

Wave Attenuation by Floating Ecosystems for Floating Photovoltaic Systems

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by

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Preface

This thesis is submitted in completion of my Master's degree in *Sustainable Energy Technology* at Delft University of Technology. The research was carried out in collaboration with Solinor, who initiated the project and provided the Willemspolder case study used throughout this work. The code developed for this thesis is publicly available at <https://github.com/Thijs-vd-Linden/floating-ecosystem-wave-barrier>.

This thesis is written for readers with a background in hydrodynamics, structural engineering, or floating photovoltaic systems. A general understanding of wave mechanics; the relevant theory is introduced in Chapter 2 for readers who are less familiar with this topic. A note on the use of AI tools in this thesis is included in Appendix C.

Readers mainly interested in the modelling approach can turn directly to Chapter 4, while those primarily interested in the results can go to Chapter 5 and Chapter 6. Readers short on time are advised to read the Conclusions in Chapter 7, which summarise the main findings of this thesis. Readers without a technical background are referred to the Non-Technical Summary in Appendix A.

This thesis marks the end of my studies in *Sustainable Energy Technology*, following a bachelor's in *Civil Engineering*. Combining aspects of both in this project has been rewarding. At the same time, the project was not without its challenges: struggling with the damping zones, getting the time-domain model to behave, and working through setbacks that come with such a large project took a fair share of patience. But the days when something finally worked felt great. Looking back on my time in Delft, I can say that I look back with a smile. These years brought challenging coursework, good friendships, and a city I felt at home in.

My gratitude goes out to my supervisors Paul, Oriol, and Rudi. Paul, thank you for making me feel at home at Solinor and for your practical perspective throughout the project. Oriol, thank you for your guidance on the modelling work and for always taking the time to dig into the details with me. Rudi, thank you for always making me reflect and keeping me on the right track.

I also want to thank Vahid for offering me the opportunity to do this thesis at Solinor, Shreyas for diving deep into the code with me, and Sathya for helping me with the energy yield implications.

To all, I hope you enjoy reading my work!

Thijs Joost van der Linden
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Summary

Floating photovoltaic systems (FPV) are systems placed on water bodies. Wave-induced motion and forces are key design challenges. They drive fatigue loading on the structure and mooring and can reduce energy yield through suboptimal module orientation. These effects can be reduced by placing a floating wave barrier alongside the FPV. Conventional floating wave barriers provide wave attenuation, but no ecological or aesthetic value. Floating ecosystems, pontoons that support vegetation above and below the waterline, are designed to improve water quality and habitat. Whether they could also serve as wave barriers and how their performance compares to conventional floating wave barriers has not previously been quantified.

This study addresses the central research question: how does the integration of floating ecosystems influence wave-structure interaction, structural response, and energy yield of FPV systems in inland water bodies? Three wave barrier representations are considered: Case 1, a freshly placed floating ecosystem modelled as a thin elastic beam; Case 2, a mature ecosystem with an added submerged porous root zone; and Case 3, a rigid hollow HDPE pipe, an established reference wave barrier. A monolithic finite-element fluid-structure interaction model, implemented in Julia/Gridap, solves the coupled fluid and structural equations simultaneously, in the frequency domain for Cases 1 and 3 and the time domain for Case 2. The model is applied to the inland wave conditions at the Willemspolder, a lake adjacent to the River Waal in the Netherlands where Solinor is implementing an FPV system with a wave barrier and floating ecosystem, characterised by a peak frequency ω_p . The wave barrier performance is quantified by the transmission coefficient K_T and the reflection coefficient K_R , and translated into wave-induced load-reduction factors for an adjacent FPV system.

Moored with stiffness tuned to ω_p , the pipe (Case 3) gives a load reduction of 99.9% at ω_p , the strongest attenuation observed, demonstrating that near-complete attenuation is achievable through resonant scattering when a wave barrier is correctly moored and tuned. This is an idealised upper bound given the assumed absence of mooring damping. For this upper bound, the drift loads are reduced to roughly one-fifth and the fatigue life by a factor of 10 to 50. A comparison against the as-built Willemspolder mooring shows that insufficient mooring stiffness performs no better than the unmoored case and that the tuning must be precise. The unmoored bare ecosystem (Case 1) performs comparably to the unmoored pipe (Case 3) across the full frequency range studied, both too small relative to the peak wavelength to obstruct the wave at ω_p , giving load reductions below 7%. The root zone in Case 2 introduces dissipation, which is governed by root density, diameter, and depth, and is expected to substantially improve attenuation as the ecosystem matures. It should be noted that the results for Case 2 remain qualitative, as a fully working damping zone could not be established; however, a visible reduction in the transmitted amplitude is observed at moderate resistance, with significant improvement expected at higher porous resistance.

The impact on energy yield is small in absolute terms. Using the more realistic structural response from Case 1 as the PV module response, the wave-induced yield loss is $\sim 0.06\%$ for a south-facing module at noon, falling to a negligible value for the east-west, low-tilt orientation used at Willemspolder. The dominant design driver is therefore structural rather than energy yield, and fatigue life extension should be the primary argument in a cost-benefit analysis. Using a floating ecosystem as a barrier additionally provides ecological and aesthetic enhancement that a conventional barrier cannot offer.

Future work should calibrate the root-zone resistance parameter against field measurements, validate the rigid-pipe and floating-ecosystem structural representations against independent benchmarks or scaled physical tests, and address the outlet-reflection limitation to enable quantitative assessment of the mature ecosystem's performance. For practical application, the calculations underlying a wave barrier recommendation should be requested and checked rather than accepted on its own. A hybrid configuration combining a moored pipe with floating ecosystem islands may offer the best balance of structural and ecological benefits.

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Nomenclature

Abbreviations

Abbreviation	Definition
AOI	Angle of Incidence
BC	Boundary Condition
CFD	Computational Fluid Dynamics
DC	Direct Current
DHI	Diffuse Horizontal Irradiance
DNI	Direct Normal Irradiance
FEM	Finite Element Method
FLS	Fatigue Limit State
FPV	Floating Photovoltaic System
FS	Free Surface
FSI	Fluid-Structure Interaction
GHI	Global Horizontal Irradiance
HDPE	High-Density Polyethylene
PDE	Partial Differential Equation
PVC	Polyvinyl Chloride
RAO	Response Amplitude Operator
ULS	Ultimate Limit State
UV	Ultraviolet
WIL	Wave-Induced Loss

Symbols

Symbol	Definition	Unit
Latin symbols		
a	Wave amplitude	[m]
a_i	Incident wave amplitude	[m]
a_t	Transmitted wave amplitude	[m]
b	Structure width	[m]
b_v	Root diameter	[m]
$B_{\omega,h}^{(n)}$	Discrete bilinear form for Case n	[-]
C	Structural damping matrix	[-]
C_d	Bulk drag coefficient	[-]
c_{dir}	Directional wind factor	[-]
c_{prob}	Probability factor	[-]
c_{season}	Seasonal wind factor	[-]
$c_r(z)$	Roughness factor	[-]
D	Pipe outer diameter	[m]
D_{in}	Pipe inner diameter	[m]

Symbol	Definition	Unit
d	Pipe draft	[m]
d_r	Root zone depth	[m]
E	Elastic modulus	[Pa]
EI	Bending stiffness per unit width	[N m ² /m]
ΔE_L	Fractional wave-induced load energy reduction	[-]
F	Fetch length	[m]
$\mathbf{F}_{\text{hyd}}$	Hydrodynamic generalised force vector	[N]
F_x	Drag force per unit volume	[N/m ³]
g	Gravitational acceleration	[m/s ²]
G_M^{tot}	Total in-plane irradiance on PV module	[W/m ²]
G_M^{dir}	Direct irradiance component	[W/m ²]
G_M^{dif}	Diffuse irradiance component	[W/m ²]
G_M^{ground}	Ground-reflected irradiance component	[W/m ²]
h	Water depth	[m]
h_b	Beam thickness	[m]
h_e	Characteristic interface length (C/DG)	[m]
H	Wave height	[m]
H_j	Modal normal projection on pipe surface	[-]
H_s	Significant wave height	[m]
I_p	Rotational inertia per unit length	[kg m]
I_{pipe}	Second moment of area of pipe	[m ⁴]
I_{yield}	Yield indicator	[-]
J	Nitsche kinematic residual	[m/s]
k	Wavenumber	[rad/m]
k_r	Terrain roughness factor	[-]
k_{heave}	Mooring stiffness in heave	[N/m/m]
k_{pitch}	Mooring stiffness in pitch	[N m/rad/m]
K	Structural stiffness matrix	[-]
K_d	Dissipation coefficient	[-]
$\mathbf{K}_{\text{hs}}$	Hydrostatic stiffness matrix	[N/m]
K_R	Reflection coefficient	[-]
K_T	Transmission coefficient	[-]
KC	Keulegan–Carpenter number	[-]
$L_h^{(n)}$	Discrete linear form for Case n	[-]
L_b	Structure length in wave direction	[m]
L_d	Damping zone length	[m]
L_f	Free fluid domain length	[m]
L_Ω	Total domain length	[m]
m	Mass per unit width	[kg/m ²]
M	Bending moment per unit width	[N m/m]
$\mathbf{M}$	Rigid-body mass/inertia matrix	[kg]
$\mathbf{n}$	Outward unit normal vector	[-]
N	Root density	[stems/m ²]
p	Dynamic pressure	[Pa]
p_0	Initial hydrostatic pressure	[Pa]
p_{in}	Inlet pressure (Case 2)	[Pa]

Symbol	Definition	Unit
$PErr$	Energy error	[%]
q_h	Pressure test function	[-]
r	JONSWAP spectral shape function	[-]
R	Pipe outer radius	[m]
R_{in}	Pipe inner radius	[m]
Re	Reynolds number	[-]
RAO_{heave}	Heave response amplitude operator	[-]
RAO_{pitch}	Pitch response amplitude operator	[rad/m]
s_h	Free-surface elevation test function	[-]
$S(\omega)$	Wave energy spectrum	[m ² s/rad]
t	Time	[s]
Δt	Time step	[s]
t_w	Pipe wall thickness	[m]
t_{end}	Simulation end time	[s]
T	Wave period	[s]
T_p	Peak wave period	[s]
$\mathbf{u}$	Fluid velocity vector	[m/s]
u_{in}	Inlet normal velocity	[m/s]
u_{out}	Outlet normal velocity	[m/s]
$\hat{u}$	Representative velocity amplitude	[m/s]
U_{10}	Mean wind speed at 10 m height	[m/s]
v_b	Basic wind velocity	[m/s]
$v_{b,0}$	Fundamental basic wind velocity	[m/s]
v_h	Beam displacement test function	[-]
$v_m(z)$	Mean wind speed at height z	[m/s]
$\mathbf{w}_h$	Fluid velocity test function (Case 2)	[-]
w_h	Fluid potential test function (Cases 1, 3)	[-]
x	Horizontal coordinate	[m]
x_c, z_c	Pipe centre coordinates	[m]
$x_{d,out}$	Start of outlet damping zone	[m]
z	Vertical coordinate	[m]
z_0	Terrain roughness length	[m]
Greek symbols		
α	JONSWAP Phillips constant	[-]
α	Albedo	[-]
$\alpha(\mathbf{x})$	Porous resistance coefficient field	[s ⁻¹]
α_f	Free-surface stabilisation parameter	[-]
α_p	Porous resistance coefficient	[s ⁻¹]
β	JONSWAP spectral parameter	[-]
β	FPV tilt sensitivity coefficient	[-]
β_h	Free-surface stabilisation scaling	[-]
Γ	Domain boundary	[-]
γ	JONSWAP peak enhancement factor	[-]
γ	C/DG interior penalty parameter	[-]
γ_N	Nitsche penalty parameter	[m ⁻¹]

Symbol	Definition	Unit
γ_0	Nitsche base penalty parameter	[-]
$\delta \xi$	Virtual rigid-body displacement	[m]
ϵ	Tilt oscillation amplitude	[rad]
η	Beam deflection	[m]
η_0	Reference wave amplitude	[m]
η_p	Pipe surface displacement	[m]
$\eta_{0,p}$	Peak wave amplitude	[m]
$\eta_{0,s}$	Significant wave amplitude	[m]
θ_M	Platform tilt angle	[rad]
θ_0	Maximum wave-induced tilt amplitude	[rad]
κ	Free-surface elevation	[m]
κ_{in}	Incident free-surface elevation	[m]
Λ	Interior interface or intersection set	[-]
λ	Wavelength	[m]
λ_p	Peak wavelength	[m]
μ_0	Maximum damping coefficient	[s ⁻¹]
μ_1, μ_2	Spatially varying damping coefficients	[s ⁻¹]
ν	Kinematic viscosity of water	[m ² /s]
ξ_1	Heave amplitude	[m]
ξ_2	Pitch amplitude	[rad]
ξ	Rigid-body DOF vector	[-]
ρ_{air}	Air density	[kg/m ³]
ρ_b	Effective beam density	[kg/m ³]
ρ_{in}	Pipe interior density	[kg/m ³]
ρ_p	HDPE pipe density	[kg/m ³]
ρ_w	Water density	[kg/m ³]
σ	JONSWAP spectral shape parameter	[-]
ϕ	Velocity potential	[m ² /s]
ϕ_{in}	Incident velocity potential	[m ² /s]
ψ_j	Rigid-body mode shape	[-]
χ_h	Beam velocity test function	[-]
Ω	Fluid domain	[-]
Ω_h	Discretised fluid domain	[-]
Ω_p	Porous subdomain	[-]
Ω_w	Water domain	[-]
ω	Wave angular frequency	[rad/s]
ω_n	Natural frequency	[rad/s]
ω_p	Peak angular frequency	[rad/s]
ζ	Beam velocity auxiliary variable	[m/s]

Introduction

According to the International Energy Agency, IEA [1], global renewable energy capacity is projected to double between 2025 and 2030, expanding by 4.6 terawatts. This increase is approximately equal to the combined total power generation capacity of China, the European Union, and Japan. Solar Photovoltaics (PV) is expected to contribute nearly 80% of this growth. The rapid expansion of solar PV is driven by factors such as declining module prices, streamlined permitting procedures, and broad public acceptance. In more than 80% of all countries, renewable energy is expected to grow more rapidly between 2025 and 2030 than during the preceding five-year period. At the same time, this rapid expansion brings challenges, including grid integration, financing, and supply chain vulnerabilities.

In parallel with this growth, end-use sectors need to become sustainable. As these sectors often require large amounts of electricity, the need for renewable production is high. The energy system of the Netherlands is undergoing rapid transformation, driven by significant growth in renewable electricity generation and the electrification of end-uses [2]. At the same time, the electricity grid did not expand at the same rate, causing a new problem: grid congestion [3].

Due to grid congestion, end users have an incentive to become less dependent on grid-provided renewable energy. Large electricity consumers seek to diversify their electricity supply to ensure reliability and room for growth. A floating photovoltaic system (FPV) could be a viable option for an end user to become more sustainable and less reliant on the grid. Energy-intensive industries such as sand extraction, ceramics, concrete production, and port operations are often located near inland water bodies, including natural lakes, artificial reservoirs, quarry lakes, retention basins, and irrigation canals, making FPV a readily accessible option.

Floating Photovoltaic Systems are a relatively new but rapidly growing segment of the PV market [4]. A major advantage of an FPV is that it does not have to compete with other land uses. The availability of land for inland solar PV deployment remains highly uncertain due to factors such as population growth, societal considerations, ecological protection, agricultural use, existing villages, and land required for conventional energy generation [5]. FPV innovation was driven by the high cost or limited availability of land in countries such as Japan, the Republic of Korea, and Singapore [4]. Currently, FPVs are being implemented worldwide. An added benefit of inland FPVs over offshore FPVs is that they can be installed close to the end user, reducing transmission infrastructure costs and even enabling an off-grid power supply.

FPVs are increasingly being deployed on inland waters, often without fully accounting for other functions these water bodies may serve, or with a desire for multifunctional use. Most water bodies suitable for FPVs also serve ecological or recreational functions. The implementation of large FPVs without considering potential negative ecological and aesthetic effects is undesirable. FPVs can cause lower dissolved oxygen levels, affect water temperature, lead to a reduction in the number of organisms and biomass, and could affect sedimentation patterns [6]. On the other hand, FPVs could reduce water evaporation, which brings benefits to drought-prone areas [7].

If an FPV is constructed on a large body of water, an additional challenge arises. Wind-induced waves could reduce the solar park's energy yield and impose loads on the structure. Wave-induced forcing governs the ultimate limit state of the mooring system and drives cyclic loading on the structure and its connections, thereby determining their fatigue life. Multiple studies have demonstrated that the tilt

angle significantly influences power output. For land-based PV, the optimal tilt angle can be determined based on geographical and environmental factors and fixed upon installation [8], [9], [10]. As waves induce oscillatory motion, the orientation of an FPV does not remain constant over time. A larger wave amplitude induces less optimal tilt and slant, larger power oscillations, and lower average power [11], [12]. To mitigate wave effects, a wave barrier can be constructed. Although this approach could mitigate wave-induced yield losses and reduce structural loading, it does not address the ecological and aesthetic problems mentioned above.

A promising concept to mitigate these problems is the use of floating ecosystems. Floating ecosystems are pontoons or modules that support vegetation above and below the waterline. They can improve a wide variety of waters and, therefore, could be a solution to the negative ecological and aesthetic effects an FPV could have [13]. In this thesis, the terms floating ecosystem and floating wetland refer to floating ecosystems used for ecological enhancement, consisting of a floating body, vegetation, and a submerged root zone. If floating ecosystems could also serve as effective wave attenuators, they could offer a solution that simultaneously improves FPV energy yield, enhances ecological value, and preserves the aesthetic quality of inland water bodies.

1.1. Research Goal

The objective of this research is to investigate the potential of floating ecosystems to act as wave-attenuating elements for floating photovoltaic systems. Using numerical modelling within a linear wave theory framework, this research analyses the wave attenuation performance of floating ecosystems, the resulting reduction in wave-induced loads on adjacent floating photovoltaic systems, and the influence on their energy yield. The ultimate goal is to provide quantitative insight into the hydrodynamic, structural, and energetic effects of integrating floating ecosystems into floating photovoltaic systems and to translate these insights into design implications for the development of multifunctional floating photovoltaic systems in inland water bodies. This research goal directly addresses the multifunctional use of inland water bodies and supports the sustainable energy transition by enabling FPV deployment without compromising the ecological and aesthetic value of the water body it occupies.

1.2. Outline

This report is structured as follows. Chapter 2 reviews the relevant literature on floating photovoltaic systems and floating ecosystems and introduces the fundamental concepts of wave theory and wave barrier performance. Chapter 3 defines the research problem and presents the research approach adopted in this study. Chapter 4 describes the formulation and implementation of the numerical model used to analyse wave attenuation and structural response. Chapter 5 presents the results of the hydrodynamic model for all three wave barrier cases and compares their performance. Chapter 6 assesses the influence of wave attenuation on the energy yield and structural design of a floating photovoltaic system. Finally, Chapter 7 and Chapter 8 present the conclusions and recommendations, respectively.

Theory and Literature Review

This chapter establishes the theoretical and contextual background for the thesis. Sections 2.1 and 2.2 introduce floating photovoltaic systems and floating ecosystems, respectively. In Sections 2.3 and 2.4, the fundamental theory of waves and wave barriers is given. Section 2.5 introduces the effect of wave-induced motion on FPV energy yield. Finally, Section 2.6 identifies the research gap that motivates the present study.

2.1. Floating Photovoltaic Systems

This section provides an overview of the key components of floating photovoltaic systems. Structural elements such as floats, mooring, PV modules, and the balance of system are introduced. It also provides the advantages and challenges of FPVs compared to land PV. An example of a floating solar installation is shown in Figure 2.1.



Figure 2.1: Aerial photograph of a floating solar installation on an inland water body [14].

2.1.1. Features

A typical FPV consists of multiple key components. The floats or pontoons, the mooring lines and anchoring, the PV modules and the balance of system [15]. These components can be seen in Figure 2.2.

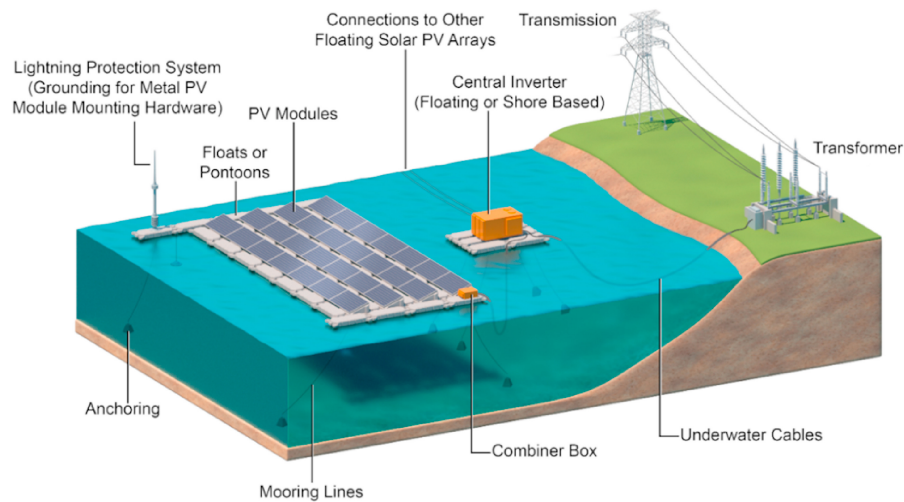


Figure 2.2: Schematic of a typical grid-connected Floating PV system and key components [16]

Floats

The floats of an FPV system provide the buoyant foundation that keeps the PV array stable on the water surface. Their primary functions are to support the weight of the PV modules, electrical components, and walkways. Most commercial floaters are manufactured from high-density polyethylene (HDPE) due to its durability, UV resistance, and low maintenance requirements [17]. Some systems use hybrid constructions combining HDPE with aluminium or steel frames to improve stiffness and long-term structural performance [18]. Mats or membranes are also used as floaters [19]. Design considerations include buoyancy capacity, interconnection method, resistance to fatigue and cracking, and compatibility with site-specific hydrodynamic loads. An example of HDPE floats is shown in Figure 2.3.



Figure 2.3: Floating solar installation during assembly, showing the modular HDPE floater structure [14].

Anchoring and Mooring

The anchoring and mooring system ensures that the FPV installation remains in its intended position despite wind forces, wave action, currents, and variations in water level. Common configurations include bottom-anchored systems (using screw anchors, concrete blocks, or piles), shore-anchored systems, and hybrid arrangements [20]. Mooring lines are typically made from polyester, polypropylene, or steel, selected based on strength requirements, elongation behaviour, and environmental exposure [21].

PV Modules

Photovoltaic modules are the main component of any solar park, as they generate the energy. They are typically based on conventional crystalline silicon technology. Bifacial photovoltaic technology is rapidly gaining market share in the global PV industry, and dual-glass bifacial modules, with high durability and performance, currently dominate the bifacial module segment [22]. Compared with land-based PV, their proximity to water results in lower operating temperatures, enhancing performance. However, there are other climate-induced risks associated with PV modules in water bodies.

Balance of System

The balance of system includes all components required to operate the FPV installation beyond the floaters and the PV modules. This generally includes the DC cabling, combiner boxes, inverters, transformers, and the electrical protection and monitoring equipment [23]. Underwater cables must be designed for mechanical protection, water-tightness, and resistance to biofouling. In many designs, inverters and transformers are located onshore to reduce weight on the platform and simplify maintenance; however, some systems place string inverters directly on the floats in dedicated housings.

2.1.2. Advantages and Challenges

The first and one of the main advantages of FPVs is the **efficient use of space**. FPV enables large-scale solar deployment on water surfaces where land availability is limited or contested. This reduces competition with agriculture, ecosystems, or urban development. Another advantage is the **enhanced energy yield**. Due to the cooling effect of water, FPV modules generally operate at lower temperatures compared to ground-mounted PV [24]. As PV modules have a negative temperature coefficient, module efficiency improves and annual energy yield increases. Furthermore, FPVs cause **reduced evaporation** and algal growth. By covering part of the water surface, FPV systems can significantly reduce evaporation losses and limit light penetration, potentially mitigating excessive algal growth in some water bodies [24].

FPV also poses challenges compared to land PV. Two of the main challenges are the **hydrodynamic loads and environmental exposure**. Wind, waves, currents, and water-level variations impose complex loads on the floats, moorings, and electrical components. Another challenge is the complexity of anchoring and mooring. In addition, cyclic wave-induced loading introduces fatigue stresses in the structure, which can accelerate structural degradation and shorten the system's lifetime. The need for reliable fixation to the bed, shores, or hybrid systems adds design challenges, especially in deep waters, soft sediments, or changing depths due to excavation. Furthermore, continuous exposure to UV radiation, humidity, and potentially corrosive water conditions increases **degradation** for polymers, metals, and electrical components [25]. This affects lifetime and maintenance needs. The **impact on the ecosystem** poses another challenge. An FPV can alter water temperature, light penetration, dissolved oxygen, and habitat conditions [24]. The ecological impacts, positive or negative, depend strongly on system design and site characteristics, and remain an active area of research. The **maintenance** of an FPV also poses several challenges. Components could be difficult to reach, and biofouling of underwater components may accelerate degradation.

2.2. Floating Ecosystems

This section discusses the features of floating ecosystems. It introduces different island types and the anchoring and mooring approaches that enable them to function effectively under varying environmental conditions. It also provides the advantages and challenges of floating ecosystems. An example of a real-world implementation of a floating ecosystem is shown in Figure 2.4.



Figure 2.4: Real-world implementation of floating ecosystems, United Kingdom [26].

2.2.1. Features

Floating ecosystems are artificial buoyant plant habitats designed to replicate the ecological functions of natural ecosystems. The floating platform typically consists of fibrous mats, buoyant frames, or polymer elements on which vegetation is planted [26]. The plant's roots grow through the platform and extend into the water, where they take up nutrients. This combination of vegetation and buoyant structure forms a dynamic biofilter that improves water quality [27]. An example of a fully grown floating ecosystem by Biomatrix Water [26], in which the roots extend through the structure, is shown in Figure 2.5.



Figure 2.5: Roots extending through a Biomatrix Water Floating Ecosystem [26].

Island Types

Floating ecosystems come in several configurations depending on structural design, ecological goals, and site conditions. The mat type consists of natural and recycled materials. Heavy-duty marine materials such as HDPE and steel can be integrated to ensure the structure remains buoyant and intact. The mat supports vegetation, and the roots extend into the water to take up nutrients [26]. Framed island systems use rigid PVC, HDPE, or metal frames filled with buoyant material and planting substrate, providing higher stability for larger installations [28]. 3D-printed variants and cages for oyster reefs are also used as floating ecosystem structures.

Anchoring and Mooring

Floating ecosystems require anchoring systems to maintain their position while allowing natural movement in response to waves and water-level fluctuations. Typical anchoring and mooring solutions are similar to those used for FPVs. The materials used must withstand biological growth and degradation while allowing sufficient slack to avoid overstressing the root systems and the island structure. Proper anchoring is essential to guarantee ecological effectiveness and long-term reliability.

2.2.2. Advantages and Challenges

The main advantage of floating ecosystems is the **improved water quality**. Floating ecosystems enhance nutrient uptake, sediment trapping, and microbial activity. They can reduce the concentration of nitrogen, phosphorus, and suspended solids, contributing to healthier aquatic ecosystems [13], [27]. Another advantage is the **creation of habitats**. Vegetation and root structures provide habitats for fish, macroinvertebrates, birds, and other wildlife. They can support or kickstart biodiversity in otherwise uniform or degraded water bodies. An added benefit could be that the islands can deter birds from the FPV, reducing bird droppings on the modules and thereby increasing the efficiency. A major developer of floating ecosystems is Biomatrix Water [26]. They claim that their ecosystems act as a safe haven for birds, improve fishing stocks and water quality, consist of 60% biomass and 30% marine engineered materials, and have a lifetime of 20 years. Furthermore, when floating ecosystems are used as a wave barrier, they could provide **wave attenuation**. The flexible structure and vegetation could dampen incoming waves, reducing the loads on the FPV and reducing the tilt and slant variations of the FPV. Other advantages are **aesthetic and social benefits**. Floating ecosystems could improve the visual quality of water bodies and FPVs and can contribute to recreational functions.

The first challenge is **structural integrity**. The islands must withstand wind, waves, and fluctuating water levels without breaking apart or losing buoyancy. Root penetration, biofouling, and material degradation can weaken the structure over time. Another important aspect are the **maintenance requirements**. Vegetation management, inspection, debris removal, and periodic plant replacement can be labour-intensive, especially in larger installations. Although floating ecosystems contribute to the ecological quality of the water body, there are some potential **ecological drawbacks**. Shading, oxygen reduction under the mats, or introduction of non-native plant species may negatively impact ecosystems. Another challenge is the **anchoring and mooring**. Poorly designed moorings can damage habitats, fail under storm conditions, and damage surrounding structures.

2.3. Wave Fundamentals

This section provides an overview of the relevant wave fundamentals. Section 2.3.1 shows the fundamental parameters of a wave. In Section 2.3.2 the linear wave theory is discussed, as well as the coefficients used to quantify wave attenuation.

2.3.1. Wave Geometry

The fundamentals of a wave are shown in Figure 2.6.

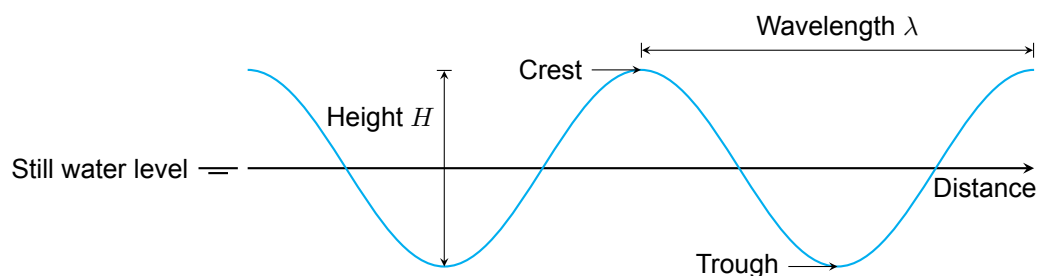


Figure 2.6: Wave fundamentals.

A surface wave can be described with the following geometry:

- **Wave height [H]** - This is the vertical distance between the crest and trough.

- **Wavelength $[\lambda]$** - This is the horizontal distance between two consecutive crests.
- **Wave period $[T]$** - This is the time between two consecutive crests passing a point.
- **Wave steepness $[H/\lambda]$** - This is an indicator of how close a wave is to breaking; high steepness waves impose higher loads.

These parameters directly govern wave kinematics and the magnitude of hydrodynamic loads acting on floating structures.

2.3.2. Wave Kinematics

One of the fundamental pillars in describing surface gravity waves is called the linear wave theory. George Biddell Airy developed this theory, also called the Airy wave theory. The derivation in this report was based on the explanation by Holthuijsen [29]. This theory is based on two equations and some boundary conditions: a *mass* balance equation and a *momentum* balance equation. The main requirement for the linear theory to apply is that the wave amplitude is small compared to the wavelength and water depth. This condition is called the *small-amplitude approximation*. Furthermore, some important assumptions are made:

- The water is assumed to be an ideal fluid. It is *incompressible*, so it has a constant *density* in space and time, is *inviscid*, and is *irrotational*.
- The fluid is assumed to consist of a single *continuous phase*, such that effects related to air entrainment, wave breaking, and cavitation are neglected.
- The *Coriolis force is ignored*.

Within the framework of linear wave theory, the motion of water particles is described using the velocity potential function $\phi(x, z, t)$. For reference, an incident monochromatic surface gravity wave propagating in the positive x -direction over constant water depth h admits the classical Airy solution for the velocity potential:

$$\phi(x, z, t) = \frac{gH}{2\omega} \frac{\cosh k(z+h)}{\cosh kh} \sin(kx - \omega t), \quad (2.1)$$

where g is the gravitational acceleration, H the wave height, k the wave number, ω the angular frequency, and h the water depth.

The dispersion relationship, given in Equation (2.2a), describes the relationship between the wavenumber k and the angular wave frequency ω . For deep water, the dispersion relationship can be simplified to Equation (2.2b) in the limit $kh \gg 1$.

$$\omega^2 = gk \tanh(kh) \quad (2.2a)$$

$$\omega^2 = gk \quad (2.2b)$$

The wavenumber is related to the wavelength λ through Equation (2.3).

$$\lambda = \frac{2\pi}{k} \quad (2.3)$$

2.3.3. Wind-Induced Waves

The wave conditions at inland water bodies are driven by wind acting over the available fetch, the uninterrupted distance over open water across which the wind blows in a given direction. An impression of the resulting wave climate under storm conditions is shown in Figure 2.7



Figure 2.7: Storm conditions at the Willemspolder, showing wind-generated waves breaking on the shore [30].

To assess wave conditions on inland lakes in the Netherlands, local wind conditions are important. Several equations are prescribed by Eurocode 1 [31] and NEN-EN 1991-1-4 [32] to calculate the mean wind speed at a height of 10 m for different wind regions in the Netherlands.

To define the significant wave height and characteristic wave period for enclosed and semi-enclosed water bodies, parametric wave growth models are employed. These models relate the fundamental wave parameters to wind speed, fetch length, exposure duration, and water depth. Both the *in situ* study by Kotaška et al. [33] and numerical studies by Nekouee et al. [34] and Etemad-Shahidi et al. [35] concluded that the Sverdrup–Munk–Bretschneider (SMB) formulation provides more accurate estimates of significant wave height and wave period under fetch-limited conditions compared to alternative empirical methods.

The SMB functions to derive the significant wave height and wave period are:

$$\frac{gH_s}{U_{10}^2} = 0.283 \tanh \left[0.0125 \left(\frac{gF}{U_{10}^2} \right)^{0.42} \right] \quad (2.4)$$

$$\frac{gT}{U_{10}} = 7.54 \tanh \left[0.077 \left(\frac{gF}{U_{10}^2} \right)^{0.25} \right] \quad (2.5)$$

where U_{10} is the mean wind speed at 10 m height and F is the fetch length.

To account for depth-limited wave growth, additional depth terms were included:

$$\frac{gH_s}{U_{10}^2} = 0.283 \tanh \left[0.530 \left(\frac{gh}{U_{10}^2} \right)^{0.75} \right] \tanh \left[\frac{0.0125 \left(\frac{gF}{U_{10}^2} \right)^{0.42}}{\tanh \left[0.530 \left(\frac{gh}{U_{10}^2} \right)^{0.75} \right]} \right] \quad (2.6)$$

$$\frac{gT}{U_{10}} = 7.54 \tanh \left[0.833 \left(\frac{gh}{U_{10}^2} \right)^{0.375} \right] \tanh \left[\frac{0.077 \left(\frac{gF}{U_{10}^2} \right)^{0.25}}{\tanh \left[0.833 \left(\frac{gh}{U_{10}^2} \right)^{0.375} \right]} \right] \quad (2.7)$$

The SMB formulation provides significant wave height and a characteristic wave period based on local wind conditions and fetch length. However, these parameters alone do not capture the irregular nature of wind-generated waves. To describe the distribution of wave energy over a range of frequencies, a spectral representation is required.

