VERIFICATION OF THE SHEARSTRESS BOUNDARY CONDITIONS IN THE UNDERTOW PROBLEM

prepared by

Marcel J.F.Stive [1] and Huib J.De Vriend [2]

[1] *Netherlands Centre for Coastal Research, Delft University of Technology, c/o Delft Hydraulics, PO Box 152, 8300 AD Emmeloord, The Netherlands*
[2] *University of Twente, Section Civil Engineering and Management, PO Box 217, 7500 AE Enschede, The Netherlands*

for

COASTAL DYNAMICS '94

Final Version April 26, 1994

VERIFICATION OF THE SHEARSTRESS BOUNDARY CONDITIONS IN THE UNDERTOW PROBLEM

Marcel J.F. Stive[1] and Huib J. De Vriend[2]

ABSTRACT: As part of a revision of the quasi-3D approach for coastal currents, the two-dimensional undertow problem is being restudied. Since the first proposal for approaches in the 1980's progress has been made by several researchers (e.g. Deigaard and Fredsoe, 1989) on the potential importance of contributions neglected initially, such as the time-mean correlation between horizontal and vertical wave-induced velocities. The effects of wave-decay, sloping bottom and oscillatory bottom boundary layer on this term, initially neglected, have now been derived formally and included in the undertow description. Before checking the effects in comparisons with measurements of undertow profiles, it is considered essential to first concentrate on an improved, or at least verified, formulation of the shearstress boundary condition at wave trough level. Since it is momentarily not feasible to make direct measurements of the shear stress condition at wave trough level, only indirect verification is possible e.g. by expressing the shearstress condition in terms of the set-up gradient. Existing small scale laboratory and recently acquired large scale laboratory results provide the set-up gradient data for this approach. The verification leads to theoretical improvements, and provides insight into possible differences between large and small scale situations.

INTRODUCTION

The importance of the modelling of undertow in studies addressing the cross-shore hydrodynamics, sediment transport and morphology is now generally accepted. Since the first attempts of Dally (1980), Svendsen (1984), Stive and Wind (1986), De Vriend and Stive (1987), Stive and De Vriend (1987) in the 1980's, new findings have been introduced in the last years which may lead to improvements. Here it is attempted to highlight and describe some of these new findings, so as to enable undertow modellers to include the new ideas in the ongoing modelling efforts.

---

1) Laboratori d'Enginyeria Marítima, Universitat Politècnica de Catalunya, Barcelona, Spain; now at Netherlands Centre for Coastal Research, Delft University of Technology, c/o Delft Hydraulics, P.O. Box 152, 8300 AD Emmeloord, The Netherlands.

2) University of Twente, Section Civil Engineering and Management, P.O. Box 217, 7500 AE Enschede, The Netherlands.

PRINCIPLES OF SOLUTION FOR THE MEAN FLOW AND THE SET-UP

The present state-of-the-art local momentum equation most commonly used to solve the undertow reads:

$$\frac{\partial}{\partial z} \nu_t \frac{\partial U}{\partial z} = g\overline{\eta}_x + (\overline{u^2})_x - (\overline{w^2})_x + (\overline{uw})_z \tag{1}$$

where:
U is the undertow,
ν_t is the eddy viscosity,
η_x is the mean water level set-up, and
u, w are the horizontal and vertical wave-orbital velocities.

This equation is derived from the time-dependent horizontal and vertical momentum equations in a Eulerian framework, neglecting the advective acceleration terms for the time-mean flow. It is assumed to be valid in the full vertical domain, except for the region near the free surface. When assuming infinitely small wave amplitudes (e.g. assuming linear wave theory) it should be valid unto mean water level, but since the wave amplitudes are finite in the surfzone it may be more correct to limit its applicability to below the wave trough level.

Let us introduce the common assumption that the wave terms can be derived independently from the mean flow, i.e. we assume that there exists a sufficiently accurate wave theory to describe the wave terms which neglects the wave-current interaction. This of course a simplification, and we may state that a true break-through here would be achieved if we could tackle the problem with a Lagrangian approach in which waves and currents would be considered simultaneously, which at the same time would allow us to deal consistently with the near surface layer (NSL). However, until such an approach is developed we rely on the experience that the suggested approach is shown to yield sufficiently accurate results for the moment.

