Design and assessment of concrete structures with strut-and-tie models and stress fields
From simple calculations to detailed numerical analysis

Lourenço, Miguel Sério; Fernández Ruiz, Miguel; Blaauwendraad, Johan; Bousias, Stathis; Hoang, Linh Cao; Mata-Falcón, Jaime; Meléndez, Carlos; Mihaylov, Boyan I.; Ferreira, Miguel Pedrosa; More Authors

DOI
10.1002/suco.202200647

Publication date
2023

Document Version
Final published version

Published in
Structural Concrete

Citation (APA)
Lourenço, M. S., Fernández Ruiz, M., Blaauwendraad, J., Bousias, S., Hoang, L. C., Mata-Falcón, J., Meléndez, C., Mihaylov, B. I., Ferreira, M. P., & More Authors (2023). Design and assessment of concrete structures with strut-and-tie models and stress fields: From simple calculations to detailed numerical analysis. *Structural Concrete*, *24*(3), 3760-3778. https://doi.org/10.1002/suco.202200647

Received: 26 August 2022 | Revised: 31 October 2022 | Accepted: 23 November 2022

DOI: 10.1002/suco.202200647

ARTICLE

fib CEB·FIP WILEY

Design and assessment of concrete structures with strut-and-tie models and stress fields: From simple calculations to detailed numerical analysis

Miguel Sério Lourenço[1,2] | Miguel Fernández Ruiz[3] | Johan Blaauwendraad[4] | Stathis Bousias[5] | Linh Cao Hoang[6] | Jaime Mata-Falcón[7] | Carlos Meléndez[8] | Boyan I. Mihaylov[9] | Miguel Pedrosa Ferreira[10] | Duarte Viúla Faria[11]

[1]JSJ Ltd, Lisbon, Portugal
[2]Instituto Politécnico de Setúbal, Setúbal, Portugal
[3]School of Civil Engineering, Universidad Politécnica de Madrid, Madrid, Spain
[4]Delft University of Technology, Delft, The Netherlands
[5]Department of Civil Engineering, University of Patras, Patras, Greece
[6]Department of Civil and Mechanical Engineering, Technical University of Denmark, Lyngby, Denmark
[7]Institute of Structural Engineering, ETH, Zurich, Switzerland
[8]Esteyco SA, Madrid, Spain
[9]Urban and Environmetal Engineering, University of Liege, Liège, Belgium
[10]Grupo NOV, Leiria, Portugal
[11]Muttoni & Fernández, Ingénieurs Conseils SA, Ecublens, Switzerland

Correspondence
Miguel Sério Lourenço, JSJ Ltd, Lisbon, Portugal.
Email: mlourenco@jsj.pt

Abstract

The introduction of strut-and-tie models in *fib* Model Code 1990 as a design basis for discontinuity regions constituted a significant step toward promoting consistent design methods for reinforced concrete structures. In *fib* Model Code 2010, the scope was broadened through the stress field method that was introduced as a complementary tool. The present article summarizes subsequent evolutions in both methods, which will be incorporated in the upcoming *fib* Model Code 2020. Besides emphasizing their suitability for the structural design and assessment, their adaptability to the "Levels-of-approximation" approach is also depicted. This article presents the theoretical ground of both methods and looks on their potential for computer-modeling implementation. With this respect, several strategies are presented by discussing their advantages and optimum field of application.

Discussion on this paper must be submitted within two months of the print publication. The discussion will then be published in print, along with the authors' closure, if any, approximately nine months after the print publication.

1 | INTRODUCTION

Since the early developments of structural concrete, designers have searched for general and comprehensive

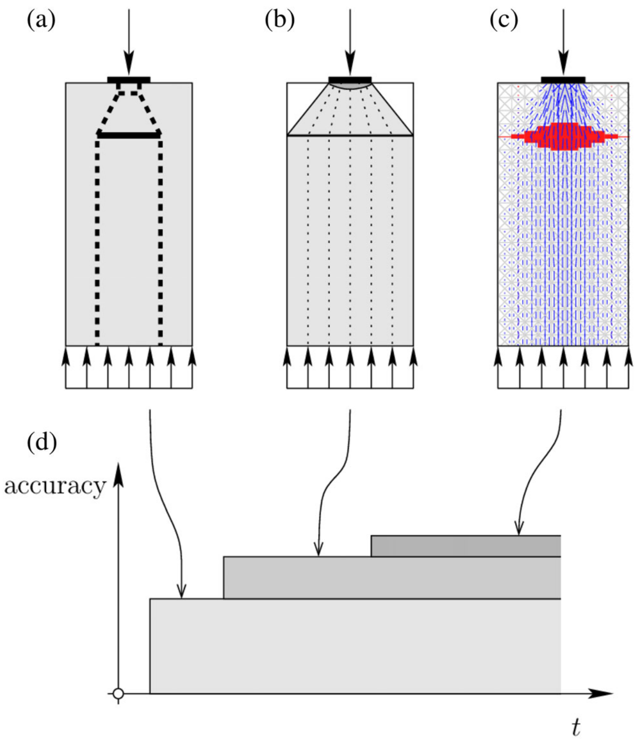


FIGURE 1 Strut-and-tie and stress fields: (a) strut-and-tie model; (b) rigid-plastic stress field; (c) elasto-plastic stress field; (d) accuracy versus time for modeling and calculations following a levels-of-approximation approach.

tools to understand its behavior and provide a consistent basis for design. Within this process, it was soon identified[1] the potential to work with fictitious truss models for concrete structures, where the compression in the concrete was represented by struts, and the tension in the reinforcement was modeled with ties. The method rapidly gained popularity and was later extended in a more general manner.[2] The advantages of designing based on truss models allowed to design structural members based on this rational approach rather than using a collection of empirical rules.

During the decades that followed, the initial studies[2] were extended to other cases,[3,4] the main thrust provided by the German school, which approached the definition of the load-carrying struts and ties to be based on elastic-uncracked stress fields (initially obtained via analytical or photo-elastic methods and later, by linear finite elements). Such improvement opened the door to design complex structural regions for which classical linear-strain-variation assumption was not applicable, and was named the "strut-and-tie method" (as an evolution of truss models). In 1984, the Canadian code[5] included for the first time the strut-and-tie approach as a general design method for regions which do not obey the classical plane-sections-remain-plane hypothesis. The approach was also soon adopted by *fib* Model Code 90[6] and gradually by most structural codes for concrete design.

In parallel to such developments, the application of the theory of plasticity to reinforced concrete[7] led to an approach with several analogies based upon its lower-bound theorem. This approach, leading to the so-called "stress fields", also defined compression fields (struts) in the concrete and tension fields (ties) in the reinforcement. However, since they were based on the lower-bound theorem of limit analysis, multiple solutions were possible and the designer had freedom to select one, overcoming most of the limitations of the strut-and-tie models based on linear elastic solutions. It was observed that some extreme solutions failed to represent the actual response of concrete, particularly when no reinforcement for crack control was provided.[8] However, systematic comparisons to test results in Denmark[9] and Switzerland[10,11] allowed the development of a safe-sided and comprehensive approach for structural concrete design.

The strut-and-tie and the stress field methods have strong analogies. As pointed out,[12] the strut-and-tie method can also be grounded on the lower-bound theorem of limit analysis. In addition, a strut-and-tie model can be generated from a given stress field by replacing the compression and tension fields with their resultants. It is an advantage to work simultaneously with both methods[13] and to develop one or the other depending on the required answer (forces or stresses), see Figures 1a,b.

Since these developments, both methods have continued to evolve, particularly taking advantage of the potential offered by computers. In this way, classical approaches based on a rigid-plastic material response have allowed the development of other approaches accounting for compatibility of deformations[14,15] (Figure 1c) or based on convex optimization.[16] These tools provide a more general framework to determine appropriate stress fields and can overcome many difficulties associated with classical approaches, such as the calculation of efficiency factors. It shall be noted that these approaches require for simple cases more time than simple rigid-plastic stress fields or strut-and-tie models. However, they are more efficient for large structures or when multiple load cases are to be analyzed. It has then to be decided by the engineer whether their application is pertinent or not. In this respect, the Levels-of-Approximation approach proposed in *fib* Model Code 2010[17,18] is perfectly in-line, and consistent refinements of analysis can be developed only when required, see Figure 1d. This article aims to summarize the fundamentals of the approaches illustrated in Figure 1, place these approaches within the framework of the Levels-of-Approximation approach, and discuss how they can be

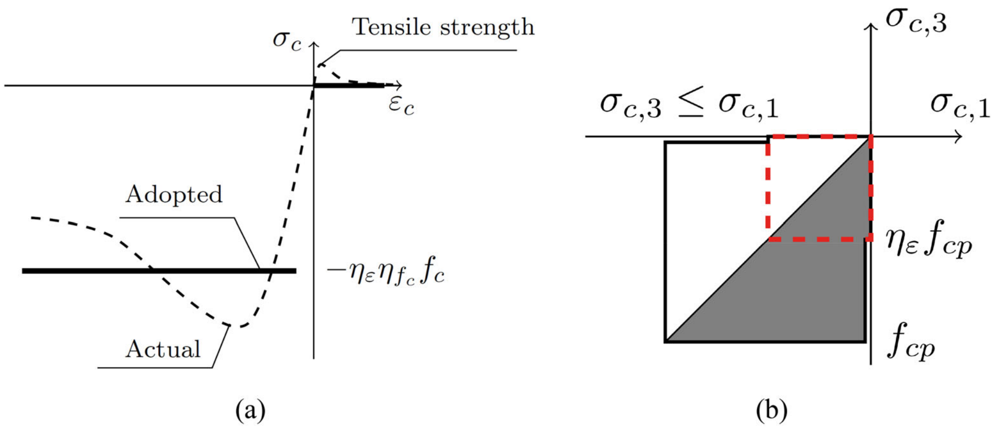


FIGURE 2 Concrete yield conditions: (a) uniaxial concrete stress–strain relationship and assumed plastic concrete resistances; (b) 2D yield condition for plain cracked concrete (adopted from *fib* bulletin 100[18]).

implemented in various tools for reliable computer modeling.

2 | BASIC IDEALIZATION OF MATERIAL STRENGTH

2.1 | General considerations

In properly modeling the response of a concrete structure with strut-and-tie-models and stress fields, the fundamental material aspects are independent of the degree of refinement of the analysis. These concepts can be translated into constitutive relationships with different levels of refinement (from rigid-perfectly plastic, to linear elastic-perfectly plastic or nonlinear), aligned with the degree of sophistication of the analysis. This section focuses on the general idealization of the material response. Details on the specific stress–strain material relationships for a particular level of approximation will be discussed in Sections 3 and 4.

