can be found using equation 4.1:

$$k_D = \frac{q_D}{u_D} = \frac{6000}{80} = 75\text{km/h} \tag{4.19}$$

The shock wave between state B and D moves backward. The speed thereof can be found by applying equation 4.9

$$w_{BD} = \frac{q_B - q_D}{k_B - k_D} = \frac{1000 - 6000}{387.5 - 75} = -16\text{km/h} \tag{4.20}$$

The negative shock wave speed means the wave moves upstream. Intuitively, this is right, since the vehicles at the head of the queue can accelerate out of the queue, and thus the head moves backwards.

4.4. STOP AND GO WAVES

On motorways, often so called stop-and-go waves occur. These short traffic jams start from local instabilities, and the speed (and flow) in the stop-and-go waves is almost zero. For a more detailed explanation, see section 6.3. Now the speed of the boundaries of the traffic states are known, we can apply this to stop-and-go waves, and understand why this typical pattern arises (figure 4.5).

Stop-and-go waves arise in dense traffic. The traffic demand is then mostly near capacity, or at approximately the level of the queue discharge rate. The upstream boundary of such a shock moves upstream. When it is in congested conditions, both the upstream state (the congested condition) and the downstream state (the standing traffic) are congested, hence the upstream boundary moves backwards with a speed equal to the wave speed of the fundamental diagram. Once it gets out of congestion, the inflow is most likely approximately equal to the capacity of the road. That means that the upstream boundary moves with a speed which is equal to the slope of the line in the fundamental diagram connecting the capacity point with the point of jam density, which is the wave speed.

The downstream boundary separates the jam state (standing traffic) with capacity (by default: vehicles are waiting to get out of the jam, hence a capacity state occurs). The speed between these two states is found by connecting the points in the fundamental diagram. The resulting speed is the wave speed of the fundamental diagram.

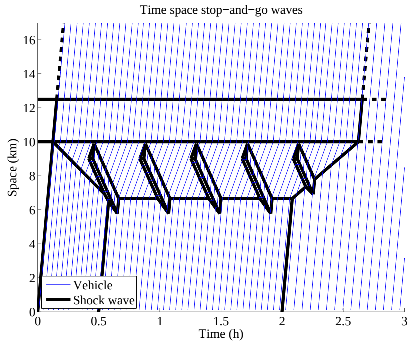


Figure 4.5: Stop and go waves in time and space

After the first stop-and-go wave has moved upstream, the inflow in the second stop-and-go wave equals the outflow of the first stop-and-go wave. These are the upstream respectively the downstream state of the second wave, which thus are the same. In between, within the wave, there is another, jammed state. According to shock wave theory the shock between state A and B (i.e., the upstream state and the state within the stop-and-go wave) and the shock between B and A (i.e. the state within the stop-and-go wave and the downstream state, which equals the upstream state) is the same. This speed is approximately equal for all roads, 15-20 km/h (Schreiter et al., 2010). Hence, the length of the stop-and-go wave remains the same.

This way, stop-and-go waves can travel long distances upstream. Section 6.4 will discuss the stop-and-go waves and their characteristics in more detail.

SELECTED PROBLEMS

This chapter is a part of what was previously a combination of fixed bottlenecks (this chapter) and moving bottlenecks (now chapter 5). The following exercises mostly start with a fixed bottleneck, but at some point include a moving bottleneck. Students are supposed to be able to answer these questions up to the moving bottleneck. These problems are: A.1.4, A.2.6, 67, 72, A.3.4, A.4.4, 101, 102, A.5.4, A.6.3, A.7.4, A.8.3, A.8.4, A.9.3, 259, 275, 287.

5

Shockwave theory: moving bottlenecks

After this chapter, the student is able to:

- Construct the traffic dynamics in space-time for a given demand profile and one (or more) stationary or moving bottleneck(s), or give properties of traffic given the traffic in space-time

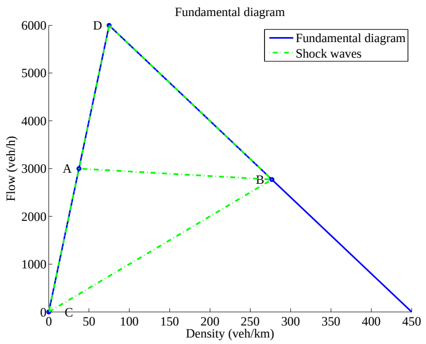
This section describes what happens if the road is blocked, either completely or not completely, by a moving bottleneck. This can be a slow moving truck or agricultural vehicle, a funeral or wedding procession. Moving bottlenecks are indeed relevant for these practical events, but is also used in a more theoretical framework, for instance applying variational theory for capacity estimates (Daganzo, 2005). In the current chapter, first the theory is explained (section 5.1), and then three examples follow (section 5.2, 5.3 and 5.4).

5.1. Theory

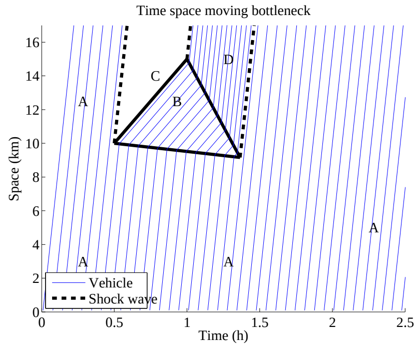
For the moving bottleneck, the same theory applies as for the fixed bottleneck. The recipe is the same: check for each bottleneck weather the demand exceeds capacity. If so, there is congestion upstream and a free flow condition (or capacity) downstream. Once again, the shock wave equation applies (equation 4.9).

Different compared to the regular, fixed bottlenecks, is the position of the congested state. For fixed bottlenecks, the flow upstream of the bottleneck equals the flow downstream of the bottleneck. In case of moving bottlenecks this differs. Consider a bottleneck which moves downstream without any overtaking opportunities. The downstream flow is zero, but vehicles can accumulate in the growing area between the considered point and the moving bottleneck, so the upstream flow is not zero.

In these types of calculations, the capacity of vehicles passing the moving bottleneck is usually an input. This gives the downstream point at the fundamental diagram. To



(a) Position of traffic states at the fundamental diagram



(b) Shock waves in space-time

Figure 5.1: The situation with a moving bottleneck without overtaking opportunities

5

find the upstream state on the fundamental diagram, one has to realise the shock wave *is* the moving bottleneck, so the speed of the shock wave *must* equal the speed of the moving bottleneck. By constructing a shock wave moving with the speed of the moving bottleneck in the fundamental diagram from the uncongested downstream point, one finds the congested point. This is at the intersection of the line segment starting at the uncongested point moving with the bottleneck speed and the congested branch of the fundamental diagram. The following examples clarify the procedure.

5.2. EXAMPLE 1: MOVING TRUCK, NO OVERTAKING POSSIBILITIES

Consider a three lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is 3000 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h. There are no overtaking opportunities. What is the traffic state at the road?

The solution of this example is given in a space-time diagram (figure 5.1b) and in the fundamental diagram (figure 5.1a). The characteristics of the states are given in table 5.1, and the characteristics of the shock waves are given in table 5.2. The explanation how these points are found follows below.

At two points states can be identified at the fundamental diagram (figure 5.1a), the intial state A (flow of 2500 veh/h, free flow speed 80 km/h) and the state downstream of the moving bottleneck (given “no overtaking possibilities” hence density zero and flow zero). In these examples, the states are identified by only two variables from the three flow, density and speed, since the third one can be calculated using equation 4.1.

The position upstream of the moving bottleneck can be determined by the intersection of two lines:

$$q_1 = q_C + v(k - k_C) \tag{5.1}$$
$$q_2 = q_D + w(k - k_c) \tag{5.2}$$

Table 5.1: The states on the road for a moving bottleneck without overtaking opportunities

Name	Flow	Density	Speed
A	3000	37.5	80
B	2769	277	10
C	0	0	NaN
D	6000	75	80

Table 5.2: The shock waves present on the road for a moving bottleneck without overtaking opportunities

State 1	State 2	shock wave speed w (km/h)
A	B	-0.96
B	C	10
A	C	80
D	C	80
B	D	-16
A	D	80

The first equation is a line in the fundamental diagram, figure 5.1a starting from point C and moving forward with the bottleneck speed v. The second line is the congested branch of the fundamental diagram, in which w is the slope of the congested branch. The intersection of these lines can be found by solving the density k from the equation $q_1 = q_2$. For this upstream density, the density of state B, we find $k_B = 277$ veh/km. The matching flow is found by filling this in either equation 5.1 or 5.2, resulting in $q_B = 2769$ veh/km. The speed is determined by the ratio of flow and density: $u = \frac{q_B}{k_B} = \frac{2769}{277} = 10$km/h. Note that this is the speed of the moving bottleneck. This must be since there are no overtaking opportunities.

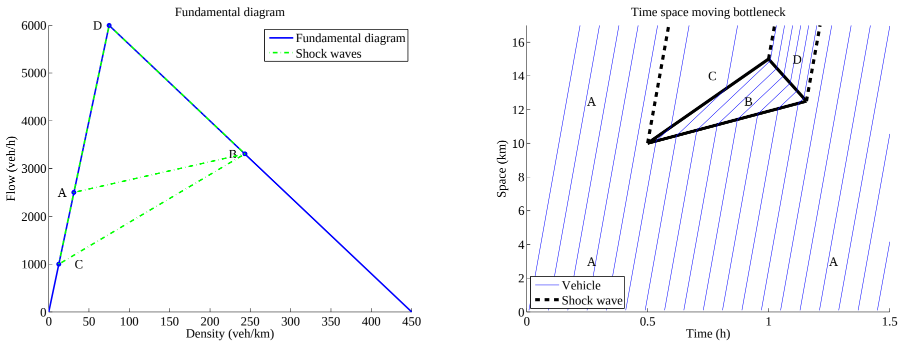
The speed of the shock wave between A and B can be calculated using the shock wave equation 4.9 applied to state A and B:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{3000 - 2769}{37.5 - 277} = -0.96\text{km/h} \tag{5.3}$$

This means the shock wave moves upstream with a low speed. With a different speed of the moving bottleneck or a different demand level, the shock wave might move faster upstream (lower bottleneck speed or higher demand), or it might move downstream (higher bottleneck speed or lower demand).

The speed of the shock wave between state B and C can also be calculated with equation 4.9. It can be determined from reasoning instead of calculations as well: it must be equal to the speed of the moving bottleneck, 10 km/h.

Once the moving bottleneck has left the road, vehicles will exit the jam. This means a boundary between the jam state, state B, at the capacity state, state D. For capacity the flow (6000 veh/h) and speed (80 km/h) can be derived from the road characteristics. The speed of the shock wave between state D and B can be determined by applying the shock wave equation (4.9) on state B and D. Alternatively, from the fundamental diagram in figure 5.1a we see that the speed of the shock wave must equal the slope of the congested



(a) Position of traffic states at the fundamental diagram (b) Shock waves in space-time

Figure 5.2: The situation at a moving bottleneck with overtaking possibilities

Table 5.3: The states on the road at a moving bottleneck with overtaking possibilities

State	Flow (veh/h)	Density(veh/km)	Speed (km/h)
A	2500	31.25	80
B	3307	243	13.6
C	1000	12.5	80
D	6000	75	80

branch of the fundamental diagram, -16 km/h. Because any shock wave between any congested state and capacity moves with this speed, this speed is also called the wave speed of the fundamental diagram.

Speeds of shock waves between A and C, C and D, and D and A all can be calculated using the shock wave equation 4.9. Moreover, all these states lie on the free flow branch of the fundamental diagram, so the shock waves between these states move at the free flow speed of 80 km/h.

5.3. EXAMPLE 2: MOVING TRUCK WITH OVERTAKING POSSIBILITIES

Now, let's consider a different situation, where overtaking of the moving bottleneck is possible. We change the conditions as follows. Consider a three lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is 2500 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h. There are overtaking opportunities, such that downstream of the bottleneck the flow is 1000 veh/h. What is the traffic state at the road?

The states and fundamental diagram, as well as the resulting traffic states are show in figures 5.2a and 5.2b respectively. Tables 5.3 and 5.4 give the properties of the states and the shock waves respectively. Below, it is explained how these states and waves are found.

Table 5.4: The shock waves present on the road at a moving bottleneck with overtaking possibilities

State 1	State 2	shock wave speed w (km/h)
A	B	3.8
A	C	80
D	C	80
B	D	-16
A	D	80

The demand is the same as in the previous example, so state A is the same. Downstream of the moving bottleneck there is a free flow traffic state (it is downstream of the bottleneck, so it is free flow). The flow is given at 1000 veh/h, and since it is in free flow, the speed is 80 km/h. Hence, the density is $k_C = \frac{q_C}{u_C} = \frac{1000}{80} = 12.5$ veh/km.

Since the demand is higher than the capacity of the moving bottleneck, upstream of the bottleneck, a congested state occurs. Is is separated from state C by a shock wave which moves with the speed of the moving bottleneck (it *is* the moving bottleneck). This means we have to find state B in the fundamental diagram which connects to state C with a line with a slope of 10 km/h; this line is indicated by q_1 (equation 5.4) Furthermore, state B has to lie on the congested branch of the fundamental diagram, indicated by q_2 (equation 5.5). The position upstream of the moving bottleneck can be determined by the intersection of two lines:

5

$$q_1 = q_C + v(k - k_C) \tag{5.4}$$
$$q_2 = q_D + w(k - k_c) \tag{5.5}$$

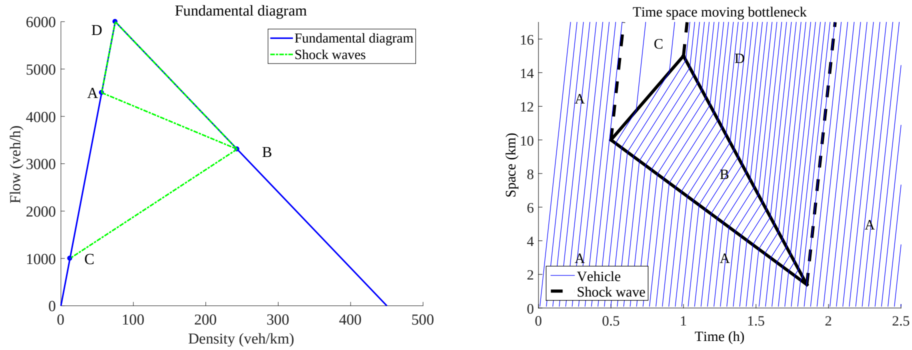
We find the density for state B by $q_1 = q_2$. This results in $k_B = 243$ veh/km. The matching flow can be found by subsituting this into equation 5.4 or equation 5.5, leading to q_B=3307 veh/h.

The shock wave speeds between B and C, as well as the shock wave speed between A and B can be calculated using the shock wave equation, equation 4.9:

$$w_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{3307 - 1000}{243 - 12.5} = 10 \text{ km/h} \tag{5.6}$$
$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 3307}{31.25 - 243} = 3.8 \text{ km/h} \tag{5.7}$$

Note that the shock wave between B and C moves with the speed of the moving bottleneck.

Once the moving bottleneck leaves the road, the method is exactly the same as in the previous example. The queued vehicles exit state B at the road capacity, state D with a flow 6000 veh/h and a speed of 80 km/h. The shock wave between state D and B moves backward with a speed which can be calculated by the shock wave equation. The shock wave speed must also be equal to the wave speed of the fundamental diagram, so using the knowledge of the previous example, we find $w_{BD} = -16$ km/h.



(a) Position of traffic states at the fundamental diagram

(b) Shock waves in space-time

Figure 5.3: The situation for the moving bottleneck with overtaking opportunities and a high demand.

5.4. EXAMPLE 3: MOVING TRUCK AND HIGH DEMAND

Consider a three lane road, where at all three lanes a triangular fundamental diagram holds. The capacity is 2000 veh/h/lane, the free flow speed 80 km/h and the jam density 150 veh/km. The demand is increased to 4500 veh/h. A truck enters the road at t=0.5h and x=10 km, and leaves the road at t=1h and x=15 km, hence driving 10 km/h, limiting the flow downstream of the moving bottleneck to 1000 veh/h. What are the conditions on the road?

The states and fundamental diagram, as well as the resulting traffic states are shown in figures 5.3a and 5.3b respectively. Tables 5.5 and 5.6 give the properties of the states and the shock waves respectively. This situation is very similar to the situation described in example 2 (section 5.3). Below, we will comment on the similarities and differences.

The downstream part is the same. States A and C, as well as the congested state upstream of the moving bottleneck are determined by the properties of the moving bottleneck, and are thus the same as in the previous example. The speed of the wave between state A and B differs, since state A is different. The speed can be calculated using the shock wave equation, equation 4.9:

$$w_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{4500 - 3307}{56.25 - 243} = -6.3 \text{ km/h} \tag{5.8}$$

So, the methodology to compute the shock wave speed is the same. However, the phenomenon is difference: contrary to the previous example, the shock wave moves upstream.

When the moving bottleneck leaves the road, the queue discharge (capacity) state D is also the same as in the previous example. Since states B and D are the same, the shock wave speed between the two is also the same. Also the shock wave speed between state A and D is the same, because both points are at the free flow branch of the fundamental diagram, hence the shock wave speed is the speed of the free flow branch of the (triangular) fundamental diagram.

Table 5.5: The states on the road for the moving bottleneck with overtaking opportunities and a high demand.

State	Flow (veh/h)	Density(veh/km)	Speed (km/h)
A	4500	56.25	80
B	3077	243	13.6
C	1000	12.5	80
D	6000	75	80

Table 5.6: The shock waves present on the road for the moving bottleneck with overtaking opportunities and a high demand

State 1	State 2	shock wave speed w (km/h)
A	B	-6.3
A	C	80
B	C	10
D	C	80
B	D	-16
A	D	80

5

SELECTED PROBLEMS

A.1.4, A.2.6, 67, 72, A.3.4, A.4.4, 101, 102, A.5.4, A.6.3, A.7.4, A.8.3, A.8.4, A.9.3, 259, 275, 287.

6

Traffic states and Phenomena

After this chapter, the student is able to:

- Comment on different levels of stability for traffic
- Recognise traffic states from traffic measurements, and derive the causes for the observed traffic states
- Comment on the capacity drop, and show this in empirical data
- Comment on the three states of traffic according to Kerner's theory

This chapter discusses some of the important phenomena in traffic.

6.1. Stability

In traffic, we can differentiate between three levels of stability: local, platoon, and traffic. This is indicated in table 6.1, and shown graphically in figure 6.1.

Table 6.1: Identification scheme for different levels of stability

Name	relevant vehicles	criterion
Local	One vehicle pair	Does the follower relax into a steady state speed after a speed change of the leader?
Platoon	A platoon of vehilces	Does the change of speed of a leader increase over the vehicle number in the platoon?
Traffic	More than one platoon	Does the speed disturbance propagate to the next platoon?

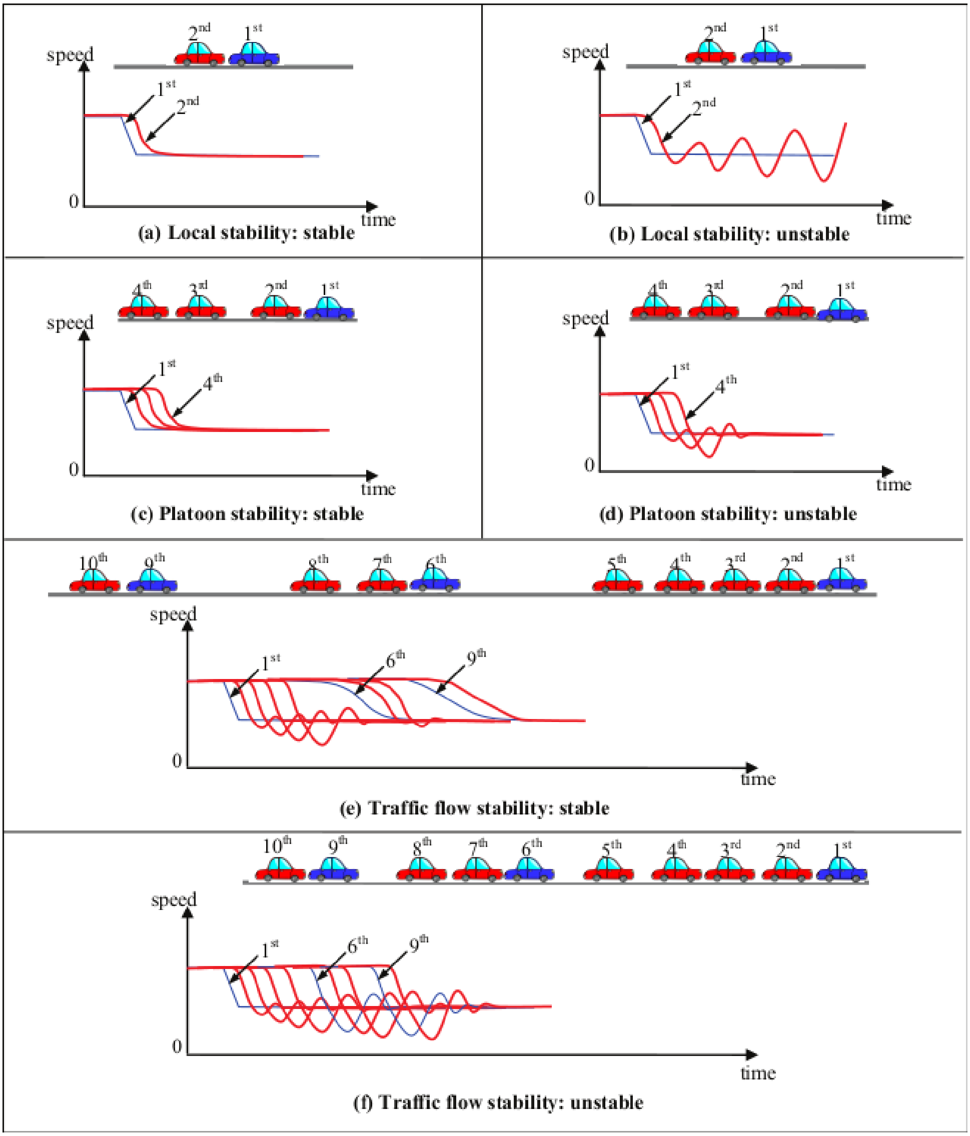


Figure 6.1: Graphical view of the three different types of stability. The graphs show the speeds as function of time for different vehicles. Figure concept from Pueboobpaphan and van Arem (2010)

6.1.1. LOCAL STABILITY

For local stability, one has to consider one vehicle pair, in which the second vehicle is following the leader. If the leader reduces speed, the follower needs to react. In case of local stability, he does so gracefully and relaxes into a new state. Some car following models in combination with particular parameter settings will cause the following vehicle to continuously change speeds. The following vehicle then has oscillatory speed with increasing amplitude. This is not realistic in real life traffic.

6.1.2. PLATOON STABILITY

For a platoon, it should be considered how a change of speed propagates through a platoon. This is called platoon stability or string stability. Consider a small change in speed for the platoon leader. The following vehicle needs to react (all vehicles in the platoon are in car-following mode). Platoon stability is whether the speed disruption will grow over the vehicle number in the platoon. In platoon stable traffic, the disruption will reduce, in platoon unstable traffic, this disruption will grow.

Note that platoon stability becomes only relevant to study for locally stable traffic. For locally stable traffic, one can have platoon instability if for a single vehicle the oscillations decrease over time, but they increase over the vehicle number, as shown in figure 6.1.

6.1.3. TRAFFIC FLOW STABILITY

The third type of stability is traffic flow stability. This indicates whether a traffic stream is stable. For the other types of stabilities, we have seen that stability is judged by the speed profile of another vehicle. For traffic flow stability it matters whether a change in speed of a vehicle will increase over a long time to any vehicle, even exceeding the platoon boundary. Traffic which is platoon stable is always traffic stable. The other way around, platoon unstable traffic is not always traffic unstable. It could happen that the unstable platoons are separated by gaps which are large enough to absorb disruptions. In this case, the traffic flow is stable, but the platoons are not. If the disruption propagates to other platoons, the traffic flow is unstable.

6.1.4. USE OF STABILITY ANALYSIS

There are mathematical tools to check analytically how disruptions are transmitted if a continuous car-following model is provided. For platoon stability this is not very realistic since all drivers drive differently. Therefore an analytical descriptions should be taken with care if used to describe traffic since one cannot assume all drivers to drive the same. In fact, driver heterogeneity has a large impact on stability. The other way around, the stability analysis could indicate whether a car-following model is realistic.

From data, one knows that traffic is locally stable and mostly platoon unstable. Depending on the traffic flows, it can be traffic unstable or stable. For large number of vehicles, traffic is generally stable (otherwise, there would be an accident for any speed disruption).

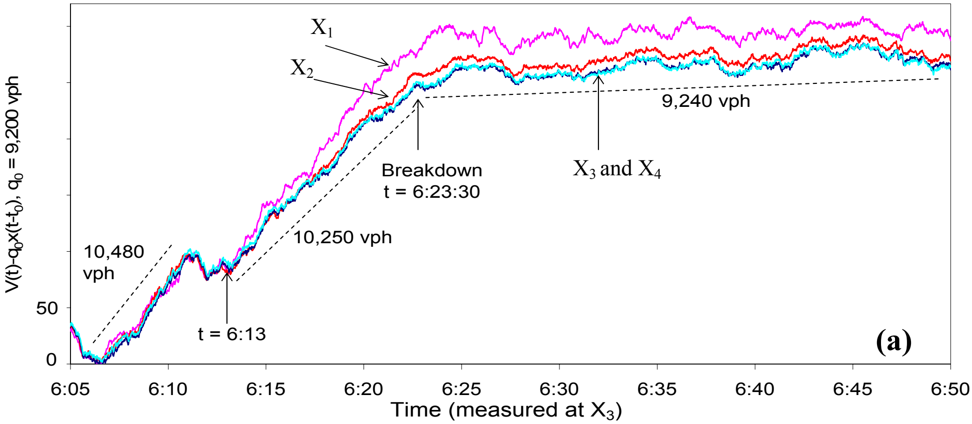


Figure 6.2: Example of an analysis with slanted cumulative curves, from Cassidy and Rudjanakanoknad (2005)

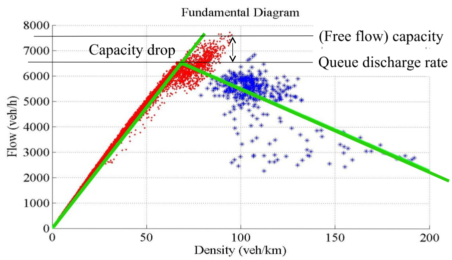


Figure 6.3: Illustration of the capacity drop in the fundamental diagram using real-world data

6.2. CAPACITY DROP

6.2.1. PHENOMENON DESCRIPTION

The capacity drop is a phenomenon that the maximum flow over a bottleneck is larger before congestion sets in than afterwards. That is, once the bottleneck is active, i.e. there is a congested state upstream of the bottleneck and no influence from bottlenecks further downstream, the traffic flow is lower. This flow is called the queue discharge rate, or sometimes the outflow capacity.

The best way to analyse this is by slanted cumulative curves. Generally, the flow during congestion would be the same. That is, the queue outflow does not depend on the length of the queue. Therefore, a natural way to offset the cumulative curves is by the flow observed during discharging conditions. One would observe an increasing value line congestion sets in and a horizontal line during congestion. Figure 6.2 shows an example with real life data (figure from Cassidy and Rudjanakanoknad (2005)).

In the fundamental diagram, one would then observe a congested branch which will not reach the capacity point. Instead, one has a lower congested branch, for instance the inverse lambda fundamental diagram. The capacity drop is the difference between the free flow capacity and the queue discharge rate. This is shown in figure 6.3.

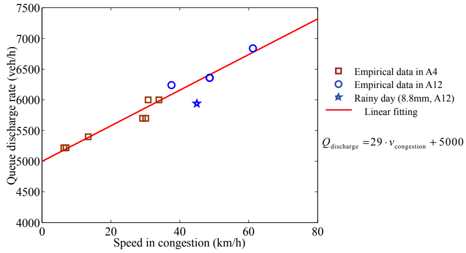


Figure 6.4: The dependency of the queue discharge rate of the speed in the queue (from Yuan et al. (2015a)

6.2.2. EMPIRICS (FROM YUAN ET AL. (2015A))

Capacity with congestion upstream is lower than the possible maximum flow. This capacity drop phenomenon has been empirically observed for decades. Those observations point out that the range of capacity drop, difference between the bottleneck capacity and the queue discharging rate, can vary in a wide range. Hall and Agyemang-Duah (1991) report a drop of around 6% on empirical data analysis. Cassidy and Bertini (1999) place the drop ranging from 8% to 10%. Srivastava and Geroliminis (2013) observe that the capacity falls by approximately 15% at an on-ramp bottleneck. Chung et al. (2007) present a few empirical observations of capacity drop from 3% to 18% at three active bottlenecks. Excluding the influences of light rain, they show at the same location the capacity drop can range from 8% to 18%. Cassidy and Rudjanakanoknad (2005) observe capacity drop ranging from 8.3% to 14.7%. Oh and Yeo (2012) collect empirical observations of capacity drop in nearly all previous research before 2008. The drop ranges from 3% up to 18%. The large drop of capacity reduces the performance of road network.

In most of observations, capacity drop at one bottleneck only exhibits a small day to day deviation (Cassidy and Bertini, 1999; Chung et al., 2007). However, it is possible to observe a large difference in the capacity drop empirically at the same location. Srivastava and Geroliminis (2013) observe two different capacity drop, around 15% and 8%, at the same on-ramp bottleneck. Yuan et al. (2) observe different discharging flows at the same freeway section with a lane-drop bottleneck upstream and estimate the outflow of congestion in three-lane section ranging from 5400 vph to 6040 vph. All of these studies show that the capacity drop can be controlled and some strategies have been described to reach the goal (Carlson et al., 2010; Cassidy and Rudjanakanoknad, 2005; Chung et al., 2007). Those control strategies strongly rely on the relation between the congestion and the capacity drop.

Recent research Yuan et al. (2015a) clearly shows this relationship with speed, see figure 6.4 (figure from Yuan et al. (2015a)).

The main cause of the capacity drop is not identified yet. Some argue it is lane changing, others argue it is the limited acceleration (not instantaneous), whereas others argue it is the *difference* in acceleration (not all the same). This remains an active field of research, both for causes of the capacity drop and for ways to control it.

6

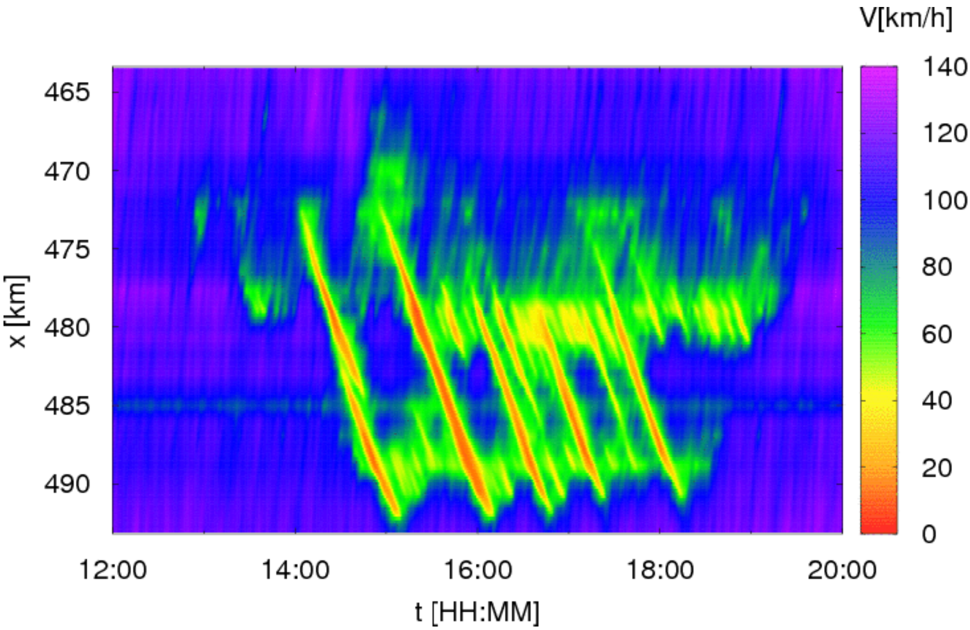


Figure 6.5: Space-time plot of the traffic operations of the German A5, 14 October 2001 – figure from traffic-states.com

6.3. STOP-AND-GO WAVE

6.3.1. PHENOMENON DESCRIPTION

Stop-and-go waves are a specific type of traffic jams. Generally, all traffic states at the congested branch are considered congested states. Sometimes traffic experiences short so-called stop-and-go waves. In these short traffic jams, vehicles come to (almost) a complete standstill. The duration of the queues is a few minutes. At the downstream end of the jam, there is no physical bottleneck. For many drivers, it is surprising they have been in the queue "for no obvious reason".

The outflow of these stop-and-go waves is at the point of the queue discharge rate.

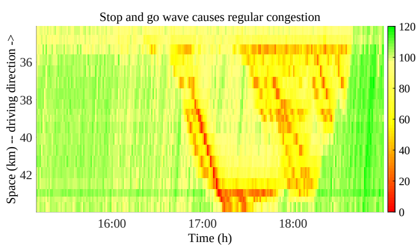


Figure 6.6: Example of a stop-and-go wave triggering a standing queue at the A4 motorway

Since the traffic in the jam is at almost standstill, the density in the stop-and-go wave is jam density. The speed at which the head moves, can be determined with shock wave theory. Using the fact that the wave speed is the difference in flow divided by the difference in density, we find that the head propagates backward with the wave speed (the slope of the congested branch of the fundamental diagram).

These short jams usually start in congestion. Only if traffic is unstable, short disturbances can grow to jams. In free traffic, the spacing between platoons is usually large enough to ensure these jams do not grow. That means that upstream of the stop-and-go wave, traffic conditions are probably close to capacity. A similar traffic state upstream and downstream of the stop-and-go wave means that the upstream and downstream boundary move at the same speed. Therefore, the jam keeps its length.

Often, these type of jams occur regularly, with intervals in the order of 10 minutes. Then, the outflow of the first stop-and-go wave (queue discharge rate) is the inflow of the second one. Then, the upstream and downstream boundary must move at the same speed. Hence, the jams move parallel in a space time figure – see also figure 6.5 (figure from traffic-states.com).

Since traffic in the jam is (almost) at standstill, no flow is possible. That means that the upstream boundary must continue propagating backwards through a bottleneck – the inflow stays at the same level and the vehicles which join the queue have no choice but join the queue keeping their desired jam spacing to their predecessor. Once the jam passes the bottleneck, the desired outflow would be the queue discharge rate of the section where the congestion is. However, the capacity of the bottleneck might be lower.

6

In such a case, the stop-and-go wave triggers a standing congestion, as is seen in figure 6.6. A stop-and-go wave propagates past a botteneck near location 43km. The outflow of the stop and go wave would be larger, but now the flow is limited. Between the stop-and-go wave and the bottleneck, a congested state occurs, of which the flow equals the flow through the bottleneck (a localised bottleneck, so at both sides the same flow). This congested state is at the fundamental diagram. For most applications where the congested branch of the fundamental diagram is considered a straight line, this congested state will lie on the line connecting the queue discharge point with the jam density. The speed of the downstream boundary of the stop-and-go wave is now determined by the slope of the line connecting the "stopped" jam state with the new congestion state. Since these are all on the same line, the speed is the same, and the stop-and-go wave continues propagating.

6.4. KERNER'S THREE PHASE TRAFFIC FLOW THEORY

In the first decade of this century, the number of traffic states were debated. Kerner (2004) claimed that there would be three states (free flow, congested, and synchronised). He claimed that all other theories, using two branches of the fundamental diagram – a free flow branch and a congested branch – assumed two states. This led to a hefty scientific debate, of which the most clear objections are presented by Treiber et al. (2000) and Helbing et al. (1999). A main criticism of them is that the states are not clearly distinguished. Whereas most scientist now agree that three phase traffic flow theory is not a fully correct description of traffic flow, it includes some features which are observed in traffic. In order to discuss these features and introduce the names in that framework, the theory is included in the course. This reader provides a short description for each of the states: an interested reader is referred to Kerner's book for full information (Kerner, 2004), or the wikipedia entry on three phase traffic flow theory for more concise information.

6.4.1. States

Kerner argues there are three different states: free flow, synchronized flow and wide moving jams.

Free flow

In free flow traffic, vehicles are not (much) influenced by each other and can move freely. In multi-lane traffic, this means that vehicles can freely overtake. Note that as consequence, the traffic in the left lane is faster than the traffic in the right lane. The description of Kerner's free flow state therefore has similarities with the description of the two-pipe regime in Daganzo's theory of slugs and rabbits (see section 8.2).

Synchronized flow

Synchronized flow is found in multi-lane traffic and is characterized by the fact that the speeds in both lanes become equal – hence the name of the state: the speeds are "synchronized". This could be seen similar to the one-pipe regime of Daganzo.

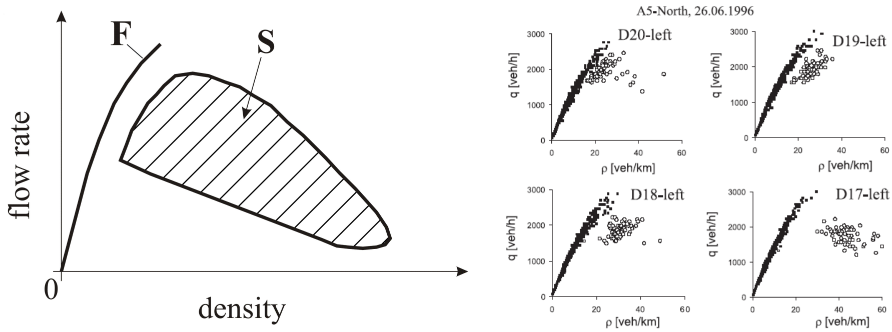
In Kerner's three phase traffic flow theory this is one of the congested states, the other one being wide moving jams. The characterising difference between the two is that in synchronised flow, the vehicles are moving at a (possibly high) speed, whereas in wide moving jams, they are at (almost) standstill. Kerner attributes several characteristics to the synchronized flow:

1. Speeds in all lanes are equal
2. Speeds can be high
3. The flow can be high – possibly higher than the maximum free flow speed. Note that in this case, the "capacity" of the road is found in the synchronized flow.
4. (Important) there is an area, rather than a line, of traffic states in the fundamental diagram.

The area indicated by an "S" in figure 6.7a is the area where traffic states with synchronised flow can be found. Note that in the figure, the free flow branch ("F") does not extend beyond the maximum flow. However, note that for a certain density, various flows are possible. The explanation by Kerner is that drivers have different equilibrium speeds, leading to different flows.

Wide moving jams

A wide moving jam is most similar to what is more often called a "stop and go wave". Stop-and-go relates to the movement an individual vehicle makes: it stops (briefly, in the order of 1-2 minutes), and then sets off again. Kerner relates the name to the pattern (it moves in the opposite direction of the traffic stream), and "wide" is related to the width



(a) Concepts, figure from Wikipedia

(b) Observations, figure from Kerner (2004)

Figure 6.7: Synchronized flow

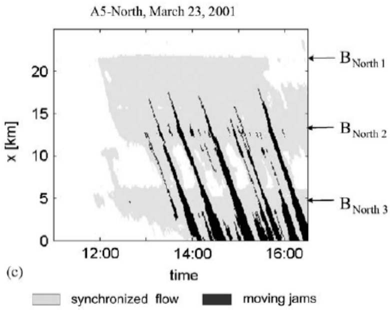


Figure 6.8: Observations of wide moving jams propagating through synchronised flow; figure from Kerner (2004)

(or more accurately, it would be called length: the distance from the tail to the head) of the queue compared to the acceleration/deceleration zones surrounding the jam. The speed in the jam is (almost) 0. In the flow-density plane, the traffic state is hence found at flow (almost) 0 and jam density.

By consequence hence the tail moves upstream with every vehicle attaching to the jam. The head also moves upstream, since the vehicles drive off from the front. Hence the pattern of the jam moves upstream. This phenomenon is purely based on the low flow inside the jam, hence it can travel upstream for long distances (dozens of kilometers). It can pass through areas of synchronised flow, and pass on and off ramps, see figure 6.8

6.4.2. TRANSITIONS

The other aspect in which Kerner's theory deviates from the traditional view, is the transitions between the states. Kerner argues there are probabilities to go from the free flow to the synchronized flow and from synchronized flow to wide moving jams. These probabilities depend on the density, and are related to the size of a disturbance.

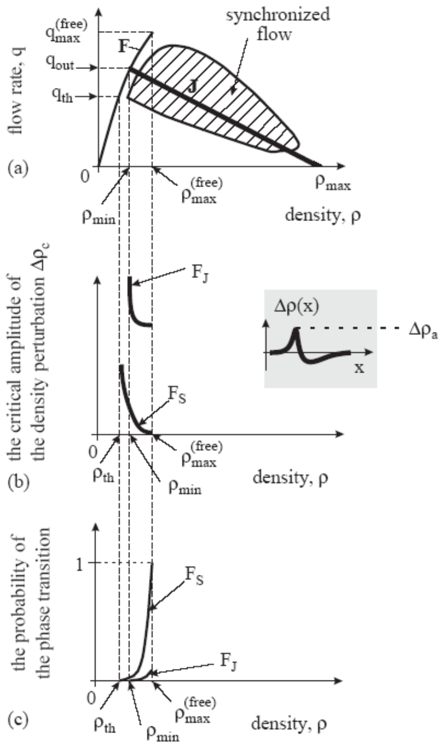


Figure 6.9: Transitions between the phases, figure from Kerner (2004)

Let's consider the transition from free flow to synchronized flow as an example. As the density approaches higher densities, the pertubation which is needed to move into another state, is reducing (since the propagation and amplification increases). One can draw the minimum size of a disturbance which changes the traffic state as function of the density, see figure 6.9. Besides, one also knows with which probability these disturbances occur. Hence, one can describe with what probability a phase transition occurs, see figure 6.9.

SELECTED PROBLEMS

For this chapter, consider problems: A.1.2, 12-13, 25, 30, 31, A.2.2, 64, 99, 107, 108, 109 110, A.5.1, A.5.2, 153, 161, 183, 180, 191, 192, 189, 218, 243, 244, A.10.2, 278, 279

7

CAR-FOLLOWING

After this chapter, the student is able to:

- Predict the vehicle's trajectory given a CF model (know Newell's model by heart, others given in eqns)
- Interpret a model/FD at the other aggregation level (microscopic and macroscopic)

A car following model describes the longitudinal action of the vehicle as function of it's leader(s). That can be, it describes its position, speed or acceleration. There are many different forms, and all have their advantages and disadvantages. They all aim to describe driving behavior, and human behavior is inconsistent and hence difficult to capture in models.

This chapter does not give an overview of all models, nor a historical overview. That is given by Brackstone and McDonald (1999). Instead, in this chapter we discuss the simplest, Newell's, and discuss some characteristics which one might include.

7.1. NEWELL'S CAR FOLLOWING MODEL

The most easy car-following model is the model presented by Newell (2002). It prescribes the position of the following x_{i+1} car as function of the position of the leader x_i. The model simply states that the position (and – by consequence – also speed, acceleration or jerk) of the follower is a distance s_j upstream of the position (or respectively speed, acceleration or jerk) of the leader of a time τ earlier

$$x_{i+1}(t) = x_i(t - \tau) - s_j \tag{7.1}$$

This means that the follower's trajectory is a copy of the leader's trajectory, translated over a vector $\{\tau, s_j\}$ in the xt-plane (see figure 7.1). This vector has also a direction, which can be computed by dividing its vertical component over the horizontal component.

$$w = \frac{s_j}{\tau} \tag{7.2}$$

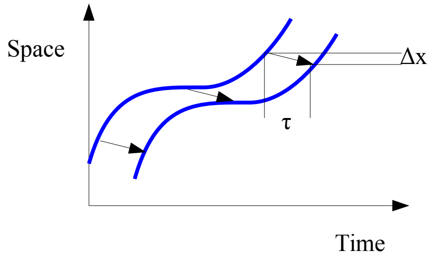


Figure 7.1: Newell's car-following model is translating a leader's trajectory over a vector $\{\tau, s_j\}$

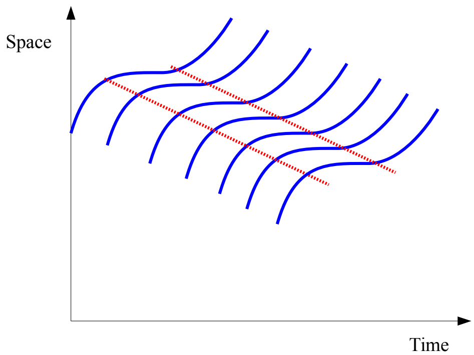


Figure 7.2: The average of the car-following behavior of drivers each with different parameters of the Newell model leads to a shock wave speed

This slope is the speed at which information travels backward in congested conditions, and hence is the slope of the congested branch of the fundamental diagram.

For each driver, different values for τ and s_j can be found. From empirical analysis (Chiabaut et al., 2010) it is shown that averaging the slopes over more than 12 drivers would yield a constant shock wave speed. This is illustrated in figure 7.2. Although the parameters of the car-following model are all different, the wave speed is an average translation of the disturbance, indicated by the red dotted line.

7.2. CHARACTERISTICS

7.2.1. DEPENDENCIES

The car-following model describes the action of the following vehicle. That of course can depend on the movement of the leading vehicle. Some elements which are often included in a car-following model are:

- Acceleration of the leader: if the leader accelerates, the follower can get closer
- Speed: a higher speed would require a longer spacing
- Speed difference: if a follower is approaching his predecessor at a high speed, he needs to brake in time

- Spacing: if the spacing is large, he might (i) accelerate and/or (ii) be less influenced by its leader
- Desired speed: (i) the faster he wants to go, the more his desire to close a gap. But also (ii) even if the predecessor is far away, the follower will not exceed its desired speed

All these elements occur in car-following models. This list combines elements used for different type of models. For instance, some models might prescribe a distance, and hence use speed as input, whereas others might prescribe a speed, and use distance as input.

Surprisingly, many of the available car-following models are "incomplete", i.e. they lack one or more of the above elements and are therefore limited in their use. Using them for a dedicated task is of course allowable. A user should ensure that the model is suited for the task.

7.2.2. REACTION TIME

Human drivers have a reaction time. Mostly, the models are evaluated at time steps, which are chosen small, often in the order of 0.1 second. A good model allows to set the reaction time of the driver separately. Then, only information which is more than a reaction time earlier can influence a driver's acceleration.

Some models will use the model time step as reaction time. In that case the speed or acceleration of the model is evaluated every time step, which then is typically 1 second. Information of the previous time step is immediately used in the next time step, and in between there are no accelerations considered. The effect of reaction time on the traffic flow is described in Treiber et al. (2006). Generally speaking, a large reaction time can make the traffic flow unstable.

7.2.3. MULTI LEADER CAR-FOLLOWING MODELS

Human drivers can anticipate on the movement of their leader by looking ahead and considering more leaders. This has empirically been shown by Ossen (2008). Generally, one would adapt the car-following model allowing n times the spacing for the nth leader. Also, one might expect that a follower acts less sensitive on inputs from leaders further away – that might be in terms of lower acceleration (i.e., closer to zero) or later (i.e., a higher reaction time).

Two "mistakes" are common, leading to multi-leader car-following models which seem to make sense, but are not realistic.

1. In the car-following behavior, only vehicles in the same platoon should be considered, or vehicles which might influence the following vehicle. A car-following model which always takes three leaders regardless of the spacings might take a leader which is a long distance away, which in reality will not influence the driver.

2. If the car following model predicts the *average* acceleration caused by each of the leaders, it might be that the follower crashes into its leader. Instead, a minimum operator is usually more suitable.

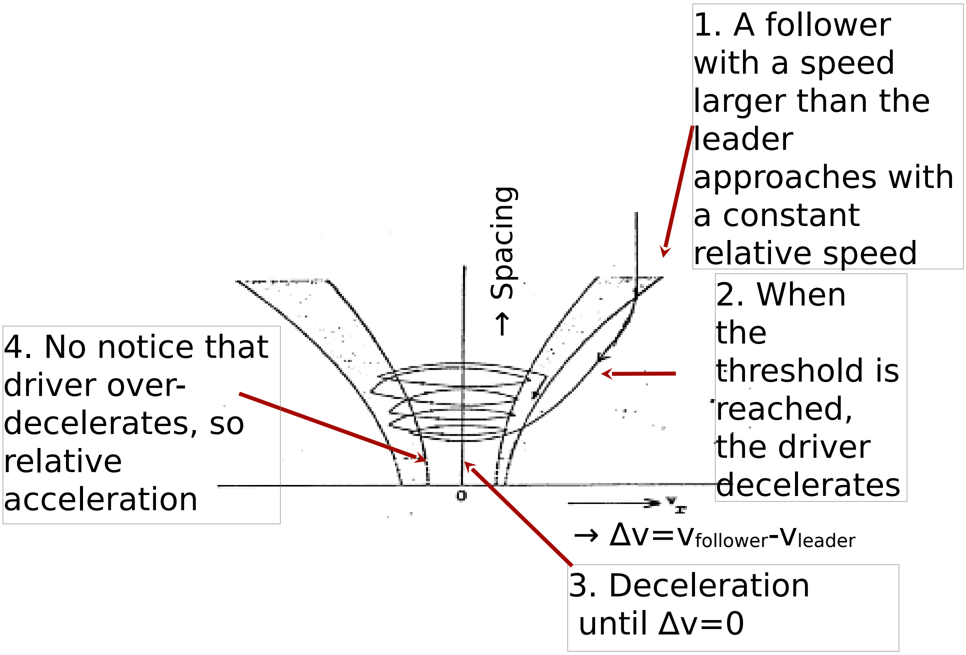


Figure 7.3: Schematic overview of the principles by Wiedemann

7

7.2.4. Insensitivity depending on distance

Most car-following models indicate drivers will adapt their speed based on the action of their leaders. Wiedemann (1974) states that drivers only react if the required action is above a certain threshold. Specifically, he indicates that within some bounds, drivers are unable to observe speed differences. One might also argue that within some boundaries, drivers are unwilling to adapt their speeds because the adaptation is too low. These bounds depend on the spacing between the leader and follower: the larger the spacing, the larger the speed difference needs to be before a driver is able to observe the speed difference. As an example, a 1 km/h speed difference might be unobservable (or unimportant to react upon) on a 200 meter spacing, but if the spacing is 10 meter, this is observable (or important).

It is relevant to draw a diagram relating the relative speed to the spacing, as is done in figure 7.3. The thresholds are indicated: within these bounds, the driver is insensitive and is considered to keep his current speed. Outside the bounds, the driver accelerates to match the traffic situation.

When the leader brakes, the follower closes in. That means that point is in the right half (the follower has a higher speed) and the line is going down (the spacing reduces because the follower is driving faster). At a certain moment, the follower comes closer to his leader and starts reacting to the speed difference, i.e. he will reduce speed. He can either not brake enough and have still a higher speed, or overreact. If the braking is not

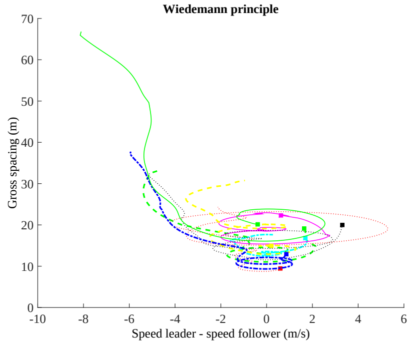


Figure 7.4: Empirical observations of leader-follower pairs in the relative speed - spacing plane

enough, the situation is the same as the starting situation, and the same situation will occur. Therefore, we will assume for the further reasoning that the follower overreacts.

The overreaction means that the follower will have a lower speed than the leader. That means that the speed difference will go through zero (the y-axis in the figure). At that moment, the speeds are equal, so the spacing remains momentarily the same. Therefore, when crossing the y-axis, the line is horizontal. After that, the leader drives at a higher speed than the follower, and the spacing will start increasing. For low speed differences, the speed difference is under the observation threshold and the follower does not adapt his acceleration. Only once the next bound is reached, he will notice the leader is shying away and start accelerating again.

If he will again overreact, the speed difference will become positive. Again, the speed difference will go through zero (the y-axis) where the line has a horizontal tangent line (a zero speed difference means that at that moment the spacing remains constant). One can continue constructing this figure, and it shows that in this phase plane (i.e., the plane relative speed - spacing) the leader-follower pair makes circles.

The wider the circles are, the wider the observation thresholds. The thresholds are considered to be higher for larger spacings. Empirical studies (e.g., Hoogendoorn et al. (2011); Knoop et al. (2009) indeed reveal these type of circles.

Caution is required in interpreting these figures. A Newell car-following model does not implement these type of observation thresholds. However, the relative speed - spacing figures will show circles due to the reaction time. Namely, once the leader brakes, the follower will not yet, and in the relative speed-spacing plane this will be shown as a part of a circle. A correct analysis tracks the car-following behaviour not at the same moments in time (vertical lines in the space time diagram), but at lines moving back with the shock wave speed. When doing so for the Newell model, one would not find any circles at all. A detailed analysis of this method is presented by Laval (2011)

Finally, some words on the principle in relation to car-following. Car-following models prescribe the position, speed or acceleration of the follower based on the position, speed, or acceleration of their leaders. The principle laid out in this section only mentions when the speed is *not* adapted, but does not specify the acceleration outside the thresholds. As such, it therefore cannot be considered a car-following model. Instead, it can be combined with another car-following model to get a complete description.

7.3. EXAMPLES

This section shows the some frequently used models, apart from the Newell car-following model described in 7.1.

7.3.1. HELLY

The first model we describe here is the Helly model (Helly, 1959). The Helly model prescribes a desired spacing s^* as function of the speed v:

$$s^* = s_0 + Tv \tag{7.3}$$

Note that this could be considered as a spacing at standstill (jam spacing) plus a dynamic part where T is the net time headway (subtracting the jam spacing from desired spacing).

Now the acceleration is determined by a desire to drive at the same speed as the predecessor and a desire to drive at the desired headway. The model prescribes the following acceleration:

$$a(t) = \alpha\,(\Delta v(t-\tau)) + \gamma\left(s(t-\tau) - s^*\right) \tag{7.4}$$

In this equation, Δv is the speed difference, t is a moment in time and τ a reaction time.

7.3.2. OPTIMAL VELOCITY MODEL

The optimal velocity model proposed by Bando et al. (1995) is a car-following model specifying the acceleration a as follows:

7

$$a = a_0(v^* - v) \tag{7.5}$$

In this equation, v is the speed of the vehicle, and a_0 a reference acceleration (tunable parameter, constant for a specific vehicle-driver combination). v^* is determined as follows:

$$v* = 16.8(\tanh(0.086(s-25) + 0.913)) \tag{7.6}$$

In this equation, s is the spacing (in meters) between the vehicle and its leader, giving the speed in m/s.

7.3.3. INTELLIGENT DRIVER MODEL

Treiber et al. (2000) proposes the Intelligent Driver Model. This prescribes the following acceleration:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_0\left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v,\Delta v)}{s}\right)^2\right) \tag{7.7}$$

with the desired spacing s^* as function of speed v and speed difference Δv:

$$s^*(v,\Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}} \tag{7.8}$$

In this equation, Δv is the speed difference between the leader and the follower, a is acceleration and b a comfortable deceleration. a_0 is a reference acceleration (parameter). Likewise, v_0 in 7.7 is a reference speed which is a parameter.

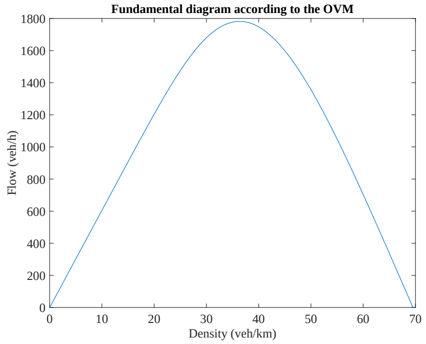


Figure 7.5: Fundamental diagram according to the OVM

7.4. RELATION TO FUNDAMENTAL DIAGRAM

The fundamental diagram gives in its usual form the relation between the density and the flow for homogeneous and stationary conditions. Also car-following models can describe homogeneous and stationary conditions. In that case, all vehicles should drive at the speed and should not change speed (stationary). Therefore, the acceleration of all vehicles must be zero.

