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MATERIAL MODEL FOR NON-LINEAR FINITE ELEMENT ANALYSES OF LARGE CONCRETE STRUCTURES

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ABSTRACT

A fully triaxial material model for concrete was implemented in a commercial finite element code. The only required input parameter was the cylinder compressive strength. The material model was suitable for non-linear finite element analyses of large concrete structures. The importance of including a crack closure algorithm was demonstrated on a case involving sequential loading. Bayesian inference was applied to results from a series of benchmark analyses, and the results indicate that the modelling uncertainty can be represented by a log-normal variable with a mean 1.10 and a coefficient of variation of 10.9%.

Keywords: Material model for concrete, non-linear finite element analyses, modelling uncertainty, Bayesian inference, large concrete structures.

1. Introduction

The design of large concrete structures like dams and offshore oil and gas platforms are normally based on global linear finite element analyses. This allows for using the principle of superpositioning in order to handle the vast number of load combinations (Brekke et al. 1994, Engen et al. 2015). For such large shell structures it is important to perform global analyses due to the interaction between global and local load effects. Solid elements are normally used due to the required accuracy in structural joints, and the elements are large compared to the sectional dimensions. Sufficient capacity is assured by performing local sectional design.

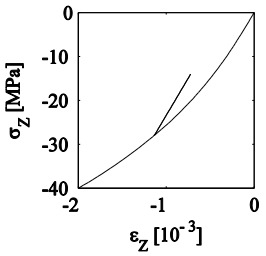
In order to take better into account the real physical behavior of reinforced concrete, non-linear finite element analyses (NLFEA) could be carried out. Due to cracking of concrete and yielding of reinforcement, a redistribution of the internal forces can be expected. All of the choices regarding force equilibrium, kinematic compatibility and constitutive relations influence the accuracy of the results obtained from NLFEA, i.e. the modelling uncertainty. However, for reinforced concrete, the material model for concrete is considered the largest source of modelling uncertainties.

The present work was based on a fully triaxial material model that has been developed since the 1970s and is still subject to improvements (Kotsovos 1979, Kotsovos 1980, Markou & Papadrakakis 2013). Engen et al. (2014) collected nearly 70 results from benchmark analyses reported in the literature which revealed a modelling uncertainty with a sample mean close to 1.0 and a sample coefficient of variation of 13%. The early versions of the material model were implemented in highly modular special purpose finite element codes, and in order to make the material model available for practicing engineers, the model was adapted to a commercial finite element code in the present work (Engen et al. 2016b).

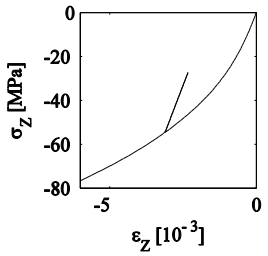
In this paper, the material model will be briefly described and one case from a series of benchmark analyses will be presented. By use of Bayesian inference, the modelling uncertainty will be quantified based on a series of 38 benchmark analyses.

2. Fully triaxial material model for concrete

The deformation of concrete subjected to general states of stress can be described by non-linear elastic moduli that are functions of the stress state (Kotsovos 1980). Such elastic moduli are normally valid up to an ultimate stress level, at which cracking is initiated. A suitable fracture criterion which is also a function of the stress level can be found in the literature (Kotsovos 1979). Due to the triaxial nature of the elastic moduli and the fracture criterion, the effect of transverse compression and tension both on the capacity and the ductility of plain concrete is automatically included. In the present work, the material model was implemented in a fixed, smeared cracking framework, allowing for both cracking and crack closure. The only required input parameter for the material model was the compressive strength f_c. Details of the material model can be found in a separate paper (Engen et al. 2016b). A brief demonstration of the behavior on material level is given in Figure 1 and 2.

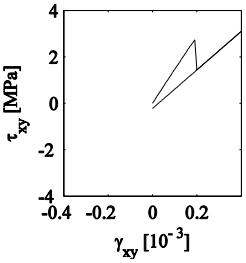


(a) Uniaxial compression: $\sigma_x : \sigma_y : \sigma_z = 0 : 0 : 1.0$.

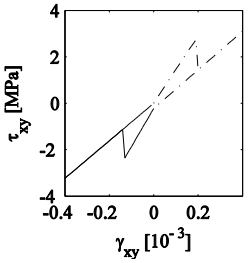


(b) Triaxial compression: $\sigma_x : \sigma_y : \sigma_z = 0.1 : 0.1 : 1.0$.