2.2 | Concrete

The concrete is assumed to carry only compressive stresses in strut-and-tie and stress field models (the tensile capacity is neglected due to its brittle response). The concrete uniaxial response is not perfectly plastic. Therefore, an equivalent concrete plastic strength should be defined to conduct a design grounded on limit analysis (theory of plasticity). The plastic strength is affected by material and structural effects. The most relevant material effect is the reduced toughness of concrete (i.e. lower capacity to dissipate energy in the post-peak phase) as its strength increases. This effect is typically accounted for by reducing the uniaxial peak compressive strength f_c with a brittleness factor η_{f_c} (Figure 2a). The most widespread formula of η_{f_c} was proposed by Muttoni (1990)[19]:

$$\eta_{f_c} = \left(\frac{30}{f_c[MPa]}\right)^{\frac{1}{3}} \leq 1 \tag{1}$$

Structural effects account for the structural response under biaxial or triaxial loading. The presence of triaxial confinement, generated, for example, in a compressed structural element subjected to plane strain conditions, allows for increasing the compressive strength above the uniaxial concrete compressive strength. The increased compressive strength can be modeled by applying the Mohr-Coulomb failure criterion. On the other hand, cracks in a compression field disturb the stress flow, which softens the response and reduces the capacity of concrete in compression. This effect is known as compression softening.[20] It is essential to provide at least a minimum amount of smeared reinforcement in the structure to control the cracks and to allow to model the compression softening in a reliable manner. Compression softening is usually considered by reducing the plastic concrete compressive strength by a factor η_ε, what results in the following yield criteria for cracked concrete under plane stress conditions (see Figure 2):

$$-\eta_\varepsilon \eta_{f_c} f_c = -\eta_\varepsilon f_{cp} \leq \sigma_c \tag{2}$$

In hand-made strut-and-tie models and stress fields (i.e. analysis using rigid-plastic material idealization), designers typically define this softening factor for each member region, based on the expected potential state of cracking and angle of the compression field[17] and even considering the amount of transverse reinforcement.[21] For cases in which stress fields and strut-and-tie models are formulated ensuring compatibility of deformations (i.e. with the aid of computer tools), an appropriate stress field for concrete can be derived assuming stress-free fictitious

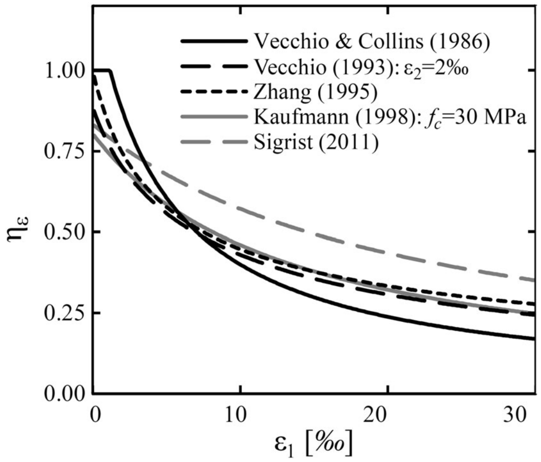


FIGURE 3 Comparison of strain-based compression softening formulations (adopted from *fib* bulletin 100[18]).

rotating cracks in which the principal directions of strains and stresses coincide. In these cases, the compression softening can be automatically computed based on the transverse tensile strain ε_1. Figure 3 shows a selection of five empirical relationships $\eta_\varepsilon(\varepsilon_1)$ proposed in the literature. Most of these relationships follow the general expression:

$$\eta_\varepsilon = \frac{1}{a + b\,\varepsilon_1} \leq 1 \tag{3}$$

Extensive validation against experimental results using the first formulation of this kind[20] (a = 0.8, b = 170 with $\varepsilon_c = -0.002$), yields satisfactory results regarding ultimate load capacity prediction.[14]

2.3 | Reinforcement

In strut-and-tie and stress field models, reinforcing bars are essentially used to carry the tensile forces. However, they can also be used as compression reinforcement, if the reinforcing bars are properly braced to avoid buckling. The yield criteria for a limit analysis should be defined as $|\sigma_s| \leq f_y$.

2.4 | Other relevant phenomena

Other material or structural effects that can influence the load-carrying capacity by modifying the strength and/or maximum deformation (e.g. sustained loading, low-cycle fatigue, cyclic loading, anchorage) should also be included in the idealization of the materials when necessary.[22]

3 | LEVELS OF APPROXIMATION FOR DESIGN / ASSESSMENT WITH STRUT-AND-TIE MODELS AND STRESS FIELDS

Strut-and-tie and stress field models are recognized as powerful tools for designing and assessing structural concrete members. They allow a rational understanding of the structural behavior by relying on the idea of following the internal flow of forces. This approach provides a unique and pedagogic insight into the behavior of the structure, which is essential for any engineer.

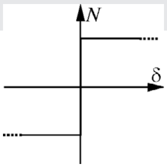
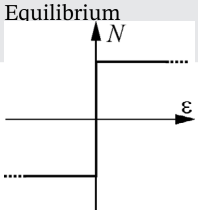
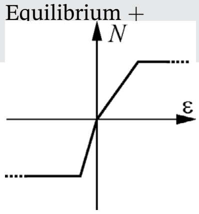
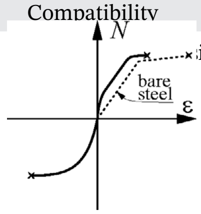
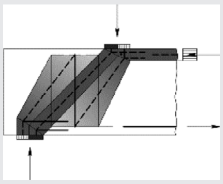
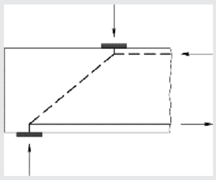
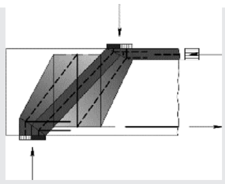
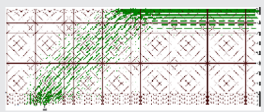
The Level-of-Approximation (LoA) approach states that the level of accuracy in predicting the structural behavior can be refined progressively by increasing the model refinement. Applying the LoA to strut-and-tie models and stress fields is extremely useful for practical purposes. The selection of the appropriate LoA should be adequately defined and may depend on several aspects: design phase of the project (preliminary design, detailed design, etc.), local or global structural complexity, the influence of local behavior on global structural response, whether assessment or design of a structure is at play, etc. The suitable design strategy and the adequate LoA for a specific case should be defined by employing the required accuracy and the time devoted to the analysis.

Applying the LoA concept in stress fields and strut-and-tie models is advised to start with simple load-carrying models, but being general enough to incorporate any relevant aspect at later stages. Thereafter, the model is refined as needed, for example, considering secondary load-carrying mechanisms or a more detailed material behavior. The objective of opting for higher-order LoA is to have a more precise representation of the actual structural behavior of the element. The designer should be aware of the simplifications when using a lower-order LoA and must ensure that those assumptions lead to a safe-sided estimate of the actual failure load.

Stress fields and strut-and-tie models may be applied to design new structures and assess existing ones. Nonetheless, the approach followed for the two applications should, in general, be different, as the scope of the tasks to be performed is also quite different:

- For the design of new structures, the task of the engineer is to produce a safe design that respects economic criteria, is simple to build and has an adequate service behavior.
- For the assessment of existing structures, the task of the designer is rather to check if the structural strength is sufficient to carry the current actions, considering present member geometry and existing reinforcement layout. Serviceability issues are not usually checked

TABLE 1 Levels-of-approximation for strut-and-tie and stress fileds

	LoA I	LoA II	LoA III	LoA IV
Main use	Design (ULS)	Design (ULS)	Assessment (ULS)	Assessment (ULS) Design (SLS)
Conditions	Equilibrium	Equilibrium	Equilibrium +	Equilibrium + Compatibility
Constitutive relationships				 ion
Model	Simple resultant model. Refinements and definition of strut widths whenever necessary (e.g. at nodes) 	Stress field model including alternative load-carrying mechanisms (if relevant)[a] 	 Compatibility-based stress field model including all relevant load-carrying mechanisms and predicting internal stress redistribution  FEM with reinforcement modeled with 1D elements.	
Output	Strut and tie forces (ULS)	Concrete stresses in stress fields; tie forces (ULS)	Concrete stresses in stress fields; tie stresses (ULS)[b]	Concrete and reinforcement stresses and strains (ULS + SLS)
Additional verifications	Relevant nodes and struts Ties Anchorage length Service behavior		Anchorage length Service behavior Ultimate strain for concrete and for reinforcement	If not direct output: Anchorage length Ultimate strain for concrete and for reinforcement
Requirements	Minimum reinforcement Detailing rules (e.g. maximum rebar spacing and confining reinforcement) Rules to limit stress redistributions to ensure adequate service behavior and ductility Rules to account for alternative load-carrying mechanisms			

[a]The load distribution between statically indeterminate or redundant models may be estimated in the LoA II based on specific prescribed rules for the structural element or linear-elastic analysis.

[b]SLS and ductility aspects may be analyzed in a simplified manner whenever tension stiffening effects are not relevant for calculation of reinforcement forces.

through analysis, as this can be done in situ (unless a significant change of the actions is expected).

Due to the differences in scope, the approach followed is also different: for new structures, simple and safe load-carrying models are to be used. To that end, the use of strut-and-tie models with some local refinements via rigid-plastic stress fields is normally adequate. This provides a lower-bound of the actual strength and yields design freedom to decide on the location of the main reinforcement, allowing ease of construction. A refined analysis is justified only when complex or critical elements are designed (for instance, accounting for compatibility of deformations). On the other hand, for the

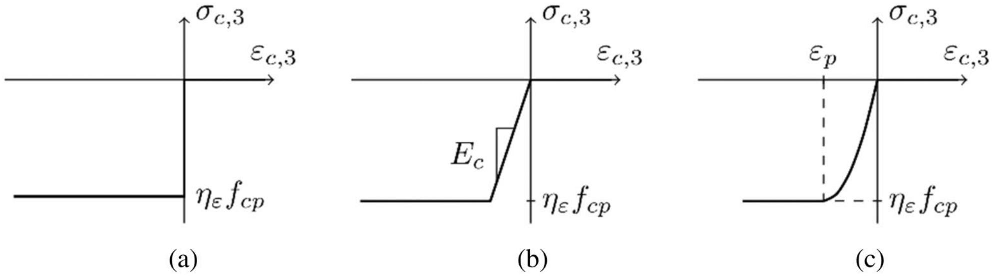


FIGURE 4 Concrete stress–strain relationships: (a) rigid-perfectly plastic (LoA I & II); (b) linear elastic-perfectly plastic (LoA III); (c) nonlinear elastic-perfectly plastic (LoA IV).

assessment of existing structures, simple models (lower-bound solutions) can be used at first to identify non-critical regions (i.e. regions where the strength is satisfied even for the simple load-carrying models). In those regions identified as critical, model refinements are justified to avoid or minimize strengthening. Note that the savings in strengthening largely compensate for the cost of additional engineering studies. Refined assessment models can be developed by accounting, for instance, for kinematic considerations. It should be noted that the deformation capacity of the structure is to be explicitly verified in case there is any doubt that the plastic stress redistribution assumed in the developed model can effectively occur.

