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Summary Report of a 2006 VKI Course on Optimization and Multidisciplinary Design ; Applications to Aeronautics and Turbomachinery

Hierarchical Methods for Shape Optimization in Aerodynamics: Multilevel parametric shape algorithms and additive preconditioners

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When one disposes of, or is able to construct a hierarchy of either physical, or purely numerical models for the same physical situation, it is natural to attempt to devise a numerical method in which the fine model is in a way to precondition the coarse. In doing this, one expects to gain efficiency, and optimally, to achieve a numerical method whose convergence characteristics are independent of certain numerical factors such as local grid size, degree of representation, etc.

A prototype for such hierarchical methods is provided by the multigrid method for solving a set of discretized partial differential equations (PDE), typically a boundary-value problem of elliptic type. There, the hierarchy is associated with a sequence of grids of various degrees of refinement and the related discretizations of the PDE problem.

In the lecture, we begin by reviewing classical results about multigrid to demonstrate the effectiveness of such multilevel hierarchical methods, which for the model problem achieve the optimal linear convergence: equivalently, the cost for solving a discrete problem with N degrees of freedom is (only) proportional to N. Here, N is typically proportional to the number of gridpoints in a fine discretization.

This leads us naturally to raise the following question: how can hierarchical concepts be used to achieve higher, if not optimal efficiency in a numerical shape optimization related to a distributed (PDE) problem, such as, aerodynamic shape optimization of a body (wing, or configuration) immersed in a compressible flow governed by the Euler equations. Several

approaches to achieve this goal have been proposed in the literature. We present two that have been particularly emphasized at INRIA.
Our efforts have been mostly concentrated on improving the convergence rate of numerical procedures both from the viewpoint of cost-efficiency and accuracy, with the perspective of reducing the design cost, but also of mastering the election and control of the design parameters, geometrical ones in particular, in a more rational way, perhaps supported by error estimates.

Technically, our efforts tend to contribute to the following challenges:
* Construct multi-level (multi-scale) shape-optimization algorithms;
* Identify critical algorithmic ingredients (transfer operators, smoothers);
* Evaluate efficiency, theorize convergence via error estimates or an appropriate modal analysis.

In a first part, we discuss the construction of self-adaptive multilevel algorithms, in the context of parametric shape optimization. Embedded search spaces are defined based on a geometrical hierarchy of nested shape parameterizations of Bezier type. We provide some details on how such multilevel geometrical representations can be used to define multilevel algorithms for shape optimization, and how parameterization adaption can be devised. We present some typical results related to a model problem in calculus of variations introduced in depth in [1], and we refer to [2]-[3]-[4] for examples of applications to aerodynamics. In particular, in these publications, the so-called "Free-Form Deformation" approach is used to extend our basic multilevel construction of parametric spaces to encompass 3D deformations in a bounding box, making our approach far more general. Second, for purpose of analysis, we present a simple conceptual model problem for shape optimization, and illustrate the corresponding eigenmodes [5]. This model allows us to discuss a central issue in multilevel algorithms: smoothing.

The second part concentrates on the case where the shape parametrisation relies on a local discretisation of the shape via the mesh. The number of parameters increases with the mesh refinement. In such case, the main challenge results from the stiffness of the numerical optimisation problem formulation implying a lower convergence. We analyse this stiffness as a lack of functional smoothness of the optimisation iteration. With this analysis, the degree of smoothness to recover is identified from an Hadamard formula. An exact compensation of this lack of smoothness is then possible and produces the best preconditioning of the optimisation descent iteration. We propose a technique for extracting a hierarchy of levels. For doing this on the unstructured meshes discretising the shapes of an aircraft, the multilevel geometrical data structure of agglomeration multigrid is used. In [6], this technique was exploited to define a hierarchical optimization algorithm. As an extension, we have also proposed certain "additive preconditioners" [7,8,9] inspired from the work by Bramble-Pasciak-Xu and able to recover the exact degree of smoothness. The descent direction is then built as follows: a gradient of the objective functional is derived from an adjoint method and Automated Differentiation [10], and the descent direction is the result of the product of the gradient by the multilevel preconditioner. Applications to aerodynamic configurations shape design in Eulerian flow are demonstrated.

