Optimization methods for integrated electricity-natural gas systems

Akshada Anand Palnitkar (5235170)

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Akshada Anand Palnitkar (5235170)

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Abstract

Energy systems have been continuously evolving with the advancement in technology. The expected result would be a smooth transition towards clean and more sustainable energy systems which work closely with one another. Conventionally electricity and natural gas have been the most commonly available resources. These two extensively used energy sources are on the forefront of this progress. The focus on using the available energy resources optimally and effectively is a need for energy providers as well as the energy consumers. However, there are different challenges in making these optimal decisions because most of the energy providers have their independent networks. The progress in energy sector has promoted the use of integrated energy systems from different geographical locations. A smart energy system that connects energy consuming sectors to the power grid to improve the synergy between energy production and consumption is referred as integrated energy system or sector coupling.

This is the point where the research on Integrated Electricity and Natural Gas System (IEGS) plays an important role. These integrated systems include several large subsystems referred as areas consisting of electricity and natural gas networks. These large systems are connected to one another through one or more connections known as tie-lines and/or tie-pipes and the energy dispatch can be controlled by the area operator. The main intention of an integrated system is that the electricity and natural gas networks are closely linked as opposed to the formerly isolated systems. The interdependence of systems adds to the complexity of the network and calls for new methods to optimally solve this multi-area IEGS problem.

The goal of this thesis report is to delve into the different optimization methods for multi-area IEGS and providing a benchmark for the possible methods based on their performance. There has been a good deal of research on single-area electricity and gas systems, but the multi-area problem is more complex. Hence it is important to understand how optimization problems are formulated in the context of such multi-area IEGS and what approaches are currently used to solve it. This study comprises of an optimization problem for an IEGS network by considering all the network constraints for these integrated systems. It also emphasizes the need of relaxation methods used to convexify the highly non-convex natural gas flow equations as they cause difficulty in finding the optimal solution.

The goal of an Optimal Energy Flow (OEF), optimization problem for an IEGS network consisting of an objective function is to minimize the system's overall operational cost while

satisfying the constraints for the electricity network, natural gas network and the coupling constraints. The coupling constraints play a critical role in finding the optimal solution since multi-area systems are being studied. In this overview, various formulations and solution methods of the optimization problem have been examined. Their performance has been compared based on certain performance metrics in order to find the best possible method to solve such complex optimization problems.

Out of the two networks, the electricity network constraints are usually either linear or convex and hence comparatively easy to deal with using existing solvers. However, the natural gas network constraints contain nonlinear, non-convex constraints which make it difficult to determine the solution of the optimization problem. Such problems cannot be easily solved using existing solvers. There are some techniques for overcoming such difficulties posed by these non-convex equations such as relaxing/approximating these non-convex constraints and reformulating the optimization problem. This study discusses a number of strategies used to reformulate the non-convex constraints into linear, quadratic, or mixed integer forms. Having a reformulated problem makes it possible to find the best possible solution for the defined problem. If this reformulation is not done, the original problem could be infeasible.

Since the integrated systems usually are extremely large, it is vital to separate them into smaller subsystems in order to solve the optimization problem efficiently. For instance, the electrical and natural gas networks can be separated based on their physical properties, leaving just the coupling constraints between these regions to be taken into account. This is known as decentralizing the network. There are different methods used for the decentralized optimization and it is important to study these algorithms for multi-area IEGS. A summary of the features and simulation results of various methods has been provided.

Finally, this thesis concludes by summarizing all the primary findings for the best possible optimization method and suggesting future research possibilities in this topic. To determine which approach best fits the OEF problem for IEGS, a thorough numerical comparison of various relaxation techniques, centralized and decentralized scheme of operation must be made. It is imperative to note that these techniques solve the approximated problem, not the original nonlinear, non-convex problem, which would leave some room for errors and hence an area of future improvement.

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Preface

The following document is prepared at the Delft Center for Systems and Control (DCSC) within Delft University of Technology as a part of my Master of Science graduation thesis. This report presents a study of Optimization methods for Integrated Electricity and Natural gas systems. The integration of different energy sources, for example natural gas network with the electricity network has provided the energy business with new opportunities and tremendous challenges.

Since electricity and natural gas are the two most commonly utilized energy sources, there has been quite some research on their independent usage. New optimization approaches are required to balance the electricity and natural gas networks as the natural gas network brings a strong non-convexity to the optimal flow problem making it difficult to solve using commercial solvers. This study attempts to provide insights into some of the available methods of relaxation/approximation, their performance, efficiency and reliability of these methods for multi-area networks.

The investigation begins with an introduction of the research problem and the rationale for the investigation. This report emphasizes the significance of relaxation techniques used to relax the non-convexity and studying optimization methods for integrated electricity natural gas systems. The mathematical formulations and optimization strategies utilized to solve the models are explained in detail. The performance of optimization algorithms is assessed based on certain performance metrics. Finally, this thesis adds to the continuing study on optimization approaches for integrated electrical and natural gas systems. It is my hope that the findings of this study would help regulators, policy-makers and industry professionals develop and operate more efficient, dependable and sustainable energy systems.

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Akshada Anand Palnitkar (5235170)

"Right knowledge is the ultimate solution, guiding us beyond illusion to the truth."
— *Inspired by the teachings of the Bhagavad Gita, Chapter 2*

Chapter 1

Introduction

Chapter 1 introduces fundamental concepts related to optimization methods for Integrated Electricity and Natural Gas System (IEGS) and builds the foundation of this report. It begins with a general overview, which contains key concepts required to understand the world of IEGS and the necessity of optimization methods for these large systems. Further it presents the research questions and the motivation for studying this topic.

1-1 Overview

In this dynamic and rapidly evolving energy landscape, there has been increasing interest in the field of energy technology. Due to the depletion of fossil fuels, many researchers claim that the world is on the verge of an energy catastrophe [7]. In this regards, it can be seen that more and more companies and individuals are focusing on energy-related technologies such as, seeking alternatives for non-renewable resources, using the existing sources as effectively as possible, etc. The investment in this topic has also been increasing since there is widespread concern about reducing the cost of energy usage and maintaining a sustainable supply of energy for increasing demands. Integrated energy systems have thus gained lot of importance and are increasingly popular in research and development. With a strong base of research, industries can use these integrated energy solutions in real life applications to provide optimal as well as more sustainable solutions to the consumers.

In the past, independent infrastructures have been responsible for supplying consumers and businesses with the majority of their energy needs. Even if the two energy systems worked in the same network, the mathematical problem formulation has been historically such that the two networks do not interfere with each other. To give an example of such a scenario where independent suppliers play a role, consider petroleum which is mostly utilized in transportation, whereas coal and natural gas are used to produce electricity and heat homes. These independent systems are still common today. With the development in the energy industry, this sector will have more interconnected systems in the future for the optimal use of resources.

Figure 1-1 shows an example of simple IEGS. The green arrows show electricity network while blue arrows refer to natural gas pipeline. Both the electricity load and gas load can be fulfilled while optimally using the resources and preventing wastage. The IEGS, refer to the interconnection of electricity grids and natural gas infrastructure, enabling the exchange of energy between these traditionally distinct systems. These two networks have been considered independently for the longest time [8], [9].

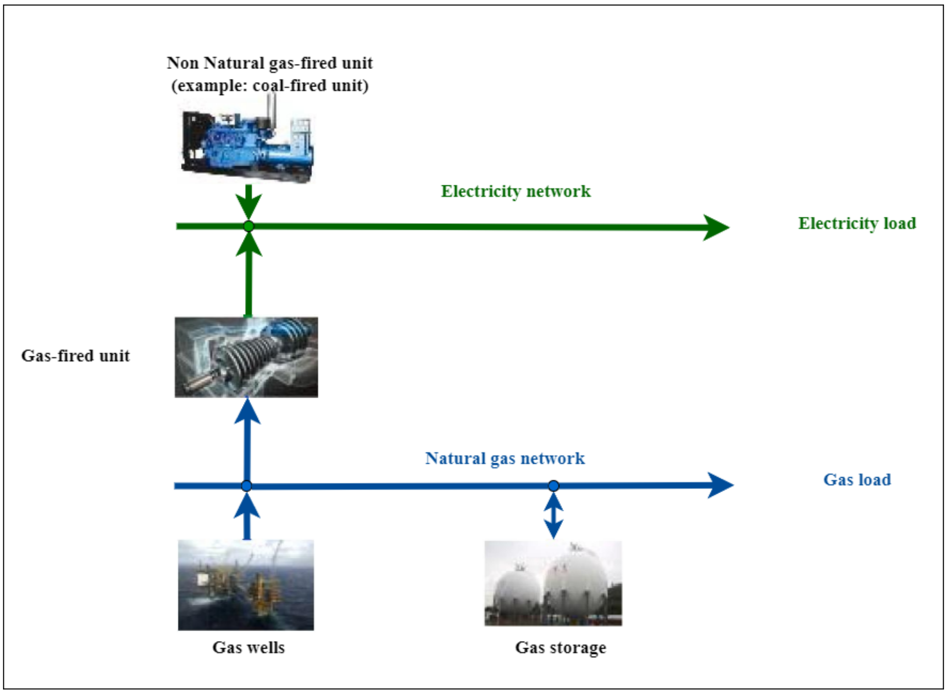


Figure 1-1: Example integrated electricity and natural gas system [1]

Modern integrated energy systems are characterized by their large scale and complex nature, comprising of multiple subsystems located across different areas and serving diverse types of users. These systems aim to efficiently meet the energy demands and enables optimization opportunities for improved reliability, enhanced flexibility, cost-effectiveness, etc. Interdependent energy infrastructure has thus become a reality and that is for the advantage of not only the consumers but also the producers. On the contrary, these systems also introduce new challenges in terms of modeling, optimization and computational loads [10]. To ensure dependable, economical and sustainable operation, the size and complexity of these systems necessitate advanced modeling, optimization and control methodologies. The ongoing advancements in technology, policy frameworks and stakeholder collaboration continues to drive the evolution and optimization of these integrated energy systems.

The focus of this study is on electricity and natural gas, which are the two most popular and widely available sources of energy [11]. The electricity grid forms the backbone of any urban energy systems since it provides energy to residential, commercial and industrial consumers. It includes power generation sources, transmission lines, distribution networks, etc. that deliver electricity to end-users. The natural gas networks also deliver natural gas to residential, commercial and industrial users for heating, cooking and industrial processes. These networks comprise pipelines, storage facilities and distribution systems to ensure the reliable supply of natural gas. Rapid development of Natural Gas-fired Unit (NGU)s in the power industry is a cause of natural gas being available at low cost and possessing higher efficiency [12].

The electricity and natural gas systems are interdependent, as seen in Figure 1-2. Despite the growing discussion around electricity and natural gas networks interactions, there is a dearth of research on multi-area IEGS in comparison to single-area studies. Typically in the case of an IEGS, the NGUs act as producers in electrical networks and consumers in natural gas networks. A notable feature of the IEGS is that the system operator can dispatch both networks and ensure optimal system operation rather than just one network [13]. The effective integration of electricity and natural gas systems has emerged as a critical challenge for ensuring reliable, efficient and sustainable energy supply.

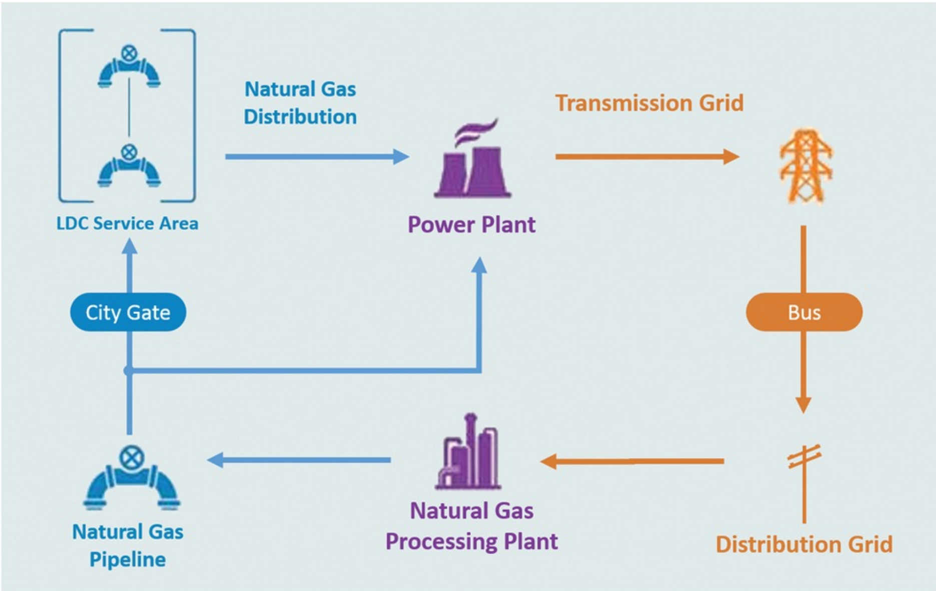


Figure 1-2: Interdependence of electricity and natural gas system [2]

Multi-area integrated electricity and natural gas networks

The IEGS in different geographical locations can be connected by electrical or gas transmission lines, thus formulating multi-area IEGS. A multi-area IEGS consists of different subsystems, thus enabling the transmission of excess energy between each other. Multi-area problems play a significant role in the optimization of these integrated systems. As electricity and natural gas networks often span multiple regions, it becomes essential to account for the interactions and dependencies between these areas. Multi-area problems involve addressing coordination, communication, and decision-making challenges among various stakeholders, including power grid operators, gas pipeline operators, market participants, and regulators.

Figure 1-3 shows a schematic of a sample multi-area IEGS where 'xxx' can be any number of areas. The interconnections between the network are referred to as tie-lines for electricity and tie-pipes for natural gas. A multi-area model differs from the traditional single-area IEGS model as it takes tie-lines and tie-pipe scheduling into account.

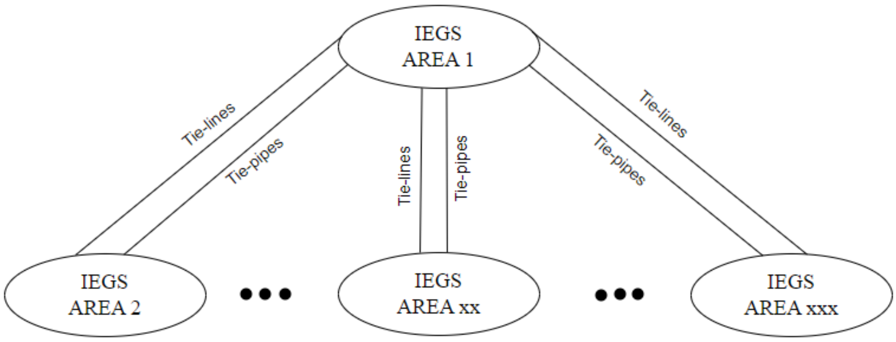


Figure 1-3: Example multi-area IEGS [1]

As the demand for energy continues to grow, optimizing the operation and planning of these interconnected networks becomes paramount. This report explores the concept of optimization methods specifically tailored to address the complexities and intricacies of integrated electricity natural gas systems. In the context of energy systems, optimization methods are employed to enhance the performance of integrated electricity natural gas networks by maximizing overall efficiency, minimizing costs and ensuring system reliability. In an integrated setup, the coordinated operation is converted to optimal operation problem. This is because the operator needs to dispatch from two networks and optimal operation of the overall system, rather than optimal operation of a single network.

Few pros and cons of IEGS are listed below:
Pros:

1. Lowering the use of primary energy sources while meeting the increasing energy demand.
2. Boost asset usage to cut down on capital expenditure.
3. Cost effective provision of flexibility in the electrical power systems.
4. Increased reliability of the electrical power system (e.g. security of supply).
5. Carbon emission can be reduced by increasing energy efficiency of the system as the operation is optimized.

Cons:

1. The fragmented institutional and market structures in the different energy sectors.
2. The increased complexity of integrated energy system necessitates the use of more robust analysis software.
3. The integration of various energy systems may result in systems that are more prone to cascading failures that compromise reliability of supply.
4. Research and development in integrated systems is multidisciplinary, which presents a difficulty because different technical knowledge requirements arise from diverse economic and market contexts.

Economic dispatch and optimal energy flow problem

When discussing a transmission system, economic dispatch refers to the process of economic optimization, given the operational limitations of the generating system, that chooses a combination of generation levels to satisfy demand at the lowest feasible cost [14]. The reduction of consumer energy costs is the primary goal of economic dispatch. The economic dispatch problem for a given network of power generators minimizes the total operating cost while determining the quantity of power generated by each unit for a given demand. In order to determine the minimum cost of power generation, such a problem requires n power generators [15].

In the past, the Economic Dispatch Problem (EDP) was formulated as a convex optimization problem or non-convex optimization problem depending on the constraints of the gas-flow equations and other components in the network. The convexity or non-convexity of the problem depends on how the components in the network are modeled. As an illustration, consider a model with energy flow and gas flow equations which are nonlinear, thus non-convex. The EDP formulated for such a model will be non-convex. This problem becomes difficult to solve hence giving rise to the need of studying different methods to solve EDP depending on the problem formulation. Some examples of traditional convex optimization methods consist of Newton method [16] and gradient search method [17]. Examples of non-convex optimization methods include genetic algorithm [18], particle swarm algorithm [19]. Most of these algorithms solve EDP in centralized way, wherein a central controller collects all the information and processes large amounts of data [17]-[19].

The optimization problem formulation can be done separately taking into account constraints for electricity and natural gas networks. The Optimal Power Flow (OPF) problem is primarily for electricity network and addresses the power flow constraints. Numerous OPF techniques have been studied to reduce cost or power loss [20]. OPF combines economic dispatch and optimal flow problems and hence can be considered as an extension of the conventional EDP [21]. For simplification, steady-state gas flow equations are used [22]. The nonlinear form of gas flow equations adds to the complexity of the optimization problem. As these systems are very large, with multiple parameters being involved, the need for computational power increases. The traditional solvers fail to provide a solution to these problems. It is therefore necessary to use some approximation techniques or relaxations to convert the problem into simpler form so that it can be put into an existing solver. In order to solve this reformulated nonlinear problem, it is necessary to study the different techniques used for approximating these nonlinear constraints.

Figure 1-4 shows an example of a model introduced by Aurangzeb *et al.* in [3], which can be considered to understand the supply and demand side of a basic power flow system whilst diving into details of Optimal Energy Flow (OEF). Industrial load, commercial load and residential loads make for the demand side, while the distribution systems, transmission network, and generation systems are part of the supply side. The control center is where the area operator makes the optimal dispatch decisions. This can be centralized or decentralized type of optimization, which will be discussed in detail further. The power flow is unidirectional, from supply side to the control center and from control center to the demand side. However, the information related to other power flow parameters is bidirectional in-case of both supply and demand side.

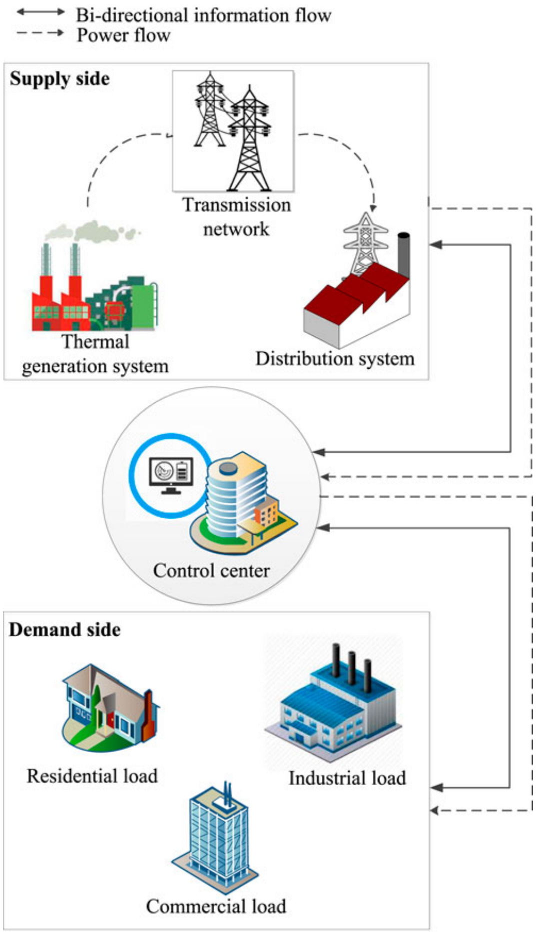


Figure 1-4: Example of power flow system [3]

In the planning and execution of an IEGS, the OEF plays a crucial role. Natural gas flow presents a challenge in solving the optimization problem when compared to fast electrical transients [23]. But as the systems get more inter-linked, the economic dispatch problem needs to be solved simultaneously. A mathematical model in the form of an optimization problem which integrates the optimal dispatch problem for electricity and natural gas networks is discussed in [24]. Numerous studies have been conducted on integrated networks, paper [25] discusses a security constrained scheduling framework.

