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RESEARCH

# Damage-mechanics insights into the relationship between upper-shelf Charpy testing and $J$ -integral testing considering varying tensile test properties

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**Abstract** Requirements on the yield-to-tensile strength ratio  $\sigma_y/\sigma_u$ , fracture elongation  $A$  and the Charpy energy  $C_v$  are used together as part of an indirect method of ensuring sufficient ductility at localised areas of stress and strain concentration in the design of steel structures. Recent studies have found that these indirect requirements could be inadequate in certain situations involving cracks or manufacturing defects. Furthermore, requirements on the  $\sigma_y/\sigma_u$  which are enforced regardless of the structural context and other material properties may unnecessarily constrain the use of steels which nonetheless have high strength, fracture toughness and ductility. In contrast to the  $\sigma_y/\sigma_u$ ,  $A$ , and  $C_v$ , a more direct measurement of a structure's ability to resist fracture is given by fracture toughness testing, such as  $J$ -integral testing, but this is less frequently used, because these tests are significantly costlier than tension and Charpy tests. More often, Charpy tests are performed and correlations between upper-shelf  $C_v$  and  $J$  values are used to estimate the fracture toughness of the material. However, the existing correlations are predominantly based on empirical findings and have not systematically accounted for the effect of variations in the  $\sigma_y/\sigma_u$ , which has been shown in recent studies to affect the fracture toughness. Using a previously validated coupled damage-mechanics model with rate- and

temperature-dependent plasticity and damage softening, this paper investigates the correlation between  $C_v$  and  $J_Q$  (the critical  $J$ ) numerically, including how it is affected by other material certificate properties such as the  $\sigma_y/\sigma_u$  and  $A$ . First, a correlation based on regression between the damage parameters and the mechanical properties from mill test certificates is found by calibrating the damage parameters for a large database of these steels. Then, the correlation between  $C_v$  and  $J_Q$  is assessed by simulating the single-edge-notch bending test for a range of varying mill test certificate properties, taking into account how the damage parameters vary with these mechanical properties. The results are analysed to give better insight into how the notch toughness correlates to the fracture toughness, taking the  $\sigma_y/\sigma_u$  and  $A$  into account. It is seen that although varying  $\sigma_y/\sigma_u$  and  $A$  has some effect on how the total notch energy  $C_v$  is correlated to  $J_Q$ , it does not reflect a significant effect on the ductile fracture initiation toughness but is rather associated with the fact that the  $C_v$  includes a significant portion of energy for stable ductile propagation and fracture occurring at the specimen's free surface, while  $J_Q$  primarily concerns the onset stage of stable ductile tunnelling behaviour at the centre of the specimen. The  $\sigma_y/\sigma_u$  and  $A$  are seen to have an even smaller effect on the correlation between  $J_Q$  and the energy ( $C_{vm}$ ) dissipated up to the occurrence of the peak force in the instrumented Charpy test, in comparison with the  $C_v$ -to- $J_Q$  correlation, especially for low  $C_{vm}$ .

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### List of symbols

$A_\varepsilon$	Engineering strain at full fracture in tensile test	$J_Q$	Critical $J$ based on the 0.2 mm crack extension offset
$A$	Fracture elongation over the proportional gauge length from pieced-together tensile test fragments	$J_{QS}$	Critical $J$ based on the 0.2 mm crack extension offset using the Schindler (2001) conversion from Charpy energy
$\Delta a$	Crack extension	$J_{QW}$	Critical $J$ based on the 0.2 mm crack extension offset using the Wallin (2001) conversion from Charpy energy
$C$	Cowper-Symonds coefficient	$K$	Strength coefficient for Swift the hardening power law
$C_1$	Normalised MMC parameter relating to the strain in the MMC surface when $c_1 = 0$ ; $\eta = 2.5$ ; $\bar{\theta} = 0$	$K_H$	Strength coefficient for the Hollomon hardening power law
$C_2$	Normalised MMC parameter relating to the maximum strain in the MMC surface when $c_1 = 0$	$l_c$	Characteristic element length
$C_3$	Normalised MMC parameter relating to the minimum strain in the MMC surface when $c_1 = 0$	$m$	Damage softening exponent
$c$	Specific heat capacity	$m_S$	Schindler $J$ - $\Delta a$ curve exponent
$c_1$	First MMC strain surface parameter	$m_W$	Wallin $J$ - $\Delta a$ curve exponent
$c_2$	Second MMC strain surface parameter	$m_x$	Thermal softening exponent
$c_3$	Third MMC strain surface parameter	$n$	Strain hardening exponent for the Swift hardening power law
$C_v$	Charpy V-notch energy	$n_H$	Strain hardening exponent for the Hollomon hardening power law
$C_{vm}$	Portion of the Charpy energy dissipated before the peak in the force-displacement plot from the instrumented Charpy test	$P$	Fitting parameters for the regression of $C_v$ on $C_2$ , $C_3$ and $\sigma_y/\sigma_u$
$D$	Damage indicator	$Q$	Fitting parameters for the regression of $A$ on $C_2$ and $C_3$
$D_c$	Critical damage	$q$	Cowper-Symonds exponent
$\mathcal{D}_p$	Internal work due to plastic deformation	$R$	Rate-dependence factor
$E$	Young's modulus	$X$	Thermal softening factor
$e$	Euler's number	$Z$	Parameter for the grouped effect of the MMC $c_1$ terms when $\eta = 2.5$ ; $\bar{\theta} = 0$
$f$	Yield function	$\beta$	Softening coefficient
$g$	Average slope for Swift hardening in the true stress versus true plastic strain space, between $\varepsilon_{sh,p}$ and $\varepsilon_{u,p}$	$\varepsilon_0$	Swift offset strain
$g_H$	Average slope for Hollomon hardening rule in the true stress versus true total strain space, between $\varepsilon_{y,t}$ and $\varepsilon_{u,t}$	$\varepsilon_f$	Ductile damage initiation strain
$J$	$J$ -integral	$\varepsilon_p$	Equivalent plastic strain
$J_{1\text{ mm},S}$	$J$ at 1mm crack extension according to Schindler's (2001) conversion from Charpy testing	$\dot{\varepsilon}_p$	Equivalent plastic strain rate
$J_{1\text{ mm},W}$	$J$ at 1mm crack extension according to Wallin's (2001) conversion from Charpy testing	$\dot{\varepsilon}_{p0}$	Equivalent plastic strain rate offset parameter
		$\dot{\varepsilon}_{p,uni}$	Quasi-static-uniaxial-testing equivalent plastic strain rate parameter
		$\ddot{\varepsilon}_p$	Rate of change of the plastic strain rate
		$\varepsilon_{sh}$	Total engineering strain corresponding to the end of the yield plateau
		$\varepsilon_{sh,p}$	Equivalent plastic strain corresponding to the end of the yield plateau
		$\varepsilon_u$	Uniform elongation, i.e. the engineering strain at the onset of necking
		$\varepsilon_{u,p}$	Equivalent plastic strain corresponding to the occurrence of necking instability
		$\varepsilon_{u,t}$	True total strain at the occurrence of necking instability

$\dot{\epsilon}_{vp}$	Viscoplastic strain rate tensor
$\epsilon_{y,t}$	True total strain at the initial yield point
$\eta$	Stress triaxiality
$\Theta$	Temperature
$\Theta_m$	Melting temperature
$\Theta_r$	Room temperature
$\bar{\theta}$	Normalised Lode angle
$\nu$	Poisson's ratio
$\xi$	Chatterjee correlation coefficient
$\rho$	Density
$\bar{\sigma}$	Quasi-static undamaged stress due to plastic strain hardening only
$\sigma_{fS}$	Average flow stress according to Schindler (2001)
$\sigma_{fW}$	Average flow stress according to Wallin (2001)
$\sigma_u$	Ultimate tensile stress from uniaxial tensile test
$\sigma_v$	von Mises equivalent stress
$\sigma_y$	Yield strength from uniaxial tensile test
$\sigma_{y,cold}$	Estimated initial flow stress at at low temperature
$\sigma_{y,room}$	Initial flow stress at room temperature
$\sigma$	Cauchy stress tensor
$\chi$	Taylor-Quinney energy coefficient
FEA	Finite-element analysis
MMC	Modified Mohr-Coulomb
SENB	Single-edge notched bending

## 1 Introduction

Structural details such as corners, holes, cut-outs, welded connections and notches are geometrical stress raisers that often experience plastic strains even before the yielding limit state of the global, structural-level member is reached. Ductility is required in these areas to allow the redistribution of these stresses and to maintain the local load bearing capacity, which is sometimes also worsened by fatigue-induced cracks or manufacturing defects. In many steel design standards (CEN 2005, 2020; AISC 2022) and maritime classification rules (LR 2024; DNV 2021), it is implied that complying with a maximum limit on the yield-to-tensile strength ratio  $\sigma_y/\sigma_u$ , a minimum required fracture elongation  $A$  and a minimum transition-temperature-based Charpy notch energy  $C_v$  is sufficient to ensure that the ductile fracture behaviour and strength assumed in the ultimate strength design calculations can be

achieved and maintained at these locations (Feldmann et al. 2020). The  $\sigma_y/\sigma_u$  limit amounts to a minimum strain hardening requirement that serves to delay necking (Considère 1885; Wong and Walters 2021), which causes the occurrence of large strains leading to fracture. The minimum fracture elongation  $A$  is a provision for delaying low-triaxiality ductile fracture (Feldmann and Schaffrath 2017). The transition-temperature-based minimum Charpy energy  $C_v$  is a provision for the avoidance of brittle fracture behaviour (Feldmann et al. 2020). In this way, these requirements are considered to act as an indirect substitute for an upper-shelf toughness requirement that has not been explicitly specified in the design codes and rules.

Recent studies (Feldmann et al. 2020; Sedlacek et al. 2008) have found that this set of indirect requirements could be inadequate for providing sufficient upper-shelf toughness in certain situations involving cracks or manufacturing defects. On the other hand, fixed requirements on the  $\sigma_y/\sigma_u$  which are enforced regardless of the structural context and other properties may, in some situations, unduly constrain the use of steels which nonetheless have high strength, fracture toughness and ductility (Feldmann et al. 2023; Feldmann and Schaffrath 2017). It is found that strain hardening, as represented by the  $\sigma_y/\sigma_u$ , has a significant effect on the ductility in plastic buckling mechanisms (Wong and Walters 2023; Studer and Taras 2023), but its effect on the local ductility at stress-concentration-inducing details is less clear (Langenberg et al. 2000). While a maximum  $\sigma_y/\sigma_u$  limit of 0.95 was indeed initially introduced (Dhalla and Winter 1974) based on the grounds of delaying necking sufficiently to ensure a minimum level of ductility, and similar values for the  $\sigma_y/\sigma_u$  limit are used to this day, post-necking behaviour and fracture toughness were neglected in these considerations. More comprehensive research on the effect of strain hardening on local ductility at cracked details under positive triaxiality is ongoing, but the implications thereof on the use of  $\sigma_y/\sigma_u$  as an indicator of local ductility at such details lack a consensus. For example, Feldmann et al. (2020) and Feldmann and Schaffrath (2017) indicate that a higher  $\sigma_y/\sigma_u$  leads to increased net section resistance relative to the ultimate tensile strength in centrally notched tensile tests, suggesting that a higher  $\sigma_y/\sigma_u$  promotes resistance at cracked details. On the other hand, Nonn et al. (2018) and Lu et al. (2015) indicate that a higher  $\sigma_y/\sigma_u$  leads to

decreased crack arrest toughness, suggesting the contrary.

In light of this, recent developments have seen the suggestion of minimum upper-shelf  $C_v$  values (CEN 2023), in addition to  $A$ , as a better primary upper-shelf toughness substitute criterion, while maintaining a minimal  $\sigma_y/\sigma_u$  for the ductility at plastic buckling localisations and the provision of a nominal minimum hardening so that plastic mechanisms can develop (Feldmann et al. 2023). Although ductile Charpy tests, when compared to fracture toughness tests, are additionally affected by strain rate effects and adiabatic heating and are more influenced by plastic energy dissipation due to extensive plastic bending, they are cheaper and easier to perform. Correlations between the upper-shelf notch toughness in terms of  $C_v$  and the fracture toughness in terms of the  $J$ -integral are often used to estimate the material's fracture toughness from Charpy tests (Wallin 2001; BSI 2019; Chaouadi and Gérard 2021). These correlations provide the basis for the use of minimum upper-shelf  $C_v$  values as an indirect upper-shelf toughness requirement for local ductility at details susceptible to crack formation and ultimate ductile fracture.