Let us further assume that the turbulence viscosity is constant over depth. Note that this is only for the clarity of the argumentation. The rationale which follows would also allow for a depth-varying viscosity, as long as this does not depend on the undertow solution, itself.

The above implies that we have one equation to solve for the undertow and the mean water level set-up. Integrating the equation twice yields:

$$\nu_t U = \frac{1}{2} g\overline{\eta}_x z'^2 + \int_0^{z'}\int_0^{z'} (\overline{u^2})_x \, dz dz - \int_0^{z'}\int_0^{z'} (\overline{w^2})_x \, dz dz + \int_0^{z'} (\overline{uw}) \, dz' + C_1 z' + C_2 \tag{2}$$

This expression contains three unknowns, viz. the two constants of integration, C_1 and C_2 remaining in this expression, and the set-up gradient. We therefore need, in addition to this equation, three boundary conditions and/or constraints:

(1) no-slip condition at the bottom ($U = 0$), from which we can find C_2;
(2) shear stress condition at the transition from the middle layer (ML) to the NSL (τ_t given), from which we can find C_1;
(3) mass balance constraint (total mass flux in the lower layers balances that in the NSL), from which we can derive the set-up gradient.

The essential information which we need here is related to the NSL. In fact, condition (2) is based on the assumption that we know the shear stress at the lower end of the NSL from the momentum balance for this layer. Similarly, when formulating the constraint (3), we assume the total mass flux in the NSL to be known.

This route is suggested by Stive and De Vriend (1987) and also followed by Deigaard et al. (1991). The former derive a formal expression through a third depth integration, while the latter use an iteration procedure which sees to it that a set-up gradient is created such that the correct depth-average mass flux is created. The result should be the same.

Alternatively, one could use the depth-averaged horizontal mass and momentum equations, with the wave-induced radiation stresses and mass fluxes properly modelled, instead of giving τ_t and imposing constraint (3). In fact, the latter is even incorrect in 3-D situations, where the mass flux in the NSL is not necessarily compensated in the lower parts of the same water column (cf. De Vriend and Kitou, 1990). In section 3 we shall use the depth-averaged momentum equation, with a new expression for the radiation stresses, to derive an expression for τ_t. Note that in either case we face the problem of describing the NSL, via τ_t and the mass flux, or via the wave-related terms in the depth-averaged mass and momentum equations.

ROLE OF THE WAVE-RELATED TERMS IN THE MEAN MOMENTUM EQUATION

In De Vriend and Stive (1987) and in Stive and De Vriend (1987) the importance of the wave terms was not fully recognized. Deigaard and Fredsoe (1989) and De Vriend and Kitou (1990) have more fully pointed out the importance of the wave terms. A possible, zero-order approach to the magnitude of the wave-related terms is given below. This approach was essentially derived by Bijker et al. (1974).

Starting point is that -under the assumption that depth dependence of the orbital motion can be neglected- the horizontal orbital motion due to harmonic progressive waves (seaward positive) and the existence of an oscillatory turbulent boundary layer can be approximated by:

$$u = A[\cos(kx + \omega t) - e^{-\phi}\cos(kx + \omega t - \phi)] \tag{3}$$

where:

A = $a\,\omega / \sinh(kh)$
k = $2\pi / L$ is the wave number
ω = $2\pi / T$ is the wave frequency
ϕ = z'/δ
δ = $\sqrt{(2\,\nu_{tb}/\omega)}$ the boundary layer thickness.

Note that the shallow water approximation is not essential to the rationale that follows. Essentially the same points can be proven without this assumption, it only takes more work (see De Vriend and Kitou, 1990).

Using the continuity equation and depth integration for the orbital motion by

$$w_z = -u_x \tag{4}$$

$$w = -\int_0^{z'} u_x\,dz \tag{5}$$

gives for the vertical orbital motion (cf. Bijker et al., 1974)

$$\begin{aligned} w = & -A_x\delta[\phi\cos(kx+\omega t) - \frac{e^{-\phi}}{2}\cos(kx+\omega t)(\sin\phi - \cos\phi) \\ & + \frac{e^{-\phi}}{2}\sin(kx+\omega t)(\sin\phi + \cos\phi) - \frac{1}{2}(\cos(kx+\omega t) + \sin(kx+\omega t))] \\ & + A(k+k_x x)\delta[\phi\sin(kx+\omega t) - \frac{e^{-\phi}}{2}\sin(kx+\omega t)(\sin\phi - \cos\phi) \\ & - \frac{e^{-\phi}}{2}\cos(kx+\omega t)(\sin\phi + \cos\phi) - \frac{1}{2}(\sin(kx+\omega t) - \cos(kx+\omega t))] \\ & + Ah_x[e^{-\phi}\cos(kx+\omega t)\cos\phi - e^{-\phi}\sin(kx+\omega t)\sin\phi - \cos(kx+\omega t)] \end{aligned} \tag{6}$$