2.3.4. Wave Spectrum

Natural sea waves are irregular, with varying heights, periods, and directions that a single sinusoidal wave cannot describe. This irregular wave condition, characterised statistically over some time interval, is referred to as a sea state. Instead, wave spectra are commonly used to represent irregular wave fields by describing how wave energy is distributed over frequency. Various spectral formulations are derived, with the Pierson-Moskowitz (PM) spectrum being commonly used. Pierson and Moskowitz derived this spectrum in 1964 by analysing fully developed waves in the North Atlantic Ocean [29]. While the PM spectrum assumes fully developed sea conditions, many practical situations are characterised by limited fetch and duration. The JONSWAP spectrum is a variation on the PM spectrum and is based on data from the Joint North Sea Wave Observation Project in 1973 by Hasselmann et al. [36]. The JONSWAP spectrum extends the PM formulation by introducing a peak enhancement factor, resulting in the following spectral definition.

$$S(\omega) = \alpha \frac{g^2}{\omega^5} \exp \left[-\beta \left(\frac{\omega_p}{\omega} \right)^4 \right] \gamma^{r(\omega)} \quad (2.8)$$

$$r(\omega) = \exp \left[-\frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2} \right] \quad (2.9)$$

$$\omega_p = \frac{2\pi}{T_p} \quad (2.10)$$

$$\sigma = \begin{cases} 0.07, & \omega \leq \omega_p \\ 0.09, & \omega > \omega_p \end{cases} \quad (2.11)$$

Where α and γ represent the Phillips constant and the peak enhancement factor, respectively. The JONSWAP spectrum for a representative fetch-limited site is shown in Figure 2.8, illustrating the concentration of wave energy around the peak frequency ω_p and the negligible spectral energy outside the range $0.5\omega_p \leq \omega \leq 2\omega_p$.

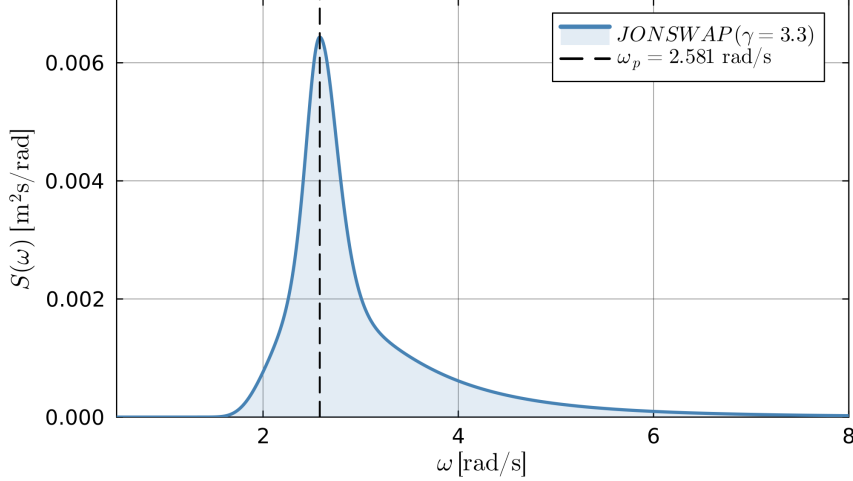


Figure 2.8: JONSWAP spectrum for a representative fetch-limited site ($\gamma = 3.3$, $\alpha = 0.0081$), showing the concentration of energy around the peak frequency ω_p .

2.4. Wave Barrier Fundamentals

This section provides an overview of the fundamentals of wave barriers. Section 2.4.2 provides the wave barrier representations used in this research, and Section 2.4.3 describes the performance coefficients.

2.4.1. Floating Wave Barriers

The function of floating wave barriers is to reflect wave energy and dissipate the kinetic energy in the incident wave. According to Sorensen [37], floating wave barriers have some distinct advantages: they are adaptable to water level fluctuations; cost does not increase rapidly with water depth; they are mobile structures; they offer little obstruction to water circulation and fish migration; and are less dependent on bottom soil conditions. To assess the effectiveness of floating wave barriers in reducing wave energy, performance coefficients are commonly employed. A schematic of a floating box-type wave barrier is shown in Figure 2.9.

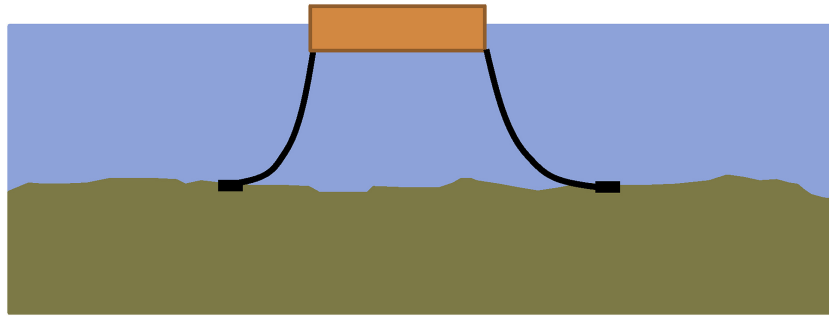


Figure 2.9: Sketch of a box-type floating wave barrier [38]

2.4.2. Wave Barrier Cases

To enable a systematic comparison of different wave barrier concepts, the relevant configurations are first described in a generalised manner. Three wave barrier cases are considered in this study: (1) a floating system that has just been placed in the water and has not yet developed vegetation, (2) a fully developed floating ecosystem with an underwater root system, and (3) a thin-walled pipe used as a wave barrier. The first two cases represent different stages in the development of a floating ecosystem, while the third presents an alternative wave-barrier concept already applied in practice.

The simplest geometry of a floating ecosystem is a rectangle. The island, before any vegetation is planted, is shown in Figure 2.10. When the floating ecosystem is first placed, the root system has not yet developed. This means that the system can initially be represented as a thin beam. This corresponds to Case 1.



Figure 2.10: The floating ecosystem from Biomatrix Water [26].

When the floating ecosystem remains at its intended location for a longer period, the island becomes more developed. Plants will have fully grown, and animals and insects will have settled. In addition, the root system will have developed, which influences the wave barrier properties. Figure 2.5 shows the same Biomatrix system as Figure 2.10, once the root system has developed.

This configuration corresponds to Case 2 in the modelling framework. In contrast to Case 1, the presence of an underwater root system introduces additional hydrodynamic effects that must be accounted for. Hu et al. [39] investigated the effect of integrating porous baffles into floating wave barriers and found that this resulted in a significant reduction in the transmission coefficient. Similarly, Li et al. [40] analysed the wave-attenuation properties of a wave barrier with kelp hanging below a floating pontoon

and reported improved wave attenuation performance compared to a traditional wave barrier. These studies demonstrate that a porous layer or submerged vegetation influences wave attenuation and cannot be ignored when modelling a floating ecosystem as a wave barrier. Consequently, the physical representation for Case 2 differs from that of Case 1.

The final configuration, Case 3, consists of a simple design: a cylindrical HDPE pipe filled with water. This type of wave barrier is attractive due to its simple construction, low material cost, and practical applicability and is already used in practice. Previous studies have shown that such pipe wave barriers can effectively attenuate surface gravity waves through a combination of wave scattering and motion-induced effects. Numerical and experimental investigations by Isaacson et al. [41] and Rossell et al. [42] demonstrated that wave attenuation depends strongly on the relative pipe size, submergence, and mooring configuration, with optimal performance achieved when the pipe is submerged close to the free surface. These studies make the thin-walled pipe a suitable and well-documented reference case against which the wave-attenuation performance of the floating ecosystem can be compared.



Figure 2.11: HDPE pipe wave barrier at a floating solar installation. (a) Overview showing the pipe as the dark floating line encircling the FPV system. (b) Close-up of the pipe and mooring connection at the water surface [14].

In contrast to Cases 1 and 2, which represent floating ecosystems at different stages of development, Case 3 serves as a reference wave barrier configuration. As such, it provides a benchmark for evaluating the wave-attenuation performance of the floating ecosystem.

2.4.3. Performance Coefficients

The effectiveness of a floating wave barrier is commonly quantified using the reflection coefficient K_R and the transmission coefficient K_T , defined as:

$$K_R = \frac{H_R}{H_i}, \quad K_T = \frac{H_T}{H_i}, \quad (2.12)$$

where H_i is the incident wave height, H_R the reflected wave height, and H_T the transmitted wave height. Under linear wave theory and neglecting viscous dissipation, conservation of wave energy flux yields:

$$K_R^2 + K_T^2 = 1. \quad (2.13)$$

For floating wave barriers incorporating porous or submerged elements, such as the root system, additional energy dissipation occurs due to drag and flow resistance. In such cases, the energy balance becomes:

$$K_R^2 + K_T^2 + K_D = 1, \quad (2.14)$$

where K_D represents the fraction of wave energy dissipated by the structure.

These coefficients provide a metric for comparing wave-attenuation performance across different wave-barrier configurations. The relative magnitudes of K_R , K_T , and K_D depend on the geometry and mechanical properties of the structure. For Case 1, the thin elastic beam has a small draft relative to the peak wavelength, so the wave field is largely undisturbed and high transmission is expected with minimal reflection and no dissipation. For Case 2, the submerged root zone introduces porous

resistance, making dissipation the dominant attenuation mechanism, with transmission decreasing as root-zone resistance increases. For Case 3, the rigid pipe interacts with the wave field through diffraction and motion-induced radiation; reflection is the primary attenuation mechanism, with effectiveness depending on the pipe diameter relative to the wavelength and the mooring stiffness.

2.5. Energy Yield of FPV

While the previous sections introduced the hydrodynamic behaviour of floating systems and the fundamentals of wave attenuation, the impact of the waves on the performance must also be considered. In contrast to ground-mounted PV systems, floating PV systems are continuously subjected to wave-induced motions, resulting in variations in module orientation. As mentioned in Chapter 1, increased wave amplitudes reduce the average power output of the system.

The performance of PV systems depends on environmental conditions and system-level effects, including incident irradiance, operating temperature, module orientation, and electrical losses. To accurately estimate the energy yield, prediction models combine optical, thermal, and electrical models. A range of tools have been developed, such as PVsyst and the System Advisor Model (SAM), which simulate PV performance under realistic conditions. Within the TU Delft Photovoltaic Materials and Devices (PVMD) group, the PVMD Toolbox has been developed as a modelling framework for advanced PV system analysis.

Recent studies have quantified the effect of wave-induced motion more precisely by introducing the concept of wave-induced loss (WIL), defined as the difference in power between a system with constant orientation and a system with changing orientation [43]. A review by Vasuki et al. [44] shows that WIL values vary significantly depending on system configuration, environmental conditions, and modelling approach.

The reduction in energy yield due to wave-induced motion can be explained by a chain of physical processes linking hydrodynamics to electrical output. Oscillatory motions lead to continuous variations in tilt and orientation, which directly affect the incident module surface irradiance. Since photovoltaic conversion depends strongly on the angle of incidence, these variations lead to suboptimal operating conditions and reduced average power output.

In addition to these effects, wave-induced motions introduce spatial variability in module orientation across the FPV system. As shown in the thesis of Tiwald [45], this variability leads to a non-uniform irradiance distribution across the array. In series-connected PV strings, the electrical current is limited by the least illuminated module, resulting in mismatch losses that reduce overall system efficiency. Under certain wave conditions, these mismatch effects can contribute significantly to the total performance loss.

2.6. Summary and Research Gap

This literature review has shown that floating photovoltaic systems are increasingly applied on inland waters and thus play an important role in the transition to a zero-emission future. The integration of floating ecosystems to increase biodiversity, enhance ecosystems, and improve water quality is increasingly used. FPVs provide renewable energy while optimising land use and the efficiency of the photovoltaic modules. On the other hand, they face challenges such as hydrodynamic loading and environmental exposure. Floating ecosystems are primarily designed to enhance water quality and ecological value, but their integration into water bodies may also influence local hydrodynamics.

Fundamental wave theory and wind-wave growth models, such as the SMB formulation, provide an established method for estimating inland wave conditions. The JONSWAP spectrum represents the irregular nature of wind-generated waves. Furthermore, the effectiveness of wave barriers is typically quantified using three performance coefficients: reflection, transmission, and dissipation. Previous studies have demonstrated that porous or submerged elements can significantly enhance wave attenuation.

However, the existing literature mainly focuses on conventional floating wave barriers or simplified porous structures. Although findings from studies on submerged vegetation, baffles, and flexible screens indicate improved wave attenuation, these findings are not directly transferable to floating

ecosystems designed for environmental purposes. Furthermore, the wave-attenuating performance of the different biological stages of a floating ecosystem has not been investigated.

As a result, there is a lack of quantitative understanding of how floating ecosystems interact with waves and how their wave-attenuation performance compares with other floating wave barrier concepts. In addition, the extent to which such wave attenuation can reduce wave-induced performance losses in FPV systems remains unclear. Addressing this knowledge gap is essential before determining whether floating ecosystems could serve as both a wave barrier for FPV systems and a functional ecosystem.

Problem Definition and Research Approach

This chapter defines the research problem and describes the approach of this report. The problem definition and corresponding research questions are presented in Section 3.1, followed by a description of the overall research approach in Section 3.2.

3.1. Problem Definition

The literature review presented in Chapter 2 has shown that floating photovoltaic systems and floating ecosystems are increasingly being applied in inland water bodies, but are typically studied as separate systems. Although previous research indicates that submerged vegetation or porous elements can contribute to wave attenuation in floating wave barriers, these findings are largely based on simplified representations of vegetation or conventional wave barrier designs. As a result, the interaction between waves and the structure of floating ecosystems at different stages of biological development has not been quantitatively assessed. Furthermore, it remains unclear how their wave-attenuation performance compares with established floating wave-barrier concepts, or how it affects the structural response and energy yield of FPV systems.

The objective of this study is therefore to quantitatively assess the wave-attenuation performance of floating ecosystems and evaluate their influence on the structural response and energy yield of floating photovoltaic systems under wind-induced wave conditions.

The central research question is formulated as follows.

How does the integration of floating ecosystems influence wave–structure interaction, structural response, and energy yield of floating photovoltaic systems in inland water bodies?

To address this main research question, the following sub-questions are defined:

- **SQ1:** What is the effect of different wave barrier representations on wave transmission, dissipation, and reflection?
- **SQ2:** What is the structural response of each wave barrier representation and the resulting reduction in wave-induced loads on an adjacent FPV system?
- **SQ3:** What are the implications of the wave-attenuation performance for the energy yield and structural design of an FPV system?

3.2. Research Approach

This study adopts a modelling-based research approach to investigate the hydrodynamic and structural integration of floating ecosystems with FPV. The methodology is structured around three analytical components: hydrodynamic modelling, structural load analysis, and energy-yield assessment. From these analyses, implications for FPV design are derived. Together, this provides the framework for answering the research questions defined in Section 3.1.

The first component focuses on **wave barrier modelling**. The objective is to quantify the extent to which floating ecosystems can attenuate wind-induced waves and thereby reduce the wave loads on an adjacent FPV. The wave-attenuation performance of different representations of floating ecosystem structures is assessed under representative inland-wave conditions. This analysis provides insight into the wave-barrier performance of floating ecosystems and serves as the basis for the following structural and energy-yield assessments.

The second component concerns the **structural response and load reduction**. Based on the results of the wave attenuation model, the structural response of each wave barrier representation is examined, including beam deflection, bending moment, and rigid-body motion. In addition, the reduction in wave-induced loads on an adjacent FPV system is quantified using the transmission coefficient, providing insight into the load-reduction potential of each wave-barrier configuration.

Finally, the results of these components are used to derive **implications for FPV design** for the integrated development of FPV-floating ecosystem systems. This section aims to translate modelling insights into engineering recommendations. In this way, the research bridges the gap between theoretical modelling and practical application to support the implementation of multifunctional FPV systems in real-world environments.

Fluid-Structure Interaction Model

This chapter presents the hydrodynamic and structural model used to simulate the fluid-structure interaction of the floating ecosystem under wave loading. This chapter does not directly answer a research question, but establishes the modelling framework required to address SQ1 and SQ2 in Chapter 5 and Chapter 6. The model is based on the formulation proposed by Colomés et al. [46] and is implemented in the Gridap finite element library in Julia¹. Three cases are considered, each representing a different wave barrier: Case 1 represents a floating ecosystem first in use, Case 2 represents a mature floating ecosystem with a root system below the structure, and Case 3 represents a rigid hollow HDPE pipe as a wave barrier.

Section 4.1 motivates the choice of the monolithic FEM formulation. Section 4.2 describes the Gridap implementation. Section 4.3 presents the shared fluid domain and frequency-domain formulation, followed by the discretisation approach in Section 4.4. The existing model formulation for Case 1 is presented in Section 4.5, and the model extensions for Cases 2 and 3 in Section 4.6. The damping zone implementation is described in Section 4.7. The model inputs for the Willemsspoolder case study are presented in Section 4.8, covering the wave conditions, numerical domain, structural parameters, and porosity parameters. Section 4.9 discusses the structural response quantities and wave-induced load reduction extracted from the model. The post-processing and wave decomposition methods are described in Section 4.10. The chapter concludes with a validation of the model in Section 4.11.

4.1. Model Choice

Fluid-structure interaction (FSI) problems are governed by coupled partial differential equations (PDEs) that describe both the motion of the fluid and the dynamic response of the structure. For most realistic FSI problems, analytical solutions are not feasible and numerical simulations are required [47].

The finite element method (FEM) divides the domain into discrete elements and approximates the solution using polynomial basis functions defined on each element [48]. This makes FEM well-suited to problems with complex geometries and higher-order differential operators. Given that the present study involves floating structures subjected to wave-induced hydrodynamic loads and structural deformation governed by fourth-order equations, FEM is the appropriate choice.

Within FEM-based fluid-structure interaction (FSI) formulations, two coupling strategies are commonly used: partitioned and monolithic. In a partitioned approach, the fluid and structural problems are solved separately, and a coupling procedure is applied to enforce compatibility between the two solutions. Although this approach allows the use of existing solvers, it may lead to instabilities or require a large number of iterations [46]. In contrast, a monolithic approach solves the fluid and structural equations simultaneously as a single coupled system. This generally results in a more stable formulation and avoids the need for an iterative coupling procedure. For this reason, the monolithic FEM formulation presented by Colomés et al. [46] is adopted in this study, implemented using the Gridap finite element library in Julia [49].

¹The implementation is publicly available at <https://github.com/Thijs-vd-Linden/floating-ecosystem-wave-barrier>.

4.2. FEM Implementation in Gridap

The monolithic formulation is implemented using the Gridap finite element library [49], which is written in the Julia programming language. Julia is designed to combine the computational performance of compiled languages with the flexibility of high-level scripting languages [50]. Gridap makes use of this by providing a syntax that closely follows the underlying mathematical formulation, resulting in a compact and readable implementation [46]. An overview of the Gridap workflow is shown in Figure 4.1. The workflow starts with the problem definition, which consists of the governing partial differential equations (PDEs), the boundary conditions (BCs), the mesh that describes the domain, and the site-specific variables. These inputs are used to construct the variational form and the finite element (FE) spaces, which together define the discrete problem. The operator is then assembled from these components and passed to a linear solver, yielding solution fields such as velocity, pressure, surface elevation, and structural response.

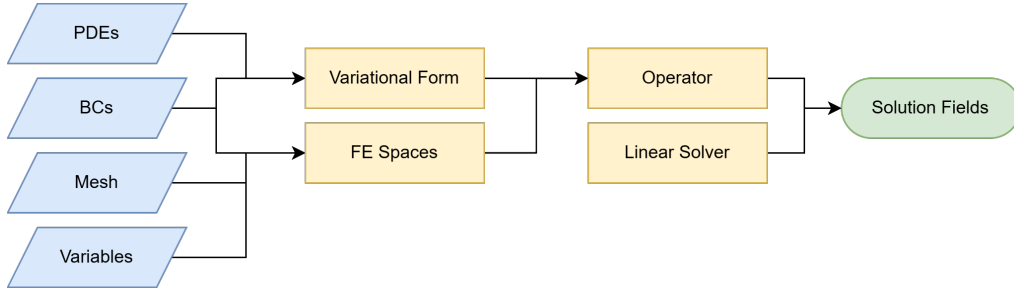


Figure 4.1: Overview of the Gridap finite element workflow, illustrating the pipeline from problem definition to solution fields.

A key requirement of the present mixed-dimensional FSI problem is the ability to assemble finite element contributions defined on domains with different spatial dimensions. Gridap supports this functionality natively, whereas many general-purpose finite element libraries do not directly provide this capability [46]. This makes Gridap a suitable choice for the formulation used in this study. In addition, using Gridap ensures consistency with the work of Colomés et al. [46], on which the present model is based.

4.3. Governing Fluid Equations

This section presents the governing fluid equations common to all three cases. Cases 1 and 3 are formulated in the frequency domain using potential flow, while Case 2 adopts a velocity–pressure formulation in the time domain.

4.3.1. Fluid Domain

The floating wave barrier is modelled as a single continuous structure, assuming that no relative motion occurs between individual structural components. A fluid domain Ω is considered, bounded by the inlet surface Γ_{in} , the outlet surface Γ_{out} , the bottom surface Γ_{b} , and the water surface. The water surface consists of the free surface Γ_{fs} and the structural or pipe boundary, denoted Γ_{str} for Cases 1 and 2 and Γ_{p} for Case 3. The surface elevation relative to the still-water level is denoted by η , $\mathbf{n}$ is the outward normal vector, ω the wave angular frequency, and λ the wavelength. The following assumptions are made:

1. Wave motion is described using linear wave theory, as detailed in Section 2.3.2.
2. The fluid is inviscid, incompressible, and irrotational, with no air entrainment and cavitation.

The inviscid, irrotational, and small-amplitude assumptions are the basis of linear wave theory [29]. The small-amplitude condition requires the wave steepness $k\eta_0 \ll 1$, which is verified in Section 4.8.1.

From the incompressibility condition, the mass continuity equation simplifies to a volume continuity equation:

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega. \quad (4.1)$$

Because the fluid is assumed to be inviscid and irrotational, a velocity potential function ϕ can be introduced such that its spatial gradient equals the fluid velocity, where ∇ denotes the gradient operator and Δ the Laplacian operator (the divergence of the gradient):

$$\mathbf{u} = \nabla \phi. \quad (4.2)$$

Combining Equations (4.1) and (4.2) yields the Laplace equation:

$$\Delta \phi = 0 \quad \text{in } \Omega. \quad (4.3)$$

The following boundary conditions apply to Ω , where $(\cdot)_t$ and $(\cdot)_{tt}$ denote the first and second time derivatives, respectively:

$$\mathbf{n} \cdot \nabla \phi = 0 \quad \text{on } \Gamma_b, \quad (4.4a)$$

$$\mathbf{n} \cdot \nabla \phi = \mathbf{u}_{\text{in}} \quad \text{on } \Gamma_{\text{in}}, \quad (4.4b)$$

$$\mathbf{n} \cdot \nabla \phi = \mathbf{u}_{\text{out}} \quad \text{on } \Gamma_{\text{out}}, \quad (4.4c)$$

$$\mathbf{n} \cdot \nabla \phi = \eta_t \quad \text{on } \Gamma_{\text{fs}} \cup \Gamma_{\text{str}}. \quad (4.4d)$$

Equation (4.4a) is the no-penetration condition at the bottom. Equations (4.4b) and (4.4c) prescribe the normal velocities at the inlet and outlet. Equation (4.4d) is the kinematic free-surface condition, enforcing that the normal fluid velocity equals the time derivative of the surface elevation, valid under the small-amplitude approximation.

The hydrodynamic pressure follows from Bernoulli's equation for inviscid, irrotational flow:

$$p = -\rho_w \phi_t - \rho_w g z - \frac{1}{2} \rho_w (\nabla \phi)^2 \quad \text{on } \Gamma_{\text{fs}} \cup \Gamma_{\text{str}}, \quad (4.5)$$

where z denotes the vertical coordinate. Under the small-amplitude assumption, the quadratic term may be neglected, yielding the linearised Bernoulli equation:

$$p = -\rho_w \phi_t - \rho_w g z \quad \text{on } \Gamma_{\text{fs}} \cup \Gamma_{\text{str}}. \quad (4.6)$$

At the free surface, imposing $p = 0$ at $z = \eta$ gives:

$$\phi_t + g\eta = 0 \quad \text{on } \Gamma_{\text{fs}}. \quad (4.7)$$

On the fluid–structure interface Γ_{str} , linearising about the still-water level $z = 0$ reduces the hydrostatic contribution to $-\rho_w g \eta$, yielding:

$$p = -\rho_w \phi_t - \rho_w g \eta \quad \text{on } \Gamma_{\text{str}}. \quad (4.8)$$

4.3.2. Frequency-Domain Formulation

Cases 1 and 3 concern steady-state wave-structure interaction under single-frequency excitation. Harmonic time dependence at angular frequency ω is assumed, with $\Re\{\cdot\}$ denoting the real part of a complex-valued quantity:

$$\phi(\mathbf{x}, t) = \Re\{\Phi(\mathbf{x}) e^{-i\omega t}\}, \quad \eta(\mathbf{x}, t) = \Re\{H(\mathbf{x}) e^{-i\omega t}\}, \quad (4.9)$$

where Φ and H are complex-valued amplitudes. The factor $e^{-i\omega t}$ cancels from all terms, reducing time derivatives to:

$$(\cdot)_t \rightarrow -i\omega(\cdot), \quad (\cdot)_{tt} \rightarrow -\omega^2(\cdot). \quad (4.10)$$

The problem is formulated directly in terms of complex amplitudes. For notational convenience, the same symbols ϕ and η are retained for the frequency-domain amplitudes in what follows. Case 2 is solved in the time domain; the reason for this is discussed in Section 4.7.

4.4. Discretisation Approach

To obtain an approximate solution, the continuous problem is transformed into a discrete one by introducing a finite set of shape functions N_i and expressing the approximate solution as a linear combination of these shape functions and the corresponding nodal degrees of freedom [48]. For the velocity potential, this gives:

$$\phi_h = \sum_{i=1}^n N_i \phi_i \quad (4.11)$$

The continuous domain Ω is thereby discretised by an FE mesh into Ω_h . The discrete boundaries are $\Gamma_{b,h}$, $\Gamma_{in,h}$, $\Gamma_{out,h}$, $\Gamma_{fs,h}$, $\Gamma_{str,h}$, $\Gamma_{p,h}$, and the interior interfaces between facets of $\Gamma_{str,h}$ are denoted $\Lambda_{str,h}$.

In the discrete formulations, the vertical displacement is split into the free-surface elevation κ on Γ_{fs} and the structural displacement η on Γ_{str} . The full derivations of the weak forms are provided in Appendix B. The governing equations for Cases 1 and 2 contain fourth-order derivatives of the beam displacement, requiring C^1 continuity across element interfaces. This requirement can be satisfied using Hermite polynomials; however, Colomés et al. [46] proposed an alternative based on a continuous/discontinuous Galerkin (C/DG) approach. In classical continuous Galerkin (CG) methods, approximation functions are continuous across element boundaries, but for fourth-order problems C^1 -continuous function spaces are required [51]. Fully discontinuous Galerkin (DG) methods relax this requirement but increase the number of unknowns and associated computational cost.

The C/DG method combines the advantages of both: discrete functions are continuous at element nodes, but the gradient is allowed to be discontinuous. Continuity of derivatives is enforced weakly using an interior penalty approach, introducing jump and mean value operators at interior interfaces:

$$[[v n]] := v^+ n^+ + v^- n^-, \quad (4.12)$$

$$\{\nabla v\} := \frac{\nabla v^+ + \nabla v^-}{2}, \quad (4.13)$$

where v^+ and v^- are the restrictions to the cells sharing a generic interior facet, and n^+ and n^- are the outward unit normals from either element [49]. The C/DG penalty terms appear in the discrete bilinear forms of Cases 1 and 2, presented in Sections 4.5 and 4.6.

4.5. Existing Model: Thin Flexible Beam (Case 1)

The formulation for Case 1 is reproduced from Colomés et al. [46]. The floating ecosystem is represented as a thin elastic beam occupying the fluid–structure interface Γ_{str} , governed by linear Euler–Bernoulli beam theory. The fundamental principles are:

- Plane cross-sections of the beam remain plane, rigid, and perpendicular to the neutral axis during bending [52].
- Shear deformation and rotatory inertia are neglected, which makes the theory most accurate for long, thin beams under small deflections [53].

The problem setting is shown in Figure 4.2, representing the bare floating ecosystem shown in Figure 2.10.

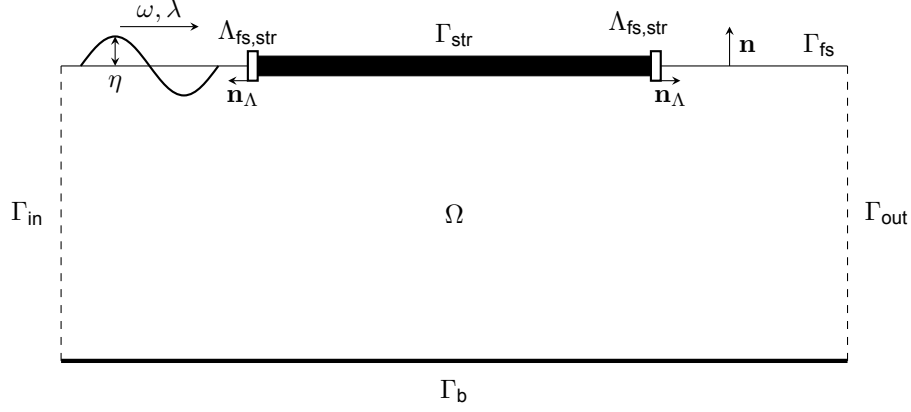


Figure 4.2: Schematic of the Case 1 problem setting, showing the fluid domain Ω bounded by the inlet Γ_{in} , outlet Γ_{out} , bottom Γ_b , free surface Γ_{fs} , and structural interface Γ_{str} . The intersections between the free surface and the structure are denoted $\Lambda_{fs,str}$, with outward normals $\mathbf{n}_\Lambda$.

The dynamic Euler–Bernoulli beam equation is given by Equation (4.14), where $(\cdot)_{xxxx}$ is the fourth-order derivative with respect to the horizontal beam coordinate x . The left-hand side represents beam inertia and elastic bending resistance, and the right-hand side is the fluid pressure load.

$$\rho_b h_b \eta_{tt} + EI \eta_{xxxx} = p. \quad (4.14)$$

For the full derivation, see Section B.1. Substituting Equation (4.6) gives:

$$\underbrace{\rho_b h_b \eta_{tt}}_{\text{beam inertia}} + \underbrace{EI \eta_{xxxx}}_{\text{elastic bending}} + \underbrace{\rho_w \phi_t}_{\text{fluid pressure}} + \underbrace{\rho_w g \eta}_{\text{hydrostatic restoring}} = 0 \quad \text{on } \Gamma_{str}. \quad (4.15)$$

The beam endpoints correspond to the intersections $\Lambda_{fs,str}$ between the free surface and the fluid–structure interface. Since both edges have free motion, the bending moment and shear force are zero:

$$\eta_{xx} = 0 \quad \text{on } \Lambda_{fs,str}, \quad (4.16a)$$

$$\eta_{xxx} = 0 \quad \text{on } \Lambda_{fs,str}. \quad (4.16b)$$

4.5.1. Discrete Formulation

To ensure stability of the coupled system, additional terms are applied to the free-surface condition following Akkerman et al. [54] and Colomés et al. [46]. The test function s is modified by adding a term proportional to the fluid test function w , scaled by α_f and multiplied by β_h . Following Colomés et al. [46], the parameters are set to $\beta_h = 0.5$ and $\alpha_f = \frac{-i\omega}{g}$, which ensures coercivity of the discrete system [54]. The continuous weak form, from which the discrete formulation is derived, is provided in Section B.2.

Find $[\phi_h, \eta_h, \kappa_h] \in \mathcal{V}_{\Omega,h} \times \mathcal{V}_{\Gamma_{str},h} \times \mathcal{V}_{\Gamma_{fs},h}$ such that:

$$B_{\omega,h}^{(1)}([\phi_h, \eta_h, \kappa_h], [w_h, v_h, s_h]) = L_h^{(1)}([w_h, v_h, s_h]) \quad \forall [w_h, v_h, s_h] \in \mathcal{V}_{\Omega,h} \times \mathcal{V}_{\Gamma_{str},h} \times \mathcal{V}_{\Gamma_{fs},h}, \quad (4.17)$$

where the bilinear form is:

$$\begin{aligned}
B_{\omega,h}^{(1)}([\phi_h, \eta_h, \kappa_h], [w_h, v_h, s_h]) = & \underbrace{\int_{\Omega_h} \nabla w_h \cdot \nabla \phi_h \, d\Omega}_{\text{Laplace (fluid interior)}} + i\omega \underbrace{\int_{\Gamma_{fs,h}} w_h \kappa_h \, d\Gamma}_{\text{kinematic FS condition}} + i\omega \underbrace{\int_{\Gamma_{str,h}} w_h \eta_h \, d\Gamma}_{\text{kinematic FSI condition}} \\
& + \underbrace{\beta_h \int_{\Gamma_{fs,h}} (s_h + \alpha_f w_h) (-i\omega \phi_h + g \kappa_h) \, d\Gamma}_{\text{free-surface stabilisation}} \\
& + \underbrace{\int_{\Gamma_{str,h}} v_h (-\omega^2 \rho_b h_b \eta_h - i\omega \rho_w \phi_h + \rho_w g \eta_h) \, d\Gamma}_{\text{beam inertia, fluid pressure, hydrostatic restoring}} \\
& + \underbrace{EI \sum_{K \in \mathcal{T}_\Gamma} \int_K v_{h,xx} \eta_{h,xx} \, d\Gamma}_{\text{beam bending (element interior)}} \\
& - \underbrace{EI \sum_{e \in \Lambda_{str,h}} \int_e (\{\eta_{h,xx}\} \llbracket v_{h,x} \rrbracket + \{v_{h,xx}\} \llbracket \eta_{h,x} \rrbracket) \, d\Gamma}_{\text{beam bending (interface consistency)}} \\
& + \underbrace{EI \sum_{e \in \Lambda_{str,h}} \int_e \frac{\gamma}{h_e} \llbracket \eta_{h,x} \rrbracket \llbracket v_{h,x} \rrbracket \, d\Gamma}_{\text{beam bending (penalty term)}},
\end{aligned} \tag{4.18}$$

where $\mathcal{T}_\Gamma$ denotes the partition of $\Gamma_{str,h}$ into one-dimensional elements, $\Lambda_{str,h}$ the set of interior interfaces, and h_e a characteristic length associated with interface e . The linear form is:

$$L_h^{(1)}([w_h, v_h, s_h]) = \underbrace{\int_{\Gamma_{in,h}} w_h u_{in} \, d\Gamma}_{\text{inlet forcing}} + \underbrace{\int_{\Gamma_{out,h}} w_h u_{out} \, d\Gamma}_{\text{outlet forcing}}, \tag{4.19}$$

4.6. Model Extensions

The finite element formulation presented in this section builds on the framework of Colomés et al. [46]. Case 2 extends the framework to a time-domain velocity–pressure formulation with a Nitsche-type kinematic coupling at the fluid–structure interface. Case 3 extends the Case 1 fluid formulation by replacing the elastic beam with a rigid-body structural model that incorporates hydrostatic stiffness and mooring.