Many car-following model prescribe the acceleration based on speed and spacing. The equilibrium conditions mean that the acceleration is zero (and one might argue the relative speeds as well). The relation between spacing and speed then gives an implicit equation for the spacing-speed diagram. Using that the density is the inverse of the (average) spacing, and the flow is (average) speed times density, one can reformulate the spacing-speed diagram from the car-following model into a fundamental diagram.

Take for example the OVM, equation 7.5 and equation 7.6. Equilibrium conditions prescribe that drivers do not accelerate, hence $a_0(v^* - v) = 0$. That means that either $a_0 = 0$, or

$$(v^* - v) = 0 \iff v^* = v \tag{7.9}$$

Since a_0 cannot be zero – in that case the vehicle would never accelerate. Therefore, in equilibrium conditions, equation 7.9 should hold. Using equation 7.6, we find:

$$v = 16.8(\tanh(0.086(s-25)+0.913)) \tag{7.10}$$

This gives a relation between speed and spacing. For a fundamental diagram in flow-density, that gives (using $q = kv$:

$$q = kv = k(16.8(\tanh(0.086(s-25)+0.913))) \tag{7.11}$$

The spacing can be changed into a density. Since all vehicles have the same spacing (equilibrium conditions), we may use

$$s = \langle s \rangle = 1/k \tag{7.12}$$

Substituting this in equation 7.11, we find an expression for the fundamental diagram.

$$q = k(16.8(\tanh(0.086(1/k-25)+0.913))) \tag{7.13}$$

7

Note that in this expression with the values as in equation 7.6, density should be in vehicles per meter and the resulting flow is given in veh/s. Converting these units, this results in the fundamental diagram as shown in figure 7.5.

SELECTED PROBLEMS

For this chapter, consider problems: 11, A.2.4, A.3.2, A.4.2, 133, 154, 179, 181, 220, 222, A.10.4, 266, 277, 280, 281, 282, 283

8

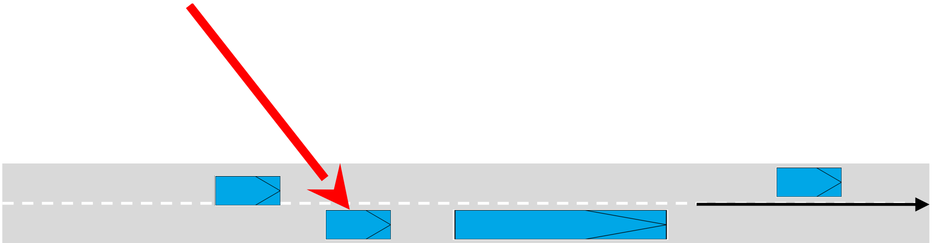
MICROSCOPIC LANE CHANGE MODELS

After this chapter, the student is able to:

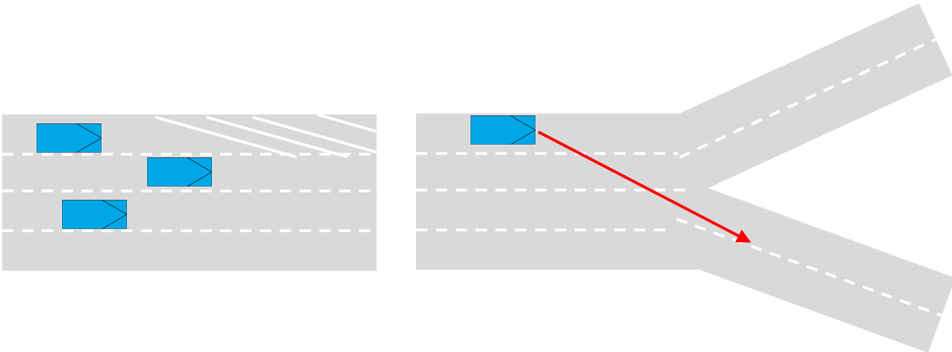
- differentiate between courtesy, mandatory and desired lane changes
- comment on the principles of the Mobil lane change model
- comment on lane selection theories, "slugs and rabbits" in particular
- explain 4 other lane change strategies
- comment and analyse lane distributions
- describe the principle of relaxation

The parts up to now are mainly focussed on the longitudinal driving behaviour and the modelling thereof. This chapter discusses lateral driving behaviour. For multi-lane roads, the lane changing plays an important role. In fact, lane changing is claimed to be the main cause for traffic breakdowns (Ahn and Cassidy, 2007). They argue that a single lane change manoeuvre might lead to a disturbance which grows and due to overreaction, leads to a stop and go wave. Moreover, the lane flow distribution (LFD) is not equal at bottlenecks, implying that in some lanes there is capacity remaining even though the combined flow gets overcritical.

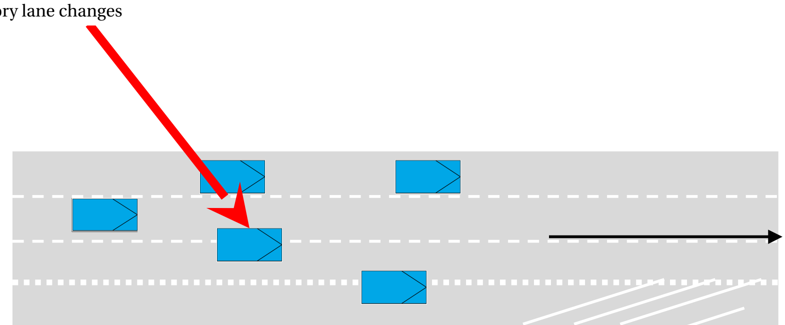
Firstly, this chapter discusses a lane selection model from a psychological perspective (section 8.2). Then, in section 8.3 a microscopic lane change model is introduced. Section 8.4 discusses more advanced lane change models, combining lateral and longitudinal movements.



(a) Desired lane changes



(b) Mandatory lane changes



(c) Coutesy lane changes

Figure 8.1: Different type of lane changes

8.1. TYPE OF LANE CHANGES

Three types of lane changes are distinguished. These are show in figure 8.1 and explained below

1. Desired lane changes are lane changes which a driver does to be better off, usually because the speed in the adjacent lane is larger.

2. Mandatory lane changes are lane changes which a driver does because he has to to continue to its destination. This might be because the lane ends and the driver has to merge, or because the road splits into roadways with different directions, see figure 8.1b

3. Courtesy lane changes are lane changes which a driver makes to help other drivers. There is no direct benefit for the driver that makes the lane change.

8.2. SLUGS AND RABBITS

This section describes a theory of lane selection, posed by Daganzo (2002a,b).

8.2.1. THEORY

This theory poses there are slugs and rabbits, two different types of drivers each with a specific type of lane selection. They are defined as follows:

Category	free speed	lane choice
Slug	Low	always right
Rabbit	High	Fastest

This means that the left lane(s) can operate at a different speed, and even in a different regime than the right lane.

8.2.2. TRAFFIC OPERATIONS

Consider a situation with low demand. Then, slugs drive at their desired speed in the right lane. This speed is lower than the desired speed of the rabbits. Rabbits hence drive their desired speed and stay in the left lane(s). Both (type of) lanes have a different speed, and operate independently. That is why this state is called a two pipe regime.

If the density in the left lane(s) increases, the speed can decrease to values below the free flow speed of the right lane. Rabbits then choose to change lanes towards the right lane, since they will change to the fastest lane. This will increase the density on the right lane, and lower the speed. The density on the left lane(s) will decrease and the speed will increase. Thus, the difference in speeds between the right and the left lane(s) decreases. This process of changing lanes continues until the speed differences between the left and the right lane have decreased to zero. Then, the complete flow operates at one state, and this flow is called a one pipe regime.

8.2.3. LOADING

The section above describes traffic operations at a continuous motorway stretch. Interesting phenomena occur at on-ramps. At an on-ramp, traffic merges onto the main road, but always into the right lane. Consequently, the rabbits are not in their desired lane.

They need a distance to perform the additional lane change into the left, and faster, lane. If the density in the left lane already is high, the traffic in the left lane might become overcritical, and speeds might reduce. Note that this is happening *downstream* of the point of merging, namely at a point where the rabbits change towards the fast lane.

8.2.4. CONSEQUENCES

Although the model of lane selection as proposed by Daganzo is simple, it does explain the following features observed in traffic.

- Boomerang effect. This effect (Helbing, 2003) says that a traffic disturbance starts at an discontinuity (for instance an onramp), than travels a while with the traffic, before traffic breaks down to very low speeds and the disturbance will travel upstream back to the discontinuity where it started.
- Uneven lane distribution. It is found that near capacity conditions, the left lane has a much higher traffic flow than the right lane (see e.g., (Knoop et al., 2010).
- Capacity drop, elaborated in section 6.2.

8.3. UTILITY MODEL

The theory of slugs and rabbits is a psychological theory which explains some phenomena. It can not yet directly be implemented in a model. The lane change model MOBIL, abbreviation for Minimizing Overall Braking Induced by Lane Changes, is a full working model which can be directly implemented in a traffic simulation. It describes for each time step whether or not a vehicle will change lanes. The model is explained in this section. The section first describes the idea of the model. Then, the model is formulated in terms of equations.

8

8.3.1. MODEL IDEA

The basic idea of the model is that for each of the lanes a *utility* is calculated. This utility is based on the following items (note: all of the utilities below have a different value for a different lane choice):

- Foreseen acceleration of the driver: the more it can accelerate the better it is
- Foreseen acceleration of the other drivers: one would prefer not to hinder others
- (For European driving:) how far right: the rules prescribe to keep right unless overtaking.

These utilities are calculated for each of the possible decisions: change left, change right or stay in current lane. Then, they are weighted per decision and summed. This gives the utility for a certain lane.

8.3.2. MODEL EQUATIONS

The model is based on utility, calculated per lane, which we indicate by U_λ. Drivers take their own utility into account, as well as the utility of others. Figure 8.2 shows the other

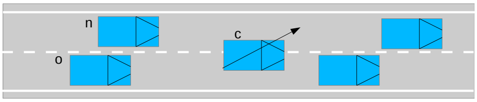


Figure 8.2: The lane changing vehicle and the surrounding vehicles. In this figure, the vehicle is considering a lane change to a left lane. If a lane change to the right lane is considered, vehicles in the other lane should be considered, which will be indicated with the same symbols

vehicles involved in a lane change. The lane changing vehicle is indicated by a c, the new follower by a n and the old follower by a o.

The total utility for a driver is considered a weighted sum of the utility for himself and the other vehicles:

$$U_{\text{tot}} = U_c + \mathscr{P} \sum_{i \in \text{other drivers}} U_i = U_c + p\,(U_o + U_n) \tag{8.1}$$

The utility for the vehicle is expressed by its instantaneous acceleration a, as computed using the IDM car-following model (see section 7.3.3). The utility U for vehicle i is then expressed as

$$U_i = a_i = a_0 \left(1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v\Delta v)}{s}\right)^2\right) \tag{8.2}$$

For the interpretation of the variables, we refer to section 7.3.3.

A lane change is performed if the utility of the driver for the other lane is at least a_{th} higher than the utility of the driver in the current lane. The variable a_{th} acts as a threshold variable in this case: as long as the utility gain is lower than that threshold, a driver will not change lanes. From a behavioral point of view, a_{th} can be seen as the cost of lane changing, which can differ per driver.

EUROPEAN DRIVING RULES

In Europe, driving rules dictate to stay right unless overtaking. In the United Kingdom, the rule is to stay left unless overtaking. In the description here, we will refer to left and right for right hand driving as in continental Europe; for changes in the UK system, change left and right in the following description.

Two changes are made to the model:

1. The utility in the right lane is maximised at the utility of the left lane. By that change, one accounts for the fact that overtaking at the right is not allowed.

2. A bias towards the right is introduced, a_{bias}. This variable has a value larger than 0, and shows how much utility people attach to keeping right. A lane change to the right is performed if

   $$U^{\text{right}} - U^{\text{left}} \geq a_{\text{th}} - a_{\text{bias}} \tag{8.3}$$

   A lane change to the left is performed if

   $$U^{\text{left}} - U^{\text{right}} \geq a_{\text{th}} + a_{\text{bias}} \tag{8.4}$$

8.4. INTEGRATED MODELLING

Usually, models for lane changing separate a desire to change lanes from the manoeuvre itself, which is for instance modelled by gap acceptance Gipps (1986). The model *MOBIL*, as described in 8.3, incorporates the longitudinal accelerations into the choice for the lane. Alternatively, the integrated model by Toledo et al. (2007, 2009) starts with the lane change decision as lead and determines the acceleration.

Intuitively, one might think that drivers prepare for a lane change in their acceleration, leave gaps for others merging into the traffic stream and only gradually adapt their headways after someone merged in front, a phenomenon which is called relaxation. These concepts are incorporated in the *LMRS* model, the Lane Change model with Relaxation and Synchronisation (Schakel et al., 2012).

Recent experimental (Keyvan-Ekbatani et al., 2016) findings show that drivers have four distinct lane change strategies:

1. Speed leading: drivers choose a speed and in order to keep that speed, they change lanes
2. Speed leading with overtaking: drivers choose a speed, and change lanes to overtake. In contrast to the speed leading strategy, drivers will speed up in order to reduce the time of overtaking.
3. Lane leading: drivers choose a certain lane and adapt their speed to the traffic in that lane
4. Traffic leading: drivers claim they "adapt to the traffic", i.e. they do not have a certain lane or speed in mind.

Surprisingly, although there are 4 distinct strategies, drivers expect that they drive "as anyone else", and they are not aware that other drivers can follow a different strategy.

SELECTED PROBLEMS

For this chapter, consider problems: 2, A.3.5, A.4.3, A.5.2, 137, 138, 182, 190, A.9.4, 249, A.11.7

9

USE OF TRAFFIC MODELS

After this chapter, the student is able to:

- choose the appropriate level of simulation
- comment on the value of calibration and validation
- (in principle) calibrate and validate a model
- comment on the number of parameters in models
- comment on stochasticity in models, and the use of random seeds

Traffic simulation models are a very useful tool to assess the impact of a road design or a new traffic management measure. In assessing measures, we differentiate between ex ante and ex post analyses. This means respectively that the analysis is carried out before the measure is implemented in practice, or after. Especially in ex ante analyses, traffic models are very useful.

The traffic flow theory and proposed simulation tools as discussed in this book can form a useful basis to perform these analyses. However, they need to be used with care. This chapter describes some steps a researcher or engineer needs to take before the models can be used. These steps are calibration (section 9.3) and validation (section 9.3) of the models. Section 9.2.2 discusses the stochasticy of models, and how to handle this. The last section discusses examples where it can go wrong. Before the steps are taken the goal of the model has to be defined, which will be explained in the following section.

9.1. GOAL OF THE MODEL

The goal of the model has to be defined in advance and the aspects the performance of the model will be judged upon also have to be defined in advance. This is called the measure of performance or measure of effectiveness. Examples of this measures of effectiveness are:

- average speed at (x,t)
- travel time
- delay

Even if the measure of performance has been set, it has to be determined how this will be measured. This is called the goodness of fit. This can be for instance the root mean square error of the travel time.

9.2. TYPE OF MODELS

Traffic models can be differentiated at different dimensions. First, we differentiate the level at which a model operates. Furthermore, the model might have one or more driver/vehicle types. Finally, another dimension is the stochasticity included.

9.2.1. LEVEL

There are different levels at which one can simulate traffic. The levels considered in this course are

- Microscopically
- Macroscopically
- (Network level)

In case of microscopic simulation, one describes all vehicles. In case of macroscopic simulation, one describes the traffic states on the road. Hence, this is one level higher than the microscopic simulation. In this book, we only consider traffic simulations with dynamics. Often, in planning, static assignment models are used; these are not considered here.

The choice of the level of simulation depends on the requirements, for instance with regard to the:

- unknown items (e.g., effect of influencing driving behavior, connected vehicles)
- network size
- allowed time for the simulation

The most important reason to choose a microscopic simulation model is if one wants to analyse the effects of individual behaviour on the traffic operations.

9.2.2. STOCHASTICITY

Many traffic simulation models are stochastic. That means that not all simulation *runs* are the same. The runs can be seen as representation of different days for which the same input conditions hold. Typical changes from run to run are:

- Moment of entry into simulation, i.e. headway.

- Car/driver specific:
  - driver type
  - routing information
  - equipped driver (connected)

To get a representative idea of the traffic situation, multiple random seeds (also called replications or iterations) are required. Generally, it is advised to use proper statistics to determine the right number of random seeds. A good way to assess this is to first test what the model needs the represent, the so called measure of effictiveness. In a few runs, determine the standard of this measure of effectiveness for various runs. Also determine the required accuracy of the measure of effectiveness for the study at hand. A very accessible approach is found in Driels and Shin (2004).

Let's consider an example. Suppose one wants to assess the average travel time in a simulation and one would like to know with 95% certainty that the calculated travel time does not differ more than 15% from the mean from the real. Several test runs show as initial values a mean travel time of 28 minutes ($\bar{x}$), and a standard deviation (variation between the different random seeds, not variation between the drivers) of 6 minutes (σ), which is the maximum allowed error (ϵ_{max}). From statistics, we know that 95% of the observations fall within a margin of 1.96 standard deviations, so we choose a z-value of 1.96.

From statistics it is known that the bandwidth for the mean depends on the number of replications:

$$\epsilon_{max} = \frac{zS}{\sqrt{n}} \tag{9.1}$$

In this equation, z is used to obtain the likelihood the observation is in between the bandwidth (1.96 in the example case). S is the standard deviation of the process, here approximated by the standard deviation of the test set σ, 6 minutes. n is the number of replications, to be determined. The maximum error is 5% of the mean of the process, here estimated by 5% of the mean of the test set, being 5% of 28 minutes, is 1.4 minutes.

Inversing equation 9.1 gives the equation for the required number of random seeds:

$$n \geq \left(\frac{zS}{\epsilon_{max}}\right)^2 \tag{9.2}$$

Filling the values for the example, we obtain:

$$n \geq \left(\frac{1.96 \times 6}{1.4}\right)^2 \approx 70 \tag{9.3}$$

This process can be used. Note that the derivation assumes that the estimation for S is accurate, so one needs already a small set of replications to estimate this. Alternatively, one can check at the end whether the used number of replications also holds if S is re-evaluated based on all runs. Moreover, it is a stochastic process and the equations assume a normal distribution. They hence can only be applied for larger sets of replications.

Finally a note from what is been used in practice. Often, due to the long computation times of traffic simulation programs – especially microscopic simulation times –

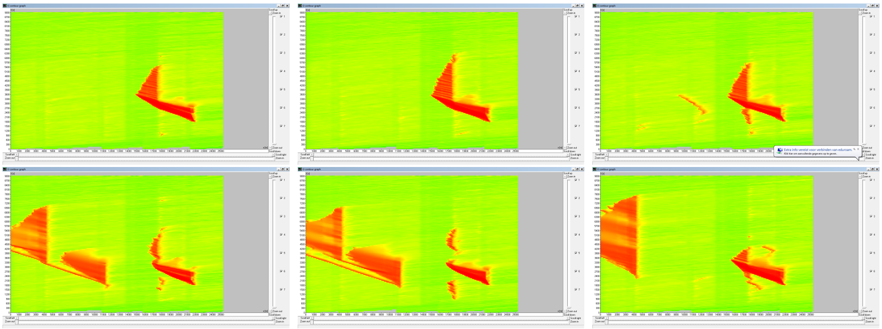


Figure 9.1: Examples of different random seeds in a simulation model. Colors indicate speed in a time-space plane (space is horizontal, time increases from bottom to top

traffic engineers consider a “manual inspection” of the typical patterns, and use 10-15 replications. This will hence not give a statistically significant outcome.

9.2.3. DIFFERENT USER CLASSES

Moreover, a traffic model can include different user classes. For instance, this could be trucks and passenger cars, but one can include more classes: different destinations, different aggressiveness of driving, different type of information.

9.3. CALIBRATION

The goal of the calibration is to make the model representative to the reality. The background will be discussed in the next section. Section 9.3.2 will provide some techniques for calibration.

9.3.1. USE OF CALIBRATION

Whereas some aspects of traffic are universal (conservation of vehicles), the driving behaviour is not constant. It might differ for instance per:

- Road layout
- Day of the week
- Weather condition
- Lighting condition
- Region/country
- Time of the day (including peak / non peak)

That also means that developers of the model cannot provide a model which can be used in any condition. Therefore, the user has to change the parameters in the model such that it fits the situation he wants to use the model for.

The calibration entails adapting the driving parameters such that the model is in agreement with reality. It is therefore essential that the user has data of traffic available to calculate the goodness of fit of the model.

For microscopic models, if the microscopic behaviour has to be represented correctly, it is needed that microscopic data is available. For an overview of the techniques and uses of microscopic calibration and validation, we refer to the thesis of Ossen (2008). Some microscopic models only aim at representing macroscopic characteristics. For these cases, the model can be calibrated using macroscopic data that uses macroscopic measures of performance. It can not be assumed, though, that the underlying microscopic properties are correct.

The end result of a calibration is a model including a parameter set which is representing the situation for which it has been calibrated. To test and check its predictive power for other situations, a validation (section 9.3) is carried out.

9.3.2. TECHNIQUES

The idea is to get the best performance of the model, i.e. optimizing the goodness of fit. In this section, two frequently used values will be discussed, being the root mean square error and the log-likelihood.

ROOT MEAN SQUARE ERROR

An often used measure to indicate the quality of the simulation is the root mean square error, or RMSE. For instance the RMSE of the speeds at each time and space interval indicates how far the speeds are off "on average". The root mean square error is the difference in speed (or in general: the indicator) squared, and then averaged over all observations, in most cases several observations over space and time. Of this average of squares, one takes the mean to get a value for the difference. This gives an indication how far the speeds are different from the base values.

Note that the RMSE of speeds is not always a good measure. For instance, speeds for the free flow branch, the traffic state can differ considerably, with an (almost) equal speed. Choosing density as predictor for a traffic state is more meaningful as representative of the traffic state. However, the core of the calibration process lies in which is the measure of performance the user wants to reproduce. If the user is satisfied if the speeds are correctly predicted and the traffic states are off, the RMSE of speeds can be a good way. Moreover, if there are stop-and-go waves which originate at random times, the RMSE of speed can be very far off if the waves are predicted in the "other phase" (i.e., start is predicted when traffic is stopped and vice versa). In that case, all speeds are wrong, and a simulation predicting no congestion would be better, although from a traffic flow perspective that is further from the truth. All in all, it lies in the requirements for the user: if the user really wants speeds, minimising the RMSE might be a good option.

LOG-LIKELIHOOD

For stochastic models, the model predicts the outcome of a certain traffic state with a probability. Let us denote the probability on a traffic state $P(S|\Pi)$. This depends on the parameters of the model, Π. The likelihood indicates how likely it is that the data is found given the model. Assuming independency between the observations, one can express the likelihood as:

$$L(\Pi) = \prod_{\text{All observed data}} P(S_{\text{observed}}|\Pi) \tag{9.4}$$

Theoretically, one likes to maximize the likelihood of the model giving the observations by adjusting the parameters. Hence, the optimal parameter set Π^* is found by:

$$\Pi^* = \arg\max_{parset} L((\Pi) \tag{9.5}$$

The likelihood is very small. That is because each datapoint contributes a multiplicative factor of 0 to 1 to the likelihood, being the probability that the model with that parameter set predicts that outcome. For many observations, that value will be very small. For computational reasons, it might therefore be advisable to maximize the logarithm of the likelihood instead, also called the loglikelihood.

The loglikelihood $\mathscr{L}$ is defined as

$$\mathscr{L} = \log L \tag{9.6}$$

Because the likelihood is a product of probabilities, the log likelihood changes into a sum of the logs of the probabilities:

$$\mathscr{L} = \log L = \sum_{\text{All observed data}} \log P(S_{\text{observed}}|\Pi) \tag{9.7}$$

9.3.3. NUMBER OF PARAMETERS

A higher number of parameters gives more degrees of freedom to fit the model to the data. However, this also comes at a risk of overfitting. One might choose the parameters to best fit the data, but the interpretation of the parameters could be wrong because the parameters are actually fitted random effects in the data rather than the underlying structure of the data.

For model developers, the advice is that a parsimonious model should be used – a model as simple as possible which represent the data. For model users, the advice is to only fit the parameters which can be estimated reliably and which have an influence on the data which are collected.

A typical example of where (too) many parameters are to be estimated is in calibrating a microscopic model, including all driver parameters, using macroscopic data. As is indicated in Ossen (2008), one cannot calibrate a microscopic model using macroscopic data since many combinations of parameters in the microscopic model lead to the same macroscopic traffic patterns. If one would like to calibrate the microscopic model nonetheless, one should refer to microscopic data.

9.4. VALIDATION

The section describes the validation of a model. In the validation, is is tested how well the model is performing for the cases it is intended for.

9.4.1. NEED OF VALIDATION

The result of a calibration is a model and parameters which are optimized. The purpose of validation is checking whether the model indeed does as it is intended to. It could be that all data from the calibration fits perfect, but for the cases in validation there are other patterns. In that case the model is calibrated, but not validated. In general, the calibration consists of finding an optimal parameter set, for which no specific quality level needs to be given. For the validation, the goal of the model must be set, as well as a validity range.

The validity range is the range for conditions under which the model is working at a quality level defined by the user on beforehand. This validity range could be the same conditions as the validation, another day, another location, another country, another number of lanes – this is to be specified by the user. However, note that the model is only validated for those conditions the validation has been carried out for. For example, if the validation is carried out for a similar day on the same road layout, it is not necessarily a correct model for another road layout.

The essence of validation is to make sure the model indeed performs to the required level. Before that is done, no statements of the quality of the model can be made.

9.4.2. DATA HANDLING FOR CALIBRATION AND VALIDATION

For calibration and validation different data sets should be used. This prevents for problems of overfitting: if the parameters are fit to the specific (random) properties of the data, this should not show in the validation.

If the validity range is different from the calibration, one data set could be used for the calibration and another dataset can be used for validation. That could for instance be two different locations. If no specific conditions could be identified, the data should at random be split in a part for calibration and validation. A typical distribution is to allow 2/3 for calibration and 1/3 for validation.

9.4.3. TECHNIQUES

As mentioned before, the first thing one needs to do is set the aims for the model: under which conditions the model needs to perform at which level. These are preferably intuitive measures; for instance a log-likelihood value is hard to interpret and hence not very useful.

The process then is straightforward: run the model with the correct input variables, and then consider the outcome in the chosen measure of performance and goodness of fit. To which extent does it meet the required (or desired) criteria.

For instance, we want to validate a macroscopic traffic flow model for another day than the day used for the calibration. Data are available for that day, showing the flows and the speeds. We require that with the right inflow, measured at each km and aggregated over 5 minutes, the RMSE of the speeds (compared to the ground truth data) is below 5 km/h. The inflow at the beginning of the road stretch is put into the model, the model predicts the speeds. These are compared with the data, and the RMSE of the speeds is calculated. If the RMSE is below 5 km/h, the model is validated. For a good overview of calibration and validation, see chapter 16 of Treiber and Kesting (2013).

9.5. OFTEN MADE MISTAKES

In traffic engineering this often goes wrong. In this section, some examples are given.

The first, and most serious mistake is that uncalibrated models are being used. That could mean that a model is developed such that some behaviour is in there. For commercial micro simulation packages and drivers on freeways, that means that typical car-following behaviour is implemented, just as a lane change model. However, drivers drive differently all over the world and in different conditions. If one would simply implement a road layout and put it in the specific package assuming that the drivers drive as the developer put into the model, the risk of having a very wrong model is large. This risk is further enhanced by the fact that the vehicle trajectories look realistic. This is because the underlying car movements make sense. However, they are not necessarily true for the case at hand.

Calibrated but unvalidated models are also common in practice. Data are being collected and parameters of the model are optimized. However, without validation, one knows the model is the best prediction possible (although a risk of overfitting exists), but one does not know how good it is. It can be very far off the reality.

In microsimulation a vehicle demand is given. It can happen that the tail of congestion in the network is spilling back to the entry of the network. In that case, the demand cannot be put on the network due to the congestion. Various simulation programs then act differently. Some will not generate new vehicles, whereas others store the vehicles in "vertical queues". Since it is generally not known what models do, it is best practice to avoid congestion spilling back to the network entry. Note that network entries also can be onramps halfway the network. In this case, one can consider elongating the onramp.

The final mistake commonly made is the validated models outside their validation range. For instance, a model could be validated for Dutch freeways with no slope at a speed limit of 100 km/h and dry weather conditions. That model, with those parameter settings, is then only validated for that range. A road in Italy, or road with slopes, or with a different speed limit or with different weather conditions would then fall outside the validation range.

SELECTED PROBLEMS

For this chapter, consider problems: 163, 219, 221

10

MACROSCOPIC DYNAMIC TRAFFIC FLOW MODELS

After this chapter, the student is able to:

- Describe the Cell Transmission Model
- Compute flows using the Cell Transmission model
- Compute propagation of traffic flows using the Cell Transmission Model
- Program the propagation of traffic flows using the Cell Transmission Model
- Comment on numerical errors and the CFL condition
- Give a physical meaning of using the N-model/Lagrangian coordinates and explain the advantages thereof
- Explain when a model is a higher order model
- Comment on how to model multi-class traffic in a macroscopic description
- Comment and physically interpret a PCE value

This chapter describes the principles which can be used in macroscopic traffic flow modelling. First, the mathematical models are presented (section 10.1). From section 10.2 several discretisation schemes are presented, i.e. models which can be implemented in a computer model.

10.1. MATHEMATICAL MODELS

Also on the macroscopic level, one can model the traffic operations. The main advantages of macroscopic modelling over microscopic modelling are

1. a lower calculation time
2. less sensitive to stochastic effects

Simulating traffic flow macroscopically means that one has to find the equations of the dynamics of the macroscopic traffic flow variables. Conservation of vehicles (realising there are no vehicles created or disappearing along a motorway stretch) yield the following equation:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{10.1}$$

This is a specific form of equation 13.1, with no source and sinks.

The most basic modelling is first order modelling. In this case, a fundamental diagram is assumed, and hence there is a relationship

$$q = q(k) \tag{10.2}$$

Substituting this equation in equation 10.1, we find

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial k}\frac{\partial k}{\partial x} = 0 \tag{10.3}$$

This can be rewritten using the characteristic speed (see chapter 13): $\frac{\partial q}{\partial k} = c(k)$

$$\frac{\partial k}{\partial t} + c(k)\frac{\partial k}{\partial x} = 0 \tag{10.4}$$

As chapter 13 already showed, the boundary conditions for the traffic flow can come from either downstream or upstream. Therefore, a combination of a downwind and an upwind scheme have been proposed, as section 10.2 will show. This model is described in the N-model (see section 15.2). The simulations also can be done in other coordinate systems. Especially the X-model is useful. Section 10.3 will discuss a numerical scheme similar to the cell transmission model of section 10.2 in the X-model. In this coordinate system, some of the mathematical disadvantages disappear

Regardless of the coordinate system, the system might contain various classes. Section 10.5 discusses this phenomenon; a way to implement it in a computer model in discrete equations is proposed in section 10.5.2.

All above methods use the fundamental diagram as starting point. This is not necessarily true. Section 10.6 discusses higher order models, i.e. models where traffic states can deviate from the fundamental diagram based on dynamics.

10.2. SPATIAL AND TEMPORAL DISCRETISATION

The most standard way to calculate traffic models is to discretise the space and time. In particular, this section will explain the cell transmission model, as introduced by Daganzo (1994). Mathematically speaking, this model is based on the model independently proposed by Lighthill and Whitham (1955) and Richards (1956) that is discretized in space and time and solved with a solution scheme proposed by Godunov (Godunov, 1959). The model is also referred to as LWR model, an asseveration of Lighthill, Williams and Richards.

10.2.1. MODEL WORKING: CELL TRANSMISSION MODEL

To start, a road is split up into segments of length L, and time is discretised in steps of Δt. The state of a cell i at time step t is determined by its density. For each cell, the fundamental diagram is specified. First the concept is explained. Then, the details and equations are given.

In uncongested conditions, the flow between upstream cell i and a downstream cell $i+1$ is determined by the demand (in veh/h) from cell i to cell $i+1$. If there are more vehicles, the demand is higher. The actual demand can be derived from the density in cell i and the fundamental diagram.

In congested conditions, the demand is high (many vehicles want to get out of the area), but the supply is the restriction of the flow. In this case, the downstream cell $i+1$ restricts the flow. The flow is hence determined by the flow in the downstream cell, which can be derived from the density and the fundamental diagram.

The CTM determines the flow over the boundary between cell i and cell $i+1$ based on demand and supply. Once this flow is determined, for each cell the difference in inflow and outflow is known, and hence how the density in the cell changes. This gives an updated density, and the process can be started again for the next time step.

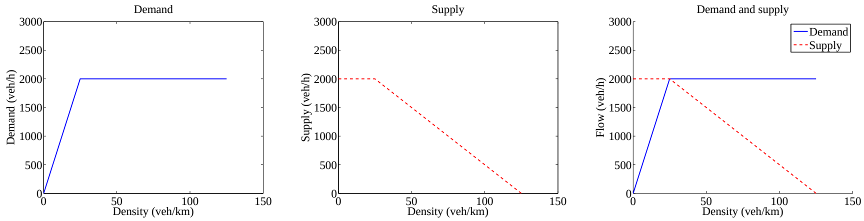
Note there is a relation between the cell length L and the time step Δt. Since the flow maximum flow is determined by the downstream cell and upstream cell, information from more than 1 cell from the boundary is not taken into account. Therefore, the time step should be small enough such that within one time step, no vehicle (or congested wave) travels more than one time step. Hence,

$$L \leqslant \Delta t v_{\max} \tag{10.5}$$

In this equation, $v_{\max}$ is the highest (absolute) speed of information in the traffic system. In freeway conditions, these are the vehicle speeds. This inequality is called the CFL condition, named after the researchers Courant, Friedrichs and Lewy (Courant et al., 1928). In physical terms, it makes sense because one cannot have vehicles passing more than one cell in a time step.

Note that equation 10.5 shows an inequality sign. In principle, shorter time steps do not form a fundamental problem. However, contrary to what one might intially think, the solution does not improve if the time step becomes shorter. The reason is that the homogeneity condition within a cell is violated stronger. Let's take an example for an empty road which fills up from the start and a short time step. In the first time step, some vehicles flow into the first cell. This cell is then considered homogeneous for determining the demand towards the next cell. At the end of the time step, there will be a demand and a flow from cell 1 to cell 2, even though in reality the vehicles did not have the opportunity to reach the boundary yet. The errors which occur in due to the discretisation scheme are called numercial errors. In this case, the wave between the empty road and the vehicles that arrive does not remain sharp, but some vehicles are propagating very fast, whereas the main part of the vehicles moves slows. The wave hence gets diffuse due to the numercial scheme, and this is also called numerical diffusion. This is in an exmaple shown in section 10.2.3.

The more formal traffic flow scheme is as follows. We start by splitting the fundamental diagram into two parts. We define a demand function D as function of the density,



(a) Demand (b) Supply (c) Demand and supply

Figure 10.1: The demand and supply curves

which equals the flow for undercritical conditions and the capacity in overcritical conditions.

$$D = \begin{cases} q(k) & \text{if } k \leq k_c \\ C & \text{if } k > k_c \end{cases} \tag{10.6}$$

This indicates how much traffic wants to go from the upstream cell to the downstream cell.

Similarly, we create a supply function S, which limits the flow at the entry of a cell in cases in which the cell is in overcritical conditions. This function equals capacity for undercritical conditions, and is equal to the flow given by the congested branch of the fundamental diagram in overcritical conditions:

$$S \begin{cases} C & \text{if} k \leq k_c \\ q(k) & \text{if} k > k_c \end{cases} \tag{10.7}$$

The demand and supply curves resulting from a triangular fundamental diagram can be found in figure 10.1. The flow from cell i to cell $i+1$ is determined by the minimum of the demand in cell i and the supply in cell $i+1$:

$$q_{i,i+1} = \min\{D_i, S_{i+1}\} \tag{10.8}$$

From figure 10.1c, it also shows how the fundamental diagram results from the minimum of demand and supply.

This can be translated into an actual number of vehicles, by multiplying this flow by its duration, i.e. the duration of a time step Δt. The effect that this flow has on the densities in the cells depends on the lengths of the cells:

$$\Delta k_i = \left(q_{i-1,i} - q_{i,i+1}\right)(\Delta t / L) \tag{10.9}$$

In implementing the CTM it is essential to realise that one determines the flow over the boundaries, and updates the cell densities iteratively. So every time, one has to change from a boundary perspective to a cell perspective. Also note that each cell is connected to two boundaries (one upstream one and a downstream one), which makes the numbering of cells and boundaries asynchronous. Consider this as warning in the implementation. There is no generic way the cells or boundaries must be numbered. In the following example, we will number cells 1-n and boundaries as well starting from 1.

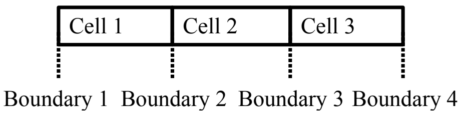


Figure 10.2: Numbering of cells and boundaries in the CTM

Cell number\Time	1	2	3	4	5	6	7
4	17.5	17.5	0	0	0	0	20
3	17.5	0	0	0	0	20	20
2	17.5	35	52.1	66	77.3	66.3	57.5
1	17.5	17.5	17.9	21.5	27.7	36.2	41.9

Table 10.1: The end result: the densities for all 7 time steps for all 4 cells

However, since the first boundary starts before the first cell, and the last boundary ends after the last cell, there is one more boundary then cells (see figure 10.2)

Note that the demand at the first boundary and the supply of the last boundary are not defined in this way. The demand at the first boundary is determined by the externally defined inflow into the network – how many vehicles want to flow in. The supply of the last boundary can be set to infinite, showing that there is no capacity restriction outside the simulated network (otherwise, that should have been in the simulation).

Finally note that the aim of the CTM is to get the densities. With the densities and the fundamental diagram (used as input), the flow in the cells and the speeds can be determined. The flows as determined over the boundary should not be used as cell flows.

10.2.2. THE CTM AT WORK: EXAMPLE

Let's consider an example. We consider the case that there is a constant inflow of 1750 veh/h into a network. All cells have the same triangular fundamental diagram, with a capacity C of 2000 veh/h, a free flow speed v_f of 100 km/h and a jam density of 125 veh/km. Consequently, the critical density is $k_c = C/v_f = 2000/100 = 20$ veh/km. After some time, a short blocking occurs in the network, which we represent here by setting the maximum capacity at one boundary temporarily (2 seconds) to 0. For the sake of illustration, we will now go over all computational steps to be made. The final traffic states are represented by the density in each of the cells, and the final result is presented in table 10.1. In this example, we use a time step of 0.5 seconds, and a cell length of 13.9 meter. Note that now the flow is blocked between cell 2 and 3, and causes an empty area downstream (cells 3 and 4), and congestion upstream. At time step 6, flow recovers again, and downstream cells are in capacity state. Later on (not shown here for space reasons), the capacity state will also move to cell 2 and 1 (boundary is moving upstream).

We will now show all the steps that are needed to get to this. In the tables below, the cells are presented in horizontally next to each other and time goes down (note that this is different than typical speed contourplots). This in in line with the cell layout as in figure 10.2. In table 10.1 we chose to present time as columns (like in a speed contour plot).

For the next iterations, we show step by step how the densities are being computed.

For reasons of understanding the steps, we will show each of the steps, which takes some space (tables) and causes redundancy.

We first start with the densities as given in the beginning, i.e. at the start all cells have a density of 17.5 veh/km. Note that in all subsequent tables, we leave out units for reasons of space. Variables are given in default units, i.e. density in veh/km, flow in veh/h and veh nr N (and hence differences in vehicles, ΔN) in veh. In the table below we also indicate the flow which matches the density according to the prescribed fundamental diagram, indicated by Q_k. This is, as argued above, *not* the flow over the boundaries. Instead, it helps in determining the demand and supply later. Moreover, after the flow Q_k it is indicated whether this state is in free-flow conditions or congested conditions with a letter f or c respectively. At the first time step we hence have this network state (table 10.2):

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	

Table 10.2: CTM: time step 1

All states are in free flow conditions, and at a density of 17.5 veh/km.

Now, we determine the demand and supply on all of the boundaries. For this, we consider for all boundaries the demand (denoted d in the table below) determined by the upstream cell and the supply (denoted s) determined by the downstream cell. The flow over the boundary (q) is the minimum of demand and supply at the boundary. The flow over the boundary can be transferred into a number of vehicles that flows over the boundary in a time step. This is indicated by ΔN, and is equal to the flow times the time step (note that one has to align units, so e.g. transform the time step to hours).

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=2000		s=2000		s=Inf
	q=1750		q=1750		q=1750		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243

Table 10.3: CTM: time step 1.5

In table 10.3 we see stationary, homogeneous, and undersaturated conditions. The demands are equal to the equilibrium flow, because the states are free flow. The supplies are equal to capacity, because the states are in free flow. The flow is hence equal to the demand, yielding 0.243 vehicles to flow over each boundary.

Now moving one time step further, we check how these vehicular flows over the boundary change the densities in the cells. We use $\Delta_c N$ to indicate how many more

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=2000		s=2000		s=Inf
	q=1750		q=1750		q=1750		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	

Table 10.4: CTM: time step 2

vehicles have flown into a cell then have flown out of the cell in the previous time step (note this is a different concept than ΔN, which we defined for boundaries). In this case, since the flows over the boundaries are equal for all boundaries, an equal amount of vehicles flows over each of the boundaries (see table 10.4)

This leaves the change in number of vehicles in each cell, $Delta_c N$ at 0, hence there is no density change, and the densities are the same as the previous time step (17.5 veh/km).

Now a blocking takes place between cell 2 and 3. This is modelled by limiting the supply of cell 3 to 0. We hence obtain the following table (table 10.5).

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=2000		s=2000		s=Inf
	q=1750		q=1750		q=1750		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=0		s=2000		s=Inf
	q=1750		q=1750		q=0		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0		ΔN=0.243		ΔN=0.243

Table 10.5: CTM: time step 2.5

As you can see, the change is the supply at boundary 3, and consequently the flow over boundary 3, and hence the number of vehicles flowing in one time step over boundary 3.

This has consequences for the density in the next time step. We follow the CTM and check the differences in flow into and out of each cell. Since the number of vehicles flowing over boundary 3 is lower than the number of vehicles flowing over boundary 2, more vehicles flow into cell 2 than flow out of it. Likewise, since the number of vehicle that flow over boundary 4 is higher than the number of vehicles that flows over bound-

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=2000		s=2000		s=Inf
	q=1750		q=1750		q=1750		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=0		s=2000		s=Inf
	q=1750		q=1750		q=0		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0.243		$\Delta_c N$=-0.243		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=17.5 k=35		Δk=-17.5 k=0		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1714c		Q_k=0f		Q_k=1750f	

Table 10.6: CTM: time step 3

ary 3, the number of vehicles in cell 3 decreases. In fact the differences in number of vehicles in one cell is indicated by $\Delta_c N$. Cell 2 has 0.243 additional vehicles compared to the previous time step, and cell 3 has 0.243 vehicles less than in the previous time step. Converting this number to a density change, we need to divide the change in vehicle number (plus 0.243 veh resp minus 0.243 veh) by the cell length (13.9 m), yielding a density difference of 17.5 veh/km. The density in cell 2 hence increases by 17.5 veh/km, to 35 veh/km, and the density in cell 3 decreases by 17.5 veh/km to 0 veh/km, see table 10.6. Note that vehicles from cell 1 can flow unhindered into cell 2 because there is no effect of the blocking yet because the density is still under critical.

Note that the equilibrium flows for these densities are also different. For (the now empty) cell 3, Q_k=0. Cell 2 gets to a congested state (the density exceeds the critical density), and with an equilibrium flow of 1714 veh/h.

Note here that cell 3 completely empties in one time step. That is because cell 3 operates at free flow speed (under critical conditions and triangular fundamental diagram) and the cell length is chosen to be the time step times the free flow speed. In physical terms, the last car just manages to drive out of the cell in one time step.

In the next step, we again determine the demand and the supply, which are being determined from the equilibrium flows. The congestion in cell 2 limits the supply to the equilibrium flow. Moreover, the supply to cell 3 is still limited to 0, representing the blocking. The demand from cell 3 is 0 because there are no vehicles, see table 10.7.

Once the demand and supply are determined for each boundary, the flow is found by taking the minimum of the two. We see a slightly lower flow over boundary 2 due to congestion in cell 2 (limiting supply). Besides, there is no flow over boundary 3 due to the blocking and no flow over boundary 4 due to no demand (no vehicles in cell 3). The flow in vehicles over boundary 2 0.238 vehicles, i.e. slightly lower than the flow over cell boundary 1.

Now, let's see how these flows cause changes in the density in the next time step.

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=0 q=0 ΔN=0		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0.243 Δk=17.5 k=35 Q_k=1714c		$\Delta_c N$=-0.243 Δk=-17.5 k=0 Q_k=0f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1714 q=1714 ΔN=0.238		d=2000 s=0 q=0 ΔN=0		d=0 s=2000 q=0 ΔN=0		d=1750 s=Inf q=1750 ΔN=0.243

Table 10.7: CTM: time step 3.5

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=0 q=0 ΔN=0		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0.243 Δk=17.5 k=35 Q_k=1714c		$\Delta_c N$=-0.243 Δk=-17.5 k=0 Q_k=0f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1714 q=1714 ΔN=0.238		d=2000 s=0 q=0 ΔN=0		d=0 s=2000 q=0 ΔN=0		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0.005 Δk=0.4 k=17.9 Q_k=1786f		$\Delta_c N$=0.238 Δk=17.1 k=52.1 Q_k=1388c		$\Delta_c N$=0 Δk=0 k=0 Q_k=0f		$\Delta_c N$=-0.243 Δk=-17.5 k=0 Q_k=0f	

Table 10.8: CTM: time step 4

Since the flow over boundary 2 is higher than the flow over boundary 3 (1714 veh/h, or 0.238 veh in one time step), the density in cell 2 increases with 0.238 vehicles. Transforming that into a density at a cell length of 13.9 meters, we have an increase in density of 17.1 veh/km. The flow over boundary 2 is slightly lower than over boundary 1, causing the number of vehicles in cell 1 to slightly increase. The difference of 0.243-0.238=0.05 veh can be converted to density by dividing over the cell length. We thus have an increase of 0.4 veh/km in density in cell 1, and the density becomes 17.9 veh/km.

Both boundaries 3 and 4 have no flow, so the density in cell 3 remains constant, at 0 veh/km. Boundary 4 and 5 have a difference in flow, of 0.243 veh. Transforming that to a density difference, we find an decrease of density of 17.5 veh/km in cell 4, emptying cell 4 completely, see table 10.8. The reasoning behind this is similar to cell 3 getting empty in the previous time step.

Looking at the equilibrium flows of the cells, we see that cell 1 has density of 17.9 veh/km, which is below the critical density of 20- veh/km and hence is in free flow. The matching equilibrium flow of is 1786 veh/km. Cell 2 gets further congested compared to previous time step, with an lower equilibrium flow of 1388 veh/km. Cell 3 and 4 are empty, and hence have a equilibrium flow of 0.

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=2000		s=2000		s=Inf
	q=1750		q=1750		q=1750		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=0		s=2000		s=Inf
	q=1750		q=1750		q=0		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0.243		$\Delta_c N$=-0.243		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=17.5 k=35		Δk=-17.5 k=0		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1714c		Q_k=0f		Q_k=1750f	
	d=1750		d=1750		d=2000		d=0		d=1750
	s=2000		s=1714		s=0		s=2000		s=Inf
	q=1750		q=1714		q=0		q=0		q=1750
	ΔN=0.243		ΔN=0.238		ΔN=0		ΔN=0		ΔN=0.243
		$\Delta_c N$=0.005		$\Delta_c N$=0.238		$\Delta_c N$=0		$\Delta_c N$=-0.243	
		Δk=0.4 k=17.9		Δk=17.1 k=52.1		Δk=0 k=0		Δk=-17.5 k=0	
		Q_k=1786f		Q_k=1388c		Q_k=0f		Q_k=0f	
	d=1750		d=1786		d=2000		d=0		d=0
	s=2000		s=1388		s=0		s=2000		s=Inf
	q=1750		q=1388		q=0		q=0		q=0
	ΔN=0.243		ΔN=0.193		ΔN=0		ΔN=0		ΔN=0

Table 10.9: CTM: time step 4.5

Now, the densities are determined again, we can go and consider the boundaries. Boundary 3 has still the supply set to 0 artificially, and no flow. Cell 1 is in free flow state, so the supply is at capacity level. With the demand lower at 1750 veh/h, the flow over boundary 1 equals 1750 veh/h, or 0.243 veh in the time step. Cell 2 is congested, leading to a supply of 1388 veh/h for boundary 2. The demand for boundary 2 is based on the free flow demand in cell 1 1786 veh/h. The flow is hence 1388 veh/h, or 0.193 veh in the time step. With no vehicles in cell 3 and cell 4, the demand over boundary 4 and 5 respectively is 0.

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=0 q=0 ΔN=0		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0.243 Δk=17.5 k=35 Q_k=1714c		$\Delta_c N$=-0.243 Δk=-17.5 k=0 Q_k=0f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1714 q=1714 ΔN=0.238		d=2000 s=0 q=0 ΔN=0		d=0 s=2000 q=0 ΔN=0		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0.005 Δk=0.4 k=17.9 Q_k=1786f		$\Delta_c N$=0.238 Δk=17.1 k=52.1 Q_k=1388c		$\Delta_c N$=0 Δk=0 k=0 Q_k=0f		$\Delta_c N$=-0.243 Δk=-17.5 k=0 Q_k=0f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1786 s=1388 q=1388 ΔN=0.193		d=2000 s=0 q=0 ΔN=0		d=0 s=2000 q=0 ΔN=0		d=0 s=Inf q=0 ΔN=0
		$\Delta_c N$=0.05 Δk=3.6 k=21.5 Q_k=1972c		$\Delta_c N$=0.193 Δk=13.9 k=66 Q_k=1123c		$\Delta_c N$=0 Δk=0 k=0 Q_k=0f		$\Delta_c N$=0 Δk=0 k=0 Q_k=0f	
	d=1750 s=1972 q=1750 ΔN=0.243		d=2000 s=1123 q=1123 ΔN=0.156		d=2000 s=0 q=0 ΔN=0		d=0 s=2000 q=0 ΔN=0		d=0 s=Inf q=0 ΔN=0
		$\Delta_c N$=0.087 Δk=6.3 k=27.7 Q_k=1852c		$\Delta_c N$=0.156 Δk=11.2 k=77.3 Q_k=909c		$\Delta_c N$=0 Δk=0 k=0 Q_k=0f		$\Delta_c N$=0 Δk=0 k=0 Q_k=0f	
	d=1750 s=1852 q=1750 ΔN=0.243		d=2000 s=909 q=909 ΔN=0.126		d=2000 s=2000 q=2000 ΔN=0.278		d=0 s=2000 q=0 ΔN=0		d=0 s=Inf q=0 ΔN=0

Table 10.10: CTM: time step 6.5

The subsequent time steps can be done similarly (see the values in table 10.10), and are left to the reader. Let's pick up the steps again once the blocking is being removed, after time step 6 so let's do step "6.5". Then, the supply of boundary 3 is no longer limited, and with an empty cell 3 the supply is 2000 veh/h. With cell 2 congested, the demand is also at capacity, at 2000 veh/h. The flow over the boundary is hence the minimum of demand and supply, being 2000 veh/h. For boundary 2, cell 2 is still in congestion and poses a limitation on the supply. For boundary 4, there is no demand since there are no vehicles in cell 3. This is shown in table 10.10.

Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5		k=17.5		k=17.5		k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=2000		s=2000		s=Inf
	q=1750		q=1750		q=1750		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1750f		Q_k=1750f		Q_k=1750f	
	d=1750		d=1750		d=1750		d=1750		d=1750
	s=2000		s=2000		s=0		s=2000		s=Inf
	q=1750		q=1750		q=0		q=1750		q=1750
	ΔN=0.243		ΔN=0.243		ΔN=0		ΔN=0.243		ΔN=0.243
		$\Delta_c N$=0		$\Delta_c N$=0.243		$\Delta_c N$=-0.243		$\Delta_c N$=0	
		Δk=0 k=17.5		Δk=17.5 k=35		Δk=-17.5 k=0		Δk=0 k=17.5	
		Q_k=1750f		Q_k=1714c		Q_k=0f		Q_k=1750f	
	d=1750		d=1750		d=2000		d=0		d=1750
	s=2000		s=1714		s=0		s=2000		s=Inf
	q=1750		q=1714		q=0		q=0		q=1750
	ΔN=0.243		ΔN=0.238		ΔN=0		ΔN=0		ΔN=0.243
		$\Delta_c N$=0.005		$\Delta_c N$=0.238		$\Delta_c N$=0		$\Delta_c N$=-0.243	
		Δk=0.4 k=17.9		Δk=17.1 k=52.1		Δk=0 k=0		Δk=-17.5 k=0	
		Q_k=1786f		Q_k=1388c		Q_k=0f		Q_k=0f	
	d=1750		d=1786		d=2000		d=0		d=0
	s=2000		s=1388		s=0		s=2000		s=Inf
	q=1750		q=1388		q=0		q=0		q=0
	ΔN=0.243		ΔN=0.193		ΔN=0		ΔN=0		ΔN=0
		$\Delta_c N$=0.05		$\Delta_c N$=0.193		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=3.6 k=21.5		Δk=13.9 k=66		Δk=0 k=0		Δk=0 k=0	
		Q_k=1972c		Q_k=1123c		Q_k=0f		Q_k=0f	
	d=1750		d=2000		d=2000		d=0		d=0
	s=1972		s=1123		s=0		s=2000		s=Inf
	q=1750		q=1123		q=0		q=0		q=0
	ΔN=0.243		ΔN=0.156		ΔN=0		ΔN=0		ΔN=0
		$\Delta_c N$=0.087		$\Delta_c N$=0.156		$\Delta_c N$=0		$\Delta_c N$=0	
		Δk=6.3 k=27.7		Δk=11.2 k=77.3		Δk=0 k=0		Δk=0 k=0	
		Q_k=1852c		Q_k=909c		Q_k=0f		Q_k=0f	
	d=1750		d=2000		d=2000		d=0		d=0
	s=1852		s=909		s=2000		s=2000		s=Inf
	q=1750		q=909		q=2000		q=0		q=0
	ΔN=0.243		ΔN=0.126		ΔN=0.278		ΔN=0		ΔN=0
		$\Delta_c N$=0.117		$\Delta_c N$=-0.151		$\Delta_c N$=0.278		$\Delta_c N$=0	
		Δk=8.4 k=36.2		Δk=-10.9 k=66.3		Δk=20 k=20		Δk=0 k=0	
		Q_k=1692c		Q_k=1117c		Q_k=2000f		Q_k=0f	

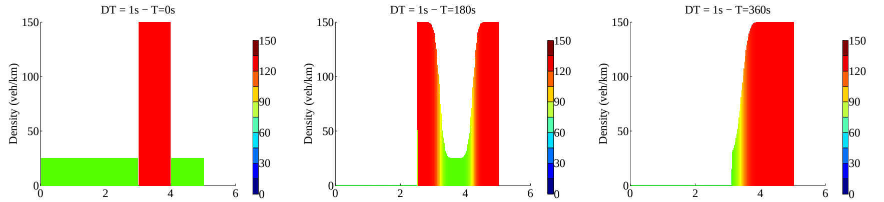
Table 10.11: CTM: time step 7

The final state of this example is shown in table 10.11, where the flows of the previous time step have lead to changes in density. One sees due to the flow over boundary 3 an increase in in vehicle number in cell 3, up to the capacity (due to the choice of the cell length in relation to the time step: the cell fills exactly up to the next boundary). Cell 2 has more outflow than inflow and its density reduces. It would take some more time steps before this cell it is completely at capacity conditions.