The existing correlations, however, are predominantly based on empirical findings and have not systematically accounted for the effect of variations in the  $\sigma_y/\sigma_u$  on the toughness noted by Nomm et al. (2018) and Lu et al. (2015). Only one existing semi-empirical correlation, proposed by Schindler (2001), which will be discussed in Section 4 below, varies with  $\sigma_y/\sigma_u$ . Relationships based on damage-plasticity modelling do not exist in the literature primarily because calibrating such models to capture the ductile fracture mechanism well for even one material alone typically involves extensive testing and numerical modelling effort (Cao 2015). Recently, a study on the modelling of the Charpy and single-edge-notched-bending (SENB) tests on AH36 and S690QL steel by the present authors (Wong and Walters 2025) has shown that, for steels with weak triaxiality dependence in the fracture locus, these two tests could be simulated well using a 2-parameter approach for the damage locus calibration, by assuming triaxiality independence. This is supported by observations made by Xue (2009, 2010) on existing exper-

iments on structural steels (Clausing 1970) and aluminium alloy (Banabic et al. 2005). By virtue of the reduced calibration effort given by this approach, this paper numerically investigates the correlation between  $C_v$  and the critical  $J$ , denoted by  $J_Q$ , predicted by the calibrated models of a database of S690Q (CEN 2019) steels. The calibration is performed based on the data obtained from the material certificate of these steels:  $\sigma_y$ ,  $\sigma_u$ ,  $A$  and  $C_v$ . Using regression analysis of the numerical results, a relationship between the damage parameters and material certificate data is proposed for this group of steels. Subsequently, SENB simulations, together with the corresponding  $J_Q$ , for the calibrated steels with varying material certificate properties are obtained. The results are analysed to study and give better insight into how the notch toughness  $C_v$  correlates to the fracture toughness  $J_Q$ , taking the  $\sigma_y/\sigma_u$  and  $A$  into account. This sheds light on the roles of  $\sigma_y/\sigma_u$  and  $A$  in the local ductility at cracked details under positive triaxiality by revealing how they affect the way notch and fracture toughness measures are correlated, presenting quantified effects of  $\sigma_y/\sigma_u$  and  $A$  on fracture toughness considerations.

## 2 Rate- and temperature-dependent damage-softening plasticity model

A rate- and temperature-dependent damage-softening flow plasticity model, as described by Wong and Walters (2025), is used. The effects are coupled multiplicatively in the yield function shown in Eq. (1), which is solved by using the consistency viscoplasticity formulation by Wang et al. (1997).

$$f = \sigma_v - \bar{\sigma} R \beta X \quad (1)$$

where  $f$  is the yield function;  $\sigma_v$  is the von Mises equivalent stress;  $\bar{\sigma}$  is the plastic strain-hardening stress;  $R$  is the rate-dependence factor;  $\beta$  is the softening coefficient; and  $X$  is the thermal softening factor.

### 2.1 Plasticity

The plastic strain-hardening stress  $\bar{\sigma}$  is described by a Swift-type (Swift 1952) stress-strain curve combined with a yield plateau:

$$\bar{\sigma} = \begin{cases} \sigma_y \left( \frac{\varepsilon_{sh} - \frac{\sigma_y}{E}}{\varepsilon_{sh,p}} \varepsilon_p + \frac{\sigma_y}{E} + 1 \right) & 0 < \varepsilon_p < \varepsilon_{sh,p} \\ K(\varepsilon_p - \varepsilon_0)^n & \varepsilon_p \geq \varepsilon_{sh,p} \end{cases} \quad (2)$$

$$\varepsilon_{sh,p} = \ln(1 + \varepsilon_{sh}) - \ln\left(1 + \frac{\sigma_y}{E}\right) \quad (3)$$

where  $\varepsilon_p$  is the true equivalent plastic strain;  $\varepsilon_{sh}$  and  $\varepsilon_{sh,p}$  are respectively the total engineering strain and the true equivalent plastic strain corresponding to the end of the yield plateau;  $K$  is the strength coefficient;  $n$  is the strain hardening exponent; and  $\varepsilon_0$  is the Swift offset parameter.

The Young’s modulus  $E$ , which is very similar for most structural steels, is here assumed to be 200 GPa (together with a Poisson’s ratio of 0.3). The  $\varepsilon_{sh}$  and  $\varepsilon_u$  are obtained from the relationships proposed by Yun and Gardner (2017), derived from regression analysis of 455 engineering stress-strain curves of structural hot-rolled steels with nominal grades up to S690, as shown in Eqs. (4) and (5):

$$\varepsilon_{sh} = \begin{cases} 0.0015 & \frac{\sigma_y}{\sigma_u} < 0.7 \\ 0.1 \frac{\sigma_y}{\sigma_u} - 0.055 & 0.7 \leq \frac{\sigma_y}{\sigma_u} \leq 0.85 \\ 0.03 & \frac{\sigma_y}{\sigma_u} > 0.85 \end{cases} \quad (4)$$

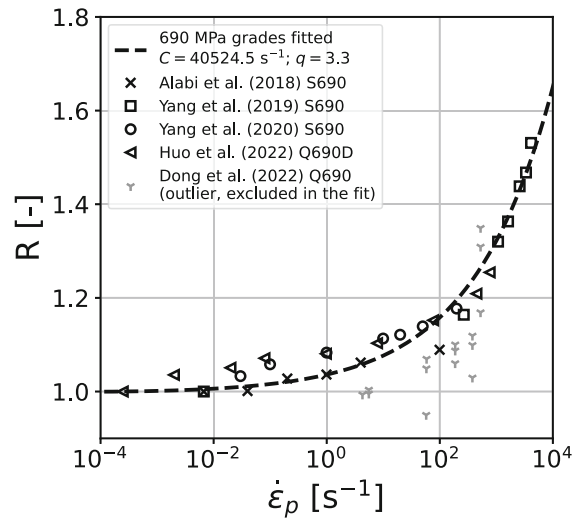
$$\varepsilon_u = \begin{cases} 0.6 \left(1 - \frac{\sigma_y}{\sigma_u}\right) & \frac{\sigma_y}{\sigma_u} \leq 0.9 \\ 0.06 & \frac{\sigma_y}{\sigma_u} > 0.9 \end{cases} \quad (5)$$

Given the known values above, assuming that the occurrence of the ultimate tensile strength  $\sigma_u$  and the associated uniform elongation  $\varepsilon_u$  satisfies the Considère (1885) criterion for necking and that the Swift power hardening curve intersects the yield plateau at  $\varepsilon_p = \varepsilon_{sh,p}$  enables the remaining parameters  $K$ ,  $n$ ,  $\varepsilon_0$  to be solved for. Thus, an estimate of the plastic strain hardening curve can be found from the  $\sigma_y$  and  $\sigma_u$  available from a material certificate.

### 2.2 Strain-rate dependence

The rate-dependence factor  $R$  is described by a Cowper-Symonds-type (Cowper and Symonds 1957) relation:

$$R = 1 - \left( \frac{\dot{\varepsilon}_{p,uni} + \dot{\varepsilon}_{p0}}{C} \right)^{\frac{1}{q}} + \left( \frac{\dot{\varepsilon}_p + \dot{\varepsilon}_{p0}}{C} \right)^{\frac{1}{q}} \quad (6)$$



**Fig. 1** Least-squares fit of  $R$  and experiments in the literature (Alabi et al. 2018; Yang et al. 2019, 2020; Huo et al. 2022; Dong et al. 2022)

where  $C$  is the Cowper-Symonds coefficient; and  $q$  is the Cowper-Symonds exponent;  $\dot{\varepsilon}_p$  is the equivalent plastic strain rate;  $\dot{\varepsilon}_{p,uni}$  is the quasi-static-uniaxial-testing equivalent plastic strain rate; and  $\dot{\varepsilon}_{p0}$  is the equivalent plastic strain rate offset parameter which serves to ensure that the gradient of  $R$  with respect to  $\dot{\varepsilon}_p$  is finite when  $\dot{\varepsilon}_p = 0$  as needed for convergence in the solution of the yield function (Eq. (1)), unlike in the original Cowper-Symonds equation where it is infinite.

Both the  $\dot{\varepsilon}_{p,uni}$  and  $\dot{\varepsilon}_{p0}$  are here assumed to be  $0.00025 \text{ s}^{-1}$ , the uniaxial testing rate according to ISO 6892-1 (ISO 2019).  $C$  and  $q$  are assumed to be  $40524.5 \text{ s}^{-1}$  and  $3.3$ , respectively, based on least-squares fitting to existing experimental data in the literature (Alabi et al. 2018; Yang et al. 2019, 2020; Huo et al. 2022), as shown in Fig. 1. The data from Dong et al. (2022) was not included in the fit due to its nature as an outlier in comparison to the data cloud formed by the rest of the experiments.

### 2.3 Thermal softening

The thermal softening factor  $X$  is based on that of the Johnson-Cook (Johnson and Cook 1985) plasticity model:

$$X = 1 - \left( \frac{\Theta - \Theta_r}{\Theta_m - \Theta_r} \right)^{m_x} \quad (7)$$

where  $\Theta$  is the temperature,  $\Theta_m$  is the melting temperature,  $\Theta_r$  is the room temperature, and  $m_x$  is the thermal softening exponent.

In the Charpy simulations, in which accounting for thermal softening effects is important (Tanguy et al. 2005; Seo et al. 2024) especially when  $C_v$  is high (Wong and Walters 2025), adiabatic conditions are assumed, such that the energy dissipated due to plastic deformation leads to an increase in the temperature of the material, according to the relationships given in Eqs. (8) and (9):

$$\dot{D}_p = \frac{\sigma : \dot{\epsilon}_{vp}}{\rho} \tag{8}$$

where  $\dot{D}_p$  is the rate of internal work done attributed to plastic deformation (Besseling and van der Giessen 1994);  $\sigma$  is the Cauchy stress tensor,  $\dot{\epsilon}_{vp}$  is the viscoplastic strain rate tensor, and  $\rho$  is the density of the material.

The rate of temperature increase is calculated from:

$$\dot{\Theta} = \frac{\chi \dot{D}_p}{c} \tag{9}$$

where  $\chi$  is the proportion of the plastic dissipation energy converted to heat (Taylor and Quinney 1933); and  $c$  is the specific heat capacity of the material.

In this study, the density  $\rho$  of the steel is assumed to be  $7850 \text{ kg m}^{-3}$ ;  $\chi$  is assumed to be 0.9, as proposed by Meyers (1994); and  $\Theta_m$ ,  $\Theta_r$ ,  $m_x$  and  $c$  are assumed to be  $1527^\circ\text{C}$ ,  $20^\circ\text{C}$ , 1.071 and  $452 \text{ J kg}^{-1}\text{C}^{-1}$ , respectively, as given by Dey et al. (2004) for Weldox 700 E steel, which is graded as S690QL (CEN 2019). The quasi-static tension and SENB simulations are assumed to be isothermal, with  $X$  (Eq. 7) always equal to one.

### 2.4 Damage

The damage-softening coefficient  $\beta$  is described by the model of Li and Wierzbicki (2010), as given in Eq. (10):

$$\beta = \begin{cases} 1 & D \leq 1 \\ \left(\frac{D_c - D}{D_c - 1}\right)^m & 1 < D < D_c \\ 0 & D = D_c \end{cases} \tag{10}$$

where  $D$  is the damage indicator;  $D_c$  is the critical damage at which the strength is fully degraded to 0; and  $m$  is the damage softening exponent.

In this study, linearity is assumed for the softening behaviour with respect to  $D$ , such that  $m = 1$ . A  $D_c$

of 1.3 is assumed, based on calibrations of the present model to S690QL and AH36 steel by Wong and Walters (2025) together with the observation that similar types of steels from different studies have similar  $D_c$ : 2.1–2.3 for low-alloy steel pipes (Sakonder et al. 2022), 1.2–1.4 for nickel-chromium Inconel alloy (Sarzos et al. 2022; Algarni et al. 2015) and 1.2–1.3 for hot-rolled low carbon structural steel (Wong and Walters 2025; Chen et al. 2023).

