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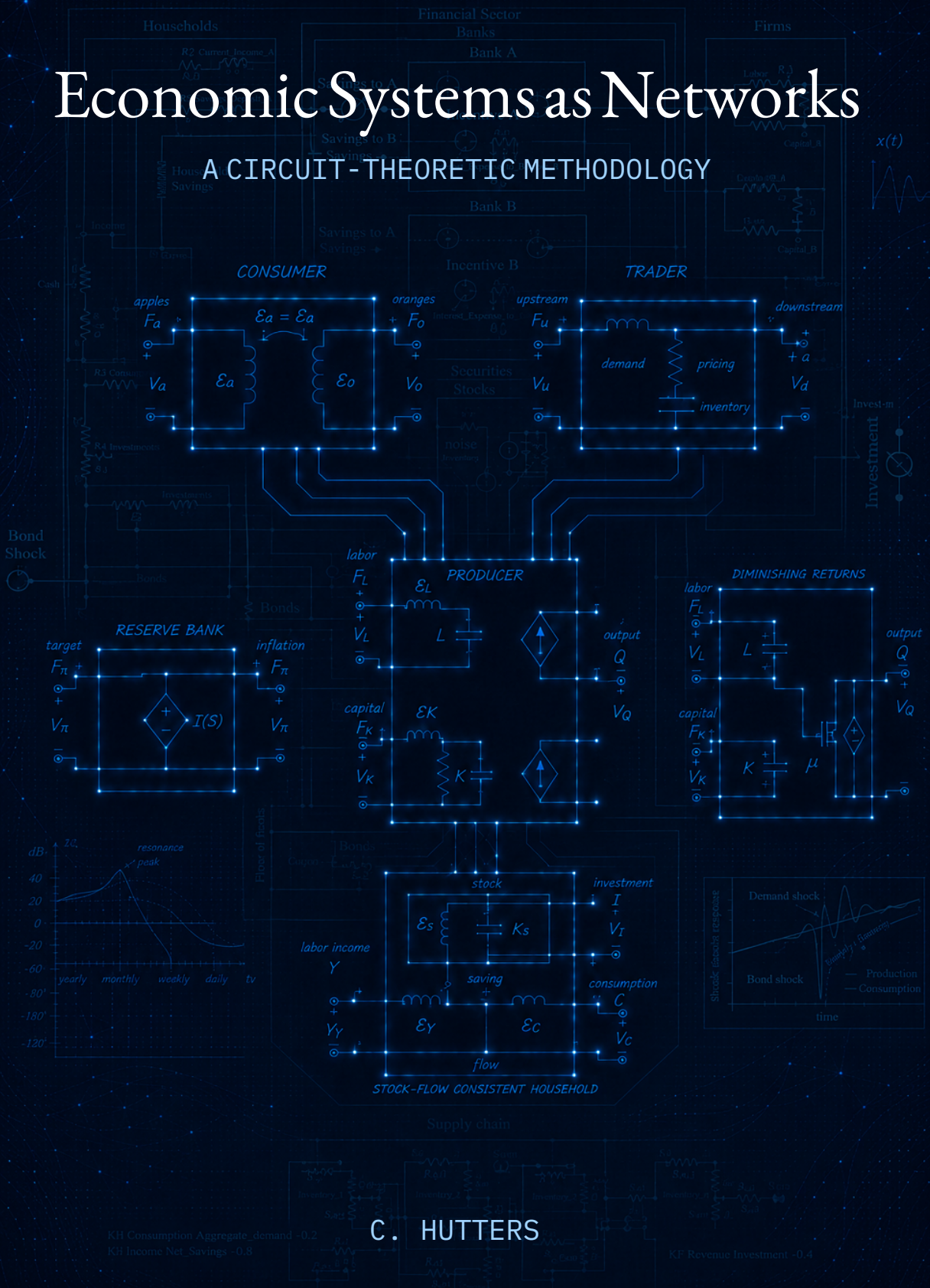
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Economic Systems as Networks

A CIRCUIT-THEORETIC METHODOLOGY



Economic Systems as Networks: A Circuit-Theoretic Methodology

Economic Systems as Networks: A Circuit-Theoretic Methodology

Dissertation

for the purpose of obtaining the degree of doctor
at Delft University of Technology
by the authority of the Rector Magnificus, prof. dr. ir. H. Bijl,
chair of the Board for Doctorates
to be defended publicly on Thursday 25, June 2026 at 15:00

by

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Summary

This dissertation develops a circuit-theoretic methodology for modeling economic systems grounded in first principles. Like electrical circuits, economic systems can be understood as networks consisting of many interacting parts whose collective behavior emerges from their interactions. Circuit theory translates physical first principles into standard components, topological rules, and graphical representations for modeling complex dynamical systems. This dissertation applies these principles to economics by developing a methodology in which economic agents are represented by electrical components and economic systems are constructed as networks of these components.

The theoretical foundation of the methodology is an economic-engineering analogy, which describes economic dynamics in terms of mechanical behavior. Since mechanical and electrical systems can be represented through equivalent analogies, this dissertation adopts an equivalent electrical representation of the economic-engineering theory. This allows complex networks of interacting economic agents to be modeled and analyzed using the principles of circuit theory and to be represented by analogous circuit diagrams. The advantage of this electrical representation is that circuit theory provides scalable tools for constructing, simulating, and analyzing interconnected dynamical systems. These tools are leveraged in this dissertation to develop the proposed modeling methodology.

The methodology is developed in two main steps. The first step introduces an economic circuit theory. Economic agents are modeled using generalized circuit elements that satisfy constitutive economic relations analogous to the laws governing resistors, inductors, and capacitors. Economic interactions are represented as network connections through which goods, analogous to currents, and incentives, analogous to voltages, are exchanged. Circuit diagrams provide both graphical and computational representations of economic systems, allowing standard circuit simulation tools such as LTspice to be used directly to simulate economic dynamics.

The second step focuses on scalability. While economic circuit theory provides a systematic way to describe interactions, large and highly interconnected economic systems quickly become difficult to manage when modeled only with elementary circuit elements. To address this, the dissertation extends economic circuit theory into a multiport network methodology. This allows complex economic systems to be constructed from modular subsystems with well-defined interfaces, while preserving interpretability and scalability.

The dissertation demonstrates the applicability of the methodology through several examples, ranging from textbook models to contemporary practical problems. A Robinson–Crusoe economy illustrates how classical microeconomic reasoning can be represented and simulated using circuit diagrams. A supply chain model shows

how frequency-domain analysis, central to engineering practice, reveals resonance and oscillatory effects in inventory behavior. An electricity market with storage demonstrates how structural changes in a market alter system dynamics. Finally, a modular macroeconomic model illustrates how the methodology can scale to systems with many interacting sectors, while preserving interpretability and allowing shocks to be traced through the network.

Beyond the dissertation, the methodology has already been applied in a range of MSc theses across domains such as energy markets, industrial competition, financial planning, and macroeconomic modeling. These applications highlight the accessibility of the methodology, particularly for students with an engineering background, and its potential to become a practical tool for research, teaching, and policy analysis.

The methodology introduces several elements from engineering modeling practice into economics: explicit dynamics, modular construction, graphical representation, and scalable system analysis. Because the models can be drawn as circuit-style diagrams and executed in standard simulation environments such as LTspice, the methodology makes it possible to explore economic dynamics computationally using established engineering tools. Taken together, these elements provide a systematic way to examine how economic behavior emerges from agent interactions and how system-wide dynamics are shaped by the underlying network structure.

Samenvatting

Dit proefschrift ontwikkelt een methodologie voor het modelleren van economische systemen op basis van elektrische netwerkanalyse, gefundeerd op grondbeginselen. Net als elektrische schakelingen kunnen economische systemen worden opgevat als netwerken van vele interacterende onderdelen, waarbij het gedrag van het geheel voortkomt uit de interacties tussen deze onderdelen. In de elektrotechniek worden fysieke grondbeginselen vertaald naar standaardcomponenten, topologische wetmatigheden en schematische weergaven voor het modelleren van complexe dynamische systemen. Dit proefschrift past deze principes toe op de economie door een methodologie te ontwikkelen waarin economische agenten worden weergegeven als elektrische componenten en economische systemen worden opgebouwd als schakelingen van deze componenten.

De theoretische basis van de methodologie is een analogie tussen de economie en de mechanica, waarin economische dynamiek wordt begrepen aan de hand van mechanische principes. Omdat mechanische systemen ook elektrisch kunnen worden weergegeven, kan deze analogie worden omgezet naar een elektrische representatie waarmee het mogelijk wordt om complexe netwerken van economische agenten te modelleren en weer te geven aan de hand van schakelschema's en te analyseren met behulp van netwerkanalyse. Omdat netwerkanalyse is ontwikkeld om het ontwerp en de analyse van elektrische systemen schaalbaar te maken, vormt zij een geschikt uitgangspunt voor de hier ontwikkelde modelleringsmethodologie.

De methodologie wordt in twee stappen ontwikkeld. De eerste stap introduceert een economische netwerktheorie. Binnen deze theorie worden economische agenten gemodelleerd met behulp van gegeneraliseerde componenten die voldoen aan economische wetmatigheden. Deze wetmatigheden zijn analoog aan de wetten die gelden voor weerstanden, spoelen en condensatoren. Economische interacties worden weergegeven als verbindingen binnen een schakeling, waarlangs goederenstromen en prikkels worden uitgewisseld. Schakelschema's bieden daarbij zowel een grafische als computationele representatie van economische systemen. Hierdoor kunnen standaard simulatieomgevingen voor elektrische schakelingen, zoals LTspice, direct worden gebruikt om economische dynamiek te simuleren.

De tweede stap richt zich op schaalbaarheid. Hoewel economische netwerktheorie een systematische manier biedt om interacties te beschrijven, worden grote en sterk onderling verbonden economische systemen al snel onoverzichtelijk en moeilijk hanteerbaar wanneer zij uitsluitend met elementaire circuitelementen worden gemodelleerd. Om dit op te lossen, breidt dit proefschrift economische netwerktheorie uit tot een multipoortnetwerkmethode. Hiermee kunnen complexe economische systemen worden opgebouwd uit modulaire subsystemen met duidelijk gedefinieerde aansluitingen, terwijl interpreteerbaarheid en schaalbaarheid behouden blijven.

Het proefschrift demonstreert de toepasbaarheid van de methodologie aan de hand van verschillende voorbeelden, variërend van klassieke modelproblemen tot hedendaagse praktijkproblemen. Een Robinson-Crusoe-economie laat zien hoe klassieke micro-economische redeneringen kunnen worden geconfigureerd en gesimuleerd met behulp van schakelschema's. Een model van een toeleveringsketen toont hoe frequentiedomeinanalyse, een centraal instrument binnen de ingenieurspraktijk, resonantie- en oscillatie-effecten in voorraadgedrag zichtbaar maakt. Een elektriciteitsmarkt met energieopslag laat zien hoe structurele veranderingen in een markt de dynamiek van het systeem beïnvloeden. Ten slotte illustreert een modulaire macro-economisch model hoe de methodologie kan worden opgeschaald naar systemen met veel interacterende sectoren, terwijl de interpreteerbaarheid behouden blijft en schokken door de economie kunnen worden gevolgd.

Ook buiten dit proefschrift is de methodologie al toegepast in verschillende afstudeerprojecten binnen domeinen zoals energiemarkten, industriële concurrentie, financiële planning en macro-economische modellering. Deze toepassingen laten zien dat de methodologie toegankelijk is, in het bijzonder voor studenten met een ingenieursachtergrond, en dat zij potentie heeft als praktisch instrument voor onderzoek, onderwijs en beleidsanalyse.

De methodologie introduceert verschillende elementen uit de ingenieurspraktijk in de economie: expliciete dynamica, modulaire opbouw, grafische representatie en schaalbare systeemanalyse. Omdat de modellen kunnen worden getekend als schakelschema's en kunnen worden uitgevoerd in standaard simulatieomgevingen zoals LTspice, maakt de methodologie het mogelijk om economische dynamiek numeriek te onderzoeken met bestaande elektrotechnische hulpmiddelen. Gezamenlijk bieden deze elementen een systematische manier om te analyseren hoe economisch gedrag voortkomt uit interacties tussen agenten en hoe systeemdynamiek wordt gevormd door de onderliggende netwerkstructuur.

Introduction

1.1. Circuit-Theoretic Modeling Methodology for Economic Systems

Circuit theory is one of the most successful modeling methodologies in engineering, and its impact is evident from the ubiquity of electrical and electronic devices in today's society [1]. Its power lies in providing engineers with a systematic way to construct, analyze, and simulate complex dynamical systems by assembling them from simpler interacting elements. Rather than working directly with the fundamental equations of electromagnetism, engineers rely on circuit theory, a methodology that translates fundamental laws—such as Maxwell's equations and Faraday's law—into practical building blocks suitable for design and analysis.

A particular strength of circuit theory is the combination of its visual organizational structure and underlying mathematical structure. Circuit diagrams simultaneously offer intuitive visual representations of how elements within a network interact and encode the differential equations that describe the evolution of all voltages and currents in the network over time. Together, this visual clarity and mathematical structure allow systems ranging from simple light-bulb circuits to highly complex electronics to be designed and analyzed systematically. As a result, engineers can rely on graphical circuit simulation environments, such as LTspice [2], to design, simulate, and analyze dynamic system models directly from their circuit representations.

The goal of this dissertation is to translate the modeling principles embodied in circuit theory into a methodology that can be applied to economic modeling.

Like electrical circuits, economic systems consist of many interacting parts whose collective behavior emerges from their interconnections [3, 4]. Economic agents (e.g., households, firms, traders, and financial institutions) continuously adjust prices, demand, inventories, and financial positions in response to incentives and constraints. These interactions result in feedback loops across markets and sectors, allowing local adjustments to influence system-wide outcomes. Over time, structural changes such as technological progress or demographic shifts alter the configuration of the system, much as redesigning an electrical network changes its behavior.

These observations motivate the use of circuit theory as a modeling framework

for economic systems. By adopting the organizational and compositional principles of circuit theory, economic models can be constructed from interacting components, represented graphically, and analyzed using well-established tools for dynamic systems. This provides a structured and scalable way to model economic systems, enabling the analysis of how interactions give rise to system-level dynamics using the mathematical and computational tools developed for circuit theory.

1.2. Economic Engineering as Theoretical Foundation

The theoretical foundation of the modeling methodology developed in this dissertation is the economic-engineering theory introduced by Mendel [5]. This theory provides a mechanical analogy of economic systems, much in the same spirit that physicists such as Maxwell [6], Helmholtz [7], and Firestone [8] used mechanical analogies to develop the principles of electrodynamics. In Mendel's formulation, prices and quantities evolve as dynamical state variables governed by internal economic forces. Phenomena such as scarcity, excess demand, and transaction frictions act analogously to mechanical forces, giving rise to adjustment dynamics similar to Newton's laws of motion. In this way, the analogy provides a dynamical extension of the classical supply–demand relationship, which is typically formulated only as a static equilibrium condition. The dissertation adopts Mendel's economic laws as a set of axioms.

The use of physical and engineering analogies in economics has a long history [9], including mechanical equilibrium models, hydraulic analogies, and early attempts to model markets using electrical circuits [10–13]. These efforts illustrated structural relationships but generally retained the static-equilibrium perspective of classical economics. The economic-engineering theory differs fundamentally by treating price as a state variable rather than an externally imposed equilibrium value, placing economic laws on a conceptual footing comparable to Newtonian dynamics.

The premise of this dissertation is that an electrical, rather than mechanical, analog is better suited for modeling when the objective is to construct and analyze complex interconnected economic systems. Electrical and mechanical systems are theoretically analogous [14], which entails that by switching to an electrical-economic analogy we can leverage circuit theory as a modeling methodology for economic networks. Just as Maxwell's equations and Faraday's law form the physical foundation on which circuit theory is built, the economic-engineering theory provides the fundamental dynamical principles on which this dissertation builds its modeling methodology.

The dissertation develops these principles into a practical modeling methodology that enables economic systems to be constructed and analyzed as networks of interacting agents. By adapting the organizational and compositional structure of circuit theory, the dissertation establishes a systematic and interpretable methodology in which economic agents and markets can be defined, interconnected, and simulated as analogous electrical circuits.

In these circuit representations, agents appear as circuit elements governed by Mendel's economic-engineering laws, and their interconnections form economic networks whose emergent behavior follows directly from the network structure. Together, the components and network structure generate the differential equations that describe the dynamical behavior of systems like markets, supply chains, and economies.

1.3. Relationship to Modeling Approaches in Economics

The methodology developed in this dissertation emphasizes the representation of economic systems as dynamical and interconnected structures. Accordingly, the discussion in Chapters 2 and 3 focuses on modeling approaches that make these aspects explicit. This focus reflects a particular modeling perspective and abstracts from other objectives that are emphasized in the economic literature. The present section provides a more elaborate positioning of the proposed methodology with respect to existing approaches, including its strengths and limitations. The section focuses on the economic modeling approaches that are most comparable in terms of their representation of dynamics, interaction, and system structure, and examine how the proposed methodology relates to them.

The most closely related modeling paradigms are Dynamic Stochastic General Equilibrium (DSGE) models, agent-based computational economic (ACE) models, and network-based models. Each of these paradigms has been developed to address particular aspects of economic systems, and reflects different modeling objectives and design choices that have emerged over time in response to specific theoretical and practical challenges.

DSGE models provide a coherent macroeconomic methodology centered on equilibrium relations among representative agents and rational expectations [15–17]. Their development can be traced to the effort to place macroeconomic analysis on firm microeconomic foundations, and to construct models that are internally consistent and suitable for quantitative policy analysis. In this context, DSGE models have been particularly successful in providing a structured framework for linking theory to aggregate data, and for analyzing the effects of policy interventions.

ACE models emphasize heterogeneous interacting agents and bottom-up emergence [18–20]. They have developed in part as a response to limitations associated with representative-agent and equilibrium-based frameworks (like DSGE modeling), particularly in capturing heterogeneity, bounded rationality, and out-of-equilibrium dynamics. By allowing agents to interact according to flexible behavioral rules, ACE captures how macroeconomic patterns emerge from the bottom up and to model complex dynamics that are difficult to capture in more aggregated frameworks.

Network-based economic models highlight how the topology of interactions shapes aggregate outcomes [3, 4, 21]. Their emergence reflects the increasing recognition that the structure of connections between agents, such as production linkages, financial exposures, or trade relationships, plays a central role in determining how shocks propagate through the economy. These models provide a natural framework for

analyzing interdependence, particularly in settings where the pattern of interactions is itself a key driver of system behavior.

The circuit-theoretic methodology developed in this dissertation shares key elements with these established modeling paradigms: like DSGE models, it provides a structured representation of economic systems grounded in well-defined relationships between variables; like ACE, it adopts a bottom-up perspective in which system-level behavior emerges from the interaction of individual components; and like network-based approaches, it represents economic systems as interconnected structures in which the topology of interactions plays a central role. At the same time, the methodology combines these elements in a distinct engineering-style framework.

Naturally, this setup implies a particular set of trade-offs between the established modeling paradigms and the methodology developed in this dissertation. In the following, we compare these trade-offs with those of existing modeling paradigms along several key dimensions.

In terms of calibration and data requirements, DSGE models currently have the most developed empirical workflow among the approaches considered here. Their structural equations are typically estimated using aggregate macroeconomic time series, and a substantial literature exists on Bayesian estimation, forecasting, and policy analysis [17, 22]. ACE models offer greater flexibility in representing heterogeneity and decentralized behavior, but their calibration and empirical validation are often more case-specific, in part because they may generate rich distributions of outcomes rather than a single tightly parameterized mapping from data to predictions[23].

The circuit-theoretic methodology takes a different position. It restricts the space of admissible models through a certain model structure: economic behavior is represented through causal, constitutive laws and network topology, rather than through an unrestricted input-output specification. This structural character is advantageous for calibration in one respect, because it gives model parameters an interpretable role within the dynamics and topology of the system, and opens the door to system-identification methods familiar from engineering. On the other hand, it also creates important practical difficulties. Some parameters introduced by the analogy may not be directly observable in available economic data, and economic data are often less informative than the controlled identification data available in engineering applications. In addition, the methodology is presently developed primarily for linear time-invariant (LTI) systems, so calibration of nonlinear or time-varying variants is likely to be more demanding.

A related strength of the methodology is that useful models can be constructed and simulated even when real-world data are limited. As several examples in the dissertation illustrate, the explicit and interpretable structure of the models allows a modeler to posit parameter values, analyze implied behavior, and study sensitivity in a transparent way. This makes the methodology well suited to exploratory structural analysis and scenario design. Its weakness, however, is that this same structural commitment rests on the axioms of the underlying economic-engineering analogy. If those axioms provide an inadequate representation of actual economic behavior, then the resulting model may be structurally elegant yet empirically misleading.

With respect to performance in analysis and forecasting, DSGE models have been

extensively used for policy analysis and macroeconomic forecasting, while agent-based and network-based models are often employed for exploratory analysis of complex dynamics and scenario-based investigations. The circuit-theoretic approach is similarly oriented toward structural and dynamical analysis, enabling the study of how interactions and network structure give rise to system-level behavior, rather than being primarily designed for predictive accuracy.

Regarding robustness, stability, and sensitivity to assumptions, DSGE models are typically analyzed around equilibrium states, with well-developed tools for local stability analysis, while agent-based and network-based models can exhibit complex and sometimes sensitive dynamics that depend on model specification. The circuit-theoretic methodology enables the use of established tools from dynamical systems and circuit theory to analyze stability and transient behavior, but, like other approaches, its results remain sensitive to modeling assumptions and parameter choices.

Finally, in terms of computational efficiency, DSGE models are generally designed to be computationally tractable at the aggregate level, whereas agent-based models can become computationally intensive as the number of agents and interactions increases. Network-based models vary in complexity depending on their level of detail and the nature of the dynamics imposed on the network. The circuit-theoretic methodology benefits from the structure of LTI dynamical systems, for which a wide range of efficient analytical and numerical methods are available. In particular, the representation of models in state-space form and their compatibility with established circuit simulation tools allow for efficient simulation and analysis, even for relatively large interconnected systems. On the other hand, this computational advantage depends on the linear and time-invariant structure of the models. For nonlinear or time-varying extensions, computational complexity can increase rapidly, as is typical for general dynamical systems, and the availability of efficient solution methods becomes more limited.

Taken together, this comparison indicates that the circuit-theoretic methodology comes with a set of trade-offs. Its strengths lie in its dynamical structure, modularity, and explicit representation of interactions, but it is grounded in the assumptions of the economic-engineering analogy, which impose a particular structure on the modeling of economic systems. As this analogy is not part of the standard economic modeling framework, it introduces an additional source of model risk. Moreover, the methodology currently lacks mature empirical calibration frameworks and extensive validation compared to more established approaches.

1.4. Scope of the Dissertation

The chapters of this dissertation translate the economic-engineering principles introduced earlier into a circuit-theoretic modeling methodology for dynamic economic systems. The development proceeds in two major steps: first by formulating economic circuit theory, and then by generalizing it into a full multiport representation suitable for large-scale interconnected agents and economies.

Chapter 2 introduces *economic circuit theory*, which embeds the economic-

engineering analogy within the structural logic of electrical circuit theory. In this formulation, elementary economic agents are represented by circuit elements with well-defined dynamical relations, and market interactions correspond to network interconnections. The chapter shows how changes in prices and quantities can be modeled analogously to voltages and currents, how interaction between agents can be captured by the network topology, and how circuit diagrams provide both a conceptual and executable representation of economic dynamics. Several examples, including a two-good market and a simplified oil market model, illustrate how dynamic behavior can be captured using the same tools that electrical engineers use to model circuits. The chapter is published as *Economic Circuit Theory: Electrical Network Theory for Dynamical Economic Systems* [24].

Chapter 3 generalizes this formulation into a full *multiport network methodology*. Multiport network theory, widely used in electrical engineering, models complex systems by representing each subsystem as a network module with ports through which power-conjugate variables are exchanged [1, 14]. As in basic circuit theory, the internal behavior of each multiport is described by constitutive relations, while the interactions among multiports follow from the network topology. A key advantage of the multiport representation is that only the behavior at the ports is required to determine the behavior of the overall system. The internal states of a multiport do not need to be tracked by the rest of the network, allowing each subsystem to be treated as a black box. This abstraction is central to the modular nature of multiport theory: modules can be designed, analyzed, and replaced independently, while still integrating consistently into arbitrarily large networks. In this way, multiport theory provides a scalable and systematic foundation for constructing complex interconnected systems

In this dissertation, these principles are adapted to economic systems. Economic agents are modeled as multiport subsystems with internal dynamics and well-defined ports that represent the exchange of goods and incentives. The chapter shows how the economic-engineering laws can be embedded into a multiport structure so that agents can be interconnected into larger systems in a systematic and scalable way. The formulation preserves the dynamical foundations established in Chapter 2 while introducing the modularity, hierarchical characteristic of engineering network models. This enables complex economic agents and systems to be constructed from reusable building blocks, analogous to how multiport models enable the construction of large electrical networks from standardized components.

Chapter 4 applies the resulting methodology to a range of illustrative systems, including a Robinson–Crusoe economy, supply chains, an electricity market with storage, and a modular macroeconomic block model. These examples show how classical and contemporary economic problems can be represented as networks, visualized through circuit-style diagrams, and simulated using standard tools such as LTspice. The chapter highlights how the methodology supports interpretability, modularity, and exploratory modeling.

Chapters 3 and 4 are submitted as a paper to the *Journal of the Franklin Institute*, see [25] for the preprint. Together, the research conducted in these chapters constitutes the central contribution of this dissertation: an engineering-style methodology

for modeling economic systems as networks.

1.5. Applications of the Methodology Beyond the Dissertation

The methodology developed in this dissertation has already been adopted in a growing body of MSc theses within the department of systems and controls at the TU Delft. These projects apply the methodology across a wide range of economic domains, including oil and hydrogen markets [26–28], power-market flexibility [29], supply chains and scheduling [30, 31], industrial competition [32], macroeconomic modeling [33, 34], housing and mortgages [35, 36], business valuation [37], labor economics [38, 39], and bank asset–liability management [40].

These applications demonstrate the methodology's applicability across different economic domains and its accessibility for engineers. All of these projects were conducted by engineering students, illustrating how the graphical and structural features enable engineers to leverage their background in systems modeling to solve problems in economics. This highlights the methodology's potential not only as a research tool, but also as a bridge between economics and engineering practice.

2

Economic Circuit Theory: Electrical Network Theory for Dynamical Economic Systems

2.1. Introduction

Understanding and modeling dynamics is a fundamental challenge in modeling economic systems (see, e.g., [41–44]). This contrasts with the engineering disciplines, which can rely on several well-established theories and methods to model physical dynamic systems [9, 14].

Traditionally, economic theory has focused on identifying the conditions for economic equilibrium, in particular that of price equilibrium. Although recently there has been a shift of focus towards modeling disequilibrium, there is currently no generally accepted economic force law to govern the dynamics of price (see [45]). Instead, economists rely on ad hoc methods. For instance, in the Dynamic Stochastic General Equilibrium (DSGE) models common in macroeconomic theory [17, 20, 46], prices are assumed to adjust instantaneously [19]. In System Dynamics (SD) modeling common in business applications, price changes are modeled through look-up tables and other ad hoc methods [47]. Agent-Based Computational Economics (ACE) models rely on various ad-hoc behavioral heuristics for price determination [19, 20, 48–50]. Finally, in complexity economics price laws are only observed at the macro scale, see e.g. [51].

