SHEAR STRESS CONCEPT IN GRANULAR FILTERS

By

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ABSTRACT

Scour is a natural phenomenon caused by the flow of water in rivers and streams and occurs as a part of the morphological changes caused by rivers or as a result of man-made structures. For the Dutch Delta Works hydraulic structures were often constructed on fines with loose packing. To guarantee the geotechnical stability of these structures the bed in their immediate vicinity had to be protected. Though several types of bed protection can be distinguished, for example concrete blocks, asphalt, and granular filters (with or without geotextiles) the scope of this paper is limited to granular filters. In the Netherlands geometrically sand-tight filters are usually used. The stability of these filters is mainly determined by the geometrical properties of the materials. Consequently, these classical filters with numerous layers are very expensive. In this study a model relation for sizing a geometrically open granular filter is discussed. Our goal is to promote discussion than rather to try to solve the many problems in the complex field of filtration in geotechnical engineering.

INTRODUCTION

Non-geometrical granular filters have a hydraulic mode of operation; i.e. the reduction of the hydraulic shear stresses on the base material is such that erosion is prevented. Available knowledge of the hydrodynamic forces, lift and drag, acting on particles in granular filters is mainly based on experience and laboratory and field measurements which has proven inadequate for the purpose of developing a highly accurate design criterion. This is due to the numerous factors that influence the stability, and to the definite probabilistic nature of the acting forces which may at times be significantly in excess of mean values and consequently cause movement. Verheij and Den Adel (1998) calibrated and validated model relations for granular filters that are based on the Navier Stokes equation for uniform flow, the so-called Forchheimer relation and the hypothesis of Boussinesq.

Figure 1 shows a horizontal one-layer filter with a thickness d above the base material. Considering uniform flow the shear stress distribution in the open flow is linear. Usually the mean flow velocity in the downstream direction can be approximated by a logarithmic function. The velocities and shear stresses in the filter layer will be briefly discussed by applying the three aforementioned equations.

In a granular filter and with uniform flow conditions the balance of forces acting on a control volume can be given by (Shimizu *et al.*, 1990)

$$\frac{\partial \tau}{\partial z} + F + \rho g i = 0 \tag{1}$$

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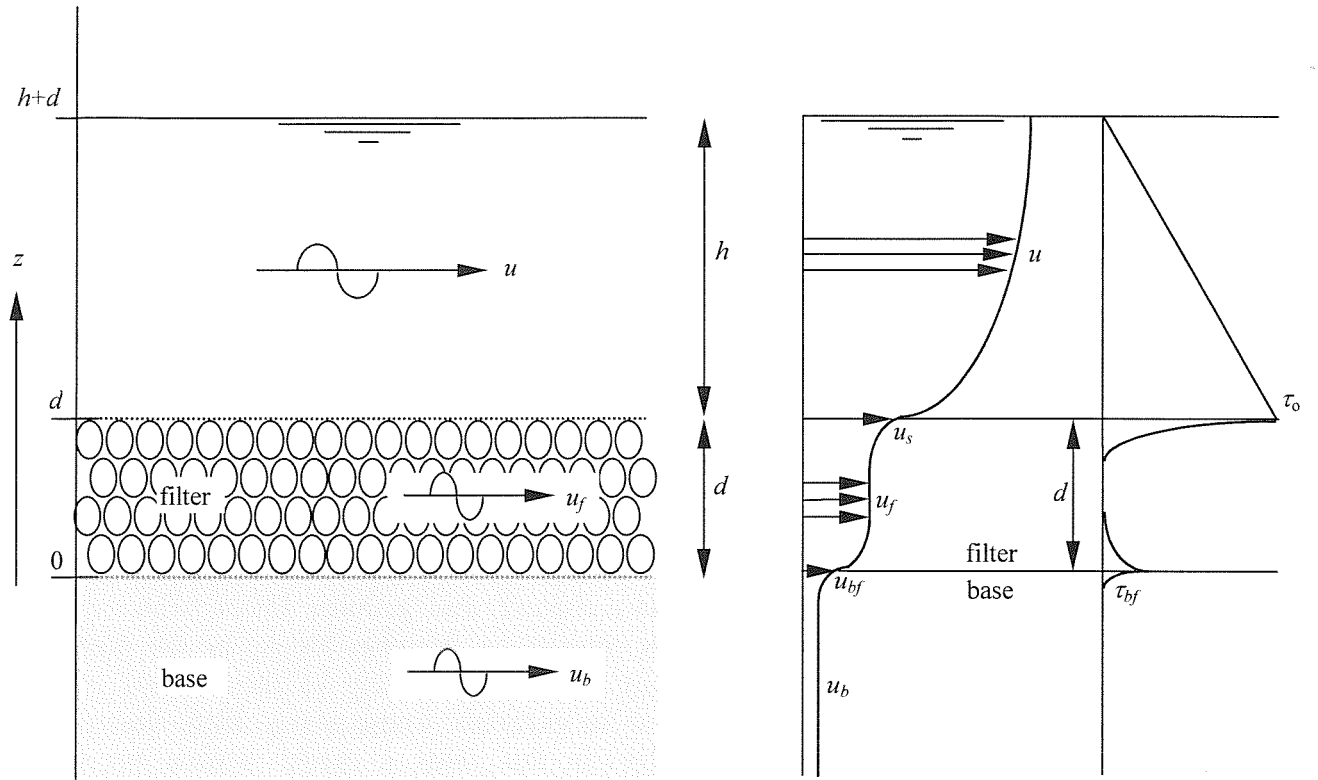[3] Delft Hydraulics, Delft, The Netherlands