where the first term is due to spatial variations in the orbital velocity amplitudes over a horizontal bottom, the second is due to the boundary layer streaming including a slope effect and the third term is due to spatial variations in the orbital velocity amplitudes over a sloping bottom. A term due to the spatial variation of the boundary layer thickness is neglected.

Based on these expressions for u and w one may -after time averaging- derive expressions for the wave terms in the momentum equation for the undertow. These results can be used to resolve the momentum equation for the undertow. We have done this according to the solution procedure described in section 1. Our analysis of these results is underway (Stive and De Vriend, 1994). Here, we concentrate on the derivation and verification of the resulting shear stress distributions.

SHEAR STRESS DISTRIBUTION OVER DEPTH AND AT THE TRANSITION TO THE NEAR SURFACE LAYER

The objective of this section is to compare the present approach with earlier results by Deigaard and Fredsoe (1989) and De Vriend and Kitou (1990). In the following we will consider two separate cases, i.e. (1) boundary layer dissipation and (2) breaking dissipation. If the problem is confined to boundary layer dissipation only, the shear stresses in the water column away from the bottom boundary layer should disappear, since the resulting wave motion can be described by potential flow (cf. Deigaard and Fredsoe, 1989 and 1992). In the case of wave dissipation due to breaking shear stresses can be generated where the energy dissipation is taking place. The latter is the most intense at the interface between the roller and the underlying body of water. We therefore can only introduce a shear stress condition at the transition between ML and NSL -as suggested in section 1- if this shear stress is related to the breaking-related energy dissipation.

CASE 1: Boundary layer dissipation, horizontal bottom, no breaking

If we consider the momentum equation outside the boundary layer, the wave-terms are to leading order:

$$\overline{u^2} = \frac{1}{2} A^2 \tag{7}$$

$$\overline{uw} = -\frac{1}{4} (A^2)_x \, z' \tag{8}$$

If we substitute this into the mean momentum equation (1) and integrate once, we find the shear stress distribution:

$$\tau(z') = \rho \nu_t \frac{\partial U}{\partial z} = \rho g \overline{\eta}_x \, z' + \frac{1}{4} \rho \, (A^2)_x z' + C_1 \tag{9}$$

Requiring the shear stress to be zero at the mean water level implies that we can solve for C_1 to yield:

$$\tau(z') = \rho g \overline{\eta}_x (z' - d_m) + \frac{1}{4} \rho (A^2)_x (z' - d_m) \tag{10}$$

This shows that no shear stresses will exist throughout the middle layer if the set-up gradient is given by:

$$\rho g \overline{\eta}_x d_m = -\frac{1}{4} \rho (A^2)_x d_m = -\frac{1}{2} E_x \tag{11}$$

(These equations are equivalent to equations (35) and (36) of Deigaard and Fredsoe, 1989). As pointed out by Deigaard and Fredsoe (1989), this is only one third of the equilibrium set-up commonly attributed to waves with boundary layer dissipation. By inserting the above result in the depth mean momentum balance equation:

$$\frac{dS_{xx}}{dx} + \rho g d_m \overline{\eta_x} + \tau_b = 0 \tag{12}$$

and using the shallow water approximation:

$$S_{xx} = S_{xx,p} + S_{xx,u} = \frac{1}{2}E + E \tag{13}$$

it follows that the bottom shear stress must balance the other two thirds of the radiation stress gradient. In the present case this can only be the shear stress due to the boundary layer streaming, which is acting in the same direction as the radiation stress gradient, indeed.