The OEF problem deals with the dispatch of optimum amount of energy through the network. This can be mathematically expressed as an optimization problem. Generally an optimization problem consists of an objective function, the decision variable (parameter to be optimized), constraints that restrict solution to certain area (equality constraints), constraints that restrict solution to certain allowed region of the parameter space (inequality constraints). The final goal of this optimization is to minimize the total cost of the network, while meeting the electricity and natural gas constraints. Since the focus of this study is only IEGS network, the objective function could be the total cost of the multi-area IEGS which includes the natural gas fuel cost of NGUs, operation cost of the non-NGUs, load shedding cost, start-up cost for units, shut-down costs, etc. This is formulated in different ways in various studies and hence different optimization algorithms are used to solve these optimization problems.

Optimization algorithms

In this report, we delve into the different types of optimization models and algorithms utilized for integrated electricity natural gas systems. Centralized or decentralized methods are used by most of the researchers. Figure 1-5 shows these two types of widely used networks. The red circle represents the controller while yellow circle represents subsystems. The black lines show connections between the subsystems and/or controllers representing the communication between them. In centralized methods, there is always a central control system, which is connected to all the subsystems. On the other hand, a decentralized scheme comprises of several controllers that communicate with each other and their individual subsystems. Thus, under decentralized control, hierarchical control can be applied.

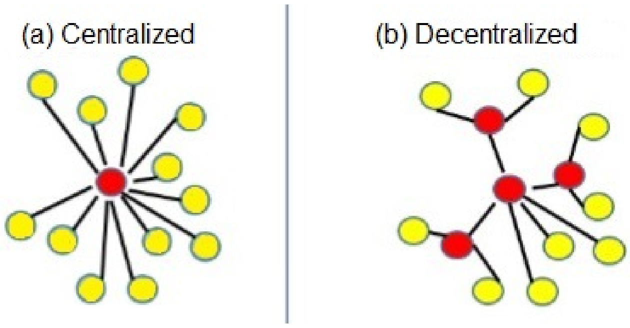


Figure 1-5: Centralized and Decentralized scheme [4]

In paper [5], a decentralized scheme for managing the operation and control of a multi-area integrated power and natural gas system is put forward. Instead of relying on a single controller to make decisions, a decentralized method distributes authority across several controllers, which are then in charge of smaller subsystems. Several benefits of decentralized approach, including faster calculations and fewer large-scale modeling requirements, are covered in [5]. When separate operators in various regions independently make optimal dispatch decision and communicate that information to the adjacent subsystems, the communication load is reduced.

The centralized model can be broken down into multiple sub-problems pertaining to distinct subsystems, allowing for the independent operation of each area while sharing information. Using decentralization model simplicity and privacy protection can be achieved. Decentralized scheme is used widely for the operation of a multi-area system [26], [27]. Some decentralized optimization methods cited in paper [26] include Lagrangian Relaxation (LR), Augmented Lagrangian Decomposition (ALD), and Alternating Direction Multiplier Method (ADMM). Even though OEF is one of the most fundamental and critical problems for IEGS, it brings obstacles caused by coupling relations, for instance, security related issues [6]. This is quite a recent area of research and the methods available for solving such problems still pose some challenges like unavailability of solvers for highly non-convex constraints from natural gas network, no convergence guarantee for the decentralized algorithm, etc.

1-2 Research question

In this section, the main research question that drives this study has been articulated alongwith a set of sub-questions. This research question will serve as a compass, providing direction to the efforts and shaping the subsequent chapters. By clearly formulating the research question, the rationale behind the chosen research question is discussed.

Main Research Question:
Which would be a better performing method for optimization of multi-area IEGS for solving an OEF problem, when different methods for relaxing the non-convex gas flow equations and optimization algorithms are implemented?

In order to find the better performing optimization method, this broad question can be broken down into smaller parts and can be considered as the steps taken during this research. The research question can be simplified by formulating following sub-questions like:

Sub-questions:

Q1) Why is it important to study different optimization methods for IEGSs?

Q2) What are the technical and operational challenges posed by IEGS? Is it possible to overcome these challenges? If so, how?

Q3) How to find which method has better performance?

Q4) What is the result when each method is implemented?

Q5) What is the result of comparison of these methods with respect to each performance metric?

Q6) What are the limitations of this work?

The answers to these sub-questions will aid in finding the answer for the main research question. All the answers found during this study will be summarized in the conclusion (Chapter 5).

1-3 Motivation

The motivation for studying different optimization methods for IEGS stems from the current need of the energy market where optimal use of energy is as crucial for producers as that for the end users. The integration of different energy sources is garnering increased attention as a possible option to accommodate the growing share of different resources like natural gas, heat, renewable energy sources like wind, solar energy, etc. The alliance among different energy carriers introduces versatility to the system and can very well compensate for the fluctuations in production and demand.

Studying optimization methods for IEGSs has practical implications for the energy transition. There is a great deal of challenges at intersection of the two domains since these systems are inherently independent. They affect the stability of the system and impact on reliability and efficiency of the network. IEGSs becomes critical as the world transitions towards cleaner energy sources. The highly non-convex and nonlinear characteristics of natural gas flow equations cause challenges. Figure 1-6 shows an example of the nonlinear nature of the natural gas flow equation (Weymouth equation).

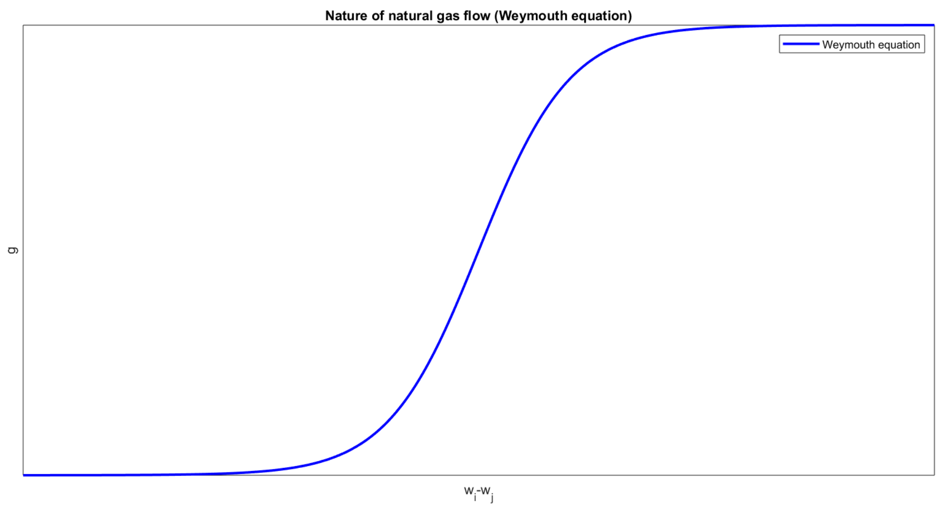


Figure 1-6: Example to illustrate nature of natural gas flow equation

Balancing electricity generation, gas supply and demand requires innovative approaches. Effective IEGS operation directly impacts costs and resource utilization. Optimization of dispatch schedules helps with economic savings. Optimization can help ensure continuity under adverse conditions like natural disasters, supply disruptions or cyberattacks. Integrated systems can enhance resilience by dynamically reallocating resources between electricity and natural gas. Studying IEGS requires collaboration across different disciplines promoting need for interdisciplinary research.

Specifically the communication between electricity and natural gas has also increased substantially due to increased electricity production from gas fired power plants. It is also interesting to study the hypothesis that coupled electricity and natural gas network is beneficial as it improves energy utilization, efficiency and provides system flexibility [28]. An addition to this is the paper [5], which uses decentralized optimization based on cone reformulation for IEGS. The nonlinear, non-convex natural gas flow equations are convexified in this study by second order cone reformulation method.

The challenges of non-convexity can be computationally expensive. Relaxations methods provide a way to approximate these non-linearities while maintaining tractability. Convex relaxations could help in flawless integration between electricity and natural gas networks by simplifying the optimization problem. Convex relaxations provide tight bounds which help ensure feasibility and stability of gas networks. This becomes an interesting area of research since we want to focus on obtaining feasible solutions for the desired optimization problem of IEGS.

The section on motivation behind studying this topic of optimization methods for IEGSs intends to answer the first sub-question introduced in the above section. The importance of studying electricity and natural gas related systems will provide to be a baseline for integrating other energy resources and thus moving towards a more sustainable future using optimum energy sources. If the energy demands are optimally met, the prices of energy can also be kept under control. With the development of newer methods studied in the research, faster ways of fulfilling customer needs can be developed and implemented for industries. Before going further, it is important to consider the different challenges involved in the implementation of these methods. These are introduced in the subsequent section.

1-4 Challenges posed by integrated electricity natural gas system

As mentioned above, IEGS poses technical as well as operational challenges. This section aims to answer the sub-question: **What are the technical and operational challenges to integrated electricity and natural gas systems? Is it possible to overcome these challenges? If so, how?**

Theoretically, it could be possible to overcome these challenges. For this, it is important to study optimization methods in order to find ways to overcome challenges and improve performance of IEGS.

Technical challenges are as follows:

1. Infrastructure differences: Electricity and natural gas systems have different infrastructures, which can make integration challenging and expensive. For example, natural gas pipelines and electricity transmission lines are completely different objects and have different materials, diameters, etc. This makes it technically difficult to use existing infrastructure and accommodate integrated systems.
2. System stability: The integration of electricity and natural gas systems can result in changes in system stability, which can impact the safety of the systems.
3. Capacity constraints: The integration of electricity and natural gas systems may have different demands and peak periods which results in capacity constraints. This can result in technical challenges in managing the supply and demand of energy across the integrated system.
4. Cybersecurity: IEGS creates new cybersecurity risks, as the integrated system becomes more complex and interconnected they require the development of new cybersecurity strategies and technologies to protect the integrated system from cyberattacks.

It may be possible to overcome these technical challenges through optimization, for instance:

a. Optimization methods can be used to identify cost-effective ways to use existing infrastructure to accommodate integrated systems. This may involve identifying the optimal locations for gas injection points or designing new pipeline that can accommodate both natural gas and electricity transmission.

b. Optimization methods can also help address capacity constraints by predicting demand and supply patterns and optimizing the distribution of energy resources. This may involve using algorithms to determine the optimal allocation of natural gas and electricity resources across the integrated system, taking into account factors such as peak periods and seasonal variations in demand.

c. Optimization methods can help address system stability by modeling and simulating the behavior of integrated electricity and natural gas systems. This can help identify potential issues and develop effective strategies for managing system stability and reliability.

Operational challenges are as follows:

1. Interoperability: Currently different area or system providers have different control and automation requirements, and may use different communication protocols and technologies. In a multi-area integrated systems, development of interoperable technologies and systems that can communicate and operate across different areas is required. This can be challenging if the current systems are incompatible and need complete replacement.

2. Control strategies and coordination: Different areas have different energy demand and supply requirements which might change the required control strategy. Different rules and regulatory policies apply to different areas. Hence, multi-area integrated systems require effective coordination and control mechanisms to manage the flow of energy across different areas.

3. Regulatory and policy barriers: Integration can be challenging because different regulatory and policy frameworks are applicable in different areas. Multi-area integrated systems is also a huge area of study in economics and finance since it may include differences in energy market structures, pricing mechanisms, and incentive structures.

4. Resilience and reliability: An integrated system needs to be resilient and reliable, with the ability to respond quickly to failures in any part of the system. This requires effective risk management strategies, contingency planning, and investment in backup and redundancy systems.

5. Data management: Storage and analysis of large amounts of data, including data on energy demand and supply, weather patterns, market prices, etc. This can be challenging, as data may be fragmented or inconsistent across different areas, making it difficult to develop accurate models and forecasts.

These operational challenges can be overcome through innovation and new technologies, for instance:

a. New technologies, standards and regulations that support the integration of electricity and natural gas systems across different areas must be introduced widely. The study of optimization methods' performance can be useful to develop such standards.

b. Optimization methods can be used to determine the optimal mix of energy sources to minimize the risk of supply disruptions, or to develop risk management strategies that take into account the uncertainties of the energy market.

c. Effective collaboration and communication among stakeholders from different sectors.

d. Effective governance structures and decision-making processes.

1-5 Thesis outline

To summarize this Chapter 1, a solid foundation of concepts has been established which encompasses the introduction to key ideas, the research question, the motivations driving this investigation. The rest of this document is organised as follows. Chapter 2 covers the background of the topic, some methodologies used by other researchers in the literature for optimization of IEGS. Additionally, it introduces a multi-area IEGS network and introduces the optimization problem formulation for the same. Further Chapter 3 consists of the nonlinear gas flow equations and different relaxation methods currently used for approximating these nonlinear optimization problems. The performance metrics are also discussed in detail. Chapter 4 shows the performance of the optimization algorithms used for IEGS. This is followed by Chapter 5 which summarizes the key points of this study, provides summary of answers to the research question and the goals for further research.

Chapter 2

Background and Optimal energy flow problem

This chapter presents a summary of comprehensive background study that synthesizes relevant literature and previous research conducted in the field of optimization of Integrated Electricity and Natural Gas System (IEGS) and relaxation techniques. It further introduces a generalized optimization problem followed by the actual optimization problem for integrated electricity and natural gas systems used for implementation during this thesis.

2-1 Literature summary

After studying relevant topics from literature, a synopsis to get insights on the existing work related power flow systems, natural gas systems, IEGS, optimization algorithms, etc. is presented here. There are differences in the problem formulations and methods implemented in each study, hence it is necessary to study them and highlight the gaps. A comprehensive research on existing literature, [29] by Enrica Raheli *et al.* examines the short-term optimal operation of IEGS. This study identifies the advantages of coordinated optimization over independent scheduling of the two sectors and concludes that fully integrated optimization solutions results in lower operational costs and greater utilization of resources including renewable resources. The work of [30] consists of a detailed gas model and further provides an Optimal Energy Flow (OEF) model for IEGS. A Mixed Integer Linear Programming (MILP) method consisting of logical programming and customized piecewise linearization is implemented by the authors to deal with the nonlinear gas flow equations.

Approaches for controlling dynamic gas flows on pipeline networks were presented for the operation of IEGS in paper [31]. The use of gas-fired generators for peak load causes variations due to high-pressure gas transmission systems leading to gas price fluctuations and supply disruptions. This affects the electric generator dispatch, electricity prices and poses a threat to the security of the power and gas networks. The authors A Zlotnik *et al.* have proposed techniques that could effectively investigate the day-ahead scheduling of power generation and gas operation and hence quantify the economic efficiency and security benefits of electricity-gas coordination. Another paper [32], discusses about a day-ahead economic dispatch model of IEGS with reserve scheduling which is helpful to manage uncertainties. It introduces a Second Order Cone (SOC) relaxation for the nonlinear gas flow equation (Weymouth equation). It further transforms the non-convex optimization problem to Mixed Integer Second Order Cone (MISOC) programming problem. The authors conclude from their simulation results that the proposed model provides more economical dispatch solution with shorter computational time than MILP models.

In paper [5], authors Yubin He *et al.* propose a decentralized OEF calculation as opposed with a centralized solution method which is more popularly used. They present the merits of decentralized OEF for a large IEGS. Their model aims at reducing the communication burden on the central controller and allowing individual area operators to make optimal dispatch decisions for their respective areas. Only required information is shared amongst the adjacent areas thus reducing the computational load as well as speeding up the process. Their model also introduces SOC reformulation to deal with the nonlinear, non-convex gas flow equation. The non-convex optimization problem is transformed into a MISOC problem. Decentralized optimization algorithm, Iterative Alternating Direction Multiplier Method (I-ADMM) is further used in order to achieve the convergence performance. The authors also emphasize that decentralization provides an advantage in terms of scalability and adaptability.

Another paper on day-ahead optimization for gas-electric systems [33], utilizes SOC programming method to solve the IEGS optimization problem. It consists of a daily operation model with electric power system, natural gas system and energy hubs and transforms the nonlinear, non-convex problem into a convex one. Convex relaxation method transforms some parts of the model from equalities to inequalities [34]. The strategic use of cone reformulation is crucial for maintaining a delicate balance in the distribution of electricity and natural gas. The crux of the mentioned relaxation method lies in employing cone reformulation, a mathematical tool that boosts the effectiveness of decentralized optimization. Distributing optimization tasks across regions enhances system resilience and responsiveness to localized changes. Decentralization minimizes the risk of widespread failures by containing disruptions within specific regions.

In the paper [6] by RP Liu *et al.*, the primary focus is on introducing a distributed operational strategy for the seamless integration of electricity and gas systems. The authors suggest an approach based on extended convex hull, leveraging mathematical concepts to optimize the coordination between these interdependent systems. The extended convex hull method, as elucidated in the paper, plays a pivotal role in achieving optimization objectives. The Extended convex hull (ECH) based relaxation method is used to convexify the nonlinear, nonconvex Weymouth equation. The ECH based constraints convexify the OEF problem without the need of introducing new binary variables. Further Jacobi Proximal Alternating Direction Multiplier Method (J-ADMM) algorithm is used to solve the convexified model. Ultimately they provide condition to check the feasibility of the optimal solution for the convexified problem. Their proposed method is able to recover an optimal solution for the original non-convex problem if the desired conditions are met. Overall, the research contributes to the ongoing discourse on integrated energy systems, presenting a distributed operational strategy that holds promise for optimizing the coordination between electric and gas networks.

Since there is quite a bit of research going on around relaxation methods for Weymouth equation, the paper [35] introduces a technique called Reformulation-Linearization Technique (RLT) for reformulation. The nonlinearity is transformed into a linear programming/quadratic programming model, leading to improved computational efficiency. The authors J Fan *et al.* discuss a new model for OEF considering gas inertia and wind power uncertainties for IEGS dispatch. The RLT method works in two stages, first it reformulates the problem by constructing a set of non-negative variable factors using the problem constraints and generates valid quadratic constraints by using pairwise products of inequality constraints or products of equality constraints with variables to generate additional nonlinear constraints. The resulting problem is then linearized by defining a set of new variables, one for each nonlinear term. The effectiveness of the method has been shown by performing different simulations.

A piecewise linear approximation method can also be employed for relaxing the nonlinear, non-convex gas flow equations. This method consists of constructing a function that fits the nonlinear objective function by adding extra binary variables, continuous variables and constraints to reformulate the original problem. A single valued function of one variable is approximated in terms of a sequence of linear segments. The paper [36] proposes Piecewise Affine (PWA) approximation method for relaxing the Weymouth equation using mixed integer linear constraints. The authors formulate the Economic Dispatch Problem (EDP) as a game equilibrium problem. They transform the nonlinear, non-convex optimization problem into a mixed integer game. Further an iterative two-stage method is used to compute the approximate generalized Nash equilibrium. Similar to the PWA relaxation, studies presented in papers [37], [38], [39] also use PWA functions to approximate gas flow equations. Here, binary variables are required to indicate the active region of the PWA functions and thus turn into mixed integer linear constraints.

