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stochastic decomposition and equilibrium

Nenad Jovanović

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Electricity markets operation planning with risk-averse agents:
stochastic decomposition and equilibium

Dissertation

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chair of the Board for Doctorates
to be defended publicly on
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by

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The doctoral research has been carried out in the context of an agreement on joint doctoral supervision between Comillas Pontifical University, Madrid, Spain, KTH Royal Institute of Technology, Stockholm, Sweden and Delft University of Technology, the Netherlands.

Keywords: decomposition techniques, market equilibrium, risk-averse agents, stochastic optimization.

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Abstract in English language

Author: Nenad Jovanović
Affiliations: Comillas Pontifical University, KTH Royal Institute of Technology, Delft University of Technology
Title: Electricity markets operation planning with risk-averse agents: stochastic decomposition and equilibrium
Language: English
Keywords: decomposition techniques, market equilibrium, risk-averse agents, stochastic optimization

The growing penetration of renewable energy sources in electricity systems requires adapting operation models to face the inherent variability and uncertainty of wind or solar generation. In addition, the volatility of fuel prices (such as natural gas) or the uncertainty of the hydraulic natural inflows requires to take into account all these sources of uncertainty within the operation planning of the generation system. Thus, stochastic optimization techniques have been widely used in this context. From the point of view of the system operation, the introduction of wind and solar generation in the mix has forced conventional generators to be subject to more demanding schedules from the technical point of view, increasing for example the number of start-up and shutdown decisions during the week, or having to face more pronounced ramps. From the point of view of the market, all these technical issues are transferred to the market prices that are subject to greater volatility. This thesis focuses on the problem of risk management using the Conditional Value at Risk (CVaR) as a coherent risk measure. The thesis presents a novel iterative method that can be used by a market agent to optimize its operating decisions in the short term when the uncertainty is characterized by a set of random variable scenarios. The thesis analyses how it is possible to decompose the problem of risk management by means of Lagrangian Relaxation techniques and Benders decomposition, and shows that the proposed iterative algorithm (Iterative-CVaR) converges to the same solution as under the direct optimization setting. The algorithm is applied to two typical problems faced by agents: 1) optimization of the operation of a combined cycle power plant (CCGT) that has to cope with the volatility in the spot market price to build the supply curve for the futures market, and 2) strategic unit-commitment model. In a second part of the thesis the problem of market equilibrium is studied to model the interaction between several generating companies with mixed generation portfolios (thermal, hydraulic and renewable). The thesis analyses how the Nash equilibrium solution is modified at different risk-aversion level of the risk of the agents. In particular, the thesis studies how the management of hydroelectric reservoirs is modified along the annual horizon when agents are risk-averse, and it is compared with the risk-neutral solution that coincides with a centralized planning when the objective is the minimization expected operational cost.

Abstract in Spanish Language (Resumen)

Autor: Nenad Jovanović
Afiliación: Universidad Pontificia Comillas, KTH Royal Institute of Technology, Delft University of Technology
Titulo: Planificación de la operación de agentes aversos al riesgo en mercados eléctricos: descomposición estocástica y equilibrio
Lengua: Inglés
Palabras claves: agentes aversos al riesgo, descomposición estocástica, equilibrio del mercado, técnicas de descomposición

La creciente penetración de fuentes de energía renovable en los sistemas eléctricos obliga a adaptar los modelos de planificación de la operación para hacer frente a la inherente variabilidad e incertidumbre de la generación eólica o solar. Además, la volatilidad de los precios de combustibles fósiles (como por ejemplo el gas natural) o la incertidumbre de las aportaciones hidráulicas obliga a que el proceso de toma de decisiones para operar las centrales se realice teniendo en cuenta todas estas fuentes de incertidumbre, de modo que las técnicas de optimización estocástica han sido ampliamente utilizadas en este contexto. Desde el punto de vista de la operación del sistema, la introducción de la generación eólica y solar en el mix de generación ha obligado a que los generadores convencionales estén sujetos a programaciones más exigentes desde el punto de vista técnico, aumentando por ejemplo el número de arranques y paradas durante la semana, o teniendo que hacer frente a rampas de programación más pronunciadas. Desde el punto de vista del mercado, todo ello se traslada al mecanismo de formación de precios que pueden estar sujetos a una mayor volatilidad. Esta tesis se centra en el problema de la gestión de riesgos desde la perspectiva de una empresa de generación utilizando como medida coherente de riesgos el Conditional Value at Risk (CVaR). La tesis propone un método iterativo que puede ser utilizado por un agente de mercado para optimizar sus decisiones de operación en el corto plazo cuando la incertidumbre está caracterizada por un

conjunto de escenarios de las variables aleatorias. La tesis analiza cómo es posible descomponer el problema de gestión de riesgos mediante técnicas de Relajación Lagrangiana y descomposición de Benders, y demuestra que el algoritmo iterativo propuesto (Iterative-CVaR) converge a la misma solución que la optimización directa. El algoritmo se aplica a dos problemas típicos a los que se enfrentan los agentes: 1) optimización de la operación de una central de ciclo combinado (CCGT) ante volatilidad en el precio del mercado spot para construir la curva de oferta para el mercado de futuros, y 2) modelo de unit-commitment estratégico. En una segunda parte de la tesis se estudia el problema del equilibrio de mercado para modelar la interacción entre varias empresas generadoras con portfolios de generación mixtos (térmicos, hidráulicos y renovables) y se analiza cómo se modifica la solución del equilibrio de Nash ante distintos niveles de aversión al riesgo de los agentes. En particular, se estudia cómo se modifica la gestión de los embalses hidroeléctricos a lo largo del horizonte anual cuando los agentes son aversos al riesgo, y se compara con la solución neutral al riesgo que coincide con una planificación centralizada donde el objetivo sea la minimización de la esperanza del coste total de explotación.

Abstract in Swedish Language (Sammanfattning)

Författare: Nenad Jovanović
Affiliering: Comillas Pontifical University, KTH Kungliga Tekniska Högskolan, Delft University of Technology
Title: Elmarknadsplanering med riskaverta aktörer: Stokastisk optimering och jämvikt
Språk: Engelska
Nyckelord: marknadsjämvikt, nedbrytningstekniker, risk-omvårdnadsmedel, stokastisk optimering

Den ökande mängden förnybara energikällor i elsystemet kräver att driftmodeller anpassas för att möta variationen och osäkerheten hos elproduktionen från vind- och solkraft. Dessutom måste volatiliteten i bränslepriser (till exempel naturgas) och osäkerheten i vattenflöden beaktas vid driftsplaneringen av elsystemet. För detta har stokastiska optimeringstekniker använts i stor utsträckning. Ur driftsynpunkt har införandet av vind- och solkraft tvingat konventionella generatorer att följa mer krävande scheman utifrån teknisk synvinkel, med t.ex. ett ökat antal start och stopp under veckan och större ramper. Utifrån marknadssynpunkt överförs dessa tekniska aspekter till marknadspriserna som blir mer volatila. Denna avhandling fokuserar på problemet med riskhantering med hjälp av det villkorliga värdet vid risk (CVaR) som ett sammanhängande riskmått. Avhandlingen presenterar en ny iterativ metod som kan användas av en marknadsagent för att optimera sina operativa beslut på kort sikt när osäkerheten präglas av en uppsättning slumpmässiga variabla scenarier. Avhandlingen analyserar hur det är möjligt att dekomponera problemet med riskhantering med hjälp av lagrangianska relaxations-tekniker och Benders dekomponering, och visar att den föreslagna algoritmen (Iterative-CVaR) konvergerar till samma lösning som under direkt optimering. Algoritmen tillämpas på två typiska problem som agenter står inför: 1) driftoptimering av ett gaskombikraftverk (CCGT) som måste

hantera volatiliteten i spotmarknadspriset för att konstruera utbudskurvan för terminsmarknaden, och 2) det strategiska unit commitment-problemet. I en andra del av avhandlingen studeras problemet med marknadsjämvikt för att modellera samspelet mellan flera aktörer med blandade produktionsportföljer (termisk, vattenkraft och förnybar). Avhandlingen analyserar hur Nash-jämviktslösningen modifieras vid olika nivåer av riskaversion hos agenterna. Speciellt studeras hur hanteringen av vattenkraftreservoarer ändras utifrån en årlig tidshorisont när agenter är riskaverta och detta fall jämförs med den riskneutrala lösningen som sammanfaller med central planering när målet är minimering av förväntad driftkostnad.

Abstract in Dutch Language (Samenvatting)

Auteur: Nenad Jovanović
Aansluiting: Comillas Pontifical University, KTH Royal Institute of Technology, Delft University of Technology
Titel: Operationele planning van elektriciteitsmarkten met risicomijdende agenten: stochastische ontbinding en evenwicht
Taal: Engels
Trefwoorden: marktevenwicht, ontledingstechnieken, risicomijdende agenten, stochastische optimalisatie

De groeiende penetratie van groene energie in elektriciteitssystemen vereist een aanpassing van operationele modellen om de inherente variabiliteit en onzekerheid van wind- of zonne-opwekking op te nemen. Bovendien vereist de volatiliteit van brandstofprijzen (zoals aardgas) of de onzekerheid van de generatie van hydraulische energie, rekening te houden met al deze bronnen van onzekerheid binnen de operationele planning van het elektriciteits-opwekkingssysteem. Stochastische optimalisatietechnieken zijn dus op grote schaal gebruikt in deze context. Vanuit het oogpunt van de werking van het systeem heeft de introductie van wind- en zonne-opwekking in de generatiemix conventionele generatoren gedwongen om vanuit technisch oogpunt aan veeleisende programma's te worden onderworpen. Tijdens de week neemt daardoor bijvoorbeeld het aantal beslissingen voor opstart en stopzetten toe van generatoren, of deze opstart wordt veel geprononceerder als normaal. Vanuit het oogpunt van de markt worden al deze technische kwesties overgebracht naar de marktprijzen die daardoor onderhevig zijn aan grotere volatiliteit. Dit proefschrift richt zich op het probleem van risicobeheer met behulp van de Conditional Value at Risk (CVaR) als een coherente risicomaatstaf. Het proefschrift presenteert een nieuwe iteratieve methode die door een marktagent kan worden gebruikt om zijn

operationele beslissingen op korte termijn te optimaliseren wanneer de onzekerheid wordt gekenmerkt door een reeks willekeurige variabele scenario's. Het proefschrift analyseert hoe het mogelijk is om het probleem van risicobeheer te ontleden door middel van Lagrangian Relaxation techniques and Benders decomposition, en toont aan dat het voorgestelde iteratieve algoritme (Iterative-CVaR) convergeert naar dezelfde oplossing als onder de directe optimalisatie-instelling. Het algoritme wordt toegepast op twee typische problemen waarmee agenten te maken hebben: 1) optimalisatie van de operaties van een stoom- en gasturbine (STEG) die moet omgaan met de volatiliteit van de spotmarktprijs om de aanbodcurve voor de termijnmarkt op te bouwen, en 2) een strategisch unit-commitment model. In een tweede deel van het proefschrift wordt het probleem van het marktevenwicht bestudeerd om de interactie tussen verschillende productiecentrales met gemengde generatieportfolio's (thermisch, hydraulisch en hernieuwbaar) te modelleren. Het proefschrift analyseert hoe het Nash-evenwicht wordt aangepast op uiteenlopende risico-aversie niveaus van agenten. In het bijzonder bestudeert het proefschrift hoe het beheer van hydroelektrische reservoirs langs een jaarlijkse horizon wordt aangepast wanneer agenten risicomijdend zijn. Dit beheer wordt vergeleken met een risico-neutrale oplossing via een gecentraliseerde planning met als doel minimalisatie van verwachte operationele kosten.

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To my wife Marija Jovanović for being my strongest support,
thank you for being my faithful companion on this journey.

Contents

List of Abbreviations

BRMP	Benders relaxed master problem
BSP	Benders sub-problem
CEN	Centralized Model
CVaR	Conditional Value at Risk
DOP	Direct optimization problem
DW	Dantzig-Wolfe
DWMP	Dantzig-Wolfe master problem
DWSP	Dantzig-Wolfe sub-problem
EMP	Extended Mathematical Programming
FPC	Future physical contracts
Gencos	Generation companies
I-CVaR	Iterative CVaR
KKT	Karush–Kuhn–Tucker
LP	Linear Programming
LR	Lagrangian Relaxation
LRMP	Lagrangian Relaxation master problem
LRSP	Lagrangian Relaxation sub-problem
MIP	Mixed Integer Problem

MO	Market Operator
NE	Nash Equilibrium
RA	Risk-averse
RES	renewable energy sources
RMIP	Relaxed Mixed Integer Problem
RN	Risk-neutral
SDDP	Stochastic dynamic dual programming
SPM	Stochastic programming models
TVaR	Tail Value at Risk
UC	Unit commitment
VaR	Value at Risk
WACC	Weighted Average Cost of Capital

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1. Introduction

1.1. Background and motivation

The pioneering deregulation process that took place in Chile in the 80s paved the way for the posterior liberalization of the electric power industry in many countries during the 90s and later. The implementation of electricity markets based on marginal pricing principles (Schweppe *et al.*, 1988) changed the way in which hydro and thermal generators were scheduled. Therefore, the centralized optimization approach was substituted in many systems by a market mechanism where the Market Operator (MO) is in charge of clearing the short-term spot market (typically a day-ahead auction complemented with real-time and balancing markets). In this context, generation companies (Gencos) are responsible for planning the optimal operation of their own assents, and for submitting the right offers to the MO that allow them to put into practice such optimal operation. In addition, the increasing level of intermittent renewable energy sources (RES), such as wind and solar, represents an additional challenge from the operation point of view (Rubin & Babcock, 2013). Therefore, the traditional models had to be adjusted taking into account the market environment.

Modeling of the electric power system is a challenging task due to the complexity of the physical assets with are subject to a variety of different technical constraints. In the case of the generation system, each available technology (nuclear, coal, gas fired units, hydro, wind generation, etc.) has its own particularities that have to be taken into account in order to obtain feasible schedules. In addition, the limits imposed by the network (transmission and distribution) condition both the planning and the operation of the whole power system. These technical aspects must be complemented by an adequate economic representation of the investment and operational costs and have to be embedded into the market models for a richer representation of agents' behavior. One of the main factors in the decision support modeling approach is the time scope:

- In the long-term period (years) the main decision variables are the investment in new generation capacity or the decommissioning or existing power plants (Wogrin *et al.*, 2011), the expansion of the transmission network (David & Wen, 2001) and, in resent years, the investment in RES (Couture & Gagnon, 2010). These decisions can be mostly driven by the market regulation, capacity markets or feed-in tariffs. Mathematical programing models can assist the decision maker to plan the expansion of both the generation and the transmission system, and these tools can be complemented by simulation models (Day & Bunn, 2001) which allow a more flexible representation of agent's strategies.

- Decisions in the medium-term (months) include the annual operation of hydropower reservoirs (Scott & Read, 1996), the fuel procurement management of conventional thermal units, the planning of maintenance activities, and the strategies to participate in the organized electricity future markets (Bessembinder & Lemmon, 2002). In this sense, the estimation of forward electricity and fuel prices, the change in the bid-ask spread of futures transactions, and the variation of currency rates in the foreign exchange market, are some examples of possible obstacles than need to be overcome in the decision making process that covers a time period of several months. In order to capture the strategic interaction among different rational participants, equilibrium models are commonly used for medium-term planning. In particular, Cournot competition (Daughety, 2005) where agents' strategies are quantities, and supply function equilibrium (Klemperer & Meyer, 1989) which defines agent's strategies in terms of their offering curves, are some examples of modeling approaches followed in the literature.

- Finally, decisions in the short-term (days) aim to find the optimal hourly scheduling of the generation units, and their corresponding optimal offering strategies for the daily spot and balancing market auctions. These decisions are made very frequently and are mostly affected by the volatility of electricity prices, demand and RES fluctuation, and the availability of generation units. Models used for the short-term planning are commonly formulated as single agent optimization models where the objective is to maximize agent's profits subject to an adequate representation of the competitors (Garcia-Gonzalez *et al.*, 2008), although some equilibrium models have been also proposed in the literature (Barroso *et al.*, 2006).

The electricity sector is subject to a constant evolution due to the emergence of new generation, transmission and distribution technologies, as well as the increasingly active role played by the consumers and storage facilities. Therefore, the above mentioned models are being reviewed constantly in order to adapt to a changing environment such as the impact of distributed generation resources (Ackermann *et al.*, 2001) and the demand response programs,(Albadi & El-Saadany, 2008).

1.1.1. The role of uncertainty in the decision making process

The process of making decisions in the electricity markets is subject to a lot of uncertainties such as the expected natural hydro inflows, the future evolution of fuel prices in the international markets, the intermitent and variable renewable energy production, etc. In this context, stochastic programming provides a general mathematical framework to represent properly all the random variables and their impact on the decisions that need to be taken throughout the temporary scope considered. Decision making under uncertainty is usually addressed to rank different available choices when the outcomes (e.g. profits or losses) depend on some random variables. Unlike the deterministic case where the optimality criterion is easier to be established (for instance the minimization of the total operational costs or the maximization of the obtained profits), in case of dealing with uncertainty, some optimization criteria needs to be defined as the outcomes are subject to some probability distributions. One possibility is to maximize/minimize the expected value of the outcome. As this could lead to unacceptable outcomes in case one of the worse scenarios occurs, the formulation of mean-risk models to ease the decision-making process has become a common practice in many power system business, (Kahneman & Tversky, 1979; Knight, 1921). These mean-risk problems are formulated including a weighted sum of two terms: the expected value of the outcome, and a risk measure. Depending on the aversion level of the decision maker, it is possible to give more priority to one or the other term and this fact makes that formulation very amenable to model real situations. Among several risk measures used in literature, this thesis focuses on the Conditional Value at Risk (CVaR).

The first step is to identify the different sources of risks, either internal or external, that represent threats for achieving the company's objectives. For all the identified risk, the agents have to analyze their likelihood of occurrence, and their impact on the company's results. Then, it is possible to include risk-management criteria within the decision processes in order to try to mitigate as much as possible their impact. Therefore, risk management has received a lot of attention in the electric power industry in order to help market participants to hedge their sources of risk for different time scopes (Denton *et al.*, 2003): short term (Conejo *et al.*, 2004; García-González *et al.*, 2007; Sheikhahmadi *et al.*, 2018), medium-term (Cabero *et al.*, 2005; Fleten *et al.*, 2002; Karavas *et al.*, 2017), and long term (Abada *et al.*, 2017; Baringo & Conejo, 2013).

As noticed by some authors, risk-averse behavior of agents that participate in liberalized electricity markets can have a big influence on their decision making process (Gérard *et al.*, 2018; Philpott *et al.*, 2016; Rodilla *et al.*, 2015; Vardanyan & Hesamzadeh, 2017). The application of the Nash equilibrium concept(Nash, 1950) to model the strategic behavior of the participants of electricity markets is the general theoretical framework proposed in the literature to capture the interdependence of their strategies, (Ventosa *et al.*, 2005). In this regard, studying the impact of risk-aversion on the Nash equilibrium solution can be very beneficial to better understand how the electricity sector can evolve in the future in a context where the risk aversion of electricity companies can substantially condition their behavior, (Engelmann & Steiner, 2007).

1.1.2. Impact of renewable energy sources on electric power systems

One of the sources of risks faced by electricity market participants is the uncertainty related to the intermittent and variable RES such as wind and solar power. The rapid growth of RES installed capacity (see Figure 1.1 which presents data from IRENA (2018)) called for the upgrade of decision support models in order to capture the effect caused by this type of energy sources. Big volatility of renewable energy production has a huge impact on the generators output, electricity prices and network investment planning. The main renewable source installed during the last decade is wind energy, although solar energy has also a strong growth momentum. The derived challenges from increasing the penetration level of such RES in ther the power system can be summarised as follows:

1. Operational point of view: The intermittent nature of wind generation makes it the most challenging generation to be incorporated into power system. This affects scheduling and adjustment of conventional generators' output (MacCormack *et al.*, 2010; Ummels *et al.*, 2007).
2. Market point of view: Available renewable energy is usually dispatched before conventional generators affecting the real-time prices, because the variable

generation costs are negligible (Klinge Jacobsen & Zvingilaite, 2010; Woo *et al.*, 2011). In some countries, mainly in Europe, wind and solar energy is supported via feed-in tariff mechanisms implying that negative electricity prices might occur (Fanone *et al.*, 2013; Martín *et al.*, 2015).

Given that the presence of RES in electric power systems is affecting the behavior of the market agents, new models need to be developed to respond to the on-going challanges. This requires to formulate and to solve complex mathematical models where the computational tractability needs to be assured. In this context, it is necessary to review the optimization models used by a single agent to make its decisions under a risk-constrained setting, and to provide solutions that are not limited to the risk of renewable sources, but general enought to deal with other traditional sources of risks such as fuel prices or natural inflows.This thesis aims to provide tools and insights regarding the behavior of Gencos when risk aversion is considered.

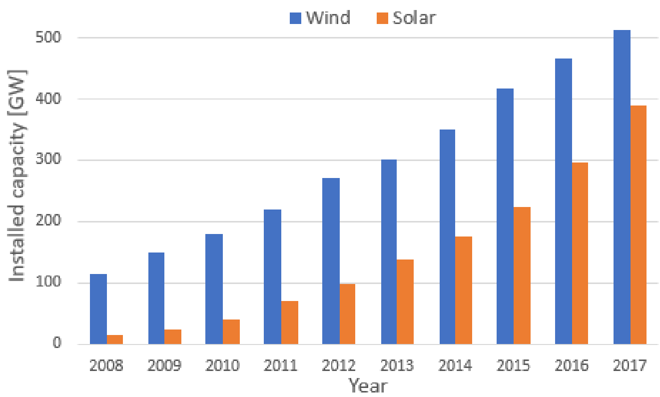


Figure 1.1.: Installed wind and solar power capacity in the world

1.1.3. Modeling challenges

In case of a single-agent approach, modeling data uncertainty via stochastic programming usually requires a very large number of scenarios to guarantee a sufficient outcome of a decision making process. Moreover, by applying a weighted mean-risk objective function in order to manage such volatile outcomes might cause computational intractability. In case of the CVaR measure, inclusion of additional coupling constraints complicates the straightforward application of stochastic decomposition techniques. Therefore, the intention of this thesis is to provide an alternative algorithm which would convert the mean-risk problem into an equivalent risk-neutral stochastic problem. The risk-aversion would be modeled via the risk-adjusted probabilities, instead of additional coupling constraints, ensuring better computational tractability

On the other side, market equilibrium approach with risk-averse has its additional complexities in terms of modeling the competitors' risk preference. Knowing that the agents' interaction leads to additional uncertainty affecting the final outcome of the problem, there is a need for a detailed analysis of the Nash equilibrium solution. This thesis proposes a multi-stage stochastic non-convex equilibrium problem where the game of agents with different risk aversion is studied. In this case, there is a demand for accommodating such complex model in a flexible way. The Extended Mathematical Programming is employed allowing automatic reformulation of the problem into a Mixed Complementarity Problem.

1.2. Objectives of the thesis

This doctoral thesis was inspired by the following research questions:

1. Is it possible to improve the computational tractability of the optimization problem of a risk-averse agent in the electricity markets?
2. Up to what extend the risk-averse agents in an electricity market change the Nash equilibrium solution?

In order to answer both research questions, the following objectives are established:

Objective1: Develop an algorithm to solve a risk-constrained optimization problem for a single agent and prove its convergence and accuracy.

The goal of this objective is to provide an alternative algorithm for the optimization of the CVaR measure which provides good computational tractability for complex optimization problems. The algorithm needs to have a general form so it can be applied to several different modeling cases.

Objective2: Apply the developed algorithm to two common problems faced by Generation Companies (single agent): forward contracting and unit-commitment.

The main aim of this objective is to identify the problems of interest to the power industry and to apply the developed risk-constrained algorithm. The proposed real-case problems shall highlight the advantages of the algorithm and inspire its application to other types of problems.

Objective3: Formulate the Nash Equilibrium in the presence of risk-averse agents and develop a model (Mixed Complementary Problem) to solve the resulting problem in a multi-agent setting.

Mathematical formulation and a modeling approach of the Nash Equilibrium are the key points of this objective, which pave the way for providing an insight of the resulting equilibrium outcomes with presence of risk-averse agents.

Objective4: Analyze the impact of risk-aversion level on the Nash Equilibrium solution.

This objective shall provide analysis of the Nash Equilibrium solution when the market agents strategies, among the usual ones, are considered to be their risk-averse levels.

1.3. Outline of the thesis

The structure of this thesis is organized to address the before-mentioned objectives. The thesis can be separated into two different main parts. The first part proposes an alternative algorithm for the CVaR modeling by means of an iterative algorithm. The the second part defines three different risk-constrained models which highlight the impact of risk aversion on the Nash equilibrium solution.

- **Chapter 2**: A state of the art literature review on risk management in electricity markets is provided in this chapter.
- **Chapter 3**: This chapter presents the developed Iterative CVaR algorithm and gives an overview of its advantages for solving risk-constrained optimization problems.
- **Chapter 4**: This chapter describes the application of the Iterative CVaR to single-agent problems: involvement in forward markets and strategic unit-commitment problem.
- **Chapter 5**: In this chapter a Nash equilibrium model with risk-averse agents is presented and the impact of different risk-averse levels on the equilibrium is analyzed and applied to a real case study.

- **Chapter 6**: This chapter provides conclusions and contributions of this thesis project and suggests future research lines.

2. State of the art literature review

2.1. Risk Management

In any decision-making process it is necessary to take into account the existence of uncertain events that can be considered as "risks" when they have a negative impact, or "opportunities" in the opposite case. Regardless of the area of study or the associated business, a proper risk modeling has to follow the next consecutive stages: 1) risk identification, 2) risk analysis and measurement, 3) risk treatment, and 4) risk monitoring and review.

The owners of power plants are exposed to a variety of uncertainties in deregulated electricity markets. These uncertainties are related to the power demand, bidding strategies of other generation companies, network contingencies, fuel costs, etc. In recent years, the penetration of renewable energy sources has increased the volatility of the spot market prices, especially in the short-term. Decision making under uncertainty requires the development of stochastic programming models (SPMs) adapted to the needs of the decision maker. Depending on the scale of the problem, the type of constraints and the number of scenarios, solving the resulting SPMs can be very challenging (Birge & Louveaux, 1997). As a consequence of such increased level of uncertainty, risk management has become a common practice in the electric power industry.

Risk modeling approach is introduced in the electricity market modeling for analyzing the exposure to a variety of uncertainties and finding the most appropriate solution for hedging against the risk. Financial instruments specially designed to manage the risk are implemented (Deng & Oren, 2006) and risk measures are introduced defining the risk preference of the agents (Oliveira *et al.*, 2006). Numerous researchers have tackled this topic from the single-agent optimization and the market equilibrium point of view. Agents are exposed to different nature of risks (operational, financial, regulatory, etc.) in the liberalized electricity markets (Denton *et al.*, 2003). Determining the exposure to a risk, that an agent is exposed to, is addressed to the *risk management models* (Eydeland & Wolyniec, 2003).

2.1.1. Risk Measures

Risk measures are mainly used to quantify a minimum amount of funds (such as cash, treasury bonds, etc.), required by the regulators in financial industry, which would be used to cover possible losses. A variety of risk measures are implemented in the power sector for risk valuation, such as: utility function, mean-variance, value-at-risk, conditional value-at-risk, etc. The applicability of those risk measures depend on the mathematical properties that they hold. For many real applications, it is desirable that the risk measure is a coherent one, Artzner *et al.* (1999). Let X and Y be the random future loss of two given portfolios. A risk measure $\rho(\cdot)$ is a mapping from a set of random variables to the real numbers, and it is considered to be a coherent risk measure if it satisfies the following axioms:

1. Monotonicity: $\rho(X) \leq \rho(Y)$ for all $X \leq Y$
2. Subaditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$ for all X and Y
3. Positive homogeneity: $\rho(\lambda X) = \lambda\rho(X)$ for all X and $\lambda > 0$
4. Translation invariance: $\rho(X+C) = \rho(X) - C$ for all X and constant $C \in \mathbb{R}$

The monotonicity axiom indicates that in case the losses are always higher for one portfolio than for the other, the risk will also be higher, i.e., the risk is higher if the portfolio is worse. The subaditivity axiom defines that a diversification of a portfolio is less risky than having individual portfolios. In the axiom 3 it is defined that a change in the portfolio by a certain coefficient changes the risk by the same coefficient (for instance, if the size of a portfolio is doubled, the risk will also be doubled). Moreover, the the joint consideration of axioms 2 and 3 imply that the risk measure is convex. Finally, axiom 4 defines that if a certain capital C is added to a portfolio, then the risk will be reduced by the same amount. For instance, Value at Risk (VaR), which is very common in the financial sector, does not hold the subaditive property, (Krokhmal *et al.*, 2011), as VaR of a given portfolio made of several elements can be higher than the sum of VaR values of each one of them. The utility function is linear but it does not hold the positive homogeneity, and the mean-variance lacks the monotonicity property. On the contrary, the Conditional Value at Risk (CVaR),Rockafellar & Uryasev (2000), holds all the properties of the coherent risk measures and has become a widely used risk measure.

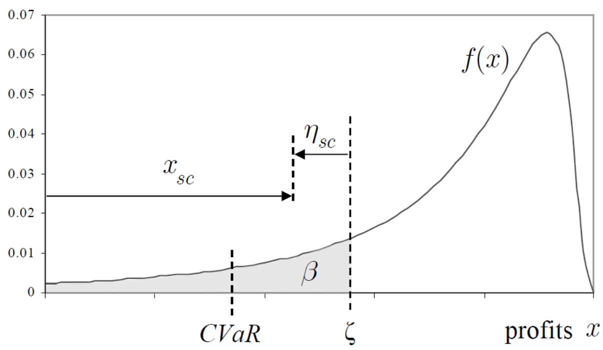


Figure 2.1.: Conditional value at risk

It is important to emphasize that in this thesis, instead of expressing the risk in terms of portfolio losses, the risk is expressed in terms of the profits. Therefore, instead of being concerned about the right tail of the distribution of losses, throughout the development of the thesis the focus will be on the left tail of the distribution of profits.

2.1.2. CVaR definition

Among all the new risk measures proposed in the literature, CVaR has gained a lot of attention during the last decade because: i) it is a convex and coherent risk measure, and ii) exhibits interesting computational properties that allow to implement it in the context of linear programming when the uncertainty is represented by a finite number of discrete scenarios. Therefore, the use of CVaR within decision support models has increased dramatically during the last years, and it is possible to find many applications, such as: the financial sector (Andersson *et al.*, 2001; Topaloglou *et al.*, 2002), the electric power industry (Eydeland & Wolyniec, 2003; García-González *et al.*, 2007), the gas and oil sector (Carneiro *et al.*, 2010; Dueñas *et al.*, 2015), the water management problem (Bjorkvoll *et al.*, 2001; Webby *et al.*, 2008), and many others.

The most common definition of CVaR is that it computes the expected value of the profits lower than the value of VaR for a given confidence level β. The concept of CVaR is graphically presented in Fig. 2.1.

