Department of Precision and Microsystems Engineering

USING TOPOLOGY OPTIMIZATION FOR ACTUATOR PLACEMENT WITHIN MOTION SYSTEMS

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Challenge the future

Using Topology Optimization for Actuator Placement within Motion Systems

Master of Science Thesis

For the degree of Master of Science in Mechanical Engineering at Delft University of Technology

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Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of Technology

PME - the Ultimate in Mechanical Engineering

“A person who never made a mistake never tried anything new.”
— *A. Einstein*

Abstract

Topology optimization is a strong approach for generating optimal designs which cannot be obtained using conventional optimization methods. Improving structural characteristics by changing the internal topology of a design domain has been fascinating scientists and engineers for years. Topology optimization can be described as a distribution of a given amount of material in a specified design domain, which is subjected to certain loading and boundary conditions. This domain can then be optimized to minimize specified objectives, for example compliance. For static problems, topology optimization is extensively used. The distribution of material, void and solid regions, can be used to solve several problems within the mechanical domain. However, this method of optimization is also used to optimize structures with respect to their resonant dynamics.

Using topology optimization preliminaries, the research first focuses on the design of supports. By taking a bridge as example, it is explained why design of supports can be helpful. When supports are not prescribed, the process of design of supports can be used to determine where these supports should be placed. The combination of topology optimization and design of supports can also be very helpful in compliant mechanisms.

Design of supports is then exploited to design of actuator placement. This new approach of optimizing focuses on design problems, where the placement of force is not prescribed. For a given material in a static domain, the optimal actuator lay-out is determined. This optimal placement of actuators can contribute to a better objective. A minimal force constraint is implemented, to avoid trivial solutions.
Topology optimization is included and combined with design of actuator placement. The simultaneous optimization process of topology and load placement is shown and explained. It is shown that topology and load placement are influencing each other which leads to even better objective results, while respecting given constraints.

Finally, a wafer stage is considered as case study. By implementing a harmonic force, dynamics are introduced. Some basic phenomena of the dynamics are introduced and explained. Then, design of actuator placement is used to ensure that certain mode shapes are not excited whereas other are. It is shown that a larger actuator design domain typically results in better dynamic performance.

After the process of design of actuators, topology optimization is included here. Topology optimization will be used to determine the most ideal placement of the actuators to comply with the requested (minimal) frequency response and lead to a better objective.
This placement of actuators can be used within certain motion systems, especially where the placement of actuators is not pre-defined by manufacturing. However, the results of the theoretical model could trigger users to reconsider their current manufacturing, in order to apply the improved actuator placements to improve their current dynamic performance.

Preface

This thesis is the result of my Master of Science graduation project. The opportunity was given by the department of Precision and Microsystems Engineering at the Delft University of Technology.

During this thesis I have been supervised by Gijs van der Veen and Matthijs Langelaar. I would like to thank my supervisors for their advice, feedback and support during my research.

Unfortunately Gijs left the University during my thesis research. Matthijs replaced him very well. Both are very supportive and their enthusiasm and knowledge of both gentlemen inspired me a lot.

I also would like to thank my friends, family and my fellow students for their support, love and patience during this project.

Stefan Broxterman
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Nomenclature

General meaning of often used symbols, unless mentioned otherwise in the context.

α	Penalty slope
γ	Support factor
β	Heavi-side filter parameter
η_i	Mode influence
$\boldsymbol{\lambda}$	Lagrange multiplier
ρ	Density
$\tilde{\rho}_e$	Filtered element density
$\bar{\rho}_e$	Projected element density
ρ_0	Self-weight density
$\boldsymbol{\rho}_n$	Node density array
$\boldsymbol{\phi}_i$	Eigenvector
ω	Actuating frequency
ω_i	Eigenfrequency
A	Support area
c	Compliance
d_{in}	Input displacement
d_{out}	Output displacement
E	Young's modulus
E_p	Penalized Young's modulus
e	Element number
f	Function value
f'	Differentiated function value
$\mathbf{f}_{sw}$	Self-weight force array
$\mathbf{f}$	Force array
$\mathbf{f}_p$	Penalized force array
G_d	Displacement gain
h	Perturbation value
i	Node number
$\mathbf{K}$	Stiffness matrix
$\mathbf{K}_e$	Element stiffness matrix
$\mathbf{K}_s$	Spring stiffness matrix
$\mathbf{K}_x$	Stiffness density displacement
$K_{s,0}$	Maximum stiffness
$\mathbf{L}$	Selection vector
m	Mass
$\mathbf{M}$	Global mass matrix
$\mathbf{M}_e$	Elemental mass matrix
N	Number of elements
N_i	Number of nodes
p	Penalty
q	Spring penalty
r	Filter radius
$\mathbf{u}$	General displacement array
$\ddot{\mathbf{u}}$	Acceleration array
$\tilde{\mathbf{u}}$	Adjoint displacement array
u_a	Displacement of selected area
$\mathbf{u}_x$	Node density displacement
V	Total volume
v	Volume
W	Weight factor
z	Support design variable

Chapter 1

Introduction

This report is a representation of my Master of Science thesis project. The aim of the research is to investigate the use of topology optimization for the optimal placement of actuators, to use within motion systems. At first, a background for this thesis is given in section 1.1, followed by the main research goals in section 1.2. The methodology is depicted next in section 1.3. This chapter is concluded by a quick outline of this thesis project in section 1.4.

1.1 Background

Nowadays, engineers are faced with structures of increasing complexity. These structures are getting smaller, lighter and more detailed. This tendency should not conflict the objective of the structure. A car, for example, would benefit from less weight for fuel cost reduction. The chassis however, should remain stiff enough to counteract deformations and provide safety for the driver. In the high-tech industry, and the equipment used there, like a wafer stage or robots, complexity is increasing. Also, the design space is getting smaller, especially in the semiconductor industry. The structure should, however, be stiff enough to not conflict its reliability. A very promising approach for these type of problems is the use of topology optimization.

Topology optimization is the process, which determines the optimal material placement within a certain design domain, in order to obtain the best possible structural performance. The process is widely used within the engineering domain, since the use of a homogenization method in topology optimization (Bendsoe and Kikuchi, 1988). Topology optimization gives the connectivity, shape and topology of elements in the design domain. The topology of elements can be described as a distribution of void and solid regions within that design domain.
Topology optimization is the newest technique in the field of structural optimization. Structural optimization is divided in three main categories, the choice of optimization is mainly based on the design variables. Three examples of this categories are depicted in Figure 1-1. Structural optimization can be used in discrete and continuum structures, depending on the

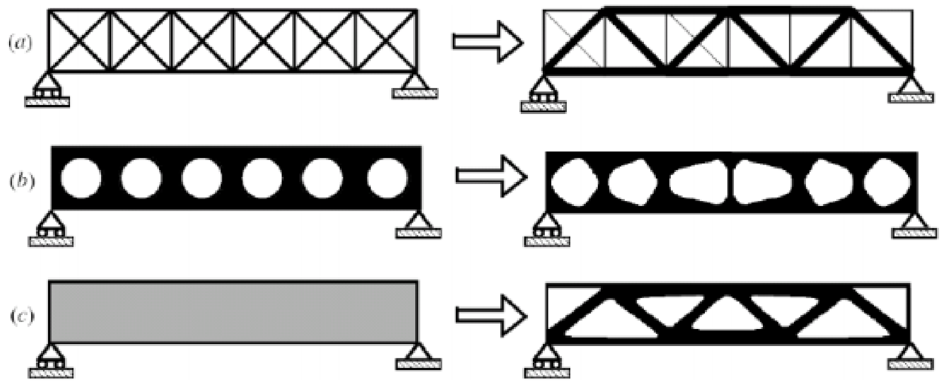


Figure 1-1: Three categories of structural optimization. a) Sizing optimization of a truss structure, b) shape optimization and c) topology optimization. The initial problems are shown at the left hand and the optimal solutions are shown at the right. (Bendsoe and Sigmund, 2003).

design properties and domain.

As can be seen in Figure 1-1a, sizing optimization is here used for a truss structure. The optimization objective is to maximize the vertical stiffness by changing the cross-sectional area of each truss element. This cross-sectional area can thus be considered as a design variable. The case depicted in Figure 1-1b is a shape optimization. Changing the geometry of the holes can provide a higher stiffness. The area and number of holes remains fixed, which is called a constraint. However, the shapes of the holes which are the design variables can be changed. In most cases, structural optimization problems are not fixed at only sizing or only shape problems. A mixture of the categories is needed, in order to achieve the most optimal result. As can be seen in Figure 1-1c a continuum structure is optimized to achieve maximum stiffness for a given amount of material. This is a typical topology optimization problem. The term topology is derived from the Greek word *topos* $(\tau\acute{o}\pi o\zeta)$, which is landscape or place. The 2D-landscape is changed, so the topology of the material is changed (Sigmund, 2000).

Current topology optimization is focusing on structural design, but there are other aspects designers have to make decisions for, like boundary conditions and load placement. These type of design problems emerge for example in the field of high-precision positioning systems, like a wafer stage. All these aspects are important in this case. In current research, there is a lot missing in this particular field. There is no research available on actuator placement, nor a combined with dynamics.

1.2 Research goals

The previous section depicts plenty of opportunity for research. The need for smaller, lighter and more complex structures can be labeled as the main reason for this research project. The first goal of this research project is to investigate the way to include the placement of supports within static topology optimization. The next step is to investigate the usage of design of supports for a variety of example problems.

The second goal of this thesis research project is to investigate the principles of actuator placement and find a way to include the best placement of these actuators. If this actuator placement is correct, topology optimization can also be included, to achieve even further improvements.

All previous investigations were in a static setting. A next step is to extend the work to a dynamic setting. This can be formulated as the third research goal of this thesis. By using a harmonic excitation the optimal actuator placement can be found. If this actuator placement can be combined with dynamic excitations, topology optimization should be implemented also in here. An interesting research goal is to optimize the actuation of a wafer stage by using topology optimization and actuator placement.

1.3 Approach

This thesis will make use of the background of topology optimization. This basics are used to achieve the research goals. In order to get familiar with topology optimization, an investigation on the process called topology optimization is done. The possibilities of this process are investigated and a user-friendly code using Matlab[1] is made, for further usage of my own and other research. The implementation process in Matlab of several features discussed in this thesis, can be found at the back of this thesis, by means of the used Matlab codes. These codes are made user-friendly to make further research more accessible.
This research focuses mostly on two-dimensional examples, where discretization sizes are held the same along the chapters, as much as possible. For consistency, the produced output pictures are shown in the same manner along this report.
The main approach of this thesis can be reflected by the partition of three different parts. First, general topology optimization preliminaries are explained. Using this gained knowledge, extensions are made in the field of boundary conditions and load placement. At last, dynamics are implemented. With these fundamentals established, a case study is used to combine all this gained knowledge.

[1] Matrix Laboratory. A numerical computing environment, using a proprietary programming language.

1.4 Outline

In this section a outline of this research project can be found. This project is divided in four parts.
Before starting this thesis an introduction to topology optimization is given in chapter 2. For readers familiar with topology optimization this chapter is optional. Next, the level of topology optimization is increased to investigate several options of topology optimization in chapter 3.

The second part of this thesis consists multiple topology optimization extensions. Design of supports, including topology optimization can be found in chapter 4. Next, in chapter 5 we translate the design of supports in the design of actuator placement.

In chapter 6, dynamics are introduced. This gives a new aspect to the method. Therefore, a case study is described in the form of a wafer stage. By the implementation of dynamics, this chapter can be seen as a complete summation and practical application of the gained knowledge. The thesis project is ended by a conclusion and recommendations for future work in chapter 7.

Appendix A contains detailed specifications of the hardware that is used. Since time is important within the topology optimization process, all the calculation results can also be found in this chapter. In Appendix B the used Matlab codes can be found. Also, for each implementation a simple and user-friendly add-in can be found in Appendix C. Using this add-in codes everyone can simply upgrade the basic code up to a desired code. Supplementary codes can be found in Appendix D, followed by a list of references.

Part I

Topology Optimization Preliminaries

Chapter 2

Topology Optimization

This chapter is dedicated to obtaining general knowledge and options using topology optimization. First it is explained what the formulation of topology optimization looks like in section 2.1. Next, it is discussed how this formulation can be solved using solution methods in 2.2. Practical applications of topology optimization can be found in section 2.3. Design of supports is slightly touched in section 2.4. Section 2.5 contains an overview of topology optimization and concludes this chapter.

2.1 Topology optimization formulation

Starting at a certain configuration the topology optimization process will optimize the structure to an objective, by varying the design variables. The structure is then optimized by creating several void and solid regions within the design domain.
A typical optimization problem is to set up a minimum compliance design. This design aims to optimize a simple mechanical structure to have a maximum stiffness, or minimum compliance ($c = k^{-1}$). Of course, the maximum stiffness will be achieved when the structure is thus a solid structure. However, for several reasons, it could be interesting to reduce the weight of the structure, while preserving its high stiffness properties. Reducing weight could reduce the material costs, save fuel costs (for example in aerodynamics), and could change intended dynamical behavior (for example in machinery).
To set up such an optimization problem, the intended volume is defined as a boundary condition. The mechanical equations should hold during this optimization problem, which can also be labeled as a boundary condition.
So now let's set up a basic topology optimization. Here we want the structure to be as stiff as possible, while it is subjected to (s.t.) a certain reduced weight value.

$$\begin{aligned} \max \quad & \text{Stiffness} \\ \text{s.t.} \quad & m \leq m_{max} \end{aligned} \tag{2-1}$$

Now assume linear elasticity, and replace stiffness by compliance to give a standard topology compliance optimization problem (Langelaar, 2012).

$$
\begin{aligned}
&\text{Equilibrium:} && \mathbf{Ku} = \mathbf{f} \\
&\text{Compliance:} && c = \mathbf{f}^T\mathbf{u} \\
&\min_{design} && \mathbf{f}^T\mathbf{u} \\
&\text{s.t.} && \mathbf{Ku} = \mathbf{f} \\
& && m \leq m_{max}
\end{aligned}
\tag{2-2}
$$

In this equation (2-2) the objective is to minimize the compliance. This objective can be achieved by varying specified design parameters, in this case there are no parameters specified, so the design parameters are free to choose. However, in most cases this does not apply. Due to external circumstances or internal properties in most cases it is necessary to specify the design variables. In many cases of topology optimization, the topology can be seen as a free variable, so the density is a design variable.

2.1.1 Compliance example

In this section a very simple compliance problem will be used, just to show how the topology optimization process is actually working. *A picture is worth a thousand words* give rise to this section. The process which leads to the optimum results will be explained further in this chapter, but for now let's focus just on the evolution of the topology optimization solution. In this particular example, the main objective is to maximize stiffness (minimize compliance), while the maximum mass of the structure is enforced. Using the formulation as depicted in (2-2), this topology optimization problem can be defined. The design parameter is the material's density.

$$
\begin{aligned}
\min_{\rho} \quad & \mathbf{f}^T\mathbf{u} \\
\text{s.t.} \quad & \mathbf{Ku} = \mathbf{f} \\
& m \leq m_{max}
\end{aligned}
\tag{2-3}
$$

The maximum allowable weight is restricted to 50% of the solid weight. So the equation $m_{max} = \frac{m_{solid}}{2}$ holds. The structure in this case is a simple cantilever beam, where the left side is clamped, while the right side is free. The height-to-width ratio is $\frac{1}{3}$, in order to give a more clear representation of iterations. A vertical point load is attached to the right lower node of the beam, as can be seen in Figure 2-1.

As can be seen, there is some evolutionary behavior showing up. The overall pattern is somewhat the same, but the details are evolving during the iterations steps. How this iteration scheme exactly looks like is depending on the solution, as well on the solution method that is being used. In the upcoming chapters this will become more clear.

2.1.2 Mesh-refinement

To actually perform a topology optimization, the optimization solver uses this discretization of elements. In order to achieve the optimum solution, the solver determines whether an

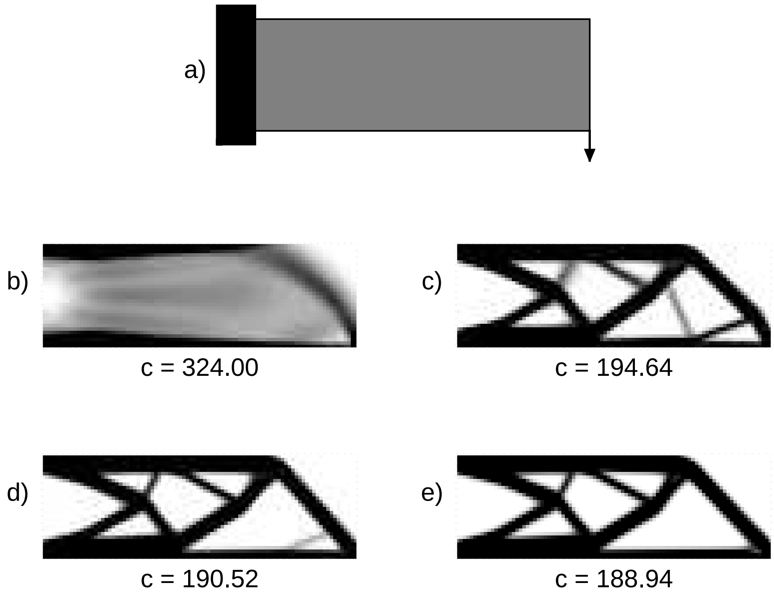


Figure 2-1: Evolution of iterations using the SIMP method. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps, c) 25% of total iteration steps, d) 50% of total iteration steps, e) final solution. The associated compliance values are shown below each figure, which represents a ratio of strain to stress (Appendix A-2).

element is a void (0) or solid (1) region. A collection of all these discretized elements then forms the topology of the complete structure.
Every topology optimization problems deals with the problem: *How should the mesh be refined?*, which refinement is fine enough to approach the reality? Of course, every continuum object needs to be discretized into a number of elements. This is basically a mesh-refinement. An increasing amount of mesh elements results in a longer computational time, but if the mesh-refinement is too rough, the solution does not represent the reality enough. In order to achieve the most optimal mesh-field, the trade-off between precision and computational time should be solved. It is very interesting to have a look at the manufacturing part of topology optimization. For example the resolution of the additive manufacturing device can be seen as the maximum amount of discretization elements, a finer mesh-refinement will from this point not lead automatically to a finer end product.
An example of the influence of different mesh-refinements is made, to show the importance of choosing a good mesh. The same configuration as defined in Figure 2-1 is used, this means the height-to-width ratio remains constant, while the mesh-refinement is changed. As can be seen in Figure 2-2 there are some notable changes in the optimization configurations. The number of elements thus influences the structural optimization result.

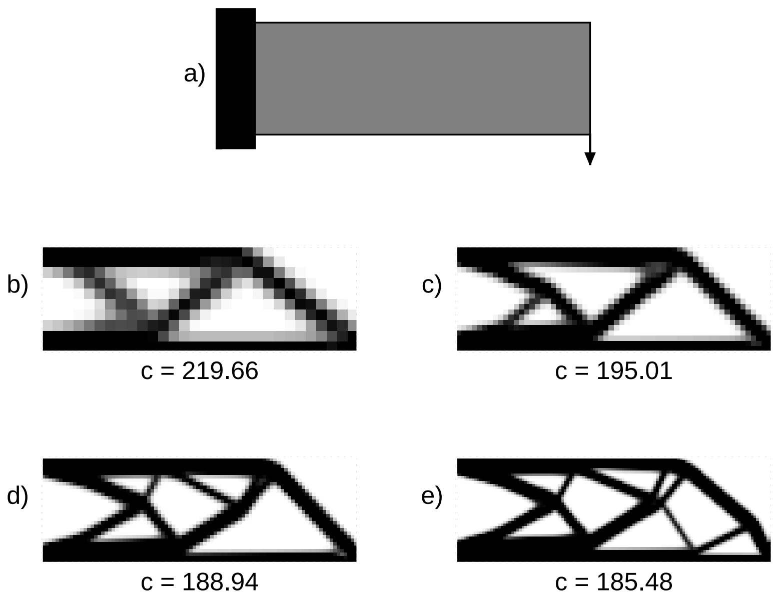


Figure 2-2: Dependency of topology on the mesh refinement using the SIMP method. The beam is discretized by b) 30 x 10, c) 60 x 20, d) 90 x 30, e) 120 x 400 elements (Appendix A-3).

2.1.3 Volume fraction

As already stated in (2.1.1) the volume fraction is restricted to 50% of the solid weight, up to here. In this section the volume fraction is noticed. This volume fraction is derived from the maximum allowable weight of the structure. As already considered before, a big advantage of topology optimization is weight reduction. Although this volume fraction is most of the time seen as the biggest restriction of the optimization, in Figure 2-3 an example of different volume fraction is shown. This picture can be used to show the differences between certain volume fraction levels. As can be seen, the structure is largely dependent on the volume fraction, the layout is heavily changed when the volume fraction increases. The associated compliances are increasing also. This is pretty clear, since more volume fraction means more available material, which leads to increasing stiffness. The compliances in this example are thus not that suitable for comparison.

2.2 Solution methods

As already mentioned before, the used method to produce the optimization is the SIMP method. This method is just a way to transfer the original structure to the optimum structure in topological perspective. A Finite Element Analysis is used for calculating the problem. This method is very useful to calculate stiffness values. Since each element is described by four nodes, shear locking can occur. Methods to overcome this problem are not within the scope of this research, however. Although in section (2.1) the method is just used, without

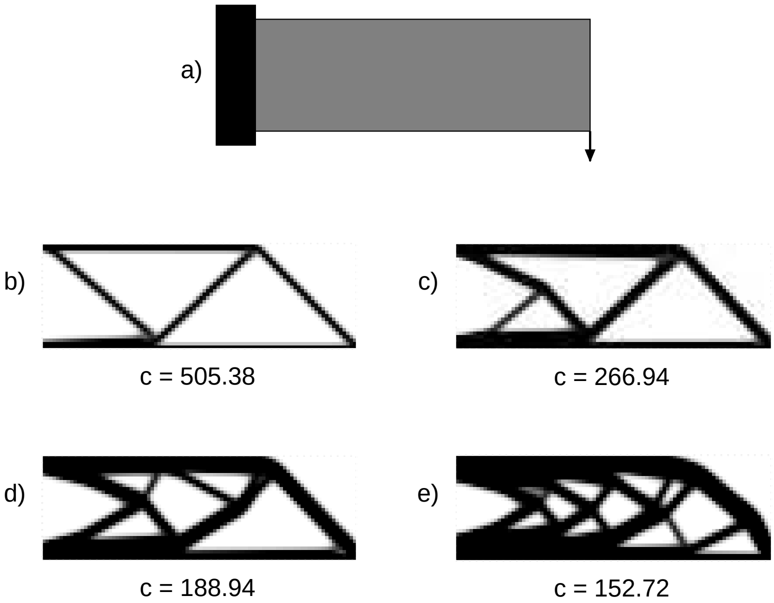


Figure 2-3: Different topologies for different volume fraction. The volume fraction is given as b) 20%, c) 35%, d) 50%, e) 65% (Appendix A-4).

any explanation; the method however, will get some more attention in this section. There are some more optimization methods around, this literature survey will only focus on the main two solution methods that are around and being used these days. Due to an overview and an example of comparison, the best method will be chosen for further usage in the literature survey.

2.2.1 SIMP method

As already can be deduced from Figure 2-2, an increasing amount of discretization elements results in a more complicated, porous, structure. But at the other hand we want many discretized elements, in order to mimic the reality. Now let's have a closer look at the Figure 2-2, it can be seen that Figure 2-2b consists of several *gray* regions, which is not desirable. Topology optimization should preferably result in a solution with only void (0) or solid (1) regions. These regions represent no or full material, respectively. Several regions in Figure 2-2b however, represent a gray region, which could be physically defined as a material with only a part of the element's density. By stating this, it can be concluded that gray regions are undesirable. To work around with this problem the *Solid Isotropic Microstructure with Penalization* (SIMP) can be used to avoid this (Rozvany et al., 1992). Reminder: only isotropic materials are considered. The SIMP approach is used to make intermediate densities unattractive, we are looking either for no (void regions) or full (solid regions) densities. Now recap (2-2), where the stiffness matrix K is mentioned. Suppose the discretization model to hold. For each jth element in the structure, a maximum stiffness K_0 can be derived, which corresponds to a fully solid element. The stiffness of the optimized element K_j is derived

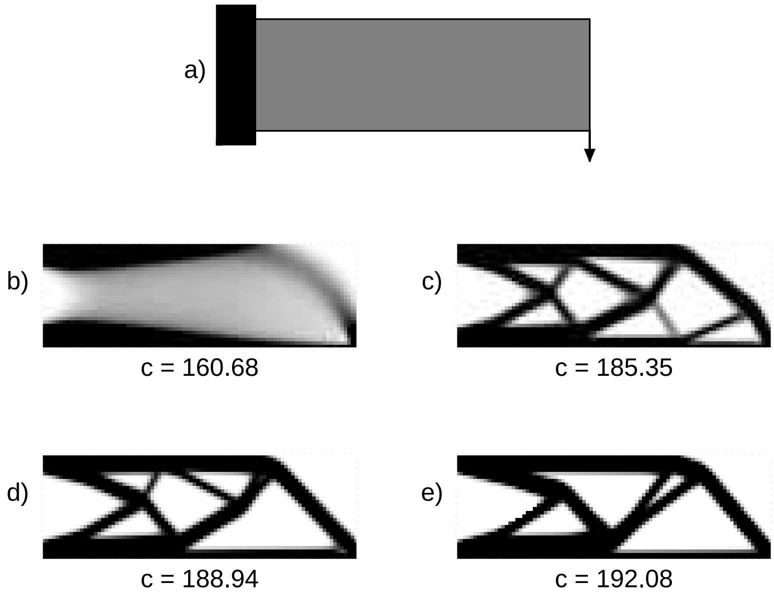


Figure 2-4: Different topologies for different penalization power. The penalty is defined by b) $p = 1$, c) $p = 2$, d) $p = 3$, e) $p = 5$ (Appendix A-5).

using the maximum stiffness and the density of this jth element. The following equation is a representation of the penalty-function of the SIMP method.

$$K_j = (\rho_j)^p K_0 \tag{2-4}$$

As can be seen, this penalty p, also named penalization power is an exponential function of the density. An example of the actual influence can be seen in Figure 2-4.

Usually, a penalty term of $p \geq 3$ should result in a void-solid division, which is a target in topology optimization. So a greater penalty results in a better result. However, the computational time is increasing also. So again for this parameter a trade-off should be made between precision and time.
In (A.9) the complete optimization scheme is shown. Each part of this scheme will be discussed later on.

2.2.2 BESO method

Besides the SIMP Method, the BESO method sure needs some attention. Using the *Evolutionary Structural Optimization* (ESO), an upgraded version of this method was found. The *Bi-directional Evolutionary Structural Optimization* (BESO) can be used within structural topology optimization (Querin et al., 2000), including compliance mechanisms (Huang and Xie, 2007).

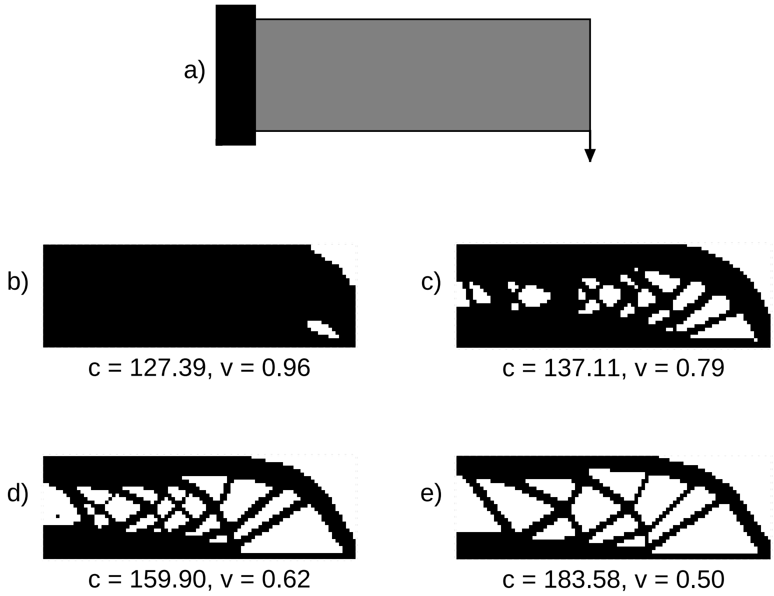


Figure 2-5: Evolution of iterations using the BESO method. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps, c) 25% of total iteration steps, d) 50% of total iteration steps, e) final solution.

The ESO concept can be seen as a process, whereby slowly inefficient material is removed, in order to achieve an optimal result. Here inefficient means not efficient toward the objective function. The BESO method uses this removal process, but also include, at the same time, an addition of material step. This explains the *bi-directional*-term. Within each iteration step, a *Constrain*-step is built in, to check whether or not the predefined constrained volume is conflicted or satisfied. A simplified flowchart of the BESO method can be found in (A.9). As already shown in (2.1.1), for this BESO method also, an evolutionary scheme can be made, just to visualize the whole method. The BESO optimization can be seen in Figure 2-5.

In contrast to the SIMP method, the BESO method starts from a solid structure. Therefore, the final solution can differ from another optimization method. This is mainly caused by the difference in the convergence approach. However, both solutions show the same kind of topology. And of course, both optimization methods can be fine-tuned, in order to achieve equal solutions. But for this time, only the standard parameters are considered.

2.2.3 Sensitivity analysis

As already depicted in the flowcharts in (A.9), the Sensitivity Analysis plays an important role in optimization processes. In this section the sensitivities of the compliance example will be explained. As can be seen in (A.9), the loop of the topology optimization starts with a sensitivity analysis. In this analysis the derivatives of the objective function are calculated, with respect to the design variables. In case of the compliance example as defined in (2-2),

the sensitivity can be seen as the derivative of the compliance over density.
In topology optimization is mainly worked with a moderate number of constraints, the *adjoint* method is used. In this method, the derivatives are not explicitly calculated, but a back-substitution is needed for each response and design variable. In order to get some more insight in the actual analysis, let's recap the formulation of (2-2), and combine it with (2-4). The minimum compliance example can now be formulated as:

$$\begin{aligned} \min_{\rho_e} \quad & \mathbf{f}^T\mathbf{u} \\ \text{s.t.} \quad & \left(\sum_{e=1}^{n} {\rho_e}^p \mathbf{K}_e\right)\mathbf{u} = \mathbf{f} \\ & \sum_{e=1}^{n} \nu_e \rho_e \leq V \\ & 0 \leq \rho_e \leq 1 \\ & e = 1, \ldots, N \end{aligned} \tag{2-5}$$

In this formulation a couple of tweaks are made, regarding the previous formulations. Substitution of (2-4) in (2-3) results in (2-5). The stiffness of the total structure is discretized by a number of elements (1 to N), as showed in Figure 2-2. The summation of these elements e results in the total stiffness. The summation of all these element volumes results in the total volume V, while the density of each element should be within the range 0 to 1.
The objective function is minimize compliance, by varying the density. To compute this minimum, the derivative of the objective function should be computed, with respect to the design variables. Using the equilibrium equation $\mathbf{Ku} = \mathbf{f}$, the derivative of the original objective function $c(\rho)$ can be computed:

$$\frac{\partial c}{\partial \rho_e} = \mathbf{f}^T \frac{\partial \mathbf{u}}{\partial \rho_e} \tag{2-6}$$

Keep in mind, the stiffness matrix $\mathbf{K}$ is typically very large. The computation needs to be done over each element e, which results in a very large computational time. In order to work around with this problem, an effective method is to define a zero function, also adjoint function, which will be added to the original compliance problem. Here, the adjoint vector $\tilde{\mathbf{u}}$ represents a fixed, real vector and satisfies the following adjoint equation.

$$\mathbf{f}^T - \tilde{\mathbf{u}}^T\mathbf{K} = 0 \tag{2-7}$$

Now adding this (2-7) to the original compliance example results in (2-8). This formulation is valid for any choice of $\tilde{\mathbf{u}}$, so we can basically take each expression we want. As long as this vector is fixed and real.

$$c(\rho) = \mathbf{f}^T\mathbf{u} - \tilde{\mathbf{u}}^T(\mathbf{Ku}\text{-}\mathbf{f}) \tag{2-8}$$

Now computing the derivative of (2-8) in a similar way of (2-6) results in

$$\frac{\partial c}{\partial \rho_e} = (\mathbf{f}^T - \tilde{\mathbf{u}}^T\mathbf{K})\frac{\partial \mathbf{u}}{\partial \rho_e} - \tilde{\mathbf{u}}^T\frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} \tag{2-9}$$

Using the property of (2-7) result in the short, low-cost equation

$$\frac{\partial c}{\partial \rho_e} = -\tilde{\mathbf{u}}^T\frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} \tag{2-10}$$

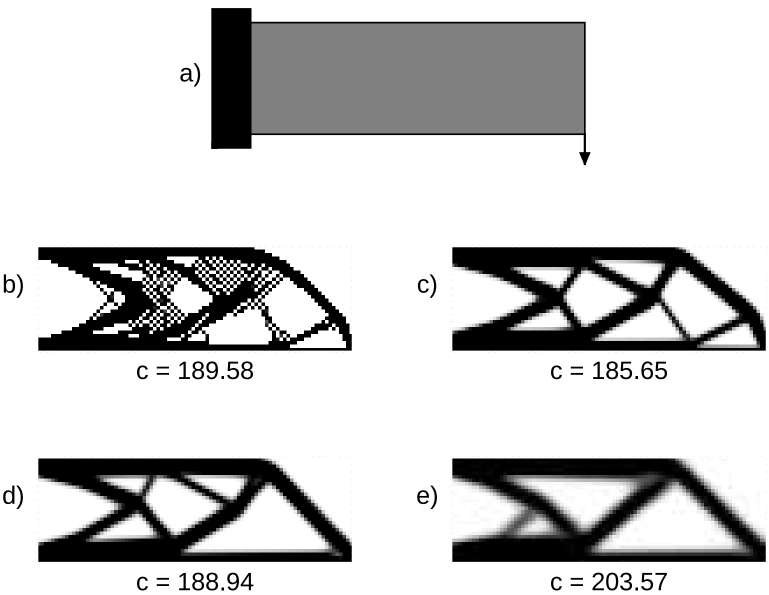


Figure 2-6: The filter radius r is here changed. The filter radius is given by b) r=1.0, c) $r = 1.25$, d) $r = 1.5$, e) $r = 3$ (Appendix A-6).

Now replacing the regular stiffness matrix $\mathbf{K}$ with the penalty-termed stiffness as derived in (2-5), results in:

$$\frac{\partial c}{\partial \rho_e} = -p({\rho_e}^{p-1})\mathbf{u}^T\mathbf{K}_e\mathbf{u} \tag{2-11}$$

Which can be seen as the sensitivity of the optimization problem. Please keep in mind; the derivatives depicted in (2-6) and the subsequently derived derivatives assuming that $\mathbf{f}$ is not dependent on the element's densities ρ_e, which is not very common.

2.2.4 Filtering

The next step in optimization, as can be seen in (A.9) is a filtering technique. The calculated sensitivities are filtered, in order to prevent so-called checkerboard patterns (Sigmund and Petersson, 1998). An increased number of elements will not automatically lead to a solution that can actually be additive manufactured. The additive manufacturing has its own resolution, in order words, the minimum thickness it can produce. To work around with this problem, a filter radius can be used in the topology optimization scheme. By modifying the element sensitivities of the compliance, using a filter radius, a weighted average of the element itself and its eight surrounded elements can be made.
Using this weighted average, the iteration scheme determines a solution, which fulfill the filter radius specification. To get some more insight of this working principle, an example is made and can be seen in Figure 2-6.

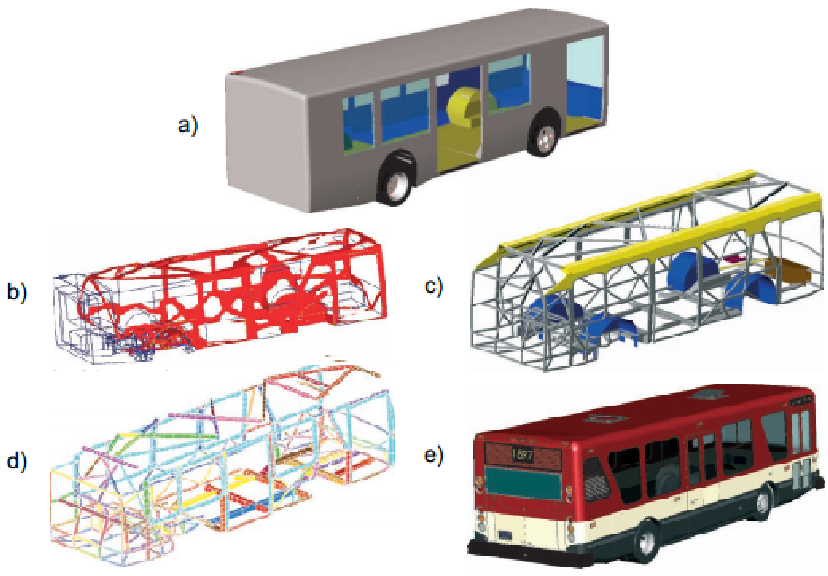


Figure 2-7: Design of a lightweight city bus. a) Initial design, b) topology optimization, c) CAD representation of the topology optimization, d) sizing optimization, e) final design (Thomas et al., 2002).

As can be concluded, a filter radius too low results in a checkerboard problem, which may be not manufacturable. By varying the filter radii this problem can be overcome. However, picking a filter radius too high can result in a non-optimal solution, since this will lead to thicker material trusses.

2.3 Applications

Topology optimization can be seen as a very effective way of creating optimum structures. As already explained in (2.1), it can be used to maximize stiffness for a lighter structure. Now let's have a look at the actual practical examples of the topology optimization. And after, the main focus of the literature survey will be explained.

2.3.1 Statics

A very interesting example is an optimization of a city bus. The main objective here is to reduce the weight of the bus, by doing this the gasoline and thus fuel costs can be reduced. Using different optimization programs the final bus design is modeled. The shape of the windows was decided by the results of the structural topology optimization. A framework of this process can be seen in Figure 2-7

Another example of the need of topology optimization is found in the MEMS industry, for example micro-scale compliant mechanisms. A common challenge in MEMS is to produce very little prescribed displacements. Using topology optimization can be very useful to fulfill this need. So in this case, the topology optimization is not mainly used to reduce weight for example, but it is used to actually achieve a certain goal. By varying the associated objective functions and constraints, a lot of possibilities can be defined in topology optimization.

2.3.2 Dynamics

The benefits of topology optimization in statics is straightforward. However, the optimization can also be of need in the dynamics.
Up to now, only statically loaded structures are considered. However, periodically loaded structures can also be optimized using structural topology optimization. Dependency of the optimum topology is shown for a structure with respect to different excitation frequencies (Ma et al., 1995).
But one can also think of the need of topology optimization to achieve a certain target in the dynamical domain. For example a structural topology optimization of vibrating structures, with specified eigenfrequencies and eigenmodes (Maeda et al., 2006). Here topology optimization is used to achieve a high eigenfrequency for example. Here this eigenfrequency can be seen as an objective function which should be maximized.

2.3.3 Other domains

Upcoming research is done in fluid design. For example the optimum structure of a channel to achieve a certain velocity and Reynolds number. Or in the (micro)fluidics, for example in micro mixers. Here topology optimization is used to optimize the mixing process of certain fluids (Andreasen et al., 2009).
Work is done in multiphysics, although there is only one physics, this term is widely used in engineering. Within this multiphysics multiple domains are coupled together to achieve a realistic behavior. While designing a micro-actuator in MEMS, thermal and electrical behavior interfere. The coupling of these domain results in a multiphysics actuator. Topology optimization can be used for both domains, and both domains can be coupled together, in order to achieve the overall optimal actuator (Sigmund, 2001b).

2.4 Design of supports

While designing compliant mechanisms we have considered a structure, with boundary conditions and objective functions. Although the boundary conditions for the support are not defined in (2-2), the compliant example does include a clamped end on the left hand, as can be seen in Figure 2-1a. However, the main objective is to maximize stiffness, minimize compliance. The position of the support can maybe changed in this example, while aiming at minimizing compliance. If this support can be varied, the support should be placed right under the load case. This results in a zero displacement and consequently infinite stiffness. Different supports will lead to different optimum structures, which is pretty straightforward. In this section, the design of supports will be discussed, which is also the main target in the upcoming literature survey and sequential thesis project.

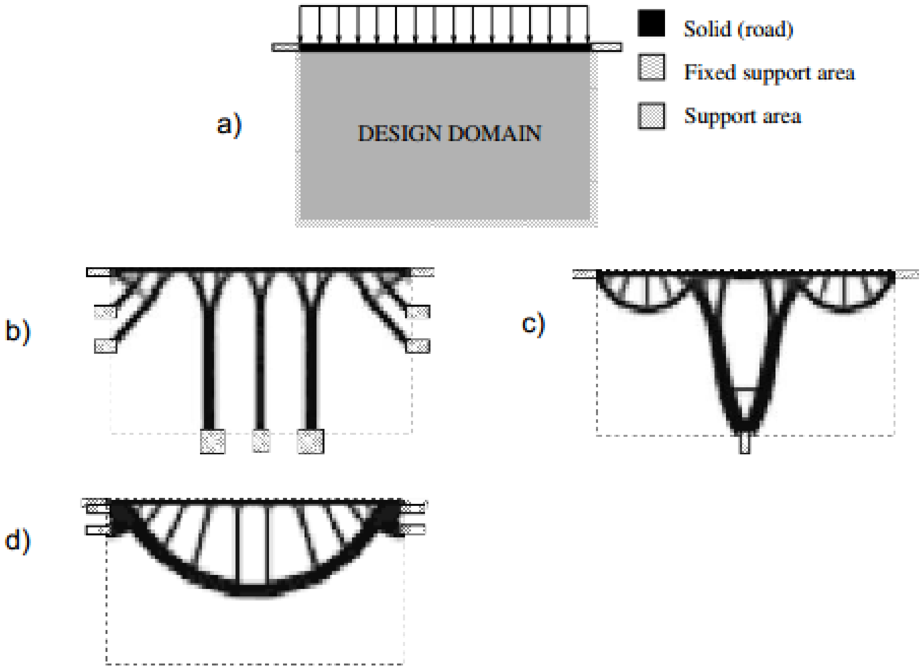


Figure 2-8: Example of design of supports with different support cost functions r_c a) Initial design, b) $r_c = 1$, c) $r_c = 10$, d) $r_c = 20$ (Buhl, 2002).

2.4.1 Optimizing supports

When optimizing the design of support, the prescribed support locations, as seen in Figure 2-1a are not longer prescribed, but interpreted as a design variable. A well-known bridge example is shown (Buhl, 2002). Here a bridge is designed and optimized to make a road in a deep canyon. In this Bridge example three cases are considered, as depicted in Figure 2-8.
In this example a pavement is modeled as a solid, clamped, side at the top of the design domain. The road experiences a distributed force as a representation of continuing traffic over the bridge. The bridge is fixed to the upper left and upper right edges. The sides and the bottom of the design domain are considered as possible support areas. A volume constraint of 20% is applied, the number of support constraint should yield maximum 20% of the total number of supports.
In Figure 2-8b, the cost of support is equally distributed for the sides and bottom. This ratio of cost $r_c = 1$, so without any other constraints, this optimization should be the perfect bridge structure with this constraints, and will result in the minimal compliance.
The pillars however, could be very expensive, or hard to place under this bridge. The second optimization Figure 2-8c is award a ratio of cost $r_c = 10$, this means a linear support cost function from 1 to 10 (top edge to bottom edge). Therefore, material at the bottom is undesirable, as can be seen only one support remains. In the third case Figure 2-8d a cost function of $r_c = 20$ is applied. By doing this, the support material at the bottom is very unlikely, and no pillar exists anymore. An application example could be a very deep canyon, where pillars are unwanted, but a maximum stiffness is wanted.
Design of supports is a promising optimization technique and can be used in a wide range of applications. This literature survey will continue in the next chapter onto this optimization

domain. A concrete working direction will be defined and further investigation will continue on this subject.

2.5 Conclusions

Structural topology optimization is a very promising way of achieving several benefits. These benefits can vary from active money saving, using less structural material (2.1), to passive money saving, the bus example (2.3.1), where removal of material results in a lighter bus and less fuel costs. Topology optimization can also be used to achieve specific targets, for example in the MEMS industry (2.3.1) and within the dynamics domain. Vibrating structures can be optimized (2.3.2) to achieve desired eigenfrequencies or eigenmodes.

We consider three different categories of optimization (1.1), namely: sizing, shape and topology optimization. Sizing optimization only changes for example cross-sections of a truss structure. Shape optimization changes the shape of the material, without removing or adding material. Topology optimization defines an optimal topology solution for a given problem, this is the main target of this literature survey.

Topology optimization can be done with several solution methods (2.2). In this survey two main methods are considered. The SIMP method (2.2.1) uses a penalization method, to prevent so-called grey regions in the optimal solution, since the material should be void or solid, and not partially present. The SIMP method optimizes with respect to the constraint, the best objective function. The BESO method (2.2.2) combines addition and removal of material until it reaches the volume constraint. An overview of both optimization processes can be found in (A.9).

There are however some considerations with topology optimization. A sensitivity analysis (2.2.3) is made, followed by associated filtering, to prevent checker-boarding (2.2.4) patterns. To remove this non-realistic solution, in terms of manufacturing, the filter radius can be tuned. Setting the radius too low results in checker-boarding, but setting the radius too high can skip other optimal solutions by not allowing fine features to emerge.

The design of supports will play a big role in the upcoming literature survey and sequential thesis project. Varying support locations results in other optimization results (2.4). This design of supports can also be used to actively achieve an optimal solution regarding the actual number and placement of supports. The bridge example (2.4.1) showed a way to improve a structure, with respect to external factors. A deep canyon could be very unlikely to support using pillars, although this will result in a stiffer construction. Using a ratio of support cost can give a mathematical insight in the relation between cost of supports and stiffness. Especially in compliance problems the exact support location may not be fixed, and some relaxation of this support location can lead to actual really good optimization results. In the upcoming chapter some deeper investigation will be done regarding this subject. Although not explicitly documented, there are some numerical results available of all the executed optimizations can be found in (A.1). These values can be used for upcoming study, in order to make a decision regarding the choice of optimization parameters.

Chapter 3

Topology Optimization for Engineers

In chapter 2 some simple cases and basic properties of topology optimization are described. In this chapter the philosophy and possibilities of topology optimization is taken a step further. Topology optimization for engineers can be used to solve a variety of mechanical problems. The simple solution method, the Optimality Criteria, can be used for simple compliance problems. When dealing with more complex problems, there is a need for a different solution method. The Method of Moving Asymptotes, as described in section 3.1 can be used to overcome this.
When solving mechanical problems, some advanced applications can be helpful, to reflect the actual design problem. In section 3.2 a number of applications are implemented and described. The main advantage of topology optimization is within the additive manufacturing domain, an example to 3D cases is given in section 3.3. In section 3.4 the use of topology optimization within compliant mechanisms is explained. Section 3.5 concludes this chapter.

Using the attached MATLAB codes (B.3 up to B.6 and C.1 up to C.8) the problems in this chapter can be solved.

3.1 Solution method: MMA

Besides the described solution method in (2.2) and up-following (2.2.3), there are some other solution methods around. For simple compliance problems, like the cantilever problems, the Optimality Criteria method from (2.2) can be used. This method is easy, fast and very cost-efficient. The Optimality Criteria method is very useful for compliance problems, since this method always wants to add material, in order to achieve a high stiffness. However, in more complex problems, this method is insufficient.
The Method of Moving Asymptotes (Svanberg, 1987), also known as MMA, is a mathematical programming algorithm which is very suitable for topology optimization. This method can be used to restrict the optimization problem to multiple constraints, and multiple design variables. In the upcoming chapter a number of applications, with the use of MMA will be

shown. The MMA program solves the following optimization problem:

$$
\begin{aligned}
\min_{x,y,z} \quad & f_0(x) + a_0 \cdot z + \sum_{i=1}^{m}(c_i y_i + \frac{1}{2} d_i {y_i}^2) & \\
\text{s.t.} \quad & f_i(x) - a_i \cdot z - yi \leq 0, & i = 1, \ldots, m \\
& {x_j}^{min} \leq x_j \leq {x_j}^{max}, & j = 1, \ldots, n \\
& y_i \geq 0, & i = 1, \ldots, m \\
& z \geq 0 &
\end{aligned}
\tag{3-1}
$$

In this formulation, f_0 is the objective function, while f_i represents the constraint functions, defined by the number of constraints m. A vector of design variables x will be updated, using y and z as positive optimization variables. This vector should be in-line with the number n of defined constraints. The programming parameters $a_0,\ a_i,\ c_i,\ d$ are so-called *magic numbers of MMA* and can be used to determine the type of optimization problem.
In order to use this method for a compliance problem, the author of (Svanberg, 1987) suggested some MMA constants: $a_0 = 1,\ a_i = 0,\ c_i = 1000,\ d = 0$. Using these constants and at the same time writing the function in terms of a compliance problem, results in:

$$
\begin{aligned}
\min_{x} \quad & \mathbf{c}(\mathbf{x}) & \\
\text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, & i = 1, \ldots, m \\
& 0 \leq x_j \leq 1, & j = 1, \ldots, n
\end{aligned}
\tag{3-2}
$$

Here, the optimization variables y and z should be zero at the optimum. The vector of design variables $\mathbf{x}$ is in this example just the density of each element.
This routine is implemented using the available MMA-code, which can be found in (D.1) and (D.2)

3.1.1 OC versus MMA

In this section a comparison between the Optimality Criteria (with density filter) and the MMA routines is made. As already stated before, the MMA-routine is very useful, dealing with multiple constraints, while the OC-routine is not handy for these types of problems. In order to make a good comparison, the simple compliance problem from (2.1.1) will be optimized using these two routines. A comparison will be made regarding the final compliance, as well as the number of iterations and total optimization time. An evolutionary scheme, related to Figure 2-1 is produced. In this problem, the Optimality Criteria is applied, using sensitivity filtering. Although this method is usable in practice, it is mathematically inconsistent. Density filtering is a solution to overcome this. As can be seen in Figure 3-1, both methods will produce a somewhat same result, but there are some differences notable. When looking at the process, it seems like Figure 3-1a goes slightly faster towards its final state, while the MMA (Figure 3-1c) needs some more time to get a slightly better result, in contrast to Figure 3-1a. The number of iterations displays some interesting results: the Optimality Criteria needed 40% more iterations to get the final result, with respect to the MMA. The computational time however, is in favor of the Optimality Criteria. The OC method is approximately four times faster than the MMA.

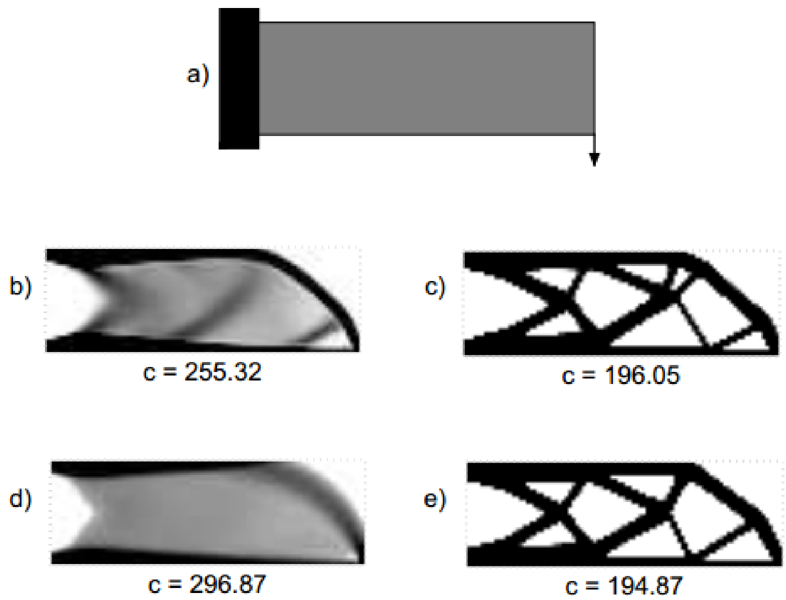


Figure 3-1: Evolution of iterations using the SIMP method. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps of Optimality Criteria, c) final solution of Optimality Criteria, d) 5% of total iteration steps of MMA approach, e) final solution of MMA approach (Appendix A-7).

So for this particular example, the MMA results in a better, stiffer result with less iterations with respect to the OC. However, the calculation time for each iteration step is a lot longer with respect to the OC. As a reference, all the calculated data can be found in (Appendix A-7).

3.2 Advanced applications

Using the now defined code, a lot of tweaks and application can be made. In this section a small amount of useful applications will be depicted. First, some words will be stated about restrictive regions, i.e. active or passive area's in the design domain. Second, an example of multiple load cases will be discussed. The next subsection results in some thoughts about self-weight of a structure. An example of a compliant mechanism synthesis will be made. And at last, but not at least, an example of a 3D problem will be displayed, just to give some more insight in topology optimization.

3.2.1 Restrictive regions

Topology optimization in general, can be used for a variety of open problems, in some cases, however, some restrictions should be implemented in the optimization problem. A particular example is to implement a so-called passive region. In this region, there should be zero

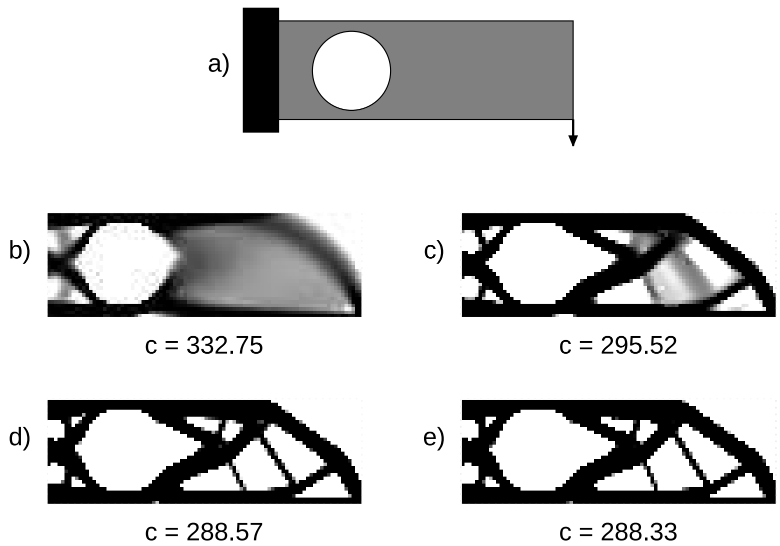


Figure 3-2: Evolution of iterations using the SIMP method and the application of a passive region. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps, c) 25% of total iteration steps, d) 50% of total iteration steps, e) final solution (Appendix A-8).

material, for example because a certain space needs to be free from material to apply a screw. As can be seen in Figure 3-2, a passive region is implemented by a circular area, which always should remain free from any material. The evolution of iteration steps give a nice view in this process.
The same procedure can be applied for active regions. In certain cases it could be very helpful to pinch material on certain places, for example for adhesive purposes. In this case, depicted in Figure 3-3 the evolutionary scheme gives a very clear view on the process.

3.2.2 Multiple load cases

Some problems can occur when defining multiple loads. A choice can be made whether to choose one or multiple load cases. Each choice will result in a different solution. When applying simultaneous load cases, the optimization solution will act as if it is an optimization of the equilibrium of the two loads. When using separated load cases, the structure will be more resistant to buckling and much stiffer when one of the loads is removed, with respect to the single load case. Of course, this will result in a longer computational time.
A small example of this load case dilemma can be found in Figure 3-4. Clearly, Figure 3-4 only make sense when both loads act simultaneously.

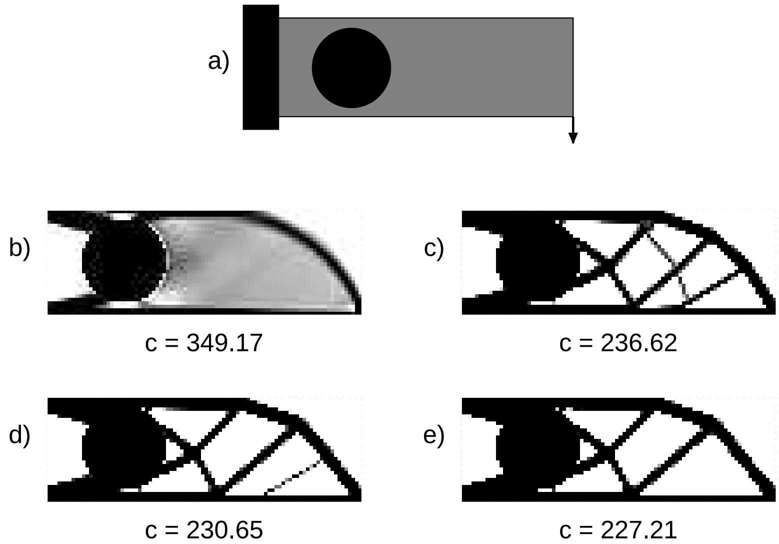


Figure 3-3: Evolution of iterations using the SIMP method and the application of an active region. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps, c) 25% of total iteration steps, d) 50% of total iteration steps, e) final solution (Appendix A-8).

3.2.3 Self-weight implementation

Up to here, the influence of gravity is not taken into account. However, when optimizing towards an optimal solution in reality, gravitational force should be implemented. By implementing this self-weight, some density ρ_0 should be used, to give each element a natural density, when the element is a complete solid region. This ρ_0 can be used to reflect the material properties of the optimization material.
When this ρ_0 is set to zero, the influence of self-weight is completely removed. The total resultant self-weight can be calculated by a summation of all the weights of the elements, as a combination of gravitational force g, the optimization density, and the material density ρ_0. This total resultant self-weight force can be compared to the external force. A weight factor W is introduced, as a ratio of the resultant gravitational force to external force. When this ratio is zero, no self-weight is taken into account. When this ratio is high, the optimization routine tends to neglect the external force, as this becomes only a fraction of the total force. In each iteration, a calculation of the current self-weight is made. Each element, combined with an element density, is divided to its four nodes. These four nodes then experiences a gravitational force of one fourth of the element density times the material density ρ_0.
This extra term of force needs an adjustment on the sensitivity analysis, as derived in (2-11). The self-weight acts like an external force, the derivative of the compliance of this force needs to be added to the original sensitivity analysis, to account for this. The updated sensitivity can be seen in (3-3).

$$\frac{\partial c}{\partial \rho_e} = 2\mathbf{u}^T \frac{\partial \mathbf{f}_{sw}}{\partial \rho_e} - p({\rho_e}^{p-1})\mathbf{u}^T \mathbf{K}_e \mathbf{u} \tag{3-3}$$

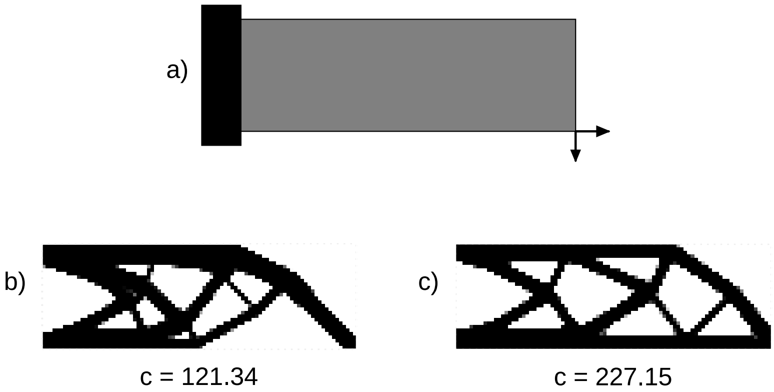


Figure 3-4: Solution regarding different load cases. a) One load case: vertical, horizontal, b) Two separated load cases (Appendix A-9).

Using the same SIMP method as before, a problem comes up. When optimizing in the lower density area, the ratio of the first and the second term in (3-3) becomes crucial and tends to prevent a complete solid/void pattern for the solution. An alternative interpolation scheme could overcome this problem. A linear profile is selected, under a certain pseudo-density ρ_c. Above this pseudo-density, a penalized E_p is calculated, just like before (Bruyneel and Duysinx, 2005). The interpolation scheme can be seen in (3-4). An example of self-weight implementation can be seen in Figure 3-5. Here, a penalty factor of $p = 5$ is used, in order to force a black-white solution.

$$E_p = \begin{cases} \rho^p E_0 & \rho_c < \rho \leq 1 \\ \rho({\rho_c}^{p-1}) E_0 & 0 < \rho \leq \rho_c \end{cases} \tag{3-4}$$

3.2.4 Continuation method

With the introduction of the self-weight, as explained in (3.2.3) some serious problem regarding the optimal solution comes up. Since the introduction of additional (self-weight) forces, the chance of getting close to the global optimum has decreased. One way to overcome this problem is by implementing a so-called continuation strategy (Groenwold and Etman, 2010). While using this continuation method, an unpenalized material distribution is used, for the first number of computational cycles. After a certain number of iterations, the penalty is increased with each iteration, up to a predefined maximum penalty. When this maximum penalty is achieved, this penalty is used along the iteration scheme; up until the convergence criterion is met.

$$p^i = \begin{cases} 1 & i \leq 20 \\ min\left\{p^{max}, 1.02 \cdot p^{i-1}\right\} & i > 20 \end{cases} \tag{3-5}$$

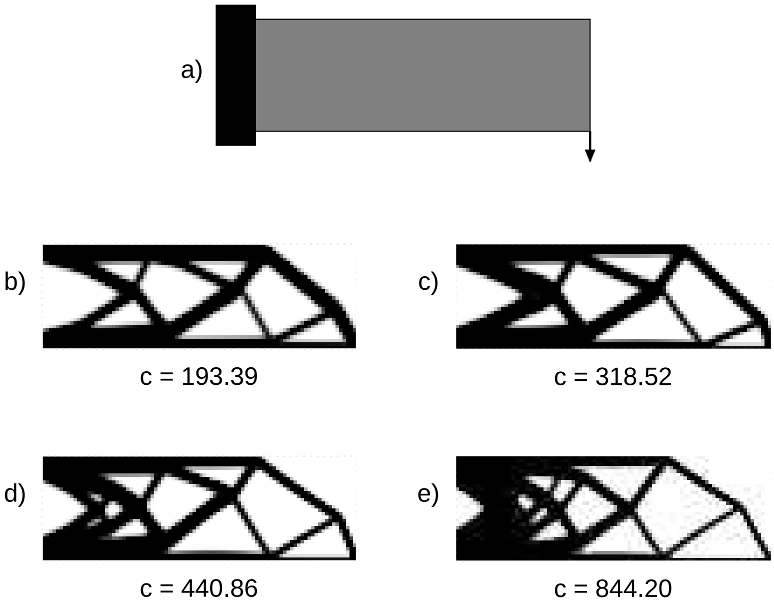


Figure 3-5: Influence of self-weight of the structure. A weight factor W can be defined as the ratio of the resultant gravitational force to external force. a) design problem, b) no self-weight ($W = 0$), c) $W = 1$, d) $W = 2$, e) $W = 5$ (Appendix A-10).

In (3-5) the increasing factor of 1.02 can be varied in the code, this value, however, seems to be a decent value for the compliance problems. The threshold value of the penalization (20) can also be varied.

3.2.5 Different filter techniques

Up until now, sensitivity filtering is used. Although this method is usable in practice, it is mathematically inconsistent. Density filtering is a solution to overcome this (Bourdin, 2001). The density filter transforms the original densities ρ_e to filtered densities $\tilde{\rho_e}$.

$$\tilde{\rho_e} = \frac{1}{\sum_{i \in N_e} H_{ei}} \sum_{i \in N_e} H_{ei} \cdot x_i \tag{3-6}$$

In this equation (3-6) the filtered density is computed by taking a weight factor H_{ei} over the set of elements N_e. This weight factor H_{ei} is zero outside the filter radius, while the operator $\Delta(e, i)$ is defined as the distance between the center of element e and the center of element i. The weight factor H_{ei} is defined as:

$$H_{ei} = max\left\{0, r - \Delta(e, i)\right\} \tag{3-7}$$

The sensitivities with respect to the design variables ρ_e can be calculated accordingly:

$$\frac{\partial c}{\partial \rho_e} = \sum_{i \in N_e} \frac{\partial f}{\partial \tilde{\rho_e}} \frac{\partial \tilde{\rho_e}}{\partial x_i} \tag{3-8}$$

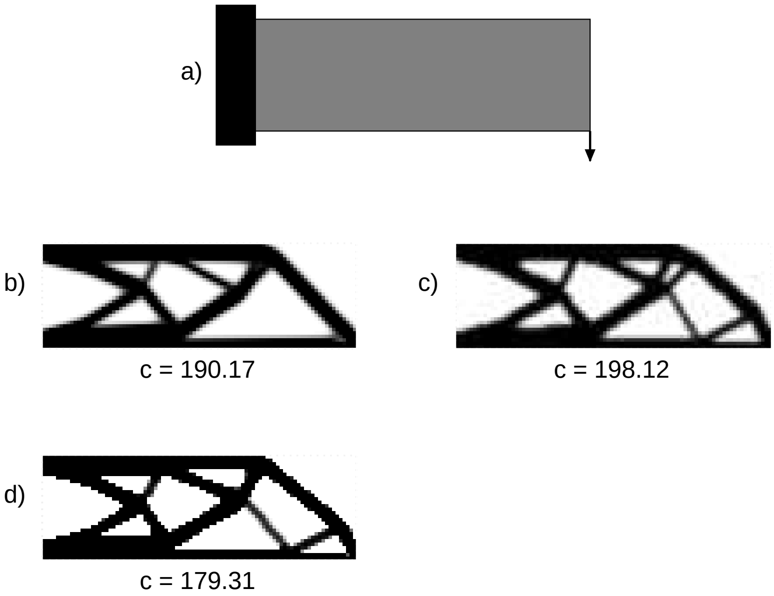


Figure 3-6: Influence of different filter techniques, using a OC solution method and continuation method. a) design problem, b) sensitivity filter, c) density filter, d) heaviside projection filter (Appendix A-11).

Another problem that could come up, when using self-weight, is the existence of gray patterns. As already discussed before, we prefer to produce a black-to-white pattern, to be able to actually produce the optimum result using additive manufacturing.
One way to obtain a black-and-white solution is using the Heaviside projection filter (Guest et al., 2004). The Heaviside filter can be seen as an upgrade of the density filter. This step function projects the filtered density $\tilde{\rho_e}$ to a projected filtered density $\bar{\rho_e}$. This $\bar{\rho_e}$ is defined as:

$$\bar{\rho_e} = \begin{cases} 1 & \text{if } \tilde{\rho_e} > 0 \\ 0 & \text{if } \tilde{\rho_e} = 0 \end{cases} \tag{3-9}$$

Since a gradient-based optimization is used, a smooth formulation for this Heaviside projection can be defined

$$\bar{\rho_e} = 1 - e^{-\beta\tilde{\rho_e}} + \tilde{\rho_e}e^{-\beta} \tag{3-10}$$

In this equation (3-10) the parameter β can be used to make a smooth approximation. This β is gradually increased from 1 to 512 by multiplying this value by 2 at every 50 iterations. Of course, this can be varied, but literature study suggests this approach. Also, this method should adjust the sensitivities of the function $f(\bar{\rho_e})$, with respect to the filtered densities $\tilde{\rho_e}$ accordingly, as can be seen in (3-11).

$$\frac{\partial f}{\partial \tilde{\rho_e}} = \frac{\partial f}{\partial \bar{\rho_e}} \frac{\partial \bar{\rho_e}}{\partial x_i} \tag{3-11}$$

A comparison of the three used filter methods can be seen in Figure 3-6.

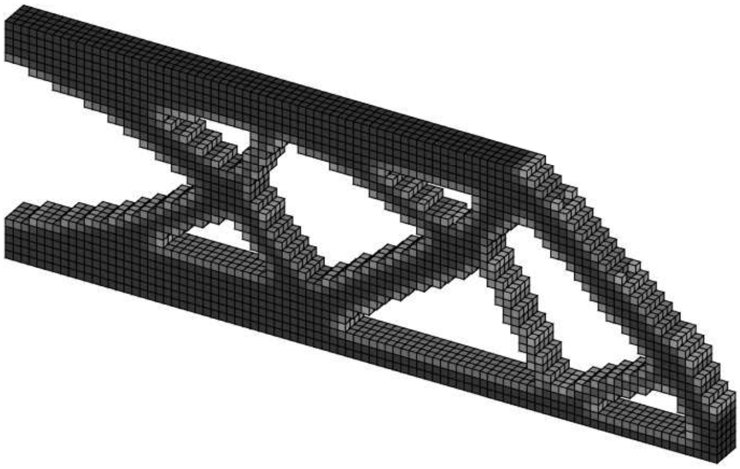


Figure 3-7: Compliance example of 3D. The design problem: clamped on all degree of freedoms on the left hand side of the beam, one downward load, attached at the right-hand side, at the bottom node in the y-direction, at the middle node in the added z-direction.

3.3 Turning 2D into 3D

While most topology optimization problems are displayed as 2D-results, the main advantage of topology optimization is found in relation to additive manufacturing. Having a 3D implementation is thus crucial. In order to work around with this additional dimension, a tweaked code is produced, which is able to calculate a 3D optimization problem.
Another dimension will add also an additional computational load. For now, only one example is depicted, just to show a working code. As can be seen in Figure 3-7, the same loads and constraints are applied. However, the third added dimension z, is now also implemented in this problem. The clamped left-hand side is fixed for all its degree of freedom, including the z-direction. A simple point load is applied to the right, in the middle of the z-direction. Because of the small number of elements in this z-direction, no difference can be found in the distribution of the elements in this z-direction. However, when discretizing in more elements, an expected discrepancy can be seen. While the example shown in Figure 3-7 is pretty clear and easy, the code actually has some more options. All the previous advanced applications are now available. An additional restrictive region is implemented, in the sense of a sphere, which can be either active or passive. The solution method can be varied, as well as the filter method.