4.6.1. Thin Elastic Beam with Submerged Root System (Case 2)

As the root system beneath the floating ecosystem does not act as an impermeable body, Case 2 introduces additional mechanisms for wave energy dissipation. The submerged root zone is represented as a porous layer, modelled by a linear Darcy resistance term α_p in the momentum equation. This requires a velocity–pressure formulation rather than potential flow, derived from the linearised incompressible Navier-Stokes equations. The full derivation is provided in Appendix B.

To enable a consistent comparison, the inlet condition in Case 2 is prescribed as a pressure Neumann condition derived from linear wave theory, ensuring both formulations describe the same incident wave, since Case 1 is defined through a velocity potential ϕ . In contrast, Case 2 is described in terms of velocity $\mathbf{u}$ and pressure p .

The problem setting is shown in Figure 4.3, representing a mature floating ecosystem shown in Figures 2.4 and 2.5.

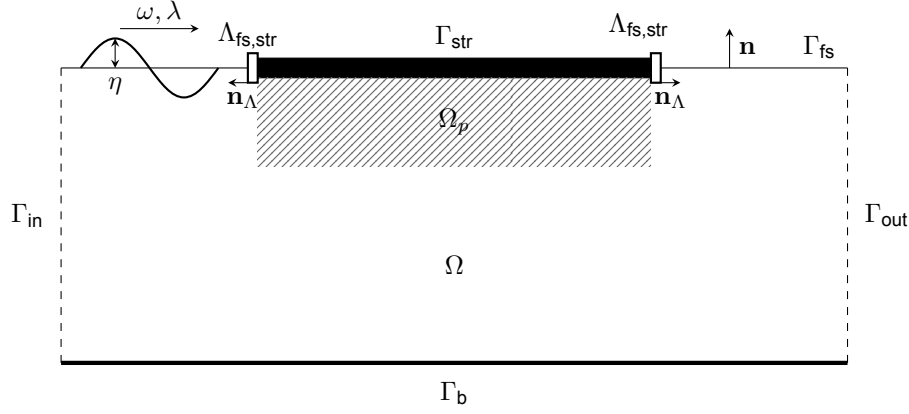


Figure 4.3: Schematic of the Case 2 problem setting, showing the fluid domain Ω with the porous subdomain Ω_p beneath the structural interface Γ_{str} . The domain is bounded by the inlet Γ_{in} , outlet Γ_{out} , bottom Γ_b , and free surface Γ_{fs} . The intersections between the free surface and the structure are denoted $\Lambda_{fs,str}$, with outward normals $\mathbf{n}_\Lambda$.

Let the fluid domain be decomposed as $\Omega = \Omega_w \cup \Omega_p$, where Ω_w denotes the clear-water region and Ω_p the porous subdomain. Under the assumptions of inviscid and irrotational flow, and neglecting convective acceleration terms in accordance with linear wave theory, the momentum equation reads:

$$\begin{aligned} \rho_w \frac{\partial \mathbf{u}}{\partial t} + \nabla p + \rho_w g \mathbf{e}_z + \rho_w \alpha \mathbf{u} &= 0 & \text{in } \Omega, \\ \nabla \cdot \mathbf{u} &= 0 & \text{in } \Omega, \end{aligned} \quad (4.20)$$

where the porous resistance coefficient is defined as:

$$\alpha(\mathbf{x}) = \begin{cases} \alpha_p & \text{in } \Omega_p, \\ 0 & \text{in } \Omega_w. \end{cases} \quad (4.21)$$

The term $\rho_w \alpha_p \mathbf{u}$ acts as a force opposing the fluid velocity within the root zone, representing the drag exerted by the root elements on the passing flow. A larger α_p corresponds to a denser root zone and extracts more energy from the wave field through dissipation. The detailed derivation is provided in Section B.3. The relevant boundary conditions for Case 2 are:

$$\mathbf{u} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_b, \quad (4.22)$$

$$\mathbf{u} \cdot \mathbf{n} = u_{in} \quad \text{on } \Gamma_{in}, \quad (4.23)$$

$$\mathbf{u} \cdot \mathbf{n} = u_{out} \quad \text{on } \Gamma_{out}, \quad (4.24)$$

$$\mathbf{u} \cdot \mathbf{n} = \eta_t \quad \text{on } \Gamma_{fs} \cup \Gamma_{str}, \quad (4.25)$$

$$p = \rho_w g \eta \quad \text{on } \Gamma_{fs}. \quad (4.26)$$

As the porous layer does not alter the beam end conditions, Equations (4.16a) and (4.16b) still apply. The structural dynamics are described by the Euler–Bernoulli beam equation in the time domain:

$$\rho_b h_b \frac{\partial^2 \eta}{\partial t^2} + \rho_w g \eta + EI \eta_{xxxx} = -p \quad \text{on } \Gamma_{str}. \quad (4.27)$$

In the time-domain implementation, the inlet, outlet, and bottom boundary conditions are enforced as strong Dirichlet conditions on the velocity field, and the free-surface elevation is prescribed at the domain edges. The inlet velocity u_{in} and free-surface elevation κ_{in} are derived from linear wave theory. In contrast, zero velocity is prescribed at the outlet and bottom boundaries. The simulation is initialised with a hydrostatic pressure field $p_0 = g(H - z)$.

Discrete Formulation

The discrete formulation uses second-order Lagrangian elements for all fields. The C/DG interior penalty terms for the beam bending operator are identical to those of Case 1, as described in Section 4.4. The full weak form is provided in Section B.3.

To reduce the second-order beam equation to a first-order system suitable for time integration, an auxiliary variable $\zeta = \eta_t$ is introduced to represent the beam velocity. The solution space is extended to $[\mathbf{u}_h, p_h, \kappa_h, \eta_h, \zeta_h] \in \mathcal{V}_{\Omega,h} \times \mathcal{Q}_{\Omega,h} \times \mathcal{V}_{\Gamma_{\text{fs}},h} \times \mathcal{V}_{\Gamma_{\text{str}},h} \times \mathcal{V}_{\Gamma_{\text{str}},h}$, with corresponding test functions $[\mathbf{w}_h, q_h, s_h, v_h, \chi_h]$.

The kinematic condition $\mathbf{u}_h \cdot \mathbf{n}_{\text{str}} = \zeta_h$ on Γ_{str} cannot be enforced strongly in the velocity–pressure setting. It is therefore imposed weakly using a Nitsche-type penalty method [55], introducing the interface residual:

$$J = \mathbf{u}_h \cdot \mathbf{n}_{\text{str}} - \zeta_h. \quad (4.28)$$

This adds a consistency term, a symmetry term, and a penalty term to the bilinear form, with penalty parameter $\gamma_N = \gamma_0/h$, where h is the characteristic mesh size along Γ_{str} . The Nitsche parameter γ_N with $\gamma_0 = 10$ is validated in Section 4.11.2.

Find $[\mathbf{u}_h, p_h, \kappa_h, \eta_h, \zeta_h]$ such that $B_h^{(2)} = L_h^{(2)} \forall [\mathbf{w}_h, q_h, s_h, v_h, \chi_h]$, where:

$$\begin{aligned}
 B_h^{(2)} = & \underbrace{\int_{\Omega_h} \mathbf{w}_h \cdot \mathbf{u}_{h,t} \, d\Omega - \int_{\Omega_h} (\nabla \cdot \mathbf{w}_h) p_h \, d\Omega + \int_{\Omega_h} q_h (\nabla \cdot \mathbf{u}_h) \, d\Omega}_{\text{momentum and continuity (fluid interior)}} \\
 & + \underbrace{\alpha_p \int_{\Omega_{p,h}} \mathbf{w}_h \cdot \mathbf{u}_h \, d\Omega}_{\text{porous resistance}} \\
 & + \underbrace{g \int_{\Gamma_{fs,h}} \kappa_h (\mathbf{w}_h \cdot \mathbf{n}) \, d\Gamma + g \int_{\Gamma_{fs,h}} s_h (\kappa_{h,t} - \mathbf{u}_h \cdot \mathbf{n}) \, d\Gamma}_{\text{dynamic and kinematic FS conditions}} \\
 & + \underbrace{\int_{\Gamma_{str,h}} p_h (\mathbf{w}_h \cdot \mathbf{n}) \, d\Gamma - \int_{\Gamma_{str,h}} q_h J \, d\Gamma + \gamma_N \int_{\Gamma_{str,h}} J (\mathbf{w}_h \cdot \mathbf{n}) \, d\Gamma}_{\text{Nitsche FSI coupling (consistency, symmetry, penalty)}} \\
 & + \underbrace{\int_{\Gamma_{str,h}} v_h \left(\frac{\rho_b h_b}{\rho_w} \zeta_{h,t} + g \eta_h - p_h \right) \, d\Gamma}_{\text{beam inertia, hydrostatic restoring, fluid pressure}} \\
 & + \underbrace{\frac{EI}{\rho_w} \sum_{K \in \mathcal{T}_\Gamma} \int_K v_{h,xx} \eta_{h,xx} \, d\Gamma}_{\text{beam bending (element interior)}} \\
 & - \underbrace{\frac{EI}{\rho_w} \sum_{e \in \Lambda_{str,h}} \int_e (\{\eta_{h,xx}\} \llbracket v_{h,x} \rrbracket + \{v_{h,xx}\} \llbracket \eta_{h,x} \rrbracket) \, d\Gamma}_{\text{beam bending (interface consistency)}} \\
 & + \underbrace{\frac{EI}{\rho_w} \sum_{e \in \Lambda_{str,h}} \int_e \frac{\gamma}{h_e} \llbracket \eta_{h,x} \rrbracket \llbracket v_{h,x} \rrbracket \, d\Gamma}_{\text{beam bending (penalty term)}} \\
 & + \underbrace{\int_{\Gamma_{str,h}} \chi_h (\eta_{h,t} - \zeta_h) \, d\Gamma}_{\text{beam velocity auxiliary equation}}
 \end{aligned} \tag{4.29}$$

and the linear form is:

$$L_h^{(2)}([\mathbf{w}_h, q_h, s_h, v_h, \chi_h]) = \underbrace{\int_{\Gamma_{in,h}} (\mathbf{w}_h \cdot \mathbf{n}) p_{in} \, d\Gamma}_{\text{inlet forcing}}. \tag{4.30}$$

4.6.2. Thin-Walled Rigid Pipe (Case 3)

In contrast to Cases 1 and 2, the thin-walled pipe is modelled as a rigid, impermeable body, free to respond in heave and pitch. Elastic deformation along its length is assumed negligible compared to its global rigid-body motion. Wave attenuation arises from a combination of diffraction, reflection, and motion-induced radiation. The problem setting is shown in Figure 4.4, representing the pipe wave barrier shown in Figure 2.11.

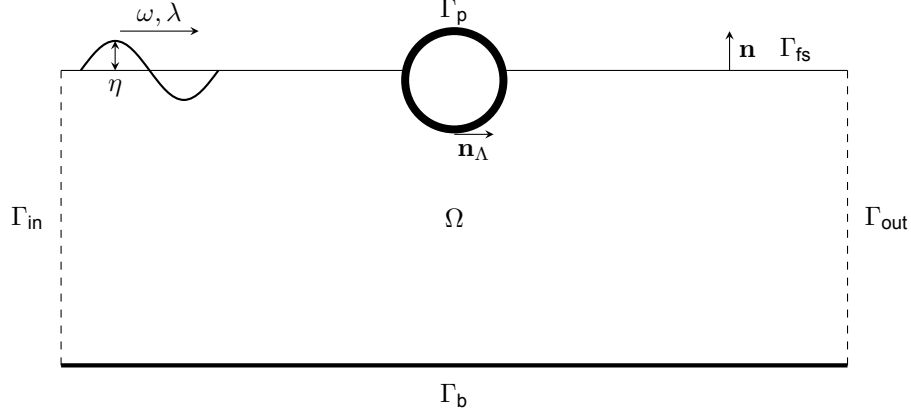


Figure 4.4: Schematic of the Case 3 problem setting, showing the fluid domain Ω with the partially submerged pipe boundary Γ_p and outward normal $\mathbf{n}_\Lambda$. The domain is bounded by the inlet Γ_{in} , outlet Γ_{out} , bottom Γ_b , and free surface Γ_{fs} .

The fluid is again described by linear potential flow, with the governing equation given by Equation (4.3). The kinematic boundary condition on the wetted pipe surface Γ_p enforces that the normal fluid velocity equals the normal velocity of the pipe boundary:

$$\mathbf{n} \cdot \nabla \phi = \mathbf{n} \cdot \dot{\mathbf{r}}_p \quad \text{on } \Gamma_p, \quad (4.31)$$

where $\mathbf{r}_p(\mathbf{x}, t)$ is the rigid-body displacement field. The pressure is given by the linearised Bernoulli equation Equation (4.6).

The rigid-body motion is described by heave ξ_1 and pitch ξ_2 , collected in $\boldsymbol{\xi} = [\xi_1, \xi_2]^T$. Surge motion is neglected. The corresponding mode shapes on Γ_p are:

$$\boldsymbol{\psi}_1(\mathbf{x}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{\psi}_2(\mathbf{x}) = \begin{bmatrix} -(z - z_c) \\ x - x_c \end{bmatrix}, \quad (4.32)$$

where (x_c, z_c) is the pipe centre. The normal projection $H_j = \mathbf{n} \cdot \boldsymbol{\psi}_j$ gives the pipe surface displacement:

$$\eta_p = \xi_1 H_1 + \xi_2 H_2. \quad (4.33)$$

Assuming harmonic time dependence, the frequency-domain rigid-body equation reads:

$$(-\omega^2 \mathbf{M} - i\omega \mathbf{C} + \mathbf{K}) \boldsymbol{\xi} = \mathbf{F}_{\text{hyd}}, \quad (4.34)$$

where $\mathbf{M}$ is the mass/inertia matrix, $\mathbf{C}$ a linear damping matrix, $\mathbf{K}$ the stiffness matrix, and $\mathbf{F}_{\text{hyd}}$ the hydrodynamic forces from the waves. This is the standard equation of motion for a damped oscillator. Near resonance, when the excitation frequency equals the system's natural frequency, the pipe's motion amplitude is large.

Discrete Formulation

The discrete formulation shares the same fluid formulation and free-surface stabilisation as Case 1, with $\beta_h = 0.5$ and $\alpha_f = \frac{-i\omega}{g}$. The full weak form is provided in Section B.4. Find $[\phi_h, \kappa_h, \boldsymbol{\xi}] \in \mathcal{V}_{\Omega, h} \times \mathcal{V}_{\Gamma_{fs}, h} \times \mathbb{R}^2$ such that:

$$B_{\omega, h}^{(3)}([\phi_h, \kappa_h, \boldsymbol{\xi}], [w_h, s_h, \delta \boldsymbol{\xi}]) = L_h^{(3)}([w_h, s_h, \delta \boldsymbol{\xi}]) \quad \forall [w_h, s_h, \delta \boldsymbol{\xi}] \in \mathcal{V}_{\Omega, h} \times \mathcal{V}_{\Gamma_{fs}, h} \times \mathbb{R}^2, \quad (4.35)$$

where the bilinear form is:

$$\begin{aligned}
B_{\omega,h}^{(3)}([\phi_h, \kappa_h, \xi], [w_h, s_h, \delta\xi]) = & \underbrace{\int_{\Omega_h} \nabla w_h \cdot \nabla \phi_h \, d\Omega}_{\text{Laplace (fluid interior)}} + \underbrace{i\omega \int_{\Gamma_{fs,h}} w_h \kappa_h \, d\Gamma}_{\text{kinematic FS condition}} + \underbrace{i\omega \int_{\Gamma_{p,h}} w_h \eta_{p,h}(\xi) \, d\Gamma}_{\text{kinematic FSI condition}} \\
& + \underbrace{\beta_h \int_{\Gamma_{fs,h}} (s_h + \alpha_f w_h) (-i\omega \phi_h + g \kappa_h) \, d\Gamma}_{\text{free-surface stabilisation}} \\
& + \underbrace{\delta\xi^T (-\omega^2 \mathbf{M} - i\omega \mathbf{C} + \mathbf{K} - \mathbf{K}_{hs}) \xi}_{\text{rigid-body dynamics}} \\
& + \underbrace{i\omega \rho_w \int_{\Gamma_{p,h}} \delta\eta_p(\delta\xi) \phi_h \, d\Gamma}_{\text{hydrodynamic force on rigid body}},
\end{aligned} \tag{4.36}$$

where the hydrostatic stiffness matrix $\mathbf{K}_{hs}$ represents the buoyancy restoring force, with entries:

$$K_{hs,ij} = \rho_w g \int_{\Gamma_p} H_i H_j \, d\Gamma. \tag{4.37}$$

The full derivation is provided in Section B.4. The linear form $L_h^{(3)}$ is identical to $L_h^{(1)}$ presented in Equation (4.19).

4.7. Damping Zones

Outgoing waves must be able to propagate away from the structure without being reflected at the domain boundaries. To prevent such reflections, damping zones are applied at the domain's inlet and outlet boundaries.

Kim et al. [56] proposed and evaluated five damping zone implementations for numerical wave tanks. Method 5, referred to as the ϕ_n - η method, was found to perform best, particularly for shorter damping zones, as it produces no wave dispersion within the damping region. The method applies a damping term to both the dynamic and kinematic free-surface boundary conditions:

$$\begin{aligned}
\frac{\partial \phi}{\partial t} &= -g\kappa - \mu_1 \left(\frac{\partial \phi}{\partial n} - \frac{\partial \phi_{in}}{\partial n} \right) & \text{on } \Gamma_d, \\
\frac{\partial \kappa}{\partial t} &= \frac{\partial \phi}{\partial z} - \mu_2 (\kappa - \kappa_{in}) & \text{on } \Gamma_d,
\end{aligned} \tag{4.38}$$

where μ_1 and μ_2 are spatially varying damping coefficients and $\partial \phi_{in}/\partial n$ and κ_{in} are the prescribed incident wave values. The damping acts only on the scattered component of the wave field, preserving the incident wave while absorbing reflected waves. The condition $\mu_2 = k\mu_1$ ensures that waves within the damping zone are not dispersed [56].

The damping coefficients increase smoothly from zero at the edge of the damping zone to a maximum value μ_0 , using the shape function proposed by Kim et al. [56]:

$$\mu_1(x) = \begin{cases} \mu_0 \left(1 - \sin\left(\frac{\pi x}{2 L_d}\right) \right) & x \in \Gamma_{d,in}, \\ \mu_0 \left(1 - \cos\left(\frac{\pi x - x_{d,out}}{2 L_d}\right) \right) & x \in \Gamma_{d,out}, \\ 0 & \text{otherwise,} \end{cases} \tag{4.39}$$

with $\mu_2 = k\mu_1$, where L_d is the damping zone length and $x_{d,out}$ is the start of the outlet damping zone.

This method is implemented in Cases 1 and 3. Assuming harmonic time dependence, the damped free-surface conditions on the inlet damping zone $\Gamma_{d,in}$ read

$$\begin{aligned} -i\omega\phi + g\kappa + \mu_1 \left(\frac{\partial\phi}{\partial n} - \frac{\partial\phi_{in}}{\partial n} \right) &= 0 \quad \text{on } \Gamma_{d,in}, \\ -i\omega\kappa - \frac{\partial\phi}{\partial z} + \mu_2 (\kappa - \kappa_{in}) &= 0 \quad \text{on } \Gamma_{d,in}, \end{aligned} \quad (4.40)$$

and for the outlet damping zone, the incident field terms are set to zero. These conditions are enforced weakly in the bilinear form, with the incident field source terms moved to the linear form $L_h^{(1)}$, ensuring that only the scattered wave component is damped.

For Case 2, the velocity–pressure formulation does not allow a direct application of Method 5 since the velocity potential ϕ is not available as a primary unknown. An alternative damping strategy based on a relaxation method was therefore explored. In this approach, each term in the weak form is weighted by a blending factor $1/(1 - \alpha)$, and the solution is simultaneously relaxed toward a prescribed target field, scaled by α . At the inlet, the target field equals the analytical incident-wave solution; at the outlet, the target field is set to zero. The blending coefficient α follows the same smooth ramp function as μ_1 in Equation (4.39), with $\mu_0 = 1.0$. A reliable solution in the frequency domain with this method could not be achieved; therefore, the model is solved in the time domain.

The same method was applied in the time domain, which reduced outlet reflections but did not eliminate them. In addition, the method reduced the incident wave amplitude to approximately $0.6\eta_0$. The absence of effective outlet damping constrains the usable time window for wave extraction, as discussed in Section 4.10.

4.8. Model Inputs

This section presents the input parameters for the three model cases. The wave conditions are derived from a site analysis of the Willemspolder location. The numerical domain, damping zone lengths, and mesh strategy are chosen to ensure accurate wave propagation and efficient computation. The structural parameters are estimated from the geometry and material properties of the Biomatrix Water Floating ecosystem, as displayed in Figure 2.10. The porosity and root zone parameters are established from the literature and treated as sensitivity parameters.

4.8.1. Wave Parameters

The wave parameters are defined separately for model validation and for the present study. The validation reproduces the benchmark of Khabakhpasheva and Korobkin [57], using the parameter values listed in Table 2 of their paper. For the present study, representative wave conditions are established for Willemspolder, a lake adjacent to the River Waal. The location is displayed in Figure 4.5 and an impression of the site is given in Figure 4.6.

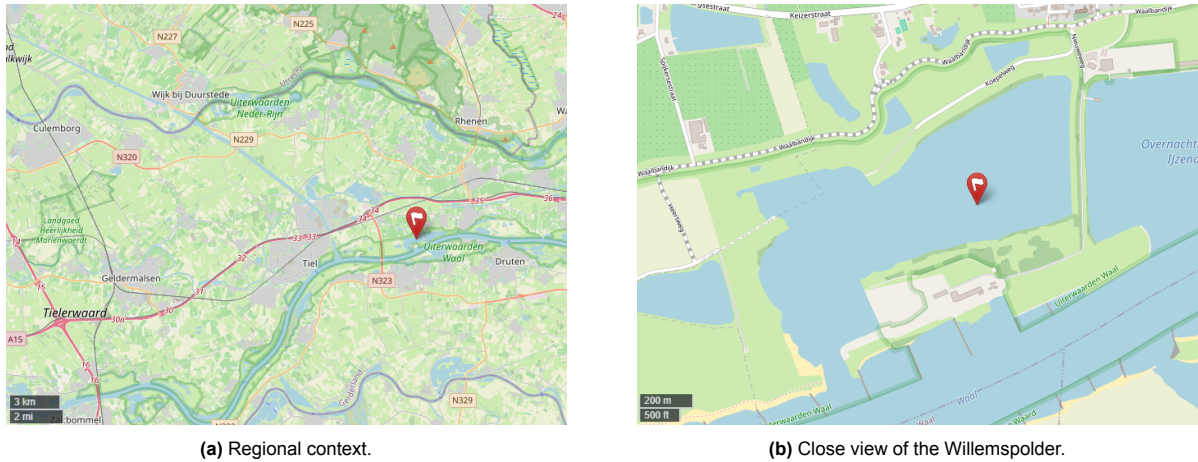


Figure 4.5: Location of the Willemspolder. Map data taken from OpenStreetMap contributors [58].



Figure 4.6: Aerial view of the Willemspolder under normal and high water conditions [30].

The conditions are displayed in Table 4.1. The wind and wave conditions are derived using the formulation discussed in Section 2.3.3. The site-specific conditions needed for the calculations were provided by Solinoor [30], and the results for the Peak wave period and significant wave height are in good agreement with site-specific estimates. The corresponding angular frequency and wave number are calculated with Equation (2.2) and the wavelength with Equation (2.3). The dimensionless depth parameter $k_p h = 12.9$ indicates deep-water conditions ($kh \gg 1$), confirming that fetch is the limiting factor.

To enable analysis of the broader response behaviour of the wave barrier representations, a frequency sweep is performed over the frequency range of the JONSWAP spectrum, as defined in Section 2.3.4, with the H_s and T_p from the Willemspolder site. The sweep covers the range $0.5\omega_p \leq \omega \leq 2\omega_p$, corresponding to $1.29 \leq \omega \leq 5.16$ rad/s, which represents the significant frequency range of the JONSWAP spectrum. This is visible from the normalised observed spectra in Holthuijsen [29], Figure 6.4, where the spectral energy is negligible outside this range. The JONSWAP spectrum computed for the Willemspolder site is shown in Figure 2.8. Thus, rather than analysing only the results at the design frequency ω_p , the sweep is used to characterise the barriers' response across the full relevant frequency range. This allows for site-specific conclusions while also revealing the response behaviour of comparable fetch-limited sites. The sweep range also includes the peak frequencies corresponding to fetch lengths between 100 and 1500 m, covering $2.29 \leq \omega_p \leq 4.38$ rad/s as computed from the SMB formulation in Section 2.3.3, confirming that results at alternative fetch lengths can be extracted directly without additional simulations.

The wave amplitude η_0 is treated separately from H_s . Under linear wave theory, all governing equations scale linearly with η_0 , meaning that the transmission and reflection coefficients K_t and K_r are independent of the chosen amplitude. In Case 2, the porous dissipation term is also linear, so K_d is likewise independent of amplitude. This makes it possible to run the frequency sweep at a small reference amplitude and recover physical results by scaling afterwards. Consistent with Colomés et al. [46], a value of $\eta_0 = 0.01$ m is chosen to ensure the small-amplitude approximation $k\eta_0 \ll 1$ is satisfied: at the design frequency, this gives $k_p\eta_0 = 6.79 \times 10^{-3} \ll 1$, satisfying the condition. For the computation of the structural loads, two physical wave heights are used: the significant and max wave heights.

Table 4.1: Summary of the site conditions of the Willemsspolder.

Parameter	Symbol	Unit	Value
Physical Constants			
Water density	ρ_w	kg/m ³	1000
Air density	ρ_{air}	kg/m ³	1.225
Gravitational acceleration	g	m/s ²	9.81
Environmental conditions			
Maximum Fetch	F	m	900
Mean 10 minute wind speed	U_{10}	m/s	19.1
Mean water depth	h	m	19.0
Peak wave period	T_p	s	2.44
Significant wave height	H_s	m	0.498
Peak angular frequency	ω_p	rad/s	2.581
Peak wave number	k_p	rad/m	0.679
Peak wave length	λ_p	m	9.26
Peak wave amplitude	$\eta_{0,p}$	m	0.249

4.8.2. Numerical Domain

The numerical domain consists of a central fluid region of length L_f , which is flanked by the inlet and outlet damping zones of length L_d , resulting in a total domain length of $L_\Omega = L_f + 2L_d$. The damping zone length is chosen to ensure the outgoing waves are not reflected into the domain. Following Colomés et al. [46], the damping coefficient for Case 1 and 3 is set to $\mu_0 = 2.5$. For Case 2, no damping zones are used; instead, the outgoing waves are prevented from reaching the structure within the extraction window by extending the free-fluid region.

For the validation case, the domain parameters are taken directly from Table 2 of Colomés et al. [46]. For the other cases, the domain is sized on the peak wavelength λ_p . The simulated structure length L_b represents the extent of the floating ecosystem in the wave propagation direction. The base case uses the width of four modules, giving $L_b = 4 \times 1.15 = 4.6$ m. At this length, $L_b/\lambda_p = 0.50$, placing the structure in the sub-wavelength regime, where rigid-body behaviour is expected to dominate over elastic deformation. To characterise the transition toward the elastic regime, a length sweep is performed over:

$$L_b \in \{4.6, 9.2, 18.4\} \text{ m} \quad (4.41)$$

corresponding to $L_b/\lambda_p \in \{0.50, 1, 2\}$, representing 4, 8, and 16 modules in width, respectively. The full three-point sweep is performed for Case 1 only. For Case 2, simulations are performed at $L_b = 4.6$ m and $L_b = 18.4$ m; the intermediate length $L_b = 9.2$ m is omitted due to the computational cost of the time-domain formulation. The domain parameters are summarised in Table 4.2.

Table 4.2: Summary of the numerical domain parameters.

Parameter	Symbol	Unit	Validation	Case 1	Case 2	Case 3
Total domain length	L_Ω	m	50	120	105	115
Free fluid domain length	L_f	m	25	60	105	35
Single damping zone length	L_d	m	12.5	30	0	40
Damping parameter	μ_0	-	2.5	2.5	-	2.5
Wavelength	λ	m	3.11	9.26	9.26	9.26
Damping zone coverage	L_d/λ	-	4.0	3.24	-	4.32

4.8.3. Mesh Strategy

All three cases use unstructured triangular meshes generated with Gmsh, with second-order (P2) Lagrangian elements and a degree of integration of 4. The meshes differ in refinement strategy per case.

Case 1 uses a selective refinement strategy with a characteristic element size of $l_{c,fs} = 0.15$ m near the free surface and beam region. The damping zones, deep region, and far boundaries are significantly coarsened to reduce the total element count:

- Free surface and beam region: $l_{c,fs} = 0.15$ m
- Damping zones: $l_{c,damp} = 1.00$ m
- Deep region ($z < -5$ m): $l_{c,deep} = 2.00$ m
- Bottom and side walls: $l_{c,far} = 2.00$ m

Case 2 uses a uniform mesh with a single characteristic element size of $l_c = 0.25$ m throughout the domain. No selective refinement is applied because the time-domain formulation requires a consistent resolution throughout the domain to accurately propagate the wave field over time.

Case 3 uses a non-uniform mesh with refinement around the pipe boundary. The characteristic element size near the pipe scales with the pipe radius, grading out to a coarser mesh away from it:

- Pipe boundary and waterline region: $l_{c,pipe} = D/5$
- Outer domain boundaries: $l_{c,far} = 0.80$ m

4.8.4. Structural parameters

The structural parameters differ between the beam cases and the pipe case. For Cases 1 and 2, the relevant parameters are the bending stiffness EI , the mass per unit width m , and the effective beam thickness h_b . Both EI and m are estimated from the geometry and material properties of the Biomatrix Water Floating Ecosystem [26], since exact manufacturer data are unavailable. The estimate is based on physical inspection of the structure, as presented in Figure 2.10. For Case 3, the relevant parameters are the pipe geometry, the material and interior densities, and the rigid body inertia properties.

Beam parameters

The structure measures 3.0 m in length, 1.15 m in width, and 0.20 m in depth. The perimeter of the structure consists of HDPE pipes that span the mat's full thickness. Two pipes run along the sides of the 3.0 m length, and two pipes run along the sides of the 1.15 m width. The interior consists of a lightweight core that provides buoyancy while keeping the mass low. Coconut coir fibre is bound around the structure and serves as the planting substrate for vegetation, providing water retention and root aeration [59].

As the fibre functions primarily as an ecological substrate rather than as part of the structural elements, its contribution to bending stiffness is neglected. As the structure is modelled as a 1D beam, the two transverse pipes closing the frame are also excluded. The bending stiffness is thereby governed by the two longitudinal HDPE pipes.

The pipe diameter equals the thickness of the structure, so the pipe centre lies on the neutral axis of the cross-section, eliminating the Steiner term in the second moment of area. The total bending stiffness is therefore:

$$EI_{\text{beam}} = 2E \cdot I_{\text{pipe}}, \quad I_{\text{pipe}} = \frac{\pi}{64} (D^4 - D_{in}^4) \quad (4.42)$$

where the factor of two represents the two pipes. The exact wall thickness of the Biomatrix pipe could not be confirmed from the manufacturer. According to ISO 4427-1 [60], the wall thickness of an HDPE pipe is defined as $t_w = D/SDR$, where SDR is the standard dimension ratio. Two standard specifications are SDR17 and SDR11, giving $t_w = 11.8$ mm and $t_w = 18.2$ mm, respectively. SDR11 is adopted, as it is the more structurally robust option. The corresponding second moments of area are:

$$\begin{aligned} I_{\text{SDR17}} &= 3.10 \times 10^{-5} \quad [\text{m}^4] \\ I_{\text{SDR11}} &= 4.34 \times 10^{-5} \quad [\text{m}^4] \end{aligned} \quad (4.43)$$

The elastic modulus of HDPE depends on both the temperature and the strain rate due to its viscoelastic nature. Amjadi and Fatemi [61] performed multiple tests at different temperatures and strain rates, resulting in values ranging from approximately 400 MPa at high temperatures and low strain rates to approximately 1400 MPa for low temperatures and high strain rates. As the structure lies in the water at temperatures typically below room temperature, and wave loading causes oscillatory strain rates, a conservative value of $E = 1000 \text{ MPa}$ is adopted, consistent with the lower range of room temperature values reported by Amjadi and Fatemi [61]. This results in:

$$\begin{aligned} EI_{\text{total, SDR17}} &= 2E \cdot I_{\text{SDR17}} = 62,020 \quad [\text{Nm}^2] \\ EI_{\text{total, SDR11}} &= 2E \cdot I_{\text{SDR11}} = 86,750 \quad [\text{Nm}^2] \end{aligned} \quad (4.44)$$

Since the hydrodynamic model represents the floating ecosystem as a one-dimensional beam, the bending stiffness needs to be normalised by its width, $b = 1.15 \text{ m}$, to obtain the value per unit width. This gives:

$$\begin{aligned} EI_{\text{total, SDR17}} &= 53,930 \quad [\text{Nm}^2/\text{m}] \\ EI_{\text{total, SDR11}} &= 75,435 \quad [\text{Nm}^2/\text{m}]. \end{aligned} \quad (4.45)$$

The mass per unit width m is derived from a direct weighing of the Biomatrix Water Floating Ecosystem, without substrate or vegetation. The area of the island is $A = 3.0 \times 1.15 = 3.45 \text{ m}^2$. The measured mass gives a mass per unit width of:

$$m_{\text{self}} = \frac{M_{\text{meas}}}{A} = \frac{43}{3.45} \approx 12.50 \quad \text{kg/m}^2 \quad \rho_b = \frac{m}{h_b} = \frac{12.50}{0.20} = 62.5 \quad \text{kg/m}^3 \quad (4.46)$$

In use, the island has substrate and vegetation loading. The properties of the substrate layer and vegetation are uncertain, as their composition is not precisely known and is site-specific. The substrate and vegetation loading therefore introduces uncertainty in the total mass per unit width. Since the mass influences the beam response through the Euler–Bernoulli equation, given in Equation (4.14), a sensitivity sweep is performed, where the lower bound corresponds to the bare measured island and the upper bound represents a heavily loaded case. The sweep is:

$$\rho_b \in \{62.5, 100, 200, 300\} \text{ kg/m}^3 \quad (4.47)$$

Rigid Pipe Parameters

Case 3 models a rigid HDPE pipe with outer diameter D , which is treated as a sweep parameter. The sweep is performed over:

$$D \in \{0.25, 0.30, 0.35, 0.40, 0.45, 0.50\} \text{ m} \quad (4.48)$$

This sweep is evaluated in two variants: an unmoored case ($k_{\text{heave}} = 0$) to assess the baseline influence from geometry alone, and a moored case where k_{heave} is tuned to maximise the attenuation at ω_p . For the moored diameter sweep, the mooring stiffness for each diameter is set to the value of k_{heave} that minimises K_T at ω_p , as identified from the parametric stiffness sweep performed for each diameter individually.