Let X be a discrete random variable describing the set of net profits of a given portfolio and x the profit values. The function $F(x)$ is the profit distribution function and $\beta \in (0,1)$ a given confidence level. The value at risk ζ for the confidence level β is defined as:

$$\zeta_\beta(X) = sup\{x \in \mathbb{R} \mid F(x) \leq \beta\} \tag{2.1}$$

which, is not a coherent risk measure (Artzner *et al.*, 1999). On the other side, the alternative known as the CVaR can be defined as:

$$CVaR_\beta(X) = E_P\left[X \mid X \leq \zeta_\beta(X)\right] \tag{2.2}$$

where it can be seen that the value of the CVaR depends on the selected value of the the confidence level β. However, according to Lüthi & Doege (2005); Ogryczak & Ruszczynski (2002); Rockafellar *et al.* (2002), the expression (2.2) refers to the expected Tail Value at Risk ($TVaR$) which may not hold the coherency. As defined in Rockafellar & Uryasev (2002), in discrete distributions (which is very common in stochastic optimization where the uncertainty is modeled by a scenario tree with discrete values of the random parameters and their corresponding discrete probabilities), the profit distribution may have a jump at $\zeta_\beta(X)$, which is referred to as a probability "atom" at the value at risk ζ for a given confidence level β. In particular, if there is a vertical discontinuity gap in $F(x)$, the same value of ζ can be obtained for the interval β^- to β^+ (see Figure 2.2) and it can be said that the $F(x)$ is not atomless. This vertical gap appears very often in the discrete distributions and splitting of the probability "atom" in a discrete distribution is the way to formulate the coherent CVaR (Sergey Sarykalin *et al.*, 2008).

By contrast, the section "Risk envelopes and Dualities" from Rockafellar *et al.* (2002) provides the coherent $CVaR$ formulation:

$$CVaR_\beta(X) = \inf_{Q \in \hat{Q}} \{E_Q\left[X\right]\} \tag{2.3}$$

where the risk envelope $\hat{Q}$ is a convex set of probability measures such as $Q \leq {}^P/_\beta$. For discrete distribution the CVaR can be formulated as:

$$\begin{aligned} CVaR_\beta(X) = \quad & \min_{q_{sc}} \sum_{sc} q_{sc} x_{sc} \\ s.t. \quad & \sum_{sc} q_{sc} = 1 \\ & 0 \leq q_{sc} \leq \frac{p_{sc}}{\beta}, \forall sc \end{aligned} \tag{2.4}$$

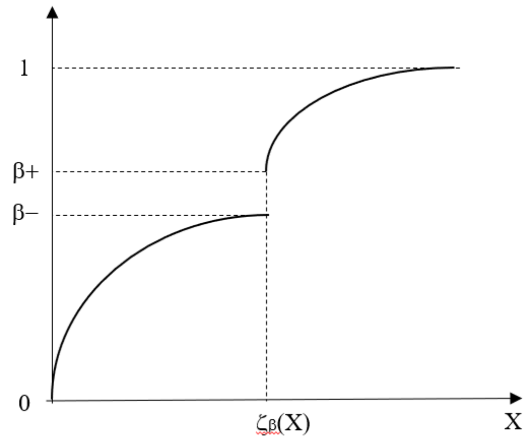


Figure 2.2.: Cumulative distribution function - vertical discontinuity gap

where, the q_{sc} are the new scenario probabilities within probability distribution $\mathbb{Q}$. These new scenario probabilities are equal to zero for a confidence level higher than a given value of β.

Developing an algorithm for computation of this new set of probability measures was a motivation for the **Objective1** of this thesis (see section 1.2).

2.1.3. The Mean-Risk-Problem

In a competitive market, a reasonable behavior of a market agent would be to maximize the expected payoffs and to manage the risk at the same time. Choosing such an optimal portfolio is known as *portfolio selection.* From the pioneering work of Markowitz (1952), the mean-risk models have become widely used where the objective represents a trade-off between the expected value $E(X)$ and the risk measure $\rho(X)$.

There are two main formulations of the mean-risk problem formulations used in literature:

1. max $E(X) + \hat{\mu} \cdot \rho(X)$, $\hat{\mu} \geq 0$
2. max $(1 - \mu)E(X) + \mu \cdot \rho(X)$, $\mu \in (0, 1)$

The main difference is in the upper value of the risk-weight parameters ($\hat{\mu}$ and μ) which sets a trade-off between the expected outcome and the risk measure. In case of a risk-neutral agent, these parameters will take the value of 0. As for the risk-averse agent, setting a value higher than 0 for the parameter $\hat{\mu}$ in the first formulation might be challenging and arguable, as theoretically there is no upper limit. The second formulation of the mean-risk problem is more appealing in this case, as setting a value of $\mu = 1$ means that a very conservative risk-averse case is taken into account, where the objective function of the optimization problem becomes the risk measure itself. Some guidelines to select the value of the parameter μ will be presented in the section 4.1.4. Besides setting a proper risk-weight parameter, choosing a risk measure has a significant impact on obtaining the optimal portfolio. The mean-variance and the value at risk have been broadly used for the mean-risk problem formulation (Alexander, 2009). On the other side, CVaR gains more attention as it allows convex problem optimization.

CVaR can be computed by using Linear Programming (LP), as defined in Rockafellar & Uryasev (2000). However, CVaR formulation introduces coupling constraints among the scenarios. With such constraints, computational tractability needs to be ensured when solving the SPMs with large number of scenarios. Alternative ways for CVaR optimization of large-scale scenario problems are defined in Conejo *et al.* (2008); García-Bertrand & Mínguez (2014); Pineda & Conejo (2010) by applying scenario reduction techniques. Scenario reduction techniques are mainly focused on computing the CVaR itself where the expected profit is totally or partially omitted. On the other side, a part of the stochastic information might be lost, such as decision to be made if the neglected scenario occurs.

The focus of the next chapter is on decomposing mean-risk models, without reduction of the original number of scenarios. These stochastic algorithms usually take into account two-stage stochastic programming problems based on Benders decomposition technique (Benders, 1962; Künzi-Bay & Mayer, 2006) and can be formulated as mono-cut (Ahmed, 2006; Fábián *et al.*, 2015; Fábián, 2008) or multi-cut (Noyan, 2012) algorithms. These methods have shown that the large-scale scenario problems can be tractable and in some cases they outperform by computational time the formulation the CVaR direct formulation Rockafellar & Uryasev (2000). Another approach, which up to our knowledge is not very common in the literature, is the application of the risk-adjusted probabilities to modeling the mean-risk problem (Abad & Iyengar, 2015; Ehrenmann & Smeers, 2011; Miller & Ruszczyński, 2008). In Miller & Ruszczyński (2008) and Ehrenmann & Smeers (2011) the case of the atomless discrete distribution of the decision variables is assumed. In order to better understand the connection of this review with the development of this thesis, an algorithm which is based on the Lagrangian relaxation decomposition technique is presented in the chapter 3. Given that this algorithm provides an optimal solution by means of an iterative method, it is refereed as the *Iterative CVaR*. For modeling the mean-risk problem with CVaR a new set of probability measures is computed. The fact that the sub-problem decouples the scenarios of the model, makes the algorithm very attractive for applying it to the large scale scenario problems.

2.2. Single risk-averse agent models

There is a significant amount of literature that implement hedging strategies from the single-agent modeling point of view (García-González *et al.*, 2007; Mo *et al.*, 2001; Oliveira *et al.*, 2006). Among the variety of topics, involvement in future markets and unit commitment models with ancillary services can be identified as the two common problems faced by Generation Companies (Gencos).

Forward contracting has been implemented in the electrical power systems to assist agents in hedging the uncertain future of the market (Fleten *et al.*, 2002). Financial instruments such as futures, put and call options can be traded in the future markets allowing Gencos to mitigate the risk of undesired scenarios. Taking into account the technical nature of the electrical power systems, the future physical contracts (FPC) are the most used instruments where the Gencos are committed to deliver the agreed amount of energy at a given price and date (Conejo *et al.*, 2008). The main benefit of the FPC in the short-term is to avoid the price volatility in the spot market and to facilitate the operation planing. In case of a unit failure, the seller (Genco) needs to buy the needed energy from the market to be delivered to the costumer, otherwise, it will be penalized for not providing energy to the system.

Unit commitment (UC) models are used for determining the optimal hourly scheduling and startup/shutdown decisions of generation plants (Padhy, 2004). In regulated power systems the objective is to minimize the total operational cost. These traditional cost-based UC models can also be applied in liberalized systems by the Market Operator in order to determine the cleared quantities (based on the received generation offers and demand bids), and the system marginal prices. There is a variety of possible UC formulations in the literature, and the need of reducing the required computational burden due to the binary variables is one of the current topics of research. In that respect, the tight and compact formulation presented in Morales-España *et al.* (2016) has proven to be very effective. By determining a precise generation unit states and current loads, the participation in the secondary

Table 2.1.: Methods for solving large scale scenario problems with CVaR

Reference	Method used	Objective function	CVaR constraints	Stochastic information
Conejo *et al.* (2008)	scenario reduction	max $E(X) + \hat{\mu} \cdot CVaR$, $\hat{\mu} \geq 0$	included	incomplete
Pineda & Conejo (2010)	scenario reduction	max $E(X) + CVaR$	included	incomplete
García-Bertrand & Mínguez (2014)	scenario reduction	min $E(X) + \hat{\mu} \cdot CVaR$, $\hat{\mu} \geq 0$	included	complete
Ahmed (2006)	two-stage decomposition	$\max\,(1-\mu)E(X) + \mu \cdot CVaR$, $\mu \in (0,1)$	included	complete
Fábián (2008),(2015)	two-stage decomposition	min $-E(X) + \hat{\mu} \cdot CVaR$, $\hat{\mu} \geq 0$	included	complete
Noyan (2012)	two-stage decomposition	min $-E(X) + \hat{\mu} \cdot CVaR$, $\hat{\mu} \geq 0$	included	complete
proposed approach (chapter 3)	two-stage decomposition	$\max\,(1-\mu)E(X) + \mu \cdot CVaR$, $\mu \in (0,1)$	excluded	complete

and tertiary reserve markets can be defined.

One of the main limitations of the single-agent approach is the absence of agents' interaction and impact of possible change in the agent's strategy forced by different behavior of its competitors.

2.3. Market equilibrium with risk-averse agents

Finding the optimal operation of the generators of a hydro-thermal system is a classic problem of the electric power industry that has deserved a lot of research in recent decades (de Queiroz, 2016). In particular, the uncertainty related to the hydro inflows has been one of the main concerns when planning the operation of hydroelectric reservoirs in the medium term (typically, one year). In this context, the application of multi-stage stochastic optimization techniques able to deal with the curse of dimensionality, such the *stochastic dynamic dual programming* (SDDP) (Pereira & Pinto, 1991), has been a common practice in many hydro-dominated systems such as Brazil, Norway, or New Zealand. In the context of a traditional vertical integrated system, the central planner (i.e., the Market Operator) is in charge of obtaining such optimal operation with the aim of maximizing the expected social welfare. On the other hand, there is a limited number of publications evaluating the agents' behavior in terms of their risk preference for the short and medium term planing. Based on the considered generation portfolio, modeling the risk in competitive environment can mainly be divided into two groups:

- Modeling risk of agents' competitive strategies in hydro-thermal generation portfolio
- Modeling risk of agents' competitive strategies in renewable-thermal generation portfolio

2.3.1. Modeling risk of agents' competitive strategies in hydro-thermal portfolio

A risk management medium-term equilibrium model for a hydro-thermal generation company is presented in Cabero *et al.* (2005). Agents' competition is computed in means of Cournot equilibrium and CVaR is introduced as a risk measure for managing the risk for selling electricity and fuel on the forward and spot market.

As demonstrated, Cournot competition has its own limits where agents' strategies are represented as quantities. This publication was later extended in Cabero *et al.* (2010) where Cournot approach is implemented and interaction between spot and forward markets is considered. However, in both mentioned papers it is assumed that only one agent in the market is risk averse and that other agents are risk neutral. As different risk factors of agents affect their strategies, obtained results may be doubtable.

In recent studies, (Rodilla *et al.*, 2015) and (Philpott *et al.*, 2016) take into account more realistic environment where two agents are risk-averse. It is proved that in a complete market where agents can trade risk products (for instance in the forward market), it is possible to achieve the same operation as central planning. However, the assumption of market completeness might not be realistic, and therefore it is necessary to assess the impact of risk aversion on the operation of the system in order to guide regulators to design additional mechanisms (such as demand procurement contracts), or to help market participants to understand the equilibrium solution for incomplete markets.

2.3.2. Modeling risk of agents' competitive strategies in renewable-thermal portfolio

A short-term equilibrium model with risk modeling of agents' strategies including wind generation in the power system is presented in Xiaoning *et al.* (2011). Duopoly is examined in this paper taken into account that all agents are risk-averse. Obtained results have shown that the increase in agent's risk aversion decreases its output, leads to an increase in the electricity prices, and lowers the risk. This study is further extended in Jing *et al.* (2012) using Cournot settings and CVaR as a risk measure. The main contribution is a study of agents' expected profit and risk reduction in the equilibrium of three asymmetric generators for 2 different cases: in the first one, only one agent is risk-averse, and in the second all agents are risk averse. In the first case, it is pointed out that the risk-averse agent faces the reduction of its expected profit as risk aversion increases while the profit of its competitors increases. For the second case, all agents gradually increase their risk factor and the results shows that they all have a higher growth in profits with a reduction of risk exposure. In respect to the contribution of Jing *et al.* (2012); Xiaoning *et al.* (2011), the obtained results might be arguable. Proposed approaches have qualitative contribution for the academic purpose. However, the main drawbacks for implementation of a real case electricity market environment are: considered time scope of only 1 hour, assumed fixed parameters of supplied function, approximation of wind generation, capacity output limits are omitted.

In Martín *et al.* (2015), a model is presented that allows analyzing the impact of regulatory measures (market design and RES support mechanisms) on the income received by conventional generation in systems with large penetration of RES, and it allows to model the risk-aversion of those agents by means of CVaR. The model is formulated as a short term market equilibrium problem, although the possibility of exercising market power is not explicitly considered. One of the most relevant conclusions is that greater requirements of regulation reserves (both the capacity and its usage) provides additional remuneration to conventional generators that can alleviate the decrease of their incomes in the energy markets.

2.3.3. Conclusion

The presented state of the art review allows identifying some gaps in the literature that would require further research (see Table 2.2). It is clear that the presence of renewable energy sources in power systems, is affecting the behavior of the participants of electricity markets. The standard methodology to model the strategic interaction of such participants is by computing the Nash equilibrium, following a number of different approaches and techniques depending on the general hypothesis, particular constraints, time scale considered, etc. Moreover, the inclusion of risk management in market equilibrium models has been a research topic in the recent years. Given that the effect of renewable energy on market prices and production levels of conventional generators is being a matter of concern, it is needed to consider this source of uncertainty in the decision making process. However, the number of previous works that address the market equilibrium problem, considering risk-averse strategic agents with renewable energy in their generation portfolio is very limited. Considering the number of generations, a realistic power system might be modeled so that the strategic behavior of market competitors can be observed in a real case study.

Table 2.2.: Market equilibrium models with risk-averse agents

Reference	Medium-term horizon	Risk measure	Generation portfolio	Risk-averse agents	Detailed hydro model
Cabero *et al.* (2005),(2010)	yes	CVaR upper bound	hydro-thermal	single agent	no
Rodilla *et al.* (2015)	yes	utility function	hydro-thermal	multi agent	no
Philpott *et al.* (2016)	no	polyhedral risk set	hydro-thermal	multi agent	no
Xiaoning *et al.* (2011)	no	mean variance	renewable-thermal	multi agent	no
Jing *et al.* (2012)	no	CVaR measure	renewable-thermal	multi agent	no
Martín *et al.* (2015)	no	CVaR measure	renewable-thermal	multi agent	no
proposed approach (chapter 5)	yes	CVaR measure	hydro-thermal-renewable	multi agent	yes

3. Iterative CVaR algorithm (I-CVaR)

3.1. The Mean-Risk-Problem and risk-adjusted probabilities

The linear programming (LP) formulation of the direct optimization problem (DOP) that models the mean-risk maximization problem, using the CVaR as the risk measure, can be formulated as follows:

$$\max_{x_{sc},s_m,CVaR,\zeta,\eta_{sc}} \quad (1-\mu)\textstyle\sum_{sc} p_{sc}x_{sc} + \mu \cdot CVaR \tag{3.1a}$$

$$s.t.$$

$$CVaR = \zeta - \frac{1}{\beta}\sum_{sc} p_{sc}\eta_{sc} \tag{3.1b}$$

$$\eta_{sc} - \zeta + x_{sc} \geq 0, \forall sc \tag{3.1c}$$

$$\eta_{sc} \geq 0 \ \ , \forall sc \tag{3.1d}$$

$$x_{sc} = \sum_{m} a_{sc,m}s_m + b_{sc}, \forall sc \tag{3.1e}$$

where $\mu \in (0,1)$ is a risk parameter that weights in the objective function (3.1a) the variable $CVaR$ and the expected profit (computed as the sum of the profits of each scenario x_{sc} multiplied by their corresponding probabilities p_{sc}). Constraints (3.1b)-(3.1d) are typically used expressions in portfolio optimization which simultaneously represent the VaR (i.e. the variable ζ) and the CVaR (Rockafellar & Uryasev, 2000). Constraint (3.1c) ensures that the profits lower or equal than the VaR are used for the CVaR computation. Variables s_m represent a set of decision variables that risk-averse decision maker can take to manage its risk and they affect the profit of each scenario x_{sc}. Equation (3.1e) represents in a generic manner the set of constraints that link the decision variables and the obtained profits. Problem (3.1) can be solved as a LP problem and this is why a linear relationship between s_m and x_{sc} has been assumed.

Notice that in problem (3.1), the maximization of the expected profit is coupled with the management of the risk exposure quantified in terms of the CVaR. Obtaining the solution of this problem can be a difficult task because of this connection: the equation (3.1b) that defines the CVaR is a complicating constraint that links the profits all the scenarios. In order to overcome this difficulty, the next section explores the possibility of decoupling the problem (3.1) by decomposing it. In this sense, the dual CVaR formulation plays a key role.

3.1.1. The concept of risk-adjusted probabilities

This subsection analyzes the possibility of substituting the mean-risk problem with CVaR (3.1a) by an objective function that can lead to the the same optimal values without the consideration of the constraints used previously to model the risk measure. Taking into account that the CVaR value can be obtained in terms of the new probability measure $\mathbb{Q}$ as follows (see section 2.1.2):

$$\begin{aligned} CVaR_{\beta}(X) = \quad & \min_{q_{sc}} \sum_{sc} q_{sc} x_{sc} \\ s.t. \quad & \sum_{sc} q_{sc} = 1 \\ & 0 \leq q_{sc} \leq \frac{p_{sc}}{\beta}, \forall sc \end{aligned} \tag{3.2}$$

the first step would be to take a look at the optimal conditions of the DOP problem. The problem (3.1) can be formulated as:

$$\max_{x_{sc}, s_m, \zeta, \eta_{sc}} \quad (1-\mu) \sum_{sc} p_{sc} x_{sc} + \mu \left(\zeta - \frac{1}{\beta} \sum_{sc} p_{sc} \eta_{sc} \right) \tag{3.3a}$$

$$s.t. \quad x_{sc} = \sum_{m} a_{sc,m} s_m + b_{sc}, \forall sc \tag{3.3b}$$

$$\eta_{sc} - \zeta + x_{sc} \geq 0 \ \perp \lambda_{sc} \geq 0, \forall sc \tag{3.3c}$$

$$\eta_{sc} \geq 0 \ , \forall sc \tag{3.3d}$$

where λ_{sc} is the corresponding Lagrangian multiplier of the inequality (3.3c). To derive the optimality conditions of this problem, a mixed form is used as only a subset of all the constraints is moved to the objective function (for the pure form see Appendix (A.1)). In this sense, the Lagrangian formulation of (3.3) can be expressed as follows:

$$\begin{aligned}\mathcal{L} = \quad & (1-\mu)\sum_{sc} p_{sc}x_{sc} + \mu\left(\zeta - \frac{1}{\beta}\sum_{sc} p_{sc}\eta_{sc}\right) \\ & +\sum_{sc}\lambda_{sc}\left(\eta_{sc} - \zeta + x_{sc}\right) \\ s.t. \quad & x_{sc} = \sum_{m} a_{sc,m}s_m + b_{sc}, \forall sc \\ & \eta_{sc} \geq 0 \ , \forall sc\end{aligned} \tag{3.4}$$

By differentiating the Lagrangian function $\mathcal{L}$ with respect to ζ and η_{sc}, the optimal conditions of (3.3) can be obtained, and these conditions provide interesting information about the Lagrangian multiplier λ_{sc}:

$$\frac{\partial \mathcal{L}}{\partial \zeta} = \mu - \sum_{sc}\lambda_{sc} = 0 \tag{3.5}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_{sc}} = -\frac{\mu p_{sc}}{\beta} + \lambda_{sc} \leq 0, \forall sc \tag{3.6}$$

The restricted optimization problem (taking into account (3.5) and (3.6)) can be written as:

$$\max_{x_{sc},s_m} \quad (1-\mu)\sum_{sc} p_{sc}x_{sc} + \sum_{sc}\lambda_{sc}x_{sc} \tag{3.7a}$$

$$s.t. \quad x_{sc} = \sum_{m} a_{sc,m}s_m + b_{sc}, \forall sc \tag{3.7b}$$

If λ_{sc} is written as $\mu{\cdot}q_{sc}$, as $\lambda_{sc} \geq 0$ and $\mu \geq 0$, then $q_{sc} \geq 0$, the restricted problem can be defined:

$$\max_{x_{sc},s_m} \quad (1-\mu)\sum_{sc} p_{sc}x_{sc} + \mu\sum_{sc} q_{sc}x_{sc} = \sum_{sc} r_{sc}x_{sc} \tag{3.8a}$$

$$s.t. \quad x_{sc} = \sum_{m} a_{sc,m}s_m + b_{sc}, \forall sc \tag{3.8b}$$

Assuming that the distribution of scenario profits x_{sc} could be estimated ex-ante, and that the corresponding *risk-adjusted probabilities* r_{sc} could be known in advance, it can be observed that the (3.3)≡(3.8). However, knowing the distribution function of the outcomes even without solving an optimization problem can be a very challenging task. One of the possible solutions can be to solve a risk-neutral problem and then to set up the probabilities q_{sc} to the worst-case profit scenarios in an iterative way. That is the intuitive idea behind the following naïve algorithm.

3.1.2. A naïve algorithm

Based on the equivalence defined in the section 3.1.1, an alternative mean-risk formulation with the risk-adjusted probabilities can be defined. The idea behind this approach is to decouple the CVaR computation. This shall be achieved by iteratively assigning probabilities to the scenarios and to reproduce the objective function in DOP.

This naïve algorithm could be specified as follows. First, the optimization problem equivalent to the DOP with no CVaR constraints should be solved (3.9). In this step, the probabilities assigned to the scenarios must be the original ones and the optimal decision variables under this setting could be computed by solving such optimization problem. Then, in a second stage, the new optimization problem (3.10) is formulated which is a dual problem of (3.3). In this second stage, the objective is to compute the new probabilities that reproduce the objective function of DOP. The CVaR information is implicitly taken into account in the iterative process by updating the probabilities of the scenarios. This results in an iterative two-stage algorithm that "jumps" from the first stage –where the optimal decisions are obtained according to the probabilities considered in that moment–, to the second stage where the last obtained profits of each scenario are considered input data, and the purpose is to compute a new set of probabilities to start a new cycle of the algorithm.

Thus, the naïve iterative algorithm first stage problem is defined mathematically as follows:

I-stage

$$\max_{x_{sc},s_m} \quad (1-\mu)\sum_{sc} p_{sc}x_{sc} + \mu\sum_{sc} q_{sc}x_{sc} \tag{3.9a}$$

$$s.t. \quad x_{sc} = \sum_{m} a_{sc,m}s_m + b_{sc}, \forall sc \tag{3.9b}$$

Notice that equation (3.9a) is the expected profit according to the risk-adjusted probabilities r_{sc} defined as $r_{sc} = (1-\mu)\sum_{sc} p_{sc} + \mu\sum_{sc} q_{sc}$ where the values of the updated probabilities for each scenario q_{sc} belong to the probability measure function $\mathbb{Q}$. As mentioned before, at this stage, the values of q_{sc} are constant and they will be iteratively updated after every cycle of this algorithm. In the second stage (3.10), computed decision variables from (3.9), are considered as constant input data and the probabilities that define the distribution probability $\mathbb{Q}$ are obtained such that the objective function (3.10a) matches the objective function in (3.9a). Note that the second stage of the algorithm assigns the higher value of q_{sc} to the worst case scenarios, and those values are updated in every iteration.

II-stage

$$\min_{q_{sc}} \quad (1-\mu)\sum_{sc} p_{sc}x_{sc} + \mu \sum_{sc} q_{sc}x_{sc} \tag{3.10a}$$

$$s.t. \quad \sum_{sc} q_{sc} = 1 \tag{3.10b}$$

$$0 \leq q_{sc} \leq \frac{p_{sc}}{\beta}, \forall sc \tag{3.10c}$$

The stopping criterion of this iterative process can be established when the maximum difference between the decision variables x_{sc} in two consecutive iterations is lower than a specified tolerance $\left|\frac{x_{sc}^{i+1}-x_{sc}}{x_{sc}}\right| \leq \varepsilon$, $\varepsilon = 10^{-4}$.

The presented iterative algorithm (3.9)-(3.10) provides the same solution as (3.1) in the case of the atomless discrete distribution of the decision variables (see section 2.1.2). However, knowing that the left tail of the distribution usually has a vertical discontinuity gap when discrete scenarios are used to model the uncertainty (see Figure 2.2), the second stage (3.10) would not provide the optimal values for the q_{sc} as it cannot handle the confidence interval from β^- to β^+. This imply that the algorithm will not converge for every given case. Therefore, the problem (3.1) needs to be treated in a way to overcome this challenge while assuring that the resulting problem allows to untangle the scenarios.

3.2. Decomposing the Mean-Risk Problem

This section emphasizes the stochastic programming problem derived from a mean-risk model. As solving such stochastic optimization problem requires considering simultaneously all the scenarios, the related computational burden can increase notably as compared with the single-scenario deterministic case. This is why it is very common to apply decomposition techniques to split the global problem into smaller ones, and to solve them independently in an iterative way. The application of decomposition techniques to the mean-risk problem can be seen in Ahmed (2006) where the Benders decomposition algorithm is modified to deal with two-stage risk stochastic programming problems. The recourse function is defined as the expectation of the second stage cost function and it is outer-approximated by means of Benders cuts. At the same time, the convex function that quantifies the risk is evaluated and approximated is the same way. Thus, a master problem can be solved, and it contains two groups of Benders cuts: those that come from the approximation of the recourse function, and those that come from the approximation of the risk function.

Despite the reported advantages of CVaR formulation, when its corresponding linear constraints are introduced in the optimization problem, the resulting model cannot be decomposed directly into scenario-based sub-problems. Therefore, CVaR coupling constraints can be a serious barrier when trying to apply scenario-based decomposition techniques in the context of stochastic programming models. To overcome this drawback, the Lagrangian relaxation decomposition technique (see section 3.2.2) can be applied to remove the coupling among the scenarios derived from the risk constraints. The Lagrangian Relaxation (LR) and the Benders decomposition (section 3.2.1) algorithms are applied to the mean risk problem, establishing the general equivalence between them in terms of their master and sub-problem mathematical formulations, and comparing their optimal solution with the one that corresponds to the direct optimization case (section 3.1).

3.2.1. Benders decomposition

In this framework of a mean-risk problem, the master problem coincides with the problem of expression (3.1):

$$\max_{x_{sc},s_m} \quad (1-\mu)\textstyle\sum_{sc} p_{sc}x_{sc} + \mu \cdot CVaR \tag{3.11a}$$

$$s.t. \quad x_{sc} = \sum_m a_{sc,m}s_m + b_{sc}, \forall sc \tag{3.11b}$$

and the first-stage variables are those of the random variable realizations together with the control variables s_m. The recourse function evaluates the risk measure for that selection of the first-stage variables and is given by the problem:

$$\mathrm{CVaR}\left(x_{sc}\right) = \max_{\zeta,\eta_{sc}} \zeta - \frac{1}{\beta}\textstyle\sum_{sc} p_{sc}\eta_{sc} \tag{3.12a}$$

$$s.t. \quad \eta_{sc} - \zeta + x_{sc} \geq 0, \forall sc \tag{3.12b}$$

$$\eta_{sc} \geq 0 \ , \forall sc \tag{3.12c}$$

The already mentioned dual problem, given in (3.2), permits the description of the recourse function as a minimum of a finite number of linear functions, omitting the variables ζ and η_{sc}. Let associate the dual variable q_{sc} with the (3.12b) and formulate the Benders sub-problem (BSP):

BSP

$$\mathrm{CVaR}\left(x_{sc}\right) = \min_{q_{sc}} \textstyle\sum_{sc} q_{sc}x^i_{sc}, i \in I \tag{3.13a}$$

$$s.t. \quad \textstyle\sum_{sc} q_{sc} = 1 \tag{3.13b}$$

$$0 \leq q_{sc} \leq \frac{p_{sc}}{\beta}, \forall sc \tag{3.13c}$$

Thus, the master problem can be reformulated as:

$$\max_{x_{sc},s_m,\theta} \quad (1-\mu)\textstyle\sum_{sc} p_{sc}x_{sc} + \mu\theta \tag{3.14a}$$

$$s.t. \quad x_{sc} = \textstyle\sum_m a_{sc,m}s_m + b_{sc}, \forall sc \tag{3.14b}$$

$$\theta \leq f^i + \sum_{sc} q^i_{sc} \cdot \left(x_{sc} - x^i_{sc}\right), \ i: 1, \ldots, I \tag{3.14c}$$

In the previous problem, known as complete master problem (3.14), the information of the recourse function is completely given by the constraints in it. These constraints are commonly known as Benders' cuts. As it might not be possible to obtain the complete set of Benders cuts when the size of the problem is large, these are iteratively obtained and added. The constraint (3.14c) can be written in a somewhat simpler way. Variables x^i_{sc} are just the proposed profits in (3.13) and as a consequence $f^i = \sum_{sc} q^i_{sc} x^i_{sc}$. Therefore, the Benders relaxed master problem (BRMP) can be defined as :

BRMP

$$\max_{x_{sc}, s_m, \theta} \quad (1-\mu) \sum_{sc} p_{sc} x_{sc} + \mu\theta \tag{3.15a}$$

$$s.t. \quad x_{sc} = \sum_{m} a_{sc,m} s_m + b_{sc}, \forall sc \tag{3.15b}$$

$$\theta \leq \sum_{sc} q^i_{sc} x_{sc},\ i : 1, \ldots, I' \subset I \tag{3.15c}$$

The Benders algorithm iterates between the relaxed master problem and the Subproblem until a first-stage variables set is repeated or a given tolerance is achieved. The algorithm is now summarized:

- **Step 1:** Initialize the iteration counter $i = 0$, the upper and lower bounds $\underline{z} = -\infty, \bar{z} = \infty$ and the convergence tolerance ε.

- **Step 2:** Solve the BRMP (if $i = 0$, then fix $\theta = 0$) to obtain the solution $\left(x^{i+1}_{sc}, \theta^{i+1}\right)$ and update the upper bound $\bar{z} = (1-\mu) \sum_{sc} p_{sc} x^{i+1}_{sc} + \mu\theta^{i+1}$.

- **Step 3:** Solve the BSP for the first-stage variables of **Step 2**. Obtain the test probabilities $\left(q^{i+1}_{sc}\right)$ and evaluate the lower bound $\underline{z} = (1-\mu) \sum_{sc} p_{sc} x^{i+1}_{sc} + \mu \sum_{sc} q^{i+1}_{sc} x^{i+1}_{sc}$.