In contrast, the engineering disciplines have traditionally focused on dynamic behavior. Especially in electrical circuit theory, this has led to an effective and sophisticated modeling tradition that can rely on a rigorous theoretical basis (e.g. [1, 9]). The ubiquitous sophisticated electronic devices that are part of contemporary life testifies to the effectiveness of electrical circuit theory to model complex and highly dynamic systems.

Our vision with economic circuit theory is to exploit this effectiveness for economic modeling. In the engineering literature, there does exist an established line

of research that attempts to model economic systems through an analogy with electrical networks [10, 11, 52–54]. The underlying idea is that electrical networks can be thought of as agent-based, with each economic agent corresponding to an electrical component. Agents exchange goods or services in the same manner as electrical components exchange electric charge, thought of as a current or flow passing through the wiring that represents the trade connections.

Our economic circuit theory builds on this line of research. However, our theory differs from this line in one crucial assumption that is essential for modeling price dynamics. This is the choice of the *magnetic flux* stored in an inductor to represent the *price* that an agent ascribes to a good. We rely on a recent publication [45], where this is first proposed and where the implied economic force law is worked out as the analog of Faraday's law. This assumption implies that the *voltage* over the inductor is interpreted as an *incentive*. Such an incentive determines the agent's price dynamics in the same manner as the voltage determines the dynamics of the flux [1].

In the aforementioned line of research, instead, price is invariably assumed to be the analog to voltage, rather than the flux. However, there is no electrical component whose constituent determines the rate of change of a voltage. Therefore, this analogy cannot be implemented in an electrical circuit to model price dynamics. Indeed, the circuits that have been proposed in these works all concern equilibrium models.

In Section 2.2 we adapt the analogy from [45] for economic circuit theory. Although many aspects of this analogy are the same as in the aforementioned line of research, we mention here several important distinctions. First and foremost, the inductor represents a location of demand rather than a source of reinvestment. Although we also use a capacitor as a storage location, it requires an incentive rather than a price to increase its inventory level. Using an inductor for the location of demand frees up the resistor, which now locates the presence of economic friction in close analogy with its physical role. Rather than a price, it requires an incentive to mediate the exchange of goods.

In Section 2.3, we introduce the main contribution of this chapter: the use of the modern topological graph-theoretic approach to electrical networks to model economic price dynamics in a unified, comprehensive, and rigorous manner for complex economic systems [1, 55–57]. Specifically, we associate the Kirchhoff loop law with a price-incentive consistency condition. It determines how, in an economic system, the incentives drive the prices of the individual agents. This contrasts with its use in the aforementioned line of research where it serves as a static balancing of prices. It complements the Kirchhoff node law serving as a stock-flow consistency requirement. Although our interpretation of the node law remains the same, their combination into the economic equivalent of Tellegen's theorem provides us with a dynamic version of Walras's law, rather than its traditional price equilibrium interpretation [54, 58].

In Section 2.4, we develop several concepts from electrical network analysis for their use in economics. These include the use of closed circuits for single-good markets and model-simplification techniques to aggregate agents into representative agents, including cooperative agents as series and competitive agents as parallel interconnections. In addition, we show how real electrical power sources can be used

to incorporate the dynamics of exogenous sources. These include the real agents such as consumers and producers who supply economic surplus to the system.

Our development of economic circuit theory culminates in Section 2.5, where we show that the price dynamics of an economic circuit can be modeled using a state-space representation. This relies crucially on the price-flux analog, used in conjunction with the price-incentive consistency condition. Since the flux linkage is a state variable of an electrical system, so is the price (say p) of an economic system within circuit theory. The state-space representation then specifies the time-rate of change of the price (i.e., $\dot{p} = \frac{dp}{dt}$). This argument would not be valid if the voltage were the analog to the price. Complementing the prices with the inventory stocks creates the state vector of a dynamical system and we detail the procedure for deriving both the state and input matrices of its state-space representation for an arbitrary economic circuit.

We then show how the state-space representation allows us to analyze the transient behavior of an economic system. We refer to this as dynamic economic scenario analysis. This is achieved in two steps. First, we establish the dynamics of the equilibrium state as a function of the exogenous sources. Then, we determine the transient behavior of the price and stock levels as they attempt to move towards this, possibly changing, equilibrium state.

In Section 2.6.1, we apply the theory to the design and analysis of several economic circuits. For a relatively simple system consisting of two competitive markets, we show that the design of the economic circuit proceeds in the same manner as an electrical engineer would design an electronic network: We identify the requisite elementary agents, simplify by aggregating them into representative agents, and then establishing the state-space representation of the dynamics. With that, we illustrate the dynamics by investigating the inventory and price responses in the markets due to a particular demand shock using the dynamic scenario analysis. Subsequently, we provide experimental verification of the theory with an application to the spot market for crude oil. We conclude the section with an application to a rather complex application to the design of a hypothetical hydrogen economy. This application shows the power of economic circuit theory to conceptualize, design, and build prototypes of complex economic systems even in situations where there is no data available.

2.2. Dynamic Structure

2.2.1. Stock and Price as Conjugate Variables

In our theory, we employ a recently published analogy [45] that enables us to incorporate price dynamics in analogous electrical networks. The economic analogs to the electrical signals resulting from this analogy are summarized in Table 2.1.

The critical distinction is the choice of the magnetic flux as the analog to price. In this interpretation, the inductor becomes an agent with a demand (or supply) function, that maintains a price level p . Its role is then conjugate to the role of the capacitor as an agent that maintains a commodity stock level (or scarcity) q .

Table 2.1: Conjugate pairs of dynamic variables and their stock-flow relationships.

	Economic	Electrical	Relation	Units
<i>Stock</i>	Commodity Stock	Electric Charge	$q = \int f dt$	#
	Price	Magnetic Flux	$p = \int v dt$	$\frac{\$}{\#}$
<i>Flow</i>	Commodity Flow	Current	$f = \frac{dq}{dt}$	$\frac{\#}{yr}$
	Incentive	Voltage	$v = \frac{dp}{dt}$	$\frac{\$}{\#yr}$

The flow variable corresponding to the price as a stock variable is what [45] refers to as a want and what we will additionally refer to as an incentive or a motive (see Section 2.3.3). In this analogy, an incentive causes a change in an agent's price level and will thereby change their flow demanded or supplied, necessitating the rest of the economic network to adapt. In the same way that a flow of commodities equals the time derivative of a commodity stock, an incentive is the time-derivative of the price. This is properly analogous to how a voltage is the time-derivative of the magnetic flux [1]. Conversely, in the same way that integrating a flow of commodities over time results in a commodity stock, integrating an incentive over time leads to a price – for example an agent's contemporaneous willingness to pay. Incorporating the effect of incentives on economic activity leads to dynamic models instead of kinematic ones.

2.2.2. Behavioral Laws

The relationships between the stock and flow variables in Table 2.1 are determined by the behavioral laws of elementary agents. We represent these agents by analogous electrical elements of which the constitutive relations [14] emulate the behavioral laws. The dynamics within an economic network are then emulated by the electrodynamical interactions of the analogous electrical elements.

Marshallian Agents

In Table 2.2, we provide a summary of the two-terminal electrical elements, their economic analogs, and behavioral laws. Two-terminal elements conduct a single current, from one terminal to the other. We refer to their economic analogs as Marshallian agents, because they are limited to processing only a single commodity flow (see Section 2.2.2 for a description of general Walrasian agents). The behavioral law pertaining to an agent with demand is the familiar Marshallian law of demand, relating the price p to the quantity demanded f . The storage law relates the stock level q of the storing agent to a desire to hold more (or less) of the commodity. The friction law specifies that the corresponding agent must be motivated with an incentive v in order to transport the flow f of the commodity. See [45] for an in-depth discussion of these relationships.

Insatiable and inelastic agents are active agents whose behavior is independent of other variables in the network. These agents are analogous to a voltage and a current

source, respectively. The incentive or the flow supplied by an active agent may be deterministic or stochastic. When deterministic, they are specified as a function of time and, when stochastic, they follow some probability distribution.

Table 2.2: Elementary Marshallian agents as electrical two-terminal elements and their behavioral laws.

Economic Agent	Electrical Component	Behavioral Law	Symbol
Demand	Inductor	$f = \varepsilon p$	
Storage	Capacitor	$v = k q$	
Friction	Resistor	$v = b f$	
Inelastic	Current Source	$f = f(t)$	
Insatiable	Voltage Source	$v = v(t)$	

Walrasian Agents

Agent that trade more than one type of commodity can be represented by electrical elements with four and more terminals. We refer to such agents as Walrasian agents.

In Figure 2.1, we show the analog to a Walrasian demander handling two separate commodities flows while maintaining a price for each. In electrical networks, this element is known as a mutual inductance. It consists of two or more coils that interact through their magnetic fields, depicted by the magnetic flux lines, and thereby couple any change in the one current (f_1) to a change in the other (f_2). The mutual inductance emulates the effect of cross-elasticity of demand where the demand for two goods are related through their relative prices. This analogy enforces our choice to model prices as magnetic flux rather than as voltages.

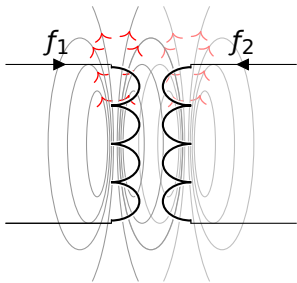


Figure 2.1: Walrasian demander for two commodities. The magnetic flux lines shared by the two Marshallian demanders quantify the price coupling.

The behavioral law of the Walrasian demander is given by a tensorial version of the

Marshallian law of demand $f = \varepsilon p$ (see Table 2.2). For a Walrasian demander trading in G commodity types, the matrix representation of the behavioral law becomes:

$$\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_G \end{pmatrix} = \begin{pmatrix} \varepsilon^{11} & \dots & \varepsilon^{1G} \\ \vdots & \ddots & \vdots \\ \varepsilon^{G1} & \dots & \varepsilon^{GG} \end{pmatrix} \begin{pmatrix} p_1 \\ \vdots \\ p_G \end{pmatrix}. \quad (2.1)$$

Here, ε^{ii} is the Marshallian price elasticity of demand and ε^{ij} , $i \neq j$ the cross-elasticity of demand. The latter is positive for substitutes, negative for complements, and zero for independent goods (see further [45] for a discussion in the context of a mechanical analog).

The other Walrasian agents can be defined in a similar manner. A Walrasian storage is equipped to store different types of commodities simultaneously, where the desirability of receiving one type is dependent on the amount of the other. Its behavioral is likewise a tensorial version of the Marshallian agent's law. Such an agent is analogous to a mutual capacitance. Walrasian frictional agents are defined similarly.

2.2.3. Causality

The dynamic structure established by the behavioral laws and the stock-flow relationships can be conveniently pictured together in what is known as a tetrahedron of state (see [14]). Figure 2.2 contains a tetrahedron pertinent to the economic analogy. The pairs of conjugate stock and flow variables are enclosed within dotted boxes on the horizontal and vertical, respectively. The stock-flow relationships and behavioral laws are contained within the dashed box on the left-leaning and the right-leaning diagonal, respectively. Friction connects the incentive directly with the flow over the vertical.

The causality enforced by a relationship is indicated in the tetrahedron by the direction in which the arrowhead points. For the stock-flow relationships, the causal direction coincides with that of the integration operator. Integration determines the present based on the past and, hence, is causal. To wit, the current stock is created by accumulating the preceding flows and the current price is arrived at by accruing the preceding incentives. Differentiation, on the other hand, compares the current stock to the stock a little time later. It is, therefore, forward looking and cannot be strictly causal.

For the behavioral laws, the causal direction is determined by the economic reality. The economic law of demand is a causal law, stipulating that the change in the flow is caused by a change in price (see e.g. [59]). For the storage law, one first registers the stock amount, which then becomes the cause for wanting to store more or less of the commodity in question. The friction law forms an exception, being bicausal. Either the flow is the cause of an incentive – as it would be for, e.g., a broker that offers price premiums or discounts based on a mismatch between supply and demand – or the incentive is the cause of a flow, which would for example be

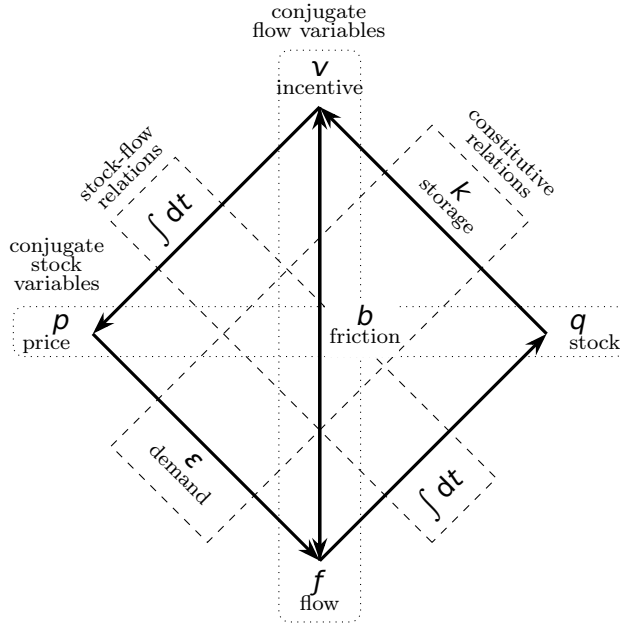


Figure 2.2: Tetrahedron of state for economic systems. The boxes highlight the main features of the dynamic structure.

the case of an inventory manager that reduces the inflow of commodities based on a storage's needs.

2.3. Topological Structure of Economic Networks

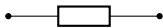
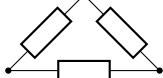
Economic networks can be structured analogously to electrical networks, achieved by “wiring” agents together to facilitate trade in a predetermined manner. In network theory, a particular wiring is specified by a topological structure. This structure prescribes the transmission of currents and the balancing of voltages in the electrical network. In economic networks, we let the topology determine how agents interact with each other by prescribing the movement of commodities and the balancing of incentives among them.

2.3.1. Topological Concepts for Economic Networks

In Table 2.3 we list the economic interpretations of the topological building blocks of a network. In the following, we consider these in detail.

Agents make up the edges of the network. We think of the commodities as flowing through the agents and incentives to be measured across agents. While in Table 2.3 we depict a generic electrical component, agents are represented by their specific analogous electrical component (Table 2.2) in a particular network.

Table 2.3: Topological concepts and their economic roles

Topology	Economics	Graph	Symbol
Edge	Agent		$\mathcal{E}$
Node	Exchange Locus		$\mathcal{N}$
Mesh	Circular Flow		$\mathcal{M}$

The nodes of the network represent the loci at which the commodities are exchanged. Each node is depicted by a thick dot between any number of components. However, in practice the node comprises the entire region (the wires) between the components. Commodities flow through a node and are distributed over the incident agents, as we show in Section 2.3.2. At each node, i.e. exchange locus, there is a particular level of desirability that results from the incident agents. The difference between the level of desirability at the two nodes at each end of an agent is equal to the agent’s incentive, as we further show in Section 2.3.3.

There is one special type of node which is referred to as a ground node in network theory. It is used as a reference desirability level, usually set to zero, to calibrate the desirability level of the remaining nodes. One can think of it as a very large reserve where the commodity is abundant such that its desirability does not change upon receipt or delivery of additional units.

In network theory, a mesh is a loop that contains no other loops. For economics, such a mesh can sustain a single circular flow of the commodity, referred to as the mesh current in network theory [55]. Agents that form a mesh can exchange commodities while having a mutual reference for the desirability level. As we show in Section 2.3.3, meshes are necessary for agents to communicate their incentives.

With the establishment of a topological structure, we can use algebraic graph theory [57] to systematically analyze and model economic networks, rather than relying on ad-hoc methods. In itself, the use of network theory for modeling economic systems is not new (see e.g. [3, 21, 60]). However, in these attempts the agents make up the nodes of a network, while the edges represent their interactions. The disadvantage of such a choice is that the flow of the goods cannot be read-off explicitly as a network current and, consequentially, the edges cannot accommodate the behavioral laws that determine the dynamic structure. By adhering to convention in network theory, its use for modeling and design in economics becomes transparent.

2.3.2. Stock-flow Consistency and the Kirchhoff Node Law

The analogy between electric charge and commodities implies that the Kirchhoff current law (KCL), also known as the node law, assures that the distributed commodities clear at each exchange locus (see e.g. [53, 61]). By using the KCL’s formulation in algebraic graph theory [55, 57], we can efficiently ensure a stock-flow consistent distribution of commodities throughout an entire network. To do so, we first introduce

the network flow vector

$$\mathbf{f} = (f_1 \quad \dots \quad f_E)^T, \quad (2.2)$$

containing a flow f_j for each agent $j \in \{1, \dots, E\}$. Then we define an incidence matrix $\mathbf{N}$ as follows: At a particular node we identify the edges that are incident to it. These represent the agents who exchange commodities through that particular locus. To keep track of the movements through the node, we define for each node a row of incidence descriptors. This yields a matrix with entries

$$\mathbf{N}_{ij} = \begin{cases} 1 & \text{if } \mathcal{E}_j \text{ provides commodities to } \mathcal{N}_i \\ -1 & \text{if } \mathcal{E}_j \text{ receives commodities from } \mathcal{N}_i \\ 0 & \text{if } \mathcal{E}_j \text{ is not incident to } \mathcal{N}_i, \end{cases} \quad (2.3)$$

where $i \in \{1, \dots, N\}$ for N exchange loci.

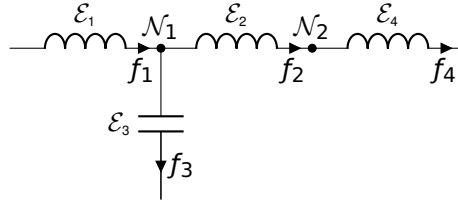


Figure 2.3: The exchange loci (nodes $\mathcal{N}_1$ and $\mathcal{N}_2$) clear the commodities among the agents incident to it. The KCL requires that $f_1 = f_2 + f_3$ and $f_2 = f_4$.

With the aid of the incidence matrix and the flow vector, the stock-flow consistency condition for an arbitrary economic network can be concisely stated as follows:

$$\mathbf{N}\mathbf{f} = \mathbf{0} \quad (2.4)$$

Here $\mathbf{0}$ is the N -dimensional zero flow vector.

To illustrate the procedure, consider node $\mathcal{N}_1$ from the network in Figure 2.3. From the choice of flow directions in the figure, we see that agent $\mathcal{E}_1$ provides commodities and therefore $\mathbf{N}_{11} = 1$. Agents $\mathcal{E}_2$ and $\mathcal{E}_3$ receive commodities, hence $\mathbf{N}_{12} = \mathbf{N}_{13} = -1$. Because agent $\mathcal{E}_4$ is not incident to $\mathcal{N}_1$, i.e. commodities do not directly flow from the storage to this exchange locus, $\mathbf{N}_{14} = 0$. Collecting the entries into a row for each node in a network and arranging the rows into a matrix gives the incidence matrix for that particular network.

2.3.3. Price-Incentive Consistency and the Kirchhoff Mesh Law

The topological structure of a network also allows us to formulate a price-incentive consistency that ensures that each price change within a network is consistent with the incentives. Its electrodynamic analog is known as the Kirchhoff voltage law (KVL), also referred to as the mesh law. This law is based on the principle of conservation of the magnetic flux linked to a mesh by the inductors [1]. For economic meshes, we

link the prices of the individual demanders (or suppliers) to the mesh. A price linked in this manner represents the personal value the agent ascribes to the commodity. In [45], it is shown that the total of these personal values in a mesh is indeed a conserved quantity, similar to the total flux linkage. In this subsection, we formulate the price clearing condition in terms of a conservation law for the total value linked to the network.

A flow of value is a want, as evidenced by the p - v stock-flow relationship (see the tetrahedron of state in Figure 2.2). To describe the various manifestations of a want within a network, we use the analogs to some specialized terminology that has been developed in electrodynamics for this purpose (see Table 2.4).

At each locus, there is a level of *desirability* that the agents incident to the locus have in common. Following custom in electrodynamics, we set the desirability of the reference node (electrical ground) to zero so that no value flows there. We measure the desirability ϕ_i at node i relative to the ground level and collect these into an N -dimensional desirability vector $\boldsymbol{\phi}$.

Table 2.4: The manifestations of a want in economic networks: incentive, motive, and desirability

Economic	Electrical	Symbol
Desirability	Electric potential	ϕ
Incentive	Voltage drop	v
Motive	Electromotive force (emf)	e

Associated to each edge, there is a unique drop v_j in desirability representing the *incentive* that the agent requires to process the commodities from one locus to the other. The flow of value associated with this drop is depicted by a curved arrow pointing in the direction of positive incentives. The curve emphasizes that the drop occurs *over* the agent (rather than *through* it as a flow). When the desirability jumps, we speak of an economic *motive*, which is generated by an active agent (Table 2.2). The difference in incentives and a motives is analogous to the difference in voltages and electromotive forces in electronics [1]. We adhere to the same sign convention as used in electrical network theory, with incentives directed opposite to the commodity flow and motives directed with the commodity flow (see Figure 2.4).

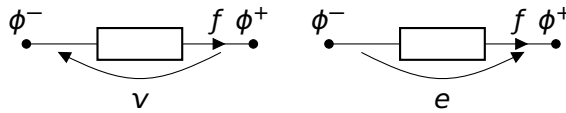


Figure 2.4: Sign convention: an incentive v measures the (passive) desirability drop across an agent and a motive e measures the desirability jump induced by an active agent.

The incidence matrix allows us to efficiently determine the network-wide incentives from the desirability levels. With $\mathbf{v}$ as an E -dimensional vector containing all

incentives within a network, we find that

$$\mathbf{v} = \mathbf{N}^T \boldsymbol{\phi}, \quad (2.5)$$

where $\boldsymbol{\phi} = (\phi_1 \dots \phi_N)^T$ is a vector containing the desirability levels of all nodes in the network and $\mathbf{N}^T$ is the transpose of the incidence matrix (2.3). The use of a matrix transpose in this connection reflects the conjugacy between the flows of value and the commodities and ensures a consistent accounting of the flows and incentives.

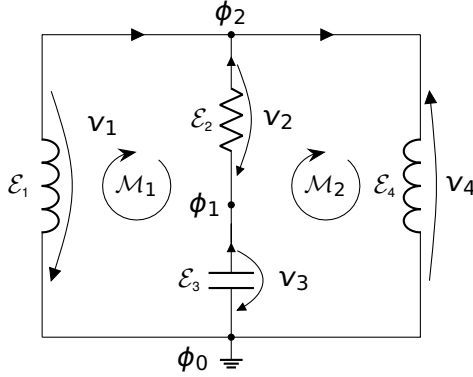


Figure 2.5: Price clearing among the agents in a circular flow. In mesh $\mathcal{M}_1$ KVL requires that $v_1 = v_2 + v_3$.

The price-incentive consistency condition can now be formulated in terms of a conservation law for economic value. To formulate the KVL for all M meshes in an economic network, we choose a direction for each circular flow and construct an $M \times E$ -dimensional matrix $\mathbf{M}$ with entries

$$\mathbf{M}_{kj} = \begin{cases} 1 & \text{if } \mathcal{E}_j \text{ is in } \mathcal{M}_k \text{ and directions agree} \\ -1 & \text{if } \mathcal{E}_j \text{ is in } \mathcal{M}_k \text{ and directions disagree} \\ 0 & \text{if } \mathcal{E}_j \text{ is not part of } \mathcal{M}_k, \end{cases} \quad (2.6)$$

where $k \in \{1, \dots, M\}$ for M exchange loci. In algebraic graph theory, $\mathbf{M}$ is known as a *branch-mesh matrix* [56]. In the economic setting, we refer to it as the agent-mesh matrix. With it, the KVL becomes

$$\mathbf{M}\mathbf{v} = \mathbf{0}, \quad (2.7)$$

where $\mathbf{0}$ is now an M -dimensional zero incentive vector. For economic networks, it gives a network-wide formulation of the price-clearing condition based on the conservation of the personal economic value of the demanders and suppliers.

We illustrate this procedure with the aid of the meshes in $\mathcal{M}_1$ and $\mathcal{M}_2$ Figure 2.5. After constructing the incidence matrix, (2.5) yields the incentives of the agents:

$v_1 = \phi_2 - \phi_0$, $v_2 = \phi_2 - \phi_1$, etc. In mesh $\mathcal{M}_1$, the incentive of agent $\mathcal{E}_1$ is against the direction of circular flow, while the incentives of $\mathcal{E}_2$ and $\mathcal{E}_3$ are along the flow. Hence, $\mathbf{M}_{11} = -1$ and $\mathbf{M}_{12} = \mathbf{M}_{13} = 1$. Agent $\mathcal{E}_4$ is not part of $\mathcal{M}_1$ and thus $\mathbf{M}_{14} = 0$. For the network in Figure 2.5, the KVL determines that $v_1 = v_2 + v_3 = -v_4$.

The use of the KVL for price-incentive consistency is new. Moreover, its formulation represents a radical departure from economic traditions concerning the role of price and, also, the existing attempts to model economic systems using circuit theory mentioned in the introduction.

In our theory, we associate a unique desirability level with a node. In economics, in contrast, it is conventional to associate a price with a locus where exchange takes place (e.g. [62]). A ground node, in particular, allows agents to dispose of the commodities at a zero desirability level, rather than at a zero price as would be understood by the traditional notion of free disposal. The use of desirability rather than price, allows us to think of the goods as flowing to where they are wanted from where they are less desired, thus establishing a cause for the change in a flow.

A price is associated with those edges that represent a demander (or a supplier). This makes the price p a property of an agent. Agents adjust their price over time following the incentives they are subject to. Subsequently, the flow supplied or demanded by such agents change according to their behavioral laws that link the flows of goods to the prices. This causal chain of dynamic responses contrasts with the typical role of price in economic models as a property of market equilibrium. In Section 2.6.1 we show that under certain conditions, the prices of individual agents align into a common market price.

2.3.4. Marshallian Markets as Closed Circuits

A Marshallian market —i.e., a market limited to a single type of commodities— can be defined in a purely topological manner, in analogy with an electrical circuit. A circuit is a network that consists of a single closed loop, possibly containing any number of meshes [1]. For a market, this definition implies that the commodities have a trade route to arrive to an agent from any other market participant. In addition, it provides the means for the market participants to communicate their incentives among each other. Typically one adds the requirement that there is also at least one active element present that drives the circuit (e.g. a battery) and we do likewise by adding at least one active agent.

The agent-mesh matrix $\mathbf{M}$ can be used to systematically identify the markets within a network. The matrix's rows represent the interactions among agents within the same mesh, while its columns reflect the inter-mesh communications between agents. Consequently, markets emerge as blocks marked off by blocks of zeros in the matrix (see Figure 2.6). This analysis is done even more efficiently if we introduce the *markets-mesh matrix*¹

$$\mathbf{T} = \mathbf{M}\mathbf{M}^T, \quad (2.8)$$

¹ In algebraic graph theory this matrix is not commonly used; however, it bears resemblance to the well-known Laplacian matrix $\mathbf{L} = \mathbf{N}\mathbf{N}^T$, where $\mathbf{N}$ denotes the incidence matrix. The Laplacian matrix is particularly notable for its role in determining the number of spanning trees within a graph.

which is a block diagonal matrix that reveals the number of markets in a network through the number of submatrices. For the network in Figure 2.6, for instance, the number of submatrices is two, both when mesh $\mathcal{M}_4$ does and does not exist. In each submatrix, the diagonal entries indicate the number of agents in each mesh, while the off-diagonal entries indicate the number of agents shared between the two corresponding meshes.

