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GRADED STRUCTURES

The design of a grid shell that uses gradual densification to enhance the performance of the overall structure.

TU Delft || Faculty of Architecture & the Built Environment

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GRADED STRUCTURES

The design of a grid shell that uses gradual densification
to enhance the performance of the structure.

by Denise Jonker

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PREFACE

This thesis is a final work as partial fulfillment for the degree of Master of Science in Building Technology at the Technical University of Delft in the Netherlands titled 'Graded Structures'.

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Denise Jonker

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1. INTRODUCTION

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1.1 Background



Figure 1 Oceanographic aquarium, design by Felix Candela (Left, Miller [2014]) and the Opera House, design by Jørn Utzon (Right, Wikipedia [2009])

1.1.1 A brief history

In the second half of the 20th-century freeform architecture started to emerge due to technological advancements in the computer industry. Computers were used by architects to aid the design process allowing for the creation, visualization and analysis of complex geometries. [Olsson, 2012]

According to Olsson [2012], freeform architecture was dominated by form finding during the late 50's until the 90's. Form finding can be defined as a process in which the form of the design is derived from the forces that act upon it. Therefore, it can be stated that form and function are strongly linked. According to Veenendaal & Block [2012], form finding can be defined as 'Finding an (optimal) shape of a [form-active structure] that is in (or approximates) a state of static equilibrium'. Structures whose shape has been determined by using form finding techniques minimize the amount of material while maximizing the strength. This technique was used in the design of the Sydney Opera House by Jørn Utzon

and his engineers, which became one of the first buildings whose design was based on form finding that was ever built. [Olsson, 2012] Examples of building designs that were based on form finding are shown in figure 1. Frei Otto, Heinz Isler, Felix Candela, Jørn Utzon and Horst Berger are all architects that were very influential during this period.

Due to further technological advancements and the development of three-dimensional Computer Aided Design (CAD) and Computer Aided Modelling (CAM) software in the 90's, computers were increasingly used in the design process. In architecture, CAD/CAM software is now widely used in the design of buildings with complex geometries. [Keough & Benjamin, 2010] These kinds of software were no longer only used to aid in the design process: they slowly became a design tool on its own. Well-known architects that use CAD software as a design tool are Zaha Hadid and Frank Gehry. According to Olsson [2012] this 'computer based sculptural architecture', in comparison to form-finding based design, 'has seemingly lost the link between form

and structural function'. According to Berger (2007), we live in a world where sensationalism is a value in itself. The creation of a freeform iconic building that is not efficient in terms of structural or other functions is often justified by its sensational value. This shows, that freeform architecture does not necessarily lead to a minimization of used materials as it does in the case of form finding.

1.1.2 Relevance

'One look around will show us that Nature knows no right angle, nor does it use straight lines. Although there are lots of similarities between species of different life forms, it uses no repetition of basic structural elements [...]. Every structural part of natural systems is unique, however big or small it is'. (Dimic, 2011) Every form in nature is freeform because its form is the result of nature's highly complex and irregular (non-uniform) forces acting upon them. Nature is not only very good at producing structural forms, but also at creating multi-functional materials and structures. A good example is the combination of form, structure and material in leaves. 'A leaf [...] can support itself structurally while at the same time converting light into sugar in the process of photosynthesis' (PopTech, 2010). The veins of a leaf support the leaf structurally while they also transport water and nutrients to the plant's cells. The overall shape of the leaf ensures the plant catches enough sunlight for the process of photosynthesis. Nature minimizes the amount of necessary material and energy in the production of its forms, structures and materials 'for it will profit the individual not to have its nutriment wasted on building up a useless structure' (Charles Darwin, 2006; cited in Dimic, 2011). Thus, nature creates very sustainable multi-functional structures.

The way mankind builds, however, is not very sustainable. As a matter of fact, construction is one of the least sustainable industries in the world. It is estimated that roughly half of the world's global energy and

water resources is used in buildings and about 60% of all global material resources is used for buildings and roads (including timber products). Additionally, the building industry is indirectly responsible for half of coral reef destruction and for a quarter of rainforest destruction. (Willmott Dixon, 2010) About half of the resources used in construction are non-renewable, or in other words finite. This means that at some point we will run out of natural resources. Desjardins (2014) predicts that, if production continues to grow at this rate, there will be no more oil, gas, uranium (all used for the production of energy), antimony, lead, indium (all used in renewable energy solutions), zinc, silver, gold and copper left by 2050 or even sooner. Therefore, recycling will become much more important, the natural resources that are left need to be used wisely and no materials and energy must be wasted in the process of constructing and maintaining buildings. The latter is an aspect that is largely dependent on the design: if we are able to optimize the design, we can reduce the amount of used material and energy.

The building industry is also responsible for a large share of global pollution. A quarter of the degradation of the air quality, half of the expulsion of climate change gases, half of the landfill waste, 40% of the drinking water pollution and half of the depletion of ozone is caused by this industry. (Willmott Dixon, 2010). The 'climate change gases' contribute largely to the so-called 'greenhouse effect' that causes the earth to warm up: solar rays warm the earth's surface to a larger temperature than they would have without these gases. These gases are produced during natural events, like forest fires. However, since the Industrial Revolution (1800) the amount of these gases in the atmosphere started to increase rapidly. 'Current concentrations of CO₂ are about 390 ppm and CH₄ levels exceed 1,770 ppb. Both numbers are much higher than at any time during the last 650,000 years.' (American Chemical Society, n.d.) Due to the excessive amounts of these gases the earth is warming up more than it should while

entailing serious climate change. As stated earlier, the building industry is responsible for 50% of the expulsion of these climate change gases. According to the American Chemical Society [n.d.] fossil fuel combustion, or in other words 'energy production', and the global manufacturing of cement are responsible for 75% of the total amount of CO2 emissions caused by humans. As stated before, roughly half of this produced energy is used in buildings. This energy is used for the production of building materials, the construction of the building, the maintenance of the building (including heating, cooling, lighting, etc.), the demolition of the building and the recycling or processing of residual materials. The energy used for the production of materials and the energy used for the operation of the building can be reduced by optimizing the design.

Apparently, nature has been significantly better at designing very sustainable multi-functional structures thus far. The numbers show that it is of great importance that the materials and energy used in/for buildings is largely reduced. In order to do so, we should learn from nature and create structures that are optimized for multiple objectives. How come we have not been able to do so up to this point?

According to Dimcic [2011] many systems that were made by people were and still are inspired by nature, 'but the degree of its simplification and abstraction always depended on knowledge, production ability and resources. Because of the lack of technology enabling the copying of Nature in an exact manner, the main principles were extracted and simplified.' This explains the large amounts of straight lines, orthogonal connections and repetition of elements. It is, for example, a lot easier to make straight steel beams opposed to curved steel beams. Nowadays, technologies have advanced to a point where we can imitate nature in a form that is much less simplified.

It can be concluded that nowadays technology has

advanced enough for the design of buildings to not only protect us from nature's influences but buildings that use these influences to our benefit in order to create sustainable designs. In fact, according to the numbers, it is essential that buildings become (a lot) more sustainable in terms of material and energy use. Since these influences are highly complex, irregular and non-uniform, we have to conclude that the simplification to straight lines and orthogonal connections will not lead to designs that use a minimum amount of material and energy. [Dimcic, 2011]

1.2 Problem Statement

The traditional function of buildings is to protect us from nature's influences and to create a space with conditions that are suitable for us [ProgramArchitecture, 2016] Nowadays, we can create buildings that do not only protect us from nature's influences, but we can create buildings that use these influences to our benefit in order to create sustainable designs.

During the sustainable design process, decisions need to be taken regarding a multitude of objectives which may be conflicting. For example, the need for a supporting structure often conflicts with the desire for a wide open view of the surroundings and the desire of minimizing the costs often suppresses many other objectives from being maximally achieved. Optimizing the design for only one of these objectives will most likely not lead to a satisfactory design solution. For the design to be satisfactory, the several objectives need to be balanced carefully. Therefore, the need for multi-objective optimization arises.

Buildings can be optimized for objectives that change gradually over their surface. These gradual changes are difficult to process using traditional building methods. The traditional building methods will result in buildings with structures that can be classified as regular grid-structures consisting of lines and points

(members and joints). [ProgramArchitecture, 2013] Gradually changing the density towards specific points on the surfaces of these (often regular quadrangular or triangular) grid-structures does not work because the grid is limited to these regular systems of lines and points. Two possible manners of densifying a quadrangular grid-structures are shown in figure 62. One way of densifying a grid is to gradually minimize the distance between the ongoing lines toward a point on the surface. This densifies the area around the point, however, it also automatically densifies other parts of the surface. Therefore, this system of densifying is not restricted to the target area. Another way of densifying a grid toward a point that does limit the densification to the target area is division of parts of the grid-structure. While restricting to the same form language in case of densification, a triangle can easily be divided into two triangles and a square can be divided into four squares. Thus, in both cases the densification of such a grid-structure happens not gradually, but incrementally. When multiple objectives are involved these effects can reduce the degree to which they are achieved. Another reason this solution often leads to results that are not desirable is because the obtained result is usually not optically acceptable. According to Dimcic [2011] 'The human eye has a remarkable pattern recognition ability and incredible precision of error detection when it comes to grid structures.' In order to create optically pleasing grid-structures, the grid must be organized in a way that the members of the structure do not form long guide lines. 'If they do, then the lines must be smooth and continuous, in order to satisfy optical conditions.' [Dimcic, 2011]

Many calculations regarding the building's performance are virtually presented using 'heat maps' that are projected on a complex surface that represents the buildings shape: such as solar radiation, wind force and the desired percentage of openings. These 'heat maps' generally show a gradual transition of colours. Ideally, the densification of the abstract grid of lines (that

represents the structure) matches this heat map. In order to do so, a graded grid of lines is necessary. This grid of lines translates to the beams of the structure, which should therefore, ideally, be a graded structure. We just established that the current regular grid structures do not allow for this gradual densification. Therefore, we can conclude that there is a discrepancy in the manner in which traditional regular grid-structures can respond to optimization for multiple objectives.

1.3 Research Design

The aim of the research is to take a step forwards in limiting the impact of the construction and energy use of buildings on the global environment by developing a workflow that integrates the results from performance analyses with the structural design.

The main research question is as follows:

How to design a grid shell for the canopy of a stadium that uses performance based densification to enhance the performance of the structure?

In addition to the main research question, the following sub questions are formulated:

1. What materials are used for grid shells?
 - 1.1 What are the properties of these materials?
 - 1.2 What connection types exist for each material?
 - 1.3 What material and connection type is most suitable for the task?
2. What kind of **graded structures** exist?
 - 2.1 What are the advantages and disadvantages of these different kinds of graded structures?
 - 2.2 Which of these graded structures is most suitable for the task?
3. How can this graded structure be generated based on an underlying **performance map**?
 - 3.1 Which of these methods is most suitable for

the task?

4. How can this method be applied to the Case Study?

4.1 How can a multiple performance based shape for the canopy be generated by using multi-objective optimization?

4.1.1 What are the objectives that will be taken into account in the multi-objective optimization?

4.2 How can the results of different performance analyses be combined in one performance map?

4.3 How is the abstract grid of lines of the graded structure translated to an actual structure for the canopy?

A graded structure can be defined as a structure that gradually changes over the surface of the building. A performance map can be defined as a gradient (or image) map which corresponds to the surface of the building that shows how different parts of the building perform for a certain objective.

Constraints

The scope of this research is limited to the design of a canopy for 'De Nieuwe Kuip' in Rotterdam by OMA. For this research it is assumed that the canopy is freestanding and has no structural connection with the stadium stand or other parts of the building. This also means that the canopy itself does not need to comply with any insulation requirements. Therefore, the insulation value of the canopy will not be considered in this research.

The model will be set up in Grasshopper. The reason for this is that Grasshopper is directly linked to Rhino, which is an advanced modeling program that is widely used. In addition to that I personally have a lot of experience with both of these programmes. Another reason for using these programs is that there are many plug-ins for Rhino and Grasshopper that allow for more extensive use of the program. These plug-ins make it possible to analyze the several objectives and optimize

for all these objectives within the same program.

The optimization of the structure will be limited to size optimization, instead of shape or topology optimization, in order to reduce calculation times. Size optimization is the least complicated and the fastest way of calculating the dimensions of the structure. Another reason for limiting this research to size optimization is that no proper plug-ins for Grasshopper were found that allow for 3D topology optimization for architectural structures. For example, in order to be able to optimize a 3D structure with Millipede, only 3D (volume) loads (N/m^3) can be applied to the structure. Within architecture volume loads are not commonly used. Point loads and 2D loads (N/m^2) are the kinds of loads that are mostly used. Only in very few cases, such as explosion resistance calculations, volume loads are applied to a structure.

Research Design

In order to find the answer to the main question and the sub questions a case study, a literature study and a quantitative research will be carried out.

Firstly, a brief literature study into grid shells is carried out to find the answer to the question: 'What materials are used for grid shells?' The findings of this literature study can be found in chapter 2 along with the general characteristics of grid shells. Then, an extensive literature study is carried out to find the answers to the questions 1.1 and 1.2. The most relevant characteristics of the materials and the corresponding connection types are summarized. The findings of this literature study can be found in chapter 3.

Thereafter, a literature study is performed to find the answer to question 2: 'What kind of graded structures exist?'. Based on the findings of the literature study into materials and connection types and based on personal preference for a specific aesthetic (biomimicry/bonestructure-like) the material FRP is

chosen in combination with Voronoi as a basis for the generation of a graded structure. These findings can be found in chapter 4.

Then, a literature study is carried out to find the answer to question 3: 'How can this graded structure be generated based on an underlying performance map?' The literature study aims to find existing methods of generating a graded Voronoi. Three methods are found. Thereafter, the existing methods are translated to a grasshopper script and their performance is tested on several imagemaps. The imagemaps for the tests are generated to test the ability of the generation method to respond to different situations that can occur in performance maps. It is tested how the generation method responds to:

1. Harsh gradients
2. Soft gradients
3. Round gradients (dots)
4. Different sizes of dots
5. The addition of a greytone to the map

In addition to that for two of the generation methods the desired sizes for the Voronoi (corresponding with black and white places of the imagemap) is varied as well to test how this affects the performance. The performance is judged by the correspondence of the generated graded Voronoi to the underlying imagemap. This is done by visually inspecting the results. The results of the tests are compared and based on this comparison the most suitable generation method is chosen. The findings can be found in chapter 4.3.

In order to answer question 4.1: 'How can a multiple performance based shape for the canopy be generated by using multi-objective optimization?' Firstly, a literature study into existing types of optimization algorithms and existing grasshopper plug-ins is performed. Based on these findings, a suitable type of optimization algorithm and a corresponding

grasshopper plug-in is chosen. In order to determine what objectives need to be taken into account (question 4.1.1) a literature study is performed to determine what functions a canopy of a stadium must fulfill. For the sake of time a selection of three objectives is made to take into account in the multi-objective optimization. A grasshopper script that also uses components of the Octopus plug-in is generated to run a multi-objective optimization for the overall shape of the canopy of the stadium. Based on personal preference and while keeping other – in the optimization neglected – functions of the canopy in account a final shape for the canopy is chosen. Then, in order to answer question 4.2: 'How can the results of different performance analyses be combined in one performance map?' a script in grasshopper is generated that combines the results of separate analyses (using different grasshopper plug-ins) and creates one performance map that will be used for the generation of graded Voronoi. Thereafter, an additional grasshopper script is created that generates the graded Voronoi with the chosen generation method based on the combined imagemap.

In order to find the answer to the last question: 'How is the abstract grid of lines of the graded structure translated to an actual structure for the canopy?' firstly a multitude of structural tests are performed with Karamba to determine what the effect of opening size and the addition of webs to the section are. Based on these findings and keeping practical things as manufactureability in mind a specific range of opening sizes for the graded Voronoi and a certain amount of webs is chosen. Then, the resulting structure is analyzed with Karamba. Thereafter, a size optimization will be executed by creating a script in grasshopper that uses the results of the structural analysis to assign specific sections to every line of the structure. The resulting structure is analyzed for maximum deflection, nodal displacement and rain water flow. Based on these results it is determined whether the structure meets the requirements. These requirements

are based on a literature study that can be found in chapter 7.3. The size optimization returns the cross section size for every member of the structure. Based on this information the final geometry is generated and technical details are drawn. In the creation of the technical details the findings of the literature study into materials, the results on the analyses of the final structure, manufactureability and maintenance are taken into account.

2. GRID SHELLS

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2.1 Types of Grid Shells

A gridshell can be defined as a structure that derives its strength from a double curvature and consists out of lattices. They are usually made out of steel or wood. They can span large distances without the need for internal load-bearing supports. Figure 2 shows the nomenclature for grid shells. There are three main types of grid shells:

1. Single layer
2. Double layer
3. Triple layer

In a single layer grid all the members lie at one surface. An example of a single layer grid shell can be found in the library pavilion (see figure 5). A double layer grid is characterized by members that lie on two surfaces. These layers are then connected by additional members, shear blocks or special node solutions. An example of a double layer grid is the roof of the Savill Garden Shell (see figure 4). Triple layer grid shells also exist. Triple layer grid shells are basically double layer grid shells with an additional layer. [Nurfariq, 2013]

2.1 Erection Methods

Grid Shells can be made out of any kind of grid. Triangular and quadrangular grids are the most common types. Some examples of grids that can be applied are:

1. Triangular
2. Quadrangular
3. Hexagonal
4. Voronoi
5. Delaunay
6. Combinations of the above

According to Aditya [2016] there are three types of erection methods for grid shells:

1. Scaffold method
2. Block assembly
3. Lift-up method

The scaffold method and the lift-up method are only suitable for surfaces that can be flattened. Therefore, all grids that contain triangles are not suitable since they can not be deformed. In addition to that, grids that

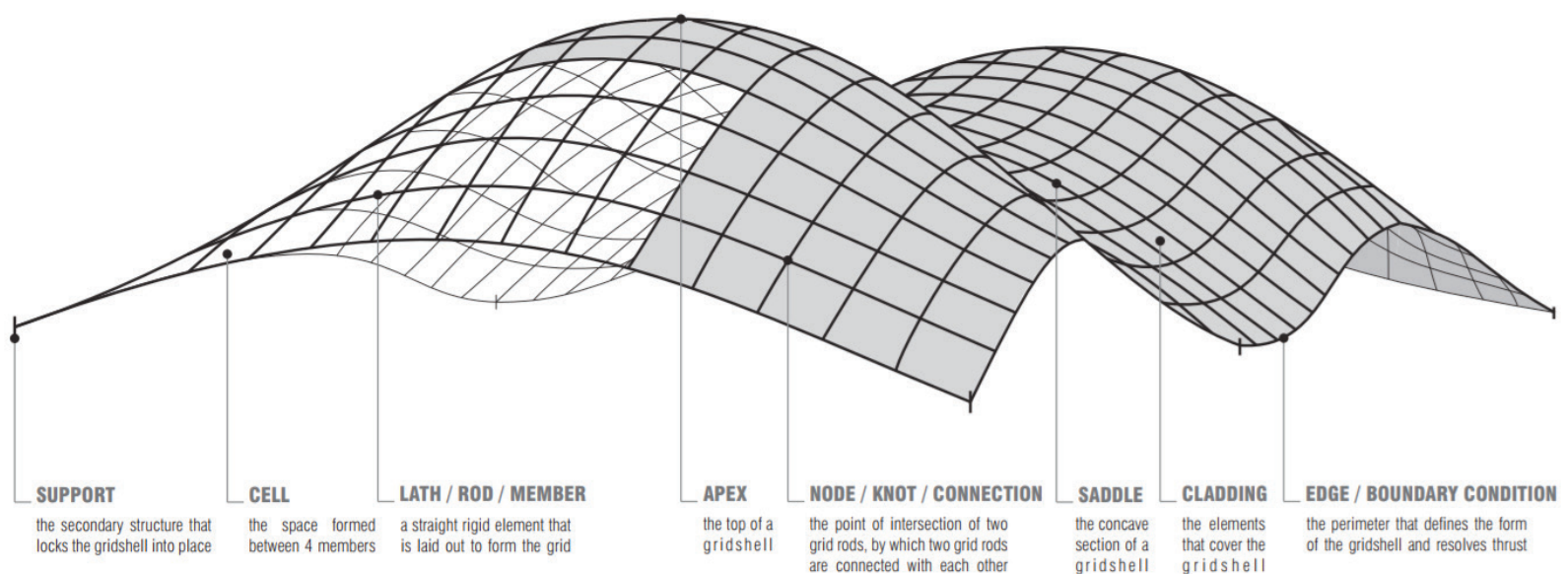


Figure 2 Nomenclature for grid shells [Alzawawi et al., 2015]

require bending stiff joints in order to maintain their shape, like hexagonal grids, can also not be flattened to a planar surface. For these types of grid the block assembly method is used. With the scaffold method, the initially flat grid is lifted onto scaffolding. Then the edges are pulled and pushed into position. Once the grid is into position, the nodes are tightened to fix the shape (see figure 6). The lift-up method starts with laying the flat grid onto the ground and pushing/pulling it up into the desired shape. Once the grid is into position the nodes are tightened to fix the shape. The block assembly method does not start with a flat mat that is pushed into shape. The grid shell is constructed member by member, cell by cell or group (of cells) by group.

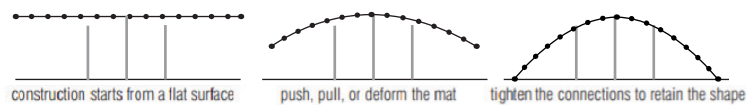


Figure 6 Scaffold Method [own image]



Figure 7 Lift-up Method [Alzawawi et al., 2015]

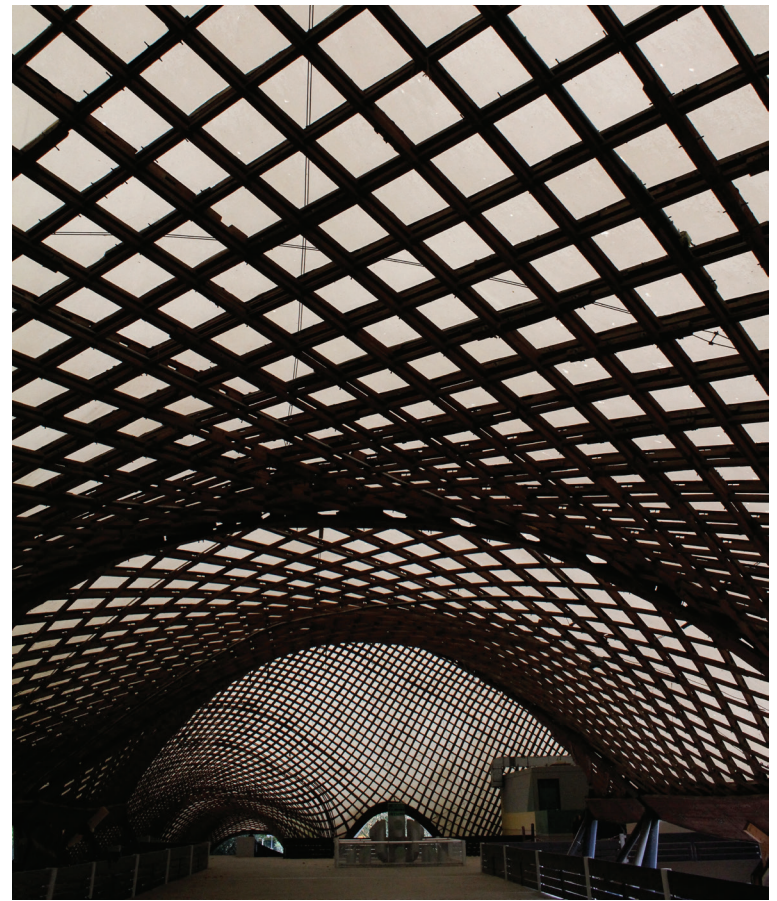


Figure 3 Interior of Multihalle Mannheim [Evolution of German Shells, 2013]



Figure 4 Interior of the Savill Garden Shell [Cullinan Studio, n.d.]



Figure 5 Library Pavillion [Architect, 2013]

3. MATERIALS

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- Polymers
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- Material Properties
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3.1 Steel

Steel is a relatively new material that only became widely available after the construction of 'The Home Insurance Building'. It was constructed in 1885 and was the first high-rise building of 10 storeys constructed in steel. Steel can be classified as a ferrous metal, a metal in which the element iron predominates. [Lyons, 2015] It is widely used in construction because of its high tensile strength.

3.1.1 Manufacturing

Steel can be produced, formed and treated in different ways. The manner in which steel is manufactured influences the properties of the material. The production of steel follows three stages: the production of pig iron, the conversion of pig iron into steel and the conversion of the steel into sections or strips. [Lyons, 2015]

Pig Iron

In order to produce pig iron, three raw materials are needed: iron ore, coke and limestone. Coke is a material that contains a large amount of carbon and has less impurities. A small amount of coke is sintered before adding it with the iron ore, coke and limestone into the top of blastfurnace. Sintering is the process of making agglomerates from a combination of iron and coke. A hot air blast is fed into the bottom of the blast furnace. As a result, the coke turns into carbon monoxide, which causes the iron oxide to turn into iron. The iron collects at the bottom of the furnace while the limestone forms a layer on top of it. Impurities of the iron are absorbed into this layer. The molten metal can be tapped from the bottom of the blastfurnace. At this point, the iron is only 90–95% pure. It contains 4–5% carbon and other impurities. As will be explained later, the amount of impurities within the metal has a significant influence on the mechanical properties of the resulting steel. The process of producing pig iron is a continuous process. [Lyons, 2015]

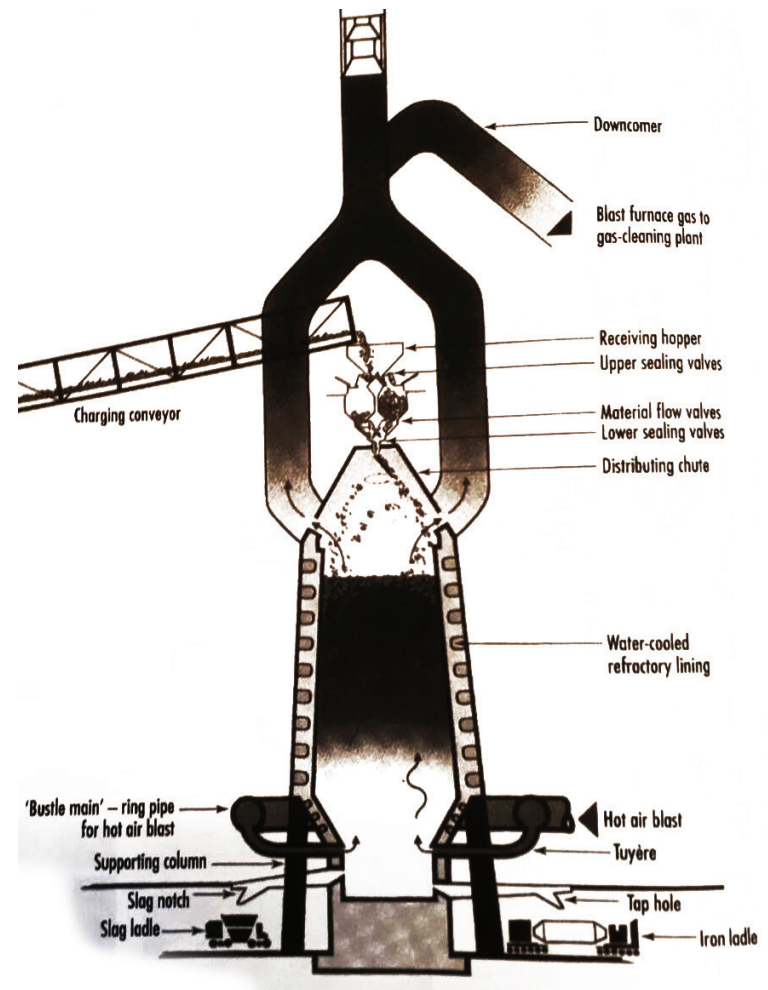


Figure 8 Blastfurnace [Lyons, 2015]

Steel

There are two processes of turning the pig iron into steel. The first process, which is called 'the basic oxygen process', is used for the manufacturing of bulk quantities of standard-grade steels. This process uses a tiltable steel furnace. The furnace is filled for one quarter with scrap metal and for three quarters with the hot metal from the blast furnace. Then, high pressure oxygen is fed into the furnace. This reduces the impurities and lowers the carbon content. Lime is added to form a layer that further reduces impurities. Lastly, other components are added to obtain the

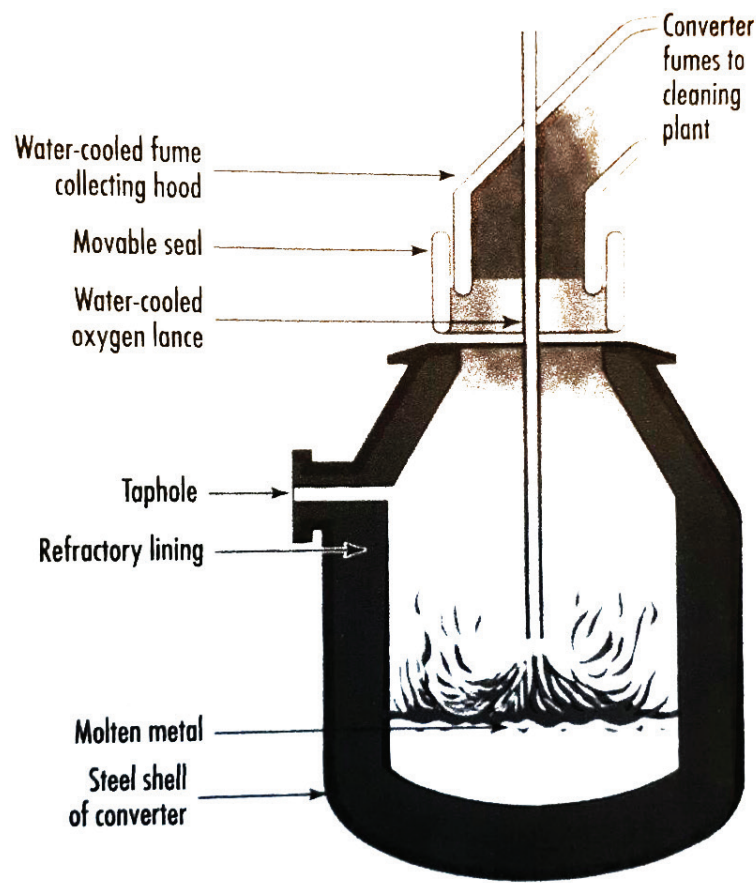


Figure 9 Basic Oxygen Furnace [Lyons, 2015]

required steel alloy. This process is not a continuous process. [Lyons, 2015]

The second process, which is used for higher quality steels, is called the 'electric arc furnace process'. A furnace with a removable lid is filled with scrap metal. Then, graphite electrodes are lowered near the surface of the scrap metal, striking a powerful electric arc between the metal and the electrodes that causes the metal to melt. Limestone is added to form a layer that absorbs impurities and oxygen is fed into the furnace to further reduce impurities. The furnace is tilted to tap of the metal once it reaches the right temperature and the chemical analysis is correct. Afterwards, alloying components may be added to the metal to obtain the desired steel alloy. [Lyons, 2015]

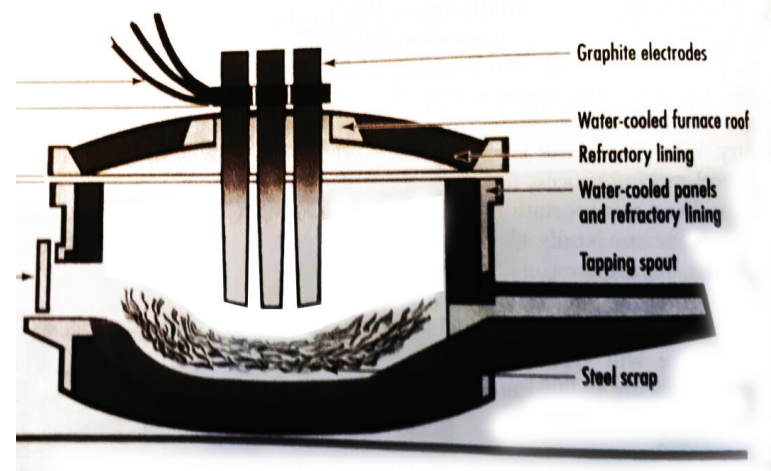


Figure 10 Electric Arc Furnace [Lyons, 2015]

Conversion into Sections or Strips

There are three main methods of processing the molten steel: casting, hot-rolling and cold-rolling. Casting is the traditional method that was used by forgers in the medieval era to transform molten steel into ingots. Nowadays, most steel is directly poured into slabs that are then cut into sections for further processing. Continuous casting is a method of producing an infinite long slab of steel that is then cut into sections. The process of casting continuously saves a lot of energy and results into steel with a better surface quality than the ingot method. [Lyons, 2015]

The hot slabs that are produced in the casting process can then be hot-rolled by putting the slabs through a series of rollers that are controlled by computers. This reduces the thickness of the slabs to a thickness between 1.5mm and 20mm. The thin slabs are then cooled by water and twisted around a coil. Universal beams, such as I-beams, are rolled from the hot slabs by putting them through a series of specially shaped rollers. The hot rolled steel can be processed further by the cold-rolling process. Cold-rolling increases the tensile strength and the surface quality of the steel. These very thin slabs can be rolled into more complex steel sections and sheets by putting them through a series of specially shaped rollers. [Lyons, 2015]

3.1.2 Mechanical Properties

The mechanical properties of steel are mostly dependent on the quantity of the element carbon within the steel alloy. The carbon content of the alloy affects the micro crystal structure within the steel. Steel can be divided into seven categories: ultra-low-carbon steels ($<0.01\%$), extra-low-carbon steels ($0.01\text{--}0.02\%$), low carbon steel ($0.05\text{--}0.15\%$), mild steel ($0.15\text{--}0.25\%$), medium-carbon steels ($0.25\text{--}0.5\%$), high-carbon steels ($0.5\text{--}0.9\%$) and carbon tool steels ($0.9\text{--}1.7\%$). Ultra-low-carbon and extra-low-carbon steels are very ductile. These steels are often used when high formability is required, f.e. in ornamental works. Low carbon and mild steels are relatively soft and are therefore suitable for cold working. The medium-carbon, high carbon steels and the carbon tool steels are getting more hard wearing and stronger with increasing carbon content. The effect of the carbon content on some of the properties of steel is shown in figure 11. [Lyons, 2015]

The speed in which steel is cooled down to room temperature also has a significant influence on the mechanical properties. Steel that is subjected to room temperature forms several crystal forms, namely ferrite, pearlite and cementite, in specific proportions. The proportion in which these crystals occur in the steel change when the temperature of the steel is altered to higher temperatures. By quickly cooling the steel these different proportions can be trapped within the steel, changing the micro crystal structure and thus changing the steel's mechanical properties. Different kind of heat treatments exist to obtain a specific range of mechanical properties. [Lyons, 2015]

Micro Structure

All metals consist of so-called grains. Usually, these grains are not visible on the surface. In order to make the grains at the surface visible, the metal surface is polished, treated with a strong acid and then treated with another mixture of chemicals (etching). The mixture

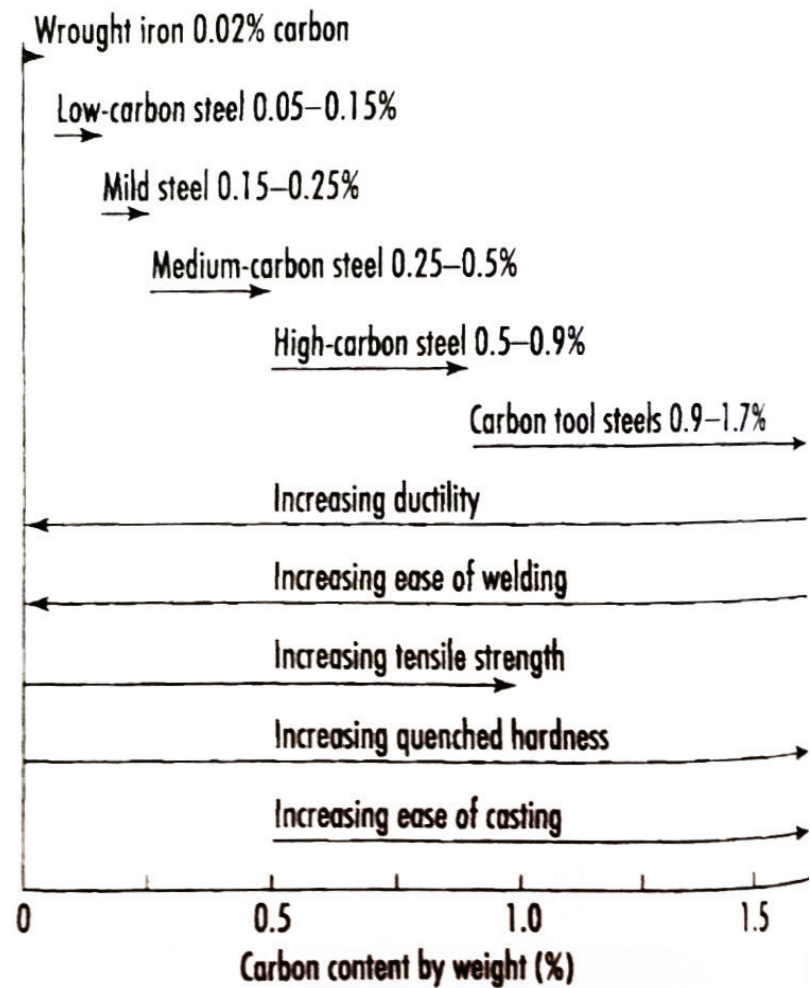


Figure 11 The effect of the carbon content of steel [Lyons, 2015]

of chemicals that is used for the etching process depends on the metal. Grains form while the metal is cooling to below its melting point. This process is called solidification. At several points in the metal crystals begin to form and grow in all directions. The crystal only stops growing once it meets a neighbouring crystal. A fully grown crystal is called a 'grain'. [Moodlemech, 2014]

Cold-rolling of metal changes the shape of the grains. The grains are elongated in the direction of rolling. Cold-rolling changes the mechanical properties of the metal. Its hardness and tensile strength increases while the

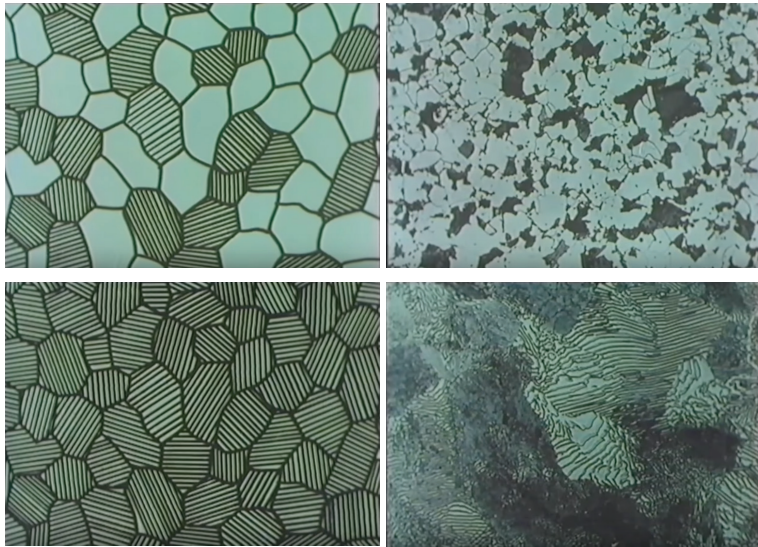


Figure 12 0.4% carbon steel (top) and 0.8% carbon steel (bottom)
[Moodlemech, 2014]

ductility decreases, resulting in a very brittle material. Usually, cold-rolled steel will receive a heat treatment to reduce the brittleness of the material. During the heat treatment, new grains form on the boundaries of the elongated grains. As a result, a new undistorted grain structure replaces the cold-rolled grain structure. This process is called 'recrystallization'. The recrystallization process restores (some of) the hardness, tensile strength and the ductility to properties that compare to the pre cold-rolled metal. The mechanical properties of the metal after recrystallization depend on the temperature under which new crystals form. When the temperature gets higher, some crystals form at the expense of their neighbours, resulting in a fairly large grain size. The resulting properties are highly undesirable for most engineering applications. According to Herring (2005) 'grain size has a measurable effect on most mechanical properties. For example, at room temperature, hardness, yield strength, tensile strength, fatigue strength and impact strength all increase with decreasing grain size. Machinability is also affected; rough machining favors coarse grain size while finish machining favors fine grain size.'

[Moodlemech, 2014]

The grains of steels are so small they can not be seen with the human eye. There are two different types of grains. The first type consists out of iron and is called 'ferrite'. Ferrite gives the steel its properties of ductility. The other type consists out of layers of iron and iron carbide and is called 'pearlite'. The latter is a chemical compound that contains both iron and carbon. Pearlite gives steel the properties of hardness and strength. For 0.4% carbon steel the two types of grains are present in roughly equal numbers. When the steel contains 0.8% of carbon, the steel contains only pearlite grains (see figure 12). The mechanical properties of steel can be changed by heat treatment.

[Moodlemech, 2014]

Heat Treatment

Heat treatment of steel changes its mechanical properties. The process of heat treatment is characterized by heating the steel to a certain temperature and recooling it under specific conditions. Heat treatments can be categorized into four processes: hardening, annealing and normalising, tempering and carburising. (Lyons, 2015)

Hardening steel can be done by rapidly quenching the steel from a high temperature. As a result the crystalline structure that the steel has at this high temperature is retained. During the heating process new crystals form on the boundaries of the grains. The grains that are formed are quite different than the former ones (austenite). During the rapid cooling, needles are formed in these grains. This crystalline structure is very hard and brittle. The higher the carbon content of the steel, the more pronounced this effect will be. Quenched steel is usually not suitable for engineering purposes. Therefore, quenched steel is usually tempered to alter the mechanical properties of the steel. (Lyons, 2015) [Moodlemech, 2014]

Annealing and normalising are both processes that soften the steel by heating it to a specific temperature

and then cooling it under specific conditions. The cooling of the steel happens more gradually than with quenching, allowing recrystallization of the steel. This relieves internal stresses in the material, resulting in a softer steel. During the annealing process the steel is heated up to 650 °C and is then slowly cooled within a controlled environment such as a furnace. This produces the softest steel possible for every amount of carbon content. During the normalising process the steel is reheated up to 830–930 °C and then more rapidly cooled in air. During the heating process new crystals form on the boundaries of the grains. The grains that are formed are quite different than the former ones (austenite). During the cooling process new grains that are similar to the initial grains (before heating) are formed. The new grains are a lot smaller and more uniform than the original grains. As a result, the steel is not as soft as with the annealing process. However, it is still soft enough to facilitate cold working.

[Lyons, 2015] [Moodlemech, 2014]

Tempering can be described as a milder form of normalising. The steel is heated to a temperature of 400–600 °C and then cooled in air. This process reduces the brittleness of the steel. The effect of the tempering is directly related to the tempering temperature. With increasing the tempering temperature, the steel becomes less brittle, more ductile and gets a lower tensile strength. [Lyons, 2015]

Carburising is a process that hardens the surface of the steel while maintaining a softer core. As a result the surface of the steel becomes hard wearing while maintaining a ductile center that is impact resistant. In order to carburize steel, the steel needs to be heated to approximately 900 °C while being surrounded by a carbon-based material (charcoal). [Lyons, 2015]

Structural Classification

Structural steels are classified in several strength grades. The gradenames for structural steels start

with an S followed by a three digit number that corresponds with the steel's minimum yield strength. In addition to that, there are several letters that can be added to this four character name to indicate sub-grades. The sub-grades JR, JO, J2 and K2 are added to the name to indicate increasing impact resistance of the steel. K has a higher impact resistance than J. The R, O and the 2 refer to the temperature at which the impact test was carried out. R stands for room temperature, O stands for 0°C and 2 stands for –20°C. N can be added to indicate that the steel is normalised or normalised rolled, M can be added to indicate that the steel is thermomechanically rolled and Q can be added to indicate quenched steels. L (low temperature impact) can be added to indicate a high impact resistance. The sub-grade W indicates that the steel is weather resistant. The letter P can be added to indicate a high phosphorus content. An example of a steel grade is S355J0WP. The name indicates that this steel is weather resistant, has a high content of phosphorus, has a minimum yield strength of 355 MPa and falls under the impact resistance category JO.

[Lyons, 2015]

Failure behaviour

Materials can be classified as either brittle or ductile. Structural steels can be classified as ductile materials as can be seen in figure 13. When a structural steel is subjected to a tensile force it passes through multiple stages before it ruptures. According to Afsar [2014] these stages are:

1. Proportional Limit
2. Elastic Limit
3. Yield Point
4. Ultimate Stress Point
5. Breaking Point

The first stage is characterized by a linear relationship between stress and strain and ends as soon as the proportional limit point has been reached. The second

phase reaches up to point E, the Elastic Limit. Up to this point the material is elastic and will return to its original shape once the force is removed. After this point the material deforms plastically. The third phase reaches up to the Yield Point. The stress that matches the yield point matches the maximum yield strength of structural steels. Shortly after this point, the steel deforms quickly with only little or no increase in force. At some point the steel hardens due to the strain. This process is called strain hardening and takes place in the Ultimate Stress Point phase. 'U' corresponds to the maximum tensile strength of the steel. The last phase ends when the material ruptures. During this phase the steel shows a phenomenon that is called 'necking': a small part of the steel decreases prominently in cross section. [Afsar, 2014]

Effect of Alloying Elements

As has been stated before, the primary alloying element for steels is carbon. Many more alloying elements can be added to the steel. Every alloy has a different effect on the steel's properties. This paragraph summarizes some frequently used alloys and their effects on the properties of the steel.

Aluminium is a deoxidizer and is added to the steel to improve the oxidation resistance of the steel alloy, especially at higher temperatures. Aluminium forms a protective layer on the surface. As a result, steels that contain a substantial amount of aluminium are often used in heat-resistant steels. Adding aluminium to the steel also results in a finer grain size resulting in improved hardness, yield strength, tensile strength, fatigue strength and impact strength. [Outokumpu.com, 2017] [Bell, 2017] [Tosec, 2016]

Chromium is added to the steel to improve oxidation and corrosion resistance. If the content of chromium is high enough (>10,5%) the steel becomes stainless. It also improves the wear resistance of the steel. A disadvantage of adding chromium is a decrease of the weldability. [Outokumpu.com, 2017] [Tosec, 2016]

Copper is not frequently used in steels since a high percentage at the steel's surface can cause breakage

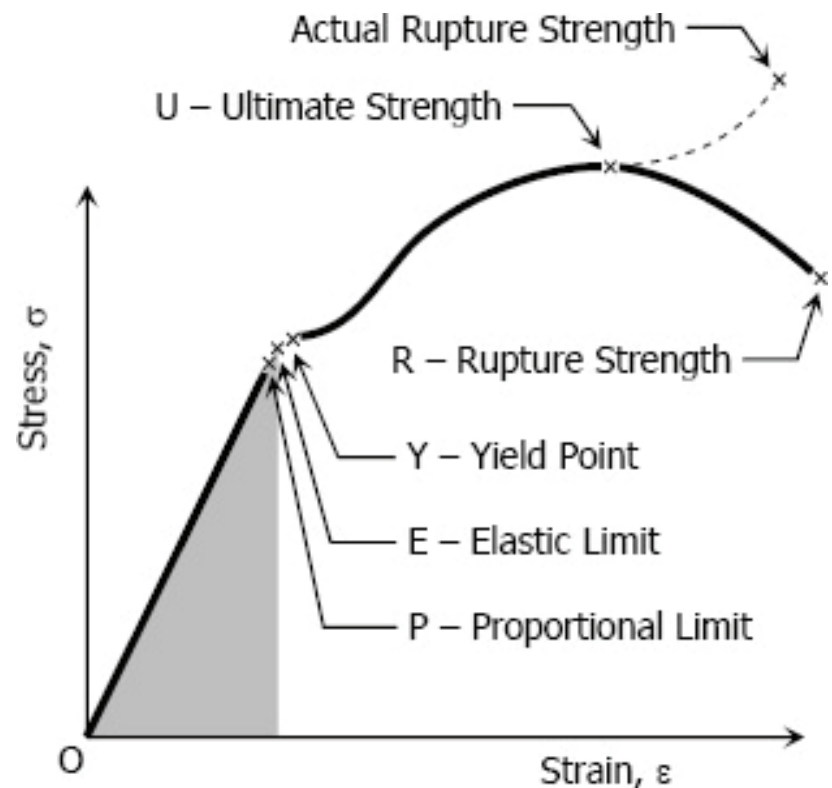


Figure 13 Stress-strain diagram for structural steels [Mathalino, 2017]

during thermal processing. When copper is added with sulphur (a natural impurity) in stainless steels, it increases resistance against sulfuric acid. [Tosec, 2016]

Manganese is added to the steel to improve the strength, ductility and hardness. Adding manganese also results in a better surface quality. Manganese can form Manganese Sulphide with Sulphur. As a result, the machinability of the steel is improved [IIMA, 2013] [Tosec, 2016]

Nickel is often added to steel to reduce the brittleness while increasing the ductility and hardness. Furthermore, it is added to stainless steels in large amounts to improve resistance to oxidization and corrosion. [Outokumpu.com, 2017] [Tosec, 2016]

Silicon - just like aluminium - is a deoxidizer. It has a positive influence on the wear resistance and the yield strength. Just like aluminium it is frequently used in heat resistant steels. A downside is the negative effect on the machinability. [Tosec, 2016]

Vanadium, Molybdeen and Tungsten result in the formation of carbides. It is added to steels to improve wear resistance. Furthermore, adding vanadium and molybdeen results in a finer grain size of the steel. Thus, improving the yield strength and the tensile strength. [Tosec, 2016]

By changing the composition of alloying elements in steel the mechanical properties can be changed. Due to the many different kinds of steel alloys, steels can have a large variety of mechanical properties (see figure 15). However, assuming that it is possible to pick desired values for each property for your steel is (usually) not possible. There is often a certain trade-off between these properties. For example, the Iron-base-superalloy (N-155, solution treated) possesses excellent qualities for fresh water, salt water, weak acids and weak alkalis but it is relatively heavy and uses more CO² and Embodied Energy then most other steels. Figure 15 has been created by using data from the CES Edupack 2016 database.

3.1.4 Connections

According to Chilton [2000] there are hundreds of space grid systems. Space grid frames can be divided into three main categories:

1. Piece-Small Systems
2. Continuous Chords
3. Prefabricated modules

The first and third method are the most commonly used methods in steel grid shells. The second method is mostly used in wooden grid shells. [Chilton, 2000]

Piece-small systems

The 'piece-small systems' can be divided into several categories. One classification that is based on the method of connecting the several parts distinguishes bolted, threaded and welded connections. However,

Chilton [2000] classifies piece-small systems by their so-called 'node-type':

1. Spherical nodes
 - a. solid
 - b. hollow
2. Cylindrical nodes
3. Prismatic nodes
4. Plates
5. Nodeless

Spherical nodes are often considered to be the most aesthetically pleasing. As the name suggests, the nodes are based on a spherical shape. This category can be further divided into solid and hollow spherical nodes. The solid spherical nodes consist out of steel spheres in which threaded holes are being drilled under the desired angles. The members are connected to these spheres by bolts that lie in the axial direction of that member. These bolts can either be a part of the member or they can be a separate part. [Chilton, 2000]

A well-known system that falls under the first category is the Mero KK system (see figure 14). This system is used for double layer grids, but the same system can

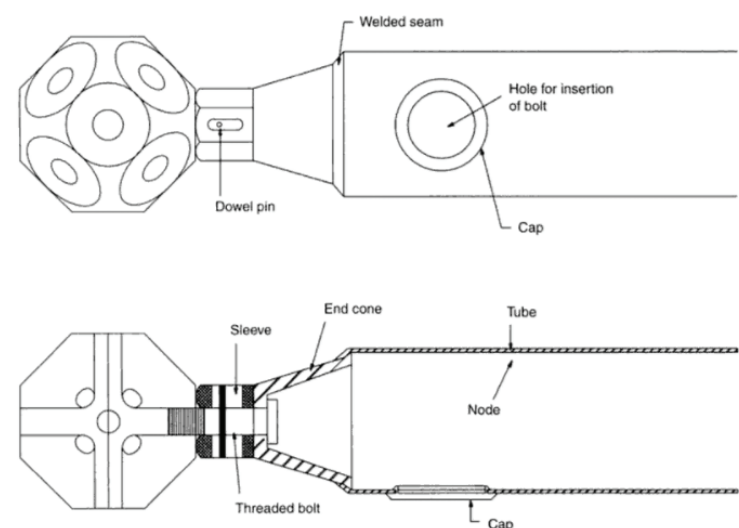
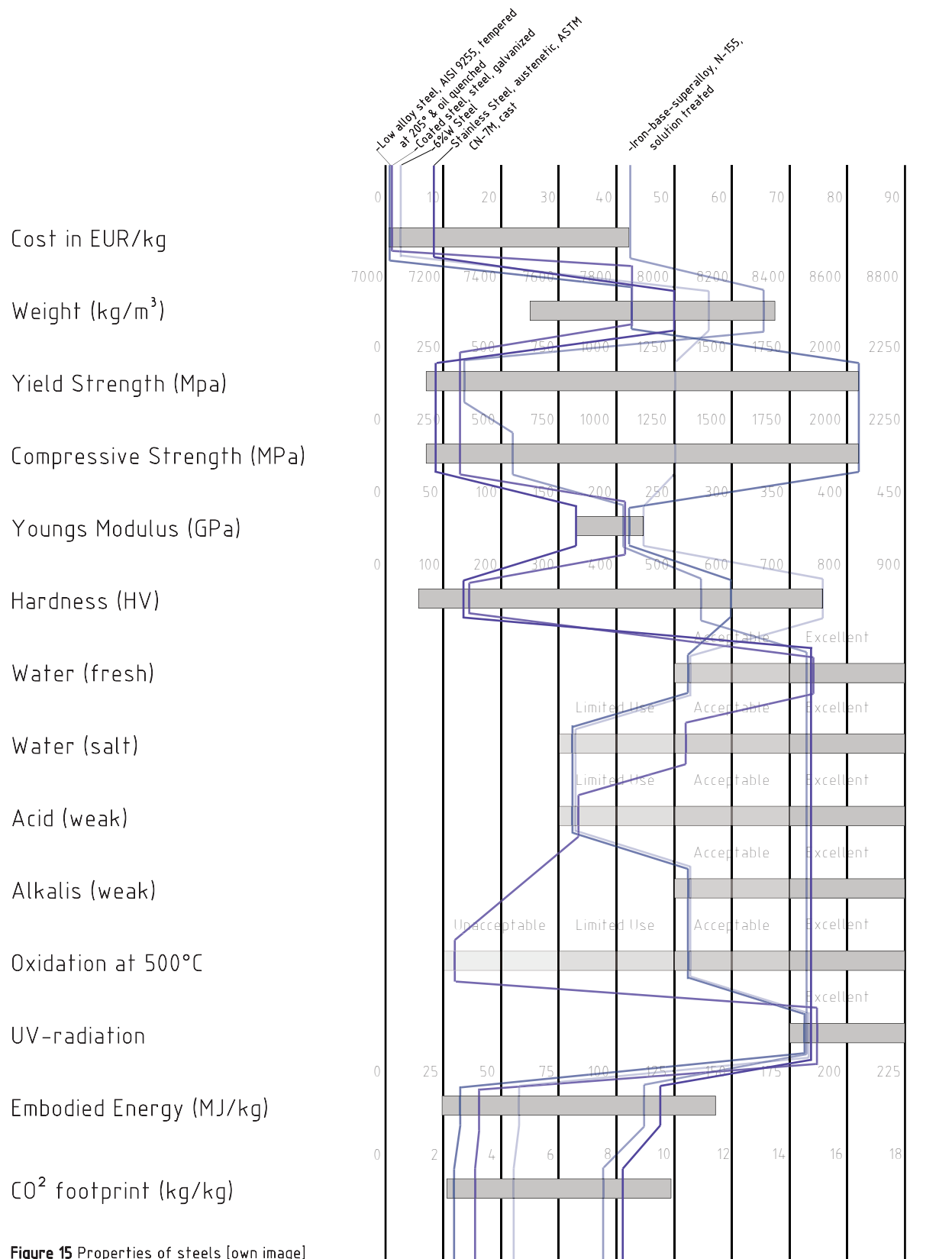


Figure 14 Mero KK system [Ramaswamy, Eekhout and Suresh, 2002]



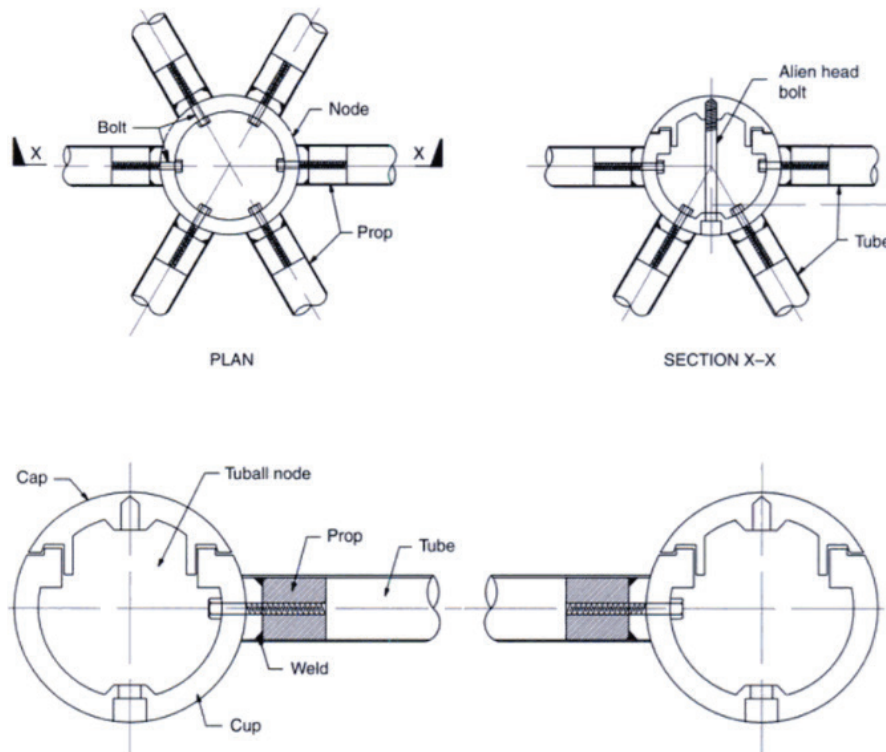


Figure 16 Tuball Connector [Ramaswamy, Eekhout and Suresh, 2002]

be applied to single layer grids. The traditional mero KK system is a universal system that contains steel solid nodes with 18 threaded holes at standard angles (45° , 60° and 90° relative to each other). By using modern computer controlled drilling techniques holes can be drilled under almost every angle, allowing the system to be used for complex double-curved space frames. However, the minimum angle between two holes should be at least 35° . The members consist out of hollow circular steel tubes with a coned end and a sleeve. Near the end of the member there is an opening through which the bolt can be inserted. The sleeve of the member contains a slotted hole and a pin to be able to tighten the bolt and to indicate whether it has been fully tightened. The opening is closed with a cap for aesthetical reasons. The connection can be described as a pinned connection. Thus, it is not able to transfer bending moments. [Chilton, 2000]

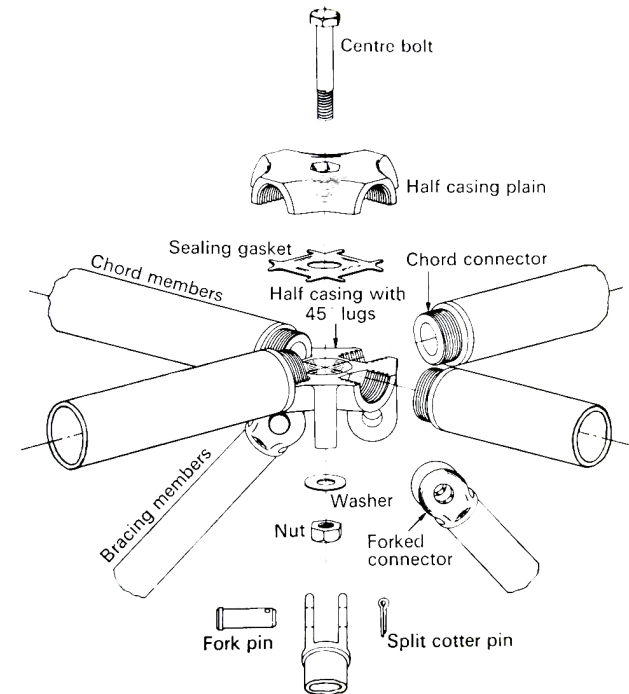


Figure 17 Nodus System [Chilton, 2000]

Hollow spherical nodes can be divided into two categories. The first category consists of an almost complete sphere, whilst the second one consists out of two hemispheres that are pressed together. An example of the first category is the Tuball Connector (see figure 16). Within the first category, bolts are inserted and tightened through an opening. The opening can be closed with a cap for aesthetical reasons. The members can be either rectangular or circular and the connections can be described as a pinned connections. The Tuball Connector is made of spheroidal graphite and is lighter and less expensive than solid connectors. An example of the second category is the Nodus System (see figure 17). The two half casings are made of spheroidal graphite. The assembly of the Nodus joint is relatively complex as it consists of many parts. One of the casings contains 45° lugs for the bracing members. The bracing members are connected with a nut and a

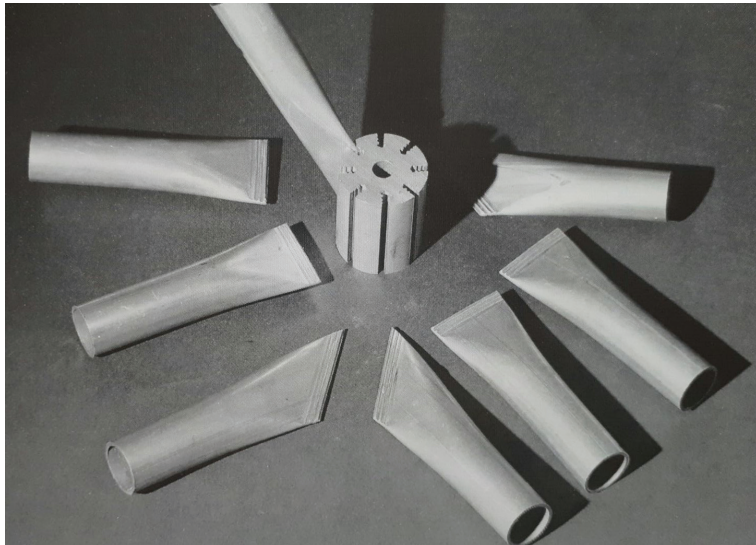


Figure 18 Triodetic system [Chilton, 2000]

bolt. The ends of the chord members are ribbed and are tightened by clamping the two half casings together. The casings are clamped by tightening a bolt that goes through the middle of these casings. These connections can be considered as clamped connections. A downside to this system is that possible grid configurations are limited to 45° and 90° degree angles. Only very small curvatures can be created with this system. [Chilton, 2000] [Ramaswamy, Eekhout and Suresh, 2002]

Cylindrical systems contain cylindrical nodes. The best known cylindrical system is the Triodetic system (see figure 18). This is the only system that is predominantly used with aluminium elements. However, steel can also be used. The system consists out of extruded sections with longitudinal ribbed slots. The members are crimped at the ends to fit into these slots. These members are fixed by clamping them in between two plates that are being held by a bolt that passes through the center of the extruded profile. These connections are capable of transferring bending moments in the direction of the slot. In addition to that, the members can be inserted in different angles and the slots can be at variable angles as well. Allowing for freedom in curvature design. [Chilton, 2000]

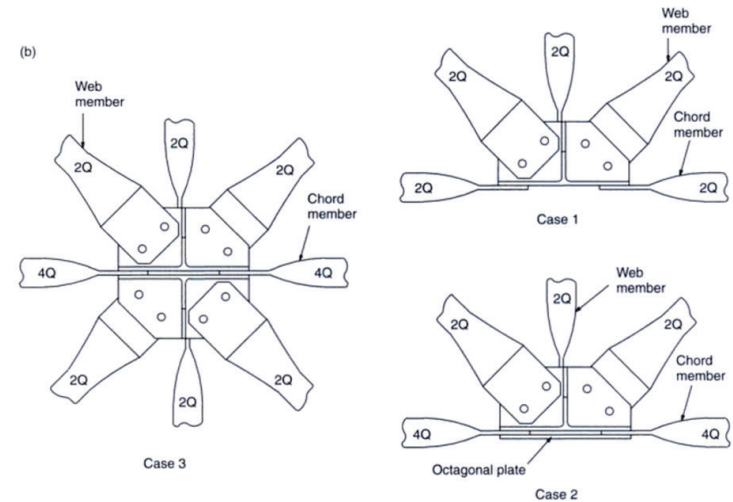


Figure 19 Octatube system [Chilton, 2000]

Plated systems consist of nodes that are made of flat or pressed plates. These types of connections are used for both steel and wood grid shells. The Octatube system (see figure 19) is an example of a plated system. The system consists out of a hexagonal plate onto which other plates are welded. The members are fixed by two bolt-nut connections. This connection is able to withstand bending moments around the axis perpendicular to the flattened part of the member. A downside to this system is that possible grid configurations are limited to 45° and 90° degree angles. Only very small curvatures can be created with this system. [Chilton, 2000] [Ramaswamy, Eekhout and Suresh, 2002]

Nodeless systems surpass the use of expensive nodal elements and directly connect the members onto each other. A downside of nodeless systems is that they tend to limit the possible configurations of the grid since the ends of members are designed to accommodate standard angles. An example of a nodeless system is the Octet Truss system (see figure 20) that was developed by Buckminster Fuller. The ends of the members consist out of an X-shaped cross section and are cut under angles that are suitable for tetrahedral and octahedral geometry. [Chilton, 2000]

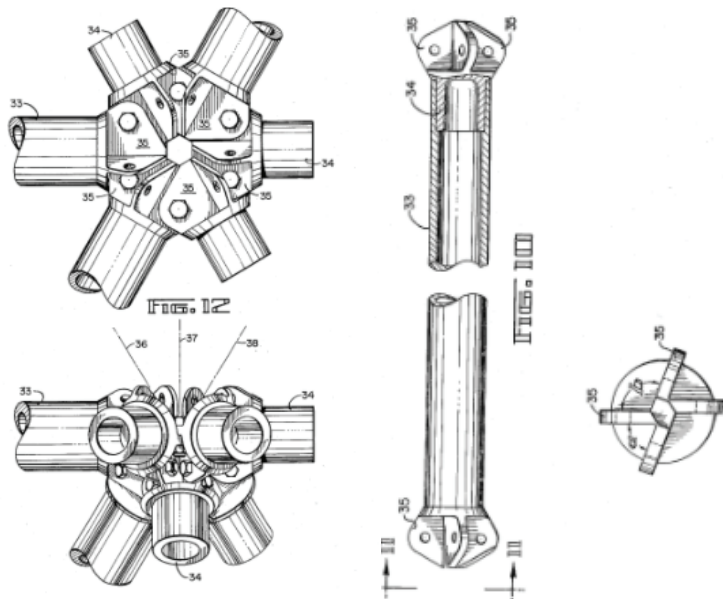


Figure 20 Octet Truss System [Buckminster Fuller, 1961]

Modules

Module systems can be described as larger scale combinations of members and joints that can be further assembled on site by bolting, threading or welding. The use of larger pre-fabricated modules reduces the amount of hours needed for on site assembly and speeds up the erection process. Another benefit of pre-fabricating off site is that a high degree of precision and quality of welds can be achieved. On the other hand, transportation costs are likely to increase as the size of modules is larger than separate joints and members. Systems can overcome this problem by modules that can be easily stacked into each other.

[Chilton, 2000]

A typical shape for modules is a pyramid. Systems like Unibat and Space Deck use pyramidal modules. The modules consist out of members and (usually) joints that are welded together. As a result, the joints form rigid connections. The Space Deck system (figure 21)

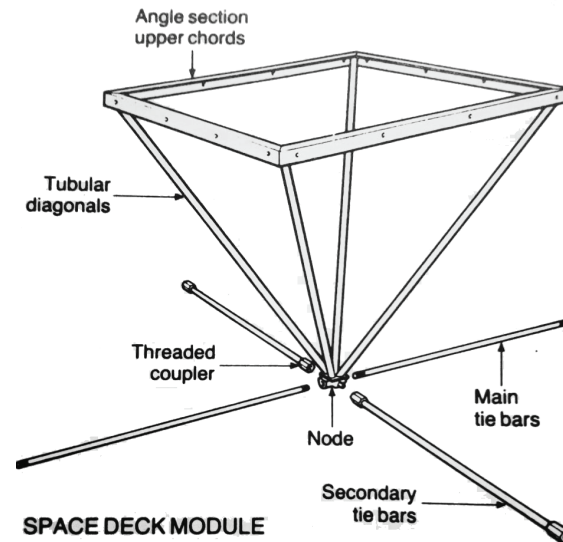


Figure 21 Space Deck System [Chilton, 2000]

consists out of a square base of steel angles that have been welded together. From the corners of this base tubular diagonals are welded to a node (top of the pyramid). The nodal element has threaded holes on two opposing sides and threaded studs on the remaining two sides. High tension steel members are used to connect the nodes of two adjacent modules whilst the square bases are directly connected to one another by bolting them together. The Unibat system works roughly the same. However, the square bases are only connected to each other in the corners. Allowing for a faster erection and a reduction in cost. [Chilton, 2000]

A downside of using standardized modules is that they restrict the possible amount of grid configurations. However, off site production of modules allows for a very accurate manufacturing of many different types of non-standardized modules. These modules can be connected on site to create complex double curved shapes.

3.2 Wood

Wood is a well-known organic construction material that has been used for construction for millennia. Arguably, it is mankind's original building material. In contrast to other frequently used construction materials, such as steel, wood is renewable. [Lyons, 2015]

3.2.1 Wood Structure

Wood consists out of three main elements: cellulose, hemicellulose and lignin. These are all natural polymers that give the wood its physical strength. The cellulose mainly contributes to the tensile strength whilst the hemicellulose and lignin mostly contribute to the elasticity and tensile strength of the wood. Cellulose is the polymer that is the main component of the tree's cell walls. The cellulose chains are made of glucose units that reach lengths of 10.000 units. The cellulose chains form a chrystalline structure. Hemicellulose is a polymer that consists of different sugar units in a chain that is shorter than the cellulose chains. As a result, this polymer is more gelatinous and forms a partially chrystalline structure. The lignin polymers

consists out of chains from benzene-like units and form a rather complex branched chain structure. These three components create so-called 'microfibrils' that form the building blocks for the cell walls. The cellulose chains are surrounded by the hemicellulose chains, which in their turn are again surrounded by cellulose chains. The combination of these three layers are then glued together with lignin. It is this composite structure that gives the wood its physical strength. [Lyons, 2015]

3.2.2 Classification

Wood can be classified into hardwoods and softwoods. These names suggest a link with the physical strength of the wood. This is true in most cases, however, this classification is based on botanical aspects.