The damage  $D$  builds up according to the accumulation of incremental equivalent plastic strains relative to the stress-state-dependent instantaneous damage initiation strain  $\epsilon_f$ , as described by Eqs. (11) and (12), the strain-based formulation of the modified Mohr-Coulomb (MMC) model (Bai and Wierzbicki 2010).

$$D = \int_0^{\epsilon_p} \frac{1}{\epsilon_f} d\epsilon_p \tag{11}$$

$$\epsilon_f = \left\{ \frac{K}{c_2} \left[ c_3 + \frac{\sqrt{3}}{2 - \sqrt{3}} (1 - c_3) \left( \sec \frac{\bar{\theta}\pi}{6} - 1 \right) \right] \times \left[ \sqrt{\frac{1 + c_1^2}{3}} \cos \frac{\bar{\theta}\pi}{6} + c_1 \left( \eta + \frac{1}{3} \sin \frac{\bar{\theta}\pi}{6} \right) \right] \right\}^{-\frac{1}{n}} \tag{12}$$

where  $\eta$  is the stress triaxiality;  $\bar{\theta}$  is the normalised Lode angle; and  $c_1$ ,  $c_2$  and  $c_3$  are the MMC model parameters.

For the ease of calibration and for obtaining a physical sense of the effect of changes in the parameters, the MMC parameters are normalised and expressed as  $C_1$ ,  $C_2$  and  $C_3$ , in the units of strains, as shown in Eqs. (13) to (16):

$$C_1 = \left( \frac{c_2}{c_3 K Z} \right)^{\frac{1}{n}} \tag{13}$$

$$C_2 = \left( \frac{2c_2}{K} \right)^{\frac{1}{n}} \tag{14}$$

$$C_3 = \left( \frac{\sqrt{3}c_2}{K c_3} \right)^{\frac{1}{n}} \tag{15}$$

where  $Z$  is given by:

$$Z = \sqrt{\frac{1 + c_1^2}{3}} + 2.5c_1 \tag{16}$$

In the triaxiality-independent instance ( $c_1 = 0$ ) of the MMC strain locus,  $C_1$  and  $C_3$  are both equal to the minimum strain in the locus, occurring at  $\bar{\theta} = 0$ , and  $C_2$  is equal to the maximum strain in the locus, occurring at  $\bar{\theta} = \pm 1$ . Furthermore,  $C_1$  is always equal

to the value of the fracture strain for ( $\eta = 2.5, \bar{\theta} = 0$ ), the perfectly plastic, plane-strain solution for the stress state at a crack tip, and the smaller  $C_1$  is than  $C_3$ , the greater the triaxiality dependence.

A cut-off for the negative-triaxiality damage initiation strains is implemented numerically at  $\eta = -1/3$  (Bao and Wierzbicki 2005) by additionally increasing the damage initiation strain  $\varepsilon_f$  gradually towards infinity as  $\eta$  drops below  $-1/3$ . For the purpose of numerical convergence in the solution of the yield function (Eq. (1)), the  $\varepsilon_f$  for  $\eta \geq 3.5$  is kept constant at its value when  $\eta = 3.5$ . Such values of  $\eta$  typically only occur when the element is already highly damaged and degraded, so this approximation is not expected to change the solution.

Using experiments on an AH36 and an S690 steel, Wong and Walters (2025) have shown that the assumption of triaxiality independence, together with the aforementioned understanding of the correlation of  $D_c$  to the steel type and the assumption of linear degradation ( $m = 1$ ), performs well in reducing the number of damage calibration parameters for weakly-triaxiality-dependent steels to only two ( $C_2$  and  $C_3$ ) while simulating the responses of the tensile, Charpy and SENB tests well. This is effectively a capitalisation on the weak triaxiality dependence of the steels in the medium-to-high triaxiality range, which is the range experienced in the tensile test as it necks and in the crack planes of the Charpy and SENB tests. This quality of Lode-angle dependence with triaxiality independence has also been noted in Xue's (2009; 2010) modelling of Clausen's (1970) experiments on several structural steels and of experiments on aluminium alloy by Banabic et al. (2005). Additionally, this effect is further compounded by the observation (Wong and Walters 2025) that, due to crack tip blunting effects, the average triaxialities  $\eta$  calculated over  $0 \leq D \leq 1$  experienced in key crack-front elements in the two tests as the damage  $D$  progresses are closer together, between around 1.1 and 1.3, than suggested by the difference in peak triaxialities ( $\eta \sim 2.2$  in the SENB test and  $\eta \sim 1.7$  in the Charpy test).

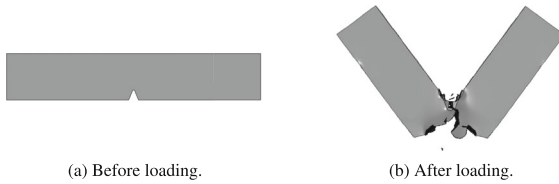
A damage locus that is described by two parameters can be calibrated using only two types of tests, given that they cover distinct stress states which are in the stress-state range of interest, and material certificates provide the results of two such tests in the form of  $A$  and  $C_v$ . Furthermore, it was observed that for key portions of the behaviour in the SENB and Charpy tests,

namely well beyond the occurrence of the critical  $J_Q$  and up to 70% of the dissipation of  $C_v$ , the stress states in the Charpy and SENB are very similar, being determined by extensive tunnelling behaviour with little or no slant fracture. (In Section 4 below, it will be seen how the difference due to the later part of the  $C_v$  dissipation contributes to variations in the correlations between  $C_v$  and the  $J_Q$  initiation toughness.) Due to the similar tunnelling behaviour in a significant portion of the tests, a calibration based on the Charpy tests provides a similar range of stress states as is provided by the calibration to a SENB test (Wong and Walters 2025). In light of this, this study adopts the above assumptions for the present database of S690 steels in order to calibrate the material models for a large number of these steels, namely calibrating  $C_2$  and  $C_3$  based on the available  $A$  and  $C_v$ . The calibrated materials are then used to simulate the SENB test in order to construct numerically, mechanistically founded correlations between  $C_v$  and  $J_Q$  which take into account the other properties, such as  $\sigma_y/\sigma_u$  and  $A$ .

It is worth mentioning that, despite the effectiveness of the assumption of triaxiality independence for the purpose of the present work, this assumption is generally not suitable for describing axisymmetric tests (Wierzbicki et al. 2005) and tests on steels with strong triaxiality dependence in the range of triaxialities of interest. Notably, even metals with low pressure sensitivity in the medium to high triaxiality ranges are generally pressure-sensitive in the very-low-triaxiality range (Xue 2009).

### 3 Relationship between material certificate data and damage parameters

A parametric study varying the  $\sigma_y/\sigma_u$  and the key damage calibration parameters ( $C_2$  and  $C_3$ ) of the aforementioned material model in finite-element simulations of the Charpy (Fig. 2) and tensile (Fig. 3) test is performed in order to obtain sets of material certificate values with their corresponding damage parameters. For each ( $\sigma_y/\sigma_u, C_2, C_3$ ) combination, both the Charpy and the tensile tests are simulated and then post-processed to obtain the  $A$  and  $C_v$ . This allows the subsequent deduction of the relationship between ( $C_2, C_3$ ) and ( $A, C_v, \sigma_y/\sigma_u$ ) using regression analysis of the numerical results.



**Fig. 2** Example Charpy simulation from the parametric study. ( $\sigma_y/\sigma_u = 0.94$ ;  $C_2 = C_3 = 0.7$ )

$A$  is here found using the definition of  $A_\varepsilon$ , the engineering strain when the force first drops below zero, which has been found (Wong and Walters 2025) to give a sufficiently close approximation of the value based on the standard  $A$  definition (ISO 2019) of summing the length of the broken gauge section and dividing the sum by the original gauge length, i.e. it is assumed that  $A = A_\varepsilon$ . The Charpy energy  $C_v$  is taken to be the total kinetic energy lost from the striker by the time the force has become stable around zero.

### 3.1 Parametric study

The simulations are performed in Abaqus Explicit 2022 (DS Simulia 2022) with the use of a user-defined material subroutine for the implementation of the material model. Eight-noded linear solid elements with reduced integration and hourglass control (C3D8R) are used. As a result of a mesh sensitivity study, elements with characteristic sizes  $l_c$  of 0.12 mm and 0.06 mm, respectively, were used in the fracture zones of the tensile and Charpy tests, where  $l_c$  is the cube root of the volume of an undeformed element. Mass scaling was applied to the Charpy and tensile test to decrease the computational time. In the simulations of the quasi-static tensile tests, the mass was increased by a factor of  $1 \times 10^9$ , and the kinetic energy was checked to be less than 1% of the total internal energy, except for the beginning of the simulation when the load application boundary accelerates to the quasi-static testing velocity. In the simulations of the dynamic Charpy tests, the mass was increased by a factor of 25, a value which is seen to produce a negligible difference in  $C_v$  in a previous mass scaling factor sensitivity study (Wong and Walters 2024). The coefficient of friction between contact surfaces in the Charpy simulations was assumed to be 0.15.

**Table 1** Range of  $\sigma_y$  and  $\sigma_y/\sigma_u$  considered

$\sigma_y$ [MPa]	730	750	770	810	850
$\sigma_y/\sigma_u$ [-]	0.92	0.93	0.94	0.96	0.97

#### 3.1.1 Parametric space selection

Table 1 gives the  $\sigma_y$  and  $\sigma_y/\sigma_u$  values that are considered in the parametric study. Although variation in the  $\sigma_y/\sigma_u$  is seen for steels with the same grade and the same  $\sigma_y$ , a higher  $\sigma_y$  is generally accompanied by a higher  $\sigma_y/\sigma_u$ . Therefore, the  $\sigma_y/\sigma_u$  is here made to vary with the  $\sigma_y$  in order to reduce the size of the parametric space while reflecting the general trend. Including a numerical dataset with varying  $\sigma_y/\sigma_u$  for fixed  $\sigma_y$  would shed further light on the separate effects of  $\sigma_y/\sigma_u$  and  $\sigma_y$ , but is outside the scope of this already computationally expensive study. It is also worth noting that, because of the range of  $\sigma_y/\sigma_u$  considered and the use of Eqs. (4) and (5),  $\varepsilon_{sh}$  and  $\varepsilon_u$  are assumed to be constant in this study. For each  $\sigma_y/\sigma_u$ , a series of simulations are run, using all the combination pairs of  $C_2$  and  $C_3$  values that can be made from [0.4, 2.6, 5.1, 7.6] and [0.4, 0.7], respectively, with the exception that  $C_2$  is set to equal  $C_3$  in pairs where  $C_2 < C_3$ , since  $C_2 < C_3$  does not align with experimental observations (see descriptions of Eqs. (14) and (15) above). Additionally, a reduced set of runs, considering a sparser set of  $\sigma_y/\sigma_u$  values, [0.92, 0.94, 0.97], is performed for the combination pairs of  $C_2$  and  $C_3$  values that can be made from [0.4, 2.6, 5.1, 7.6] and [0.5], to provide more numerical data while keep the computing costs within reasonable limits.

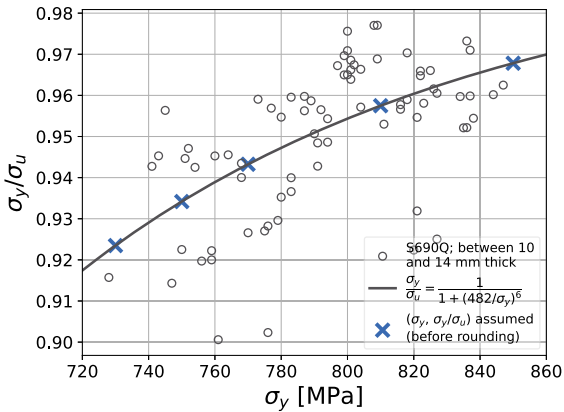
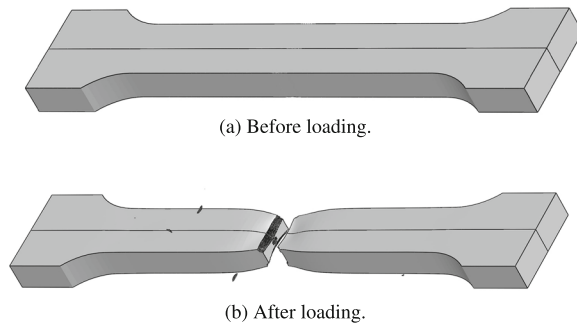
The values chosen for  $\sigma_y$  and  $\sigma_y/\sigma_u$  are based on the database of material certificates provided by Huisman BV, from which 77 steels are used to form a parametric space, by discretising the  $\sigma_y$  domain into intervals, and taking the  $\sigma_y/\sigma_u$  predicted by a least-squares fit to these 77 steels, as shown in Fig. 4 and Eq. (17):

$$\frac{\sigma_y}{\sigma_u} = \frac{1}{1 + \left(\frac{482}{\sigma_y}\right)^6} \quad (17)$$

where  $\sigma_y$  and  $\sigma_u$  are expressed in megapascals.