CASE 2: Wave breaking dissipation, horizontal bottom, no boundary layer dissipation

Again we use the local mean momentum balance equation (1) as the starting point, but now choose a shear stress τ_t at the transition between ML and NSL as a boundary condition. The idea is that in this case the shear stress can be maintained due to the NSL related dissipation due to wave breaking. Furthermore, we adopt the leading order approximations for the wave terms away from the bottom boundary layer:

$$\overline{u^2} = \frac{1}{2}A^2 \tag{14}$$

$$\overline{uw} = -\frac{1}{4}(A^2)_x z' \tag{15}$$

Assuming ν_t constant over the layers yields:

$$\tau(z') = \rho\nu_t \frac{\partial U}{\partial z} = \rho g \overline{\eta_x}(z' - d_m) + \tau_t + \frac{1}{4}\rho(A^2)_x(z' - d_m) \tag{16}$$

and

$$\tau_b = -\rho g d_m \overline{\eta_x} + \tau_t - \frac{1}{4}\rho(A^2)_x d_m \tag{17}$$

Since we consider wave breaking dissipation only, the mean bottom shear stress τ_b should be zero. Note that for the moment we assume that the middle layer is actually having an upper boundary at the mean water level d_m.

The shear stress τ_t may be resolved between the τ_b equation and the depth mean horizontal momentum balance equation. In order to do this we need to introduce an expression for the radiation stress which at least needs to be extended with the roller effect. Following Svendsen (1984) Deigaard and Fredsoe (1989) and De Vriend (1993) suggest the shallow water approximation

$$S_{xx} = S_{xx,p} + S_{xx,u} = \frac{1}{2}E + [E + \rho\frac{Rc}{T}] \tag{18}$$

where R is the roller area, empirically approximated by $R=0.9H^2$.

The above implies that the set of equations available to resolve the shear stress τ_t reads:

$$\frac{dS_{xx}}{dx} + \rho g d_m \overline{\eta}_x = 0 \tag{19}$$

$$\frac{dS_{xx}}{dx} = \frac{3}{2} E_x + \frac{\rho}{T}(Rc)_x \tag{20}$$

$$\tau_b = -\rho g \overline{\eta}_x d_m + \tau_t - \frac{1}{4}\rho (A^2)_x d_m = -\rho g \overline{\eta}_x d_m + \tau_t - \frac{1}{2} E_x = 0 \tag{21}$$

which yields:

$$\tau_t = -E_x - \frac{\rho}{T}(Rc)_x \tag{22}$$

and

$$\tau(z') = -E_x (1 + \frac{d_m - z'}{2d_m}) - \rho g \overline{\eta}_x (d_m - z') - \frac{\rho}{T}(Rc)_x \tag{23}$$

These equations are equivalent to equations (56) and (58) of Deigaard and Fredsoe (1989), when introducing the relation

$$E_x = \frac{1}{c}(E_f)_x = -\frac{D}{c} \tag{24}$$

where E_f is the energy flux and D is the dissipation due to wave breaking.

In the above the level of transition between ML and NSL is assumed to be at mean water level. In practice, the finite amplitudes in the surf zone make it more convenient for us to apply τ_t at the trough level, which will change the above derivations somewhat to yield:

$$\tau_t (1 + \frac{3(d_m - d_t)}{2d_t}) = -E_x - \frac{\rho}{T}(Rc)_x \tag{25}$$

EXPRESSING THE SHEAR STRESSES IN TERMS OF THE DISSIPATION

Since from the above we have shown that shear stresses above the wave boundary layer can only be generated in wave breaking related dissipative waves, it is useful and practical to express the shear stresses in terms of the dissipation rather than in terms of the energy density gradients. This will prevent driving mechanisms in the vorticity free situation, i.e. in the shoaling region possibly in combination with the boundary layer induced dissipation.

We start with noting that the now commonly used wave decay models, either for periodic or random waves, are calibrated and validated by using the observed decay of potential energy. In the stationary case these models solve the energy flux equation

$$\frac{\partial E c_g}{\partial x} + D_f = 0 \tag{26}$$

where D_f is a fictitious dissipation balancing the potential energy decay. So, effectively these models describe the potential energy decay due to wave breaking, in which we assume that this includes the potential energy due to the existence of the roller.

If we now adopt the common assumption that the kinetic energy flux due to the organized wave motion equals that of the potential energy flux we may express the energy balance equation including the roller related kinetic energy flux as

$$\frac{\partial 2E_p c_g}{\partial x} + \frac{\partial E_r c}{\partial x} + D_r = 0 \tag{27}$$

where D_r is now the real dissipation and the kinetic roller energy $E_r = {}^1/_2\ \rho\ Rc/T$.