Botanical Classification

Structural wood is defined as hardwood or softwood based on its botanical classification. Deciduous trees with broad leaves are classified as hardwoods. These trees loose their leaves in autumn when they are in a temperate climate. In tropical climates the trees continually renew their leaves. Conifers are classified as softwoods. Most of these trees are evergreens

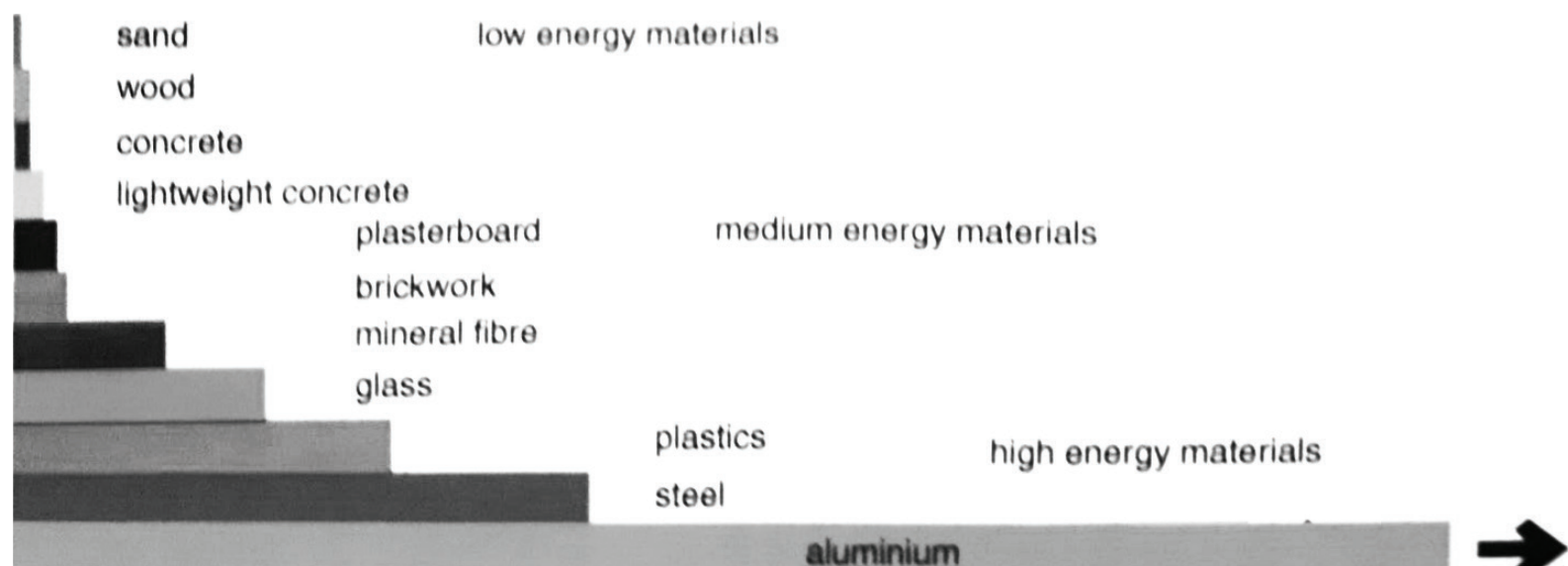


Figure 22 Relative embodied CO₂ in building materials by weight [Lyons, 2015]

and have needle shaped leaves. They predominantly grow in the northern hemisphere in temperate climates. Generally, hardwoods are stronger than softwoods. However, there are a few exceptions on this rule of thumb. For example, balsawood, which is a very soft and flexible wood that is often used for model making, is botanically classified as a hardwood. [Lyons, 2015]

Microscopically, softwoods show only one type of cells whilst hardwoods have a more complex cell structure. The cells of softwoods vary in size according to the seasons. In spring and early summer the tree grows rapidly, forming a relatively large-celled 'early wood'. During autumn and winter the cells form more slowly, forming the denser 'late wood'. Hardwoods can be divided into two categories according to the size of the vessels in the wood. They consist out of smaller cells that provide mechanical support and of larger cells that form vessels for the conduction food and water. The first category, the diffuse-porous hardwoods, contain vessels of approximately the same size. These vessels are evenly distributed throughout the wood. Beech, birch and most tropical woods can be grouped under this category. The second category, ring-porous hardwoods, have different sizes of vessels. The large vessels are located in the early wood whilst the late wood contains only small vessels. Oak, ash and elm are well-known ring-porous hardwoods. [Lyons, 2015]

Structural Classification

According to the standard both softwoods and hardwoods are listed with a four-letter code. The first two letters refer to the genus whilst the last two letters refer to the specific species. For example, a typical softwood like the western red cedar (*Thuja Plicata*) is referred to as THPL. Structural timbers are specified by either a strength classification or a combination of strength classification and timber species. The specification should also 'include lengths and cross-section sizes, surface finish or tolerance class, moisture content and any preservative or special treatments'. [Lyons, 2015]

Strength grading of timbers can be done in two ways: visual strength grading and machine strength grading. Visual strength grading requires a lot of skill from the person that does the visual inspection. Every piece of timber is visually inspected for defects that are mentioned in the chapter 'timber defects'. Based on the number of defects the timber is assigned to a grade. Visually graded softwoods can be graded as SS (Structural Grade), GS (General Structural Grade) or as rejected. Visually graded hardwoods can be graded as THA or THB (Heavy Structural Temperate Hardwood), TH1 or TH2 (General Structural Temperate Hardwood), or as rejected. Tropical hardwoods can only be graded as (HS) or as rejected. The grades THA and TH1 have fewer

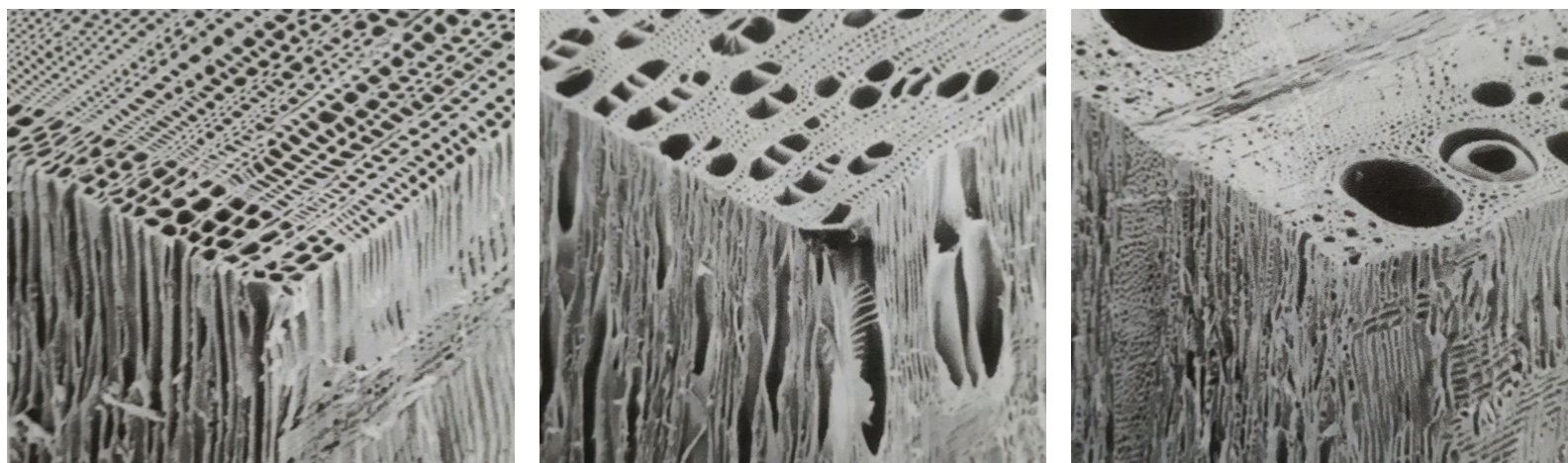


Figure 23 Softwood, Scots Pine (left) | Diffuse-porous hardwood, Birch (middle) | Ring-porous hardwood, Oak (right) [Lyons, 2015]

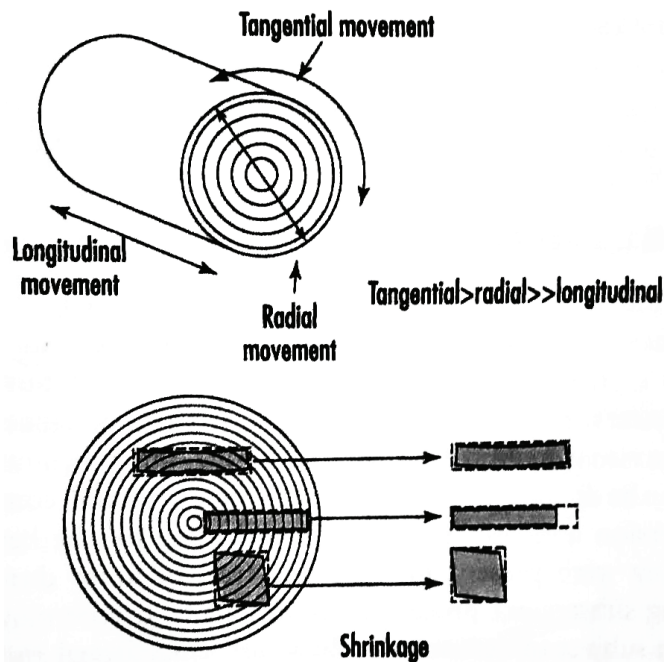


Figure 24 Moisture movement and drying shrinkage [Lyons, 2015]

natural defects as opposed to THB and TH2. For machine testing, there is usually a quick visual inspection as well that may immediately grade the timber as rejected. If the timber passes this visual test it is then tested by a special grading machine that moves the timber through a series of rollers. Based on the measured load that is needed to obtain a certain amount of deflection or based on the deflection produced by a standard load the timber is classified as a certain grade. There are four strength grades for machine graded timbers: C27, C24, C22 and C16. Sometimes the timber is graded with a corresponding colour for the grade. In the same order as the lettercodes that were mentioned: red, purple, blue and green. The number after the 'C' corresponds to the bending stiffness of the wood in GPa. C refers to coniferous softwoods. A 'D' in combination with a number can also be used. In this case the 'D' stands for deciduous softwoods. The C strength classes range from C14 to C50 and the D strength classes range from D18 to D70. Thus it can be concluded that structural woods have bending stiffness between 14 and 70 GPa. [Lyons, 2015]

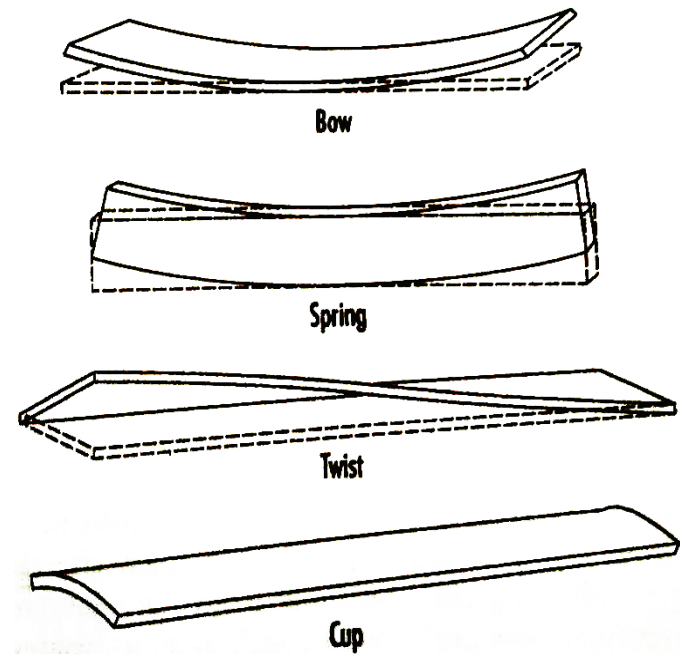


Figure 25 Warping of timbers [Lyons, 2015]

3.2.3 Manufacturing

The process of cutting logs into sections is called conversion. Further cutting these sections into usable sizes is called manufacturing. Timber that is used for solid structural sections is sawn. Timber can be sawn from logs in several types of cuts. There are two main ways of cutting logs referring to the angle between the face of the timber and the growth rings:

1. Plain sawn
2. Quarter sawn

If this angle is smaller than 45° the timber can be grouped under the category 'plain sawn'. If the angle is larger than the timber is quarter sawn. The angle between the face of the timber and the growth rings has a significant influence on the properties of the timber. Plain sawn timbers have a tendency to 'cup' (see figure 25). However, their appearance tends to be more aesthetically pleasing. Quarter sawn woods tend to be more weather resistant, hard wearing and

are more resistant to flaking. Typical cuts for the conversion of timber are shown in figure 26. By cutting a log through and through a mixture of plain sawn and quarter sawn timber is obtained. This type of cutting is the most economical since it does not require the log to be repositioned for each cut. The latter is the case in quarter sawing of logs. Sometimes the center of the tree is removed because it is soft and often obtains defects. The remaining center is called a 'boxed heart'. After the conversion of logs they are 'seasoned'. The primary aim of seasoning is to stabilize the timber to a moisture content that is compatible with the equilibrium conditions under which it is to be used' [Lyons, 2015].

Moisture Content

The moment trees are cut, they have a certain moisture content. Firstly, the tree will lose the water from the cell cavities whilst the cell walls contain their water content. As a result, no shrinking occurs. The point at which all the water from the cavities has disappeared is called the 'fibre saturation point'. After this point, water from the cell walls will be removed as well, resulting in shrinkage and hardening of the wood. Wood has different moisture movements along the wood: tangential movement, longitudinal movement and radial movement. Tangential movement is the fastest, followed by radial movement and longitudinal movement. When there is a large ratio between tangential and radial moisture movements the timber distorts (see figure 24). The shrinkage of the wood can cause distortion of the timber (see figure 24 and figure 25). The process that is called 'seasoning' controls the loss of moisture from timber. Moisture evaporates from the surface first followed by moisture from the center. The moisture loss needs to be controlled. If the surface dries out too quickly whilst the center is still wet several problems can arise. There are two main types of seasoning:

1. Air seasoning
2. Kiln drying

During the air seasoning process, the timber is protected from the ground and from precipitation. The timber is stacked in layers with strips of wood in between. Due to the strips of wood air is capable of passing through the stack. The ends of the timber are protected by coatings to ensure no rapid moisture loss occurs. The process of acquiring the desired moisture content last up to a few months for softwoods and up to years for hardwoods. The second type of seasoning uses a closed chamber in which the temperature and moisture content is controlled. By following a specific programme for this temperature and moisture content timber can be seasoned without significant degradation of the timber. For economical reasons, the timber is often air seasoned up to the fibre saturation point and kiln seasoned up to the desired moisture content. According to Lyons

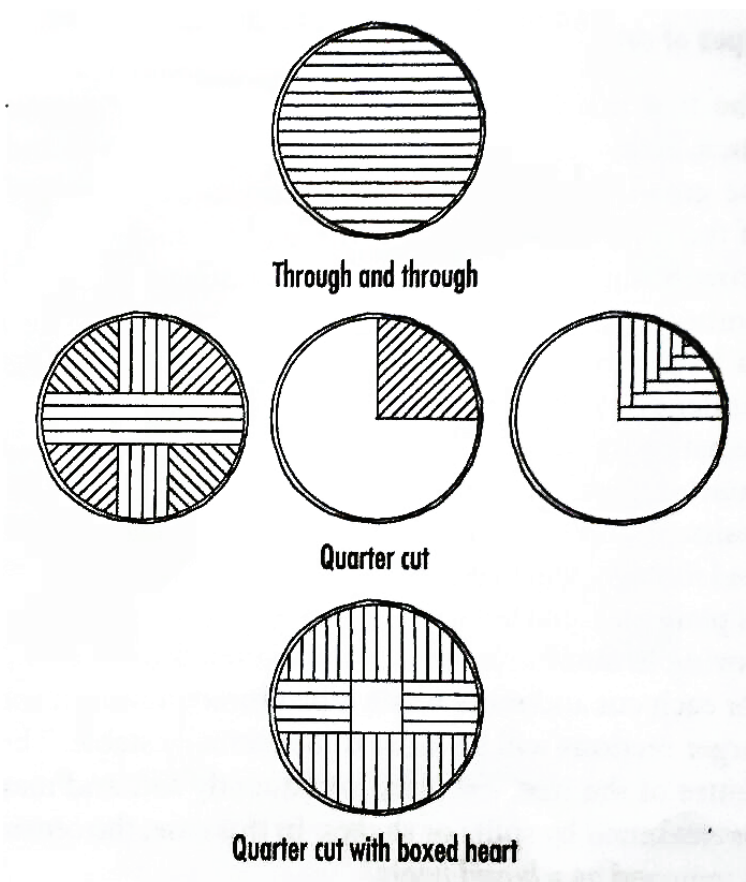


Figure 26 Conversion of timber [Lyons, 2015]

[2015] this saves roughly half of the kiln time and fuel. Kiln seasoning from the fibre saturation point up to the desired moisture content usually takes a few days for softwoods and about three weeks for hardwoods. [Lyons, 2015]

Timber Defects

Timber can show several so-called defects. Most of these defects enhance the visual quality of the wood like knots. Timber defects can be grouped in three categories:

1. Natural Defects
2. Conversion Defects
3. Seasoning Defects

From these defects the seasoning defects have been discussed already (distortion and cracks). There are four types of natural defects: knots, natural inclusions, compression and tension wood and lastly abnormal growth rings. Knots are formed at places where branches of the tree meet the trunk. Live knots are formed by living branches whilst dead knots are formed by dead branches. Dead knots are often very loose and cause structural weakness in the timber. Natural inclusions include 'bark pockets', 'pitch pockets' and 'resin streaks'. Bark pockets occur when pieces of bark are enclosed by timber. This is caused by earlier damage of the growth layer. Pitch pockets and resin streaks are parts of the timber that contain fluid resin and can often be found along the grain of softwoods. Compression and tension wood is formed in trees that are subjected to excessive forces (strong winds, slopes, etc.) trees produce these types of wood to counteract these forces. Compression wood can be produced in softwoods and has a larger content of lignin and a darker colour. Tension wood is formed in hardwoods and contains an extra layer of cellulose in the wood's cell walls. It is lighter of colour. Compression and tension woods have a very high longitudinal shrinkage upon seasoning, causing

significant distortions. Lastly, abnormal growth rings can affect the timber strength. Ideally timber contains five rings per centimeter for softwoods and three rings per centimeter for hardwoods. Very slow growth and very fast growth forms timber that is weaker. [Lyons, 2015]

Conversion defects, are as the name suggests defects that are caused by the conversion process. There are two types of conversion defects: sloping grain and wane. Ideally the timber should be straight grained. If the slope of the grain increases, the bending strength of the timber decreases. The slope in hardwoods needs to be limited to 1 in 11 whilst the slope in softwoods needs to be limited to 1 in 6. Wane is when a corner of the timber is cut off due to the cutting of a piece of bark or curved surface. A small degree of wane is acceptable in structural timbers. [Lyons, 2015]

Laminated Timber

Large timber pieces are restricted by their availability and there are restrictions to the maximum size in which structural wooden members can be sawn. In addition to that, the strength of the sections is based on the weakest part of the material. The fabrication of laminated timber eliminates these restrictions. Laminated timber is manufactured by gluing several smaller timber sections together with resin. The use of very large laminated timber members allows the architect to design large spans in wood. [Lyons, 2015]

3.2.4 Properties

Woods can have a variety of properties depending on the species. Figure 27 shows the range in values for different (mechanical) properties. The specific properties for Oak, Balsa and Kaneelhart are shown in this figure as well.

Mechanical Properties

When comparing the properties of woods and steels several conclusions can be drawn. Woods weigh

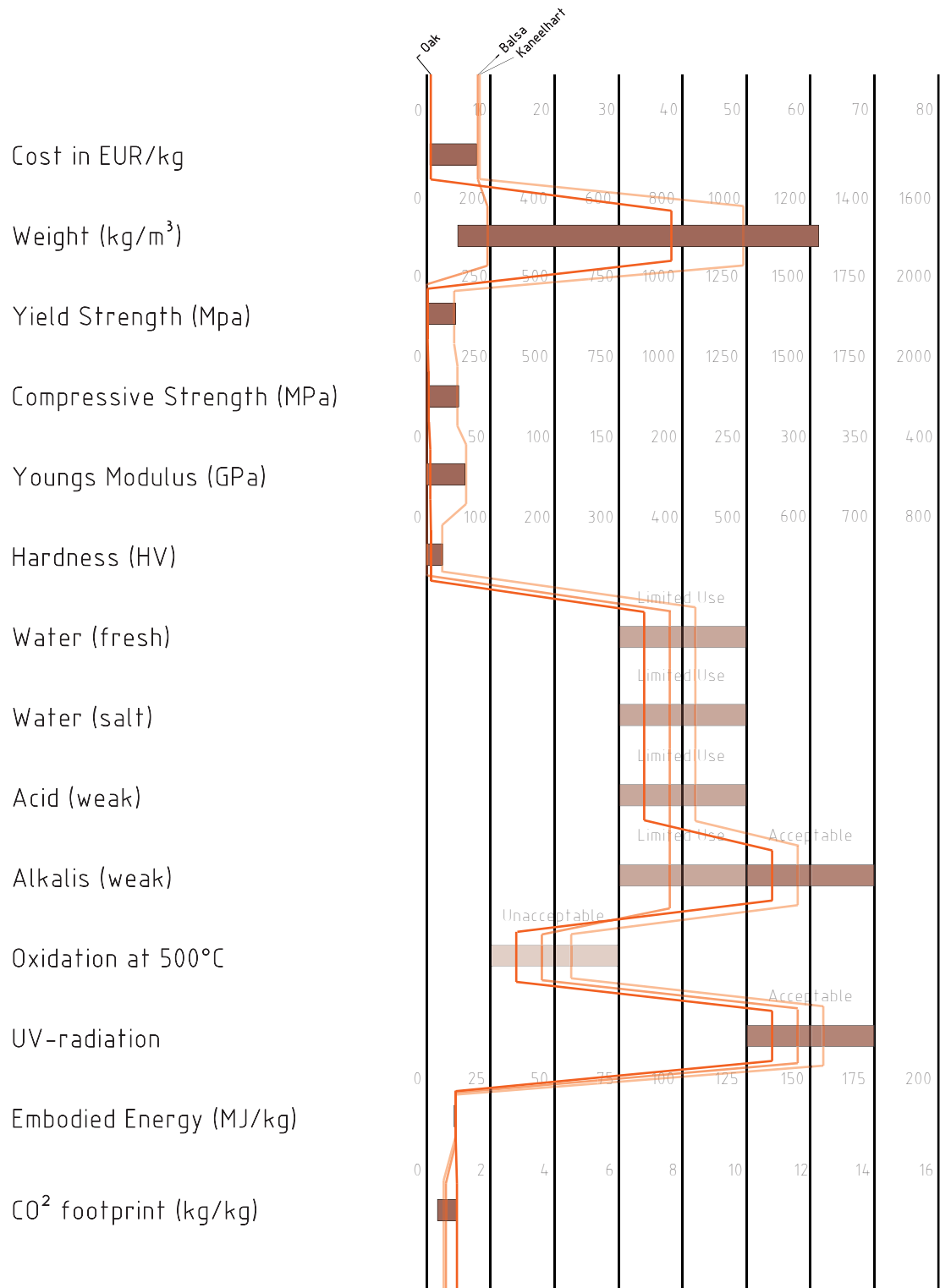


Figure 27 Properties of woods [own image]

approximately 11,5 times less than steels. In addition to that, the yield strength, compressive strength, young's modulus and hardness all lie (significantly) higher for steels. However, they also use (a lot more) energy and produce a lot more CO² per kg in the primary production. Furthermore, there are steels that have excellent resistance against water (fresh and salt), weak acids, weak alkalis, oxidation at 500°C, and UV-radiation. On the contrary, all woods have a limited resistance against fresh water, salt water and weak acids and perform badly at high temperatures. Lastly, woods are usually cheaper than steels depending on the wood species and the steel alloy.

Durability and Sustainability

In comparison with other commonly used building materials, wood uses very little energy per kilogram in its process of extraction, manufacturing and construction process. According to Lyons (2015) concrete, plasterboard, brickwork, mineral fibre, glass, plastics, steel and aluminium all use more energy within these processes (see figure 22).

In addition to that, the material is renewable. However, in order for the extraction of wood from nature to be durable and sustainable, the extraction process needs to be properly managed. Deforestation has two major effects on nature. Firstly, deforestation allows wind and rain to corrode the soil layer that is needed for plants and trees to grow on. As a result, nothing is capable of growing within these areas, leaving desert-like conditions. Secondly, the reduction of the amount of (rain)forests world wide has a significant effect on the greenhouse effect. The most important greenhouse gas is CO² and trees are capable of extracting these CO² gases from the air. Deforestation causes a reduction of the rate in which CO² gases are extracted, leaving more CO² in the air. As a result, the greenhouse effect increases. For these reasons, immediate replanting and cutting trees in the same rate (or a slower rate) as trees are growing is necessary. Wood that is certified

with the FSC (Forest Stewardship Council) or the PEFC (Programme for Endorsement of Forest Certification) certification ensures the wood comes from sustainably managed sources. [Lyons, 2015]

The durability of wood is greatly affected by its deterioration. The major causes for deterioration in timber are:

1. Weathering
2. Fungi
3. Insects
4. Fire

Timber weathers under the influence of rain, sunlight and wind. Timbers slowly lose their colour and turn grey, especially if they are used in exterior conditions. Under the influence of sunlight and oxygen cellulose and lignin slowly break down into materials that can be dissolved in water. This causes the loss of colour of the timber. If timber repeatedly gets wet it can start to show (bigger) cracks. In addition to that, it increases the risk of deterioration caused by fungi. Fungi are simple plants that are not capable of the process of photosynthesis. As a result, they have to obtain their nutrients from other organic materials. Fungi can grow in timber if the moisture content is 20% or higher. Most fungi species thrive at a moisture content between 20% to 30% with a temperature between 20°C and 30°C. Some timbers are resistant to fungi because their heartwood contains elements that are poisonous to fungi. Timber can be classified into five categories that indicate the resistance of the wood against fungi:

1. Very Durable
2. Durable
3. Moderately Durable
4. Slightly Durable
5. Not Durable

Fungi can be classified in two different categories:

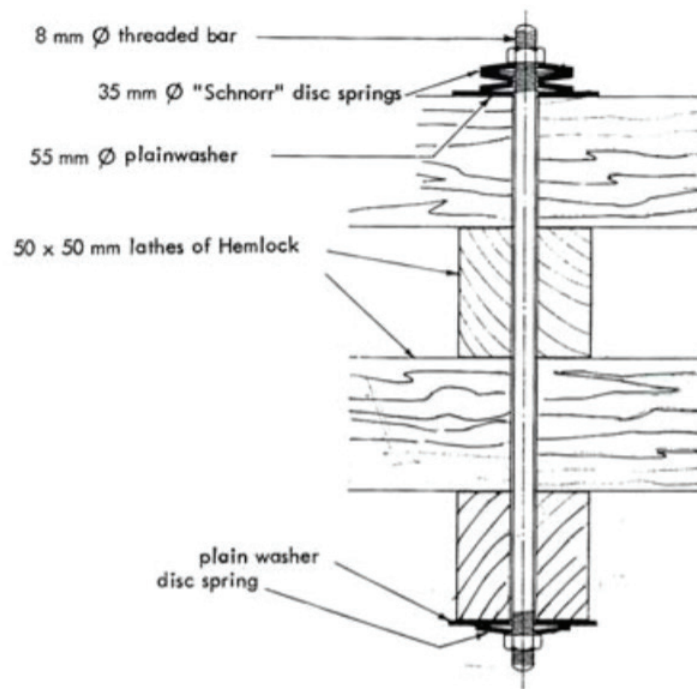


Figure 28 Nodal detail of the Multihalle Mannheim [Toussaint, 2007]

moulds and stains; and wet and dry rots. Moulds and stains usually do not have a significant effect on the strength of timber. Softwoods are more prone to attack than hardwoods. Kiln drying of timber greatly reduces the risk of attack by moulds and stains as it quickly reduces the moisture content at the surface of the timber. Wet and dry rots are very destructive fungi that greatly affect the strength of the timber. They can be classified as soft rots, brown rots or white rots. Soft rots mainly affect the surface of the timber and usually only occurs at timbers that are buried in the ground. Brown rots attack the cellulose within timber, causing the wood to get a darker colour. White rot attacks the cells itself, causing the wood to get lighter. Dry rots are a major cause of deterioration of woods. Dry rots can grow at a moisture content of 20% and higher. They form cotton-wool like patches over the surface of the timber and the mycelium strands can reach a diameter of 20mm. An attack of dry rot causes the timber to break both along and across the grain into dry and brittle parts. Wet rot grows a moisture content

between 40% and 50%. Therefore, it usually only occurs in case of leaks or condensation problems. The timber cracks along the grain. Fungal attack is mainly an issue in colder wet climates. [Lyons, 2015]

Fungal attack is less of an issue in warmer climates. However, in these warmer climates insect attacks pose a threat for timbers. Several beetle species and termites can attack timber which reduces the strength of the material. Both beetle larvae and adult beetles bore through the timber causing a reduction in mechanical strength. Insects that are common in the UK and its surroundings are the Common Furniture Beetle, the Death Watch Beetle, the House Longhorn Beetle, the Powder Post Beetle and the Ambrosia Beetle. Termites used to only pose a threat in tropical climates. However, certain species now pose a threat in temperate climates as well. [Lyons, 2015]

3.1.4 Connections

According to Chilton [2000] there are hundreds of space grid systems. Space grid frames can be divided into three main categories:

1. Piece-Small Systems
2. Continuous Chords
3. Prefabricated modules

The second method is the most commonly used method for wooden grid shells. [Chilton, 2000]

Multihalle Mannheim

A great example of a wooden grid shell that uses continuous chords as a nodal joint solution is the Multihalle Mannheim (see figure 28 and figure 29). The roof was constructed by pushing the flat grid up towards its final shape. This is a common method for erecting wooden grid shells. A wooden grid shell that is made of straight members is called a strained shell. Shells that are constructed by using pre-bent members

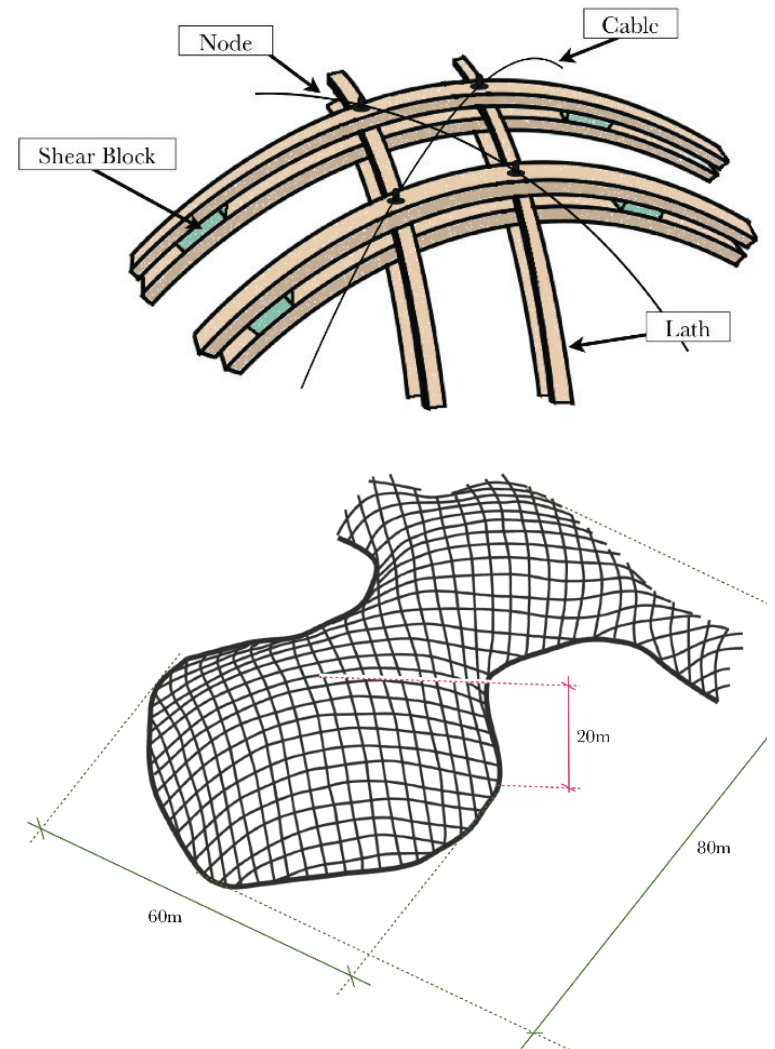
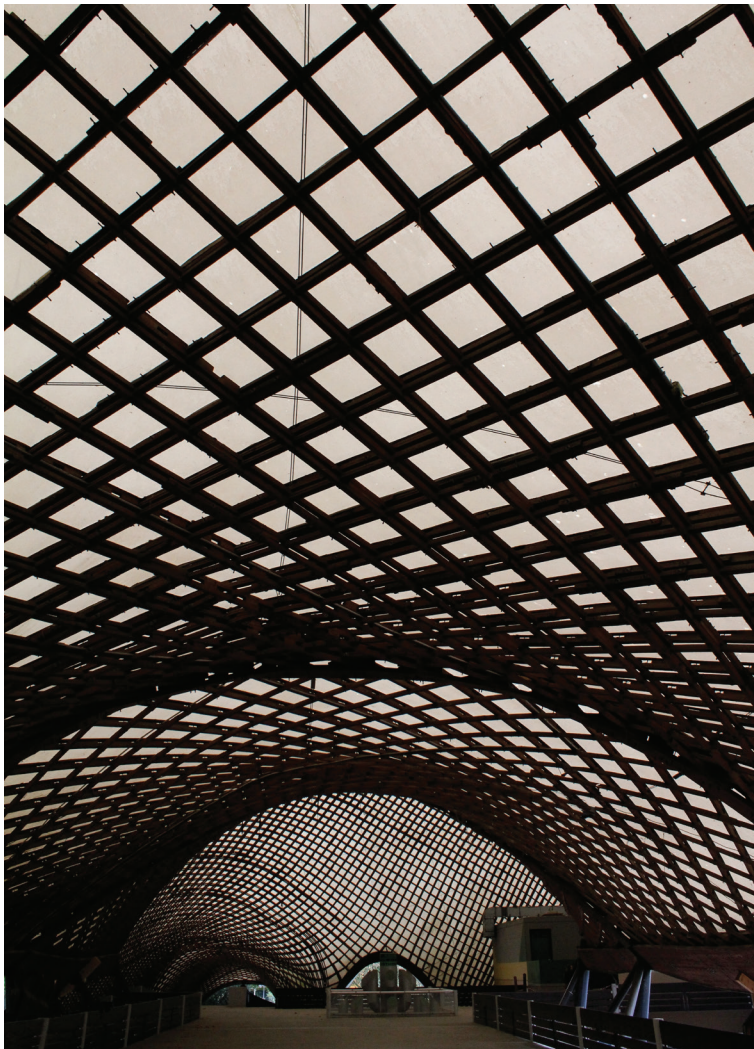


Figure 29 Interior of Multihalle Mannheim (left), structural detailing principles (upper right) and the overall dimensions (bottom right) [Evolution of German Shells, 2013]

are called unstrained shells. The transition from flat grid to curved grid sets certain requirements for the detailing of the nodal connections. This is especially true for the Multihalle Mannheim grid that consists of a double layer of laths for increased stiffness of the overall structure. During the erection the outer layer slides more than the inner layer. Thus, the nodal connection should allow for this movement by including slotted holes. In addition to that, the shape of the grid transfers from a square grid to similar parallelograms. This means the nodal connections should allow for

rotation as well. A pinned connection allows for this rotational freedom. After the erection process, the bolts are tightened to create friction to prevent slipping of the laths. Tightening of the bolts increases the stiffness of the structure when subjected to shear stress. In order to further increase the stiffness so-called shear blocks were introduced (see figure 29). The tightened bolts and shear blocks provide in out of plane shear stiffness. Diagonal steel cables were needed to provide the structure with its diagonal stiffness. These cables are connected to every sixth

node. [Toussaint, 2007] [Evolution of German Shells, 2013]

Weald and Downland Gridshell

The Weald and Downland Gridshell is a double layer gridshell made from oak laths. The high moisture content of green oak gives the timber great flexibility during the erection process. Natural drying of the oak once in its final position strengthens the entire structure. In order to obtain laths of great lengths, shorter laths were fingerjointed using special glue technology. On site, scarf joints were used to joint 6m long laths together. Unlike the Multihalle Mannheim, this gridshell is pulled down into its final shape from a flat position. The nodal connection needs to allow for sliding and for rotation. The solution for this gridshell's nodal connections is shown in figure 30. The nodal connector consists out of three plates. The middle plate has a pin in its center that keeps the two middle laths in place. The two outer laths can slide freely during the erection. After the erection the bolts are tightened. Two of the four bolts can be used to fasten the diagonal bracing laths. Shear blocks were added to increase the stiffness of the overall structure. The diagonal bracing laths of the structure also support the wall and roof cladding. [Slavid, 2009] [Toussaint, 2007]



Figure 30 Nodal Connection of the Weald and Downland Gridshell [Cullinan Studio, n.d.]

Savill Garden Shell

At first sight, the detailing of the Savill Garden Shell (see figure 31) and the Multihalle Mannheim seem to be very equal. However, with the Savill Garden Shell the two layers of the double layer grid were not bent at the same time. Therefore, no slotted holes or other solutions to allow for sliding during the erection were needed. Firstly, the bottom layer was bent. Then, shear blocks were screwed onto the bottom layer. Lastly, the last layer was screwed on top of these shear blocks. By separately bending the layers into shape, a larger spacing between the two layers became possible. As a result, this shell has a greater out of plane strength and stiffness. Also, the Savill Garden Shell uses a continuous layer on top of the grid for diagonal stiffness instead of diagonal laths or cables. [Harris and Roynon, n.d.] [Toussaint, 2007]

Oguni Dome

All wooden grid shells that were previously discussed can be grouped under the continuous chord category. The Oguni Dome nodal connections can be grouped under the category Piece-Small Systems. Nodal connectors



Figure 31 Interior of the Savill Garden Shell [Cullinan Studio, n.d.]

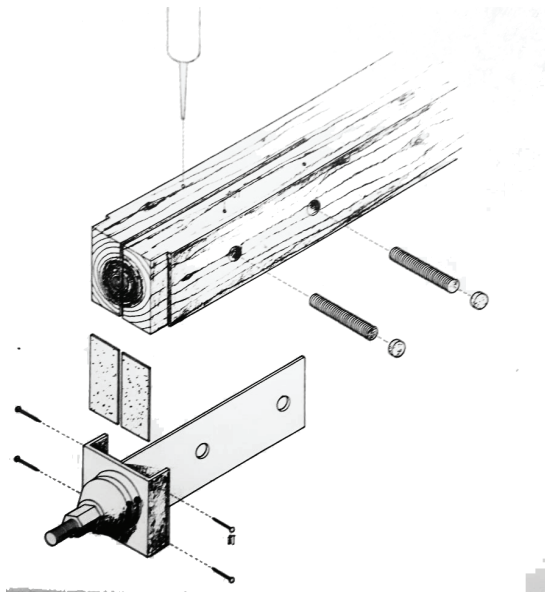


Figure 32 Connection Principal Oguni Dome [Chilton, 2000]

for piece small systems that can be used for wooden grid shells are very similar to the ones used in steel gridshells. However, were steel sections for grid shells are usually hollow, wooden sections for grid shells are solid. Thus, a different method of connecting the wooden members to the nodal connectors needs to be found. A standard steel truss system with spherical joints was adapted to be suitable for timber members. (see figure 32 and figure 33). [Chilton, 2000]



Figure 33 Oguni Dome [Chilton, 2000]

Library Pavillion

The Library Pavillion (see figure 34) by students of the Singapore University of Technology and Design can be grouped under the category 'Prefabricated modules'. All elements from this gridshell were fabricated from lasser-cutted flat plywood and flat steel sheets. The nodal connectors have a star shape and are completely flat. Off site several members and connectors are screwed together. These pre-assembled pieces are further assembled by screwing them together on site. [Architect, 2013]

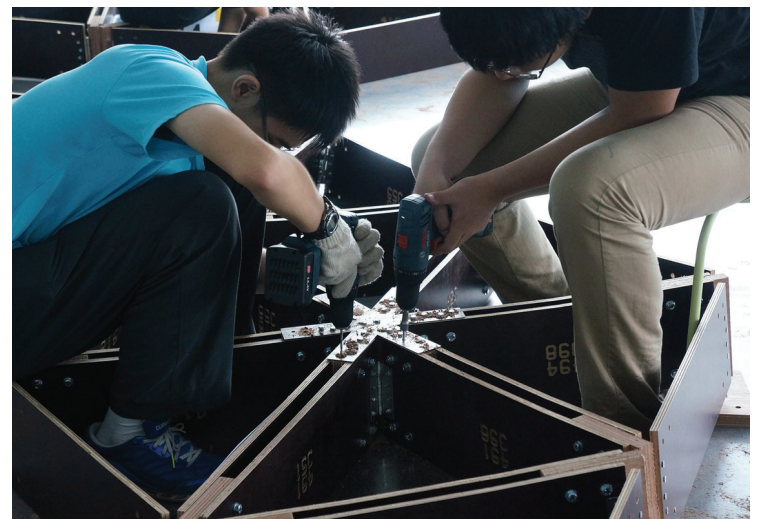


Figure 34 Library Pavillion [Archinect, 2013]

3.3 FRP

The use of fibre-reinforced polymers (FRP) has only fairly recently been introduced in the building industry. The first building of glass fibre-reinforced polymers (GFRP) was constructed in 1955. The building served as a radiostation. Enclosures of radiostations need to be as free from metals as possible to prevent interference with the signals. Thus, the need for metal free enclosures led to the introduction of FRP in the building industry. FRP has some advantages compared to wood and steel. It does not rot or rust and it does not get attacked by fungi or insects. In addition to that, FRP has the strength of steel but weighs 4 times less.

[Cremers et al., 2011]

3.3.1 Polymers

FRP consists, as the name suggests, of both fibres and polymers. Polymers can be classified in three groups based on the manner in which molecules are arranged:

1. Thermoplastics
2. Elastomers
3. Thermosets

The three categories differ in the amount of cross-links that exist between molecular chains within the material (see figure 35). The amount of cross-links have a significant influence on the characteristics of the polymer. Thermoplastics have no cross-links between the chains. As a result, they are very meltable and mouldable. However, the mechanical properties of thermoplastics are inferior and undesirable for structural use. A benefit is that thermoplastics can be easily recycled by melting and remoulding them. Elastomers, or rubbers, have weak cross-links between the molecular chains. As a result, they can not be melted after the cross-linking. Elastomers can be stretched too large extents. Due to their low strength they are not suitable to be used as construction materials. However, elastomers are frequently used as seals within the building industry. Thermosets have a very strong cross-linking between the molecular chains. They can not be melted after cross-linking and have very good mechanical properties. In addition to that, they have a high heat resistance. Hybrid forms of two of the three groups exist as well, combining the favourable characteristics of both groups. [Cremers et al., 2011]

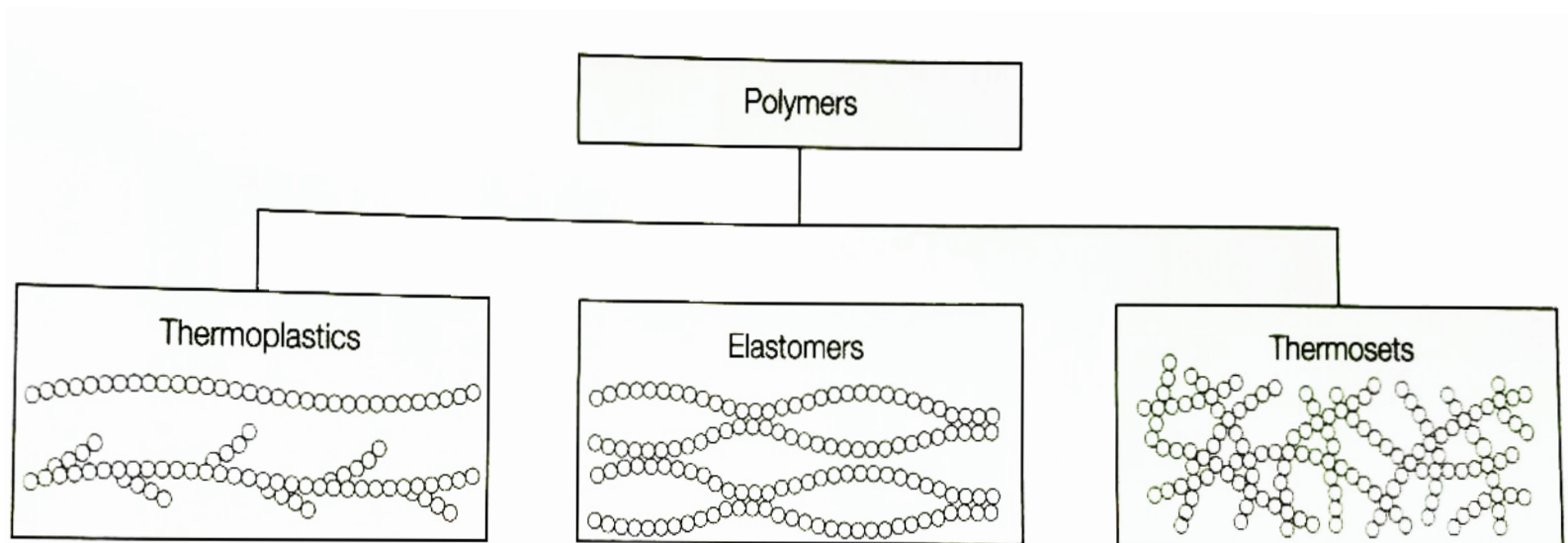


Figure 35 Chemical structure of thermoplastics, elastomers and thermosets [Cremers et al., 2011]

+ to +++ Improvement in properties - Reduction in properties		Tensile strength	Compressive strength	Elastic modulus	Impact strength	Reduced shrinkage	Better heat resistance	Chemicals resistance	Economy
Spherical fillers	Sawdust	+			+	+			+
	Carbon black			+	+	+			
	Metal oxides			+	-	+	+		
	Calcium carbonate (chalk)		+	+	-+	+	+		++
	Kaolinite				-	+	+	+	+
	Silica			+	-	+			+
	Sand/quartz powder		+	+	-	+			++
Flake-like fillers	Graphite					+		+	+
	Talcum		+	+	-	+	+		+
	Mica	+		++	-+	+	+	+	+
Fibrous fillers and reinforcing materials	Polymer fibres				++				
	Carbon fibres	+++	+	+++	-	+	++		-
	Glass fibres	++	+	++	-+	+	++		+

Figure 36 Fillers/Additives and their effect [Cremers et al., 2011]

Manufacturing

Polymers are manufactured in chemical plants where three different reaction mechanisms take place in order to form the polymer:

1. Polymerization
2. Polycondensation
3. Polyaddition

During the polymerization reaction, so-called monomers are connected to form longer polymers with the help of a catalyst that increases the speed of the polymerization process. This reaction does not produce any byproducts. Polycondensation combines two different types of molecules while producing water, hydrogen chloride, ammonia or alcohol as a byproduct. Polyaddition does roughly the same as polycondensation with the main difference that it does not produce any byproduct.

[Cremers et al., 2011]

Thermoplastics are manufactured in granular or powder form. On further processing, they are melted and moulded to create the final product. Elastomers are produced from crude rubber, which does not contain any cross-links. Firstly, the crude rubber is melted and moulded. Then, the vulcanization process is carried out to form the cross-links in the polymer. The vulcanization process uses sulphur, high temperature and high pressure to form the cross-links. Thermosets are produced from a liquid product (resin) that does not contain cross-links. The cross-links are formed during a chemical reaction that takes place when a hardener is added to the resin. [Cremers et al., 2011]

Fillers and Additives

The (mechanical) properties of polymers are influenced by the fillers and additives that are mixed in with the basic material. Fillers are substances that are added

to the mixture with the sole purpose of 'extending' the polymer (to increase its overall volume). Additives are substances that are added to the mixture in order to improve properties or function as catalyst or reactant. Usually it is the additives that make the polymer useful for its purpose. Additives do not only change the (mechanical) properties of the polymer, they also change its workability. According to Cremers et al. [2011] the following materials are often used as fillers and additives:

1. Cost-reducing fillers (f.e. chalk)
2. Colourants (dyes and pigments)
3. Catalysts and reactants
4. Stabilizers for improved durability (f.e. resistance against UV)
5. Plasticisers (in thermoplastics to prevent brittleness)
6. Flame retardants
7. Thixotropic materials (for improved coating properties with thermosets)

Dyes and pigments are used to change the opacity and colour of the polymer since polymers are usually transparent. Dyes are soluble in polymers, whilst pigments are insoluble and added in the form of tiny particles. Dyes should be insoluble in water if the colour should not fade during usage. They produce a transparent colour as opposed to pigments that can produce an opaque colour depending on the size of the particles. A list of commonly used fillers and additives and their effects is shown in figure 36. [Cremers et al., 2011]

3.3.2 Fibres

Fibres have a very small cross-section in relation to their length. Therefore, they have a very distinct orientation. Fibres are flexible and strong and can be used for a variety of purposes. They can be either natural or man-made (synthetic). Man-made fibres are preferred for structural applications because

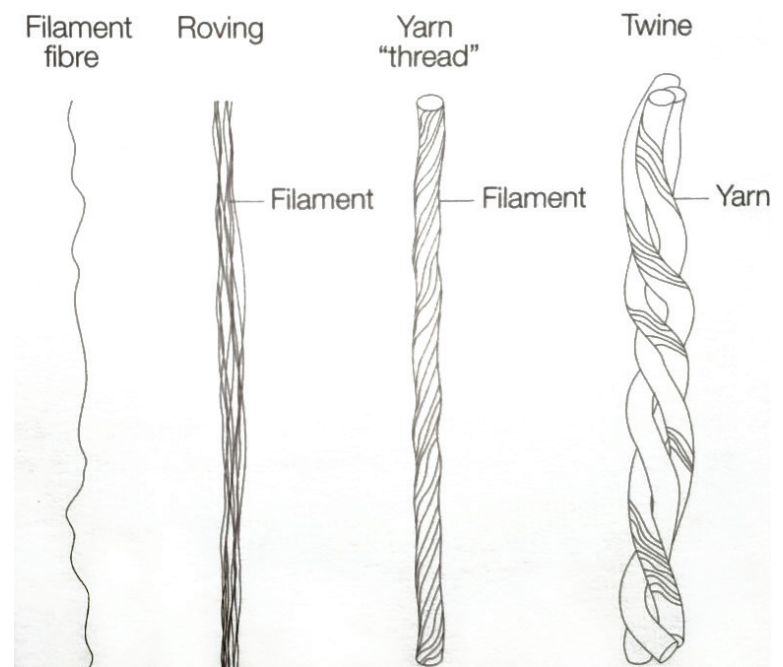


Figure 37 Processing of individual fibres [Cremers et al., 2011]

only these fibres possess the necessary strength and durability. Fibres can be grouped under four categories:

1. Inorganic Fibres
2. Polymer Fibres
3. Metal Fibres
4. Natural Fibres

The majority of fibres consists of bundles of tiny fibres. Glass and metal fibres form an exception to this rule. Individual fibres are called filaments. These filaments are usually too fine for further processing. Thus, they are grouped in larger bundles after production. A bundle of parallel filaments is called a 'roving' whilst a bundle of twisted filament is called a 'yarn thread'. Twisting multiple yarn thread together is called a 'twine' (see figure 37). Fibres have a strength that lies significantly higher than that of their respective raw material's strength. The melting and stretching of the material to form the fibres aligns the internal structure in the longitudinal direction, making it much stronger. [Cremers et al., 2011]

Inorganic fibres

All fibres that are not based in carbon compounds are inorganic fibers. Carbon fibers that are made purely from carbon are considered to be inorganic fibres as well. Based on this description, metal fibres are inorganic as well. However, metal fibres are usually treated as a separate category. Inorganic fibers have a few major advantages as opposed to organic fibers: they do not display creep, they can be used under much higher temperatures and they are a lot stiffer. There are two kinds of inorganic fibres:

1. Glass Fibres
2. Carbon Fibres

Glass fibres come in several additional categories. E-glass (electric glass) is the standard product for glass fibres. AR-glass is alkaline resistant, thus the name AR. C-glass (corrosion) has a good chemical resistance. And then there are the high strength R-glass (resistance) and S-glass (strength) types. Glass fibres have an amorphous structure, making them isotropic. In other words, the material has identical properties in both the longitudinal and the transverse direction. Glass fibres do not burn, show almost no creep and show no decrease in mechanical properties up to 250 °C. Carbon fibres can be grouped under three categories: standard modulus fibres with high tensile strength, high modulus fibres with a high material stiffness and intermediate modulus fibres with moderate tensile strength and material stiffness. Carbon fibers are anisotropic. The tensile strength and stiffness of the material lies much lower for the transversal directions. Carbon fibres are prone to buckling. They weigh very little, have a very high corrosion resistance and show no creep. A downside of carbon fibres is that they are very expensive since the process of creating them requires a lot of heat. [Cremers et al., 2011]

Polymer Fibres

Polymer fibres are lighter and stronger than inorganic

fibres. In addition to that, these types of fibers tend to creep. Polymer fibres are highly anisotropic. Their strength and stiffness in the transversal direction lies significantly lower than in the longitudinal direction. Most polymer fibres are not used in fibre reinforced composites as they are often too flexible for structural uses. There are two exceptions to this rule:

1. Aramid Fibres
2. Polyethylene Fibres

Aramid fibres are very lightweight. The compressive strength of these fibres lies a lot lower than its tensile strength. Aramid fibres have some important disadvantage that explain the limited use in the building industry: they have a low resistance against UV-light and they are difficult to machine. Polyethylene fibres are so light that they can float on water. The compressive strength of these fibres is almost equal to zero. Polyethylene fibres have a very good impact resistance. [Cremers et al., 2011]

Natural Fibres

Natural fibres are not used in the building industry very often due to their large scatter of mechanical properties and moisture sensitivity. It is expected to see an increase in the use of natural fibres due to the sustainability aspect. Natural fibres can be divided into three categories:

1. Mineral Fibres
2. Animal Fibres
3. Vegetable Fibres

Only vegetable fibres can be used within the building industry due to their more desirable mechanical properties. Hemp and Flax fibres have good mechanical properties. However, they are still inferior to glass fibres. The production of these fibres use a significantly lower amount of embodied energy and CO² in the primary production. [Cremers et al., 2011]

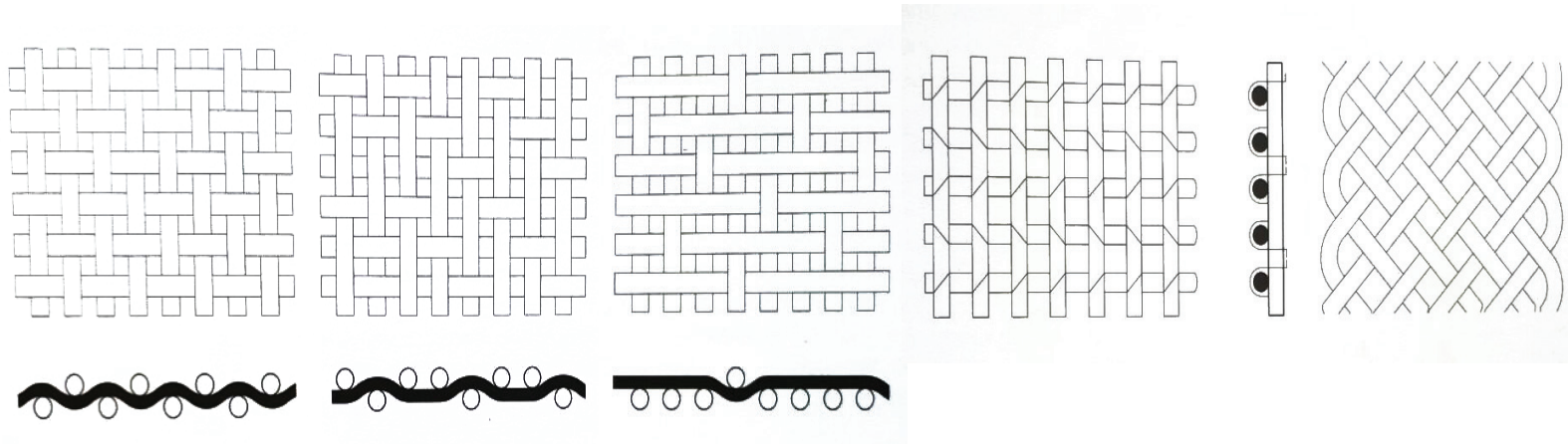


Figure 38 From left to right: plain weave, twill weave, satin weave, non-crimp fabric and braid fabrics [Cremers et al., 2011]

Metal Fibres

These fibres have a relatively large diameter. Therefore, they are not suitable to be used within a polymer composite. Metal fibres are mostly used to reinforce concrete, creating fibre reinforced concrete. [Cremers et al., 2011]

3.3.3 Fibre Fabrics

Threads of filaments can be woven into fabrics before they become the reinforcement of a polymer composite. Types of fibre fabrics that are used to reinforce polymers are:

1. Woven Fabrics
2. Non-Crimp Fabrics
3. Braids
4. Chopped Strand Mats

Woven fabrics are made from yarn threads that cross each other at 90°. There are different types of weaving patterns that all have a different influence on the mechanical properties in the so-called 'warp' and 'weft' direction. In other words, depending on the weaving pattern the fabric can have different properties in its x and y direction. Figure 38 shows, from left to right, a plain weave, a twill weave and a satin weave. Twill

weave is stronger and more stiff than the plain weave. The satin weave is mainly characterized by its good drapeability which makes it a good weave to be used for components with small radii. [Cremers et al., 2011]

Non-crimp fabrics are made from layers of thread that lie on top of each other instead of being woven. A very thin sewing thread keeps the fibre threads in place. A non-crimp fabric has a higher strength than woven fabrics because the fabric is completely flat, as opposed to the undulating woven fabrics. In addition to that, non-crimp fabrics can be adapted to the loading directions because the angle between the two layers does not have to be 90°. [Cremers et al., 2011]

Braids have a very good impact resistance. Due to the criss cross shape of the braided fabric, the fibers produce a greater resistance in case of fracturing. [Cremers et al., 2011]

Chopped strand mats are, as their name suggests, made from pieces of fibre that are randomly arranged across the fabric. The fabric has no dominant direction and has therefore roughly equal mechanical properties in every direction. Chopped strand mats are used when there are no high demands for the mechanical properties. Very fine chopped mats are called fleeces. These are

often used as an outer layer in polymer composites as they result in smooth surfaces. [Cremers et al., 2011]

3.3.4 Core Materials

Fibre-reinforced polymers can have a sandwich-like structure with a different material in the core. Due to the lower self-weight of these materials, larger spans can be reached than with solid FRP components. The core is capable of transferring shear forces and functions as thermal insulation as well. There are three types of core materials that can be used to form a FRP composite:

1. Polymer Foams
2. Honeycombs
3. Balsa Wood

Generally speaking, composites with polymer foam cores exhibit the best properties in both mechanical and building physics terms. In addition to that, they can be shaped in complex 3D forms, allowing a much larger design form than with honeycomb cores. The latter has a poor drapeability and has a lower insulation value than foam cores. Balsa wood can also be used as a core material. Its weight lies in the midrange of foam cores. However, its mechanical properties lie below foam cores and they are sensitive to moisture and rots. It can be concluded that foam cores are the most promising for structural use in FRP composites. [Cremers et al., 2011]

Polymer foams either have an open cell structure or a closed cell structure and can be divided into three groups:

1. Flexible Foams
2. Tough Foams
3. Brittle Foams

Flexible foams and brittle foams are not sufficient as load bearing cores for FRP composites. Flexible

foams are too unstable whilst brittle foams damage easily. Usually, closed celled foams are preferred for FRP composites since they do not absorb the resin. For vacuum insulation panels, however, open cells are necessary to be able to completely vacuumize the panel. There are three types of foams that can be used as cores for structural FRP composites:

1. Polysterene Foams (EPS and XPS)
2. Polyurethane Foams (PUR)
3. Polyvinyl Chloride Foams (PVC)

Polysterene foams are tough foams with good thermal insulation properties. However, their load-bearing capacity is quite low. Extruded Polysterene foams (XPS) are heavier and have better mechanical properties than Expanded Polysterene foams (EPS). In addition to that, XPS have closed cells which make them more suitable for FRP composites than EPS. The largest benefit of Polysterene foams is that they are very affordable. Polyurethane foams (PUR) exist in both flexible and tough foams. PUR has excellent load bearing properties, has a very good thermal insulation. Rigid PUR foams tend to absorb resin. However, they have a very good price-strength ratio and can be easily milled with a CNC machine. Thus, they are commonly used as core materials. Polyvinyl Chloride Foams (PVC) possess the best mechanical properties of the three categories. In addition to that, they hardly absorb resin. However, their insulation value lies lower than PUR foams and the production costs are high. [Cremers et al., 2011]

3.3.5 Fibre-reinforced Polymers

Fibres can be further processed to form a composite with a polymer. Within this composite the fibres function as reinforcement. The polymer in which the fibres are embedded is called the 'matrix'. For application in architecture fibres with a minimum elongation are preferred. Usually the tensile strength is less important because the deformation of the structure is

more decisive than the internal stresses. Thus, glass and carbon fibres are usually the ones that are used in practice. In addition to that, the thermosets that are often used as matrix can only elongate a little amount, causing the matrix to crack before the full tensile strength of the fibers is reached. When comparing glass fibres and carbon fibres, the strengths turn out to be roughly equal. However, the Youngs Modulus of carbon fibres is a lot higher. As a result, Carbon Fibre Reinforced Polymers (CFRP) are a lot stronger than Glass Fibre Reinforced Polymers (GFRP) because a higher strength of the carbon fibers can be reached without cracking of the matrix. [Cremers et al., 2011]

Properties

Machine-made GFRP components can reach strengths equal of structural steels. However, their elastic modulus lies at a tenth of the values for steels. CFRP on the other hand, has an elastic modulus that is similar to steels and a strength that surpasses the strength of steels. The properties of FRP based on the CES Edupack database are shown in figure 39. [Cremers et al., 2011]

Manufacturing

There are several production methods that can be used depending on the size, batch size and complexity of the geometry. Manual methods are suitable for small batches, for batches in which every part is unique and for parts that have a complex geometry. Automated methods are suitable for large batches. The following methods exist for FRP products:

1. Hand lay-up
2. Resin Infusion and Vacuum Methods
3. Compression- and Transfermoulding Methods
4. Pultrusion
5. Wrapping
6. Braiding
7. Fibre Spraying

Hand lay-up is suitable for irregular shapes, shapes

with a complex geometry and very large shapes. The hand lay-up method requires a mould. A rigid foam core can also function as 'mould'. In this case, however, the mould becomes part of the element. Moulds can be made from several materials, all having their advantages and disadvantages (see chapter 'Mould Making'). First, a release agent is applied to the mould. Then, the surface of the mould is covered with a ground layer of resin. Once this layer has dried, the reinforcement is placed, impregnated with resin and pressed into place with a roller. Rolling the resin into the reinforcement fibres also eliminates air bubbles. Additional layers can be added using the same method. Figure 40 shows this process of manually laminating the FRP. The first layer of reinforcement usually exists of a fine fleece, resulting in a good surface finish. The maximum fibre content that can be achieved using hand lay-up is 45% of the volume. When a flame retardant is added to the resin, this value is lower. [Cremers et al., 2011]

Resin infusion and vacuum methods still require hand lay-up of the reinforcement layers. However, the resin is applied later in the process. These methods result in a very smooth surface and very few air inclusions. Once the reinforcement layers have been placed the laminate is closed off with a porous air tight release film and a so-called breather cloth. With the resin infusion method, resin is inserted with pressure at one side while air is removed from the other side. These methods are illustrated in figure 41. With the vacuum method the resin is sucked into the mould by using a low pressure. Therefore, this method takes longer, but also achieves better results. The final product is then cured at a temperature of approximately 180 °C under pressure. Curing the laminate under pressure increases the density of the laminate. [Cremers et al., 2011]

Compression- and transfermoulding methods are automated methods. The production of the moulds for these methods is rather expensive. Thus, these methods are used for industrialized products from which

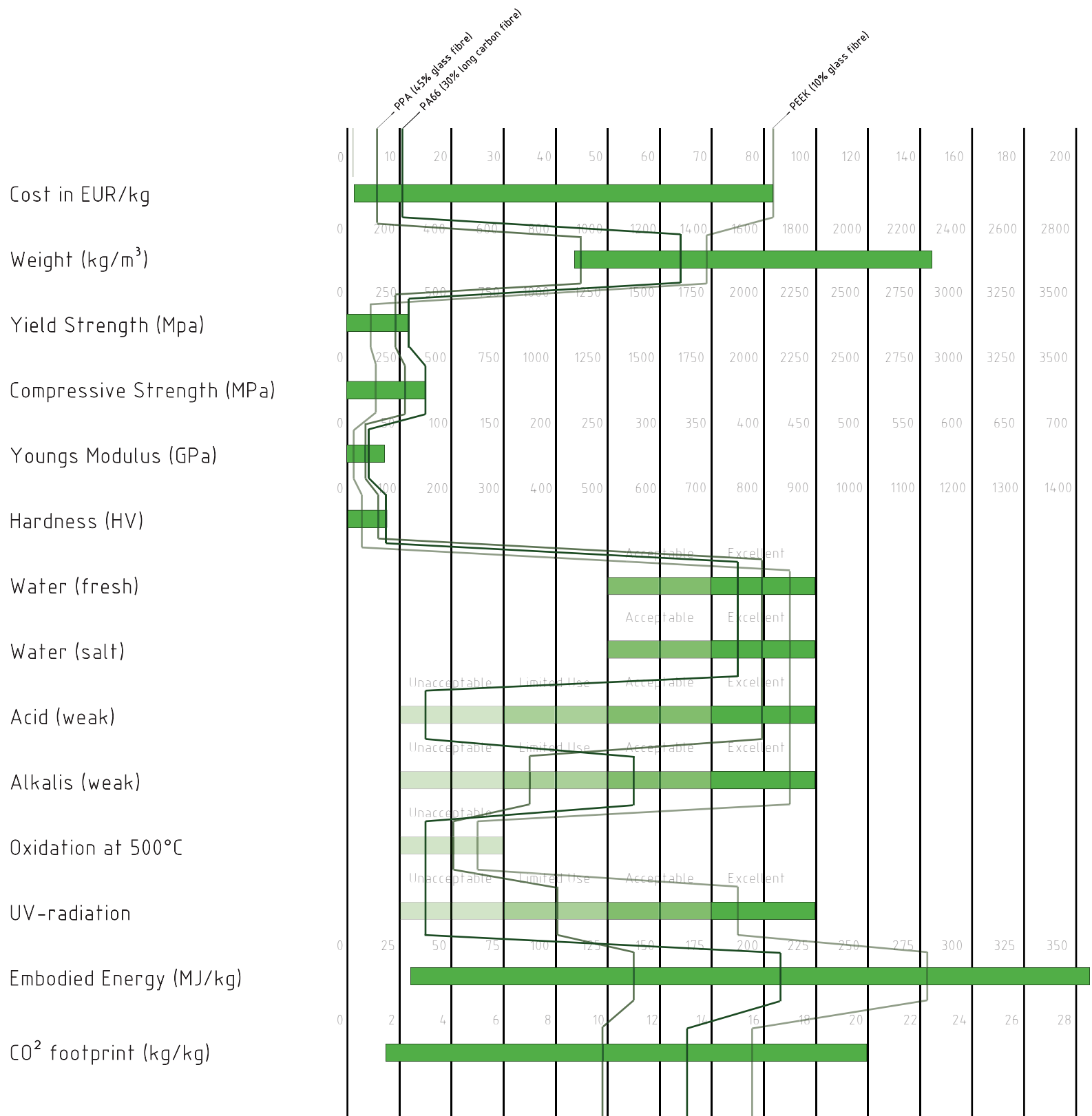


Figure 39 Properties of FRP's [own image]

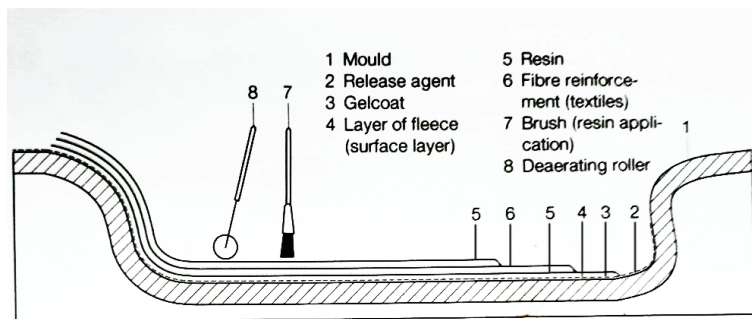


Figure 40 Manual laminating (hand lay-up) [Cremers et al., 2011]

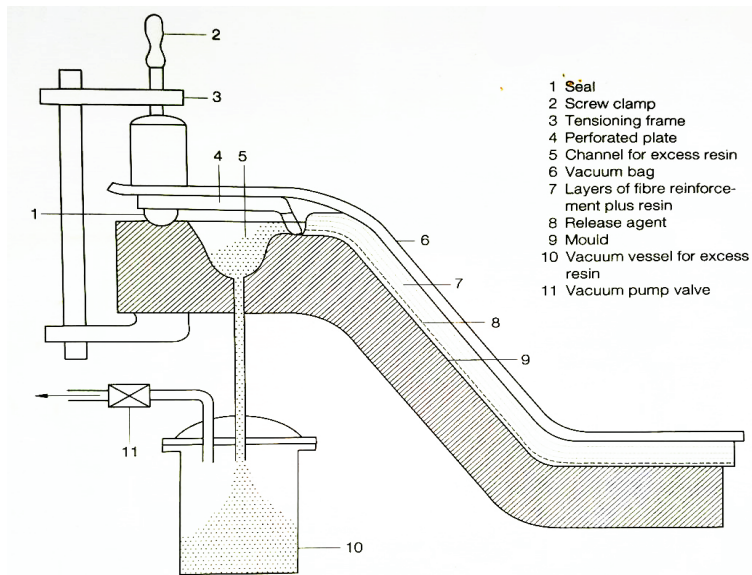


Figure 41 Resin infusion and vacuum methods [Cremers et al., 2011]

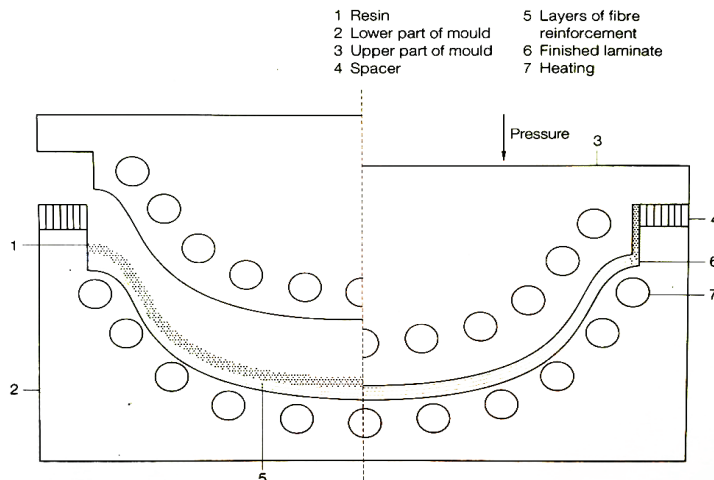


Figure 42 Compression- and transfermoulding methods [Cremers et al., 2011]

large batches will be made. These methods can be applied with both fluid and semi-finished products. The latter are called prepregs. By pressing the two steel or aluminium moulds together, fibre contents of up to 65% can be reached. These methods are shown in figure 42. [Cremers et al., 2011]

Pultrusion is used to create simple sections and sheets with a high fibre content. Long fibres (rovings) impregnated with resin are pulled through a heated mould that cures the resin within minutes. Textile can be included as well to reinforce the surface of the product. The longitudinal rovings are responsible for the flexural strength of the member. The properties perpendicular to the roving are mainly determined by the textile. A fibre content up to 70% can be reached by using pultrusion. Their are limits on the possible shapes. For example, ribs are not possible in the transverse direction and the sections are usually straight. Setting up the pultrusion is quite expensive. Therefore, it is mostly used for large batches. The pultrusion method is shown in figure 43. [Cremers et al., 2011]

Wrapping is also known as filament winding. As the name suggest, the process involves winding the filament around the mould. It can be used for all shapes that are rotationally symmetric (and hollow). With this method, the fibres can be packed densely. In order to ease demoulding, the shape should be slightly conical. The fibres cross at a degree between 15° and 75° of the longitudinal direction. This method is shown in figure 44. [Cremers et al., 2011]

Braiding is quite similar to wrapping. This method is used for the production of highly stressed components in the aerospace industry. It is possible to get fibres to lie parallel to the longitudinal direction. The method allows a larger variety in shapes: varying cross sections are possible to some extent. This method is shown in figure 45. [Cremers et al., 2011]

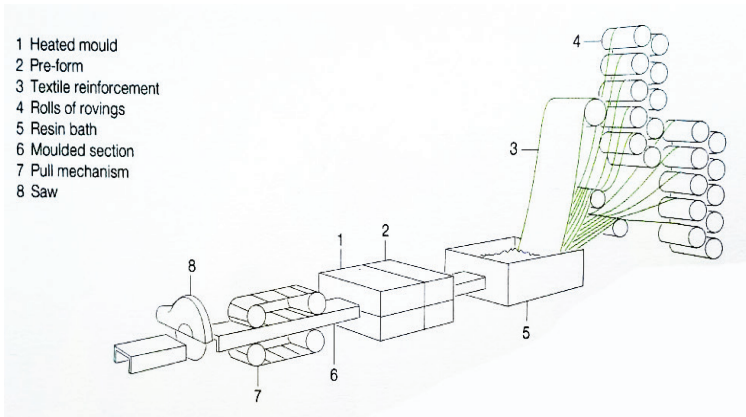


Figure 43 Pultrusion [Cremers et al., 2011]

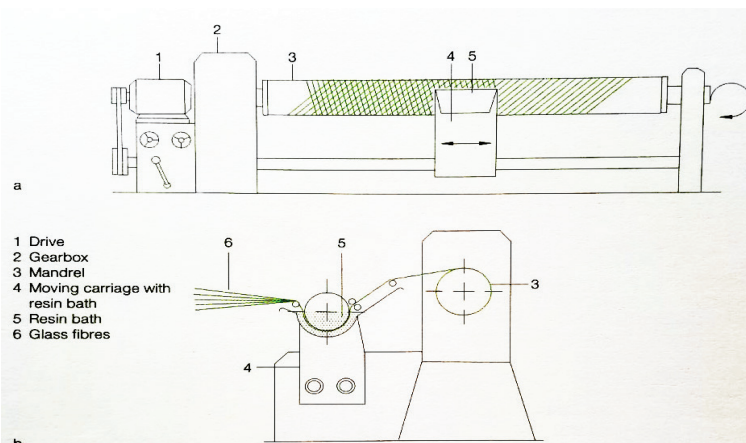


Figure 44 Wrapping [Cremers et al., 2011]

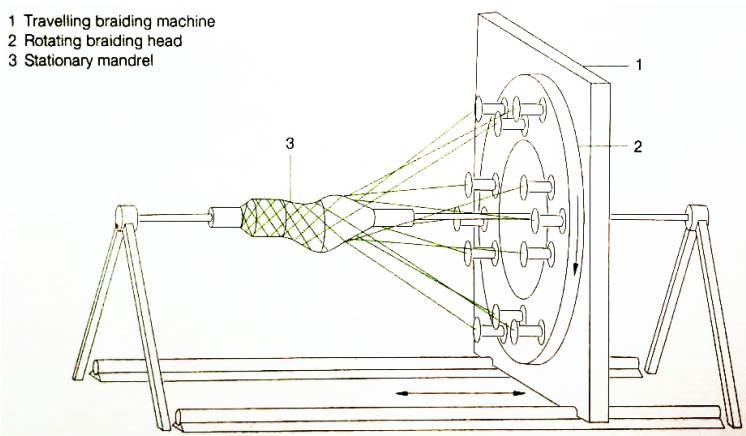


Figure 45 Braiding [Cremers et al., 2011]

Fibre spraying can be used for complex shapes and is inexpensive. However, it can only be used for parts with low load bearing capacities. Thus, it will not be discussed further. [Cremers et al., 2011]

Mould Making

Moulds can be made from a variety of materials. Depending on how often one plans to use a mould, cost and durability are aspects that need to be considered. The production of moulds can be more expensive than the final product itself. The desired surface quality can also play an important role in determining the material for the mould. Possible materials are:

1. Gypsum and Clay
2. Rigid Foams
3. FRP
4. Metals
5. Wood
6. Polymers

Gypsum and clay are very cheap and can be shaped into complex forms. Gypsum is only suitable for small moulds because of its low strength. In addition to that it is easily damaged during the demoulding process, making it usable for only once. Rigid foams can be used as a mould (or core) as well. The cost is relatively high and when it is not used as a core, it needs to be sealed with a special compound and wax in order to prevent bonding between the foam and the FRP. Foam moulds are only reusable for a few times. The smaller the pores of the foam the better it can be shaped with a CNC milling machine. GFRP can be used as a mould for other FRP products as well. The advantage of this is that GFRP moulds can be used for more than 100 times. Moulds that are made from steel or aluminium can be used for very large batches since they are very durable. However, these moulds are very expensive to produce. When the demands for dimensional accuracy are rather low, wood moulds can be a solution for large products. Wood can also be CNC milled to form a very

accurate complex geometry. Polymer moulds are used when there is a demand for high precision. The use of a polymer mould also results in a product with an excellent surface finish. When using silicone moulds, no release agents are required. [Cremers et al., 2011]

3.3.5 Building with Freeform Polymers

The thermal expansion of unreinforced polymers is quite high. The expansion coefficients for FRP is lower. Carbon fibres can be used as reinforcement in such a way that they do not expand with higher temperatures at all. When a product is restrained at both ends, so-called restraint stresses occur in the material. For completely restrained FRP products, the maximum allowed utilization lies a lot lower than 100%, which indicates material breakage. For GFRP the maximum utilization with a temperature difference of 50 °C lies at 21%. However, this is a simplified derivation that cannot be directly transferred to structures. The combined stress of all loads applied and the expansion stresses must not exceed 100%. [Cremers et al., 2011]

Connections

Several aspects need to be taken into account when designing connections. The transfer of forces, the robustness, watertightness, the need for detachability, visual impact, etc. are all important aspects. Since FRP is a relatively new material within the building industry, the development of connection methods for FRP is still ongoing. The following connection types have been used so far:

1. Pin-type Fasteners
2. Glued Joints
3. Push-Fit systems
4. Laminated Joints
5. Built-in Metal Fittings

Pin-type fasteners are easy to assemble and are detachable. However, due to the brittle nature of

most polymers redistribution of the high stresses that occur in the materials around bolts, screws or rivets is limited. For this reason, there is usually a large amount of fasteners required to avoid issues. Another disadvantages of this method is that by drilling holes in the FRP, the fibers are cut, making load transferring around the hole through fibers impossible. In addition to that, the drilling damages the surface layer, making the connection prone to moisture damage. [Cremers et al., 2011]

Six **bolted detail connections** are shown in figure 46. Connection A shows a simple lap joint which results in a visible offset. Due to the eccentric connection there will be a bending moment. To overcome the visual offset, one of the members can be bent as in connection B. However, the local bending moment remains. Connection C overcomes the local bending moment by using splice plates in order to create a symmetrical joint. However, this reintroduces the visible offset. Another solution that results in a less notable connection (at least on one side) is by bending the ends 90° as is shown in connection D and E. However, due to the bending moments in these bent edges, within these connections the force transfer lies lower than with the previous solutions. Connection F shows a solution where fasteners have been embedded in the FRP. Resulting in a smooth finish on one side and in a symmetric connection. **Rivets** have load-bearing capacity that is very similar to bolts. They are usually applied in large numbers to form a connection. Unlike bolts, rivets are normally not detachable. [Cremers et al., 2011]

Glued joints have not been used often for load-bearing joints because the quality of the joint cannot be checked afterwards. Failure of a glued joint in case of poor workmanship is usually abrupt and brittle. One of the advantages of glued joints is that they allow for an even transfer of forces, resulting in a better load-bearing capacity than with pin-type fasteners. In addition to that, glued joints seal the connection making it watertight. Also, soft adhesives can be used

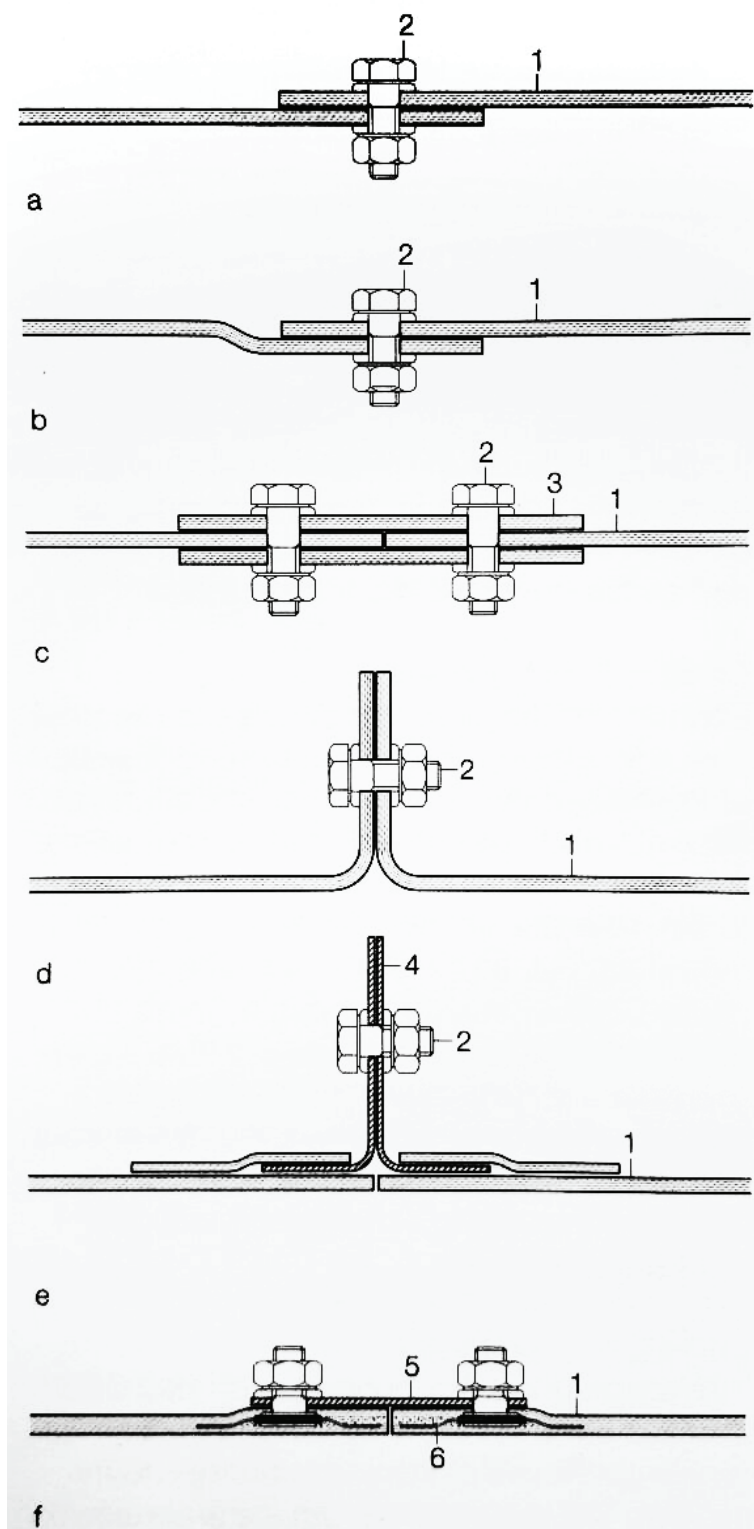


Figure 46 Bolted Connections [Cremers et al., 2011]

to allow for thermal expansion and to some extent for dimensional tolerances. Ideally, the surface area of the glued joint should be as large as possible. An important note is that a glued joint should always be loaded in shear and not in tension. Possible connection details for glued joints are shown in figure 47. Connection D shows an example that is not very suitable since the area of the glued joint is very small. Connections with bent edges are also less desirable because there will be tensile stresses in the glued joint. [Cremers et al., 2011]

Push-fit systems do not need as much bolts or glued joints or even none of these additional joint principles at all. However, these connections are only used for parts that are not primary load-bearing such as cladding. [Cremers et al., 2011]

Laminated joints can be made in several ways (see figure 48 and figure 49). With the first way as displayed in figure 48 A two FRP components are but jointed and are then laminated on both sides. A similar method can be used, however, with this method there are protruding fibres in the two components that are but jointed. The protruding fibres are then laminated together (see figure 48 B). Both of these methods leave a thicker joint. The third method uses so-called peel-ply fabric. The steps for making such a joint are displayed in figure 49. [Cremers et al., 2011]

Built-in metal fittings as shown in figure 50 need to be as thin as possible to interrupt the laminated layers as little as possible. The use of a perforated metal allows looping of fibers through the holes and a better penetration of the resin during production. This results in a higher load-bearing capacity. Threaded sleeves can also be built in. These type of connections are especially useful for individual connections with a concentrated load transfer. [Cremers et al., 2011]

Pultruded FRP sections (rectangular or circular) can also be connected using nodal connectors by slightly adapting the typical systems for steel.