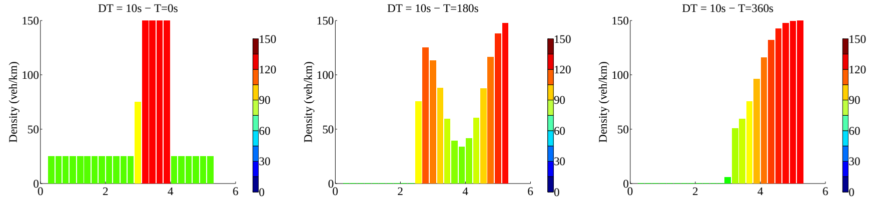
Cell		1		2		3		4	
Bdry	1		2		3		4		5
		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f		k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1000 q=1000 ΔN=0.139		d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0.104 Δk=7.5 k=25 Q_k=1905c		$\Delta_c N$=-0.104 Δk=-7.5 k=10 Q_k=1000f		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1905 q=1750 ΔN=0.243		d=2000 s=1000 q=1000 ΔN=0.139		d=1000 s=2000 q=1000 ΔN=0.139		d=1750 s=Inf q=1750 ΔN=0.243
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0.104 Δk=7.5 k=32.5 Q_k=1762c		$\Delta_c N$=0 Δk=0 k=10 Q_k=1000f		$\Delta_c N$=-0.104 Δk=-7.5 k=10 Q_k=1000f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1762 q=1750 ΔN=0.243		d=2000 s=1000 q=1000 ΔN=0.139		d=1000 s=2000 q=1000 ΔN=0.139		d=1000 s=Inf q=1000 ΔN=0.139
		$\Delta_c N$=0 Δk=0 k=17.5 Q_k=1750f		$\Delta_c N$=0.104 Δk=7.5 k=40 Q_k=1619c		$\Delta_c N$=0 Δk=0 k=10 Q_k=1000f		$\Delta_c N$=0 Δk=0 k=10 Q_k=1000f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1750 s=1619 q=1619 ΔN=0.225		d=2000 s=1000 q=1000 ΔN=0.139		d=1000 s=2000 q=1000 ΔN=0.139		d=1000 s=Inf q=1000 ΔN=0.139
		$\Delta_c N$=0.018 Δk=1.3 k=18.8 Q_k=1881f		$\Delta_c N$=0.086 Δk=6.2 k=46.2 Q_k=1501c		$\Delta_c N$=0 Δk=0 k=10 Q_k=1000f		$\Delta_c N$=0 Δk=0 k=10 Q_k=1000f	
	d=1750 s=2000 q=1750 ΔN=0.243		d=1881 s=1501 q=1501 ΔN=0.208		d=2000 s=2000 q=2000 ΔN=0.278		d=1000 s=2000 q=1000 ΔN=0.139		d=1000 s=Inf q=1000 ΔN=0.139
		$\Delta_c N$=0.035 Δk=2.5 k=21.3 Q_k=1975c		$\Delta_c N$=-0.069 Δk=-5 k=41.2 Q_k=1596c		$\Delta_c N$=0.139 Δk=10 k=20 Q_k=2000f		$\Delta_c N$=0 Δk=0 k=10 Q_k=1000f	

Table 10.12: CTM working for a partial blocking

As a final note, a similar example could be given for a state with a partial blocking, allowing 1000 veh/h rather than 0. The full computational scheme of that case is given in table 10.12. The reasoning behind the steps is entirely the same. The only difference is that the (artifical) supply restriction during the blocking is set to 1000 veh/h rather than 0 veh/h. The interpretation thereof is left to the reader.



(a) time step 1 s (b) time step 1 s (c) time step 1 s



(d) time step 10 s (e) time step 10 s (f) time step 10 s

Figure 10.3: An example of the numerical diffusion in an CTM model. Without numerical diffusion, in this example all traffic states would either be at capacity or at jam density.

10.2.3. NUMERICAL DIFFUSION

The main disadvantage of the the CTM is the discretisation scheme leading to so called numerical diffusion. This means that the shocks as described with shock wave theory do not stay sharp. The numerical diffusion is lowest if the cell length are as short as possible, but if the cell lengths should never be smaller than the time step times the free flow speed, so no vehicle can travel further than one cell in a time step.

Figure 10.3 shows this numerical diffusion in an example. The situation starts with a high density (jam density) between km 3 and 4. At km 5, the road is blocked. Once the simulation starts, shock wave theory predicts that the shock remains intact and travels upstream. The analytical solution is as follows. When there is no further inflow, it should reduce from the tail whereas the head travels backward. A second queue at jam density should start growing with the head at km 5. This is what should be modelled by the method.

The numerical models show different effect based on their time step. The figure shows the results at time 0 (one queue, initial conditions), 180 seconds and 360 seconds. The cell lengths were chosen to match the temporal resolution, so to minimize numerical diffusing. For the simulation, we chose a fundamental digram with k_c=25 veh/km, k_j= 150 veh/km, and C =2000 veh/h.

The figure shows that whereas we would ideally see a sharp shock between waves, this starts to differ. With the short time step, due to the numerical scheme of the solution, the shock is smoothed. If a non matching cell length would be chosen, this effect would have been larger. For larger time steps, the inaccuracy in absolute spatial terms is largers (the shock takes a larger space). For very large time steps (the right column), due to the large spatial areas to match so, the patterns is not longer recognizable due to aggregation of all vehicles into too large cells.

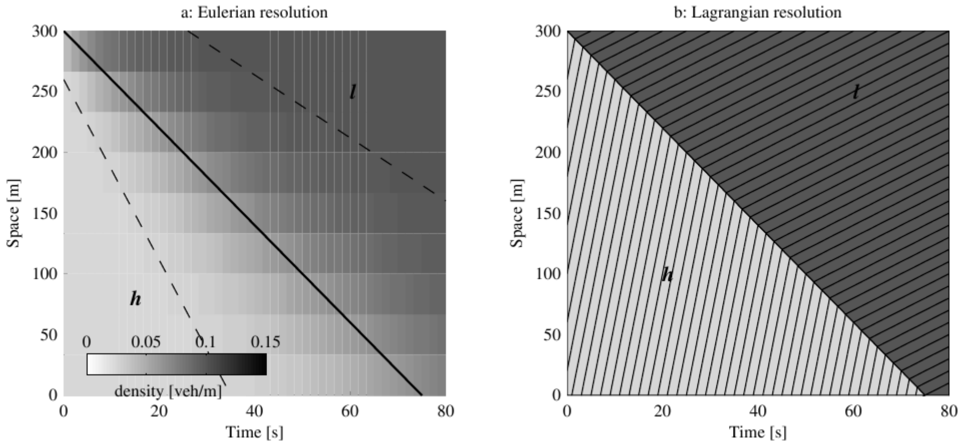


Figure 10.4: Simulation results of the backward propagation of a congestion tail, simulated in Lagrangian and Eulerian coordinates. Figure from Leclercq et al. (2007)

With other discretisation schemes, or other coordinates, for instance lagrangian coordinates (see section 10.3), this numerical error can be less.

10.3. SIMULATION OF THE N-MODEL

An alternative to the cell transmission model could be a model which does not have cells which are fixed in space, but which has cells of a block of vehicles. Basically, the system now predicts the movement of each ΔN vehicle, where its speed is governed by the $V(s)$ fundamental diagram. The difference in space between the Nth vehicle and the $(N-\Delta N)$th vehicle determines the spacing, which in turn determines the speed. Assume a time step of Δt seconds. After applying the speed for Δt seconds, new positions are assigned, leading to new spacings etc.

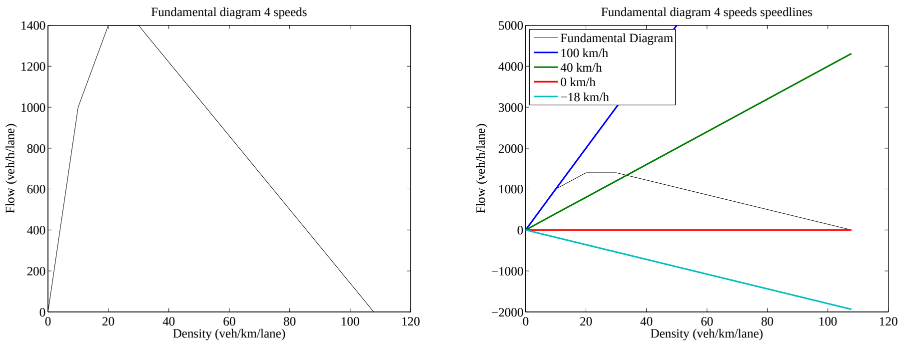
A main advantage is that numerical diffusion is not or less present with a well chosen discretisation scheme in lagrangian coordinates. Figure 10.4 shows how the numerical schemes can work.

A mathematical description of a multiclass model in Lagrangian coordinates is given in section 10.5.2. For more information on Lagrangian macroscopic models, see van Wageningen-Kessels et al. (2010).

10.4. EVENT BASED

In event based models, time is not approached by infinitely small time steps. Instead, the traffic is assumed to stay the same up to the next event.

A simple event based traffic model is the shock wave theory discussed in section 4. There, the traffic states propagate until two waves intersect, and the next wave will be created. Then again, the next event can be calculated, but there is no need to check how the waves propagate in the mean time.



(a) The fundamental diagram

(b) The fundamental diagram and the speed curves

Figure 10.5: The fundamental diagram in the flow density plane

10.4.1. Link Transmission Model

The link transmission model is a model developed by Yperman et al. (2006). It considers nodes as base elements, and using cumulative curves it is determined how much traffic can flow into the link and out of the link. Just like the cell transmission model, a demand and a supply are determined; the difference being that this has to be determined *only once* per node. A link hence does not need to be cut in different cells, which makes the computation more efficient.

It is based on Newell (1993), which gives limits for the number of vehicles which will have passed a specific point. That is maximized by the traffic, but limited by either demand or supply.

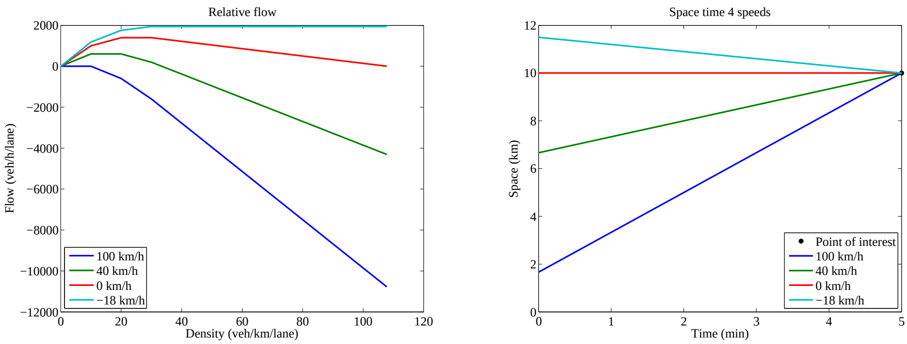
Suppose the road has a piecewise linear fundamental diagram, as in figure 10.5a. Each of the pieces of the fundamental diagram corresponds with a wave speed. For several moving observers, passing with a speed v one can calculate the maximum flow passed this observer, which can be calculated by (see equation 3.11). The flow for a certain density is given by the fundamental diagram. However, by moving there is a flow kv subtracted from it. This flow difference which should be subtracted is indicated in the colored lines in figure 10.5b. The flow relative to the observer is then the distance between that line and the fundamental diagram, or in an equation (equal to equation 3.11):

$$q_{\mathrm{rel}} = k\,(v - v) \tag{10.10}$$

This maximum flow is plotted in figure 10.5b. This relative flow for each of the observers is plotted in figure 10.6a. It is important to note that these relative flows are maximum for the density range in which the characteristic speed of the fundamental diagram matches the speed of the corresponding moving observer.

These flows are measured along a moving path, as indicated in figure 10.6b. For the point of interest, indicated in the figure, one can calculate how many vehicles have passed. This depends on the conditions earlier in time. The limits of the number of vehicles that can have passed are given by relative flows passing the moving observer.

In our case for four different parts of the fundamental diagram, there are four paths



(a) The relative flows passing the different moving observers (b) The trajectories of the moving observers

Figure 10.6: Calculating the flow using moving observers

along which the relative flow should be checked, as shown in figure 10.6b. If the initial conditions (at t=0) are known, the cumulative vehicle number at each of the starting points of these lines is known (indicated by $N_{0,i}$). To this initial cumulative vehicle number we should add the relative flow times the time to have the restriction on the vehicle number for that particular moving observer, indexed i

$$N^i_{\text{lim}} \leq N_{0,i} + q_{\text{rel}} \Delta t \tag{10.11}$$

Because the relative flow is maximum if the characteristic speed is the speed of the moving observer, we know that we can only consider the maximum flow pass the moving observer. Or in other words, the lower flows for other densities do not need to be considered. Hence the vehicle number in the point of interest can be calculated:

$$N = \min_i \left(N^i_{\text{lim}} \right) \tag{10.12}$$

10.5. MULTI-CLASS

10.5.1. PRINCIPLES

Another dimension, which in principle can be combined with all discretisations mentioned above, is the number of classes in the model. Usually, these are trucks and passenger cars, since these have distinct characteristics. The three main differences are:

1. Trucks have a lower speed
2. Trucks have a longer length
3. Trucks have a lower acceleration

In first-order traffic flow modelling, accelerations are not considered as such, and thus difference three is ignored. The first two differences lead to a different fundamental diagram for trucks.

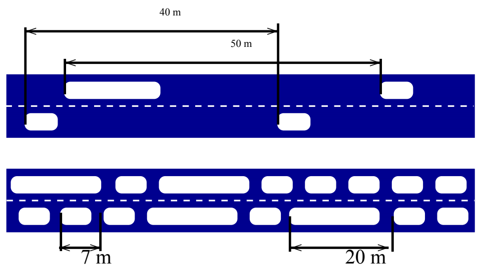


Figure 10.7: The influence of the vehicle length on the density and the remaining net spacing for different speeds

Figure 10.7 shows what the influence is of different speeds at the gross headways. For high speeds, the gross distance mainly consists of net headway, and thus the truck does not occupy much more space than a passenger car. For low speeds, this difference is the same, but relatively, this difference is larger.

The capacity is the maximum number of vehicles that can pass a cross section per unit of time. As is clear from the above, this number will be less if the traffic stream consists of more trucks. The effect of trucks is expressed as passenger car equivalent, or PCE. The PCE value for vehicles in class J is indicated by $\mathscr{P}_J$. In this description, the flow is limited by the capacity:

$$q \leq \sum_J \mathscr{P}_J q_J \tag{10.13}$$

In this equation, q_J, is the flow of vehicle class J. PCE values for trucks can be found in hand books, for instance Heikoop (2011); Transportation Research Board, (2000). Typically, the PCE for trucks is around 2. If we consider a road with a capacity of 2000 pcu/h (passenger car unit, but often simply quoted as 2000 veh/h), a demand of 2000 passenger cars per hour is critical. A demand of 1800 passenger cars per hour and 200 trucks per hour leads to an effective demand of 1800+2*200=2200 pce/h (see equation 10.13), which is overcritial. Given the vehicle demand of 1800 veh/h and the capacity of 2000 pcu/h, there is a remaining capacity of 200 pcu per hour. This equals a flow of 200/2=100 trucks/h.

1. [PCE calculation]Exercise: calculate the PCE for trucks for the fastlane discription for 10% and 20% trucks. Assume reasonable numbers for the unknown variables.

10.5.2. FASTLANE

The two classes can be mixed, and then the classes should interact. Fastlane (van Lint et al., 2008) is a model where this mixture is incorporated. The model is based on a class-specific fundamental diagram which gives the relation between the *net* distance headway and the speed. Note that this relation does not directly work with the density, i.e., the inverse of the gross distance headway. The reasoning is that the speed of both classes in congestion is equal and depends only on the net distance headway. Because trucks are longer, the gross distance headway for the same speeds is longer for trucks. By separating the headways from the length of the vehicle, only one speed-headway relationship for congestion needs to be specified.

The independent variable for the model could be “empty space”. This can be uniquely changed into a generalized density, assuming a uniform vehicle of one passenger car vehicle.

Using these principles, as well as the class-specific fundamental diagrams and the model equations for the Fastlane model can be derived. The resulting model equation can be found in Van Wageningen-Kessels (2013), from which the remainder of this section is taken.

Before we reformulate the multi-class conservation equation into Lagrangian coordinates (see section 10.3), we need some definitions and preliminaries. We define class 1 as the reference class for the coordinate system. This implies that only vehicles of class 1 are numbered. Therefore, coordinate n refers to the number of vehicles of class 1 that have passed location x until time t. Furthermore, the coordinate system moves with the vehicles of class 1, i.e. with velocity v_1. Therefore, the Lagrangian time derivative in the multi-class model is defined by:

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial x} \tag{10.14}$$

Finally, the class-specific spacing of class 1 can be expressed as the partial derivative:

$$s_1 = -\frac{\partial x}{\partial n} \tag{10.15}$$

Realising that $s_u = 1/k_u$, this can be substituted into the Eulerian conservation equation (10.1), the quotient rule is applied and the result is rewritten:

$$\frac{\partial}{\partial t}\left(\frac{1}{s_u}\right) + \frac{\partial}{\partial x}\left(\frac{v_u}{s_u}\right) = 0 \quad \Rightarrow \quad \frac{\partial s_u}{\partial t} - s_u \frac{\partial v_u}{\partial x} + v_u \frac{\partial s_u}{\partial x} = 0 \tag{10.16}$$

Subsequently, substituting the multi-class Lagrangian time derivative (10.14) yields:

$$\frac{\mathrm{D}s_u}{\mathrm{D}t} - s_u \frac{\partial v_u}{\partial x} + (v_u - v_1)\frac{\partial s_u}{\partial x} = 0 \tag{10.17}$$

Finally, (10.15) is substituted to find the Lagrangian multi-class conservation equation:

$$\frac{\mathrm{D}s_u}{\mathrm{D}t} + \frac{s_u}{s_1}\frac{\partial v_u}{\partial n} + \frac{v_1 - v_u}{s_1}\frac{\partial s_u}{\partial n} = 0 \tag{10.18}$$

We note that for class 1 the conservation equation (10.18) reduces to a simpler form:

$$\frac{\mathrm{D}s_1}{\mathrm{D}t} + \frac{\partial v_1}{\partial n} = 0 \tag{10.19}$$

Yet an alternative formulation of the conservation equation in Lagrangian coordinates is proposed in (Yuan, 2013; Yuan et al., 2015b):

$$\frac{\mathrm{D}s_u}{\mathrm{D}t} + \frac{\partial v_u}{\partial n_u} = 0 \tag{10.20}$$

with Lagrangian time derivative:

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + v_u \frac{\partial}{\partial x_u} \tag{10.21}$$

The main difference is that in this model a different coordinate system is introduced for each class. The coordinate systems are coupled via the fundamental relation and the effective density function. Yuan (2013); Yuan et al. (2015b) argue that this formulation may lead to more efficient simulations if there are only few vehicles in the reference class.

10.6. HIGHER ORDER MODELLING

There are other macroscopic models, which include more terms and can include traffic states that are not on the fundamental diagram. That way, hysteresis can be included. One of the most used higher order models is the Metanet model. In this model, the speed increase is limited, representing a reaction time and a limited acceleration. In this reader we do not discuss these models. For more information on the disadvantages and advantages, see Daganzo (1995b) and Aw and Rascle (2000). For more information on the Metanet model, see Kotsialos et al. (2002), which is a macroscopic equivalent of the Payne car-following model (Payne, 1971).

10.7. SUGGESTED PROBLEMS

28, A.2.1, A.3.1, 88, 134, 135, 136, 193, 240, 241, 242, 291, 292

11

An Introduction to Node Models

Acknowledgment This chapter is partially based on the text by Lin Xiao and Jeroen P.T. van der Gun

After this chapter, the student is able to:

- Build, interpret, and comment on the equal demand merge model the capacity based merge model
- Explain the 7 golden rules for node models (when given, no need to learn by heart)

Links and nodes are the essential elements of network topology. No matter we consider people or goods transported in physical network or data delivered in logical network, the structure of network is determined by the arrangement of these components. Each network, regardless of its complexity, can be schematized as an abstract topological network with links and nodes only. So does the traffic network.

In traffic simulation, road network are generally built up by connected links and nodes. Links are considered as the roads where the vehicles travel on. And nodes are the intersections of these links, which passing flows from and to different segments. Therefore, the connected links and nodes form a network to support vehicles' movements from each origin to various destinations.

Important for traffic simulation is the ability to mimic the propagation of vehicle flows along the network. In the texts so far, we have mainly considered the traffic operations at links, ignoring what is happening at nodes. In this chapter, we discuss the node models.

The node model is the mechanism of receiving flow and sending flow at a node. It reflects the movement of flow from upstream to downstream links and determines how much flow can be actually carried out. Flow is constrained by traffic demand and supply. In general, traffic demand are the flows which intend to move from incoming links to outgoing links. And traffic supply is defined as the maximum flows that outgoing

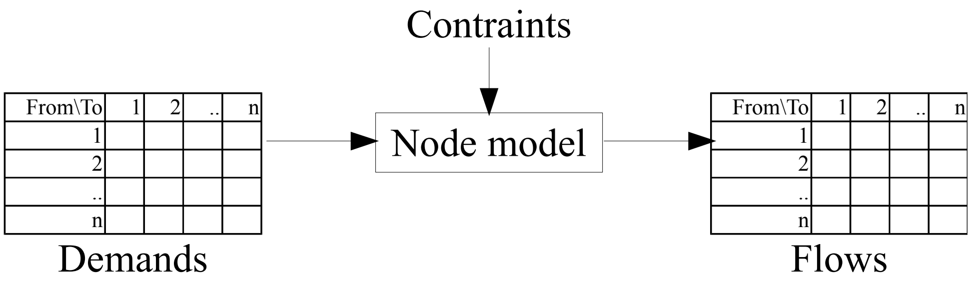


Figure 11.1: OD matrix and flow matrix as input and output of the node model

links can receive. Because of the possible imbalance between demand and supply, actual flows are not as same as the demand flows. Therefore, node model is expected to be a distribution process of traffic flow, transforming an origin-destination matrix (with origins and destinations the inflow and outflow links of a node, repectively) into a flow matrix (see figure 11.1).

The general structure of a node model is shown in figure 11.2. Each node model is connected with one or several incoming links (1,2... n_{in}) and outgoing links (1,2... $n$out), which are sending and receiving flow from and to the node.

Outgoing link j receives flows from each incoming link, as the q_i^j in jth column of flow matrix. (Note that we will use subscripts to indicate incoming links and superscripts to indicate outgoing links) Outflow q^j is the total flows towards outgoing link j, which is restricted by the ability of the downstream link to accept more vehicles, indicated by maximum receiving flow R^j. We thus have a supply constraint:

$$q^j = \sum_i q_i^j \leq R^j \tag{11.1}$$

Likewise, the sum of the flows from a incoming link cannot exceed the capacity of that link S_i:

$$q_i = \sum_j q_i^j \leq S_i \tag{11.2}$$

The turning fraction f_i^j is defined as the fraction of the total flow q_i turning to outflow link j. At the scale we are considering here, congestion leaves this ratio invariant (i.e., we do not assume the vehicles chose a different route due to congestion). Therefore, the node model should leave the turning ratio invariant.

In Tampère et al. (2011) two functions of node models are described. First, a node model imposes constraints on outflow of incoming links because imbalanced demand and supply. If node demand is larger than node supply (for any of the outflow links – or possibly for an internal contraint), not all the demand flow can be satisfied and flow is then limited to the minimum flow that can be achieved. Therefore outflow of upstream links are constrained.

The second function of node model is to see consistency between demand and supply constraints. The constraints imposed only on link inflow and outflow leads to the

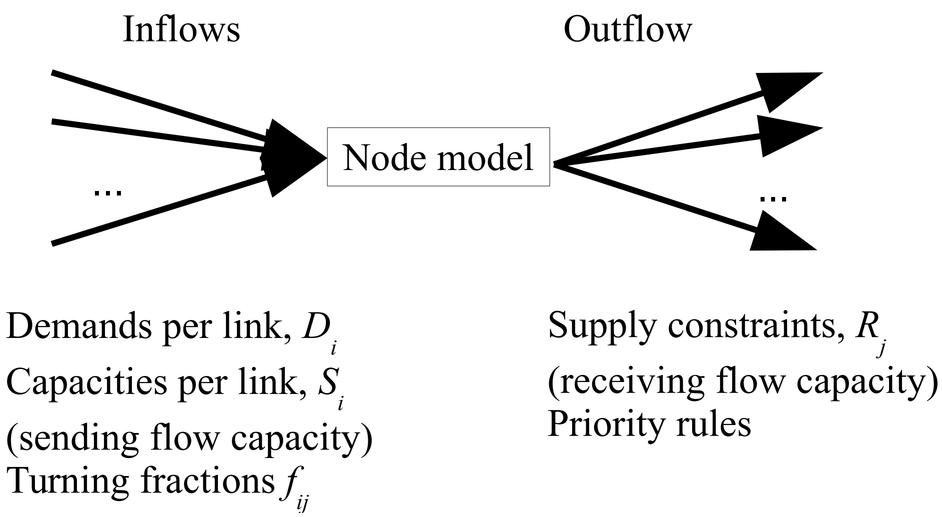


Figure 11.2: Node model structure

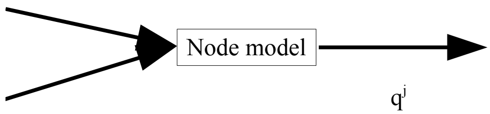


Figure 11.3: Merge model

interactions between each flows. It means any change of q_i^j may lead to changes of the other flow. Thus the node model distributes flow under the guarantee of consistency, and it becomes a flow optimization problem with demand and supply constraints.

We will in the sequel first show a simplified case to get the model to work for a merge or diverge with a the cell transmission model, and then comment on the generic requirements for the node models.

11.1. NODE MODELS IN THE CELL TRANSMISSION MODEL

Daganzo (1995a) first defines the two basic forms of a node model he considers: a merge model and a diverge model. Restricting the attention to networks with three-legged junctions, Daganzo described a merge model as two links enter the cell and only one link leave it, while diverge model has one incoming link and two outgoing links. Valid node representations are outlined in paper and it should be stressed that links can only belong to either the merge model or diverge model. Junctions with more legs are not considered here.

11.1.1. MERGE MODEL

PRINCIPLES

Considering three-legged junction, we have two incoming links (i=1,2) and one outgoing link (j=1) in merge model (figure 11.3). The demands at the two the incoming links are d_1 and d_2. The outgoing link has a supply constraint as R. More importantly, we define a capacity priority fractions p_1 and p_2, which indicate the relative priority to traffic coming from link 1 and link 2. The priorities are defined as fraction of the total flow, hence $p_1 + p_2 = 1$. Then, because of demand and supply constraints, the flow should satisfy:

$$q_1 \leq d_1 \tag{11.3}$$
$$q_2 \leq d_2 \tag{11.4}$$
$$q_1 + q_2 \leq R \tag{11.5}$$

There are now 3 possible combination of demand, which are also graphically shown in figure 11.4. That figure shows the flows on both axes (demands from the incoming links are denoted as d_1 and d_2, as well as the demand (d) per incoming link. We list them in order of increasing demand, or alternatively seen as order of decreasing maximum receiving flow.

1. A high maximum receiving flow R, or $d_1 + d_2 \leq R$. In this case, the demand is lower than the supply and all traffic can be handled. That results in $q^j = q_1 + q_2$. In this case, traffic is free flow on both incoming links, and $q_1 = d_1$ and $q_2 = d_2$.

2. If the $R < q_1 + q_2$, there is congestion at at least one of the two links. The outflow in this case is the capacity ($q^j = R$). Based on this flow, the maximum permissible flow per incoming link is $p_1 q_j$ for link 1 and $p_2 q_j$ for link 2. (At least one of the two exceeds this value, otherwise we are at condition (1)) Condition (2) holds if only one of the 2 exceeds this value, condition (3) if both exceed this value. The situation as depicted in figure 11.4 shows that $p_1 q^j > d_1$. Hence, all flow from link 1 can be served, so $q_1 = d_1$. We then can derive that $q_2 = R - q_1$. Graphically, in figure 11.4, that means that the flows are found at q^*, at the intersection of the demand of the inflow link 1 and the supply of the outflow link.

3. If the supply is restricted $R < q_1 + q_2$, and neither of the two incoming links can output his demand to the outflow link, the flows are distributed by priority fractions. We hence have: $q_1 = p_1 R$ and , $q_2 = p_2 R$. Graphically, in figure 11.4, these flows are on intersection of the line with the right turn fractions and the line of the maximum supply (3).

In a static traffic assignment model, flows from an incoming link can be sustained at a flow lower than capacity of that link. In a dynamic model, as soon as congestion occurs, the demand from the incoming link at the node immediately goes to the capacity of that link (there is namely congestion at the downstream end). In this model, that means that once there is congestion at both links, the ratio between the flows from each incoming link is fixed, and it is *not longer dependent on the flow further upstream on that link*. Therefore, in practice, the priority ratio will apply quite often. In practice, this ratio is determined by infrastructural design.

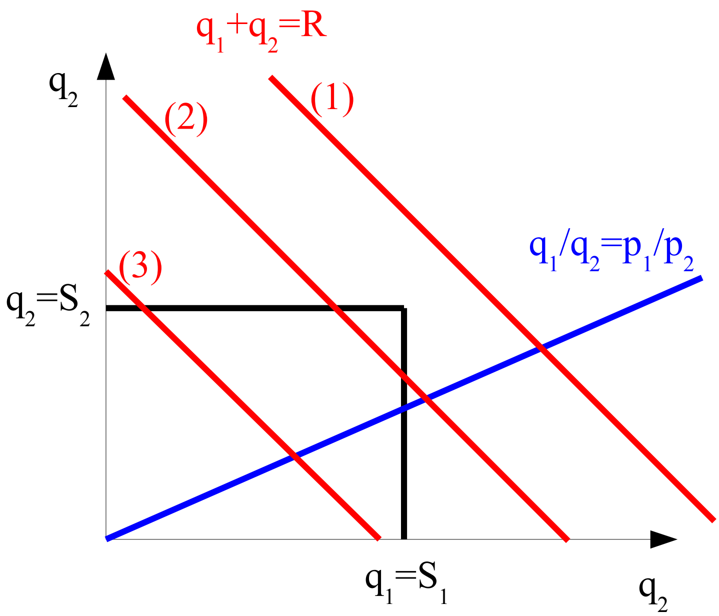


Figure 11.4: Diagram of feasible flows for a merge junction

NUMERICAL EXAMPLE

Let's now consider a numerical examples. Let's assume a road with a demand d_1=3500 veh/h, d_2=1500 veh/h. The priority of the roads in oversaturated conditions is 4:1 (for instance due to 4 lanes to the intersection in road 1 and 1 lane for road 2). We consider this system in 3 scenarios for the capacity of the outgoing link: 8000 veh/h, 4500 veh/h and 2000 veh/h. The matching situations are illustrated in figure 11.5

1. If the capacity of the outgoing link is 8000 veh/h, the full demand can be served. We hence have $q_1 = d_1$ =3500 veh/h and $q_2 = d_2$ =1500 veh/h.

2. If the capacity of the outgoing link is 4500 veh/h, that is not sufficient for all demands: $d_1 + d_2 = 3500 + 1500 > 4500$. If we consider how much of the capacity of the outgoing link would be available for inflow 1, that is $\frac{4}{1+4}4500 = 3600$ veh/h, which is higher than the demand of 1, being 3500 veh/h. Hence, the full demand of 1 is served: $q_1 = 3500$ veh/h. The remaining capacity is allocated to direction 2: $q_2 = 4500 - 3500 = 1000$ veh/h.

3. If the capacity of the outgoing link is 2000, neither of the two streams can be fully served while respecting the priority. Hence, we divide the capacity of the outgoing link over the priority rules: $q_1 = 2000 * \frac{4}{4+1} = 1600$, and $q_2 = 2000 * \frac{1}{4+1} = 400$

11.1.2. DIVERGE MODEL

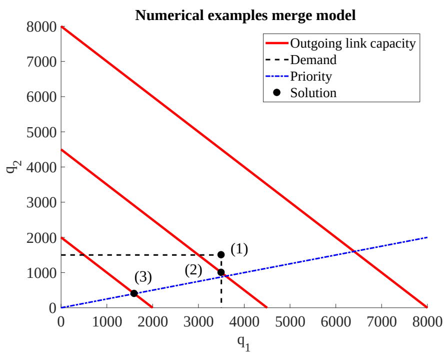


Figure 11.5: Numerical examples of the merge model

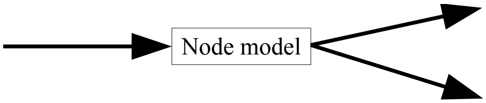


Figure 11.6: Diverge model

PRINCIPLES

In the diverge model, there is one incoming link (i=1) and two outgoing links (j=1,2). The objective is to maximize the total outflow q of the incoming link, and hence to maximize the total inflow to the downstream links. There is a demand from the incoming link equal to d. The supply constraints are R^1 and R^2. The fraction heading towards link 1 and 2 are denoted f^1 and f^2 respectively. With the turning fractions given, flows should satisfy:

$$q^1 = f^1 q \leq R^1 \tag{11.6}$$
$$q^2 = f^2 q \leq R^2 \tag{11.7}$$
$$q^1 + q^2 \leq S \tag{11.8}$$

It is assumed that a vehicle unable to exit the incoming link hinder the vehicle behind, which implies that the first-in-first-out rule (FIFO) is applied in the diverge model, which is also known as conservation of turning fractions (CTF) in this context. One can interpret this as that 1 out of n vehicles is turning, so if the flow to the exit is limited, also the flow to the main line is limited.

One can first determine whether all both are possible. If $f^1 d < R^1$ and $f^2 d < R^2$, all flow can pass, so $q^1 = df^1$ and $q^2 = df^2$.

If not, the most restrictive outflow link will determine the relative flow. Let's determine the fraction of demand towards a particular outflow link that can continue: $\phi^i = f^i d / R^i$. The minimum of these factors is $\Phi = min(\phi^1, \phi^2)$. Now, the overall flow should be reduced to a factor Φ of the demand:

$$q^1 = \Phi d f^1 \tag{11.9}$$
$$q^2 = \Phi d f^2 \tag{11.10}$$
$$\tag{11.11}$$

Note that the turning fractions can be specified explicitly, potentially as a function of time, but can also be derived by modelling traffic flows heterogeneously. In Daganzo (1995a), traffic is for example disaggregated by destination, and we can then specify the turning fractions per destination. E.g. Yperman (2007) further generalizes this into the concept of multi-commodity modelling. Van der Gun et al. (2015) for example instead use disaggregation by origin to specify turning fractions.

Finally, note that whereas this has been formulated as a two-link outflow model, this is easily transferred to a system with multiple outflow links.

NUMERICAL EXAMPLE

Consider a freeway with a capacity of 4000 veh/h (direction 1) and an offramp of 1000 veh/h (direction 2). The demand is 3000 veh/h. Now we consider two different situations:

1. If the turn fraction to the offramp (ϕ_2) is 1/6, the demand to the offramp is 1/6 * 3000 = 500 veh/h<1000 veh/h, so no congestion occurs. For the main line, the demand is 5/6 * 3000 = 2500 so no congestion occurs there either. We hence have $q^1 = 2500$ veh/h, and $q^2 = 500$ veh/h.

2. if the turn fractino to the offramp (ϕ_2 is 2/3, the demand to the offramp is 2/3 * 3000 = 2000 veh/h. This is twice the capacity of the onramp. The demand to the main line can fit into the main line (1/3 * 3000 = 1000 < 4000 veh/h), $\phi_1 = 1$. The minimum of the two fractions is $\Phi = \phi_2 = \frac{1}{2}$. That means that half of the demand can go through. We have $q^1 = \Phi 1000 = 500$ veh/h, and $q^2 = \Phi 2000 = 1000$ veh/h. Note that the total flow out of the node is $q = q^1 + q^2 = 1500 veh/h$. This means congestion occurs, and will grow on the upstream link.

11.2. GENERAL NODE MODELS

The previously discussed node models only apply to simple merges and diverges. General node models with an arbitrary number of incoming and outgoing links are a lot more difficult to formulate. Tampère et al. (2011) cite Bliemer (2007) as a typical general model for which they provide a numerical example.

An important problem with the Bliemer model, that also occurs in the previously described Daganzo (1995a) merge model, is that it does not satisfy the so-called invariance principle. Introduced by Lebacque (2005), this principle states that flows across an intersection should not change during an infinitesimal time step if demand and supply remain constant. In practice, that would happen if one assigns an outflow proportional to the overall inflow demand. One should consider that at one location the queue forms, and hence take – as explained in section 11.1.1 – a infrastructural proportionality constant (e.g., based on capacities

Before formulating a node model themselves, Tampére et al. first formulate the list of requirements it should satisfy:

- Support arbitrary numbers of incoming and outgoing links.
- Maximize flows. The final solution should not leave capacity underused if there is demand. There can also be internal constraints within the node.

- Produce non-negative flows.
- Conserve vehicles.
- Satisfy demand and supply constraints of the connected links. There may be additional internal node supply constraints, e.g. to account for crossing flows that do not have an incoming or outgoing link in common. The previously mentioned SCIRs should also be satisfied.
- Conserve turning fractions (CTF).
- Satisfy the invariance principle. (Discussed above)

Then, they mathematically formulate node models for unsignalized and signalized intersections satisfying these requirements. The employed solution to satisfy the invariance principle is to define priorities proportional to the capacities (i.e. maximum demands) of incoming links rather than the current demands of incoming links. The unsignalized intersection model does not include node supply constraints, whereas the signalized intersection model includes node supply constraints representing the green phases of the traffic lights. The solution algorithm is too complex to describe here, so the interested reader is referred to the paper for details. Despite its complexity, it is guaranteed to converge to the solution within a fixed, finite number of iterations.

However, the solutions are not necessarily unique, as shown by Corthout et al. (2012). There can be a case with through traffic and left turning traffic, where through traffic has priority. If a situation is triggered which makes one direction flow (say, direction bottom-up), then that limits the turning flow in the opposite direction (top-down), and hence also the through flow in the opposite direction (top-down). That in turn allows a higher turning flow from bottom to top, strengthening the unequal situation. The same situation could also be reversed, with a higher flow bottom to top. Note that the problem is symmetrical, and the priority rules are not different from one side or the other.

11.3. DISCUSSION

One may wonder whether there exist other approaches to solve a general node model. As an alternative to the Tampère et al. (2011) algorithm, Flötteröd and Rohde (2011) propose an incremental procedure and extend that to include node supply constraints, finding the same solution in a different way. There seems plenty of work to do to operationalize and validate node models for all kinds of complex intersection shapes that exist in reality, including e.g. roundabouts. For example, Smits et al. (2015) show that while the invariance principle can be satisfied without assuming capacity-proportional priorities, the solution algorithms of the such node models may not fully converge within finite time.

As final note: there is an analogy between the node models and a macroscopic multilane framework extending the cell transmission model. That lane change model can be expressed in terms of a network, where each link represents a lane for the length of some segment and each node between segments represents an opportunity to change lanes Laval and Daganzo (2006); Nagalur Subraveti et al. (2019)

12

MACROSCOPIC FUNDAMENTAL DIAGRAM

After this chapter, the student is able to:

- Explain the concept of the MFD
- Explain the differences between the MFD and FD
- Comment on the influence of density heterogeneity on the MFD
- Give examples of control concepts based on the MFD
- Explain the value of perimeter control, for instance using cumulative curves

The idea of a Macroscopic Fundamental Diagram (MFD) or Network Fundamental Diagram (NFD) is that rather than at the level of a road, at the level of an *area* there exists a relationship between the number of travellers on the road and the average speed of these travellers. Moreover, it could be argued that the noise in the measurements might be less. For individual detectors, one will find deviations from the fundamental diagram, (both upward and downward) which creates high scatter. Geroliminis and Daganzo (2008) showed that if one averages all detectors over a large area almost all scatter disappears (see figure 12.1; figure from Geroliminis and Daganzo (2008)). From a mathematical point, this makes sense since it simply is the law of large numbers. But although the concept seems simple, the effect might be large.

The essence of an MFD is that a high density affects the traffic flow to even under capacity. This is in contrast to a single road with a bottleneck, where in case of a high demand, the outflow will be at capacity (or queue discharge rate). This internal congestion can only take place if the (tail of) the congestion influences the throughput. This is the case with spill back effects, e.g. a tail of the queue growing backwards and thereby influencing drivers which want to take an exit which does not pass the bottleneck. A very

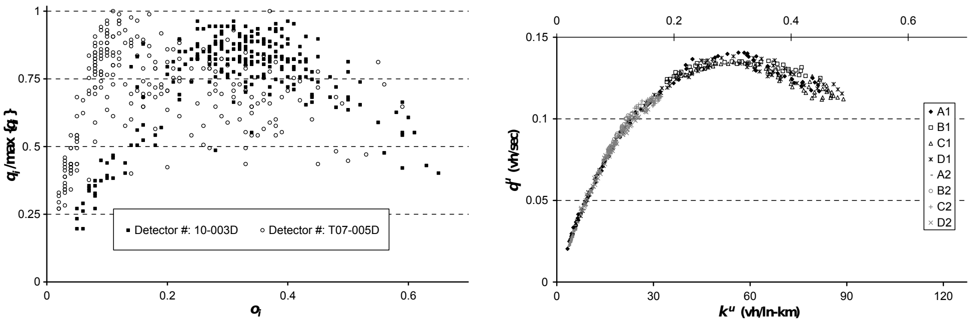


Figure 12.1: The first Macroscopic Fundamental Diagram constructed from data, from Geroliminis and Daganzo (2008).

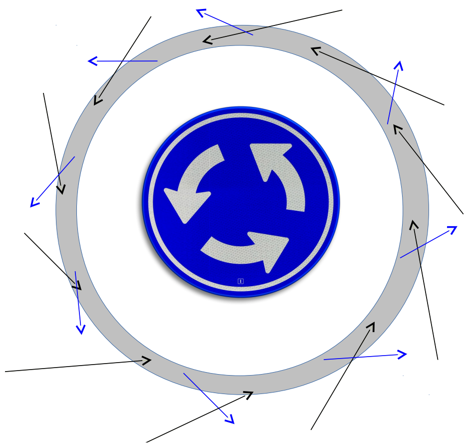


Figure 12.2: A simplified network congestion influences the outflow.

Table 12.1: Variables for the MFD

Name	symbol	meaning
Accumulation	A	Number of vehicles in the network (veh, or veh/km)
Production	P	Internal flows in the network (veh/h)
Performance	$\mathscr{P}$	Outflow out of the network (veh/h)

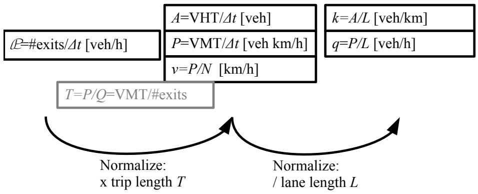


Figure 12.3: The relation between the different quantities related in the MFD

much simplified example of this is given by Daganzo (2007), which proposed a single ring road with entrances and exits everywhere, see fig 12.2.

In MFDs, some terminology is different (see table 12.1 for an overview). Accumulation (A) is the total number of vehicles in a network, which can be expressed as total number of vehicles or number of vehicles per roadway length. Note that since the roadway length is fixed, these two are proportional. Some scientific works separate accumulation from the average density, with the network length (preferably the lane-kms) as proportionality factor. It can be argued that for comparison of the traffic state between networks the scaled version, average density, is more relevant. In the sequel of this work, we will hence use accumulation also as scaled variable.

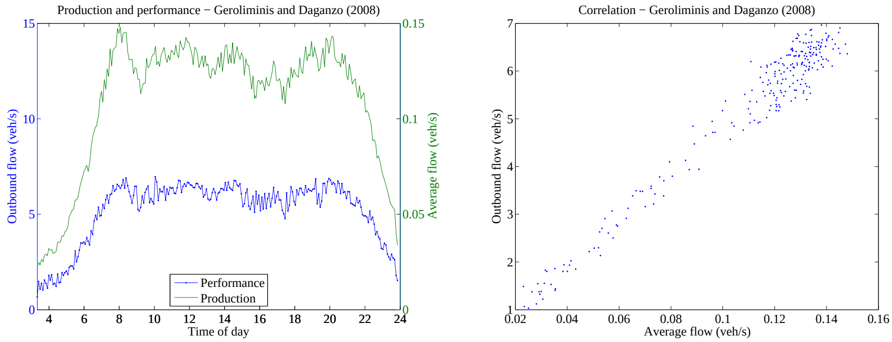
Production (P) is the internal flow in the network. The average flow can be computed by Edie's definitions for all links in the network (equation 1.8). If this is not corrected for the network length (only total travel distance divided by the aggregation time), one gets the total production. Similar to accumulation and average density, it can be argued that a scaled version would be more useful to compare between networks, hence here we will be using production for the average flow (i.e., the total distance divided by the aggregation time *and* the network length). Performance ($\mathscr{P}$) is the outflow of the network, which is the sum of the flow of the outgoing links. Geroliminis and Daganzo (2008) show there is a strong relationship between the production and the performance. This relationship for empirical data is shown in figure 12.4. The ratio of production and performance is the length of the trip within the zone. (As a thought experiment: if one increases the trip length inside the zone by a factor of two, the internal flow, and hence the production, needs to increase by a factor of two before the same outflow, performance, is reached).

This chapter first discusses what could be a possible control application for the MFD, before continuing to conditions under which the MFD holds (section 12.2). Then, section 12.3 discusses a way to include the MFD into a simulation model, and the chapter ends with some recent developments.

12.1. PERIMETER CONTROL

The idea behind perimeter control is as follows. As the production decreases with an increasing accumulation, it might be better to limit the accumulation in a zone, the so called protected network. Let's consider for the sake of simplicity the situation, with one zone which has this limitation, and storing vehicles outside the network will not affect traffic operations in the neigbouring zones (of course the waiting vehicles themselves are affected).

12



(a) timeseries

(b) correlation

Figure 12.4: The relation between the production and the performance

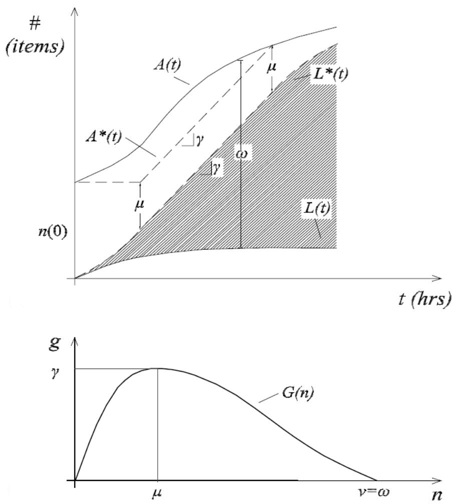


Figure 12.5: The effect of perimeter control in cumulative curves, figure from Daganzo (2007).

If we assume a constant trip length, for instance by assigning random exits around the ring road, the production will decrease as well. This causes an escalating effect of further increasing accumulation and a further decreasing outflow. An example can be found in figure 12.5 (figure from Daganzo (2007)). This principle will now be discussed.

The top figure shows the cumulative inflow curve (upper curve, indicated $A(t)$ – note that in this example, the original notation from (Daganzo, 2007) will be used), and the cumulative outflow curve without ($L(t)$) and with perimeter control ($L(t)$). The inflow curve is externally given, as is the MFD showing the production (outflow, G in this example) as function of accumulation (lower figure, n in the example, so the line is indicated $G(n)$). The accumulation in the system can be found by the difference between the inflow and outflow. Only the outflow at t=0 is specified, which equals 0, leading to the accumulation $n(0)$, which happens to be greater than the critical accumulation μ.

The dynamics of the outflow can now be predicted. In case of no perimeter control, the performance is governed by the MFD, which is the derivative of the outflow function. In the example $n(0) > \mu$, so the traffic is in a congested state, and the outflow can be read from $G(n)$. This outflow is lower than the inflow into the system, and as a consequence, the accumulation increases, leading to a decreasing outflow. Hence, the cumulative outflow curve $L(t)$ flattens. Once the accumulation increases to ω, the maximum number of vehicles in the road, the outflow reduces to 0.

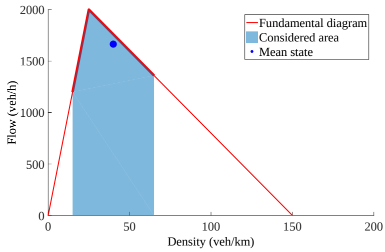
Perimeter control entails limiting the inflow into the ring, and keep this at the level of the critical accumulation where the outflow is highest, i.e. not exceed μ. The inflow curve with the restriction is indicated with $A^*(t)$ in the figure. It starts with not letting any vehicles in until the accumulation is equal to μ. By consequence, the outflow increases over this time, since the accumulation becomes closer to mu, and hence the outflow closer to the maximum outflow γ. Once the accumulation is reduced to μ, the outflow is γ; to maintain this number of vehicles, the same amount of vehicles are left in the system. Once the queue is solved, i.e. no more vehicles have to wait to get into the system, the perimeter control stops. The outflow curve constructed in this way is indicated as $L^*(t)$.

The perimeter control comes at a cost, the waiting time outside the network. This can be indicated in the figure as the area between lines A^* and A. The benefit is the higher outflow, which is the shaded area between L^* and L. Note that the benefits exceed the costs.

12.2. TRAFFIC DYNAMICS

A very strict claim on the application of the MFD is that it only holds if the traffic conditions are homogeneous throughout the network. In that case, all links in the area have a similar density, and the MFD simply is an average of all the same traffic states; the average then is that traffic state. In Cassidy et al. (2011) it has been shown what the averaging effects are in case a triangular fundamental diagram is assumed for the underlying links.

Figure 12.6 shows the effect of averaging two traffic states. If both states lie on the free flow branch, the average also lies on the free flow branch. If both lie on the congested branch, the average also lies on the congested branch. Only if one point lies on the congested branch and one point at the free flow branch, the average does not lie on either branch, but under the fundamental diagram. With a similar reasoning it can be shown that the average traffic state always lies *under* the fundamental diagram for one



12

Figure 12.6: The construction of the average traffic state for two states on a triangular fundamental diagram.

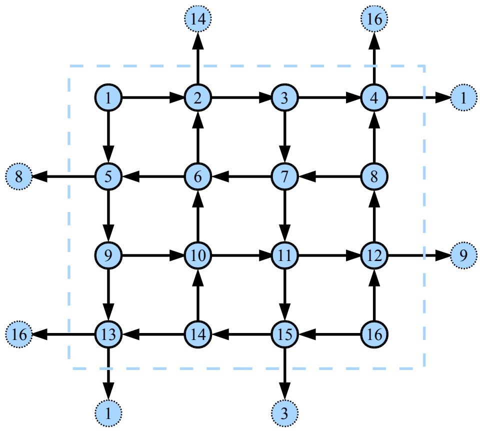


Figure 12.7: Illustration of a 4x4 grid network with periodic boundary conditions

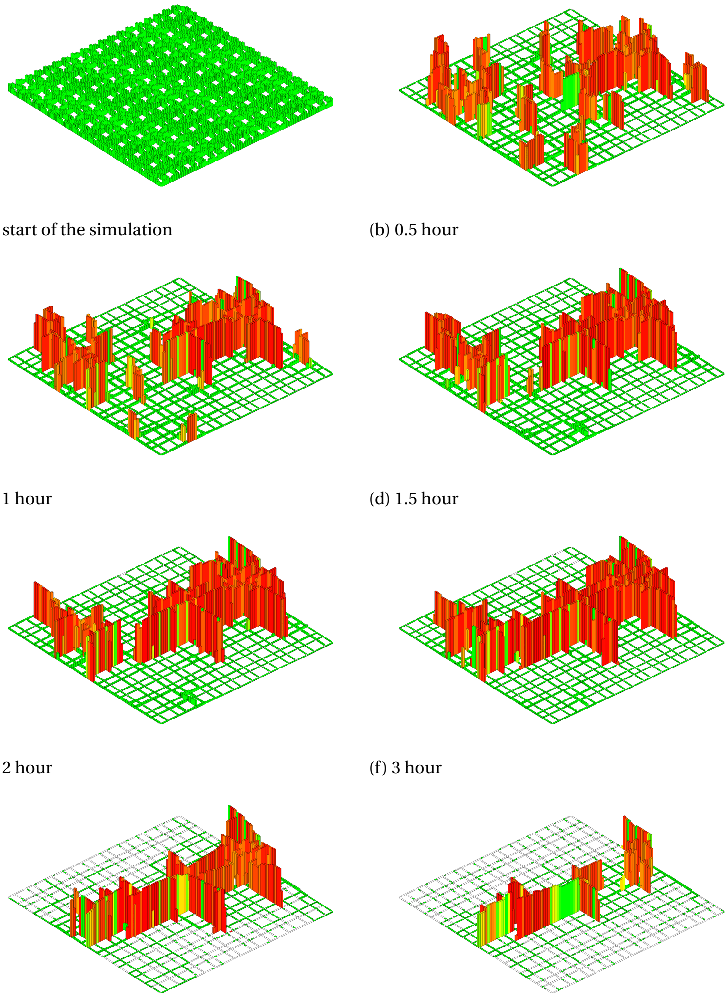
link if the fundamental diagram is concave.

One can continue this arguing and find that the larger the spread is, the further the average flow deviates from the flow which would match the average density according to the fundamental diagram. In an extreme case, some links are empty (q=0, k=0), and other links are jammed (q=0, k=k_j). All links then have a flow of 0, so the average flow also equals 0. However, the flow matching the average density, somewhere between $0 < k < k_j$ in the fundamental diagram is larger than 0.

12.2.1. Approaches to include the standard deviation

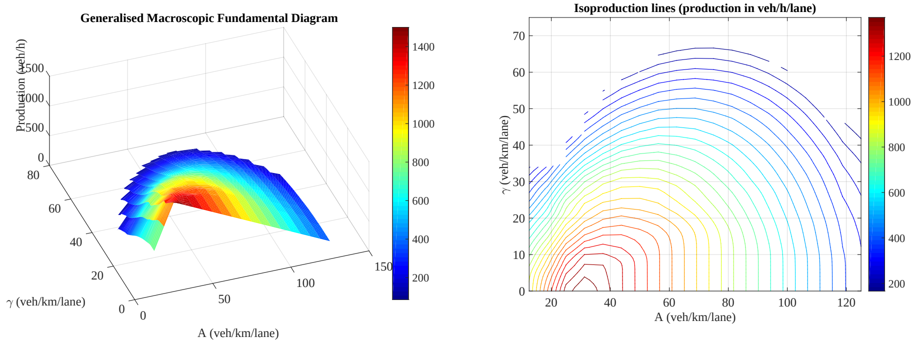
It hence makes sense to include the spread of the densities as second explanatory variable besides the average density.

One can show the effects of the network dynamics by performing a traffic simulation study and analysing the results (Knoop et al., 2015). One of the simplest network setups is a Manhattan grid network (one way streets in a square grid) with periodic boundary conditions. Periodic boundary conditions mean that traffic exiting at the right of the network enters at the left and traffic from exiting at the top enters at the bottom and vice versa. The network structure is shown in figure 12.7.



(a) start of the simulation
(b) 0.5 hour
(c) 1 hour
(d) 1.5 hour
(e) 2 hour
(f) 3 hour
(g) 3.5 hour
(h) 4 hour

Figure 12.8: **Evolution of the densities (bar heights) and speeds (colours) in the network**



(a) The generalised macroscopic fundamental diagram

(b) Iso-production lines in the accumulation-inhomogeneity of density plane

Figure 12.9: The generalised macroscopic fundamental diagram

When the traffic starts to run, various distributed bottlenecks become active. This is shown in figure 12.8b. After some time (figure 12.8d-f), traffic problems concentrate more and more around one location. The number of vehicles in the rest of the network reduces, ensuring free flow conditions there. This complete evolution can be found in figure 12.8a-f.