Table 1 summarizes the primary uses of the different LoA for strut-and-tie and stress field models, the conditions to be fulfilled by the models, the output obtained and the additional verifications that may be required in each case.

4 | GENERAL CONSIDERATIONS FOR COMPUTER MODELING

4.1 | Introduction

Classical strut-and-tie models and stress fields assume rigid-plastic material response. However, taking advantage of available computational power, new developments can cover a wide range of approaches with different assumptions and levels of refinement. This section provides an overview of the different material constitutive relations and types of modeling used for computer analysis with strut-and-tie models and stress fields, followed by a discussion on their limits of applicability.

4.2 | Material constitutive relations

The idealization of the ultimate material response presented in Section 2 is implemented into stress–strain laws that can be tailored to the Level of Approximation of the analysis (Table 1), ranging from simple hand-made calculations based only on equilibrium, to refined analyses accounting for compatibility conditions. In general, it is possible to define three levels of refinement:

- Rigid-perfectly plastic (LoA I and II).
- Linear elastic-perfectly plastic (LoA III).
- Nonlinear elastic-perfectly plastic (LoA IV).

A clearly defined plastic plateau should always be provided, even when nonlinear constitutive relationships are employed (Section 2). In this way, the modeling approach remains grounded on the lower-bound theorem of limit analysis. For concrete, these levels of refinement lead to the stress–strain relationships shown in Figure 4. Strain limits might be considered whenever an estimate of the deformation capacity is required (LoA IV).

The transfer of forces between concrete and reinforcement is ensured by bond. This interaction is relevant for anchorage verifications and tension stiffening. In simple cases, e.g. when anchorage of a tie occurs outside of the nodal region, the anchorage may not be included in the numerical model (i.e. perfect bond is assumed) but considered in the detailing of the reinforcement (by extending the reinforcement length or evaluating the maximum force that reinforcement can support, in case of assessment). In elastic–plastic stress fields, a simple procedure to satisfy the yield conditions of bond is to reduce the area of the reinforcement to limit the axial force below the maximum force that is possible to transfer due to bond. An explicit model for bond stress can alternatively be used to correctly evaluate the stiffness and load-carrying capacity of a structure. In case an FE model is employed, the beneficial effect of the transverse pressure can be explicitly incorporated via a finite element (0D) with three nodes, two for surrounding concrete and one for reinforcement.[23]

Bond is also responsible for activating concrete tensile stresses between two stress-free cracks. Since part of the force carried by the reinforcement is transferred to concrete, the response of bonded reinforcement is stiffer than the material behavior of bare steel. This effect, named "tension stiffening" influences the load-deformation behavior of the analyzed structure. Therefore, it should be considered in LoA IV when assessing the serviceability behavior (namely deformations and cracking) and/or the deformation capacity. A suitable approach to account for tension stiffening in Compatibility-based Stress Fields is discussed in Section 5.2.

4.3 | Types of modeling

There exist the following modeling types in the context of strut-and-tie models and stress fields:

- Separate modeling of concrete and reinforcement.
- Composite modeling of reinforced concrete elements.

The first approach is used in classical strut-and-tie models and stress fields. This approach requires modeling each reinforcing bar. Its results provide a high level of detail, which is particularly interesting when analyzing details of a structure with static or geometric discontinuities. The meshing for concrete and reinforcement might share nodes. Multi-point constraint (MPC) elements should be introduced to connect independent meshes.

In the second modeling approach, the structure is divided into reinforced concrete elements, with concrete and reinforcement modeled as a composite. Typical elements used in this modeling are (i) membrane elements (also known as panels), resisting shear and normal forces, and (ii) stringers (also known as chords), resisting normal forces. Instead of modeling each reinforcing bar (or layers of bars), this approach only requires the amount of reinforcement for each part of the structure. While this modeling approach might yield less detailed information than when separately modeling concrete and reinforcement, it is computationally very efficient and well-suited for analyzing large structures. The Finite Element Limit Analysis (Section 5.3) and the Stringer-Panel Method (Section 5.4) are examples of this modeling strategy.

4.4 | Limits of applicability

Strut-and-tie models and stress fields with a Level-of-Approximation I, II, and III assume the materials to behave in a perfectly plastic manner (i.e. the materials require sufficient deformation capacity to develop the plastic stress redistributions needed in the element). However, the deformation capacity of concrete and even that of reinforcement is limited. To ensure the safe applicability of strut-and-tie models and stress fields, it is essential to (i) model the material response according to Section 2 and (ii) ensure sufficient deformation capacity by providing a minimum amount of distributed reinforcement.

There is still no consensus about the minimum amount of reinforcement required to ensure the necessary deformation capacity. Current recommendations are mostly based on experience and empirical observations with normal strength concrete and reinforcement (a value of around 0.10% is typically provided in beam regions and 0.20%–0.35% in disturbed regions). A refinement of this value shall attract research efforts in the future by clarifying the role of bond, material ductility,[24,25] size effect, and energy balance at the moment of cracking.

Employing the highest level of approximation (LoA IV) allows for verifying the deformation capacity of the structure and capturing failures due to insufficient ductility of the materials (e.g. in elements with small amounts of low ductility reinforcement), as discussed in Section 4.1. The broader applicability of this refined model goes hand-in-hand with increased complexity and probability of modeling errors. Therefore, it is essential to use a level of refinement appropriate for each problem and to verify the results of numerical approaches with simple hand calculations.

5 | OVERVIEW OF COMPUTER MODELING APPROACHES

5.1 | Existing approaches

There exist a variety of computer modeling approaches for reinforced concrete structures with a wide range of fundamental assumptions and levels of complexity. On the spectrum of complexity, the simplest approaches can be considered those which implement strut-and-tie models for design or analysis of concrete structures. While strut-and-tie models are in principle simple as they only use one-dimensional elements, they are also unique for each load case on the structure, and therefore require significant effort to formulate and calculate. In this regard, automated approaches, such as Computer Aided Strut-and-Tie (CAST),[26] can significantly help for the day-to-day application of strut-and-tie models. In the other extreme on the spectrum of complexity one may find nonlinear finite element and discrete element analysis tools (FEM and DEM respectively). In DEM, the cracks are modeled in a discrete manner as they propagate between the discrete elements linked by nonlinear springs. The discrete crack approach has also been implemented in FEM, even though finite element formulations are typically based on the smeared crack approach, with fixed or rotating cracks, where concrete is treated as a continuum with equivalent

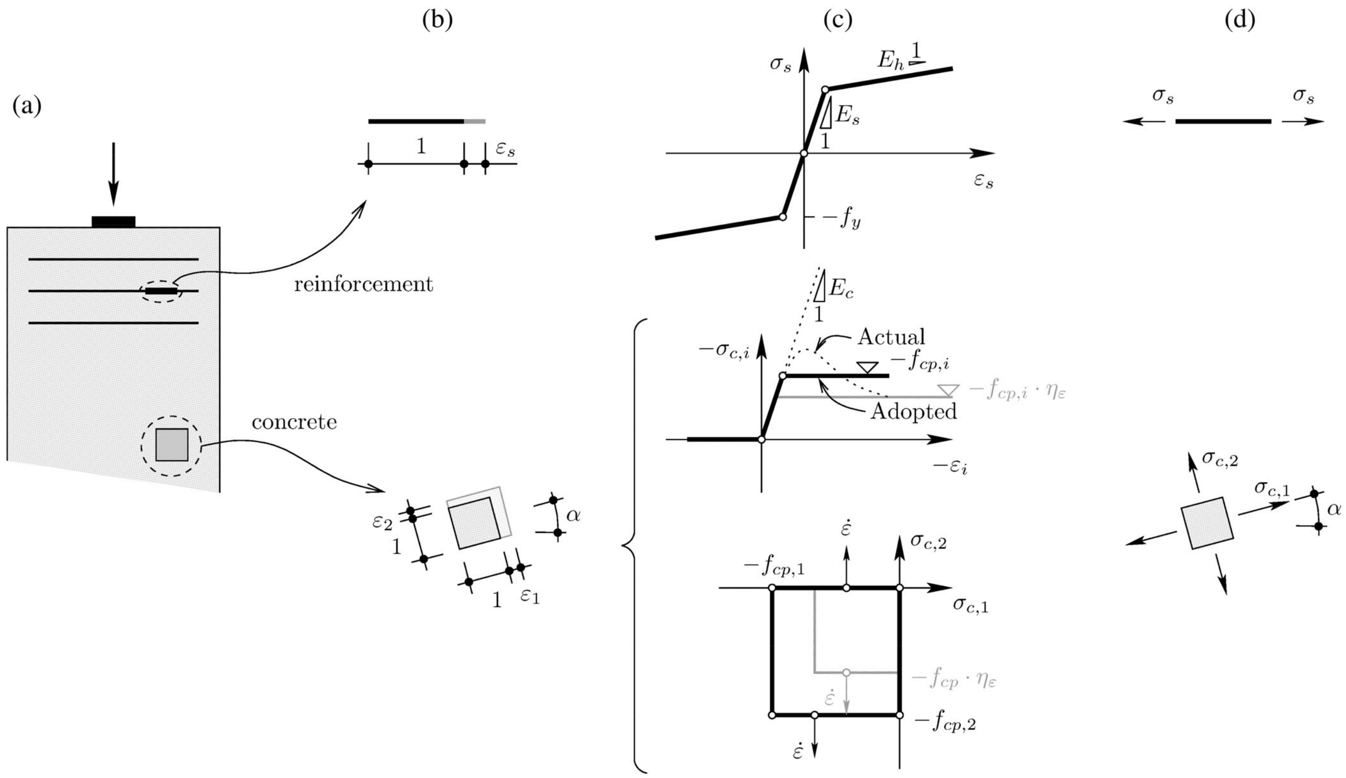


FIGURE 5 Elasto-plastic stress fields: (a) reinforced concrete element; (b) strain fields at the elements; (c) constitutive laws; (d) corresponding stress fields at the elements.

stress–strain relationships under bi-axial or tri-axial loading. In terms of stress–strain relationships (or constitutive models), a number of approaches exist based on damage-plasticity,[27] fracture mechanics,[28] modified compression field theory,[20] and disturbed stress field model[29] among others. In general, these approaches can account for a wide range of effects such as concrete cracking, tension stiffening, compression softening, aggregate interlock, confinement, bond, reinforcement buckling, etc.