The same two SDR classes used for the beam bending stiffness are considered, SDR11 and SDR17. SDR11 with wall thickness $t_w = D/11$ and SDR17 with $t_w = D/17$, giving inner diameter $D_{\text{in}} = D - 2t_w$. The pipe floats with a draft of $d = 2D/3$, so two-thirds of its cross-section is submerged. The pipe centre depth below the waterline is:

$$z_c = -(d - R) = -\left(\frac{2D}{3} - \frac{D}{2}\right) = -\frac{D}{6} \quad (4.49)$$

where $R = D/2$ is the outer radius.

According to ISO 4427-1 [60] the density of HDPE $\rho_p = 930 \text{ kg/m}^3$. The pipe interior is assumed to be homogeneous and estimated with $\rho_{\text{in}} = \frac{2}{3}\rho_w + \frac{1}{3}\rho_{\text{air}}$. The mass per unit length is:

$$m = \rho_p \pi (R^2 - R_{\text{in}}^2) + \rho_{\text{in}} \pi R_{\text{in}}^2 \quad (4.50)$$

where R_{in} is the inner radius, ρ_p the density of the HDPE pipe.

The rotational inertia per unit length about the pipe centre z_c is:

$$I_p = \frac{\pi}{2} [\rho_p (R^4 - R_{in}^4) + \rho_{in} R_{in}^4] \quad (4.51)$$

The mooring system of the floating pipe provides restoring forces in heave and pitch. The mooring stiffness in heave, k_{heave} , depends on the mooring line pretension, the rope angle, and the characteristics of the elongation under load of the elastic elements. As these values vary with site characteristics, a parametric sweep over $k_{heave} \in [10^{-3}, 10^2] \times K_{hs,11}$ is performed. To identify the optimal stiffness with higher precision, a refined sweep is performed over a narrow range around the coarse optimum for each diameter individually. This assesses the sensitivity of the wave attenuation to the mooring stiffness, where $K_{hs,11}$ is the hydrostatic heave stiffness given by Equation (4.37). Pitch mooring stiffness is set to zero to simplify the problem. Although the damping matrix C is set to zero, radiation damping is implicitly accounted for through the fluid-structure coupling terms. Mooring is not considered in Cases 1 and 2 because the structure is a continuous elastic beam that deforms but does not translate or rotate as a rigid body. Since mooring restrains rigid-body motion, it is neglected in the beam cases.

Summary

The structural parameters are summarised in Table 4.3.

Table 4.3: Summary of the structural parameters of the Cases.

Parameter	Symbol	Unit	Value
Case 1 & 2			
Beam thickness	h_b	m	0.2
Mass per unit width (bare)	m	kg/m ²	12.50
Effective beam density (bare)	ρ_b	kg/m ³	62.5
Effective beam density sweep	ρ_b	kg/m ³	{62.5–300}
Bending stiffness SDR17	EI	Nm ² /m	53,930
Bending stiffness SDR11	EI	Nm ² /m	75,435
Case 3			
HDPE density	ρ_p	kg/m ³	930
Inner density	ρ_{in}	kg/m ³	667
Outer diameter	D	m	{0.25–0.50}
Wall thickness SDR17	t_w	m	D/17
Wall thickness SDR11	t_w	m	D/11
Draft	d	m	2D/3
Mooring stiffness heave	k_{heave}	N/m/m	empirical, per diameter
Mooring stiffness pitch	k_{pitch}	Nm/rad/m	0
Mooring damping	C	-	0

4.8.5. Porosity and Root Zone

The parameters corresponding to the root zone depend on the planted vegetation. As native aquatic vegetation is used, the choice of vegetation is site-specific. This means no general geometry or density is available. The geometry of the porous subdomain Ω_p is defined by the island dimensions and root length, and the resistance parameter α_p is established from the literature and treated as a sensitivity parameter.

The horizontal extent of the root zone equals the length of the simulated floating ecosystem, assuming root growth along its full length. The vertical extent d_r represents the depth to which the roots extend

into the water and also depends on the type of vegetation. The experimental study of Stefani et al. [62] measured root growth in several plants that grow in floating ecosystems native to northern Italy and the Netherlands. Plants used are, for example, *Phragmites australis* and *Typha latifolia*, also known as common reed and bulrush. The results showed that the roots extended 75-115 cm at a mature stage of the floating ecosystem. The base case of $d_r = 0.75$ m is adopted as a conservative representative of the fully grown root zone.

The porous resistance term $\rho_w \alpha_p \mathbf{u}$ in Equation (4.20) represents the linearised drag force induced by the root zone on the fluid. Keeping only the linear Darcy term and neglecting the quadratic Forchheimer term is needed to keep consistent with the small-amplitude assumption and ensures that the transmission and reflection coefficients remain independent of the wave amplitude.

Since an accurate α_p cannot be established without site-specific experimental data, a physically motivated range has to be derived. The drag force applied by the root zone on the fluid is modelled using the Morison equation, which describes the hydrodynamic force on a cylindrical body in oscillatory flow [63]. Although the roots are flexible rather than rigid, the Morison framework remains applicable because the bulk drag coefficient C_d accounts for plant motion, stem interactions, and other unresolved processes [64]. The root zone geometry, consisting of flexible elements hanging under the floating ecosystem, is comparable to the suspended canopy studied by Zhu et al. [65]. Their results show that the energy dissipation is largest when the canopy is placed near the water surface, which supports the shallow root zone depth adopted here. Following Dalrymple et al. [63] and [64], individual roots are approximated as cylindrical elements, and the drag per unit volume of the root zone is expressed as:

$$F_x = \frac{1}{2} \rho_w C_d b_v N |u| u \quad (4.52)$$

where C_d is the depth-averaged bulk drag coefficient, b_v is the root diameter, N is the root density in stems per m^2 , and u is the horizontal fluid velocity. For vertically oriented cylindrical elements, only the horizontal component of the velocity contributes to the drag force, since the drag acts perpendicular to the cylinder axis.

To maintain consistency with the linear domain of the governing equations, the non-linear term $|u|u$ is linearised by replacing the $|u|$ term with a representative velocity amplitude $\hat{u}$, evaluated at the mid-depth of the root zone, $z = -d_r/2 = -0.375$ m below the surface, under the Willemsspoolder design conditions. The porous resistance term in Equation (4.20) acts on the full velocity vector, but due to the cylindrical argument above it is reduced to the horizontal component. Equating with Equation (4.52), yields:

$$\alpha_p = \frac{1}{2} C_d b_v N \hat{u}. \quad (4.53)$$

As α_p is treated as a sensitivity parameter, the ranges are established as follows. Under deep water approximation, the horizontal velocity amplitude follows from the linear wave theory and is shown in Equation (4.54) [29]. Evaluating at the root zone mid-depth with the values provided in Table 4.1 gives:

$$\hat{u} = \omega_p \eta_0 e^{-k_p d_r/2} \approx 0.50 \text{ m/s}. \quad (4.54)$$

To characterise the flow regime around the root elements and, with that, establish an appropriate range for C_d , the Reynolds number and the Keulegan-Carpenter number are evaluated. The Reynolds number describes the ratio of inertial and viscous forces in fluid flow, and the Keulegan-Carpenter number characterises the relative importance of the drag forces over inertia forces for a body in an oscillatory fluid flow [64]. These are calculated with Equations (4.55) and (4.56).

$$Re = \frac{\hat{u} b_v}{\nu} \quad (4.55)$$

$$KC = \frac{\hat{u} T_p}{b_v} \quad (4.56)$$

Root diameters for aquatic plants such as *Phragmites australis* and *Typha latifolia* are not directly reported in the available literature for floating ecosystem configurations. A range of $b_v = 0.001 - 0.005$

m is assumed and is part of the uncertainty absorbed in the α_p sensitivity sweep. For root diameters in the range of 0.001 – 0.005 m:

$$Re \approx 500 - 2500$$

$$KC \approx 250 - 1200$$

where $\nu = 10^{-6} \text{ m}^2/\text{s}$ is the kinematic viscosity of water.

The large KC values indicate that the drag forces dominate over the inertia. For smooth cylinders in the subcritical Reynolds number regime, 500 – 2500, the drag coefficient ranges from $C_d \approx 1.0 - 1.2$, as read from the $C_d - Re$ curve for a smooth cylinder [66]. As root flexibility reduces the effective drag, a conservative range of $C_d = 0.5 - 1.2$ is adopted, with the lower bound accounting for the flexibility and is based on the research of Zhu et al. [65], who report bulk drag coefficients of 0.15–0.55 for suspended flexible canopies. The root density N represents the number of root elements per unit of horizontal area. No direct measurements of N were found in the literature for aquatic plants in floating ecosystem configurations. Thus, a large range of $N = 100 - 3500 \text{ stems/m}^2$ is adopted, where the upper bound accounts for potentially dense and mature root zones. For reference, Abdolali et al. [64] report a density range $N = 200 - 400 \text{ stems/m}^2$ for submerged salt marsh vegetation, which is likely less dense than a mature root zone. Combining these bounds via Equation (4.53) yields:

$$0.0125 \lesssim \alpha_p \lesssim 5 \text{ s}^{-1} \quad (4.57)$$

Thus, a sensitivity sweep is performed over:

$$\alpha_p \in \{0.01, 0.1, 0.5, 1.0, 2.5, 5.0\} \text{ s}^{-1} \quad (4.58)$$

4.9. Structural Response

The structural response is extracted from the FEM solution and scaled to the physical wave amplitudes. Under linear wave theory, all response quantities scale linearly with the incident wave amplitude η_0 as discussed in Section 4.8.1. The physical response is thereby recovered by the scaling factor $\eta_{0,s}/\eta_0$, where $\eta_{0,s} = H_s/2$ is the significant incident wave amplitude.

For Cases 1 and 2, the relevant structural response quantities are beam deflection and bending moment. The deflection amplitude is obtained from the FEM solution as $|\eta_h|$, which is then multiplied by the scaling factor to obtain the physical deflection. The bending moment per unit width is computed as:

$$M = EI \cdot \eta_{xx} \quad (4.59)$$

where η_{xx} is the second derivative of the beam deflection along the beam axis, extracted from the FEM solution.

For Case 3, the structural response is characterised by the response amplitude operators (RAO) for heave and pitch, defined as:

$$\text{RAO}_{heave} = \frac{|\xi_1|}{\eta_0}, \quad \text{RAO}_{pitch} = \frac{|\xi_2|}{k\eta_0} \quad (4.60)$$

where ξ_1 and ξ_2 are the complex heave and pitch amplitudes extracted from the rigid-body solution. The physical heave and pitch responses can be obtained by multiplying the RAO with $\eta_{0,s}$. The RAO describes the structural response amplitude per unit wave amplitude, and characterises how strongly the structure responds to a wave of given amplitude at each frequency.

4.9.1. Load Reduction

The reduction in wave-induced loads on the FPV can be quantified by the transmission coefficient K_T . For a harmonic load $F(t) = |F| \cos(\omega t)$, the time averaged energy is:

$$E \propto |F|^2. \quad (4.61)$$

Since $|F| \propto \eta_0$, it follows that $E \propto \eta_0^2$. Behind the barrier $\eta_0 \rightarrow K_T \eta_0$, so:

$$E_T \propto K_T^2 \eta_0^2 = K_T^2 E_{in} \quad (4.62)$$

The fractional reduction in wave-induced load energy is therefore:

$$\Delta E_L = 1 - K_T^2 \quad (4.63)$$

where $\Delta E_L = 0$ corresponds to no reduction in wave-induced load energy, while $\Delta E_L = 1$ corresponds to complete attenuation of the wave-induced loads.

4.10. Post-processing and Wave Decomposition

The wave transmission and reflection coefficients K_T and K_R , introduced in Section 2.4, are extracted from the free surface elevation field after solving each case. The extraction method differs between the frequency-domain cases (Cases 1 and 3) and the time-domain case (Case 2).

For Cases 1 and 3, K_T and K_R are extracted directly from the complex free surface elevation κ . Three probe locations are placed upstream and three downstream of the structure, outside the damping zones. Since the wavenumber k is known from the dispersion relation, the incident, reflected, and transmitted amplitudes are separated at each probe set by decomposing the free surface into forward- and backwards-propagating components. The energy error is defined as:

$$\text{PErr} = (1 - K_T^2 - K_R^2) \times 100\%, \quad (4.64)$$

and quantifies the deviation from energy conservation. For Cases 1 and 3, $|\text{PErr}| < 1\%$ across the full frequency sweep.

For Case 2, the solution is obtained in the time domain with a time step of $\Delta t = T/40$ and a simulation duration of $t_{\text{end}} = 17T$. The pressure field is initialised to the hydrostatic profile $p_0 = g(H - z)$ at $t = 0$ to reduce initial transients. Due to the absence of fully effective outlet damping, the usable analysis window is restricted to $[12T, 17T]$. Within this window, the incident wave amplitude measured at upstream probes is approximately $0.62 \eta_0$, reduced by the relaxation-based damping method. Reliable quantitative extraction of K_T and K_R is therefore not possible. Wave attenuation is assessed through signed snapshots of the free surface profile κ/η_0 and beam deflection profile η/η_0 at $t = 17T$.

4.11. Validation and Limitations

This section presents the validation of the cases in Sections 4.11.1 and 4.11.2 and the limitations of the cases in Section 4.11.3.

4.11.1. Validation of Cases

The formulation is validated against the benchmark of Khabakhpasheva and Korobkin [57], following the approach of Colomés et al. [46]. The benchmark considers a floating elastic beam of $L = 12.5$ m in a fluid domain of depth $H = 1.1$ m, with a rigid connection between two beam segments of rigidities $D_1 = 47,100$ Nm and $D_2 = 471$ Nm. The wavelength-to-beam ratio is $\alpha = 0.249$.

Figure 4.7 shows the normalised beam displacement $|\eta|/\eta_0$ along the beam length x/L , compared against the benchmark data of Khabakhpasheva and Korobkin [57] and the numerical results of Riyan-syah et al. [67]. The figure shows that the model for Case 1 correctly implements the monolithic FEM formulation proposed by Colomés et al. [46].

The validation is performed for Case 1, which shares the fluid-structure coupling formulation with Case 3. The rigid-body structural model of Case 3 is a simplification of the elastic beam formulation, and the formulation for the fluid domain is identical to Case 1.

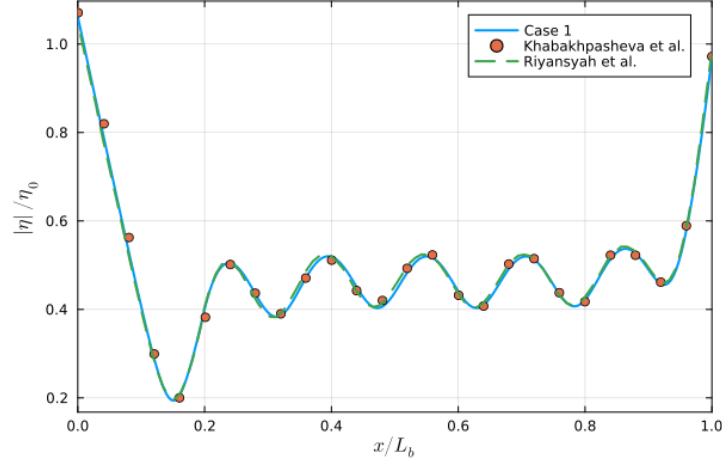


Figure 4.7: Normalised beam displacement $|\eta|/\eta_0$ along the beam length x/L for the Khabakhpasheva and Korobkin [57] benchmark, rigid connection ($\xi = 625$). The present model (solid blue) is compared against the benchmark data (orange dots) and the results of Riyansyah et al. [67] (green dashed).

The time-domain velocity–pressure formulation of Case 2 is validated against the same benchmark of Khabakhpasheva and Korobkin [57], using the same parameters listed above with $\alpha_p = 0$, reducing the formulation to the same fluid–structure interaction problem as for Case 1. The key difference is that Case 2 uses a velocity–pressure formulation in the time domain, whereas Case 1 uses a potential flow formulation in the frequency domain. Since the relaxation-based damping zone method does not fully absorb the reflections, a large domain of $L_\Omega = 150$ m is used to delay the arrival of reflected wave energy at the beam and ensure a steady state can be formed. The simulation is run for $t_{\text{end}} = 80T$, and snapshots are extracted in a clean window before outlet reflections contaminate the solution.

Figure 4.8 shows the normalised beam deflection envelope $|\eta|/\eta_0$ along the beam length, formed by five snapshots within one period. The envelope closely reproduces the shape and amplitude of the benchmark data in Khabakhpasheva and Korobkin [57]. The amplitude is slightly underestimated, but follows the correct number of bending nodes and is in good agreement across the interior. The validation confirms that the time-domain velocity–pressure formulation correctly reproduces the hydroelastic response of a floating elastic beam.

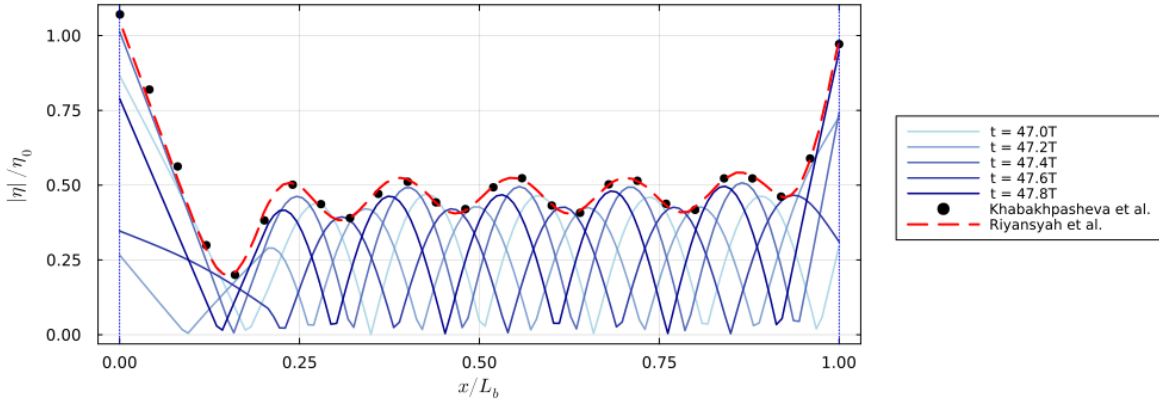


Figure 4.8: Normalised beam deflection envelope $|\eta|/\eta_0$ along the beam length x/L_b for the Khabakhpasheva and Korobkin [57] benchmark, $\alpha_p = 0$. Five snapshots spaced by $T/5$ within one period are overlaid. The present model (blue lines) is compared against the benchmark data of Khabakhpasheva and Korobkin [57] (black dots) and the numerical results of Riyansyah et al. [67] (red dashed).

4.11.2. Nitsche Parameter Validation

The velocity–pressure formulation of Case 2 enforces fluid-structure coupling through a Nitsche-type penalty method. This condition requires that the fluid's normal velocity equals the beam velocity at the interface. The interface residual, or kinematic error, is defined in Equation (4.28). Under strong enforcement, J would be exactly zero, but under weak enforcement, a small residual would remain. The L^2 norm of J measures the root-mean-square mismatch integrated over the boundary Γ_{str} , where a small value confirms that the fluid normal velocity $\mathbf{u}_h \cdot \mathbf{n}_{\text{str}}$ matches the beam velocity ζ_h pointwise along the interface. A well-chosen parameter ensures that $J \approx 0$ at steady state, while an insufficient value leads to model inconsistency.

The validation is performed by running a Case 2 simulation of $15T$ at ω_p , with $\alpha_p = 0$. The normalised L^2 residual is monitored over time; the penalty parameter is set to $\gamma_0 = 10$, corresponding to $\gamma_N = \gamma_0/h$ where h is the element size. A threshold value of 10^{-2} is adopted as a practical criterion: a normalised residual below this value indicates that the kinematic error is less than 1

Figure 4.9 shows the evolution of the normalised residual over time. During the transient phase, the residual grows as the wave field builds up in the domain. Steady state is reached from approximately $8T$, after which the residual oscillates around a constant level. The mean normalised residual between $12T$ and $15T$ is 6.3×10^{-3} , with a maximum of 8.6×10^{-3} , both below the target threshold of 10^{-2} . The kinematic constraint is therefore satisfied for $\gamma_0 = 10$.

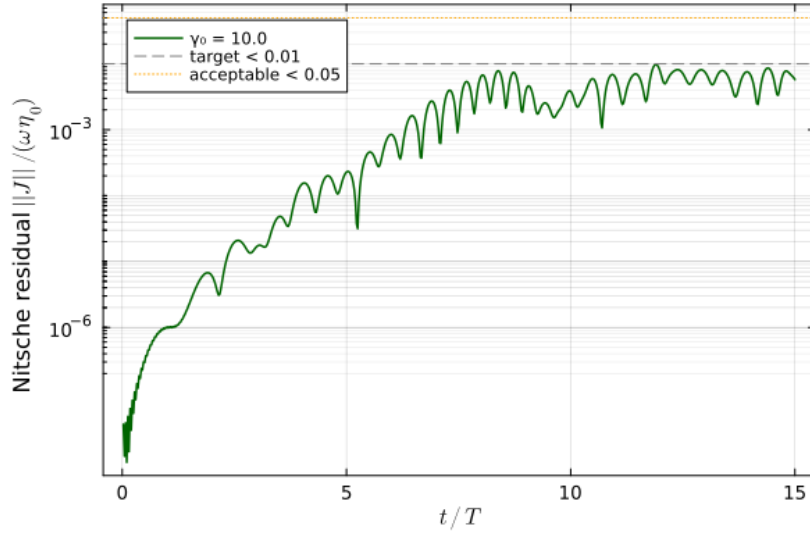


Figure 4.9: Normalised Nitsche kinematic residual over time for Case 2 with $\gamma_0 = 10.0$ and $\alpha_p = 0$. The residual reaches a steady-state level below the target threshold of 10^{-2} (dashed line) after approximately $t = 8T$.

4.11.3. Limitations

The three cases are formulated under linear wave theory, assuming small-amplitude waves and irrotational, inviscid flow. The model is formulated in two dimensions, representing a cross-section of the barrier under normally incident waves. Three-dimensional effects, including wave diffraction around barrier ends, oblique wave incidence, and the finite length of the barrier, are not captured. Nonlinear effects such as wave breaking, viscous boundary layers, and higher-order free-surface deformations are also not captured. At the reference amplitude $\eta_0 = 0.01$ m the nonlinear parameter $k_p \eta_0 = 6.79 \times 10^{-3} \ll 1$, confirming the assumption is well satisfied. At the significant wave amplitude $\eta_{0,s} = 0.249$ m, $k_p \eta_{0,s} = 0.169$, which remains within the range where linear theory provides a reasonable first-order approximation.

Cases 1 and 3 are solved in the frequency domain, assuming steady-state harmonic conditions at a single frequency. Transient effects are not resolved.

For Case 2, the relaxation-based damping zone does not fully absorb outlet reflections, which contaminate the solution over time. The usable analysis window is restricted to $[12T, 17T]$. Furthermore, the

relaxation method reduces the incident wave amplitude to approximately $0.62 \eta_0$, as discussed in Section 4.7. Together, these effects prevent reliable quantitative extraction of K_T and K_R , which could in principle be achieved by using a sufficiently long domain to delay the arrival of reflected wave energy, as demonstrated in the validation in Section 4.11. However, this is computationally expensive for a full parametric frequency sweep. The results for Case 2 are therefore presented qualitatively, and the load reduction ΔE_L is not reported for this case.

For Case 3, the mooring damping is set to zero ($C = 0$), meaning the only energy dissipation at resonance is wave radiation. For a pipe small relative to the wavelength, radiation damping is small, resulting in unphysically large heave amplitudes at resonance. In practice, mooring damping and viscous effects would limit the motion to physically realistic values. This is consistent with experimental floating breakwater studies: Isaacson et al. [41] report that potential-flow models tend to over-predict the response near resonance, and Rossell et al. [42] find that cable-moored cylindrical breakwaters remain vulnerable to the effects of true resonance. The transmission coefficient at ω_p should therefore be interpreted as an upper bound on performance for an undamped resonant system rather than a realisable design target. Furthermore, the mooring is simplified to a linear heave stiffness only, with pitch mooring stiffness set to zero. The optimal stiffness is identified empirically from a parametric sweep rather than from a physical mooring design. The influence of mooring geometry, line pretension, and elastic properties on the heave and pitch response is therefore not captured.

4.12. Chapter Summary

This chapter has presented the fluid-structure interaction model used to simulate wave attenuation by the three wave barrier representations. The monolithic FEM formulation, implemented in Julia/Gridap, provides a consistent framework for Cases 1, 2, and 3, covering potential flow in the frequency domain for Cases 1 and 3 and a velocity–pressure formulation in the time domain for Case 2. The model inputs for the Willemsspoolder site have been established, the formulation has been validated against the benchmark of Khabakhpasheva and Korobkin [57], and the key limitations have been identified. With the modelling framework in place, Chapter 5 presents the results of the three cases and addresses SQ1: the effect of different wave barrier representations on wave transmission, dissipation, and reflection, and the structural response component of SQ2. The resulting reduction in wave-induced loads on an adjacent FPV system is then derived in Chapter 6, completing SQ2.

Results of Hydrodynamic Model

This chapter represents the results of the hydrodynamic model described in Chapter 4 and addresses SQ1 and the structural response component of SQ2. The resulting reduction in wave-induced loads is developed further in Chapter 6. The three cases are evaluated under the Willemspolder site conditions summarised in Table 4.1, with the frequency sweep covering $0.5\omega_p \leq \omega \leq 2\omega_p$. For Cases 1 and 3, the transmission coefficient K_T , the reflection coefficient K_R , and energy error PErr are reported. For Case 2, reliable extraction of K_T and K_R is not possible due to outlet boundary reflections; therefore, results are presented as free-surface and beam-deflection profiles. Where relevant, the heave response amplitude operator (RAO) is also reported.

The results are discussed per case in Sections 5.1 to 5.3, followed by a comparison of the cases in Section 5.4.

5.1. Case 1: Thin Elastic Beam

Case 1 represents the bare floating ecosystem as a thin elastic beam in the frequency domain, as described in Section 4.5. The results are presented for the unmoored configuration across three parameter sweeps: bending stiffness, beam density, and beam length. The influence of mooring on the wave attenuation performance of Case 1 is discussed in Section 5.4.

5.1.1. Flow Field at Peak Frequency

Figure 5.1 shows the free surface elevation η_h and velocity potential ϕ_h at $\omega = \omega_p = 2.581$ rad/s for the base case $L_b = 4.6$ m, SDR17, $\rho_b = 62.5$ kg/m³, as derived in Section 4.8.4. The real part, imaginary part, and absolute values are shown. In the absolute value panel, the upstream standing wave pattern is visible, resulting from partial wave reflection by the beam. Downstream of the beam, the transmitted wave amplitude is constant, showing a clean transmitted wave with no reflection from the outer boundary. The velocity potential decays with depth throughout the domain, consistent with the linear wave theory discussed in Section 2.3.2.

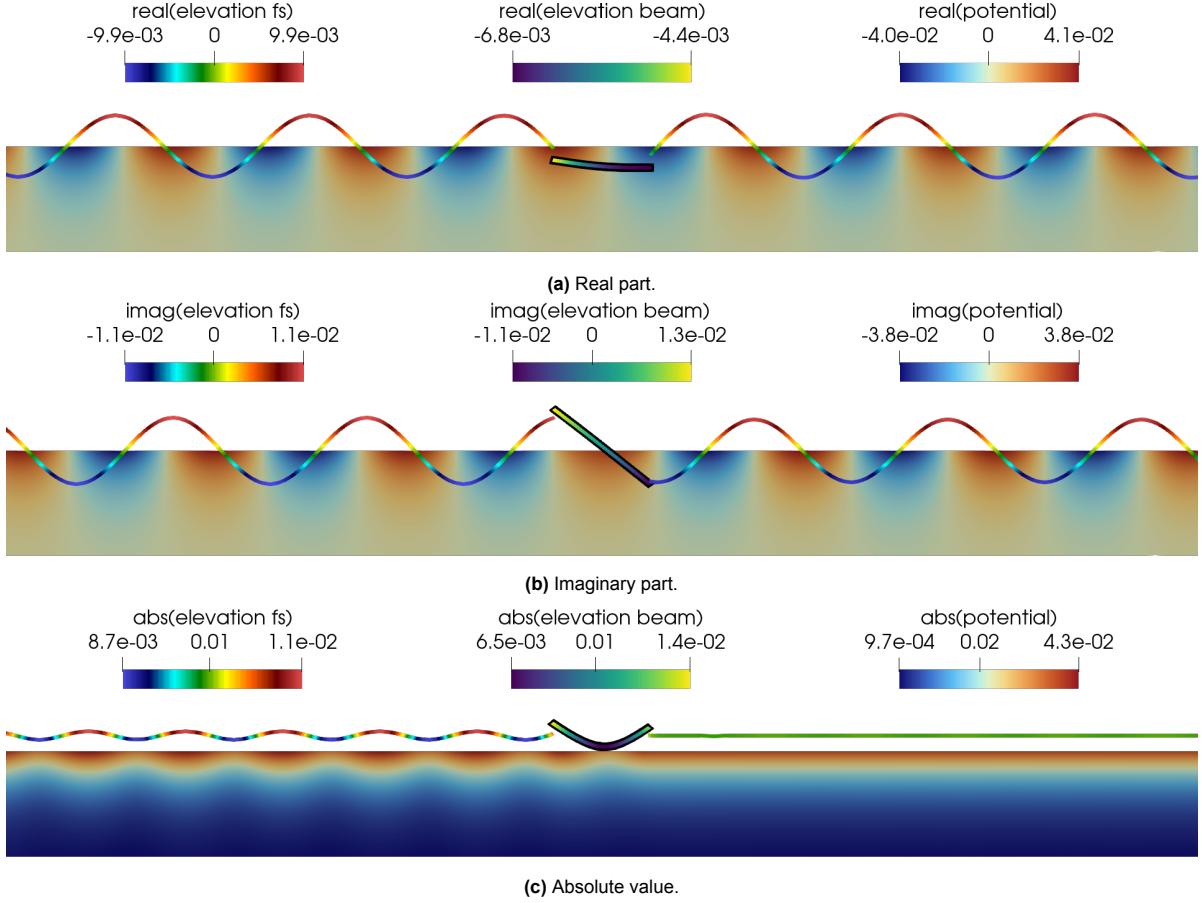


Figure 5.1: Free surface elevation (left), beam elevation (middle) and velocity potential ϕ_h (right) at $\omega = \omega_p = 2.581$ rad/s, $L_b = 4.6$ m, SDR17, $\rho_b = 62.5$ kg/m³. The beam lies at the centre of the domain and is shaded in black. The vertical direction of the elevation is scaled by a factor of 150 for visibility. The visualised region spans $x = 30$ m to $x = 90$ m (60 m) within the total computational domain, with a water depth of 5 m.

5.1.2. Wave Amplitude at Peak Frequency

Figure 5.2 shows the normalised wave amplitude $|\kappa|/\eta_0$ along the free surface at ω_p , together with the incident component $|\kappa_{in}|/\eta_0$, reflected component $|\kappa_r|/\eta_0$, and beam displacement $|\eta|/\eta_0$, for $L_b = 4.6$ m and $L_b = 18.4$ m. For the short beam, the small upstream amplitude oscillations confirm a small reflection of the wave by the beam. The downstream amplitude remains uniform at $|\kappa|/\eta_0 \approx 1$, indicating negligible attenuation. The long beam shows stronger oscillations, although the differences between the beams are small. The short beam ($L_b = 4.6$ m) yields $K_T = 0.992$ and $K_R = 0.129$ at ω_p , while the long beam ($L_b = 18.4$ m) yields $K_T = 0.983$ and $K_R = 0.180$, confirming that neither configuration provides meaningful wave attenuation at the peak frequency.

For $L_b = 4.6$ m, the beam displacement profile is nearly linear, indicating almost rigid behaviour with negligible elastic deformation. For $L_b = 18.4$ m, the deflection profile shows multiple nodes, consistent with elastic bending of the beam under wave loading.

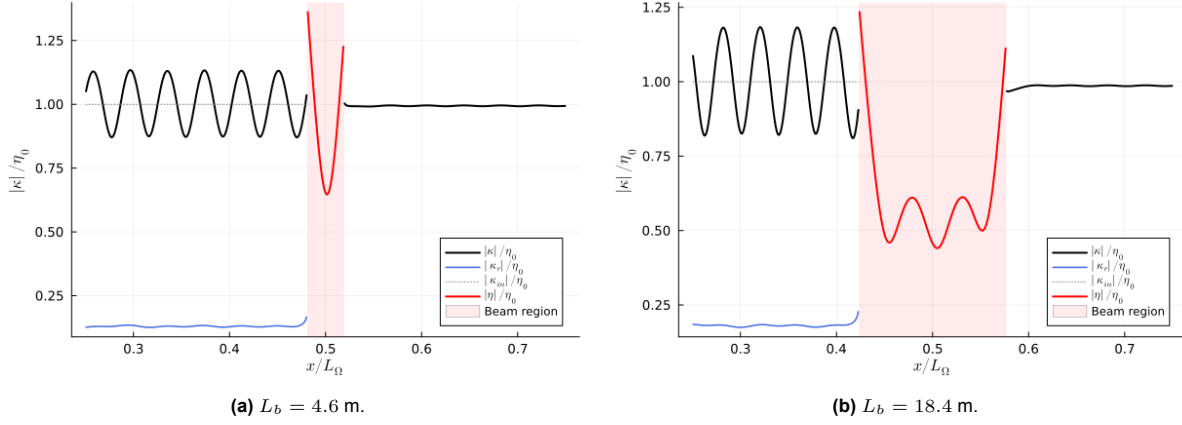


Figure 5.2: Normalised wave amplitude along the free surface at $\omega_p = 2.581$ rad/s, SDR17, $\rho_b = 62.5$ kg/m³. Shown are the total amplitude $|\kappa|/\eta_0$, the incident component $|\kappa_{in}|/\eta_0$, the reflected component $|\kappa_r|/\eta_0$, and the beam displacement $|\eta|/\eta_0$. The shaded region indicates the beam.