- **Step 4:** Check the convergence: if $\frac{|\bar{z}-\underline{z}|}{|\bar{z}|} \leq \varepsilon$, then stop the algorithm. Otherwise, add a new Benders' cut using the test probabilities and augment the relaxed master problem. Increase the iteration counter $i = i + 1$ and go to **Step 2**.

3.2.2. Lagrangian relaxation

Consider the direct optimization problem (3.1) set in the next form:

$$
\begin{aligned}
\max_{x_{sc}, s_m, CVaR, \zeta, \eta_{sc}} \quad & (1-\mu)\sum_{sc} p_{sc} x_{sc} + \mu \cdot CVaR \\
s.t. \quad & CVaR = \zeta - \frac{1}{\beta}\sum_{sc} p_{sc}\eta_{sc} \perp \lambda_0 \\
& \eta_{sc} - \zeta + x_{sc} \geq 0 \perp \lambda_{sc}, \forall sc \\
& \eta_{sc} \geq 0 \ , \forall sc \\
& x_{sc} = \sum_{m} a_{sc,m} s_m + b_{sc}, \forall sc
\end{aligned}
\tag{3.16}
$$

For the dual variables $\lambda_{sc} \geq 0$, $\lambda_0 \leq 0$, let define the dual function $\omega(\lambda_0, \lambda_{sc})$ via the optimization of the Lagrangian sub-problem given as:

$$
\begin{aligned}
\omega(\lambda_0, \lambda_{sc}) = \quad & \max_{\substack{x_{sc}, s_m, \eta_{sc} \\ \text{CVaR}, \zeta}} (1-\mu)\sum_{sc} p_{sc} x_{sc} + \mu \text{CVaR} \\
& + \lambda_0 \left(CVaR - \zeta - \frac{1}{\beta}\sum_{sc} p_{sc}\eta_{sc} \right) \\
& + \sum_{sc} \lambda_{sc} (\eta_{sc} - \zeta + x_{sc}) \\
s.t. \quad & x_{sc} = \sum_{m} a_{sc,m} s_m + b_{sc}, \forall sc \\
& \eta_{sc} \geq 0 \ , \forall sc
\end{aligned}
\tag{3.17}
$$

The dual problem consists on minimizing the dual function:

$$
\begin{aligned}
\min_{\lambda_0, \lambda_{sc}} \quad & \omega(\lambda_0, \lambda_{sc}) \\
s.t. \quad & \lambda_0 \leq 0 \\
& \lambda_{sc} \geq 0, \forall sc
\end{aligned}
\tag{3.18}
$$

It is well known from linear programming theory, that the optimum of the dual problem coincides with that of the primal problem. Among all the techniques to optimize this dual problem, we focus on an exterior approximation method, that resembles the Benders decomposition method and establishes the connection with it. Prior to the discussion of the method, we observe, that the Lagrangian sub-problem is simplified when arranging the variables in the objective function:

$$\begin{aligned}
\omega\left(\lambda_0, \lambda_{sc}\right) = \quad & \max_{\substack{x_{sc}, s_m, \eta_{sc} \\ \text{CVaR}, \zeta}} (1-\mu) \sum_{sc} p_{sc} x_{sc} + \sum_{sc} \lambda_{sc} x_{sc} \\
& + (\mu + \lambda_0)\,\text{CVaR} - \left(\lambda_0 + \sum_{\text{sc}} \lambda_{\text{sc}}\right) \zeta \\
& + \sum_{sc} \left(\frac{\lambda_0 p_{sc}}{\beta} + \lambda_{sc} \right) \eta_{sc} \\
s.t. \quad & x_{sc} = \sum_m a_{sc,m} s_m + b_{sc}, \forall sc \\
& \eta_{sc} \geq 0 \;\; , \forall sc
\end{aligned} \tag{3.19}$$

Due to the fact that the above problem has infinite solutions for many values of the dual variables, and that our final goal is to minimize the dual function, we restrict the dual variables to those satisfying:

$$\begin{aligned}
& \mu + \lambda_0 = 0 \\
& \lambda_0 + \sum_{sc} \lambda_{sc} = 0 \\
& \frac{\lambda_0 p_{sc}}{\beta} + \lambda_{sc} \leq 0 \quad \forall sc
\end{aligned} \tag{3.20}$$

With these conditions (3.20), the Lagrangian sub-problem adopts the next structure:

$$\begin{aligned}
\omega\left(\lambda_{sc}\right) = \quad & \max_{x_{sc}, s_m} (1-\mu) \sum_{sc} p_{sc} x_{sc} + \sum_{sc} \lambda_{sc} x_{sc} \\
s.t. \quad & x_{sc} = \sum_m a_{sc,m} s_m + b_{sc}, \forall sc
\end{aligned} \tag{3.21}$$

and the dual problem, after substituting $\lambda_0 = -\mu$ is now formulated as follows:

$$\begin{aligned}
\min_{\lambda_{sc}} \quad & \omega\left(\lambda_{sc}\right) \\
s.t. \quad & \sum_{sc} \lambda_{sc} = \mu \\
& 0 \leq \lambda_{sc} \leq \frac{\mu p_{sc}}{\beta}, \forall sc
\end{aligned} \tag{3.22}$$

Once it has been defined the dual function, it can be observed that it is given as the maximum of a finite number of linear functions. For this reason, the dual function is a convex function and the dual problem can be reformulated as the next linear problem, named master problem of the LR:

$$\begin{aligned}
&\min_{\lambda_{sc},\omega} \omega \\
&s.t. \quad \omega \geq (1-\mu)\sum_{sc} p_{sc} x_{sc}^{i} + \sum_{sc} \lambda_{sc} x_{sc}^{i} \quad i:1,\ldots,I \\
&\sum_{sc} \lambda_{sc} = \mu \\
&0 \leq \lambda_{sc} \leq \frac{\mu p_{sc}}{\beta}, \forall sc
\end{aligned} \tag{3.23}$$

where (x_{sc}^{i}) are the collection of extreme values of problem (3.21). Note the similarity of the Lagrangian sub-problem (3.21) with the Relaxed master problem of the Benders decomposition (3.15). The similarity is even greater if we redefine $q_{sc} = {}^{\lambda_{sc}}/_{\mu}$ because the Lagrangian Relaxation sub-problem (LRSP) is now given as:

LRSP

$$\begin{aligned}
\omega(q_{sc}) = \quad & \max_{x_{sc}, s_n} (1-\mu)\sum_{sc} p_{sc} x_{sc} + \mu \sum_{sc} q_{sc}^{i} x_{sc} \\
s.t. \quad & x_{sc} = \sum_{m} a_{sc,m} s_m + b_{sc}, \forall sc
\end{aligned} \tag{3.24}$$

and the master problem of the Lagrangian Relaxation (LRMP) as:

LRMP

$$\begin{aligned}
&\min_{q_{sc},\omega} \omega \\
&s.t. \quad \omega \geq (1-\mu)\sum_{sc} p_{sc} x_{sc}^{i} + \mu \sum_{sc} q_{sc} x_{sc}^{i}, \; i:1,\ldots,I \\
&\sum_{sc} q_{sc} = 1 \\
&0 \leq q_{sc} \leq \frac{p_{sc}}{\beta}, \forall sc
\end{aligned} \tag{3.25}$$

Note that this reformulation recovers the constraints for the test probabilities of the conditional value at risk definition. The LR algorithm iterates between the Lagrangian sub-problem and the Lagrangian master problem until certain tolerance is achieved. Instead of a complete Lagrangian master problem, a relaxed one is formulated that is augmented as the algorithm proceeds and more Lagrangian cuts are computed. This relaxed master problem adopts the same structure as (3.25) with the exception that not all the extreme points have been computed. This is, we consider a reduced set of Lagrangian cuts (I' $< I$). The algorithm is now summarized.

- **Step 1:** Initialize the iteration counter $i = 0$, the upper and lower bounds $\underline{z} = -\infty, \bar{z} = \infty$ and the convergence tolerance ε.
- **Step 2:** Solve the LRMP (if $i = 0$, then fix $q_{sc} = 0$) to obtain the test probabilities $(q_{sc}^{i+1})$and update the lower bound $\underline{z} = \omega$.
- **Step 3:** Solve the LRSP for the test probabilities obtained in **Step 2**. Obtain the (x_{sc}^{i+1}) and evaluate the upper bound $\bar{z} = (1-\mu)\sum\limits_{sc} p_{sc} x_{sc}^{i+1} + \mu \sum\limits_{sc} q_{sc}^{i+1} x_{sc}^{i+1}$.
- **Step 4:** Check the convergence: if $\frac{|\bar{z}-\underline{z}|}{|\bar{z}|} \leq \varepsilon$, then stop the algorithm. Otherwise, add a new Lagrangian cut using the test probabilities and augment the relaxed master problem. Increase the iteration counter $i = i + 1$ and go to **Step 2**.

3.2.3. Theoretical comparison between Benders and LR

Notice that both algorithms present the similarity of augmenting the master problems as the algorithms proceed. The master problem of the Benders decomposition presents a similar structure to the LRSP. Even more, these two problems would be identical if the BRMP had just one Benders cut. In a parallel manner, the BSP has a similar structure to the LRMP. Again, notice that these two problems would be also identical if the LRMP had just one Lagrangian cut. Some differences should also be outlined. Whether the BRMP proposes random variable realization, the LRMP proposes test probabilities. On the contrary, the BSP gives back test probabilities, the LRSP returns a random variable realization. The Benders cuts are formed using the test probabilities whether the Lagrangian cuts are formed using the variable realizations. Note that both Benders decomposition and Lagrangian relaxation algorithms must converge to the solution of original problem (3.1) as both algorithms have been proven generally to solve linear problems.

Figure 3.1 summarizes both algorithms and presents in a simple way the iteration procedures for each one.

3.2.4. Drawbacks of Benders decomposition

The formulation of the Benders decomposition algorithm permits the optimization of the mean-risk problem by iterating between two problems of smaller size. However, it does not avoid the coupling of the maximization of the expected profit and

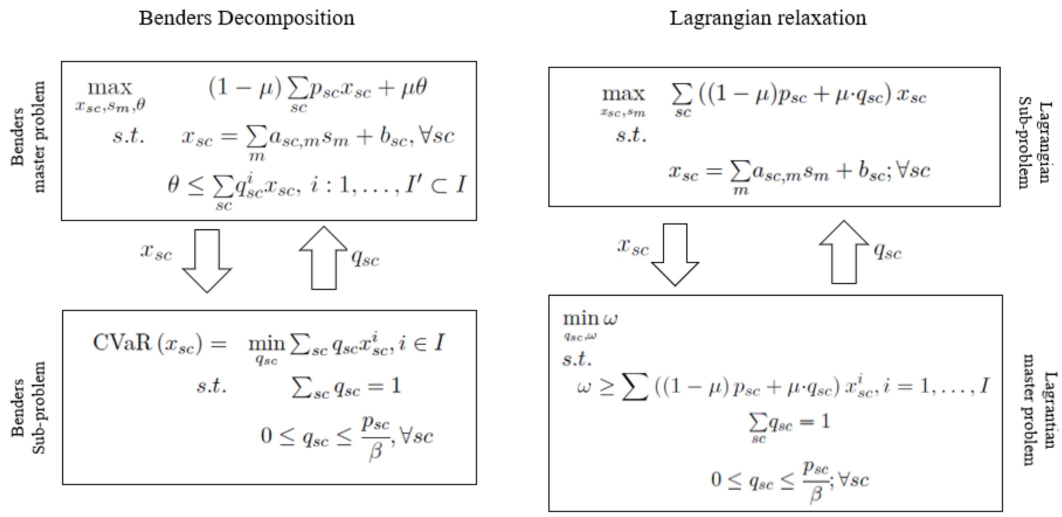


Figure 3.1.: Graphical representation of Benders Decomposition and Lagrangian Relaxation algorithms

the management of the risk, because the Benders cuts describe in an alternative manner, all the risk constraints. Another computational burden must be considered regarding the decoupling of the problem. The Benders decomposition algorithm does not decouple the profit maximization from the risk management part of the problem, because its master problem incorporates both the constraints of the profit computation and the information about the risk constraints in the form of Benders cuts. Moreover, in case of a complex optimization problem, it is very challenging to further decompose the BRMP due to its coupling constraints.

As a primal decomposition approach, the Benders algorithm provides the primal solution values equal to the ones that can be obtained from DOP. However, the values of the test probabilities q_{sc} would have to be recomputed taking into account all the active Benders cuts and their dual variables. An insight on the primal optimal value recovery is given in the section 3.2.6.

3.2.5. Drawbacks of Lagrangian relaxation

The Lagrangian relaxation algorithm does decouple the problem in these two parts, because the LRMP carries the optimization of the risk exposure while the LRSP optimizes the expected benefit. Due to this fact, it is expected to be more difficult to solve the Benders method than the LR one, as shown with the test case (section

3.2.7). An important aspect of the LR algorithm is the reconstruction of the optimal primal variables' values after the execution of the algorithm. Note that the Lagrangian sub-problem solves each iteration an optimization problem where the risk constraints have been relaxed, so that the intermediate primal solutions do not necessarily satisfy those risk constraints. Therefore, the not so effective solution to this problem would be to solve of the extended Lagrangian sub-problem.

It is natural to wonder whether the primal solution produced by the Lagrangian function x^*_{sc} is an optimal solution of the complete problem (3.1). This is the situation when the pair $(x^*_{sc}, \lambda^*_{sc})$ is a saddle point of the Lagrangian sub-problem (Minoux, 1986). However, being a saddle point implies all the problem constraints hold, even those that are relaxed in the Lagrangian function. So, it cannot be concluded that the primal values produced by the Lagrangian problem (even when the optimal dual variable are used), are the optimal solution of the problem. Nevertheless, an extra constraint can be included in the Lagrangian sub-problem that guarantees that the produced primal solution is an optimal solution of (3.1). Let λ^*_{sc} be the set of optimal dual values. Those dual variables with positive value indicate that the corresponding risk constraints are active constraints, so they can be considered to be $\eta_{sc} - \zeta + x_{sc} = 0$ while the rest of constraints can be neglected form the problem. Additionally, the sense of the optimization of problem (3.19) will draw the term $\left(-\frac{\mu p_{sc}}{\beta} + \lambda_{sc}\right)\alpha_{sc}$ to zero. For those dual values such that $-\frac{\mu p_{sc}}{\beta} + \lambda_{sc} = 0$, no additional limitation is imposed over the variable η_{sc}. However, if $-\frac{\mu p_{sc}}{\beta} + \lambda_{sc} < 0$, the positiveness of α_{sc} implies that this variable has zero value in the optimal solution. Thus, the constraint $\eta_{sc} - \zeta + x_{sc} = 0$ turns to be $\zeta = x_{sc}$ for each scenario whose dual variable is not bidding $0 < \lambda_{sc} < \frac{\mu p_{sc}}{\beta}$. Those scenarios are precisely the scenarios whose profit value coincide with the VaR. That set of scenarios is denoted as $VaR(sc)$. The above result paths the way to construct a Lagrangian sub-problem that can be used after the LR algorithm to recover the optimal primal solution. The constraints that force the scenarios with non-bidding dual variables to have the same value are introduced in (3.26). This problem is denoted as the extended Lagrangian sub-problem:

$$\begin{array}{lc} \omega\left(q_{sc}\right) = & \max\limits_{x_{sc}, s_m} \left(1-\mu\right) \sum\limits_{sc} p_{sc} x_{sc} + \mu \sum\limits_{sc} q_{sc} x_{sc} \\ s.t. & x_{sc} = \sum\limits_{m} a_{sc,m} s_m + b_{sc}, \forall sc \\ & x_{sc_j} = x_{sc_k} \, \forall \left\{sc_j, sc_k\right\} \in VaR(sc) \end{array} \tag{3.26}$$

In order to overcome the before mentioned drawbacks, next section presents recovering of the primal variables from Jovanović *et al.* (2017).

3.2.6. Recovering the primal solution: a DW approach

The challenge of decomposing the mean-risk problem with CVaR is highlighted with the algorithms presented in the previous sections. Although it can be computed by using Linear Programming (LP), as defined in Rockafellar & Uryasev (2000), the computational tractability needs to be ensured when solving the stochastic programming models with large number of scenarios. As the dual decomposition (LR algorithm) of the DOP problem, from section 3.2.2, shows some promising properties, the rational approach would be to explore the possible benefits of applying the primal decomposition technique to the mean-risk problem in terms of the Dantzig-Wolfe (DW) decomposition.

The primal solution recovering is developed in two steps. First, the Dantzig-Wolfe decomposition technique is applied to the DOP problem (3.1). It results in a Dantzig-Wolfe master problem (DWMP) and sub-problem (DWSP). It is observed that constraints (3.1b) and (3.1c) appear in the DWMP but not in the DWSP. Therefore, a DWMP dual can be derived in order to obtain a LRMP . Second, Dantzig-Wolfe iterations between DWMP and DWSP are equivalent to iterations between LRMP and LRSP, and once the convergence is achieved the primal optimal values can be achieved by a straightforward multiplication.

Specifically, let the pair of $\{x_{sc}, s_m\}$ be the extreme points of the feasible region of the polyhedron X that defines the feasible region of (3.1e). It can be expressed as the convex combination of the extreme points of such polyhedron:

$$x_{sc} = \sum_i \lambda^i x^i_{sc} \tag{3.27}$$

$$s_m = \sum_i \lambda^i s^i_m \tag{3.28}$$

where $\lambda^i \geq 0$ and $\sum_i \lambda^i = 1$. Hence, by substituting (3.1b) in (3.1a), the DWMP can be formulated as:

DWMP

$$\max_{\lambda^i,\zeta,\eta_{sc}} \quad (1-\mu)\sum_{sc} p_{sc}\sum_i \lambda^i x^i_{sc} + \mu\left(\zeta - \frac{1}{\beta}\sum_{sc} p_{sc}\eta_{sc}\right) \tag{3.29a}$$

$s.t.$

$$\sum_i \lambda^i x^i_{sc} - \zeta + \eta_{sc} \geq 0 \perp \lambda_{sc}, \forall sc \tag{3.29b}$$

$$\sum_i \lambda^i = 1 \perp \omega \tag{3.29c}$$

$$\lambda^i \geq 0, i = 1, \ldots, I \tag{3.29d}$$

$$\eta_{sc} \geq 0, \forall sc \tag{3.29e}$$

where $\lambda_{sc} \geq 0$ and ω are the dual variables of (3.29b) and (3.29c). Note that, according with DW methodology, only coupling constraints (3.1b) and (3.1c) are kept in the DWMP, whereas constraint (3.1e) is left out from it. DWSP takes care of this constraint. It maximizes the objective function keeping constraint (3.1e) and taking into account (3.1b) and (3.1c) through their multipliers. Specifically:

DWSP

$$\max_{x_{sc},s_m} \quad \sum_{sc} \left(\left(1-\mu\right) p_{sc} + \lambda_{sc}\right) x_{sc} - \omega \tag{3.30a}$$

$s.t.$

$$x_{sc} = \sum_m a_{sc,m} s_m + b_{sc}, \forall sc \tag{3.30b}$$

DW algorithm works by iterating between DWSP and DWMP. In each iteration, DWMP proposes a Lagrangian multiplier value λ_{sc} to DWSP. DWMP does not use the whole set of extreme points that define X, but a subset of them ($i = 1, ...I$; possibly just one in the first iteration). DWSP computes a solution $(x^{i+1}_{sc}, s^{i+1}_m)$. If this point is not in the previous subset, it is added to it and a new iteration performed.

DWMP is a simpler problem than DOP. However, its dual (LR algorithm) happens to be significantly simpler and much more transparent.

Incidentally, Dantzig-Wolf decomposition algorithm is also known to converge to the solution of linear programs. As a consequence, the proposed algorithm will converge to the solution of problem (3.1) (see Luenberger *et al.* (1984)). After the convergence is achieved and optimal values of x^*_{sc} and s^*_m are known from (3.27) and (3.28), the value of $CVaR$ can be obtained:

$$CVaR = \sum_{sc} q^*_{sc} x^*_{sc} \tag{3.31}$$

In contrast to the optimization problem (3.26), used as an extension to the Lagrangian sub-problem for reconstruction the optimal primal variables in section 3.2.2, the proposed recovery of the primal variables is a straightforward multiplication. From (3.27) it can be concluded that in case of ending up with only one active Lagrangian cut (3.25) at $i = I$, then its corresponding dual variable will be $\lambda^{I*} = 1$ and therefore $x^*_{sc} = x^{I*}_{sc}$ at the optimum. The real-case application of the Iterative CVaR (I-CVaR) algorithm is presented in the following chapter.

3.2.7. Illustrative example of Benders and Lagrangian comparison

In order to illustrate the performance of the two decomposition approaches explained in sections 3.2.1 and 3.2.2, this section presents an example case that can be completely reproducible and that presents some interesting features. For the sake of simplicity, it is assumed that the uncertainty can be modeled with just 20 scenarios $sc \in \{sc_1, sc_2, \ldots, sc_{20}\}$, and that the profits x_{sc} follow the next expression:

$$x_{sc} = x_{A,sc} + s \cdot (x_{B,sc} - x_{A,sc}) \ \forall sc \tag{3.32}$$

where $0 \leq s \leq 1$ represents the *here and now* decision which is unique. The samples $x_{A,sc}$ and $x_{B,sc}$ have been generated by sampling two normal distributions with the following mean and standard deviation values:

$$\begin{aligned} \bar{x}_A = 10 \quad \sigma_A = 3 \\ \bar{x}_B = 8 \quad \sigma_B = 1 \end{aligned}$$

The obtained samples have the mean and the standard deviation as in (3.33).

$$\begin{aligned} \bar{x}_A = 8.499 \quad \sigma_A = 2.906 \\ \bar{x}_B = 7.782 \quad \sigma_B = 0.801 \end{aligned} \tag{3.33}$$

Table 3.1 shows profit samples $x_{A,sc}$ and $x_{B,sc}$.

Table 3.1.: Profit samples

sc	$x_{A,sc}$	$x_{B,sc}$	sc	$x_{A,sc}$	$x_{B,sc}$
1	12.273	8.292	11	9.239	8.746
2	5.269	7.23	12	7.939	6.866
3	8.421	8.721	13	12.006	9.413
4	6.581	7.537	14	12.74	8.871
5	5.837	6.828	15	6.454	7.905
6	8.376	8.634	16	15.026	8.57
7	9.295	7.154	17	8.987	7.15
8	3.68	6.965	18	7.894	7.188
9	10.509	7.554	19	8.181	7.796
10	4.596	7.232	20	6.69	6.982

Therefore, if $s = 0$, the obtained profits will be distributed as x^A_{sc}, and if $s = 1$, the obtained profits will be distributed as x^B_{sc}. A risk neutral agent would choose a mean risk function with $\mu = 0$, so that the value of s would be 0, as this is the choice that maximizes the expected profits, despite having a higher volatility. A pure risk averse agent would choose a mean risk function with $\mu = 1$, and the value of its action would be 1, so her/his benefits would be given by distribution x_b despite having a lower value of the expected benefit. An intermediate risk averse agent would choose a level of risk exposure by means of parameter μ, and would decide the optimal value of s by solving the mean risk problem. Notice that increasing the value of s decreases the expected value of the profits, but increases the CVaR as the volatility is smaller. Therefore, it is not possible to establish a priori the optimal mapping between the risk aversion parameter μ and the optimal risk management decision (i.e., the optimal value of control variable). Figure 3.2 shows the shape of the objective function with respect to control variable for the particular case of $\mu = 0.3$ and a risk level of $\beta = 0.3$.

Results from DOP are presented in Table 3.2 for different values of the risk weight parameter μ. It can be observed that the greater the parameter, the lower the expected benefit and the greater the CVaR value.

Same results are depicted in Figure 3.3 by means of the probability density function of the profit outcome.

The already described results are also obtained with the Benders and the LR algorithms. Upper and lower bounds for both methods are presented in Table 3.3. The Benders algorithm converges in the 5th iteration with $\varepsilon = 0$ and LR converges in the

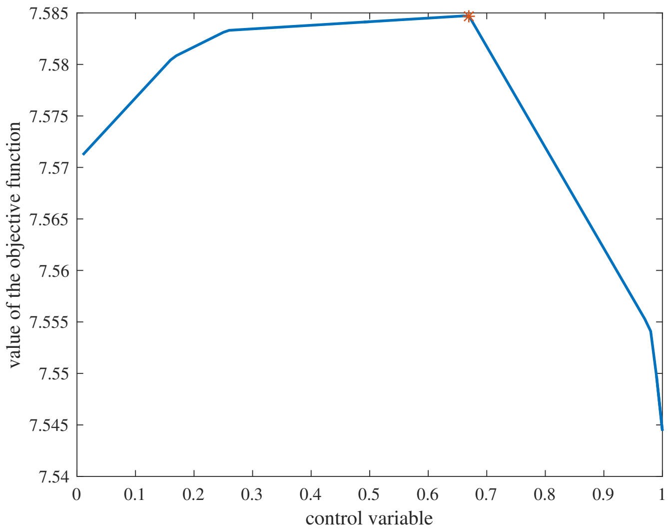


Figure 3.2.: The shape of objective function vs. the control variable s

Table 3.2.: Results from DOP

	μ			
	0	0.1	0.3	0.5
$\bar{x}_{sc}$	8.499	8.499	8.019	7.797
σ_x	2.906	2.906	1.378	0.827
o.f.	8.499	8.189	7.585	7.393
$CVaR$	0	5.403	6.571	6.989
s	0	0	0.669	0.979

3th iteration with the $\varepsilon = 0$. The Benders method returns after the 5th iteration the optimal value of s. The LR method does not necessarily provide the correct values of the primal variables once the convergence has been reached. Thus, after the 3th iteration of the LR algorithm, the values of the optimal *test probabilities* are given and they can be used to solve the extended Lagrangian sub-problem that returns the optimal primal values. In this example, the test probabilities, given in Table 3.4.

In order to reconstruct the optimal value of the primal variable s, it is necessary to solve the extended Lagrangian sub-problem given as (3.34).

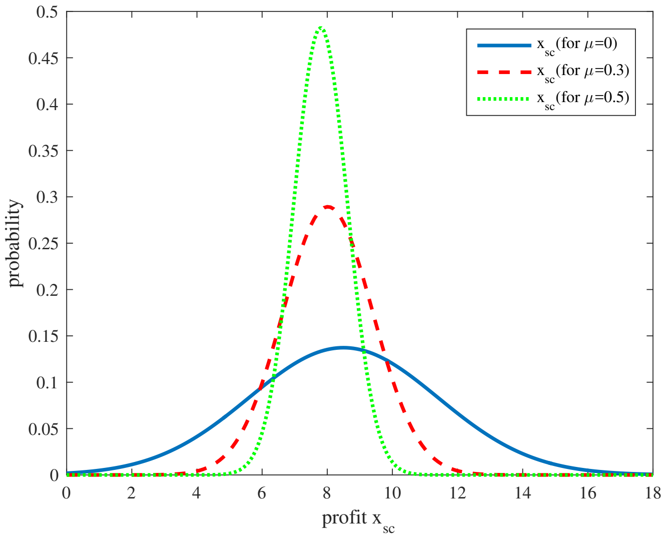


Figure 3.3.: Distribution of profits for $\mu = 0$, $\mu = 0.3$ and $\mu = 0.5$

Table 3.3.: Upper and Lower bounds for Benders and Lagrangian

$\mu = 0.3$				
i	Benders		Lagrangian	
	$\bar{z}$	$\underline{z}$	$\bar{z}$	$\underline{z}$
1	8.949	7.571	8.287	1
2	7.632	7.544	7.632	7.571
3	7.622	7.566	7.585	7.585
4	7.601	7.584	/	/
5	7.585	7.585	/	/

$$\begin{array}{ll} \max\limits_{x_{sc},s} & (1-\mu)\sum\limits_{sc} p_{sc}x_{sc} + \mu \sum\limits_{sc} q_{sc}^{LR} x_{sc} \\ s.t. & x_{sc} = x_{A,sc} + s \cdot (x_{B,sc} - x_{A,sc}), \forall sc \\ & x_4 = x_{12} \end{array} \tag{3.34}$$

with q_{sc}^{LR} being the test probabilities obtained in the last iteration LR algorithm. On the other hand, a more efficient way to obtain the optimal values is to recompute them by using (3.27) and (3.28).

The value of $CVaR$ can be computed by the next expression,

$$\text{CVaR} = \sum_{sc} q_{sc}^{LR} x_{sc}^{D} \tag{3.35}$$

Table 3.4.: Test probabilities Lagrangian Relaxation

$\mu = 0.3$							
sc	$\underline{q_{sc}}$	q_{sc}	$\overline{q_{sc}}$	sc	$\underline{q_{sc}}$	q_{sc}	$\overline{q_{sc}}$
1	0	0	0.1667	11	0	0	0.1667
2	0	0.1667	0.1667	12	0	0.0057	0.1667
3	0	0	0.1667	13	0	0	0.1667
4	0	0.1609	0.1667	14	0	0	0.1667
5	0	0.1667	0.1667	15	0	0	0.1667
6	0	0	0.1667	16	0	0	0.1667
7	0	0	0.1667	17	0	0	0.1667
8	0	0.1667	0.1667	18	0	0	0.1667
9	0	0	0.1667	19	0	0	0.1667
10	0	0.1667	0.1667	20	0	0.1667	0.1667

being x_{sc}^D the input data obtained as the solution of the problem (3.34).

4. Application of the I-CVaR to single-agent problems

4.1. Short-term risk management models

In order to show the performance of the proposed Iterative CVaR algorithm, two different models are implemented based on Jovanović *et al.* (2017). The first one (Model I) addresses the problem faced by a thermal plant that has the opportunity to sign a future physical contract to reduce its risk exposure due to the volatile spot market prices during a time horizon of one week (168 hours). The second one (Model II) is a risk-constrained UC model that can be used by a generation company owning a portfolio of several thermal plants in order to decide the optimal startup and shutdown decisions taking into account the uncertain energy spot prices and secondary reserve prices. This section is devoted to the description and formulation of these models.

The operation of thermal power producers in a context of high wind generation and demand reduction can be very challenging. For instance, in the case of the Iberian electricity market, the combined cycle gas turbines have suffered a substantial reduction of their load factors. As defined in the previous chapter, the Iterative CVaR algorithm is applied to the short-term management problem of thermal plants exposed to volatile electricity prices. The prices have been taken from real historical data of the Iberian electricity market. Two different problems are taken into account, where the decision makers are risk-averse price takers, and therefore the influence of their actions on the resulting market price is beyond the scope of this study. The first model (denoted as Model I) deals with the hedging problem of one thermal power producer. In current electricity markets there is a wide range of financial instruments allowing the producers to hedge their positions. As forward contracts are common, it is assumed that the decision maker has three possible options: trading on the spot market, signing a future physical contract (FPC), or both. For hedging the risk, a

FPC can be signed with a fixed forward price per megawatt hour. The FPC is a type of contract where the producer is obliged to physically deliver an agreed base-load production. Results obtained by the Iterative CVaR algorithm are compared with the standard CVaR formulation from Rockafellar & Uryasev (2000). For the analyzed study case, a substantial reduction in the computational time is achieved with the proposed case study. This computational advantage is more remarkable in the case of building the forward market offering strategy where many problems need to be solved by changing the possible forward market prices (see section 4.2.2). The second model (denoted as Model II) presents a very complex formulation of a risk-constrained unit commitment (UC) scheduling based on Morales-España *et al.* (2014), which has been modified in order to introduce the stochasticity (energy and reserve market prices), and to represent the agent perspective. The applied UC formulation considers a ramp-based representation of instantaneous power profiles (including startup and shutdown trajectories) that overcomes the traditional energy blocks representation. This very detailed modeling of the available reserves and the real ramping capability of the plants allows to asses in an accurate manner the incomes from selling both the energy and the secondary regulation reserve. By implementing the Iterative CVaR on the Model II shows the applicability of the the proposed algorithm on the detailed power system problem.