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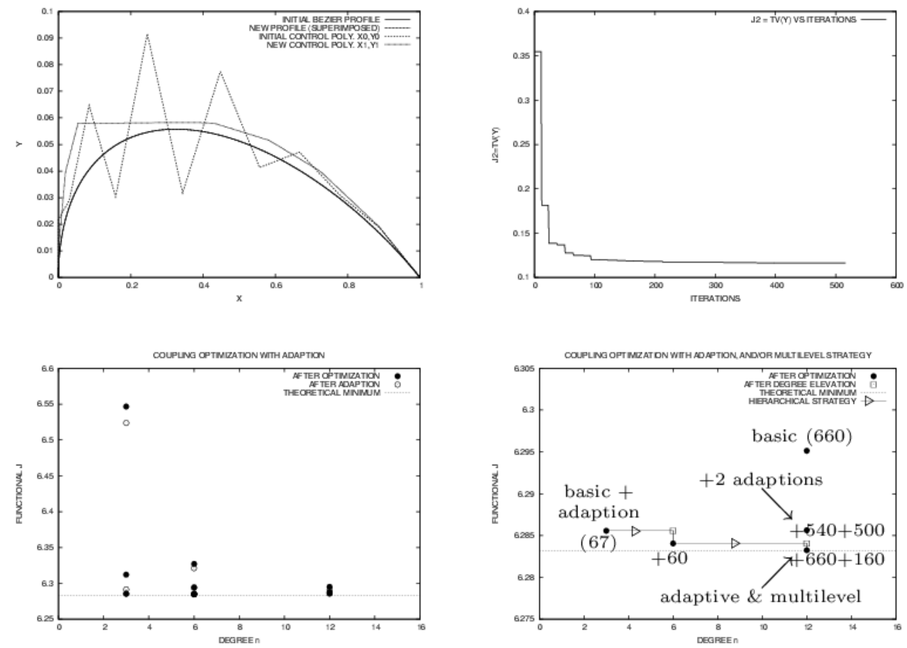

Figure 1: Iterative convergence of three methods; top: standard algorithm (left) and progressive degree-elevation (right) with proper/improper transfers; bottom: basic, progressive and FMOSA over 200 iterations (left) and 60 iterations (right) (from [3]).

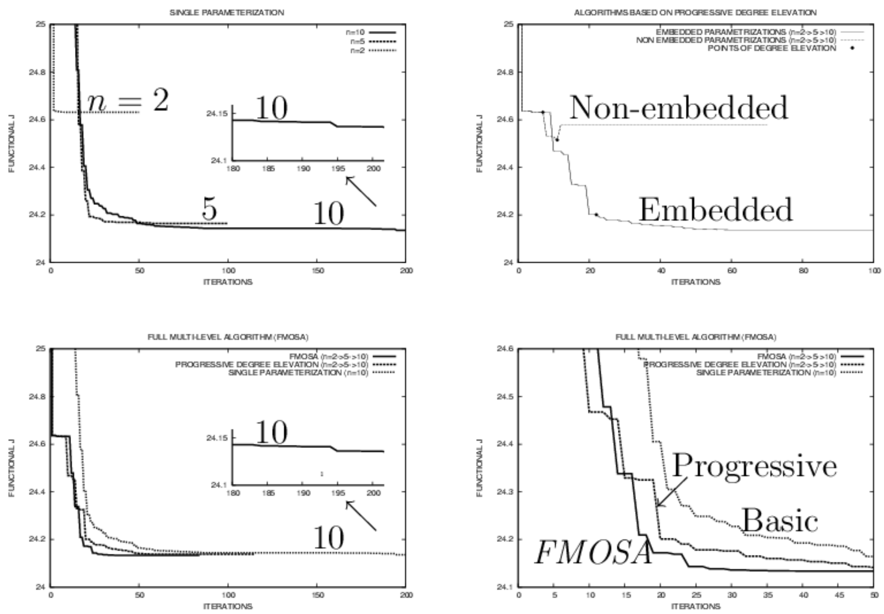

Figure 2: Parameterization adaption; top: regularizing effect on control polygon (left:

polygons with and without adaption), and iterative convergence of solver (right); bottom: accuracy versus degree: basic method and adaption (left), and adaption and hierarchy (right) (from [3]).