3.3.1 Gray-scale filter

A new filter is introduced, namely a gray-scale filter. This gray-scale filter is a very powerful filter overlay to enable white-black regions (Groenwold and Etman, 2009). Because of its easy implementation and proven effectiveness for 3D applications (Liu and Tovar, 2014), this new filter is introduced. Gray-scale filter is used to further achieve black-white regions, by

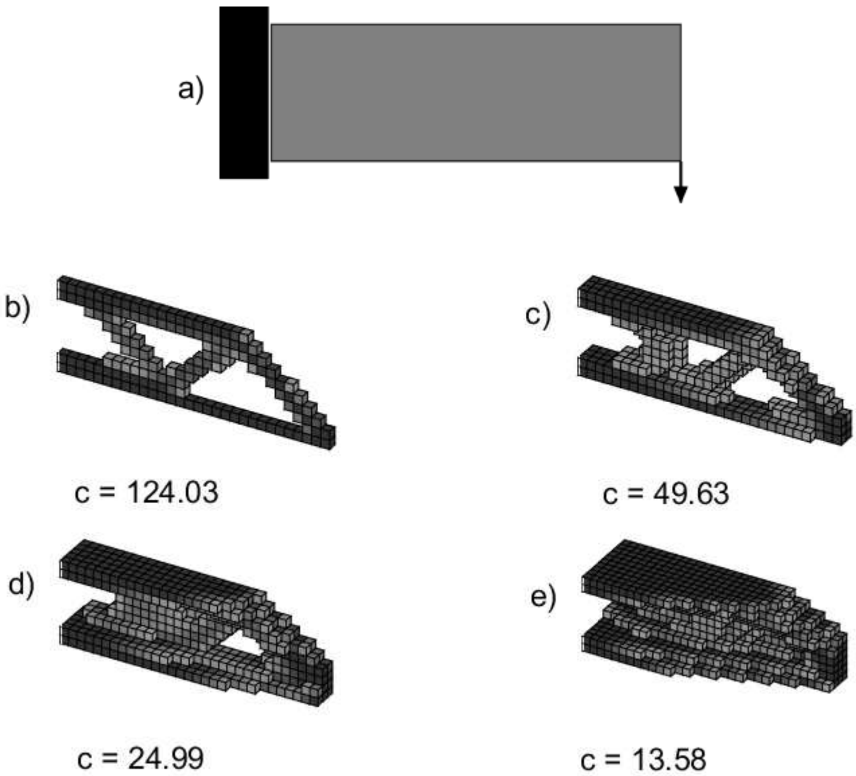


Figure 3-8: Influence of lateral elements of the structure. The number of lateral elements in the z-direction are varied. A continuation method is used, as well as a sensitivity filter with gray-scale filtering. The beam is discretized by b) 30 x 10 x 1, c) 30 x 10 x 3, d) 30 x 10 x 5, e) 30 x 10 x 10 elements (Appendix A-12).

introducing an exponent q. The working principle of gray-scale filtering can be seen in (3-12). The standard Optimality Criteria is a special case of gray-scale filtering with $q = 1$.

$$x_i{}^{new} = \begin{cases} max(0, x_i - m) & x_i B_i{}^n \leq max(0, x_i - m) \\ min(1, x_i + m) & x_i B_i{}^n \geq min(1, x_i - m) \\ (x_i B_i{}^n)^q & otherwise \end{cases} \tag{3-12}$$

The main advantage of the three dimensional optimization is of course the third dimension. This number of lateral elements can be varied, to see some interesting results. In Figure 3-8 this variation of lateral elements can be found. The force is pointed downwards, just like the simple 2D cases. This force however, is kept at the same spot each variation, in order to actually see some really nice results.

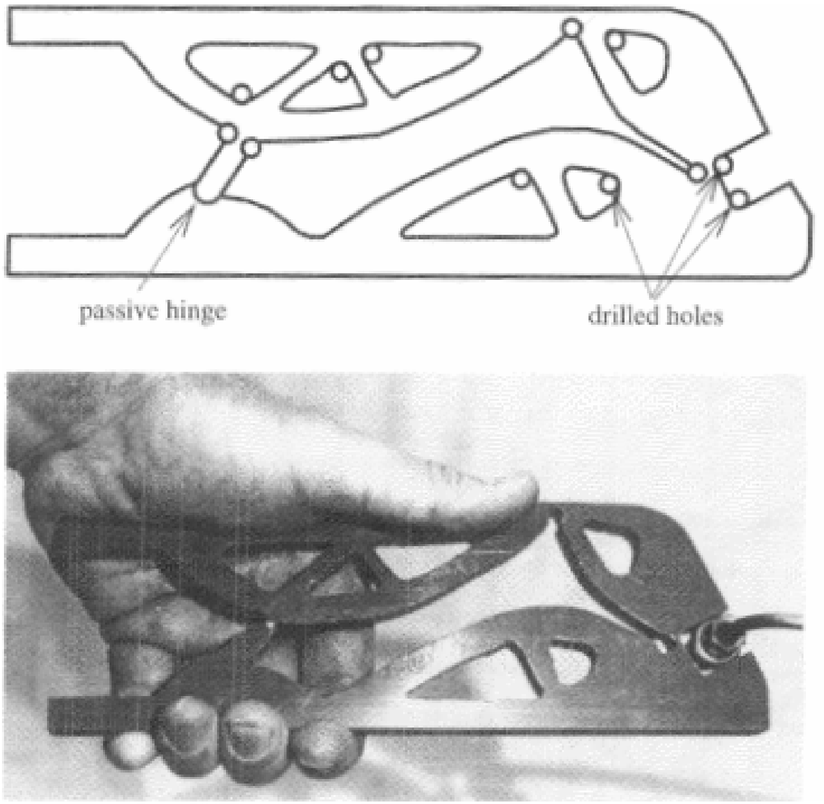


Figure 3-9: Interpretation and realization of a hand tool (Sigmund, 1997)

3.4 Compliant mechanisms

Compliant mechanisms are very popular nowadays. Besides to the compliance examples, which mainly rely on their stiffnesses; compliant mechanisms are used for their mobility. This mobility comes mainly from the flexibility of the mechanisms. These mechanisms can be manufactured easily with 3D printing, so the need for topology optimization is quite big. Especially within the MEMS-domain, these compliant mechanisms can be helpful. Using topology optimization, the optimal structure of very small compliant mechanisms can be designed. A typical example of the use of compliant mechanism design can be seen in Figure 3-9.

3.4.1 Inverter and amplifier

As can be seen in Figure 3-9 an inverter can be useful within the domain of small compliant mechanisms. This inverter can be used to invert an input displacement to a reversed output displacement, while maintaining almost the same amount of movement.
While designing these compliant mechanisms, it is very common to have an in- and output displacement, in stead of an in- and output force load. Therefore, a small spring is introduced to the in- and output nodes. These springs will convert the force into a direct displacement. By varying these spring stiffness, one can achieve different inputs and consequentially designs. In order to solve some compliance problems, a design problem is formulated in Figure 3-10. As

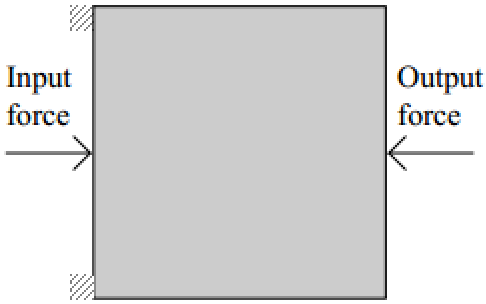


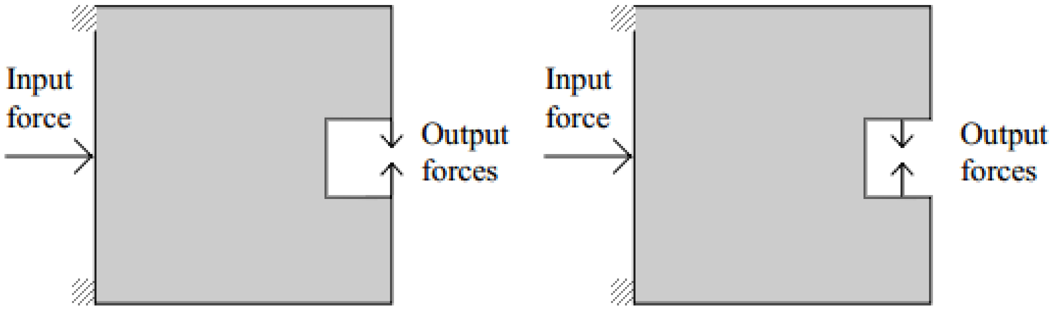
Figure 3-10: Compliant mechanism design problem

(a) Optimal lay-out of an inverter

(b) Optimal lay-out of an amplifier

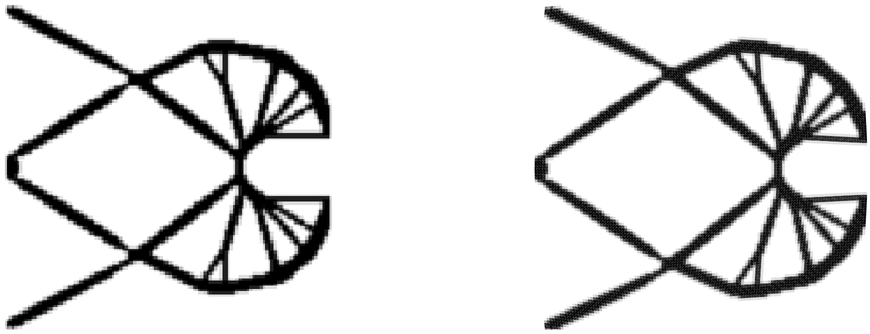
Figure 3-11: Two compliant mechanisms. The inverter inverts a input of $d_{in} = 891$ into an output of $d_{out} = -899$, which results in a displacement gain of $G_d = -1.01$. The amplifier converts an input of $d_{in} = 18.42$ into an output of $d_{out} = -40.89$, which results in a mechanical displacement gain of $G_d = -2.22$ (Appendix A-13).

can be seen in Figure 3-10, these input and output forces are attached to predefined springs, in order to describe a displacement field. Using the boundary conditions at the upper and lower left corners, and using the prescribed input displacement; the optimal topology can be determined for maximizing the output displacement. In Figure 3-11a the optimal topology can be seen for an inverter mechanism. Here, the main objective is to convert a positive input into a negative output, while maintaining the same absolute displacement. In Figure 3-11b an example of an amplifier can be found. This mechanism also converts a positive input into a negative input, but also doubles the amount of displacement at the output side. Keep in mind that these compliant mechanisms are flexible and only can be used for very small displacements. The displacement patterns of both the inverter and amplifier are correct, however, not included in this report. Due to the geometry of the structures, as well as using small displacements, the deformed shape of the structure looks almost the same as the optimal topology lay-out. In this section there is no need to display the deformed shapes. In the next section however, some displacements are plotted for a gripper problem.



(a) Output force at the right of the design domain **(b)** Output force in the middle of the empty gripper domain

Figure 3-12: Design problem for micro-gripper

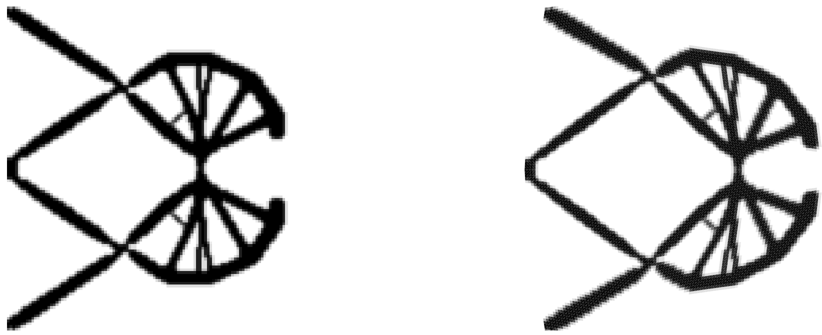


(a) Optimal gripper topology **(b)** Deformed shape

Figure 3-13: Optimal topology and displacement pattern for design problem Figure 3-12a, a horizontal input on the left, a vertical output on the outer right. The gripper inverts a input of $d_{in} = 76.48$ into an output of $d_{out} = 28.06$, which results in a displacement gain of $G_d = 0.73$.

3.4.2 Micro-gripper

In (3.4.1) only horizontal displacements are taken into account. However, when taking a look at Figure 3-9 there is also a conversion step needed to translate horizontal action into vertical output. Two simple design cases are depicted in Figure 3-12. For this particular design problem, the white area can be seen as a restricted area, where no material is allowed, a void region. In Figure 3-12a an output force at the right is requested, in example for gripping a small sphere. In Figure 3-12b the same void region is considered. However, an output force is requested in the middle of the area, for example grabbing a small cube.
The optimal topology for design problem Figure 3-12a can be seen in Figure 3-13a. Here, a maximum volume of 20% is allowed, while respecting the fixtures as described in the problem statement. A displacement field is plotted in Figure 3-13b, where a small displacement input is given. As can be seen, the jaws are slightly pulled together, just enough to grab a sphere. The same solutions, but now for design problem Figure 3-12b, can be found in Figure 3-14a and the subsequent displacement field in Figure 3-14b.



(a) Optimal gripper topology **(b)** Deformed shape

Figure 3-14: Optimal topology and displacement pattern for design problem Figure 3-12b, a horizontal input on the left, a vertical output on the middle of the void region. The gripper inverts a input of $d_{in} = 75.21$ into an output of $d_{out} = 30.48$, which results in a displacement gain of $G_d = 0.81$.

3.5 Conclusions

In chapter 3, a variety of engineering problems are described. The Method of Moving Asymptotes (3.1) seems to be a very effective solution pattern. The computational time is usually somewhat longer, but the final result is more accurate and more importantly, this method is able to solve a wider range of problems, as the OC method is restricted to simple compliance problems. When dealing with predefined regions within the design domain, an option to implement restrictive region (3.2.1) can be very helpful. When solving a physical design problem, the existence of gravity needs to be taken into account, a self-weight implementation (3.2.3) can be used to deal with this.
On the computational side, the continuation method (3.2.4) can be used to gain some speed and accuracy. Varying with different filter techniques (3.2.5) can be helpful to force the optimal solution into a strict black-and-white solution.
Adding a third dimension (3.3) can be used to mimic actual design problems, the computational time however will increase exponentially.
Small compliant mechanisms can be optimized for different objectives. Using different in- and output requirements results in different optimal topology designs. For small displacements only, the displacement field for this elastic material can be plotted on the go, in order to check whether or not the optimal result is working.
All of these described features need to be used for creating an optimal bridge (2.4), which will be the main focus for the next chapter and further research.

Part II

Topology Optimization Extensions: Design of Supports and Loads

Chapter 4

Design of Supports

Design of supports has already been touched in subsection 2.4.1, which will be recalled in Figure 4-1. Support design can be used in a variety of domains, especially when the placement of the support is not prescribed. Also, when a part of the support should be fixed, topology optimization can be used to create additional supports within the design domain, in order to optimize towards the prescribed objective.

In section 4.1 the basic fundamentals of support design are described. The bridge example as shown in Figure 4-1 is solved in section 4.2, including a variety of adaptions and possibilities. The integration of supports in existing layouts is described in subsection 4.3.3.

Compliant mechanisms as described before are also in big interest for design of supports, as discussed in section 4.4. Practical applications of support design are shown in section 4.5. The topic design of supports is concluded in section 4.6.

All the described problems in this chapter can be solved using the attached MATLAB codes (B.7, B.8 and C.9).

4.1 Support design formulation

In order to work with this method of support design, a new set of design variables can be introduced. For all the possible support area, springs are attached on the four nodes of the elements within that area, in vertical and horizontal direction. So each element is now supported by eight springs as depicted in Figure 4-2 This new set of support design variables **z** can now be used to calculate a new stiffness. Just like the SIMP-method (2.2.1), the spring stiffness matrix K_s can be deduced from the maximum stiffness $K_{s,0}$:

$$K_s = (z_j)^q K_{s,0} \tag{4-1}$$

Where q can be seen as a penalization factor of the new design variables, corresponding to the

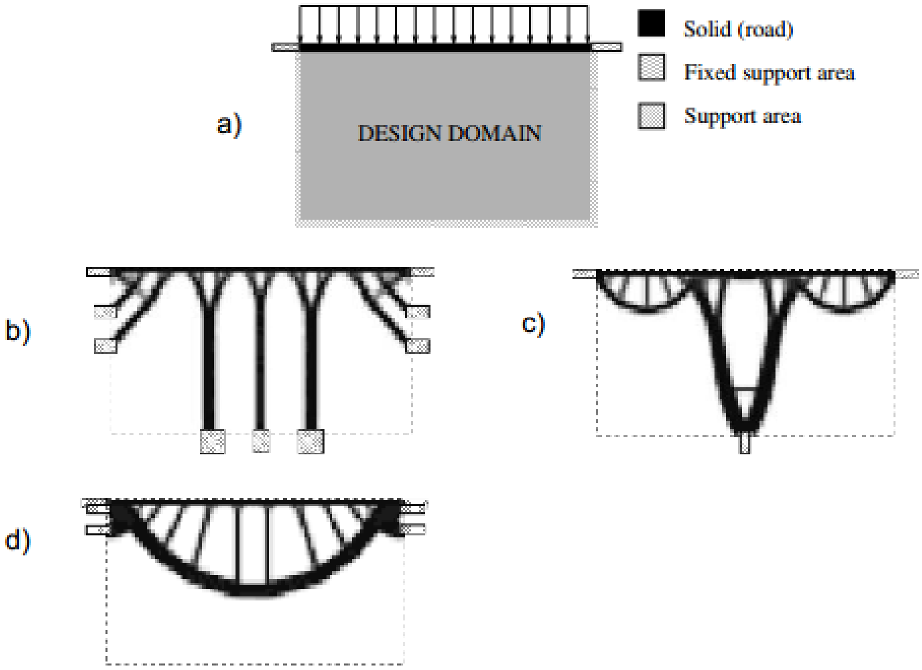


Figure 4-1: Example of design of supports with different support cost functions γ a) Initial design, b) $\gamma = 1$, c) $\gamma = 10$, d) $\gamma = 20$ (Buhl, 2002).

penalty factor p, from the SIMP method. To actually get the total stiffness, the mechanical stiffness and the spring stiffness should be added, which results in a global stiffness matrix **K**.

$$\mathbf{K} = \sum_{e=1}^{n} \rho_e{}^p \mathbf{K}_e + \sum_{e=1}^{n} z_e{}^q \mathbf{K}_{s,e} \tag{4-2}$$

While dealing with support design, a large risk of obtaining local optima can be labeled as a significant issue. A lower bound of the spring design variable **z** can be used to overcome this problem. In order to prevent extreme structures, for example creating supports only in one direction, it might be helpful to combine each spring of the element to each other, and thus creating one spring design variable for one element. To work with the support cost function, as depicted in Figure 4-1 a support factor γ can be imposed to the spring design variables, attaching a certain cost to the support of an element. The total amount of weighted support area, should be attached to a certain constraint A. This can be seen and formulated as the material distribution. Just like in (2-5) a simple compliance minimization problem can be

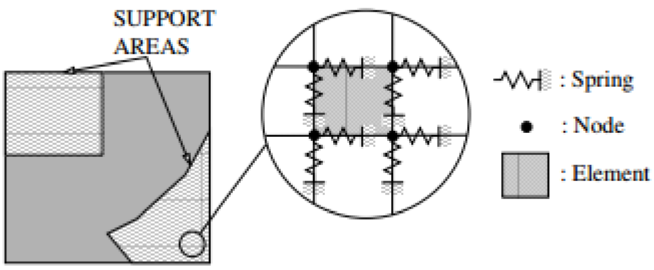


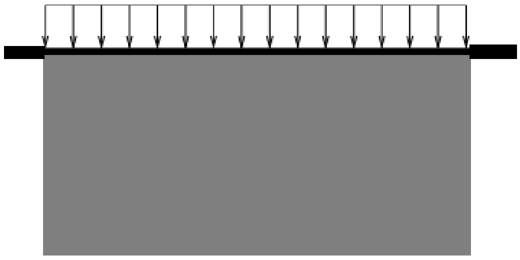
Figure 4-2: Example of support springs, each elements is supported by eight springs (Buhl, 2002).

formulated.

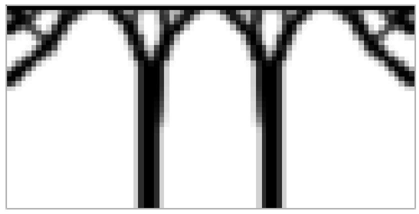
$$
\begin{aligned}
\min_{\rho_e} \quad & \mathbf{f}^T\mathbf{u} \\
\text{s.t.} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \\
& \mathbf{K} = \sum_{e=1}^{n} {\rho_e}^p \mathbf{K}_e + \sum_{e=1}^{n} {z_e}^q \mathbf{K}_{s,e} \\
& \sum_{e=1}^{n} \nu_e \rho_e \leq V \\
& \sum_{e=1}^{n} \gamma_e z_e \leq A \\
& 0 \leq \rho_e \leq 1 \\
& e = 1, \ldots, N
\end{aligned}
\tag{4-3}
$$

The associated sensitivity, with respect to the spring design variables can be calculated accordingly:

$$
\frac{\partial c}{\partial z_e} = -q({z_e}^{q-1})\mathbf{u}^T\mathbf{K}_{s,e}\mathbf{u} \tag{4-4}
$$



(a) Design problem of simple bridge



(b) Optimal lay-out of simple bridge

Figure 4-3: A simple bridge example. The bridge is discretized by 80 x 40 elements. a) design problem, b) Optimal solution.

4.2 The bridge

Consider a simple bridge example, as depicted in Figure 4-3a. For now, let's ignore the design of supports. So in this simple example a discretized bridge of 80 by 40 elements is optimized. A distributed force is exerted on the top of the bridge, which can be seen as a total self-weight of the upper road of the bridge. For this optimization, the upper elements are described as a restrictive region, being fully solid elements. The bridge is fixed at the upper left and upper right node. A volume fraction of 20% is given as constraint. The objective is to minimize the total compliance. The design problem with the associated optimal solution can be found in Figure 4-3b. Since this case represents a practical problem, some notes on the load case should be made. Since the road is dominant among the external loads, the distributed load can be designed as 1 loadcase. In this section, and the following chapters this consideration is implemented. For certain simple cases, there is no need for design of supports. Especially in straightforward theoretical cases this result will be sufficient.
However, when imposing restrictions or variations of costs within the support design domain, it could be a good idea to implement support design. A practical example of the need of support design is described in 4.2.1.

4.2.1 The optimal bridge

A simple bridge design is already described in Figure 4-3. Let's consider a practical example. The bridge in this example is used to make a pavement road across a deep valley. The valley can be seen as two parallel vertical walls, which can be used to create supports. Big and long pillars will be very expensive, especially when the valley becomes deeper. This is a very good example where design of support can be very useful. A volume fraction of 20% of the design domain is a constraint. Only 20% of the support design area (ie. the wall and floor of the valley) can be used to create support. In Figure 4-4b these constraints are built in. As can be seen, the supports are created at the sides and at the bottom; almost the same result as the simple bridge example in Figure 4-3. Now a cost distribution is imposed on the same design. A linear cost distribution is created along the y-axis, this variation is varying linearly from 1 at the top edge, to a certain γ at the bottom edge. In Figure 4-4 some variations of this γ is made. As can be seen, when increasing the upper limit of the support cost γ, the structure tends to support itself towards the pavement road. This is pretty straightforward, since the

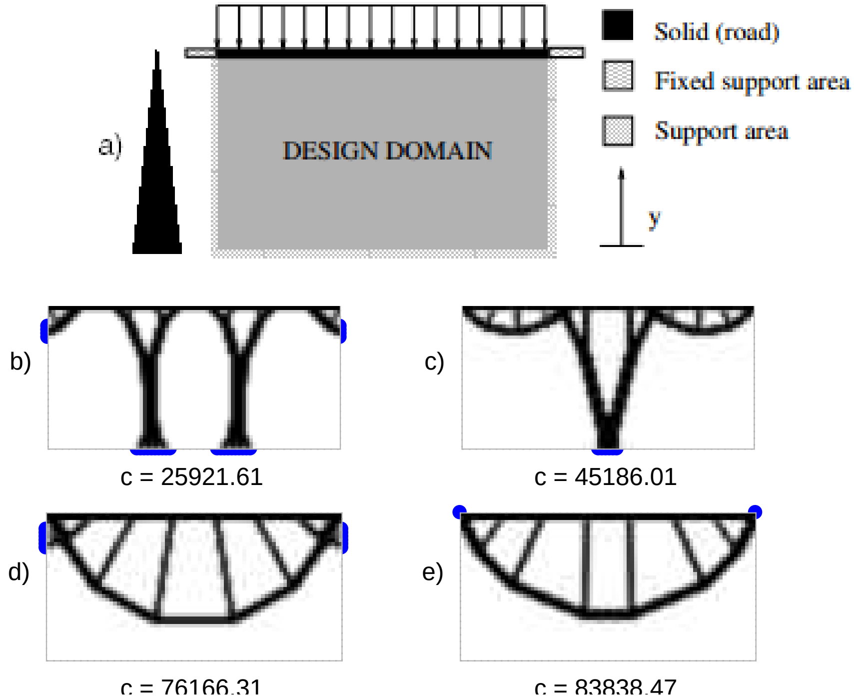


Figure 4-4: The bridge including design of supports with a varying cost of support design. The cost is linearly varying in the vertical direction from 1 (top edge) to γ (bottom edge). The bridge is discretized by 80 x 40 elements. a) design problem, b) $\gamma = 1$, c) $\gamma = 5$, d) $\gamma = 10$, e) $\gamma = 50$ (Appendix A-14).

cost of placing supports is increasing at the bottom of the design domain. In Figure 4-4e the cost of placing supports became to high to even place supports. The optimal structure in this case is of course less stiff than the original one. As can be seen in Figure 4-4, the result is in line with the result as depicted in Figure 4-1.

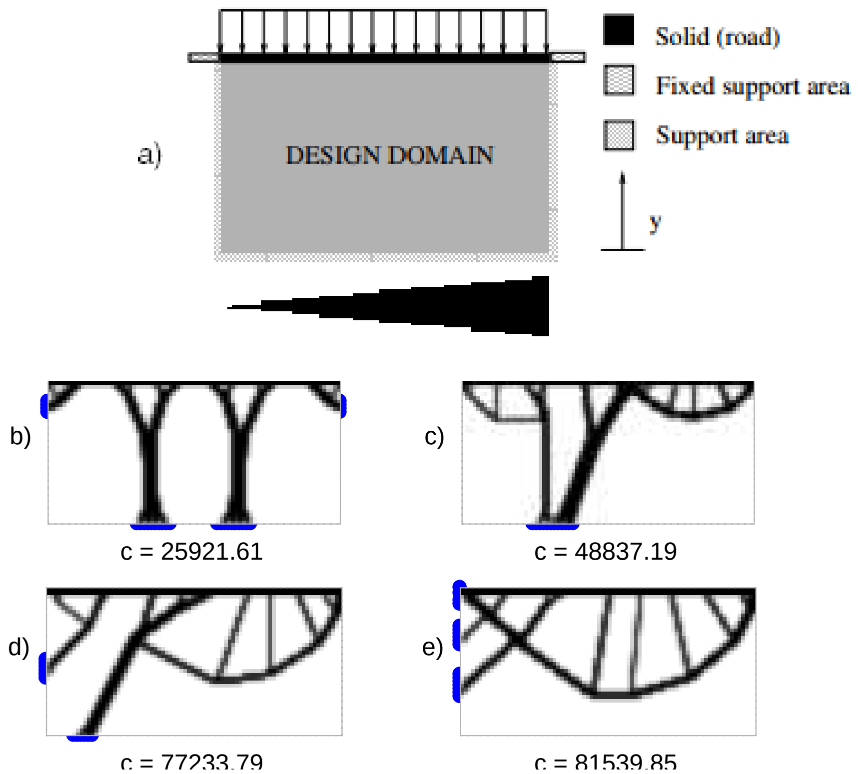


Figure 4-5: The bridge including design of supports with a varying cost of support design. The cost is linearly varying in the horizontal direction from 1 (left) to γ (right), from . The bridge is discretized by 80 x 40 elements. a) design problem, b) $\gamma = 1$, c) $\gamma = 5$, d) $\gamma = 10$, e) $\gamma = 20$ (Appendix A-15).

4.3 Advanced bridge designs

A variation of cost distribution in the vertical support domain is described in 4.2.1. However, there are some more variations possible. Think about the same bridge example as before. Now, a lake exists in the bottom right edge of the design domain. Creating pillars within the lake is expensive and unwanted. To overcome this design problem, a horizontal cost distribution is imposed on the bridge. Designing a support on the left hand side will cost 1, while the right hand side costs γ. This ratio of costs is varying linearly and results in Figure 4-5. The result is kind of similar to the results of Figure 4-4, which means in this case that the supports are forced to the left side of the design domain. As depicted in Figure 4-6, let's move this lake problem into the middle of the bottom of the valley. By doing this, a cost variation can be introduced varying from 1 to γ to 1, which corresponds from left to middle to right. As can be seen in Figure 4-6, a higher value of γ results in a greater tendency of the structure to move the support to the outer regions. Which in this case means a greater tendency to avoid placing supports within the lake.

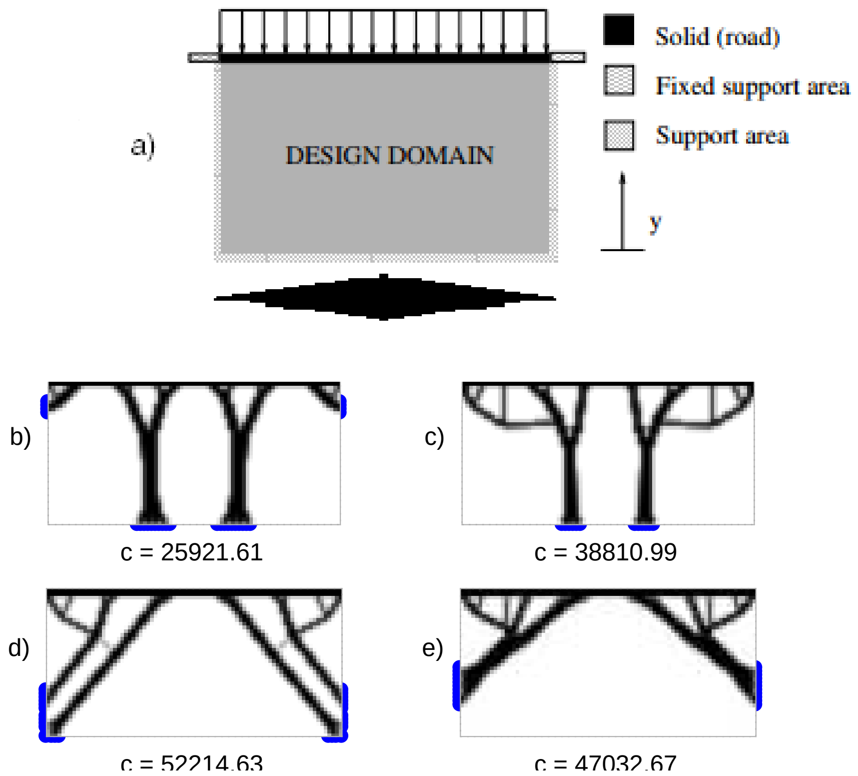


Figure 4-6: The bridge including design of supports with a varying cost of support design. The cost is varying in the horizontal direction from 1 (left) to γ (center) to 1 (right), from . The bridge is discretized by 80 x 40 elements. a) design problem, b) $\gamma = 1$, c) $\gamma = 3$, d) $\gamma = 5$, e) $\gamma = 10$ (Appendix A-16).

4.3.1 Hanging bridge

In this the design domain of Figure 4-4 is doubled. The force application and the fixed support locations remains the same. By the expansion of the design domain however, the support area is expanded also. As can be seen in Figure 4-7a, the sides can be used for support location, as well as the ground. Since the design domain is doubled, the volume constraint is scaled twice also. In order to compare this result with Figure 4-4, the volume constraint is divided by two. The upper limit of the total weight of the hanging bridge is now the same as the optimal (normal) bridge. The support area is almost doubled as well, compared to the optimal bridge, the support constraint is, however, kept at 20% of the total support area.
As can be seen in Figure 4-7, the support cost ratio γ is varying linearly from the road edge to the bottom edge. The cost from the top edge to the road edge remains 1 in each case. The support cost of the upper half of the design domain is thus very cheap, while the support cost of the lower half can be varied, which can be helpful in case of deep valleys, as already explained in 4.2.1.
An increasing support cost function γ results in a tendency to lift the bridge up. In the same time, the supports at the bottom of the design domain are reduced. As can be seen in Figure 4-7d, the two pillars of Figure 4-7b are replaced with only one pillar. In Figure 4-7e the pillars are completely vanished. The "upper" supports are increased at the same time, in

order to remain a low compliance.

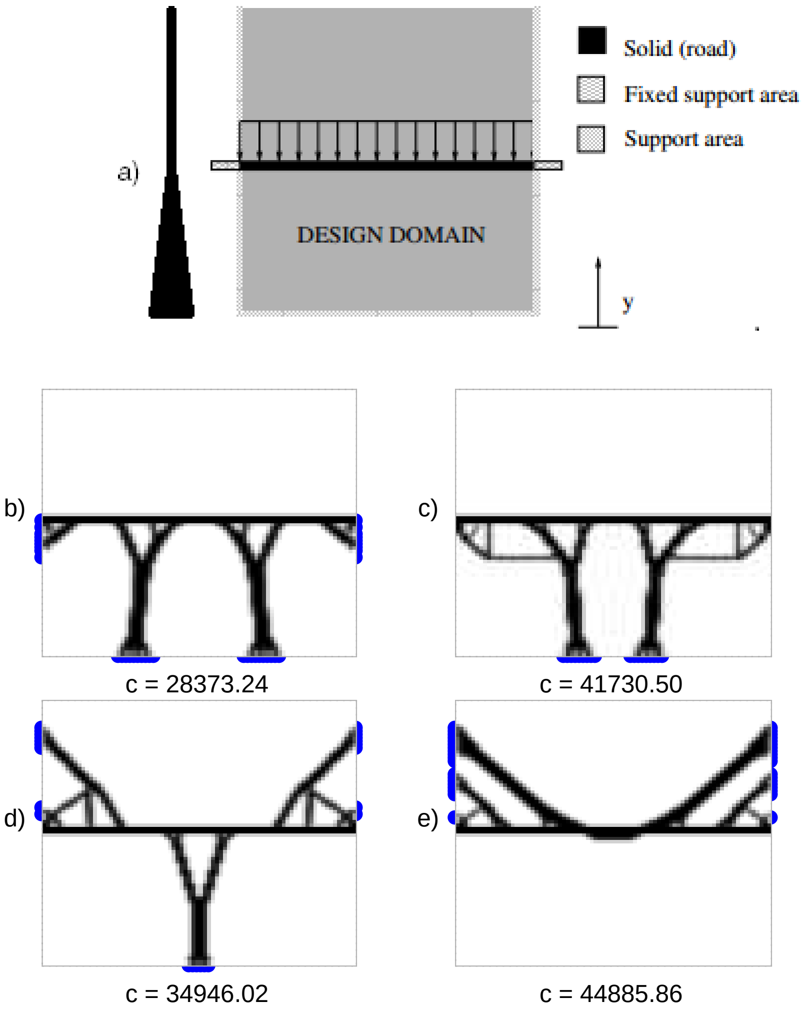


Figure 4-7: The hanging bridge including design of supports with a varying cost of support design. The cost is varying in the vertical direction from 1 (top road edge) to γ (bottom edge). The cost from the top edge to road edge remains 1 in each case. The bridge is discretized by 80 x 80 elements. a) design problem, b) $\gamma = 1$, c) $\gamma = 2$, d) $\gamma = 4$, e) $\gamma = 6$ (Appendix A-17).

4.3.2 Train tunnel

In 4.2.1 an example of a lake in the middle of the design domain is already explained and used to demonstrate the need of implementing ratio of support cost design. In this case the lake is exchanged by a train tunnel, since trains are also a starting point of bridge design. The main difference between the lake and the train is the possibility of placing supports within that particular area. The train tunnel is designed in Figure 4-8a. In this figure two cases are considered. In case 1 the space for the train can be seen as an open space. This space needs

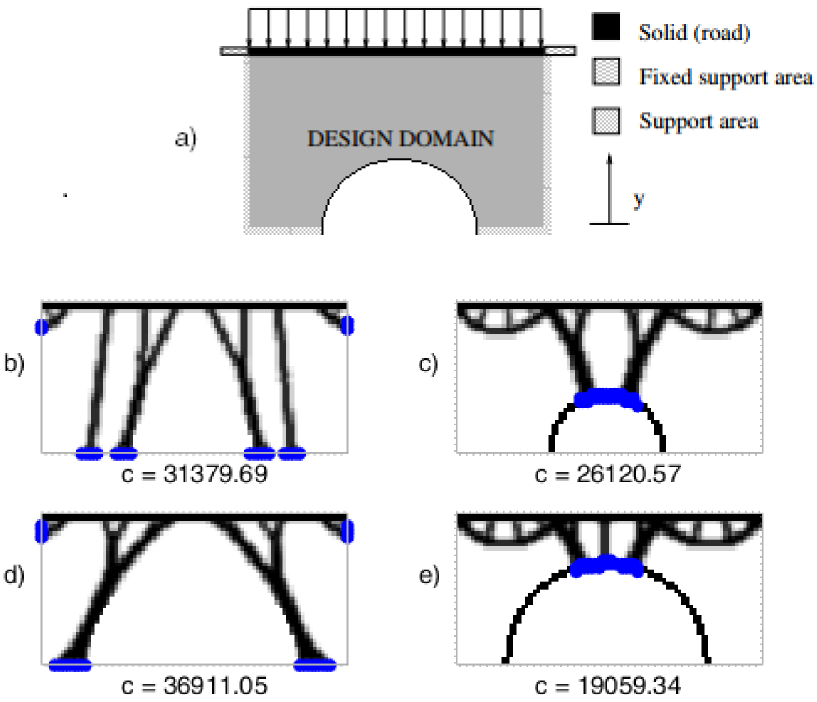
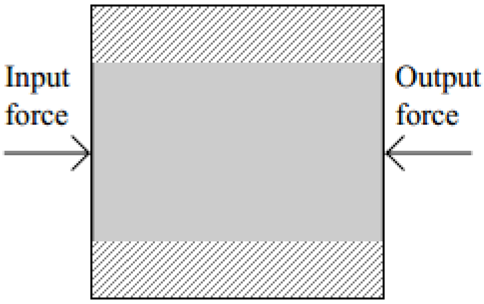


Figure 4-8: The train tunnel shows four different examples of train tunnel designs. The cost is varying in the vertical direction from 1 (top road edge) to γ (bottom edge). The cost from the top edge to road edge remains 1 in each case. The bridge is discretized by 80 x 40 elements. a) design problem, b) open space, small gap, c) tunnel, small gap , d) open space, big gap, e) tunnel, big gap (Appendix A-18).

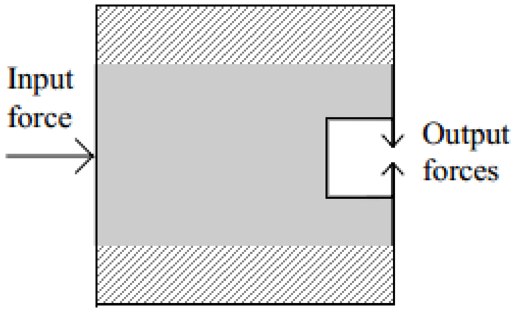
to be avoided by the bridge design (Figure 4-8b and Figure 4-8d). In the second case the train space is covered by a train tunnel. Space within the tunnel needs to be void, the outside of the tunnel is solid region and can be used to place supports (Figure 4-8c and Figure 4-8e). Also, the radius of the train tunnel/space is varied. Figure 4-8b and Figure 4-8c shows an example of a small train crossing, while Figure 4-8d and Figure 4-8e demonstrate the optimization process of a big train crossing. As can be seen, the open air train bridge design is slightly weaker compared to the tunnel train bridge design, which is expected before. A bigger open air train space results in a higher compliance, since the topology is forced to the outwards of the design domain. However, a bigger tunnel results in a lower compliance, compared to the small tunnel, since this tunnel on itself provide a lot of additional stiffness to the bridge.

4.3.3 Integration of layout design in supports

In 4.2 the optimal layout of a variety of bridge designs is depicted. The supports are designed by using the expressions as shown in 4.1. These supports can be seen as topology, which is attached to the support area. A practical support however, should be a variation of the topology, since the shape and size of the support is very important, to use the support location of the topology as actual real support. Some research is already done within this field (Zhu



(a) Compliant mechanism amplifier design problem



(b) Output force at the right of the design domain

Figure 4-9: Design problem for micro-gripper

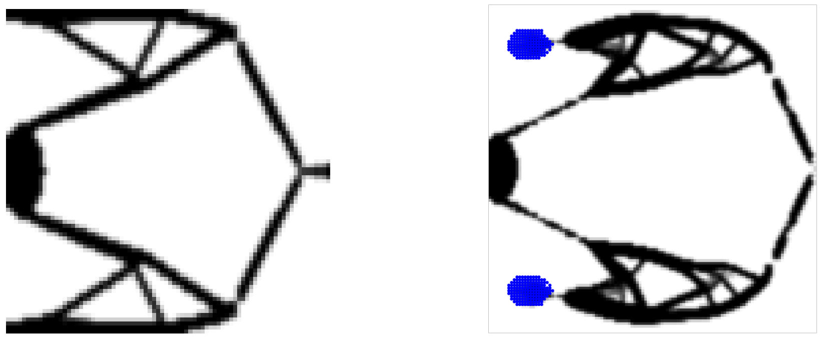
and Zhang, 2010), where complex shapes of supports are implemented within the topology optimization problem. In this report this subject is only slightly touched in this section, since the practical implementation of the supports is outside the scope of this research project.

4.4 Design of compliant mechanisms

In 3.4 compliant mechanisms are already described. The topology of the compliant mechanisms is optimized in order to maximize outputs. In Figure 3-11b an example of topology optimization for an amplifier is shown. In Figure 3-13a the optimal topology of a micro-gripper example is shown. For these two compliant mechanisms the support locations are fixed, while the topology is the free variable. In this chapter, design of supports is explained. In this section an example of the use of support design within compliant mechanisms is made and compared to the previous results.
As a recap, the amplifier and micro-gripper design problems are depicted in Figure 4-9, the main difference with the previous design problem is the implementation of the support design area. The support design domain is modeled as an upper and lower band with a total area of one third of the whole design domain. This support design domain can be used to place supports of the structure, with an upper limit of 20 % of the total support design domain.

4.4.1 The optimal amplifier

In Figure 4-9a a positive amplifier is topologically optimized. The result of this optimization is already shown and explained in 3.4.1. The result can be seen in Figure 4-10a. As already explained before, design of support is now included in the design domain, to achieve an even better amplifier, by means of a higher amplification gain (G_d). The design of support formulation as described in 4.1 is used here and implemented in the compliant mechanism code. The result can be found in Figure 4-10b. As can be seen, the design is slightly different, the supports are design within the support design domain. The supports are represented by the blue dots. The input displacement is within the same range, while the output displacement of Figure 4-10b is two times higher, which eventually results in a amplification gain of almost



(a) Optimal lay-out of an amplifier **(b)** Optimal lay-out with DoS

Figure 4-10: Optimal topology and displacement pattern for design problem Figure 4-9a, a horizontal input on the left is amplified to the right. a) the original amplifier converts an input of $d_{in} = 18.42$ into an output of $d_{out} = -40.89$, which results in a mechanical displacement gain of $G_d = -2.22$. b) the amplifier including design of supports converts an input of $d_{in} = 14.15$ into an output of $d_{out} = -83.40$, which results in a mechanical displacement gain of $G_d = -5.89$ (Appendix A-19).

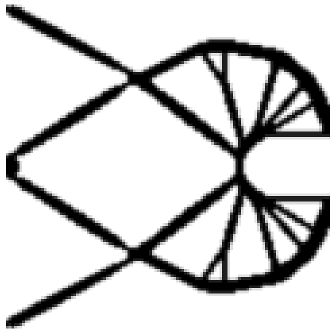
six times the input displacement. From this can be concluded, that implementing design of supports within the design domain results in a higher amplification factor and thus a better result.

4.4.2 The optimal micro-gripper

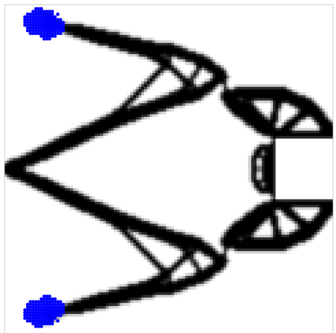
In 3.4.2 two different micro-grippers are shown. In this section the first one is chosen to optimize with the use of design of supports. The design problem can be found in Figure 4-9b. Again, the upper and lower band represents support area, which can be used for support placement, with an upper limit of 20 %. In Figure 4-11a the optimal gripper is depicted, with fixed supports. In Figure 4-11b this gripper is optimized with the use of design of supports, and can be seen as the solution of the design problem Figure 4-9b. As can be seen, the optimal topology is still a small pliers. The supports however, are pushed from the outer edges from the design domain. This topology results in a total displacement gain of $G_d = 1.97$, which is almost three times higher as the original topology result.

4.5 Application of support design

As can be seen, application of support design can be used for a variety of application, big structures like bridges 4.2, as well as very small structures like micro-grippers 4.4. In this section some words about the application of support design will be said.
Design of supports can be easily used to connect components of a multi-component structure to each other (Chickermane and Gea, 1997). Also, when using the connection locations as joints, the optimal support lay-out can be used to achieve the optimal result (Qian and Ananthasuresh, 2004). Within the assembly of aircraft structures, not only the structure



(a) Optimal gripper topology



(b) Optimal lay-out with DoS

Figure 4-11: Optimal topology and displacement pattern for design problem Figure 4-9b, a horizontal input on the left, a vertical output on the outer right. a) the original gripper inverts a input of $d_{in} = 76.48$ into an output of $d_{out} = 28.06$, which results in a displacement gain of $G_d = 0.73$. b) the gripper including design of supports inverts input of $d_{in} = 52.23$ into an output of $d_{out} = 51.40$, which results in a displacement gain of $G_d = 1.97$ (Appendix A-19).

is optimized for minimizing compliance, but also the shear loads on the fastener joints are controlled. By using design of supports the optimal locations of the fastener joints can be found to achieve maximum overall strength (Zhu et al., 2014).

In the domain of dynamics, maximization of natural frequency can be used to maximize the bandwidth of the structure. Design of supports can be used to achieve the optimal support lay-out to tune dynamic behavior (Jihong and Weihong, 2006). This same technique can be used to achieve certain dynamic behavior, like changing eigenmodes and eigenfrequencies, or maximizing transmission. Another promising development lays within the thermodynamic domain. In this field, heating can be seen as force application, while isolation prevents heating. This prevention is a counteract of the applied force, so a support location. So design of supports can be seen as design of isolation location.

4.5.1 Actuator locations

As already mentioned in 4.5, design of supports can be used in dynamics to tune and tweak frequency response. Support locations are fixed, with no external loads. However, it could be interesting to apply a force on these support locations, which will make the supports act as actuators. By applying this, the optimal placement of actuators can be found, to achieve a certain frequency response. A very promising application within the manufacturing of microchips, where frequency response is very important. The main concern will be to reformulate the support location into an actuator location. Since a support is a fixed location, with no external loads applied, while an actuator location does apply a load to the structure.

In the next section these concerns will be explained even further.

4.6 Conclusions

In chapter 3, a variety of complex problems are described and solved using topology optimization. In this chapter the optimization process is taken a step further and is used to solve problems using variable support designs.
When determining the optimal place of supports, a new set of variables needs to be defined and optimized (4.1). By using this additional set of variables the program determines the optimal solution for both the topology, as well as the support lay-out.
An example of a simple bridge is shown (4.2), this bridge is optimized by using variable supports (4.2.1). This bridge example is extended for some advanced bridge design problems (4.3), where ratio of support cost is varied in different ways. Some more examples of bridge problems are explained, for example a hanging bridge (4.3.1), where supports can be placed on the upper sides of the design domain. Also, when placing a bridge with restricted areas, design of supports can be used to calculate the most ideal configuration (4.3.2).
Some compliant mechanism examples, as already described in this report (3.4) are optimized including design of supports, by placing the support domain within the original, topology design domain. Usage of design of supports is very promising in this area, since the displacement gains can be improved (4.4).
Design of supports can be helpful in a lot of different fields, some practical examples are given (4.5) and the idea of changing fixed supports to actuator locations is slightly touched (4.5.1). The optimal way of actuator placement will be the focus for the next chapter and will be used further on this research.

Chapter 5

Design of Actuator Placement

In chapter 4 several cases of design of supports are explained. In this chapter the philosophy of extending the optimization problems is taken a step further. Instead of designing the best support locations, this chapter aims to design the optimal force locations. Design of actuator placement can also be used in combination with topology optimization, for example when volume constraints are involved.

In section 5.1 the basics of actuator placement are described. The need of sensitivity checking is explained in subsection 5.1.3. A simple cantilever beam is used to show the design of force distribution in section 5.2. Some advanced applications for actuator placement are shown in section 5.3.

Up to here, the topology of the previous examples remained fixed. An implementation of topology optimization is explained in section 5.4. Practical applications of actuator design are shown in section 5.5. Section 5.6 concludes this chapter.

The implementation and working MATLAB codes are attached (B.9, B.10, C.10 and C.11) and can be used to reproduce the problems of this chapter.

5.1 Actuator design formulation

This chapter has some similarities with respect to 4, where supports can be placed within a certain support design area, in order to achieve the best objective. In chapter 4 the support design is thus considered as design variable. In this chapter however, the support area remains fixed. The applied force is now considered as a free design variable.

There are some similarities between design of support and design of actuator placement. However, the behavior of both design variables are quite different, a fixed support cannot generate force, while an actuator does generate an active force.
This chapter will mainly focus on minimizing vertical displacement. In order to compare these results, in section 5.2 a simple cantilever beam will be optimized towards minimal compliance. Since the previous chapters mostly rely on compliance minimization, this knowledge can be used to create and solve a simple cantilever beam problem.
The minimal compliance problem is solved the same way using the SIMP method, as described in 2.2.1. This method is solved with a different continuation method for the penalty than described before in 3.2.4. More on this new approach can be found in subsection 5.1.2. The same approach will be used further in this chapter.
Another big difference, with respect to chapter 4 is the constraint function. When topology is not included, the constraint on the volume can be dropped. The supports remains fixed, so this constraint will also drop out. There is however, a new constraint function introduced on the force. Since in all cases the compliance and displacement will be minimized, the optimizer will tend to use as little force as possible. A minimum force should be introduced, in order to force the optimizer to use this minimum of actuator force (f_{min}). This is opposed to the previously used constraints on the volume and supports, where the optimizer wants to use as much volume and/or supports as possible.

In order to minimize the compliance by varying the actuator placement f_i, a minimization problem can be formulated.

$$
\begin{aligned}
\min_{f_i} \quad & \mathbf{f}^T \mathbf{u} \\
\text{s.t.} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \\
& \mathbf{K} = \sum_{e=1}^{n} \rho_e{}^p \mathbf{K}_e \\
& \sum_{i=1}^{n} f_i \leq f_{min} \\
& -1 \leq f_i \leq 0 \\
& i = 1, \ldots, N_i
\end{aligned}
\tag{5-1}
$$

Where N_i reflects the number of nodes. The associated sensitivities can then be calculated, by differentiating the objective to the design variables. In this chapter, the force can be varied from $f_i = -1$ to $f_i = 0$, since the actuators are pointed downwards. In the upcoming figures, the mean absolute displacement is depicted in each figure.

5.1.1 Sensitivity selection

The same adjoint approach, as described in 2.2.3 will be used to evaluate the associated sensitivities. In this section, the sensitivity approach is used to calculate the sensitivity of the compliance problem, towards the actuator placement. This is done to make a good comparison with the previously calculated sensitivities.

Again, to prevent the calculation of the derivatives of the displacement explicitly, an adjoint method is used to achieve the correct sensitivity.
The minimum compliance problem can be rewritten by adding a zero function, including the Lagrange multiplier $\boldsymbol{\lambda}$.

$$c(f_i) = \mathbf{f}^T\mathbf{u} + \boldsymbol{\lambda}^T(\mathbf{Ku} \text{ - } \mathbf{f}) \tag{5-2}$$

Now the corresponding sensitivity can be calculated by derivating to the design variable f_i.

$$\frac{\partial c}{\partial f_i} = (\mathbf{f}^T + \boldsymbol{\lambda}^T\mathbf{K})\frac{\partial \mathbf{u}}{\partial f_i} + (\mathbf{u}^T - \boldsymbol{\lambda}^T)\frac{\partial \mathbf{f}}{\partial f_i} \tag{5-3}$$

Where $\boldsymbol{\lambda} = -\mathbf{Kf}$, which on his turn is equal to $\boldsymbol{\lambda} = \textbf{-u}$, due to the equilibrium $\mathbf{Ku} = \mathbf{f}$. This will eventually lead to the sensitivity

$$\frac{\partial c}{\partial f_i} = 2\mathbf{u}^T\frac{\partial \mathbf{f}}{\partial f_i} \tag{5-4}$$

Where $\frac{\partial \mathbf{f}}{\partial f_i}$ can be seen as a selection vector of the participating force. This can be simply calculated by a column vector which yields a zero for non-participating force, which corresponds to a node that cannot design a force; and a value one for the nodes that corresponds to the actuator design area.

5.1.2 Arching continuation approach

The continuation method is a very powerful approach to prevent the optimizer getting local optima (3.2.4). There is however a big consideration for applying this continuation method in the force design domain. Since density can typically vary from zero to one, either void or solid a power factor will result in a tendency to create void or solid regimes. Since the optimizer wants to create a lot of density, a positive penalty factor $p > 1$ can be used to prevent the optimizer from creating gray regions.
When dealing with this force design variable in combination with minimizing compliance or displacements, the best result will be no force, the optimizer will thus remove as much force as possible. A penalty factor $p > 1$ will help the optimizer with this process, which is not desirable. A positive penalty factor $p < 1$ will send the optimizer towards the biggest value of the design variable.
When applying a power factor in this case, where force can vary from -1 to 0, there is a problem to overcome. When factorizing a negative value of $\mathbf{f}$ with a power factor $p < 1$, it will result in imaginary values, which is not desirable in this case. A simple way to overcome this would be to penalize the absolute value, this absolute values however, cannot be differentiated. A solution is using a new way of continuation, which is labeled as Arching continuation approach. This approach can deal with negative and positive input values and will give real penalized outputs. This Arching continuation approach can produce a penalized value by

$$\mathbf{f}_p = \frac{\arctan(\alpha\mathbf{f})}{\arctan(\alpha)} \tag{5-5}$$

The numerator will penalize the function, while the denominator creates a normalization. The variable α can be chosen, to increase or decrease the slope of the arches. A visual representation of this new continuation method can be seen in Figure A-12. As can be seen, an increase of α will result in a higher slope, so a more aggressive penalization. The best way to use this Arching continuation is to exponentially increase α. The optimizations in this chapter are made using a starting value of $\alpha = 0.5$, which is then exponentially increased by a factor 1.06 each iteration, until the final value of α is achieved. $\alpha = 5$ seems to be a good number for force penalization. These values are found to somewhat mimic the continuation method as described before (3.2.4). This approach can also be used, when a design variable can vary from positive to negative, for example in compliant mechanisms. Also, the differential of this Arching continuation approach can be easily derived and used in further sensitivity analysis.

5.1.3 Finite difference method

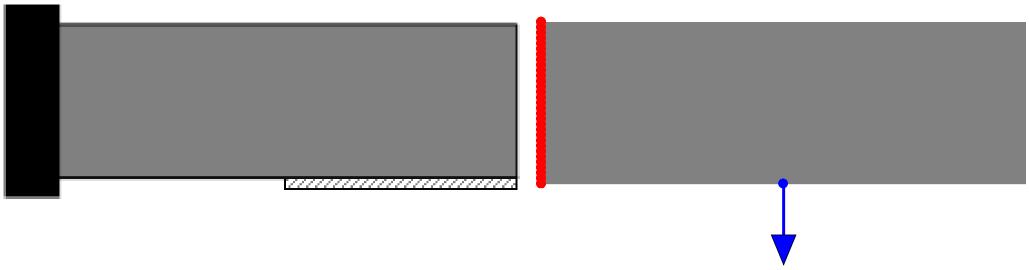
Up to the design of actuator placement, sensitivities seem to be pretty straight-forward. In this section however, the sensitivity analysis becomes a bit more challenging. In order to deal with sensitivities with respect to different physical quantities, it is a good idea to check whether or not the applied sensitivities are well calculated and derived. One way to check this is using the Finite difference method. This is done by introducing a small perturbation h, which can typically vary between $h = 10^{-2}$ to $h = 10^{-6}$.
The function value f is used as starting point. The sensitivity can now be calculated. At the design variable point a, where the sensitivity yields the maximum value, a small perturbation h is added to this design variable point a. The function value f is then calculated again. The difference between these function values f are now subtracted and divided by h, which is basically the slope of the function. This slope can then be compared to the calculated sensitivities. This difference should be very small to confirm a correct sensitivity function. This finite difference method can be found in (5-6).

$$f'(a) \approx \frac{f(a+h) - f(a)}{h} \tag{5-6}$$

Where $h = 10^{-6}$ seems to be a good value. This finite difference can thus easily be used to check the sensitivities of the objective. But it can also be used to check the constraint sensitivity.

5.2 Simple cantilever beam

A simple cantilever beam is used to show the working principle of the design of optimal loading. Recap: the topology remains fixed. By introducing a force design domain and a minimum force value the MMA optimizer (3.1) can now solve the optimal actuator placement. Consider a simple cantilever beam as used before. The force design domain is in this example chosen to be from the down right point to the down middle point, as can be seen in Figure 5-1a. By using the optimization problem as explained in (5-1) and the corresponding sensitivities as described in (5-3) the optimal actuator placement can be found, as shown in Figure 5-1b. This results is in line with the preliminary thoughts. The most optimal place of force placement will be at the nearest point from the fixed support (left-hand side), to minimize the generated moment. The minimum force constraint is active, so the total force equals the minimum force, which is also expected. A minimum amount of force will result in a minimal compliance, or maximal stiffness.



(a) Design problem of simple cantilever beam **(b)** Optimal actuator placement

Figure 5-1: A simple cantilever beam example for actuator placement. The red dots indicated the fixed supports, the blue arrow corresponds to the force. The mean vertical displacement in the direction of the force is displayed below each solution. The beam is discretized by 90 x 30 elements. a) design problem, b) optimal solution.

5.2.1 Minimal displacement

In contrast to a minimal compliance problem, as described in 5.2, the focus will from now on changed to a minimum of vertical displacement. The optimal results should not differ much, with respect to the minimal compliance problem, since the displacement and compliance are correlated to each other. The minimization problem can be formulated again, but now for minimum displacement.

$$\begin{aligned} \min_{f_i} \quad & u_a \\ \text{s.t.} \quad & u_a = \mathbf{L}^T\mathbf{u} \\ & \mathbf{K}\mathbf{u} = \mathbf{f} \\ & \mathbf{K} = \sum_{e=1}^{n} {\rho_e}^p \mathbf{K}_e \\ & \sum_{i=1}^{n} f_i \leq f_{min} \qquad -1 \leq f_i \leq 0 \\ & i = 1, \ldots, N_i \end{aligned} \tag{5-7}$$

Where $\mathbf{L}$ is the selection vector of the displacement. If only the vertical displacement is involved, which is the case in this section, the selection vector will have the form $\mathbf{L} = [0\ 1\ 0 \ldots 1\ 0\ 1]^T$. The minimum vertical displacement can be rewritten including an adjoint function.

$$u_a(f_i) = \mathbf{L}^T\mathbf{u} + \boldsymbol{\lambda}^T(\mathbf{K}\mathbf{u} \text{ - } \mathbf{f}) \tag{5-8}$$

The sensitivity analysis of this minimization function is in line with the previously analysis (5-3), but is a bit more complicated.

$$\frac{\partial u_a}{\partial f_i} = (\mathbf{L}^T + \boldsymbol{\lambda}^T\mathbf{K})\frac{\partial \mathbf{u}}{\partial f_i} - \boldsymbol{\lambda}^T\frac{\partial \mathbf{f}}{\partial f_i} \tag{5-9}$$

Where $\boldsymbol{\lambda} = -\mathbf{K}^{-1}\mathbf{L}$. In contrast to (5-4) this $\boldsymbol{\lambda}$ will not vanish and need to be calculated each iteration. Therefore the total running time for solving minimal displacement typically

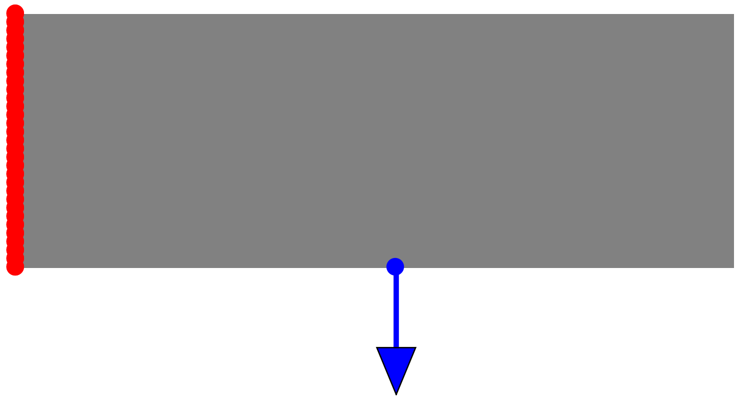


Figure 5-2: Example of minimal vertical displacement including actuator placement.

will be longer than that of solving minimal compliance problems. The final sensitivity can thus be rewritten as

$$\frac{\partial u_a}{\partial f_i} = -\boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial f_i} \tag{5-10}$$

As can be seen, the result of the actuator placement for minimal displacement (Figure 5-2) does not differ from the result for minimal compliance (Figure 5-1b). The displayed U reflects the mean displacement in the direction of the force.

5.3 Advanced applications

Design of actuator placement can be used to minimize a simple cantilever beam, while respecting a minimum force. This minimum force should be provided by an actuator. However, what would happen when the minimum applied force cannot provided by a single actuator, due to its limit of power? An additional constraint should be included in the optimization, to deal with this problem. In Figure 5-3a a result of this problem statement is provided. In this problem, the same objective and design variables are provided, as described in 5.2.1, but a single actuator can only provide one fifth of the total minimal force. As can be seen, the best way to place the actuators is place them five in line, at the most left point of the actuator design area. This should be okay, since the actuators together want to minimize the moment exerted on the cantilever.

5.3.1 Maximal displacement

Up to here, only minimal compliance and displacement was considered. Most of the time a minimization is the best way to optimize, and most of the time we are looking for a minimum, think of minimum cost, minimal weight, minimal displacement etc. There are some cases arguable where a maximum of displacement is desirable. A simple actuator system within the manufacturing domain is a good example. There we want to maximize displacement with

(a) Constraint on force per actuator

(b) Maximal displacement

Figure 5-3: Optimal actuator placement of advanced applications for a) minimal displacement, with a constraint on the force per actuator, b) maximal displacement.

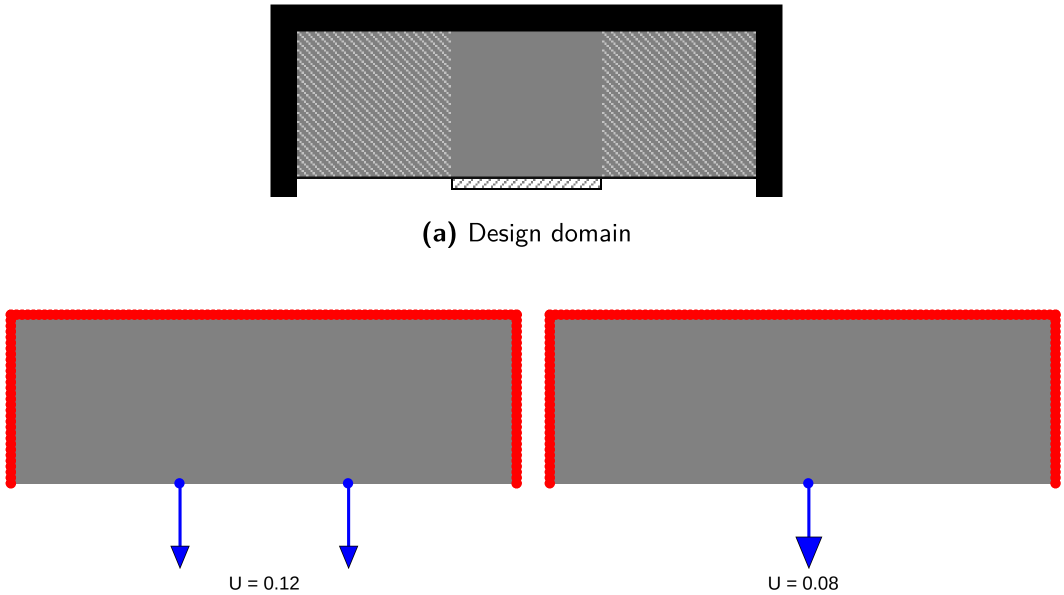
a minimum force, additional stiffness demands can be included as constraints. In Figure 5-3b a schematic of maximum displacement of a beam can be found. The force is pointed at the most right point of the design domain. This is in line with the preliminary thoughts, since a large distance between force and fixtures results in a maximum moment acting on the cantilever beam, which on his turn will result in maximum displacement.

5.3.2 Triple fixed beam

Design of Actuator placement is pretty straightforward for a simple cantilever beam, especially when the topology remains fixed. Therefore, a new design problem is considered and solved. Let's consider a triple fixed beam, which can be seen as a bridge structure, which is also completely fixed on the left and right sides. A schematic of this design problem can be seen in Figure 5-4a. As can be seen, the actuator design domain consist one third of the bottom row. The optimal solution for force placement will now be calculated. This is done by using the same objective as used before (5-7) and the same sensitivity analysis (5-10). A minimum force constraint is implemented with a minimum value of $\mathbf{f} = 1$. The result of the optimization can be found in Figure 5-4b. As can be seen, the most optimal solution is two distributed forces (each consist of $\mathbf{f} = 0.5$). This is a correct result, since the force needs to be placed as close as possible to the supports, which is in this case two points.

5.3.3 Minimal area displacement

Up to here, the main focus was to minimize the overall vertical displacement. In this section some words are spend on the ability of optimizing the actuator placement towards minimal displacement in a certain area. This is very useful in manufacturing technology, since most of the time engineers are interested in local effects. To demonstrate the working principle, the same optimization as in 5.3.3 is done, but now with a different selection vector $\mathbf{L}$, which will only select the vertical displacement of the striped area in Figure 5-4a. This selection vector is also used in the sensitivity analysis. The optimal actuator placement for this solution is depicted in Figure 5-4c. As can be seen, the force is placed at the center of the actuator design domain, which would be probably the worst solution of the regular minimal displacement



(a) Design domain

(b) Minimal overall displacement

(c) Minimal displacement striped area

Figure 5-4: Triple Fixed beam, clamped at three sides. Optimal actuator placement of advanced applications for a) design domain, b) minimal displacement all grey area, c) minimal displacement of striped area only. The associated deformed geometry can be found in (Appendix A.6.1).

problem (5.3.2). Of course, the average displacement is somewhat lower ($U = 0.08$ vs $U = 0.12$), which can be explained by the fact that in the overall displacement, the displacements above the actuator application also contribute to the average, while in Figure 5-4c only the striped area is used to calculate the mean value.