In addition, while there is a lot of research that includes the CVaR framework in the optimization portfolio, up to the knowledge of the author, there is a lack of literature in the electricity sector explaining the selection of the CVaR parameters. That is to say, to which degree the decision maker should be risk averse. Selecting these parameters is of a great importance as they influence operational and the financial decisions. An approach to select a sensible value for these parameters is provided in section 4.1.4.

4.1.1. Nomenclature

Indexes and Sets

h:	Hours
sc, Ω:	Scenarios, $sc \in \Omega$
i, I:	Iterations $i \in I$

Parameters

$\pi s_{sc,h}$: Electricity spot prices
πf: Price of the future physical contract
cv: Variable fuel consumption cost
cnl: None-load cost
csu: Fuel consumption at startup
com: Operation and maintenance cost
p_{sc}: Probabilities within probability distribution $\mathbb{P}$
μ: Risk weight factor
β: Confidence interval
g^{max}: Maximum power output
g^{min}: Minimum stable load
E^{min}: Minimum weekly energy production
ε: Maximum weekly energy production
U_0: Initial commitment status
g_0: Initial output power

Positive variables

$g_{sc,h}$: Power produced
$gn_{sc,h}$: Net power dispatched above g^{min}
gf: Power sold through the future physical contract

Binary variables

$U_{sc,h}$: Unit commitment variable
$SU_{sc,h}$: Startup decision

Variables

x_{sc}: Net profit obtained
$c_{sc,h}$: Net profit obtained
$gs_{sc,h}$: Power sold on the spot market
$CVaR$: Conditional value at risk
ζ: Value at risk
η_{sc}: Auxiliary variable for CVaR computation

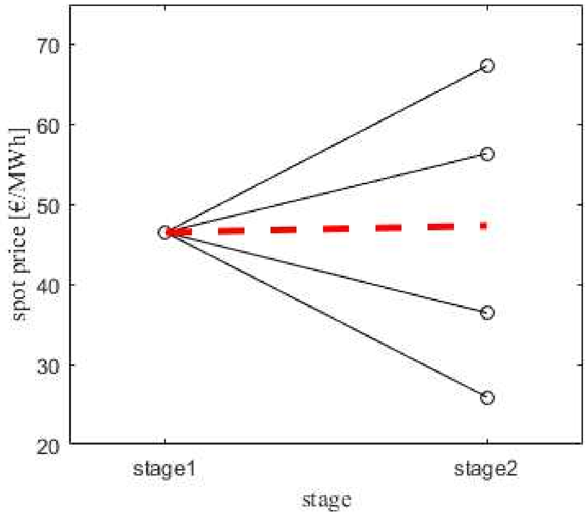
q_{sc}: Risk-adjusted probability measures within probability distribution $\mathbb{Q}$
$SD_{sc,h}$: Shut-down decision
ω: Lagrangian optimality cut
z: The value of objective function
$\overline{z}^i$: Upper bound
$\underline{z}^i$: Lower bound

4.1.2. Model I: Determining the optimal forward market offer

In the Model I the thermal plant faces the problem of building its optimal offer for the forward market in the one-week time horizon. Decision making process is influenced by huge volatility of electricity prices due to the dynamics of electricity spot market. Therefore, the decision maker is aware of the risk exposure and can hedge against possible low profits by signing a FPC with fixed quantity of power per hour and fixed forward price. This action is a *here-and-now* decision which is made at $stage1$ (see Fig. 4.1), i.e. before the thermal plant operation on the spot market. In case the forward price is slightly lower than the average electricity spot price (presented with dashed line) it would make no sense for a risk-neutral decision maker to sell its production through the FPC. However, a risk-averse decision maker might prefer to sell a part of its production at such lower price if this entails being protected against the lowest price scenarios. Another decision at $stage1$ is the unit commitment for the first 24 hours are common for all the scenarios, as the flexibility to change the startup and shutdown decision for the next day could be limited. The remaining thermal plant capacity can be traded (bought or sold) on the spot market (*wait-and-see* decision). Note that the FPC has a direct impact on the decisions made at $stage2$, such as: 1) limited capacity for trading on the spot market and 2) the plant will decide to sell/buy if the spot price is higher/lower than the forward price. For the sake of simplicity we neglect transaction costs.

The mean-risk objective function of the problem (4.1a) is formulated as (3.1a). Following the problem description (see subsection 4.1.2) we can define profits x_{sc} and decision variables s_m by replacing (3.1e) with a set of constraints that include: incomes, cost and technical characteristics (4.1e)-(4.1p). This set of constraints belongs to the group of the complicating constraints. The profit (4.1e) of the thermal plant consists of incomes made from electricity sold/bought in the spot market and through the FPC subtracted by the generation cost.

Figure 4.1.: Decision making process



$$z = \max\left(1-\mu\right)\sum_{sc} p_{sc}x_{sc} + \mu \cdot CVaR \tag{4.1a}$$

$s.t.$

$$\zeta - \frac{1}{\beta}\sum_{sc} p_{sc}\eta_{sc} - CVaR = 0 \tag{4.1b}$$

$$\eta_{sc} - \zeta + x_{sc} \geq 0, \forall sc \tag{4.1c}$$

$$\eta_{sc} \geq 0; \forall sc \tag{4.1d}$$

$$x_{sc} = \sum_{h}\left(\pi s_{sc,h} gs_{sc,h} + \pi f \cdot gf - c_{sc,h}\right), \forall sc \tag{4.1e}$$

$$c_{sc,h} = cnl \cdot U_{sc,h} + csu \cdot SU_{sc,h}$$

$$+\left(cv + com\right) g_{sc,h}, \forall sc, h \tag{4.1f}$$

$$U_{sc,h} = U_{sc,h-1} + SU_{sc,h} - SD_{sc,h}, \forall sc, h \tag{4.1g}$$

$$U_{sc,h} = U_{sc',h}, \forall sc, sc' \in \Omega, \forall h \in [1, 24] \tag{4.1h}$$

$$g_{sc,h} = gs_{sc,h} + gf, \forall sc, h \tag{4.1i}$$

$$g_{sc,h} = U_h g^{min} + gn, \forall sc, h \tag{4.1j}$$

$$g_{sc,h} \leq U_h\left(g^{max} - g^{min}\right), \forall sc, h \tag{4.1k}$$

$$RU \geq gn_{sc,h} - gn_{sc,h-1}, \forall sc, h \tag{4.1l}$$

$$RD \geq gn_{sc,h-1} - gn_{sc,h}, \forall sc, h \tag{4.1m}$$

$$E^{min} \leq \sum_h g^{min}, \forall sc \tag{4.1n}$$

$$E^{max} \geq \sum_h g^{max}, \forall sc \tag{4.1o}$$

$$-g^{max} \leq gs_{sc,h} \leq g^{max}, \forall sc, h \tag{4.1p}$$

Equations (4.1b)-(4.1c) are linear constraints used for CVaR optimization, and they represent the group of coupling constraints. Cost of electricity production (4.1f) is a linear function including: no-load cnl, startup csu, variable cv, operation and maintenance cost com. The logic coherence between commitment status, startup and shutdown decisions is expressed in (4.1g). Notice that for the first hour it is assumed that $U_{sc,h-1} = U_0$, i.e., the initial commitment status which is an input data. Commitment decisions are unique for the first day of the week, i.e., they are the same for the first 24 hours for all scenarios (4.1h). Equation (4.1i) expresses the total generation of the plant which is the sum of the power quantity for the spot and forward market. Minimum and maximum generation limits are defined with (4.1j) and (4.1k). Note that $gn_{sc,h}$ is the net power dispatched above minimum generation. Limits for upward (4.1l) and downward ramp (4.1m) prevent the rapid change in power production by the plant. For the first period, those ramp constraints have to take into account the initial conditions of the previous hour. Therefore, it is assumed that for $h = 1$, the term $gn_{sc,h-1}$ would represent the power produced above the minimum stable load in the previous hour of the scheduling horizon (input data): $g_0 - g^{min}$ in case $U_0 = 1$, and 0 otherwise . Thermal plant needs to satisfy the minimum energy production(4.1n) and the maximum energy production (4.1o) during the whole time horizon. In order to prevent the plant to sell or buy electricity on the spot market more than its maximum capacity we impose a limit (4.1p).

|In comparison to the mean-risk problem formulation from the section (3.1), expression (3.1e) as an equality constraint is replaced with equality (4.1e)-(4.1j) and inequality constraints (4.1k)-(4.1p), which define the polyhedron X (see (3.27)). Problem formulation (4.1) has the same mathematical structure as (3.1) and application of the Iterative CVaR is a straightforward process.

4.1.3. Model II: Risk-constrained unit commitment problem

Increasing penetration of renewable energy sources represents a major challenge to the UC problem. The impact of the stochasticity and variability of wind and solar power can be very high for isolated systems. This fact motivated the research presented in Asensio & Contreras (2016) where a risk-averse UC model is formulated. The structure of such model resembles the DOP shown in (3.1), but instead of expressing the risk in terms of profit, it is expressed in terms of total cost. The authors show that the UC solution depends notably on the selected risk-aversion level, and the included analysis of the efficient frontier can be very helpful for the decision maker.

In this section the UC problem from the perspective of a generation company is studied (self-UC). Therefore, the demand balance equations, and the constraints to guarantee the fulfillment of upward and downward reserve requirements for the whole system, are removed. The objective is to maximize the difference between market incomes and operational costs, under the assumption of a price-taker agent that participates both at the day-ahead energy market, and at the secondary regulation reserve market. As stated in Asensio & Contreras (2016), the flexibility provided by each unit in one hour depends on its output power during the previous hour, and on its ramping capabilities. In order to avoid the inaccuracies that could be derived from the standard representation of hourly energy blocks, Model II applies the methodology presented in Morales-España *et al.* (2014) where generation is modeled as piecewise-linear functions that represent the instantaneous power trajectories. The main advantage of this approach is that the obtained regulation reserves are very accurate as their computation takes into account the evolution of the instantaneous power within the hour, and the ramping capability. In addition, the formulation takes into account the following issues:

- Minimum up and down times
- Different startup costs depending on the number of hours the unit has been off.
- Detailed startup and shutdown power trajectories
- Consideration of different types or regulation reserves: up and down secondary reserve (15 min), up and down tertiary reserve (30 min), and off-line tertiary up and down reserve.

As including here the complete mathematical formulation of Model II would compromise the readability of this chapter, the reader is referred to Morales-España *et al.* (2014) for the detailed formulation of the tech-economical characteristics and constraints of the generation units (equations (7)-(14) and (21)-(45) given in Morales-España *et al.* (2014)). Thus, Model II has been built on the basis of such UC formulation, but with the following major improvements:

- The deterministic formulation of Morales-España *et al.* (2014) is transformed into a two-stage stochastic model in a similar manner as in Asensio & Contreras (2016). The first stage variables are the UC decisions that are unique for any possible scenario realization. The recourse functions are the instantaneous power of each thermal unit at the end of each hour, and the mentioned three types of reserves. The model computes the hourly energy generation assuming a linear power trajectory between the instantaneous power at the beginning and at the end of each hour.

- The demand balance equation is removed, as the generation company is not obliged to produce any particular load profile.

- Similarly, the system reserve requirements for each type of reserve are also removed.

- For each hour and for each scenario, market incomes are computed by summing: (i) the sales of the aggregated energy produced by all the committed generators, and (ii) the sales of the aggregated secondary reserve procured by the units.

- The problem is formulated as a risk-averse model by computing the CVaR.

- The objective function is the weighed sum of the expected profit, and the CVaR, where the selected weight factor allows to model different risk aversion levels.

The resulting Model II can be solved both by a direct optimization, or by applying the Iterative CVaR algorithm presented in section 3.2. This entails to build the corresponding master and sub-problem of the algorithm adapted to this particular formulation in an analogous way as it was explained for Model I.

4.1.4. Selection of CVaR parameters

Taking into account the objective function of the of mean-risk problem presented in section 2.1.3, it can be seen that the level of risk-aversion can be quantified in terms of μ, i.e. the relative weight that the risk measure $\rho(X)$ has in the objective function. In case $\mu = 0$, the risk measure would not be considered in the objective function, leading to the risk-neutral case where the objective is just to maximize the expected profit $E(X)$. The other extreme ($\mu = 1$) would represent the case of a decision maker who is a pure risk-averse agent. Any intermediate value $\mu \in [0, 1]$ would implicitly represent a certain risk level of the decision maker. In addition, in case the used risk-measure is the $\text{CVaR}_\beta(X)$, the value of the chosen confidence level β also has a direct impact on the solution of the mean-risk problem. For instance, $\beta = 0.05$ means that the risk measure included in the objective function would be the average value of the profits obtained in all those scenarios whose benefit is lower than the 5% percentile. In case that $\beta = 0.01$, the decision maker would only be concerned about 1% of the scenarios located in the left tail of the probability density function. Therefore, the two parameters that must be specified in order to set up the risk-averse mean-risk problem are the following ones

- The risk weight factor $\mu \in [0, 1]$
- The confidence level β

To the best knowledge of the author, there is little research published in determining the value of these CVaR parameters applied to the electric power industry and in all the reviewed research works, the value of μ and β are assumed to be known. In this section we present the approach proposed in this thesis to establish the values of these parameters. The objective here is not so much to establish an exact algorithmic procedure to set univocally both parameters, as to open a discussion about what the interpretation of their values might be, by trying to transfer to the electric power industry some common practices and concepts widely used in the financial sector, (Chatterjee, 2014).

The 2007-2008 financial crisis uncovered the vulnerability of the banking system at global level and gave place to new regulations and supervision procedures in order to strengthen its resilience to absorb adverse economic shocks. In this sense, the implementation of the Basel III framework in EU has resulted in the adoption by Member States of a set norms such as the Capital Requirement Regulation (EU, 2013b) and the Capital Requirement Directive IV (EU, 2013a). As a result of this regulation, in order to cover unexpected losses, financial institutions are obliged to set aside an amount of capital which is higher when the assets of the bank are riskier. This *own funds requirement* is computed as a percentage of the risk-weighted assets of the bank. Similarly, banks have to hold a certain level of liquidity to cover stressed events of outflows, or additional capital buffers to ensure their capability to absorb the losses related to a crisis episode, to ensure the bank capital, or to guarantee that during the low economic cycle, the bank is able to continue offering its lending products.

From the perspective of an electric generation company that sells its production in the market subject to uncertainty, one could ask to what extent poor market results could compromise the development of its activity. If there was a scenario where market revenues were scarce (for example, because of a high presence of almost zero variable cost renewable generation that would lead to a reduction in the marginal price of the system), the company would see a reduction in the amount of money available to cover all its payment commitments.The Working Capital of a company is the difference between its current assets (cash, accounts receivable, and inventories of finished goods and raw materials), and its current liabilities (accounts payable), (Smith, 1973). Therefore, an insufficient Working Capital reduces the financial health of the company as the capability to payback its creditors could be compromised. Let assume that the company has a *base fund* ($FBase$) which represents such available assets (cash or similar) to deal with the payment commitments. The capital yield of $FBase$ is normally very low due to its required very high liquidity, and therefore its total volume has to be carefully established: on the one hand, if the fund $FBase$ were too small, the company could not be able to meet its short-term payment commitments; on the other hand, if the fund were larger than necessary, the company would be incurring an opportunity cost since the excess of the fund not strictly necessary could be allocated somewhere else in order to obtain a higher return. It should also be borne in mind that the company's activity is subject to multiple sources of uncertainty and, therefore, the predictability of cash flows is not absolute. If the company is faced with an unlikely scenario of very bad market results, that amount of $FBase$ could be insufficient. One possible solution would be to allocate an additional capital fund $FRisk$ that guaranties liquidity in such kind of extreme events of very bad market outcomes. Under this setting, the company's objective function should be the following one:

$$\max \quad \sum_{sc} p_{sc} x_{sc} \quad - CBase \cdot FBase - Crisk \cdot FRisk \tag{4.2}$$

where the first term is the sum of the operational profits for all the considered scenarios weighted by their corresponding probabilities, followed by the financing cost of the mentioned funds $FBase$ and $FRisk$, where $CBase$ and $CRisk$ are the unitary financing costs (expressed in % per unit of time). Without loss of generality, we can assume that the base fund $FBase$ and its associated cost $CBase$ are known values, and therefore they can be eliminated from the objective function resulting in the following expression:

$$\max \quad \sum_{sc} p_{sc} x_{sc} \quad - Crisk \cdot FRisk \tag{4.3}$$

The difficulty here lies precisely in determining the value of the amount of money $FRisk$ that the company must immobilize to be protected against the market risk. In that respect, risk measures presented in section 2.1.1 are commonly used in the financial industry to that purpose.

In order to illustrate the underlying rational of the proposed method, let assume that the company follows a pure risk-neutral criterion when planning the operation of its generation portfolio to sell the produced energy in the electricity market. The company hires a new risk manager and the first thing that it does is to measure the risk associated with the activity of selling energy in the spot market. For this discussion we can assume that the manager uses a given risk measure $\rho()$ based on the potential losses (or disbenefits[1]), and therefore it will take a higher value when the expected outcomes of the market are riskier. After doing the calculation, she obtains $\rho(X_o)$, where X_o is a discrete random variable describing the set of net disbenefits that corresponds to the risk-neutral operation.

Following the reasoning, one might ask what would happen if the company planned the operation of its power plants by applying a risk aversion criterion. In that case, the risk-manager would compute the updated risk measure under this setting, i.e. $\rho(X_r)$. Assuming that the risk management strategy works properly, i.e. that it reduces the market risk, it must be satisfied that $\rho(X_r) \leq \rho(X_o)$ as the goal is to push to the left the losses located at the right tail of the loss distribution function, which is equivalent to the next expression:

[1]If X represents the profit, the disbenefit would be $-X$.

$$\rho(X_r) = \rho(X_o) - a \tag{4.4}$$

with $a \geqslant 0$. Assuming that $\rho()$ is a coherent risk measure, the translation invariance property, see section 2.1.3, implies the following relationship:

$$\rho(X_r) = \rho(X_o) - a = \rho(X_o + a) \tag{4.5}$$

The expression (4.5) states that when adding a certain amount of capital to a portfolio, the risk is reduced by the same amount. In this case it can be interpreted as follows: in terms of risk, introducing the risk-aversion criterion in the operation planning of the power plants is equivalent to continue operating the plants under a risk-neutral approach but putting aside an additional very liquid and secure capital a. Let $FRisk_o$ and $FRisk_r$ be the amounts to be kept in the fund when the operation is carried out following a risk-neutral and a risk-averse criterion respectively. As the risk has been reduced, the amount of money that should be kept in the fund would be smaller and the reduction in the fund thanks to carry out a risk-averse operation should be equal to the reduction of the money that is at risk:

$$FRisk_r = FRisk_o - a = FRisk_o - (\rho(X_o) - \rho(X_r)) \tag{4.6}$$

Once that the amount to be deposited in the fund has been obtained, we can substitute it in (4.3):

$$\begin{aligned} \max \quad & \textstyle\sum_{sc} p_{sc} x_{sc} - Crisk \cdot (FRisk_o - (\rho(X_o) - \rho(X_r))) \\ & = \textstyle\sum_{sc} p_{sc} x_{sc} - Crisk \cdot (FRisk_o - \rho(X_o)) + Crisk \cdot \rho(X_r))) \\ & = \textstyle\sum_{sc} p_{sc} x_{sc} - K_o + Crisk \cdot \rho(X_r))) \end{aligned} \tag{4.7}$$

where $K_o = Crisk \cdot (FRisk_o - \rho(X_o))$. One can expect that the quantity deposited in the fund is equal to the amount that is subject to risk, i.e. $FRisk_o = \rho(X_o)$ which yields to $K_o = 0$. If that is not the case, K_o would be a constant term that could be removed from the objective function, leading to the following expression:

$$\max \quad \textstyle\sum_{sc} p_{sc} x_{sc} - \quad Crisk \cdot \rho(X_r) \tag{4.8}$$

Throughout all this thesis, the risk measure has been defined in terms of profits instead of losses. Thus, if X_r stands for the discrete random variable describing the set of net profits that corresponds to the risk-averse operation, the objective would be to push to the right the profits located at the left tail of the profit distribution function. Thus, if the risk measure is defined in terms of profits instead of losses, it must be changed the negative sign of the previous expression, leading to:

$$\max \quad \textstyle\sum_{sc} p_{sc} x_{sc} + \quad Crisk \cdot \rho(X_r) \tag{4.9}$$

Using the CVaR as the coherent risk-measure (defined in terms of profits as in section 2.1.2), it results in the following expression:

$$\max \quad \textstyle\sum_{sc} p_{sc} x_{sc} + \quad Crisk \cdot CVaR \tag{4.10}$$

If (4.10) is multiplied by $(1-\mu)$ the objective function is given by:

$$\max\,(1-\mu)\sum_{sc} p_{sc} x_{sc} + (1-\mu)\,Crisk \cdot CVaR \tag{4.11}$$

and in comparison to (3.1a), it is now obvious that $(1-\mu)\,Crisk = \mu$. Solving for the variable μ, we obtain:

$$\mu = \quad \tfrac{Crisk}{Crisk+1} \tag{4.12}$$

As typical values of the fund cost satisfy that $Crisk \ll 1$, it can be derived the next expression:

$$\mu \approx \quad Crisk \tag{4.13}$$

Notice that the fund amount $FRisk_r$ can be reduced if expected bad outcomes are better, that is, if $CVaR$ is greater.

Regarding to the fund cost $Crisk$, for the weekly unit-commitment problem, it could be computed as the company's weighted average cost of capital (WACC) deducted by risk-free spread per week. The WACC can differ substantially between different countries and companies, (Ondraczek *et al.*, 2015). For instance, assuming that WACC is 18% per year and that the risk-free rate is 2% per year, the weekly fund cost is:

$$Crisk = \frac{18\% - 2\%}{52} \approx 0.003\% \tag{4.14}$$

In conclusion, we have seen that one way to look at CVaR is as a tool for setting the capital fund that guaranties liquidity in the event of bad market outcomes. The fund will be used to provide liquidity whenever a bad market outcome occurs. The cost of setting that fund is the opportunity cost of the allocated capital, that can be quantified as the spread between the and the risk-free rate. By setting such a additional fund the company's risk is reduced to a low level, and it makes sense to optimize the expected profit.

Regarding to the other parameter to be determined, i.e. the confidence level β, in the banking sector it is common for the regulator to set the confidence interval for which the risk measures must be calculated. For instance, the official confidence level at which banks are supposed to absorb their annual losses according to The Basel Committee on Banking Supervision is set at 99.9%, (Zimper, 2014). However, in the context of this thesis, the generation company is not subject to such kind of external requirements. Implicit in the previous rational is that the fund is expected not to be used very frequently. Otherwise, management and additional operational costs should be added. In order words, by setting the fund something equivalent to the insurance that has been acquired. Therefore, β should be set in order that recourse to the fund is infrequent enough as it would not be efficient to have to replenish the fund continuously. In the case of the unit-commitment problem, a sensible figure might be, e.g., once per year up to three times per year. In the study case of Model I the time horizon is one week. Therefore, for a period of one week:

$$\beta \in [\tfrac{1}{52}, \tfrac{3}{52}] = [0.019, 0.058] \tag{4.15}$$

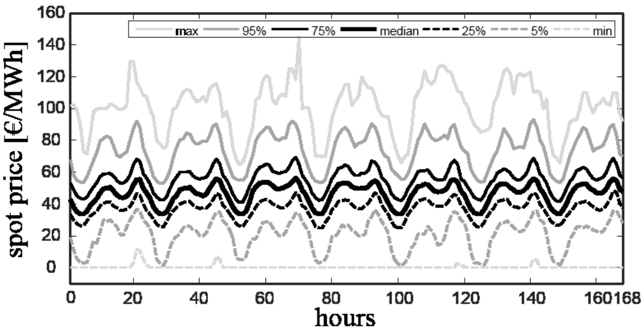
The importance of CVaR parameters selection is highlighted in the section 4.2, which analyzes the impact of the risk weight factor $\mu = 0.003$ on the mean value of profits and the $CVaR$ value. Taking into account that the value of μ can be very small, a large number of scenarios might be needed for determining a sensible value of $CVaR$. In section 4.3 the same risk weight factor is used.

4.2. Study case: Optimal forward market offer (Model I)

The study case presented in this section is implemented in GAMS (Brooke *et al.*, 1996), and solved with CPLEX 12 on a personal computer with eight core processor at 3.6 GHz and 16 GB of RAM. The problem presented in (4.1) has the structure of (3.1). Therefore, it is referred to it as Direct Optimization Problem (DOP). The proposed iterative algorithm (denoted as I-CVaR) requires solving sequentially (3.25) - (3.24) for the particular case of the presented Model I. Given that the duality gap of linear problems is null, the optimal solution of the I-CVaR will be exactly the same as the one obtained with a convex and linear DOP. Therefore, in order to illustrate the performance of the presented I-CVaR algorithm in section 4.2.1 and 4.2.2, the binary variables of the problem have been relaxed to ensure the convexity of the model. In section 4.2.3 the results are presented of the non-convex case for different risk weight factor μ.

The stochastic model (4.1) consists of 520 scenarios. For electricity price modeling, a simple method chosen based on the 10 year public historical data from the Iberian electricity market OMIE (2015). Starting from Saturday, 1st of January 2005, every 7 days represent one spot price scenario in our problem. Each scenario consists of 168 hourly spot prices which are exogenous stochastic variables and they all have the same probability $p_{sc} = 1/520$. Figure (4.2) shows the median of all the scenarios, together with the percentiles 95%, 75%, 25% and 5% in order to illustrate the variability of the prices used in the example case. The maximum and minimum price are also shown.

Figure 4.2.: Spot price variability



Note that there are many events of null prices that mainly correspond to events of high wind generation and low demand. Regarding the confidence interval, the selected value is $\beta = 0.05$. The values of the thermal plant parameters can be seen in Table 4.1. The price of the FPC is set to $\pi f = 46.55$ €$/MWh,$ which is lower than the average spot price ($\pi s^{avg}_{sc,h} = 46.578$ €$/MWh$). Minimum and maximum energy production are $E^{min} = 6300\,MWh$ and $E^{max} = 75600\,MWh$, respectively. The generation cost parameters can be seen in Table 4.2. It is assumed that the initial state of the thermal unit is ON. The convergence tolerance is set to $\varepsilon = 10^{-4}$.

Table 4.1.: Generator parameter data

g^{max} $[MW]$	g^{min} $[MW]$	RU $[MW/h]$	RD $[MW/h]$
500	150	200	200

Table 4.2.: Cost parameter data

cv [€$/MW$]	35
cnl [€/h]	2200
csu [€]	40000
com [€$/MW$h]	5

4.2.1. Results

The results of this study case are presented in the following order: (i) the computational burden of implementing the CVaR methodology and the importance of the CVaR parameter selection, (ii) the efficiency of using the Iterative CVaR over DOP, and (iii) the primal variables optimal value recovery.

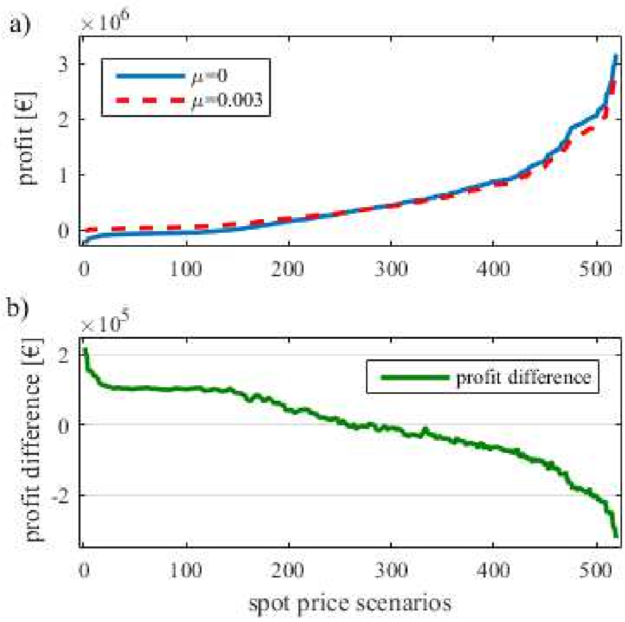
First, the computational challenge of the CVaR constraints (4.1b)-(4.1c) is highlighted. In Table 4.3 the optimal solution of the expected profit z, the mean value of the profits $\overline{x}_{sc}$, the value of $CVaR$, the power sold through the future physical contract gf, and the CPU time can be seen. Obtained values from DOP are compared for the risk-neutral ($\mu = 0$) and the risk-averse ($\mu = 0.003$) cases. Notice that for the risk-neutral case, the CVaR constraints are not active, meaning that the mathematical structure of (3.1) adopts the same form as (3.24) and the computational burden is decreased. On the other hand, the coupling constraints in DOP are activated by setting a not-null value of μ and the CPU time increases (see Table 4.3 for $\mu = 0.003$). The CPU time of the risk-averse approach is more than 3 times grater than the risk-neutral approach. The value of $CVaR$ for $\mu = 0$ is computed as the sub-product by accounting for the 5% lowest profit scenarios.

Table 4.3.: Optimization results for DOP and I-CVaR

	$\mu = 0$	$\mu = 0.003$	
	DOP	DOP	I-CVaR
z [k€]	532.528	530.719	530.719
$\overline{x}_{sc}$ [k€]	532.528	532.287	532.287
$CVaR$ [k€]	-127.102	9.625	9.625
gf [MW]	0	50.965	50.965
CPU time [s]	54.741	171.710	134.753
iterations	/	/	3

As expected, the risk-averse case leads to a lower mean profit $\overline{x}_{sc}$ and higher value of $CVaR$. It is important to point out the importance of the CVaR parameter selection from section (4.1.4). In particular, the thermal plant sells 50.965 MW as base-power on the forward market which significantly increases the obtained value of the $CVaR$ (by 136.727 k€) with a negligible influence on the $\overline{x}_{sc}$ (just a small decrease of 241 €). Obtained profit distributions for $\mu = 0$ and $\mu = 0.003$ can be seen on Fig. 4.3-a. As expected, for the risk-averse case, the profit values for the left tail of the distributions are higher than in the risk-neutral case. Conversely, the scenarios endowed with higher gains have lower profits in the risk-averse case than in the neutral case. (see Fig. 4.3-b).

Figure 4.3.: Profit distribution comparison



Second, the results of I-CVaR are compares with the risk-averse approach of DOP to indicate the efficiency and consistency of the proposed algorithm. The CPU time of I-CVaR is 134.753 seconds, which is 21.5% faster than the DOP. Table 4.3 shows that the obtained values of z, $\overline{x}_{sc}$, $CVaR$ and gf are equal to the values obtained with the DOP. In this problem setting, the implementation of I-CVaR overcomes the DOP in terms of computational time even though it requires 3 iterations to converge (see Table 4.4). The values of the upper and lower bounds in the third iteration are equal, meaning that the algorithm converges with $\varepsilon = 0$. Notice that the objective function of the LRSP can be interpreted as the maximization of the expected profit under a risk-adjusted probability: $(1-\mu)p_{sc} + \mu \cdot q_{sc}$. Regarding the LRMP (3.25), as it is a small linear problem, it does not require too much time to be solved. In addition, per each new iteration the solver has a better "starting point" which is closer to the optimal solution. Providing such starting solution makes the LRSP less time consuming in every following iteration and improves the computational efficiency of the whole iterative algorithm.