This indicates that with this second explanatory variable, the traffic flow can be approximated reliably. We thus define a concept, the generalised fundamental diagram (GMFD) which predicts the production as function of the accumulation and the standard deviation of density (Knoop et al., 2015)

$$P = P(A, \sigma_A). \tag{12.1}$$

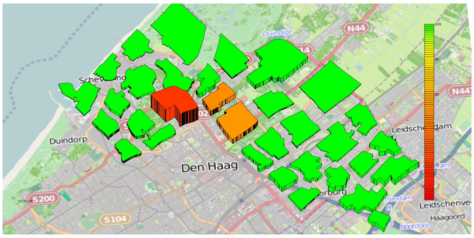
Figure 12.9a shows the GMFD. Figure 12.9b shows the same surface, but now in iso-production lines. The production decreases once the inhomogeneity increases, and that this holds for every accumulation.

12.3. SIMULATION

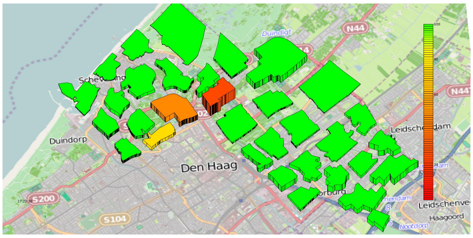
Since there is a relation between the number of vehicles in a zone and the outflow, also a dynamic simulation model can be developed. This is a recent development, see e.g. Knoop and Hoogendoorn (2015) for the Network Transmission Model. In this model, an area is divided into zones, and the traffic flow is calculated using the MFDs.

In each of the MFDs, the principle as described in the beginning of this section is implemented, where the accumulation determines the outflow. The interesting part of this concept is that, contrary to the isolated example in 12.1, the queuing outside the protected network also influences the traffic flow in that zone. Hence, there is a trade-off between letting vehicles into a protected zone, delaying the protected zone, or letting them wait outside, delaying the vehicles outside. Also, routing around the protected zone is an option, at the cost of more traffic outside (Hajiahmadi et al., 2013).

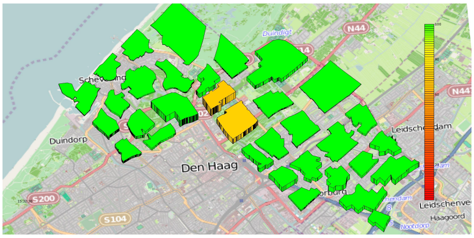
This model has been applied for the city of The Hague (Knoop et al., 2016), with a good result, showing that the rough approach works in giving the approximately right



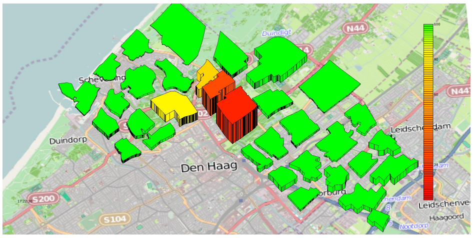
(a) Initial parameters, first part of simulation



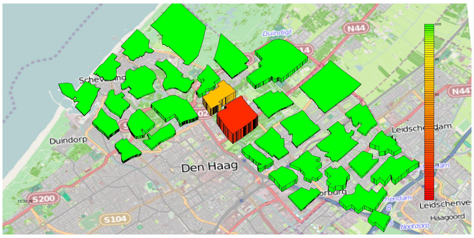
(b) Initial parameters, last part of simulation



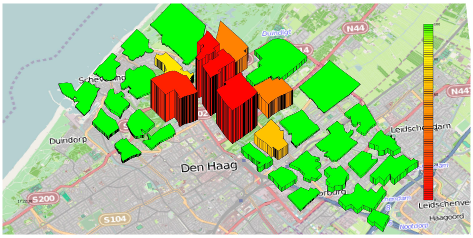
(c) Adapted parameters, first part of simulation



(d) Adapted parameters, last part of simulation



(e) Adapted parameters, accident, first part of simulation



(f) Adapted parameters, accident, last part of simulation

Figure 12.10: Screen shots of the simulation. The color indicates the speed relative to the free flow speed, and the height the accumulation, figure from Knoop et al. (2016)

results for traffic states. The model output with speeds (color) and accumulations (bar height) is shown in figure 12.10 (from Knoop et al. (2016)).

12.4. RECENT INSIGHTS

The field of the MFD is actively studied. The simulation models of which one example is described above in section 12.3 are field of studies, but there are other approaches as well. The division of space into different zones is also currently being studied, since one would like to have homogeneous areas, see Ji and Geroliminis (2012).

An interesting development is to estimate the MFD from scratch, without measurements. For fundamental diagrams, such estimates are possible. Laval and Castrillón (2015) uses an interesting technique to do the same for a zone, simplified to a ring road with random traffic lights. This paper follows other work, for instance by Leclercq and Geroliminis (2013).

Finally, the route length is not constant. Leclercq et al. (2015) indicates that the traffic patterns might lead to people using a different, non-shortest, route, thereby influencing the relationship performance-production. All in all, some aspects of the MFD still need studying, but the concept is perceived as promising.

SELECTED PROBLEMS

For this chapter, consider problems: 26, 27, 86, 87, A.5.5, 157, 158, 159, 164, 202, 214, A.10.6, 268

13

METHOD OF CHARACTERISTICS

The kinematic wave model provides a relatively simple approach to reproduce and predict the spatio-temporal characteristics of traffic congestion by means of a macroscopic model, either by simulation (numerical solution approaches) or, as we will discuss in this chapter, analytically. The so-called *method of characteristics* (MoC) is a well known solution method for partial differential equations.

The kinematic wave model is, however, a bit special. Since in many cases, the kinematic wave model does not have a classical solution (solutions which are continuously differentiable) due to the occurrence of shocks. As a result, we will look for so-called *generalised solutions* (weak solutions that are piece-wise continuously differentiable). However, since there is an infinite number of generalised solutions, we will have to look for a special solution which satisfies the *entropy condition* (i.e. local maximisation of the flow).

This chapter describes the MoC approach to determine solutions of the kinematic wave model. We will start with briefly explaining the general concept and the mathematics. Then, section 13.2 will show two examples.

13.1. MATHEMATICAL CONSTRUCTION

This section first describes the construction of the curves, then applies the theory to a road with no entering and exiting traffic Section 13.1.3 gives the interpretation.

13.1.1. CONSTRUCTION

The conservation of vehicles yields that change of the number of vehicles in a section ($\frac{\partial k}{\partial t}$) is explained by the difference in flow at both ends of the section $\frac{\partial q}{\partial x}$, the inflow within that section (r, also called a source) and the outflow within that section (s, also called sink term). Put together, we get the conservation equation:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = r - s \tag{13.1}$$

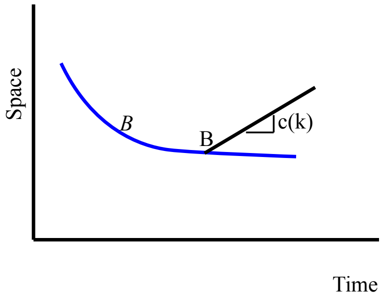


Figure 13.1: A boundary with known traffic states in the space-time diagram, and a characteristic curve

Note that the flow and density are both functions of time t and location x. We furthermore have:

$$q = Q(k, t, x) \tag{13.2}$$

Note that we allow spatial and temporal fluctuations in the fundamental diagram.

Now suppose that we know the traffic conditions (i.e. the traffic density) for a boundary $\mathscr{B}$ in the (t, x) plane. These can either reflect initial conditions (e.g. the density on the road at some initial time $t = t_0$), boundary conditions (inflow into the considered roadway section; outflow restrictions), etc.

Together with the boundary conditions, the kinematic wave model can be used to determine the traffic conditions influence by the traffic conditions described at the boundary $\mathscr{B}$ (see figure 13.1).

The idea of the method of characteristics is to transform the kinematic wave model (which is a partial differential equation) into a set of ordinary differential equations, which can be solved with standard techniques. To see how to do this, we fill in Eq. (13.2) into Eq. (13.1) yielding:

$$\frac{\partial k}{\partial t} + \frac{\partial Q}{\partial k}\frac{\partial k}{\partial x} = \frac{\partial k}{\partial t} + c\frac{\partial k}{\partial x} = r - s - \frac{\partial Q}{\partial x} \tag{13.3}$$

Note that $c = c(k, t, x) = \frac{\partial Q}{\partial k}$ is the slope of the fundamental diagram at (t, x), describing the speed of a very small shock (a so-called kinematic wave).

Let us now consider a point B on the boundary $\mathscr{B}$. This point (t_B, x_B) describes the initial point of the characteristic curve. Next to the initial point, the curve is defined by its slope, which is by definition equal to the kinematic wave speed $c = c(k, t, x)$ (see figure 13.1). This gives us the following expression:

$$\frac{dx}{dt} = c(k, t, x) \tag{13.4}$$

with:

$$x(t_B) = x_B \tag{13.5}$$

Note that on the boundary we know the density k. If we are able to determine the change in the density along the characteristic from that point onward, we can determine the density along the characteristic. That is, we want to establish $\frac{dk}{dt}$. We get:

$$\frac{dk}{dt} = \frac{\partial k}{\partial t} + \frac{\partial k}{\partial x}\frac{dx}{dt} \tag{13.6}$$

Since along the characteristic, we have $\frac{dx}{dt} = c$, we can rewrite:

$$\frac{dk}{dt} = \frac{\partial k}{\partial t} + c\frac{\partial k}{\partial x} = r - s - \frac{\partial Q}{\partial x} \tag{13.7}$$

with initial condition $k(t_B, x_B) = k_B$ given. This ordinary differential equation shows how the density changes along the characteristic. Considering all characteristics emanating from the boundary should allow us to construct the density everywhere within the interior of the boundary, at least at all points where the density is determined by the conditions in the boundary (so-called region of influence of $\mathscr{B}$).

13.1.2. APPLICATION TO HOMOGENEOUS ROADS

To further interpret these results, let us now consider a homogeneous road, i.e. $Q(k, t, x) = Q(k)$. Furthermore, no traffic enters or exits the road. In this case, Eq. (13.7) becomes:

$$\frac{dk}{dt} = 0 \tag{13.8}$$

This means that the derivative of the density along the characteristic is equal to 0, i.e. the density is constant along the characteristic. Since $k(t_B, x_B) = k_B$, along the entire characteristic we have $k(t, x) = k_B$. In other words, the density is conserved along the characteristic curve.

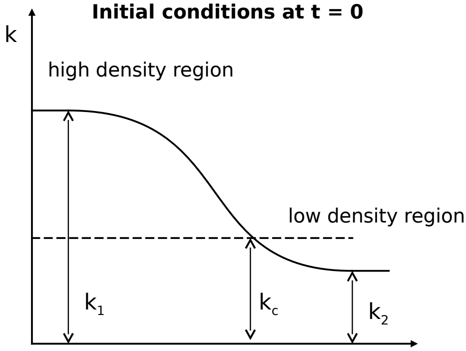
Furthermore, the density is constant along the curve, so the kinematic wave speed is $c = c(k) = \frac{\partial Q}{\partial x}$. Since by definition the characteristic curve satisfies $\frac{dx}{dt} = c$, the slope of the curve is under these conditions constant meaning that the curve is in fact a straight line.

13.1.3. INTERPRETATION

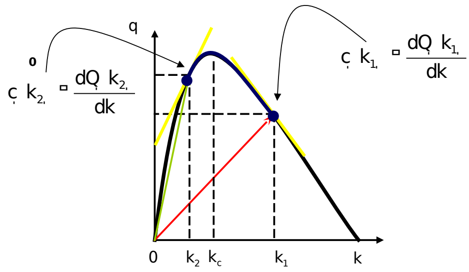
The consequence from the mathematics described in section 13.1.1 is that traffic states always propagate on lines in space-time moving with the speed of the derivative of the fundamental diagram. This makes the construction of the characteristics very simple, and the characteristics can be used to track back or forward traffic conditions in space and time.

13.2. APPLICATION

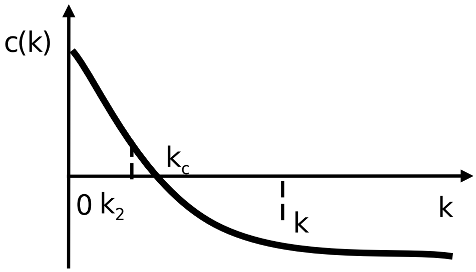
Let us now see an application of the method of characteristics in the prediction of traffic conditions. For this case, we will take the boundary $\mathscr{B}$ to be the initial conditions on an infinite length roadway section. Two cases will be considered here: the first one where traffic is dense upstream and in low density downstream (e.g., the moment a traffic light turns to green). Also the different situation is considered, where traffic upstream is in a low density condition and traffic downstream is in a high density condition.



(a) initial condtions



(b) Fundamental diagram



(c) Characteristic speeds

Figure 13.2: Traffic states and their characteristic speeds for an acceleration fan

13.2.1. ACCELERATION FAN

Consider a situation where upstream traffic is in an overcritical state (i.e., density is higher than the critical density) and downstream the traffic is in an undercritical state, see figure 13.2a. The characteristic speeds depend on the traffic density. For a concave fundamental diagram (see figure 13.2b) they are sketched in figure 13.2c.

For each of the states at the initial position the density is determined. For that density, the characteristic speed is determined, i.e., the derivative of the fundamental diagram at that density. Note once more that these speeds are not the speeds of the vehilces in that traffic state. Figure 13.2c sketches the characteristic speed. The traffic states now propagate with that speed. This means that states in the free flow condition propagate in the forward direction, whereas congested traffic states move in the backward direction. The capacity state moves at a speed zero, so remains at the same location.

The characteristic curves and their speeds are shown in figure 13.3. Due to the concavity of the fundamental diagram, these lines spread out. The area over which the density changes hence take place increases. Because the lines are straight, the increase is linear over time. Because of the pattern, this phenomenon is also called an acceleration fan.

Note that the analysis here is based on a concave fundamental diagram. If a triangular fundamental diagram would be used, one might argue there are only two waves: one moving forward with the free flow speeed and one moving backward with the wave

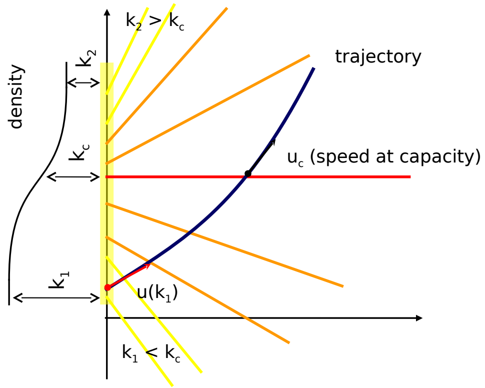


Figure 13.3: The kinematic waves (or characteristic curves) for an traffic situtation where the upstream traffic state is overcritical and the downstream traffic state is undercritical

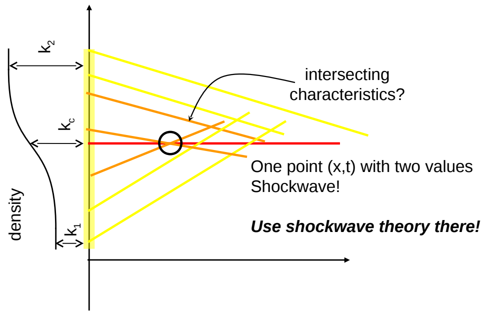


Figure 13.4: Illustration of the characteristics in space-time for an deceleration

speed. This would leave a void in conditions at the area in between these waves. It is better to consider a triangular fundamental diagram as a continuously differentiable fundamental diagram which happens to change curvature very slowly on the branches and very quickly at the top. In that case, the same analysis can be done.

In case of a traffic light, the density drop can be sudden, where upstream of the traffic light the traffic is in jam density and downstream the road is empty. Then, this could be approximated by a smooth curve with a very sharp decrease of density with an increase of space at the stop line. Then, one could apply the same analysis, yielding the same characteristic curves expanding, now all from the location of the stop line.

13.2.2. DECELERATION

A similar construction can be made for a deceleration wave. The difference is that in this case the upstream traffic states have a higher density than the downstream ones. The traffic states with higher densities, in this case the downstream states, have lower characteristic speeds (see figure 13.2b). See figure 13.4 shows the characteristics in space time. In case of a deceleration, the upstream characteristics travel faster than the downstream characteristics, leading to intersecting characteristics. Since each characteristic curve represent a traffic state, crossing characteristics would mean having two traffic states at the same place at the same time.

If the characteristics cross, one has to revert to shockwave theory. Each of the characteristics indicates a certain traffic state. If these two traffic states meet, one has a shock for which the speed is known (see section 4).

SELECTED PROBLEMS

For this chapter, consider problems: 145, 146, 147, 216, 269

14

HEADWAY MODELS

This section discusses headway models. In this chapter, the headway is always the time headway.

14.1. RELATION BETWEEN HEADWAY AND CAPACITY

The headway distribution describes which headways can be observed with which probability. One might argue that this is related to the flow, since the average headway is the flow. The flow is determined by the demand. However, in the bottleneck the flow is determined by the minimum headway at which drivers follow each other. Or in other words, the headway distribution determines the capacity in the bottleneck. Suppose the headway distribution is given by $P(h)$. Then, the average headway can be determined by

$$\langle h \rangle = \int_0^{\text{inf}} hP(h)dh \tag{14.1}$$

This is a mathematical way to describe the average headway once the distribution is given. The flow in the active bottleneck, hence the capacity is the inverse of the mean headway, see table 1.1:

$$C = 1/\langle h_{\text{bottleneck}} \rangle \tag{14.2}$$

The number of vehicles arriving in a certain period could be a useful measure. This holds for instance for traffic lights, where the number of arrivals per red period is relevant. As illustrated in figure 14.1, there could be different lanes for different directions at a traffic light. The idea is that the traffic towards one direction will not block the traffic to other directions, hence, the length should be long enough to allow the number of vehicles in the red period. The average number of vehicles in a red period can be determined from the flow. However, mostly requirements are that in p% of the red times (under a constant demand) the queue should not exceed the dedicated lane. In that case, the distribution of number of arrivals in that period can form the basis for the calculations.

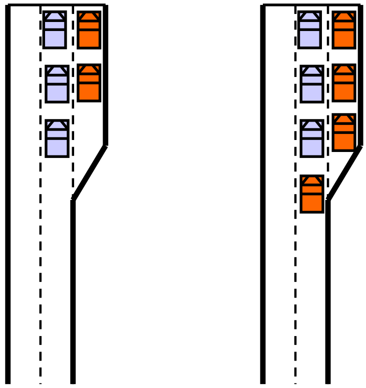


Figure 14.1: A queuing area. The orange coloured vehicles turn right, whereas the blue ones continue straight

Table 14.1: The different processes and the underlying assumptions

Process characteristic	Headway dist	Dist of nr of arrivals per interval
Independent arrivals	Exponential	Poisson
Correlated arrivals		Binomial
Negatively correlated arrivals		Negative binomial

Table 14.2: Overview of the means and variances of the different distribtions. In this table, q is the flow in the observation period, p is the probability of including the observation in the period.

Distribution	Mean	Variance	
Poisson	q	q	=mean
Binomial	np	$np(1-p)$	<mean
Negative binomial	$n(1-p)/p$	$n(1-p)/p^2$	>mean

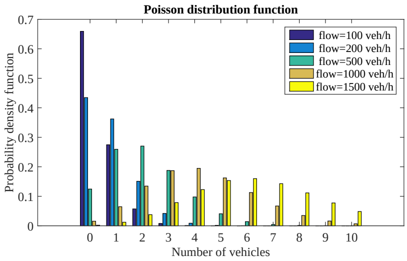


Figure 14.2: Example of the Poisson distribution for different flow values; the flow is indicated in veh/h, and these are the probabilities for arrivals in 15 seconds

14.2. ARRIVALS PER INTERVAL

This section describes the number of arrivals per time interval. For different conditions, this distribution is different. Table 14.2 gives an overview of the distributions described in this section, and gives some characteristics. One should differentiate between the probability of number of arrivals (a macroscopic characteristic, based on aggregating over a certain duration of time), described in this section, and the probability of a headway (a microscopic characteristic, described in section 14.3).

14.2.1. POISSON

The first distribution function described here is the Poisson distribution. One will observe this distribution function once the arrivals are independent (see also section 14.3.1). The resulting probability is described by a so-called Poisson distribution function. Mathematically, this function is described by:

$$P(X = k) = \frac{q^k}{k!}e^{-q}. \tag{14.3}$$

This equation gives the probability that k vehicles arrive if the average arrival rate *per period* is q. Hence note that one needs to rescale q to units of number of vehicles per aggregation period!

As example, consider a flow of 600 veh/h with independent arrivals. What is the probability to have 7 vehicles arriving in a period of 30 seconds? Answer: we first rescale the flow to the same interval as the period of interest, i.e. 30 seconds. We find q=600 veh/h = 5 veh/30 seconds. Now, applying equation 14.3, we find the probability of finding 7 vehicles in the observation interval:

$$P(X = 7) = \frac{5^7}{7!}e^{-5} = 0.104 \tag{14.4}$$

So in this example, there is a 10.4% chance that 7 vehicles arrive in this period.

Figure 14.2 shows examples of the Poisson distribution. Note that for low values of the flow (expected value smaller than 1), the probability is decreasing. If the flow is higher, there is a maximum probability at a number of arrivals which is at a higher value than 1.

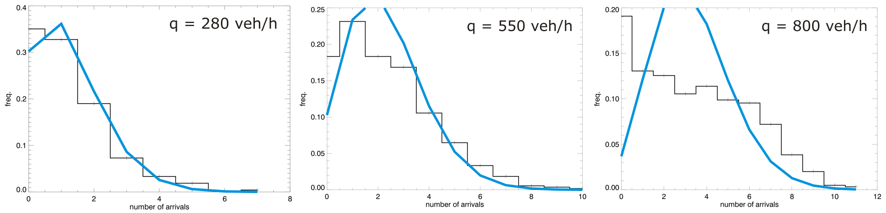


Figure 14.3: Illustration of the number of arrivals from real world data

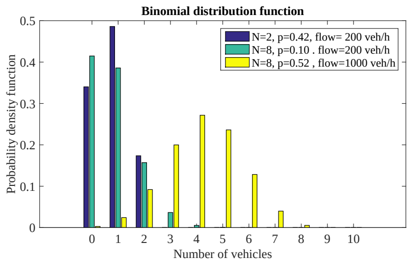


Figure 14.4: Examples of the binomial distribution function for the number of arrivals in an aggregation period, conceptual idea from Hoogendoorn (2005)

Figure 14.3 shows the best fits of this distribution on real life data. This distribution is accurate if the flow is low, and is not so good if the flow increases. This is because once the flow is high, the assumption of independent arrivals does not hold any more. Once vehicles are bound by the minimum headway, the arrivals are not independent any more. This restriction come more into play once the flow is high.

14.2.2. BINOMIAL

The binomial distribution can be used if there are correlations between the arrivals. For instance on busy roads, one can expect that more vehicles drive at a minimum headway. Whereas in case of the Poisson distribution, the variance of the distribution was equal to the mean, in this case the variance is smaller (since more drivers drive at a certain headway). The mathematical equation describing the function is:

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{14.5}$$

The idea behind the distribution is that one does n trial, each with an independent success rate of p. The number of successes is k. The mean of the distribution is np, see table 14.2. A certain flow specifies the mean of the arrivals, which hence determines np. This gives a freedom to choose n or p, by which one can match the spread of the function. The number of observations in the distribution can never exceed n, so an reasonable choice of n would be the interval time divided by the minimum headway. Figure 14.4 shows examples of the binomial distribution function.

Note that the variance of the binomial function is smaller than the Poisson distribution for the same flow. This can be a reason to choose this function.

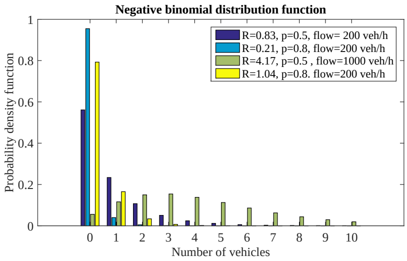


Figure 14.5: Example of the negative binomial function for the number of arrivals in an aggregation period

14.2.3. NEGATIVE BINOMIAL

The negative binomial distribution can be used if there are negative correlations between the arrivals. In traffic this happens for instance downstream of a signalised intersection. If one observes several vehicles at a short headway, one gets a larger probability that net next headway will be large because the traffic light will switch to red.

The probability distribution function for the number of observed vehicles in an aggregation interval is:

$$P(X=k)=\binom{k+r-1}{k}p^k(1-p)^r \tag{14.6}$$

This distribution describes when one observes individual and independent process with a success rate of p. One observes so until r failures are observed. X is the stochast indicating how many successes are observed.

Figure 14.5 shows the value of this function for different parameter sets. Note that the variance (and hence standard deviation) can be set independently from the mean, like in the binomial distribution function. For this distribution (see table 14.2), the mean is given by $n(1-p)/p$, and the variance is given by $n(1-p)/p^2$, which is the mean divided by p. Since p is a probability and has a value between 0 and 1, we can derive that the variance is larger than the mean. A larger variance is what one would intuitively expect downstream of a signalised intersection. This characteristic can be used to have an idea of the distribution to use.

14.3. HEADWAY DISTRIBUTIONS

In this section the exponential headway distribution is described, used with independent arrivals, and composite headway models.

14.3.1. EXPONENTIAL

The first distribution is the exponential distribution, shown in figure 14.7. This is defined by:

$$P(h|q)=q\exp-qh \tag{14.7}$$

Note that this equation has a *single* parameter, being the flow q. That is the inverse of the average headway. The underlying assumption of this distribution is that all drivers can choose their moment of arrival independently. Consider that each (infinitesimal small)

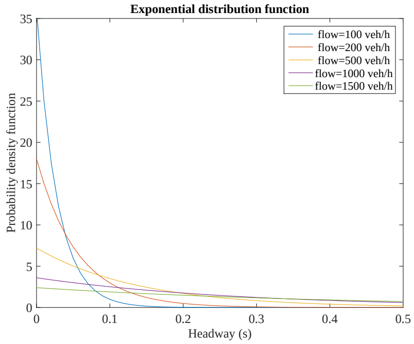


Figure 14.6: The probability density function for headways according to the exponential distribution for different flow values

time step a driver considers to leave with a fixed probability. One then gets a exponential distribution function for the headway.

This is a very good assumption on quiet roads, when there are no interactions between the vehicles. The interactions occur once vehicles are limited in choosing their headway, mostly indicated by the minimum headway. An illustration how this works out for a real life case is shown in figure 14.3 for the number of arrivals (see section 14.2.

The only good way to test whether the exponential data describes the data one observes is to do a proper statistical test (e.g., a Kolmogorov-Smirnov test). There are also rules of thumb. A characteristic for this distribution is that the standard deviation is equal to the mean. If one has data of which one thinks the arrivals are independent and this criterion is satisfied, the arrivals are very likely to be exponentially distributed.

This distribution function for the headways matches the Poisson distribution function for the number of arrivals.

14.3.2. Composite headway models

Whereas the exponential distribution function works well for low flows, for higher flows the distribution function is not very good. For these situations, so called composite headway distributions are being used (see for more information Hoogendoorn (2005)). The basic idea is that a fraction of the traffic Φ is driving freely following a headway distribution function $P_{\text{free}}(h)$. The other fraction of the traffic $1-\Phi$ is driving constraint, i.e. is following their leader, and have a headway distribution function $P_{\text{constraint}}(h)$.

In a composite headway distribution, these two distribution functions are combined. The combined distribution function can hence be expressed as

$$P(h) = \Phi P_{\text{free}}(h) + (1-\Phi) P_{\text{constraint}}(h) \tag{14.8}$$

Given the reasoning in section 14.3.1, it would make sense to choose an exponential distribution function for the free headways.

A plot of the headway distribution function (in fact, a survival function of the headway, i.e. 1- the cumulative distribution function) is shown in figure 14.7. The vertical axis is a logarithmic axis. Note that in these axis the exponential distribution function is

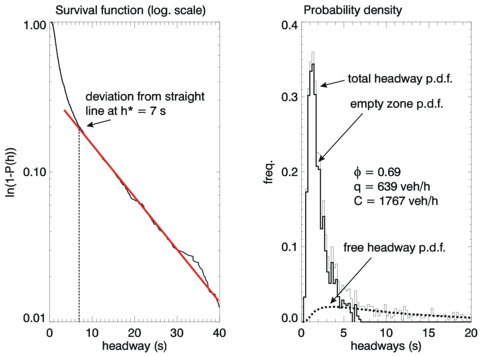
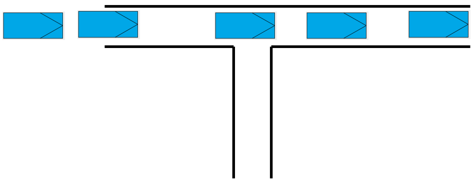
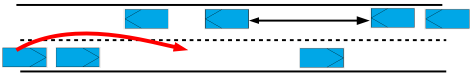


Figure 14.7: Example of the composite headway distribution and its estimation for real life data



(a) Turning onto a road which has priority



(b) Overtaking

Figure 14.8: Examples of critical headways

a straight line. That is what is observed for the large headways. For the smaller headways (in the figure, for less then 7 seconds) this does not hold any more. This is due to the limitation of following distance. As the figure shows, this can be determined graphically. In the right figure, you find the observation frequency of the constraint drivers (dark line), and all drivers (the light line). Note how the distribution for all drivers is higher for larger headways and (almost) all short headways are for constraint drivers.

14.4. CRITICAL GAP

The critical gap is the smallest gap (time gap) that a vehicle accepts to go into. Some examples are given in section 14.4.1. The relation with the inflow capacity is discussed in section 14.4.2.

14.4.1. SITUATIONS

Examples of situations where the critical gap is relevant, are vehicles turning onto a road where one has to give priority, overtaking, merging into the traffic stream or lane changing. Some of them are shown in figure 14.8.

The critical gap is defined in the flow. Note that the critical gap differs per person, as well as per situation. One might have a small critical gap to merge into a traffic stream moving at slow speed.

For overtaking, the critical gap (measured in headway in the opposite traffic stream) needs to be larger because the traffic is moving in the opposite direction. For instance, if a driver spends 3 seconds in lane for traffic in the direction for an overtaking manoeuvre, during that time the opposing vehicles also move in the direction of the overtaking vehicle. To find the critical (time) gap, one needs to consider the distance the overtaker uses at the other lane (overtaking time times speed) – this is the space the overtaker would need if the opposing vehicles were stationary. To this, one should *add* the distance the opposing traffic moves in the overtaking time (overtaking time times the speed of the opposing vehicles). This combined distance gap (plus arguably a safety margin) can be translated into a time gap. Since one measures the gap as headway in the opposing traffic, one needs to divide the required space gap by the speed of the opposing traffic.

14.4.2. INFLOW CAPACITY

Consider the situation as in figure 14.8a. Consider that the main line traffic (traffic from left to right) has a headway distribution of $P(h)$, and there is an infinite line of vehicles waiting to enter the road, all with an critical gap gc. One can calculate how many vehicles can enter the road per unit of time, as will be shown below.

We know that for certain gaps, vehicles can enter. The first step is to find the number of gaps which pass per unit of time. That equals the number of vehicles (each vehicle causes one new gap). The number of vehicles per unit of time is the flow, which is the inverse of the mean of the headway, which in turn can be calculated from the headway distribution.

Then, for all headways larger than the critical gap, but smaller than twice the critical gap one vehicle can enter. The frequency that this happens is the flow (=gap rate) times the probability that the gap is this size, determined from integrating the probability density function of the headway over the right headways:

$$q_{\text{in}_{1\text{ vehicle}}} = q \int_{gc}^{2gc} P(h) dh \tag{14.9}$$

The probability that in one gap two vehicles can enter is the integral of the probability density function for the headways from twice the critical gap to three times the critical gap. The rate at which these gaps occur is the flow q, so the rate at which these gaps occur is $q \int_{2gc}^{3gc} P(h) dh$. Per gap, two vehicles enter, so the matching inflow is

$$q_{\text{in}_{2\text{ vehicles}}} = 2q \int_{2gc}^{3gc} P(h) dh \tag{14.10}$$

One can continue this reasoning and find an equation for the inflow:

$$q_{\text{in}_{\text{total}}} = \sum_n \left(q_{\text{in}_{\text{n vehicles}}}\right) = \sum_n nq \int_{ngc}^{(n+1)gc} P(h) dh \tag{14.11}$$

This can numerically be evaluated.

Typically, one would expect the headway in the order of twice the minimum headway. Namely, after merging, one would like to have at least a minimum headway upstream (lag gap) and a minimum headway downstream (lead gap).

In the calculation of the maximum inflow, the headway distribution function plays an important role. As an extreme example, suppose that all headways are equal at 99% of

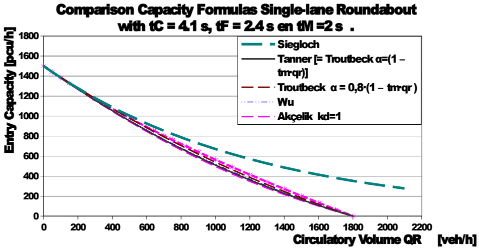


Figure 14.9: The inflow onto a road (or roundabout) as function of the main road (roundabout) traffic – figure from Fortuijn and Hoogendoorn (2015)

the critical gap. Then, the flow is at about half the capacity of the road, but the inflow is 0. For different distribution functions it can be calculated what is the maximum inflow on the main road. This is shown in figure 14.9 for different functions. It shows that the inflow capacity decreases with the main road flow. However, the precise type of distribution matters less: the graphs for the different distribution functions are quite similar.

SELECTED PROBLEMS

For this chapter, consider problems: 38, 39, 40, 160, 186, 187, 231, 239

15

TRAFFIC STATE DYNAMICS IN THREE REPRESENTATIONS

Traffic can be described by a fixed relation between X, N and T. The most common way to describe traffic is the N-model using Eulerian coordinates, which describes the number of vehicles (N) that have passed location x at time t. Another well-known representation is the X-model in Lagrangian coordinates, which describes the position (X) of vehicle n at time t. The third and least common representation is the T-model, which describes the time (T) at which vehicle n crosses location x (Laval and Leclercq (2013)). All three models describe the same traffic state, for example the situation shown in figure 15.1. The example displays the journey of around 75 vehicles on a single lane road. The 5th vehicle stops for around 60 time steps and creates a jam, which slowly dissolves. With use of this example, the describing parameters, characteristics and shockwave theory in the 3 different approaches are described in the following sections.

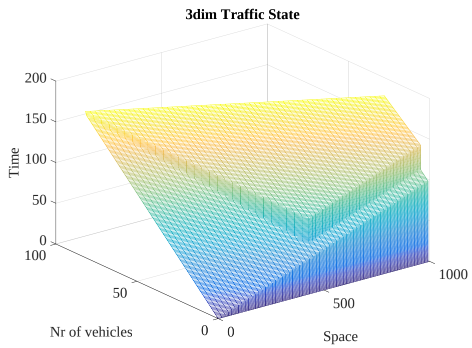


Figure 15.1: Example of a traffic situation, expressed in x, n and t

Table 15.1: Variables used in different coordinate systems

	$N(x,t)$	$X(n,t)$	$T(n,x)$
$/dt$	$q(x,t)$	$v(n,t)$	
$/dx$	$-k(x,t)$		$p(n,x)$
$/dn$		$-s(n,t)$	$h(n,x)$

Table 15.2: Overview of parameters

	$N(x,t)$	$X(n,t)$	$T(n,x)$
1st explanatory variable	x	n	n
2nd explanatory variable	t	t	x
Independent variable	$-k$	$-s$	p
Dependent variable	q	v	h
FD	$Q(k)$	$V(s)$	$H(p)$
Trajectories	iso-n	iso-x	iso-t
slope trajectories	$\frac{dx}{dt} = v$	$\frac{dn}{dt} = q$	$\frac{dx}{dn} = -s$
vertical distance	$\frac{\Delta x}{\Delta N} = -\frac{1}{k}$	$\frac{\Delta n}{\Delta X} = -k$	$\frac{\Delta x}{\Delta T} = v$
horizontal distance	$\frac{\Delta t}{\Delta N} = \frac{1}{q}$	$\frac{\Delta t}{\Delta X} = p$	$\frac{\Delta n}{\Delta T} = q$

15.1. DESCRIBING PARAMETERS

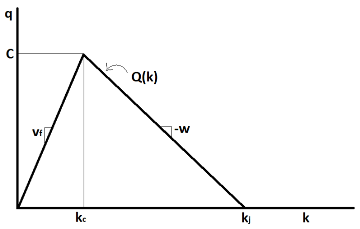
In the different models, the traffic state are explained in different combinations of x, n and t. The derivatives of these parameters give a first insight to the important variables. For example, the change in vehicle number n with time t is the flow q and the change of vehicle number n with space x is the density k. An overview of the other variables are presented in table 15.1. The pitfall in determining the derivatives are the correct signs, which originate from the convention of the scales. The convention of space can be either positive or negative, but time is always positive. For vehicle number, the convention is that the first vehicle on a road has the lowest n number. As a result, the higher vehicle numbers correspond to lower x values, and the derivative dN/dx has a minus sign.

In the sequel of this chapter, the equivalencies of the representations will be shown, as well as how they can be used. We will be using that similar equations allow to use the same answers and computation schemes. Table 15.2 summarizes the comparisons which will be explained in the following sections.

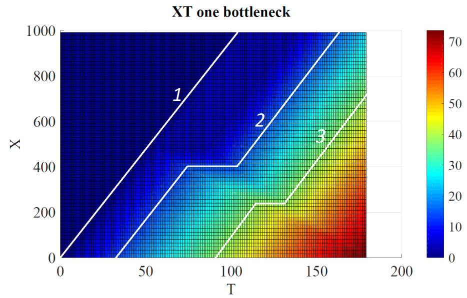
15.2. N-MODEL

The most common way to describe traffic is the N-model. In this model the flow q is proportional to the density k and speed u, with $q = ku$. A fundamental diagram can be drawn for flow where density is the main variable to determine the flow. In the remainder of this chapter we will be using the triangular fundamental diagram.

Trajectories in the N-model are iso-n lines which represent the movement of an individual vehicle in time and space. The XT-diagram in Figure 15.2b shows model output of around 75 vehicles driving on a 1 lane road, which is the same situation as in Figure 15.1. The 5th vehicle stops at $x = 450$ for around 60 timesteps, causing a jam. The trajecto-



(a) Fundamental diagram for the N-model



(b) XT-diagram and iso-n lines, with 3 trajectories. The colors indicate the vehicle number.

Figure 15.2: Representation of traffic in the N-model

ries of 3 cars are isolated, the first having an undisturbed path, the second trajectory is stationary for around 40 timesteps before continuing its path, and the third trajectory is stationary at a smaller x-location for a shorter time period. This indicates that the jam grows in the $-x$ direction, and that the queue length dissolves with time. In this model data, but also in observed trajectory data, the vertical distance between two trajectories provides information about the density, using $\frac{\Delta x}{\Delta N} = -\frac{1}{k}$. The horizontal distance between trajectories is a measure for the flow $\frac{\Delta t}{\Delta N} = \frac{1}{q}$.

Assuming that all vehicles that enter a certain road stretch also have to exit the road stretch within a certain amount of time, it can be stated that the number of vehicles are conserved. Hence, with no sinks and sources, the conservation equation (earlier generally stated in equation 13.1) is given by:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{15.1}$$

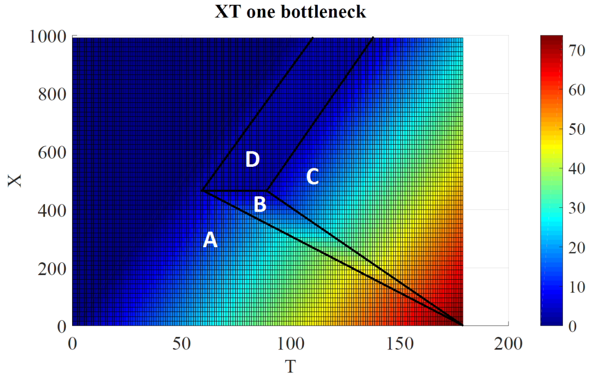
This states that the change in density k with time t and the change of flow q with distance x should equal out to zero.

15.2.1. Shockwave Theory in the XT-plane

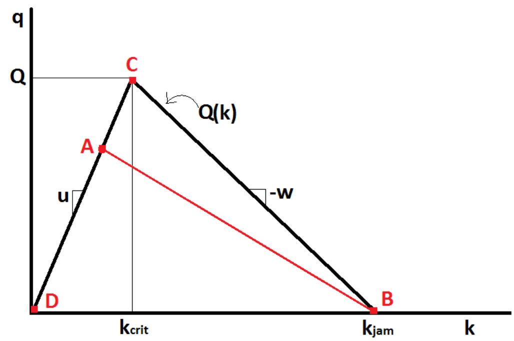
By combining trajectory data and the fundamental diagram, it is possible to identify traffic states. These traffic states are a combination of density and flow, for example, jam state or capacity state. Figure 15.3 shows an example the traffic states which are identified using the fundamental diagram. Here, the example is given. A full explanation is given in chapter 4.

15.3. X-model

The most useful description mathematically is the X-model. In this case, X is being described as function of n and t. In this representation, the fundamental diagram describes the speed V based on the spacing s. The shape of $V(s)$ is presented in Figure 15.4a and has the following properties:



(a) XT-diagram with different traffic states



(b) Fundamental diagram with different traffic states

Figure 15.3: Shockwave Theory in the N-model, with different traffic states, A = inflow, B = jam, C = Capacity, D = empty road

- A speed of 0 between the origin $s = 0$ and the jam spacing s_j
- A congestion branch where speed increases with spacing. This state occurs when $s_j < s \leq s_c$, the angle of increase is equal to $\tau = 1/(wk)$
- A free flow branch where the speed is maximum and continuous (v_f). This state occurs when spacing exceeds the critical value $s > s_c$
- A critical spacing s_c which is the minimum value for spacing with maximum speed (v_f)

From the perspective of the X-model, the traffic situation of Figure 15.1 is visualized in the NT diagram 15.4b. Looking horizontally in the plot, there is a continuous increase in distance (colors) for the first vehicles, indicating that they have an undisturbed journey. Vehicle nr 20 is positioned on 1 location for a period of time, so it experiences a delay due to a jam. The length of the jam is not visible in this representation, only the delay it causes. The delay is largest for the lower vehicle numbers (but >5) and decreases with time for higher vehicle numbers, indicating that the jam is slowly dissolving. Furthermore, the delay starts at a distance around 400m (green color) and changes to blue colors with time, indicating that the jam is moving in the $-x$ direction.

Trajectories in the X-model are iso-x lines. It tracks how many vehicles have crossed a fixed location with time, in other words, every trajectory is cumulative curve. The three indicated trajectories in Figure 15.4b are cumulative curves for approximately $x = 100$, $x = 400$ and $x = 600$.

15

From the NX diagram, also other traffic variables can be estimated. The vertical distance between two trajectories is a measure for density $\frac{\Delta n}{\Delta X} = -k = -\frac{1}{s}$ and the horizontal distance can be used to estimate the pace $\frac{\Delta t}{\Delta X} = p = \frac{1}{v}$.

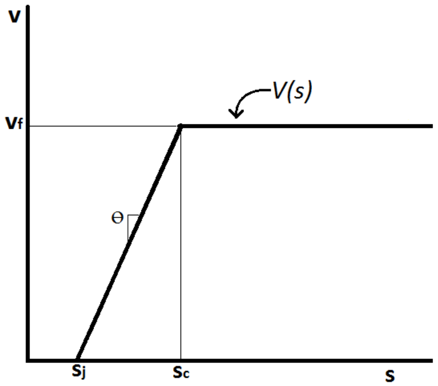
Based on the assumption that vehicles cannot disappear from the road, a continuity equation can be retrieved. The continuity equation for the X-model is:

$$\frac{\partial s}{\partial t} + \frac{\partial v}{\partial n} = 0 \tag{15.2}$$

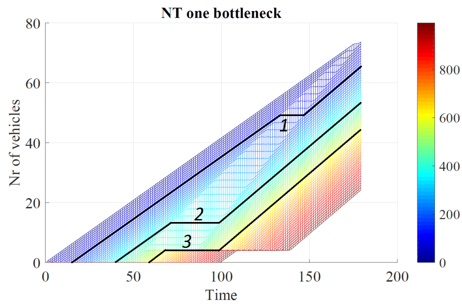
which states that the change in spacing s with time t and the change of speed v with vehicle number n summed should be to zero.

The description in the X-model is also called a Lagrangian coordinate system. In this system, the coordinates move with the traffic. This is opposed to a system where coordinates are fixed, which we call a Eulerian coordinate system. In this representation, we describe the position of the Nth vehicle; for a discretisation of ΔN=1, we hence have a microscopic model. Also for other values of ΔN, there is an easy linking of the microscopic to the macroscopic variables, since the vehicle movements are described. Table 15.3 shows the comparison of the two coordinate systems.

Since the equations are similarly formed, similar solution techniques can be done in this representation as in the N-model. As with the other representations, shockwave theory will be discussed. Due to the importance and the mathematical advantages of a numerical scheme, for this representation also numerical simulation scheme is presented, similar to the cell transmission model: see section 10.3.



(a) Fundamental diagram for the X-model

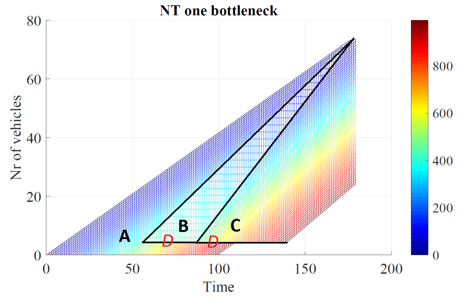


(b) NT-diagram and iso-x lines, with 3 trajectories. The colors indicate the location x.

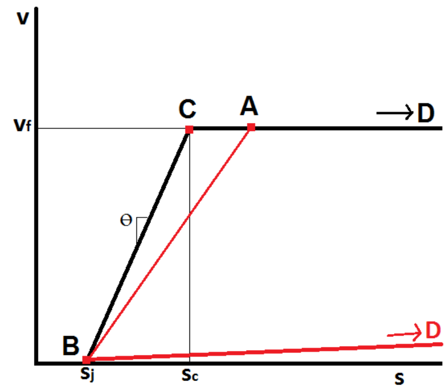
Figure 15.4: Representation of traffic in the X-model

Table 15.3: The variables in the Lagrangian formulation

Variable	Euler	Lagrange
Traffic state	Flow q	Speed v
	Density k	Spacing s
Variable	space	vehicle number
	time	time



(a) NT-diagram with different traffic states



(b) Fundamental diagram with different traffic states

Figure 15.5: Shockwave Theory in the X-model, with different traffic states, A = inflow, B = jam, C = Capacity, D = empty road. Traffic state D is infinitely small.

15.3.1. Shockwave Theory in the NT plane

Similar to Shockwave Theory in the N-model, it is possible to also identify traffic states in the X-model. This is done by combining the trajectory data in the NT-plane with the fundamental diagram, as shown in Figure 15.5. Four states can be identified: Inflow A, jam B, capacity C and empty road D. The steps to identify traffic states is similar to the shockwave theory in the XT-plane, with 1 exception. The spacing on an empty road can be infinitely large, so state D does not have an exact position in the fundamental diagram. Since the spacing at D is infinitely large, the shock between jam B and empty road D is horizontal in the limit. Furthermore, the size of the shock is infinitely small, so it is invisible in the NT-diagram. The connecting line between inflow state A and jam state B, indicates how fast the jam is growing, whereas the connection line between jam state B and capacity state C, gives information on how the jam dissolves.

In summary these are the main advantages of the Lagrangian formulation are:

- “Easy” numerical discretization (simulation)
- Accurate simulation results
- Fast simulation
- Trajectories of (individual) vehicles come out by default

15.4. T-model

The variables used to describe traffic states in the T-model are pace p and headway h. The properties of the fundamental diagram for headway as function of pace $H(p)$ are:

- There is a capacity state at a minimum pace p_c where headway is minimum H_c at $H = 1/C$

15

- Headway increases from H_c to infinity in the free flow branch, so it is a vertical line at $p = 1/v_f$
- Headway increases with pace in the congested branch with angle ϵ which equals $1/k_j$
- For high values of pace (=low speed), the headway increases to infinity

The traffic state in the T-model is represented in the NX-plane, see Figure 15.6. In this graph, the colors represent the time, so a jump in colouring indicates the boundary to a different traffic state. A horizontal line in the NX diagram gives the time at which all vehicles cross a certain location, so the vertical color jump indicates that there is an empty road. A vertical line in the NX diagram follows 1 vehicles and give the time at which that vehicle passes a certain position. The color jump in vertical direction indicates that the vehicle takes a longer time than usual before reaching the next position, so it is caught in a jam. The extent of the color jump indicates the magnitude of the jam. A large jump equals a large delay.

Trajectories in the T-model are iso-t lines which are basically snapshots. It provides the position of all vehicles on the road at a certain time step. The orientation of the line is from the upper left to the lower right, which is opposite to the trajectories in the other two models. The different orientation is a result of the convention of scale in vehicle number, as mentioned in section 15.1. The example trajectory nr 2 in Figure 15.6b indicates that the first 5 vehicles are situated around 800 distance, followed by an empty road. Then, approximately 5 vehicles are going at capacity flow between 550 to 450 distance, while 10 vehicles are trapped in a jam around distance 400. Finally, 10 vehicles are situated between distance 0 and 375 in the 'normal' or initial traffic state. It is difficult to see in this example, but the slope of trajectory is smallest (least negative) in the capacity state. In general, the vertical distance between two trajectories is an estimation of speed $\frac{\Delta x}{\Delta T} = v = \frac{1}{p}$ while the horizontal distance is a measure for the flow $\frac{\Delta n}{\Delta T} = q = \frac{1}{h}$.

Assuming that no vehicles disappear from the road, the continuity equation for the T-model is given by:

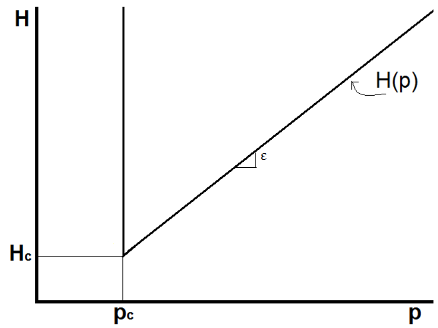
$$\frac{\delta p}{\delta n} - \frac{\delta h}{\delta x} = 0$$

stating that the change of pace p with vehicle number n should be equal to the change in headway h with distance x.

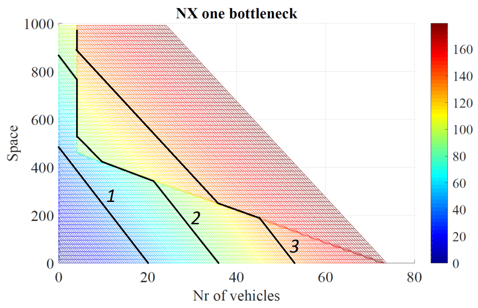
15.4.1. SHOCKWAVE THEORY IN THE NX-PLANE

Due to the opposite orientation of the trajectories, the fundamental diagram needs to be adjusted before shockwave theory can be applied. Since the free flow branch of the $H(p)$ is vertical, it is sufficient to only reflect the congested branch in the line $H = H_c$, resulting in a line with angle $-q$. This line is partially drawn in Figure 15.7b, with jam state B' located at minus infinity.

Returning to the XN-diagram, two traffic states are easily recognized due to the jumps in color, with initial state A in the lower part and the capacity state C in the upper right part. In between these states, two infinitely small traffic states exists. State D indicates the empty road, and state B indicates the jam state. The headway for an empty road and



(a) Fundamental diagram for the T-model



(b) XN-diagram and iso-t lines, with 3 trajectories. The colors indicate the time.

Figure 15.6: Representation of traffic in the T-model

at stand still are both infinitely large, therefore the states B and D are not physical points in the fundamental diagram, but indicated with the arrows. Because the jam state is infinitely large, the shockwave of the jam (B) with the inflow (A) and outflow (C) are parallel, with an infinitely small area in between. A similar reasoning holds for the empty road state D, which is situated in between the initial state A and capacity state C.

Only the tail of the queue is visualized in the NX-diagram, and not the queue itself. This is because stopped vehicles do not move in X and therefore are not visualized. Only in the time step where they start to move again, the vehicles are visualized in the next state. The tail of the jam moves in $-x$ direction with slope $-\epsilon = -\frac{1}{k_j}$, so it depends on the jam density only. In this example, the tail of the jam grows till it reaches the beginning of the road. If the jam had dissolved sooner, there would have been another vertical shock between capacity state C and inflow state A, at the vehicle number that does not experience any delay anymore.

Although shockwave theory can be applied in this representation, it does not provide extra information. The occurrence of a jam can be identified from this graph and the growth of the jam in space can be estimated. However, it is impossible to identify when the congestion will be dissolved. This is a disadvantage of this representation.

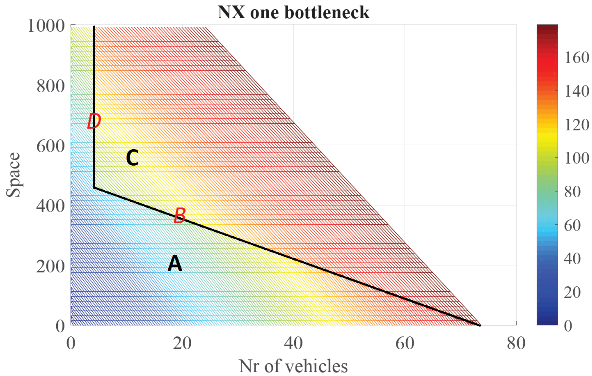
15.5. DISCUSSION

In summary, the N-model is more intuitive and easy to understand. The X-model is mathematically easier but more difficult to interpret. The X-model is difficult to grasp and does not yet have clear benefits. Table 15.2 provides an overview of the main parameters used in the different representation.

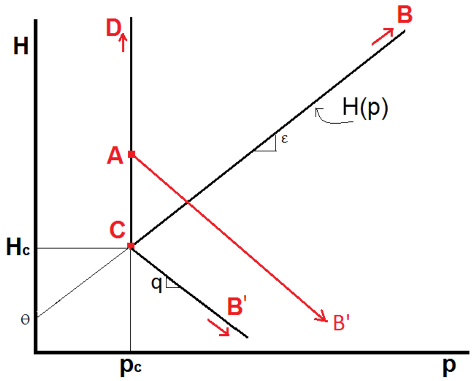
SELECTED PROBLEMS

For this chapter, consider problem: 276.

15



(a) XN-diagram with different traffic states



(b) Fundamental diagram with different traffic states

Figure 15.7: Shockwave Theory in the T-model, with different traffic states, A = inflow, B = jam, C = Capacity, D = empty road. Traffic state B and D are infinitely small.

References

Ahn, S. and Cassidy, M. (2007). Freeway traffic oscillations and vehicle lane-change maneuvers. In Allsop, R. E., Bell, M. G. H., and Heydecker, B. G., editors, *Proceedigs of the International Symposium of Transportation and Traffic Theory*, pages 691–710. Elsevier, Amsterdam.

Aw, A. and Rascle (2000). Resurrection of the "second order" models of traffic flow. *SIAM Journal on Applied Mathematics*, 60:916–938.

Bando, M., Hasebe, K., Nakayama, A., Shibata, A., and Sugiyama, Y. (1995). Dynamical model of traffic congestion and numerical simulation. *Physical Review E*, 51(2):1035. Copyright (C) 2007 The American Physical Society Please report any problems to prola@aps.org PRE.

Bliemer, M. (2007). Dynamic queuing and spillback in analytical multiclass dynamic network loading model. *Transportation Research Record: Journal of the Transportation Research Board*, (2029):14–21.

Brackstone, M. and McDonald, M. (1999). Car-Following: A Historical Review. *Transportation Research F*, 2:181 – 186.

Carlson, R. C., Papamichail, I., Papageorgiou, M., and Messmer, A. (2010). Optimal motorway traffic flow control involving variable speed limits and ramp metering. *Transportation Science*, 44(2):238–253.