5.1.3. Bending Stiffness Sweep

Figure 5.3 shows K_T , K_R , and PErr as a function of ω for the bending stiffness sweep, performed on the $L_b = 18.4$ m mesh. Two distinct local K_R minima and K_T maxima are visible for both SDR classes, corresponding to their wet natural frequencies. The stiffer SDR11 specification shifts the minima and maxima to higher frequencies compared to SDR17, consistent with the higher natural frequencies of a stiffer beam. At ω_p , SDR17 gives $K_T = 0.983$ and $K_R = 0.180$, while SDR11 gives $K_T = 0.997$ and $K_R = 0.060$. The higher transmission of SDR11 at ω_p is consistent with the wet natural frequency of the beam lying closer to ω_p , where K_T reaches a local maximum. The larger PErr at low frequencies is consistent with the longer wavelengths approaching the damping-zone length limit, thereby reducing the effectiveness of wave absorption.

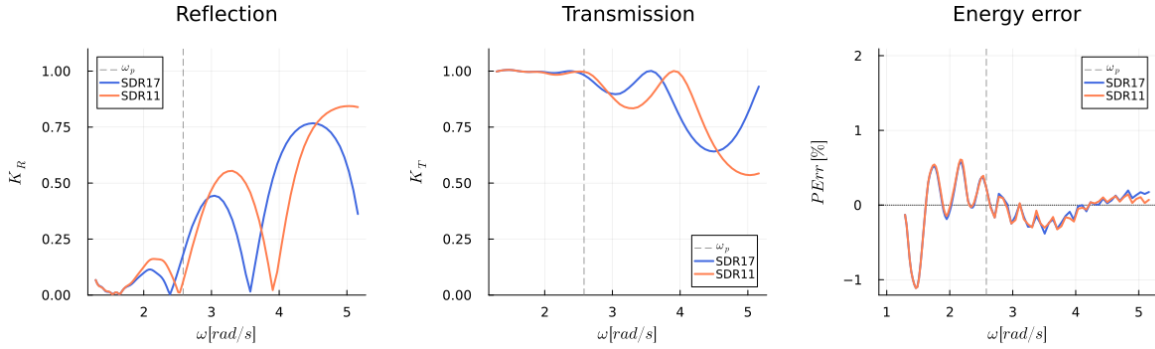


Figure 5.3: Reflection coefficient K_R (left), transmission coefficient K_T (centre), and energy error PErr (right) as a function of ω for the bending stiffness sweep on the $L_b = 18.4$ m mesh. SDR17 ($EI = 53,930$ Nm²/m) and SDR11 ($EI = 75,435$ Nm²/m) are shown. The vertical dashed line indicates $\omega_p = 2.581$ rad/s.

5.1.4. Beam Density Sweep

Figure 5.4 shows K_T , K_R , and PErr as a function of ω for the beam density sweep, performed on the $L_b = 18.4$ m mesh. The four curves are closely spaced, with the heavier beams shifted slightly towards lower frequencies, consistent with the lower natural frequencies of higher-mass beams. At ω_p , K_T ranges from 0.983 for the bare island ($\rho_b = 62.5$ kg/m³) to 0.971 for the heavily loaded case ($\rho_b = 300$ kg/m³), a difference of 0.012. This confirms that substrate and vegetation loading have a negligible influence on wave attenuation performance.

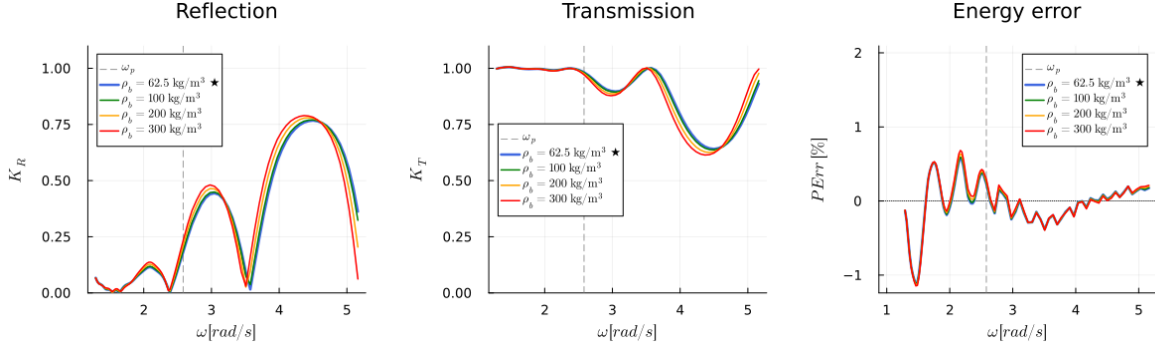


Figure 5.4: Reflection coefficient K_R (left), transmission coefficient K_T (centre), and energy error PErr (right) as a function of ω for the beam density sweep on the $L_b = 18.4$ m mesh. The base case $\rho_b = 62.5$ kg/m³ is highlighted. The vertical dashed line indicates $\omega_p = 2.581$ rad/s.

5.1.5. Beam Length Sweep

Figure 5.5 shows K_T , K_R , and PErr as a function of ω for the beam length sweep. At ω_p , the base case $L_b = 4.6$ m gives $K_T = 0.992$ and $K_R = 0.129$, the intermediate case $L_b = 9.2$ m gives $K_T = 0.972$ and $K_R = 0.238$, and the longest case $L_b = 18.4$ m gives $K_T = 0.983$ and $K_R = 0.180$. All three lengths give $K_T > 0.97$ at ω_p , confirming negligible attenuation at the peak frequency regardless of length. All beams show stronger attenuation at higher frequencies, the number of local K_R minima increasing with L_b . This reflects the transition from the rigid-body behaviour of the shorter beam to the elastic behaviour of the longer beam.

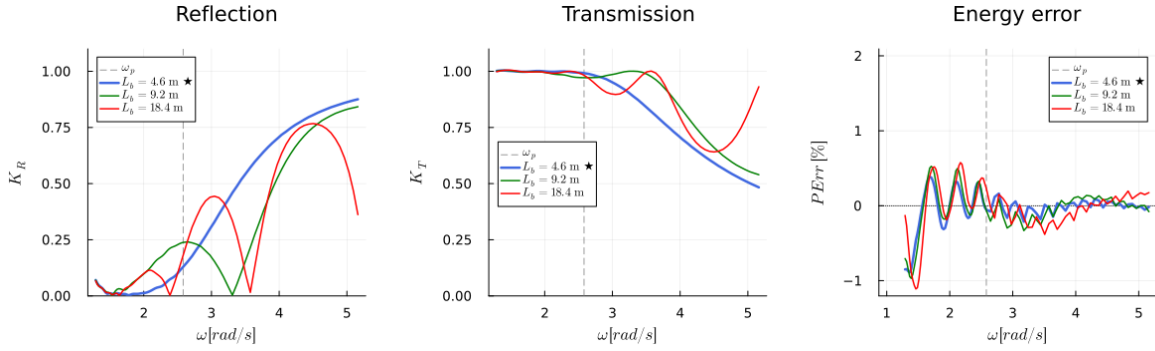


Figure 5.5: Reflection coefficient K_R (left), transmission coefficient K_T (centre), and energy error PErr (right) as a function of ω for the beam length sweep. $L_b \in \{4.6, 9.2, 18.4\}$ m, corresponding to $L_b/\lambda_p \in \{0.50, 0.99, 1.99\}$. The base case $L_b = 4.6$ m is highlighted. The vertical dashed line indicates $\omega_p = 2.581$ rad/s.

The bending moment envelope along the beam at $\omega = \omega_p$ is shown in Figure 5.6. For $L_b = 46$ m, the distribution is smooth and single-peaked, consistent with rigid-like behaviour in which the beam deflects as a whole without internal nodes. For $L_b = 92$ m, the peak magnitude increases and the distribution broadens, indicating the beginning of flexible behaviour. For $L_b = 184$ m, two distinct peaks are visible at $x/L_b \approx 0.35$ and $x/L_b \approx 0.70$, reflecting the multi-node bending shape of the fully flexible regime. Peak magnitudes increase with beam length, consistent with the larger curvatures of longer, more flexible beams. For all lengths, the free-end boundary conditions are satisfied, with $|M| \rightarrow 0$ at both endpoints.

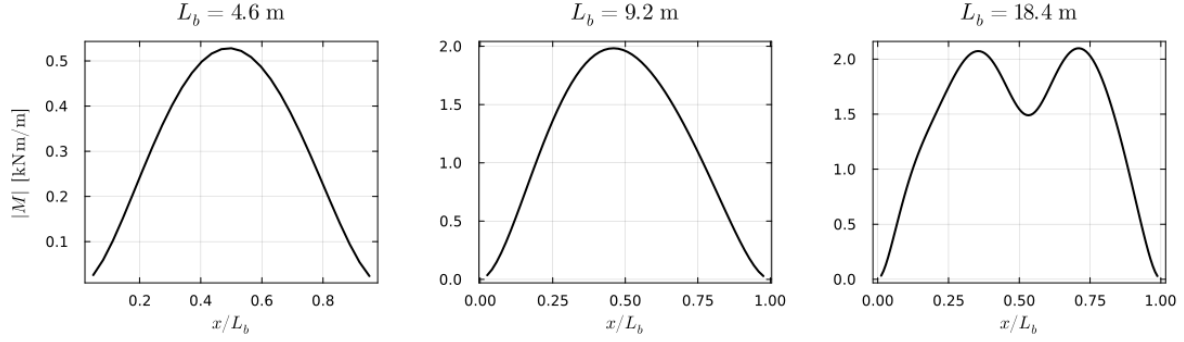


Figure 5.6: Bending moment envelope $|M|$ along the beam at $\omega = \omega_p$, scaled to the significant wave amplitude $\eta_{0,s} = 0.249$ m, for $L_b = 4.6$ m, 9.2 m, and 18.4 m.

Table 5.1 summarises K_T , K_R , and the load reduction ΔE_L at ω_p across all three sweeps. The load reduction remains below 6% for all configurations, confirming that the thin elastic beam provides negligible wave attenuation at the peak frequency regardless of structural parameters. Although attenuation improves at higher frequencies, these conditions carry less spectral energy under the Willemspolder JONSWAP spectrum and contribute less to the annual wave-induced loading than conditions near ω_p .

Table 5.1: Summary of wave attenuation performance at $\omega_p = 2.581$ rad/s for all Case 1 parameter sweeps.

Sweep	Configuration	K_T [-]	K_R [-]	ΔE_L [-]
Base case	$L_b = 4.6$ m, SDR17, $\rho_b = 62.5$ kg/m ³			
		0.992	0.129	0.016
El sweep	$L_b = 18.4$ m, $\rho_b = 62.5$ kg/m ³			
	SDR17	0.983	0.180	0.034
	SDR11	0.997	0.060	0.006
ρ_b sweep	$L_b = 18.4$ m, SDR17			
	$\rho_b = 62.5$ kg/m ³	0.983	0.180	0.034
	$\rho_b = 100$ kg/m ³	0.981	0.188	0.038
	$\rho_b = 200$ kg/m ³	0.977	0.209	0.046
	$\rho_b = 300$ kg/m ³	0.971	0.232	0.057
L_b sweep	SDR17, $\rho_b = 62.5$ kg/m ³			
	$L_b = 4.6$ m	0.992	0.129	0.016
	$L_b = 9.2$ m	0.972	0.238	0.055
	$L_b = 18.4$ m	0.983	0.180	0.034

5.2. Case 2: Thin Elastic Beam with Submerged Root System

Case 2 extends Case 1 by adding a submerged porous root zone, as described in Section 4.6.1, introducing dissipation as an additional attenuation mechanism. The results are obtained using a time-domain formulation and presented for a sweep of porous resistance values α_p at two beam lengths. Due to the absence of a fully effective damping zone, reflected waves from the inlet and outlet boundaries contaminate the solution over time. As a result, quantitative extraction of K_T and K_R is unreliable, and the results are presented qualitatively. As with Case 1, the beam is unmoored; the influence of mooring is discussed in Section 5.4.

5.2.1. Flow Field

Figure 5.7 shows the velocity and pressure fields at $t = 36.27$ s for $\alpha_p = 1.0 \text{ s}^{-1}$ and $L_b = 18.4$ m, illustrating the two-dimensional flow structure and the attenuation of wave energy downstream of the root zone. The free surface elevation κ_h shown in the figure is visibly irregular rather than a clean sinusoid. This irregularity is already present at early simulation times, before reflected waves from the domain boundaries can have reached the beam, suggesting a cause unrelated to the outlet-reflection contamination discussed above. The exact source has not been identified and is left for future investigation. The wave field is further assessed through snapshots of the free surface profile κ/η_0 at $t = 17T$, and the beam response through the spatial deflection profile η/η_0 at the same instant.

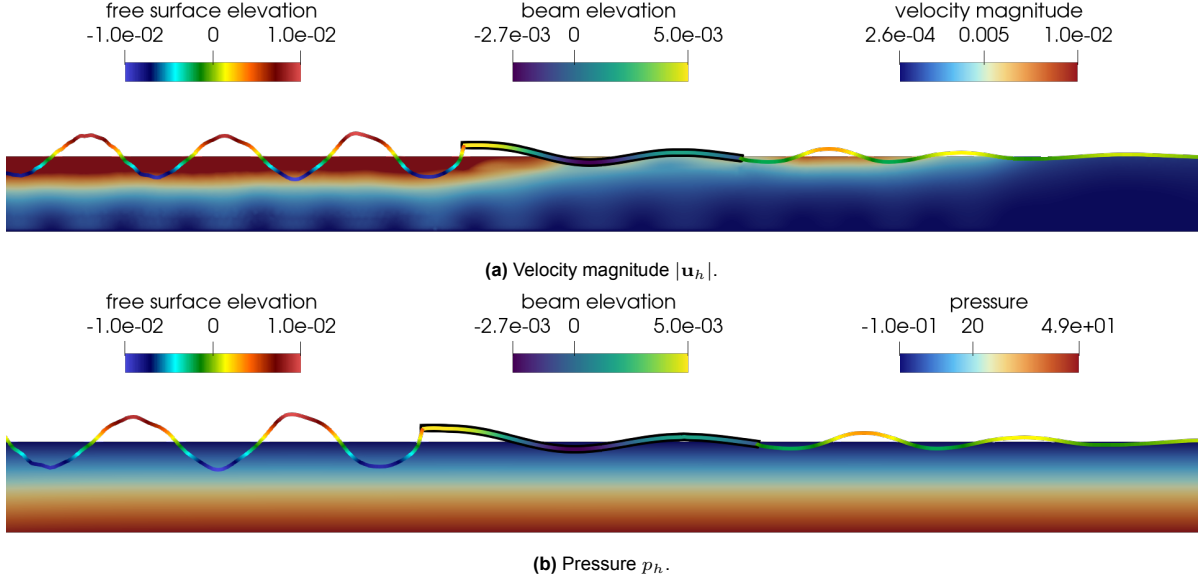


Figure 5.7: Velocity magnitude $|u_h|$ (top) and pressure p_h (bottom) at $t = 36.27$ s for $\alpha_p = 1.0 \text{ s}^{-1}$, $L_b = 18.4$ m. The free surface elevation κ_h and beam deflection η_h are shown as the coloured lines at the surface. The beam lies at the centre of the domain and is shaded in black. Wave attenuation downstream of the root zone is clearly visible in both fields. The vertical direction is scaled by a factor of 150 for visibility. The visualised region spans $x = 10.5$ m to $x = 94.5$ m (84 m) within the total computational domain, with a water depth of 5 m.

5.2.2. Porous Resistance Sweep

Figures 5.8 and 5.9 show the free surface amplitude κ/η_0 at $t = 17T$ for $\alpha_p \in \{0.00, 0.01, 0.1, 0.5, 1.0, 2.5, 5.0\} \text{ s}^{-1}$, for $L_b = 4.6$ m and $L_b = 18.4$ m, respectively. Upstream of the structure, the amplitude reaches approximately η_0 for all α_p values, consistent with the prescribed incident wave amplitude.

Downstream of the structure, the wave amplitude decreases with increasing α_p . For $\alpha_p = 0.01 \text{ s}^{-1}$, the downstream amplitude is nearly indistinguishable from the $\alpha_p = 0$ case, confirming that a very sparse root zone has negligible effect on wave transmission. For $\alpha_p = 0.5 \text{ s}^{-1}$, a visible reduction in downstream amplitude is observed. For the highest resistance value $\alpha_p = 5.0 \text{ s}^{-1}$, the downstream amplitude is near zero across both beam lengths, indicating near-complete dissipation of the transmitted wave energy by the root zone.

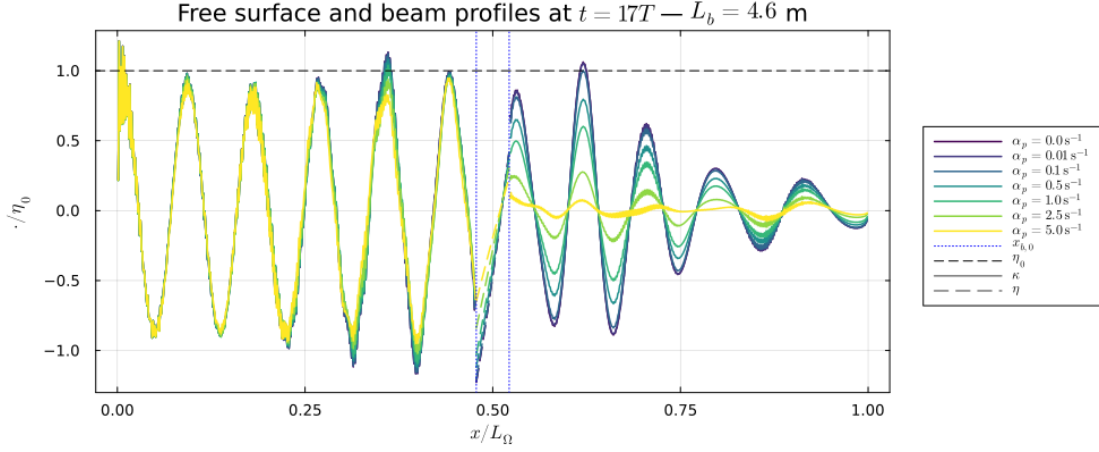


Figure 5.8: Normalised free surface amplitude $|\kappa|/\eta_0$ and beam deflection $|\eta|/\eta_0$ (dashed) at $t = 17T$ for $\alpha_p \in \{0.00, 0.01, 0.1, 0.5, 1.0, 2.5, 5.0\} \text{ s}^{-1}$, $L_b = 4.6$ m. Vertical dotted lines indicate the beam edges $x_{b,0}$ and $x_{b,1}$. The dashed horizontal line indicates η_0 .

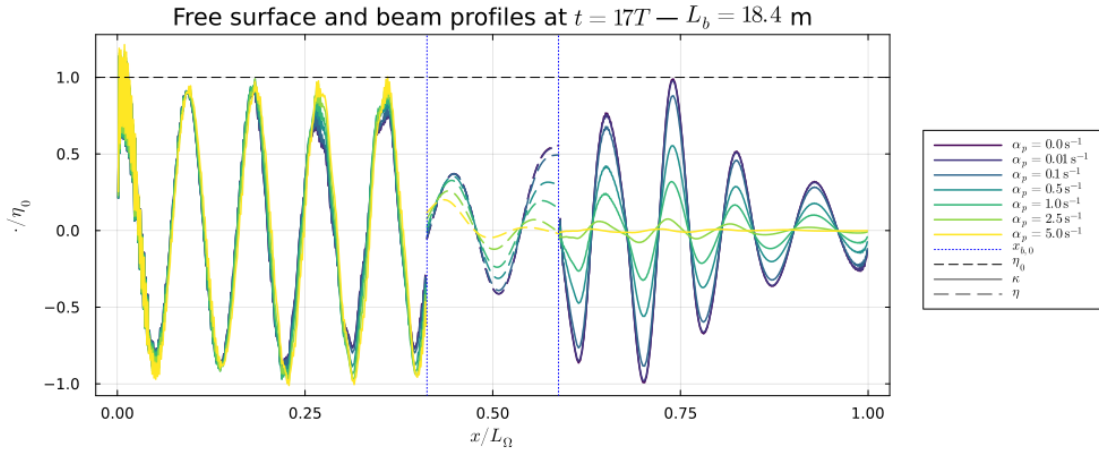


Figure 5.9: Normalised free surface amplitude $|\kappa|/\eta_0$ at $t = 17T$ for $\alpha_p \in \{0.00, 0.01, 0.1, 0.5, 1.0, 2.5, 5.0\} \text{ s}^{-1}$, $L_b = 18.4$ m. Vertical dotted lines indicate the beam edges $x_{b,0}$ and $x_{b,1}$. The dashed horizontal line indicates η_0 .

Figure 5.10 shows the beam deflection profile η/η_0 at $t = 17T$ for all α_p values and both beam lengths. For $L_b = 4.6$ m, the deflection profile is nearly linear, indicating the same rigid behaviour as in Case 1. The deflection amplitude at the upstream edge is approximately $-1.25 \eta_0$ for $\alpha_p = 0$ and decreases to approximately $-0.65 \eta_0$ for $\alpha_p = 5.0 \text{ s}^{-1}$. For $L_b = 18.4$ m, clear elastic behaviour is visible, consistent with the Case 1 response. The deflection amplitude decreases strongly with α_p . For $\alpha_p = 5.0 \text{ s}^{-1}$ the profile is nearly flat, indicating near-complete attenuation of the incoming wave amplitude. The bending moment distributions for Case 2 follow the same spatial patterns as Case 1, shown in Figure 5.6, but with reduced peak magnitudes, consistent with the lower deflection amplitudes due to wave energy dissipation in the porous root zone.

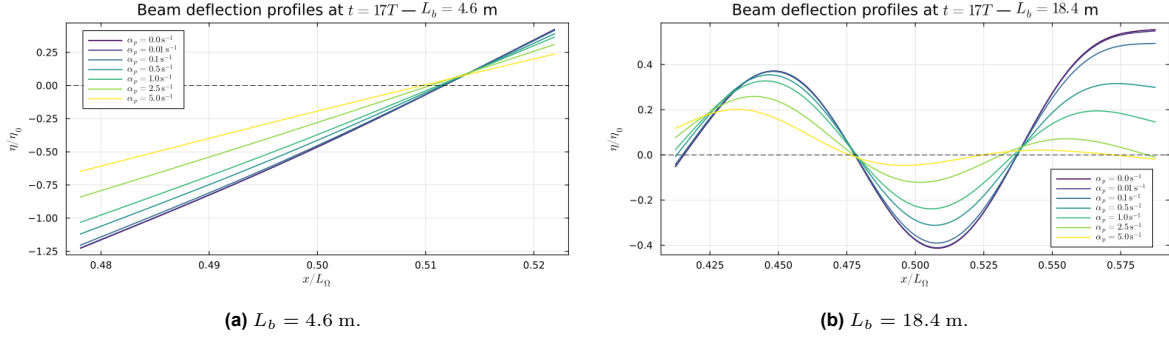


Figure 5.10: Beam deflection profile η/η_0 at $t = 17T$ for $\alpha_p \in \{0.00, 0.01, 0.1, 0.5, 1.0, 2.5, 5.0\} \text{ s}^{-1}$. (a) $L_b = 4.6$ m and (b) $L_b = 18.4$ m.

The results confirm that the porous root zone is the primary mechanism for wave attenuation in Case 2. A visible reduction in transmitted wave amplitude is already observed at $\alpha_p = 0.5 \text{ s}^{-1}$, and a realistic range for a mature root zone, corresponding to moderate values of C_d , b_v , and N as derived in Section 4.8.5, is estimated at $\alpha_p \approx 1\text{--}2 \text{ s}^{-1}$. The floating ecosystem, in its mature configuration, therefore has significantly greater wave-attenuation potential than the bare mat in Case 1.

5.3. Case 3: Thin-walled Rigid Pipe

Case 3 represents a rigid hollow HDPE pipe in the frequency domain, as described in Section 4.6.2, free to respond to heave and pitch. The present analysis focuses on the heave response, considering two configurations: unmoored ($k_{heave} = 0$) and moored with an empirically optimised stiffness. The diameter is treated as a sweep parameter over $D \in \{0.25 - 0.50\} \text{ m}$. At ω_p , the unmoored pipe is small relative to the peak wavelength ($D/\lambda_p \leq 0.054$), and the wave field is largely undisturbed by the pipe cross-section. With optimal mooring stiffness, the system resonates at ω_p , and the resulting pipe motion radiates a wave field that interferes with the transmitted wave.

5.3.1. Flow Field at Peak Frequency

Figure 5.11 shows the free surface elevation η_h and velocity potential ϕ_h at $\omega = \omega_p = 2.581 \text{ rad/s}$ for the base case $D = 0.45 \text{ m}$, unmoored ($k_{heave} = 0$). The real part, imaginary part, and absolute value are shown. In the absolute value panel, the incident wave amplitude is uniform upstream of the pipe, consistent with the near-zero reflection of the unmoored case. Downstream, the transmitted wave amplitude is largely unchanged, confirming the negligible attenuation reported below.

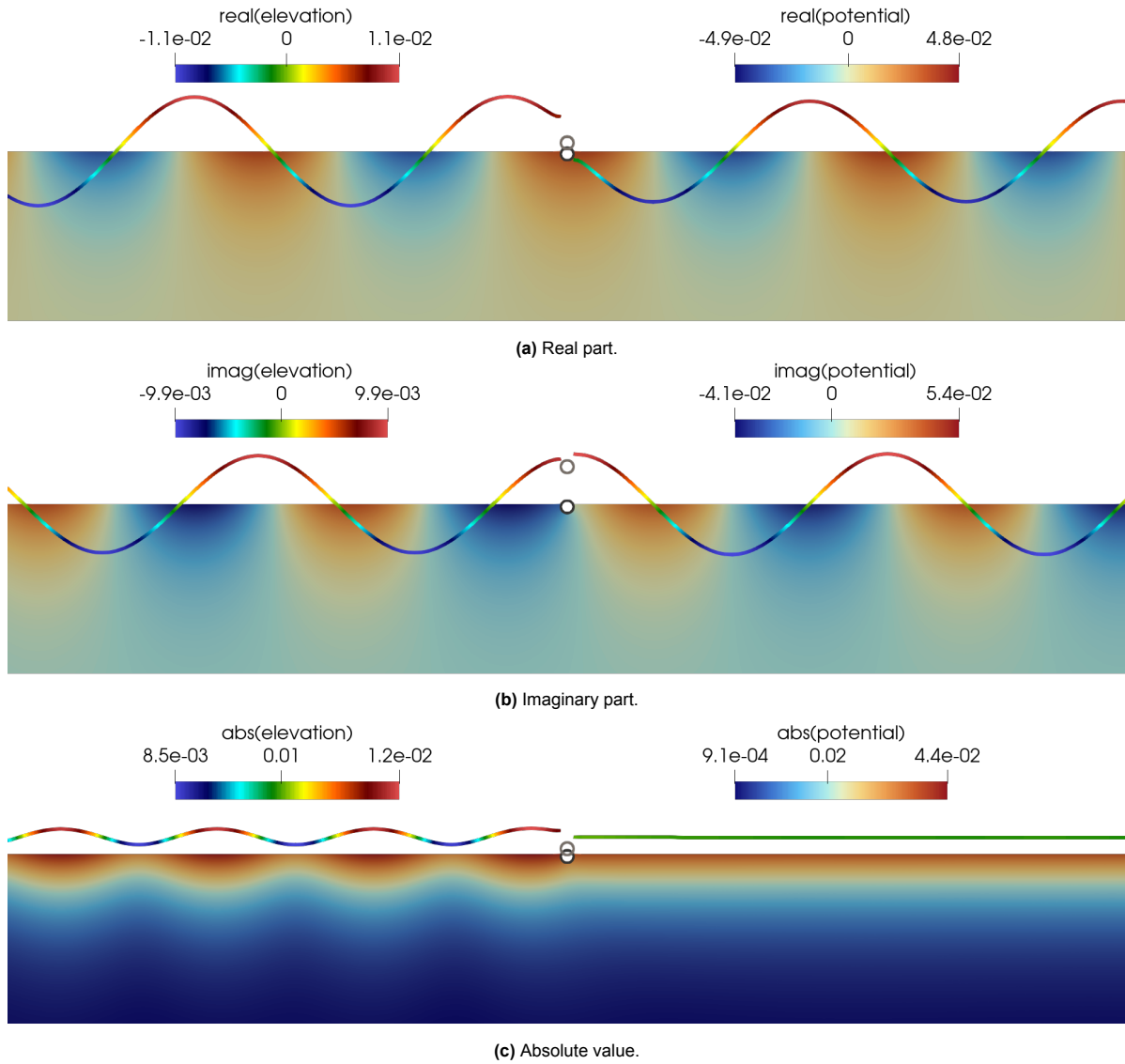


Figure 5.11: Surface elevation η_h (left) and velocity potential ϕ_h (right) at $\omega = \omega_p = 2.581$ rad/s, base case $D = 0.45$ m, unmoored ($k_{heave} = 0$). The pipe cross-section is indicated in black at rest and in gray at its heave-displaced position, scaled by the same factor of 150 as the elevation. The vertical direction of the elevation is scaled by a factor of 150 for visibility. The visualised region spans $x = 40$ m to $x = 75$ m (35 m) within the total computational domain, with a water depth of 5 m.

5.3.2. Wave Amplitude at Peak Frequency

Figure 5.12 shows the normalised wave amplitude $|\kappa|/\eta_0$ along the free surface at ω_p , together with the decomposed incident component $|\kappa_{in}|/\eta_0$ and reflected component $|\kappa_r|/\eta_0$. As in Figure 5.11, this is the unmoored base case $D = 0.45$ m. The small upstream oscillation of the envelope confirms a weak reflected wave. Downstream of the pipe, the amplitude is uniform, indicating a clean transmitted wave. The pipe waterline gap is indicated by the shaded region. The marked heave response at ω_p , $|\xi_1|/\eta_0 = 0.82$, is smaller than the incident wave amplitude, consistent with the pipe being too small relative to the peak wavelength to scatter the wave significantly. The unmoored base case yields $K_T = 0.99$ and $K_R = 0.15$, confirming low attenuation of the unmoored pipe.

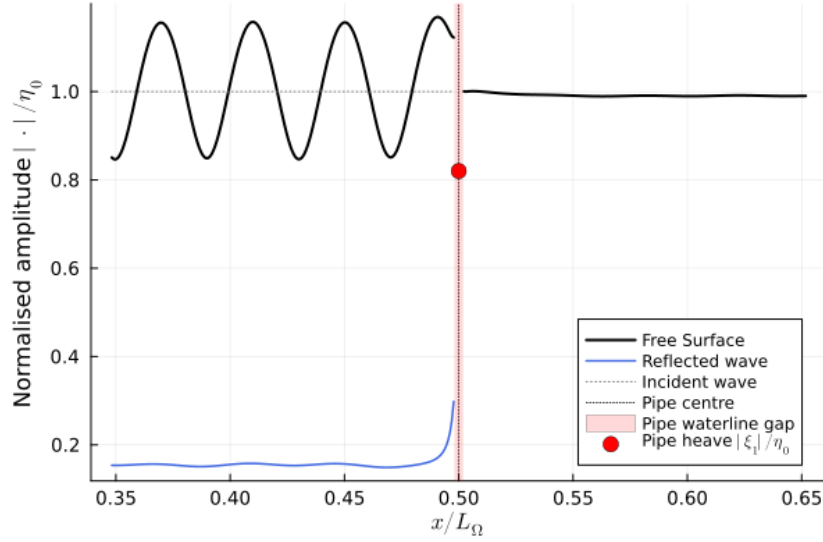


Figure 5.12: Normalised wave amplitude along the free surface at $\omega_p = 2.581$ rad/s, base case $D = 0.45$ m, unmoored ($k_{heave} = 0$). Shown are the total amplitude $|\kappa|/\eta_0$, the incident component $|\kappa_{in}|/\eta_0$, and the reflected component $|\kappa_r|/\eta_0$. The shaded region indicates the pipe waterline gap. The red marker at the pipe centre indicates the heave response, $|\xi_1|/\eta_0 = 0.82$. The horizontal axis is normalised by the total domain length L_Ω .

5.3.3. Unmoored Baseline

Figure 5.13 shows K_T , K_R , and the heave response amplitude operator (RAO) as a function of ω for the unmoored diameter sweep. At ω_p , all diameters give a low K_R and high K_T , confirming the negligible attenuation reported in Figure 5.12. The summary of the values is reported in Table 5.2. Both coefficients show a strong frequency dependence: K_T decreases, and K_R increases with ω , consistent with the increasing ratio D/λ at shorter wavelengths. At ω_p the ratio $D/\lambda_p \leq 0.054$ for all diameters, confirming that the pipe is small relative to the peak wavelength.

The heave RAO shows a broad peak near $\omega \approx 1.5$ rad/s for all diameters, identifying the wet natural frequency $\omega_n \approx 1.51$ – 1.56 rad/s. Above ω_n the RAO decreases with increasing frequency. The larger diameters show lower K_T values throughout the sweep, consistent with a larger cross-section relative to the wavelength.