The dataset of 77 steels is a result of the following filtering criteria applied to the database. First, only steels with the equivalent grades of StE690 and S690Q (CEN 2019), which have the same minimum tough-

**Fig. 3** Example tensile simulation from the parametric study, with plane of symmetry. ( $\sigma_y/\sigma_u = 0.94$ ;  $C_2 = C_3 = 0.7$ )



**Fig. 4**  $\sigma_y/\sigma_u$  and  $\sigma_y$  assumed in the parametric study, based on fit to database

ness standard, are included in the dataset. Second, only steels with a thickness between 10 and 14 mm were included in the dataset. In the tensile test simulations, the steel is assumed to be 12 mm and modelled with a standard (IACS 2023) 25 mm width and proportional gauge length. The relatively small range of thicknesses in the dataset, centred around the value assumed in the simulations, is used so that more materials are included in the dataset while limiting any potential thickness effects that might exist despite the proportionality of the specimens (ASTM 2024). Finally, only steels for which all the three  $C_v$  repetitions reported on the material certificate were larger than 100 J, and for which all three repetitions did not differ from each other by more than 50 J, were included in the dataset. This is to ensure that the Charpy tests considered in the dataset involved fully ductile behaviour even though these tests performed at  $-20\text{ }^\circ\text{C}$  according to the corresponding material standard (CEN 2019), since the simulations do not account for any transition to unstable cleavage fracture.

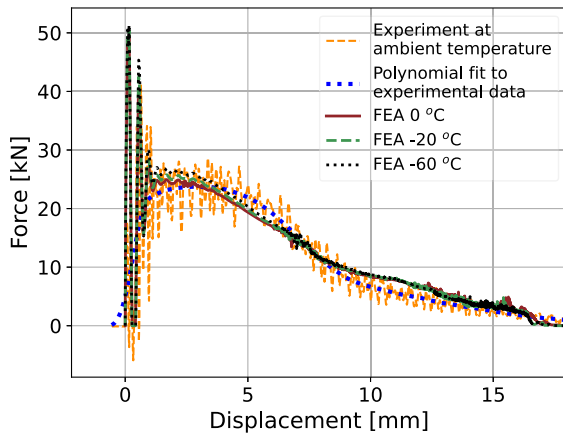
### 3.1.2 Charpy testing temperature assumed in the parametric study

The tensile tests, which are used to deduce the parameters describing the plastic strain-hardening behaviour assumed in the Charpy and tensile simulations, were performed at ambient temperature, while the Charpy tests were performed at  $-20\text{ }^\circ\text{C}$ . The temperature affects the plastic flow stress of materials, but this temperature difference of  $-40\text{ }^\circ\text{C}$  could be expected to make a relatively small difference in the flow stress (Yan et al. 2014). To ascertain the significance of this difference, Charpy simulations accounting for temperatures of  $-20\text{ }^\circ\text{C}$  and  $-60\text{ }^\circ\text{C}$  were additionally performed for the calibrated S690QL steel investigated in Wong and Walters (2025), which was one of the materials for which detailed experimental comparisons were carried out as part of the validation of the present material modelling approach. Since the Johnson-Cook-based model described above (Eq. 7) is suitable only for thermal softening at temperatures above room temperature, the effect of the lower temperature is included here by directly modifying the entire plastic strain-hardening curve according to the BS 7910 (BSI 2019) equation recommended for assessment temperatures below room temperature and above  $-196\text{ }^\circ\text{C}$ , given the room temperature yield strength:

$$\sigma_{y,cold} = \sigma_{y,room} + \frac{10^5}{491 + 1.8\Theta} - 189 \quad (18)$$

where  $\sigma_{y,cold}$  [MPa] is the estimated flow stress at low temperature,  $\sigma_{y,room}$  [MPa] is the flow stress at room temperature, and  $\Theta$  [ $^\circ\text{C}$ ] is the temperature.

For simplicity, this relationship is assumed to be applicable directly to the true plastic strain hardening stress  $\bar{\sigma}$  (Eq. (2)), by setting  $\bar{\sigma} = \sigma_{y,room}$ , such that the temperatures of  $-20\text{ }^\circ\text{C}$  and  $-60\text{ }^\circ\text{C}$  result in a uniform increase to the  $\bar{\sigma}$  of 31 MPa and 72 MPa, respectively,



**Fig. 5** Charpy simulations of S690QL steel at  $-60\text{ }^{\circ}\text{C}$ ,  $-20\text{ }^{\circ}\text{C}$  and  $20\text{ }^{\circ}\text{C}$

for all  $\varepsilon_p$ . This assumption of uniform increase is in line with the observed effect of lower temperatures on the shape of the stress-strain curves as seen in Yan et al. (2014). The other material parameters, including the thermal softening parameters, are kept the same (Table 2), and the simulations start at room temperature with the uniformly increased plastic strain-hardening curve due to the lower temperature, such that the thermal softening model (Eq. 7) is assumed to act based on the temperature difference relative to the starting temperature.

The simulation results (Fig. 5 and Table 3) indicate that a testing temperature of  $-20\text{ }^{\circ}\text{C}$ , and the associated 31 MPa increase in flow stress relative to room temperature conditions, makes a negligible difference to both the Charpy force-displacement curve and the Charpy energy, assuming that the Charpy test behaviour is in the fully ductile regime. In contrast, the temperature rise around the notch due to adiabatic effects, which could be as much as  $350\text{ }^{\circ}\text{C}$  (Tanguy et al. 2005), would lead to a 20 % decrease in the flow stress, given the present thermal softening parameters (Table 2). Therefore, the Charpy simulations in the parametric study, which do take the thermal softening effects into account, are performed assuming room temperature plastic strain-hardening properties, although the Charpy tests were performed at  $-20\text{ }^{\circ}\text{C}$ .

### 3.1.3 Significance of the yield-to-tensile strength ratio $\sigma_y/\sigma_u$

Classically, the  $\sigma_y/\sigma_u$  is related to the amount of strain hardening through the application of the Considère

(1885) uniaxial necking criterion to a Hollomon (1945) power curve (Eq. (19)) that passes through the yield point in the true stress versus true total strains plot, which results in Eqs. (20) and (21) (Zhu and Leis 2005):

$$\bar{\sigma} = K_H \varepsilon_t^{n_H} \tag{19}$$

$$n_H = \varepsilon_{u,t} \tag{20}$$

$$\frac{\sigma_y}{\sigma_u} = \frac{1}{1 + \varepsilon_y} \left( \frac{e \ln(1 + \varepsilon_y)}{n_H} \right)^{n_H} \tag{21}$$

where  $\bar{\sigma}$  is the true stress,  $K_H$  is the Hollomon strength coefficient,  $n_H$  is the Hollomon hardening exponent,  $\varepsilon_y$  is the engineering yield strain,  $e$  is the Euler’s number,  $\varepsilon_t$  is the true total strain, and  $\varepsilon_{u,t}$  is the true total strain corresponding to the occurrence of necking instability.

This enables the theoretical hardening curve (i.e. strength coefficient  $K_H$ , hardening exponent  $n_H$ , and hence its slope) up to the point of necking for Hollomon-type materials to be found, given only  $\sigma_y/\sigma_u$ ,  $\sigma_y$  and  $E$ , thereby relating  $\sigma_y/\sigma_u$ , a ratio of stresses only, to the strain hardening slope. This forms the basis for the use of  $\sigma_y/\sigma_u$  as an indicator for the strain hardening behaviour in some design rules concerning failure modes sensitive to the hardening slope, such as the plastic buckling of beams (Wong and Walters 2023; Studer and Taras 2023).

Although  $\sigma_y/\sigma_u$  could function adequately as an indirect indicator for strain hardening in specifying reference limits for specific design cases, the theoretically derived Hollomon-based relationship between the necking strain and  $\sigma_y/\sigma_u$  is not generally applicable, and a range of uniform elongation values is observed for a given  $\sigma_y/\sigma_u$  in typical structural steels (Yun and Gardner 2017), in violation of Eqs. (20) and (21). This can be attributed to the restrictions imposed on the shape of the strain hardening curve when assuming the Hollomon rule. Therefore, a more versatile plastic strain hardening rule (Eq. (2)), which consists of a yield plateau followed by a Swift curve, is adopted in the present study.

Due to the presence of the Swift offset parameter in Eq. (2),  $n$  alone is no longer sufficient to fully describe the hardening slope. To quantify the relationship between  $\sigma_y/\sigma_u$  and the amount of hardening for the present hardening rule, an expression for the average slope  $g$  in the true stress versus true plastic strain space, between the end of the yield plateau  $\varepsilon_{sh,p}$  and the occurrence of necking instability  $\varepsilon_{u,p}$ , is derived:

**Table 2** Calibrated material parameter values from Wong and Walters (2025) for an S690QL steel, used in combination with Eq. (18) for the low-temperature ductile simulations

		S690QL
(a) Plasticity and damage		
$\sigma_y$	[MPa]	836
$\varepsilon_{sh,p}$	[-]	0.0438
$\varepsilon_0$	[-]	0.0413
$K$	[MPa]	984.1
$n$	[-]	0.0192
$C_1$	[-]	0.47
$C_2$	[-]	8.50
$C_3$	[-]	0.47
$m$	[-]	1
$D_c$	[-]	1.3
(b) Rate dependence, thermal effects and elasticity.		
$C$	[s <sup>-1</sup> ]	40524.5
$q$	[-]	3.3
$\dot{\varepsilon}_{p,uni}$	[s <sup>-1</sup> ]	0.00025
$\dot{\varepsilon}_{p0}$	[s <sup>-1</sup> ]	0.00025
$\Theta_m$	[°C]	1527
$\Theta_r$	[°C]	20
$m_x$	[-]	1.071
$c$	[J kg <sup>-1</sup> °C <sup>-1</sup> ]	452
$E$	[MPa]	206064
$\nu$	[-]	0.3

**Table 3** Charpy energy values in simulations with flow stress modified to account for lower temperatures

		Test	Simulations		
$\Theta$	[°C]	~ 20	-60	-20	20
$C_v$	[J]	204.2	224.7	219.6	217.1

$$g = \frac{\int_{\varepsilon_{sh,p}}^{\varepsilon_{u,p}} \frac{d\bar{\sigma}}{d\varepsilon_p} d\varepsilon_p}{\varepsilon_{u,p} - \varepsilon_{sh,p}} \tag{22}$$

$$= \frac{\sigma_y(1 + \varepsilon_{sh})}{(\varepsilon_{sh,p} - \varepsilon_{u,p} + n)^n (\varepsilon_{u,p} - \varepsilon_{sh,p}) \cdot (n^n - (\varepsilon_{sh,p} - \varepsilon_{u,p} + n)^n)}$$

where  $n$  is related to  $\sigma_y/\sigma_u$  by:

$$\frac{\sigma_y}{\sigma_u} = \frac{(1 + \varepsilon_u)(\varepsilon_{sh,p} - \varepsilon_{u,p} + n)^n}{n^n (1 + \varepsilon_{sh,p})} \tag{23}$$

Eqs. (22) and (23) show that the average strain hardening slope becomes dependent on  $\varepsilon_{sh}$  and  $\varepsilon_u$ , in addition to  $n$ ,  $\sigma_y$ ,  $\sigma_u$  and  $E$ . On the other hand, the average

slope  $g_H$  for the Hollomon case depends only on  $n_H$ ,  $\sigma_y$ ,  $\sigma_u$  and  $E$ , as given by Eq. (24):

$$g_H = \frac{\int_{\varepsilon_{y,t}}^{\varepsilon_{u,t}} \frac{d\bar{\sigma}}{d\varepsilon_t} d\varepsilon_t}{\varepsilon_{u,t} - \varepsilon_{y,t}} \tag{24}$$

$$= \frac{\sigma_u e^{n_H} (n_H^{n_H} - \varepsilon_{y,t})}{n_H^{n_H} (n_H - \varepsilon_{y,t})}$$

where  $\varepsilon_{y,t}$  is the true total strain at the initial yield point, and  $e$  is Euler’s number.