5.4 Topology optimization for actuator placement

Up to here, the topology of the beam remains constant, namely completely solid. This was done to verify and demonstrate the working principle of design of actuator placement. It will become much more interesting if the topology is included in the design problem, while the placement of actuators can be optimized at the same time. The same objective holds, but an additional set of design variables is added to the problem. Also, an additional constraint is added to the problem, to limit the volume V that can be used. The optimization problem can now be described as

$$
\begin{aligned}
\min_{f_i,\rho_e} \quad & u_a \\
\text{s.t.} \quad & u_a = \mathbf{L}^T\mathbf{u} \\
& \mathbf{K}\mathbf{u} = \mathbf{f} \\
& \mathbf{K} = \sum_{e=1}^{n} {\rho_e}^p \mathbf{K}_e \qquad\qquad e = 1, \ldots, N \\
& \sum_{i=1}^{n} f_i \leq f_{min} \qquad\qquad i = 1, \ldots, N_i \\
& \sum_{e=1}^{n} \nu_e \rho_e \leq V \\
& -1 \leq f_i \leq 0
\end{aligned}
\tag{5-11}
$$

Where i still denotes the node numbers, and e denotes the number of elements. For optimizing this problem, the arching continuation method (5.1.2) is used to penalize the forces, the regular continuation method (3.2.4) is used to penalize the density.
The sensitivities of this problem will not change for the force design variables, but will change for the density design variable. By using the adjoint method again, the sensitivity from the displacement u_a (5-8) to the density variable can be calculated now

$$
\frac{\partial u_a}{\partial \rho_e} = (\mathbf{L}^T + \boldsymbol{\lambda}^T\mathbf{K})\frac{\partial \mathbf{u}}{\partial \rho_e} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{K}}{\partial \rho_e} \tag{5-12}
$$

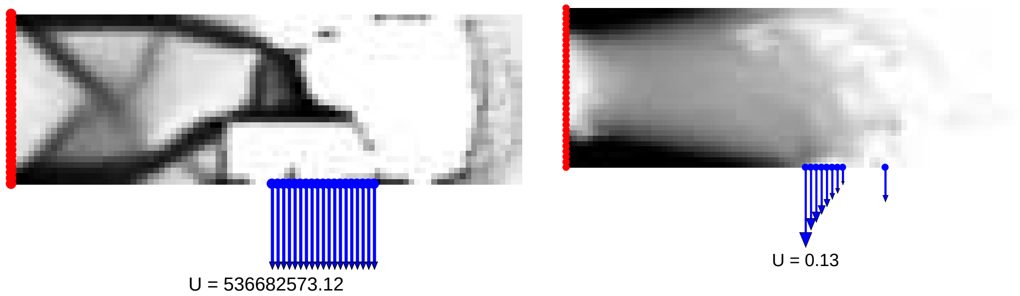
By choosing again $\boldsymbol{\lambda} = -\mathbf{K}^{-1}\mathbf{L}$, the $\frac{\partial u_a}{\partial \rho_e}$ does not have to be calculated explicitly

$$
\frac{\partial u_a}{\partial \rho_e} = \boldsymbol{\lambda}^T \frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} \tag{5-13}
$$

Now formulate $\frac{\partial \mathbf{K}}{\partial \rho_e}$ as the derivative of the penalty-termed stiffness as derived in (2-5)the final sensitivity can be made as

$$
\frac{\partial u_a}{\partial \rho_e} = p({\rho_e}^{p-1})\boldsymbol{\lambda}^T\mathbf{K}_e\mathbf{u} \tag{5-14}
$$

The tolerance is updated to a summation of the difference of the force and density, with respect to the last iteration. Due to this tolerance update, and the addition of another set of design variables, the computational time will rise exponential.
The optimal result for a minimization of the overall displacement, so the selection vector will have the form $\mathbf{L} = [1\ 1\ 1 \ldots 1\ 1\ 1]^T$, can be seen in Figure 5-5a. The result can be labeled as quite remarkable. A deeper investigation can explain this weird behavior. By minimizing the displacement in the direction of the force, the optimizer also wants to maximize in the opposite direction. That's exactly what happens in this problem. The optimizer discovers a maximization of the opposite direction can be achieved by adding force. However, since the force is pointed downwards, displacement in the opposite direction is not expected. This result is probably caused by a numerical issue of the optimizer linked by the FEM method.



(a) Minimal displacement including topology **(b)** Minimal squared displacement

Figure 5-5: Optimal actuator placement including topology optimization for a) minimal displacement, b) minimal squared displacement. The associated displacement plots can be found in (Appendix A.6.2) and (Appendix A.6.4).

5.4.1 Displacement consideration

One way to overcome the problem of 5.4 is by simply minimizing the squared displacement. This will skip the tendency to maximize the opposite direction . It will change the minimization problem from (5-8) into a new formulation:

$$u_a(f_i, \rho_e) = (\mathbf{L}^T\mathbf{u})^2 + \boldsymbol{\lambda}^T(\mathbf{Ku - f}) \tag{5-15}$$

With the corresponding sensitivities:

$$\frac{\partial u_a}{\partial f_i} = (2\mathbf{L}^T\mathbf{u}\mathbf{L}^T + \boldsymbol{\lambda}^T\mathbf{K})\frac{\partial \mathbf{u}}{\partial f_i} - \boldsymbol{\lambda}^T\frac{\partial \mathbf{f}}{\partial f_i} \tag{5-16}$$

$$\frac{\partial u_a}{\partial \rho_e} = (2\mathbf{L}^T\mathbf{u}\mathbf{L}^T + \boldsymbol{\lambda}^T\mathbf{K})\frac{\partial \mathbf{u}}{\partial \rho_e} + \boldsymbol{\lambda}^T\frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} \tag{5-17}$$

Where $\boldsymbol{\lambda} = -2\mathbf{K}^{-1}\mathbf{L}\mathbf{u}^T\mathbf{L}$ to remove the $\partial\mathbf{u}$ terms.
The corresponding result of this optimization can be seen in Figure 5-5b. The result is obviously a lot better then Figure 5-5a. We see an expected behavior, namely, the force is placed most left of the force design domain. The topology is then be used to counteract this force and make a stiff structure. Even after a maximum number of iterations, still a gray pattern remains for the topology. It seems like the optimizer simply does not want to create a black-and-white pattern. This behavior is in collaboration with the distribution of the force. A trade-off between counteracting the force by placing material and minimizing the moment exerted on the beam is made. This trade-off seems to be difficult to solve. However, as little force as possible is used, which makes the force constraint f_{min} an active constraint, which is in line with the theory. The figures does not converge to black and white regimes, in the next section a possible solution is explained.

5.4.2 Compliance constraint

The result in Figure 5-5b is quite good, but not perfect at all. It seems like the optimizer creates little material near to the force, and the structure is a little leaned over. Perhaps the

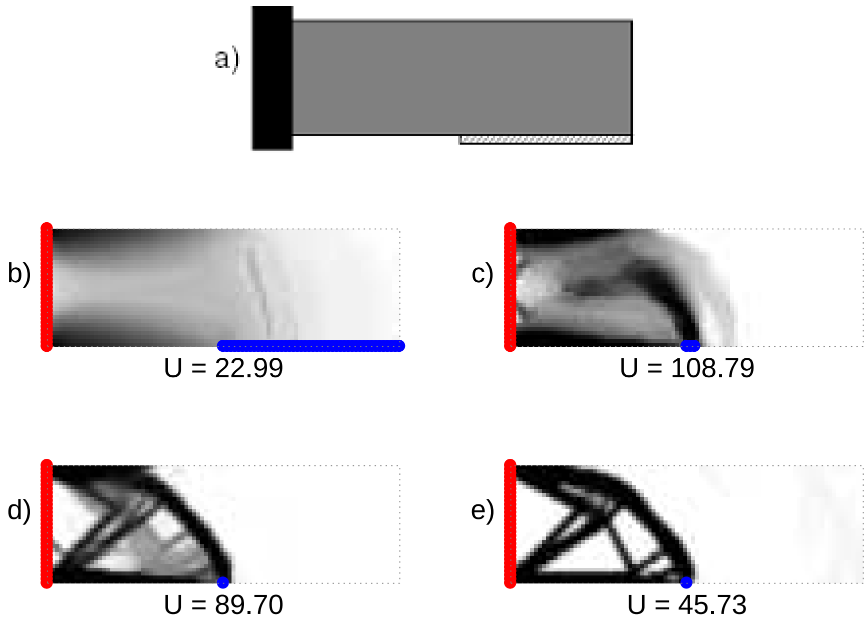
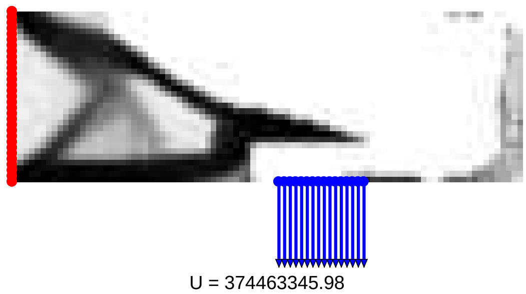


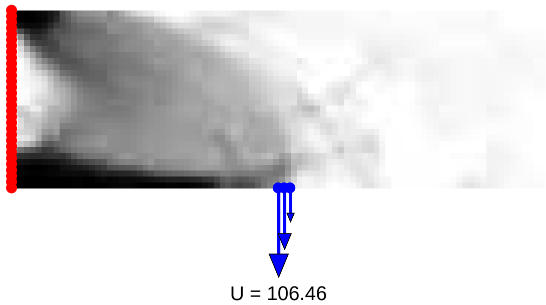
Figure 5-6: Design evolution history of the optimization process for minimal displacement using actuator placement and topology optimization. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps, c) 25% of total iteration steps, d) 50% of total iteration steps, e) final solution. The associated mean displacements are shown under each figure. The associated displacement plots can be found in (Appendix A.6.2) and (Appendix A.6.4).

FEM method could transport the external force through void regions to the structure. This results that a certain area, very close to the force, is displaced very much. This force however, is absorbed by the void regions, so the other solid regions are little effected by displacements due to the external force. The objective isn't determined as a minimal displacement for a local area near the force, but the objective is to minimize the overall displacement. When little elements are displaced very much, while the rest of the element displaces only a little, the overall displacement could be labeled as relatively low.
One way to overcome this problem is by adding another constraint function. It would be nice to implement within the optimization problem a compliance constraint ($c = \mathbf{f}^T\mathbf{u}$) , which should not be too high. By using an arbitrary upper limit value of total compliance, we can prevent the optimization process to let the force pass through less dense regions.

During optimization it seems like the force and density constraints need to be upscaled, to make these two constraints more important than the compliance constraint. This is done due to the fact that the main constraints for this problem are the force and density constraints, while the compliance constraint is just added to prevent the optimizer creating non-physical solutions.
A design evolution history of this optimization process is depicted in Figure 5-6. As can be seen, the optimization process seems very nice and decent. The force is gradually placed to the left hand side of the actuator design domain, the topology is gradually optimized in a black-and-white structure, which is in line with the previous compliance problems. The additional constraint does result in a longer computational time, however.



(a) Minimal displacement including topology and density dependency



(b) Minimal squared displacement including density dependency

Figure 5-7: Optimal actuator placement including topology optimization with density dependency for a) minimal displacement, b) minimal squared displacement The associated displacement plots can be found in (Appendix A.6.3) and (Appendix A.6.5).

5.4.3 Objective refinement

A pretty nice result of the design evolution history can be found in Figure 5-6. It seems like the result is very optimal. There is however one issue that can be improved. The calculation time takes very long. This can probably be ascribed to the formulation of the objective. In (5-15) the objective is described as minimizing the absolute displacement for the whole design domain. However, since the topology is included, the main point of interest is not the displacements of the design domain, but mostly the displacement field of the constructed structure itself. To refine the prescribed objective, it can be very interesting to include the density distribution in the objective. A simple multiplication of the topology distribution by the associated displacement field will result in a new objective. This objective will tend to minimize the displacement of the solid regions. So basically, it will minimize the actual displacements of constructed area. By applying this refinement, a speed improvement can be made. Since the optimizer is no longer interested in minimization of void regions, the computational load can be used for solid regions, which eventually will lead to shorter computation time. The updated minimization problem can now be re-formulated as:

$$u_a(f_i, \rho_e) = (\mathbf{L}^T \mathbf{u}_x)^2 + \boldsymbol{\lambda}^T (\mathbf{K}\mathbf{u} - \mathbf{f}) \tag{5-18}$$

Where $\mathbf{u}_x$ can be seen as a Hadamard product of the node displacement and the node density. Since density is always element based (ρ_e), and node density does not have any physical interpretation, a transformation from the element density should be made into the virtual node density value ρ_n. This Hadamard product can be written as:

$$\mathbf{u}_x = \mathbf{u} \odot \boldsymbol{\rho}_n \tag{5-19}$$

By using this formulation, the corresponding sensitivities can be calculated as:

$$\frac{\partial u_a}{\partial f_i} = (2\mathbf{L}^T \mathbf{u}_x \mathbf{L}^T + \boldsymbol{\lambda}^T \mathbf{K}_x) \frac{\partial \mathbf{u}_x}{\partial f_i} - \boldsymbol{\lambda}^T \frac{\partial \mathbf{f}}{\partial f_i} \tag{5-20}$$

$$\frac{\partial u_a}{\partial \rho_e} = (2\mathbf{L}^T \mathbf{u}_x \mathbf{L}^T + \boldsymbol{\lambda}^T \mathbf{K}_x) \frac{\partial \mathbf{u}_x}{\partial \rho_e} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{u} \tag{5-21}$$

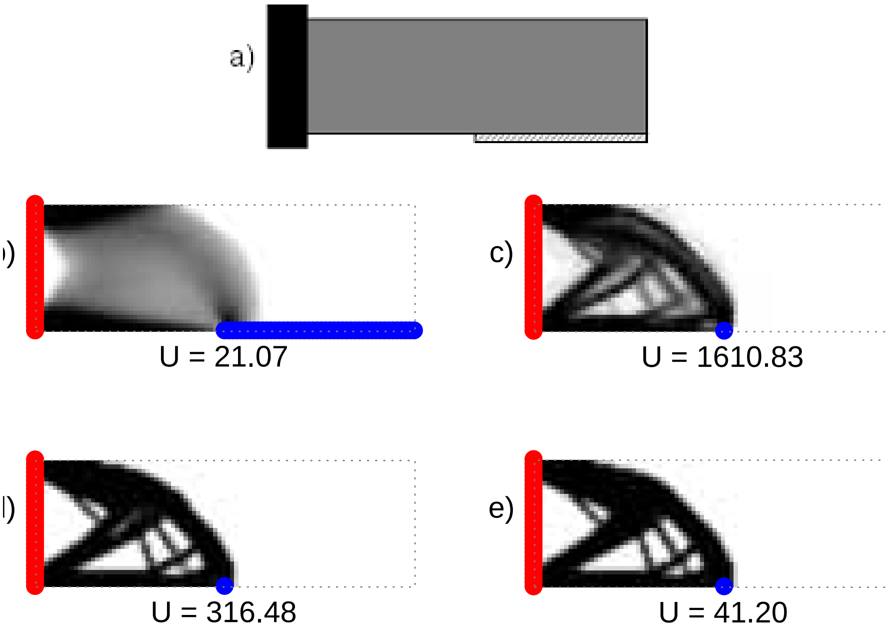


Figure 5-8: Design evolution history of the optimization process for minimal displacement using actuator placement, topology optimization and density dependency. The beam is discretized by 90 x 30 elements. a) design problem, b) 5% of total iteration steps, c) 25% of total iteration steps, d) 50% of total iteration steps, e) final solution. The associated mean displacements are shown under each figure. The associated displacement plots can be found in (Appendix A.6.3) and (Appendix A.6.5).

Where $\boldsymbol{\lambda} = -2\mathbf{K}^{-1}\mathbf{L}{\mathbf{u}_x}^T\mathbf{L} \odot \boldsymbol{\rho}_n$ to remove the $\partial\mathbf{u}_x$ terms, and $\mathbf{K}_x = \mathbf{K} \odot \frac{1}{\boldsymbol{\rho}_n}$. The optimal results are depicted in Figure 5-7. The previous optimal results of Figure 5-7 are now updated, including the new objective from (5-18). In order to compare the difference between both formulations, the same lay-out is used in Figure 5-7, as in Figure 5-5.

As can be seen, the results have different solutions. There is less gray area and the calculation time is improved. Because of the implementation of the density dependency, the optimizer neglects void regions, which eventually leads to shorter computational time and better design of actuator placement. However, both results are still not optimal. One way to overcome this problem is to implement the compliance constraint, as explained in 5.4.2. The optimal result of this optimization problem, including this compliance constraint and including density dependency is depicted in Figure 5-8. As can be seen, the optimizer seems to reach its final stage much faster, than without using density dependency (Figure 5-6). Also, when looking at the deformed geometry, it can be concluded that the optimizer priorities minimal displacement of the solid regions. Therefore, the solution of Figure 5-8 differs from the previous solution and is better physically interpretable. The overall mean displacement is also improved by 10%.

5.5 Application of actuator placement

Design of actuator placement can be used in a variety of domains. Simple cantilever problems can be optimized using the combination of actuator design (Begg et al., 1997) and topology optimization. Especially within the manufacturing domain, actuator placement can be very promising. An optimal actuator layout can be used to minimize internal deformations, which contributes to a more reliable system. Besides to minimizing displacements and minimizing compliance, it can also be used to achieve dynamic performance using a harmonic response (Barboni et al., 2000). For example the frequency spectrum can be tuned using an actuator optimization model as mechanical filter, to ensure that certain mode shapes are not excited whereas other are. Altering eigenmodes can also be done by using actuator placement, in combination with topology optimization, for example to extend the bandwidth. These optimizations can be taken a step further by including control of these actuators (Alves da Silveira et al., 2015). By including this control functionality, it could be very promising to use optimal actuator placement in combination with piezoelectric materials (Foutsitzi et al., 2013). In this field, it should be possible to tune certain dynamic behavior of the material by optimizing the applied voltage to the piezoelectric elements.
Also in the thermal domain actuator placement can be used. In this field, heating can be seen as force application. The perfect heat locations can be found by using actuator placement, in order to maximize the thermal performance of a certain model (Sheng and Kapania, 2001).
In the next chapter a complete case study will be made, which could be very promising in the nearby future. A wafer stage will be optimized to enhance its dynamical performance

5.6 Conclusions

In chapter 4, some words are spend on the design of supports. In this chapter the focus is changed to variable force applications. Optimization can be used to find the perfect actuator placement, in order to achieve an objective.
When determining the optimal place of actuators, the force is used as a set of design variables (5.1). A penalization problem comes up when using negative forces, or forces that are pointing downwards. A way to overcome this, is by using the new introduced Arching Continuation Method (5.1.2), for penalizing negative and positive forces.
A simple solid cantilever beam can be optimized for actuator placement, by minimizing compliance (5.2) or minimizing displacement (5.2.1). Also, design of actuator placement can be used to optimize a variety of advanced applications (5.3), by tweaking the objective and associated sensitivities.
Topology optimization can also be added to the problem. The placement of actuators will cooperate with the topology in order to achieve the best objective (5.4). Some changes should be made to the objective, however (5.4.1), to prevent the optimizer from searching for unwanted optima. A third constraint is sometimes needed, to force the structure being physically interpretable (5.4.2). It can also be helpful to include density dependency into the objective (5.4.3) for even better interpretable results.
Design of actuator locations can be promising in a lot of different fields, from mechanical to thermal problems (5.5). In the next chapter a case study is dedicated to this current chapter, where a wafer stage will be optimized, in order to maximize its dynamical behavior.

Part III

Dynamic Topology Optimization

Chapter 6

Case Study: Wafer Stage

In chapter 5 the design of actuator placement is studied for static problems. In this chapter this approach is taken further by considering dynamics. This placement of dynamic actuator force can also be used in combination with topology optimization, for example to reduce the applied dynamic load.

In section 6.1 an introduction of a wafer stage is made. Dynamics are introduced in section 6.2, where several dynamical aspects are investigated. These phenomena are demonstrated using three different examples. In section 6.3 the design of actuators is investigated to achieve better (dynamical) performance.

Up to here, the design domain is considered as a complete solid region. In section 6.4 topology optimization is included besides the design of actuators in a dynamical spectrum. The solid case examples are now all solved with topology optimization introduced. In section 6.5 a final, optimal solution is given, by making multiple sides of the domain available for actuator design.

In section 6.6 a nice lateral 2D case is made, with a nice 3D graphical representation. Section 6.7 concludes this chapter.

6.1 Case introduction

This section is dedicated to a dynamic actuation of a structure. For example a wafer stage. This stage is used as a driver for a wafer. This wafer is a thin slice of a semiconductor, for example a thin plate of high pure crystalline silicon, which is used in electronics for the

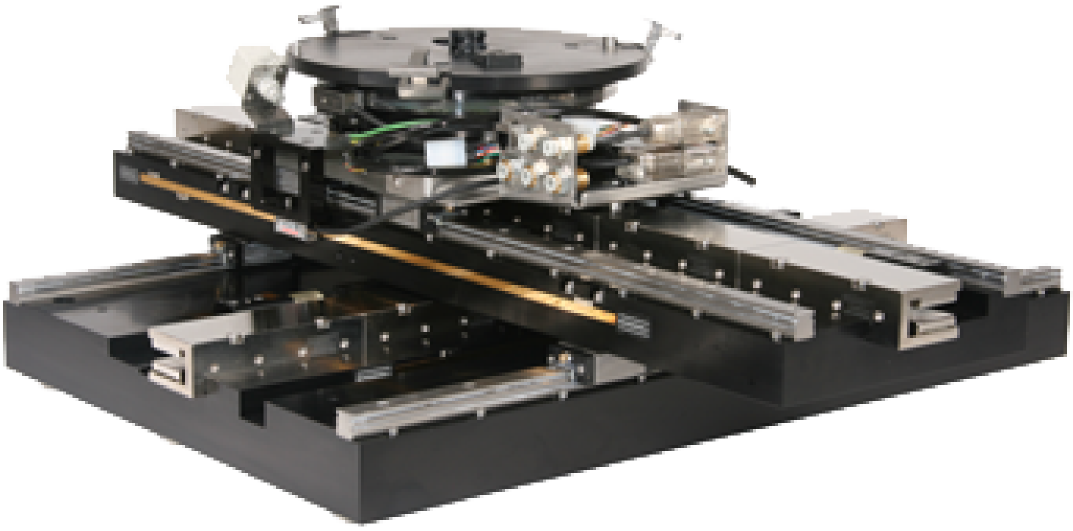


Figure 6-1: A wafer stage and its surrounding complexity (Reliant Systems Inc., 2017)

manufacturing of integrated circuits, as seen in electronic chips. A picture of a wafer stage and its corresponding complexity can be found in Figure 6-1

This wafer stage is actuated and accelerated in order to create a certain motion pattern. This motion pattern can then be used with highly precision positioning to expose the wafer to ultra-violet light, in order to create a certain etching pattern. Extreme precision is required, and any unwanted displacement can result in errors that deteriorate the performance of the electronic circuits. Displacements can arise from small deformations within the wafer stage or from the heat production by the actuator, which results in thermal expansion of the wafer stage. Due to the small size of the integrated circuits, very small deformations in the material can have a big impact. The aim is therefore on a reliable system. The bandwidth and speed of the wafer stage is also a big challenge these days. Time is money, so a faster system will result in more cashflow.

This chapter investigates a new approach to make an improvement on the current wafer stages, by making use of actuator design and topology optimization.

6.2 Dynamics

To understand the way placement of actuators is working in combination with dynamics, let us first have a closer look at the dynamics of this system. The stage should move from left to right, by using an actuator. In this research, we assume the stage to be actuated harmonically, as a model for a cyclic production process. The general dynamic equilibrium equation is given by:

$$\mathbf{Ku} + \mathbf{M\ddot{u}} = \mathbf{f}\sin(\omega t) \tag{6-1}$$

Figure 6-2: Design domain of single force case example. The striped area indicated the objective area. The associated (absolute) vertical displacement of the objective area U is depicted below the figure.

Where $\mathbf{M}$ is the global mass matrix, which is a combination of all elemental mass matrices $\mathbf{M}_e$. In the model we use a lumped mass matrix. This is an easy and fast way of building up a mass matrix, by simply placing a quarter of the element mass along the eight degrees of freedom of that element.
This dynamic equation of motion is only correct when neglecting damping, which is the case in the considered application. The harmonic excitation will result in a harmonic response. By choosing a harmonic oscillation for the displacement vector $\mathbf{u}$, the second derivative can be calculated accordingly:

$$\begin{aligned} \mathbf{u} =& \quad \mathbf{u}\sin(\omega t) \\ \dot{\mathbf{u}} =& \quad \omega\mathbf{u}\cos(\omega t) \\ \ddot{\mathbf{u}} =& \quad -\omega^2\mathbf{u}\sin(\omega t) \end{aligned} \tag{6-2}$$

Substituting these expressions in (6-1) and removing the $\sin(\omega t)$ terms yields:

$$\begin{aligned} \mathbf{Ku} + \mathbf{M}(-\omega^2\mathbf{u}) &= \mathbf{f} \\ (\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} &= \mathbf{f} \end{aligned} \tag{6-3}$$

6.2.1 Single force actuator

For a given desired acceleration of the stage mass, the minimum applied force can be calculated accordingly. By making use of Newton's second law ($\mathbf{f} = m \cdot \mathbf{a}$), where m indicates the total mass of the body, the minimum force which should be applied to the body is found. We add this as a constraint to force optimization problem, which will prevent the optimizer from creating a zero force. Without this constraint, the zero force solution is an attractive solution for the optimizer, as it results in minimal (zero) displacements.
To understand the behavior of the dynamic force optimization problem, we deliberately start with a solid stage, so the topology cannot change here. Additionally, a force can be attached to the middle of the right hand side of the body, to let the body move harmonically. A schematic of the first investigation is drawn in Figure 6-2. As can be seen, the stage consist of a solid body and is actuated with one force on the side. The striped area on the top of the design domain represents the area of the objective. In the simplified stage example the objective is to minimize vertical displacement on the top of the wafer stage, where the thin plate of silicon lays. To be able to minimize this value further on in this thesis research, the displacement is squared, as also described in 5.4.1. However, to compare the several cases

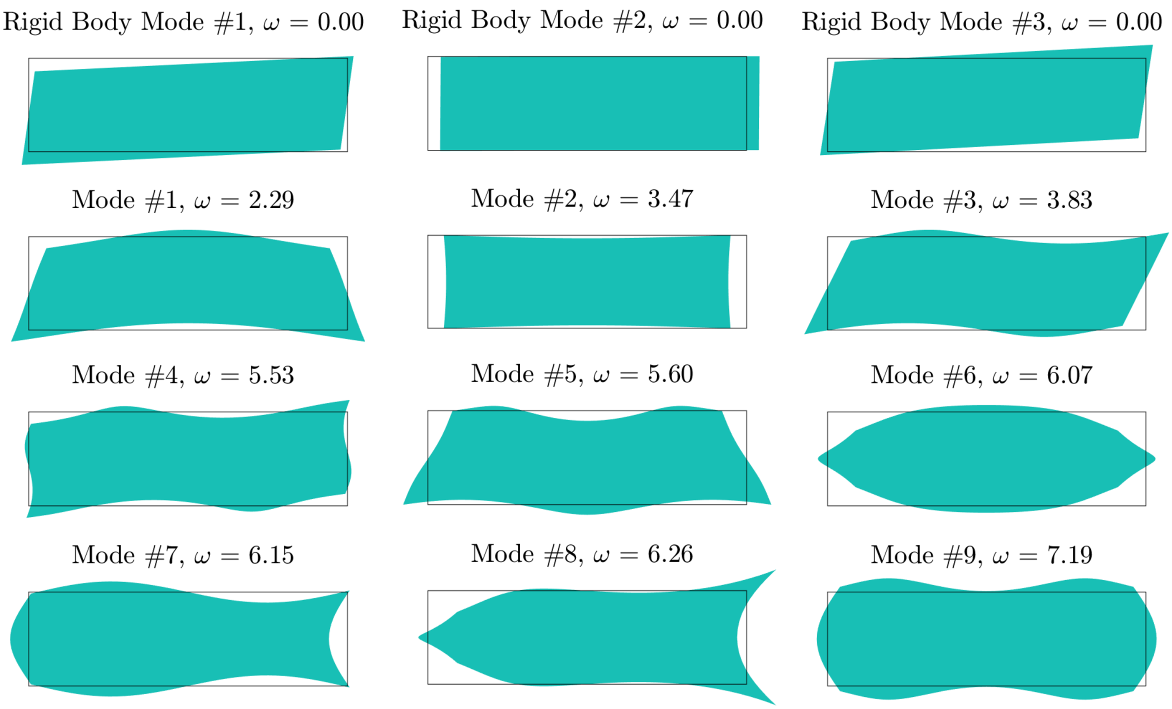


Figure 6-3: Eigenmodes for the first twelve eigenvectors. The upper three eigenmodes are rigid body modes, where zero or very little deformations are involved. The mode shapes in this figure should be combined with Figure 6-4 to get the correct insight in the behavior of each eigenmode.

in this chapter, the sum of the absolute displacements of the top layer, U, is depicted below each figure.

Starting with (6-3), the stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$ remain constant, since the density, and thus the stiffness and mass distribution, will not change. This means the displacement solution can only be solved using the actuation frequency ω.

6.2.2 Eigenmodes

For a set of frequencies ω the expression $(\mathbf{K} - \omega^2\mathbf{M})$ can result in zero. In this case the displacement solution for a nonzero excitation does not exist. This corresponding frequencies are called eigenvalues, or in this particular case, eigenfrequencies. Each eigenfrequency has its own characteristic displacement field, called its eigenmode. This eigenmode can be seen as a natural vibration of the system, where all parts move together at the same frequency, the eigenfrequency. The corresponding shape of the behavior can be depicted by a so-called mode shape. Additionally, the following equation (6-4) can be solved:

$$(\mathbf{K} - \omega_i^2\mathbf{M})u_i = \mathbf{0} \tag{6-4}$$

This equation will result in a set of eigenvectors u_i and corresponding eigenvalues where this equation holds. This set of solved displacement vectors $\mathbf{u}$ are called eigenvectors and will be displayed as $\boldsymbol{\phi}$ in the remainder of this thesis.
The mode-shapes can be divided in rigid body modes and structural modes. Rigid body modes

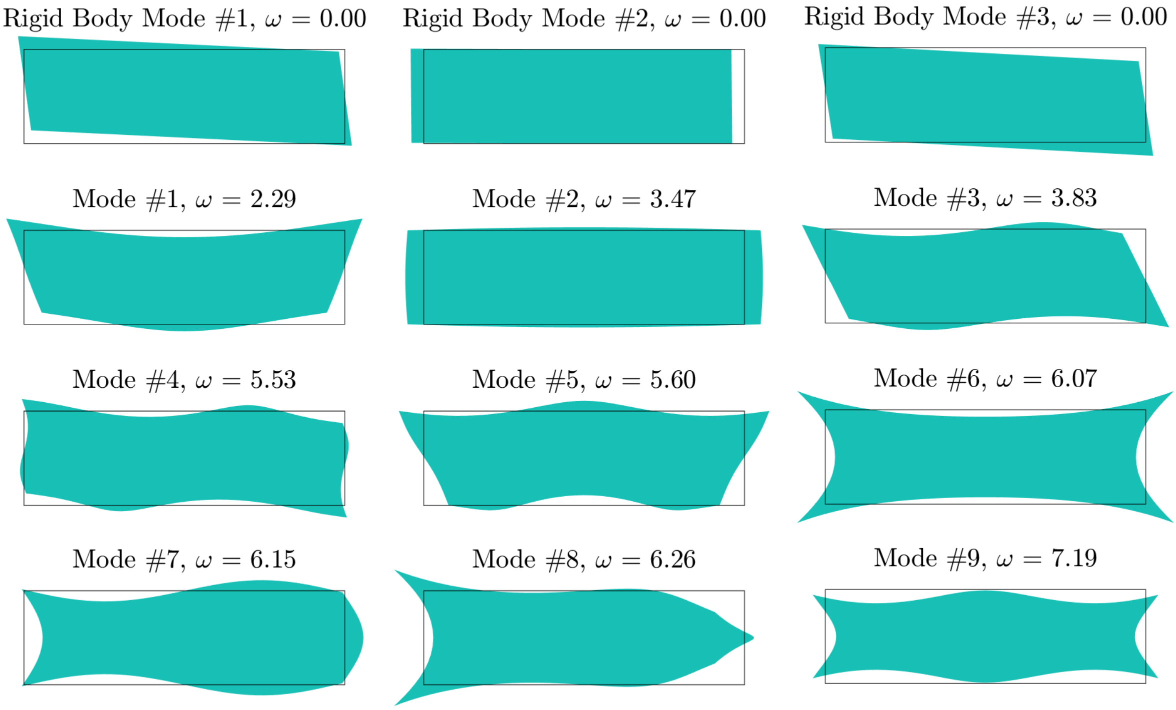


Figure 6-4: Eigenmodes for the first twelve eigenfrequencies. The upper three eigenmodes are rigid body modes, where zero deformations are involved. The mode shapes in this figure should be combined with Figure 6-3 to get the correct insight in the behavior of each eigenmode.

seem to show deformations, but in fact this is rigid rotation. In this case three rigid body modes are involved. When tuning the frequency up, at certain levels, the frequency equals an of eigenfrequency. The associated eigenmodes, the natural vibrations can be described by the modeshapes as depicted in Figure 6-3 and Figure 6-4. Since the rigid body modes do not involve structural deformations, it is very common to start counting eigenfrequencies and eigenmodes from the first structural eigenfrequencies.

Some of the eigenmodes, described in Figure 6-3 and Figure 6-4 are typical bending modes; these eigenmodes exist in beam examples. For example mode number 1, 3, 4 and 5.

6.2.3 Frequency response

A frequency response can be seen as quantitative measure of the output spectrum in response to a certain input. This frequency response is very helpful to characterize the dynamics. In this case, the input is the exerted force. The output of interest is the displacement field. A characteristic way of displaying a frequency response is by using a Bode plot. A Bode plot of this case is depicted in Figure 6-5. In this figure the horizontal output displacement of a point, just above the force application point, is plotted to a certain frequency spectrum. This point, just above the force is chosen, since this point is most of the time displaced. The bottom of the bode plot describes the phase behavior of the system. Zero degree phase means the system is in-phase, the body moves in the same direction as the force. $-180°$ phase means

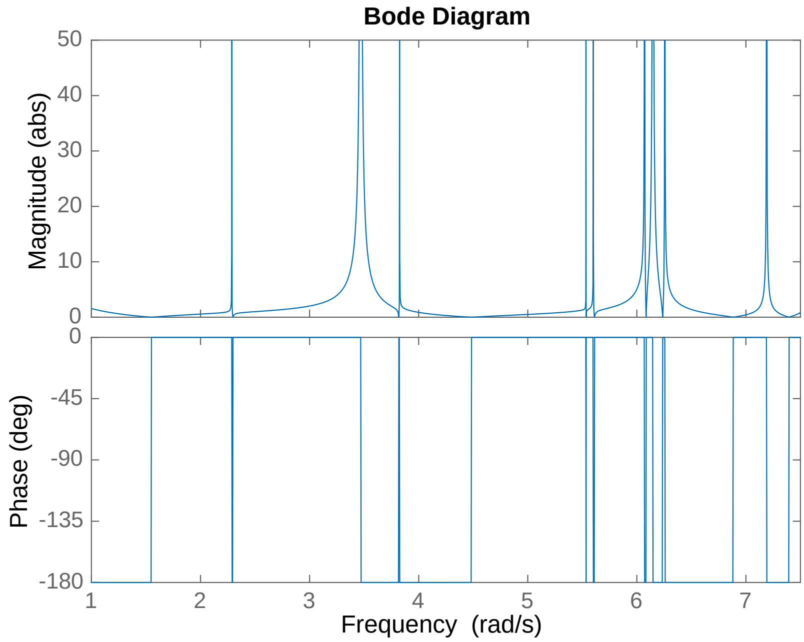


Figure 6-5: Frequency response of a solid body, excited by one harmonic force. The output point is chosen just above the point the force attaches.

the system is completely out of phase and the body moves in the opposite direction of the applied force.

6.2.4 Dynamic mode dependency

Another point of interest in optimization with dynamics, is the influence per mode on the final result. This mode influence can be calculated. By taking the dot product for each eigenvector $\boldsymbol{\phi}_i$, as described in (6-4) and the applied force vector $\mathbf{f}$, the degree to which each mode is excited the force placement can be calculated.
Now, by implementing a weight factor, the influence per mode η_i for the applied excitation frequency ω_i can be calculated accordingly (Rixen, 2008).

$$\eta_i = \frac{\boldsymbol{\phi}_i{}^T \mathbf{f}}{(\omega_i^2 - \omega^2)} \tag{6-5}$$

In this equation (6-5) can be seen, that modes (actuated at their corresponding eigenfrequency ω_i) far away from the excitation frequency ω, will result in a larger denominator and thus in a smaller contribution η_i of this mode. On the other hand, the closer the excitation frequency ω approaches an eigenfrequency ω_i, the smaller the denominator get and thus the influence on this corresponding mode will be larger. In this section an excitation frequency of $\omega^2 = 8$ is used. This frequency lies just between the first and second eigenmode of the solid stage and thus can give us a good view on the dynamic behavior.
The mode contribution for the case depicted in Figure 6-2 is displayed in Table 6-1. In this first

column the mode number can be seen, the second column holds the associated eigenfrequency. In the third column the mode contribution $\boldsymbol{\phi}_i{}^T\mathbf{f}$, followed by the scaled contribution η_i, as described in (6-5). This is done, so the difference between $\boldsymbol{\phi}_i{}^T\mathbf{f}$ and the scaling of the mode contribution can be seen very clearly. In the last column the relative contribution of this mode influence can be found. This contribution is normalized by taking the sum of these first twelve eigenmodes.
As can be seen, the central force placement, is mostly affecting the second (structural) mode, and the second rigid body mode. This means, that the current placement of the single force will result in a displacement field which largely consists of these two modes. A corresponding mode contribution for the six most important modes, over a spectrum of frequencies can be found in Figure A-19. In this schematic it can perfectly be seen which mode contributes how much on every frequency. When the excitation frequency approaches an eigenfrequency, the corresponding mode will be actuated the most and will thus take the largest relative contribution of the total modes. When using this graph and take for example $\omega^2 = 8$, which is used for producing the objective function and also for producing Table 6-1, this frequency can be chosen and the relative contribution values can be seen accordingly, these are in line with Table 6-1.

6.2.5 Double actuator

In the previous example Figure 6-2 only one force is considered. In this subsection however, the force is divided by two and placed at the lower-right and upper-right corner of the objective area (striped area). The distance from the top to the upper force application point is the same as the distance between the bottom and the lower force application point. attachment Since the force is divided by two, the total force remains the same. A schematic of this case is depicted in Figure 6-6. Keep in mind, since the design domain is still solid, the stiffness and mass matrices will not change. The modeshapes in this case are thus the same as in the single force problem (Figure 6-2). As can be seen in Figure 6-6, the objective is improved by almost 30%, with respect to Figure 6-2. The associated mode contribution can be found in

Mode	**Eigenfrequency**	$\boldsymbol{\phi}_\mathbf{i}{}^\mathbf{T}\mathbf{f}$	**Mode contribution $\boldsymbol{\eta_i}$**	**Contribution (%)**
Rigid #1	1.24e-07	0.20	-0.03	0.82
Rigid #2	1.34e-07	5.05	-0.63	20.40
Rigid #3	2.63e-07	0.07	-0.01	0.30
#1	2.29	0	0	0
#2	3.47	-7.36	-1.82	58.89
#3	3.83	0	0	0
#4	5.53	0	0	0
#5	5.60	0	0	0
#6	6.07	3.89	0.13	4.36
#7	6.15	7.44	0.25	8.07
#8	6.26	-3.82	-0.12	3.96
#9	7.19	4.33	0.10	3.20

Table 6-1: Mode contribution of single force case as described in (Figure 6-2) and taking a frequency of $\omega^2 = 8$

Figure 6-6: Design domain of two forces case example. The striped area indicates the objective area.

Figure 6-7: Design domain of distributed force case example. The striped area indicates the objective area.

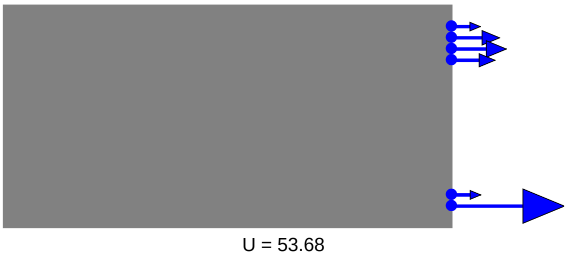
Table A-34. By compare this table with the Table 6-1 conclusions on the mode actuation can be made. By placing the force away from the middle, the second mode is less actuated (η_i). Since this mode is close to the actuation frequency, the total displacement will be lower. The associated mode contribution plot can be found in Figure A-20.

6.2.6 Distributed actuators

In this section the actuation force is distributed on the right side of the domain. Since this distributed load is placed on the elements, the upper and lower nodes only have only one contribution from the elements.

As can be seen, the depicted objective in Figure 6-7 is somewhat worse than using two actuators (6.2.5), but better than only using one force (6.2.2). The corresponding mode contribution can be found in Table A-35. By turning the two force case to a distributed force case, the second mode is actuated more, and since this mode is dominant for this excitation frequency $\omega^2 = 8$, which can be the cause for the larger objective value. The mode contribution graph can be found in Figure A-21.
Now using the information of these three force cases, perhaps an even better solution is possible, by making a mixture of the two force and distributed force cases. In the next section design of actuators will be used to optimize the case.

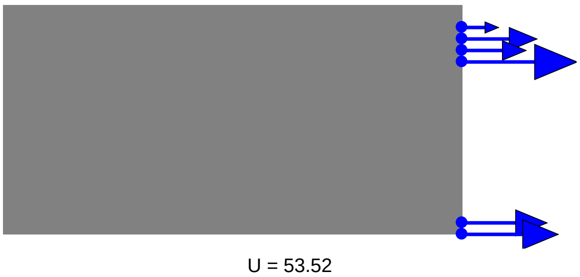
(a) Design domain



(b) Optimal actuator layout

Figure 6-8: Design domain and optimal actuator layout. The gray striped area indicates the objective area. The white striped area indicates the (positive) actuator design domain. The size and placement of the arrows represent the location and magnitude of the optimized force layout. The associated mode contribution can be found in Table A-36.

(a) Design domain



(b) Optimal actuator layout

Figure 6-9: Design domain and optimal actuator layout. The gray striped area indicates the objective area. The white striped area indicates the (positive) actuator design domain. The size and placement of the arrows represent the location and magnitude of the optimized force layout. The associated mode contribution can be found in Table A-37.

6.3 Design of actuators

Up to here, the force is applied at a certain location(s). This section is dedicated to the design of actuator placement, which is already explained in Chapter 5. By making a combination of this design of actuator placement with the described dynamics in 6.2 a solution of the best placement of actuators can be determined, while respecting the dynamics.
The design domain looks like before (Figure 6-2), but now it includes a design domain for actuators on the right hand side. This new design case is depicted in Figure 6-8a. Of course, the main target will be to improve the previous objective. In line with the previous chapters, a new optimization formulation (6-6)can be made, including the design of actuators. As already described in 5-15, the objective is minimizing the squared displacement. The total mass of the structure can be calculated by making a summation of elemental mass. This elemental mass is made of the element's density ρ_e and the material density ρ_0. The total applied force should be enough to move the body with a pre-defined acceleration vector a, in this case it would be a horizontal acceleration of $\omega^2 = 8$, in order to compare the results with the previous examples. The force that can be used to meet this constraint can vary between 0 and 10 for each actuator location (depicted by the white striped area in Figure 6-8a).

$$
\begin{aligned}
\min_{f_i,\rho_e} \quad & u_a \\
\text{s.t.} \quad & u_a = (\mathbf{L}^T\mathbf{u})^2 \\
& (\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{f} \\
& \mathbf{K} = \sum_{e=1}^{n} {\rho_e}^p \mathbf{K}_e \\
& \mathbf{M} = \sum_{e=1}^{n} \mathbf{M}_e \\
& \sum_{e=1}^{n} \nu_e \rho_e \leq V \\
& m \cdot \mathbf{a} \leq \sum_{i=1}^{n} f_i \qquad\qquad i = 1, \ldots, N_i \quad a = \omega^2 \\
& m = \sum_{e=1}^{n} \nu_e \rho_e \rho_0 \qquad\quad e = 1, \ldots, N \\
& 0 \leq f_i \leq 10
\end{aligned}
\tag{6-6}
$$

This optimization problem is then solved using the MMA-solver, which is used along this report. The minimum vertical displacement of the top layer can now be rewritten including an adjoint function.

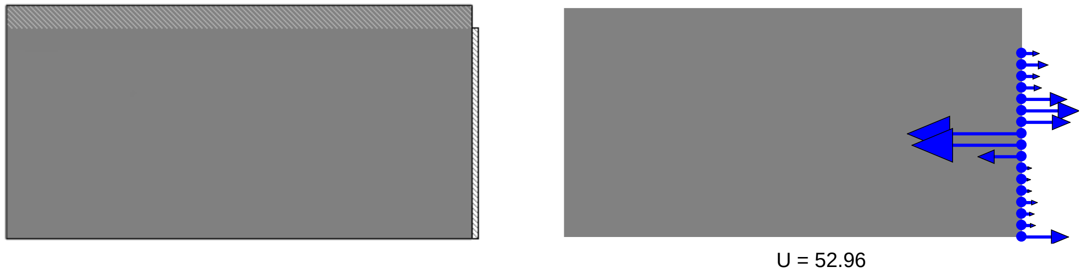
$$
u_a(f_i) = (\mathbf{L}^T\mathbf{u})^2 + \boldsymbol{\lambda}^T\Big[(\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} - \mathbf{f}\Big] \tag{6-7}
$$

Where $\mathbf{L}$ is the selection vector of the displacement, in this case the top layer of the design domain.
The corresponding objective sensitivities can now be calculated accordingly.

$$
\frac{\partial u_a}{\partial f_i} = \Big[2\mathbf{L}^T\mathbf{u}\mathbf{L}^T + \boldsymbol{\lambda}^T(\mathbf{K} - \omega^2\mathbf{M})\Big]\frac{\partial \mathbf{u}}{\partial f_i} - \boldsymbol{\lambda}^T\frac{\partial \mathbf{f}}{\partial f_i} \tag{6-8}
$$

Where $\boldsymbol{\lambda} = -2(\mathbf{K} - \omega^2\mathbf{M})^{-1}\mathbf{L}\mathbf{u}^T\mathbf{L}$ to remove the $\partial\mathbf{u}$ terms.
The optimization result is depicted in Figure 6-8b. The blue arrows indicates the force applications, where the magnitude of the force is proportional to the size of the arrowhead and the total length of the arrow. A threshold value of 0.05 is chosen. This means that a force arrow is only displayed when its value represents a minimum of five percent of the maximum force applied.

As can be seen in Figure 6-8b, a big force is attached at the bottom right, and some cluster of forces at the top right. The objective is slightly better than the two forces case, as described in Figure 6-6. The total designed force equals the $m \cdot \mathbf{a}$ term, which means the optimizer does not use more force than strictly needed to meet the constraint. This is in line with the preliminary thoughts, since additional force will result in additional stresses and thus additional deformations.
The improvement on the objective is made, but some more improvement should be possible. By extending the actuator design domain to include the bottom right corner as actuator design domain this improvement could be possible. The updated design domain is depicted in Figure 6-9a. The optimal actuator layout can be seen in Figure 6-9b. As can be seen, the



(a) Design domain **(b)** Optimal actuator layout

Figure 6-10: Design domain and optimal actuator layout, while enabling negative forces design. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The associated mode contribution can be found in Table A-38.

extension of the actuator design domain leads to force attachment at that additional area and thus a very different force layout. Thanks to this extension, an improvement to the objective can be made. A very small improvement, but an improvement.

6.3.1 Design of negative forces

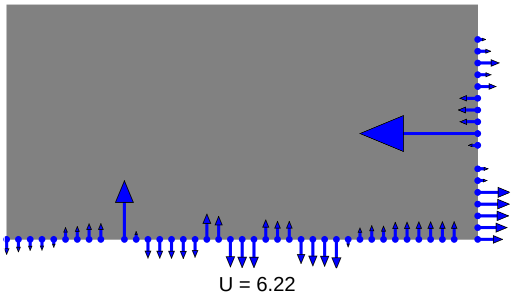
In 6-9 an improvement for the objective value is made available. An even further improvement should be possible. By enabling the possibility to create negative forces (forces in the opposite direction of the acceleration) an improvement can be made, which may seem counterintuitive at first. The same design domain as depicted in Figure 6-9a and almost the same formulation as 6-6. Only one adjustment should be made here, by changing the magnitude of the actuator design domain to $-10 \leq f_i \leq 10$. The updated optimization result can be found in Figure 6-10b. As can be seen, there is a big negative force at the mid-half. In general, it seems a bit unexpected the optimal result would even use negative forces. Since the total force should still at least equal the ($\mathbf{f} = m \cdot \mathbf{a}$) term, a negative force will thus also lead to larger positive forces. The reason to create negative forces is to counteract the dynamic eigenmodes. A negative force can be used to counteract or reduce the dynamical effects, although a larger amount of forces should be used.

It can be concluded that improvements in the objective can be made by placing forces in other direction than the acceleration force. In the next section this approach is taken a step further.

6.3.2 Design of force at multiple sides

Up to here, the force application could only be attached at the right-half side of the design body. However, improvements can be made by making multiple sides of the body available for actuator placement. In this section, bottom force can be applied at the bottom-side of the design domain. These forces can be upwards (positive) or downwards (negative). Of course, these vertical forces are not contributing to the ($\mathbf{f} = m \cdot \mathbf{a}$) expression. But these forces can be used to reduce mode excitations. An updated design domain can be found in Figure 6-11a,

(a) Design domain

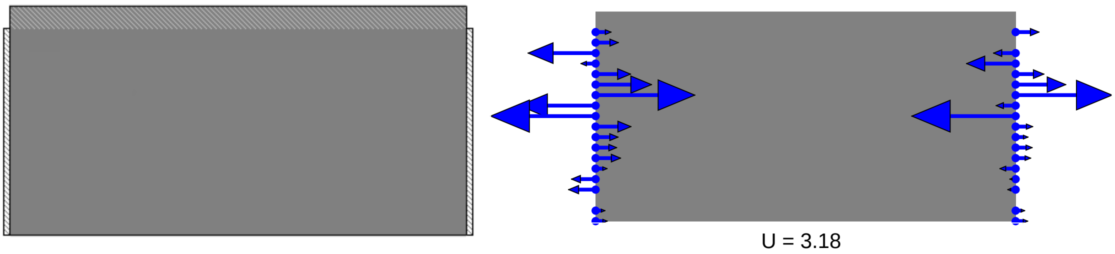


(b) Optimal actuator layout

Figure 6-11: Design domain and optimal actuator layout, while enabling negative forces design. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The associated mode contribution can be found in Table A-39.

the corresponding optimal actuator location layout can be found in Figure 6-11b. As can be seen, a huge improvement can be made to the objective value. The extension of the actuator design domain indeed leads to a very big improvement of this optimization problem.
Another possibility could be to include the left-half side of the body in the actuator design domain. An even larger improvement of the objective value U can be made. The updated design domain can be seen in Figure 6-12a. The optimal actuator layout is depicted next to it in Figure 6-12b. As can be seen, a variety of forces are applied to the body. The total applied force is almost exactly the minimal force needed, to accelerate the body with an acceleration of $w^2 = 8$. Some big forces at the mid-half of both sides are pointing left, which is in opposite direction of the acceleration, to reduce the excitations caused by the dynamic behavior. Another two big forces are needed to actually achieve the minimal total horizontal force. Another point of interest is the steadily decreasing of the contribution of the second mode, which can be seen in Table A-40. For the design result depicted in Figure 6-11b this contribution is almost zero. This means the optimizer want to make a design which has very little impact from this second mode. The fact it almost hit zero means the optimizer did a very good job at this one.
By enabling the left-half side of the body for actuator design domain, a very nice objective improvement can thus be achieved.

An even better solution could be to combine Figure 6-11 and Figure 6-12. The result is depicted in Figure A-24. Here, a big problem when optimizing this type of design problem, is the possibly overfitting of the model. The optimizer has just too many variables and the optimizer is more likely to approach a (high) local optimum. The result depicted in Figure A-24 shows a distribution along all sides of the design domain. The horizontal force is almost twice the minimum needed force to achieve the prescribed acceleration. This could also be a symptom of the overfitting of the model. It can be concluded that, in order to achieve a maximal optimization result, the design domain should not be too vague or too big.
Another option to optimize, is actuating at the natural frequency. Of course, it is not common to actuate at or near an eigenfrequency. But in some cases, when the material and frequency are given, it could be possible we need to optimize the actuator layout in order to trigger the modes as little as possible.



(a) Design domain **(b)** Optimal actuator layout

Figure 6-12: Design domain and optimal actuator layout, while enabling negative forces design. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The associated mode contribution can be found in Table A-40.

6.4 Topology optimization for dynamic performance

Up to here, the stage consisted of a completely solid stage. This thesis research is based on topology optimization, however. By enabling the possibility of changing the topology in the design domain, even better results can probably be achieved. The combination of actuator placement and topology optimization is already touched in 5.4 for static problems. By enabling dynamics (6.2), a more advanced optimization problem can be set up.
The objective in this section remains the same, namely minimizing vertical displacement of the top layer. This top layer should be a solid area. Since in this section the topology can be changed, a restrictive area (3.2.1) is introduced at the top of the design domain.
The main focus will be to look for a better performing wafer stage example, in terms of the vertical displacements of the top layer. Eigenmodes and frequency response are already described in 6.2.2 and 6.2.3. These dynamic properties depend on the stiffness and mass distribution. Since the topology can now be changed, the eigenfrequencies, eigenmodes and frequency response will also change during the optimization process.

6.4.1 Topology optimization for fixed force

To understand the behavior of topology optimization, in this section a topology optimization example for a fixed force case is considered. Since the two force example (6.2.5) seems to be a good starting point, we will use this example for topology optimization. In this example the force remains the same, the magnitude is based on moving a solid stage ($\mathbf{f} = m \cdot \mathbf{a}$). This means the optimizer could make a complete solid stage. On the other hand, removing material does not contribute to a lower force application in this example. This example is created to see whether or not the optimizer wants to remove material and what regions should be void. The design domain can be found in Figure 6-6. As can be seen, two problems seem to come up. At first, the lower force is not directly connected to the structure by solid regions. This means the force attachment has no physical interpretation. This problem is already seen in Figure 5-5b, with a possible solution as described in 5.4.2, to overcome this problem.

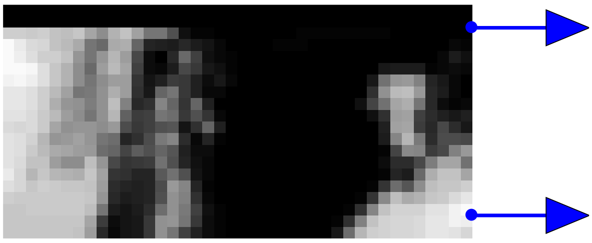


Figure 6-13: Optimal topology for minimal displacement using static forces. The associated total vertical displacement of the top layer is depicted under the figure. The total horizontal force used is $\mathbf{f} = 3.20$.

A compliance constraint should be implemented, in order to ensure the force is attached to the structure. By implementing a compliance constraint, the optimizer is prohibited from creating very large displacements at the point of force attachment.
Another topology phenomenon comes up in Figure 6-13, namely gray regions. Gray regions also have no physical interpretation. A penalty-term of $p = 3$ is already implemented, but still a lot of gray regions exist. Since the optimization problem depends on the topology and subsequently on the frequency response, it could be possible the optimizer only wants part of the stiffness (and mass) of certain elements, in order to reduce certain mode excitations.

6.4.2 Topology optimization for double actuator

Enabling topology optimization can indeed enhance the result and could contribute to a smaller objective value, hence less displacements. In Figure 6-13 an example of this topology optimization result is depicted. Here, the force remained fixed. Note that the applied force here, does not change, while the weight is reduced. Smaller mass means less force required ($\mathbf{f} = m \cdot \mathbf{a}$).
It could therefore be helpful, to implement this equation in the optimization routine. A weight reduction could therefore result in a force reduction. This force reduction could lead to less deformation in the material and therefore in a smaller displacement field of the top layer. Design of actuators (6.3) could be very helpful also, to calculate the optimal actuator layout.
As already described in 6.4.1, two problems should be overcome. In this section a compliance constraint is implemented, the same way as introduced before in 5.4.2. As can be seen in Figure 6-14, the force seems to be attached to the structure. Since the minimum force needed is from now on coupled to the mass, a mass reduction could thus lead to a force reduction. Note that the minimum force to accelerate the solid body is $\mathbf{f} = 3.20$. To get an insight in the force reduction that can be achieved, the associated applied total horizontal force is depicted in the legend of each optimization case.
The second problem, gray regions, is also investigated in Figure 6-14. Here, the top layer is still solid material and the displacements of this top layer should be reduced. The topology can be varied in the design domain and actuators can be designed at the two points as depicted in Figure 6-6. Although the density distribution is different for the cases as depicted in Figure 6-14, the volume fraction is around the same value. This also holds for the minimum horizontal applied force. Typically, it can be concluded, that weight reduction results in force reduction.

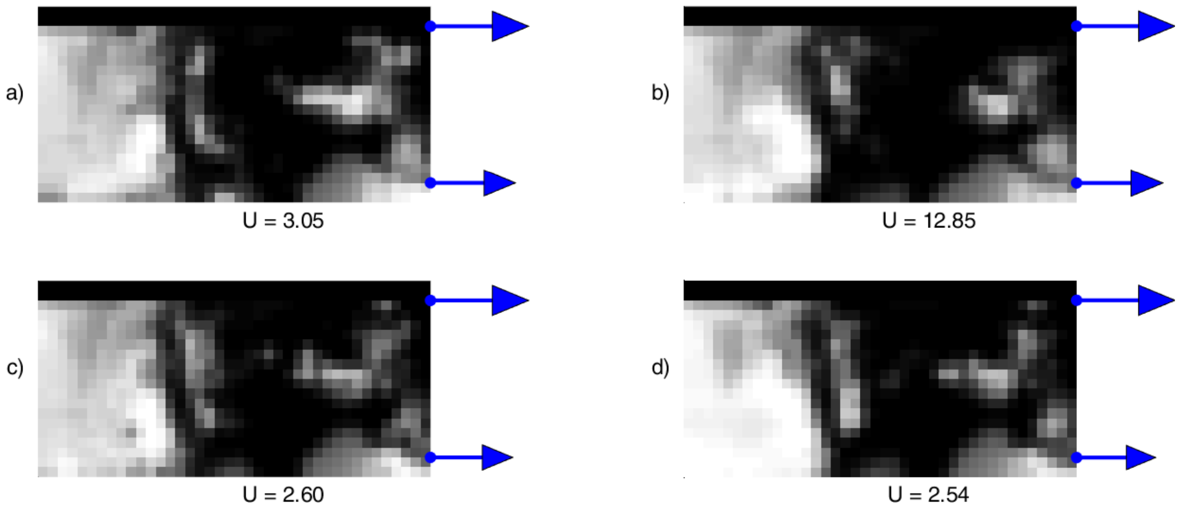


Figure 6-14: Optimal actuator placement including topology optimization for minimal displacement using two forces, including a compliance constraint. The penalty is defined by a) $p = 3$, b) $p = 4$, c) $p = 5$, d) $p = 6$. The associated total vertical displacements of the top layer are shown under each figure. The total horizontal force used is a) $\mathbf{f} = 2.16$, b) $\mathbf{f} = 2.16$, c) $\mathbf{f} = 2.17$, d) $\mathbf{f} = 2.13$.

The same optimization problem as stated in 6-6 holds, with the notation that ρ_e can now be varied. This means the same vertical displacement of the top layer is considered. The objective thus remain the same.

$$u_a(f_i, \rho_e) = (\mathbf{L}^T\mathbf{u})^2 + \boldsymbol{\lambda}^T\Big[(\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} - \mathbf{f}\Big] \tag{6-9}$$

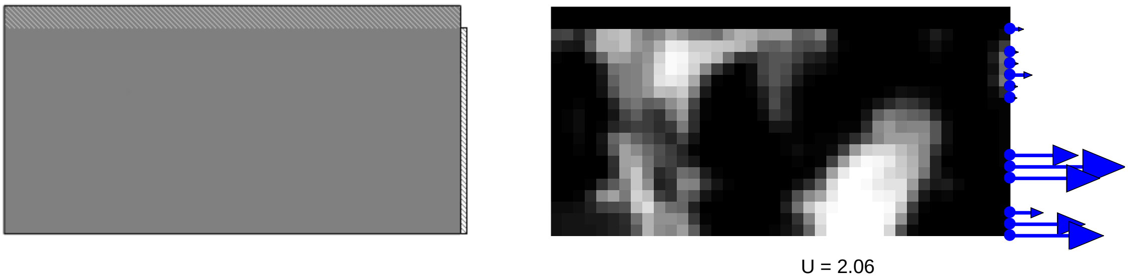
This equation should also be differentiated to the density variable ρ_e. This sensitivity can be calculated as:

$$\frac{\partial u_a}{\partial \rho_e} = \Big[2\mathbf{L}^T\mathbf{u}\mathbf{L}^T + \boldsymbol{\lambda}^T(\mathbf{K} - \omega^2\mathbf{M})\Big]\frac{\partial \mathbf{u}}{\partial \rho_e} + \boldsymbol{\lambda}^T\frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} - \omega^2\boldsymbol{\lambda}^T\frac{\partial \mathbf{M}}{\partial \rho_e}\mathbf{u} \tag{6-10}$$

Where $\boldsymbol{\lambda} = -2(\mathbf{K} - \omega^2\mathbf{M})^{-1}\mathbf{L}\mathbf{u}^T\mathbf{L}$ to remove the $\partial\mathbf{u}$ terms.
By varying the SIMP penalty-term, some insight in the behavior of the structure can be achieved. While increasing the penalty-term from $p = 3$ (Figure 6-14a), to $p = 6$ (Figure 6-14d), it can be clearly seen that the behavior tend to optimize towards a black-and-white solution, which is better physically interpretable. Although a high penalty is implemented at Figure 6-14d, the structure still wants to create gray regions. This means the optimizer want some stiffness in that particular region, even when this will have a big trade-off. It can be concluded, the total vertical displacement of the top layer is decreasing by the implementation of topology optimization, when compared to the massive stage from Figure 6-6.

6.4.3 Side force and topology optimization

As already concluded in 6.3, design of actuators along the side can be helpful in achieving lower displacements. In this section the complete righthand side of the design domain can be used



(a) Design domain **(b)** Optimal solution

Figure 6-15: Design domain and optimal actuator layout, including topology optimization. The gray striped area indicates the objective area. The white striped area indicates the (positive) actuator design domain. The size and placement of the arrows represent the location and magnitude of the optimized force layout. The total horizontal force used is $\mathbf{f} = 2.51$.

for force actuation. By enabling this option in combination with topology optimization, it allows the optimizer to avoiding low eigenmodes and actuating at points where less excitation is experienced. The topology can be used to avoid certain modes, the force can be used to avoid excitations of certain modes. The combination can be used to create an efficient frequency response for the particular case. An example of this problem is depicted in Figure 6-15. In the design domain (Figure 6-15a) the design domain for actuators can be found. The optimal topology and actuator distribution is depicted in Figure 6-15b. As can be seen from this solution, enabling topology optimization can enhance performance, compared to the massive stage example without topology optimization (Figure 6-9b). Also, by creating a bigger force design domain, reducing displacements of the top layer can be achieved, compared to the double actuator design case (Figure 6-14).

6.4.4 Negative forces and topology optimization

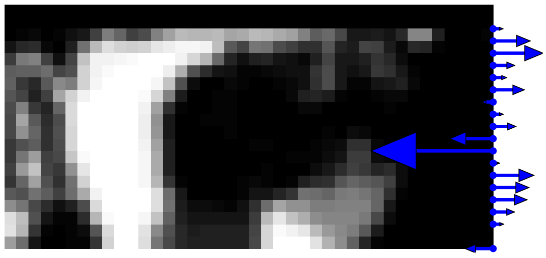
Up to here, this section (6.4) only includes (design of) positive forces. However, as can be seen in 6.3.1, enabling the possibility for creating negative forces could counteract or reduce certain mode excitations.
A problem comes up here, when implementing the compliance constraint (5.4.2). This compliance constraint is defined as:

$$c = \mathbf{f}^T \mathbf{u} \tag{6-11}$$

This formula is pretty straightforward, but a problem comes up when creating negative forces. The point of negative force attachment, could have a positive displacement at that particular point. This is especially true in this case, since the body needs to move to the right. The negative contribution could make it easier to meet the compliance constraint, and allow again forces that act on gray/void elements. To overcome this problem, we want to calculate the compliance as the absolute values of f and u. A simple multiplication of these absolute values gives a problem, since this function is not differentiable. Note the definition of an absolute value:

$$|x| = \sqrt{x^2} \tag{6-12}$$

(a) Design domain



(b) Optimal solution

Figure 6-16: Design domain and optimal actuator layout, while enabling negative forces design and using topology optimization. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The total horizontal force used is **f** $= 2.35$.

A possible solution is to introduce a very small value ϵ to the value that needs to become absolute. This ϵ can be implemented in 6-12 and subsequently in 6-11:

$$\begin{aligned} c &= |\mathbf{f}|^T|\mathbf{u}| \\ &= \left(\sqrt{\mathbf{f}^{\,2}+\epsilon}\right)^T\left(\sqrt{\mathbf{u}^2+\epsilon}\right) \end{aligned} \tag{6-13}$$

When choosing the value ϵ small enough, the influence will become very small and can be neglected. The main problem is here the multiplication of vectors f and u. These vectors should be squared element-wise, by making use of the Hadamard product, which was already introduced in 5-19. The compliance in correct vector notation can now be rewritten as:

$$c = \left(\sqrt{\mathbf{f}\odot\mathbf{f}+\epsilon}\right)^T\left(\sqrt{\mathbf{u}\odot\mathbf{u}+\epsilon}\right) \tag{6-14}$$

This compliance constraint is differentiable to the force and density variable. By adding again an adjoint vector, the sensitivities can be calculated.

$$c(f_i,\rho_e) = \left(\sqrt{\mathbf{f}\odot\mathbf{f}+\epsilon}\right)^T\left(\sqrt{\mathbf{u}\odot\mathbf{u}+\epsilon}\right) + \boldsymbol{\lambda}^T\left[(\mathbf{K}-\omega^2\mathbf{M})\mathbf{u}-\mathbf{f}\right] \tag{6-15}$$

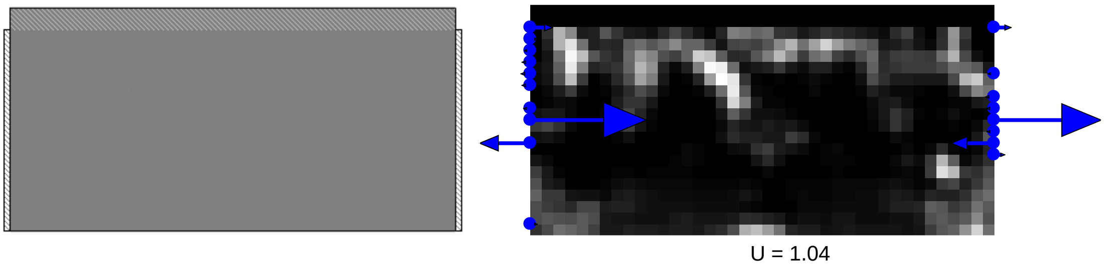
with the corresponding sensitivities:

$$\frac{\partial c}{\partial \rho_e} = \left[\frac{\mathbf{u}\sqrt{\mathbf{f}\odot\mathbf{f}+\epsilon}}{\sqrt{\mathbf{u}\odot\mathbf{u}+\epsilon}} + \boldsymbol{\lambda}^T(\mathbf{K}-\omega^2\mathbf{M})\right]\frac{\partial \mathbf{u}}{\partial \rho_e} + \boldsymbol{\lambda}^T\frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} - \omega^2\boldsymbol{\lambda}^T\frac{\partial \mathbf{M}}{\partial \rho_e}\mathbf{u} \tag{6-16}$$

$$\frac{\partial c}{\partial f_i} = \left[\frac{\mathbf{u}\sqrt{\mathbf{f}\odot\mathbf{f}+\epsilon}}{\sqrt{\mathbf{u}\odot\mathbf{u}+\epsilon}} + \boldsymbol{\lambda}^T(\mathbf{K}-\omega^2\mathbf{M})\right]\frac{\partial \mathbf{u}}{\partial f_i} + \left[\frac{\mathbf{f}\,\sqrt{\mathbf{u}\odot\mathbf{u}+\epsilon}}{\sqrt{\mathbf{f}\odot\mathbf{f}+\epsilon}} - \boldsymbol{\lambda}^T\right]\frac{\partial \mathbf{f}}{\partial f_i} \tag{6-17}$$

Where $\boldsymbol{\lambda} = -\left(\mathbf{K}-\omega^2\mathbf{M}\right)^{-1}\left(\frac{\mathbf{u}\sqrt{\mathbf{f}\odot\mathbf{f}+\epsilon}}{\sqrt{\mathbf{u}\odot\mathbf{u}+\epsilon}}\right)$ to remove the $\partial\mathbf{u}$ terms.

This compliance constraint is now used to optimize the case depicted in Figure 6-16a. As



(a) Design domain **(b)** Optimal solution

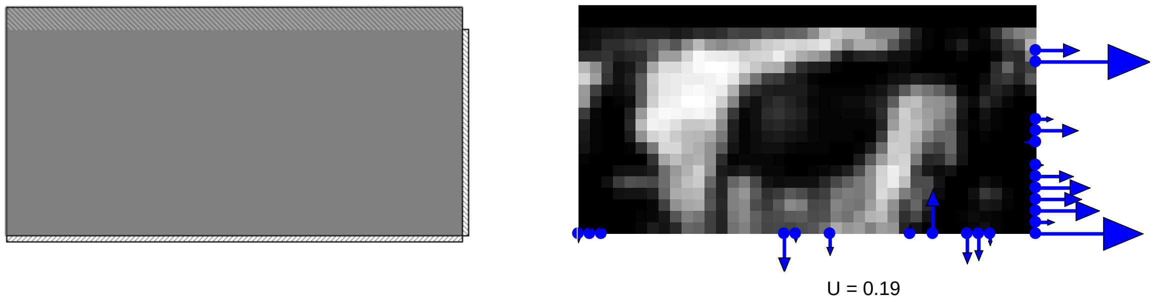
Figure 6-17: Design domain and optimal actuator layout, while enabling negative forces design and using topology optimization. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The total horizontal force used is $\mathbf{f} = 2.89$.

already stated, the force design can vary from positive to negative values. The optimal actuator layout and corresponding topology can be found in Figure 6-16b. As can be seen here, the total vertical displacement of the top layer is again decreased, when compare to the previous case in Figure 6-15b where only positive forces can be created along the right side of the design domain. Also, an big improvement with respect to the massive case Figure 6-12b can be achieved.