2-1-1 Summary of relaxation methods for nonlinear gas flow equations

As discussed in the previous section, the nonlinear gas flow equation (Weymouth equation) is relaxed using different methods in various studies. This part provides a summary of the relaxation methods used in literature for IEGS in the form of Table 2-1.

Table 2-1: Relaxation methods

Sr. No.	Method	Auxiliary variables	Resulting optimization problem	Scalability*
1.	Second Order Cone (SOC) [5]	Yes	Mixed integer second order cone	Yes
2.	Extended convex hull (ECH) [6]	No	Convex	Yes
3.	Reformulation Linearization Technique (RLT) [35]	Yes	Linear/quadratic	Yes
4.	Piecewise linear approximation (PWA) [36]	Yes	Mixed integer linear	Yes

Note: * scalability must be checked when the actual relaxation method is implemented.

2-1-2 Summary of optimization algorithms

As introduced in Chapter 1, different optimization algorithms exist in the fields of science, engineering and mathematics. The goal of this thesis is to study the performance of different optimization methods for IEGS. During the implementation of the methods both centralized and decentralized optimization algorithms have been implemented. In summary, while centralized optimization relies on a central coordinator, decentralized optimization leverages distributed collaboration among nodes to achieve optimal solutions.

This chapter summarizes few of the distributed algorithms used in literature for IEGS. From the papers studied, following observations have been noted. Table 2-2 shows the summary of some of the important features of the methods discussed.

Table 2-2: Features of algorithms

Sr. No.	Method	Iterative	Gas flow equation approximation method	Type of problem solved	Convergence	Number of layers or loops in the algorithm
1.	Standard ADMM [5]	No	SOC	MISOC	Not guaranteed	1
2.	I-ADMM [5]	Yes	SOC	MISOC	Guaranteed when used with Sequential Cone Programming (SCP)	2
3.	J-ADMM [6]	No	ECH	Convex	Guaranteed	2
4.	Two-Stage method [36]	No	PWA	Mixed integer linear	Guaranteed	1

Since the optimization problem in each paper has been reformulated using different methods, it seems that all of the algorithms are scalable. However, it is necessary to check this before making any final remarks.

2-2 Generalized optimal energy flow problem

It is possible to formulate the optimal energy flow problem as an optimization problem as shown in equation 2-1. Typically, an optimization problem has an objective function, a decision variable which is the parameter to be optimized, constraints that limit the solution to a particular area of the parameter space (equality constraints), and constraints that limit the solution to a specific area of the parameter space that is allowed (inequality constraints). The solution of such an optimization problem is a set of values for the decision variables that satisfy all the constraints and optimize (minimize or maximize) the objective function. This solution can be a single point for continuous problems or a set of points for other problems.

In case of optimization problems for IEGS, the ultimate objective is to reduce the overall cost while still adhering to the limitations on electricity and natural gas systems. All the constraints must be satisfied in order to have a feasible solution. In case of a multi-area IEGS model, a generalized OEF model can be formulated as follows:

$$\begin{aligned} & \min_{x_a} f(x_a) \\ \text{subject to} \quad & g_k(x_a) = 0, \quad k = 1, \ldots\ldots, n_e \\ & h_l(x_a) \leq 0, \quad l = 1, \ldots\ldots, n_{ie}, \end{aligned} \tag{2-1}$$

where,
a: subsystem considered for analysis, where a=1, $\cdots$, N
$f(x_a)$: objective function, comprising the total cost of the network
x_a: the parameter vector used to optimize the objective function f (decision variable)
$g_k(x_a) = 0$: equality constraint which correspond to coupling constraints
$h_l(x_a) \leq 0$: inequality constraint represent local constraints depending on electricity and gas network
n_e, n_{ie}: number of equality and inequality constraints, respectively.

The objective function f for an IEGS network may be the entire cost of the multi-area IEGS, which would include the natural gas fuel cost of Natural Gas-fired Unit (NGU)s, operation cost of the non-NGUs, the cost of load shedding, start-up cost for units, shut-down costs, etc. Various authors formulate this differently and hence it becomes difficult to directly compare the results. In this study we focus on one specific formulation and try to implement different relaxation methods and optimization algorithms for the same problem.

2-3 Optimization problem formulation for integrated electricity and natural gas network

After carefully reviewing various studies, it was observed that different studies take into account different network components in order to formulate the optimization problem for OEF. In order to compare the results of different methods, one specific model and specific datasets must be used to obtain accurate results. The formulation chosen in this thesis is similar to the one discussed in the paper [5]. It differs from the conventional single area IEGS models as there are tie-lines and tie-pipes due to multi-area model. The objective function of this OEF for a multi-area IEGS is shown in equation (2-2). It represents the total cost of the multi-area IEGS, which includes the natural gas fuel cost of NGUs, operation cost of non-NGUs and penalty cost for load shedding in each subsystem.

$$f(x_a) = \sum_{a=1}^{N} \left\{ \sum_{i \in W} \mu_g v_{a,i} + \sum_{i \in N_{NG}} F_{a,i}\left(P_{a,i}\right) + V^p \cdot \sum_{j=1}^{M} L_{a,j} \right\}, \tag{2-2}$$

where,
a: subsystem under analysis, where a=1, $\cdots$, N
μ_g: natural gas price
$v_{a,i}$: natural gas production of well W
$F_{a,i}$: natural gas consumption of gas fired unit i
NG: non gas fired unit
$P_{a,i}$: real power generation
V^p: penalty price for load shedding
L_j: load shedding at bus j, where j=1, $\cdots$, M.

Equivalent to the paper [5], the decision variable x_a includes generator outputs (P_i), bus phase angles (θ_i, δ_i), load shedding (L_i), natural gas well outputs (v_i), nodal pressures (ω_i) and natural gas flows on tie-pipes ($g_{(i,j)}$).

$$\begin{aligned} x_a = \{ &\{P_i\}_{i \in N_G^a \cup N_{NG}^a}, \{\theta_i\}_{i \in B^a}, \{\delta_i\}_{i \in \bar{B}^a}, \{L_i\}_{i \in B'^a}, \{v_i\}_{i \in W^a}, \\ &\{\omega_i\}_{i \in N_G^a}, \left\{g_{(i,j)}\right\}_{(i,j) \in E_{gas}^a} \}, \quad \forall a, \end{aligned} \tag{2-3}$$

where,
N_G^a: set of gas fired units
N_{NG}^a: set of non-gas fired units
B^a: set of inner buses
$\bar{B}^a$: set of boundary buses
B'^a: set of buses with load shedding
W^a: set of natural gas production wells
E_{gas}^a: set of all gas flow between nodes (i, j)

{Note that the superscript 'a' for these sets can be skipped for ease of notation.}

2-3-1 Electricity network constraints

An electricity network mainly consist of generators, transformers, transmission lines and supply electricity to consumers. Before introducing constraints, here Figure 2-1 is a representation of the electrical network as an undirected graph $\mathcal{G}^e = (B, \mathcal{E})$, where $B = \{b_1, b_2, \ldots, b_B\}$, the set of busses (nodes) and $\mathcal{E} \subset \mathcal{B} \times \mathcal{B}$, is the set of power lines.

The optimization of an electric network refers to the deduction of the necessary network reinforcements that guarantee the supply to electrical loads. This optimization is basically minimizing the cost for both distribution and transmission systems [40]. The nodal balance is extremely crucial for any

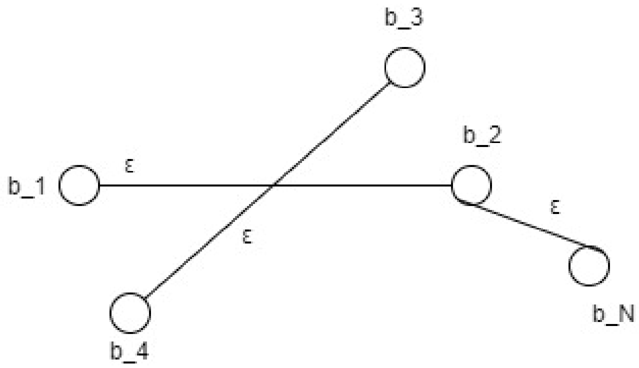


Figure 2-1: Electrical network

network. Therefore, the nodal balance equation for electricity network is also an important constraint as it represents the fundamental principle of power flow conservation in electrical networks and must be satisfied for all nodes in the system. The constraints represent physical limitations, operational requirements that must be satisfied for the reliable and efficient operation of the electric grid.

The electrical network constraints discussed in paper [5], consist of generation limits of generators, equation (2-4), nodal balance equation for electric network, equation (2-5), the DC power flow function for inner-lines and tie-lines, equation (2-6), transmission capacity constraints of electricity lines, equation (2-7), phase angle limits for reference bus, equation (2-8). Load shedding component is constrained for maintaining the security of system operation, equation (2-9).

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad \forall i \in N_G, \tag{2-4}$$

where,
$P_i^{\min}$ and $P_i^{\max}$: lower and upper limits on the power generated

The nodal power balance equation is:

$$\sum_{i \in G_j} P_i - \sum_{h \in \mathcal{N}_i^g} p_{(i,h)}^f - D_j + L_j = 0, \tag{2-5}$$

where,
$\mathcal{N}_i^g = \{j | (i,j) \in \mathcal{E}\}$: set of transmission lines
G_j: set of all units connected to bus j
$p_{(i,j)}^f$: power flow on transmission line (i,j)
D_j: electricity demand at bus j
L_j: load shedding at bus j.

The DC power flow function of inner lines and tie lines:

$$\begin{aligned} p_{(h,j)}^f &= (\theta_h - \theta_j)/x_{hj}, \quad h \in \mathcal{N}_j^g, j \in B^a \\ p_{(h,j)}^f &= (\theta_h - \delta_j)/x_{hj}, \quad h \in B^a, j \in \bar{B}^a \\ p_{(h,j)}^f &= (\delta_h - \delta_j)/x_{hj}, \quad h, j \in \bar{B}^a, \end{aligned} \tag{2-6}$$

where,
θ_h: phase angle of inner bus h

θ_j: phase angle of inner bus j
δ_j: phase angle of boundary bus j
δ_h: phase angle of boundary bus h.

Transmission capacity of electricity lines:

$$-F_i^{\max} \leq p_{(i,h)}^f \leq F_i^{\max}, \quad \forall i \in N_{NG} \tag{2-7}$$

where,
F_i: cost function of non-natural gas unit.

Phase angle limits for reference bus:

$$\theta_j = 0, \quad j = \mathrm{R}^{\mathrm{e}}, \tag{2-8}$$

where,
R^{e}: index of reference electric bus.

Load shedding component:

$$0 \leq L_i \leq L_i^{\max}, \quad \forall i \in B'^a. \tag{2-9}$$

2-3-2 Natural gas network constraints

The natural gas system exhibits dynamic characteristics which increases complexity of the mathematical model thus making it difficult to solve the optimization problem. Therefore steady-state natural gas flow is commonly used in dispatch problems. Since the natural gas flow system can be modeled using the Weymouth equation which is nonlinear, non-convex. It characterizes the relationship between natural gas flow and pressure at the inlet and outlet of a gas pipeline. Consider the gas network as an undirected graph $\mathcal{G}^g = (\mathcal{N}, \mathcal{P})$, where $\mathcal{N} = \{n_1, n_2, \ldots, n_N\}$, the set of gas nodes and $\mathcal{P} \subset \mathcal{N} \times \mathcal{N}$, is the set of edges, with both the edges $(i,j),(j,i) \in \mathcal{P}$ representing the pipeline that connects nodes i and j. Figure 2-2 shows an example of a natural gas network with nodes and edges.

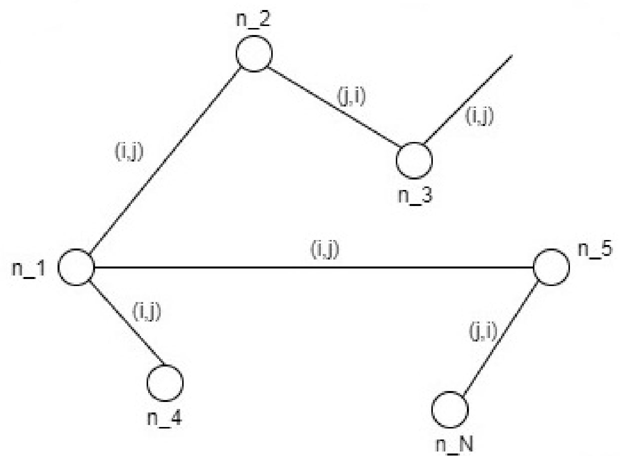


Figure 2-2: Natural gas network

The natural gas network constraints represent the physical, operational and safety limitations that must be satisfied for the effective management of natural gas flow within the network. The network constraints discussed in [5] describe the static characteristics of natural gas system, equation (2-10)-(2-16). The natural gas fuel consumption function of NGUs is shown in equation (2-10). The steady state natural gas flow function of inner pipeline, defined by difference of pressures on two end nodes is referred in equation (2-11). The constraint due to the boundary for natural gas nodal pressure is shown in equation (2-12). The capacity constraint of tie-pipe, equation (2-13). Equation (2-14) is nodal natural gas balance for natural gas network. Equation (2-15) is nodal natural gas demand, including residential and NGU gas demands. Equation (2-16) represents the production limit of natural gas wells.

$$F_i = \alpha_i + \beta_i P_i + \gamma_i \left(P_i\right)^2, \quad i \in \mathcal{N} \tag{2-10}$$

where,
α_i, β_i, γ_i: Fuel coefficients of natural gas-fired unit i

$$g_{(i,j)} = \operatorname{sgn}\left(\omega_i, \omega_j\right) \cdot C_{(i,j)} \sqrt{\left|\omega_i^2 - \omega_j^2\right|}, \quad (i,j) \in I^P, \tag{2-11}$$

where,
$g_{(i,j)}$: Natural gas flow on pipeline (i,j)
$C_{(i,j)}$: the Weymouth constant which depends on the characteristics of the pipeline
(i,j): represents gas pipeline in which the first and last node is i and j respectively
I^P: Set of inner pipes
ω_i: the pressure of initial node
ω_j: the pressure of end node,

The operator sgn represents the direction of natural gas flow:

$$\operatorname{sgn}(\omega_\mathrm{i}, \omega_\mathrm{j}) = \begin{cases} 1 & \text{if} \quad \omega_i > \omega_j \\ 0 & \text{if} \quad \omega_i = \omega_j \\ -1 & \text{if} \quad \omega_i < \omega_j \end{cases}$$

Then the nodal pressure of the upper and lower boundary:

$$\omega_i^{\min} \leq \omega_i \leq \omega_i^{\max} \quad \forall i \in N_G \tag{2-12}$$

$$-g_{(i,j)}^{\max} \leq g_{(i,j)} \leq g_{(i,j)}^{\max}, \quad (i,j) \in T^P \tag{2-13}$$

where,
T^P: Set of tie-pipes

The nodal balance for natural gas network:

$$\sum_{s^p \in G(i)} v_{s^p} - Q_i - \sum_{(i,j) \in G_i^{PF}} g_{(i,j)} + \sum_{(i,j) \in G_i^{PE}} g_{(i,j)} = 0 \tag{2-14}$$

where,
s^p: index of natural gas well
$G(i)$: set of thermal units

Q_i: Nodal natural gas demand
G_i^{PF}: set of natural gas pipelines from node i
G_i^{PE}: set of natural gas pipelines to node i.

$$Q_i = D_i^G + \sum_{i \in C^U(i)} F_i^g, \tag{2-15}$$

where,
D_i^G: residential natural gas demand at node i
$C^U(i)$: set of natural gas-fired units connected to node i.

$$v_i^{\min} \leq v_i \leq v_i^{\max} \quad \forall i \in W \tag{2-16}$$

2-3-3 Coupling constraints

When a large system is partitioned, there are constraints which couple multiple subsystems and are referred to as coupling constraints. A coupling constraint comes into play when it is no longer possible to split up a large problem. These constraints represent the interactions and dependencies between the electricity and natural gas networks in this case. These constraints make sure that the operation of one network does not compromise the performance or safety of the other. Coupling constraints are useful in cases where a large number of nodes are concerned. It is important to find the proper coupling constraints among connected subsystem areas. Figure 2-3 shows decomposition strategy for coupling variables of multi-area IEGS.

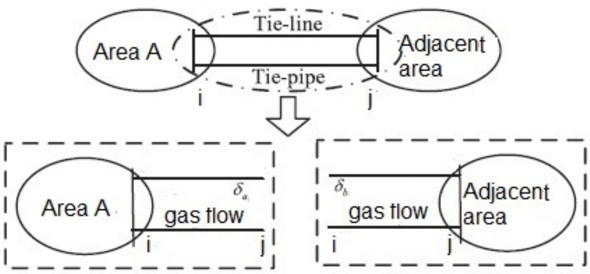


Figure 2-3: Decomposition of coupling variables [5]

In the model discussed in [5], phase angle of boundary bus and natural gas flow on the tie-pipe are selected as coupling variables. Equation (2-17) indicates that the phase angles of an end bus perceived by its connected subsystems should be identical. Similarly, equation (2-18) states that the natural gas flows on the tie-pipe perceived by its connected subsystems are also identical.

$$\delta_{\psi(h),h} = \delta_{\psi(j),h}, \delta_{\psi(h),j} = \delta_{\psi(j),j}, \quad (h, j) \in T^L, \tag{2-17}$$

where,
$\psi(h)$: subsystem that electricity bus h belongs to adjacent area
$\psi(j)$: subsystem that electricity bus j belongs to in area a.

$$g_{\varphi(i),(i,j)} = g_{\varphi(j),(i,j)}, \quad (i,j) \in \mathcal{P}, \tag{2-18}$$

where,
$\varphi(i), \varphi(j)$: subsystems such that natural gas nodes, i from area a and j from adjacent area belong to respectively.

By incorporating coupling constraints into optimization models for IEGS, planners and operators can effectively manage the complex interactions between the two networks. They can optimize their joint operation to achieve objectives such as cost minimization, emissions reduction and system reliability. Failure to consider the coupling constraints can lead to suboptimal solutions, inefficiency in operation and increased risks in the operation of IEGS. Hence accurate modeling and enforcement of constraints are important for successful integration.

Chapter 3

Methodology

In this Chapter 3, a detailed explanation of techniques used for reformulation of the nonlinear, non-convex constraints and various optimization methods that are implemented to find optimal solutions have been presented. Further the list of performance metrics which would be used to analyze the results in the following chapters has been defined along-with their definitions.

3-1 Methods implemented

Moving further after introduction of the optimization problem, an overview of the methodologies that will be employed to address the identified mathematical problem is presented below. These methodologies encompass reformulation techniques for Weymouth equation and optimization algorithms. A detailed explanation of each method has been summarized. Here is a list of relaxation methods and optimization algorithms that have been implemented during the course of this study:

1. Second Order Cone (SOC) relaxation method
2. Centralized optimization algorithm
3. Standard Alternating Direction Multiplier Method (ADMM) algorithm
4. Iterative Alternating Direction Multiplier Method (I-ADMM) optimization algorithm
5. Extended convex hull (ECH) relaxation method
6. Jacobi Proximal Alternating Direction Multiplier Method (J-ADMM) optimization algorithm

These methods might have a different representation while explaining the method in this chapter but during this study, the optimization algorithm is implemented for the optimization problem with defined objective function in equation (2-2) along-with all the constraints and the relaxation method is mainly for the nonlinear, non-convex Weymouth equation (2-11).

3-1-1 Second order cone relaxation method

Conic optimization problem has been increasingly studied in different fields recently [41]. SOC is an effective way used to approximate the non-convex constraints (equations (2-10) to (2-12)). The SOC is also known as Lorentz cone, shown in Figure 3-1. The SOC uses a slack variable which is added to an inequality constraint to transform it into an equality constraint and introduces a non-negativity constraint on the slack variable. The main idea of this technique is to relax the Weymouth function into a SOC form and use an auxiliary function to maintain the accuracy of slack variables. Note that convexification must be exact to ensure that the feasible region remains the same as the original primal problem.