To illustrate the implication of this difference between  $g$  and  $g_H$ , the empirically obtained predictions for  $\varepsilon_{sh}$  (Eq. (4)) and  $\varepsilon_u$  (Eq. (5)) from Yun and Gardner (2017) are used to calculate  $g$ , which is then plotted together

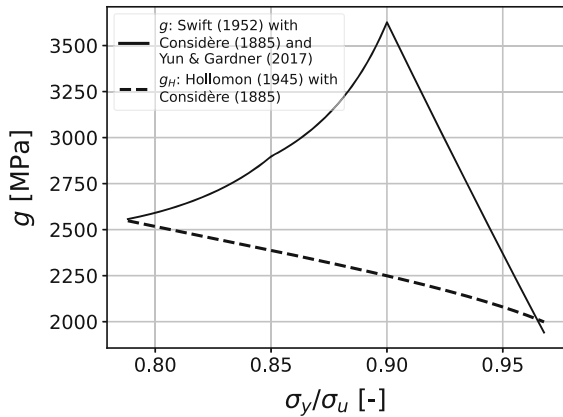


Fig. 6 Average hardening slope  $g$  against  $\sigma_y/\sigma_u$

with  $g_H$ , as shown in Fig. 6. The correlation between  $\sigma_y/\sigma_u$  and  $\sigma_y$  given in Eq. 17 is also assumed. Due to the dependence of  $g$  on  $\varepsilon_{sh}$  and  $\varepsilon_u$ , the relationship between the strain hardening slope  $g$  and  $\sigma_y/\sigma_u$  is not monotonic, in contrast with  $g_H$ . In the present study, the range of  $\sigma_y/\sigma_u$  is limited to  $0.92 \leq \sigma_y/\sigma_u \leq 0.97$ , which corresponds to values observed in the reference database of material certificates of S690 steels (discussed in Section 3.1.1). In this regime,  $\varepsilon_{sh}$  and  $\varepsilon_u$  remain constant according to Eqs. (4) and (5), and an increasing  $\sigma_y/\sigma_u$  is associated with a decreasing hardening slope (Fig. 6), as is normally expected. This is confirmed by the plot (Fig. 7) of the present hardening rule (Section 2.1) together with the Considère necking point, which illustrates how the hardening curve changes in the present study: the strength increases, the average hardening slope decreases, and the yield plateau length and necking strain stays constant, as  $\sigma_y/\sigma_u$  is increased from 0.92 to 0.97. However, it is important to note the range for which this applies and the presence of scatter observed in  $\varepsilon_{sh}$  and  $\varepsilon_u$  values in practice.

### 3.2 Regression of $A$ and $C_v$ on $C_2$ and $C_3$

The results of the parametric runs, together with a fitted relationship based on least-squares regression, are plotted in Figs. 8 to 11. The proposed regression model is given in Eq. (25) and is a result of fitting different trial models based on the trends observed in the data and choosing the one that best reflect the trends.

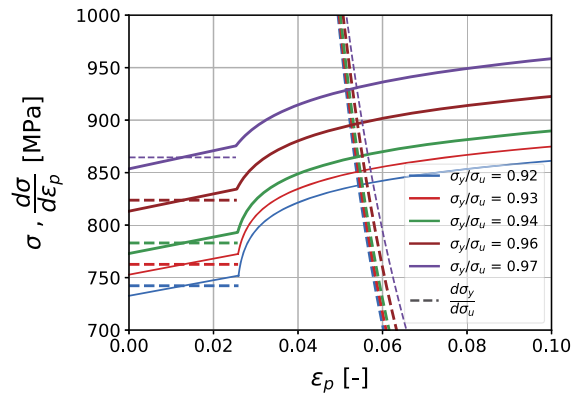


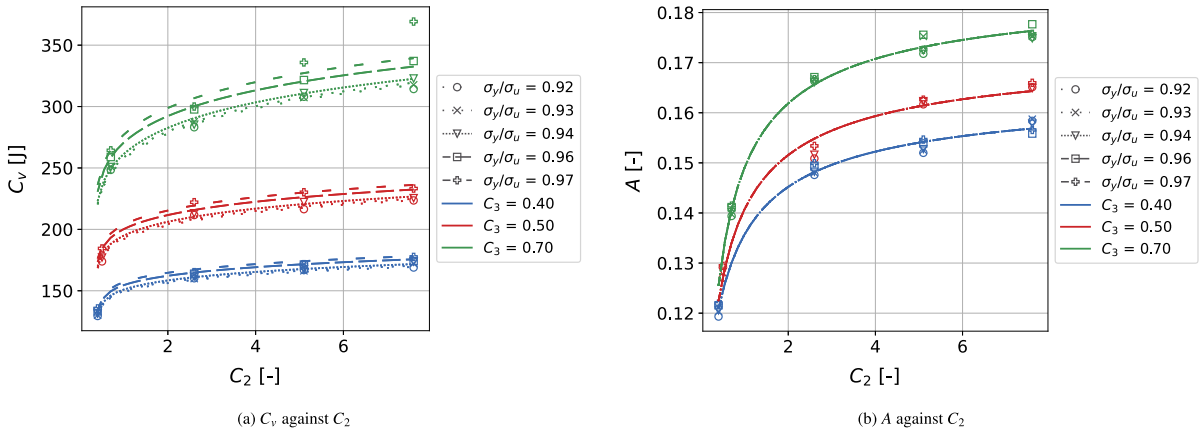
Fig. 7 Effect of varying  $\sigma_y/\sigma_u$  on the plastic strain hardening curve

$$\left\{ \begin{array}{l} C_v(C_2, C_3, \frac{\sigma_y}{\sigma_u}) = P_1 P_2 \left(\frac{\sigma_y}{\sigma_u} - 1\right) C_2^{P_3} \\ \quad + P_4 P_5 \left(\frac{\sigma_y}{\sigma_u} - 1\right) \frac{\sigma_y}{\sigma_u} C_3^{P_6} \\ \quad + P_7 P_8 \left(\frac{\sigma_y}{\sigma_u} - 1\right) C_2^{P_9} C_3^{P_{10}} \\ A(C_2, C_3) = Q_1 C_2^{Q_2} + Q_3 C_3^{Q_4} + Q_5 C_2^{Q_6} C_3^{Q_7} \end{array} \right. \quad (25)$$

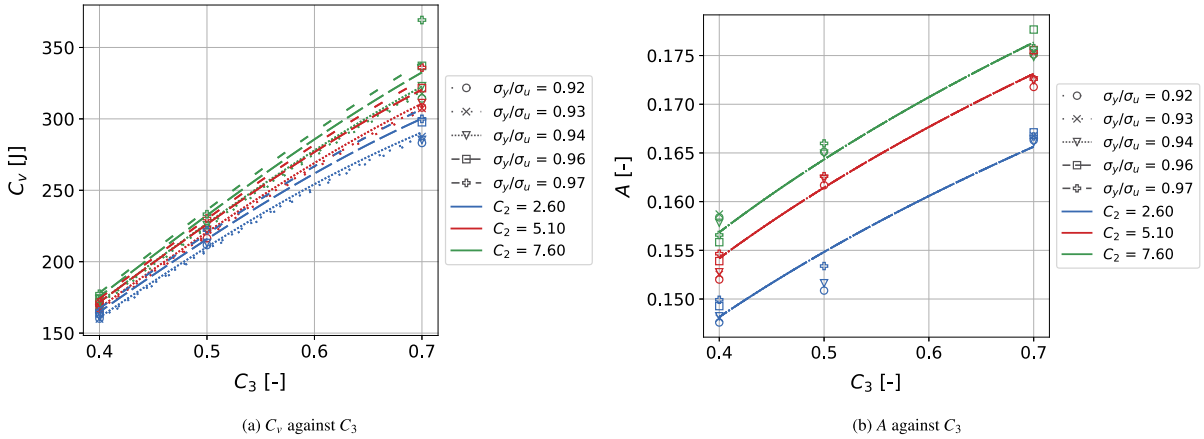
where  $P_i$  and  $Q_i$  are the fitting parameters, as shown in Table 4.

The data points in Figs. 8b, 9b and 10b show that  $\sigma_y/\sigma_u$  has no appreciable effect on  $A$ . Because  $\sigma_y/\sigma_u$  is assumed to vary monotonically with  $\sigma_y$ , this also implies that  $\sigma_y$  has little effect on  $A$ . The tiny variations seen in  $A$  with respect to changes in  $\sigma_y/\sigma_u$  seen in these plots appear as scatter with no consistent trend. Hence, it is assumed that  $\sigma_y/\sigma_u$  has no effect on  $A$  in this regime and does not appear as a variable in the equation for  $A$ . From Figs. 8 and 9, both  $A$  and  $C_v$  appear to have a power-law relation with respect to  $C_2$  and  $C_3$ , although this is relatively weak between  $A$  and  $C_3$ , especially when  $C_2$  is high. Additionally, Fig. 10a suggests that  $C_v$  has an exponential dependence on  $\sigma_y/\sigma_u$ . The proposed nature of the dependencies between the individual variables are summarised in Table 5. These determine the form of constituent terms chosen in the Eq. 25, i.e.  $P_i \left(\frac{\sigma_y}{\sigma_u} - 1\right)$  for exponential relations and  $P_i C_i^{P_j}$  or  $Q_i C_i^{Q_j}$  for power relations.

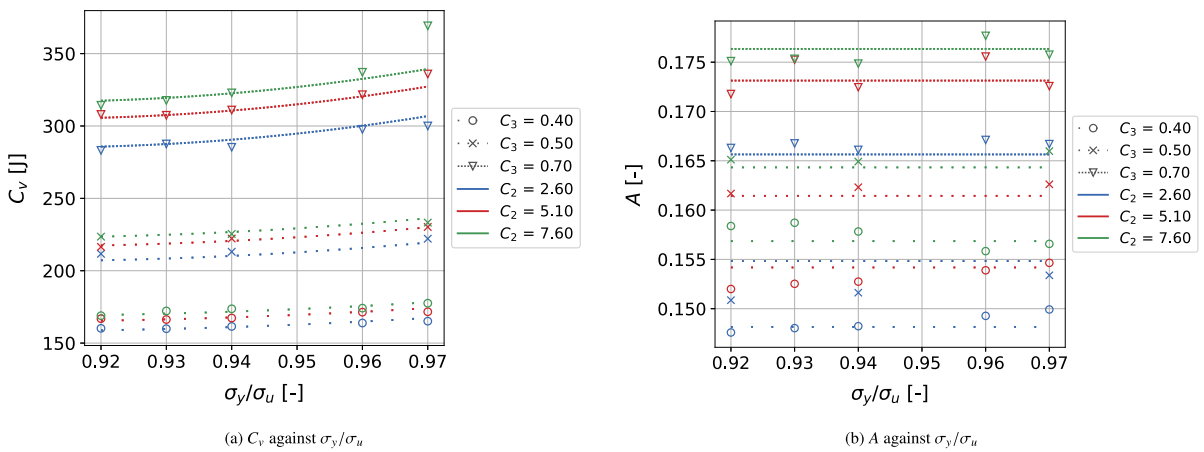
The data points in Figs. 8 to 10 also reveal information about the coupling between the plasticity parameter  $\sigma_y/\sigma_u$  and the damage parameters  $C_2$  and  $C_3$ . It can be seen that changes in  $C_3$  non-linearly affects the



**Fig. 8**  $C_v$  and  $A$  against  $C_2$  from the parametric simulations



**Fig. 9**  $C_v$  and  $A$  against  $C_3$  from the parametric simulations



**Fig. 10**  $C_v$  and  $A$  against  $\sigma_y/\sigma_u$  from the parametric simulations

**Table 4** Fitted calibration parameters

(a) $P_i$ parameters		
$i$	units	$P_i$
1	[J]	-0.89626
2	[-]	494.46
3	[-]	-3.2177
4	[J]	-22127
5	[-]	0.55185
6	[-]	2.0040
7	[J]	22481
8	[-]	1.6325
9	[-]	0.0027501
10	[-]	1.9707
(b) $Q_i$ parameters		
$i$	units	$Q_i$
1	[-]	5.0212
2	[-]	-0.84337
3	[-]	0.20165
4	[-]	0.21133
5	[-]	-5.0639
6	[-]	-0.84132
7	[-]	0.0026558