6.5 Topology optimization for actuator placement

In this section a combination of all previously described knowledge, examples and case studies will come together. The main focus is still improving the objective value by minimizing vertical displacements of the top layer. We have already seen an example of using multiple sides for actuator placement in 6.3.2. By adding the design of density, by terms of topology optimization (6.4) and using the compliance constraint described in 6-15 some promising improvements are already shown. In this section, design of forces at multiple sides is combined with topology optimization. Preliminary thoughts tells us that a combination of these options can improve the objective even further.
In Figure 6-12 an example of using both sides of the design domain for actuator placement is shown. This same actuator design domain is used, but now enabling topology optimization. The result is depicted in Figure 6-17b. The force distribution is somewhat different from the massive case (Figure 6-12). The objective improvement is made, however. The volume fraction used to achieve this can be labeled as large, compared to the previous topology examples in 6.4.

Another option could be to design at the right side of the domain and the bottomside of the domain. This example for a massive stage is already depicted in Figure 6-11. Now by implementing topology optimization perhaps even better results can be achieved. The design domain is depicted in Figure 6-17a, with the corresponding optimized result depicted in Figure 6-17b. As can be seen here, the design of actuators differs from the massive stage example with the same actuator design domain (Figure 6-11). Also, the objective value, the vertical displacement of the top layer is reduced even further, compared to Figure 6-11 and



(a) Design domain

(b) Optimal solution

Figure 6-18: Design domain and optimal actuator layout, while enabling negative forces design and using topology optimization. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The total horizontal force used is $\mathbf{f} = 2.52$.

Figure 6-16. Although the penalty term is set to a high value ($p = 6$), the optimal result still shows some gray regions, even after a high number of iterations, the gray regions still have the preference.

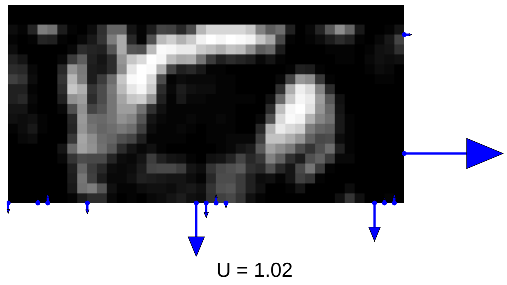
The overall vertical displacement of the top layer is reduced to $U = 0.19$, as can be seen in Figure 6-18b. When a comparison is made to the original model, a massive stage with two point forces ($U = 54.07$), the change is substantial. The total reduction of the displacements is -99.6%. So it can be stated, using topology optimization and actuator placement can have a huge impact on reducing displacements. An overview of all the produced examples in this chapter can be found in Table 6-2.

6.5.1 Improving gray regions

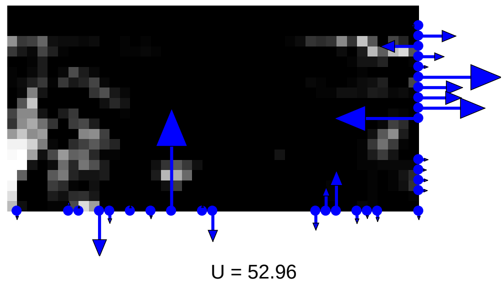
As already stated before, the best solution as depicted in Figure 6-18 includes gray regions. One way to improve this result for manufacturing, it could be a good idea to increase the penalty term even further. In Figure 6-19a an example of an increasing penalty ($p = 11$) is given. As can be seen, there is some improvement in terms of black-and-white solutions. This results however, in a larger displacement field, since the optimizer is even more forced to create black-and-white solution.
The results as achieved in this chapter are filtered during the optimization process using a density filter (3.2.5), using another filter, in this case the Heaviside filter (3.2.5) is used to force the optimizer towards black-and-white solutions. The result of using this Heaviside filter is depicted in Figure 6-19b. As can be seen, the solution is improved even more, in terms of black-and-white regions. This also results in a larger displacement field of the top layer, but it is better physically interpretable.

6.5.2 Changing conditions

Up to here, we let the optimizer choose the best solution, with no maximum weight restrictions (Although the solution is limited to use 100% of the material). In some case however, it could



(a) High penalty example



(b) Heaviside filter example

Figure 6-19: Design domain and optimal actuator layout, while enabling negative forces design and using topology optimization for two different solving situations: a) high penalty example ($p = 11$), b) Heaviside filter example. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout.

be possible that weight reductions are required. In Figure A-26 some examples with these restrictions are solved. As can be seen, a weight reduction as constraint results in larger displacement fields. This is in line with the preliminary thoughts, since these additional restrictions causes less stiffness properties.
Another changing condition could be changing actuation frequency. When the actuation frequency is pre-required, the optimal solution will be different. In Figure A-27 two examples for different actuation frequencies can be found. The examples consists of taking the half of the original actuation frequency ($\omega^2 = 4$) and taking the double of the original actuation frequency ($\omega^2 = 16$). The behavior of the optimizer will be the same, namely placing as much as eigenfrequencies in front of the actuation frequency, in order to reduce the mode excitations. The optimal result is, however, heavily dependent on the mass and the applied force of the structure. As can be seen, the volume fraction that is used by the optimizer is around the same. Changing actuation frequency however can thus result in more or less force needed to achieve the desired acceleration ($\omega^2 = a$). This same approach can also be used to optimize the problem sketched in A.8.1.
It can be concluded that the optimizer can handle multiple restrictions, for example a weight restriction, or a desired actuation frequency. Both can be implemented and the algorithm can calculate the optimal solution for each particular case.

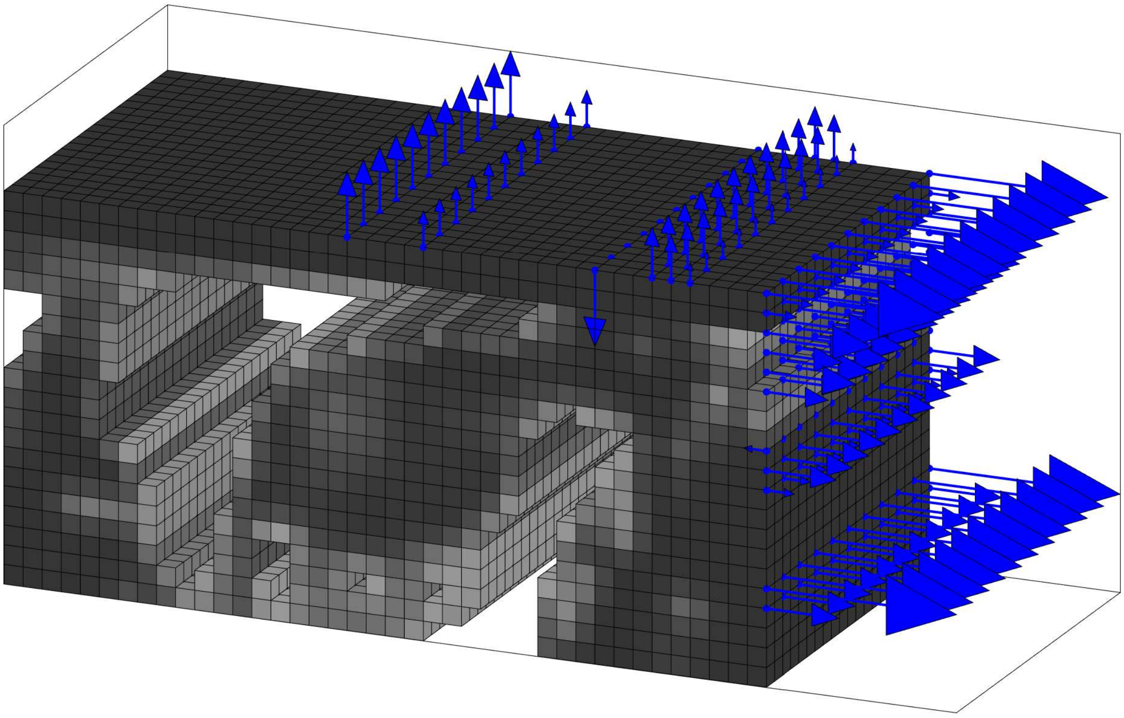


Figure 6-20: A 2D lateral extrusion of the optimal wafer stage as depicted in Figure 6-18b.

6.6 3D extrusion

Up to here, only 2D situations are considered. There is no step taken towards a three dimensional case. The main reason for this is the computational time. In this section however, a 3D extrusion is made. This extrusion is just a 2D lateral case in another dimension. A threshold value of 0.5 is chosen. This means all densities below this threshold value are displayed as void regions, for a better visual representation. A same threshold method is implemented for the force distribution. Note that the lateral extrusion contains one half of the width of the wafer stage. This is only done for a better view to the user. As already explained in 6.1 this wafer stage should be used to produce circular wafers. The width and depth of the wafer should thus be the same size. A full representation of this lateral extrusion can be found in A.8.5.

6.7 Conclusions

In chapter 5, some words are spent on the design of actuator placement. A simple cantilever beam was optimized using design of actuators. Later on, topology optimization was included also.
In this chapter a wafer stage, as described in 6.1 is introduced. This wafer stage is simplified in a 2D example, which is used as design domain. This wafer stage is harmonically actuated by the implementation of dynamics (6.2). By investigating dynamical phenomena like eigenmodes (6.2.2) and frequency responses (6.2.3) these dynamics are investigated for three

simple cases. The double actuator case (6.2.5) seems to be the best solution of these three considered cases.
An interesting field is the design of actuators in this dynamic spectrum. By taking several design cases (6.3), a distributed force on the left- and righthand side of the design domain (Figure 6-12) seems to be very promising.
Since dynamics are involved in this wafer stage, the design is heavily dependent on its frequency response. This frequency response is determined by its stiffness and mass properties. It could therefore be very helpful to have a look at the volume distribution this material by the implementation of topology optimization (6.5). Several cases are considered and the best solution to this dynamic wafer stage design problem is by designing actuators on the righthand side and the bottomside of the spectrum (Figure 6-18). A problem that could come up is the translation to additive manufacturing. For example the existence of gray regions should be solved (6.5.1). A translation to this additive process also needs to be made by introducing a third dimension (6.6), although this is not explicitly solved in this chapter.
Using topology optimization in combination with actuator placement can be very promising in dynamic problems. In this chapter a displacement reduction of 99.6% is made. Although a translation to the real world and additive manufacturing needs to be made, still a lot of benefits can be achieved by the combination of these optimizations.

Design problem	**Displacement top**	**Applied force**	**Relative improvement**
Figure 6-2	61.92	3.20	+14.5%
Figure 6-6	54.07	3.20	0
Figure 6-7	56.30	3.20	-4.1%
Figure 6-8	53.68	3.20	-0.1%
Figure 6-9	53.52	3.20	-0.1%
Figure 6-10	52.96	3.20	-0.2%
Figure 6-11	6.22	3.20	-88.5%
Figure 6-12	3.18	3.20	-94.1%
Figure 6-13	19.14	3.20	-64.6%
Figure 6-14d	2.54	2.13	-95.3%
Figure 6-15	2.06	2.51	-96.2%
Figure 6-16	1.95	2.35	-96.4%
Figure 6-17	1.04	2.89	-98.1%
Figure 6-18	0.19	2.52	-99.6%

Table 6-2: Displacement overview for all considered cases from Chapter 6. The first column states the design result, the second column states the total absolute vertical displacement of the top layer. The third column states the used force (note that the solid stages all used the minimum required force **f** to achieve the desired acceleration ω^2). The last column shows the relative change, as compared to the massive case, with two forces (Figure 6-6), since this seems to be a good starting solution in prior.

Part IV

Closure

Chapter 7

Conclusions and Recommendations

This chapter concludes this Master of Science graduation project. All the chapters are concluded in 7.1. Next, some recommendation for future research are made in 7.2.

7.1 Conclusions

In this thesis, topology optimization is used for a variety of design problems. At first, design of supports is considered. When the placement of supports is not prescribed, design of supports tells us the best support layout. By making the combination with topology optimization, the design of supports cooperates with the topology to create a structure which optimizes static behavior.
The classical approach of constructing a bridge does not include support design. By the implementation of a support cost function, the best support layout can be determined, while respecting the surroundings. Design of supports can for example be used to minimize environmental damage, without conflicting the objective of a bridge.

Design of actuator placement can be used to determine the best actuator layout for a given objective. When optimizing towards minimal displacement or compliance problems, a minimum force constraint should be introduced. This minimum introduction is necessary, to prevent the optimizer creating trivial solutions by placing zero force, which can be the best solution (in a static domain), for the minimization of displacements.
The combination with topology optimization has shown cooperation between both design variables. When optimizing in a certain design domain, it can be helpful to introduce density dependency. It has shown that density dependency is very effective in minimizing displacements of the constructed area only.

In chapter 6, dynamics are considered by introducing a harmonic excitation. Design of actuators is shown to be very effective in reduction of a certain displacement field. The frequency response, caused by the harmonic excitation at a certain frequency, can be used to place forces in a smart way, in order to reduce the objective. The actuator placement can be placed in a such a manner, that modes are exerted in a way that contributes to improving an objective. Implementation of Newton's second law is necessary, to ensure the applied force is large enough to excite the body with the desired frequency. This is done by introducing an additional acceleration constraint. In general, more design space for actuator placement results in better objectives. There are some situations however, where overfitting occurs. When giving the optimizer too much freedom, the result is more likely stuck at a local minimum.

The combination of design of actuator placement with topology optimization is performed in a dynamic domain. Since the topology can change, the frequency response can change. This change of frequency response is combined with actuator placement, to get even better results, with respect to the objective. The optimizer can efficiently place and remove material on places which contributes to the objective, while the force excitations at the same time help reducing unwanted behavior. The combination of optimizing both design variables was shown to be very effective in reducing a certain displacement field.

7.2 Recommendations

Although this thesis contributes to reduction of a certain displacement field, there are numerous of challenges to consider for future research. The implementation of design of supports is demonstrated in a static domain. It will be interesting to expand this implementation to a dynamic setting. The shown examples of bridge challenges are all based on static loads. If there is some traffic crossing this bridge however, some additional dynamic forces will be exerted on the road. This dynamic force should be included here. Another example of design of supports is shown for compliant mechanisms, also here, dynamics should be included to represent the physics better.

Design of force in a static domain is investigated. In these examples the supports remain at the same locations. It could be interesting to investigate the optimization process of both design of supports and design of actuators simultaneously. This could be helpful regarding design of compliant mechanism.

This thesis has shown reduction of a certain displacement field, some challenges need to be investigated, before this approach can be implemented for the design of accurate wafer stages. The model does not contain any damping, which should be implemented accordingly, in order to represent a physical example. The implementation of damping could lead to phase differences and additional behavior. This should be investigated also.

The case study in chapter 6 is made using a 2D element which is discretized by $40x20$ elements. This discretization could be made much bigger. By enhancing this mesh, more details can be displayed, since the resolution becomes larger. This will lead unfortunately to a larger computational time, in order to solve the desired problem. Also the introduction of a third dimension should be helpful. Not only for better visual insight in the behavior, but also to make the model physically better interpretable. However, as we have already shown in A.4, this introduction will lead to an even larger computational time.
The existence of gray regions should be investigated even further, for example by taking a

bigger resolution. Also, looking for different filter techniques could be helpful towards this problem. Note that both solutions could lead to larger need of computational sources. Although the MMA solver seemed to help me quite well with my design problems, investigation of other solving techniques should be considered also.
A different interpolation scheme, for example the RAMP approach, could be investigated to achieve better black and white regions. Although the SIMP approach is a very effective interpolation scheme for solving static topology optimization problems, for dynamic cases the RAMP method results possibly in a better black and white solution.

There are some challenge regarding the actuator layout. Instead of creating a distributed force, it can be interesting to investigate the possibility to cluster forces (gradually) into a few points, in order to give a more realistic actuator placement.
The overfitting case as shown in A.8.2 and A.8.3 can maybe be solved by taking the optimal result from 6.5 and then gradually add different locations of actuator placement to this design. By using this different approach of solving, even better dynamic behavior could be achieved.

The dynamic force is implemented using a harmonic excitation. It can be interesting to investigate the optimal actuator placement for a transient response. This transient response could also lead to an investigation of a dynamic actuator pattern. Forces are turned on and off at a certain time. This will lead to an even larger computational load, but could be really helpful in the high-precision industry. Finally, Instead of looking for minimal displacements, it could be interesting to solve different objectives. For example using actuator placement and topology optimization to achieve a certain displacement field at a certain frequency, with a given weight.

Appendix A

Appendix

In this chapter all additional information for this research project can be found. First, the computer configuration is shown in (A.1).
This configuration is used to produce the wanted optimizations, which are represented by figures during this report. The numerical results of all these figures can be found in (A.2). For the first chapters convergence is shown, in order to get more insight in different optimization methods, which can be found in (A.3). A graphical representation of the implementation of a third dimension can be seen in (A.4).
The introduced arching continuation method is graphically represented by (A.5). Deformed geometry for some interesting examples are depicted in (A.6).
Wen dynamics are introduced, mode contributions can be found in (A.7). Additional examples of the wafer stage are described in (A.8).
A graphical representation of the difference between a SIMP and BESO method can be found in (A.9).

A.1 Computational setup

Although not explicitly documented, there are some numerical results available of all the executed optimizations.
These calculation results are derived from my personal computer. The associated computer and program specification can be found in A-1. For these numerical results, the draw and output options are set to disabled, to increase speed.

Program	**MATLAB R2016b 64-bit (9.1)**
Operating System	Windows 10 Pro 64-bit
CPU	Intel Core i7 @ 2.30GHz
RAM	16.0 GB DDR3 @ 1600MHz
Hard-Disc	Samsung 840 EVO 250GB SSD

Table A-1: Computer resources

A.2 Numerical results

First, let's have a look at the evolutionary example, and focus on the final result, as depicted in Figure 2-1e. The following parameters are here used, and will for this section considered as standard.
The chosen penalty $p = 3$, the mesh is discretized by 90 x 30 elements, the filter radius $r_{min} = 1.5$. The volume constraint is kept at 50% of the original design. Using this parameters, the following table can be made, just to get a clear vision on the numerical results. In A-2 the parameters from the associated example are depicted. Followed by the number of iterations, the optimization time (in seconds) and the final compliance. In the upcoming tables, some numerical results of the depicted examples in the report can be seen.

A.2.1 Chapter 2 results

Numerical results for chapter 2.

Parameter	**Figure 2-1e**
mesh	90 x 30
vol	0.5
p	3
r_{min}	1.5
iter	87
time	8.0
comp	188.9

Table A-2: Standard compliance example

Parameter	Figure 2-2b	Figure 2-2c	Figure 2-2d	Figure 2-2e
mesh	30 x 10	60 x 20	90 x 30	120 x 40
vol	0.5	0.5	0.5	0.5
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
iter	104	61	87	118
time	0.8	1.0	3.3	8.3
comp	219.7	195.0	188.9	185.5

Table A-3: Mesh refinement example

Parameter	Figure 2-3b	Figure 2-3c	Figure 2-3d	Figure 2-3e
mesh	90 x 30	90 x 30	90 x 30	90 x 30
vol	0.5	0.5	0.5	0.5
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
iter	187	106	87	179
time	7.3	4.1	3.3	7.0
comp	505.4	266.9	188.9	152.7

Table A-4: Volume fraction example

Parameter	Figure 2-4b	Figure 2-4c	Figure 2-4d	Figure 2-4e
mesh	90 x 30	90 x 30	90 x 30	90 x 30
vol	0.5	0.5	0.5	0.5
p	1	2	3	5
r_{min}	1.5	1.5	1.5	1.5
iter	15	158	87	187
time	0.9	6.0	3.3	7.3
comp	160.7	185.3	188.9	192.1

Table A-5: Penalty example

Parameter	Figure 2-6b	Figure 2-6c	Figure 2-6d	Figure 2-6e
mesh	90 x 30	90 x 30	90 x 30	90 x 30
vol	0.5	0.5	0.5	0.5
p	3	3	3	3
r_{min}	1.0	1.25	1.5	3.0
iter	54	158	87	66
time	2.4	6.2	3.3	2.5
comp	189.6	185.7	188.9	203.6

Table A-6: Filter example

A.2.2 Chapter 3 results

Numerical results for chapter 3.

Parameter	Figure 3-1c	Figure 3-1e
mesh	90 x 30	90 x 30
vol	0.5	0.5
p	3	3
r_{min}	1.5	1.5
iter	205	118
time	8.6	34.6
comp	196.0	195.0

Table A-7: OC vs MMA

Parameter	Figure 3-2	Figure 3-3
mesh	90 x 30	90 x 30
vol	0.5	0.5
p	3	3
r_{min}	1.0	1.25
iter	155	305
time	83.1	190.3
comp	288.3	227.2

Table A-8: Passive and active examples

Parameter	Figure 3-4b	Figure 3-4c
mesh	90 x 30	90 x 30
vol	0.5	0.5
p	3	3
r_{min}	1.5	1.5
iter	167	134
time	97.9	52.1
comp	121.3	227.2

Table A-9: Multiple load cases

Parameter	Figure 3-5b	Figure 3-5c	Figure 3-5d	Figure 3-5e
mesh	90 x 30	90 x 30	90 x 30	90 x 30
vol	0.5	0.5	0.5	0.5
p	5	5	5	5
r_{min}	1.5	1.5	1.5	1.5
ρ	0	$7.6 \cdot 10^{-5}$	$1.5 \cdot 10^{-4}$	$3.8 \cdot 10^{-4}$
iter	190	166	256	238
time	140.7	141.0	193.3	202.7
comp	193.4	318.5	440.9	844.2

Table A-10: Self-weight example

Parameter	Figure 3-6b	Figure 3-6c	Figure 3-6d
mesh	90 x 30	90 x 30	90 x 30
vol	0.5	0.5	0.5
p_{max}	3	3	3
r_{min}	1.5	1.5	1.5
iter	80	109	500
time	5.7	15.0	85.5
comp	190.2	18.1	179.3

Table A-11: Different filters example

Parameter	Figure 3-8b	Figure 3-8c	Figure 3-8d	Figure 3-8e
mesh	30 x 10 x 1	30 x 10 x 3	30 x 10 x 5	30 x 10 x 10
vol	0.5	0.5	0.5	0.5
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
iter	106	95	115	75
time	15.6	32.7	77.9	117.1
comp	124.0	49.6	25.0	13.6

Table A-12: 3D mesh refinement example

Parameter	**Figure 3-11a**	**Figure 3-11b**	**Figure 3-13a**	**Figure 3-14a**
mesh	80 x 80	80 x 80	120 x 120	120 x 120
vol	0.5	0.5	0.5	0.5
p	4	4	4	4
r_{min}	1.5	1.5	1.4	1.4
iter	1000	316	147	1000
time	114.42	31.93	342.1	253.2
d_{in}	891.35	18.42	76.48	75.21
d_{out}	-898.99	-40.89	-28.06	-30.48
G_d	-1.01	-2.22	-0.73	-0.81

Table A-13: Complaint mechanism example

A.2.3 Chapter 4 results

Numerical results for chapter 4.

Parameter	Figure 4-4b	Figure 4-4c	Figure 4-4d	Figure 4-4e
mesh	80 x 40	80 x 40	80 x 40	80 x 40
vol	0.2	0.2	0.2	0.2
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
vol_z	0.2	0.2	0.2	0.2
q	5	5	5	5
r_c	1	5	10	50
iter	157	211	200	192
time	43.8	58.2	68.8	79.2
comp	25921.6	45186.0	76166.3	83838.5

Table A-14: Optimal bridge

Parameter	Figure 4-5b	Figure 4-5c	Figure 4-5d	Figure 4-5e
mesh	80 x 40	80 x 40	80 x 40	80 x 40
vol	0.2	0.2	0.2	0.2
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
vol_z	0.2	0.2	0.2	0.2
q	5	5	5	5
r_c	1	5	10	20
iter	157	206	249	268
time	43.8	64.4	90.4	99.0
comp	25921.6	48837.2	77233.8	81539.9

Table A-15: Optimal bridge example 1

Parameter	Figure 4-6b	Figure 4-6c	Figure 4-6d	Figure 4-6e
mesh	80 x 40	80 x 40	80 x 40	80 x 40
vol	0.2	0.2	0.2	0.2
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
vol_z	0.2	0.2	0.2	0.2
q	5	5	5	5
r_c	1	3	5	10
iter	157	233	238	200
time	43.8	60.0	65.8	56.1
comp	25921.6	38811.0	52214.6	47032.7

Table A-16: Optimal bridge example 2

Parameter	Figure 4-7b	Figure 4-7c	Figure 4-7d	Figure 4-7e
mesh	80 x 80	80 x 80	80 x 80	80 x 80
vol	0.1	0.1	0.1	0.1
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
vol_z	0.2	0.2	0.2	0.2
q	5	5	5	5
r_c	1	2	4	6
iter	226	244	199	250
time	137.1	148.7	121.7	160.2
comp	28373.2	41730.5	34946.0	44885.9

Table A-17: Hanging bridge

Parameter	Figure 4-8b	Figure 4-8c	Figure 4-8d	Figure 4-8e
mesh	80 x 40	80 x 40	80 x 40	80 x 40
vol	0.2	0.2	0.2	0.2
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
vol_z	0.2	0.2	0.2	0.2
q	5	5	5	5
d_{tunnel}	32	32	53.3	53.3
iter	299	278	283	173
time	88.1	71.9	85.7	45.8
comp	31379.7	26120.6	36911.1	19059.3

Table A-18: Train tunnel example

Parameter	Figure 4-10b	Figure 4-11b
mesh	120 x 120	120 x 120
vol	0.2	0.2
p	3	3
r_{min}	1.5	1.5
vol_z	0.2	0.2
q	3	3
iter	135	145
time	327.5	262.5
d_{in}	14.15	52.23
d_{out}	-83.40	51.40
G_d	-5.89	1.97

Table A-19: Optimal compliant mechanisms

A.2.4 Chapter 5 results

Numerical results for chapter 5.

Parameter	Figure 5-1b
mesh	90 x 30
vol	1.0
p	3
r_{min}	1.5
F	-1.0
comp	44.3

Table A-20: Minimal compliance beam

Parameter	Figure 5-2	Figure 5-3a	Figure 5-3b
mesh	90 x 30	90 x 30	90 x 30
vol	1.0	1.0	1.0
p	3	3	3
r_{min}	1.5	1.5	1.5
F	-1.0	-1.0	-1.0
U	8.9	9.5	22.5

Table A-21: Simple cantilever beam

Parameter	Figure 5-4b	Figure 5-4c
mesh	90 x 30	90 x 30
vol	1.0	1.0
p	3	3
r_{min}	1.5	1.5
F	-1.0	-1.0
U	0.12	0.08

Table A-22: Triple fixed beam

Parameter	Figure 5-5b	Figure 5-6e	Figure 5-7b	Figure 5-8e
mesh	90 x 30	90 x 30	90 x 30	90 x 30
vol	0.25	0.30	0.24	0.30
p	3	3	3	3
r_{min}	1.5	1.5	1.5	1.5
F	-1.06	-1.05	-1.04	-1.01
U	0.13	45.73	106.46	41.20

Table A-23: Cantilever beam with topology optimization

A.2.5 Chapter 6 results

Numerical results for chapter 6.

Parameter	Figure 6-2	Figure 6-6	Figure 6-7
mesh	40 x 20	40 x 20	40 x 20
vol	1.0	1.0	1.0
p	3	3	3
r_{min}	1.5	1.5	1.5
ω^2	8	8	8
F_{tot}	3.20	3.20	3.20
F_{hor}	3.20	3.20	3.20
U	61.92	54.07	56.30

Table A-24: Dynamic solid beam

Parameter	Figure 6-8b	Figure 6-9b
mesh	40 x 20	40 x 20
vol	1.0	1.0
p	3	3
r_{min}	1.5	1.5
ω^2	8	8
F_{tot}	3.20	3.20
F_{hor}	3.20	3.20
U	53.68	53.52

Table A-25: Design of dynamic actuator placement

Parameter	Figure 6-10b	Figure 6-11b	Figure 6-12b
mesh	40 x 20	40 x 20	40 x 20
vol	1.0	1.0	1.0
p	3	3	3
r_{min}	1.5	1.5	1.5
ω^2	8	8	8
F_{tot}	3.20	2.41	3.20
F_{hor}	3.20	3.20	3.20
U	52.96	6.22	3.18

Table A-26: Design of dynamic actuator placement

Parameter	Figure 6-13	Figure 6-14a	Figure 6-14b
mesh	40 x 20	40 x 20	40 x 20
vol	0.72	0.66	0.68
p	3	3	4
r_{min}	1.5	1.5	1.5
ω^2	8	8	8
F_{tot}	3.20	2.16	2.16
F_{hor}	3.20	2.16	2.16
U	19.14	3.05	12.85

Table A-27: Dynamic actuator placement and topology

Parameter	Figure 6-14c	Figure 6-14d	Figure 6-15b
mesh	40 x 20	40 x 20	40 x 20
vol	0.67	0.65	0.78
p	5	6	6
r_{min}	1.5	1.5	1.5
ω^2	8	8	8
F_{tot}	2.17	2.13	2.51
F_{hor}	2.17	2.13	2.51
U	2.60	2.54	2.06

Table A-28: Dynamic actuator placement and topology

Parameter	Figure 6-16b	Figure 6-17b	Figure 6-18b
mesh	40 x 20	40 x 20	40 x 20
vol	0.73	0.90	0.74
p	6	6	6
r_{min}	1.5	1.5	1.5
ω^2	8	8	8
F_{tot}	2.35	2.89	2.89
F_{hor}	2.35	2.89	2.52
U	1.95	1.04	0.19

Table A-29: Dynamic actuator placement and topology

Parameter	Figure 6-19a	Figure 6-19b	Figure A-26a
mesh	40 x 20	40 x 20	40 x 20
vol	0.82	0.93	0.77
p	11	6	6
r_{min}	1.5	1.5	1.5
ω^2	8	8	8
F_{tot}	5.03	2.04	3.53
F_{hor}	2.68	3.01	2.51
U	1.02	1.37	1.59

Table A-30: Dynamic actuator additional cases

Parameter	Figure A-26b	Figure A-27a	Figure A-27b
mesh	40 x 20	40 x 20	40 x 20
vol	0.49	0.87	0.80
p	6	6	6
r_{min}	1.5	1.5	1.5
ω^2	8	4	16
F_{tot}	1.83	1.31	4.60
F_{hor}	1.51	1.49	5.23
U	15.28	0.98	3.19

Table A-31: Dynamic actuator additional cases

Parameter	Figure A-24	Figure A-25
mesh	40 x 20	40 x 20
vol	1.0	0.94
p	3	6
r_{min}	1.5	1.5
ω^2	8	8
F_{tot}	5.33	4.25
F_{hor}	4.19	3.00
U	13.71	1.95

Table A-32: Dynamic actuator overfitting cases

A.3 Convergence graph

To achieve more insight in the convergence process, for a number of examples, convergence graphs are plotted. The associated examples can be found in the legend of each picture. Using these graphs, some conclusions can be made regarding the need of using many iterations, which results into only a very minor benefit, with respect to the compliance.

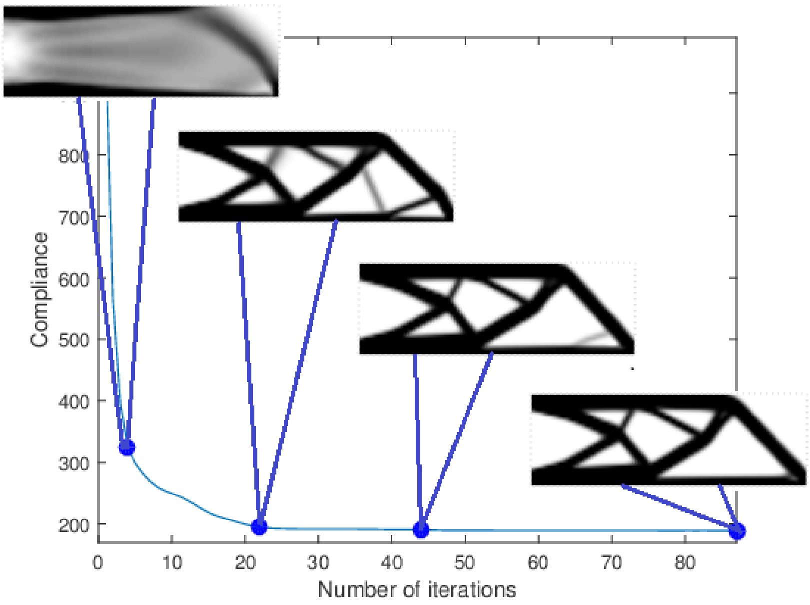


Figure A-1: Convergence plot of Figure 2-1

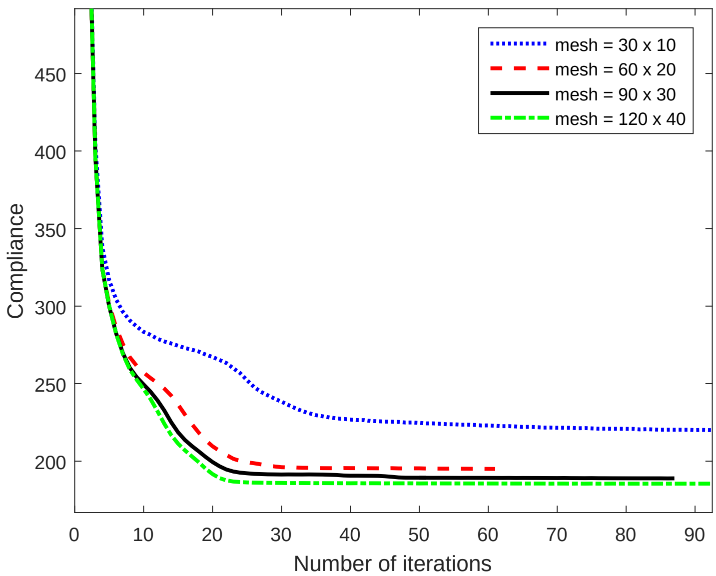


Figure A-2: Convergence plot of Figure 2-2

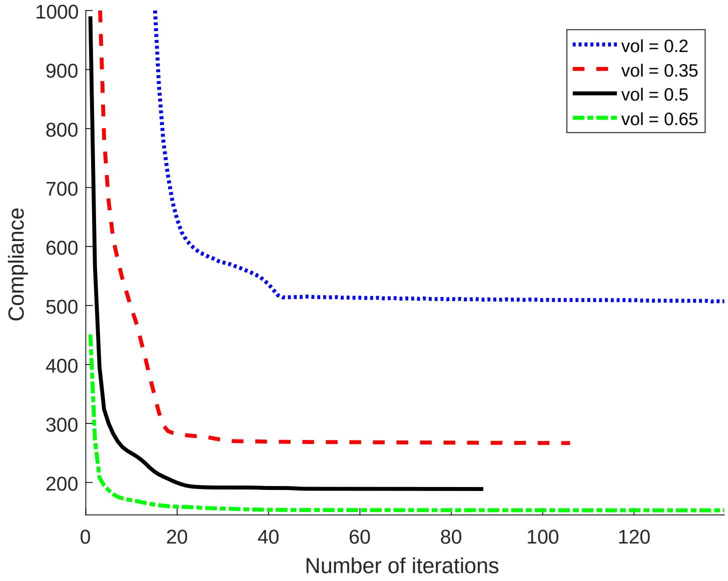


Figure A-3: Convergence plot of Figure 2-3

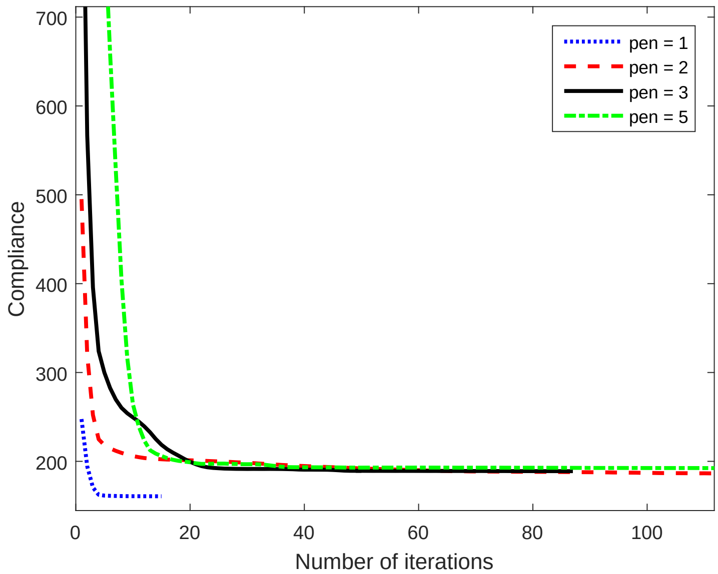


Figure A-4: Convergence plot of Figure 2-4

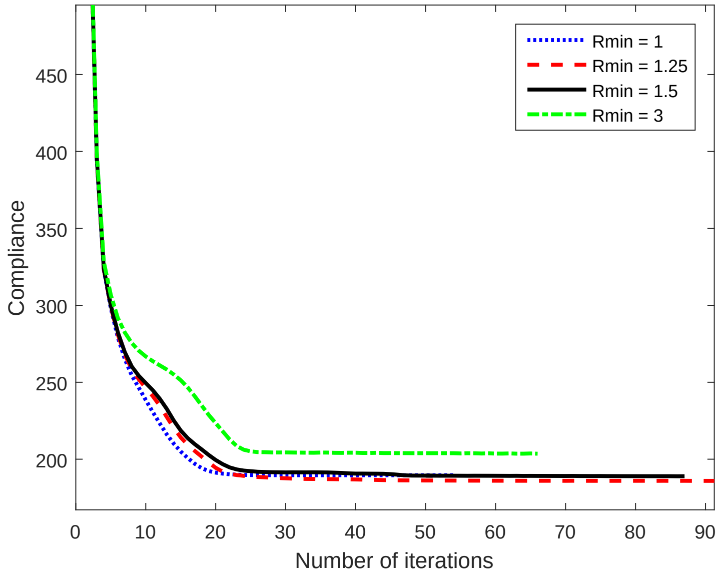


Figure A-5: Convergence plot of Figure 2-6

A.3.1 Chapter 3 graphs

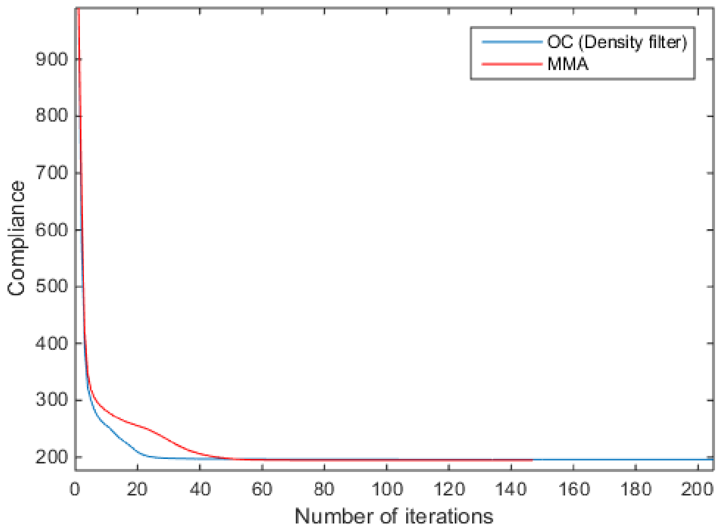


Figure A-6: Convergence plot of Figure 3-1c vs Figure 3-1e

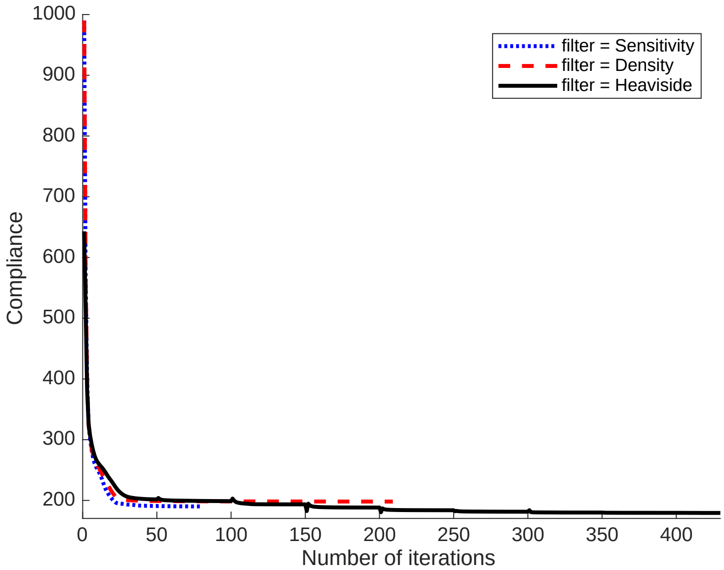


Figure A-7: Convergence plot of Figure 3-6

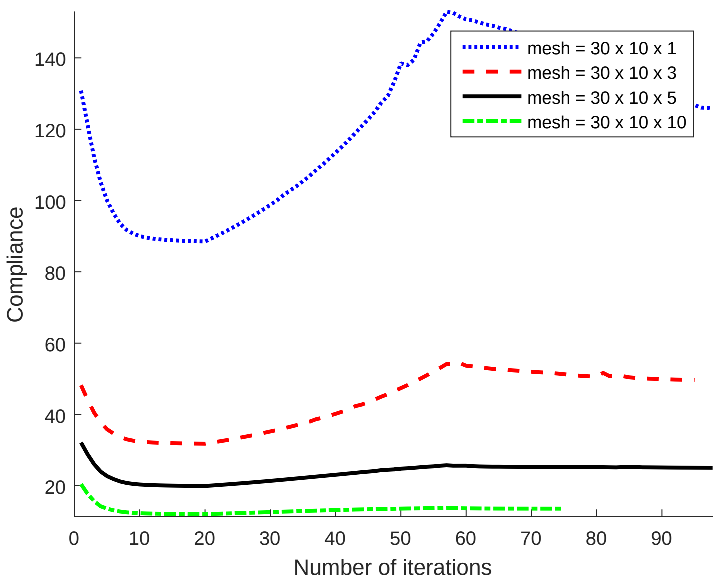


Figure A-8: Convergence plot of Figure 3-8

A.3.2 Chapter 4 graphs

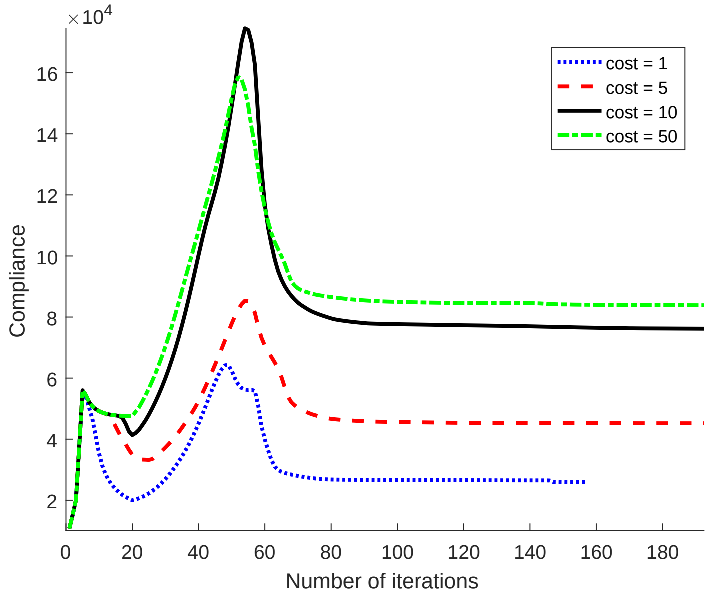


Figure A-9: Convergence plot of Figure 4-4

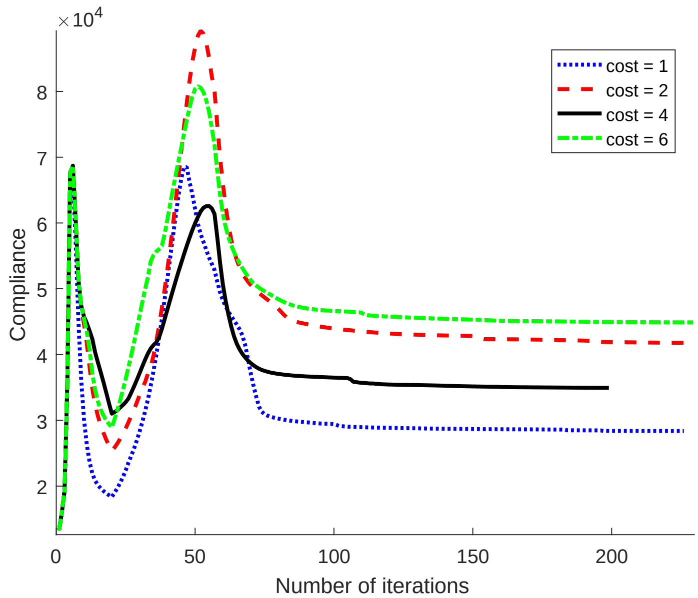


Figure A-10: Convergence plot of Figure 4-7

A.4 Computational graph

The third dimension is pretty shiny, but the computational time seems to increase exponentially, in this section a graph is shown, which includes a computational time comparison of a simple compliance problem, discretized by $30x10xnz$ elements. This number of lateral elements nz is varied, to see the differences in computational time.
This timing example is done with two different types of outputs, namely, the No Output option (draw = 0, dis = 0) and the newly introduced Partial Output option (draw = 2, dis = 2).
After, an exponential fit seems to fit the best results. This is created using the Curve Fitting tool in MATLAB, after which this graph is made using the outputted parameters.

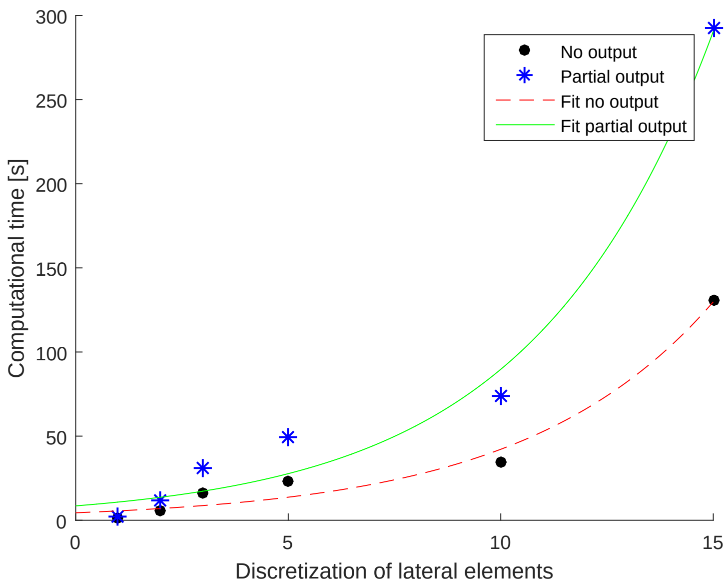


Figure A-11: Computational Example by variation of lateral elements.

A.5 Arching continuation

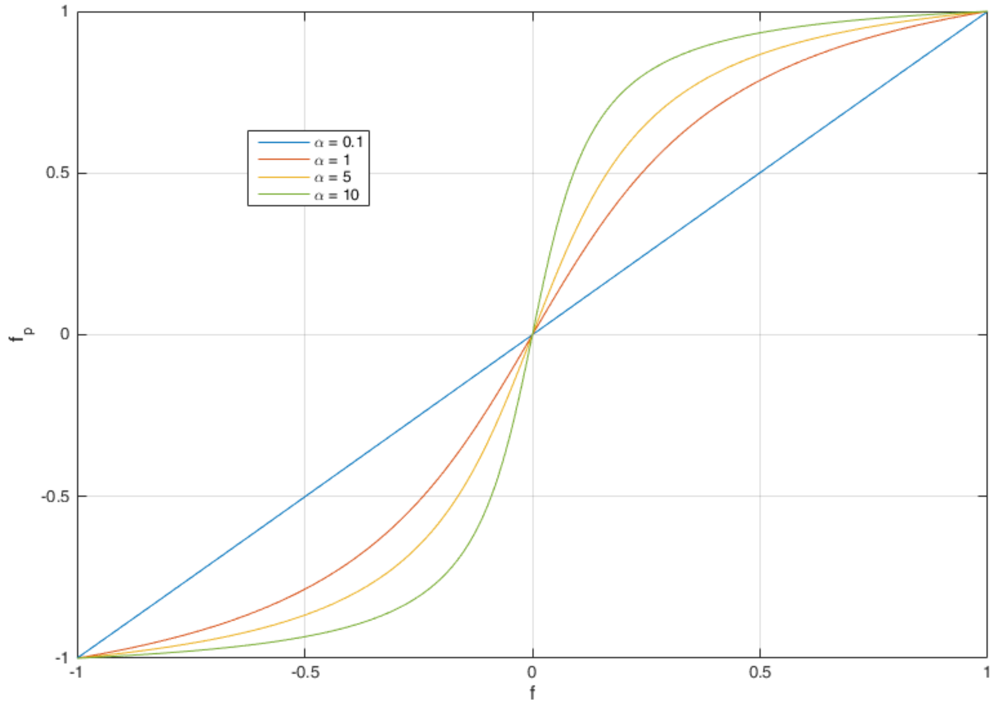


Figure A-12: Example of the arching continuation method. Equation 5-5 is displayed for different values of α.

A.6 Deformed geometry

This section displays the deformed geometry of calculated structures. The colors represent the associated displacements. The displacement of each element is calculated by taking an average of its eight surrounding node displacements.

A.6.1 Deformed triple fixed beam

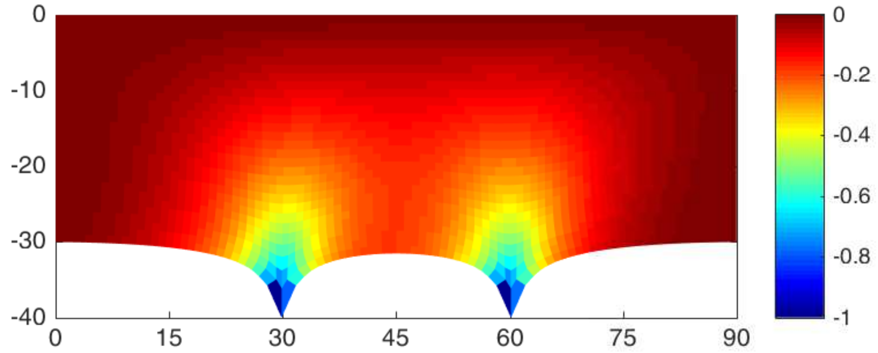


Figure A-13: Deformed geometry of Figure 5-4b. Displacements are normalized by taking the maximum absolute displacement as 1.

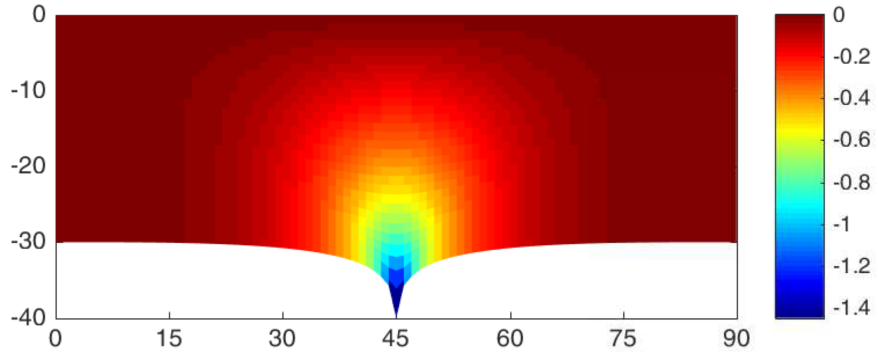
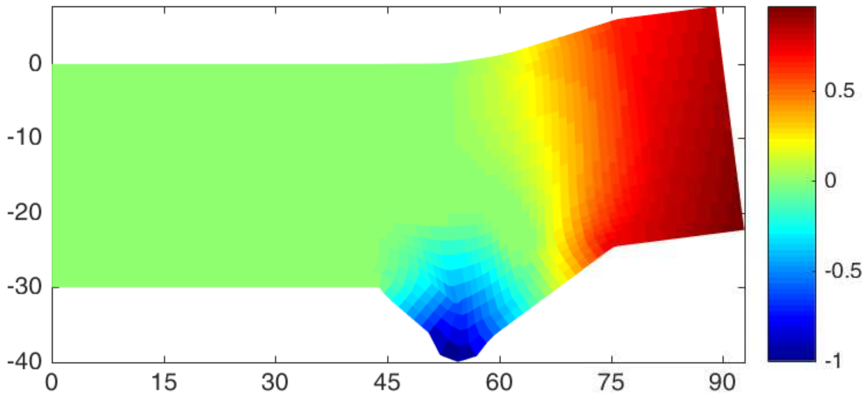
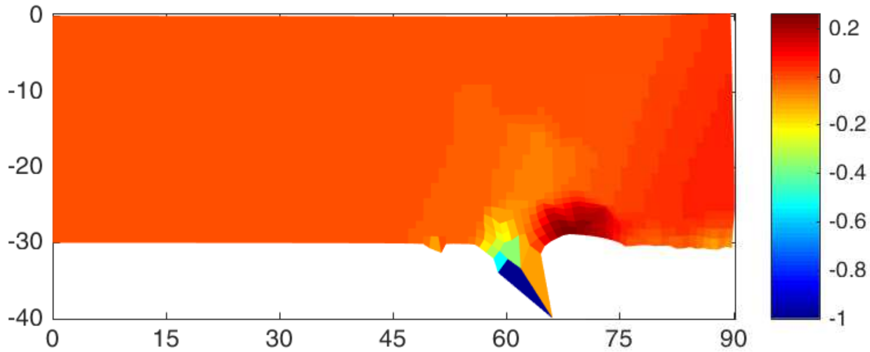


Figure A-14: Deformed geometry of Figure 5-4c. Displacements are normalized by taking the maximum absolute displacement of Figure A-13 as 1. The displacement in this figure are above 1, which means more displacement in the exerted area; however, the overall displacement is smaller than displayed in Figure A-13.

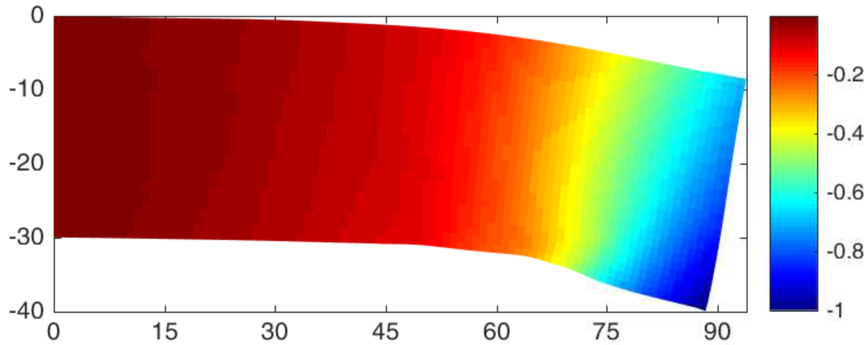
A.6.2 Deformed cantilever beam



(a) Deformed geometry of Figure 5-5a.



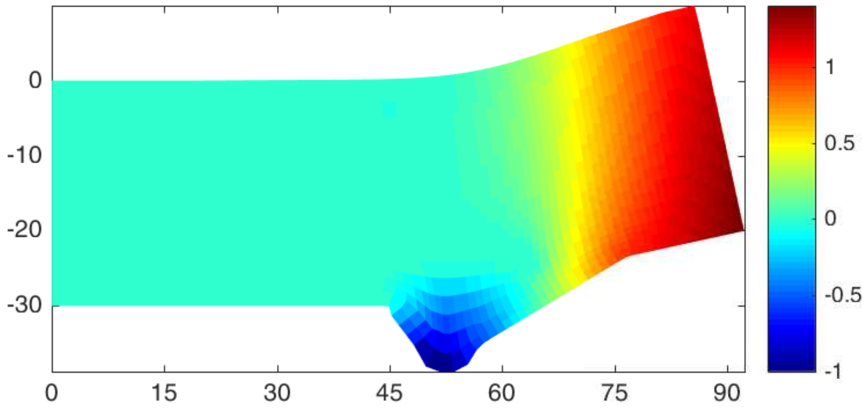
(b) Deformed geometry of Figure 5-5b.



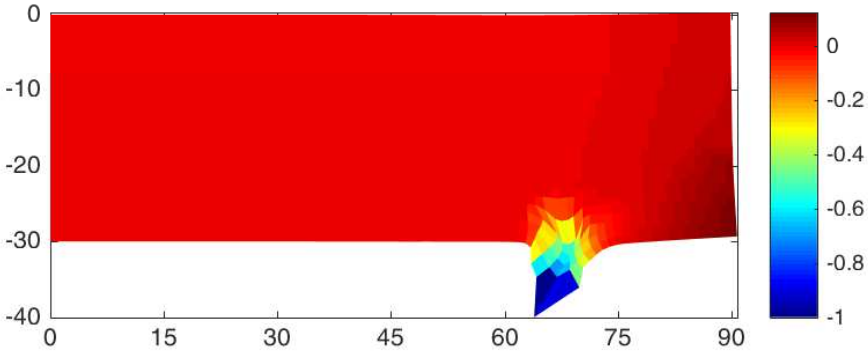
(c) Deformed geometry of Figure 5-6e.

Figure A-15: Deformed geometry of cantilever beam examples from Chapter 5. Displacements are normalized for each plot, by taking the maximum displacement of each structure as 1.

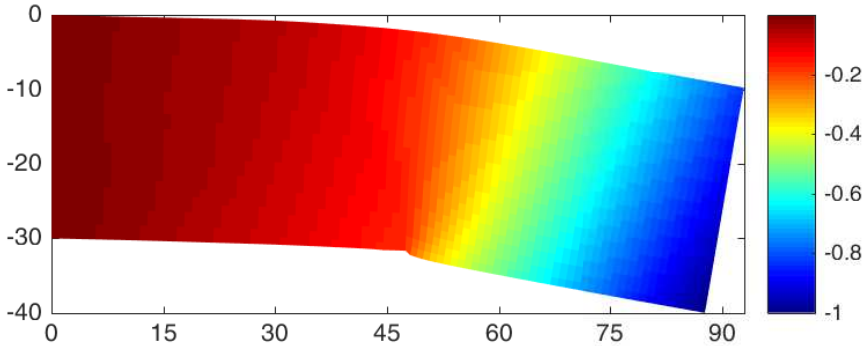
A.6.3 Deformed cantilever beam with density dependency



(a) Deformed geometry of Figure 5-7a.



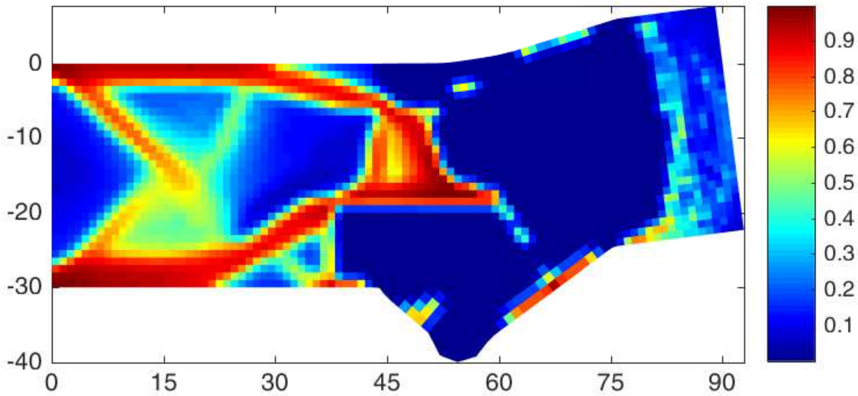
(b) Deformed geometry of Figure 5-7b.



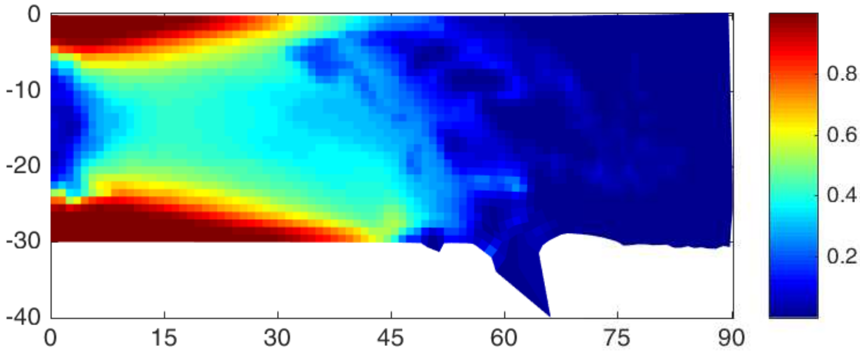
(c) Deformed geometry of Figure 5-8e.

Figure A-16: Deformed geometry of cantilever beam examples from Chapter 5. The optimal result is achieved using the object refinement from 5.4.3. Displacements are normalized for each plot, by taking the maximum displacement of each structure as 1.

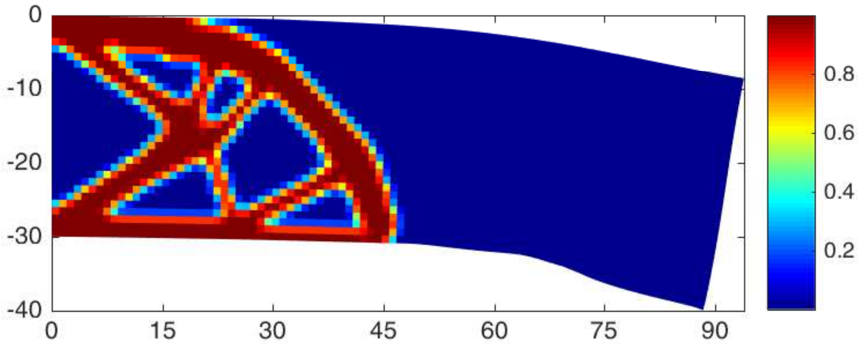
A.6.4 Deformed cantilever beam topology



(a) Deformed geometry of Figure 5-5a.



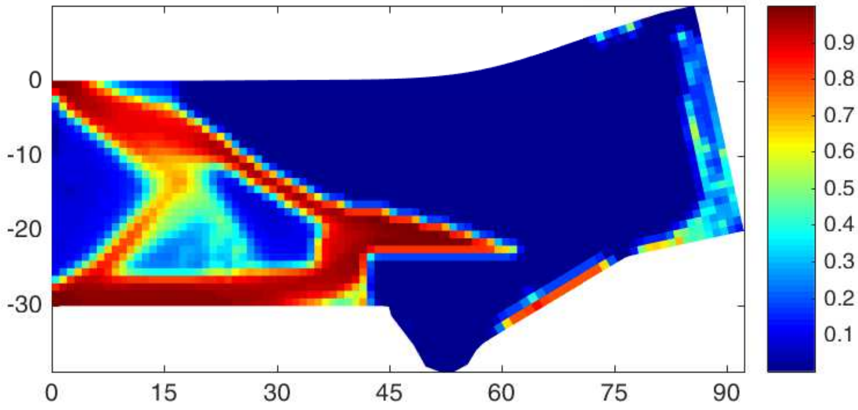
(b) Deformed geometry of Figure 5-5b.



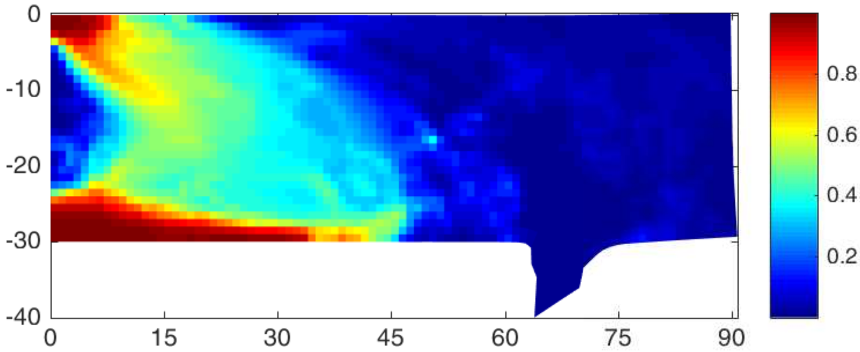
(c) Deformed geometry of Figure 5-6e.

Figure A-17: Deformed geometry of cantilever beam examples from Chapter 5. In these figures the topology is used as color reference, while the displacements represent deformed geometry of the design domain.

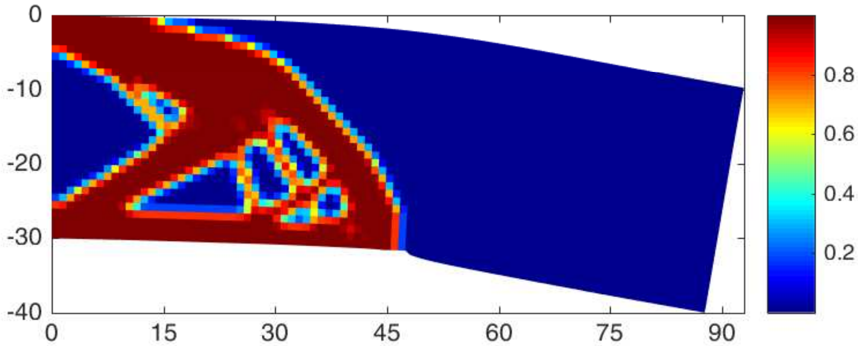
A.6.5 Deformed cantilever beam topology with density dependency



(a) Deformed geometry of Figure 5-7a.



(b) Deformed geometry of Figure 5-7b.



(c) Deformed geometry of Figure 5-8e.

Figure A-18: Deformed geometry of cantilever beam examples from Chapter 5. The optimal result is achieved using the object refinement from 5.4.3. In these figures the topology is used as color reference, while the displacements represent deformed geometry of the design domain.

A.7 Mode contribution

This section gives a tabular and graphical representation of the contribution of modes. The tables shows the mode contribution and some additional values, while the graphics display the mode dependency of a frequency spectrum.

A.7.1 Mode contribution tables

The mode contribution for several cases is displayed over here. In this first column the mode number can be seen, the second column holds the associated eigenfrequency. In the third column the mode contribution $\boldsymbol{\phi}_i{}^T\mathbf{f}$, followed by the scaled contribution η_i, as described in (6-5). This is done, so the difference between scaling and the scaling of the mode can be seen very clearly. In the last column a weight factor of this mode influence can be found. This weight factor is normalized by taking the sum of these first twelve eigenmodes.