By using relation (34) we may now express the real dissipation as follows

$$\frac{D_r}{c} = -E_x - \frac{\rho}{T}(Rc)_x = \frac{D_f}{c} - \frac{\rho}{T}(Rc)_x \tag{28}$$

Using the empirical result for the roller area, the roller decay related contribution can be expressed as:

$$\frac{\rho}{T}(Rc)_x \simeq 0.57k\,(Ed_m)_x \tag{29}$$

These results may be used to express the shear stresses in terms of the dissipation as follows

$$\tau_t = \frac{D_r}{c} = \frac{D_f}{c} - \frac{\rho}{T}(Rc)_x = \frac{D_f}{c} - 0.57k\,(E^* d_m)_x \tag{30}$$

and

$$\tau(z') = \frac{D_f}{c}\,(1 + \frac{d_m - z'}{2d_m}) - \rho g \overline{\eta}_x (d_m - z') - 0.57k\,(E^* d_m)_x \tag{31}$$

where E^* is the energy density of those waves which are breaking, i.e. which are effectively contributing to the breaking related dissipation. How to implement this in situations for random waves and for periodic waves will be addressed furtheron.

VERIFICATION OF THE SHEAR STRESS AT THE TRANSITION TO THE NEAR SURFACE LAYER

Since it is momentarily not feasible to make direct measurements of the shear stress at the transition between ML and NSL, only indirect verification is possible, e.g. by using relation (30) between τ_t and the mean wave set-up gradient. We emphasize that these two quantities are strongly related, which is not surprising since in both quantities the momentum properties of the NSL are embedded. Where τ_t translates the momentum decay of the NSL to the lower layers, the mean set-up includes the momentum decay of the NSL in the total decay. We may thus conclude that our ability to predict the mean water level set-up correctly indicates our ability for the prediction of the NSL momentum decay and therewith of τ_t. This is the topic addressed in this section both for random and periodic waves.

As a starting point for both cases we take the mean momentum balance and the expression derived for S_{xx}:

$$\frac{dS_{xx}}{dx} + \rho g d_m \overline{\eta}_x = 0 \qquad (32)$$

$$\frac{dS_{xx}}{dx} = \frac{3}{2}E_x + \frac{\rho}{T}(Rc)_x = \frac{3}{2}E_x + 0.57k\,(E^*d_m)_x \qquad (33)$$

Without the contribution of the roller to the radiation stress as expressed by the second term in equation (42) the problem reduces to the classical solution. For this it is well-known that the observed delay in the set-up gradients compared to the wave height gradients is not resolved. The roller term can be introduced to resolve this deficiency.

CASE 1: Periodic waves

In the case of periodic waves breaking on a slope there is a sharp transition in the fictitious energy flux as detected by the potential energy or wave height decay. If we would interpret this to be an instantaneous transition to a fully dissipative breaking mode, this implies that an instantaneous increase would be introduced in the radiation stress through the roller contribution to the radiation stress in expression (42). As described by Nairn et al. (1990) the physical mechanism to prevent this is the existence of a transition region, where the rollers are developed gradually as the potential energy decay sets in. It is only after the transition region that the fully developed breaking waves create rollers of which the properties, such as their kinetic energy, are proportional to the potential energy decay.

Here, we will not address possible modelling approaches to describe the transition zone phenomenon. Instead we will show that for random breaking waves the need to tackle the transition zone phenomenon is less important.

CASE 2: Random waves

In the case of random breaking waves there exists a gradually increasing fraction of breaking waves as the depth limitation starts to exert its influence on the wave field. Physically, this implies that while the total potential energy is still undergoing shoaling effects a fraction of the waves is dissipative due to wave breaking. If we assign a roller related kinetic energy to the breaking wave fraction, we will only gradually increase the radiation stress and therewith prevent an instantaneous increase of the radiation stress, an effect that would exist if all waves would be assumed to be in a full breaking mode. It so appears that to a first approximation there is no need to tackle the transition zone effect, because of the gradual increase of the dissipation.

So, in the case of random waves, we suggest to introduce $E^* = Q_b\, E_{br}$, so that:

$$\frac{dS_{xx}}{dx} = \frac{3}{2}E_x + k(Q_b d_m E)_x \qquad (34)$$

where Q_b is the breaking wave fraction[1], and E_{br} is the energy density of the breaking waves.