Cassidy, M. and Bertini, R. (1999). Some traffic features at freeway bottlenecks. *Transportation Research Part B: Methodological*, 33(1):25 – 42.

Cassidy, M., Jang, K., and Daganzo, C. (2011). Macroscopic fundamental diagram for freeway networks: Theory and observation. In *Proceedings of the 90th Annual Meeting of the Transportation Research Board.*

Cassidy, M. J. and Rudjanakanoknad, J. (2005). Increasing the capacity of an isolated merge by metering its on-ramp. *Transportation Research Part B: Methodological*, 39(10):896 – 913. ISSN 0191-2615.

Chiabaut, N., Leclercq, L., and Buisson, C. (2010). From heterogeneous drivers to macroscopic patterns in congestion. *Transportation Research Part B: Methodological*, 44(2):299 – 308. ISSN 0191-2615.

Chung, K., Rudjanakanoknad, J., and Cassidy, M. J. (2007). Relation between traffic density and capacity drop at three freeway bottlenecks. *Transportation Research Part B: Methodological*, 41(1):82–95.

Corthout, R., Flötteröd, G., Viti, F., and Tampère, C. M. (2012). Non-unique flows in macroscopic first-order intersection models. *Transportation Research Part B: Methodological*, 46(3):343–359.

Courant, R., Friedrichs, K., and Lewy, H. (1928). Über die partiellen differenzengleichungen der mathematischen physik. *Mathematische annalen*, 100(1):32–74.

Daganzo, C. (2007). Urban gridlock: Macroscopic modeling and mitigation approaches. *Transportation Research Part B: Methodological*, 41(1):49–62.

Daganzo, C. F. (1994). The Cell Transmission Model: a Dynamic Representation of Highway Traffic Consistent With the Hydrodynamic Theory. *Transportation research part B*, 28B(4):269–287.

Daganzo, C. F. (1995a). The cell transmission model, part ii: network traffic. *Transportation Research Part B: Methodological*, 29(2):79–93.

Daganzo, C. F. (1995b). Requiem for second-order fluid approximations of traffic flow. *Tranportation Research Part B: Methodological*, 29:277–286.

Daganzo, C. F. (1997). *Fundamentals of Transportation and Traffic Operations*. Pergamon.

15

Daganzo, C. F. (2002a). A behavioral theory of multi-lane traffic flow. part i: Long homogeneous freeway sections. *Transportation Research Part B: Methodological*, 36(2):131 – 158. ISSN 0191-2615.

Daganzo, C. F. (2002b). A behavioral theory of multi-lane traffic flow. part ii: Merges and the onset of congestion. *Transportation Research Part B: Methodological*, 36(2):159 – 169. ISSN 0191-2615.

Daganzo, C. F. (2005). A variational formulation of kinematic waves: basic theory and complex boundary conditions. *Transportation Research Part B: Methodological*, 39(2):187–196.

Drake, J. S., Schöfer, J. L., and May, A. (1967). A statistical analysis of speed density hypotheses. In Edie, L. C., Herman, R., and Rothery, R., editors, *Proceedings of the Third International Symposium on the Theory of Traffic Flow*. Elsevier North-Holland, New York.

Driels, M. R. and Shin, Y. S. (2004). Determining the number of iterations for monte carlo simulations of weapon effectiveness.

Dynasmart (2003). DYNAMSMART-X – User's Guide. Technical report, University of Maryland.

Flötteröd, G. and Rohde, J. (2011). Operational macroscopic modeling of complex urban road intersections. *Transportation Research Part B: Methodological*, 45(6):903–922.

Fortuijn, L. G. H. and Hoogendoorn, S. P. (2015). Turbo roundabouts: Comparing capacity estimation on gap and flow level. In *Proceedings of the 94th Annual Meeting of the Transportation Research Board*.

Geroliminis, N. and Daganzo, C. F. (2008). Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. *Transportation Research Part B: Methodological*, 42(9):759–770.

Gipps, G. P. (1986). A model for the structure of lane-changing descisions. *Tranportation Research Part B: Methodological*, 20:403–414.

Godunov, S. K. (1959). A difference scheme for numerical computation of of discontinuous solutions of equations of fluid dynamics. *Math. Sb.*, 47:271 – 290.

Greenshields, B. D. (1934). A Study of Traffic Capacity. *Proceedings Highway Research Board*, 14:448–477.

Hajiahmadi, M., Knoop, V. L., De Schutter, B., and Hellendoorn, H. (2013). Optimal dynamic route guidance: A model predictive approach using the macroscopic fundamental diagram. In *Intelligent Transportation Systems-(ITSC), 2013 16th International IEEE Conference on*, pages 1022–1028. IEEE.

Hall, F. L. and Agyemang-Duah, K. (1991). Freeway Capacity Drop and the Definition of Capacity. *Transportation Research Record: Journal of the Transportation Research Board No.1320*, pages 91–98.

Heikoop, H., editor (2011). *Capaciteitswaarden Infrastructuur Autosnelwegen*. Dienst Verkeer en Scheepvaart.

Helbing, D. (2003). A section-based queueing-theoretical traffic model for congestion and travel time analysis in networks. *Journal of Physics A: Mathematical and General*, 36(46):L593.

Helbing, D., Hennecke, A., and Treiber, M. (1999). Phase diagram of traffic states in the presence of inhomogeneities. *Physical Review Letters*, 82(21):4360.

Helly, W. . (1959). Simulation of bottlenecks in single lane traffic flow. In *Proceedings of the Symposium on Theory of Traffic Flow*, pages 207– 238. General Motors Research Laboratories, Elsevier, New York.

Hoogendoorn, S., Hoogendoorn, R. G., and Daamen, W. (2011). Wiedemann revisited. *Transportation Research Record: Journal of the Transportation Research Board*, 2260(1):152–162.

Hoogendoorn, S. P. (2005). Unified approach to estimating free speed distributions. *Transportation Research Part B: Methodological*, 39(8):709–727.

Ji, Y. and Geroliminis, N. (2012). On the spatial partitioning of urban transportation networks. *Transportation Research Part B: Methodological*, 46(10):1639–1656.

Kerner, B. S. (2004). *The Physics Of Traffic: Empirical Freeway Pattern Features, Engineering Applications, And Theory*. Springer, Berlin.

Keyvan-Ekbatani, M., Daamen, W., and Knoop, V. (2016). Categorization of the lane-change decision process on freeways. *Transportation Research part C*.

Knoop, V., Tamminga, G., , and Leclercq, L. (2016). Network transmission model: Application to a real world city. In *proceedings of the 95th Annual Meeting of the Transportation Reseach Board*.

Knoop, V. L., Duret, A., Buisson, C., and Van Arem, B. (2010). Lane distribution of traffic near merging zones – influence of variable speed limits. In *Proceedings of IEEE Intelligent Transportation Systems*.

Knoop, V. L. and Hoogendoorn, S. P. (2015). An area-aggregated dynamic traffic simulation model. *European Journal of Transport and Infrastructure Research (EJTIR), 15 (2), 2015*.

Knoop, V. L., Hoogendoorn, S. P., and Van Zuylen, H. J. (2007). Empirical Differences between Time Mean Speed and Space Mean Speed. In *Proceedings of Traffic and Granular Flow 07*. Springer, Paris, France.

Knoop, V. L., van Lint, H., and Hoogendoorn, S. P. (2015). Traffic dynamics: Its impact on the macroscopic fundamental diagram. *Physica A: Statistical Mechanics and its Applications*, 438:236–250.

Knoop, V. L., Van Zuylen, H. J., and Hoogendoorn, S. P. (2009). *Proceeding of the International Symposium of Transportation and Traffic Theory*, chapter Microscopic Traffic Behavior near Accidents. Springer, New York.

Koshi, M., Iwasaki, M., and Ohkura, I. (1981). Some findings and an overview on vehicular flow characteristics. In *Proceedings of the 8th International Symposium on Transportation and Traffic Theory*, pages 403–426. Univ. of Toronto Press, Toronto.

Kotsialos, A., Papageorgiou, M., Diakaki, C., Pavlis, Y., and Middelham, F. (2002). Traffic flow modeling of large-scale motorway networks using the macroscopic modeling tool metanet. *Intelligent Transportation Systems, IEEE Transactions on*, 3(4):282–292.

Laval, J. and Castrillón, F. (2015). Stochastic approximations for the macroscopic fundamental diagram of urban networks. In *Transportation Research Procedia*, volume 7, page 615–630.

Laval, J. A. (2011). Hysteresis in traffic flow revisited: An improved measurement method. *Transportation Research Part B: Methodological*, 45(2):385 – 391.

Laval, J. A. and Daganzo, C. F. (2006). Lane-changing in traffic streams. *Transportation Research Part B: Methodological*, 40(3):251–264.

Laval, J. A. and Leclercq, L. (2013). The hamilton–jacobi partial differential equation and the three representations of traffic flow. *Transportation Research Part B: Methodological*, 52:17–30.

Lebacque, J.-P. (2005). First-order macroscopic traffic flow models: Intersection modeling, network modeling. In *Transportation and Traffic Theory. Flow, Dynamics and Human Interaction. 16th International Symposium on Transportation and Traffic Theory*.

Leclercq, L., Chiabaut, N., Laval, J. A., and Buisson, C. (2007). Relaxation Phenomenon After Lane Changing. *Transportation Research Record: Journal of the Transportation Research Board, No. 1999*, pages 79–85.

Leclercq, L. and Geroliminis, N. (2013). Estimating mfds in simple networks with route choice. *Transportation Research Part B: Methodological*, 57:468–484.

Leclercq, L., Parzani, C., Knoop, V. L., Amourette, J., and Hoogendoorn, S. P. (2015). Macroscopic traffic dynamics with heterogeneous route patterns. *Transportation Research Part C: Emerging Technologies*, 59:292–307.

Lighthill, M. J. and Whitham, G. B. (1955). On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads,. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 229(1178):317 – 345.

Nagalur Subraveti, H. H. S., Knoop, V. L., and van Arem, B. (2019). First order multi-lane traffic flow model–an incentive based macroscopic model to represent lane change dynamics. *Transportmetrica B: Transport Dynamics*, 7(1):1758–1779.

Newell, G. (2002). A simplified car-following theory: a lower order model. *Transportation Research Part B: Methodological*, 36(3):195–205.

Newell, G. F. (1993). A simplified theory of kinematic waves in highway traffic, part ii: Queueing at freeway bottlenecks. *Transportation Research Part B: Methodological*, 27(4):289 – 303.

Oh, S. and Yeo, H. (2012). Estimation of capacity drop in highway merging sections. *Transportation Research Record: Journal of the Transportation Research Board*, 2286(1):111–121.

Ossen, S. J. L. (2008). *Longitudinal Driving Behavior: Theory and Empirics*. Trail thesis series, Delft University of Technology.

Payne, H. J. (1971). Models of freeway traffic and control, mathematical models of public systems. *Simulation Council Proceedings Series*, 28(1):51–61.

Pueboobpaphan, R. and van Arem, B. (2010). Understanding the relation between driver/vehicle characteristics and platoon/traffic flow stability for the design and assessment of cooperative cruise control. In *Proceedings of the 89th Annual Meeting of the Transportation Research Board*.

Richards, P. I. (1956). Shock waves on the highway. *Operations Research 4*, 4:42 – 51.

Schakel, W. J., Van Arem, B., and Knoop, V. (2012). Lmrs: An integrated lane change model with relaxation and synchronization. In *Proceedings of the 91st Annual Meeting of the Transportation Research Board*.

Schreiter, T., Van Lint, H., Yuan, Y., and P., H. S. (2010). Propagation wave speed estimation of freeway traffic with image processing tools. In *Proceedings of the 89th Annual Meeting*.

Smits, E.-S., Bliemer, M. C., Pel, A. J., and van Arem, B. (2015). A family of macroscopic node models. *Transportation Research Part B: Methodological*, 74:20–39.

Smulders, S. (1989). Control of freeway traffic flow. *CWI Tract*, 80.

Srivastava, A. and Geroliminis, N. (2013). Empirical observations of capacity drop in freeway merges with ramp control and integration in a first-order model. *Transportation Research Part C: Emerging Technologies*, 30:161–177.

Tampère, C. M., Corthout, R., Cattrysse, D., and Immers, L. H. (2011). A generic class of first order node models for dynamic macroscopic simulation of traffic flows. *Transportation Research Part B: Methodological*, 45(1):289–309.

Toledo, T., Koutsopoulos, H. N., and Ben-Akiva, M. (2007). Integrated driving behavior modeling. *Transportation Research Part C: Emerging Technologies*, 15(2):96 – 112. ISSN 0968-090X.

Toledo, T., Koutsopoulos, H. N., and Ben-Akiva, M. (2009). Estimation of an integrated driving behavior model. *Transportation Research Part C: Emerging Technologies*, 17(4):365 – 380. ISSN 0968-090X.

Transportation Research Board, (2000). Highway Capacity Manual. Technical report.

Treiber, M., Hennecke, A., and Helbing, D. (2000). Congested traffic states in empirical observations and microscopic simulations. *Physical Review E*, 62(2):1805.

Treiber, M. and Kesting, A. (2013). *Traffic Flow Dynamics; Data, Models and Simulation.* Springer.

Treiber, M., Kesting, A., and Helbing, D. (2006). Delays, inaccuracies and anticipation in microscopic traffic models. *Physica A: Statistical Mechanics and its Applications*, 360(1):71–88.

Van der Gun, J. P., Pel, A. J., and Van Arem, B. (2015). A general activity-based methodology for simulating multimodal transportation networks during emergencies. In *ICEM 2015: 3rd International Conference on Evacuation Modeling and Management, Tainan, Taiwan, 1-3 June 2015.*

Van Lint, J., Bertini, R. L., and Hoogendoorn, S. P. (2014). Data fusion solutions to compute performance measures for urban arterials. In *Symposium Celebrating 50 years of Traffic Flow Theory 2014 FTF Summer Meeting, Portland (USA), 11-13 August, 2014.* TRB.

van Lint, J. W., Hoogendoorn, S. P., and Schreuder, M. (2008). Fastlane: New multiclass first-order traffic flow model. *Transportation Research Record: Journal of the Transportation Research Board*, 2088(1):177–187.

15

van Wageningen-Kessels, F., van Lint, H., Hoogendoorn, S., and Vuik, K. (2010). Lagrangian formulation of multiclass kinematic wave model. *Transportation Research Record: Journal of the Transportation Research Board*, (2188):29–36.

Van Wageningen-Kessels, F. L. M. (2013). *Multi-class continuum traffic flow models: Analysis and simulation methods*. Ph.D. thesis, Delft University of Technology.

Wiedemann, R. (1974). *Stimulation des Strassenverkehrsflusses*. Heft 8 der schriftenreihe des ifv, Universitâ€šÃ„â^žt Karlsruhe.

Wu, N. (2002). A new approach for modeling of Fundamental Diagrams. *Transportation Research Part A: Policy and Practice*, 36(10):867–884. Doi: DOI: 10.1016/S0965-8564(01)00043-X.

Yperman, I. (2007). The link transmission model for dynamic network loading.

Yperman, I., Logghe, S., Tampère, C. M. J., and Immers, B. (2006). Multicommodity link transmission model for dynamic network loading. In *Proceedings of the 85th Annual Meeting of the Transportation Research Board*.

Yuan, K., Knoop, V., and Hoogendoorn, S. (2015a). Capacity drop: A relation between the speed in congestion and the queue discharge rate. In *Proceedings of the 94th Annual Meeting of the Transportation Research Board*.

Yuan, Y. (2013). *Lagranian Multi-Class traffic State Estimation*. Ph.D. thesis, Delft University of Technology.

Yuan, Y., Van Wageningen-Kessels, F., Van Lint, H., and Hoogendoorn, S. (2015b). *Proceedings of Traffic and Granular Flow 2011*, chapter Two modeling and discretization choices for Lagrangian multi-class first-order traffic flow model and their related (dis-)advantages. Springer.

Zhang, H. M. (1999). A mathematical theory of traffic hysteresis. *Transportation Research Part B: Methodological*, 33(1):1 – 23.

A

TEST QUESTIONS

The following pages contain exams with relevant questions. Problem set 1-11 are sets for a full exam of “traffic flow theory and simulation”. The level of the questions is the same as can be expected for the course “Traffic Flow Modelling and Control”; note that some of the content of these questions falls in part 1, and some of the content falls in part 2. There is an example quesiton on Urban intersection control. From problem set 12, the exams are full exams of Traffic Flow Modelling and Control part 1. Overall, a 3 hour exam is expected to have approximately 55 to 60 points.

A.1. PROBLEM SET 1

A.1.1. SHORT QUESTIONS

A road has a capacity of 2000 veh/h, a free flow speed of 120 km/h, a critical density of 25 veh/km and a jam density of 150 veh/km.

Exercise 1. *Draw a realistic fundamental diagram in the with the above properties, both in the flow-density plane and in the speed-density plane; indicate how speed can be found in the flow-density diagram. (4 points)*

Exercise 2. *Describe briefly (100 words) the assumptions in Daganzo’s theory of slugs and rabbits (3 points)*

Exercise 3. *What are cumulative curves, and how are they constructed? (1 point)*

Exercise 4. *What is a vertical queuing model and how is this related to cumulative curves (2 points)*

Exercise 5. *What is higher, the space mean speed of the time mean speed? Why? (1 point)*

Exercise 6. *How can the space mean speed be calculated from individual local speed observations v_i. Give an equation. (1 point)*

A

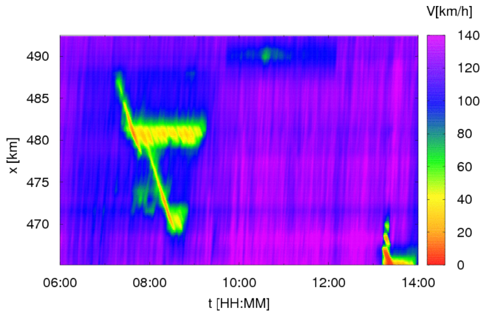
A.1.2. STATE RECOGNITION

Exercise 7. *What are the phases according to three-phase traffic flow theory? How are they characterised? (2 points)*

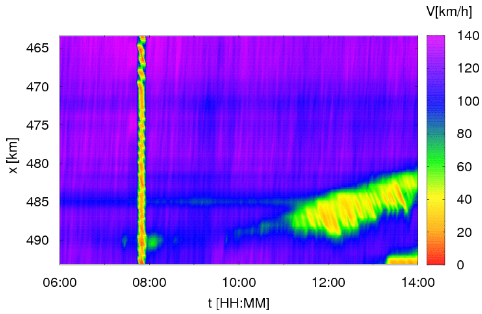
Below, you find some traffic state figures from www.traffic-states.com. For each figure, indicate:

1. The driving direction (top-down or bottom-up) and explain why based on traffic flow theory (0.5pt per figure, spread over b-d) and

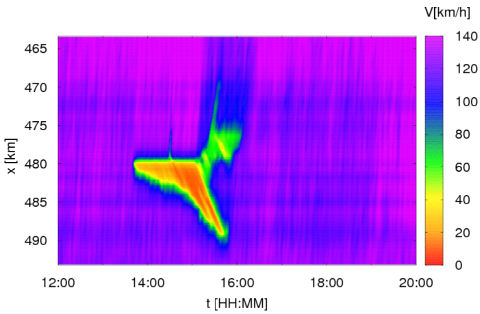
2. the traffic characteristics present, and the most likely causes for these (6 pt, spread over b-d)



Exercise 8.
(up to 10.00 am) (2 points)



Exercise 9.
(3 points)



Exercise 10.
(3.5 points)

A.1.3. SIMULATION MODEL

Exercise 11. *Explain briefly (indication: 100 words) Newells car-following model. Also plot a space-time diagram indicating these properties. (3 points)*

Exercise 12. *Name the three types of (in)stability in traffic flow, and explain briefly their effect (50 words and 1 graph per type) (3 points)*

Exercise 13. *Can multi-anticipation improve stability? Which type and how? (3 points)*

Exercise 14. *How can lane-changing cause a breakdown? (50 words) (1 point)*

Exercise 15. *How can lane-changing prevent a breakdown? (Hint: use the lane distribution.) (2 points)*

A.1.4. MOVING BOTTLENECK

Imagine a truck drivers strike in France on a 3-lane motorway from time t=t0. For the road you may assume a triangular fundamental diagram with a capacity of 2000 veh/h/lane, a critical density of 25 veh/km/lane and a jam density of 125 veh/h/lane. Suppose there is no traffic jam at t<t0, and a constant demand of 5000 veh/h. Assume the truck drivers drive at 30 km/h, not allowing any vehicle to pass.

Exercise 16. *Calculate is the maximum flow on the road behind the trucks? (3 points)*

Exercise 17. *Sketch the traffic flow operations in a space-time diagram. Pay attention to the direction of the shockwave, and note the propagation speeds of the other waves. (3 points)*

After a while, the trucks leave the road.

Exercise 18. *Sketch again the traffic operations in the space time diagram. Do this until the traffic jam is solved (if applicable), or until the traffic states propagate linearly if demand does not change. (3 points)*

Exercise 19. *Calculate the speed at which the tail of the queue propagate backwards? (1 point)*

Exercise 20. *In the plot of question 2, draw the trajectory of the vehicle arriving at the tail of the queue at the moment the strike ends (Use a different colour or style than the shockwaves). (1 point)*

Exercise 21. *What is the speed of the vehicle in the jam? (1 point)*

A.1.5. MARATHON DELFT

Imagine a marathon organised in Delft. There are 30.000 participants, starting at a roadway section which is 25 meters wide.

Exercise 22. *Give a realistic estimate for the capacity of the roadway in runners per hour. Base your answer on the width and headway of a runner*

Exercise 23. *Calculate how long it would take for all runners to start? (1 point)*

Suppose the runners have uniform distribution of running times from 2.5 to 4.5 hours, and all run at a constant speed.

Exercise 24. *Given your earlier assumptions, calculate the minimum required width of the road halfway the track to avoid congestion? For reasons of simplicity, you may assume that they all start at the same moment (instead of your answer at question 5b). (5 points)*

A.2. PROBLEM SET 2

A.2.1. SHORT OPEN QUESTIONS

Exercise 25. *Explain why Kerner claims that in his three phase theory there is no fundamental diagram (indication: 25 words) (1 point)*

Exercise 26. *What is a Macroscopic Fundamental Diagram (indication: 25 words)? (1 point)*

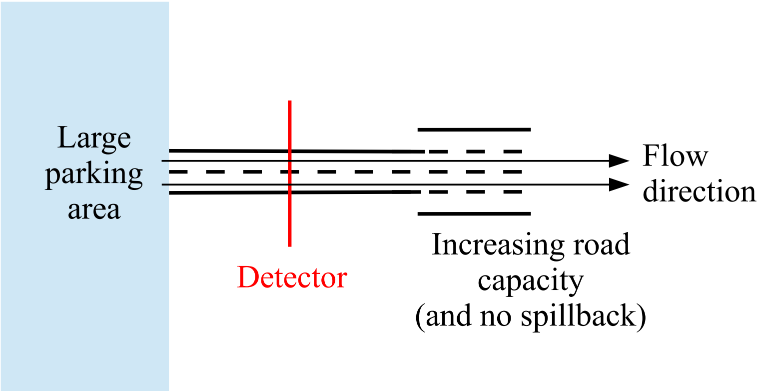
Exercise 27. *What are similarities and differences of a Macroscopic Fundamental Diagram compared to a normal fundamental diagram? Explain based on the underlying phenomena (indication: 75 words). (2 points)*

Exercise 28. *Explain in words the how the flow in from one cell to the next is calculated according to the Cell Transmission Model (indication: 100 words; equations can be useful, but need to be explained). (2 points)*

Exercise 29. *What do we mean with Lagrangian coordinates in macroscopic traffic flow simulation? Which variables are used? Why is this an advantage over other systems? (indication: total 100 words) (3 points)*

A.2.2. LEAVING THE PARKING LOT

Consider the situation where many cars are gathered at one place, and they can only leave over one road.



The above figure shows a simplified representation of the road layout. There is a detector at the red line, in the two-lane road stretch. Downstream of the detector the road widens and there are no further downstream bottlenecks.

A

Exercise 30. *Explain what the capacity drop is (be precise in your wording!). (1 point)*

Exercise 31. *How large is the capacity drop? Give a typical interval bound. (1 point)*

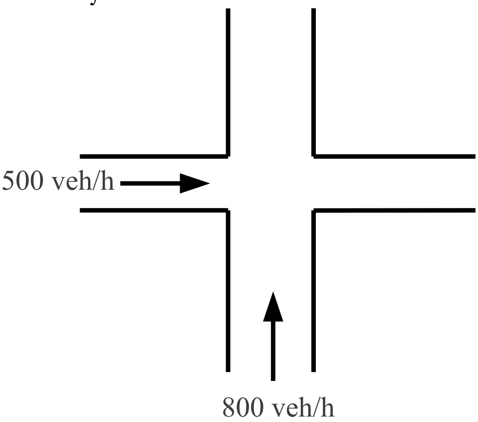
Exercise 32. *Sketch the traffic flow at the detector (indicated in the figure) as function of the demand (in veh/h, ranging from 0 to three times the road capacity) from the parking lot. (2 points)*

Exercise 33. *Explain the general shape you draw in the previous question. (1 point)*

Exercise 34. *Give some rough estimates for values in your graph. (1 point)*

A.2.3. TRAFFIC LIGHTS

Consider a junction with a traffic light with equal green time g, and a clearance time of 2 seconds (i.e., the time needed to clear the junction; during this time, both directions are red). If the traffic light turns green, the first vehicle needs 3 seconds to cross the line. Afterwards, vehicles waiting in the queue will follow this vehicle with a 2 second headway.



Exercise 35. *What is the fraction of time traffic is flowing over the stop line per direction as function of the cycle time c? (2 points)*

Exercise 36. *What is the maximum flow per direction as function of the cycle time? In your equation, what are the units for the variables? (3 points)*

Suppose traffic flow from direction 1 is 500 veh/h and traffic from direction 2 is 800 veh/h, and suppose a uniform arrival pattern.

Exercise 37. *What is the minimum cycle time to ensure that no queue remains at the end of the cycle? Does an approach with vertical queuing models yield the same result as shockwave theory? Why? (3 points)*

This cycle time is fixed now at 120 seconds. We relax the assumption of uniform distribution to a more realistic exponential arrival pattern.

Exercise 38. *To which distribution function for the number of arrivals per cycle (N) does this lead? Name this distribution (1 pt). (1 point)*

The probability distribution function of X is given by:

$$f(k;\lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \tag{A.1}$$

with $e \sim 2.71828...$) and $k!$ is the factorial of k. The positive real number λ is equal to the expected value of X, which also equals the variance.
Assume there is no traffic waiting when the traffic light turns red at the beginning of the cycle.

Exercise 39. *Express the probability p that there are vehicles remaining in the queue for direction 2 when the traffic light turns red at the end of the cycle in an equation (p = ...). Write your answer as mathematical expression in which you specify the variables. Avoid infinite series. There is no need to calculate the final answer as a number (3 points)*

Exercise 40. *Argue whether this probability is higher or lower than if a uniform arrival process is assumed (1 point)*

A.2.4. CAR-FOLLOWING MODEL

Consider the IDM car-following model, prescribing the following acceleration:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_0\left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v,\Delta v)}{s}\right)^2\right) \tag{A.2}$$

with the desired distance s^* as function of speed v and speed difference Δv:

$$s^*(v,\Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}} \tag{A.3}$$

Exercise 41. *Explain in words the working of this car-following; i.e. comment on the acceleration. (3 points)*

Another car-following model is the relatively simple car-following of Newell.

Exercise 42. *How does the Newell car-following model work? (indication: 50 words) (1 point)*

Exercise 43. *Name two weak points of both car-following models (IDM and Newell) (2 points)*

Exercise 44. *Give two reasons to choose the IDM model over Newell's model (2 points)*

Exercise 45. *Give two reasons to choose Newell's model over the IDM model (2 points)*

A

A.2.5. PEDESTRIANS IN A NARROW TUNNEL

We consider a pedestrian flow through a narrow tunnel, which forms the bottleneck in the network under high demand. The tunnel is 4 meters wide and 20 meters long. You may assume that the pedestrians are distributed evenly across the width of the tunnel. The pedestrian flow characteristics are described by a triangular fundamental diagram, with free speed $v_0 = 1.5 m/s$, critical density $k_c = 1 P/m^2$ and jam density $k_j = 6 P/m^2$ (P stands for pedestrian).

Exercise 46. *Draw the fundamental diagram for the tunnel. What is the capacity of the tunnel expressed in P/min? (2 points)*

Between 11-12 am, the average flow through the tunnel is q = 180 P/min.

Exercise 47. *Assuming stationary conditions, what is the density in the tunnel expressed in P/m^2 ? (2 points)*

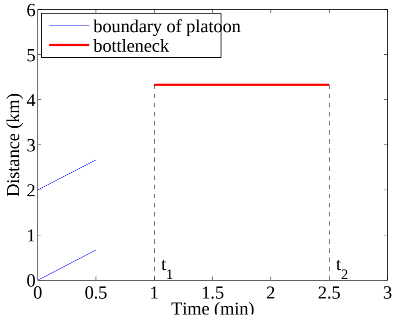
A.2.6. MOVING BOTTLENECK WITH DIFFERENT SPEEDS

Consider a two-lane road with traffic in opposing directions. Assume a triangular fundamental diagram with a free speed of 80 km/h, a critical density of 20 veh/km and a jam density of 120 veh/h. At t=0 there is a platoon of 30 vehicles on the road with equal spacing in the section x=0 to x=2km. There are no other vehicles on the road. In this question, we will consider the effect of a speed reduction to 15 km/h.

Exercise 48. *Calculate the density on the road in the platoon. Indicate the conditions in the fundamental diagram. Make a clear, large drawing of the fundamental diagram such that you can reuse it for indicating states in the remaining of the question. (1 point)*

In each of the following subquestions, consider these initial conditions and no other bottlenecks than introduced in that subquestion – i.e., there never is more than one bottleneck.

Suppose there is a local, stationary bottleneck where drivers have to pass at 15 km/h from time $t1$ to $t2$. The figure below shows the position and the duration compared to the platoon in the space time plot.



Exercise 49. *Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style.* Indicate how you construct the diagram *(i.e., indicate how you get the lines, angles, and distances in the plot). There is no need to calculate the exact traffic states. Additional information: the combination of speeds and the fundamental diagram will lead to congestion. (4 points)*

Suppose there is a temporal speed reduction to 15 km/h for a short period of time over the whole length of the road at the same time. You may assume the platoon to be completely on the road.

Exercise 50. *Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style.* Indicate how you construct the diagram *(i.e., indicate how you get the lines, angles, and distances in the plot). (2 points)*

Suppose there is a large tractor moving in the opposite direction of the traffic at 5 km/h from x1 to x2. This wide vehicle causes vehicles that are next to it to reduce speed to 15 km/h. The speed (5 km/h) is lower than the speed of the shock wave at the tail of the queue in question b

Exercise 51. *Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style.* Indicate how you construct the diagram *(i.e., indicate how you get the lines, angles, and distances in the plot). (4 points)*

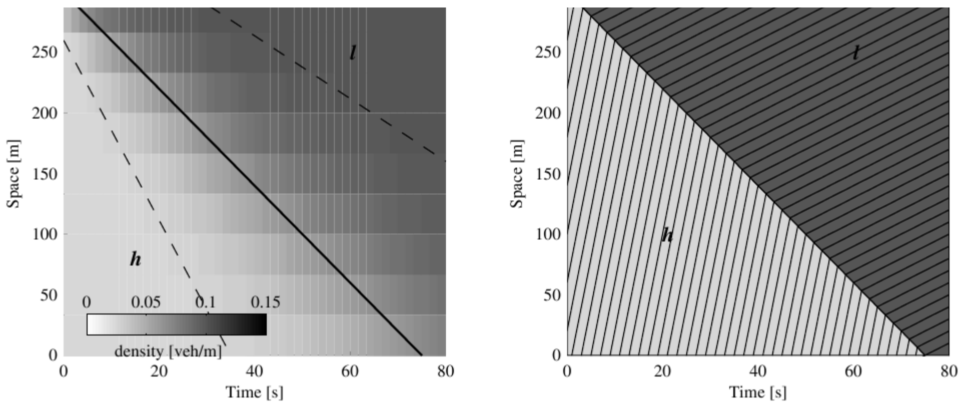
Suppose there is a large tractor moving in the opposite direction of the traffic at 50 km/h. This wide vehicle causes vehicles that are next to it to reduce speed to 15 km/h. The tractor speed (50 km/h) is higher than the speed of the shock wave of the tail of the queue in question b.

Exercise 52. *Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style.* Indicate how you construct the diagram *(i.e., indicate how you get the lines, angles, and distances in the plot). (Hint: it can help to consider a finite tractor length, but it is not necessary) (5 points)*

A.3. PROBLEM SET 3

A.3.1. SHORT OPEN QUESTIONS

Below, two simulation results of a homogeneous congested area spilling back upstream are plotted. You see a high density, congested area, indicated with *h* downstream of x=0 at t=0, and a low density, uncongested area, indicated by *l*. The driving direction is up. One of the figures plots the results of a simulation in Eulearian coordinates and one of a simulation in Lagrangian coordinates.



Exercise 53. *Explain which of the two is Eulerian and which one Lagrangian.(1 point)*

Exercise 54. *Give the two main advantages of using Lagrangian coordinates. (2 points)*

Exercise 55. *Explain what a pce value is. (30 words) (1 point)*

Exercise 56. *How does the pce value of trucks depend on the speed, qualitatively (i.e., does it increase, decrease, ...). Give your reasoning*

Exercise 57. *How does the pce value of trucks depend on the speed, qualitatively (i.e., does it increase, decrease, ...). Give your reasoning (2 points)*

A.3.2. MULTI-LEADER CAR-FOLLOWING MODELS

Consider the IDM car-following model, prescribing the following acceleration:

$$\frac{dv}{dt} = a_0 \left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right) \tag{A.4}$$

In this equation, v is the speed of the following vehicle, v_0 a reference speed, s the distance headway and s^* a reference distance headway as function of speed v and the difference in speed with the leader, Δv:

$$s^*(v, \Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}} \tag{A.5}$$

In this equation s_0 is a desired distance at standstill, and a is a reference acceleration and b is the maximum comfortable braking.

Exercise 58. *Explain in words the working of this car-following model; i.e. comment on the acceleration. Only comment on equation A.4. (indication: 50 words) (2 points)*

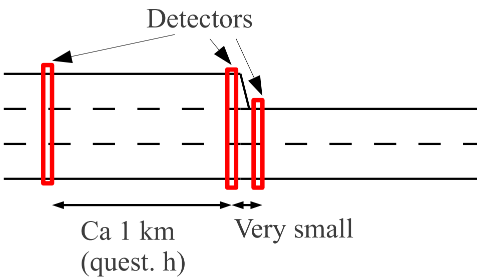
Exercise 59. *Explain in words what is meant with multiple-leader car-following models. Only comment on the multi-leader part. (indication: 50 words) (1 point)*

Exercise 60. *Reformulate the IDM model into a multiple-leader car-following model by including two leaders. Give your reasoning, and, for all points, formulate those reasonings into equations. (3 points)*

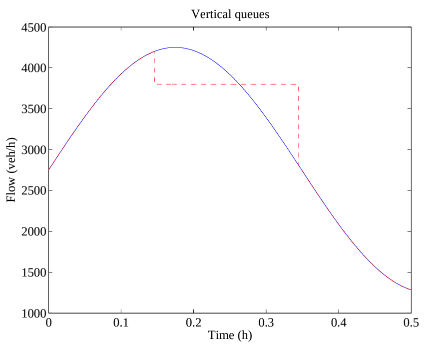
Exercise 61. *Formulate the generalised car-following IDM model considering n leaders (3 points)*

A.3.3. MEASURING THE SPEED AT A CROSS SECTION

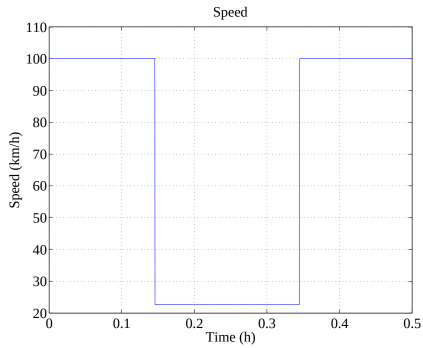
Consider the following road layout with a lane drop from 3 to 2 lanes; traffic is flowing left to right.



The demand, as well as the flow at the downstream detector, is given in the figure below. The speed at the detector, located just upstream (you may assume no spacing between the bottleneck and the detector) of the bottleneck, is as follows.



(a) subfigure[Flow and demand



(b) subfigure[Speed

In the question, you might need values from the graph. Slightly misreading the graph is not a problem, but *mention the values you directly read from the graph and where you find these.*

Exercise 62. *In figure a with the flow and the demand, which line is the demand and which line the flow. Argue why (1 point)*

Exercise 63. *Give the free flow capacity (and the reasoning how you find it) (2 points)*

Exercise 64. *Give the queue discharge rate. (and the reasoning how you find it) (2 points)*

Assume the fundamental diagram per lane is the same for all lanes, at all locations in this setting.

Exercise 65. *Explain why can this situation not be described with a triangular fundamental diagram. (1 point)*

Exercise 66. *Draw the simplest fundamental diagram possible for the three lane section (aggregated over all lanes). Explain how you find the values for the relevant points, and give calculate them. (5 points)*

Exercise 67. *On you answer sheet, sketch the demand as function of time (i.e., copy the above figure, indicate the times of speed change in the graph - no points given) and in the same graph, sketch the resulting flow at the detector. Pay attention to the times at which the flow changes compared to the time of the speed changes. (2 points)*

Now the delay is analysed by slanted cumulative curves.

Exercise 68. *Explain the offset you choose. (1 point)*

Exercise 69. *Sketch the slanted cumulative curve of the flow and the demand for the situation at hand. (2 points)*

Exercise 70. *Indicate in the figure how you can determine the delay. (1 point)*

Now a second detector is constructed approximately 1 km upstream of the bottleneck.

Exercise 71. *Sketch the demand as function of time (i.e., copy the second figure of this question, indicate the times of speed change in the graph - no points given) and in the same graph, sketch the resulting flow at this second detector. Pay attention to the times at which the flow changes compared to the time of the speed changes. (2 points)*

We now relax the assumption of all traffic states at the fundamental diagram to a more realistic situation (i.e., including demand and supply variations). In this situation, We measure the one-minute aggregated (harmonically averaged) speeds during one month.

Exercise 72. *Sketch a probability distribution or histogram of these speeds. Explain the shape (2 points)*

A.3.4. MOVING BOTTLENECK

Consider a two-lane motorway. Assume a triangular fundamental diagram with a free flow speed of 80 km/h, a critical density of 25 veh/h/lane and a jam density of 150 veh/km/lane. The inflow is stationary at 2000 veh/h.

Exercise 73. *Draw the fundamental diagram and calculate the capacity of the road stretch2*

Consider a wide vehicle (special transport) driving slowly is entering the road. There are no overtaking possibilities. This creates congestion, *of which the tail happens to stay at the same position.*

Exercise 74. *Calculate the flow in the jam. (Hint: you can use the speed of the shockwave at the tail of the queue)1*

Exercise 75. *Calculate the density in the jam.2*

Exercise 76. *Calculate the speed of the special transport.2*

After 5 km, the truck leaves the road.

Exercise 77. *Construct the space-time diagram of this situation, from before the moment the truck enters the road to after the moment the traffic situation is stationary. Explain how you find the speed of the solving of congestion; you may re-use the figure you created in a.4*

Exercise 78. *At the maximum queue length, how many vehicles are in congestion1*

A.3.5. MULTI-LANE TRAFFIC FLOW

Exercise 79. *Give the names of two regimes according to Daganzo's theory of slugs and rabbits (only names are required). (1 point)*

Below, you find a Google Earth image of the A1 motorway near Bathmen. Consider the right to left (east to west) direction

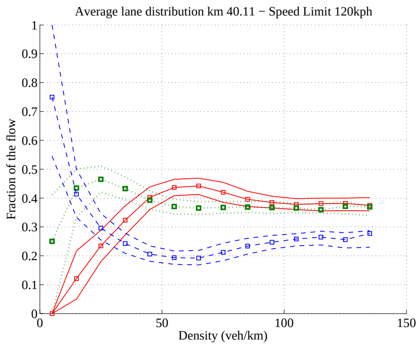
Exercise 80. *To which of the regimes of question a) does the situation in the figure resemble most. Explain why (2 points)*

There are 4 trucks and 7 passenger cars (including one delivery van) in the image. The fraction of passenger cars in the density is hence 7/11.

Exercise 81. *What would you expect from the fraction of passenger cars in the flow: is this higher, lower or equal to 7/11. Motivate your answer 2 points*

Exercise 82. *Calculate the fraction of the is passenger cars in the flow. Assume reasonable speeds for the different vehicle types in your answer – state your assumptions clearly. (4 points)*

Below, a figure of a lane flow distribution for a three lane road in the Netherlands is given.



Exercise 83. *In the figure, which color matches which lane. Argue why (1 point)*

Exercise 84. *Describe briefly (100 words) the assumptions in Daganzo's theory of slugs and rabbits (3 points)*

Exercise 85. *Using Daganzo's theory of slugs and rabbits, explain the differences in flow between most used lane (blue) and the least used lane (red) at a density of 75 veh/km. (2 points)*

A.4. PROBLEM SET 4

A.4.1. SHORT QUESTIONS

Exercise 86. *What does a Macroscopic Fundamental Diagram describe? Make sure your answer shows clearly the differences between an MFD and a fundamental diagram.(2 points)*

Exercise 87. *What is qualitatively the effect of inhomogeneity in the network on the MFD? (No explanation needed) (1 points)*

Exercise 88. *In Lagrangian coordinates, the fundamental diagram is often expressed in spacing (horizontal) - speed (vertical) form. Sketch a fundamental diagram in the spacing-speed plane (2 points)*

A.4.2. FROM CAR-FOLLOWING TO A FUNDAMENTAL DIAGRAM

The optimal velocity model is a car-following model specifying the acceleration a as follows:

$$a = a_0(v^* - v) \tag{A.6}$$

In this equation, v is the speed of the vehicle, and a_0 a reference acceleration (tunable parameter, constant for a specific vehicle-driver combination). v^* is determined as follows:

$$v* = 16.8(\tanh(0.086(\Delta x - 25) + 0.913)) \tag{A.7}$$

In this equation, Δx is the distance (in meters) between the vehicle and its leader.

Exercise 89. *Explain qualitatively the working of this car-following model; i.e. comment on these two equations. (2 points)*

Exercise 90. *What are the conditions for which a fundamental diagram holds? (1 point)*

Exercise 91. *Derive the expression a fundamental diagram (flow as function of density) for these conditions using the OVM model4*

Exercise 92. *What are the values capacity, free speed and jam density (either derive the value or use you graphical calculator to determine this – you may round numbers to the precision of your liking...) (1 point)*

Exercise 93. *Argue whether the values in the previous question are reasonable for the spacing/speed relationship? If so, for what conditions (urban, freeway...?) If not, can you change the parameters (numbers) of the model (A.6 and A.7) to get a proper fundamental diagram (if so: what are reasonable numbers). Show in your answer that you understand the traffic operations, and/or the model working.*

A.4.3. MOBIL LANE CHANGE MODEL

Consider the mobil lane change model. A driver c has several options: change lanes to the left, to the right or stay in his current lane. For each of the options a total utility (denoted U_{tot}) can be calculated.

$$U_{\text{tot}} = U_c + p \sum_{i \in \text{other drivers}} U_i \tag{A.8}$$

The utility for the driver i U_i is assumed to be its instantaneous acceleration, as computed using a car-following model (the Intelligent Driver Model – although the specific model is not relevant for the question). The driver is assumed to take the option with the highest utility.

Exercise 94. *Explain the working of the model in words (2 points)*

Exercise 95. *What value for p can be expected (1 point)*

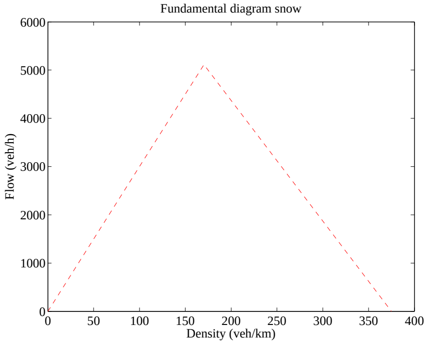
A situation like this can be observed in practice

Exercise 96. *Does the MOBIL lane change model explain this? Reasoning from the model, explain how (if it does explain) or why (if it does not explain). (2 points)*

A.4.4. Snow plow

During a winter night, a 30 cm snow covered a three lane motorway fell. Traffic is still moving.

This changes the traffic operations. Assume a triangular fundamental diagram. The free flow speed reduces to 30 km/h, the jam density to 125 veh/km/lane. The capacity is 5000 veh/h.



A truck spins and cannot move further, thereby blocking the road completely, not allowing other vehicles to pass. This leads to a traffic jam. The inflow is 1000 veh/h.

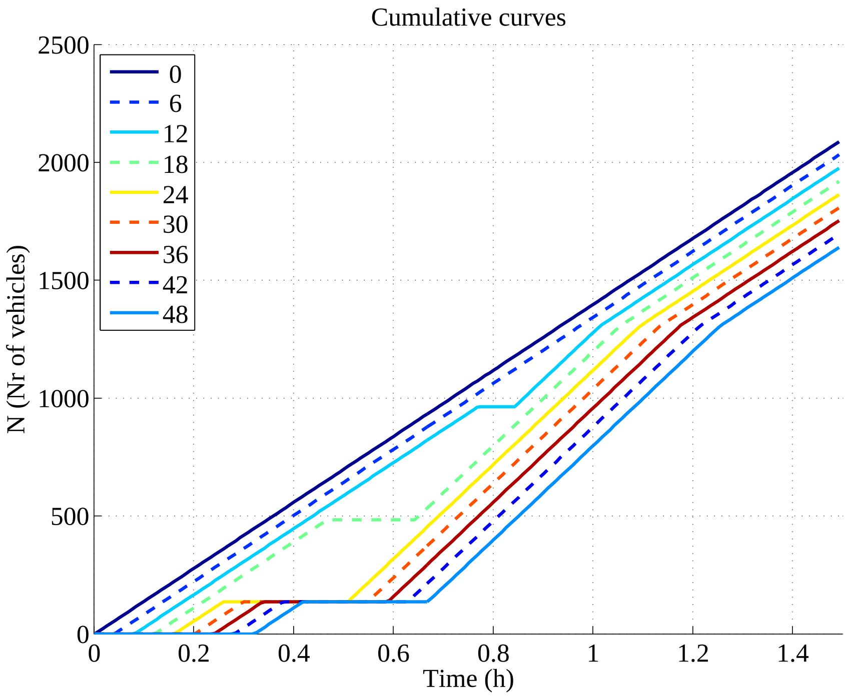
Exercise 97. *Draw the resulting traffic operations in a space time plot and in the fundamental diagram. Calculate all shock wave speeds. (4 points)*

A snow plow comes to free the vehicles that are stuck. After freeing the vehicles, the snow plows clear two of the three lanes of the motorway. Thereby, they drive at 5 km/h on the motorway. The capacity of vehicles passing the snow plough on the left lane is 2000 veh/h. The inflow on the road is 1000 veh/h.

Exercise 98. *Sketch the traffic operations in a space-time plot and in the fundamental diagram. Explain how you construct the graphs. (No point given here for the beginning, discussed in question b) (5 points)*

A.4.5. CUMULATIVE CURVES

The graphs show cumulative curves for different locations along the road. The legends shows the distances in km from the beginning (i.e., upstream end) of a road. For the remainder of the question, reasoning is more important than exact readouts from the graph. When using graph readouts, please state so explicitly and *note the values you read from the graph.*



Exercise 99. *Explain the traffic state, mention a possible cause (e.g., "different speed limits for different sections", "peak hour jam") and explain why. (3 points)*

Exercise 100. *Estimate is the total delay encountered here (3 points)*

Exercise 101. *Sketch the traffic situation in space-time (shock waves – no trajectories needed).* Estimate the the location of changes in traffic states. *(3 points)*

Assume a triangular fundamental diagram.

Exercise 102. *Estimate, from the given curves, give the free speed, capacity, critical density and jam density (4 points)*

A.4.6. CROWN JEWELS IN THE TOWER

We consider an exhibition with the most important piece an object in a small glass show case in the middle of the room. Visitors do not have a preference to see a particular side of the piece. The room is 15 meters long and 6 meters wide.

Exercise 103. *Explain why a larger glass show case can increase the capacity of the exhibition room, in terms of visitors per unit of time2*

The exhibitor has chosen for a show case of 3x1 meters.

Exercise 104. *Estimate the capacity of the room, measured in visitors per minute; base your answer on a reasonable watching time for the piece of art and state the assumptions explicitly4*

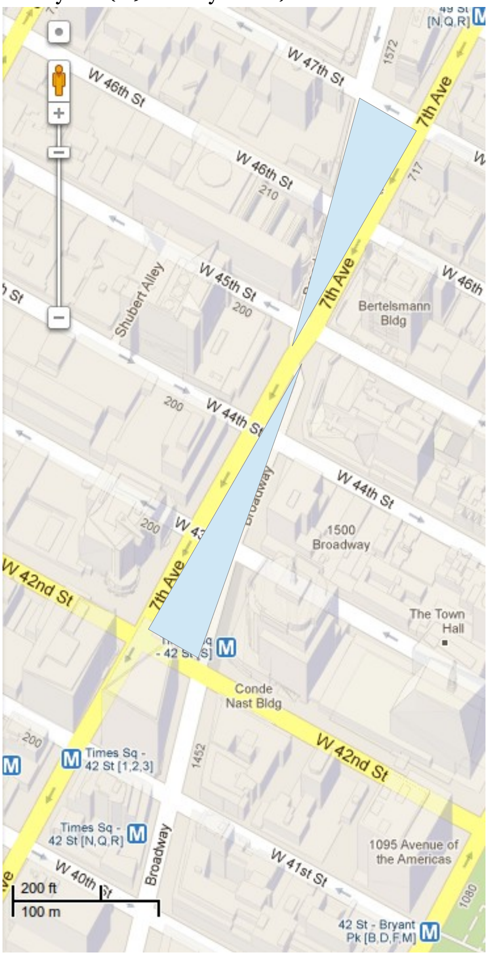
An alternative design is considered. Instead of the visitors walking by the art, the visitors can stand on a moving walkway (like in the airport), which is constructed at each side of the glass. Visitors step at the moving walkway at the beginning of the room, and step off at the end. They are not allowed to walk backwards on the moving walkway.

Exercise 105. *What is the capacity of the room in this case. Base your answer on (explicitly stated) reasonable assumptions on distance and speed.3*

A.5. PROBLEM SET 5

A.5.1. SHORT QUESTIONS

Below, you find a google maps image of New York; Times Square is shaded in blue. The municipality of NY claimed that 1,000,000 people were at Times Square at the start of the new year (1 January 0h00).



Exercise 106. *From a traffic flow perspective, comment on this claim. (2 points)*

A

Exercise 107. *How can synchronized flow be recognized? (1 point)*

Exercise 108. *Sketch the three phases of traffic flow in a flow-density diagram, and give the names (2 points)*

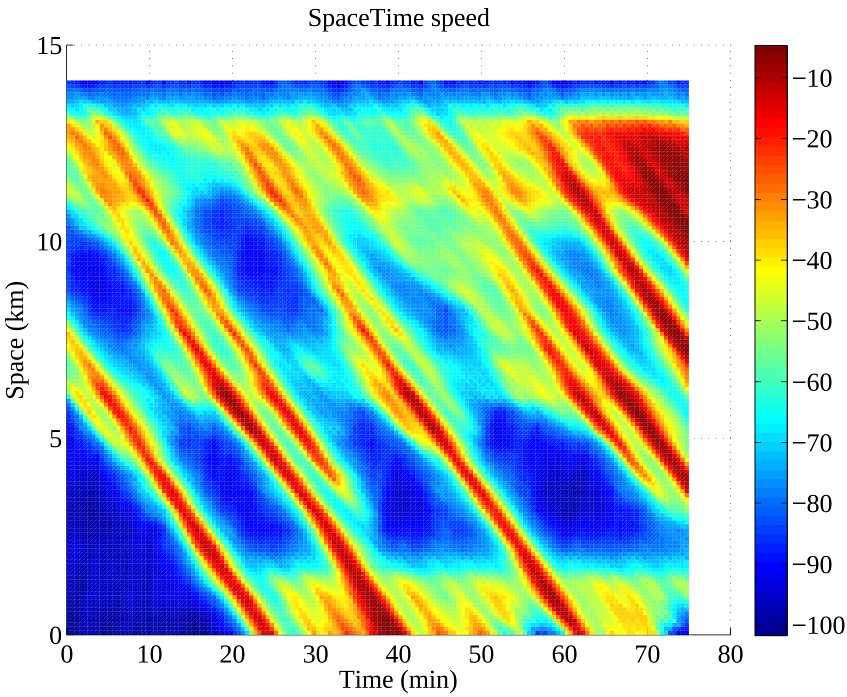
Exercise 109. *Comment on the concept of capacity in the three phase traffic flow theory (1 point)*

Exercise 110. *How are Lagragian coordinates for traffic defined (i.e., what is special on the the definition of the coordinates; how do they differ from Eulerian coordinates?) (1 point)*

Exercise 111. *What the main mathematical advantage of the Lagrangian coordinate system? (1 point)*

A.5.2. MOTORWAY TRAFFIC OPERATIONS

Below, you find a speed contour plot of traffic operations at a motorway (the negative speeds are plotted for color representation: units are in km/h, so blue is 100 km/h and red is (almost) standstill). (if you are color blind and cannot see the colors – please ask!)



Exercise 112. *Explain which direction traffic is flowing from the graphs1*

Exercise 113. *Name the two main traffic phenomena you see - and briefly describe what is happening. (2 points)*

Exercise 114. *What are the three different levels of stability, and what do they mean (2 points)*

Exercise 115. *Comment on the stability of traffic for this day (3 points)*

Exercise 116. *What traffic states can occur according to the theory of slugs and rabbits. Give the names and a short description (10-20 words per state) (2 points)*

Exercise 117. *Modelling a multi lane Dutch motorway, how does this theory in resulting traffic differ from the Mobil lane change model for moderately dense (undercritical) traffic? (2 points)*

A

A.5.3. CAR-FOLLOWING

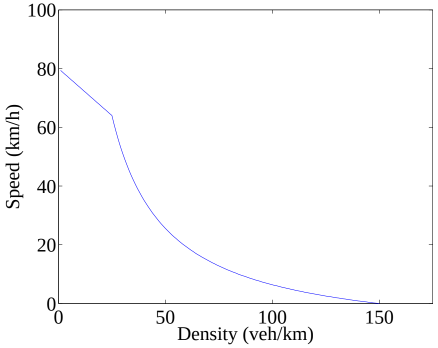
A fundamental diagram only holds in stationary and homogeneous traffic conditions.

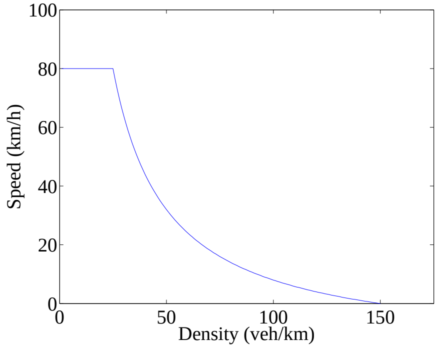
Exercise 118. *Explain the terms stationary and homogeneous (indication: 20 words each) (2 points)*

From a car-following model the fundamental diagram can be derived.