In Figure 2.6 we show a network with two separate Marshallian markets. In both markets an active agent is present. One verifies indeed that in Market 1, agents ε_1 through ε_4 are able to, directly or indirectly, deliver the goods to one another through the nodes and communicate their incentives through the meshes. Agents ε_5 and ε_6 are prevented from trading with Market 1 and form separate market on their own. Agent ε_7 is not incorporated into any closed circuit, excluding it from market activities. This changes when the switch between nodes $\mathcal{N}_C$ and $\mathcal{N}_D$ closes and the agent enters the Market 2, forming a new mesh mesh $\mathcal{M}_4$ (the switch enables us to model the entrance or exit of agents). The corresponding matrices are displayed in 2.9. The dotted lines represent the separation of the markets and the gray entries apply when the switch between $\mathcal{N}_C$ and $\mathcal{N}_D$ is closed.

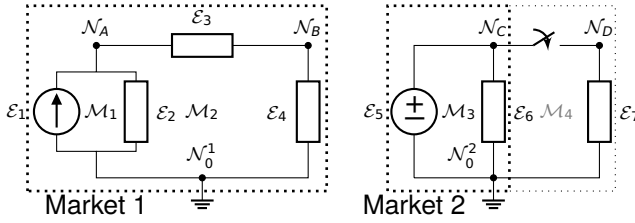


Figure 2.6: Each closed circuit represents a market

$$\mathbf{M} = \begin{matrix} & \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 & \varepsilon_5 & \varepsilon_6 & \varepsilon_7 \\ \begin{matrix} \mathcal{M}_1 \\ \mathcal{M}_2 \\ \mathcal{M}_3 \\ \mathcal{M}_4 \end{matrix} & \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \end{matrix}, \quad (2.9)$$

and

$$\mathbf{T} = \begin{matrix} & \mathcal{M}_1 & \mathcal{M}_2 & \mathcal{M}_3 & \mathcal{M}_4 \\ \begin{matrix} \mathcal{M}_1 \\ \mathcal{M}_2 \\ \mathcal{M}_3 \\ \mathcal{M}_4 \end{matrix} & \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \end{matrix}. \quad (2.10)$$

2.3.5. Network Connectivity as Market Resilience

The resilience of a market due to agents exiting can be determined by inspecting the circuit's connectivity. In network theory, the connectivity is defined to be the minimum number of edges that must be removed to disconnect two nodes [63]. In an economic network, these are the minimum number of agents that need to exit the market to disrupt the connection between two exchange loci.

Markets with a high degree of competition are naturally resilient, while markets with a high degree of cooperation are particularly vulnerable. Competitive agents trade in parallel (Section 2.4) and all agents have to be removed for the trade to cease. On the contrary, when agents cooperate, which corresponds to agents in series interconnection, the exit of but a single agent severs the flow of the goods between the loci. This highlights the critical role of network topology for market resilience analysis.

2.3.6. Walrasian Markets and Magnetic Coupling

Walrasian demanders couple separate Marshallian markets into a single Walrasian market by virtue of their ability to maintain prices for two distinct types of commodities. The effect of one price on another is analogous to the magnetic coupling of topologically uncoupled circuits in electrical networks [1]. This analogy underscores the necessity of modeling price as the analog of magnetic flux and not as the analog of voltage.

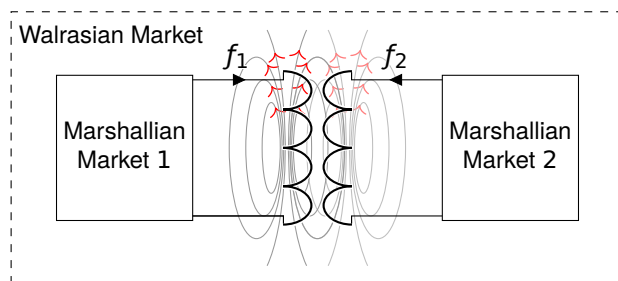


Figure 2.7: Walrasian market as two Marshallian market coupled through demand. The field lines permeating both inductors represent the price coupling between the two commodities.

The price coupling between two markets is visualized in Figure 2.7 in the same manner as magnetic coupling is visualized. The magnetic field lines permeating the coil at a market represent the demander's price for the commodity traded on it. The density of field lines permeating both coils represent the degree to which the two prices influence each other. The behavioral law 2.1, then specifies how any price changes then lead to adjustments in the quantities demanded on both markets.

2.3.7. General Market Clearing and Tellegen's Theorem

Both physical and price clearing conditions can be combined into a single general clearing condition for any economic network. In electrical network theory, the corresponding condition is known as Tellegen's theorem [57]. Using the network flow vector $\mathbf{f}$ (2.2) and the incentives vector $\mathbf{v}$ (2.5), it can be concisely formulated as:

$$\mathbf{f}^T \mathbf{v} = 0 \quad (2.11)$$

The product of the flows and their conjugate incentives is the rate at which the agents allocate the economic *surplus* among each other in the network (see [45]). In electrical networks, this is the electrical *power* that is being transferred between the elements in a network. Tellegen's theorem can thus be seen to assert a fundamental principle of conservation of economic surplus within an economic network, analogous to the conservation of energy in a electrical network. At a node, where the desirability level is equal among the agents, this manifests itself as physical clearing. In a mesh, where the mesh flow is shared among the agents, this manifests itself as price clearing. Together, they form a more fundamental principle of clearing of the economic surplus, consistent with the conservation law of surplus and energy (see, further, [45]).

Tellegen's theorem serves as an efficient framework for maintaining stock-flow consistency in Walrasian markets. By allowing the examination of the entire network of interconnected markets, it ensures that the sum of the products of flows and incentives around any market is zero, streamlining the process of confirming consistency across the market system without repetitive individual checks.

2.4. Aggregate Agents as Circuit Modules

Network simplification allows multiple components to be represented by a single equivalent component, reducing the complexity of a network [1]. In economic circuits, this allows us to aggregate the behavior of a group of elementary agents into a single representative behavioral law. In this section, we analyze two types of such aggregate agents.

2.4.1. Active Marshallian Agents

By aggregating a passive and an active agent, we obtain what we refer to as active Marshallian agents or real agents, which include consumers, producers, and traders. Active agents are analogous to real power sources in electronics, where the addition of a passive element to the ideal source is designed to capture the non-ideal behavior of the device. Active agents drive economic activity by allocating economic surplus [45] to the economic network.

In Figure 2.8, we illustrate how a consumer can be configured as an active Marshallian agent by combining the price-inelastic demand (current source) with price-elastic demand (inductor). The effective behavioral law of the consumer follows the behavioral laws of the elementary agents combined with the KCL (2.4):

$$f_d = u - \varepsilon_d p_d, \quad (2.12)$$

where u is the inelastic demand and $\varepsilon_d p_d$ is the elastic demand. This behavioral law yields the familiar downward-sloping demand line of a consumer.

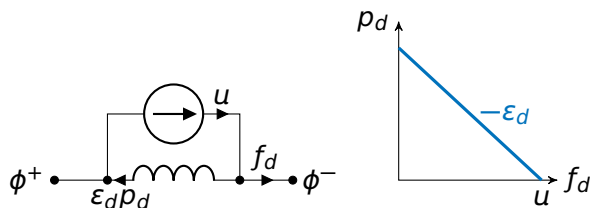


Figure 2.8: Active demander consisting of a current source and an inductor (left). The behavioral law $f_d = u - \varepsilon_d p$ results in the familiar downward sloping demand schedule of a consumer (right).

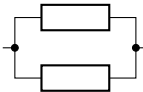
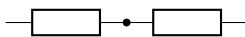
Numerous other configurations of active agents are possible. For instance, another type of consumer can be configured by taking insatiable incentives in parallel with an elastic demander and a frictional element that encapsulates the consumption itself [45]. Similarly, producers can be configured by reversing the flow direction of u , forcing the demander to act as a supplier. This leads to a flow quantity supplied of $f_s = u + \varepsilon_s p_s$ and the familiar upward sloping supply line.

2.4.2. Representative Agents as Equivalent Networks

Elementary agents of the same type can be aggregated into a single representative agent with an equivalent behavioral law. We consider two specific topologies within which the agents are aggregated: a *competitive* and a *cooperative* configuration. In a network, we configure competitive agents in parallel and cooperative agents in series (see Table 2.5). When in parallel—as, e.g., in a competitive market—the agents compete for the total flow available while being subjected to a common incentive and when in series—as, e.g., in a linear supply chain—the agents cooperate to establish a common flow while their incentives add up to that of a representative agent.

In Table 2.5 we summarize the behavioral laws for the various representative agents that can be constituted with each of the elementary Marshallian agents. In particular, it is seen that competitive demanders (or suppliers) act like a single representative demander whose price elasticity is the sum of the individual price elasticities. This is shown graphically in Figure 2.9 for the case of three suppliers and corresponds to the standard analysis of market supply (see e.g. [59]). The more individual demanders, the more price elastic the equivalent representative demander. Conversely, when cooperating, the resultant price elasticity is the inverse of the sum of the inverse price elasticities, which is shown graphically in Figure 2.10. There, the equivalent supplier has a lower price elasticity, thus rendering it less and less elastic as agents are added.

Table 2.5: Behavioral laws of representative agents

Type	Competitive	Cooperative
		
Demand	$f = (\sum_i \varepsilon_i) p$	$f = (\sum_i \varepsilon_i^{-1})^{-1} p$
Storage	$v = (\sum_i k_i^{-1})^{-1} q$	$v = (\sum_i k_i) q$
Friction	$v = (\sum_i b_i^{-1})^{-1} f$	$v = (\sum_i b_i) f$
Inelastic	$f = \sum_i f_i$	$f = f_1 = \dots = f_i$
Insatiable	$v = v_1 = \dots v_i$	$v = \sum_i v_i$

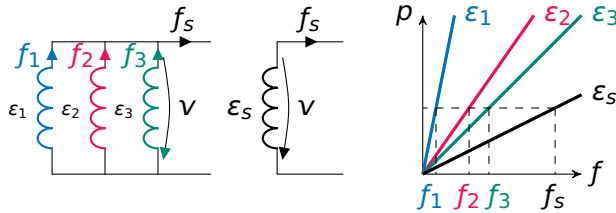


Figure 2.9: Competitive suppliers as parallel inductors (left). The three suppliers can be represented by a single equivalent inductor (center). The market supply aggregates over the individual supply schedules at the contemporary market price (right).

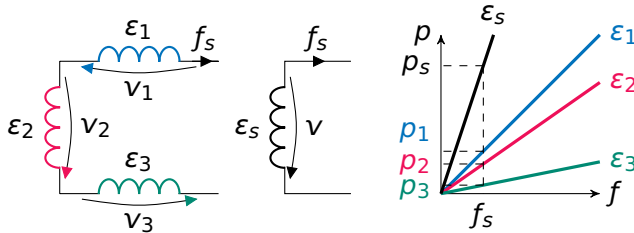


Figure 2.10: Cooperative suppliers as series inductors (left) and single equivalent supplier (center). The price p is aggregated over the supply schedules of the suppliers (right).

2.5. Economic Networks as Dynamical Systems

The imposed analogy enables us to formulate economic networks as abstract dynamical systems and leverage the methods thereof. In this section we demonstrate how this contributes to the analysis and computation of the dynamics of economic networks.

2.5.1. State-Space Representation

By combining the dynamic structure presented in Section 2.2 with the topological structure presented in Section 2.3, we obtain a dynamical systems model [14]. Such a model specifies how the prices and stock levels within the economic network change in time. Here we consider behavioral laws that are linear and whose parameters do not change over time (see Table 2.2), leading to linear time invariant (LTI) models [64].

The assumption that the behavioral laws are LTI allows us to express the dynamics of an economic network in a state-space representation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{w}, \quad (2.13)$$

where the overdot forms the standard notation for a time derivative. This representation simultaneously captures the endogenous ($\mathbf{A}\mathbf{x}$), exogenous ($\mathbf{B}\mathbf{u}$), and stochastic ($\mathbf{w}$) influences on the dynamics of the state variables within the network, see Figure 2.11. As we show in Section 2.5.3, this perspective enables us to conduct dynamic economic scenario analyses. In the following, we demonstrate how the state-space representation of an economic network is derived.

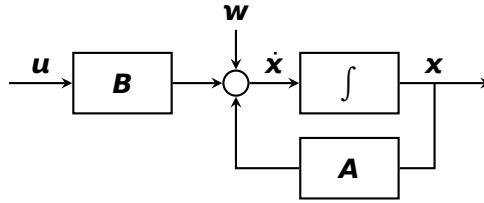


Figure 2.11: Block diagram of the state-space representation in (2.13).

The state of an economic network is determined by the stock levels of the storages and the price levels maintained by the demanders. We collect these into a single state vector

$$\mathbf{x} = (q_1 \ \cdots \ q_K \ p_1 \ \cdots \ p_L)^T, \quad (2.14)$$

where K is the number of storages and L the number of demanders. Of the passive agents, the frictional agents act as intermediaries in the trade activity and they do not contribute a state variable to the state vector (see Table 2.2). While the particular arrangement of the state variables is arbitrary, it is often convenient to arrange them into conjugate pairs (see (2.27)).

The inputs are determined by the active agents (see Table 2.2). Supposing that there are I active agents in a network, we have an input vector

$$\mathbf{u} = (u_1 \ \cdots \ u_I)^T, \quad (2.15)$$

of which each component is either an insatiable motive or an inelastic flow depending on the nature of the agent. Let I be the set of inelastic agents (current sources), and

S the set of insatiable agents (voltage sources). Then, for agent j

$$u_j = \begin{cases} f_j(t) & \text{when } j \in I, \\ v_j(t) & \text{when } j \in S \end{cases} \quad (2.16)$$

The particular arrangement of $\mathbf{u}$ is arbitrary and can be chosen for convenience. The behavior of the active agents depends on time alone. For this reason, they are properly considered to be exogenous variables, consistent with their role of inputs.

The input and state vectors determine the deterministic part of the rate of change,

$$\dot{\mathbf{x}} = (\dot{q}_1 \ \cdots \ \dot{q}_K \ \dot{p}_1 \ \cdots \ \dot{p}_L)^T, \quad (2.17)$$

of the state vector. Its components are the rates $\dot{q}_k$ at which the commodities are stocked up and the rates $\dot{p}_l$ with which the prices are adjusted. These equal the flow rate into the k th storage and the incentive over the l th demander, respectively, i.e.:

$$\dot{q}_k = f_k \quad \text{and} \quad \dot{p}_l = v_l$$

We use the topological structure of the network to formulate the contribution of the agents participating in the network to the rate of change $\dot{\mathbf{x}}$ of the state. This involves that for the stock up rate $\dot{q}_k$, we consider either one of the exchange loci, say $\mathcal{N}_n$, to which the storage is connected and clear this with the flows from the other agents into this locus consistent with the KCL. For the price movement $\dot{p}_l$ we consider the mesh $\mathcal{M}_m$ in which that demander is adjusting its price and clear it with the incentives of the remaining agents contributing to this mesh consistent with the KVL.

Next, we use the dynamic structure to express the flows and incentives, either as a function of the state $\mathbf{x}$ or of the time t . This is achieved by substituting the behavioral laws for the flows and incentives until we recover either a function of $\mathbf{x}$ or of t and arranging these obtained functions in the order in which they appear in the state vector. The procedure is summarized in (2.18).²

The result is the state-space representation of the dynamical-systems model of the network. The procedure makes it evident that it is determined by the topological structure—as given by the matrices $\mathbf{N}$ and $\mathbf{M}$ —together with dynamical structure—as given by behavioral laws f_j and v_j .

Because the behavioral laws are linear, the dependence of the vector $\dot{\mathbf{x}}$ on the state can be summarized by a single matrix equation $\mathbf{Ax}$, which captures the endogenous dynamics of the network. The matrix $\mathbf{A}$ is known as the state matrix and it contains the parameters of the behavioral laws of the agents within the network. The flows and incentives that depend on time are inputs, conform (2.16), and hence the dependence of $\dot{\mathbf{x}}$ on time can be summarized by the equation $\mathbf{Bu}$, which captures it as a dependence on the exogenous inputs. The matrix $\mathbf{B}$ is known as the input matrix. The sum of $\mathbf{Ax}$ and $\mathbf{Bu}$ then gives the total of the deterministic effects on $\dot{\mathbf{x}}$.

The stochastic agents determine the uncertainty in the dynamics of the state. We collect their contribution into a noise vector

$$\mathbf{w} = (w_1 \ \cdots \ w_{K+L})^T,$$

²Note that in this procedure each particular f_j or v_j is either a function of $\mathbf{x}$ OR a function of t .

which has the same dimension as the state vector. The influence of a stochastic agent on a storage or a price is determined by placing the components at the appropriate location in the noise vector. In the absence of any noise, a zero is entered. This arrangement allows us to express the contribution of the noise on the rate of change of the state vector as $\dot{\mathbf{x}} = \mathbf{w}$, which is added to the deterministic solution to give the state space representation in (2.13).

$$\begin{aligned}\dot{q}_k &= \frac{1}{\mathbf{N}_{nk}} \sum_{j \neq k} \mathbf{N}_{nj} f_j = \frac{1}{\mathbf{N}_{nk}} \sum_{j \neq k} \mathbf{N}_{nj} f_j(\mathbf{x}) + \frac{1}{\mathbf{N}_{nk}} \sum_{j \neq k} \mathbf{N}_{nj} f_j(t) \\ \dot{p}_l &= \frac{1}{\mathbf{M}_{ml}} \sum_{j \neq l} \mathbf{M}_{mj} v_j = \frac{1}{\mathbf{M}_{ml}} \sum_{j \neq l} \mathbf{M}_{mj} v_j(\mathbf{x}) + \frac{1}{\mathbf{M}_{ml}} \sum_{j \neq l} \mathbf{M}_{mj} v_j(t) \quad (2.18)\end{aligned}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

2.5.2. Equilibrium

Equilibrium for an economic network requires that both the prices and the stocks remain stationary in time (see e.g. [64]). The requirement for equilibrium is formulated by requiring that either

$$\dot{\mathbf{x}} = 0 \quad \text{or} \quad E[\dot{\mathbf{x}}] = 0 \quad (2.19)$$

The first formulation suffices for deterministic systems, while the second formulation generalizes to stochastic systems.

The value of an equilibrium state, $\mathbf{x}^*$, can be found under the condition that the state matrix is invertible in the following manner:

$$\mathbf{x}^* = -\mathbf{A}^{-1}(\mathbf{B}\mathbf{u} + E[\mathbf{w}]) \quad (2.20)$$

When the state matrix is not invertible, alternative approaches such as null-space analysis, numerical techniques, parameterized solutions, or the use of a pseudoinverse can be used to find equilibrium states (see e.g. [65]). The input-output perspective allows us to consider equilibria that change in time, depending on the nature of the exogenous influences ($\mathbf{u}$) and the mean of the stochastic influences ($E[\mathbf{w}]$). For instance, if the network is subjected to an exogenous sustained shock in inelastic demand, it will have two equilibrium states, an initial one before the shock and a final to which it attempts to move. In the theory of demand, this is interpreted as a shift in the demand curve (see Figure 2.12 in the next subsection, wherein the actual movement in time to equilibrium is analyzed). More drastically, if the network is instead subjected to a periodically changing inelastic demand, its equilibrium state changes continually over time. A changing $E[\mathbf{w}]$ on the other hand will result in a drift of the equilibrium state.

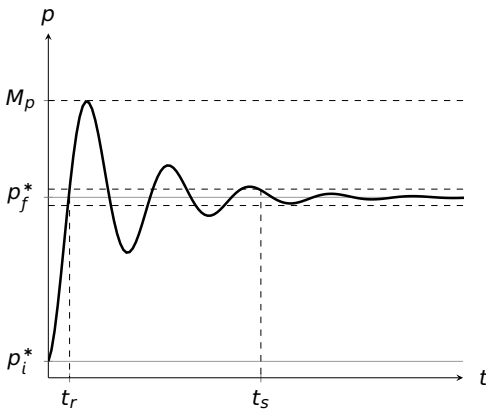
The existence of an equilibrium state does not guarantee that the economic network will achieve this equilibrium. When the eigenvalues of $\mathbf{A}$ are negative, the network is unstable and the prices and inventories will diverge from the equilibrium (see e.g.

[66]). In addition, when the time-period of a varying exogenous input (such as a cyclical inelastic demand) is shorter than the time it takes the network to adjust to the changing equilibrium, the economic network finds itself chasing but not reaching equilibrium.

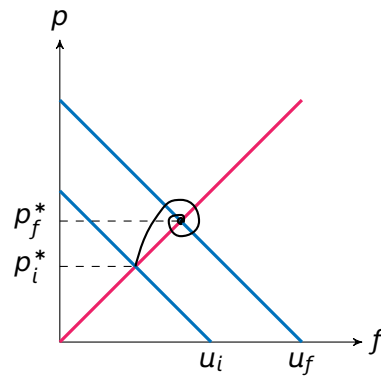
2.5.3. Transient Analysis for Dynamic Economic Scenario Analysis

The state-space representation of the dynamics allows us to analyze how the network transitions from one equilibrium state to another after being subjected to a shock. In control theory, this is known as transient analysis. For simple economic networks, transient responses can be determined analytically, while for more complex models they can be approximated or determined numerically [64]. For economics, the methods of transient analysis allow us to conduct *dynamic* scenario analyses, where the emphasis is on the precise movements of price levels and stock quantities within an economic network.

The plot in Figure 2.12 shows a typical response of a second-order system to a step input consisting of a sudden sustained shock. A corresponding second-order economic network consists of a single representative demander, storage, and frictional agent as passive agents. Such systems represent, e.g., traders who maintain both a price and an inventory (see also [45]). Under a demand shock, the price response exhibits the damped cyclical passage to the equilibrium value p_f^* from the initial equilibrium value p_i^* .



(a) Time graph of the transient response between an initial price equilibrium p_i^* to a final one, p_f^* . Also included are the rise time t_r , settling time t_s , and maximum overshoot M_p .



(b) Equilibrium states pictured as a shift in the demand curve (due to a shift from u_i to u_f) and a hypothetical transient response between these.

Figure 2.12: Transient response to a demand shock of a network consisting of a demander, storage, and trade friction.

From control theory [64], we can adapt several metrics to quantify the price

movement. In Figure 2.12, we include the rise time t_r , the maximum overshoot M_p over the final equilibrium price, and the time t_s at which the price settles to within a certain error of the final equilibrium price. The rise time quantifies the short-run price rigidity and the overshoot shows how much the price rises above its new equilibrium before settling to it. The settling time provides a point in time separating the short run from the long run. A related concept used in econometrics is the speed-of-adjustment parameter [67]. In the absence of any friction, the system exhibits an indefinite cycle around the equilibrium state, never settling.

Figure 2.12b contains an illustrative picture the transient as a time-parameterized curve in the price-flow plane wherein demand and supply lines are typically graphed. The actual movement consists of a simultaneous damped oscillations along the supply and demand curves.

2.6. Applications

Economic network theory enables us to design and analyze economic systems models in the same manner as an electrical engineer would design an analyze an electronic system. In this section, we demonstrate this design process for several economic systems.

2.6.1. Design and Analysis of a Two-Commodity Walrasian Market

In this subsection, we illustrate the application of the theory by analyzing a two-commodity Walrasian market shown in Figure 2.13. The market model serves to demonstrate the various analogies between the electrical components and agents and between the topologies and interactions introduced in this chapter.

Design of the Network

To model the trade in two commodities we consider two Marshallian markets formed by the two circuits in Figure 2.13. We label the agents with a superscript $i \in \{1, 2\}$ to indicate the market in which they are active. The markets are identical with the exception of the stochastic agent $\mathcal{V}^1$ which is active in Market 1 alone.

In each market, a cooperative monopoly of, say a wholesaler S_1^i and a retailer S_2^i , delivers the commodity. In addition, two storages C_1^i and C_2^i compete for the inventory stock and two brokers $\mathcal{R}_1^i$ and $\mathcal{R}_2^i$ compete to benefit from any mismatches between the flows supplied and demanded by offering premiums or discounts on the price level. Two Walrasian demanders, $\mathcal{D}_1$ and $\mathcal{D}_2$, compete for both commodities. By choosing between the commodities from both markets based on their cross-elasticity, they establish the price coupling between the two markets. The demanders are also configured in an active role as consumers by being paired with a source of inelastic demand in each market, $\mathcal{U}_1^i$ and $\mathcal{U}_2^i$, providing economic surplus. In the market for commodity 1, there is a stochastic agent $\mathcal{V}^1$ that introduces noise in the form of a stochastic insatiable motive.

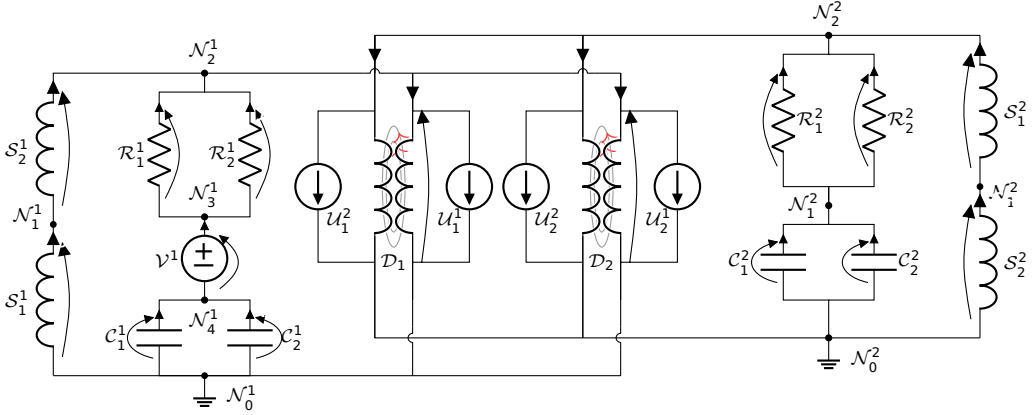


Figure 2.13: Network for a Walrasian market in two commodities. Each Marshallian submarket consists of two suppliers (inductors on the outside), two consumers (inductor-source combinations in the middle), brokers (resistors), and storages (capacitors). The Walrasian nature of the demanders is indicated by the overlapping flux lines (red arrowheads).