- 1 Polymer component
- 2 Adhesive
- 3 Splice plate both sides
- 4 Splice plate on one side
- 5 Channel section

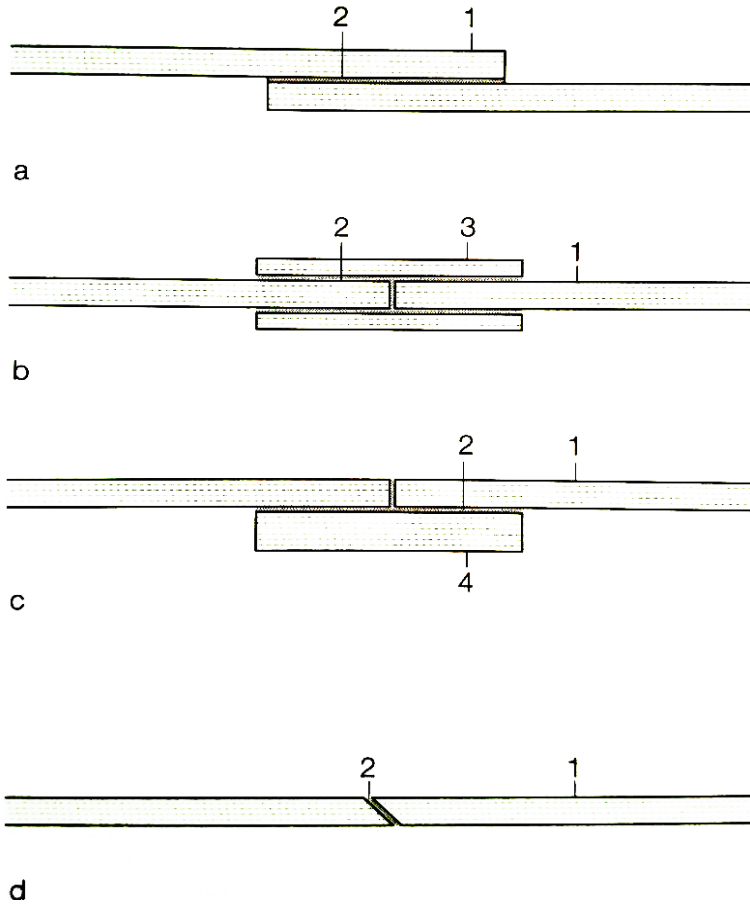


Figure 47 Glued Connection [Cremers et al., 2011]

- 1 Fibre-reinforced polymer
- 2 Lug made from thin sheet steel
- 3 Long fibres (rovings), looped through perforations
- 4 Normal planar fibre reinforcement

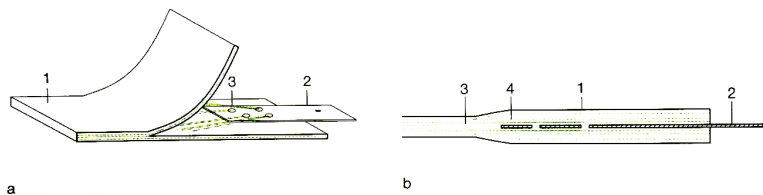


Figure 48 Laminated Connection [Cremers et al., 2011]

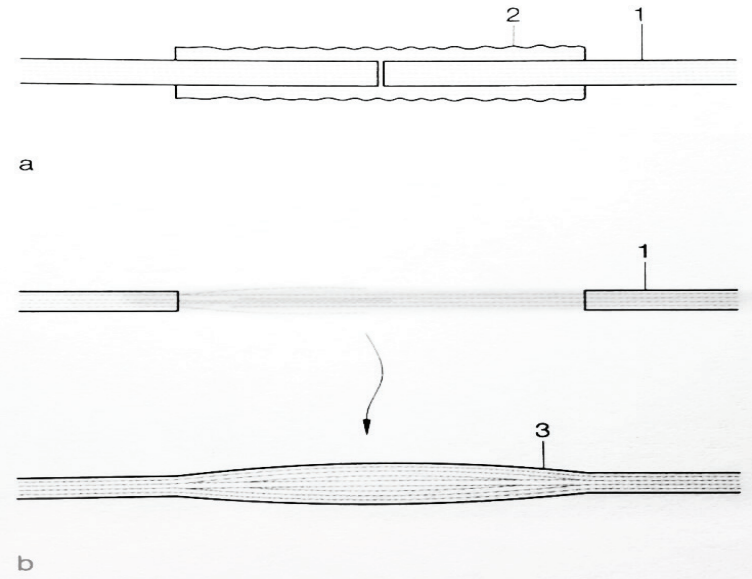


Figure 49 Laminated Connection [Cremers et al., 2011]

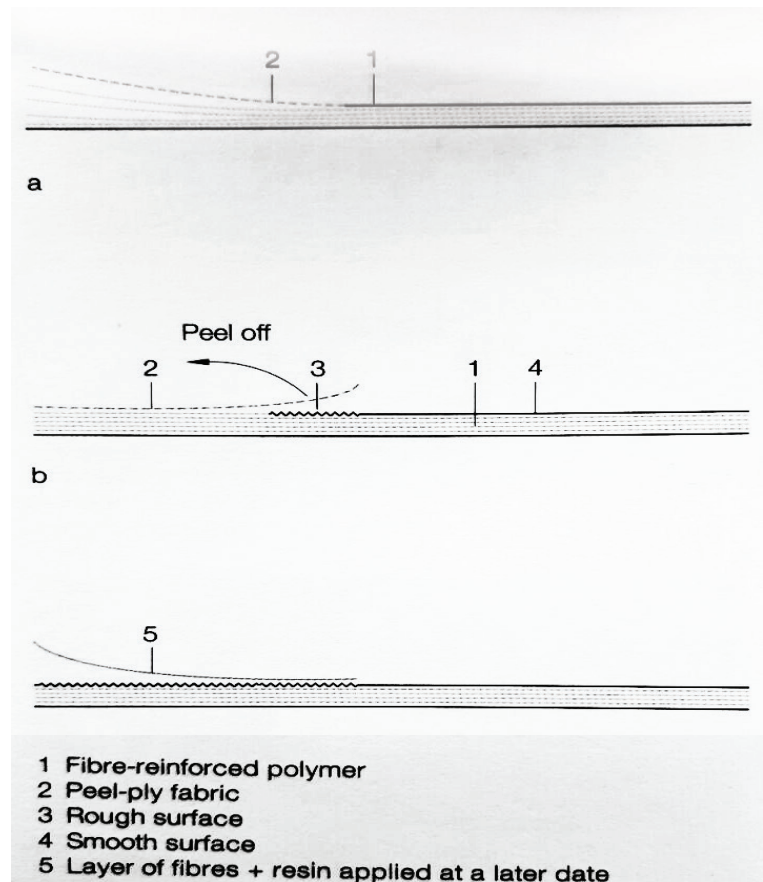


Figure 50 Laminated Connection [Cremers et al., 2011]

- 1 GFRP facing ply
- 2 Rigid foam core
- 3 Steel built-in part
- 4 Bolt
- 5 Plug of material to conceal connection

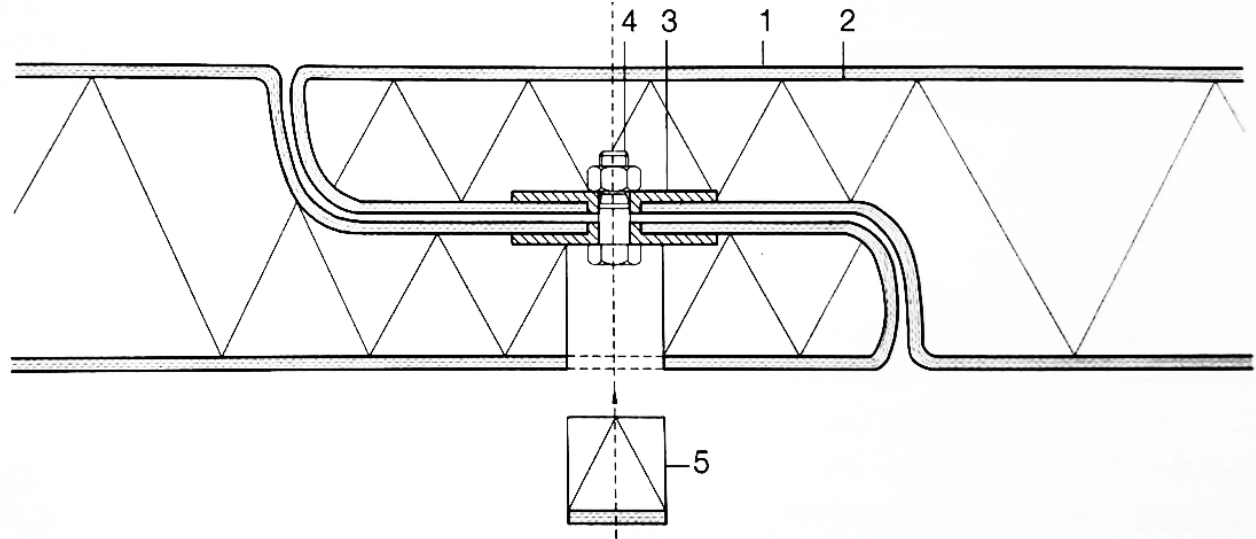


Figure 50 Laminated Connection [Cremers et al., 2011]

3.4 Summery

This chapter summarizes the most important characteristics of the previously discussed materials and the connection types.

3.4.1 Materials

For the assessment of materials only the structurally used materials are judged. Tool steels, for example, are left out of the assessment. It needs to be noted that the very high values for Yield Strength and Young's Moduli for the FRP's are only reachable with pultruded profiles with a unidirectional laminate. It can be concluded that wood is the most sustainable solution. However, the Yield strength and Young's

Modulus of woods are a lot lower in comparison with steel and FRP, making it the least suitable option for such a large span as needed for a stadium ($\pm 200\text{m}$). In addition to that, wood has the highest maintenance of all three options. Steels form a sound option for such a large span structure as they are strong, stiff and relatively cheap. The FRP types that are stronger and stiffer than steel are either pultruded profiles or contain the very expensive carbon fibres. FRP's are in most cases more expensive than steels, but they are also about 6 times less heavy and can be considered to be maintenance-free. Carbon fibers are very expensive and are therefor not considered to be an option. It can be concluded that both steels as GFRP would be suitable for the construction of the canopy of a stadium.

	Structural characteristics	Durability	Sustainability (-- to ++)	Cost
Steel	Yield strength: 185 – 960 MPa Young's Modulus: 200 – 230 GPa Density: 7400 – 8400 kg/m ³	- Stainless steels are corrosion resistant - Alloying elements can be added to increase weather resistance - Low maintenance	CO ² emission: – €0.60 –	€6
Wood	Yield strength: 7 – 16 MPa Young's Modulus: 7 – 16 GPa Density: 350–550 kg/m ³	- Susceptible to: 1. Fungi 2. Insects 3. Weathering 4. Fire 5. Natural defects - High maintenance	CO ² emission: ++	€0.60 – €10
FRP	Yield strength: GFRP: 25 – 1760 MPa CFRP: 32 – 2430 MPa Young's Modulus: GFRP: 4.5 – 48 GPa CFRP: 5.5 – 375 GPa Density: GFRP: 1090 – 2220 kg/m ³ CFRP: 1450 – 1670 kg/m ³	- Corrosion resistant - Very low maintenance	CO ² emission: +/-	GFRP: €1.50 – €33 CFRP: €14.50 – €312

Figure 51 Summery of material characteristics based on CES Edupack 2017 and Lyons [2015]

3.4.2 Connection Types

	Connection Type	Characteristics
Steel	- Spherical Nodes - Mero KK - Hollow Spherical - Cylindrical Nodes - Triodetic System - Plate Connections - Octatube System - Nodeless Connections - Octet truss System	- Allows the creation of complex double curvatures - Pin-type connection that is not capable of transferring bending moments. Only suitable for triangular grids - Suitable for single & double layer grids - Only allows the creation of very small curvatures - Pin-type connection that is not capable of transferring bending moments. Only suitable for triangular grids - Suitable for single & double layer grids - Allows the creation of complex double curvatures - Is only capable of transferring out-of-plane bending moments. Only suitable for triangular grids - Only suitable for single layer grids - Only allows the creation of very small curvatures - Somewhat capable of resisting in-plane bending moments. Quadrangular and hex-based grid may also be possible. - Suitable for single & double layer grids - Does not allow the creation of curvatures - Only suitable for tetrahedral or octahedral geometries - Only suitable for double layer grids - Final shape must be a shape that can be generated by pushing a flat surface down or upwards.
Wood	- Multihalle Mannheim - Weald & Downland - Savill Garden - Piece-small Systems	- Double layered, transfer of forces between layers with shear blocks - Continuous members - Diagonal steel bracing cables needed for stiffness - Slotted holes in joints - Only suitable for rectangular grids with continuous chords - Final shape must be a shape that can be generated by pushing a flat surface down or upwards. - Double layered, transfer of forces between layers with shear blocks - Continuous members - Diagonal laths are needed for stiffness - Only suitable for rectangular grids with continuous chords - Final shape must be a shape that can be generated by pushing a flat surface down or upwards. - Double layered, transfer of forces between layers with shear blocks - Continuous members - No slotted holes in joints - Only suitable for rectangular grids with continuous chords - The same types of nodal connections that are commonly used for steel members can be adapted to work with wooden members as well.
FRP	- Pin-type Fasteners - Glued Joints - Pin-type Fasteners - Laminated Joints - Built-in Metal fittings - Piece-small Systems	- Easy to assemble and disassemble - Redistribution of high stresses around bolts, screws or rivets may cause problems. A high amount of fasteners is needed. - Not often used for load-bearing joints as the quality of a joint cannot be checked afterwards - Only capable of transferring shear forces. Not capable of transferring tension - Even transfer of forces - Only used for non-loadbearing connections - Leave a thicker joint - Perforated parts in the metal fitting allow for the fibers to loop through the holes, creating a higher load-bearing capacity than with pin-type fasteners - Particularly useful for individual connections with a concentrated load-transfer. - The same types of nodal connections that are commonly used for steel members can be adapted to work with pultruded FRP profiles as well.

Figure 52 Summary of the characteristics of the connection types.

4. GRADED STRUCTURES

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Regular grids

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Performance based gradient

Lloyd's algorithm

Spring force system

Smart Circle Packing Algorithm

4.1 Background

4.1.1 Nature

Gradient structures can be found ubiquitously in nature. According to Wu et al. [2014] 'a typical gradient structure is characterized with a microstructural gradient at a macroscopic scale'. Nature developed these structures over millions of years because their properties exceed the properties of homogeneous material structures while minimizing the amount material. An example of a graded structure in nature can be found in bamboo. 'The distribution of load-bearing fibers in bamboo is not uniform but hierarchical so that fiber density is highest at the outer periphery where the stresses are largest [...]. By combining this density gradient with an optimized cellular microstructure tubular macroscopic shape, bamboo is two-to-four times more mechanically efficient than a beam with the same bending stiffness made from solid cell-wall material [Yarris, 2014]. Another well-known graded structure can be found within bones. Bones have the capability to remove material where it is not needed. Parts of the bone that are not fully utilized are being removed. The resulting material follows the principal stress lined within the bone (see figure 54). As a result the bone is light and strong.

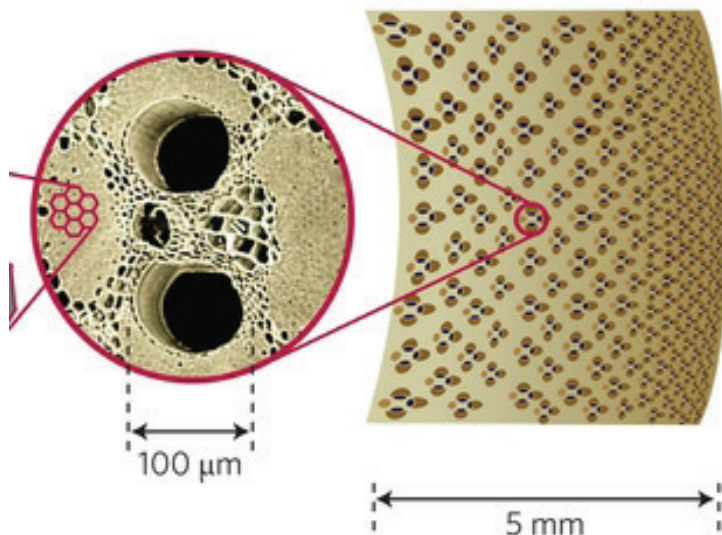


Figure 53 Bamboo's graded structure [Yarris, 2014]

4.1.2 Man-made Application

Within architecture there is much interest in the application of gradient structures for engineering materials because 'the advantage of gradient structures is their maximization of physical and mechanical performance while minimizing material cost' [Wu et al., 2014]. Thus far, only the structural benefits of gradient structures have been mentioned. An example of a gradient structure on nanostructure level that combines additional functions can be found within the Dailies Total1 water gradient contact lens (see figure 55). Pruitt and Baumann [2013] realized that certain properties of a contact lens could be controlled by the core of the material while other properties were only necessary at the surface of the lens. The core should be responsible for the transmission of oxygen and for the lens handling while the surface mostly needs to be wet and lubricious. So instead of using a homogeneous material they combined a highly breathable silicone hydrogel at the core with a non-silicone hydrophilic polymer structure at the surface.

So far, only applications of graded structures on nanostructure level were discussed. Is it possible to scale these principles up? Joris Laarman used the

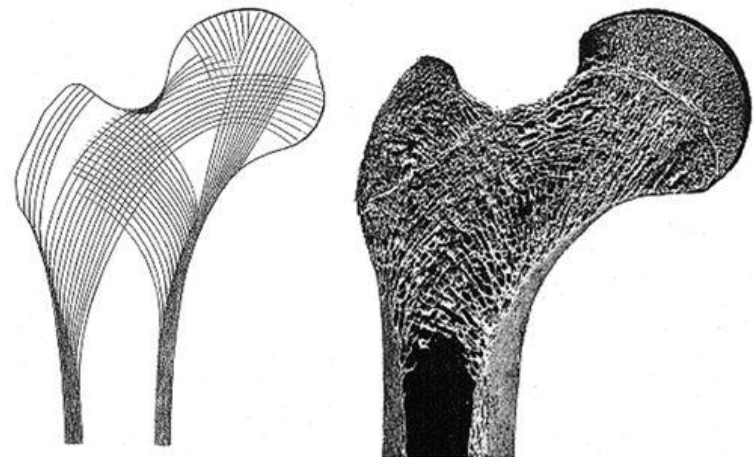


Figure 54 Bone's graded structure [Hanne, Teefer and Cole, n.d.]

bonestructure principle to design the 'Bone Chair' by making use of a topology optimization programme that removes the material that is not strictly necessary. This principle is also used by engineers to optimize car parts, such as chassis frames, and airplane parts in order to minimize weight while maximizing the mechanical properties. The applications of topology design are slowly becoming a topic of research within the field of bridge design and architecture, taking the principle to larger and larger scales.

Within architecture, graded structures have been applied at façades of several designs by gradually changing the geometry that is created within the regular grid cells. In most of these designs there does not appear to be a performance based reason why the openings are larger or smaller at specific places. It appears to purely serve as an aesthetical factor. In some cases however, the functions of the rooms adjacent to the façade need different amounts of lighting. This translates to larger openings at the places that require more sunlight. This principle has been applied to the design of the Omotesando Commercial Complex in Tokyo, by UNStudio. A property that all of the graded façades seem to have in common is that they restrict the gradient to the geometry of the filling panel within a boundary that is the same over the entire façade instead of gradually changing these boundaries (as well).

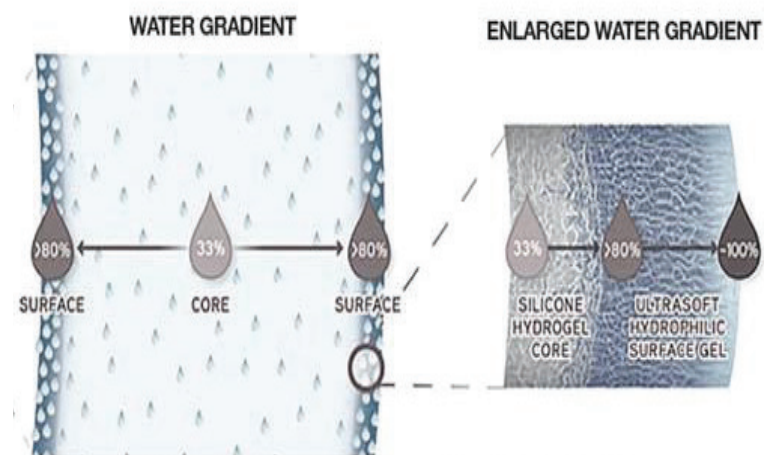


Figure 55 Cross section of the Dailies Total1 water gradient contact lens [Pruitt and Baumann, 2013]

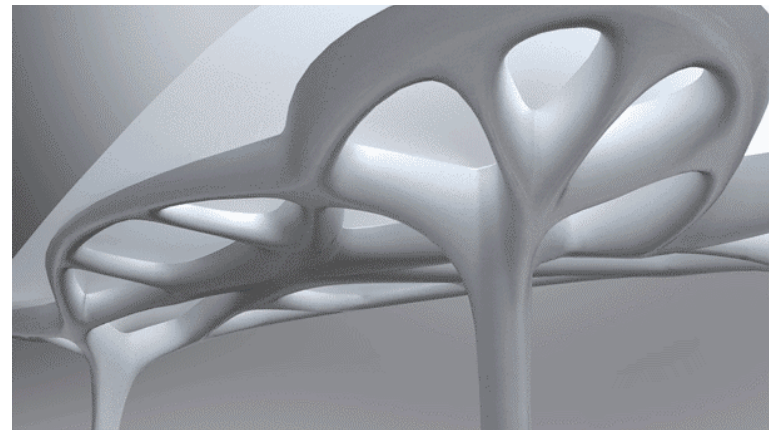


Figure 56 Topology optimized columns [Fluxstructures, n.d.]



Figure 57 The Bone Chair [Laarman, 2006]

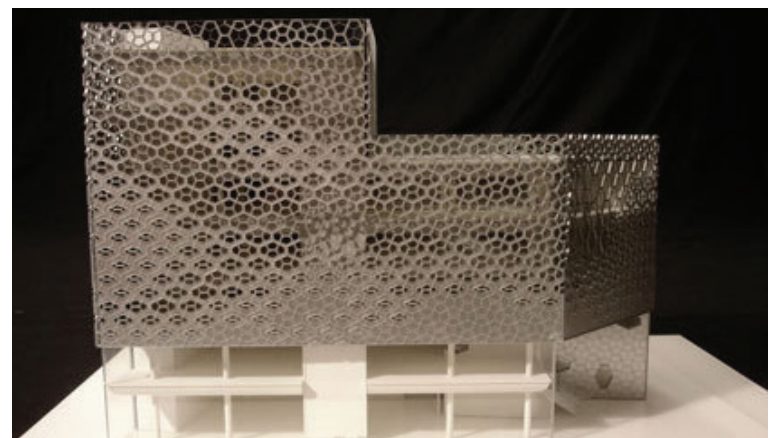


Figure 58 Graded façades. Left: Louis Vuitton Store, Shenzhen [From up North, 2014]. Top: ArboSkin Pavillion, Stuttgart [Arch20, n.d.]. Right: Omotesando Commercial Complex, Tokyo [Schwaller, 2008].

4.2 Types of Grids

The definition of a graded structure in this research is a structure that changes gradually over the building. These gradual changes can be of many different natures: size of members, size of openings, composition, etc. This research focuses specifically on the gradual densification of the structure over its surface. Graded structures can be classified in roughly two categories: irregular grids and regular grids. Within these categories, gradual change can often be achieved by more than one method. Structures in the first category can be created from any distribution of points. Voronoi and Delaunay can be subsumed under this category. Structures in the second category are derived from regular grid structures. This chapter will discuss the main categories and corresponding densification methods.

4.2.1 Irregular Grids

Voronoi diagram

A Voronoi diagram is the partitioning of a plane in regions based on a specific set of generator points or sites. 'Each region corresponds to one of the [...] generators, and all of the points in one region are closer to the corresponding site than to any other site' [Dobrin,

2005]. Edges of the diagrams occur on places where two generators are equidistant, vertices occur on places where three or more generators are equidistant. An example of a Voronoi diagram is shown in figure 59. Voronoi diagrams can be spotted everywhere in nature, from the distribution of cells to the coat patterns of giraffes. Voronoi diagrams have many application in several fields, like physics, astronomy, robotic, social geography, etc. [de Berg et al., 2008] According to Dobrin (2005) and de Berg et al. (2008) Voronoi diagrams have the following additional properties:

1. The average number of the vertices of a region is less than six
2. All Voronoi regions are convex
3. For each vertex there exists an empty circle with that vertex as a center that passes through at least three generator points
4. A Voronoi diagram is considered non-degenerate if the number of edges that meet at a vertex is equal to three throughout the diagram

Voronoi diagrams can be regular and irregular. Careful placement of the generator points will result in a Voronoi diagram that looks like a regular traditional grid while a more random placement of points will

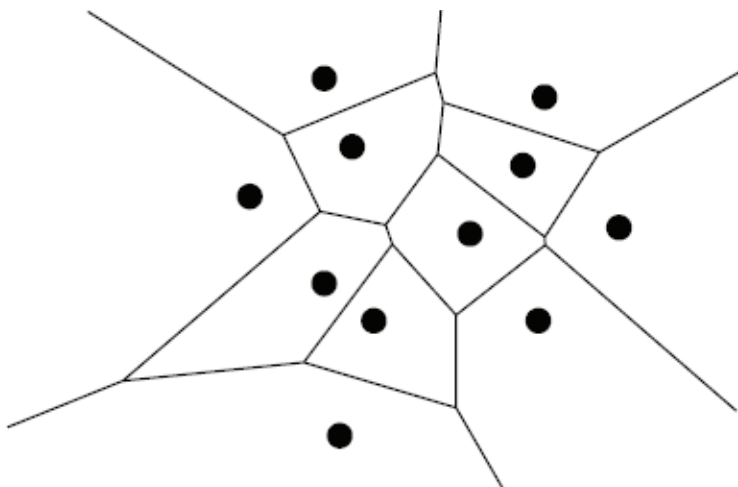


Figure 59 Voronoi Diagram (de Berg et al., 2008)



Figure 60 Delaunay Diagram [Wikipedia, 2017]

result in the irregular look that is so characteristic for Voronoi diagrams. The densification of the diagram is established by placing the generator points closer together. The closer the points are to each other, the denser the resulting diagram. The densification can happen locally without affecting other regions.

Delaunay diagram

The Delaunay diagram is the so-called dual of the Voronoi diagram. It connects each generator point of a Voronoi diagram with the generator points of their neighbouring cells. According to Dobrin (2005) Delaunay diagrams have the following properties:

1. A Delaunay diagram will be a triangulation if all vertices of the diagram are non-degenerate
2. In case of non-degeneracy Voronoi edges and Delaunay edges are orthogonal
3. If all vertices of the Delaunay diagram cross the vertices of the Voronoi diagram (assuming non-degeneracy), the Delaunay diagram is a Pitteway triangulation
4. Each generator point of a Voronoi diagram is a vertex in the Delaunay diagram
5. The circumcentre of each Delaunay triangle corresponds to a Voronoi vertex
6. The number of Delaunay edges is equal to the number of Voronoi edges

The densification of the diagram is established by placing the generator points closer together. The closer the points are to each other, the denser the resulting diagram. The densification can happen locally without affecting other regions.

4.2.2 Regular Grids

According to Bautista (2017) there are only three regular 2D tessellations. Namely, tessellations that use either equilateral triangles, squares or regular hexagons. The hexagonal grid consists of cells with six sides and at each point in the grid three vertices meet. The quadrangular grid consists of cells with four sides and at each point in the grid four vertices meet. And the triangular grid consists of cells with three sides and at each point of the grid six vertices meet.

There are four ways of densifying a regular grid: changing the distance between ongoing lines, subdividing, morphing and varying geometry within the cells. The first way of densifying a regular grid is to gradually minimize the distance between the ongoing lines toward a point on the surface. This densifies the area around the point, however, it also automatically densifies other parts of the surface. Therefore, this system of densifying is not restricted to the target area (see figure 62).

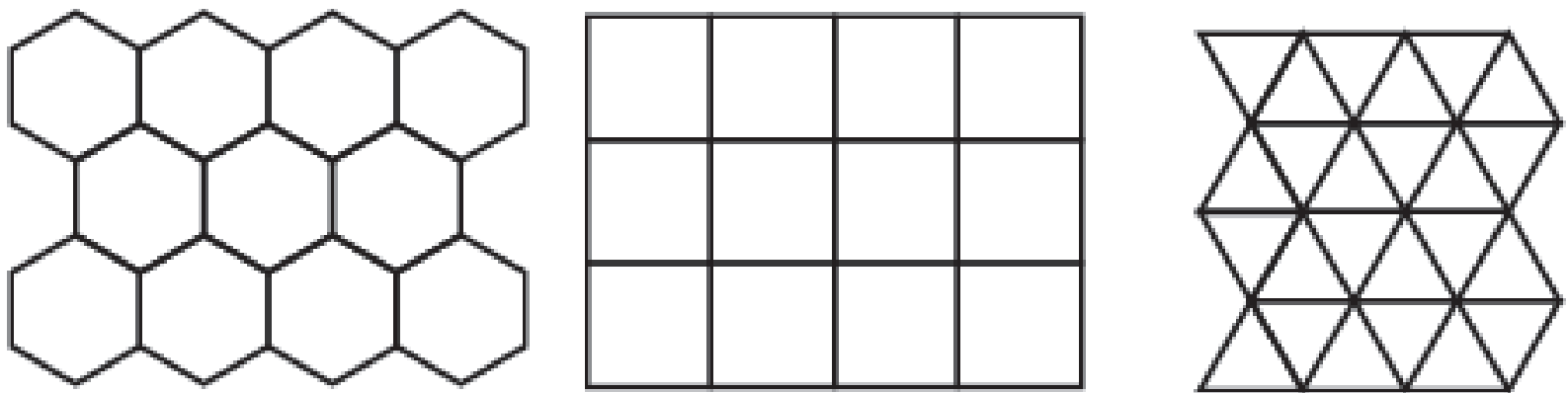


Figure 61 The three regular tessellations [Wolfram MathWorld, n.d.]

Another way of densifying a grid toward a point that does limit the densification to the target area is division of parts of the grid-structure. While restricting to the same form language in case of densification, a triangle can easily be divided into two triangles and a square can be divided into four squares. A hexagon, however, cannot be divided into smaller hexagons. The only way to densify a regular hexagon by subdivision is by departing from the hexagonal form language. It is, for example, possible to subdivide a hexagon into six equal triangles. By subdividing the cells the grid at that specific area get denser by the amount of a certain integer. It is impossible to make parts of the grid 1.5 times denser. Therefore, the result of densification by using this method appears 'patchy' (see figure 62). Another reason this solution often leads to results that are not desirable is because the obtained result is usually not optically acceptable. According to Dimcic (2011) 'The human eye has a remarkable pattern recognition ability and incredible precision of error detection when it comes to grid structures.' In order to create optically pleasing grid-structures, the grid must be organized in a way that the members of the structure do not form long guide lines. 'If they do, then the lines must be smooth and continuous, in order to satisfy optical conditions.' [Dimcic, 2011]

A third method of densifying a regular grid is by morphing the grid. This can be done by placing points in the grid that either attract or repel points of the grid. As a result the lines within the grid start to curve

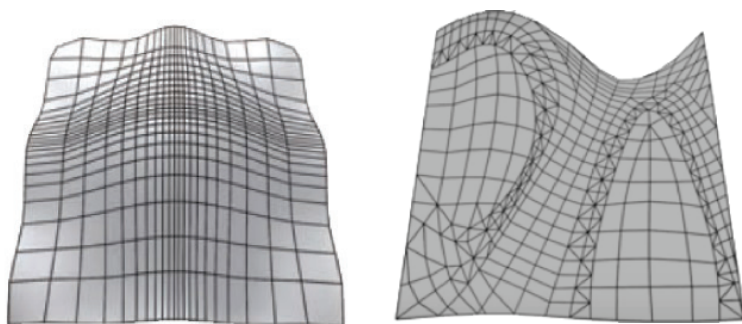


Figure 62 Densification of traditional grids [source: Dimcic (2011) and ProgramArchitecture (2013)]

towards (attractor) or away (repeller) from this point (see figure 63). This method has the same disadvantage as the first method: it also affects other areas than the target area. For example: in case of repeller points, the area closest to the point gets less dense by pushing the curves away from it. As a result the areas surrounding the arising gaps get densified as a side effect.

The last method of densifying regular grids is by altering the geometry that is created within the cells. Strictly speaking, the resulting geometry can no longer be seen as a grid structure since it does not consist out of beams, trusses and connections. Nevertheless, it has been listed in this research for the sake of completeness. The resulting structure can be defined as a panelized surface. Within this category, the size of the holes in the panels is determined by either the proximity to a line/point or the underlying colour of a gradient map (see figure 64). The maximum sizes of the holes is determined by the size of the cell. In addition to that, the position of the hole is fixed.

4.2.3 Assessment

It can be concluded that regular grid structures are the least suitable for gradual densification. For two of the methods for densifying regular grid structures a side effect is that when the target area is densified, other areas are effected as well. One of the methods does not allow gradual densification but only incremental densification, resulting in a patchy look that is not optically acceptable. The last option that was named in the previous paragraph does not result in a grid structure but in a panelized surface, and was therefore immediately eliminated.

Even though they are stronger, Delaunay triangulations are not seen in nature, while Voronoi structures are quite common. There are two important reasons for this phenomenon. First of all, the space in between the

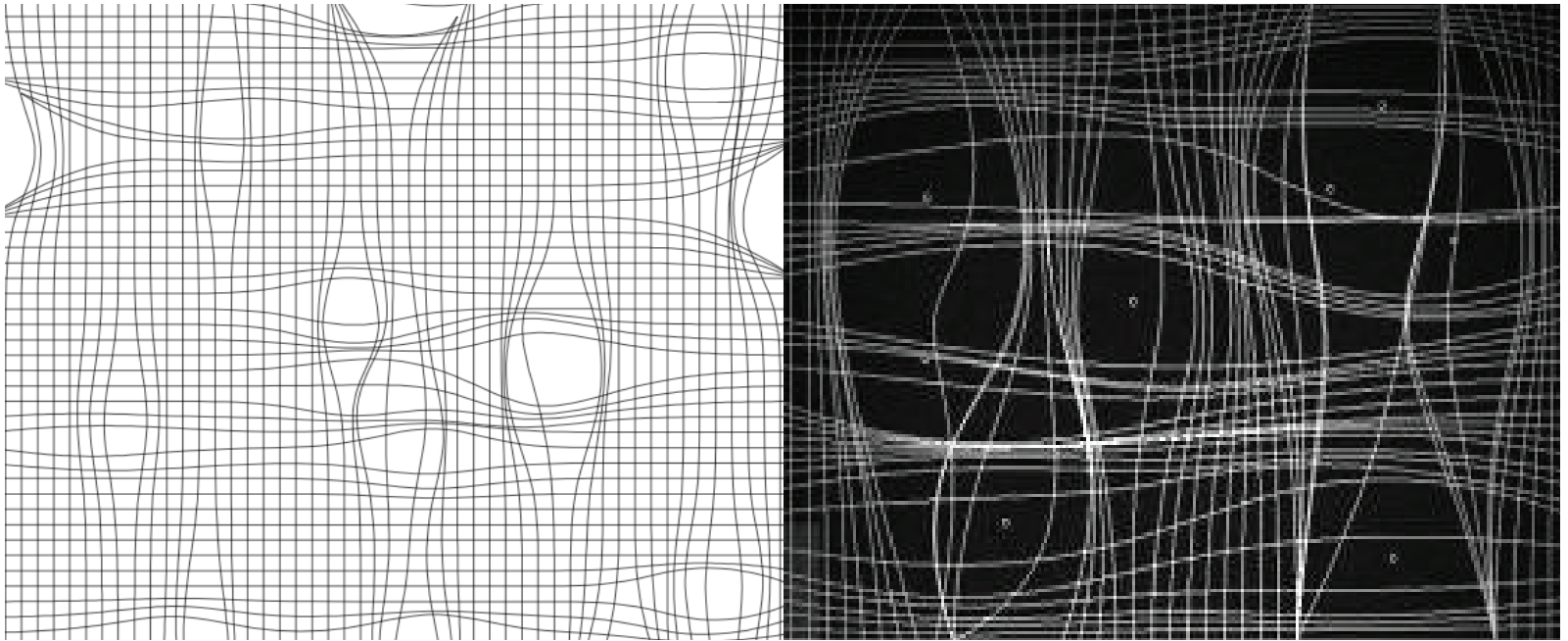


Figure 63 Densification of traditional grids by morphing [3.bp.blogspot.com (n.d.) and Pinterest, (n.d.)]

structural edges (the cells) is just as important as the structural properties of the grid. A shape without the sharp edges that consist in triangles, such as a convex polyhedron or a circle, is usually more functional. Imagine if the cells of a beehive would be rectangular instead of hexagonal: the height and width of each cell would need to be much larger for a bee to fit in and there would be a relatively large amount of unused space in the corners. And secondly, Delaunay grids use more material than Voronoi grids and therefore require more material and energy to be constructed. Voronoi grids form an efficient trade-off solution between functionality and strength.

Delaunay grid structures and Voronoi grid structures both have their advantages and disadvantages from an architectural and structural point of view. Delaunay grid structures can be described as irregular triangular structures. According to Pottman et al. (2015) triangular grid structures have two major advantages as opposed to quadrangular and polyhedral hex-dominant grid structures, such as Voronoi structures. Firstly, by choosing a triangular grid all the panels are by

definition planar. Additional measures need to be taken to assure the panels of quadrangular and polyhedral hex-dominated structures are planar. Planarity of the panels is a major advantage because it is a lot cheaper to produce planar glass panes/panels as opposed to single or even double curved glass panes/panels. Secondly, triangular grids have very good structural properties as no point of the triangle can be moved without lengthening and/or shortening two of the edges. This causes the triangle to be very stable as opposed to quadrangular structures, which easily shear. Polyhedral hex-dominated structures perform better than quads as they are 'more rigid and not as susceptible to deformation [...] as quads' (Pottmann et al., 2015).

Pottman et al. (2015) also mention three disadvantages of triangular structures as opposed to quadrangular and polyhedral hex-dominant meshes. A major disadvantage is that within a Delaunay triangulation, each vertex is incident with approximately six edges as opposed to quadrangular structures (four incident edges) and polyhedral hex-dominant structures (approximately three incident edges). This results in relatively

complex and expensive connections. In addition to that, the fabrication of triangular glass panes or panels generates more waste than quadrangular or polyhedral panels. Lastly, Pottman et al. (2015) state that triangle structures generally require more material than quadrangular or hex-dominant structures. Polyhedral hex-dominant structures use the least amount of material of the three. An additional major disadvantage to irregular triangular structures that has not been mentioned by Pottman et al. (2015) is that the resulting grid usually results in a grid that is not optically acceptable because guidelines are formed that are not smooth and

continuous (see image 62). Voronoi tessellations do not form these guidelines and therefore result in a grid that optically outperforms the Delaunay grid.

It can be concluded that Delaunay outperforms the Voronoi only on the structural aspect while Voronoi has a lower connection complexity, requires less material, results in a lower amount of waste for glass panes/panels and results in a look that is optically pleasing. In addition to that the aesthetic look that I am trying to achieve is a bonestructure/biomimicry like look. A Voronoi grid structure corresponds to the

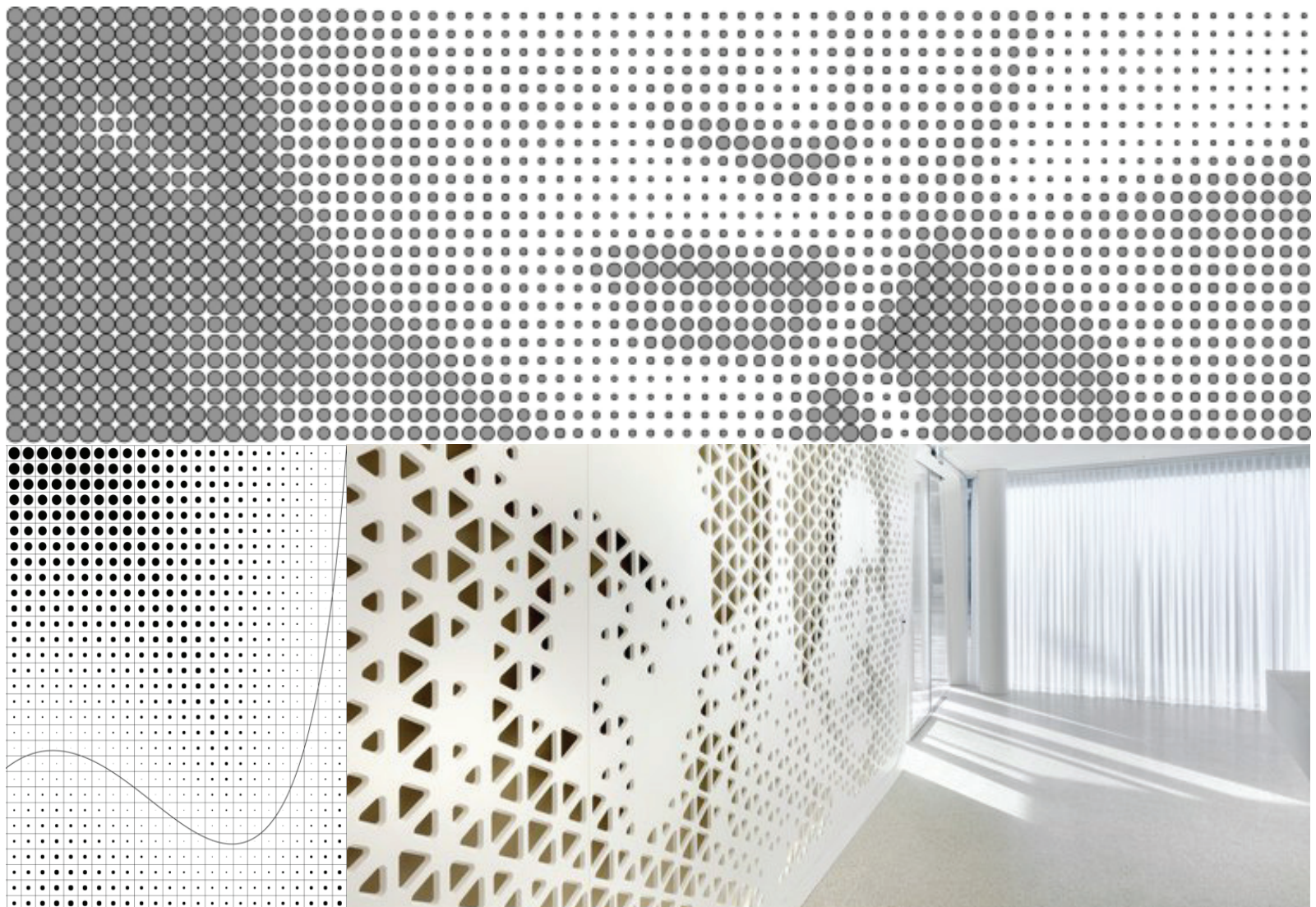


Figure 64 Densification of traditional grids by altering the cell geometry [Madigan (2012), Madigan (2012) and Architonic (2011)]

image that I am trying to achieve. Thus, the decision to use a Voronoi pattern for the canopy is also based on personal preference. To me, this outweighs the fact that delaunay is the best option from a structural point of view.

The polygons in a Voronoi pattern need stiff corners in order to maintain their shape (unlike triangles). Thus, every connection/node need to be capable of handling bending moments. According to the scheme on page 65 this means that all steel connections and all piece-small connection types are no longer an option. The remaining wood type connections are only suitable for square grids with continuous members. Thus, only FRP remains as a suitable option. FRP also has the largest freedom of shape, making it possible to create a bone-like look.

4.3 Generation of a Graded Voronoi

4.3.1 Performance Based Gradient

The gradient of the envelope structure can be determined by many different aspects. Ideally, the structure of the envelope matches the desired performance. Many calculations regarding the building's performance are virtually presented using 'image maps' that are projected on a complex surface that represents the building's shape. These image maps can represent many different performances such as the desired acoustical performance in regards to the adjacent functions, the received solar radiation, the desired solar radiation, wind pressure, stresses within the structure, deflections of the structure, etc.

Ideally, the densification of the Voronoi grid structure matches the resulting image map. In order to do so, a grasshopper script that generates the graded Voronoi must be capable of handling a variety of different cases. When buildings do not have a shape with a fluent surface, like building 'blocks', the performance analysis usually returns an image map with very harsh colour changes (see figure 65). The script also needs to be capable of handling both subtle and stronger gradients. In addition to that the script needs to be sensitive to different shades of grey as well and it is important to know to what 'dot size' the script can respond. Several grasshopper scripts/generation methods will be tested on their performance based on the translation of the images in figure 66 to a graded Voronoi. There are

multiple ways to generate a graded Voronoi. There are roughly six methods that are used in scripts that generate the basic points that serve as input for a graded Voronoi:

1. Methods that add or remove points were needed (f.e. probability based culling)
2. Methods that use a relaxation method to redistribute the points
3. Methods that use a Circle Packing algorithm based on attracting and repelling forces to distribute points
4. Methods that use a Circle Packing algorithm that is based on mathematical theorems
5. Methods that use remeshing techniques
6. Combinations of the methods above

There is a very large amount of algorithms that can generate a graded structure. For the sake of time a selection was made to test all the categories that were named above. Firstly, Lloyd's algorithm, in combination

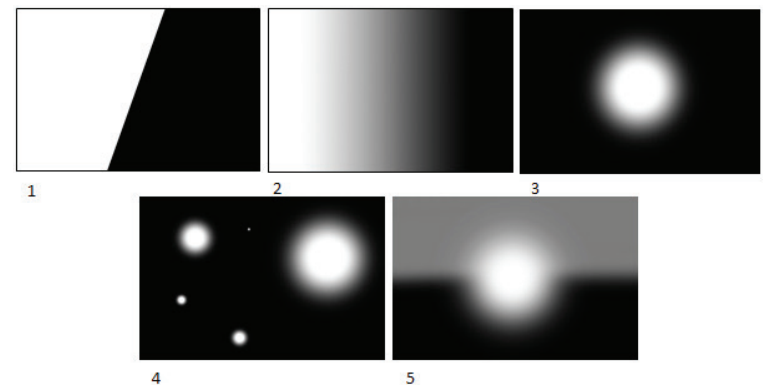


Figure 66 Images that are used as an image map for the graded Voronoi algorithms [own image]

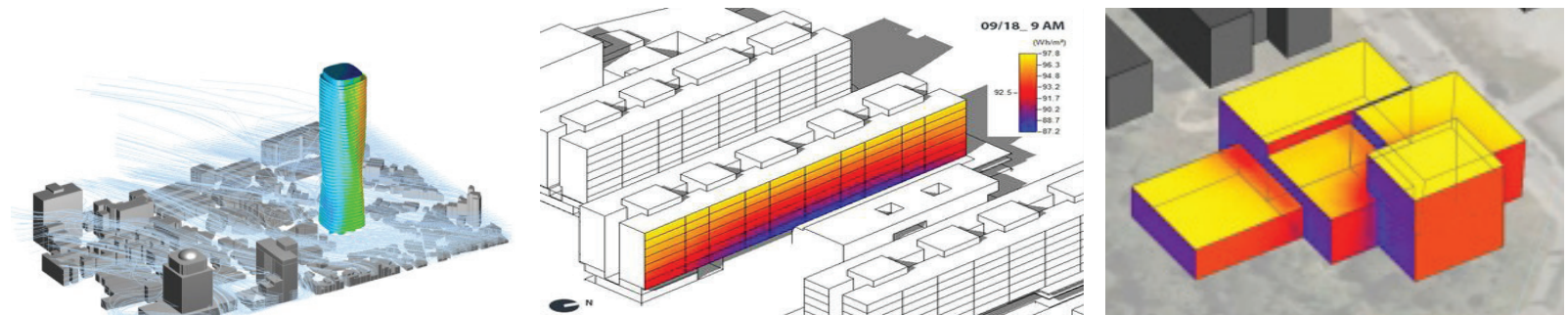


Figure 65 Left: wind pressure analysis [Fudala, 2017]. Middle: solar radiation analysis [Building Performance Analysis, 2015]. Right: solar radiation analysis [Auworkshop, n.d.]

with probability based culling was tested (category 1&2). Then, a spring force system was tested that can be grouped under category 3. Lastly, a smart circle packing algorithm that uses mathematical theorems and remeshing techniques was tested (category 4&5).

4.3.2 Lloyd's Algorithm

The first script is a script of Daniel Piker that was found on the grasshopper 3D forum. It allows for the generation of an image based graded Voronoi by using Lloyd's Algorithm, also known as Voronoi Relaxation. By running Lloyd's Algorithm the Voronoi tessellation will converge towards a centroidal Voronoi tessellation (CVT). A CVT is a Voronoi tessellation from which the generation point of each cell is also its center of mass. Lloyd's algorithm is closely related to the k-means clustering algorithm. However, Lloyd's Algorithm uses the center of mass from a geometric region (the cell) instead of the center of mass of a set of points within the region to relocate the generation points of the Voronoi. According to Du, Emelianenko and Ju (2006) Lloyd's Algorithm can be described as follows:

ALGORITHM 1.1 (Lloyd algorithm for computing CVTs).

Input:

Ω , the domain of interest; ρ , a density function defined on Ω ;
 k , number of generators; $\{\mathbf{z}_i\}_{i=1}^k$, the initial set of generators.

Output:

$\{V_i\}_{i=1}^k$, a CVT with k generators $\{\mathbf{z}_i\}_{i=1}^k$ in Ω .

Iteration:

1. Construct the Voronoi tessellation $\{V_i\}_{i=1}^k$ of Ω with generators $\{\mathbf{z}_i\}_{i=1}^k$.
2. Take the mass centroids of $\{V_i\}_{i=1}^k$ as the new set of generators $\{\mathbf{z}_i\}_{i=1}^k$.
3. Repeat procedures 1 and 2 until some stopping criterion is met.

The image below shows the operation of Lloyd's algorithm. Firstly, a Voronoi tessellation is constructed with the first set of points. The center of mass from all the generated cells is calculated. Then, these centers of mass are used as new generation points. This process repeats until the center of mass and the generation points are within a certain distance to each other. The algorithm tends to return cells that are approximately the same size.

In order to create an image based Voronoi, the cells have to be different sizes. The first set of generation points is based on a black and white image map. Points at the white parts are subtracted by using probability based culling. These points are then used as input for Lloyd's algorithm. Grasshopper normally does not support loops, but by using the hoopsnake component introducing loops in the grasshopper script becomes possible.

The Script

The first part of the grasshopper script generates a grid of points over the domain of the image that will later be inserted. The domain on the x-axis is 0 to 15 and the domain on the y-axis is 0 to 10.0. The domain values are inserted into two domain range components that create a range of numbers within these domains. The number value is also inserted into the these range component. This value determines how many numbers are created within this domain. The upper range component

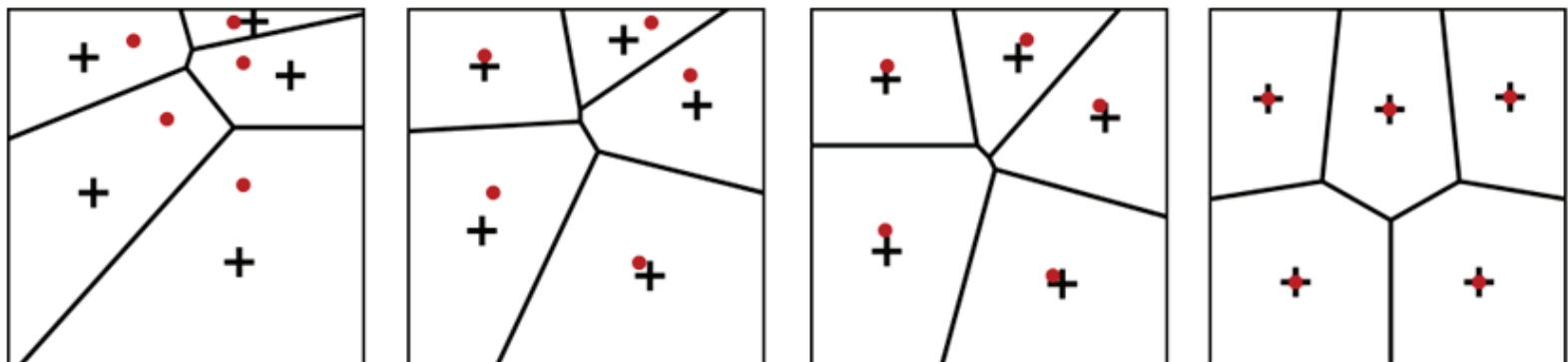


Figure 67 Four iterations of Lloyd's Algorithm [Wikipedia, 2017]

generates the x-values of the points and the bottom one generates the y-values of the points. The number of points that is generated on the x domain is multiplied by 1.5, since the length of the image in the x direction is 1.5 times larger than the length in the y direction. In the image above, the number value at the beginning of the script was set on 20, resulting in a point grid with 20 points in the y direction and 30 points in the x direction.

The second part of the grasshopper script removes points from the point grid based on the image map. This part needs to remove points at the lighter parts of the image. The points from the grid that were generated in the first part of the script are inserted into the Image Sampler component. This component returns the brightness value at the location of each point. A value of 1 represents a white colour and a value of 0 represents a black value. The resulting values are extracted from 1 to inverse the image. If this step would be skipped, the points would be removed at the darker parts of the image, which is not desirable. The resulting values are inserted into a domain component that constructs a numeric domain from two numeric extremes. The A input is set to zero. An additional value (standard: 0.2) is added to the B input of this component by adding an expression. This value is added so that there is always a chance that not all points within white areas are removed. The combination of the two Random components generate a random value between the domains that have been generated by the Domain component. For example: if the value of 1 point was 0.5 (grey) the generated domain would be 0 to 0.7 and the Random components would return a random number within this range. The resulting numbers are inserted into the Larger Than component, which returns a 'true' value if the value is larger than a specific threshold that is inserted into the B component. The output of this component is then used as a pattern to cull some of the grid points that have been generated in the first part of the script by connecting both the generated points and the output of the Larger Than component

to the Cull Pattern component. As a result, there are more points removed in the white areas than in the dark areas of the image map.

The last part of the grasshopper script is the part that performs the actual algorithm. The remaining points from the second part of the script are inserted into the 'Starting Data' input from the Hoopsnake component. The 'Feedback Output' of the Hoopsnake component is then inserted into the Voronoi component that generates a 2D Voronoi based on a collection of points and gives the Voronoi cells as output. These cells are then inserted into the Polygon Center component that calculates the center of mass from all of the cells. These centers have to form the new input for the Voronoi component. This can be done by inserting the output of the Polygon Center component into the 'Data' input from the Hoopsnake component. In order to control the amount of iterations that the Hoopsnake component performs, a Smaller Than component is connected to the Iterations Counter output of the Hoopsnake component. As long as this value is smaller than the desired amount of iterations, it returns a true value that is used as input for the 'Boolean' input for the Hoopsnake component. If the value becomes equal to the desired amount of iterations it returns a false value, which causes the algorithm to stop.

Testing

The performance of the script was tested on all of the images that are shown in figure 66. The results of the tests are shown in the images on the next pages. During the tests the input number value of the first part of the script was varied as well as the number of iterations. There are several conclusions that can be drawn after analyzing the results and working with the script. The first conclusion that can be drawn is that the resulting sizes of the Voronoi cells are highly dependent on the amount of points that were generated in the first part of the script and the amount of iterations that have been performed. The user has no direct influence

over how large the largest openings and how small the smallest openings will be. A second conclusion is that the script is not good at handling hard gradients. By performing only a few iterations (10) it is clear where the line between the black and the white part is, however, the cells on the white part of the image vary largely in size, while they should be approximately the same size if they would be a true representation of the image map. The cause lies in the nature of Lloyd's algorithm. Initially, it only generates points at the black part of the image. Lloyd's algorithm causes cells to move more towards the white part of the image (see figure 68). As a result, the cells at the black part are roughly the same size, but the cells at the white part are not. Thirdly, the script returns better results with softer gradients as long as the bright 'spots' are relatively large. The script has a tendency to ignore small spots as can be seen in the images on the next pages, even though most of these patches are larger than the size of the smallest cells. Lastly, the script returns very unfavourable results when multiple shades of grey are used in an overlay (test image 5). With the standard settings the script completely ignores the grey parts of the image and sees it as a black colour. Tweaking the settings of the Larger Than slider and/or the expression from the Domain component returned another unfavourable result: it still ignores the grey colour, but sees it as a white colour instead of a black one.

It can be concluded that both probability based culling (category 1) and Lloyd's Algorithm (category 2) are not sufficient for image based Voronoi generation. The user has no control over the minimum and maximum sizes of the cells, the end result is highly dependent on the amount of iterations, the script it is not capable of handling a hard gradient and the script returns unfavourable results when being used for more complex gradient maps that use an overlay of several shades of grey.

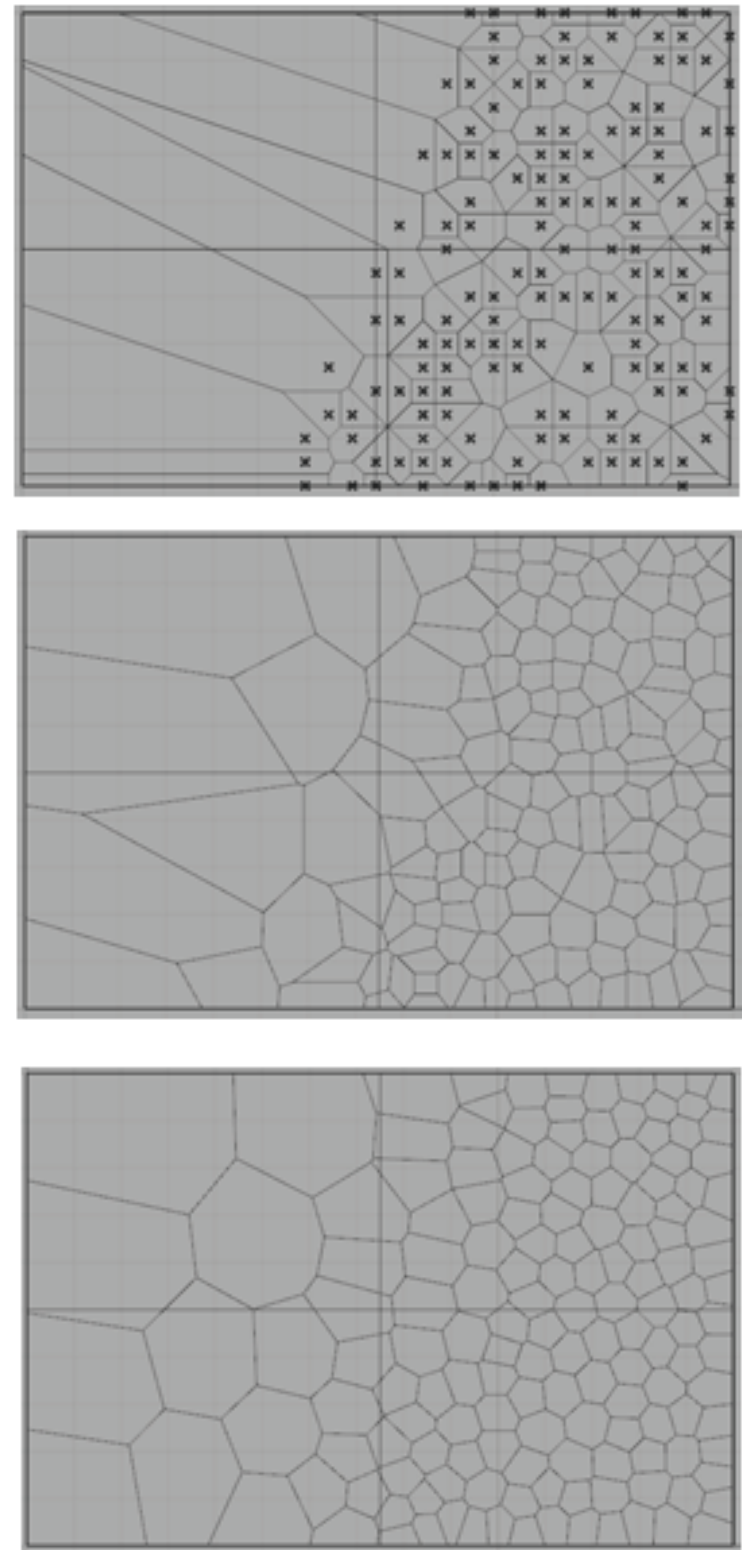
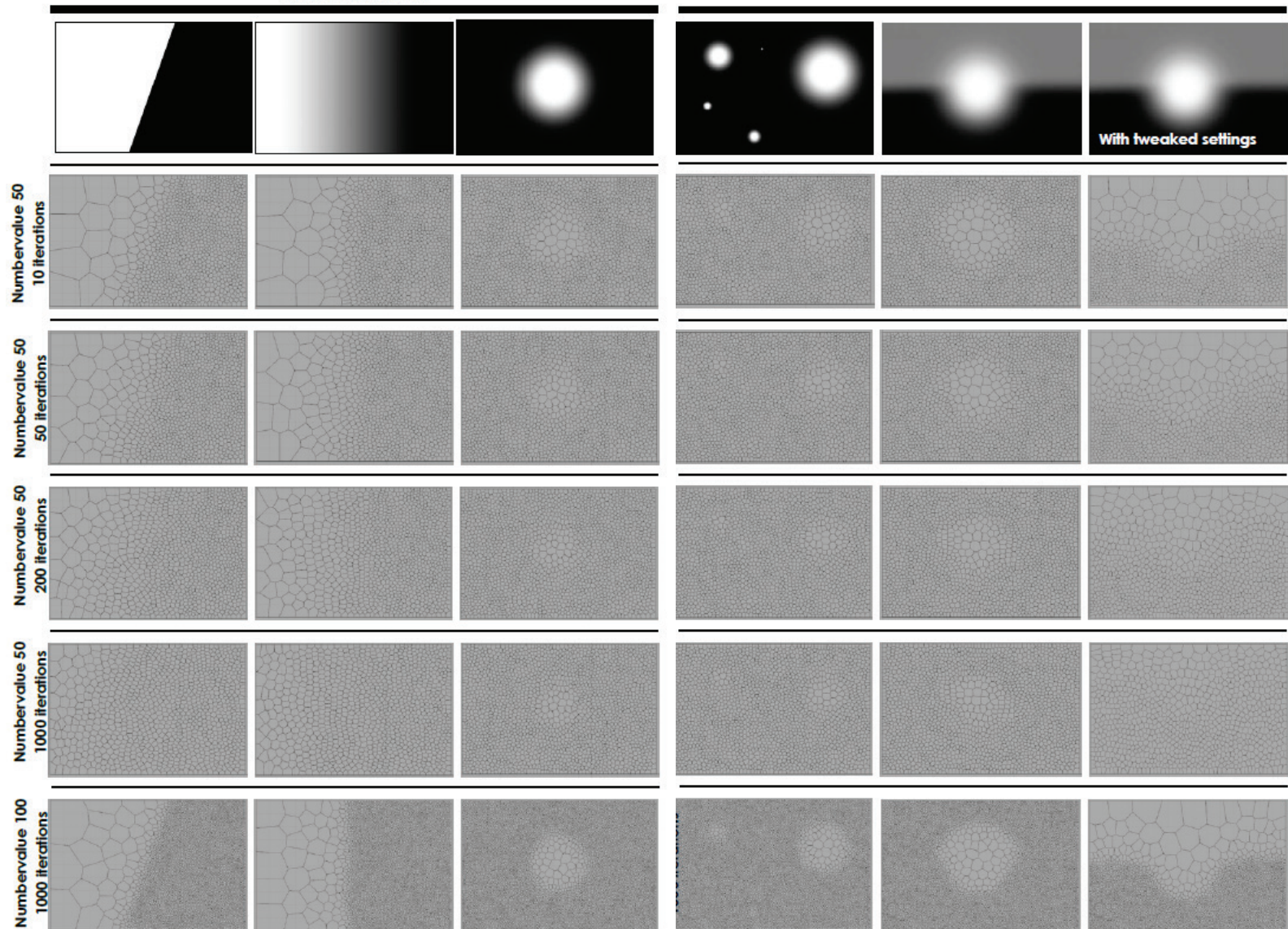


Figure 68 Resulting Voronoi with Lloyds Algorithm for the image map with a harsh gradient after 0 (left), 1 (middle) and 10 (right) iterations with a starting number value of 20

Test Results



4.3.3 Spring Force System

The second script is another script of Daniel Piker that was found on the grasshopper 3D forum. The script uses collision forces between circles with a fixed radius to create a circle packing. The forces between circles are represented as springs that obey Hooke's law. This law states that the amount of force required to extend or compress the spring a certain amount is proportional to the length of the amount. The rest length of the spring that connects two circles is equal to the radius of the first circle plus the radius of the second circle. When the spring is in its relaxed state the circles touch each other. When the circles collide (overlap) the spring is in an compressed state and pushes the circles out of each other until the spring is back at its rest length (see figure 69). When the circles are not touching and there is space in between them, the spring is in an extended state and pulls the circles back to each other until the spring is at its rest length.