Mode	Eigenfrequency	$\boldsymbol{\phi}_\mathbf{i}{}^\mathbf{T}\mathbf{f}$	Mode contribution $\boldsymbol{\eta_i}$	Contribution (%)
Rigid #1	1.24e-07	0.20	-0.03	0.82
Rigid #2	1.34e-07	5.05	-0.63	20.40
Rigid #3	2.63e-07	0.07	-0.01	0.30
#1	2.29	0	0	0
#2	3.47	-7.36	-1.82	58.89
#3	3.83	0	0	0
#4	5.53	0	0	0
#5	5.60	0	0	0
#6	6.07	3.89	0.13	4.36
#7	6.15	7.44	0.25	8.07
#8	6.26	-3.82	-0.12	3.96
#9	7.19	4.33	0.10	3.20

Table A-33: Mode contribution of single force case as described in (Figure 6-2) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i{}^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	0.18	-0.02	0.78
Rigid #2	1.34e-07	5.06	-0.63	21.50
Rigid #3	2.63e-07	0.08	-0.01	0.32
#1	2.29	0	0	0
#2	3.47	-6.74	-1.67	56.87
#3	3.83	0	0	0
#4	5.53	0	0	0
#5	5.60	0	0	0
#6	6.07	7.21	0.25	8.50
#7	6.15	1.64	0.06	1.88
#8	6.26	-7.63	-0.25	8.34
#9	7.19	-2.32	-0.05	1.80

Table A-34: Mode contribution of two forces case as described in (Figure 6-6) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i{}^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	0.18	-0.02	0.85
Rigid #2	1.34e-07	5.06	-0.63	23.55
Rigid #3	2.63e-07	0.07	-0.01	0.35
#1	2.29	0	0	0
#2	3.47	7.15	1.77	66.01
#3	3.83	0	0	0
#4	5.53	0	0	0
#5	5.60	0	0	0
#6	6.07	-0.27	-0.01	0.34
#7	6.15	-5.38	-0.18	6.72
#8	6.26	-0.46	-0.01	0.55
#9	7.19	1.90	0.04	1.62

Table A-35: Mode contribution of distributed force case as described in (Figure 6-7) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	4.97	-0.62	20.68
Rigid #2	1.34e-07	0.68	-0.08	2.83
Rigid #3	2.63e-07	63	-0.08	2.63
#1	2.29	-0.05	0.02	0.55
#2	3.47	6.87	1.7	56.65
#3	3.83	0.01	0	0.03
#4	5.53	-0.39	-0.02	0.57
#5	5.60	-0.18	-0.01	0.25
#6	6.07	-5.19	-0.18	5.98
#7	6.15	2.79	0.09	3.11
#8	6.26	5.54	0.18	5.92
#9	7.19	-1.05	-0.02	0.8

Table A-36: Mode contribution of distributed force case as described in (Figure 6-8) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	0.18	-0.02	0.76
Rigid #2	1.34e-07	5.06	-0.63	21.25
Rigid #3	2.63e-07	.04	-0.01	0.19
#1	2.29	-0.05	0.02	0.6
#2	3.47	-6.8	-1.69	56.67
#3	3.83	0.1	0.01	0.49
#4	5.53	-0.99	-0.04	1.47
#5	5.60	0.53	0.02	0.76
#6	6.07	-6.06	-0.21	7.07
#7	6.15	-2.21	-0.07	2.5
#8	6.26	-6.45	-0.21	6.97
#9	7.19	-1.66	-0.04	1.27

Table A-37: Mode contribution of distributed force case as described in (Figure 6-9) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	5.04	-0.63	15.39
Rigid #2	1.34e-07	0.46	-0.06	1.4
Rigid #3	2.63e-07	37	-0.05	1.14
#1	2.29	0.03	-0.01	0.31
#2	3.47	6.27	1.55	38.02
#3	3.83	0.92	0.14	3.41
#4	5.53	-5	-0.22	5.41
#5	5.60	-3.27	-0.14	3.42
#6	6.07	-15.17	-0.53	12.86
#7	6.15	-2.66	-0.09	2.18
#8	6.26	15.86	0.51	12.45
#9	7.19	-7.18	-0.16	4.02

Table A-38: Mode contribution of distributed force case as described in (Figure 6-10) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	-0.99	0.12	1.5
Rigid #2	1.34e-07	5.13	-0.64	7.74
Rigid #3	2.63e-07	-2.99	0.37	4.52
#1	2.29	-1.35	0.49	5.89
#2	3.47	-6.62	-1.64	19.84
#3	3.83	6.59	0.99	12
#4	5.53	-12.09	-0.53	6.46
#5	5.60	-29.86	-1.28	15.44
#6	6.07	-22.62	-0.78	9.47
#7	6.15	6.36	-0.21	2.58
#8	6.26	14.08	0.45	5.46
#9	7.19	32.96	0.75	9.11

Table A-39: Mode contribution of distributed force case as described in (Figure 6-11) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i{}^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	5.05	-0.63	45.42
Rigid #2	1.34e-07	0.42	-0.05	3.78
Rigid #3	2.63e-07	33	-0.04	2.96
#1	2.29	-0.03	0.01	0.89
#2	3.47	0.06	0.01	1.03
#3	3.83	1.04	0.16	11.28
#4	5.53	-2.71	-0.12	8.61
#5	5.60	0.57	0.02	1.74
#6	6.07	1.7	0.06	4.25
#7	6.15	2.91	0.1	7.02
#8	6.26	4.9	0.16	11.32
#9	7.19	1.03	0.02	1.69

Table A-40: Mode contribution of distributed force case as described in (Figure 6-12) and taking a frequency of $\omega^2 = 8$

Mode	Eigenfrequency	$\phi_i{}^T f$	Mode contribution η_i	Contribution (%)
Rigid #1	1.24e-07	6.62	-0.83	8.08
Rigid #2	1.34e-07	6.44	-0.81	7.86
Rigid #3	2.63e-07	.13	0.77	7.48
#1	2.29	2.14	-0.77	7.55
#2	3.47	-19.06	-4.72	46.13
#3	3.83	-2.03	-0.31	2.99
#4	5.53	6.23	0.28	2.69
#5	5.60	26.74	1.14	11.17
#6	6.07	0.45	0.02	0.15
#7	6.15	0.14	0	0.05
#8	6.26	9.58	0.31	3
#9	7.19	-12.74	-0.29	2.85

Table A-41: Mode contribution of distributed force case as described in (Figure A-24) and taking a frequency of $\omega^2 = 8$

Mode	**Eigenfrequency**	$\boldsymbol{\phi_i}^{\mathbf{T}}\mathbf{f}$	**Mode contribution** $\boldsymbol{\eta_i}$	**Contribution (%)**
Rigid #1	1.24e-07	5.14	-0.43	20.26
Rigid #2	1.34e-07	1.39	-0.12	5.47
Rigid #3	2.63e-07	1.45	-0.12	5.72
#1	2.29	-0.05	0.01	0.34
#2	3.47	0	0.08	3.99
#3	3.83	-2.49	-0.96	45.66
#4	5.53	0.91	0.05	2.33
#5	5.60	0.12	0.01	0.29
#6	6.07	-0.24	-0.01	0.46
#7	6.15	3.11	0.12	5.74
#8	6.26	5.46	0.2	9.57
#9	7.19	-0.15	0	0.17

Table A-42: Mode contribution of distributed force case as described in (Figure A-23) and taking a actuation frequency very close to the second eigenfrequency, $\omega = 3.47$

A.7.2 Mode contribution graphics

A corresponding mode contribution for the six most important modes, over a spectrum of frequencies can be found here. In this schematic it can perfectly be seen which mode contributes how much on every frequency. When the excitation frequency approaches an eigenfrequency, the corresponding mode will be actuated the most and will thus take the most relative contribution of the total modes.

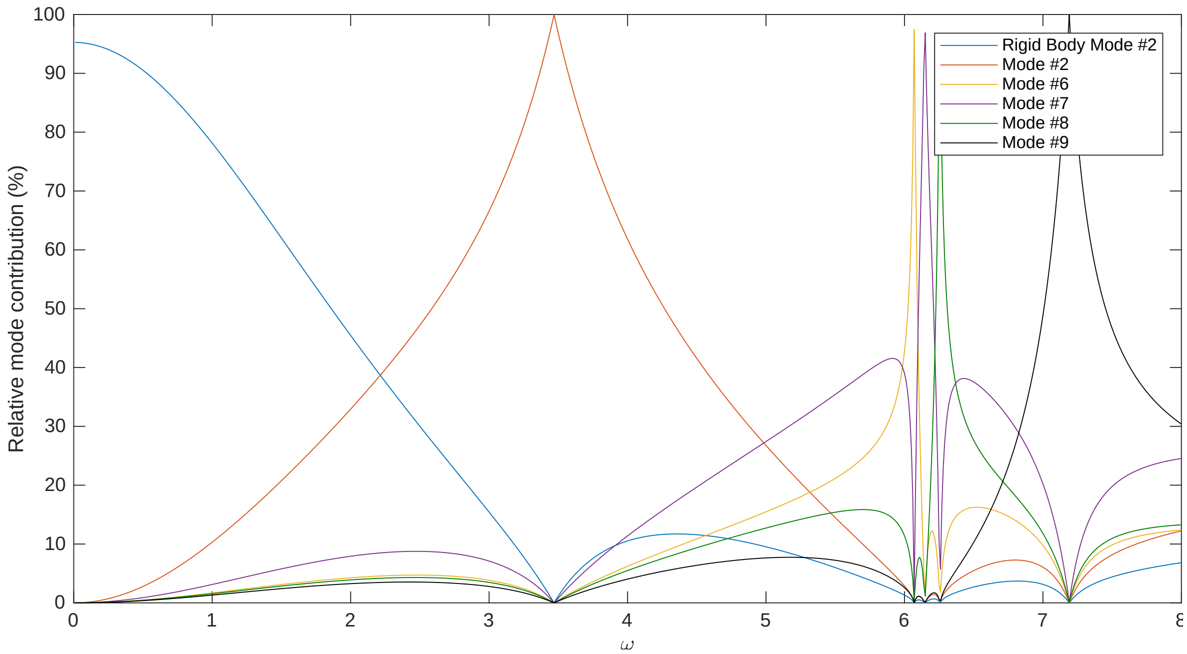


Figure A-19: Mode contribution for the six most important modes, using a single force case as depicted in (Figure 6-2) for a frequency spectrum.

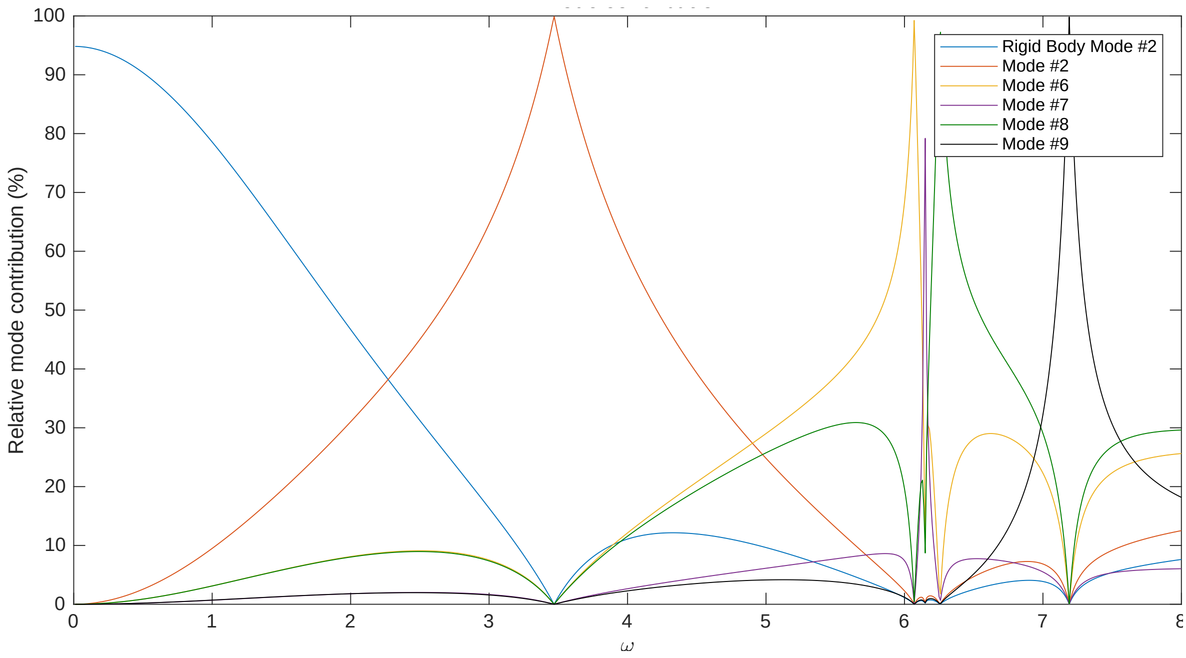


Figure A-20: Mode contribution for the six most important modes, using a two forces case as depicted in (Figure 6-6) for a frequency spectrum.

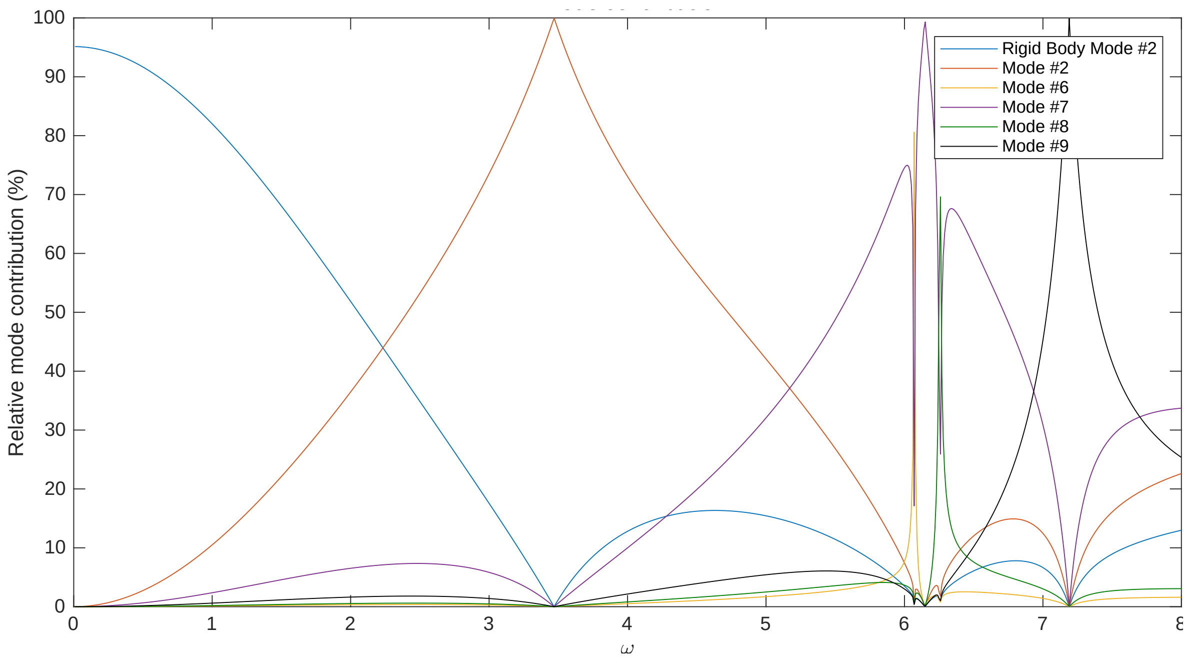


Figure A-21: Mode contribution for the six most important modes, using a distributed force case as depicted in (Figure 6-7) for a frequency spectrum.

A.7.3 Mode contribution progress plots

In this section a mode contribution progress plot can be found. This is basically a progress plot from the optimization problem depicted in Figure A-23a towards the optimal solution as depicted in Figure A-23b.
In this figure (Figure A-22) the (absolute) mode contribution $\boldsymbol{\phi}_{\mathbf{i}}{}^{\mathbf{T}}\mathbf{f}$ is plotted over time.
The mode contribution is plotted on a log-scale, for better visual reasons. As can be seen, the sum of the five most important modes is decreasing over time, although the second mode is increasing. This means, when iterations increasing, the total mode contribution is decreasing, which could benefit the objective value to minimize.
To help the optimizer a little bit, to win some time, after 20 iterations, the total placed force is scaled down to its minimum force value ($\mathbf{f} = m \cdot \mathbf{a}$). This manipulation process is only performed once during the optimization. The manipulation leads to a faster optimization result.

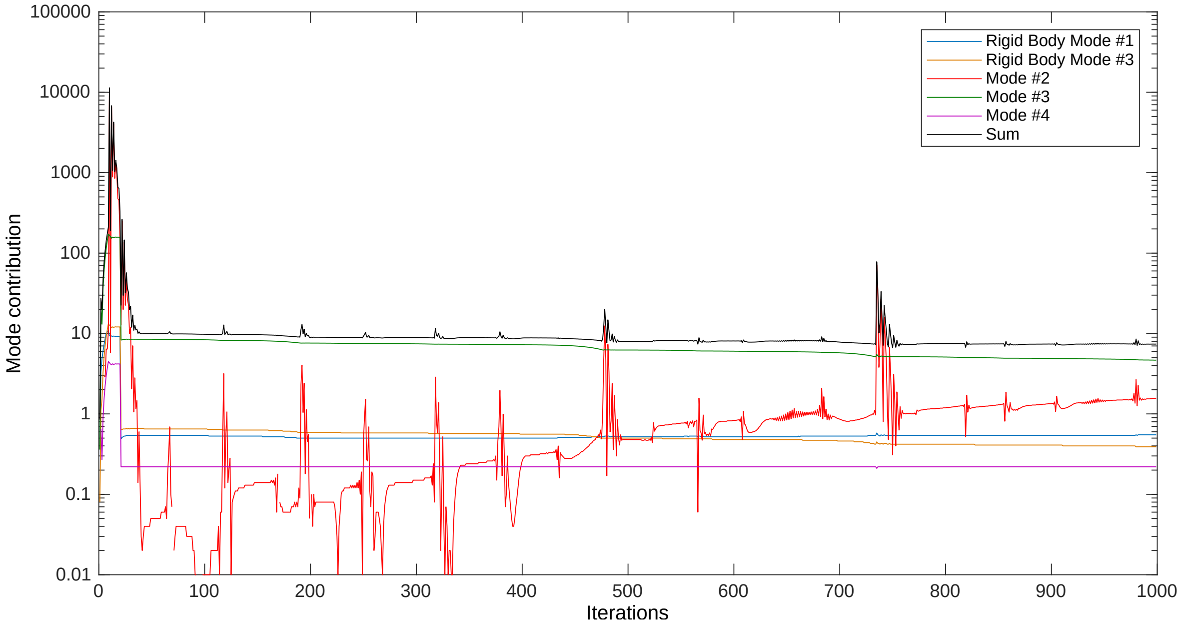


Figure A-22: Mode contribution of the five most important modes. The black line indicates the sum of the mode contribution $\boldsymbol{\phi}_{\mathbf{i}}{}^{\mathbf{T}}\mathbf{f}$ of these five mode contributions values. These mode contributions are made using the optimal actuator layout as depicted in (Figure A-23).

A.8 Additional stage examples

In this section some additional variations of the stage examples from 6 are given. For references please review 6 to gain some knowledge on the background of these examples and problems.

A.8.1 Optimizing at eigenfrequency

Up to here, the actuation frequency ω was taken at a value $\omega^2 = 8$, which is just between the first and second eigenfrequencies. It is most of the time a good thing to refrain from actuating near or at a particular eigenfrequency. However, in some cases, it could be necessary to actuate near a certain eigenfrequency. For example, when the target frequency is close to an eigenfrequency. A change of the material distribution can be made, to change the dynamic response. If this change is not allowable, the only way to deal with this problem, is by changing the force actuation. This change of force layout can reduce or suppress certain dynamic behavior, to counteract that particular mode shape. This type of situation is investigated in this section.
In the example depicted in Figure A-23a the body is actuated at the second eigenfrequency. This second eigenfrequency seems to have a big influence on the total dynamic spectrum, so it is the most interesting frequency to investigate. The actuation frequency is very close to this eigenfrequency, because actuation exactly at the eigenfrequency is not solvable.
The same force design domain as explained in Figure 6-12 is used, so on the both sides, positive and negative forces are allowed. The optimal actuator layout can be found in Figure A-23b. Note that this particular eigenmode example has a big objective value. This is caused by the fact we are actuating almost at an eigenmode itself, which has very large displacements at that particular frequency. Additionally, the actuation frequency is increased, so more force is needed to fulfill this constraint. This is another reason why it is not a fair comparison to Figure 6-12b.
The left-hand side and right-hand side depicted in Figure A-23b show almost the same behavior, but some very little change in the values can be found. This is possibly caused by the very small interval between the eigenfrequency and the actuation frequency.
The optimization process starts with an initial distributed force on the left- and right-hand side of the design domain. The sum of this distributed force equals the minimum required force ($\mathbf{f} = m \cdot \mathbf{a}$). The optimizer then looking for an optimal force application. A schematic of this process can be found in Figure A-22.

(a) Design domain

(b) Optimal actuator layout

Figure A-23: Design domain and optimal actuator layout, while enabling negative forces design. The gray striped area indicates the objective area. The white striped area indicates the actuator design domain. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout. The actuation frequency is very close to the second eigenfrequency $\omega = 3.47$. The associated mode contribution can be found in Table A-42.

A.8.2 Overfitting design of actuators

The result of the optimization for the design domain as depicted in Figure 6-12a, while also enabling actuator design at the bottom of the body, in positive (upward) and negative (downward) direction is depicted in Figure A-24. Here, a big problem when optimizing this type of design problem, is the possibly overfitting of the model. The optimizer has just too much variables and the optimizer is more likely to approach a (high) local optimum. The result depicted in Figure A-24 shows a distribution along all sides of the design domain. The horizontal force is almost twice the minimum needed force to achieve the prescribed acceleration. This could also be a symptom of the overfitting of the model. It can be concluded that, in order to achieve a maximal optimization result, the design domain should not be too vaguely or too big.

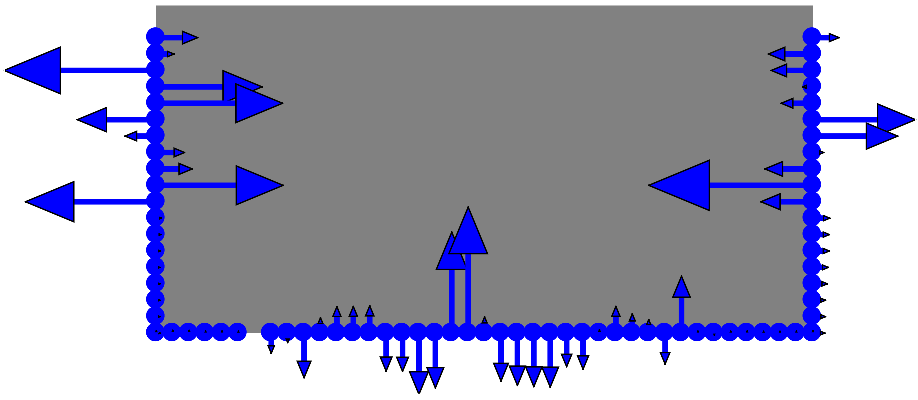


Figure A-24: Optimal actuator layout as depicted in Figure 6-12a, while also enabling actuator design at the bottom of the body, in positive (upward) and negative (downward) direction. The size and placement of the arrows represent the location and magnitude of the optimized force layout. The associated mode contribution can be found in Table A-41.

A.8.3 Overfitting design of actuators with topology optimization

The result of the optimization for the design domain as depicted in Figure 6-12a, while also enabling actuator design at the bottom of the body, in positive (upward) and negative (downward) direction and enabling topology optimization. The result is depicted in Figure A-25. Here, a big problem when optimizing this type of design problem, is the possibly overfitting of the model. The optimizer has just too much variables and the optimizer is more likely to approach a (high) local optimum. The result depicted in Figure A-25 shows a distribution along all sides of the design domain.
Some better results can be achieved, for example only using the left- and righthand side of the domain (Figure 6-17b). The result depicted over there is even better than the result depicted in Figure A-25.

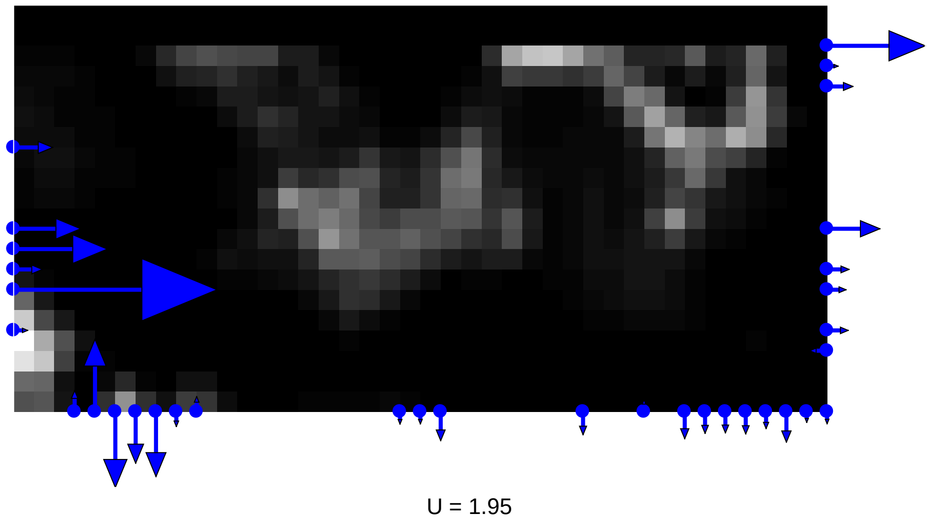
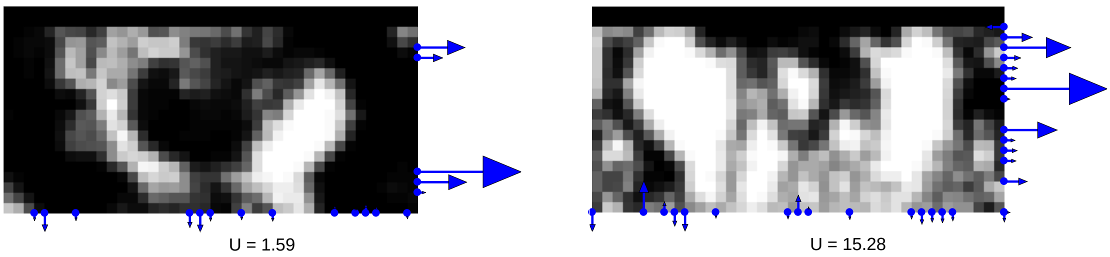


Figure A-25: Optimal actuator layout as depicted in Figure 6-12a, while also enabling actuator design at the bottom of the body, in positive (upward) and negative (downward) direction. The size and placement of the arrows represent the location and magnitude of the optimized force layout. The total horizontal force used is $\mathbf{f} = 3.00$.

A.8.4 Changing conditions representations

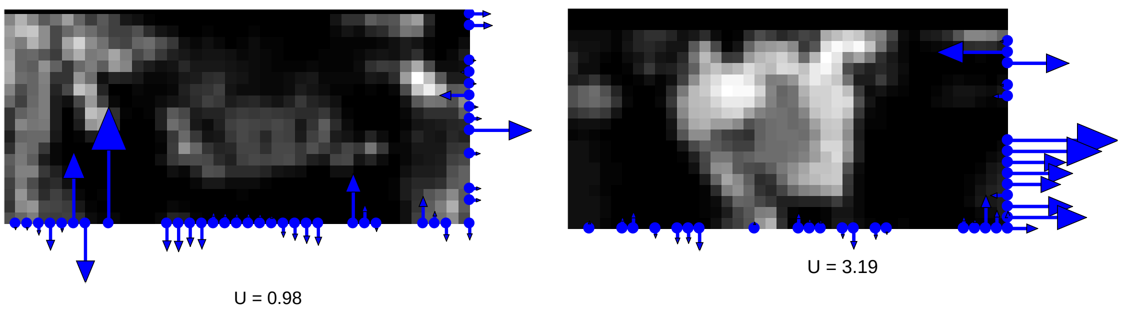
In this section some examples of changing conditions on the solved problems from 6 are given. These cases are described in 6.
For example changing the maximum volume:



(a) Optimal solution **(b)** Optimal solution

Figure A-26: Design domain and optimal actuator layout, while enabling negative forces design and using topology optimization with two different volume constraints, with a maximum volume of: a) 80%, b) 50%. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout.

And the solution for different actuation frequencies:



(a) Optimal solution **(b)** Optimal solution

Figure A-27: Design domain and optimal actuator layout, while enabling negative forces design and using topology optimization for two different actuation frequencies: a) $\omega^2 = 4$, b) $\omega^2 = 16$. The size, placement and direction of the arrows represent the location, magnitude and orientation of the optimized force layout.

A.8.5 3D Extrusion

In this subsection however, a 3D extrusion is made. This extrusion is just a 2D lateral case in another dimension. A threshold value of 0.5 is chosen. This means all densities below this threshold value are displayed as void regions, for a better visual representation. As already explained in 6.1 this wafer stage should be used to produce circular wafers. The width and depth of the wafer should thus be the same size.

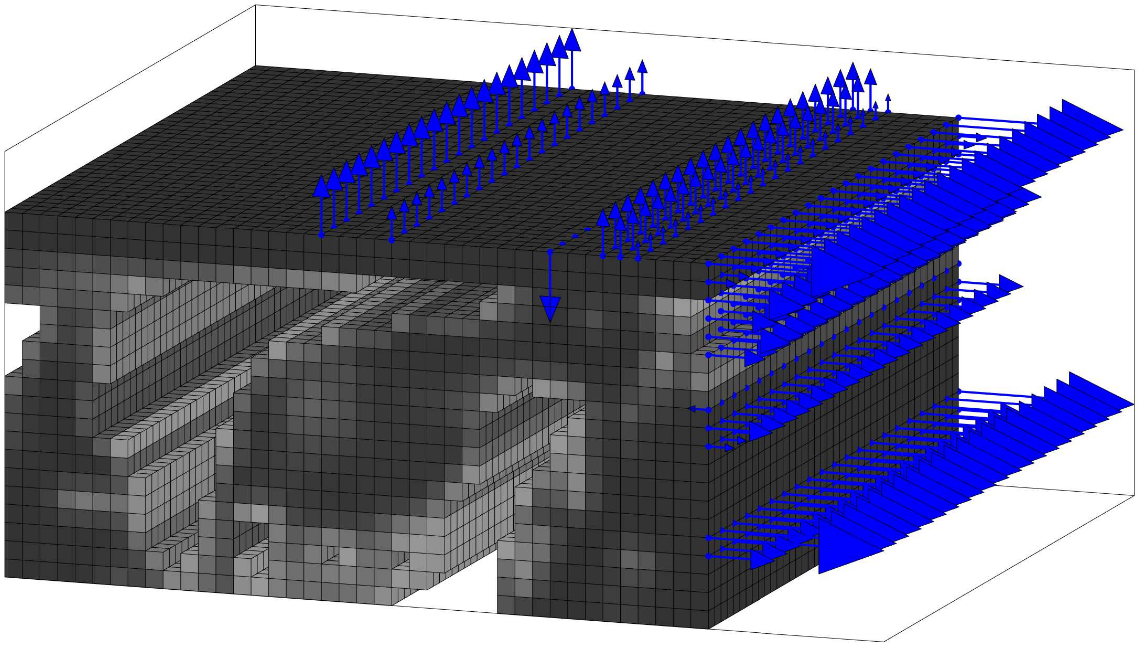
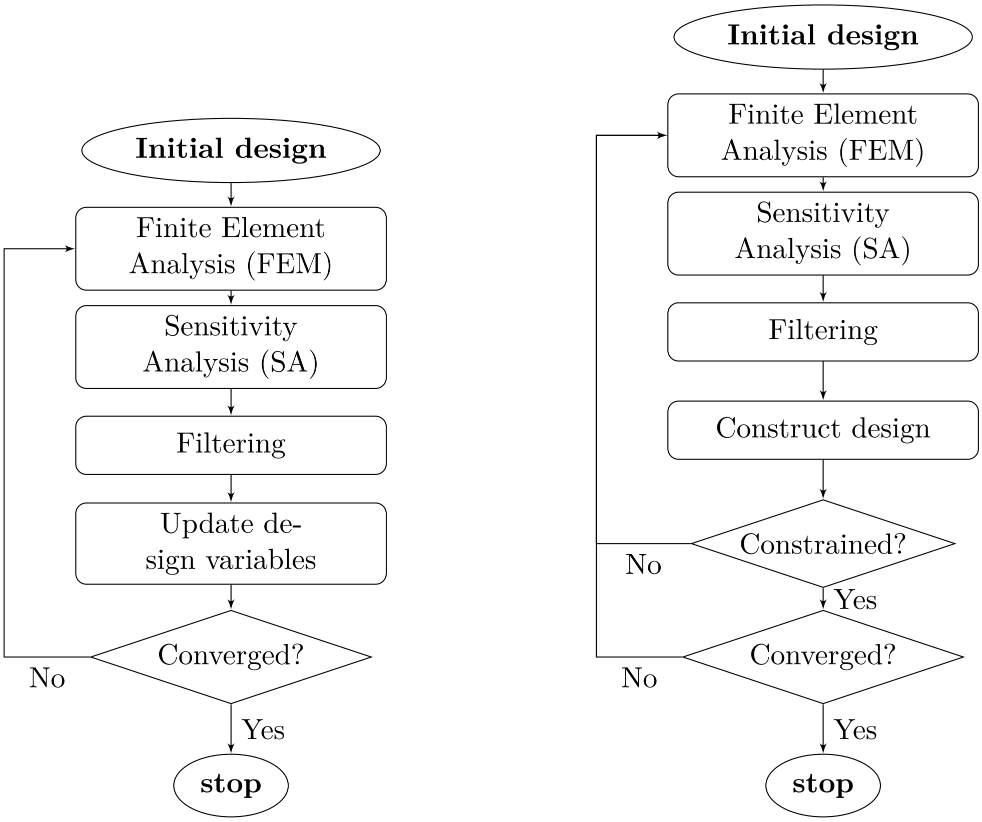


Figure A-28: A 2D lateral extrusion of the optimal wafer stage as depicted in Figure 6-18b.

A.9 Flowcharts



(a) SIMP approach **(b)** BESO approach

Figure A-29: Flowchart of topology optimization methods

Appendix B

Matlab Codes

In this and upcoming sections the created and used MATLAB codes are provided. In this Appendix the complete codes can be found. In Appendix C add-ins can be found. These add-ins can be used to create certain functionality. In Appendix D supplementary codes can be found.

In this section Basic.m (B.1) can be found. Using Basic.m a typical problem can be solved easily. All lines consists of helpful comments. The Advanced.m can be used to plot progression pictures, as shown in Chapter 1 of the Literature Survey. A big reference should be made to (Sigmund, 2001a) and (Andreassen et al., 2011) for providing a kick-start of the codes in this appendix.

By implementing the add-ins from (C.1) to (C.6), the final M-codes BASIC.m (B.3) and ADVANCED.m (B.2) can be made.

A third dimension could be added and so the matlab code also need some extensions. A simple add-in to extend 2D to 3D is made available in (C.7), with a reference to (Liu and Tovar, 2014). The final code of using the BASIC-code for three dimensional cases can be found in (B.4), and also, the ADVANCED version of this 3D code can be found in (B.5).

When dealing with compliant mechanisms. The BASIC-code needs to be updated using the simple add-in (C.8) for an inverter case. When producing a micro-gripper, one can also grab the final code immediately (B.6).

By making use of (C.9) a complete code of the implementation of design of supports can be made (B.7), with the associated advanced code for design of supports (B.8).

The implementation of design of actuator placement can be made (B.9) using the provided code. When also introducing topology optimization, one can grab the final code right away (B.10).

B.1 Basic.m

The working principle of the Basic-code will be explained in this section.
At first, it can be specified whether or not the Advanced.m code is used [line 15]. When this value is zero, the Basic-code continue as just one optimization problem, without any comparison calculations and plots. When this value equals zero, a number of design variables can be defined [line 16-23]. Some basic options for the calculation can be defined also [line 24-27].The output options can be defined in [line 28-30], which can be used to gain some speed on the optimization process, as outputting and plotting can take a lot of time. The element properties can be defined [line 31-34]. The force and supports needs to be defined next [line 35-40].
From this line on, the user input is not necessary anymore, the elemental stiffness matrix is build up in [line 41-49], followed by the building of the nodes matrix [line 50-54]. To gain optimization speed a preparation scheme for the filter is made up [line 55-75]. After building up the load vector and some initialization [line 76-90] the main optimization loop starts at [line 91].
While the convergence of the optimization loop is above the minimum convergence, and the maximum number of iterations is not exceeded, the loop keeps running and assign a new loop number [line 92-93]. Each loop consists of a finite element analysis, where the stiffness matrix is built up and updated according to each node number, followed by an update of the element's displacements and associated compliance values [line 94-104]. Now, a sensitivity analysis is performed for each element and filtered accordingly. [line 105-107].
The design variables are updated using the Optimality Criteria method, where the Lagranian multipliers for the volume constraint are calculated. Eventually, the design variable x is updated [line 108-122]. Each element value of x is stored in a massive matrix X, which contains each value of x for every iteration; the same holds for the compliance c [line 123-127]. The final results are displayed in the MATLAB command window, and the iterations and final result is plotted, if this is specified in the pre-amble [line 128-175]. For speed improvements, an additional option is made, to just optimize without iteration output and drawings [line 176-225]. The tic-toc commands displaying the total run time of the code [line 226].

```
1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %                                                                    %
3  % Topology Optimization Using Matlab                                 %
4  % BasicRep.m                                                         %
5  %                                                                    %
6  % Delft University of Technology, Department PME                     %
7  % Master of Science Thesis Project                                   %
8  %                                                                    %
9  % Stefan Broxterman                                                  %
10 %                                                                    %
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12 %
13 tic                       % start timer
14 %% DEFINE PARAMETERS
15 adv = 1;                  % use advanced function [0 = off , 1 = on]
16 if adv == 0               % define parameters at behalf of the advanced
       function
```

```
    nx = 90;              % numer of elements horizontal
    ny = 30;              % number of elements vertical
    vol = 0.5;            % volume fraction [0-1]
    pen = 3;              % penalty
    rmin = 1.5;           % filter size
    clc; clf; close all;
end
%% DEFINE CALCULATION
tol = 0.01;               % tolerance for convergence criterion [0.01]
move = 0.2;               % move limit for lagrange [0.2]
miter = 1000;             % maximum number of iterations [1000]
%% DEFINE OUTPUT
draw = 1;                 % plot iterations [0 = off, 1 = on]
dis = 1;                  % display iterations [0 = off, 1 = on]
%% DEFINE MATERIAL
E = 1;                    % young's modulus of solid [1]
Emin = 1e-9;              % young's modulus of void [1e-9]
nu = 0.3;                 % poisson ratio [0.3]
%% DEFINE FORCE
Fe = 2*(nx+1)*(ny+1);     % element of force application [2*(nx+1)*(ny+1)]
Fn = 1;                   % number of applied force locations [1]
Fv = -1;                  % value of applied force [-1]
%% DEFINE SUPPORTS
fix = 1:2*(ny+1);         % fixed elements [1:2*(ny+1)]
%% PREPARE FINITE ELEMENT
N = 2*(nx+1)*(ny+1);      % total element nodes
all = 1:2*(nx+1)*(ny+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12]; % fem
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6]; % fem
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4]; % fem
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2]; % fem
Ke = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]); % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node numer matrix
dofvec = reshape(2*nodes(1:end-1,1:end-1)+1,nx*ny,1); % create dof vector
dofmat = repmat(dofvec,1,8)+repmat([0 1 2*ny+[2 3 0 1] -2 -1],nx*ny,1); % create dof matrix
iK = reshape(kron(dofmat,ones(8,1))',64*nx*ny,1); % build sparse i
jK = reshape(kron(dofmat,ones(1,8))',64*nx*ny,1); % build sparse j
%% PREPARE FILTER
iH = ones(nx*ny*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));      % create sparse vector of ones
kH = zeros(size(iH));     % create sparse vector of zeros
m = 0;                    % index for filtering
for i = 1:nx              % for each element calculate distance between ...
    for j = 1:ny          % elements' center for filtering
        r1 = (i-1)*ny+j; % sparse value i
        for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx)  % center of element
            for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % center of element
                r2 = (k-1)*ny+l; % sparse value 2
```

```
                m = m+1; % update index for filtering
                iH(m) = r1; % sparse vector for filtering
                jH(m) = r2; % sparse vector for filtering
                kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)); % weight
                    factor
            end
        end
    end
end
H = sparse(iH,jH,kH);    % build filter
Hs = sum(H,2);           % summation of filter
%% DEFINE STRUCTURAL
x = repmat(vol,ny,nx);   % initial material distribution
xF = x;              % set filtered design variables
Fsiz = size(Fe,2);       % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
U = zeros(size(F));      % pre-allocate space displacement
c = zeros(miter,1);      % pre-allocate objective vector
%% INITIALIZE LOOP
iter = 0;                % initialize loop
diff = 1;                % initialize convergence criterion
%% START LOOP
while (diff > tol) && iter < miter  % convergence criterion not met
    iter = iter+1;       % define iteration
    p = pen;         % set penalty
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xF(:)'.^pen*(E-Emin)),64*nx*ny,1); % create
        sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;        % build stiffness matrix
    U(free,:) = K(free,free)\F(free,:); % displacement solving
    c(iter) = 0;         % set compliance to zero
    Sens = 0;            % set sensitivity to zero
    %% Calculate compliance and sensitivity
    c0 = reshape(sum((U(dofmat)*Ke).*U(dofmat),2),ny,nx); % initial
        compliance
    c(iter) = c(iter) + sum(sum((Emin+xF.^p*(E-Emin)).*c0)); % calculate
        compliance
    Sens = Sens -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
    Senc = ones(ny,nx); % set constraint sensitivity
    Sens(:) = H*(x(:).*Sens(:))./Hs./max(1e-3,x(:)); % update filtered
        sensitivity
    %% Update design variables Optimality Criterion
    l1 = 0;          % initial lower bound for lagranian mulitplier
    l2 = 1e9;        % initial upper bound for lagranian multiplier
    while (l2-l1)/(l1+l2) > 1e-3; % start loop
        lag = 0.5*(l1+l2); % average of lagranian interval
```

```
        xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
        xF = xnew; % updated result
        if sum(xF(:)) > vol*nx*ny; % check for optimum
            l1 = lag; % update lower bound to average
        else
            l2 = lag; % update upper bound to average
        end
    end
    diff = max(abs(xnew(:)-x(:))); % difference of maximum element change
    x = xnew;             % update design variable
    %% Store results into database X
    X(:,:,iter) = xF;    % each element value x is stored for each iteration
    C(iter) = c(iter);   % each compliance is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    %% Results
    if dis == 1           % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
    end
    if draw == 1          % plot iterations
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','w','ycolor','w')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        drawnow;
        hold on
        if iter == 1
            axis equal; axis tight;
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
```

```
            end
            plot(npfx,npfy,'g.','MarkerSize',20)
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0                 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c) ...
        ' Con:' sprintf('%6.3f',diff)]);
end
if draw == 0                % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','w','ycolor','w')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    %% Plot coloured dots for constraints
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
    %% Plot coloured dots for force application
    for i = 1:length(Fe)
        npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
        nplot = ceil(Fe(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npfy(i) = nplot-0.5;
    end
    plot(npfx,npfy,'g.','MarkerSize',20)
    %% Plot compliance plot
    if adv == 0
        figure(1)
        subplot(2,1,2)
```

```
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    end
end
toc                         % stop timer
```

B.2 ADVANCED.m

As an addition to (B.1), this advanced code can be used to plot progression pictures for multiple situations. In this Advanced code the optimization variables can be varied automatically. First, change the value of adv in Basic.m to one, in order to enable the program to vary the design variables. For upcoming add-ins the same Advanced function could be used at any time.

In the Advanced.m code the variation of the design variable needs to be chosen [line 13-14]. The next lines can be used to make vectors, which consist the values of the design variables, which are willing to be compared [line 15-21]. As up to now, it can only hold a maximum of four values per run. Only one variable can be varied per run, in order to hold the order variables constant, a default value can be defined in [line 22-28]. The program now write some values and pre-allocate spaces, user input is not needed from this line on [line 29-40]. The loop is starting, and makes a call to Basic.m for each design configuration, the programs determines whether or not the users made a design variation, or just want to plot an evolutionary scheme, as defined by var = 6 [line 41-69]. The figures and progression plots are made in the coming lines [line 70-97]. Each calculation run time is collected and stored. After completion of each variation of the design, a matrix Y is displayed in the command windows. Which consist the number of run, the design variation vector, the number of loops needed for that configuration, and the associated objective and run time [line 98-124]. Compliance values are stored and plotted in one graph [line 125-211]. Next, values of the variations are stored into the workspace for further usage and finally the design problem is drawn [line 212-238].

```
1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %                                                                     %
3  % Topology Optimization Using Matlab                                  %
4  % ADVANCED.m                                                          %
5  %                                                                     %
6  % Delft University of Technology, Department PME                      %
7  % Master of Science Thesis Project                                    %
8  %                                                                     %
9  % Stefan Broxterman                                                   %
10 %                                                                     %
11 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12 clc; clf; close all; clear X;
13 %% DEFINE OPTIMIZATION VARIABLES
14 var = 4;                % [1 = mesh, 2 = penalty, 3 = filter radius, 4 =
     volume fraction, 5 = filter method, 6 = evolution]
15 nxvec = [30,60,90,120]; % horizontal elements vector
16 nyvec = [10,20,30,40];  % vertical elements vector
17 volvec = [0.2 0.35 0.5 0.65]; % volume fraction vector
18 rminvec = [1,1.25,1.5,3]; % filter size vector
19 penvec = [1, 2, 3, 5];  % penalty vector
20 filvec = [0, 1, 2];     % filter vector
21 evolvec = [0.05, 0.25, 0.5, 1]; % evolution fraction vector
22 %% SET DEFAULT VALUES
23 nx = nxvec(3);          % default number of horizontal elements
24 ny = nyvec(3);          % default number of vertical elements
25 vol = volvec(3);        % default number of volume fraction
```

```
pen = penvec(3);          % default penalty
rmin = rminvec(3);        % default filter radius
fil = filvec(1);          % default filter method
%% SET OPTIMIZATION VALUES
ex = [30,60,90,120];      % vector size for pre-allocating space
figend = 4;               % set total of varying values
label = ['a','b','c','d','e']; % graphic label
%% PRE-ALLOCATE SPACE
loops = zeros(1,size(ex,2)); % initial loops matrix
obj = zeros(1,size(ex,2)); % initial ojective matrix
t = zeros(1,size(ex,2)); % initial time matrix
Y = zeros(size(ex,2),5); % initial results matrix
if var == 6               % for evolution scheme, BasicK.m only needs to ...
    BASIC                 % run one time only
end
%% START LOOP
for fig = 1:figend        % start itertation loop
    tic;                  % start timer
    if var ~= 6           % for non-evolution scheme, run below
        clear X; clear C; % clear results matrix for each run
        if var == 1           % differentiation on number of elements
            nx = nxvec(fig); % pick each horizontal value
            ny = nyvec(fig); % pick each vertical value
        elseif var == 2       % differentiation on penalty
            pen = penvec(fig); % pick each penalty
        elseif var == 3       % differentiation on filter radius
            rmin = rminvec(fig); % pick each rmin
        elseif var == 4       % differentiation on filter method
            vol = volvec(fig); % pick each filter method
        elseif var == 5       % differentiation on filter method
            fil = filvec(fig); % pick each filter method
        end
        BASIC             % run Basic.m
        loops(fig) = size(X,3); % number of iterations used
        obj(fig) = c(iter); % store objective function
        prog = X(:,:,loops(fig)); % store densities for progression drawing
    elseif var == 6       % store compliance for evolution vector
        loops = size(X,3); % for evolutionary scheme, calculate rounded ...
        loop(1) = round(evolvec(1)*loops); % values of loops and store ...
        loop(2) = round(evolvec(2)*loops); % this loop number
        loop(3) = round(evolvec(3)*loops);
        loop(4) = round(evolvec(4)*loops);
        prog = X(:,:,loop); % progression picture for each evolution fraction
    end
    %% Set graphics
    if draw == 1          % check for drawing
        H = get(gcf,'Position'); % get position of figure
    else
```

```
        H = [680,558,560,420]; % set size of figure(2) plot windows
    end
    H2 = figure(2);      % plot window for progression pictures
    set(H2,'position',[H(1)+H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
    %% Draw progression plots
    subplot(3,2,fig+2)  % plot each differentiation
    colormap(gray);      % grayscale
    if var == 6          % evolution needs different plotting
        imagesc(1-prog(:,:,fig)); % plot progression picture
        xlabel(sprintf('c = %.2f',C(loop(fig))),'color','k')
    else
        imagesc(1-prog); % plot progression picture
        xlabel(sprintf('c = %.2f',obj(fig)),'color','k')
    end
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','w','ycolor','w')
    axis equal; axis tight; % set additional options
    if var == 6          % evolution needs different plotting
        xlabel(sprintf('c = %.2f',C(loop(fig))),'color','k')
    else
        xlabel(sprintf('c = %.2f',obj(fig)),'color','k')
    end
    ylabel(sprintf('%s      ',(label(fig+1))),...
        'rot',0,'color','k','FontSize',11)
    %% Store compliance
    if var ~= 6            % store compliance for further plotting
        if fig == 1
            C1 = C;
        elseif fig == 2
            C2 = C;
        elseif fig == 3
            C3 = C;
        elseif fig == 4
            C4 = C;
        end
    end
    %% Draw graphics
    xbox = get(gca,'XLim');
    ybox = get(gca,'YLim');
    xwidth = xbox(2)-xbox(1);
    ywidth = ybox(2)-ybox(1);
    rectangle('Position',[xbox(1),ybox(1),xwidth,ywidth],...
        'EdgeColor',[0.5 0.5 0.5],'LineStyle',':');  drawnow;
    t(fig) = toc;
    %% Output
    if var ~= 6  % output results for non-evolutionary schemes
        Y(fig,:) = [fig ex(fig) loops(fig) obj(fig) t(fig)];
        if fig == figend
            Y
        end;
    end
    %% Compliance graphs
```

```
    if var ~= 6
        H3 = figure(3);
        set(H3,'position',[H(1)-H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
        hold on
        switch fig
            case 1      % first variable
                plot(1:length(C1),C1,'b:','LineWidth',2)
                xaxmax = mean(length(C1));
                yaxmax = max(max(C1));
                yaxmin = min(C1);
                if var == 1
                    legend(sprintf('mesh = %g x %g ',nxvec(1),nyvec(1)))
                elseif var == 2
                    legend(sprintf('pen = %g',penvec(1)))
                elseif var == 3
                    legend(sprintf('Rmin = %g',rminvec(1)))
                elseif var == 4
                    legend(sprintf('vol = %g',volvec(1)))
                elseif var == 5
                    legend(sprintf('filter = Sensitivity'))
                end
            case 2      % second variable
                plot(1:length(C2),C2,'r--','LineWidth',2)
                xaxmax = mean([length(C1) length(C2)]);
                yaxmax = max([max(C1) max(C2)]);
                yaxmin = min(min([C1 C2]));
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)),sprintf('mesh = %g x %g',nxvec(2),nyvec(2)))
                elseif var == 2
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)))
                elseif var == 3
                    legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)))
                elseif var == 4
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)))
                elseif var == 5
                    legend(sprintf('filter = Sensitivity'),sprintf('filter = Density'))
                end
            case 3      % third variable
                plot(1:length(C3),C3,'k','LineWidth',2)
                xaxmax = mean([length(C1) length(C2) length(C3)]);
                yaxmax = max([max(C1) max(C2) max(C3)]);
                yaxmin = min(min([C1 C2 C3]));
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)),sprintf('mesh = %g x %g',nxvec(2),nyvec(2)),sprintf('mesh = %g x %g',nxvec(3),nyvec(3)))
                elseif var == 2
```

```
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)),sprintf('pen = %g',penvec(3)))
                elseif var == 3
                    legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)),sprintf('Rmin = %g',rminvec(3)))
                elseif var == 4
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)),sprintf('vol = %g',volvec(3)))
                elseif var == 5
                    legend(sprintf('filter = Sensitivity'),sprintf('filter = Density'),sprintf('filter = Heaviside'))
                end
            case 4      % fourth variable
                plot(1:length(C4),C4,'g-.','LineWidth',2)
                xaxmax = mean([length(C1) length(C2) length(C3) length(C4)]);
                yaxmax = max([max(C1) max(C2) max(C3) max(C4)]);
                yaxmin = min(min([C1 C2 C3 C4]));
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)),sprintf('mesh = %g x %g',nxvec(2),nyvec(2)),sprintf('mesh = %g x %g',nxvec(3),nyvec(3)),sprintf('mesh = %g x %g',nxvec(4),nyvec(4)))
                elseif var == 2
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)),sprintf('pen = %g',penvec(3)),sprintf('pen = %g',penvec(4)))
                elseif var == 3
                    legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)),sprintf('Rmin = %g',rminvec(3)),sprintf('Rmin = %g',rminvec(4)))
                elseif var == 4
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)),sprintf('vol = %g',volvec(3)),sprintf('vol = %g',volvec(4)))
                end
        end
        xlabel('Number of iterations')
        ylabel('Compliance')
        if exist('pcon','var') == 0
            yaxmax = mean([yaxmin yaxmax]);
        elseif pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        axis([0 xaxmax 0.95*yaxmin yaxmax])
    elseif var == 6
        H3 = figure(3);
        set(H3,'position',[H(1)-H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
        hold on
        plot(C)
        xlabel('Number of iterations')
```

```
        ylabel('Compliance')
        axis([0 length(C) 0.9*min(C) max(C)])
    end
end
%% STORE RESULTS
disp('Y = i, penalty, loops, objective, time')
if var == 1                 % mesh refinement
    Ymesh = Y;              % store result matrix
    save('MeshRefinementY.mat','Y');
elseif var == 2             % penalty
    Ypenal = Y;             % store result matrix
    save('PenaltyY.mat','Y');
elseif var == 3             % filter radius
    Yfilter = Y;            % store result matrix
    save('FilterY.mat','Y');
elseif var == 4            % volume fraction
    Yvolume = Y;            % store result matrix
    save('VolumeY.mat','Y');
end
%% DRAW DESIGN PROBLEM
figure(2)
subplot(3,2,(1:2))          % plot the initial mechanical problem
rectangle('Position',[xbox(1),ybox(1),xwidth,ywidth],...
    'FaceColor',[0.5 0.5 0.5])
axis equal; axis tight;
set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
    'YTicklabel',[],'xcolor','w','ycolor','w')
ylabel(sprintf('%s)      ',(label(1))) ,'rot',0,'color','k','FontSize',11)
draw_arrow([xbox(2) ybox(1)],[xbox(2) -0.25*ywidth],1)
rectangle('Position',[-0.1*xwidth,ybox(1)-0.1*ywidth,...
    0.1*xwidth,1.2*ywidth],'FaceColor',[0 0 0],'LineWidth',3)
```

B.3 BASIC.m

The final M-code, including all previous described functionality can be found here

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                        %
% Topology Optimization Using Matlab                                     %
% BASIC.m                                                                %
%                                                                        %
% Delft University of Technology, Department PME                         %
% Master of Science Thesis Project                                       %
%                                                                        %
% Stefan Broxterman                                                      %
%                                                                        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
tic                         % start timer
%% DEFINE PARAMETERS
adv = 1;                    % use advanced function [0 = off, 1 = on]
if adv == 0                 % define parameters at behalf of the advanced function
    nx = 90;                % numer of elements horizontal
    ny = 30;                % number of elements vertical
    vol = 0.5;              % volume fraction [0-1]
    pen = 3;                % penalty
    rmin = 1.5;             % filter size
    fil = 0;                % filter method [0 = sensitivity filtering, 1 = density filtering, 2 = heaviside filtering]
    clc; clf; close all;
end
%% DEFINE SOLUTION METHOD
sol = 0;                    % solution method [0 = oc(sens), 1 = mma]
pcon = 0;                   % use continuation method [0 = off, 1 = on]
%% DEFINE CALCULATION
tol = 0.01;                 % tolerance for convergence criterion [0.01]
move = 0.2;                 % move limit for lagrange [0.2]
pcinc = 1.03;               % penalty continuation increasing factor [1.03]
piter = 20;                 % number of iteration for starting penalty [20]
miter = 1000;               % maximum number of iterations [1000]
%% DEFINE OUTPUT
draw = 1;                   % plot iterations [0 = off, 1 = on]
dis = 1;                    % display iterations [0 = off, 1 = on]
%% DEFINE MATERIAL
E = 1;                      % young's modulus of solid [1]
Emin = 1e-9;                % young's modulus of void [1e-9]
nu = 0.3;                   % poisson ratio [0.3]
rho = 0e-3;                 % density [0e-3]
g = 9.81;                   % gravitational acceleration [9.81]
%% DEFINE FORCE
Fe = 2*(nx+1)*(ny+1);       % element of force application [2*(nx+1)*(ny+1)]
Fn = 1;                     % number of applied force locations [1]
Fv = -1;                    % value of applied force [-1]
%% DEFINE SUPPORTS
```

```
fix = 1:2*(ny+1);         % fixed elements [1:2*(ny+1)]
%% DEFINE ELEMENT RESTRICTIONS
shap = 0;                 % [0 = no restrictions, 1 = circle, 2 = custom]
area = 0;                 % [0 = no material (passive), 1 = material (active)]
nodr = (round(ny/2)+(0:ny:(nx-1)*ny)); % custom restricted nodes
%% PREPARE FINITE ELEMENT
N = 2*(nx+1)*(ny+1);      % total element nodes
all = 1:2*(nx+1)*(ny+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12]; % fem
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6]; % fem
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4]; % fem
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2]; % fem
Ke = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]); % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node numer matrix
dofvec = reshape(2*nodes(1:end-1,1:end-1)+1,nx*ny,1); % create dof vector
dofmat = repmat(dofvec,1,8)+repmat([0 1 2*ny+[2 3 0 1] -2 -1],nx*ny,1); % create dof matrix
iK = reshape(kron(dofmat,ones(8,1))',64*nx*ny,1); % build sparse i
jK = reshape(kron(dofmat,ones(1,8))',64*nx*ny,1); % build sparse j
%% PREPARE FILTER
iH = ones(nx*ny*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));      % create sparse vector of ones
kH = zeros(size(iH));     % create sparse vector of zeros
m = 0;                    % index for filtering
for i = 1:nx              % for each element calculate distance between ...
    for j = 1:ny          % elements' center for filtering
        r1 = (i-1)*ny+j; % sparse value i
        for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx)  % center of element
            for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % center of element
                r2 = (k-1)*ny+l; % sparse value 2
                m = m+1; % update index for filtering
                iH(m) = r1; % sparse vector for filtering
                jH(m) = r2; % sparse vector for filtering
                kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)); % weight factor
            end
        end
    end
end
H = sparse(iH,jH,kH);     % build filter
Hs = sum(H,2);            % summation of filter
%% DEFINE ELEMENT RESTRICTIONS
x = repmat(vol,ny,nx);    % initial material distribution
if shap == 0              % no restrictions
    efree = (1:nx*ny)'; % all elements are free
    eres= [];             % no restricted elements
elseif shap == 1          % restrictions
    rest = zeros(ny,nx); % pre-allocate space
```

```
    for i = 1:nx          % start loop
        for j = 1:ny      % for each element
            if sqrt((j-ny/2)^2+(i-nx/4)^2) < ny/2.5 % circular restriction
                rest(j,i) = 1; % write restriction
                if rest(j,i) == area % check for restriction
                    x(j,i) = area; % store restrictions in material distribution
                end
            end
        end
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;               % set filtered design variables
elseif fil == 2           % heaviside filter
    beta = 1;             % hs filter
    xTilde = x;           % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design space
end
xFree = xF(efree);        % define free design matrix
%% DEFINE STRUCTURAL
Fsiz = size(Fe,2);        % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% DEFINE MMA PARAMETERS
m = 1;                    % number of constraint functions
n = size(xFree(:),1);     % number of variables
xmin = zeros(n,1);        % minimum values of x
xmax = ones(n,1);         % maximum values of x
xold1 = zeros(n,1);       % previous x, to monitor convergence
xold2 = xold1;            % used by mma to monitor convergence
df0dx2 = zeros(n,1);      % second derivative of the objective function
dfdx2 = zeros(1,n);       % second derivative of the constraint function
low   = xmin;             % lower asymptotes from the previous iteration
upp   = xmax;             % upper asymptotes from the previous iteration
a0 = 1;                   % constant a_0 in mma formulation
a = zeros(m,1);           % constant a_i in mma formulation
cmma = 1e3*ones(m,1);     % constant c_i in mma formulation
d = zeros(m,1);           % constant d_i in mma formulation
subs = 200;               % maximum number of subsolv iterations
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
U = zeros(size(F));       % pre-allocate space displacement
c = zeros(miter,1);       % pre-allocate objective vector
%% INITIALIZE LOOP
iter = 0;                 % initialize loop
diff = 1;                 % initialize convergence criterion
```

```
loopbeta = 1;             % initialize beta-loop
%% START LOOP
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    loopbeta = loopbeta +1; % iteration loop for hs filter
    iter = iter+1;        % define iteration
    if pcon == 1          % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;       %... set penalty 1
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
        end
    elseif pcon == 0      % not using continuation method
        p = pen;          % set penalty
    end
    %% Selfweight
    if rho ~= 0           % gravity is involved
        xP=zeros(ny,nx); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny     % for each element, set gravitational...
            Fsw(dofmat(i,2:2:end))=Fsw(dofmat(i,2:2:end))-xF(i)*rho*9.81/4;
        end               % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0       % no gravity
        xP = xF.^p;       % penalized design variable
        Fsw = 0;          % no selfweight
    end
    Ftot = F + Fsw;       % total force
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xP(:)'*(E-Emin)),64*nx*ny,1); % create sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;         % build stiffness matrix
    U(free,:) = K(free,free)\Ftot(free,:); % displacement solving
    c(iter) = 0;          % set compliance to zero
    Sens = 0;             % set sensitivity to zero
    %% Calculate compliance and sensitivity
    for i = 1:size(F,2) % for number of load cases
        Ui = U(:,i);      % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        c(iter) = c(iter) + sum(sum((Emin+xF.^p*(E-Emin)).*c0)); % calculate compliance
        Sens = Sens + reshape(2*Ui(dofmat)*repmat([0;-9.81*rho/4],4,1),ny,nx) -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
    end
    Senc = ones(ny,nx); % set constraint sensitivity
    if fil == 0           % optimality criterion with sensitivity filter
        Sens(:) = H*(x(:).*Sens(:))./Hs./max(1e-3,x(:)); % update filtered sensitivity
```

```
    elseif fil == 1     % optimality criterion with density filter
        Sens(:) = H*(Sens(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:)./Hs); % update filtered sensitivity of constraint
    elseif fil == 2     % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:).*dx(:)./Hs); % update filtered sensitivity of constraint
    end
    %% Update design variables Optimality Criterion
    if sol == 0         % use optimality criterion method
        l1 = 0;         % initial lower bound for lagranian mulitplier
        l2 = 1e9;       % initial upper bound for lagranian multiplier
        while (l2-l1)/(l1+l2) > 1e-3; % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
            if shap == 1 % restriction is on
                xF(rest==1) = area; % set restricted area
            end
            if sum(xF(:)) > vol*nx*ny; % check for optimum
                l1 = lag; % update lower bound to average
            else
                l2 = lag; % update upper bound to average
            end
        end
        %% Method of moving asymptotes
    elseif sol == 1     % use mma solver
        xval = xFree(:); % store current design variable for mma
        if iter == 1    % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = Sens(efree)*cscale; % store sensitivity for mma
        f = (sum(xF(:))/(vol*nx*ny)-1); % normalized constraint function
        dfdx = Senc(efree)'/(vol*ny*nx); % derivative of the constraint function
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
```

```
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = xFree(:); % previous x, to monitor convergence
        xnew = xF;      % update result
        xnew(efree) = xmma; % include restricted elements
        xnew = reshape(xnew,ny,nx); % reshape xmma vector to original size
        if fil == 0     % sensitivity filter
            xF = xnew;  % update design variables
        elseif fil == 1 % density filter
            xF(:) = (H*xnew(:))./Hs; % update filtered densities result
        elseif fil == 2 % heaviside filter
            xTilde(:)= (H*xnew(:))./Hs; % filtered result
            xF(:)=1-exp(-beta*xTilde)+xTilde*exp(-beta); % update design variable
        end
        if shap == 1    % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end

    end
    xFree = xnew(efree); % set non-restricted area
    diff = max(abs(xnew(:)-x(:))); % difference of maximum element change
    x = xnew;               % update design variable
    if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;      % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
    %% Store results into database X
    X(:,:,iter) = xF;   % each element value x is stored for each iteration
    C(iter) = c(iter);  % each compliance is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    %% Results
    if dis == 1         % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
    end
    if draw == 1        % plot iterations
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','w','ycolor','w')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        drawnow;
        hold on
```

```
        if iter == 1
            axis equal; axis tight;
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            plot(npfx,npfy,'g.','MarkerSize',20)
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0             % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c) ...
        ' Con:' sprintf('%6.3f',diff)]);
end
if draw == 0            % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','w','ycolor','w')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    %% Plot coloured dots for constraints
```