Network Simplification

Before deriving a state-space representation, we simplify the network by identifying any representative agents, consolidating the elementary agents based on whether they act competitively or cooperatively. From Figure 2.13, we immediately see that the storages C^i , brokerages $\mathcal{R}^i$, and the inelastic agents U^i act competitively. We simplify each of those into a representative agent with the parameters

$$k_i = \frac{k_{C_1^i} k_{C_2^i}}{k_{C_1^i} + k_{C_2^i}}, \quad b_i = \frac{b_{\mathcal{R}_1^i} b_{\mathcal{R}_2^i}}{b_{\mathcal{R}_1^i} + b_{\mathcal{R}_2^i}}, \quad u_i = u_{U_1^i} + u_{U_2^i}, \quad (2.21)$$

respectively. We consolidate the demanders and suppliers into a single representative Walrasian agent. This requires several steps. First, we consolidate the cooperative suppliers and competitive demanders in each Marshallian market into the representative suppliers S^i and demanders D^i with respective price elasticities

$$\varepsilon_{S^i} = \frac{\varepsilon_{S_1^i} \varepsilon_{S_2^i}}{\varepsilon_{S_1^i} + \varepsilon_{S_2^i}}, \quad \text{and} \quad \varepsilon_{D^i} = \varepsilon_{D_1^i} + \varepsilon_{D_2^i}. \quad (2.22)$$

Then, we consolidate these into a single representative Walrasian demander/supplier $\mathcal{D}S$. To determine its elasticity tensor ε , we note that the representative supplier and demander act competitively in each Marshallian market and that the price coupling between the markets follows from the demand alone. These two observations lead to the following expressions for the principal and cross-elasticities of demand, respectively:

$$\varepsilon^{ii} = \varepsilon_{S^i}^{ii} + \varepsilon_{D^i}^{ii}, \quad \varepsilon^{12} = \varepsilon_{D^1}^{12} + \varepsilon_{D^2}^{12} \quad (2.23)$$

These form the components of the elasticity tensor ε , which is the parameter of the behavioral law of the representative agent $\mathcal{DS}$. The law relates the vector of excess flows in the markets to the vector of markets' price levels.

Consolidating the elementary agents in the network of Figure 2.13 into representative ones leads to the simplified network given in Figure 2.14. Its dynamics are equivalent to those of the original design.

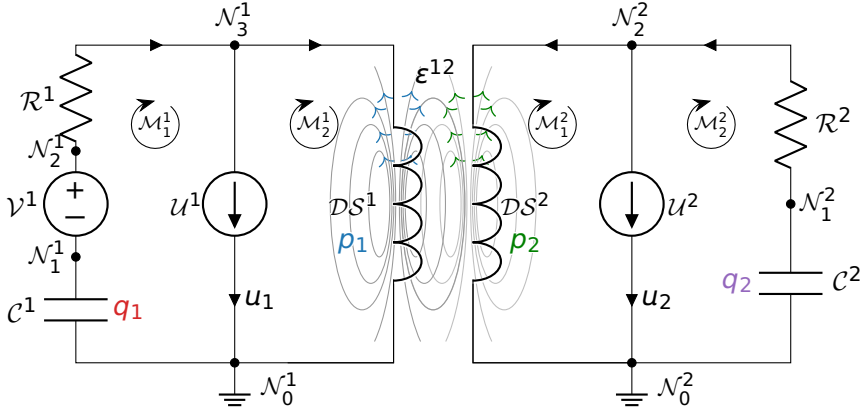


Figure 2.14: Reduced network with representative agents.

State-Space Representation

To determine the state-space representation of the dynamics of the reduced network, we first establish its incidence and agent-mesh matrices. Following the procedure outlined in Section 2.3, we find:

$$\mathbf{N} = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 & : & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & : & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & : & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & : & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & : & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & : & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & : & -1 & -1 & 0 & 1 \end{pmatrix}, \quad (2.24)$$

and

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & -1 & 0 & : & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & : & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & : & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & : & 0 & 1 & -1 & -1 \end{pmatrix}. \quad (2.25)$$

Here, the rows of $\mathbf{N}$ in top-down order represent $\mathcal{N}_0^1, \mathcal{N}_1^1, \mathcal{N}_2^1, \mathcal{N}_3^1, \mathcal{N}_0^2, \mathcal{N}_1^2$, and $\mathcal{N}_2^2$; the rows of $\mathbf{M}$ represent $\mathcal{M}_1^1, \mathcal{M}_2^1$, and $\mathcal{M}_2^2$; and the columns of both matrices

represent, from left to right, $\mathcal{C}^1$, $\mathcal{V}^1$, $\mathcal{R}^1$, $\mathcal{U}^1$, $\mathcal{DS}^1$, $\mathcal{DS}^2$, $\mathcal{U}^2$, $\mathcal{C}^2$, and $\mathcal{R}^2$. The dotted lines indicate the separation between the markets. This is further corroborated by the markets-mesh matrix

$$\mathbf{T} = \left(\begin{array}{cc|cc} 4 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ \hline 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 3 \end{array} \right), \quad (2.26)$$

consisting of two diagonal blocks.

With the incidence and branch-mesh matrices established, our next step is to follow the procedure in Section 2.5.1. In total, there are two price levels and two stock quantities (see Figure 2.14), making the state vector four-dimensional. We organize it (for the purpose of the following analysis) into two consecutive conjugate price-stock pairs, one for each market, as follows:

$$\mathbf{x} = (\mathbf{q}_1 \quad \mathbf{p}_1 \quad \mathbf{q}_2 \quad \mathbf{p}_2)^T \quad (2.27)$$

We have color-coded the state variables to assist with the interpretation of the simulation results in the next subsection. Figure 2.14 depicts the agents to which the state variables pertain.

The input vector consists of the inelastic-flow components of the consumers and the noise vector has a single component containing the motives of the stochastic agent. Following the procedure, we find the state-space model:

$$\begin{pmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{p}}_1 \\ \dot{\mathbf{q}}_2 \\ \dot{\mathbf{p}}_2 \end{pmatrix} = \begin{pmatrix} 0 & -\varepsilon^{11} & 0 & -\varepsilon^{12} \\ k_1 & -b_1\varepsilon^{11} & 0 & -b_1\varepsilon^{12} \\ \hline 0 & -\varepsilon^{12} & 0 & -\varepsilon^{22} \\ 0 & -b_2\varepsilon^{12} & k_2 & -b_2\varepsilon^{22} \end{pmatrix} \begin{pmatrix} \mathbf{q}_1 \\ \mathbf{p}_1 \\ \mathbf{q}_2 \\ \mathbf{p}_2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ b_1 & 0 \\ \hline 0 & 1 \\ 0 & b_2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \mathbf{w} \\ 0 \\ 0 \end{pmatrix} \quad (2.28)$$

The state-space model allows us to analyze various aspects of the market's dynamics, including its causality and the conjugacy between the state variables. We consider the effects of the stochastic and inelastic agents first and then how the state matrix determines how these effects are propagated in time by the network's endogenous dynamics.

The stochastic motives are configured to directly contribute to the time rate $\dot{\mathbf{p}}_1$ at which the price in Market 1 changes. It requires the representative demander to constantly readjust its price level in a random manner.

We next consider the input matrix that captures the exogenous effect of the inelastic agents on the dynamics. In each market, the inelastic demand directly relieves the

inventory stock of the commodity it is demanding (first and third row of the input matrix). Concurrently, they adjust the market price levels, a process intermediated by the frictional representative agent who broker the flow. The latter is captured by the second and fourth row of the input matrix.

Finally, we consider the endogenous effects captured by the state matrix. The state matrix consists of four 2×2 blocks and the network consists of two coupled Marshallian second-order markets. The diagonal blocks determine the internal endogenous dynamics of the Marshallian markets and the off-diagonal blocks the coupling between them. Indeed, if we remove the price coupling by setting the cross-elasticity of demand $\varepsilon^{12} = 0$, the off-diagonal blocks contain only zeroes and the Marshallian markets become dynamically independent. For non-zero values of ε^{12} , the price in one markets affects both the flow and the price in the other. Positive values of ε^{12} lead to the quantity in the other market decreasing and the commodities are substitutes of one another. Negative values, on the other hand, characterize commodities that are complements. The zeros in the off-diagonal blocks acting on the inventory state variables show that the markets are physically disconnected.

The market friction is given by the diagonal elements of the blocks. In this position, they determine the influence of one price on itself or on the price in the other market. The internal market friction is given by the rate of discount $b_i \varepsilon^{ii}$, in units of inverse time [45], on the price that is required to adequately incentivize the intermediating broker to pass the commodities from the storage to the rest of the market. For the intermarket price coupling, the required discount rate depends on the cross-elasticity of demand instead.

The conjugacy of the state variables is borne out by the off-diagonal elements of the blocks. The price level in a market places downward pressure on the conjugate stock-up rate due to reduced demand. Consistent with a linear law of demand, the effect is proportional to the price elasticity $-\varepsilon^{ii}$. The stock level, in turn, exercises an upward pressure on the conjugate price due to the increased incentive to hold the commodity. This incentive gives a measure to the convenience of holding the commodity and it is proportional to the storage preference k_i . The stock-up rate in one market is also pressured by the price level in the other market. This effect is proportional to the cross elasticity of demand $-\varepsilon^{12}$, consistent with the Walrasian nature of the demanders.

Dynamic Scenario Analysis

To demonstrate the influence that the agents in the Walrasian market have on the dynamics, we subject the network to a sustained inelastic demand shock in Market 1 and analyze how it adjusts to this. Specifically, for the inputs in the state-space representation, we set:

$$u_1(t) = \begin{cases} 30 \text{ \#}_1/\text{dy}, & t \geq 1 \text{ dy} \\ 20 \text{ \#}_1/\text{dy}, & t < 1 \text{ dy} \end{cases} \quad \text{and} \quad u_2(t) = 20 \text{ \#}_2/\text{dy}, \quad (2.29)$$

where $\text{\#}_i$ denotes the unit of the commodity traded in Market i . The shock induces both markets to move from an initial equilibrium state to a final one. In the following,

we analyze the transient response of the markets under different variations of the parameter values. The results are contained in Figure 2.15. In addition, we have made an interactive version of the source code available.³

From Figure 2.15a we see that when $k = 0$ and there is no preference to store the commodity, Market 1 remains understocked and Market 2 remains overstocked in the long run. However, when $k > 0$ and there is a convenience to holding the stock, inventories settle at their desired level. Prices in both markets, on the other hand, remain at elevated levels in the long run, regardless of a preference to store. This in reaction to the permanent increase in quantity demanded.

Figure 2.15c shows the effect of the stochastic agent on the equilibrium state. The stochastic motive follows a normal distribution with zero mean and variance σ^2 . We observe that the state variables have no deterministic equilibrium and only achieve an equilibrium in expectation. The intensity of the random behavior increases with increasing variance. Its effects are most pronounced on p_1 since the stochastic motive acts directly on it. It causes its conjugate storage to be affected and then, through the inter-market price coupling, the price and storage levels in Market 2. Each causal step in this chain attenuates the intensity of the random behavior.

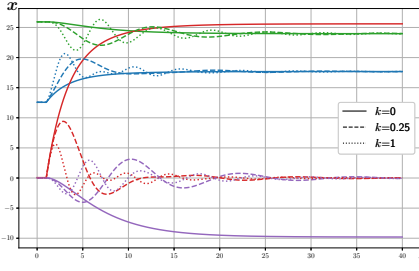
From Figure 2.15b we see how the market friction influences the settling time. It determines the separation of the short run from the long run. When $b = 0$ and there are no trade frictions, neither stocks nor prices ever settle and, instead, remain volatile by cycling around their equilibrium values. For values of $b > 0$, the signals settle, with the settling time itself shortening with increasing trade friction, as the available surplus is consumed more and more rapidly.

Other factors that influence the settling time are the preference to store and the price elasticity. Figure 2.15a shows that increasing k ensures shorter settling times, as the agents are motivated by the perceived convenience of holding the commodity. In general, settling times improve with higher price elasticities, as market participants are increasingly motivated to adjust their price. Figure 2.15d illustrates the effect of the cross-elasticity of demand ε^{12} . We see that independent goods have the shorter settling times and that complementary and substitutions are comparable in the long run.

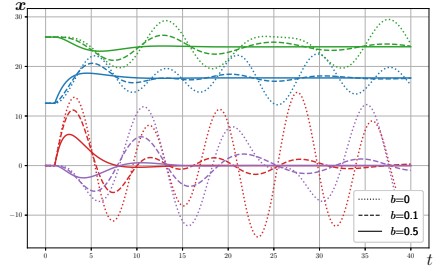
As concerns the rise time, both the storage and the friction are important factors in its determination. We see that when the commodity is convenient to hold, rise times shorten and, as a result, prices appear less sticky and inventories are more responsive. A similar effect occurs with increasing friction, extending rise times and making prices stickier due to the reduction in market liquidity.

Figure 2.15a shows that, although higher storage preferences lead to shorter rise and settling times, this comes at the expense of increased price volatility, both in frequency and maximum overshoot. An increased convenience of storage induces the agents to turn over their inventories at a higher rate. Friction, instead, reduces volatility. Whereas rapid stock turnover rates would be attractive to benefit from anticipated changes in demand when costs are low, the added costs of trading causes the market participants to slow down their trade cycles and also reduces

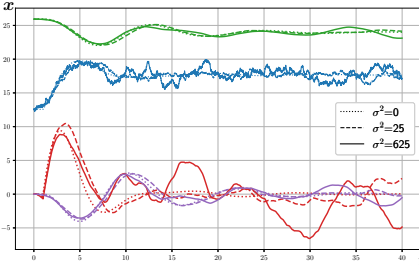
³https://colab.research.google.com/drive/1wOQ6LXt1PW74byJQyo_UBV2xnxLv1mPO?usp=sharing



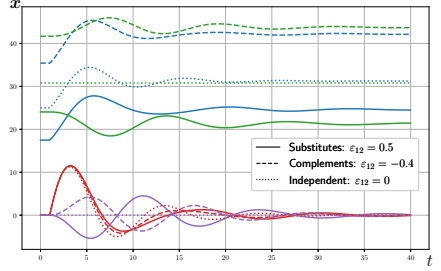
(a) Varying storage preferences. When there is no preference (solid plots), the backlogs persist. Increasing the preference leads to less stickiness but more volatility.



(b) Varying the market friction. In the absence of friction, stocks and prices do not settle but sustain a cycle. Higher friction leads to shorter settling times and lower volatility.



(c) Influence of the variance of the disturbance on the randomness of the stocks and prices. The influence reduces as it propagates through the causal chain of the dynamics, from p_1 through q_1 to p_2 and finally q_2 .



(d) Varying the cross-elasticity of demand. The price of substitute commodities move counter (solid), complements (dashed) along, and independent (dotted) do not effect each other.

Figure 2.15: Transient responses of the state variables to a step demand shock in Market 1 for various choices of the model parameters. State variables are q_1 (red), p_1 (blue), q_2 (purple), and p_2 (green). Parameters $k = k_1 = k_2$ and $b = b_1 = b_2$ for all figures and, unless indicated otherwise, $\varepsilon^{11} = 2.15$, $\varepsilon^{12} = 0.5$, $\varepsilon^{22} = 1.3$, $k = 0.25$, and $b = 0.2$.

their desire to hold the stock too far from the desired levels.⁴

The cyclical nature of the responses allows to also identify which state variables are leading, lagging, and contrarian indicators of others. In network theory, the phase shifts between the signals is used for this purpose. From Figure 2.15a we see that the stock level in Market 1 leads its conjugate price by a relatively small phase shift. This is because the demand shock is met at the onset by the available inventory. The price in Market 1 then leads the price in Market 2 and finally the latter's conjugate

⁴Although this is not shown in the figure, the overshoot and the frequency disappear entirely when the friction is high enough. Instead of volatility, one observes a hyperbolic discounting of the price (see [45]).

stock level. Each step in the causal chain adding to the phase shift with the stock level in Market 1. As a result, the price in Market 2 is almost half a turn out of phase with the stock level in Market 1, with the two variables become close to being contrarian indicators of each other. The leading, lagging, and contrarian nature of the state variables is congruent to the causality embedded in the state-space model.

2.6.2. System Identification of a Price-Dynamical Oil Market Model

The use of circuit theory has the advantage that established system identification techniques for determining the parameters of dynamic models can be exploited for economic modeling. In this section, we summarize its application to a model for the spot market for crude oil. The material is taken from [68], to which the reader is referred to for the details.

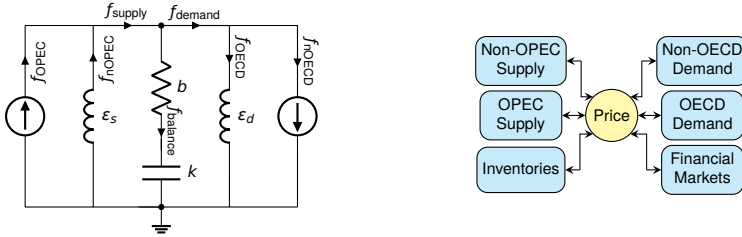


Figure 2.16: Left: Economic circuit for the oil-market model. Right: Schematic of the IEA model from [68].

Figure 2.16 shows a circuit representing the spot market for crude oil. The circuit is based on the bond-graph model in [68]. It contains the price drivers identified by the US Energy Information Administration (EIA) [69], also shown in the figure. These include the OPEC (current source) and non-OPEC (inductor) supply on the left, the financial markets (resistor) and inventory (capacitor) effects in the center, and the OECD (inductor) and non-OECD (current source) demand components on the right.

The circuit can be reduced to a two-state system with market price p and inventory q with the following state-space representation:

$$\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -b(\epsilon_s - \epsilon_d) & -k \\ \epsilon_s - \epsilon_d & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} -b \\ 1 \end{pmatrix} u, \quad (2.30)$$

where the input $u = f_{OPEC} - f_{NOECD}$ is the net inelastic supply. The parameters (b , k , ϵ_s and ϵ_d) were estimated using grey-box identification, thus incorporating the structure of the circuit. For comparison, a third-order black-box model was trained on the same dataset.

The circuit was subjected to the historical net supply for oil during the period July 2002 to May 2003, during which a substantial supply shock occurred due to the Venezuelan oil strike. The input and the price responses are graphed in Figure 2.17

against the actual realized price in the period. Despite being of a lower order than the black-box model, the price response of the circuit provided a closer fit with the actual price movement (VAF=83% vs. VAF=55%). This should be contributed to the causal structure of the circuit model.

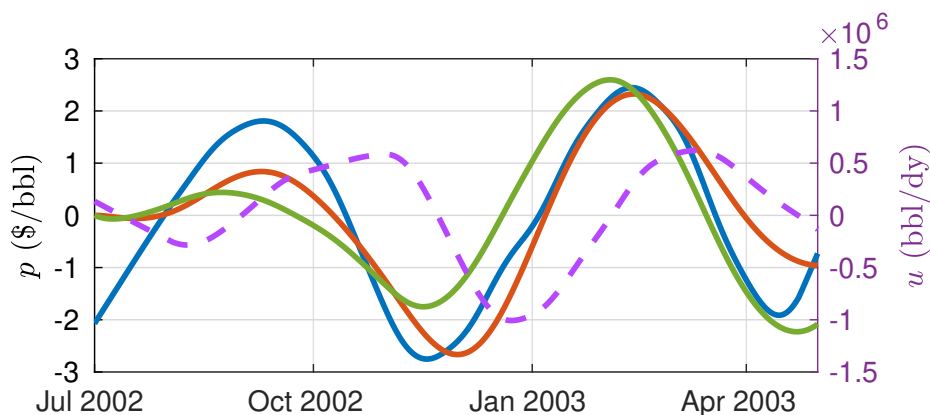


Figure 2.17: Price response as determined by the economic circuit (red) and by black-box model (green) vs. the historical price data (blue), together with the supply input (purple dashed). (From [68].)

The conjugacy between price and stock variables allows us to validate the grey-box model on historical inventory data, since this dataset has not been used for identification. The results, plotted in Figure 2.18, show that the circuit is capable of following the dominant frequency component of the stock. Such a validation step would not be possible with the black-box model.

The performance of the circuit can be improved by modeling additional price drivers, thereby increasing the order of the model beyond two. For instance, the dip in measured inventory stocks during the winter of 2002-2003 appears to be due to the increase in demand during cold weather. By including such effects in the design of the circuit, its performance can be systematically improved.

2.7. Conclusions and Further Research

In this chapter, we propose an economic circuit theory. It is based on electrical circuit theory and, similar to its electrical counterpart, it is particularly applicable to economic systems that are out of an equilibrium state, where prices are volatile and demand is continuously changing. In addition, its basis in economic laws and its causal structure make the models predictive and interpretable. This allows for an engineering perspective on economic modeling, focusing on the design of new systems in addition to describing existing ones.

The distinguishing feature of an economic circuit model is that it combines a physical stock-flow with a price-incentive consistency condition. This combination yields a causal description of the network dynamics, with prices levels driving the

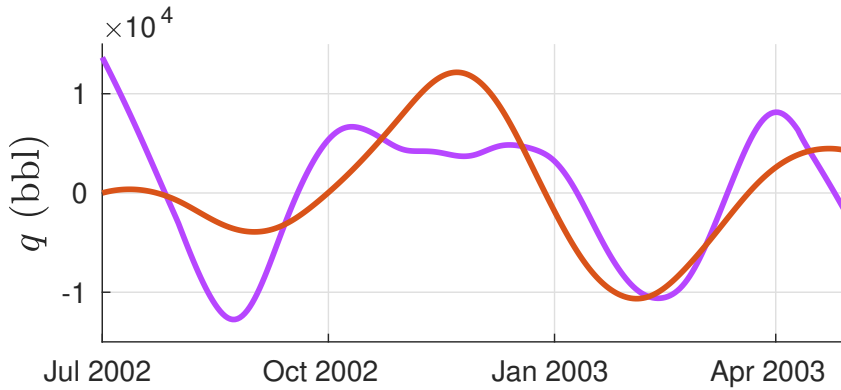


Figure 2.18: Out-of-sample prediction of inventory levels (red) vs. historical inventory data (purple). (From [68].)

flows and the conjugate stock quantities driving the incentives. This allows us to derive our main result: a state-space representation of the dynamics, i.e., a set of differential equations specifying the rate of change of the price levels and stock quantities of the market participants. The representation accounts for both endogenous and exogenous influences, the latter both deterministic and stochastic.

The economic circuit theory presented in the chapter has several shortcomings and potentials for further development.

In our derivation we assume that the agents' behavioral laws are linear and time invariant. In practice, this implies that the models are valid only for relatively small variations in the price levels and stock quantities and that the parameters are stationary in the relatively short run. Nevertheless, applications to electrical systems have shown that complex dynamic behavior can be described with linear models alone, simply by increasing the order of the model. Alternatively, one can resort to nonlinear and time-varying network theories.

To test the validity of the models in real-world applications, its parameters have to be estimated. This issue is not addressed in the chapter. It should be noted that the assessment burden is comparatively light, as the structure strongly constrains the class of possible models. (To wit, the example market in Section 2.6.1 requires only six real parameters to be estimated, despite the complexity of the response.) Although efficient methods for system identification exist in the engineering literature (e.g. [70]) and identification has been applied to a comparative model in [68], more research is needed to determine what type of data sets are needed for economic systems and to determine whether these are available.

Although our theory is agent-based, its reliance on elementary agents (i.e., as electrical two-terminal components) limits its potential for agent-based modeling. Specifically, the representative agents defined in the chapter consist of elementary agents of the same type. To develop agent-based systems, a more general definition of representative agents that combines several types of elementary behavior, together with the rules specifying their interaction, is required. In electrical network theory,

this matter is resolved by taking a port-based approach. In a forthcoming publication, we show how it allows one to configure representative agents (known as multiports) that exhibit highly complex emergent behavior and interact through their ports. Using network analysis methods, systems can be designed in a bottom-up fashion. Using network synthesis methods, one can proceed in a top-down fashion, by prescribing the behavior and then determining an actual network that behaves accordingly.

An opportunity presented by economic circuits is that they are amenable to being controlled using methods from control theory. One thinks of a government, policy makers, or a regulating agency as separate from the system as represented by the network. With a state-space representation, an output representing some endogenous variable can be fed back and compared to a desired value. The loop is closed by the controller who provides the appropriate incentives at the input for the system to adjust its behavior. For linear systems, efficient methods exist for their control. An additional theoretical benefit is that the behavior of the system and the policy makers are not commingled, enabling the designer to circumvent the Lucas critique. We consider the exploitation of control theory a particularly fruitful avenue of investigation and we are currently actively researching it.

3

Modeling Economic Systems as Multiport Networks

3.1. Introduction

Multiport network theory has proven to be a powerful tool in electronics and mechatronics, especially when this concerns the design and analysis of highly complex systems. The theory is based on electrical circuit theory, to which it adds several key features. These include, in particular, the ability to represent the system at different levels of abstraction, its modular nature, and its high level of computational efficiency [1, 14, 71].

In this chapter, we leverage these features to model, design, and analyze economic systems.

It builds on the previous chapter that developed economic circuit theory. Economic circuits are electronic circuits whose variables and parameters are replaced by their economic analogs. Their primary advantage in this context is that circuits provide an explicitly dynamic representation, with differential equations that describe the flow of goods and their price [45]. A critical disadvantage of circuit theory that we have found is that they are inefficient for modeling complex systems. Economic circuits are built by interconnecting elementary agents or components into networks. As such, they are built exclusively at the micro level and, therefore, rapidly become unmanageable when they grow in size.

Using multiport theory, models can be designed at any level of abstraction. It provides for modules called multiports or n -ports, which internally contain entire circuits but are treated like “black boxes.” A multiport can represent any economic unit or agent that is appropriate to a particular level of abstraction: from the macro level of an entire economy, down through sectors, subsectors, individuals, to the micro level of components. The theory thus fits in a more general trend in economics to account for micro-level heterogeneity at the macro level [72–75].

By modeling in a top-down fashion, going from the macro level through intermediate levels down to the micro level, the design of complex systems becomes manageable. In the economic tradition, the Macroeconomic Dynamic Stochastic

General Equilibrium (DSGE) models [17, 22, 76] are built in this fashion. It is assumed that macro-level sectors can be represented by what are known as representative agents.

The concept of representative agents has come under criticism in the economic literature for failing to account for heterogeneity at lower levels of abstraction [20, 49, 77, 78]. However, by treating a representative agent as a multiport, a fully dynamic description of its micro-level behavior at the component level is obtained. Moreover, the heterogeneity of representative agents can be specified at various levels of abstraction between the micro and macro levels. We demonstrate this in Chapter 4, where we compare a DSGE model from the literature to a multiport implementation.

Inversely, by working in a bottom-up fashion, the behavior at the macro level is treated as emergent from its specification as a circuit at the micro level. This is comparable to how Agent-Based Computational Economics (ACE) models are built [19, 79–81].

These, however, have been criticized for lacking a formalism for aggregating agents, instead relying on ad-hoc methods [82]. In addition, the models are notoriously computationally inefficient: Their reliance on Monte-Carlo simulations renders them relatively computationally intensive, difficult to calibrate, replicate, or extend systematically [23, 83].

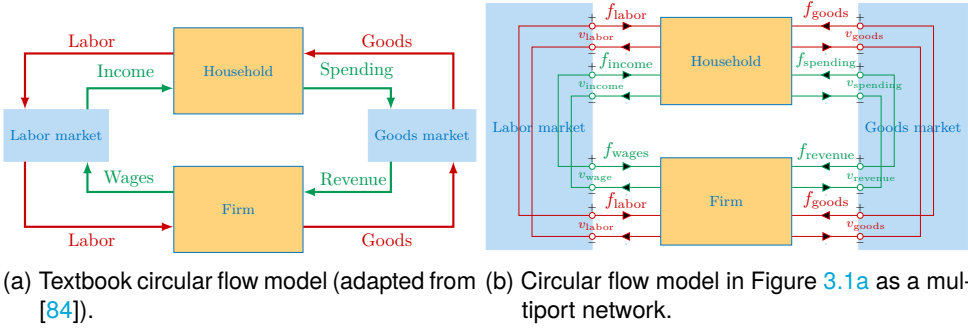
In contrast, multiport theory provides a structured formalism for aggregating micro-level agents into higher-level economic units, while constraining the way in which such aggregation is specified through its underlying network representation. We detail this in Section 3.3. In addition, a multiport network is readily implemented on modern circuit simulators like LTSpice, which are known to be computationally efficient and are capable of handling complex networks. This is demonstrated in Section 4 for a wide variety of applications.