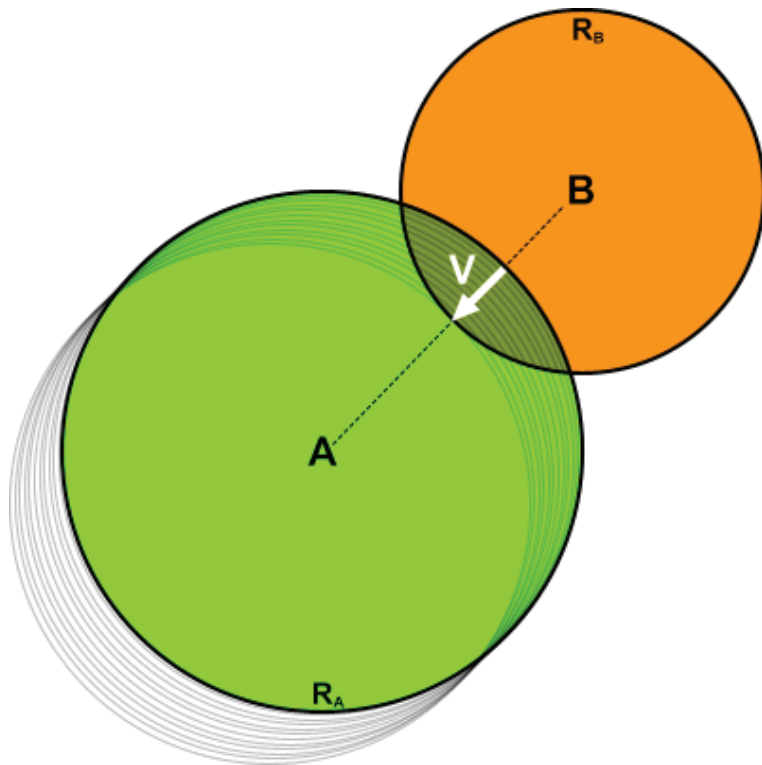


Figure 69 Collision forces of two circles [McNeel wiki, n.d.]

The script was adapted so that it would use the underlying image map to determine the radii of the circles instead of using a fixed radius for all of the circles. This version of the script moves the circles without updating the radius of the circles to its new position on the image map. A variation of this script that updates the radii of circles in between iterations was written as well.

The scripts are very similar. However, the second script uses the HoopSnake component to update the circle radii in between kangaroo's iterations. The centers of the circles are used to generate the resulting graded Voronoi.

The script

The first part of the algorithm generates circles with a radius that matches the underlying image map. The image, has a length of 15 units in the x-direction and a length of 10 units in the y-direction. A rectangular surface that matches these measurements is used as the first input for the script. This surface is connected to the Populate Geometry component along with a count slider that determines how many points will be generated on the surface. The resulting points are then inserted to the Image Sampler component that returns the brightness value at the location of each point. The values of the brightness all lie on a domain of 0 to 1.0. In order remap the outer boundaries to match the desired minimum and maximum circle radii, these values are inserted into the Remap Numbers component. The desired domain is constructed by inserting two sliders that represent the minimum and maximum radius size into the Construct Domain component. The new desired domain is also plugged into the Remap Domain component as the 'Target Domain'.

In the second part of the script, the circle packing forces for the kangaroo solver are generated. The rectangular surface and the generated points are inserted into the PullToSurf component. This

component generates a force that keeps the points on the surface at all times. The pull strength is set on a very high value of 9000. In order to generate the spring forces, all points are inserted into both of the inputs from the Cross Reference component. Next, the outputs of this component are used as inputs for the Line component, that generates a line between two points. Because of the Cross Reference component, lines that connect each circle to every other circle are created. The combination of the other Cross Reference component and the Mass Addition component calculate the sum of the radii of all of the connected circles.

The generated lines are inserted into the Connection input of the SpringsFromLine component and the sum of the radii is inserted into the Rest Length input and the UpperCutoff input of the SpringsFromLine component. Lastly, the Spring Forces and the Pull Forces are inserted into the Force Object input along with an on/off toggle and a timer.

Spring Forces and the Pull Forces are inserted into the Force Object input along with an on/off toggle and a timer. This is the part of the algorithm that calculates how the points (the centers of the circles) are being

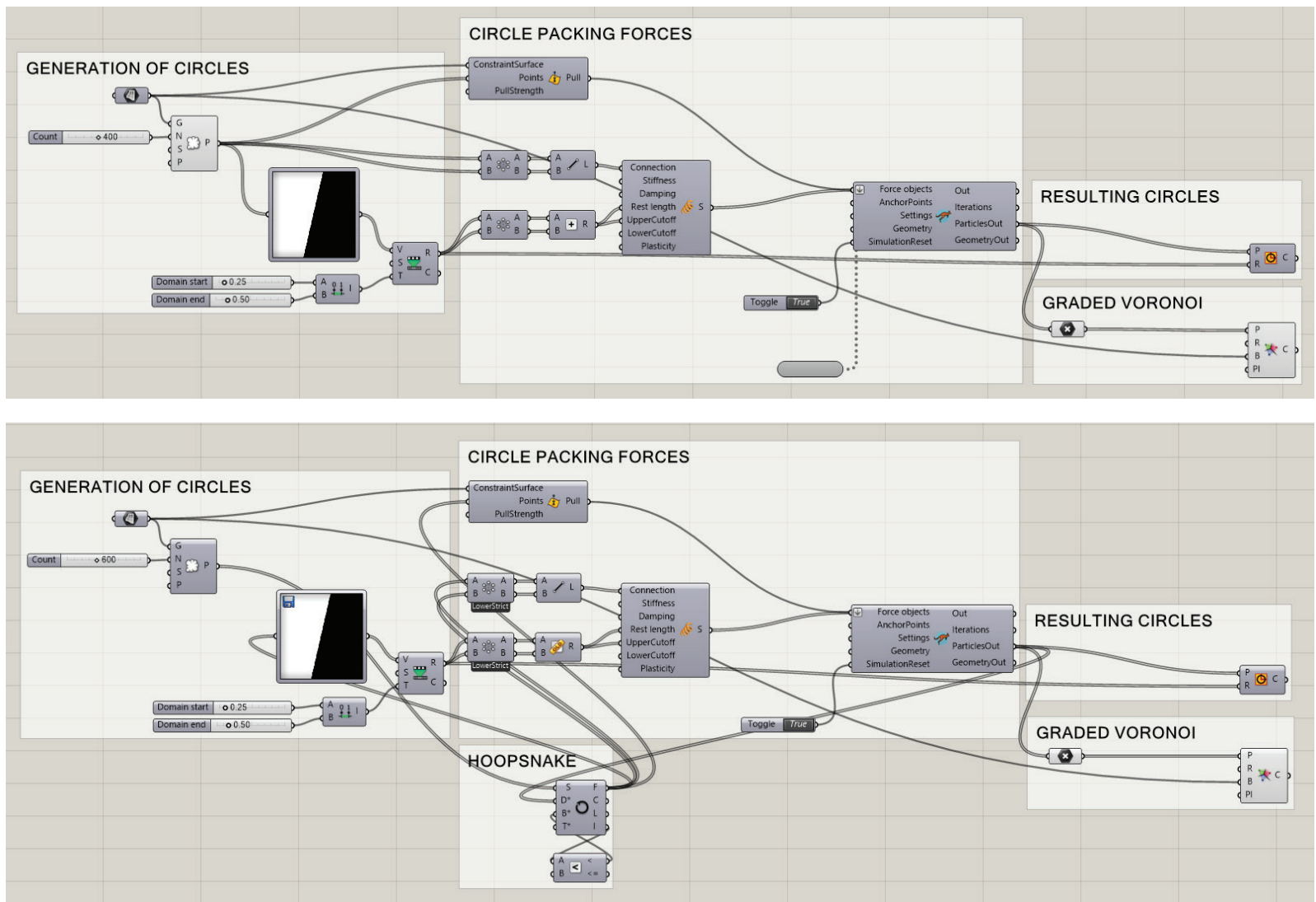


Figure 70 Script without (above) and with (bottom) hoopsnake

moved over the surface. The ParticlesOut output is connected to a Circle component along with the output of the Remap Domain component, that determines the radii of the circles. The ParticlesOut output is also connected to the Voronoi component that generates the final 2D Voronoi based on these points.

Testing

The performance of the script was tested on all of the images that are shown in figure 66. The results of the tests are shown in the images on the next pages. During the tests the amount of generated points was varied as well as the minimum and maximum radii for the

circles. There are several conclusions that can be drawn after analyzing the results and working with the script. Firstly, in contrast to Lloyd's algorithm, whether the algorithm returns a favourable result or not is highly dependent on the number of circles that are generated on the surface. If there are too little circles to pack the entire surface, the algorithm returns strange results with deviating Voronoi cells that do not match the desired cell size (see figure 72). This requires quite some tweaking of the script. This holds true for both the normal variant and the HoopSnake variant of the script. A smart circle packing algorithm that can edit and remove points when necessary would be preferred.

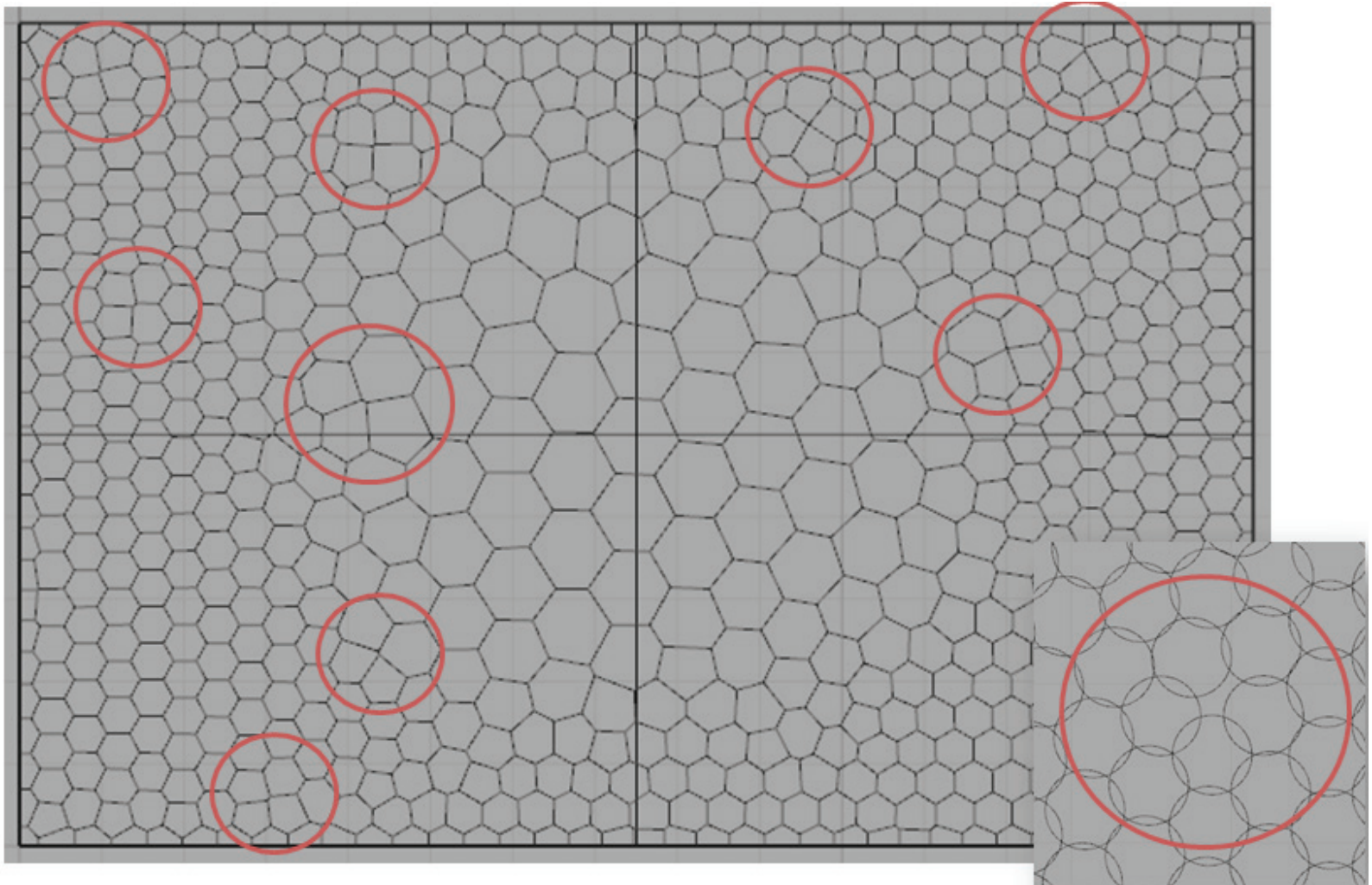
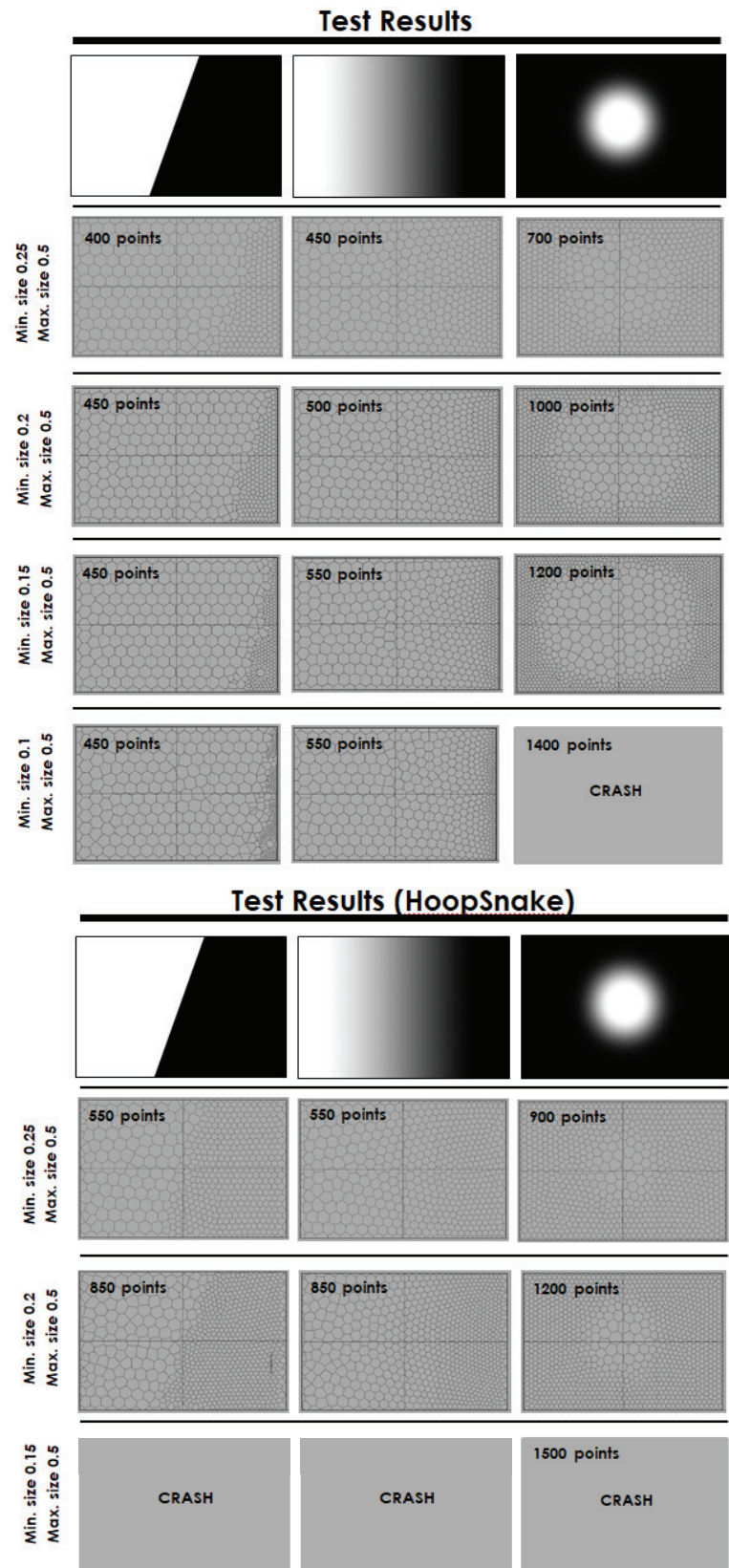


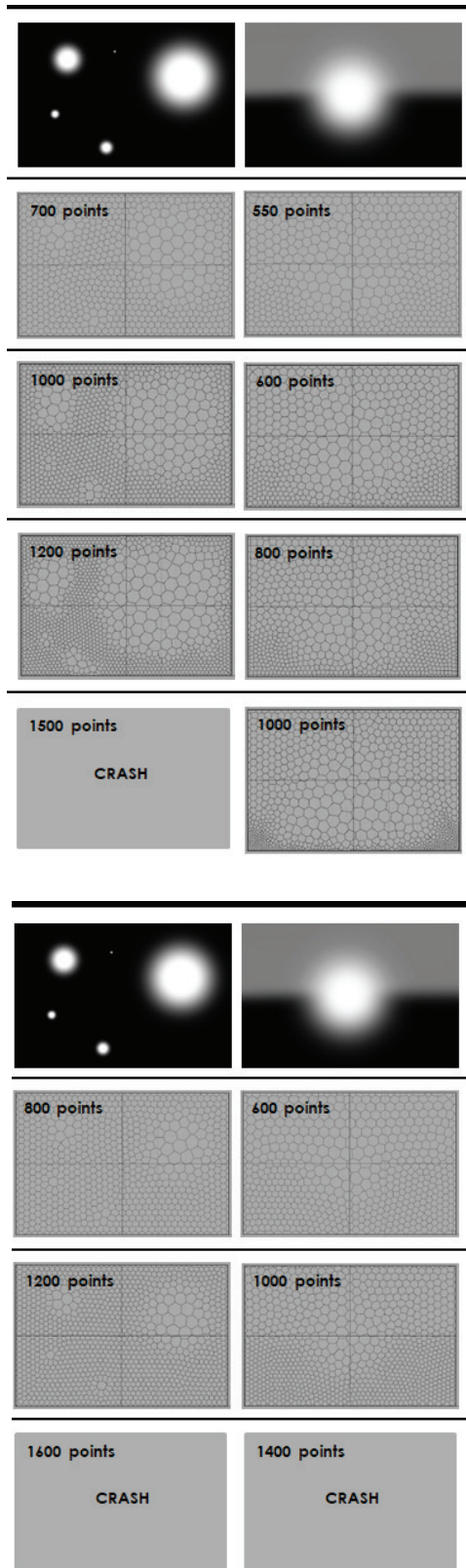
Figure 72 Collision forced Circle Packing, run with 700 circles, with image map 3.

A second conclusion that can be drawn is that in order for this script to return results that properly match the underlying image map, it is highly important that the radii of the circles are updated in between iterations. This is most evident when comparing the results of both of the algorithms for the image with the harsh gradient. This can be explained easily. The initial points are evenly spread over the surface. As a result there is approximately an equal amount of big circles and small circles. When the script is run, the large circles will push all the small circles to the side. A downside to this is that the script with HoopSnake is significantly slower than the regular script. However, it outperforms the regular algorithm and Lloyd's algorithm on all of the test images. It even acknowledges very small light spots, as long as they are larger than the minimal circle radius.

When the resulting Voronoi diagrams are closely analyzed several patches of cells with deviating shapes can be found. It appears that even when there are so many circles that all circles overlap in the final result, there are still some places where there is a relatively large space in between the circles. At these places, the deviating shaped cells occur. This problem could be resolved by a script that converges towards a tangent circle packing.

It can be concluded that the HoopSnake variant of the algorithm works better than both Lloyd's Algorithm and the regular collision force circle packing script, although it is significantly slower. However, the performance of the script is highly dependent on a proper choice of amount of circles. A smart circle packing algorithm that can edit and remove points when necessary would be preferred. In addition to that, both of the scripts show several patches of cells with deviating shapes, caused by spaces between the circles. A script that returns a tangent circle packing overcomes this issue.





4.3.4 Smart Circle Packing Algorithm

The first two scripts that have been tested both pose a couple of critical problems. Lloyd's algorithm does not allow the user to set a minimum and maximum cell size; the result is highly dependent on the amount of points that have been inserted and on the amount of iterations of the script; the script is not good at handling hard gradients (image map 1), gradients with multiple shades of grey on top of each other (image map 5) and it tends to ignore smaller spots in the image map (4). The Spring Force based script (with HoopSnake) overcomes some of these problems. However, this script is very slow, still very dependent on the amount of points that are inserted and returns a result that contains patches with cells that are oddly shaped.

The reason why the Spring Force system is a slow script is because 'although the spheres only exert force on each other when they are closer than a certain distance, they still need to be constantly checked against all the other spheres to see if they are within this distance' [Piker, 2012]. If every circle needs to be checked against all the other circles, and there are a 100 circles, then 99 checks need to be performed per circle. This equals an amount of $100 \times 99 = 9900$ calculations in total. One can see how this becomes a very slow method once the amount of circles starts to increase further. Another question that remains to be solved is how to add or subtract points where necessary.

The last script solves all of these problems by using the initial mesh connectivity. This mesh, that exist of vertices and edges, is iteratively updated locally based on the underlying image map by using edge split and vertex collapse principles. In addition to that, it uses edge flip to reduce the deviation of the overall mesh from the average degree (which is six for Voronoi). Additionally, the script does not use Spring Forces as the driving force for the circle packing. Essentially, it is using a condition on the sum of the length of the edges

from two adjacent triangles. It iteratively changes the lengths of the edges to meet this condition. The script also has a built in Laplacian smoother, which is used to generate relatively regular meshes. These principles are further explained in the next chapters.

Split/Collapse/Flip

The script determines whether an edge needs to split or collapse based on the values of an underlying image map. In order to do so, the image map is translated to a grid of points with values that match the underlying image map. The target edge length of each edge is determined by using an inverse distance weighted interpolation that uses the values of nearby points to make an assumption of the value at that specific edge. The closer to a certain point, the more influence the value of that point has on the resulting target length. When the edge length is within a certain tolerance of the target length, no changes are made to that edge. If the edge is shorter than the target length, the edge

collapses, by merging the end points (vertices) of that edge together (see figure 73). If the edge is longer than the target length, the edge is being split in half, creating an extra vertex and two new edges.

When all the edges are compared to their target length and either stayed the same, split or collapsed. It is checked whether flipping the edge results in a smaller deviation from the average degree (which is six for Voronoi). If this is the case, the edge is flipped. If it results in larger or equal deviation it stays the same. The script also used the Laplacian Smoothing principle that is illustrated in figure 74. Each vertex is connected to its neighbours. The center of mass of these neighbours is determined and the vertex is moved to this point. The Laplacian Smoothing part of the script may not have been strictly necessary, however it results in a rather regular mesh.

Circle Packing

After the script checked whether an edge collapse/split/flip has to be performed the script determines how the edge length compares to a specific condition on the sum of the length of the edges from two adjacent triangles. It iteratively changes the lengths of the edges to meet this condition. Due to this part of the script, the script returns a tangent circle packing. According to Schiffner et al. [2009] this script only returns precise tangent circle packings when the surface is either a

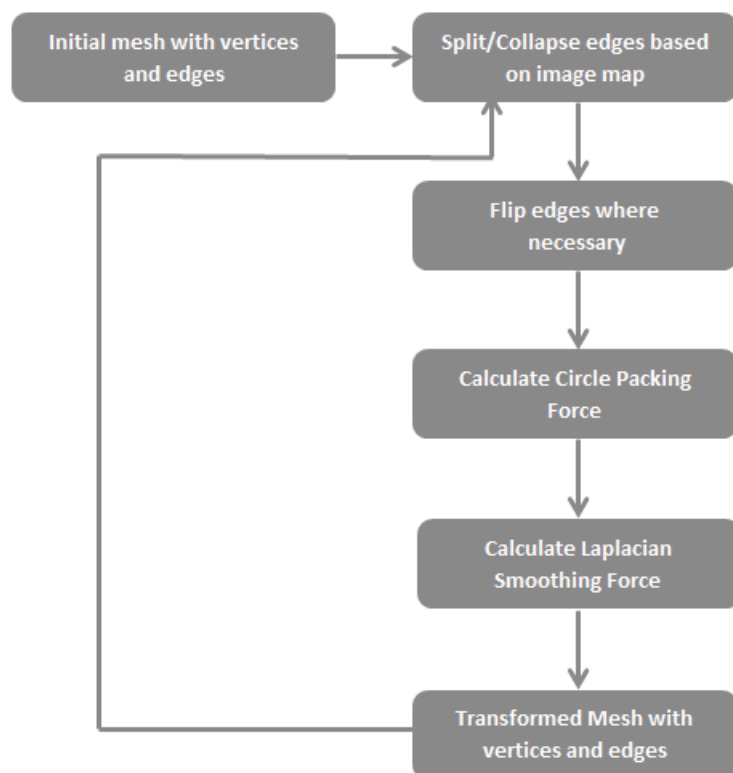


Figure 73 Scheme that represents the steps of the Smart Circle Packing Algorithm

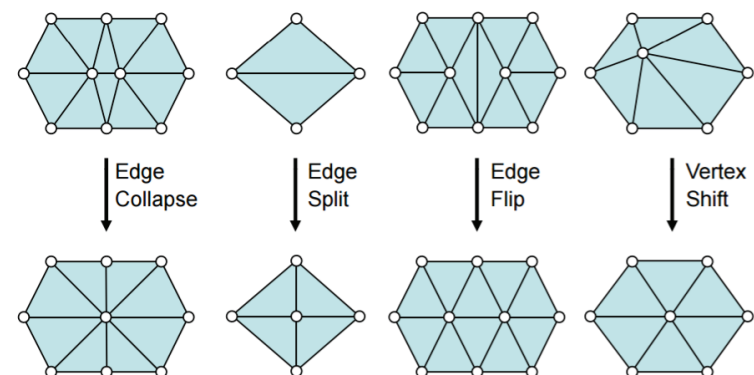


Figure 74 Illustration of the Edge Collapse, Edge Split, Edge Flip and Laplacian Smoothing principles [Stanford, n.d.]

plane or a sphere. In all other cases the script returns a good approximation of a tangent circle packing. This is because circles that touch each other, by definition, either lie on a common sphere or lie on a planar surface. [Schiftner et al., 2009]. As soon as the target surface deviates from this, the circles of the packing will no longer touch each other in one point, thus it results in an approximate tangent circle packing.

The condition that the four boundary edges of two adjacent triangles have to meet is as follows:

$$L_{ij} = R_i + R_j \quad \text{and} \quad L_{jk} = R_j + R_k \quad \text{and} \quad L_{il} = R_i + R_l \quad \text{and} \quad L_{kl} = R_k + R_l$$

This means that:

$$L_{ij} + L_{kl} = R_i + R_j + R_k + R_l \quad \text{and} \quad L_{jk} + L_{il} = R_j + R_k + R_i + R_l$$

Or in other words:

$$L_{ij} + L_{kl} = L_{jk} + L_{il}$$

A formula to determine the radii of the tangent circles can also be derived:

$$L_{ij} = R_i + R_j \quad \text{and} \quad L_{jk} = R_j + R_k \quad \text{and} \quad L_{ik} = R_i + R_k$$

Substituting these formulas into each other gives us the formula of the radius based on the edges of the triangle:

$$L_{ij} = R_i + L_{jk} - R_k$$

$$L_{ij} = R_i + L_{jk} - (L_{ik} - R_i)$$

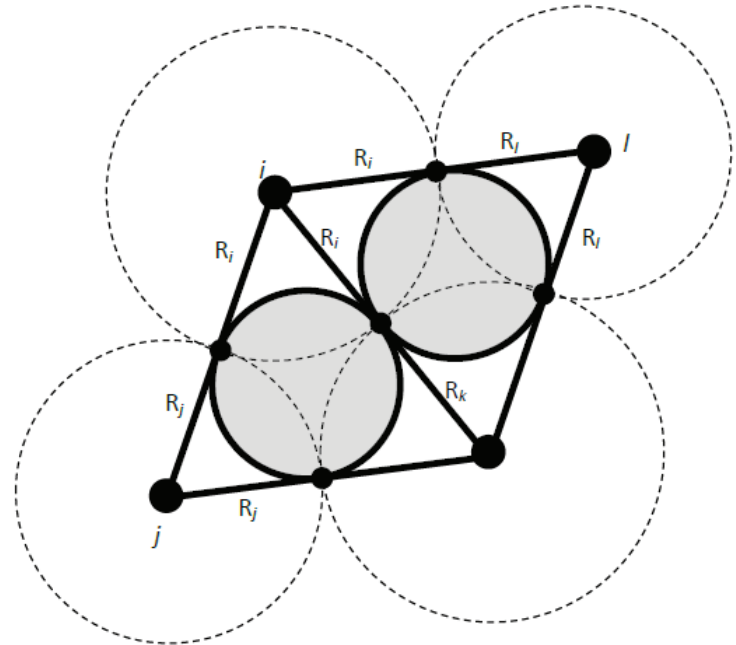
$$L_{ij} = R_i + L_{jk} - L_{ik} + R_i$$

$$2R_i = L_{ij} + L_{ik} - L_{jk}$$

$$R_i = \frac{L_{ij} + L_{ik} - L_{jk}}{2}$$

Testing

The first part of the grasshopper script generates a grid of points over the domain of the image that will later be inserted. The domain on the x-axis is 0 to 15 and the domain on the y-axis is 0 to 10.0. The domain



values are inserted into two domain range components that creates a range of numbers within these domains. The number value is also inserted into the these range component. This value determines how many numbers are created within this domain. The upper range component generates the x-values of the points and the bottom one generates the y-values of the points. The number of points that is generated on the x domain is multiplied by 1.5, since the length of the image in the x direction is 1.5 times larger than the length in the y direction. In the image above, the number value at the beginning of the script was set on 50, resulting in a point grid with 50 points in the y direction and 75 points in the x direction.

The resulting points are then inserted to the Image Sampler component that returns the brightness value at the location of each point (Part 2). The values of the brightness all lie on a domain of 0 to 1.0. In order to remap the outer boundaries to match the desired minimum and maximum length of the edges of the triangles in the resulting mesh, these values are inserted into the Remap Numbers component. The desired domain is

constructed by inserting two sliders that represent the minimum and maximum edge length size into the Construct Domain component. The new desired domain is also plugged into the Remap Domain component as the 'Target Domain'.

Part 3 of the script translates the target surface into a mesh. The Brep component (that contains this surface) is inserted into the 'Mesh Surface' component that divides the surface into mesh faces. The amount of faces that needs to be generated in the v-direction of the surface (y-direction in world) is set on 50. This value is multiplied by 1.5 and inserted into the u input to create 75 faces in the u direction (x-direction in world).

Part 4 of the grasshopper script contains the C# component that contains the script that performs the actual tangent circle packing algorithm. This component has quite a lot of inputs. The first input 'M1' requires the target mesh. If this mesh contains any quads it is turns these quads into triangles. The second input 'boundary' requires the edge curve of the mesh (if it has any). The script can only handle meshes that contain one edge curve. This means that it cannot, for example, solve a triangular surface with a hole in it. In order to extract the boundary curve from the mesh, the mesh is inserted to 'Weaverbird's Naked boundary' component. The next input from the script is the 't' input. The slider that is connected to this input sets the tolerance for the edge collapse/split thresholds. The next three inputs 'smooth', 'circlepack' and 'pull', determine the strengths for the smooth, circle packing and pull to surface forces. The pull force serves to keep all the vertices of the mesh to the surface at all times. The input 'SizePoints' requires all the points that were generated in the first part of the script as input. The input 'Sizes' requires the target size of edge lengths at all of these points. Therefore, the output of the Remap Numbers component is used as value for this input. The input 'falloff' influences the inverse distance weighted interpolation. A low number for this

input results in a smoother blend between points, while a high number gives more sharply distinguished regions. The algorithm can be run by setting the toggle that is connected to the 'reset' input on 'false' and by turning on the timer. The output of the C# component is the resulting mesh with vertices and edges.

Based on this mesh, the tangent circles can be drawn by using the formula for radii that was given on page 89 (part 5). The vertices of the mesh with its adjacent vertices can be extracted from the mesh by plugging the mesh into 'NakedVertices' component and by inserting the 'Clothedl' that contains all the points with neighbours, into the 'VertexNeighbours' component along with the mesh. This component returns the central vertices(output 'C'), the matching vertices of its neighbours (output 'N') and the normal of the central vertex (output 'V'). The formula that calculates the radius of a circle with a specific vertex as its center requires the length of the three edges of one adjacent triangle. In order to do so, the output 'N' is inserted to the 'List Item' component, that extracts the first and second point neighbour of each central vertex. Along with the central vertex, these points create one of the adjacent triangles of the central vertex. By adding these points into the 'Distance' component, the distances of each of the edges are determined. The combination of the 'Mass Addition', 'Multiplication' (x0.5) and the 'Subtraction' component perform the calculation of the radii of the circles based on the formula given on page 36. Lastly, the circles are created by inserting the calculated radii, along with the base planes for each circle (derived from the output 'N' from the 'VertexNeighbours' component) into the 'Circle' component.

Lastly, the generated Voronoi is created based on the centerpoints of each circle, that match the vertices of the mesh. This part starts with a toggle that, in combination with the 'Stream Filter' component, can turn this part of the script on and off because it is

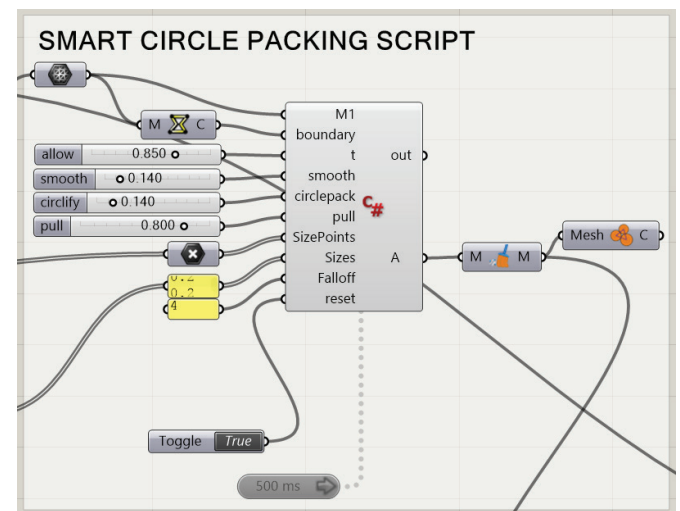
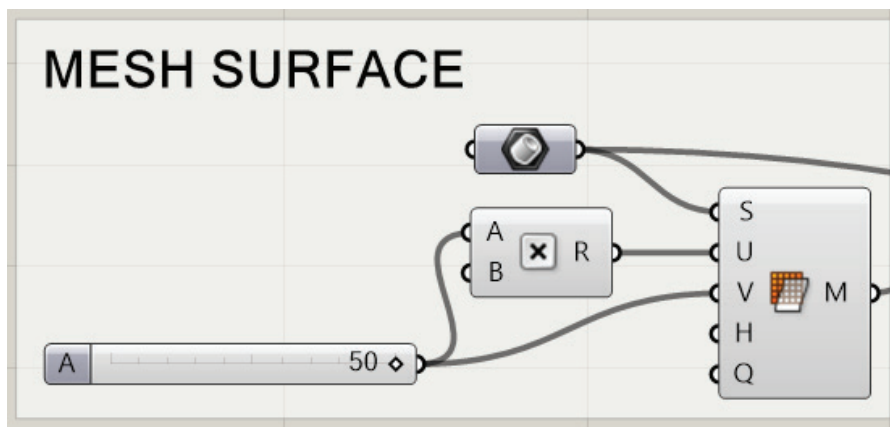
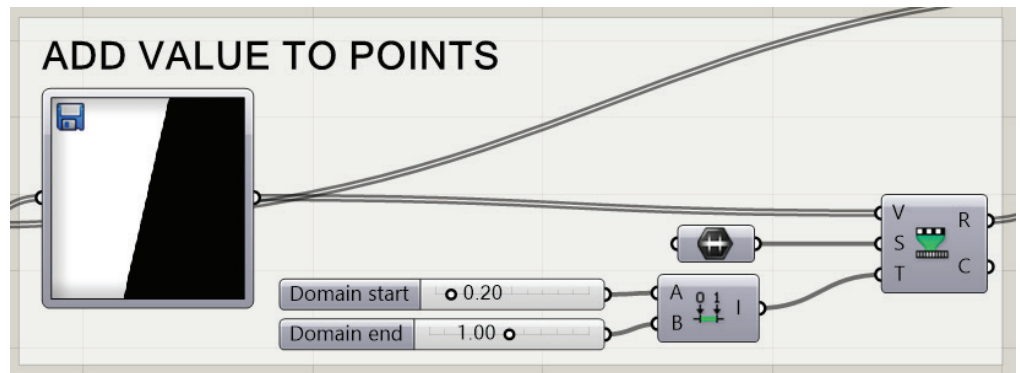
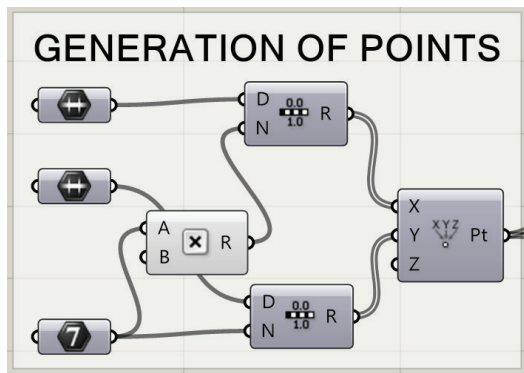
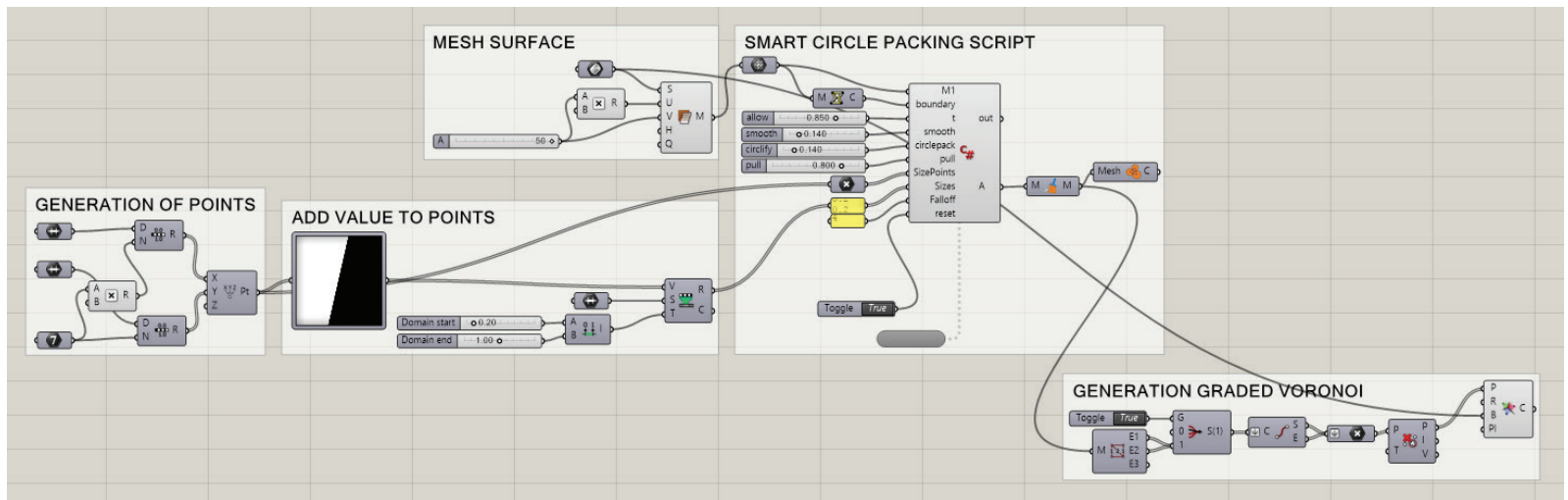
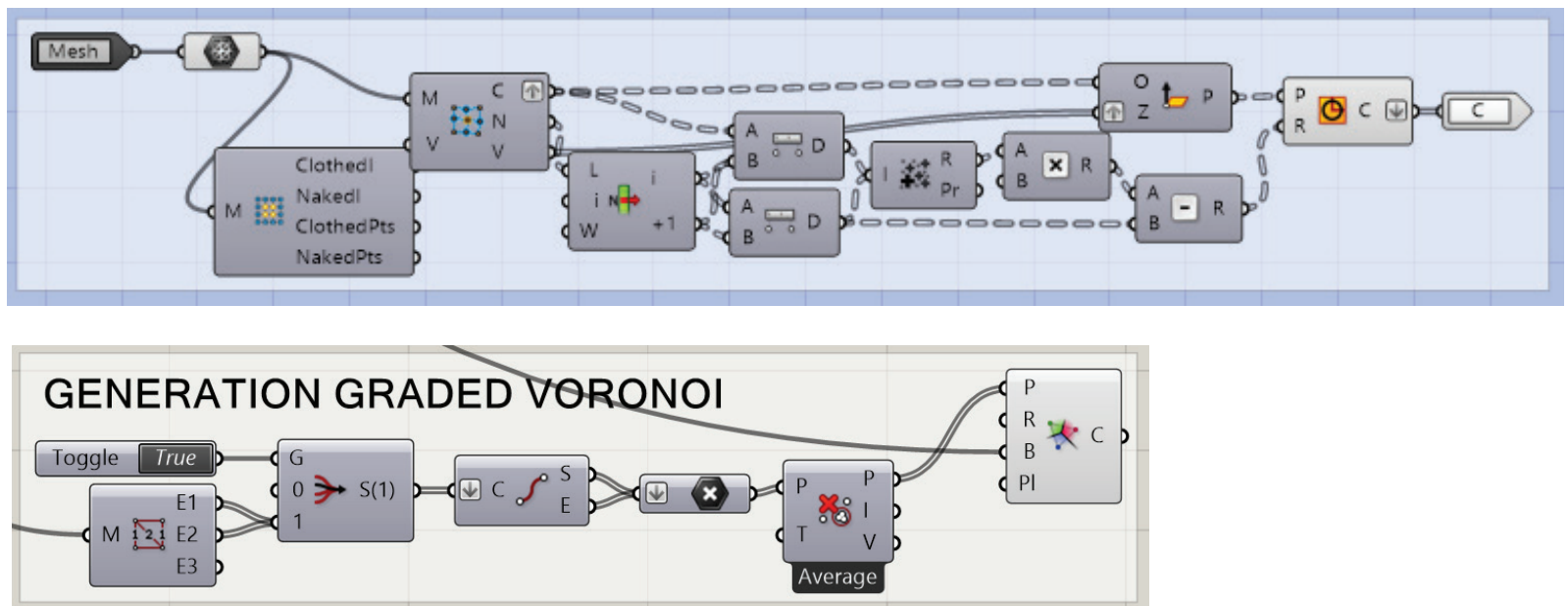


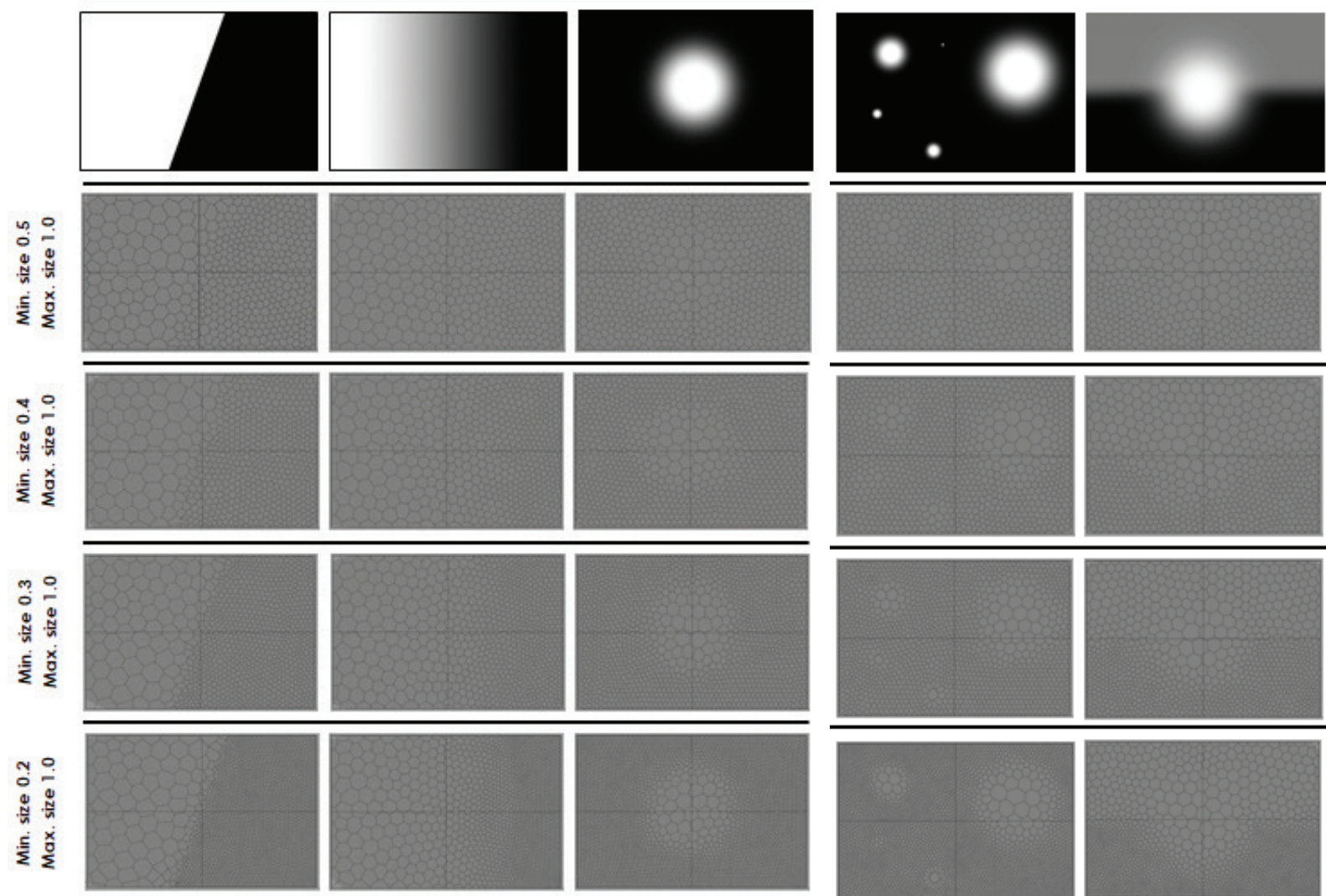
Figure 75 Smart circle packing script overview (top), part 1 (middle left), part 2 (middle right), part 3 (bottom left), part 4 (bottom right)

rather time consuming to calculate the Voronoi for each iteration of the C# script. The resulting mesh from the C# component is plugged into the 'Mesh Edges' components that gives us all the edges of the mesh triangles. The resulting lines are then inserted into the 'End Points' component that extract the endpoints of these lines. These points are then inserted into the 'Cull Duplicates' component that removes all the duplicate points. Based on these points, the graded Voronoi is created by inserting them into the 'Voronoi' component.

The performance of the script was tested on all of the images that are shown in figure 66. The results of the tests are shown in the images on the next pages. During the tests the minimum and maximum opening sizes were varied. There are several conclusions that can be drawn after analyzing the results and working with the script. Firstly, in comparison to Lloyd's Algorithm the smart circle packing script matches the underlying image map a lot better. When this aspect is compared to the Spring Force (with Hoopsnake) script, it can be noticed that the smart algorithm performs slightly better, since it does not have the issue with circles that are



Test Results



5. CASE STUDY

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5.1 Background [OMA, 2016]

5.1.1 Masterplan

'De Kuip' was opened in 1937 and holds a special place in the hearts of the citizens of Rotterdam. The stadium is characterized by its intimate atmosphere that arises from the short distance from the stand to the field. Despite several refurbishments, the iconic stadium is no longer lucrative enough to be able to financially support a team that continuously wants to compete for the title. Additionally, the stadium does not meet the modern requirements that have been stated by the UEFA and FIFA. As a result, the stadium is no longer appealing for the accommodation of large international competitions such as interlands, UEFA Cup competitions and Champions League competitions. Therefore, it has been decided that a new stadium will be built on the riverside of 'De Nieuwe Maas'.

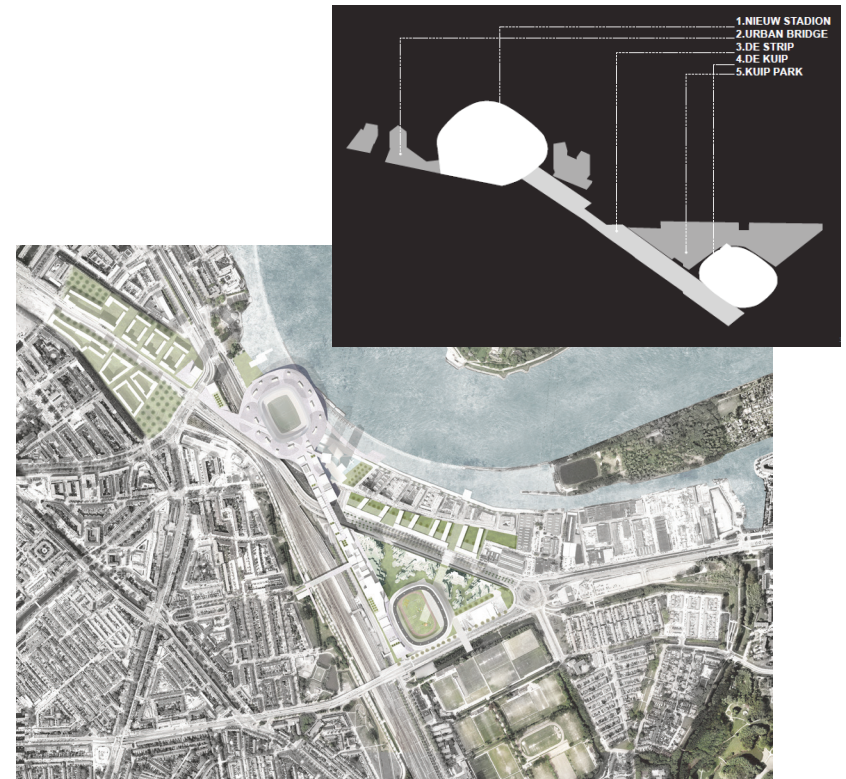


Figure 77 Plan of the Feyenoord City Masterplan by OMA [OMA, 2016]



Figure 78 Artist Impression of the new Feyenoord stadium by OMA [OMA, 2016]

The new stadium is part of a bigger masterplan called 'Feyenoord City' in which the new stadium will function as a driving force for further development in Rotterdam-Zuid. The municipality of Rotterdam has created an area concept called 'Stadionpark' that roughly covers the area between the Van Brienoord-bridge (slightly more to the right than the area shown in figure 77) and the railway between Rotterdam and Dordrecht (see figure 77). Within this area, the municipality strives towards a coherent area in which the sub regions are connected with each other and in which the theme 'Sports and Exercise' is key. The masterplan consists out of three main elements:

1. A new stadium for Feyenoord
2. Redevelopment of the old stadium
3. The additional programme of Feyenoord City

The additional programme of Feyenoord City consists out of three elements: the urban bridge, the Strip and the Kuip Park. The urban bridge will be the entrance for slow traffic coming from the city center. It is a boulevard that starts at the 'Laan op Zuid', passes through the public part of the new iconic stadium and ends at De Veranda. Along the Urban Bridge residential housing and commercial functions will be developed. The Strip will be a new elevated promenade for cyclist and pedestrians between the old and the new stadium. It exists out of an attractive public space with a mixture of urban sport pockets, hospitality industry and retailing. The Kuip Park will become a green area with

residential housing. This area will provide in a right balance between apartments, housed within parklike surroundings.

5.1.2 Stadium

The chosen site for the new stadium lies at a prominent spot on the riverside of 'De Nieuwe Maas'. The new stadium has to have an iconic character and has to contribute to the vivacity of the riverside. Fitting to this vision is a sustainable stadium with an open character and plenty of additional programme.

Concept for the stadium

The design of the stadium must respond to the different contexts it is surrounded by: the water on one side and the city on the other. The waterside has to have a more open character than the side that faces the city. The stadium will have a stand that has optimal sightlines and is as close to the playfield as possible. The bottom ring of the stand is sunken into the ground and is surrounded by a concourse that seamlessly blends into the bottom ring (see section in figure 79). Both the bottom ring and the concourse are publicly available for events and activities on non-match days. The two upper rings of the stand float on top of the concourse and together they form a perfect stadium bowl whose design is based on FIFA regulations (see figure 79).

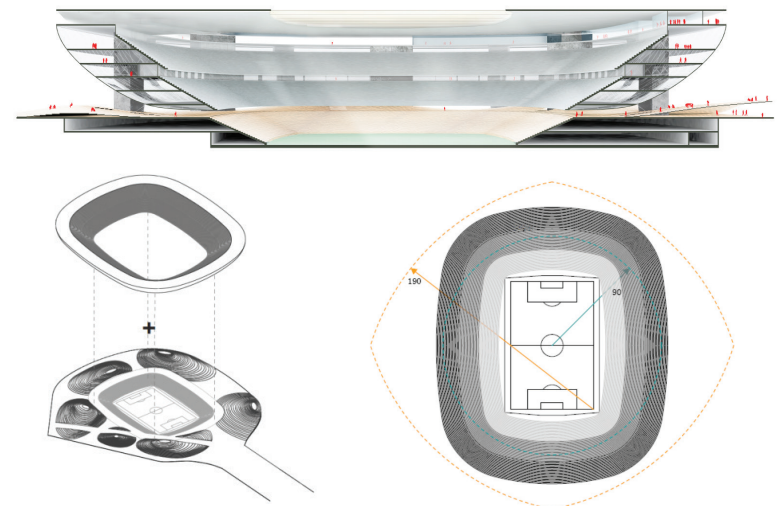
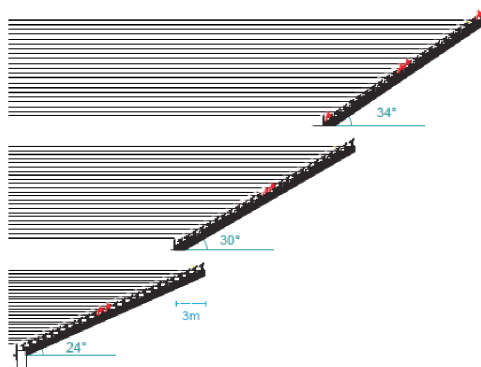


Figure 79 Above: A section of the new stadium | Left: Optimal stand angles according to FIFA regulations | Middle: Exploded view showing the public area with the sunken lower ring of the stand and the basic volume of the stadium | Right: Optimal distance to field (blue) and maximum allowed distance to field (orange) according to FIFA regulations. [OMA, 2016]

5.2 Functions of a Canopy

There are many factors that need to be taken into account while designing a stadium. This chapter states the functions that a canopy of a stadium needs to fulfill and explains how these functions can be influenced by the design of the stadium. According to Burgoyne and Silcock (2008) the three main user groups to consider while designing a canopy are:

1. Spectators
2. Owners and operators
3. Contestants

All of these groups have different needs and wishes. The extent to which these needs and wishes are met are greatly influenced by the design of the canopy. Spectators' wishes and needs differ from the needs of the owners or the contestants. Having an unobstructed view is one of the most important factors for a spectator, just like safety is. Shading from sun, wind and rain and a cool and well ventilated space add to the comfort of the spectator while watching the game. An aesthetically pleasing stadium and a design that gives the spectators a sense of identity are also important. Contestants' wishes and needs are mainly focused on the ability to properly play soccer. Thus, a good quality of the playing surface is very important for the contestants. In addition to that, floodlighting is essential for matches that are being held in the evenings. Proper ventilation and a good atmosphere

are additional wishes for contestants that allow them to excel. The wishes and needs of the owner(s) and operators are mainly focused on benefits. The stadium needs to be cost effective, durable, energy efficient, easy to maintain, flexible and needs to have good broadcasting facilities. The needs and wishes of the three main groups are listed in the scheme below.

The impact of the stadium on its surroundings is also an important factor to take into consideration. Sound and light pollution need to be restricted by a proper design. In addition to that, stadiums are very large buildings that have a huge visual impact on its surroundings. They are inward oriented and often have tall backs that appear very intimidating. All of these needs and wishes can influence the design of the canopy. The next paragraphs will discuss how the design of the stadium can influence these factors.

5.2.1 Unobstructed View

Spectators come to the stadium to watch the match. According to UEFA guides (Fenwick et al., 2017) a 'good view clearly depends on how far the seat is from the action. A tight bowl configuration will aim to bring even the most distant seats as close as possible to the pitch, increasing viewing quality and helping to create a 'cauldron' effect. The aim should always be to keep the seats within the maximum distances set out by UEFA and FIFA regulations.' It is highly important that they have an unobstructed view on the pitch. According to UEFA

Spectators	Owners/Operators	Contestants
- Unobstructed view - Safety - Shading from the sun - Shading from wind and rain - Cool and well ventilated - Aesthetically pleasing - Sense of identity	- Cost effective - Durable - Energy efficient - Easy to maintain - Flexible - Good broadcasting facilities	- Good quality of playing surface - Floodlighting - Ventilation - Good atmosphere

Figure 80 Needs and wishes of users of the stadium [Burgoyne and Silcock, 2008]

guidelines it is even a hard demand: 'All spectators should have an unobstructed and complete view of the field of play' [Fenwick et al., 2017]. In order to provide this, very large spans need to be realized. According to Burgoyne and Silcock (2008), most canopy types for stadiums can be divided into four categories:

1. Goal post structures
2. Cantilever structures
3. (Space) frame structures
4. Tensile structures

The initial choice for one of these categories has a very large influence on the design and cost of the stadium. This research focuses on a specific kind of (space) frame structure, namely, frame structures whose density change gradually over their surface (as opposed to regular grid structures). The next paragraphs briefly summarize the main characteristics of each category.

Goal post structures

The cheapest and simplest of these categories are the goal post structures. They have a column on either side of the stand and therefore the view of the spectators is not entirely unobstructed. A beam spans between these columns and supports the weight of the entire canopy. In order to prevent failure, the beam needs regular maintenance. According to Burgoyne and Silcock (2008) it requires the highest level of maintenance when compared to the other three categories. By using goal post structures, one also limits the design to a rectangular shape, since it does not allow for curves [Burgoyne and Silcock, 2008]. In contrast to goal post structures, the other types of structures do provide the spectator with an unobstructed view. However, these categories have other downsides.

Cantilever structures

Cantilever structures provide an unobstructed view and allow for a stadium with a circular or elliptical shape. The canopy is supported by fixing one end to a (vertical)

support with a rigid connection while the other end is unsupported. Sometimes counterweights or cables are introduced on the other side of the canopy to work as a counterpart to the weight of the canopy. According to Burgoyne and Silcock (2008) cantilever structures are characterized by a tall back, that cause them to appear intimidating from the outside. A cantilever roof is more expensive than a goal post structure, but is cheaper than space frame structures and tensile structures. Also, it is the easiest structure to maintain in comparison to the other categories.

(Space) frame structure

A spaceframe is a 'complex three-dimensional structural framework, capable of spanning and containing very large volumes, constructed using pyramidal, hexagonal, and other geometrical figures, often made of lightweight tubing. It has further advantages of behaving as a single integral unit, and can resist loads in any direction' [Curl, 2006]. The entire structure is lightweight and capable of spanning large distances. Additionally, aesthetically

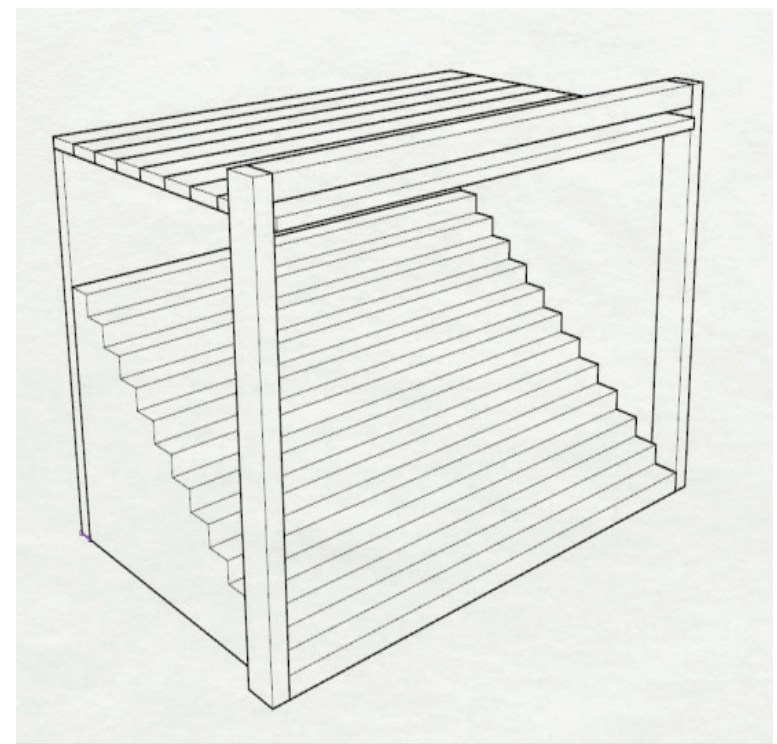


Figure 81 Goal post structure [own image]

impressive curvatures can be created by using this type of structure. Spaceframe structures are expensive but relatively easy to maintain. Prefabrication of smaller parts of the frame can reduce construction time. Frame structures, or in other words, grid structures, can be described as the 2D version of space frame structures.

Tensile Structures

Tensile structures are structures from which the structural members mostly consist out of members that undergo tension (instead of pressure) like cables or fabric. Tension structures are lightweight and their design can be easily adapted to different stadium shapes and sizes. The structure needs to be designed carefully to ensure the rain and snow cannot collect in a small area and cause the fabric to fail. According to Burgoyne and Silcock (2008) it is the most expensive structure of all four categories.

5.2.2 Safety

Often large amounts of people are being accommodated in the stadium. Therefore, safety is a major aspect of the design. The stadium needs to have a sufficient amount of emergency exits that are capable of handling large amounts of people. This affects the design of the canopy by the amount of entrances/exits it contains. In addition to that, the canopy of the stadium must be structurally sound in order to prevent failure and large deflection.

'To increase the safety of the large span structure it is important that the design includes also the existence of redundancy. This means that the structure doesn't collapse immediately after a certain structural element fails' [Romeijn, 2006]. Fire safety is also an important factor. In case of fire, the structure must endure the harsh conditions long enough for all people to safely escape the building.

5.2.3 Roof Opening & Orientation

The orientation of the stadium, the shape of the roof opening and the size of the roof opening affect many of the factors that have been listed in figure 80. For the contestants the quality of the pitch is highly important. According to To [2017] the quality of the surface is influenced by the air movement, sunlight patterns, daily temperatures and humidity levels. The speed of the air movement should be around 1 to 2 m/s for optimal turf health. According to Burgoyne and Silcock [2008] the amount of shading on the pitch needs to be minimized as this will have a large impact on the quality of the turf. Therefore, the shadow that the roof casts on the pitch needs to be evaluated at several times of the day throughout the year. Determining whether the conditions within the stadium ensure that the pitch will be of exceptional quality is a very specialized task that is being executed by a so called 'turf consultant'. The air movement within the stadium can be calculated by doing CFD simulations with a detailed model of the final design of the stadium. The air movement is not only important for the quality of the turf, but also for the comfort of the spectators. Thus, an evaluation of the final design by using CFD calculations is of great importance.

The shadow that the roof casts is not only important for the quality of the field, but is also important for the spectators. 'When planning the orientation of the pitch, the primary consideration is its position in relation to the sun and prevailing wind. In Europe, a north-south orientation is generally considered best, as it means that, in the evening, the setting sun will not hinder the vision of one team more than the other' [Fenwick et al., 2017]. In addition to that, the main stand of the stadium is usually oriented towards the east (west side of the stadium), in order to reduce the risk of spectators vision being hindered by the sun during matches that occur during the afternoon or sunsets. The canopy also functions as a shelter for rain and wind.

Nowadays, all important soccer matches are broadcasted. The shade that the roof casts on the pitch influences the quality of the TV coverage of the match: the contrast between areas that are brightly lit by the sun and the shaded parts can prevent good TV coverage [Fenwick et al., 2017]. The cameras are positioned on the west side of the stadium, above the main stand, to avoid issues with glare. For matches that are played in the evenings, floodlighting is essential to ensure that contestants and spectators have good vision but also to ensure proper TV coverage [Burgoyne and Silcock, 2008].

5.2.4 Lighting

According to UEFA standards [Fenwick et al., 2017] 'specialist lighting consultants are required to design and certify the floodlighting. This is a complex and sensitive process, as stadium lighting needs to be configured in such a way that the entire playing surface is evenly lit, with no sections in shadow, and it must also provide lighting levels that meet TV broadcasting requirements.' In addition to that, floodlights can negatively influence the surroundings of the stadium. Care needs to be taken to minimize the 'visual contamination' caused by the lights of the stadium. The same applies for lighting that illuminates the entire structure. Correct placement of the floodlights is also necessary to avoid glare. In order to do so, the floodlights need to be placed at a specific height and specific positions around the field. A more detailed description can be found in the 'UEFA Stadium Lighting Guide 2016' [UEFA.com, 2016].

5.2.5 Shape of Roof

The shape of the canopy mainly influences the air movement within the stadium and the acoustic properties. According to Burgoyne and Silcock [2008], a shape that slopes inwards reduces the impact on the surroundings in terms of sound while creating an 'exhilarating atmosphere'. However, this is only a guideline and 'a

detailed acoustic assessment is essential to ensure that the stadium design is configured with optimal sound dynamics' [Fenwick et al., 2017]. Burgoyne and Silcock [2008] also state that circular or elliptical shapes usually have a calming effect on the air movement inside a stadium that contestants and spectators experience as comfortable. In addition to that, roofs with open gaps at the corners can be beneficial for the grass pitch, as it causes the air movement over the pitch to increase, which is beneficial for the pitch [Burgoyne and Silcock, 2008].

It is important to be aware of the fact that a stadium is a very big building that has a huge visual impact on its surroundings. Additionally, stadiums are in its nature buildings that are 'inward looking' resulting in a very tall façade that has a significant impact at street level. One approach is to limit the impact at street level by limiting the height of the roof by a design that has a pitch that is lower than the street level. The opposite approach is to create an aesthetically pleasing iconic building that stands out and makes a statement. This also increases the sense of identity that spectators have with the club and the stadium.

5.2.6 Cost Effective

For the owner(s) and operators the most important factor is the cost effectiveness of the stadium. According to Burgoyne and Silcock [2008] and Fenwick et al. [2017] modern stadiums need to depend on other means besides matches in order for them to be financially sound, especially during the off-season period. In order to do so, it is important that the stadium is designed in such a way that it can be used for a variety of functions, such as concerts and restaurants.

Sustainability is becoming increasingly important. According to Burgoyne and Silcock [2008] funding is difficult to obtain when the design of the stadium does not take sustainability into account. They state that minimizing the energy use of the stadium is one of the

most important aspects and it needs to be considered as early as possible in the design phase. On the one hand, this can be implemented by limiting the amount of energy that the building uses. On the other hand, the total energy use can be limited by letting the stadium generate (a part) of the energy it uses.

The first can be implemented by designing a canopy that allows plenty of daylight to enter the stadium in order to limit the energy use of artificial lighting. Care should also be taken in order to make sure the luminance levels will not be too high or show large contrasts, as this can be received as hindering. Precise luminance studies need to be performed in order to evaluate the luminance levels. In addition to that, sun that enters the stadium causes the temperature of surfaces and the air inside the stadium to rise. In winter this is beneficial, because it saves energy on heating areas. However, in summer the areas tend to overheat which means more energy needs to be used to cool the areas to the desired temperature. Finding a balance between the amount of sun that enters the stadium, the amount of energy that is needed for heating and the amount of energy needed for cooling is a complex task that requires the evaluation of the energy use and the conditions related to comfort levels, such as air temperature and air movement, by performing complex calculations. The latter can be implemented by generating electricity from the wind, sun, or geothermal energy. According to Fenwick et al. (2017) the installation of PV-panels on stadium roofs has proven to be very effective, since they generate energy whenever light shines on them without creating pollution. In addition to that, they require no mechanical operation. Solar panels are still a rather expensive investment from which the economic benefits will only be felt after a longer period of time, since they greatly reduce the energy cost.

5.3 Selection of Functions

It can be concluded that there are many factors that need to be considered for the design of the canopy, from which some are conflicting. For example, a good quality pitch needs as much sun as possible – this translates to a very large roof opening (or no roof at all) – while the spectators do not want to be hindered by glare, rain and wind, which imposes restrictions on the maximum size of the roof opening. In order to design a stadium, a large team in which many different kinds of expertise are represented is needed. However, optimizing the shape of the canopy for all of these factors is not within the scope of this research. Therefore, the shape of the canopy is optimized for a small selection of these factors that will most likely result in a representative shape to build upon within a rather short time frame. It is important to note that, in a situation in which the stadium will actually be built, under no circumstances any of these factors can be ignored during the design phase.

It was decided to optimize the overall shape of the canopy by using an optimization algorithm. Other options were considered as well. A first option is to simply design a shape that is 'pretty' and then test whether it is capable of fulfilling all the required functions. The downside to this option is that a shape that meets the requirements is (usually) not the best possible solution. Another option is to tweak the parameters of the design (roof opening size, etc.) by hand and see how that influences the performance of the overall shape for the several required functions. The downside to this is that it takes a lot of time and the best solutions may still not be found (since the manner in which the parameters affect the several objectives is usually very complex and even unpredictable). A third option is to try all the possible configurations for the parameters. This is extremely time expensive and takes years to complete. Genetic algorithms are a suitable option to overcome these disadvantages.

In total, three objectives are taken into account in the multiple objective optimization. The first objective is closely related to the structural strength of the overall shape of the canopy. This objective was chosen because it is the most important factor. If the structure is not capable of withstanding all the forces that act upon it, it cannot be built or – even worse – might collapse which entails disastrous consequences. The structural strength will be measured by calculating the deflection of the overall shape by treating it as a shell. This is a simplification that measures the performance of the overall shape as opposed to the final structure with all its beams and connections.

The second objective that is taken into account in the multiple objective optimization is the maximization of the solar gains. As stated before, minimizing the energy use and cost is one of the most important factors of a stadium design and placing solar panels on stadium roofs has proven to be very successful in doing so. It also functions as a counterpart to the first objective since optimizing for solar gains results in a very large roof area that is oriented towards the sun, which is not supported anywhere to provide the spectators with an unobstructed view. The larger the unsupported roof area, the more it will deflect. This objective is measured by calculating the amount of radiation that reaches the roof throughout a year. The plug-in 'Ladybug' is used to perform these calculations.