Figure 1   Overview of definitions for a one-layer filter

in which τ is the shear stress, z is the vertical co-ordinate, F is the seepage resistance, ρ is the fluid density, g is the acceleration of gravity and i is the energy gradient. The first term represents the momentum transfer from the free surface flow to the filter bed. To solve equation (1) the seepage resistance and the shear stress have to be related to parameters that express the loading in granular filters. The Forcheimer relation reads:

$$F = -\rho g\left(au + bu^2\right) \tag{2}$$

where u is the (mean) filter velocity and a and b are constants (with a dimension). The first term in (2) represents the Darcy's law and is applicable in laminar flow conditions, whereas the second term characterises the resistance in turbulent flow conditions. The hypothesis of Boussinesq can be given by:

$$\tau = \mu\frac{\partial u}{\partial z} \quad \text{with} \quad \mu = \rho v_t \tag{3}$$

where v_t is the eddy viscosity. It should be remembered that the shear stress in granular layers is not a shear stress following the definition of the shear stress in open channel flow. The shear stress in this study must be considered as a loading parameter.

Most of the turbulence-model-development and application work was carried out in the area of mechanical and aeronautical engineering. In the early eighties Rodi (1984) assessed the applicability of turbulence models to hydraulic flow problems. However, these models have not been extensively validated for flow in porous mediums such as granular filters. Therefore in this study some assumptions have been made about the eddy viscosity. In general, the eddy viscosity is related to a representative length scale and to a representative flow velocity. The length scale has been determined by the open space or by the magnitude of the particle sizes. Here the eddy viscosity is approximated by:

$$v_t = \alpha u D_{f,15} \tag{4}$$

where α ($\cong 0.9$) is a constant and $D_{f,15}$ is the diameter of the filter material (exceeded by 85% of the weight percentage). Combining equations (1), (2) and (3) and assuming turbulent conditions ($a = 0$) the distribution of the shear stress in a one-layered filter can be modelled by (Verheij et al., 2000):

$$\tau = \tau_{bf}\,e^{-\xi z} + \tau_0\,e^{\xi(z-d)} \quad \text{with} \quad \xi = \sqrt{\frac{2gb}{\alpha D_{f,15}}} \quad \text{and} \quad b = \frac{2.2}{n^2 g D_{f,15}} \tag{5}$$

where τ_{bf} is the shear stress at the interface of the filter layer and the base layer, τ_0 is the bed shear stress (or shear stress at the interface of the flow and the filter layers), d is the thickness of the filter material and n ($= 0.4$) is the porosity. For turbulent flow conditions the damping parameter is approximately equal to $\xi = 5.5/D_{f,15}$. Since the damping parameter ξ is smaller than the characteristic length scale in the filter material, the continuum assumption in Forcheimer's equation is violated. Therefore the applicability of equation (5) is limited. Nevertheless the relation between τ and z will hold. Though the relation between ξ and $D_{f,15}$ may be quantitatively invalid, there is experimental evidence that ξ is inversely proportional to $D_{f,15}$. The filter velocity as function of the vertical co-ordinate can be written as:

$$u(z) = \sqrt{C_1 e^{z\xi} + C_2 e^{-z\xi} + \frac{1}{b}} \quad \text{with} \quad C_1 = \frac{2(\tau_0 - \tau_{bf}e^{-\xi d})}{\alpha\rho D_{f,15}\xi(e^{\xi d} - e^{-\xi d})} \quad C_2 = \frac{2(\tau_0 - \tau_{bf}e^{\xi d})}{\alpha\rho D_{f,15}\xi(e^{\xi d} - e^{-\xi d})} \tag{6}$$