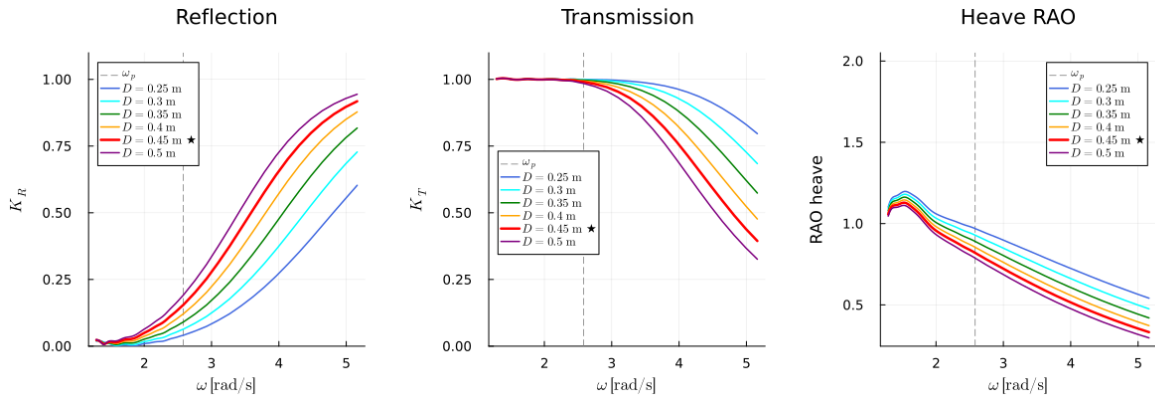


Figure 5.13: Reflection coefficient K_R (left), transmission coefficient K_T (centre), and heave RAO (right) as a function of ω for the unmoored diameter sweep, $D \in \{0.25, 0.30, 0.35, 0.40, 0.45, 0.50\}$ m. The base case $D = 0.45$ m is highlighted. The vertical dashed line indicates $\omega_p = 2.581$ rad/s.

Table 5.2: Case 3 unmoored diameter sweep results at $\omega_p = 2.581$ rad/s.

D [m]	K_T [-]	K_R [-]	ΔE_L [%]
0.25	0.9996	0.0409	0.08
0.30	0.9988	0.0637	0.23
0.35	0.9973	0.0906	0.53
0.40	0.9946	0.1210	1.07
0.45	0.9906	0.1548	1.87
0.50	0.9849	0.1914	3.00

5.3.4. Mooring Stiffness Sweep

Figure 5.14 shows K_T and K_R at ω_p as a function of $k_{heave}/K_{hs,11}$ for all diameters. The optimal mooring stiffness is identified empirically by evaluating K_T at ω_p for 50 log-spaced stiffness values over $k_{heave} \in [10^{-3}, 10^2] \cdot K_{hs,11}$, and selecting the value that minimises K_T . This approach is chosen because placing the resonance frequency exactly at ω_p would require running the full frequency sweep for each stiffness value, which is computationally expensive. The analytical alternative, which uses the effective mass extracted from the unmoored RAO to compute the required stiffness directly, is unreliable because the added mass varies with frequency.

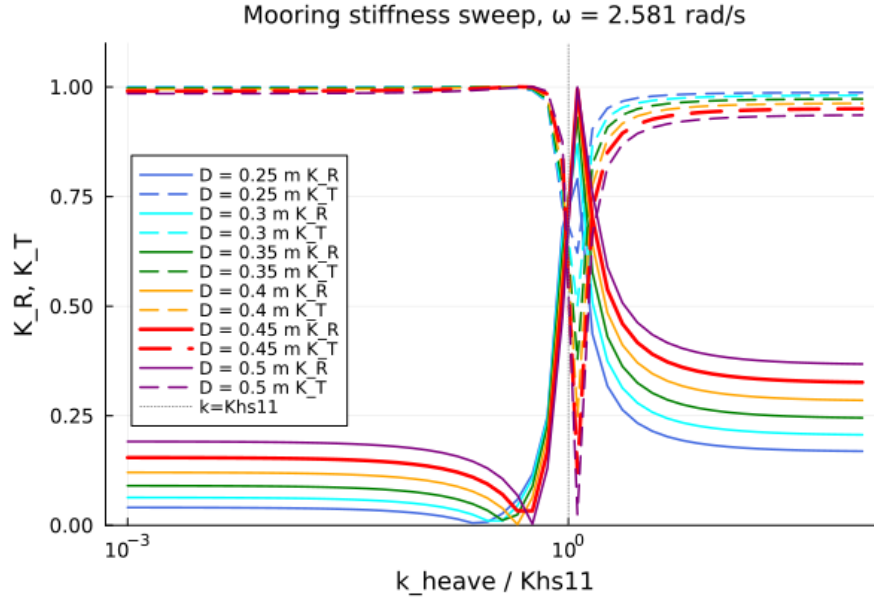


Figure 5.14: Transmission coefficient K_T (dashed) and reflection coefficient K_R (solid) at ω_p as a function of normalised mooring stiffness $k_{heave}/K_{hs,11}$, for all diameters $D \in \{0.25, 0.30, 0.35, 0.40, 0.45, 0.50\}$ m. The optimal stiffness $k_{heave}/K_{hs,11} = 1.151$ is identical for all diameters.

To identify the optimal stiffness per diameter with higher precision, a refined sweep based on the outcomes of Figure 5.14 is performed over $k_{heave}/K_{hs,11} \in [0.9, 1.3]$ at 50 linearly spaced points. Figure 5.15 shows the resulting K_T curves and Table 5.3 lists the optimal stiffness ratio and corresponding $K_{T,min}$ per diameter. The optimal ratio increases with D , from $k_{heave}/K_{hs,11} = 1.031$ for $D = 0.25$ m to $k_{heave}/K_{hs,11} = 1.153$ for $D = 0.50$ m. With the refined optimal stiffness, all diameters achieve $K_{T,min} < 0.035$ at ω_p .

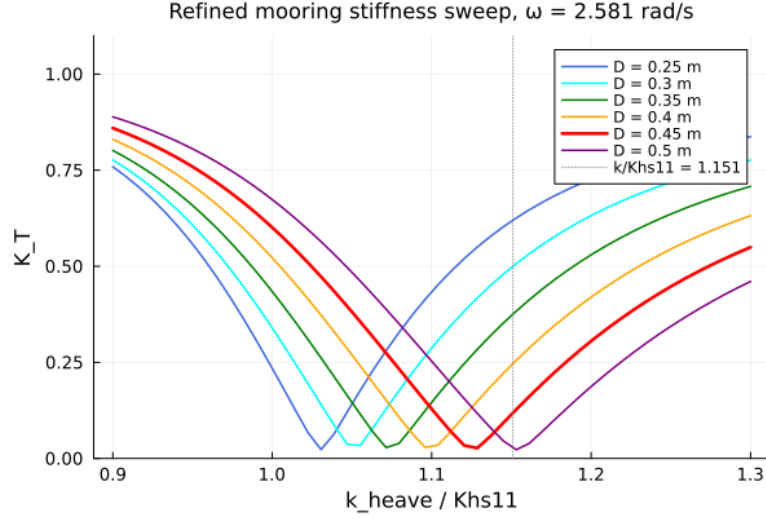


Figure 5.15: Refined mooring stiffness sweep over $k_{heave}/K_{hs,11} \in [0.9, 1.3]$ at $\omega_p = 2.581$ rad/s, for $D \in \{0.25, 0.30, 0.35, 0.40, 0.45, 0.50\}$ m. The optimal stiffness ratio increases with D .

Table 5.3: Refined optimal mooring stiffness per diameter at $\omega_p = 2.581$ rad/s.

D [m]	$k_{heave}/K_{hs,11}$ [-]	$K_{T,min}$ [-]
0.25	1.031	0.023
0.30	1.055	0.034
0.35	1.071	0.028
0.40	1.096	0.029
0.45	1.129	0.027
0.50	1.153	0.023

5.3.5. Moored Diameter Sweep

Figure 5.16 shows K_T , K_R , and the heave RAO as a function of ω for the moored diameter sweep, with the refined optimal stiffness per diameter from Table 5.3. The results at ω_p are summarised in Table 5.4. With the optimal mooring applied, all diameters show strong attenuation with a low K_T minimum, confirming that the stiffness correctly tunes the system resonance to the design frequency for each diameter.

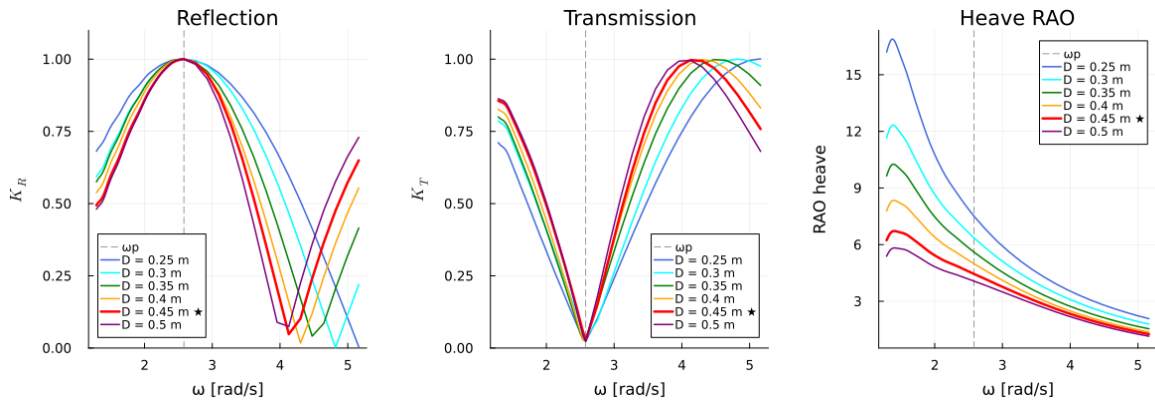


Figure 5.16: Reflection coefficient K_R (left), transmission coefficient K_T (centre), and heave RAO (right) as a function of ω for the moored diameter sweep, $D \in \{0.25, 0.30, 0.35, 0.40, 0.45, 0.50\}$ m, with refined optimal stiffness per diameter from Table 5.3. The base case $D = 0.45$ m is highlighted. The vertical dashed line indicates $\omega_p = 2.581$ rad/s.

All diameters almost achieve full reflection at ω_p , with $K_R > 0.999$ and $K_T < 0.034$ for all cases. The attenuation at ω_p is high for all diameters, with load reductions $\Delta E_L > 99.9\%$ for all diameters. The energy balance is satisfied across the full sweep, with $|\text{PErr}| < 0.1\%$ for all moored cases.

The heave RAO at ω_p ranges from 7.5 for $D = 0.25$ m to 4.1 for $D = 0.50$ m. Scaled to the significant wave amplitude $\eta_{0,s} = 0.249$ m, this corresponds to physical heave amplitudes of 1.0–1.9 m at the design frequency. These amplitudes are large relative to the incident wave amplitude. With no mooring damping ($C = 0$), the only energy dissipation at resonance is wave radiation, which is small for a pipe of this size; thus, the model overestimates the motion. The resulting resonance peak is therefore unphysically large. In practice, mooring damping and viscous effects would limit the actual heave amplitude to physically realistic values. The heave RAO also shows a pronounced peak at the wet natural frequency $\omega_n \approx 1.39\text{--}1.41$ rad/s, with maximum values ranging from 5.8 for $D = 0.50$ m to 16.9 for $D = 0.25$ m. The wave attenuation results at ω_p are not affected by this limitation, as the energy balance is well satisfied with $|\text{PErr}| < 0.1\%$ for all diameters.

Table 5.4: Case 3 moored diameter sweep results at $\omega_p = 2.581$ rad/s, with refined optimal stiffness per diameter.

D [m]	$k_{heave}/K_{hs,11}$ [-]	K_T [-]	K_R [-]	ΔE_L [%]
0.25	1.031	0.023	0.9999	99.9
0.30	1.055	0.034	0.9999	99.9
0.35	1.071	0.028	0.9996	99.9
0.40	1.096	0.029	0.9996	99.9
0.45	1.129	0.027	1.0000	99.9
0.50	1.153	0.023	0.9999	99.9

5.4. Case Comparison

Table 5.5 summarises K_T , K_R at $0.5\omega_p$, ω_p , $1.5\omega_p$, and $2\omega_p$ for the base configurations of Cases 1 and 3. Case 2 is excluded from the table as reliable extraction of K_T and K_R was not possible; its results are discussed in Section 5.2.

At ω_p , both the elastic beam and the unmoored pipe transmit nearly all incident wave energy, with $K_T = 0.992$ and $K_T = 0.991$, respectively. Both structures move with the incident wave rather than obstructing it. Additionally, the pipe is too small relative to the peak wavelength ($D/\lambda_p = 0.049$) to scatter the wave significantly. At higher frequencies, the attenuating performance increases for both cases. At $2\omega_p$, the beam reaches $K_T = 0.483$ and the unmoored pipe $K_T = 0.394$. This change can be related to the increasing ratio D/λ_p and higher relative stiffness for the beam. The attenuation performance of the unmoored pipe and the $L_b = 4.6$ m beam is similar across the frequency spectrum.

The moored pipe ($D = 0.45$ m) demonstrates the effectiveness of resonant scattering: at ω_p it achieves a load reduction of 99.9%. Away from the resonance, however, it performs comparably to or worse than the unmoored cases, with $K_T = 0.976$ at $1.5\omega_p$ and $K_T = 0.758$ at $2\omega_p$, confirming the narrow character of the mooring-tuned pipes. It should be noted that this result assumes no damping ($C = 0$), which is unphysical: in practice, mooring damping would limit the heave amplitude at resonance and reduce the peak attenuation. The 99.9% load reduction should not be interpreted as a realisable design target.

Case 1 is evaluated only unmoored. The pipe shows that tuning the heave stiffness to ω_p shifts the resonance and converts the structure from wave-transparent to a near-total reflector. The same mechanism is expected to apply to the beam's rigid-heave response. Mooring and tuning the beam should therefore increase reflection and reduce transmission near ω_p . The beam's elastic bending modes are governed by EI and mass, not by mooring stiffness. These modes could still dominate the beam's response independently of the tuned heave resonance, particularly for the longer beams. The magnitude of this effect cannot be quantified without a moored Case 1 model and is identified as a direction for future research in Chapter 8.

Case 2 extends Case 1 by adding a porous root zone, which introduces dissipation as a third mechanism. Section 5.2 shows that the root zone can perform as the dominant contributor to the wave

attenuation: at $\alpha_p = 5.0 \text{ s}^{-1}$ near-complete downstream attenuation is observed for both beam lengths. The structural response of Case 2 remains largely the same as for Case 1. The root zone therefore provides greater attenuation potential than the elastic beam alone. As with Case 1, Case 2 is evaluated only unmoored. Mooring and tuning the heave stiffness to ω_p would be expected to add resonant reflection to Case 2 as well. This would supplement the root-zone dissipation rather than replace it, since dissipation is already the dominant mechanism. Whether the two mechanisms reinforce or interfere has not been investigated.

At $0.5\omega_p$, Case 1 and the unmoored Case 3 show near-complete transmission, with $K_T \approx 1.00$. The reported values slightly exceed 1, a numerical effect caused by the longer wavelength reaching the damping zone length limit, making the wave absorption less effective. The moored pipe at $0.5\omega_p$ shows $K_T = 0.856$ and $K_R = 0.494$, as at this frequency the tuned mooring stiffness still reduces wave transmission.

Table 5.5: Wave attenuation performance at $0.5\omega_p$, ω_p , $\approx 1.5\omega_p$, and $2\omega_p$ for the base configurations of Cases 1 and 3.

Case	Configuration	$0.5\omega_p = 1.291 \text{ rad/s}$		$\omega_p = 2.581 \text{ rad/s}$		$1.5\omega_p = 3.87 \text{ rad/s}$		$2\omega_p = 5.162 \text{ rad/s}$	
		K_T [-]	K_R [-]	K_T [-]	K_R [-]	K_T [-]	K_R [-]	K_T [-]	K_R [-]
Case 1	$L_b = 4.6 \text{ m}$, SDR17	1.002	0.071	0.992	0.129	0.731	0.682	0.483	0.876
Case 3	$D = 0.45 \text{ m}$, unmoored	1.002	0.023	0.991	0.155	0.768	0.641	0.394	0.918
	$D = 0.45 \text{ m}$, moored	0.856	0.494	0.027	1.000	0.976	0.206	0.758	0.650

5.5. Chapter Summary

The chapter has presented the results addressing SQ1 and the structural response component of SQ2. The three representations show fundamentally different attenuation mechanisms: elastic scattering dominates at higher frequencies in Case 1, porous dissipation is the primary mechanism in Case 2, and resonant scattering governs Case 3 when moored. For Case 1, ΔE_L at ω_p remains below 6 % across all parameter configurations studied. For Case 2, quantitative extraction of ΔE_L was not possible due to outlet boundary reflections, though qualitative results indicate significant dissipation potential at higher α_p values. For the optimally moored pipe, ΔE_L reaches 99.9 % at ω_p , establishing the upper bound on wave-induced load reduction on an adjacent FPV system. The implications of these results for FPV design are addressed in Chapter 6.

Implications for FPV Systems

The hydrodynamic model presented in Chapter 4 delivers the frequency-dependent transmission coefficient $K_T(\omega)$ for Cases 1 and 3. Together with the JONSWAP spectrum, its spectral integral $\Delta E_{L, \text{spec}}$ is computed and used throughout this chapter to translate the modelling results into practical implications for FPV design. Case 2 was excluded from the quantitative analysis as reliable extraction of $K_T(\omega)$ was not possible due to outlet boundary reflections, as discussed in Section 4.11.3. This chapter addresses SQ3 and completes SQ2 by translating the structural response of Chapter 5 into the resulting reduction in wave-induced loads on an adjacent FPV system.

Section 6.1 introduces the wave-induced loading mechanisms. Section 6.2 derives the load reduction factors. Section 6.3 translates these into structural design consequences for the ZIM Float system. Section 6.5 assesses the effect of wave attenuation on energy yield. Section 6.8 provides integrated design recommendations.

6.1. Wave-Induced Loading on FPV systems

The ZIM Float FPV system [18] consists of steel boats with a corrosion-resistant coating, carrying HDPE floats and photovoltaic modules. For the Willemspolder installation, the system contains 450 solar boats arranged in rigid floats of 6, 12, or 15 boats each. Within a float, the boats are rigidly connected; adjacent floats are coupled by flex-element connectors that allow relative rotational motion between floats, shown in Figure 6.1. The flex elements are designed to absorb differential motion under wave loading and have a 25-year design life. Cyclic loading of these connectors governs the fatigue limit state of the structure.

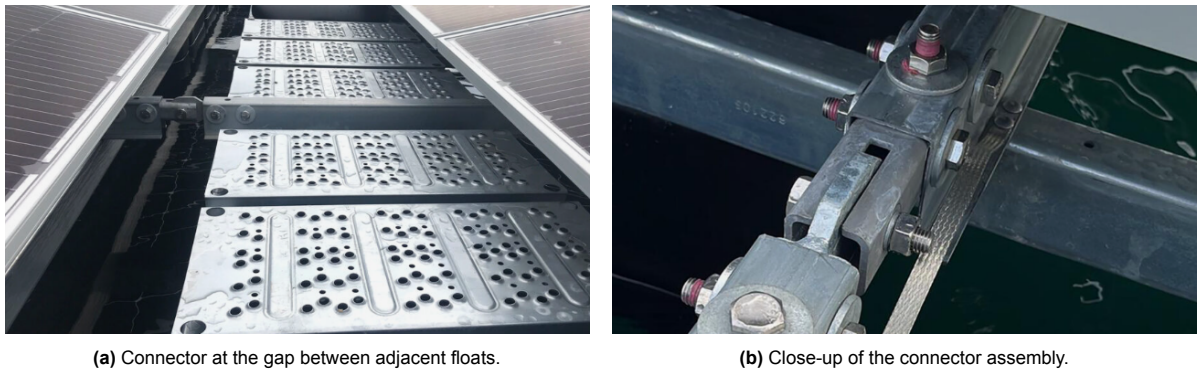


Figure 6.1: Connector permitting relative motion between adjacent floats on the ZIM Float system. The pinned joint and tie element allow rotation while transferring load between floats under wave-induced differential motion. Photos: Zimmermann PV-Floating [18].

Wave loading acts on the system through two main mechanisms. First, the front row of floats receives direct hydrodynamic forcing from the incident waves. This load is transferred through the mooring system to the anchors, thereby setting the governing load for the ultimate limit state (ULS). Given the scale and mass of a large FPV array, it is expected that the array itself progressively damps the wave field as it propagates across successive rows, in which case rows further from the incident wave front

would experience smaller structural loads and smaller energy yield loss than the front row. This effect is not captured by the factors derived below, which assume a uniform incident wave field across the array, and would require a dedicated array-scale hydrodynamic analysis to quantify. Second, waves propagating across the array produce a differential vertical motion between the floats, cyclically loading the flex-element connectors. This mechanism governs the fatigue limit state (FLS); the governing load case for Willemspolder and its relation to the present results are discussed in Section 6.4.

The load types considered here are driven by the incident wave field. This study operates in the linear regime, where the response of each float is described by its Response Amplitude Operator (RAO), which is independent of the barrier. The RAO describes the structural response amplitude per unit wave amplitude and characterises how strongly the structure responds to a wave of given amplitude at each frequency. The barrier reduces the incident wave amplitude, and the RAO operates on the reduced field. In the ratio of load with a barrier to load without a barrier, the RAO cancels, leaving dimensionless load-reduction factors derived directly from the spectral transmission results. This is the same principle used in Section 6.6, where the platform sensitivity coefficient β cancelled in the loss ratio. The three load-reduction factors and their derivations are presented in Section 6.2.

6.2. Load Reduction Factors

The wave-attenuation performance across the full sea state is characterised by the spectrally weighted load reduction, defined as:

$$\Delta E_{L,\text{spec}} = 1 - \frac{\int S(\omega) K_T^2(\omega) d\omega}{\int S(\omega) d\omega} \quad (6.1)$$

where $S(\omega)$ is the JONSWAP spectral energy density and $K_T(\omega)$ is the transmission coefficient obtained from the frequency sweep. The integral is evaluated numerically over $\omega \in [0.5\omega_p, 2\omega_p]$. $\Delta E_{L,\text{spec}}$ represents the fraction of wave-induced losses eliminated by the barrier, averaged over the full sea state. Since the significant wave height satisfies $H_s = 4\sqrt{m_0}$, where $m_0 = \int_0^\infty S(\omega) d\omega$ is the zeroth spectral moment of the wave spectrum [29], the transmitted significant wave height ratio follows directly as:

$$R_{load} = \frac{H_{s,\text{T}}}{H_{s,\text{i}}} = \sqrt{1 - \Delta E_{L,\text{spec}}} \quad (6.2)$$

In the linear regime, hydrodynamic forcing on a floating body scales with wave amplitude, so R_{load} is the ratio of wave-induced load on the FPV system with versus without a wave barrier. The mean second-order wave drift force scales with wave energy rather than wave amplitude, giving:

$$R_{drift} = \frac{F_{drift,\text{T}}}{F_{drift,\text{i}}} = 1 - \Delta E_{L,\text{spec}} \quad (6.3)$$

The drift force is a steady horizontal load on the FPV array, resisted by the mooring lines and transferred to the anchors. Reducing the incident wave energy by the factor R_{drift} therefore reduces the mooring line design load and the anchor holding capacity. Since $R_{drift} = R_{load}^2 < R_{load}$ for $0 \leq R_{load} \leq 1$, the mooring system always benefits more from the same barrier than the structural members do.

Under the FLS, damage grows according to an S-N relationship $N_f \propto \sigma_s^{-n}$, where σ_s is the stress amplitude, n the S-N exponent, and N_f the number of cycles. n is typically in the range of 3 to 5 for steel connections [68]. For a narrowband sea state spectrum, wave amplitudes follow a Rayleigh distribution with parameter $\sigma = \sqrt{m_0}$ [29]. Since $\sigma = \sqrt{m_0} = H_s/4$, the ratio $\sigma_{\text{T}}/\sigma_{\text{i}}$ equals R_{load} , and the fatigue damage rate ratio follows as:

$$R_{fatigue} = \frac{\sigma_{\text{T}}^n}{\sigma_{\text{i}}^n} = \left(\frac{\sigma_{\text{T}}}{\sigma_{\text{i}}} \right)^n = \left(\frac{H_{s,\text{T}}}{H_{s,\text{i}}} \right)^n = R_{load}^n \quad (6.4)$$

This ratio is independent of the FPV's annual exposure to the sea state, since the same exposure time appears in the numerator and denominator, cancelling out. The absolute fatigue life requires S-N data for the flex element connectors. The wave barrier extends that life by a factor R_{load}^{-n} .

The three factors are summarised in Table 6.1 for the configurations studied in Chapter 5.

Table 6.1: Load reduction factors for the configurations studied in Chapter 5.

Configuration	$\Delta E_{L,spec}$	R_{load}	R_{drift}	R_{load}^3	R_{load}^5
Case 1, $L_b = 4.6$ m SDR17	12.4 %	0.936	0.876	0.820	0.718
Case 3, unmoored, $D = 0.45$ m	10.9 %	0.944	0.891	0.841	0.750
Case 3, moored optimal, $D = 0.45$ m	78.2 %	0.467	0.218	0.102	0.022

The spectrally weighted load reduction factors are calculated using the JONSWAP spectrum for the dominant fetch from the west of 900 m. The frequency sweep covers the peak frequencies of shorter fetches, but the spectral weighting reflects the governing wave direction. For other wave directions with shorter fetches, the JONSWAP spectrum shifts to higher peak frequencies. For the unmoored elastic beam and unmoored pipe, this generally improves attenuation, as both perform better at higher frequencies. For the moored pipe, the mooring stiffness would need to be retuned to the new peak frequency to achieve comparable attenuation.

6.3. Structural Design Consequences

Case 1 and the unmoored Case 3 pipe perform similarly, with $\Delta E_{L,spec}$ of 12.4 % and 10.9 % respectively and R_{load} of 0.936 and 0.944. The bare floating ecosystem transmits nearly all incident wave energy at ω_p , consistent with the high K_T at peak frequency reported in Chapter 5. The unmoored pipe is too small relative to the peak wavelength to reflect significant energy at ω_p . Neither of the two configurations provides a large reduction of the structural loads: wave-induced loads are reduced by less than 7 %, drift loads by less than 11 %, and fatigue damage rate by less than 18 % for $n = 3$. For structural design purposes, neither the size reduction, the mooring specification, nor the evaluation of the fatigue of the flex-element justifies a reduction in member size. These results represent a lower bound on attenuation performance: in practice, both configurations would be moored, and mooring stiffness is expected to improve attenuation, as Section 5.4 shows that tuning to ω_p consistently reduces K_T .

Note that Case 1 and Case 2 represent the same physical floating ecosystem at different stages of its service life: Case 1 is the bare floating ecosystem without a root zone, as placed initially, and Case 2 represents the same structure once a mature root zone has developed beneath it. Case 2 shows that this mature root zone introduces porous dissipation as an additional attenuation mechanism, capable of near-complete downstream attenuation at sufficiently high resistance values. A mature floating ecosystem, therefore, has greater attenuation potential than the bare mat results suggest and would be expected to outperform the unmoored pipe at frequencies where root-zone dissipation is significant. Unlike the moored pipe, whose performance depends critically on precise mooring stiffness tuning to ω_p , the root zone dissipation is broadband and requires no tuning. Moreover, the choice between the two configurations is not governed by attenuation performance alone: the floating ecosystem provides ecological and aesthetic value beyond wave attenuation, a relevant consideration for permits and stakeholder acceptance at inland FPV sites, which the pipe does not offer. The two configurations therefore address different practical constraints, and a hybrid combining a moored pipe with floating ecosystem islands, as recommended in Section 6.8, may provide the best outcome.

The optimally moored pipe at $D = 0.45$ m provides greater attenuation. $R_{load} = 0.467$ means the wave-induced load on the FPV system is reduced by more than half across the full sea state, relative to the case without barrier. $R_{drift} = 0.218$ shows that the mean drift load on the mooring system is approximately one-fifth. This reduction directly affects the anchor specification and mooring line characteristics. For the flex-element connectors and the front row boats, $R_{load}^3 = 0.102$ and $R_{load}^5 = 0.022$ show that the fatigue damage rate is reduced to 10 % and 2 % of the unprotected rate for $n = 3$ and $n = 5$ respectively. Equivalently, the fatigue life of these components is extended by a factor of approximately 10 for $n = 3$ and 50 for $n = 5$ relative to the case without a barrier. These results represent an upper bound on attenuation performance: as stated in Section 5.3, the model overestimates performance by assuming zero mooring damping, thereby neglecting energy dissipation in the mooring lines and anchors. In practice, mooring damping can reduce attenuation relative to the idealised result.

These factors can be used for a cost-benefit analysis. The load history of an FPV without barrier is

multiplied by R_{load} for ULS checks and by R_{load}^n for fatigue life assessment. The mooring line tension is multiplied by R_{drift} to size the anchor and line. The barrier cost is then weighed against the extended fatigue life of the flex-element connectors and front row floats, the potential reduction in structural material and mooring specification driven by the lower design loads, and, where applicable, the yield benefit quantified in Section 6.6.

6.4. Comparison with Willemspolder Design

The mooring design report for the Willemspolder installation identifies the North-West direction (fetch 900 m, $H_s = 0.60$ m, $T_p = 2.3$ – 2.5 s) as the governing load case for the mooring system and recommends a 450 mm diameter pipe wave barrier [30]. The reported T_p range of 2.3–2.5 s corresponds to $\omega = 2.513$ – 2.732 rad/s, which brackets the modelled peak frequency $\omega_p = 2.581$ rad/s, confirming that the frequency sweep used in this comparison is representative of the governing load case identified in the mooring design report. The wave conditions in the present study use $H_s = 0.498$ m for the same fetch, derived from the mean 10-minute wind speed of 19.1 m/s, while the mooring report applies the base wind speed $v_{b,0} = 24.5$ m/s directly to calculate the wave height estimation. The dimensionless load reduction factors derived in Section 6.2 are independent of the absolute wave height; the wave height cancels in the ratio, and the factors therefore scale directly to the project design loads. However, the factors do depend on the mooring stiffness through its effect on $K_T(\omega)$: the values in Table 6.1 correspond to the optimally tuned configuration and will not be achieved if the as-built mooring stiffness differs from the optimal value, as the comparison below shows. The mooring report explicitly identifies fatigue reduction as the primary function of the wave barrier, consistent with the conclusion of Section 6.3 that the dominant design driver is structural.

To assess whether the as-specified mooring achieves the required stiffness, the performance of the design stiffness specified in the mooring design report is compared with that of the optimally moored and unmoored pipe with a diameter of 450 mm. The derivation of the Willemspolder design stiffness is derived from the mooring geometry and is provided in Section B.7. The as-built mooring provides a total heave stiffness of approximately 167 kN/m, roughly 24 % of the 705 kN/m required for resonant tuning to ω_p . The as-specified mooring is therefore substantially softer in heave than the optimally moored case.

To show the attenuation performance with the design mooring characteristics, a frequency sweep was performed for $D = 0.45$ m with the as-built heave stiffness of 1116 N/m/m per unit length. The results are shown in Figure 6.2 alongside the unmoored and optimally moored cases. The as-built mooring is insufficient to shift the pipe resonance to ω_p ; the resonance remains near 1.5 rad/s, where the JON-SWAP spectrum contains little energy. The as-built wave barrier therefore performs near-identically to the unmoored pipe across the energy-carrying frequency range of the spectrum, confirming the stiffness estimate from Section B.7.

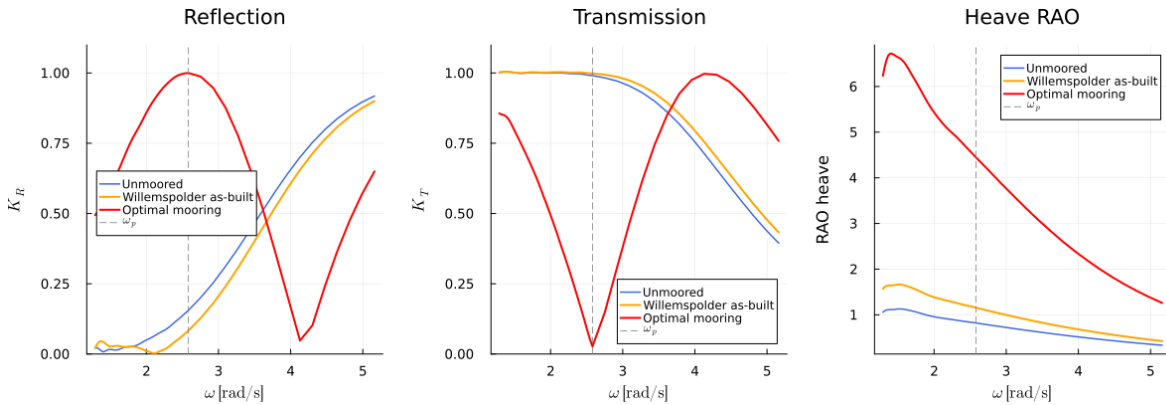


Figure 6.2: Reflection coefficient K_R (left), transmission coefficient K_T (centre), and heave RAO (right) as a function of ω for the unmoored, Willemspolder as-built mooring, and optimally moored configurations, $D = 0.45$ m. The vertical dashed line indicates $\omega_p = 2.581$ rad/s.

The spectrally weighted load reduction of the as-built configuration is $\Delta E_{L,\text{spec}} = 8.4\%$, with $R_{load} = 0.957$, $R_{drift} = 0.916$, $R_{load}^3 = 0.877$, and $R_{load}^5 = 0.804$. This is comparable to the lower-bound configurations in Table 6.1 and well below the optimally moored upper bound. The as-built wave barrier provides no meaningful improvement over an unmoored pipe. The slight decrease relative to the unmoored case reflects the as-built mooring shifting the pipe resonance, but by far too little to reach optimal mooring. Increasing the stiffness without tuning to ω_p does not improve performance; an intermediate stiffness can even result in worse attenuation. To achieve the upper-bound attenuation, the mooring stiffness must be tuned precisely to ω_p .

6.5. Effect on the Energy Yield

A direct prediction of FPV energy yield would require a fully coupled model combining hydrodynamics, time-dependent motion, irradiance transposition, and electrical system behaviour. Such approaches are used in recent studies on WIL [44]. However, the present study employs a frequency-domain hydrodynamic model and does not resolve time-dependent motion or electrical processes. To relate wave attenuation to PV performance, a physics-based, dimensionless performance indicator is introduced. This indicator provides a first-order estimate of the reduction in wave-induced losses, assuming that platform motion scales linearly with wave amplitude and that the associated performance losses scale with the square of the motion amplitude.

The key quantity obtained from the wave barrier model is the transmission coefficient:

$$K_T = \frac{a_t}{a_i} \quad (6.5)$$

which describes the ratio of the transmitted to the incident wave amplitudes.

The relationship between wave amplitude and platform motion is based on the linearity of the present modelling approach. The hydrodynamic problem is formulated using linear wave theory, and the structural response is obtained within a linear frequency-domain framework. Under these assumptions, the governing equations are linear with respect to the wave amplitude.

As a result, the response of the FPV system scales linearly with the excitation. This implies that the FPV platform's motion amplitude is proportional to the local wave amplitude. Consequently, a reduction in wave amplitude due to the floating ecosystem results in a proportional reduction in platform motion.