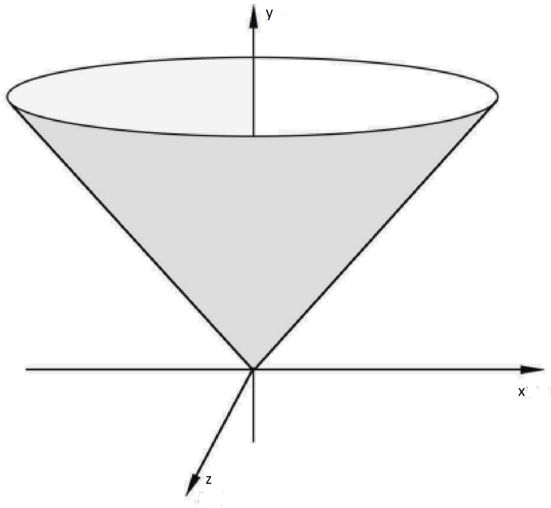


Figure 3-1: Second order cone

The solution proposed in paper [5] for independent subsystems is converting nonlinear, non-convex constraints, equations (2-10) to (2-12) into mixed-integer SOC constraints. Constraint in equation (2-10) can be directly converted into following SOC form. This constraint is always tight since the unnecessary natural gas consumption by Natural Gas-fired Unit (NGU)s will lead to higher operating costs.

$$F_i^g \geq \alpha_i + \beta_i P_i + \gamma_i (P_i)^2, \quad i \in \mathcal{N}. \tag{3-1}$$

The equations (2-11), (2-12) show highly nonlinear steady state pipeline flow which can be converted to Mixed Integer Non Linear Programming (MINLP) form as shown in equations (3-2) to (3-5), where π_i is the squared nodal pressure.

$$\left(I_{(i,j)}^{+} - I_{(i,j)}^{-}\right)(\pi_i - \pi_j) = \left(1/C_{(i,j)}\right)^2 g_{(i,j)}^2, \tag{3-2}$$

$$-\left(1 - I_{(i,j)}^{+}\right) g_{(i,j)}^{\max} \leq g_{(i,j)} \leq \left(1 - I_{(i,j)}^{-}\right) g_{(i,j)}^{\max}, \tag{3-3}$$

$$I_{(i,j)}^{+} + I_{(i,j)}^{-} = 1, \tag{3-4}$$

where,
$I_{(i,j)}^{+}, I_{(i,j)}^{-}$: binary indicators of natural gas flow direction on pipeline (i,j).

$$\pi_i^{\min} \leq \pi_i \leq \pi_i^{\max}, \tag{3-5}$$

where,
$\pi_i^{\min}$:Minimum value of squared nodal pressure of node i,
$\pi_i^{\max}$:Maximum value of squared nodal pressure of node i.

Equation (3-2) can be replaced by further relaxing the constraint into Mixed Integer Second Order Cone (MISOC) constraints form as shown below:

$$\Gamma_{(i,j)} \geq \left(1/C_{(i,j)}\right)^2 g_{(i,j)}^2, \tag{3-6}$$

$$\Gamma_{(i,j)} \geq \pi_j - \pi_i + \left(I_{(i,j)}^{+} - I_{(i,j)}^{-} + 1\right)\left(\pi_i^{\min} - \pi_j^{\max}\right), \tag{3-7}$$

$$\Gamma_{(i,j)} \geq \pi_i - \pi_j + \left(I_{(i,j)}^{+} - I_{(i,j)}^{-} - 1\right)\left(\pi_i^{\max} - \pi_j^{\min}\right), \tag{3-8}$$

$$\Gamma_{(i,j)} \leq \pi_j - \pi_i + \left(I_{(i,j)}^{+} - I_{(i,j)}^{-} + 1\right)\left(\pi_i^{\max} - \pi_j^{\min}\right), \tag{3-9}$$

$$\Gamma_{(i,j)} \leq \pi_i - \pi_j + \left(I_{(i,j)}^{+} - I_{(i,j)}^{-} - 1\right)\left(\pi_i^{\min} - \pi_j^{\max}\right). \tag{3-10}$$

Here, $\Gamma_{(i,j)}$ is the auxiliary variable for SOC relaxation.

Further, the SOC constraints, equations (3-6) to (3-10) are equivalent to equation (3-2) when equation (3-6) is tight (an inequality constraint is tight at a certain point if the point lies on the corresponding hyperplane).

The pros and cons of SOC are discussed below,

Pros:

- The SOC reformulation presented in paper [5] obtains a reliable solution.
- The SOC programming method shows high computational efficiency [33].
- The accuracy and computational speed of SOC is high.
- This approach seems scalable to large-scale instances.

Cons:

- The solution time is higher since additional iterations are required to drive the cone constraints tight.
- The operation cost is slightly increased due to narrow feasible region.

3-1-2 Centralized optimization algorithm

As introduced in Chapter 1, centralized optimization algorithm is used to optimize systems where all decision making processes and computations are conducted by a single central entity. This entity is the decision maker responsible for formulating and solving the optimization problem. The central control center gathers information, makes decisions and coordinates actions to achieve a desired objective. It has visibility over the entire system, irrespective of how big or small the system is. It has access to global information about the system in order to make informed decisions.

Figure 3-2 shows the flowchart for centralized algorithm.

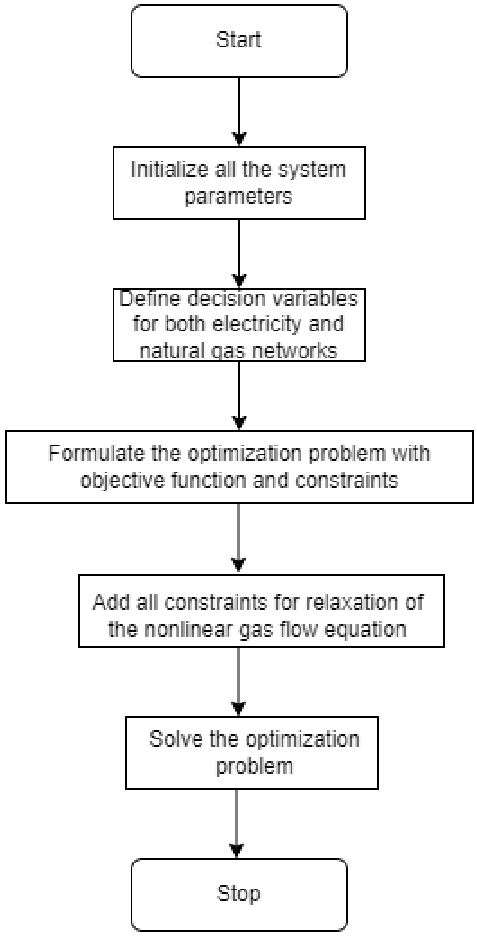


Figure 3-2: Centralized algorithm

Here a pseudo-code for centralized algorithm for Integrated Electricity and Natural Gas System (IEGS) is introduced in Algorithm 1.

Algorithm 1 Centralized optimization

Step 1:
Initialize all the system parameters and constraints.
Define electricity demands, gas demands, operational constraints like power generation limits, gas pipeline limits, etc.
Step 2:
Define decision variables for both electricity and natural gas networks.
Step 3:
Formulate the objective function, here minimization of the operational cost including natural gas fuel cost and penalty cost of load shedding is considered.
Step 4:
Set up constraints for both electricity network and natural gas network. Include the power balance equations and gas balance equations. The constraints for operational limits for generators, gas pipelines, should be considered. The nonlinear, non-convex gas flow equation is relaxed by either of the relaxation methods. The constraints to suffice for this must be included.
Step 5:
Solve the optimization problem using solver like Gurobi to obtain optimal solution.

Some challengess posed by centralized optimization are that it considers all information is available centrally. It may be difficult to solve large-scale problems due to increased computational complexity and it might require lot of time to solve such problems. Hence distributed or decentralized or hybrid approaches are also being studied specifically for large-scale problems like the case here for IEGS.

3-1-3 Standard Alternating Direction Multiplier Method algorithm

The most commonly used method when considering decentralized optimization is the method of ADMM. ADMM breaks down the large, complex problem into smaller, more manageable sub-problems. It alternates between updating variables with each sub-problem and does not affect the consistency across them. It provides a lot of advantages specially to handle large-scale problems. This is a versatile algorithm and can be applied for parallel and distributed computing. Before focusing on Standard ADMM algorithm, it is important to to establish the foundation for this method. The ADMM algorithm was originally proposed in the 1970's by Glowinski & Marrocco (1975) and Gabay & Mercier (1976). This algorithm is used for solving particular types of convex optimization problems. ADMM is becoming popular because it is a simple and powerful algorithm which often allows for solving distributed optimization. It takes the form of a decomposition-coordination procedure, in which the solutions to small local sub-problems are coordinated to find a solution to a large global problem [42].

ADMM can be considered as a combination of the dual decomposition method and the Augmented Lagrangian Decomposition (ALD) method. It combines the benefits of both and demonstrates superior convergence properties. ADMM particularly deals with optimization problems which have a separable objective function, meaning that the objective function can be split into multiple small parts. Several different types of convex optimization problems can be framed as an ADMM. The paper [42] provides a detailed explanation of how ADMM works and that is explained here before studying its application in IEGS optimization problems.

General optimization problem formulation for ADMM:

ADMM solves problems of the form presented as follows:

$$\begin{aligned} \min_{x,z} \quad & f(x) + g(z) \\ \text{s.t.} \quad & Ax + Bz = c, \end{aligned} \tag{3-11}$$

with $x \in \mathbf{R}^n$ and $z \in \mathbf{R}^m$, where $A \in \mathbf{R}^{p \times n}, B \in \mathbf{R}^{p \times m}$ and $c \in \mathbf{R}^p$.

The optimal value of the problem equation (3-11) will be:

$$p^\star = \inf\{f(x) + g(z) \mid Ax + Bz = c\}.$$

The functions f and g are assumed to be convex. The only difference from the general optimization problem equation (2-1) is that the decision variable, has been split into two parts, called x and z here, with the objective function separable across this splitting. Note that for ease of notation we consider the decision variable x and split it into x and z, whereas decision variable in equation (2-1) is $x_a \forall a = 1, \cdots, N$.

ADMM algorithm:

To improve the convergence, ALD introduces a quadratic penalty term ($\rho > 0$) to the objective function in equation (3-11). The augmented Lagrangian is defined as follows:

$$L_\rho(x, z, \theta) = f(x) + g(z) + \theta^T(Ax + Bz - c) + \frac{\rho}{2}\|Ax + Bz - c\|_2^2, \tag{3-12}$$

where,
$\rho > 0$: penalty parameter
θ: dual variable.

The ADMM algorithm consists of steps for x-minimization (3-13), z-minimization (3-14) and dual variable (θ) update (3-15), shown as follows:

$$x^{k+1} = \underset{x}{\operatorname{argmin}} L_\rho\left(x, z^k, \theta^k\right), \tag{3-13}$$

$$z^{k+1} = \underset{z}{\operatorname{argmin}} L_\rho\left(x^{k+1}, z, \theta^k\right), \tag{3-14}$$

$$\theta^{k+1} = \theta^k + \rho\left(Ax^{k+1} + Bz^{k+1} - c\right). \tag{3-15}$$

The dual variable update uses a step size equal to the augmented Lagrangian parameter ρ. The name alternating direction boils down to the fact that x and z are updating in an alternating way.

ADMM convergence:

Many convergence results have been discussed in literature for ADMM. Prior to studying the convergence results for ADMM, two assumptions must be considered as discussed in [42]:

Assumption 1: The function f and g are closed, proper and convex.
This assumption implies that the sub-problems arising in the x-update ((3-13)) and z-update ((3-14)) are solvable, that is, there exist x and z, that minimize the augmented Lagrangian.

Assumption 2: The unaugmented Lagrangian L_0 has a saddle point.
There exist $L_0(x^\star, z^\star, \theta^\star)$, not necessarily unique. From assumption 1, $L_0(x^\star, z^\star, \theta^\star)$ is finite for any saddle point $(x^\star, z^\star, \theta^\star)$. This implies that $(x^\star, z^\star)$ is a solution to equation (3-11).

The ADMM satisfies the following, under assumptions 1 and 2:

1) Residual convergence: $r^k \to 0$ as $k \to \infty$, i.e. the iterates approach feasibility.
2) Objective convergence: $f\left(x^k\right) + g\left(z^k\right) \to p^\star$ as $k \to \infty$, i.e. the objective function approaches the optimal value.
3) Dual variable convergence: $\theta^k \to \theta^\star$ as $k \to \infty$, where $\theta^\star$ is a dual optimal point.

Note that x^k and z^k need not converge to optimal value.

In practice, ADMM can be very slow to converge. Often it converges to modest accuracy (sufficient for most applications) within a few tens of iterations. This is usually acceptable for large scale problems. In some cases it is possible to combine ADMM with another method for obtaining better accuracy.

To extend the discussed ADMM algorithm in the applications for IEGS, different studies provide various ways, with the goal of achieving convergence. Following subsection discusses the standard ADMM approach before actually focusing on the dedicated methods used in literature for multi-area IEGS. It explains how the steps for x-minimization (3-13) and z-minimization (3-14) occur in the standard ADMM algorithm proposed in [5].

Standard ADMM for IEGS:

Standard ADMM can be applied for an Optimal Energy Flow (OEF) problem of a multi-area IEGS. Let us first recall the problem defined in equation (2-2) (also presented in paper [5]),

$$\min_{x_a} \sum_{a=1}^{N} \left\{ \sum_{i \in W} \mu_g v_{a,i} + \sum_{i \in N_{NG}} F_{a,i}\left(P_{a,i}\right) + V^p \cdot \sum_{j=1}^{M} L_{a,j} \right\}.$$

The coupling constraints are phase angles of boundary buses and natural gas flows on tie-pipes perceived by connected subsystems. First the areas are decoupled by relaxing the coupling constraints equation (2-17) and (2-18) by an augmented Lagrangian function of each subsystem. The area subproblem (SP_a) for IEGS, which is equivalent to equation (3-11) is given as:

$$\begin{aligned} \min_{x_a} & \sum_{a=1}^{N} \left\{ \sum_{i \in W} \mu_g v_{a,i} + \sum_{i \in N_{NG}} F_{a,i}\left(P_{a,i}\right) + V^p \cdot \sum_{j=1}^{M} L_{a,j} \right\} \\ & + \sum_{j \in \bar{B}^a} \left[\lambda_{e,a,j}\left(\delta_{a,j} - \bar{\delta}_j\right) + 0.5 \cdot \rho_{e,j}\left(\delta_{a,j} - \bar{\delta}_j\right)^2 \right] \\ & + \sum_{(i,j) \in T_a^P} \left[\lambda_{g,a,(i,j)}\left(g_{a,(i,j)} - \overline{g}_{(i,j)}\right) \right. \\ & \left. + 0.5 \cdot \rho_{g,(i,j)}\left(g_{a,(i,j)} - \overline{g}_{(i,j)}\right)^2 \right] \end{aligned} \tag{3-16}$$

s.t. (2-4) to (2-9) and (2-10) to (2-16) holds.

Here, $\bar{\delta}_j$ and $\overline{g}_{(i,j)}$: average values of coupling variables perceived by their connected areas given as:

$$\bar{\delta}_j = \left(\sum_{a \in \phi(j)} \delta_{a,j} \right) / |\phi(j)|. \tag{3-17}$$

and

$$\overline{g}_{(i,j)} = \left(\sum_{a \in \phi((i,j))} g_{a,(i,j)} \right) / |\phi((i,j))|, \tag{3-18}$$

where,
$\phi(j)$: set of areas connected to boundary bus j.
Function $|\cdot|$ denotes the number of areas connecting to bus j or tie-pipe (i,j).
$\phi((i,j))$: set of areas connected to tie-pipe (i,j).

The coordination among subsystems is achieved by updating Lagrangian multipliers. This is equivalent to θ-update (3-15). The corresponding multipliers $\lambda_{e,a,j}$ and $\lambda_{g,a,(i,j)}$ are given as follows:

$$\lambda_{e,a,j}^{k+1} = \lambda_{e,a,j}^{k} + \rho_{e,j} \left(\delta_{a,j}^{k} - \bar{\delta}_j^{k} \right), \tag{3-19}$$

where,
$\lambda_{e,a,j}$: ADMM multipliers for electricity network and

$$\lambda_{g,a,(i,j)}^{k+1} = \lambda_{g,a,(i,j)}^{k} + \rho_{g,(i,j)} \left(g_{a,(i,j)}^{k} - \overline{g}_{(i,j)}^{k} \right), \tag{3-20}$$

where,
$\lambda_{g,a,(i,j)}$: ADMM multipliers for natural gas network.

Each subsystem calculates the regional sub-problem with updated Lagrangian multipliers. The decentralized algorithm can be terminated when all coupling variables perceived by their connected subsystems are close enough. The paper [5] claims that the proposed decentralized operation scheme is highly efficient because the coupling constraints $\delta_{a,j}$ and $g_{(i,j)}$ is the only information shared among subsystems. The regional privacy is protected and communication burden is reduced.

Figure 3-3 shows the flowchart for standard ADMM algorithm.

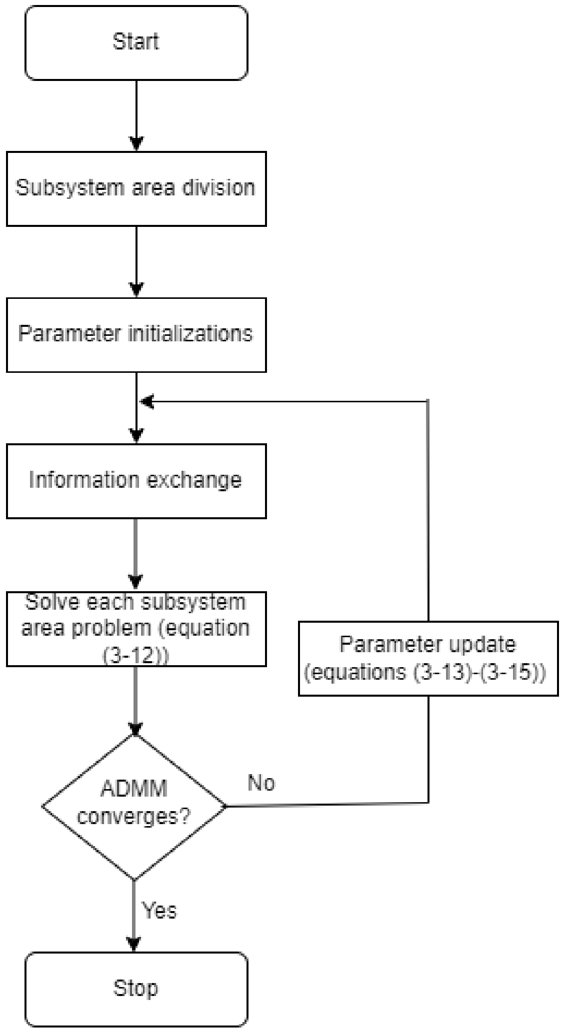


Figure 3-3: Standard ADMM for decentralized scheme [5]

Here a pseudo-code for a decentralized algorithm for IEGS is introduced in Algorithm 2.