The parameters C_1 and C_2 can be expressed by the velocities as well: u_{bf} and u_s (with $u_{bf} = u(z = 0)$ and $u_s = u(z = d)$)

$$C_1 = \frac{\left(u_s^2 - \frac{1}{b}\right) - e^{-\xi d}\left(u_{bf}^2 - \frac{1}{b}\right)}{\left(e^{\xi d} - e^{-\xi d}\right)} \qquad C_2 = \frac{e^{\xi d}\left(u_{bf}^2 - \frac{1}{b}\right) - \left(u_s^2 - \frac{1}{b}\right)}{\left(e^{\xi d} - e^{-\xi d}\right)} \tag{7}$$

When the influence of the permeability ratio of the filter and base material is included, the boundary condition for the filter velocities at $z = 0$ and at $z = d$ can respectively be given by:

$$u(z = 0) = \sqrt{u_f u_b} \qquad u(z = d) = \sqrt{\frac{2\tau_0}{\alpha\rho D_{f,15}\xi} + u_f^2} \tag{8}$$

For uniform and laminar flow conditions ($b = 0$) Verheij et al. (2000) derived similar relations for both the filter velocities and shear stresses. The damping parameter for laminar flow is approximately 6 times larger than for turbulent flow ($\xi = 30/D_{f,15}$). For a two-layer filter also an analytical solution can be derived.

CONCEPT OF GRASS

Particle transport occurs when there is no balance between load (shear stress) and strength (inter particle friction). When the load is less than some critical value, the bed material remains motionless. Then the bed can be considered as fully stable. But when the load over the bed attains or exceeds its critical value, particle motion begins. The beginning of motion is difficult to define and this can be ascribed to phenomena that are random in time and space. In 1936, Shields published his experimental results for the initiation of movement of uniform granular material on a flat bed, later known as the Shields-criterion although Rouse proposed the well-known curve.

In the Shields diagram, the influence of fluctuating shear stresses on bed particles is not directly specified. Though the distribution of the instantaneous bed shear stress is unknown, there are indications that this distribution has to be asymmetrical owing to sweeps and ejection (Lu and Willmarth, 1973). When dealing with the concept of Grass, the exact shape of the distribution is irrelevant because a characteristic bed shear stress can be defined, this being a time-averaged

value and a fluctuating term that originates from the turbulence near the bed. The characteristic value is a value that is higher or lower than the time-averaged value. Usually characteristic values are expressed as a mean value and a fraction or manifold of the standard deviation. In fact, the problem of bed stability will now be transferred to the magnitude of this fluctuation. In addition to the random nature of the load, another random variable in the process of initial instability is determined by the strength of the particles close to the bed.

To make an adaptation to non-uniform flow it is useful to analyse the influence of the turbulence in the vicinity of the bed for uniform flow. For this exercise the concept of Grass (1970) can be applied, this being based on statistical assumptions for both the loading and strength parameters (Figure 2). The characteristic bed shear stress ($\tau_{0,k}$) and the characteristic strength, which is the critical bed shear stress ($\tau_{c,k}$) can be respectively written as:

$$\tau_{0,k} = \tau_0 + \gamma\sigma_0 \qquad \text{and} \qquad \tau_{c,k} = \tau_G - \gamma\sigma_c \qquad\qquad (9)$$