This section aims to present computer modeling approaches that, in terms of complexity, are situated between automated strut-and-tie models and complex nonlinear FEM and DEM approaches. The main feature of these approaches is that they neglect the tension in the concrete when considering the equilibrium of the structure and model the compression behavior in a simplified manner. Concrete is considered to reach a plastic resistance in compression (see Section 2.1). Therefore, the presented approaches are consistent, in terms of ultimate resistance, with the lower-bound approach of the theory of plasticity. Even though these approaches target mainly at the ultimate limit state, some simple adjustments can be made to perform also serviceability calculations. In general, the presented approaches aim to strike a practical balance between simplicity and accuracy, allowing engineers to use them safely and to develop a good understanding of the behavior of the structure at hand.

5.2 | Elasto-plastic stress fields

The concept of Elasto-Plastic Stress Fields (EPSF) was formulated[13] as a simple computational approach to obtaining suitable stress fields. The original idea was to ensure the fulfillment of equilibrium and yield conditions and thereby satisfy lower-bound solutions' requirements according to limit analysis. To that end, the method assumes coaxiality of principal directions of stress and strain tensors. This assumption makes it possible to establish the stress field from a given strain field by applying material laws and yield conditions for concrete and reinforcing steel (Figure 5). The associated displacement field may thus be determined using a Newton–Raphson algorithm, ensuring the equilibrium conditions between the stress field and the external actions (or boundary conditions). The EPSF was identified to have a number of strengths, namely:

- A limited number of material parameters are required (only moduli of elasticity and yield strength/plastic compression strength).
- It can be implemented in an efficient manner following a finite-element approach.[13]
- The calculated state of strain at any load step enables the implementation of a strain-dependent formulation of the efficiency factor for the concrete,[29] thus allowing

for a detailed assessment of how the magnitude of the plastic strength of the concrete varies from point to point and from load step to load step.

- Since the compatibility conditions are ensured, the resulting displacement field at maximum load automatically leads to a kinematically-admissible failure mechanism. Considering that the stress field (i) is in equilibrium with the actions, (ii) respects the yield conditions of the materials (lower-bound of the strength), and (iii) is associated with a licit mechanism at failure (where the yield conditions develop at the plastic regions, upper-bound of the strength), it can be interpreted as an exact solution in the sense of the theory of plasticity (provided that efficiency factors were constant[18]). This feature is particularly useful for assessing existing structures[14] (exact solution, maximum capacity of a structure).

The idealization of the material response follows the principles presented in Section 2, Section 3, Section 4. EPSF assumes a linear elastic-perfectly plastic material response. The contribution of concrete in tension is neglected, both when modeling the stress and the deformation states (tension stiffening effects are ignored). These assumptions require smeared cracking conditions for a consistent application of EPSF. Hence, EPSF is only valid for members provided with an amount of (distributed) reinforcement above the minimum required for crack control and where brittle failure of the reinforcement is not expected. Special applications for brittle reinforcement[30] or members with poor control of cracking[31] have been developed. Nevertheless, these may be considered special cases requiring dedicated use of the EPSF or considering tension stiffening effects (Section 5.2). Furthermore, simplified estimates of the crack widths are possible (also in discontinuity regions[15]), even though the primary aim of the EPSF is to assess the load-carrying capacity at the ultimate limit state.

It is important to highlight that the assumption of coaxiality between the principal direction of strain- and stress tensors is a deviation from the classical theory of plasticity, which assumes coaxiality between the stress tensor and the increments of plastic strains (and not of total strain).[32,33] An extensive review of this topic can be found elsewhere.[18,34] However, the direction of the increment of plastic deformations in a perfectly plastic material, converges to that of the total strains for large deformation demands and proportional loading, thus ensuring theoretical consistency between EPSF and the plastic solution.[35]

The results of EPSF have been extensively validated, showing consistent and robust agreement with available experimental data[14,36] (comparison to more than 200 beams and discontinuity regions). In addition, the simplicity of the method has encouraged its use in practice and guidelines for its application in such cases (particularly bridges) have been developed.[14] In this respect, the consistency of the partial safety factor format has been verified[14,37] and ensures a safe-sided application at ultimate limit state (which may be more problematic or complex for approaches relying on more material parameters, such as the tensile strength of concrete).

The EPSF has inspired further developments[15,23,30] which consider more sophisticated material constitutive relationships, to allow for a refined analysis of the load-carrying structural capacity (Level-of-Approximation IV[18]). The following section gives an overview of these approaches covering serviceability and deformation capacity aspects.

5.3 | Compatibility-based stress fields

According to *fib* Bulletin 100,[18] a Level-of-Approximation IV stress field analysis is a refinement of the Level-of-Approximation III stress field analysis, presented in Section 5.2. Hence, the same principles of the LoA III are applicable in this case. The main improvement of a LoA IV analysis is to provide direct information about the serviceability behavior and deformation capacity of the structure. To this end, proper constitutive relationships for concrete and reinforcement should be considered, including suitable strain limits. The bond between the reinforcement and the surrounding concrete may be included in the analysis if needed. The advantages of a LoA IV are:

- The analysis provides information about steel and concrete strains at all load stages.
- It is possible to check the redistribution of internal forces and evaluate if the materials rupture before reaching the plastic solution predicted by a LoA III (i.e. assuming perfectly-plastic material response). This is particularly useful for assessing the capacity of structures with low ductility reinforcement or with low amounts of conventional steel reinforcement (e.g. existing structures).
- The analysis yields serviceability verifications (deflections and crack widths), even for discontinuity concrete regions.

Taking advantage of these benefits requires considering more complex constitutive relationships than in an LoA III analysis (i.e. nonlinear stress–strain laws for

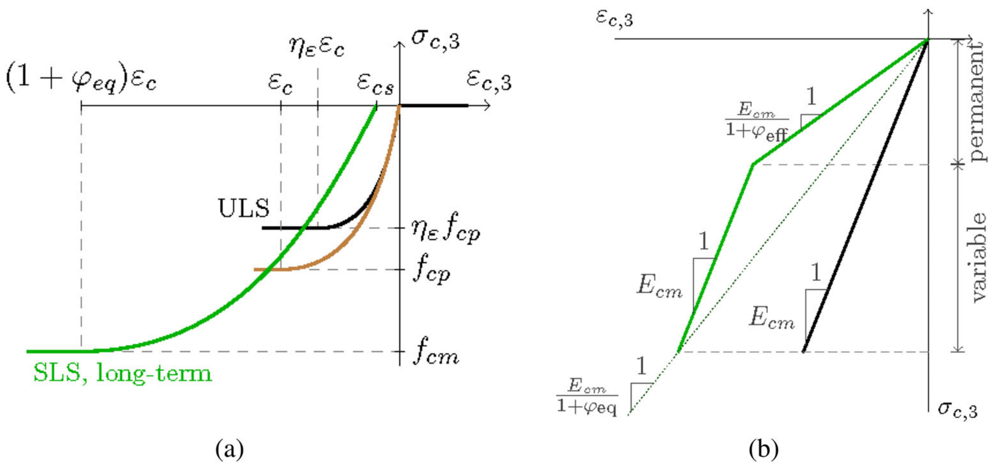


FIGURE 6 Constitutive behavior of concrete (a) in compression; (b) long-term behavior in compression for service loads.

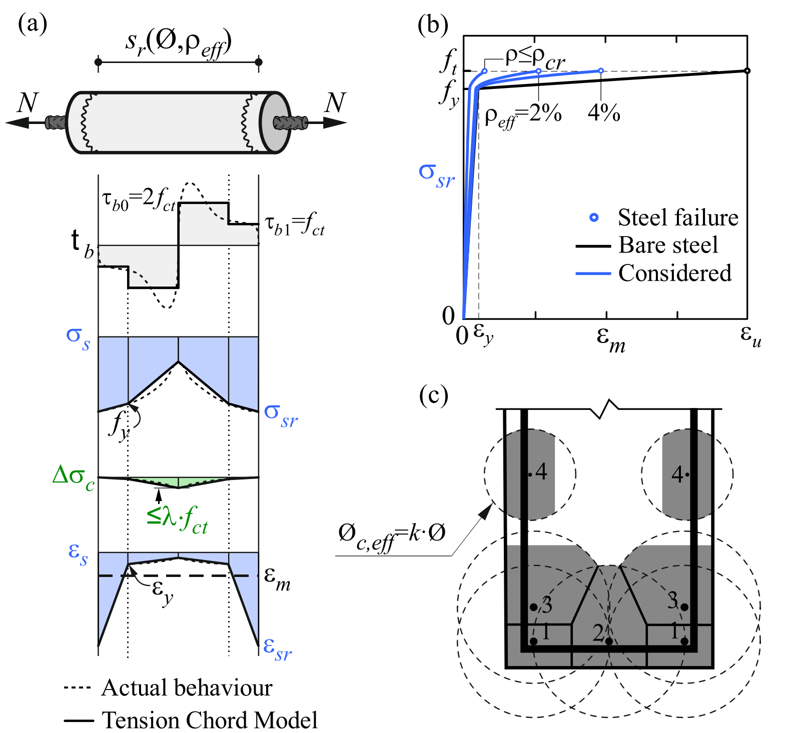


FIGURE 7 Modeling of tension stiffening in compatibility-based stress fields: (a) tension chord model[39] for stabilized cracking with distribution of bond shear (τ_b), steel (σ_s) and concrete stresses ($\Delta\sigma_c$), and steel strains between cracks (ε_s); (b) resulting tension stiffening behavior in terms of reinforcement stresses at the cracks (σ_{sr}) and average strains (ε_m) for European B500B steel with bilinear idealization and average cracks spacing; (c) effective area of concrete in tension for stabilized cracking[15,25]

concrete, Figure 4c, reinforcing steel and bond, including strain limits). Due to creep and shrinkage effects and partial safety factors, different material properties are to be considered for ULS and SLS, as shown in Figure 6a. A long-term serviceability analysis can be performed as (i) a calculation in two steps (variable plus permanent loads) with different stiffnesses or (ii) in a calculation with a single step using an equivalent stiffness per Figure 6b. The increased model complexity with respect to a LoA III analysis increases model uncertainty and the probability of modeling mistakes. Therefore, it is essential to verify the quality of the results against those from analysis with a lower LoA.

Although when formulating equilibrium in strut-and-tie model and stress fields the concrete tensile strength should be neglected (for consistency with the provisions of the theory of plasticity, i.e. the concrete constitutive behavior should always have a tension cut-off), tension stiffening (activation of concrete tensile stresses between two stress-free crack surfaces) has a significant impact on the stiffness of the reinforcement, with its magnitude depending on the (effective) reinforcement ratio. A straightforward and effective technique to account for this phenomenon, while respecting the assumption of stress-free cracks, consists of modeling the reinforcement in terms of average- rather than maximum strains at the

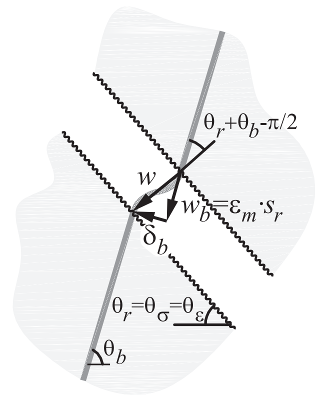


FIGURE 8 Crack width calculation from reinforcement average strains provided by compatibility-based stress fields.