Fig. 1. Examples of loading, unloading and reloading of concrete with f_c = 40 MPa.



(a) Loading beyond cracking followed by unloading.



(b) Reversed loading with crack closure followed by unloading. The load pattern from a) is shown as dash-dotted.

Fig. 2. Examples of loading in shear of unreinforced concrete with f_c = 40 MPa.

3. Kinematic compatibility and force equilibrium

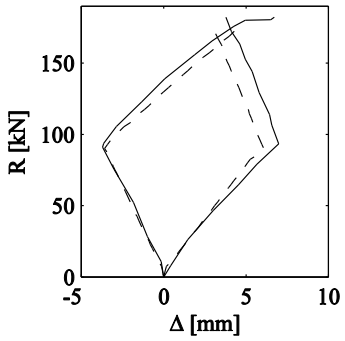
Relatively large 8-noded solid elements were used for the concrete, and the reinforcement was represented by embedded elements assuming no relative displacement between concrete and reinforcement. Modified Newton-Raphson with a force convergence criterion of 1% in combination with line search was used for solving the non-linear equilibrium equations.

4. Validation by benchmark analyses

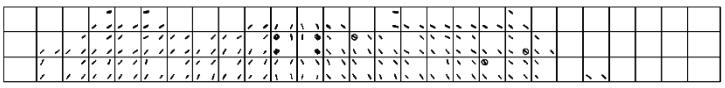
The material model was validated on structural level by performing analyses of experiments from the literature. Validation on material level can be found elsewhere (Kotsovos 1979, Kotsovos 1980, Engen et al. 2015). Results from one selected benchmark analysis are summarized in this paper. The focus of the present work is on the global behavior and the ultimate limit capacity, and the results are thus limited to crack plots at selected load steps and a global force-displacement curve.

Jelic et al. (2004) performed experiments on simply supported rectangular beams subjected to sequential loading. Beam LDCB3 was selected for the present paper. The layout and loading arrangement are shown

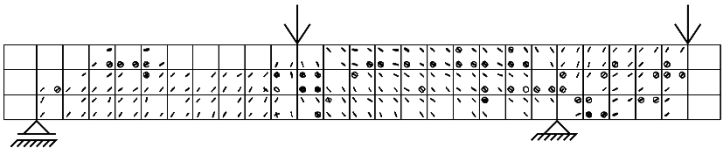
in Figure 3c. The load at midspan was applied and kept constant at a level of 90 kN while applying the load at the tip of the cantilever until failure. In Figure 3a, the total load is plotted against the displacements below the point loads. Positive deflections are downward, i.e. in the direction of the applied loads. Crack plots for two selected load steps are shown in Figure 3b and 3c. It can be observed that several cracks were closing during application of the second load. The brittle failure mode was characterized by diagonal cracking extending between the point load at the tip of the cantilever and the right support.



(a) Experimental (solid) and predicted (dashed) force-displacement curves. The total vertical load is plotted against the displacements measured below the point loads.



(b) Crack plot for full intensity of the load at midspan.



(c) Crack plot at failure.

Fig. 3. Results from analyses of a rectangular beam with an overhang subjected to sequential loading.

5. Quantification of the modelling uncertainty

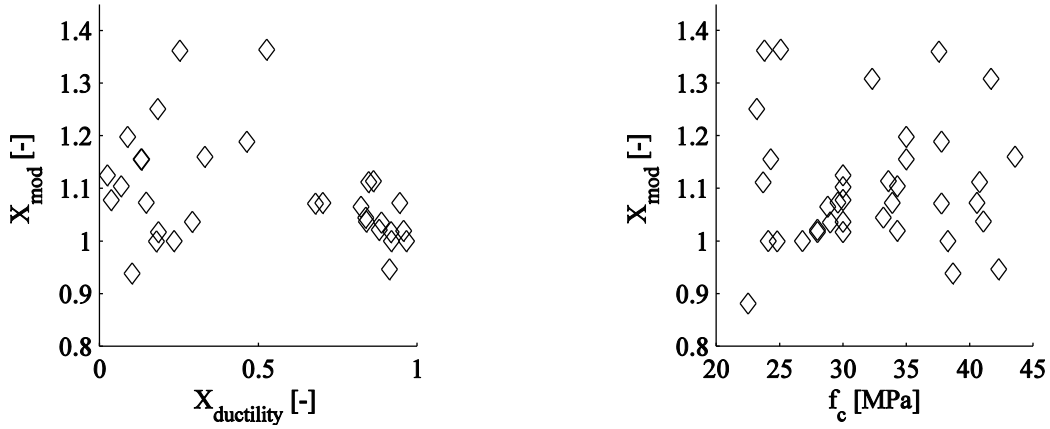
According to the *Probabilistic Model Code* (JCSS 2001), the modelling uncertainty was defined as the ratio of the experimental and the predicted capacity, $X_{\text{mod}} = R_{\text{exp}}/R_{\text{NLFEA}}$. In Bayesian data analysis both the variable in question and the parameters of its probability distribution are treated as random variables (Gelman et al. 2014). This allows for incorporating both prior knowledge and observed data, and also serves as a tool to quantify the statistical uncertainty of the parameters of the probability distribution. By assuming no prior information, the following equations (1) and (2) can be developed and used to estimate the mean μ and the variance σ^2 of the variable in question given n observations collected in the vector $\boldsymbol{y}$. E refers to the expected value, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ is the sample mean and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ is the sample variance. The equations are adapted from the work by Gelman et al. (2014), and details of the derivations can be found in the work by Engen et al. (2016a).