where γ is determined by an allowable transport of the bed material, σ_0 is the standard deviation of the instantaneous bed shear stress and τ_0 is the time-averaged bed shear stress, σ_c is the standard deviation of the instantaneous critical bed shear stress and τ_G is the time-averaged critical bed shear stress according to Grass. A specific transport will occur if $\tau_{0,k} = \tau_{c,k}$ which will be elucidated later.
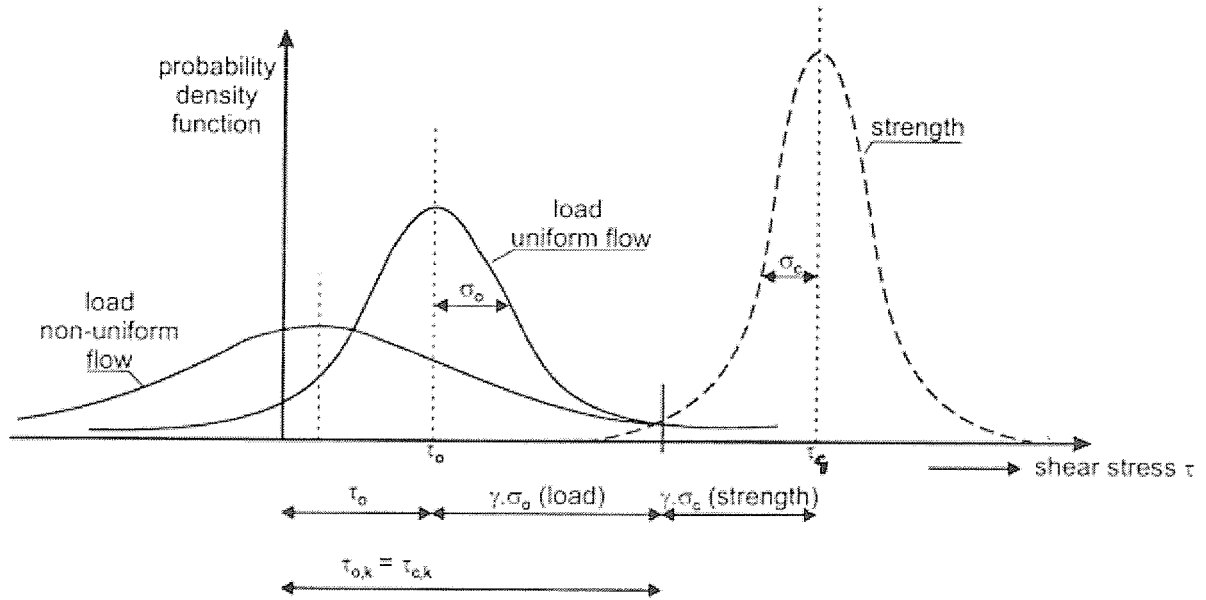

Figure 2. Probability functions of the loading and strength parameters (Grass, 1970)

If the characteristic loading near the bed is equal to the characteristic strength (thus $\tau_{0,k} = \tau_{c,k}$) and if $\sigma_c = \alpha_{c,f}\tau_G$ with $\tau_G = \Psi_{c,G,f}\Delta_f\rho g D_{f,50}$ (analogous to the Shields concept) and assuming $\gamma_{strength} = \gamma_{load} = \gamma$, a general relation for the filter layer follows:

$$\Delta_f D_{f,50} = \frac{\tau_0 + \gamma\,\sigma_0}{\Psi_{c,G,f}\,\rho g\left(1 - \alpha_{c,f}\gamma\right)} \qquad\qquad (10)$$

where $\alpha_{c,f}$ is a coefficient representing the variation of the material characteristics of the filter layer. For uniform flow Grass found that a bed of nearly uniform sand ($\alpha_c = 0.3$) was completely stable for $\gamma = 1$ and for $\gamma = 0$ a significant transport of sediment particles was observed. Based on his experiments, he reported that for $\gamma = 0.625$ the criterion of Shields was met for the initial movement of sands up to a size of 250 µm. In his opinion the $\gamma = 0.625$ criterion was also in

agreement with observations of Vanoni and Tison when using the Rouse curve as a basis for the critical shear stress prediction.

The critical bed shear stress τ_G is approximately 1.5 times higher than the time-averaged bed shear stress and thus 1.5 times higher than the mean critical value according to Rouse.