Exercise 119. *Can a car-following model be derived from the fundamental diagram? If so, explain how. If not, explain why not. (2 points)*

The two speed-density relationships shown below relate to a triangular fundamental diagram and Smulders fundamental diagram





Exercise 120. *Which figure is which fundamental diagram. Why? (1 point)*

Exercise 121. *Draw the corresponding flow-density and speed-flow curves for Smulders fundamental diagram (in your drawing, put attention to (or indicate) the values or slopes at several critical points at the ends of lines or at turning points)3*

A.5.4. PARTIAL ROAD BLOCKING NEAR A SIGNALISED INTERSECTION

Upstream of a signalised intersection the road is partially blocked by a crane (see below figure for illustration). Assume that between the crane and the intersection, all four lanes can (and will) be used if needed. *That means that the length between the crane and intersection is longer than in the images.*

During the red phase, the tail of the queue spills back further than the crane. During the green phase, the queue dissipates completely. Throughout this question, assume a fundamental diagram which is concave (not triangular).

Exercise 122. *Sketch the fundamental diagrams. Give typical values for the characterizing points. (2 points)*

Use this diagram throughout this question.

Exercise 123. *Sketch the traffic operations in a space time plot, starting at the moment the traffic light turns red and ending at the moment the traffic light turns red again. Also indicate all states in the fundamental diagram. Only consider traffic states upstream of the traffic light.8*

Exercise 124. *Draw the (moved) cumulative curves upstream of the traffic jam and downstream of the traffic light (2 points)*

Exercise 125. *How could you get the total delay from this for all vehicles together (explain, but no need for calculations) (1 point)*

Exercise 126. *Comment qualitatively on the effect of the distance between the intersection and the crane on the delay (2 points)*

The workers move the crane downstream in order to reach a building further downstream. They can only do so when there is no queue downstream. Assume they start when the queue is dissolving and traffic flows past the crane, but the queue is not dissolved completely yet, and assume a speed for the movement. The fundamental diagram does not change due to the movement (i.e., you may use the FDs of question a)

Exercise 127. *Sketch the traffic states in the flow-density plane, from the moment the crane moves until the queue is completely dissipated (you may assume the traffic light stays green, and assume the distance to the traffic light is large enough that the crane can keep moving until the traffic jam is solved). (4 points)*

A

A.5.5. Network Fundamental Diagram

Exercise 128. *What variables are related to each other in the Network Fundamental Diagram. Give the names and explain the terms (more than one answer possible) (2 points)*

Exercise 129. *Sketch a typical NFD (1 point)*

Exercise 130. *How is this shape different from a regular fundamental diagram. Explain briefly why (20 words). (1 point) –*

Exercise 131. *Comment on the traffic mechanism behind the right hand side. Show in your answer that you understand the difference in traffic mechanism with the fundamental diagram (1 point)*

A traffic engineer suggests to limit the access to the central business district, a part of the road network with many companies.

Exercise 132. *Explain using the NFD why this can be beneficial even though the stopped vehicles need to wait and experience delay. (3 points)*

A.6. Problem set 6

Exercise 133. *Explain what a car-following model does in general (1 point)*

A.6.1. Short questions

The pce value of a truck is claimed to be 2 for a road without queue warning systems.

Exercise 134. *Explain what a pce value is (indication: 30 words). (1 point)*

A study shows that by installing queue tail warning systems the capacity of the road increases. The factor by which it increases is 1.05 for passenger cars and 1.09 for trucks, meaning it can handle 5% more cars and 9% more trucks

Exercise 135. *Based on the information above, calculate the new pce value for trucks for the road with queue warning systems. (3 points)*

Exercise 136. *Traffic simulation in Lagrangian coordinates has the advantage that there is less numerical diffusion. Explain in your own words what this means – indication 50 words. (1 point)*

The total utility U for the mobil lane change model is given by:

$$U_{\text{tot}} = U_{\text{own vehicle}} + p \sum_{i \in \text{other drivers}} U_i \tag{A.9}$$

In this equation, the utility for each driver is determined by its instanteneous acceleration, given by a car following model. This utility can be calculated for the situation with lane change ($U^{(\text{lane change})}$), as well as without lane change, ($U^{(\text{no lane change})}$). For European driving, vehicle will change to the right if $U^{(\text{change to the right}}) > (U^{(\text{no lane change})} - a_{th}$ and to the left if $U^{(\text{change to the left})} > (U^{(\text{no lane change})} + a_{th}$. In these equations, $a_t h$ is a threshold which should be exceeded to perform a lane change.

Exercise 137. *What is a reasonable value for* a_{th} *– explain why. (1 point)*

Exercise 138. *What would be a reasonable range for the parameter* p *– explain why. (1 point)*

Consider a fundamental diagram for pedestrian flow in the flow-density plane.

Exercise 139. *What are the units, as well as the minimum and maximum values at the axes? (2 points)*

A

A.6.2. BRIDGE OPENING

Consider a two lane motorway, with a capacity of 5000 veh/h and a queue discharge rate of 4500 veh/h. On this motorway, the free flow speed differs from the critical speed.

Exercise 140. *Sketch a fundamental diagram for the motorway in the flow-speed (!) plane.2*

Exercise 141. *Indicate reasonable values for the (i) free flow speed, (ii) critical speed, (iii) critical density and (iv) jam density.3*

Assume the fundamental diagram holds. The demand is constant at 3500 veh/h. The bridge opens at t=1h and is opened for 15 minutes.

Exercise 142. *Sketch the cumulative curves for the inflow and outflow.2*

Exercise 143. *Calculate the maximum number of vehicles in the queue.1*

Exercise 144. *Calculate the total delay of all vehicles combined.3*

In the remainder of the question, assume a Greenshields fundamental diagram with a capacity of 5000 veh/h, and a critical density of 60 veh/km.

A further detailed description of the traffic situation uses a so called *acceleration fan.*

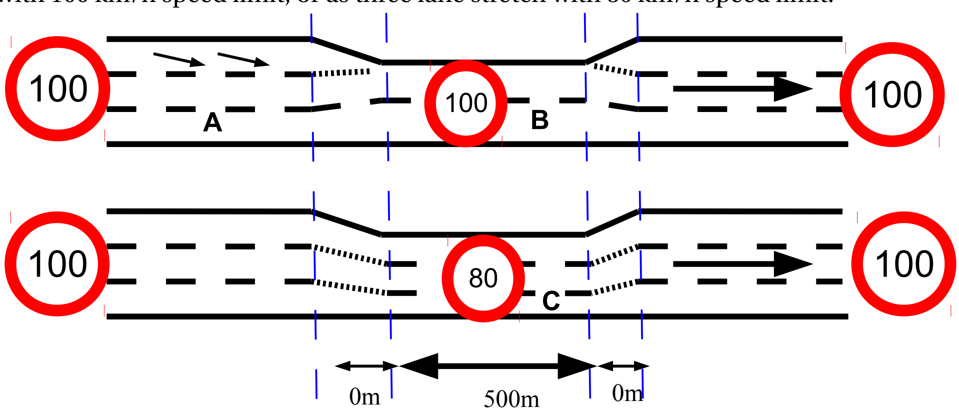
Exercise 145. *Explain in your own words what is an acceleration fan.1*

Exercise 146. *Under this assumption, sketch the flow 500 meters downstream of the bridge as function of time from the moment of the bridge closing (i.e., when the vehicles can flow again); also provide a reference value at the vertical axis.3*

Exercise 147. *Calculate when the flow starts to change in the graph above (hard question: 2 instead of 4 points).2*

A.6.3. VARIABLE ROAD LAYOUT

Consider a 3 lane motorway with a 100 km/h speed limit. The road has a narrower part of a limited length (a bridge, 500 meters long), which can be used as two lane stretch with 100 km/h speed limit, or as three lane stretch with 80 km/h speed limit.



Throughout the question you may assume a triangular fundamental diagram for all road types. For the wide lanes, the critical density is 25 veh/km/lane and the jam density is 125 veh/km/lane. Assume the width of the lanes does not influence the congested branch.

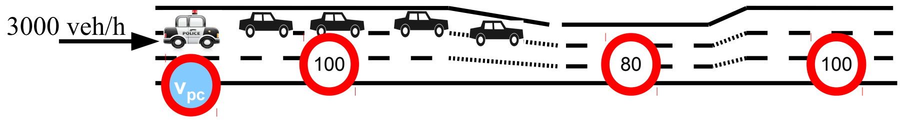
Exercise 148. *Sketch the fundamental diagrams (aggregated over all lanes) for the three parts (all in one figure): three wide lanes, two wide lanes and three narrow lanes: parts A, B and C in the figure above. (2 points)*

Exercise 149. *Calculate the capacity of the three lane narrow road section. (2 points)*

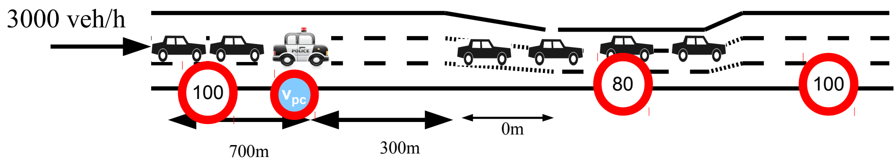
The road authority switches dynamically between the three narrow lanes and the two wide lanes.

Exercise 150. *Explain why this dynamic switching is beneficial for the delays (i.e., why does this give less delays than choosing one layout permanently) (indication: 50 words). (1 point)*

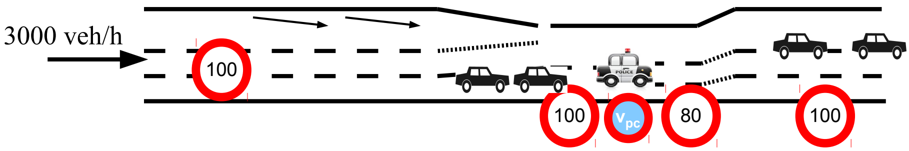
During switching, the road needs to have an empty stretch of 300 meters (because one cannot change the lanes at which vehicles are driving). The road authority wants to do so by insert a *pace car* into the traffic stream starting 1000m upstream of the switching part. This should drive at an appropriate speed (v_{pc}) to create this gap (the gap should be downstream of the pace car after this has travelled 700m). Consider the situation of switching from 3 lanes to 2 lanes. The demand is 3000 veh/h. For illustration purposes, the process is also shown in figures below.



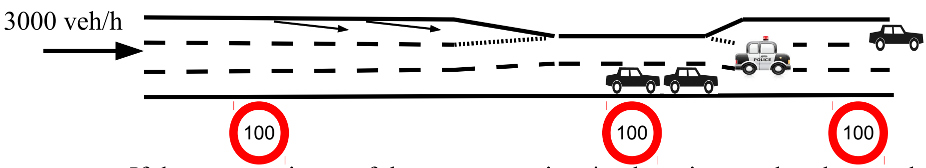
In normal conditions a pace car is introduced, driving at a speed v_{pc}



After 700 m driving, the pace car has a gap in front of 300 m. The last vehicle of the platoon downstream of the pace car is 300 downstream of the pace car, at the location of the change of layout



Upstream of the pace car, the road switches to a 2 lane road with a 100 km/h speed limit



If the pace car is out of the narrow section, it adapts its speed to the speed limit if not limited by traffic. Otherwise, it adapts it speed to the vehicles in front.

Exercise 151. *Calculate the speed at which the pace car needs to drive. (1 point)*

The pace car acts hence as moving bottleneck, and will continue driving at this speed throughout the bridge section, and then (instanteneously) speed up to the free speed.

Exercise 152. *Indicate the traffic states in the flow density diagram and sketch traffic states in the space-time diagram (and relate them to each other using shock wave theory). (no vehicle trajectories needed) (hard question: 6 points awarded for 9.5 points). Tip for sketching: choose the demand low in your graph (for simplicity you may assume other flows to be higher). If you did not find an answer for the question above, assume it is driving 60 km/h. (6 points)*

A.6.4. TRAFFIC STABILITY

Exercise 153. *Give the name of the mild congestion located at a bottleneck location in Kerner's three phase traffic flow theory (1 point)*

In the flow-density plan, this is an area rather than a line.

Exercise 154. *Link this to the Wiedeman car-following principle. (indication: 100 words) (2 points)*

A.6.5. LEVELS OF DESCRIPTION

One can argue that there are three levels at which traffic can be described: microscopic, macroscopic, and network-level. On the microscopic level, traffic behaviour can be indicated by a spacing-speed diagram.

Exercise 155. *Sketch this diagram. (2 points)*

Exercise 156. *Sketch the matching diagram in the flow-density plane; indicate how the values for this graph can be obtained from the previous graph.(2 points)*

Exercise 157. *Give is the name for the flow aggregated on network level (1 point)*

Exercise 158. *Give the name for the density aggregated on network level (1 point)*

Exercise 159. *Sketch the matching Network Fundamental Diagram for the network level in the same graph as the fundamental diagram of question b, and briefly comment on the difference (50 words).2*

A.7. PROBLEM SET 7

A.7.1. SHORT OPEN QUESTIONS

Upstream of an junction is a controlled intersection.

Exercise 160. *Which distribution is likely to describe the number of arrivals per time interval (i.e., flow) on the main road near this junction (and downstream of the controlled intersection)? Name and explain briefly why (no need to comment on the equations of the distribution functions). (2 points)*

Exercise 161. *Argue why traffic instability can only occur under platoon instability. (2 points)*

Traffic dynamics can be described by linking three variables, usually indicated by x, N, and t.

Exercise 162. *Describe the physical meaning of N(x,t). (1 point)*

Exercise 163. *Describe the difference between calibration and validation of a traffic model. (Indication: 40 words) (2 points)*

Exercise 164. *Which relationship is described by the macroscopic fundamental diagram. In your answer, make sure to be specific enough that the answer does not apply for the fundamental diagram. (There is no need to comment on the shape, traffic process leading to the shape or to name specific terms – indication: 25 words) (1 point)*

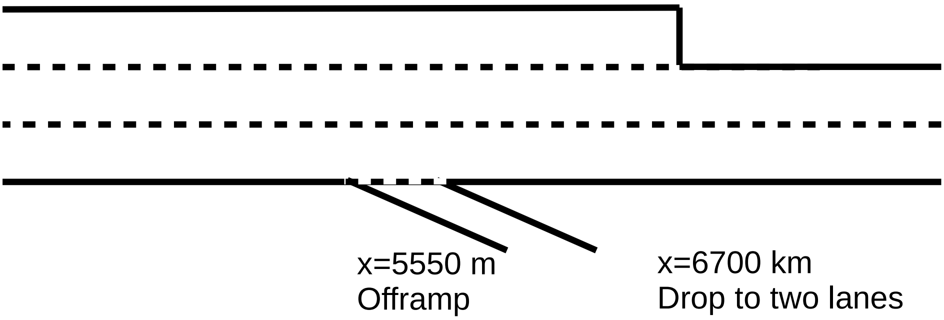
For pedestrian traffic, a typical value for the capacity which can be found in a handbook is 1.22.

Exercise 165. *What are the units for this capacity. (1 point)*

Exercise 166. *What is the physical meaning of these units? (1 point)*

A.7.2. CUMULATIVE CURVES

Consider the following road layout:

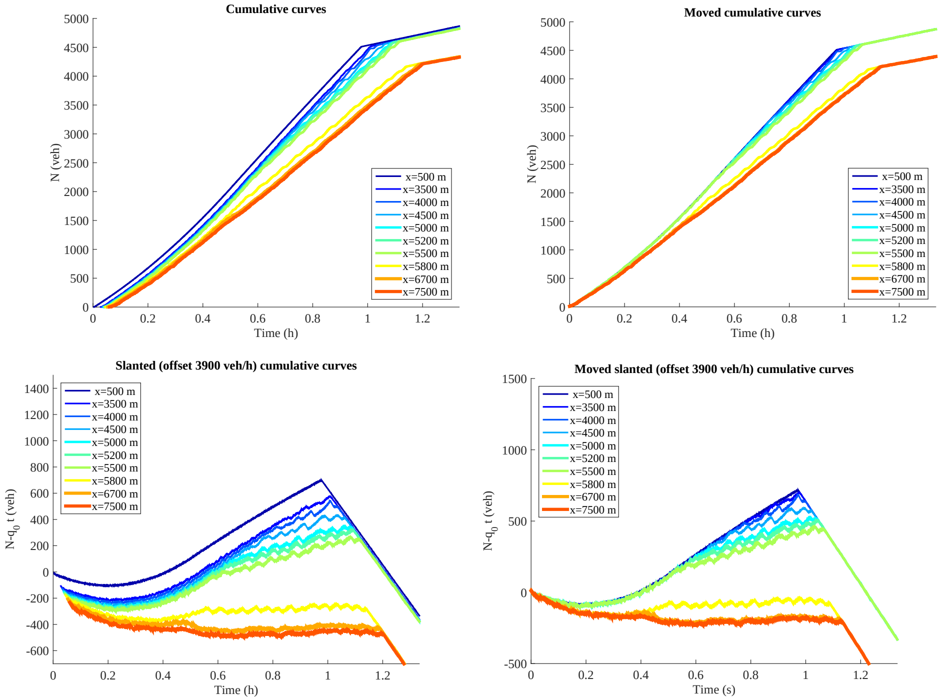


A three lane road has an offramp just downstream of 5500m, and a lane drop at approximately 6700m.

Cumulative counts are obtained from Fosim.

Exercise 167. *Name the two main limitations of using cumulative curves for delay determination in real world conditions. (2 points)*

On the next page, you find the cumulative curves, the moved cumulative curves (i.e., corrected for the free flow travel time), the slanted cumulative curves (offset 3900 veh/h), and the slanted moved cumulative curve (i.e., corrected for the free flow travel time). (If you cannot differentiate colors, raise your hand to get explanation on the lines!)



Exercise 168. *Describe the traffic operations (or sketch the traffic operations in space-time): This should show: is there an temporal bottleneck? Why does congestion end (demand or supply)? Where is the head and tail of the queue approximately. Also indicate what information you use to get to this conclusions. (3 points)*

Exercise 169. *Estimate the (free flow) capacity of the bottleneck in veh/h from the curves. (2 points)*

Exercise 170. *Estimate the capacity drop in veh/h from the curves. (2 points)*

The total delay due to the queuing is determined by summing the delays for all vehicles.

Exercise 171. *Argue whether the delay would differ if a vertical queuing model for the bottleneck was used, and if so, would the delay be less or more with a vertical queuing model. (2 points)*

A.7.3. MACROSCOPIC TRAFFIC VARIABLES IN CASE OF DIFFERENT VEHICLE CLASSES

Suppose there are two classes of vehicles, fast vehicles driving 120 km/h and slow vehicles moving at 90 km/h. At the entrance to the road (x=0), an equal number of each type of vehicles is measured under a total flow of 1000 veh/h (so 500 veh/h each). *For some subquestions you might need answers of earlier subquestions, which you could not answer. In that case, assume an answer, state so on your answer sheet and continue with the assumed answers.*

Exercise 172. *Calculate the density on the road. Base your answer on the density of the slow vehicles and the fast vehicles. (3 points)*

Exercise 173. *Calculate the space mean speed. (2 points)*

Exercise 174. *Calculate the time mean speed. (2 points)*

Exercise 175. *Give the equation for Edie's generalised definition of density. (1 point)*

There is an observer moving at 60 km/h.

Exercise 176. *Calculate the flow of fast moving vehicles passing this observer. (3 points)*

Exercise 177. *What is the time mean speed (absolute to the road, not relative to the observer) of the vehicles moving this observer (hard question: 3.5 points, yielding 2 points) (2 points)*

Exercise 178. *Will the flow of slow moving vehicles passing the moving observer be larger than, equal to or lower than the flow of the fast moving vehicles passing the moving observer. Argue (rather than calculate) why. (2 points)*

The capacity of the road is, independent of the vehicle class, 2000 veh/h, and the jam density is, independent of the vehicle class, 125 veh/km. Assume the car-following behavior of both classes can be described by Newell's car-following model.

Exercise 179. *Draw (not sketch) the fundamental diagrams for the two vehicle classes (2 points)*

Exercise 180. *If we take the fast vehicles as reference, what is the pce value (or fce, "fast car equivalent") of the slow vehicle (so expressed in units of fast vehicles) (1 point)*

Exercise 181. *Give the car-following model equation for the fast vehicle, including the values of the parameters for the fast vehicles (only the free flow branch is needed). (3 points)*

Consider these traffic operations at a two lane road. According to a lane selection model, all fast vehicles stay left and all slow vehicles stay right.

Exercise 182. *What is the name of this model (only the name is needed – no reasoning). (1 point)*

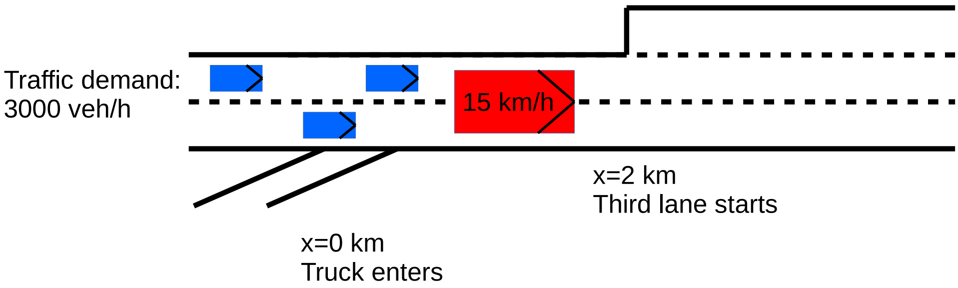
Now consider a different traffic state. Once traffic goes into the congested state, the fast cars move to the fastest lane.

Exercise 183. *Give the name of this state called according to* three phase traffic flow theory. *(1 point)*

A

A.7.4. SLOW TRUCK ON THE MOTORWAY

Consider a road for which on all of the lanes a triangular fundamental diagram holds with a critical density of 25 veh/km, a capacity of 2000 veh/h and a jam density of 125 veh/km. There is a two-lane road for a length of x<2km, and with an extra third lane for x>2km. The traffic demand is 3000 veh/h. At t=0 a wide truck (special transport) joins the road from x=0, driving at 15 km/h. This truck blocks two lanes, so in the two lane section, there are no overtaking possibilities: see figure.



Exercise 184. *Sketch the traffic operations (both in space-time and in flow-density) for the time up to the moment the truck reaches the three lane section. (5 points)*

After the truck reaches the downstream section, vehicles can overtake. The capacity point lies at k=60veh/km and q=2600 veh/h for the whole roadway.

Exercise 185. *Sketch the traffic operations (both in space-time as in flow-density) from the moment the truck reaches the three lane section. Sketch the slopes of the waves which occur from that moment, but* do not consider possible waves which start later.*(5 points)*

A.8. PROBLEM SET 8

A.8.1. SHORT OPEN QUESTIONS

Exercise 186. *Give the name of the distribution describing the headway distribution if the arrival process of vehicles is independent. (1 point)*

A particular distribution function describes the number of vehicles per aggregation interval well if the flow on the road is high.

Exercise 187. *Give the name of this distribution. (1 point)*

The lane flow distribution is the ratio of the flow in a lane over the total flow.

Exercise 188. *Express the relative flow in lane i f_i as function of the average headways $< h_i >$, which are given for all lanes i on a motorway (3 points)*

The theory of slugs and rabbits states there are two types of cars/drivers on the road. Assume they both have a pce value of 1.

Exercise 189. *Give the meaning of the letters pce (give the full words). (1 point)*

Consider the theory of slugs and rabbits. The demand (in veh/h) of slugs is equal to the demand of rabbits. Assume a two-lane road operating below critical density at all lanes.

Exercise 190. *What is the distribution of the flow over the lanes in free flow conditions (qualitatively – no numbers are needed)? Give your reasoning. (2 points)*

Exercise 191. *Explain how stop-and-go waves emerge. Use, name, and describe the two most relevant levels of stability in your answer. (3 points)*

Exercise 192. *Explain why in the three phase theory of Kerner there is not one value for capacity. (1 point)*

One of the advantages of using Lagrangian coordinates is that traffic predictions do not have "numerical diffusion".

Exercise 193. *Explain in your own words what "numerical diffusion" is. (1 point)*

A.8.2. MICROSCOPIC EFFECTS ON THE FUNDAMENTAL DIAGRAM

Exercise 194. *Give the conditions under which the fundamental diagram holds. (1 point)*

When aggregating traffic, a better fundamental diagram can be found if instead of a rectangular area in space-time a parallelogram is used.

Exercise 195. *Explain why, discussing free flow conditions as well as congested conditions. In your answer, also indicate the slopes of the edges of the parallelogram. (3 points)*

Traditionally, the density is calculated at one moment in time over a road section, and the flow is calculated over a time period at one location.

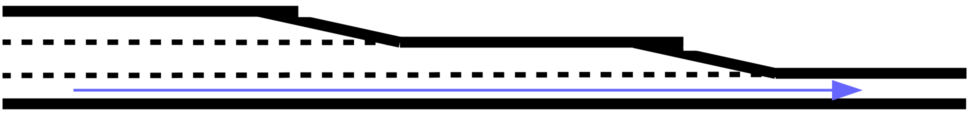
Exercise 196. *Give the name for the definitions used to express flow and density for an area in space-time. Equations are not needed. (1 point)*

During acceleration, one could observe deviations from the fundamental diagram. Consider multi-anticipation, i.e. drivers adapting their speed not only on the leader, but also on vehicles further downstream. This influences amongst others transitions from a congested state to capacity.

Exercise 197. *Describe for this transition the effect of multi-anticipation on hysteresis and sketch the effect of these changes in the flow-density plane (also show the normal FD in this graph as reference).4*

A.8.3. PREDICTION OF DOWNSTREAM CONDITIONS

The number of lanes on a freeway reduces from 3 to 2 to 1, as sketched below.



The one lane section forms an active bottleneck. Assume throughout this question a triangular fundamental diagram, equal for each of the lanes, with a free flow speed of 100 km/h.

In the three lane part, a driver is driving in congestion due to the reduction to one lane. His speed is 8 km/h and he estimates the gross spacing to his predecessor in the same lane at 11m. *(If you are at one part unable to find the correct answer, assume one, state so clearly, and continue with the next part of the question).*

Exercise 198. *Assuming homogeneous conditions, calculate the flow through the bottleneck. (4 points)*

Exercise 199. *Calculate the jam density for the three lane part. Hint: use the capacity of the three lane part. (4 points)*

Exercise 200. *Calculate the speed of the vehicles in the two lane section. (3 points)*

Changes of inflow can lead to a changing traffic state. Adapting inflow can then make the traffic state stationary (i.e. constant over time). Consider one of these stationary situations, with congestion spilling back to somewhere in the three lane section

Exercise 201. *Sketch the flow (aggregated over all lanes) as function of space for this situation. (1 point)*

Consider all possible stationary situations, which have a different accumulation (i.e., nr of vehicles in the area).

Exercise 202. *Sketch the traffic production (here, average internal flow in veh/h) as function of the accumulation. Values are not needed in the sketch. (2 points)*

A.8.4. Demonstration of police cars

The police are planning a demonstration by which they drive 30 km/h on all lanes of the motorway. In practice, the police cars travel as normal vehicles in the traffic stream before the demonstration. And at t=0, they slow down instantaneously to 30 km/h. During the demonstration (the slow driving), there are no overtaking possibilities. In this question, the traffic state upstream of the police cars is referred to as "congestion". See the figure below for an idea.

In this question we consider the following variables:

1. The duration of the demonstration d
2. the demand (assumed constant throughout the demonstration), q_{in}.

After the demonstration, the police cars instantaneously accelerate to the free flow speed.

Assume a triangular fundamental diagram with a free flow speed of 80 km/h, a capacity of 2000 veh/h/lane (indicated C) and a jam density of 150 veh/h/lane. Consider a two lane road with no on and off ramps.

Exercise 203. *Sketch the traffic states in space-time and in the flow density plane for a high demand (demand is higher than the flow in congestion). (6 points)*

Exercise 204. *Calculate the flow in the congestion. (3 points)*

Exercise 205. *Sketch the moved cumulative curves for the inflow and the moved cumulative curves for the outflow with the demonstration. Consider the inflow point upstream of any queue, and the outflow point downstream of any queue. Consider time frame from well before the demonstration to well after any queue. (4 points)*

Exercise 206. *Show that the total delay (D) is given by the equation* $D = (1/2) * d^2 * q_{in} * C/(C - q_{in})$. *(4 points)*

Exercise 207. *What happens to the delay if the demand approaches capacity? Explain from a traffic flow perspective why. (3 points)*

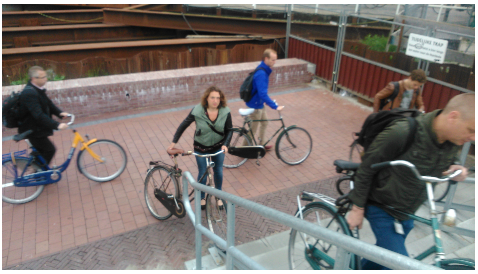
Exercise 208. *Sketch the traffic states in space-time and in the flow density plane for a demand lower than the flow in congestion (2 points)*

Exercise 209. *Argue whether the equation for the delay still holds in this case (1 point)*

Exercise 210. *Give the reasoning why the delay is quadratic as function of duration of the demonstration (3 points)*

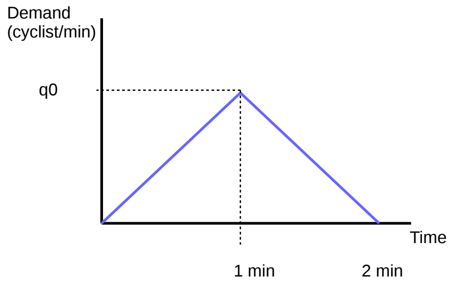
A.8.5. STAIRS FOR CYCLISTS AT DELFT TRAIN STATION

Currently, the station in Delft has an underground bike parking. To get to the street level, the cyclists need to dismount, and walk their bicycle up the stairs. See the picture below. Assume all cyclists do so with the bike at their right. The figures below give an impression of the site, although not at peak demand

Exercise 211. *Calculate the capacity for this site based on an assumption of headway between people walking their bike. (3 points)*

In the peak hour, a train arrives. Suppose 300 passengers leave the train station by bike using these stairs. Suppose the difference between the first passenger leaving the bike parking and the last one is 2 minutes. Assume a triangular demand pattern at the bottom of the stairs:



Exercise 212. *Calculate the value for q0 in this pattern. (3 points)*

Assume the capacity C is lower than q0. Number the cyclists 1...N in order of leaving the bike parking. The cyclist with the highest delay due to queuing is N_E.

Exercise 213. *Derive the equation for N_E as function of C and $q0$. Give your reasoning. (If you are unable to give the equation, points can be awarded for a description of the construction.) (4 points)*

A.9. PROBLEM SET 9

A.9.1. SHORT QUESTIONS

The Macroscopic Fundamental Diagram relates the average density to the average flow.

Exercise 214. *Give the name for the term used to indicate the average flow. (1 point)*

Exercise 215. *On a particular road the Greenshields fundamental diagram holds, with a free flow speed of 80 km/h and a jam density of 100 veh/km. Does this determine the capacity with no further assumptions? If so, calculate. If not, why not. (3 points)*

Exercise 216. *Describe what an acceleration fan is. (2 points)*

On a road, traffic is measured using loop detectors at a cross section. For each aggregation period the flow is given as output, as well as the time mean speed of traffic passing that detector. Then, densities are computed from the speed and flow, and a relation between speed and flow is plotted.

Exercise 217. *Explain whether there is a bias in the densities; if yes, explain which and why; if no, explain why not. (3 points)*

Exercise 218. *Traffic in a model is locally stable. Can the platoons be unstable? Explain why or why not. (2 points)*

Exercise 219. *Explain when and why "random seeds" are needed (approximately 100 words, no equations needed). (2 points)*

A.9.2. CAR-FOLLOWING IN FOG

Helly's car-following model is given by the following equations:

$$a = \alpha \Delta v + \beta \left(s - s^*\right) \tag{A.10}$$
$$s^* = x_0 + vT \tag{A.11}$$

In this equation, a is the acceleration, v denotes the speed, Δv the difference in speed between the leader and its predecessor, s the spacing. Parameters in the model are α, β, x_0 and T. The interpretation of the parameters, and the meaning of s^* is left to the students. For reasons of simplicity, the time dependence and reaction time are ignored in this question.

Exercise 220. *Derive the equation of the fundamental diagram, q=q(k), from the above equation* for the congested branch. *Hint: use the relation v=v(s) in the derivation.4*

The model can be calibrated and validated.

Exercise 221. *Explain what is the value of validating the model (i.e., what can a validated model be used for, which cannot be done with a non-validated model).(1 point)*

The following parameters are found to fit the traffic best:
$\alpha = 0.3\text{s}^{-1}, \beta = 0.08\text{s}^{-2}, x_0 = 20\text{m}, T = 1\text{s}$. (In this line, s means second and m meter). Besides the car-following part, also a free speed is required to derive a full fundamental diagram. This is found to be 30 m/s.

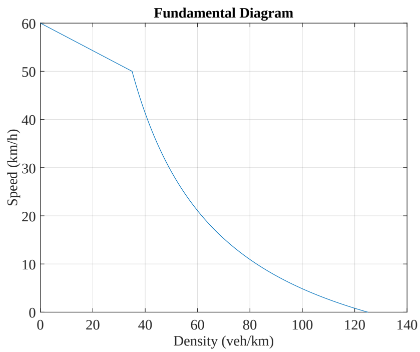
Exercise 222. *Compute the capacity of the road in veh/h* (hard question) *(2 points)*

The Dutch government recommends drivers in fog: "Half the speed, double the space headway to the predecessor". Suppose everyone takes this advice.

Exercise 223. *Compute the capacity of the road in the fog. You may (but do not need to) relate this to question 2c; in case you did not find an answer to that question, assume an answer and state so explicitly.(3 points)*

A.9.3. CLEANING THE ROAD

Consider a one-lane road for which the following fundamental diagram holds:



This is a well-known fundamental diagram.

A

Exercise 224. *Give the name of the shape of this fundamental diagram.(1 point)*

Exercise 225. *Compute the capacity of the road.(2 points)*

Exercise 226. *Sketch the fundamental diagram in the flow-density plane.(1 point)*

A cleaning car enters the road to clean the road, driving at 15 km/h.

Exercise 227. *Compute the maximum flow upstream of the cleaning car using the given fundamental diagram.(2 points)*

The inflow to the road is 1200 veh/h; the cleaning car enters the road at x=1km and leaves at x=2 km. Overtaking is not possible.

Exercise 228. *Construct the traffic state using shockwave theory: i.e., sketch the traffic operations in the flow-density plan and in space-time(6 points)*

A.9.4. MOBIL LANE CHANGING MODEL

The Mobil lane change model describes whether drivers change lanes or not. The equations for a European system (keep right unless overtaking) are given by the following equations. The utility for a driver is given by:

$$U_{\text{tot}} = U_c + \mathscr{P}(U_o + U_n) \tag{A.12}$$

The utility is calculated for each lane. Acceleration is considered as utility, and subscripts indicate the current vehicle (c), the old follower (o) and the (potential) new follower (n). A lane change to the right is performed if

$$U_{\text{tot}}^{\text{right}} - U_{\text{tot}}^{\text{left}} \geq a_{\text{th}} - a_{\text{bias}} \tag{A.13}$$

A lane change to the left is performed if

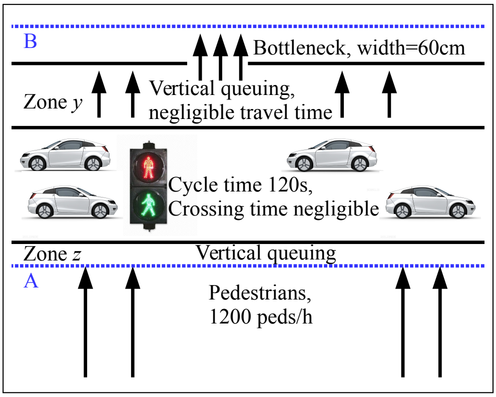
$$U_{\text{tot}}^{\text{left}} - U_{\text{tot}}^{\text{right}} \geq a_{\text{th}} + a_{\text{bias}} \tag{A.14}$$

Exercise 229. *Explain the working of the Mobil lane change model in words (approx 100 words); in your answer, include an interpretation for $\mathscr{P}$. (3 points)*

Exercise 230. *Give approximate values (including units) for a_{th} and a_{bias}, and reason why (3 points)*

A.9.5. PEDESTRIANS AT A TRAFFIC LIGHT

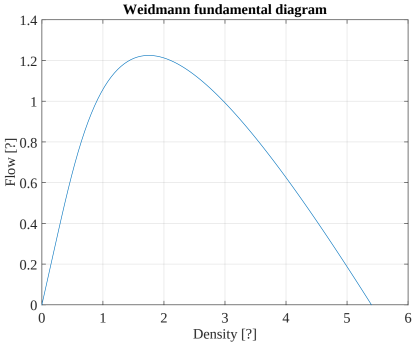
Consider pedestrians approaching the traffic light at a busy intersection. The pedestrians arrive independently at the traffic light at a flow of 1200 peds/h. The cycle time is 120 seconds. Assume all pedestrians cross the road simultaneously (next to each other) in negligible time and then the traffic light turns red again. (So the pedestrian traffic light is red for 120 seconds, then all pedestrians cross instantaneously and the traffic light turns red again instantaneously). A sketch of the situation is found below:



Exercise 231. *Give the name of the distribution of the number of pedestrians arriving per red time. (1 point)*

Exercise 232. *How many pedestrians cross during a green phase. (2 points)*

Weidmann proposed the following fundamental diagram for pedestrians.



Exercise 233. *Give the units for the values at the axes. (2 points)*

The pedestrians cross (simultaneously) during the green phase and arrive at the other side. After a short (negligible) distance, the sidewalk has a bottleneck of 60 cm wide. Assume that the Weidmann fundamental diagram holds. You may assume this bottleneck has no length (e.g., a door on the sidewalk).

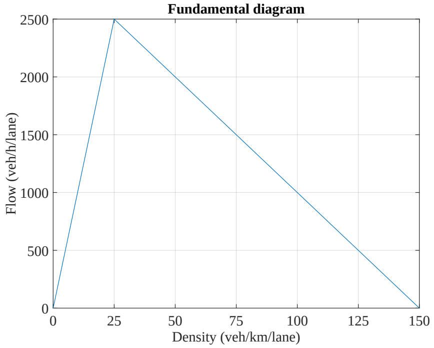
Exercise 234. *Calculate how long would it take the pedestrians to pass the bottleneck. (3 points)*

Exercise 235. *Sketch the moved cumulative curves for this situation (inflow at the blue dashed line "A", outflow at the blue dashed line "B") for a period of one cycle. Start your sketch just before the moment the traffic light is green, i.e. the pedestrians are about to cross the road. Hint: at that moment, how many pedestrians have then crossed line A and not line B? (3 points)*

A

A.9.6. HYSTERESIS

On a road the following fundamental diagram holds under equilibrium conditions.



However, due to the delayed reaction of drivers, hysteresis occurs. Copy the fundamental diagram to your answer sheet.

Exercise 236. *In the fundamental diagram, sketch which path the traffic states make in the transition from congestion to capacity and back. Argue why. (2 points)*

Exercise 237. *Draw (not sketch) the fundamental diagram in the speed-spacing plane. (3 points)*

Exercise 238. *Sketch the hysteresis of question a in the fundamental diagram in the speed-spacing plane. (1 point)*

A.10. PROBLEM SET 10

A.10.1. SHORT QUESTIONS

Consider a road where vehicles are arriving independently.

Exercise 239. *Give the name for the distribution of the number of vehicles arriving within a time interval on this road. (1 point)*

The pce value of a truck is claimed to be 2 for a road without queue warning systems.

Exercise 240. *Explain in your own words what a pce value of two means. (1 point)*

A study shows that by installing queue tail warning systems the capacity of the road increases. The factor by which it increases is 1.05 for passenger cars and 1.09 for trucks, meaning it can handle 5% more or and 9% more trucks.

Exercise 241. *Based on the information above, calculate the new pce value for trucks for the road with queue warning systems (3 points)*

Traffic simulation in Lagrangian coordinates has the advantage that there is less numerical diffusion.

Exercise 242. *Explain in your own words what numerical diffusion in traffic simulation is. (1 point)*

Exercise 243. *Explain what is meant by local instability in traffic. (1 point)*

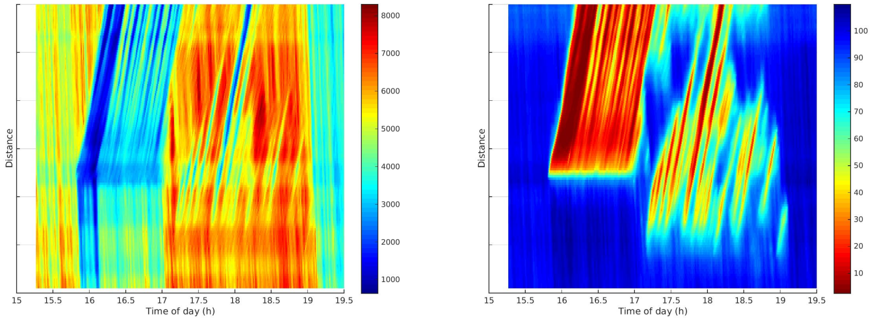
Exercise 244. *Can a stationary tail of the queue occur* due to *a moving bottleneck? If so, explain how. If not, explain why not. (2 points)*

Someone claims traffic on a road is behaving according toa Greenshields fundamental diagram with free flow capacity=2000 veh/h, critical density = 20 veh/km and kjam=150 veh/km.

Exercise 245. *Is this possible? If so, sketch the fundamental diagram in the speed-density plane and comment on the construction; if not possible, argue why. (2 points)*

A.10.2. TRAFFIC FLOW PROPERTIES

Consider the following flow contour plot (left) and speed contour plots (right):



Exercise 246. *Derive in which direction traffic is flowing (2 points)*

The spatial scale is lacking.

Exercise 247. *Reason what is the length of the road shown in the figure. Base your answer on traffic properties. (2 points)*

Exercise 248. *Does traffic at this day exhibit a capacity drop? If so, estimate the capacity drop, and explain how you find it. If not, explain how a capacity drop would show.* For clarity, hand in the graphs in which you indicate the various observation points. *(2 points)*

A.10.3. SPEEDS

Consider a road with slugs and rabbits

Exercise 249. *What are the behavioural assumption(s) with regard to lane choice* for rabbits *in Daganzo's theory of slugs and rabbits. (1 point)*

Traffic is in a two-pipe regime, and the speed of rabbits is 100 km/h and the speed of slugs is 80 km/h. Instantaneously on the the road, the amount of slugs and rabbits is equal.

Exercise 250. *Compute which fraction of flow is rabbits. (2 points)*

Exercise 251. *Compute the time mean speed. (2 points)*

Exercise 252. *Explain how the mean speed can be computed using Edie's generalised definitions.* Equations only do not suffice; explanation of the working of the equations is needed. *(2 points)*

A.10.4. ANALYSE A CAR-FOLLOWING MODEL

For car-following, the following model is being proposed (Helly model):

$$a(t) = \alpha\,(v_i(t-\tau) - v_{i-1}(t-\tau)) + \beta\,(x_{i-1}(t-\tau) - x_i(t-\tau) - s_0 - T\,v_i(t-\tau))$$

In this model, x denotes the position, v denotes the speed and a denotes the acceleration, all evaluated at a time t or $t-\tau$ (τ >0 representing a reaction time); subscripts i and $i-1$ indicate the current vehicle or its leader, respectively. Further parameters of the model are s_0 and T, as well as sensitivity parameters α and β.

Exercise 253. *Explain the working of this model in words.(2 points)*

Exercise 254. *Sketch the congested branch of the fundamental diagram in the speed-spacing plane. To do so, first derive this relationship in mathematical expressions.(5 points)*

Due to hysteresis not all traffic states are on the fundamental diagram.

Exercise 255. *Sketch in the speed-spacing plane how traffic states move in the fundamental diagram, i.e. indicate hysteresis. Motivate your answer.(2 points)*

The fundamental diagram can be transferred to the flow-density plane.

Exercise 256. *Derive this relationship mathematically.(2 points)*

A.10.5. TRAFFIC LIGHT

Consider a road at an intersection. The traffic can be described with a triangular fundamental diagram with a capacity of 1500 veh/h, a free flow speed of 50 km/h and a wave speed of -15 km/h. The inflow is 1000 veh/h.

Exercise 257. *Sketch the fundamental diagram in the flow-density plane.(1 point)*

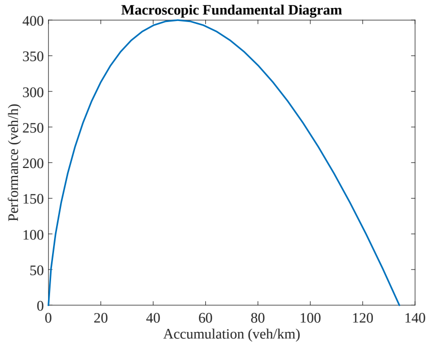
Exercise 258. *Compute the jam density. (2 points)*

At t=0, the traffic light turns red for 2 minutes. Then, the traffic light turns green for 0.5 minute, after which the traffic light turns red again for 1 minute. Afterwards, the traffic light stays green.

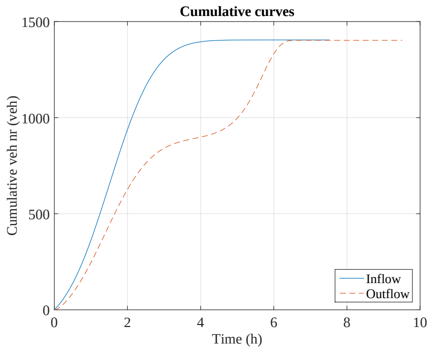
Exercise 259. *Using shockwave theory, sketch the traffic states in the space-time diagram and show the states and waves in the flow density diagram. Comment on how you find the lines. (9 points)*

A.10.6. MACROSCOPIC FUNDAMENTAL DIAGRAM

For a zone the following Macroscopic Fundamental Diagram holds:



The performance on the vertical axis indicates the outflow out of the zone; the total road length in the network is 3.8 km. Assuming homogeneous and stationary traffic conditions, the following cumulative inflow and outflow curves are obtained:



The inflow curve is a result of (exogenously given) demand.

Exercise 260. *Sketch the inflow pattern as flow over time; estimate the maximum flow into the road. (2 points)*

Exercise 261. *Explain the shape of the outflow curve; in particular, include a description of the process underlying the shape from t=2h to t=5h. (3 points)*

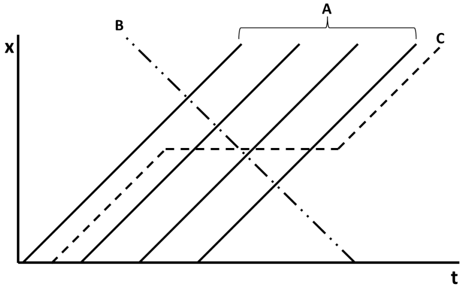
Exercise 262. *Explain how perimeter control can reduce delay. (2 points)*

Exercise 263. *Sketch in the graph the amount of total delay that can be reduced; comment on your construction. (2 points)*

A

A.10.7. NON-VEHICULAR TRAFFIC

Consider the trajectories of pedestrians in this figure:



The trajectory of pedestrian C is temporarily horizontal.

Exercise 264. *Does this imply he is standing still? Argue why. (1 point)*

Pedestrian traffic can be described with the social force model

Exercise 265. *Give a conceptual description of the model (equations are not needed); in your answer, include whether the model is microscopic or macroscopic. (3 points)*

A.11. PROBLEM SET 11

A.11.1. SHORT QUESTIONS

Exercise 266. *Describe what characterizes the difference between a cellular automaton model and another car-following model. (1 point)*

Exercise 267. *Give a reasonable value (with unit) for the capacity of pedestrians. (1 point)*

Exercise 268. *Explain why the MFD does not have the same "pointy" top as a fundamental diagram for a road. (1 point)*

Exercise 269. *Explain why in the method of characteristics, characteristics cannot cross each other in the xt=plane. Comment on an intersection of characteristics of the same traffic state and of different traffic states. (2 point)*

In a traffic stream with independent arrivals, both the exponential probability distribution function as well as the Poisson distribution function play a role.

Exercise 270. *Give the name of the variable of which the distribution is quantified by a Poisson distribution (be precise). (1 point)*

A.11.2. BIKING QUEUES

A group of 50 people approaches a traffic light with a flow of 1 cyclist per 2 seconds. A traffic light is red when they approach, and turns green 10 seconds after the last person has arrived at the traffic light. The capacity is 1 cyclist per second.

Exercise 271. *Sketch the cumulative curves upstream and downstream of the traffic light (assuming vertical queuing). (2 points)*

Exercise 272. *Indicate how the delay can be computed with cumulative curves. (1 point)*

One can approximate the traffic operations with a triangular fundamental diagram. Then, the same questions could be computed with shockwave theory.

Exercise 273. *Argue whether the delay computations would be different using shockwave theory, and if so, whether they would be smaller or larger. (2 points)*

A.11.3. SPEED AVERAGING

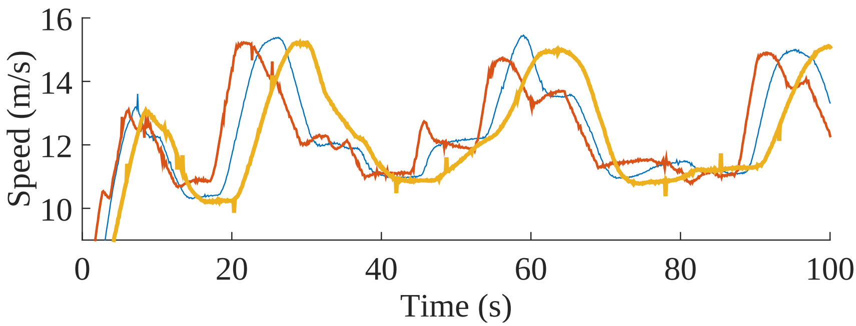
Consider a group of cyclists all following the same circular route: 80% rides a normal bike, and 20% a bike with electrical support motor. Cyclists at a normal bike move at 15 km/h and those with an electrical support motor move at 25 km/h

Exercise 274. *Argue whether the time mean speed is higher or lower than the space mean speed.(2 points)*

Exercise 275. *Compute the average speed on the track using Edie's generalised definitions of traffic. Show the steps you take. (4 points)*

A.11.4. EXPERIMENT OF ACC EQUIPPED CARS

An experiment was carried out with vehicles driving an adaptive cruise control, i.e., the vehicle adapts the speed to its predecessor, which is measured by radar. The graph below shows the resulting speeds.



The platoon consists of these three vehicles

Exercise 276. *Give the convention of numbering vehicles: upstream to downstream or downstream to upstream. (1 point)*

Exercise 277. *Indicate which line relate to each of the vehicles 1 to 3. Argue why. (2 points)*

Exercise 278. *Comment on the local stability for these vehicles. (1 point)*

Exercise 279. *Comment on the platoon stability for these vehicles. (2 points)*

A

A.11.5. ACCIDENT DOWNSTREAM OF A BRIDGE

Consider a road with a bridge. Drivers do not exceed the speed limit. In congestion, they act according to the Helly car following model. The acceleration a at moment t is given by:

$$a(t) = \alpha\left(\Delta v(t-\tau)\right) + \gamma\left(s(t-\tau) - s^*\right) \tag{A.15}$$

t is the time, v is speed, and Δv is the speed difference with the leader. s is the spacing, s^* is given by:

$$s^* = s_0 + T v(t-\tau) \tag{A.16}$$

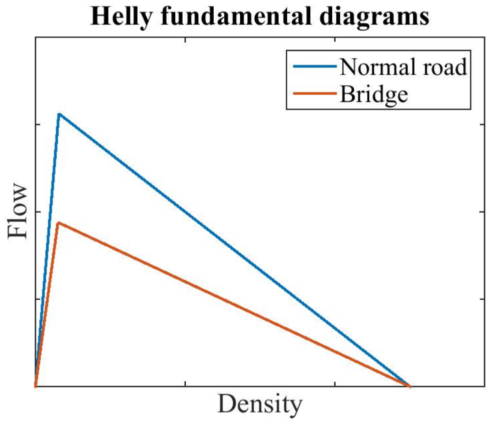
τ, s_0, and T, as well as α and γ are parameters on driving behavior.

Exercise 280. *Explain the working of this model. (2 points)*

Exercise 281. *Give a realistic value for τ (include units in your answer where relevant). (1 point)*

Exercise 282. *Reason which parameters determine the fundamental diagram. (2 points)*

On the road, there is a bridge At the bridge, driver behavior changes due to narrower lanes. The fundamental diagram hence changes. Both fundamental diagrams are shown.



Note both branches change.

Exercise 283. *Reason which model parameter(s) change(s). Only comment on parameters which can be derived from the fundamental diagram.2*

On a particular day, there are free flow traffic operations until an accident happens downstream of the bridge which temporarily blocks the road completely. The tail has spilled back further upstream than the bridge when the incident is cleared. Consider constant inflow.

Exercise 284. *Sketch the traffic operations in the time-space diagram, using a construction in the density-flow diagram. Show the construction steps. For simplicity, you may assume the bridge has a length of 0 (but the capacity constrain remains). (8 points)*

A.11.6. MOVING BOTTLENECK PRINCIPLES

Consider a two-lane road with a Greenshields fundamental diagram. The free flow speed is 60 km/h and the jam density is 100 veh/km/lane, hence 200 veh/km for the roadway.

Exercise 285. *Argue whether this information is sufficient to specify the fundamental diagram. If so, determine the roadway capacity. If not, indicate which information is missing, and state a reasonable value for this. (3 points)*

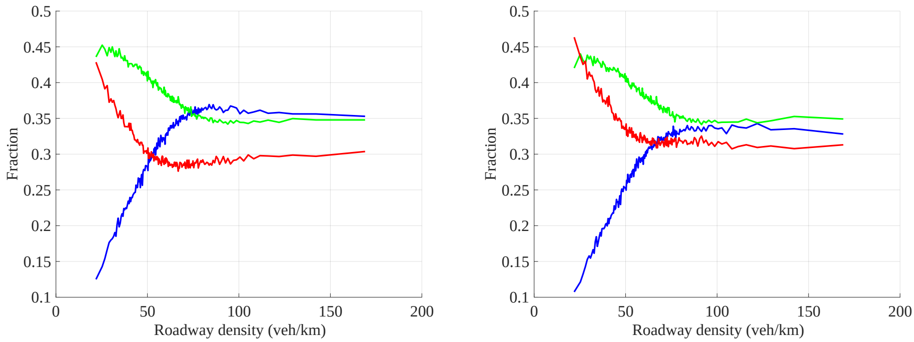
A truck is driving slowly at 15 km/h. Traffic can overtake the truck. The capacity point at the (moving) bottleneck is characterised by a density of 50 veh/km, and a flow of 1000 veh/h. This will yield congestion.

Exercise 286. *Sketch the fundamental diagram in the flow-density plane (if needed with the additional estimates).(1 points)*

Exercise 287. *Sketch how to find the states upstream and downstream of the moving bottleneck in the fundamental diagram. (2 points)*

A.11.7. MULTI-LANE TRAFFIC

Consider the following graphs, representing the fraction of flow and the fraction of density at each of the three lanes on the three lane part of the A13 motorway near Delft in the evening peak.

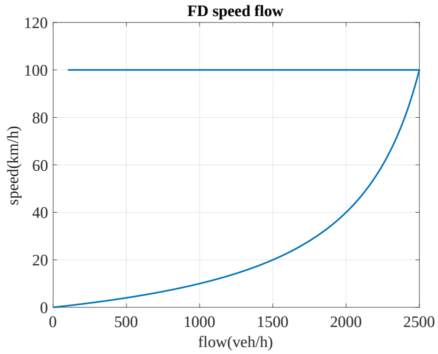


Exercise 288. *Argue which line denotes the left lane, which the center lane and which one the right lane.(2 points)*

Exercise 289. *Argue which graph denotes the fraction of flow and which the fraction of density. (3 points)*

A.11.8. MACROSCOPIC TRAFFIC SIMULATION

A road is split into cells of length Δx. For each of the cells, the same triangular fundamental diagram holds. This fundamental diagram is depicted in the speed-flow plane.