**Table 5** Dependencies of  $C_v$  and  $A$  on  $C_2$ ,  $C_3$  and  $\sigma_y/\sigma_u$

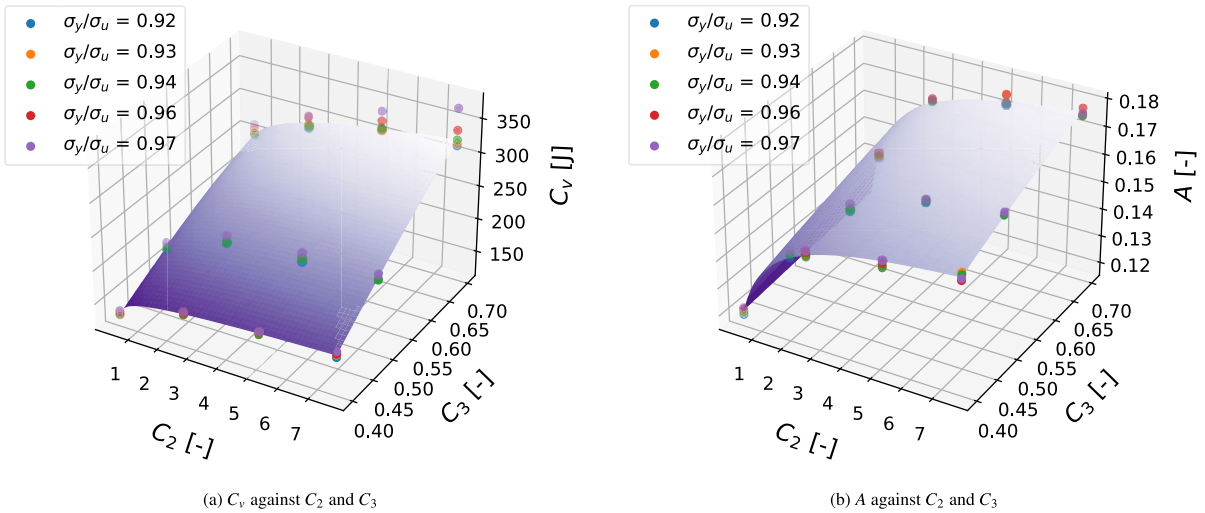
	$C_2$ [-]	$C_3$ [-]	$\sigma_y/\sigma_u$ [-]
$C_v$ [J]	Power	Power	Exponential
$A$ [-]	Power	Power	None

way  $C_v$  is related to  $C_2$  (Fig. 8a) and changes in  $C_2$  non-linearly affects the way  $C_v$  is related to  $C_3$  (Fig. 9a). The same can be said of the relationship between  $A$ ,  $C_2$  and  $C_3$ . Therefore, the terms  $P_i C_2^{P_j} C_3^{P_k}$  and  $Q_i C_2^{Q_j} C_3^{Q_k}$  are included in the equation for  $C_v$  and  $A$ , respectively. From Fig. 8a, 9a and 10a, the coupling effect is also seen to act between  $\sigma_y/\sigma_u$ ,  $C_2$  and  $C_3$ . So, multiplicative terms between  $P_i \left(\frac{\sigma_y}{\sigma_u} - 1\right)$ ,  $P_i C_2^{P_j}$  and  $P_i C_3^{P_j}$  are used in the equation for  $C_v$ .

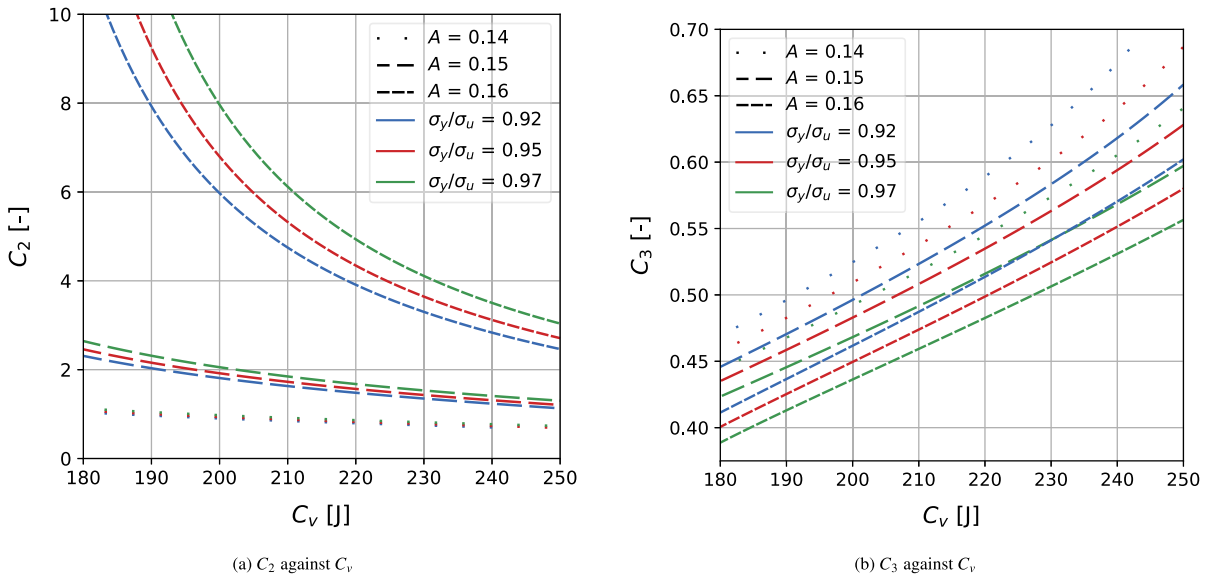
Since  $C_2$  and  $C_3$  are normalised parameters representing, respectively, the maximum (at  $\bar{\theta} = \pm 1$ ) and minimum (at  $\bar{\theta} = 0$ ) strains in the triaxiality-independent strain-based MMC damage locus (see sec-

tion on Damage above), some physical sense could be derived from these findings.

From Figs. 9 and 11, the effect of  $C_3$  is stronger on  $C_v$  than on  $A$ . This could be related to how the tunneling effects in the Charpy test, which have been shown to have a dominant effect in the behaviour (Wong and Walters 2025), begin at mid-thickness in the specimen, where  $\bar{\theta} = 0$ . In contrast,  $\bar{\theta}$  in the tensile test starts at 1 and gradually drops as necking occurs. The difference in the proportion of effects caused by  $C_2$  and  $C_3$  on  $C_v$  and  $A$  sheds light on the role played by each damage parameter in the process of calibration. While it is obvious that materials with high  $A$  and  $C_v$  are characterised by both high  $C_2$  and high  $C_3$ , these results reveal infor-



**Fig. 11**  $C_v$  and  $A$  against  $C_2$  and  $C_3$  from the parametric simulations together with fitted surface (using  $\sigma_y/\sigma_u = 0.94$  as an example)



**Fig. 12**  $C_2$  and  $C_3$  against  $C_v$  obtained from solving Eq. 25

mation about the trends that characterise materials with high notch toughness  $C_v$  but low uniaxial-test fracture elongation  $A$ . Fig. 11 suggests that for a material with triaxiality-independent damage behaviour, the combination of high  $C_v$  and low  $A$  is characterised by combinations of high damage initiation strains at  $\bar{\theta} = 0$  and low damage initiation strains at  $\bar{\theta} = 1$ . Conversely, a combination of low notch toughness and high uniaxial-test fracture elongation is characterised by low  $C_2$  and high  $C_3$ .

It is also worth noting that the effect of  $C_2$  (the maximum strain of the damage initiation locus) on  $C_v$ , which was identified as being very weak in previous work by the present authors (Wong and Walters 2024), could in fact become significant if the variations in  $C_2$  become very large (Fig. 8a).

Finally, given a set of material certificate values for a steel within the range of the parametric study, Eq. (25) can be solved numerically to find an estimation of the calibrated damage parameters  $C_2$  and  $C_3$ . Natu-

rally, the range of  $A$  and  $C_v$  for which solutions exists has its bounds, as can be guessed from the surfaces shown in Fig. 11. Furthermore, the solution should not involve extrapolation beyond the range investigated by the parametric study. Fig. 12 shows the solutions of Eq. 25 for different  $C_v$ ,  $\sigma_y/\sigma_u$  and  $A$  within the investigated range. It reveals that  $A$  has a stronger effect on the correlation between  $C_2$  and  $C_v$  and the correlation between  $C_3$  and  $C_v$  than does  $\sigma_y/\sigma_u$ , even when the coupling between plasticity and damage has been accounted for. To evaluate the consequences of this trend, observed in the internal damage parameters, for measured fracture properties, Eq. (25) is used in the next section to estimate the damage parameters and simulate the SENB fracture toughness test to obtain critical ductile fracture  $J$ -integral values,  $J_Q$ , for given combinations of  $A$ ,  $C_v$  and  $\sigma_y/\sigma_u$ .

#### 4 Correlation between $C_v$ and $J_Q$

A total of 15 simulations of the SENB test (Fig. 13) were performed, for  $A$  varying between 0.14 and 0.16,  $\sigma_y/\sigma_u$  varying between 0.92 and 0.97, and  $C_v$  varying between 150J and 250J, as shown in Table 6. The results are plotted in Fig. 14, along with the upper-shelf  $J_Q - C_v$  conversion developed by Wallin (2001), which has been adopted in BS7910 (BSI 2019), and a similar correlation developed by Schindler (2001). Comparison to other similar upper-self correlations (Gioielli et al. 2000; Merkle and Johnson 1985; Eason and Nelson 1989; Eason et al. 1991; Kussmaul and Roos 1985; Sreenivasan 2014) is omitted due to differences in testing conditions and fracture toughness definitions used to obtain those correlations. A review on differences between the other proposed correlations can be found in Wallin (2001, 2021).

The Wallin (2001) correlation is obtained by solving Eqs. (26) to (29):

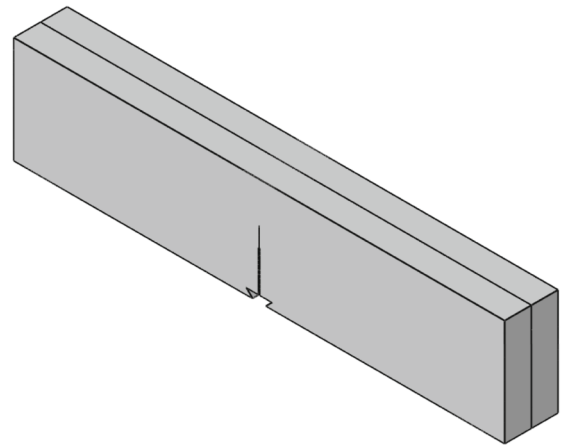
$$J_{1\text{mm},w} = 0.53C_v^{1.28} \tag{26}$$

$$m_w = 0.133C_v^{0.256} - \frac{\sigma_y}{4664} + 0.03 \tag{27}$$

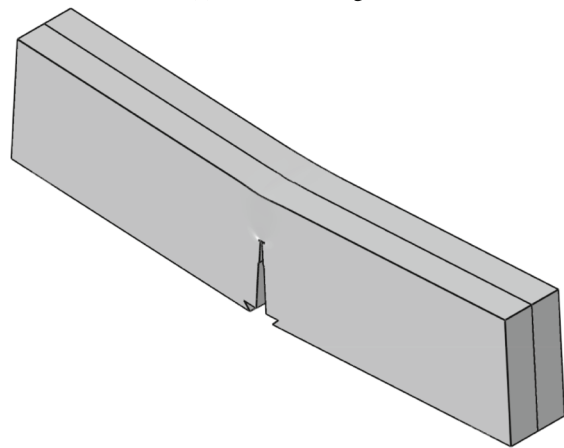
$$\sigma_{fW} \approx \sigma_y \left( 1 + \frac{150}{\sigma_y} \right) \tag{28}$$

$$\frac{J_{QW}}{2\sigma_{fW}} + 0.2 = \left( \frac{J_{QW}}{J_{1\text{mm},w}} \right)^{\frac{1}{m_w}} \tag{29}$$

where  $J_{1\text{mm},w}$  [kJ m<sup>-2</sup>] is Wallin’s  $J$ -integral at 1 mm crack extension;  $C_v$  [J] is the Charpy impact energy,



(a) Before loading.



(b) After loading.

**Fig. 13** Example SENB simulation from the parametric study (corresponding to  $\sigma_y/\sigma_u = 0.95$ ;  $C_v = 200$  J;  $A = 0.15$ ), with plane of symmetry

$m_w$  [-] is the Wallin  $J$ - $\Delta a$  curve exponent,  $\sigma_y$  [MPa] is the yield strength,  $\sigma_{fW}$  [MPa] is the average flow stress according to Wallin (1995); and  $J_{QW}$  [kJ m<sup>-2</sup>] is the critical  $J$  based on the 0.2mm crack extension offset using the Wallin (2001) conversion from Charpy energy.