FILTER MODEL RELATIONS

In a similar way model relations can be derived for filters at the interface of the filter and the base layer (Figure 3). Using equation (5) the mean load parameter at the interface is ($z = 0$):

$$\tau(z = 0) = \tau_{bf} + \tau_0 e^{-\xi d} \tag{11}$$

Assuming $\tau_{bf} = \eta \tau_0$ and applying the concept of Grass, the characteristic load can be given by:

$$\tau_k(z = 0) = (\tau_0 + \gamma \sigma_0)(\eta + e^{-\xi d}) \tag{12}$$

The characteristic strength of the base material is:

$$\tau_{c,k}(z = 0) = \tau_G - \gamma \sigma_{c,b} = \Psi_{c,G,b} \Delta_b \rho g D_{b,50}(1 - \gamma \alpha_{c,b}) \tag{13}$$

By combining equations (10), (11) and (13), the following model relation for open granular filters will be obtained:

$$\frac{D_{f,50}}{D_{b,50}} = \frac{1}{\eta + e^{-\xi d}} \frac{1 - \gamma \alpha_{c,b}}{1 - \gamma \alpha_{c,f}} \frac{\Psi_{c,G,b}}{\Psi_{c,G,f}} \frac{\Delta_b}{\Delta_f} \tag{14}$$
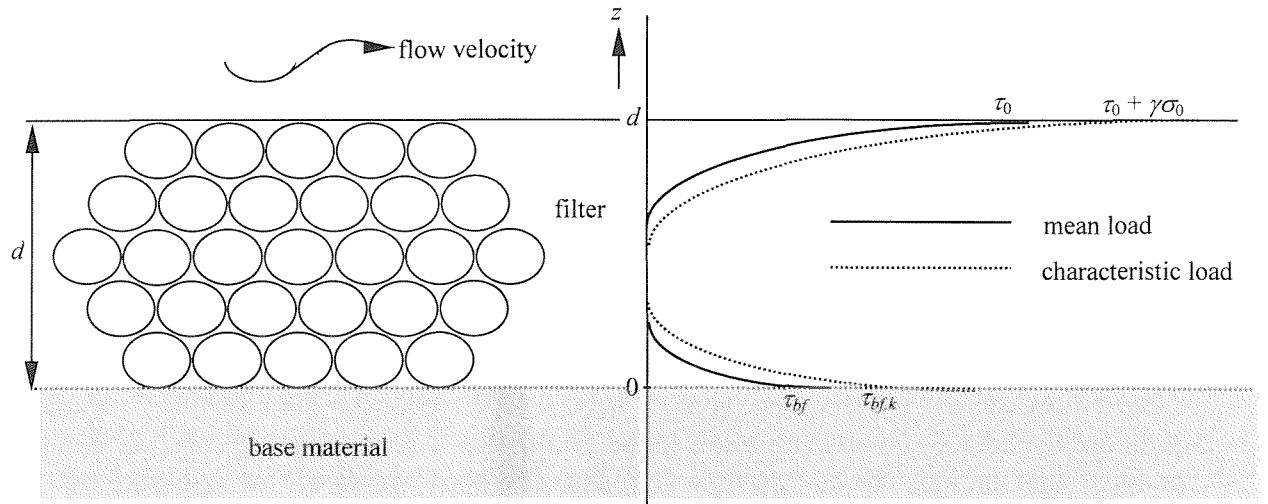

Figure 3   Distribution of the mean and characteristic load

Although characteristic values for both loading and strength are included, the resulting ratio of D_f/D_b is independent of the fluctuations in the loading. There are two reasons for this rather unexpected result: First it is assumed that both filter and base material will display initial movement under the same loading conditions. Second, fluctuations in the load exert a load on the filter material similar to that on the base material. The effects of non-uniform flow have been taken into account by applying equation (10) and can be represented by the standard deviation of the instantaneous bed shear stress (Hoffmans, 1992,1996).

With equation (14) the influence of particle gradation on the stability of the base material can be explained in a qualitative way. For example, when the base material is more graded than the filter material, $\alpha_{c,b}$ is greater than $\alpha_{c,f}$. Consequently, the required ratio D_f/D_b is less when this

value is compared to situations where base and filter materials do have the same gradation. If only the filter material is broadly graded, $\alpha_{c,f}$ is greater than $\alpha_{c,b}$, so the maximum value of D_f/D_b is higher than for similarly graded materials. These predictions correspond with observations in flume experiments. A broadly graded base material has more fines than a more uniform material. The material in the filter layer has to prevent the erosion of the fines. This can only be achieved by reducing the filter velocities or by putting more fines into the filter layers. A broadly graded material in the filter layer has relatively more fines, which reduce the pore velocity in the filter and so also the loading on the base material. Hence, the broadly graded filter material is allowed to have an average grain size that is larger than for uniform material.