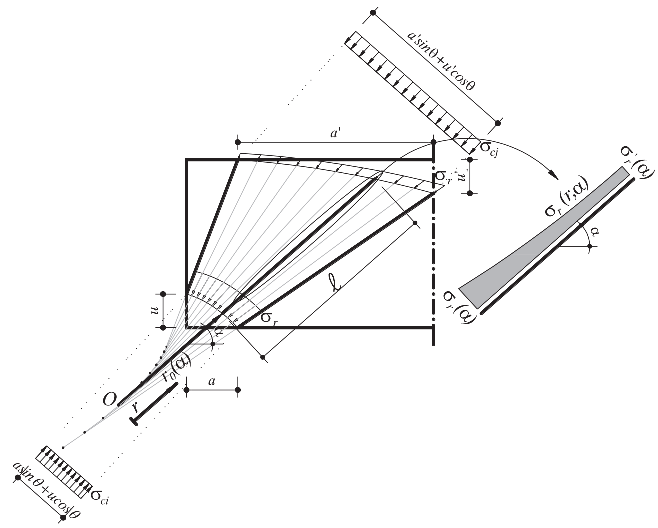


FIGURE 9 Stresses of a non-centred fan.

cracks,[25] thus leading to increased reinforcing bar stiffness and accounting for the presence of surrounding concrete. While this is a complex phenomenon, the consideration of simplified bond shear stress-slip relationships (as e.g. proposed in the Tension Chord Model,[38] Figure 7a) allows stiffening of reinforcement and crack spacing to be linked to two factors only: concrete strength class and effective amount of reinforcement of each reinforcing bar segment. Application of this approach, together with a reinforcement response that includes hardening behavior,[39] is essential to obtain the deformation capacity of the structure since reinforcement embedded into concrete ruptures at an average strain much smaller than the failure strain of a bare reinforcement (Figure 7b).

The main challenge to determining tension stiffening in a general manner is to assign to each reinforcing bar an appropriate concrete area acting in tension between the cracks (i.e. define the effective reinforcement amount). Provisions for setting this effective concrete area based on concrete cover and bar spacing are given in design codes, albeit only for specific cases. To solve the lack of generality of these provisions, a numerical procedure has been proposed[15] to calculate this effective reinforcement ratio for any reinforcement configuration (Figure 7c).

Crack widths can be easily estimated after performing a Compatibility-based Stress Field analysis that accounts for tension stiffening and long-term effects, following the approach in Figure 8. The crack width (w) depends on the crack spacing (s_r) provided by the tension stiffening model, the average reinforcement strain (ε_m) and the relative angle between the crack (θ_r, as defined by the principal directions of stresses and strains) and the reinforcement direction (θ_b):

$$w = \frac{w_b}{\cos\left(\theta_r + \theta_b - \pi/2\right)} = \frac{\varepsilon_m \cdot s_r}{\cos\left(\theta_r + \theta_b - \pi/2\right)} \tag{4}$$

5.4 | Adaptive stress field method

The energy-based approach of Adaptive Stress Field Models (ASFM)[49] takes advantage of the stress field method to perform simple nonlinear analysis of structural concrete regions. In addition, the interpretation of the flow of forces derived from this method allows for an intuitive understanding of the structural behavior during the complete loading process. The adaptive stress field method considers that the principle of minimum complementary energy governs internal stress redistribution caused by the nonlinear response of the materials. On that basis, the load-carrying elements (compression fields and ties) are adapted iteratively. This method does not have a stiffness matrix governing the system, like the EPSF and CSFM methods presented before. The solution is obtained by an optimization process in which the variables are the stress field geometry or the nodes location connecting the struts and ties. This stress based optimization concept is in a sense similar to the FELA approach (presented in the next chapter), but it is simpler as it uses discrete elements and not a continuous domain. The mechanical properties of the elements under compression and tension are obtained directly from the geometry of the stress fields, accounting for the nonlinear constitutive relationships of the materials.[49]

With respect to the consideration of the concrete response, the fan-shaped compression stress fields are defined according to the equilibrium conditions of an infinitesimal strut within the fan (Figure 9). The radial stresses at each boundary are defined according to

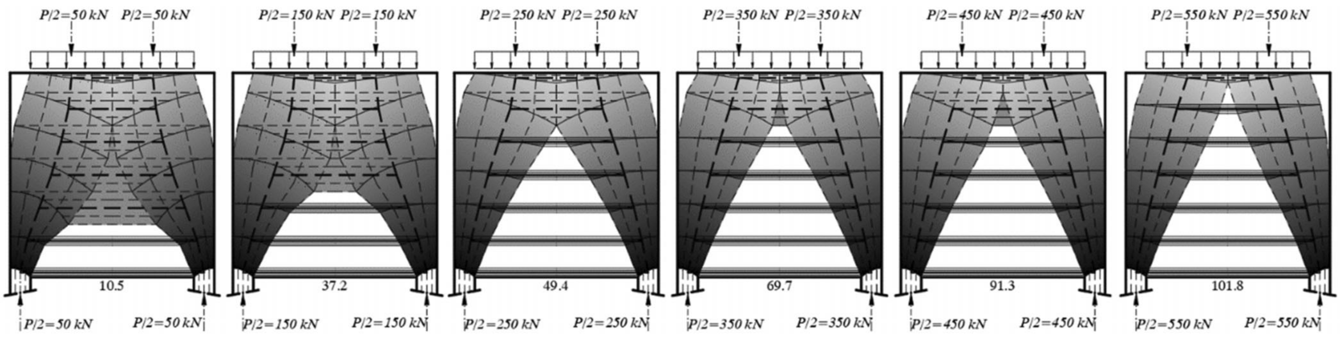


FIGURE 10 Deep beam adaptive stress field model at several steps of loading.

Equations 5, where $\sigma_I, \sigma_{II}, \sigma'_I, \sigma'_{II}$ are the principal stresses in the end nodes of the fan.

$$\sigma_r = \sigma_{II} \frac{1 + tan^2\alpha}{\frac{\sigma_{II}}{\sigma_I} + tan^2\alpha} \qquad \sigma'_r = \sigma'_{II} \frac{1 + tan^2\alpha}{\frac{\sigma'_{II}}{\sigma'_I} + tan^2\alpha} \tag{5}$$

Prismatic stress fields can be derived as a particular case of fan-shaped stress field. The distribution of concrete strains is obtained considering a suitable stress–strain curve[49] and the compression softening, which depends on the presence of transversal tensile strains.[20] The compression softening is calculated by computing the strains in the tension elements and if a compression stress field crosses the ties, the transversal strain is calculated following the Mohr's circle for strains.

It is noted that the total internal energy of the system (adaptive structure) is mainly influenced by its ties, particularly once the concrete cracks. To suitably simulate the response of a structural concrete region (both under service loads and after yielding of the reinforcement), detailed models for the tension stress fields are required. To that end, constitutive relationships for reinforced concrete ties can be used.[49] When smeared cracking conditions can be ensured, assuming a stabilized cracking pattern is suitable. However, such condition does not apply in many discontinuity regions (e.g. load near support, dapped-end beam, etc). In such cases, the cracking pattern is usually characterized by a main crack that has a relevant effect on the global structural behavior. To properly simulate a reinforced concrete tie element, the approach presented in the Tension Chord Model[38] is considered within the frame of the adaptive stress field method (Figure 7a).

The predictions of the Adaptive Stress Field Method are presented in a simple deep beam in Figure 10. The numerical model included the horizontal web reinforcement. The adaptive variables were the relative horizontal coordinates of compression stress fields, leading to either compression or tension in the horizontal elements. The model adequately simulated a large stress redistribution, also observed in deep beam tests,[3] with a significant increase in the global inner lever arm after concrete cracking. The available ductility allowed taking advantage of the beam's effective depth fully. The failure was reached by concrete crushing at the supports and yielding of the bottom reinforcement.

5.5 | Finite element limit analysis

As stated in the Introduction, multiple solutions to the same problem are possible when stress fields are established on the basis of lower-bound theorem of limit analysis. Each stress field is related to a specific lower bound to the load-carrying capacity. This essentially means that the task of determining the load carrying capacity based on limit analysis can be formulated as an optimization problem. For complex structures, however, it can be very time consuming, or even impossible, for an engineer to determine, via manual calculations, a suitable stress field. Furthermore, an engineer would in most cases not be able to assess his/her choice of design against the optimal solution.

To overcome these challenges, engineers may adopt the method of Finite Element Limit Analysis (FELA), which provides a general framework for numerical determination of the optimal stress fields in structures with rigid-plastic material behavior. Basically, the FELA concept combines a stress-based formulation of the finite element method with optimization algorithms to determine the load-carrying capacity. The method ensures that, for a given model, the optimal solution is always found. The first framework for FELA appeared in the 1970s,[41,42] where simple problems were solved by use of linear programming. In contemporary works,[16,43–48] more advanced nonlinear optimization technics are employed, significantly enhancing the efficiency and ability of the method to solve large-scale problems.

Similar to the above-mentioned EPSF-approach, FELA uses a mesh discretization known from the finite

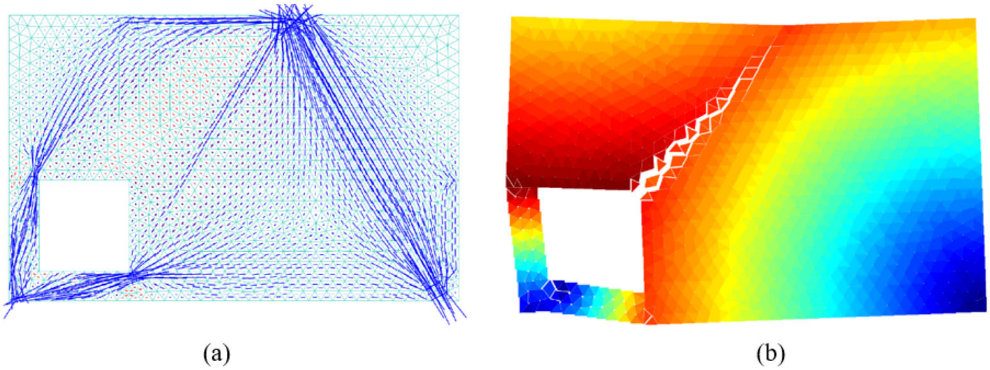


FIGURE 11 Finite element limit analysis of deep RC beam with opening: (a) optimal stress field (principal stresses); (b) corresponding collapse mechanism[18]

element method to translate the continuum mechanical problem into a numerical format. However, except from this similarity, the analysis strategy required in FELA is distinctly different from the (incremental and displacement based) solution strategy of the classical finite element method. Because of the assumption of rigid-plastic material, it is not possible in a lower bound formulation of FELA to base the response of each finite element on an approximation of the displacement field. Instead, equilibrium elements are used. In simple terms, this means that instead of having a stiffness matrix, *equilibrium elements* are formulated by directly establishing the equilibrium equations that relate the forces acting on the element to the stress field assumed within the element. Several types of lower bound elements have been developed.[44]

The fact that the lower bound theorem only involves equilibrium and yield conditions has the consequence that in a FELA formulation, the number of stress variables to be determined will, in general, exceed the number of available equations, leading to problems with multiple solutions. Therefore, the challenge here is to formulate the problem in a manner that, even for large-scale problems, is computationally efficient. For the type of yield criteria (optimization constraints) encountered in reinforced concrete, convex optimization techniques, such as second-order cone programming and semidefinite programming, have shown to be extremely effective and recent research and development works have shown that FELA has now matured to a point where applications to large-scale problems are possible.[46,47]

As a simple example, Figure 11a shows the optimal stress field obtained by FELA for a deep beam with an opening. The result agrees well with the solution found by use of EPSF - detailed discussions and comparisons may be found elsewhere.[18] For a model of the deep beam containing approx. 8000 equilibrium elements, the optimization takes less than 1 min on a standard desktop computer. It is noted that in FELA, computational time is approximately proportional to the problem size (i.e. number of elements), which is an important feature when dealing with large scale problems.