Algorithm 2 Standard ADMM

Step 1:
Initialize values of shared information $\delta^0_{a,j}, g^0_{a,mn}$.
Set iteration index k=0.
Initialize Lagrangian multipliers $\lambda^0_{e,a,j}, \lambda^0_{g,a,mn}$ for electricity and gas network respectively for each area.
Set ADMM residuals tolerance $\varepsilon^P, \varepsilon^D$.
Step 2:
Latest shared information $\delta^k_{a,j}, g^k_{a,mn}$ of each area is sent to adjacent area.
Step 3:
The average shared information is updated by each area operator by following equations, where $\bar{\delta}_j$ and $\overline{g}_{mn}$ are the average values of coupling variables perceived by their connected areas and function $|\cdot|$ denotes the number of areas connecting to bus j or tie-pipe mn.

$$\bar{\delta}_j = \left(\sum_{a \in \phi(j)} \delta_{a,j} \right) / |\phi(j)|$$

$$\overline{g}_{mn} = \left(\sum_{a \in \phi(mn)} g_{a,mn} \right) / |\phi(mn)|$$

Step 4:
Each area operator solves it's own sub-problem with average shared information and latest Lagrangian multipliers. The updating process of Lagrangian multipliers is as follows:

$$\lambda^{k+1}_{e,a,j} = \lambda^k_{e,a,j} + \rho_{e,j} \left(\delta^k_{a,j} - \bar{\delta}^k_j \right)$$

$$\lambda^{k+1}_{g,a,mn} = \lambda^k_{g,a,mn} + \rho_{g,mn} \left(g^k_{a,mn} - \overline{g}^k_{mn} \right)$$

For each subsystem, optimal solutions χ^k_a are obtained.
Step 5:
Each subsystem checks if the convergence residuals are within tolerances:
if Both residuals are within tolerance
then End ADMM procedure

$$gap^P_a = \max \left\{ \left\| \left(\delta^k_{a,j} - \bar{\delta}^k_j \right) / x_{hj} \right\|^2_2, \left\| g^k_{a,mn} - \overline{g}^k_{mn} \right\|^2_2 \right\} \leq \varepsilon^P, \forall a$$

$$gap^D_a = \max \left\{ \left\| \rho_{e,j} \left(\bar{\delta}^k_j - \bar{\delta}^{k-1}_j \right) / x_{hj} \right\|^2_2, \| \rho_{g,mn} \left(\overline{g}^k_{mn} - \ \overline{g}mn^{k-1} \right) \|^2_2 \right\} < \varepsilon^D, \forall a$$

else Each area operator updates its multipliers by equations in Step 4.
endif
Step 6:
Set k=k+1. Continue the same process steps 2 to 4 until the stopping criteria are met.

3-1-4 Iterative Alternating Direction Multiplier Method optimization algorithm

Iterative ADMM as the name suggests extends the basic ADMM by introducing an iterative process for solving the smaller sub-problems obtained by decomposing large-scale problems. Since the standard ADMM may not guarantee convergence in case of non-convex MISOC problem, an iterative ADMM algorithm is proposed in paper [5] and it is studied here. The authors state that this algorithm improves the convergence performance by removing integers through fixing continuous and binary variables in each iteration. This means that, during each I-ADMM iteration, the standard ADMM and SOC reformulation are utilized with fixed integer variables to perform decentralized operation of the multi-area IEGS. The shared information between each area is fixed by the Area Operator (AO), equivalent to the latest ADMM values. The iterative procedure stops when integer variables between two adjacent iterations do not change significantly. The iterative framework contributes a lot to enhance the convergence, although this algorithm is heuristic and cannot guarantee global optimality.

Figure 3-4 shows the flowchart for iterative ADMM algorithm.

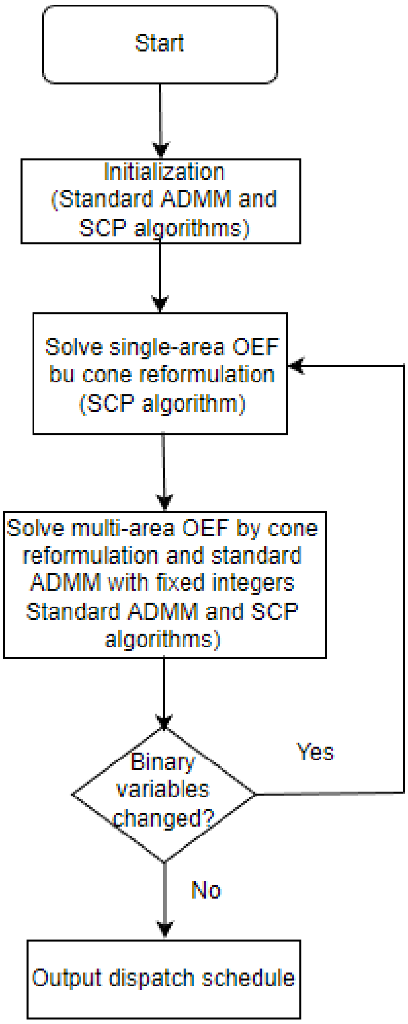


Figure 3-4: Iterative ADMM for IEGS [5]

Here a pseudo-code for iterative ADMM algorithm for IEGS is introduced in Algorithm 3.

Algorithm 3 Iterative ADMM

Step 1:
Represent the multi area OEF problem as Mixed Integer Second Order Cone Programming (MISOCP) and relax integer variables to continuous variables [0], [1], for converting the model into Second Order Cone Programming (SOCP).
Apply Standard ADMM for decentralized operation.
This initialization step provides the initial values of shared information.
Set iteration index k=0.
Step 2:
Solve each single-area OEF with integers which are not fixed.
Each subsystem area models its OEF problem as MISOCP and fix its shared information.
Each AO solves its OEF problem by SOC reformulation and obtains the optimal value of binary variables I_a^k.
Step 3:
Solve each multi-area OEF with integers fixed.
Apply Standard ADMM (Algorithm 2) to the multi-area IEGS with fixed I_a^k.
The convexity of the SOCP guarantees the convergence of ADMM.
This step updates the shared information P_a^k of each subsystem.
Step 4:
Check if the integer variables change between two adjacent iterations:

$$\mathbf{GAP}^I = \mathbf{I}^k - \mathbf{I}^{k-1}$$

if $\mathbf{GAP}^I = 0$
then End I-ADMM procedure
Return P_a^k to obtain the final schedule.
else Go to Step 5.
endif
Step 5:
Set k=k+1. Continue steps 2 to 4 until the stopping criteria are met.

In the paper [5], an additional constraint (3-21) is included to make equation (3-6) tight. This gives a MISOC problem with concave constraints, making it hard to find global optimum.

$$\Gamma_{(i,j)} - \left(1/C_{(i,j)}\right)^2 g_{(i,j)}^2 \leq 0 \tag{3-21}$$

The idea of Sequential Cone Programming (SCP) is to approximate (3-21) by a first order Taylor expansion with respect to $g_{(i,j)}^{k-1}$ obtained in the last iteration.

$$\begin{aligned} \Gamma_{(i,j)}^k - \left(1/C_{(i,j)}\right)^2 \left[\left(g_{(i,j)}^{k-1}\right)^2 + 2g_{(i,j)}^{k-1} \cdot \left(g_{(i,j)}^k - g_{(i,j)}^{k-1}\right)\right] \\ \leq s_{(i,j)}^k \end{aligned} \tag{3-22}$$

where,
$s_{(i,j)}$: non-negative slack variable

A penalty factor, ψ is introduced. The penalty function method can effectively bring the $\Gamma_{(i,j)}$ close to $\left(1/C_{(i,j)}\right)^2 g^2_{(i,j)}$ without increasing the computational burden. ψ has a small value in the beginning so as to find a good enough solution quickly. It increases the value as the sequential procedure proceeds which brings the value of $s_{(i,j)}$ to zero.

Figure 3-5 shows the flowchart for SCP algorithm.

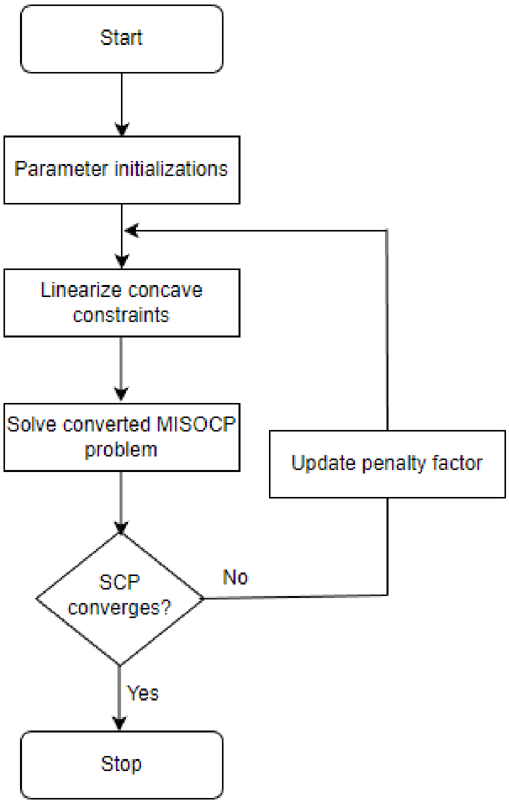


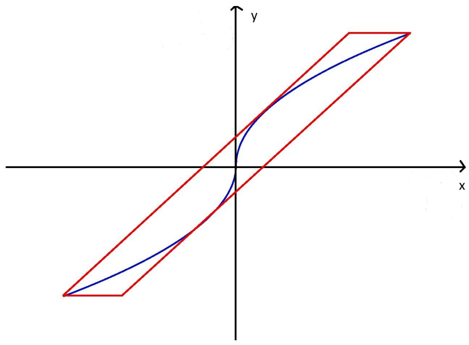
Figure 3-5: Sequential Cone Programming for SOC reformulation [5]

3-1-5 Extended Convex Hull relaxation method

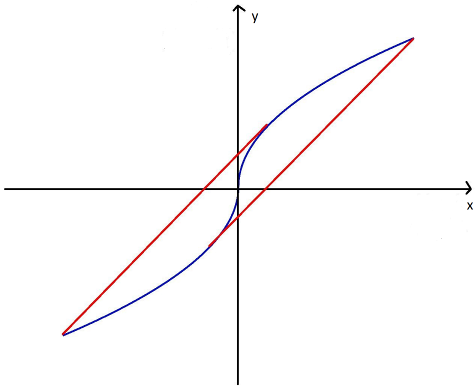
An ECH based relaxation technique has been proposed in paper [6] to convexify the Weymouth equations. The ECH-based constraints lead to a convexified OEF problem without introducing new binary variables. Before studying what an ECH is, let us first define the convex hull. From the definition in the book [41], the set S is convex if the line segment connecting any two points in S lies in S. The convex hull of a set S is denoted by $\operatorname{conv} S$ and defined as a set of all convex combinations of points in S:

$$\operatorname{conv} S = \{\theta_1 x_1 + \cdots + \theta_k x_k \mid x_i \in C, \theta_i \geq 0, i = 1, \ldots, k, \theta_1 + \cdots + \theta_k = 1\},$$

where,
x_i: set of points
θ: parameter value.



(a) Extended convex hull (ECH) for the Weymouth equation



(b) Convex hull for the Weymouth equation

Figure 3-6: Extended convex hull and convex hull [6]

As the name suggests, the ECH of a set is convex. It contains the convex hull of this set. It should also contain less redundant elements and have a simple analytical form. Denote the convex hull and the ECH of the set Y as Y_c and Y_e respectively. Y_e is convex and $Y_c \subset Y_e$ as per the above discussion. The redundant elements are those which belong to Y_e but not Y_c. The proposed ECH is problem dependent.

The ECH and convex hull of Weymouth equation is shown in Figure 3-6. The horizontal axis, x denotes the difference between the pressure squares of gas nodes i and j $(\omega_i^2 - \omega_j^2)$. The vertical axis, y is the gas flow $g_{(i,j)}$. The blue curve depicts the Weymouth equation. In Figure 3-6a, the region surrounded by red lines is the ECH and the red lines and the outermost blue lines in Figure 3-6b consist of the convex hull boundary.

The convex hull is the tightest convex relaxation, but the proposed ECH in Figure 3-6a is comparatively more prone to being characterized mathematically, although it contains a larger area consisting of redundant elements. Additionally, the ECH does not introduce any binary variables to the gas block model whilst preserving the bi-directional property of the Weymouth equation. In the paper [6], Weymouth equations are replaced by ECH-based constraints, and the mathematical formulation is as follows:

$$\omega_i^{\min} \leqslant \omega_i \leqslant \omega_i^{\max}, \tag{3-23}$$

$$\mathrm{a}_l^{\mathrm{L}} \cdot \left(\omega_{i(l)} - \omega_{j(l)}\right) + \mathrm{b}_l^{\mathrm{L}} \leqslant \omega, \quad l \in \mathcal{P}, \tag{3-24}$$

$$\omega_l \leqslant \mathrm{a}_l^{\mathrm{U}} \cdot \left(\omega_{i(l)} - \omega_{j(l)}\right) + \mathrm{b}_l^{\mathrm{U}}, \quad l \in \mathcal{P}. \tag{3-25}$$

Constraint (3-23) sets the upper and lower bounds of the gas flow in a gas passive pipeline. It also states the upper and lower bounds of the ECH (Figure 3-6a). Constraints (3-24), (3-25) represent the left and right bounds of the proposed ECH (Figure 3-6b), respectively, where a_l^L, a_l^U, b_l^L, and b_l^U are constants.

Therefore, the gas model along with constraints equations (2-11) to (2-16) can be approximated as a convex by replacing the Weymouth equation (2-11) by ECH-based constraints.

The pros and cons of ECH are discussed here,

Pros:

- The resulting convex problem is comparatively easier to solve.
- No additional binary variables are required.
- Bi-directional property of gas pipelines is respected, which increases the gas transmission flexibility.
- This approach seems scalable to large-scale instances.

Cons:

- The values of constants in the left and right bounds (constraints (3-24), (3-25)) must be chosen carefully.

In order to centrally solve the optimization problem, this ECH relaxation method can be used to relax the nonlinear, non-convex gas flow equations and further solved using Algorithm 1.

3-1-6 Jacobi Proximal Alternating Direction Multiplier Method optimization algorithm

Jacobi Proximal Alternating Direction Multiplier Method is an extension of the basic ADMM and is also an iterative optimization technique. It can be used where variables are updated in parallel with an advantage of faster processing time compared to other iterative techniques. The J-ADMM algorithm has been implemented in paper [6] to solve the convexified model wherein the Weymouth equation is relaxed by ECH based constraints as discussed in Section 3-1-5. The multi-block distributed optimization problems does not necessarily converge. The J-ADMM algorithm [43], can be proved to converge when solving multi-block optimization problems. The J-ADMM algorithm, allows parallel computing and can provide the unique optimal operation policy for the multi-block IEGS with guaranteed convergence and hence seems quite applicable and thus adopted to solve the distributed OEF problem.

In this paper [6], the power network and gas network are decoupled based on physical differences and power network is further divided into blocks (r). The objective function for OEF problem defined in equation (2-2) is therefore split into separate functions corresponding to blocks. The compact form of convexified OEF problem, in which the Weymouth equation is reformulated by its ECH based constraints is represented below:

$$\min_{\mathbf{x}_1,\cdots,\mathbf{x}_N} f_1(\mathbf{x}_1) + \cdots + f_N(\mathbf{x}_N)$$

$$\text{s.t. } \mathbf{A}_1 \cdot \mathbf{x}_1 + \cdots + \mathbf{A}_N \cdot \mathbf{x}_N = \mathbf{c} \tag{3-26}$$

$$\mathbf{x}_1 \in \Omega_1, \cdots, \mathbf{x}_N \in \Omega_N$$

where,
N: number of blocks ($N \geq 2$)
x_r: variables belonging to block r with $r = 1, ..., N$
Ω_r: convex feasible regions for block r (which is similar to Ω_a in (2-3))
f_r: convex objective functions for block r i.e.

$$\min_{x_a} \sum_{a=1}^{N} \left\{ \sum_{i \in W} \mu_g v_{a,i} + \sum_{i \in N_{NG}} F_{a,i} \left(P_{a,i} \right) + V^p \cdot \sum_{j=1}^{M} L_{a,j} \right\}_r .$$

$\mathbf{A}_r$: constant matrices
c: constant vector.

The procedure for solving the multi-block optimization problem using J-ADMM algorithm as presented in the paper [6] consists of initializing the penalty parameter d, damping parameter γ, matrix $\mathbf{P}^r$ with $r = 1, .., N$ and Lagrangian multiplier λ_0. The next step is to solve the objective function (3-27) for blocks 1 to N in parallel.

$$\mathbf{x}_r^{k+1} = \arg \min_{\mathbf{x}' \in \Omega'} f_r \left(\mathbf{x}_r \right) + \left(\frac{\mathrm{d}}{2} \right) \cdot \left\| \mathbf{A}_r \cdot \mathbf{x}_r + \sum_{j \neq r} \mathbf{A}_j \cdot \mathbf{x}_j^k - \mathbf{c} - \left(\frac{\lambda^k}{\mathrm{d}} \right) \right\|_2^2 + \left(\frac{1}{2} \right) \cdot \left\| \mathbf{x}_r - \mathbf{x}_r^k \right\|_{P^r}^2 , \quad (3\text{-}27)$$

with,
$r = 1, \cdots, N$
$\mathbf{x}_r^k$: optimal solutions for block r at k-1 iteration
P: positive semi-definite matrix.

Next step is to obtain the updated values of $\mathbf{x}_r^{k+1}$ for $r = 1, ..., N$ from equation (3-27). Then update the Lagrangian multiplier as follows:

$$\lambda^{k+1} = \lambda^k - \gamma \cdot \mathrm{d} \cdot \left(\sum_{r=1}^{N} \mathbf{A}_r \cdot \mathbf{x}_r^{k+1} - \mathbf{c} \right) . \quad (3\text{-}28)$$

According to the algorithm presented in paper [6], J-ADMM converges to its global optimum if matrix P^r and damping parameter γ satisfy following conditions.

$$P^r \succ \mathrm{d} \cdot \left(\frac{1}{\sigma_r} - 1 \right) \cdot \mathbf{E}_r^{\mathrm{T}} \cdot \mathbf{E}_r, \quad r = 1, \cdots, N. \quad (3\text{-}29)$$

where,
$\mathbf{E}_r$: constant matrix.

$$\sum_{r=1}^{N} \sigma_r < 2 - \gamma, \quad r = 1, \cdots, N \quad (3\text{-}30)$$

where,
σ_r: constant value.

If all $\sigma_r < \frac{2-\gamma}{N}, \quad r = 1, \cdots, N$, the conditions (3-29)-(3-30) can be simplified as follows:

$$P^r \succ \mathrm{d} \cdot \left(\frac{N}{2-\gamma} - 1 \right) \cdot \mathbf{E}_r^{\mathrm{T}} \cdot \mathbf{E}_r, \quad r = 1, \cdots, N. \quad (3\text{-}31)$$

Figure 3-7 shows the flowchart of J-ADMM algorithm.

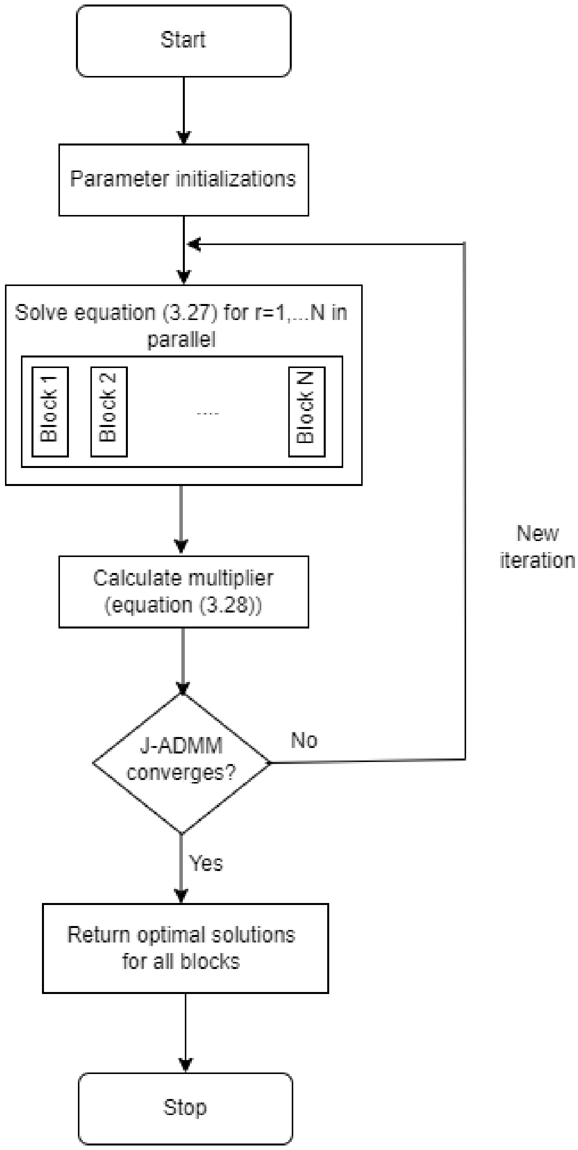


Figure 3-7: J-ADMM algorithm [6]

Here a pseudo-code for Jacobi proximal ADMM algorithm for IEGS is introduced in Algorithm 4.