---

1) An initial parameterization of Q_b which we have used for a long time, and also here, reads:

$Q_b = 7\ Q_{b,B\&J}$ for $Q_{b,B\&J} \leq 0.1$

$Q_b = 1. - 548. (.3 - Q_{b,B\&J})^{4.67}$ for $0.1 < Q_{b,B\&J} < 0.3$

$Q_b = 1.$ for $Q_{b,B\&J} \geq 0.3$

where $Q_{b,B\&J}$ is the breaking wave fraction according to the Battjes and Janssen (1978) model, of which it is well-known that this significantly underestimates the breaking wave fraction.

Implementation of this in the mean horizontal momentum balance and applying this to existing small scale laboratory and recently acquired large scale laboratory results is being undertaken. An initial result is shown in Figure 1 (Battjes and Janssen, testcase 15). The upper part of Figure 1 indicates the increase of and shift in gradient change of the radiation stress due to the existence of the roller. As a consequence the set-up gradient undergoes a spatial delay leading to an improved prediction of the mean water level gradient, which indirectly implies a verification of the trough level shear stress (lower part of Figure 1).

Our result seems to contradict the findings of Nairn et al. (1990), who studied the so-named transition zone effects in detail. They found it necessary to introduce transition zone effects to arrive at a similar result. We suspect, however, that they haven't determined the relative effect of an approach without and with transition zone effects for the random wave case. Apparently, in the random wave case the present approach leads already to an important improvement. We note that the essential assumption in the present approach is that the roller energy can be expressed in locally determined variables, such as the fraction of breaking waves and the heights of the breaking wave fraction. Obviously, further improvements could be expected if nonlocal modelling approaches are introduced, such as the transition zone approximation of Nairn et al. (1990). Our present preference, however, would be to directly address the improvement of the breaking wave fraction with "transition" or memory effects such as addressed by Lippmann and Thornton (1993, these proceedings).

ACKNOWLEDGEMENTS

A large part of this paper is based on work in the "G8 Coastal Morphodynamics" research programme, which is funded partly by the Commission of the European Communities in the framework of the Marine Science and Technology Programme (MAST), under contract no. MAS2-CT92-0027. The work is co-sponsored by Delft Hydraulics, in the framework of the Netherlands Centre for Coastal Research.

REFERENCES

Battjes, J.A. and Janssen, J.P. (1978). Energy Loss and Set-up due to Breaking of Random Waves. Proc. 16th Conf. on Coastal Eng, ASCE, pp 569-588.

Bijker, E.W., Kalkwijk, J.P.Th. and Pieters, T. (1974). Mass transport in gravity waves on a sloping bottom. Proc. 14th Conf. on Coastal Eng, ASCE, pp 447-465.

Dally, W.R. (1980). A numerical model for beach profile evolution. M.Sc. Thesis, Dep. Civil Eng., Univ. Delaware.

Deigaard, R. and Fredsoe, J. (1989). Shear Stress Distribution in Dissipative Water Waves. Coastal Engineering, 13, 357-378.

Deigaard, R., Justesen, P. and Fredsoe, J. (1991). Modelling of undertow by a one-equation turbulence model. Coastal Engineering, 15, 431-458.

De Vriend, H.J. (1993). Quasi-3D nearshore currents revisited. Workshop Quasi-3D flow modelling, Barcelona.

De Vriend H.J. and Kitou, N. (1990). Incorporation of wave effects in a 3D hydrostatic mean current model. Proc. 22nd Coastal Eng. Conf., ASCE, 1005-1018.

De Vriend, H.J. and Stive, M.J.F. (1987). Quasi-3D Modelling of Nearshore Currents. Coastal Engineering, 11, 565-601.

Fredsoe, J. and Deigaard, R. (1992). Mechanics of coastal sediment transport. Advanced Series on Ocean Engineering, Vol. 3, World Scientific, pp 369.

Lippmann, T.C. and Thornton, E.B. (1994). The spatial distribution of wave rollers and turbulent kinetic energy on a barred beach. Proceedings Coastal Dynamics 1994.

Nairn, R.B., Roelvink, J.A. and Southgate, H.N. (1990). Transition zone width and implications for modelling surfzone hydrodynamics. Proc. 22nd Conf. on Coastal Eng., ASCE, pp 68-81.