6.6. Derivation of the Yield Indicator

Physical Context

To interpret the yield indicator derived in the following subsections, the assumptions and scene are first described. An FPV array consists of modules mounted at a fixed tilt θ_{ref} and azimuth A_{ref} , chosen to match the purpose of the produced energy. This could be to maximise energy yield or to provide steady energy throughout the day, for example. Waves cause the platform to oscillate about this reference orientation. The wave-induced motion relevant to irradiance is pitch, induced by waves propagating normally to the module rows, which directly changes the module tilt. Waves propagating from other directions induce roll, which has a smaller effect on the in-plane irradiance and is not considered further.

The effect of the wave-induced oscillations on the yield over time depends on the relationship between the reference orientation and the irradiance optimum. The irradiance optimum is the module orientation that maximises the total in-plane irradiance G_M^{tot} at a given moment, and varies with the position of the sun throughout the day and year. Two configurations illustrate this behaviour.

For a south-facing module at a moderate tilt, it operates near its optimum around solar noon. The irradiance function $G_M^{tot}(\theta_M)$, shown in Equation (6.9), is near its maximum. Oscillation in either direction will reduce the time-averaged in-plane irradiance. Wave-induced motion is a loss in this configuration.

Another commonly used orientation in FPV systems is an east-west orientation of the modules, with low tilt. The low tilt is constrained by wind loads on the structure, and east-west is chosen to distribute the yield more evenly throughout the day. As a result, the modules rarely operate near their irradiance optimum. Wave-induced oscillations in this configuration alternately increase and decrease the in-plane irradiance, and the net time-average effect will be small. It can be a gain or a loss.

A further consideration is the correlation between wave conditions and irradiance. As these are inland water bodies, the largest wave amplitudes occur during storms, which come with clouded skies and low irradiance. Thus, the largest oscillations occur at low irradiance levels, reducing their contribution to the annual energy yield.

The yield indicator derived below consequently provides an upper bound on the annual yield impact of wave attenuation. First, the wave surface slope is used to establish an upper bound on the platform tilt. Second, the sensitivity to tilt and azimuth oscillations is derived analytically for both wave directions. Third, the contribution of the mismatch loss is established. Together, these lead to the yield indicator. To make the indicator concrete, a numerical example is provided in Section 6.7 using the Willemsspolder site conditions.

Wave Slope as Tilt Scale

The platform tilt angle θ_M depends on the hydrodynamic response of the floating structure to the waves. In a complete model, this would be obtained through the RAO, incorporating system-specific factors such as geometry, mass distribution, and wave frequency. Such computation is out of the scope of the present frequency-domain wave barrier model, which does not model the FPV system itself.

Instead, the wave surface slope is used as a reference magnitude for the tilt excitation. Under linear wave theory, the free surface elevation of a wave with amplitude a and wavenumber k is:

$$\eta = a \cos(kx - \omega t). \quad (6.6)$$

Differentiation with respect to x gives the surface slope, which oscillates between $-ka$ and $+ka$ as the wave passes. Thus the relevant quantity is the amplitude of this slope:

$$\theta_0 = ka \quad (6.7)$$

This represents the maximum tilt a small floating structure would experience if it moved entirely with the local wave surface. For real platforms, the actual response is smaller, due to inertia, structural stiffness, and mooring constraints. The maximum tilt θ_0 therefore is the upper bound on the platform amplitude, and serves as the scale of the tilt excitation.

Substituting the transmitted wave amplitude $a_t = K_T \cdot a_i$ into the wave slope expression, the transmitted tilt scale becomes:

$$\theta_t = K_T \cdot ka_i = K_T \cdot \theta_0 \quad (6.8)$$

The barrier, therefore, reduces the tilt in direct proportion to the transmission coefficient, regardless of the specific platform characteristics within the linear wave theory framework adopted here.

Irradiance Sensitivity to Tilt

The total in-plane irradiance received by a PV module is composed of three contributions [69]:

$$G_M^{tot} = G_M^{dir} + G_M^{dif} + G_M^{ground} \quad (6.9)$$

Each component depends on the module's orientation relative to the sun. A schematic of the contributions is shown in Figure 6.3.

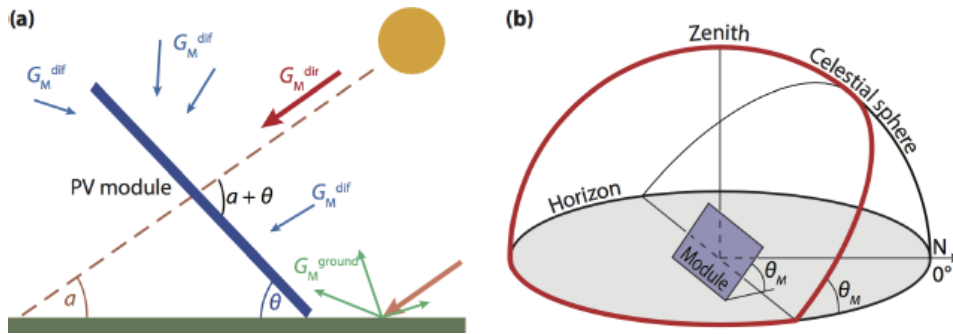


Figure 6.3: (a) The three contributions to the irradiance on a PV module G_M . (b) Illustrating the definition of the sky view factor (SVF), which is the fraction of the celestial hemisphere enclosed by the thick red line. Taken from Smets et al. [69].

The sensitivity of the time-averaged irradiance to tilt oscillations ϵ about θ_{ref} is derived below; the full derivations are provided in Section B.6. For the Willemspolder conditions, the upper-bound tilt is $\theta_0 = 0.169$ rad, at which the small-angle approximation $\cos(\epsilon) \approx 1 - \epsilon^2/2$ is justified.

The diffuse irradiance depends on the sky view factor $SVF = (1 + \cos \theta_M)/2$. For a module oscillating about θ_{ref} with tilt amplitude ϵ , averaging over the two extreme positions and applying the small-angle expansion gives the time-averaged change in diffuse irradiance, where DHI is the diffuse horizontal irradiance:

$$\Delta \bar{G}_M^{dif} = -\frac{DHI \cdot \cos(\theta_{ref})}{4} \cdot \epsilon^2 \propto \epsilon^2 \quad (6.10)$$

The coefficient is negative for any mounting tilt, so tilt oscillation always reduces the time-averaged diffuse irradiance. The ground-reflected component follows by the same argument with opposite sign, but since $DHI > GHI \cdot \alpha$ for open water, the net effect of both is a reduction scaling as ϵ^2 . Here, GHI denotes the global horizontal irradiance.

For the direct beam component, the irradiance depends on the angle of incidence (AOI):

$$AOI = \cos^{-1} [\sin(\theta_M) \cos(a_s) \cos(A_M - A_s) + \cos(\theta_M) \sin(a_s)] \quad (6.11)$$

with a_s the solar altitude and A_M, A_s the module and solar azimuth. A second-order Taylor expansion of $\cos(AOI)$ about θ_{ref} gives a quadratic dependence on ϵ . Near the irradiance optimum, the curvature is negative, and tilt oscillation is a net loss, as discussed in Section 6.6. The total time-averaged irradiance change scales as:

$$\Delta \bar{G}_M^{tot} \propto \epsilon^2 \quad (6.12)$$

Mismatch Losses

In a series-connected PV string, the current is limited by the least-illuminated module. When wave-induced motion causes non-uniform orientations across the array, modules within the same string receive different levels of irradiance, reducing the output of the full string below the average module output. This is referred to as mismatch losses.

The magnitude of this mismatch is governed by the spread in module orientation across the array. For a sinusoidal wave field, the tilt at position x along the array is: $\theta(x) = \theta_M \sin(kx)$. Figure 6.4 illustrates this: as the wave passes beneath the string, modules near a zero-crossing of the surface experience the largest tilt, while modules near a crest or trough are closer to level. The spatial variance of the tilt deviation across the array scales as $\langle \theta^2 \rangle = \theta_M^2/2$. A larger spatial variance indicates a wider spread in module orientations and, therefore, a larger mismatch loss. The mismatch loss therefore scales as:

$$\delta_{\text{mismatch}} \propto \theta_M^2 \quad (6.13)$$

This is the same quadratic scaling as the irradiance loss. Unlike irradiance loss, mismatch loss always reduces the string output regardless of the reference orientation or the sun's position. For the Willemspolder configuration, where modules are mounted at low tilt in an east-west orientation, the mismatch loss is expected to be the dominant contributor to wave-induced yield losses. Mitigating these losses requires string- or module-level measures, as discussed in Section 6.8.

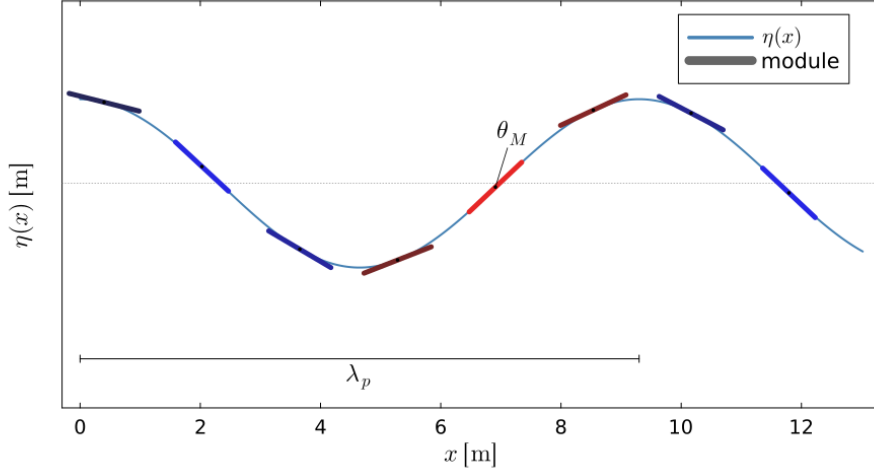


Figure 6.4: Schematic of module tilt variation along a string as a wave of wavelength λ_p passes beneath it. Modules are positioned on the wave surface profile $\eta(x)$ and tilted to match the local surface slope. Tilt is coloured by direction and magnitude (red: positive, blue: negative); the steepest sampled tilt, θ_M , occurs near a zero-crossing of the surface, while modules near a crest or trough are comparatively level. The wavelength λ_p is marked for reference.

Yield Indicator

Under linear wave theory, described in Section 2.3.2, wave energy scales with the square of the wave amplitude:

$$E \propto a^2 \quad (6.14)$$

Which implies that the transmitted wave energy scales as:

$$\frac{E_T}{E_i} \propto K_T^2 \quad (6.15)$$

Both the irradiance loss and the mismatch loss are assumed to scale quadratically with the platform tilt θ_M , and under the linear framework θ_M scales linearly with the local wave amplitude. The total wave-induced loss without wave attenuation L_0 can be described as:

$$L_0 = \beta \cdot \theta_M^2 \propto \beta \cdot a_i^2 \quad (6.16)$$

where β is a dimensionless coefficient that represents the combined sensitivity of the FPV system to the tilt, including both irradiance and mismatch effects; its precise value depends on system-specific factors and is not calculated in the present study. Behind the wave barrier the wave amplitude reduces to $a_t = K_T \cdot a_i$ and since $\theta_m \propto a$ the loss becomes:

$$L_t = \beta \cdot \theta_{M,t}^2 \propto \beta \cdot K_T^2 a_i^2 = K_T^2 \cdot L_0 \quad (6.17)$$

The relative reduction in wave-induced losses is obtained by comparing the loss behind the barrier to the loss without a barrier:

$$\frac{L_t}{L_0} = K_T^2 \quad (6.18)$$

where β and a_i cancel out in this ratio. The dimensionless yield indicator is therefore defined as the fractional reduction in wave-induced losses:

$$I_{\text{yield}} = 1 - \frac{L_t}{L_0} = 1 - K_T^2 \quad (6.19)$$

where a value of $I_{\text{yield}} = 0$ corresponds to no reduction in the wave-induced losses and a value of $I_{\text{yield}} = 1$ corresponds to complete elimination of the wave-induced losses.

6.7. Application to Willemspolder

Numerical Example of Yield Indicator

To illustrate the physical significance of the indicator, an example is worked out for the conditions at the Willemspolder site. Two tilt estimates are compared. The first is the upper bound $\theta_0 = k_p \eta_{0,s} = 0.169$ rad ($= 9.7^\circ$), corresponding to a module that follows the wave surface perfectly. The second is the pitch response of the short elastic beam from Case 1 at ω_p , giving $\theta_M = 0.039$ rad ($= 2.2^\circ$). The beam does not represent the FPV platform, but provides a more realistic estimate of the wave-induced tilt for a small floating structure. The beam pitch is a factor of 4.4 smaller than the upper bound.

The irradiance loss is evaluated for a south-facing module at the PVGIS-recommended optimal tilt of 37° for the Willemspolder site, under clear-sky conditions near solar noon on July 15, 2023, obtained from the PVGIS-SARAH3 database [70]. The in-plane irradiance at the reference orientation is 890 W/m^2 , derived analytically at the reference tilt. To illustrate the effect of wave attenuation, a transmission coefficient of $K_T = 0.7$ is assumed, which represents moderate attenuation.

The results are summarised in Table 6.2. Without a barrier, the upper-bound tilt gives an irradiance loss of 10.9 W/m^2 (1.22%), while the realistic beam response gives 0.57 W/m^2 (0.064%). With the barrier both losses reduce by 51%, consistent with $I_{\text{yield}} = 1 - 0.7^2 = 0.51$. The example confirms two key properties of the indicator: the fractional reduction scales correctly with K_T^2 , and the absolute wave-induced irradiance losses are small under the sheltered conditions at Willemspolder. The 1.22% figure is an upper bound, computed for a south-facing module near its irradiance optimum, assuming the platform follows the wave surface perfectly. For the actual Willemspolder installation, where modules are mounted at a low tilt in an east-west orientation and the platform tilt is smaller than the realistic beam estimate, wave-induced yield losses are negligible and no corrective measures are required.

Table 6.2: Irradiance loss for upper bound and realistic beam tilt, with and without a wave barrier of $K_T = 0.7$. Willemspolder site conditions, near solar noon on July 15, south-facing module at $\theta_{\text{ref}} = 37^\circ$.

Scenario	ϵ [°]	Loss [W/m^2]	Loss [%]	Reduction [%]
Upper bound, no barrier	9.7	10.9	1.22	—
Upper bound, $K_T = 0.7$	6.8	5.3	0.60	51
Realistic beam, no barrier	2.2	0.57	0.064	—
Realistic beam, $K_T = 0.7$	1.6	0.28	0.031	51

Spectral Weighting Method

The yield indicator $I_{\text{yield}} = 1 - K_T^2$ is derived in Section 6.6 for a single frequency. In reality, the wave field at a site contains energy distributed across a range of frequencies, and the wave barrier performs differently at each frequency. To obtain a site-specific estimate that accounts for the full frequency content of the sea state, the indicator is integrated over the JONSWAP spectrum defined in Section 2.3.4. The spectrally weighted load reduction $\Delta E_{L,\text{spec}}$, defined in Equation (6.1), represents the fraction of wave-induced losses eliminated by the barrier, averaged over the full sea state. A value of 0 means the barrier provides no yield benefit and a value of 1 means complete elimination of wave-induced losses across all frequencies.

The results are computed for Cases 1 and 3 using the results from the frequency sweeps from Chapter 5 and are summarised in Table 6.3. For Case 2, reliable extraction of $K_T(\omega)$ across the frequency sweep was not possible due to outlet reflections, as discussed in Section 4.11.3, and $\Delta E_{L,\text{spec}}$ is therefore not reported. Cases 1 and 3 unmoored show similar spectrally weighted performance, consistent with the comparable $K_T(\omega)$ curves. The moored pipe achieves a substantially higher $\Delta E_{L,\text{spec}}$ of 78.2%, due to having near complete attenuation at ω_p , where the JONSWAP spectrum concentrates most of its energy.

Table 6.3: Spectrally weighted yield indicator $\Delta E_{L,spec}$ for Cases 1 and 3 at the Willemspolder site.

Case	Configuration	$\Delta E_{L,spec}$ [%]
Case 1	$L_b = 4.6$ m, SDR17	12.4
Case 3	unmoored, $D = 0.45$ m	10.9
Case 3	moored optimal, $D = 0.45$ m	78.2

Limitations and Assumptions

The yield indicator is derived under several simplifying assumptions. The main ones are discussed below.

The platform motion is assumed to scale linearly with the local wave amplitude. This follows from the linear wave theory framework used throughout this study and is consistent with the wave conditions at Willemspolder. For larger wave amplitudes, non-linear effects would break the derived proportionality.

The wave-induced irradiance loss is assumed to scale quadratically with the tilt amplitude, based on a second-order Taylor expansion of the irradiance function about the reference orientation. Higher terms are neglected, which is justified for the small tilt amplitudes at the Willemspolder. For larger tilt amplitudes, such as those induced by waves in offshore environments, the approximation becomes less accurate.

The indicator assumes normally incident regular waves. The effect of waves coming from different directions or irregular wave conditions is not accounted for.

Whether the wave-induced oscillations cause a yield gain or loss depends on the module orientation relative to the irradiance optimum. For the east-west orientation with low tilt, the modules are not in the irradiance-optimal configuration. The irradiance contribution to the loss is less significant in this orientation, and the mismatch loss is expected to be the dominant loss contributor. The indicator therefore provides a conservative upper bound for this configuration.

The largest wave amplitudes occur during extreme weather, which is associated with overcast skies and low irradiance. The indicator does not account for the correlation and treats all sea states equally. The actual annual yield impact is therefore smaller than the indicator predicts.

Finally, the sensitivity coefficient β is not calculated in this study. Its value depends on system-specific factors, such as location, structure geometry, string length, and module orientation. The indicator gives the fractional loss reduction regardless of β , making it applicable to any linear FPV system. To calculate the absolute yield improvement in energy terms requires knowledge of β .

6.8. Integrated Design Recommendations

The load reduction factors in Table 6.1 provide the range of the expected performance of the Willemspolder barrier. Case 1 and the unmoored Case 3 represent the lower bounds, and the optimally moored Case 3 provides the upper bound. The actual attenuation of a moored wave barrier depends critically on the mooring stiffness relative to the site peak frequency. With proper tuning to ω_p , performance approaches the upper bound; without it, performance can fall at or below the lower bound, as the Willemspolder comparison in Section 6.4 demonstrates. The range is large: R_{load} varies from 0.936 to 0.467 and R_{load}^5 from 0.718 to 0.022. At the lower bound, it provides limited structural benefit. At the upper bound, the reduction is substantial, with drift loads reduced to one-fifth and fatigue life extended by a factor of 10 to 50. If the barrier is economically justified at the lower bound, the investment remains viable across the performance range, provided the mooring is tuned to ω_p . Without proper tuning, performance can fall below the lower bound, and the barrier provides no meaningful benefit over the unmoored case. If only justified at the upper bound, a project-specific dynamic model with calibrated mooring stiffness and damping is needed before making a final investment decision.

These factors translate directly into design inputs for the project-specific cost-benefit analysis. The unshielded load data from the project assessment is scaled by R_{load} for the ULS checks, by R_{load}^n for the fatigue assessment, and by R_{drift} for the anchor and mooring line sizing. The dominant design

driver is structural: a fatigue life extension by a factor of 10 to 50 has a larger impact than eliminating wave-induced yield losses that are small in absolute terms at this site, as shown in Section 6.7, and should be the primary argument in the cost-benefit analysis.

For the moored wave barrier, the mooring stiffness should be tuned to the peak frequency ω_p of the site-specific wave spectrum through a stiffness sweep in the numerical model. The analytical formula based on the undamped natural frequency is unreliable due to frequency-dependent added mass, so empirical minimisation of K_T at ω_p is the recommended approach. The optimal stiffness ratio $k_{heave}/K_{hs,11}$ falls in the range 1.03–1.15 across all diameters studied and provides a practical starting point. Mooring damping is not included in the present model but should be included in the final mooring design.

A hybrid configuration combining a moored pipe with floating ecosystem islands could provide an optimal outcome. The pipe provides the dominant structural attenuation at ω_p at relatively low material cost, while the floating ecosystem provides additional dissipation and adds ecological and aesthetic value. This is a relevant consideration for permits and stakeholder acceptance for inland FPV systems.

Mismatch losses, identified in Section 6.6 as the dominant wave-induced loss for the east-west, low-tilt Willemspolder configuration, can be mitigated independently of the structural measures above. The relevant length scale is the wavelength: $\lambda_p = 2\pi/k_p \approx 9.3$ m at Willemspolder. To reduce the spread in tilt within a string, strings should span a small fraction of λ_p and therefore keep δ_{mismatch} low. The downside of shorter strings is the increase in the number of inverters or combiner boxes required. An alternative to shorter strings is module-level power optimisers: a small DC-DC converter mounted on each module lets that module operate at its own maximum power point, rather than being limited by the weakest module in the string. This does increase the cost and maintenance of the FPV.

6.9. Chapter Summary

This chapter has addressed SQ3 and completed SQ2 by translating the structural response in Chapter 5 into the resulting reduction in wave-induced loads, with implications for FPV energy yield and structural design. For Cases 1 and 3, the spectrally weighted load reduction factors show that the unmoored bare ecosystem and unmoored pipe provide limited structural benefit, with wave-induced loads reduced by less than 7 % and fatigue damage rates by less than 18 % for $n = 3$. The optimally moored pipe provides substantially greater attenuation, reducing drift loads by a factor of 5 and extending fatigue life by a factor of 10 to 50. The practical performance depends critically on mooring tuning: with the mooring stiffness tuned to ω_p , performance approaches the upper bound; without it, performance can fall at or below the lower bound, as shown for the Willemspolder as-built configuration in Section 6.4. The bare floating ecosystem and unmoored pipe represent a lower bound; a mature ecosystem with root zone is expected to outperform this bound through porous dissipation, which acts independently of the mooring configuration and requires no frequency tuning. The impact on energy yield is small in absolute terms at the Willemspolder site. The 1.22 % upper-bound irradiance loss assumes a south-facing module at optimal tilt at noon, following the wave surface perfectly. For the east-west-oriented, low-tilt installation at Willemspolder, wave-induced yield losses are negligible, and no corrective measures are required. The dominant design driver is therefore structural, and the extension of fatigue life is the primary argument in a cost-benefit analysis. The central research question is addressed in Chapter 7.

Conclusions

This chapter answers the research questions defined in Chapter 3. The central research question is repeated below.

How does the integration of floating ecosystems influence wave–structure interaction, structural response, and energy yield of floating photovoltaic systems in inland water bodies?

Section 7.1 answers SQ1, Section 7.2 answers SQ2, and Section 7.3 answers SQ3. Section 7.4 combines these answers into a conclusion to the main research question.

7.1. Wave Transmission, Dissipation, and Reflection

SQ1: What is the effect of different wave barrier representations on wave transmission, dissipation, and reflection?

Effectiveness depends mainly on the active attenuation mechanism, not the representation itself. The unmoored bare ecosystem (Case 1) and the unmoored pipe (Case 3) are both too small relative to the peak wavelength λ_p to obstruct the wave at ω_p , giving load reductions at or below 6 % and 3 % respectively, worsening further at lower frequencies. Adding a submerged porous root zone (Case 2) introduces dissipation independent of structural size, governed instead by root density, diameter, and depth; outlet reflections limited the results to qualitative, but a visible reduction in transmitted amplitude is observed at moderate resistance, with significant further improvement expected for a mature root zone. Moored with stiffness tuned to ω_p , the pipe achieves a load reduction of 99.9 % through resonant scattering, the strongest attenuation observed, though narrow-band and an idealised upper bound given the assumed absence of mooring damping.

SQ1 is answered as follows: elastic scattering and diffraction are both insufficient at the site peak frequency. Porous dissipation, requiring no frequency tuning, is the most promising mechanism studied. Resonant scattering achieves the strongest attenuation, but only when the pipe is moored and precisely tuned.

7.2. Structural Response and Load Reduction

SQ2: What is the structural response of each wave barrier representation and the resulting reduction in wave-induced loads on an adjacent FPV system?

The elastic representations (Cases 1 and 2) deflect nearly rigidly at short beam lengths and exhibit clear elastic bending with multiple nodes at longer lengths; the porous root zone reduces deflection and bending-moment amplitudes without altering this spatial pattern. The rigid pipe (Case 3) responds through heave, achieving near-total reflection when moored and tuned to ω_p , though the assumed absence of mooring damping makes this an upper bound rather than a realisable target.

These responses translate into wave-induced load reductions on the FPV system. For the unmoored bare ecosystem and unmoored pipe, the reduction is limited: wave-induced loads are reduced by less than 7 %, drift loads by less than 11 %, and fatigue damage rates by less than 18 % for $n = 3$. The optimally moored pipe reduces drift loads by roughly one-fifth and extends fatigue life by a factor of

10 to 50, but only when the mooring stiffness is tuned to the site's peak frequency; the Willemsspolder comparison shows that an as-built stiffness well below this requirement instead performs close to the lower bound.

SQ2 is answered as follows: without mooring tuning, the wave barrier provides no meaningful reduction in wave-induced or drift loads; with tuning, the upper-bound benefit established above is achieved.

7.3. Implications for Energy Yield and Structural Design

SQ3: What are the implications of the wave-attenuation performance for the energy yield and structural design of an FPV system?

The impact of wave attenuation on energy yield is small in absolute terms at Willemsspolder. The 1.22 % figure is an upper bound, assuming a south-facing module at its irradiance optimum and following the wave surface perfectly; a more realistic estimate, using the beam pitch response from Case 1, gives 0.064 %. For the east-west, low-tilt orientation actually used at Willemsspolder, this loss is negligible. Mismatch losses, which reduce string output regardless of module orientation, are expected to be the dominant contributor to wave-induced yield losses for this configuration.

The structural implications are larger. The load reduction established in addressing SQ2 translates into a substantial fatigue life extension at the upper bound, against a negligible yield benefit. The dominant design driver is therefore structural rather than energy yield, and fatigue life extension should be the primary argument in a cost-benefit analysis. This benefit is achieved only when the moored pipe's stiffness is precisely tuned to the site's peak frequency. The Willemsspolder comparison shows that an as-built mooring falls short of this requirement and instead performs close to the lower bound, with no meaningful fatigue benefit.

SQ3 is answered as follows: wave attenuation reduces structural loads and energy yield losses by comparable proportions, but the absolute yield loss at Willemsspolder is already small. In contrast, the structural benefit at the upper bound is substantial. The dominant design driver is therefore structural, not energy yield.

7.4. Answer to the Main Research Question

The moored pipe demonstrates that strong attenuation is achievable in this setting through resonant scattering, but only when the mooring stiffness is precisely tuned to the site's peak frequency; without tuning, performance reverts to the lower bound. The bare floating ecosystem, in its initial configuration, performs comparably to the unmoored pipe over the full frequency range studied: both are too small relative to the peak wavelength to scatter the incident wave, and neither meaningfully obstructs it. Since the bare ecosystem and the unmoored pipe behave equivalently, a correctly moored floating ecosystem could achieve a similar structural benefit to that of the moored pipe, while a mature ecosystem adds further attenuation through the porous root zone, a broadband mechanism that requires no frequency tuning.

The resulting structural benefit ranges accordingly: negligible at the lower bound, but a fatigue life extension by a factor of 10 to 50 and a reduction of drift loads to roughly one-fifth at the upper bound, achieved by the pipe when moored and precisely tuned to the site's peak frequency. The energy yield loss remains negligible in absolute terms for the east-west, low-tilt module orientation used at Willemsspolder. Integration of a floating ecosystem with FPV systems therefore offers a structural rather than an energetic benefit, with the magnitude of that benefit depending on the ecosystem's maturity. Using a floating ecosystem as the wave barrier additionally provides ecological and aesthetic enhancement beyond this structural benefit.

Recommendations

This chapter presents recommendations following from the limitations and findings of this study. Section 8.1 recommends directions for future research. Section 8.2 translates the findings of this study into practical recommendations for Solinoor.

8.1. Recommendations for Future Research

This section recommends directions for future research arising from the model assumptions and scope limitations of this study.

- Resolve the outlet-reflection contamination not fully suppressed by the Case 2 damping zone formulation (Section 4.11.3) to enable reliable quantitative extraction of $K_T(\omega)$ and $K_R(\omega)$, rather than the qualitative assessment presented in this study. On top of that, the irregularity in the free surface elevation κ_h (Figure 5.7) should also be investigated.
- Calibrate the porous resistance parameter α_p against physical measurements of a mature water-plant root zone. Stem density N , root diameter, and root zone depth were not available as field data in this study and were therefore estimated rather than measured.
- Examine the sensitivity of the Case 2 wave attenuation performance to the root zone depth d_r , varying around the base case of $d_r = 0.75$ m adopted in this study.
- Include mooring damping in the Case 3 model to obtain a more realistic upper bound on attenuation performance, rather than the present undamped result.
- Investigate the effect of a non-zero pitch mooring stiffness on Case 3 attenuation performance, currently set to zero to simplify the mooring optimisation.
- Validate the Case 3 rigid-body formulation against an independent benchmark, using the fixed-cylinder tests of Rossell et al. [42]. This requires only a rigidly restrained boundary condition on Γ_p within the existing diameter and frequency sweeps, compared against D/λ rather than absolute scale, since the present validation (Section 4.11.1) confirms only the shared fluid formulation via the Case 1 beam benchmark and not the rigid-body dynamics independently.
- Validate the Case 1 and Case 2 structural and porous-resistance representations against a scaled physical model in a wave flume, measuring wave transmission, reflection, structural deflection, and root-zone damping directly. The present validation (Section 4.11.1) confirms the fluid-structure coupling formulation against a generic hydroelastic beam benchmark, not the specific floating ecosystem and root-zone representation studied here.
- Investigate the effect of mooring on the floating ecosystem (Cases 1 and 2). Whether mooring could improve attenuation for the floating ecosystem, as it does for the rigid pipe, remains to be established.
- Extend the model to irregular waves and assess the spectral attenuation performance under a realistic sea state, rather than a single-frequency sweep.
- Investigate a hybrid configuration combining the moored pipe with floating ecosystem islands, assessing the combined attenuation across the full JONSWAP spectrum.

- Conduct a project-specific dynamic model of the FPV system with calibrated mooring stiffness and damping, to narrow the range between the lower- and upper-bound load reduction factors presented here.
- Investigate the long-term degradation of the floating ecosystem under sustained wave loading and environmental exposure, including biofouling, UV degradation of the HDPE beam elements, and changes in root-zone density and stiffness over the structure's service life, paralleling the fatigue-life consideration already applied to the FPV flex connectors in Section 6.3.
- Perform *in situ* mismatch loss measurements at an operating FPV park, to determine whether mitigation measures such as shorter strings or module-level power optimisers are cost-justified.
- Develop a fully coupled hydrodynamic-motion-irradiance-electrical model to obtain an absolute energy yield prediction, resolving the sensitivity coefficient β left uncalculated by the dimensionless yield indicator used in this study.

8.2. Recommendations for Solinoor

This section provides practical recommendations for Solinoor, building on the project-specific cost-benefit recipe already established in Section 6.3 and the integrated design recommendations of Section 6.8.

Wave barrier mooring recommendations from the mooring contractor currently arrive without visibility into the underlying derivation. The Willemspolder case shows the consequence: reconstructing the as-built stiffness from the mooring geometry (Section B.7) gives a heave stiffness of 167 kN/m against the 705 kN/m optimal value for resonant tuning, a shortfall not visible from the contractor's recommendation itself. The underlying derivation should be requested as standard practice for any project where wave-barrier performance is part of the design case.

In client and stakeholder conversations, fatigue life extension should lead, not energy yield. The yield benefit quantified in Section 6.7 is negligible at sites like Willemspolder. The fatigue life extension of Section 6.3 is the stronger argument.

The factors R_{load} , R_{drift} , and R_{load}^n derived in this study are dimensionless ratios, not absolute fatigue lives. Converting the upper-bound fatigue case above into a number that can be acted on requires S-N data for the relevant flex connector, available from the FPV system provider. In collaboration with the FPV system provider, the exact fatigue life extension could be derived from the factors above to complete the cost-benefit case. With the underlying mooring derivation, the load reduction factors above, and the material data from the system provider, Solinoor has the inputs needed for a complete project-specific cost-benefit analysis.

Claims about the floating ecosystem's attenuation performance should distinguish the qualitative trend from a quantitative figure. The bare-mat results of Case 1 are the only quantitatively supported figures; the root-zone dissipation mechanism in Case 2 is established qualitatively but was not reliably quantified in this study, as discussed in Section 4.11.3. The qualitative trend, that attenuation improves as the root system matures, is supported and can be stated. A specific performance number for a mature ecosystem is not yet supported and should wait for the pilot data discussed below.

A pilot installation at an existing FPV park, placing a small number of floating ecosystem islands and monitoring wave attenuation as the root system matures, could provide the quantitative figure the claim above is missing. This would also directly support the α_p calibration recommended in Section 8.1, providing the stem density, root diameter, and attenuation measurements this study could not obtain.

The hybrid configuration should be positioned as more than an attenuation strategy. Combining a moored pipe with floating ecosystem islands, as discussed in Section 6.8, adds ecological and aesthetic value that addresses permitting and stakeholder acceptance at inland sites in a way the pipe alone cannot, independent of any attenuation benefit. This positioning may be more valuable in practice than the attenuation gain itself.

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Non Technical Summary

Floating solar parks place solar panels on water rather than on land. This is attractive in places where land is scarce, such as the Netherlands. Waves are a problem for floating solar parks. They push and rock the panels, which causes wear on the structure and the mooring lines. Waves can also reduce the amount of electricity the panels generate, since a panels tilted away from the sun captures less sunlight.

A wave barrier placed along the solar park can reduce this wave action. Conventional barriers are purely functional. They add no ecological or visual value to the water body and focus on attenuating waves. A common example is a simple HDPE pipe, floating along the park. This matters for permits and public acceptance, especially for solar parks built on inland lakes and canals.

A floating ecosystem is a different option. This is a vegetated mat with roots hanging into the water, also called a floating wetland. Once mature, it has fully developed vegetation above the waterline and a root system below. Floating ecosystems are already used to improve water quality and create habitat for birds and fish. This thesis investigates whether a floating ecosystem could also serve as a wave barrier, thereby providing a dual function for the adjacent floating solar park. A simple HDPE pipe was studied for comparison.

Three configurations were modelled. The first two represent the same floating ecosystem at different points in its life: a freshly placed mat with no roots yet developed, and the same structure once a mature root system has grown beneath it. This root system resists water flow and dissipates wave energy. The third is the reference pipe, modelled both with and without a mooring line. Each configuration was tested under wave conditions representative of a Dutch inland water body. The resulting wave reduction was translated into load reductions on a nearby floating solar park and into the effect on its energy yield.