```
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
    %% Plot coloured dots for force application
    for i = 1:length(Fe)
        npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
        nplot = ceil(Fe(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npfy(i) = nplot-0.5;
    end
    plot(npfx,npfy,'g.','MarkerSize',20)
    %% Plot compliance plot
    if adv == 0
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    end
end
toc                         % stop timer
```

B.4 BASIC 3D.m

In the previous section, an add-in is given to produce a simple 3D optimization. However, for certain cases, it could be helpful to have the same different options as described in the add-in sections. The following code includes the same functionality as the BASIC code (B.3), but now for three dimensions. This code is tested and working. A small reminder should be made regarding the restrictions option. In the 2D optimization code a simple circle could be made, to describe circular restricted area. In this 3D code however, this circular restricted area is replaced by two options. Shape option 1 describes a cylindrical restrictive regions, shape option 2 describes a spherical restrictive region.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                      %
% Topology Optimization Using Matlab                                   %
% BASIC3D.m                                                            %
%                                                                      %
% Delft University of Technology, Department PME                       %
% Master of Science Thesis Project                                     %
%                                                                      %
% Stefan Broxterman                                                    %
%                                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
tic                        % start timer
%% DEFINE PARAMETERS
adv = 1;                   % use advanced function [0 = off, 1 = on]
if adv == 0                % define parameters at behalf of the advanced function
    nx = 30;               % numer of elements horizontal
    ny = 10;               % number of elements vertical
    nz = 4;               % number of elements lateral
    vol = 0.5;             % volume fraction [0-1]
    pen = 3;               % penalty
    rmin = 1.5;            % filter size
    fil = 0;               % filter method [0 = sensitivity filtering, 1 = density filtering, 2 = heaviside filtering]
    clear X ;
    clc; clf; close all;
end
%% DEFINE SOLUTION METHOD
sol = 1;                   % solution method [0 = oc(sens), 1 = mma]
pcon = 1;                  % use continuation method [0 = off, 1 = on]
%% DEFINE CALCULATION
tol = 0.01;                % tolerance for convergence criterion [0.01]
move = 0.2;                % move limit for lagrange [0.2]
pcinc = 1.03;              % penalty continuation increasing factor [1.03]
piter = 20;                % number of iteration for starting penalty [20]
miter = 1000;              % maximum number of iterations [1000]
graysc = 1;                % use gray-scale filter [0 = off, 1 = on]
q = 1;                     % gray-scale parameter
qmax = 2;                  % maximum gray-scale parameter
```

```
plotiter = 5;              % gap of iterations used to plot or draw iterations
%% DEFINE OUTPUT
draw = 1;                  % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 1;                   % display iterations [0 = off, 1 = on, 2 = partial]
%% DEFINE MATERIAL
E = 1;                     % young's modulus of solid [1]
Emin = 1e-9;               % young's modulus of void [1e-9]
nu = 0.3;                  % poisson ratio [0.3]
rho = 0e-3;                % density [0e-3]
g = 9.81;                  % gravitational acceleration [9.81]
%% DEFINE FORCE
Fe = (3*(nx+1)*(ny+1)-1)+(3*(nx+1)*(ny+1))*(0:nz)'; % element of force application
Fn = 1;                    % number of applied force locations [1]
Fv = -1;                   % value of applied force [-1]
%% DEFINE SUPPORTS
fix = repmat((1:3*(ny+1))',1,nz+1)+repmat((0:nz)*3*(nx+1)*(ny+1),length((1:3*(ny+1))),1); % fixed elements
fix = fix(:);
%% DEFINE ELEMENT RESTRICTIONS
shap = 2;                  % [0 = no restrictions, 1 = cylinder, 2 = sphere]
area = 0;                  % [0 = no material (passive), 1 = material (active)]
nodr = (round(ny/2)+(0:ny:(nx-1)*ny)); % custom restricted nodes
%% PREPARE FINITE ELEMENT
N = 3*(nx+1)*(ny+1)*(nz+1); % total elements nodes
all = 1:3*(nx+1)*(ny+1)*(nz+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A = [32 6 -8 6 -6 4 3 -6 -10 3 -3 -3 -4 -8;
    -48 0 0 -24 24 0 0 0 12 -12 0 12 12 12]; % fem
k = 1/72*A'*[1; nu];      % simple stiffness matrix
%% GENERATE SIX SUB-MATRICES AND THEN GET KE MATRIX
K1 = [k(1) k(2) k(2) k(3) k(5) k(5);
     k(2) k(1) k(2) k(4) k(6) k(7);
     k(2) k(2) k(1) k(4) k(7) k(6);
     k(3) k(4) k(4) k(1) k(8) k(8);
     k(5) k(6) k(7) k(8) k(1) k(2);
     k(5) k(7) k(6) k(8) k(2) k(1)]; % stiffness matrix
K2 = [k(9)  k(8)  k(12) k(6)  k(4)  k(7);
     k(8)  k(9)  k(12) k(5)  k(3)  k(5);
     k(10) k(10) k(13) k(7)  k(4)  k(6);
     k(6)  k(5)  k(11) k(9)  k(2)  k(10);
     k(4)  k(3)  k(5)  k(2)  k(9)  k(12)
     k(11) k(4)  k(6)  k(12) k(10) k(13)]; % stiffness matrix
K3 = [k(6)  k(7)  k(4)  k(9)  k(12) k(8);
     k(7)  k(6)  k(4)  k(10) k(13) k(10);
     k(5)  k(5)  k(3)  k(8)  k(12) k(9);
     k(9)  k(10) k(2)  k(6)  k(11) k(5);
     k(12) k(13) k(10) k(11) k(6)  k(4);
     k(2)  k(12) k(9)  k(4)  k(5)  k(3)]; % stiffness matrix
K4 = [k(14) k(11) k(11) k(13) k(10) k(10);
```

```
    k(11) k(14) k(11) k(12) k(9)  k(8);
    k(11) k(11) k(14) k(12) k(8)  k(9);
    k(13) k(12) k(12) k(14) k(7)  k(7);
    k(10) k(9)  k(8)  k(7)  k(14) k(11);
    k(10) k(8)  k(9)  k(7)  k(11) k(14)]; % stiffness matrix
K5 = [k(1) k(2)  k(8)  k(3) k(5)  k(4);
    k(2) k(1)  k(8)  k(4) k(6)  k(11);
    k(8) k(8)  k(1)  k(5) k(11) k(6);
    k(3) k(4)  k(5)  k(1) k(8)  k(2);
    k(5) k(6)  k(11) k(8) k(1)  k(8);
    k(4) k(11) k(6)  k(2) k(8)  k(1)]; % stiffness matrix
K6 = [k(14) k(11) k(7)  k(13) k(10) k(12);
    k(11) k(14) k(7)  k(12) k(9)  k(2);
    k(7)  k(7)  k(14) k(10) k(2)  k(9);
    k(13) k(12) k(10) k(14) k(7)  k(11);
    k(10) k(9)  k(2)  k(7)  k(14) k(7);
    k(12) k(2)  k(9)  k(11) k(7)  k(14)]; % stiffness matrix
Ke = 1/((nu+1)*(1-2*nu))*...
    [ K1  K2  K3  K4;
    K2'  K5  K6  K3';
    K3' K6  K5' K2';
    K4  K3  K2  K1'];    % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node number matrix
nodes2 = reshape(nodes(1:end-1,1:end-1),ny*nx,1); % create node number matrix
nodes3 = 0:(ny+1)*(nx+1):(nz-1)*(ny+1)*(nx+1); % create node number matrix
nodes4 = repmat(nodes2,size(nodes3))+repmat(nodes3,size(nodes2)); % create node number matrix
dofvec = 3*nodes4(:)+1; % create dof vector
dofmat = repmat(dofvec,1,24)+repmat([0 1 2 3*ny+[3 4 5 0 1 2] -3 -2 -1 3*(ny+1)*(nx+1) + [0 1 2 3*ny + [3 4 5 0 1 2] -3 -2 -1]],nx*ny*nz,1); % create dof matrix
iK = kron(dofmat,ones(24,1))'; % build sparse i
jK = kron(dofmat,ones(1,24))'; % build sparse j
%% PREPARE FILTER
iH = ones(nx*ny*nz*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));    % create sparse vector of ones
kH = zeros(size(iH));   % create sparse vector of zeros
m = 0;                  % index for filtering
for h = 1:nz            % for each element calculate...
    for i = 1:nx        % distance between elements'...
        for j = 1:ny    % centre for filtering
            r1 = (h-1)*nx*ny + (i-1)*ny+j; % sparse value 1
            for k2 = max(h-(ceil(rmin)-1),1):min(h+(ceil(rmin)-1),nz)  % centre of element
                for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx) % centre of element
                    for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % centre of element
                        r2 = (k2-1)*nx*ny + (k-1)*ny+l; % sparse value 2
                        m = m+1;      % update index for filtering
                        iH(m) = r1;   % sparse vector for filtering
```

```
                        jH(m) = r2;   % sparse vector for filtering
                        kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)+(h-k2)^2); % weight factor
                    end
                end
            end
        end
    end
end
H = sparse(iH,jH,kH);   % build filter
Hs = sum(H,2);          % summation of filter
%% DEFINE STRUCTURAL
x = repmat(vol,ny,nx,nz); % initial material distribution
if shap == 0            % no restrictions
    efree = (1:nx*ny*nz)'; % all elements are free
    eres= [];           % no restricted elements
elseif shap == 1 || shap == 2       % restrictions
    rest = zeros(ny,nx,nz); % pre-allocate space
    for i = 1:nx        % start loop
        for j = 1:ny    % for each element
            for k = 1:nz % for lateral element
                if sqrt((j-ny/2)^2+(i-nx/3)^2) < ny/2.5 % circular restriction
                    if shap == 1 % cyilindrical restriction
                        rest(j,i,k) = 1; % write restriction
                        if rest(j,i,k) == area % check for restriction
                            x(j,i,k) = area; % store restrictions in material distribution
                        end
                    elseif shap == 2 % spherical restriction
                        if sqrt((j-ny/2)^2+(k-nz/2)^2) < nz/2.5 % spherical restriction
                            if sqrt((k-nz/2)^2+(i-nx/3)^2) < nz/2.5 % spherical restriction
                                rest(j,i,k) = 1; % write restriction
                                if rest(j,i,k) == area % check for restriction
                                    x(j,i,k) = area; % store restrictions in material distribution
                                end
                            end
                        end
                    end
                end
            end
        end
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;             % set filtered design variables
elseif fil == 2         % heaviside filter
```

```
    beta = 1;              % hs filter
    xTilde = x;            % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design space
end
xFree = xF(efree);         % define free design matrix
%% DEFINE STRUCTURAL
Fsiz = size(Fe,2);         % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% DEFINE MMA PARAMETERS
m = 1;                     % number of constraint functions
n = size(xFree(:),1);      % number of variables
xmin = zeros(n,1);         % minimum values of x
xmax = ones(n,1);          % maximum values of x
xold1 = zeros(n,1);        % previous x, to monitor convergence
xold2 = xold1;             % used by mma to monitor convergence
df0dx2 = zeros(n,1);       % second derivative of the objective function
dfdx2 = zeros(1,n);        % second derivative of the constraint function
low   = xmin;              % lower asymptotes from the previous iteration
upp   = xmax;              % upper asymptotes from the previous iteration
a0 = 1;                    % constant a_0 in mma formulation
a = zeros(m,1);            % constant a_i in mma formulation
cmma = 1e3*ones(m,1);      % constant c_i in mma formulation
d = zeros(m,1);            % constant d_i in mma formulation
subs = 200;                % maximum number of subsolv iterations
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npz = zeros ( length ( fix ) ,1)' ; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
npfz = zeros(length(Fe),1)'; % pre-allocate force dots
U = zeros(size(F));        % pre-allocate space displacement
c = zeros(miter,1);        % pre-allocate objective vector
%% INITIALIZE LOOP
iter = 0;                  % initialize loop
diff = 1;                  % initialize convergence criterion
loopbeta = 1;              % initialize beta-loop
%% START LOOP
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    loopbeta = loopbeta +1; % iteration loop for hs filter
    iter = iter+1;         % define iteration
    if pcon == 1           % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;         %... set penalty 1
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
        end
    elseif pcon == 0       % not using continuation method
        p = pen;           % set penalty
    end
    if graysc == 1           % if grayscale is enabled
```

```
        if iter <= 15   % within 15 iterations
            q = 1;      % don't use grayscale
        else            % after 15 iterations
            q = min(qmax,1.01*q); % use continuation method to pick a gray-scale factor
        end
    end
    %% Selfweight
    if rho ~= 0         % gravity is involved
        xP=zeros(ny,nx,nz); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny*nz % for each element, set gravitational...
            Fsw(dofmat(i,2:3:end))=Fsw(dofmat(i,2:3:end))-xF(i)*rho*9.81/4;
        end             % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0     % no gravity
        xP = xF.^p;     % penalized design variable
        Fsw = 0;        % no selfweight
    end
    Ftot = F + Fsw;     % total force
    %% Finite element analysis
    kK = Ke(:)*(Emin+xP(:)'*(E-Emin)); % create sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;       % build stiffness matrix
    U(free,:) = K(free,free)\Ftot(free,:); % displacement solving
    c(iter) = 0;        % set compliance to zero
    Sens = 0;           % set sensitivity to zero
    %% Calculate compliance and sensitivity
    for i = 1:size(F,2) % for number of load cases
        Ui = U(:,i);    % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx,nz); % initial compliance
        c(iter) = c(iter) + sum(sum(sum((Emin+xF.^p*(E-Emin)).*c0))); % calculate compliance
        Sens = Sens + reshape(2*Ui(dofmat)*repmat([0;-9.81*rho/4; 0],8,1),ny,nx,nz) -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
    end
    Senc = ones(ny,nx,nz); % set constraint sensitivity
    if fil == 0         % optimality criterion with sensitivity filter
        Sens(:) = H*(x(:).*Sens(:))./Hs./max(1e-3,x(:)); % update filtered sensitivity
    elseif fil == 1     % optimality criterion with density filter
        Sens(:) = H*(Sens(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:)./Hs); % update filtered sensitivity of constraint
    elseif fil == 2     % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:).*dx(:)./Hs); % update filtered sensitivity of constraint
```

```
    end
    %% Update design variables Optimality Criterion
    if sol == 0          % use optimality criterion method
        l1 = 0;          % initial lower bound for lagranian mulitplier
        l2 = 1e9;        % initial upper bound for lagranian multiplier
        while (l2-l1)/(l1+l2) > 1e-3; % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            if graysc == 0       % don't use grayscale
                xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
            elseif graysc == 1   % use grayscale
                xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag)).^q))); % update element densities
            end
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
            if shap == 1 || shap == 2% restriction is on
                xF(rest==1) = area; % set restricted area
            end
            if sum(xF(:)) > vol*nx*ny*nz; % check for optimum
                l1 = lag; % update lower bound to average
            else
                l2 = lag; % update upper bound to average
            end
        end
        %% Method of moving asymptotes
    elseif sol == 1      % use mma solver
        xval = xFree(:); % store current design variable for mma
        if iter == 1     % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = Sens(efree)*cscale; % store sensitivity for mma
        f = (sum(xF(:))/(vol*nx*ny*nz)-1); % normalized constraint function
        dfdx = Senc(efree)'/(vol*ny*nx*nz); % derivative of the constraint function
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = xFree(:); % previous x, to monitor convergence
        xnew = xF;      % update result
```

```
        xnew(efree) = xmma; % include restricted elements
        xnew = reshape(xnew,ny,nx,nz); % reshape xmma vector to original size
        if fil == 0     % sensitivity filter
            xF = xnew;  % update design variables
        elseif fil == 1 % density filter
            xF(:) = (H*xnew(:))./Hs; % update filtered densities result
        elseif fil == 2 % heaviside filter
            xTilde(:)= (H*xnew(:))./Hs; % filtered result
            xF(:)=1-exp(-beta*xTilde)+xTilde*exp(-beta); % update design variable
        end
        if shap == 1 || shap == 2   % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end

    end
    xFree = xnew(efree); % set non-restricted area
    diff = max(abs(xnew(:)-x(:))); % difference of maximum element change
    x = xnew;           % update design variable
    if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;     % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
    %% Store results into database X
    X(:,:,:,iter) = xF;   % each element value x is stored for each iteration
    C(iter) = c(iter);  % each compliance is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    %% Results
    if dis == 1         % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
         ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
    elseif dis == 2     % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
            disiter = plotiter;
            elseif iter == disiter
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
        end

```

```
    end
    if draw == 1         % plot iterations
        figure(1)
        subplot(2,1,1)
        [nely,nelx,nelz] = size(xF);
        hx = 1; hy = 1; hz = 1;            % User-defined unit element size
        face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
        for k = 1:nelz
            z = (k-1)*hz;
            for i = 1:nelx
                xplot = (i-1)*hx;
                for j = 1:nely
                    y = nely*hy - (j-1)*hy;
                    if (xF(j,i,k) > 0.5)  % User-defined display density threshold
                        vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z; xplot+hx y z; xplot y z+hx;xplot y-hx z+hx; xplot+hx y-hx z+hx;xplot+hx y z+hx];
                        vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -vert(:,2,:);
                        patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k))]);
                        hold on;
                    end
                end
            end
        end
        axis equal; axis tight;
        set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
            'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','zcolor','w')
        view([30,30]);
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                nplotx = ceil(fix(i)/(3*(ny+1)));
                while nplotx > (nx+1)
                    nplotx = nplotx -(nx+1);
                end
                npx(i) = nplotx-1;
                nplot = ceil(fix(i)/3);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-1;
                npz(i) = 1-ceil(fix(i)/(3*(nx+1)*(ny+1)));
            end
            plot3(npx,npz,npy,'r.','MarkerSize',20)
```

```
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                nplotx = ceil(Fe(i)/(3*(ny+1)));
                while nplotx > (nx+1)
                    nplotx = nplotx -(nx+1);
                end
                npfx(i) = nplotx-1;
                nplot = ceil(Fe(i)/3);
                while nplot > (ny)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot;
                npfz(i) = 1-ceil(Fe(i)/(3*(nx+1)*(ny+1)));
            end
            plot3(npfx,npfz,npfy,'g.','MarkerSize',20)
            drawnow;
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    elseif draw == 2        % plot parts of iterations
        if iter == 1 || iter == drawiter
            if iter == 1
            drawiter = plotiter;
            elseif iter == drawiter
                drawiter = drawiter + plotiter;
            end
        figure(1)
        subplot(2,1,1)
        [nely,nelx,nelz] = size(xF);
        hx = 1; hy = 1; hz = 1;            % User-defined unit element size
        face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
        for k = 1:nelz
            z = (k-1)*hz;
            for i = 1:nelx
                xplot = (i-1)*hx;
                for j = 1:nely
                    y = nely*hy - (j-1)*hy;
                    if (xF(j,i,k) > 0.5)  % User-defined display density threshold
                        vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z; xplot+hx y z; xplot y z+hx;xplot y-hx z+hx; xplot+hx y-hx z+hx;xplot+hx y z+hx];
                        vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -vert(:,2,:);
```

```
                        patch('Faces',face,'Vertices',vert,'FaceColor'
                            ,[0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k))
                            ,0.2+0.8*(1-xF(j,i,k))]);
                        hold on;
                    end
                end
            end
        end
        axis equal; axis tight;
        set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
            'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','
            zcolor','w')
        view([30,30]);
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                nplotx = ceil(fix(i)/(3*(ny+1)));
                while nplotx > (nx+1)
                    nplotx = nplotx -(nx+1);
                end
                npx(i) = nplotx-1;
                nplot = ceil(fix(i)/3);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-1;
                npz(i) = 1-ceil(fix(i)/(3*(nx+1)*(ny+1)));
            end
            plot3(npx,npz,npy,'r.','MarkerSize',20)
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                nplotx = ceil(Fe(i)/(3*(ny+1)));
                while nplotx > (nx+1)
                    nplotx = nplotx -(nx+1);
                end
                npfx(i) = nplotx-1;
                nplot = ceil(Fe(i)/3);
                while nplot > (ny)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot;
                npfz(i) = 1-ceil(Fe(i)/(3*(nx+1)*(ny+1)));
            end
            plot3(npfx,npfz,npfy,'g.','MarkerSize',20)
            drawnow;
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
```

```
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
        end
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0 || dis == 2                 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
end
if draw == 0 || draw == 2               % plot final result
    figure(1)
    subplot(2,1,1)
    [nely,nelx,nelz] = size(xF);
    hx = 1; hy = 1; hz = 1;                % User-defined unit element size
    face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
    for k = 1:nelz
        z = (k-1)*hz;
        for i = 1:nelx
            xplot = (i-1)*hx;
            for j = 1:nely
                y = nely*hy - (j-1)*hy;
                if (xF(j,i,k) > 0.5)  % User-defined display density threshold
                    vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z; xplot+hx y z; xplot y z+hx;xplot y-hx z+hx; xplot+hx y-hx z+hx;xplot+hx y z+hx];
                    vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -vert(:,2,:);
                    patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k))]);
                    hold on;
                end
            end
        end
    end
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
        'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','zcolor','w')
    view([30,30]);
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    % Plot coloured dots for constraints
    for i = 1:length(fix)
        nplotx = ceil(fix(i)/(3*(ny+1)));
```

```
        while nplotx > (nx+1)
            nplotx = nplotx -(nx+1);
        end
        npx(i) = nplotx-1;
        nplot = ceil(fix(i)/3);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-1;
        npz(i) = 1-ceil(fix(i)/(3*(nx+1)*(ny+1)));
    end
    plot3(npx,npz,npy,'r.','MarkerSize',20)
    % Plot coloured dots for force application
    for i = 1:length(Fe)
        nplotx = ceil(Fe(i)/(3*(ny+1)));
        while nplotx > (nx+1)
            nplotx = nplotx -(nx+1);
        end
        npfx(i) = nplotx-1;
        nplot = ceil(Fe(i)/3);
        while nplot > (ny)
            nplot = nplot-(ny+1);
        end
        npfy(i) = nplot;
        npfz(i) = 1-ceil(Fe(i)/(3*(nx+1)*(ny+1)));
    end
    plot3(npfx,npfz,npfy,'g.','MarkerSize',20)
    % Plot compliance plot
    figure(1)
    subplot(2,1,2)
    plot(c(1:iter))
    xaxmax = c(iter);
    yaxmax = max(c);
    yaxmin = min(c(1:iter));
    ylim([0.95*yaxmin yaxmax])
    xlim([0 iter+10])
end
toc                     % stop timer
```

B.5 ADVANCED 3D.m

By the inspiration of the ADVANCED (B.2) for 2D-problems, an 3D-adapted code is made available. The changes are quite big, so it's recommended to just run this new file, instead of writing an add-in code.
By the introduction of this code, it can be very interesting to vary the number of discretization of the lateral elements and see how it behaves.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                       %
% Topology Optimization Using Matlab                                    %
% ADVANCED3D.m                                                          %
%                                                                       %
% Delft University of Technology, Department PME                        %
% Master of Science Thesis Project                                      %
%                                                                       %
% Stefan Broxterman                                                     %
%                                                                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; clf; close all; clear X; clear prog;
%% DEFINE OPTIMIZATION VARIABLES
var = 6;                    % [1 = mesh, 2 = penalty, 3 = filter radius, 4 =
    volume fraction, 5 = filter method, 6 = evolution]
nxvec = [30,60,90,120]; % horizontal elements vector
nyvec = [10,20,30,40];  % vertical elements vector
nzvec = [1,2,3,5];          % lateral elements vector
volvec = [0.2 0.35 0.5 0.65]; % volume fraction vector
rminvec = [1,1.25,1.5,3]; % filter size vector
penvec = [1, 2, 3, 5];  % penalty vector
filvec = [0, 1, 2];     % filter vector
evolvec = [0.05, 0.25, 0.5, 1]; % evolution fraction vector
%% SET DEFAULT VALUES
nx = nxvec(1);              % default number of horizontal elements
ny = nyvec(1);              % default number of vertical elements
nz = nzvec(3);              % default number of lateral elements
vol = volvec(3);            % default number of volume fraction
pen = penvec(3);            % default penalty
rmin = rminvec(3);          % default filter radius
fil = filvec(1);            % default filter method
q = 1;                      % gray-scale parameter
qmax = 2;                   % maximum gray-scale parameter
%% SET OPTIMIZATION VALUES
ex = [30,60,90,120];        % vector size for pre-allocating space
figend = 4;                 % set total of varying values
label = ['a','b','c','d','e']; % graphic label
%% PRE-ALLOCATE SPACE
loops = zeros(1,size(ex,2)); % initial loops matrix
obj = zeros(1,size(ex,2)); % initial ojective matrix
t = zeros(1,size(ex,2)); % initial time matrix
Y = zeros(size(ex,2),5); % initial results matrix
if var == 6                 % for evolution scheme, BasicK.m only needs to
    ...
```

```
    BASIC3D              % run one time only
end
%% START LOOP
for fig = 1:figend       % start itertation loop
    tic;                 % start timer
    if var ~= 6          % for non-evolution scheme, run below
        clear X; clear C; % clear results matrix for each run
        if var == 1      % differentiation on number of elements
            nx = nxvec(fig); % pick each horizontal value
            ny = nyvec(fig); % pick each vertical value
            nz = nzvec(fig);
        elseif var == 2 % differentiation on penalty
            pen = penvec(fig); % pick each penalty
        elseif var == 3 % differentiation on filter radius
            rmin = rminvec(fig); % pick each rmin
        elseif var == 4 % differentiation on filter method
            vol = volvec(fig); % pick each filter method
         elseif var == 5 % differentiation on filter method
            fil = filvec(fig); % pick each filter method
        end
        BASIC3D          % run Basic.m
        loops(fig) = size(X,4); % number of iterations used
        obj(fig) = c(iter); % store objective function
        prog = X(:,:,:,loops(fig)); % store densities for progression drawing
    elseif var == 6      % store compliance for evolution vector
        loops = size(X,4); % for evolutionary scheme, calculate rounded ...
        loop(1) = round(evolvec(1)*loops); % values of loops and store ...
        loop(2) = round(evolvec(2)*loops); % this loop number
        loop(3) = round(evolvec(3)*loops);
        loop(4) = round(evolvec(4)*loops);
        prog = X(:,:,:,loop); % progression picture for each evolution fraction
    end
    %% Set graphics
    if draw == 1 || draw == 2        % check for drawing
        H = get(gcf,'Position'); % get position of figure
    else
        H = [680,558,560,420]; % set size of figure(2) plot windows
    end
    H2 = figure(2);      % plot window for progression pictures
    set(H2,'position',[H(1)+H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
    %% Draw progression plots
    subplot(3,2,fig+2)   % plot each differentiation
    if var == 6          % evolution needs different plotting
        [nely,nelx,nelz] = size(prog(:,:,:,fig));
        hx = 1; hy = 1; hz = 1;             % User-defined unit element size
        face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
        for k = 1:nelz
```

```
            z = (k-1)*hz;
            for i = 1:nelx
                xplot = (i-1)*hx;
                for j = 1:nely
                    y = nely*hy - (j-1)*hy;
                    if (prog(j,i,k,fig) > 0.5)  % User-defined display
                        density threshold
                        vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z;
                             xplot+hx y z; xplot y z+hx;xplot y-hx z+hx;
                            xplot+hx y-hx z+hx;xplot+hx y z+hx];
                        vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -
                            vert(:,2,:);
                        patch('Faces',face,'Vertices',vert,'FaceColor'
                            ,[0.2+0.8*(1-prog(j,i,k,fig)),0.2+0.8*(1-prog(
                            j,i,k,fig)),0.2+0.8*(1-prog(j,i,k,fig))]);
                        hold on;
                    end
                end
            end
        end
    else                    % plot advanced graphs
        [nely,nelx,nelz] = size(prog);
        hx = 1; hy = 1; hz = 1; % User-defined unit element size
        face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
        for k = 1:nelz
            z = (k-1)*hz;
            for i = 1:nelx
                xplot = (i-1)*hx;
                for j = 1:nely
                    y = nely*hy - (j-1)*hy;
                    if (prog(j,i,k) > 0.5)  % User-defined display
                        density threshold
                        vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z;
                             xplot+hx y z; xplot y z+hx;xplot y-hx z+hx;
                            xplot+hx y-hx z+hx;xplot+hx y z+hx];
                        vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -
                            vert(:,2,:);
                        patch('Faces',face,'Vertices',vert,'FaceColor'
                            ,[0.2+0.8*(1-prog(j,i,k)),0.2+0.8*(1-prog(j,i,
                            k)),0.2+0.8*(1-prog(j,i,k))]);
                        hold on;
                    end
                end
            end
        end
    end
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
        'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','zcolor
            ','w')
    view([30,30]);
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
```

```
    hold on
    if var == 6          % evolution needs different plotting
        xlabel(sprintf('c = %.2f',C(loop(fig))),'color','k')
    else
        xlabel(sprintf('c = %.2f',obj(fig)),'color','k')
    end
    zlabel(sprintf('%s)      ',(label(fig+1))),...
        'rot',0,'color','k','FontSize',11)
    %% Store compliance
    if var ~= 6            % store compliance for further plotting
        if fig == 1
            C1 = C;
        elseif fig == 2
            C2 = C;
        elseif fig == 3
            C3 = C;
        elseif fig == 4
            C4 = C;
        end
    end
    %% Draw graphics
    xbox = [0.5 nx+0.5];
    ybox = [0.5 ny+0.5];
    xwidth = xbox(2)-xbox(1);
    ywidth = ybox(2)-ybox(1);
    t(fig) = toc;
    %% Output
    if var ~= 6          % output results for non-evolutionary schemes
        Y(fig,:) = [fig ex(fig) loops(fig) obj(fig) t(fig)];
        if fig == figend
            Y
        end;
    end
    %% Compliance graphs
    if var ~= 6
        H3 = figure(3);
        set(H3,'position',[H(1)-H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
        hold on
        switch fig
            case 1      % first variable
                plot(1:length(C1),C1,'b:','LineWidth',2)
                xaxmax = mean(length(C1));
                yaxmax = max(max(C1));
                yaxmin = min(C1);
                if var == 1
                    legend(sprintf('mesh = %g x %g ',nxvec(1),nyvec(1)))
                elseif var == 2
                    legend(sprintf('pen = %g',penvec(1)))
                elseif var == 3
                    legend(sprintf('Rmin = %g',rminvec(1)))
                elseif var == 4
                    legend(sprintf('vol = %g',volvec(1)))
```

```
                elseif var == 5
                    legend(sprintf('filter = Sensitivity'))
                end
            case 2       % second variable
                plot(1:length(C2),C2,'r--','LineWidth',2)
                xaxmax = mean([length(C1) length(C2)]);
                yaxmax = max([max(C1) max(C2)]);
                yaxmin = min(min([C1 C2]));
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)),sprintf('mesh = %g x %g',nxvec(2),nyvec(2)))
                elseif var == 2
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)))
                elseif var == 3
                    legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)))
                elseif var == 4
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)))
                elseif var == 5
                    legend(sprintf('filter = Sensitivity'),sprintf('filter = Density'))
                end
            case 3       % third variable
                plot(1:length(C3),C3,'k','LineWidth',2)
                xaxmax = mean([length(C1) length(C2) length(C3)]);
                yaxmax = max([max(C1) max(C2) max(C3)]);
                yaxmin = min(min([C1 C2 C3]));
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)),sprintf('mesh = %g x %g',nxvec(2),nyvec(2)),sprintf('mesh = %g x %g',nxvec(3),nyvec(3)))
                elseif var == 2
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)),sprintf('pen = %g',penvec(3)))
                elseif var == 3
                    legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)),sprintf('Rmin = %g',rminvec(3)))
                elseif var == 4
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)),sprintf('vol = %g',volvec(3)))
                elseif var == 5
                    legend(sprintf('filter = Sensitivity'),sprintf('filter = Density'),sprintf('filter = Heaviside'))
                end
            case 4       % fourth variable
                plot(1:length(C4),C4,'g-.','LineWidth',2)
                xaxmax = mean([length(C1) length(C2) length(C3) length(C4)]);
                yaxmax = max([max(C1) max(C2) max(C3) max(C4)]);
                yaxmin = min(min([C1 C2 C3 C4]));
```

```
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)),sprintf('mesh = %g x %g',nxvec(2),nyvec(2)),sprintf('mesh = %g x %g',nxvec(3),nyvec(3)),sprintf('mesh = %g x %g',nxvec(4),nyvec(4)))
                elseif var == 2
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)),sprintf('pen = %g',penvec(3)),sprintf('pen = %g',penvec(4)))
                elseif var == 3
                    legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)),sprintf('Rmin = %g',rminvec(3)),sprintf('Rmin = %g',rminvec(4)))
                elseif var == 4
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)),sprintf('vol = %g',volvec(3)),sprintf('vol = %g',volvec(4)))
                end
        end
        xlabel('Number of iterations')
        ylabel('Compliance')
        if exist('pcon','var') == 0,
            yaxmax = mean([yaxmin yaxmax]);
        elseif pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        axis([0 xaxmax 0.95*yaxmin yaxmax])
    elseif var == 6
        H3 = figure(3);
        set(H3,'position',[H(1)-H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
        hold on
        plot(C)
        xlabel('Number of iterations')
        ylabel('Compliance')
        axis([0 length(C) 0.9*min(C) max(C)])
    end
end
%% STORE RESULTS
disp('Y = i, penalty, loops, objective, time')
if var == 1                 % mesh refinement
    Ymesh = Y;              % store result matrix
    save('MeshRefinementY.mat','Y');
elseif var == 2             % penalty
    Ypenal = Y;             % store result matrix
    save('PenaltyY.mat','Y');
elseif var == 3             % filter radius
    Yfilter = Y;            % store result matrix
    save('FilterY.mat','Y');
    elseif var == 4         % volume fraction
    Yvolume = Y;            % store result matrix
    save('VolumeY.mat','Y');
end
```

```
%% DRAW DESIGN PROBLEM
figure(2)
subplot(3,2,(1:2))       % plot the initial mechanical problem
rectangle('Position',[xbox(1),ybox(1),xwidth,ywidth],...
    'FaceColor',[0.5 0.5 0.5])
axis equal; axis tight;
set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
    'YTicklabel',[],'xcolor','w','ycolor','w')
ylabel(sprintf('%s)      ',(label(1))) ,'rot',0,'color','k','FontSize',11)
draw_arrow([xbox(2) ybox(1)],[xbox(2) -0.25*ywidth],1)
rectangle('Position',[-0.1*xwidth,ybox(1)-0.1*ywidth,...
    0.1*xwidth,1.2*ywidth],'FaceColor',[0 0 0],'LineWidth',3)
```

B.6 BASIC COMPLIANT MECHANISMS.m

In this section, the complete code of producing a micro-gripper is made available. Using the predefined discretization, a void region is declared as restrictive region, to allow a gripper mechanism. Using this boundary condition, the design problem of Figure 3-12a can be calculated. Displacement field is plotted on the go.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                       %
% Topology Optimization Using Matlab                                    %
% BASIC_COMPLIANCE.m                                                  %
%                                                                       %
% Delft University of Technology, Department PME                        %
% Master of Science Thesis Project                                      %
%                                                                       %
% Stefan Broxterman                                                     %
%                                                                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
tic                         % start timer
%% DEFINE PARAMETERS
adv = 0;                    % use advanced function [0 = off, 1 = on]
if adv == 0                 % define parameters at behalf of the advanced function
    nx = 120;               % numer of elements horizontal
    ny = 60;                % number of elements vertical
    vol = 0.2;              % volume fraction [0-1]
    pen = 4;                % penalty
    rmin = 1.4;             % filter size
    fil = 1;                % filter method [0 = sensitivity filtering, 1 = density filtering, 2 = heaviside filtering]
    clc; clf; close all; clear X;
end
%% DEFINE SOLUTION METHOD
sol = 0;                    % solution method [0 = oc(sens), 1 = mma]
pcon = 1;                   % use continuation method [0 = off, 1 = on]
%% DEFINE CALCULATION
tol = 0.01;                 % tolerance for convergence criterion [0.01]
move = 0.1;                 % move limit for lagrange [0.2]
pcinc = 1.03;               % penalty continuation increasing factor [1.03]
piter = 20;                 % number of iteration for starting penalty [20]
miter = 1000;               % maximum number of iterations [1000]
sym = 2;                    % symmetry [0 = off, 1 = x-axis, 2 = y-axis]
def = 1;                    % plot deformations [0 = off, 1 = on]
%% DEFINE OUTPUT
draw = 1;                   % plot iterations [0 = off, 1 = on]
dis = 1;                    % display iterations [0 = off, 1 = on]
%% DEFINE MATERIAL
E = 1;                      % young's modulus of solid [1]
Emin = 1e-9;                % young's modulus of void [1e-9]
nu = 0.3;                   % poisson ratio [0.3]
```

```
rho = 0e-3;               % density [0e-3]
g = 9.81;                 % gravitational acceleration [9.81]
Kin = 0.01 ;              % spring stiffness at input force [5e-4]
Kout = 0.01;              % spring stiffness at output force [5e-4]
%% DEFINE FORCE
Uin = 2*(ny+1)-1;         % input force node
Uout = 2*(nx+1)*(ny+1)-round((2/6)*ny)-2; % output force
Fe = [Uin Uout];          % element of force application [Uin Uout]
Fn = [1 2];               % number of applied force locations [1 2]
Fv = [1 -1];              % value of applied force [1 -1]
%% DEFINE SUPPORTS
fix = [1:4 (Uin+1):2*(ny+1):round((5/6)*(Uout+1))]; % create symmetry
%% DEFINE ELEMENT RESTRICTIONS
shap = 1;                 % [0 = no restrictions, 1 = circle, 2 = custom]
area = 0;                 % [0 = no material (passive), 1 = material (active)]
nodr = (round(ny/2)+(0:ny:(nx-1)*ny)); % custom restricted nodes
%% PREPARE FINITE ELEMENT
N = 2*(nx+1)*(ny+1);      % total element nodes
all = 1:2*(nx+1)*(ny+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12]; % fem
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6]; % fem
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4]; % fem
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2]; % fem
Ke = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]); % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node numer matrix
dofvec = reshape(2*nodes(1:end-1,1:end-1)+1,nx*ny,1); % create dof vector
dofmat = repmat(dofvec,1,8)+repmat([0 1 2*ny+[2 3 0 1] -2 -1],nx*ny,1); % create dof matrix
iK = reshape(kron(dofmat,ones(8,1))',64*nx*ny,1); % build sparse i
jK = reshape(kron(dofmat,ones(1,8))',64*nx*ny,1); % build sparse j
%% PREPARE FILTER
iH = ones(nx*ny*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));      % create sparse vector of ones
kH = zeros(size(iH));     % create sparse vector of zeros
m = 0;                    % index for filtering
for i = 1:nx              % for each element calculate distance between ...
    for j = 1:ny          % elements' center for filtering
        r1 = (i-1)*ny+j; % sparse value i
        for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx)  % center of element
            for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % center of element
                r2 = (k-1)*ny+l; % sparse value 2
                m = m+1; % update index for filtering
                iH(m) = r1; % sparse vector for filtering
                jH(m) = r2; % sparse vector for filtering
                kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)); % weight factor
            end
        end
```

```
    end
end
H = sparse(iH,jH,kH);   % build filter
Hs = sum(H,2);          % summation of filter
%% DEFINE ELEMENT RESTRICTIONS
x = repmat(vol,ny,nx);  % initial material distribution
if shap == 0            % no restrictions
    efree = (1:nx*ny)'; % all elements are free
    eres= [];           % no restricted elements
elseif shap == 1        % restrictions
    rest = zeros(ny,nx); % pre-allocate space
    %     for i = 1:nx        % start loop
    %         for j = 1:ny    % for each element
    %             if sqrt((j-ny/2)^2+(i-nx/4)^2) < ny/2.5 % circular restriction
    %                 rest(j,i) = 1; % write restriction
    %                 if rest(j,i) == area % check for restriction
    %                     x(j,i) = area; % store restrictions in material distribution
    %                 end
    %             end
    %         end
    %     end
    for i = round((5/6)*nx):nx
        for j = round((5/6)*ny):ny
            rest(j,i) = area;
            x(j,i) = area;
        end
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;             % set filtered design variables
elseif fil == 2         % heaviside filter
    beta = 1;           % hs filter
    xTilde = x;         % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design space
end
xFree = xF(efree);      % define free design matrix
%% DEFINE STRUCTURAL
Fsiz = size(Fe,2);      % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% DEFINE MMA PARAMETERS
m = 1;                  % number of constraint functions
n = size(xFree(:),1);   % number of variables
xmin = zeros(n,1);      % minimum values of x
xmax = ones(n,1);       % maximum values of x
xold1 = zeros(n,1);     % previous x, to monitor convergence
xold2 = xold1;          % used by mma to monitor convergence
df0dx2 = zeros(n,1);    % second derivative of the objective function
dfdx2 = zeros(1,n);     % second derivative of the constraint function
```

```matlab
low   = xmin;             % lower asymptotes from the previous iteration
upp   = xmax;             % upper asymptotes from the previous iteration
a0 = 1;                   % constant a_0 in mma formulation
a = zeros(m,1);           % constant a_i in mma formulation
cmma = 1e3*ones(m,1);     % constant c_i in mma formulation
d = zeros(m,1);           % constant d_i in mma formulation
subs = 200;               % maximum number of subsolv iterations
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
U = zeros(size(F));       % pre-allocate space displacement
c = zeros(miter,1);       % pre-allocate objective vector
%% INITIALIZE LOOP
iter = 0;                 % initialize loop
diff = 1;                 % initialize convergence criterion
loopbeta = 1;             % initialize beta-loop
%% START LOOP
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    loopbeta = loopbeta +1; % iteration loop for hs filter
    iter = iter+1;        % define iteration
    if pcon == 1          % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;        %... set penalty 1
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
        end
    elseif pcon == 0      % not using continuation method
        p = pen;          % set penalty
    end
    %% Selfweight
    if rho ~= 0           % gravity is involved
        xP=zeros(ny,nx); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny     % for each element, set gravitational...
            Fsw(dofmat(i,2:2:end))=Fsw(dofmat(i,2:2:end))-xF(i)*rho*9.81/4;
        end               % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0       % no gravity
        xP = xF.^p;       % penalized design variable
        Fsw = 0;          % no selfweight
    end
    Ftot = F + Fsw;       % total force
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xP(:)'*(E-Emin)),64*nx*ny,1); % create sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;         % build stiffness matrix
```

```
    K(Uin,Uin) = K(Uin,Uin) + Kin; % add input spring stiffness
    K(Uout,Uout) = K(Uout,Uout) + Kout; % add output spring stiffness
    U(free,:) = K(free,free)\Ftot(free,:); % displacement solving
    c(iter) = 0;          % set compliance to zero
    %% Calculate compliance and sensitivity
    U1 = U(:,1); U2 = U(:,2);
    c0 = reshape(sum((U1(dofmat)*Ke).*U2(dofmat),2),ny,nx);
    c(iter)  = U(Uout,1);
    Sens = p*(E-Emin)*xF.^(p-1).*c0;
    Senc = ones(ny,nx); % set constraint sensitivity
    if fil == 0           % optimality criterion with sensitivity filter
        Sens(:) = H*(x(:).*Sens(:))./Hs./max(1e-3,x(:)); % update filtered sensitivity
    elseif fil == 1       % optimality criterion with density filter
        Sens(:) = H*(Sens(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:)./Hs); % update filtered sensitivity of constraint
    elseif fil == 2       % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:).*dx(:)./Hs); % update filtered sensitivity of constraint
    end
    %% Update design variables Optimality Criterion
    if sol == 0           % use optimality criterion method
        l1 = 0;           % initial lower bound for lagranian mulitplier
        l2 = 1e9;         % initial upper bound for lagranian multiplier
        while (l2-l1)/(l1+l2) > 1e-4 && l2 > 1e-40; % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            xnew = max(0,max(x-move,min(1,min(x+move,x.*(max(1e-10,-Sens./lag)).^0.3)))); % update element densities
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
            if shap == 1 % restriction is on
                xF(rest==1) = area; % set restricted area
            end
            if sum(xF(:)) > vol*nx*ny; % check for optimum
                l1 = lag; % update lower bound to average
            else
                l2 = lag; % update upper bound to average
            end
        end
        %% Method of moving asymptotes
    elseif sol == 1       % use mma solver
        xval = xFree(:); % store current design variable for mma
```

```
        if iter == 1    % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = Sens(efree)*cscale; % store sensitivity for mma
        f = (sum(xF(:))/(vol*nx*ny)-1); % normalized constraint function
        dfdx = Senc(efree)'/(vol*ny*nx); % derivative of the constraint function
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = xFree(:); % previous x, to monitor convergence
        xnew = xF;      % update result
        xnew(efree) = xmma; % include restricted elements
        xnew = reshape(xnew,ny,nx); % reshape xmma vector to original size
        if fil == 0     % sensitivity filter
            xF = xnew;  % update design variables
        elseif fil == 1 % density filter
            xF(:) = (H*xnew(:))./Hs; % update filtered densities result
        elseif fil == 2 % heaviside filter
            xTilde(:)= (H*xnew(:))./Hs; % filtered result
            xF(:)=1-exp(-beta*xTilde)+xTilde*exp(-beta); % update design variable
        end
        if shap == 1    % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end

    end
    xFree = xnew(efree); % set non-restricted area
    diff = max(abs(xnew(:)-x(:))); % difference of maximum element change
    x = xnew;               % update design variable
    if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;     % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
    %% Store results into database X
    X(:,:,iter) = xF;   % each element value x is stored for each iteration
    C(iter) = c(iter);  % each compliance is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    %% Results
    if dis == 1         % display iterations
```

```matlab
        disp([' Iter:' sprintf('%4i',iter) ' Uin:' sprintf('%6.2f',U(Uin)) ...
            ' Uout:' sprintf('%6.2f',c(iter)) ' Con:' sprintf('%6.2f',diff) ' Vol:' sprintf('%6.2f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
    end
    if draw == 1          % plot iterations
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','w','ycolor','w')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        drawnow;
        hold on
        if iter == 1
            axis equal; axis tight;
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            plot(npfx,npfy,'g.','MarkerSize',20)
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
        figure(2)
        if sym ~= 0          % apply symmetry
            if sym == 1      % symmetry around x-axis
```

```
                xFlip = fliplr(xF);
                xFliplot = [xFlip xF];
            end
            if sym == 2      % symmetry around y-axis
                xFlip = flip(xF);
                xFliplot = [xF; xFlip];
            end
            colormap gray
            imagesc(1-xFliplot)
            axis equal
            axis off
        end
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0                  % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Uin:' sprintf('%6.2f',U(Uin)) ...
        ' Uout:' sprintf('%6.2f',c(iter)) ' Con:' sprintf('%6.2f',diff) ' Vol:' sprintf('%6.2f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
end
if draw == 0                 % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','w','ycolor','w')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    %% Plot coloured dots for constraints
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
    %% Plot coloured dots for force application
    for i = 1:length(Fe)
        npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
        nplot = ceil(Fe(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npfy(i) = nplot-0.5;
    end
    plot(npfx,npfy,'g.','MarkerSize',20)
    %% Plot compliance plot
```

```
    if adv == 0
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    end
    figure(2)
    if sym ~= 0          % apply symmetry
        if sym == 1      % symmetry around x-axis
            xFlip = fliplr(xF);
            xFliplot = [xFlip xF];
        end
        if sym == 2      % symmetry around y-axis
            xFlip = flip(xF);
            xFliplot = [xF; xFlip];
        end
        colormap gray
        imagesc(1-xFliplot)
        axis equal
        axis off
    end
end
%% PLOTTING DISPLACEMENT (COMPLIANT MECHANISMS)
if def == 1
    figure(2)
    xaxis = get(gca,'XLim');
    yaxis = get(gca,'YLim');
    figure(3)
    clear mov
    colormap(gray);
    Umov = 1;                 % Start movie counter
    Umax = 0.05;              % Define maximum displacement
    for Udisp = linspace(0,Umax,1); % Vary input displacement
        clf
        for ely = 1:ny       % plot displacements...
            for elx = 1:nx   % for each element...
                if xF(ely,elx) > 0 % exclude white regions for plotting purposes
                    n1 = (ny+1)*(elx-1)+ely;
                    n2 = (ny+1)* elx +ely;
                    Ue = -Udisp*U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
                    ly = ely-1; lx = elx-1;
                    xx = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx]';
```

```
                    yy = [-Ue(2,1)-ly -Ue(4,1)-ly -Ue(6,1)-ly-1 -Ue(8,1)-ly-1]';
                    patch([xx xx],[yy+ny -yy-ny],[-xF(ely,elx) -xF(ely,elx)],'LineStyle','none');
                end
            end
        end
        xlim(xaxis)
        ylim(yaxis-ny)
        drawnow
        mov(Umov) = getframe(3); % movie
        Umov = Umov +1;     % update counter
    end

    movlip = flip(mov);     % create symmetry
    movull = [mov movlip];  % create symmetry
    FileName = ['Compliant_',datestr(now, 'ddmm_HHMMSS'),'.avi']; % dynamic filename
    movie2avi(movull, FileName, 'compression', 'None', 'FPS', 10); % save video
end
toc                         % stop timer
```

B.7 Design of Supports.m

In this section, the complete code of producing bridge examples is available. A distributed vertical force at the top, and a user-friendly configuration interface can be used to calculate design of support, including a pre-defined cost distribution. The produced picture in Figure 4-4 can be made immediately by running this code.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                         %
% Topology Optimization Using Matlab                                      %
% BRIDGE.m                                                                %
%                                                                         %
% Delft University of Technology, Department PME                          %
% Master of Science Thesis Project                                        %
%                                                                         %
% Stefan Broxterman                                                       %
%                                                                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
tic                         % start timer
%% DEFINE PARAMETERS
adv = 0;                    % use advanced function [0 = off, 1 = on]
if adv == 0                 % define parameters at behalf of the advanced function
    nx = 80;                % numer of elements horizontal
    ny = 40;                % number of elements vertical
    vol = 0.2;              % volume fraction [0-1]
    pen = 3;                % penalty
    rmin = 1.5;             % filter size
    fil = 1;                % filter method [0 = sensitivity filtering, 1 = density filtering, 2 = heaviside filtering]
    clc; clf; close all; clear X; clear Z; % clear workspace
end
%% DEFINE SOLUTION METHOD
sol = 1;                    % solution method [0 = oc(sens), 1 = mma]
pcon = 0;                   % use continuation method [0 = off, 1 = on]
%% DEFINE CALCULATION
tol = 0.01;                 % tolerance for convergence criterion [0.01]
move = 0.2;                 % move limit for lagrange [0.2]
pcinc = 1.03;               % penalty continuation increasing factor [1.03]
piter = 20;                 % number of iteration for starting penalty [20]
miter = 1000;               % maximum number of iterations [1000]
plotiter = 5;               % gap of iterations used to plot or draw iterations [5]
def = 0;                    % plot deformations [0 = off, 1 = on]
zplot = 0.99;               % define treshold plotting supports [0.99]
%% DEFINE OUTPUT
draw = 2;                   % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 2;                    % display iterations [0 = off, 1 = on, 2 = partial]
%% DEFINE MATERIAL
```

```
E = 1;                    % young's modulus of solid [1]
Emin = 1e-9;              % young's modulus of void [1e-9]
nu = 0.3;                 % poisson ratio [0.3]
rho = 0e-3;               % density [0e-3]
g = 9.81;                 % gravitational acceleration [9.81]
%% DEFINE FORCE
Fe = 2:2*(ny+1):2*(ny+1)*(nx+1); % element of force application [2:2*(ny+1):2*(ny+1)*(nx+1)]
Fn = 1;                   % number of applied force locations [1]
Fv = -1;                  % value of applied force [-1]
%% DEFINE SUPPORTS
fix = [1:2 2*(ny+1)*nx+(1:2)]; % define fixed locations [1:2 2*(ny+1)*nx+(1:2)]
%% DEFINE DESIGN OF SUPPORTS
supp = [1:ny (1:ny)+(nx-1)*ny ny:ny:nx*ny]; % support area [1:ny (1:ny)+(nx-1)*ny ny:ny:nx*ny]
supp = unique(supp);      % create unique support area
zvol = 0.2;               % maximum support area [0.2]
cost = 1;                 % set maximum cost of supports [1]
k0 = 0.01;                % spring stiffness for support stiffness [0.01]
q = 5;                    % penalty for support design [3]
zmin = 1e-4;              % minimum support design variable [1e-4]
dist = 2;                 % cost distribution [0 = off, 1 = x-distributed, 2 = y-distribution]
%% DEFINE ELEMENT RESTRICTIONS
shap = 2;                 % [0 = no restrictions, 1 = circle, 2 = custom]
area = 1;                 % [0 = no material (passive), 1 = material (active)]
nodr = [1:ny:nx*ny 2:ny:nx*ny]; % custom restricted nodes [1:ny:nx*ny]
%% PREPARE FINITE ELEMENT
N = 2*(nx+1)*(ny+1);      % total element nodes
all = 1:2*(nx+1)*(ny+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12]; % fem
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6]; % fem
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4]; % fem
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2]; % fem
Ke = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]); % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node numer matrix
dofvec = reshape(2*nodes(1:end-1,1:end-1)+1,nx*ny,1); % create dof vector
dofmat = repmat(dofvec,1,8)+repmat([0 1 2*ny+[2 3 0 1] -2 -1],nx*ny,1); % create dof matrix
iK = reshape(kron(dofmat,ones(8,1))',64*nx*ny,1); % build sparse i
jK = reshape(kron(dofmat,ones(1,8))',64*nx*ny,1); % build sparse j
%% PREPARE FILTER
iH = ones(nx*ny*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));      % create sparse vector of ones
kH = zeros(size(iH));     % create sparse vector of zeros
m = 0;                    % index for filtering
for i = 1:nx              % for each element calculate distance between ...
    for j = 1:ny          % elements' center for filtering
        r1 = (i-1)*ny+j; % sparse value i
```

```
        for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx)  % center of element
            for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % center of element
                r2 = (k-1)*ny+l; % sparse value 2
                m = m+1; % update index for filtering
                iH(m) = r1; % sparse vector for filtering
                jH(m) = r2; % sparse vector for filtering
                kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)); % weight factor
            end
        end
    end
end
H = sparse(iH,jH,kH);   % build filter
Hs = sum(H,2);          % summation of filter
%% DEFINE ELEMENT RESTRICTIONS
x = repmat(vol,ny,nx);  % initial material distribution
if shap == 0            % no restrictions
    efree = (1:nx*ny)'; % all elements are free
    eres= [];           % no restricted elements
elseif shap == 1        % circular restrictions
    rest = zeros(ny,nx); % pre-allocate space
    for i = 1:nx        % start loop
        for j = 1:ny    % for each element
            if sqrt((j-ny/2)^2+(i-nx/4)^2) < ny/2.5 % circular restriction
                rest(j,i) = 1; % write restriction
                if rest(j,i) == area % check for restriction
                    x(j,i) = area; % store restrictions in material distribution
                end
            end
        end
    end
elseif shap == 2        % custom restrictions
    rest = zeros(ny*nx,1); % pre-allocate space
    for i = 1:length(nodr) % write restriction
        resti = nodr(i); % write restriction
        rest(resti) = 1; % write restriction
    end
    rest = reshape(rest,ny,nx);
    for i = 1:nx            % start loop
        for j = 1:ny        % for each element
            if rest(j,i) == area % check for restriction
                x(j,i) = area; % store restrictions in material distribution
            end
        end
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
```

```
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;                % set filtered design variables
elseif fil == 2            % heaviside filter
    beta = 1;              % hs filter
    xTilde = x;            % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design space
end
xFree = xF(efree);         % define free design matrix
%% DEFINE STRUCTURAL
Fsiz = size(Fe,2);         % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% DESIGN OF SUPPORT DISTRIBUTION
xsiz = size(xFree(:),1); % size of design variables
zsiz = size(supp,2);       % size of support design variables
xzer = zeros(xsiz,1);      % empty row of zeros for mma usage
zzer = zeros(zsiz,1);      % empty row of zeros for mma usage
z = zeros(ny,nx);          % create design of support domain
z(supp) = 1; %zvol;            % plugin initial support design variables
zval = z';                 % create vector of design variables
Si = 1;                    % counter
if dist == 1               % x-axis cost distribution
    Scos = [linspace(1,cost,nx/2) linspace(cost,1,nx/2)]; % x-axis cost distribution
    Scost = zeros(nx,nx); % create multiplication matrix
    for i = 1:nx           % create weighted cost matrix
        Scost(Si,i) = Scos(i); % plug-in cost values
        Si = Si+1;         % update counter
    end
elseif dist == 2           % y-axis cost distribution
    Scos = [linspace(cost,1,ny/2) linspace(1,cost,ny/2)]; % y-axis cost distribution
    Scost = zeros(ny,ny); % create multiplication matrix
    for i = 1:ny           % create weighted cost matrix
        Scost(Si,i) = Scos(i); % plug-in cost values
        Si = Si+1;         % update counter
    end
end
Adofsup = dofmat(supp,:); % degrees of freedom for support locations
Asup = unique(Adofsup(:)); % unique support locations
zF = z;                    % set design of support
zval = zval(zval ~= 0); % create configurable design of support vector
k1 = k0*eye(8);            % reshape scalar to diagonal matrix
%% DEFINE MMA PARAMETERS
m = 2;                     % number of constraint functions
n = xsiz+zsiz;             % number of variables
xmin = [1e-4*ones(xsiz,1);zmin*ones(zsiz,1)]; % minimum values of x
xmax = ones(n,1);          % maximum values of x
xold1 = zeros(n,1);        % previous x, to monitor convergence
xold2 = xold1;             % used by mma to monitor convergence
df0dx2 = zeros(n,1);       % second derivative of the objective function
dfdx2 = zeros(m,n);        % second derivative of the constraint function
low   = xmin;              % lower asymptotes from the previous iteration
```

```matlab
upp   = xmax;           % upper asymptotes from the previous iteration
a0 = 1;                 % constant a_0 in mma formulation [1]
a = zeros(m,1);         % constant a_i in mma formulation
cmma = 1e3*ones(m,1);   % constant c_i in mma formulation
d = zeros(m,1);         % constant d_i in mma formulation
subs = 200;             % maximum number of subsolv iterations [200]
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
npdx = zeros(length(nodes),1)'; % pre-allocate force dots
npdy = zeros(length(nodes),1)'; % pre-allocate force dots
U = zeros(size(F));     % pre-allocate space displacement
c = zeros(miter,1);     % pre-allocate objective vector
%% INITIALIZE LOOP
iter = 0;               % initialize loop
diff = 1;               % initialize convergence criterion
loopbeta = 1;           % initialize beta-loop
%% START LOOP
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    loopbeta = loopbeta +1; % iteration loop for hs filter
    iter = iter+1;      % define iteration
    if pcon == 1        % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;      %... set penalty 1
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
        end
    elseif pcon == 0    % not using continuation method
        p = pen;        % set penalty
    end
    %% Selfweight
    if rho ~= 0         % gravity is involved
        xP=zeros(ny,nx); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny   % for each element, set gravitational...
            Fsw(dofmat(i,2:2:end))=Fsw(dofmat(i,2:2:end))-xF(i)*rho*9.81/4;
        end             % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0     % no gravity
        xP = xF.^p;     % penalized design variable
        Fsw = 0;        % no selfweight
    end
    Ftot = F + Fsw;     % total force
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xP(:)'*(E-Emin)),64*nx*ny,1); % create sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
```

```
    K = (K+K')/2;        % build stiffness matrix
    Kfvec = zeros(2*(ny+1)*(nx+1),1); % build zeros support vector
    for i = 1:length(supp) % for each support element...
        dofsup = dofmat(supp(i),:); %...find the corresponding dof
        for j = 1:length(dofsup) % calculate new stiffness vector
            Kfvec(dofsup(j)) = Kfvec(dofsup(j))+(zF(supp(i))^q)*k0;
        end
    end
    Kf = spdiags(Kfvec,0,2*(ny+1)*(nx+1),2*(ny+1)*(nx+1)); % create diagonal Kf
    Kt = K+Kf;           % update total force
    U(free,:) = Kt(free,free)\Ftot(free,:); % displacement solving
    c(iter) = 0;         % set compliance to zero
    comp(iter) = 0;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%TEMP%%%
    Sens = 0;            % set sensitivity to zero
    Senz = 0;            % set constraint sensitivity to zero
    %% Calculate compliance and sensitivity
    for i = 1:size(Fn,2) % for number of load cases
        Ui = U(:,i);     % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        cz0 = reshape(sum((Ui(dofmat)*k1).*Ui(dofmat),2),ny,nx); % initial support compliance
        c(iter) = c(iter) + sum(sum((Emin+xF.^p*(E-Emin)).*c0)) + sum(sum((zF.^q).*cz0)); % calculate compliance
        comp(iter) = comp(iter) + sum(sum((Emin+xF.^p*(E-Emin)).*c0));%%%%%TEMP%%%
        Sens = Sens + reshape(2*Ui(dofmat)*repmat([0;-9.81*rho/4],4,1),ny,nx) -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
        Senz = Senz + -q*zF.^(q-1).*cz0; % calculate sensitivity to support variable
    end
    Senc = ones(ny,nx); % set constraint sensitivity
    if dist == 0
        Sencz = ones(ny,nx);
    elseif dist == 1
        Sencz =ones(ny,nx)*Scost; % set weighted cost constraint sensitivity
    elseif dist == 2
        Sencz = Scost*ones(ny,nx); % set weighted cost constraint sensitivity
        %Sencz = ones(ny,nx); % set weighted cost constraint sensitivity
    end
    if fil == 0          % optimality criterion with sensitivity filter
        Sens(:) = H*(x(:).*Sens(:))./Hs./max(1e-3,x(:)); % update filtered sensitivity
    elseif fil == 1      % optimality criterion with density filter
        Sens(:) = H*(Sens(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:)./Hs); % update filtered sensitivity of constraint
    elseif fil == 2      % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
```

```
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:).*dx(:)./Hs); % update filtered sensitivity of constraint
    end
    %% Update design variables Optimality Criterion
    if sol == 0          % use optimality criterion method
        l1 = 0;          % initial lower bound for lagranian mulitplier
        l2 = 1e9;        % initial upper bound for lagranian multiplier
        while (l2-l1)/(l1+l2) > 1e-3; % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
            if shap == 1 % restriction is on
                xF(rest==1) = area; % set restricted area
            end
            if sum(xF(:)) > vol*nx*ny; % check for optimum
                l1 = lag; % update lower bound to average
            else
                l2 = lag; % update upper bound to average
            end
        end
        %% Method of moving asymptotes
    elseif sol == 1      % use mma solver
        xval = [xFree(:);zval(:)]; % store current design variable for mma
        if iter == 1     % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
            cscale = 5.0131e-6;
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = [Sens(efree)*cscale; Senz(supp)'*cscale]; % store sensitivity for mma
        if dist == 0     % no cost distribution
            Scosts = zF; % cost-funcion no influence
        elseif dist == 1 % x-axis cost distribution
            Scosts = zF*Scost; % update weighted constraint function
        elseif dist == 2 % y-axis cost distribution
            Scosts = Scost*zF; % update weighted constraint function
        end
        f = [(sum(xF(:))/(vol*nx*ny)-1);(sum(Scosts(supp))/(zvol*size(supp,2))-1)]; % normalized constraint function
```

```
        dfdx = [Senc(efree)'/(vol*ny*nx) zzer'; xzer' Sencz(supp)/(zvol*size(supp,2))]; % derivative of the constraint function
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = [xFree(:);zval(:)]; % previous x, to monitor convergence
        xnew = xF;      % update result
        xnew(efree) = xmma(1:xsiz); % include restricted elements
        znew = zF;      % update design result
        znew(supp) = xmma(xsiz+1:end); % include mma solved supports
        xnew = reshape(xnew,ny,nx); % reshape xmma vector to original size
        znew = reshape(znew,ny,nx); % reshape support vector to original size
        if fil == 0     % sensitivity filter
            xF = xnew;  % update design variables
        elseif fil == 1 % density filter
            xF(:) = (H*xnew(:))./Hs; % update filtered densities result
        elseif fil == 2 % heaviside filter
            xTilde(:)= (H*xnew(:))./Hs; % filtered result
            xF(:)=1-exp(-beta*xTilde)+xTilde*exp(-beta); % update design variable
        end
        if shap == 1 || shap == 2   % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end
        zF(:) = znew(:); % update support variables
        zval = znew(supp); % update support variables
    end
    xFree = xnew(efree); % set non-restricted area
    diff = max(abs(xnew(:)-x(:))); % difference of maximum element change
    x = xnew;              % update design variable
    z = znew;              % update support design variable
    if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;  % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
    %% Store results into database X
    X(:,:,iter) = xF;   % each element value x is stored for each iteration
    C(iter) = c(iter);  % each compliance is stored for each iteration
    Z(:,:,iter) = zF;   % each support variable is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    assignin('base', 'Z', Z); % each iteration (3rd dimension).
    %% Results
```

```
    if dis == 1          % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff) ' ZVol:' sprintf('%6.3f',mean(Scosts(supp)))]);
    elseif dis == 2      % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
                disiter = plotiter;
            elseif iter == disiter
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
                ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff) ' ZVol:' sprintf('%6.3f',mean(Scosts(supp)))]);
        end
    end
    if draw == 1         % plot iterations
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        axis equal; axis tight
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            plot(npfx,npfy,'g.','MarkerSize',20)
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for design of supports
```

```
        for i = 1:nx*ny
            if zF(i) > zplot % treshold for plotting supports
                if ceil(i/ny) == nx
                    npdx(i) = ceil(i/ny) + 0.5;
                elseif ceil(i/ny) == 1
                    npdx(i) = ceil(i/ny) -0.5;
                else
                    npdx(i) = ceil(i/ny);
                end
                nplot = i;
                while nplot > ny
                    nplot = nplot-ny;
                end
                if nplot == ny
                    npdy(i) = nplot+0.5;
                elseif nplot == 1
                    npdy(i) = nplot-0.5;
                else
                    npdy(i) = nplot;
                end
            end
        end
        if iter > 1
            delete(Dos)
        end
        if exist('npdx') %#ok<EXIST>
            Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize',20);
            clear npdx; clear npdy;
            uistack(Dos,'bottom')
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        set(gca,'YTick',[],'YTicklabel',[])
        xlabel('Iterations')
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    elseif draw == 2    % plot parts of iterations
        if iter == 1 || iter == drawiter
            if iter == 1
                drawiter = plotiter;
            elseif iter == drawiter
                drawiter = drawiter + plotiter;
            end
```

```
figure(1)
subplot(2,1,1)
colormap(gray); imagesc(1-xF);
set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
    'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
axis equal; axis tight
drawnow;
hold on
if iter == 1
    % Plot coloured dots for force application
    for i = 1:length(Fe)
        npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
        nplot = ceil(Fe(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npfy(i) = nplot-0.5;
    end
    plot(npfx,npfy,'g.','MarkerSize',20)
    % Plot coloured dots for constraints
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
end
% Plot coloured dots for design of supports
for i = 1:nx*ny
    if zF(i) > zplot % treshold for plotting supports
        if ceil(i/ny) == nx
            npdx(i) = ceil(i/ny) + 0.5;
        elseif ceil(i/ny) == 1
            npdx(i) = ceil(i/ny) -0.5;
        else
            npdx(i) = ceil(i/ny);
        end
        nplot = i;
        while nplot > ny
            nplot = nplot-ny;
        end
        if nplot == ny
            npdy(i) = nplot+0.5;
        elseif nplot == 1
            npdy(i) = nplot-0.5;
        else
            npdy(i) = nplot;
        end
```

```
                end
            end
            if iter > 1
                delete(Dos)
            end
            if exist('npdx') %#ok<EXIST>
                Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize',20);
                clear npdx; clear npdy;
                uistack(Dos,'bottom')
            end
            % Plot compliance plot
            figure(1)
            subplot(2,1,2)
            plot(c(1:iter))
            set(gca,'YTick',[],'YTicklabel',[])
            xlabel('Iterations')
            ylabel('Compliance')
            xaxmax = c(iter);
            yaxmax = max(c);
            yaxmin = min(c(1:iter));
            if pcon == 0
                yaxmax = mean([yaxmin yaxmax]);
            end
            ylim([0.95*yaxmin yaxmax])
            xlim([1 min(iter+10,miter)])
        end
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0 || dis == 2 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
        ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff) ' ZVol:' sprintf('%6.3f',mean(Scosts(supp)))]);
end
if draw == 0 || draw == 2 % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    % Plot coloured dots for force application
    for i = 1:length(Fe)
        npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
        nplot = ceil(Fe(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
```

```
            end
            npfy(i) = nplot-0.5;
        end
        For = plot(npfx,npfy,'g.','MarkerSize',20);
        uistack(For,'bottom')
        % Plot coloured dots for constraints
        for i = 1:length(fix)
            npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
            nplot = ceil(fix(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npy(i) = nplot-0.5;
        end
        plot(npx,npy,'r.','MarkerSize',20)
        % Plot coloured dots for design of supports
        for i = 1:nx*ny
            if zF(i) > zplot % treshold for plotting supports
                if ceil(i/ny) == nx
                    npdx(i) = ceil(i/ny) + 0.5;
                elseif ceil(i/ny) == 1
                    npdx(i) = ceil(i/ny) -0.5;
                else
                    npdx(i) = ceil(i/ny);
                end
                nplot = i;
                while nplot > ny
                    nplot = nplot-ny;
                end
                if nplot == ny
                    npdy(i) = nplot+0.5;
                elseif nplot == 1
                    npdy(i) = nplot-0.5;
                else
                    npdy(i) = nplot;
                end
            end
        end
        if exist('Dos(1)') %#ok<EXIST>
            delete(Dos(1))
        end
        if exist('npdx') %#ok<EXIST>
            Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize',20);
            clear npdx; clear npdy;
            uistack(Dos,'bottom')
        end
        % Plot compliance plot
        if adv == 0
            figure(1)
            subplot(2,1,2)
            plot(c(1:iter))
            set(gca,'YTick',[],'YTicklabel',[])
            xlabel('Iterations')
```

```
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    end
end
%% PLOTTING DISPLACEMENT (COMPLIANT MECHANISMS)
if def == 1
    figure(1)
    subplot(2,1,1)
    xaxis = get(gca,'XLim');
    yaxis = get(gca,'YLim');
    figure(3)
    clear mov
    colormap(gray);
    Umov = 1;             % start movie counter
    Umax = -0.005;          % define maximum displacement
    for Udisp = linspace(0,Umax,10); % vary input displacement
        clf
        for ely = 1:ny % plot displacements...
            for elx = 1:nx % for each element...
                if xF(ely,elx) > 0 % exclude white regions for plotting purposes
                    n1 = (ny+1)*(elx-1)+ely;
                    n2 = (ny+1)* elx +ely;
                    Ue = Udisp*U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
                    ly = ely-1; lx = elx-1;
                    xx = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx]';
                    yy = [-Ue(2,1)-ly -Ue(4,1)-ly -Ue(6,1)-ly-1 -Ue(8,1)-ly-1]';
                    subplot(2,1,1)
                    patch([xx xx],[yy yy],[-xF(ely,elx) -xF(ely,elx)],'LineStyle','none');

                end
            end
        end
        xlim(xaxis)
        ylim([-yaxis(2) yaxis(1)])
        axis equal; axis tight;
        set(gca,'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
        drawnow
        mov(Umov) = getframe(3); % movie
        Umov = Umov +1; % update counter
    end
    movlip = flip(mov); % create symmetry
```

