SUBSPACE RECYCLING TECHNIQUES FOR THE ACCELERATED SOLUTION OF IMPLICIT DISCRETE FIELD FORMULATIONS

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ABSTRACT

Modeling slowly varying transient electric high voltage fields, magneto-quasistatic fields or transient temperature fields within coupled thermo-/electrodynamic field simulations using geometric discretization methods such as the Finite Integration Technique or the Whitney Finite Element method results in stiff nonlinear systems of ordinary differential equations or of nonlinear differential-algebraic equations of index 1, respectively [1]. For their solution adaptive embedded implicit time integration schemes can be used. Within multi-stage singly diagonal implicit Runge-Kutta type schemes, where several nonlinear systems of equations have to be solved for each time step using Newton or quasi-Newton methods and in multi-stage linear-implicit Rosenbrock time integration schemes the problem of a repeated and successive solution of high-dimensional linear algebraic systems of equations with (near) identical system matrix and different righthand side vectors occurs. For the solution of these systems preconditioned conjugate gradient (PCG) schemes are used, where the preconditioned vector subspaces resulting from a previous PCG iteration run are recycled. These subspaces are kept deliberately low-dimensional when using (algebraic) multigrid preconditioners (e.g. [2]). They can be reused within a subspace projection extrapolation start value generation scheme [3] acting as an implicit deflation of the following PCG iteration process. Alternatively, within an Augmented PCG scheme [4] the residual vectors of the AugPCG are kept orthogonal with respect to the recycled vector subspace using an additional projection step within each iteration. Numerical results for three-dimensional electric and magnetic simulations are presented and the efficiency of the new schemes is compared to that of standard non-multiple righthand side schemes.

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