The pipe becomes a much stronger barrier once its mooring line is tuned to the right stiffness for the local wave conditions. This can cut wave-induced loads by roughly half, and extend the fatigue life of the panels' connectors and mooring lines by a factor of ten or more. This tuning has to be precise: at the real Willemspolder site studied in this thesis, the installed mooring was only about a quarter as stiff as the optimal value and, as a result, performed no better than if it had no mooring at all. The freshly placed floating ecosystem and the unmoored pipe both have a more limited effect, reducing wave-induced loads only modestly.

The mature floating ecosystem showed that its root system can dissipate wave energy on its own, without needing any mooring tuning. This result is qualitative rather than a confirmed number, since the simulations of this case could not be fully resolved. The effect on solar energy output was small across all configurations studied. At this site, the panels lose more from structural wear and risk than from reduced sunlight capture. The main benefit of a wave barrier is therefore to protect the structure, not to increase energy yield.

Combining the two barrier types may offer the best outcome in practice. A moored pipe can provide the bulk of the wave reduction. Floating ecosystem islands can add the ecological and aesthetic value a plain pipe cannot offer. The findings of this thesis support further research into this combination, more precise mooring design, and field measurements in an actual floating ecosystem before drawing firm conclusions about its performance as a stand-alone wave barrier.

B

Derivations

B.1. Derivation of the Euler-Bernoulli Beam Equation

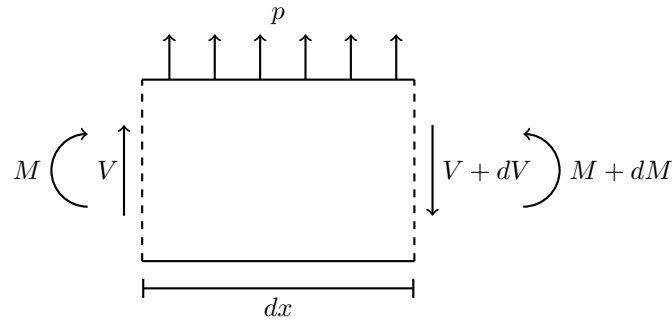


Figure B.1: Free body diagram of an infinitesimal beam element under distributed load $p(x, t)$.

The vertical force equilibrium on a differential beam element yields

$$\frac{\partial V}{\partial x} + p(x, t) = \rho_b h_b \eta_{tt}(x, t), \quad (\text{B.1})$$

where V is the shear force, p is the distributed wave load, and $\rho_b h_b$ is the mass per unit length of the beam.

The moment equilibrium on the same element gives the classical relation between shear force and bending moment,

$$V = \frac{\partial M}{\partial x}. \quad (\text{B.2})$$

Differentiating (B.2) with respect to x yields

$$\frac{\partial V}{\partial x} = \frac{\partial^2 M}{\partial x^2} = M_{xx}. \quad (\text{B.3})$$

Substituting (B.3) into (B.1) gives

$$M_{xx} + p = \rho_b h_b \eta_{tt}. \quad (\text{B.4})$$

Next, the bending moment is related to the deflection via the Euler-Bernoulli constitutive law. For small slopes, the curvature is approximated as $\kappa \approx \eta_{xx}$, and the bending moment becomes

$$M = -EI \eta_{xx}, \quad (\text{B.5})$$

where EI is the bending rigidity of the beam.

Differentiating (B.5) twice with respect to x ,

$$M_{xx} = -EI \eta_{xxxx}. \quad (\text{B.6})$$

Finally, substituting (B.6) into (B.4) yields the dynamic Euler-Bernoulli beam equation,

$$\rho_b h_b \eta_{tt} + EI \eta_{xxxx} = p(x, t). \quad (\text{B.7})$$

This is the form used in hydroelasticity, where p is the hydrodynamic pressure acting on the beam.

B.2. Weak form derivation of Case 1

The weak form of the fluid problem is derived following the monolithic formulation of Colomés et al. [46]. Multiplying Equation (4.3) by the scalar test function w and integrating over Ω gives

$$\int_{\Omega} w \Delta \phi \, d\Omega = 0. \quad (\text{B.8})$$

Integration by parts gives

$$\int_{\Omega} w \Delta \phi \, d\Omega = - \int_{\Omega} \nabla w \cdot \nabla \phi \, d\Omega + \int_{\partial\Omega} w (\mathbf{n} \cdot \nabla \phi) \, d\Gamma. \quad (\text{B.9})$$

Substituting the first equation gives

$$\int_{\Omega} \nabla w \cdot \nabla \phi \, d\Omega = \int_{\partial\Omega} w (\mathbf{n} \cdot \nabla \phi) \, d\Gamma. \quad (\text{B.10})$$

Since $\partial\Omega = \Gamma_b \cup \Gamma_{in} \cup \Gamma_{out} \cup \Gamma_{fs} \cup \Gamma_{str}$ and using the boundary conditions Equation (4.4), the boundary integral can be split as

$$\int_{\partial\Omega} w (\mathbf{n} \cdot \nabla \phi) \, d\Gamma = \int_{\Gamma_{in}} w u_{in} \, d\Gamma + \int_{\Gamma_{out}} w u_{out} \, d\Gamma + \int_{\Gamma_{fs}} w \kappa_t \, d\Gamma + \int_{\Gamma_{str}} w \eta_t \, d\Gamma. \quad (\text{B.11})$$

Rearranging gives the fluid weak form:

$$\int_{\Omega} \nabla w \cdot \nabla \phi \, d\Omega - \int_{\Gamma_{fs}} w \kappa_t \, d\Gamma - \int_{\Gamma_{str}} w \eta_t \, d\Gamma = \int_{\Gamma_{in}} w u_{in} \, d\Gamma + \int_{\Gamma_{out}} w u_{out} \, d\Gamma. \quad (\text{B.12})$$

Multiplying the free-surface condition Equation (4.7) by test function s and integrating over Γ_{fs} gives

$$\int_{\Gamma_{fs}} s (\phi_t + g\kappa) \, d\Gamma = 0. \quad (\text{B.13})$$

For the structural equation, applying integration by parts twice to the fourth-order bending term along the beam coordinate x gives

$$\int_{x_i}^{x_{i+1}} v \eta_{xxxx} \, dx = [v \eta_{xxx}]_{x_i}^{x_{i+1}} - \int_{x_i}^{x_{i+1}} v_x \eta_{xxx} \, dx, \quad (\text{B.14a})$$

$$= [v \eta_{xxx} - v_x \eta_{xx}]_{x_i}^{x_{i+1}} + \int_{x_i}^{x_{i+1}} v_{xx} \eta_{xx} \, dx. \quad (\text{B.14b})$$

Applying the free-end boundary conditions Equations (4.16a) and (4.16b) eliminates the boundary terms, yielding

$$\int_{\Gamma_{str}} v \eta_{xxxx} \, dx = \int_{\Gamma_{str}} v_{xx} \eta_{xx} \, dx. \quad (\text{B.15})$$

Thus, the weak form of the structural equation for Case 1 is

$$\int_{\Gamma_{str}} v (\rho_b h_b \eta_{tt} + \rho_w \phi_t + \rho_w g \eta) \, d\Gamma + EI \int_{\Gamma_{str}} v_{xx} \eta_{xx} \, d\Gamma = 0. \quad (\text{B.16})$$

Applying the frequency-domain substitutions $(\cdot)_t \rightarrow -i\omega(\cdot)$ and $(\cdot)_{tt} \rightarrow -\omega^2(\cdot)$, as introduced in Section 4.3.2, yields the following frequency-domain weak form, which is implemented in the discrete formulation in Section 4.5.1. Combining the fluid weak form, the free-surface condition, and the structural weak form, the monolithic weak problem for Case 1 reads: find $[\phi, \eta, \kappa] \in \mathcal{V}_\Omega \times \mathcal{V}_{\Gamma_{\text{str}}} \times \mathcal{V}_{\Gamma_{\text{fs}}}$ such that

$$B_\omega^{(1)}([\phi, \eta, \kappa], [w, v, s]) = L^{(1)}([w, v, s]) \quad \forall [w, v, s] \in \mathcal{V}_\Omega \times \mathcal{V}_{\Gamma_{\text{str}}} \times \mathcal{V}_{\Gamma_{\text{fs}}}, \quad (\text{B.17})$$

where the bilinear form is given by

$$\begin{aligned} B_\omega^{(1)}([\phi, \eta, \kappa], [w, v, s]) = & \int_\Omega \nabla w \cdot \nabla \phi \, d\Omega + i\omega \int_{\Gamma_{\text{fs}}} w \kappa \, d\Gamma + i\omega \int_{\Gamma_{\text{str}}} w \eta \, d\Gamma \\ & + \int_{\Gamma_{\text{fs}}} s (-i\omega \phi + g\kappa) \, d\Gamma \\ & + \int_{\Gamma_{\text{str}}} v (-\omega^2 \rho_b h_b \eta - i\omega \rho_w \phi + \rho_w g \eta) \, d\Gamma + EI \int_{\Gamma_{\text{str}}} v_{xx} \eta_{xx} \, d\Gamma, \end{aligned} \quad (\text{B.18})$$

and the linear form is

$$L^{(1)}([w, v, s]) = \int_{\Gamma_{\text{in}}} w u_{\text{in}} \, d\Gamma + \int_{\Gamma_{\text{out}}} w u_{\text{out}} \, d\Gamma. \quad (\text{B.19})$$

B.3. Case 2: Velocity-Pressure Weak Form

In Case 2, the fluid is described by the velocity field $\mathbf{u}$ and the dynamic pressure p , rather than a velocity potential. The porous root layer is represented by a volumetric linear Darcy resistance term in the momentum equation. The structural unknown remains the beam displacement η , and the free-surface elevation is denoted by κ on Γ_{fs} .

B.3.1. From Navier-Stokes to the linearised momentum equation

The starting point is the incompressible Navier–Stokes equations:

$$\rho_w \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p_{tot} + \mu_w \Delta \mathbf{u} + \rho_w \mathbf{g} + \mathbf{f}_{drag}, \quad \nabla \cdot \mathbf{u} = 0. \quad (\text{B.20})$$

The total pressure is decomposed into a hydrostatic and dynamic part, $p_{tot} = p_h + p$, where $\nabla p_h = \rho_w \mathbf{g}$. Substituting this relation eliminates the gravity body force from the momentum equation, with gravity instead entering through the boundary conditions: the free-surface dynamic condition $p = \rho_w g \kappa$ on Γ_{fs} and the hydrostatic restoring term $\rho_w g \eta$ in the beam equation on Γ_{str} . In what follows, p denotes the dynamic pressure only.

In accordance with linear wave theory, the convective acceleration term $(\mathbf{u} \cdot \nabla) \mathbf{u}$ and the viscous term $\mu_w \Delta \mathbf{u}$ are neglected for small-amplitude wave motion. The porous drag force is modelled as a linear Darcy resistance $\mathbf{f}_{drag} = -\rho_w \alpha(\mathbf{x}) \mathbf{u}$. This yields the linearised momentum equation:

$$\rho_w \mathbf{u}_t + \nabla p + \rho_w \alpha(\mathbf{x}) \mathbf{u} = \mathbf{0}, \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega. \quad (\text{B.21})$$

Assuming harmonic time dependence $e^{-i\omega t}$ yields the frequency-domain form:

$$-i\omega \rho_w \mathbf{u} + \nabla p + \rho_w \alpha(\mathbf{x}) \mathbf{u} = \mathbf{0}, \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega. \quad (\text{B.22})$$

B.3.2. Strong form

Let the fluid domain be decomposed as $\Omega = \Omega_w \cup \Omega_p$, where Ω_w denotes the clear-water region and Ω_p the porous region. The porous resistance coefficient is defined as:

$$\alpha(\mathbf{x}) = \begin{cases} \alpha_p, & \mathbf{x} \in \Omega_p, \\ 0, & \mathbf{x} \in \Omega_w. \end{cases} \quad (\text{B.23})$$

The free-surface conditions are:

$$\mathbf{u} \cdot \mathbf{n} = \kappa_t, \quad p + \rho_w g \kappa = 0 \quad \text{on } \Gamma_{fs}. \quad (\text{B.24})$$

The kinematic coupling condition at the structure boundary is:

$$\mathbf{u} \cdot \mathbf{n} = \eta_t \quad \text{on } \Gamma_{str}. \quad (\text{B.25})$$

The Euler–Bernoulli beam equation in the time domain reads:

$$\rho_b h_b \eta_{tt} + \rho_w g \eta + EI \eta_{xxxx} = p \quad \text{on } \Gamma_{str}. \quad (\text{B.26})$$

B.3.3. Weak form of the momentum equation

Multiplying the momentum equation by test function $\mathbf{w}$ and integrating over Ω , then integrating the pressure-gradient term by parts, gives:

$$\int_{\Omega} \mathbf{w} \cdot \mathbf{u}_t \, d\Omega - \int_{\Omega} (\nabla \cdot \mathbf{w}) p \, d\Omega + \rho_w \alpha_p \int_{\Omega_p} \mathbf{w} \cdot \mathbf{u} \, d\Omega + \int_{\partial\Omega} p(\mathbf{w} \cdot \mathbf{n}) \, d\Gamma = 0. \quad (\text{B.27})$$

B.3.4. Weak form of the continuity equation

Multiplying the continuity equation by pressure test function q and integrating over Ω gives:

$$\int_{\Omega} q (\nabla \cdot \mathbf{u}) \, d\Omega = 0. \quad (\text{B.28})$$

B.3.5. Boundary terms and free-surface conditions

The boundary integral is split over Γ_{fs} , Γ_{str} , Γ_b , Γ_{in} , and Γ_{out} . The seabed term vanishes by impermeability. On the free surface, substituting $p = -\rho_w g \kappa$ gives:

$$\int_{\Gamma_{\text{fs}}} p(\mathbf{w} \cdot \mathbf{n}) \, d\Gamma = -\rho_w g \int_{\Gamma_{\text{fs}}} \kappa(\mathbf{w} \cdot \mathbf{n}) \, d\Gamma. \quad (\text{B.29})$$

The free-surface kinematic condition is imposed weakly:

$$\int_{\Gamma_{\text{fs}}} s(\mathbf{u} \cdot \mathbf{n} - \kappa_t) \, d\Gamma = 0. \quad (\text{B.30})$$

The structure kinematic coupling is imposed weakly:

$$\int_{\Gamma_{\text{str}}} v(\mathbf{u} \cdot \mathbf{n} - \eta_t) \, d\Gamma = 0. \quad (\text{B.31})$$

B.3.6. Weak form of the beam equation

Multiplying the beam equation by v and integrating over Γ_{str} , then applying integration by parts twice with the free-end boundary conditions Equations (4.16a) and (4.16b), gives:

$$\int_{\Gamma_{\text{str}}} v(\rho_b h_b \eta_{tt} + \rho_w g \eta - p) \, d\Gamma + EI \int_{\Gamma_{\text{str}}} v_{xx} \eta_{xx} \, d\Gamma = 0. \quad (\text{B.32})$$

B.3.7. Assembled frequency-domain weak form

Collecting all contributions and applying the frequency-domain substitutions $(\cdot)_t \rightarrow -i\omega(\cdot)$ and $(\cdot)_{tt} \rightarrow -\omega^2(\cdot)$, as introduced in Section 4.3.2, the monolithic frequency-domain weak problem reads: find $[\mathbf{u}, p, \kappa, \eta] \in \mathcal{V}_{\Omega} \times \mathcal{Q}_{\Omega} \times \mathcal{V}_{\Gamma_{\text{fs}}} \times \mathcal{V}_{\Gamma_{\text{str}}}$ such that $B_{\omega}^{(2)} = L^{(2)} \forall [\mathbf{w}, q, s, v]$, where:

$$\begin{aligned} B_{\omega}^{(2)} = & -i\omega\rho_w \int_{\Omega} \mathbf{w} \cdot \mathbf{u} \, d\Omega - \int_{\Omega} (\nabla \cdot \mathbf{w}) p \, d\Omega + \int_{\Omega} q (\nabla \cdot \mathbf{u}) \, d\Omega \\ & + \rho_w \alpha_p \int_{\Omega_p} \mathbf{w} \cdot \mathbf{u} \, d\Omega - \rho_w g \int_{\Gamma_{\text{fs}}} \kappa(\mathbf{w} \cdot \mathbf{n}) \, d\Gamma \\ & + \int_{\Gamma_{\text{str}}} p(\mathbf{w} \cdot \mathbf{n}) \, d\Gamma + \int_{\Gamma_{\text{fs}}} s(\mathbf{u} \cdot \mathbf{n} + i\omega\kappa) \, d\Gamma \\ & + \int_{\Gamma_{\text{str}}} v(\mathbf{u} \cdot \mathbf{n} + i\omega\eta - \omega^2\rho_b h_b \eta + \rho_w g \eta - p) \, d\Gamma \\ & + EI \int_{\Gamma_{\text{str}}} v_{xx} \eta_{xx} \, d\Gamma, \end{aligned} \quad (\text{B.33})$$

and the linear form is:

$$L^{(2)} = \int_{\Gamma_{\text{in}}} (\mathbf{w} \cdot \mathbf{n}) p_{\text{in}} \, d\Gamma. \quad (\text{B.34})$$

B.3.8. Assembled time-domain weak form

The time-domain weak form, which is the formulation implemented in the present model, retains the time derivatives directly:

$$\begin{aligned}
 B^{(2)} = & \int_{\Omega} \mathbf{w} \cdot \mathbf{u}_t \, d\Omega - \int_{\Omega} (\nabla \cdot \mathbf{w}) p \, d\Omega + \int_{\Omega} q (\nabla \cdot \mathbf{u}) \, d\Omega \\
 & + \rho_w \alpha_p \int_{\Omega_p} \mathbf{w} \cdot \mathbf{u} \, d\Omega + g \int_{\Gamma_{fs}} \kappa (\mathbf{w} \cdot \mathbf{n}) \, d\Gamma \\
 & + \int_{\Gamma_{str}} p (\mathbf{w} \cdot \mathbf{n}) \, d\Gamma + g \int_{\Gamma_{fs}} s (\kappa_t - \mathbf{u} \cdot \mathbf{n}) \, d\Gamma \\
 & + \int_{\Gamma_{str}} v \left(\frac{\rho_b h_b}{\rho_w} \eta_{tt} + g\eta - p \right) \, d\Gamma \\
 & + \frac{EI}{\rho_w} \int_{\Gamma_{str}} v_{xx} \eta_{xx} \, d\Gamma,
 \end{aligned} \tag{B.35}$$

and the linear form is:

$$L^{(2)} = \int_{\Gamma_{in}} (\mathbf{w} \cdot \mathbf{n}) p_{in} \, d\Gamma. \tag{B.36}$$

The frequency-domain weak form derived above was not implemented in the final model. A reliable solution could not be achieved due to limitations of the relaxation-based damping zone method in the velocity–pressure setting, as discussed in Section 4.7. The C/DG interior penalty terms and Nitsche kinematic coupling terms are added in the discrete formulation presented in Section 4.6.1.

B.4. Derivation of the weak rigid-body equation via virtual work

This appendix derives the weak (variational) form of the rigid-body equations of motion used for the floating pipe in Case 3. The derivation is based on the principle of virtual work. For any kinematically admissible virtual displacement, the virtual work of external forces equals the virtual work of inertial and restoring forces.

B.4.1. Strong form of the rigid-body equation

Let $\xi(t) \in \mathbb{R}^m$ collect the retained rigid-body degrees of freedom (DOFs), e.g. heave and pitch in a two-dimensional setting ($m = 2$). The time-domain rigid-body equation of motion is:

$$\mathbf{M}\ddot{\xi}(t) + \mathbf{C}\dot{\xi}(t) + \mathbf{K}\xi(t) = \mathbf{F}_{\text{hyd}}(t), \quad (\text{B.37})$$

where $\mathbf{M}$ is the rigid-body mass/inertia matrix, $\mathbf{C}$ a linear damping matrix, $\mathbf{K}$ the linear restoring stiffness matrix, and $\mathbf{F}_{\text{hyd}}$ the hydrodynamic generalised force vector.

B.4.2. Principle of virtual work

For a system described by generalised coordinates $\xi(t)$, the dynamic equilibrium condition in variational form reads:

$$\delta\xi^T (\mathbf{M}\ddot{\xi} + \mathbf{C}\dot{\xi} + \mathbf{K}\xi - \mathbf{F}_{\text{ext}}) = 0 \quad \forall \delta\xi, \quad (\text{B.38})$$

where $\delta\xi$ denotes an arbitrary virtual displacement.

B.4.3. Virtual work statement and weak form

Introduce an arbitrary virtual rigid-body displacement vector $\delta\xi \in \mathbb{R}^m$. Multiplying (B.37) by $\delta\xi^T$ yields the virtual work statement:

$$\delta\xi^T (\mathbf{M}\ddot{\xi} + \mathbf{C}\dot{\xi} + \mathbf{K}\xi - \mathbf{F}_{\text{hyd}}) = 0 \quad \forall \delta\xi. \quad (\text{B.39})$$

B.4.4. Hydrodynamic generalised forces from pressure work

Let Γ_p denote the wetted boundary of the rigid pipe and $\mathbf{n}$ the outward unit normal to the fluid domain. The virtual work of pressure is:

$$\delta W_{\text{hyd}} = \int_{\Gamma_p} p(\mathbf{x}, t) \mathbf{n}(\mathbf{x}) \cdot \delta \mathbf{r}_p(\mathbf{x}, t) d\Gamma, \quad (\text{B.40})$$

where $\delta \mathbf{r}_p$ is the virtual displacement field of the rigid-body boundary.

The rigid-body displacement field on Γ_p is expressed in terms of modal contributions:

$$\mathbf{r}_p(\mathbf{x}, t) = \sum_{j=1}^m \xi_j(t) \psi_j(\mathbf{x}), \quad \delta \mathbf{r}_p(\mathbf{x}, t) = \sum_{j=1}^m \delta \xi_j \psi_j(\mathbf{x}). \quad (\text{B.41})$$

Substituting (B.41) into (B.40) and comparing with $\delta W_{\text{hyd}} = \delta \xi^T \mathbf{F}_{\text{hyd}}$ yields the generalised hydrodynamic forces:

$$F_{\text{hyd},j}(t) = \int_{\Gamma_p} p(\mathbf{x}, t) \mathbf{n}(\mathbf{x}) \cdot \psi_j(\mathbf{x}) d\Gamma, \quad j = 1, \dots, m. \quad (\text{B.42})$$

B.4.5. Weak rigid-body equation including fluid coupling

Substituting (B.42) into (B.39) yields:

$$\delta \xi^T (\mathbf{M}\ddot{\xi} + \mathbf{C}\dot{\xi} + \mathbf{K}\xi) - \int_{\Gamma_p} p(\mathbf{x}, t) \mathbf{n}(\mathbf{x}) \cdot \delta \mathbf{r}_p(\mathbf{x}, t) d\Gamma = 0 \quad \forall \delta \xi. \quad (\text{B.43})$$

B.4.6. Frequency-domain form and hydrostatic stiffness

Under linear potential flow, the hydrodynamic pressure is obtained from the linearised Bernoulli equation:

$$p(\mathbf{x}, t) = -\rho_w \phi_t(\mathbf{x}, t) - \rho_w g z(\mathbf{x}, t). \quad (\text{B.44})$$

Substituting (B.44) into (B.43) and using the rigid-body kinematics (B.41) gives:

$$\delta \boldsymbol{\xi}^T (\mathbf{M} \ddot{\boldsymbol{\xi}} + \mathbf{C} \dot{\boldsymbol{\xi}} + \mathbf{K} \boldsymbol{\xi}) + \rho_w \int_{\Gamma_p} (\phi_t + g \eta_p) \mathbf{n} \cdot \delta \mathbf{r}_p \, d\Gamma = 0 \quad \forall \delta \boldsymbol{\xi}. \quad (\text{B.45})$$

The term involving $g \eta_p$ represents the hydrostatic restoring force. Substituting the modal expansion $\eta_p = \sum_j \xi_j H_j$ and $\delta \eta_p = \sum_j \delta \xi_j H_j$ yields the hydrostatic stiffness matrix:

$$K_{\text{hs},ij} = \rho_w g \int_{\Gamma_p} H_i H_j \, d\Gamma. \quad (\text{B.46})$$

Applying the frequency-domain substitutions $(\cdot)_t \rightarrow -i\omega(\cdot)$ and $(\cdot)_{tt} \rightarrow -\omega^2(\cdot)$, as introduced in Section 4.3.2, the weak rigid-body equation in frequency domain reads:

$$\delta \boldsymbol{\xi}^T (-\omega^2 \mathbf{M} - i\omega \mathbf{C} + \mathbf{K} + \mathbf{K}_{\text{hs}}) \boldsymbol{\xi} - \rho_w \int_{\Gamma_p} (-i\omega \phi + g \eta_p) \delta \eta_p \, d\Gamma = 0 \quad \forall \delta \boldsymbol{\xi}, \quad (\text{B.47})$$

which corresponds to the bilinear form $B_\omega^{(3)}$ presented in Section 4.6.2.

B.5. Derivation of the Mean 10-Metre Wind Speed

The mean wind speed at 10 m height is derived following Eurocode 1 [31] and NEN-EN 1991-1-4 [32]. The Willemspolder is located in wind area III of the Netherlands, for which the fundamental basic wind velocity is $v_{b,0} = 24.5$ m/s. The basic wind velocity is given by:

$$v_b = c_{dir} \cdot c_{season} \cdot c_{prob} \cdot v_{b,0} \quad (B.48)$$

The directional factor $c_{dir} = 1$ and seasonal factor $c_{season} = 1$ are adopted following NEN-EN 1991-1-4 [32]. The probability factor c_{prob} accounts for the design lifetime and is calculated as:

$$c_{prob} = \left(\frac{1 - K \cdot \ln(-\ln(1 - p))}{1 - K \cdot \ln(-\ln(0.98))} \right)^N \quad (B.49)$$

with $K = 0.281$ and $N = 0.5$ [32]. For a design lifetime of 25 years, the annual exceedance probability is $p = 0.04$, giving $c_{prob} = 0.95$ and $v_b = 23.3$ m/s.

The mean wind speed at height z is obtained from:

$$v_m(z) = c_r(z) \cdot c_o(z) \cdot v_b \quad (B.50)$$

The orography factor is $c_o = 1$ following Annex A.3 of Eurocode 1 [31]. The roughness factor is:

$$c_r(z) = k_r \cdot \ln \left(\frac{z}{z_0} \right) \quad (B.51)$$

where the terrain factor is:

$$k_r = 0.19 \cdot \left(\frac{z_0}{0.05} \right)^{0.07} \quad (B.52)$$

For terrain category II, the roughness length is $z_0 = 0.2$ m, giving $k_r = 0.215$. Evaluating at $z = 10$ m yields a mean wind speed of $U_{10} = v_m(10) = 19.1$ m/s, consistent with the value listed in Table 4.1.

B.6. Irradiance Sensitivity Derivation

The diffuse irradiance is $G_M^{dif} = DHI \cdot SVF$ where the sky view factor for a free-horizon location is:

$$SVF = \frac{1 + \cos(\theta_M)}{2} \quad (B.53)$$

For a module oscillating about θ_{ref} with tilt amplitude ϵ , averaging over the two extreme positions gives:

$$\overline{SVF} = \frac{1}{2} (SVF(\theta_{ref} + \epsilon) + SVF(\theta_{ref} - \epsilon)) = \frac{1 + \cos(\theta_{ref}) \cos(\epsilon)}{2} \quad (B.54)$$

Applying the small-angle approximation $\cos(\epsilon) \approx 1 - \epsilon^2/2$:

$$\overline{SVF} \approx SVF(\theta_{ref}) - \frac{\cos(\theta_{ref})}{4} \epsilon^2 \quad (B.55)$$

The time-averaged change in diffuse irradiance is therefore:

$$\Delta \overline{G}_M^{dif} = -\frac{DHI \cdot \cos(\theta_{ref})}{4} \cdot \epsilon^2 \propto \epsilon^2 \quad (B.56)$$

This result holds for any reference tilt θ_{ref} .

The ground-reflected irradiance is:

$$G_M^{ground} = GHI \cdot \alpha \cdot \frac{1 - \cos(\theta_M)}{2} \quad (B.57)$$

By the same symmetry argument:

$$\Delta \overline{G}_M^{ground} = \frac{GHI \cdot \alpha \cdot \cos(\theta_{ref})}{4} \cdot \epsilon^2 \propto \epsilon^2 \quad (B.58)$$

Since $DHI > GHI \cdot \alpha$ for open water, the combined effect of diffuse and ground-reflected components is a net reduction scaling as ϵ^2 .

For the direct beam component, define $f(\theta_M) = \cos(AOI(\theta_M))$. A second-order Taylor expansion about θ_{ref} gives:

$$f(\theta_{ref} + \epsilon) \approx f(\theta_{ref}) + f'(\theta_{ref})\epsilon + \frac{1}{2}f''(\theta_{ref})\epsilon^2 \quad (B.59)$$

Averaging over $\pm\epsilon$ the linear term cancels and:

$$\Delta \overline{G}_M^{dir} = \frac{1}{2} DNI \cdot f''(\theta_{ref}) \cdot \epsilon^2 \propto \epsilon^2 \quad (B.60)$$

where $f''(\theta_{ref})$ is the second derivative of $\cos(AOI)$ with respect to θ_M evaluated at the reference orientation. Near the irradiance optimum $f''(\theta_{ref}) < 0$ and tilt oscillation is a net loss.

B.7. Mooring Stiffness Comparison for Willemspolder Wave Barrier

This appendix derives the required heave stiffness for resonant tuning of the 450 mm pipe wave barrier at Willemspolder. It estimates the heave stiffness provided by the as-specified mooring geometry.

Required Heave Stiffness

The hydrostatic heave stiffness per unit length of the pipe is:

$$K_{hs,11} = \rho_w g \cdot 2b_{wl} \quad (B.61)$$

where b_{wl} is the half-width of the pipe at the waterline. For a pipe with outer radius $R = D/2$ floating at a draft of $d = 2D/3$, the pipe centre lies at $z_c = -D/6$ below the waterline. The mooring design report specifies a draft of approximately 80 % of the pipe diameter [30], deeper than the $d = 2D/3 \approx 67\%$ draft assumed in the present hydrostatic stiffness derivation. Using $d = 0.8D$ instead reduces the waterline

half-width b_{wl} by approximately 15 %, and with it $K_{hs,11}$ and the required heave stiffness $k_{heave,opt}$ by the same proportion. The shortfall identified above is therefore conservative: the actual required stiffness, and hence the shortfall relative to the as-built mooring, would be somewhat smaller under the as-built draft. The half-width at the waterline follows from the pipe geometry:

$$b_{wl} = \sqrt{R^2 - \left(\frac{D}{6}\right)^2} \quad (B.62)$$

For $D = 0.45$ m: $b_{wl} = 0.212$ m, giving $K_{hs,11} = \rho_w g \cdot 2 \times 0.212 \text{ m} = 4.16 \text{ kN/m/m}$. For a pipe length of $L_{pipe} = 150$ m:

$$K_{hs,11} = 4.16 \text{ kN/m/m} \times 150 \text{ m} = 624 \text{ kN/m} \quad (B.63)$$

The optimal mooring stiffness ratio from the refined stiffness sweep in Section 5.3 is $k_{heave}/K_{hs,11} = 1.129$ for $D = 0.45$ m, giving:

$$k_{heave,opt} = 1.129 \times 624 \text{ kN/m} = 705 \text{ kN/m} \quad (B.64)$$

Provided Heave Stiffness

The heave stiffness contribution of a single anchor rope is:

$$k_{vert,i} = \frac{EA}{L_i} \sin^2 \bar{\alpha}_{v,i} \quad (B.65)$$

where EA is the axial rope stiffness, L_i the rope length, and $\bar{\alpha}_{v,i} = (\alpha_{v,min} + \alpha_{v,max})/2$ the mean vertical rope angle between the high and low water level conditions.

The axial stiffness EA of the Gleistein X-Permanent Flex 28 mm rope is derived from the manufacturer datasheet [30]. At 10 % of the breaking load of 307.5 kN the elongation is 0.3 %:

$$EA = \frac{F}{\varepsilon} = \frac{0.1 \times 307.5 \text{ kN}}{0.003} = 10.250 \text{ kN} \quad (B.66)$$

Table B.1 lists the rope lengths, mean vertical angles, and resulting heave stiffness contributions for each of the seven wave barrier anchor points.

Table B.1: Heave stiffness contribution per anchor rope for the Willemspolder wave barrier, based on rope lengths and vertical angles from the mooring design [30] and $EA = 10.250$ kN.

Anchor	L_i [m]	$\alpha_{v,min}$ [°]	$\alpha_{v,max}$ [°]	$\bar{\alpha}_v$ [°]	$k_{vert,i}$ [kN/m]
ap-w1	48.11	13.3	24.0	18.6	21.8
ap-w2	54.70	14.9	24.3	19.6	21.1
ap-w3	50.49	11.6	21.9	16.8	16.9
ap-w4	57.50	15.8	24.7	20.2	21.4
ap-w5	35.78	8.8	23.4	16.1	22.0
ap-w6	46.24	14.0	25.1	19.6	24.8
ap-w7	18.65	1.4	29.7	15.5	39.5
Total $k_{provided}$					167.4

The wave barrier mooring consists of seven anchor points, designated ap-w1 through ap-w7 in the mooring design [30]. The four additional anchor points, ap-c1 through ap-c4, listed in the same table, anchor the electrical cable bridge connecting the FPV system to shore and are not part of the wave barrier mooring.

The total provided heave stiffness of 167 kN/m is approximately 24 % of the required 705 kN/m, a shortfall of a factor of 4.2. Note that ap-w7 contributes disproportionately due to its short rope length of 18.65 m; its minimum vertical angle of 1.4° at high water implies the rope is nearly horizontal in that condition and contributes negligible heave stiffness, making the estimate optimistic.



Use of AI Tools in this Thesis

Throughout the development of this thesis, the AI assistant Claude (Anthropic) and Grammarly were used to support the writing and code preparation process. Their role was strictly supportive, aimed at improving the clarity, structure, and consistency of the thesis and its accompanying codebase, without compromising its academic integrity.

Grammarly was used during the writing phase to identify grammatical errors and phrasing inconsistencies. **Claude** was used in two ways. First, it supported the cleanup and documentation of the hydrodynamic model's Julia codebase. Second, it was used during writing and editing, and to assist with formatting and language refinement of the text.

All technical content, modelling decisions, results, and conclusions in this thesis are the author's own. AI tools were not used to generate research findings, analysis, or interpretation of results.