Figure 11b shows the corresponding collapse mode, which can be found by solving the so-called dual optimization problem in FELA.[49] Note that the collapse mechanism determined by FELA does not contain information about absolute displacements, but only the displacement rates. This means that when using FELA, it is not possible to assess the deformation capacity of the structure, but only assess qualitatively the nature of the collapse mode (i.e. ductile or brittle).

Recently, the FELA approach has been extended to include elasto-plastic material behavior in order to model, not only the load carrying capacity, but also the complete load–displacement response of the structure. The extension of the original (rigid-plastic) optimization problem is performed by adopting a stress-based finite element formulation and considering energy principles (principle of minimum complementary elastic energy). This allows taking into consideration the nonlinear material response[54] and is useful to estimate the strain field of the member (and thus the associated efficiency factors of concrete). As it can be noted, such approach is in fact similar to the optimization criterion in the above-mentioned ASFM approach.

5.6 | Stringer-panel-models

The stringer-panel-model (SPM)[50] is a particular application of stress fields for modeling plane wall-type structures or structural details. This method uses two types of elements: two-dimensional shear panels and one-dimensional stringers (see Figure 12). The stringers are located at the interfaces between the panels and the outer boundary of the structure. The crossing points of stringers are nodes, where external loading and supports can be applied. Loads can also consist of homogenously distributed forces along the path of the stringers. Panels transfer uniform shear membrane forces, and the

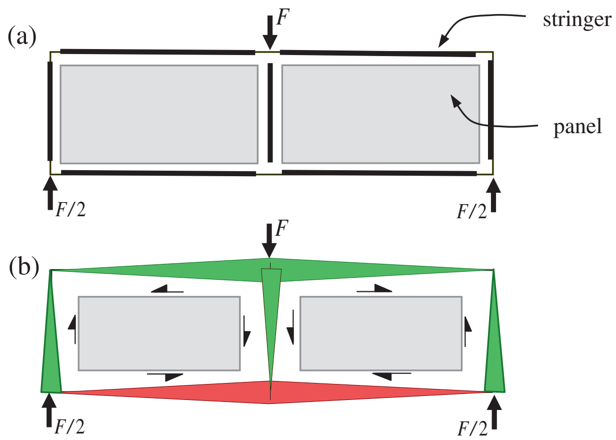


FIGURE 12 Concept of the stringer panel method: (a) stringers and panels; (b) forces in the elements (figure adapted from[18])

stringers transfer normal forces. Because of the uniform shear forces in the panels, the normal force varies linearly over the path of a stringer. Stringer forces can be tensile and compressive, and lumped forces in the nodes. Although rectangular panels are considered most often,[18,50] the approach is also suitable for quadrilateral panels and can be extended to compositions of planes in three dimensions (e.g. box beams). The method is particularly handy for D-regions, but can also be applied to a whole structure.

Simple, coarse models are often statically determinate, so that panel shear forces can be solved by equilibrium considerations alone (and from the shear forces the stringer forces are derived). The applied loads on the model must constitute a set of external equilibrating forces. Such models are a typical LoA I application, especially useful for preliminary design. The evaluation can be performed by hand and is adequate for many structural regions of discontinuity.

The calculated tensile stringer forces require bundled reinforcement and compressive stringer forces are subjected to a verification in order not to exceed the available compressive strength. In the panels, distributed reinforcement in two orthogonal directions is required, and concrete compressive stresses in diagonal directions must be verified. Typical design examples in this category are: corners of portal frames, corbels, abrupt changes of beam depth, dapped beam ends, and openings in girder webs.

More complicated models will become statically indeterminate. An example is a cantilever deep beam with an opening (*fib* Bulletin 61[52]). Statically indeterminate models can be handled in two different ways. In the first way, a LoA I approach is adopted, starting from plasticity considerations. The structural engineer transfers the model into a structural determinate one, by simply assigning shear values to a sufficient number of elements (thus allowing an equilibrium-based solution through hand-calculation). The second way for statically indeterminate models is to execute a stiffness analysis, accounting not only for equilibrium alone, but also for compatibility and relevant constitutive laws. In this case, the structural engineer operates in the domain of LoA III, and typically requires adequate software. If used in combination with material nonlinearity, the approach tends to an LoA IV.

The capabilities of the SPM are shown in the following for a wall example (Figure 13) presented in *fib*'s Bulletin 100.[18] The selected mesh is almost the coarsest possible. Because of symmetry, the analysis can be restricted to the left half of the wall. The choice of the shear force in four panels transforms the problem into a statically determinate one. Three different analyses are shown here. In the first one, a homogeneous distribution of the support reaction at the base is assumed (Figure 13a). The shear force of four panels is chosen zero (light shading). The analysis can be performed by hand. In the second analysis (Figure 13b) another group of four zero-shear panels is chosen, with a very concentrated distribution of the base reactions (significantly different from the homogeneous distribution of that shown in Figure 13a). Again, a hand analysis can be performed. In the third analysis (Figure 13c) a stiffness analysis has to be performed. For this analysis a rigid base is assumed. Now, appropriate geometric and stiffness data of the stringers and shear panels must be provided.

Here, only the resulting reinforcement sketches of the three analyses are shown. In cases (a) and (c), dominant stringer reinforcement is located above the opening, whereas in case (b) the position of the dominant stringer reinforcement is, on the contrary, below. The required two-way mesh reinforcement can be shaped by two different meshes, a light one (actually minimum reinforcement) and a heavier one. Again, case (b) deviates from cases (a) and (c). The plasticity choice of case (b) evidently strongly violates the linear-elastic stress solution (c), where the plasticity choice of case (a) keeps close to it. With the choice of case (b) heavy stress redistribution must be expected.

5.7 | 3D stress field models - FESCA 3D

The application and scope of strut-and-tie and the stress field models has evolved since the early days of these methods. Originally, its application focused on relatively simple reinforced concrete elements. With time, while researchers contributed to refine the methods, designers

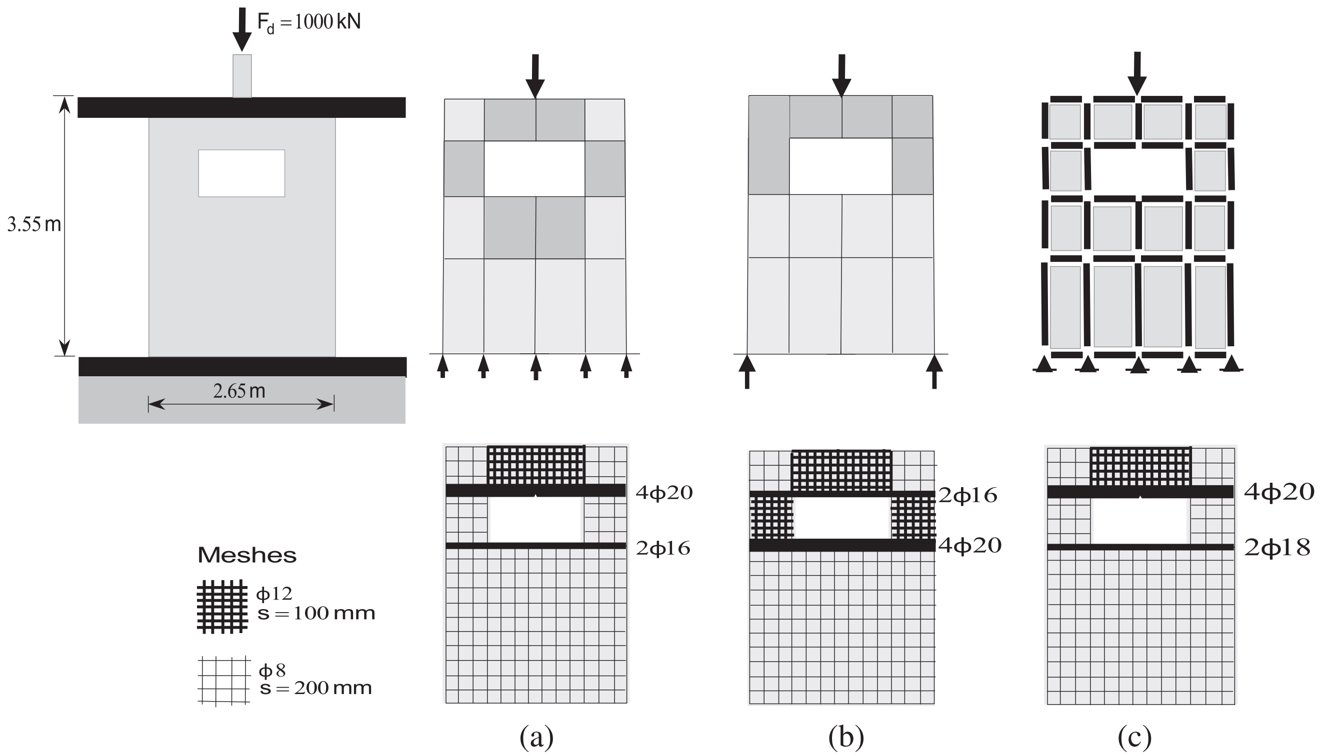


FIGURE 13 Application of the stringer panel method to the design of a wall with opening under column load: (a)-(b) hand calculation introducing plasticity; (c) computational linear-elastic stiffness analysis.

gained confidence on its use, applying them to increasingly complex elements. Today, the strut-and-tie and the stress field methods are well-recognized tools for analyzing and designing concrete elements with a 2D behavior. Although this covers a wide number of practical situations, 3D behavior is required for others. This is for instance the case of a pile cap foundation, as well as others (as anchorage blocks, wind tower turbines, etc). One main step for a complete generalization of the methods is its extension to 3D.