The last objective that is taken into account is the minimization of the surface area of the canopy. Optimizing for the second objective will lead to a very large surface area of the entire canopy. This implies that lots of materials need to be used (glass, roof covering, etc.). Optimizing for the first objective results in a smaller surface area, but it still does not provide us with a minimal surface area that is closely related to reducing material cost.

When looking back at the factors presented in figure 80, it can be concluded that two of these objectives are directly implemented into the multiple objective optimization: (structural) safety and energy efficiency. A couple of the factors presented in this table are closely related to the shape and position of the roof opening. Firstly, the quality of the pitch is closely related to the amount of sun that the playing field receives. This means the roof needs a large opening to allow as much sunlight as possible into the stadium. At the same time, the roof needs to provide shading from the sun in order to prevent hinder caused by glare for the spectators and for the cameras needed for broadcasting. In addition to that, the roof needs to protect the spectators from wind and rain. These aspects are taken into account in the generation of the model that will be used for the optimization. Naturally, the performance of the final design in terms of these factor would still need to be evaluated by complex calculations executed by experts in their field. In addition to that, the shape that will be generated by the model implies an 'unobstructed view'.

Some of the factors in the table require very complex calculations, these factors are not being taken into account. This holds true for factors that require CFD calculations ('ventilation' and 'good quality of the playing surface'), for the floodlighting placement ('broadcasting', 'floodlighting' and 'light pollution') and for the factors regarding acoustics ('good atmosphere' and 'reducing sound pollution'). The remaining factors ('aesthetically pleasing' and 'sense of identity') will be considered during the evaluation of the solutions that come out of the algorithm.

6. OPTIMIZATION ALGORITHMS

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Mathematical optimization

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6.1 Types of Optimization

There are many kinds of optimization algorithms that can be divided into roughly three categories: mathematical optimization algorithms, gradient algorithms and stochastic algorithms. This chapter provides a taxonomy of different kinds of optimization algorithms and compares them in order to determine what algorithm is most suitable for our optimization task. Determining what algorithm suits our problem best is highly important.

One strategy in finding the optimal solution to a problem with certain objectives is to simply try all the possible values. This is called exhaustive or enumerative search. When the search space is small, this method works just fine. However, for complex problems the search space is so large that a computer will need an extremely long time (longer than a century) to check all the possible

solutions. This is simply not feasible.

Another strategy is to simply use an algorithm that one already knows and force the problem to fit into the constraints that are accompanied by that specific algorithm. By using this strategy the problem itself is simplified to fit the algorithm. By using the words of Michalewicz and Fogel [2004] 'Armed with just a hammer, everything looks like a nail. Similarly, armed with only a small subset of classic problem-solving techniques, all [...] functions are magically transformed into smooth, linear, convex mappings'. It is often hard to determine how far the optimal solution that was found using this simplified problem is removed from the optimal solution to the actual non-simplified problem. A better strategy is to determine what algorithm fits the problem best. The optimal strategy would be to create an algorithm from scratch that perfectly suits the problem. However, given the time frame for this research, this is not feasible. Therefore, this chapter aims to determine what algorithm suits our problem best.



Figure 82 The artist's problem represented in a two-dimensional coordinate system where x_1 is on the horizontal axis and x_2 is on the vertical axis [source: own image]

6.1.1 Mathematical Optimization

A whole series of optimization algorithms can be ranged under 'Mathematical Optimization' (MO). These algorithms are also being referred to as 'calculus based' or 'programming' algorithms. They can usually be recognized by the name of the algorithm, that ends with 'programming'. According to Hoffstein [2013] the types of problems that can be solved with programming have a 'canonical form that requires the minimization [or maximization] of some function [...] subject to some [...] constraints.' These kinds of algorithms are proven to give the one optimal solution and will give the same solution every time as long as the algorithm is run with the exact same input values. There are several well-known MO algorithms which are often named after the form of the used formulas and/or variables:

1. Linear programming
2. Integer programming
3. Quadratic programming
4. Dynamic Programming
5. Non Linear Programming
6. Topology Optimization

In order to better understand how MO works, the algorithm is best illustrated on a simple problem that can be solved using linear programming. Consider an artist that sells two types of art: small canvases that contain a very colourful detailed image and large canvases that contain a very abstract image. The artist wants to know how many copies she has to make of each canvas in order to optimize her profit. The profit of a small canvas is 15 euro's and the profit of a large canvas is 30 euro's. The production of a small canvas requires one unit of paint and four hours of labour, whereas the production of a large canvas requires 5 units of paint and two hours of labour. Every two weeks the artist has 80 available hours of labour and 110 units of paint. This problem can be written in its canonical form:

$$\begin{array}{ll}\text{maximize} & P = 15x_1 + 30x_2 \\ \text{subject to} & x_1 + 5x_2 \leq 110 \\ \text{Restriction in terms of units of paint} & \\ \text{and} & 4x_1 + 2x_2 \leq 80 \\ \text{Restriction in terms of hours of labour} & \\ \text{and} & x_1 \geq 0 \\ \text{and} & x_2 \geq 0\end{array}$$

The first formula is called the 'evaluation function'. The graphs associated with these formulas can be drawn in a two-dimensional coordinate system that has x_1 on the horizontal axis and x_2 on the vertical axis. As can be seen in figure 82, the graphs enclose a specific region, all the points within these region represent feasible solutions.

Within linear programming, since the evaluation function is linear the optimal solution lies on one of the vertexes of the region, or, if the slope of the evaluation function is equal to the slope of one of the restricting functions, anywhere along a boundary. In this specific case, the optimal solution lies on a vertex. The artist will make the most profit if she makes 10 small canvases and 20 large canvases every two weeks.

6.1.2 Gradient Algorithms

Gradient algorithms (GA) can be envisioned by a marble on a curved surface: the marble will roll to a point where it is stable (see figure 83). 'This classification of algorithm is best used with smooth, convex functions without many local minima. While these restrictions are unrealistic for most data-driven problems, which tend to have coarse and complex surfaces, the process is well suited for deriving parameters in theoretical frameworks where we know the functional form' [Hoffstein, 2013] According to Michalewicz and Fogel [2004] most gradient algorithms have a couple of disadvantages. Usually they give a solution that is only locally optimal. Furthermore, the algorithm does not give an indication of how much the given local optimum differs from the

global optimum. Another weakness is that the found optimum highly depends on the initial configuration. The luring advantage of these kinds of algorithms is that they are fairly easy to use and understand. Some well-known examples of gradient algorithms are:

1. Hill-climbing algorithms
2. Tabu Search
3. Simulated Annealing

Hill-climbing methods can, in contrast to what the name suggest, be both ascending and descending algorithms. This depends on whether the maximum or the minimum of a function needs to be found. Hill-climbing methods use one point as a starting point. With every iteration of the algorithm, a new point is selected that lies in the neighbourhood of that point. If this point provides a better value than the original point, that point becomes the new starting point. This process keeps repeating until no point with a better value is found. There are several kinds of hill-climbing algorithms. These algorithms differ in the manner in which new points are picked. A well-known example of a hill-climbing algorithm is the steepest ascent/descent algorithm. This algorithm does not just pick one point within the neighbourhood of the starting point, but considers all the points within the starting point's neighbourhood. The best point of all of these points is picked as the new starting point. Since all hill-climbing methods stop when the gradient is zero, or in other words, when there is no better solution to be found within the neighbourhood of the starting point, the algorithm can easily get stuck at a local optimum. [Michalewicz and Fogel, 2004]

There are several ways to overcome this problem with gradient algorithms. Tabu search and simulated annealing are two examples that provide methods that help to overcome the problem of getting stuck at a local optimum. Tabu search uses a 'memory' to keep track of what parts of the search space have already been investigated using a hill-climbing method. These parts

become forbidden areas (tabu) for selecting a next point within the area for further evaluation. According to Michalewicz and Fogel [2004] tabu search is deterministic in contradiction to simulated annealing that is probabilistic. With simulated annealing algorithms there is an additional factor, the so-called temperature, that changes during the procedure. This factor changes 'the probability of moving from one point of the search space to another' [Michalewicz and Fogel, 2004]. The new point does not have to be better than the initial point, but it has to be 'acceptable'. Whether a new solution is acceptable or not is dependent on the temperature. In the beginning of the procedure the temperature has quite a large number. Therefore, the search in the beginning of the procedure resembles a random search. The smaller the temperature gets, the more the algorithm starts to act as a hill-climbing algorithm. Simulated annealing can be seen as a hybrid between stochastic algorithms and gradient algorithms. According to Hoffstein [2013] 'it is fairly common to include a stochastic element to gradient methods, either randomly resetting the particle's location, randomly perturbing the direction the particle is moving in, or randomly dropping multiple particles, to help overcome the risk of local minima.'

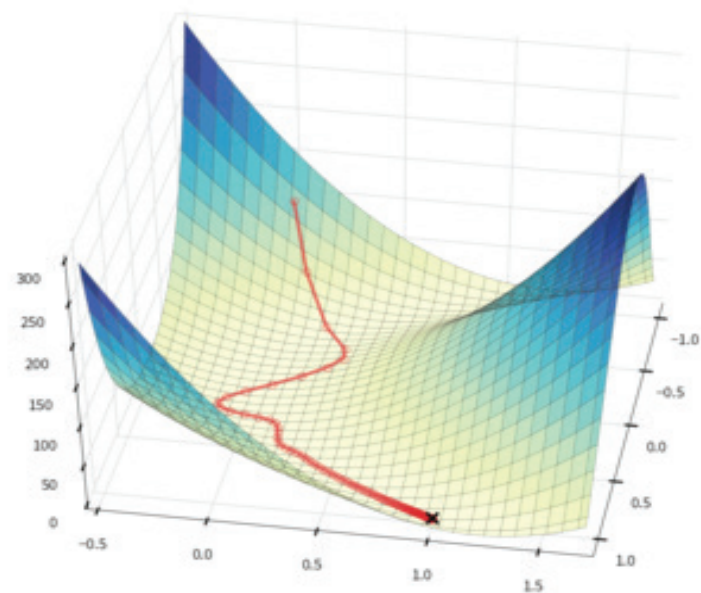


Figure 83 Visual representation of a gradient algorithm [Quora, 2016]

6.1.3 Stochastic Algorithms

A synonym for 'stochastic' is 'random' or 'probabilistic'. According to Hoffstein [2013] 'randomness is injected into the search procedure to accelerate progress, helping to overcome local minima and noise from modeling error. This process allows us to rapidly learn about our search space without making any assumptions.' [Hoffstein, 2013] In the previous chapter it has already been mentioned that stochastic elements can be combined with gradient algorithms to create a hybrid. Two well-known examples of purely stochastic algorithms are:

1. Particle Swarm Optimization
2. Evolutionary Algorithms

Swarm optimization and evolutionary algorithms are alike in mainly two ways: the algorithm starts with an entire generation of starting points instead of just one and searches for the global optimum by creating new generations. [Hu, 2006] The difference between the algorithms lies in the manner in which new generations are created.

Evolutionary algorithms makes use of so-called evolutions operators that are inspired by Darwin's evolution theory. Members (solutions) of the initial generation will become the parents of the next generation's members. These new members are created by operators such as crossover and mutation out of the features of the parent. The members of the next generation have to 'fight' to survive, in biology this is called 'natural selection' and 'survival of the fittest'. Members that are better, according to a certain fitness function, are more likely to survive. The survivors of this generation will then become the parents of a new generation, etc. Evolutionary algorithms can provide the user with near optimal results. However, correctly setting the operators of an evolutionary algorithm is crucial to the attainment of a good solution and doing so might prove quite difficult; every problem requires a

different setting of the parameters for the algorithm to run properly.

Swarm optimization algorithms are inspired by the foraging behaviour of flocking birds. The area which is available for birds to find food can be compared to the search space and the places where food can be found can be compared to the optima within the search space. 'The best strategy to find the food [...] is to follow the bird which is nearest to the food' [Hu, 2006]. The initial generation of 'birds' (particles) in the search space is random. They are also randomly assigned a certain beginning velocity. With every iteration the particles will cluster more and more around the optima within the search space. [Brownlee, 2015]

6.1.4 Multiple-Objective Optimization

'The process of optimizing systematically and simultaneously a collection of objective functions is called multi-objective optimization (MOO)' [Marler and Arora, 2004]. Sometimes the objectives complement each other. For example, using as little material as possible for a construction also means minimizing the cost for that construction (assuming using less material does not make the construction itself more difficult or, in other words, more labour intensive). But for most real world problems, two or more objectives are conflicting. For example, the need for a supporting structure often conflicts with the desire for a wide open view of the surroundings and the desire of minimizing the costs often suppresses many other objectives from being maximally achieved. In situations like this, one cannot (further) improve for one objective without degrading the other. Thus, for most multiple objective optimizations there is typically more than one optimal solution.

A solution within a search space is called 'pareto optimal' if there is no other point that improves at least one objective function without detriment to

another function' [Marler and Arora, 2004], in other words, Pareto optimality of a solution means an optimal trade-off between two or more contradicting objectives [Vierlinger, 2013]. All pareto optimal solutions lie on a curve, surface or hypersurface within the search space that is called the pareto front and all these solutions are performing equally well regarding the fitness function. Ideally, an algorithm for MOO finds solutions that are evenly distributed near or onto the pareto front. When optimizing for two objectives, the pareto front is a curve, when optimizing for three objectives the pareto front is a surface and when optimizing for more than three objectives the pareto front it is a hypersurface.

According to Marler and Arora [2004] there are three major types of MOO methods: 'methods with a priori articulation of preferences, methods with a posteriori articulation of preferences, and methods with no articulation of preferences'. A priori articulation of preferences usually incorporate scalarization methods that make use of parameters to add a 'weight' to each objective. These methods cast the multi-objective problem into one single-objective optimization formula [Grodzevich and Romanko, 2006] that is similar to a mathematical optimization problem. Just like mathematical optimization, these methods give us one solution. Examples of priori methods that incorporate 'weights' according to Marler and Arora [2004] are:

1. Weighted sum method
2. Weighted global criterion method
3. Weighted min-max method
4. Exponential weighted criterion method
5. Weighted product method

These methods have several drawbacks according to Mundhekar [2011]. Small changes in the weight factors may result in very different results with respect to the different objectives, whereas very large changes may result in almost unnoticeable changes. This implies that even for an even spread of weights the results may be

(very) unevenly distributed over the pareto front. Also, many of the methods listed above are only capable of finding solutions on convex parts of the pareto front. There are also methods of introducing preferences that do not involve weights. These methods use a certain hierarchical order of the objectives. An example of this is the lexicographic method. This methods optimizes the objectives in an hierarchical order that has been determined by the user. The algorithm first optimizes for the most important objective, then for the second and so forth.

Posteriori articulation of preferences contains all methods that generate multiple solutions that are pareto optimal. The user (decision maker) chooses one of these solutions as the final optimal solution. This allows the user to take into account objectives that are very hard, if not impossible, to describe numerically, such as 'beauty' or 'fitting into the environment'.

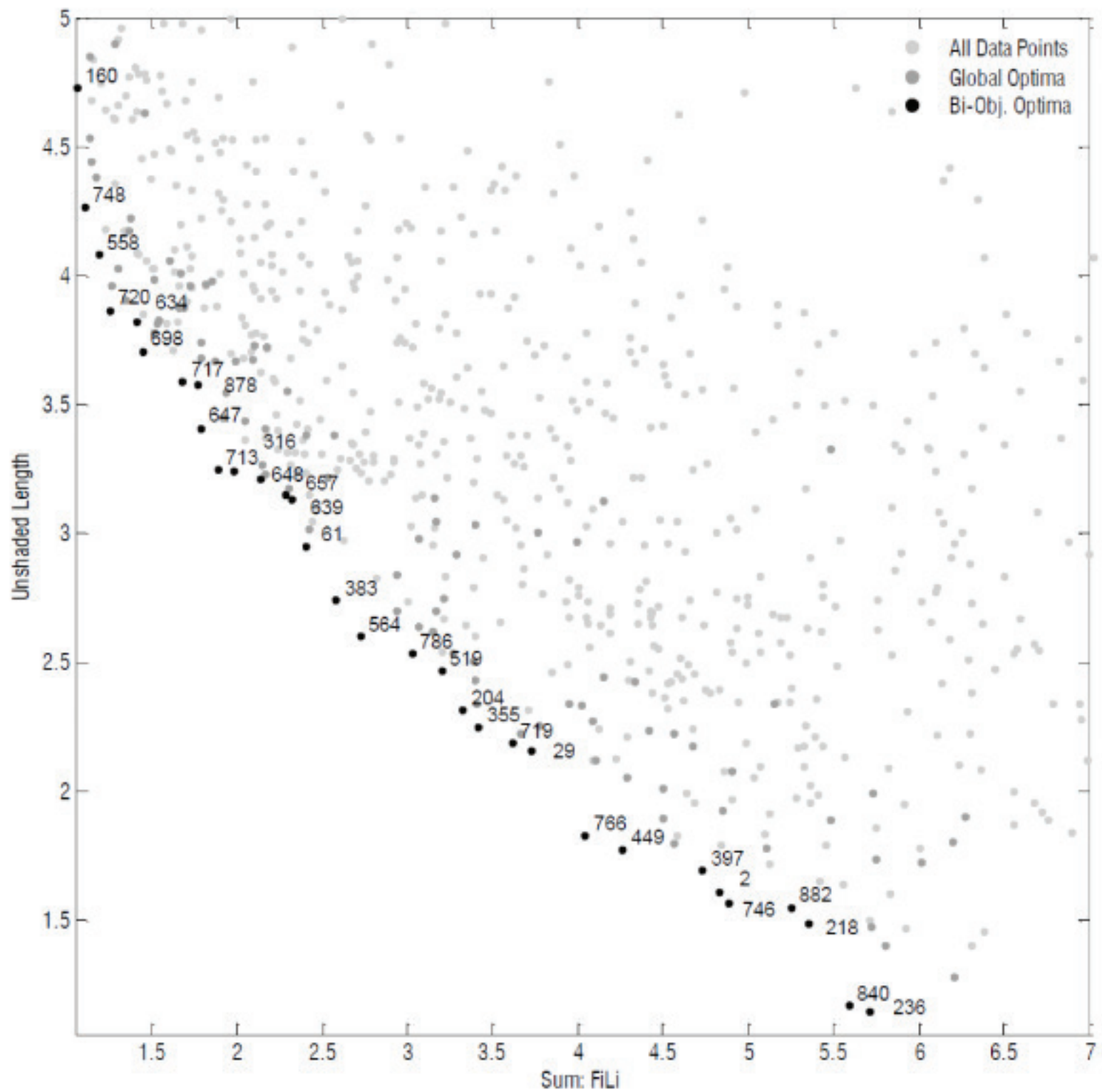


Figure 84 Visual representation of a pareto front found by using an Evolutionary Algorithm

6.2 Grasshopper Plug-ins

There are several grasshopper plug-ins for optimization purposes. The plug-ins and its most relevant properties are listed in the table below.

6.2.1 Assessment

Mathematical optimization algorithms have a large advantage relative to all the other algorithms that have been described above: they are proven to give the optimal solution. Another advantage of these kinds of algorithms is that it gives the same solution every time. Disadvantages of MO algorithms are that they may take a very long time to converge to the optimal solution and that they are only capable of giving one optimal solution at all times. In case of multiple objectives that include some conflicting ones, this means that the user determines how strong the objectives are weighed in the evaluation function. In order to find more than one solution that lie on the pareto front, the user has

to run the algorithm multiple times with a different fitness function. As stated previously, small changes in the weight factors may result in very different results with respect to the different objectives, whereas very large changes may result in almost unnoticeable changes. This implies that even for an even spread of weights the results may be (very) unevenly distributed over the pareto front. In order to be able to solve a problem by using mathematical optimization, one needs to know the functional form of the problem and be able to translate this problem to its canonical form. This requires a very specific kind of knowledge and set of skills that many do not possess. Also, for complex problems this may be very time consuming. Furthermore, for real-life problems the functional form of relations between two or more objectives is often unknown. In other words, the user may not know how two objectives relate to one another. For example, if the shape of the building is changed in such a way that the solar panels on the roof produce 10% more solar energy, how does this influence the amount of material that is needed for the construction? Additionally, there might be

Plugin	Main Category	Sub category	MOO	Solutions	Notes
Opossum	Stochastic	- Genetic Algorithm - Swarm Particles	yes	one	Similar to galapagos, but finds solutions with a small number of evaluations
Goat	Calculus	- unknown	yes	one	Perfectly complements galapagos
Silvereye	Stochastic	- Particle Swarm Optimization	no	one	Single-objective
Dodo	Calculus	- Nonlinear Optimization - Constrained Optimization	yes	one	
Octopus	Stochastic	- Genetic Algorithm	yes	multiple	Produces a range of trade-off solutions
Galapagos	Stochastic	- Simulated Annealing - Genetic Algorithm	yes	one	Combines multiple objectives into one fitness function
MOO	Stochastic	- Genetic Algorithm (NSGA-II)	yes	multiple	The component allows the user to input sliders, objectives, and other settings, and it returns an approximation of the performance Pareto front for the design

several ways to change the shape of the building in such a way that the solar panels produce 10% more sunlight. How can the formula account for that? This example illustrates that it may be very difficult or even impossible to describe a problem by using a formula. Also, these kind of algorithms give no or poor results when one or more of the objectives has a function that contains discontinuities [Michalewicz and Fogel, 2004]. For these objectives the formulas can be simplified by approximating the original function with, for example, a linear or a quadratic formula. However, in this case an exact optimum is found for a simplified problem. It is difficult to determine how much this optimal result differs from one that would have been found if the problem was not simplified.

Gradient algorithms have the advantage that they are fairly easy to use and to understand. However, gradient algorithms have a tendency of getting stuck at local optima. Even modern versions of GA, like tabu search and simulated annealing, are not proven to find the global optimum. Also, the found optimum highly depends on the initial configuration of the algorithm. In addition to that, just like with MO algorithms, the several objectives need to be combined within one fitness function. Therefore, the user has to determine the weights of the objectives. For gradient algorithms it is also the case that small changes in the weight factors may result in very different results with respect to the different objectives, whereas very large changes may result in almost unnoticeable changes. This implies that even for an even spread of weights the results may be (very) unevenly distributed over the pareto front, assuming that the found solution was a global optimum instead of a local optimum. If the latter is not the case, then the found solution is not a pareto optimal solution.

Stochastic algorithms are not proven to find the optimal solution. However, they are capable of finding near optimal solutions. These kinds of algorithms are also capable of producing more than one solution

every time that the algorithm is run. Therefore, the algorithm is capable of finding solutions that are equally good without any priori interference of the user. A disadvantage of stochastic algorithms which is especially true for evolutionary algorithms is that correctly setting the operators is crucial to the attainment of a good solution and doing so might prove quite difficult.

The optimal algorithm for our problem must be capable of handling multiple objectives and be capable of producing a range of solutions that lie on the pareto front, preferably in one run. These solutions must be evenly distributed across the pareto front in order to properly represent this front. Additionally, the algorithm must be relatively easy to understand and to apply by architects and architecture students. The advantages and disadvantages of the three main categories of optimization algorithms are listed in the table on the next page.

Based on the desired capabilities of the algorithm and the advantages and disadvantages that have been listed above, stochastic algorithms provide the best fit to the stated problem. These algorithms are the only ones that are capable of producing a range of solutions that lie on or near to the pareto front in one run. Even though these solutions are not proven to be optimal, as is the case with MO algorithms, they are capable of producing near optimal solutions. Also, for these algorithms the user does not have to know the functional form of relations between the objectives. Grasshopper plug-ins that are suitable for our problem are Octopus and MOO. The MOO plug-in is not available for downloading (yet?). Thus, Octopus is the plug-in that will be used for the multiple objective optimization.

	Advantages	Disadvantages
Mathematical Optimization	- Gives a proven optimal solution - Gives the same solution every time	- Produces only one solution - Complicated - May take a long time to converge - Getting an evenly spread distribution on the pareto front is very difficult
Gradient Algorithms	- Easy to use and understand	- Produces only one solution - Chance of getting stuck at local optima - Found optimum is very much dependent on initial configuration - Not proven to find the global optimum - Getting an evenly spread distribution on the pareto front is very difficult
Stochastic Algorithms	- Capable of finding near optimal solutions - Can produce multiple solutions that are equally good	- correctly setting the operators of is crucial to the attainment of a good solution and doing so might prove quite difficult - Not proven to find optimal solutions

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7.1 Overall Shape

7.1.1 Parameters for the Design

The overall shape of the canopy is generated by lofting three curves: the curve that represents the opening of the roof, a curve in the middle and a curve at ground level. These curves are displayed as thick white lines in figure 85. The curve that represents the opening of the roof and the curve in the middle have been assigned the following parameters:

1. Scale in the X-direction
2. Scale in the Y-direction
3. Rotation around the X-axis
4. Rotation around the Y-axis
5. Translation in the Z-direction

The curve at the bottom has the following parameters assigned:

1. Scale in the X-direction
2. Scale in the Y-direction

As stated before, there are a couple of factors that are closely related to the position of the roof opening: the quality of the pitch is closely related to the amount of sun it receives through the opening of the roof and the roof provides the spectators (and cameras) with shelter from sun, wind and rain. Therefore, the minimum size of the curve that represents the roof opening is restricted. When looking at the top view of stadiums with natural pitches, it can be noticed that there are two common characteristics regarding the shape and size of roof openings (examples can be seen in the figure 86). Firstly, most stadiums have a roof opening whose shape matches the shape of the pitch. Secondly, often the roof opening is a little larger than the pitch.

The final shape of the roof opening is a trade-off between allowing as much sun light as possible to reach the pitch and between sheltering as much spectators as possible from sun, wind and rain. Therefore, the minimum size of the roof opening for the model has been restricted to a roof opening that matches the shape of the pitch, but is slightly larger in size. This has been shown in the top view in figure 89. The size of

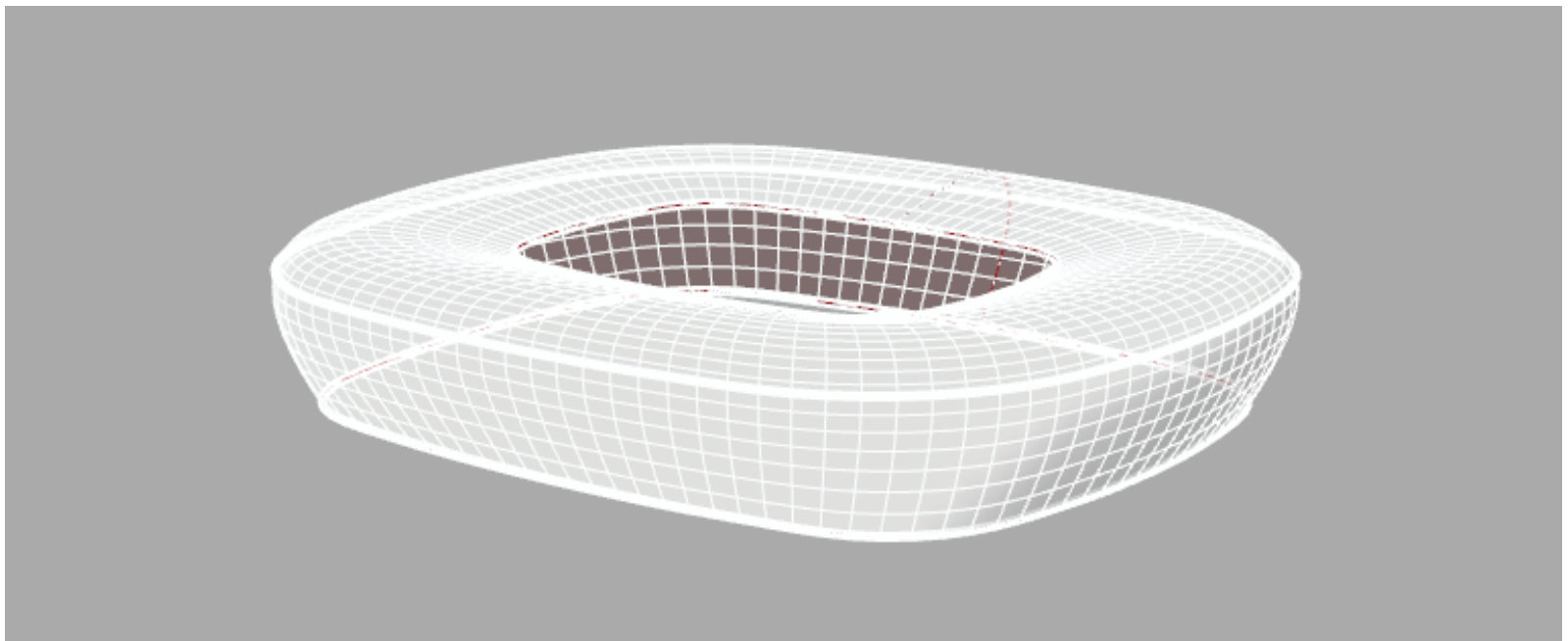


Figure 85 The basic shape for the model with the three curves shown in thick white lines [own image]



Figure 86 From left to right: Nàtional Stadium Brazil, Stadium Octávio Mangabeira, Arena Castelão, Stadium des Lumières, Matmut Atlantique and Allianz Riviera [AIRBUS, (n.d.) and Cité de l'espace, (2016)]

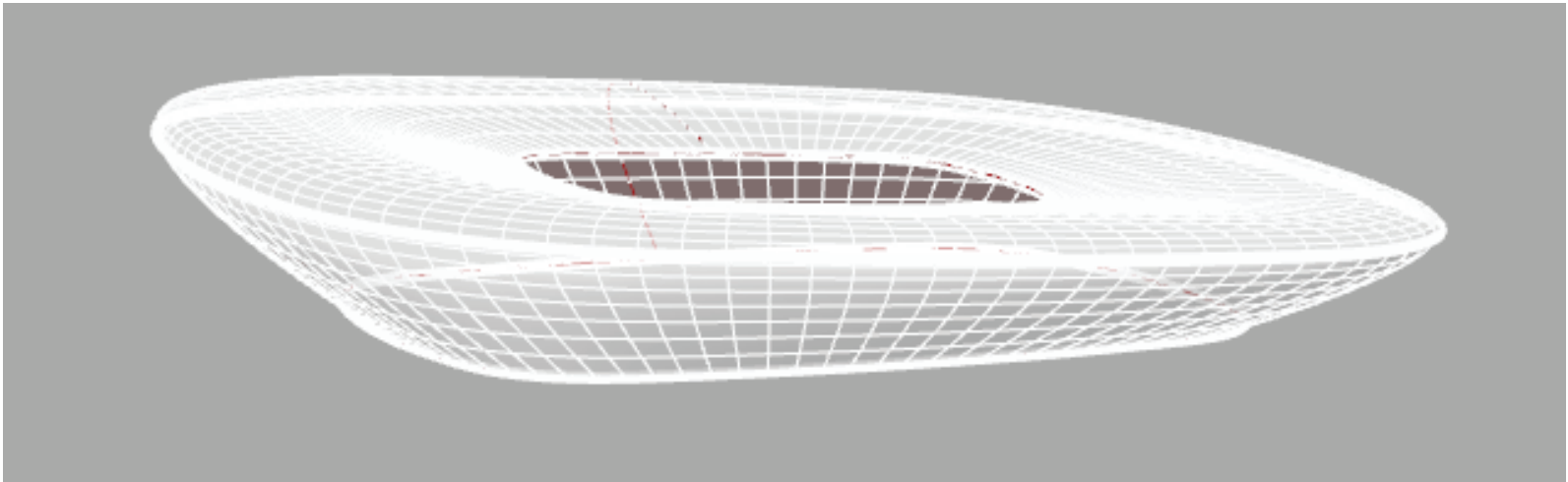


Figure 87 Solution favourable in terms of solar gains [own image]

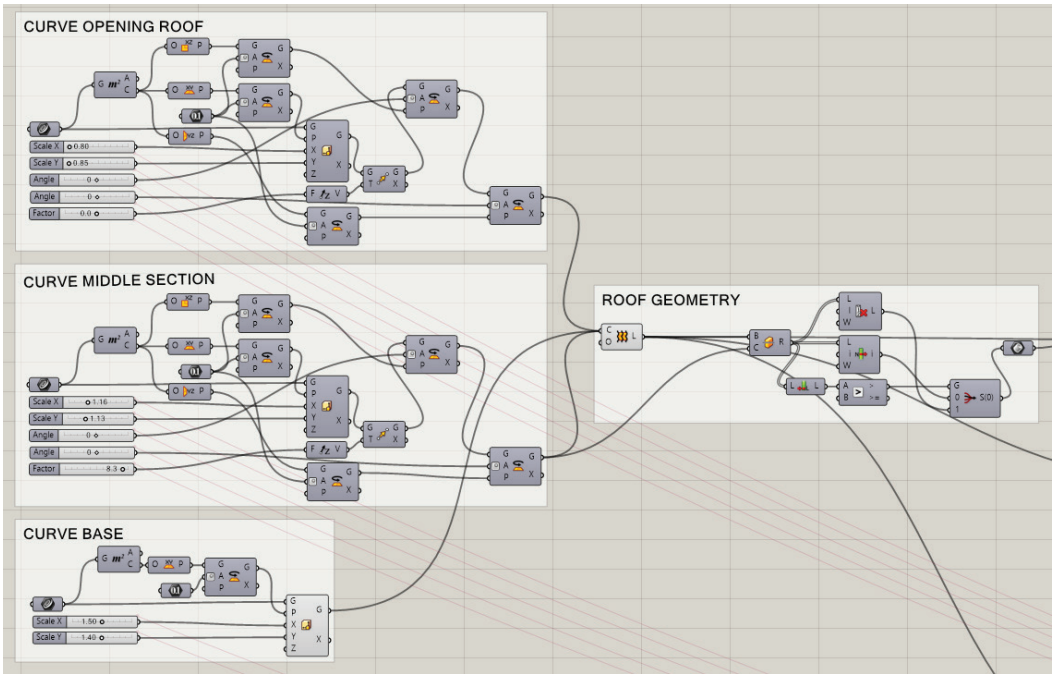


Figure 88 The basic shape for the model with the three curves shown in thick white lines [own image]

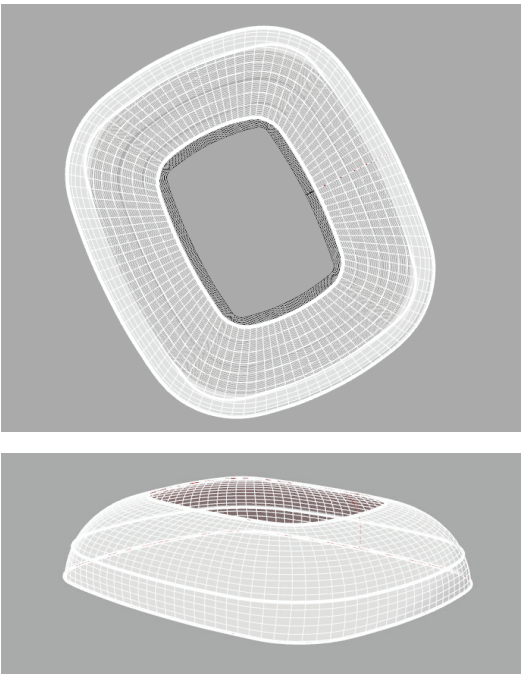


Figure 89 Minimum size of the roof opening (top) and a solution favourable in terms of structural performance (bottom) [own image]

the curve at ground level is constrained to a size that does not exceed the size of the platform it stands on (see figure 79). This size has been dictated by OMA's masterplan. Furthermore, the constraints of all the curves allow the generation of shapes that approach a dome-like shape that is structurally favourable as well as shapes with large roofs that are tilted towards the sun that are favourable in terms of solar gains (see figure 87 and 89).

7.1.2 Grasshopper Model

Part 1 of the scripts shows the components that generates the curve that represents the opening of the roof. The 'curve' components is connected to the 'area' component to extract the center of the curve. With the center point, three planes are created: the xz-plane, the xy-plane and the yz-plane. All of these planes are then rotated around the z-axis by 27 degrees to align the orientation of the planes with the orientation of the stadium. The curve is first scaled in the x and the y direction by inserting the curve, the rotated xy-plane and the scale sliders into the 'Scale non-uniform' component. Then, the non-uniformly scaled curve is inserted in the 'move' component along with a 'unit Z' that is connected to the 'factor slider'. If this slider is set to one, the curve moves one meter upwards. The moved curve is then firstly rotated around the xy-plane and then rotated around the yz-plane. The resulting curve is inserted in the 'loft' component.

Part 2 of the script shows the components that generate the curve in the middle of the shape that represents the canopy. This curve is generated in the same way the roof opening curve is generated. Part 3 shows the components that generate the curve at ground level. This curve is not rotated around any axis and is not moved upwards or downwards.

After the curves have been lofted to form the overall shape of the canopy, the roof part of the lofted

shape is extracted to insert into the solar radiation calculator. (Part 4) The lofted shape itself is inserted into the karamba shell calculation and into the 'area' component. The results of the three calculations are then inserted into the Octopus multiple objective optimization. The rectangular group that is shown in Part 4 contains toggles that can activate or deactivate the three calculators. These toggles have been inserted to be able to deactivate the calculators while changing something in the scripts or parameters that precede these calculations. Otherwise grasshopper performs all calculations again after every change, however small: running all these calculations takes at least 10 seconds. Having to wait 10 seconds every time a minor change has to be made is not desirable.

In order to cut off the part of the geometry that represents the roof, the 'loft' is inserted into the 'Split Brep' component together with the 'middle section curve'. (part 5) This component sees this curve as a planar surface that cuts through the lofted shape. If the 'middle section curve' is lower than the 'curve opening roof' it cuts the shape into two parts. The two sections then go into the 'List Item' component that picks the last item on the list, since this last item is the section that represents the roof part. The output of this component goes into the '0' input from the 'Stream Filter' component. If the 'middle section curve' is higher than the 'curve opening roof' it cuts the shape into three parts. The roof part consists out of two of these parts. The three parts go into the 'Cull Index' component that removes the second item from the list, since this is the part that does not form the roof. The output of this component goes into the '1' input from the 'Stream Filter' component. The output from the 'Split Brep' component also is inserted into the 'List Length' component that tests how many parts are in the list (two or three in this case). The output goes into the 'Larger Than' component. This component returns a 'true' (1) if the list contains more than two parts and a 'false' (0) when the list contains two parts. This output

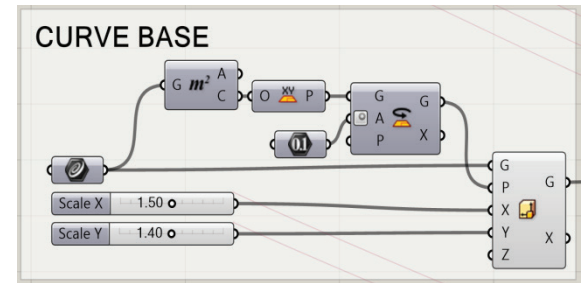
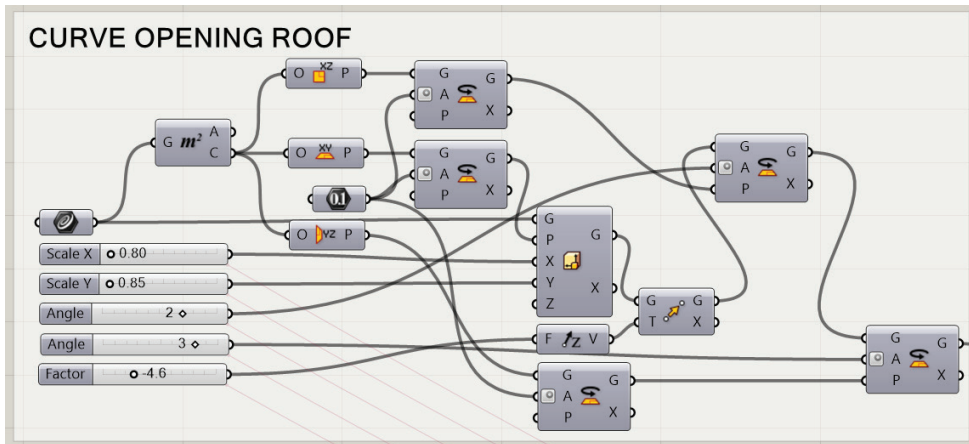
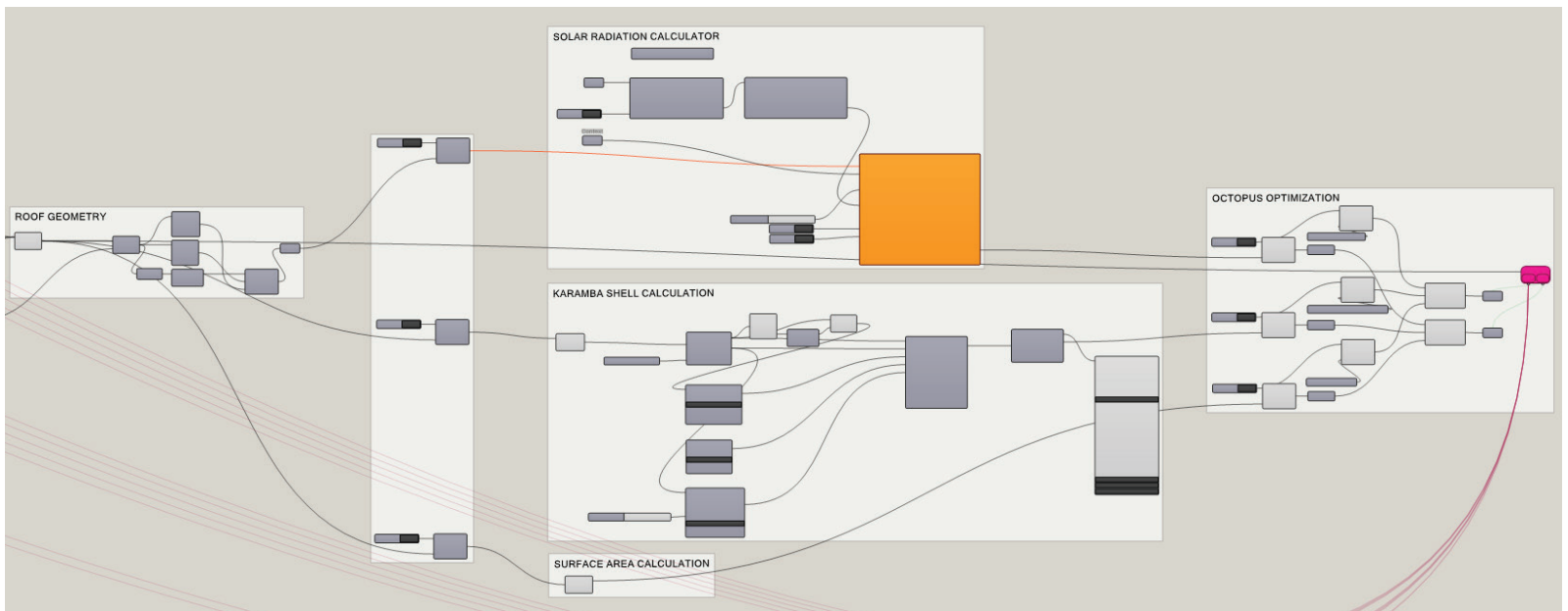
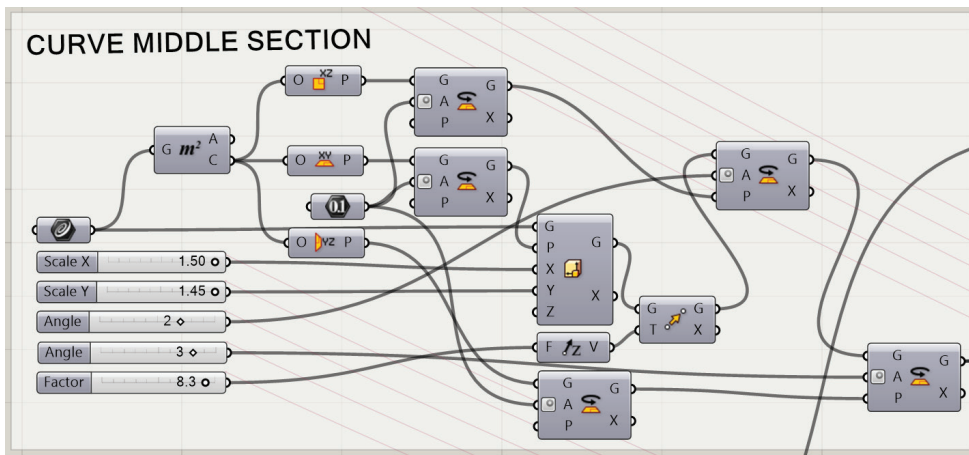


Figure 90 Part 1: Top left
 Part 2: Middle right
 Part 3: Top right
 Part 4: Bottom



is used to determine what input of the 'Stream Filter' components will be the output. If the list contains three parts it uses the '1' input and if the list contains two parts it uses the '0' input. In that way, the resulting 'Brep' always contains the part(s) that represent the roof of the shape.

Ladybug's (grasshopper plug-in) 'GenCumulativeSkyMtx' component uses an .epw data file that contains climate data, in order to calculate the sky's radiation for each hour of the year. (Part 6) In order for the calculation to run, the toggle that is connected to the component needs to be set to 'true'. The output of this component is needed for the radiation analysis. The output of this component is inserted into the 'selectSkyMtx' component to convert the data to something the 'radiationAnalysis' component can process. The 'radiationAnalysis' component also uses the geometry and the geometry of the buildings that may cause a shadow on the roof as input. The 'distance from base' is set to 0.01 to ensure that the calculations are performed on the right side of the geometry. The 'total radiation' is then inserted into the Octopus optimization as an objective that needs to be maximized.

The output from the 'loft' component is inserted into the 'Area' component in order to calculate the surface area of the entire shape (Part 7). The output of this calculation is then inserted into the Octopus optimization as an objective that needs to be maximized.

For the Karamba Shell calculation, the lofted surface is first translated to a mesh by the 'Mesh Brep' component. That mesh is then converted into a shell by Karamba's 'MeshToShell' component (Part 8). In order to extract the points from the mesh that are at ground level, the height from each point is checked (the z-value). If this value is smaller than 0.01 it is at ground level. All points that are not smaller than 0.01 are then removed from the list by the 'Cull Pattern' component. The points that are left, i.e. the points that

are at ground level, are then inserted into Karamba's 'Support' component. Within this component the 'Tx', 'Ty' and the 'Tz' values are checked to create supports that cannot move in any direction, but can rotate around all three axes. These supports are then inserted into Karamba's 'Assemble Model' component. The element-output from the 'MeshToShell' component is inserted into Karamba's 'Cross Sections' component in order to give the shell a thickness of 20 centimeters. The resulting shell is then used as another input for the 'Assemble Model' component, along with a gravity load that is generated by Karamba's 'Loads' component. The resulting model is inserted into Karamba's 'Analyse' component that calculates the deflection of the shell. The maximum displacement is then used as input for the Octopus optimization as an objective that needs to be minimized.

The combination of toggles and stream filters function as toggles that determine what objectives are taken into account in the Octopus optimization (Part 9). When a toggle is set to 'true' the objective is taken into account. The upper toggle activates or deactivates the 'solar radiation' objective, the middle one the 'Karamba Shell Calculation' and the last one the 'Surface Area' calculations. Since Octopus is only capable of minimizing an objective, the objective that needs to be optimized (Solar Radiation) gets a negative expression, i.e. the value that comes out of the solar radiation calculation becomes negative instead of positive. All the objectives are inserted into the Octopus component, along with the lofted shape (the phenotype) and all the parameter sliders (genotype).

7.1.3 Results of Optimization

As has been stated before, there are many objectives that can be taken into account for the optimization of the overall canopy shape. In no means does the selection of a few aspects mean that the other aspects can be neglected.

The next pages show the results from the multiple objective optimization run with Octopus with a population size of 100 and with 90 generations. The Elitism was set on 0.500, the Mutation Probability on 0.100, the Mutation Rate on 0.500 and the Crossover Rate at 0.800. These are the standers settings from the Octopus component. For an explanation of the meaning and the operation of these settings is referred to the Octopus Manual in the Appendices.

The results of the optimization are shown in the images on the following pages. Octopus displays its solutions in a coordinate system. The closer a solution lies to the smallest value of an axis, the more optimal this solution is in term of the matching objective. Octopus displays solution that lie on the pareto front as dark red dots, pink dots show the elitism solutions (solutions that are bred out of the elite instead of the entire population) and the green dots show the other solutions that have been evaluated by the algorithm (the history).

After analyzing the results, a couple of conclusions can be drawn. The most notable conclusion is that there appears to be a very strong correlation between the Total Radiation and the Minimal Surface resulting in a closely packed distribution of points (see figure 93): the larger the surface area of the overall shape, the larger the received amount of Total Radiation on the roof of the shape. In addition to that, this image gives a good approximation of the pareto front for these two objectives. Secondly, there appears to be a weak (linear) correlation between the Minimal Surface and the maximum displacement (see figure 94): the larger the maximum displacement of the shell that represents the shape, the larger the surface area of the shape. In terms of these two objectives, there does not seem to be a pareto front. There appears to be one best solution when only these two objectives are taken into account, like in the artist's problem that was explained at page 107. Lastly, there appears to be a weak correlation between the Total Radiation

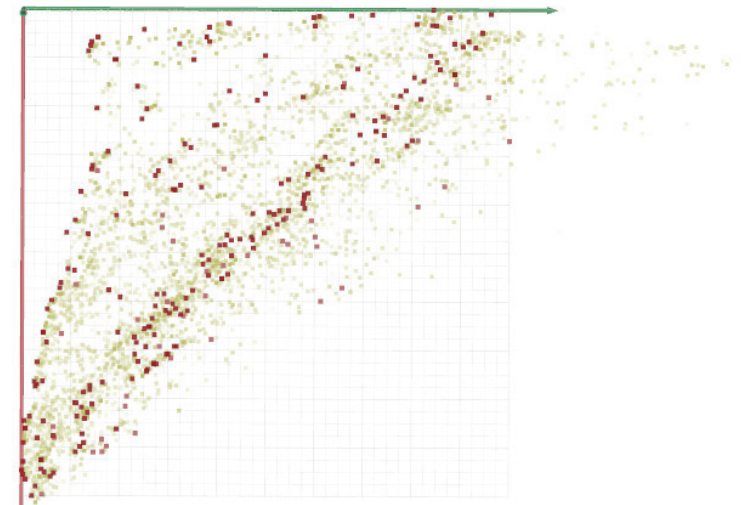


Figure 92 Display from the multi-objective optimization projected on the xy-plane, with Total Radiation on the red-axis, Maximum Displacement on the green axis

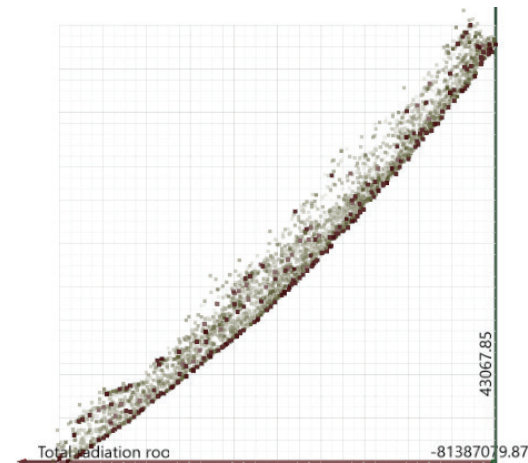


Figure 93 Display from the multi-objective optimization projected on the xz-plane, with Total Radiation on the red-axis, Minimal Surface on the green axis.

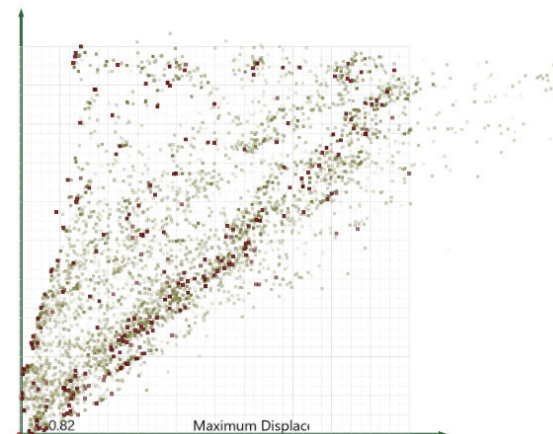


Figure 94 Display from the multi-objective optimization projected on the yz-plane, with Minimal Surface on the dark green axis, Maximum Displacement on the light green axis.

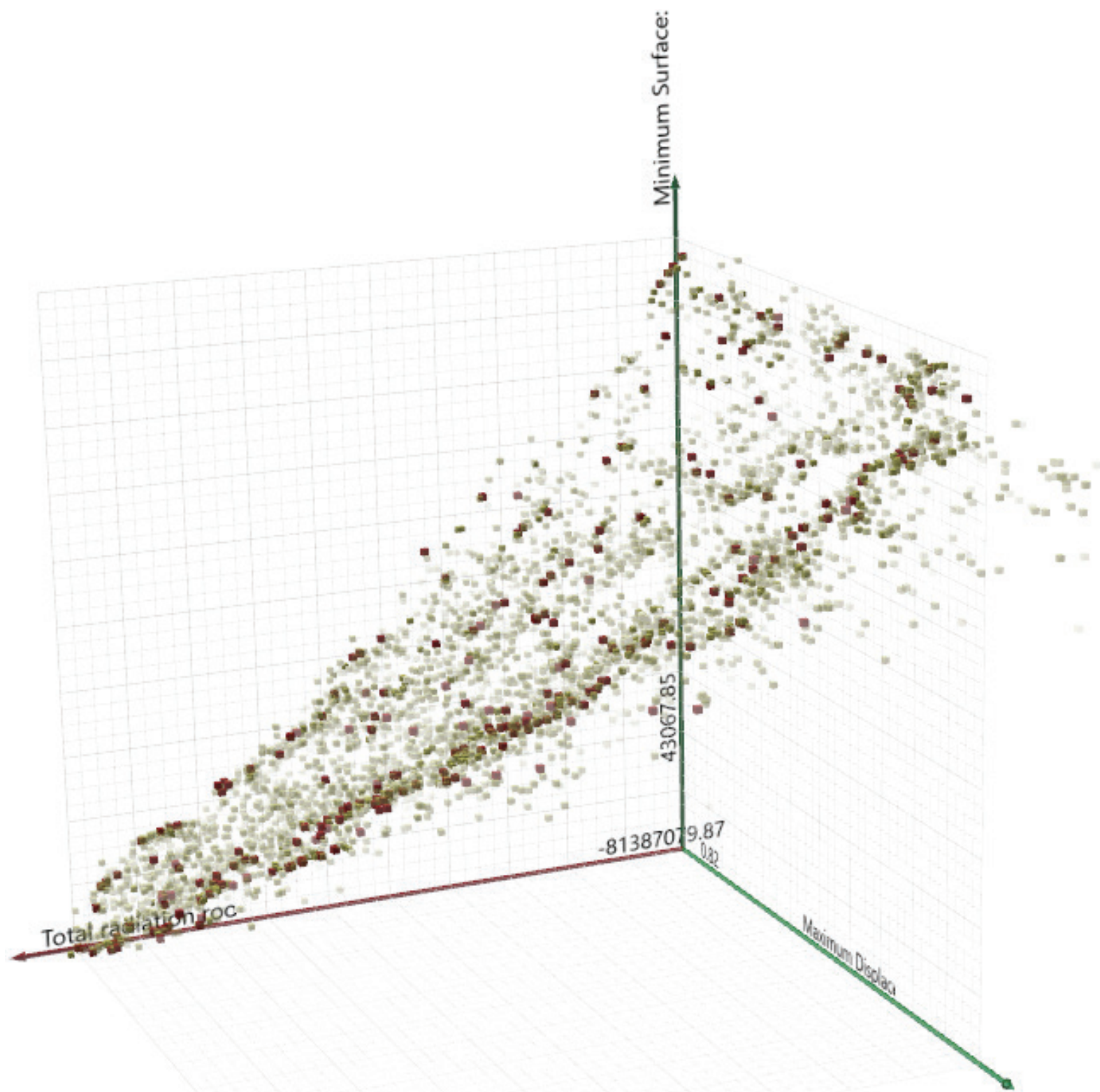


Figure 95 3D display from the multi-objective optimization run with Octopus, with Total Radiation on the red (x) axis, Maximum Displacement on the light green (y) axis and Minimal Surface on the dark green (z) axis.

and the Maximum Displacement: the more radiation the roof receives the larger the maximum displacement of the shell that represents the shape of the canopy. In addition to that, figure 92 gives a vague impression of where the pareto front of these two objectives would approximately be.

Choosing the Final Shape

Figure 96 shows all the meshes of the canopy shapes that belong to the 200 non-dominated solutions that have been found. This is the moment where the user chooses the final solution. As stated before, the factors that this decision will be based on are: 'Aesthetically Pleasing' and 'Sense of Identity'. These two factors are both very subjective.

It turns out that most of the shapes have a lot of similarities. Most of the solutions have a shape that is built out of a middle curve that is tilted towards the south and a roof opening curve that is tilted towards the north. This effect is caused by the objective that optimizes for the amount of solar radiation that the roof receives. Apparently, this configuration of curves has a positive influence on obtaining this objective. The final shape that was chosen has an about equal weighting of the three objectives and, in my opinion, has an aesthetically pleasing shape that has a high iconic value.

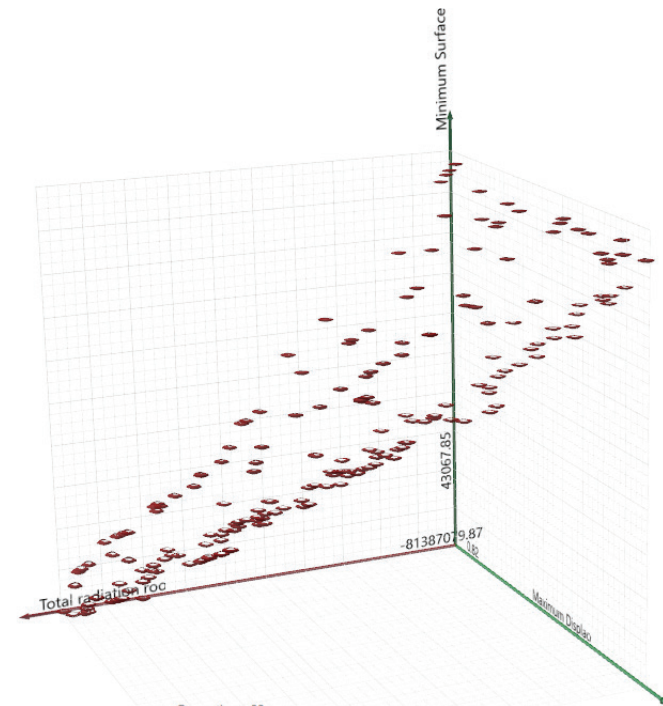


Figure 96 Display from the multi-objective optimization projected with only the pareto solutions displayed.

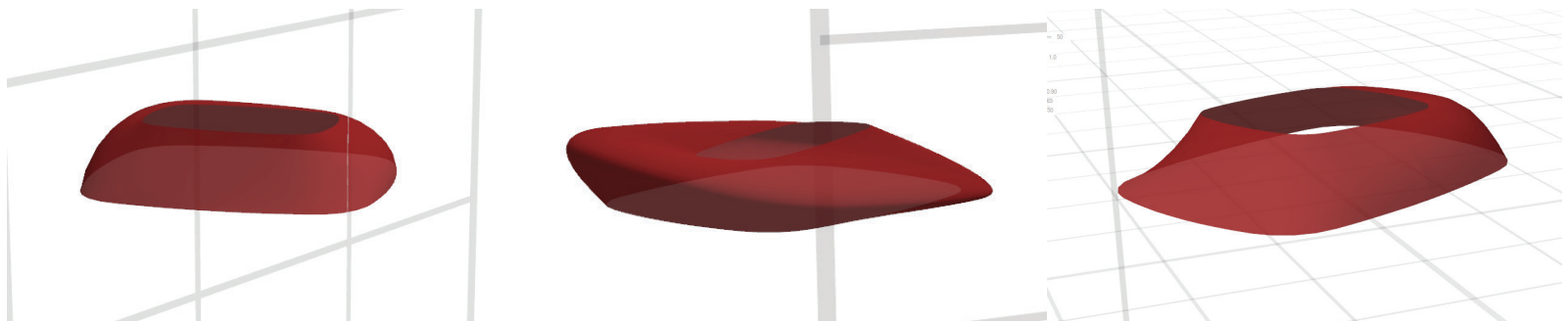
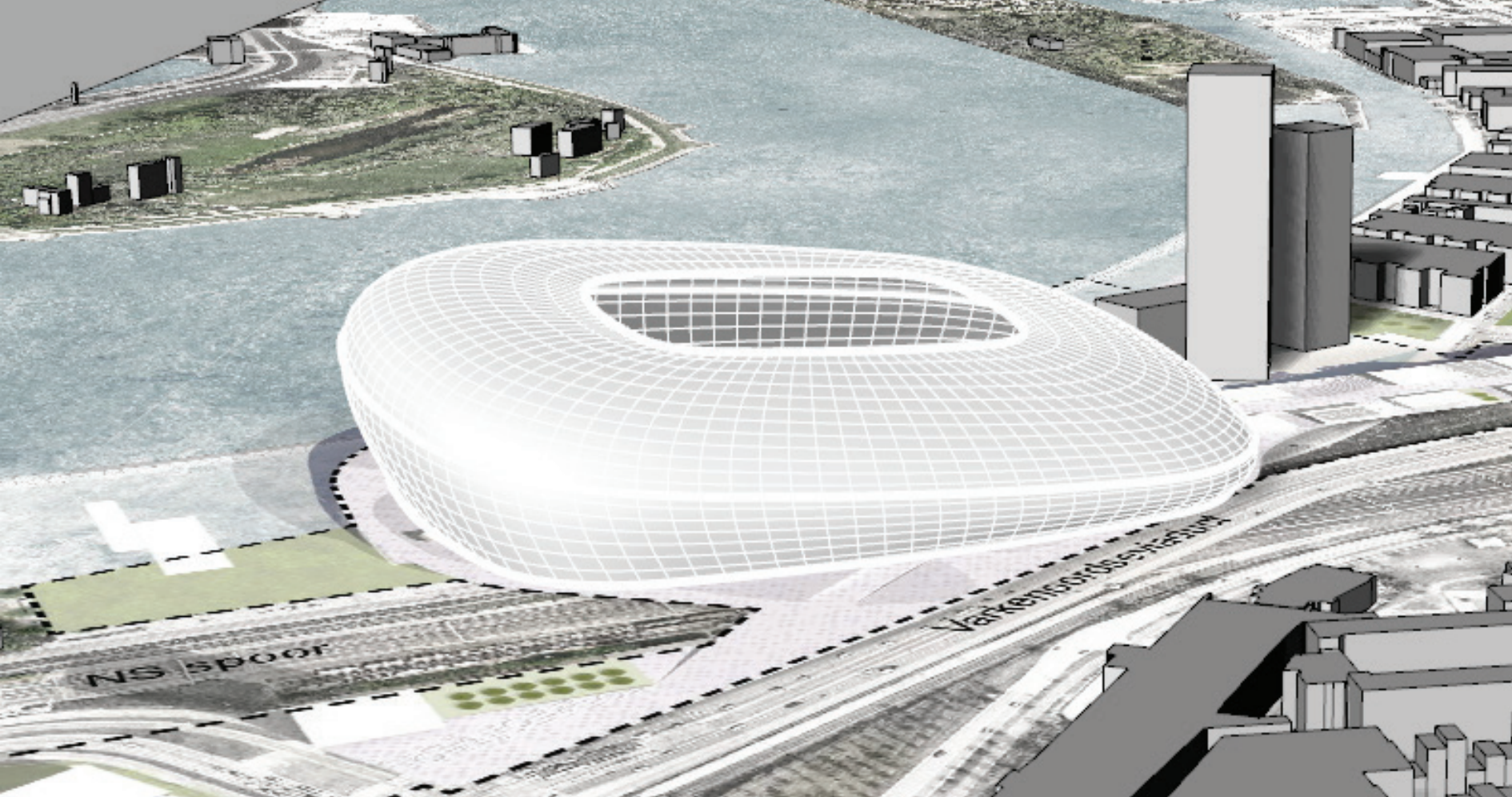


Figure 97 From left to right: a solution in which the Maximum Displacement is weighted very strongly, a solution in which the Total Radiation is weighted very strongly and a solution in which the minimal surface is weighted very strongly

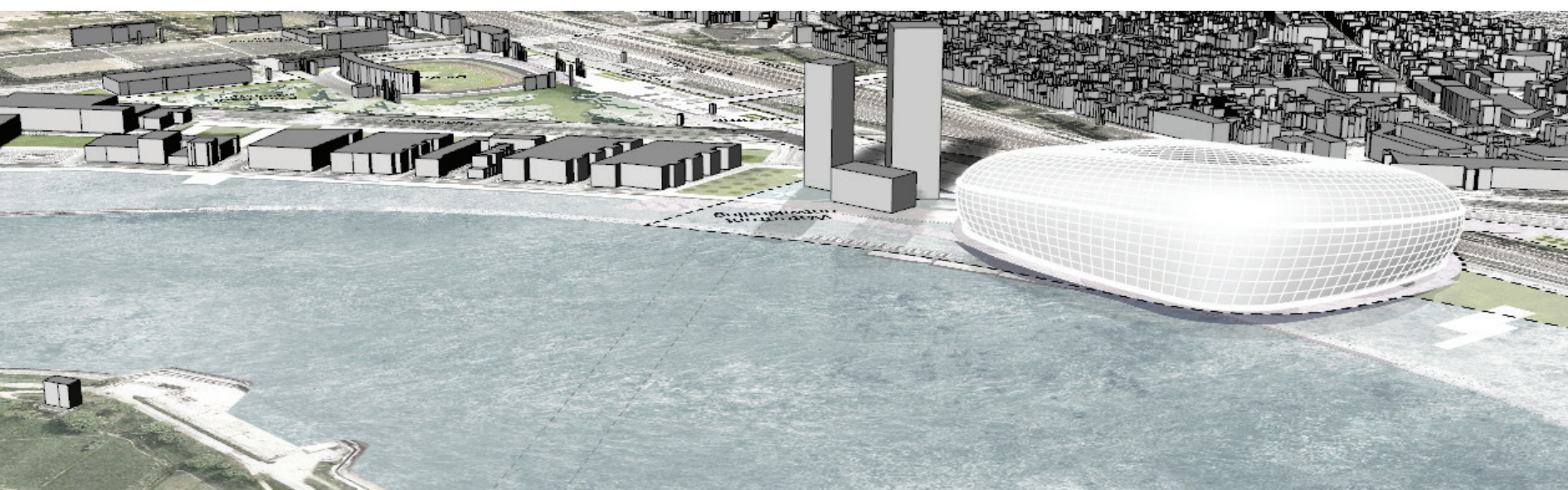
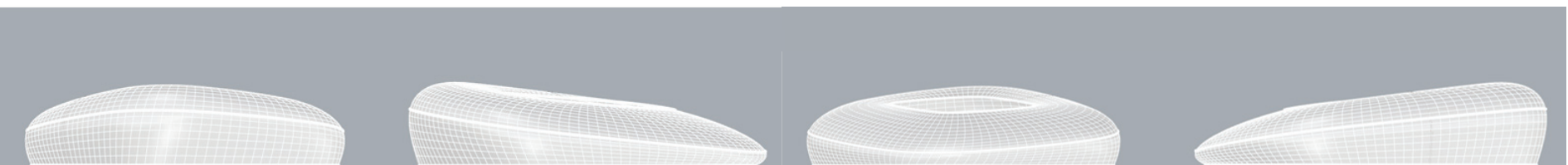


NORTH

WEST

SOUTH

EAST



7.2 Graded Voronoi

7.2.1 Image Map

The openings of the structure (the windows) of the canopy will vary in size over the surface. There are a multitude of factors that determine gradual change of the size of these openings. The desired size of the openings over the surface can be translated to a black and white image: the lighter the colour in the image, the larger the size of the desired openings. In other words, if we would paint a surface that represents the overall shape of the canopy in black, white and greytone, we would give parts where we want large openings a white colour. The smaller we want the openings to be at a certain area, the darker we paint that area. The resulting black and white image is based on both architectural and functional assumptions. There are four aspects that are taken into account in the generation of this gradient map:

1. The size of the openings must be related to the surroundings
2. The sizes of the bottom openings need to be functional
3. The size of the openings respond to the stresses in the structure
4. The size of the openings respond to the amount of sunlight it receives

Ideally, the openings on the waterside are larger than the openings on the side of the city to match the different characters on both sides. Also, parts of the stadium that provide a view on a landmark, historic building, etc. may ask for a larger size of openings on that part of the canopy. Secondly, from a functional perspective the openings of the canopy near the ground need to be big enough in order to maintain an open connection between the bottom ring of the stand with its surrounding concourse and the surrounding public space. Thirdly, the black and white map for the

opening size will also take stresses in the structure into account. Parts that undergo high stresses will be darker, representing small holes, and parts that undergo low stresses will be lighter, corresponding to larger openings. This will be measured by calculating the stresses on a shell that represents the basic shape of the canopy. Lastly, the sizes of the openings over the surface of the canopy will also respond to the amount of solar radiation that it receives. The southern side of the stadium will receive more sunlight than the northern side. The openings on the northern side of the stadium will be relatively bigger than the south side to allow as much light as possible to enter the building. While at the same time the openings at the south should not be too big to prevent overheating of the rooms that border the southern façade in summer.

The functions from spaces adjacent to the canopy have been studied in order to determine whether there are specific functions that require a lot of light or some that require a lot less. If this is the case, it may be an important factor for deciding where the openings of the canopy need to be larger and where they need to be smaller. However, the functions adjacent to canopy all require roughly the same amount of daylight.

As has been stated before, there are many aspects that can be taken into account for the generation of the image map. Since the scope of this research is limited to the translation of any performance map to a graded structure, the generated image map functions merely as an example. In no means does the selection of a few aspects mean that the other aspects can be neglected.

Part 1 of the script generates the layer of the performance map that contains the wishes of the architect. In this case, the canopy of the stadium needs to have bigger openings on the places where sightlines from the surroundings reach the stadium. This part of the script has two inputs: the shape of the canopy (chosen shape from the optimization) as a surface and

the points on the surface where the architect wants to have bigger openings. The surface is inserted into the 'Divide Domain²' component that divides the surface into segments (controlled by the U and V inputs). The U and V values are controlled by two sliders that are set on 19 (U) and 100 (V). The resulting segments go into the 'Isotrim' component along with the surface. This component splits the original surface into segments. The resulting surfaces are inserted into the 'Area' component to extract the centroid of each surface. In order to determine the distance from each centroid to the input points, the centroids and the input points are inserted into the 'Closest Point' component. The distance values are inserted into the 'Bounds' component to

create a numeric domain that encompasses all these distance values. The resulting domain is inserted into the 'Deconstruct Domain' component to extract the lower and upper bounds of the domain. The lower bound becomes the input for the 'Lower Limit' input of the 'Gradient' component and the upper bound goes into the 'Upper Limit' input of this component. The distance values go into the 'Parameter' input (t). The 'Gradient' component determines the colour of each of the surface segments based on their distance to the input points. The closer they are to the points, the brighter the colour.