Exercise 290. *Draw (precisely, no sketch; use graph paper for this question) this fundamental diagram in the flow-density plane. Comment on the construction.3*

At a certain moment, the cells have the following densities:

Direction	⟶				
Cell number	1	2	3	4	5
Density (veh/km)	5	20	100	20	125

Exercise 291. *Calculate the flow (in veh/h) from cell 2 to cell 3 according to the cell transmission model.2*

Exercise 292. *Calculate the flow from (in veh/h) cell 3 to cell 4 according to the cell transmission model.3*

B

SOLUTIONS TO TEST QUESTIONS

1 Note that there is a capacity drop here, so the fundamental diagram should either contain a capacity drop (e.g., inverse lambda) or be curved.

2 Slugs stay always in the right lane (1) Rabbits choose the fastest lane (1) Free speed of rabbits is higher than the free speed of slugs (1)

3 A count of the number of vehicles over time at one location (1).

4 A model describing the inflow and outflow of the vehicles, with a restriction on the maximum outflow rate. The queues will not occupy any horizontal space (1). This is basically two cumulative curves, inflow and outflow, at one location, in which the angle of the outflow is maximized at capacity (1).

5 Time mean speed, since the faster cars are weighted higher (1)

6 $u = 1/\sum_i (1/v_i)$

7
1. Free flow => free driving + speed
2. Synchonized flow => equal speeds in both lanes, speeds <70 km/h
3. Wide Moving Jams => backwards travelling waves, v very low.
   (1 point for names, 1 for definitions)

8 Down-top => stop-and-go wave travelling backwards (0.5) Stop -and-go wave (0.5) and a local bottleneck at 482 (0.5) causing an area of synchronised flow (congestion) (0.5)

9 down-top. Finer structure within the congested part. (0.5) Moving bottleneck (0.5) => structure moves upstream (1) Data error at 8am (1)

10 down-top, from the shockwave speeds (0.5) Incident (1) Synchronized flows (and stop-and-go waves) (1) Moving bottleneck after resolving (1)

11 Trajectory of vehicle is its leaders' trajectory delayed in time (1) and moved backwards in space (1).

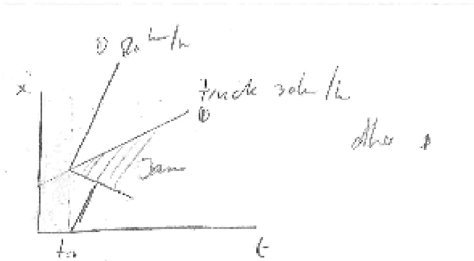
12
1. Follower-leader (local) => over reaction on disturbance in one leader-follower pair
2. Platoon => disturbance grows in a platoon
3. Traffic flow => space between platoons is insufficient, so a disturbance in a platoon causes a delay in the next platoon

13 Yes. Smoothing (1) of disturbances improves platoon stability (1). Thereby, the amplitudes of a disturbance are reduced (1).

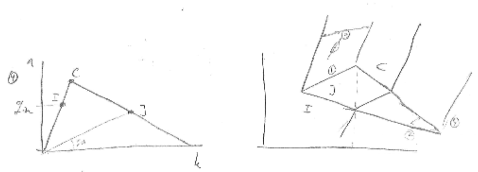
14 A slower vehicle can merge into another lane, which causes a disturbance, growing to a breakdown (1)

15 Now the lane distribution is uneven (1). If vehicles change towards lanes with lower densities, a jam might be prevented (1).

16 Use the triangular fundamental diagram. For the congested branch, the equation is q=2000-2000/100*(k-25). From the speed we q=kv=30k. Solving this set of equations (1), we find k=50, and thus the flow of 30*50 = 1500 veh/h (1)



17
Starting with shockwave theory can be rewarded with 1 pt.



18

19 in I: flow 5000 veh/h, and density 62.5 veh/km, using the equation for the congested branch used in e (1 point). In J: q=4500 veh/h, k=150 veh/km. Wij=-500/87.5=-5.7. So it moves with 4.7 km/h upstream (1).

20 Already in drawing

21 Speed is equal to the truck speed, 30 km/h. (1)

22 Width of a runner 75 cm (1 point for 50-100 cm), headway 1.5 sec (1 point for 0.9 to 2 seconds). 25 meters / 0.75 = 33,3 runners per roadway (both this, and the rounded values are correct). The capacity is 33,3/0,75 x 3600 = 79,200 pedestrians per hour.

23 30,000/79,200 * 60 min/h = 23 min. (1)

24 Halfway: 1 hour duration of the passing of all runners (1) 30,000 runners in 1 hour => 30,000/60 = 500 runners per min. (1) Assumed earlier: 75 com width, 1.5 second headway, so capacity is 1/(0.75 * 1.5) = 0,89 ped/m/s = 53 ped/m/min (1) The required width: 500/53 = 9,4 meters. (1)

25 The congested branch is not a line, but an area (1)

26 An MFD relates the *average* density in an area, the accumulation, to the *average* flow in the area

27 The MFD is flattened at the top compared to the regular FD (1). This is due to averaging effects: the top can only be reached if *all* links are operating at capacity. If some have a over-critical density and some an under-critical, the average density can be critical, but the flow is less than at capacity (1).

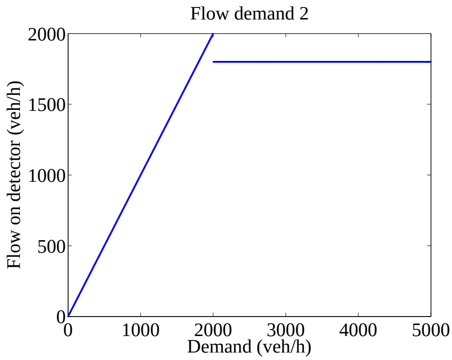
28 The CTM is a numerical scheme to calculate traffic flows in a macroscopic way (0.5), based on demand of the upstream cell and supply of the downstream cell(0.5). The flow is the minimum of both (0.5); demand is increasing with density up to the critical density and remains constant afterwards, supply starts at capacity and starts decreasing from capacity (0.5)

29 Lagrangian coordinates are used in macroscopic traffic flow description. Instead of fixing coordinates at moments in space, the coordinates go with the traffic (1). Variables used in this system are hence time, *vehicle number* (1) and speed. This is an advantage since the representation of traffic flow equations is more accurate (no numerical diffusion) (1)

30 The maximum flow on the road before the onset of congestion is higher than the flow of vehicles driving out of congestion. (0.5) This phenomenon of drivers keeping a larger headway after a standstill is called the capacity drop. (0.5)

31 Approximately a few to sometimes 30% is claimed

B

32 

(1 for shape + 1 for capacity drop):

33 the flow equals the demand (0.5), but is never higher than the capacity, and it will not decrease (no over-critical densities) (0.5). Capacity drop explained if present.

34 Capacity is approximately 2000 veh/h/lane (1500-2500: 0.5 pt, possibly lower if urban roads are assumed and explained), so 4000 veh/h for the roadway (0.5)

35 Effective green: $(g-3)$ seconds per direction (0.5). The cylce time is $c = 2(g+2)$ (0.5). Thus, $g = (c/2)-2$ (0.5). Relative green time: $2\frac{g-3}{c} = 2\frac{c/2-5}{c}$ (0.5) $= 1-\frac{10}{c}$.

36 Relative green time per direction is half that of the total relative green time: $\frac{1}{2}-\frac{5}{c}$ (1). The flow when effective green is 3600(sec/h)/2(s/veh)=1800 vehicles per hour, or 1/2 vehicle per second (either flow value: 1 point). The flow per direction is thus $1/2(\frac{1}{2}-\frac{5}{c})$ with flow in veh/s and c in seconds or $1800(\frac{1}{2}-\frac{5}{c})$ with flow in veh/h and c in seconds. (1 point)

37 Both methods will yield the same answer, since not the queue length, but the flow is asked. (1) Most vehicles come from direction 2, so the green time (still equal) should be based on direction 2 (1). Using the equation from the last question, we have $1800(\frac{1}{2}-\frac{5}{c}) = 800$ (0.5). Solving this for c yields $\frac{1}{2}-\frac{5}{c} = \frac{8}{18} = \frac{4}{9}$. So $\frac{5}{c} = \frac{1}{10}$, so $c = 50$ seconds(0.5).

38 Poisson distribution

39 The cycle time is 120 seconds. The expected number of vehicles in a cycle is $120/60 * 800/60 = 160/6 = 26.7$ (0.5) so $\lambda = 26.7$ (0.5) 120 seconds cycle time, so 60 seconds per direction. 5 seconds are lost (2 s clearance time and 3 s startup loss), so 55 seconds leading to floor(55/2)=27 vehicles at maximum through a green phase (0.5). The probability of an overflow queue is the probability that the number of cars arriving is 28 or larger, $P(X >= 28)$ (0.5); this can be calculated by $1-P(X <= 27)$ (0.5). This is calculated as $p = \sum_{k=0}^{27} \frac{26.7^k e^{-26.7}}{k!}$ (0.5)

40 The spread of a Poisson arrival process is larger(0.5), so the probability of having overflow queues is larger(0.5; only with correct reasoning)

41 Vehicles accelerate in principle with acceleration a, but this reduces when they approach their desired speed (1) or if they approach their desired distance (1). The desired distance increases with speed.

42 It is a translation of the leaders' trajectory (0.5), translated forward in time by a fixed value τ and back in space by Δx (0.5).

43 Choose two:

- Drivers are considering multiple vehicles ahead
- Drivers are unable to judge perfectly speed and gap
- Drivers are not changing acceleration continuously
- There is also interaction with lane-changing

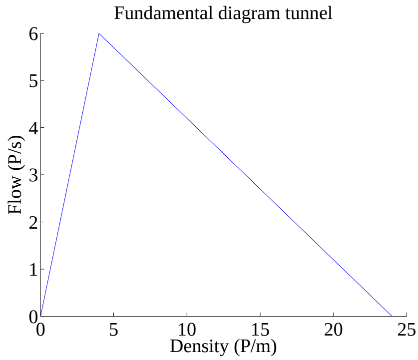
44 Choose 2:

- It better captures the non-equilibrium conditions.
- There is a possibility that stop-and-go waves form (instabilities).
- Drivers have a finite acceleration

45 Choose 2 (1 point per good answer):

- In the IDM, drivers will, unrealistically, never reach their desired speed, not even if the distance is larger than the desired distance
- It has less parameters to calibrate – more realistic
- It is better understandable

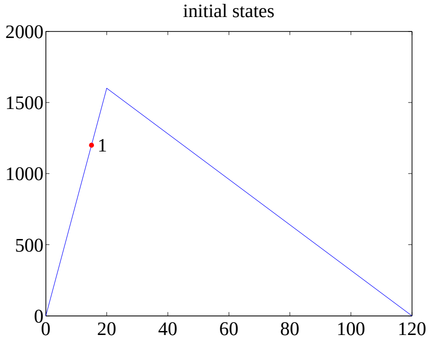
46 Convert the densities to tunnel-wide properties. The critical density then is 4P/m and the jam density $24P/m$. (1) The capacity is vk=6 P/s. (0.5)



(0.5)

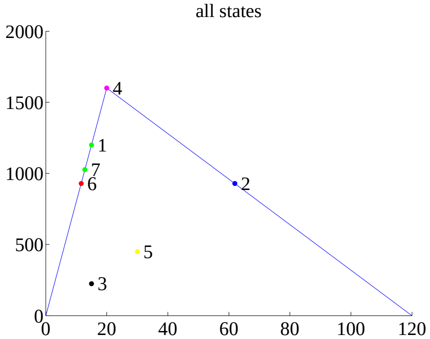
47 180P/min = 3 P/s (unit conversion: 0.5). Undercritical (of course, the tunnel is the bottleneck!), so v=vf. $k = q/v$=3/1.5=2 P/m (0.5). That is 2 pedestrians per meter length, so 0.5 pedestrians per m^2 (1)

48 30 vehicles in a 2 km section, so the density is 15 veh/km. (0.5)



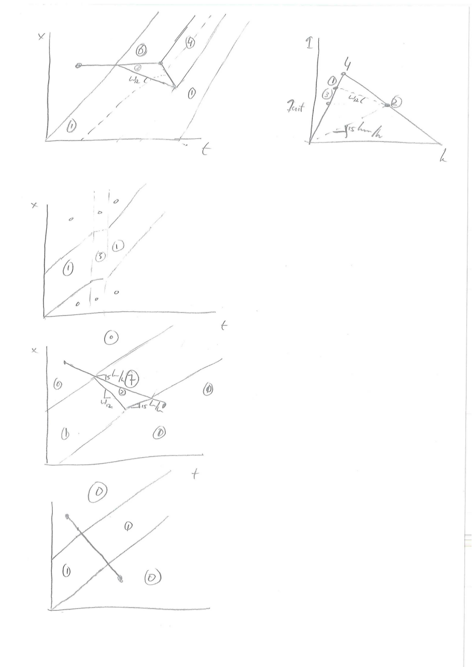
(figure: 0.5);

49 Draw a line with an slope of 15 km/h up in the fundamental diagram. You find the intersection with the congested branch at point 2. (0.5 point) The flow is lower than the initial conditions, so congestion occurs. (given)



(0.5 points)

For the xt-plot we start with the bottleneck at a constant x from t1 to t2. This causes traffic state 2 to occur. The boundary moves backwards with $w_{1,2}$, which is the same as the angle in the fundamental diagram between state 1 and 2. (0.5). Downstream, the flow is the same as the capacity in 2, but in free flow (state 6) (0.5). After removal of the bottleneck, there are capacity conditions (0.5) and the shock waves propagate with speeds equal to the angle 2-4 and 1-4 (0.5 combined). From $t1$ there also is a shock wave separating state 1 and 6, moving with a speed equalling the slope of line 1-6 in the FD (0.5). Trajectories: 0.5.
All xt-plots are found in the next figure



50 Since it is temporal, the density remains constant (1 point). This leads to the construction of point 3 a the intersection of the line with a slope of 15 km/h and the density of 15 veh/km (0.5, including showing this point in the graph). Trajectories 0.5.

51 It is given that the tractor moves back very slowly, so we start at the situation comparable to b/ That means there will be a congestion state at the fundamental diagram where people drive 15 km/h (0.5), state 2 in the graph. The moving vehicle forms a boundary in shockwave theory (1), separating state 2 from another state where traffic moves out of the queue. By using the speed of the tractor, we find state 7 (0.5) as outflow state. After the tractor leaves the road, traffic is at capacity (0.5) and restores just like in question c (0.5).
fundamental diagram (0.5) & trajectories: (0.5)

52 The tractor forms a moving bottleneck, and the boundary between two traffic states (0.5). It separates the initial traffic state 1 from another traffic state where vehicles travel at 50 km/h (1). Constructing this in the fundamental diagram is done by drawing a line with a slope of 50 km/h down from point 1. We find the traffic state next to the tractor at this point (0.5), drawing point 5 in fundamental diagram: 0.5. The end of the tractor has a similar effect of a boundary (0.5) moving with 50 km/h. Constructing this in the fundamental diagram gives back point 1 (0.5). So at both ends, we have traffic state 1 (1), and next to the truck state 5 (no points for that). There are no other shock waves. Trajectories move either straight on, or are delayed a bit next to the tractor. (0.5)

B

53 The left one is Eulerian. Is shows numerical diffusion. Moreover, the grid structure in time and space is clearly visible, whereas in Lagrangian coordinates the grids follow a fixed time and the vehicles (trajectories are visible in the right figure) (1 point for one of the explanations)

54 The representation of traffic flow equations is more accurate (no numerical diffusion) (1); also, the calculations are more efficient since it is not needed to consider both the upstream and downstream cell as with Eulerian coordinates (e.g., CTM) (1)

55 The relative amount of space (in time! -0.5 if not mentioned) that a non-passenger car vehicle occupies on the road

56 Trucks are longer, which influences the pce value. If the pce is the ratio of the length + net headway, assuming the same net headway the pce value increases with deceasing speed.

57 Trucks are longer, which influences the pce value. If the pce is the ratio of the length + net headway, assuming the same net headway the pce value increases with deceasing speed.

58 Vehicles accelerate in principle with acceleration a, but this reduces when they approach their desired speed (1) or if they approach their desired distance (1). The desired distance increases with speed.

59 In regular cf-models, the drivers react on actions by their leader. In multiple-leader models, drivers take several leaders into account in determining their action

60 Calculate for both leaders the desired distance:

$$s_1^*(v,\Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}} \tag{B.1}$$

and

$$s_2^*(v,\Delta v) = s_0 + 2 * vT + \frac{v\Delta v}{2\sqrt{ab}} \tag{B.2}$$

Note that the desired distance to the second leader has twice the speed component.(1) Each of these will lead to an acceleration:

$$a_1 = a_0\left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s_1^*(v,\Delta v)}{s}\right)^2\right) \tag{B.3}$$

$$a_2 = a_0\left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s_2^*(v,\Delta v)}{x_2 - x_0}\right)^2\right) \tag{B.4}$$

(1) (Note the desired distance to the second leader is compared with the distance between the second leader and current vehicle) These have to be combined into one acceleration (0.5). For safety, we choose the minimum acceleration rather than for instance the mean:

$$\frac{dv}{dt} = min\{a_1, a_2\} \tag{B.5}$$

(0.5)

61 This basically is the same as the 2-leader model. First formulate the desired distance to the n-th leader

$$s_n^*(v, \Delta v) = s_0 + nvT + \frac{v\Delta v}{2\sqrt{ab}} \tag{B.6}$$

(1) Then calculate the resulting acceleration for each of them:

$$a_n = a_0 \left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s_n^*(v, \Delta v)}{x_n - x_0}\right)^2\right) \tag{B.7}$$

These have to be combined into one acceleration (0.5). For safety, we choose the minimum acceleration rather than for instance the mean:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \min\{a_n\} \tag{B.8}$$

(0.5)

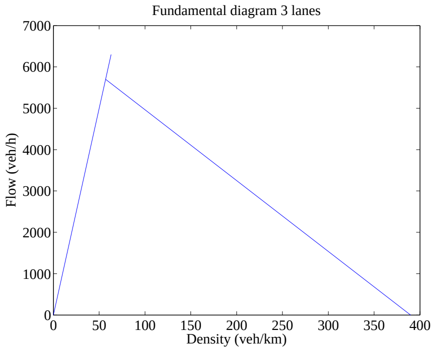
62 The edgy line is the flow, since at the start the demand will exceed the flow before this is reversed. (Also: the flow will be capped at capacity (queue discharge rate) and is expected to be constant for a while). Alternatively: the integral these curves are cumulative curves, and the cumulative curve of the demand cannot exceed the one of the flow. Therefore, the one reducing earlier is the flow)

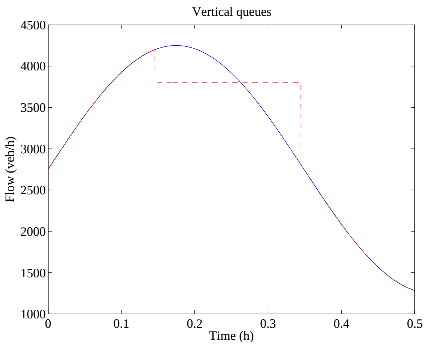
63 The free flow capacity can be found by the flow at the upstream detector before congestion sets in (1), so 4200 veh/h (read from the graph at the time congestion sets in)(1)

64 The queue discharge rate can be found by the flow after congestion has set in (1), so 3800 veh/h (1)

65 The free flow capacity is higher than the queue discharge rate, i.e., there is a capacity drop (1 for either explanation)

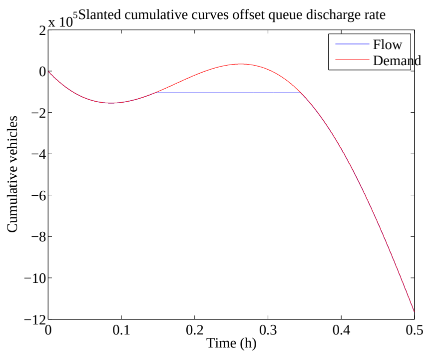
66 The FD should have a capacity drop (see previous questions), so the simplest is to assume an inverse lambda shaped FD (0.5 pt either by naming or in drawing) The free flow capacity is found in question a: 3x2100 veh/h. Also the free flow speed is known (from the speed figure): 100 km/h (0.5 for both combined). This gives the free flow branch (0.5 point for drawing). The density matching the queue outflow rate can be found on the free flow branch, by looking up the density for the queue outflow rate (0.5). Kc=qc/vfree=1900/100=19 veh/km. (multiply by 3 for the 3-lane section (1)). Furthermore, it is known that with a flow of 2 lanes x 1900 veh/h/lane (the flow through the bottleneck), the speed is 22,6 km/h (read from graph) (1 pt), which gives a density of 3800/22.6=168 veh/km (0.5 pt). Now, the congested branch can be constructed. We find the wave speed w=$\Delta q/\Delta k$=((3*1900)-(2*1900))/(3*19 -168)=-17 km/h. The jam density is found by kj=Kc-qc/w=3*(19-1900/-17.1)=390 veh/km (0.5 pt).





67
Points for the general shape (1) and for the correct times of flow change (1)

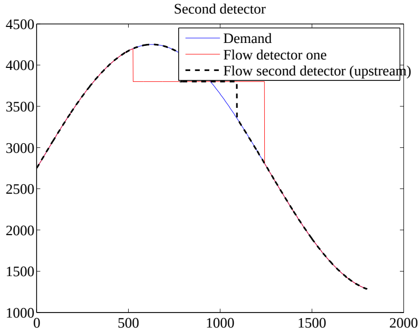
68 The most useful offset is the maximum flow of the road which is maintained for a while, i.e. the queue discharge rate (1 pt). (Alternative: the free flow capacity: 1/2)



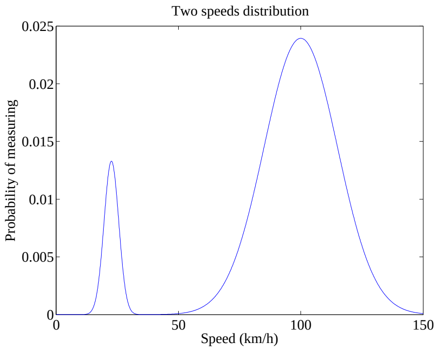
69

70 This is the area between the curves

71 It takes a while before the congestion spills back to the second detector. Similarly, congestion resolves from the tail, so the flow starts following the demand already earlier in time



72 If we do not relax the assumption of the fundamental diagram, there are only two speeds: the free flow speed and the speed for the state in the congested branch where the flow matches the queue discharge rate for the 2 lane stretch. (1) The frequency at which they are measured depends on the relative time of congestion (typically, 2 peaks at each 1 hour of congestion, so approximately 10% of the time congestion) (0.5). Relaxing the assumption of the fundamental diagram now, we do not find two exact values of the speed, but a distribution centered around two values:



(0.5)

73 Capacity is maximum flow, q=ku=25*80=2000 veh/h/lane. For a two lane section, this is 4000 veh/h. (1 for points) plus 1 for graph

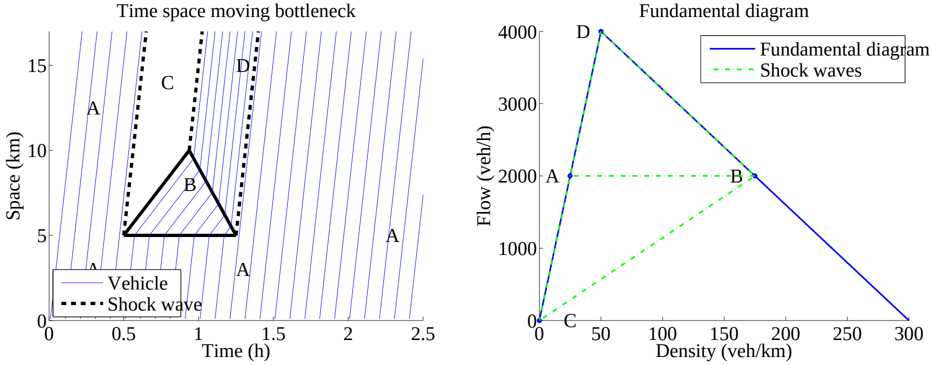
74 Since the tail is at the same place, the flow in the queue should be equal to the inflow (1), so 2000 veh/h.

75 The fundamental diagram is described by q=C(1-((k-kc)/(kj-kc))); use kj=300;C=4000;kc=50; (1) Use the fundamental diagram to find the density at q=2000 veh/h. (0.5) Solving q=1000 veh/h gives k=175 veh/km (0.5)

76 The speed of the vehicle is equal to the speed in the congested state (1). Using q=ku (0.5), this gives v=2000/175=11,4 km/h (0.5)

B

77 The truck drives at 11,4 km/h, and forms the moving bottleneck for 5 km. This can be plotted in the space time diagram (1). The tail of the queue remains stationary (1).



For the solution of the traffic jam, construct the line in the FD between the jam point and the capacity point (1). Finishing the figure: 1 point.

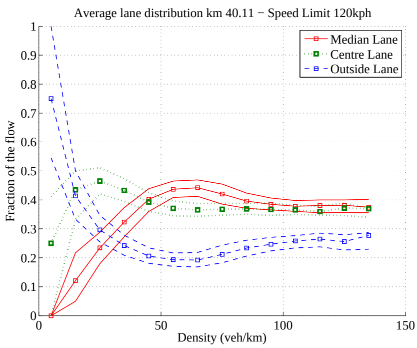
78 The queue has at maximum a length of 5 km, and the density is 175 veh/km – so 5*175= 875 veh.

79 Two-pipe regime and one-pipe regime

80 Two-pipe regime (1). Trucks drive in one lane, and passenger cars in the other, at higher speeds (based on the density) (1).

81 The speed of the passenger cars is higher, so they will pass the stationary detectors more "often" (1). Hence, the fraction of the flow will be higher (1).

82 Requested is the fraction of the flow of passenger cars, (class 1): $\rho = q_1/q_{tot} = q_1/(q_1 + q_2) = (k_1 v_1)/(k_1 v_1 + k_2 v_2)$. (1 point for q=kv per class).
From the image we know k2=4/7*k1 (0.5 point), so $\rho = (k_1 v_1)/(k_1 v_1 + 4/7 k_1 v_2) = v_1/(v_1 + 4/7 v_2)$ (0.5 point) in which v indicates the speeds. Assuming v_1=120 km/h and v_2=80 km/h (1 point – assuming equal speeds maximizes the nr of points for this question to 1), we find: $\rho = 120/(120 + 4/780) = ...$ (1)



83 If it is really quiet on the road, people keep right (hence blue is right). When it gets busier, more people go towards the middle lane, and then towards the left lane. (0.5 for the right reasoning, 0.5 for drawing the right conclusions; no points for only conclusions; 0.5 if middle and median lane exchanged.)

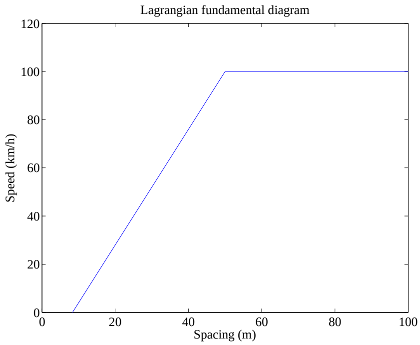
84 Slugs stay always in the right lane (1) Rabbits choose the fastest lane (1) Free speed of rabbits is higher than the free speed of slugs (1)

85 Near capacity, the left lanes travel still faster then the right lane (1). All rabbits are hence in the left lane, leaving gaps in the right lane (0.5) which hence has a lower flow (0.5)

86 It averages the flow and densities (0.5) *for an area* (by which it differs from the regular FD) (1) and relates them to each other (0.5)

87 The average flow decreases for the same average density if the traffic is distributed less homogeneously over the network.

88 The speed is increasing with increasing spacing (0.5). There is a minimum spacing (0.5) and a maximum speed (0.5). Plot: 0.5



89 Vehicles have a desired speed v^* which is determined by the distance to their leader (1). The acceleration is proportional to the difference between their speed and the desired speed.

90 Traffic must be stationary and homogeneous

91 If traffic is stationary, the acceleration is 0 (1), so $v = v^*$ (1). Realising that $k = 1/\Delta x$ (0.5), or correcting for the units (density is in veh/km!) we find $k = 1000/\Delta x$ (0.5), leading to:

$$v = 16.8(tanh(0.086((1/k) - 25) + 0.913)) \tag{B.9}$$

(0.5) This is the speed in m/s, which should be translated into km/h, so:

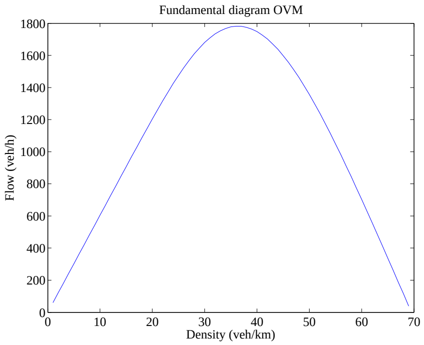
$$v = 3.616.8(tanh(0.086((1/k) - 25) + 0.913)) \tag{B.10}$$

(0.5) Now applying q=ku=kv (0.5), we find

$$q = k3.616.8(tanh(0.086((1/k) - 25) + 0.913)) \tag{B.11}$$

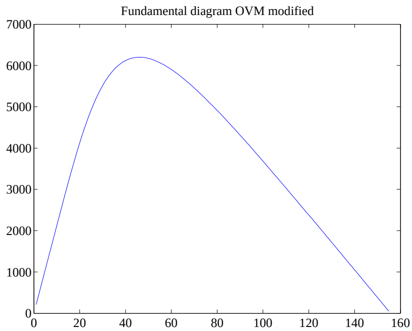
(0.5)

92 The fundamental diagram is shown below:



From this, we read a capacity of approximately 1750 veh/h, the free flow speed is approximately 60 km/h and a critical density of approximately 70 veh/km(1).

93 A free speed of 60 km/h could match a provincial road. However, the capacity for a provincial road is usually not so high. For sure, the jam density would be more in line with the jam density of motorways, i.e. well over 100 veh/km. Therefore, the fundamental diagram is not very accurate. It would be better to have a higher jam density, so scale the graph on the horizontal axis , meaning the number 0.086 should be lower (by about a factor 125/75=1.75 – increasing the jam density to 140). Then, the speed drops even further, so that should be corrected (increase a factor 1.75), and then once more increased by a factor of almost 2 to get a motorway speed . This means increasing 16.8 to 1.75 x 2 x 16.8=... While this would give a proper free speed and jam density, the capacity increases to extreme values (1), so a proper parameter set is not possible



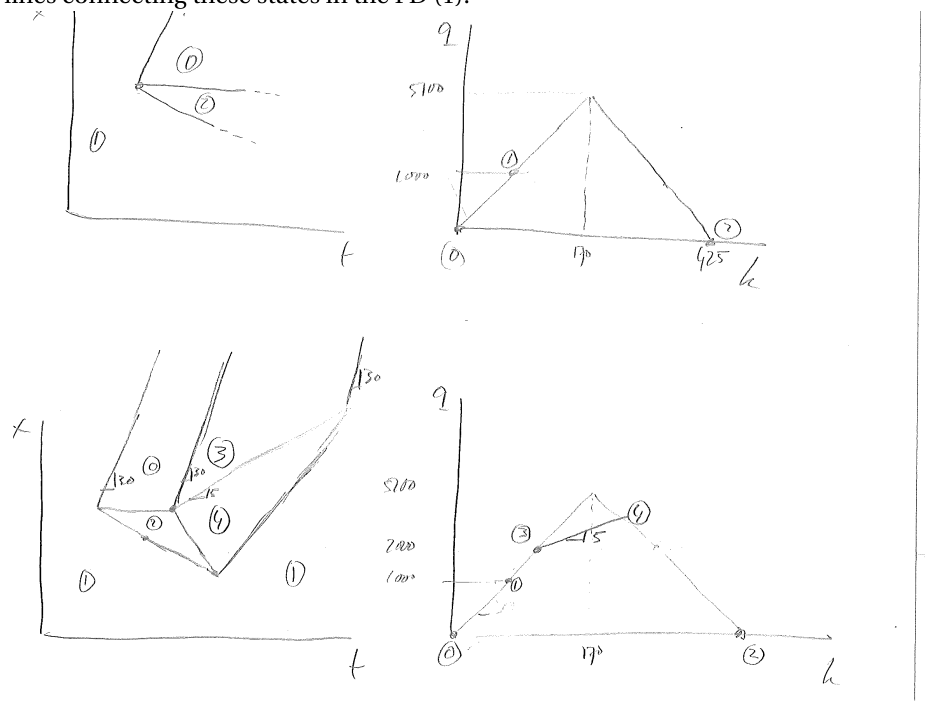
94 Drivers are expected to change lanes whenever it is beneficial for them (measured in terms of acceleration) (1). They do take the other drivers' benefit into account, but less then their own (at factor p) (1).

95 It can be expected that drivers take other drivers into account (p>0), but value their benefits less then their own (p<1).

96 No, it does not. The drivers are all at the left lanes. There is no more acceleration in the left lane, so the model does not give higher utility to the left lane.

97 Once the trucks come to a complete stop, upstream an congested area at jams density will be created (0.5); downstream, there will be an empty road (0.5). There are three shock waves. One with the free speed (30 km/h) separating the empty road with from the initial state (1). One at the location of the stopped trucks, separating the empty road from the jam state, at 0 km/h (1). Finally, there is a wave at the tail of the queue, propagating at a speed of $\Delta q/\Delta k$=1000/((1000/30)-425)=-2,5 km/h (upstream). (1 point)

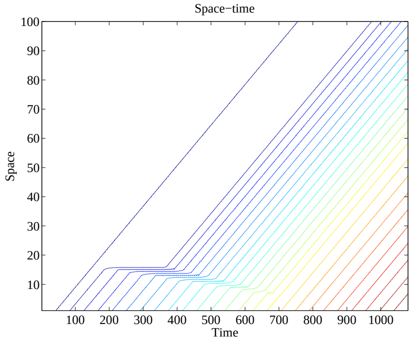
98 The start is the same as described in b. Then, at the head of the queue, vehicles are freed. The fastest vehicles, passing the moving bottleneck, will drive at a free flow speed of 30 km/h (1, including drawing in xt). The snow plows will form a moving bottleneck driving at 5 km/h (1, including drawing in xt). Downstream of this moving bottleneck there will be a free flow (0.5) traffic state with a flow of 2000 veh/h (given, 0.5) – this is state 3. The plows will form a moving bottleneck, and hence the separation of two traffic states, a congested upstream (state 4, unknown yet) and an uncongested downstream (state 3). (0.5 point). Plotting: 0.5 point. State 4 can be found by plotting the shock wave in the fundamental diagram, from state 3 upwards with a slope of 5 km/h (1). The shock wave speeds between 4 and 1, as well as between 3 and 1 can be found by the slope of the lines connecting these states in the FD (1).



99 The inflow is constant (constant raise of N at x=4 - 1 point), but the outflow is temporarily zero (no extra vehicles temporarily in the graph (1). Hence, there is a temporal blocking (0.5) completely blocking the flow. (0.5)

100 The total delay can be derived from the area between the cumulative curves (1). In this case, we compare the cumulative curves where there is no delay (at x=4km) and that where the delay is maximum (near x=25 km) (1). The surface between the lines is (calculate: 1 point).

101 There is a temporal blocking, so the general pattern is as follows:



(1)

The head of the queue, and thus the location of the blocking is between location 16 and 20 (1). The tail is between km 8 and 12, since there is no delay (1)

102 The capacity is the derivative of cumulative curve during the outflow (0.5), here 2000 veh/h (0.5). The jam density can be derived from the number of vehicles between the cumulative curves in standstill: here 120 veh/km (480 veh in 4 km) (1). The slope of the congested branch van be determined by the speed at which the head of the queue (0.5) moves backward, e.g. at 16 km at t=0.6 and 12 km at t=0.8, i.e. 4 km in 0.2h = 20 km/h (0.5). The critical density then is the jam density minus the capacity divided by the shock wave speed (120-2000/20=120-100=20veh/km - 0.5 pt). The free speed then is the capacity divided by the critical density, i.e. 2000/20=100 km/h.

103 The limiting point for the flow is not the amount of space in the room, but the visitors watching the piece of art (1). Because then there are more visitors that can have a look at the art at the same time, because they do not stand crowded around a single object but have a large circumference to stand around (1).

104 I assume every visitor wants to be in the front row (1 point) for the assumed watching time of 30 seconds per person (10 s - 2 minutes: 1 point). Finally, I assume the width of a pedestrian to be 75 cm (50-110 cm: 1 point). The circumference of the show case is 2x3+2x1=8 meter, which means 8/0.75=10 2/3 visitors can stand at the circumference. (Both rounding to an integer number of visitors, as continuing with the real number is OK – arguing that the spots are not predefined, but dynamically filled). The capacity is 10 visitors per 30 seconds = 20 visitors per minute.(1)

105 When stepping on the moving walkway, visitors keep a gross distance headway of approximately 75 cm (50-110 cm: 1 point; net=gross-20cm); assume a single lane of visitors (for the best view) per walkway. The walkway moves with an assumed 0.5 m/s (0.2-1.5: 1 point). The flow is then 1/0.75*.5=2/3 visitor per second = 40 visitors per minute. (0.5) Since there are walkways at both sides, the capacity is increased to 80 visitors per minute. (0.5)

106 The figure shows the size of the square approximately 500x50/2=12,500 square meters(0.5) The maximum density for pedestrians is approximately 5 ped/square meter (3-10: 0.5 pt). So, at times square at maximum 75,00 people can stand (0.5). The claim cannot be strictly right.(0.5)

107 Speeds in both lanes are equal(1). Alternatives with speeds etc: 0.5 points

108 Sketch showing the areas in more or less the right place (0.5), with appropriate names (0.5). Note that synchronized flow is an area (1)

109 There is no capacity(0.5), since there is a probability of breakdown from free flow to synchronised flow(0.5). This is an increasing function with flow. (Remarks on the area in synchronize flow: no points)

110 The coordinates move with the traffic, rather than that they are fixed at a certain location

111 No numerical diffusion, so more accurate results

112 Traffic is flowing from bottom because the congested patterns propagate in the opposite direction.

113 Bottleneck at km 14 (0.5), creating congestion and instability (1 point for this causality) leads to wide moving jams (0.5 for name)

114 Local: a change of speed of the vehicle cause oscillations with increasing amplitude in speeds with the direct follower (or not)
String: the amplitude of speed disturbances grows (or not) over a platoon,
Traffic: the disturbances within a platoon are propagated (or not) to the next platoon
(one point for the names, one point for the explanation)

115 The stability of traffic is one of the levels of above – we only need to consider that level. We see wide moving jams (or stop and go waves) emerging from an area of congestion, so traffic is unstable in the congested branch. (1) That means traffic is also platoon unstable. (1). Traffic is in general locally stable (only instable in simulations)

116 In the two pipe traffic state (0.5) traffic has different speeds in each lane / slugs and rabbits are separated (0.5); in one pipe traffic state (0.5) slugs and rabbits mix and the speeds are the same (0.5)

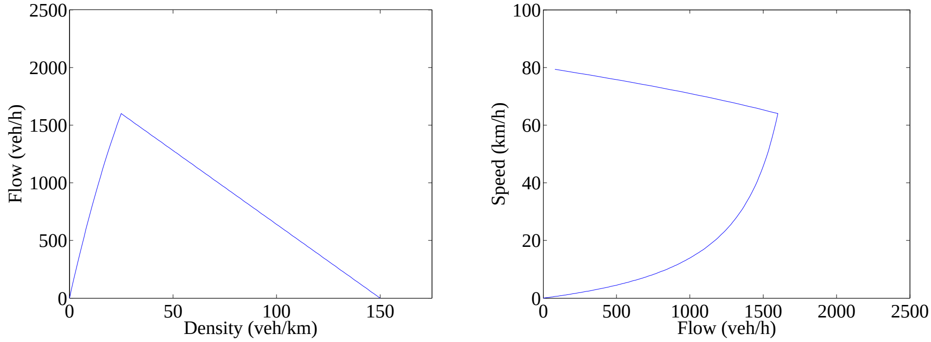
117 According to slugs and rabbits, the traffic is more present in the left lane (1), whereas for the Mobil lane change model there is a stronger tendency for drivers to take the right lane if there is no other vehicle.(1)

118 Stationary means that the traffic stream does not change over time (1). Homogeneous means that the traffic state does not change with space (the same at different locations) (1).

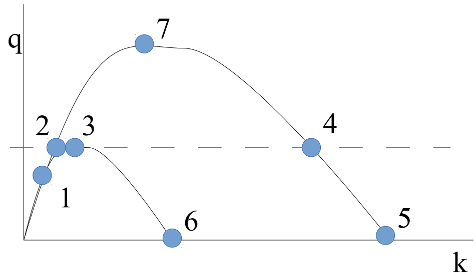
119 It cannot, since the FD gives the equilibrium conditions (1), and drivers have different ways to reach these equilibrium conditions. (1)

120 The right one is the triangular, since there is no influence of density in the speed in the free flow part

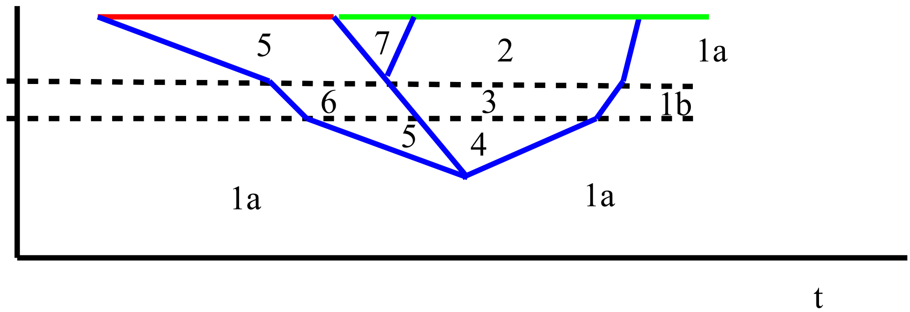
121 2 plots - for each one point. 1 point for the values



122 This is an urban area, so the free flow speed is around 50 km/h. (0.5). The jam density is hardly dependent on the conditions, so around 125 veh/km/lane (0.5) The capacity is a bit lower than in motorway conditions, at around 1200 veh/h/lane (1000-2000 veh/h/lane: 0.5 point). Draw this for a 4 and 1 lane part (0.5 point)



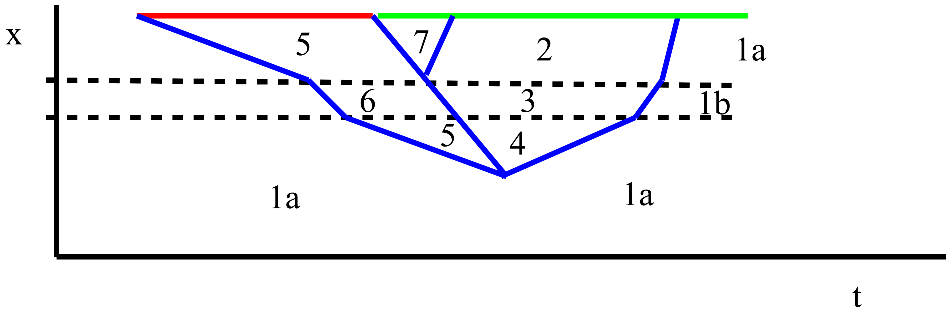
123

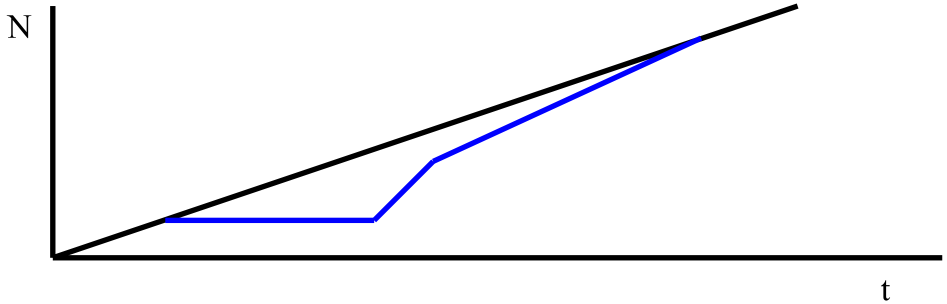


If during the green phase the queue completely dissipates, the inflow should be lower than the capacity for the one lane part (1). Starting with the traffic light in green/red: 1 point. During the red phase, the flow is zero, so we get jam density in the 3 lane part

(0.5) and the 1 lane part (0.5). Once the traffic light turns green, there is capacity outflow (state 7 - 0.5 point). Once the backward shockwave reaches the part where the crane is, the flow in the part where the crate is, is now limited to the of one lane (0.5). In the downstream part, we now have the same flow in the free flow branch (0.5). An congested traffic state with the same flow occurs upstream of the crane area (1 point) (5.5 points for the traffic states). Now, connect all traffic states (2.5 points).

124 Upstream, the inflow is equal to the flow in 1 (0.5). Downstream, the flow is zero (during the red phase)(0.5), the flow of 7 (0.5), the flow of 2 and the flow of 1 once the queue has been dissolved. (0.5)

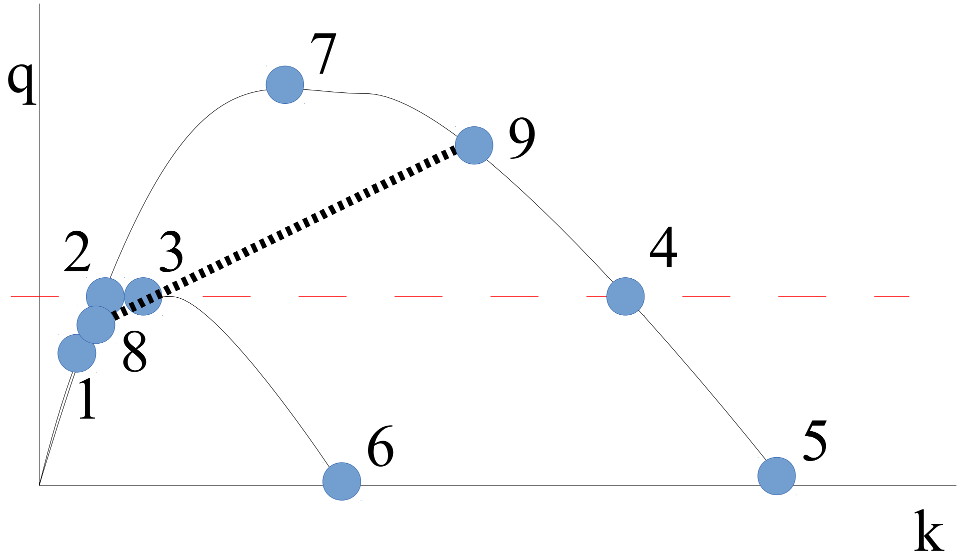


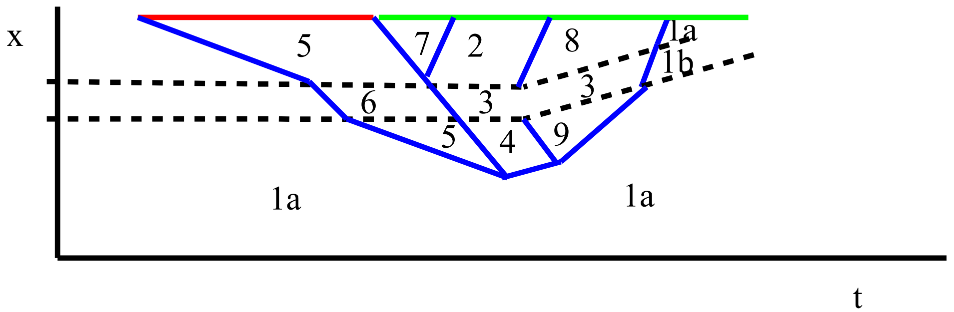


125 The delay is the area between the blue and the black line

126 If the crane is further from the intersection, state 7 lasts longer (1). Hence, the farther upstream the crane is, the longer the steep increase of the blue line (0.5) and the smaller the delay (0.5).

127 Still, the traffic operations next to the moving crane are at capacity conditions as long as there is a queue waiting. (0.5) Hence, we have point 3 (0.5). The crane forms a moving bottleneck and these are shocks which connect to the free flow and congested states on the fundamental diagram downstream and upstream (1 point for mentioning, 1 point for drawing correctly in FD). The remainder of the shocks can now be drawn (1 point for finishing)





128 The Network Fundamental Diagram describes the production (0.5) average flow (0.5) as function of the accumulation (0.5) , i.e. the number of vehicles in a network (0.5) Also possible: the performance (0.5), i.e. the outflow (0.5), as function of the accumulation. Wrong combination of terms and meaning costs 1 point.

129 NFD plot with flattened top (top: 0,5 point).

130 The NFD goes up and down, just like the regular FD, but the top is flattened (1 pt, including graph). The flat top is caused by the fact that not all roads can be operating at capacity (1)

131 Once the roads get more congested, the blocking back causes the traffic to slow down.

132 See the course slides. If the inflow is limited, the production can be higher, and hence the outflow can be higher (1). Without control, the inflow into the protected network can be higher than the outflow out of the protected network(1), thereby steadily increasing the accumulation(1). The higher the accumulation, the lower the outflow, until finally a gridlock situation occurs.

B

133 A car-following model describes how the position, speed or acceleration of a follower depends on the movement of the leader

134 The relative amount of space (in time! -0.5 if not mentioned) that a non-passenger car vehicle occupies on the road

135 The road could handle 1 passenger car per unit of time (without loss of generality, choose time unit appropriately such that this is correct) or 0.5 truck (0.5 for this reasoning, or any other leading anywhere). Now, this would be 1.05 passenger car of 1.09*0.5=0.545 truck (1). To get the pce value, one has to divide the maximum truck flow over the passenger flow (1): 1.05/0.545 = 1.92.

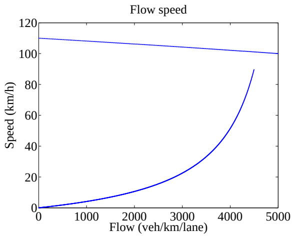
136 This means that the the solution is more exact; a platoon or stop-and-go wave will for instance spread in a Eulerian coordinate system, whereas this will stay together (resp the wave will remain sharp) in the Lagrangian coordinates.

137 Comfortable accelerations lie in the order of 1 m/s2, strong braking in the order of 6 m/s2. A threshold value to accelerate would hence be around 1 m/s2 (0.3 to 2: 1 point)

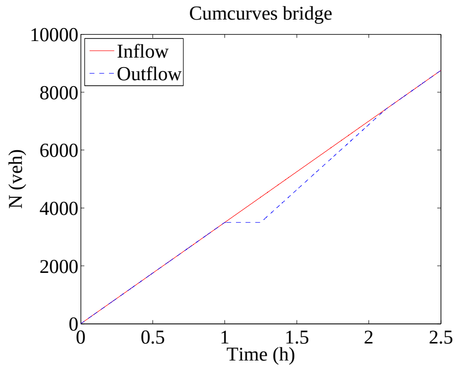
138 It is the politeness – how people value the acceleration of others. This can be assumed to be between irrelevant (value: 0) and just as important as themselves (value: 1)

139 Flow in ped/s/m (max: 0.6-2), density in ped/m2 (max: 4-12 1 point) Base for instance the flow on a 1.5s headway per lane (similar to car traffic), leading to 1/1.5=0.67 ped/s/lane. In 1 meter, at 60cm width per ped, 1,67 fit next to each other (leading to 1 or two lanes). Being partly next to each other, the lane capacity is multiplied by 1.67: cap=.97*1.67=1.1 ped/m/s

140 Capdrop: 0.5 point. Decreasing speed): 0.5 point. General shape: 0.5



141 Free flow speed (100-120 km/h), critical speed(75-95 km/h), critical density(17-30 veh/km/lane), jam density (100-200 veh/km/lane). 1 point for the densities, 1 for the speeds. 1 point for matching numbers (q=ku) at capacity (free flow / critical)



142

143 In 15 minutes 3500/4=875 vehicles arrive, which are in the queue. After the bridge opens, the queue length reduces (3500 veh/h in 4500 veh/h out)

144 See the cumulative curves. The total delay is the total area between the lines.(1) In this case, the congestion reduction phase lasts 875/(4500-3500)*60 = 52,5 min, (1) so in total congestion lasts 67,5 minutes. (0.5). This is a triangle, which surface is being determined by the height times the width times divided by two. The surface then is 67,5 min * 875 veh/ 2 /60 (min/hr) = 492 veh h. (0.5)

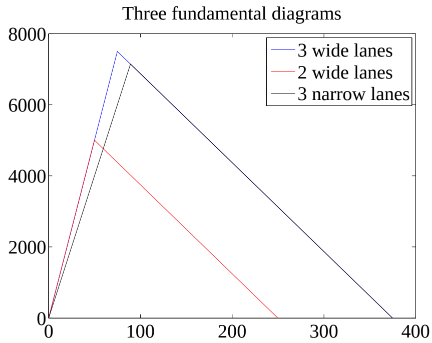
145 From one point different waves occur: a fast one to high speeds, and a backward moving one to lower speeds.

146 At the beginning there is no flow (the first vehicles need some time to arrive - 0.5) and it then gradually increases (0.5) to the capacity (1) which it will asymptotically reach (so no touching!) - 1.

147 This is downstream. The fastest wave travels at free flow speed (0.5). This is derived from the fundamental diagram (0.5). The Greenshields fundamental diagram is a parabola (0.5). To find the parameter (1 pt): the speed decreases linearly with the density. The capacity is found halfway the density range, so at 60 veh/km. The speed there can be determined by v=q/k=5000/60=83 km/h. Since the speed-density relation is linear, the speed at k=0 is twice as high: v0=83*2=166 km/h Other possibility: for the sake of calculation, flip the parabola to a form upside down, and move it to such that the top at (0,0) and increasing flow. The parameters are found by $a(60)^2 = 5000$ (0.5), so $a = 5000/3600 = 1.39 veh/h/(veh/km)^2 = 1.39 km^2/veh/h$ (0.5). The speed is the derivative (0.5), i.e. 2ax at x=60. That means v=2*1.39*60 = 167 km/h. (0.5) Note that it is unlikely that a wave will travel that fast in Dutch motorways. So the wave arrives at 200 m downstream 500/(167/3.6)=11 seconds after the bridge closes.

Because this is a hard question, the final points are divided by 2.

148 For each of the FDs one point. Note the congested branch is uninfluenced for the congested part, so is overlapping with the three lane FD

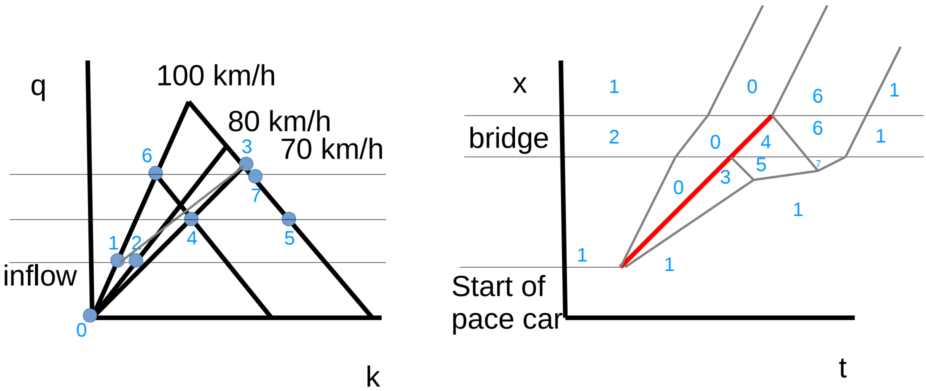


149 The question relates to: where do the free flow line for 80 kms/h and the congested line cross. (0.5) The free flow line is: q=80k (0.5). The congested line is q=100*25-25*(k-25). (0.5) Setting q equal gives kc=29.8 veh/km/lane and q=2381 veh/h/l/lane. (0.5) For three lanes this gives 7129 veh/h. (0.5)

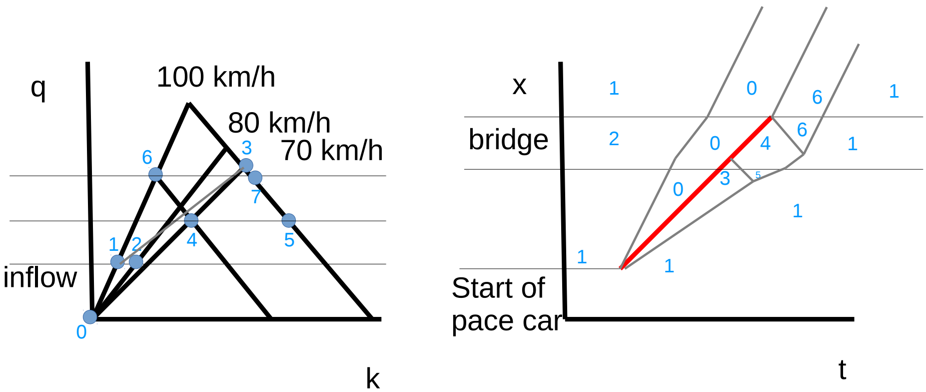
150 In busy conditions, the increased capacity causes less queuing, and hence a lower travel time (0.5). In free flow conditions, the higher speed causes lower travel times. (0.5)

151 The last vehicle downstream of the pace car needs to travel 1000 meters, where the pace car travels 700 meters. Upstream, the traffic has a speed of 100 km/h (0.5), so the time is 1000/(100/3.6)=10*3.6=36 seconds (0.5). The pace car hence has to drive 700 meter in 36 seconds (0.5), hence the speed is 700/36 * 3.6 = 70 km/h. (0.5) (the factor 3.6 is for unit conversion to km/h.