Using the assumptions $\alpha_{c,b} = \alpha_{c,f}$ and multiplying both sides by $D_{f,15}/D_{f,50,}$ equation (14) reduces for high values of ξd into:

$$\frac{D_{f,15}}{D_{b,50}} = \frac{D_{f,15}}{D_{f,50}} \frac{1}{\eta} \frac{\Psi_{c,G,b}}{\Psi_{c,G,t}} \frac{\Delta_b}{\Delta_t}$$

(15)

The value of η has been calibrated by using experimental results obtained by Van Huijstee *et al.* (1991). In these 9 flume experiments the instability of the filter layer and the base layer was simultaneously observed. The mean value of η is about 0.01 with the boundaries $0.005 < \eta < 0.025$. Hoffmans (1996) also found a value of 0.01 on the basis of the Japanese tests of Shimizu *et al.* (1990). Remark that the calibration and validation of η was based on uniform-flow experiments.

The resemblance to traditional relations derived by Terzaghi is surprising. The stability between filter and base layer for geometrically sand-tight filters is:

$$\frac{D_{f,15}}{D_{b,85}} < 5 \quad \vee \quad \frac{D_{f,50}}{D_{b,50}} < 10$$

(16)

which means $\eta \cong 0.1$.

The differences between equations (15) and (16) can be ascribed to a safety factor that varies from 4 to 20. This analysis shows that for uniform flow the relations for geometrical sand-tight materials are strongly oversized. When the turbulence intensities are much higher, for example downstream of sills, the value of η ($\eta \cong 0.01$) might be questionable. Under these conditions the ratio D_f/D_b probably tends to the geometrical value of about $\eta \cong 0.1$. It should be remarked that in this study, equation (15) has not been validated for non-uniform flow conditions.
Bakker *et al.* (1995) and Stephenson (1979) discussed filter model relations, which predict similar ratios between particle sizes of filter and base material. Although the prediction potential of these relations is reasonable for the experiments investigated, they depend on the ratio R/D_f, which is not realistic for uniform flow conditions.

DAMPING PARAMETER

The damping parameter (ξ) is related to material properties both for laminar and turbulent flow. For laminar flow ξ is approximately $30/D_{f,15}$. Following Ikeya (1991) the damping parameter varies from $14/D_{f,50}$ to $30/D_{f,50}$. In both cases the length scale of the damping is much smaller than the particle diameter of the grains in the filter layer. Consequently, the influence of the boundary layer is practically negligible in the case of laminar flow.

For turbulent flow conditions Verheij *et al.* (2000) found $\xi \cong 5{,}5/D_{f,15}$ whereas Ikeya (1991) arrived at the following: $1/D_{f,50} < \xi < 6/D_{f,50}$. Ikeya discussed a suggestion made by Stephenson (1979) that the turbulent boundary layer in the filter layer is approximately, $1.5D_{f,50}$ which was later independently confirmed by the measurements of Suzuki (1992). Summarising equation (15) is valid for both laminar and turbulent flow.

The difference between results of the Dutch and Japanese researchers can be attributed to a different way of modelling the eddy viscosity and to different values for the coefficients in the so-called Forcheimer relation. The Japanese assumed a constant eddy viscosity in the filter layer. In this study the eddy viscosity is related to the varying filter velocity (see equation 4). Note that the eddy viscosity is not a physical parameter, but a parameter that helps us to relate velocities to shear stresses. Since no measurements of filter velocities in relation to loading parameters are available no conclusions can be drawn at present.

CONCLUSIONS

In this study model relations for both the filter velocity and the shear stress at the interface filter-base material are presented. Although the exact relation between the damping parameter in a filter material and its material properties is disputable, the type of relation between characteristic length scale and particle size will hold, in spite of the fact that the assumptions for a continuum approach are violated.

It should be noted that the term shear stress is somewhat misleading. In fact the distribution of the shear stress in filter layers has to be considered as a distribution of a loading parameter. A model relation has been discussed for geometrically open filters, which can be used for both uniform and non-uniform flow. This relation is based on simultaneous instability of filter and base material. The influence of the grading effects of the filter and base materials has been shown qualitatively. This relation corresponds closely to the traditional stability relation of Terzaghi for geometrical filter design and represents the range of the magnitude of the safety factor.

To increase the accuracy of the model presented here more detailed information is needed, in particular the value of η, which may be found by carrying out experiments with non-uniform flow conditions. It is necessary to use sophisticated equipment to measure filter velocities and loading parameters.

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