```
    movull = [mov movlip]; % create symmetry
    FileName = ['Compliant_',datestr(now, 'ddmm_HHMMSS'),'.avi']; % dynamic filename
    movie2avi(movull, FileName, 'compression', 'None', 'FPS', 10); % save video
end
toc                      % stop timer
    max(K(:))
    max(Kf(:))
```

B.8 ADVANCED DOS.m

By the inspiration of the ADVANCED (B.2) and the Design of Supports plug-in (C.9) a complete advanced and enhanced code is made. This code includes displaying support design and can be used to easily vary in cost distribution functions, in order to produce the figures as depicted in 4.3. The changes are quite big, so it's recommended to just run this new file, instead of writing an add-in code.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                      %
% Topology Optimization Using Matlab                                   %
% ADVANCED_DOS.m                                                       %
%                                                                      %
% Delft University of Technology, Department PME                       %
% Master of Science Thesis Project                                     %
%                                                                      %
% Stefan Broxterman                                                    %
%                                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; clf; close all; clear X;
%% DEFINE OPTIMIZATION VARIABLES
var = 5;                    % [1 = mesh, 2 = penalty, 3 = filter radius, 4 = volume fraction, 5 = support cost, 6 = evolution]
nxvec = [30,60,80,120]; % horizontal elements vector
nyvec = [10,20,40,40];  % vertical elements vector
volvec = [0.2 0.35 0.5 0.65]; % volume fraction vector
rminvec = [1,1.25,1.5,3]; % filter size vector
penvec = [1, 2, 3, 5];  % penalty vector
filvec = [0, 1, 2];     % filter vector
costvec = [1, 5, 10, 50]; % cost vector
evolvec = [0.05, 0.25, 0.5, 1]; % evolution fraction vector
%% SET DEFAULT VALUES
nx = nxvec(3);              % default number of horizontal elements
ny = nyvec(3);              % default number of vertical elements
vol = volvec(1);            % default number of volume fraction
pen = penvec(3);            % default penalty
rmin = rminvec(3);          % default filter radius
fil= filvec(2);             % default filter method
cost = costvec;             % default cost distribution
%% SET OPTIMIZATION VALUES
ex = [30,60,90,120];        % vector size for pre-allocating space
figend = 4;                 % set total of varying values
label = ['a','b','c','d','e']; % graphic label
%% PRE-ALLOCATE SPACE
loops = zeros(1,size(ex,2)); % initial loops matrix
obj = zeros(1,size(ex,2)); % initial ojective matrix
t = zeros(1,size(ex,2)); % initial time matrix
Y = zeros(size(ex,2),5); % initial results matrix
if var == 6                 % for evolution scheme, BasicK.m only needs to ...
    BRIDGE                  % run one time only
```

```
end
%% START LOOP
for fig = 1:figend      % start itertation loop
    tic;                % start timer
    if var ~= 6         % for non-evolution scheme, run below
        clear X; clear C; % clear results matrix for each run
        if var == 1         % differentiation on number of elements
            nx = nxvec(fig); % pick each horizontal value
            ny = nyvec(fig); % pick each vertical value
        elseif var == 2     % differentiation on penalty
            pen = penvec(fig); % pick each penalty
        elseif var == 3     % differentiation on filter radius
            rmin = rminvec(fig); % pick each rmin
        elseif var == 4     % differentiation on filter method
            vol = volvec(fig); % pick each filter method
         elseif var == 5    % differentiation on support cost
            cost = costvec(fig); % pick each support cost
        end
        figure(1)
        clf
        BRIDGE              % run Basic.m
        loops(fig) = size(X,3); % number of iterations used
        obj(fig) = c(iter); % store objective function
        prog = X(:,:,loops(fig)); % store densities for progression drawing
    elseif var == 6     % store compliance for evolution vector
        loops = size(X,3); % for evolutionary scheme, calculate rounded ...
        loop(1) = round(evolvec(1)*loops); % values of loops and store ...
        loop(2) = round(evolvec(2)*loops); % this loop number
        loop(3) = round(evolvec(3)*loops);
        loop(4) = round(evolvec(4)*loops);
        prog = X(:,:,loop); % progression picture for each evolution fraction
    end
    %% Set graphics
    if draw == 1        % check for drawing
        H = get(gcf,'Position'); % get position of figure
    else
        H = [680,558,560,420]; % set size of figure(2) plot windows
    end
    H2 = figure(2);     % plot window for progression pictures
    set(H2,'position',[H(1)+H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
    %% Draw progression plots
    subplot(3,2,fig+2)  % plot each differentiation
    colormap(gray);     % grayscale
    if var == 6         % evolution needs different plotting
        imagesc(1-prog(:,:,fig)); % plot progression picture
        xlabel(sprintf('c = %.2f',C(loop(fig))),'color','k')
    else
        imagesc(1-prog); % plot progression picture
```

```
        xlabel(sprintf('c = %.2f',obj(fig)),'color','k')
    end
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
    axis equal; axis tight; % set additional options
    if var == 6          % evolution needs different plotting
        xlabel(sprintf('c = %.2f',C(loop(fig))),'color','k')
    else
        xlabel(sprintf('c = %.2f',obj(fig)),'color','k')
    end
    ylabel(sprintf('%s)      ',(label(fig+1))) ,...
        'rot',0,'color','k','FontSize',11)
    hold on
    % Plot coloured dots for design of supports
    for i = 1:nx*ny
        if zF(i) > zplot % treshold for plotting supports
            if ceil(i/ny) == nx
                npdx(i) = ceil(i/ny) + 0.5;
            elseif ceil(i/ny) == 1
                npdx(i) = ceil(i/ny) -0.5;
            else
                npdx(i) = ceil(i/ny);
            end
            nplot = i;
            while nplot > ny
                nplot = nplot-ny;
            end
            if nplot == ny
                npdy(i) = nplot+0.5;
            elseif nplot == 1
                npdy(i) = nplot-0.5;
            else
                npdy(i) = nplot;
            end
        end
    end
    if exist('npdx') %#ok<EXIST>
        Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize',20);
        clear npdx; clear npdy;
        uistack(Dos,'bottom')
    end
    %% Store compliance
    if var ~= 6           % store compliance for further plotting
        if fig == 1
            C1 = C;
        elseif fig == 2
            C2 = C;
        elseif fig == 3
            C3 = C;
        elseif fig == 4
            C4 = C;
        end
```

```matlab
    end
    %% Draw graphics
    xbox = get(gca,'XLim');
    ybox = get(gca,'YLim');
    xwidth = xbox(2)-xbox(1);
    ywidth = ybox(2)-ybox(1);
    rectangle('Position',[xbox(1),ybox(1),xwidth,ywidth],...
        'EdgeColor',[0.5 0.5 0.5],'LineStyle',':');  drawnow;
    t(fig) = toc;

    %% Output
    if var ~= 6  % output results for non-evolutionary schemes
        Y(fig,:) = [fig ex(fig) loops(fig) obj(fig) t(fig)];
        if fig == figend
            Y
        end;
    end
    %% Compliance graphs
    if var ~= 6
        H3 = figure(3);
        set(H3,'position',[H(1)-H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
        hold on
        switch fig
            case 1      % first variable
                plot(1:length(C1),C1,'b:','LineWidth',2)
                xaxmax = mean(length(C1));
                yaxmax = max(max(C1));
                yaxmin = min(C1);
                if var == 1
                    legend(sprintf('mesh = %g x %g ',nxvec(1),nyvec(1)))
                elseif var == 2
                    legend(sprintf('pen = %g',penvec(1)))
                elseif var == 3
                    legend(sprintf('Rmin = %g',rminvec(1)))
                elseif var == 4
                    legend(sprintf('vol = %g',volvec(1)))
                elseif var == 5
                    legend(sprintf('cost = %g',costvec(1)))
                end
            case 2      % second variable
                plot(1:length(C2),C2,'r--','LineWidth',2)
                xaxmax = mean([length(C1) length(C2)]);
                yaxmax = max([max(C1) max(C2)]);
                yaxmin = min(min([C1 C2]));
                if var == 1
                    legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)), sprintf('mesh = %g x %g',nxvec(2),nyvec(2)))
                elseif var == 2
                    legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)))
                elseif var == 3
```

```
                        legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)))
                    elseif var == 4
                        legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)))
                    elseif var == 5
                        legend(sprintf('cost = %g ',costvec(1)),sprintf('cost = %g',costvec(2)))
                    end
                case 3       % third variable
                    plot(1:length(C3),C3,'k','LineWidth',2)
                    xaxmax = mean([length(C1) length(C2) length(C3)]);
                    yaxmax = max([max(C1) max(C2) max(C3)]);
                    yaxmin = min(min([C1 C2 C3]));
                    if var == 1
                        legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)), sprintf('mesh = %g x %g',nxvec(2),nyvec(2)), sprintf('mesh = %g x %g',nxvec(3),nyvec(3)))
                    elseif var == 2
                        legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)),sprintf('pen = %g',penvec(3)))
                    elseif var == 3
                        legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)),sprintf('Rmin = %g',rminvec(3)))
                    elseif var == 4
                        legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)),sprintf('vol = %g',volvec(3)))
                    elseif var == 5
                        legend(sprintf('cost = %g ',costvec(1)),sprintf('cost = %g',costvec(2)),sprintf('cost = %g',costvec(3)))
                    end
                case 4       % fourth variable
                    plot(1:length(C4),C4,'g-.','LineWidth',2)
                    xaxmax = mean([length(C1) length(C2) length(C3) length(C4)]);
                    yaxmax = max([max(C1) max(C2) max(C3) max(C4)]);
                    yaxmin = min(min([C1 C2 C3 C4]));
                    if var == 1
                        legend(sprintf('mesh = %g x %g',nxvec(1),nyvec(2)), sprintf('mesh = %g x %g',nxvec(2),nyvec(2)), sprintf('mesh = %g x %g',nxvec(3),nyvec(3)), sprintf('mesh = %g x %g',nxvec(4),nyvec(4)))
                    elseif var == 2
                        legend(sprintf('pen = %g ',penvec(1)),sprintf('pen = %g',penvec(2)),sprintf('pen = %g',penvec(3)), sprintf('pen = %g',penvec(4)))
                    elseif var == 3
                        legend(sprintf('Rmin = %g ',rminvec(1)),sprintf('Rmin = %g',rminvec(2)),sprintf('Rmin = %g',rminvec(3)),sprintf('Rmin = %g',rminvec(4)))
                    elseif var == 4
```

```
                    legend(sprintf('vol = %g ',volvec(1)),sprintf('vol = %g',volvec(2)),sprintf('vol = %g',volvec(3)),sprintf('vol = %g',volvec(4)))
                elseif var == 5
                    legend(sprintf('cost = %g ',costvec(1)),sprintf('cost = %g',costvec(2)),sprintf('cost = %g',costvec(3)),sprintf('cost = %g',costvec(4)))
                end
        end
        xlabel('Number of iterations')
        ylabel('Compliance')
        if exist('pcon','var') == 0
            yaxmax = mean([yaxmin yaxmax]);
        elseif pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        axis([0 xaxmax 0.95*yaxmin yaxmax])
    elseif var == 6
        H3 = figure(3);
        set(H3,'position',[H(1)-H(3) H(2) H(3) H(4)]); % place figure(2) next to (1)
        hold on
        plot(C)
        xlabel('Number of iterations')
        ylabel('Compliance')
        axis([0 length(C) 0.9*min(C) max(C)])
    end
end
%% STORE RESULTS
disp('Y = i, penalty, loops, objective, time')
if var == 1                 % mesh refinement
    Ymesh = Y;              % store result matrix
    save('MeshRefinementY.mat','Y');
elseif var == 2             % penalty
    Ypenal = Y;             % store result matrix
    save('PenaltyY.mat','Y');
elseif var == 3             % filter radius
    Yfilter = Y;            % store result matrix
    save('FilterY.mat','Y');
elseif var == 4            % filter radius
    Yvolume = Y;            % store result matrix
    save('VolumeY.mat','Y');
end
%% DRAW DESIGN PROBLEM
figure(2)
subplot(3,2,(1:2))          % plot the initial mechanical problem
rectangle('Position',[xbox(1),ybox(1),xwidth,ywidth],...
    'FaceColor',[0.5 0.5 0.5])
axis equal; axis tight;
set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
    'YTicklabel',[],'xcolor','w','ycolor','w')
ylabel(sprintf('%s)     ',(label(1))) ,'rot',0,'color','k','FontSize',11)
draw_arrow([xbox(2) ybox(1)],[xbox(2) -0.25*ywidth],1)
```

```
rectangle('Position',[-0.1*xwidth,ybox(1)-0.1*ywidth,...
    0.1*xwidth,1.2*ywidth],'FaceColor',[0 0 0],'LineWidth',3)
```

B.9 Design of Actuator Placement.m

In this section, the complete code of designing optimal actuator placement is available. Here, topology is not yet involved and remains fixed. By running this code, the produced picture in Figure 5-2 can be made immediately.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                      %
% Topology Optimization Using Matlab                                   %
% Design of Actuator Placement                                         %
%                                                                      %
% Delft University of Technology, Department PME                       %
% Master of Science Thesis Project                                     %
%                                                                      %
% Stefan Broxterman                                                    %
%                                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
tic                     % start timer
%% DEFINE PARAMETERS
adv = 0;                % use advanced function [0 = off, 1 = on]
if adv == 0             % define parameters at behalf of the advanced function
    nx = 90;            % numer of elements horizontal
    ny = 30;            % number of elements vertical
    vol = 1;            % volume fraction [0-1]
    pen = 3;            % penalty
    rmin = 1.5;         % filter size
    fil = 1;            % filter method [0 = sensitivity filtering, 1 = density filtering, 2 = heaviside filtering]
    clc; clf; close all; clear X; clear W; % clear workspace
end
%% DEFINE SOLUTION METHOD
sol = 1;                % solution method [0 = oc(sens), 1 = mma]
pcon = 1;               % use continuation method [0 = off, 1 = on]
fincheck = 1;           % finite difference check [0 = off, 1 = on, 2 = break]
%% DEFINE CALCULATION
tol = 0.001;            % tolerance for convergence criterion [0.01]
move = 0.2;             % move limit for lagrange [0.2]
pcinc = 1.03;           % penalty continuation increasing factor [1.03]
piter = 20;             % number of iteration for starting penalty [20]
miter = 1000;           % maximum number of iterations [1000]
plotiter = 5;           % gap of iterations used to plot or draw iterations [5]
def = 0;                % plot deformations [0 = off, 1 = on, 2 = play video]
wplot = 0.20;           % define treshold factor of Fmax for force plot [0.20]
h = 1e-6;               % perturbation value for finite difference method [1e-6]
%% DEFINE OUTPUT
```

```
draw = 0;                   % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 0;                    % display iterations [0 = off, 1 = on, 2 = partial]
%% DEFINE MATERIAL
E = 1;                      % young's modulus of solid [1]
Emin = 1e-9;                % young's modulus of void [1e-9]
nu = 0.3;                   % poisson ratio [0.3]
rho = 0e-3;                 % density [0e-3]
g = 9.81;                   % gravitational acceleration [9.81]
%% DEFINE FORCE
Fe = 2*(ny+1)+45*2*(ny+1):2*(ny+1):2*(ny+1)*(nx+1); % element of force application [2*(ny+1)+45*2*(ny+1):2*(ny+1):2*(ny+1)*(nx+1)]
Fn = 1;                     % number of applied force locations [1]
Fv = -1/length(Fe);         % value of applied force [-1]
%% DEFINE SUPPORTS
fix = 1:2*(ny+1);           % fixed degrees of freedom [1:2*(ny+1)]
%% DEFINE DESIGN OF ACTUATOR
Fmaxnode = 1;               % define max force per node [1]
Fmin = -1;                  % minimal force constraint [1]
sen = 5;                    % penalty for actuator design [5]
if abs(Fmaxnode) > abs(Fmin) % check for force model
    Fmma = -Fmin;           % use Fmin as maximum xmma value
else
    Fmin = Fmin/Fmaxnode; % use fraction for constraint function
    Fmma = Fmaxnode;        % use maximum force per node as maximum xmma value
end
Uarray = 1:2*(nx+1)*(ny+1); % define objective area
%% DEFINE ELEMENT RESTRICTIONS
shap = 0;                   % [0 = no restrictions, 1 = circle, 2 = custom]
area = 1;                   % [0 = no material (passive), 1 = material (active)]
nodr = [1:ny:nx*ny 2:ny:nx*ny]; % custom restricted nodes [1:ny:nx*ny]
%% PREPARE FINITE ELEMENT
N = 2*(nx+1)*(ny+1);        % total element nodes
all = 1:2*(nx+1)*(ny+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12]; % fem
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6]; % fem
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4]; % fem
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2]; % fem
Ke = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]); % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node numer matrix
dofvec = reshape(2*nodes(1:end-1,1:end-1)+1,nx*ny,1); % create dof vector
dofmat = repmat(dofvec,1,8)+repmat([0 1 2*ny+[2 3 0 1] -2 -1],nx*ny,1); % create dof matrix
iK = reshape(kron(dofmat,ones(8,1))',64*nx*ny,1); % build sparse i
jK = reshape(kron(dofmat,ones(1,8))',64*nx*ny,1); % build sparse j
%% PREPARE FILTER
iH = ones(nx*ny*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));        % create sparse vector of ones
kH = zeros(size(iH));       % create sparse vector of zeros
```

```
m = 0;                    % index for filtering
for i = 1:nx              % for each element calculate distance between ...
    for j = 1:ny          % elements' center for filtering
        r1 = (i-1)*ny+j; % sparse value i
        for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx)  % center of element
            for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % center of element
                r2 = (k-1)*ny+l; % sparse value 2
                m = m+1; % update index for filtering
                iH(m) = r1; % sparse vector for filtering
                jH(m) = r2; % sparse vector for filtering
                kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)); % weight factor
            end
        end
    end
end
H = sparse(iH,jH,kH);    % build filter
Hs = sum(H,2);            % summation of filter
%% DEFINE ELEMENT RESTRICTIONS
x = vol*ones(ny,nx);     % initial material distribution
if shap == 0              % no restrictions
    efree = (1:nx*ny)'; % all elements are free
    eres= [];             % no restricted elements
elseif shap == 1          % circular restrictions
    rest = zeros(ny,nx); % pre-allocate space
    for i = 1:nx          % start loop
        for j = 1:ny      % for each element
            if sqrt((j-ny/2)^2+(i-nx/4)^2) < ny/2.5 % circular restriction
                rest(j,i) = 1; % write restriction
                if rest(j,i) == area % check for restriction
                    x(j,i) = area; % store restrictions in material distribution
                end
            end
        end
    end
elseif shap == 2          % custom restrictions
    rest = zeros(ny*nx,1); % pre-allocate space
    for i = 1:length(nodr) % write restriction
        resti = nodr(i); % write restriction
        rest(resti) = 1; % write restriction
    end
    rest = reshape(rest,ny,nx);
    for i = 1:nx              % start loop
        for j = 1:ny          % for each element
            if rest(j,i) == area % check for restriction
                x(j,i) = area; % store restrictions in material distribution
            end
        end
```

```
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;                % set filtered design variables
elseif fil == 2            % heaviside filter
    beta = 1;              % hs filter
    xTilde = x;            % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design space
end
xFree = xF(efree);         % define free design matrix
%% DEFINE STRUCTURAL
Fsiz = size(Fe,1);         % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% DESIGN OF ACTUATOR DISTRIBUTION
wsiz = size(Fe,2);         % size of actuator variables
wzer = zeros(wsiz,1);      % empty row of zeros for mma usage
wF = F;                    % plugin initial force distribution
wval = F(Fe);              % create vector of design variables
%% DEFINE MMA PARAMETERS
m = 1;                     % number of constraint functions
n = wsiz;                  % number of variables
xmin = -1*ones(n,1);       % minimum values of x
xmax = -(1e-9/Fmma)*ones(wsiz,1); % maximum values of x
xold1 = zeros(n,1);        % previous x, to monitor convergence
xold2 = xold1;             % used by mma to monitor convergence
df0dx2 = zeros(n,1);       % second derivative of the objective function
dfdx2 = zeros(m,n);        % second derivative of the constraint function
low   = xmin;              % lower asymptotes from the previous iteration
upp   = xmax;              % upper asymptotes from the previous iteration
a0 = 1;                    % constant a_0 in mma formulation [1]
a = zeros(m,1);            % constant a_i in mma formulation
cmma = 1e3*ones(m,1);      % constant c_i in mma formulation
d = zeros(m,1);            % constant d_i in mma formulation
subs = 200;                % maximum number of subsolv iterations [200]
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
npdx = zeros(length(nodes),1)'; % pre-allocate force dots
npdy = zeros(length(nodes),1)'; % pre-allocate force dots
U = zeros(size(F));        % pre-allocate space displacement
c = zeros(miter,1);        % pre-allocate objective vector
L = zeros(N,1);            % pre-allocate selection tensor
labda = zeros(N,1);        % pre-allocate lagrange multiplier
Fi = zeros(1,N);           % pre-allocate force selection vector
Cons = zeros(miter,1);     % pre-allocate constraint vector
%% DEFINE SELECTION TENSOR
for j = Uarray             % for each iteration..
    if mod(j,2) == 0       % ...check for horizontal or vertical
```

```matlab
            L(j) = 1;          % vertical selection value
        else
            L(j) = 1;          % horizontal selection value
        end
end
%% INITIALIZE LOOP
iter = 0;                      % initialize loop
diff = 1;                      % initialize convergence criterion
loopbeta = 1;                  % initialize beta-loop
%% START LOOP
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    loopbeta = loopbeta +1; % iteration loop for hs filter
    iter = iter+1;             % define iteration
    if pcon == 1               % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;             %... set penalty 1
            s = 0.5;           %... set penalty 0.5 for actuator design
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
            s = min(sen,1.06*s); % ... set continuation penalty actuator design
        end
    elseif pcon == 0           % not using continuation method
        p = pen;               % set penalty
        s = sen;               % set penalty actuator design
    end
    %% Selfweight
    if rho ~= 0                % gravity is involved
        xP=zeros(ny,nx); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny          % for each element, set gravitational...
            Fsw(dofmat(i,2:2:end))=Fsw(dofmat(i,2:2:end))-xF(i)*rho*9.81/4;
        end                    % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0            % no gravity
        xP = xF.^p;            % penalized design variable
        Fsw = 0;               % no selfweight
    end
    wP = atan(s*wF)/atan(s); % penalized actuator variable
    Ftot = Fmma*(wP) + Fsw; % total force
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xP(:)'*(E-Emin)),64*nx*ny,1); % create sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;              % build stiffness matrix
    U(free,:) = K(free,free)\Ftot(free,:); % displacement solving
    c(iter) = 0;               % set compliance to zero
    Sens = 0;                  % set sensitivity to zero
    Senw = 0;                  % set constraint sensitivity to zero
```

```
    Cons(iter) = 0;      % set constraint to zero
    Senc = ones(1,N);    % set constraint sensitivity
    %% Calculate compliance and sensitivity
    for i = 1:size(Fn,2) % for number of load cases
        Ui = U(:,i);     % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        c(iter) = c(iter) - sum(sum(Ui)); % objective
        labda(free) = -K(free,free)\L(free); % calculate lagrange multiplie
        Fi(Fe) = (Fmma*s./((s^2*wF(Fe).^2+1)*(atan(s))));% force selection vector
        FFi = spdiags(Fi',0, N,N); % force selection vector
        Sens = Sens + FFi(Fe,Fe)*labda(Fe); % calculate sensitivity
        Cons(iter) = Cons(iter) + Fmma*(Fmin/sum(sum(wF)))-1; % calculate constraint
        dCdf = Senc(Fe)'*Fmma*full(Fmin)/-(sum(sum(full(wF))))^2; % constraint sensitivity
        if iter == 2   % finite difference method
            wF1 = wF;  % store first force vector
            [~,S1] = max(abs(Sens(:))); % calculate maximum sensitivity value
            Sens1 = Sens(S1); % store maximum sensitivity value
            [~,S2] = max(abs(dCdf(:))); % calculate maximum sensitivity value
            Sens2 = dCdf(S2); % store maximum sensitivity value
        end
    end

    if fil == 0          % optimality criterion with sensitivity filter
        Sens(:) = Sens; % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity
    elseif fil == 1      % optimality criterion with density filter
        Sens(:) = Sens; % update filtered sensitivity of constraint
        Sencw(:) = Senc; % update filtered sensitivity of constraint
    elseif fil == 2      % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity of constraint
    end
    %% Update design variables Optimality Criterion
    if sol == 0          % use optimality criterion method
        l1 = 0;          % initial lower bound for lagranian mulitplier
        l2 = 1e9;        % initial upper bound for lagranian multiplier
        while (l2-l1)/(l1+l2) > 1e-3 % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
```

```
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
            if shap == 1 % restriction is on
                xF(rest==1) = area; % set restricted area
            end
            if sum(xF(:)) > vol*nx*ny % check for optimum
                l1 = lag; % update lower bound to average
            else
                l2 = lag; % update upper bound to average
            end
        end
        %% Method of moving asymptotes
    elseif sol == 1     % use mma solver
        xval = wval(:); % store current design variable for mma
        if iter == 1    % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = Sens*cscale; % store sensitivity for mma
        f = Cons(iter); % normalized constraint function
        dfdx = dCdf;    % derivative constraint function
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = wval(:); % previous x, to monitor convergence
        xnew = xF;      % update density result
        wnew = wF;      % update force result
        wnew(Fe) = xmma(1:end); % include mma result
        if fil == 0     % sensitivity filter
            xF = xnew;  % update design variables
        elseif fil == 1 % density filter
            xF(:) = (H*xnew(:))./Hs; % update filtered densities result
        elseif fil == 2 % heaviside filter
            xTilde(:)= (H*xnew(:))./Hs; % filtered result
            xF(:)=1-exp(-beta*xTilde)+xTilde*exp(-beta); % update design variable
        end
        if shap == 1 || shap == 2   % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end
        wF(:) = wnew(:); % update support variables
        wval = wnew(Fe); % update support variables
    end
    diff = max(abs(full(Fmma*wnew(:))-full(F(:)))); % difference of maximum element change
    F = Fmma*wnew;      % update design variable
```

```
    if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;  % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
    %% Finite difference method
    if (fincheck == 1 || fincheck == 2) % check for finite difference method
        if iter == 2    % on first findif iteration
            wF = wF1;   % store first findif result...
            wF(Fe(S1)) = wF1(Fe(S1))+h; %...and add a small pertubation
        elseif iter == 3 % on second findif iteration
            findif = (c(3)-c(2))/h; % calculate finite difference method
            Sensdif = abs(max((findif-Sens1)/Sens1,(Sens1-findif)/findif)); % maximum difference
            if Sensdif > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = wF1;     % store first findif result...
            wF(Fe(S2)) = wF1(Fe(S2))+h; %...and add a small pertubation
        elseif iter == 4  % on third findif iteration
            findif2 = (Cons(4)-Cons(2))/h; % calculate finite difference method
            Sensdif2 = abs(max((findif2-Sens2)/Sens2,(Sens2-findif2)/findif2)); % maximum difference
            if Sensdif2 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif2)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
        end
    end
    %% Store results into database X
    X(:,:,iter) = xF;  % each element value x is stored for each iteration
    C(iter) = c(iter);  % each compliance is stored for each iteration
    W(:,:,iter) = full(wF);   % each force variable is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    assignin('base', 'W', W); % each iteration (3rd dimension)
    %% Results
```

```
    if dis == 1          % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Ftot:' sprintf('%6.3f',sum(full(wP(:)))) ' Diff:' sprintf('%6.3f',diff)]);
    elseif dis == 2      % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
                disiter = plotiter;
            elseif iter == disiter
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
                ' Ftot:' sprintf('%6.3f',sum(full(wP(:)))) ' Diff:' sprintf('%6.3f',diff)]);
        end
    end
    if draw == 1         % plot iterations
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        axis equal; axis tight
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for force application
        Fmaxplot = min(min(full(F)));
        for i = 1:length(Fe)
            if F(Fe(i)) < wplot*Fmaxplot
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
        end
```

```
        if iter > 1
            delete(Dof)
        end
        if exist('npfx','var')
            Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
            clear npfx; clear npfy;
            uistack(Dof,'top')
        end
        % Plot coloured arrows for force application
        if (((diff < tol) && iter >= piter+1) || iter >= miter)
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            for i = 1:length(Fe)
                if F(Fe(i)) < wplot*Fmaxplot
                    headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*Fmaxplot)));
                    if mod(Fe(i),2)
                        arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                    else
                        arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                    end
                end
            end
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        set(gca,'YTick',[],'YTicklabel',[])
        xlabel('Iterations')
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    elseif draw == 2    % plot parts of iterations
        if iter == 1 || iter == drawiter
            if iter == 1
                drawiter = plotiter;
            elseif iter == drawiter
```

```
        drawiter = drawiter + plotiter;
    end
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    axis equal; axis tight
    drawnow;
    hold on
    if iter == 1
        % Plot coloured dots for constraints
        for i = 1:length(fix)
            npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
            nplot = ceil(fix(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npy(i) = nplot-0.5;
        end
        plot(npx,npy,'r.','MarkerSize',20)
    end
    % Plot coloured dots for force application
    Fmaxplot = min(min(full(F)));
    for i = 1:length(Fe)
        if F(Fe(i)) < wplot*Fmaxplot
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
    end
    if iter > 1
        delete(Dof)
    end
    if exist('npfx','var')
        Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
        clear npfx; clear npfy;
        uistack(Dof,'top')
    end
    % Plot coloured arrows for force application
    if (((diff < tol) && iter >= piter+1) || iter >= miter)
        for i = 1:length(Fe)
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
```

```
                    npfy(i) = nplot-0.5;
                end
                for i = 1:length(Fe)
                    if F(Fe(i)) < wplot*Fmaxplot
                        headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*
                            Fmaxplot)));
                        if mod(Fe(i),2)
                            arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe
                                (i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                        else
                            arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)
                                +0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0
                                 1])
                        end
                    end
                end
            end
            % Plot compliance plot
            figure(1)
            subplot(2,1,2)
            plot(c(1:iter))
            set(gca,'YTick',[],'YTicklabel',[])
            xlabel('Iterations')
            ylabel('Compliance')
            xaxmax = c(iter);
            yaxmax = max(c);
            yaxmin = min(c(1:iter));
            if pcon == 0
                yaxmax = mean([yaxmin yaxmax]);
            end
            ylim([0.95*yaxmin yaxmax])
            xlim([1 min(iter+10,miter)])
        end
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0 || dis == 2 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter))
        ...
        ' Ftot:' sprintf('%6.3f',sum(full(wP(:)))) ' Diff:' sprintf('%6.3
           f',diff)]);
end
if draw == 0 || draw == 2 % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]'
           )
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
```

```
    % Plot coloured dots for constraints
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
    % Plot coloured dots for force application
    Fmaxplot = min(min(full(F)));
    for i = 1:length(Fe)
        if F(Fe(i)) < wplot*Fmaxplot
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
    end
    if iter > 1
        delete(Dof)
    end
    if exist('npfx','var')
        Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
        clear npfx; clear npfy;
        uistack(Dof,'top')
    end
    % Plot coloured arrows for force application
    if (((diff < tol) && iter >= piter+1) || iter >= miter)
        for i = 1:length(Fe)
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
        for i = 1:length(Fe)
            if F(Fe(i)) < wplot*Fmaxplot
                headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*Fmaxplot)));
                if mod(Fe(i),2)
                    arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                else
                    arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                end
            end
        end
    end
```

```
    % Plot compliance plot
    if adv == 0
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        set(gca,'YTick',[],'YTicklabel',[])
        xlabel('Iterations')
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    end
end
%% PLOTTING DISPLACEMENT
if (def == 1 || def == 2)
    FileName = ['Displacement_',datestr(now, 'ddmm_HHMMSS'),'.avi']; %
        dynamic filename
    vidObj = VideoWriter(FileName);
    vidObj.FrameRate = 3;
    figure(1)
    subplot(2,1,1)
    xaxis = get(gca,'XLim');
    yaxis = get(gca,'YLim');
    open(vidObj);
    figure(2)
    clear mov
    colormap(gray);
    Umov = 1;                   % start movie counter
    Uim = zeros(5642,1);
    Uim(2:2:end) = Ui(2:2:end);
    Uim(1:2:end) = -Ui(1:2:end);
    Umax = -10/max(abs(Uim)); % define maximum displacement
    steps = 1;                  % number of displacement steps
    set(gca,'nextplot','replacechildren');
    Upatch = zeros(nx*ny,1);
    for i = 1:ny*nx
        Uindex = 2*(i+floor((i-1)/ny))-1+[1 2 2*(ny+1)+1 2*(ny+1)+3];
        Upatch(i,1) = mean(U(Uindex));
    end
    Upatch = reshape(Upatch,ny,nx);
    Upatchmin = min(min(Upatch));
    Upatchnorm = -Upatch/Upatchmin;
    for Udisp = linspace(Umax/steps,Umax,steps) % vary input displacement
        clf
        for ely = 1:ny      % plot displacements...
            for elx = 1:nx  % for each element...
                if xF(ely,elx) > 0 % exclude white regions for plotting
                    purposes
```

```
                    n1 = (ny+1)*(elx-1)+ely;
                    n2 = (ny+1)* elx +ely;
                    Ue = Udisp*Uim([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2
                        +2; 2*n1+1;2*n1+2],1);
                    ly = ely-1; lx = elx-1;
                    xx = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx
                         ]';
                    yy = [-Ue(2,1)-ly -Ue(4,1)-ly -Ue(6,1)-ly-1 -Ue(8,1)-
                        ly-1]';
                    patch([xx xx],[yy yy],[Upatchnorm(ely,elx) Upatchnorm
                        (ely,elx)],'LineStyle','none');

                end
            end
        end
        colormap jet        % for better interpation...
        axis tight
        axis equal
        xticks([0 15 30 45 60 75 90])
        box on
        colorbar
        drawnow             % ...draw coloured densities
        currFrame = getframe; % get current frame...
        writeVideo(vidObj,currFrame); % ... write to video file
    end
    close(vidObj);
end
if def == 2                 % when def equals 2...
    implay(FileName)            % ...open Matlab Movie Player
end
toc
```

B.10 Design of Actuator Placement Including Topology Optimization.m

In this section, the complete code of designing optimal actuator placement, including topology optimization is made available. By running this code, the produced picture in Figure 5-6 can be made immediately. The constraints are scaled, in order to prioritize the compliance constraint.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                      %
% Topology Optimization Using Matlab                                   %
% Design of Actuator Placement                                         %
%                                                                      %
% Delft University of Technology, Department PME                       %
% Master of Science Thesis Project                                     %
%                                                                      %
% Stefan Broxterman                                                    %
%                                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
tic                         % start timer
%% DEFINE PARAMETERS
adv = 0;                    % use advanced function [0 = off, 1 = on]
if adv == 0                 % define parameters at behalf of the advanced function
    nx = 90;                % numer of elements horizontal
    ny = 30;                % number of elements vertical
    vol = 0.2;              % volume fraction [0-1]
    pen = 3;                % penalty
    rmin = 1.5;             % filter size
    fil = 1;                % filter method [0 = sensitivity filtering, 1 = density filtering, 2 = heaviside filtering]
    clc; clf; close all; clear X; clear W; % clear workspace
end
%% DEFINE SOLUTION METHOD
sol = 1;                    % solution method [0 = oc(sens), 1 = mma]
pcon = 1;                   % use continuation method [0 = off, 1 = on]
fincheck = 1;               % finite difference check [0 = off, 1 = on, 2 = break]
%% DEFINE CALCULATION
tol = 0.001;                % tolerance for convergence criterion [0.01]
move = 0.2;                 % move limit for lagrange [0.2]
pcinc = 1.03;               % penalty continuation increasing factor [1.03]
piter = 20;                 % number of iteration for starting penalty [20]
miter = 1000;               % maximum number of iterations [1000]
plotiter = 5;               % gap of iterations used to plot or draw iterations [5]
def = 0;                    % plot deformations [0 = off, 1 = on, 2 = play video]
wplot = 0.20;               % define treshold factor of Fmax for force plot [0.20]
```

```
h = 1e-6;                   % perturbation value for finite difference method [1e-6]
%% DEFINE OUTPUT
draw = 1;                   % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 1;                    % display iterations [0 = off, 1 = on, 2 = partial]
%% DEFINE MATERIAL
E = 1;                      % young's modulus of solid [1]
Emin = 1e-9;                % young's modulus of void [1e-9]
nu = 0.3;                   % poisson ratio [0.3]
rho = 0e-3;                 % density [0e-3]
g = 9.81;                   % gravitational acceleration [9.81]
%% DEFINE FORCE
Fe = 2*(ny+1)+45*2*(ny+1):2*(ny+1):2*(ny+1)*(nx+1); % element of force application [2*(ny+1)+45*2*(ny+1):2*(ny+1):2*(ny+1)*(nx+1)]
Fn = 1;                     % number of applied force locations [1]
Fv = -1/length(Fe);         % value of applied force [-1]
%% DEFINE SUPPORTS
fix = 1:2*(ny+1);           % fixed degrees of freedom [1:2*(ny+1)]
%% DEFINE DESIGN OF ACTUATOR
Fmaxnode = 1;               % define max force per node [1]
Fmin = -1;                  % minimal force constraint [1]
sen = 5;                    % penalty for actuator design [5]
if abs(Fmaxnode) > abs(Fmin) % check for force model
    Fmma = -Fmin;           % use Fmin as maximum xmma value
else
    Fmin = Fmin/Fmaxnode; % use fraction for constraint function
    Fmma = Fmaxnode;        % use maximum force per node as maximum xmma value
end
Uarray = 2:2:2*(nx+1)*(ny+1);          % define objective area
%% DEFINE ELEMENT RESTRICTIONS
shap = 0;                   % [0 = no restrictions, 1 = circle, 2 = custom]
area = 1;                   % [0 = no material (passive), 1 = material (active)]
nodr = [1:ny:nx*ny 2:ny:nx*ny]; % custom restricted nodes [1:ny:nx*ny]
%% PREPARE FINITE ELEMENT
N = 2*(nx+1)*(ny+1);        % total element nodes
all = 1:2*(nx+1)*(ny+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12]; % fem
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6]; % fem
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4]; % fem
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2]; % fem
Ke = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]); % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node numer matrix
dofvec = reshape(2*nodes(1:end-1,1:end-1)+1,nx*ny,1); % create dof vector
dofmat = repmat(dofvec,1,8)+repmat([0 1 2*ny+[2 3 0 1] -2 -1],nx*ny,1); % create dof matrix
iK = reshape(kron(dofmat,ones(8,1))',64*nx*ny,1); % build sparse i
jK = reshape(kron(dofmat,ones(1,8))',64*nx*ny,1); % build sparse j
%% PREPARE FILTER
```

```
iH = ones(nx*ny*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));     % create sparse vector of ones
kH = zeros(size(iH));    % create sparse vector of zeros
m = 0;                   % index for filtering
for i = 1:nx             % for each element calculate distance between ...
    for j = 1:ny         % elements' center for filtering
        r1 = (i-1)*ny+j; % sparse value i
        for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx)  % center of element
            for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % center of element
                r2 = (k-1)*ny+l; % sparse value 2
                m = m+1; % update index for filtering
                iH(m) = r1; % sparse vector for filtering
                jH(m) = r2; % sparse vector for filtering
                kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)); % weight factor
            end
        end
    end
end
H = sparse(iH,jH,kH);    % build filter
Hs = sum(H,2);           % summation of filter
%% DEFINE ELEMENT RESTRICTIONS
x = vol*ones(ny,nx);     % initial material distribution
if shap == 0             % no restrictions
    efree = (1:nx*ny)'; % all elements are free
    eres= [];            % no restricted elements
elseif shap == 1         % circular restrictions
    rest = zeros(ny,nx); % pre-allocate space
    for i = 1:nx         % start loop
        for j = 1:ny     % for each element
            if sqrt((j-ny/2)^2+(i-nx/4)^2) < ny/2.5 % circular restriction
                rest(j,i) = 1; % write restriction
                if rest(j,i) == area % check for restriction
                    x(j,i) = area; % store restrictions in material distribution
                end
            end
        end
    end
elseif shap == 2         % custom restrictions
    rest = zeros(ny*nx,1); % pre-allocate space
    for i = 1:length(nodr) % write restriction
        resti = nodr(i); % write restriction
        rest(resti) = 1; % write restriction
    end
    rest = reshape(rest,ny,nx);
    for i = 1:nx             % start loop
        for j = 1:ny         % for each element
            if rest(j,i) == area % check for restriction
```

```
                x(j,i) = area; % store restrictions in material distribution
            end
        end
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;             % set filtered design variables
elseif fil == 2         % heaviside filter
    beta = 1;           % hs filter
    xTilde = x;         % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design space
end
xFree = xF(efree);      % define free design matrix
%% DEFINE STRUCTURAL
Fsiz = size(Fe,1);      % size of load vector
F = sparse(Fe,Fn,Fv,N,Fsiz); % define load vector
%% DESIGN OF ACTUATOR AND TOPOLOGY DISTRIBUTION
xsiz = size(xFree,1);   % size of topology variables
wsiz = size(Fe,2);      % size of actuator variables
xzer = zeros(xsiz,1);   % empty row of zeros for mma usage
wzer = zeros(wsiz,1);   % empty row of zeros for mma usage
wF = F;                 % plugin initial force distribution
wval = F(Fe);           % create vector of design variables
%% DEFINE MMA PARAMETERS
m = 3;                  % number of constraint functions
n = xsiz+wsiz;          % number of variables
xmin = [1e-9*ones(xsiz,1); -1*ones(wsiz,1)];    % minimum values of x
xmax = [ones(xsiz,1); -(1e-9/Fmma)*ones(wsiz,1)]; % maximum values of x
xold1 = zeros(n,1);     % previous x, to monitor convergence
xold2 = xold1;          % used by mma to monitor convergence
df0dx2 = zeros(n,1);    % second derivative of the objective function
dfdx2 = zeros(m,n);     % second derivative of the constraint function
low   = xmin;           % lower asymptotes from the previous iteration
upp   = xmax;           % upper asymptotes from the previous iteration
a0 = 1;                 % constant a_0 in mma formulation [1]
a = zeros(m,1);         % constant a_i in mma formulation
cmma = 1e3*ones(m,1);   % constant c_i in mma formulation
d = zeros(m,1);         % constant d_i in mma formulation
subs = 50;             % maximum number of subsolv iterations [200]
%% PRE-ALLOCATE SPACE
npx = zeros(length(fix),1)'; % pre-allocate constraint dots
npy = zeros(length(fix),1)'; % pre-allocate constraint dots
npfx = zeros(length(Fe),1)'; % pre-allocate force dots
npfy = zeros(length(Fe),1)'; % pre-allocate force dots
npdx = zeros(length(nodes),1)'; % pre-allocate force dots
npdy = zeros(length(nodes),1)'; % pre-allocate force dots
U = zeros(size(F));     % pre-allocate space displacement
c = zeros(miter,1);     % pre-allocate objective vector
L = zeros(N,1);         % pre-allocate selection tensor
```

```
labda = zeros(N,1);      % pre-allocate lagrange multiplier
labda2 = zeros(N,1);     % pre-allocate second lagrange multiplier
Fi = zeros(1,N);         % pre-allocate force selection vector
Ua = zeros(N,1);         % pre-allocate displacement vector
Cons = zeros(miter,1);   % pre-allocate constraint vector
Cons2 = zeros(miter,1); % pre-allocate constraint #2 vector
Cons3 = zeros(miter,1); % pre-allocate constraint #3 vector
%% DEFINE SELECTION TENSOR
for j = Uarray           % for each iteration..
    if mod(j,2) == 0     % ...check for horizontal or vertical
        L(j) = 1;        % vertical selection value
    else
        L(j) = 0;        % horizontal selection value
    end
end
%% INITIALIZE LOOP
iter = 0;                % initialize loop
diff = 1;                % initialize convergence criterion
loopbeta = 1;            % initialize beta-loop
%% START LOOP
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    loopbeta = loopbeta +1; % iteration loop for hs filter
    iter = iter+1;       % define iteration
    if pcon == 1         % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;       %... set penalty 1
            s = 0.5;     %... set penalty 0.5 for actuator design
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
            s = min(sen,1.06*s); % ... set continuation penalty actuator design
        end
    elseif pcon == 0     % not using continuation method
        p = pen;         % set penalty
        s = sen;         % set penalty actuator design
    end
    %% Selfweight
    if rho ~= 0          % gravity is involved
        xP=zeros(ny,nx); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny    % for each element, set gravitational...
            Fsw(dofmat(i,2:2:end))=Fsw(dofmat(i,2:2:end))-xF(i)*rho*9.81/4;
        end              % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0      % no gravity
        xP = xF.^p;      % penalized design variable
        Fsw = 0;         % no selfweight
    end
    wP = atan(s*wF)/atan(s); % penalized actuator variable
```

```
    Ftot = Fmma*(wP) + Fsw; % total force
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xP(:)'*(E-Emin)),64*nx*ny,1); % create
        sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;         % build stiffness matrix
    Kt = K;               % update total force
    U(free,:) = Kt(free,free)\Ftot(free,:); % displacement solving
    c(iter) = 0;          % set compliance to zero
    Sens = 0;             % set sensitivity to zero
    Senw = 0;             % set constraint sensitivity to zero
    Cons(iter) = 0;       % set constraint to zero
    Senc = ones(1,N);     % set constraint sensitivity
    %% Calculate compliance and sensitivity
    for i = 1:size(Fn,2) % for number of load cases
        Ui = U(:,i);      % displacement per load case
        Ua(Uarray) = Ui(Uarray); % selection of displacement
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial
            compliance
        c(iter) = c(iter) + sum(Ua.^2); % objective
        labda(free) = -sparse(Kt(free,free))\sparse(2*Ua(free)); %
            calculate lagrange multiplier
        labda2(free) = 2*Ua(free); % calculate second lagrange multiplier
        c00 = reshape(sum((labda(dofmat)*Ke).*Ui(dofmat),2),ny,nx); %
            initial labda compliance
        Fi(Fe) = (Fmma*s./((s^2*wF(Fe).^2+1)*(atan(s))));% force
            selection vector
        FFi = spdiags(Fi',0, N,N); % force selection vector
        Sens = Sens + p*(E-Emin)*xF.^(p-1).*c00; % calculate density
            sensitivity
        Senw = Senw - FFi(Fe,Fe)*labda(Fe); % calculate force sensitivity
        Cons(iter) = Cons(iter) + 10*(sum(xF(:))/(vol*nx*ny)-1); %
            calculate constraint
        dCdx = 10*Senc(efree)/(vol*ny*nx); % constraint sensitivity
        Cons2(iter) = Cons2(iter) + 10*(Fmin/sum(sum(wF)))-1; % calculate
            constraint
        dCdf = 10*Senc(Fe)*Fmin/-(sum(sum(full(wF))))^2; % constraint
            sensitivity
        Cons3(iter) = Cons3(iter) + (sum(sum((Emin+xF.^p*(E-Emin)).*c0))
            -50); % compliance constraint
        dCCdx = -p*(E-Emin)*xF.^(p-1).*c0; % constraint sensitivity
        dCCdf = labda2(Fe)'*FFi(Fe,Fe); % constraint sensitivity
        if iter == 2    % finite difference method
            F1 = wF;    % store force vector
            X1 = xF;    % store density vector
            [~,S1] = max(abs(Sens(:))); % calculate maximum sensitivity
                value
            Sens1 = Sens(S1); % store maximum sensitivity value
            [~,S2] = max(abs(Senw(:))); % calculate maximum sensitivity
                value
            Sens2 = Senw(S2); % store maximum sensitivity value
            [~,S3] = max(abs(dCdx(:))); % calculate maximum sensitivity
                value
```

```
            Sens3 = dCdx(S3); % store maximum sensitivity value
            [~,S4] = max(abs(dCdf(:))); % calculate maximum sensitivity value
            Sens4 = dCdf(S4); % store maximum sensitivity value
            [~,S5] = max(abs(dCCdx(:))); % calculate maximum sensitivity value
            Sens5 = dCCdx(S5); % store maximum sensitivity value
            [~,S6] = max(abs(dCCdf(:))); % calculate maximum sensitivity value
            Sens6 = dCCdf(S6); % store maximum sensitivity value
        end
    end
    if fil == 0          % optimality criterion with sensitivity filter
        Sens(:) = Sens; % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity
    elseif fil == 1      % optimality criterion with density filter
        Sens(:) = H*(Sens(:)./Hs); % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity of constraint
    elseif fil == 2      % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity of constraint
    end
    %% Update design variables Optimality Criterion
    if sol == 0          % use optimality criterion method
        l1 = 0;          % initial lower bound for lagranian mulitplier
        l2 = 1e9;        % initial upper bound for lagranian multiplier
        while (l2-l1)/(l1+l2) > 1e-3 % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
            if shap == 1 % restriction is on
                xF(rest==1) = area; % set restricted area
            end
            if sum(xF(:)) > vol*nx*ny % check for optimum
                l1 = lag; % update lower bound to average
            else
                l2 = lag; % update upper bound to average
            end
        end
        %% Method of moving asymptotes
    elseif sol == 1      % use mma solver
```

```
        xval = [xFree(:);wval(:)]; % store current design variable for mma
        if iter == 1    % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = [Sens(efree);Senw]*cscale; % store sensitivity for mma
        f = [Cons(iter);Cons2(iter);Cons3(iter)]; % normalized constraint function
        dfdx =[dCdx wzer'; xzer' dCdf; dCCdx(efree)' dCCdf]; % derivative constraint functions
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = [xFree(:);wval(:)]; % previous x, to monitor convergence
        xnew = xF;      % update density result
        wnew = wF;      % update force result
        xnew(efree) = xmma(1:xsiz); % update mma to density
        wnew(Fe) = xmma(xsiz+1:end); % update mma to force
        if fil == 0     % sensitivity filter
            xF = xnew;  % update design variables
        elseif fil == 1 % density filter
            xF(:) = (H*xnew(:))./Hs; % update filtered densities result
        elseif fil == 2 % heaviside filter
            xTilde(:)= (H*xnew(:))./Hs; % filtered result
            xF(:)=1-exp(-beta*xTilde)+xTilde*exp(-beta); % update design variable
        end
        if shap == 1 || shap == 2   % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end
        wF(:) = wnew(:); % update support variables
        xFree = xnew(efree); % update density variable
        wval = wnew(Fe); % update support variables
    end
    diff = (max(abs(full(Fmma*wnew(:)-full(F(:)))))+max(abs(xnew(:)-x(:)))); % difference of maximum element change
    x = xnew;           % update design variable density
    F = Fmma*wnew;      % update design variable force
    if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;  % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
    %% Finite difference method
    if (fincheck == 1 || fincheck == 2) % check for finite difference method
```

```
        if iter == 2    % on first findif iteration
            xF = X1;    % store first findif result...
            xF(S1) = X1(S1)+h; %...and add a small pertubation
            wF = F1;   % store first findif result
        elseif iter == 3 % on second findif iteration
            findif = (c(3)-c(2))/h; % calculate finite difference method
            Sensdif = abs(max((findif-Sens1)/Sens1,(Sens1-findif)/findif)); % maximum difference
            if Sensdif > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;   % store first findif result...
            wF(Fe(S2)) = F1(Fe(S2))+h; %...and add a small pertubation
            xF = X1;   % store first findif result
        elseif iter == 4  % on third findif iteration
            findif2 = (c(4)-c(2))/h; % calculate finite difference method
            Sensdif2 = abs(max((findif2-Sens2)/Sens2,(Sens2-findif2)/findif2)); % maximum difference
            if Sensdif2 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif2)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;   % store first findif result
            xF = X1;   % store first findif result...
            xF(S3) = xF(S3)+h; %...and add a small pertubation
        elseif iter == 5  % on fourth findif iteration
            findif3 = (Cons(5)-Cons(2))/h; % calculate finite difference method
            Sensdif3 = abs(max((findif3-Sens3)/Sens3,(Sens3-findif3)/findif3)); % maximum difference
            if Sensdif3 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif3)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;   % store first findif result
            wF(S4) = wF(S4)+h; % store first findif result...
            xF = X1; %...and add a small pertubation
        elseif iter == 6  % on fifth findif iteration
            findif4 = (Cons2(6)-Cons2(2))/h; % calculate finite difference method
```

```
            Sensdif4 = abs(max((findif4-Sens4)/Sens4,(Sens4-findif4)/findif4)); % maximum difference
            if Sensdif4 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif4)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;   % store first findif result
            xF = X1;   % store first findif result...
            xF(S5) = xF(S5)+h; %...and add a small pertubation
        elseif iter == 7  % on sixth findif iteration
            findif5 = (Cons3(7)-Cons3(2))/h; % calculate finite difference method
            Sensdif5 = abs(max((findif4-Sens5)/Sens5,(Sens5-findif5)/findif5)); % maximum difference
            if Sensdif5 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif5)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;    % store first findif result...
            wF(Fe(S6)) = F1(Fe(S6))+h; %...and add a small pertubation
            xF = X1;   % store first findif result
        elseif iter == 8  % on second finidif iteration
            findif6 = (Cons3(8)-Cons3(2))/h; % calculate finite difference method
            Sensdif6 = abs(max((findif6-Sens6)/Sens6,(Sens6-findif6)/findif6)); % maximum difference
            if Sensdif6 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif6)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
        end
    end
    %% Store results into database X
    X(:,:,iter) = xF;   % each element value x is stored for each iteration
    C(iter) = c(iter);  % each compliance is stored for each iteration
    W(:,:,iter) = full(wF);    % each force variable is stored for each iteration
    assignin('base', 'X', X); % each iteration (3rd dimension)
    assignin('base', 'C', C); % each iteration (3rd dimension)
    assignin('base', 'W', W); % each iteration (3rd dimension)
```

```
    %% Results
    if dis == 1           % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(
            iter)) ...
             ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('%6.3f'
              , ...
             sum(full(F))) ' Diff:' sprintf('%6.3f',diff)]);
    elseif dis == 2      % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
                disiter = plotiter;
            elseif iter == disiter
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c
                (iter)) ...
                 ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('
                    %6.3f', ...
                 sum(full(F))) ' Diff:' sprintf('%6.3f',diff)]);
        end
    end
    if draw == 1          % plot iterations
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7
              0.7]')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        axis equal; axis tight
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for force application
        Fmaxplot = min(min(full(F)));
        for i = 1:length(Fe)
            if F(Fe(i)) < wplot*Fmaxplot
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
```

```
                npfy(i) = nplot-0.5;
            end
        end
        if iter > 1
            delete(Dof)
        end
        if exist('npfx','var')
            Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize'
                ,20);
            clear npfx; clear npfy;
            uistack(Dof,'top')
        end
        % Plot coloured arrows for force application
        if (((diff < tol) && iter >= piter+1) || iter >= miter)
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            for i = 1:length(Fe)
                if F(Fe(i)) < wplot*Fmaxplot
                    headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*Fmaxplot)
                        ));
                    if mod(Fe(i),2)
                        arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))
                            /Fmaxplot npfy(i)],headsize,2,[0 0 1])
                    else
                        arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*
                            F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                    end
                end
            end
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        set(gca,'YTick',[],'YTicklabel',[])
        xlabel('Iterations')
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    elseif draw == 2    % plot parts of iterations
        if iter == 1 || iter == drawiter
```

```
        if iter == 1
            drawiter = plotiter;
        elseif iter == drawiter
            drawiter = drawiter + plotiter;
        end
        figure(1)
        subplot(2,1,1)
        colormap(gray); imagesc(1-xF);
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        axis equal; axis tight
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for force application
        Fmaxplot = max(max(full(F)));
        for i = 1:length(Fe)
            if F(Fe(i)) > wplot*Fmaxplot
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
        end
        if iter > 1
            delete(Dof)
        end
        if exist('npfx','var')
            Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
            clear npfx; clear npfy;
            uistack(Dof,'top')
        end
        % Plot coloured arrows for force application
        if (((diff < tol) && iter >= piter+1) || iter >= miter)
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
```

```
                    while nplot > (ny+1)
                        nplot = nplot-(ny+1);
                    end
                    npfy(i) = nplot-0.5;
                end
                for i = 1:length(Fe)
                    if F(Fe(i)) > wplot*Fmaxplot
                        headsize = 1/sqrt(length(nonzeros(F(Fe)>0.5*
                            Fmaxplot)));
                        if mod(Fe(i),2)
                            arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe
                                (i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                        else
                            arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)
                                +0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0
                                1])
                        end
                    end
                end
            end
            % Plot compliance plot
            figure(1)
            subplot(2,1,2)
            plot(c(1:iter))
            set(gca,'YTick',[],'YTicklabel',[])
            xlabel('Iterations')
            ylabel('Compliance')
            xaxmax = c(iter);
            yaxmax = max(c);
            yaxmin = min(c(1:iter));
            if pcon == 0
                yaxmax = mean([yaxmin yaxmax]);
            end
            ylim([0.95*yaxmin yaxmax])
            xlim([1 min(iter+10,miter)])
        end
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0 || dis == 2 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter))
        ...
        ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('%6.3f',
            ...
        sum(full(wP(:)))) ' Diff:' sprintf('%6.3f',diff)]);
end
if draw == 0 || draw == 2 % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
```

```
        'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    % Plot coloured dots for constraints
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
    % Plot coloured dots for force application
    Fmaxplot = max(max(full(F)));
    for i = 1:length(Fe)
        if F(Fe(i)) > wplot*Fmaxplot
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
    end
    if iter > 1
        delete(Dof)
    end
    if exist('npfx','var')
        Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
        clear npfx; clear npfy;
        uistack(Dof,'top')
    end
    % Plot coloured arrows for force application
    if (((diff < tol) && iter >= piter+1) || iter >= miter)
        for i = 1:length(Fe)
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
        for i = 1:length(Fe)
            if F(Fe(i)) > wplot*Fmaxplot
                headsize = 1/sqrt(length(nonzeros(F(Fe)>0.5*Fmaxplot)));
                if mod(Fe(i),2)
                    arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                else
```

```
                    arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                end
            end
        end
    end
    % Plot compliance plot
    if adv == 0
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        set(gca,'YTick',[],'YTicklabel',[])
        xlabel('Iterations')
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    end
end
%% PLOTTING DISPLACEMENT
if (def == 1 || def == 2)
    FileName = ['Displacement_',datestr(now, 'ddmm_HHMMSS'),'.avi']; % dynamic filename
    vidObj = VideoWriter(FileName);
    vidObj.FrameRate = 3;
    figure(1)
    subplot(2,1,1)
    xaxis = get(gca,'XLim');
    yaxis = get(gca,'YLim');
    open(vidObj);
    figure(2)
    clear mov
    colormap(gray);
    Umov = 1;                % start movie counter
    Uim = zeros(5642,1);
    Uim(2:2:end) = Ui(2:2:end);
    Uim(1:2:end) = -Ui(1:2:end);
    Umax = -10/max(abs(Uim)); % define maximum displacement
    steps = 1;               % number of displacement steps
    set(gca,'nextplot','replacechildren');
    Upatch = zeros(nx*ny,1);
    for i = 1:ny*nx
        Uindex = 2*(i+floor((i-1)/ny))-1+[1 2 2*(ny+1)+1 2*(ny+1)+3];
        Upatch(i,1) = mean(U(Uindex));
    end
    Upatch = reshape(Upatch,ny,nx);
    Upatchmin = min(min(Upatch));
    Upatchnorm = -Upatch/Upatchmin;
```

```
    for Udisp = linspace(Umax/steps,Umax,steps) % vary input displacement
        clf
        for ely = 1:ny      % plot displacements...
            for elx = 1:nx  % for each element...
                if xF(ely,elx) > 0 % exclude white regions for plotting
                   purposes
                    n1 = (ny+1)*(elx-1)+ely;
                    n2 = (ny+1)* elx +ely;
                    Ue = Udisp*Uim([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2
                        +2; 2*n1+1;2*n1+2],1);
                    ly = ely-1; lx = elx-1;
                    xx = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx
                        ]';
                    yy = [-Ue(2,1)-ly -Ue(4,1)-ly -Ue(6,1)-ly-1 -Ue(8,1)-
                        ly-1]';
                    patch([xx xx],[yy yy],[Upatchnorm(ely,elx) Upatchnorm
                        (ely,elx)],'LineStyle','none');

                end
            end
        end
        colormap jet        % for better interpation...
        axis tight
        axis equal
        xticks([0 15 30 45 60 75 90])
        box on
        colorbar
        drawnow             % ...draw coloured densities
        currFrame = getframe; % get current frame...
        writeVideo(vidObj,currFrame); % ... write to video file
    end
    close(vidObj);
end
if def == 2                 % when def equals 2...
    implay(FileName)            % ...open Matlab Movie Player
end
toc
```

Appendix C

Add-in Codes

In this section some add-ins can be found. Keep in mind: it is highly recommended to not just copy and paste the code, but type it yourself. By this way, the user could actually achieve some knowledge over the changes made, and also overcome copy-paste problems.
The add-ins are split up in the following parts: making use of the MMA solution (C.1), using restrictive regions (C.2), solving multiple load cases (C.3), implementing self-weight (C.4), using the continuity method (C.5) and using different filtering techniques (C.6).
Up to here, all functions for two dimensional cases are described. A third dimension can be introduced by applying (C.7). An add-in to be able to calculate compliant mechanisms can be seen in (C.8). Design of supports can be implemented by following (C.9).
Design of actuator placement can be implemented by following the regime (C.10). When also implementing topology optimization, besides the actuator placement, make sure to implement (C.11).

C.1 Basic MMA Add-in.m

As a follow-up from (B.1), an extra implementation of the MMA solution is added (3.1). In the preamble, the solution method can be implemented [between line 23-24]:

```
%% DEFINE SOLUTION METHOD
sol = 1;            % solution method [0 = oc(sens), 1 = mma]
```

The parameters of this MMA can be implemented accordingly [between line 80-81]:

```
%% DEFINE MMA PARAMETERS
m = 1;                      % number of constraint functions
n = size(xF(:),1);     % number of variables
xmin = zeros(n,1);          % minimum values of x
xmax = ones(n,1);           % maximum values of x
xold1 = zeros(n,1);         % previous x, to monitor convergence
xold2 = xold1;              % used by mma to monitor convergence
df0dx2 = zeros(n,1);        % second derivative of the objective function
dfdx2 = zeros(1,n);         % second derivative of the constraint function
low   = xmin;               % lower asymptotes from the previous iteration
upp   = xmax;               % upper asymptotes from the previous iteration
a0 = 1;                     % constant a_0 in mma formulation
a = zeros(m,1);             % constant a_i in mma formulation
cmma = 1e3*ones(m,1);       % constant c_i in mma formulation
d = zeros(m,1);             % constant d_i in mma formulation
subs = 200;                 % maximum number of subsolv iterations
```

A simple if-loop needs to be implemented [between line 108-109]:

```
    if sol == 0             % use optimality criterion method
```

The actual MMA algorithm can now be implemented [between line 120-121]:

```
        %% Method of moving asymptotes
    elseif sol == 1         % use mma solver
        xval = x(:); % store current design variable for mma
        if iter == 1     % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = Sens(:)*cscale; % store sensitivity for mma
        f = (sum(xF(:))/(vol*nx*ny)-1); % normalized constraint function
```

```
10          dfdx = Senc(:)'/(vol*ny*nx); % derivative of the constraint function
11          [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
12              mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
13              f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
14          xold2 = xold1;  % used by mma to monitor convergence
15          xold1 = x(:); % previous x, to monitor convergence
16          xnew = xF;       % update result
17          xnew(:) = xmma; % include restricted elements
18          xnew = reshape(xnew,ny,nx); % reshape xmma vector to original size
19          xF = xnew;  % update design variables
20      end
```

Some external function are called, which are attached to this report (D.1) and (D.2). [line 12 from above]

C.2 Basic Restrictions Add-in.m

As a follow-up from (B.1), an extra implementation of restricted regions is added (3.2.1). In the preamble, the restricted element parameters is implemented [between line 40-41]:

```
%% DEFINE ELEMENT RESTRICTIONS
shap = 0;                % [0 = no restrictions, 1 = circle, 2 = custom]
area = 0;                % [0 = no material (passive), 1 = material (active)]
nodr = (round(ny/2)+(0:ny:(nx-1)*ny)); % custom restricted nodes
```

In order to make a work-around for the restrictive regions, an add-in needs to be implemented, before the loop, by making this addition a number of lines needs to be replaced [replace line 76-78]:

```
%% DEFINE ELEMENT RESTRICTIONS
x = repmat(vol,ny,nx);   % initial material distribution
if shap == 0             % no restrictions
    efree = (1:nx*ny)'; % all elements are free
    eres= [];            % no restricted elements
elseif shap == 1         % restrictions
    rest = zeros(ny,nx); % pre-allocate space
    for i = 1:nx         % start loop
        for j = 1:ny     % for each element
            if sqrt((j-ny/2)^2+(i-nx/4)^2) < ny/2.5 % circular restriction
                rest(j,i) = 1; % write restriction
                if rest(j,i) == area % check for restriction
                    x(j,i) = area; % store restrictions in material distribution
                end
            end
        end
    end
    efree = find(rest ~= 1); % set free elements
    eres = find(rest == 1); % set restricted ellements
end
xF = x;               % set filtered design variables
xFree = xF(efree);       % define free design matrix
%% DEFINE STRUCTURAL
```

Only when using the MMA method, and already implemented all add-ins as described in (C.1), the restrictions vector needs to be initialized by the MMA method by replacing one variable [replace line 85]:

```
n = size(xFree(:),1);    % number of variables
```

To set restricted area on, using the Optimality Criteria, a small add-in is made [between line 114-115]:

```
        if shap == 1 % restriction is on
            xF(rest==1) = area; % set restricted area
        end
```

Only when using the MMA method, and already implemented all add-ins as described in (C.1) a smart implementations is made, while using the MMA solution, it can be helpful to skip all restrictive regions from the design space, and after the optimization simple plug them into the design space. This work-around should gain some time performance. The MMA code needs to be replaced [replace line 140-159]:

```
        %% Method of moving asymptotes
    elseif sol == 1     % use mma solver
        xval = xFree(:); % store current design variable for mma
        if iter == 1    % for the first iteration...
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = Sens(efree)*cscale; % store sensitivity for mma
        f = (sum(xF(:))/(vol*nx*ny)-1); % normalized constraint function
        dfdx = Senc(efree)'/(vol*ny*nx); % derivative of the constraint function
        [xmma,~,~,~,~,~,~,~,~,low,upp] = ...
            mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...
            f0,df0dx,df0dx2,f,dfdx,dfdx2,low,upp,a0,a,cmma,d,subs); % mma solver
        xold2 = xold1;  % used by mma to monitor convergence
        xold1 = xFree(:); % previous x, to monitor convergence
        xnew = xF;      % update result
        xnew(efree) = xmma; % include restricted elements
        xnew = reshape(xnew,ny,nx); % reshape xmma vector to original size
            xF = xnew;  % update design variables
        if shap == 1    % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end
    end
    xFree = xnew(efree); % set non-restricted area
```

C.3 Basic Load Cases Add-in.m

When applying multiple load cases, a small adjustment should be implemented (3.2.2). To adapt the program to perform the compliance and sensitivity analysis for the number of pre-defined load-cases, these lines should be replaced. [replace line 102-105]:

```
    %% Calculate compliance and sensitivity
    for i = 1:size(F,2) % for number of load cases
        Ui = U(:,i);    % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        c(iter) = c(iter) + sum(sum((Emin+xF.^p*(E-Emin)).*c0)); % calculate compliance
        Sens = Sens -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
    end
```

When performing multiple load cases, the force vector needs to be defined accordingly. The example shown in (Figure 3-4b) can be build by replacing the force vector. [replace line 35-38]:

```
%% DEFINE FORCE
Fe = [2*(nx+1)*(ny+1) 2*(nx+1)*(ny+1)-1];    % element node of applied force
Fn = [1 2];
Fv = [-1 1];
```

C.4 Basic Self-weight Add-in.m

When adding self-weight to the optimization problem some additions can be implemented (3.2.3). The self-weight parameters can be defined [between line 34-35]:

```
rho = 0e-3;              % density [0e-3]
g = 9.81;                % gravitational acceleration [9.81]
```

When gravity is involved, the force of the gravity needs to be added to the external force [between line 94-95]:

```
    %% Selfweight
    if rho ~= 0          % gravity is involved
        xP=zeros(ny,nx); % pre-allocate space
        xP(xF>0.25) = xF(xF>0.25).^p; % normal penalization
        xP(xF<=0.25) = xF(xF<=0.25).*(0.25^(p-1)); % below pseudo-density
        Fsw = zeros(N,1); % pre-allocate self-weight
        for i=1:nx*ny    % for each element, set gravitational...
            Fsw(dofmat(i,2:2:end))=Fsw(dofmat(i,2:2:end))-xF(i)*rho
                *9.81/4;
        end              % force to the attached nodes
        Fsw=repmat(Fsw,1,size(F,2)); % set self-weight for load cases
    elseif rho == 0      % no gravity
        xP = xF.^p;      % penalized design variable
        Fsw = 0;         % no selfweight
    end
    Ftot = F + Fsw;      % total force
```

To adapt the finite element analysis to the added self-weight, some replacements are needed [replace line 95-99]:

```
    %% Finite element analysis
    kK = reshape(Ke(:)*(Emin+xP(:)'*(E-Emin)),64*nx*ny,1); % create
       sparse vector k
    K = sparse(iK,jK,kK); % combine sparse vectors
    K = (K+K')/2;        % build stiffness matrix
    U(free,:) = K(free,free)\Ftot(free,:); % displacement solving
```

The sensitivity analysis needs to be adapted accordingly by replacing one line [replace line 105]:

```
        Sens = Sens + reshape(2*Ui(dofmat)*repmat([0;-9.81*rho/4],4,1),ny
            ,nx) -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
```

C.5 Basic Continuity Add-in.m

To implement the continuity approach, a number of changes needs to be made to the program (3.2.4). First, one line of specifying whether or not the user want the continuity method [between line 23-24]:

```
pcon = 0;            % use continuation method [0 = off, 1 = on]
```

Next, the continuity parameters can be defined [between line 26-27]:

```
pcinc = 1.03;            % penalty continuation increasing factor [1.03]
piter = 20;              % number of iteration for starting penalty [20]
```

The iteration loop needs to be adapted, in order to fulfill the maximum "continuity" iterations, these replacements should be made [replace line 91-94]:

```
while ((diff > tol) || (iter < piter+1)) && iter < miter  % convergence criterion not met
    iter = iter+1;       % define iteration
    if pcon == 1         % use continuation method
        if iter <= piter % first number of iterations...
            p = 1;       %... set penalty 1
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
        end
    elseif pcon == 0     % not using continuation method
        p = pen;         % set penalty
    end
```

In order to display the compliance of the iterations at a correct scale, one adaption should be made to the plotting code. [between line 171-172]:

```
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
```

The exact same addition should be added further on [between line 218-219]:

```
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
```

C.6 Basic Filters Add-in.m

During the report some filtering techniques are introduced. (3.2.5) both the sensitivity, density and the heaviside projection filter are implemented in this section. At first, determine the type of filter [between line 21-22]:

```
    fil = 0;              % filter method [0 = sensitivity filtering, 1 =
        density filtering, 2 = heaviside filtering]
```

Set the design space accordingly to the filter technique specified [between line 75-76]:

```
if fil == 0 || fil == 1 % sensitivity, density filter
    xF = x;             % set filtered design variables
elseif fil == 2         % heaviside filter
    beta = 1;           % hs filter
    xTilde = x;         % hs filter
    xF = 1-exp(-beta*xTilde)+xTilde*exp(-beta); % set filtered design
        space
end
```

Initialize the loop-number of β [between line 90-91]:

```
loopbeta = 1;           % initialize beta-loop
```

Make sure this β is updated during each loop [between line 92-93]:

```
    loopbeta = loopbeta +1; % iteration loop for hs filter
```

The sensitivity analysis needs to be adapted to the new filter inputs, by replacing the original sensitivity line [replace line 107]:

```
    if fil == 0         % optimality criterion with sensitivity filter
        Sens(:) = H*(x(:).*Sens(:))./Hs./max(1e-3,x(:)); % update
            filtered sensitivity
    elseif fil == 1     % optimality criterion with density filter
        Sens(:) = H*(Sens(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:)./Hs); % update filtered sensitivity of
            constraint
    elseif fil == 2     % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
```

```
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Senc(:) = H*(Senc(:).*dx(:)./Hs); % update filtered sensitivity of constraint
    end
```

After the Optimality Criteria, the design variables needs to be filtered accordingly by replacing the update step [replace line 114]:

```
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
```

Only when the MMA method is implemented (C.1), and only this implemented is done, replace the same line as above [replace line 158]:

```
            if fil == 0 % sensitivity filter
                xF = xnew; % updated result
            elseif fil == 1 % density filter
                xF(:) = (H*xnew(:))./Hs; % updated filtered density result
            elseif fil == 2 % heaviside filter
                xTilde(:)= (H*xnew(:))./Hs; % set filtered density
                xF(:) =1-exp(-beta*xTilde)+xTilde*exp(-beta); % updated result
            end
```

After the optimization step, update the β of the heaviside projection filter. [between line 122-123]:

```
if fil == 2 && beta < 512 && pen == p(end) && (loopbeta >= 50 || diff <= tol) % hs filter
        beta = 2*beta;    % increase beta-factor
        fprintf('beta now is %3.0f\n',beta) % display increase of b-factor
        loopbeta = 0;   % set hs filter loop to zero
        diff = 1;       % set convergence to initial value
    end
```

C.7 3D Add-in.m

Up to here, only two dimensions are take into account. However, there is an add-in available in order to upgrade the Basic (B.1) to three dimensions (3.3). This add-in consist of a lot of manipulations, replacements and additions. At first, define the number of lateral elements [between line 18-19]:

```
    nz = 5;              % number of elements lateral
```

Because the discretization can vary over time, it is helpful to clear the big X-matrix each run [between line 21-22]:

```
    clear X;             % clear the big X matrix
```

In order to force a black-white solution, a so-called gray-scale filter is implemented, the associated parameter and the option itself can be implemented to enable or disable the filter technique. [between line 27-28]:

```
graysc = 1;                 % use gray-scale filter [0 = off, 1 = on]
q = 1;                      % gray-scale parameter
qmax = 2;                   % maximum gray-scale parameter
plotiter = 5;               % gap of iterations used to plot or draw
    iterations
```

Because the third dimension cost a lot of computational time, it can be helpful to plot the iterations and graphics only partially by introducing the plotiter, which is a definition of the output steps of the iterations [replace line 34-36]:

```
plotiter = 5;             % gap of iterations used to plot or draw
    iterations
%% DEFINE OUTPUT
draw = 1;                 % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 1;                  % display iterations [0 = off, 1 = on, 2 =
    partial]
```

The number of elements is increased, by the introduction of a third dimension. As an example, to reproduce the example shown in Figure 3-7, change the load [replace line 36]:

```
Fe = (3*(nx+1)*(ny+1)-1)+(3*(nx+1)*(ny+1))*(0:nz)'; % element of force
    application
```

The number of elements is increased, by the introduction of a third dimension. As an example, to reproduce the example shown in Figure 3-7, change the fixed locations [replace line 40]:

```
fix = repmat((1:3*(ny+1))',1,nz+1)+repmat((0:nz)*3*(nx+1)*(ny+1),length
    ((1:3*(ny+1))),1);
fix = fix(:);                % fixed elements
```