Stive, M.J.F. and Wind, H.G. (1986). Cross-shore mean flow in the surf zone. Coastal Engineering, 10, 325–340.
Stive, M.J.F. and De Vriend, H.J. (1987). Quasi-3D nearshore current modelling: wave-induced secondary currents. Proc. Special Conf. on Coastal Hydrodynamics, ASCE, 356-370.
Svendsen, I.A. (1984). Wave Heights and Set-up in a Surf Zone. Coastal Engineering, 8, 303-330.

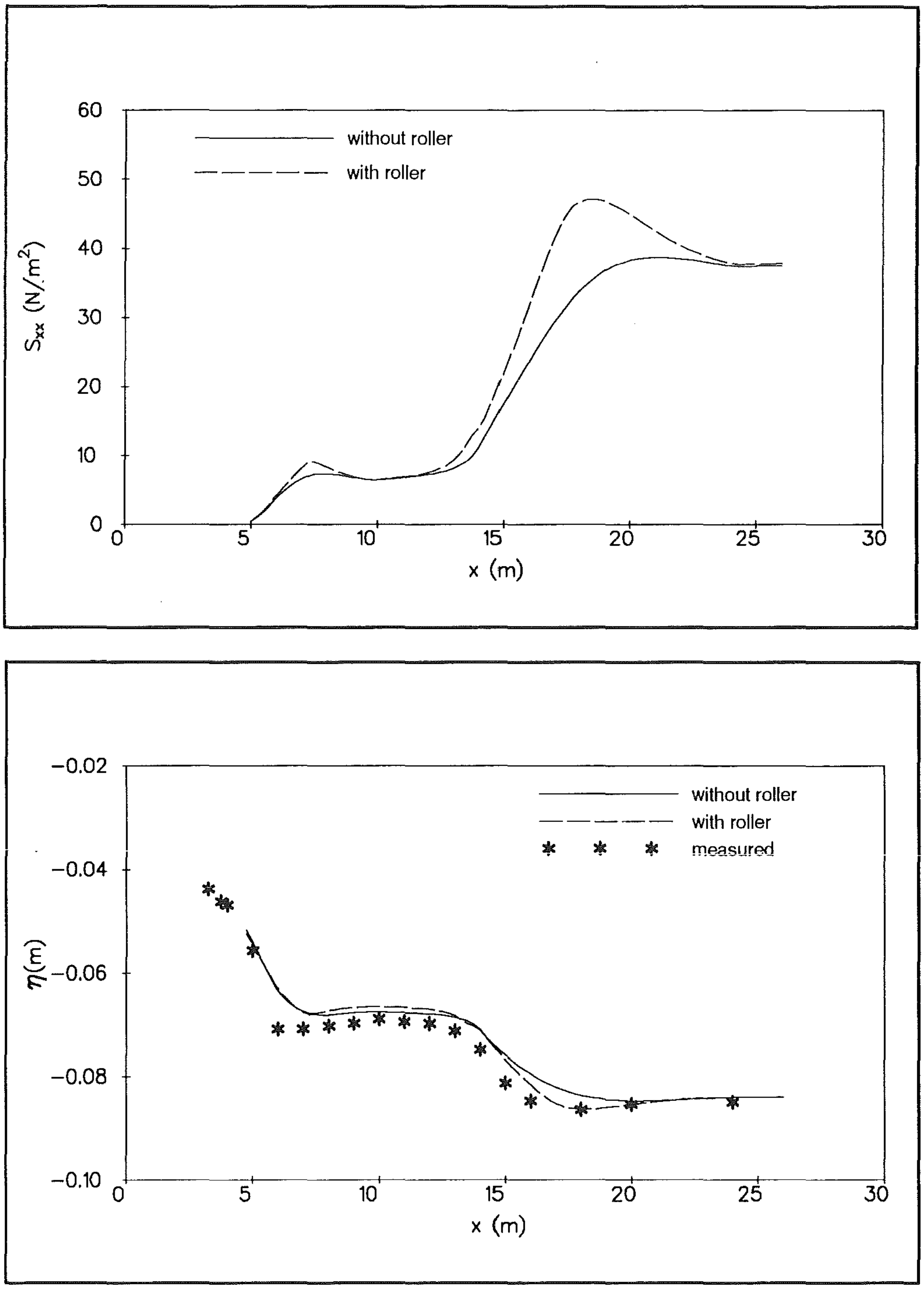

Figure 1 Radiation stress and set-up variation for test case 15 of Battjes and Janssen (1978)