Algorithm 4 Jacobi Proximal ADMM

Step 1:
Initialize all the parameters: penalty parameter d, damping parameter γ, matrix P^r (r=1,...,N), stopping criteria ε_1 and ε_2, maximum number of iterations k_{max}.
Initialize variables x_0^r (r=1,...,N) and Lagrangian multiplier λ_0.
Set iteration index k=0.
Step 2:
Solve equation (3-27).
Obtain the updated values $x_{k+1}^r (r = 1, ..., N)$
After obtaining x_{k+1}^r for r=1,...,N, update Lagrangian multiplier λ_{k+1}
Step 3:
Check whether both stopping criteria are satisfied
if Yes
then Stop and return x_{k+1}^r (r=1,...,N)
else if $k = k_{max}$
Stop and return NULL (fail to converge).
else Set k=k+1.
Go to Step 2.
endif

Solution feasibility and recovery method

The optimal solution obtained by solving the convexified problem may not be feasible for the original non-convex problem because the feasible region for the original problem is expanded. Therefore, the optimal value of the problem (3-26) may be smaller than that of the original problem. The paper [6] proposes the following to check whether the two optimum solutions are equal without solving the original non-convex problem.

Proposition: The original and the convexified problems have the same optimum if the problem:

$$\begin{aligned}
&\min_{\pi,\delta^+,\delta^-} \mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{\delta}^+ + \mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{\delta}^- \\
&\text{s.t. } \left(g_l^*\right)^2 \cdot \operatorname{sgn}\left(g_l^*\right) = \mathrm{W}_l \cdot \left(\pi_{i(l)} - \pi_{j(l)}\right), \quad l \in \mathcal{P} \\
&\operatorname{sgn}\left(g_l^*\right) = \begin{cases} 1 & g_l^* \geqslant 0 \\ -1 & g_l^* < 0 \end{cases}, \quad l \in \mathcal{L} \\
&\left(1-\delta_i^-\right) \cdot \mathrm{G}_i^{\min} \leqslant \pi_i \leqslant \left(1+\delta_i^+\right) \cdot \mathrm{G}_i^{\max}, i \in \mathcal{N} \\
&\pi_{j(c)} \leqslant \alpha_c \pi_{i(c)}, \\
&\delta_i^+, \delta_i^- \geqslant 0,
\end{aligned} \tag{3-32}$$

is feasible and its objective value is equal to zero,
where, $\boldsymbol{\delta}^+ = \left(\delta_1^+, \cdots, \delta_M^+\right)^{\mathrm{T}}$ and $\boldsymbol{\delta}^- = \left(\delta_1^-, \cdots, \delta_M^-\right)^{\mathrm{T}}$ are slack variables.

The paper [6] also provides a proof for the above proposition. It can be further used to recover the feasible optimal solution for the original problem from the convexified problem.

3-2 Performance metrics

Performance metrics can be defined as quantitative measures used to assess the performance of a system or a method based on specific criteria. These metrics can include measures such as accuracy, speed, reliability, scalability, efficiency, cost, etc. The metrics used to evaluate a system or a method should be relevant to the goals and objectives of the system or the method, and they should be measurable and quantifiable. To assess the effectiveness of proposed solutions, it is essential to establish performance metrics that allows for objective comparison. Benchmarking involves comparing one or more optimization algorithms based on such a set of performance metrics.

Performance metrics are often used in engineering, science, and business to evaluate the effectiveness and efficiency of systems, processes, and products. They help make informed decisions, to identify areas for improvement, and to compare different methods or systems. They can also be used to establish benchmarks and to track progress over time. In this section, performance metrics tailored to the specific requirements of our problem have been introduced. These performance metrics will serve as a benchmark for evaluating and comparing the results obtained in Chapter 4.

Two main classifications of metrics can be given as theoretical metrics and simulation based metrics. This is based on the way in which the metric is found. Following Table 3-1, provides some of commonly studied metrics and provides the type to which it belongs.

Table 3-1: Types of metrics

Sr.No.	Metric	Type
1.	Optimal cost	Simulation
2.	Number of iterations	Simulation
3.	CPU time	Simulation
4.	Convergence	Theoretical & Simulation
5.	Solution feasibility	Theoretical & Simulation
6.	Scalability	Simulation
7.	Adaptability	Simulation
8.	Communication burden	Theoretical
9.	Stability	Simulation
10.	Algorithmic complexity	Theoretical & Simulation
11.	Robustness	Simulation
12.	Energy efficiency	Theoretical

Here a description of some of the most commonly used performance metrics has been provided to better understand their function and reason for the choice of these metrics in our comparative study. Not all the performance metrics mentioned in Table 3-1 have been considered under the scope of this study.

1. **Optimal cost (optimal value)**: The best possible solution to an optimization problem is known as the optimal value. It is obtained from values of the decision variables that attain the minimum (or maximum) value of the objective function over the feasible region. In an optimization problem where the objective function is to be maximized the optimal value is the least upper bound of the objective function values over the entire feasible region. If there is no upper bound, then we say that the optimal value is $+\inf$, while if the feasible region is the empty set, we define the optimal value of a maximization problem to be $-\inf$. Conversely, in an optimization problem where the objective function is to be minimized the optimal value is the greatest lower bound of the objective function values over the entire feasible region. If there

is no lower bound, then we say that the optimal value is $-\inf$, while if the feasible region is the empty set, we define the optimal value of a minimization problem to be $+\inf$. Therefore, every optimization problem has a well-defined optimal value. But the important note is that not every optimization problem has an optimal solution.

2. **Number of iterations**: Number of function evaluations or number of iterations refers to the count of repetitive steps taken by an algorithm to find an optimal solution. These iterations depend on the optimization problem convexity, algorithm used, most importantly convergence criteria like upper/lower bounds, tolerance, etc.

3. **CPU time**: The total computational time or time required for entire execution of a program is referred here as CPU time. It includes the time required for computations, memory access and other CPU-related activities. In other words, the length of time that a central processing unit takes to process instructions for a particular program or task. The CPU time is used to quantify the overall empirical efficiency of functionally similar algorithms. Better efficiency algorithms are those which have minimum CPU time.

4. **Convergence**: Convergence can be defined as a process by which a sequence of values approaches a specific limit or target value. As a fundamental concept in mathematics, property (exhibited by certain infinite series and functions) of approaching a limit more and more closely as an argument (variable) of the function increases or decreases or as the number of terms of the series increases. In the context of optimization, convergence implies that an algorithm is getting closer to an optimal solution. Algorithms are considered convergent if they approach the optimal solution as the number of iterations increases. The speed at which an algorithm converges is also important and is known as convergence rate.

5. **Solution feasibility**: Optimization problems may have feasible or infeasible solutions. Solution feasibility refers to whether the obtained solution satisfies all the constraints and requirements for a given problem. A solution is considered feasible if it meets all the necessary requirements without violating any constraints. Feasibility ensures that the solution is practically usable and aligns with the problems objectives. An infeasible solution is the one which violates one or more constraints. In most of the optimization algorithms, first an attempt is made to find the feasible solution and then another attempt is made to locate another feasible solution which will improve the objective function value. This ensures that the solution remains valid and the best optimum value can be found at the end.

6. **Scalability**: Scalability can be explained as the ability of a system to handle bigger demands or larger problem sizes or increased complexity in the network efficiently. Scalable algorithms can handle problems with a large number of variables, constraints or nodes. As the problem size increases, computation time or memory usage should not degrade significantly for a scalable algorithm. It is important to balance accuracy and efficiency while achieving scalability of optimization algorithms.

7. **Adaptability**: It is important to consider if an optimization algorithm is capable to handle changing conditions, for example, load changes, network size or network topology changes, etc. Adaptability can be explained as the ability of the system to adjust, modify or respond efficiently to changes in the environment, thus implying flexibility. Adaptive algorithms are more preferable in dynamic environments since it is easier to keep the same algorithm and extend it to a different condition as compared to devising an entirely new algorithm for including a change in condition.

8. **Communication burden**: In simple words, communication burden refers to the cost or effort required for different parts of the algorithm (for instance nodes in a network) to exchange information with each other. It can be understood as a balance of efficiency, scalability and convergence. Excessive communication burden can slow down the convergence. In order to achieve optimal solutions, efficient communication strategies are crucial which can be obtained by keeping the communication burden low.

9. **Stability**: This metric could be used to evaluate how well the algorithm maintains stability during execution. It provides an intuition to understand if an algorithm can be scalable. Stable algorithms are considered to achieve good general performance. Unstable algorithms may oscillate or diverge.

10. **Algorithmic complexity**: Algorithmic complexity or computational complexity refers to the quantity of computational resources (such as time, memory etc.) needed by an algorithm. Complexity analysis can be useful in understanding if the algorithm is scalable with the problem size. This metric can provide an insight into the efficiency of the algorithm. Efficient algorithms have lower complexity.

11. **Robustness**: The metric which ensures that the performance of an optimization algorithm remains stable and reliable under uncertainty or variations is referred to as robustness. Robust algorithms maintain good performance even with variations.

12. **Energy efficiency**: While evaluating the performance of an optimization algorithm, computational tasks are usually resource intensive and consume energy. Thus, energy efficiency becomes an important metric to minimize power consumption, reduce energy usage meaning lowering operational costs, etc. Energy-efficient algorithms minimize resource usage by minimizing unnecessary computations. Such algorithms aim to reduce the environmental impact.

To conclude Chapter 3 has provided a comprehensive description of the methodologies employed for relaxing the non-convexity as well as the optimization algorithms used to solve the OEF problem for IEGS. The later half provides the need of performance metrics and describes them in detail. Some of these metrics are important for evaluating the proposed solutions. By establishing a solid foundation in the theoretical aspects of our research, we can now move into the realm of practical implementations. In Chapter 4, we shift our focus to the empirical findings and analysis obtained through experimentation. These results will allow us to gain valuable insights into the effectiveness and viability of these methodologies.

Chapter 4

Results

In this Chapter 4, the results of experimentation and analysis obtained through the application of different methods have been presented. This chapter serves as a bridge between theoretical foundations established in the previous chapters and practical implementation. Deeper insights into the performance and effectiveness of the various methods can be established by examining and interpreting these results.

4-1 Case study

Simulations carried out on system with specifications as follows:

- Numerical tests are performed by a Matlab 2021b platform
- Gurobi Optimizer version 10.0.2 build v10.0.2rc0 (win64)
- Processor: AMD Ryzen 7 4700U with Radeon Graphics
- Installed RAM: 16 GB
- System type: 64-bit operating system, x64-based processor

The data sets used for this case study:

The test cases used for this study are similar to those used in paper [5] which comprise of the required system parameters. Table 4-1 provides a summary of the systems in each test system used in respective case studies.

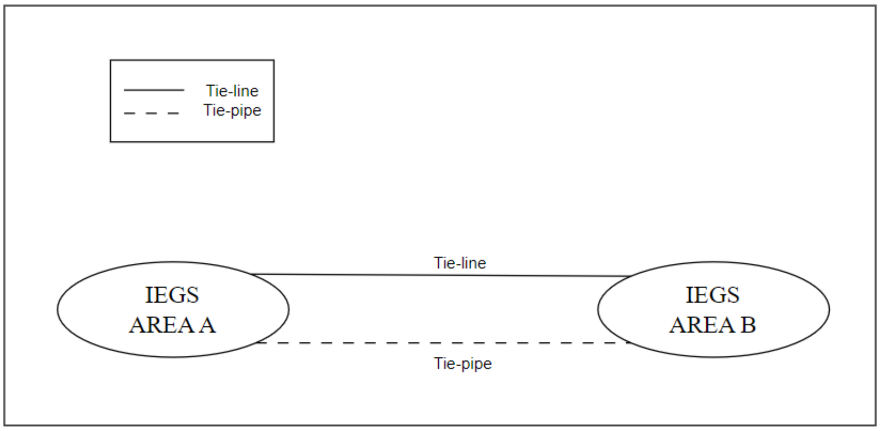
- One-area 6-bus-6-node Integrated Electricity and Natural Gas System (IEGS) : Gastranssmion6_1.xlsx
- Two-area 12-bus-12-node IEGS (2A-IEGS) : `motor.ece.iit.edu/data/Gastranssmion12_multi-area.xlsx`
- Three area 73-bus-30-node IEGS (3A-IEGS) : `motor.ece.iit.edu/data/Gastranssmion73_multi-area.xlsx`
- Four area 472-bus-40-node IEGS (large) (4A-IEGS): `motor.ece.iit.edu/data/Gastranssmion472_multi-area.xlsx`

Table 4-1: Summary of test systems in Case study

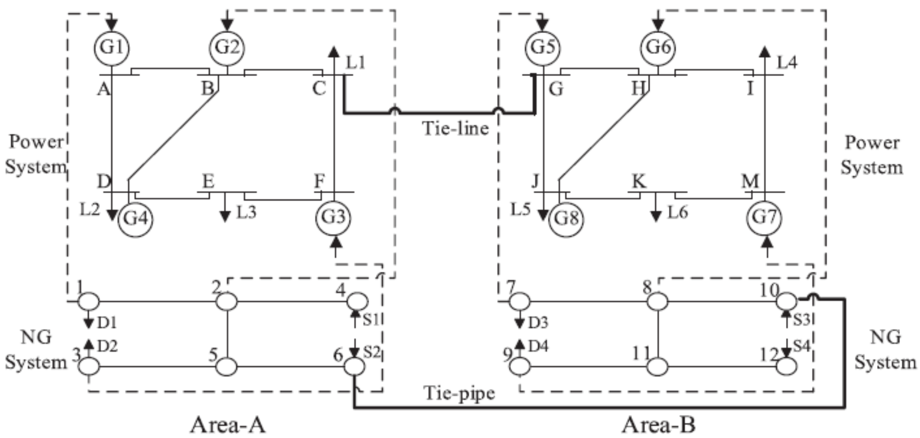
System	No. of non-Natural Gas-fired Unit (NGU)	No. of NGU	No. of gas wells	No. of tie-lines (electricity)	No. of tie-pipes (gas)
1A-IEGS	1	3	2	0	0
2A-IEGS	2	6	4	1	1
3A-IEGS	75	24	9	5	3
4A-IEGS	184	32	12	5	4

The topology for different cases is as follows:

The 2A-IEGS is composed of two exactly same 6-bus-6-node integrated energy systems connected by one tie line and one tie pipe as shown in Figure 4-1a.



(a) Example 2A-IEGS topology



(b) Detailed 2A-IEGS configuration [5]

Figure 4-1: 2A-IEGS topology and configuration

A detailed configuration of the 2A-IEGS is shown in Figure 4-1b, where it shows the locations of power generators, loads, gas wells. The dashed lines show connections within the same area while the bold line shows the interconnection between Area A and Area B (referred to as tie lines and tie pipes).

Similarly, for a 3A-IEGS is composed of the integrated energy systems connected by 5 tie lines and 3 tie pipes as summarized in Table 4-1 and other system parameters are given in data set provided above. Topology of 3A-IEGS is shown in Figure 4-2.

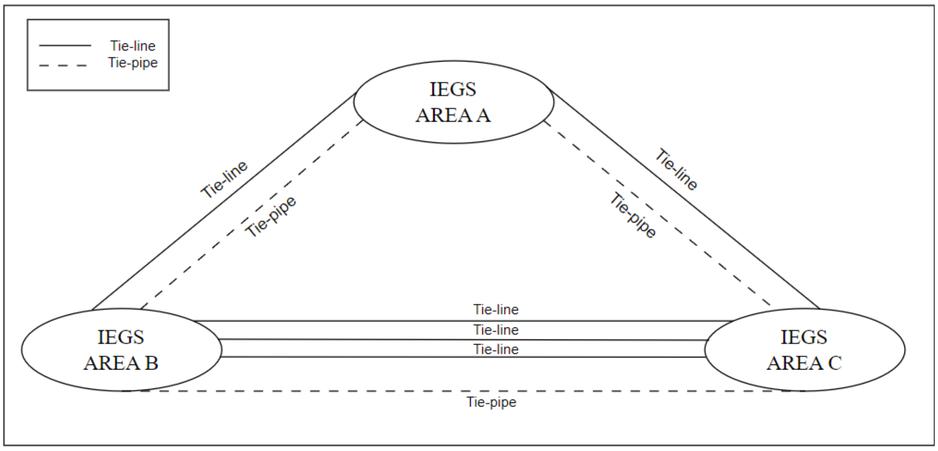


Figure 4-2: Example 3A-IEGS topology

Extending to a lager multi-area integrated system, the 4A-IEGS is composed of 472-bus-40-node IEGS and consists of 4 tie lines and 4 tie pipes as summarized in Table 4-1. Other system parameters are given in the data set provided above. The system topology is shown in Figure 4-3.

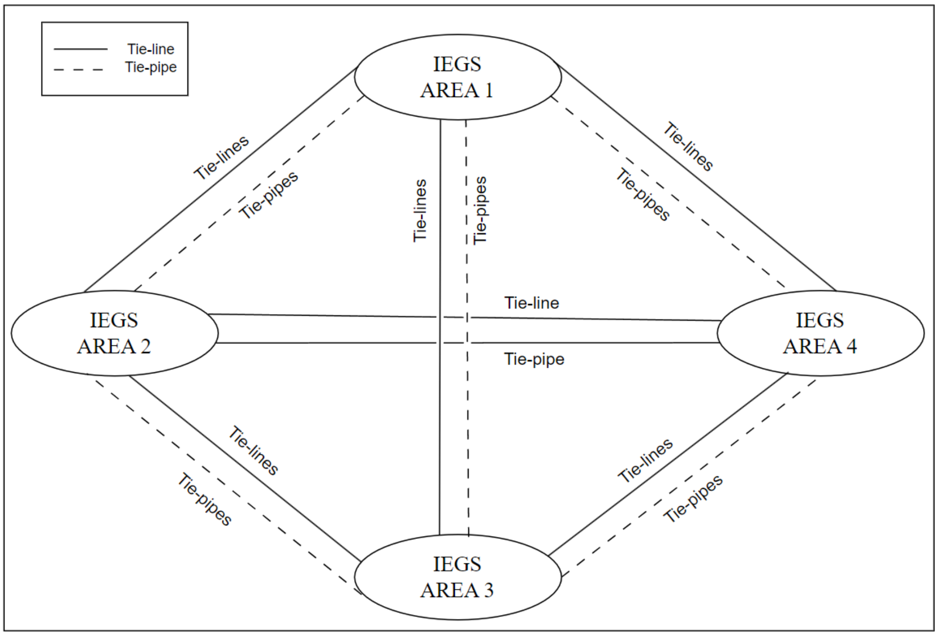


Figure 4-3: Example 4A-IEGS topology

4-2 Necessity of Iterative Alternating Direction Multiplier Method (I-ADMM) algorithm

Before moving on to the results for each method, this section explains why I-ADMM algorithm is used by presenting the plots of maximum residual values of Standard Alternating Direction Multiplier Method (ADMM) and I-ADMM when Second Order Cone (SOC) relaxation method is used to replace the nonlinear constraints.

4-2-1 SOC relaxation; Standard ADMM algorithm

As the relaxed optimization problem is solved by using standard ADMM algorithm, for 2A-IEGS, evolution of maximum residuals is presented in Figure 4-4. The iterative solution process shows the nature of standard ADMM algorithm. It is evident that the maximum residuals do not converge but keeps oscillating around the same value even after 100's of iterations. This shows that the standard ADMM method is not capable of providing an optimal solution for this optimization problem. The computation time required is too high. Similarly, the test cases for 3A-IEGS and 4A-IEGS also do not give an optimal solution. It has a similar nature as shown for 2A-IEGS. Hence it can be referred as a non-convergent as defined in the performance metrics.