The finite element analysis needs to be rebuild. The number of changes is big, so the best way is by making a replacement of the current preparation of the finite elements [replace line 41-48]:

```
%% PREPARE FINITE ELEMENT
N = 3*(nx+1)*(ny+1)*(nz+1); % total elements nodes
all = 1:3*(nx+1)*(ny+1)*(nz+1); % all degrees of freedom
free = setdiff(all,fix); % free degrees of freedom
A = [32 6 -8 6 -6 4 3 -6 -10 3 -3 -3 -4 -8;
    -48 0 0 -24 24 0 0 0 12 -12 0 12 12 12]; % fem
k = 1/72*A'*[1; nu];    % simple stiffness matrix
```

The next lines are ment to replace the element stiffness matrix and eventually introduce a new way to define the nodes and dof vectors and matrices [replace line 49-54]:

```
%% GENERATE SIX SUB-MATRICES AND THEN GET KE MATRIX
K1 = [k(1) k(2) k(2) k(3) k(5) k(5);
    k(2) k(1) k(2) k(4) k(6) k(7);
    k(2) k(2) k(1) k(4) k(7) k(6);
    k(3) k(4) k(4) k(1) k(8) k(8);
    k(5) k(6) k(7) k(8) k(1) k(2);
    k(5) k(7) k(6) k(8) k(2) k(1)]; % stiffness matrix
K2 = [k(9)  k(8)  k(12) k(6)  k(4)  k(7);
    k(8)  k(9)  k(12) k(5)  k(3)  k(5);
    k(10) k(10) k(13) k(7)  k(4)  k(6);
    k(6)  k(5)  k(11) k(9)  k(2)  k(10);
    k(4)  k(3)  k(5)  k(2)  k(9)  k(12)
    k(11) k(4)  k(6)  k(12) k(10) k(13)]; % stiffness matrix
K3 = [k(6)  k(7)  k(4)  k(9)  k(12) k(8);
    k(7)  k(6)  k(4)  k(10) k(13) k(10);
    k(5)  k(5)  k(3)  k(8)  k(12) k(9);
    k(9)  k(10) k(2)  k(6)  k(11) k(5);
    k(12) k(13) k(10) k(11) k(6)  k(4);
    k(2)  k(12) k(9)  k(4)  k(5)  k(3)]; % stiffness matrix
K4 = [k(14) k(11) k(11) k(13) k(10) k(10);
```

```
    k(11) k(14) k(11) k(12) k(9)  k(8);
    k(11) k(11) k(14) k(12) k(8)  k(9);
    k(13) k(12) k(12) k(14) k(7)  k(7);
    k(10) k(9)  k(8)  k(7)  k(14) k(11);
    k(10) k(8)  k(9)  k(7)  k(11) k(14)]; % stiffness matrix
K5 = [k(1) k(2)  k(8)  k(3) k(5)  k(4);
    k(2) k(1)  k(8)  k(4) k(6)  k(11);
    k(8) k(8)  k(1)  k(5) k(11) k(6);
    k(3) k(4)  k(5)  k(1) k(8)  k(2);
    k(5) k(6)  k(11) k(8) k(1)  k(8);
    k(4) k(11) k(6)  k(2) k(8)  k(1)]; % stiffness matrix
K6 = [k(14) k(11) k(7)  k(13) k(10) k(12);
    k(11) k(14) k(7)  k(12) k(9)  k(2);
    k(7)  k(7)  k(14) k(10) k(2)  k(9);
    k(13) k(12) k(10) k(14) k(7)  k(11);
    k(10) k(9)  k(2)  k(7)  k(14) k(7);
    k(12) k(2)  k(9)  k(11) k(7)  k(14)]; % stiffness matrix
Ke = 1/((nu+1)*(1-2*nu))*...
    [ K1  K2  K3  K4;
    K2'  K5  K6  K3';
    K3' K6  K5' K2';
    K4  K3  K2  K1'];   % element stiffness matrix
nodes = reshape(1:(nx+1)*(ny+1),1+ny,1+nx); % create node number matrix
nodes2 = reshape(nodes(1:end-1,1:end-1),ny*nx,1); % create node number matrix
nodes3 = 0:(ny+1)*(nx+1):(nz-1)*(ny+1)*(nx+1); % create node number matrix
nodes4 = repmat(nodes2,size(nodes3))+repmat(nodes3,size(nodes2)); % create node number matrix
dofvec = 3*nodes4(:)+1; % create dof vector
dofmat = repmat(dofvec,1,24)+repmat([0 1 2 3*ny+[3 4 5 0 1 2] -3 -2 -1 3*(ny+1)*(nx+1) + [0 1 2 3*ny + [3 4 5 0 1 2] -3 -2 -1]],nx*ny*nz,1); % create dof matrix
iK = kron(dofmat,ones(24,1))'; % build sparse i
jK = kron(dofmat,ones(1,24))'; % build sparse j
```

Now, the filter needs to redefined to account for the third dimension, here again, the number of changes is big, so a complete replacement is recommended [replace line 55-77]:

```
%% PREPARE FILTER
iH = ones(nx*ny*nz*(2*(ceil(rmin)-1)+1)^2,1); % build sparse i
jH = ones(size(iH));     % create sparse vector of ones
kH = zeros(size(iH));    % create sparse vector of zeros
m = 0;                   % index for filtering
for h = 1:nz             % for each element calculate...
    for i = 1:nx         % distance between elements'...
        for j = 1:ny     % centre for filtering
            r1 = (h-1)*nx*ny + (i-1)*ny+j; % sparse value 1
            for k2 = max(h-(ceil(rmin)-1),1):min(h+(ceil(rmin)-1),nz)  % centre of element
```

```
                for k = max(i-(ceil(rmin)-1),1):min(i+(ceil(rmin)-1),nx) % centre of element
                    for l = max(j-(ceil(rmin)-1),1):min(j+(ceil(rmin)-1),ny) % centre of element
                        r2 = (k2-1)*nx*ny + (k-1)*ny+l; % sparse value 2
                        m = m+1;      % update index for filtering
                        iH(m) = r1;   % sparse vector for filtering
                        jH(m) = r2;   % sparse vector for filtering
                        kH(m) = max(0,rmin-sqrt((i-k)^2+(j-l)^2)+(h-k2)^2); % weight factor
                    end
                end
            end
        end
    end
end
H = sparse(iH,jH,kH);   % build filter
Hs = sum(H,2);          % summation of filter
%% DEFINE STRUCTURAL
x = repmat(vol,ny,nx,nz); % initial material distribution
```

The constraint dots are pre-allocated, with the introduction of a third dimensions, the pre-allocations should be implemented also [between line 83-84]:

```
npz = zeros(length(fix),1)'; % pre-allocate constraint dots
```

The force dots are pre-allocated, with the introduction of a third dimensions, the pre-allocations should be implemented also [between line 85-86]:

```
npfz = zeros(length(Fe),1)'; % pre-allocate force dots
```

The loop is starting, when the gray-scale filter is enabled, this implementation uses a continuation method, in order to apply the correct gray-scale filter parameter [between line 94-95]:

```
    if gray == 1          % if grayscale is enabled
        if iter <= 15     % within 15 iterations
            q = 1;        % don't use grayscale
        else              % after 15 iterations
            q = min(qmax,1.01*q); % use continuation method to pick a gray-scale factor
        end
    end
```

The stiffness matrix is reshaped to account a third dimension, therefore the sparse vector k needs to replaced [replace line 96]:

```
kK = Ke(:)*(Emin+xF(:)'.^p*(E-Emin)); % create sparse vector k
```

The compliance and sensitivity analysis needs to be replaced also to account for the newly introduced dimension. This can be done by a simple replacement of the lines [replace line 102-106]:

```
%% Calculate compliance and sensitivity
c0 = reshape(sum((U(dofmat)*Ke).*U(dofmat),2),ny,nx,nz); % initial compliance
c(iter) = c(iter) + sum(sum(sum((Emin+xF.^p*(E-Emin)).*c0))); % calculate compliance
Sens = Sens -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
Senc = ones(ny,nx,nz); % set constraint sensitivity
```

When the gray-scale filter is enabled, the Optimality Criteria method should be used, to filter the calculated elements to achieve a black-white solution [replace line 113]:

```
    if q == 0        % don't use grayscale
        xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag))))); % update element densities
    elseif q == 1    % use grayscale
        xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-Sens./Senc/lag)).^q))); % update element densities
    end
```

The optimum solution within the Optimality Criteria should also account for the third dimension [replace line 115]:

```
    if sum(xF(:)) > vol*nx*ny*nz; % check for optimum
```

The third dimension should be stored also in the big X-matrix, by replacing the existing line [replace line 124]:

```
X(:,:,:,iter) = xF;    % each element value x is stored for each iteration
```

In order to show a graphical output of the results, a complete rewritten part of the code is needed. The changes from plot to plot3d are that big, all the results lines should be replaced with the following collection of lines [replace line 128-223]:

```
    %% Results
    if dis == 1            % display iterations
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
    elseif dis == 2        % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
                disiter = plotiter;
            elseif iter == disiter
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
                ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff)]);
        end

    end
    if draw == 1           % plot iterations
        figure(1)
        subplot(2,1,1)
        [nely,nelx,nelz] = size(xF);
        hx = 1; hy = 1; hz = 1;            % User-defined unit element size
        face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
        for k = 1:nelz
            z = (k-1)*hz;
            for i = 1:nelx
                xplot = (i-1)*hx;
                for j = 1:nely
                    y = nely*hy - (j-1)*hy;
                    if (xF(j,i,k) > 0.5)  % User-defined display density threshold
                        vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z; xplot+hx y z; xplot y z+hx;xplot y-hx z+hx; xplot+hx y-hx z+hx;xplot+hx y z+hx];
                        vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -vert(:,2,:);
                        patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k))]);
                        hold on;
                    end
                end
```

```
            end
        end
        axis equal; axis tight;
        set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
            'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','zcolor','w')
        view([30,30]);
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                nplotx = ceil(fix(i)/(3*(ny+1)));
                while nplotx > (nx+1)
                    nplotx = nplotx -(nx+1);
                end
                npx(i) = nplotx-1;
                nplot = ceil(fix(i)/3);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-1;
                npz(i) = 1-ceil(fix(i)/(3*(nx+1)*(ny+1)));
            end
            plot3(npx,npz,npy,'r.','MarkerSize',20)
            % Plot coloured dots for force application
            for i = 1:length(Fe)
                nplotx = ceil(Fe(i)/(3*(ny+1)));
                while nplotx > (nx+1)
                    nplotx = nplotx -(nx+1);
                end
                npfx(i) = nplotx-1;
                nplot = ceil(Fe(i)/3);
                while nplot > (ny)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot;
                npfz(i) = 1-ceil(Fe(i)/(3*(nx+1)*(ny+1)));
            end
            plot3(npfx,npfz,npfy,'g.','MarkerSize',20)
            drawnow;
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        ylim([0.95*yaxmin yaxmax])
        xlim([0 iter+10])
    elseif draw == 2          % plot parts of iterations
```

```
        if iter == 1 || iter == drawiter
            if iter == 1
                drawiter = plotiter;
            elseif iter == drawiter
                drawiter = drawiter + plotiter;
            end
            figure(1)
            subplot(2,1,1)
            [nely,nelx,nelz] = size(xF);
            hx = 1; hy = 1; hz = 1;            % User-defined unit element size
            face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
            for k = 1:nelz
                z = (k-1)*hz;
                for i = 1:nelx
                    xplot = (i-1)*hx;
                    for j = 1:nely
                        y = nely*hy - (j-1)*hy;
                        if (xF(j,i,k) > 0.5)  % User-defined display density threshold
                            vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z; xplot+hx y z; xplot y z+hx;xplot y-hx z+hx; xplot+hx y-hx z+hx;xplot+hx y z+hx];
                            vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -vert(:,2,:);
                            patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k))]);
                            hold on;
                        end
                    end
                end
            end
            axis equal; axis tight;
            set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
                'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','zcolor','w')
            view([30,30]);
            xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
            drawnow;
            hold on
            if iter == 1
                % Plot coloured dots for constraints
                for i = 1:length(fix)
                    nplotx = ceil(fix(i)/(3*(ny+1)));
                    while nplotx > (nx+1)
                        nplotx = nplotx -(nx+1);
                    end
                    npx(i) = nplotx-1;
                    nplot = ceil(fix(i)/3);
                    while nplot > (ny+1)
```

```
                        nplot = nplot-(ny+1);
                    end
                    npy(i) = nplot-1;
                    npz(i) = 1-ceil(fix(i)/(3*(nx+1)*(ny+1)));
                end
                plot3(npx,npz,npy,'r.','MarkerSize',20)
                % Plot coloured dots for force application
                for i = 1:length(Fe)
                    nplotx = ceil(Fe(i)/(3*(ny+1)));
                    while nplotx > (nx+1)
                        nplotx = nplotx -(nx+1);
                    end
                    npfx(i) = nplotx-1;
                    nplot = ceil(Fe(i)/3);
                    while nplot > (ny)
                        nplot = nplot-(ny+1);
                    end
                    npfy(i) = nplot;
                    npfz(i) = 1-ceil(Fe(i)/(3*(nx+1)*(ny+1)));
                end
                plot3(npfx,npfz,npfy,'g.','MarkerSize',20)
                drawnow;
            end
            % Plot compliance plot
            figure(1)
            subplot(2,1,2)
            plot(c(1:iter))
            xaxmax = c(iter);
            yaxmax = max(c);
            yaxmin = min(c(1:iter));
            ylim([0.95*yaxmin yaxmax])
            xlim([0 iter+10])
        end
    end
end
%% ONLY DISPLAY FINAL RESULT
if dis == 0 || dis == 2                % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter))
        ...
        ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',
            diff)]);
end
if draw == 0 || draw == 2              % plot final result
    figure(1)
    subplot(2,1,1)
    [nely,nelx,nelz] = size(xF);
    hx = 1; hy = 1; hz = 1;                % User-defined unit element size
    face = [1 2 3 4; 2 6 7 3; 4 3 7 8; 1 5 8 4; 1 2 6 5; 5 6 7 8];
    for k = 1:nelz
        z = (k-1)*hz;
        for i = 1:nelx
            xplot = (i-1)*hx;
            for j = 1:nely
```

```
                y = nely*hy - (j-1)*hy;
                if (xF(j,i,k) > 0.5)  % User-defined display density threshold
                    vert = [xplot y z; xplot y-hx z; xplot+hx y-hx z; xplot+hx y z; xplot y z+hx;xplot y-hx z+hx; xplot+hx y-hx z+hx;xplot+hx y z+hx];
                    vert(:,[2 3]) = vert(:,[3 2]); vert(:,2,:) = -vert(:,2,:);
                    patch('Faces',face,'Vertices',vert,'FaceColor',[0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k)),0.2+0.8*(1-xF(j,i,k))]);
                    hold on;
                end
            end
        end
    end
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'ZTick',[],'XTicklabel',[],...
        'YTicklabel',[],'ZTicklabel',[],'xcolor','w','ycolor','w','zcolor','w')
    view([30,30]);
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    % Plot coloured dots for constraints
    for i = 1:length(fix)
        nplotx = ceil(fix(i)/(3*(ny+1)));
        while nplotx > (nx+1)
            nplotx = nplotx -(nx+1);
        end
        npx(i) = nplotx-1;
        nplot = ceil(fix(i)/3);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-1;
        npz(i) = 1-ceil(fix(i)/(3*(nx+1)*(ny+1)));
    end
    plot3(npx,npz,npy,'r.','MarkerSize',20)
    % Plot coloured dots for force application
    for i = 1:length(Fe)
        nplotx = ceil(Fe(i)/(3*(ny+1)));
        while nplotx > (nx+1)
            nplotx = nplotx -(nx+1);
        end
        npfx(i) = nplotx-1;
        nplot = ceil(Fe(i)/3);
        while nplot > (ny)
            nplot = nplot-(ny+1);
        end
        npfy(i) = nplot;
        npfz(i) = 1-ceil(Fe(i)/(3*(nx+1)*(ny+1)));
    end
```

```
    plot3(npfx,npfz,npfy,'g.','MarkerSize',20)
    % Plot compliance plot
    figure(1)
    subplot(2,1,2)
    plot(c(1:iter))
    xaxmax = c(iter);
    yaxmax = max(c);
    yaxmin = min(c(1:iter));
    ylim([0.95*yaxmin yaxmax])
    xlim([0 iter+10])
end
toc                     % stop timer
```

C.8 Complaint Mechanisms Add-in.m

In this section, an add-in is made available to compute a variety of complaint mechanisms, as described in (3.4).
Using the previously described BASIC-code (B.3) as basis, the following lines should upgrade the code to calculate compliant mechanisms. At first, change the move limit for the Optimality Criteria, to allow for smaller steps in the optimization [replace line 30]:

```
move = 0.1;             % move limit for lagrange [0.1]
```

Since compliance mechanisms often consist of a symmetric problem, a new symmetry-function is build in. Also, an option for plotting small displacement is implemented [between line 33-34]:

```
sym = 2;                % symmetry [0 = off, 1 = x-axis, 2 = y-axis]
def = 1;                % plot deformations [0 = off, 1 = on]
```

In compliance mechanisms it is helpful to describe and calculate a displacement, in stead of a force. Therefore a stiffness for the input and output load can be defined [between line 42-43]:

```
Kin = 5e-2;             % spring stiffness at input force [5e-4]
Kout = 5e-4;            % spring stiffness at output force [5e-4]
```

The compliant mechanism case as described in 3-11a can be created by changing the force and supports [replace line 44-48]:

```
%% DEFINE FORCE
Uin = 2*(ny+1)-1;       % input force node
Uout = 2*(nx+1)*(ny+1)-1; % output force node
Fe = [Uin Uout];        % element of force application [Uin Uout]
Fn = [1 2];             % number of applied force locations [1 2]
Fv = [1 -1];            % value of applied force [1 -1]
%% DEFINE SUPPORTS
fix = [1:4 (Uin+1):2*(ny+1):(Uout+1)]; % fixed elements
```

In order to implement the stiffness at the input and output nodes, the predefined spring stiffness needs to be added to the existing stiffness matrix [between line 177-178]:

```
    K(Uin,Uin) = K(Uin,Uin) + Kin; % add input spring stiffness
    K(Uout,Uout) = K(Uout,Uout) + Kout; % add output spring stiffness
```

The adjoint load cases, as well as the new objective needs to be defined by replacing the original line [replace line 182-187]:

```
    U1 = U(:,1); U2 = U(:,2);
    c0 = reshape(sum((U1(dofmat)*Ke).*U2(dofmat),2),ny,nx);
    c(iter)  = U(Uout,1);
    Sens = p*(E-Emin)*xF.^(p-1).*c0;
```

When using the Optimality Criteria method, the convergence criteria is changed [replace line 203-205]:

```
        while (l2-l1)/(l1+l2) > 1e-4 && l2 > 1e-40; % start loop
            lag = 0.5*(l1+l2); % average of lagranian interval
            xnew = max(0,max(x-move,min(1,min(x+move,x.*(max(1e-10,-Sens
                ./lag)).^0.3)))); % update element densities
```

With this compliant mechanisms, in order to compare the in- and output displacements, it could be helpful to display these displacement in the workspace [replace line 269-271]:

```
disp([' Iter:' sprintf('%4i',iter) ' Uin:' sprintf('%6.2f',U(Uin)) ...
            ' Uout:' sprintf('%6.2f',c(iter)) ' Con:' sprintf('%6.2f',
                diff) ' Vol:' sprintf('%6.2f',mean(xF(:))) ' Diff:'
                sprintf('%6.3f',diff)]);
```

When symmetry case is implemented, this requires an additional figure, which displays the symmetric case scenario, using the live optimization [between line 316-317]:

```
        if sym ~= 0           % apply symmetry
            if sym == 1       % symmetry around x-axis
                xFlip = fliplr(xF);
                xFliplot = [xFlip xF];
            end
            if sym == 2       % symmetry around y-axis
                xFlip = flip(xF);
                xFliplot = [xF; xFlip];
            end
            colormap gray
```

```
            imagesc(1-xFliplot)
            axis equal
            axis off
        end
```

Also display the final results, when not using the display output [replace line 320-322]:

```
disp([' Iter:' sprintf('%4i',iter) ' Uin:' sprintf('%6.2f',U(Uin)) ...
            ' Uout:' sprintf('%6.2f',c(iter)) ' Con:' sprintf('%6.2f',
                diff) ' Vol:' sprintf('%6.2f',mean(xF(:))) ' Diff:'
                sprintf('%6.3f',diff)]);
```

As defined before, also an implementation for the fast implementation, without live optimization needs to be implemented [between line 367-368]:

```
        if sym ~= 0         % apply symmetry
            if sym == 1     % symmetry around x-axis
                xFlip = fliplr(xF);
                xFliplot = [xFlip xF];
            end
            if sym == 2     % symmetry around y-axis
                xFlip = flip(xF);
                xFliplot = [xF; xFlip];
            end
            colormap gray
            imagesc(1-xFliplot)
            axis equal
            axis off
        end
```

In case displacement needs to be plotted, a small add-in to create a movie for different input displacement is made available [between line 368-369]:

```
%% PLOTTING DISPLACEMENT (COMPLIANT MECHANISMS)
figure(2)
xaxis = get(gca,'XLim');
yaxis = get(gca,'YLim');
if def == 1
    figure(3)
    clear mov
    colormap(gray);
    Umov = 1;                   % start movie counter
    Umax = 0.0025;              % define maximum displacement
    for Udisp = linspace(0,Umax,10); % vary input displacement
```

```
        clf
        for ely = 1:ny      % plot displacements...
            for elx = 1:nx  % for each element...
                if xF(ely,elx) > 0 % exclude white regions for plotting purposes
                    n1 = (ny+1)*(elx-1)+ely;
                    n2 = (ny+1)* elx +ely;
                    Ue = -Udisp*U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
                    ly = ely-1; lx = elx-1;
                    xx = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx]';
                    yy = [-Ue(2,1)-ly -Ue(4,1)-ly -Ue(6,1)-ly-1 -Ue(8,1)-ly-1]';
                    patch([xx xx],[yy+ny -yy-ny],[-xF(ely,elx) -xF(ely,elx)],'LineStyle','none');
                end
            end
        end
        xlim(xaxis)
        ylim(yaxis-ny)
        drawnow
        mov(Umov) = getframe(3); % movie
        Umov = Umov +1;     % update counter
    end
    toc                     % stop timer
    movlip = flip(mov);     % create symmetry
    movull = [mov movlip];  % create symmetry
    FileName = ['Compliant_',datestr(now, 'ddmm_HHMMSS'),'.avi']; % dynamic filename
    movie2avi(movull, FileName, 'compression', 'None', 'FPS', 10); % save video
end
```

C.9 Design of Supports Add-in.m

In this section, an add-in is made available to include design of supports, as described in (4.1). Using the previously described BASIC-code (B.3) as basis, the following lines should upgrade the code to include computation of the optimal support design. At first, change the pre-amble to clear a big Z matrix, which stores the support design each iteration. [replace line 23]:

```
    clc; clf; close all; clear X; clear Z; % clear workspace
```

Because the design of support costs a lot of computational time, it can be helpful to plot the iterations and graphics only partially by introducing the plotiter, which is a definition of the output steps of the iterations [replace line 34-36]:

```
plotiter = 5;           % gap of iterations used to plot or draw iterations [5]
%% DEFINE OUTPUT
draw = 1;               % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 1;                % display iterations [0 = off, 1 = on, 2 = partial]
```

Design of support implementation require some additional input parameters. By implementing the following inputs. Additionally, the force as shown in Figure 4-4 needs to be changed. [replace line 43-44]:

```
%% DEFINE DESIGN OF SUPPORTS
supp = [1:ny (1:ny)+(nx-1)*ny ny:ny:nx*ny]; % support area [1:ny (1:ny)+(nx-1)*ny ny:ny:nx*ny]
supp = unique(supp);    % create unique support area
zvol = 0.2;             % maximum support area [0.2]
cost = 1;               % set maximum cost of supports [1]
k0 = 0.01;              % spring stiffness for support stiffness
q = 5;                  % penalty for support design [3]
zmin = 1e-9;            % minimum support design variable [1e-9]
dist = 0;               % cost distribution [0 = off, 1 = x-distributed, 2 = y-distribution]
%% DEFINE FORCE
Fe = 2:2*(ny+1):2*(ny+1)*(nx+1); % element of force application [2:2*(ny+1):2*(ny+1)*(nx+1)]
```

To implement the case as shown in Figure 4-4, the fixed supports need to be changed. The same example includes a solid road at the upper size, so an element restriction needs to be defined. [replace line 47-52]:

```
%% DEFINE SUPPORTS
fix = [1:2 2*(ny+1)*nx+(1:2)]; % define fixed locations [1:2 2*(ny+1)*nx+(1:2)]
%% DEFINE ELEMENT RESTRICTIONS
shap = 2;              % [0 = no restrictions, 1 = circle, 2 = custom]
area = 1;              % [0 = no material (passive), 1 = material (active)]
nodr = 1:ny:nx*ny;     % custom restricted nodes [1:ny:nx*ny]
```

To handle the element restriction, the following lines need to be inserted. [between line 104-105]:

```
elseif shap == 2          % custom restrictions
    rest = zeros(ny*nx,1); % pre-allocate space
    for i = 1:length(nodr) % write restriction
        resti = nodr(i); % write restriction
        rest(resti) = 1; % write restriction
    end
    rest = reshape(rest,ny,nx);
    for i = 1:nx              % start loop
        for j = 1:ny          % for each element
            if rest(j,i) == area % check for restriction
                x(j,i) = area; % store restrictions in material distribution
            end
        end
    end
```

The design of support implementation needs some more actions, which needs to be defined. Also, a distribution of costs can be defined over here. [between line 118-119]:

```
%% DESIGN OF SUPPORT DISTRIBUTION
xsiz = size(xFree(:),1); % size of design variables
zsiz = size(supp,2);     % size of support design variables
xzer = zeros(xsiz,1);    % empty row of zeros for mma usage
zzer = zeros(zsiz,1);    % empty row of zeros for mma usage
z = zeros(ny,nx);        % create design of support domain
z(supp) = zvol;          % plugin initial support design variables
zval = z';               % create vector of design variables
Si = 1;                  % counter
if dist == 1             % x-axis cost distribution
    Scos = linspace(1,cost,nx); % x-axis cost distribution
    Scost = zeros(nx,nx); % create multiplication matrix
    for i = 1:nx         % create weighted cost matrix
        Scost(Si,i) = Scos(i); % plug-in cost values
        Si = Si+1;       % update counter
    end
```

```
elseif dist == 2            % y-axis cost distribution
    Scos = linspace(1,cost,ny); % y-axis cost distribution
    Scost = zeros(ny,ny); % create multiplication matrix
    for i = 1:ny            % create weighted cost matrix
        Scost(Si,i) = Scos(i); % plug-in cost values
        Si = Si+1;          % update counter
    end
end
Adofsup = dofmat(supp,:); % degrees of freedom for support locations
Asup = unique(Adofsup(:)); % unique support locations
zF = z;                     % set design of support
zval = zval(zval ~= 0); % create configurable design of support vector
```

The number of constraints is increased, by the introduction of the design of supports. Consequentially, the number of variables is increased, since the design space is enlarged. A minimum variable of the support density is introduced, to prevent the solution to lock in a local optimum. [replace line 120-122]:

```
m = 2;                      % number of constraint functions
n = xsiz+zsiz;              % number of variables
xmin = [zeros(xsiz,1);zmin*ones(zsiz,1)]; % minimum values of x
```

A small error is fixed, to include the number of constraint functions. [replace line 127]:

```
dfdx2 = zeros(m,n);         % second derivative of the constraint function
```

A pre-allocation step is required for plotting purposes, as explained further on. [between line 139-140]:

```
npdx = zeros(length(nodes),1)'; % pre-allocate force dots
npdy = zeros(length(nodes),1)'; % pre-allocate force dots
```

Design of supports require an additional calculation of the springs stiffnesses of the supports. This stiffness tensor needs to be calculated each loop and is added to the external stiffness tensor, resulting in a final stiffness matrix for that iteration. [replace line 178]:

```
    Kfvec = zeros(2*(ny+1)*(nx+1),1); % build zeros support vector
    for i = 1:length(supp) % for each support element...
        dofsup = dofmat(supp(i),:); %...find the corresponding dof
        for j = 1:length(dofsup) % calculate new stiffness vector
```

```
            Kfvec(dofsup(j)) = Kfvec(dofsup(j))+(zF(supp(i))^q)*k0;
        end
    end
    Kf = spdiags(Kfvec,0,2*(ny+1)*(nx+1),2*(ny+1)*(nx+1)); % create diagonal Kf
    Kt = K+Kf;            % update total force
    U(free,:) = Kt(free,free)\Ftot(free,:); % displacement solving
```

The added constraint requires an additional sensitivity analysis. Also, the objective is changed, since it includes now the design of supports factor. By replacing the following lines the compliance and subsequent sensitivities are correctly calculated. [replace line 181-188]:

```
    Senz = 0;             % set constraint sensitivity to zero
    %% Calculate compliance and sensitivity
    for i = 1:size(Fn,2) % for number of load cases
        Ui = U(:,i);     % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        cz0 = reshape(sum((Ui(dofmat)*k0).*Ui(dofmat),2),ny,nx); % initial support compliance
        c(iter) = c(iter) + sum(sum((Emin+xF.^p*(E-Emin)).*c0)) + sum(sum((zF.^q).*cz0)); % calculate compliance
        Sens = Sens + reshape(2*Ui(dofmat)*repmat([0;-9.81*rho/4],4,1),ny,nx) -p*(E-Emin)*xF.^(p-1).*c0; % sensitivity
        Senz = Senz + -q*zF.^(q-1).*cz0; % calculate sensitivity to support variable
    end
    Senc = ones(ny,nx); % set constraint sensitivity
    if dist == 0
        Sencz = ones(ny,nx);
    elseif dist == 1
        Sencz =ones(ny,nx)*Scost; % set weighted cost constraint sensitivity
    elseif dist == 2
        Sencz = Scost*ones(ny,nx); % set weighted cost constraint sensitivity
    end
```

The MMA solver as described in C.1 needs some changes. The design variable space is enlarged, by the introduction of the support design. The sensitivities of the support constraint is now added to the MMA solver. A cost distribution is used in the MMA-solver, to get the optimal result with respect to the cost function objective. [replace line 225-232]:

```
        xval = [xFree(:);zval(:)]; % store current design variable for mma
        if iter == 1    % for the first iteration...
```

```
            cscale = 1/c(iter); % ...set scaling factor for mma solver
        end
        f0 = c(iter)*cscale; % objective at current design variable for mma
        df0dx = [Sens(efree)*cscale; Senz(supp)'*cscale]; % store sensitivity for mma
        if dist == 0    % no cost distribution
            Scosts = zF; % cost-funcion no influence
        elseif dist == 1 % x-axis cost distribution
            Scosts = zF*Scost; % update weighted constraint function
        elseif dist == 2 % y-axis cost distribution
            Scosts = Scost*zF; % update weighted constraint function
        end
        f = [(sum(xF(:))/(vol*nx*ny)-1);(sum(Scosts(supp))/(zvol*size(supp,2))-1)]; % normalized constraint function
        dfdx = [Senc(efree)'/(vol*ny*nx) zzer'; xzer' Sencz(supp)/(zvol*size(supp,2))]; % derivative of the constraint function
```

The output of the MMA solver is changed, so different commands are needed to get the right results. The output form the MMA solver is received into density and support design. [replace line 237-240]:

```
        xold1 = [xFree(:);zval(:)]; % previous x, to monitor convergence
        xnew = xF;      % update result
        xnew(efree) = xmma(1:xsiz); % include restricted elements
        znew = zF;      % update design result
        znew(supp) = xmma(xsiz+1:end); % include mma solved supports
        xnew = reshape(xnew,ny,nx); % reshape xmma vector to original size
        znew = reshape(znew,ny,nx); % reshape support vector to original size
```

To enable the restriction, after the filtering step, as well as update the new design of support, requires an additional step, which is found here. [replace line 249-253]:

```
        if shap == 1 || shap == 2   % if restrictions enableed
            xF(rest==1) = area; % set restricted area
        end
        zF(:) = znew(:); % update support variables
        zval = znew(supp); % update support variables
    end
```

Substitute here the correct support design variables. [between line 256-257]:

```
    z = znew;           % update support design variable
```

Store each support variable in a big Z-matrix for each iteration. [between line 265-266]:

```
    Z(:,:,iter) = zF;   % each support variable is stored for each iteration
```

Make sure the big Z-matrix is stored to the workspace. [between line 267-268]:

```
    assignin('base', 'Z', Z); % each iteration (3rd dimension)
```

The introduced plotiter, needs some different output setting. The output is hold and outputted each plotiter iteration. Also, for each display setting, the amount of support material is shown. [replace line 270-271]:

```
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff) ' ZVol:' sprintf('%6.3f',mean(Scosts(supp)))]);
    elseif dis == 2     % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
                disiter = plotiter;
            elseif iter == disiter
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
                ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',diff) ' ZVol:' sprintf('%6.3f',mean(Scosts(supp)))]);
        end
```

The support design elements can be plotted using blue dots. Using a threshold value of 0.99 to determine whether or not to plot a support design element. [between line 304-305]:

```
        % Plot coloured dots for design of supports
        for i = 1:nx*ny
            if zF(i) > 0.99 % treshold for plotting supports
                if ceil(i/ny) == nx
                    npdx(i) = ceil(i/ny) + 0.5;
```

```
                elseif ceil(i/ny) == 1
                    npdx(i) = ceil(i/ny) -0.5;
                else
                    npdx(i) = ceil(i/ny);
                end
                nplot = i;
                while nplot > ny
                    nplot = nplot-ny;
                end
                if nplot == ny
                    npdy(i) = nplot+0.5;
                elseif nplot == 1
                    npdy(i) = nplot-0.5;
                else
                    npdy(i) = nplot;
                end
            end
        end
        if exist('Dos(1)') %#ok<EXIST>
            delete(Dos(1))
        end
        if exist('npdx') %#ok<EXIST>
            Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize',20);
            clear npdx; clear npdy;
            uistack(Dos,'bottom')
        end
```

When enabling partial drawing, the following lines needs to be added into the code, to work around with this method. [between line 316-317]:

```
    elseif draw == 2    % plot parts of iterations
        if iter == 1 || iter == drawiter
            if iter == 1
                drawiter = plotiter;
            elseif iter == drawiter
                drawiter = drawiter + plotiter;
            end
            figure(1)
            subplot(2,1,1)
            colormap(gray); imagesc(1-xF);
            set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
                'YTicklabel',[],'xcolor',[0.7 0.7 0.7],'ycolor',[0.7 0.7 0.7])
            xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
            axis equal; axis tight
            drawnow;
            hold on
            if iter == 1
                % Plot coloured dots for force application
```

```
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            plot(npfx,npfy,'g.','MarkerSize',20)
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for design of supports
        for i = 1:nx*ny
            if zF(i) > 0.99 % treshold for plotting supports
                if ceil(i/ny) == nx
                    npdx(i) = ceil(i/ny) + 0.5;
                elseif ceil(i/ny) == 1
                    npdx(i) = ceil(i/ny) -0.5;
                else
                    npdx(i) = ceil(i/ny);
                end
                nplot = i;
                while nplot > ny
                    nplot = nplot-ny;
                end
                if nplot == ny
                    npdy(i) = nplot+0.5;
                elseif nplot == 1
                    npdy(i) = nplot-0.5;
                else
                    npdy(i) = nplot;
                end
            end
        end
        if exist('Dos(1)') %#ok<EXIST>
            delete(Dos(1))
        end
        if exist('npdx') %#ok<EXIST>
            Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize',20);
            clear npdx; clear npdy;
            uistack(Dos,'bottom')
        end
        % Plot compliance plot
```

```
            figure(1)
            subplot(2,1,2)
            plot(c(1:iter))
            set(gca,'YTick',[],'YTicklabel',[])
            xlabel('Iterations')
            ylabel('Compliance')
            xaxmax = c(iter);
            yaxmax = max(c);
            yaxmin = min(c(1:iter));
            if pcon == 0
                yaxmax = mean([yaxmin yaxmax]);
            end
            ylim([0.95*yaxmin yaxmax])
            xlim([1 min(iter+10,miter)])
        end
```

When disabling outputs, the design of support volume still needs to be displayed, at the end of the optimization process. [replace line 320-324]:

```
if dis == 0 || dis == 2 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter))
        ...
        ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Diff:' sprintf('%6.3f',
            diff) ' ZVol:' sprintf('%6.3f',mean(Scosts(supp)))]);
end
if draw == 0 || draw == 2 % plot final result
```

When disabling outputs, the support design elements still can be plotted using blue dots. Using a threshold value to determine whether or not to plot a support design element. [between line 353-354]:

```
        % Plot coloured dots for design of supports
        for i = 1:nx*ny
            if zF(i) > 0.99 % treshold for plotting supports
                if ceil(i/ny) == nx
                    npdx(i) = ceil(i/ny) + 0.5;
                elseif ceil(i/ny) == 1
                    npdx(i) = ceil(i/ny) -0.5;
                else
                    npdx(i) = ceil(i/ny);
                end
                nplot = i;
                while nplot > ny
                    nplot = nplot-ny;
                end
                if nplot == ny
                    npdy(i) = nplot+0.5;
```

```
                elseif nplot == 1
                    npdy(i) = nplot-0.5;
                else
                    npdy(i) = nplot;
                end
            end
        end
        if exist('Dos(1)') %#ok<EXIST>
            delete(Dos(1))
        end
        if exist('npdx') %#ok<EXIST>
            Dos = plot(nonzeros(npdx),nonzeros(npdy),'b.','MarkerSize'
                ,20);
            clear npdx; clear npdy;
            uistack(Dos,'bottom')
        end
```

C.10 Design of Actuator Placement Add-in.m

In this section, an add-in is made available to include design of forces, as described in (5). Using the previously described BASIC-code (B.3) as basis, the following lines should upgrade the code to include computation of the optimal placement of actuator design.
At first, in order to disable topology optimization, the volume can be fixed at a total void regime. [replace line 19]:

```
    vol = 1;                % volume fraction [0-1]
```

Next, change the pre-amble to clear a big W matrix, which stores the force design for each iteration. [replace line 23]:

```
    clc; clf; close all; clear X; clear W; % clear workspace
```

Now to enable finite difference check, and a way to enable or disable this finite difference check method. In the pre-amble a variable for this check can be build in. [between line 27-28]:

```
fincheck = 1;               % finite difference check [0 = off, 1 = on, 2 = break]
```

Because the design of actuator placement take a lot of computational time, it can be helpful to plot the iterations and graphics only partially by introducing the plotiter, which is a definition of the output steps of the iterations. Also, it is possible to create a deformed shape after the optimization. A threshold factor for plotting the forces, as well as a small perturbation value are implemented. [replace line 34-36]:

```
plotiter = 5;               % gap of iterations used to plot or draw iterations [5]
def = 0;                    % plot deformations [0 = off, 1 = on, 2 = play video]
wplot = 0.20;               % define treshold factor of Fmax for force plot [0.20]
h = 1e-6;                   % perturbation value for finite difference method [1e-6]
%% DEFINE OUTPUT
draw = 1;                   % plot iterations [0 = off, 1 = on, 2 = partial]
dis = 1;                    % display iterations [0 = off, 1 = on, 2 = partial]
```

The actual implementation of the design of actuator placement is made here. The minimal force constraint and maximum force per actuator can be defined here, as well as setting an area for the objective. [between line 48-49]:

```
%% DEFINE DESIGN OF ACTUATOR
Fmaxnode = 1;            % define max force per node [1]
Fmin = -1;               % minimal force constraint [1]
sen = 5;                 % penalty for actuator design [5]
if abs(Fmaxnode) > abs(Fmin) % check for force model
    Fmma = -Fmin;        % use Fmin as maximum xmma value
else
    Fmin = Fmin/Fmaxnode; % use fraction for constraint function
    Fmma = Fmaxnode;     % use maximum force per node as maximum xmma
        value
end
Uarray = 1:2*(nx+1)*(ny+1); % define objective area
```

The initial distribution of the actuator lay-out is implemented here. Also, the MMA parameters are changed to handle with the force, since the force should be negative. [replace line 119-123]:

```
%% DESIGN OF ACTUATOR DISTRIBUTION
wsiz = size(Fe,2);       % size of actuator variables
wzer = zeros(wsiz,1);    % empty row of zeros for mma usage
wF = F;                  % plugin initial force distribution
wval = F(Fe);            % create vector of design variables
%% DEFINE MMA PARAMETERS
m = 1;                   % number of constraint functions
n = wsiz;                % number of variables
xmin = -1*ones(n,1);     % minimum values of x
xmax = -(1e-9/Fmma)*ones(wsiz,1); % maximum values of x
```

Additional pre-allocation of variables is needed, to speed up the optimization program. [between line 139-140]:

```
npdx = zeros(length(nodes),1)'; % pre-allocate force dots
npdy = zeros(length(nodes),1)'; % pre-allocate force dots
L = zeros(N,1);          % pre-allocate selection tensor
labda = zeros(N,1);      % pre-allocate lagrange multiplier
Fi = zeros(1,N);         % pre-allocate force selection vector
Cons = zeros(miter,1);   % pre-allocate constraint vector
```

To be able to make a selection of certain area, which needs to be optimized, a selection vector can be defined. This vector makes it easy to switch between horizontal and vertical displacements. [between line 141-142]:

```
%% DEFINE SELECTION TENSOR
for j = Uarray            % for each iteration..
    if mod(j,2) == 0      % ...check for horizontal or vertical
        L(j) = 1;         % vertical selection value
    else
        L(j) = 1;         % horizontal selection value
    end
end
```

To be able to penalize the force, a newly introduced penalty approach is made. The calculation of the continuation method should be changed, to include correct penalization of the force. [replace line 153-157]:

```
            s = 0.5;      %... set penalty 0.5 for actuator design
        elseif iter > piter % after a number of iterations...
            p = min(pen,pcinc*p); % ... set continuation penalty
            s = min(sen,1.06*s); % ... set continuation penalty actuator
                design
        end
    elseif pcon == 0      % not using continuation method
        p = pen;          % set penalty
        s = sen;          % set penalty actuator design
```

To actually calculate the penalization of the force, and subsequently scale the force to force the MMA solver to search the optimal value between 0 and 1, some adjustments should be made. [replace line 173]:

```
    wP = atan(s*wF)/atan(s); % penalized actuator variable
    Ftot = Fmma*(wP) + Fsw; % total force
```

Since the finite difference is built in inside the calculation loop, some pre-allocation steps are needed inside this loop. [between line 180-181]:

```
    Senw = 0;             % set constraint sensitivity to zero
    Cons(iter) = 0;       % set constraint to zero
    Senc = ones(1,N);     % set constraint sensitivity
```

The actual finite difference check inside the loop needs to be included. Also, the objective is updated here to optimize towards minimum displacement. The associated sensitivities are calculated and filtered accordingly. [replace line 182-198]:

```
    for i = 1:size(Fn,2) % for number of load cases
        Ui = U(:,i);    % displacement per load case
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        c(iter) = c(iter) - sum(sum(Ui)); % objective
        labda(free) = -K(free,free)\L(free); % calculate lagrange multiplie
        Fi(Fe) = (Fmma*s./((s^2*wF(Fe).^2+1)*(atan(s))));% force selection vector
        FFi = spdiags(Fi',0, N,N); % force selection vector
        Sens = Sens + FFi(Fe,Fe)*labda(Fe); % calculate sensitivity
        Cons(iter) = Cons(iter) + Fmma*(Fmin/sum(sum(wF)))-1; % calculate constraint
        dCdf = Senc(Fe)'*Fmma*full(Fmin)/-(sum(sum(full(wF))))^2; % constraint sensitivity
        if iter == 2   % finite difference method
            wF1 = wF;  % store first force vector
            [~,S1] = max(abs(Sens(:))); % calculate maximum sensitivity value
            Sens1 = Sens(S1); % store maximum sensitivity value
            [~,S2] = max(abs(dCdf(:))); % calculate maximum sensitivity value
            Sens2 = dCdf(S2); % store maximum sensitivity value
        end
    end
    if fil == 0          % optimality criterion with sensitivity filter
        Sens(:) = Sens; % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity
    elseif fil == 1      % optimality criterion with density filter
        Sens(:) = Sens; % update filtered sensitivity of constraint
        Sencw(:) = Senc; % update filtered sensitivity of constraint
    elseif fil == 2      % optimality criterion with heaviside filter
        dx = beta*exp(-beta*xTilde)+exp(-beta); % update hs parameter
        Sens(:) = H*(Sens(:).*dx(:)./Hs); % update filtered sensitivity
        Sencw(:) = Senc; % update filtered sensitivity of constraint
    end
```

The MMA solver can here be adjusted to store the current force distribution as design variable. [replace line 225]:

```
        xval = wval(:); % store current design variable for mma
```

Since the sensitivity and constraint values are calculated inside the loop, some adjustments should be made inside the MMA loop. [replace line 230-232]:

```
        df0dx = Sens*cscale; % store sensitivity for mma
        f = Cons(iter);      % normalized constraint function
        dfdx = dCdf;         % derivative of normalized constraint function
```

The MMA solver should be updated to update the force design. [replace line 237-240]:

```
        xold1 = wval(:); % previous x, to monitor convergence
        xnew = xF;      % update density result
        wnew = wF;      % update force result
        wnew(Fe) = xmma(1:end); % include mma result
```

The update of the force distribution is here built in. Also, an adjustment is made for the tolerance, to check the difference between force vectors. [replace line 253-256]:

```
        wF(:) = wnew(:); % update force variables
        wval = wnew(Fe); % update force variables
    end
    diff = max(abs(full(Fmma*wnew(:))-full(F(:)))); % difference of maximum element change
    F = Fmma*wnew;      % update design variable
```

The actual finite difference check is made after each loop. Here, the values are changed using a small perturbation. The pre-allocation of fincheck can be used to check, stop, or skip the finite difference method. [between line 262-263]:

```
    %% Finite difference method
    if (fincheck == 1 || fincheck == 2) % check for finite difference method
        if iter == 2    % on first findif iteration
            wF = wF1;   % store first findif result...
            wF(Fe(S1)) = wF1(Fe(S1))+h; %...and add a small pertubation
        elseif iter == 3 % on second findif iteration
            findif = (c(3)-c(2))/h; % calculate finite difference method
            Sensdif = abs(max((findif-Sens1)/Sens1,(Sens1-findif)/findif)); % maximum difference
            if Sensdif > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif)])
                if fincheck == 2 % when fincheck is not accomplished...
```

```
                    break %... break the loop and stop the code
                end
            end
            wF = wF1;      % store first findif result...
            wF(Fe(S2)) = wF1(Fe(S2))+h; %...and add a small pertubation
        elseif iter == 4  % on third findif iteration
            findif2 = (Cons(4)-Cons(2))/h; % calculate finite difference method
            Sensdif2 = abs(max((findif2-Sens2)/Sens2,(Sens2-findif2)/findif2)); % maximum difference
            if Sensdif2 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif2)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
        end
    end
```

Make a separate value index, which stores each force distribution for each iteration. [between line 265-266]:

```
    W(:,:,iter) = full(wF);    % each force variable is stored for each iteration
```

Additionally, store this variable to the workspace. [between line 267-268]:

```
    assignin('base', 'W', W); % each iteration (3rd dimension).
```

The introduced plotiter, needs some different output setting. The output is hold and outputted each plotiter iteration. Also, for each display setting, the amount of total force is shown. [replace line 270-271]:

```
        disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
            ' Ftot:' sprintf('%6.3f',sum(full(wP(:)))) ' Diff:' sprintf('%6.3f',diff)]);
    elseif dis == 2     % display parts of iterations
        if iter == 1 || iter == disiter
            if iter == 1
                disiter = plotiter;
            elseif iter == disiter
```

```
                disiter = disiter + plotiter;
            end
            disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
                ' Ftot:' sprintf('%6.3f',sum(full(wP(:)))) ' Diff:' sprintf('%6.3f',diff)]);
        end
```

The force distribution can be plotted by blue dots and attached arrows. Using a threshold value of *wplot* to determine whether or not to plot a force application. [replace line 277-304]:

```
        set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
            'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
        xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
        axis equal; axis tight
        drawnow;
        hold on
        if iter == 1
            % Plot coloured dots for constraints
            for i = 1:length(fix)
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for force application
        Fmaxplot = min(min(full(F)));
        for i = 1:length(Fe)
            if F(Fe(i)) < wplot*Fmaxplot
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
        end
        if iter > 1
            delete(Dof)
        end
        if exist('npfx','var')
            Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
            clear npfx; clear npfy;
```

```
            uistack(Dof,'top')
        end
        % Plot coloured arrows for force application
        if (((diff < tol) && iter >= piter+1) || iter >= miter)
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            for i = 1:length(Fe)
                if F(Fe(i)) < wplot*Fmaxplot
                    headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*Fmaxplot)));
                    if mod(Fe(i),2)
                        arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                    else
                        arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                    end
                end
            end
        end
```

When enabling partial drawing, the following lines needs to be added into the code, to work around with this method. [between line 316-317]:

```
    elseif draw == 2    % plot parts of iterations
        if iter == 1 || iter == drawiter
            if iter == 1
                drawiter = plotiter;
            elseif iter == drawiter
                drawiter = drawiter + plotiter;
            end
            figure(1)
            subplot(2,1,1)
            colormap(gray); imagesc(1-xF);
            set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
                'YTicklabel',[],'xcolor',[0.7 0.7 0.7],'ycolor',[0.7 0.7 0.7])
            xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
            axis equal; axis tight
            drawnow;
            hold on
            if iter == 1
                % Plot coloured dots for constraints
                for i = 1:length(fix)
```

```
                npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
                nplot = ceil(fix(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npy(i) = nplot-0.5;
            end
            plot(npx,npy,'r.','MarkerSize',20)
        end
        % Plot coloured dots for force application
        Fmaxplot = min(min(full(F)));
        for i = 1:length(Fe)
            if F(Fe(i)) < wplot*Fmaxplot
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
        end
        if iter > 1
            delete(Dof)
        end
        if exist('npfx','var')
            Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
            clear npfx; clear npfy;
            uistack(Dof,'top')
        end
        % Plot coloured arrows for force application
        if (((diff < tol) && iter >= piter+1) || iter >= miter)
            for i = 1:length(Fe)
                npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
                nplot = ceil(Fe(i)/2);
                while nplot > (ny+1)
                    nplot = nplot-(ny+1);
                end
                npfy(i) = nplot-0.5;
            end
            for i = 1:length(Fe)
                if F(Fe(i)) < wplot*Fmaxplot
                    headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*Fmaxplot)));
                    if mod(Fe(i),2)
                        arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                    else
                        arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                    end
                end
```

```
            end
        end
        % Plot compliance plot
        figure(1)
        subplot(2,1,2)
        plot(c(1:iter))
        set(gca,'YTick',[],'YTicklabel',[])
        xlabel('Iterations')
        ylabel('Compliance')
        xaxmax = c(iter);
        yaxmax = max(c);
        yaxmin = min(c(1:iter));
        if pcon == 0
            yaxmax = mean([yaxmin yaxmax]);
        end
        ylim([0.95*yaxmin yaxmax])
        xlim([1 min(iter+10,miter)])
    end
```

When disabling outputs, the design of force placement still needs to be displayed, at the end of the optimization process. [replace line 320-353]:

```
%% ONLY DISPLAY FINAL RESULT
if dis == 0 || dis == 2 % display final result
    disp([' Iter:' sprintf('%4i',iter) ' Obj:' sprintf('%10.4f',c(iter)) ...
        ' Ftot:' sprintf('%6.3f',sum(full(wP(:)))) ' Diff:' sprintf('%6.3f',diff)]);
end
if draw == 0 || draw == 2 % plot final result
    figure(1)
    subplot(2,1,1)
    colormap(gray); imagesc(1-xF);
    axis equal; axis tight;
    set(gca,'XTick',[],'YTick',[],'XTicklabel',[],...
        'YTicklabel',[],'xcolor','[0.7 0.7 0.7]','ycolor','[0.7 0.7 0.7]')
    xlabel(sprintf('c = %.2f',c(iter)),'Color','k')
    drawnow;
    hold on
    % Plot coloured dots for constraints
    for i = 1:length(fix)
        npx(i) = ceil(fix(i)/(2*(ny+1)))-0.5;
        nplot = ceil(fix(i)/2);
        while nplot > (ny+1)
            nplot = nplot-(ny+1);
        end
        npy(i) = nplot-0.5;
    end
    plot(npx,npy,'r.','MarkerSize',20)
```

```
    % Plot coloured dots for force application
    Fmaxplot = min(min(full(F)));
    for i = 1:length(Fe)
        if F(Fe(i)) < wplot*Fmaxplot
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
    end
    if iter > 1
        delete(Dof)
    end
    if exist('npfx','var')
        Dof = plot(npfx(npfx(:)>0),npfy(npfy(:)>0),'b.','MarkerSize',20);
        clear npfx; clear npfy;
        uistack(Dof,'top')
    end
    % Plot coloured arrows for force application
    if (((diff < tol) && iter >= piter+1) || iter >= miter)
        for i = 1:length(Fe)
            npfx(i) = ceil(Fe(i)/(2*(ny+1)))-0.5;
            nplot = ceil(Fe(i)/2);
            while nplot > (ny+1)
                nplot = nplot-(ny+1);
            end
            npfy(i) = nplot-0.5;
        end
        for i = 1:length(Fe)
            if F(Fe(i)) < wplot*Fmaxplot
                headsize = 1/sqrt(length(nonzeros(F(Fe)<0.5*Fmaxplot)));
                if mod(Fe(i),2)
                    arrowz([npfx(i) npfy(i)],[npfx(i)+0.5*ny*F(Fe(i))/Fmaxplot npfy(i)],headsize,2,[0 0 1])
                else
                    arrowz([npfx(i) npfy(i)],[npfx(i) npfy(i)+0.5*ny*F(Fe(i))/Fmaxplot],headsize,2,[0 0 1])
                end
            end
        end
    end
```

To show deformed shape of the structure, please add-in the following lines. [between line 368-369]:

```
%% PLOTTING DISPLACEMENT
if (def == 1 || def == 2)
    FileName = ['Displacement_',datestr(now, 'ddmm_HHMMSS'),'.avi']; %
```

```
        dynamic filename
    vidObj = VideoWriter(FileName);
    vidObj.FrameRate = 3;
    figure(1)
    subplot(2,1,1)
    xaxis = get(gca,'XLim');
    yaxis = get(gca,'YLim');
    open(vidObj);
    figure(2)
    clear mov
    colormap(gray);
    Umov = 1;                % start movie counter
    Uim = zeros(5642,1);
    Uim(2:2:end) = Ui(2:2:end);
    Uim(1:2:end) = -Ui(1:2:end);
    Umax = -10/max(abs(Uim)); % define maximum displacement
    steps = 1;               % number of displacement steps
    set(gca,'nextplot','replacechildren');
    Upatch = zeros(nx*ny,1);
    for i = 1:ny*nx
        Uindex = 2*(i+floor((i-1)/ny))-1+[1 2 2*(ny+1)+1 2*(ny+1)+3];
        Upatch(i,1) = mean(U(Uindex));
    end
    Upatch = reshape(Upatch,ny,nx);
    Upatchmin = min(min(Upatch));
    Upatchnorm = -Upatch/Upatchmin;
    for Udisp = linspace(Umax/steps,Umax,steps) % vary input displacement
        clf
        for ely = 1:ny       % plot displacements...
            for elx = 1:nx   % for each element...
                if xF(ely,elx) > 0 % exclude white regions for plotting
                    purposes
                    n1 = (ny+1)*(elx-1)+ely;
                    n2 = (ny+1)* elx +ely;
                    Ue = Udisp*Uim([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2
                        +2; 2*n1+1;2*n1+2],1);
                    ly = ely-1; lx = elx-1;
                    xx = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx
                        ]';
                    yy = [-Ue(2,1)-ly -Ue(4,1)-ly -Ue(6,1)-ly-1 -Ue(8,1)-
                        ly-1]';
                    patch([xx xx],[yy yy],[Upatchnorm(ely,elx) Upatchnorm
                        (ely,elx)],'LineStyle','none');

                end
            end
        end
        colormap jet         % for better interpation...
        axis tight
        axis equal
        xticks([0 15 30 45 60 75 90])
        box on
        colorbar
```

```
        drawnow                % ...draw coloured densities
        currFrame = getframe; % get current frame...
        writeVideo(vidObj,currFrame); % ... write to video file
    end
    close(vidObj);
end
if def == 2                    % when def equals 2...
    implay(FileName)               % ...open Matlab Movie Player
end
toc
```

C.11 Topology Add-in.m

In this section, an add-in is made available to include topology, to work with design of actuator placement, as described in (5.4). Using the previously described Actuator Placement-code (C.10) as basis, the following lines should upgrade the code to include computation of the optimal placement of actuator design. The implementation of topology optimziation will result in a much longer computational time.
First, the volume constraint can now be used, so a change to the volume should be made. [replace line 19]:

```
    vol = 0.2;           % volume fraction [0-1]
```

Since the topology is now included, the sizes should be calculated and included. [replace line 149-150]:

```
%% DESIGN OF ACTUATOR AND TOPOLOGY DISTRIBUTION
xsiz = size(xFree,1);    % size of topology variables
wsiz = size(Fe,2);       % size of actuator variables
xzer = zeros(xsiz,1);    % empty row of zeros for mma usage
```

The number of constraints should be updates, and so does the size of the number of variables. [replace line 155-158]:

```
m = 3;                   % number of constraint functions
n = xsiz+wsiz;           % number of variables
xmin = [1e-9*ones(xsiz,1); -1*ones(wsiz,1)];     % minimum values of x
xmax = [ones(xsiz,1); -(1e-9/Fmma)*ones(wsiz,1)]; % maximum values of x
```

The topology optimization add-in results in an additional number of calculations inside the loop, so extra allocation steps are needed. [replace line 181-182]:

```
labda2 = zeros(N,1);     % pre-allocate second lagrange multiplier
Fi = zeros(1,N);         % pre-allocate force selection vector
Ua = zeros(N,1);         % pre-allocate displacement vector
Cons = zeros(miter,1);   % pre-allocate constraint vector
Cons2 = zeros(miter,1);  % pre-allocate constraint #2 vector
Cons3 = zeros(miter,1);  % pre-allocate constraint #3 vector
```

In this case, only the vertical selection is included, so no need for horizontal. [replace line 188]:

```
        L(j) = 0;        % horizontal selection value
```

The change of design variables will result in a huge amount of changes inside the loop. Additional sensitivities needs to be calculated. An additional compliance constraint is added, in order to create physically possible structures. Also, the finite difference method is extended for all sensitivities. [replace line 240-253]:

```
        Ua(Uarray) = Ui(Uarray); % selection of displacement
        c0 = reshape(sum((Ui(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial compliance
        c(iter) = c(iter) + sum(Ua.^2); % objective
        labda(free) = -sparse(Kt(free,free))\sparse(2*Ua(free)); % calculate lagrange multiplier
        labda2(free) = 2*Ua(free); % calculate second lagrange multiplier
        c00 = reshape(sum((labda(dofmat)*Ke).*Ui(dofmat),2),ny,nx); % initial labda compliance
        Fi(Fe) = (Fmma*s./((s^2*wF(Fe).^2+1)*(atan(s))));% force selection vector
        FFi = spdiags(Fi',0, N,N); % force selection vector
        Sens = Sens + p*(E-Emin)*xF.^(p-1).*c00; % calculate density sensitivity
        Senw = Senw - FFi(Fe,Fe)*labda(Fe); % calculate force sensitivity
        Cons(iter) = Cons(iter) + 10*(sum(xF(:))/(vol*nx*ny)-1); % calculate constraint
        dCdx = 10*Senc(efree)/(vol*ny*nx); % constraint sensitivity
        Cons2(iter) = Cons2(iter) + 10*(Fmin/sum(sum(wF)))-1; % calculate constraint
        dCdf = 10*Senc(Fe)*Fmin/-(sum(sum(full(wF))))^2; % constraint sensitivity
        Cons3(iter) = Cons3(iter) + (sum(sum((Emin+xF.^p*(E-Emin)).*c0))-50); % compliance constraint
        dCCdx = -p*(E-Emin)*xF.^(p-1).*c0; % constraint sensitivity
```

```
        dCCdf = labda2(Fe)'*FFi(Fe,Fe); % constraint sensitivity
        if iter == 2    % finite difference method
            F1 = wF;    % store force vector
            X1 = xF;    % store density vector
            [~,S1] = max(abs(Sens(:))); % calculate maximum sensitivity value
            Sens1 = Sens(S1); % store maximum sensitivity value
            [~,S2] = max(abs(Senw(:))); % calculate maximum sensitivity value
            Sens2 = Senw(S2); % store maximum sensitivity value
            [~,S3] = max(abs(dCdx(:))); % calculate maximum sensitivity value
            Sens3 = dCdx(S3); % store maximum sensitivity value
            [~,S4] = max(abs(dCdf(:))); % calculate maximum sensitivity value
            Sens4 = dCdf(S4); % store maximum sensitivity value
            [~,S5] = max(abs(dCCdx(:))); % calculate maximum sensitivity value
            Sens5 = dCCdx(S5); % store maximum sensitivity value
            [~,S6] = max(abs(dCCdf(:))); % calculate maximum sensitivity value
            Sens6 = dCCdf(S6); % store maximum sensitivity value
```

The MMA solver should work with more design variables. [replace line 294]:

```
        xval = [xFree(:);wval(:)]; % store current design variable for mma
```

The MMA solver is here updated to extend the number of constraints and additional sensitivities. [replace line 299-301]:

```
        df0dx = [Sens(efree);Senw]*cscale; % store sensitivity for mma
        f = [Cons(iter);Cons2(iter);Cons3(iter)]; % normalized constraint function
        dfdx =[dCdx wzer'; xzer' dCdf; dCCdx(efree)' dCCdf]; % derivative constraint functions
```

The previous design variable is here updated to include topology design. [replace line 306]:

```
        xold1 = [xFree(:);wval(:)]; % previous x, to monitor convergence
```

The MMA result is split to update the topology and actuator placement. [replace line 309]:

```
        xnew(efree) = xmma(1:xsiz); % update mma to density
        wnew(Fe) = xmma(xsiz+1:end); % update mma to force
```

The density design variables should be updated. [between line 321-322]:

```
        xFree = xnew(efree); % update density variable
```

The tolerance is updated, as a summation of changes in force and changes in density. [replace line 324]:

```
    diff = (max(abs(full(Fmma*wnew(:)-full(F(:))))+max(abs(xnew(:)-x(:))))); % difference of maximum element change
    x = xnew;           % update design variable density
```

The finite difference method checks six different sensitivities. This results in an extension of the code. [replace line 335-354]:

```
            xF = X1;    % store first findif result...
            xF(S1) = X1(S1)+h; %...and add a small pertubation
            wF = F1;   % store first findif result
        elseif iter == 3 % on second findif iteration
            findif = (c(3)-c(2))/h; % calculate finite difference method
            Sensdif = abs(max((findif-Sens1)/Sens1,(Sens1-findif)/findif)); % maximum difference
            if Sensdif > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;   % store first findif result...
            wF(Fe(S2)) = F1(Fe(S2))+h; %...and add a small pertubation
            xF = X1;   % store first findif result
        elseif iter == 4  % on third findif iteration
            findif2 = (c(4)-c(2))/h; % calculate finite difference method
            Sensdif2 = abs(max((findif2-Sens2)/Sens2,(Sens2-findif2)/findif2)); % maximum difference
            if Sensdif2 > 0.01 % when difference between sensitivity and findif is too much display
```

```
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif2)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;    % store first findif result
            xF = X1;    % store first findif result...
            xF(S3) = xF(S3)+h; %...and add a small pertubation
        elseif iter == 5  % on fourth findif iteration
            findif3 = (Cons(5)-Cons(2))/h; % calculate finite difference method
            Sensdif3 = abs(max((findif3-Sens3)/Sens3,(Sens3-findif3)/findif3)); % maximum difference
            if Sensdif3 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif3)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;    % store first findif result
            wF(S4) = wF(S4)+h; % store first findif result...
            xF = X1; %...and add a small pertubation
        elseif iter == 6  % on fifth findif iteration
            findif4 = (Cons2(6)-Cons2(2))/h; % calculate finite difference method
            Sensdif4 = abs(max((findif4-Sens4)/Sens4,(Sens4-findif4)/findif4)); % maximum difference
            if Sensdif4 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif4)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
            end
            wF = F1;    % store first findif result
            xF = X1;    % store first findif result...
            xF(S5) = xF(S5)+h; %...and add a small pertubation
        elseif iter == 7  % on sixth findif iteration
            findif5 = (Cons3(7)-Cons3(2))/h; % calculate finite difference method
            Sensdif5 = abs(max((findif4-Sens5)/Sens5,(Sens5-findif5)/findif5)); % maximum difference
            if Sensdif5 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif5)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
                end
```

```
            end
            wF = F1;    % store first findif result...
            wF(Fe(S6)) = F1(Fe(S6))+h; %...and add a small pertubation
            xF = X1;   % store first findif result
        elseif iter == 8  % on second finidif iteration
            findif6 = (Cons3(8)-Cons3(2))/h; % calculate finite difference method
            Sensdif6 = abs(max((findif6-Sens6)/Sens6,(Sens6-findif6)/findif6)); % maximum difference
            if Sensdif6 > 0.01 % when difference between sensitivity and findif is too much display
                disp(['Warning: Sensitivity needs to be checked, max difference:' sprintf('%10.2f',Sensdif6)])
                if fincheck == 2 % when fincheck is not accomplished...
                    break %... break the loop and stop the code
```

An update is made, to include topology in the output window. [replace line 369]:

```
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('%6.3f', ...
            sum(full(F))) ' Diff:' sprintf('%6.3f',diff)]);
```

An update is made, to include topology in the output window. [replace line 378]:

```
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('%6.3f', ...
            sum(full(F))) ' Diff:' sprintf('%6.3f',diff)]);
```

An update is made, to include topology in the output window. [replace line 549]:

```
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('%6.3f', ...
            sum(full(F))) ' Diff:' sprintf('%6.3f',diff)]);
```

An update is made, to include topology in the output window. [replace line 549]:

```
            ' Vol:' sprintf('%6.3f',mean(xF(:))) ' Ftot:' sprintf('%6.3f', ...
            sum(full(F))) ' Diff:' sprintf('%6.3f',diff)]);
```

Appendix D

Supplementary Codes

In this section some supplementary MATLAB codes can be found. The prescribed MMA solution (C.1) method calls two external functions, in order to calculate the optimal solution. ~~These functions can be found in (D.1) and (D.2).~~ A function to create arrows can be found in (D.3).

D.3 Arrowz.m

In order to be able to plot force application as an arrow representation, a new code is written. This code can be used to draw a certain arrow from start- to endpoint, with an adjustable shaft- and headsize. Also, the color of these arrows can be adjusted (Broxterman, 2016).

```
function arrowz(startpair,endpair,varargin)
% Written in Sep 2016 by Stefan Broxterman (TU Delft)
%
% ARROWZ draws an easily adjustable arrow from startpair to endpair. These
% pairs should be vectors of length 2. The input of ARROWZ can vary from 2
% to 6 inputs.
%
% ARROWZ(starpair,endpair) creates an easy arrow, a line plot from
% startpair to endpair, with an additional head within the direction of the
% endpair. Input format is [x y],[x y].
%
% ARROWZ(starpair,endpair,headsize) is able to adjust the size of the head.
% Default size is 1.
%
% ARROWZ(starpair,endpair,headsize,shaftsize) sets the thickness of the
% shaft to the desired size. Default size is 1.
%
% ARROWZ(starpair,endpair,headsize,shaftsize,color) specifies the color of
% the total arrow. These values should be provided as RGB. Default is black
% [0 0 0].
%
% ARROWZ(starpair,endpair,headsize,shaftsize,headcolor,shaftcolor) colors
% the shaft of the arrow into a seperate color. Default is black [0 0 0].
%
% Many thanks to Ryan Molecke
switch nargin                              % Check number of inputs
    case 2
        headsize = 1;
        shaftsize = 1;
        headcolor = [0 0 0];
        shaftcolor = [0 0 0];
    case 3
        headsize = varargin{1};
        shaftsize = 1;
        headcolor = [0 0 0];
        shaftcolor = [0 0 0];
    case 4
        headsize = varargin{1};
        shaftsize = varargin{2};
        headcolor = [0 0 0];
        shaftcolor = [0 0 0];
```

```
    case 5
        headsize = varargin{1};
        shaftsize = varargin{2};
        headcolor = varargin{3};
        shaftcolor = varargin{3};
    case 6
        headsize = varargin{1};
        shaftsize = varargin{2};
        headcolor = varargin{3};
        shaftcolor = varargin{4};
end
% Begin drawing
v1 = headsize*(startpair-endpair)/2.5;  % Create drawing vector

alfa = pi/8;                            % 45*pi/360
R = [cos(alfa) -sin(alfa) ; sin(alfa) cos(alfa)]; % Rotational matrix
R1 = [cos(-alfa) -sin(-alfa) ; sin(-alfa) cos(-alfa)]; % Reverse Rot mat

v2 = v1*R;                              % Create right-hand vector
v3 = v1*R1;                             % Create left-hand vector
x1 = endpair;                           % Top of the arrow
x2 = x1 + v2;                           % Right-hand arrowhead point
x3 = x1 + v3;                           % Left-hand arrowhead point
x4 = 0.5*(x2+x3);                       % Create endpoint of shaft
hold on;
% Begin plot
plot([startpair(1) x4(1)],[startpair(2) x4(2)],...
    'linewidth',shaftsize,'color',shaftcolor);
fill([x1(1) x2(1) x3(1)],[x1(2) x2(2) x3(2)],headcolor);
```

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