The limited progress experienced so far in applying the strut-and-tie and stress field methods to 3D is not by chance. Firstly, treating many concrete elements as 2D entities is a sound assumption. Secondly, adding a third dimension significantly complicates the analysis, problem solution, and visualization of flow of forces. And thirdly, experimental campaigns focusing on the characterization of the 3D behavior of concrete are scarce, especially in cracked states. This fact ultimately leads to lack of guidance and uncertainties in the determination of the strength of 3D struts and nodal zones.

The strut-and-tie and the stress field models were originally used as hand calculation tools. The development of 3D strut-and-tie models by hand (as truss-models) is feasible, at least for understanding the flow of forces, although the verification of nodal zones still represents difficulties. However, the development and calculation of stress fields in 3D by hand calculations is impractical. It has been shown that computer modeling might overcome the drawbacks of these methods. The power of simplified, 2D nonlinear finite element analysis neglecting the tensile strength of concrete was shown by the Elastic–Plastic Stress Field method.[13] This approach was extended to 3D with the development of FESCA 3D tool (Finite Elements for Simplified Concrete in 3D[56]).

Different FE software packages allow for the development of 3D FE models, considering linear and nonlinear behavior. On the one hand, linear elastic analysis does not represent the real behavior of reinforced concrete structures, but stress fields derived from these analyses were originally seen as a sound baseline to orientate the truss elements in strut-and-tie models. However, 3D linear stress fields are not easy to use as a basis for strut-and-tie models, as stresses generally become disperse in the concrete volume. On the other hand, and similarly to 2D, full nonlinear analyses are not suitable for design purposes (require a high level of expertise as results can be very sensitive to the adopted material parameters). FESCA is an intermediate approach, where the level of nonlinearity is sufficient to greatly simplify the material response (no tensile strength, plastic response, consideration of compression softening due to transverse cracking in 3D models[56] or adapting planar models, for example,[20] accounting for the sum of principal tensile strains[54]) and provides results that are robust and can be easily exploited.

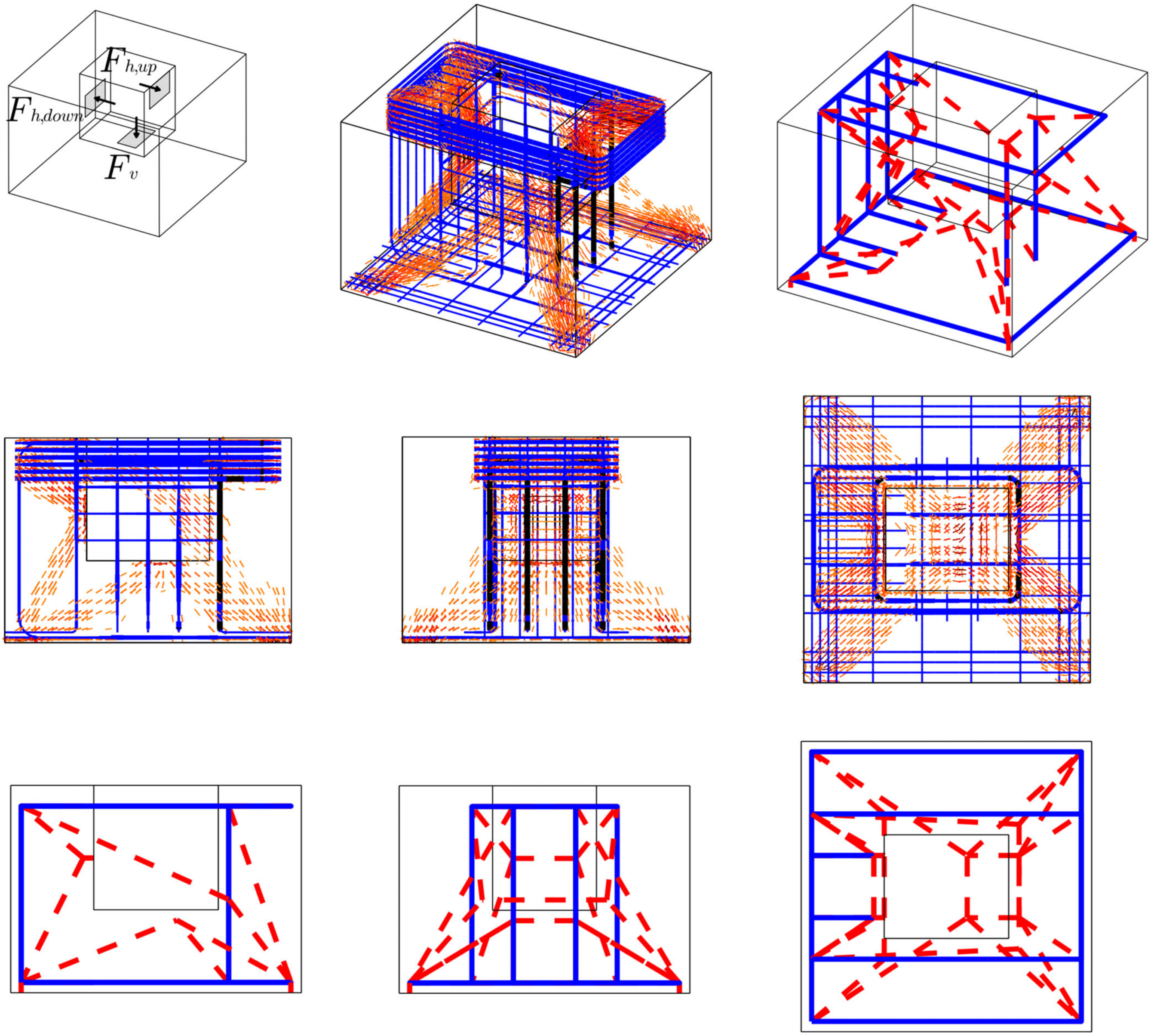


FIGURE 14 Application of FESCA 3D for the development of 3D stress fields of a socket base column-to-foundation connection supported on four corner piles[52,53]

Figure 14 shows an instance of an application to a socket base column-to-foundation connection. In the figure, the derivation of a 3D strut-and-tie model compatible with the arranged reinforcement based on the obtained stress field is also presented. Regarding the capacity check of struts and nodal zones, finite elements allow assessment of the concrete stress at any integration point, which can then be compared against the maximum allowable value. The latter is determined by the adopted concrete failure criteria, generally considering the multi-axial stress state. Compared to 2D, a 3D analysis allows for a more complete consideration of the influence of the confinement/cracked states on the effective compressive strength as the stress state in all spatial directions is known. However, further experimental research is needed for a comprehensive definition of a concrete failure surface including all potential 3D stress/strain state combinations, as the current experimental database is mainly derived from 2D stress/strain states.

6 | CONCLUSIONS

Strut-and-tie models and stress fields are recognized as a reliable, rational framework for designing and assessing reinforced concrete structures. In order to exploit their full potential in view of modern computational capabilities, this article discusses how these approaches can be used efficiently under different numerical implementations and levels of refinement. The main advantages and limitations of these methods can be summarized as follows:

- They offer conceptual clarity and simplicity, allowing the engineer to develop a sound understanding of the ultimate behavior of the structure.
- Strut-and-tie models and stress fields are well-suited for the Levels-of-Approximation (LoA) approach, where the complexity and accuracy of the model can be selected according to the task at hand (e.g. design phase of the project, local or global structural complexity, the influence of local behavior to global structural response, design, or assessment), while relying on the same fundamental principles.
- They produce solutions consistent with the lower-bound approach of limit analysis and, therefore, can be used within the safety formats of current design codes (e.g. partial safety factors).
- While strut-and-tie models and stress fields are mainly applicable to ultimate limit state design/assessment, they can also be extended to evaluate serviceability and deformation capacity, taking advantage of computer modeling.
- As strut-and-tie models and stress fields are grounded on the theory of plasticity, their application is limited to members/structures which possess sufficient deformation capacity. Such capacity can be ensured with minimum distributed reinforcement as required by current design codes, even though further research is needed to determine the amounts of this reinforcement on a rational basis.

Finally, in terms of levels of approximation, it is always recommended to perform a simple analysis, even when the task requires higher level of refinement and computer modeling. In this manner, the engineer can develop an in-depth and sound understanding of the structure, decide where to allocate the main reinforcement (ties), and avoid/identify errors when using more complex approaches.

ACKNOWLEDGMENTS

The authors of this article would like to express their gratitude to all members of fib's WP 2.2.4, particularly to Stein Atle Haugerud and Quentin Roubaty, for the fruitful and stimulating conversations held during writing of fib's Bulletin number 100, whose content is the basis of the present manuscript.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ORCID

Miguel Fernández Ruiz https://orcid.org/0000-0001-6720-8162
Jaime Mata-Falcón https://orcid.org/0000-0001-8701-4410
Miguel Pedrosa Ferreira https://orcid.org/0000-0002-1990-3598

REFERENCES

1. Ritter W. The Hennebique Construction Method (in German, "Die Bauweise Hennebique"), Schweizerische Bauzeitung, 1899. XXXIII, No. 7, 41–61.
2. Mörsch E. Reinforced concrete construction, theory and application (in German, "Der Eisenbetonbau, seine Theorie und Anwendung"). 3rd ed. Stuttgart: Verlag von Konrad Wittwer; 1908. p. 376.
3. Leonhardt F, Walther R. Deep beams (in German, "Wandartige Träger"), Deutscher Ausschuss für Stahlbeton, 1966, Heft 178, Ther Ber, 159.
4. Schlaich J, Weischede D. A practical method for methodical design and building in reinforced concrete (in German, "Ein praktisches Verfahren zum methodischen Bemessen und Konstruieren im Stahlbetonbau"), bulletin d'Information n° 150. Geneva: Comité Euro-International du Béton; 1982. p. 163.
5. Collins MP, Mitchell D. Rational approach to shear design--the 1984 Canadian code provisions. J Proc. 1986;83(6), p. 925–933.
6. CEB-FIP. Model Code 90, CEB-FIP Bulletins N°213/214. London: Thomas Telford Ltd.; 1993. p. 460.
7. Drucker DC. On structural concrete and the theorems of limit analysis, publications, International Association for Bridge and Structural Engineering, Zurich: IABSE Publications. Vol 21; 1961. p. 49–59.
8. Nielsen MP, Hoang LC. Limit analysis and concrete plasticity. 3rd ed. Boca Raton: CRC Press; 2016. p. 796.
9. Nielsen MP, Braestrup MW, Bach F. Rational analysis of shear in reinforced concrete be.ms, IABSE Colloquium Proceedings, Bergamo, Italy P-15; Vol. 2, 16, 1978.
10. Thürlimann B. Plastic Analysis of Reinforced Concrete Beams. Vol 28. Copenhagen, Denmark: IABSE Colloquium; 1979. p. 71–90.
11. Marti P. Truss models in detailing. Concrete International. 1985;7:66–73.
12. Schlaich J, Schäfer K, Jennewein M. Toward a consistent design of structural concrete. Prestressed Concrete Institute Journal, 1987.
13. Fernández RM, Muttoni A. On development of suitable stress fields for structural concrete. ACI Struct J. 2007;104(4): 495–500.
14. Muttoni A, Fernández Ruiz M, Niketić F. Design versus assessment of concrete structures using stress fields and strut-and-tie models. Am Concrete Inst Struct J. 2015;112(5):605–15.
15. Mata-Falcón J, Tran DT, Kaufmann W, Navrátil J. Computer-aided stress field analysis of discontinuity concrete regions, proceedings of the computational modelling of concrete and concrete structures conference (EURO-C 2018). Bad Hofgastein, Austria: CRC Press; 2018. p. 641–50.
16. Krabbenhøft K, Lyamin AV, Sloan SW. Formulation and solution of some plasticity problems as conic programs. Int J Solids Struct. 2007;44:1533–49.

