

SCALING ANALYSIS BASED ON EXTREMAL POINTS TOPOLOGY

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Abstract The interaction of different scales is among the most interesting and challenging features in turbulence research. Existing approaches used for scaling analysis such as structure-function and Fourier spectrum method have their respective limitations, for instance scale mixing, i.e. the so-called infrared and ultraviolet effects. To make improvement in this regard, a new method, multi-level segment analysis (MSA) based on the local extrema statistics, has been developed. Data test results show that MSA can successfully reveal different scaling regimes in complex systems such as Lagrangian and two-dimensional turbulence, which have been remaining controversial in turbulence research. In principle MSA can generally be applied for various analyses.

METHOD DEFINITION

Turbulence is a complex system where a broad range of spatial/temporal scales interacts. Typically such interaction generates different scaling relations in the respective scale ranges. The conventional analysis approach to understand the scaling properties is structure-function (SF). However, the average operation in SF mixes regions with different correlations [4], i.e. the filtering (as infrared and ultraviolet) effect. Other alternatives such as Fourier analysis in the frequency have the same limitation.

A new view on the field structure is based on the topological features of the extremal points, which in principle inherit the imprint of turbulence dynamics. Considering a one-dimensional physical process $f(x)$ with a prescribed s , we can collect all the local extremal points with respect to scale s (or within a window with width of s). Here we define segment as the part of $f(x)$ between two adjacent extremal points. At a specific s level, denote the corresponding extremal point set as $x_{s,i}$, $i = 1, 2, \dots$ (along the coordinate increasing direction). From the segments and extremal points the scale and functional structure can be determined by $f(x_{s,i}) - f(x_{s,i-1})$ and the length scale, i.e., $\ell = x_{s,i} - x_{s,i-1}$, which are by nature from the physical process itself without arbitrariness. Scan over different s and collect all the segment characteristics, the functional statistics can be determined. Phenomenologically varying the s value reflects the turbulence (scale) hierarchy structure. In this sense this procedure is named as multi-level segment analysis (MSA).

MAIN RESULTS AND CONCLUDING REMARKS

It has been remaining controversial to understand the scaling relations in Lagrangian Turbulence and two-dimensional (2D) turbulence, which have some theoretical predictions, however, can not be convincingly verified. In studying the Lagrangian velocity structure, the recent progresses [3, 2] have indicated the validity of the classical dimensional argument. The present MSA method can reproduce the same results as well, with a broader inertial range.

2D turbulence is another important test case. Both for the theoretical study and real applications 2D turbulence functions as an ideal model. One of the most fascinating features hereof is the dual-cascade mechanism, i.e. the inverse energy and forward enstrophy cascades co-existing in different scale regimes, from which it is straightforward to show by the dimensional arguments the different velocity structure function dependence on the separation scale l . Many efforts [1] have been devoted to detect this important problem. The results based on MSA are shown in Fig. 1. We can see clearly the two inertial regimes with reasonable scaling exponents.

In summary MSA differs from other methods by nature is that the length scale and functional structures are determined by turbulence physics, i.e. statistics of extremal points, without arbitrary and artificial input. Therefore in principle MSA is generally applicable. A serious deficit of the conventional SF is the strong mixing of different correlation regions due to sample averaging. To define the segment structure helps to annihilate such filtering and extract the possible scaling relations in respective scale regimes. Numerical tests have proved successful in studying the controversial Lagrangian and 2D turbulent systems. Moreover MSA does not have any special requirement on the data itself, such as periodicity, noise perturbation and unsmoothness structures.

References

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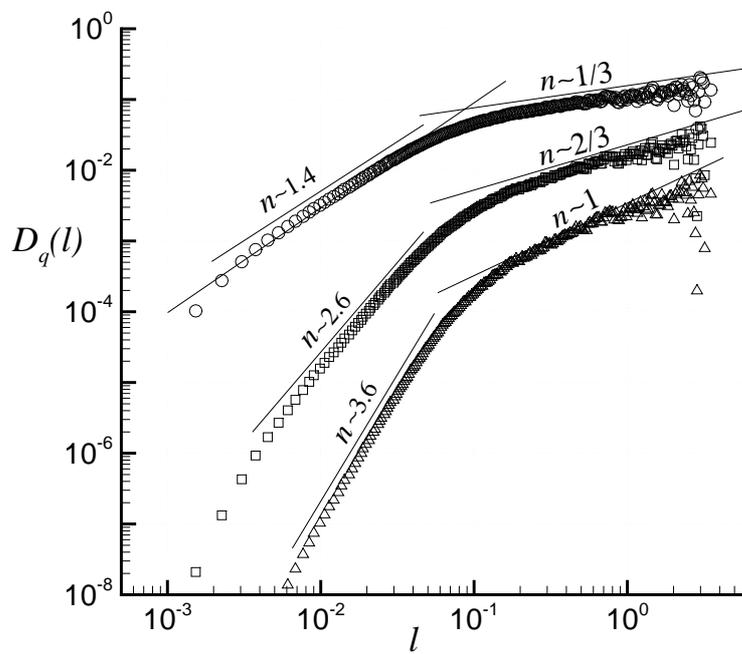


Figure 1. Longitudinal velocity structure functions by MSA in 2D turbulence.