Table 4.4.: Upper and lower bound convergence

	$\mu = 0.003$		
iterations	$\bar{z}^i$ [k€]	$\underline{z}^i$ [k€]	$\dfrac{\mid \bar{z}^i - \underline{z}^i \mid}{\mid \bar{z}^i \mid}$
1	532.052	0	1.002
2	537.472	528.742	0.016
3	530.719	530.719	0

Finally, the recovery of the optimal primal variables is shown. Table 4.5 shows the values of x_1^i and gf^i per each iteration and its corresponding multiplier λ^i. The optimal values are obtained using the following expressions:

$$x_1 = \sum_i \lambda^i x_1^i$$
$$gf = \sum_i \lambda^i gf^i$$

which are $x_1^* = 242.192\,\text{k€}$ and $gf^* = 50.965\,MW$ for $\mu = 0.003$. Optimal primal variable recovery can be used for all the decision variables.

Table 4.5.: Values of gf^i and λ^i per iteration

	$\mu = 0.003$		
iteration	λ^i $[p.u.]$	x_1^i $[k€]$	gf^i $[MW]$
1	0.102	222.814	500
2	0.898	16.786	0
3	0	16.786	0

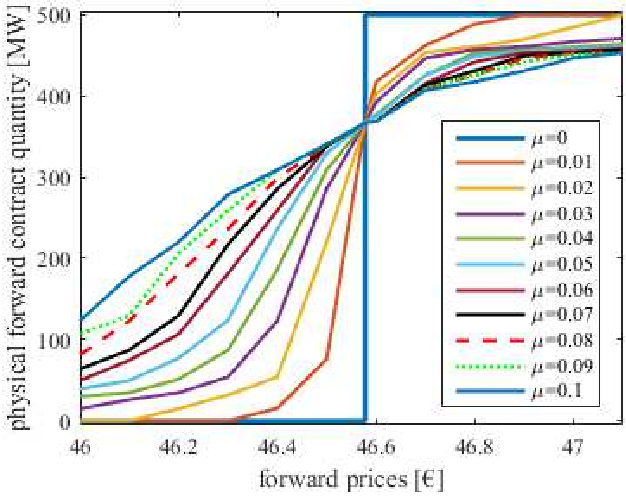
4.2.2. Forward market offering strategy

Previous results correspond to a given fixed price of the forward contract and a given risk-averse case. However, it would be interesting to analyze the impact of both parameters. In a real setting, the πf might not be known in advance as it depends on the interaction among all the traders. Therefore, the decision maker needs to define an optimal offering strategy to sell its power in the forward market. This strategy adopts the form of a supply function that expresses how much quantity should be sold for each possible forward market price. For a detailed sensitivity analysis with respect to the change in πf and μ, the offering strategy for the PFC is shown in Fig. 4.4. The risk preference range $0 \leq \mu \leq 0.1$ and the forward price range $46 \leq \mu \leq 47.1$ $[€/MWh]$ is taken into account. As can be seen, with risk-neutral case ($\mu = 0$) the thermal plant does not sell its capacity through the PFC if the forward price is lower than the average spot price. On the contrary, the power plant commits its maximum capacity when the forward price is higher than the average spot price. For the risk-averse cases ($\mu > 0$) a variety of supply functions can be observed. Note that all these risk-averse supply functions intersect at the same point ($gf = 367.106\,MW$) where the future electricity price is equal to the average spot price ($\pi f = \pi s_{sc,h}^{avg}$). This outcome is also related to the equilibrium position where the plant does not benefit from a possible deviation of its decision unilaterally. It is important to notice that building the set of the supply functions for different risk profiles with the proposed I-CVaR is very efficient in terms of computational time for this problem.

4.2.3. Non-convex problem

In case the mentioned model is formulated considering binary variables (commitment and the startup variables) the resulting problem is non-convex. For the non-convex problem binary variables are used in (4.1g). Table 4.6 shows the CPU time and the relative optimality gap (ROG), compared with the DOP. Here the term of the ROG

Figure 4.4.: Forward market offering strategy



is included which addresses the relative difference between the objective function of the DOP and I-CVaR (4.16). Although the CPU time is in favor of the DOP for this case, the relative optimality gap is negligible. The I-CVaR converges after three iterations with $\varepsilon = 0$ and the obtained solution for the physical forward contract (gf) is the same as in DOP. As the Iterative CVaR sub-problem (3.24) does not depend on the CVaR measure, it allows one to further decompose the optimization problem.

$$ROG = |\frac{z^*_{DOP} - z^*_{I-CVaR}}{z^*_{DOP}}| \tag{4.16}$$

Table 4.6.: Non-convex case results

μ	CPU time [s]		i	ROG
	DOP	I-CVaR		
0.003	217	417	3	$2.72 \cdot 10^{-6}$
0.02	202	394	3	$2.17 \cdot 10^{-13}$
0.04	228	405	3	$2.68 \cdot 10^{-6}$
0.06	215	395	3	$2.65 \cdot 10^{-6}$
0.1	218	388	3	$2.6 \cdot 10^{-6}$

4.3. Study case: Risk-constrained Unit Commitment (Model II)

Model II is applied for a hypothetical generation company that owns the same 10 thermal plants used in the example case of Morales-España *et al.* (2014) for a time horizon of 24 hours. The company faces the price uncertainty that has been modeled by 520 equally probable scenarios ($p_{sc} = {}^{1}/_{520}$). Each one of them contains a realization of 24 hour energy spot market and secondary reserve market prices. A sample of one day per each week in the year, based on the 10 year public historical data OMIE (2015) was used to build the price scenarios.

4.3.1. Results

The used CVaR parameters are $\mu = 0.003$ and $\beta = 0.05$. Two different computational approaches are presented. First, binary variables of Model II are relaxed and the model is solved as a Relaxed Mixed Integer Problem (RMIP). In this way the convexity is assured and the obtained values of the DOP and the I-CVaR are the same (see RMIP column in Table 4.7). It should be highlighted that the I-CVaR converges in 2 iterations with $\varepsilon = 0$ and it is 5% faster than the DOP. These results show the applicability of the Iterative CVaR for a detailed power system representation.

Table 4.7.: RMIP and MIP optimization results

	RMIP	MIP
z [k€]	928.001	927.610
$\overline{x}_{sc}$ [k€]	930.684	930.295
$CVaR$ [k€]	36.476	35.202
iterations	2	2

Second, Model II is solved as Mixed Integer Problem (MIP) taking into account the nature of the the binary variables. As expected, based on the results from section 4.2.3, the I-CVaR is 54% slower than the DOP and the algorithm converges after 2 iterations. However, note that the ROG is equal to zero, which means that the results obtained with the I-CVaR are exactly the same as the results obtained with the DOP in contrast to results from the subsection 4.2.3. The solutions obtained with RMIP and MIP are compared. Both mean profit $\overline{x}_{sc}$ and the value of $CVaR$ have close values, as can be seen from Table 4.7. Table 4.8 represents the commitment of the plants for the 24 hours. Thermal plants 1 to 7 are the ones that are operating during the day and the plants 8 to 10 are down due to the high operation cost in comparison with the price scenarios. Note that plants number 3, 4 and 5 are not committed in the first 3 hours of the day. There are 2 main reasons that justify this solution: (i) due to the low demand the hourly prices are very low, and (ii) the initial down state of the plants are -5, -5 and -6 hours, respectively, which drastically increases the startup cost of the thermal plants. The UC schedule in the case of the RMIP defers from the MIP only in the 4th hour for the plants 3, 4 and 5, and the startup variable takes the value around 0.51 instead of 1. This relaxation of the binary variable reduces the startup cost of the mentioned plants and therefore increases the overall mean profit.

Table 4.8.: MIP UC schedule

plants	Hours (1-24)
1	111111111111111111111111
2	111111111111111111111111
3	000111111111111111111111
4	000111111111111111111111
5	000111111111111111111111
6	111111111111111111111111
7	111111111111111111111111
8	000000000000000000000000
9	000000000000000000000000
10	000000000000000000000000

5. Electricity market Nash Equilibrium with risk-averse agents

5.1. Medium term market equilibrium model

It is well-known that a perfectly competitive market results in the same operation as the theoretical social-welfare optimum. However, even in the absence of strategic behavior, the possible risk-aversion of market participants can lead to a different solution. Therefore, the fact that risk-aversion can lead to a market inefficiency is of paramount importance.

This chapter presents a market equilibrium model from Jovanović *et al.* (2018) where the players are Gencos that own typical generation portfolios (coal plants, combined cycle gas turbines, hydro units, and RES) and compete to supply the demand in the medium term taking into account that they can be endowed with different risk aversion profiles. The model is formulated as a multi-stage stochastic equilibrium problem. The presented model is more general than the previous work Rodilla *et al.* (2015) as it takes into account a more realistic representation of the generation system. Thus, instead of a very stylized representation of the hydro system and a single thermal generator, this model uses a more detailed representation of the reservoirs during the whole planning horizon and it includes multiple thermal plants that can belong to different agents. In addition, instead of using utility functions to model risk aversion, the model implements the Conditional Value at Risk (CVaR) of Rockafellar & Uryasev (2000) due to its suitability to be embedded within optimization models. Regarding Philpott *et al.* (2016), the presented model does not require to build in advance the extreme points of the polyhedron that define the risk set of each agent, and it takes into account the net-head dependency. In addition, the implemented model in the example-case section is not just a two-stage scenario tree, but a multi-stage scenario tree with 12 time periods (months) that can be used to analyze the impact of the risk aversion on the annual evolution of the main variables. Finally, this chapter analyzes how the market equilibrium solution changes for different risk aversion levels of the involved agents. To do so, the results are compared with the outcome of the centralized planer model (section 5.3).

5.2. Nomenclature

Sets and indexes

T:	Set of time periods t
H:	Set of hydro generators h
J:	Set of thermal generators j
R:	Set of renewable energy sources (RES) generators r
S:	Set of scenarios sc
M:	Set of generation companies (market participants) m
N:	Set of all nodes of the multistage stochastic tree
N_T:	Set of terminal nodes
$\overline{N}_T$:	Set of all the nodes that are not terminal nodes, i.e., $\overline{N}_T = N \backslash N_T$
N_{sc}:	Set of all nodes included in scenario, sc
J_m:	Subset of thermal generators that belong to generation company $m \in M$
H_m:	Subset of hydro generators that belong to generation company $m \in M$
$F(t,i)$:	Father node of node (t,i) in the multistage stochastic tree
$D(t,i)$:	Descendant nodes of node (t,i) in the multistage stochastic tree
$\Omega_{t,i}$:	Set of all the scenarios that include the node (t,i) in their path

Parameters

$DEM_{t,i}$:	Demand at node (t,i)
$L_{t,i}$:	Duration of the time that corresponds to node (t,i)
$I^h_{t,i}$:	Natural inflows at the reservoir of hydro generator $h \in H$ in node (t,i)

P_{sc}: Probability of scenario $sc \in S$
$P_{t,i}$: Probability of node (t, i)
$R^r_{t,i}$: Power generation of renewable energy sources at node (t, i)
V^h: Initial volume of water stored at the reservoir of hydro generator $h \in H$
V^h_f: Target volume of water at the end of the time horizon for hydro generator $h \in H$
$\overline{V}^h$: Maximum reservoir capacity for hydro generator $h \in H$
$\underline{V}^h$: Minimum reservoir capacity for hydro generator $h \in H$
$\underline{F}^h_{t,i}$: Minimum water flow for hydro generator $h \in H$ at node (t, i)
$\overline{F}^h$: Maximum water flow for hydro generator $h \in H$
$\overline{G}^j$: Maximum output power of thermal generator $j \in J$
$\beta^m = [0, 1]$: Confidence interval of each generation company $m \in M$
$\mu^m = [0, 1]$: Risk aversion level of each generation company $m \in M$

Functions

$C^j_{t,i}(g^j_{t,i})$: Cost function of thermal generator $j \in J$, at node (t, i)
$E^h(v^h_{t,i})$: Energy coefficient to translate water flow into output power of hydro unit $h \in H$

Variables

x^m_{sc}: Profit obtained by generation company $m \in M$, in scenario $sc \in S$
$v^h_{t,i}$: Volume of water stored in $h \in H$ at the end of the time period of node (t, i)
$f^h_{t,i}$: Flow of water released by the reservoir of hydro generator $h \in H$, in node (t, i)
$s^h_{t,i}$: Water spillage of hydro generator $h \in H$, in node (t, i)
$g^h_{t,i}$: Output power generated by hydro generator $h \in H$, in node (t, i)
$g^j_{t,i}$: Output power of thermal generator $j \in J$, in node (t, i)
$\pi_{t,i}$: Spot price in node (t, i)

$CVaR^m$:	Conditional Value at Risk of each generation company $m \in M$
ζ^m:	Value at Risk of each generation company $m \in M$
η^m_{sc}:	Auxiliary variable used for $CVaR$ computation

5.3. Benchmark Model: Centralized Stochastic Hydrothermal Coordination Model

This section presents the benchmark model against which the operation of the generation system in the presence of risk-averse agents is compared. This model is formulated as a centralized hydrothermal coordination problem where the objective function is to minimize the expected operation cost subject to satisfy the demand and all the technical constraints of the generators. The existence of uncertain parameters makes it necessary to formulate a multi-stage stochastic problem as explained hereafter.

5.3.1. Modeling the Uncertainty Using a Stochastic Tree

Many of the parameters used to model the generation system can be considered as known input data such as the nominal technical characteristics of the generating units (maximum power, input-output curves, etc.) or the initial value of the hydraulic reserves. However, there are many other parameters which are subject to uncertainty. Among them, the ones linked to the meteorology are crucial. On the one hand, the amount of rain or snow affects the level of hydroelectric energy stored in the reservoirs. In fact, the possible scenarios of natural hydraulic inflows constitute one of the most significant concerns when planning the medium term operation of the system. Other examples are the wind speed that determines the power that can be produced in wind farms, or solar radiation for electrical production in photovoltaic or thermo-solar installations. On the other hand, meteorological factors also affect the electricity demand, which also depends on the economic activity and working patterns. Finally, the fuel prices of conventional thermal generators play a key role when determining the optimal dispatch of the generators, and the fluctuations in the coal and natural gas markets are also important sources of uncertainty.

A common approach for taking into account this uncertainty within an optimization model is to adopt a discrete representation of the probability distribution of all the random parameters in the form of a multistage scenario tree that considers the non-anticipative criterion of the decisions, (see Birge & Louveaux (1997)). Figure 5.1 shows an example of a multistage scenario tree that consists of N nodes.

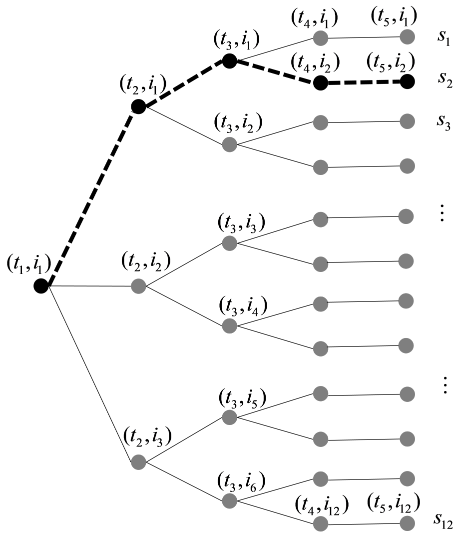


Figure 5.1.: Example of stochastic tree representation used to model the uncertainty

Each one the nodes is denoted by (t, i) where t is the time stage and i is the position of the node at that time period. Given that every node as a single predecessor, it is common to use the term '*father node*' is indicated by $F(t, i)$. For instance $F(t_3, i_2) = (t_2, i_1)$. In addition, the *descendants* of each node is denoted by $D(t, i)$. For example $D(t_2, i_2) = \{(t_3, i_3), (t_3, i_4)\}$.

Notice that the first node, i.e., the root node denoted as (t_1, i_1), is unique and it represents a *here and now* decision node as the value of the decision variables corresponding to it are the same for all the scenarios. A typical example of this kind of decisions is the problem of determining the level of the reservoirs at the end of the first time-period knowing that the future natural inflows are uncertain. The posterior decision variables are *recourse functions* as they can be adapted as far as the uncertainty is being unveiled. All the branches can be characterized by a certain probability of occurrence, which leads to a total probability of occurrence of each scenario P_{sc} that satisfies that $\sum_{sc \in S} P_{sc} = 1$. Notice that each scenario sc is made of a whole set of nodes starting at the root node, and finishing in a terminal node, i.e., a node without any descendant. The set of nodes that belong to a given scenario sc is denoted as N_{sc}. For instance, the nodes that correspond to scenario s_2 have been highlighted in the figure. In addition, every node can also be characterized by a probability of occurrence denoted by $P_{t,i}$.

5.3.2. Hydroelectric Generation

The power generated by a hydro unit depends on the water flow impacting the blades of the turbine and the net head, i.e., the difference between the elevation of the stored water and the elevation of the turbine drain, minus energy losses within the pipeline. In addition, the output power also depends on the efficiency of the turbine, the drive system and the generator. In order to model these relationships it is common to use a family of input–output curves relating the water flow and the output power for each possible net-head value (Wood & Wollenberg, 1984). These input–output curves are very common in short-term hydro scheduling models (Conejo *et al.*, 2002) where the hourly generation of the hydro plants needs to be carefully modeled. However, in medium-term models it is usual to aggregate the production of many hours into a representative time period (for instance, all the peak hours of the working days of a given week). In this case, instead of the instantaneous relationship between power and flow rate, it is more relevant to model the ratio between the energy produced during such aggregated time-periods and the total volume of water released through the hydro turbine. This ratio is named the *energy coefficient*, and for a given hydro plant h, it is denoted as E^h. Instead of assuming a static representation (as in Philpott *et al.* (2016)), its dependency on the stored volume of water is considered given that the net head changes with respect to the volume of water stored accordingly to the shape of the pond.

5.3.3. Mathematical Formulation of the Centralized Model

The objective of a centralized planner is to find the operation of the system that minimizes the expected cost while satisfying the demand balance equation and all the technical constraints of the system. The objective function can be formulated as:

$$\min \sum_{(t,i)\in N} (P_{t,i} \cdot L_{t,i} \cdot \sum_{j\in J} C^j_{t,i}(g^j_{t,i})) \tag{5.1}$$

Notice that in (5.1), the costs of the thermal generators $j \in J$ at each node are multiplied by the probability $P_{t,i}$ and the duration $L_{t,i}$ of the corresponding node.

In addition, the per-hour cost functions of each generator $C^j_{t,i}(\cdot)$ depend on the power produced by them $g^j_{t,i}$, and they can be different for each node in order to allow different fuel-price scenarios.

The optimization problem is subject to the following set of constraints:

$$\sum_{j\in J} g^j_{t,i} + \sum_{h\in H} g^h_{t,i} + \sum_{r\in R} R^r_{t,i} = DEM_{t,i} \perp \lambda_{t,i} \ , \quad \forall (t,i) \in N \tag{5.2a}$$

$$v^h_{t,i} = v^h_{F(t,i)} + (I^h_{t,i} - f^h_{t,i} - s^h_{t,i}) \cdot L_{t,i} \perp \gamma^h_{t,i} \ , \forall h \in H, \forall (t,i) \in N \tag{5.2b}$$

$$g^h_{t,i} = f^h_{t,i} \cdot E^h(v^h_{t,i}) \perp \theta^h_{t,i} \ , \quad \forall h \in H, \forall (t,i) \in N \tag{5.2c}$$

$$\underline{F}^h_{t,i} \leq f^h_{t,i} \leq \overline{F}^h \perp \underline{\eta}^h_{t,i}, \bar{\eta}^h_{t,i} \ , \forall h \in H, \forall (t,i) \in N \tag{5.2d}$$

$$0 \leq g^j_{t,i} \leq \bar{G}^j \perp \underline{\eta}^j_{t,i}, \overline{\eta}^j_{t,i} \ , \forall j \in J, \forall (t,i) \in N \tag{5.2e}$$

$$\underline{V}^h \leq v^h_{t,i} \leq \overline{V}^h \perp \underline{\kappa}^h_{t,i}, \overline{\kappa}^h_{t,i} \ , \forall h \in H, \forall (t,i) \in \overline{N}_T \tag{5.2f}$$

$$0 \leq s^h_{t,i} \perp \psi^h_{t,i} \ , \forall h \in H_m, \forall (t,i) \in N \tag{5.2g}$$

Equation (5.2a) establishes that the sum of the production of all the thermal generators ($j \in J$) plus the sum of all hydroelectric units ($h \in H$) plus the generation of renewable energy sources ($r \in R$) must satisfy the demand $DEM_{t,i}$ at every node. Notice that for each constraint, its corresponding Lagrange multiplier is shown after the symbol $\perp$ that indicates complementarity. The Lagrange multiplier of (5.2a) is represented by $\lambda_{t,i}$ and it measures what would be the impact on the objective function if the demand in this particular node increases one unit.

Equation (5.2b) establishes the water balance equation: the volume $v^h_{t,i}$ of stored water at the reservoir of hydro unit h at the end of the time stage that corresponds to the node (t,i) is equal to the volume of water at the end of the previous period defined by the father node, $v^h_{F(t,i)}$, plus the amount of water that corresponds to the natural inflows $I_{t,i}^{h}$, minus the amount of water due to the water flow released to generate hydro power, $f_{t,i}^{h}$, minus the possible spillages, $s_{t,i}^{h}$. In the particular case of the root-node, the volume of the father node is the initial volume, V^{h}, which is input data. In addition, for the particular case of the terminal nodes, N_T, the volume stored is a variable of the problem but the predefined target value at the end of the planning horizon, i.e., $v_{t,i}^{h} = V_{f}^{h}, \forall h \in H, \forall (t,i) \in N_T$.

The relationship between the water flow discharged from the hydro turbine, $f_{t,i}^{h}$, and the output power generated, $g_{t,i}^{h}$, is expressed in (5.2c) where $E^{h}(v_{t,i}^{h})$ is the energy coefficient that depends on the volume of water stored at the reservoir. The water flow limits of hydro units are established in (5.2d) to model the physical limits of the intake of the turbine or any other water right such as minimum ecological flows. Notice that the generation limits of the hydro plant are a result of the joint consideration of Constraints (5.2c) and (5.2d). However, in case the electric generator had a more restrictive power limit, it would be possible to add a new constraint imposing such limit. The maximum and minimum limits for thermal generators are taken into account in (5.2e). Notice that, for the sake of simplicity, the existence of minimum stable loads for thermal generators has not been considered as this would require the usage of binary variables. As the presented model is intended to illustrate the impact of risk aversion levels on the operation of the hydro reservoirs in the medium term, all the issues related to a detailed modeling of thermal generators and their intertemporal constraints (such as ramps), have been neglected. The limits of the reservoirs are included in (5.2f) which are formulated for all the nodes of the tree except for the terminal nodes, given that as it was mentioned before, at the last stage the volumes are fixed to the target level which is supposed to be feasible without any loss of generality. Finally, the non-negativity constraint of spillages is formulated in (5.2g).

5.4. Market Equilibrium Model with Risk-Averse Agents

5.4.1. Market Equilibrium Concept with Risk Aversion Agents

One of the consequences of the deregulation of the electric power industry in the late 90s and the implementation of wholesale electricity markets in many countries around the world is that traditional operation and planning models had to be adapted to represent the competition of market participants. Among the variety of possible techniques, equilibrium models emerged as a solid framework to model the new situation (Ventosa *et al.*, 2005). The basic idea is to find the Nash Equilibrium (NE) of rational agents that are competing in a market where their payoff functions are interdependent. Let $u_m(s_m, s_{-m})$ be the utility function of market participant m that depends on its own strategies, s_m, and on the ones of its competitors, s_{-m}. The NE solution (s^*_m, s^*_{-m}) satisfies that no one is incentivized to deviate unilaterally from that solution, i.e., $\forall m \quad u_m(s^*_m, s^*_{-m}) \geq u_m(s_m, s^*_{-m}) \forall s_m \in S_m$, where it has been taken into account that the selected strategies must belong to the set of feasible ones, S_m . This is equivalent to formulate the simultaneous maximization of the utility functions of all the market participants, taking into account the interdependency of their actions. Therefore, the NE has the form of MOPEC (multiple optimization problems with equilibrium constraints). In the case of a set of M generation companies that compete to supply the demand, the NE could be formulated as

$$\begin{aligned} &s^*_m \in argmax u_m(s_m, s_{-m}, \pi) \ , \quad \forall m \in M \\ &0 \leq H(s_1, ...s_m, .., s_M) \perp \pi \geq 0 \end{aligned} \tag{5.3}$$

where $H(s_1, ...s_m, .., s_M) \perp \pi$ represents the spot market clearing where quantities are traded at price π (see Ferris *et al.* (2009)).

In an electricity market it is common to assume that the utility function of generation companies is the obtained profit, i.e., the difference between market incomes and the generation costs. However, given the inherent uncertainties of the power system, it is necessary to define such utility taking into account that market profits depend on a stochastic process. Let us assume that all market participants share the same information about such stochastic process, where $\mathbb{P}$ is the probability distribution that is common knowledge. Let $\mathbf{x}^m$ be a vector whose components are the profits obtained by agent m for all the finite scenarios used in the stochastic tree representation. One possible way to define the utility function of the market participant is to compute the expected value of the obtained profits, $\mathbf{E}_{\mathbb{P}}(\mathbf{x}^m)$. However, for a risk-averse agent, it is possible to define its utility in terms of a coherent risk measure such as the $CVaR$. The linear combination of the expected profit and the $CVaR$ is also a coherent risk measure, and therefore the objective function of a risk averse agent can be defined in general terms as:

$$(1 - \mu^m)\mathbf{E}_{\mathbb{P}}(\mathbf{x}^m) + \mu^m \cdot CVaR^m \tag{5.4}$$

Notice that in (5.4), the parameter μ^m satisfies $0 \leq \mu^m \leq 1$ and it represents the risk-aversion level of the agent. The particular case of $\mu^m = 0$ is equivalent to the risk-neutral setting. In the following section, the change of the market equilibrium solution is analyzed with respect to the values of μ^m.

5.4.2. Mathematical Formulation of the Market Equilibrium Model

The market equilibrium solution can be obtained by formulating the simultaneous optimization of the utility function of all the market participants, subject to the market clearing constraints where sellers and buyers agree to trade electricity at a given price. According to what has been explained in section 5.4.1, the market equilibrium can be formulated by (5.5a)–(5.5k) (which are replicated for every agent) plus the market clearing formulated in (5.6):

For all agents $m \in M$

$$\max(1-\mu^m)\sum_{sc\in S} P_{sc}\cdot x_{sc}^m + \mu^m \cdot CVaR^m \tag{5.5a}$$

s.t.

$$\begin{aligned} x_{sc}^m = & \sum_{(t,i)\in N_{sc}} L_{t,i}\cdot(\pi_{t,i}\cdot(\sum_{j\in J_m} g_{t,i}^j + \sum_{h\in H_m} g_{t,i}^h + \sum_{r\in R_m} R_{t,i}^r) \\ & - \sum_{j\in J_m} C_{t,i}^j(g_{t,i}^j)))\perp\chi_{sc}^m \ ,\forall sc\in S \end{aligned} \tag{5.5b}$$

$$v_{t,i}^h = v_{F(t,i)}^h + (I_{t,i}^h - f_{t,i}^h - s_{t,i}^h)\cdot L_{t,i}\perp\gamma_{t,i}^h \ ,\forall h\in H_m, \forall(t,i)\in N \tag{5.5c}$$

$$g_{t,i}^h = f_{t,i}^h\cdot E^h(v_{t,i}^h)\perp\theta_{t,i}^h \ , \quad \forall h\in H_m, \forall(t,i)\in N \tag{5.5d}$$

$$\underline{F}_{t,i}^h \le f_{t,i}^h \le \overline{F}^h\perp\underline{\eta}_{t,i}^h, \overline{\eta}_{t,i}^h \ ,\forall h\in H_m, \forall(t,i)\in N \tag{5.5e}$$

$$0 \le g_{t,i}^j \le \overline{G}^j\perp\underline{\eta}_{t,i}^j, \overline{\eta}_{t,i}^j \ ,\forall j\in J_m, \forall(t,i)\in N \tag{5.5f}$$

$$\underline{V}^h \le v_{t,i}^h \le \overline{V}^h\perp\underline{\kappa}_{t,i}^h, \overline{\kappa}_{t,i}^h \ ,\forall h\in H_m, \forall(t,i)\in \overline{N}_T \tag{5.5g}$$

$$0 \leq s^{h}_{t,i} \perp \psi^{h}_{t,i} \ \ , \forall h \in H_m, \forall (t,i) \in N \tag{5.5h}$$

$$\zeta^m - x^m_{sc} \leq \eta^m_{sc} \perp \nu^m_{sc} \ \ , \forall sc \in S \tag{5.5i}$$

$$CVaR^m = \zeta^m - \frac{\sum\limits_{\forall sc \in S} P_{sc} \cdot \eta^m_{sc}}{\beta^m} \perp o^m \tag{5.5j}$$

$$-\eta^m_{sc} \leq 0 \perp \delta^m_{sc} \ \ , \ \forall sc \in S \tag{5.5k}$$

Equation (5.5a) establishes the objective function of each agent. Notice that in order to compute the expected profit, all the market participants share the same probability functions. The profit of market participant m in scenario sc as the sum of the market incomes minus the generation costs is defined in (5.5b) (only thermal). Market incomes are computed by adding all the thermal, hydro, and RES generation that belong to such market agents, remunerated at the market price $\pi_{t,i}$ for all the nodes that comprise the scenario and taking into account the probability and the duration of each node. Equations (5.5c)–(5.5h) are analogous to the (5.2b)–(5.2g) as explained in section 5.3.3, with the only difference that they are applied only to the generation units that belong to their corresponding market participant m. The set of linear constraints (5.5i)–(5.5k) that allow to compute the $CVaR$ where ζ^m represents the Value at Risk for agent m, and η^m_{sc} is an auxiliary variable to account only for the positive values of the difference between the ζ^m and the profit of each scenario, x^m_{sc}.

The previous equations must be replicated for all market participants and complemented with the spot-market clearing where all the market participants participate selling the production of their thermal, hydro, and renewable generators at price $\pi_{t,i}$:

$$\sum_{m \in M} \Big(\sum_{j \in J_m} g^{j}_{t,i} + \sum_{h \in H_m} g^{h}_{t,i} + \sum_{r \in R_m} R^{r}_{t,i} \Big) = DEM_{t,i} \perp \pi_{t,i} \ \ , \quad \forall (t,i) \in N \tag{5.6}$$

5.5. Relationship between the Centralized and the Market Equilibrium Models

The objective of a centralized planner in charge of planning the operation of the electric power system is to maximize the total welfare of producers and consumers. For the case of an inelastic demand, this welfare maximization is equivalent to minimizing the total operational cost while satisfying the demand at every time stage as presented in section 5.3.3. The idea that the same solution can be obtained by a market mechanism relies on Adam Smith's "invisible hand" hypothesis that states that an efficient allocation of resources can be achieved by competitive markets, supported by the welfare theorems of microeconomics. In this sense, regulatory bodies in charge of designing the electricity market mechanisms aim to ensure that market functioning maximizes the total welfare of producers and consumers in the same manner as in an hypothetical perfect centralized planning. However, there are several reasons why market results can deviate from the theoretical social optimum. The most obvious one is the possible exercise of market power due to a strategic behavior of generation companies that might not reflect the true generation costs in their offers. However, even in the absence of such strategic behavior, the existence of uncertainty can lead also to a market inefficiency as demonstrated in Philpott *et al.* (2016); Rodilla *et al.* (2015). In both works, it is proved that risk trading (for instance, by signing forward markets with risk-neutral agents such as the aggregated demand), can lead again to the optimum centralized solution even in case of risk-averse agents.