The chapter is organized as follows: First, in Section 3.2, we outline the economic analogs to the basic concepts of the theory, like ports, interconnections, and the multiports themselves. Then, in Section 3.3, we examine the multiport agent, how its endogenous dynamic behavior is modeled by an internal circuit, and how this is summarized by a behavioral model. In Section 3.4.1, we show how the different multiport interconnection patterns allow us to aggregate agents to higher levels of abstraction.

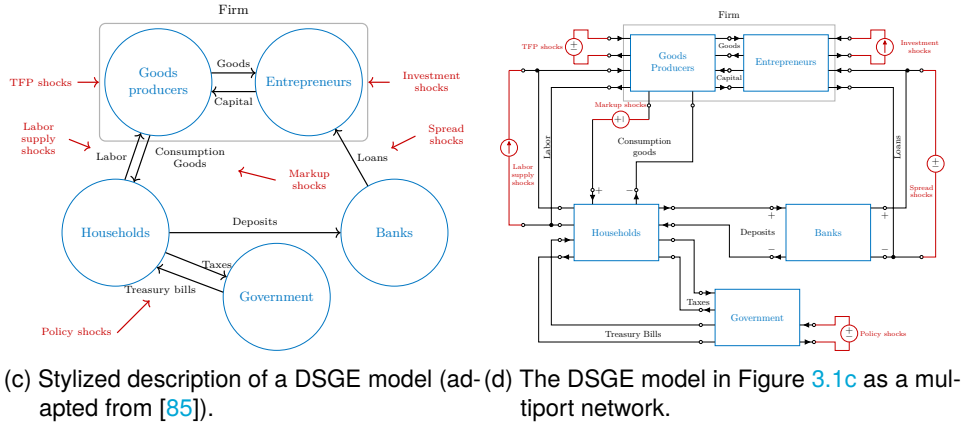
3.2. Economic Models as Multiport Networks

In the economic literature, authors draw schematic representations of their models to emphasize the logical structure. Figures 3.1a and 3.1c are typical examples, the former of a circular flow model, as given in a popular macro text [84], and the latter of a DSGE model by the New York Federal Reserve Bank [85]. The blocks or bubbles represent economic agents, which could be individuals, companies, or entire sectors of the economy. The arrows connecting them represent the direction of the flow of either money, goods, or services.

Engineers draw similar representations when designing multiport models. In Figures 3.1b and 3.1d, we give the multiport networks corresponding to these models.



(a) Textbook circular flow model (adapted from [84]). (b) Circular flow model in Figure 3.1a as a multiport network.



(c) Stylized description of a DSGE model (adapted from [85]). (d) The DSGE model in Figure 3.1c as a multiport network.

Figure 3.1: Traditional economic models (left) and their multiport adaptations (right).

The agents correspond to specific multiports, which are similarly pictured as boxes. The connections, however, appear as loops rather than arrows. They enter a multiport at a pair of terminals rather than a single point of entry. Such a pair is known as a port. A port allows an agent to communicate the incentive it requires for supplying the flow, together with the flow itself. This is further detailed in Section 3.3.3. Table 3.1 reproduces the relationship between the electrical and economic terminology from [24].

This is an important departure from the economic models in Figures 3.1a and 3.1c. These show what flows where, but not why. With conjugate incentives at its ports, an agent's endogenous dynamics —as specified by its internal economic circuit— determine the economic forces that cause the flows.

Although in general markets can be viewed as sophisticated multiport agents in their own right, we have modeled them in Figure 3.1b as flow-through entities (blue). They pair two associated flows, of which one is a good or service, and the other is the money provided as compensation: payment for the goods and remuneration for the labor. This makes a compelling connection with business practice. For instance, the revenue and goods ports at the firm can, together, be thought of as a point of sale (POS). We develop this connection further in Section 3.3.2.

Table 3.1: Analogy between economic and electrical signals [24].

Economic	Electrical	Relation	Units
Flow	Current	$f = \frac{dq}{dt}$	$\frac{\#}{\text{yr}}$
Incentive	Voltage	$v = \frac{dp}{dt}$	$\frac{\$}{\# \text{yr}}$
Stock	Electric Charge	$q = \int f dt$	$\frac{\#}{\#}$
Reservation Price	Magnetic Flux	$p = \int v dt$	$\frac{\$}{\#}$
Surplus Allocation Rate	Power	$P = f v$	$\frac{\$}{\text{yr}^2}$

3.3. Agents as multiports

3.3.1. Communication with Signals

In the circular flow model of Figure 3.1a, a flow is given by a single number specifying its equilibrium value. In the multiport network model of Figure 3.1b, a flow is, instead, given by a signal specifying the flow's behavior as a function $f = f(t)$ of time.

In a multiport network, the interaction between agents at their ports also includes an incentive v . Similar to how f can be seen as a rate of change in goods, v can be seen as a rate of change in price [45] and is therefore also regarded as a signal, i.e., a function $v = v(t)$.

Agents communicate the quantities they demand or the incentives they require by sending entire signals. A single equilibrium value constitutes the special case of a constant or DC signal. Although in general these can take on a wide variety of shapes, two fundamental types of signals are of particular interest in economics, as they are in engineering.

First are sinusoidal or AC signals. These appear in cyclical behavior which manifests itself in business cycles [86], credit cycles [87], Kondratieff waves [88], and other seasonalities [89]. Second are exponentially decaying or growing signals. These correspond to exponential discounting or exponential growth [45]. Arbitrary signals can be constructed by superimposing seasonal, exponential, and stationary signals.

Like their engineering counterparts, the combination of cyclical and discounting signals is more conveniently represented in the frequency domain. Laplace transforming f and v gives, respectively, signals F and V which are functions $F = F(s)$ and $V = V(s)$ of the complex frequency $s = \sigma + i\omega$. For economics, we think of σ as the discount rate (see, e.g., [59]) and ω as the frequency of the cyclical component of the signal. A particular economic cycle or discount rate can then be represented by a single value in the spectrum.

3.3.2. Interactions and Transactions

An interaction between agents involves a conjugate pair (f, v) of signals, where both the flow f and the corresponding incentive v are communicated. This is a significant departure from economic models, where only the flow is communicated. It is the

presence of a conjugate incentive that allows us to accurately represent the dynamic behavior as emergent from the micro level.

This definition of an interaction also allows us to keep track of the flow of economic surplus to or from an agent, i.e., the rate at which the surplus is allocated in time to the agent. This corresponds to the power in an electrical network (see further [45] and also [24]). It is the product of the flow of goods with the incentive:

$$P = v f \quad (3.1)$$

A transaction involves two interactions: one for the goods or services, and one for the accompanying money. In Figure 3.1b, we indicate two types of transactions: one for the purchase and sale of the goods and another for the hiring and firing of labor. Whereas interactions involve only an exchange of either a good or a service, in a transaction, this is paired with the exchange of money.

The definition of a transaction allows us to make the connection with accounting and business practice. Two quantities are important:

- *Transaction Price*: Ratio of the flow of money to the flow of the good or service. For example, in Figure 3.1b the ratio $f_{\text{spending}}/f_{\text{goods}}$ gives the current purchase price, in \$ per item, in the goods market.
- *Accounting Profit*: Difference between incoming and outgoing flow of money from two markets. For example, in Figure 3.1b the difference $f_{\text{revenue}} - f_{\text{wages}}$ gives the rate of profit, in \$ per time, for the firm.

The accounting notions of price and profit are distinct from the economic notions of the reservation price and surplus allocation rate, as they are defined in Table 3.1. The latter concepts are endogenous to the agent, and a notion of a transaction is not required for their definition.

3.3.3. Interactions with Ports

In circular flow or DSGE models, the arrows giving the inter-agent interactions are drawn in a rather informal manner. In multiport networks, in contrast, these are formalized through the notion of a port. When designing a network, we first enumerate the distinct goods, services, or types of money with which an agent interacts. Then, we open a port for each of them. An agent exchanging n types of goods, services, or types of money is thus represented by an n -port.

Schematically, ports consist of two terminals, which are indicated by a pair of white bullets in Figure 3.1b. In the economic interpretation of these terminals, the + and – signs give the polarity of the incentive [24], indicating whether it is a proper incentive (positive) or a desire (negative). The arrows specify the positive direction of the flow and allow us to distinguish between receipt and delivery. In contrast to the circular flow model, there is no implied direction for the goods or money, and, like currents in circuit theory, they can be either positive or negative.

A valid port is one to which the port condition applies [90]. The condition dictates that the incoming flow at a terminal equals the outgoing flow at the other terminal. In

electrical networks, it ensures that the network does not violate Kirchhoff's current law. In economic applications, it serves to impose the physical clearing of the goods within the network (see [24]).

3.3.4. Exogenous-Endogenous Shocks with Independent-Dependent Signals

In the economic literature, a distinction is made between exogenous variables, which originate from outside of the model, and endogenous variables, which are determined by the model [91]. In a typical analysis, exogenous shocks are imposed, and then the response of endogenous variables of interest is investigated.

In multiport modeling, we generalize this concept and apply the distinction to each agent individually, at any level of abstraction. Multiports have a well-defined system boundary. Exogenous signals are, thus, necessarily imposed at a port. In multiport theory, such signals are known as independent signals. The endogenously determined response is referred to as a dependent signal.

There is a limit on the number of independent signals which can be exogenously imposed on a multiport. For an arbitrary n -port, there are $2n$ signals consisting of an incentive and a flow for each port. At a maximum, n of these can be chosen to be independent. It does not matter which ports these signals belong to. The remaining n signals are then necessarily dependent and form the endogenously generated response of the agent. If there are fewer than n exogenously imposed signals, one has the freedom to treat some signals as if they were independent.

Using this distinction at a particular level of abstraction allows us to analyze networks in a causal manner. Starting off with an exogenous shock to a particular agent, we can causally track the response by taking its dependent signal and treating it as an independent signal of a counterparty agent. This process then repeats until we arrive at the agent whose dependent variable forms the response of interest. Any freedom to choose which variables are dependent can be exploited to investigate different causal chains leading to the response. Section 4 provides several demonstrations of this technique.

3.3.5. Agent Dynamic Behavior with Parameter Models

To determine the response of an agent to exogenously imposed signals, we must establish the relationship between the dependent and independent signals. Formally, such a relationship is an operator between function spaces. We refer to it as the agent's dynamic behavioral operator.

In this chapter, we limit ourselves to linear dynamics and, hence, the behavioral operator becomes a linear operator. For these, a frequency domain representation is the most convenient, because the operator's action can be described using linear algebra as follows: For a general n -port, we collect the Laplace transforms of n independent signals into a vector $\mathbf{U}$ and the n dependent ones into a vector $\mathbf{Y}$. The relationship is then given by

$$\mathbf{Y} = \mathbf{e} \mathbf{U} \quad (3.2)$$

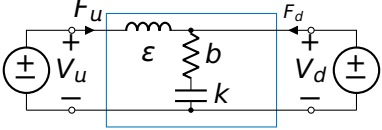
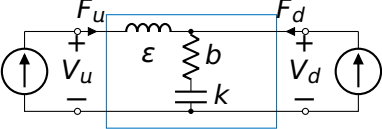
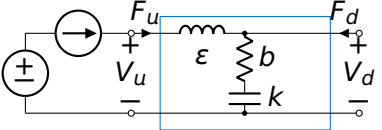
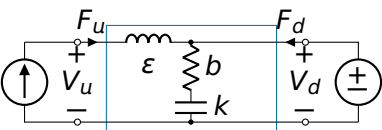
where $\mathbf{e}$ is a $n \times n$ complex matrix whose entries are also functions of s . In multiport theory, these are commonly referred to as parameters. The behavioral operator is also referred to as a parameter model. For applications to economics, we can think of the parameters as elasticities and the matrix $\mathbf{e}$ as giving the $\mathbf{U}$ elasticity of $\mathbf{Y}$, although other interpretations are often relevant (see Section 3.4).

In multiport theory, a convenient classification of parameter models exists for 2-ports (see, e.g. [1]). In this case, the behavioral operator is represented by a 2×2 complex matrix. For a 2-port, there are four distinct choices of independent variables, which lead to the four models shown in Table 3.3. In the following subsections, we interpret these from an economic perspective with the aid of the supply chain intermediary in Figure 4.6 of the example in Section 4.3. In addition, we show how the dynamic behavioral operator emerges from the dynamics at the circuit level. We refer the reader to [24] for how this is done.

Table 3.2: Economic elements and their electrical analogs with their behavior in the frequency domain (adapted from [24]). ε , k , and b are the elasticities of demand, storage capacity, and economic friction, respectively.

Economic	Electrical	Behavior	Symbol
Demand	Inductor	$V = \frac{1}{\varepsilon} s F$	
Storage	Capacitor	$V = \frac{1}{s k} F$	
Friction	Resistor	$V = \frac{1}{b} F$	

Table 3.3: Parameter models for a 2-port agent. The agent acts as a middleman between its upstream and downstream counterparties. It maintains a price at the inductor, an inventory at the capacitor, and accrues transaction costs at the resistor (see Table 3.2). Upstream and downstream signals are subscripted with a u and d , respectively.

Type	Description	Circuit	Parameter Model
Admittance	Flexibility: Agent “admits” the upstream supply downstream.		$\begin{pmatrix} F_u \\ F_d \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{\varepsilon}{s} & -\frac{\varepsilon}{s} \\ -\frac{\varepsilon}{s} & \frac{s^2 + \varepsilon bs + \varepsilon k}{bs^2 + ks} \end{pmatrix}}_{\mathbf{y} = \mathbf{z}^{-1}} \begin{pmatrix} V_u \\ V_d \end{pmatrix}$
Impedance	Stickiness: Agent “impedes” the upstream supply from reaching downstream		$\begin{pmatrix} V_u \\ V_d \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{s^2 + \varepsilon bs + \varepsilon k}{\varepsilon s} & \frac{bs + k}{\frac{s}{bs + k}} \\ \frac{\varepsilon s}{bs + k} & \frac{bs + k}{s} \end{pmatrix}}_{\mathbf{z} = \mathbf{y}^{-1}} \begin{pmatrix} F_u \\ F_d \end{pmatrix}$
Transmission	Surplus Distribution: Agent “transmits” the upstream surplus downstream		$\begin{pmatrix} V_d \\ F_d \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \frac{bs^2 + ks}{\varepsilon bs + \varepsilon k} \\ \frac{s}{bs + k} & \frac{bs + k}{\varepsilon} \end{pmatrix}}_{\mathbf{t}} \begin{pmatrix} V_u \\ -F_u \end{pmatrix}$
Hybrid	Agent “impedes” the upstream supply and “admits” the downstream supply (and vice versa).		$\begin{pmatrix} V_u \\ F_d \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{bs^2 + ks}{\varepsilon bs + \varepsilon k} & 1 \\ -1 & \frac{s}{bs + k} \end{pmatrix}}_{\mathbf{h} = \mathbf{g}^{-1}} \begin{pmatrix} F_u \\ V_d \end{pmatrix}$

(a) Admittance or $\mathbf{y}$ -parameters

The first row of Table 3.3 contains what are known as admittance or $\mathbf{y}$ parameters in multiport theory. They loosely correspond to the notion of flexibility in economics. The incentives are imposed, and the agent responds with the flows demanded or supplied. A specific example is displayed in Figure 3.2a and discussed in the accompanying text.

The parameter values follow readily from the circuit. The circuit acts as an “incentive divider,” the economic analog to a voltage divider. Integrating the net incentive $V_d - V_u$ gives the agent’s reservation price, i.e., $p = \frac{1}{s}(V_d - V_u)$. A subsequent multiplication with the inductor’s price elasticity gives the upstream flow, i.e., $F_u = \varepsilon p$. This completes the first row of the matrix. The second row gives the amount F_d that is “admitted” downstream. This consists of the negative of the upstream flow adjusted with the amount coming out of or going into storage (the capacitor). This latter amount depends on the downstream incentive alone and modifies the bottom right parameter to the given expression.

(b) Impedance $\mathbf{z}$ -parameters

The impedance or $\mathbf{z}$ parameter matrix in the second row of Table 3.3 is the matrix inverse $\mathbf{z} = \mathbf{y}^{-1}$ of the admittance parameters. Stickiness is a general term used in economics as an antonym for flexibility.

The central bank in Figure 3.2c is a natural fit for $\mathbf{z}$ -parameters. It is subjected to two exogenous flows, a target and realized inflation rate, and responds by incentivizing the market to change these.

The parameters for the example circuit are established by reasoning as follows: The imbalance $F_u - F_d$ between the exogenous upstream and downstream flows accumulates in the storage into a stock $q = 1/s(F_u - F_d)$ at the capacitor. Multiplication with the storage elasticity k gives the incentive kq measuring the relative scarcity of the goods. The premiums required to overcome the economic friction are added to this. These are proportional to the flows with constant b . For the top left parameter, we also need to take into account the incentive that represents the force of demand for the upstream supply.

(c) Transmission or $\mathbf{t}$ -parameters

The transmission, chain, cascade, or $\mathbf{t}$ parameters are used to model how the economic surplus can propagate through a network. They are characterized by having both the independent signals on a single port called the input port. The pair of dependent signals appears on the remaining output port. This condition allows one to calculate the rate at which surplus is delivered at the ports as the convolutions

$$F_u * V_u \quad \text{and} \quad F_d * V_d. \quad (3.3)$$

In the time domain, the convolutions revert to a regular product (3.1).

In electrical networks, $\mathbf{t}$ parameters are useful in analyzing power transmission systems. The analogy between energy and surplus makes them useful for economics when analyzing the allocation of surplus between consumers and producers.

In particular, as the alternate name “chain parameters” might already suggest, they are the appropriate choice when modelling supply chains; in fact, the example circuit was taken from the chain in Figure 4.6. When one thinks of the chain as being supply-driven, the upstream port should be thought of as the input port and the downstream one as the output. (For demand-driven chains, the inverse choice is appropriate.) When chained together with a producer at the upstream end and a consumer at the downstream end, the example explicitly calculates the dynamics of how the surplus is distributed between the two, and how much of it is lost in the chain itself.

(d) Hybrid h and g -parameters

Hybrid parameters are used when independent signals consist of the flow on one port and the incentive on the other port. For 2-ports, there are two types of parameter models: h parameters are typically used when the incentive is thought of as the cause, and $g = h^{-1}$ parameters when it is the flow.

In electrical networks, hybrid parameters are useful for amplifiers, transistors, and controlled sources. When these elements are applied to economics, they are likewise useful, as evidenced by the examples in Figures 3.2d and 3.2e.

The agent in Table 3.5 incentivizes its upstream supplier by passing along its customer’s incentive V_d , together with its own incentives for storing, overcoming friction, and demand. This gives the dependent V_u . The other dependent signal, F_d , is determined in a similar manner.

3.3.6. Micro-Level Dynamical Behavior

We specify the dynamical behavior of an agent at the micro level, i.e., the lowest level of abstraction. This is the level where economic circuit theory applies. Economic circuits are the economic analogs to electronic circuits, and a broad range of dynamic economic behavior can be modeled in this manner. Table 3.2 contains the analogy between elementary passive electrical components and economic behavior and the corresponding behavioral law in the frequency domain. We refer the reader to [24] for a comprehensive treatment of the theory in the time domain.

In this section, we illustrate how economic circuit theory determines the dynamical behavior of an agent as it appears at the agent’s ports. In the following subsections, we do this by means of the example agents in Figure 3.2 and refer the reader to [24] for further examples.

(a) Consumer with Substitute Goods

Figure 3.2a represents a consumer who demands apples and oranges at quantities F_a and F_o , respectively. Because the consumer is active in two markets simultaneously, its demand schedule is implemented as two inductors configured with a mutual inductance. In this way, the y -parameter matrix in (3.4) features the principal price elasticities on the diagonal and the cross-elasticities on the off-diagonal ele-

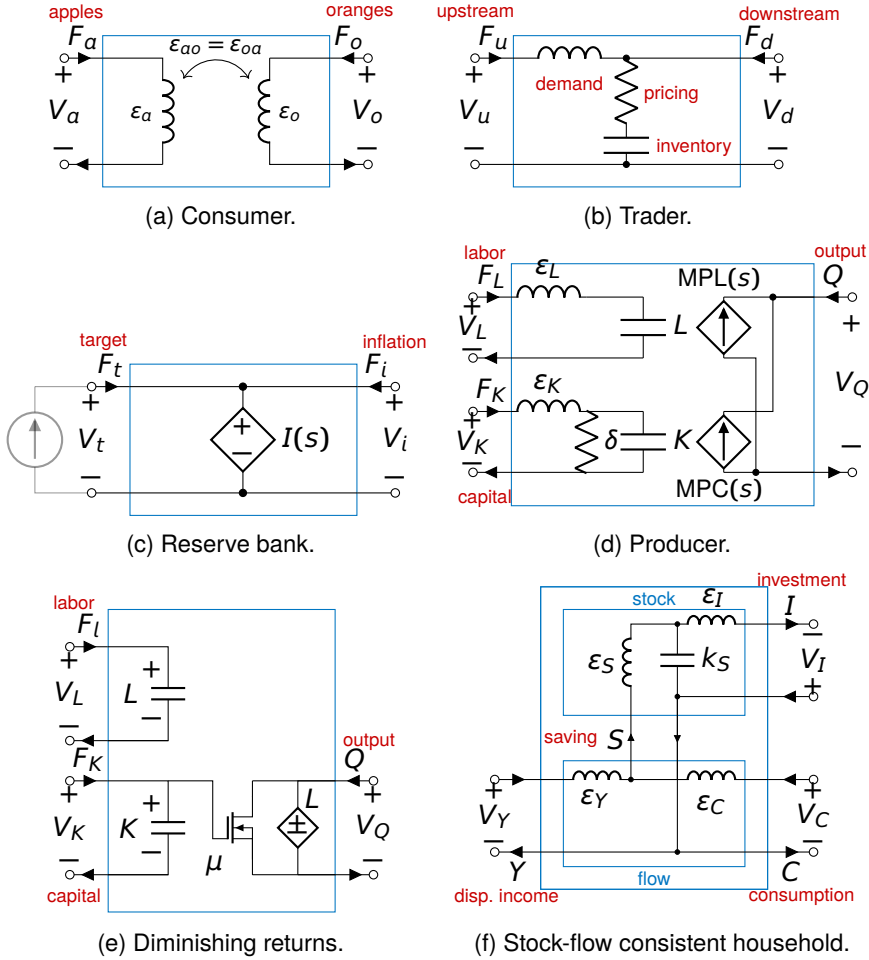


Figure 3.2: Example agent circuits.

ments [24]. Assuming that the incentives V_a and V_o are exogenous, the consumer's demand is given by a $\mathbf{y}$ -parameter model:

$$\mathbf{y} = \frac{1}{s} \begin{pmatrix} \epsilon_{aa} & \epsilon_{ao} \\ \epsilon_{oa} & \epsilon_{oo} \end{pmatrix} \begin{pmatrix} V_a \\ V_o \end{pmatrix} \quad (3.4)$$

price elasticity apples $\downarrow$ cross-elasticity orange $\downarrow$
 ϵ_{aa} ϵ_{ao} price on apples
 ϵ_{oa} ϵ_{oo}
 cross-elasticity apple $\uparrow$ price elasticity oranges $\uparrow$
 price on oranges

The frequency-domain integrator $1/s$ transforms the exogenously imposed incentives into the agent's endogenous prices for the goods. The elasticities then transform these into the demanded flows F_a and F_o that make up the consumer's

response. The principal elasticities determine the influence of the good's own price on the demand, and the cross elasticities determine the influence of the other goods' price. (See, further, [45] and [24].)

(b) Trader as RLC circuit

Figure 3.2b contains a model of a trader, or other type of intermediary, for a particular good. It appears in Figure 4.6 of Section 4.3 in the context of a supply chain, where it moves goods from upstream to downstream. The trader maintains a price in the inductor, an inventory in the capacitor, and generates trade friction in the resistor.

The choices for the parameter models are given in Figure 3.3. In a supply chain, $\mathbf{t}$ parameters are the natural choice. In a supply-driven scenario, the upstream port is the input, and the trader responds to the exogenously imposed incentives and flows of the supplier. In a demand-driven scenario, it is the consumers who apply the incentives and flows at the downstream port, and the trader passes these on to the supplier.

In either case, any goods that are not cleared as they move along the supply chain flow in or out of the inventory held in the capacitor. Any mismatch between the incentives at the ports is balanced by the incentives that drive the price on the inductor (see also [24]).

(c) Reserve Bank as Controlled Source

In Figure 3.2c, we use a current-controlled voltage source to model an institution, such as a Federal Reserve Bank, that is tasked with controlling the rate of interest. The committee, such as the Federal Open Market Committee (FOMC), provides it a target rate F_t . The observed actual rate F_a is also imposed exogenously. The bank responds by communicating incentives to the economy to adjust the actual rate, and to the committee to adjust the target. This leads to a $\mathbf{z}$ -parameter model for the execution of monetary policy.

$$\begin{pmatrix} V_t \\ V_a \end{pmatrix} = I(s) \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} F_t \\ F_a \end{pmatrix}. \quad (3.5)$$

The matrix is chosen so that the bank reacts to the spread, i.e., the deviation $F_t - F_a$ of the actual rate from the target. We refer to $I(s)$ as the policy function. We let it be a function of the complex frequency s . This allows the bank to let its policy depend on the discount rate and the frequency of the seasonality. Whenever convenient, this can be transformed to the time domain to describe the dynamical behavior.

With this model, the bank can be thought of as a feedback controller. The target rate serves as the setpoint, the spread serves as the error, and the policy function serves as the control law. An interesting example is to set the policy function equal to the control law of a PID controller, i.e.,

$$I(s) = K_p + \frac{K_i}{s} + K_d s$$

The proportional gain K_p measures how aggressively the bank should act at the moment. The integral gain K_i does this for the drift that accumulates over time. The derivative gain K_d allows the bank to anticipate and can be useful when there is a tendency to strongly overshoot or oscillate.