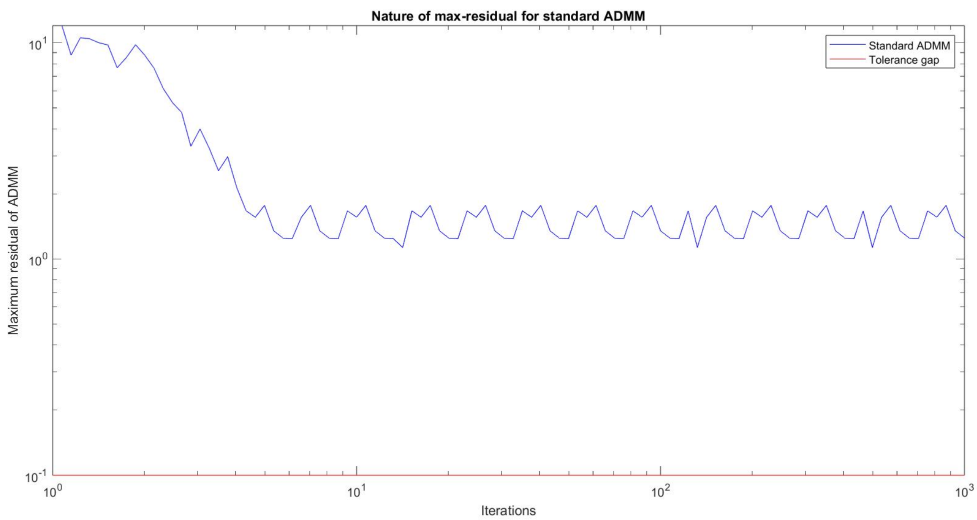


Figure 4-4: Nature of standard ADMM for 2 area network

Summary of the parameters used for the algorithm is presented below in Table 4-2.

Table 4-2: Parameters used for ADMM algorithm

ADMM	ρ_e	ρ_g	ε^P	ε^D	
	$1.5e+5$	15	0.1	0.1	
SCP	ψ^0	$\psi^{\max}$	V	ε^Z	ε^S
	0.1	1000	2	1	0.1

4-2-2 SOC relaxation; I-ADMM algorithm

Following Figure 4-5 shows the nature of max-residuals for I-ADMM implemented on the 2A-IEGS. It can be clearly seen that the blue line converges not once but twice. For this 2A-IEGS, during the initialization of I-ADMM, the standard ADMM takes about 31 iterations to converge. Further the integer variables are calculated by using the fixed variables which were passed on to each subsystem. There is a sharp jump after 32 iterations and again a decrease in the value of the solution as number of iterations increases. The jump is around the same moment as the fixed binary variables problem begins. This takes around 36 iterations to converge and the whole algorithm stops after that. This can be seen from the nature of the blue line Figure 4-5. Finally, the I-ADMM algorithm converges after 68 iterations. This algorithm is referred as a convergent algorithm. Iterative method provides a better result as compared to the previously described standard ADMM method.

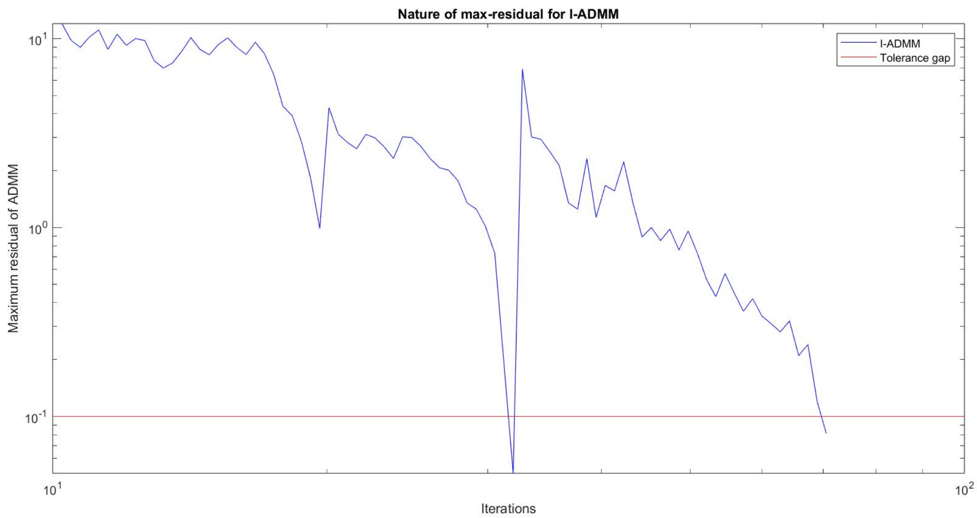


Figure 4-5: Nature of I-ADMM for 2 area network

Similarly tests were conducted on 3A-IEGS and 4A-IEGS data sets. Results for optimal cost are discussed in the following sections. Convergence is obtained in those cases as well and only 1 iteration in I-ADMM is needed, which reduces the communication as compared to centralized algorithm. The 3A-IEGS has 74-bus-30-nodes and 5 tie-lines and 3 tie-pipes. The ADMM tolerance is 0.1 for these cases. The 4A-IEGS is a large 4 area network with 472-bus-40-nodes connected by 4 tie-lines and 4 tie-pipes. The computational time is very high, hence the tolerance of ADMM is set at 0.5 for faster convergence. As the connecting lines provide energy sharing between areas, uniform distribution of energy flow can be achieved if natural gas network is congested.

4-3 Result for 1 Area IEGS

The result of employing different relaxation methods SOC relaxation and Extended convex hull (ECH) relaxation implemented on a small data set (1 area) with 6 nodes and 6 buses is presented here.

4-3-1 Cost versus iterations

The optimization problem is solved centrally and nature of the cost versus iterations is shown in Figure 4-6. Both the relaxed optimization problems converge to a solution. The red line representing the ECH relaxation method is faster than the SOC relaxation method. This is possible because ECH relaxation gives convex constraints while SOC relaxation needs more iterations to solve the mixed integer problem. The optimal cost is mentioned in Table 4-3.

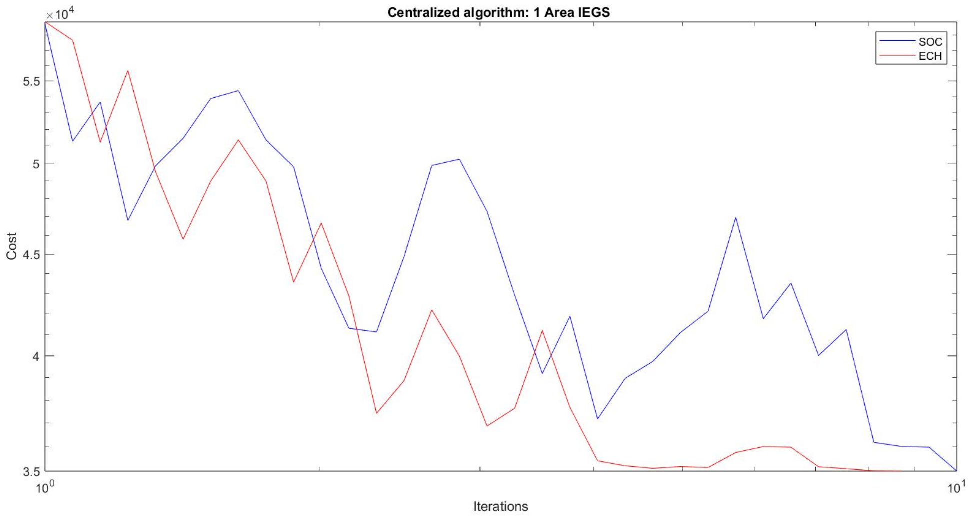


Figure 4-6: Result for 1A-IEGS

Table 4-3: Solution for 1 Area IEGS

Sr.No.	Metric	SOC; Centralized	ECH; Centralized
1.	Optimal cost ($)	$3.50 \cdot 10^4$	$3.49 \cdot 10^4$
2.	No. of iterations	10	8

A feasible solution for relaxed optimization problem is obtained for this test case. Optimal cost is in the same range for the relaxed optimization problem.

4-3-2 Cost versus CPU time

Here, the optimization problem is solved centrally and nature of the cost versus CPU time is shown below in Figure 4-7. The red line representing the ECH relaxation method takes less time compared to the SOC relaxation method. The reason is same as described before, where ECH relaxation gives convex constraints while SOC relaxation needs more time to solve the mixed integer problem. The CPU time (computational time) is mentioned in Table 4-4.

Convex optimization gives faster convergence as compared to Mixed Integer Second Order Cone (MISOC) optimization.

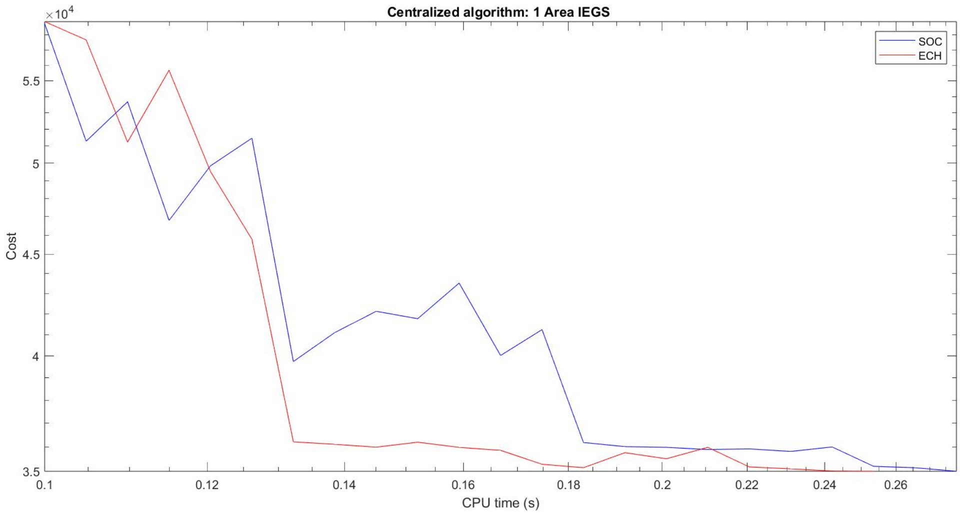


Figure 4-7: Result for 1A-IEGS CPU time

Table 4-4: Solution for 1 Area IEGS CPU time

Sr.No.	Metric	SOC; Centralized	ECH; Centralized
1.	Optimal cost ($)	$3.50 \cdot 10^4$	$3.49 \cdot 10^4$
2.	CPU time (s)	0.276	0.26

4-4 Result for 2 Area IEGS

As multiple areas are included, the comparison of centralized algorithm and decentralized algorithm can be seen here along with different relaxation methods SOC relaxation and ECH relaxation. The optimization problem is solved centrally for SOC relaxation method, ECH relaxation method and decentralized I-ADMM algorithm with SOC relaxation. The results for cost versus iterations are for centralized method only, while CPU time comparison is made for decentralized method.

4-4-1 Cost versus iterations

The relaxed problem is solved centrally and optimal solution is obtained for test case with 2 Area IEGS. The nature of cost versus iterations is shown in Figure 4-8. The number of iterations required for each method is different. The blue and the black lines for SOC relaxation with centralized algorithm and ECH relaxation with centralized method show similar nature as that for 1 Area. The black line reaches the solution is less number of iterations. The numerical results are shown in Table 4-5.

Table 4-5: Solution for 2 Area IEGS

Sr.No.	Metric	SOC; Centralized	ECH; Centralized
1.	Optimal cost ($)	$6.90 \cdot 10^4$	$6.90 \cdot 10^4$
2.	No. of iterations	12	11

Feasible solution obtained for both the relaxation methods. Optimal cost is in similar range for the relaxed optimization problem.

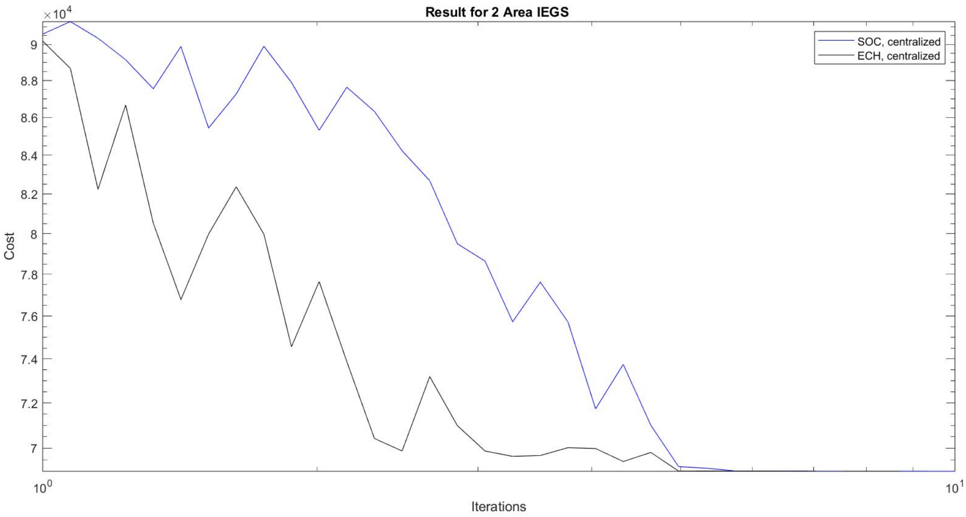


Figure 4-8: Result for 2A-IEGS

4-4-2 Cost versus CPU time

The Figure 4-9 shows the time required for each method to find a solution is different. The blue and the black lines for SOC relaxation with centralized algorithm and ECH relaxation with centralized method show similar nature as that for 1 Area. The red line represents the I-ADMM algorithm, which has dependency on binary variables, adds complexity but provides an accurate solution. The numerical results are shown in Table 4-6.

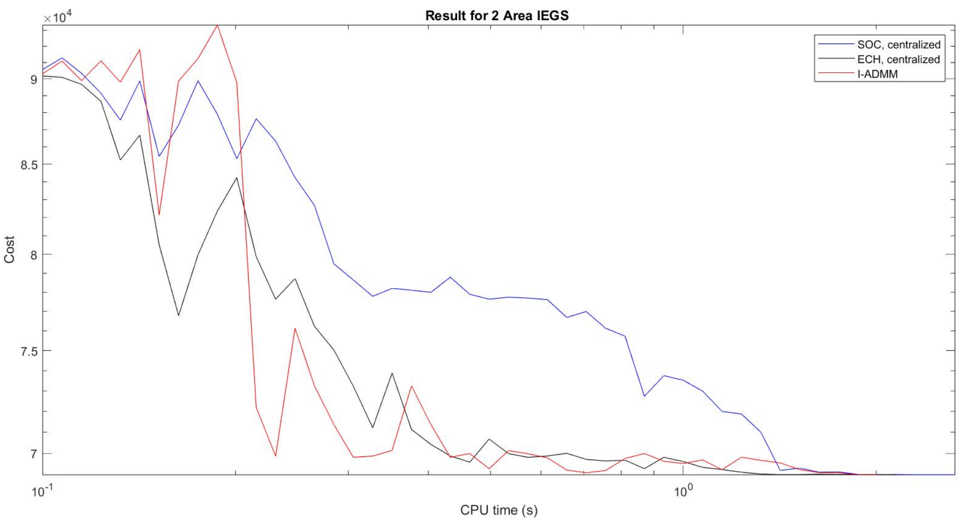


Figure 4-9: Result for 2A-IEGS CPU time

The ECH based relaxed optimization problem converges faster than SOC relaxation but slightly slower than I-ADMM.

Table 4-6: Solution for 2 Area IEGS CPU time

Sr.No.	Metric	SOC; Centralized	ECH; Centralized	SOC; I-ADMM
1.	Optimal cost ($)	$6.90 \cdot 10^4$	$6.90 \cdot 10^4$	$6.98 \cdot 10^4$
2.	CPU time (s)	2.6	2.15	2.01

4-5 Result for 3 Area IEGS

Similar to the 2 Area system, the results for employing different relaxation methods, SOC relaxation and ECH relaxation in centralized and decentralized method for cost versus iterations and cost versus CPU time are discussed here. It is expected that the result would be similar to that of 2 Area IEGS.

4-5-1 Cost versus iterations

For the centralized relaxation problem for 3 Area IEGS, optimal solution is obtained and result is shown in Figure 4-10. The nature of the plot is quite similar as expected, the only difference being the number of iterations required for convergence. Since the network is bigger, more time is required for the algorithm. The numerical results are shown in Table 4-7.

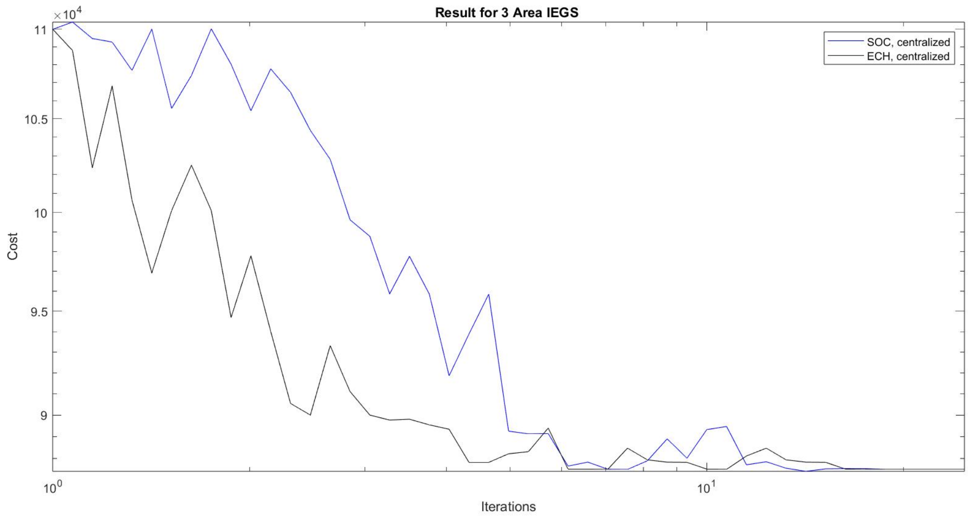


Figure 4-10: Result for 3A-IEGS

Table 4-7: Solution for 3 Area IEGS

Sr.No.	Metric	SOC; Centralized	ECH; Centralized
1.	Optimal cost ($)	$8.75 \cdot 10^4$	$8.75 \cdot 10^4$
2.	No. of iterations	25	23

Feasible solution obtained for 3 Area IEGS. Optimal cost is in the same range for the relaxed optimization problem.

4-5-2 Cost versus CPU time

Similarly, the results for employing different relaxation methods SOC relaxation and ECH relaxation in centralized and decentralized method is shown in Figure 4-11. As the system is bigger, more time is required for the algorithm. The numerical results are shown in Table 4-8.