$$\mathrm{E}[\mu|\boldsymbol{y}] = \bar{y} \tag{1}$$

$$\mathrm{E}[\sigma^2|\boldsymbol{y}] = \frac{n-1}{n-3}s^2 \tag{2}$$

By analyzing a series of 38 benchmark analyses, it was suggested to represent the modelling uncertainty with a log-normally distributed variable. By applying (1) and (2) the mean of the modelling uncertainty was estimated to 1.10 and the variance was estimated to 0.120 with a resulting coefficient of variation of 10.9%. The details of the calculations can be found in a separate paper (Engen et al. 2016a).

Engen et al. (2016a) defined the ductility index $X_{\text{ductility}}$ as the ratio of the plastic dissipation of the reinforcement and the total plastic dissipation at failure. The ductility index is thus regarded as a characterization of the failure mode, where the limiting values $X_{\text{ductility}} = 0$ and $X_{\text{ductility}} = 1$ would correspond to a fully brittle and fully ductile failure mode respectively. Figure 4a shows the correlation between the modelling uncertainty and the ductility index. A slight dependency of the modelling uncertainty on the failure mode can be observed. Figure 4b shows the correlation between the modelling uncertainty and the compressive strength, and no simple linear correlation can be observed.



(a) Ductility index. (b) Compressive strength.

Fig. 4. Correlation between modelling uncertainty X_{mod} and a) ductility index $X_{\text{ductility}}$ and b) compressive strength f_c.

6. Discussion

The detail level which is needed for the material model depends on the phenomena that are to be studied in the analysis. In the present study, the ultimate limit capacity was sought, and for this purpose, the simple material model was appropriate. The ability of the material model to allow for crack closure was shown to be important in order to analyze cases involving sequential loading.

The modelling uncertainty got the largest contribution from the benchmark analyses with brittle failure modes. Brittle failure modes that are governed by the concrete are often more difficult to predict with a high degree of accuracy than ductile failure modes governed by the reinforcement (Schlune et al. 2012). Also, the brittle failure modes have a higher inherent uncertainty due to the higher physical uncertainty of concrete compared to the reinforcement (JCSS 2001). Due to this it was suggested that the numerator of the modelling uncertainty unintentionally contributed to the estimated parameters of the probability distribution of the modelling uncertainty. The simple multiplicative definition of the modelling uncertainty that was used in the present work might not be optimal for separating the modelling uncertainty from other uncertainties in engineering analyses.

Compared to results presented in the literature (e.g. Schlune et al. 2012) the estimated parameters for the modelling uncertainty were considered adequate and comparable to the global safety factor for modelling uncertainty for well validated numerical models proposed in fib *Model Code 2010* (*fib* 2013).

7. Conclusions

The fully triaxial material model was implemented in a commercial finite element code. Despite the simple behavior and the relatively large finite elements that were used, a series of 38 benchmark analyses resulted in a modelling uncertainty with a mean 1.10 and a coefficient of variation of 10.9%. The results support the findings from a recent paper (Engen et al. 2015) where a shift of the attention from a detailed description of the post-cracking tensile behavior of concrete to a rational description of the pre-cracking compressive behavior was advised in cases where large elements are used and the ultimate limit capacity is sought.

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Multiconsult, for having the courage to initiate the research project. The material model is presently being implemented in DIANA.

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