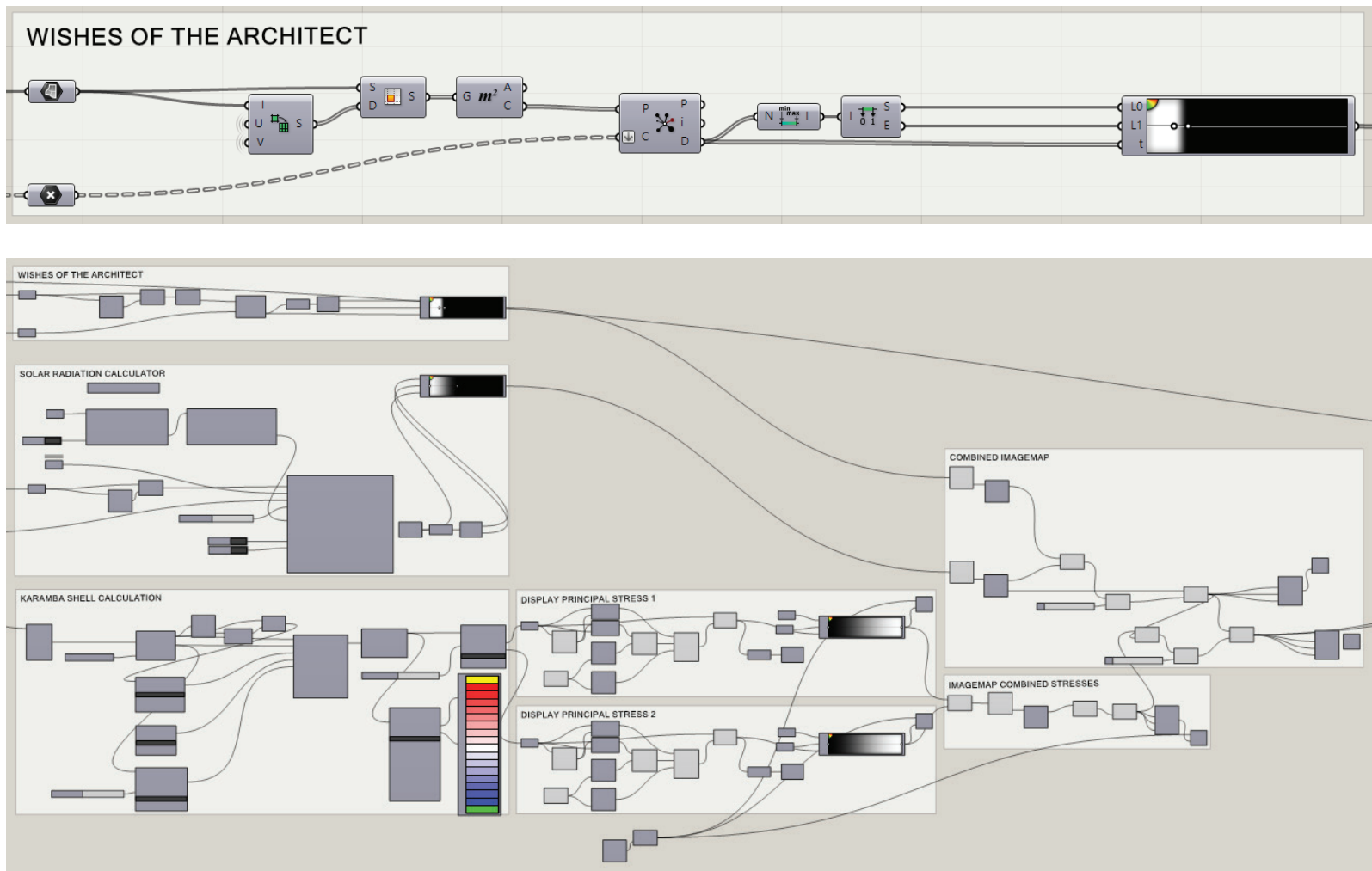


Figure 98 Top: Part 1 | Bottom: overview of the entire imagemap generating script

Part 2 of the script calculates the amount of solar radiation the entire canopy receives. The largest part of this script functions the same as is explained on page 120. However, the resolution is changed so that it matches the subdivision from the 'Wishes of the Architect' part of the script. The values of the Radiation Analysis correspond to the sub surfaces. The output is flattened so that all the values are within one list and then they are translated to a gradient in the same manner the distance values in the previous part of the script were translated. The higher the value, the darker the colour.

Part 3 of the script calculates the stresses within the shell. It functions in the same way as the part of the script that has been described on page 120 but it has an extra component added to the end of the script: the 'Result Vectors on Shells' component. This component provides us with two values of the principal stresses per principal stress for every sub surface from the canopy.

Part 4 of the script translates the principle stress values to gradient maps. The first part (within the rectangle) extracts the highest value of the two values that are given for both the first and the second principal stress. If the first stress value of a subsurface is higher, then the second needs to be removed and vice versa. To do so, the values from the stresses are inserted to the 'Shift List' component, to shift all values in the list one place upwards on the list. Then, the original list is inserted in the 'A' input of the 'Larger Than' component and the shifted list is inserted in the 'B' component. By doing so, it is checked whether the first value is bigger. If this is the case, the component returns a 'true'. The same is done a second time, however, this time the inputs are inversed. Thus, this component returns a 'true' if the second value of a subsurface is higher. Since all the values are in one list (instead of a separate list for each surface, containing two items), and being compared to one long shifted list, it does not just compare the 1st to the 2nd value of one sub surface and vice versa, but it also compares one of the values to a value of another subsurface.

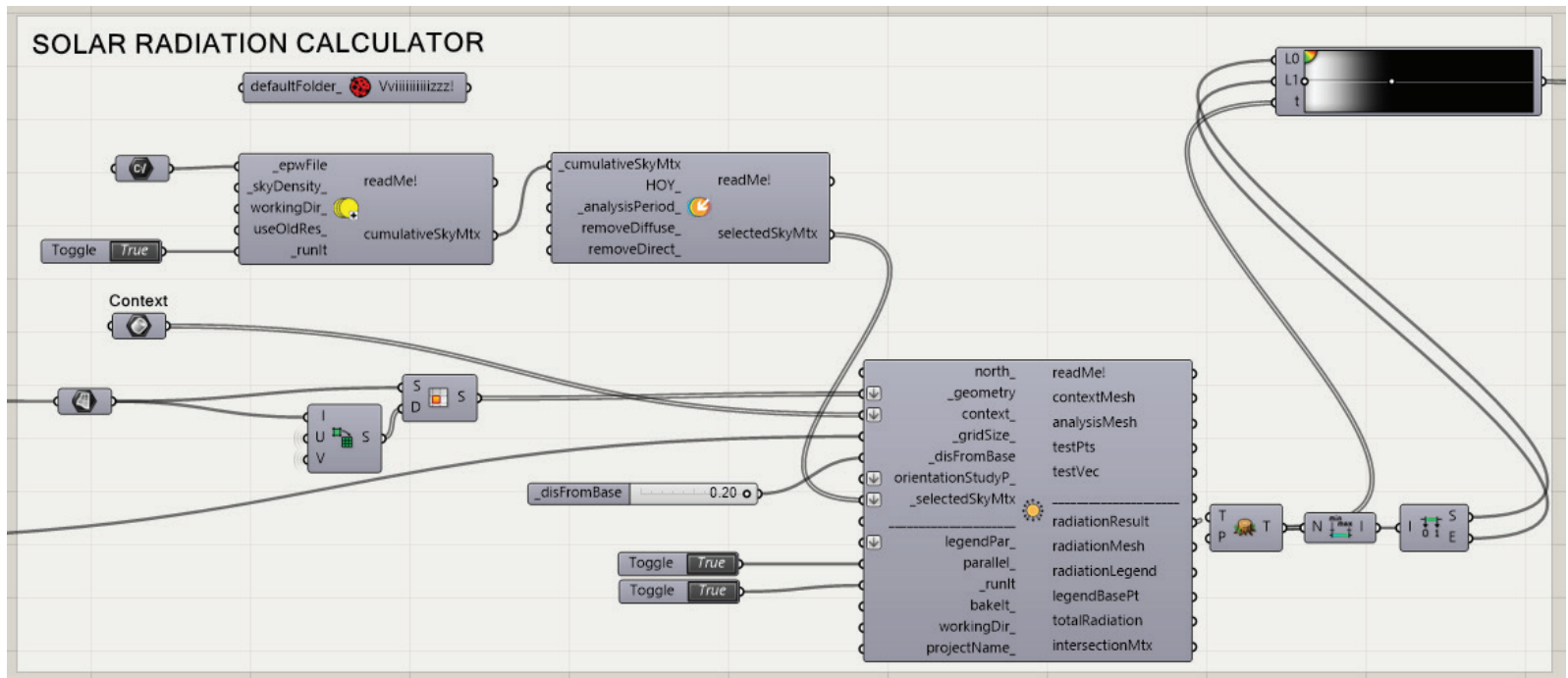


Figure 99 Part 2

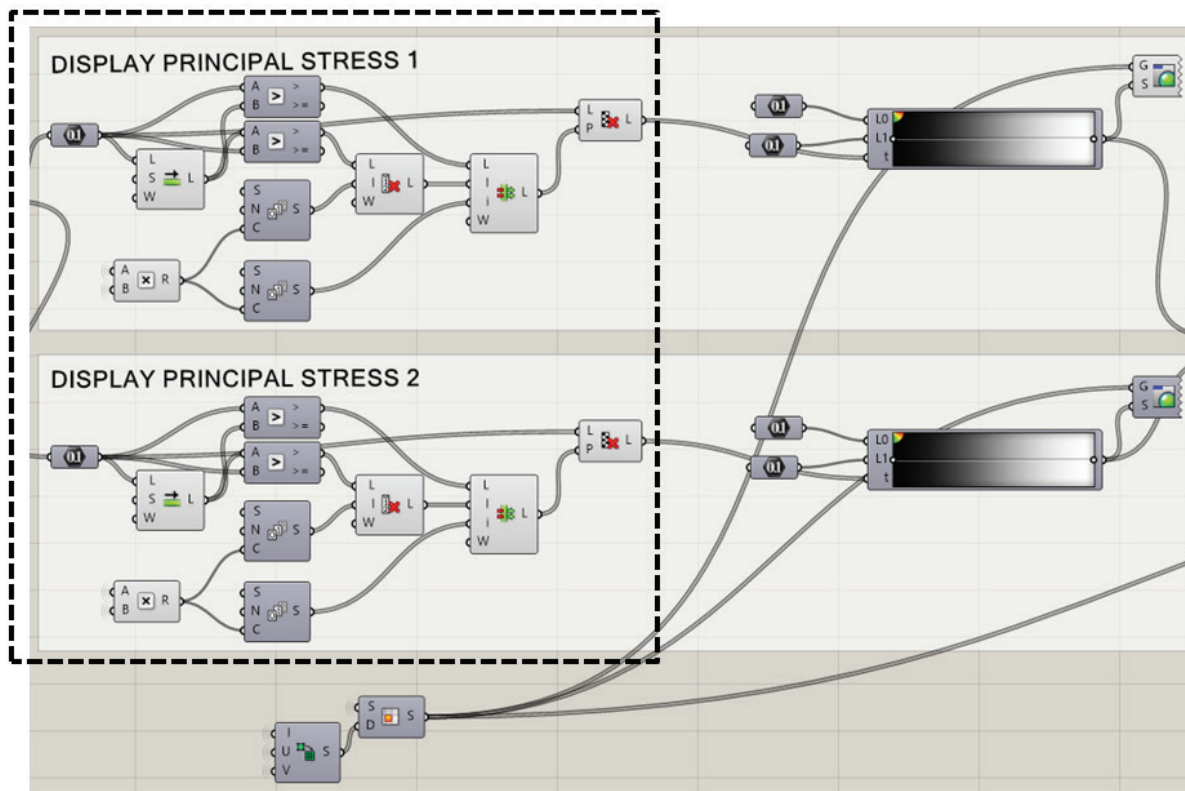
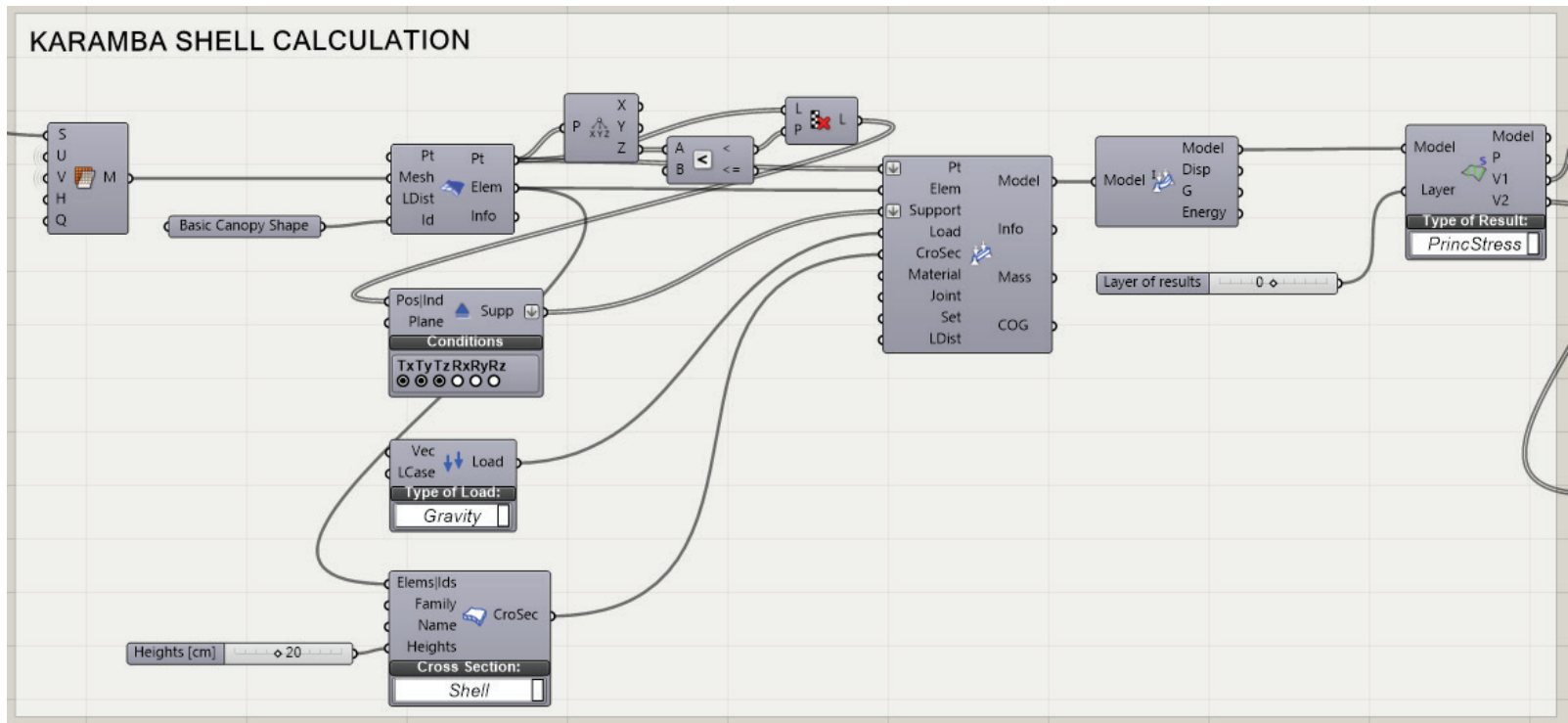


Figure 100 Top: Part 3 | Bottom: Part 4

These are the 'true' or 'false' values given at every second item of the list. Since this information is not needed, every second value of the second 'Larger Than' component is removed by using a 'Cull Index' component in combination with a 'Series' component. Then, every second value from the first 'Larger Than' component is replaced by the remaining values of the 'Cull Index' component by using the 'Replace Items' component. The result is a culling pattern that culls the first value if it is smaller than the second and vice versa. The culling is performed by the 'Cull Pattern' component and the largest stress value per subsurface remains. These values are inserted into the 'Gradient' component along with two numeric values that determine the lower limit and the upper limit of the gradient. The lower limit is zero and the upper limit is determined by the highest measured value of the two principal stresses. The resulting image maps display a brighter colour for higher stresses.

Part 5 of the script combines the two principal stress maps. This is not done by using a two layer principle in which the upper layer has a certain transparency (as happens in Photoshop, for example) because that would darken brighter parts in the underlying map that need to stay that bright. Instead, the bright patches of the principal stress 2 map need to be added to the principal stress 1 map. This can be done by adding the values of both the 'Gradient' components. This principle does not work for adding dark patches to the 1st map. Then, the image map needs to be inverted so that the highest stress values gets the darkest colour. In order to do so, the RGB colours first need to be inverted to a brightness value from 0 (black) to 255 (white). This can be done by plugging them into the 'Deconstruct' component that gives the R value in the 'x' output, the G value in the 'y' output and the B value in the 'z' output. All of these values are on a domain from -1.0 to 1.0. These values are remapped on a 0 to 255 domain by plugging them into the 'Remap Numbers' component. Then, the values are inverted by a series

of 'Subtraction' components. If the original value was 0, it needs to become 255 and vice versa. And if the original value was 128, it needs to become 129 and vice versa. To do so, firstly 127.5 is subtracted from the remapped value, then the resulting value is subtracted from 127.5.

Lastly, all the image maps are combined using a two layer principle in which the upper layer has a certain transparency (as happens in Photoshop, for example) (Part 6). First, the 'Wishes of the Architect' map and the 'Solar Radiation' map are combined. Then, this resulting map is combined with the 'Combined Stresses' map. In order to do so the difference between the values of the first map and the second map are calculated by subtracting the values of the second map from the values of the first map. The resulting value is multiplied by a number on a scale of 0 to 1.00, in which a value of 1 results in a 100% transparency of the second image map, so the second image map does not show. The resulting value is added to the value of the second image map. This process is repeated to add the third layer 'Combined Stresses' to the image map.

7.2.2 Voronoi

Part 1 of the script remaps the values from the combined image map that have a domain of 0 to 255 to a domain that matches the minimum and maximum cell size of the Voronoi.

Part 2 of the script generated a graded Delaunay. The explanation of this part of the script matches the explanation that can be found on page 90.

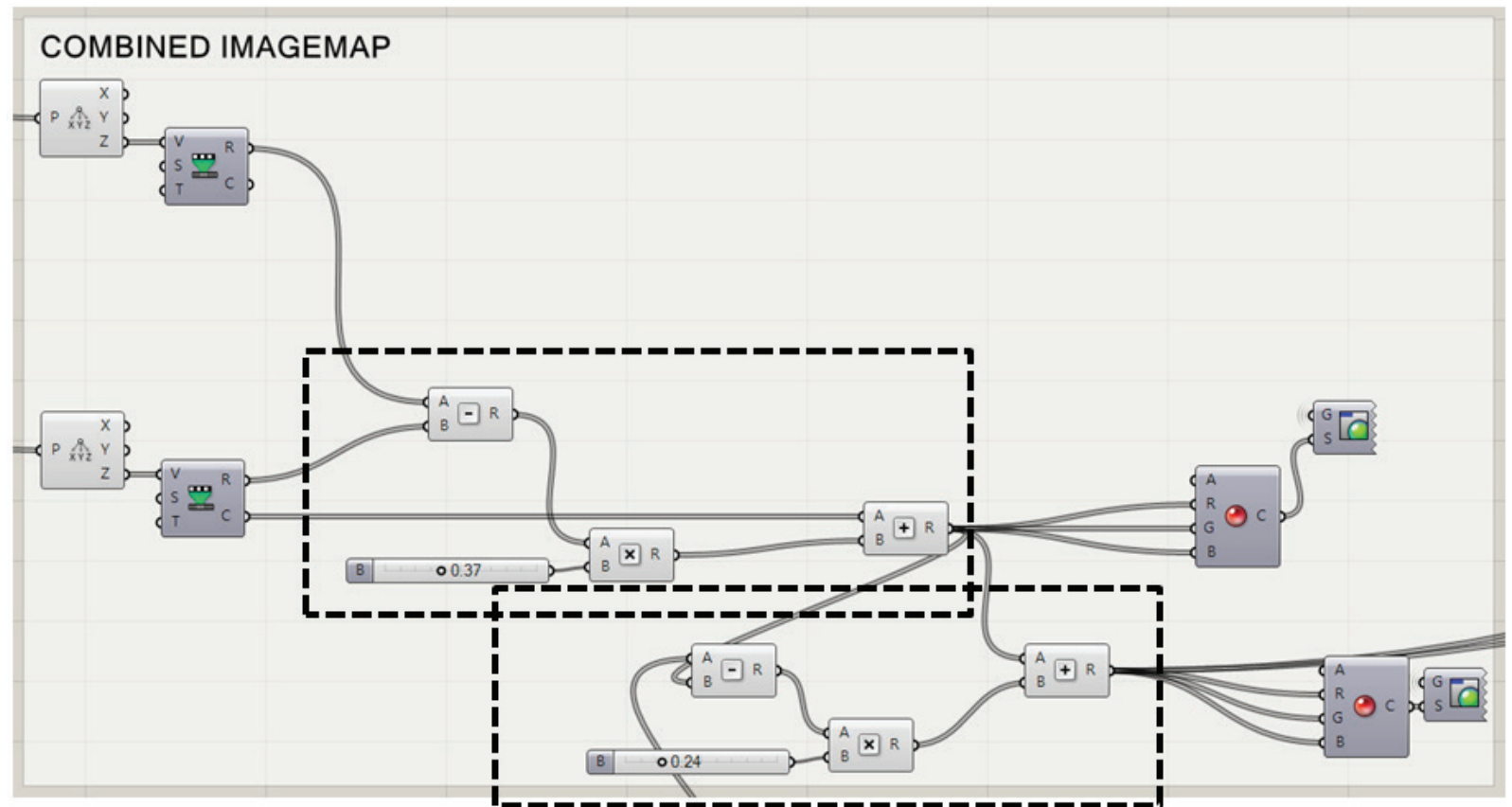
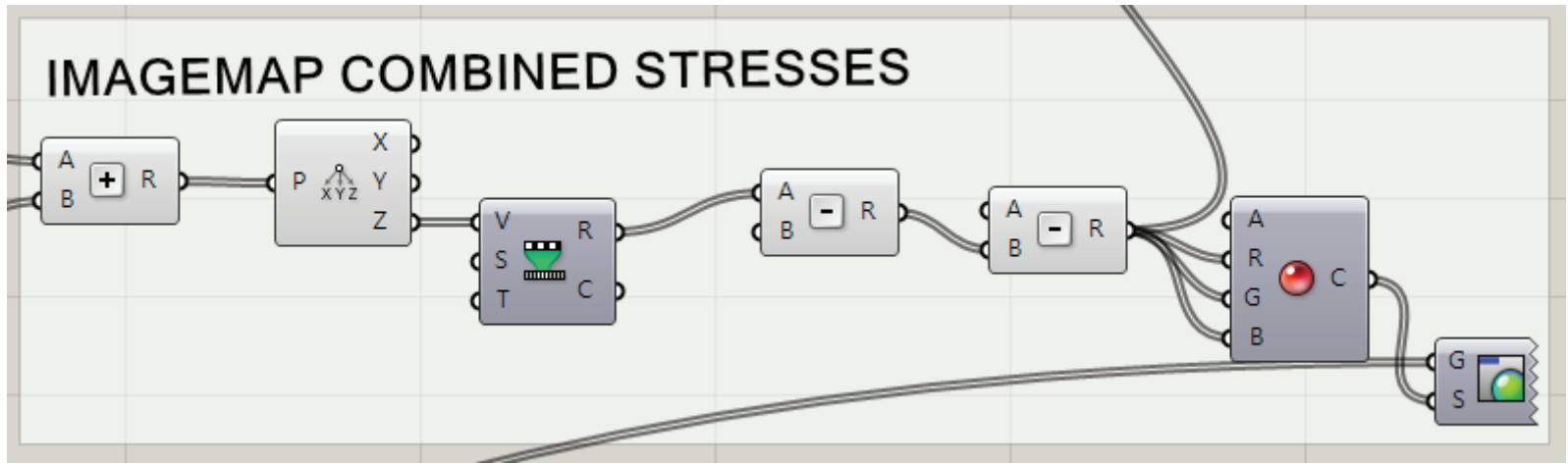


Figure 101 Top: Part 5 | Bottom: Part 6

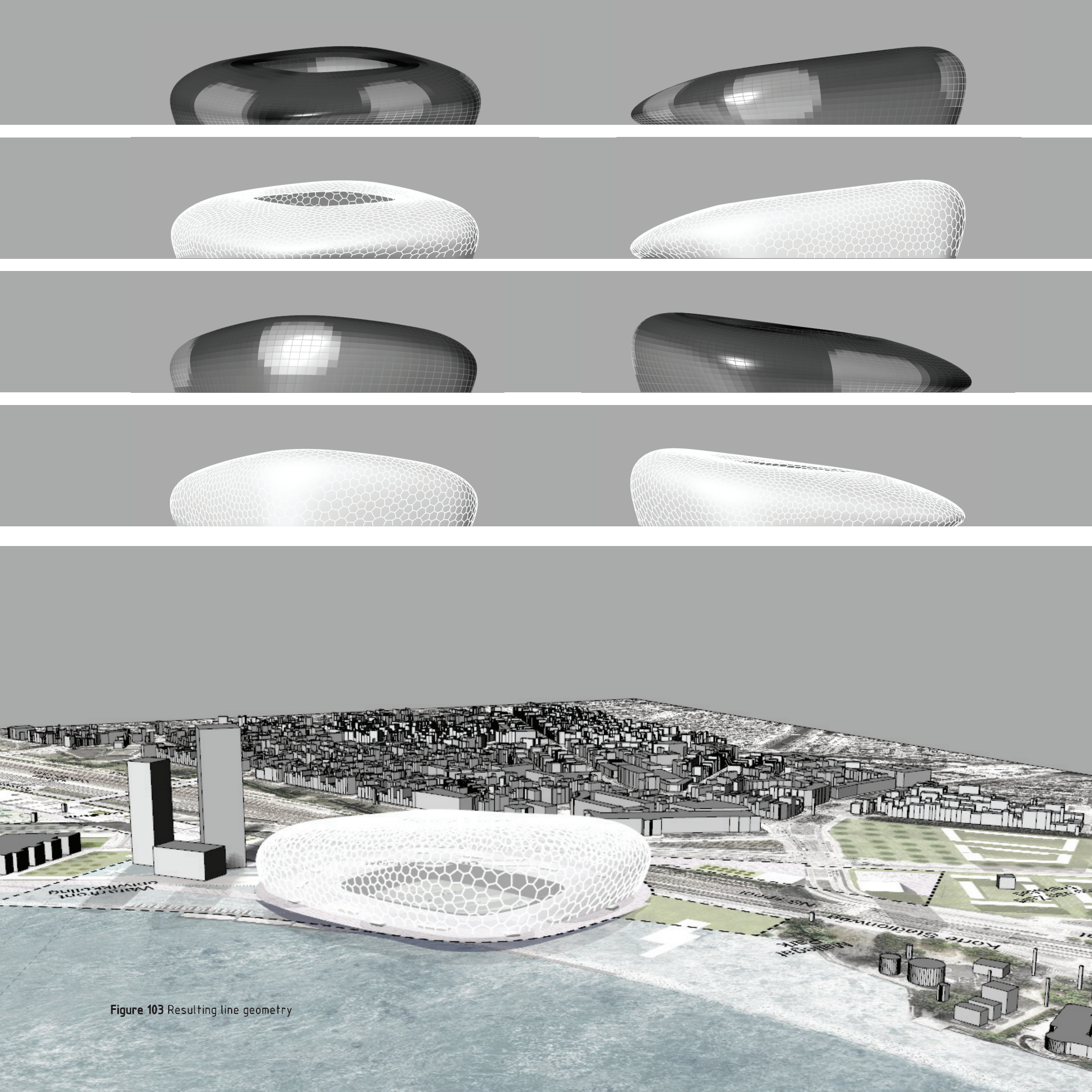


Figure 103 Resulting line geometry

7.3 Structural Testing

Now that a method of generating a graded Voronoi on the overall shape of the canopy is established, several tests regarding the structural performance can be performed. What is the effect of the size of the openings on the structural performance? If the Voronoi pattern is densified in places where higher stresses occur in the structure, does the structural performance get better? Does the structural performance of a structure get better when additional webs are added to the FRP-foam beam? These test were performed with gravity loading. In other words, the deflection that occurs in the structure due to its own weight.

7.3.1 Opening Sizes

Several tests were performed to analyze the effect of the opening size of Voronoi patterns on the structural performance. Both regular patterns and irregular (graded) patterns were tested. A regular Voronoi pattern is a pattern in which all the openings are roughly the same size. Whilst the irregular Voronoi pattern matches an underlying image map that represents the stress values occurring in the structure. This stress map was generated by analyzing the stresses on a shell that matches the overall shape of the canopy. The first set of tests was performed by changing the opening sizes while remaining a constant total mass of the structure. Since the (average) opening size of the structure changes for each calculation, so does the total member length. Thus, the section of the members must change for each solution to keep the total mass constant. In other words, when the average opening size increases, the total member length decreases and the size of the section increases. For these calculations a simple square cross section was used and the mass of the total structure is kept on $1.55e^6$ kg (with a deviation of max. $0.01e^6$ kg). The results for six of the regular Voronoi patterns are shown in figure 104. The results of four of the irregular voronoi patterns are shown

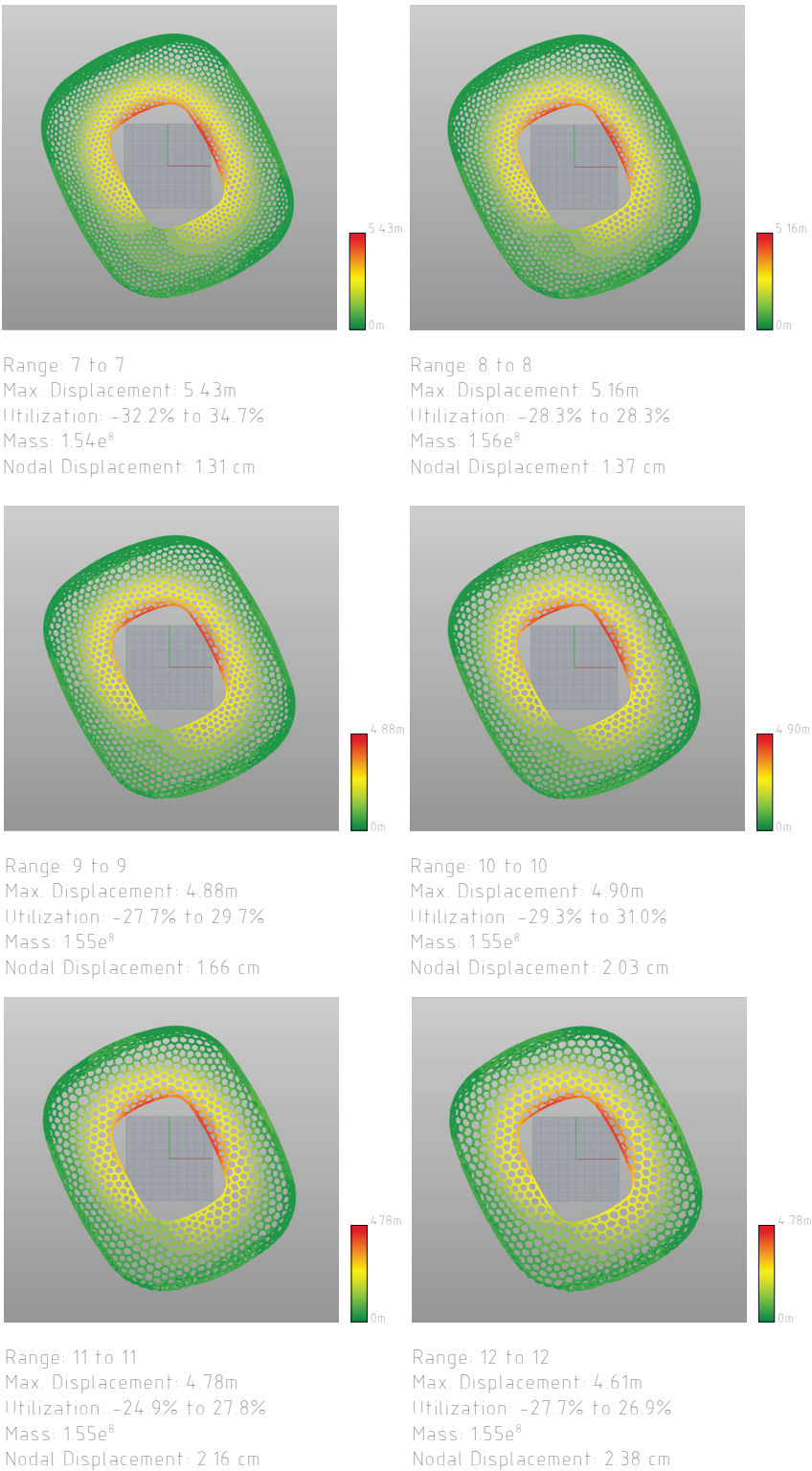


Figure 104 Results of first test regarding opening sizes, showing six regular Voronoi patterns

in figure 105. All the individual results can be found in the appendices. As can be seen in these results, the utilization lies well within the allowed domain of -100% to 100%. If the utilization of a beam exceeds these values, it will most likely fail. Usually the strength of FRP structures is not restrictive. In most cases the deflection of the structure is the limiting factor. The utilization values from these tests show that the same applies to this structure. Therefore, the deflection will be the most important in evaluating the results. The results of all the calculations of the first test are shown in figure 106 and figure 107. Based on these

results it can be concluded that when the average opening size increases, the deflection decreases. This can be explained by taking a closer look at the manner in which a constant mass was obtained. Namely, by changing the size of the square beams. The larger the openings, the higher the beams, the larger the bending stiffness ($E \times I$) and the smaller the deflection. Thus, the smaller deflection is caused by the larger height of the beams. In order to eliminate this factor, a second test is performed in which the density of the material is changed to maintain a constant mass for all the options. As a result, the mass distribution is the only

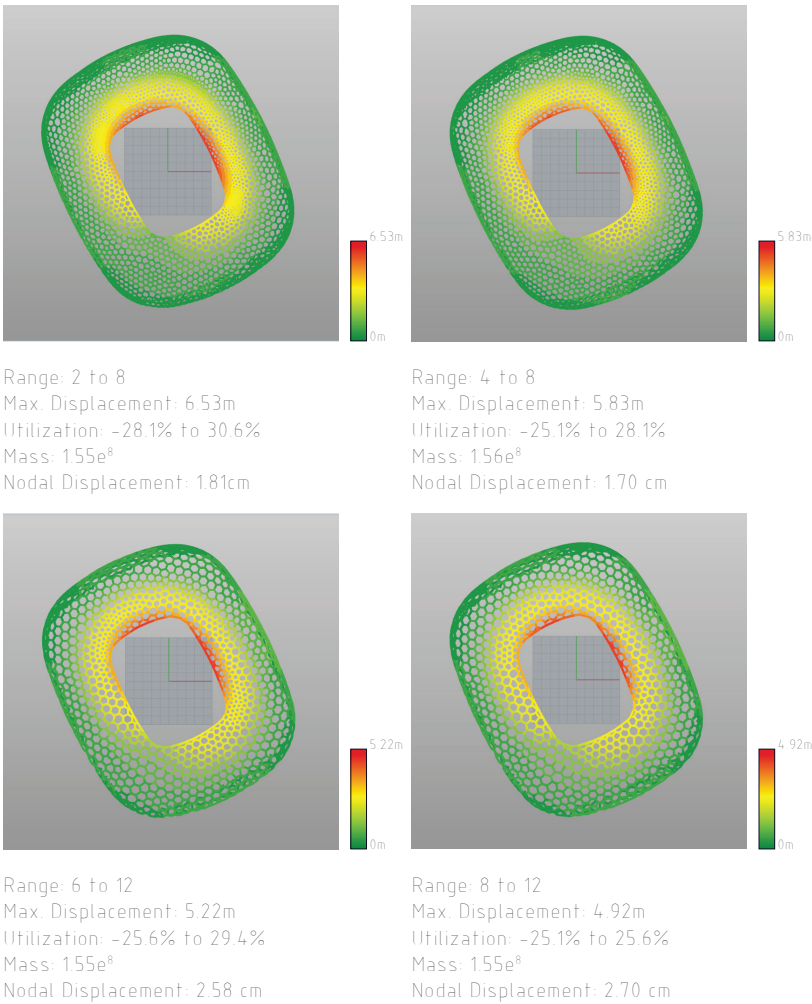


Figure 105 Results of first test regarding opening sizes, showing four irregular Voronoi patterns

		Largest Openings							
		5	6	7	8	9	10	11	12
Smallest Openings	2	6,36	6,81	6,44	6,53	6,32	6,39	6,38	6,43
	3		6,36	5,98	6,23	5,68	6,17	5,67	6,15
	4			5,99	5,83	5,39	5,77	5,4	5,77
	5	5,81			5,61	5,34	5,51	5,11	5,53
	6		5,62			5,34	5,32	4,96	5,22
	7			5,43			5,27	4,94	5,08
	8	5,3			5,16			4,93	4,92
	9	5,29	4,86			4,88			4,82
	10	5,3	4,86	4,79			4,9		
	11	5,3	4,85	4,8	4,7			4,78	
12	5,29	4,81	4,8	4,7	4,69			4,61	

Figure 106 Results of first tests in a table (in meters)

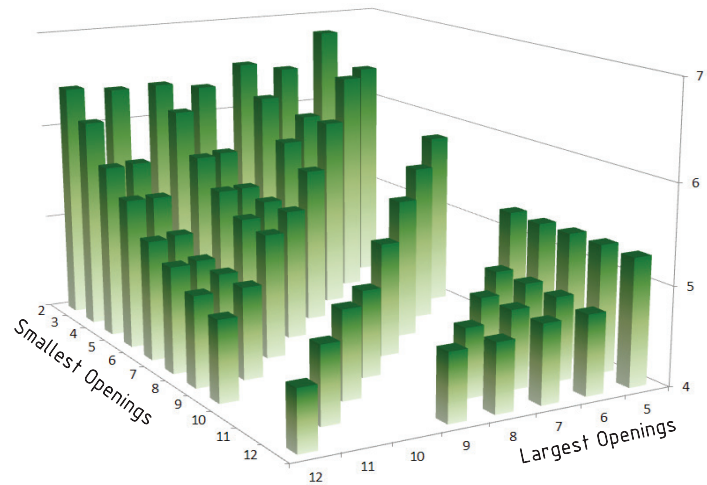


Figure 107 Results of first tests in a 3D bar graph (in meters)

remaining factor. The results for six of the regular Voronoi patterns are shown in figure 108. The results of eight of the irregular voronoi patterns are shown in figure 109. All the individual results can be found in the appendices. The results of all the calculations of the second test are shown in figure 110 and figure 111. After performing the second test, several conclusions can be drawn. It can be noticed that almost all irregular patterns perform better than their regular partner. It can also be concluded that the solutions with smaller opening sizes perform better. In addition to that, it can be concluded that a small variety in opening sizes (f.e. 6 to 10) outperforms solutions with a large variety in opening sizes in most cases (f.e. 2 to 10).

Based on the results of both tests a decision needed to be made for the range of openings of the final design. According to the first test, larger openings result in a smaller overall deflection under deadload. According to the second test, smaller variety in opening sizes perform better than large varieties in opening sizes. However, the variety in opening sizes is not only used to increase the structural performance, but is also used for creating bigger windows at the north side and smaller windows at the south side. In addition to that, the openings also need to respond to the wishes of the architect. On locations where strong axes from the surroundings reach the canopy, larger openings are desired. In order to reach a noticeable effect on these aspects a certain variety in opening sizes is required. For all these reasons, a range of openings from 5 to 12 was chosen. This results in a relatively small overall deflection compared to other solutions (according to test 1) while still outperforming its regular partner by 82cm's (according to test 2). The resulting line geometry for the final solution is shown in figure 112. This figure also shows the imagemap on which the gradation of the Voronoi pattern is based. When comparing the final result to these image maps it becomes clear that the graded Voronoi pattern matches.

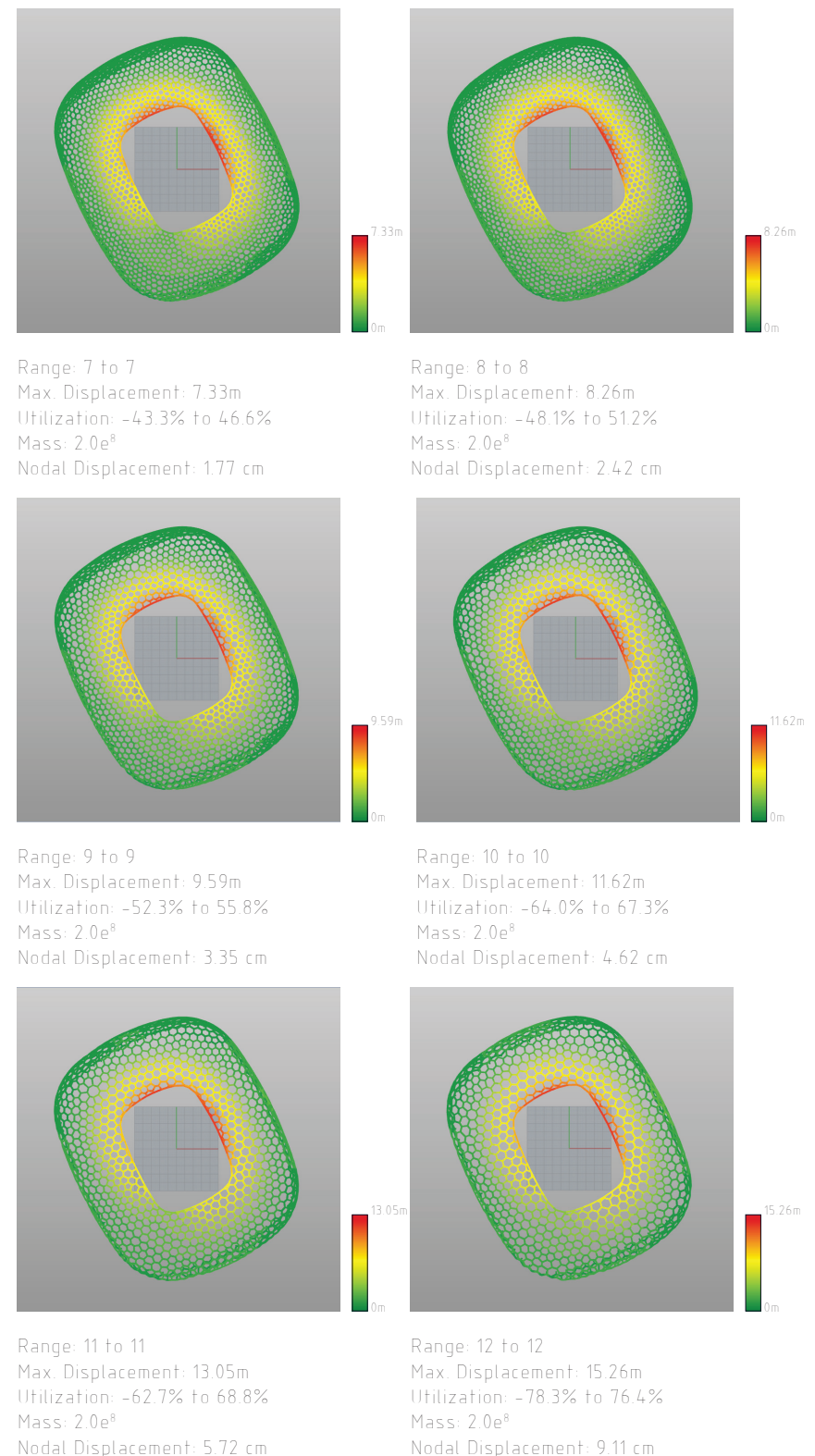


Figure 108 Results of second test regarding opening sizes, showing six regular Voronoi patterns

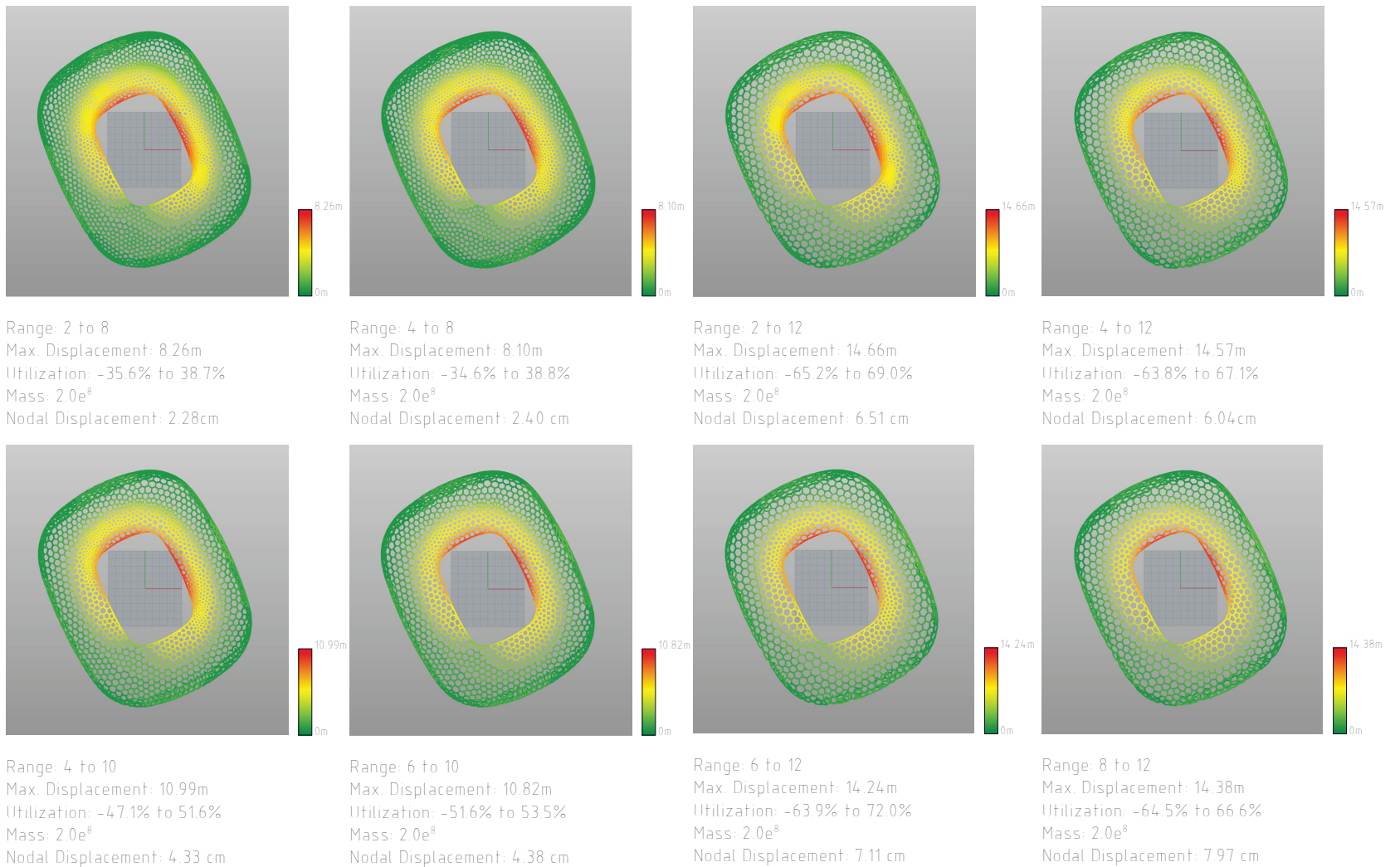


Figure 109 Results of second test regarding opening sizes, showing eight irregular Voronoi patterns

		Largest Openings							
Smallest Openings		5	6	7	8	9	10	11	12
	2	4,32	5,50	6,78	8,26	9,92	11,04	13,21	14,66
	3		5,41	6,58	8,16	9,62	11,09	12,88	14,78
	4			6,61	8,10	9,53	10,99	12,80	14,57
	5	4,46			8,02	9,58	10,85	12,72	14,44
	6		5,54			9,58	10,82	12,58	14,24
	7			7,33			10,98	12,55	14,30
	8				8,26			12,50	14,38
	9					9,59			14,35
	10						11,62		
	11							13,05	
	12								15,26

Figure 110 Results of second tests in a table (in meters)

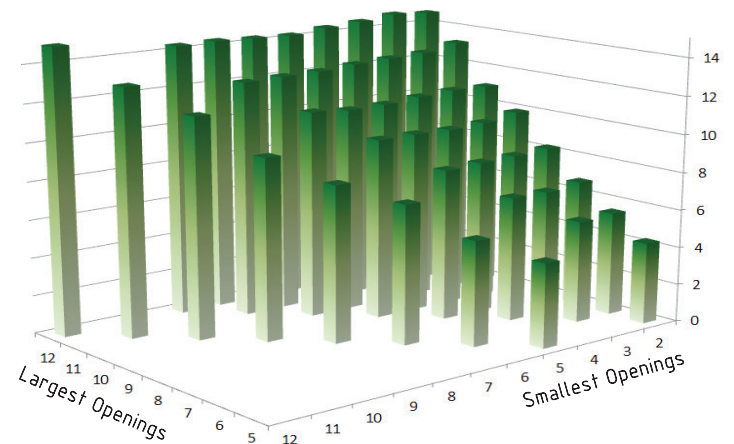


Figure 111 Results of second tests in a 3D bar graph (in meters)

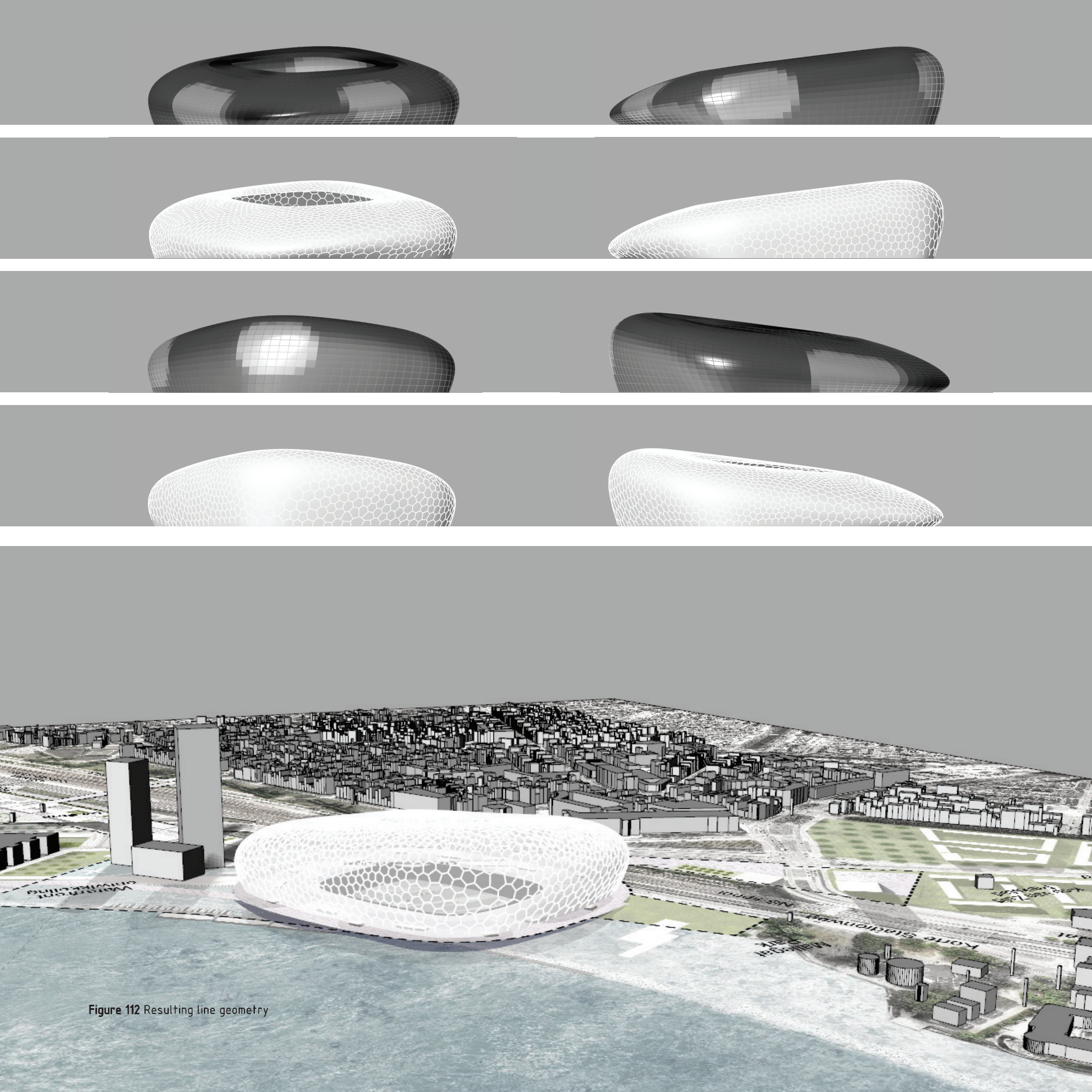


Figure 112 Resulting line geometry

7.3.2 Requirements

The resulting lines from the Voronoi grid are translated to a structure with beams and supports by Karamba. The resulting structure will be analyzed for five loadcases:

- Load case 0: only gravity load (reference)
- Load case 1: north western wind load
- Load case 2: south western wind load
- Load case 3: south eastern wind load
- Load case 4: north eastern wind load

The deflection resulting from the gravity load will be the standard deflection of the entire structure (permanent load). The structure will be built in such a way that the structures standard deflected shape is equal to the designed shape. The difference in deflection between load case 0 and the other load cases are the deformations that can make spectators feel unsafe when they are in the stadium. According to Nijssse [2012], this maximum deformation must be within 0.004 times the length of the span, which in this case is 200 meters for the width of the entire structure.

$$0.004 \times 200 = 0.8 \text{ m deflection.}$$

According to Nijssse [2012], the following loads need to be considered for roofs:

1. A live load: 1.0 kN/m^2
2. The stadium lies within windzone II in an urban area and has a maximum height of 75 meters:
 -1.37 kN/m^2 windload

In order to determine how the windload affects the structure, it was simplified to a rectangular shape as can be seen in figure 113. The basic calculations are performed with the same beam section for every beam. These cross section have also been simplified to rectangular sections. The beams are made from GFRP whose material properties were found in the CES

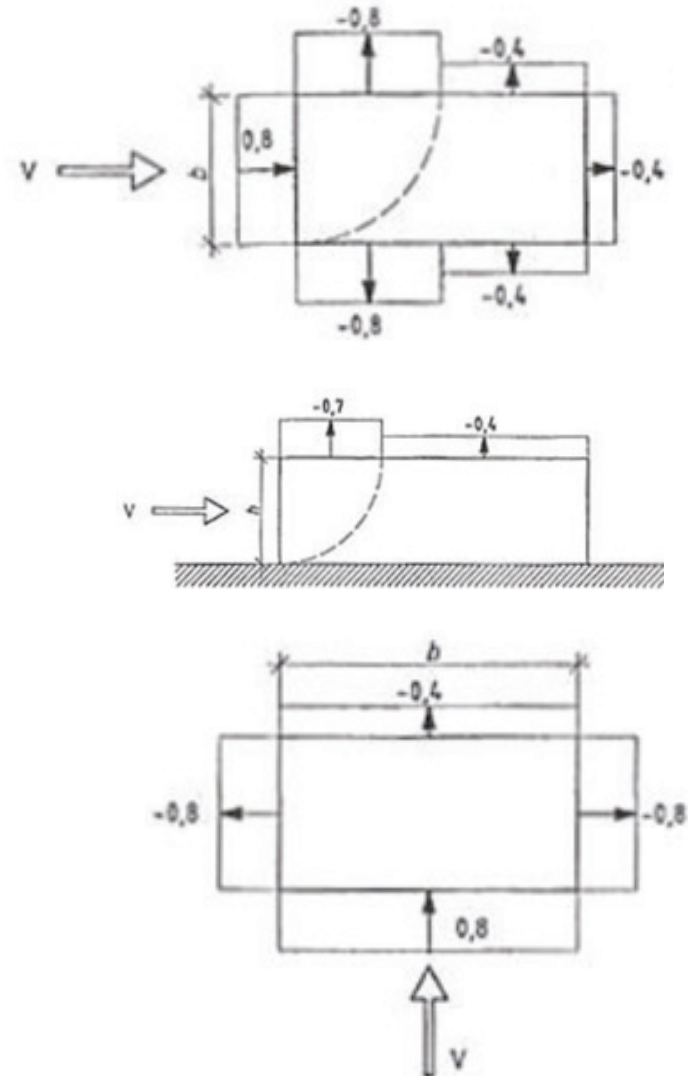


Figure 113 Wind form factors for rectangular buildings according to Nijssse [2012]

Edupack 2016 programme. Figure 114 shows a graph for polymers that have glass fibers in a woven fabric. This restriction was made based on findings of the literature study into FRP (see chapter 3.3). Since the deflection is restricting (instead of the allowed stress) the value of the Young's Modulus was considered as well as the price of the material. If only the Young's Modulus was considered, the phenolic/E-glass fiber, woven prepreg, biaxial lay-up would be the winner. However, both of the phenolic/E-glass polymers are more than 10 times as expensive as the polyester/E-glass fiber polymers.

When a graph is made in which the Young's Modulus is divided by the price for one of the axes (see figure 114) it can be concluded that for its price the Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up has the best Young's Modulus and the best Yield Strength. Therefore, this polymer was chosen. This material has the following properties:

Young's Modulus = 21 GPa
Yield Strength = 370 MPa
Poisson's ratio = 0.16
Price = 2 €/kg
Density = 1850 kg/m³
Shear Modulus = 3.9 GPa

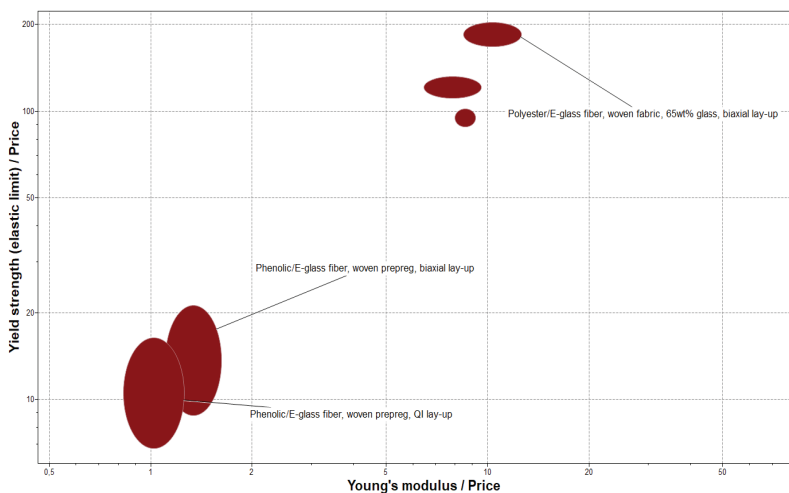
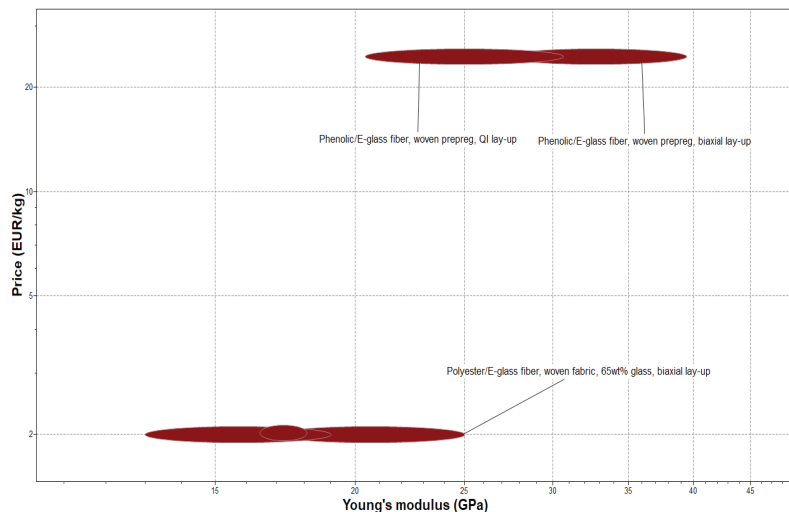


Figure 114 CES 2016 graph of polymers with glass fibers and woven fabric

In order to find a suitable material for the core material, CES Edupack was used as well. A limit was made that only allows foams that can be CNC-milled. An additional limit was made to only allow PS, PU and PVC. This is based on the literature study into FRP (See chapter 3.3) Since foams have a very low Yield Strength and Young's Modulus compared to polymers, this material does not significantly contribute to the bending stiffness of the section. However, the heavier the foam, the larger the deadload of the structure and the larger the overall deflection. For this reason, price and density were decisive. Figure 115 shows a CES graph with price on the y-axis and density on the x-axis. The price of all of the foams is the same. The weight, however, varies significantly. The lightest material was chosen: Polyurethane foam (rigid, closed cell, 0.062). The material has the following properties:

Young's Modulus = 0.015 GPa
Yield Strength = 0,25 MPa
Poisson's ratio = 0.32
Price = 9.38 €/kg
Density = 61.6 kg/m³
Shear Modulus = 0.004 GPa

These properties were used for the karamba calculations that are discussed in chapter 7.3.3 and 7.3.4.

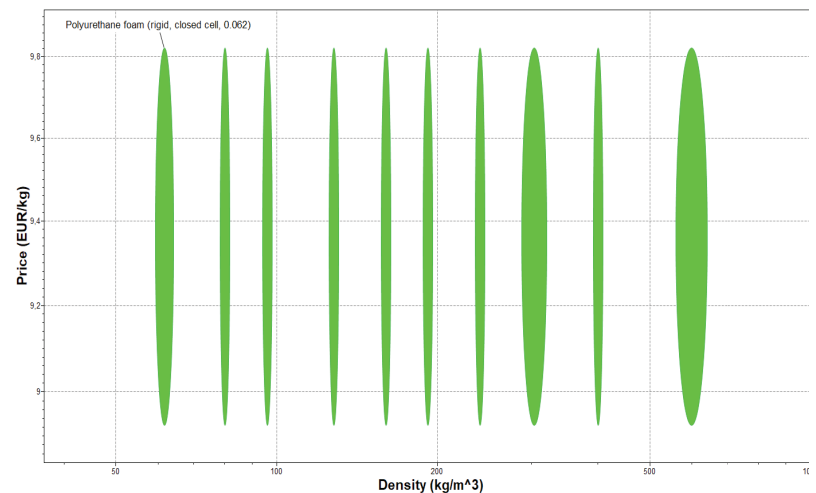


Figure 115 CES 2016 graph of PS, PUR and PVC foams that are suitable for milling

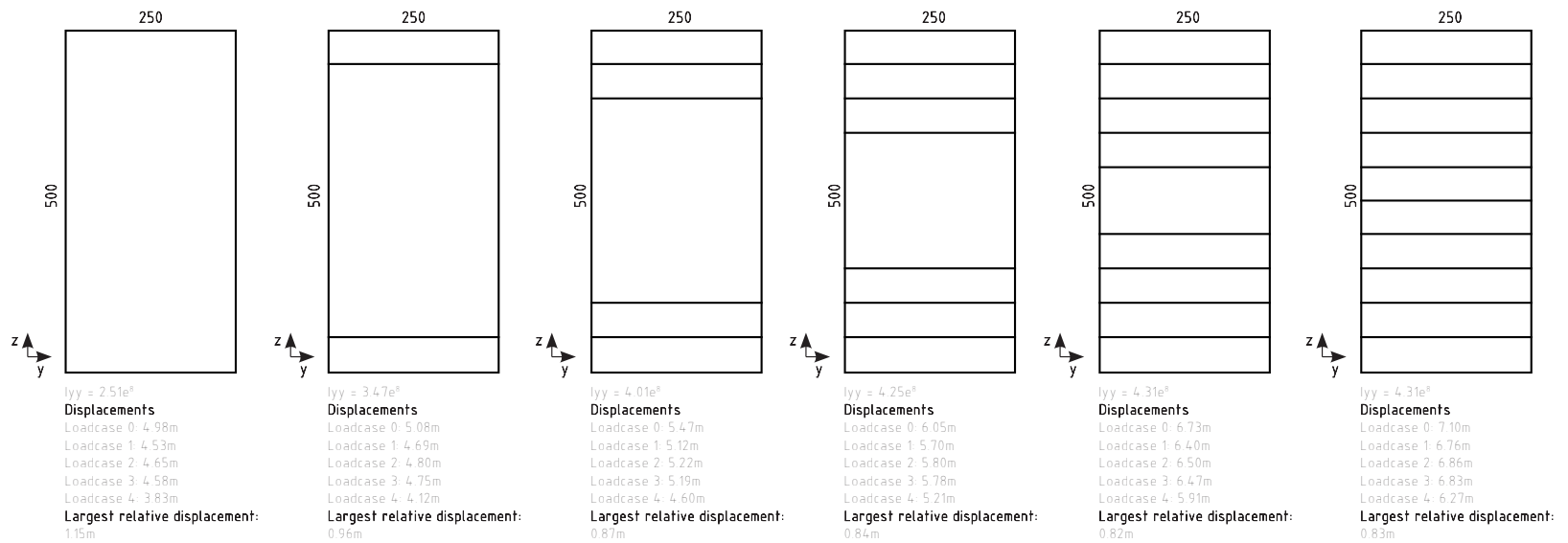


Figure 116 Five 250x500mm sections with different amount of webs. Thickness of the polymer layer: 50 mm, distance between webs: 50mm

7.3.3 Addition of Webs

With Karamba it was tested how the addition of webs affects the deflection. The options that were evaluated are shown above. The webs are not only needed to limit the relative deflection of the structure. They are also needed to reduce the size of the foam blocks that need to be produced. Every polygon of the structure will become one element. All these separate elements will be stacked and combined to form the entire structure. The largest polygon has a height and width of 12 meters. Creating this polygon of one block of foam is not desirable. Introducing extra polymer webs divides the entire element into separate smaller foam blocks. This is less expensive, results into less material waste and makes the production of the large elements manageable. From these calculations it can be concluded that mainly the outer webs contribute to a large reduction of the relative deflection. In addition to that, it can be noticed that the overall deflection for every separate loadcase increases with the addition of every web. The more webs are added, the closer the structure comes to meeting the requirements of a maximum relative deflection of 80cm. The addition of a web in the middle of the section makes the relative deflection slightly worse.

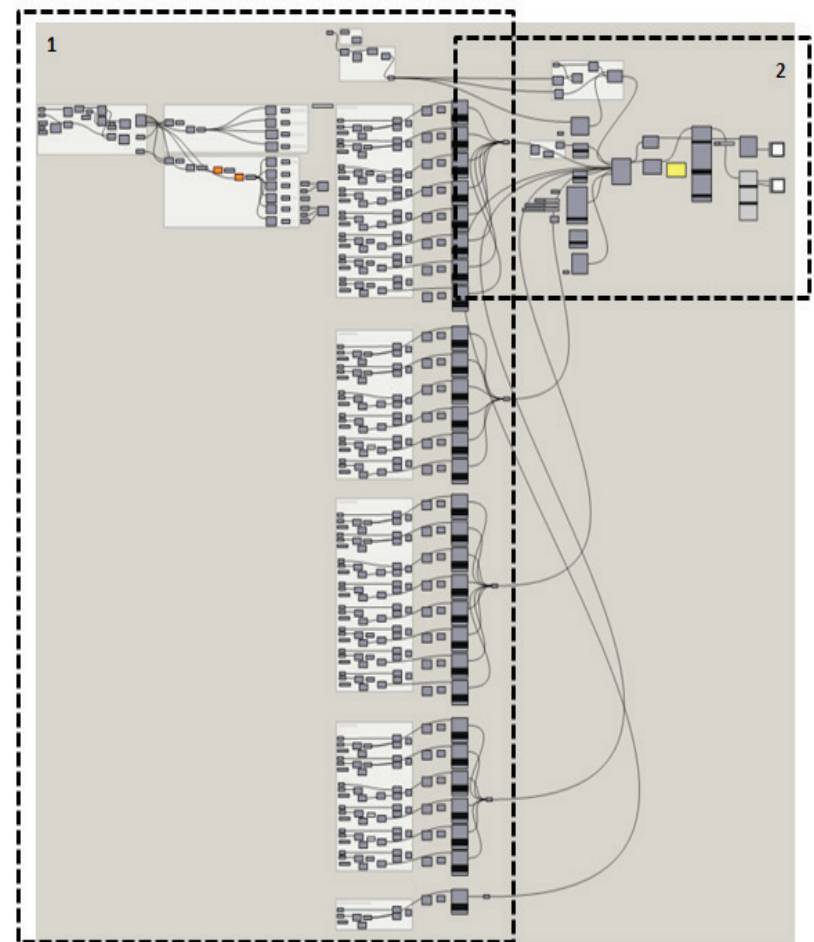
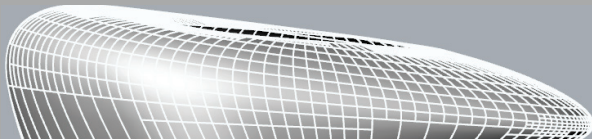
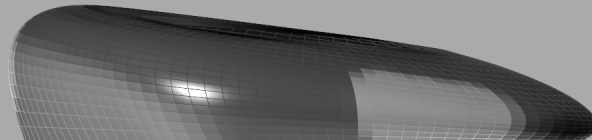
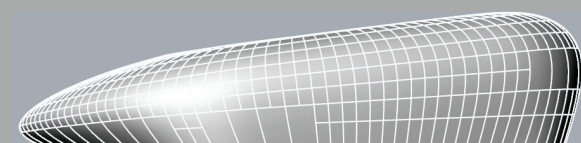
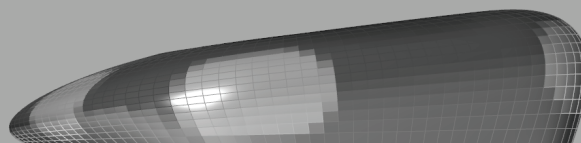


Figure 117 Grasshopper script for the Karamba calculations with rectangle 1 that sets up the load cases and rectangle 2 that constructs the model and performs the calculations



7.3.4 Comparison Traditional Grid

The same loadcases were applied to a quadrangular grid that was densified using a traditional method: dividing the quad in four when a certain threshold of the underlying imagemap is exceeded. The total member length of the quadrangular structure is almost equal to the Voronoi structure. Section number 6 from figure 116 is used. This led to the following results:

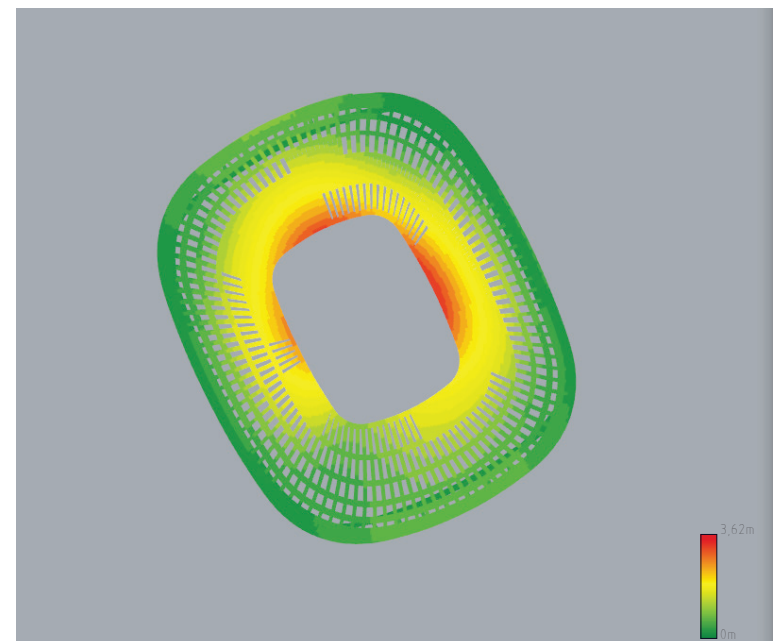
Total member length: 25.735m
Total mass: $8.6e^7$
Loadcase 1: 3.62m
Loadcase 2: 3.49m
Loadcase 3: 3.54m
Loadcase 4: 3.48m
Loadcase 5: 3.24m
Max. Relative Displacement: 0.38m
Utilization: -28.4% tot 17.2%

In comparison to the Voronoi structure with section number 6:

Total member length: 25.658m
Total mass: $8.6e^7$
Loadcase 1: 7.10m
Loadcase 2: 6.76m
Loadcase 3: 6.86m
Loadcase 4: 6.83m
Loadcase 5: 6.27m
Max. Relative Displacement: 0.83m
Utilization: -38.2% tot 36.9%

It can be concluded that the traditionally densified grid performs better from a structural point of view. However, the images on the right show that the traditionally densified structure is only a very coarse approximation of the underlying imagemap. It does not show gradual transitions, instead it shows very harsh differences with an aesthetic look that does not meet the requirements for the error detection rules as have been

explained before: the structure shows long guidelines that abruptly stop at several points throughout the structure. Since the resulting structure is not a very good approximation of the underlying performance map, it will automatically perform worse on all these performances than a graded Voronoi structure – and in the case of this specific performance map leaving either too much or too little sunlight into the rooms adjacent to the façade (this differs for different places of the façade). This can cause rooms to overheat in summer, rooms to be too cold in winter, hinder by too much light and extra energy use because not enough light enters the room. Thus, the traditionally densified grid may perform better from a structural point of view, but it performs worse on all the objectives that were accounted for in the imagemap.



7.3.5 Size Optimization

From the previous calculations it can be concluded that the addition of webs, with the exception of the web in the middle of the cross section, is beneficial for the relative deflection of the structure. However, the relative deflection does not meet the requirement yet. In addition to that, the same cross section has been applied to the entire structure. Optimizing the size of the cross section for every separate beam not only reduces the amount of material that is used but also reduces the deflections. Within the size optimization that was performed for this structure, the width of the cross sections was kept constant for every beam. Only the height of the section changes over the structure. The different height for each beam was determined by analyzing the bending moments for every beam. The larger the bending moment, the larger the height of the cross section. Figure 118 shows the analysis of the entire structure. Green colours correspond to low bending moments and red colours correspond to high bending moments. The height of the allowed cross sections for

the optimization lies between 50cm and 800cm. Figure 120 shows several ranges of cross sections and the amount of beams in the optimized structure to which these cross sections are applied. A total number of 6536 out of 6671 beams have a smaller cross section than the unoptimized structure. For the unoptimized solution:

Relative Deflection : 0.82 m
Mass Polymer: $8.07e^7$ kg
Mass Foam: $1.71e^7$ kg
Cost: 325.5 million

For the optimized solution:

Relative Deflection : 0.75 m
Mass Polymer: $4.36e^7$ kg
Mass Foam: $9.89e^6$ kg
Cost: 180 million

The optimized solution meets the requirement of a maximum allowed relative deflection of 80cm.