The Schindler (2001) correlation is obtained by solving Eqs. (30) to (33):

$$J_{1\text{mm},s} = \frac{0.0250}{8m_s} (1000C_v - 7.97U_f) \left( \varepsilon_u - \frac{\sigma_u}{E} \right)^{1-m_s} \tag{30}$$

$$m_s = \frac{3}{4} \left( 1 + \varepsilon_u - \frac{\sigma_u}{E} \right)^{-1} \tag{31}$$

**Table 6** Values of  $\sigma_y/\sigma_u$  for different combinations of  $A$  and  $C_v$  used for generating the simulations in the parametric study on fracture toughness from SENB testing

$C_v$ [J]	150	180	190	200	210	220	250
$A$ [-]							
0.14	0.97	0.97	-	-	0.97	-	-
0.15	0.92, 0.95, 0.97	-	-	0.92, 0.95, 0.97	-	-	0.92, 0.95, 0.97
0.16	-	-	0.97	-	-	0.97	0.97

$$\sigma_{fS} = \frac{\sigma_y + \sigma_u}{2} \tag{32}$$

$$\frac{J_{QS}}{2\sigma_{fS}} + 0.2 = \left( \frac{J_{QS}}{J_{1\text{ mm},S}} \right)^{\frac{1}{m_S}} \tag{33}$$

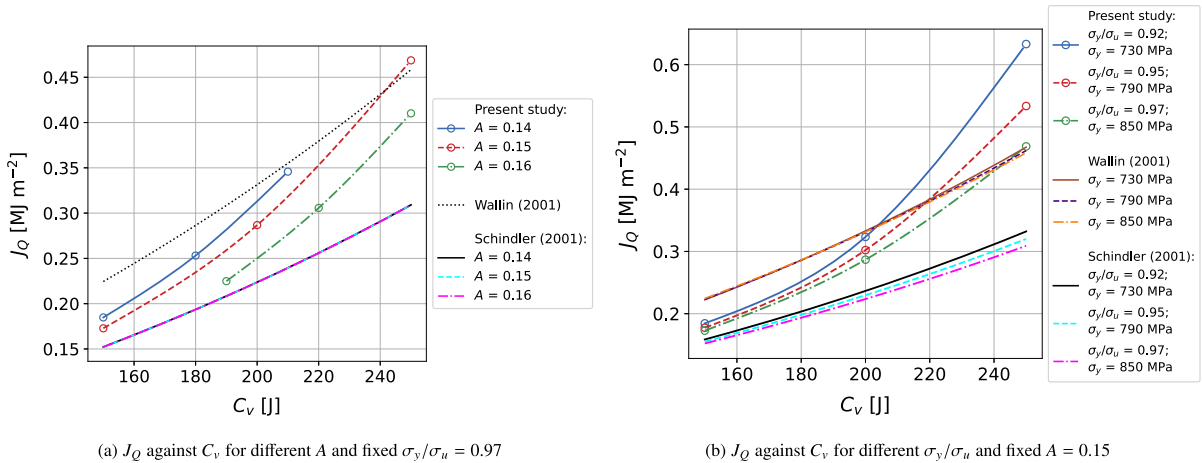
where  $J_{1\text{ mm},S}$  [kJ m<sup>-2</sup>] is Schindler’s  $J$ -integral at 1 mm crack extension;  $m_S$  [-] is the Schindler  $J$ - $\Delta a$  curve exponent;  $C_v$  [J] is the Charpy impact energy;  $U_f$  [N mm<sup>-2</sup>] is the specific energy or strain energy density of the tensile test, here approximated by the area under the engineering stress-strain curve;  $\varepsilon_u$  [-] is the uniform elongation expressed as a fraction;  $\sigma_u$  is the ultimate tensile strength;  $\sigma_y$  [MPa] is the yield strength,  $\sigma_{fS}$  [MPa] is the average flow stress according to Schindler (2001); and  $J_{QS}$  [kJ m<sup>-2</sup>] is the critical  $J$  based on the 0.2 mm crack extension offset using the Schindler (2001) conversion from Charpy energy.

#### 4.1 Implications of changes in $A$

Figure 14a shows how the correlation between  $J_Q$  and  $C_v$  varies for different  $A$ , when the  $\sigma_y/\sigma_u$  and  $\sigma_y$  are fixed. The parametric study shows that a higher  $J_Q$  is correlated with a lower  $A$  for a given  $C_v$  in this case. The Wallin (2001) correlation does not include  $A$ , but the Schindler (2001) correlation does show the same trend with  $A$  seen the parametric study, although its effect is so small it appears negligible. The  $A$  enters the correlation indirectly through  $U_f$  in Eq. (30), the specific energy of then tensile test, that is strongly correlated to  $A$  when the other engineering tensile test properties and, by extension, the plastic strain hardening properties are kept constant. From Eqs. (30) and (33), it can be seen that a decrease in  $U_f$  caused by a decrease in  $A$  leads to a increase in  $J_{QS}$  when all else is kept constant, even though this difference is so small that it is not visible in the plot (Fig. 14a).

Although both the present numerical results and Schindler (2001) indicate a weak negative correlation between  $A$  and  $J_Q$ , many sources report a positive correlation between some measure of ductile fracture initiation toughness (such as equivalent critical plane strain fracture toughness  $K_{IC}$ ) and either the tensile test strain energy density (associated with the area under the stress-strain curve) or the tensile test fracture elongation, for various ductile metals including steels (Li et al. 2022; Shashank Dutt et al. 2025; Alexopoulos and Tiryakioglu 2009; Hirth and Froes 1977). Taking into consideration the mechanisms assumed in Schindler’s model, in which  $\varepsilon_u$  accounts for strain hardening and  $U_f$  is used based on empirical grounds to account for the difference in the energies associated with cracked and uncracked geometries, an increase in  $A$  and  $U_f$  with the other engineering test properties kept constant can be considered to be associated with an increase in the portion of the energy dissipation associated only with the initial deterioration of the uncracked notch in the Charpy test, which is not reflected in  $J_Q$  values obtained from fracture toughness testing of a the pre-cracked test, such that the remaining portion of the  $C_v$ , which is correlated to the  $J_Q$ , decreases, reflecting a decrease in  $J_Q$ . However, due to the empirical nature of this use of  $U_f$  to account for the effect of the Charpy notch and the difficulty in accounting for these energies separately, it is difficult to confirm the validity of this hypothesis using Schindler’s model.

On the other hand, in the present numerical study, this trend of increasing  $A$  being accompanied by decreasing  $J_Q$  can be explained, for the assumed triaxiality-independent damage model, using mechanistic considerations relating to the fracture initiation strains and energetic considerations through partitioning of the Charpy impact energy. First, considering how all the energy attributed to ductile fracture initiation is



**Fig. 14**  $J_Q$  against  $C_v$ ; circles represent simulated data points

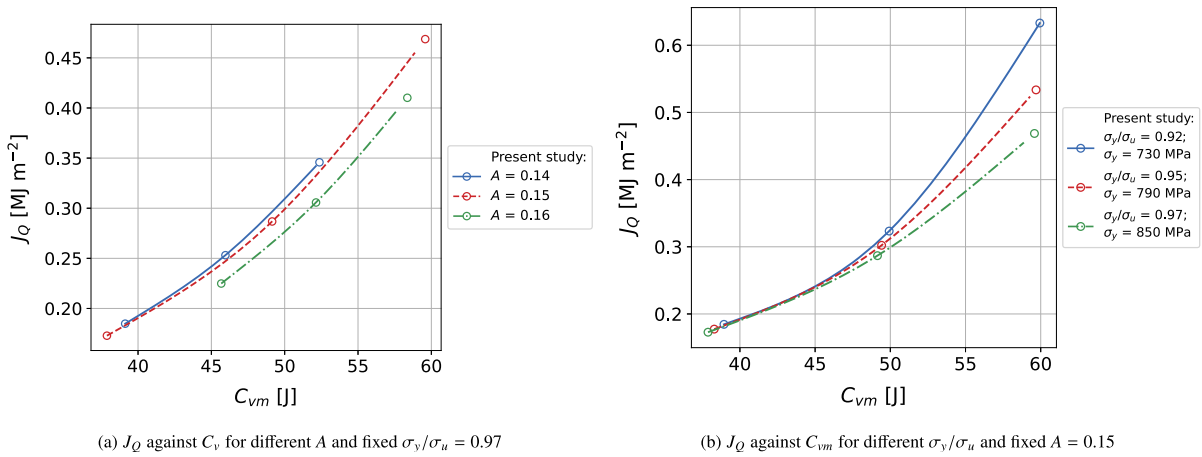
dissipated before the force peak in a ductile Charpy impact test (Fabry et al. 1993; Chaouadi and Fabry 2002) and how  $J_Q$  acts as a ductile fracture initiation measure evaluated close to the onset of stable crack extension in a fracture toughness test (ASTM 2023),  $J_Q$  is plotted in Fig. 15a against  $C_{vm}$ , the portion of the Charpy energy dissipated before the peak in the force-displacement plot from the instrumented Charpy test, instead of the total  $C_v$ . In this plot, variations due to  $A$  in the correlation between the Charpy measure and the  $J_Q$  measure dwindle. This reveals that differences in the  $C_v$  to  $J$  correlations observed for different  $A$  are related to differences in specifically the post-peak energy. This can in turn be related to the trend observed in the regression of  $A$  and  $C_v$  on  $C_2$  and  $C_3$  (Section 3).

The regression obtained in Section 3 (Fig. 12) indicates that maintaining a constant  $C_v$  while decreasing  $A$  involves decreasing  $C_2$ , the damage initiation strain at  $\bar{\theta} = 1$ , and increasing  $C_3$ , the damage initiation strain at  $\bar{\theta} = 0$ , in the assumed triaxiality-independent damage locus.  $A$  is strongly associated with  $\bar{\theta} = 1$  because damage starts building up with  $\bar{\theta} = 1$ , the stress state induced by the pre-necking uniaxial testing conditions. ( $\bar{\theta}$  gradually drops as the neck develops and damage accumulates.) The pre-peak  $C_{vm}$  is mainly influenced by  $C_3$  because the initial cracking in a Charpy test starts with tunnelling at the mid-thickness of the specimen, which is characterised by a plane-strain state of  $\bar{\theta} = 0$ . The post-peak Charpy behaviour and hence the total  $C_v$  are, in contrast, also affected by  $C_2$  in addition to

$C_3$ , because as the tunnelling progresses outward from mid-thickness towards the free surface and as shear lips develop, damage build-up involving the  $\bar{\theta} = 1$  of the free surface occurs too. As for the fracture toughness test, cracking begins with tunnelling at the mid-thickness like the Charpy test, and the stable ductile crack initiation value  $J_Q$  corresponds to only a small amount of crack extension.  $J_Q$  is therefore much more strongly affected by  $C_3$ , just like the pre-peak  $C_{vm}$  from the Charpy test. The trends observed in the influence of  $A$  on the  $C_v$  to  $J_Q$  correlations (Fig. 14a) and the lack thereof in the  $C_{vm}$  to  $J_Q$  correlations (Fig. 15a) are therefore related to the prevailing stress states involved in the determination of each of the engineering properties. A lower  $A$  for constant  $C_v$  is associated with lower damage strains for  $\bar{\theta} = 1$  and higher damage strains for  $\bar{\theta} = 0$ , which is tied to an increase in both the  $C_{vm}$  and the  $J_Q$ .

As a conceptual aid for the above discussion, a simplified qualitative summary of the strength of the interactions between the different variables is given in Table 7, and the evolution of the normalised Lode angle  $\bar{\theta}$  at key locations in the tensile, Charpy and SENB simulations for the model calibrated to S690QL steel from Wong and Walters (2025) is shown in Fig. 16.