(d) Marginal Product with Controlled Current Sources

Economists use production functions to determine the marginal product of the factors of production. Typically, a function of the form $Q = Q(K, L)$ is chosen. The quantity of output Q is a flow of goods and the inputs K and L are stocks representing the physical capital and labor, respectively (see, e.g., [59]). The marginal products of capital and labor are then defined, respectively, as follows:

$$\text{MPC} = \frac{\partial Q}{\partial K}, \quad \text{MPL} = \frac{\partial Q}{\partial L}. \quad (3.6)$$

Figure 3.2d contains the design of a multiport agent representing a producer or productivity sector of the economy. It has three ports: two for the factor inputs and one for the output. The inductors on the inputs represent the demand for the factor flows, i.e., hiring of workers and acquisition rates of capital equipment. The capacitors record the total stock of the factors, and a parallel resistor on the capital stock represents its depreciation. The production function is encoded in the two controlled current sources. When connected in parallel, the circuit gives the following function:

$$Q = \text{MPC}(s)K + \text{MPL}(s)L. \quad (3.7)$$

From an economic perspective, this is a linear production function with coefficients equal to the marginal productivities. From a network perspective, the productivities are the elements of the agent's parameter matrix. They are the transfer functions between the factor inputs and the output of goods:

$$\text{MPC}(s) = \frac{Q(s)}{K(s)} \quad \text{MPL}(s) = \frac{Q(s)}{L(s)}. \quad (3.8)$$

Unlike the productivities in (3.6), the transfer functions in (3.8) depend on the complex frequency s . For economic applications, their dependence on the frequency of the seasonality and the own rate of discount provides an important generalization in our opinion. The poles of the transfer function, in particular, are critical to analyzing the dynamic response of the producer when the inputs are not static flows. In Section 4.3, we show the use of Bode plots to compare the productivity as it depends on the frequency of the seasonality of the demand.

(e) Diminishing Returns with Field-Effect Transistors

Typically, production functions are assumed to be nonlinear due to the diminishing marginal productivity of inputs (see e.g. [59]). As we show in Figure 3.3, the characteristics of such production functions appear similar to the drain current of a MOSFET when graphed.

Figure 3.2e contains an example circuit that uses a MOSFET to implement a rudimentary production function with diminishing productivity of labor and a minimum required capital. Like the network in Figure 3.2d, the multiport concerns labor, capital, and output.

The MOSFET is configured so that its drain current (I_D) represents the output Q . The capacitance of the capital stock is set to unity so that the gate voltage (V_G) of the MOSFET equals the capital stock itself, i.e., $V_K = K$. Similarly, a controlled source then sets its drain voltage (V_D) equal to the labor L . This results in the following nonlinear production function:

$$Q = \mu \left((K - K_{th})L - \frac{L^2}{2} \right), \quad (3.9)$$

which corresponds to the DC drain current of the MOSFET. The transconductance μ of the MOSFET serves to quantify the factor productivity. It is straightforward to check that the function is diminishing in labor. For capital, it requires a minimum amount K_{th} , given by the threshold gate-source voltage, for production to start.

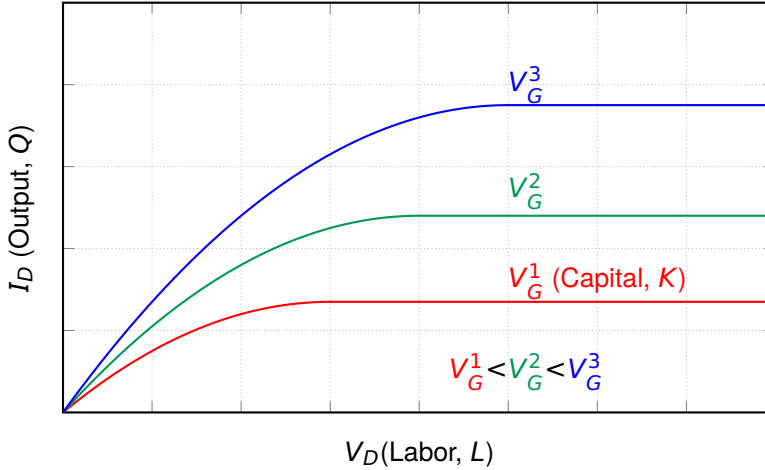


Figure 3.3: MOSFET output characteristics [92] as production functions with diminishing returns [93].

(f) Stock-Flow Consistent Household

Stock-flow consistent modeling is a relatively new technique in economics that enforces a strict accounting for all relevant flows and their stocks [94]. Using multiport networks, stock-flow consistency is automatically enforced. In [24], we show how Kirchhoff's current law enforces it at the economic circuit level. For multiport networks, it is the port condition, as described in Section 3.3.3, that guarantees stock-flow consistency among the multiport agents in a network.

The example in Figure 3.2f models a household as a combination of a balance sheet 2-port and a flow statement 3-port. It is taken from [40] where it forms part of a stock-flow consistent multiport model of an economy. The flow statement has a port for disposable income Y , consumption C , and saving S . These flows are consistent with the GDP identity $Y = C + S$ that forms the household's budget constraint. Saving S enters the balance sheet, which also has a port for investment I . The difference $S - I$, which the household holds as cash on hand, is kept in the capacitor. By combining a flow statement with a balance sheet, the household multiport is rendered stock-flow consistent over both income and capital flows.

The inductors add dynamic behavior to the household. At the external facing ports, they communicate the household's desire for income and consumption, as well as the incentive required to invest. At the internal saving port, the household consolidates its books, and the inductor determines the transactional motive for holding cash.

3.4. Agents as Equivalent 2-ports

It is often convenient to add a level of abstraction between the multiport agent and its internal circuit. For this, we exploit the concept of an equivalent circuit. This is a circuit that contains a component for each parameter in the parameter matrix. It allows a designer to rapidly establish a working network that exhibits the correct dynamical behavior, postponing the precise implementation of the circuit to a later stage.

In Table 3.4, we present the equivalent circuits for the 2-port parameter models introduced in Table 3.3. These contain four components. Two are passive (rectangle shapes), i.e., they do not require surplus to operate. They correspond to the diagonal entries of the matrix, where the principal effects of the operator are determined. The other two are active (diamond shapes), i.e., they do require surplus, and correspond to the antidiagonal elements which determine the cross effects.

Their placement in the matrix suggests certain economic interpretations for the parameters. These are indicated in the last column of Table 3.5. In addition to the familiar price elasticity and its inverse, the price inelasticity, we recognize two other concepts. Price transmission is a concept in the economic literature (see, e.g., [95]), where the effect of one price on the other is quantified. This is, for instance, the case for $\mathbf{t}_{11}$ parameter of the transmission model in Table 3.4. It is a dimensionless quantity. A related concept, similarly dimensionless, that emerges in this context is what we refer to as flow throughput. In the table, the $\mathbf{t}_{22}$ parameter forms an example.

3.4.1. Hierarchical Modeling and Aggregation

In the introduction, we already considered the problems in determining the emergent dynamics for aggregate agents, as they are documented in both the macroeconomic and agent-based economic traditions. In this section, we show how the multiport framework provides a structured and mathematically rigorous approach to these problems.

Table 3.4: Parameter-equivalent agent abstractions.

Type	Circuit	Parameter Model
Admittance		price elasticity good 1 ↓ $\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$ cross-elasticity of price 1 on good 2 cross-elasticity of price 2 on good 1 ↓ $\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$ price elasticity good 1 $\underbrace{\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}}_y$
Impedance		price elasticity good 1 ↓ $\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$ cross-elasticity of price 1 on good 2 cross-elasticity of price 2 on good 1 ↓ $\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$ price elasticity good 1 $\underbrace{\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}}_z$
Transmission		price transmission ↓ $\begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}$ cross-elasticity of price 1 on good 2 cross-inelasticity of price 1 on good 2 ↓ $\begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}$ flow throughput $\underbrace{\begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}}_t$
Hybrid		price elasticity good 1 ↓ $\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$ flow throughput price transmission ↓ $\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$ price elasticity good 2 $\underbrace{\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}}_h$

In multiport theory, the representation of a model at a higher level of abstraction is created by interconnecting several multiports into a single aggregate multiport. As long as the port condition is fulfilled, this process leads to a valid multiport network for the aggregate agent [1]. Therefore, its dynamical behavior remains emergent from underlying circuit-level descriptions.

The aggregation process can be repeated to create a hierarchy of levels of abstraction. The port condition guarantees that the internal consistency of the model is preserved and the correct dynamical behavior emerges, independent of the heterogeneity of the constituent agents.

Table 3.5: Equivalent operators for various modes of interaction and their interconnection topologies [1].

Topology		Algebra	
Interaction	Interconnection	Aggregate	Representative
Competition	Parallel	$\sum_{k=1}^n \mathbf{y}_k$	$\frac{1}{n} \sum_{k=1}^n \mathbf{y}_k$
Cooperation	Series	$\sum_{k=1}^n \mathbf{z}_k$	$\frac{1}{n} \sum_{k=1}^n \mathbf{z}_k$
Franchise	Series-Parallel	$\sum_{k=1}^n \mathbf{h}_k$	$\frac{1}{n} \sum_{k=1}^n \mathbf{h}_k$
Chain	Cascade	$\prod_{k=1}^n \mathbf{t}_k$	$\left(\prod_{k=1}^n \mathbf{t}_k\right)^{\frac{1}{n}}$
Cartel	Parallel-Series	$\sum_{k=1}^n \mathbf{g}_k$	$\frac{1}{n} \sum_{k=1}^n \mathbf{g}_k$

3.4.2. The Topology and Parameters of Interaction Modes

Multiport networks allow for a topological visualization of the economic interactions among agents. The particular mode in which agents interact determines how they are aggregated into a single aggregate agent. Each interaction mode corresponds to a particular topology in which the multiports are interconnected to create a single multiport.

For 2-ports, we distinguish between the five modes of interaction listed in the first column of Table 3.5. The corresponding interconnection topologies are listed in the second column of the table.

In the third and fourth columns, we list the formulae for the parameter models of the corresponding aggregate agent and representative agent, respectively. They are particularly intuitive. Aggregation is done using matrix addition, except for the chaining parameters, which are multiplied. The representative agent is found by averaging: a geometric average for chains and a straightforward arithmetic average for all others. These models provide the emergent dynamical behavior at the aggregate level of abstraction in terms of the behavior at the lower level of abstraction.

In Figure 3.4, we diagram the interconnection topologies for the interaction modes for pairs of 2-port agents. In the following five subsections, respectively, we analyze the given five interaction modes and their emergent dynamics in detail.

(a) Competition

When competing for a limited provision f of goods, the participating agents each receive a fraction of the total. In addition, they must share the available economic surplus. This means that the power $\mathbf{v}f$, i.e., the surplus allocation rate, must also be conserved. These two conditions imply that competing agents exhibit the same

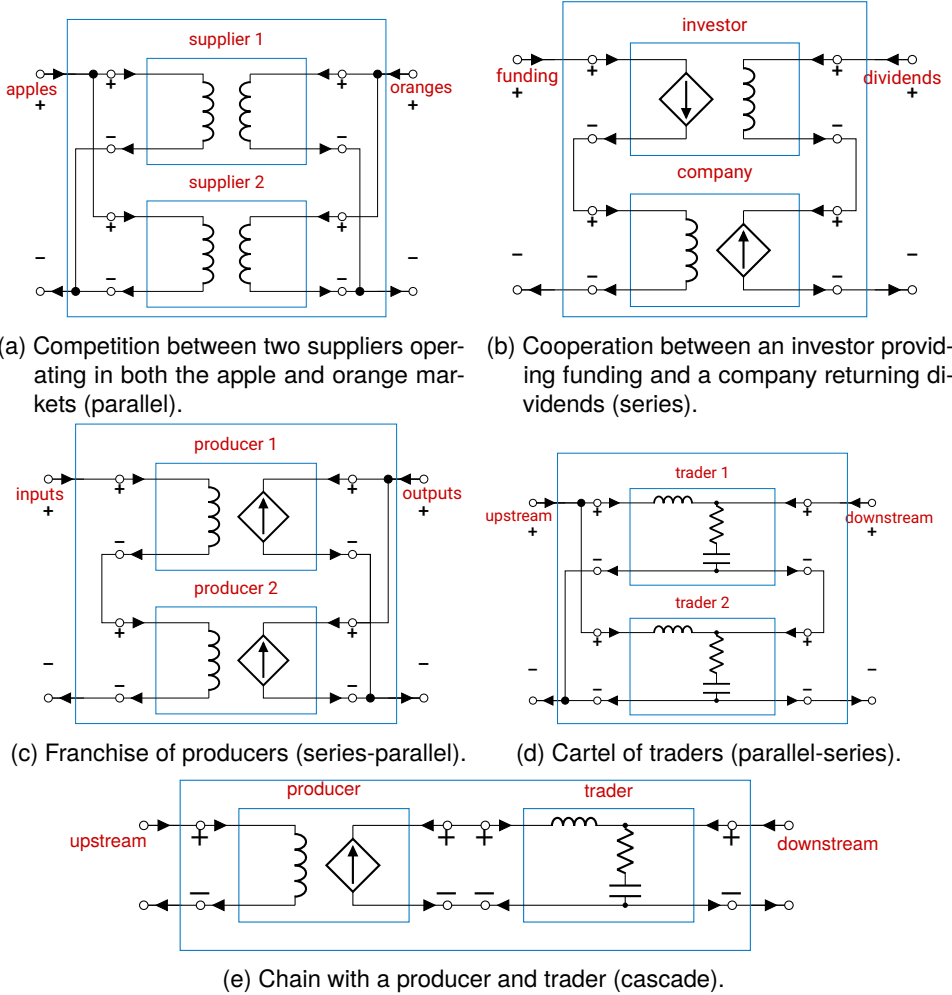


Figure 3.4: Interconnection topologies for common interaction modes.

incentive. Therefore:

$$f = \sum_{i=1}^m f_i \quad \text{and} \quad v = v_1 = \dots = v_n. \quad (3.10)$$

These are the conditions for market clearing (see [24]).

Figure 3.4a shows how two agents can be interconnected to enforce these equalities. In multiport theory, this is known as a parallel interconnection. The figure considers two suppliers competing for trade in the apple and orange markets. For each good, the market-clearing equalities have to hold separately.

The figure shows that the aggregate supplier is also a 2-port agent, who is similarly

active in both markets. Its $\mathbf{y}$ -parameter matrix is found by adding the $\mathbf{y}$ -parameter matrices of the individual suppliers. A representative agent corresponds to the average of the two.

(b) Cooperation

We consider agents to be cooperating if the goods have to go through both at the same time. They will agree to do this when incentivized adequately, i.e., to the degree that the market clears. This implies that

$$f = f_1 = \cdots = f_n \quad \text{and} \quad v = \sum_{i=1}^m v_i. \quad (3.11)$$

These equations are conjugate to those of competitive agents in (3.10). Here, the total provided incentive, rather than the flow, is balanced among the agents, and it is the flows that are equal.

In Figure 3.4b, we show the interconnection that is consistent with (3.11). The example concerns an investor and a stock company in a mutually beneficial relationship, with the investor supplying the funds for which the company returns a dividend stream to the investor.

Together, they form a single aggregate agent representing the financial sector, with parameters $\mathbf{z}_{\text{inv}} + \mathbf{z}_{\text{co}}$. The representative agent is found by taking the average instead.

(c) Franchise and Cartel Formation

Agents may cooperate in some markets and compete in others. In such a case, some of the ports are interconnected in parallel, while the others are interconnected in series.

Figures 3.4c and 3.4d contain two examples of common business models. The former shows what is known as a franchise, where producers cooperate in procuring input goods but compete in supplying their outputs to the market. The latter shows a cartel formed by two traders, where they compete for the upstream supply, but they cooperate to manage the downstream distribution.

Given the appropriate choice of parameters (see Table 3.3), the equivalent and representative operators are determined by addition and averaging, respectively.

(d) Chaining

In Porter's [96] concept of a value chain, an organization is viewed as a chain of business units, where the output of a unit is connected to the input of the subsequent unit. In a multiport network rendition of this, we implement the business units as 2-ports and configure them in cascade.

This is implemented in Figure 3.4e for a portion of a supply chain. A supplier sources raw material upstream, transforms these inputs into goods with value added,

which are output to a trader who subsequently adds more value and outputs them downstream.

The transformation process of a business unit is modeled by its behavioral operator, making this process fully dynamic. The $\mathbf{t}$ -parameters specify this in the frequency domain. (In the case study in Section 4.3, we show how natural the latter is to analyze the bullwhip effect.) They quantify the process of value adding by determining the relative increase in desirability $V_{\text{out}}/V_{\text{in}}$ between the input and output of each unit.

To find the $\mathbf{t}$ -parameters for the entire chain, we therefore multiply the $\mathbf{t}$ -parameters of the individual units. The representative agent is found by taking a geometric average of the latter. In the time domain, these are determined using the convolution operator. This contrasts with the other interaction modes, since addition and averaging are preserved when transforming between domains.

Rather than aggregating, it is perhaps more appropriate to refer to the process as compounding in the case of chains. Therefore, the determination of the emergent dynamics of value chains appears to be relatively more challenging.

The rate at which the surplus is allocated over a chain can also be found using $\mathbf{t}$ -parameters. This works analogously to how amplifiers are chained to increase the power gain from input to output. This is useful to determine profits at each business and enhance the chain's efficiency to control overall profits.

3.4.3. Aggregation and the Operator Algebra

The general process of determining the behavioral operator of aggregate or representative agents is done using an efficient operator algebra. Table 3.5 shows it explicitly for the parameter models in the frequency domain, where the operations reduce to regular addition and multiplication. The algebra provides designers with a structured technique for bottom-up modeling of networks.

The procedure is to first aggregate into sub-aggregations of agents that follow the same parameter model. Then these are aggregated again until the final network is reached.

We illustrate the aggregation procedure with the supply chain in Figure 3.5. In the supply-chain literature, this particular chain is known as a diamond-shaped supply chain [97]. It consists of an original equipment manufacturer, or OEM, at the source, who delivers products through three intermediary tiers to an end user.

The intermediaries within each tier compete, and we thus model them using $\mathbf{y}$ parameters. To obtain the equivalent parameter model for a tier, we sum the constituent matrices. Then, to place them in the chain, they are converted to $\mathbf{t}$ parameter models. For a generic tier's $\mathbf{y}$ parameter, this is done as follows (see [1]):

$$\mathbf{t} = \frac{1}{\mathbf{y}_{12}} \begin{pmatrix} -\mathbf{y}_{11} & 1 \\ \det(\mathbf{y}) & -\mathbf{y}_{22} \end{pmatrix}.$$

Doing this for all three tiers leads to parameter matrices $\mathbf{t}_{\text{T1}}$, $\mathbf{t}_{\text{T2}}$, and $\mathbf{t}_{\text{T3}}$. Finally, the $\mathbf{t}$ -parameter matrix for the entire supply chain is found by multiplying these together with the OEM's parameter as follows:

$$\mathbf{t}_{\text{SC}} = \mathbf{t}_{\text{OEM}} \cdot \mathbf{t}_{\text{T1}} \cdot \mathbf{t}_{\text{T2}} \cdot \mathbf{t}_{\text{T3}}.$$

The chain's parameter matrix unambiguously determines the behavioral operator of the entire chain. It does not matter that the individual intermediary or tiers have entirely dissimilar behavioral operators and, thus, are strongly heterogeneous. If necessary, a representative tier or intermediary can be found using the methods outlined in the previous section. The aggregate behavior of the chain, however, is not limited to a scaled-up version of a single, homogeneous agent, as [98] has criticized the concept for.

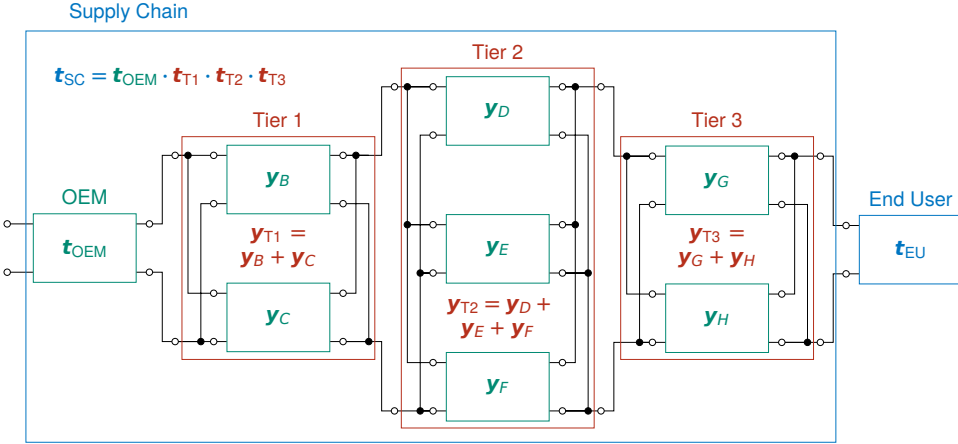


Figure 3.5: Diamond-shaped supply chain network and the determination of its equivalent parameter model t_{SC} .

3.5. Conclusions

This chapter extended the economic circuit theory into a multiport formulation to enable structured modeling of complex economic agents and networks. By representing each agent as a multiport, defined by a set of ports through which it exchanges goods and incentives with its environment, the modeler can design agent behavior independently of the surrounding economic system. Furthermore, this modularity allows agents to be built, validated, and reused across different modeling contexts without requiring redesign.

The multiport framework also ensures that all interconnections between agents are stock-flow consistent and that price and quantity dynamics are preserved across the network. This makes it possible to model economic systems graphically, using circuit-style diagrams that reflect both the behavioral structure of agents and the topology of their interactions.

Importantly, the framework supports different levels of abstraction. Agents can be kept simple or made arbitrarily complex, and groups of agents can be aggregated into higher-level subsystems. This makes the modeling approach scalable and adaptable, suitable not just for stylized textbook economies, but for real-world systems involving many interacting agents and layers of decision-making.

With the multiport formulation, we now have a practical and systematic method for building complex, dynamic economic models that remain interpretable and adaptable as they grow in scope and detail. In the next chapter, we demonstrate the applicability in a number of examples.

4

Applications

4.1. Introduction

In this concluding chapter, we apply the theory presented in Chapter 3 to build four network models of increasing complexity in the following sections:

- 4.2 A Robinson Crusoe economy to show how a classic economic textbook problem is reformulated and rendered fully dynamic by the theory.
- 4.3 A multi-level supply chain to show how a frequency domain analysis can provide insight into the bullwhip effect.
- 4.4 An electricity market, where we add a battery storage system to show how the modularity of theory allows designers to build hypothetical scenarios for analysis of the resulting duck curves.
- 4.5 A complete network model of an economy, featuring multiple layers of abstraction down to the micro level. We generate theoretical GDP, inflation, and interest rate forecasts.

For each application, we design the system model in a top-down fashion. At the top level of abstraction, we take the entire model as a single multiport, whose ports, if any, accommodate the exogenous inputs to it. At the bottom level lies the component level where the elementary agents make up an economic circuit. All models feature at least one intermediate level. The economy, in particular, features four additional levels to show how a complex model can be built this way. These include going from the top downward, a macro level with its sectors, then one with subsectors, and a level with the individual agents, before finally reaching the micro level.

To analyze the emergent dynamics, we work bottom-up. Starting at the micro level, we aggregate until we arrive at the level of abstraction where the signals of interest are intercommunicated. The emergent dynamical behavior is obtained by simulating the models on the LTSpice circuit simulator [2]. Flows or incentives are readily measured by, respectively, placing ammeters or voltmeters at the relevant

interconnection. We interpret the dynamical behavior by tracing causal paths, consisting of dependent/independent signal links, that lead from an imposed shock to some response of interest.

The use of existing network simulators for economic networks has proven to be highly efficient, both in the design and execution phases. They have been developed to design and run simulations of highly complex electronic networks efficiently, and this does carry over to economic networks. Using an ordinary mid-level laptop, the simulations are practically instantaneous except for the model of the entire economy, which completes within a few seconds.

4.2. The Robinson Crusoe Economy

The Robinson Crusoe economy, see e.g. [59], is a highly simplified model used in economics to illustrate how the effects of scarcity, production, consumption, and the labor-leisure tradeoff on the economic equilibrium. In this section, we design such a multiport network model that includes these effects. We design top down and focus on the dynamics of the progression to equilibrium.

A Robinson Crusoe economy features a single individual on a deserted island. There is no trade with the outside world, and only time is provided for. This means that the whole economy is represented by a 1-port having a single port where time flows in at a constant rate. Figure 4.1 shows an LTSpice implementation with the flow of time given by a current source.

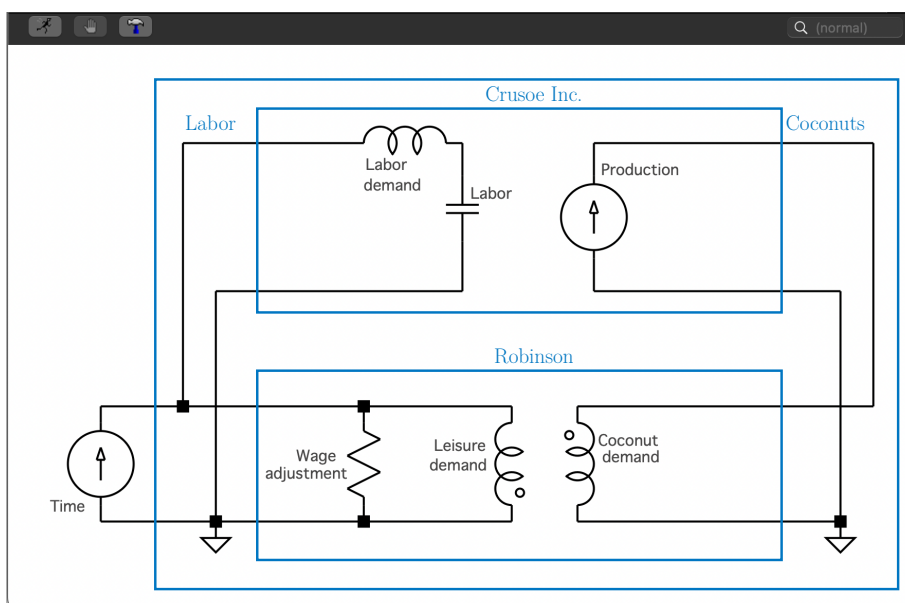


Figure 4.1: Robinson Crusoe Economy as a multiport network in LTSpice.

One abstraction level lower, Robinson Crusoe is split into a production and con-

sumption sector called Robinson and Crusoe Inc., respectively. Robinson supplies labor and demands coconuts, while Crusoe Inc. hires labor to produce those coconuts. The sectors are thus both 2-ports, each with a port for labor and one for coconuts. The interconnections given in the figure reflect that the sectors compete for the available time and that the economic surplus is divided between the two.

Robinson's trade-off between leisure and coconuts is modeled as in the consumer example in Section 3.3.3 and Figure 3.2a. Here, it models how Robinson alters his demand for leisure based on the price of coconuts and vice versa. The labor provided by Robinson to Crusoe Inc. is the available time less the leisure demand.

Crusoe Inc.'s production function is modeled as a simplified version of the producer example in Section 3.3.6 and Figure 3.2d, featuring only the labor factor.