152 Traffic is in free flow conditions (0.5), at a flow of 3000 veh/h (0.5), giving a state in high speeds (state 1) (0.5) and at 80 km/h in the narrow lane section (state 2) (0.5 – lower speeds). At a certain moment, the pace car enters, driving at 70 km/h leaving a gap in front (state 0) (0.5) Upstream there will be a congested state at the fundamental diagram for three wide lanes (0.5) at 70 km/h (0.5) (state 3). The capacity for the two lane part at 70 km/h (the pace car still determines the speed) is point 4 (0.5). This is lower than the local demand, state 3. Hence, there will a congested state upstream (state 5 - 1 pt) with the same flow as the congested state in the two lane section. Once the pace car leaves the road, a flow equal to the capacity of the two lane section exits the congested state 4. The head of this tail is moving backward (just like a head of a queue once the traffic light turns green). Downstream in the 3 lane section, the same flow (capacity flow of the two lane section) is found. Then, two possibilities arise (based on your sketch – no need for exact computation): (1) backward wave gets to the end of the bridge before the forward bridge reaches it. Then, in the three lane section a congested state with the same flow as the capacity in the two lane section is arises. Possibility (2) is that the forward wave is earlier, and in that case the congestion solves from the tail. Drawing the space time diagram: 2 points: (cumulative: 9.5, divide points by 1.5 because it is a hard question)



Solution 1: backward wave gets to the end of the bridge before the forward bridge reaches it

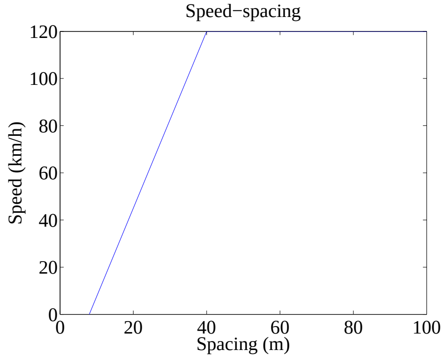


Solution 2: backward wave gets to the end of the bridge after the forward bridge reaches it

153 Synchronized flow

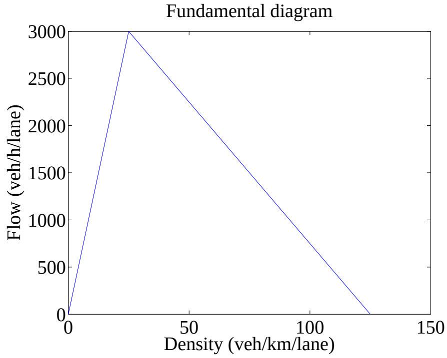
154 Wiedeman states that drivers will not react to minor speed differences if they are below a certain threshold (1). That is similar to the synchronize flow area (0.5) which states that drivers are willing to accept various time headways at different speeds (0.5)

155 Speed increases with increasing spacing (0.5). No values for smaller spacings than a critical value, and for higher spacings the speed is constant (0.5) (at the desired speed).



(1 for graph)

156 Free flow speed is slope (0.5), critical density is 1/critical spacing (0.5), jam density = 1/min spacing. (0.5)

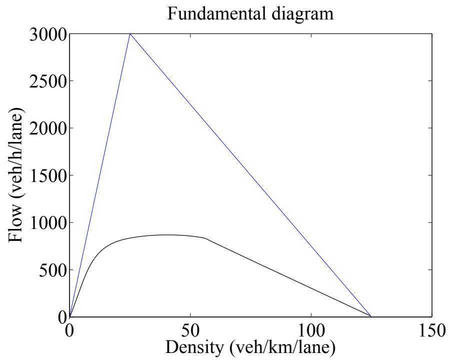


A piecewise linear speed-spacing diagram leads to a triangular fundamental diagram (0.5 – not asked in wording, but a wrong shape will get a reduction in points).

157 Production

158 Accumulation

159 (it is lower and it has a flattened top. (1) Plotting: (1). The right part is mainly theoretical, since one cannot reach that state (not needed for points).



Good reasoning as well: in a network, the speeds are lower, so the overall NFD is lower.

160 The controlled intersection causes headways to be more spread (1). In this case, the negative binomial (1) function is a good way to describe arrivals.

161 If the platoon is stable, it means traffic disturbances damp out within the platoon (1). That means that they cannot grow from platoon to platoon, which is the definition for traffic instability (1)

162 N(x,t) indicates how many vehicles have passed location x at time t

163 With calibration, one finds the best parameter settings for a model (1), whereas in validation, one checks how good this (optimized) model works for another situation (1)

164 An MFD indicates the relation between the outflow out of (or internal flows in) *an area* to the number of vehicles in that area

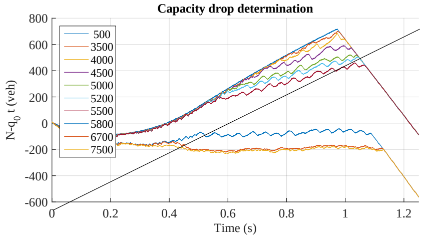
165 Capacity is given in ped/m/s

166 The capacity indicates the number of pedestrians per unit of time per unit of width.

167 (1) One has to estimate the number of vehicles in the road section at the start, (2) there is a increasing error from miscounts (3 – less of an issue) one does not have observations from the vehicles leaving or entering the road section (one has not here in this case either – therefore 0.5 point)

168 The queue starts at the bottleneck, since there is delay between x=6700 and x=5800 (there is a distance between the moved slanted cumulative curves of 6700 and 5800) (1). The delay spills back to approximately 3500m (there is still a little delay between 3500 and 4000m) (1). The congestion solves by a lower demand, seen from the demand curve (only that: 0.5 pt) and the fact that the delay at the more upstream locations solves earlier (add that: 0.5).

169 The most accurate reading is from the slanted curves.



Before congestion it has a slope of approximately (650- -400)/(1.2 - 0.2)=1050 veh/h (1). This should be added to the offset to get the flow (0.5): 1050+3900 = 5050 veh/h (0.5) – reading from the non slanted curves 1 pt. Reading the maximum flow from the highest value on the three lane section: 0.5 point (it is a flow, but not related to the capacity)

170 The difference between the flow before congestion sets in and after it has set in is the capacity drop (1). After congestion has set in, the slope is approximately 0 (0.5) (or equalling a flow of 3900 veh/h). The flow 1050 veh/h lower than before congestion, which is the value for the capacity drop (0.5)

171 Yes, the delay would differ since the delay of the vehicles leaving at the offramp would not be taken into account (1). Hence, the delay computed with a vertical queuing model would be less (1)

172 The partial densities can be calculated by k=q/v per class (1), leading to k1=500/120=4.2 veh/km and k2=500/90=5.6 veh/km (1). The total density is the sum of these two (0.5), 9.7 veh/km (0.5)

173 For the space mean speed, one can (a) average the paces or (b) Put weight on the speed values the speeds using the densities (either method: 1 point). (a) The result is 1/((1/90+1/120)/2) (0.5) = 102.8 km/h (0.5) (b) The result is 4.2 * 120+ 5.6 * 90 /(4.2+5.6) (0.5 point) = 102.8 km/h

174 (i) For the time mean speed, one observes as many slow as fast vehicles (equal flows). (1) Hence, the mean speed is (90+120)/2 (0.5 pt) =105 km/h (0.5 pt). If b is the correct answer to c and c the correct answer to b: 1 point total.

175 $k_{\text{Edie}} = \frac{\text{Total time spent}}{\text{Area in space-time}}$ (1 point)

176 Use the equation qrel=kfast*vrel (1). kfast=4.2 veh/km (0.5) and vrel=120-60=60 km/h (1). Therefore,qrel=4.2*60=250 veh/h

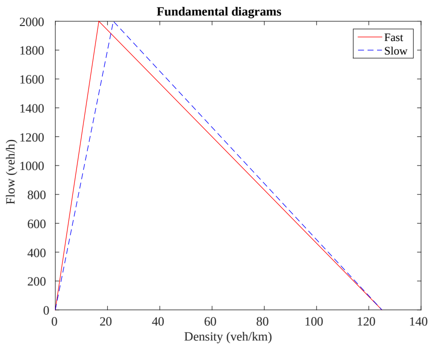
177 Calculate – in the same way as above – the flow of the slow moving vehicles: qslow=5.6*20=113 veh/h (2). Now, average the speeds:

$$v_{\text{mean}} = (q_{\text{slow}} * 80 + q_{\text{fast}} * 120)/(q_{\text{fast}} + q_{\text{slow}}) \tag{B.12}$$

(1 point) = 107.6 (0.5)

178 The flows passing a fixed observer are equal, and relative speed difference with the fast vehicles is larger. (1) Hence, using qrel=k*vrel, the flow of the fast vehicles becomes relative larger (1).

B



179
1 point for each line

180 1 since the capacities of both vehicle classes are equal

181 Newell's car-following model is described by the equation

$$x_{\text{i}}(t) = x_{\text{i-1}}(t-\tau) - s_{\text{jam}} \tag{B.13}$$

(1 point) τ can be found as 1/capacity (1) 1/2000 (/h), or 3600/2000=1.8 s (0.5). s_{jam} can be found as 1/jam density (1), or 1/125 (/km), or 1000/125=8 m (0.5)

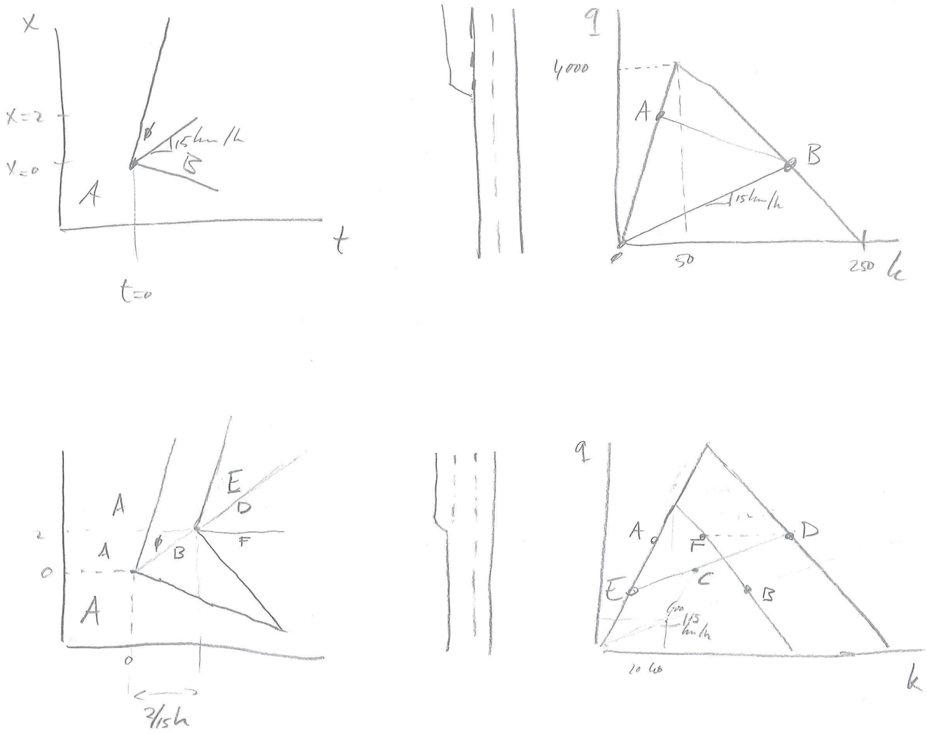
182 Slugs and Rabbits

183 Synchronised flow

184 Only the two lane part is relevant here, so draw that fundamental diagram (1 point for the FD with the correct axes (per roadway, rather than lane). The initial conditions (A) can be indicated in the FD (0.5). Now, the traffic upstream of the truck is driving at 15 km/h (0.5), hence we get a congested state at the line q=15*k, leading to B (0.5 point). The trajectory of the truck can be put in xt, driving at 15 km/h (1 point). The congested state B spills backwards with a speed of which the slope equals the slope of A-B in the fundamental diagram (1 point). Downstream of the truck, there is an empty road (0.5) and the matching shockwave between that and state A (0.5)

185 Draw the fundamental diagrams (1 point), and the capacity point C (0.5). The truck forms a bottleneck, and hence moving at 15 km/h, (0.5) through point C, there is a line with a slope of 15 km/h (0.5), connecting downstream point E (0.5) and upstream point D (0.5). Shockwaves 0-E (0.5) and ED (0.5) can now be drawn. In the two lane section, state is at the two-lane fundamental diagram: a congested state at the same flow of D –

point F (1 point). Shock wave BF at the speed of the line BF in the FD (0.5 point).



B

187 Binomial distribution

188

$$f_i = \frac{q_i}{q} = \frac{q_i}{\sum_{\text{all lanes}} q_i} = \frac{1/\langle h_i \rangle}{\sum_{\text{all lanes}} 1/\langle h_i \rangle} \tag{B.14}$$

(The step at each equal sign is 1 point)

189 Passenger car equivalent

190 The demands are equal, hence the flows of slugs and rabbits are equal (0.5). They are separated in free flow conditions (1), so the flow in both lanes is equal (0.5)

191 If a platoon is unstable (0.5) a small disturbance can grow over the platoon (0.5). If the disturbance also grows beyond the platoon into the next platoon (0.5) this is called platoon instability (0.5). A small disturbance hence can grow until vehicles come to a complete stop (0.5).

192 Breakdowns occur stochastically, and with a higher flow, the probability of a breakdown is higher. Then, there is not a single value for capacity.

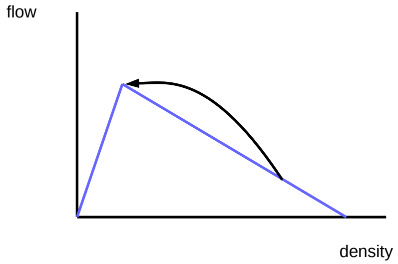
193 It means that the predicted traffic states are not exact; shock waves get spread out over space if time progresses (1)

194 Traffic must be stationary and homogeneous

195 Traffic characteristics move with the slope of the fundamental diagram. (1 pt, word characteristics not needed) Simplifying to a triangular fundamental diagram, they move either forward with the free flow speed or backward with the wave speed. A parallelogram with slopes free speed and wave speeds (1) aggregates hence a more homogeneous traffic states.

196 Edie's (generalised) definitions

197 Due to this multi-anticipation, drivers will accelerate earlier out of the congested state (1) (even before the density will go down as much as usual – i.e., the FD). Hence, the speed and thus the flow go up faster than normal (1). The end situation (capacity) is the same(1). We hence get a transition line above the congested branch (1: sketch).



198 The flow through the bottleneck is equal to the flow in the jam (1). This is found by q=ku (1). k=1000/11*3=273 (0.5 + 0.5 for the *3 part) and u=8 (0.5). Hence, q= 273*8=2182 veh/h (0.5)

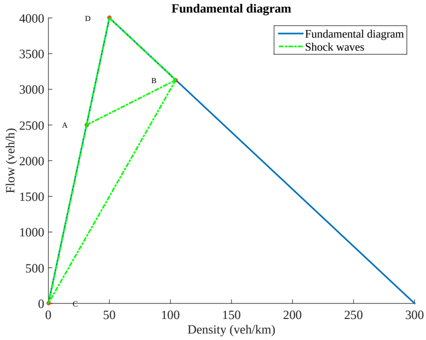
199 The capacity point for the three lane part is three times the capacity of the one lane part (0.5), which is the flow through the bottleneck (0.5). So the capacity is 3*2128=6545 veh/h (0.5). The critical density is hence 6545/cfree=65 veh/km (1). The wave speed can be found by realising the point q=2182 and k=272 lied on the FD (0.5). Now, the jam density can be found in various ways. For instance: the wave speed is dq/dk, with dq=-(6545-2182)=-4364 and dk=272-65=207 w=-21km/h. kj=kc+C/w=67+3*2182/21=375 veh/km (1)

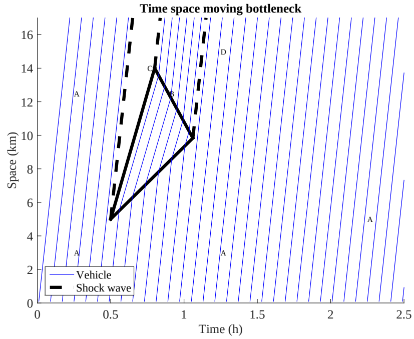
200 This question relates to the flow in the two lane section in a congested state (0.5) for a flow of 2182 (0.5). The density is exactly between the critical density (2/3*65=44 veh/km) and the jam density (2/3*375 = 250 veh/km), so at 294/2=150-3=147 veh/km. (1) The speed then is u=q/k=2182/147=15 km/h (1).

201 Since the traffic state is stationary, the shock waves must have slope zero, so the flows must be equal. (1)

202 For all stationary states, the flow is the same for all locations. Hence, the average flow is the same as the outflow. (1) This increases as in the FD up to the critical density, and then remains at that maximum (1).

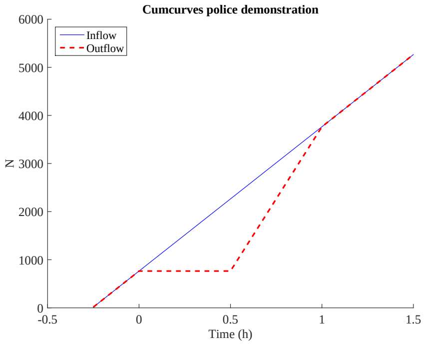
203 Sketch the FD (1). Draw a line with slope - speed - 30 km/h from the origin (0.5). This is the line where the congested state must lie, i.e. at the intersection of this line and the FD – indicate in FD (1). The original traffic situation is at a demand level higher than this flow (given) at the free flow branch (0.5). After the police cars speed up, the state returns to capacity (1). Indicate the connections in the FD (1) and in space-time (1)
The following graphs are sketches, and the units are not matching the question at hand.





204 Calculate the intersection point of the line with slope v0 and the FD (1) Description of the congested FD: q=C-C*(k-kc)/(kj-kc) (1). Fill in values, and solve:
$C - C * (k - kc)/(kj - kc) = v0k$
k=104 veh/km (1) q=v0 k (0.5)= 30*104 = 3130 veh/h (0.5)

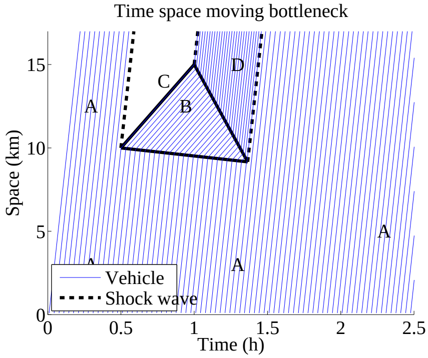
205 Inflow is constant (0.5), so a straight line with slope of demand (0.5). Outflow is equal to the inflow until the moment that the police cars enter (0.5). From that moment in time, the outflow hence slope is 0 (1). After they leave, the flow is the capacity (not asked), so steeper outflow (0.5), up to the moment they are at the same vehicle level (0.5), after which they are at the same line (0.5)

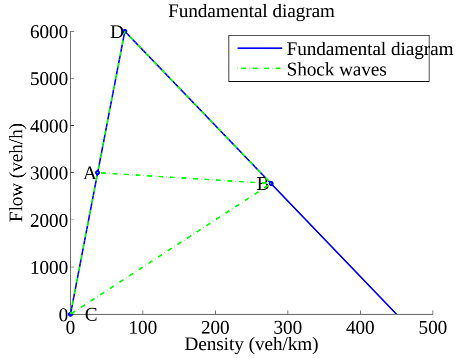


206 The delay is given by the area between the two lines in the graph above (1). The max nr of vehicles in the queue ($N_{\max}$) is $d * q\text{in}$. The time to solve is then $t_{\text{solve}} = N_{\max}/(C - q\text{in}) = d * q\text{in}/(C - q\text{in})$ (1). The area is $1/2 * N_{\max} * (d + t_{\text{solve}}$ (0.5). Replacing the terms gives$D = d * q\text{in} * (d + d * q\text{in}/(C - q\text{in})$. Further simplification (proof) leads to

207 The delay approaches infinity if the demand approaches the capacity. (1). That is because the jam will never solve if the demand is equal to the outflow (1), and all following vehicles will get a delay (1).

208 This is basically the same as the question b, with the difference that the demand is above the congested flow on the free flow branch of the FD (0.5). The segment connecting this point to the congested traffic state hence goes down (0.5), and this means the tail of the queue moves back (0.5). In graphs:





209 It does – it is based on the flows and the gaps, and they remain the same.

210 See the cumulative curves: the area is the delay (no points - awarded earlier). This increases proportional with both the number of vehicles in the queue, as well as with the delay for the first vehicle (1), which are both proportional to the duration of the demonstration (1), so in total that means a quadratic relationship (1)

211 Expected headway: 5 seconds. (1). Flow per lane: 3600/5 peds/h. (1) Flow total: 3600/5*2 = 1440 ped/h. (1)

212 In total the number of cyclists N is the area under the graph (1), calculated by

$$N = \int_0^2 q(t)dt \tag{B.15}$$

. By geometry, this equals leading to: 2 (min) * q0 *1/2 = q0 [cyclist/min] (1). This equals 300 cyclists (0.5). Therefore, q0=300 cyclist/min

213 The queue grows as long as q is larger than C (0.5), hence we need to find the number of cyclists that have passed at the moment q reduces to a value lower than C (0.5). It is (1-(C/q0)) minutes (0.5) *after the peak of the flow* (0.5) at one minute, so $t_E = (1-(C/q0)) + 1 = 2 - C/q0$. The amount of cyclist which then have passed is $N_E = \int_0^{t_E} q(t)dt$, (1) which equals (or from geometry)

$$\begin{aligned} N_E &= 1/2 * q0 + (t_E - 1) * C + 1/2 * (t_E - 1) * (q0 - C) && \text{(B.16)} \\ &= 1/2 * q0 + (1 - C/q0) * C + 1/2 * (1 - C/q0) * (q0 - C) && \text{(B.17)} \\ &= 1/2 * q0 + (1 - C/q0)(C + 1/2(q0 - C)) && \text{(B.18)} \end{aligned}$$

(1)

214 Production

215 Yes, we can. The critical density for the Greenshields fundamental diagram is halfway the density range (0.5), hence at 50 veh/km (0.5). The speed at that density is half the free flow speed (linear relationship, 0.5 pt), being 40 km/h (0.5 pt). The capacity hence is q=ku (0.5 pt) = 40*50 = 2000 veh/h (0.5 pt)

216 An acceleration fan occurs at the head of a jam. Since characteristics of the lowest density move upstream, and characteristics of highest density move downstream, the sharp shock at the head of the queue (1) will spread out (1).

217 The density should be derived from the quotient of the flow and the space mean speed (1). The time mean speed overestimates the space mean speed (1), hence the density is underestimated (1)

218 Locally stable traffic means a vehicle will reduce the amplitude of it speed variation over time; from vehicle to vehicle, the amplitude can still grow, so platoons can be unstable.

219 In a stochastic simulation package (1 point), the traffic situation is depending on a random draw. You should do multiple simulations to check to which extent the results depend on the random draw.

220 FD holds in equilibrium, hence Δv equals 0, and a equals zero. This means (first equation) that $s = s^*$. (1). The second equation gives a relation between speed and density. We can rewrite this to

$$v = (s - x_0)/T \tag{B.19}$$

(1 pt) Now substituting s=1/k gives:

$$v = (1/k - x_0)/T \tag{B.20}$$

(1 pt) Multiplying both sides with k gives the relation q=q(k)

$$vk = q = k(1/k - x_0)/T = 1/T - kx_0/T \tag{B.21}$$

(1 pt)

221 After validation, it is known to which extent the model can be applied in other conditions and what it's quality is

222 Since in the congested branch (equation above) q decreases with k, the capacity is found at the intersection of the congested branch and the free flow speed. We hence have to solve

$$v = q/k = k(1/k - x_0)/T/k = v_f = (1/k - x_0)/T \tag{B.22}$$

(1) Solving this to k gives

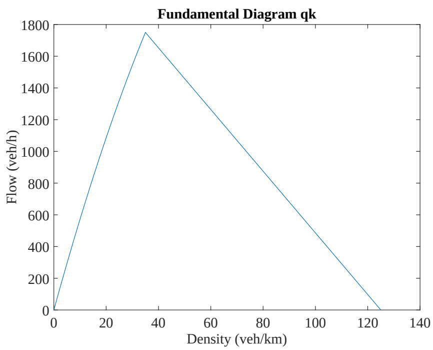
$$k = 1/(Tv_f + x_0) \tag{B.23}$$

(1) Using the parameters, we find k=1/(1*30+20)=1/50 veh/m. (0.5) Computing q from this gives q=kv=1/50 * 30 = 30/50 =3/5 veh/s (1) = 2160 veh/h (0.5)

223 Consider the FD in speed-density. Its form does not change, only the distance changes hence the values at the density axis are halved (0.5 pt). Moreover, the speed reduces to half, hence the values on the speed axis are also halved (0.5 pt). The effect on capacity is that the product of density and speed (1 point) is half times half = 1 quarter of the original capacity (1 point). The capacity is hence 2160/4=1080/2=540 veh/h

224 Smulders fundamental diagram

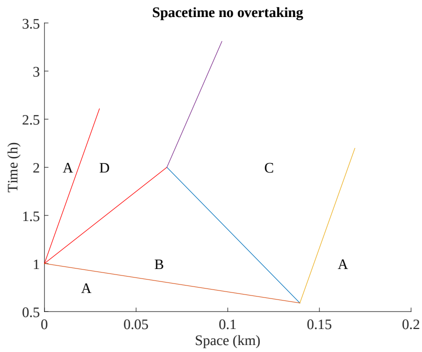
225 Capacity is at the point where the congested branch starts (0.5); reading out the graph gives 35 veh/km and 50 km/h (0.5), so a capacity of (q=ku, 0.5 point), 35*50 = 1750 veh/h

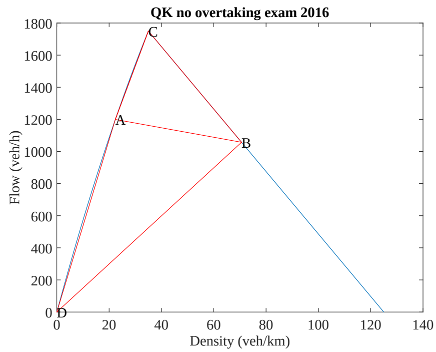


226
There is a slight curvature in the free flow branch, and a straight congested branch

227 This is the traffic state at 15 km/h (0.5), for which the FDs can be used (0.5). Hence, read out the FD at 15 km/h (0.5), yielding a density of approximately 70,5 veh/km (0.5). Using q=ku (0.5), this gives a flow of approximately 1060 veh/h

228 First, find the traffic state upstream of the moving bottleneck, which is found by constructing the intersection point of the line q=15k and the fundamental diagram (1 point, point B). Downstream of the MB is an empty road (state D, 1 point). Inflow point is at the free flow branch (0.5 point), and after the MB leaves the road, the state is at capacity (1 point).
Constructing the space time: start with the MB (1 point), construct AB, AC and CB (1 point), finishing 0.5 point.





229 A driver will change lanes whenever the utility for changing lanes is higher then staying in the current lane (1). The utility consists of the accelerations (no points, mentioned), for the driver itself and for the vehicles directly influenced (n and o) (1), where the latter two are discounted using a politeness factor $\mathscr{P}$ (1)

230 The values are accelerations so should be given in m/s^2 (1 point). a_{bias} should be higher than a_{th}, otherwise no lane change to the right. Accelerations in the order of 1 m/s^2 are sizeable (i.e., quite some throttle opening), so values should be in the order of one thenth of that. Values between 0.01 and 1 are given 1 point (Treiber and Kesting propose 0.3 and 0.1 m/s^2 for a_{bias} and a_{th} respectively.)

231 Poisson

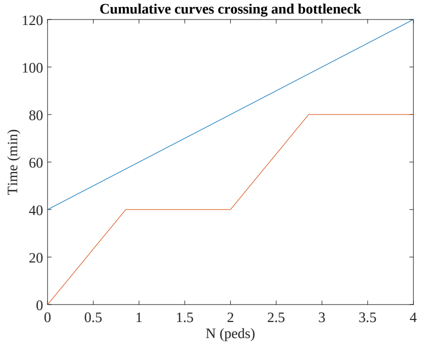
232 1200 peds/h = 1200/60 peds/min = 20 peds/min. In 120 seconds = 2 mins 2*20 = 40 peds arrive (and cross) (1 point for flow times cycle time, 0.5 point for unit conversions, 0.5 point for end answer)

233 Flow in peds/m/s and density in peds/m^2 (1 point for each)

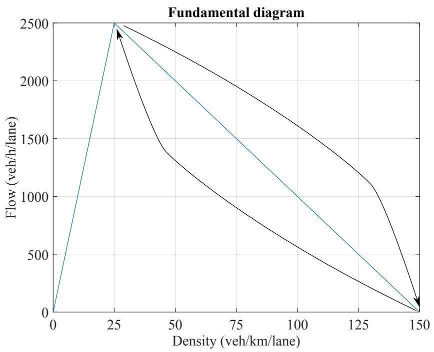
234 The capacity (read from graph) is 1.22 peds/m/s (1 point for using the capacity value), so the capacity for the bottleneck is 0.6*1.22 = 0.732 peds/s. (1). It will hence take 40 peds/ 0.732 peds/s (0.5) = 54 seconds (0.5)to pass the bottleneck.

B

235 Starting with the hint: at the start, 40 pedestrians have crossed A and not B (see question b). (0.5) So start the curves with 40 pedestrians difference (0.5). Then, the inflow steadily increases with 1200 peds/h, or 40 peds/cycle (1). For the outflow, the queue discharges at 0.732 peds/s up to the moment that 40 pedestrians have exited, and remains constant then (1). (Indeed, by that time other pedestrians are waiting at the traffic light again, so the lines do not touch.

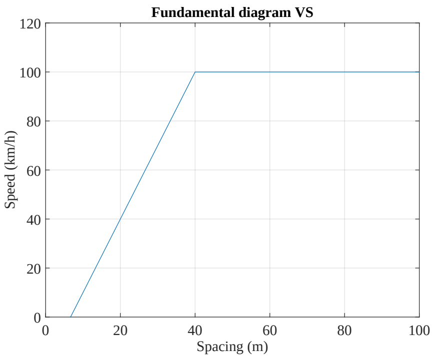


236 Consider for instance an acceleration from low speeds to high speeds. Traffic starts right bottom, and then the density decreases. However, since the speed of the vehicle is not adapted, it is still low, so it curves counter-clockwise to the top. (1) The other way round, under deceleration, consider a point near the top. Then, first the density increases (traffic states moves to the right), and the speed is still higher than for the newly increased density. Hence, the traffic state moves counter-clockwise down.(0.5 for a second reasoning)



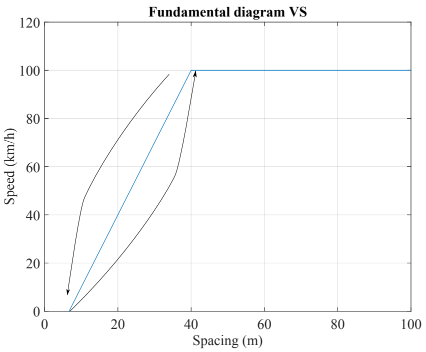
237 Triangular, so the free flow speed is fixed (at 2500/25=100 km/h). Straight line in congested branch means q and k are linear, say form q=1/r-wk. Divide both sides by k, and use substitution s=1/k: q/k=v=1/k 1/r-w = 1/r s - w, so also v and s are linear. From the fundamental diagram, find 1/r= 3000 veh/h (point where the congested line would intersect the y-axis) and w= 2500/125=20km/h. (Also accepted: triangular fundamental diagram in qk leads to a piecewise linear fundamental diagram in vs, with smin=1/kjam, scrit=1/kcrit and vmax=qcrit/kcrit). Plotting the minimum of congested speed and free flow speed gives:

B



(1pt)

238



239 Poisson distribution

240 Every truck counts for two vehicles in determining the capacity.

241 If we relate it to time headways (1) of a passenger car without queue warning systems, we get for cars a pce=1/1.05 and for trucks 2/1.09. (1). The new pce value is the ratio of the two (0.5): 2/1.09/(1/1.05)=1.05/1.09*2=1.92 (0.5)

242 It means that the edge of a traffic jam flattens (contrary to predicted with shockwave theory)(1)

243 It means that after a braking manouever of a leader, a follower will not find the same speed, but keeps continuing adjusting speed (0.5) in an increasing amplitude, i.e. going to higher and lower speed for every iteration (0.5).

244 Yes, if the inflow is equal to the flow in the queue behind the moving bottleneck, the tail remains stationary

245 This is not possible. For Greenshields, the critical density is half the jam density (1). Since this is not the case here, we cannot draw. (1)
A linear FD in VK is awarded 1/2 point.

246 Stop-and-go waves travel in the opposite direction of traffic, so traffic is moving top-down. No point rewarded if the tail of the queue of the accident is indicated.

B

247 Given one specific traffic jam and its slope (choose one, 1 point), we find the length of the shown road is around the distance travelled in 2/3 of an hour. Since stop-and-go waves travel with approximately 18 km/h, this equals or 12 km (1).

248 The figure shows an accident at the road at the second tick from the bottom. This is not the fixed bottleneck. The fixed bottleneck is located at around the first tick from the bottom. The capacity drop would be visible at a location just downstream of the fixed bottleneck (0.5). There, the flow would be higher before congestion has set than after congestion has set in (1). In the current graph, this is not visible. (0.5)

249 Rabbits take the lane where the speed is the highest.

250 The density is equal (from question,0.5), and the flow is proportional to density times speed (0.5), i.e. a normalized flow of 80 of slugs and 100 of rabbits(0.5). That means 100/180 = 5/9 of the flow is rabbits(0.5).

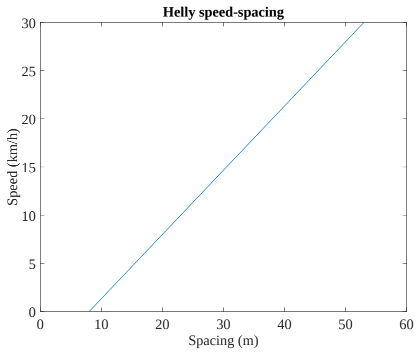
251 Weighted average speed proportional to the flows (1): 5/9*100+4/9*80=91 km/h. Computing space mean speed, or assuming flows are equal gives no points.

252 Edie's generalised definitions suggest that the speed is the total distance covered divided by the total time spent. TTD is the sum of all distance covered by all vehicles within a predefined area in space-time; TTS is the sum of all time spent of all vehicles within a predefined area in space-time.
Only the equations do not give any points.

253 The model prescribes an acceleration based on a speed difference (first term, sensitivity α) (1) and a difference between the spacing $(x_{i-1}(t-\tau)-x_i(t-\tau)$ and a desired spacing $(s_0+Tv_i(t-\tau))$ (second term, sensitivity β.) (1) The desired spacing is linearly increasing with the speed of the vehicle.

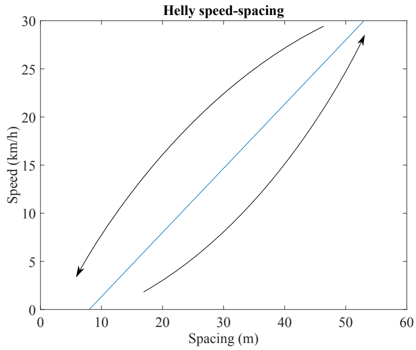
254 FD is homogeneous and stationary conditions (0.5), meaning $v_i = v_{i-1} = u$ (homogeneous, 0.5), and a=0 (stationary: 0.5). Hence, $x_{i-1}(t-\tau)-x_i(t-\tau)-s_0-Tu$ (0.5 – 2 points up to now). Spacing can be obtained by $s(t-\tau) = x_{i-1}(t-\tau)-x_i(t-\tau)$ (0.5) and stationarity $s(t) = s(t-\tau) = s$ (0.5). Then, $a = 0 = 0+\beta(s-s_0-Tu)$, or $u = (s-s_0)/T$. (1)
Plotting gives a linear increase of u as function of spacing

Explicitly making the assumptions is essential. A non-motivated sketch in vt gives no points; starting from $u = (s-s_0)/T$ and plotting that will provide 1 point in total.



255 The delay τ causes a delayed reaction. If the leader accelerates (going right in the spacing), the follower goes later up in speed (1); similar reasoning for decelerating (0.5), so we obtain a anti-clockwise hysteresis loop (0.5).

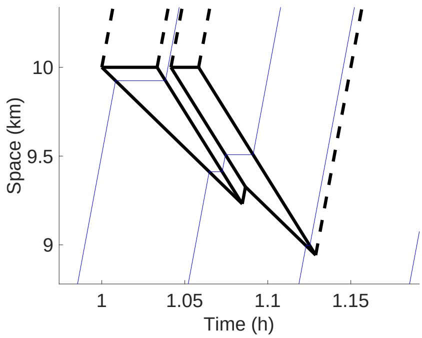
Plotting arrows based on an assumption delayed/anticipation, or providing both will give 1 point.

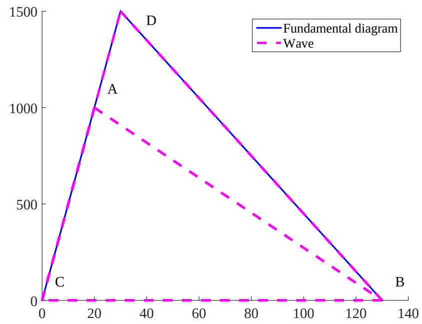


256 Starting from $s = s_0 + Tu$, we apply $s = 1/k$ (0.5) and $u = q/k$ (0.5). We obtain $1/k = s_0 + Tq/k$, rewritten to $1 = ks_0 + Tq$, or $q = (1 - ks_0)/T$ (1).

258 The critical density is 1500/50=30veh/km (0.5); The jam density is 1500/15=100 veh/km higher than the critical density (1), hence the jam density is 30+100=130 veh/km (0.5)

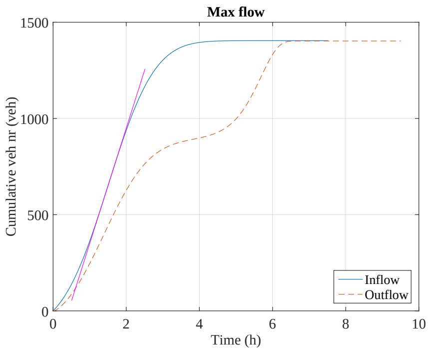
259 Indicate the inflow state in FD (A, 0.5). Then, the first light causes an empty congested state downstream (state C, 0.5) and a state at jam density upstream (B, 0.5). This creates three waves: AB, AC, BC (0.5 each; 1.5 total). When the traffic light turns green, a capacity state D (0.5) emerges, with shock waves CD and BD (0.5 point each – total so far: 4.5). The next red phase causes again states B (0.5) and C (0.5). However, this now gives waves BD (0.5) and CD (0.5). When waves AB and BD of the first traffic light join, wave AD emerges, up to the moment that this intersects with BD (0.5). Then, a wave AB continues (1). Finishing goes in the same way as finishing the first red phase: state D, waves CD, BD (0.5). When wave DB intersects with BD, the wave AD occurs and moves forward (0.5).

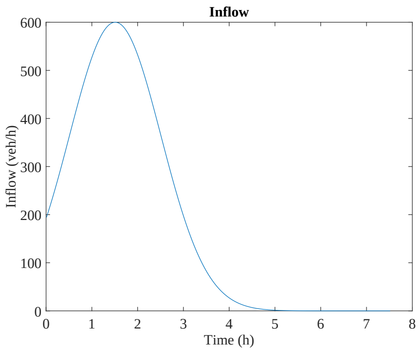




For this answer, the graphs have been constructed automatically. In your answer, provide the reasoning as above. Moreover, place letters in within the space-time diagram. The dotted shock waves should be drawn just as the others. Trajectories are not needed (not part of the question)

260 The flow is the slope of the cumulative inflow, which first increases (0.5) and then decreases, see graph above (0.5). The maximum flow is the highest slope, estimated at 600 veh/h (0.5).
Half a point reduction if the line does not increase at the beginning.

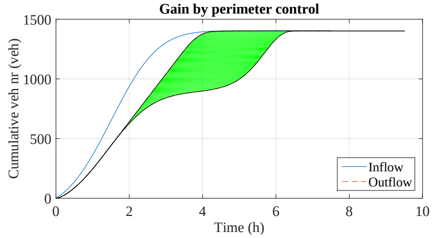




261 The outflow curve increases when traffic starts to accumulate (1). The slope of the line is the outflow, given by the accumulation. If the accumulation increases to values over capacity, the outflow decreases, yielding a flatter outflow curve (1). Only when the inflow reduces to values under the low outflow, the accumulation decreases and the outflow increases again (1).

262 By limiting the inflow and not exceeding the critical accumulation, an outflow of 600 veh can be maintained (1); the remaining queuing is outside the network (1).

263 Whenever there are too many vehicles in the network, we assume they can be queued outside the network (perimeter control), and the outflow will be 400 veh/h, i.e. the capacity outflow from the MFD (0.5). The cumulative outflow under perimeter control hence keeps increasing with 400 veh/h whenever there is an excess number of vehicles between cumulative inflow and outflow(0.5). The amount of travel time gained is the area between the old and new outflow curve, indicated in green here.



264 No, he can be moving in the y-direction

265 The SFM describes for individual pedestrians (hence, microscopic, 1) how they move. A "force" changes their direction: they are attracted by their destination (1), and repelled by obstacles (0.5) and other pedestrians (0.5).

266 In a cellular automaton model, the space is discretized (1).

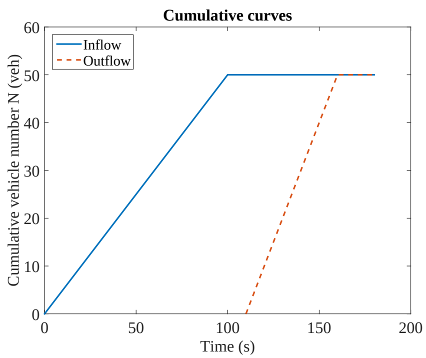
267 1.22 peds/m/s. Unit, 0.5 point. Value between 0.6 and 2: 0.5 point.

268 One can only see the top they all operate at capacity. (0.5) Whereas this is theoretically possible, in practice this will not happen (0.5). [Additional/alternative: it can only happen if all roads have the same FD, which is unlikely to be the case (1) – 1 point max]

269 A characteristic indicate a traffic state. At one moment in time, a location can only have one traffic state, hence characteristics of different traffic states cannot cross. (1) For the same traffic state: they have the same speed (because fully determined by the state), hence they do not get to each other; moreover, once they are at the same location, they will leave as one at one speed (because fully determined by the traffic state)(1).

270 The number of arriving vehicles in a period of time (1 point; no points for headways; no points for mentioning independent process, or "independent arrivals", since this was given in the intro).

271



272 The delay is the area between the two lines (1).

273 Shockwave theory adds a spatial extent, but this is not relevant(1), hence the delay is the same(1). (In case of a non-triangular fundamental diagram, delays would increase since the speeds would remain low – this goes beyond the required answer.)

274 The faster cyclists are observed more frequently (0.5), and hence have a higher weight (0.5). Hence, the time mean speed is higher (1)

275 We would need to compute total travelled distance (TTD) in an area of space-time and total time spent (TTS); both should be divided by the area in space time, and then the quotient should be taken (1). We consider the full track (asked), and a time T (one could for simplicity take 1 hour). Assume n cyclists (or for simplicity 100 cyclists). (0.5) Since the quotient should be taken, the area in space-time drops out and is is not needed to compute the area and: v=TTD/TTS. (0.5)

$$TTS = n * T \tag{B.24}$$

(there are n cyclists each present during the full time T, 0.5 point)

$$TTD = 0.8 * n * 15 * T + 0.2 * n * 25 * T \tag{B.25}$$

(There are 0.8*n cyclist on a normal bike each travelling 15 T in time T, and 0.2*n cyclists each travelling 25 T in time T, 1 point). Now, we compute: $v = TTD/TTS = 0.8 * 15 + 0.2 * 25 = 17$ km/h (0.5) This is indeed the same as the space mean speed, and could be obtained directly. That was not the question, and hence would receive no points.

276 The most downstream vehicle has the lowest number

277 The vehicles *react* on a leader, hence the information comes later to vehicles further upstream in the platoon (1). Hence, red=1, blue=2, and yellow=3. (1 for correct numbers, 0.5 subtracted for a mistake)

278 Changes in speed dampen out quite nicely, so there is local stability

279 The speed reductions of the first vehicle are exaggerated by the following vehicles, e.g. at the first speed decrease, the first vehicle reduces speed to around 10.8 m/s and the last vehicle to 10.2 m/s. (1) If this is the case, this is called platoon unstable (1).

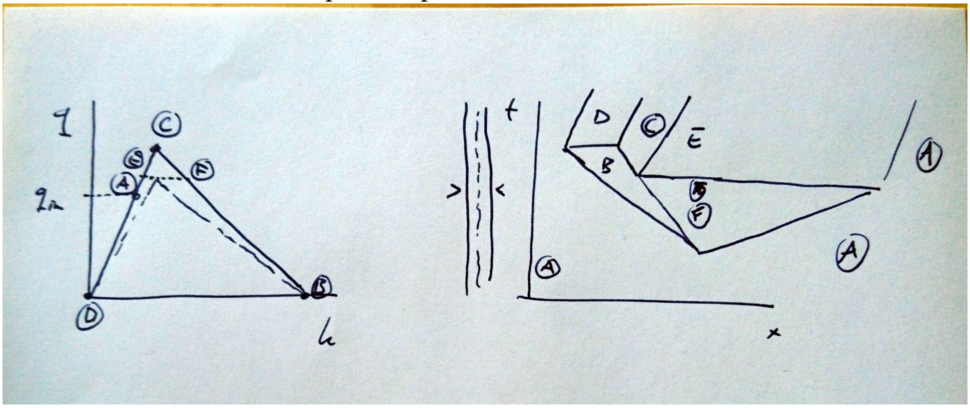
280 A desired distance is s^*, computed increasing (linearly) with speed v. (1) The acceleration is a linear combination of the speed difference of the leader and the difference of spacing and desired spacing, (1) each with its own sensitivity parameter

281 τ can be compared to a reaction time, of around 1 second (0-2 seconds: 1 point; one might also argue about anticipation which reduces reaction time).

282 The FD is based found in equilibrium, hence Δv equals 0, and acceleration equals 0, so γ and τ is irrelevant (1). The FD is hence determined by $s = s_0 + Tv$ (0.5); so with parameters s_0 and T. (0.5) (s and v are the variables, and no parameters)

283 The jam density stays the same, hence s_0 is the same (1). The congested branch is lower, hence T changes.(1)

284 Draw the FDs and the xt plane.(0 points).

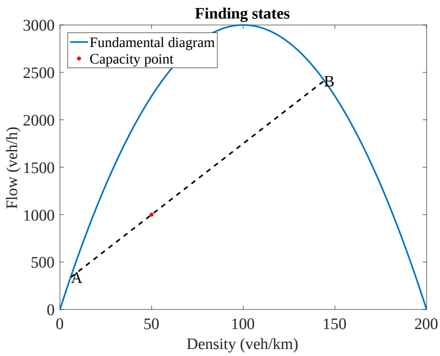


Consider an inflow lower than the capacity of road (no congestion, given), leading to state A on the free flow branch (1). A road block will lead to empty state D downstream (0.5) and jam density upstream, state B (0.5). Waves AD (0.5) and AB (0.5) follow the speeds of the connections in the FD. Once the incident is removed, there is capacity outflow (0.5), with wave DC (0.5). When DC reaches the bridge, no more outflow is possible then the capacity of the bridge, leading to state E downstream (1) and F upstream (1). Waves CE (0.5), EF (0.5) and AF (0.5) can now be constructed, followed by EA (0.5)

285 It suffices: Greenshields assumes a linear relation between speed and flow, so with two given points it is fully determined (1). The capacity is found halfway the density range (1 for knowing or deriving). The speed is half the free flow speed, so the capacity is 30*100=3000 veh/h. (60*1000, ignoring the speed reduction would not give the third point.)

286 Any convex FD will do... We can draw exactly, see next question. Not curved FD: -0.5 point.

287 The fundamental diagram is as follows:



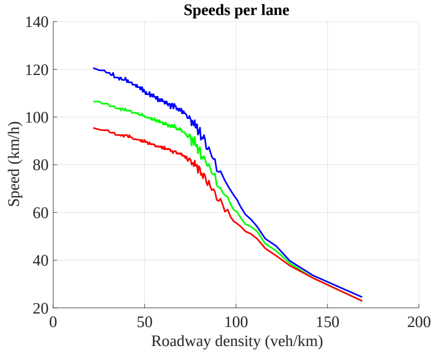
(1 point)

One can draw a line through the capacity point with a speed equal to the speed of the moving bottleneck (1 point). This connects the (free flow) state downstream of the moving bottleneck (state A) and the (congested) state B upstream of the moving bottleneck.

B

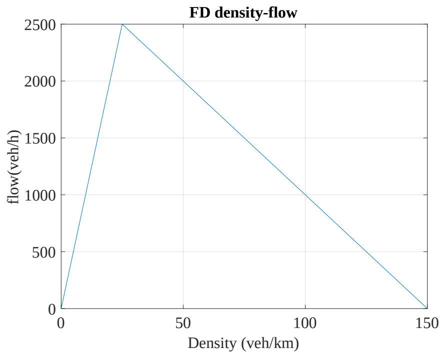
288 The blue line starts from 0 at the low densities, which is the left lane (keep right rule prevents people from using the left lane if not needed) (1). The red line is the right lane, since it starts (like the green one) high, but reduces more quickly and to lower flows than the green one (1).

289 The speeds differ per lane, see the fundamental diagrams:



Speeds in the left lane are highest(1), so the graph with the highest fraction in the left lane is the flow (1), so the left figure is the flow (1). Similar reasoning (but the other way around) for right lane.

290 Given is that the fundamental diagram is triangular. The free flow branch is a straight line with a slope of the free speed, 100 km/h, (0.5) up to the maximum flow (2500 veh/h), yielding the critical density of 25 veh/km. (0.5). The congested branch is a straight line down (0.5), hence one point on the congested branch suffices (0.5). We read a flow of 2000 veh/h and a speed of 40 km/h, yielding a density of 2000/40=50 veh/km (0.5). The fundamental diagram is finilized by drawing a line through the capacity point and the found point (0.5)



291 We take the minimum of demand of cell 2 and supply of cell 3. Cell 2 is undercritical, and has a demand read from the fundamental diagram at k=20 veh/km (0.5), being

2000 veh/h. Cell 3 is overcritical and has a supply read from the fundamental diagram at k=100 veh/km (0.5), being 1000 veh/h. The flow is determined by the minimum, here the supply at 1000 veh/h. (1)

292 We take the minimum of demand of cell 3 and supply of cell 4. Cell 3 is overcritical, hence demand equals capacity, 2500 veh/h (1). Cell 4 is undercritical, hence supply equals capacity, 2500 veh/h (1). The minimum of both is the flow (0.5), hence 2500 veh/h (0.5).

C

MATLAB CODE FOR CREATING (SLANTED) CUMULATIVE CURVES

```
function cumcurves()
%This function will give the cumulative curves for a flow which
%has (in time) three values, and one bottleneck halfway
q0=[3600;5000;2000];%the three demands
Tchange=[60;90];%times in minutes at which the demands change
c=4000;%capacity
T=0:200;%minutes
dt=1/60;%time steps (in hours: time step is 1 min)
dem=q0(end)*ones(size(T));%pre-allocate demand function to the last ...
    demand value
for(i=numel(Tchange):-1:1)
    dem(T<Tchange(i))=q0(i);%adapt the demand function
end

figure;
plot(dem,'linewidth',2)
hold on
plot(repmat(c,size(dem)),'r--','linewidth',3)
ylim([0 6000])
legend('Demand','Capacity','location','Northeast')
ylabel('Flow (veh/h)')
xlabel('Time (min)')
exportfig('Demand')
%%

Nin=dt*cumsum(dem);
qout=zeros(size(Nin));
qout(1)=dem(1);%in veh/h
queued=zeros(size(dem));
for(t=2:numel(T))
    qout(t)=1/dt*min(dt*c, dt*dem(t)+queued(t-1));
    queued(t)=queued(t-1)+dt*dem(t)-dt*qout(t);
```

```
end
Nout=dt*cumsum(qout);
figure;plot(Nin,'linewidth',2,'color',[0.5 0.5 1]);hold ...
    on;plot(Nout,'r--','linewidth',3)
legend('N_{in}','N_{out}','location','Northwest')
ylabel('Cumulative flow')
xlabel('Time (min)')
exportfig('Cumulative curves')
%%

%compute total delay:
TotalD=dt*sum(queued)%then the total delay in hours
NrVeh=Nin(end);
AvgDelay=TotalD/NrVeh%then the total delay in hours
AvgDelayMin=60*AvgDelay;%then the total delay in hours

%%
Tin=interp1(Nin,T,1:NrVeh);%time to enter for each vehicle -- ...
    interpolation
Tout=interp1(Nout,T,1:NrVeh);%time to exit for each vehicle -- ...
    interpolation
DT=Tout-Tin;%additional travel time
figure;
plot(1:NrVeh,DT,'linewidth',2)
xlabel('Vehicle number')
ylabel('Delay (min)')
exportfig('Delay per vehicle')

figure;
plot(Tin,DT,'linewidth',2)
xlabel('Entry time (min)')
ylabel('Delay (min)')
exportfig('Delay as function of time')
```

Index

Traffic Flow Theory: An introduction with exercises

Victor L. Knoop

Traffic processes cause several problems in the world. Traffic delay, pollution are some of it. They can be solved with the right road design or traffic management (control) measure. Before implementing these designs of measures, though, their effect could be tested. To this end, knowledge of traffic flow theory is needed.

This book teaches students in the field of traffic flow theory. It takes an approach from physical phenomena, focussing on observable and tangable quantities. It uses examples and storytelling to educate future traffic engineers. It also contains over 250 example questions on the study material, allowing students to practice.

Dr. Victor L. Knoop
TU Delft | Civil Engineering

Dr. Victor Knoop is associate professor at the Department of Transport & Planning. He has a background in flows, with a master degree in physics. Since his PhD his main research interest lies in traffic dynamics. His research focuses on how driver movements create effects at the level of a traffic stream.

He is author of over 200 scientific publications in the field of Traffic Flow Theory. He is a very enthousiastic lecturer, and has taught courses on Traffic Flow Theory for over a decade.

TU Delft

ISBN 978-94-6366-378-6
DOI https://doi.org/10.5074/t.2021.002

textbooks.open.tudelft.nl