It is understood that the trend with which  $C_2$  and  $C_3$  changes for changing  $C_v$  or  $A$ , on which the preceding discussion is based, is limited by the assumption of triaxiality independence in the damage locus. If triaxiality dependence were included in the damage locus, the higher peak triaxialities in the SENB test

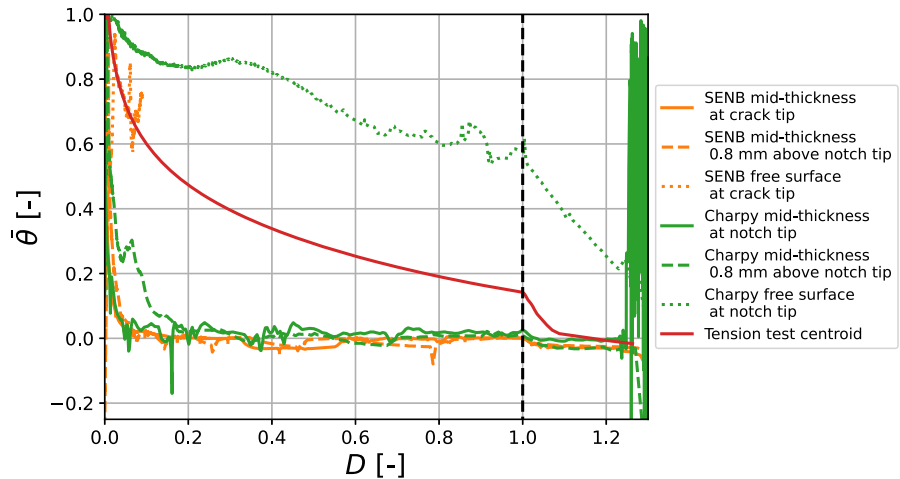


**Fig. 15**  $J_Q$  against  $C_{vm}$ ; circles represent simulated data points

**Table 7** Qualitative description of the strength of the interaction between the engineering parameters and material parameters as a conceptual aid

	$C_{vm}$ [J]	$C_v$ [J]	$A$ [-]	$J$ [MJ m <sup>-2</sup> ]
$C_2$ [-]	Weak	Strong	Strong	Weak
$C_3$ [-]	Strong	Strong	Weak	Strong
$\sigma_y/\sigma_u$ [-]	Strong for high $C_3$ ; Weak for low $C_3$	Strong for high $C_3$ ; Weak for low $C_3$	None	Weak

**Fig. 16** Evolution of the normalised Lode angle  $\bar{\theta}$  as damage  $D$  builds up at key locations in the tensile, Charpy and SENB simulations for a calibrated S690QL steel (from Wong and Walters (2025))



( $\eta \sim 2.2; \bar{\theta} = 0$ ) compared to the Charpy test ( $\eta \sim 1.7; \bar{\theta} = 0$ ) could be associated with lower damage initiation strains, giving combinations of damage parameters and simulations that result in a decreased  $J_Q$  and constant  $C_v$  for decreased  $A$  by virtue of decreased damage strains at high triaxiality ( $\eta \sim 2.2; \bar{\theta} = 0$ ), despite the higher damage strains at peak Charpy triaxialities ( $\eta \sim 1.7; \bar{\theta} = 0$ ) that would accompany the lower

$\bar{\theta} = 1$  damage strains described above. This accounts for the discrepancy between the presently observed trend and that which is described in the literature (Li et al. 2022; Shashank Dutt et al. 2025; Alexopoulos and Tiryakioglu 2009; Hirth and Froes 1977) for  $A$  and  $J_Q$ . The presently observed negative correlation is one that is specific to the triaxiality-independent assumption. Nonetheless, the considerations relating to the

stress states and energy attributions behind these trends remain the same, and it remains that the correlation of  $J_Q$  with  $A$  is weak (Chatterjee (2020) correlation coefficient  $\xi = -0.02$ ), and the  $J_Q$  is correlated to  $C_{vm}$  so strongly in comparison ( $\xi = 0.79$ ) that changes in  $A$  have only a very weak effect on the  $J_Q$ -to- $C_{vm}$  correlation.

#### 4.2 Implications of changes in $\sigma_y/\sigma_u$

Fig. 14b shows how the correlation between  $J_Q$  and  $C_v$  varies for increasing  $\sigma_y/\sigma_u$ , with a correspondingly increasing  $\sigma_y$  (Eq. (17)), when the  $A$  is fixed. The Wallin (2001) correlation does not account for  $\sigma_y/\sigma_u$ , but predicts a higher  $J_Q$  for a lower  $\sigma_y$ , when all else is constant. The parametric study suggests that this trend still holds when the  $\sigma_y$  is accompanied by a higher  $\sigma_y/\sigma_u$ , with the difference that a slightly larger effect on the  $J_Q$  is predicted when the  $C_v$  is large. The effect of the increasing  $\sigma_y/\sigma_u$  and  $\sigma_y$  on  $J_Q$  observed here also agrees with the trend seen in the Schindler (2001) correlation. Nonetheless, the change in  $J_Q$  due to changing  $\sigma_y/\sigma_u$  and  $\sigma_y$  is small in comparison to that due to the change in  $C_v$ . At a  $C_v$  of 200J, a change in  $\sigma_y/\sigma_u$  from 0.92 to 0.97, a range which spans 60% of the  $\sigma_y/\sigma_u$  range of the considered dataset (Fig. 4; Section 3.1.1), and includes 80% of the number of observations in the dataset, leads to only a 15% change in  $J_Q$ . The same change is effected by a change in  $C_v$  corresponding to a mere 5% of the range spanned by the  $C_v$  values in the considered dataset.

Comparing Figs. 14b and 15b, it is worth noting that although varying  $\sigma_y/\sigma_u$  has a smaller effect on the correlation between  $J_Q$  and  $C_{vm}$  than on the correlation between  $J_Q$  and  $C_v$  for low  $C_{vm}$  and low  $C_v$ , this does not apply to high  $C_{vm}$ . This is unlike the effect of variations in  $A$  on the  $C_{vm}$ -to- $J_Q$  correlation, where the effect of the variation in  $A$  is seen to be small for all  $C_{vm}$ . Nonetheless, considering  $C_{vm}$  instead of  $C_v$  still has the effect of reducing the influence of variations in  $\sigma_y/\sigma_u$  on the prediction of fracture toughness from the Charpy data.

## 5 Conclusions

A numerical investigation of the correlation between  $J_Q$  and  $C_v$  and how it is affected by variations in

$A$ ,  $\sigma_y/\sigma_u$  and  $\sigma_y$  has been performed using a rate- and temperature-dependent damage-softening plasticity model. To this end, a parametric study, involving a range spanned by a set of material certificate data and some empirical assumptions deduced from the dataset, was carried out, in which the SENB fracture toughness test was modelled for varying  $A$  and  $C_v$ , assuming correspondingly varying damage parameters which are calibrated based on the  $C_v$  and the  $A$ . The calibration was obtained from a regression analysis of  $C_v$  and  $A$  on material parameters  $C_2$ ,  $C_3$  and  $\sigma_y/\sigma_u$ , performed on a series of Charpy and tensile simulations for each of 52 combinations of  $C_2$ ,  $C_3$  and  $\sigma_y/\sigma_u$  values spanning the parametric space. The following conclusions are drawn from this study.

The pre-peak energy from the force-displacement plot in an instrumented ductile Charpy test  $C_{vm}$  has a stronger ( $\xi = 0.79$ ) and more independent correlation with  $J_Q$  than does  $C_v$  ( $\xi = 0.57$ ; Figs. 14 and 15).  $J_Q$  is predominantly related to the onset of ductile crack extension, but  $C_v$  includes a significant amount of energy attributed to continued ductile crack propagation. On the other hand,  $C_{vm}$  includes less of this crack propagation energy but includes all the energy up to the onset of ductile fracture and is thus more closely tied to  $J_Q$ . Developing and making use of correlations between  $C_{vm}$  and  $J_Q$  would result in more consistent predictions of ductile fracture initiation toughness than given by correlations between  $C_v$  and  $J_Q$ .

The  $\sigma_y/\sigma_u$  ratio has a weak correlation with  $C_{vm}$  ( $\xi = 0.17$ ) and with  $J_Q$  ( $\xi = 0.17$ ), especially for low-value ranges of  $C_{vm}$  and  $J_Q$ . The correlation between  $C_{vm}$  and  $J_Q$  themselves is far stronger ( $\xi = 0.79$ ), such that changes in  $C_{vm}$  can be seen to give predictions of far more significant changes in  $J_Q$  than changes in  $\sigma_y/\sigma_u$  could be related to (Fig. 15b). The  $\sigma_y/\sigma_u$  ratio alone, without the information  $C_{vm}$  or  $C_v$ , is not useful for ductile fracture initiation toughness prediction.

For a model with a triaxiality-independent, Lode-dependent strain-based damage locus, the  $A$  is weakly and negatively correlated to  $J_Q$  for a constant  $C_v$ . Changes in  $A$  appear to affect the correlation between  $C_v$  and  $J_Q$  because  $A$  is correlated to the later-stage portion of the  $C_v$  energy, broadly associated with  $\bar{\theta} = 1$ , corresponding to both the uniaxial tensile pre-necking stress state and the free-surface stress state which becomes relevant with significant ductile crack propagation in the Charpy specimen. For a constant  $C_v$ , a decrease in  $A$  is correlated with a decrease in

that later-stage proportion of the energy, such that the  $C_{vm}$  and  $J_Q$  associated with fracture initiation increase. Even though the negative trend of the correlation between  $C_v$  and  $J_Q$  seen in the numerical results due to the assumption of triaxiality independence does not agree with experimental observations (Li et al. 2022; Shashank Dutt et al. 2025; Alexopoulos and Tiryakioglu 2009; Hirth and Froes 1977), it remains that  $J_Q$  is much more strongly correlated to  $C_{vm}$  than to  $A$ . Differences in  $A$  only have a small effect on the correlation (Fig. 15a) between  $J_Q$  and  $C_{vm}$ , which acts as a more reliable indicator than  $C_v$ . Due to this, the  $A$  alone, without  $C_{vm}$  or  $C_v$ , is in general not useful for ductile fracture resistance considerations.

Although varying  $\sigma_y/\sigma_u$  and  $A$  has some effect on how the total notch energy  $C_v$  is correlated to  $J_Q$ , it does not reflect a significant effect on the ductile fracture initiation toughness but is rather associated with the fact that the  $C_v$  includes a significant portion of energy for stable ductile propagation and fracture occurring at the specimen's free surface, while  $J_Q$  primarily concerns the onset stage of stable ductile tunnelling behaviour at the centre of the specimen. The  $\sigma_y/\sigma_u$  and  $A$  are seen to have an even smaller effect on the correlation of  $C_{vm}$ -to- $J_Q$  in comparison with the  $C_v$ -to- $J_Q$  correlation, especially for low  $C_{vm}$ . When predicting a fracture toughness for the sake of assessing the limit state associated with the onset of stable ductile crack propagation using alternative tests in situations where direct fracture toughness testing is not an option, the advice is to use  $C_{vm}$  instead of  $C_v$ ,  $\sigma_y/\sigma_u$  or  $A$ .

## 6 Recommendation for further work

In this paper, it has been proposed that Charpy-based measures perform better than the tensile test properties for the purpose for incorporating sufficient upper-shelf toughness into design, and a damage-mechanics-based model is used to estimate the relationship between Charpy energy and fracture toughness. However, the relationship obtained in this work is based on a range of properties typical for S690QL steel and involves the assumption of triaxiality independence which less applicable to some steels. Further research is necessary to describe a more general relationship between Charpy energy and fracture toughness for a broader range of steels before the present findings can be translated into industrial practice. This could be done through further

damage-mechanics modelling studies with the use of a more nuanced description for the different stress-state dependence, rate dependence and plastic strain hardening behaviours observed across a range of steel grades or through statistical treatment of large datasets of the identified key mechanical test properties.

Furthermore, this paper proposes that imposing a  $\sigma_y/\sigma_u$  limit is not a useful measure for ensuring sufficient local ductility against fracture in the upper-shelf as assumed in industry practice. This raises the question of discontinuing its current use as a blanket material ductility requirement for high-strength steels and including it only in the design for failure modes (Wong and Walters 2021) in which it plays a role, such as plastic buckling (Wong and Walters 2023). However, the present study has considered only purely ductile fracture, and the role of the  $\sigma_y/\sigma_u$  in the upper ductile-to-brittle transition, where cleavage fracture might occur after significant ductile tearing, still needs to be considered, due to the potential effects of plastic straining and plastic zone stress redistribution on cleavage fracture (Ruggieri and Dodds 2015; Ruggieri et al. 2015), considering how  $\sigma_y/\sigma_u$  is correlated to plastic strain hardening behaviour (Section 3.1.3). Further research is necessary to resolve this question because the literature on the upper-transition from the perspective of macroscopic approaches (Lindqvist and Wallin 2019; Walters and Wallin 2013) is sparse relative to the work done for the lower transition, and as for local approaches, significant improvements are necessary and still being developed for combined ductile and cleavage models (e.g. Seo et al. 2025; Tanguy et al. 2005).

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**Data Availability Statement** Data will be made available on request.

## Declarations

**Competing interests** The authors declare no competing interests.

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