We investigate the dynamics of the economy as it moves from its initial conditions to its equilibrium state. In Figure 4.2, we plot the demand for leisure, labor, and coconuts in time coming from the LTSpice implementation. The results are readily understood by analyzing the dynamics at the component level as follows:

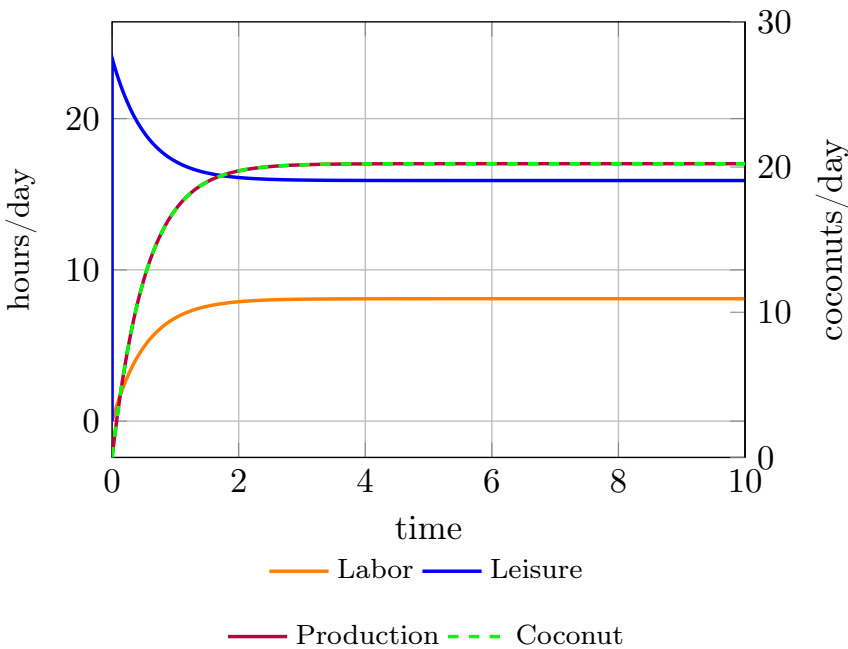


Figure 4.2: Subsequent to Robinson Crusoe's arrival on the island, the economy exhibits an exponential movement to equilibrium.

Before Robinson Crusoe's arrival, there was no demand for labor, leisure, or coconuts. Upon arrival, Robinson initially places no value on his leisure time (blue) and flows it all through the shunt resistor to the ground at zero desirability. Proportional to this flow, an incentive is built up over the resistor to get to work. This incentive is communicated to Crusoe Inc., which hires Robinson to work, causing the demand

for labor to ramp up (orange). It also incentivizes Robinson to raise his price for leisure time and demand more of it. The labor hired by Crusoe Inc. is used to produce (red) the coconuts demanded by Robinson (green). The mutual inductance between Robinson's demand for leisure and coconuts determines the equilibrium state observed in the figure.

We next complete the model by including the effects of scarcity on the dynamics. This is done by adding a capacitor that represents a storage where coconuts are held in inventory (see Figure 4.3). When coconuts are scarce, inventories are low, and vice versa. In Figure 4.4, we plot the dynamics of the various demand flow quantities. These exhibit second-order dynamics. The presence of an LC circuit in the coconut market results in strong seasonal fluctuations (green). Due to the cross-elasticities of demand for leisure and coconuts, the effect spreads out to the labor market while dampening out in the process due to wage adjustments.

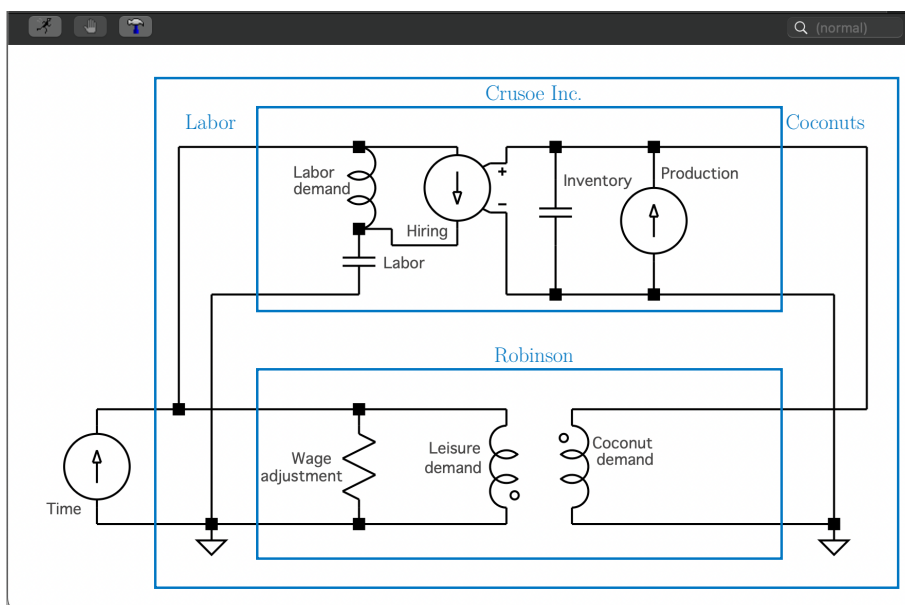


Figure 4.3: Robinson Crusoe Economy with extended Crusoe Inc. 2-port.

In practice, to mitigate such fluctuations in demand, Crusoe Inc. might hire labor when inventories are low and dismiss labor when they are high. In Figure 4.3, we do this by including a controlled current source that actively regulates the demand for labor based on the inventory level. The results in Figure 4.5 shows that this significantly reduces the fluctuations. By adjusting the sensitivity of the controlled source, Crusoe Inc.'s hiring policy can be tuned to yield a desired demand profile.

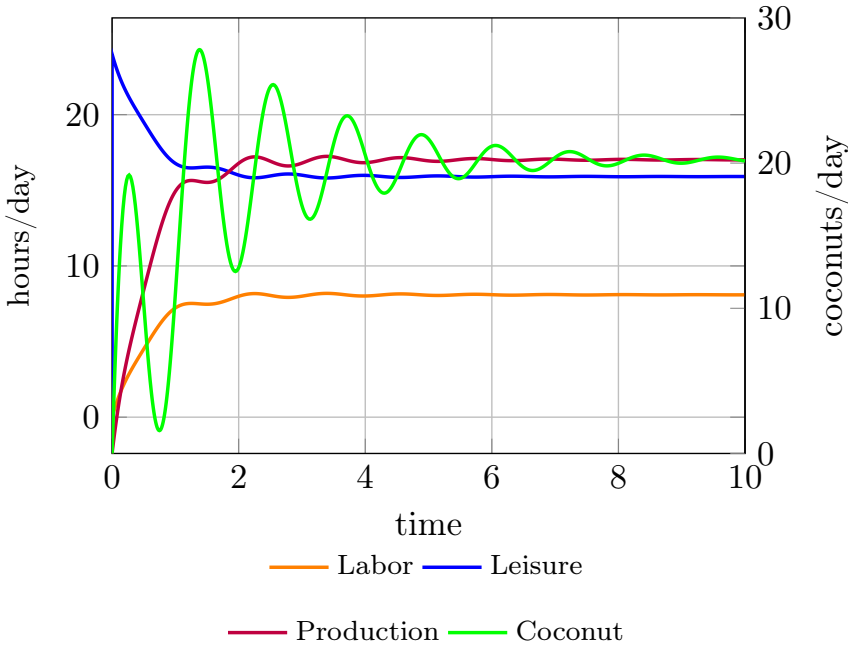


Figure 4.4: Effect of inventory on economic dynamics.

4.3. Filtering Effect of Intermediaries in Supply Chains

In this example application, we analyze a supply chain in the frequency domain. Supply chains often exhibit the bullwhip effect (see e.g. [99]), where small fluctuations in supply or demand grow larger as they move downstream. Such effects are naturally analyzed in the frequency domain. Here we show how the multiport framework allows us to study the chain as a cascade of signal processing filters and to use tools like Bode plots to analyze how signals are transmitted, delayed, or amplified across the supply chain.

The entire chain is modeled as a 1-port, whose port is facing upstream to an exogenous producer, see Figure 4.6. The chain has a lower level consisting of three tiers: a supplier, an intermediary, and a customer. Both the supplier and intermediary are 2-ports, receiving the goods from upstream and delivering them downstream. The customer is a 1-port and terminates the chain. The multiports are interconnected in cascade. The chain thus functions to divide the economic surplus by passing it along from producer to consumer.

To investigate the dynamical behavior, consider first the supply chain without the supplier and the intermediary, i.e., direct delivery of the upstream supply to the customer. The producer is modeled as an AC current source, and the customer by an inductor that describes its demand schedule. The Bode diagram in Figure 4.7 shows

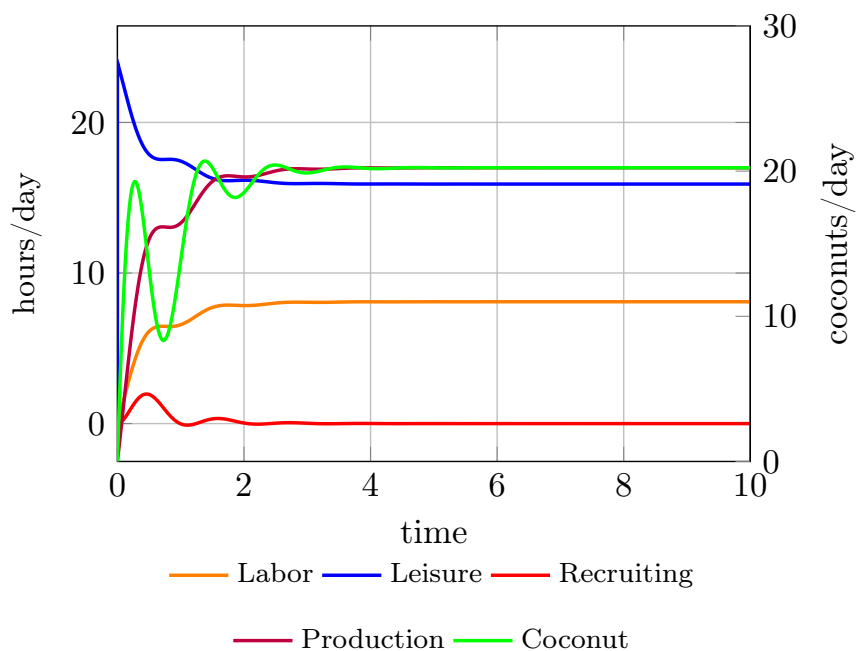


Figure 4.5: Active recruitment damps economic dynamics.

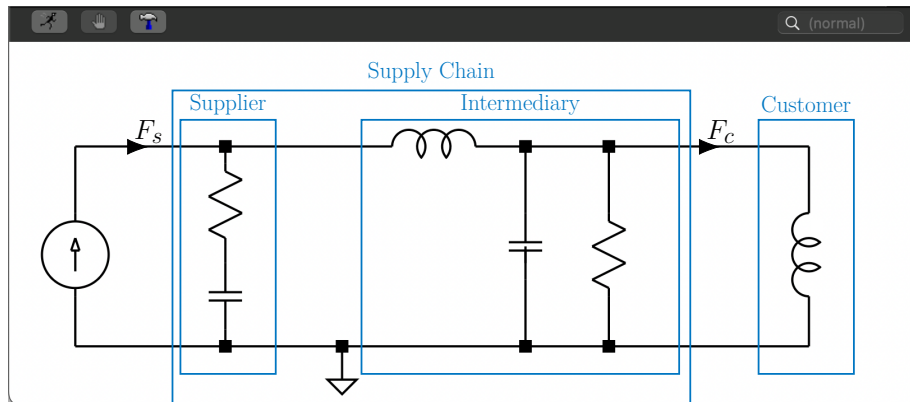


Figure 4.6: Rudimentary supply chain as a multiport.

0 dB amplification and 0-degree phase shift across the entire frequency spectrum (green). This means that a change in upstream supply results in an immediate and unamplified change in downstream supply to the customer. In practice, this would correspond to a situation of volatile prices and inventories.

Next, we include the supplier. It has a capacitor for local storage and a resistor for

carrying costs. With it, we obtain the behavior given by the blue plots. The magnitude plot shows that the volatility in production on a weekly or higher frequency are filtered out, which means that the customer faces more stable prices and inventory levels. The phase plot shows, in addition, that a delay of up to a quarter of the cycle period is to be expected for high-frequency production shocks. The fluctuations in the flow to the customer will lag those in the flow from the external production.

Last, we include an intermediary. It is modeled conform the trader example in Section 3.3.6 and Figure 3.2b. The corresponding Bode plots are given in red and show the profile of a second-order system. Below the monthly rate, the fluctuations are mostly passed along to the customer. At the monthly rate, there is a sharp resonance peak at which they are strongly amplified, and deliveries become suddenly delayed. At higher frequencies, the response of the chain rapidly falls off, with delays increasing gradually.

The resonance peak is the rate at which we should expect effects like the bullwhip to occur. Its location is mostly determined by the intermediary's storage elasticity as given by the capacitor elastance. Higher elasticities correspond to more aggressive inventory management, and this shifts the resonance peak to the right. This makes the bull whip less likely to occur. However, this may subject the customer to undesirable high-frequency noise as the chain filters out a smaller band of high frequencies.

The intensity of the bullwhip effect is given by the height of the resonance peak. This is determined by the damping ratio [64]. By adjusting its price, storage, and friction elasticities, the intermediary can decrease the peak and help to mitigate the effect.

The general technique for achieving these desiderata is known as loop shaping in control theory [64]. With it, supply chains can be tuned to track the reference output rate given by the producer, while rejecting unwanted disturbances and attenuating any spurious noise. In this way, the chain provides customers with a smooth, shock-free supply while minimizing returns to the producer.

In Figure 4.8, the time-domain plots illustrate how the intermediary alters the propagation of cyclical supply variations. In panel (a), representing a weekly input cycle, the downstream supply (red) closely tracks the upstream supply (blue) with only a slight lag and reduced amplitude, indicating that high-frequency fluctuations are effectively filtered. In panel (b), corresponding to a monthly input cycle near the resonance frequency, the downstream response is not only delayed but also amplified relative to the upstream signal—a hallmark of the bullwhip effect. The phase lag visible in both cases is consistent with the negative phase shifts in the Bode plot, but the magnitude and sign of the response differ sharply depending on the driving frequency.

It is straightforward to extend this analysis to multi-tiered supply chains with multiple intermediaries connected in cascade. In such cases, the overall Bode plot of the chain is obtained by summing the Bode plots, i.e., both the magnitude and phase plots, of the individual tiers.

The procedure allows us to understand the bullwhip effect. If the tiers are not too dissimilar in their dynamic behavior, the location of the resonance peak will not

differ too much. In this case, if a tier gets excited at its resonance frequency, the subsequent tiers will further amplify the fluctuations as they pass through the chain.

In Figure 4.9, we show the measurements of the flow through each tier in the LTspice simulation of a three-tiered supply chain. This chain extends the chain in Figure 4.6 by inserting an identical intermediary as an additional tier. The bullwhip effect is evident from the increase in the amplitude of the fluctuations as we go down the tiers. Interestingly, the analysis uncovers a related bullwhip effect on the time lags. This is evident from the accompanying phase shift of the signals. The entire bullwhip effect, thus, occurs in both magnitude and phase.

4.4. The Impact of Battery Energy Storage Systems on Electricity Markets

Contemporary electricity markets are becoming more complex due to the integration of renewable energy sources and rapid technological advancements (see, e.g. [100]). To examine supply, curves are drawn that give the electricity generated by each source at each moment in the day. These curves show a common problem in energy systems with high solar penetration: Because of a timing imbalance between peak solar production at midday and peak demand during the evening hours, renewable energy sources need to be curtailed while fossil generators need to step in to meet peak demand. This is borne out by the duck-like shape [101] of the graph for natural gas (gray) in Figure 4.11a.

To increase the market's efficiency and reduce the carbon footprint, energy storage systems are added. With these, excess renewable production can be shifted to the times when demand peaks. Figure 4.11b shows the electricity mix¹ in California on the same day in 2024, after such systems have been added. Excess solar production is now stored during the day and made available during peak demand, reducing the natural gas production during the evening.

However, rather than analyzing post hoc, it is advantageous to predict what, in general, the effects are of adding actors to the market. Electricity supply per source can be modeled analytically with multiport networks. They, therefore, present an opportunity to fulfill that purpose.

Figure 4.10 contains our multiport network model as it appears in LTSpice. We model the supply market as a single 1-port, on which the demand is exogenously imposed at its port. Demand, which we assume to be inelastic, is modeled by a current source that is programmed to follow the typical pattern of electricity demand during a day.

At a lower level of abstraction, the market is split into an energy generation sector and a system operator that transmits the energy from the generators to the exogenous demand. Proceeding further downward, the generator 1-port consists of four individual generators interconnected to be in competition with each other: gas, wind, solar, and a Battery Energy Storage System (BESS). Total production appears

¹For the purpose of presentation we have excluded data on electricity sources other than natural gas, wind, solar, and batteries.

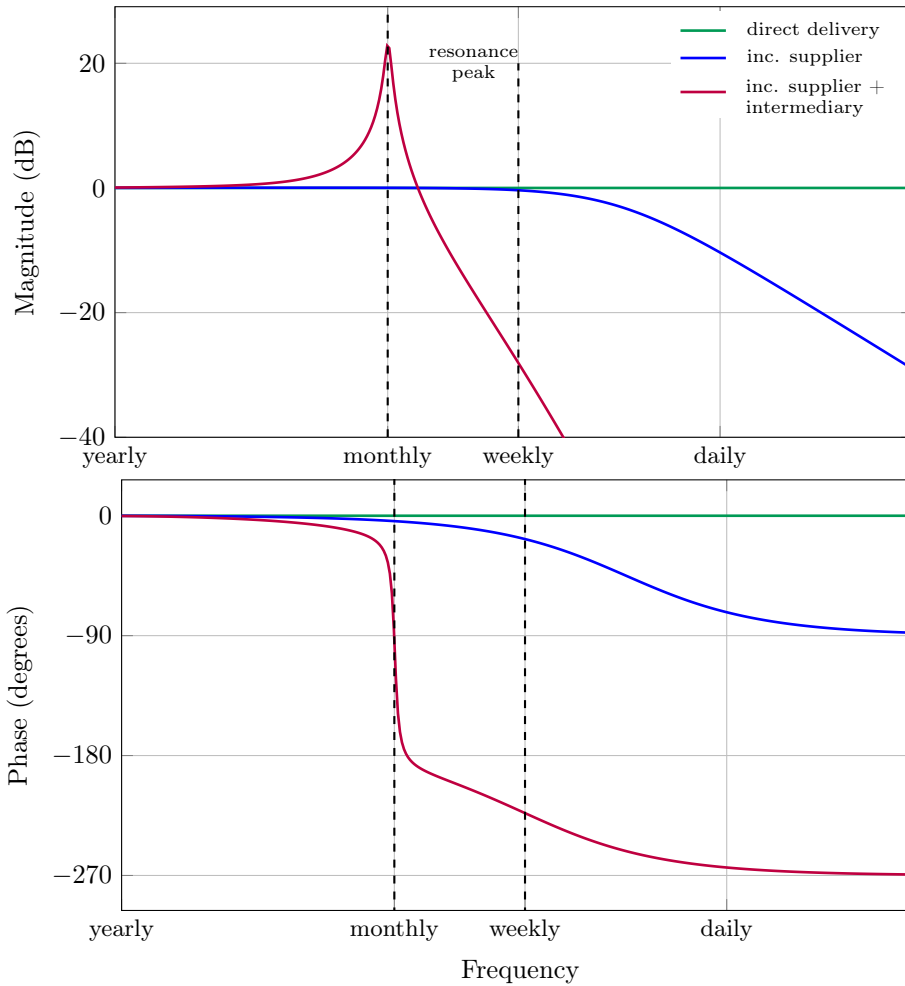
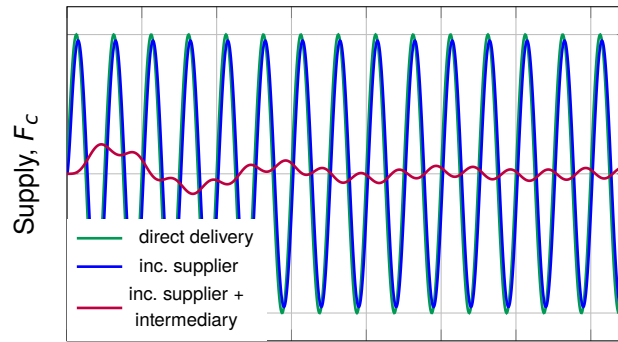


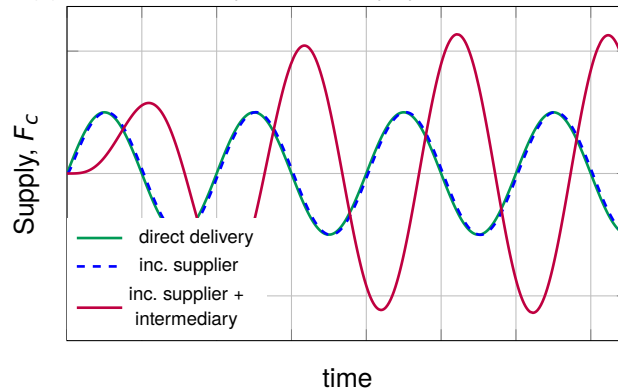
Figure 4.7: Bode plots of the elasticity from upstream, F_S , to downstream, F_C , supply. The intermediary acts as a low-pass filter, with a cut-off at monthly supply cycles. The resonance peak at this cycle is the cause of the bullwhip effect.

at the port, where it is transmitted to the supply-facing port of the operator. The latter is a 2-port which, at its demand-facing port, distributes the produced electricity to the consumers.

We first consider the market without a BESS. Figure 4.12a displays the electricity production per source during a particular day generated by the network. Notice that gas-powered production takes on the distinct duck-like shape seen in empirically determined curves, as in Figure 4.11a. Our analytical solution, however, puts us in a position to investigate the causes.



(a) The intermediary filters weekly cycles



(b) The intermediary amplifies monthly cycles.

Figure 4.8: Time-domain analysis of the effect of the intermediary in the supply chain.

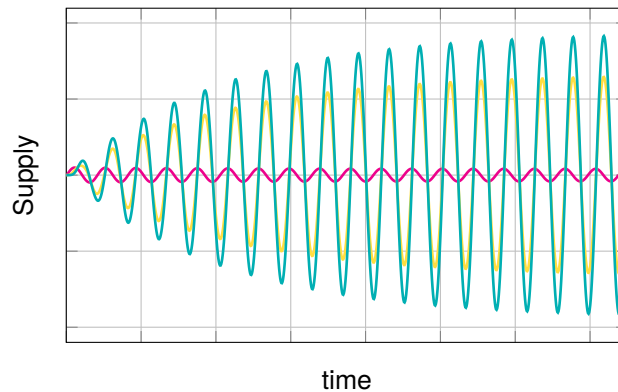


Figure 4.9: Bullwhip effect: seasonality amplifies across the supply chain.

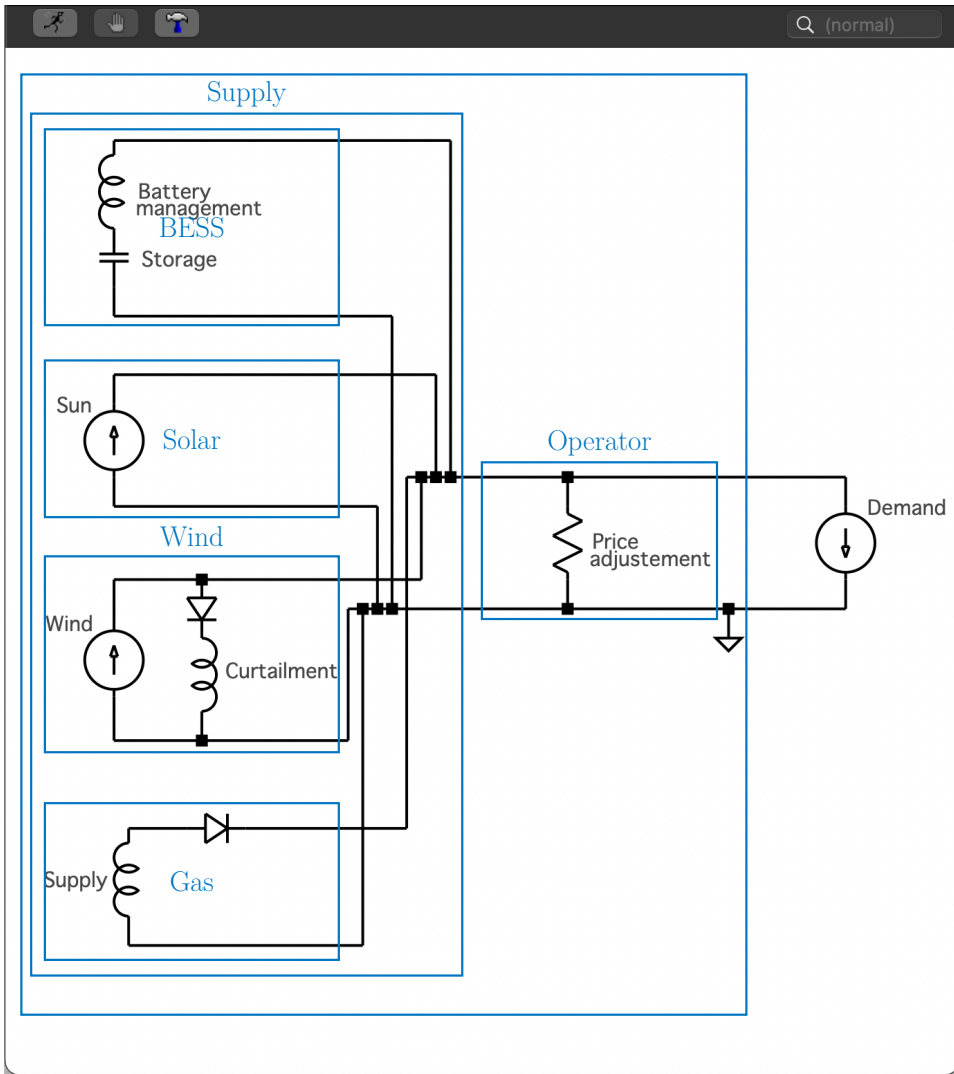


Figure 4.10: Electricity-market multiport model in LTspice.

To analyze this dynamic behavior, we examine the internal circuitry of the agents that determines the underlying dynamic behavior. The current source for solar energy is programmed to follow the available sunlight during the day, showing the distinctive peak at midday (orange). The correct wind curtailment (red) is achieved by shunting its current source with an inductor and a diode in series. This ensures that no wind (blue) is delivered when the price on the inductor drops below zero. The inductor in gas makes it purely price-driven, with the diode ensuring that it engages in production only. The curve shows (gray) how gas ramps up sharply in the evening when prices

are high.

As a dynamical system, the network is almost trivially simple, having only two real-valued poles. Yet, it is able to give a surprisingly accurate reproduction of comparable empirical curves.

Coming back to our goal, we now add a BESS to the supply sector for comparison. It is modeled as an LC circuit. The capacitor represents the battery where the energy is stored. The inductor records its internal reservation price and provides the appropriate incentives to store or discharge. This adds two complex-conjugate poles, making the model fourth order.

The behavior of the resulting electricity market is shown in Figure 4.12b. Comparing it with that in Figure 4.12a, we see that the wind curtailment bump at midday has disappeared, and gas production during peak demand is lowered. This effect is also corroborated by the empirically determined curve in Figure 4.11b. The excess renewable energy during the afternoon hours is stored by the BESS (pink). At peak demand in the evening, the BESS discharges, partly replacing the gas-powered production and thereby avoiding emissions.