To study the possible equivalence between the centralized optimum planning and the market equilibrium in the context of the multi-stage stochastic framework presented previously, the Karush–Kuhn–Tucker (KKT) for both settings is derived and compared in the following section.

5.5.1. Optimality Conditions of the Centralized Model

The first step to derive the KKT conditions of the centralized problem (5.1)–(5.2g) is to build the Lagrangian function as shown in (5.7):

$$
\begin{aligned}
\mathcal{L} \; = \; & \sum_{(t,i)\in N} (P_{t,i} \cdot L_{t,i} \cdot \sum_{j\in J} C^j_{t,i}(g^j_{t,i})) \\
& + \sum_{(t,i)\in N} \lambda_{t,i} \cdot (DEM_{t,i} - \sum_{j\in J} g^j_{t,i} - \sum_{h\in H} g^h_{t,i} - \sum_{r\in R} R^r_{t,i}) \\
& + \sum_{h\in H} (\sum_{(t,i)\in N} \gamma^h_{t,i} \cdot (v^h_{t,i} - v^h_{F(t,i)} - (I^h_{t,i} - f^h_{t,i} - s^h_{t,i}) \cdot L_{t,i})) \\
& + \sum_{h\in H} (\sum_{(t,i)\in N} \theta^h_{t,i} \cdot (g^h_{t,i} - f^h_{t,i} \cdot E^h(v^h_{t,i}))) \\
& + \sum_{h\in H} (\sum_{(t,i)\in N} \underline{\eta}^h_{t,i} \cdot (\underline{F}^h_{t,i} - f^h_{t,i}) + \bar{\eta}^h_{t,i}(f^h_{t,i} - \bar{F}^h))) \\
& + \sum_{j\in J} (\sum_{(t,i)\in N} \underline{\eta}^j_{t,i} \cdot (-g^j_{t,i}) + \bar{\eta}^j_{t,i}(g^j_{t,i} - \bar{G}^j))) \\
& + \sum_{h\in H_m} (\sum_{(t,i)\in \bar{N}_T} \underline{\kappa}^h_{t,i}(\underline{V}^h - v^h_{t,i})) + \sum_{h\in H_m} (\sum_{(t,i)\in \bar{N}_T} \bar{\kappa}^h_{t,i}(v^h_{t,i} - \bar{V}^h)) \\
& + \sum_{h\in H} (\sum_{(t,i)\in N} \psi^h_{t,i} \cdot (-s^h_{t,i})
\end{aligned}
\tag{5.7}
$$

Equaling to zero the first derivative of the Lagrangian with respect to the primal variables allows us to write the first set of the KKT optimality conditions:

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial g^j_{t,i}} = P_{t,i} \cdot L_{t,i} \frac{dC^j_{t,i}(g^j_{t,i})}{dg^j_{t,i}} - \lambda_{t,i} - \underline{\eta}^j_{t,i} + \bar{\eta}^j_{t,i} = 0 \;\; , \forall j \in J, \forall (t,i) \in N \tag{5.8a}
$$

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial g^h_{t,i}} = -\lambda_{t,i} + \theta^h_{t,i} = 0 \;\; , \forall h \in H, \forall (t,i) \in N \tag{5.8b}
$$

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial f^h_{t,i}} = \gamma^h_{t,i} \cdot L_{t,i} - \theta^h_{t,i} \cdot E^h(v^h_{t,i}) - \underline{\eta}^h_{t,i} + \bar{\eta}^h_{t,i} = 0 \;\; , \quad \forall h \in H, \forall (t,i) \in N \tag{5.8c}
$$

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial v^h_{t,i}} = \gamma^h_{t,i} - \sum_{n\in D(t,i)} \gamma^h_n - \theta^h_{t,i} \cdot f^h_{t,i} \frac{dE^h(v^h_{t,i})}{dv^h_{t,i}} - \underline{\kappa}^h_{t,i} + \bar{\kappa}^h_{t,i} = 0 \;\; , \forall h \in H, \forall (t,i) \in \bar{N}_T \tag{5.8d}
$$

$$
\frac{\partial \mathcal{L}(\cdot)}{\partial s^h_{t,i}} = \gamma^h_{t,i} \cdot L_{t,i} - \psi^h_{t,i} = 0 \;\; , \forall h \in H, \forall (t,i) \in N \tag{5.8e}
$$

The rest of KKT have been omitted for the sake of simplicity (equality constraints, inequality constraints, and slackness complementarity conditions).

A well-known result, which is in the core of the marginalist theory, is that in case the thermal generator is operating within its limits (i.e., $\underline{\eta}^j_{t,i} = \overline{\eta}^j_{t,i} = 0$, by (5.8a) if follows that the dual variable of the demand balance constraint is the marginal cost of that generator (affected in this case by the duration and the probability due to the stochastic formulation with not-hourly time periods)

$$\lambda_{t,i} = P_{t,i} \cdot L_{t,i} \frac{dC^j_{t,i}(g^j_{t,i})}{dg^j_{t,i}} \tag{5.9}$$

Another interesting remark is related to the interpretation of the Lagrange multiplier $\gamma^h_{t,i}$, that represents the marginal water value as it measures the savings in the objective function if one extra unit of natural inflows, $I^h_{t,i}$, were available in that period. In case of a deterministic formulation (a single scenario where every node has a single descendant), if the reservoir is being operated within its limits, i.e., $\underline{\kappa}^h_{t,i} = \overline{\kappa}^h_{t,i} = 0$, and if the net head effect could be neglected, i.e., $dE^h(v^h_{t,i})/dv^h_{t,i} = 0$, then the marginal water value in both time periods (the node and its descendant) would be the same.

5.5.2. Optimality Conditions of the Market Equilibrium Model

The Lagrangian function of the optimization problem (5.5a)–(5.5k) solved by a particular agent m is:

$$
\begin{aligned}
\mathcal{L}^m \quad &= -(1-\mu^m) \sum_{sc \in S} P_{sc} \cdot x_{sc}^m - \mu^m \cdot CVaR^m \\
&+ \sum_{sc \in S} \chi_{sc}^m \cdot (x_{sc}^m - \sum_{(t,i) \in N_{sc}} L_{t,i} \\
&\cdot (\pi_{t,i} \cdot (\sum_{j \in J_m} g_{t,i}^j + \sum_{h \in H_m} g_{t,i}^h + \sum_{r \in R_m} R_{t,i}^r) - \sum_{j \in J_m} C_{t,i}^j(g_{t,i}^j))) \\
&+ \sum_{h \in H_m} (\sum_{(t,i) \in N} \gamma_{t,i}^h \cdot (v_{t,i}^h - v_{F(t,i)}^h - (I_{t,i}^h - f_{t,i}^h - s_{t,i}^h) \cdot L_{t,i})) \\
&+ \sum_{h \in H_m} (\sum_{(t,i) \in N} \theta_{t,i}^h \cdot (g_{t,i}^h - f_{t,i}^h \cdot E^h(v_{t,i}^h))) \\
&+ \sum_{h \in H_m} (\sum_{(t,i) \in N} \underline{\eta}_{t,i}^h \cdot (\underline{F}_{t,i}^h - f_{t,i}^h) + \bar{\eta}_{t,i}^h (f_{t,i}^h - \bar{F}^h))) \\
&+ \sum_{j \in J_m} (\sum_{(t,i) \in N} \underline{\eta}_{t,i}^j \cdot (-g_{t,i}^j) + \bar{\eta}_{t,i}^j (g_{t,i}^j - \bar{G}^j))) \\
&+ \sum_{h \in H_m} (\sum_{(t,i) \in \bar{N}_T} \underline{\kappa}_{t,i}^h (\underline{V}^h - v_{t,i}^h)) + \sum_{h \in H_m} (\sum_{(t,i) \in \bar{N}_T} \bar{\kappa}_{t,i}^h (v_{t,i}^h - \bar{V}^h)) \\
&+ \sum_{h \in H_m} (\sum_{(t,i) \in N} \psi_{t,i}^h \cdot (-s_{t,i}^h) \\
&+ \sum_{sc \in S} \nu_{sc}^m \cdot (\zeta^m - x_{sc}^m - \eta_{sc}^m) \\
&+ o^m \cdot (CVaR^m - \zeta^m + \tfrac{1}{\beta^m} \sum_{\forall sc \in S} P_{sc} \cdot \eta_{sc}^m \ ) \\
&+ \sum_{sc \in S} \delta_{sc}^m \cdot (-\eta_{sc}^m)
\end{aligned}
\tag{5.10}
$$

The first order conditions for market participant m, can be formulated as:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}^m(\cdot)}{\partial g_{t,i}^j} = -(\sum_{sc \in \Omega_{t,i}} \chi_{sc}^m) \cdot L_{t,i} \cdot \pi_{t,i} \\
&+ (\sum_{sc \in \Omega_{t,i}} \chi_{sc}^m) \cdot L_{t,i} \frac{dC_{t,i}^j(g_{t,i}^j)}{dg_{t,i}^j} - \underline{\eta}_{t,i}^j + \bar{\eta}_{t,i}^j = 0 \ \ , \forall j \in J_m, \forall (t,i) \in N
\end{aligned}
\tag{5.11a}
$$

$$
\frac{\partial \mathcal{L}^m(\cdot)}{\partial g_{t,i}^h} = -(\sum_{sc \in \Omega_{t,i}} \chi_{sc}^m) \cdot L_{t,i} \cdot \pi_{t,i} + \theta_{t,i}^h = 0 \ \ , \forall h \in H_m, \forall (t,i) \in N \tag{5.11b}
$$

$$
\frac{\partial \mathcal{L}^m(\cdot)}{\partial f_{t,i}^h} = \gamma_{t,i}^h \cdot L_{t,i} - \theta_{t,i}^h \cdot E^h(v_{t,i}^h) - \underline{\eta}_{t,i}^h + \bar{\eta}_{t,i}^h = 0 \ \ , \quad \forall h \in H_m, \forall (t,i) \in N \tag{5.11c}
$$

$$\frac{\partial \mathcal{L}^m(\cdot)}{\partial v^h_{t,i}} = \gamma^h_{t,i} - \sum_{n \in D(t,i)} \gamma^h_n \\ - \theta^h_{t,i} \cdot f^h_{t,i} \frac{dE^h(v^h_{t,i})}{dv^h_{t,i}} - \underline{\kappa}^h_{t,i} + \overline{\kappa}^h_{t,i} = 0 \ \ , \forall h \in H_m, \forall (t,i) \in \overline{N}_T \tag{5.11d}$$

$$\frac{\partial \mathcal{L}^m(\cdot)}{\partial s^h_{t,i}} = \gamma^h_{t,i} \cdot L_{t,i} - \psi^h_{t,i} = 0 \ \ , \forall h \in H_m, \forall (t,i) \in N \tag{5.11e}$$

$$\frac{\partial \mathcal{L}^m(\cdot)}{\partial x^m_{sc}} = -(1 - \mu^m) \cdot P_{sc} + \chi^m_{sc} - \nu^m_{sc} = 0 \ \ , \forall s \in S \tag{5.11f}$$

$$\frac{\partial \mathcal{L}^m(\cdot)}{\partial CVaR^m} = -\mu^m + o^m = 0 \tag{5.11g}$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \zeta^m} = \sum_{sc \in S} \nu^m_{sc} - o^m = 0 \tag{5.11h}$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \eta^m_{sc}} = -\nu^m_{sc} + o^m \frac{P_{sc}}{\beta^m} - \delta^m_{sc} = 0 \tag{5.11i}$$

5.5.3. Impact of Risk Aversion Level

This subsection analyses the impact on the equilibrium solution of the risk-aversion level of market participants.

To start with, let assume that all market participants are risk-neutral. In this case, $\mu^m = 0, \forall m \in M$. By, it follows that $o^m = 0, \forall m \in M$, and by (5.11h) it can be written that $\sum_{sc \in S} \nu^m_{sc} = 0$. As ν^m_{sc} are non-negative multipliers, it follows that $\nu^m_{sc} = 0, \forall sc \in S$. In this case, (5.11f) can be rewritten as:

$$\chi^m_{sc} = P_{sc} \ \ , \forall s \in S \tag{5.12}$$

The interpretation of (5.12) is that in the risk-neutral equilibrium, the Lagrange multipliers of (5.5b), formulated for all the market participants for every scenario, must coincide with the probability of that scenario. Then, (5.11a) and (5.11b) become:

$$\frac{\partial \mathcal{L}^m(\cdot)}{\partial g^j_{t,i}} = -(\sum_{sc \in \Omega_{t,i}} P_{sc}) \cdot L_{t,i} \cdot \pi_{t,i}$$
$$+ (\sum_{sc \in \Omega_{t,i}} P_{sc}) \cdot L_{t,i} \frac{dC^j_{t,i}(g^j_{t,i})}{dg^j_{t,i}} - \underline{\eta}^j_{t,i} + \bar{\eta}^j_{t,i} = 0 \ \ , \forall j \in J_m, \forall (t,i) \in N \tag{5.13}$$

Given that the probability of one node can be computed as the sum of the probabilities of all the scenarios that cross that node in their paths from the root node until the terminal nodes, it can be written:

$$\sum_{sc \in \Omega_{t,i}} P_{sc} = P_{t,i} \tag{5.14}$$

In addition, if the following relationship is defined as:

$$\lambda_{t,i} = L_{t,i} \cdot \pi_{t,i} \cdot P_{t,i} \tag{5.15}$$

then replication of (5.11a) for all the market participants leads to the same ones as the defined by (5.11a). Applying the same idea to all the remaining equations, the same set of equations as the optimality conditions of the centralized problem would be obtained. Therefore, it can be concluded that the operation of the generation system that corresponds to the risk-neutral market equilibrium is exactly the same one as in the centralized optimal operation.

However, in case there is at least one risk-averse agent, i.e., $\mu^m \neq 0$, then the operation of the system could differ from the centralized one given that (5.11a) would not be satisfied. In this case, from (5.11f) it follows that:

$$\chi^m_{sc} = (1 - \mu^m) \cdot P_{sc} + \nu^m_{sc} \ \ , \forall m \in M, \forall s \in S \tag{5.16}$$

The Lagrange multipliers χ^m_{sc} can be interpreted as the risk-adjusted probabilities $\mathbb{Q}^m$ which are different for each agent. These risk-adjusted probabilities are the ones that market agents should assign to the scenarios so that the maximization of their expected profits according to them, would result in the same solution as the risk-averse operation. However, this interpretation requires a demonstration that χ^m_{sc} satisfies the requirements of a probability measure. Equation (5.16) forces χ^m_{sc}

to be positive, given the non-negativity constraint of ν_{sc}^m. In addition, summing for all scenarios both sides of (5.16), it results in the following expression:

$$\begin{aligned} \sum_{sc\in S}\chi_{sc}^m &= \sum_{sc\in S}(1-\mu^m)\cdot P_{sc} + \sum_{sc\in S}\nu_{sc}^m \ ,\forall m\in M \\ &= (1-\mu^m)\sum_{sc\in S}P_{sc} + \sum_{sc\in S}\nu_{sc}^m, \forall m\in M \end{aligned} \tag{5.17}$$

By (5.11g) and (5.11h), it follows that $\sum_{s\in S}\nu_{sc}^m = o^m = \mu^m$, and therefore it leads to the next result:

$$\sum_{sc\in S}\chi_{sc}^m = 1-\mu^m + \mu^m = 1, \forall m\in M \tag{5.18}$$

Once that it has been proven that χ_{sc}^m can be interpreted as the risk-adjusted probabilities, it is interesting to provide some additional insights. Taking the (5.11g) and (5.11i) into account, it follows that:

$$\chi_{sc}^m = (1-\mu^m)\cdot P_{sc} + \mu^m\frac{P_{sc}}{\beta^m} + \delta_{sc}^m \ ,\forall m\in M, \forall s\in S \tag{5.19}$$

Assume that the profit for a given scenario x_{sc}^m is strictly lower than the value at risk ζ^m,constraints (5.5i) and (5.5k) would be active and not-active respectively. Therefore, for that scenario, it would be satisfied that $\nu_{sc}^m \neq 0$ and $\delta_{sc}^m = 0$. Then (5.19) for this particular case would be:

$$\chi_{sc}^m = (1-\mu^m)\cdot P_{sc} + \mu^m\frac{P_{sc}}{\beta^m} \ ,\forall m\in M, \forall sc\in S \ /x_{sc}^m < \zeta^m \tag{5.20}$$

On the other hand, for a scenario where the profit exceeds ζ^m it would be satisfied that $\nu_{sc}^m = 0$. Then, by (5.16), it can be derived:

$$\chi_{sc}^m = (1-\mu^m)\cdot P_{sc} \ ,\forall m\in M, \forall s\in S \ /x_{sc}^m > \zeta^m \tag{5.21}$$

Both expressions cannot be computed ex-ante, unless a reasonable estimation of which scenarios give place to the worst profits is available.

Finally, for a given node, the risk-adjusted probabilities of each node can be computed as:

$$Q^m_{t,i} = \sum_{sc \in \Omega_{t,i}} \chi^m_{sc}$$

5.6. Results

This sections presents the obtained results of the centralized model (CEN), market equilibrium risk-neutral (RN) and the risk-averse model (RA). The main aim is to identify the impact of risk aversion on the operation of hydro reservoirs in the presence of renewable energy sources. Moreover, the mutual influence of agents' different risk aversion levels on the results is also studied.

The models presented in sections 5.3 and 5.4 are implemented in GAMS (Brooke *et al.*, 1996). Model CEN is solved via the CONOPT solver due to the nonlinearity introduced by the net-head dependency (see section 5.6.1). Models RN and RA are coded using the Extended Mathematical Programming (EMP) feature of GAMS (Ferris *et al.*, 2009), and the resulting mixed-complementary problems are solved with PATH. For solving the presented non-linear RA problem and achieving a feasible solution, a "warm start" is modeled in GAMS where the results from the CEN model are taken as a starting point (see Appendix B). Then the value of the risk aversion level (μ^m) is gradually increased in small intervals for every next run of the model until a desired μ^m value is reached.

Regarding the input data, the power system presented in the case study is derived from the Spanish mainland electricity market. To simplify the system, the data of demand, RES, and hydro and thermal capacity is scaled down in the same proportion. Nuclear energy is excluded from the generation mix. Four years of data from the Spanish Transmission System Operator "Red Eléctrica de España" (REE, 2018) is used to replicate the real system as accurate as possible from year 2013 to 2016.

5.6.1. System Description

The power system presented in this case study is a scaled-down version of the Spanish mainland electricity market (see section 5.6). In order to represent the uncertainty, a stochastic tree has been implemented where after period $t1$ (the 'here and now' decision) there are 3 new nodes in $t2$, followed by 9 nodes in $t5$, and finally 27 nodes from $t6$ to $t12$. Therefore, the uncertainty is modeled with 27 scenarios of the following input data: demand, RES, water inflow and the fuel cost. The time horizon is considered to be 12 months, where the first period ($t1$) starts in October as in Spain the hydrological year goes from October until September. Each month has its corresponding duration ($L_{t,i}$) in hours depending on the number of days. In Figure 5.2, the maximum and minimum values of the RES and demand are presented per each time period. Notice that the highest share of the RES in the demand per month is 28% ($t2$) and the lowest one is 12% ($t12$).

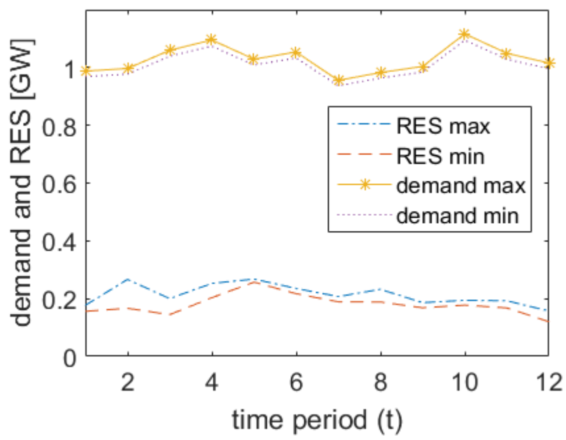


Figure 5.2.: Demand and renewable energy sources (RES) forecast.

Regarding the power generation, the system consists of four thermal units (see Table 5.1) and two hydro plants (see Table 5.2), which belong to two Gencos. The first agent Genco1 owns two thermal units ($J1$ and $J2$), and the hydro unit $H1$. The second agent Genco2 owns the other two thermal units ($J3$ and $J4$) and the second hydro unit $H1$. Both agents share a half capacity of the RES production and a half of the water inflow forecast. The thermal generation represents coal units ($J1$ and $J3$) and combined cycle gas turbines ($J2$ and $J4$). These thermal generators do not behave strategically, and their cost functions have been tuned to cover a representative range of real marginal costs. In particular, the cost function of a given thermal generator j is a quadratic polynomial which results in a linear marginal cost function:

$$C^j_{t,i}(g^j_{t,i}) = \left[C_1 g^j_{t,i} + C_2 \left(g^j_{t,i}\right)^2\right] FC_{t,i} \Rightarrow \frac{\partial C^j_{t,i}(g^j_{t,i})}{\partial g^j_{t,i}} = \left(C_1 + 2C_2 g^j_{t,i}\right) FC_{t,i}$$

For example, the marginal cost of the generator $J1$ at its maximum production is 42 €/MWh. The factor *fuel cost coefficient* allows to increase or reduce the total cost by assigning different values to $FC_{t,i}$ at each time period. Taking into account the uncertainty of the fuel prices in the time horizon of 12 months, different scenarios in Figure 5.3 represent the price volatility that agents can face. Notice that some of the 27 scenarios share the same values of $FC_{t,i}$ and this is why in the figure one can only see 13 final leaves (each scenario considers a different combination of all the uncertain parameters).

Table 5.1.: Parameter data of thermal units

Thermal Units	$\overline{G}$ (GW)	C_1 (k€/(GW · h))	C_2 (k€/(GW^2 · h))
$J1$	0.2	30	30
$J2$	0.5	45	15
$J3$	0.2	35	30
$J4$	0.5	40	15

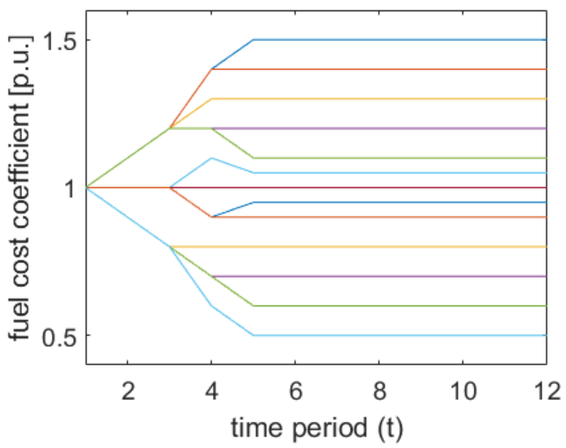


Figure 5.3.: Fuel cost coefficient scenarios, $FC_{t,i}$.

Table 5.2 shows the parameter data of the hydro units. Note that the hydro unit $H1$ has a maximum water flow two times larger than $H2$, and the hydro inflows are considered to be the same for hydro reservoirs. The uncertainty of the hydro inflows is modeled with 27 different scenarios and the maximum and the minimum ranges can be seen in Figure 5.4.

Table 5.2.: Parameter data of hydro units

Hydro Units	$\overline{F}$ (m^3/s)	$V_0 = V_f$ (hm^3)	$\underline{V}$ (hm^3)	$\overline{V}$ (hm^3)	$E^h_{\max}$ (MW/m^3/s)	$E^h_{\min}$ (MW/m^3/s)
$H1$	400	1000	500	4000	1.3	1.00
$H2$	200	800	400	3000	1.56	1.20

The energy coefficient to translate water flow into output power of the hydro unit is expressed as:

$$E^h(v^h_{t,i}) = E^h_{\min} + \frac{E^h_{\max} - E^h_{\min}}{\overline{V}^h - \underline{V}^h}\left(v^h_{t,i} - \underline{V}^h\right)$$

where it can be seen that when the volume of the reservoir is at the minimum level, the energy coefficient corresponds to the minimum one, $E^h_{\min}$, as the net head is going to be the lowest. When the reservoir is at its maximum capacity, the energy coefficient is the maximum one, $E^h_{\max}$. Any intermediate volume stored at the reservoir results in the linear interpolation between those extreme values. It is important to notice that this representation allows to take into account that with the same amount of released water, the obtained energy (and therefore market incomes) can be different.

5.6.2. CVaR parameters

Taking into account what is stated in section 4.1.4, the characterization of the level of risk of each of the market participant m requires determining the following two parameters:

- The risk weight factor $\mu^m \in [0, 1]$
- The confidence level β^m

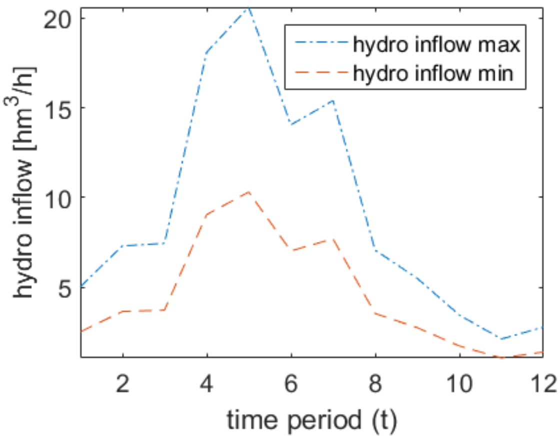


Figure 5.4.: Hydro inflow range

As the WACC might be different for each company, the financing cost of the risk fund could also be different for each market participant, i.e. $Crisk^m$. For the sake of simplicity, assume that all the companies share the same WACC value (for instance 18% per year) and that the risk-free rate is 2% per year. In this case, the annual fund costs would be:

$$Crisk^m = 18\% - 2\% = 16\% \tag{5.22}$$

As shown in section 4.1.4, the risk weight factor should satisfy:

$$\mu = \frac{Crisk}{Crisk+1} \tag{5.23}$$

Therefore, for the annual hydrothermal coordination problem a sensible value for the risk weight factor would be the following one:

$$\mu^m = \frac{Crisk^m}{Crisk^m+1} = \frac{0.16}{0.16+1} \simeq 0.138 \tag{5.24}$$

As in this chapter one of the objectives is to analyze the impact of μ^m on the equilibrium solution, a wider range of this parameter will be used in the simulations.

Regarding to the confidence level, as the total number of discrete scenarios that can be considered might be limited due to computational constraints, the minimum value of β^m is also related to the cardinal of total scenarios. In addition, if the companies do not want to access the fund more than once or twice for each period of ten years, it should be satisfied that $\beta^m \in [0.1, 0.2]$. In the practical implementation of the model, the selected value has been $\beta^m = 0.2, \forall m$.

5.6.3. Numerical Solution of the Models

In this section, the following results are presented: comparison of the centralized and risk-averse models, the impact of risk aversion on the hydro reservoir operation, and finally the expected profit and the $CVaR$ value of two market agents for different risk profiles.

As demonstrated in the section 5.5.1, both CEN and RN models lead to the same optimal results: identical operation of the thermal units, identical operation of the hydro reservoirs, identical generation costs, identical prices taking into account (5.8a), etc. Given that the amount of output variables is too large to be presented here, Figure 5.5 shows two illustrative examples: the coverage of the demand through the whole year for the two scenarios with the more extreme inflows (Scenario 1 for high water inflows, and Scenario 27 for very low ones). It is noticeable how the system utilizes differently the available hydro energy, where, for example, the hydro reservoir of the $H1$ has a level of 4000 hm^3 in Scenario 1 and a level of 2543.356 hm^3 in Scenario 27 at the end of the period $t3$. However, the operation for the time period $t1$ is exactly the same due to the non-anticipative criterion at the first stage.

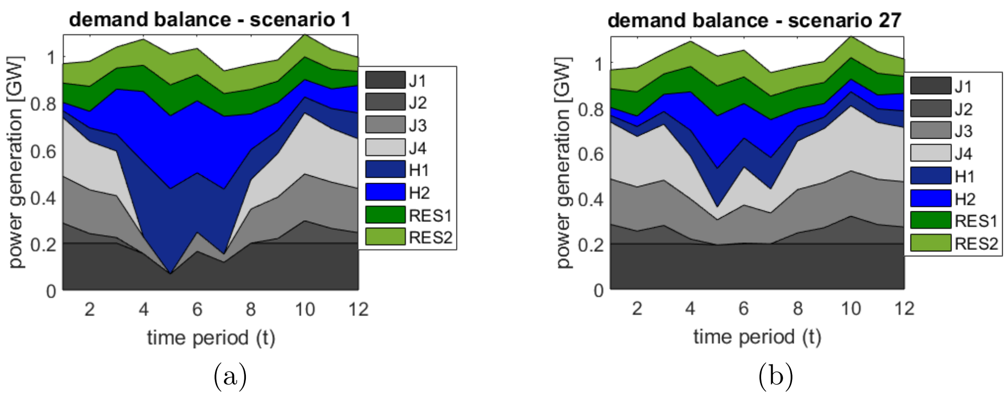


Figure 5.5.: Demand balance for Scenario 1 (a) and Scenario 27 (b) for the centralized case

Another interesting result is related with the social welfare. Summing the expected profit of the two agents from the RN model (Genco1 99,148.51 k€ and Genco2

103,892.21 k€) and adding the total cost of the operation (190,141.82 k€) provides the same 'demand payment' that would be obtained in the CEN case if the marginal cost were used to charge the consumption (393,182.54 k€).

In order to decrease the risk of the uncertain parameters, the agents may deviate from the centralized hydro management planning. This is illustrated in Table 5.3 that shows the relative difference between the hydro reservoir levels ($(v^{RA} - v^{RN})/v^{RN}$) at the end of each time period for each Genco and assuming that both agents are pure risk averse agents ($\mu = 1$ for both).

Table 5.3.: Relative change of the hydro reservoir levels between the risk neutral and risk averse cases

t1		t2		t3		t4	
v^{H1}	v^{H2}	v^{H1}	v^{H2}	v^{H1}	v^{H2}	v^{H1}	v^{H2}
						0%	0%
		-15.31%	0%	-6.34%	0%	0%	0%
						0%	0%
						0%	0.61%
-26.23%	0%	-16.60%	0%	-8.44%	0.64%	0%	0%
						0%	0%
						-18.72%	0%
		-39.57%	-12.72%	-56.14%	-24.94%	-23.58%	0%
						-20.34%	0%

The following results emphasize the agents' risk aversion influence on the hydro reservoir levels, particularly in the period $t3$ when the RES decreases by 16% and the demand increases by 6% in average, compared to the period $t2$ (see Figure 5.2). Figure 5.6 shows the relative difference between the average reservoir level in period $t3$ for the RN and the RA case of the hydro reservoirs belonging to Genco1 (a) and Genco2 (b) for different risk-aversion levels.