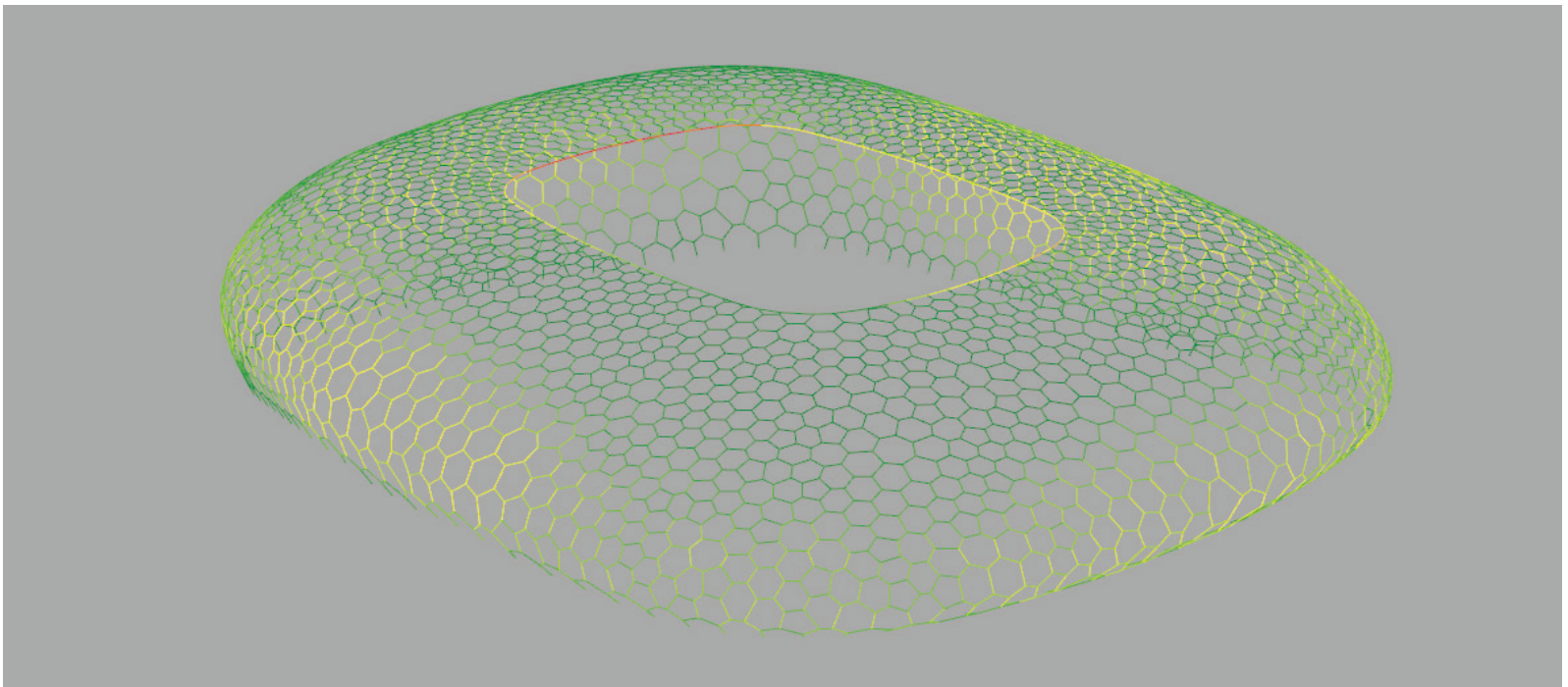


Figure 118 Karamba analysis of the bending moments that occur in every beam as a basis for the cross section optimization

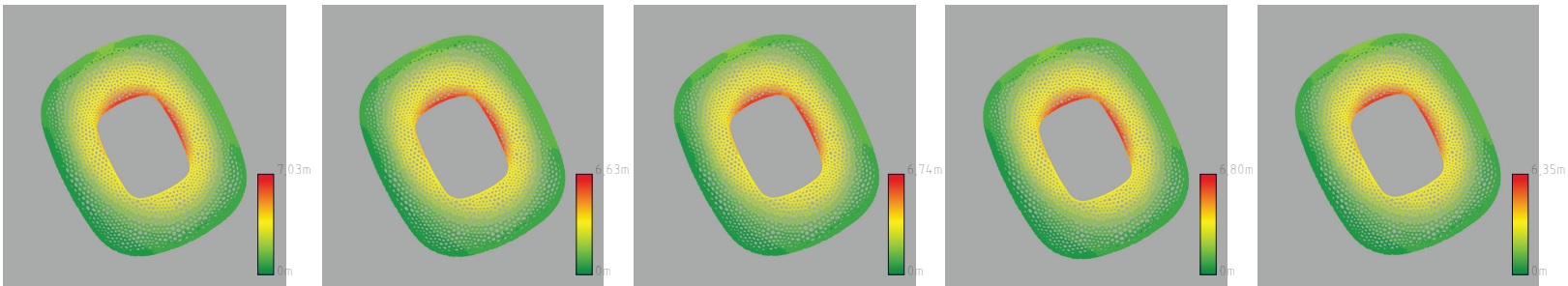


Figure 119 From left to right the deflections of: Loadcase 0, Loadcase 1, Loadcase 2, Loadcase 3, Loadcase 4

Profile Heights	Amount of beams
750-800	3
700-750	5
650-700	9
600-650	7
550-600	7
500-550	104
450-500	262
400-450	578
350-400	975
300-350	1101
250-300	1967
200-250	1653

Figure 120 Occuring profile heights in structure

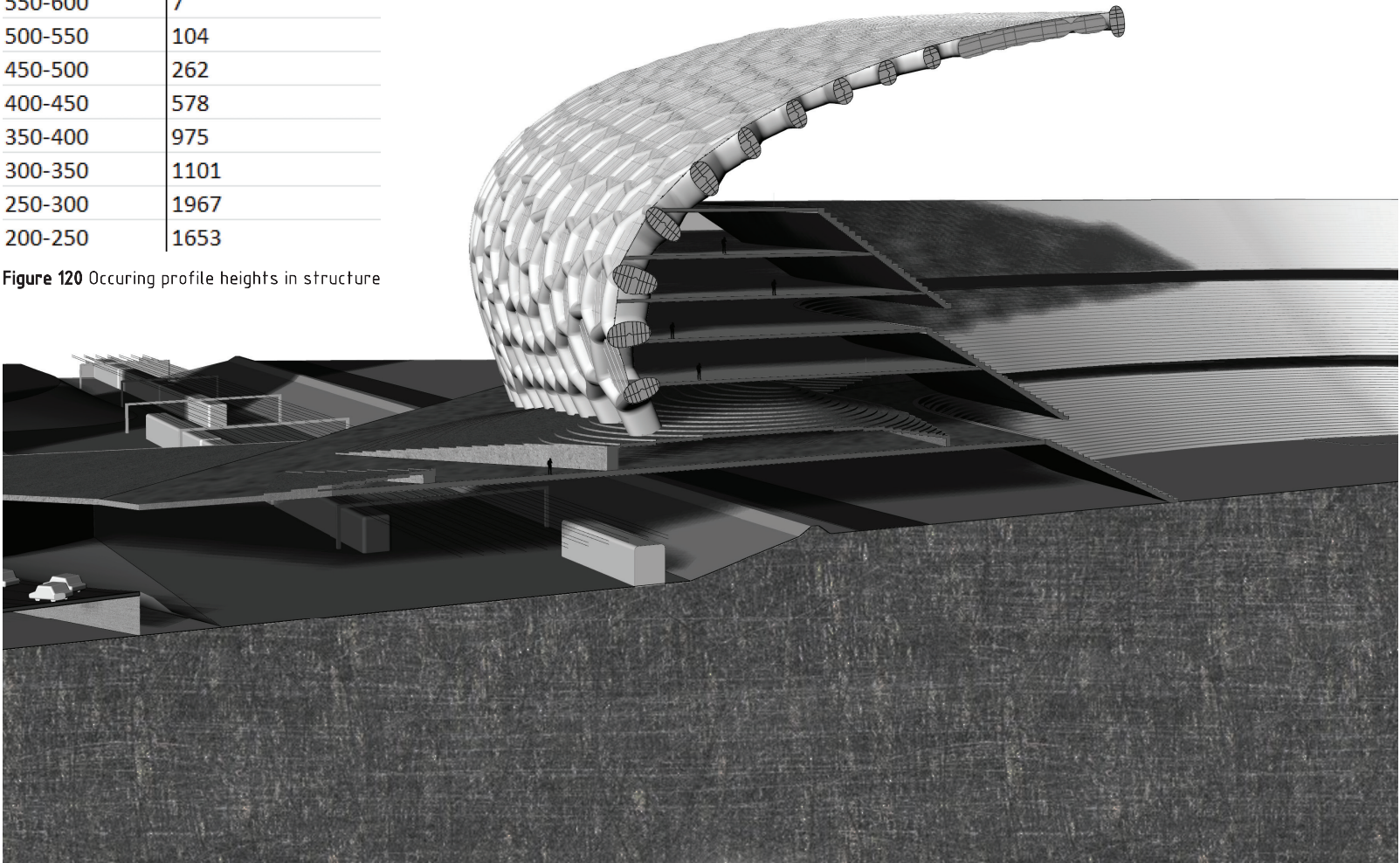


Figure 121 3D section of the Canopy

7.4 Detailing

With the detailing of such a large complex structure several aspects need to be taken into account: desired aesthetics, water flow, nodal displacement, thermal expansion and manufactureability.

7.4.1 Water Flow

With the detailing and the design of buildings the discharge of rainwater needs to be taken into account. Rainwater should not gather in parts of the structure to prevent leakage and fungus growth. In order to check whether this is the case, the water flow needs to be analyzed. In order to do so, the Eve rain grasshopper plug-in was used. Figure 122 shows the water flow analysis of the canopy. The blue line show locations where rainwater flows of the edge of the canopy onto the tribune and the soccerfield. This is highly undesirable. A second analysis was performed to see whether an addition of a ridge along the opening of the canopy would solve the problem. This analysis is shown in figure 123. It can be noticed that this solves the problem. The water that cause problems earlier now flows along the ridge and flows off the building on the south side. Thus, along the edge of the opening

of the canopy a ridge needs to be present. The ridge is formed by a higher cross section along the edge than elsewhere in the roof (see detail 3).

In order for the water to freely flow all the way to the ground the water must not gather on top of mullions or parts of the FRP elements. For this reason, the depth of the glass panes in each element varies depending on the angle of the glass pane at that point. Thus, at flatter parts (like the roof) the glass panes lie on top of the FRP elements (see detail 2) whilst at more vertical parts the glass lies deeper (see detail 1). This allows the FRP elements to protrude and emphasizes the graded Voronoi pattern. This desired effect is the reason that not all glass panes lie on top of the FRP elements.

7.4.2 Nodal Displacement

Due to all the loads that work upon the structure the structure does not only deform globally, but also locally. Global displacement is measured in maximum displacement and is measured by determining what point of the structure moves the most in a global coordinate system. Nodal displacement is a local displacement of a beam between its two end points. When no loads apply this beam is straight. When loads are applied the beam

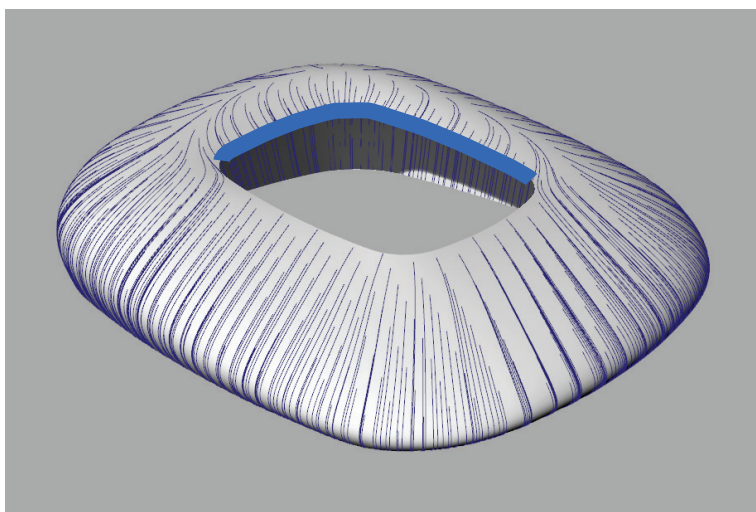


Figure 122 Rainflow without ridge along opening

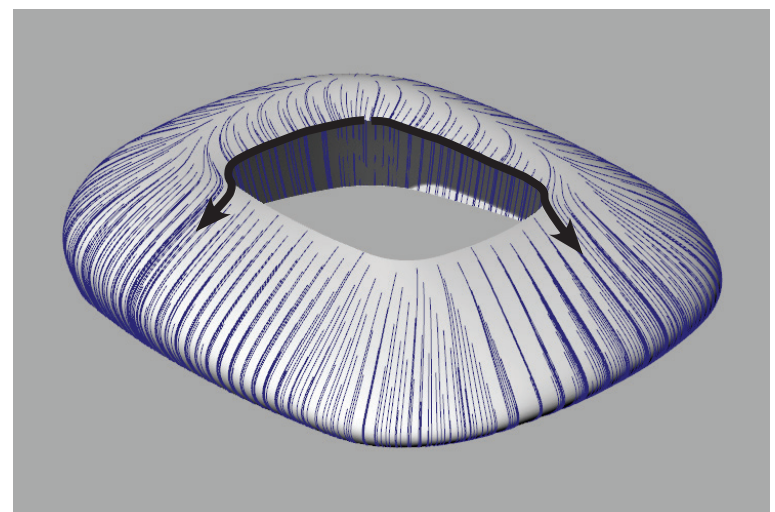


Figure 123 Rainflow ridge along opening

itself deforms. With the measurement of the nodal displacement the global displacement of the entire beam itself is neglected, only the deviation of several points on the beam from the original straight line is measured (see figure 125). This nodal displacement is an important factor for the following reason. Within each FRP element there is a glass façade consisting of mullions and glasspanes. When the nodal displacement is larger than the tolerances between the glass and the mullions, the glass breaks due to the stresses. Figure 124 shows an analysis from the nodal displacements of the structure. The maximum nodal displacement that occurs in the structure is 1.31 mm. At most places however, the nodal displacement is so small it is neglectable. This 1.31 mm is a small displacement that can easily be accounted for. The glass pane has a small tolerance to move within the mullions (see detail 1A). This tolerance is large enough to allow for nodal displacements.

7.4.3 Thermal Expansion

In large structures thermal expansion could cause large stresses if there are no expansion bearings or dilatations. Therefore, the detailing must account for thermal expansion. The thermal expansion coefficient

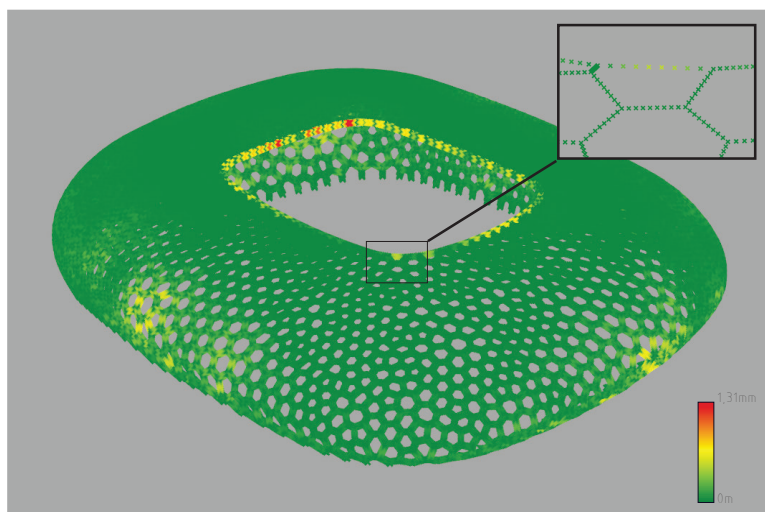


Figure 124 Nodal Displacements of the structure

for the polymer that was chosen is $13 \mu\text{strain}/^\circ\text{C}$. In other words, for every degree of temperature change, the FRP expands 0.000013 m per meter. The length of the structure is 235 m. It is assumed that the temperature of the FRP will not exceed the following boundaries: -20°C to 60°C . As a result the thermal expansion over the entire building will be 25cm. Thus, if all the bearings of the structure would not allow for thermal expansion, large internal stresses would occur. For this reason, expansion bearings were designed (see detail 4). Figure 126 shows an exaggerated example of thermal expansion for the canopy and indicates in what direction every expansion bearing should slide.

7.4.4 Manufacturability

As has been stated before, the webs do not only increase the bending stiffness but they also ensure that even the largest elements of $12\text{m} \times 12\text{m}$ can be manufactured. By dividing each element in several foam blocks that are divided by webs, the foam blocks become a lot smaller and more manageable. Chapter 7.5 shows the production of a panel.

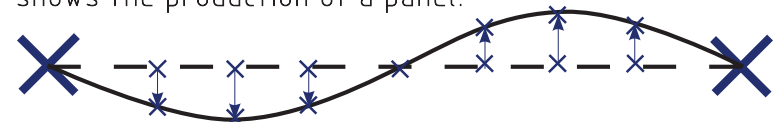


Figure 125 Principle of measuring nodal displacement

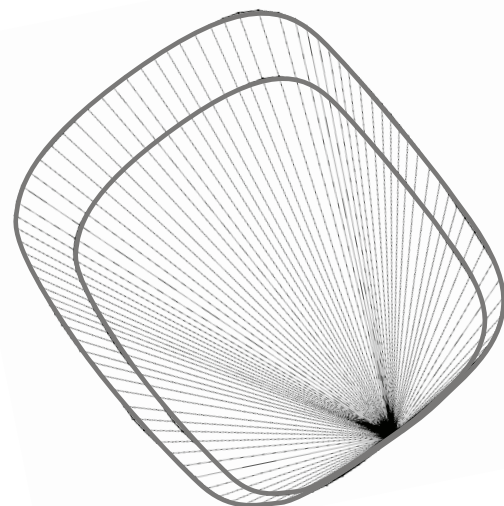


Figure 126 Exaggeration of the thermal expansion to indicate in what direction every bearing needs to allow for expansion

7.4.5 Technical Details

Detail 1 shows a typical cross section close at the openings at floor level. The bottom element does not contain glass and functions as a gateway. Every polygon is one separate element with FRP and foam (see black line in figure 127). The separate elements are joined by both gluing them and adding tension tubes with treaded ends and bolts that press the two element together. The tubes are necessary because glue is only capable of transferring shear forces. Since the overall shape of the building is not one in which

only pressure forces occur, tension will occur within the structure as well. Therefore, the tension tubes are necessary. The glue ensures an even distribution of force transfer across the area. The holes where the tension tubes are fastened are being closed by a foam cap. The fibers across the hole are protruding from the FRP. Once the cap is placed, the protruding fibers are used to laminate (see detail 1A) over the cap and create a seamless look. Neoprene rubbers are clenched between the two elements to create water barriers. These rubbers are added as an extra safety since the structural sealant should also be watertight.

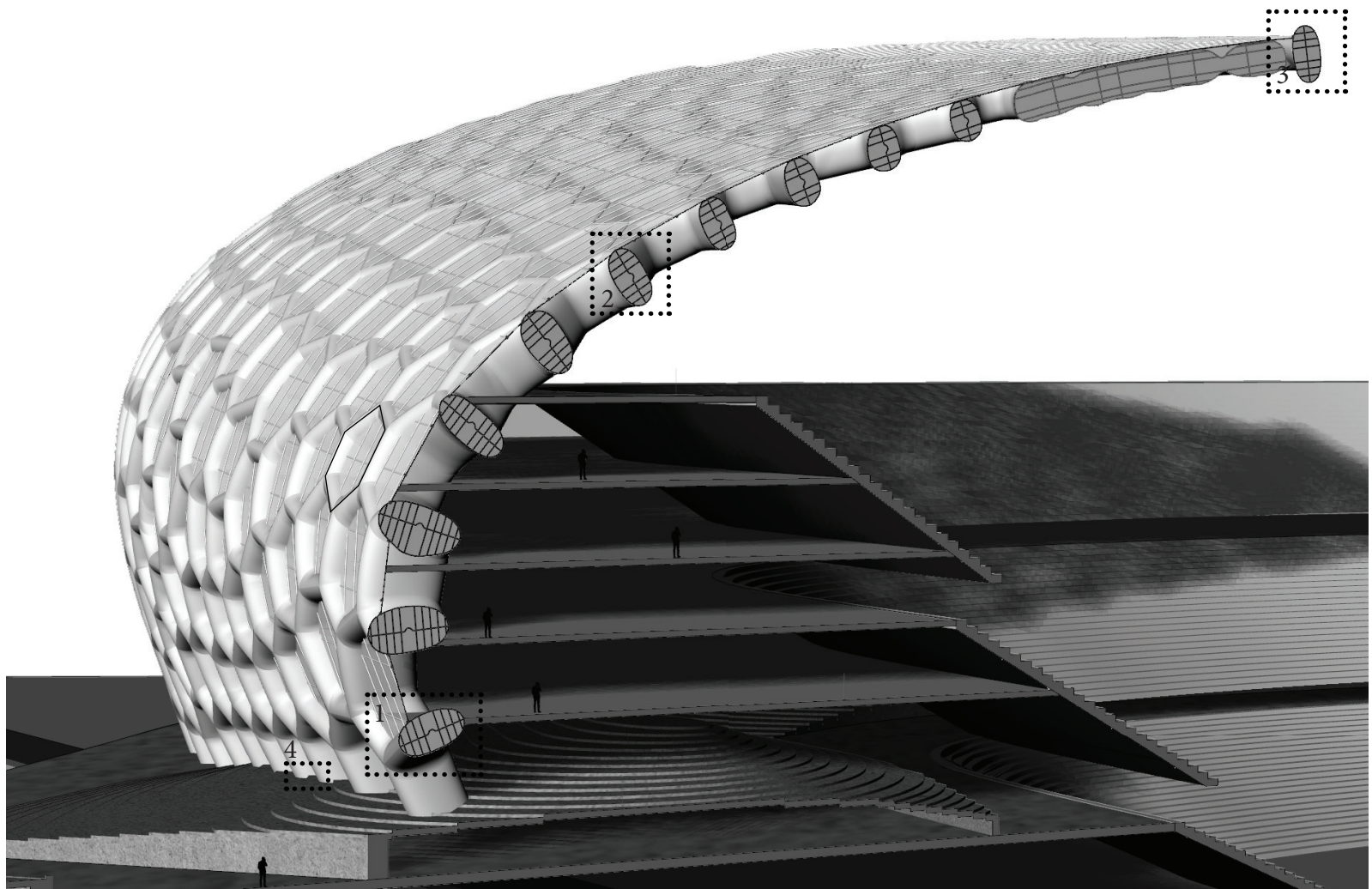


Figure 127 3D section of the canopy

Detail 1A shows how the glass part of the façade is attached to the FRP elements. Aluminium members with threaded holes are embedded in the polymer. The aluminium members are attached to the FRP element by attaching a screw through the mullion to the embedded member. In detail 1A it can also be seen that the glass has roughly a cm of extra space before it touches the mullion. Thus, it is more than capable of resisting the nodal displacement that occur in the structure. Silicone sealant is added between the mullion and the FRP element on the outside as a 1st water barrier and a layer of neoprene is clenged between the mullion and the FRP element to create a 2nd barrier on the inside.

Detail 2 has no neoprene in between the two FRP elements as the water tightness has already fully been solved in the windows that lie on top of the elements. For these windows a cover with click system is not possible as it protrudes from the glass pane and water will gather behind the mullion. Detail 2A shows the application of a different system that has successfully been applied in Het Scheepvaartmuseum in Amsterdam. An aluminium profile with an integrated socket joint is attached to aluminium members that are embedded within the FRP. Due to the socket joint the profile is freely tiltable. Ongoing glass rubbers are attached to the profile to form the 2nd water barrier. These rubbers are flexible enough to respond to the segmentation

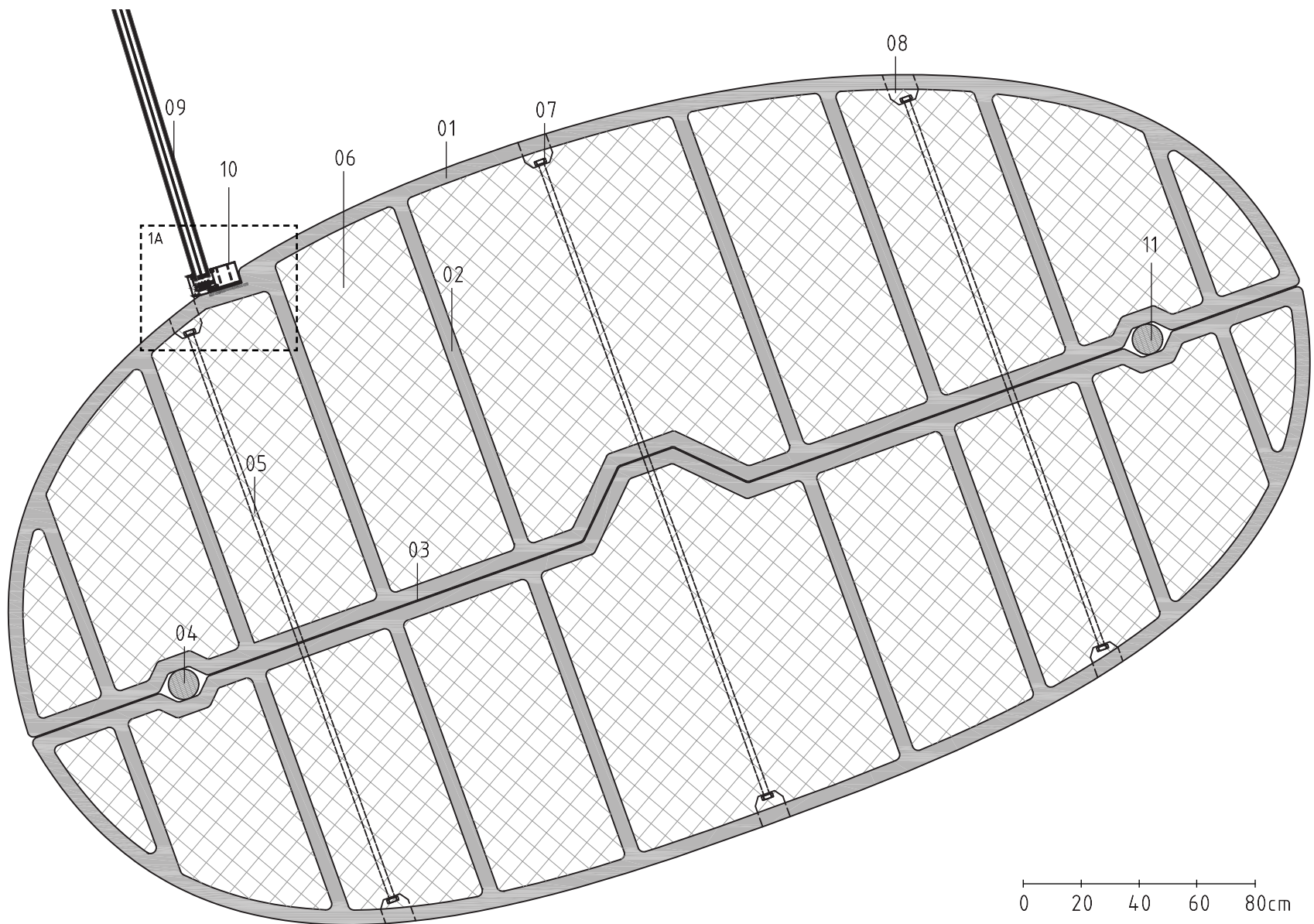
between the separate glass panes. The glass panes are kept into place by butterfly clamps in the rabbet of the triple glazing. In order to make this possible, the spacers of the glass pane are moved to the inside by a small amount. Between the two glass panes a silicon sealant is applied to form the 1st water barrier.

Detail 3 shows the edge of the roof opening of the canopy. The detailing is very similar to detail 1 and 1A. With the addition of an anchor for rope access. These anchors are applied all over the roof to allow for maintenance rope access. As can be seen in detail 3, the outermost 'beam' forms a ridge that ensures the rainwater will not fall of the canopy onto the tribune and the soccer field.

Detail 4 and detail 4A show a section of an expansion bearing. A stainless steel profile with two slotted holes and a layer of PTFE (teflon) is added to another stainless steel profile with two pins with threaded ends. These pins slide through the slotted holes, allowing for expansion of the entire structure. This expansion joint is connected to the FRP element by attaching it to the embedded steel members. The elements with the expansion bearing are then attached to the concrete by attaching it through the steel threaded anchors with bolts.

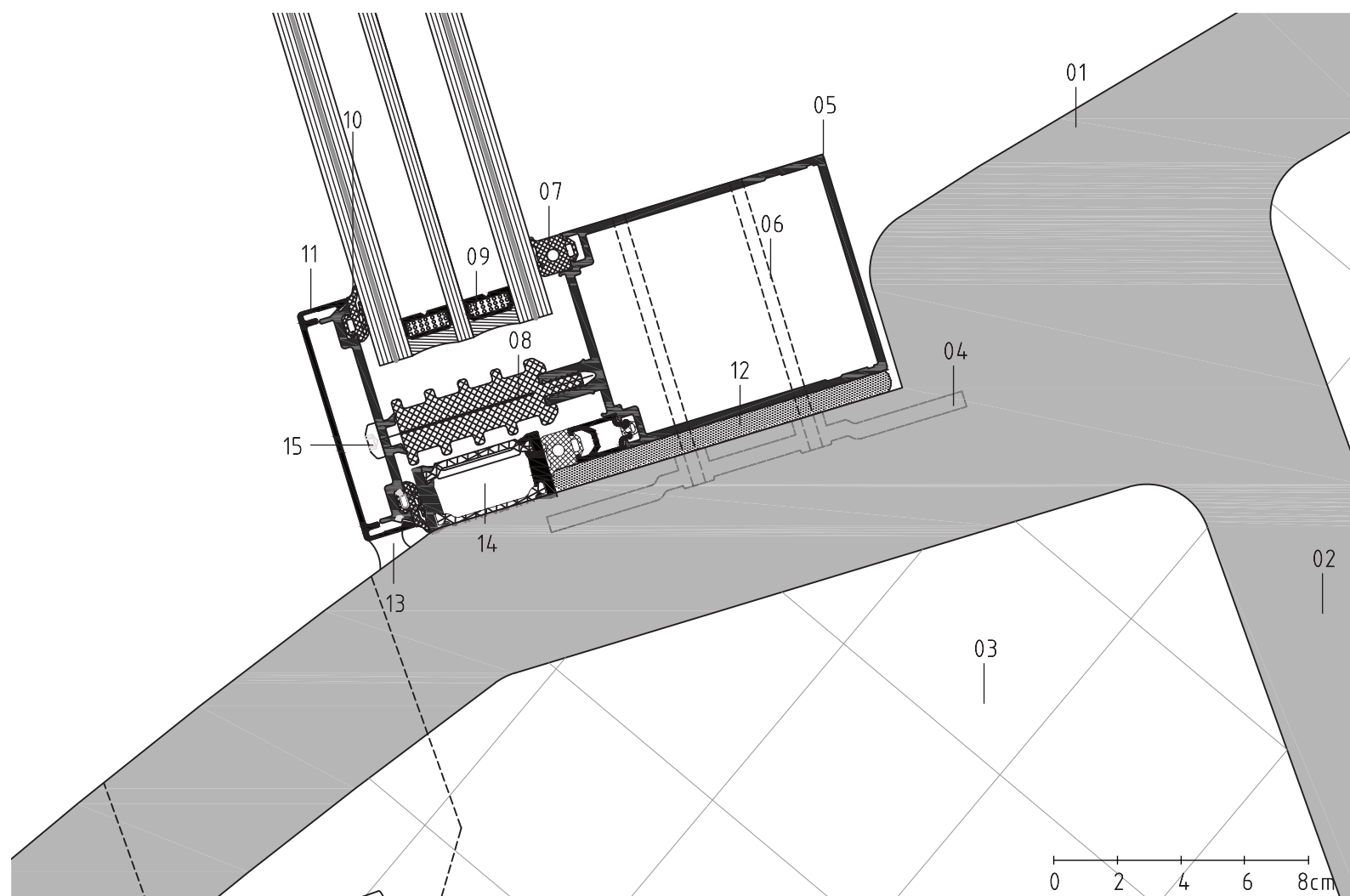
- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Web of the FRP component
- 03 Structural adhesive
- 04 Neoprene (1st water barrier)
- 05 Steel tension tube with threaded ends
- 06 Polyurethane foam (rigid, closed cell, 0.062)
- 07 Tightening bolt
- 08 Polyurethane foam cap (rigid, closed cell, 0.062)
- 09 Triple glazing 10-15-5-15-10
- 10 Aluminium mullion
- 11 Neoprene (2nd water barrier)

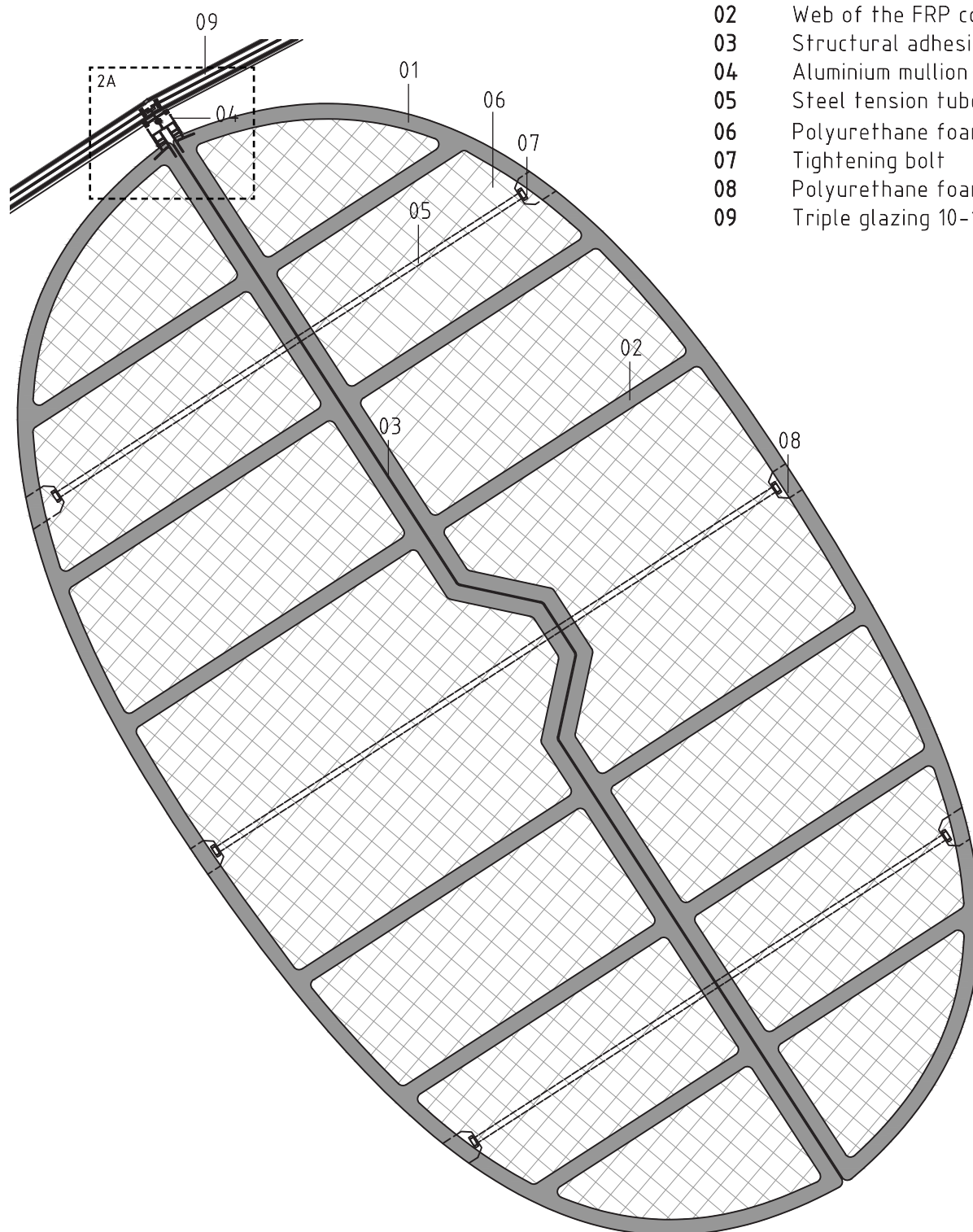
1



1A

- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Web of the FRP component
- 03 Polyurethane foam (rigid, closed cell, 0.062)
- 04 Aluminium member with threaded holes, embedded in polyester
- 05 Aluminium mullion
- 06 Screw
- 07 Rubber (2nd water barrier)
- 08 Thermal break
- 09 Spacer
- 10 Rubber (1st water barrier)
- 11 Cover with click system
- 12 Neoprene
- 13 Silicone sealant
- 14 Aluminium spacer with thermal break
- 15 Screw



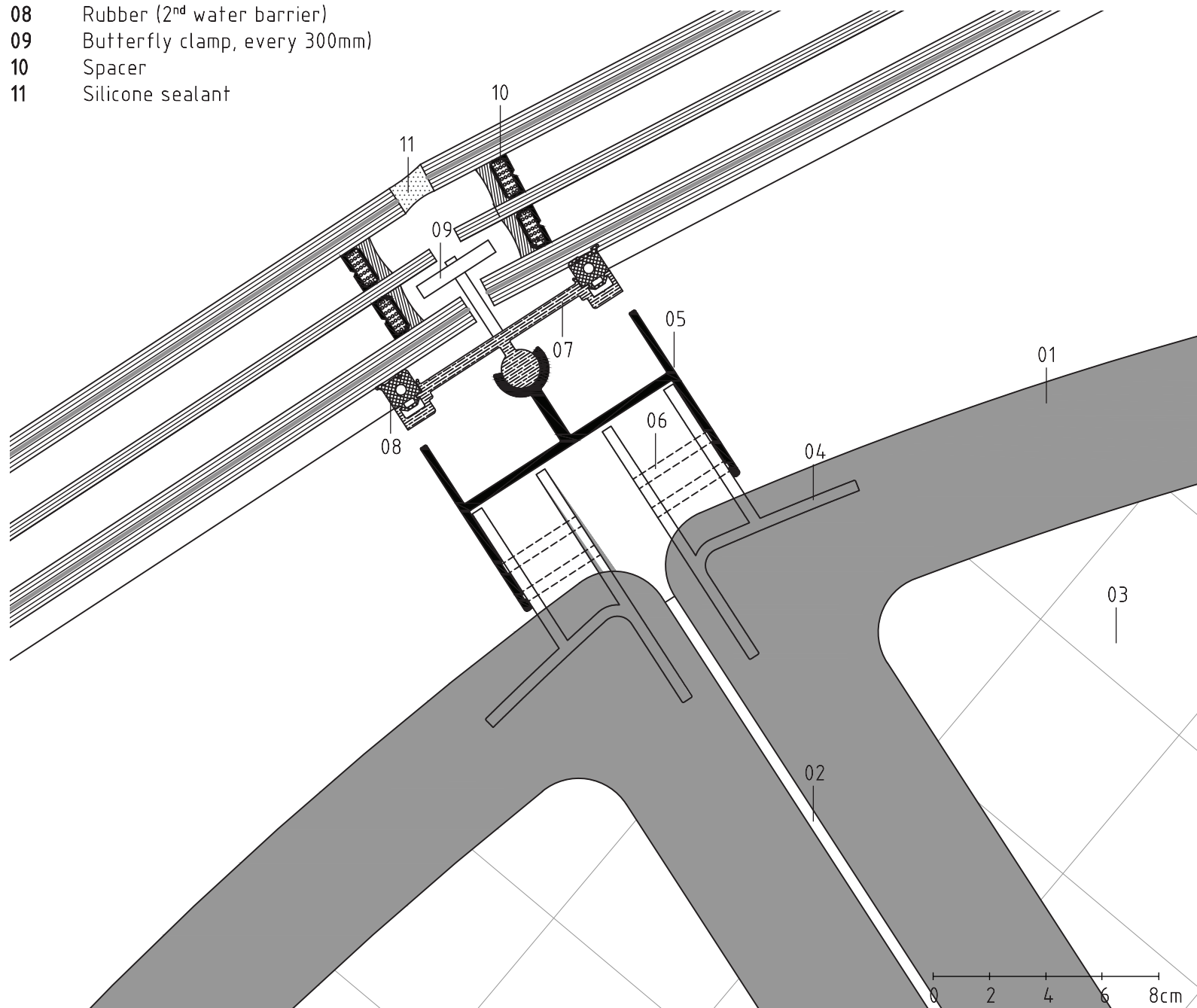


- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Web of the FRP component
- 03 Structural adhesive
- 04 Aluminium mullion
- 05 Steel tension tube with threaded ends
- 06 Polyurethane foam (rigid, closed cell, 0.062)
- 07 Tightening bolt
- 08 Polyurethane foam cap (rigid, closed cell, 0.062)
- 09 Triple glazing 10-15-5-15-10

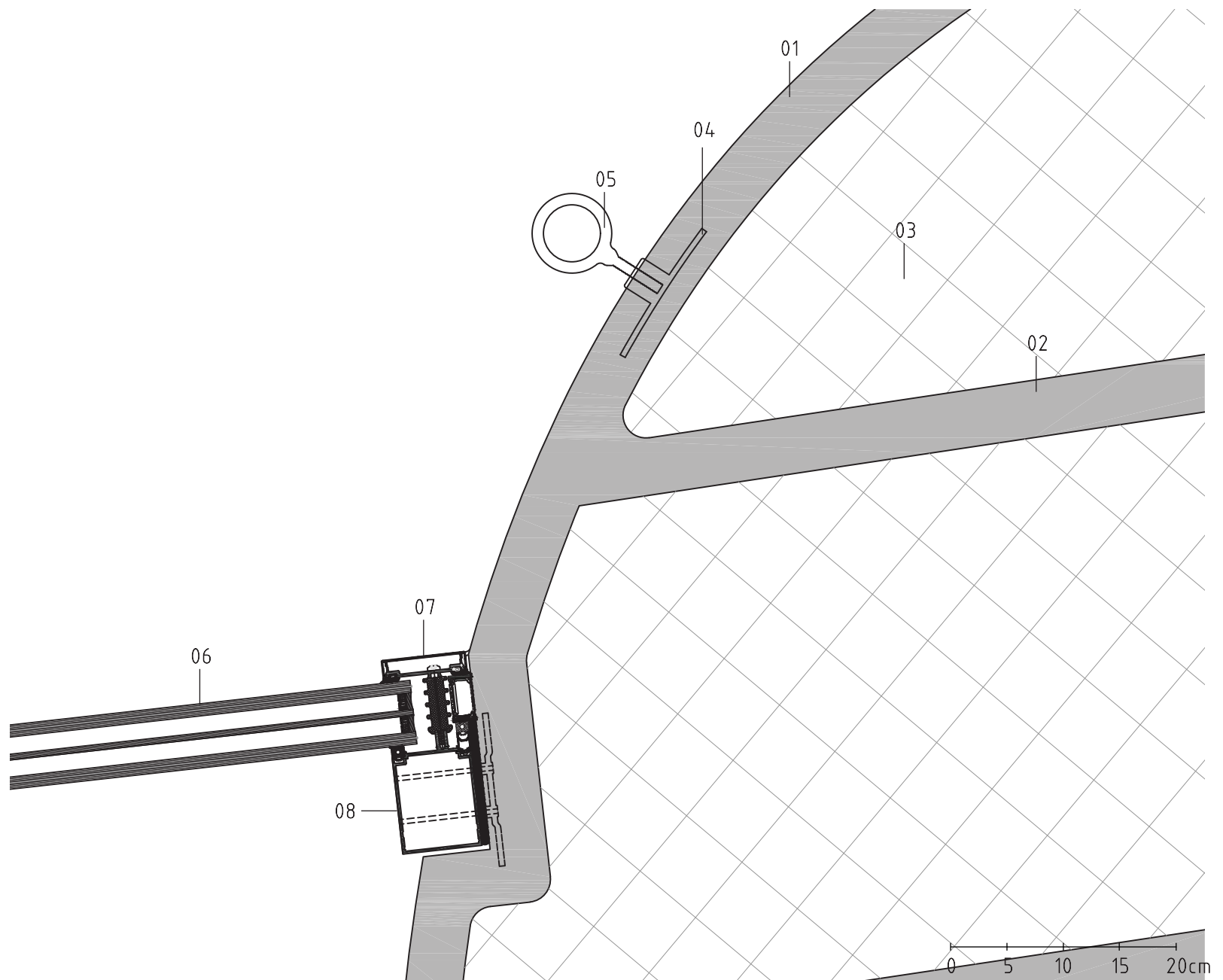
0 20 40 60 80cm

2A

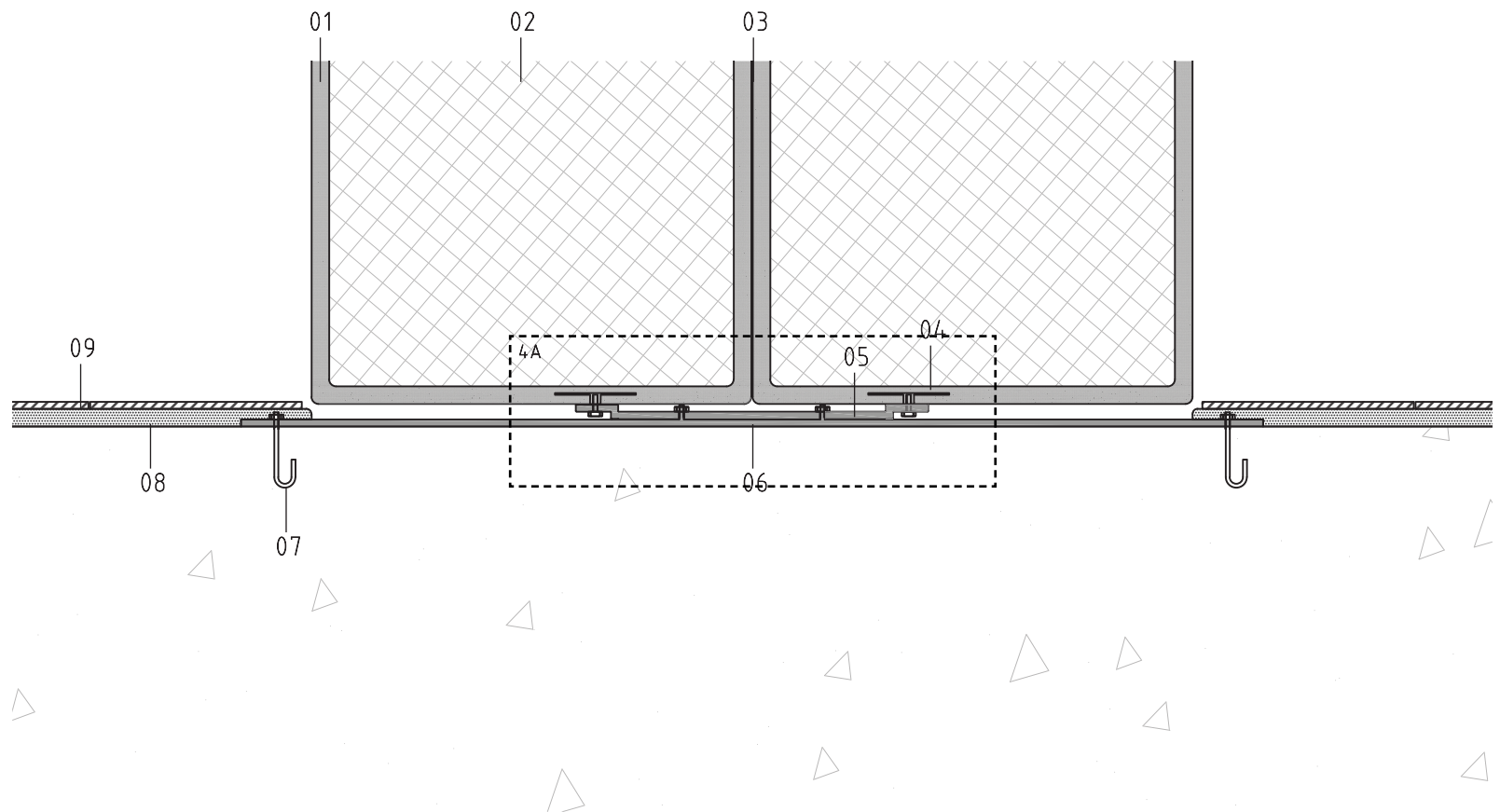
- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Structural adhesive
- 03 Polyurethane foam (rigid, closed cell, 0.062)
- 04 Aluminium member with threaded holes, embedded in polyester
- 05 Aluminium mullion with socket joint
- 06 Screws
- 07 Aluminium profile
- 08 Rubber (2nd water barrier)
- 09 Butterfly clamp, every 300mm)
- 10 Spacer
- 11 Silicone sealant



- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Web of the FRP component
- 03 Polyurethane foam (rigid, closed cell, 0.062)
- 04 Aluminium member with threaded holes, embedded in polyester
- 05 Aluminium bracket with threaded end for rope access
- 06 Triple glazing 10-15-5-15-10
- 07 Cover with click system
- 08 Aluminium mullion

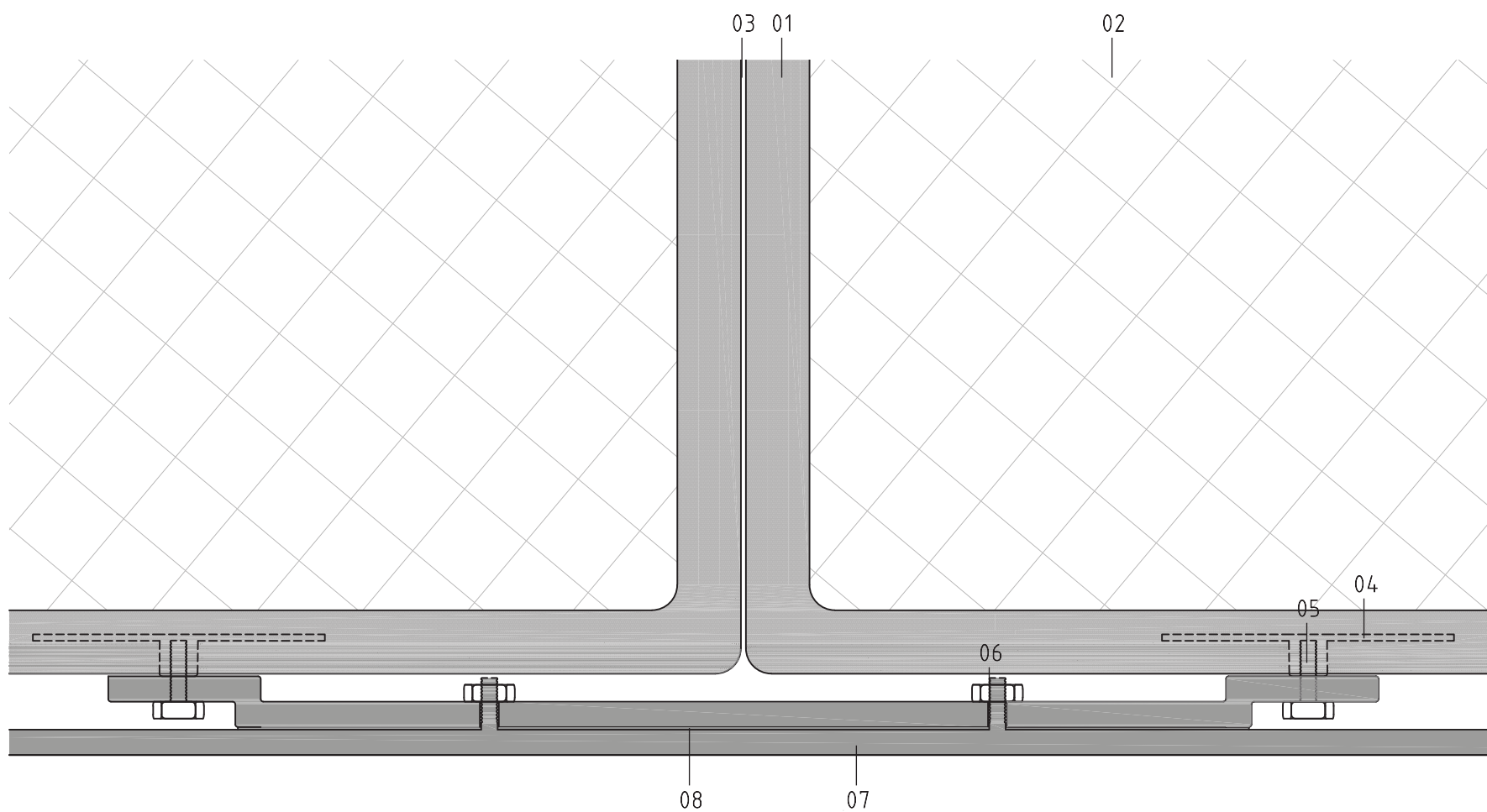


- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Polyurethane foam (rigid, closed cell, 0.062)
- 03 Structural adhesive
- 04 Stainless steel member with threaded holes, embedded in polyester
- 05 Stainless steel member with a layer of PTFE
- 06 Stainless steel member with two extruding pins
- 07 Steel reinforcement anchors with threads
- 08 Sand
- 09 Tiles



- 01 Polyester/E-glass fiber, woven fabric, 65wt% glass, biaxial lay-up, 50mm thickness
- 02 Polyurethane foam (rigid, closed cell, 0.062)
- 03 Structural adhesive
- 04 Stainless steel member with threaded holes, embedded in polyester
- 05 Bolt
- 06 Slotted holes
- 07 Stainless steel member with two extruding pins
- 08 Layer of PTFE

4A















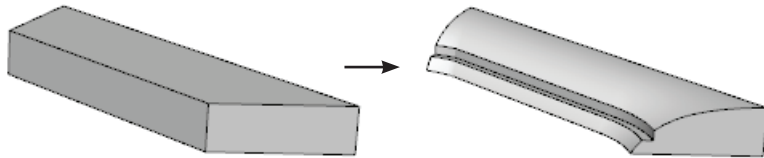




7.5 Production of a Panel

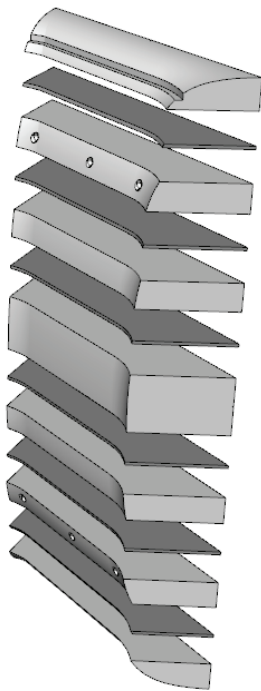
Step 1

All the foam blocks will be milled into the desired shape.



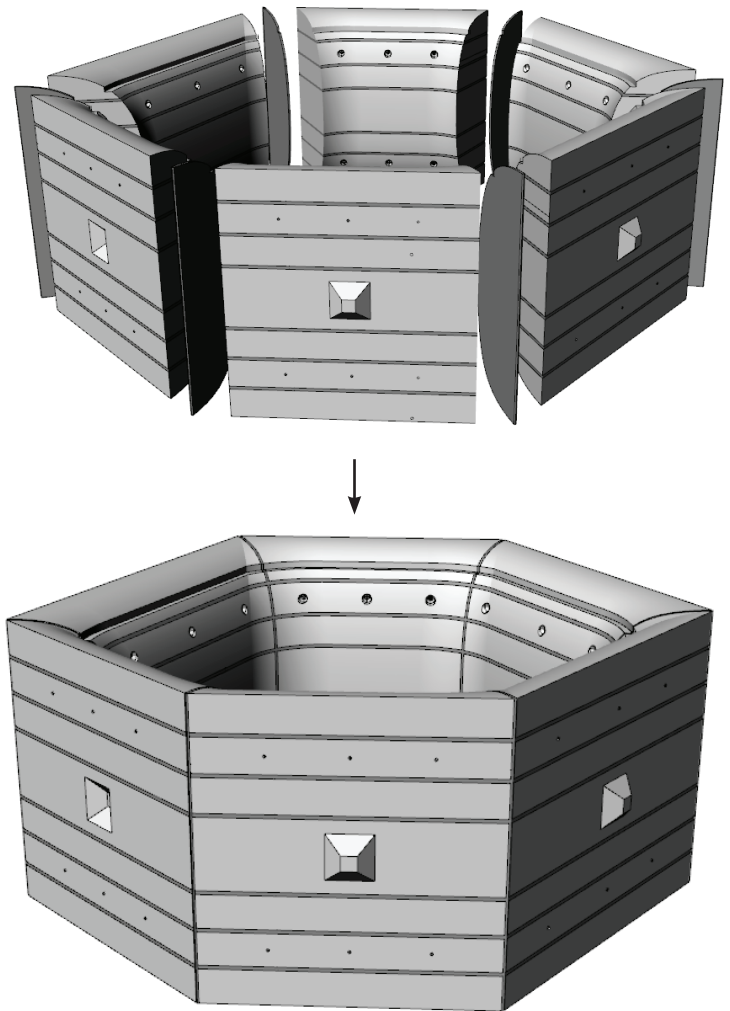
Step 2

The foam blocks of one 'member' are stacked together with layers of woven E-glass fabric in between.



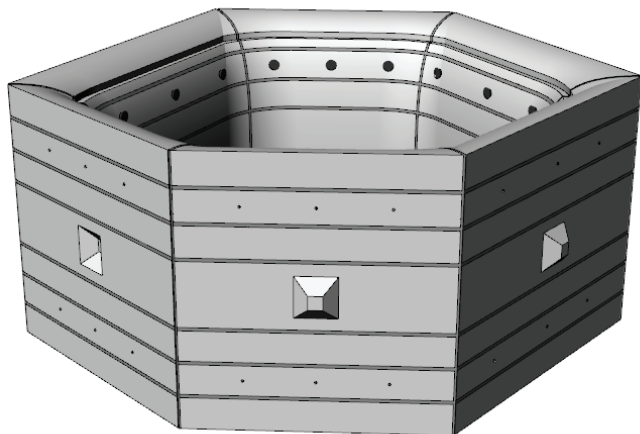
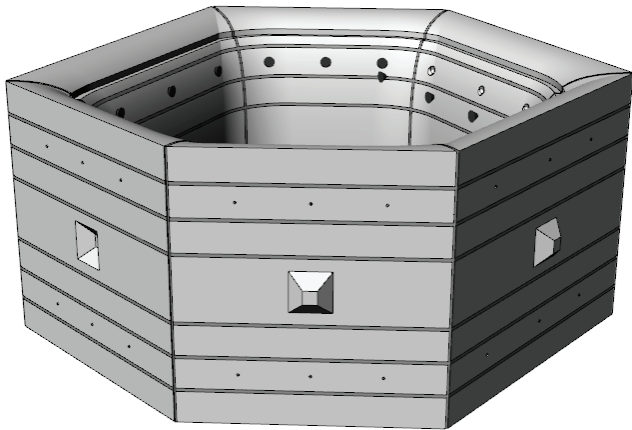
Step 3

All the members of one element are combined with layers of woven E-glass fabric in between.



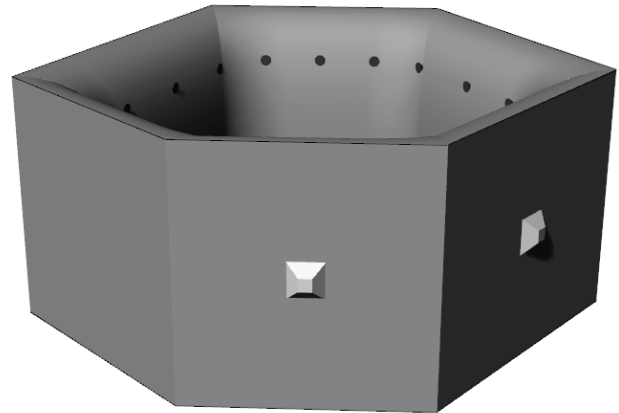
Step 4

Caps are placed in the openings to avoid resin getting into them during the vacuum injection.



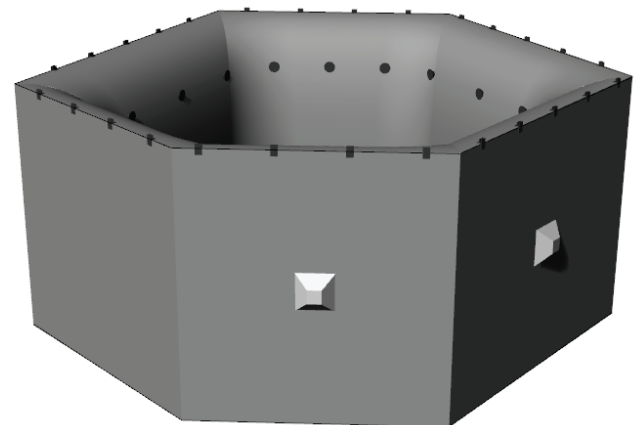
Step 5

First layers of fiber are added onto the element.



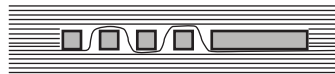
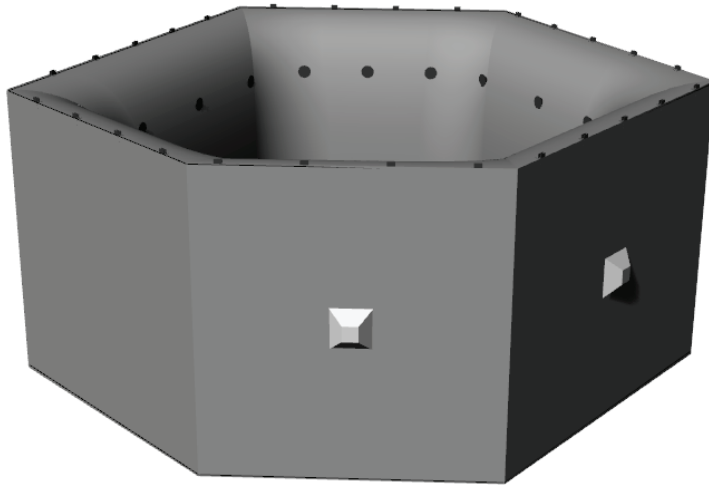
Step 6

Aluminium members are embedded into the upper layer of fibers. Fibers are pulled through the perforated parts of the aluminium.



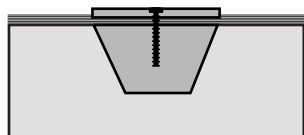
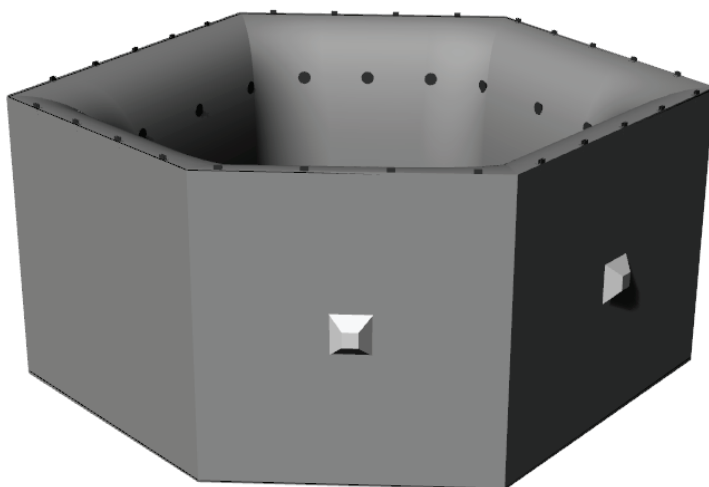
Step 7

Add final layers of woven fabric to the element.



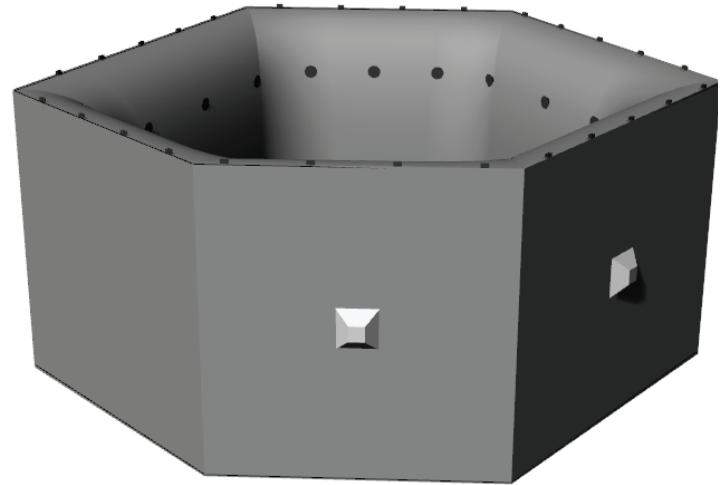
Step 8

Screw covers onto the opening covers and clench the fibers in between. To ensure they stay dry during the vacuum injection process.



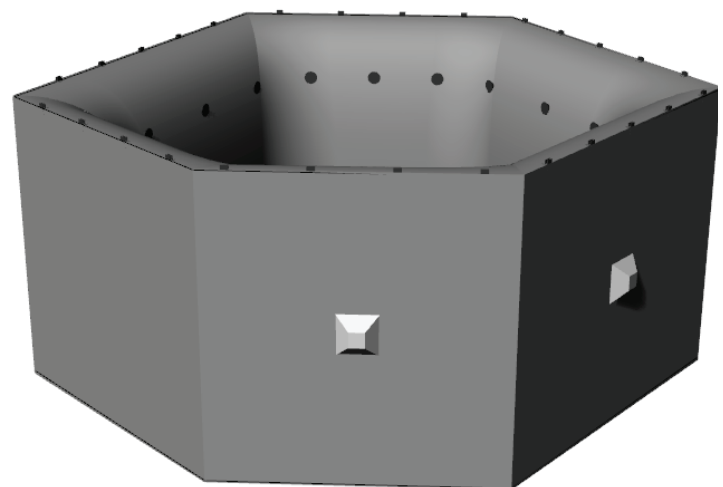
Step 9

Vacuum injection



Step 10

Once the resin is cured the resulting element has to be sanded and polished to get rid of any imperfections that were created during the vacuum injection process



8. CONCLUSIONS

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8.1 Conclusions

This research aimed to find an answer to the question: *'How to design a grid shell for the canopy of a stadium that uses performance based densification to enhance the performance of the structure?'*

In order to answer this question a literature study and a quantitative research method have been carried out.

A literature research was performed into steel, wood, FRP and their possible connection types for grid shells. For the assessment of materials only the structurally used materials were judged. Tool steels, for example, were left out of the assessment. It can be concluded that wood is the most sustainable solution. However, the Yield strength and Young's Modulus of woods are a lot lower in comparison with steel and FRP, making it the least suitable option for such a large span as needed for a stadium ($\pm 200\text{m}$). In addition to that, wood has the highest maintenance of all three options. Steels form a sound option for such a large span structure as they are strong, stiff and relatively cheap. The FRP types that are stronger and stiffer than steel are either pultruded profiles or contain the very expensive carbon fibres. FRP's are in most cases more expensive than steels, but they are also about 6 times less heavy and can be considered to be maintenance-free. Carbon fibers are very expensive and are therefore not considered to be an option. It can be concluded that both steels as GFRP would be suitable for the construction of the canopy of a stadium.

A literature research was performed into types of graded structures. There are two main categories for graded structures: irregular grid structures and regular grid structures. The literature research shows that regular grid structures are the least suitable for gradual densification. These structures have a side effect that when the target area is densified, other areas are effected as well. There are only two types of irregular grids: Voronoi and Delaunay. Delaunay outperforms the Voronoi only on the structural aspect. Voronoi has a lower connection complexity, requires less material, results in

a lower amount of waste for glass panes/panels and results in a look that is optically pleasing. In addition to that the aesthetic look that I am trying to achieve is a bone structure/biomimicry-like look. A Voronoi grid structure corresponds to the image that I am trying to achieve. Thus, the decision to use a Voronoi pattern for the canopy is also based on personal preference. To me, this outweighs the fact that Delaunay is the best option from a structural point of view.

Based on the choice for a Voronoi pattern, a final decision regarding the material could be made since the polygons in a Voronoi pattern need stiff corners in order to maintain their shape (unlike the triangles from a Delaunay pattern). Thus, every connection/node needs to be capable of handling bending moments. This means that all steel connection types for grid shells and all piece-small connection types for grid shells are no longer an option. The remaining wood type connections are only suitable for square grids with continuous members. Thus, only FRP remains as a suitable option. FRP also has the largest freedom of shape making it possible to create the desired bone-like look.

Three scripts that are capable of translating an image map to a graded Voronoi have been tested: A script that uses Lloyd's Algorithm and probability based culling, a script that uses a circle packing algorithm that is based on the spring force system and a Smart Circle Packing script that combines remeshing techniques with a circle packing algorithm based on mathematical theorems. The first two scripts that have been tested both pose a couple of critical problems. The Lloyd's algorithm script does not allow the user to set a minimum and maximum cell size; the result is highly dependent on the amount of points that have been inserted and on the amount of iterations of the algorithm; the algorithm is not good at handling hard gradients, gradients with multiple shades of grey on top of each other and it tends to ignore smaller spots in the image map. The Spring Force based algorithm (with HoopSnake) overcomes some of these problems. However, this script is very slow, still very dependent on the amount of points that are inserted and returns a result that

contains patches with cells that are oddly shaped. The Smart Circle Packing script solves all of these problems and is therefore the most suitable algorithm for the task.

A literature study into different types of multiple objective optimization algorithms, to generate the basic shape of a building, have been performed. The study shows that stochastic algorithms are most suitable for the task. They are fairly easy to apply and do not require an understanding of the mathematical relations between objectives. In other words, the user does not need any knowledge about how two objectives relate to one another. Stochastic algorithms are capable of finding near optimal solutions. These kinds of algorithms are also capable of producing more than one solution every time that the algorithm is run. Therefore, the algorithm is capable of finding solutions that are equally good without any priori interference of the user. Octopus is the only plug-in for grasshopper that is capable of performing a stochastic multiple objective optimization.

The results of different performance maps for a building can be combined into one image map by using a method that is comparable with the method that Photoshop uses to combine several layers. This can be done by calculating how far the colour values from two layers differ and then subtracting a certain percentage of this difference of the top layer. This percentage equals the transparency of the top layer.

The research has been applied to a Case Study: a new design for a soccer stadium De Nieuwe Kuip in Rotterdam. Two tests regarding the opening sizes of both regular and graded Voronoi patterns were performed. In the first test the mass of every solution was kept constant by changing the size of the section. With the second test the mass of every solution was kept constant by changing the density of the material. According to the first test, larger openings result in a smaller overall deflection under deadload. According to the second test, smaller variety in opening sizes perform better than large varieties in opening sizes. However, the variety in opening sizes is not only used to increase the structural performance, but is

also used for other objectives. For all these reasons, a range of openings from 5 to 12 was chosen. This results in a relatively small overall deflection compared to other solutions (according to test 1) while still outperforming its regular partner by 82cm's (according to test 2).

Both the traditional densification method as the graded Voronoi have been tested on this case study. It can be concluded that the traditionally densified grid performs better from a structural point of view. However, the images show that the traditionally densified structure is only a very coarse approximation of the underlying imagemap. It does not show gradual transitions, instead it shows very harsh differences with an aesthetic look that does not meet the requirements for the error detection rules as have been explained before: the structure shows long guidelines that abruptly stop at several points throughout the structure. Since the resulting structure is not a very good approximation of the underlying performance map, it will automatically perform worse on all these performances than a graded Voronoi structure – and in the case of this specific performance map leaving either too much or too little sunlight into the rooms adjacent to the façade (this differs for different places of the façade). This can cause rooms to overheat in summer, rooms to be too cold in winter, hinder by too much light and extra energy use because not enough light enters the room. Thus, the traditionally densified grid may perform better from a structural point of view, but it performs worse on all the objectives that were accounted for in the imagemap.

The last step to design a canopy of a stadium that uses performance based densification to enhance the performance of the structure is to optimize the size of the beams in order to reduce the amount of materials and the material cost. By using the size of the bending moment in every beam to assign a specific section to that beam the cost of the material was reduced from €325.5 million euros to €180 million euros. This is a reduction of 55% (!).

8.2 Future Research

There are several notes that need to be made regarding the results. The generation of graded structures within grasshopper turned out to be a relatively new and unexplored area. All the scripts that were found have been created by the same person: Daniel Piker. However, he created the algorithms based on the findings of a literature research into meshing methods. Graded structures are a relatively unexplored field within architecture and they could be applied in many different ways besides basing them on a performance map. For example, the smart circle packing algorithm could be used in combination with a structural calculation to add more beams where needed to boost the overall structural performance in an iterative way. More research in this area is needed to further explore the possibilities of applying graded structures within architecture.

In addition to that, the generation of a proper performance map that, for example, accurately represents the desired size of window openings based on complex interior climate calculations was not within the scope of this research. What exactly is the added value of using graded structures for the reduction of energy used for lighting and heating is another topic for further research.

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Marler, R.T. and Arora, J.S. (2004) 'Survey of multi-objective optimization methods for engineering', *Structural and Multidisciplinary Optimization*, 26(6), pp. 369–395. doi: 10.1007/s00158-003-0368-6.

Michalewicz, Z. and Fogel, D.B. (2004) *How to solve it: Modern heuristics*. 2nd edn. Berlin: Springer-Verlag Berlin and Heidelberg GmbH & Co. K.