17. fib. Model Code for Concrete Structures 2010. Lausanne, Switzerland: Ernst & Sohn; 2013. p. 434.
18. fib. Working group 2.2.4. Design and assessment with strut-and-tie models and stress fields: from simple calculations to detailed numerical analysis. fib Structural Concrete Federation, Bulletin No 100, 2021, 235.
19. Muttoni A. The applicabilty of the theory of plasticity to design of reinforced concrete (in German: "die Anwendbarkeit der Plastizitätstheorie in der Bemessung von Stahlbeton"), No 176. Basel, Switzerland: Birkhäuser Verlag, Institut für Baustatik und Konstruktion ETH Zürich; 1990. p. 164.
20. Vecchio FJ, Collins MP. The modified compression-field theory for reinforced concrete elements subjected to shear. ACI J Proc. 1986;83(2):219–31.
21. ACI Committee. ACI 318–14 building code requirements for structural concrete and commentary, ACI Committee 318. Detroit: American Concrete Institute; 1724.
22. Kostic N. Topology of stress fields for the design of reinforced concrete structures, Doctoral dissertation. EPFL Lausanne, Switzerland; 2009.
23. Ferreira M, Almeida JF, Lourenço M. Modelling bond within elasto-plastic stress fields (EPSF) models. *Fib* symposium, innovations in materials, design and structures. In Derkowski W, et al. Proceedings of the Symposium Krakow: Gdańsk University of Technology; 2019.
24. Kaufmann W, Mata-Falcón J, Beck A. Future directions for research on shear in structural concrete. *Fib* bulletin 85: towards a rational understanding of shear in beams and slabs, Lausanne, Switzerland: fib publications; 2018.
25. Kaufmann W, Mata-Falcón J, Weber M, Galkovski T, Tran, Kabelac J, Konecny M, et al. Compatible stress field design of structural concrete: principles and validation, Zurich, Switzerland: ETH Zurich and IDEA StatiCa s.r.o, ISBN 978–3–906916-95-8; 2020. p. 158.
26. Kuchma D, Tjhin TN. CAST (computer aided strut-and-tie) design tool structures, structures congress, Washington: ASCE, 2001. https://doi.org/10.1061/40558(2001)142
27. Maekawa KH, Okamura H, Pimanmas A. Non-linear mechanics of reinforced concrete. Boca Raton: CRC Press,; 2003.
28. Bažant Z, Adley MD, Carol I, Jirásek M, Akers SA, Rohani B, et al. Large-strain generalization of microplane model for concrete and application. ASCE J Eng Mech. 2000;126(9), p. 971–980.
29. Vecchio FJ. Disturbed stress field model for reinforced concrete: formulation. ASCE J Struct Eng. 2000;126(9):1070–7.
30. Valeri P, Fernández Ruiz M, Muttoni A. Modelling of textile reinforced concrete in bending and shear with elastic-cracked stress fields. Eng Struct. 2020;215:110664.
31. Campana S, Muttoni A. Analysis and design of an innovative solution for tunnels using elastic-plastic stress fields, proceeding of the 8th *fib*-PhD symposium. Copenhagen, Denmark: Technical University of Denmark DTU; 2010. p. 75–80.
32. de Saint-Venant B. Comptes Rendus. 1870;70:473.
33. Lévy M. Mémoire sur les équations générales des mouvements intérieurs des corps solides ductiles au delà des limites où l'élasticité pourrait les ramener à leur premier état. CR Acad Sci Paris. 1870;70:1323–5.
34. Garbelini C. Advances in the soil-structure interaction analysis – from surface footings to thermoactive deep foundations, PhD thesis. École Polytechnique Fédérale de Lausanne, Switzerland; 2020. p. 303.
35. Prager W. The theory of plastic flow versus theory of plastic deformation. J Appl Phys. 1948;19:540–3.
36. Argirova G, Fernández Ruiz M, Muttoni A. How simple can nonlinear finite element modelling be for structural concrete? Vol 66, Extra 1, m013. Spain: Informes de la Construcción, IETCC-CSIC; 2014. p. 1–8.
37. Yu Q, Valeri P, Fernández Ruiz M, Muttoni A. A consistent safety format and design approach for brittle systems and application to textile reinforced concrete structures. Eng Struct. 2021;249:113306.
38. Marti P, Alvarez M, Kaufmann W, Sigrist V. Tension chord model for structural concrete. Struct Eng Int. 1998;8(4):287–98.
39. Sigrist V. Zum Verformungsvermögen von Stahlbetonträgern. Vol 210. Zurich, Switzerland: ETH Zurich; 1995.
40. Grierson DE, Gladwell GML. Collapse load analysis using linear programming. ASCE J Struct Div. 1971;97:1561–73.
41. Anderheggen E, Knöpfel H. Finite element limit analysis using linear programming. Int J Solid Struct. 1972;8(1972):1413–31.
42. Makrodimopoulos A, Martin CM. Lower bound limit analysis of cohesive-frictional materials using second-order cone programming. Int J for Num Meth In Eng. 2006;66(4):604–34.
43. Bispos C, Pardalos P. Second-order cone and semi-definite representation of material failure criteria. J Optimiz Theory Appl. 2007;134(2):275–301.
44. Larsen, KP. Numerical limit analysis of reinforced concrete structures. Ph.D. thesis, DTU, Dept. of Civil Eng., 2010.
45. Herfelt MA. Numerical limit analysis of precast concrete structures. PhD thesis, DTU, Copenhagen, Denmark: Dept. of Civil Eng., 2017.
46. Herfelt MA, Poulsen PN, Hoang LC, Jensen JF. Lower bound plane stress element for modelling 3D structures. Proc Inst Civil Eng Eng Comput Mech. 2017;170(3):107–17.
47. Jensen TW, Poulsen PN, Hoang LC. Layer model for finite element limit analysis of concrete slabs with shear reinforcement. Eng Struct. 2019;195:51–61.
48. Krenk S, Damkilde L. Høyer, O limit analysis and optimal design of plates with equilibrium elements. J Engng Mech. 1994;120:1237–54.
49. Andersen MEM, Poulsen PN, Olesen JF. Finite-element limit analysis for solid modeling of reinforced concrete. J Struct Eng. 2021;147(5):2021.
50. Lourenço M, Almeida J. Adaptive stress field models: assessment of design models. Closure ACI Struct J. 2013;110(6), p.1109–1124.
51. Blaauwendraad J. Stringer-panel models in structural concrete: Applied to D-region Design. Berlin, Germany: Springer Briefs in Applied Sciences and Technology; 2018. p. 99.
52. fib. Working party 1.1-3, design examples for strut-and-tie models, fib, structural concrete federation, Bulletin No 61, 2011, 219.
53. Meléndez C, Miguel PF, Pallarés L. A simplified approach for the ultimate limit state analysis of three-dimensional reinforced concrete elements. Eng Struct. 2016;123:330–40.
54. Meléndez C. A finite element-based approach for the analysis and design of 3D reinforced concrete elements and its application to D-regions, PhD Dissertation. Valencia, Spain: Universitat Politècnica de València; 2017.
55. Vestergaard D, Larsen KP, Hoang LC, Poulsen PN, Feddersen B. Design oriented elasto-plastic analysis of reinforced concrete structures with in-plane forces applying convex optimization. Struct Concre. 2021;22(6):3272–87.
56. Vecchio FJ, Selby R. Toward compression-field analysis of reinforced concrete solids. J Struct Eng. 1991;117(6):1740–58.

AUTHOR BIOGRAPHIES

Miguel Sério Lourenço, Senior Structural Engineer, JSJ Ltd, Lisbon, Portugal; Invited Professor, Instituto Politécnico de Setúbal, mlourenco@jsj.pt.

Miguel Fernández Ruiz, Professor, School of Civil Engineering, Universidad Politécnica de Madrid, Spain, miguel.fernandezruiz@upm.es.

Johan Blaauwendraad, Professor-Emeritus, Delft University of Technology, info@blaauwendraad.online.

Stathis Bousias, Professor, Department of Civil Engineering, University of Patras, Greece, sbousias@upatras.gr.

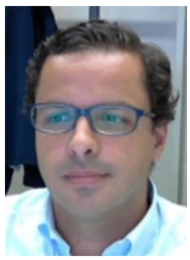
Linh Cao Hoang, Professor, Department of Civil and Mechanical Engineering, Technical University of Denmark, linho@byg.dtu.dk.

Jaime Mata-Falcón, Senior Researcher, Institute of Structural Engineering, ETH Zurich, mata-falcon@ibk.baug.ethz.ch.

Carlos Meléndez, Senior Structural Engineer, Esteyco SA, Madrid, Spain, carlos.melendez@esteyco.com.

Boyan I. Mihaylov, Assistant Professor, Urban and Environmetal Engineering, University of Liege, boyan.mihaylov@uliege.be.

Miguel Pedrosa Ferreira, Grupo NOV, Portugal, miguelpedrosaferreira@gmail.com.



Duarte Viúla Faria, Senior Structural Engineer, Muttoni & Fernández, Ingénieurs Conseils SA, Ecublens, Switzerland, duarte.viulafaria@mfic.ch.

How to cite this article: Lourenço MS, Fernández Ruiz M, Blaauwendraad J, Bousias S, Hoang LC, Mata-Falcón J, et al. Design and assessment of concrete structures with strut-and-tie models and stress fields: From simple calculations to detailed numerical analysis. Structural Concrete. 2023. https://doi.org/10.1002/suco.202200647