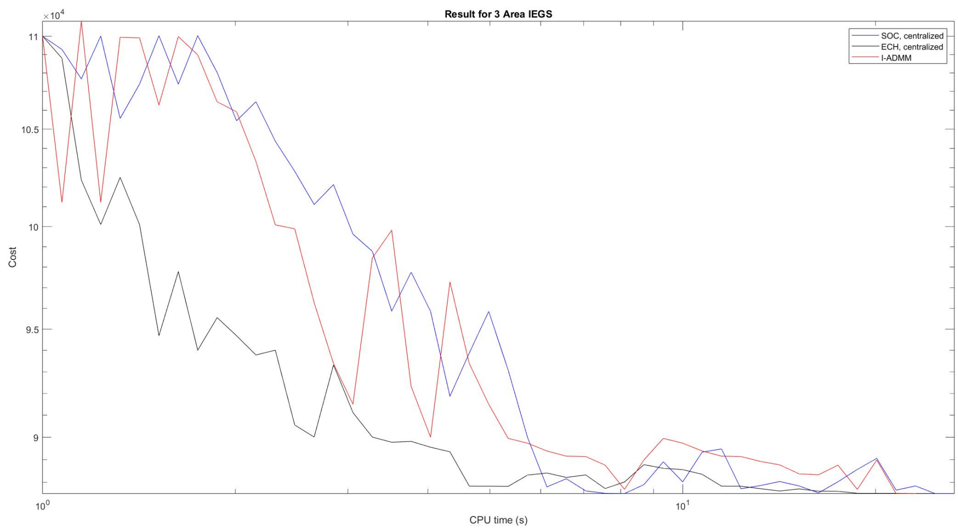


Figure 4-11: Result for 3A-IEGS CPU time

Table 4-8: Solution for 3 Area IEGS CPU time

Sr.No.	Metric	SOC; Centralized	ECH; Centralized	SOC; I-ADMM
1.	Optimal cost ($)	$8.75 \cdot 10^4$	$8.75 \cdot 10^4$	$8.75 \cdot 10^4$
2.	CPU time (s)	26.51	24.7	23.12

Optimal cost is in the same range for the relaxed optimization problem using centralized as well as decentralized methods. CPU time for decentralized method is smaller as compared to centralized method for the relaxed problem.

4-6 Result for 4 Area IEGS

Extending the data set to a 4 Area system, the results after SOC relaxation and ECH relaxation in centralized and decentralized manner here.

4-6-1 Cost versus iterations

Figure 4-12 shows the nature of the plot of cost versus iterations for each method. As expected, the nature of the plots is similar to that of other areas. This means that the algorithm works in a similar way for lager network and hence can be called as scalable. For a bigger network, more time and energy is utilized in finding the optimal solution. The numerical results are shown in Table 4-9.

It can be seen that feasible solution obtained. Optimal cost for centrally solving the problem is in the same range for both relaxation methods.

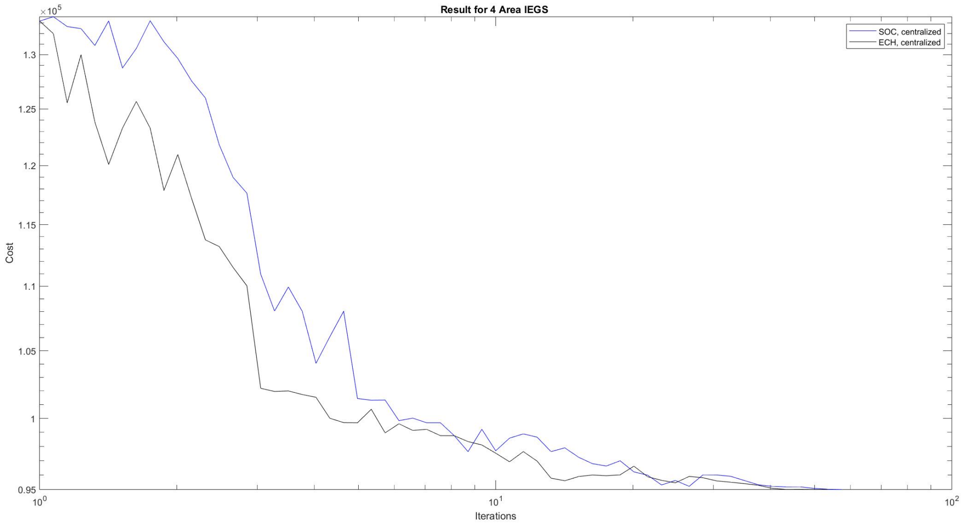


Figure 4-12: Result for 4A-IEGS

Table 4-9: Solution for 4 Area IEGS

Sr.No.	Metric	SOC; Centralized	ECH; Centralized
1.	Optimal cost ($)	$9.50 \cdot 10^4$	$9.50 \cdot 10^4$
2.	No. of iterations	57	53

4-6-2 Cost versus CPU time

Similarly, a plot for cost versus CPU time can be seen in Figure 4-13. As expected, the nature of the plots is similar to that of other areas. More time and energy is utilized in finding the optimal solution for larger network. The numerical results are shown in Table 4-10.

Table 4-10: Solution for 4 Area IEGS CPU time

Sr.No.	Metric	SOC; Centralized	ECH; Centralized	SOC; I-ADMM
1.	Optimal cost ($)	$9.50 \cdot 10^4$	$9.50 \cdot 10^4$	$9.65 \cdot 10^4$
2.	CPU time (s)	81.1	75.6	200.9

It can be seen that feasible solution obtained. Optimal cost for centrally solving the larger system is in the same range. Higher optimal cost when decentralized optimization method was implemented, more like sub-optimal solution. Time required for decentralized method is larger than centralized method in this case.

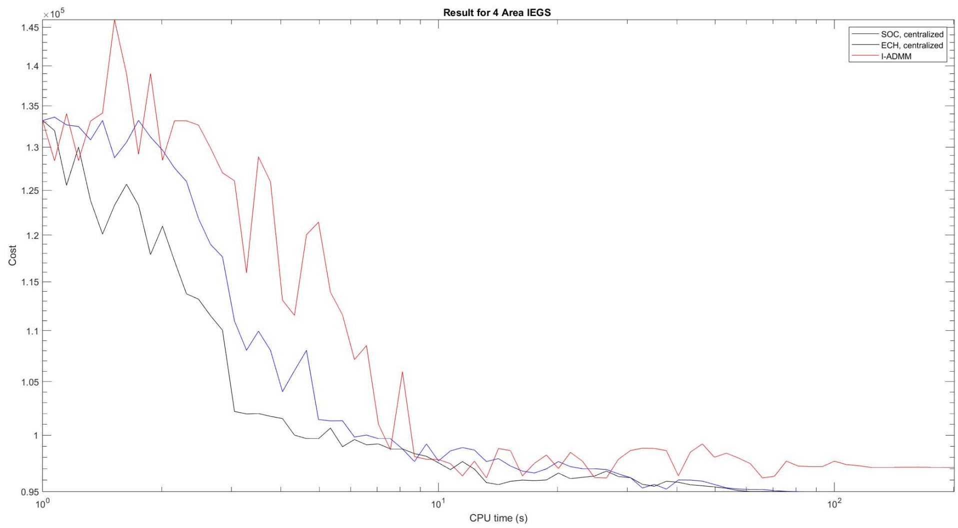


Figure 4-13: Result for 4A-IEGS CPU time

4-7 ECH relaxation; Jacobi Proximal Alternating Direction Multiplier Method (J-ADMM) algorithm

This section, the results obtained from the J-ADMM algorithm were suppossed to be summarized. Compared to other ADMM based methods, the J-ADMM algorithm allows for parallel computing. Since IEGS consists of multiple areas, it can be considered as multiple blocks and solving this with J-ADMM is supposed to be more efficient due to parallel computing. During the course of parallel computing, the J-ADMM would update all the blocks simultaneously, in parallel. The parallel computing does not incorporate the updates of Lagrangian multiplier λ. The method proposed in paper [6] promises that the optimal solution is unique because of the additional proximal term, $(1/2) \cdot \|\mathbf{x}^r - \mathbf{x}_k^r\|^2_{\mathbf{P}^r}$, which ensures the mathematical formulation is strictly convex.

The application of same algorithm to different data sets, namely 2A-IEGS, 3A-IEGS and 4A-IEGS that are considered for this study. However, it is difficult to obtain a feasible solution for the defined optimization problem since the constraints are not exactly satisfied. The problem gives an infeasible solution and the reason could be that the solution falls outside the extended convex hull for this specific problem. The solution obtained in paper [6], has a different objective function and a different data set But for this study, it is crucial to use the same optimization problem and data set as that used for the I-ADMM method, for the sake of comparison.

4-8 Discussion

All the results provided in this chapter show that the methods provide approximately the same optimal solution for the reformulated optimization problem. The number of iterations required for each method are different. The decentralized method of I-ADMM is definitely better because it solves the problem in a single iteration and is computationally more efficient. It reduces the computational load as compared to the centralized algorithm because individual Area Operator (AO) make decisions and the central controller is not loaded.

It can also be seen that all of the methods show convergence towards an optimal solution and hence they are all called convergent. All the methods that provide feasible solutions for the defined constraints are discussed. Scalability is also checked since the methods have been checked on test cases of different sizes from small to large.

By considering factors such as CPU time, solution feasibility, and scalability, the aim is to identify the most promising approaches for solving this complex optimization problem. Following list of observations is summarized from analysing the results discussed above:

- Complexity varies based on the type of constraints: SOC (MISOC) > ECH (Convex) reformulated problem.
- Optimal cost is in similar range for different relaxation methods in case of all test cases.
- SOC and ECH relaxed problems are scalable and adaptable to changing network.
- Centralized method provides optimal cost with both SOC relaxation, ECH relaxation.
- Decentralized optimization method is more efficient as it reduces the computational load.
- Decentralized method helps solve the issue of single point of failure caused by centralized controller.
- Data privacy is respected in decentralized method since only the necessary information is passed to the neighboring area.
- The solutions obtained in the chapter are for the relaxed optimization problem. The original nonlinear, non-convex problem may or may not have the same feasible solution.

Chapter 5

Conclusion and future work

The main objective of this thesis is to study the different optimization methods used for solving the optimal energy flow problem of Integrated Electricity and Natural Gas System (IEGS). Since the natural gas flow equations add to the complexity in solving the mathematical problem, relaxation methods are used to reformulate the non-convex constraints. The multi area optimization problem is solved in two ways: centralized and decentralized methods. The results of these methods have been seen in Chapter 4. This chapter presents the conclusion and few recommendations for future work related to this project.

5-1 Conclusion

A comprehensive study of performance metrics has been conducted as part of this thesis. In order to compare different optimization methods, it was necessary to first study how each method works and then find which method leads to better performance results. It would consist of solving an optimization problem for multi-area IEGS with constraints for both electricity and natural gas networks and corresponding coupling constraints. This would comprise of the crucial part of implementing the methods to relax the non-convexity due to gas flow equations and then implementation of different algorithms used for solving the Optimal Energy Flow (OEF) problem for multi-area IEGS. The results of this work should provide sort of a benchmark for the optimization methods used for solving the OEF problem for multi-area IEGS.

Here is a summary of the answers to the sub-questions which aid in answering the main research question defined in chapter 1.

1. Why is it important to study different optimization methods for IEGSs?
   The most important reason for studying optimization methods for any system begins from the goal of minimizing the cost or maximizing the output. Similarly, the goal of studying optimization methods for IEGS stems from the idea of minimizing the operational cost of the integrated network. It is important to reduce the cost for the producers as well as the consumers. It is also important because it fulfils the supply-demand gap. Since the storage of natural gas is a challenge, integrating it with an electricity networks reduces wastage of excess energy which could be very beneficial. Studying optimization methods for IEGS is also crucial for efficient

energy usage. Hence, before implementing the integrated networks in the field, it is necessary to study the feasibility using different mathematical problem formulations and finding the best performing behavior.

2. What are the technical and operational challenges posed by IEGS? Is it possible to overcome these challenges? If so, how?
As discussed in chapter 1, the challenges posed by IEGS have been described in great detail. This makes it necessary for us to consider not only technical but also the operational challenges which can be mitigated by proper use of optimization methods. Some of the ways (atleast theoretical), have also been presented which can be useful. The importance of studying optimization methods for integrated electricity and natural gas systems also paves the way for including more and more resources and driving the network towards sustainability. This will be beneficial for all the involved entities like producers, consumers, policy makers, etc.

3. How to find which method has better performance?
First an optimization problem has been defined in chapter 2. Then an extensive explanation of relaxation methods to linearize the nonlinear natural gas flow equation has been provided. Further different optimization methods has been provided and the algorithms have been provided. To find which method performs better, there should be comparative parameters and these quantitative parameters are referred to as performance metrics. Multiple performance metrics have been described but the methods are compared based only on selected few metrics like optimal cost, number of iterations, CPU time.

4. What is the result when each method is implemented?
The results obtained after implementation of the methods discussed in the previous chapter have been presented in chapter 4. The centralized algorithm provides almost similar results for both types of relaxation methods. The decentralized method Iterative Alternating Direction Multiplier Method (I-ADMM) is better than the centralized algorithm as it is computationally fast. However, it may provide suboptimal solution as seen in the larger test case where the centralized solution is better. The Jacobi Proximal Alternating Direction Multiplier Method (J-ADMM) algorithm should have been better as it is capable of parallel computing. However it is a bit difficult to find a feasible solution and the reason could be that the local optimal solution falls outside the extended convex hull.

5. What is the result of comparison of these methods with respect to each performance metric?
As discussed in the final section of chapter 4, all the results obtained for each method have been presented. The goal of this study was to compare the performance of different methods and hence it is important to solve the same optimization problem and test it on same data sets. From the results, it seems obvious that the better performing method as of now seems to be the decentralized I-ADMM algorithm. It converges in less number of iterations and CPU time required is less than that of the other methods with one exception of 4 Area IEGS.

6. What are the limitations of this work?
Optimization methods for IEGS studied in this thesis have certain limitations. First and foremost, the relaxed solution might not be the best solution for the original nonlinear, non-convex problem. It is difficult to determine a solution for such a problem when solvers are not available. The complexity of networks is another main point to be considered since multiple interconnected components including power grids, natural gas networks, other sources maybe involved. Here, noise is not considered but considering uncertainties and noise could also pose a different issue in the actual field applications. The dimensions of variables and constraints in the optimization problem also makes it complex. Another limitation could be balancing the cost, reliability and efficiency while reducing the environmental impact. Additionally modelling inaccuracy is a major drawback. It is crucial to have an accurate model of IEGS including all the components. Excessively simplified problems may lead to suboptimal solutions. Furthermore the obvious limitation is the computational efficiency while solving large-scale optimization problems and decentralized or distributed algorithms are definitely a possible solution to this issue.

5-2 Future work

Some recommendations for future work are presented here after having reflected on the results of this thesis.

- Including different kinds of resources like heat, thermal, renewable energy sources into the integrated network could be a logical next step to study different optimization problem formulations and methodologies used. There is quite some work in the energy domain and there are definitely opportunities to integrate different energy sources.
- Developing enhanced models with more accuracy and detailed models of IEGS components. It should be close to the real-time systems used in the industry and simulation should be able to consider dynamic behavior, transient effects, etc.
- Uncertainty management is a good starting point for future work. It would be interesting to investigate robust approaches which can handle uncertainties due to renewable energy generation, demand and fuel prices.
- Cybersecurity related challenges are of utmost priority in every field not just the energy markets. Addressing the cybersecurity related challenges by identifying potential vulnerabilities and threats and trying to protect the IEGS from these threats is very important. It is necessary to develop protocols and regulations to safeguard these integrated infrastructure.
- Integrating the demand side management strategies into IEGS could be a possibility. This could be dne by shifting loads, demand responses,etc. to enhance the efficiency of the system. It could also be beneficial to implement predictive analytics and adaptive control to improve the overall performance.

Glossary

List of Acronyms

IEGS	Integrated Electricity and Natural Gas System
NGU	Natural Gas-fired Unit
EDP	Economic Dispatch Problem
OPF	Optimal Power Flow
OEF	Optimal Energy Flow
LR	Lagrangian Relaxation
ALD	Augmented Lagrangian Decomposition
ADMM	Alternating Direction Multiplier Method
SOC	Second Order Cone
SOCP	Second Order Cone Programming
SCP	Sequential Cone Programming
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Non Linear Programming
MISOC	Mixed Integer Second Order Cone
MISOCP	Mixed Integer Second Order Cone Programming
PWA	Piecewise Affine
ECH	Extended convex hull
RLT	Reformulation-Linearization Technique
AO	Area Operator
I-ADMM	Iterative Alternating Direction Multiplier Method
J-ADMM	Jacobi Proximal Alternating Direction Multiplier Method

List of Symbols

(i, j)	Represents gas pipeline in which the first and last node is i and j respectively.
α_i, β_i, γ_i	Fuel coefficients of natural gas-fired unit i.
$\bar{\xi}_i$	Shutdown cost.

$\bar{B}^a$	Set of boundary buses.
δ_h	Phase angle of boundary bus h.
δ_j	Phase angle of boundary bus j.
$\Gamma_{(i,j)}$	Auxiliary variable for Second Order Cone (SOC) relaxation.
$\mathrm{a}_l^{\mathrm{L}}, \mathrm{a}_l^{\mathrm{U}}, \mathrm{b}_l^{\mathrm{L}}, \mathrm{b}_l^{\mathrm{U}}$	Constant for left and right bounds of Extended convex hull (ECH).
$\mathrm{R^e}$	Index of reference electric bus.
μ_g	Natural gas price.
ω	Pressure of natural gas nodes.
ω_i	The pressure of initial node.
ω_i	Nodal pressures.
ω_j	The pressure of end node.
$\phi(j)$	Set of areas connected to boundary bus j.
$\phi(mn)$	Set of areas connected to tie-pipe mn.
$\pi_i^{\max}$	Maximum value of squared nodal pressure of node i.
$\pi_i^{\min}$	Minimum value of squared nodal pressure of node i.
$\psi(h)$	Subsystems that electricity bus h belongs to adjacent area.
$\psi(j)$	Subsystems that electricity bus j belongs to in area a.
$\rho > 0$	Penalty parameter.
θ_h	Phase angle of inner bus h.
θ_i, δ_i	Bus phase angles.
θ_j	Phase angle of inner bus j.
$\varphi(i)$, $\varphi(j)$	Subsystems such that natural gas nodes, i from area a and j from adjacent area belong to respectively.
ξ_i	Startup cost.
a	Subsystem considered for analysis, where a=1, $\cdots$, N.
B'^a	Set of buses with load shedding.
B^a	Set of inner buses.
$C^U(i)$	Set of natural gas-fired units connected to node i.
$C_{(i,j)}$	The Weymouth constant which depends on the characteristics of the pipeline.
D_i^G	Residential natural gas demand at node i.
D_j	Electricity demand at bus j.
E_{gas}^a	Set of all gas flow between nodes (i, j)
$f(x_a)$	Objective function, comprising the total cost of the network.
$F_{a,i}$	Natural gas consumption of gas fired unit i.
F_i	Cost function of non-natural gas unit.
G_i^{PE}	Set of natural gas pipelines to node i.
G_i^{PF}	Set of natural gas pipelines from node i.
$g_{(i,j)}$	Natural gas flows on tie-pipes.
G_j	Set of all units connected to bus j.
$g_k(x_a) = 0$	Equality constraint which correspond to coupling constraints.
$h_l(x_a) \leq 0$	Inequality constraint represent local constraints depending on electricity and gas network.
i, j	Indices of natural gas nodes or electricity buses.
I^P	Set of inner pipes.
$I_{(i,j)}^+, I_{(i,j)}^-$	Binary indicators of natural gas flow direction on pipeline (i,j).
l	Passive gas pipeline.
L_i	Load shedding.
L_j	Load shedding at bus j.
N_G^a	Set of gas fired units.
N_{NG}^a	Set of non-gas fired units.

n_e, n_{ie}	Number of equality and inequality constraints, respectively.
P	Generation dispatch of units.
$p^f_{(i,j)}$	Power flow on transmission line (i, j).
P^W	Forecast of renewable energy units.
$P_{a,i}$	Real power generation.
P_i	Generator outputs.
$P_i^{\min}$, $P_i^{\max}$	Lower and upper limits on the power generated.
Q_i	Nodal natural gas demand.
r	Renewable energy units.
s^p	Index of natural gas well.
T^P	Set of tie-pipes.
V^p	Penalty price for load shedding.
$v_{a,i}$	Natural gas production of well W.
v_i	Natural gas well outputs.
W^a	Set of natural gas production wells.
x_a	The parameter vector used to optimize the objective function f (decision variable).
x_{g1}, x_{g2}, z_g	Auxiliary variables.
y, z, u	Decision variables.
z	Indicator for shutdown of thermal units.
Ω_a	The feasible operation region of area a, described by the electricity and natural gas network constraints.

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