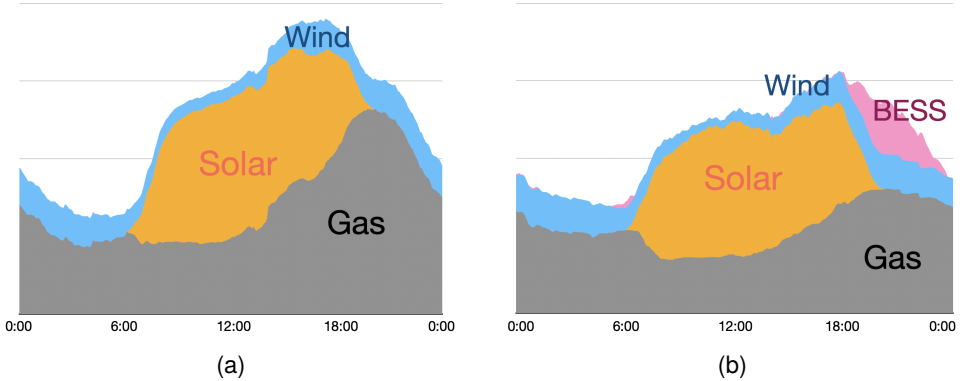


Figure 4.11: Electricity production per selected energy source in California on July 17, 2020 (a) and July 17, 2024 (b). Data from [102].

4.5. A Multiport Network Model of the Economy

4.5.1. Introduction

In this final application, we design and analyze the behavior of relatively complex model of an entire economy. The underlying network is taken from [40], to which we refer the reader for the details. This concerns a rather comprehensive model that combines the circular flow, stock-flow consistent, ISLM, and Solow growth models from macroeconomic texts, together with a model of a commercial bank based on accounting principles. To it, we have added a model for the security markets from [103] and the diamond-shaped supply chain presented in the preceding.

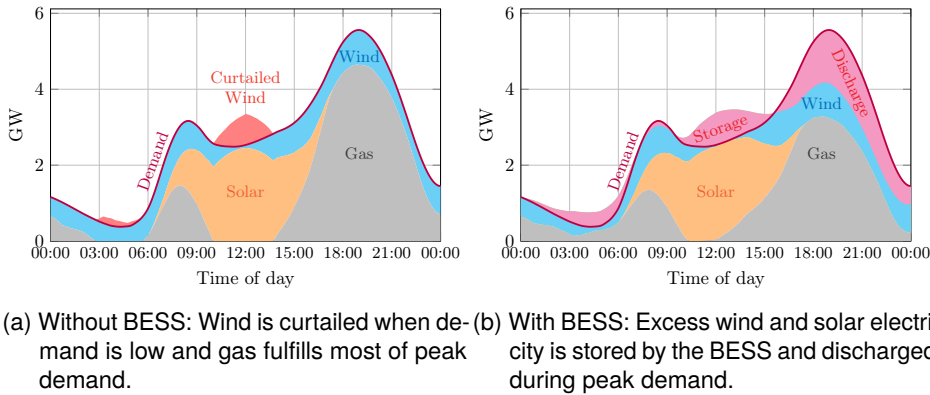


Figure 4.12: Duck curve simulations.

4.5.2. The Model

The final LTSpice layout is displayed in Figure 4.14, in the background of the plots of the flows measured by the corresponding amp meters. We have indicated these by a circular icon marked with an A. We have color-coded them to correspond to the graph of the corresponding signals that overlay the network.

We have represented the economy itself as a 2-port. There is a port for the flow of foreign capital to the securities market (Green), and one for the flow of goods to the productive sector (Red). Otherwise, the economy is closed.

At one abstraction level lower, the network contains five multiports. This includes the sectors typical of a macro model: household, financial, firms, and government. To this, we have added a supply chain as a demonstration of the modularity of the theory.

We have modeled the financial sector to have two further levels of abstraction. At one level lower, it consists of a banking sector and a sector representing the securities industry. We have designed it to be heterogeneous, as we show below. If needed, the methods in Section 3.4.3 can be exploited to determine the appropriate representative agents for the sector and its subsectors. Here we focus on the LTSpice simulations.

Further down the level, the banking sector contains two 5-port banks. They are in competition with one another for assets and liabilities by offering interest payments on the former and charging interest on the latter. The assets are loans to firms, and the liabilities consist of both household savings and current account deposits. They have distinctly dissimilar policies: Bank A is an aggressive discounter, while Bank B offers households the traditional banking experience. This is borne out by the graph containing the incentive that each bank provides the households to save: Bank A rapidly adjusts this based on circumstances (Magenta), while Bank B holds it steady (Plum). The circuit for this follows the example in Section 3.3.6 and Figure 3.2c.

The securities industry consists of a stock market and a bond market. Both are 2 ports. At one port, they compete for household investment. At the other port, they

offer dividends and coupon payments, respectively, in return.

The stock market adds significant volatility to its returns. Although it is configured with a noise source (TealBlue), in reality, this is done in an entirely endogenous manner; see [103] for the details. The price volatility causes the flow of funds to also become volatile, both to the stock market (BrickRed) and somewhat less to the bond market (Orange). At the flows of savings (BlueViolet and ProcessBlue), it is still noticeable, and also Bank A's policy is slightly affected. Beyond that, at the firms' capital investment (Sepia), the rate of production (OliveGreen), and demand (LimeGreen), the effect ceases to be visible.

The bond market, in contrast, provides a stable return on investment. It maintains a current price for bonds at the inductor. The current source makes the coupon payments based on the total face value of the bonds issued.

An external port is connected to domestic investment to accommodate foreign investment. After 1.5 years, the appetite of foreign investors to purchase bonds suddenly steps up (Green). As a result, the domestic flow of bonds steps down (Orange), and is redirected to the stock market (BrickRed).

The shock then further propagates through the network. Bank A, the aggressive discounter, reacts to the increase in savings and makes a downward revision to its savings incentive (Magenta).

Theoretically, this change then further reverberates through the economy, as the change in demand influences consumer expenditure and so on. However, it is not productive to keep follow this, as the output from the LTSpice simulation contains the combined effect.

We model the government as a 2-port without any further lower levels of abstraction. It taxes household income and adds government expenditure to the firms.

Going beyond the typical textbook macro model, we add a separate market for goods and services. Households demand goods and services, and firms produce them, and delivery is done through a supply chain.

The price coupling between the goods and services versus the various flows of money is achieved through mutual inductances, comparable to the substitution effect in the example in Section 3.3.6. The circuitry is designed to operate in conformance with the IS-LM model² to account for the various money flows and interest rates provided by the banking sector. We refer the reader to [40] for the details.

The supply chain is a 2-port system, with one port interconnected with the households and the other with the firms. Like the supply chain in Figure 3.5, it is of diamond shape. The intermediaries are modeled as RLC circuits, identical to the trader in Figure 3.2b. Each contributes two complex poles and, hence, the chain adds significantly to the complexity of its dynamic behavior.

This is evidenced by the economy's response to the shock in demand. At year 3, foreign demand shows a short impulse in its incentive to acquire goods. Its suddenness causes massive imbalances in the supply chain, causing prices to drop in reaction to the inventory buildup. The drop in price causes demand to return, and the cycle repeats itself somewhat less markedly, leading to the damped oscillatory motion towards equilibrium characteristic of stable higher-order systems.

²The acronym stands for Investment Savings vs Loanable Funds and Money; see [84]

The fluctuations then propagate through the entire economy. Households, when spending less on consumption, increase the rate at which they save (BlueViolet and ProcessBlue) and invest (Orange and BrickRed), and vice versa. Bank A immediately takes advantage of this, adjusts its client rates (Magenta), and increases its share of total savings, only to reverse course when households increase consumption (LimeGreen) again. They thus follow the oscillations, lagging with a relatively small delay in phase. A similar effect, albeit less noticeable, is seen in the flow of funds to the securities market. Firms reduce capital investment (Sepia) when production (OliveGreen) is low and vice versa. Fluctuating production, finally, leads to oscillations in capital investment.

Due to their interconnectedness, households and firms additionally influence one another's responses. The actual responses, as provided by the LTSpice simulation, are therefore the results of a highly complex set of interactions and communications between agents with all manner of internal feedback loops. The fact that the simulation runs within seconds is a testament to the potential of circuit theory for modeling the dynamics of complex economic systems.

4.5.3. Forecasting and Analysis

We conclude by showing how the foregoing model can be used to measure the typical variables of interest to economists. With their DSGE model in [85] (reproduced here in Figure 3.1c), the authors focus on forecasting GDP growth, inflation, and the rate of interest. For the network model, these forecasts are determined simply by measuring the relevant flows in the LTSpice simulation.

GDP To determine GDP, economists use the national income identity, which, for an open economy, reads $Y = C + I + G - NX$. Y is the flow of money that represents GDP. The constituent flows are readily measured at the appropriate ports. Consumption expenditure C and investment I at the households, government expenditure G at its port, and net exports NX at the economy's external port. Summing these measurements gives the projection of GDP shown in Figure 4.13.

At initiation, GDP grows steadily, but the rate slows down after the bond investment shock. This is because the resulting increase in household investment in stocks decreases consumption expenditure C . In addition, the accompanying decrease in tax revenue reduces government spending G . These effects outweigh the increase in capital investment I .

The effects of the demand shock on GDP are significantly more dramatic. Immediately after the shock, GDP plummets due to the sharp decrease in consumption. Due to the momentary nature of the shock, growth recovers on average after this. However, due to the massive supply chain disruptions, GDP oscillates around its mean value, initially wildly so, which gradually dampens out to resume the previous steady growth rate. At the onset of the COVID-19 pandemic, a similar demand shock took place, and its observed effects on the economy are comparable to these results.

Inflation The addition of a separate goods market to the network allows the network to determine the price level and the inflation rate. Economists typically conflate physical output with GDP, but this is unnecessary here. To measure the price level, we pair the flow of goods with that of the corresponding money. For retail price levels, we measure the ratio of household expenditure to the flow of goods to the households. For wholesale price levels, we take the revenue against the flow to the firms. Inflation is determined by taking the time derivative of the price levels. To obtain the rate of inflation, we take the logarithmic derivative instead. Higher-order derivatives give the dynamics of inflation.

Figure 4.13 shows the retail price level, as measured in the simulation. Shortly after its initiation, inflation disappears after a brief rise. Then, due to the sudden availability of foreign funds following the bond investment shock, the inflation rate ticks up and remains steady until the onset of the demand shock. The sudden drop in demand that follows has a strong deflationary effect, with prices falling rapidly. As with GDP, inflation ultimately goes back to its previous level. Supply chain dynamics do cause significant volatility, albeit relatively less pronounced than GDP.

Interest Rate For the prevailing rate of interest, we use the banking sector's Funds Transfer Pricing or FTP Rate. This is the average of the rate offered on savings and that charged on loans. A more complete model might include a central bank within the sector that influences rates.

Figure 4.13 contains the network's forecast for the FTP rate. The response reflects the heterogeneity of the sector. The overall downward trend is due to Bank B's intransigence on its savings rate, while being forced to compete for the issue of loans to the firms, thus bringing down the overall FTP rate. Bank A's aggressive policies allow it to adjust both its client rates to closely follow the overall state of the economy. Because of its actions, the interest rate serves to counteract the changes in the price level. Comparing the FTP rate with the inflation rate in Figure 4.13, we see that the former is 180° out of phase with the latter, thus acting countercyclically.

Unlike GDP and the price level, the volatility in stock prices does influence the FTP rate. This is particularly noticeable when the rate is relatively constant. The banking sector competes for funds with the securities industry, and Bank A's aggressive policies subject the sector to any noise emanating from the stock market.

4.5.4. Comparison with a DSGE Model

We conclude by comparing the network with the DSGE model of the Federal Reserve Bank of New York [85] (see Figure 3.1c). In their publication, the authors likewise concentrate on forecasting GDP growth, inflation, and the interest rate. We focus on the individual letter combinations in the DSGE acronym.

We first turn to the DS of Dynamic Stochastic and consider the fluctuations that appear in the forecasts. In the DSGE model, they are directly attributed to the stochastic shocks and are considered to be *exogenous* (see, further, [73]). In the network, they are mainly caused by the supply chain and, hence, are *endogenously* generated. In fact, they point to the presence of second and higher-order dynamics in the chain

(see Section 4.3). The shocks merely serve to excite the various frequencies: The Brownian noise on the stock market is responsible for exciting the high-frequency fluctuations on the FTP rate and the demand shock the lower frequency oscillations in the various forecasts. The latter shock, like the bond investment shock, is not even required to be stochastic and can potentially be some known disturbance.

Turning to the GE for General Equilibrium, we note that for both models the equilibria are general in the sense that they both have separate financial and goods markets. However, whereas the existence of an equilibrium state is *assumed* in a DSGE model, in the network, it is an *emergent* phenomenon, arising from the interactions among the agents. Therefore, the existence of an equilibrium state is not a precondition for the functioning of a network and even chaotic systems or those with multistability can be analyzed.

To model a sector, the DSGE model relies on the concept of a *representative* agent. The network model, instead, relies on a description in terms of two *levels of abstraction*. This allows us to understand and interpret the sector's macro-level behavior as a synthesis of the behavior at the lower levels of abstraction. In the network, the financial sector has two levels between macro and micro. First, the entire sector is made up of a banking sector whose behavior is manifestly distinct from the securities market. Then, the banking sector consists of two different types: traditional and discount banks. Similarly, the securities market consists of the stock and bond markets, with only the former exhibiting price volatility. If needed, a representative agent for the sector, or any subsectors for that matter, is then recovered by aggregating back up (see Section 3.4.1).

4.6. Conclusions

This chapter demonstrated how economic systems can be constructed and analyzed following the multiport modeling framework. In each case, the recipe was the same: determine which agents are involved, represent their behavior as an internal circuit, and create a network by interconnecting their ports in the topology that represents their interaction. The resulting network diagrams provide a graphical representation of the economic problem at hand but also encode the economic dynamics directly. We have shown that the framework is applicable to textbook concepts, policy scenarios, and even sector-wide dynamics can be translated into interactive models, offering both insight and intuition into how economic systems behave.

Using LTspice as a simulation environment, we showed how these models can be brought to life. Modelers can tune parameters, experiment with structural changes, and explore dynamic behavior across time and frequency domains, just as electrical engineers do when designing electrical devices. The representation as a network diagram provides a transparent way to observe and interpret economic activity, making it easy to trace how trade flows, prices, and other signals emerge and propagate through the system.

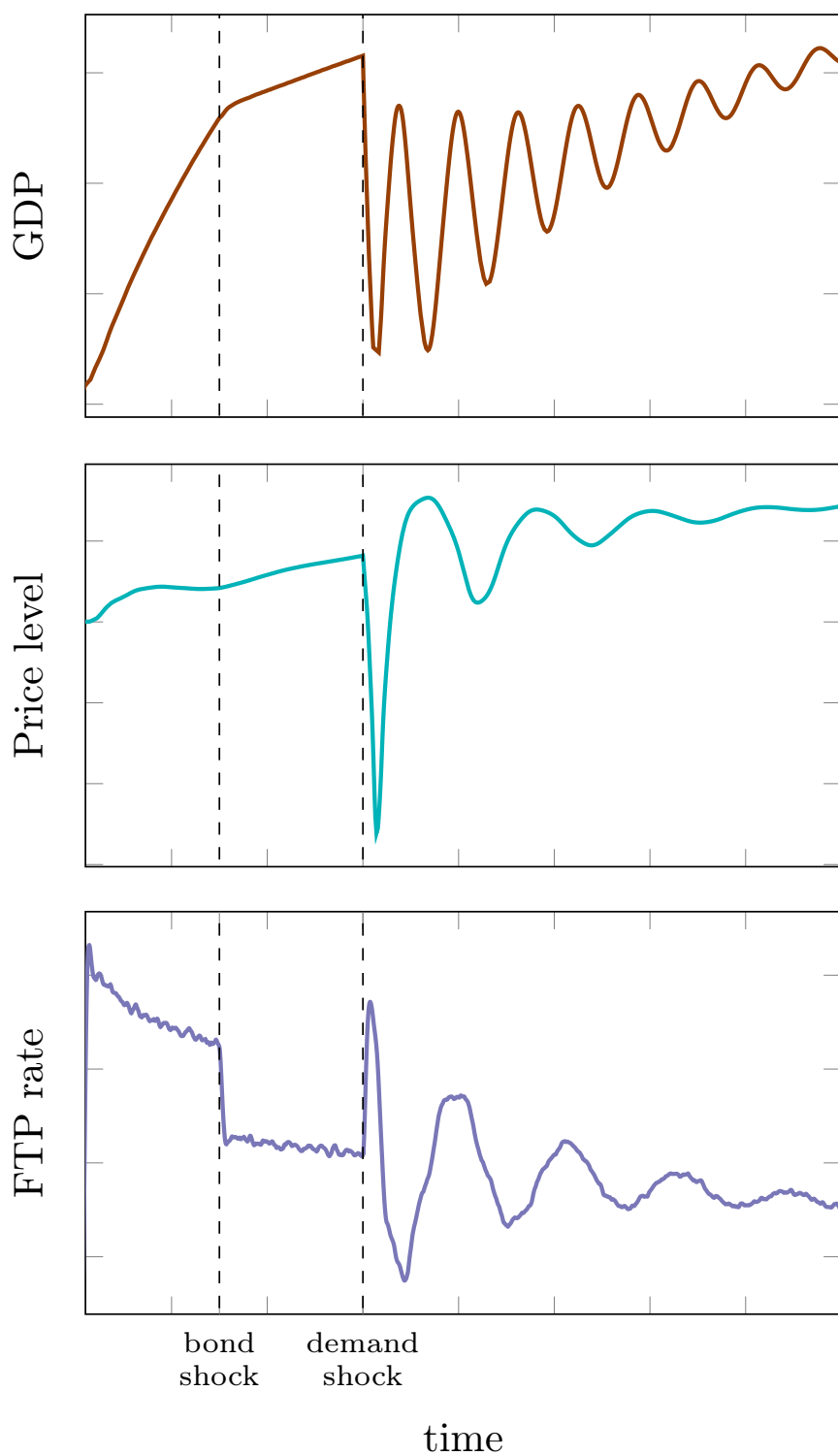
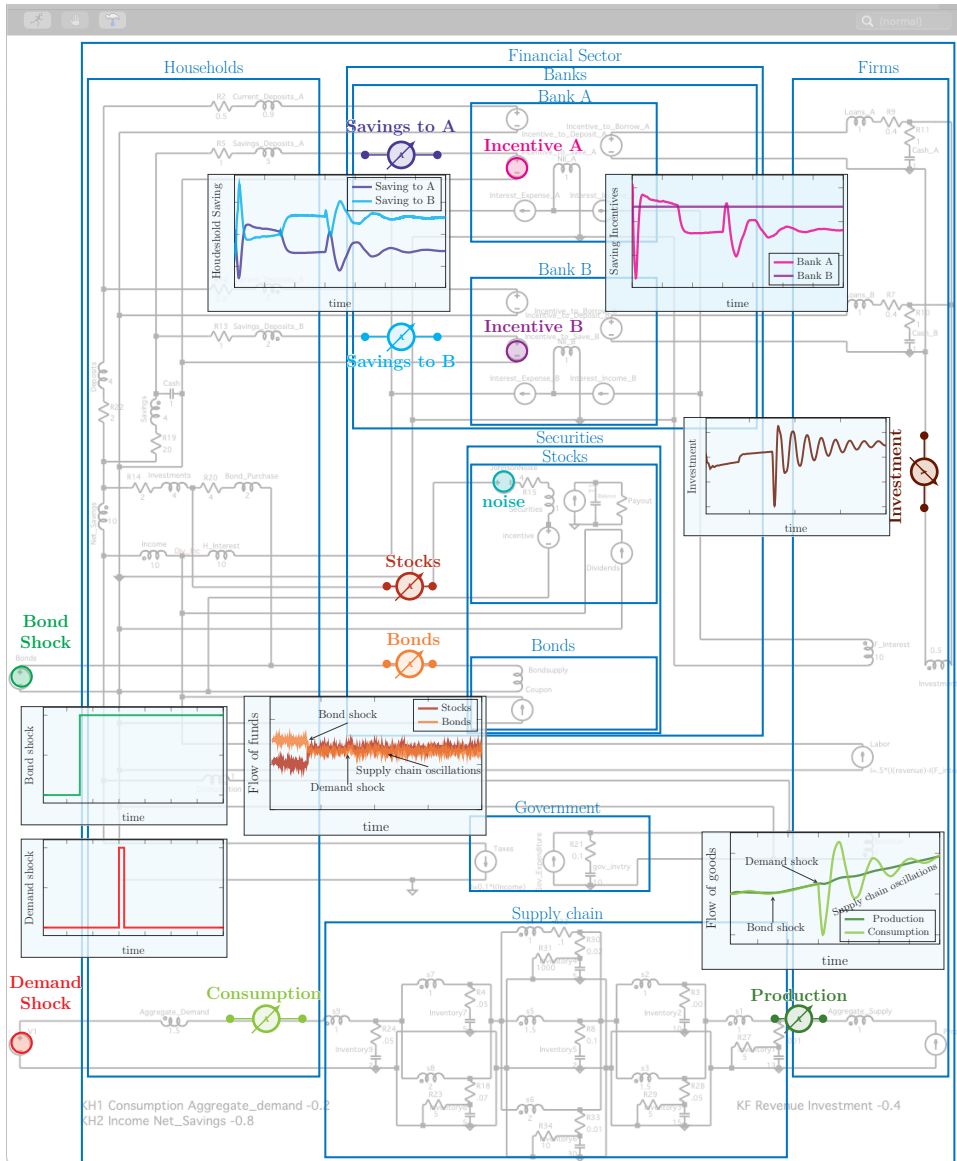


Figure 4.13: Measurements of the GDP, the price level, and the FTP rate.



5

Conclusions and Recommendations

5.1. Contributions of the dissertation

The goal of this dissertation was to develop a methodology that enables Mendel's economic engineering theory to be applied to the modeling of complex economic systems. This is achieved by representing economic agents as network components governed by constitutive behavioral laws, and economic interactions as interconnections that define how these components are coupled. System-level economic dynamics then arise directly from the interaction between the representative components and the Kirchhoff laws. In doing so, the methodology turns Mendel's principles into an operational modeling framework, in the same way that circuit theory transforms Maxwell's and Faraday's laws into an engineering methodology for electrical systems.

The methodology makes it possible to build models of economic systems as analogous electrical networks. Agents and sectors are constructed as modular, hierarchical multiport components, whose ports serve as explicit interfaces through which flows of goods, resources, or assets are exchanged under the influence of economic incentives. At each port, incentives play a role analogous to electrical potentials: together with flows, they determine how economic activity propagates through the system. In this way, ports encode both the dynamical behavior of agents and the topology of interactions, making them central to the construction of economic networks.

By organizing models around ports and interconnections, agents and sectors can be analyzed and validated independently before being assembled into larger systems. Because models take the form of executable circuit representations with explicit dynamical structure, economic dynamics can be simulated directly using established engineering tools such as LTspice. This opens the door to exploring economic dynamics, transient behavior, feedback effects, and even frequency analysis.

These capabilities are demonstrated through a diverse set of applications spanning multiple scales. Classical textbook economic problems are brought to life as

dynamical systems that can be simulated, expanded, and analyzed interactively. Through supply-chain and market models, it was shown how frequency-domain analysis naturally applies to the economic domain. Bode plots, for example, reveal how economic systems amplify or dampen shocks at different time scales, how delays shape adjustment dynamics, and where resonances and instabilities arise. This makes it possible to address economic problems through the powerful tools developed for engineering purposes can be used to tackle economic problems.

The methodology also enables explicit analysis of structural change. As illustrated by the energy system example, changes in market organization can be incorporated directly into the model, after which their dynamic consequences follow transparently from the same underlying principles. This kind of reasoning is difficult to achieve with data-driven regression models, which are tied to historical observations and offer limited insight into alternative system designs. In contrast, the engineering-based methodology supports forward-looking analysis based on first principles.

Finally, the applications show that the methodology scales naturally to large systems. The modular macroeconomic model demonstrates how economies with many interacting sectors can be assembled hierarchically while preserving clarity and interpretability. Because the framework rests on explicit dynamical laws, it readily accommodates feedback control, stochastic disturbances, and noise, allowing economic systems to be analyzed with the same breadth of tools that engineers routinely apply to complex physical systems.

5.2. Limitations of the modeling methodology

While the dissertation develops an engineering style modeling methodology for economic systems, several limitations should be acknowledged. These limitations clarify the scope of the work and identify opportunities for further refinement.

First, the methodology inherits the assumptions of the underlying economic engineering theory. Just as circuit theory is an engineering representation of Maxwell's laws rather than a full description of electromagnetic phenomena, the modeling choices made here reflect an engineering focus on structure, dynamics, and interconnection, rather than a complete economic theory.

Second, the methodology represents agents as dynamical systems with well defined interfaces. This abstraction aids modularity and analysis but may omit aspects of real economic behavior such as institutional constraints, strategic decision making, or discontinuous adjustments. Extensions may be required to accommodate such features.

Third, the practical use of the methodology depends on the availability of suitable models for individual agents and markets. As in engineering, the usefulness of the methodology grows with the development of validated component models, and building such a library is an important task for future work.

Finally, while the methodology is designed for simulation and structural analysis, it does not guarantee predictive accuracy. Its contribution lies in organization, interpretability, and dynamical consistency rather than empirical forecasting.

5.3. Recommendations for future work

The modeling methodology developed in this dissertation opens several promising directions for future research.

A central topic is the incorporation of empirical data. Although one example using oil market data was included, the primary focus of the dissertation was methodological. A systematic study of how to calibrate and estimate multiport economic models is a natural next step. Such work involves both validating the model structures and determining how available economic data can be used to identify system parameters. Unlike engineering systems, economic systems cannot usually be subjected to controlled excitation to generate identification data, complicating the inference problem.

The modular structure of the methodology may provide a route forward. Because agents and subsystems have clearly defined interfaces and interpretable internal dynamics, individual components could potentially be estimated in isolation before being combined into larger networks. This may help mitigate identification difficulties that arise in highly aggregated or black box econometric models while preserving the structural interpretability that is central to the methodology.

Another direction is the integration of control theory. The models developed here are dynamical systems, which makes it natural to apply both classical and modern control methods. Agents can be represented as feedback controllers, such as firms as adaptive optimizers, governments as regulators, or households as forward looking decision makers. Several MSc theses have already explored this direction, and it would benefit from further formal development.

Developing dedicated software tools is another practical recommendation. While LTspice has proven useful, it remains designed for electrical engineers. A graphical environment tailored to economic circuit and multiport modeling would increase accessibility and usability for economists and policy analysts. Such a tool could support model construction, specification, and simulation in a domain relevant manner.

Related to this is the need to develop standard libraries of reusable economic components. Libraries for consumers, producers, intermediaries, and sector level modules would accelerate model building, enhance reproducibility, and support collaboration across research groups. Over time, such an ecosystem could substantially change how economic models are constructed and shared.

The methodology also lends itself to educational applications. Its graphical and network based nature helps students visualize economies as dynamic systems of interacting agents. Unlike static equilibrium models, the methodology emphasizes causality and adjustment processes, offering students a deeper understanding of economic dynamics. Future work could involve developing textbooks or instructional materials based on this perspective.

Overall, the dissertation proposes a way to model economic systems with the mindset of an engineer: structurally, visually, and with explicit attention to dynamics. Although the models take the form of electrical circuits, the systems they represent are real economic networks. The modeling methodology introduced here opens new possibilities for simulating, analyzing, and understanding economic systems.

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