Mundhekar, A. (2011) *Multiobjective optimization and trade offs using pareto optimality*. Available at: <http://www.slideshare.net/AmoghMundhekar/multiobjective-optimization-and-trade-offs-using-pareto-optimality> (Accessed: 7 February 2017).

Quora. (2016). *In what ways is Randomness important to Deep Learning*. [online] Available at: <https://www.quora.com/In-what-ways-is-Randomness-important-to-Deep-Learning> [Accessed 28 Sep. 2017].

Vierlinger, R. (2013). *Multi Objective Design Interface*. [online] Researchgate. Available at: <https://www.researchgate.net/publication/283073414> [Accessed 28 Sep. 2017].

6.2 Grasshopper Plugins

Michalewicz, Z. and Fogel, D.B. (2004) *How to solve it: Modern heuristics*. 2nd edn. Berlin: Springer-Verlag Berlin and Heidelberg GmbH & Co. K.

7 DESIGN

7.1 Overall Shape

AIRBUS. (n.d.). *Stadium construction work in Brazil viewed by Pléiades*. [online] Available at: <http://www.intelligence-airbusds.com/en/4752-stadium-construction-work-in-brazil-viewed-by-pleiades> [Accessed 28 Sep. 2017].

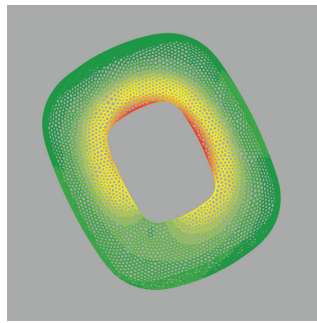
Cité de l'espace. (2016). *The Euro 2016 From Space*. [online] Available at: <http://en.cite-espace.com/space-news/the-euro-2016-stadiums-from-space/> [Accessed 28 Sep. 2017].

7.3 Structural Testing

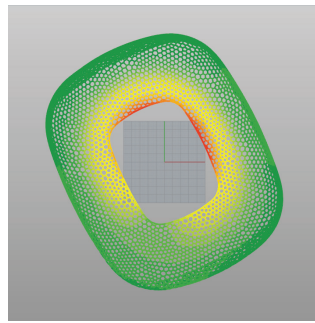
Nijssen, R. (2012). *Dictaat draagconstructies I* [BK1043]. 1st ed. Delft: TU Delft.

APPENDIXES

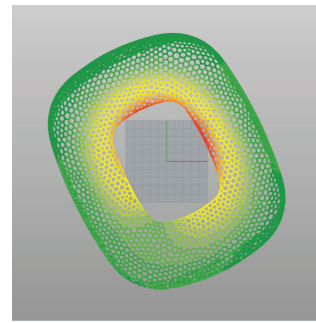
Test 1



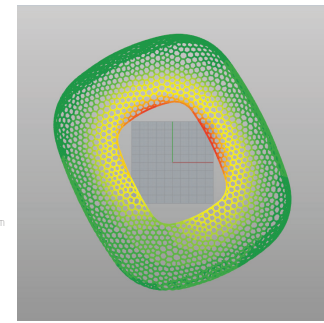
Range: 5 to 5
Max. Displacement: 5.81m
Utilization: -36% to 36.9%
Mass: $1.55e^8$
Nodal Displacement: 1.15 cm



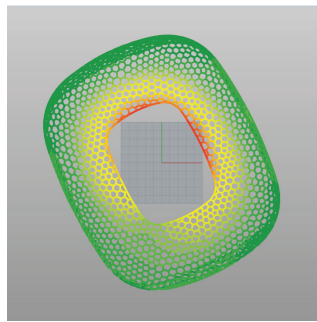
Range: 6 to 6
Max. Displacement: 5.62m
Utilization: -31.8% to 34.2%
Mass: $1.56e^8$
Nodal Displacement: 1.27 cm



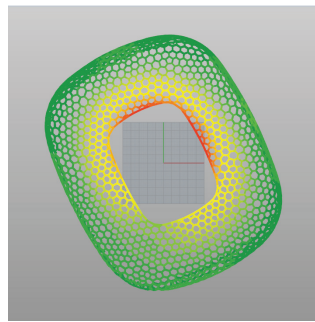
Range: 7 to 7
Max. Displacement: 5.43m
Utilization: -32.2% to 34.7%
Mass: $1.54e^8$
Nodal Displacement: 1.31 cm



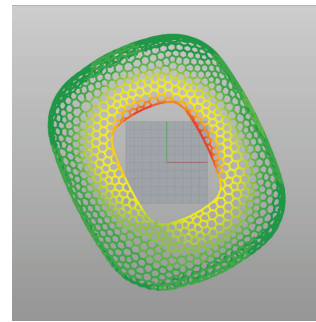
Range: 8 to 8
Max. Displacement: 5.16m
Utilization: -28.3% to 28.3%
Mass: $1.56e^8$
Nodal Displacement: 1.37 cm



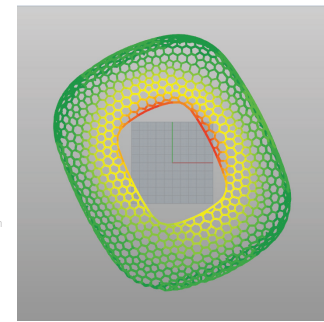
Range: 9 to 9
Max. Displacement: 4.88m
Utilization: -27.7% to 29.7%
Mass: $1.55e^8$
Nodal Displacement: 1.66 cm



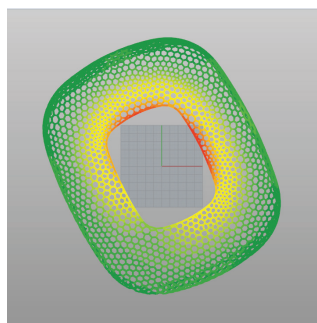
Range: 10 to 10
Max. Displacement: 4.90m
Utilization: -29.3% to 31.0%
Mass: $1.55e^8$
Nodal Displacement: 2.03 cm



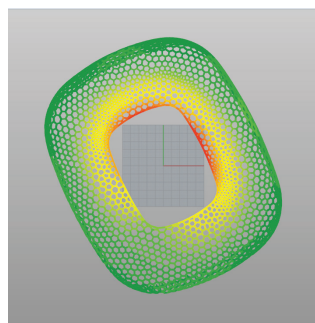
Range: 11 to 11
Max. Displacement: 4.78m
Utilization: -24.9% to 27.8%
Mass: $1.55e^8$
Nodal Displacement: 2.16 cm



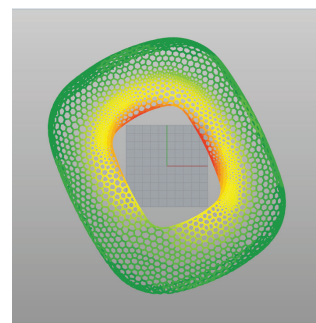
Range: 12 to 12
Max. Displacement: 4.61m
Utilization: -27.7% to 26.9%
Mass: $1.55e^8$
Nodal Displacement: 2.38 cm



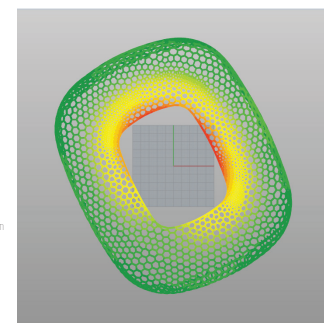
Range: 4 to 9
Max. Displacement: 5.39m
Utilization: -27% to 31.6%
Mass: $1.54e^8$
Nodal Displacement: 2.05 cm



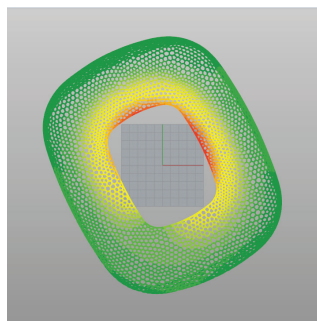
Range: 3 to 9
Max. Displacement: 5.68m
Utilization: -27.4% to 29.7%
Mass: $1.55e^8$
Nodal Displacement: 2.20 cm



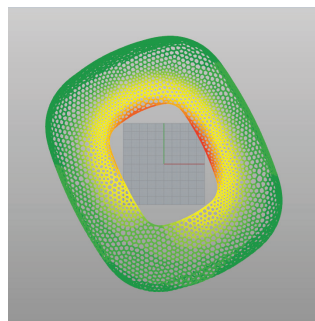
Range: 2 to 9
Max. Displacement: 6.32m
Utilization: -28.2% to 32.9%
Mass: $1.55e^8$
Nodal Displacement: 2.45cm



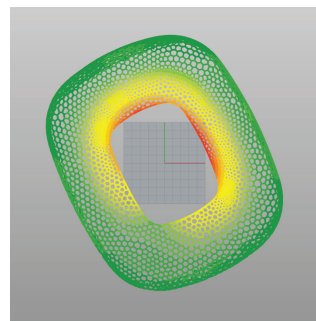
Range: 2 to 10
Max. Displacement: 6.39m
Utilization: -29.6% to 31.6%
Mass: $1.55e^8$
Nodal Displacement: 2.39 cm



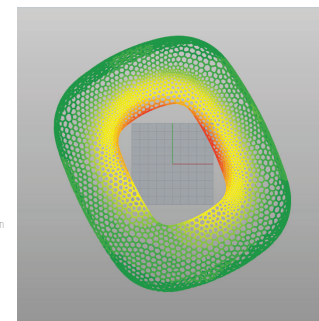
Range: 4 to 6
Max. Displacement: 5.95m
Utilization: -31.4% to 36.1%
Mass: $1.54e^8$
Nodal Displacement: 0.99 cm



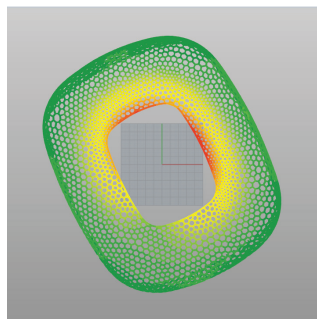
Range: 3 to 7
Max. Displacement: 5.98m
Utilization: -29.3% to 34.0%
Mass: $1.55e^8$
Nodal Displacement: 1.29 cm



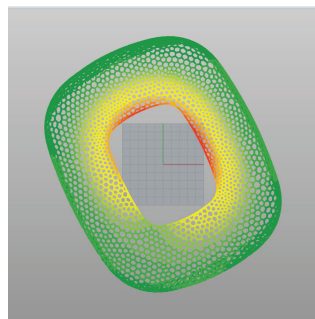
Range: 2 to 8
Max. Displacement: 6.53m
Utilization: -28.1% to 30.6%
Mass: $1.55e^8$
Nodal Displacement: 1.81cm



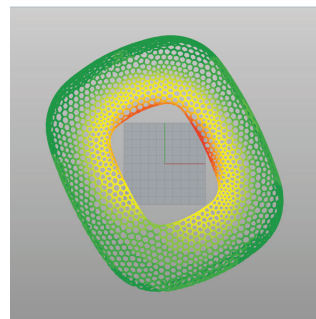
Range: 3 to 8
Max. Displacement: 6.23m
Utilization: -27.5% to 29.6%
Mass: $1.56e^8$
Nodal Displacement: 1.75 cm



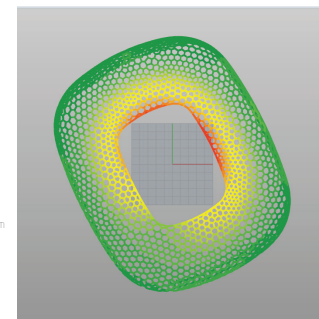
Range: 4 to 8
Max. Displacement: 5.83m
Utilization: -25.1% to 28.1%
Mass: $1.56e^8$
Nodal Displacement: 1.70 cm



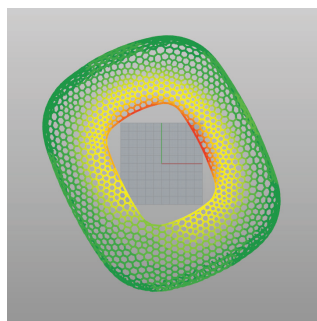
Range: 5 to 8
Max. Displacement: 5.61m
Utilization: -28.8% to 33.3%
Mass: $1.54e^8$
Nodal Displacement: 1.66 cm



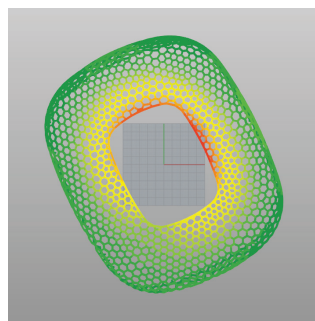
Range: 6 to 9
Max. Displacement: 5.34m
Utilization: -28.7% to 31.8%
Mass: $1.55e^8$
Nodal Displacement: 2.01 cm



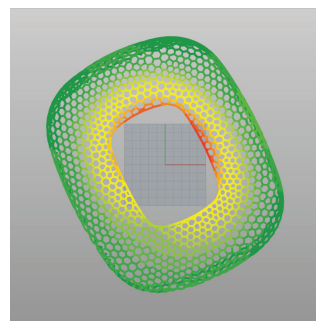
Range: 5 to 9
Max. Displacement: 5.34m
Utilization: -28.7% to 31.8%
Mass: $1.55e^8$
Nodal Displacement: 2.02 cm



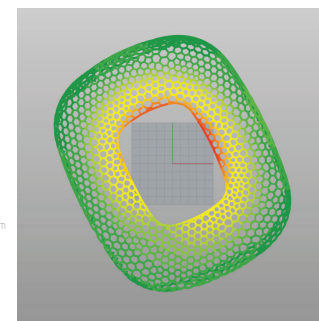
Range: 7 to 10
Max. Displacement: 5.27m
Utilization: -25.9% to 26.9%
Mass: $1.55e^8$
Nodal Displacement: 1.99 cm



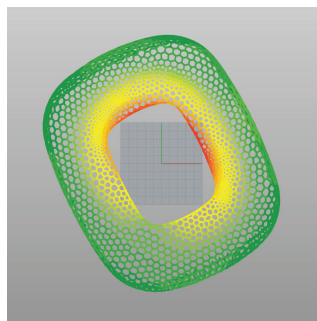
Range: 8 to 11
Max. Displacement: 4.93m
Utilization: -31.1% to 31.9%
Mass: $1.55e^8$
Nodal Displacement: 2.37 cm



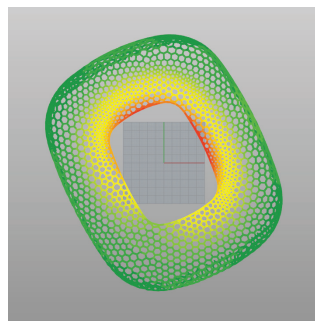
Range: 7 to 11
Max. Displacement: 4.94m
Utilization: -32.6% to 34.7%
Mass: $1.55e^8$
Nodal Displacement: 2.44cm



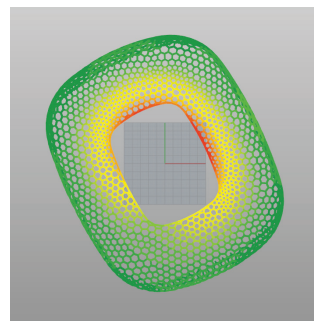
Range: 6 to 11
Max. Displacement: 4.96m
Utilization: -29.4% to 30.3%
Mass: $1.55e^8$
Nodal Displacement: 2.46 cm



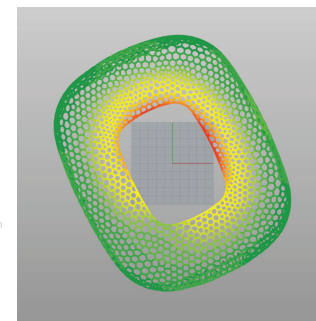
Range: 3 to 10
 Max. Displacement: 6.17m
 Utilization: -27.5% to 29.1%
 Mass: 1.55e⁸
 Nodal Displacement: 2.28 cm



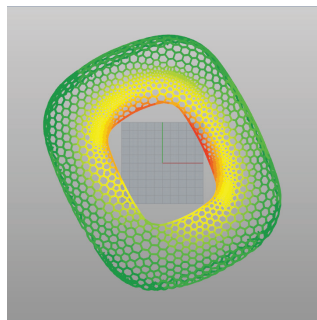
Range: 4 to 10
 Max. Displacement: 5.77m
 Utilization: -26.2% to 28.4%
 Mass: 1.55e⁸
 Nodal Displacement: 2.13 cm



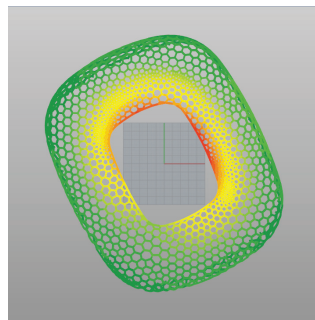
Range: 5 to 10
 Max. Displacement: 5.51m
 Utilization: -26.3% to 27.6%
 Mass: 1.55e⁸
 Nodal Displacement: 1.97 cm



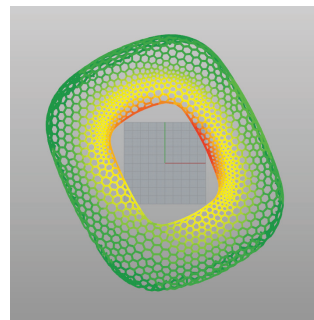
Range: 6 to 10
 Max. Displacement: 5.32m
 Utilization: -26.3% to 27.3%
 Mass: 1.55e⁸
 Nodal Displacement: 1.96 cm



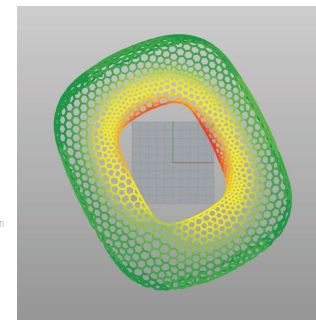
Range: 2 to 12
 Max. Displacement: 6.43m
 Utilization: -31.4% to 33.4%
 Mass: 1.55e⁸
 Nodal Displacement: 2.62 cm



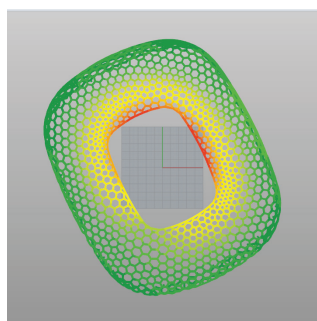
Range: 3 to 12
 Max. Displacement: 6.15m
 Utilization: -29.5% to 31.4%
 Mass: 1.55e⁸
 Nodal Displacement: 2.49 cm



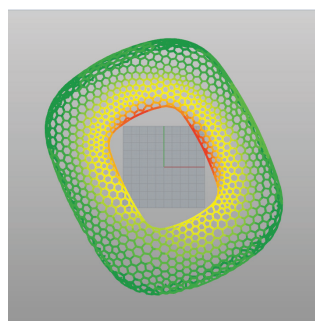
Range: 4 to 12
 Max. Displacement: 5.77m
 Utilization: -27.2% to 28.9%
 Mass: 1.55e⁸
 Nodal Displacement: 2.33cm



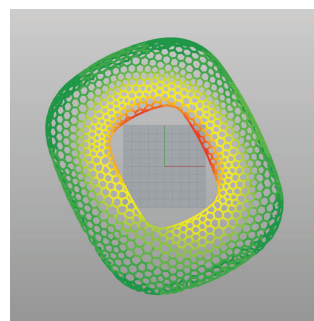
Range: 5 to 12
 Max. Displacement: 5.53m
 Utilization: -27.0% to 31.0%
 Mass: 1.55e⁸
 Nodal Displacement: 2.13 cm



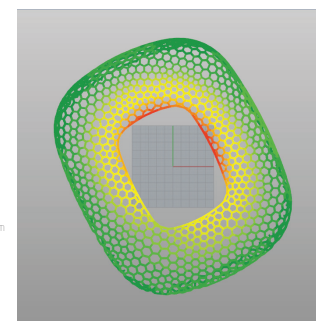
Range: 6 to 12
 Max. Displacement: 5.22m
 Utilization: -25.6% to 29.4%
 Mass: 1.55e⁸
 Nodal Displacement: 2.58 cm



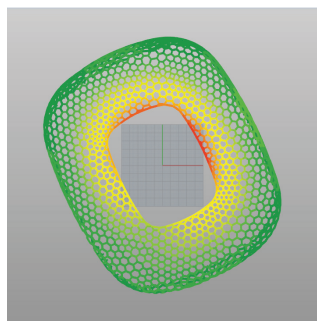
Range: 7 to 12
 Max. Displacement: 5.08m
 Utilization: -26.2% to 26.8%
 Mass: 1.55e⁸
 Nodal Displacement: 2.78 cm



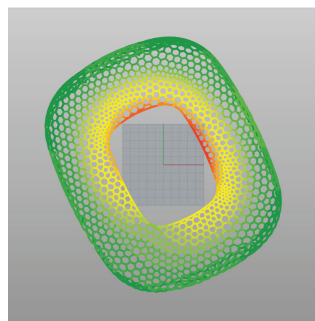
Range: 8 to 12
 Max. Displacement: 4.92m
 Utilization: -25.1% to 25.6%
 Mass: 1.55e⁸
 Nodal Displacement: 2.70 cm



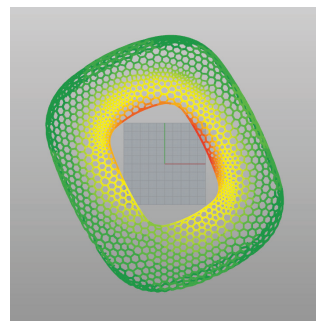
Range: 9 to 12
 Max. Displacement: 4.82m
 Utilization: -27.9% to 29.4%
 Mass: 1.55e⁸
 Nodal Displacement: 3.01 cm



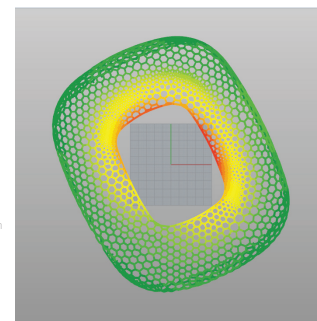
Range: 5 to 11
Max. Displacement: 5.11m
Utilization: -29.8% to 31.4%
Mass: $1.56e^8$
Nodal Displacement: 2.21 cm



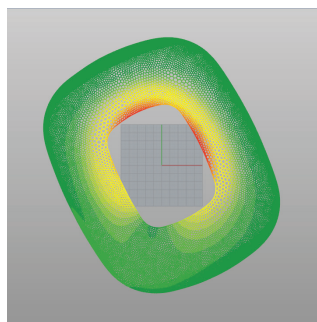
Range: 4 to 11
Max. Displacement: 5.40m
Utilization: -28.4% to 31.0%
Mass: $1.55e^8$
Nodal Displacement: 2.26 cm



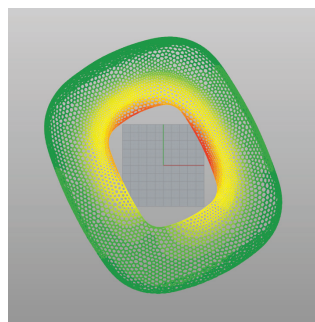
Range: 3 to 11
Max. Displacement: 5.67m
Utilization: -28.1% to 30.9%
Mass: $1.55e^8$
Nodal Displacement: 2.32 cm



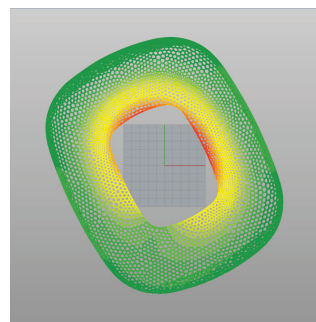
Range: 2 to 11
Max. Displacement: 6.38m
Utilization: -28.1% to 33.5%
Mass: $1.56e^8$
Nodal Displacement: 2.57 cm



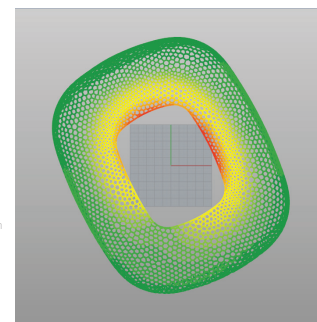
Range: 2 to 5
Max. Displacement: 7.66m
Utilization: -53.8% to 55.8%
Mass: $1.54e^8$
Nodal Displacement: 1.23 cm



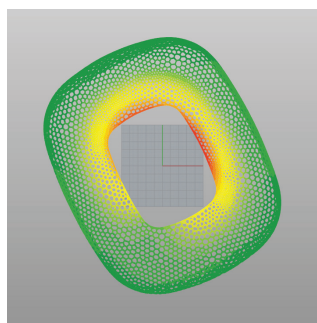
Range: 2 to 6
Max. Displacement: 6.81m
Utilization: -33.2% to 35.3%
Mass: $1.54e^8$
Nodal Displacement: 1.22 cm



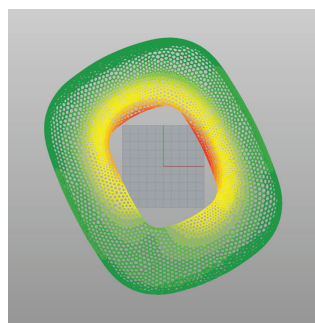
Range: 3 to 6
Max. Displacement: 6.36m
Utilization: -29.6% to 33.9%
Mass: $1.56e^8$
Nodal Displacement: 1.23cm



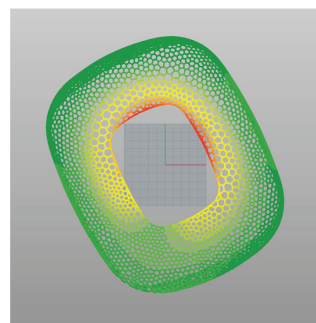
Range: 4 to 7
Max. Displacement: 5.99m
Utilization: -30.5% to 29.7%
Mass: $1.54e^8$
Nodal Displacement: 1.42 cm



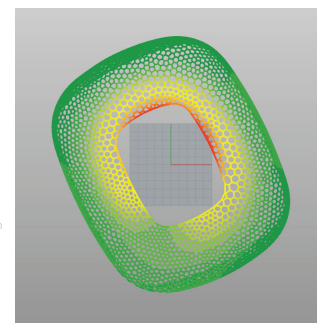
Range: 2 to 7
Max. Displacement: 6.44m
Utilization: -30.7% to 32.0%
Mass: $1.55e^8$
Nodal Displacement: 1.40 cm



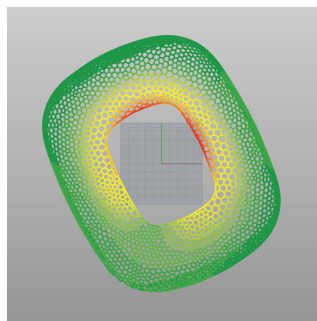
Range: 5 to 2
Max. Displacement: 6.81m
Utilization: -33.2% to 35.3%
Mass: $1.54e^8$
Nodal Displacement: 1.22 cm



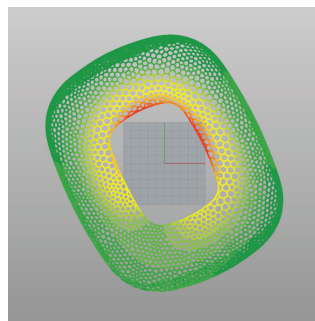
Range: 12 to 5
Max. Displacement: 5.29m
Utilization: -32.7% to 36.1%
Mass: $1.55e^8$
Nodal Displacement: 2.01cm



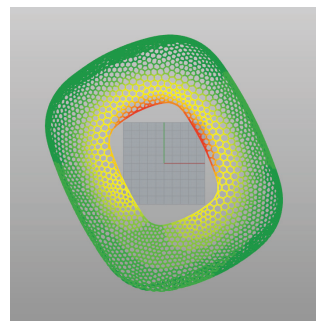
Range: 11 to 5
Max. Displacement: 5.30m
Utilization: -32.5% to 36.2%
Mass: $1.55e^8$
Nodal Displacement: 2.02 cm



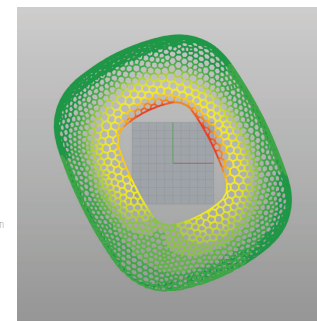
Range: 10 to 5
 Max. Displacement: 5.30m
 Utilization: -32.6% to 36.4%
 Mass: 1.56e⁸
 Nodal Displacement: 1.95 cm



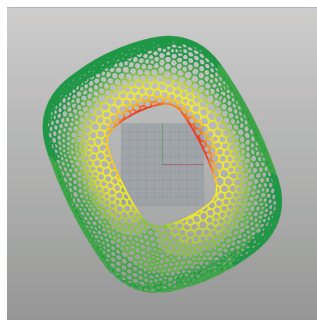
Range: 9 to 5
 Max. Displacement: 5.29m
 Utilization: -33.6% to 36.2%
 Mass: 1.56e⁸
 Nodal Displacement: 1.98 cm



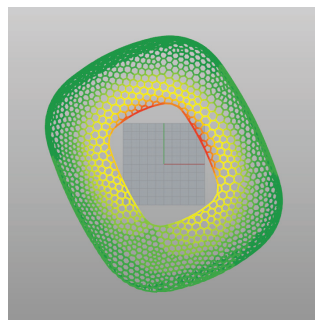
Range: 8 to 5
 Max. Displacement: 5.30m
 Utilization: -31.6% to 32.4%
 Mass: 1.56e⁸
 Nodal Displacement: 2.00 cm



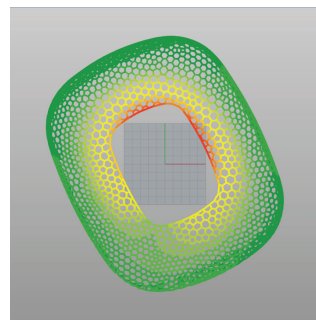
Range: 9 to 6
 Max. Displacement: 4.86m
 Utilization: -29.0% to 31.8%
 Mass: 1.55e⁸
 Nodal Displacement: 2.20 cm



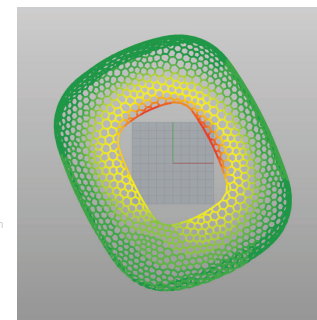
Range: 10 to 6
 Max. Displacement: 4.86m
 Utilization: -29.0% to 31.6%
 Mass: 1.55e⁸
 Nodal Displacement: 2.16 cm



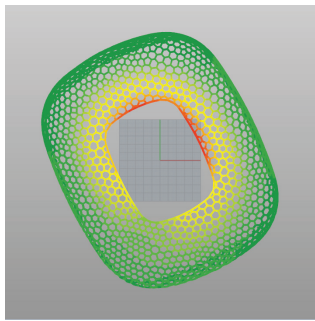
Range: 11 to 6
 Max. Displacement: 4.85m
 Utilization: -29.1% to 31.6%
 Mass: 1.55e⁸
 Nodal Displacement: 2.03 cm



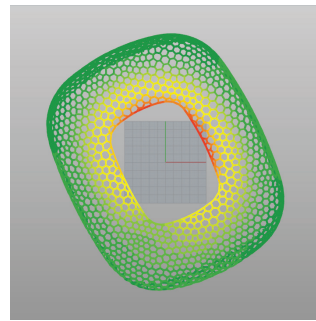
Range: 12 to 6
 Max. Displacement: 4.81m
 Utilization: -29.2% to 32.0%
 Mass: 1.56e⁸
 Nodal Displacement: 2.07cm



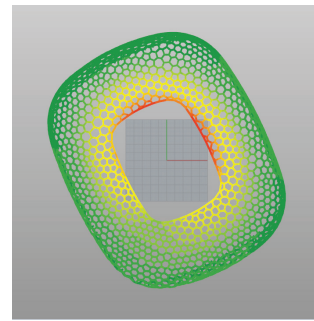
Range: 12 to 7
 Max. Displacement: 4.79m
 Utilization: -27.5% to 28.5%
 Mass: 1.55e⁸
 Nodal Displacement: 2.17 cm



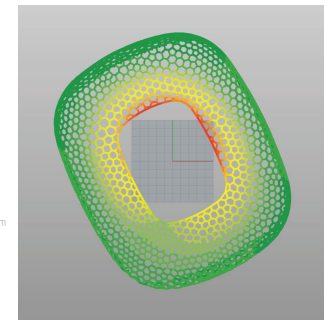
Range: 11 to 7
 Max. Displacement: 4.80m
 Utilization: -27.6% to 28.6%
 Mass: 1.55e⁸
 Nodal Displacement: 2.60 cm



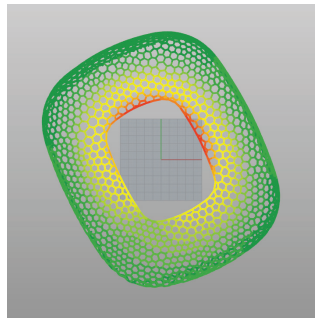
Range: 10 to 7
 Max. Displacement: 4.80m
 Utilization: -27.9% to 28.4%
 Mass: 1.55e⁸
 Nodal Displacement: 2.48 cm



Range: 11 to 8
 Max. Displacement: 4.70m
 Utilization: -35.4% to 34.8%
 Mass: 1.55e⁸
 Nodal Displacement: 2.79cm

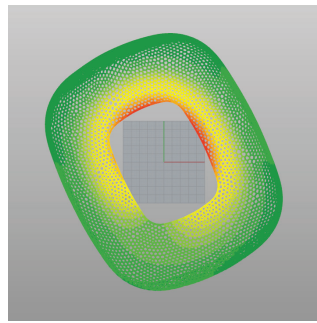


Range: 12 to 8
 Max. Displacement: 4.70m
 Utilization: -35.4% to 34.7%
 Mass: 1.55e⁸
 Nodal Displacement: 2.74 cm

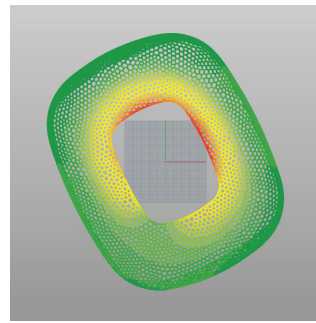


Range: 12 to 9
 Max. Displacement: 4.69m
 Utilization: -35.2% to 34.4%
 Mass: 1.55e⁸
 Nodal Displacement: 2.67 cm

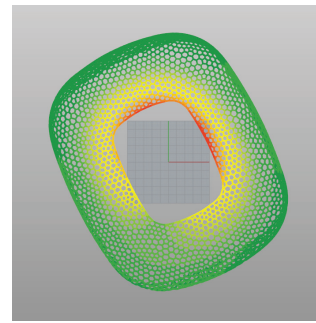
Test 1



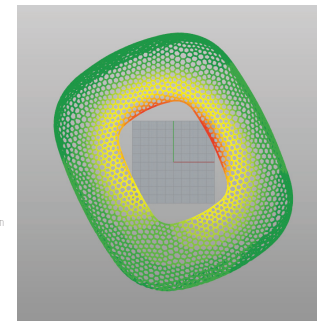
Range: 5 to 5
Max. Displacement: 4.46m
Utilization: -28.6% to 29.3%
Mass: $2.0e^8$
Nodal Displacement: 0.83 cm



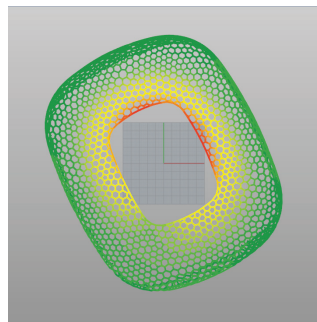
Range: 6 to 6
Max. Displacement: 5.54m
Utilization: -31.8% to 34.1%
Mass: $2.0e^8$
Nodal Displacement: 1.22 cm



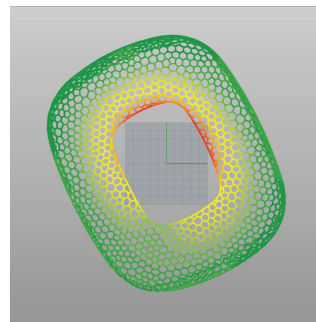
Range: 7 to 7
Max. Displacement: 7.33m
Utilization: -43.3% to 46.6%
Mass: $2.0e^8$
Nodal Displacement: 1.77 cm



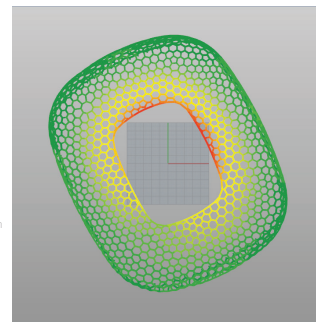
Range: 8 to 8
Max. Displacement: 8.26m
Utilization: -48.1% to 51.2%
Mass: $2.0e^8$
Nodal Displacement: 2.42 cm



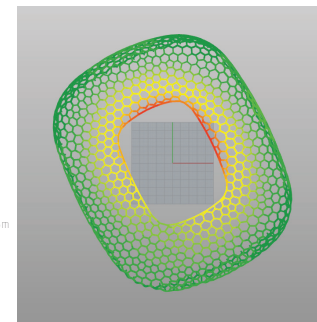
Range: 9 to 9
Max. Displacement: 9.59m
Utilization: -52.3% to 55.8%
Mass: $2.0e^8$
Nodal Displacement: 3.35 cm



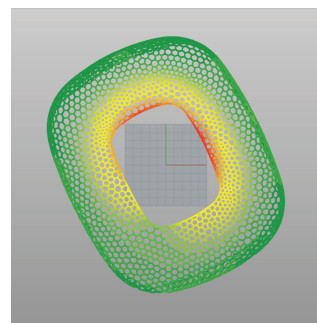
Range: 10 to 10
Max. Displacement: 11.62m
Utilization: -64.0% to 67.3%
Mass: $2.0e^8$
Nodal Displacement: 4.62 cm



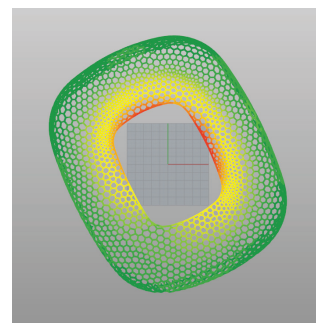
Range: 11 to 11
Max. Displacement: 13.05m
Utilization: -62.7% to 68.8%
Mass: $2.0e^8$
Nodal Displacement: 5.72 cm



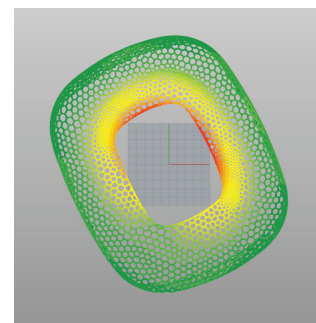
Range: 12 to 12
Max. Displacement: 15.26m
Utilization: -78.3% to 76.4%
Mass: $2.0e^8$
Nodal Displacement: 9.11 cm



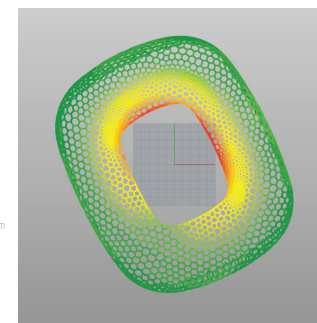
Range: 4 to 9
Max. Displacement: 9.53m
Utilization: -46.5% to 54.1%
Mass: $2.0e^8$
Nodal Displacement: 3.85 cm



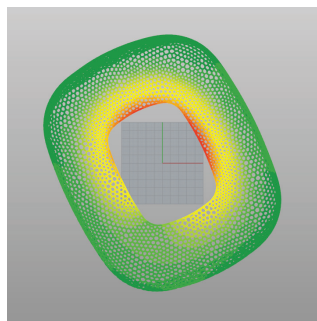
Range: 3 to 9
Max. Displacement: 9.62m
Utilization: -45.2% to 48.8%
Mass: $2.0e^8$
Nodal Displacement: 3.92 cm



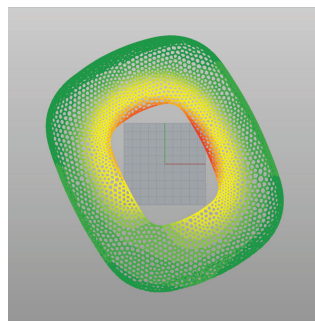
Range: 2 to 9
Max. Displacement: 9.92m
Utilization: -43.7% to 50.6%
Mass: $2.0e^8$
Nodal Displacement: 3.99cm



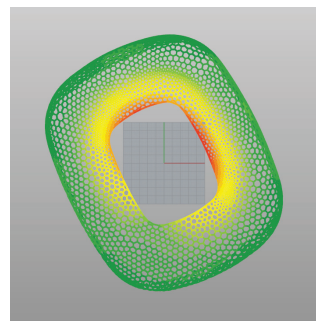
Range: 2 to 10
Max. Displacement: 11.04m
Utilization: -49.1% to 52.9%
Mass: $2.0e^8$
Nodal Displacement: 4.37 cm



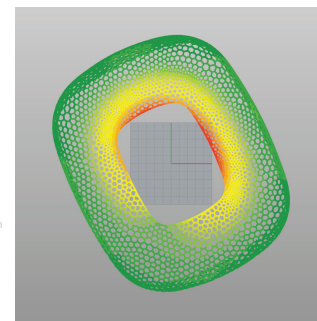
Range: 4 to 6
Max. Displacement: 5.68m
Utilization: -30.2% to 35.0%
Mass: $2.0e^8$
Nodal Displacement: 0.89 cm



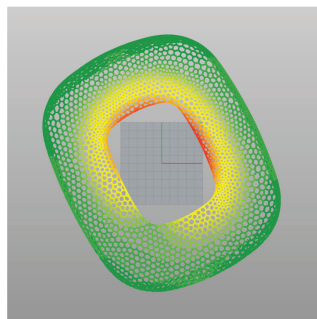
Range: 3 to 7
Max. Displacement: 6.58m
Utilization: -32.4% to 37.7%
Mass: $2.0e^8$
Nodal Displacement: 1.38 cm



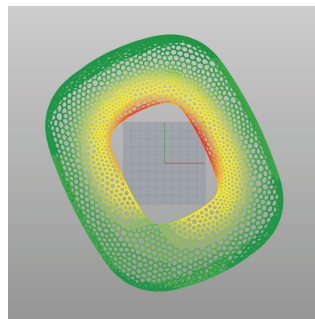
Range: 2 to 8
Max. Displacement: 8.26m
Utilization: -35.6% to 38.7%
Mass: $2.0e^8$
Nodal Displacement: 2.28cm



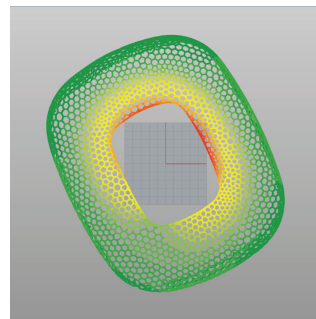
Range: 3 to 8
Max. Displacement: 8.16m
Utilization: -36.0% to 38.7%
Mass: $2.0e^8$
Nodal Displacement: 2.30 cm



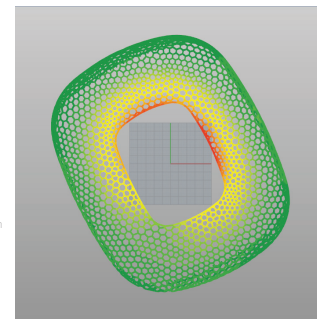
Range: 4 to 8
Max. Displacement: 8.10m
Utilization: -34.6% to 38.8%
Mass: $2.0e^8$
Nodal Displacement: 2.40 cm



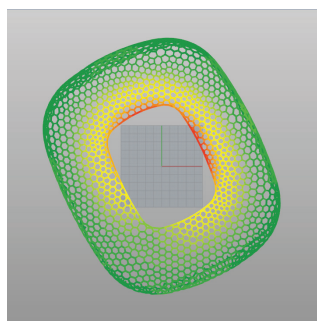
Range: 5 to 8
Max. Displacement: 8.02m
Utilization: -41.0% to 47.2%
Mass: $2.0e^8$
Nodal Displacement: 2.42 cm



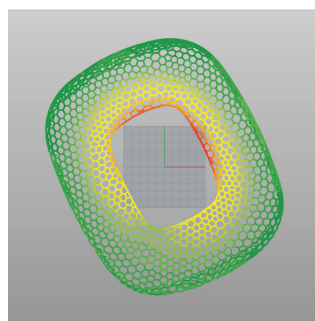
Range: 6 to 9
Max. Displacement: 9.58m
Utilization: -50.2% to 55.3%
Mass: $2.0e^8$
Nodal Displacement: 3.84 cm



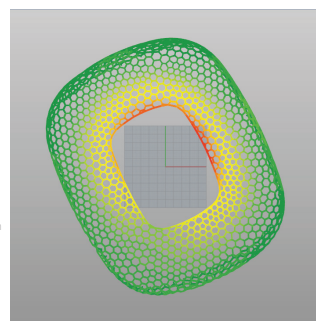
Range: 5 to 9
Max. Displacement: 9.58m
Utilization: -50.1% to 55.2%
Mass: $2.0e^8$
Nodal Displacement: 3.86 cm



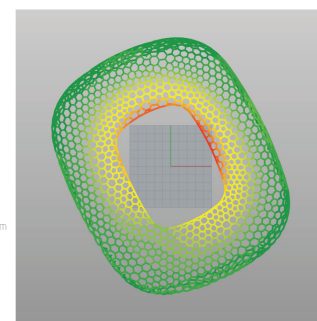
Range: 7 to 10
Max. Displacement: 10.98m
Utilization: -52.0% to 54.0%
Mass: $2.0e^8$
Nodal Displacement: 4.55 cm



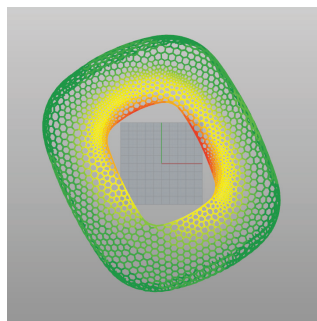
Range: 8 to 11
Max. Displacement: 12.50m
Utilization: -74.6% to 74.2%
Mass: $2.0e^8$
Nodal Displacement: 6.06 cm



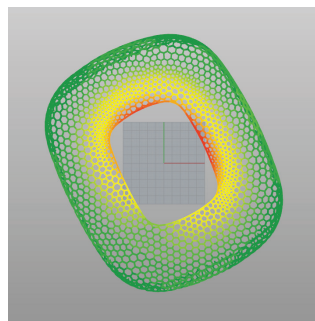
Range: 7 to 11
Max. Displacement: 12.55m
Utilization: -76.1% to 80.4%
Mass: $2.0e^8$
Nodal Displacement: 6.24cm



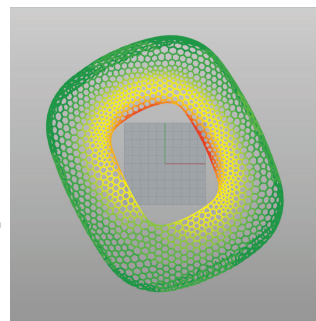
Range: 6 to 11
Max. Displacement: 12.58m
Utilization: -68.9% to 70.8%
Mass: $2.0e^8$
Nodal Displacement: 6.27 cm



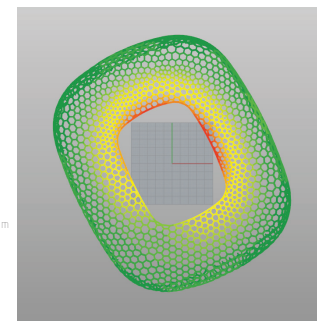
Range: 3 to 10
Max. Displacement: 11.09m
Utilization: -47.3% to 50.5%
Mass: $2.0e^8$
Nodal Displacement: 4.37 cm



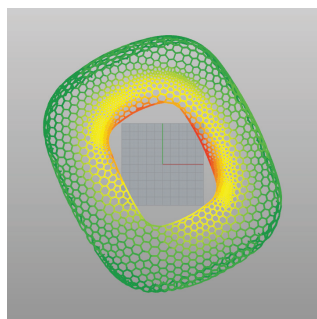
Range: 4 to 10
Max. Displacement: 10.99m
Utilization: -47.1% to 51.6%
Mass: $2.0e^8$
Nodal Displacement: 4.33 cm



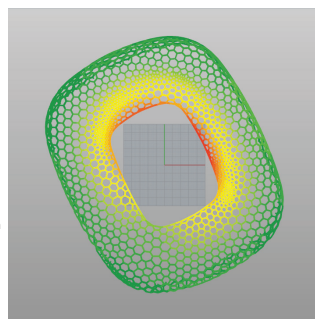
Range: 5 to 10
Max. Displacement: 10.85m
Utilization: -48.8% to 52.4%
Mass: $2.0e^8$
Nodal Displacement: 4.21 cm



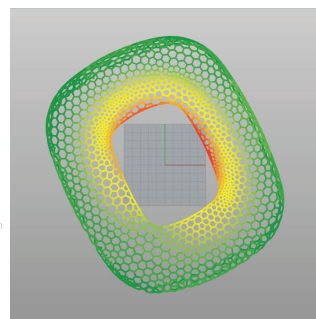
Range: 6 to 10
Max. Displacement: 10.82m
Utilization: -51.6% to 53.5%
Mass: $2.0e^8$
Nodal Displacement: 4.38 cm



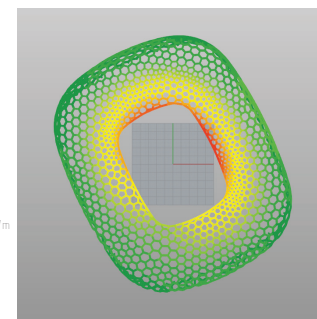
Range: 2 to 12
Max. Displacement: 14.66m
Utilization: -65.2% to 69.0%
Mass: $2.0e^8$
Nodal Displacement: 6.51 cm



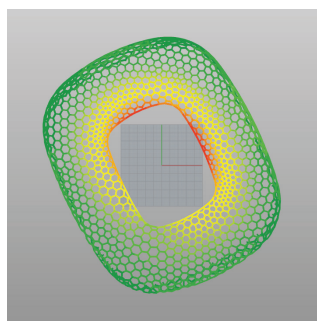
Range: 3 to 12
Max. Displacement: 14.78m
Utilization: -64.3% to 67.9%
Mass: $2.0e^8$
Nodal Displacement: 6.22 cm



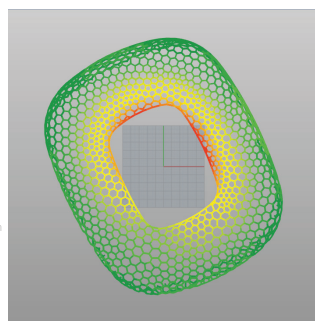
Range: 4 to 12
Max. Displacement: 14.57m
Utilization: -63.8% to 67.1%
Mass: $2.0e^8$
Nodal Displacement: 6.04 cm



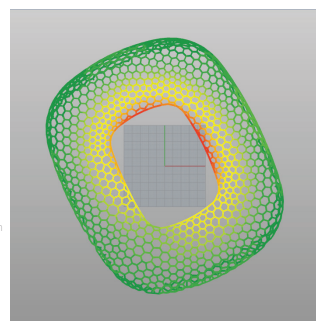
Range: 5 to 12
Max. Displacement: 14.44m
Utilization: -65.4% to 73.6%
Mass: $2.0e^8$
Nodal Displacement: 5.89 cm



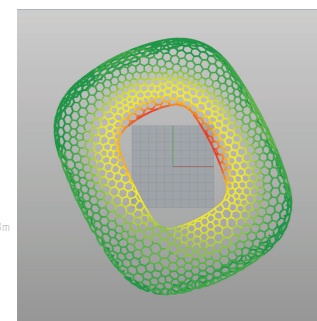
Range: 6 to 12
Max. Displacement: 14.24m
Utilization: -63.9% to 72.0%
Mass: $2.0e^8$
Nodal Displacement: 7.11 cm



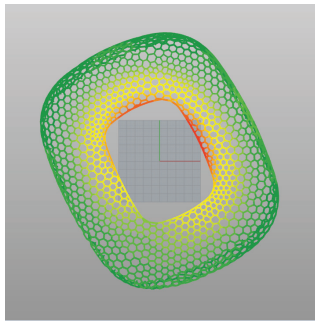
Range: 7 to 12
Max. Displacement: 14.30m
Utilization: -66.3% to 67.7%
Mass: $2.0e^8$
Nodal Displacement: 7.95 cm



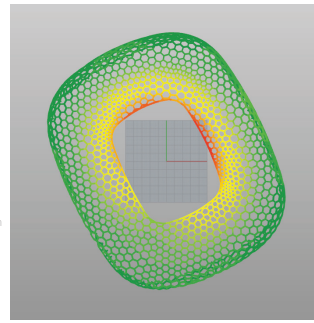
Range: 8 to 12
Max. Displacement: 14.38m
Utilization: -64.5% to 66.6%
Mass: $2.0e^8$
Nodal Displacement: 7.97 cm



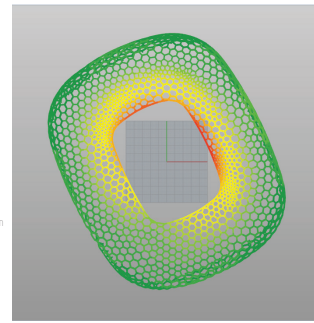
Range: 9 to 12
Max. Displacement: 14.35m
Utilization: -74.5% to 77.9%
Mass: $2.0e^8$
Nodal Displacement: 9.10 cm



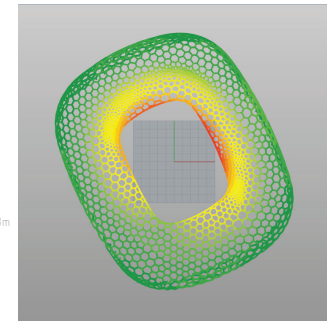
Range: 5 to 11
 Max. Displacement: 12.72m
 Utilization: -68.4% to 71.0%
 Mass: $2.0e^8$
 Nodal Displacement: 5.50 cm



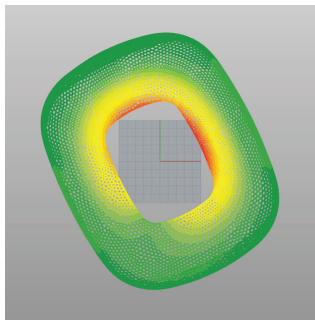
Range: 4 to 11
 Max. Displacement: 12.80m
 Utilization: -62.5% to 67.5%
 Mass: $2.0e^8$
 Nodal Displacement: 5.34 cm



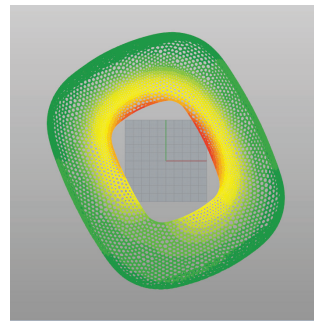
Range: 3 to 11
 Max. Displacement: 12.88m
 Utilization: -59.3% to 66.1%
 Mass: $2.0e^8$
 Nodal Displacement: 5.29 cm



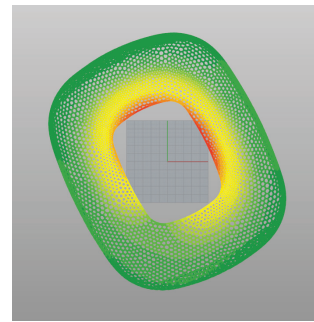
Range: 2 to 11
 Max. Displacement: 13.21m
 Utilization: -54.7% to 65.6%
 Mass: $2.0e^8$
 Nodal Displacement: 5.51 cm



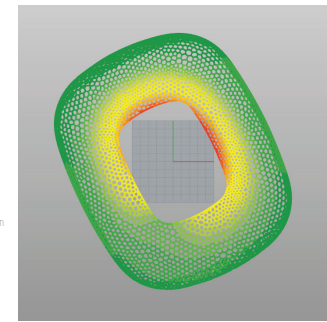
Range: 2 to 5
 Max. Displacement: 4.32m
 Utilization: -22.1% to 24.3%
 Mass: $2.0e^8$
 Nodal Displacement: 0.52 cm



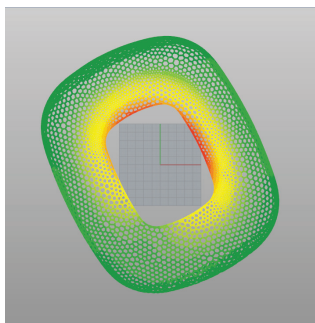
Range: 2 to 6
 Max. Displacement: 5.50m
 Utilization: -29.7% to 29.1%
 Mass: $2.0e^8$
 Nodal Displacement: 0.90 cm



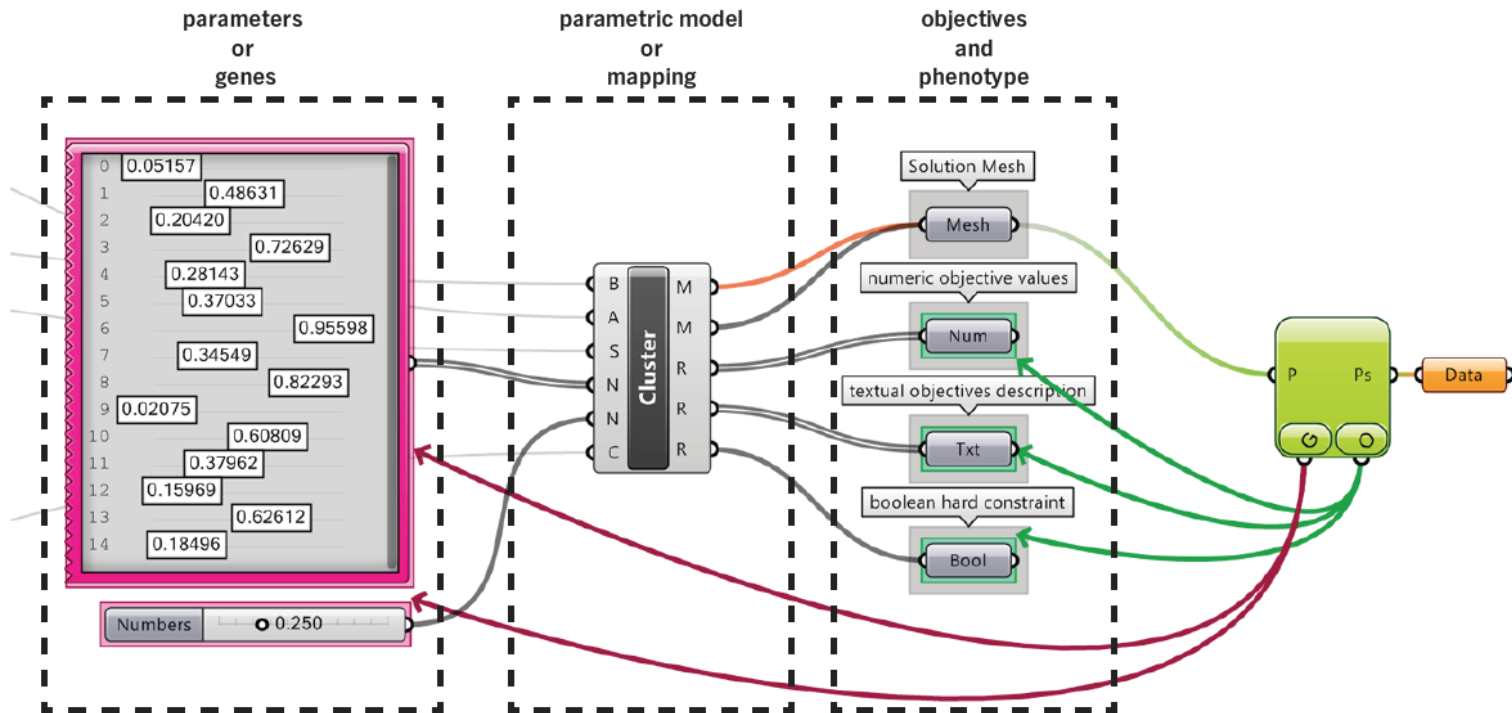
Range: 3 to 6
 Max. Displacement: 5.41m
 Utilization: -26.8% to 29.4%
 Mass: $2.0e^8$
 Nodal Displacement: 0.96cm



Range: 4 to 7
 Max. Displacement: 6.61m
 Utilization: -34.1% to 33.0%
 Mass: $2.0e^8$
 Nodal Displacement: 1.52 cm



Range: 2 to 7
 Max. Displacement: 6.78m
 Utilization: -33.1% to 34.4%
 Mass: $2.0e^8$
 Nodal Displacement: 1.42 cm



parameters or genes: Octopus can connect to multiple GenePool and NumberSlider components. Like for any other components, hold shift to connect additional wires, hold Ctrl to remove single connections. Octopus will explore possible values of the connected Sliders according to their individual range settings [0 to 100, 5 to 9, or 3.1 to 62.3, etc.]

parametric model or mapping: Takes some input parameters and does some calculations / analysis / generation .. How long this step takes is important for the overall performance of the search, since a lot of evaluations with different parameter values have to be performed.

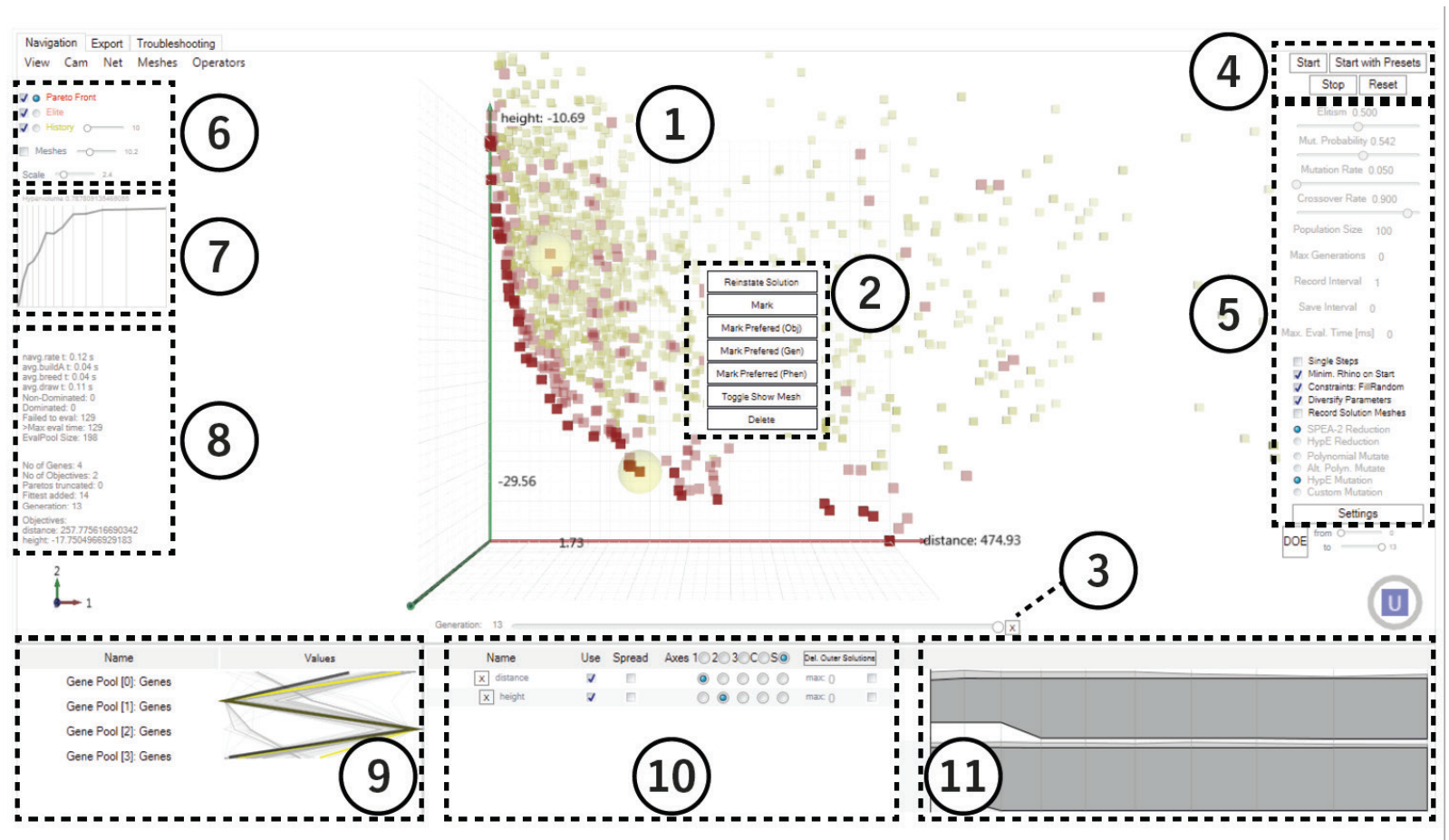
objectives and phenotype:

Solution Mesh: optionally, a 3D mesh representation of the solution can be fed into Octopus for visual assessment. Meshes with high number of faces will increase the memory usage of Octopus and increase the filesize of the definition when saved.

Numeric objective values [Number parameter]: The fitness values of a solution. Minimum two, maximum is theoretically unlimited.

Textual objectives description [Text parameter]: Simple short names describing the objective dimensions

Boolean hard constraints [Boolean parameter]: Optional, a boolean parameter can be connected. Octopus expects a 'true' value for every valid solution, otherwise the solution is thrown away.



- 1 Main viewport / viewcube. Opaque cubes indicate the non-dominated pareto-front, transparent cubes are dominated solutions still belonging to the Elite. Transparent yellow cubes are elite-solutions from previous generations [history], the more transparent the older. Transparent yellow spheres indicate a simple marking. Those marked solutions are shown all the time. On the coordinate axes, and, if used, the additional color- and size-dimensions, a minimum and maximum value is shown for the respective objective dimension - corresponding to the set which is currently selected to fit into the viewcube.
- 2 Context menu when left-clicking a solution. **'Reinstate Solution'** restores the solutions' parameters [genes] in the Grasshopper setup and recalculates it. **'Mark'** adds a yellow sphere around the solution to keep it alive and visible until unmarked, also a yellow polyline is added at the parameter-graph [item 9]. **'Mark Preferred (Obj)'** adds a user preference at this region in the objective space, meaning the weighting of objectives is said to be preferred. Totally different parameters of two solutions can lead to similar objective values, so another level of preference articulation is needed to favor parametrically [genetically] similar solutions [and hence, often formally similar]: **'Mark Preferred (Gen)'**, which introduces an additional objective dimension, no matter how many genetic preferences are set. **'Mark Preferred (Phen)'** is of no function at the moment. **'Toggle Show Mesh'** toggles between the cube- and mesh-view of a single solution. **'Delete'** deletes the solution.
- 3 History Slider, which allows you to scroll through the history of the search process. All history except the last generation can be deleted with the 'X' - button. The search can be resumed from any point in history, the generations of the new [parallel] timeline are just added at the end of the history. Also, markings and preferences can be done in history.
- 4 Process control: **'Start'** starts the process with 2 x PopulationSize of random solutions in the beginning, or resumes a process that has been stopped. **'Start with Presets'** can be used to incorporate the Slider-Setup currently in Grasshopper. When a search is started for the first time, 2 x PopulationsSize solutions are generated from the current Slider-Setup by using the current settings for mutation [see 5]. When a search is resumed, just one solution with the current slider setup is added to the pool. Both start buttons will open a little additional window that runs in a separate thread to stop heavy calculations anytime. **'Stop'** stops a search. Stopped searches are always resumable. **'Reset'** clears all solutions, preferences, history, etc., but not the settings for Octopus itself.
- 5 Algorithm Settings: **'Elitism'** gives the percentage of new solutions that are bred out of the Elite instead of the entire pool. When set high, more local optimization is performed. **'Mut. Probability'** is the probability of each parameter /gene to become mutated with the **'Mutation Rate'**. A low Mutation Rate means little changes to the parameters' values, a high rate means big changes. This can be tested with one of the commands in the menu Operators - TestMutation. **'Crossover Rate'** is the probability of two subsequently generated solutions to exchange parameter values.
'Population Size' is the number of solutions per generation. The Elite size is set accordingly, so a total of 2 x PopulationSize number of solutions are in each generations' pool. This size should be set according to the complexity of the problem, since a lot of solutions at the same time can maintain a lot of different alternatives. **'Max. Generations'** is set to zero by default, meaning there is no end to the search. Otherwise Octopus will stop after this number of generations. **'Record Interval'** is the interval of generations in which a history record is stored. Can reduce the memory footprint for long searches. **'Save Interval'** gives the interval of generations after which the Grasshopper file is saved to prevent data loss when Rhino crashes during search for whatever reason. **'Max. Eval Time [ms]'** - if a solution takes longer than this to compute, it is added to a special collection of solutions that can be debugged afterwards, by reinstating them in the 'Troubleshooting' tab. Can be used to track special unwanted situations for specific parameter values. **'Single Steps'** lets the search stop after each generation.
'Minim. Rhino on Start' minimizes Rhino and Grasshopper when a search is started, because performance of the evaluation can go up 20 times when the screen updates don't have to be performed. However this might not work for non-english installations of Grasshopper, then you should minimize manually. **'Constraints: Fill Random'** is a setting only relevant when hard constraints are specified by connecting Octopus Objectives to a boolean-component. When fill random is enabled, the search stays within the first generation until it has found enough valid random solutions to fill the first generation up to 2 x PopulationSize. Otherwise, just a minimum number of 6 solutions is required and the pool will eventually fill up with valid solutions over time. Enabling is recommended to have a good spread of different solutions from the beginning on. **'Diversify Parameters'** introduces an additional objective dimension which favors solutions very different from others in their parameter-configuration (gene-space). Eventually, the Checkbox has to be unchecked and checked to work. **'Record Solution Meshes'**: when checked, the mesh-representation is also stored for history solution which greatly increases memory usage. By default only the last generation stores solution meshes.
'SPEA-2 or HypE reduction' lets you choose between two different strategies of how a pareto non-dominated front should be truncated to fit the archive size when it is too big. **'Polynomial, Alternative Polynomial, HypE, or Custom Mutation'** lets you choose between different mutation strategies.

- 6** Display Settings: '**ParetoFront, Elite, History**': The row of Checkboxes determines if a set is shown at all. The row of Radio-Buttons determines the scale of the scene by fitting the respective set into the viewcube. Additionally, history has a slider for how many history solutions to show. '**Meshes**' means the optional solutions meshes that can be plugged into Octopus. When supplied, this checkbox is a global toggle between cube- and mesh-view of the solutions. The slider determines the scale of the meshes in the viewport. '**Scale**' controls the scale of the solutions' cubes.
- 7** Hypervolume Graph: The Hypervolume is a mathematical measure for the spread of solutions and is used by the algorithm, but the implementation here as a history graph is experimental.
- 8** Statistics during the process
- 9** Genetic distance graph: Each row represents a parameter [gene], where the corners of the polylines represent values of that parameter. Each solution shown in the main viewport has a polyline in the genetic distance graph. This can give an overview of the convergence of a search.
- 10** List of objectives by their name and in the order of how they are supplied to Octopus in Grasshopper. The 'Use' column can be used to neglect objectives within the algorithm. This way they are just calculated and can be shown, but are not considered in comparing solutions to each other. 'Spread' has no function at the moment. The 'Axes' group is used to assign objectives to be shown on different axes [either the three spatial axes, the coloring, or the sizing of solutions]. One objective can be shown on multiple axes, but of course one axis can only show one objective. At 'max:', a maximum value for the respective objective dimension can be set. The main viewport is then scaled to this maximum. For all future evaluations, solutions exceeding this maximum are thrown away. The button 'Del. outer solutions' deletes all existing solutions exceeding a maximum.
- 11** Convergence Graphs: One graph for each objective dimension, showing the upper- and lower-bounds of the pareto-front (dark gray) and the Elite (light gray, background) for the number of history solutions specified in the Display Settings [6].

View

☒	ShowCoordinateSystem
☒	ShowGridLines
☐	ShowCameraInfo
☐	ShowTriangleCountInfo
☐	ShowFieldOfView
☐	ShowFrameRate
☒	Enable Clipping Planes
☒	Enable Billboard Labels
☐	Show Delaunay Front Mesh
☐	Show 2D Front Line
☐	Show Genetic Relations
☐	Show All Genetic Relations
☒	Show Context Menu On Click
☐	Zoom On Click
☐	Front Mesh Smoothing Level ▶
☐	Title ▶
☐	SubTitle ▶
☐	Info

Cam

☐	Orthographic
☐	InfiniteSpin
☒	IsInertiaEnabled
☐	RotationSensitivity ▶
☐	ZoomSensitivity ▶
☐	CameraInertiaFactor ▶
☐	Reset

Meshes

means the optional solutions meshes

Recalc Current Eval Pool
Recalc Front
Toggle All Individual Visibility
Toggle Visible Individuals' Visibility
Set All Individual Visibility False
Set All Individual Visibility True
Clear History
Clear All
Clear All Hidden
Toggle Overlapping

Operators

some test commands

Test Mutation without Recomputation
Test Mutation with Recomputation
Dynamic Seeding
Make generation out of all history records

MASTER DISSE