In the equilibrium based models, the risk aversion of the one agent affects the payoff outcome of the other agent. As an example, the case where both agents have extreme risk aversion ($\mu1 = \mu2 = 1$) is compared with the risk neutral case in Table 5.4 that shows the expected profit and the $CVaR$ value. The RN value of $CVaR$ is computed

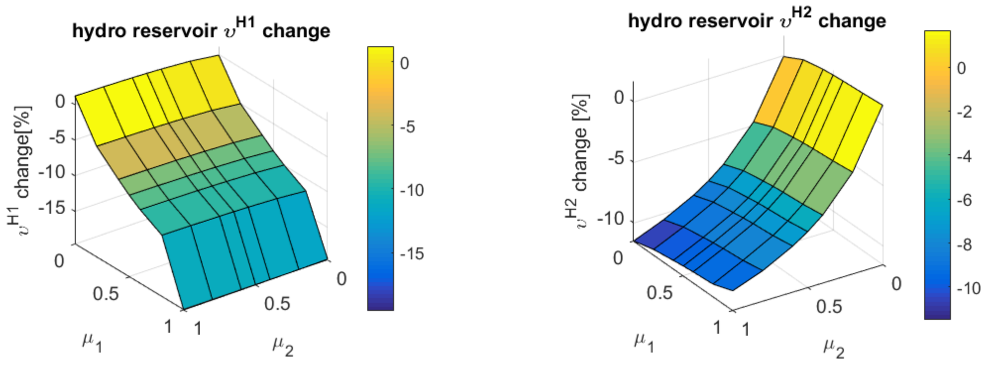


Figure 5.6.: Hydro reservoir relative change in $t3$ for Genco1 (a) and Genco2 (b).

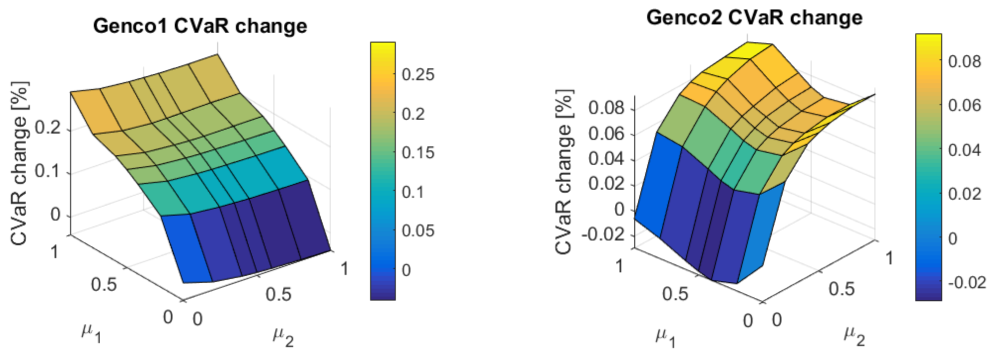
as the simple average of the β worse profit scenarios. Notice that Genco1 increases its $CVaR$ value while the expected profit decreases. At the same time, Genco2 has a higher $CVaR$ value and a higher expected profit.

Table 5.4.: Expected profit and $CVaR$ values ($\mu1 = \mu2 = 1$)

Model	Genco1		Genco2	
	$\mathbf{E}_{\mathbb{P}}(x^1)$(k €)	$CVaR^1$(k €)	$\mathbf{E}_{\mathbb{P}}(x^2)$(k €)	$CVaR^2$(k €)
RN	99,148.51	56,208.25	103,892.21	58,705.98
RA	99,025.92	56,355.53	103,974.62	58,755.82

Figure 5.7 shows the change of the $CVaR$ value for both agents when they change their risk aversion from 0 to 1. The z-axis shows the relative change of the $CVaR$ value in comparison to the RN case ($(CVaR^{RA} - CVaR^{RN})/CVaR^{RN}$). Although the changes in percentages have a low magnitude, in real terms the impact is not so negligible (for instance, a percentage of 0.05% corresponds, in this setting, to 30,000 €). Note that the Genco1 risk aversion has an interesting impact on the $CVaR$ value of the Genco2 (see Figure 5.7b).

To better understand the impact of risk-averse agents in the equilibrium, the graphical data from Figure 5.7a,b are shown in Tables 5.5 and 5.6, respectively.



(a) (b)

Figure 5.7.: $CVaR$ value relative change for Genco1 (a) and Genco2 (b)

Table 5.5.: Relative change of $CVaR$ values for Genco1

$\mu_1 \backslash\ \mu_2$	0	0.2	0.4	0.6	0.8	1
0	0%	-0.021%	-0.031%	-0.036%	-0.039%	-0.041%
0.2	0.125%	0.106%	0.099%	0.095%	0.093%	0.092%
0.4	0.168%	0.152%	0.146%	0.143%	0.141%	0.140%
0.6	0.203%	0.188%	0.182%	0.179%	0.179%	0.179%
0.8	0.223%	0.208%	0.202%	0.202%	0.203%	0.202%
1	0.291%	0.275%	0.268%	0.264%	0.262%	0.262%

Table 5.6.: Relative change of $CVaR$ values for Genco2

$\mu_1 \backslash\ \mu_2$	0	0.2	0.4	0.6	0.8	1
0	0%	0.057%	0.077%	0.084%	0.087%	0.087%
0.2	-0.023%	0.038%	0.059%	0.067%	0.070%	0.070%
0.4	-0.029%	0.033%	0.055%	0.063%	0.066%	0.067%
0.6	-0.022%	0.041%	0.063%	0.071%	0.076%	0.078%
0.8	-0.013%	0.050%	0.072%	0.084%	0.090%	0.092%
1	-0.006%	0.053%	0.073%	0.081%	0.083%	0.085%

5.7. Discussion

Given the previous results, it can be pointed out that in the equilibrium setting, risk-aversion can have a significant impact on the hydro reservoirs' operation that can deviate from the centralized planning. Notice that the relative change between the reservoir levels corresponding to the ideal centralized operation (which are the same ones as in the risk-neutral case), and the market equilibrium case where both agents are pure risk-averse players, take place since the first period. This is more relevant for the reservoir of unit $H1$ which has the highest decrease in the third branch of the stochastic tree in $t3$. It is interesting to highlight that the operation of the first month for the reservoir of $H1$ (which is a *here and now* decision), leaves the reservoir more empty (-26.23%) than in the centralized operation. The interpretation of this fact is clear: the Genco1 prefers to use the available water in the first month where the uncertainty of market prices is null in order to reduce the future risk of more volatile incomes in posterior months. Moreover, a similar analysis could be carried out to analyze the impact on the other *here and now* variables: the power generated by all the thermal and hydro units, and the water released or spilled by hydro plants in the first stage, $t1$.

Regarding the analysis of how the risk-aversion level of each agent affects the hydro operation, Figure 5.6 shows that higher risk aversion leads to a higher deviation in relative terms of the reservoir levels of both agents. In the case of Genco1, the relative difference is almost -20% and it can be noted that the risk aversion of the other agent does not have any significant impact (see Figure 5.6a). On the other side, the hydro reservoir of the Genco2 is mostly influenced by its own risk preference (decreasing by 12%), but also by the risk aversion of the Genco1 (see Figure 5.6b). The results are mainly driven by the maximum output of $H1$. Therefore, for the particular structural conditions of each electricity market, the impact for each agent can be different and a specific assessment is necessary. The methodology presented can help to carry out this kind of analysis. In any case, it is necessary to clarify that the fact that the operation of reservoirs in the case of risk-averse agents leads to greater water discharges at the reservoirs during the first periods is something that can not be generalized since these results depend on the particular data of this example case. The data used to construct the fuel price scenarios have a volatility that is growing substantially throughout the year. The objective here is not so much to reproduce real scenarios of this volatility (which is certainly not so pronounced), but to configure an example case that allows observing the effect of risk aversion in the solution of the equilibrium. Other values of the parameters subject to uncertainty would lead to a different operation of the reservoirs, and it could theoretically occur that instead of producing more hydroelectric generation in the early stages, the hydro production were delayed to final stages. The important conclusion is that the introduction of risk aversion modifies the equilibrium solution to a greater or lesser extent depending on the level of uncertainty. Despite this finding is not new as other authors have reached a similar conclusion with a more stylized model, the equilibrium model developed in this thesis has a more realistic representation of the hydro-thermal generation system that allows to study the impact of the risk-averse level of the market participants on the annual operation (twelve monthly periods) of their assets. In addition, the usage of the CVaR brings to the model all the benefits related to the use of a coherent risk measure.

Finally, it is common to assume that the risk-aversion level of each participant is something that depends only on its own characteristics. Therefore, the parameters μ_1 and μ_2 should be considered as input data and reflect the genuine values according to the individual risk attitude of each Genco. However, the results shown in Table 5.5 and Table 5.6, where the market equilibrium has been obtained for multiple combinations of μ_1 and μ_2, can motivate an alternative interpretation. If Genco1 chooses a risk-aversion level of $\mu 1 = 0.4$, taking into account that the Genco2 is risk neutral ($\mu 1 = 0$), it achieves an increase of its $CVaR$ value of +0.168% with respect to the risk neutral case. However, as Genco2 can also choose the same μ, it decreases the predicted $CVaR$ value of the Genco1 down to 0.146%. This numerical example raises the question of what would be the most expected risk-attitude of market participants in case the risk level were the strategies to be followed by the agents. In other words, if playing in the market with different risk-aversion levels than the genuine ones provides market participants with better equilibrium payoffs, one could expect that Gencos will behave accordingly to this solution. This is one of the findings of this chapter that opens the question of whether agents could hide behind alleging that they have a certain level of risk (different from what could be considered reasonable) if they obtained a better result. For instance, in the example case, the best strategy for the Genco1 would be to select a higher risk-aversion level than the Genco2. Given the values from Table 5.5 and Table 5.6, the best strategy would be $\mu = 1$ for both Gencos but based on (5.3), only when $\mu 1 = \mu 2 = 1$ Genco1 is not going to deviate unilaterally from its strategy and neither Genco2. Therefore, in case the payoff is defined as the maximum relative increment of the $CVaR$ value with respect to the centralized operation, the Nash equilibrium would correspond to the pure risk aversion profile of both agents.

6. Conclusions

6.1. Thesis summary and main conclusions

This thesis tackles the problem of optimizing the operation of a risk-averse generation company that participates in the electricity market and it studies the impact of the risk-aversion level of market participants in the equilibrium solution under a risk constrained setting. This is achieved by (i) providing an algorithm to solve a risk-constrained optimization problem, (ii) the application of the developed algorithm to two relevant short-term problems, (iii) formulating the Nash equilibrium model in the presence of risk-averse agents, and (iv) analyzing the impact of risk-averse level on the equilibrium solution.

First, a new algorithm (Iterative CVaR) is proposed (Chapter 3). The main advantage is that it transforms the mean-risk problem into a sequence of successive simpler optimization problems, where the original probabilities are substituted by new risk-adjusted ones. The convergence of the algorithm is guaranteed, as shown in this thesis, given that it can be explained in terms of the Dantzig-Wolfe decomposition of the original problem whenever the hypothesis of linearity is fulfilled. At each iteration of the algorithm, the values of such risk-adjusted probabilities are re-computed until the convergence is reached. A small and completely reproducible example has been used to present the new CVaR formulation.

Second, the Iterative CVaR algorithm has been applied successfully to two short-term risk management problems: a single thermal plant that has to find its hedging strategy subject to a high volatility of spot prices and a company owning ten thermal plants that wants to find the optimal hourly scheduling taking into account the risk of volatile market prices (Chapter 4). These applications show that the proposed Iterative CVaR can deal with real problems. In addition, it provides the risk-adjusted probabilities practical use to solve a mean-risk problem that can be cast in a risk-neutral manner. Real data taken from the Iberian electricity market has been used leading to the same results than the direct optimization case, but in a more efficient way in terms of computation time. Moreover, a systematic way of selecting the risk-weight parameter and confidence interval has been discussed. This allows to balance the improvement of the worst case scenario outcomes with a minor reduction of the expected mean value. Despite the fact that solving the non-convex versions of both problems (i.e. when considering the presence of binary variables) with the Iterative CVaR has not proven to be beneficial in terms of computational time, for this specific cases, the sub-problems of the algorithm could be object of a further decomposition by applying traditional techniques developed risk-neutral formulations.

Third, the hydro-thermal operation planning with risk-averse generation companies in the presence of RES has been analyzed by a multi-stage stochastic equilibrium model (Chapter 5). The impact on the hydro reservoir levels can be noticeable even in the early stages of the decision making process, such as the first stage of the scenario tree. Different uncertainties in the medium term (demand forecast, RES, fuel prices and hydro inflows) induce the market risk-averse agents to hedge against the risk of the worst scenarios.

And fourth, the market equilibrium solution is analyzed for different risk preference levels of the involved market agents (Chapter 5). It is highlighted that the strategic behavior of the risk-averse generation companies can diverge from the centralized planner optimum due to the lack of market mechanisms (market incompleteness). The agents' payoffs are very dependent on their own behavior, but also on the action of the other agents. In order to define their own risk preference for achieving a certain payoff outcome, the agents need to make an estimation of the competitors' risk-averse level as a part of their strategies. This suggest to define the risk level as the strategic decision in a game theory framework, where agents choose the risk level that corresponds to the Nash equilibrium.

The main conclusions that can be drawn from this research are the following ones:

- It is possible to apply an Iterative CVaR algorithm where risk constraints that model the CVaR can be eliminated, solving at each stage a "risk-neutral" type optimization where the true scenario probabilities are substituted by the risk-adjusted ones. For the linear setting, this thesis provides the practical proof that the iterative algorithm converges and that both the primal and dual variables of the problem can be obtained.
- The application of the Iterative CVaR can be beneficial in comparison to the direct optimization. In fact, during the elaboration of this thesis it has been found problems where the Iterative CVaR outperforms the direct optimization,

while in other examples this was not the case. Therefore a specific analysis has to be carried out for each specific problem.

- As the risk constraints that tangle all scenarios are removed at each iteration, it would be possible to apply standard decomposition techniques in the sub-problem.
- This thesis provide a guidance to select the parameters that define the mean-risk problem: the relative weight in the optimization problem of the expected profit (or loss) and the CVaR, and the confidence interval that defines the worst scenarios.
- Despite that convergence has only been proved for the linear case, the practical application to non-convex problem has been positive.
- The presence of risk-averse agents changes the Nash equilibrium solution even in the case the market prices represent the thermal marginal cost. Therefore, it is necessary to assure a liquid risk market in order to help the agent to hedge their risk.
- As the risk level is a subjective decision of each agent, the analysis of how the outcomes of each agent changes for different coefficients of risk, makes it possible to define the game in terms of these strategies.

6.2. Contributions

The main contributions of this thesis are the following:

1. Decomposition of the mean-risk problem - The mean-risk problem has been successfully decomposed by implementing the Lagrangian Relaxation (LR) and Benders decomposition algorithms and a general equivalence between them is established in terms of their Master and Sub-problem mathematical formulations. Both algorithms provide the same solution as the direct optimization. It has been shown that the LR decomposition allows to decouple the problem scenarios being an advantage in comparison to the Benders decomposition.
2. New algorithm for CVaR modeling - The two stage algorithm, called Iterative CVaR, based on the LR and Dantzig-Wolfe decomposition techniques, is developed providing a good computational tractability for solving the mean-risk problems. It has been numerically proven that the algorithm converges in a

linear setting. Two methods to compute the primal variables are proposed once the convergence has been reached in the LR algorithm.

3. The application of Iterative CVaR on the real example cases in the electricity market - Two most common problems in a short-term are identified and the proposed Iterative CVaR algorithm is successfully applied. It is shown that the improved computational efficiency for large-scale scenario stochastic optimization problems can be achieved. These results have been published in the paper Jovanović *et al.* (2017). Furthermore, a novel method, up to the authors knowledge, for constructing a forward offer curves is presented providing an insight on the quantities to be submitted for a given future market price and risk-averse preference.

4. The guidance to select the risk associated with the CVaR formulation. This discussion is one of the contributions of this thesis, and it makes the CVaR consistent with the general rational of risk measures as used by the financial industry when setting the amount of liquids funds (e.g. Treasury bonds or other very low-risk/high liquidity assets) to be protected against very bad market outcomes. This way, the value of the risk measure can be thought as a quantity of money that is under risk, and therefore the company could be hedged by depositing that quantity in a very liquid fund. This way, if market results are good, the company does not need to make use of that fund, incurring just the corresponding opportunity cost. However, if market results are very bad, company could make use of the available fund, which should be replenished as soon as possible. A brief description of this approach has been also published in the Jovanović *et al.* (2017).

5. The risk averse equilibrium model which allows to consider a more realistic representation of the generation system than previous research works - The Extended Mathematical Programming approach is used to formulate the equilibrium problem. Instead of a very stylized representation of the hydro system and a single thermal generator, this model uses a more detailed representation of the reservoirs during the whole planning horizon and it allows to consider multiple thermal plants that can belong to different agents. In addition, instead of using utility functions to model risk aversion, the model implements the CVaR measure due to its suitability to be embedded within optimization models. The proposed model does not require to build in advance the extreme points of the polyhedron that define the risk set of each agent, and

it takes into account the net-head dependency. Moreover, the implemented model is a multi-stage scenario tree with 12 time periods (months) that can be used to analyze the impact of the risk aversion on the annual evolution of the main variables. Finally, this thesis shows how the market equilibrium solution changes for different risk-aversion levels of the involved agents, and uses an example case to quantify those differences. These results have been published in the paper Jovanović *et al.* (2018).

6.3. Future work

The findings of this thesis can be further extended and improved in new lines of future work.

Given that this thesis has shown that it is possible to transform the mean-risk problem faced by a risk-averse agent into a risk-neutral sequence of problems, the first line of future work would be to study if this approach allows solving large-size problems where the constraints used to model the risk prevents the application of decomposition strategies by scenarios. This way, the strategies used in the scientific literature to decompose the problem by scenarios might be applicable at each stage of Iterative CVaR algorithm. This opens a range of possibilities for future applications.

The second line of future work is focused on studying, from the mathematical point of view, what should be the conditions for the proposed algorithm to work for the non-convex case. In this thesis, the algorithm has been applied to solve the non-convex problem (including the binary variables that allow to consider the existence of the minimum stable load of thermal plants, or the discrete start-up and shutdown decisions), and the results have been satisfactory in the sense of reproducing the same solution as by direct optimization (although, at a higher computational cost in this case). Given that the execution time can be very case-dependent, it would be worth analyzing from the theoretical point of view if some conclusion can be drawn about the applicability of this algorithm for the non-convex case.

Following the current trends of the power systems and market development, the proposed models in Chapter (4) could be further improved. For example, the energy storage problem is a century-old burden of the power electricity systems. However, with the rise of all-electric vehicles and the reduction in the battery production cost, it is very likely that the energy storage will have an effect on the day-ahead and intraday energy markets, and also in the requirement of ancillary services such as regulation reserves as the storage flexibility might reduce the influence of the RES intermittent power generation. Thus, the fourth future work line would be to incorporate storage modeling and analyze the impact on the risk management strategies of the agents, as well as their impact on the equilibrium solution.

From the point of view of risk management, the fifth line of work proposed would be to incorporate the concept of time-consistency and to use a risk-measure compatible with this approach. In this thesis it has been just considered the distribution of total profits for the whole time horizon. However, the manner in which the profits are distributed along the intermediate temporal stages can affect the solution to the problem. This topic has received great interest lately, and research works like Philpott *et al.* (2016) and Vardanyan & Hesamzadeh (2017) constitute pioneering works that can guide future developments in this line.

Another line of future work is to study how the level of risk of agents could constitute their market strategy, regardless of their true level of risk aversion. In this same line, it is proposed to investigate if the iterative algorithm used to solve the problem of an agent, could have a fit to solve the equilibrium problem.

Finally, it is proposed to study to what extent the risk-aversion degree of market participants at the operational level could condition their investment decisions in the future. This could be achieved by incorporating the developed model in chapter 5 as a sub-module of an expansion planning model. From the regulatory point of view, this analysis might bring interesting discussions on whether it is necessary to design new products so that both at the investment and operational level, the decisions taken in a liberalized environment with risk-averse agents are equivalent to those that lead to the maximization of the social welfare.

A. Appendix

The objective of this annex is to provide some mathematical background material that is relevant to this thesis. In particular it includes:

1. Constrained optimization

2. Benders decomposition

3. Dantzig-Wolfe decomposition

A.1. Constrained optimization

The material presented hereafter is based on Bertsekas (1999), where only the most relevant issues for the thesis have been included. Mathematical proofs have been omitted as they can be checked in the reference.

Consider the following constrained optimization problem:

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & \\ & h_i(x) = 0, \quad i = 1, \ldots, m \\ & g_j(x) \leq 0, \quad j = 1, \ldots, r \end{aligned} \tag{A.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $h_i : \mathbb{R}^n \to \mathbb{R}, i = 1, ..., m$, $g_j : \mathbb{R}^n \to \mathbb{R}, j = 1, ..., r$ are continuously differentiable functions. For convenience, it is possible to define the following constraint functions $h : \mathbb{R}^n \to \mathbb{R}^m$, and $g : \mathbb{R}^n \to \mathbb{R}^r$:

$$h = (h_i, \ldots, h_m), \; g = (g_i, \ldots, g_r),$$

so it can be expressed as

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & \\ & h(x) = 0 \\ & g(x) \leq 0 \end{aligned} \tag{A.2}$$

Any feasible point x of problem (A.2) has to satisfy all the equality and inequality constraints. For the inequality constraints, it can happen that only a subset of them is active at that point. This set of active inequality constraints can be denoted as $A(x) = \{j \mid g_j(x) = 0\}$.

The corresponding Lagrangian function of problem (A.1) can be defined as follows:

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{j=1}^{r} \mu_j g_j(x) \tag{A.3}$$

where the scalars $\lambda_i, \ldots, \lambda_m$ and $\mu_i, \ldots, \mu_r$ are the so called *Lagrange multipliers* for equality and inequality constraints respectively.

A feasible vector x is called *regular* if the constraint gradients of equality constraints $\nabla h_i(x), \ldots, \nabla h_m(x)$, and the constraint gradients of the active inequality constraints $\nabla g_j(x), j \in A(x)$, are linearly independent.

A.1.0.1. Karush-Kuhn-Tucker Necessary Conditions (KKT)

Let x^* be a local optimum of problem (A.1) which is regular. Then there exist unique values of the multipliers $\lambda_i^*, \ldots, \lambda_m^*$ and $\mu_i^*, \ldots, \mu_r^*$ that satisfy:

$$\nabla_x L(x^*, \lambda^*, \mu^*) = 0 \tag{A.4}$$

$$\mu_j^* \geq 0, j = 1, \ldots, r \tag{A.5}$$

$$\mu_j = 0, \forall j \notin A(x) \tag{A.6}$$

The meaning of (A.6) is that the Lagrange multipliers of the non-active inequality constraints are null, i.e. there would be no change in the objective function by relaxing them. An alternative way of expressing this idea is by means of the so called *complementary slackness condition* $\mu_j^* \cdot g_j(x^*) = 0$.

The necessary KKT optimality conditions can therefore be summarized as follows:

$$\nabla_x L(x, \lambda, \mu) = \nabla f(x) + \sum_{i=1}^{m} \lambda_i \nabla h_i(x) + \sum_{j=1}^{r} \mu_j \nabla g_j(x) = 0 \; (\textbf{stationarity}) \tag{A.7}$$

$$\begin{array}{l} h_i(x) = 0, i = 1, \ldots, m \\ g_j(x) \leq 0, j = 1, \ldots, r \end{array} \; (\textbf{primal feasibility}) \tag{A.8}$$

$$\mu_j^* \geq 0, j = 1, \ldots, r \; (\textbf{dual feasibility}) \tag{A.9}$$

$$\mu_j \cdot g_j(x) = 0, j = 1, \ldots, r \; (\textbf{complementary slackness}) \tag{A.10}$$

It is common to use the symbol $\perp$ to indicate complementarity. Therefore, it would be simpler to write the KKT necessary conditions as:

$$\begin{array}{c} \nabla_x L(x, \lambda, \mu) = 0 \\ \nabla_\lambda L(x, \lambda, \mu) = 0 \\ 0 \leq \mu \perp g(x) \leq 0 \end{array} \tag{A.11}$$

where $\nabla_\lambda L(x, \lambda, \mu) = 0$ is equivalent to write the equality constraints.

One possible way to obtain local optima of (A.2) is to find the solution of the necessary KKT conditions in (A.11) and to check additional second order conditions. In case f is a convex function, and the feasible region defined by all the constraints is a convex set, then the previous KKT conditions are also sufficient.

A.2. Benders decomposition review

Let consider a two-stage linear programming problem PL-2 (Taşkin, 2011), which has the following generic formulation:

$$\begin{aligned} \min_{x,y} \quad & c^T x + g^T y \\ s.t. \quad & Ax = b \\ & Tx + Wy = h \\ & x, y \geqslant 0 \end{aligned} \tag{A.12}$$

where x and y represent the set of variables of the first and second stage respectively. The problem (A.12) can be expressed also as follows:

$$\begin{aligned} \max_{x,y} \quad & c^T x + \theta(x) \\ s.t. \quad & Ax = b \\ & x \geqslant 0 \end{aligned} \tag{A.13}$$

where the recourse function $\theta(x)$ represents the value of the objective function of the second stage in terms of the decisions made in the first stage, i.e.:

$$\begin{aligned} \theta(x) = \quad & \min_{y} g^T y \\ s.t. \quad & Wy = h - Tx, : \pi \\ & y \geqslant 0 \end{aligned} \tag{A.14}$$

In the literature, (A.13) is known as the master problem, and (A.14) is known as sub-problem of the Benders decomposition. Notice that the sub-problem (A.14) assess the recourse function for a given decision x of the first stage. The dual variables π “measure” the sensibility of the recourse function with respect the considered values of x.

The dual representation of the sub-problem is the next one:

$$\begin{aligned} \theta(x) = \quad & \max_{\pi} (h - Tx)^T \pi \\ s.t. \quad & W^T \pi \leq g \end{aligned} \tag{A.15}$$

Defining $\Pi = \left\{ \pi^1, \pi^2, \ldots, \pi^I \right\}$ as the finite set of all the vertices of the convex polyhedron defined by $W^T \pi \leq g$, and taking into account that the optimal solution of a linear programming problem is one of the vertices of the convex polyhedron that defines the feasibility region, the recourse function could be found by enumerating all the possible vertices: $\theta(x) = \max_{\mu} \left\{ (h - Tx)^T \pi^i \right\}$, $i = 1, \ldots, I$. So the sub-problem can be expressed as:

$$\begin{aligned} \theta(x) = \quad & \min_{\theta} \theta \\ s.t. \quad & \theta \geq (h - Tx)^T \pi^1 \\ & \vdots \\ & \theta \geq (h - Tx)^T \pi^I \end{aligned} \tag{A.16}$$

The constraints in (A.16) are called *Benders cuts* and they represent an outer approximation of the recourse function. Notice that θ is a free variable. Taking into account (A.16), an equivalent formulation of the original problem (A.12) is the next one, which is known as *complete master problem* because it contains the entire set of cuts that define perfectly the recourse function:

$$\begin{aligned} & \min_{x,\theta} c^T x + \theta \\ s.t. \quad & Ax = b \\ & \theta \geq (h - Tx)^T \pi^1 \\ & \vdots \\ & \theta \geq (h - Tx)^T \pi^I \\ & x \geq 0 \end{aligned} \tag{A.17}$$

The formulation of (A.17) requires to compute *ex ante* the complete set of Benders cuts. As this might be not possible when the size of the problem is large, the Benders decomposition algorithm is based on the idea of adding these cuts in an iterative way. Thus, in an intermediate stage of the algorithm after j iterations, the problem would be the so called *relaxed master problem*:

$$\begin{aligned} & \min_{x,\theta} c^T x + \theta \\ s.t. \quad & Ax = b \\ & \theta \geq (h - Tx)^T \pi^i, \, i = 1, \ldots, I \\ & x \geq 0 \end{aligned} \tag{A.18}$$

In order to ease the practical implementation of the algorithm, the cuts can be arranged as follows, which is the standard formulation of the relaxed master problem:

$$
\begin{aligned}
& \min_{x,\theta} c^T x + \theta \\
s.t. \quad & Ax = b \\
& \theta \geq f^i + \pi^{iT} T \left(x^i - x \right), \, i = 1, \ldots, I \\
& x \geq 0
\end{aligned} \tag{A.19}
$$

The cuts in this formulation can be interpreted as a linear approximation around an exact value of the recourse function (note that if $x = x^i$ then $\theta = f^i$). Notice that x^i and f^i store the solution values of first stage variables and the recourse function in iteration i.

In general, given a proposal x^i of the master problem in iteration i, the value of f^i can be obtained by solving the so called *Benders sub-problem*, which also provides the dual variables π^i necessary to build the mentioned linear approximation of the Benders cuts.

$$
\begin{aligned}
f^i = \quad & \min_{y} g^T y \\
s.t. \quad & Wy = h - Tx^i, : \pi^i \\
& y \geq 0
\end{aligned} \tag{A.20}
$$

The previous formulation assumes that the Benders sub-problem is feasible and bounded for any proposal of the master problem. In case it is not bounded, the general problem is neither bounded, so it has no practical interest. In case it is not feasible, the optimality cut is replaced by a infeasibility cut that eliminate such unfeasible proposal from the feasibility region of the master problem. In this paper we will assume that the sub-problem is always feasible given. In this case, the Benders algorithm can be summarized as follows:

- **Step 1:** Initialize the iteration counter $i = 0$, the upper and lower bounds $\bar{z} = \infty$, $\underline{z} = -\infty$, and the convergence tolerance ϵ.
- **Step 2:** Solve the relaxed master problem (A.19) (if $i = 0$, then fix $\theta = 0$) to obtain the solution x^{i+1}, θ^{i+1}, and evaluate the lower bound $\underline{z} = c^T x^{i+1} + \theta^{i+1}$.
- **Step 3**: Solve the sub-problem (A.20) considering the proposal of the first stage variables obtained in **Step 2** (x^{i+1}) and obtain $y^{i+1}$to update the upper bound as $\bar{z} = c^T x^{i+1} + g^T y^{i+1}$.
- **Step 4**: Check the convergence: if $^{|\bar{z}-\underline{z}|}/_{|\bar{z}|} \leq \epsilon$, then stop the algorithm; otherwise, add a new Benders cut using the data contained in the solution o

the sub-problem of **Step 3** (f^{i+1} and π^{i+1}), increase the iteration counter $i = i + 1$ and go to **Step 2.**

A.3. Dantzig-Wolfe decomposition principle

Let consider a two-stage linear programming problem:

$$\begin{aligned} \min_{x} \quad & c^T x \\ s.t. \quad & Ax = b \\ & x \geqslant 0 \end{aligned} \tag{A.21}$$

where the $Ax = b$ represents the coupling constraint. The aim of the Dantzig-Wolfe approach George B. Dantzig & Philip Wolfe (1960) is to decouple this constraint in such a way that it never solves all the possible sub-problems $Ax = b$. An alternative is to solve a series of smaller sub-problems individually, in which the master problem focuses on the coupling constraint. Defining a bounded region of the problem (A.21) as $X = \{x|Ax = b,\, x \geq 0\}$, a convex combination of the extreme points x^i can be defined as:

$$\begin{gathered} x = \sum_i \lambda^i x^i \\ \sum_i \lambda^i = 1 \\ \lambda^i \geqslant 0, i = 1, \ldots, I \end{gathered} \tag{A.22}$$

Now the Dantzig-Wolfe master problem, also known as the restricted master problem is as follows:

$$\begin{gathered} \min_{\lambda^i} \sum_i \lambda^i \left(c^T x^i \right) \\ \sum_i \lambda^i \left(A^i x^i \right) = b^i, : \pi^i \\ \sum_i \lambda^i = 1, : \omega^i \\ \lambda^i \geqslant 0, i = 1, \ldots, I \end{gathered} \tag{A.23}$$

where the π^i and ω^i represent the dual multipliers. The corresponding sub-problem can be formulated as:

$$\begin{aligned} \min_{x} \quad & \left[c - \left(A^{i}\right)^{T} \pi^{i}\right]^{T} x - \omega^{i} \\ s.t. \quad & Ax = b \\ & x \geqslant 0 \end{aligned} \tag{A.24}$$

Convergence of the algorithm is achieved when $\left[c - \left(A^{i}\right)^{T} \pi^{i}\right]^{T} x - \omega^{i} \leq \varepsilon.$, where ε is a very small positive number. The dual problem of the (A.23) - (A.24) represents the Lagrangian relaxation algorithm, which provides the exact solution.

B. Appendix - GAMS EMP code

This appendix provides a GAMS code for modeling the problems, using Extended Mathematical Programming, which results are presented in Chapter 5.

```
$Title "Electricity Market Equilibrium with Hydro-Thermal and RES generation"

 $ontext
 Instituto de Investigación Tecnológica (IIT)
 Universidad Pontificia Comillas
 $offtext
 ;
 * global options:  allow empty and multiple definition of sets
 $OnEmpty OnMulti
 $Phantom null
 * options
 option QCP = cplex;
 option solprint=on
 option Limrow=24;
 ;

* ========================

* Definition of SETS

SETS

 m                    market agents
 gt                   thermal generators
 gh                   hydro generators
 res                  renewable energy sources (RES) generators
 d                    demand aggregators
 t                    time periods
 s                    scenarios
 i                    index of nodes for each time t
```

```
 stoch               index of stochastic parameters
 am                  attributes of the market agents
 mod                 equilibrium models
 acom                attributes in the reports

 mact (m)            market agents active in the market equilibrium formulation
 mg (m)              market agents that are generation companies GENCOs
 md (m)              market agents that are demand agents ESCOs
 ma (m)              market agents that are arbitrageurs

 mg_gt (m,gt)        ownership of each thermal generator by each GENCO
 mg_gh (m,gh)        ownership of each hydro generator by each GENCO
 mg_res (m,res)      ownership of each hydro generator by each GENCO
 md_d (m,d)          ownership of each demand aggregator by each ESCO

 agt                 attributes of thermal units
 agh                 attributes of hydro units
 aghti               attributes of the hydro plant that depend on the node
 aresti              attributes of RES generators that depend on the node

 adti                attributes of demand aggregators that depend on the node
 at                  attributes of each time stage
 iter                iterations

;

* ========================

* Definition of alias

 alias (s,ss), (t,tt), (gt,ggt), (gh,ggh), (i,ii), (d,dd), (res,rres)
 ;

* ========================

* Definition of dynamic SETS

SETS
```

```
 tf(t)                time periods where forwards can be signed (except the last)
 tf(t)                time periods where forwards can be signed (except the last)
 tlast(t)             last period

 sa(s)                active scenarios
 n (t,i)              nodes of the stochastic tree
 nt (tt,t,i)          nodes (t i) that belong to stage tt

 ns (t,i,s)            nodes (t i) that belong to scenario s
 father(t,i,tt,ii)     father node of node (t i)

;

* ========================

* Definition of parameters

PARAMETERS

 factor                     factor to change easily the cost functions
 flagCen                    option to run the Cen model
 flagMrNe                   option to run the MrNe model
 flagMrAv_CVaR              option to run the MrAv_CVaR model

 flagMrAvCVaR               option to run the MrAv_CVaR model
 nsce                       number of scenarios
 niter                      number of iterations that will be run

 ProbS(s)                   probability of each scenario
 ProbN(t,i)                 probability of each node
 ProbS_RISK(s,mod,m)        risk modified probability of each scenario seen by
                            agent m
 ProbN_RISK(t,i,mod,m)      risk Modified Probability of each node seen by agent m

 DM(m,am)                   data of each market agent
 TREE(s,t)                  stochastic tree clustering of scenarios for each time
                            stage
 DT(t,at)                   data of each time stage
 DGT(gt,agt)                data of thermal generators
```

```
 DGH(gh,agh)                 data of hydro generators
 DGHTI(gh,t,i,aghti)         data of hydro generators that depend on the node
 DRESTI(res,t,i,aresti)      data of RES generators that depend on the node
 RESti(t,i)                  total renewables per node
 INFLti(t,i)                total hydro inflow per node
 DEMti(t,i)                 total nominal demand per node
 FCti(t,i)                  factor to multiply the generation costs
 A(s)                       auxiliary parameter used to sort the profits for each
                            scenario when initializing the CVaR model
 AIndex(s)                  auxiliary parameter used to sort the profits for each
                            scenario when initializing the CVaR model
 ASorted(s)                 auxiliary parameter used to sort the profits for each
                            scenario when initializing the CVaR model
 sol_gent (mod,gt,s,t)         solution data for thermal generators
 sol_genh (mod,gh,s,t)         solution data for hydro generators
 sol_genres(mod,res,s,t)       solution data for RES
 sol_volh(mod,gh,s,t)          solution data for volume of water
 sol_outflowh(mod,gh,s,t)      solution data for hydro outflows
 sol_pricespot(mod,s,t)        solution data for spot prices
 sol_dem (mod,s,t)             solution data for demand
 sol_profit(mod,s,m)           solution data for profits
;

* ========================

* Definition of scalars

SCALARS

 aux0                             auxiliary scalar
 epsilon                          small number /1e-6/
 from_m3_per_s_to_hm3_per_h       conversion factor (to multiply) /0.036/
 mumin                            minimum value of mu /0.001/
;
```

```
* ========================

* Definition of variables

VARIABLES

 v_fobj_market_agent(m)       objective function of market participant [kEur]
 v_profit (m, s)              profit of market participant [kEur]
 v_pi (t, i)                  price of the spot market [kEur per GWh]
 v_cost(gt,t,i)               generation cost of thermal unit gt [kEur]

 v_CVaR(m)                    conditional value at risk [kEur]
 v_VaR(m)                     value at risk [kEur]

POSITIVE VARIABLES

 v_gent (gt, t,i)          power generation of thermal unit [GW]
 v_genh (gh, t,i)          power generation of hydro unit [GW]
 v_genres(res,t,i)         power generation of res unit [GW]
 v_dem (d,t ,i)            power consumed by demand [GW]

 v_qh(gh,t,i)              water outflow of hydro unit gh [hm3 per h]
 v_volume(gh,t,i)          volume of water at the reservoir of gh at the
                           beginning t [hm3]
 v_qspillage(gh,t,i)       spillage outflow of hydro unit gh [hm3 per h]
 v_profit_aux(m,s)         auxiliary variable for CVaR computation [kEur]

;

* ========================

* Declaration of equations

EQUATIONS

 e_FobjCen                    objective function of the centralized operation [kEur]
 e_DemBala(t,i)               demand balance [GW]
 e_ThermCost(gt,t,i)          definition of thermal cost [kEur]
 e_WaterBala (gh,t,i)         water balance at the reservoir [hm3]
```

```
 e_FobjCen                    objective function of the centralized operation [kEur]
 e_DemBala(t,i)               demand balance [GW]
 e_ThermCost(gt,t,i)          definition of thermal cost [kEur]
 e_WaterBala(gh,t,i)          water balance at the reservoir [hm3]

 e_HydroGen(gh,t,i)           hydro input-output generation function [GW]
 e_fobj_market_agent(m)       objective function for market risk neutral [kEur]
 e_def_profit_nof(m,s)        profit equation without forward [kEur]
 e_fobj_market_agent_mean     objective function for market models risk averse
 _risk(m)                     [kEur]

 e_def_profit_aux(m,s)        equation to model the auxiliary variable for CVaR
                              [kEur]
 e_def_CVaR(m)                equation to model the CVaR [kEur]

;

* Begin *************INCLUDE OF DATA FROM EXCEL*************

;

$setglobal namemodel RESEQ

$onecho > tmp_%namemodel%_%gams.user1%.txt

 r1 = sets
 o1 = sets_%namemodel%_"%gams.user1%".txt
 r2 = parameters
 o2 = parameters_%namemodel%_"%gams.user1%".txt

 r3 = time
 o3 = time_%namemodel%_"%gams.user1%".txt
 r4 = thermal
 o4 = thermal_%namemodel%_"%gams.user1%".txt

 r5 = hydro
 o5 = hydro_%namemodel%_"%gams.user1%".txt
 r6 = agents
 o6 = agents_%namemodel%_"%gams.user1%".txt
```

```
 r7 = tree
 o7 = tree_%namemodel%_"%gams.user1%".txt
 r8 = stoch
 o8 = stoch_%namemodel%_"%gams.user1%".txt

$offecho


$call xls2gms m i="%gams.user1%".xlsm @"tmp_%namemodel%_%gams.user1%.txt"


sets


 $include sets_%namemodel%_%gams.user1%.txt
 table DT(t,at)
 $include time_%namemodel%_%gams.user1%.txt
 table DGT(gt,agt)
 $include thermal_%namemodel%_%gams.user1%.txt
 table DGH(gh,agh)
 $include hydro_%namemodel%_%gams.user1%.txt
 table DM(m,am)
 $include agents_%namemodel%_%gams.user1%.txt
 table TREE(s,t)
 $include tree_%namemodel%_%gams.user1%.txt
 table DSTOCH(stoch,s,t)
 $include stoch_%namemodel%_%gams.user1%.txt
 ;
 $include parameters_%namemodel%_%gams.user1%.txt

;


* End *************INCLUDE OF DATA FROM EXCEL*************


* ========================


* Centralized planning (Cen)
```

```
 e_FobjCen ..                        v_fobj_cen =E= SUM[(t,i)$n(t,i), ProbN(t,i)
                                     *DT(t,'dur') * [-sum(gt,v_cost(gt,t,i))]] ;

 e_DemBala(t,i)$n(t,i) ..            sum(gt,v_gent(gt,t,i)) + sum(gh,v_genh(gh,t,i))
                                     + sum(res,v_genres(res,t,i)) =E= DEMti (t,i) ;

 e_ThermCost(gt,t,i)$n(t,i) ..       v_cost(gt,t,i) =E= (DGT(gt,'c1')*v_gent(gt,t,i)
                                     + DGT(gt,'c2')*power(v_gent(gt,t,i),2) +
                                     DGT(gt,'c3')*power(v_gent(gt,t,i),3))*FCti(t,i);

 e_WaterBala(gh,t,i)$n(t,i) ..       v_volume(gh,t,i) =E=
                                     sum((tt,ii)$(father(t,i,tt,ii)),
                                     v_volume(gh,tt,ii))$[ord(t)>1] +
                                     DGH(gh,'Vo')$[ord(t)=1]
                                     +DT(t,'dur')*(- v_qh(gh,t,i) -
                                     v_qspillage(gh,t,i) +DGHTI(gh,t,i,'inflows') ) ;

 e_HydroGen(gh,t,i)$n(t,i) ..        v_genh(gh,t,i) =E= v_qh(gh,t,i) * (
                                     DGH(gh,'coefmin') +
                                     (DGH(gh,'coefmax')-DGH(gh,'coefmin'))/(DGH(gh,'Vmax')-
                                     DGH(gh,'Vmin')) *
                                     (v_volume(gh,t,i)-DGH(gh,'Vmin')) );

* ========================

* Market with risk neutral agents (MrNe)

 e_fobj_market_agent(m)$mact(m) ..      v_fobj_market_agent(m) =E= SUM[s$sa(s),
                                        ProbS(s)* v_profit(m,s)] ;

 e_def_profit_nof(m,s)$(mact(m)         v_profit(m,s) =E= SUM[(t,i)$ns(t,i,s),
 and sa(s)) ..                          DT(t,'dur')*(
                                        SUM[gt$mg_gt(m,gt), v_gent(gt,t,i) * v_pi
                                        (t,i)-v_cost(gt,t,i) ]
                                        + SUM[gh$mg_gh(m,gh), v_genh(gh,t,i) * v_pi
                                        (t,i)]
                                        + SUM[res$mg_res(m,res), v_genres(res,t,i) *
                                        v_pi (t,i) ] ) ];
```

```
 * e_DemBala same as in Cen)
 * e_ThermCost same as in (Cen)
 * e_WaterBala same as in (Cen)
 * e_HydroGen same as in (Cen)

* ========================

* Market with risk averse agents (CVaR) and only spot market (MrAv_CVaR)

 e_fobj_market_agent_mean_risk(m)      v_fobj_market_agent(m) =E= (1-DM(m,'muiter'))
 $mact(m) ..                           * SUM[s$sa(s), ProbS(s)* v_profit(m,s)] +
                                       DM(m,'muiter') * v_CVaR(m) ;

 * e_def_profit_nof(m,s) same as in MrNe
 * e_DemBala same as in Cen)
 * e_ThermCost same as in (Cen)
 * e_WaterBala same as in (Cen)
 * e_HydroGen same as in (Cen)

 e_def_profit_aux(m,s)$(mact(m) and sa(s) and DM(m,'mu') <> 0 and DM(m,'beta')<>0)..
 v_profit_aux(m,s) =G= v_VaR(m)-v_profit(m,s) ;

 e_def_CVaR(m)$( mact(m) and DM(m,'mu') <> 0 and DM(m,'beta')<>0 ) ..
 v_CVaR(m)-[v_VaR(m)-SUM[s,ProbS(s)*v_profit_aux(m,s)]/DM(m,'beta')]=E=0;

* ========================

* Definition of Models

model

 Model_Cen
 /
 e_FobjCen
 e_DemBala
 e_ThermCost
 e_WaterBala
 e_HydroGen
 /
```

```
 Model_MrNe
 /
 e_fobj_market_agent
 e_def_profit_nof
 e_ThermCost
 e_DemBala
 e_WaterBala
 e_HydroGen
 /
 MrAv_CVaR
 /
 e_fobj_market_agent_mean_risk
 e_def_profit_nof
 e_DemBala
 e_ThermCost
 e_WaterBala
 e_HydroGen
 e_def_profit_aux
 e_def_CVaR
 /
***************************************************************************

* ========================

* Management of Input Data

;

set g all (thermal and hydro) generation units / set.gt, set.gh, set.res / ;

* ========================

* Configuration of the stochastic tree

tf(t) = yes$(ord(t) < card (t));
```

```
tlast(t) = yes$(ord(t) = card (t));

sa(s) = yes$(ord(s)<= nsce);

* Identification of nodes based on the information provided in the tree

n(t,i) = yes$sum(sa,TREE(sa,t)=ord(i));

* Identification of the father of each node based on the information provided in the
tree

loop(t$(ord(t)> 1),

loop(s$sa(s),

father(t,i,t-1,ii)$ ( TREE(s,t) =ord(i) and TREE (s,t-1)=ord(ii)) = yes ;

 );
 );

nt(tt,t,i) = yes$(ord(tt)=ord(t) and n(t,i));

ns(t ,i,s) = yes$(tree(s,t)=ord(i));

* We assume that equiprobable scenarios.

* The probabilities of each node are computed accordingly

ProbS(sa) = 1/nsce;

ProbN(t,i) = sum(s$(ns(t,i,s) and sa(s)),ProbS(s));

* Mapping between the general stochastic parameters that depend on scenario and time

* with the defined data that depend on nodes:
```

```
RESti (t,i)$(sum(sa$(TREE(sa,t)=ord(i)), 1)) = sum(sa$(TREE(sa,t)=ord(i)),
DSTOCH('stoch01',sa,t))/sum(sa$(TREE(sa,t)=ord(i)), 1) ;

DEMti (t,i)$(sum(sa$(TREE(sa,t)=ord(i)), 1)) = sum(sa$(TREE(sa,t)=ord(i)),
DSTOCH('stoch02',sa,t))/sum(sa$(TREE(sa,t)=ord(i)), 1) ;

INFLti(t,i)$(sum(sa$(TREE(sa,t)=ord(i)), 1)) = sum(sa$(TREE(sa,t)=ord(i)),
DSTOCH('stoch03',sa,t))/sum(sa$(TREE(sa,t)=ord(i)), 1) ;

FCti (t,i)$(sum(sa$(TREE(sa,t)=ord(i)), 1)) = sum(sa$(TREE(sa,t)=ord(i)),
DSTOCH('stoch04',sa,t))/sum(sa$(TREE(sa,t)=ord(i)), 1) ;

* Splitting the total stochastic parameters among the items according to
proportional criteria

DRESTI (res,t,i,'forecast' ) = (RESti (t,i)/sum(rres,1))$sum(rres,1);

DGHTI (gh ,t,i,'inflows' ) =
(INFLti(t,i)*DGH(gh,'Vmax')/sum(ggh,DGH(gh,'Vmax')))$sum(ggh,DGH(gh,'Vmax'));

* Scaling the input parameters

* [MW/(m3/s)] -> [GW/(Hm3/h)]

DGH(gh, 'coefmin') = DGH(gh, 'coefmin')*1e-3/from_m3_per_s_to_hm3_per_h;

DGH(gh, 'coefmax') = DGH(gh, 'coefmax')*1e-3/from_m3_per_s_to_hm3_per_h;

* (m3/s)] -> (Hm3/h)

DGH (gh, 'qmax' ) = DGH(gh, 'qmax' )*from_m3_per_s_to_hm3_per_h;

* Inflows are directly in hm3/h

* Bounds of decision variables
```

```
v_volume.up(gh,t,i)$n(t,i) = DGH(gh,'Vmax');

v_volume.lo(gh,t,i)$n(t,i) = DGH(gh,'Vmin');

v_qh.up (gh,t,i)$n(t,i) = DGH(gh,'qmax');

v_gent.up (gt,t,i)$n(t,i) = DGT(gt,'pmax');

v_volume.fx(gh,t,i)$(n(t,i) and tlast(t)) = DGH(gh,'Vf');

* Fix the renewables to avoid curtailments of RES

v_genres.fx (res, t,i)$n(t,i) = DRESTI (res,t, i,'forecast');

v_qh.lo (gh,t,i)$n(t,i) = DGHTI(gh,t,i,'inflows')*.5;

*****************************************************************

* Solving the Models

*==============================(1)==============================

* Solve the centralized problem (Cen)

if( flagCen = 1,

solve Model_Cen using NLP maximizing v_fobj_cen ;

sol_gent ('Cen', gt , s, t ) = SUM[i$ns(t,i,s), v_gent.l(gt,t,i) ] ;

sol_genh ('Cen', gh , s, t ) = SUM[i$ns(t,i,s), v_genh.l(gh,t,i) ] ;

sol_genres ('Cen', res, s, t ) = SUM[i$ns(t,i,s), v_genres.l(res,t,i) ] ;

sol_volh ('Cen', gh , s, t ) = SUM[i$ns(t,i,s), v_volume.l(gh,t,i) ] ;
```

```
sol_outflowh ('Cen', gh , s, t ) = SUM[i$ns(t,i,s), v_qh.l(gh,t,i) ] ;

sol_dem ('Cen', s , t ) = SUM[i$ns(t,i,s), DEMti(t,i) ] ;

sol_pricespot('Cen', s , t ) = SUM[i$ns(t,i,s),
-e_DemBala.m(t,i)/ProbN(t,i)/DT(t,'dur') ] ;

);

*==============================(2)==============================

* Solve the Risk Neutral Market Equilibrium problem (MrNe)

file info / '%emp.info%' /;

if( flagMrNe = 1,

* Initialization of the variables of the MCP problem with the ones

* corresponding to Cen to help finding the solution

v_pi.l (t,i)$n(t,i) = -e_DemBala.m(t,i)/ProbN(t,i)/DT(t,'dur');

v_cost.l(gt,t,i) = (DGT(gt,'c1')*v_gent.l(gt,t,i) +
DGT(gt,'c2')*power(v_gent.l(gt,t,i),2) +
DGT(gt,'c3')*power(v_gent.l(gt,t,i),3))*FCti(t,i);

v_profit.l(m,s)$(mact(m) and sa(s)) = SUM[(t,i)$ns(t,i,s), DT(t,'dur')*(

SUM[gt$mg_gt(m,gt), v_gent.l(gt,t,i) * v_pi.l (t,i)-v_cost.l(gt,t,i) ]

+ SUM[gh$mg_gh(m,gh), v_genh.l(gh,t,i) * v_pi.l (t,i) ]

+ SUM[res$mg_res(m,res), v_genres.l(res,t,i) * v_pi.l (t,i) ] )];

v_fobj_market_agent.l(m)$mact(m) = SUM[s$sa(s), ProbS(s)* v_profit.l(m,s)] ;
```

```
* . . . . . . . . . . . . . . . . . . . . . . . . . . .

put info;

put / 'equilibrium';

loop(m$mact(m), put / 'max ' v_fobj_market_agent(m);

loop(s$sa(s), put / v_profit.tn(m,s));

loop((gt,t,i)$(n(t,i) and mg_gt(m,gt)) , put / v_gent.tn(gt,t,i) ' '
v_cost.tn(gt,t,i) );

loop((gh,t,i)$(n(t,i) and mg_gh(m,gh)) , put / v_genh.tn(gh,t,i) ' ' v_qh(gh,t,i) '
' v_volume.tn(gh,t,i) ' ' v_qspillage.tn(gh,t,i) );

loop((res,t,i)$(n(t,i) and mg_res(m,res)) , put / v_genres.tn(res,t,i) );

put / e_fobj_market_agent.tn(m) ;

loop(s$sa(s), put / e_def_profit_nof.tn(m,s) );

loop((gt,t,i)$(n(t,i) and mg_gt(m,gt)), put / e_ThermCost.tn(gt,t,i) );

loop((gh,t,i)$(n(t,i) and mg_gh(m,gh)), put / e_WaterBala.tn(gh,t,i) ' '
e_HydroGen(gh,t,i));

);

put /;

loop((t,i)$n(t,i), put / 'vi ' e_DemBala.tn(t,i) ' ' v_pi.tn(t,i));

putclose info /;

Model_MrNe.optfile = 1;
```

```
solve Model_MrNe using emp ;

sol_gent ('MrNe', gt , s, t ) = SUM[i$ns(t,i,s), v_gent.l(gt,t,i) ] ;

sol_genh ('MrNe', gh , s, t ) = SUM[i$ns(t,i,s), v_genh.l(gh,t,i) ] ;

sol_genres ('MrNe', res, s, t ) = SUM[i$ns(t,i,s), v_genres.l(res,t,i) ] ;

sol_volh ('MrNe', gh , s, t ) = SUM[i$ns(t,i,s), v_volume.l(gh,t,i) ] ;

sol_outflowh ('MrNe', gh , s, t ) = SUM[i$ns(t,i,s), v_qh.l(gh,t,i) ] ;

sol_dem ('MrNe', s , t ) = SUM[i$ns(t,i,s), DEMti(t,i) ] ;

sol_pricespot('MrNe', s , t ) = SUM[i$ns(t,i,s), v_pi.l (t,i) ] ;

sol_profit ('MrNe', s , m ) = v_profit.l(m,s) ;

);

*============================== (3)==============================

Solve the Risk Averse Market (CVaR) Equilibrium problem (MrAvCVaR)

if( flagMrAvCVaR = 1,

* Initialization of the variables of the MCP problem with the ones

* corresponding to the previous runs (Cen) to help finding the solution

v_pi.l (t,i)$n(t,i) = SUM[s$ns(t,i,s), sol_pricespot('Cen', s , t)]/SUM[s$ns(t,i,s),
1];

v_cost.l(gt,t,i) = (DGT(gt,'c1')*v_gent.l(gt,t,i) +
DGT(gt,'c2')*power(v_gent.l(gt,t,i),2) +
DGT(gt,'c3')*power(v_gent.l(gt,t,i),3))*FCti(t,i);
```

```
v_profit.l(m,s)$(mact(m) and sa(s)) = SUM[(t,i)$ns(t,i,s), DT(t,'dur')*(

SUM[gt$mg_gt(m,gt), v_gent.l(gt,t,i) * v_pi.l (t,i)-v_cost.l(gt,t,i) ]

+ SUM[gh$mg_gh(m,gh), v_genh.l(gh,t,i) * v_pi.l (t,i) ]

+ SUM[res$mg_res(m,res), v_genres.l(res,t,i) * v_pi.l (t,i) ] )];

* In order to initialize the CVaR it is necessary to compute the VaR.

* For each agent the sorted profits are computed in ASorted (using the library
gdxrank)

loop(m$(mact(m) and DM(m,'mu') <> 0 and DM(m,'beta')<>0),

A(s)$sa(s)=v_profit.l(m,s) ;

execute_unload "rank_in.gdx", A;

execute 'gdxrank rank_in.gdx rank_out.gdx > %system.nullfile%';

execute_load "rank_out.gdx", AIndex=A;

ASorted(s + (AIndex(s)- Ord(s)))$sa(s) = A(s);

* I assume that all elements in s are active scenarios, and I extract the position
of the beta% percentile (using floor)

v_VaR.l(m)$(mact(m) and DM(m,'mu') <> 0 and DM(m,'beta')<>0) =
sum(s$(ord(s)=floor(DM(m,'beta')*card(s))), ASorted(s));

);

v_profit_aux.l(m,s)$(mact(m) and sa(s) and DM(m,'mu') <> 0 and DM(m,'beta')<>0) =
max(0, v_VaR.l(m)-v_profit.l(m,s)) ;
```

```
v_CVaR.l(m)$( mact(m) and DM(m,'mu') <> 0 and DM(m,'beta')<>0 ) =
v_VaR.l(m)-SUM[s,ProbS(s)*v_profit_aux.l(m,s)]/DM(m,'beta');

v_fobj_market_agent.l(m)$mact(m) = (1-DM(m,'muiter')) * SUM[s$sa(s), ProbS(s)*
v_profit.l(m,s)] + DM(m,'muiter') * v_CVaR.l(m) ;

* . . . . . . . . . . . . . . . . . . . . . . . . . . .

put info;

put / 'equilibrium';

loop(m$mact(m), put / 'max ' v_fobj_market_agent(m);

put / v_CVaR.tn(m) ' ' v_VaR.tn(m)

loop(s$sa(s), put / v_profit.tn(m,s) ' ' v_profit_aux(m,s) );

loop((gt,t,i)$(n(t,i) and mg_gt(m,gt)) , put / v_gent.tn(gt,t,i) ' '
v_cost.tn(gt,t,i) );

loop((gh,t,i)$(n(t,i) and mg_gh(m,gh)) , put / v_genh.tn(gh,t,i) ' ' v_qh(gh,t,i) '
' v_volume.tn(gh,t,i) ' ' v_qspillage.tn(gh,t,i) );

loop((res,t,i)$(n(t,i) and mg_res(m,res)) , put / v_genres.tn(res,t,i) );

put / e_fobj_market_agent_mean_risk.tn(m) ' ' e_def_CVaR(m) ;

loop(s$sa(s), put / e_def_profit_nof.tn(m,s) ' ' e_def_profit_aux.tn(m,s) );

loop((gt,t,i)$(n(t,i) and mg_gt(m,gt)), put / e_ThermCost.tn(gt,t,i) );

loop((gh,t,i)$(n(t,i) and mg_gh(m,gh)), put / e_WaterBala.tn(gh,t,i) ' '
e_HydroGen(gh,t,i));

);
```

```
put /;

loop((t,i)$n(t,i), put / 'vi ' e_DemBala.tn(t,i) ' ' v_pi.tn(t,i));

putclose info /;

MrAv_CVaR.optfile = 1;

loop(iter$(ord(iter) <= niter),

DM(m,'muiter')=min(1, mumin + DM(m,'mu')*(ord(iter)-1)/card(iter) );

solve MrAv_CVaR using emp ;

);

sol_gent ('MrAvCVaR', gt , s, t ) = SUM[i$ns(t,i,s), v_gent.l(gt,t,i) ] ;

sol_genh ('MrAvCVaR', gh , s, t ) = SUM[i$ns(t,i,s), v_genh.l(gh,t,i) ] ;

sol_genres ('MrAvCVaR', res, s, t ) = SUM[i$ns(t,i,s), v_genres.l(res,t,i) ] ;

sol_volh ('MrAvCVaR', gh , s, t ) = SUM[i$ns(t,i,s), v_volume.l(gh,t,i) ] ;

sol_outflowh ('MrAvCVaR', gh , s, t ) = SUM[i$ns(t,i,s), v_qh.l(gh,t,i) ] ;

sol_dem ('MrAvCVaR', s , t ) = SUM[i$ns(t,i,s), DEMti(t,i) ] ;

sol_pricespot('MrAvCVaR', s , t ) = SUM[i$ns(t,i,s), v_pi.l (t,i) ] ;

sol_profit ('MrAvCVaR', s , m ) = v_profit.l(m,s) ;

);

*****************************************************************
```

```
*---------Declaration of auxiliary file-------

file TMP_output /tmp_output.txt/

put TMP_output putclose

'par=sol_gent rng=generationT!C2 rdim=3 '/

'par=sol_genh rng=generationH!C2 rdim=3 '/

'par=sol_genres rng=generationRES!C2 rdim=3 '/

'par=sol_volh rng=reservoirsV!C2 rdim=3 '/

'par=sol_outflowh rng=reservoirsQ!C2 rdim=3 '/

'par=sol_dem rng=DEM!C2 rdim=2 '/

'par=sol_pricespot rng=price!C2 rdim=2 '/

'par=sol_profit rng=profits!C2 rdim=2 '/

*---------GAMs loading data-------------------

execute_unload "results.gdx"

*---------GAMs loading data-------------------

execute 'gdxxrw.exe results.gdx o="Output.xlsx" SQ=n Squeeze=0 EpsOut=0
@tmp_output.txt'

*---------Deleting Auxiliary Files--

execute 'del tmp_output.txt results.gdx';
```

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Curriculum Vitae

Nenad Jovanović was born on the 23rd of August 1986, in Ćuprija, in a country was then called the Socialist Federal Republic of Yugoslavia (central Serbia). He grew up in Negotin, in the east of Republic of Serbia, where he finished in parallel his primary and specialist music schools in 2001, and Technical High School in 2005.

After moving to the city of Niš in south of Serbia, he enrolled at the Faculty of Electronic Engineering in order to continue his education in the field of the Electrical Power Engineering. He has received his master degree in Electrical Engineering and Computes Science at the University of Nis in 2012.

In October 2012, Nenad was awarded the Erasmus Mundus Joint Doctorate in Sustainable Energy Technologies and Strategies (SETS), a joint program between Comillas Pontifical University in Spain, Delft Technical University (TUDelft) in the Netherlands and KTH Royal Institute of Technology in Sweden. He joined the Institute of Technological Research as a research assistant at the Comillas Pontifical University, Madrid, Spain. During the SETS program he has spent time as a visiting researcher at the Institute of the Applied Mathematics, at the TUDelft. The research conduced during the PhD studies was published in journals with the JCR impact factor.

Nenad's fields of interest include stochastic optimization, power system modeling and energy economics. In February 2017 he relocated to Serbia to start a family and where he currently works as an Energy Consultant.

List of doctoral candidate's publications

Journal Articles with JCR factor:

[A1] Jovanović, Nenad, García-González, Javier, Cerisola, Santiago, & Barquín, Julián. 2018. Impact of Risk Aversion on the Operation of Hydroelectric Reservoirs in the Presence of Renewable Energy Sources. Energies, 11(6), 1389.

[A2] Jovanović, Nenad, García-González, Javier, Barquín, Julián, & Cerisola, Santiago. 2017. Electricity market short-term risk management via risk-adjusted probability measures. IET Generation, Transmission & Distribution, 11(10), 2599–2607.

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